# Improved Layered Space-Time Architecture with Unequal Transmit Power Allocation and Multi-Stage Decoding

MHD. Dherar Rezk

A Thesis

in

The Department

of

Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements

for the Degree of Masters of Applied Science at

Concordia University

Montreal, Quebec, Canada

December 2006

*Your file Votre référence*
*ISBN: 978-0-494-28926-6*
*Our file Notre référence*
*ISBN: 978-0-494-28926-6*

# ABSTRACT


## Improved Layered Space-Time Architecture with Unequal Transmit Power Allocation and Multi-Stage Decoding

MHD. Dherar Rezk

The use of multiple antennas at the transmitter and the receiver can significantly increase the data rate and reliability of communications over wireless channels. With constraints in practical implementations, transmission techniques are designed to achieve a certain trade-off between transmission rates and implementation complexity. Layered space time (LST) architecture achieves high data transmission rates in multiple-input multiple-output (MIMO) systems with reasonable complexity through separate multi-user detection and decoding. However, LST architecture is suboptimal since the redundancy in error correcting codes is not fully exploited in detection. This suggests improvements to the existing LST design are possible. Multi-level coding (MLC), which achieves the channel capacity based on the information chain rule, has been applied in MIMO communications for improved performance. In practice, however, rate optimization is difficult to implement. In this thesis, we consider improved LST architecture that provides better performance without increasing the implementation complexity.

For improved performance, we propose the use of multi-stage decoding (MSD) in the LST receiver. The use of MSD exploits the inherent redundancy in error correcting codes in data detection and effectively applies the idea of multi-level coding (MLC). In addition, we propose unequal transmit power allocation to achieve equal capacities among layers. The introduction of transmit power as an additional dimension in design adds more flexibility to the design process and improves performance without increasing implementation complexity.

Based on the notion of unequal power allocation, an improved LST architecture is proposed. The power allocation required to achieve equal capacities among layers in LST architecture is derived. The theoretical analysis of the proposed unequal power allocation is carried out for both fast and quasi-static fading channels based on different criteria. Performance analysis of the proposed approach is conducted and its practical implementation is discussed.

It is shown that the proposed architecture is flexible in terms of implementation and offers a convenient trade-off between capacity and implementation complexity. The difference between achievable capacity in the proposed architecture and theoretical limits is negligible and converges to a constant at high SNR. Simulation results demonstrate that the proposed architecture provides significant performance gain as compared to existing LST architectures and approaches the near optimum bit-interleaved coded modulation (BICM) within a fraction of 1 dB.

Dedicated to my family and to the loving memory of my father............

# ACKNOWLEDGEMENT

I would like to express my sincere appreciation to my supervisor Dr. Xiaofeng Wang for his support and guidance. Dr. Wang's support has been enriching and an invaluable source of help and encouragement throughout my research work.

Special thanks go to Dr. Yousef Shayan for his support during the last stages of my research, in spite of his busy schedule and various commitments. I would like also to thank all those who helped and taught me during my studies leading to the achievements of this thesis.

# TABLE OF CONTENTS

# LIST OF FIGURES

# LIST OF TABLES

# LIST OF ACRONYMS

| | |
|---|---|
| BCJR | Bahl-Cocke-Jelinek-Raviv |
| BICM | Bit Interleaved Coded Modulation |
| CDF | Cumulative Distribution Function |
| DBLAST | Diagonal Bell Laboratories Space-Time |
| HBLAST | Horizontal Bell Laboratories Space-Time |
| HCM | Hybrid Coded Modulation |
| LST | Layered Space-Time |
| MIMO | Multiple-Input Multiple-Output |
| MLC | Multi-Level Coding |
| MLC-SM | Multi-Level Coding with Spatial Multiplexing |
| MSD | Multi-Stage Decoding |
| SISO | Single-Input Single-Output |
| STBC | Space-Time Block Codes |
| STTC | Space-Time Trellis Codes |
| TLST | Threaded Layered Space-Time |
| VBLAST | Vertical Bell Laboratories Space-Time |

