Ultimate Bearing Capacity of Shallow Foundations on Layered Soils

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ABSTRACT

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The ultimate bearing capacity of shallow foundations subjected to axial vertical loads and resting on soil consisting of two layers has been investigated for the case of strong cohesionless soil overlying weak deposit. In the literature, several theories can be found using simplified failure mechanisms together with a reduced level of the shear strength mobilization on the assumed punching shear zone. It can be reported that large discrepancies between the measured and the predicted values of the ultimate bearing capacities were observed.

In this thesis, stress analysis was performed on the actual failure planes observed in the laboratory. In this analysis, full mobilization of the shear strength on the failure planes was considered. New bearing capacity equation was derived as a function of the properties of the upper and lower soil layers, the thickness of the upper layer, the footing depth/width ratio and the angle of the failure surfaces with respect to the vertical. The available experimental data in the literature were used to validate the proposed theory. A design procedure is presented for practicing use.

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α = Angle between the vertical and the assumed failure surface.

γ = Unit weight of the soil.

φ = Angle of shearing resistance.

C = Cohesion.

δ = The mobilized angle of shearing resistance on the assumed failure planes.

Pp = The total passive earth pressure.

B = Width of the footing.

D = Depth of the footing in the upper soil layer.

H = Thickness of the upper soil layer below the footing's base.

Z = Depth of the slice in the upper soil layer.

σ_{zz} = Vertical stress acting on the slice.

W = Weight of the slice.

Kp = Coefficient of passive earth pressure given by Caquot and Kerisel (1949).

q_u = Ultimate bearing capacity.

q_b= Ultimate bearing capacity of a strip footing on a very thick bed of the lower layer.

q_1 = Ultimate bearing capacity of a strip footing on a very thick bed of layer 1.

q_2 = Ultimate bearing capacity of a strip footing on a very thick bed of layer 2.

$N_{\gamma 2}, N_{q2}, N_{C2}$ = Bearing capacity factors corresponding to plane-strain angle of shearing resistance ϕ_2 of the lower layer.

F, an assumed parameter for simplification =

$$\ln(B + 2H\tan\alpha) - \ln B = \ln\left[\frac{B + 2H\tan\alpha}{B}\right].$$

ρ = A defined term function of H/B and D/B = $\rho = \lambda*(H/B) + \theta$.

λ = A defined parameter having the values in Table 3.9.

θ = A defined parameter having the values in Table 3.9.

μ = A defined term = $b = 45 + \frac{\phi_1}{2}$.

CHAPTER ONE
INTRODUCTION

Foundation design consists of two distinct parts: the ultimate bearing capacity of the soil under the foundation, and the tolerable settlement that the footing can undergo without affecting the superstructure. The ultimate bearing capacity aims at determining the load that the soil under the foundation can handle before shear failure; while, the calculation of the settlement caused by the superstructure should not exceed the limits of the allowed deformation for stability, function and aspects of construction.

Research on the ultimate bearing capacity problems can be carried out using either analytical solutions or experimental investigations. The former could be studied through theory of plasticity or finite element analysis, while the latter is achieved through conducting prototype, model and full-scale tests. A satisfactory solution is found only when theoretical results agree with those obtained experimentally.

A literature survey on the subject shows that the majority of the bearing capacity theories involve homogeneous soils under the foundations. Soil properties were assumed to remain constant for the bearing capacity analysis, and therefore analytical solutions, like Terzaghi's bearing capacity theory, matched with the experimental results. However, in cases where the soil properties vary with depth, most of these theories cannot be implemented, and the analytical solutions that take into consideration the non-homogeneity of the soils are approximations, and hence the results are inaccurate.

Layered soil profiles are often encountered whether naturally deposited or artificially made. Within each layer, the soil may be considered as homogeneous. The ultimate load failure surface in the soil depends on the shear strength parameters of the soil layers such as; the thickness of the upper layer; the shape, size and embedment of footing; and the ratio of the thickness of the upper layer to the width of the footing. Therefore, it is important to determine the soil profile and to calculate the bearing capacity accordingly.

In recent years, approximate solutions for the bearing capacity of shallow foundations on layered soil have been presented for a number of commonly encountered non-homogeneous soil profiles. Two cases of layered soil profiles have been considered: a strong layer overlying a weak layer, and vice versa. This thesis will focus on the case of a dense sand layer overlying a weak deposit of cohesive or non-cohesive soil.

The assumption involved in predicting the theoretical ultimate bearing capacity is that, at the ultimate load a soil mass in the upper sand layer, roughly a truncated pyramid in shape is pushed into the lower soil layer. At the ultimate load, the punching shear failure mechanism developed in the upper sand layer has a parabolic shape and a Prandtl-type failure in the lower layer. This parabolic shear failure surface makes the theoretical calculations very complex. To avoid this complexity, investigators assumed that the punching shear failure surface consists of two vertical planes at the edges of the footings.

In this thesis, a new punching shear failure mechanism is considered. It suggests that the failure surface consists of two inclined planes having an angle α with the vertical and passing through the edges of the footing with a Prandtl-type failure in the lower soil layer. The inclined planes represent an accurate approximation of the parabolic surfaces.

The objectives of this research are:

1. To develop a rational theory for the ultimate bearing capacity of strong cohesionless layer overlying deep natural deposit using the actual failure plane.
2. To develop design procedure to be recommended for practicing use.

CHAPTER TWO
LITERATURE REVIEW

2.1 General

In the literature, over the last four decades, several reports can be found dealing with the problem of foundations resting on layered soils. At first, researchers based their studies on the results of prototype laboratory model testing in order to develop empirical formulae to predict the ultimate bearing capacity of these footings. Recently, theories based on finite element and numerical analyses were presented that gave more rational solutions as compared to the previous ones. In this chapter a brief review of some of these reports followed by discussions are presented in chronological order.

2.2 Historical review

Button, 1953 (5) analyzed the bearing capacity of a strip footing resting on two layers of clay. He assumed that the cohesive soils in both layers are consolidated approximately to the same degree. In order to determine the ultimate bearing capacity of the foundation, he assumed that the failure surface at the ultimate load is cylindrical, where the curve lies at the edge of the footing. The bearing capacity factor used depends on the upper soil layer and on the ratio of the cohesions of the lower/upper clay layers.

Reddy and Srinivasan, 1967 (25) extended the work of Button to include the effect of the non-homogeneity and anisotropy of soil with respect to the shear strength. The basic assumptions involved in determining the ultimate bearing capacity are: the failure surface is cylindrical, the coefficient of anisotropy is the same at all points in the foundation medium, the soil in each layer is either homogeneous with respect to the shear strength or the shear strength in each layer varies linearly with depth and for the soil at failure, the $\phi = 0$ condition is valid.

In both papers, the assumption of cylindrical potential failure surface led to values of Nc 7% higher than the values obtained by the Prandtl solution in the case of homogeneous subsoil. In the case of an-isotropic and non-homogeneous subsoil, the values are even higher and the error increases with increasing non-homogeneity of the two layers.

Brown and Meyerhof, 1969 (2) investigated foundations resting on a stiff clay layer overlying a soft clay layer deposit, and the case of a soft layer overlying a stiff layer. They assumed that the footing fails by punching through the top layer for the first case, and with full development of the bearing capacity of the lower layer in the second case. Equations and charts giving the appropriate modified bearing capacity factors were given, derived from the empirical relationships obtained based on the experimental results. The results of the investigation are summarized in charts, which may be used in evaluating the bearing capacity of layered clay foundations, but these results are essentially experimental, and therefore are strongly affected by the characteristics of the clay tested.

The purpose of this paper is to present the results of a series of model footing tests carried out on two-layered clay soils, and the models have many limitations. First, they are limited to one type of clay, although the strength of the clay was varied, the deformation properties remained constant. Second, studies were limited to surface loading only, using rigid strip and circular footings with rough bases. Third, all studies were made in terms of the undrained shear strength of the clay, using the "$\phi = 0$" analyses.

They also conducted a series of tests on footings in homogeneous clay. They observed that the pattern of failure beneath a footing is a function of the physical mode of rupture of the clay, which is strongly dependent on the structure of the clay. The failure mechanism of the structure of the clay is not adequately defined by conventional Mohr-coulomb concepts of cohesion and friction.

Meyerhof, 1974 (19) investigated the case of sand layer overlying clay: dense sand on soft clay and loose sand on stiff clay. The analyses of different modes of failure were compared with the results of model test results on circular and strip footings and field data.

In the case of dense sand overlying a soft clay deposit, the failure mechanism was assumed as an approximately truncated pyramidal shape, pushed into the clay so that, in the case of general shear failure, the friction angle ϕ of the sand and the undrained cohesion c of the clay are mobilized in the combined failure zones. Based on this theory, semi-empirical formulae were developed to calculate the bearing capacity of strip, and circular footings resting on dense sand overlying soft clay. He conducted model tests on strip and circular footings on the surface and at shallow depths in the dense sand layer overlying clay. The results of these tests, and the field observations were found to agree with the theory developed.

In the case of loose sand on stiff clay, the sand mass beneath the footing failed laterally by squeezing at an ultimate load. Formulae for the ultimate bearing capacity of strip and circular footings were developed. Model tests were carried out on strip and circular footings, and the results also agreed with the theory developed.

Theory and test results showed that the influence of the sand layer thickness beneath the footing depends mainly on the bearing capacity ratio of the clay to the sand, the friction angle ϕ of the sand, the shape and depth of the foundation.

This paper is limited to vertically loaded footings, and does not include eccentric or inclined loads, it is also limited to sand over clay, and has no solution for clay over sand. In the case of dense sand on soft clay, the theory considers simultaneous failure of the sand layer by punching, and general shear failure in the clay layer, which is not always the case.

Meyerhof and Hanna, 1978 (20) considered the case of footings resting in a strong layer overlying weak deposit and a weak layer overlying strong deposit. The analyses of different soil failure were compared with the results of model tests on circular and strip footings on layered sand and clay. They developed theories to predict the bearing capacity of layered soils under vertical load and inclined loading conditions.

In the case of a strong layer overlying a weak deposit, considering the failure as an inverted uplift problem, an approximate theory of the ultimate bearing capacity was developed, At failure, a soil mass, roughly shaped like a truncated pyramid, of the upper layer is pushed into the underlying deposit in the approximate direction of the applied load. The forces developed on the actual punching failure surface in the upper layer are the total adhesion force and a total passive earth pressure inclined at an average angle δ acting upwards on an assumed plane inclined at an angle α to the vertical. The analysis for strip footings was extended to circular and rectangular footings, and approximate formulae for the bearing capacity of strip, rectangular, and circular footings were developed, taking into consideration the case of eccentric and inclined loading as well. Model tests on rough strip and circular footings under central inclined loads at varying angles α were made on the surface and at shallow depth in different cases of two layered soils of sand and clay, where good agreement was found between the theoretical and experimental results

In the case of a weak layer overlying a strong deposit, considering that the weak soil mass beneath the footing may fail laterally by squeezing, which is the same theory as from the previous paper developed the theory of the ultimate bearing capacity. The bearing capacity can be estimated by the approximate semi-empirical formula. Model tests were also carried out on strip and circular footings under vertical and inclined loads, and the results of the tests were compared to the theoretical ones.

The authors concluded that the ultimate bearing capacity of footings on a dense layer overlying a weak layer can be expressed by inclination factors in conjunction with punching shear coefficients, which depend on the shear strength parameters and bearing

capacity ratio of the layers under vertical loads. Formulae and design charts were developed and introduced in this paper.

This paper is a development of the previous theory (Meyerhof 1974), taking into consideration all possible cases of two different layers of subsoil, and also including the effect of inclined and eccentric loading on the ultimate bearing capacity of strip, rectangular, and circular footings. This theory and the failure mechanism considered are approximations of the real failure mechanism, which depends on many factors.

Hanna and Meyerhof, 1979 (10) extended their previous theory of the ultimate bearing capacity of two-layer soils to the case of three-layer soils. The analysis compared well with the results of model tests of strip and circular footings on a three-layer soil. Only one case was considered in this paper, that for footings subjected to vertical loads and resting on a subsoil consisting of two strong layers overlying a weak deposit.

