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On the Quinean-analyticity of mathematical propositions*

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Abstract

This paper investigates the relation between Carnap and Quine's views on analyticity on the one hand, and their views on philosophical analysis or explication on the other. I argue that the stance each takes on what constitutes a successful explication largely dictates the view they take on analyticity. I show that although acknowledged by neither party (in fact Quine frequently expressed his agreement with Carnap on this subject) their views on explication are substantially different. I argue that this difference not only explains their differences on the question of analyticity, but points to a Quinean way to answer a challenge that Quine posed to Carnap. The answer to this challenge leads to a Quinean view of analyticity such that arithmetical truths are analytic, according to Quine's own remarks, and set theory is at least defensibly analytic.

Introduction

Quine described his differences with Carnap as principally involving analyticity and ontology. This paper will deal only with the first of these problems — leaving their different views on ontology for another occasion. Concerning their views on analyticity, it will be shown that their respective positions on

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this issue stem from the differences in their views on what constitute a successful explication. Not only this, but each side's failure to fully understand his opponent is also explained by the differences in their respective views on explication. Each side mistakenly believed that they agreed on the notion of an explication.

After having established the above points, I turn to the question of whether one can give an explication of analyticity that satisfies Quine's views on explication. I argue that Quine's views on explication themselves suggest a way of defining analyticity. The resulting notion I call *Quinean-analyticity*. Finally, I argue that essentially all of arithmetic and set theory come out as Quinean-analytic.

In the first section I discuss Carnap's explicit remarks on explication, and in the second, those of Quine. In the third section, I discuss the differences between their accounts and how these differences are of crucial importance for understanding the debate over analyticity. In the fourth section, I examine Quine's own attempted explication of analyticity. The best of these is a notion that is far too restrictive to be of much interest and, by Quine's own diagnosis, is 'epistemically insignificant'. I propose the concept *Quinean-analytic*, which is inspired from Quine's views on explication. In the fifth and final section, I argue that much of mathematics comes out as Quinean-analytic in this sense.

1 Carnap on Philosophical Analysis

In 1932, Carnap published 'Uberwindung der Metaphysik durch Logische Analyse der Sprache'. In this paper, Carnap is mainly concerned with demonstrating the thesis that metaphysical statements are meaningless. However, the paper includes a brief discussion of a positive thesis as well. The positive thesis is the claim that over and above the elimination of meaningless statements, the task of a scientific philosophy is to "clarify meaningful concepts and propositions, to lay logical foundations for factual science and for mathematics". (Carnap, 1996, p. 27) Here we get a glimpse of Carnap's fundamental beliefs about what the role of philosophy is. Philosophy is the activity of clarifying concepts (or propositions) and making clear the logical relations that hold between them. This positive thesis emphasizes the relationship between the new scientific philosophy and the tradition from which it emerges. Even speculative metaphysicians were engaged in little, if anything, more than logic and conceptual analysis. It is the negative thesis, that some classes of statements have no empirical content and so sufficient clarification is impossible, which separates logical positivism from much of the tradition. These quite general views about the role of philosophy first articulated in 1932 were never given up by Carnap. This is evidenced by Carnap, in the 1957 English translation, merely appended some clarificatory remarks to the end of the article.

Let us now, then, try to pin down exactly what Carnap's views on explication are. Carnap is explicit in his views on conceptual analysis in two places. He gives a detailed account of conceptual analysis in *Logical Foundations of Prob-*

ability (esp. chapter I) and he considers and compares successful analyses in *Meaning and Necessity* (esp. §2 and §§7-16). Carnap begins *Logical Foundations of Probability* by stating that the purpose of the book is to give a clarification (this term is used by Carnap along with 'explication' as interchangeable with 'philosophical analysis') of the notion of probability. He considers an analysis successful if it fulfills the following requirements:

> 1. The explicatum is to be *similar to the explicandum* in such a way that, in most cases in which the explicandum has been so far used, the explicatum can be used; however, close similarity is not required and considerable differences are permitted.
>
> 2. The characterization of the explicatum, that is the rules of its use (for instance, in the form of a definition), is to be given in an *exact* form, so as to introduce the explicatum into a well-connected system of scientific concepts.
>
> 3. The explicatum is to be is to be a *fruitful* concept, that is, useful for the formulation of many universal statements (empirical laws in the case of a nonlogical concept, logical theorems in the case of a logical concept).
>
> 4. The explicatum should be as *simple* as possible; this means as simple as the more important requirements (1), (2), and (3) permit. (Carnap, 1950, §3, original italics)

As an example of a successful analysis, Carnap mentions the term 'fish' as used by biologists. He imagines (as probably was the case) that there was a prescientific term 'fish' that meant roughly 'animal that swims around in water'. This term included marine mammals as fish. The modern scientific concept 'fish' applies to most but not all of the same cases as the prescientific concept, thus (1) is satisfied. The remaining requirements are satisfied by the definition as well. Carnap stresses the situation is not adequately described by the statement 'The previous belief that whales (in German even called 'Walfische') are also fish is refuted by zoology.' (Carnap, 1950, §3) The new concept replaces the prescientific concept. Sentences predicating 'fish' of a whale are not incorrect if 'fish' is used in the prescientific sense. We are free to use either language but we have good reason to use the scientific language instead of the prescientific language.

> What kind of correspondence is required here between the first concept, the explicandum, and the second, the explicatum?
>
> Since the explicandum is more or less vague and certainly more so than the explicatum, it is obvious that we cannot require the correspondence between the two concepts to be a complete coincidence. But one might perhaps think that the explicatum should be as close to or as similar with the explicandum as the latter's vagueness permits. However, it is easily seen that this requirement would be too strong, that the actual procedure of scientists is often not in agreement with it, and for good reasons. (Carnap, 1950, §3)

We see then two features stand out concerning Carnap's account of explication. First, Carnap rejects any question of correctness. The explicatum is to replace the explicandum. Second, proposals are to be assessed on practical, that is pragmatic, grounds. Carnap is well aware of the paradox of analysis. The phrase 'paradox of analysis' comes from C. H. Langford, but it is most closely associated with G. E. Moore. The paradox of analysis is that no analysis can be both informative and correct. For, if the analysans simply says the same thing as the analysandum, then it is uninformative. If they say something different, it is incorrect. After mentioning both Moore and Langford, Carnap rejects the question of the correctness of an explication.

