THREE ESSAYS ON THE ENVIRONMENT WITH STRATEGIC INTERACTIONS

ELEFTHERIOS FILIPPIADIS

A THESIS
IN THE DEPARTMENT
OF
ECONOMICS

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY (ECONOMICS) AT
CONCORDIA UNIVERSITY
MONTREAL, QUEBEC, CANADA

October 2011

CONCORDIA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

This is to certify that the thesis prepared

By: **Eleftherios Filippiadis**

Entitled: **Three Essays on the Environment with Strategic Interactions**

and submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY (Economics)

complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the final examining committee:

____ Chair
Dr. S. Shaw

____ External Examiner
Dr. H. Benchekroun

____ External to Program
Dr. Z. Patterson

____ Examiner
Dr. P. Gomme

____ Examiner
Dr. S. Papai

____ Thesis Supervisor
Dr. E. Diamantoudi

Approved by ____
Dr. E. Diamontoudi, Graduate Program Director

December 9, 2011 ____
Dr. B. Lewis, Dean, Faculty of Arts and Science

Abstract

Three Essays on the Environment with Strategic Interactions

Eleftherios Filippiadis, Concordia University

This thesis is a study on the environment with strategic interactions. Chapter 1 examines the income-pollution relationship in a multicountry framework. A general static model is proposed that guarantees existence and uniqueness of the solution. It is shown that Environmental Kuznets Curves (EKC) can be obtained for a rich set of parameters, while it does not exclude more complicated forms of income-pollution relationship. Also, it is shown that, given sufficient world income, equal income distribution benefits the environment.

Chapter 2 is a dynamic extension of Chapter 1. An overlapping generations model is proposed. We derive analytical solutions for the case of a single country and it is shown that pollution is more likely to decrease with income when the steady state value of capital is relatively high. For the case of two countries, this model preserves the uniqueness result of Chapter 1. This allows us to explore regularities in the income-pollution relationship with the use of simulations. Our results suggest that high productivity differences and/or high differences in initial endowments can drive the poor country into an environmental poverty trap.

Finally, Chapter 3 examines the effect of ad hoc subsidies on the environment and on the optimal effluence tax. We show in a general equilibrium set up with market imperfections that the existence of ad hoc subsidies reduces total output. Their effect on emissions and on the total welfare depends on the parameters. We derive the conditions under which ad hoc subsidies can be welfare-improving. Finally, it is shown that the second-best environmental policy is first-best equivalent.

... to my father.

Acknowledgements

I would like to express my sincere gratitude to my supervisor, Dr. Effrosyni Diamantoudi, for her great guidance, ingenious advice, and continual support she provided me over the entire course of this thesis. This work would not have been completed without her strong commitment to excel in her part as my thesis supervisor.

Moreover, I would like to thank the External Examiner, Dr. H. Benchekroun, Department of Economics, McGill University, the Chair of the Examining Committee, Dr. S. Shaw (Education), and the members of the Examining Committee, Dr. Z. Patterson (Geography, Planning & Environment), Dr. P. Gomme (Economics), and Dr. S. Papai (Economics). Their comments and suggestions are extremely valuable.

I am indebted to the former Graduate Program Directors during my Ph.D. studies, Dr. Gordon Fisher, and Dr. Nikolay Gospodinov, as well as Dr. William Sims, Chair of the Department of Economics. I would also like to thank the administrative staff in the Department of Economics, Lise Gosselin, Lucy Gilson, Bonnie Janicki, and Elise M. Melancon, who were always very supportive and extremely helpful during the past six years.

Finally, I want to express my gratitude to my family and friends who, on many occasions, had to share with me the burden of undergoing a very challenging Ph.D. program.

Contents

List of Figures

List of Tables

Chapter 1

The Environmental Kuznets Curve in Multicountry Setting[1]

1.1 Introduction

Ever since Malthus published his "Essay on the Principle of Population" (1798), the question of sustainable growth has been raised: Can human civilization grow without limits or are there obstacles on the way which will bring a stop to that growth? Malthus' pessimistic approach that "*...the power of population is indefinitely greater than the power in the earth to produce subsistence for man*" was refuted and disproved by the impending Industrial Revolution. Technological advancement and the use of machines in both agriculture and manufacturing led to a growth in production that exceeded population growth proving that economic growth and prosperity are strongly connected. The arguments, however, against the Malthusian theory were to be tested again in a different terrain. The Industrial Revolution soon brought up the problem of pollution and the exhaustion

[1]This Chapter is based on joint work with my supervisor Dr. Effrosyni Diamantoudi (Associate Professor, Department of Economics, Concordia University).

of natural resources. John Stuart Mill addressed this issue in his "Principles of Political Economy" (1848), recognizing the dangers of unlimited growth: destruction and exhaustion of natural resources, and depletion of the environment due to pollution.

The "Club of Rome," a group of macrotheorists in the 1970s, revived these early theories, claiming that growth is limited due to environmental constraints. Natural resources and clean environment cannot be sustained for ever, if the economies continue to grow without limit. This view is expressed by Georgescu-Roegen (1971) and Meadows *et al.* (1972). The latter provided general predictions about the exhaustion dates of resources like chromium, gold, and petroleum, under the assumption that the population and the use of natural resources increase exponentially while discovery and renewal of natural resources increase linearly. For the economists of the "Club of Rome," zero steady state growth is suggested as the only solution to the environmental problem. Responding to the "Club of Rome," other studies including Malenbaum (1978), Williams *et al.* (1987), and Tilton (1990) have shown a decrease in the intensity of use of some natural resources or, even more, an absolute decline in the use of some natural resources as economies grow. In the next decade, starting in early 1990s, the debate over the predictions of "The Limits to Growth" focused on the dynamic behavior of pollutants: a group of environmentalists claimed that growth generates pollution and that there is no adequate level of absorption and regeneration in the ecological system leading to a global natural disaster.[2] Concerns have also been raised about the globalization of economies: free trade increases output, thus stimulating growth, which leads to more pollution.

The inverse U-shaped relationship between pollution and per capita income was first pointed out in the empirical studies of Grossman and Krueger (1991), Shafik and Bandyopadhyay (1992), and Panayotou (1993). These studies use cross-country data of local air and water pollutants (such as CO, NO_x, SO_x, sus-

[2]For references on this subject one can check Grosmman Krueger and (1991)

pended particulate matter, municipal waste, lead) and for some of these pollutants the conclusion is that at early stages of economic development pollution rises until a turning point beyond which pollution steadily decreases as per capita income rises. Grossman and Krueger (1991) verify this relationship for SO_2 and smoke while for some of the pollutants the relationship between per capita income and pollution is monotonic (positive for municipal waste and negative for suspended particulate matter, for example). They are the first that decomposed pollution change into three effects: (1) the *scale effect*, where the higher the income the higher the production, thus the higher the pollution, (2) the *technological effect*, where the higher the income the more environmentally friendly the technology is, resulting in lower pollution, and (3) the *composition effect*, where higher income implies a change in the composition of production towards "greener" products. So, as income rises, the composition and technological effects may cancel out the scale effect resulting in an EKC. Using data from Mexico's maquiladora sector, they claim that trade liberalization can benefit the environment by enhancing the composition and the technological effect. Shafik and Bandyopadhyay (1992) for the World Bank and the "1992 World Development Report," and Panayotou (1993) for the International Labor Organization, have also reached similar conclusions about some local pollutants. The term Environmental Kuznets Curve is introduced by the latter.

Selden and Song (1994), and Grossman and Krueger (1995) use cross-country data and examine the relationship between per capita income and many different air and water pollutants. Their findings seem to support the aforementioned earlier studies. Recognizing that the process of improving environmental conditions with economic growth is not automatic but rather requires government regulation, they suggest three intuitive explanations for the downward-sloping part of the EKC: first, as income rises "*... citizens demand that more attention be paid to the noneconomic aspects of their living conditions,*" thus increasing the demand for cleaner environment; second, richer countries tend to lower the production of pollution-intensive industries and import these goods from less developed countries, thus changing the composition of production and consumption; third, as economies progress, cleaner technologies of production and more efficient abatement technologies are available. The explanation that the income elasticity of environmental quality is more than one (or just positive) seems to be the dominant one in the related literature while the environmental "dumping" - or the pollution haven hypothesis - constitutes the most persuasive counterargument to the EKC theory.

Among those who discuss the role of a positive income elasticity for environmental quality one can find Arrow *et al.* (1995) and Carson *et al.* (1997). Arrow *et al.* (1995) claim that if people spend on average more when their income increases, they will be willing to spend more for the environment as well. Carson *et al.* (1997) uses data from 50 U.S. states and finds that per capita emissions of seven major air pollutants (including among others CO_x, NO_x, and air toxics) decrease with increasing per capita income. The underlying explanation is that the income elasticity for environmental quality is greater than one.

However, the positive income elasticity of pollution is a result of different primary forces and, therefore, it cannot be used to explain the EKC. McConnell (1997), for example, shows that environmental quality being a luxury or even a necessary good is neither necessary nor sufficient condition for the existence of EKC. It is shown that preferences consistent with a positive income elasticity can coexist with lower willingness to pay for abatement, resulting in monotonically increasing pollution with income. Lopez (1994) shows that the downward-sloping part of the EKC can be explained if the production sector fully internalizes the "stock feedback effects on production." In the absence of such internalization, the inverse relationship between income and pollution can be explained by the high elasticity of substitution between pollution and conventional inputs and the high degree of relative risk aversion of the utility function. Stokey (1998) provides a theoretical model that derives an inverted V-shaped EKC. In the spirit of Lopez

(1994), the primary explanation for the EKC is the high elasticity of consumption: when the marginal utility of consumption changes slowly, pollution as a production factor and conventional inputs (represented by national income) are substitutes, thus an increase in one decreases the other. An important implication of this model is that all types of pollutants must exhibit the EKC property, though the turning points might be different. Lieb (2002) extends the work of McConnell and shows that when preferences over consumption are satiated there will be a turning point in the pollution-per capita income relationship. This result holds partially even if the preferences over consumption are asymptotically satiated. It also shows that previous works like those of McConnell (1997) and Stokey (1998) are actually a special case of this model.

Others, like Roca (2003), deviate even farther from the positive income elasticity of pollution as an explanation for the EKC. Roca (2003) reviews theoretical models of EKC introduced earlier in the literature, and claims that the income elasticity alone cannot explain the observed patterns of pollution as a function of income. Since pollution causes external effects, income and power distribution can affect the curvature, the turning point, and even the shape of an EKC. Along these lines, one finds Torras and Boyce (1998) and Magnani (2000) who claim that a more equitable distribution of power and income can benefit the environment, thus explaining the downward-sloping part of an EKC. Finally, Andreoni and Levinson (2001) proposed a model where the EKC is a result of increasing returns (IRS) to abatement. The existence of IRS in the abatement process of some pollutants is verified empirically in their study.

In their seminal paper, Grossman and Krueger (1991), as well as in Grossman and Krueger (1995), provide evidence that, although there is a displacement of polluting industries from developed to developing countries, the magnitude of this shift is insignificant. Arrow *et al.* (1995) and Ekins (1997) examine the role of international trade and the EKC: "cleaner" production in the developed world does not coincide with higher demand for "greener" products. Therefore, in the presence of international trade, the demand for pollution-intensive goods on behalf of developed countries is satisfied by the production of these goods in the developing world. Despite deriving an EKC, these models are pessimistic: higher income does not imply lower pollution but rather a "transfer" of pollution from the developed to the developing world (environmental dumping). Empirical studies examining the role of international trade in the derivation of an EKC offer a blurred image. Some researchers, like Liddle (1996) tend to agree with the claim of Grossman and Krueger that the role of international trade and specialization is not significant in producing an EKC, while others, like Suri and Chapman (1997) are more sceptical and tend to disagree.

Transboundary and global pollutants are subject to analysis in the more recent EKC literature. Local pollutants like SO_2, NO_x, and heavy metals tend to have immediate effects on the environment and, consequently, on public health. As a result, these pollutants are more likely to exhibit the inverse-U pattern in the income-pollution relationship. Transboundary pollutants, on the other hand, such as CO_2 and other greenhouse gases, do not seem to follow the path expected by the EKC theory. Instead of an inverted U-shaped curve, global pollutants often exhibit a strictly monotonic relationship between per capita income and emissions with the two being positively related. The reason is that transboundary pollutants do not seem to have an immediate and easily recognized effect on people's well-being. Moreover, these emissions are tied to energy consumption, when current technology cannot support the exponentially increasing demand for energy with solely environmentally friendly production processes. As a result, even if transboundary pollutants exhibit the EKC property, the turning point is expected at higher per capita income compared to the case of local pollutants. Due to the reasons presented above, emissions of transboundary pollutants remained unregulated for longer compared to local pollutants. Even when scientists presented their findings about the damage that global pollutants can cause (e.g. ozone layer depletion, global warming), regulating these emissions required action to be taken at an international level. International environmental agreements are slow processes, they usually fail to involve all the major polluters, and their targets are often missed by far (*e.g.* the Kyoto Protocol).

However, many of these problems have been mitigated in recent years. Scientific research on the results of global pollution has improved public awareness, international negotiations have resulted in many bilateral and multilateral environmental agreements, and innovative green production processes are becoming cost-efficient leading to more and more countries adopting them. In the recent literature about the CO_2 emissions, the existence of an EKC path cannot be confirmed or rejected with certainty. Galeotti *et al.* (2006) estimated an EKC with a reasonable turning point for the developed countries (OECD members) but not for the developing countries. The robustness of their findings was checked with the use of different data sets, and with the use of different functional forms of the regressions. Yaguchi *et al.* (2007) used data from China and Japan to investigate the dynamics of SO_2 and CO_2 with the EKC hypothesis to be clearly rejected for the case of CO_2 emissions. Wagner and Muller-Furstenberger (2007) question the results of the empirical research on CO_2 emissions claiming that the econometric techniques used are often inappropriate.

The review of the EKC's literature shows that the theoretical models proposed are inadequate to describe the cases of transboundary and/or global pollutants. It is a common characteristic of these models that are single-country models with two immediate implications: (i) countries that share the same technological and demand parameters should follow the same income-emission path independently of their initial income, and (ii) the pollution externality is restricted within the borders of a single country.

The present study introduces a mutlicountry model for the analysis of global pollutants. By doing so, we are allowing for interdependence in the countries' pollution decisions, thus changing the pollution problem from that of optimal control to a pollution game. We propose a fairly general set up where emissions can be considered a production factor and under relatively weak assumptions on preferences and technology we show that the pollution game has a unique solution. Comparative static analysis shows that an inverted U-shaped can be derived for the case of symmetric countries. However, when the symmetry is not extended to the initial income of the countries, various forms of income-pollution paths can be derived. These paths are country specific, thus being in accordance with the empirical literature on CO_2.

In what follows, the basic set up of the model is presented in Section 1.2. Section 1.3 shows that our model can be described as a Potential Game. Using the theory of Potential Games, existence and uniqueness of the model's solution is proven. Comparative statics for the symmetric case of n countries and the 2-country asymmetric case are presented in Section 1.4. Section 1.5 presents a special case with specific functional forms and Section 1.6 concludes.

1.2 The model

1.2.1 Basic setup

We are assuming an environment of n countries, each one producing a single good with pollution, $x_i \in X_i \subset \mathbb{R}_+$, being a by-product of the production process, where $i \in N = \{1, 2, ..., n\}$. Pollution is assumed global. For each country $i \in N$ a social utility function is given by

$$V_i = u_i(c_i) - h(\sum_{i \in N} x_i), \tag{1.1}$$

where u_i is a country-specific utility function, twice continuously differentiable, that is increasing and concave in consumption, c_i, and h is a damage function, also twice continuously differentiable and common for all countries, that is strictly increasing and convex in total pollution $\mathbf{x} = \sum_{i \in N} x_i$.

Individual pollution is bounded from above at every level of income. This upper bound is an increasing function of income, *i.e.,*

$$\overline{x}_i = \phi(y_i), \tag{1.2}$$

with $\partial\phi/\partial y_i \geq 0$, $\partial^2\phi/\partial y_i^2 \leq 0$, $\lim_{y\to\infty}\phi(\cdot) = \infty$. Under this formulation, improvements in abatement technology will reduce the maximum level of pollution at any given level of income. However, in our model there is no cost associated with improvements in abatement technology, thus abatement is considered exogenous.

Finally, assuming no trade, consumption possibilities are fully defined by the production process. The production function[3] is expressed as

$$c_i = y_i\sigma(x_i), \tag{1.3}$$

where $y_i \in \mathbb{R}_{++}$ is the potential income, that is the income when the dirtiest technology is used, and $\sigma(x_i)$ is a technology index that converts potential output into actual consumption with

$$\sigma(x_i) = \frac{x_i}{\overline{x}_i} = \frac{x_i}{\phi(y_i)}. \tag{1.4}$$

It is obvious that $\sigma(x_i) \in [0,1]$, $\sigma_x > 0$, and $\sigma_{xx} = 0$. Using this definition of technology index, the consumption possibilities are now defined as

$$c_i = y_i\frac{x_i}{\phi(y_i)}. \tag{1.5}$$

We can now define the following:

Definition 1 *Let $G = \left\langle N, X, \{V_i\}_{i\in N}\right\rangle$ the pollution game, where the set of players is $N = \{1, 2, ..., n\}$, the strategy space is $X = X_1\times X_2\times ...\times X_n$, and $V_i : X \to R$*

[3] Note that the term production function is abused here. This function does not describe the production process, but rather it expresses the consumption possibilities relative to the intensity of pollution. A similar function is used by Stokey (1995).

is the payoff function of player i.

In this setup, the maximization problem for a social planner in country i is given by

$$\max_{x_i \in [0,\phi(y_i)]} V_i = u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \sum_{k \in N/\{i\}} x_k), \tag{1.6}$$

with first order and Kuhn-Tucker conditions being

$$\begin{cases} \frac{\partial u_i}{\partial c_i} \frac{y_i}{\phi(y_i)} - \frac{\partial h}{\partial x_i} - \lambda_i = 0 \\ \phi(y_i) - x_i \geq 0 \\ \lambda_i \geq 0 \\ \lambda_i (\phi(y_i) - x_i) = 0 \end{cases}, \tag{1.7}$$

where λ_i is the Langrangean multiplier for country i. Country i's reaction function is fully characterized by the above conditions: at the interior, the reaction function is given implicitly by the equation $(\partial u_i/\partial c_i)(y_i/\phi(y_i)) - (\partial h/\partial x_i) = 0$ with $\partial x_i/\partial x_j < 0$ implying strategic substitutability; at the corner, the reaction function is simply $x_i = \phi(y_i)$ suggesting that a dominant strategy may exist for country i. The reaction functions of all n countries constitute an $n \times n$ system of equations to be solved.

1.2.2 Relative risk aversion

The role of relative risk aversion in the income-pollution path has been examined in Lopez (1994) among others. The use of the relative risk aversion in a deterministic model should not be a surprise: it can be thought of as the Frisch coefficient that expresses the relative curvature of the utility function. Intuitively, the Frisch coefficient captures the rate at which the marginal utility declines as income rises. In what follows, we concentrate on the interior solution of our model where the reaction function of country i is given implicitly by

$$\frac{\partial u_i}{\partial c_i}\frac{y_i}{\phi(y_i)} = \frac{\partial h}{\partial x_i}.$$

Note that the term on the left-hand side represents the marginal benefit of pollution (MBP) with $\partial MBP/\partial x_i < 0$, while the term to the right represents the marginal cost of pollution (MCP) with $\partial MCP/\partial x_i > 0$. It is interesting to notice that the marginal cost of pollution does not depend on the level of income but only on the choice of domestic and foreign countries' pollution. The marginal benefit of pollution, however, depends on the level of income. The latter relationship depends on the degree of relative risk aversion[4] (RRA) that the utility function exhibits at a specific level of pollution x_i. It follows from differentiating the MBP with respect to income, *i.e.*,

$$\frac{\partial MBP}{\partial y_i} = \left(\frac{\phi(y_i) - y_i\phi'(y_i)}{[\phi(y_i)]^2}\right)\frac{\partial u_i}{\partial c_i} + \left(\frac{\phi(y_i) - y_i\phi'(y_i)}{[\phi(y_i)]^2}\right)\frac{\partial^2 u_i}{\partial c_i^2}\frac{y_i}{\phi(y_i)}x_i \Rightarrow$$

$$\frac{\partial MBP}{\partial y_i} = \left(\frac{\phi(y_i) - y_i\phi'(y_i)}{[\phi(y_i)]^2}\right)\frac{\partial u_i}{\partial c_i}\left[1 - RRA_i\right],$$

where $RRA_i = -\left(c_i u_i^{''}/u_i^{'}\right)$. Given our assumptions on $\phi\left(y_i\right)$, the first two factors on the right hand-side are non-negative. Therefore, the sign of the left-hand side depends on whether RRA exceeds unit or not. For RRA less than one, the MBP is increasing in income, while for RRA more than one the MBP is decreasing in income.