The same theoretical failure mechanism was assumed by considering a soil mass of the upper two layers is pushed into the lower layer, and the same forces acting on the failure surface was assumed as well. Formulas and charts were developed and can be used in designing foundations having the same conditions. Model tests on rough strip and circular footings under central vertical loads were made on the surface of three-layer sand consisting of two dense upper layers and a loose lower one. By comparing the results of the model tests with the results of the punching theory, good agreement was found. Briefly, this paper is an extension of the previous theory in order to include the case of the three-layer soil. But, it is restricted to only one case of three-layer soil, and it needs more development to include all possible cases of three-layer soils.

Pfeifle and Das, 1979 (23) presented laboratory model tests results for the case of rough rectangular footings in sand with a rigid rough base located at a limited depth. The results were compared to the predicted results of Mandel and Salencon (1972) and Meyerhof

(1974). The authors concluded that the critical depth of location of the rough rigid base beyond which it has no effect on the value of the ultimate bearing capacity is about 50%-75% higher than that predicted by the theory. And the previous theories do not predict correctly the bearing capacity for the case when the rigid base is located at shallow depth. This experimental investigation is very limited to one case of layered soils, and the friction angle ϕ of the sand used varies in a small range (42°-45°), and the conclusion may be valid only for this range of ϕ.

Hanna and Meyerhof, 1980 (11) extended the previous theory to cover the case of footings resting on subsoil consisting of a dense layer of sand overlying a soft clay deposit, and they presented the results of this analysis in the form of design charts. It is a kind of revision of the assumptions previously used in the punching theory of the previous papers in order to reduce their effects on the analysis.

Thus, in the previous theory of punching, the bearing capacity formula depends to a large extent on the value of the average mobilized angle of shearing resistance δ on the assumed failure planes, which was previously estimated as $2\phi/3$. The exact value of δ is difficult to calculate, and approximate value will affect the results of the calculation of the bearing capacity. However, these difficulties may be overcome by expressing the angle δ in the dimensionless form ($\delta/\phi 1$), where $\phi 1$ is the friction angle of the upper sand layer. The design charts presented in this paper, together with the punching theory previously developed by the authors, can be utilized to predict the ultimate bearing capacity of footings on a dense sand layer overlying a soft clay deposit. These charts give more accurate results than the previous formulas and charts, but still estimation of the real bearing capacity since they are based on experimental results, the properties of sand and clay and the method used to model the tests can affect the charts and therefore the results.

Hanna and Meyerhof, 1981 (12) investigated experimentally the ultimate bearing capacity of footings subjected to axially inclined loads by conducting tests on model strip and circular footings on homogeneous sand and clay. The results were analyzed to determine the inclination factors, depth factors and the shape factors incorporated in the general bearing capacity equation for shallow foundations. These values were compared with the recommended values given in the *Canadian Foundation Engineering Manual.* The values of these factors given in the manual agree reasonably well with the experimental ones, except for the depth and shape factors, for which the theoretical values are on the conservative side when applied to inclined loads.

Hanna, 1981 (13) extended his previous theory to cover the case of footings resting on subsoil consisting of a strong sand layer overlying a weak sand deposit. Applying the same theory that at ultimate load, a soil mass of the upper layer is pushed to the lower sand layer, and by calculating the forces on the assumed vertical punching failure surface, the ultimate bearing capacity can be calculated theoretically. Charts are presented in this paper and can be used in the design of footings. In order to verify the theory presented, model tests on strip and circular footings resting on a dense sand layer overlying loose sand layer were done, and the results of the tests agreed well with the theory presented.

Hanna, 1981 (15) conducted an experimental investigation on the ultimate bearing capacity of strip and circular model footings on a two-layered soil in order to verify the validity of the empirical method proposed by Satyanarayana and Grag (1980) to predict numerically the ultimate bearing capacity of footings on layered soils. Summary of the results was presented in the form of comparative charts in order to compare the experimental and the theoretical results. The author concluded that by extensive comparisons between the observed ultimate bearing capacity values and those calculated by the method reveal discrepancies ranging between 70% and 85%. Thus, the method needs more refinement and further investigation before it can be recommended for practical applications.

Hanna, 1982 (14) investigated the case of footings resting on subsoil consisting of a weak sand layer overlying a dense sand deposit. Based on model tests of strip and circular footings, the author extended the classical equation of bearing capacity to cover cases of these footings in layered sand; consisting of weak sand layer overlying a dense sand deposit. In order to calculate the ultimate bearing capacity of these footings, the author proposed to used the classical equation of homogeneous sand in conjunction with the modified bearing capacity factors. These factors depend on the relative strength of the upper and lower layers and the thickness of the upper weak sand layer, and are calculated from the model tests results conducted on similar soil profiles. Design charts were presented as an aid in design.

According to the theory presented in this paper, the failure mechanism of the upper layer is the same as if the footing was in an homogeneous deep sand layer, and the influence of the layered soil is restricted to the difference in the bearing capacity factors, which were calculated experimentally from model tests. In my opinion, it is a simple method to overcome the complexity of finding the real failure mechanism, and it gives fairly accurate results. But the values of the bearing capacity factors depend on the kind of sand used in the tests, and they may change by using different kind of sands taken from different places.

Georgiadis and Michalopoulos, 1985 (9) presented a numerical method for evaluating the bearing capacity of shallow foundations on layered soil, which may contain any combination of cohesive and non-cohesive layers. Several potential failure surfaces were analyzed and the minimum material factor for which the foundation is stable was determined. Comparisons between the results obtained with this method, a number of semi-empirical solutions for homogeneous and two-layers soil profiles, experiments and other numerical methods including finite elements, demonstrated the validity of the proposed method.

Semi-empirical methods for the evaluation of the bearing capacity of shallow foundations on two-layer systems are primarily based on the results of experimental investigations. Most of them are restricted to a number of limited cases, and cannot cover any layered soil profiles; moreover, the bearing capacity computed with the various semi-empirical formulas are usually scattered. In the case of more than two layers in a soil profile, the bearing capacity can be computed with a finite element analysis or numerical analysis.

Das, 1988 (8) presented a technique to improve the ultimate bearing capacity and settlement conditions of shallow foundations on soft clay soil, which consists of placing the footings over a compact granular fill laid over the clay layer. Placing geotextile at the interface of the clay layer and the sand layer can further increase the bearing capacity. The purpose of placing the granular layer is to distribute the load on a larger area of the clay layer, and the purpose of placing the geotextile mesh is to reduce the depth of the sand layer required to distribute the load. The objective of this research was primarily to present the results of model tests conducted on a strip foundation resting on a sand layer overlying a weak clay layer, and compares the results with the theory of Meyerhof and Hanna (1978). Secondly, to compare results of the bearing capacity of footings on layered soil with and without the use of the geotextile mesh at the interface of the two layers in order to evaluate any advantage derived from the inclusion of the geotextile.

A number of laboratory model test results for the ultimate bearing capacity of strip footings resting on a sand layer underlain by a weak clay layer with and without the inclusion of geotextile at the interface of the two layers have been presented in this paper. Based on the experimental results; first, without the inclusion of geotextile, the results were consistent with the theory of Meyerhof and Hanna (1978). Second, the inclusion of the geotxtile at the interface of the layers increase the bearing capacity, and at the same time, reduce the depth of the sand layer to be placed over the clay layer. Third, the most economical width of the geotextile layer to be used as determined from the study is about four times the width of the strip footing.

This paper is experimental and the conclusions deduced are strictly related to the model tests done, so the results may vary with the type of geotextile mesh used, its strength, dimensions, and the depth at which the geotextile is placed. More investigation and experiments are needed regarding the use of geotextile for increasing the bearing capacity of shallow foundations on weak soils.

Oda and Win, 1990 (21) investigated the ultimate bearing capacity of footings on a sand layer overlying a clay layer in order to study its influence on the ultimate bearing capacity of footings. Twelve tests were carried out on sand beds with an interstratified clay layer. For this purpose, the thickness and the depth of the clay layer were variables. It was found experimentally that the clay layer reduces the bearing capacity of the footing even at a depth five times greater than the width of the footing. Thus, the author concluded that the plastic flow, which occurs in the lateral direction in a clay layer, exerts drag force on an upper sand layer, and this drag force results in the loss of bearing capacity.

It is obvious and experimentally proven that the presence of a thin clay layer even at a great depth reduces the bearing capacity of footings resting on granular soil.

Madhav and Sharma, 1991 (17) derived the ultimate bearing capacity of footings resting on a sand layer over a soft clay layer using a punching shear mechanism (Meyerhof 1974; Meyerhof and Hanna 1978). This theory implies that the net ultimate bearing capacity of the clay layer is independent of its location. It is the same whether it is at the surface or beneath the sand layer. Thus, the effect of vertical stress at the interface on the ultimate bearing capacity of the lower clay layer has been neglected. This paper attempts to include the effect of load distribution at the interface on the ultimate bearing capacity of the underlying soft clay.

The vertical stresses due to the upper sand layer have been replaced by distributed loads on the interface of the two layers, three cases of load distribution are considered in this paper: uniform load with linear decrease, uniform load with exponential decrease, and triangular loading. Based on this theory, the bearing capacity of footings with variable surcharge is found to increase by about 20%-30% in case the surcharge stress extends to a distance five times the footing width. A similar increase is also found in the case of a triangular load as well.

Abdel-Baki et al., 1993 (1) investigated the effect of a single strong reinforcement layer, placed within granular soil, on the bearing capacity of footings subjected to eccentric, inclined and concentric loads. The effect of the reinforcement on the bearing capacity was investigated experimentally.

The results of the experimental tests proved that the reinforcement had a considerable effect on the bearing capacity of the footings. The bearing capacity of a reinforced layer was about three times the unreinforced one. The effect of the length of the reinforcement was also investigated, and it was found that there is no significant effect on the bearing capacity if the reinforcement length is extended over 1.25 times the width of the footing. The effect of reinforcement continuity below the footing was also investigated in this paper, and it was found that the bearing capacity from the reinforcement decreased as the gap in the reinforcement increased and reached zero when the gap width was equal to the footing width.

Radoslaw, Michalowski and Shi, 1995 (24) used the kinematics approach of limit analysis to calculate the average limit pressure under footings in order to find the bearing capacity of footings resting on two-layer soil. This method is applicable to any combination of parameters of the two layers, however the results presented in this paper were limited to the case when a footing is placed on a layer of granular soil resting on clay.

The theory presented yields the upper bound to the true limit loads. Once reasonable collapse mechanisms are considered, the theory yields the failure load that is very close to the true collapse loads of elastic-perfectly plastic bodies. The upper-bound theorem states that the rate of energy dissipation is larger than or equal to the rate of work done by external forces in any kinematically admissible mechanism. Thus, if the material properties and geometry of a collapse mechanism are known, one can find an upper bound to the true limit load through equating the rate of work of external forces to the rate of internal energy dissipation. The collapse mechanism is constructed in such a way that the velocity discontinuities at the edge of the footing are bent at the interface between the layers. The angle at which they bend is equal to the difference in the internal friction angle of the soils in the two layers. The depth of the collapse mechanism was found to be dependent on the strength of the clay layer.

The results are presented as limit pressures and not as bearing capacity factors, and they are strongly dependent on the internal friction angle of the sand, the thickness of the sand layer, cohesion of the clay, and the surcharge pressure. Results are presented in the form of dimensionless charts for different internal friction angles of sand. Comparison charts between the results obtained by using this method to the results calculated by the method of Meyerhof and Hanna (1980) are also presented in this paper. Results of this investigation were also compared to the results obtained by Griffith (1982).

Burd and Frydman, 1996 (3) presented an admirable discussion of the analysis of the bearing capacity of layered soils. The purpose of this discussion is twofold. First, a comparison is given between the results of independent numerical analyses performed by the discussers' and the authors' design charts. Second, the ideas presented in the paper are used as the basis for a discussion of the analytical framework proposed by Hanna and Meyerhof (1980) for this type of problem.

