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# STUDENTS' PRAXEOLOGIES OF ROUTINE AND NON-ROUTINE LIMIT FINDING TASKS: NORMAL BEHAVIOR VS. MATHEMATICAL BEHAVIOR

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RESUMEN

Este trabajo presenta evidencia empírica de cómo el *posicionamiento* de los estudiantes frente a tareas del tipo "calcule el límite" cambia de estudiantes que se *comportan como sujetos de la institución* – ateniéndose a las *normas* – a estudiantes que se *comportan como sujetos cognitivos* responsables por sus discursos teóricos, cuando las tareas cambian de rutinarias a no-rutinarias. El foco de investigación es en el tema *límite de funciones*, como es presentado en cursos de Cálculo de nivel post-segundario en norte América. La actividad de los estudiantes es analizada utilizando la noción de *praxeología* de la Teoría Antropológica de la Didáctica. Para analizar las diferencias entre las praxeologías construidas por los estudiantes y el origen de las diferentes actitudes que muestran los estudiantes frente al saber matemático, se complementa la TAD con un marco teórico para el análisis institucional desarrollado en ciencias políticas

RESUME

Cet article présente l'évidence empirique d'un phénomène observé dans le contexte d'un cours de Calcul offert dans une institution Nord-Américaine postsecondaire. Il s'agit du changement de *positionnement* chez les étudiants lorsqu'ils passent du travail sur les tâches routinières à propos des limites des fonctions aux tâches non-routinières. Face aux tâches de routine, les élèves *se positionnent en tant que sujets de l'institution* – se conformant selon ses *normes*. Face aux tâches non-routinières, *ils prennent la position du sujet cognitif* responsable de son discours théorique. L'activité des étudiants est analysée en utilisant la notion de *praxéologie* de la Théorie Anthropologique du Didactique. Pour analyser les différences entre les praxéologies construites par les étudiants et les sources de leurs différents positionnements envers le savoir mathématique, nous avons complémenté la TAD par un cadre théorique pour l'analyse des institutions.

## 1. INTRODUCTION

The purpose of this paper is twofold. On the one hand, the research aims at showing that while students' technological and theoretical discourses (in the sense of the components of the theoretical block of a praxeology) that support *routine*-practical blocks seem to be strongly influenced by *institutional norms*, the corresponding discourses supporting *non-routine*-practical blocks do reflect mathematical thinking. On the other hand, the analysis aims at showing that combining the Anthropological Theory of the Didactic (ATD; Chevallard, 1999, 2002) with a framework for institutional analysis meaningfully contributes to the understanding of students' behavior in an institutional setting.

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This paper presents partial results of a research project aimed at investigating the influence of institutional practices on students' and instructors' perceptions of the *knowledge to be learned* about limits of functions in a large, multi-section Calculus course in a North American college. Other partial results of this research have been presented in Hardy (2009ab).

Nowadays in North America there are large post-secondary educational institutions in which enrollment in a Calculus course can surpass 250 students and even double this number. On a given semester, there can be 10 or more sections of one Calculus course taught by at least 5 different instructors. The structure of these institutions has called my attention on the existence of practices that occur *outside* the classroom and that may have a strong influence on didactic phenomena; practices such as those related to preparing the course outline, choosing a textbook, preparing the final examination by a committee of instructors, studying for the final examination by students, etc. In the college studied in this research, some of these institutional practices emphasize certain mathematical tasks making them *routine*. My focus is on tasks related to the topic limit of functions, routinized by their repetitive occurrence in the final examinations. Reports on the (negative) effect of the routinization of tasks – in textbooks and in the classroom – on students' learning, can be found, for example, in Lithner (2004) and Selden, Selden, Hauk & Mason (1999). The absence of theoretical content or the dissociation between theoretical discourses and technical discourses in relation to the topic limits of functions has been reported by different authors. For example, Barbé, Bosch, Espinoza and Gascón (2005) report on the absence of a theoretical block in the knowledge to be taught about algebra of limits in Spanish high schools; they analyze how this absence imposes didactic constraints on the teachers' practice. The work of Lithner (2004) and Raman (2004) shows that in many Calculus textbooks intuitive ideas about limits, the definitions and techniques to find limits are presented in different self-contained sections and that links among these sections are often absent. In Hardy (2009a) I study some of the possible effects that the routinization of tasks and the absence of theoretical content in instructors' models of the knowledge to be learned might have on students' praxeological organizations to deal with limit finding tasks. The analysis carried out in the present paper seems to stress the negative effect of the routinization process and of the absence of theoretical discourses in instructors' expectations of students' performance.