For the case of a single country, the above result shows that if at the optimal pollution level (where MBP equals MCP), RRA is greater than one, any increase in potential income will decrease MBP and, therefore, call for a decrease in pollution. If, on the other hand, the RRA at optimal pollution is less than one, an increase in potential income will increase MBP and call for a decrease in emissions. This result is best illustrated in the figure below. The optimal pollution, x_1, corresponds to point A where MBP_1 equals the MCP. When income

[4] We are using the Arrow-Pratt measurement of relative risk aversion, that is $RRA = -\frac{c_i u_i^{''}}{u_i^{'}}$.

increases, MBP rotates clockwise around the pivotal point B where RRA equals one. This results in a new higher equilibrium level of pollution, x_2, corresponding to the intersection of MBP_2 and MCP at point C.

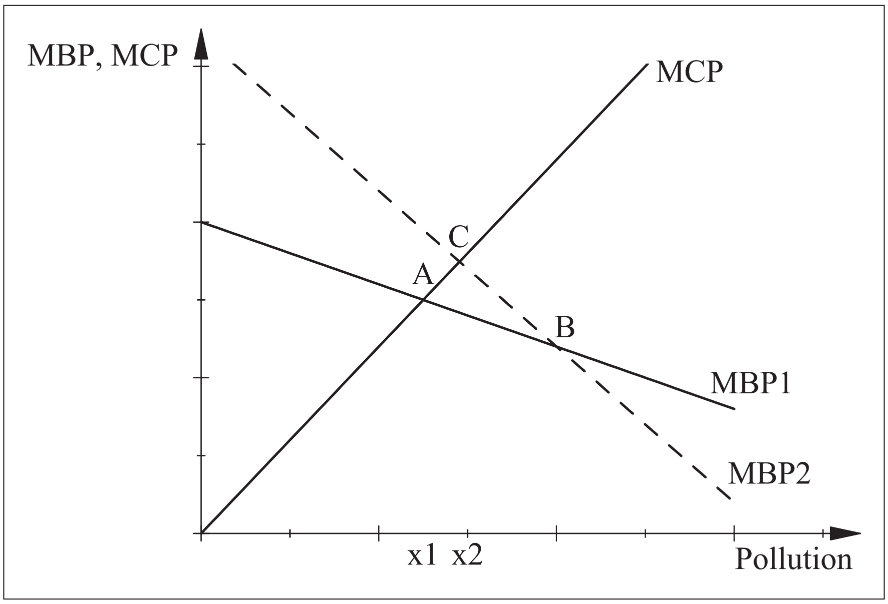


Figure 1.1: Equilibrium when RRA < 1.

What is interesting is that, as long as the utility function does not exhibit constant relative risk aversion, the pivotal point, B, is not fixed but it depends on the values of x and y. Most importantly, when the utility function exhibits increasing relative risk aversion,[5] the pivotal point moves northwest in the graph as y increases. Recall that at point B in the above figure, RRA equals one. Since income increases affect RRA as well, a change in pollution is called for, so that RRA remains equal to one. In particular, totally differentiating RRA and setting $dRRA$ equal to zero yields

$$\frac{\partial RRA}{\partial y_i} dy_i + \frac{\partial RRA}{\partial x_i} dx_i = 0,$$

where with a slight abuse of notation

$$\frac{\partial RRA}{\partial y_i} = -x_i \left(\frac{\phi(y_i) - y_i \phi'(y_i)}{\phi(y_i)^2} \right) \left(\frac{u_i'' u_i' + c_i u_i''' u_i' - c_i (u_i'')^2}{(u_i)^2} \right) > 0$$

[5]The assumption of increasing relative risk aversion with respect to consumption has implications on the third derivative of the utility function, u_i.

and

$$\frac{\partial RRA}{\partial x_i} = -\left(\frac{y_i}{\phi(y_i)}\right)\left(\frac{u_i''u_i' + c_i u_i''' u_i' - c_i (u_i'')^2}{(u_i)^2}\right) > 0.$$

The positivity of the above derivatives stems from the increasing RRA assumption. As a result, after a significant increase in y, we get Figure 1.2, where now the pivotal point B is to the right of the original equilibrium at point A. In this case, an increase in income results in a new equilibrium at point C with decreased level of pollution. Intuitively, any increase in income has two effects: first, it increases the marginal product of pollution, thus asking for an increase in pollution; second, it decreases the marginal benefit of consumption, and thus calling for a decrease in pollution. When RRA is less than one, the former is greater than the latter, while for RRA greater than one the opposite is true.

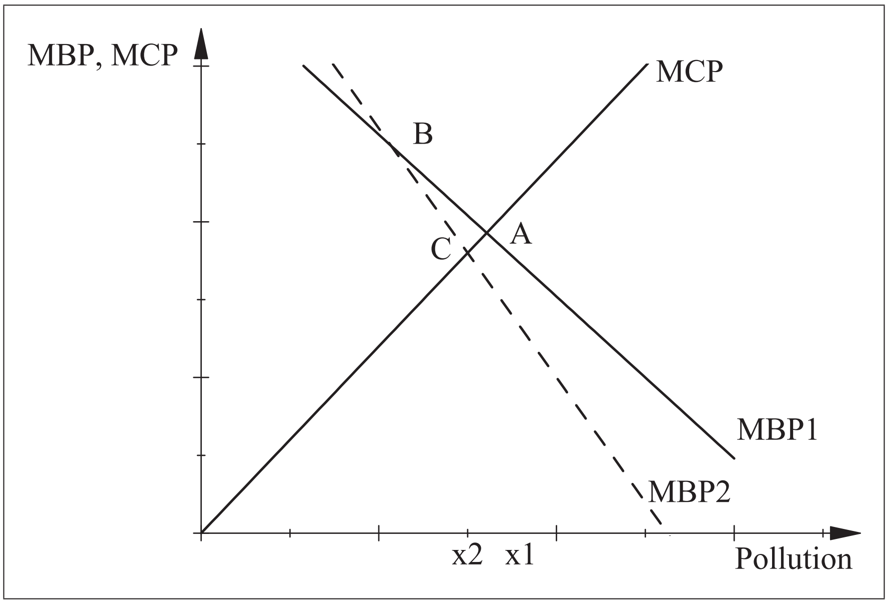


Figure 1.2: Equilibrium when RRA >1.

Finally, in the case of a constant RRA utility function, the pivotal point is not in the positive orthant and MBP changes monotonically with income. Therefore, as long as RRA is less than one, pollution increases with income, while it decreases otherwise. In the latter case, an inverted V-shaped EKC can be derived as the economy transits from a corner to an interior solution. This proves that our analysis is a generalization of Stokey (1996).

These results can be extended for multiple countries under some restrictions.

In what follows, we first show when our model has a unique solution. Moreover, for the case of increasing or constant but greater than unit RRA, the solution will be interior for some income level, resulting in an inverted U-shaped income pollution path.

1.3 The competition in pollutants as a Potential Game

Solving an $n \times n$ system of non-linear reaction functions can be complicated. Most importantly, neither the existence nor the uniqueness of a solution can be guaranteed. As a result the pollution-income path might not be tractable. In what follows, we show that when the competition in pollutants takes the form defined in our model, it can be thought of as a Potential Game.

Definition 2 (*Monderer and Shapley, 1996*) *Let* $\Gamma = \left\langle N, X, \{V_i\}_{i \in N} \right\rangle$ *be a strategic form game with a finite number of players. The set of players is* $N = \{1, 2, ..., n\}$*, the strategy space is* $X = X_1 \times X_2 \times ... \times X_n$*, and* $V_i : X \to R$ *is the payoff function of player* i*. A function* $P : X \to R$ *is an ordinal potential for* Γ*, if for every* $i \in N$ *and for every* $x_{-i} \in X_{-i}$*,*

$$V_i(x, x_{-i}) - V_i(z, x_{-i}) > 0 \Leftrightarrow P(x, x_{-i}) - P(z, x_{-i}) > 0,$$

for every x*,* $z \in A_i$*. A game* Γ *is an ordinal potential game if it admits an ordinal potential.*

We apply the concept of Potential Game to prove existence and uniqueness of the solution to this problem. A simple inspection of the specifics of the model described in section 1.1 shows that the competition in pollutants satisfies the following conditions:

(i) individual strategy spaces are compact as intervals of real numbers, *i.e.* $X_i = [0, \phi(y_i)] \subset \mathbb{R}$,

(ii) the payoff functions are continuously differentiable, and

(iii) the cross-partial derivatives of any two payoff functions are equal, *i.e.*

$$\frac{\partial^2 V_i}{\partial x_i \partial x_j} = \frac{\partial^2 V_j}{\partial x_j \partial x_i} = -\frac{\partial^2 h(x_i + \sum_{k\neq 1}^{n} x_k)}{\partial x_i \partial x_j}.$$

Therefore, according to *Theorem 4.5* in Monderer and Shapley (1996) the Pollution Game G is a Potential Game.

Finding a Potential Function is not always immediate. In our setup, differentiability of individual social utilities is the key element for the algorithm used to find the Potential Function. The following Lemma identifies a potential function for the competition in pollutants game that is strictly concave, and thus it possesses a global maximum, in the strategy space X. The proof of this lemma is straightforward and it is presented in the appendix.

Lemma 3 *The following is a potential function of G:*

$$P(\mathbf{x}) = \sum_{i=1}^{n} u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \textstyle\sum_{k\neq 1}^{n} x_k). \tag{1.8}$$

Moreover, this potential function is strictly concave.

The proof of Lemma 3 is provided in the Appendix. Note that, by definition, the maximization of the Potential Function has the same solution as the maximization problems expressed by (1.6) since both maximization problems yield the same first order and Kuhn-Tucker conditions. The maximization of the potential function is formally expressed as

$$\max_{\mathbf{x}\in\bigcup_{i=1}^{n}[0,\phi(y_i)]} P(\mathbf{x}) = \sum_{i=1}^{n} u_i \left(y_i \frac{x_i}{\phi(y_i)} \right) - h(x_i + \textstyle\sum_{k\neq 1}^{n} x_k), \tag{1.9}$$

where $\mathbf{x} \in \bigcup_{i=1}^{n} [0, \phi(y_i)]$ is the union of the individual constraints and it is a compact set. The first order and Kuhn-Tucker conditions are

$$\begin{cases} \frac{\partial u_i}{\partial c_i} \frac{y_i}{\phi(y_i)} - \frac{\partial h}{\partial x_i} - \lambda_i = 0 \\ \phi(y_i) - x_i \geq 0 \\ \lambda_i \geq 0 \\ \lambda_i (\phi(y_i) - x_i) = 0 \end{cases}, \text{ for all } i \in N \tag{1.10}$$

Under the strict concavity of the objective function and the compactness of the constraints' set, the maximization of the Potential Function with respect to every individual country's pollution level yields a global maximum. Note that, in general, a global maximum of the Potential Function does not guarantee a unique solution for the Potential Game. However, in our case we get the following lemma:

Proposition 4 *Strict concavity of the Potential Function (1.8) implies a unique Nash equilibrium in the pollution game G.*

Proof. The Potential Function is continuous and it is defined over a compact set. According to Weierstrass Theorem it possesses at least one maximum. Strict concavity of the Potential function guarantees the uniqueness of a maximum. It suffices to show that the first order conditions (1.7) that correspond to the pollution game, and the first order conditions (1.10) of the Potential Function's maximization are the same. Therefore, if the maximization of the Potential Function has a unique solution, so does the system of reaction functions defined by (1.7).

■

This result is of great significance since it excludes multiple equilibria and/or indeterminacy in the case of global pollutants.

1.4 Comparative statics

Assuming that the unique solution is interior, the system of reaction functions is given by

$$\frac{\partial u_i}{\partial c_i}\frac{y_i}{\phi(y_i)} - \frac{\partial h}{\partial x_i} = 0, \quad i = 1, 2, ..., n.$$

To simplify notation we have,

$$\frac{\partial u_i}{\partial c_i} = u'$$

and

$$\frac{y_i}{\phi(y_i)} = g(y_i),$$

where by assumption $g' \geq 0$. Moreover note that $\forall i \neq j \in N$ we have

$$\frac{\partial^2 h}{\partial x_i^2} = \frac{\partial^2 h}{\partial x_j^2} = \frac{\partial^2 h}{\partial x_i \partial x_j} = h''.$$

Therefore, the system of reaction functions that solves the pollution game can be written as

$$u_i' g(y_i) - \frac{\partial h}{\partial x_i} = 0, \quad i = 1, 2, ..., n.$$

Taking total derivatives yields

$$\left[u_i'' g(y_i)^2 - h''\right] dx_i + \left[u_i' g'(y_i) + u_i'' g(y_i) x_i\right] dy_i - \left[\sum_{j \neq i}^{n} h''\right] dx_j = 0.$$

The first term in brackets is the slope of country i's reaction function and it is negative. The second term in brackets shows how changes in own income affect the optimal choice of country i. As shown before, this term depends on the degree of relative risk aversion. The last term in the brackets shows the interdependency of country i's pollution decision with the choices of the rest of the world. Denote

$$\begin{aligned} &u_i'' g(y_i)^2 - h'' = R_i' < 0, \\ &u_i' g'(y_i) + u_i'' g(y_i) x_i = u_i' g'(y_i)(1 - RRA_i). \end{aligned}.$$

Therefore, we get

$$R_i' dx_i + u_i' g'(y_i)(1 - RRA_i) dy_i - \left[\sum_{j \neq i}^{n} h''\right] dx_j = 0 \Rightarrow$$

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dy_i} = 0 \Rightarrow$$

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dx_i}\frac{dx_i}{dy_i} = 0$$

1.4.1 The n-country symmetric case

Assume that all n countries share the same utility function, they have the same initial income, and they all grow at the same rate, *i.e.* $\forall i, j \in N$ we have $dy_i = dy_j$. Therefore, in equilibrium all countries should choose the same pollution level, *i.e.*, $x_i^* = x_j^*$. If the solution is interior the following is true:

Lemma 5 *Assume n symmetric countries, where $n \in \mathbb{N}$, competing in pollutants. Then*

$$\frac{dx^*}{dy} \gtreqless 0 \text{ if } RRA \lesseqgtr 1$$

Moreover, x^* can be expressed as a function of y, i.e., $x^* = F(y)$ where $F : Y \to X$ is continuous.

Proof. For the proof of the above Lemma recall that

$$R_i' \frac{dx_i}{dy_i} + u_i' g'(y_i)(1 - RRA_i) - \left[\sum_{j \neq i}^{n} h''\right] \frac{dx_j}{dx_i}\frac{dx_i}{dy_i} = 0$$

Dropping the subscripts, the above equation becomes

$$R' \frac{dx}{dy} + u' g'(y)(1 - RRA) - (n-1) h'' \frac{dx}{dx}\frac{dx}{dy} = 0 \Rightarrow$$

$$\frac{dx^*}{dy} = -\frac{u'g'(y)(1 - RRA)}{R' - (n-1)h''}$$

Note that $R' - (n-1)h'' < 0$, thus this derivative is well defined over Y. Therefore, x^* can be expressed as a continuous function of y. Moreover, by assumption we have $u' \geq 0$, $g' \geq 0$. Thus, $\forall n \in \mathbb{N}$, $RRA \lesseqgtr 1 \Rightarrow \frac{dx}{dy} \gtreqless 0$. ■

This shows that, at the interior, pollution increases with income when the degree of relative risk aversion is low, while pollution decreases with income otherwise. Moreover, the optimal pollution level is a continuous function of the level of income eliminating the possibility for big changes in pollution for infinitesimal changes in income. However, the solution need not be interior. We can generalize the result of Lopez (1994) to include n symmetric countries and an upper bound in pollution at each level of income, with the following proposition:

Proposition 6 *Assume n countries, where $n \in \{1, 2, ..., N\}$, competing in pollutants. Assume that*

(i) $u''$is bounded from below

(ii) either $\forall x > 0$, $\frac{\partial RRA}{\partial c} > 0$, or $\frac{\partial RRA}{\partial c} = 0$ and $RRA > 1$

Then pollution increases at first, but eventually decreases with income, thus generating an EKC.

Figure 1.3 provides a sketch of the proof for the case of increasing relative risk aversion. Curve F represents the interior solution that, according to Lemma 2, is continuous. Under increasing RRA, F has a global maximum at y_0. Pollution's upper bound, $\phi(y)$, is by assumption unbounded from above, increasing, and concave. Then the pollution-income path is the lower envelope of F and ϕ generating an EKC. A full proof of proposition 2 is presented in the appendix.

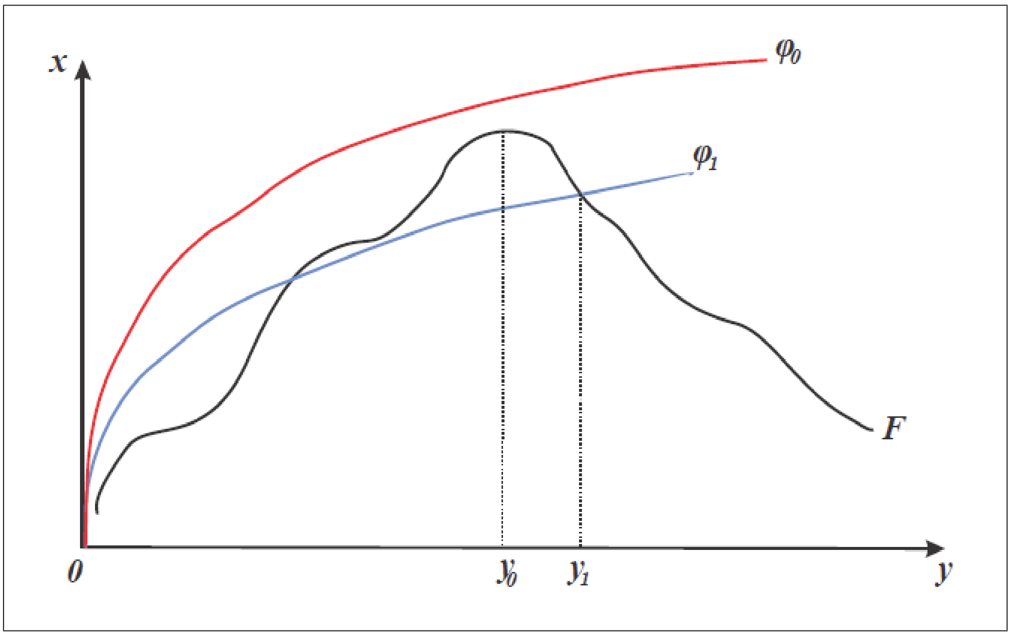


Figure 1.3: Proof of Proposition 2.

1.4.2 The 2-country asymmetric case

In **the** case of two asymmetric countries we get (from the perspective of country 1),

$$R_1' \frac{dx_1}{dy_1} + u_1' g'(y_1)(1 - RRA_1) - h'' \frac{dx_2}{dx_1}\frac{dx_1}{dy_2} = 0,$$

where from the first order conditions of country 2 we get,

$$\frac{dx_2}{dx_1} = \frac{h''}{R_2'} - \frac{u'g'(y)(1 - RRA)}{R_2'}\frac{dy_2}{dx_1}.$$

Substituting in the above expression and solving yields

$$\frac{dx_1}{dy_1} = \left[R_1' - \frac{(h'')^2}{R_2'}\right]^{-1}\left[u_1' g'(y_1)(1 - RRA_1) + \frac{h''}{R_2'}u_2' g'(y_2)(1 - RRA_2)\frac{dy_2}{dy_1}\right].$$

Note that the fist term in brackets is negative, thus

$$sign\left[\frac{dx_1}{dy_1}\right] = sign\left[u_1' g'(y_1)(1 - RRA_1) + \frac{h''}{R_2'}u_2' g'(y_2)(1 - RRA_2)\frac{dy_2}{dy_1}\right].$$

It is worth noticing that the pollution-income relationship for country 1 depends not only on its own income, but also on country 2's income as well as on the income distribution. We can distinguish three cases, assuming that in each case country 1 is wealthier than country 2, and that both countries experience income growth and increasing relative risk aversion. In the first case, both countries have relatively low income so that $RRA_i < 1$, $\forall i = 1,2$. The first term of the RHS is positive while the second term is negative. Depending on the size of the derivative dy_2/dy_1, the latter exceeds the former the higher this derivative is, resulting in a decreasing pollution-income path. Intuitively, when low-income countries converge in income, the wealthier countries among them have a lower turning point income compared to the symmetric case. In the second case, country 1 has relatively high income so that $RRA_1 > 1$, while country 2 has relatively low income so that $RRA_2 < 1$. Both terms of the RHS are negative and country 1 definitely decreases pollution with income. Intuitively, richer countries reduce pollution a lot faster when facing pollution-aggressive poor countries. Finally, in the third case, both countries have relatively high income so that $RRA_i > 1$, $\forall i = 1,2$. The first term of the RHS is negative while the second term is positive. The latter exceeds the former the bigger the derivative dy_2/dy_1 is. Intuitively, when high-income countries converge in income, the wealthier countries among them reduce pollution at a slower rate compared to the symmetric case. In extreme cases, wealthier countries may even start increasing pollution with income (resulting in an N-shaped pollution-income path).