> In a problem of explication the datum, [...] viz., the explicandum, is not given in exact terms; if it were, no explication would be necessary. Since the datum is inexact, the problem itself is not stated in exact terms; and yet we are asked to give an exact solution. This is one of the puzzling peculiarities of explication. It follows that, if a solution for a problem of explication is proposed, we cannot decide in an exact way whether it is right or wrong. Strictly speaking, the question whether the solution is right or wrong makes no good sense because there is no clear-cut answer. The question should rather be whether the proposed solution is satisfactory, whether it is more satisfactory than another one, and the like. (Carnap, 1950, §2)

In *Meaning and Necessity*, Carnap looks at proposals on how to interpret definite descriptions as put forward by Hilbert and Bernays, Russell, and Frege. Hilbert and Bernays propose using definite descriptions only after the uniqueness condition (that is, when there is a unique object satisfying the description) is provably satisfied. Frege proposes having 'the Φ' always stand for an object — an arbitrary one when uniqueness is not satisfied. Russell's theory is of course the well-known theory put forward in his 'On Denoting'. Concerning these three views, Carnap writes:

> The different interpretations of descriptions are not meant as assertions about the meaning of phrases of the form 'the so-and-so' in English, but as proposals for an interpretation and, consequently, for deductive rules, concerning descriptions in symbolic systems. Therefore, there is no theoretical issue of right or wrong between these various conceptions, but only the practical question of the convenience of the different methods. (Carnap, 1956, p. 33)

Here again we see Carnap expounding the same view on analysis. An analysis is meant as a proposal for replacing a certain notion. The proposal is then to be judged on pragmatic grounds. Questions of correctness are to be replaced by questions concerning suitability of the new notion for serving as a replacement of the old notion. We will now see that Quine had a very similar view of explication, but a view that differs from Carnap's in one crucial respect.

2 Quine on Philosophical Analysis

Quine acknowledges Carnap's account of explication in many of his most important works. Far from being critical of Carnap's views on explication, Quine repeatedly endorses them:

> There is also, however, a variant type of definitional activity which does not limit itself to the reporting of pre-existing synonymies. I have in mind what Carnap calls explication — an activity to which philosophers are given, and scientists also in their more philosophical moments. In explication the purpose is not merely to paraphrase the definiendum into an outright synonym, but actually to improve upon the definiendum by refining or supplementing its meaning. (Quine, 1953, p. 25)[1]

More evidence that Quine agreed with Carnap's views on explication is given in a referee report for *Logical Foundations of Probability* written by Quine. The report is published in Quine & Carnap (1990). On the review form, Quine is asked about the 'incidental achievements' of the book, and he writes "Clarifies nature of philosophical explication in general." (Quine & Carnap, 1990, p. 400)

The importance of explication is also stressed in *Word and Object* (see esp. §53). Both Carnap and Quine thought explication was an important philosophical task. However, they held different views on what constitutes a proper philosophical analysis. It is not that Carnap thought explication (including an explication of the conditions for scientific knowledge) was important and Quine did not. Rather, they both thought this was an important task but held different standards for successful explications. By examining the difference in their views on explication we can explain their disagreement over analyticity.

In §53 of *Word and Object*, Quine lays out the qualities that he believes a successful philosophical analysis should have. This is Quine's most detailed, but certainly not his only, discussion of the features a proper explication. His other comments are entirely in keeping with the view presented here. The account, as mentioned, is remarkably similar to that of Carnap (whom he acknowledges). After presenting his own views on philosophical analysis Quine writes "Philosophical analysis, explication, has not always been seen in this way", and then in a footnote he adds "by Carnap, yes, see *Meaning and Necessity* p. 7 f." However, there is an important difference between Carnap and Quine's views on explication that becomes apparent in §53 (and elsewhere). The section is titled 'The ordered pair as a philosophical paradigm'. As the title suggests, Quine believes that the various definitions of the ordered pair (e.g. as $\langle x, y\rangle = \{\{x\}, \{x, y\}\}$ or $\langle x, y\rangle = \{\{x\}, \{\emptyset, y\}\}$) satisfy, in an exemplary way, the conditions for a proper philosophical analysis. Concerning this he states:

> This construction is paradigmatic of what we are most typically up to when in a philosophical spirit we offer an analysis or explica-

[1] Quine does, however, claim that even this rests ultimately on synonymy. This remark is discussed later in this paper.

> tion of some hitherto inadequately formulated idea or expression. We do not claim synonymy. We do not claim to make clear and explicit what users of the language had in mind all along. We do not expose hidden meanings, as the words 'analysis' and 'explication' would suggest; we supply lacks. *We fix on the particular functions of the unclear expression that make it worth troubling about*, and then devise a substitute, clear and couched in terms of our liking, that fills those functions beyond those conditions of partial agreement, dictated by our interests and purposes, any traits of the explicans come under the head of 'don't cares'. (Quine, 1960, §53, my italics)

The ordered pair is a paradigm of philosophical analysis because the function that any analysis of 'ordered pair' must satisfy, can itself be made precise. This condition is that $\langle x, y \rangle = \langle w, z \rangle$ only if x=w and y=z.[2] All set theoretic definitions of 'ordered pair' satisfy this condition.[3] Therefore, on Quine's view, although different explications (e.g. $\langle x, y \rangle = \{\{x\}, \{x, y\}\}$ or $\langle x, y \rangle = \{\{x\}, \{\emptyset, y\}\}$) may contradict one another, they can both be correct analyses because they both satisfy the relevant condition. It is only by assuming synonymy between the original notions and the various clarifications that any contradiction arises. Explication, for Quine, is elimination. We eliminate the vague and poorly understood class of ordered pairs, and replace talk of ordered pairs with talk of classes of sets of the appropriate kind.[4] We may refer to this class of sets as the 'class of ordered pairs', but this is just for expedience and no claim to synonymy is made.