Individuals may behave different in different contexts and in front of different tasks. One could say that an individual *behaves* as a cognitive subject when he or she behaves as a scholar seeking the truth, he or she wants to know for the sake of knowing – in brief, he or she behaves as a *Homo sapiens*. But if we don't take into account the fact that the learner is not only *Homo sapiens,* but also *Homo economicus* and *Homo institutionalis,* then his or her behavior may appear inconsistent to us. Understanding the different positions that individuals may take in a certain context and in front of a given task, and the reasons of him or her being at this position is key for our understanding of teaching and learning phenomena as they occur in institutional settings. In the present research I have found that while students dealing with the typical tasks proposed to them by the institution seem to behave as *institutional subjects* who rely on the institution's responsibility towards the validity of knowledge (Hardy, 2009a), students dealing with non-routine tasks seem to behave as *cognitive subjects*, interested in mathematical knowledge per se and preoccupied by consistency and theoretical explanations. Hence, is not that the students are unable to behave *mathematically*; it is that the institution does not provide them with the opportunity to do so.

To analyze the differences between students' behavior in front of *routine* and *non-routine* tasks I conducted 28 interviews with students who have successfully completed a Calculus course. Students' performance in the interview seems to support the conjecture that when dealing with routine tasks (or any task that resembles a routine task), students' choices of techniques are based on what the institution emphasizes as *normal* behavior. Non-routine tasks, however, seem to help students in taking the position of cognitive subjects responsible for their theoretical explanations. In both cases, students' activities are described in terms of praxeological organizations; I have tried to capture the differences between them complementing the notion of praxeology with the notions of *norms*, *rules* and *strategies* and the notion of *position* in an institution, provided by the Institutional Analysis and Development framework (IAD; Ostrom, 2005).

The combination of ATD and IAD has already been used in Sierpinska, Bobos & Knipping (2008) in a study of students' frustration in pre-requisite mathematics courses at the university level, and in Hardy (2009ab) in studying students' and instructors' perceptions of the knowledge to be learned about limits of functions at the college level.

This paper is structured as follows. In the next section I present my theoretical framework; in particular, I present the notions of rules, norms, strategies and position developed in IAD, and explain how I have combined these with the notion of praxeology developed in ATD. Relevant features of the college institution and the Calculus course studied in this paper are also presented in the next section. The methodology is presented in section 3. Results and their analysis are in section 4. The paper ends with a discussion of the results and some conclusions.

## 2. THEORETICAL FRAMEWORK

The main components of ATD and in particular the notion of praxeology have been thoroughly presented in several recent papers and thus I do not go through them here (see, for example, Chevallard, 2002; Barbé, Bosch, Gascón & Espinoza, 2005; see also Hardy, 2009a).

In my research, the notion of praxeology provided me with a structure to study the epistemological nature of students (mathematical) activities as they occur within an institution. To study the institutional status of these activities, however, the incorporation of some elements of the IAD framework proved to be useful. In particular, I am interested in understanding the mechanisms that regulate institutional (mathematical) activity and the relation between these regulatory mechanisms and students' construction and appropriation of mathematical knowledge.