1.5 A special case

In order to investigate the subject further we adopt specific functional forms. The functional forms adopted satisfy all the assumptions made in Section 1.1. More specifically, the utility received from consumption is represented by a truncated quadratic function, *i.e.*

$$u_i(c_i) = \begin{cases} c_i - \frac{1}{2}\beta c_i^2, & \text{if } c_i \leq 1/\beta \\ 1/2\beta, & \text{otherwise} \end{cases},$$

with $\beta \in (0,1)$. One can interpret β as a risk aversion indicator, with higher values of β corresponding to a higher degree of relative risk aversion[6]. The production function is assumed to be

$$c_i = y_i \sigma\left(x_i\right) = y_i \left(\frac{x_i}{\overline{x}_i}\right),$$

where $\sigma\left(x_i\right) = \left(x_i/\overline{x}_i\right)$. Substituting the production function in the utility function yields

$$u_i(c_i) = y_i \left(\frac{x_i}{\overline{x}_i}\right) - \frac{1}{2}\beta y_i^2 \left(\frac{x_i}{\overline{x}_i}\right)^2.$$

The social damage function, h, is quadratic in total pollution, *i.e.*

$$h_i = \frac{\rho}{2}\left(x_i + \sum_{j \neq i}^{n} x_j\right)^2,$$

where $\rho \in R_+$ is a scale parameter that shows how pollution is perceived by country i. Finally, we assume that pollution is bounded from above at every level of income. This upper bound is an increasing function of income, *i.e.*

$$\overline{x}_i = \phi(y_i) = y_i^{\alpha},$$

where $\alpha \in (0,1)$. This parameter, α, incorporates technological improvement in the abatement process implicitly into our model. Note that, although $\alpha < 1$, it is still positive, meaning that any given degree of technological advancement in the abatement process is not enough to generate an EKC.

[6]Using the Arrow-Pratt measurement of relative risk aversion for this utility function we get $RRA = -\frac{c_i u_i''}{u_i'} \Rightarrow RRA = \frac{c_i\beta}{1-c_i\beta}$ and $\partial RRA/\partial\beta = c_i/\left(1-c_i\beta\right)^2 > 0$.

Under this setup, the maximization problem for a social planner in country i is given by

$$\max_{x_i\in[0,\overline{x}_i]} V_i \;=\; y_i\left(\frac{x_i}{\overline{x}_i}\right)-\frac{1}{2}\beta y_i^2\left(\frac{x_i}{\overline{x}_i}\right)^2-\frac{\rho}{2}\left(x_i+\sum_{j\neq i}^{n}x_j\right)^2 \Rightarrow$$

$$\max_{x_i\in[0,y_i^{\alpha}]} V_i = y_i^{1-\alpha}x_i-\frac{1}{2}\beta y_i^{2-2\alpha}x_i^2-\frac{\rho}{2}\left(x_i+\sum_{j\neq i}^{n}x_j\right)^2. \qquad (1.11)$$

Solving this model requires, first, finding the best responses for each country, and second, identifying the possible Nash Equilibria.[7] Given the definitions of the objective functions, country i's best response is a function of all other countries' choices of pollutants. Therefore, the solution of this model can be found by solving the system of the best response functions (*i.e.* reaction functions).

Taking the first order condition of the maximization problem defined by (1.11), yields

$$y_i^{1-\alpha}-\beta y_i^{2-2\alpha}x_i-\rho\left(x_i+\sum_{j\neq i}^{n}x_j\right)\geq 0,$$

with the equality to hold $\forall\ x_i \in [0, y_i^{\alpha}]$. The reaction function of a country is then given by

$$x_i=\begin{cases}\dfrac{y_i^{1-\alpha}}{\rho+\beta y_i^{2-2\alpha}}-\dfrac{\rho\sum\limits_{j\neq i}^{n}x_j}{\rho+\beta y_i^{2-2\alpha}}, & \text{if } \sum\limits_{j\neq i}^{n}x_j\geq\dfrac{(\rho+\beta y_i^{2-2\alpha}-y_i^{1-2\alpha})y_i^{\alpha}}{\rho}\\[2ex] y_i^{\alpha}, & \text{otherwise}\end{cases}.$$

Note that, at the interior, the competition in pollutants is competition in strategic substitutes yielding downward-sloping reaction functions, *i.e.*

$$\frac{\partial x_i}{\partial x_j}=-\frac{\rho}{\rho+\beta y_i^{2-2\alpha}}<0,\ \forall j\neq i.$$

[7] Starting from a completely symmetric case with n countries, it is natural to assume that their decisions on pollution are taken simultaneously.

In what follows, we solve this model for the case of (i) n symmetric countries, and (ii) two asymmetric countries. Comparative static analysis is conducted for the parameters of the model regarding the pollution behavior and the turning point income.

1.5.1 Solution and comparative statics

The symmetric case

We first examine the totally symmetric case where all countries share the same income, **i.e.,** $y_i = y$, and the same parameter values α, β, and ρ. Given these assumptions and for $n \in \{1, 2, 3, ...\}$ the optimal pollution is

$$x^* = \frac{y^{1-\alpha}}{\rho n + \beta y^{2-2\alpha}},$$

where $x^* = x_i^*$. Note that, given $x^* \leq \overline{x}$, the pollution-income path follows the EKC pattern, with

$$\begin{cases} \dfrac{\partial x^*}{\partial y} \geq 0, & \text{if } y \leq \left(\dfrac{\beta}{\rho n}\right)^{-\left(\frac{1}{2-2\alpha}\right)} \\ \dfrac{\partial x^*}{\partial y} < 0, & \text{otherwise.} \end{cases}$$

It is also interesting to see that $\partial x^*/\partial \beta < 0$, $\partial x^*/\partial \rho < 0$, and $\partial x^*/\partial n < 0$. That is, any increase in the degree of RRA, the degree of pollution dispersion and perception, or the number of countries, is associated *ceteris paribus* with a lower level of individual pollution. The explanation lies in the nature of the game: pollutants are strategic substitutes. For any given level of income, factors that make a country less aggressive in the pollution game will result in lower emissions. No clear conclusion can be drawn for the relationship between pollution and the abatement technology.

The optimal pollution level has a single peak (turning point) since

$$\frac{\partial x}{\partial y} = \frac{(1-\alpha)\left(\rho n - \beta y^{2-2\alpha}\right)}{y^{\alpha}\left(\rho n + \beta y^{2-2\alpha}\right)^{2}} = 0 \Rightarrow \rho n - \beta y^{2-2\alpha} = 0,$$

with

$$y_{TP} = \left(\frac{\beta}{\rho n}\right)^{-\left(\frac{1}{2-2\alpha}\right)},$$

where y_{TP} is the turning point income. Furthermore, $\partial y_{TP}/\partial \beta < 0$, $\partial y_{TP}/\partial \rho > 0$, and $\partial y_{TP}/\partial n > 0$.

Depending on specific values of α, β, ρ, and n, we get $[\rho n + \beta y^{2-2\alpha} - y^{1-2\alpha} \leq 0]$, and pollution reaches its upper bound, i.e. $x_i = \overline{x}$. At the corner, the pollution-income relationship follows an increasing and concave path since,

$$\begin{cases} \frac{\partial x^*}{\partial y} > 0 \Leftrightarrow \dfrac{\partial\left[y^{\alpha}\right]}{\partial y} = \alpha y^{\alpha-1} > 0, \text{ and} \\ \\ \frac{\partial^2 x^*}{\partial y^2} > 0 \Leftrightarrow \dfrac{\partial\left[\alpha y^{\alpha-1}\right]}{\partial y} = \alpha\left(\alpha-1\right) y^{\alpha-2} < 0. \end{cases}$$

At the corner, pollution does not depend on β, ρ, or n, while it increases monotonically with α.

Graphically, the comparative static analysis at the interior is represented by the following graphs. Figure 1.4 shows the relationship between EKC and the number of competing countries, while Figure 1.5 shows the relationship between EKC and environmental awareness. Figure 1.6 describes the relationship between RRA and the EKC. We observe that the EKC becomes smoother when the number of countries increases, when there is a hike in environmental awareness, and/or when the RRA increases.

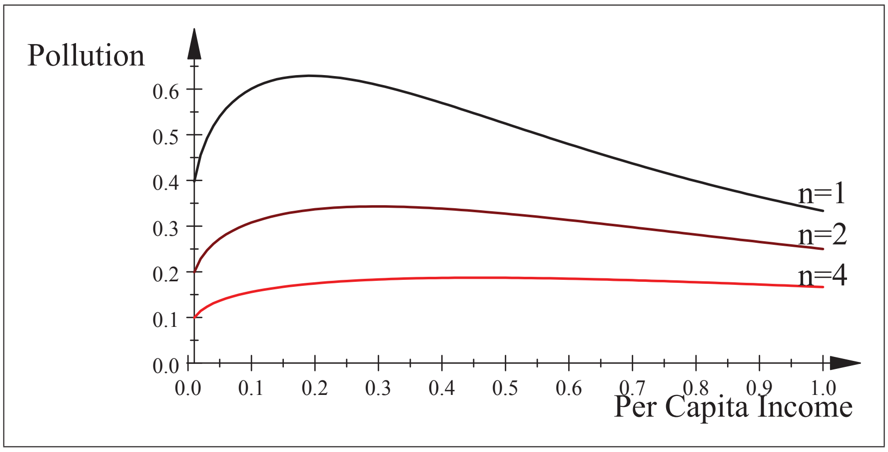


Figure 1.4: Number of countries and EKC.

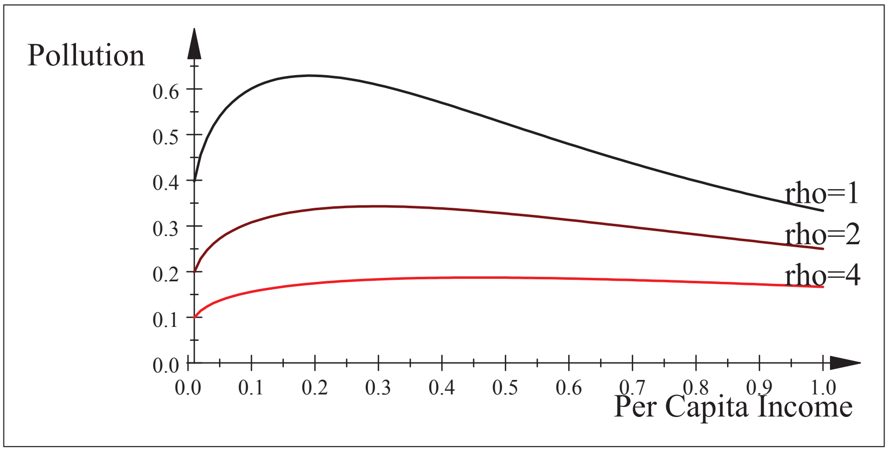


Figure 1.5: Environmental awareness and EKC.

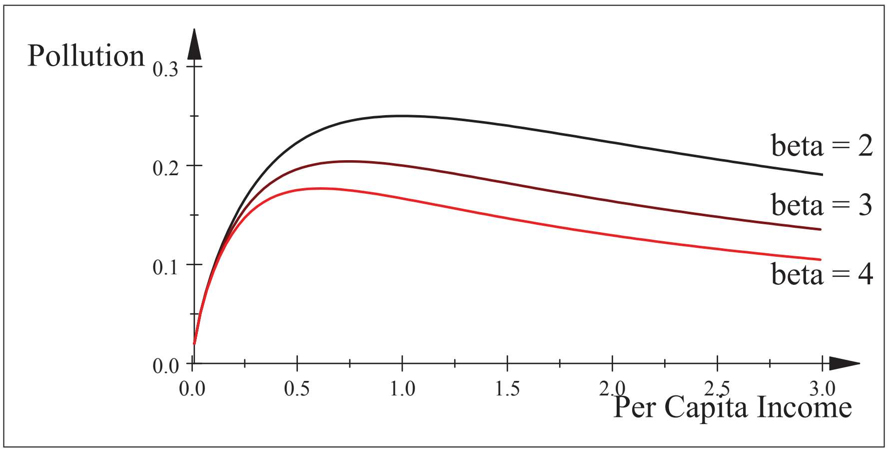


Figure 1.6: RRA and EKC.

The asymmetric case

For $n = 2$ we get

$$x_i^* = \begin{cases} 0, & \text{if } y_i < l\left(y_i\right) \\ \dfrac{\rho y_i^{1-\alpha} - \rho y_j^{1-\alpha} + \beta y_i^{1-\alpha} y_j^{2(1-\alpha)}}{\rho\beta y_i^{2(1-\alpha)} + \rho\beta y_j^{2(1-\alpha)} + \beta^2 y_i^{2(1-\alpha)} y_j^{2(1-\alpha)}}, & \text{if } y_i \geq l\left(y_j\right), \text{ and } y_j \geq l\left(y_i\right) \\ \dfrac{y_i^{1-\alpha}}{\rho + \beta y_i^{2-2\alpha}}, & \text{if } y_j < l\left(y_i\right) \end{cases}$$

where

$$l\left(y_k\right) = \left(\frac{\rho}{\rho + \beta y_k^{2(1-\alpha)}}\right)^{\frac{1}{1-\alpha}} y_k.$$

Under extreme income inequalities the poor country does not pollute, *i.e.*, $y_i << y_j \Rightarrow x_i^* = 0$. This situation is referred to as environmental poverty trap. At the interior, i.e. $x_i^* < \overline{x}$, the pollution-income path follows the EKC pattern for various values of α, β, and ρ. But most importantly, (i) the closer the incomes, y_i and y_j, are, and/or (ii) the greater the values of α and ρ are, and/or (iii) the lower the value of β is, the greater the value interval that yields EKC patterns:

$$\begin{cases} \dfrac{\partial x_i^*}{\partial y_i} \geq 0, & \text{if } 2\rho y_i^{1+\alpha} y_j^{1+\alpha} - \rho y_i^2 y_j^{2\alpha} + y_i^{2\alpha} y_j^2 - \beta y_i^2 y_j^2 \geq 0 \\ \dfrac{\partial x_i^*}{\partial y_i} < 0, & \text{otherwise.} \end{cases}$$

In Figure 1.7 one can see the EKC income-pollution path for country i (domestic country), for different values of country j's income (foreign country).

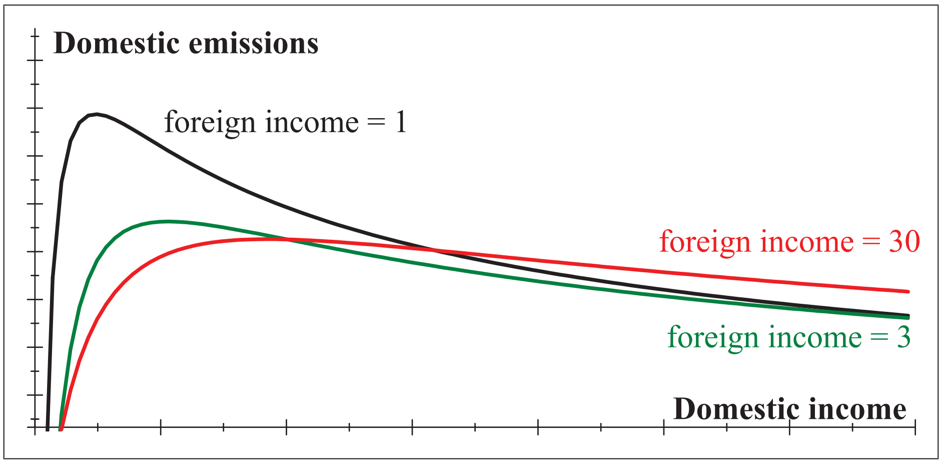


Figure 1.7: EKC and foreign income.

For specific values of α and β, *i.e.*, $\left(\beta y_j^{2(1-\alpha)}+1\right) y_i^{1-\alpha} - \beta\left(\beta y_j^{2(1-\alpha)}+1\right) y_i^{2-\alpha} + \left(\beta y_j^{2(1-\alpha)}\right) y_i^{\alpha} - y_j^{1-\alpha} \leq 0$, x_i^* reaches the upper bound $\overline{x}_i$. If $x_i^* = \overline{x}_i$ then the pollution-income path follows an increasing and concave path:

$$\begin{cases} \frac{\partial x_i^*}{\partial y_i} > 0 \Leftrightarrow \frac{\partial\left[y_i^{\alpha}\right]}{\partial y_i} = \alpha y_i^{\alpha-1} > 0 \\ \\ \frac{\partial^2 x_i^*}{\partial y_i^2} < 0 \Leftrightarrow \frac{\partial\left[\alpha y_i^{\alpha-1}\right]}{\partial y_i} = \alpha\left(a-1\right) y_i^{\alpha-2} < 0. \end{cases}$$

If x_i^* is interior then the poorer country pollutes more if both countries' incomes are already high. Whereas if both countries are poor, then the richer one pollutes more. This observation is consistent with (cross-sectional) EKC patterns:

$$x_i > x_j \Leftrightarrow y_i < y_j \text{ and } y_i y_j > \left(\frac{2}{\beta}\right)^{\frac{1}{1-a}}$$

Income distribution and pollution Denote total income $y = y_1 + y_2$. Then, for any given share $t \in [0,1]$[8] of world income for country 1 we get $y_1 = ty$ and $y_2 = (1-t)\,y$. Country 1's pollution can be written as

$$x_1^* = \frac{(ty)^{1-\alpha} - (1-t)^{1-\alpha} y^{1-\alpha} + \beta\,(ty)^{1-\alpha}\left[(1-t)\,y\right]^{2(1-\alpha)}}{\beta\,(ty)^{2(1-\alpha)} + \beta\left[(1-t)\,y\right]^{2(1-\alpha)} + \beta^2\,(ty)^{2(1-\alpha)}\left[(1-t)\,y\right]^{2(1-\alpha)}}$$

[8] Note that we actually require $t \in \left[\underline{t}, \overline{t}\right]$ where $\underline{t} > 0$, $\overline{t} < 1$ such that interior solution is guaranteed.

and total pollution becomes

$$x^*_{TOTAL} = x^*_1 + x^*_2 = \frac{t\,(1-t)\left[t\,(1-t)^{\alpha} + t^{\alpha}\,(1-t)\right] y^{1-\alpha}}{t^2\,(1-t)^{2\alpha} + t^{2\alpha}\,(1-t)^2 + \beta t^2\,(1-t)^2\, y^{2(1-\alpha)}}.$$

In Figure 1.8 we can see the behavior of total pollution as world income increases for different shares t of country 1. Note that if the share t of a country remains relatively constant, an EKC is more likely to be obtained. However, this observation cannot be extended to n countries. When $n = 2$, fixing one country's income share also sets the share of its rival. For $n > 2$, this is no longer true: even if a country grows steadily over time, variations on the relative shares of the rest of the world might have a great impact on that country's income-pollution relationship.

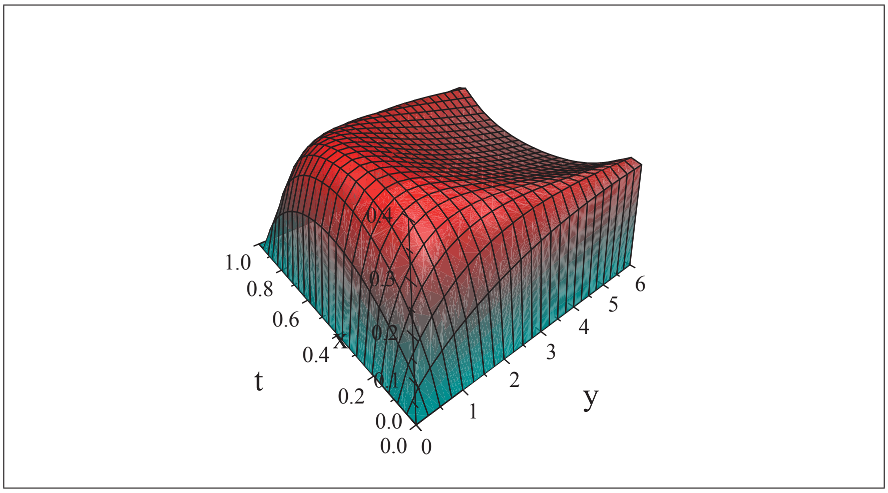


Figure 1.8: Income distribution and EKC.

It is worth noticing that for low levels of global income, equal income distribution seems to hurt the environment, while as global income rises, income equality is optimal from a global perspective. Intuitively, when the world is very poor, the marginal benefit from pollution is relatively high. An unequal income distribution decreases the marginal benefit from pollution of the relatively wealthier countries, leading them to decrease the rate at which they increase pollution. At the same time, the relatively poorer countries are experiencing a lower marginal benefit from pollution compared to the symmetric case. Their reaction is to decrease pollution at an increasing rate. The latter exceeds the former, thus unequal distribution is beneficial for the environment. For relatively higher global income the opposite is true. In Figure 1.9, we see the total pollution for different shares of global income for country 1. The red line represents high global income, while the black line represents low global income.