A numerical study of the bearing capacity of thin sand layers overlying soft clay deposits has been considered using two separate approaches. One set of results was

obtained using a finite-element method; the other set was obtained using the finite-difference code FLAC. By comparing the results of the two methods, the authors found that, for the range of parameters considered, the kinematic approach may over-estimate the bearing capacity by a significant amount.

They also discuss the analytical procedures mainly the one proposed by Hanna and Meyerhof (1980), which was based on the equilibrium analysis of a granular soil below the footing. To calculate the bearing capacity of the system using this approach, it is necessary to estimate the value of shear force acting on the sides of this block of soil. This was achieved by introducing the coefficient of punching shear Ks, which is related to a set of parameters including the unit weight γ and the depth t of the sand layer. And these two parameters are not considered in the design charts presented by Hanna and Meyerhof; therefore, their design charts are only appropriate for the particular values of γ and t on which they are based, and cannot be relied upon for a range of values of sand unit weight, although the limit-equilibrium approach on which they are based appears to be acceptable.

Burd and Frydman, 1997 (4) presented the case of bearing capacity of a rigid plane-strain footing placed on the surface of a soil consisting of a uniform sand layer overlying thick, homogeneous bed of clay. The study is restricted to cases where the thickness of the sand layer is comparable to the footing width, and in all cases the ground surface and the interface between the two soil layers are horizontal. The assumption is made that the clay layer is considered undrained, and the sand layer is drained.

A discussion is given of the various theories previously proposed to solve this case of bearing capacity on two layered soils, followed by the numerical parametric study proposed in this paper. Starting with the load spread models, then the punching shear models proposed by Meyerhof (1974), to the kinematic analysis methods proposed by Flockiewicz (1989) and Michalouski and Shi (1995), and then the numerical models using finite element or finite difference methods in which the soil is subdivided into a

mesh of discrete elements. In order to assess the various analytical procedures reviewed and to provide data of direct use in design, a parametric study has been carried out using two distinct numerical modelling procedures. The finite element calculations were carried out using the program OXFEM, and the finite difference calculations were performed using the program FLAC.

This study highlighted the importance of the no dimensional group $Su/\gamma D$ in determining the mechanics of the system, which is closely related to the over consolidation ratio of the clay and allows a realistic range of clay shear strength values to be adopted in the study. The results of the study have been used to produce charts of bearing capacity that may be used directly in design. The data have also been used to study the effectiveness of the sand in spreading the load applied by the footing. It was found that the sand layer is more effective in spreading the footing load when the clay is normally or lightly over consolidated, than when the over consolidation ratio of the clay is large.

Kenny and Andrawes, 1997 (16) presented a theoretical model for the case of footings resting on a sand layer overlying clay deposit. Model tests were carried out in the laboratory to evaluate the stress-settlement relationship of the sand alone, clay sub grade alone, and the sand overlying clay. The stress-settlement relationships for all tests are then presented in non-dimensional form, and the results of this investigation are compared with experimental data reported by other researchers and presented in a chart.

For the case of a clay sub grade which exhibits the characteristics of local shear failure, the present study suggests that use of the Terzaghi or Vesic shear strength reduction factors will provide a more reliable bearing capacity prediction for sand over clay. Local spreading methods are often considered to give conservative prediction of the bearing capacity, but it is apparent from the present study that this is not the case. Moreover, the method will only yield reasonable estimates of the bearing capacity if a smaller load-spreading angle than previously proposed is used. Hanna & Meyerhof 1978 method is

found to give reasonably accurate bearing capacity estimates for smaller sand depths. However, the accuracy of the predictions can be improved if local shear failure of the clay is taken into account.

Merifield and Sloan, 1999 (18) studied the ultimate bearing capacity of surface strip footings resting on a horizontally layered clay profile. Many empirical and semi-empirical formulas can be used, which give approximate solutions to the problem. More recently, Florkiewicz (1989) presented an upper bound method proposing a kinematically admissible failure mechanism. Although this method is useful, but limited results were produced. The upper bound method has been widely used to estimate the bearing capacity of layered clays, but it may lead to a lower factor of safety for design than the real one. A more desirable solution is a lower bound estimate, as it results in a safe design and, if used in conjunction with an upper bound solution, serves to bracket the actual collapse load from above and below.

The purpose of this paper is to take advantage of the ability of the limit theorems to bracket the actual collapse load by computing both types of solution for the bearing capacity of footings on a two-layered clay profile. These solutions are obtained using the numerical techniques developed by Sloan (1988) and Sloan & Kleeman (1995), which are based upon the limit theorems of classical plasticity and finite elements.

The lower bound solution is obtained by modeling a statically admissible stress field using finite elements with stress nodal variables, where stress discontinuities can occur at the interface between adjacent elements of the mesh representing the soil profile and geometry. An upper bound on the exact collapse load can be obtained by modelling a kinematecally admissible velocity field. To be kinematically admissible, such a velocity field must satisfy the set of constraints imposed by compatibility, velocity boundary conditions and the flow rule.

The computed upper and lower bound estimates of the bearing capacity factors for layered clay soils are given in a table. These results indicate that the true collapse load

typically being bracketed to within 12 % or better. Also comparison figures compare the numerical bounds and the available upper bound solutions, the empirical solutions, and the semi-empirical solutions are presented in this paper.

Finally, the following conclusions can be made based on the limit analysis results:

a) For a strong over soft clay profile, a number of different failure mechanisms exist that are functions of both the upper layer thickness and its strength relative to the underlying weaker layer. For this reason, existing upper bound solutions and the semi-empirical solutions are unable to model the failure mode.
b) Existing upper bound, empirical and semi empirical solutions can differ from the bound solutions by up to ± 20%. The error is great when the top layer is very strong compared the lower layer and/or its depth is greater than half the footing width.

CHAPTER THREE
THEORETICAL MODEL

3.1 General

In this chapter, the case of bearing capacity of a shallow foundation subjected to vertical loading and resting on a strong sand layer overlying a weak deposit was considered. In this study the cases of dense sand overlying medium dense sand and dense sand overlying soft clay were analysed.

The failure mechanism observed by Meyerhof and Hanna, 1978 was idealized in developing the proposed theoretical model to predict the ultimate bearing capacity of these footings.

3.2 Theory

Referring to Figure 3.1, the case of a strip footing having a width, B and depth D, in a dense sand layer overlying a medium dense sand deposit was considered. The dense sand layer has a thickness of H below the footing base. The unit weight and angle of shearing resistance of the upper sand layer were γ_1, ϕ_1 and γ_2, ϕ_2, C_U for the lower layer. The assumptions involved in developing the theoretical model are that at ultimate load, failure occurs by punching, the footing pushes the soil mass below the footing in the upper sand layer, in a roughly truncated parabolic shape, into the lower layer.

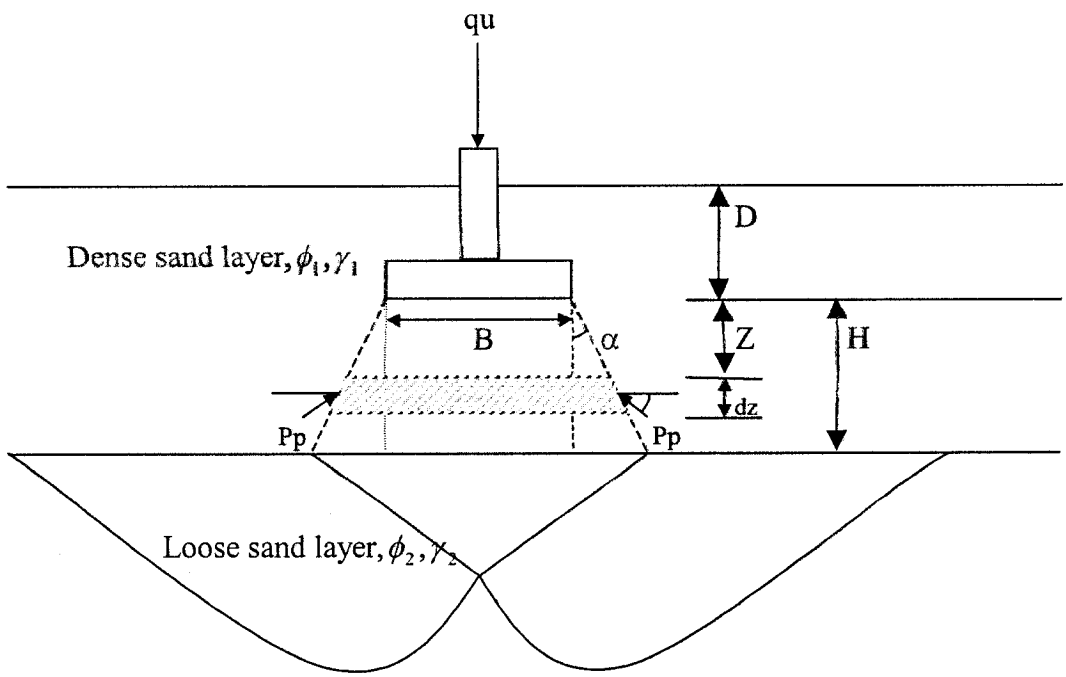

Figure 3.1. Punching shear model of strip footing over two layered soil.

Referring to Figure 3.2, the actual failure plane, which is parabolic in shape, was replaced with the best fitting line inclined at an angle α with the vertical.

Consider the stresses acting on a horizontal slice at a depth z and a thickness of dz (Figure 3.3):

a. The total passive earth pressure Pp, inclined at an average angle δ, acting upwards,

b. The vertical stress (σ_{zz}) acting on the slice,

c. The vertical stress $\sigma_{zz} + d\sigma_{zz}$ acting upward on the bottom of the slice,

d. The weight of the slice, W.

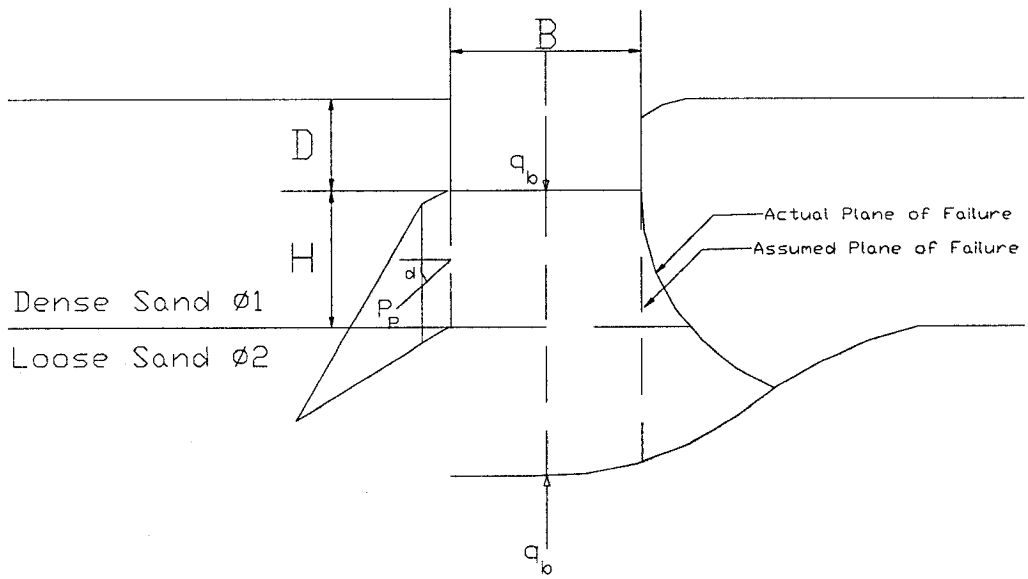

Figure 3.2 Hanna (1981) mode of failure: strip footing on dense sand overlying loose sand deposit