# NOTATIONS

Some notations used in the thesis are listed below

| | |
|---|---|
| $a$ | A variable in italic lower-case letter |
| $\mathbf{a}$ | A column vector in bold-face lower-case letter |
| $\mathbf{A}$ | A matrix in bold-face upper-case letter |
| $\mathbf{I}_n$ | An $n$ x $n$ identity matrix |
| $\mathbf{A}^H$ | Hermitian of matrix A |
| det($\mathbf{A}$) | The determinant of matrix A |
| diag( ) | The operator that makes a vector into a matrix with the elements of the vector as the diagonal entries of the matrix. |
| $E[\ ]$ | The expectation of a certain entity |
| $\nabla(f(x))$ | The Lagrangian of a function $f(x)$ |
| $(\mathbf{A})^{-1}$ | The inverse of a matrix **A** |
| $(\mathbf{A})^{1/2}$ | The element-wise square root of matrix **A** |

# Chapter 1

# Introduction

## 1.1 Research Background

The area of wireless communication has been in the focus of active research over the past few decades. The trend of emerging technologies and applications puts a high demand for mobile communication systems that offer reliable, high data rate transmission with reasonable implementation complexity. In order to satisfy these demands, modern wireless communication systems have to overcome various obstacles. First, wireless channels have a highly regulated, scarce and precious radio spectrum. Secondly, transmission over wireless channels is characterized by the phenomena of random fluctuations in the received signal envelope, defined as *fading*, which results in severe degradation of performance. This has geared the search for efficient signalling techniques that can combat fading and achieve high transmission rates without bandwidth expansion. In this regard, *multiple-input multiple-output* (MIMO) technology [1, 2] has received a lot of attention and has been the prevalent choice in increasing spectral efficiency over wireless channels.

In MIMO, multiple antennas are employed at both the transmitter and the receiver. MIMO technology provides increased diversity by delivering multiple copies of the transmitted signal over flat, often independent channel paths to the receiver end of the channel. This, coupled with advanced signal separation techniques employed at the receiver, works as a remedy to signal fading, and increases spectral efficiency without expanding required bandwidth. It was proven in [3] and [4] that, for a MIMO system with $n_t$ transmit and $n_r$ receive antennas, the maximum achievable *spatial diversity gain* [6] is equivalent to $n_t n_r$, assuming flat independent Rayleigh fading between pairs of transmit and receive antennas. Also, it was shown in [1-5] that the capacity of MIMO channels improves by a factor equal to the minimum of $(n_t, n_r)$ compared to capacity of *single-input single- output* (SISO) channels, and that capacity of MIMO channels grows linearly with number of employed antennas.

Efficient signal transmission schemes are needed to realize the enormous capacity and performance gains promised by MIMO technology. For this purpose, *space-time codes* [8, 9] were invented and have received tremendous research effort ever since. Space-time coding exploits the available diversity in MIMO systems, by introducing controlled redundancy of transmitted signals in both temporal and spatial domains through a joint design of error control coding and modulation. Coupled with advanced signal processing algorithms employed at the receiver, Space-time codes were found capable of achieving huge capacities and improved reliability when compared to communication over SISO channels.

Space-time codes vary in their design objectives. In a broad sense, there have been two main approaches to space-time code design. One approach aims at improving transmission reliability through maximizing diversity gain. *Space-time block codes* (STBCs) [3, 10, 11] and *space-time trellis codes* (STTCs) [8] are examples of this approach. The other approach aims at maximizing achievable capacity through increasing *spatial multiplexing gain* [6]. *Layered space-time* (LST) codes, first introduced by Foschini [5] are of the well known architectures that adopt this approach.

In STBCs, two-dimensional coding is applied on a block of input symbols, producing a transmission matrix whose columns represent time and rows represent antennas. The key advantage of STBCs is that it can achieve full diversity gain with orthogonal designs, and has a simple linear maximum likelihood receiver. In [3], Alamouti presented a two transmit, two receive antenna scheme with a simple maximum likelihood decoder that was historically the first scheme to achieve full diversity gain. However, STBCs do not achieve coding gain. Furthermore, certain STBCs introduce loss in spectral efficiency as they require excess bandwidth [12].