According to the IAD framework, an institution is an organization of repetitive human interaction whose aim is to achieve certain goals (Ostrom, 2005). The institution defines who its participants are and what mechanisms will regulate and make possible the accomplishment of the goals.

In the framework of the educational system considered in this paper, students finish high school at the age of 16 and have to attend a 2-year college program in order to be entitled to university studies. The high school curriculum does not include Calculus; students take a Calculus course for the first time at college level. In the college studied in this research, Calculus is taught as a multi-section course. In particular, in the studied semester, there were 19 sections taught by 14 different instructors. There is a common outline, prepared by a committee appointed by the mathematics department, a common textbook, chosen by a textbook-committee appointed by the mathematics department, and a common final examination prepared, administered and corrected by a committee made of all the instructors teaching the course. The teaching of Calculus is an institution in itself, which I call *College-Calculus institution*. It is linked to some supra-institutions such as the department of mathematics, the science committee, the ministry of education, and to some sub-institutions such as the curriculum committee, the classrooms, the final examination committee, etc.

Taking into account the existence of a common final examination – that indeed defines a practice since its general structure has not changed over the years (constants in the questions change but *types* of questions do not change) – and that this final examination is the only practice enforced by the College-Calculus institution (midterms, assignments and practices *in* the classroom are the section instructors' business), I could characterize the College-Calculus institution's practices about limits by looking at the types of tasks related to limits of functions proposed over the last seven years (2002 to 2008) and their model solutions available in the textbooks and in the so-called "correction sheets" for the final examinations, written by teachers and made available to students. All problems related to limits in the final examinations are of the "find the limit" type. The model solutions contain no theoretical justifications or proofs of validity. Instructors are free to provide theoretical justifications of the techniques in their lectures, but they cannot require students to provide such justifications in the final examinations. Therefore, while instructors' classroom practices could be modeled with complete mathematical praxeologies, containing both the practical and the theoretical blocks, the final examination practice, which defines, for students, the knowledge to be learned, must be modeled with praxeologies containing only practical blocks: types of tasks and techniques for solving them. The praxeologies corresponding to these problems are described in Hardy (2009a).

### *2.1 Routine and non-routine practical blocks*

I call *routine tasks* and *routine techniques* those belonging to any of the practical blocks routinized by its repeated occurrence in the final examinations. Non-routine tasks are those that have not appeared in the final examinations of the past seven years. Among the non-routine tasks, I distinguished those that strongly resemble routine tasks but differ from them on the conceptual level from those that are easily recognized – by the students – as non-routine. For example, a routine task in the final examinations is to find the limit at a constant of a rational function such that is an indetermination of the type "zero over zero", the polynomials in the numerator and denominator are easily factorable by a standard school-algebra technique, and the value of the limit is a constant (e.g., $\lim_{x\to 2}\frac{x^2-4}{x^2-x-2}$). A task with a similar setting but in which the value of the limit is infinity or does not exist is a non-routine task that strongly resembles the routine task just described (e.g., $\lim_{x\to 2}\frac{x^2-4}{x^2-4x+4}$). A limit finding task involving non-algebraic functions, for example, are *essentially* non-routine; they are easily recognized by the students as non-routine.

### *2.2 Rules, norms and strategies*

The IAD framework distinguishes three mechanisms that regulate human activity within institutions: rules, norms and strategies. Rules are explicitly stated by a recognized legal authority and their violation carries a sanction: "you have to do so and so, if not so and so will happen to you". Norms are implicit and are part of the moral fabric of the institution – they refer to custom and tradition; they do not carry (legal) sanctions and they cannot be (legally) enforced. Strategies refer to courses of action to be taken by a participant of an institution to achieve certain goals. Because an institution is an organization of repetitive interaction, participants of an institution get to learn the norms and the strategies by observing patterns of interactions and their outcomes.