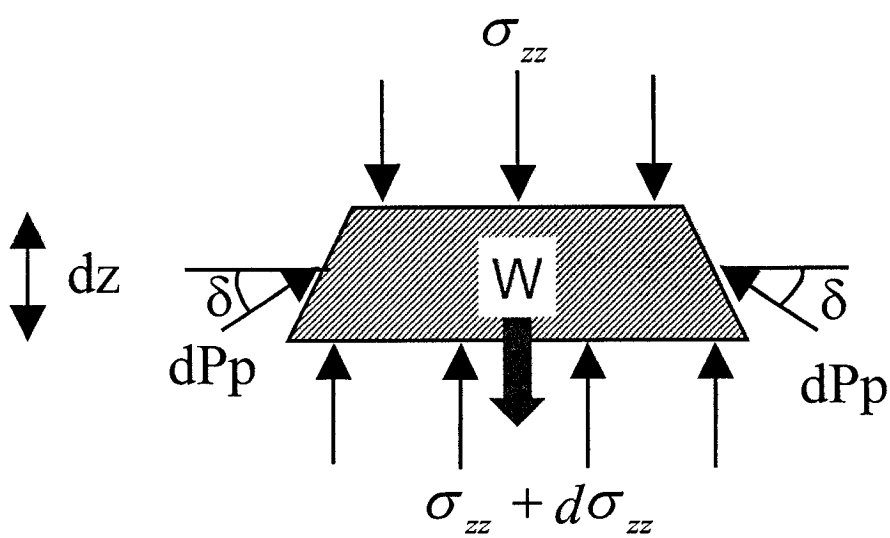

Figure 3.3. Applied forces on a strip dz of the failure zone at depth z

From the limit equilibrium of the vertical forces, the following equation can be written;

$$\sigma_{zz} * (B + 2z\tan\alpha) - (\sigma_{zz} + d\sigma_{zz}) * (B + 2(z + dz)\tan\alpha) - 2 * dPp_v * dz$$
$$+ \gamma_1 * \left(B + 2\left(z + \frac{dz}{2} \right) \tan\alpha \right) * dz = 0 \qquad (3.1)$$

Where

- σ_{zz}: the vertical stress acting on the top of the slice in the failure zone.
- $\sigma_{zz} + d\sigma_{zz}$: the vertical stress acting upward on the bottom of the slice in the failure zone.
- B: the width of the footing.
- D: the depth of the footing in the upper layer.
- Z: the depth of the slice below the footing.
- α: the angle of the assumed failure plane with the vertical.
- δ: mobilized angle of shearing resistance on the assumed failure plane.
- γ_1: the unit weight of the upper sand layer.
- dPp: the passive earth pressure, which is acting on the side of the punching slice

$$dPp = \gamma_1 * \left(D + z + \frac{dz}{2} \right) * Kp \qquad (3.2)$$

- Kp: the coefficient of passive earth pressure given by Caquot and Kerisel (1949).

After simplification, Eq.3.1 can be written as follows:

$$-\sigma_{zz} * (2dz\tan\alpha) - d\sigma_{zz} * (B + 2(z + dz)\tan\alpha) - 2dPp_v * dz + \gamma_1 \left(B + 2\left(z + \frac{dz}{2} \right) \tan\alpha \right) * dz = 0 \qquad (3.3)$$

Furthermore

$\sigma_{zz} * (2dz\tan\alpha)$ is too small and can reasonably be assumed to equal zero.

Then Eq.3.3 becomes:

$$-d\sigma_{zz}(B+2(z+dz)\tan\alpha)-2dPp_v * dz+\gamma_1\left(B+2\left(z+\frac{dz}{2}\right)\tan\alpha\right)*dz=0 \qquad (3.4)$$

By substituting (Eq.3.2) in (Eq.3.4), the equation of equilibrium can be expressed as follows

$$d\sigma_{zz}(B+2(z+dz)\tan\alpha)=-2\left(\gamma_1 * Kp\left(D+z+\frac{dz}{2}\right)\right)\sin\delta * dz+\gamma_1\left(B+2\left(z+\frac{dz}{2}\right)\tan\alpha\right)dz \qquad (3.5)$$

Multiplying and rearranging the factors, Eq.3.5 can be rewritten as follows:

$$d\sigma_{zz}(B+2(z+dz)\tan\alpha)=-2\gamma_1 KpD(\sin\delta)dz-2\gamma_1 Kpz(\sin\delta)dz-\gamma_1 Kp(\sin\delta)dzdz+\gamma_1\left(B+2\left(z+\frac{dz}{2}\right)\tan\alpha\right) \qquad (3.6)$$

The following components of Eq.3.6 are too small and can be neglected:

$\gamma_1 Kp * (\sin\delta)dzdz$, and

$\gamma_1 * \frac{dz}{2}\tan\alpha$

Thus Eq.3.6 can be written as

$$d\sigma_{zz}=\frac{-2\gamma_1 KpD(\sin\delta)}{B+2z\tan\alpha} * dz+\frac{-2\gamma_1 Kp(\sin\delta)}{B+2z\tan\alpha} * zdz+\gamma_1 dz \qquad (3.7)$$

Integrating Eq.3.7;

$$\int d\sigma_{zz} = \int \frac{-2\gamma_1 KpD(\sin\delta)}{B+2z\tan\alpha} * dz + \int \frac{-2\gamma_1 Kp(\sin\delta)}{B+2z\tan\alpha} * zdz + \int \gamma_1 dz \tag{3.8}$$

Let $A_1 = -2\gamma_1 KpD(\sin\delta)$, (3.9)

and $A_2 = -2\gamma_1 Kp(\sin\delta)$ (3.10)

$$\sigma_{zz} = \int \frac{A_1 * dz}{B+2z\tan\alpha} + \int \frac{A_2 * zdz}{B+2z\tan\alpha} + \int \gamma_1 dz \tag{3.11}$$

(I) (II) (III)

The stress σ_{zz} is the sum of the three integrals I, II, and III, which can be solved separately as follows:

$$\text{Integral I} \Rightarrow A_1 * \int \frac{dz}{B+2z\tan\alpha} \tag{3.12}$$

In order to solve this integral let $u = B + 2z\tan\alpha$, then $du = 2\tan\alpha * dz$, and $dz = \frac{du}{2\tan\alpha}$,

Substitute the value of dz, and then integral I can be written as:

$$A_1 * \int \frac{du}{2\tan\alpha} * \frac{1}{u} = \frac{A_1}{2\tan\alpha} * \ln|u| + c = \frac{A_1}{2\tan\alpha} * (\ln(B + 2z\tan\alpha)) + c \tag{3.13}$$

Where c is the integration constant

Integral II $\Rightarrow$

$$\int \frac{A_2}{B+2z\tan\alpha} * zdz = A_2 * \int \frac{zdz}{B+2z\tan\alpha} = \frac{A_2}{(2\tan\alpha)^2} * [B + 2z\tan\alpha - B(\ln(B + 2z\tan\alpha))] + c \tag{3.14}$$

And integral III $\Rightarrow \int \gamma_1 dz = \gamma_1 * z + c$ (3.15)

Substituting Eq.3.13, Eq.3.14 and Eq.3.15 in Eq.3.11 can be writing as follows:

$$\sigma_{zz} = \frac{-\gamma_1 KpD \sin\delta}{\tan\alpha} * \ln(B + 2z\tan\alpha) + \frac{-2\gamma_1 Kp \sin\delta}{(2\tan\alpha)^2} * [B + 2z\tan\alpha - B\ln(B + 2z\tan\alpha)] + \gamma_1 z + c \quad (3.16)$$

In order to determine the value of the constant c, the boundary conditions were considered; accordingly, z varies from 0 to H
where H is the depth of the upper layer below the footing base.

At z = 0 (the slice is just below the footing) the stress $\sigma_{zz} = q_u$, and Eq.3.16 can be written as:

$$qu = \frac{A_1}{2\tan\alpha}\ln(B) + \frac{A_2}{(2\tan\alpha)^2}(B - B\ln(B)) + c \quad (3.17)$$

Where q_u is the ultimate bearing capacity of the footing on two-layer soil

Replacing A_1 and A_2 by their values;

$$qu = \frac{-\gamma_1 KpD \sin\delta}{\tan\alpha}\ln(B) - \frac{\gamma_1 Kp \sin\delta}{2(\tan\alpha)^2} * B(1 - \ln B) + c \quad (3.18)$$

The value of the integration constant c was then calculated as:

$$c = qu + \frac{\gamma_1 Kp \sin\delta}{\tan\alpha} * \left[D\ln B + \frac{B(1 - \ln B)}{2\tan\alpha} \right] \quad (3.19)$$

At z = H; (the slice is at the interface of the two layers), the stress $\sigma_{zz} = qb$

Where q_b is the ultimate bearing capacity of a strip footing on a very thick bed of the lower layer, q_b can be evaluated as follows:

$$qb = \frac{1}{2}\gamma_2 B N_{\gamma 2} + \gamma_1 (H + D) N_{q2} \qquad \text{for sand layer} \qquad (3.20)$$

$$qb = C_U N_{C2} + \gamma_1 (H + D) \qquad \text{for clay layer} \qquad (3.21)$$

Where $N_{\gamma 2}, N_{q2}, N_{C2}$ are the bearing capacity factors corresponding to plane-strain angle of shearing resistance ϕ_2 of the lower layer.

By replacing z with H and σ_{zz} with qb, Eq.3.16 can be written as:

$$\sigma_{zz} = \frac{A_1}{2\tan\alpha}\ln(B + 2H\tan\alpha) + \frac{A_2}{(2\tan\alpha)^2}\left[B + 2H\tan\alpha - B\ln(B + 2H\tan\alpha)\right] + \gamma_1 H + c \qquad (3.22)$$

By substituting A_1, A_2 and c by their values in Eq.3.22,

$$qb = \frac{-2\gamma_1 KpD\sin\delta}{2\tan\alpha}\ln(B + 2H\tan\alpha) + \frac{-2\gamma_1 Kp\sin\delta}{(2\tan\alpha)^2}\left[B + 2H\tan\alpha - B\ln(B + 2H\tan\alpha)\right]$$
$$+ \gamma_1 H + qu + \frac{\gamma_1 Kp\sin\delta}{\tan\alpha}\left[D\ln B + \frac{B(1 - \ln B)}{2\tan\alpha}\right] \qquad (3.23)$$

After rearranging and simplifying, Eq.3.23 can be written as:

$$qu = qb - \gamma_1 H + \frac{\gamma_1 DKp\sin\delta}{\tan\alpha}\left[\ln(B + 2H\tan\alpha) - \ln B\right]$$
$$+ \frac{\gamma_1 Kp\sin\delta}{(2\tan\alpha)^2}\left[B + 2H\tan\alpha - B\ln(B + 2H\tan\alpha) - B + B\ln B\right] \qquad (3.24)$$

Assuming that

$$F = \ln(B + 2H\tan\alpha) - \ln B = \ln\left[\frac{B + 2H\tan\alpha}{B}\right] \tag{3.25}$$

After simplification, Eq.3.24 can be written as:

$$qu = qb - \gamma_1 H + \frac{\gamma_1 Kp\sin\delta}{\tan\alpha}\left[DF + \frac{2H\tan\alpha - BF}{2\tan\alpha}\right] \tag{3.26}$$

Or in a dimensionless form by dividing both sides by $\gamma_1 B$:

$$\frac{qu}{\gamma_1 B} = \frac{qb}{\gamma_1 B} + \frac{Kp\sin\delta}{\tan\alpha}\left[\frac{DF}{B} + \frac{H}{B} - \frac{F}{2\tan\alpha}\right] - \frac{H}{B} \tag{3.27}$$

The parameters used in Eq.3.26 are as described above. Furthermore:
Kp = coefficient of passive earth pressure. This coefficient is taken from the tables produced by Caquot and Kerisel (1949). According to Caquot and Kerisel, Kp depends on the angle of shearing resistance ϕ_1 and the ratio δ/ϕ_1, where the angle δ is the mobilized angle of shearing resistance on the assumed failure planes.

The following arguments are considered:

1- If the analysis is made on the actual curved failure planes, the angle δ will be equal to ϕ_1, when the analysis is made on the assumed (diagonal) planes, the angle δ, mobilized on the assumed failure planes, must be less than ϕ_1, as failure has not taken place on the real failure planes.