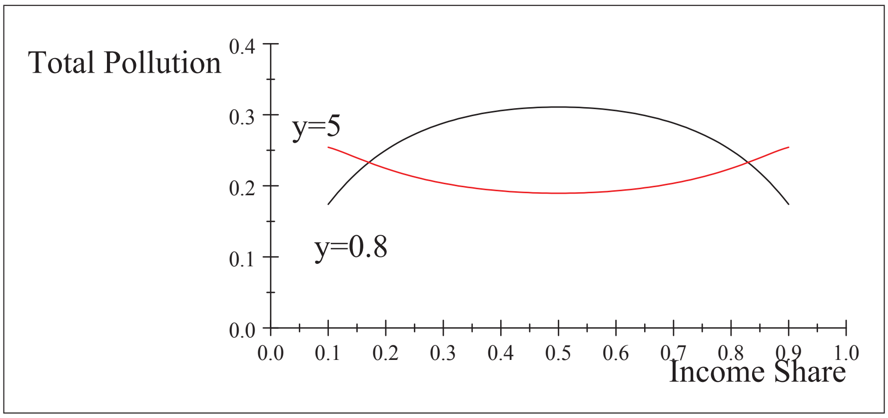


Figure 1.9: Income distribution and total emissions.

1.6 Conclusions

In this paper, we show that the competition in pollutants has a unique solution, thus giving rise to a tractable income-pollution path. Moreover, a hump shaped pollution-income path is generated for multiple symmetric countries under relatively weak assumptions and without any stringent conditions on technology. The setup of the model is fairly general and it can be modified to include production where pollution can be thought of as a factor of production. It is a generalization of Lopez (1994) in that it considers multiple countries and it assumes an upper bound on maximum pollution. It also generalizes Stokey (1998) in that it applies to a larger family of utility functions. Our model, in the presence of income asymmetries alone, can generate an N-shaped pollution income path, consistent with recent empirical studies. Finally, always for the asymmetric cases, it can give rise to different EKCs for different countries. This is solely a result of the externality caused from pollution and it is not related to environmental dumping.

1.7 Appendix

Proof of Lemma 3. First, note that $\forall i \in N$, the welfare functions are defined as $V_i : X_i \to \mathbb{R}$, where $X_i \subset \mathbb{R}$ and they are continuously differentiable. Function $P : X_i \to \mathbb{R}$ is also continuously differentiable and $\forall i \in N$ we have

$$\frac{\partial P}{\partial x_i} = \frac{\partial V_i}{\partial x_i}.$$

Therefore, according to *Lemma 4.4* in Monderer and Shapley (1996), (1.8) is a potential function of G. Furthermore, note that $\forall i \in N$

$$\frac{\partial^2 P}{\partial x_i^2} = \frac{\partial^2 V_i}{\partial x_i^2} = \left[\frac{\partial^2 u_i}{\partial c_i^2}\left(\frac{y_i}{\phi(y_i)}\right)^2\right] - \frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_i^2} = A_i - \frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_i^2},$$

where $A_i = \left[\frac{\partial^2 u_i}{\partial c_i^2}\left(\frac{y_i}{\phi(y_i)}\right)^2\right] < 0$. *Therefore, by definition $P(x)$ is a potential function. Moreover, the Hessian of $P(x)$ is given by*

$$H = \begin{bmatrix} \frac{\partial^2 P}{\partial x_1^2} & \frac{\partial^2 P}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 P}{\partial x_1 \partial x_n} \\ \frac{\partial^2 P}{\partial x_1 \partial x_2} & \frac{\partial^2 P}{\partial x_2^2} & \cdots & \frac{\partial^2 P}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 P}{\partial x_1 \partial x_n} & \frac{\partial^2 P}{\partial x_2 \partial x_n} & \cdots & \frac{\partial^2 P}{\partial x_n^2} \end{bmatrix} =$$

$$= \begin{bmatrix} A_1 & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_1 \partial x_2} & \cdots & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_1 \partial x_n} \\ -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_2 \partial x_1} & A_2 & \cdots & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_2 \partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_n \partial x_1} & -\frac{\partial^2 h\left(x_i + \sum_{k\neq i}^n x_k\right)}{\partial x_n \partial x_2} & \cdots & A_n \end{bmatrix}$$

It worth noticing that the second derivatives of the social damage function - own and cross partial derivatives - are the same, *i.e.* for any k, l, m, $n \in N$ we have

$$\frac{\partial^2 h\left(\sum_{i=1}^n x_i\right)}{\partial x_k \partial x_l} = \frac{\partial^2 h\left(\sum_{i=1}^n x_i\right)}{\partial x_m \partial x_n} = B\left(\sum_{i=1}^n x_i\right) \geq 0$$

Consider $\mathbf{z} \in \mathbb{R}^n \diagup \{\mathbf{0}_{n\times 1}\}$ *and construct the quadratic form*

$$\mathbf{z}^T H \mathbf{z} =$$

$$= \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}^T \begin{bmatrix} A_1 & -B\left(\sum_{i=1}^n x_i\right) & \cdots & -B\left(\sum_{i=1}^n x_i\right) \\ -B\left(\sum_{i=1}^n x_i\right) & A_2 & \cdots & -B\left(\sum_{i=1}^n x_i\right) \\ \vdots & \vdots & \ddots & \vdots \\ -B\left(\sum_{i=1}^n x_i\right) & -B\left(\sum_{i=1}^n x_i\right) & \cdots & A_n \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} =$$

$$= \sum_{i=1}^n \left(z_i^2 A_i\right) - B\left(\sum_{i=1}^n x_i\right) \left(\sum_{i=1}^n z_i\right)^2 < 0$$

Thus the Hessian matrix of the potential function is negative definite and, therefore, the potential function $P(\mathbf{x})$ *is strictly concave.* ■

Proof of Proposition 6. Denote the interior solution as $x^* = F(y)$, while the corner solution is expressed by $x^* = \phi(y)$. Given the uniqueness of solution the relationship between income and pollution can be expressed as follows:

$$x^* = \min\{F(y), \phi(y)\}$$

This equation is by construction continuous since its arguments are continuous functions, function $\phi(y)$ by assumption and function $F(y)$ due to Lemma 5.

We first check the case of increasing RRA. Note that under the first part of assumption (ii) we get $\frac{\partial RRA}{\partial c} > 0 \Rightarrow \frac{\partial RRA}{\partial y} > 0$, while due to assumption (i) we get

$\lim_{y\to 0} RRA = 0$, and

$\lim_{y\to\infty} RRA = \infty$

Therefore, and since RRA is obviously continuous in y, $\exists$ some $y_0 \in R_{++}$ such that $RRA = 1, \forall x \in R_{++}$. Due to Proposition 2 we know that $y_0 = \arg\max F(y)$. Moreover, $F(y_0) < \infty$, since $\forall y < \infty$, $\lim_{x\to\infty}[u'(y/\phi(y))] = 0 < \lim_{x\to\infty} h' = \infty$. At $y = y_0$ we distinguish two cases:

(a) $F(y_0) < \phi(y_0)$. Therefore, $\forall y > y_0$, $x^* = \min\{F(y), \phi(y)\} = F(y)$ and due to Proposition 2, $dx^*/dy < 0$.

(b) $F(y_0) \geq \phi(y_0)$. Recall that $\phi(y)$ is continuous and note that $\lim_{y\to\infty} \phi(y) = \infty$. Therefore, according to the intermediate value theorem, $\exists$ some $y_1 \in [y_1, \infty)$ such that $\phi(y_1) = F(y_0)$. Since $\phi(\cdot)$ is strictly increasing and unbounded form above, $\forall y > y_1 \Rightarrow \phi(y) > \phi(y_1) = F(y_0) \Rightarrow \phi(y) > F(y) \Rightarrow x^* = \min\{F(y), \phi(y)\} = F(y)$ and due to Proposition 2, $dx^*/dy < 0$.

We now turn to the case where RRA is constant and greater than unit. As a result $F(y)$ is strictly decreasing and since $\phi(y)$ is strictly increasing and unbounded from above, $\exists$ some $y_1 \in [y_1, \infty)$ such that $\phi(y_1) = F(y_0)$. Therefore, $\forall y > y_1$ $\Rightarrow \phi(y) > \phi(y_1) = F(y_1) \Rightarrow x^* = \min\{F(y), \phi(y)\} = F(y) \Rightarrow dx^*/dy < 0$. ■

Chapter 2

A Dynamic Analysis of the EKC in a Multicountry Setting

2.1 Introduction

Income growth and environmental quality are two very important determinants of social welfare that are linked through the Environmental Kuznets Curve hypothesis (EKC). The Environmental Kuznets Curve postulates that at the early stages of economic development the environment deteriorates. However, there is a critical income level beyond which income growth is associated with improvements in environmental quality. Graphically, this can be represented by an inverted U-shaped curve. But does the EKC exist? This hypothesis has been supported by numerous studies and rejected by many others. The prolonged controversy over the EKC is not surprising since its acceptance has very important policy implications. The environment can benefit from growth-stimulating policies only when the income-pollution relationship turns negative beyond a reasonable level of income. Otherwise, social welfare may decrease in the long run due to catastrophic effects of growth on the natural environment. However, the relationship between income and pollution is far more complex than responding to the above question with a simple yes or no. The present paper stands in middle ground, as the existence of an EKC is not guaranteed.

The literature on EKC starts in the early 90s with the seminal papers of Grossman and Krueger (1991) and Shafik and Bandyopadhyay (1992) showing that the EKC describes the evolution of some pollutants. Many theoretical models, both static and dynamic, have been developed subsequent to these studies, and different explanations have been offered. The most prominent ones are high-income elasticity for environmental quality (Selden and Song (1994)), increasing returns to abatement (Adreoni and Levinson (2001)), and pollution havens due to international trade (Copeland and Taylor (1995)). An analytical but not exhaustive review of the related literature has been provided in Chapter 1 of the present thesis.

The majority of the models proposed in the literature are single-agent models. This naturally raises the question: How well do these models picture the income-pollution relationship when the international externalities of transboundary and global pollutants are disregarded? Even in the absence of trade, countries interact since the emissions of one hurt the others and vice versa. Therefore, emissions are strategic complements and countries engage in a pollution game. Recently, a few studies have tried to shed some light on that subject. The first two chapters of the present thesis investigate this subject. Chapter 1 shows that, under certain conditions, the pollution game has a unique solution. Income is treated as an exogenous variable and even when countries are asymmetric only with respect to their endowment, the countries will follow different income-pollution paths. The factors that affect the trajectory of a country's pollution over time are: (i) the income level of the country, (ii) the income levels of the other countries, and (iii) the growth rates of all incomes. As a result, many different income-pollution paths, not identical for all countries, can be derived. Finally, Chapter 1 uses specific functional forms to show that when the world income is relatively low, unequal income distribution may be beneficial for the environment. Hutchinson and Kennedy (2010) derive very similar results using specific functional forms in their model.

The present study is an extension from Chapter 1. We modify the model proposed in Section 1.5 to allow for endogenous structure in the evolution of output. More specifically, we use an overlapping generations (OLG) model where households live for two periods. In the first period of their life, households decide over the pollution technology to be used in the production and their savings for the next period. They receive disutility from current pollution. Output is produced according to an alternative AK process. This model allows for differences in total factor productivities and the effects of TFPs on the income-pollution relationship are analyzed. The present model predicts an EKC in the case of a single country only when the steady state value of capital is relatively high. The effect of changes in productivity and/or in environmental awareness on the steady state values of capital and emissions are also analyzed. In the case of two countries, it is shown that the existence and uniqueness result of Proposition 1.4 can be preserved, thus avoiding indeterminacies in the numerical analysis.

The dynamic analysis of income growth and pollution can be challenging. However, overlapping generations (OLG) models allow the separation of intergenerational aspects from intertemporal choices. OLG models have often been used in the Environmental Kuznets Curve literature. John and Pecchenino (1994) use a general equilibrium OLG model to analyze the income-pollution relationship. Their model predicts the existence of EKC while under certain conditions it gives rise to Pareto-ranked multiple equilibria. Their model has been challenged by Prieur (2009) who extends it by introducing irreversibility of the stock pollution. It is shown that the emergence of the EKC can be questioned. Unlike John and Pecchenino (1994), our model is not general equilibrium but we are analyzing it from the perspectives of a social planner.

The remainder of this chapter is organized as follows: Section 2.2 provides the basic setup of the model. Section 2.3 analyzes the case of a single country. In Section 2.4 we discuss theoretically the case of two countries. Section 2.5 offers numerical analysis. Section 2.6 concludes.

2.2 The model

We consider a two-country overlapping generations model where the production process is described by a modified version of the AK growth model. Specifically, we define the potential output, $y'_{i,t}$, as country i's output ($i = 1, 2$) associated with the highest level of emissions, *i.e.*,

$$y'_{i,t} = A'_i k_{i,t}, \tag{2.1}$$

where $A'_i > 0$ denotes the total factor productivity and $k_{i,t}$ is the available capital of country i at time t. The actual output is then defined as

$$y_{i,t} = y'_{i,t}\sigma = A'_i k_{i,t}\sigma, \tag{2.2}$$

where $\sigma \in [0, 1]$ is a pollution technology index, identical for both countries, with $\sigma = 1$ denoting the dirtiest production method. We specify this index as the ratio of actual emissions, $x_{i,t}$, over the maximum emissions, $\overline{x}_{i,t}$, *i.e.*,

$$\sigma = \frac{x_{i,t}}{\overline{x}_{i,t}}. \tag{2.3}$$

Maximum emissions are an increasing and concave function of the potential output, that is

$$\overline{x}_{t,i} = \left(y'_{t,i}\right)^a = B\left(A'_i k_{i,t}\right)^a, \tag{2.4}$$

where $a \in (0, 1)$, and $B > 0$. The latter is a scale parameter allowing for different emissions per unit of output ratios, while the former is considered as a policy parameter where lower α implies more strict environmental regulation. However, to ensure analytical solutions, we set $\alpha = 1/2$. Substituting equations (2.3) and (2.4) in (2.2) and assuming $a = 1/2$ we get

$$y_{t,i} = A_i' k_{i,t} \frac{x_{i,t}}{B\left(A_i' k_{i,t}\right)^{1/2}} = A_i k_{t,i}^{1/2} x_{i,t}, \tag{2.5}$$

where $A_i = \left(A_i'\right)^{1/2}/B$ denotes the TFP adjusted for pollution.

Households in these economies live for two periods. For simplicity we normalize the initial populations to unity while assuming no population growth. They receive utility from consumption in both periods of their lives and disutility from global pollution while young. Denoting $x_{t,j}$ the rival country's emissions, generation t's utility is defined as

$$U\left(c_{i,t}^y, c_{i,t+1}^o\right) = u\left(c_{i,t}^y\right) + \rho u\left(c_{i,t+1}^o\right) - d\left(x_{i,t} + x_{j,t}\right) \tag{2.6}$$

where $c_{i,t}^y$ is their consumption when young, $c_{i,t+1}^o$ denotes the consumption when old, $u(\cdot)$ is non-decreasing and concave, $d(\cdot)$ is increasing and convex, and $\rho \in [0,1]$ is a parameter of time preference. To simplify our analysis we set $\rho = 1$. During the first period of their life, while young, people have to decide the pollution technology to be used. Given the stock of capital at the beginning of the period, the choice of pollution technology determines the actual output according to equation (2.5). Once generation $t-1$ has claimed their savings, $s_{i,t-1} = c_{i,t}^o$, out of the total output produced in period t, generation t has to decide how to allocate their disposable output between current consumption and savings. Note that there is no borrowing in this economy, so that $s_{i,t} \geq 0$. Assuming full depreciation of capital, the capital's law of motion is

$$k_{i,t+1} = s_{i,t}. \tag{2.7}$$

Therefore, households' per-period budget constraints are

$$c^y_{i,t} = y_{i,t} - s_{i,t} - s_{i,t-1} = y_{i,t} - s_{i,t} - k_{i,t},$$

and

$$c^o_{i,t+1} = s_{i,t},$$

for the first and the second period, correspondingly. Assuming quadratic utility from consumption[1] and quadratic disutility from pollution, the optimization problem of the representative household is

$$\begin{aligned}
\max_{x_{i,t}\in[0,\overline{x}_{i,t}]} \quad & U\left(c^y_{i,t}, c^o_{i,t+1}\right) = c^y_{i,t} - \tfrac{\beta}{2}\left(c^y_{i,t}\right)^2 + \left(c^o_{i,t+1} - \tfrac{\beta}{2}\left(c^o_{i,t+1}\right)^2\right) - \tfrac{\gamma}{2}\left(x_{i,t} + x_{j,t}\right)^2 \\
\text{s.t.} \quad & c_{i,t} = y_{i,t} - s_{i,t} - k_{i,t}, \\
& c_{i,t+1} = s_{i,t}, \\
& y_{i,t} = A_i k_{i,t}^{1/2} x_{i,t},
\end{aligned}$$

where $\beta > 0$ is a parameter that is positively related to the degree of relative risk aversion (RRA), and $\gamma > 0$ is an index of environmental awareness. Equivalently, the optimization problem can be expressed as

$$\begin{aligned}
\max \quad & L_i\left(x_{i,t}, s_{i,t}, \lambda_{i,t}\right) = \left(A_i k_{i,t}^{1/2} x_{i,t} - s_{i,t} - k_{i,t}\right) - \\
& -\tfrac{\beta}{2}\left(A_i k_{i,t}^{1/2} x_{i,t} - s_{i,t} - k_{i,t}\right)^2 + \left(s_{i,t} - \tfrac{\beta}{2} s_{i,t}^2\right) - \tfrac{\gamma}{2}\left(x_{i,t} + x_{j,t}\right)^2 \\
& +\lambda_{i,t}\left(A_i B^2 k_{i,t}^{1/2} - x_{i,t}\right) + \mu_{i,t}\left(A_i k_{i,t}^{1/2} x_{i,t} - s_{i,t} - k_{i,t}\right)
\end{aligned} \tag{2.8}$$

For $i = 1, 2$, the first order Kuhn-Tucker conditions are

$$A_i k_{i,t}^{1/2} - \gamma\left(x_{i,t} + x_{j,t}\right) - A_i \beta k_{i,t}^{1/2}(A_i k_{i,t}^{1/2} x_{i,t} - s_{i,t} - k_{i,t}) = \lambda_{i,t} - \mu_{i,t} A_i k_{i,t}^{1/2}, \tag{2.9}$$

$$\beta(A_i k_{i,t}^{1/2} x_{i,t} - s_{i,t} - k_{i,t}) - \beta s_{i,t} = \mu_{i,t}, \tag{2.10}$$

[1] Here we consider only the part of the utility that corresponds to $c_{t,1}$ and $c_{t+1,2}$ such that $u' > 0$. It can be shown that the optimal values of current and future consumption are such that this condition is not violated.

$$\lambda_{i,t}\left(A_iB^2k_{i,t}^{1/2}-x_{i,t}\right)=0, \tag{2.11}$$

$$\mu_{i,t}\left(A_ik_{i,t}^{1/2}x_{i,t}-s_{i,t}-k_{i,t}\right)=0, \tag{2.12}$$

$$\lambda_{t,i}\geq 0, \tag{2.13}$$

$$\mu_{t,i}\geq 0, \tag{2.14}$$

$$h_x\left(x_{i,t},s_{i,t}\right)=A_iB^2k_{i,t}^{1/2}-x_{i,t}\geq 0, \tag{2.15}$$

and

$$h_s\left(x_{i,t},s_{i,t}\right)=A_ik_{i,t}^{1/2}x_{i,t}-s_{i,t}-k_{i,t}\geq 0, \tag{2.16}$$

where $h_i\left(x_{i,t},s_{i,t}\right)$, $i=x,s$, denote the respective constraint functions. Note that the objective function is defined over the compact set $\left(x_{i,t},s_{i,t}\right)\subset\mathbb{R}_+^2$. Moreover, $U_i\left(\cdot\right)$ is everywhere continuous. Therefore, according to the Weierstrass Theorem, this problem has a global minimum and a global maximum. Furthermore, the constraint qualification is satisfied since the rank of the matrix below equals to the number of the constraints:

$$rank\begin{bmatrix}\dfrac{\partial h_x}{\partial x_{i,t}} & \dfrac{\partial h_x}{\partial s_{i,t}}\\ \dfrac{\partial h_s}{\partial x_{i,t}} & \dfrac{\partial h_s}{\partial s_{i,t}}\end{bmatrix}=rank\begin{bmatrix}-1 & 0\\ A_ik_{i,t}^{1/2} & -1\end{bmatrix}=2.$$

According to Proposition 6.5 in Sundaram (1996), this ensures that conditions (2.9)-(2.16) define the solution $\left(x_{i,t}^*,s_{i,t}^*\right)$ in the case of a single country, or the best responses $\left(x_{i,t}^*\left(x_{j,t},s_{j,t}\right),s_{i,t}\left(x_{j,t},s_{j,t}\right)\right)$ in the case of two countries.