STTCs, pioneered by Tarokh, Sechadri and Calderbank, combine error control coding and modulation in a joint design process. Binary bits are mapped to modulation symbols using a trellis structure. At the receiver, decoding is carried out by a Viterbi decoder. STTCs were found capable of achieving substantial diversity and coding gains. However, the complexity of the Viterbi decoder grows exponentially with number of antennas, making the scheme prohibitively complex for high data rate applications [12].

LST or *Bell laboratories layered space time* (BLAST) architecture was found capable of achieving enormous capacities through spatial multiplexing of data while maintaining moderate implementation complexity [12]. In LST, input data is multiplexed into sub-streams called 'layers'. Within each layer, data is independently channel coded, interleaved and modulated. Modulated symbols from a layer are last space-time mapped to transmit antennas according to the LST mapping scheme employed. *Horizontal* [13], *vertical* [14], *diagonal* [5] and *threaded* [15] layered space-time codes (HBLAST, VBLAST, DBLAST and TLST, respectively) are different layered space-time coding schemes. Often, suboptimal receivers that perform detection and decoding separately are employed in LST schemes [16]. In these suboptimal schemes, detection of data layers is carried out using a multi-user detection algorithm [7]. The complexity of the LST receiver increases linearly with the number of antennas, making the scheme suitable for high data rate applications [5]. However, conventional LST receivers do not exploit redundancy in channel codes in data detection, thereby resulting in performance degradation.

*Multi-level codes* (MLC) [17, 18], can be viewed as a direct application of the *information chain rule* [19]. In a MIMO system that employs $n_t$ transmit antennas and $M$-ary modulation, $l = n_t \log_2(M)$ binary bits are mapped to the $n_t$ transmit antennas in each signalling interval. In MLC, the vector of $l$ bits is decomposed into $l$-levels. Each level is channel-encoded independently from other levels. According to the information chain rule, MLC achieves channel capacity if channel code rates in individual levels are optimized to be equal to the capacities of the corresponding sub-channels. At the receiver, data levels are decoded using *multi-stage decoding* (MSD) [17]. In MSD, a complete level is decoded and decoding decisions are used to cancel the level contribution in the received signal, prior to the detection of subsequent levels. This operation continues successively until all levels are decoded. Apparently, the MSD receiver exploits the redundancy in channel coding to improve the detection of the rest data layers. The MLC was found capable of achieving the channel capacity while it employs a moderate complexity decoding technique. However, the need for $n_t \log_2(M)$ channel encoders that must support different rates with fine granularity at the transmitter increases implementation complexity. In addition, channel variations in practice often make an exact rate optimization impossible and hence lead to losses in performance.

*Bit-interleaved coded modulation* (BICM) [20] achieves near optimum performance with a relatively simple transmitter implementation. BICM uses a single channel code followed by a bit inter-leaver. Interleaved bits are then divided into $n_t$ branches, modulated and sent simultaneously over the channel. At the receiver, detection and decoding of data are carried out jointly using a turbo receiver [21]. BICM offers a convenient alternative to MLC. However, the high computational complexity of the BICM receiver makes the scheme undesirable for high data rate transmission.

The aforementioned discussion highlights the fact that the need for transmission schemes that achieve high spectral efficiencies with reasonable implementation complexity still exists. LST and MLC schemes have drawn a lot of attention as they have proven to be efficient with moderate complexity through the use of multi-user detection algorithms at the receiver. However, multi-user detection algorithms suffer the inherent drawback of unequal protection of transmitted signals against interference. In LST, early detected layers suffer greater levels of interference from layers not yet detected. Since the detection of a layer can be undermined by the detection errors in previously detected layers, this could increase error propagation through layers and worsen the overall error performance. Rate optimization in MLC seems a suitable cure to this dilemma, as the transmission rate in each level can be chosen as not to exceed the capacity of the corresponding sub channel. This, combined with the MSD receiver, minimizes error propagation and improves performance. However, as mentioned before, rate optimization in MLC leads to various implementation difficulties.