The similarities with Carnap's account should be apparent. Quine, as we saw, himself explicitly acknowledges the agreement. Both accounts reject the ordinary language account of philosophical analysis and insist that to analyze a concept is to replace the concept with another that is more precise and useful. We have a good deal of freedom in giving analyses and the choice between competing analyses is usually not a question of which is the correct one. There is, however, an important difference between their views on explication. Carnap's requirements for a proper explication will (in general) be satisfied, at least to a degree, by any explication, and therefore of use mainly for ordering explications according to which is better, rather than ruling any out. Quine, on the other hand, provides *a necessary condition* that any analysis must meet in order to be a proper explication. In order for an explication to be acceptable, on Quine's view, we must be able to identify some particularly important function of the explanandum that ought to be preserved by any analysis. On Carnap's view, although there should be similarity between the analyzed notion and the explanandum, we are not responsible for preserving any single aspect

[2]Since Quine uses the conditional instead of the biconditional, Quine must be thinking of $\langle x, y \rangle$ as a function of x and y. In this way the other direction of the biconditional becomes unnecessary.

[3]Quine also notes that ordered pairs need not be defined as sets at all. In arithmetic the ordered pair $\langle x, y \rangle$ can be defined as $2^x * 3^y$.

[4]On Quine's view explication is also elimination in the sense that in general we also eliminate a type of entity. For example when we define ordered pairs as sets we have eliminated the class of ordered pairs. The present essay will, however, not deal with ontological questions.

of the unanalyzed notion. So, though both Carnap and Quine have views on philosophical analysis that explicitly reject the ordinary language conception of analysis, there is an important difference between their accounts.

In fact, the rejection of the ordinary language conception of an explication (or analysis), as exposing hidden meanings, appears to be the main point of agreement between Carnap and Quine's views on analysis. The view that we replace an unclear notion with a clear one, is seen by both as an inevitable consequence of the paradox of analysis. However, Quine unlike Carnap takes explanation to involve the identification and preservation of important features of the explanandum.

> Showing how the useful purposes of some perplexing expression can be accomplished through new channels would seem to count as explication just in case the new channels parallel the old ones sufficiently for there to be a striking if partial parallelism of function between the old troublesome form of expression and some form of expression figuring in the new method. (Quine, 1960, §53)

It will be seen in the next section that this difference in their views on explication is sufficient to explain their positions on the question of analyticity.

3 Carnap, Quine, Analysis and Analyticity

As is well known, in 'Two Dogmas of Empiricism', Quine argues against proposals for an analysis (or explication) of 'analytic'. The article contains many arguments against such proposals, but only one of those arguments applies to Carnap's analysis. The argument in question states that Carnap has only provided an analysis of 'analytic in L_1' or 'analytic in L_2' rather than an analysis of 'analytic in X' where 'X' is a variable.[5] Quine argues that because of this lack, Carnap has no real justification for using the string of letters 'analytic in' in both predicates. Quine later acknowledges that he came to understand Carnap's motivation for using the phrase 'analytic in' for both predicates and that motivation is that they have the same explicandum. (Quine & Carnap (1990), 'Letter to Carnap, March 29, 1951')[6] Despite this situation, and despite Carnap's several attempts to explicate the notion of analyticity, Quine, throughout his philosophical career, continued to reject the notion of analyticity.[7]

[5]There is also a point about state-descriptions being insufficient to define the general notion of analyticity, but this is not an argument against Carnap, since, as Quine acknowledges, state-descriptions were never meant to play this role.

[6]Grice & Strawson (1956) tentatively attribute to Quine the view that there is no clear explicandum. They do not more than *tentatively* attribute this view to Quine, since they think it is highly implausible that our use of the term 'analytic' is not guided by some coherent informal notion. But Quine need not deny that there is a coherent notion to be explained. What he denies, as will be argued, is that we can identify an important feature of this notion that needs to be preserved by any explication. I think Grice and Strawson are right in giving prominence to the task of giving an analysis. However, without a detailed understanding of what Carnap and Quine each meant by giving an analysis (explication), the exact nature of the problem is unclear.

[7]Except in a very restricted sense which will be discussed below.

It is perhaps easy to see this debate between Carnap and Quine as being very one-sided. One could side with Carnap and hold that Carnap gave sufficiently clear — in fact, mathematically precise — definitions of analyticity, but that Quine quite dogmatically refused to accept any of them. On the other hand, one could see Quine as having made reasonable demands for 'behavioral criteria' that Carnap stubbornly never accepted and thus never met. There have been recently several attempts (Creath (2007), George (2000), and Stein (1992)) to give more neutral reading of this debate, such that neither side is being unreasonable in the face of the opposing position.[8] As valuable as these attempts are, I do not think they are sufficient in explaining how Carnap and Quine spent so much time talking past each other on this issue. To fully understand this debate, the notion of explication needs to be moved to the center, and the differences in their views on explication need to be taken into account. The debate, after all, concerns the question of whether analyticity can be clearly explicated. In what follows I will give a fully neutral account of the debate where the concept of explication plays a central role. Each side has his own view of what constitutes a successful explication and is unaware that this view differs from that of his opponent in the debate.

How then are we to see this debate as centrally concerning the notion of explication? Quine's famous attack on the notion of analyticity is, of course found in 'Two Dogmas ...'. In fact, 'Two Dogmas...' contains a brief discussion of explication:

> In explication the purpose is not merely to paraphrase the definiendum into an outright synonymy, but actually to improve upon the definiendum by refining or supplementing its meaning. But even explication, though not merely reporting a preëxisting synonymy between definiendum and definiens, does rest nevertheless on other pre-existing synonymies. The matter may be viewed as follows. Any word worth explicating has some contexts which, as wholes, are clear and precise enough to be useful; *and the purpose of explica-*

[8] I mentioned George's reading of the Carnap and Quine debate. I would like, now, to make a brief point about George on the analytic/synthetic distinction. George shows that Carnap can easily adopt a Carnapian attitude towards the A/S distinction. That is, Carnap can see it as a matter of choice whether we accept a system that includes the analytic/synthetic distinction. Quine, however, on George's view, is in a very difficult position. George dubs the rejection of the A/S distinction a 'double-edged sword'. Quine can neither hold that the existence of the A/S distinction is an empirical question — since he claims that there has been no empirical meaning given to the A/S distinction — nor can he claim with Carnap that it is a matter of choice. Quine's seems then to have cut his own legs out from beneath him, and fallen into incoherence. George then claims that Quine avoids incoherence by relying on a delicate balance between his somewhat obscure notion of 'linguacentrism' and his empiricism. George concludes that "[d]ebate about whether the dispute is empty or instead substantive is, for Quine, itself lacking content." (George, 2000, p. 22) Despite, George's best efforts, the position he saddles Quine with is, if not incoherent, dangerously close to incoherent. The problem I see with George's analysis is that he takes the participants to be addressing the question 'Is there an A/S distinction?' instead of the question 'Can *analyticity* be successfully analyzed?' As we will see, once the participants in the debate are seen as addressing the second question (as their writings on the subject suggest) the problems that George struggles with simply do not arise.