2- The assumed failure planes are considered the best-fit straight line to the actual failure planes (Fig. 3.1).

3- The angle δ varies with depth, decreases as the assumed failure plane diverges from the actual failure plane (parabolic), so that δ is equal to ϕ_1 when both assumed and actual failure planes coincide with each other.

In order to facilitate the calculations, an assumed constant value of 0.9 was given to the ratio δ / ϕ_1 since the assumed failure plane is the best-fit line to the actual parabolic one. Accordingly, the values of Kp, and $\sin\delta$ depend only on the value of ϕ_1.

The non-dimensional ratio $\frac{qb}{\gamma_1 B}$ can be written as follows:

$$\frac{qb}{\gamma_1 B} = \frac{1}{2} \times \frac{\gamma_2}{\gamma_1} \times N_{\gamma 2} + \frac{H+D}{B} \times N_{q2} \qquad \text{for lower sand layer} \qquad (3.28)$$

$$\frac{qb}{\gamma_1 B} = \frac{C_U N_{C2}}{\gamma_1 B} + \frac{H+D}{B} \qquad \text{for lower clay layer} \qquad (3.29)$$

and

$$\mathrm{F} = \ln(B + 2H\tan\alpha) - \ln B = \ln\left[\frac{B + 2H\tan\alpha}{B}\right] \qquad (3.30)$$

3.3 Experimental values of the angle α

The predicted values of the ultimate bearing capacity of footing on layered soils using Eq.3.27 depends on the angle α previously defined, and the parameters related to the soil profile and the geometry of the footing.

In order to establish a correlation between the angle α and these parameters, the experimental data available in the literature was used. The experimental data used in analysis was divided into two categories: strong sand overlying a weak sand or clay deposit and are given below.

3.3.1 Dense sand layer overlying loose sand deposit

The experimental bearing capacity of footings on layered soil and subjected to vertical loading reported by Hanna (1978) was analysed to determine the corresponding angle α. The results are presented in Table 3.1 and Figure 3.4. In these tests, the ratio H/B varied between 0.5 and 5, and the ratio q2/q1 remained constant for any soil layer combination; i.e.

$$q2/q1 = \gamma_2 N_{\gamma 2} / \gamma_1 N_{\gamma 1} = 87.8 * 41.06/104 * 468.3 = 0.074$$

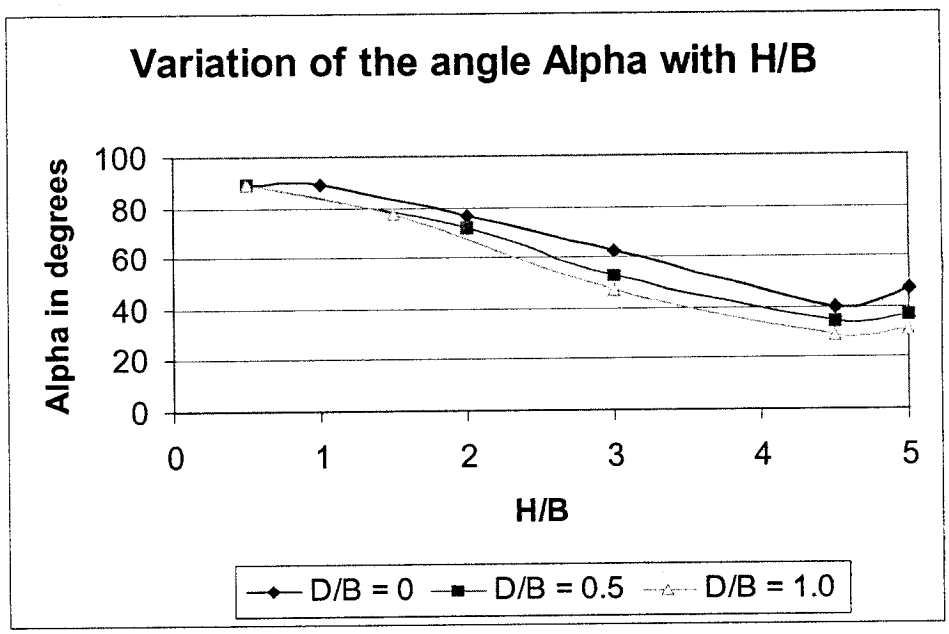

Figure 3.4 Experimental values of the angle α vs. H/B (Hanna 1978, table 1)

Strip footing in dense sand overlying loose sand under vertical loads

assuming that $\delta / \phi = -0.9$

Experimental Data					
test	(H+D)/B	D/B	H/B	qu	Calculated
no				Kn/m^2	alpha
18	0.5	0	0.5	25.44	89
19	1	0	1	36.68	89
20	2	0	2	72.74	76.9
21	3	0	3	120.94	62.2
22	4.5	0	4.5	231.74	40.1
23	5	0	5	237.88	46.6
25	1	0.5	0.5	36.27	89
26	2.5	0.5	2	101.43	71.5
27	3.5	0.5	3	170.03	52.4
28	5	0.5	4.5	303.45	34.6
29	5.5	0.5	5	323.86	36.9
31	1.5	1	0.5	48.40	89
32	2.5	1	1.5	99.84	77
33	4	1	3	219.61	47.1
34	5.5	1	4.5	391.77	28.7
35	6	1	5	412.46	31.2

Having:

$$\phi_1 = 47.7^o, \gamma_1 = 104\ (pcf), N_{\gamma 1} = 468.3, N_{q1} = 211.8$$
$$\phi_2 = 34.0^o, \gamma_2 = 87.8(pcf), N_{\gamma 2} = 41.06, N_{q2} = 29.44$$

q2/q1 = 0.07

Table 3.1 Experimental values of the angle α (Hanna 1978, table 1)

In table 4.4 of Hanna's thesis (1978), only one set of tests corresponding to surface footings were considered. In this set, the ratio H/B varies between 0.5 and 2. The ratio q2/q1 is constant in the whole set of tests.

$$q2/q1 = \gamma_2 N_{\gamma 2} / \gamma_1 N_{\gamma 1} = 95.5 * 167.95/104 * 468.3 = 0.33$$

By fixing the ratios q2/q1 and D/B and varying H/B, the ultimate bearing capacity was calculated and a certain value of the angle α was found. Figure 3.5 is plotted below for the angle α versus H/B ratio.

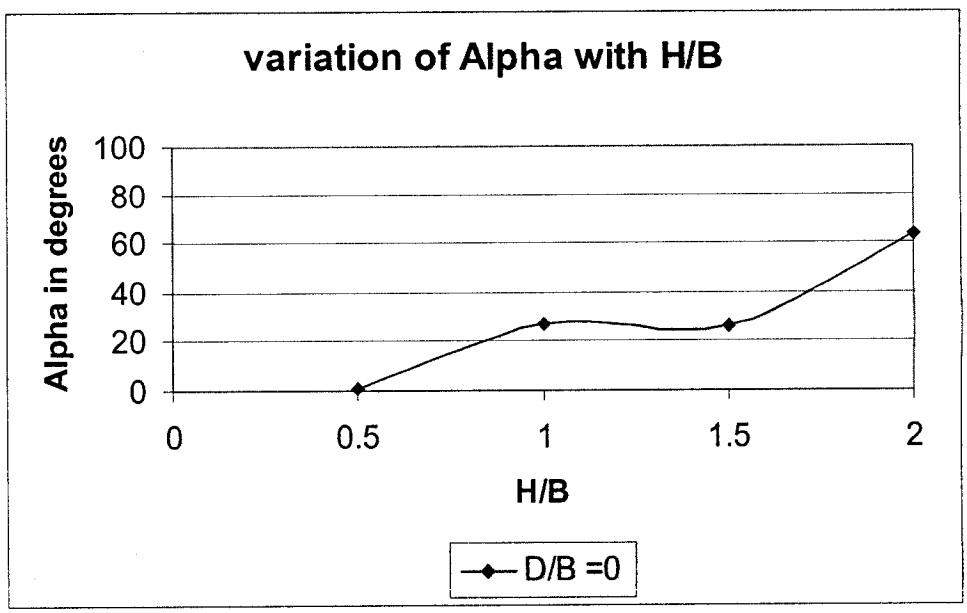

Figure 3.5 Experimental values of the angle α vs. H/B (Hanna 1978, table 2)

Surface strip footing on dense sand overlying compact sand under vertical loads

Assuming $\delta / \phi = -0.9$

Experimental Data					
test	(H+D)/B	D/B	H/B	qu	Calculated
no				Kn/m^2	alpha
36	0.5	0	0.5	111.15	1
37	1	0	1	156.31	26.7
38	1.5	0	1.5	210.16	25.9
39	2	0	2	234.09	63.5

q2/q1 = 0.33

$$\phi_1 = 47.7^o, \gamma_1 = 104\ (pcf),\ N_{\gamma 1} = 468.3,\ N_{q1} = 211.8$$
$$\phi_2 = 42.4^o, \gamma_2 = 95.5(pcf),\ N_{\gamma 2} = 167.95,\ N_{q2} = 90.84$$

Table 3.2 Experimental values of the angle α (Hanna 1978, table 2)

The experimental data of Das & Mundoz (1982) was analyzed to determine the values of the angle α. This article considers only surface foundations. The ratio H/B varies between 0.5 and 2.5. The ratio q2/q1 is kept constant in the set of tests.

$$q2/q1 = \gamma_2 N_{\gamma 2} / \gamma_1 N_{\gamma 1} = 97 * 56.31/108.52 * 186.54 = 0.27$$

By keeping the ratios q2/q1 and D/B constant and varying H/B, the ultimate bearing capacity was calculated and the angle α was found.

Strip footing in dense sand overlying loose sand under vertical loads

Assuming that $\delta / \phi = -0.9$

Experimental Data						
Series no	test no	(H+D)/B	D/B	H/B	qu (psi)	Calculated alpha
1	8	0.5	0	0.5	8.563	89
1	9	1	0	1	12.3	89
1	10	1.5	0	1.5	18.155	89
1	11	2	0	2	22.835	89
1	12	2.5	0	2.5	25.55	89

$$\phi_1 = 43^\circ, \gamma_1 = 108.52\,(pcf),\ N_{\gamma 1} = 186.54,\ N_{q1} = 99.02$$
$$\phi_2 = 36^\circ, \gamma_2 = 97\,(pcf),\ N_{\gamma 2} = 56.31,\ N_{q2} = 37.75$$

q2/q1 = 0.27

Table 3.3 Experimental values of the angle α (Das & Munoz 1982)

3.3.2 Dense sand layer overlying a weak clay deposit

In the case of dense sand overlying a weak clay deposit, the equation of bearing capacity is applicable, but q_b of the lower clay layer requires a different equation. The lower clay layer is always assumed to be an undrained saturated clay, so that the clay's internal friction angle is always considered equal to zero ($\phi_2 = 0$).

Das (1988) performed many series of tests on this type of soil profile. Some were performed with a geotextile at the sand-clay interface, but only tests without a geotextile at the interface of the two layers are considered in this investigation.

The experimental values of the ultimate bearing capacity are not found in tabular form, but rather in graphs, the values of the experimental bearing capacity were extracted from Fig. 8 of Das (1988) article.

The value of the ratio D/B was taken as 0 and 0.5 respectively. The ratio H/B varies from 0.25 to 3 in each set of tests, and the ratio q2/q1 is constant in both sets of tests.

$$q2/q1 = C_U N_{C2}/0.5\gamma_1 BN_{\gamma 1} = 5.7*2.03*12^3/0.5*108.2*3*205.59 = 0.6$$

By keeping the ratios q2/q1 and D/B constant and varying H/B, the ultimate bearing capacity was calculated and the angle α was found. The variation of the angle α with the ratio D/B and H/B is presented in Figure 3.6.