Mathematical activity in institutions is also governed by rules, norms and strategies. Axioms or logic connections can be understood as rules whose violation leads to contradiction (the sanctions). Mathematical techniques can be understood as strategies to accomplish a certain goal (proving a theorem, finding a limit, etc.). Etiquette in writing papers follows norms; the elegance of the writing, however, cannot replace the validity of a proof.

The main contribution of combining these elements with the notion of praxeology is to provide a framework to investigate the mechanisms that regulate students' and instructors' construction of the praxeologies: *how tasks are classified into types of tasks, how techniques are chosen, and what the sources of explanatory discourses are*.

### *2.3 Positioning*

Participants of an institution are assigned – or assign themselves – to different positions. Positions are the anonymous slots into and out of which participants can move (Ostrom, 2005: 40). A position carries in its definition "the set of authorized actions and limits on actions that the holder of the position can take" to achieve a certain goal (ibid. 41).

In my research, I have identified two positions that students can take in the institution College-Calculus and that correspond to two of the four positions identified by Sierpinska et al. (2008: 291) in their study of students' frustrations in pre-requisite mathematics courses. These are "Student" and "Learner" [1]. In the position of "Student" a participant abides by the rules and norms of the College-Calculus institution addressed to its students; in particular, he or she studies because there is a test coming, not out of disinterested curiosity or passion for the subject. A student takes the position of "Learner" when he or she behaves as a cognitive subject interested in knowing Calculus; his or her goal in the institution is to learn.

I have characterized students' positioning by analyzing their theoretical discourses – *how they explain their choices of a technique, how they support these techniques, and how they justify the validity of these techniques*; i.e., by analyzing the theoretical blocks of the praxeologies they have built.

[1] The other two positions identified by Sierpinska et al. are that of a "Person" and that of a "Client" (2008: 291); it might well be that these positions are available in the institution I have studied – none of the interviewed students, however, seemed to have occupied one of these two positions while dealing with the tasks proposed in the interviews.

## 3. METHODOLOGY

To study students' behavior in front of routine and non-routine tasks I conducted, in the 2008 winter semester, twenty eight interviews with college level students who had successfully passed a Calculus course in the 2007 fall semester. Procedures regarding subjects' recruitment and their distribution with respect to the teachers are thoroughly explained in Hardy (2009ab). Interviews were task-based; this type of methodology is described in Goldin (1997).

Interviews consisted in a one hour one-to-one encounter with the subject and the protocol included a variety of tasks related to limits (classifying limit expressions, finding limits, finding limits from graphs, and graphing functions). In this paper I discuss some striking differences in students' behavior when dealing with two of the limit finding tasks. These tasks – from now on Task R and Task NR – consisted on finding limits that strongly resemble routine tasks but differ from them on the conceptual level and on finding limits that were easily identify by the students as non-routine, respectively. Students were asked to think aloud while working on both tasks. When they were finished, I asked them questions about their calculations, failed attempts, answers, etc.

The choice of problems in Task R is justified in Hardy (2009a). While problems in both Task R and Task NR are non-routine, problems in Task NR are *essentially* non-routine in the sense that they are non-routine *and* they do not resemble routine tasks. The functions involved (cosine and exponential) are different from the standard functions that students are given when asked to find limits in the final examination. Nevertheless, the interviewed students were taught, at some point in their pre-Calculus and Calculus courses, the graphs of cosine and exponential functions. In addition, techniques to conjecture the value of a limit by means of a calculator and reading limits from graphs are listed in the course outline of Calculus. This is to say that, though problems in Task R and NR are non-routine, the Calculus course includes techniques that can help students to guess or conjecture the proposed limits. Task NR has a double purpose: on the one hand, it serves as a control of students' abilities to combine the different things they have learned to find limits – as it is discussed below, students were able to combine different approaches when dealing with Task NR but they were unable to do so when dealing with Task R. On the other hand, it allowed me to observe students' behavior in front of limit finding tasks that are for the students themselves, clearly non-routine.