2.3 Single country

In this section we analyze the case of a single country. For simplicity, the country-denoting subscripts are dropped. Given conditions (2.9)-(2.16) and setting $x_{j,t}$, the optimal values of emissions and savings are:

$$x_t^* = \begin{cases} \dfrac{(2+\beta k_t)\, A k_t^{1/2}}{2\gamma + \beta A^2 k_t}, & \text{if } k_t \geq \Psi \\ \\ A B^2 k_t^{1/2}, & \text{otherwise} \end{cases} \tag{2.17}$$

and

$$s_t^* = \begin{cases} \dfrac{(A^2-\gamma)\, k_t}{2\gamma + \beta A^2 k_t}, & \text{if } k_t \geq \Psi \\ \\ \frac{1}{2}\left(A^2B^2 - 1\right) k_t, & \text{otherwise} \end{cases} \tag{2.18}$$

where $\Psi = \dfrac{2\left(\gamma B^2 - 1\right)}{\beta\left(1 - B^2A^2\right)}$ is the boundary condition under the assumption that $1 - B^2A^2 < 0$. Note that the optimal emissions are always positive. As borrowing is not allowed in our model and savings can never exceed the disposable income in the current period, we require $0 \leq s_t^* \leq A k_t^{1/2} x_t^* - k_t$. For the interior solution this condition is expressed by $0 \leq (A^2 - \gamma) \leq (2A^2 - \gamma)$ which is true for any $A^2 \geq \gamma$. For the corner solution this condition is expressed fully by $0 \leq \left(A^2B^2 - 1\right)/2 \leq \left(A^2B^2 - 1\right)$ which is true for any $A^2B^2 - 1 \geq 0$. In summary, the conditions imposed are

$$\begin{cases} A^2 \geq \gamma, \\ \\ A \geq \dfrac{1}{B}. \end{cases} \tag{2.19}$$

2.3.1 Steady state analysis

There are two steady state values for capital when looking at the interior solution, with one of them being zero. We are interested in the non-zero steady state:

$$k^{SS} = \frac{A^2 - 3\gamma}{\beta A^2} \tag{2.20}$$

We require that this value is strictly positive. This implies that $A^2 > 3\gamma$. Note that since the steady-state value of the interior case exists, a country can never experience perpetual growth. The reason is that, even if a country starts at the corner, growing constantly, eventually the boundary condition $k_t \geq \Psi$ will bind and it will enter an interior solution where growth comes to an end when the economy reaches its steady state. The non-zero steady state values for emissions and savings are

$$x^{SS} = \frac{3}{A^2}\left(\frac{A^2 - 3\gamma}{\beta}\right)^{1/2}, \tag{2.21}$$

$$s^{SS} = k^{SS} = \frac{A^2 - 3\gamma}{\beta A^2}. \tag{2.22}$$

2.3.2 Steady states: Comparative statics

We first examine the effects ofTFP, environmental awareness and relative risk aversion on the long-run capital. Differentiating (2.20) with respect to these parameters we get:

$$\frac{\partial k^{SS}}{\partial A} = \frac{6}{A^3\beta}\gamma > 0,$$

$$\frac{\partial k^{SS}}{\partial \gamma} = -\frac{3}{A^2\beta} < 0,$$

and

$$\frac{\partial k^{SS}}{\partial \beta} = \frac{3\gamma - A^2}{A^2\beta^2} < 0.$$

As $s^{SS} = k^{SS}$ the same results hold for savings in the steady state. Note that an increase in TFP will increase the steady state value of capital as expected. It is also interesting to note that increased public environmental awareness will result in a lower steady-state value of capital. When γ is increased, it is optimal to reduce emissions in the current period. This will result in lower total output and reduced savings, thus lowering the amount of capital. Finally, an increase in β decreases the marginal utility of consumption for any given level of capital. As households have preferences over consumption and clean environment, this implies that emissions will decrease, hence total output and savings will decrease as well.

The effects of the model's parameters on emissions are also analyzed. Differentiating (2.21) with respect to these parameters we get:

$$\frac{\partial x^{SS}}{\partial A} = \frac{3\left(6\gamma - A^2\right)}{A^3\left(A^2 - 3\gamma\right)}\sqrt{\frac{1}{\beta}\left(A^2 - 3\gamma\right)} < 0,$$

$$\frac{\partial x^{SS}}{\partial \gamma} = -\frac{9}{2A^2\beta\sqrt{\frac{1}{\beta}\left(A^2 - 3\gamma\right)}} < 0,$$

and

$$\frac{\partial x^{SS}}{\partial \beta} = -\frac{3\left(A^2 - 3\gamma\right)}{2A^2\beta^2\sqrt{\frac{1}{\beta}\left(A^2 - 3\gamma\right)}} < 0.$$

As expected, an increase in environmental awareness decreases steady-state emissions. Changes in β work similarly. It is worth noticing that under condition (2.19) the steady-state level of emissions depends monotonically on productivity. This is an immediate result of the model's formulation that allows for emissions to be viewed as a factor of production (see Lopez (1994)) that is substitutable with capital. We see that when the TFP is sufficiently high to guarantee positivity of the solutions, the marginal product of emissions is sufficiently low and the trade-off between capital and emissions in the production process works in favour of capital accumulation. Therefore, the economy ends up with higher capital and lower emissions.

2.3.3 Transitional dynamics

Substituting equation (2.18) in (2.7) yields two first-order difference equations that describe the evolution of capital when optimal emissions are interior and corner solution, correspondingly:

$$k_{t+1} = \begin{cases} \dfrac{\left(A^2 - \gamma\right)k_t}{2\gamma + A^2\beta k_t}, & \text{if } k_t \geq \Psi \\ \\ \left(A - 1\right)k_t, & \text{otherwise} \end{cases}$$

For $k_t(0) = k_0$ the solutions of these difference equations are:

$$k(t) = \begin{cases} \dfrac{(A^2 - 3\gamma) k_0}{A^2 \beta k_0 + 2^t \left(\dfrac{\gamma}{A^2 - \gamma} \right)^t (A^2 (1 - \beta k_0) - 3\gamma)}, & \text{if } k_t \geq \Psi \\ \\ \left(\dfrac{A^2 B^2 - 1}{2} \right)^t k_0, & \text{otherwise} \end{cases} \tag{2.23}$$

Non-oscillating convergence to a steady-state value for the interior case implies that $\gamma / (A^2 - \gamma) > 0$. Perpetual growth for the corner solution is guaranteed under the assumption that $A^2 B^2 - 1 > 0$. Both these conditions are met by (2.19). For the remainder of this paper we require that the parameter values are such that the economy's capital monotonically converges to a positive steady state. We are motivated by the fact that almost all countries are experiencing positive long-run growth. By substituting the solution in the optimal values of emissions and savings we get

$$x^*(t) = \begin{cases} \dfrac{2 + \beta k(t)}{2\gamma + \beta A^2 k(t)} A \sqrt{k(t)}, & \text{if } k(t) \geq \Psi \\ \\ A B^2 \sqrt{k(t)}, & \text{otherwise} \end{cases} \tag{2.24}$$

and

$$s_t^* = \begin{cases} \dfrac{(A^2 - \gamma) k(t)}{2\gamma + \beta A^2 k(t)}, & \text{if } k(t) \geq \Psi \\ \\ \frac{1}{2} (A^2 B^2 - 1) k(t), & \text{otherwise} \end{cases} \tag{2.25}$$

Despite the analytical form of emission dynamics, it is not possible to derive results about local optima based on that. Thus the actual shape of emissions through time can be derived analytically. However, we can show the following:

Proposition 7 *Given that k_0 is relatively small and allowing for convergence to a positive steady state, the income pollution relationship is described by an EKC. However, given a positive growth of capital, the evolution of emissions over time* *can take three possible forms:*

(i) emissions are monotonically increasing in time,

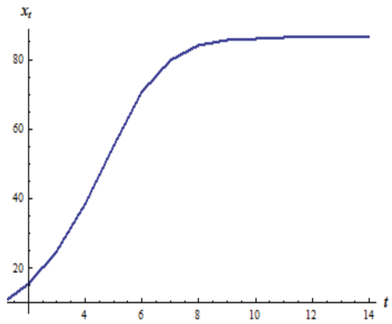
(ii) emissions increase but then decrease in time,or

(ii) emissions monotonically decrease in time.

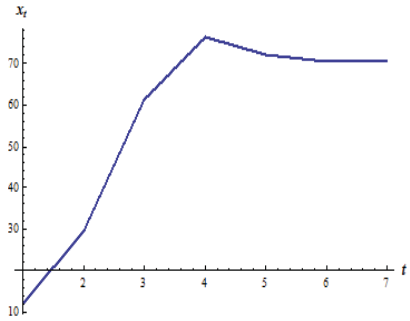
The proof of this proposition is in Appendix II. A sketch of the proof is provided here. First, we differentiate (2.24) with respect to $k(t)$. It can be shown that there are two values of k_t, say $\underline{k}_t < \overline{k}_t$, that satisfy $\partial x^* [k(t)] / \partial k(t) = 0$. These are potential optima. When checking for the second derivative of optimal emissions, *i.e.*, $\partial^2 x^* [k(t)] / / \partial k(t)^2$, one can verify the existence of a saddle point. For values of $k(t)$ lower than the saddle point $x^* [k(t)]$ is convex, while for values of $k(t)$ higher than the saddle point $x^* [k(t)]$ is concave. Therefore, the $\underline{k}(t)$ is a local maximum and $\overline{k}(t)$ is a local minimum. However, it can be proven that over the entire parameter range $\underline{k}(t) < k^{SS} < \overline{k}(t)$. Therefore the emissions can never have a local minimum but they always have a maximum with respect to $k(t)$. Note that this analysis does not consider the dynamic change in capital but it rather treats $k(t)$ as an exogenous parameter. Once the dynamics of capital are taken into consideration we have

$$\partial x^* (t) / \partial t = \partial x^* [k(t)] / \partial k(t) \times \partial k(t) / \partial t$$

Note that the sign of the second part of the RHS has been assumed non-negative, being zero at the steady state. The possible trajectories of the emissions over time follow immediately. The emissions will be monotonically increasing in time if $\partial x^* [k(t)] / \partial k(t)$ does not turn negative before $k(t)$ reaches its steady state. Otherwise, two cases are possible depending on the level of the initial endowment. Finally, one should notice that we focus only on the interior solution. This is due to the fact that the boundary value of Ψ is finite, while at the corner capital grows indefinitely. Therefore, taking into consideration the corner solution does not change our results. Figure 2.1 shows the possible cases. Part (a) corresponds to the case where the economy reaches its steady state before the turning point, thus the strictly increasing income-pollution relationship. Part (b) depicts an EKC where emissions increase with income but start decreasing before the steady state.



(a)



(b)

Figure 2.1: Emissions over time

2.4 Two countries

2.4.1 Existence and uniqueness of the solution

When considering two countries, conditions (2.9)-(2.16) for $i = 1, 2$ are used to derive the reaction functions. However, neither existence nor uniqueness of the solution can be guaranteed. First, we show that there is an optimization problem that is equivalent to (2.8). In this problem, emissions is the only control variable. The corresponding Kuhn-Tucker conditions of the two countries describe the pollution game. It is then easy to show that this game satisfies the conditions of Monderer and Shapley (1995) for a Potential Game. Given the specific functional forms of our model, *Lemma 1* and *Proposition 3* can be applied, thus ensuring a unique solution in the pollution game.

Proposition 8 *For any positive values of domestic and foreign emissions, country i' s optimal savings equals half the disposable income, i.e.,*

$$s^*_{i,t} = \frac{1}{2}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right).$$

Therefore, the following optimization problem is equivalent to problem (2.8):

$$\begin{aligned} \max_{x_{t,i}} \quad & L_i\left(x_{i,t}, \lambda_{i,t}\right) = \left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right) - \tfrac{\beta}{4}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right)^2 \\ & -\tfrac{\gamma}{2}\left(x_{i,t} + x_{j,t}\right)^2 + \lambda_{i,t}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right). \end{aligned} \tag{2.26}$$

Proof. Given that $c^y_{i,t} = A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t} - s_{i,t}$, and $c^o_{i,t+1} = s_{i,t}$ we evaluate (2.6) at $s^*_{i,t} = \frac{1}{2}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right)$, thus yielding:

$$U^*_i\left(x_{i,t}, x_{j,t}\right) = \left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right) - \frac{\beta}{4}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t}\right)^2 - \gamma\left(x_{i,t} + x_{j,t}\right)^2. \tag{2.27}$$

Let $\left(x^*_{i,t}, s^{**}_{i,t}\right)$ and $\left(x^*_{j,t}, s^*_{j,t}\right)$ denote the solution of the pollution game, where $s^{**}_{i,t} \neq s^*_{i,t}$. The indirect utility for these solutions becomes

$$\begin{aligned} U^{**}_i \quad = \quad & \left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t} - s^{**}_{i,t}\right) - \frac{\beta}{2}\left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t} - s^{**}_{i,t}\right)^2 \\ & + \left(s^{**}_{i,t} - \frac{\beta}{2}\left(s^{**}_{i,t}\right)^2\right) - \gamma\left(x_{i,t} + x_{j,t}\right)^2 \end{aligned}$$

Then it must be the case that for all $s_{t,i}$ we have $U^{**}_i - U_i\left(x^*_{i,t}, s_{i,t}, x^*_{j,t}, s^*_{j,t}\right) \geq 0$. Since this is true for all $s_{i,t}$ it must also be the case that $U^{**}_i - U^*_i\left(x^*_{i,t}, x^*_{j,t}\right) \geq 0$. However,

$$U^{**}_i - U^*_i\left(x^*_{i,t}, x^*_{j,t}\right) = -\frac{\beta}{4}(k_{i,t} + 2s^{**}_{i,t} - A_i k_{i,t}^{1/2} x_{i,t})^2 < 0.$$

This contradicts the assumption that $s^{**}_{i,t} \neq s^*_{i,t}$. Therefore, $s^*_{i,t}$ is the optimal savings. When using $U^*_i\left(x_{i,t}, x^*_{j,t}\right)$ as an objective function, country i's maximization problem is described by (2.26). ■

Given the equivalent utility function (2.26), we can now show that:

(i) individual strategy spaces are compact as intervals of real numbers, *i.e.*, $x_{t,i} = [0, \overline{x}_{t,i}] \subset \mathbb{R}$,

(ii) the payoff functions are continuously differentiable, and the cross-partial derivatives of any two payoff functions are equal, *i.e.*

$$\frac{\partial^2 U_i}{\partial x_{i,t}\partial x_{j,t}} = \frac{\partial^2 U_j}{\partial x_{i,t}\partial x_{j,t}} = -2\gamma.$$

Therefore, the pollution game described in the present paper is a Potential Game. Lemma 1 provides a Potential Function which is concave and Proposition 4 proves that the Potential Game has a unique solution.

2.4.2 Optimal emissions

Maximization of problem (2.26) requires the following conditions:

$$A_i k_{i,t}^{1/2} - \frac{\beta}{2} A_i k_{i,t}^{1/2} \left(A_i k_{i,t}^{1/2} x_{i,t} - k_{i,t} \right) - \gamma \left(x_{i,t} + x_{j,t} \right) = \lambda_{i,t}, \tag{2.28}$$

$$A_i B^2 k_{i,t}^{1/2} - x_{i,t} \geq 0, \tag{2.29}$$

$$\lambda_{t,i} \left(A_i B^2 k_{i,t}^{1/2} - x_{i,t} \right) = 0, \tag{2.30}$$

and

$$\lambda_{i,t} \geq 0. \tag{2.31}$$

Conditions (2.28)-(2.31) for $i = 1, 2$ characterize the solution of the pollution game. The optimal emissions can be shown to be

$$x_{t,i}^* = \begin{cases} A_i B^2 k_{i,t}^{1/2}, \\ \\ \dfrac{\left(\beta A_i^2 k_{i,t} + 2\gamma\right)\left(2 + \beta k_{j,t}\right) A_j k_{j,t}^{1/2} - 2\gamma A_i k_{i,t}^{1/2} \left(2 + \beta k_{i,t}\right)}{\beta \left(2\gamma A_j^2 k_{j,t} + A_i^2 k_{i,t} \left(\beta A_j^2 k_{j,t} + 2\gamma\right)\right)}, \\ \\ \dfrac{A_i k_{i,t}^{1/2} \left(2 + \beta k_{i,t}\right) - 2\gamma B^2 A_j k_{j,t}^{1/2}}{\beta A_i^2 k_{i,t} + 2\gamma}, \end{cases} \tag{2.32}$$

where the top case represents maximum emissions for country i, the middle case corresponds to country i's emissions when both countries emit less than maximum, and the bottom case represents emissions of country i when the rival country emits at the maximum. A final observation regarding $x^*_{i,t}$ is that (2.32) does not guarantee non-negativity of the emissions. We assume that the parameter values along with the current levels of domestic and foreign capital are such that the optimal emissions are non-negative. For the numerical simulation results provided in the following section, if at some point t, optimal emissions $x^*_{i,t}$ becomes negative they are set equal to zero and it remains zero thereafter.

Note that according to Proposition 8 the optimal savings can be described as

$$s^*_{i,t} = \frac{1}{2}\left(A_i k_{i,t}^{1/2} x^*_{i,t} - k_{i,t}\right),$$

where $x^*_{i,t}$ is appropriately defined by (2.32). Therefore, economy i's capital law of motion is

$$k_{i,t+1} = s^*_{i,t} = \frac{1}{2}\left(A_i k_{i,t}^{1/2} x^*_{i,t} - k_{i,t}\right). \tag{2.33}$$

Unfortunately, no analytical solutions about the dynamics or the steady state of the model can be derived unless we assume the same productivities and initial capital for both countries. However, given (2.32) and (2.33), we are able to conduct numerical analysis.

2.5 Numerical analysis

For the numerical analysis presented in this section we used Mathematica 8.0. The emissions of a single country can be interior (*i.e.*, $x^*_{i,t} \in (0, \overline{x}_{i,t})$), maximum (*i.e.*, $x^*_{i,t} = \overline{x}_{i,t}$), or zero (*i.e.*, $x^*_{i,t} = 0$). Given that there are two countries, this gives rise to $3^2 = 9$ different combinations to be considered when running simulations. However, we can immediately exclude the case where both countries emit zero as this is only possible in the trivial case where both economies start with zero capital. For all the remaining cases the proper boundary conditions have been considered. The parameter values have been chosen so that (i) when starting from the same initial conditions, the two countries will converge to a positive steady state, and (ii) when a country follows the upper boundary path on emissions it is strictly increasing. The former implies that $A_1 = A_2 \geq 6\gamma$ when $\gamma < 0.25$, or $A_1 = A_2 \geq 1 + 2\gamma$ otherwise. The second condition implies that $A_i > 1/B$. Table 2.1 in the appendix presents the eight possible cases while Figure 2.4 shows how the code works. First, we analyze the symmetric case with differences only on the endowments. The analysis with respect to TFP differences follows.

2.5.1 Symmetric case: differences in endowments

As pointed out in Chapter 1, when considering multiple countries the income-pollution relationship has no clear "shape" even if the countries are completely symmetric in everything but their endowments. In this subsection we ask the following questions: *(i)* Does the relatively rich country have a lower/higher emission trajectory over time? *(ii)* When EKCs are observed, does the relatively rich country have a higher/lower critical income beyond which its emissions decrease? *(iii)* Which factors of the model increase the probability that the relatively poor country will experience an environmental poverty trap? *(iv)* Does equal income distribution benefit the environment? To investigate these questions, we considered three levels of total initial endowments, namely low, average, and high. We have set the initial endowments in absolute terms as well as in percentage $d < 100\%$ of the steady state-value of capital. The latter were simulated in order to control for the growth rate. The critical assumption made is that with an equal income distribution between the two countries, capital will converge to a positive steady state.

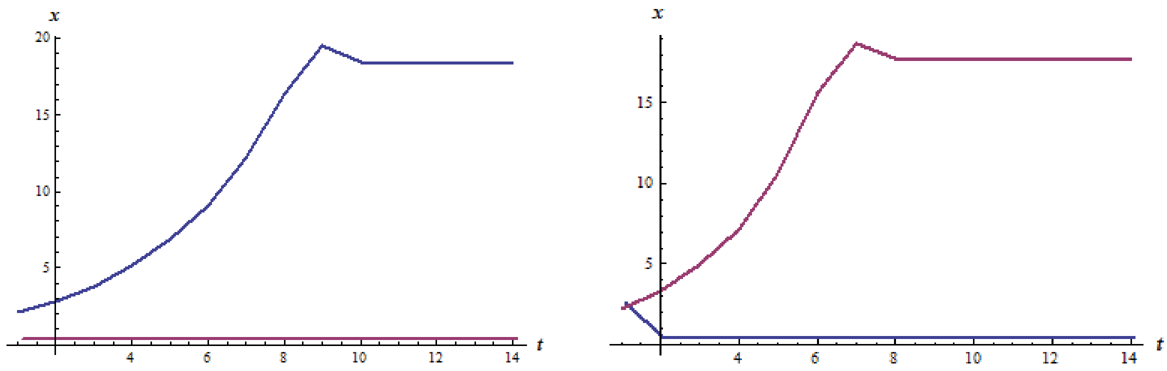
Our simulations were inconclusive on answering *(i)* and *(ii)*. To study the third question we looked for the critical percentage difference in initial endowments, say d_C, above which the poor country will be caught into a poverty trap. Table 2.2, (a), (b) and (c) show these critical values for three different levels of the productivity index and of the environmental awareness. For the results of this table, parameters B and β are fixed. We observe the following:

Remark 9 *Large income differences can lead the poor country into an environmental poverty trap. The probability of a poverty trap is decreasing in total initial endowments and in productivity, while it is increasing in environmental awareness.*

Comparing parts (a), (b), and (c) of Table 2.2 confirms this Remark. An explanation of this fact is the presence of increasing returns to scale with respect to capital and emissions. Increasing returns to scale clearly benefit the rich country. However, as TFP and/or the total initial endowments increase, the benefits from the IRS are reduced and the rich country loses its strategic advantage. Note that sensitivity analysis showed that changes in β do not affect the numerical results, while there is a critical value of B below which there is a low probability for the poverty trap to occur. No significant differences in our results were found when we controlled for the growth rates. This indicates that in our model, if endowment levels and their growth rates do not work in the same direction, the former have greater impact on the income-pollution relationship than the latter. Finally, our results cannot confirm that income inequalities might be beneficial for the environment.