Strip footing in dense sand overlying a clay layer under vertical loads

Assuming that $\delta / \phi = -0.9$

Experimental Data						
Series	test	(H+D)/B	D/B	H/B	qu	Calculated
no	no				Kn/m^2	alpha
1	2	0.25	0	0.25	75.02	89
1	3	0.5	0	0.5	79.98	89
1	4	1	0	1	93.29	89
1	5	1.5	0	1.5	109.98	86.2
1	6	2	0	2	133.28	63.2
1	7	2.5	0	2.5	131.63	71.4
1	8	3	0	3	139.97	69.7
2	2	0.75	0.5	0.25	83.29	89
2	3	1	0.5	0.5	99.98	89
2	4	1.5	0.5	1	158.31	22.1
2	5	2	0.5	1.5	209.95	11.2
2	6	2.5	0.5	2	216.64	25.1
2	7	3	0.5	2.5	214.99	37.5
2	8	3.5	0.5	3	216.64	45.1

$$\phi_1 = 43.5^o, \gamma_1 = 108.2(pcf), N_{\gamma 1} = 205.59, N_{q1} = 107.165$$
$$C_U = 2.03(psi), \gamma_2 = 127.98(pcf), N_{q2} = 1.0$$

q2/q1 = 0.60

Table 3.4 Experimental values of the angle α (Das 1988)

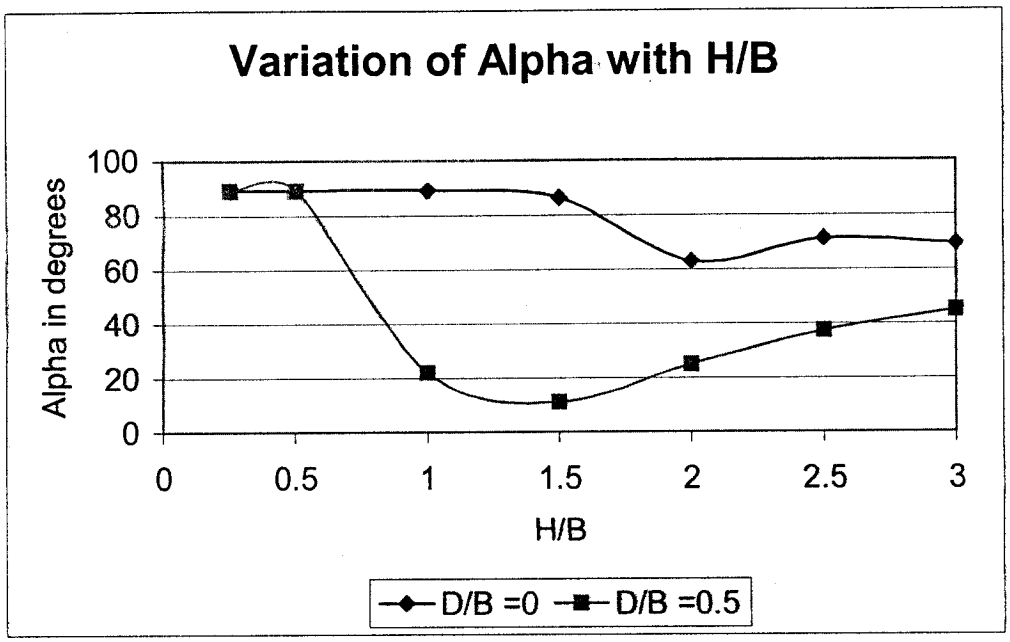

Figure 3.6 Experimental values of the angle α vs. H/B (Das 1988)

3.4 Consideration for the Angle α

After analysing the back calculations, the first step towards predicting the behaviour of the angle α is to determine the parameters on which it depends, which are as follows:

1. The ratio H/B (depth of the upper sand layer over the footing's width).
2. The ratio q2/q1 (ultimate bearing capacity of the lower layer over the ultimate bearing capacity of the upper layer taken as homogenous)
3. The angle of shearing resistance of the upper sand layer ϕ_1
4. The ratio δ / ϕ_1 (the mobilized angle of shearing resistance on the assumed failure planes; δ over the angle of shearing resistance of the upper sand layer ϕ_1).
5. The ratio D/B (depth of the footing in the upper sand layer over the footing's width)

3.4.1 First trial

In the first trial to predict the angle α, a certain function for the angle α was assumed while the rest of the parameters were varied in order to calculate the ratio δ / ϕ_1, since it varies between 0 and 1.

The ratio q2/q1 also varies between 0 and 1 since the case of a strong upper layer overlying a weak deposit is considered in this study. A value of q2/q1 equal to 1 refers to the homogenous case, where according to Terzaghi, the failure below the footing occurs with an angle α equal to $\left(45 + \frac{\phi_1}{2}\right)$ with the vertical.

A value of q2/q1 equal to 0 refers to either case scenario, q2 tends to zero or q1 tends to ∞. In the first scenario, if the upper layer is overlying a fluid, the punching occurs rapidly and vertically, and the corresponding angle α equals 0. In the second scenario, if the footing is lying on a very strong upper layer, like rock or concrete, there will be no punching, and according to Meyerhof 1953 at ultimate load failure occurs horizontally in the upper strong layer, the angle α tends to 90°.

The first trial to assume a function for the angle α was by assuming a parabolic equation in function of the ratio q2/q1 (Figure 3.7).

$$\text{Angle } \alpha = 148.86*(q2/q1)^2-163.3*(q2/q1)+86.377 \qquad (3.31)$$

This parabolic equation is approximately equal to 90 for q2/q1 = 0 and 70 for q2/q1 = 1 (the homogeneous case).

By using the experimental data available, calculating the angle α by using the previous equation, it is possible to back calculate for the ratio δ / ϕ_1. A sample of the calculation is presented in Table 3.5.

The deduced values of the ratio δ / ϕ_1 are not consistent with the condition that the value of the ratio lays between 0 and 1. Therefore the assumed equation of the angle α is not valid and must be changed.

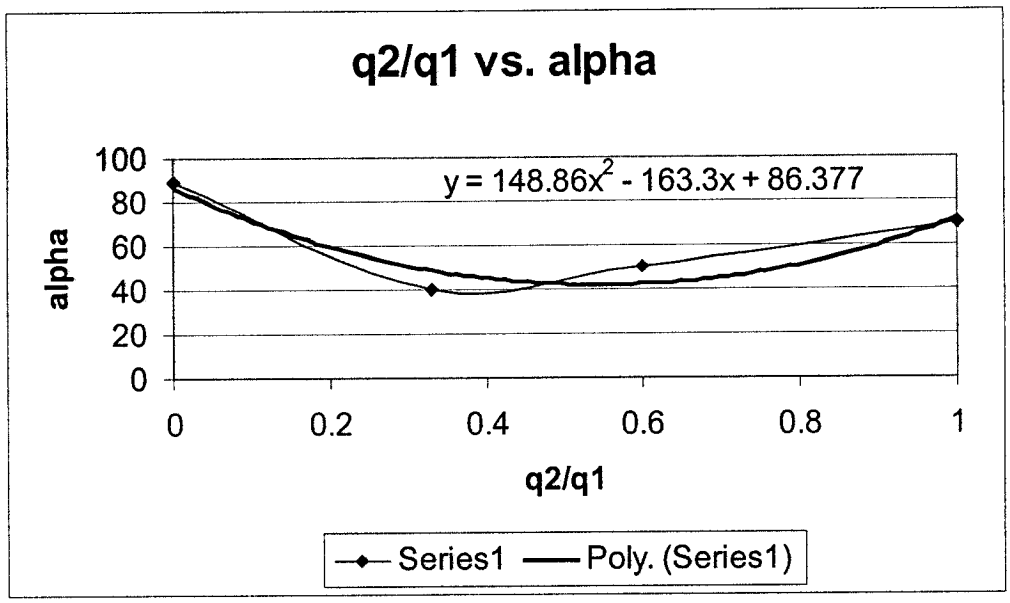

Figure 3.7. Angle α vs. q2/q1

Das Strip footing in dense sand overlying clay layer under vertical loads

Assuming that $\delta / \phi = -1$

Experimental Data						Calculated Data								
Series	test	(H+D)/B	D/B	H/B	qu	$qb / \gamma B$	F	qu/Bg	Kpsind	assumed	delta	Sin d	Kp	kp*sind
no	no				(psi)					delta/phi				
1	2	0.25	0	0.25	10.88	61.82	0.81	43.31	-518.9	0.001	0.044	0.0008	31.9	0.0242
1	3	0.5	0	0.5	11.6	62.07	1.25	46.18	-153.9	1	43.5	0.6884	31.9	21.965
1	4	1	0	1	13.53	62.57	1.79	53.86	-29.95	1	43.5	0.6884	31.9	21.965
1	5	1.5	0	1.5	15.95	63.07	2.14	63.49	4.4834	0.19	8.265	0.1438	31.9	4.587
1	6	2	0	2	19.33	63.57	2.39	76.95	25.219	1.2	52.2	0.7902	31.9	25.213
1	7	2.5	0	2.5	19.09	64.07	2.6	75.99	18.166	0.8	34.8	0.5707	31.9	18.211
1	8	3	0	3	20.3	64.57	2.77	80.81	19.612	0.87	37.85	0.6135	31.9	19.577

Table 3.5 Validation of the assumed angle α with the experimental data (First trial)

3.4.2 Second trial

The second trial assumption of the angle α was a straight line, assuming that the angle α = 0 for q2/q1 = 0 and $\left(45+\frac{\phi_1}{2}\right)$ for q2/q1 = 1, the equation of the angle α is as follows:

$$\alpha = \left(45+\frac{\phi_1}{2}\right) * q2/q1 \tag{3.32}$$

By using the experimental data available, calculating the angle α by using the previous equation, it is possible to back calculate for the ratio δ/ϕ_1. A sample of the calculation is presented in Table 3.6.

$\delta / \phi = -1$

Experimental Data														
Series	test	(H+D)/B	D/B	H/B	qu	$qb / \gamma B$	F	qu/Bg	Kpsind	assu.	delta	Sin d	Kp	kp*sind
no	no				(psi)					delta/phi				
2	2	0.75	0.5	0.25	12.08	62.32	0.35	48.09	-54.2	0.001	0.044	0.0008	31.91	0.0242
2	3	1	0.5	0.5	14.50	62.57	0.61	57.72	-8.255	0.001	0.044	0.0008	31.91	0.0242
2	4	1.5	0.5	1	22.96	63.07	0.99	91.4	27.18	1.35	58.73	0.8547	31.91	27.272
2	5	2	0.5	1.5	30.45	63.57	1.26	121.2	35.98	2.1	91.35	0.9997	31.91	31.9
2	6	2.5	0.5	2	31.42	64.07	1.47	125.1	28.44	1.45	63.08	0.8916	31.91	28.45
2	7	3	0.5	2.5	31.18	64.57	1.65	124.1	22.23	1.01	43.94	0.6938	31.91	22.14
2	8	3.5	0.5	3	31.42	65.07	1.8	125.1	18.7	0.83	36.11	0.5893	31.91	18.803

phi1= 43.5

q2/q1 = 0.60

alpha1 = 40

tan alpha 40 0.8391

Table 3.6 Validation of the assumed angle α with the experimental data (Second trial)

The deduced values of the ratio δ / ϕ_1 are not consistent with the condition that the value of the ratio lays between 0 and 1, thus the assumed equation of the angle α is not valid and must be changed.

3.4.3 Third trial

The third trial assumption of the angle α takes into consideration all parameters involved and gives a series of equations for α. The ratio δ / ϕ_1 equal to 0.9 (constant) as previously mentioned is a good assumption.

The equation of the angle α has the following form:

$$\text{Angle } \alpha = \rho * \ln (q2/q1) + \mu \tag{3.33}$$

Where:

a) The term ρ is function of the ratios H/B and D/B.

b) The term μ is function of the angle of shearing resistance of the upper sand layer ϕ_1, $\mu = 45 + \frac{\phi_1}{2}$.

c) The ratio q2/q1 is as previously defined.

Back calculations using the available experimental data of Hanna and Das give an indication of the behaviour of the ρ term. An assumed equation for ρ as a function of the ratio H/B for every value of the ratio D/B can then be established.