> *tion is to preserve the usage of these favored contexts* while sharpening the usage of other contexts. In order that a given definition be suitable for the purpose of explication, therefore, what is required is not that the definiendum in its antecedent usage be synonymous with the definiens, but just that each of these favored contexts of the definiendum, taken as a whole in its antecedent use, be synonymous with the corresponding context of the definiens. (Quine, 1953, p. 25, my italics)

Notice that with this talk of 'favored contexts', Quine is here expressing the same view on explication that he later spells out in more detain in *Word & Object*. Although Quine explicitly rejects the ordinary language conception of philosophical analysis, we are, on Quine's view, responsible for preserving some pre-identified aspects of ordinary usage. We can ignore any aspects of ordinary use that are of little or no practical utility. However, if we cannot isolate within ordinary usage valuable distinguishing properties, then any proposed analysis will be deemed an inadequate explication. So, Quine is here asking about the possibility of giving an *explication* of analyticity in exactly the sense explored in the previous section.

Now that we have considered in some detail their views on explication, let us investigate further how these positions are related to their views on analyticity. The problem Quine poses in 'Two Dogmas ...' is to define a notion of analyticity that is wider than logical truth. That is, the goal is to define analyticity such that it includes logical truths such as 'all unmarried men are unmarried', but also includes such truths as 'all bachelors are unmarried'. If you hold Carnap's views on explication, then what you seek is a precise, simple, fruitful notion that is sufficiently similar to the ordinary notion. Concerning giving an explication of analyticity, Carnap says:

> Our explication [...] will refer to semantic language systems, not to natural languages. It shares this character with most of the explications of philosophically important concepts given in modern logic, e.g., Tarski's explication of truth. It seems to me that the problems of explicating concepts of this kind for natural language are of an entirely different nature. (Carnap, 1956, pp. 222-3, Appendix B)

Since Carnap is merely trying to define, in the context of formal languages, a concept that is sufficiently similar to our ordinary notion of analyticity, he sees the task as trivially easy. All that is needed is to supplement the logical system with *meaning postulates*. Given the notion of logical truths, the wider class of sentences are all of the logical consequences of the meaning postulates.

We saw above an important difference between Carnap's view of explication and Quine's. Quine, like Carnap, sees explication as replacing a poorly understood concept with a clearer more precise one. However, for Quine we must begin by identifying a feature (or possibly features) of the ordinary notion that we wish to preserve. Judged according to Quine's criteria for successful explication, Carnap's analysis of analyticity in 'Meaning Postulates' is clearly inadequate. Interpreted as identifying a property of the term in ordinary language that is to be preserved it is simply silly. Thus Quine's sarcastic comment: "Semantic rules are distinguishable, apparently, only by the fact of appearing on a page under the heading 'Semantic Rules' ". (Quine, 1953, p. 33) Or:

> Carnap's present position is that one has specified a language quite rigorously only when one has fixed, by dint of so-called meaning postulates, what sentences are to count as analytic. The proponent is supposed to distinguish between those of his declarations which count as meaning postulates, and thus engender analyticity, and those that do not. This he does, presumably, by attaching the the label 'meaning postulate'. (Quine, 1963, p. 404)

What Quine is doing in this passage makes perfect sense in light of the present analysis. Quine takes it to be a criterion of successful analysis that it identify an important feature of the ordinary notion. This criterion is completely missing from Carnap's notion of explication. Carnap does not at all take himself to have identified a property of our ordinary notion of analyticity. I will call an explication that satisfies Quine's standards a *Quinean-explication* and one that satisfies Carnap's more liberal standards a *Carnapian-explication*. Both philosophers are addressing the question 'can we give a successful explication of the concept *analyticity*?' However, each interprets this question differently. Carnap interprets this question as asking if a successful Carnapian-explication can be given, and takes the answer to be a trivially obvious *yes*. Quine, does not see how to give a successful Quinean-explication of analyticity, sees Carnap as attempting to provide one, and thus sees Carnap as obviously failing. Quine and Carnap could not agree then on whether a successful explication of analyticity was possible, and therefore did quite a bit of talking past one another. This had to do more with the notion of explication than it did with the notion of analyticity.

It was shown above that Quine seems to have been completely unaware that his view of explication differs from Carnap's. Quine, as we saw, enthusiastically cites Carnap as first articulating the notion of explication that Quine wishes to defend. But it was not only Quine who failed to appreciate the difference between their views on explication. Carnap was completely unaware of this difference as well. In his 'Quine on Analyticity' (which remained unpublished until it was translated and included in Quine & Carnap (1990)), Carnap repeatedly accuses Quine of being unclear over whether he is talking about the explicandum or the explicatum. However, Quine is not confusing properties of the explicandum with those of the explicatum. Quine is looking for features of the ordinary notion which the explication *is meant to preserve*. Since this aspect is entirely missing from a Carapian-explication, Carnap can make little sense of Quine's demands. Consider the following passage:

> Later Quine says: "Semantic rules <determining the analytic statements of an artificial language are of interest only insofar as we already understand the notion of analyticity; they are of no help

> in gaining this> understanding" (op. cit., p. 34). This is the same obscurity again [confusing properties of the explicatum with those of the explicandum]. The answer is the same too: we have an understanding of the notion of analyticity, in practice clear enough for application in many cases, but not exact enough for certain cases or for theoretical purposes. The semantic rules give us an exact concept; we accept it as an explication if we find by comparison with the explicandum that it is sufficiently in accord with this. It seems to me that this demand is fulfilled for the two concepts under consideration here with respect to the simple, limited language systems treated thus far: (1) for the concept L-truth as an explication of logical truth in the narrow sense, (2) for the concept, based on meaning postulates, of L-truths as an explicatum for analyticity, truth in virtue of meaning (in the broad sense). (Quine & Carnap, 1990, p. 431 (the portion in angle brackets was added by Creath to Carnap's notes for the sake of the reader.))