2.5.2 Asymmetric case: differences in TFPs

When allowing for different initial endowments, the various results are derived. First, if the rich country has an advantage in TFP, the probability of a poverty trap increases. Depending on the initial conditions, it is more likely that the rich country with higher productivity will eventually emit less with income. On the other hand, when the TFP of the rich country is lower, the poor country will eventually take its place. Moreover, significant productivity advantage of the poor country combined with a small difference in endowments, are the necessary conditions that might create a poverty trap for the initially rich country. What can be surprising, though, is that even if the poor country has higher productivity it might end up in the poverty trap when its initial endowment is not sufficiently large. Figure 2.2 shows a turnover case: in part (a), the poor country (red line) is in a poverty trap. It is the case where the benefits from higher income exceed those of the higher TFP for the first periods. However, as the TFP of the poor country increases, the opposite might happen, as depicted on the second part of figure 2.2. Note that this result is verified over a large range of the parameter values



(a) (b)

Figure 2.2: Poverty trap.

Finally, it can be shown that in some cases the rich country will have emissions that are increasing in income after the turning point is reached. This is the case of the inverted S-shaped income-pollution curve. Figure 2.3 provides an example where the rich country's emissions, depicted in blue color, are described by an inverted S-shaped income-pollution relationship.

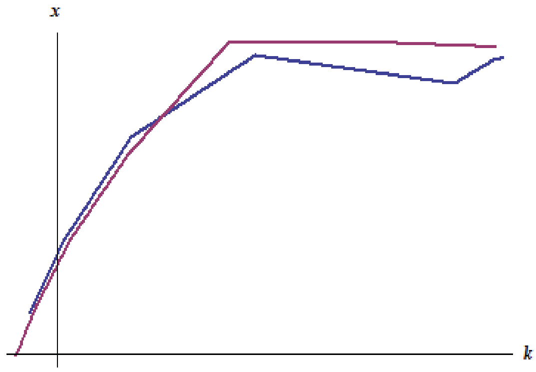


Figure 2.3: Inverted S-shaped EKC

2.6 Conclusions

The present paper discusses the income-pollution relationship in a multi-country dynamic framework. It follows and completes the work of Chapter 1 in that it introduces a dynamic structure in the evolution of capital. This is done by adopting a properly modified overlapping generations model that allows for intergenerational and intertemporal issues to be disconnected. In the case of a single country, it is shown that the income-pollution relationship can be either monotonic or it can exhibit the EKC shape. When allowing for two countries, the model described satisfies the conditions of Chapter 1 for existence and uniqueness of the solution. This allows us to analyze numerically the effects of *(i)* endowment differences, and *(ii)* productivity differences on the income-pollution relationship. More specifically, our numerical analysis indicates that, even for otherwise symmetric countries, late starters are facing the danger of a poverty trap. This danger is more imminent the lower the TFPs, the lower the global income, and the higher the environmental awareness. For the asymmetric case, our simulations have shown no regularities. However, inverted S-shaped income-emission path can be derived in our model. This analysis confirms the argument that the relationship between income and pollution is more complex than the EKC hypothesis postulates.

In our analysis we have assumed that for every $i = 1, 2$ a social planner corresponds to each generation t. One can also derive the cooperative solution of the two countries given generation t and compare the two cases. Full parameterization of the model to allow for country-specific environmental awareness and degree of relative risk aversion is considered. A far more daring step will be to convert this model into an infinite horizon model.

2.7 Appendix I

Case Code	Emissions of Country 1	Emissions of Country 2
CS 1	$x^*_{t,1} \in (0, \bar{x}_{t,1})$	$x^*_{t,2} \in (0, \bar{x}_{t,2})$
CS 2	$x^*_{t,1} \in (0, \bar{x}_{t,1})$	$x^*_{t,2} = B^2 A_2 k^{1/2}_{t,2}$
CS 3	$x^*_{t,1} \in (0, \bar{x}_{t,1})$	$x^*_{t,2} = 0$
CS 4	$x^*_{t,1} = B^2 A_1 k^{1/2}_{t,1}$	$x^*_{t,2} \in (0, \bar{x}_{t,2})$
CS 5	$x^*_{t,1} = 0$	$x^*_{t,2} \in (0, \bar{x}_{t,2})$
CS 6	$x^*_{t,1} = B^2 A_1 k^{1/2}_{t,1}$	$x^*_{t,2} = 0$
CS 7	$x^*_{t,1} = 0$	$x^*_{t,2} = B^2 A_2 k^{1/2}_{t,2}$
CS 8	$x^*_{t,1} = B^2 A_1 k^{1/2}_{t,1}$	$x^*_{t,2} = B^2 A_2 k^{1/2}_{t,2}$

Table 2.1: Combinations of emission intervals.

↓ A γ →	100	110	150
100	13.4	14.0	0.3
102	18.1	10.0	0.6
103.5	22.0	13.1	1.0

(a) Low endowment

↓ A γ →	100	110	150
100	58.0	33.0	3.0
102	77.0	42.0	4.0
103.5	100.0	54.0	6.0

(b) Average endowment

↓ A γ →	100	110	150
100	-	-	27
102	-	-	43
103.5	-	-	68

(c) High endowment

Table 2.2: The effects of A and γ on the d_C.

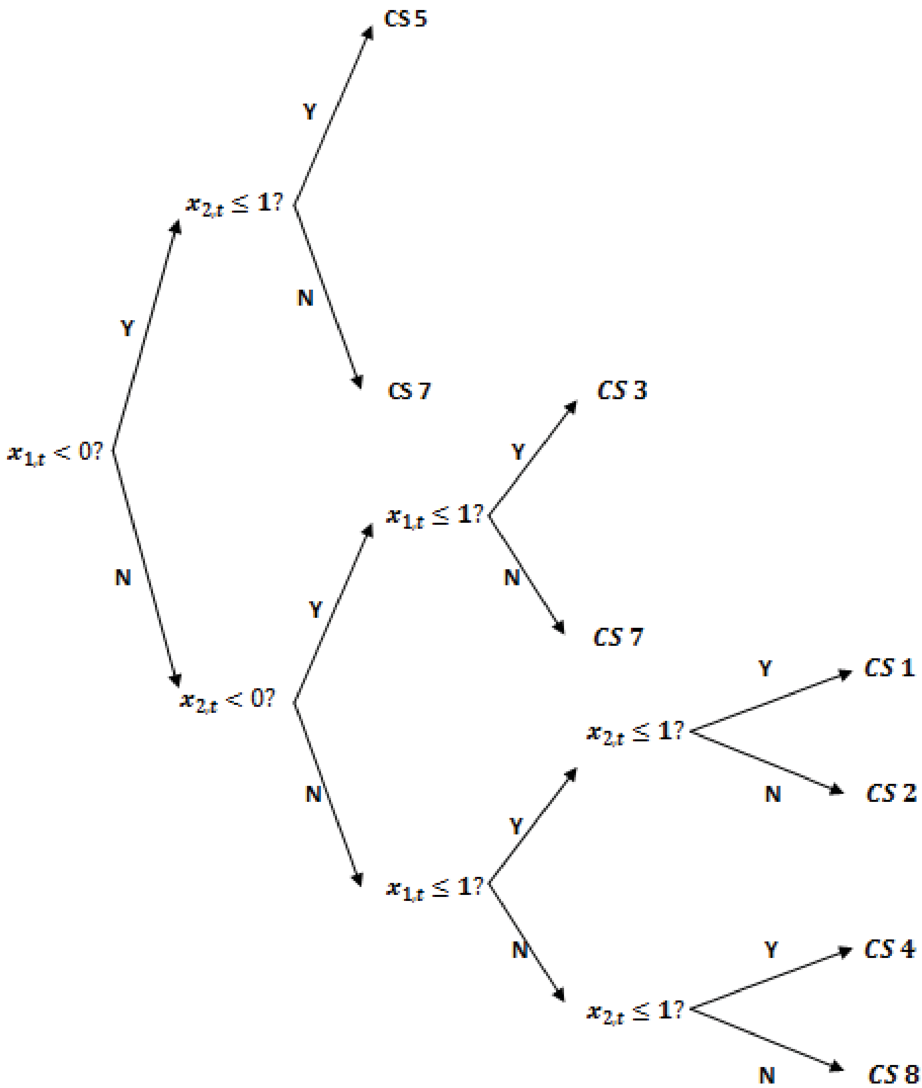


Figure 2.4: Graphical description of the code.

2.8 Appendix II

Proof of Proposition 7. For the proof of this proposition we have to examine the sign of

$$\partial x^{*}(t)/\partial t=\partial x^{*}[k(t)]/\partial k(t)\times\partial k(t)/\partial t.$$

We start with the sign of the $\partial x^{*}[k(t)]/\partial k(t)$. Differentiating equation (2.24) with respect to the capital yields

$$\frac{\partial x^{*}[k(t)]}{\partial k(t)}=\frac{A\left(A^{2}\beta^{2}k(t)^{2}-2A^{2}\beta k(t)+6\gamma\beta k(t)+4\gamma\right)}{2\sqrt{k(t)}\left(\beta k(t)A^{2}+2\gamma\right)^{2}}$$

Setting the derivative equal to zero under the assumptions that

$$\begin{aligned}\beta &\neq 0\wedge\frac{1}{A^{2}\beta}\left(-3\gamma+\sqrt{9\gamma^{2}-10A^{2}\gamma+A^{4}}+A^{2}\right)\in\mathbb{C}\setminus\left\{0,-\frac{2}{A^{2}\beta}\gamma\right\}\wedge\\ \wedge A &\neq 0\wedge-\frac{1}{A^{2}\beta}\left(3\gamma+\sqrt{9\gamma^{2}-10A^{2}\gamma+A^{4}}-A^{2}\right)\in\mathbb{C}\setminus\left\{0,-\frac{2}{A^{2}\beta}\gamma\right\},\end{aligned}$$

yields two solutions:

$$\begin{aligned}\underline{k}(t) &= \frac{1}{A^{2}\beta}\left(A^{2}-3\gamma-\sqrt{9\gamma^{2}-10A^{2}\gamma+A^{4}}\right),\\ \overline{k}(t) &= \frac{1}{A^{2}\beta}\left(A^{2}-3\gamma+\sqrt{9\gamma^{2}-10A^{2}\gamma+A^{4}}\right).\end{aligned}$$

Taking the second derivative yields

$$\begin{aligned}\frac{\partial^{2}x^{*}[k(t)]}{\partial k(t)^{2}} &= -\frac{A}{4k(t)^{\frac{3}{2}}\left(\beta k(t)A^{2}+2\gamma\right)^{3}}A^{4}\beta^{3}k(t)^{3}-6A^{4}\beta^{2}k(t)^{2}+\\ &\quad +12A^{2}\beta^{2}\gamma k(t)^{2}+24A^{2}\beta\gamma k(t)-12\beta\gamma^{2}k(t)+8\gamma^{2}\end{aligned}$$

Setting it equal to zero, only one real valued solution exists, that is:

$$\begin{aligned}\widehat{k}(t)=20\gamma^{2}+A^{4}\beta^{2}\left(\tfrac{1}{A^{6}\beta^{3}}(H)\right)^{\frac{2}{3}}-\\ -24A^{2}\gamma+2A^{4}\beta\sqrt[3]{H}+4A^{4}-4A^{2}\beta\gamma\sqrt[3]{H}\end{aligned}$$

where

$$H = 152A^2\gamma^2 - 88\gamma^3 - 72A^4\gamma + 8A^6 +$$

$$16A^6\beta^3\sqrt{-\frac{1}{A^{12}\beta^6}\left(A^8\gamma^2 - 8A^6\gamma^3 + 14A^4\gamma^4 - 8A^2\gamma^5 + \gamma^6\right)}.$$

Despite the length of this expression we are able to verify that $\widehat{k}(t) \in \left(\underline{k}(t), \overline{k}(t)\right)$. It is also straightforward to verify that for $k(t) = 0$, we get $\partial^2 x^* [k(t)] / \partial k(t)^2 < 0$. For values of $k(t) \leq \widehat{k}(t)$ optimal emissions are a concave function of capital, otherwise it is a convex function. Therefore, $\underline{k}(t)$ is a local maximum and $\overline{k}(t)$ is a local minimum of the $x^* [k(t)]$. We compare the extrema to the steady state value of capital defined by equation (2.20). Note that $k^{SS} - \overline{k}(t) = (A^4 - 10A^2\gamma + 9\gamma^2)^{1/2} / \beta A^2 < 0$. This shows that the local minima can never be reached. On the other hand $k^{SS} - \overline{k}(t) = (A^4 - 10A^2\gamma + 9\gamma^2)^{1/2} / \beta A^2 < 0$, so that a maximum is always reached. We now turn on the sign of $\partial k(t) / \partial t$ that by assumption is non-zero. There are two cases. When the k^{SS} is reached before $\partial x^* [k(t)] / \partial k(t)$ turns negative, the dynamics of emissions over time are positively monotonic. If, on the other hand, k^{SS} is reached once $\partial x^* [k(t)] / \partial k(t) < 0$ the dynamics of $x^*(t)$ will depend only on the initial condition. ■

Chapter 3

Subsidies, emissions, and optimal environmental taxation in general equilibrium with imperfect competition[1]

3.1 Introduction

Governments use various forms of subsidies to support specific sectors of their economies. Different types of subsidies have been used in practice and analyzed theoretically, such as price supports, low-interest financing, reduced regulation, and tax reliefs among others. As subsidies are often used in environmentally sensitive sectors, it is crucial to examine their effects on overall emissions. Arguments made on the effectiveness of environmental policy that ignore the existence of ad

[1]The present chapter is based on joint work with my supervisor Dr. Effrosyni Diamantoudi (Associate Professor, Department of Economics, Concordia University), and Dr. David Kelly (Associate Professor, Department of Economics, University of Miami).

hoc subsidies in national economies can be misleading.

Many empirical studies confirm the importance of awarded subsidies in national economies and in sectors that have a significant environmental footprint. According to Barde and Honkatukia (2004), agriculture, fishing, water, transport, manufacturing, and energy are industries with significant environmental impact that are heavily subsidized. Using OECD data they estimate the total amount of subsidies per aforementioned industry. Table 3.1 taken from Kelly (2009) summarizes their findings. Moreover, van Beers and van den Bergh (2001) estimate world subsidies account for 3.6% of global GDP, while van Beers and de Moor (2001) provide more data about OECD countries, non-OECD countries, and the world. Table 3.2 taken from Pearce (2001) summarizes the results of van Beers and de Moor (2001).

The effects of subsidies on the environment are analyzed in Barde and Honkatukia (2004): input price subsidization tends to increase the use of dirty inputs; tax reliefs have a technology lock-in effect by keeping in the industry inefficient firms that otherwise would have exited; reduced regulation directly increases emissions. Bajona and Kelly (2006) show that China's emissions were reduced when subsidies were eliminated in the country's effort to comply with WTO directives. Using a partial equilibrium analysis, van Beers and van den Bergh (2001) find that subsidies increase total output and emissions in small open economies while at the same time market failure is reinforced. On the other hand, Kelly (2009) uses a general equilibrium model and derives conditions under which subsidies hurt the environment both in the short-run and in the steady state.

This paper analyzes the effects of subsidies on the environment and on the optimal effluence tax. We consider an economy where households have preferences over two consumption goods, one of them treated as numeraire. Households' optimal decisions give rise to output demands. On the production side, two firms engage in imperfect competition (à la Cournot) in the output market and behave competitively in the input markets. Firms are asymmetric with respect to their total factor productivities (TFP) and their emission intensities. Finally, the government awards an ad hoc output subsidy to one of the firms. We are motivated here by the empirical observation that bailouts and subsidies tend not to go to already profitable firms or industries, but instead to firms facing lower cost competitors. Examples include (i) state owned enterprises (SOEs), which often compete with lower cost private firms, and (ii) bailouts in which a high cost competitor is saved from bankruptcy, while profitable low cost competitors receive nothing. An alternative explanation is that subsidized firms must divert resources to lobbying activities which lowers overall TFP. We then optimally determine the level of a unified effluence tax in order to maximize social welfare given the subsidy. We show that in a general equilibrium setup subsidies awarded to the least productive firm result in reduced total production. This result is in accordance with existing results in the literature in a general equilibrium context (see, for example, Kelly (2009)) and contradicts results derived from partial equilibrium analysis. The effect of subsidies on emissions depends on two ratios: the emission intensities ratio and the TFPs ratio. It is shown that when the subsidized sector has relatively higher pollution intensity than the non-subsidized sector, total emissions will increase. Both results combined show that subsidies are welfare reducing.[2]

It is well established in the literature of optimal environmental taxation that the corrective power of the Pigouvian tax, that is a per unit of emission tax equal to the marginal social damage from pollution, exceeds any other environmental policy when the market of the dirty good is perfectly competitive. It has also been shown that in the case of market imperfections the optimal effluence tax might be lower than the Pigouvian tax (see Barnett (1980)). This is due to the fact that a high emission tax will tend to reinforce the underproduction due to the market imperfection. The optimal emission tax for the case with or without subsidy is

[2] Given that empirical studies show that subsidies are mostly awarded to the less efficient firms, the probability of subsidies being welfare improving depends on whether the subsidized firms have much lower emission intensities than the non-subsidized firms.

derived in the present paper. As in Simpson (1995) it is shown that the optimal environmental tax in both cases might even exceed the Pigovian tax. Moreover, in our setup, such a policy is first best equivalent.

Finally, this paper analyzes the effect of subsidies on emissions in a general equilibrium context with market imperfections. Due to the analytical difficulties we encountered in solving for the general equilibrium under market imperfections, we make use of specific functional forms. Moreover, simplifying assumptions on the competitive behavior of the firms in the input markets have been made. Otherwise, according to Bonanno (1990) profit maximization might be inadequate to describe firms' behavior.

In what follows, Section 2 provides the basic setup of households, firms, and the role of the government. Section 3 describes the equilibrium concept. Section 4 provides the solutions without any government intervention, and Section 5 derives the general solution and the optimal tax with or without subsidies, while it examines the roles of ad hoc subsidies and an effluence tax. Section 6 concludes.

3.2 The model

3.2.1 Households

We consider a continuum of households with measure one that values the consumption of two goods, a clean numeraire good, c_0, and a dirty good, c. Households receive disutility from pollution emissions E caused by the production of the dirty good. Preferences are quasi-linear in leisure:[3]

$$U = c_0 + \alpha c - \frac{\beta}{2}c^2 - \zeta E^2, \tag{3.2.1}$$

[3]This assumption has been widely used in the non-linear taxation literature (see for example Boone and Bovenberg, 2007), and it allows the derivation of closed-form solutions in our model.

where α, β, $\zeta > 0$; α and β are demand parameters while ζ is a parameter of perceived pollution.

Households derive income from renting capital K and labor L at rental rates r and w respectively. We also assume households supply K units of capital inelastically to the production of the dirty good, while their available time is normalized to unity. Households also receive profits π from the non-competitive firms producing the dirty good, and receive lump sum net transfers from the government, NT. Let p be the price of the dirty good, and w be the opportunity cost of leisure, then the household budget constraint is:

$$c_0 + pc = rK + w + NT + \pi. \tag{3.2.2}$$

As it is often assumed in the related literature, households do not internalize the negative pollution externality when optimizing. This is due to the fact that households cannot link their consumption decision to the production process, while the negligible effect of their consumption on emissions is ignored. Therefore, the consumption of the dirty good has a negative pollution externality. Maximization of household utility (3.2.1) subject to the budget constraint (3.2.2) leads to linear inverse demand for the consumption good:

$$p = \alpha - \beta c^*,$$

where c^* is optimal consumption. Equating demand c with supply Q in the inverse demand for the consumption good gives:

$$p = \alpha - \beta Q. \tag{3.2.3}$$

The household's demand for the numeraire can be found as a residual using the budget constraint:

$$c_0^* = rK + w + NT + \pi - pc^*. \tag{3.2.4}$$

It can be derived immediately that in equilibrium the optimal amount of the numeraire is zero. To verify this, note that the term in the parenthesis on the right-hand side of equation (3.2.4) is identically equal to zero. Profits are the difference between total revenues and total costs. The sum of total revenues for the two firms is $pQ_P + pQ_G = pQ$ while the sum of total costs includes the production costs $c_P(Q_P)$, $c_G(Q_G)$ and the net taxes (emission tax minus subsidies) imposed by the government, i.e., $T_P + T_G = T$. Balanced government budget implies $NT = T$, while output market clearing implies $pc^* = pQ$. Finally, note that for $i = P, G$, we have $c_i(Q_i) = rK_i(r, w, Q_i) + wL_i(r, w, Q_i)$ where $K_i(\cdot)$ and $L_i(\cdot)$ are firm i's input demands. When the input markets clear we get $c_P(Q_P) + c_G(Q_G) = (rK_P + wL_P) + (rK_G + wL_G) = rK + w$. Therefore, we obtain:

$$\pi = pQ - c_P(Q_P) - c_G(Q_G) - T \Rightarrow$$

$$\pi = pc^* - rK - w + NT.$$

Substituting this expression in (3.2.4) yields $c_0^* = 0$.