3.4.3.1 Equation of ρ

The equations of ρ are straight-line functions of the ratio H/B having the following form:

$$\rho = \lambda * (H/B) + \theta \tag{3.34}$$

The constants λ and θ are assumed according to the back calculations and were found to have the values presented in Table 3.9 for every value of the ratio D/B. The equation of ρ for D/B = 0 is illustrated in Figure 3.8.

D/B	λ	θ
0	4.108	-9.159
0.5	4.577	-9.42
1	4.513	-9.96

Table 3.7 Values of the constants λ and θ.

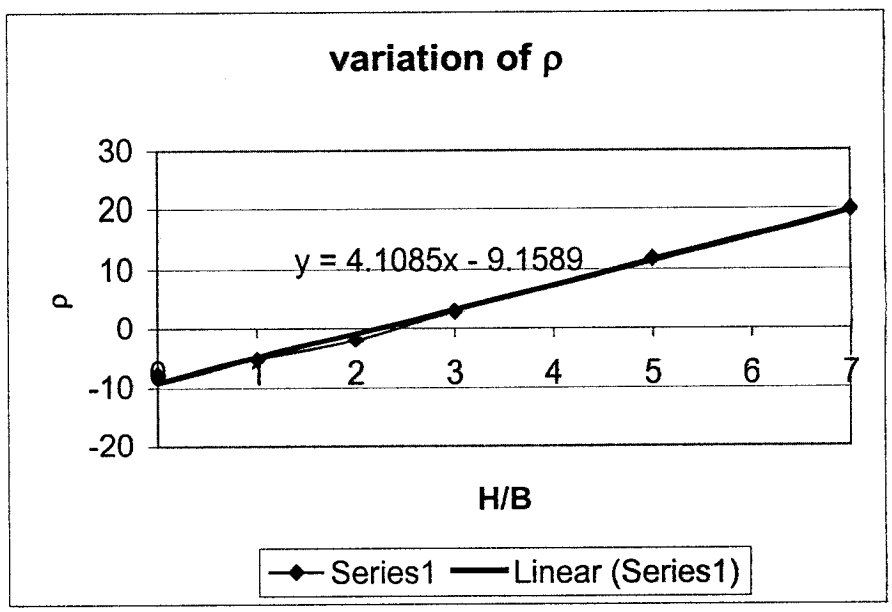

Figure 3.8. The parameter ρ vs. the ratio H/B for D/B = 0 (Third trial)

3.4.4 Validation of the present theory

Since the experimental data available belong to two cases of soil profiles, the variations of the angle α with respect to the ratio H/B are presented according to the soil profile. This section is divided in two parts each corresponding to a different case of soil profile.

3.4.4.1 Dense sand layer overlying loose sand deposit

The first part is the case of a dense sand layer overlying a loose sand deposit; three tables of experimental and calculated data represent the soil profile of the first case. Table 3.8 (Hanna (1978), table 1) contains data for different series of tests, where H/B varies from 0.5 to 5, and D/B varies from 0 to 1. The present theory is applied to the experimental data and the calculated ultimate bearing capacity found is compared to the experimental one. All calculations are presented in Table 3.8 and the variation of the ultimate bearing capacity qu with H/B and D/B (experimental and present theory data) are presented in Figure 3.9.

Hanna Strip footing in dense sand overlying loose sand under vertical loads

Assuming that $\delta / \phi = -0.9$

Experimental Data					Given	Calculated Data							
Series	test	(H+D)/B	D/B	H/B	qu	A	Alpha	Tan	$qb / \gamma B$	F	qu/GB	qu	%
no	no				kn/m^2			Alpha				Kn/m^2	Accu.
1	19	1	0	1	36.68	-5.05	82.00	7.11	46.77	2.72	49.74	41.28	12.55
1	20	2	0	2	72.74	-0.94	71.30	2.95	76.21	2.55	92.76	76.98	5.84
1	21	3	0	3	120.9	3.17	60.61	1.78	105.65	2.46	148.10	122.91	1.66
1	22	4.5	0	4.5	231.7	9.33	44.56	0.98	149.81	2.29	263.77	218.91	-5.52
1	23	5	0	5	237.9	11.38	39.21	0.82	164.53	2.21	315.56	261.90	10.09
2	25	1	0.5	0.5	36.27	-7.13	87.42	22.16	46.77	3.14	49.42	41.02	13.09
2	26	2.5	0.5	2	101.4	-0.27	69.54	2.68	90.93	2.46	125.06	103.80	2.36
2	27	3.5	0.5	3	170	4.31	57.63	1.58	120.37	2.35	193.36	160.48	-5.60
2	28	5	0.5	4.5	303.4	11.18	39.75	0.83	164.53	2.14	340.02	282.20	-6.99
2	29	5.5	0.5	5	323.9	13.47	33.80	0.67	179.25	2.04	409.01	339.46	4.80
3	31	1.5	1	0.5	48.4	-7.70	88.90	52.31	61.49	3.98	63.95	53.08	9.66
3	32	2.5	1	1.5	99.84	-3.19	77.15	4.39	90.93	2.65	120.09	99.67	-0.17
3	33	4	1	3	219.6	3.58	59.53	1.70	135.09	2.42	228.84	189.93	-13.51
3	34	5.5	1	4.5	391.8	10.35	41.90	0.90	179.25	2.21	388.05	322.07	-17.80
3	35	6	1	5	412.5	12.61	36.03	0.73	193.97	2.11	460.98	382.59	-7.25

Having:

$\phi_1 = 47.7^o, \gamma_1 = 104\ (pcf), N_{\gamma 1} = 468.3, N_{q1} = 211.8$

$\phi_2 = 34.0^o, \gamma_2 = 87.8\ (pcf), N_{\gamma 2} = 41.06, N_{q2} = 29.44$

phi	delta	mod Kp	Kp*sind
47.7	42.9	51.315	34.95

Phi 1= 47.7 b1= 68.85

q2/q1 = 0.07

Table 3.8 Calculation of the ultimate bearing capacity using the present theory (Hanna 1978, table 1)

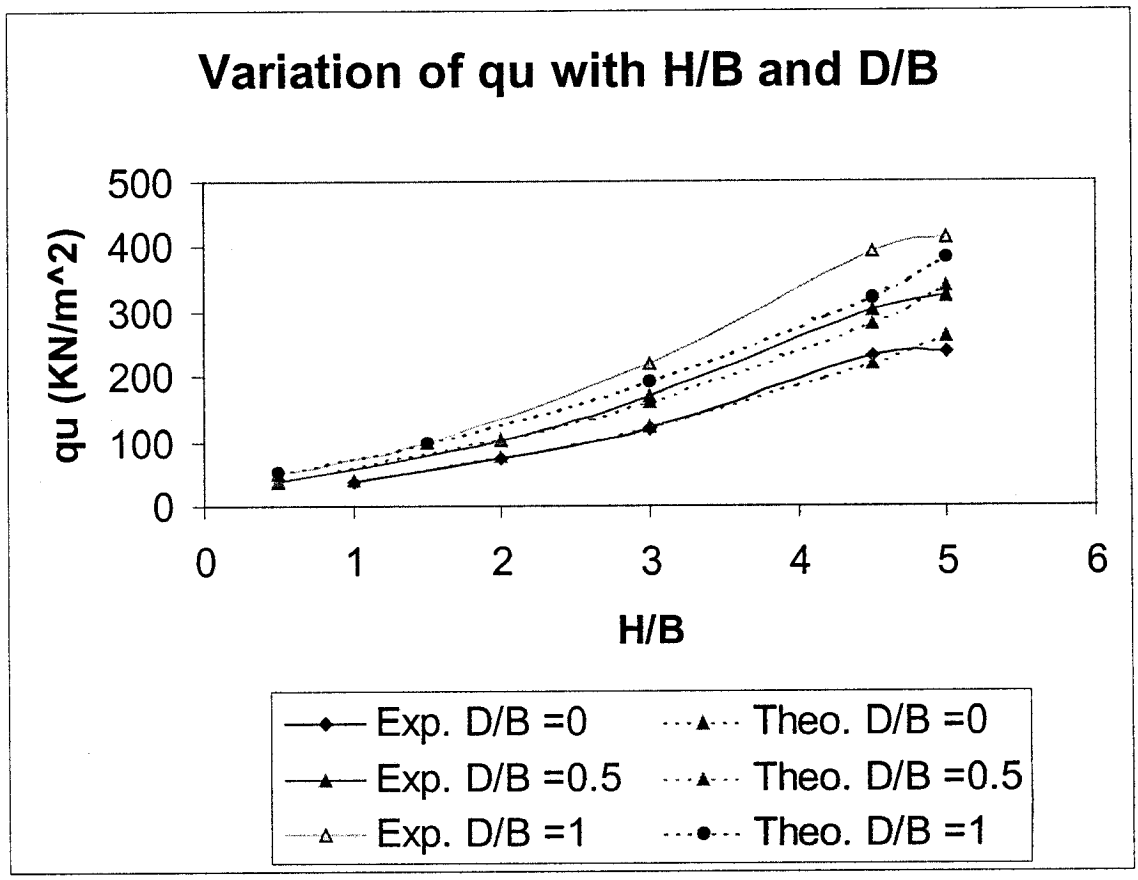

Figure 3.9 Comparison of the experimental and theoretical values of qu (Hanna 1978, table 1)

Table 3.9 (Hanna (1978), table 2) contains data for one set of tests, where H/B varies from 0.5 to 2 for D/B equals 0. The present theory is applied to the experimental data and the calculated ultimate bearing capacity is compared to the experimental one. All calculations are presented in Table 3.9 and the variation of the ultimate bearing capacity qu with H/B (experimental and present theory data) is presented in Figure 3.10.

Surface strip footing on dense sand overlying compact sand under vertical loads

Assuming $\delta / \phi = -0.9$

Experimental Data				Given	Calculated							
test	(H+D)/B	D/B	H/B	qu	A	Alpha	Tan	$qb / \gamma B$	F	qu/Bg	qu	%
no			KN/m^2				Alpha				KN/m^2	Accu.
36	0.5	0	0.5	111	-7.1	76.73	4.24	122.5	6.85	119.47	99.15	-10.68
37	1	0	1	156	-5.05	74.45	3.59	167.9	7.02	167.15	138.72	-11.08
38	1.5	0	1.5	210	-3	72.17	3.11	213.3	7.18	215.74	179.05	-14.74
39	2	0	2	234	-0.94	69.89	2.73	258.8	7.15	265.6	220.44	-5.79

$\phi_1 = 47.7^o, \gamma_1 = 104\ (pcf), N_{\gamma 1} = 468.3, N_{q1} = 211.8$

$\phi_2 = 42.4^o, \gamma_2 = 95.5\ (pcf), N_{\gamma 2} = 167.95, N_{q2} = 90.84$

Kp*sind= 34.95
phi = 47.7
b = 68.85
q2/q1 = 0.33

Table 3.9 Calculation of the ultimate bearing capacity using the present theory (Hanna 1978, table 2)

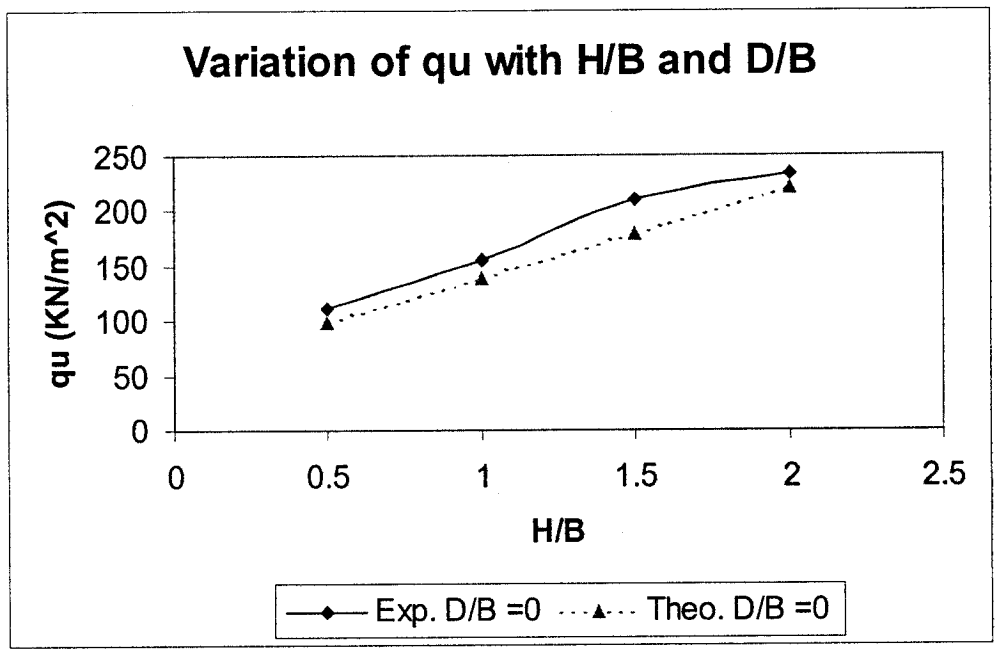

Figure 3.10 Comparison of the experimental and theoretical values of qu (Hanna 1978, table 2)

Table 3.10 (Das & Munoz 1982) contains data for one set of tests, where H/B varies from 0.5 to 2.5 for D/B equals 0. The present theory is applied to the experimental data and the calculated ultimate bearing capacity is compared to the experimental one. All calculations are presented in Table 3.10 and the variation of the ultimate bearing capacity qu with H/B (experimental and present theory data) is presented in Figure 3.11.