## 1.2 Problem Statements and Research Objectives

The ever increasing demand for MIMO systems that provide high spectral efficiencies and a fair trade-off between performance and implementation complexity keeps the door open for the search for new MIMO transmission schemes. Of particular interest among schemes that were found to achieve high spectral efficiencies are LST and MLC schemes. The LST scheme offers enormous capacities while it maintains a moderate implementation complexity level through the use of sub-optimal multi-user detection methods at the receiver. However, layers in LST scheme are not equally immune against interference. As discussed earlier, this decreases the robustness of the scheme and makes it susceptible to error propagation. In MLC, channel code rates in levels are optimized to be equal to the capacities of the corresponding sub-channels. The use of MSD at the receiver better exploits redundancy in coding for improved performance. However, optimized coding rates are sensitive to channel variations, thus making the system inflexible and adding to the implementation complexity.

In this thesis, we propose a new LST scheme with improved performance but with relatively similar complexity. We investigate the effects of transmit power allocation among layers on performance. The introduction of power allocation as a design parameter offers more flexibility and mitigates the conventional, often harsh trade-off between implementation complexity and performance. We also propose the employment of MSD at the receiver. The use of MSD takes advantage of the inherent redundancy in error correcting codes in improving detection, and effectively applies the mutual information chain rule in LST schemes. When combined with MSD, the goal of transmission power allocation is to equalize ergodic capacities among layers, and offer higher immunity against interference to early detected layers. This results in simplified implementation and improved robustness of the LST scheme over fast fading channels.

However, in quasi-static fading channels, the ergodic capacity is no longer achievable with reasonable code delay. In this case, a more appropriate capacity measure is the outage capacity. Hence, different power optimization criteria shall be taken for different channels. Below, we present our research objectives for fast and quasi-static fading channels, respectively.

- Develop an improved LST architecture based on transmission power allocation among layers combined with MSD. Find the optimal transmission power allocation that ensures equal capacities among layers in an LST architecture over flat fast Rayleigh fading channels. Find the losses in capacity due to unequal power allocation. Compare the proposed scheme with existing schemes over fast flat Rayleigh fading channels in terms of performance and complexity.

- Generalize the proposed architecture to flat quasi-static Rayleigh fading channels on basis of outage capacity. Derive the power allocation that maximizes outage capacity for a given maximum outage probability. Derive the power allocation that maximizes outage capacity for a given maximum outage probability in the proposed architecture under the constraint of equal rates among layers. Analyze losses in outage capacity due to the adopted constraint. Compare the proposed scheme and existing LST schemes over quasi-static Rayleigh fading channels in terms of performance and complexity.

## 1.3 Contributions

In this thesis, we first introduce an improved LST architecture with unequal transmit power allocation among layers combined with an MSD receiver. The power allocation that ensures equal ergodic capacities among layers over fast flat Rayleigh fading MIMO channels is derived. The optimal power allocation as a function of channel SNR is investigated and results show that the power allocation quickly converges to its asymptotic solution. This is important as it suggests that a constant power allocation can be used in practice whenever the SNR is relatively high. The anticipated loss in capacity due to unequal power allocation is also investigated and is provided in closed form for the asymptotic case, (i.e.), when the loss is at its maximum level. Results demonstrated that the loss is negligible and converges to a constant independent of transmit power. Simulation results show that the proposed scheme significantly outperforms the conventional LST schemes without increasing the complexity.

The proposed architecture is extended to quasi-static Rayleigh fading channels. We derive power allocation that maximizes outage capacity under the constraint of equal rates among layers for a given maximum outage probability. We show that the derivation is the solution of an optimization problem with nonlinear constraints. For comparison purpose, the power allocation that maximizes the outage capacity without the constraint of equal rate among layers is also derived. We show that equal power allocation maximizes outage capacity for a given maximum outage probability. Given this, we investigate the losses in outage capacity due to unequal power allocation for different values of maximum outage probability. Results show that, similar to fast fading channels, outage capacity loss is marginal and converges to a constant value at high channel SNR. Simulation results show that the proposed scheme significantly outperforms the conventional LST schemes over quasi-static fading channels.

## 1.4 Thesis Outline

The rest of the thesis is organized as follows:

Chapter 2 presents a review of key concepts and transmission schemes related to the problem under investigation. First, the MIMO channel model and the concepts of ergodic and outage capacity are introduced. Then, the LST architecture and LST mapping schemes are presented. After that, MLC – based schemes and BICM are introduced.

Chapter 3 presents the proposed architecture. Derivation and analysis of power allocation are presented for flat fast Rayleigh fading channels. Capacity loss is then investigated. Lastly, simulation results are provided.