3.2.2 Firms

We consider two types of firms. The first type we denote "subsidized firms" (G), and the second type "private firms," (P).[4] The total output of the economy

[4] Although we refer to "types" of firms, in the remainder of the paper we restrict our analysis to just two firms: a private and a subsidized firm. Since the issue of endogenous market structure is not analyzed in the present paper, the comparative statics with respect to the number of firms that belong to either type is identical with those **conducted** with respect to the TFP ratio.

consists of the sum of the firms' productions, i.e.,

$$Q = Q_P + Q_G \tag{3.2.5}$$

Both firms have access to production technologies that differ only in their TFPs and emission intensities. The private firm has TFP equal to A_P whereas the subsidized firm has technology $A_G < A_P$. Without loss of generality and to simplify our results, we set $A_G = 1$ and $A_P = A > 1$. As mentioned in the introduction, empirical observations confirm that the subsidy recipients are usually low-efficiency firms.

The corresponding emission intensities are σ_i, $i = P, G$, thus the total emissions are

$$E = \sigma_P Q_P + \sigma_G Q_G. \tag{3.2.6}$$

Finally, an emission tax, τ, is imposed on both firms while a per-unit subsidy, s, is awarded to the productively inefficient firm as mentioned above. Since in this model individual emissions are linearly dependent on the firms' outputs, the model allows for the examination of non-unified environmental taxation. The argument in favour of firm-specific effluence tax becomes more appealing in the related literature. In the present paper the amount of subsidies is not defined optimally. However, it will be shown that in the present context a unified environmental tax is non-inferior to a firm-specific effluence tax.

Production costs

Firms use capital K and labor L to produce a homogenous good. We assume a Cobb-Douglas production function $Q_i = A_i K_i^{\gamma} L_i^{1-\gamma}$, $i = G, P$. Cost minimization yields the following input demands:

$$K_i = \frac{\gamma}{r} c_i' Q_i, \tag{3.2.7}$$

$$L_i = \frac{1-\gamma}{w} c_i' Q_i. \tag{3.2.8}$$

Given these input demands, the cost functions are:

$$c_i\left(Q_i\right) = \left(\frac{r}{\gamma}\right)^{\gamma} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \frac{Q_i}{A_i} \equiv c_i' Q_i, i = G, P. \tag{3.2.9}$$

Cournot duopoly

The private and subsidized firms compete in a Cournot duopoly. The problem of the private firm is to optimally choose Q_P in order to maximize its profits

$$\pi_P = p\left(Q_P + Q_G\right) Q_P - c_P\left(Q_P\right) - \tau \sigma_P Q_P. \tag{3.2.10}$$

The first order condition is:

$$p'\left(Q_P + Q_G\right) Q_P + p\left(Q_P + Q_G\right) = c_P' + \tau \sigma_P Q_P.$$

Due to the specific functional form of the inverse demand (3.2.3) the above condition becomes:

$$\alpha - 2\beta Q_P - \beta Q_G = c_P' + \tau \sigma_P. \tag{3.2.11}$$

We assume the subsidized firm receives a subsidy s per unit of output produced. The subsidy can then be thought of either as a production subsidy or a price support. Given the subsidy, the profit function of the subsidized firm is:

$$\pi_G = p\left(Q_P + Q_G\right) Q_G + sQ_G - c_G\left(Q_G\right) - \tau \sigma_G Q_G. \tag{3.2.12}$$

The first order condition when maximizing with respect to Q_G is:

$$p'\left(Q_P + Q_G\right) Q_G + p\left(Q_P + Q_G\right) = c_G' - s + \tau \sigma_G Q_G.$$

Due to the specific functional form of the inverse demand (3.2.3) it becomes:

$$\alpha - 2\beta Q_G - \beta Q_P = c'_G - s + \tau\sigma_G. \tag{3.2.13}$$

Combining the reaction functions (3.2.11) and (3.2.13) gives the Cournot-Nash output levels:

$$Q_P = \frac{\alpha - 2c'_P + c'_G - s + (\sigma_G - 2\sigma_P)\,\tau}{3\beta}, \tag{3.2.14}$$

$$Q_G = \frac{\alpha - 2c'_G + c'_P + 2s - (2\sigma_G - \sigma_P)\,\tau}{3\beta}. \tag{3.2.15}$$

As expected in a partial equilibrium context, the output subsidy causes the subsidized firm to increase output, and the private firm reacts by decreasing output. The overall output will increase. On the other hand, the effect of the effluence tax on the optimal quantities is not clear. For relatively small differences in emission intensities, the emission tax seems to decrease the output of both firms. However, for relatively large emission intensities, the effluence tax raises the rival's cost, thus benefiting the relatively greener firm.

In the present model, the output subsidy also affects input demands by both firms, leading to changes in the rental rates and costs, c'_i, $i = P, G$. The overall effect that subsidies have on outputs is analyzed in Section 5.2, while the overall effects of an effluence tax on outputs are analyzed in Section 5.1.

Input demands

Substituting the optimal outputs (3.2.14) and (3.2.15) into the input demands (3.2.7) and (3.2.8) generates input demands that are functions of parameters, input prices, and the subsidy:

$$K_G = \frac{\gamma}{r} c'_G \frac{\alpha - 2c'_G + c'_P + 2s - (2\sigma_G - \sigma_P)\,\tau}{3\beta} \tag{3.2.16}$$

$$K_P = \frac{\gamma}{r} c'_P \frac{\alpha - 2c'_P + c'_G - s + (\sigma_G - 2\sigma_P)\,\tau}{3\beta}, \tag{3.2.17}$$

$$L_G = \frac{1-\gamma}{w} c'_G \frac{\alpha - 2c'_G + c'_P + 2s - (2\sigma_G - \sigma_P)\tau}{3\beta}, \tag{3.2.18}$$

$$L_P = \frac{1-\gamma}{w} c'_P \frac{\alpha - 2c'_P + c'_G - s + (\sigma_G - 2\sigma_P)\tau}{3\beta}, \tag{3.2.19}$$

where

$$c'_i = \left(\frac{r}{\gamma}\right)^{\gamma} \left(\frac{w}{1-\gamma}\right)^{1-\gamma} \frac{1}{A_i}, i = G, P. \tag{3.2.20}$$

3.2.3 The government

The government grants a per-unit subsidy to the subsidized firm. Given the amount of awarded subsidies, the government tries to mitigate the pollution externality with the use of an emission tax. It is important to point out that in our model we examine the effect of ad hoc subsidies: the government does not choose the per-unit subsidy optimally. However, we are allowing for the government to use a second-best tax policy, given the subsidy, in order to maximize social welfare. Therefore, the government's optimization problem when emission taxes are imposed is:

$$\max_{\tau} U = l^*(\tau, s) + \alpha Q^*(\tau, s) - \frac{\beta}{2}\left[Q^*(\tau, s)\right]^2 - \zeta\left[E^*(\tau, s)\right]^2, \tag{3.2.21}$$

where the asterisk denotes the optimal values of leisure, consumption, and emissions given the subsidy and for any level of emission tax τ.

The net taxes collected from the firms are then redistributed to the consumers in the form of lump sum transfers.[5] The net transfers from the government to the households are therefore

$$T = (\sigma_P Q_P + \sigma_G Q_G)\tau - sQ_G \tag{3.2.22}$$

[5] Obviously, in the case where the amount of awarded subsidies exceeds the amount of taxes collected, the consumers are incurring this cost.

3.2.4 Market-clearing conditions

The market-clearing conditions set input demand equal to supply:

$$K_G + K_P = K, \tag{3.2.23}$$

$$L_G + L_P = 1. \tag{3.2.24}$$

Dividing equation (3.2.16) by (3.2.18), equation (3.2.17) by (3.2.19), and using (3.2.23) and (3.2.24) yields:

$$w = \frac{1-\gamma}{\gamma} rK. \tag{3.2.25}$$

The output subsidy does not favor one input over another and so the ratio of factor prices does not depend on the subsidy. Moreover, the input prices are linearly dependent as the input price ratio is fixed.

3.3 Equilibrium

In this model the equilibrium is characterized as follows:

1. Households maximize their utility by optimally choosing l and c, given the good and input prices, the dividends, and the net transfers from the government.

2. Firms are profit maximizers acting strategically in the good's market given the input prices, the awarded subsidies and the imposed taxes.

3. The government budget is balanced.

4. The input markets clear.

5. In the goods market, the solution concept is Cournot-Nash: each firm optimally chooses its production level, given the optimal choice of its rival.

Given the input demands defined by (3.2.16)-(3.2.19), and the marginal costs defined by (3.2.20), equations (3.2.3), (3.2.5), (3.2.6), (3.2.10)-(3.2.13), and (3.2.23)-(3.2.25), comprise a system of equations that fully defines the optimal values of r, w, Q, Q_i, p, π_i, and E.[6]

3.4 Benchmark case: zero subsidies, no taxes

To simplify notation we will denote the values of the endogenous variables when $s = \tau = 0$ with an upper bar. It is straightforward to show that the input prices are

$$\overline{r} = \frac{\alpha\left(1+A\right) - 3A\beta K^{\gamma}}{2\left(1-A+A^{2}\right)}\gamma A K^{\gamma-1},$$

and

$$\overline{w} = \frac{\alpha\left(1+A\right) - 3A\beta K^{\gamma}}{2\left(1-A+A^{2}\right)}\left(1-\gamma\right) A K^{\gamma},$$

while outputs are

$$\overline{Q}_{G} = \frac{\left(1-A\right)\alpha + \left(2A-1\right)A\beta K^{\gamma}}{2\beta\left(1-A+A^{2}\right)},$$

$$\overline{Q}_{P} = \frac{A\left(A-1\right)\alpha - \left(A-2\right)A\beta K^{\gamma}}{2\beta\left(1-A+A^{2}\right)},$$

and

$$\overline{Q} = \overline{Q}_{G} + \overline{Q}_{P} = \frac{\left(A-1\right)^{2}\alpha + \left(A+1\right)A\beta K^{\gamma}}{2\beta\left(1-A+A^{2}\right)}.$$

Given these quantities the corresponding price is

$$\overline{p} = \frac{\left(A^{2}+1\right)\alpha - \left(A+1\right)A\beta K^{\gamma}}{2\left(1-A+A^{2}\right)},$$

and thus the profits are

$$\overline{\pi}_{G} = \beta\overline{Q}_{G}^{2} = \beta\left(\frac{\left(1-A\right)\alpha + \left(2A-1\right)A\beta K^{\gamma}}{2\beta\left(1-A+A^{2}\right)}\right)^{2},$$

[6] Although there are 11 equations and only 10 endogenous variables, equations (3.2.23) and (3.2.24) are linearly dependent through equation (3.2.25). Thus, the system is exactly identified.

and

$$\overline{\pi}_P = \beta \overline{Q}_P^2 = \beta \left(\frac{A\,(A-1)\,\alpha - (A-2)\,A\beta K^{\gamma}}{2\beta\,(1-A+A^2)} \right)^2 .$$

Finally, the resulting emissions are given by:

$$\overline{E} = \frac{(1-A)\,(\sigma_G - A\sigma_P)\,\alpha + (\sigma_G\,(2A-1) - \sigma_P\,(A-2))\,A\beta K^{\gamma}}{2\beta\,(1-A+A^2)}.$$

For the remainder of this paper we are making all the necessary assumptions regarding the parameter values to ensure positivity of all the benchmark values of the variables.[7] First, note that the denominators in all optimal values are positive and that profits are always positive as $\beta > 0$ and $1 - A + A^2 > 0$. Total output and total emissions, being weighted sums of firms' outputs, are positive when at least one firm is producing. Therefore, it suffices to require the numerators of the optimal interest rate, the subsidized firm's output, the private firm's output and the output price to be positive. It can be shown that these conditions are summarized by:[8]

$$\frac{3A}{(1+A)}\beta K^{\gamma} < \alpha < \frac{(2A-1)\,A}{A-1}\beta K^{\gamma}. \tag{3.4.1}$$

3.5 General solution

In this section we allow for non-zero subsidies, i.e., $s > 0$, and the possibility of an effluence tax. In this general case the input prices are:

$$\widetilde{r} = \overline{r} + \frac{((2A-1)\,s + (\sigma_G\,(1-2A) + (A-2)\,\sigma_P)\,\tau)}{2\,(1-A+A^2)}\gamma A K^{\gamma-1}, \tag{3.5.1}$$

and

$$\widetilde{w} = \overline{w} + \frac{((2A-1)\,s + (\sigma_G\,(1-2A) + (A-2)\,\sigma_P)\,\tau)}{2\,(1-A+A^2)}\,(1-\gamma)\,A K^{\gamma}, \tag{3.5.2}$$

[7] We are not considering the case where either of the firms' outputs becomes zero as in this case the other firm will be acting as a monopoly. Our results might then change significantly.

[8] A detailed derivation of (3.4.1) can be found in Appendix I.

while outputs are

$$\widetilde{Q}_G = \overline{Q}_G + \frac{s - (\sigma_G - A\sigma_P)\,\tau}{2\beta\,(1 - A + A^2)}, \tag{3.5.3}$$

$$\widetilde{Q}_P = \overline{Q}_P - \frac{As - A\,(\sigma_G - A\sigma_P)\,\tau}{2\beta\,(1 - A + A^2)}, \tag{3.5.4}$$

and

$$\widetilde{Q} = \overline{Q}_G + \overline{Q}_P + (A-1)\,\frac{(\sigma_G - A\sigma_P)\,\tau - s}{2\beta\,(1 - A + A^2)}. \tag{3.5.5}$$

Given these outputs, the corresponding price and profits are:

$$\widetilde{p} = \overline{p} - (A-1)\,\frac{(\sigma_G - A\sigma_P)\,\tau - s}{2\,(1 - A + A^2)}, \tag{3.5.6}$$

$$\widetilde{\pi}_G = \beta \widetilde{Q}_G^2 = \beta\left(\overline{Q}_G + \frac{s - (\sigma_G - A\sigma_P)\,\tau}{2\beta\,(1 - A + A^2)}\right)^2, \tag{3.5.7}$$

and

$$\widetilde{\pi}_G = \beta \widetilde{Q}_P^2 = \beta\left(\overline{Q}_P - \frac{As - A\,(\sigma_G - A\sigma_P)\,\tau}{2\beta\,(1 - A + A^2)}\right)^2, \tag{3.5.8}$$

Finally, total emissions are given by:

$$\widetilde{E} = \overline{E} + \frac{(\sigma_G - A\sigma_P)\,s - (\sigma_G - A\sigma_P)^2\,\tau}{2\beta\,(1 - A + A^2)}. \tag{3.5.9}$$

3.5.1 Optimal emission tax under zero subsidies

In this section we allow the government to impose an effluence tax, given that $s = 0$. The overall effect of an effluence tax on the variables of the model can be then analyzed. Unlike the partial equilibrium analysis in Section 3.2.2, where the effect of an emission tax on input demands and, consequently, on input prices is ignored, the analysis here incorporates these effects. We can then compare the results of our model with existing results about emission taxes in the literature. By differentiating the above expressions with respect to the emission tax it is straightforward to show the following proposition. The proof of Proposition 10 is presented in Appendix I.

Proposition 10 *Given $A > 1$*

(a) if $\frac{\sigma_G}{\sigma_P} > A$ an emission tax will increase Q_P, and Q, and it will decrease r, w, Q_G, and p.

(b) if $\frac{\sigma_G}{\sigma_P} < A$ an emission tax will decrease Q_P, and Q, and it will increase Q_G, and p. Furthermore, input prices increase if $A > 2$ and $0 < \frac{\sigma_G}{\sigma_P} < \dfrac{A-2}{2A-1}$.

(c) E always decreases.

According to this proposition, although the effluence tax increases the cost of both firms, it gives a strategic advantage to the relatively "greener" firm. First, as the emission tax increases, firms respond by reducing their outputs. This results in lower input demands and, consequently, lower input prices. Lower input prices make the two firms more aggressive and they tend to increase their outputs. The overall result on output depends on the responsiveness of a firm's marginal cost on the emission tax. The firm with the relatively more elastic marginal cost will benefit from the hike on the emission tax. It can be shown that in our model, the private firm's marginal cost is more responsive to tax changes when $\sigma_G > A\sigma_P$.[9] Note that this comparison does not refer to pure emission intensities, but rather refers to comparing TFP-enhanced emission intensities, i.e., $A_i\sigma_i$, $i = P, G$. Since

[9] Given that the marginal cost of firm i is $c_i' + \tau\sigma_i$, $i = P, G$, we define the tax elasticity of the marginal cost as

$$\varepsilon_{\tau,i} = \frac{\partial\left(c_i' + \tau\sigma_i\right)}{\partial\tau}\frac{\tau}{c_i' + \tau\sigma_i} = \left(\frac{\partial c_i'}{\partial\tau} + \tau\right)\frac{\tau}{c_i' + \tau\sigma_i}.$$

Using (3.2.20) one can show that $c_G' = Ac_P'$. Substituting in the above formula for $i = P, G$, and dividing by parts we get

$$\frac{\varepsilon_{\tau,P}}{\varepsilon_{\tau,G}} > 1 \Rightarrow \sigma_G > A\sigma_P.$$

the private firm is the most efficient firm in terms of production, total output always changes on the same direction with Q_P. Finally, it is worth noticing that the higher the TFP ratio and the lower the emission intensities ratio, the more likely that the input prices will increase. This is the case where the effluence tax does not award a very significant advantage to the private firm and the demand for inputs by the subsidized firm does not decrease significantly.

To find the optimal tax we substitute the optimal aggregate production (3.5.5) and the total emissions (3.5.9) evaluated at $s = 0$ in the welfare maximization problem (3.2.21):

$$\begin{aligned}\max_{\tau} U &= \alpha\left(\overline{Q} + \frac{(\sigma_G - A\sigma_P)(A-1)}{2\beta(1-A+A^2)}\tau\right) - \\ &\quad -\frac{\beta}{2}\left(\overline{Q} + \frac{(\sigma_G - A\sigma_P)(A-1)}{2\beta(1-A+A^2)}\tau\right)^2 - \zeta\left(\overline{E} - \frac{(\sigma_G - \sigma_P A)^2}{2\beta(1-A+A^2)}\tau\right)^2\end{aligned}$$

The optimal tax is therefore,

$$\begin{aligned}\tau^* &= \frac{\alpha\beta(A-1)(A^2+1) - 2\alpha\zeta(\sigma_G - A\sigma_P)^2(A-1)}{(\sigma_G - A\sigma_P)\left(\beta(A-1)^2 + 2\zeta(\sigma_G - A\sigma_P)^2\right)} + \\ &\quad + \frac{2\zeta(\sigma_G(2A-1) + (2-A)\sigma_P)(\sigma_G - A\sigma_P)\beta AK^{\gamma}}{(\sigma_G - A\sigma_P)\left(\beta(A-1)^2 + 2\zeta(\sigma_G - A\sigma_P)^2\right)} - \\ &\quad - \frac{\beta(A^2-1)\beta AK^{\gamma}}{(\sigma_G - A\sigma_P)\left(\beta(A-1)^2 + 2\zeta(\sigma_G - A\sigma_P)^2\right)}\end{aligned} \tag{3.5.10}$$

We can now show the following:

Proposition 11 *Given $A > 1$, the optimal emission tax exceeds the Pigouvian tax when $\sigma_G - A\sigma_P > 0$. If, however, $\sigma_G - A\sigma_P < 0$ then the optimal emission tax might be greater or lower than the Pigouvian tax.*

Proof. Substituting the optimal tax given by equation (3.5.10) in equation (3.5.9) and setting $s = 0$ we get the total emissions under optimal tax, i.e.,

$$E^* = \frac{(1-A)\left((\sigma_G - A\sigma_P)\,\alpha - (\sigma_G - \sigma_P)\,A\beta K^\gamma\right)}{\beta\,(A-1)^2 + 2\zeta\,(\sigma_G - A\sigma_P)^2}$$

Given the total emissions, the optimal tax can be expressed as a function of the Pigouvian tax, $\tau_P = 2\zeta E^*$. Adding and subtracting this term in the optimal tax (3.5.10) and simplifying yields:

$$\tau^* = \frac{(A-1)\left((A^2+1)\,\alpha - (A+1)\,A\beta K^\gamma\right) + 2\zeta\,(\sigma_G - A\sigma_P)\,(\sigma_P + A\sigma_G)\,AK^\gamma}{(\sigma_G - A\sigma_P)\left(\beta\,(A-1)^2 + 2\zeta\,(\sigma_G - A\sigma_P)^2\right)}\beta + 2\zeta E^*$$

The sign of the first term of the RHS can be positive or negative. However, under the non-negativity assumptions along with $A > 1$ and $\sigma_G - A\sigma_P > 0$ the optimal tax exceeds the Pigouvian tax. ■

This result is in accordance with Simpson (1995), that first derived the conditions for the optimal emission tax to exceed the Pigouvian tax in a partial equilibrium framework.