Strip footing in dense sand overlying loose sand under vertical loads

Assuming that $\delta / \phi = -0.9$

Experimental Data				Given	Calculated							
test	(H+D)/B	D/B	H/B	qu	A	Alpha	Tan	$qb / \gamma B$	F	qu/Bg	qu	%
no				KN/m^2			Alpha				KN/m^2	Accu.
8	0.5	0	0.5	59.04	-7.10	75.81	3.95	44.05	1.60	45.038	78.01	24.32
9	1	0	1	84.81	-5.05	73.12	3.29	62.92	2.03	66.088	114.47	25.91
10	1.5	0	1.5	125.2	-3.00	70.42	2.81	81.80	2.24	88.059	152.52	17.91
11	2	0	2	157.4	-0.94	67.73	2.44	100.67	2.38	110.96	192.19	18.10
12	2.5	0	2.5	176.2	1.11	65.04	2.15	119.55	2.46	134.83	233.53	24.55

Having

$$\phi_1 = 43^o, \gamma_1 = 108.52\ (pcf),\ N_{\gamma 1} = 186.54,\ N_{q1} = 99.02$$
$$\phi_2 = 36^o, \gamma_2 = 97\ (pcf),\ N_{\gamma 2} = 56.31,\ N_{q2} = 37.75$$

phi	delta	mod Kp	Kp*sind
43.00	38.7	31.712	19.83

b = 66.5

q2/q1 = 0.27

Table 3.10 Calculation of the ultimate bearing capacity using the present theory (Das & Munoz 1982)

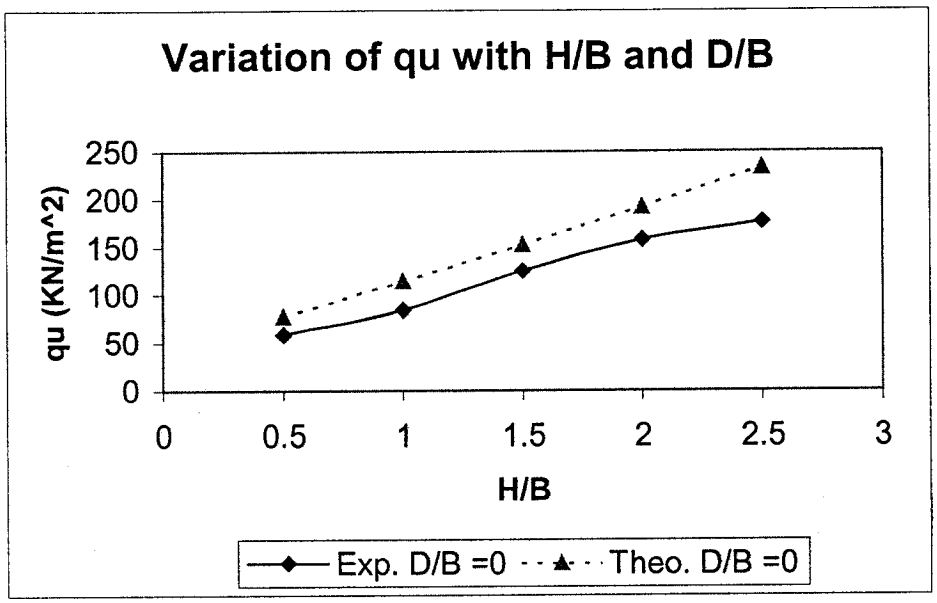

Figure 3.11 Comparison of the experimental and theoretical values of q_u (Das & Munoz 1982)

3.4.4.2 Dense sand layer overlying weak clay deposit

The second part is the case of a dense sand layer overlying a weak clay deposit; a table of experimental and calculated data represents the soil profile of the second case. Table 3.11 (Das 1988) contains data for two series of tests, where H/B varies from 0.25 to 3 and D/B varies from 0 to 0.5. The present theory is applied to the experimental data and the calculated ultimate bearing capacity is compared to the experimental one. All calculations are presented in Table 3.11 and the variation of the ultimate bearing capacity qu with H/B (experimental and present theory data) is presented in Figure 3.12.

Strip footing in dense sand overlying clay layer under vertical loads

Assuming that $\delta / \phi = -0.9$

Experimental Data					Given	Calculated							
Series	test	(H+D)/B	D/B	H/B	qu	A	Alpha	Tan	$qb / \gamma B$	F	qu/Bg	qu	%
no	no				KN/m^2			Alpha				KN/m^2	Accu.
1	2	0.25	0	0.25	75.02	-8.13	70.91	2.89	53.06	0.89	53.50	69.30	-7.63
1	3	0.5	0	0.5	79.98	-7.10	70.39	2.81	53.31	1.34	54.77	70.94	-11.30
1	4	1	0	1	93.29	-5.05	69.34	2.65	53.81	1.84	58.00	75.12	-19.48
1	5	1.5	0	1.5	110	-3.00	68.28	2.51	54.31	2.14	61.81	80.06	-27.22
1	6	2	0	2	133.3	-0.94	67.23	2.38	54.81	2.35	66.13	85.65	-35.75
1	7	2.5	0	2.5	131.6	1.11	66.18	2.27	55.31	2.51	70.91	91.84	-30.21
1	8	3	0	3	140	3.17	65.13	2.16	55.81	2.63	76.15	98.62	-29.55
2	2	0.75	0.5	0.25	83.29	-8.276	70.99	2.9023	53.559	0.897	57.26	74.16	-10.96
2	3	1	0.5	0.5	99.98	-7.132	70.4	2.8087	53.809	1.337	60.29	78.09	-21.90
2	4	1.5	0.5	1	158.30	-4.843	69.23	2.6367	54.309	1.836	65.85	85.29	-46.12
2	5	2	0.5	1.5	210.00	-2.555	68.06	2.4824	54.809	2.134	71.45	92.54	-55.93
2	6	2.5	0.5	2	216.60	-0.266	66.89	2.3429	55.309	2.339	77.32	100.15	-53.76
2	7	3	0.5	2.5	215.00	2.023	65.71	2.2162	55.809	2.492	83.58	108.25	-49.65
2	8	3.5	0.5	3	216.60	4.311	64.54	2.1005	56.309	2.61	90.26	116.91	-46.03

phi1= 43.5 b1 = 66.75

q2/q1 = 0.60

phi	delta	mod Kp	Kp*sind
43.5	39.2	33.378	21.07

Table 3.11 Calculation of the ultimate bearing capacity using the present theory (Das 1988)

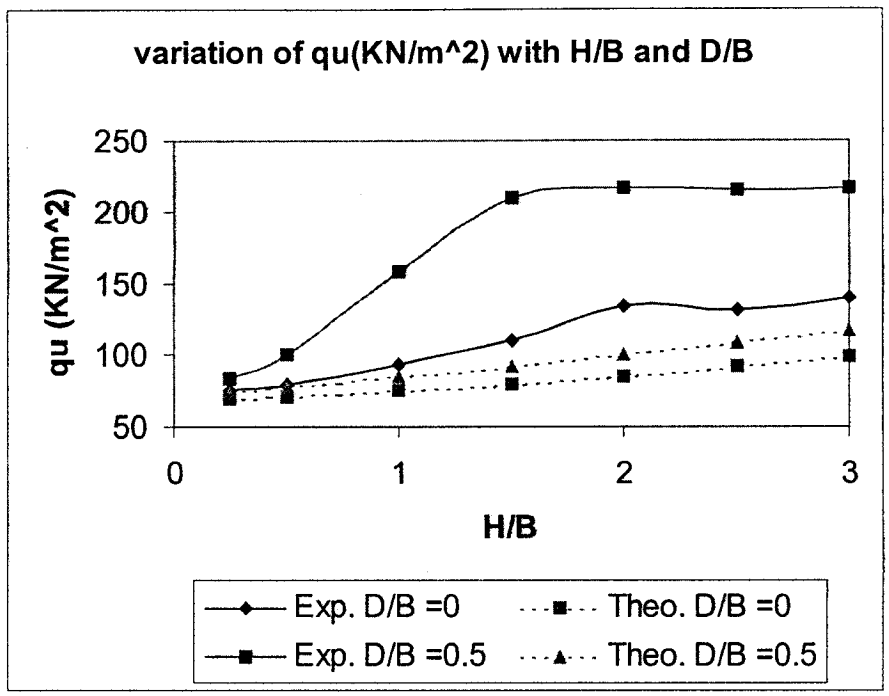

Figure 3.12 Comparison of the experimental and theoretical values of q_u (Das 1988)

CHAPTER FOUR
CONCLUSIONS AND RECOMMENDATIONS

4.1 General

The ultimate bearing capacity of shallow foundations subjected to axial vertical loads and resting on soil consisting of two layers has been investigated for the case of strong cohesionless soil overlying weak deposit. Stress analysis was performed on the actual failure planes and design theory was presented. The following can be concluded:

4.1.1 Dense sand layer overlying loose sand deposit

Stress analysis was performed on the actual failure planes observed in the laboratory. In this analysis, full mobilization of the shear strength on the failure planes was considered. New bearing capacity equation was derived as a function of the properties of the upper and lower soil layers, the footing depth/width ratio and the angle of the failure surfaces with respect to the vertical.

In general, the predicted values of the bearing capacity using the present theory compared well with available experimental data in the literature. The theoretical values of the present theory varied between 1% and 13% for lower values of H/B, and it reaches 17% for values of the ratio H/B of 4.5 and 5, where the case of homogeneous case prevails. Design theory and formulae were presented for practicing use.

4.1.2 Dense sand layer overlying weak clay deposit

In general, the predicted values of the bearing capacity using the present theory compared well with available experimental data in the literature. The theoretical values of the

present theory varied between -7% to -20% for small value of the ratio H/B, but the error increased for higher values of H/B. Further investigations are needed in order to find a better equation of the angle α.

Design theory and formulae were presented for practicing use.

4.2 Recommendations for future research

Additional research in the subject is suggested in order to expand the new punching theory for all practical cases of foundations on layered soils. It should include:

1. Validating the proposed theory with experimental data produced for the case of strip footings under vertical loads in a dense upper sand layer overlying loose sand deposit.
2. Validating the proposed theory and refining the empirical equation of the angle α for the case of strip footings under vertical loads in a dense upper sand layer overlying a weak clay deposit.
3. Extending the punching theory for the case of an upper strong clay layer overlying loose sand or weak clay layers.
4. Extending the punching theory for all shapes of foundations, circular and rectangular foundations, in order to develop the respective shape factors.
5. Extending the punching theory to foundations under inclined loads in order to develop the respective inclination factors.
6. Extending the punching theory to foundations under eccentric loads.

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