Here we see Carnap showing some frustration. He takes there to be fairly clear cases where the explicandum applies (or not), and takes the notion he has outlined to be clear and precise and sufficiently similar in extension. He has thus shown that he has given a successful Carnapian-explication of the notion.[9] He does not see what more Quine is asking for. Quine of course need not disagree with any of this. Quine sees Carnap as having failed to identify which feature of the explicandum is preserved by the explicatum. That is, Quine seeks a Quinean-explication of this notion.

Quine's demands for a Quinean-explication of analyticity were often phrased in terms of giving behavioral criteria for analyticity. These behavioral criteria were supposed to play the role of linking a precise definition of analyticity with an important feature of the ordinary notion. Of course, on Carnap's view, no such link is required — all that is required is a certain similarity between the explicatum and explicandum. It is for this reason that Carnap says "I do not think that a semantical concept, in order to be fruitful, must necessarily have a pragmatical counterpart." (Carnap, 1956, p. 235, appendix D)

Carnap, as we saw, did not understand the nature of Quine's misgivings about the notion of analyticity. In particular, Carnap could not understand how Quine could reject his analysis of analyticity for certain formalized languages without also rejecting Tarski's analysis of truth. We are again, given the present interpretation in a perfect position to understand this. Quine can accept Tarski's analysis as a successful explication of truth because Tarski identifies the feature of our ordinary notion that the analysis is meant to preserve. That is, for any sentence 'P', 'P' is true if and only if P. This feature is preserved between the ordinary notion and the formalized version. On this ground Tarski's analysis can qualify as a successful Quinean-explication. It seems Carnap never really understood what exactly it was about his explications that failed to sat-

[9] Of course, he also thinks it can be fruitful. For instance if it could be shown that all mathematical truths are analytic.

isfy Quine. However, as we saw, Quine did not realize where their differences lay either.

4 Towards a Quinean-explication of *Analyticity*

In the last section it was argued that the disagreement, between Carnap and Quine, over whether a successful explication of analyticity is possible, had more to do with their views on explication than with the notion of analyticity. It may now be asked which (if either) of these views on explication we ought to accept. Carnap's account offers more freedom while giving an analysis. Conceivably, the fidelity to ordinary notions that is required by Quine's analysis may, in some cases, stand in the way of scientific progress. On the other hand, many of the most clearly successful analyses do meet Quine's more strict requirements. We have already seen how Tarski's analysis of truth satisfies Quine's criteria. The same can be said of Turing's analysis of a computation and Einstein's analysis of simultaneity. Both of these analyses begin by identifying important features involved in our ordinary applications of the concepts.

I have no desire to settle the issue of which (if either) is the best or proper view on what constitutes a successful explication. Instead, what I wish to do is to address the question of whether it is possible to give a Quinean-explication of analyticity. That is, is it possible to identify an important feature of our ordinary notion of *analytic* such that giving an explication that preserves this feature allows us to do those things that make it 'worth troubling about' analyticity in the first place?[10]

Let me be explicit in what I am doing here. My goal is not to argue that Quine could have, and therefore should have, defined the notion of analyticity. Quine is interested in outlining a philosophical view that does not include such notions as 'analytic'. My point is not at all that there is something illegitimate about this. After all, 'analyticity' is a term used by philosophers, it is neither part of our commonsense view of the world nor our scientific theories. However, Quine often, throughout his philosophical career, made the stronger claim that one cannot give a successful analysis of 'analyticity'. It is this stronger claim that I wish to address here. I will argue that given Quine believed Quinean-explications are possible, he cannot hold the stronger thesis.

Quine outlines a notion very close to our ordinary notion of analyticity in at least three places — *Word and Object*, *Roots of Reference*, and *Pursuit of Truth*.[11] In *Word and Object*, Quine outlines the concept of a *socialized stimulus-analytic* sentence. A sentence fits into this class if almost all speakers of the language would assent to it given any stimulus.

> But analyticity in this improved sense will apply as well to 'there have been black dogs' as to '2 + 2 = 4' and 'No bachelor is married.'

[10]The talk of identifying what it is about the notion that makes it *worth troubling about* comes from §53 of *Word and Object* (as quoted above).

[11]Dummett (1978) discusses the example of *Word & Object* and Creath (2007) discusses the remaining examples.

> Let us face it: our socialized stimulus-synonymy and stimulus-analyticity are not behavioral reconstructions of intuitive semantics, but only behavioristic ersatz. (Quine, 1960, §14)

Quine's complaint here is not that he has not defined this notion in behavioristic terms, nor is it that the notion as he defines it has *no relation* to the ordinary notion of analytic truth. To see what exactly the problem is here, consider this evidently related passage from 'Epistemology Naturalized':

> This attribute [community-wide acceptance] is of course no explication of analyticity. The community would agree that there have been black dogs, yet none who talk of analyticity would call this analytic. My rejection of the analyticity notion just means drawing no line between between what goes into the mere understanding of the sentences of a language and what else the community sees eye-to-eye on. I doubt that a distinction can be made between meaning and such collateral information as is community wide. (Quine, 1969, pp. 86)

In 'Two dogmas in retrospect' Quine clearly points out that he does not reject that there is a coherent notion of analyticity, as used by philosophers, that includes such sentences as 'No bachelor is married'. "Analyticity undeniably has a place at a common-sense level, and this has made readers regard my reservations as unreasonable." (Quine, 2008, p. 395) In response to this he mentions his definition of analyticity given in *Roots of Reference*:

> Here then we may at last have a line on the concept of analyticity: a sentence is analytic if *everybody* learns that it is true by learning its words. Analyticity, like observationality, hinges on social uniformity. (Quine, 1973, p.79 original italics)

However, he points out that this definition does not extend beyond a handful of clear cases:

> For the past five minutes I have been expressing a generous attitude toward analyticity. In fact my reservations over analyticity are the same as ever, and concern the tracing of any demarcation, even an approximate one, across the domain of sentences in general. The crude criterion of *Roots of Reference*, based on word learning, is no help; we don't in general know how we learned a word, nor what truths were learned in the process. Nor do we have any reason to expect any uniformity in this regard from speaker to speaker, and there is no reason to care. Elementary logic and the bachelor example are clear enough cases, but there is no going on from there. (Quine, 2008, p. 396)

So we see that Quine's own attempts to come up with a Quinean-explication of analyticity either included far too much or far less than more traditional accounts of analyticity. Community-wide acceptance is far too broad to count as an analysis of analyticity. This may be a feature of the ordinary notion, but it is insufficient to pin down the notion of analyticity. It is akin to identifying $\langle x, y\rangle = \langle w, z\rangle$ only if x=w as *all* that is needed to be preserved of the notion of ordered pair. Although this is true of ordered pairs, it is not yet sufficient to pin down what we mean beyond the 'don't cares'.