The results and conclusions presented in this paper are based on an analysis of students' approach to the problem R[2]: find $\lim_{x\to 5}\frac{x^2-4}{x^2-25}$ in task R and to the two problems in task NR: NR1 find $\lim_{x\to\infty} e^x\cos(x)$ and NR2 find $\lim_{x\to-\infty} e^x\cos(x)$. As was pointed before, my goal is to compare students' behavior in front of routine and non-routine limit finding tasks. Problems R and NR1/NR2 serve to this purpose because, on the one hand, while the rational expression in R strongly resembles those appearing in the final examinations (the limit is taken at a constant that is a zero of the denominator and both polynomials are easily factorable using school-algebra techniques), none

[2] This problem corresponds to problem 2.3 in Hardy (2009a).

of the routine techniques help in finding the limit, and on the other hand, the three limits R, NR1 and NR2 can be found using similar ideas (e.g., graphing, table of values, etc.).

## 4. RESULTS

In the interviews, students have shown that their choice of a technique to find a limit is not based on Calculus criteria, but on certain aspects of the functions involved in the limit expressions – aspects that are not relevant from the point of view of Calculus but are *made* relevant by their *normal* occurrence in final examinations. A detailed analysis of students' performance in problem R is given in Hardy (2009a: section 5). In this paper I focus on some striking differences between the ways in which some students approached problem R and the ways in which the same students approached problems NR1/2. In particular, I analyze the behavior of two groups of students: Group 1, consisting of 5 students who stated that they couldn't remember how to deal with a problem such as R, and Group 2, consisting of 3 students who gave an answer for problem R in the form of a question: "is it undefined?" or "infinity, right?".

### *4.1 GROUP 1*

Students in Group 1 did not consider the possibility of using non-routine techniques when dealing with problem R – they perceived it as a routine task and their focus of attention was on trying to *remember* the right routine technique. For example, one of the students in Group 1, student S6, failed to find limit R; he said:

> S6: My first reflex was, is always, I put the five here so I saw it was over zero. [...] Again I factor out but... [...] I can't remember how to treat it. [...]

When dealing with the essentially non-routine task, however, these students made use of different techniques, combining graphs with the use of the calculator and with mathematical notions such as "bounded function". For example, student S6 succeeded in finding the limit NR2. First he said that the value of the limit was the same as the one he gave for NR1 (infinity) but after I made a brief intervention he found the correct limit by reasoning about the behavior of the two functions involved in the expression and their product. Our exchange was the following:

> I: What about *e* to the *x*? [My intervention.]
>
> S6: Oh, no, it would all go towards zero, yes, that's true. *e* to the *x* would be closer and closer to zero. So this would go towards... but I do not know how to treat the *cos* in that situation. I know this goes towards zero [for the exponential function] and *cos* ... Because even if this is going towards zero, if my *cos* is getting bigger and smaller... but it is always the same sequence multiplied by that, I guess I could assume it is going towards zero.

He even made the non-trivial reflection that, if a function is going towards zero and it is multiplied by a function that is getting bigger and smaller, that does not guarantee by itself that the limit would be zero. He convinced himself that the limit was zero when he realized that the behavior of cosine was "always the same sequence".

Two students in Group 1 spontaneously thought about using the calculator to deal with problems NR1/2 but they didn't consider this possibility when dealing with problem R – in front of which they froze just stating "I am stuck" or "I am not sure [what to do]".

Furthermore, one of the students, after using the calculator to conjecture the value of limits NR1/2, tried to support her conjectures by means of mathematical arguments. Her reasoning went along the following line: because the cosine function is bounded, what matters for the limit is the behavior of the exponential function.

### *4.2 Group 2*

The students in Group 2 provide an answer for problem R in the form of a question but they did not feel compelled to justify or further explore their conjectures. The same students, however, when dealing with task NR, were not satisfied with just stating a conjecture; without being prompted to, they tried to justify it. For example, when given problem R, student S21 said: "could it be undefined?". When given problems NR1/2, however, he said:

> S21: Well, there's an *e* that would keep growing. Cosine goes between one and negative one. Well, the cosine you can't really find it, it keeps going up and down so the limit does not exist. […] I think the limit... Well, since *cos* is either one or negative one it would be positive infinity or negative infinity. But since it is to infinity, it can't really exist.