3.5.2 Non-zero subsidies

It is also straightforward to show the following:

Proposition 12 *Given $A > 1$, an ad hoc awarded output subsidy to the less efficient firm*

(a) always increases input prices, r, w, the output of the subsidized firm, Q_G, and the output price, p, while it decreases the output of the subsidized firm, Q_P, and the total output Q.

(b) increases the total emissions in the economy if $\sigma_G - A\sigma_P > 0$.

The proof of the above Proposition 3 is provided in Appendix I. The effect of ad hoc subsidies on the interest rate is of some importance when considering capital accumulation. Although this model is static, one can assume, as in Kelly (2009), that ad hoc subsidies that increase the interest rate can lead to over-accumulation of capital. Moreover, the effect of ad hoc subsidies on total output and emissions gives rise to the following:

Corollary 13 *Given $\sigma_G/\sigma_P > A > 1$, (ad hoc) subsidies are welfare-reducing.*

What is important here is that for the subsidies to be welfare-reducing the subsidized firm must have a much larger emission intensity than the private firm. In such a case, our findings are aligned with Kelly (2009). However, if the emission intensity of the subsidized firm is relatively low, and depending on parameter values, this result can even be reversed. The reduction in emissions due to subsidies can be significant enough to outweigh the negative welfare effects of reduced output.

Optimal emission tax under non-zero subsidies

To find the optimal tax we substitute the optimal aggregate production (3.5.5) and the total emissions (3.5.9) into the welfare maximization problem (3.2.21). By taking the first order condition with respect to the tax, we get:

$$\begin{aligned}\tau^{**} &= \frac{\beta\left(A-1\right)\left(\left(A-1\right)s+\alpha+A^2\alpha\right)-2\zeta\left(\sigma_G-A\sigma_P\right)^2\left(-s-\alpha+A\alpha\right)}{\left(\sigma_G-A\sigma_P\right)\left(2\zeta\left(\sigma_G-A\sigma_P\right)^2+\beta\left(A-1\right)^2\right)} \\ &+\frac{\left[2\zeta\left(\sigma_G\left(2A-1\right)+\left(2-A\right)\sigma_P\right)\left(\sigma_G-A\sigma_P\right)-\beta\left(A^2-1\right)\right]\beta AK^\gamma}{\left(\sigma_G-A\sigma_P\right)\left(2\zeta\left(\sigma_G-A\sigma_P\right)^2+\beta\left(A-1\right)^2\right)}\end{aligned}$$

Therefore, emissions under the optimal tax are

$$E^{**}=\frac{\left(1-A\right)\left(\left(\sigma_G-A\sigma_P\right)\alpha-\left(\sigma_G-\sigma_P\right)A\beta K^\gamma\right)}{\beta\left(A-1\right)^2+2\zeta\left(\sigma_G-A\sigma_P\right)^2}=E^*$$

As before, we can express the optimal tax as a function of the Pigouvian tax,

$\tau_P = 2\zeta E^*$ that is

$$\tau^{**} = \tau^* + \frac{s}{(\sigma_G - A\sigma_P)} + 2\zeta E^*$$

Note that the first and second terms have the same sign when $\sigma_G - A\sigma_P > 0$. Therefore, in the presence of subsidies, the result of proposition 2 is reinforced: in the presence of ad hoc subsidies, it is more likely that the optimal emission tax exceeds the Pigouvian tax.

It is the layman's presumption that the first-best policy is to eliminate any inefficient subsidies and then impose the optimal effluence tax, τ^*. This is verified in this model as the second term of the right-hand side in the optimal tax equation above serves exactly this purpose. What is interesting though is that it is proven that a unified emission tax is superior to any firm specific-tax in the present model.

3.5.3 Existence of firms

In this section we examine the non-negativity conditions of the model's variables (*i.e.*, quantities and prices must be non-negative) in the presence of the subsidy given that (3.4.1) holds. In light of Proposition 3, the subsidy cannot change the sign of r, w, Q_G, and p as these variables are positive according to (3.4.1) and positively related to s. Furthermore, the non-negativity condition for Q_P will suffice for positive total output and emissions, as these are just weighted averages of the firms' quantities. Therefore, we focus on the condition for the non-negativity of Q_P. It can be shown that, as long as the subsidy is not too high, the private firm will remain in the market. The related condition then is:

$$s < (A-1)\,\alpha + \frac{(A+1)}{(A-1)}A\beta K^\gamma$$

In summary we have 2 cases:

1. Both firms produce:

$$(A-1)\,\alpha - (2A-1)\,A\beta K^\gamma < s < (A-1)\,\alpha + \frac{(A+1)}{(A-1)}A\beta K^\gamma.$$

2. Only the subsidized firm produces:

$$s > (A-1)\,\alpha + \frac{(A+1)}{(A-1)} A\beta K^{\gamma}.$$

Therefore, the subsidy can drive out the private firm completely. This would affect the comparative statics discussed above, but mostly in the same direction. An increase in subsidies which drives out the private firm would still increase subsidized output, subsidized profits, subsidized input usage, and probably factor prices. Private output, inputs, and profits would decrease in the subsidy until zero output is reached, after which they would remain constant in the subsidy. However, the condition for emissions to increase when subsidy increases would vary as would total output.

3.6 Conclusions

Using a general equilibrium framework with imperfect competition, we show that subsidies allocated to the firms with the relatively low total factor productivity can be welfare-reducing as both total output and environmental quality decrease. A necessary condition for this is that the subsidized sector has relatively higher emission intensity. The reduction in total output can be decomposed into two parts. First, the subsidy tends to increase the output of the subsidized firm and decrease that of the private firm. In the absence of market interactions, the result in total output would have been positive. However, subsidies also distort input demands. More specifically, the increase in the input demands of the subsidized firms exceeds that of the private firms, resulting in an increase in input prices, namely the wage and interest rate. Increased costs of production tend to decrease the production of both the subsidized and the private firms. The overall result is a decrease in total output.

The effect of subsidies on emissions can be also decomposed into two parts. First, due to lower total output, emissions tend to decrease. However, reallocation of resources to the relatively high emission-intensity firm tends to increase emissions. Under the conditions specified in the model, the latter exceeds the former and total emissions increase.

Finally, we derive the second-best optimal environmental tax given subsidies. It is shown that this tax is increasing in subsidies. As this is a case of imperfect competition, market inefficiency requires the tax to be lower than the Pigouvian tax. However, to correct the distorting effect of the subsidies in reallocating resources to the least productively efficient firms, the tax should be higher than the Pigouvian tax. The higher the given subsidies in the economy, the more likely it is that the second-best optimal tax will exceed the tax equal to the marginal social damage.

3.7 Appendix I

Deriving the existence condition (3.4.1). Given the benchmark values of the variables, positivity of Q_G, Q_P, p, and r is implied if by

$$\alpha < \frac{(2A-1)}{(A-1)} A\beta K^{\gamma}, \tag{3.7.11}$$

$$\alpha > \frac{(A-2)}{(A-1)} \beta K^{\gamma}, \tag{3.7.12}$$

$$\alpha > \frac{(A+1)}{(A^2+1)} A\beta K^{\gamma}, \tag{3.7.13}$$

and

$$\alpha > \frac{3}{(1+A)} A\beta K^{\gamma}, \tag{3.7.14}$$

correspondingly. Note that (3.7.14) is stronger constraint than (3.7.13) as $\frac{3}{(1+A)} A\beta K^{\gamma} - \frac{(A+1)}{(A^2+1)} A\beta K^{\gamma} = \frac{2A\beta K^{\gamma}}{A^3+A^2+A+1} (A^2 - A + 1) > 0$. Moreover, (3.7.14) is also stronger than (3.7.12) as $\frac{3}{(1+A)} A\beta K^{\gamma} - \frac{(A-2)}{(A-1)} \beta K^{\gamma} = \frac{2\beta K^{\gamma}}{A^2-1} (A^2 - A + 1) > 0$. Finally, (3.7.14) is weaker than (3.7.11) as $\frac{(2A-1)}{(A-1)} A\beta K^{\gamma} - \frac{3}{(1+A)} A\beta K^{\gamma} = \frac{2A\beta K^{\gamma}}{A^2-1} (A^2 - A + 1)$

■

Proof of Proposition 10. Taking the first derivatives of (3.5.1)-(3.5.9) with respect to τ yields

$$\partial\widetilde{r}/\partial\tau = \frac{\sigma_G (1-2A) + (A-2)\sigma_P}{2(1-A+A^2)} \gamma A K^{\gamma-1},$$

$$\partial\widetilde{w}/\partial\tau = \frac{\sigma_G (1-2A) + (A-2)\sigma_P}{2(1-A+A^2)} (1-\gamma) A K^{\gamma},$$

$$\partial\widetilde{Q}_G/\partial\tau = -\frac{\sigma_G - A\sigma_P}{2\beta(1-A+A^2)},$$

$$\partial\widetilde{Q}_P/\partial\tau = \frac{A(\sigma_G - A\sigma_P)}{2\beta(1-A+A^2)},$$

$$\partial\widetilde{Q}/\partial\tau = (A-1) \frac{(\sigma_G - A\sigma_P)}{2\beta(1-A+A^2)},$$

$$\partial\widetilde{p}/\partial\tau = -(A-1) \frac{(\sigma_G - A\sigma_P)}{2(1-A+A^2)},$$

$$\partial \widetilde{E}/\partial \tau = -\frac{(\sigma_G - A\sigma_P)^2}{2\beta\left(1 - A + A^2\right)}.$$

By inspecting the sign of the above expressions one can immediately verify part (c) as $\partial \widetilde{E}/\partial \tau < 0$. Regarding part (a) note that when $\sigma_G - A\sigma_P > 0$, we get $\partial \widetilde{Q}_G/\partial \tau < 0$, $\partial \widetilde{Q}_P/\partial \tau > 0$, $\partial \widetilde{Q}/\partial \tau > 0$, and $\partial \widetilde{p}/\partial \tau < 0$. Regarding the input prices note that the signs of their respective derivatives depend on the sign of $\sigma_G(1-2A) + (A-2)\sigma_P$. Note that $\sigma_G - A\sigma_P > 0 \Rightarrow \sigma_G - A\sigma_P + (A-1)\sigma_G + 2\sigma_P > 0 \Rightarrow \sigma_G(1-2A) + (A-2)\sigma_P < 0$. This completes the proof of part (a). For part (b) note that when $\sigma_G - A\sigma_P < 0$, we get $\partial \widetilde{Q}_G/\partial \tau > 0$, $\partial \widetilde{Q}_P/\partial \tau < 0$, $\partial \widetilde{Q}/\partial \tau < 0$, and $\partial \widetilde{p}/\partial \tau$. For the input prices, note that $\sigma_G(1-2A) + (A-2)\sigma_P < 0 \Rightarrow \frac{\sigma_G}{\sigma_P} < \dfrac{A-2}{2A-1}$. Since the emission intensities are positive, the last inequality can only be true if $A > 2$. ■

Proof of Proposition 12. Taking the first derivatives of (3.5.1)-(3.5.9) with respect to s yields

$$\partial \widetilde{r}/\partial s = \frac{(2A-1)}{2\left(1 - A + A^2\right)}\gamma A K^{\gamma-1},$$

$$\partial \widetilde{w}/\partial s = \frac{(2A-1)}{2\left(1 - A + A^2\right)}(1-\gamma) A K^{\gamma},$$

$$\partial \widetilde{Q}_G/\partial s = \frac{1}{2\beta\left(1 - A + A^2\right)},$$

$$\partial \widetilde{Q}_P/\partial s = -\frac{A}{2\beta\left(1 - A + A^2\right)},$$

$$\partial \widetilde{Q}/\partial s = -\frac{(A-1)}{2\beta\left(1 - A + A^2\right)},$$

$$\partial \widetilde{p}/\partial s = \frac{(A-1)}{2\left(1 - A + A^2\right)},$$

$$\partial \widetilde{E}/\partial s = \frac{(\sigma_G - A\sigma_P)}{2\beta\left(1 - A + A^2\right)}.$$

By inspecting the sign of the above expressions one can immediately verify part (a). Regarding part (b) note that when $\sigma_G - A\sigma_P > 0$, we get $\partial \widetilde{E}_G/\partial s < 0$. ■

3.8 Appendix II

Industry	**Total OECD Subsidies**	**Year**	**Source**
Agriculture	\$318 Billion	2002	OECD (2003)
Fishing	\$6 Billion	1999	OECD (2001)
Energy	\$20-30 Billion	1999	Barde and Honkatukia (2004)
Manufacturing	\$43.7 Billion	1993	OECD(1998b)
Transport	\$40 Billion	1998	Nash, Bickel, Rainer, Link, and Steward (2002)
Water	\$10 Billion	n.a.	Barde and Honkatukia (2004)

Table 3.1: Total OECD subsidies in environmentally sensitive industries, in the most recent year available, as reported in Table 7.2 and text of Barde and Honkatukia (2004).*Source: Kelly (2007)*

Industry	**OECD**	**Non-OECD**	**World**	**OECD as % of world**
Natural Resource sectors				
Agriculture	335	65	400	84
Water	15	45	60	25
Forestry	5	30	35	4
Fisheries	10	10	20	50
Mining	25	5	30	83
Energy and industry sectors				
Energy	80	160	240	33
Road Transport	200	25	225	89
Manufacturing	55	negligible	55	100
Total	**725**	**340**	**1065**	**68**
Total as % GDP	3.4	6.3	4.0	

Table 3.2: Estimates of World subsidies (in USD billion). *Source: Pearce (2002)*

Bibliography

[1] Andreoni, J., Levinson, A., 2001. The simple analytics of the Environmental Kuznets Curve. *Journal of Public Economics,* 80 (2), 269– 286.

[2] Antweiler, W., Copeland, B.R., Taylor, M.S., 2001. Is free trade good for the environment? *American Economic Review,* 91 (4), 877–908.

[3] Arrow, K., Bolin, B., Costanza, R., Folke, C., Holling, C.S., Janson, B., Levin, S., Maler, K., Perrings, C., Pimental, D., 1995. Economic growth, carrying capacity, and the environment. *Science, 15*, 91– 95.

[4] Bajona, C., Kelly, D. L., 2006. Trade and the Environment with Pre-Existing Subsidies: A Dynamic General Equilibrium Analysis. University of Miami Department of Economics Working Paper.

[5] Barde, J.-P., Honkatukia, O., 2004. Environmentally Harmful Subsidies. In Tom Tietenberg, and Henk Folmer (ed.), The International Yearbook of Environmental and Resource Economics 2004/2005 . chap. 7, 254-288, Edward Elgar, Northampton, MA.

[6] Bernard, J., Gavin, M., Khalaf, L., 2011. The Environmental Kuznets Curve: Tipping Points and Uncertainty. Manuscript.

[7] Barnett, A.J, 1980. The Pigouvian tax rule under monopoly. *American Economic Review*, 70, 1037–1041.

[8] Bonanno, G., 1990. General equilibrium theory with imperfect competition. *Journal of Economic Surveys,* 4, 297-328.

[9] Boone, J., Bovenberg, L., 2007. The Simple Economics of Bunching: Optimal Taxation with Quasi-Linear Preferences. *Journal of Public Economic Theory,* 9 (1), 2007, 89–105.

[10] Carson, R.T., Jeon, Y., McCubbin, D.R., 1997. The relationship between air pollution emissions and income : US data. *Environment and Development Economics,* 2, 433– 450.

[11] Copeland, B.R., Taylor, M.S., 1995. Trade and Transboundary Pollution. *American Economic Review*, 85, 716-737.

[12] Ekins, P., 1997. The Kuznets Curve for the environment and economic growth: examining the evidence. *Environment and Planning*, A 29, 805– 830.

[13] Galeoti, M.,Lanza, A., Pauli, F., 2006. Reassessing the environmental Kuznets curve for CO2 emissions: A robustness exercise. *Ecological Economics*, 57, 152– 163.

[14] Georgescu-Roegen, N., 1971. The Entropy Law and the Economic Process. Harvard University Press, Cambridge, Massachusetts.

[15] Grossman, G.M., Krueger, A.B., 1991. Environmental impacts of the North American Free Trade Agreement. NBER Working paper 3914.

[16] Grossman, G.M., Krueger, A.B., 1993. Environmental impacts of the North American Free Trade Agreement. In: Garber, P. (Ed.), The U.S. – Mexico Free Trade Agreement. MIT Press, Cambridge, pp. 13–56.

[17] Grossman, G.M., Krueger, A.B., 1995. Economic growth and the environment. *Quarterly Journal of Economics*, 110 (2), 353– 377.

[18] Hutchinson, E., Kennedy P. W., 2010. Income Growth and Equilibrium Dynamics for a Global Pollutant. Manuscript.

[19] John, A., Pecchenino, R., 1994. An overlapping generations Model of Growth and the Environment. *Economic Journal*, 104, 1393-1410.

[20] Kelly, D. L., 2009. Subsidies to Industry and the Environment. Discussion Paper 14999, NBER Working Paper.

[21] Kuznets, S., 1955. Economic growth and income inequality. *American Economic Review*, 45, 1– 28.

[22] Liddle, B., 2001. Free trade and the environment – development system. *Ecological Economics*, 39 (1), 21– 36.

[23] Lieb, C.M., 2002. The Environmental Kuznets Curve and satiation: A simple static model. *Environment and Development Economics*, 7, 429– 448.

[24] Lopez, R., 1994. The environment as a factor of production: the effects of economic growth and trade liberalization. *Journal of Environmental Economics and management*, 27, 163–184.

[25] Magnani, E., 2000. The Environmental Kuznets Curve, environmental policy and income distribution. *Ecological Economics*, 32, 431–443.

[26] Malenbaum, W., 1978. World demand for raw materials in 1985 and 2000. McGraw-Hill, New York.

[27] McConnell, K.E., 1997. Income and the demand for environmental quality. *Environment and Development Economics*, 2, 383–399.

[28] Meadows, D. H., Meadows, D. L., Randers, J., Behrens, W. W., 1971. The Limits to Growth. Universe Books, New York.

[29] Monderer, D., Shapley, L., 1996. Potential games, *Games and Economic Behaviour*, 14, 124-143.

[30] Panayotou, T., 1993. Empirical tests and policy analysis of environmental degradation at different stages of economic development, ILO, Technology and Employment Programme, Geneva.

[31] Pearce, D., 2002. Environmentally Harmful Subsidies: Barriers to Sustainable Development. Paper Presented at the OECD Workshop on Environmentally Harmful Subsidies, 7–8 November, 2002, Paris.

[32] Prieur, F., 2009. The environmental Kuznets curve in a world of irreversibility. *Economic Theory*, 40 (1), 57-90.

[33] Roca, J., 2003. Do individual preferences explain Environmental Kuznets Curve? *Ecological Economics*, 45 (1), 3– 10.

[34] Selden, T., Song, D., 1994. Environmental quality and development: is there a Kuznets Curve for air pollution emissions? *Journal of Environmental Economics and Management*, 27, 147– 162.

[35] Simpson, R. D., 1995. Optimal Pollution Taxation in a Cournot Duopoly. *Environmental and Resource Economics,* 6, 359–369.

[36] Stokey, N.L., 1998. Are there limits to growth? *International Economic Review*, 39 (1), 1– 31.

[37] Suri, V., Chapman, D., 1998. Economic growth, trade and the environment: implications for the environmental Kuznets curve. *Ecological Economics*, 25, 195– 208.

[38] Tilton, J.E., 1990. The OECD countries: Demand trend setters. In: Tilton, J.E. (ed.), World Metal Demand: Trend and Prospects. Resource for the Future, Washington, DC, 35-76.

[39] Torras, M., Boyce, J.K., 1998, "Income, inequality, and pollution: a reassessment of the Environmental Kuznets Curve," Ecological Economics 25, 147–160.

[40] van Beers, C., de Moor, S., 2001. Public Subsidies and Policy Failures: How Subsidies Distort the Natural Environment, Equity and Trade and How to Reform Them. Edward Elgar, Cheltenham.

[41] van Beers, C., van den Bergh, J. C., 2001. Perseverance of Perverse Subsidies and their Impact on Trade and Environment. *Ecological Economics*, 36, 475-86.

[42] Wagner, M., Müller-Fürstenberger, G., 2008. The Carbon Kuznets Curve: A cloudy picture emitted by bad econometrics? *Resource and Energy Economics,* 30 (3), 388-408.

[43] WIlliams, R.H., Larson, E.D., Ross, M.H., 1987. Materials affluence and industrial energy use. *Annual Review of Energy*, 12, 99-144.

[44] World Bank, 1992. World Development Report. Oxford Univ. Press, New York.

[45] Yaguchi, Y., Sonobe, T., Otsuka, K., 2007. Beyond the Environmental Kuznets Curve: a comparative study of SO2 and CO2 emissions between Japan and China. *Environment and Development Economics*, 12, 445–470.