Chapter 4 presents the derivation and analysis of the proposed power allocation for flat quasi-static Rayleigh fading channels. Simulation results are then provided.

Chapter 5 concludes the thesis including a summary of the presented research work and an outline for the future work.

# Chapter 2

# Preliminary

In this chapter, background knowledge and review of literature closely related to the research subject are presented to provide the right context for the problem under investigation. The chapter is organized as follows: The MIMO channel under consideration, its model, and concepts of ergodic and outage capacity are discussed in section 2.1. Several popular MIMO transmission schemes that are designed to achieve high spectral efficiencies are discussed in the sections that follow. In section 2.2, LST architecture, its advantages and drawbacks are reviewed. Then, MLC, various MLC - based schemes and BICM are reviewed in sections 2.3, 2.4 and 2.5 and contrasted with respect to their achievable capacity, flexibility and complexity of implementation. The discussion will highlight the needs in transmission schemes design and will establish the validity of the proposed approach detailed in subsequent chapters.

## 2.1 Channel Model

Wireless communication is characterized by multi-path propagation of the transmitted signal between the transmitter and the receiver. Depending on the relative size and texture, objects in the transmission medium such as vehicles, buildings, trees and foliage might act as shaders, scatterers or reflectors of the transmitted signal. Consequently, multiple copies of the transmitted signal arrive at the receiver from different directions and with different propagation delays. As these copies have different phases due to different propagation delays and angles of arrivals, they might add constructively or destructively. Possible variations in the transmission medium such as movement of objects and weather conditions cause changes in the attributes of the received signal. These unpredictable time-variations in propagation paths induce random fluctuations in the received signal envelope, which is defined as *fading*.

The randomness in the received signal envelope raises the need for a statistical model of the channel. In the case of a large number of propagation paths, the central limit theorem states that the received signal might be modelled as a complex Gaussian random process with zero mean. Thus, the received envelope can be modelled as a Rayleigh-distributed random variable [12]. In the presence of a static channel path between the transmitter and the receiver, (i.e.), line of sight (LOS) communications, the random process is no more zero-mean and thus the received signal envelope is modelled by a Rician –distributed random variable.

Fading channels are categorized according to different criterions. With respect to their frequency response, fading channels can be classified into frequency selective, and frequency non-selective, or flat fading channels. In the latter, the bandwidth of the transmitted signal is much smaller than the coherence bandwidth of the channel. Thus, all frequency components of the signal undergo the same fading pattern. Conversely, frequency components in the signal bandwidth suffer different fading patterns over frequency-selective channels which results in heavy signal distortion.

With respect to the rate of change, fading channels are categorized as slow, and fast fading channels. In slow fading channels, the channel is assumed constant over a number of successive transmission periods. In fast fading channels, channels are assumed to change rapidly with respect to symbol time.

In this report, we assume transmission over flat, fast and quasi-static Rayleigh fading MIMO channels. The channel state information is assumed to be perfectly known at the receiver but unknown at the transmitter. In fast fading channels, the channel is assumed to change from symbol to symbol. While in quasi-static channels, the channel is assumed constant over the period of a whole transmission frame, but changes from frame to frame.

Consider a MIMO channel with $n_t$ transmit and $n_r$ receive antennas as the one represented in Fig. 2.1. $h_{ij}$ denotes the path gain between the $i$th transmit antenna and the $j$th receive antenna.