Student S12, when given problem R, said "infinity, right?" and did not display any further interest in the problem. When given problem NR1 she also formulated her answer in the form of a question: "can't you say it is just infinity?"; however, she immediately continued, trying to justify her statement:

> S12: […] Ok, so this is infinity and cos of… can't you say it's just infinity? Because this is infinity and cos is… bounded; so infinity.

To deal with problem NR2 she spontaneously started using her calculator to conjecture the value of the limit. The third student in Group 2, who had also answer problem R with a question and no further justification, when given problems NR1/2 said:

> S28: Oh, ok. Well, this goes to plus infinity [for the exponential function in NR1], this goes to… plus one, minus one [for the cosine function]. I could say… because this is plus or minus infinity. Well, my teacher would say that this limit won't exist, but I say it's plus infinity and minus infinity. And this is of course zero because this goes to zero [the exponential function in NR2] and this is plus one and minus one [the cosine function], this goes very rapidly to zero [the exponential], so you don't care if it's minus one of plus one. [He drew a very good sketch of the function $f(x) = e^x \cos(x)$.]

## 5. Discussion and conclusions

Independently of their performance, all students recognized problems NR1/2 as non-routine. Their discourse related to this recognition was based on unfamiliarity: they claimed that these were problems that they have not often – or ever – done ("I don't remember doing limits with $e^x$", "do we cover this in Calculus I?"). In this sense, I surmise that, in the most general setting, students immediately classify tasks they are confronted with into two types: *familiar tasks* and *unfamiliar tasks*. The set of familiar tasks contains the set of routine tasks and those that strongly resemble them. The set of unfamiliar tasks contains those problems they think they have never seen or done before. These two types of tasks define two different praxeologies related to limit finding tasks: a familiar or routine praxeology and an unfamiliar or non-routine praxeology. The features that guide the classification of tasks, however, are not based on criteria characteristic of Calculus. To distinguish familiar from unfamiliar tasks, interviewed students appeared to focus on the function of which a limit was being taken: if the function is a rational function (or a function involving square radicals), then it is a familiar task, otherwise it is an unfamiliar task. Among familiar tasks, they distinguish tasks in which the polynomials involved are "easily" factorable from tasks in which the polynomials are not "easily" factorable (see Hardy, 2009a: section 6). Criteria typically belonging to Calculus such as whether the limit is taken at a constant or at infinity, types of indeterminations, existence of the limit, etc., were not considered as classifying features.

Techniques to accomplish familiar tasks are those they have learned in the context of accomplishing routine tasks: factoring and cancelling common factors, substitution, and rationalization. To accomplish unfamiliar tasks, it is clear to them that none of the routine techniques apply; this fact seems to trigger the use of other techniques they have learned: making tables of values, graphing, studying the behavior of the involved functions separately, etc.

In (Hardy, 2009a) is shown that students' technological discourse when dealing with familiar tasks refers to what's *normal* in the institution. Interviewed students supported their use of a technique to deal with a familiar task (such as problem R) by saying "we do this because that's what we usually do under the circumstances" (ibid., section 6). The corresponding theoretical discourse seems to be absent. It is my conjecture that in the praxeology corresponding to unfamiliar tasks, the justification of the technique (the technology) is that *routine techniques do not apply to these problems* – for example, the expressions cannot be factored or rationalized – and therefore one must resort to unorthodox means. This justification is a *cognitive norm*. On the theory level, the students made use of *mathematical rules* (e.g., the limit of a bounded function multiplied by a function that tends to zero is zero), and *mathematical strategies* (e.g., making a table of values to conjecture the value of a limit).