The analysis of analyticity in terms of what everyone learns is true by learning the component words is an acceptable explication of analyticity, on Quine's view. It identifies a feature of the ordinary notion that is to be preserved. It is just that this feature gives us very little. We get 'there are seven days in a week' and 'if *P* and *Q*, then *P*' as analytic, but not much more than these obvious examples of analytic truths. Of course, Quine is perfectly satisfied with this explication:

> In short, I recognize analyticity in its obvious and useful but epistemically insignificant applications. The needs that Carnap felt for this notion in connection with mathematical truth are better met through holism. (Quine, 2008, p. 397)

The purpose of this section was to tackle the question of whether a Quinean-explication of analyticity is possible. Quine's final answer to this question is essentially 'yes, but the notion turns out to be so elementary that it is not of much interest'. So we may now ask if we can do somewhat better than this. That is, can we identify a feature of sentences that we call analytic, and use this feature to define a class that is wider than Quine's 'epistemically uninteresting' class but not so wide so as to include everything that is widely believed? The goal is not to arrive at the ultimate and authoritative definition of *analytic*, but to show that there is at least one way to define analyticity that satisfies Quine's demands and can be shown to be of philosophical interest. Given what has been shown in the preceding sections, a particular account suggests itself. What feature of our the notion of *analyticity* should we seek to preserve? I suggest the following feature: analytic truths are what is identified in the process of giving an analysis. To make sure the definition cannot be rejected outright by Quine, the notion of analysis I will use is that of a Quinean-explication.

In §53 of *Word and Object*, Quine has essentially two points. First, there is an important and interesting activity that we refer to as giving an explication of some notion. Secondly, the various explications of the ordered pairs exemplify this perfectly. When examining how we use the term 'ordered pair', we notice that beyond satisfying the principle (that Quine calls (1)):

(1) $\langle x, y\rangle = \langle w, z\rangle$ only if x=w and y=z

any other features of ordered pair fall into the category of 'don't cares'. I suggest we build an account of analyticity around the notion of successful Quinean-explication. After all, these two ideas have been traditionally linked. Since Quine's own account of analyticity is, in his own estimation, of no particular interest, I will use the term *Quinean-analytic* to refer to the notion of analyticity suggested by Quine's account of successful analysis. The feature of the ordinary notion of analyticity we will preserve is that the analytic truths are those identified in the course of giving an analysis. Let me now be explicit about this:

> Quinean-Analyticity: A statement is Quinean-analytic if (and only if) it is either a statement identified as what must preserved by an acceptable Quinean-explication, or is a logical consequence of such statements.

I should now say something about the use of the word 'acceptable' in the above definition. Obviously, it would not do to count as analytic *anything* which *anyone* identifies as something that needs to preserve in giving an analysis of some notion. Some proposed Quinean-analyses will clearly be unacceptable. Quine himself certainly thinks so. After all, he denies that Carnap gave an acceptable analysis of analyticity. On the other hand, Quine focuses on the example of ordered pair, because he thinks in this case it is clear what any acceptable analysis must preserve. "The ordered pair has had illustrative value because of the crispness of requirement (1) and because of the multiplicity and the conspicuous artificiality of the explications. But what it illustrates as to the nature of philosophical explication applies more widely." (Quine, 1960, §53) Here, Quine speaks of this as involving a Wittgensteinean dissolution of the problem of whether the ordered pairs are *really* the Wiener or Kuratowski version. In this case we can clearly separate what is important (and thus needs to be preserved by any explication) and what is an artificial property of the explication. Quine takes the ordered pair to be a paradigm of philosophical analysis. What Quine called (1), therefore, is a paradigm example of a Quinean-analytic sentence in the sense just defined.

It might now be pointed out that, although there are clear cases where the concept applies and clear cases in which it does not, there may still be many borderline cases. But this only shows that analyticity, as so defined, is no more and no less vague that the notion of Quinean-explication. Quine could not, therefore, hold that Quinean-explication (what he called philosophical analysis) is important and philosophically illuminating, while analyticity is hopelessly unclear. Neither can Quine object by claiming that an explication ought to have no borderline cases at all. Certainly what everyone learns is true by learning the component words is not completely devoid of vagueness. Another objection might be that the 'must' in the definition adds further vagueness. This, of course, is not true, since giving an acceptable Quinean-explication already involves identifying what must be preserved about the ordinary notion being examined.

Carnap's definition of 'analytic' is certainly related to his account of explication. In fact, this may be seen as one of the most unsatisfying features of Carnap's philosophy. In terms of explications, as we saw, his attitude is that no explications are really incorrect, but are simply to be judged on pragmatic grounds. Likewise, any set of sentences (closed under logical consequence) is a perfectly acceptable candidate for the class of analytic truths and we are, again, to judge between these on pragmatic grounds. As I have argued elsewhere (Lavers, 2008), Carnap, when engaged in actually explicating a notion like classical mathematical truth, does not propose simply replacing our ordinary notion with some sufficiently similar copy. He does not advocate, for instance, replacing our ordinary notion of *truth of number theory* with the property of being a provable sentence in first-order Peano arithmetic. Instead, he seeks to defend the the notion of classical mathematical truth by showing that although it cannot be captured in terms of deducibility in some particular formal system, it can, nonetheless, be sufficiently clearly defined. Although there may not be a contradiction between his views on explication and his attempt to defend our ordinary notion of mathematical truth, it certainly seems that the view of analysis guiding him in this case is something other than an attitude of *anything goes and is to be judged on pragmatic grounds*. If this is right, then even those sympathetic to a Carnapian point of view would welcome a more interesting account of analysis (or explication) and analyticity.