The following table summarizes the analysis above:

| Practical Blocks | | Theoretical Blocks | |
|---|---|---|---|
| Types of tasks | Techniques | Technology | Theory |
| Familiar | Techniques that apply to routine tasks: factoring, cancelling common factors, substitution, rationalization, etc. | This is what we usually do under the circumstances. | Absent. |
| Unfamiliar | Unorthodox techniques: graphing, making tables of values, studying the involved functions separately, etc. | Routine techniques do not apply to these problems. | *Based on mathematical rules and strategies.* |
| *Classification of tasks and choice of techniques based on institutional (non-mathematical) norms.* | | *Discourse based on institutional (non-mathematical) norms.* | |

TABLE 1. Students' praxeologies to deal with familiar and unfamiliar tasks.

Students in Group 1 perceived limit R as a routine task and could not overcome the fact that the typical techniques that apply to routine tasks did not seem to work in this case. They claimed that they did not remember how to deal with a limit of this kind or that they could not see how to use the standard techniques. Though being unable to find the limit, they did not consider the possibility of using non-routine techniques. Nevertheless, when given the essentially non-routine tasks, they proved to have other resources to deal with limits; they either spontaneously thought of using the calculator, or they reasoned about the behavior of a function from the behavior of its factors. Unfamiliar tasks seem to trigger the recovery of knowledge that has been hindered by the routinization process.

Students in Group 2 solve problem R by stating a question, e.g., "Infinity, right?", and not showing any further interest on the problem. When given problems NR1/2, however, their answer-solution was followed by an explanatory discourse, e.g., "Infinity, right? Because so and so …".

In the studied college, instructors' model of the *knowledge to be learned* about limits is compound only of practical blocks. Fragments of corresponding theoretical blocks, however, do form part of the *knowledge to be taught* as defined in curricular documents by the College-Calculus institution (Hardy, 2009b). This, together with the institutional context, suggests two different explanations to why students, when dealing with tasks that they relate to those exemplifying the knowledge to be learned (familiar tasks), provide a solution without justification, but when dealing with an unfamiliar task, they do try to justify their statements. On the one hand, it might be that when dealing with familiar tasks, students just "copy" the format of solutions expected in the final examinations: a solution without any justification. When dealing with unfamiliar tasks, however, they have to search for an approach that belongs to the knowledge they have been taught, and in that type of knowledge, theoretical justifications do exist. On the other hand, it might be that when dealing with familiar tasks, students rely on the authority of the institution – it is the institution, embodied in the persons responsible for the knowledge to be taught and learned, and in the official documents and texts, and not the students, who is responsible for the validity of this knowledge. Theory in the mathematical sense is not the students' responsibility (Chevallard, 1985: 75). Unfamiliar tasks, however, might be identified by the students as "outside" the jurisdiction of the institution; theoretical explanations and consistency is indeed a concern and students take responsibility over them. These two possible explanations are not mutually exclusive; they might complement each other in explaining students' behavior in front of routine and essentially non-routine tasks.

Students in Groups 1 and 2 have shown (different forms of) a *normal behavior* when dealing with familiar tasks: abiding by the norms of the institution, not going any further than what they believe is expected from them. This suggests that when dealing with familiar tasks students' positioning was that of "Students". When dealing with essentially non-routine tasks, however, they behaved as cognitive subjects and they based their explanatory discourses on mathematical rules and strategies – they combined different techniques to tackle a problem, thinking *outside the box* and providing theoretical justifications. In other words, students positioned themselves as "Learners".

Institutions vary substantially in the degree to which participants control their own entry into or exit from a position. Questions that arise from this are, on the one hand, what is the degree of freedom that students have to enter or exit a position; and, on the other hand, whether there are particular actions that can be taken, by the different institutions taking part in the didactic phenomena, to favor students taking the position of "Learners".