Let us, therefore, return to the suggestion made above that we construct a notion of analyticity around what Quine considered to be a clear case of successful analysis. First, it should be noted that this differs from Quine's own attempts to define analyticity. On the present account '$\langle x, y\rangle = \langle w, z\rangle$ only if x=w and y=z' is the paradigmatic example of an analytic statement. But it is not true of this statement that it is universally acknowledged by all speakers of the English language. If one were to conduct a survey to determine what portion of people would assent to this statement, one is bound to get a variety of answers in response. So it does not satisfy Quine's first attempt at an analysis of analyticity — that of community-wide acceptance.[12] Nor does it satisfy Quine's final definition of analyticity. Remember, on this account a truth is analytic if everyone learns it is true by learning its words. Although anyone with a competent grasp of the notion of an ordered pair can recognize that this sentence is true, it is not the case (as a historical matter of fact) that for each of us we recognized the truth of this claim by learning the words. So, the above notion of a Quinean-analytic sentence is a notion that differs from the attempts at an explication of analyticity made by Quine himself. As we saw above, Quine himself declared his very narrow definition of analyticity as unsuitable to play any significant role in the foundations of mathematics. Of course, this is as Quine wished, since he wanted to present an alternative epistemological picture to that of Carnap. But in the next section we will explore the status of mathematics in relation to the notion of a Quinean-analytic sentence.

[12] Of course, if we could choose our community as we wish this could come out as accepted by the entire community. But the notion of Quinean-analyticity remains different from community-wide acceptance. If we select a small enough community a feature that is not Quinean-analytic but a 'don't care' might be accepted by the entire community.

5 Quinean-Analyticity and Mathematical Truth

The last section introduced the concept of a Quinean-analytic statement. It was shown that this notion is more substantial than Quine's own of 'epistemically insignificant' definition of analyticity. We now turn to the question of the status of mathematical axioms with respect to this newly introduced notion. Let us begin with arithmetic, and then later we will turn to set theory.[13] Quine is quite clear that we can identify what must be preserved by any analysis of the natural numbers:

> The condition upon all acceptable explications of number (that is, the natural numbers $0, 1, 2, \ldots$) can be put almost as succinctly as (1): any progression i.e., any infinite series each of whose members has only finitely many precursors — will do nicely. (Quine, 1960, §54)

Given that the above-defined notion of Quinean-analyticity applies to anything that is preserved by all explications, the Quinean-analytic truths of arithmetic are all those statements that are true of any such progression. What two such progressions may disagree over goes under the head 'don't cares'.

> One uses Frege's version or von Neumann's or yet another, such as Zermelo's, opportunistically to suit the job in hand, if the job is one that calls for providing a version of number at all. (Quine, 1960, §54)

One of the motivations for the account of analyticity provided in the last section is that it suggests itself in light of what was discussed earlier.[14] The other motivation is the connection with mathematical truth. Remember, after Quine presents his own 'epistemically insignificant' concept of analyticity he points out that it does not meet Carnap's needs in terms of mathematical truth. But here we have Quine saying what must be preserved by any analysis of the natural numbers can be made perfectly clear. That is, he is saying that the *truths of arithmetic*, what are true of all such progressions, are Quinean-analytic. So, by Quine's own lights, the truths of arithmetic count as Quinean-analytic.

One may now see a connection between what has just been discussed and Benacerraf's famous paper 'What numbers could not be'. In this paper Benacerraf discusses the case of Ernie and Johnny who have each had an unusual mathematical education. They each learned set theory before they learned

[13]Since these are the examples Quine himself discusses.

[14]I am not claiming that this is the only possible definition of analyticity. Others, for instance Juhl & Loomis (2010) or Russell (2008), have recently put forward their own definition of analyticity. I have no interest in arguing that the definition given here is better than theirs. My goal is merely to show that there is at least one interesting definition of analyticity that satisfies Quine's demands for a successful explication. Showing that it counts much of mathematics as analytic is the way I intend to show that the definition provided above is of interest. If the reader feels that I have given merely one interesting definition of analyticity among many possible, I will be sufficiently satisfied.

arithmetic. One was then taught arithmetic by being given a definition of the numbers as the Zermelo ordinals and a rule for associating numbers with the cardinality of sets. The other was taught to identify the numbers with the Von Neumann ordinals, and is similarly given a rule for associating numbers with the cardinality of sets. Benacerraf argues that each will be able to begin to derive truths of arithmetic. Of course:

> Comparing notes, they soon became aware that something was wrong, for a dispute immediately ensued about whether or not 3 belonged to 17. Ernie said that it did, Johnny that it did not. Attempts to settle this by asking ordinary folk (who had been dealing with numbers as numbers for a long time) understandably brought only blank stares. (Benacerraf, 1965, p. 54)

We see here that Quine and Benacerraf agree on quite a bit. They both agree that the important truths about the numbers are precisely what we have been calling the Quinean-analytic truths. But they diverge after this point. Quine takes it that what is left undecided by these privileged truths we may fill in as we wish — so long as we don't lose sight that these properties fall under the heading 'don't cares'. Benacerraf concludes that since there is no particular sequence singled out as the series of natural numbers, that the natural numbers are not any particular series at all.

Interestingly, Benacerraf, here, makes an explicit comparison with Quine's view from *Word and Object*. He points out two more differences between himself and Quine. Quine considers and rejects, as a requirement on an explication of number, that it ought to tie claims of pure arithmetic to our judgement of cardinality. Benacerraf disagrees with Quine on this: "It will then be from its place in that sequence that is, from its relation to other members of the sequence, and from the rules governing the use of the sequence in counting that it will derive its individuality. It is for this last reason that I urged, contra Quine, that the account of cardinality must explicitly be included in the account of number." [p. 72] Benacerraf also holds that the less-than relation must be recursive in order for a progression to count as the numbers. This is not a disagreement over *whether* it is the Quinean-analytic truths that are the important ones, but about *what counts* as Quinean-analytic.[15] These differences about what features ought to be preserved by any explication do not show the notion of Quinean-analyticity irremediably vague. It shows that we are perhaps fallible in respect to identifying Quinean-analytic truths but there is no reason to abandon hope that such disputes could be settled with debate. Let us leave aside, then, these specific differences of what all accounts of number must have in common and return to the more global features of their views.