Balacheff (1999) points out the difference between the norms in a didactic contract, which act locally, in relation to a particular classroom activity, as a temporary didactic strategy, and the more permanent normative mechanisms of classroom custom. A question that arises from this distinction is how these different types of norms complement each other (or crash together) in shaping students' (and instructors') behavior in the classroom. The norms discussed in this paper are of the second type; they refer to custom and tradition in the College-Calculus institution. Students' performance in the interviews shows that these are the norms that regulate students' behavior when facing the typical limit finding tasks proposed by the institution.

In the interviews, the introduction of non-routine tasks seemed to act as a disruptive element, breaking the norm, and thus creating a space for creativity and requiring explicit theoretical justification.

In my research, ATD provided me with a framework to characterize the different types of knowledge taking part in the process of didactic transposition and the epistemological nature of students' and instructors' practices – in relation to the topic limits of functions. An analysis in terms of praxeologies shows precisely what types of tasks the participants identify, what are the techniques they choose to accomplish the tasks, and what their explanatory discourses are. The notions of norms, rules, strategies, and position, defined in IAD, allowed me to identify and analyze the (institutional) sources of students' behavior in those practices and to attempt an explanation of students' different attitudes towards mathematical knowledge. This analysis, in terms of the mechanisms that regulate human behavior in institutions, contributes to the understanding of *how* and *why* participants classify tasks, choose techniques, and provide explanatory discourses. It is in this sense, that the combination of ATD and a framework for institutional analysis can contribute to our understanding of didactic phenomena occurring in institutions.

## REFERENCES


Balacheff, N. (1999). Contract and custom: Two registers of didactical interactions. *The Mathematics Educator*, *9*(2), 23-29.

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: the case of limits of functions in Spanish high schools. *Educational Studies in Mathematics, 59,* 235-268.

Bosch, M., Chevallard, Y. and Gascón, J. (2005). 'Science or Magic?' The use of models and theories in didactics of mathematics. *Fourth Congress of the European Society for Research in Mathematics Education*. (http://cerme4.crm.es/Papers%20definitius/11/Bosch%20Chevall.pdf)

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en Didactique des Mathématiques*, **19**(2), 221–266.

Chevallard, Y. (2002). Organiser l'étude. 1. Structures et fonctions. In J.-L. Dorier, M. Artaud, M. Artigue, R. Berthelot, R. Floris (Eds.), *Actes de la 11e Ecole d'Eté de Didactique des Mathématiques* (pp. 3-22). Grenoble: La Pensée Sauvage.

Goldin, G.A. (1997). Observing mathematical problem solving through task-based interviews. *Journal for Research in Mathematics Education*. Monograph 9, Qualitative Research Methods in Mathematics Education, 40-177.

Hardy, N. (2009a). Students' perceptions of institutional practices: the case of limits of functions in college level Calculus courses. *Educational Studies in Mathematics*, *Online first* publication number DOI 10.1007/s10649-009-9199-8; Published online June 19, 2009.

Hardy, N. (2009b). Students' models of the knowledge to be learned about limits in college level Calculus courses. The influence of routine tasks and the role played by institutional norms. PhD Thesis, Concordia University, Montreal, QC, Canada.

Lithner, J. (2004). Mathematical reasoning in calculus textbook exercises. *Journal of Mathematical Behavior,* **23**, 405-427.

Ostrom, E. (2005). *Understanding institutional diversity*. Princeton, NJ: Princeton University Press.

Raman, M. (2004). Epistemological messages conveyed by three college mathematics textbooks. *Journal of Mathematical Behavior*, **23**, 389-404.

Selden, A., Selden, J., Hauk, S. and Mason, A. (1999). *Do Calculus students eventually learn to solve non-routine problems*. Technical report, Department of Mathematics, Tennessee Technological University.

Sierpinska, A., Bobos, G. and Knipping, Ch. (2008). Source of students' frustration in pre-university level, prerequisite mathematics courses. *Instructional Science,* **36**, 289-320.