As close as their positions are, I think there is good reason, still, to prefer Quine's way of viewing things. For Quine the important thing is to identify those features that are preserved by any acceptable analysis and to differentiate those things from the 'don't cares'. While Benacerraf is largely in agreement

[15]Of course, they do not use the term Quinean-analytic, but are arguing about what features any specific account of number must have. This is exactly what we have been calling Quinean-analytic.

with this, he wants to make the further conclusion that numbers are not any particular progression, but that the properties of the numbers are what all such progressions have in common. This of course brings with it a certain well-known incoherence. 'Numbers are not any particular progression' is *certainly not true* no matter what progression we use to represent the numbers — in fact, it is false on all of them. Such difficulties may not be insurmountable, but I think Quine's view is able to satisfy many of our structuralist instincts without even skirting incoherence. Instead of saying 'Numbers are not any particular progression, but what is true of number is what is true of all such progressions' we say 'that the numbers are progression X is not Quinean-analytic, but what is true of all progressions is just what is Quinean-analytic of the notion of number.'

Quine held that, in general, a specific progression proposed as an explication of our concept of number will consist of sets. Quine also generally views explication as a kind of ontological reduction. As mentioned before questions of ontology will not be dealt with in the current paper. It is nonetheless important, now, to turn our attention to sets in order to identify what might be called Quinean-analytic in this case. Sets (or classes) are certainly an important case in Quine's view:

> Classes can do the work of ordered pairs and therefore relations (§53), and they can do the work of the natural numbers (§54). They can do the work of richer sorts of number too — rational, real, complex; for these can be variously explicated on the basis of natural numbers by suitable constructions of classes and relations. Numerical functions, in turn, can be explicated as relations of numbers. All the universe of classes leaves no further objects to be desired for the whole of classical mathematics.(Quine, 1960, §55)

What pushes strongly in favour of nominalism, on Quine's view, is the lack of naturalness in the conception of set. As Quine sees things, all of the various set theories that exist today restrict comprehension in an unnatural way in order to avoid the paradoxes. On Quine's view, then, all such systems, serve as practical tools, but because of their unnaturalness are not explications of some pre-existing notion. This view on the history of set theory, which sees Zermelo's axioms, for instance, as making arbitrary restrictions on comprehension to avoid paradox, has today fallen out of favour. It is certainly now not taken for granted that there is one natural (although inconsistent) notion of set, Frege and Russell's, and then arbitrary and *ad hoc* ones. Many philosophers of mathematics now recognize that, for instance, Cantor and Zermelo did not share the same notion of set as Frege and Russell. It has furthermore been argued that Zermelo's axioms are obtained by an analysis of Cantor's notion of a set. I will not make this case here, but simply refer the reader to, for instance, Kreisel (1967) and Hallett (1984). If this claim, that the Zermelo axioms are obtained by an analysis of a certain conception of set, can be defended, then, as a consequence, these axioms (and their consequences) become Quinean-analytic of an important conception of sets.

So far, things have turned out quite happily for us. We seem to get, as coming out analytic, exactly those sentences we would want. There is of course some bad news. We are not, given our postponing ontological questions for another occasion, able to claim that all Quinean-analytic statements are true. I think this view can be defended and have argued for it in my (Lavers, 2009).[16] Regardless of whether all Quinean-analytic claims come out true, I believe the notion of a Quinean-analytic truth is more interesting than Quine's own attempts to define analyticity.[17]

There is one last point that needs to be made about the notion of Quinean-analyticity. Recall Quine's criticism of community-wide acceptance as an analysis of analyticity. Here he thinks the analysis fails because 'there have been black dogs' would come out analytic. One could at least conceive of someone giving an analysis of some notion that includes a quite clearly empirical claim. We might, to use Quine's own example, imagine an explication of the concept of 'dog' that saw it as an important feature of this concept that there have been black dogs. However, it was mentioned above, in connection with Benacerraf's criticism of Quine, that we are of course not infallible in identifying the Quinean-analytic features of some concept. There is always room for debate. In this case, it might be shown, for example, that an account that does not include the claim that there have been black dogs does just as well in explicating how we usefully employ the term 'dog'.[18] We could then eliminate the empirical claim as an inessential part of our explication. It seems to me that this will always be possible, whenever an explication includes an empirical claim, but I will not provide a detailed argument here. However, the mere possibility that what we identify as a Quinean-analytic claim might include empirical knowledge, only shows that what we have (fallibly) identified as Quinean-analytic is not guaranteed to be *a priori*. As we saw, Quine himself saw important truths of arithmetic as having the feature we are calling 'Quinean-analytic'. We may have less confidence in the *a priori* status of arithmetical truths following from their Quinean-analyticity than if we provided a gapless derivation from purely logical laws. In this case, we may have reason to believe the laws of arithmetic are *a priori* even if we have not rigorously demonstrated them to be so.

Conclusions

We saw above that Carnap had a more liberal notion of explication than Quine. Quine's view of explication (a Quinean-explication) holds that in providing an explication the goal is to identify important features of the explicandum

[16] I don't use the terminology of 'Quinean-analytic' in this article, but I do argue that what we identify by analyzing our informal mathematical notions is true.

[17] Of course, Quine wanted to put forward an epistemic view that did not include the notion of analyticity. It is certainly not being claimed that Quine himself would be completely satisfied with the views presented in this paper.

[18] That is we might give an account according to which this sentence is not analytic — not an account according to which it is not true.

that ought to be preserved. There is no parallel in the notion of a Carnapian-explication. We saw that neither Carnap, nor Quine was aware of this difference between their views on explications. Thus when they were addressing the question, of whether the concept of analyticity can be successfully explicated, they each interpreted the question as involving their own views on explication. It is for this reason that neither side in the debate could properly understand the other. We then saw that an interesting notion of analyticity (Quinean-analyticity) can be constructed out of the notion of a Quinean-explication. According to Quine's own words, arithmetic has the feature we have called Quinean-analytic. Quine did not think that, for instance, the Zermelo axioms were arrived at by analyzing an already existing conception of set but simply by *ad hoc* stipulation. Subsequent work in the history and philosophy of set theory has made Quine's position here seem very dubious. Thus, it is likely defensible that all the recognized truths of classical mathematics have the property we have called Quinean-analytic. Of course, we closed by a discussion of the remaining problems for the notion of Quinean-analyticity. Those problems are that we cannot be sure that a Quinean-analytic claim is true, nor that it is *a priori* if it is true. I believe that both of these problems can be addressed. The first involves getting into problems of ontology that were bracketed from the present discussion. The second would involve addressing the question of whether there is a Quinean-explication that ineliminably involves empirical claims. But these two further questions will be left to be addressed elsewhere.

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