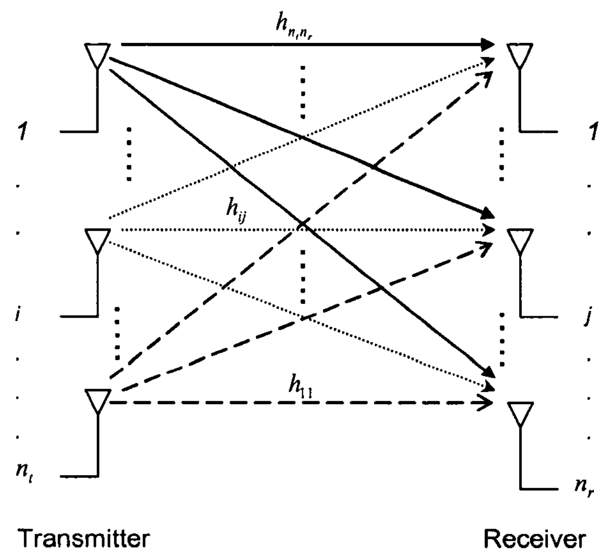



**Fig. 2.1: MIMO channel model**

Assuming uncorrelated transmissions, path gains are modelled as independent, identically distributed (i.i.d) Rayleigh random variables in magnitude with uniform phase over the range [0-2π]. The received signal vector $\mathbf{r} = [r_1, r_2, ..., r_{n_r}]^T$ is given by

$$\mathbf{r}=\mathbf{Hx}+\mathbf{n} \tag{2.1}$$

where $\mathbf{H}$ is the $n_r \times n_t$ channel matrix, composed of channel path gains between transmit and receive antennas, $\mathbf{x} = [x_1, x_2, ..., x_{n_t}]^T$ is the $n_t \times 1$ transmitted vector, and $\mathbf{n}$ is the circularly symmetric complex Gaussian distributed $n_r \times 1$ AWGN vector with $E[\mathbf{nn}^H] = \sigma_n^2 I_{n_r}$.

### 2.1.1 MIMO Channel Capacity

Ergodic Capacity is the maximum transmission rate achievable at an arbitrarily small probability of error. The MIMO channel ergodic capacity is given by [2]

$$C = \log_2 \det(I_m + \frac{P}{n_t \sigma_n^2}\mathbf{Q}) \qquad (2.2)$$

where $P$ is the total transmission power, $\sigma_n^2$ is the variance of the Gaussian noise and $\mathbf{Q}$ is given by

$$\mathbf{Q} = \begin{cases} \mathbf{H}.\mathbf{H}^H : n_r < n_t, m = n_r \\ \mathbf{H}^H.\mathbf{H} : n_r \geq n_t, m = n_t \end{cases}$$

Channel ergodic capacity is achieved when data vector $\mathbf{x}$ is zero mean circularly symmetric complex Gaussian with covariance matrix $E[\mathbf{x}\mathbf{x}^H] = \frac{P}{n_t} I_{n_t}$ [2]. It was proven in [1] that ergodic capacity is achieved with equal allocation of transmission power among antennas.

For certain types of channels, the rate of change of channel realization cannot be modeled as an ergodic process, i.e., the channel is random but remains constant over a frame long enough from information theoretic point of view. Quasi-static channels are an example. In quasi-static channels, a channel realization is represented by a random matrix $\mathbf{H}$ which remains constant over a whole data transmission frame but changes from frame to frame. Consequently, the ergodic capacity is zero since there is a non-zero probability that the channel realization $\mathbf{H}$ does not support transmission at an arbitrarily small probability of error, regardless of the rate of transmission. In this case, a more convenient measure is *outage capacity* [2].

Channel capacity $C$ can be modeled as a random variable since it is a function of $\mathbf{H}$. A channel outage occurs if, for a certain channel realization, channel capacity $C$ falls below the transmission rate $r$. Channel outage occurs with probability $P_{out} = P(C \leq r)$, which can be found from the cumulative distribution function (CDF) of $C$. The channel outage capacity $C_\varepsilon$ is defined as the maximum rate at which arbitrarily reliable transmission is possible for a minimum

(1-ε) x100 percent of channel realizations [2]. That is

$$C_\varepsilon = \max(r) : P_{out} \le \varepsilon \tag{2.3}$$

Spectrally efficient transmission schemes are required to realize the enormous capacities promised by MIMO technology. Initially, MIMO technology has been used to provide diversity through delivery of multiple independent copies of the same signal at the receiver. This improves reliability and combats fading. However, a different approach in code design suggests that MIMO can be used to improve spectral efficiency through increasing the multiplexing gain. That is, independent data are delivered over parallel channels between pairs of transmit and receive antennas. There exists a trade-off between achievable diversity gain and multiplexing gain [2, 6] in space-time code design. Certain code designs are aimed at maximizing the multiplexing gain in order to achieve higher capacities. Several popular examples are discussed in the upcoming sections.

## 2.2 Layered Space-Time (LST) Codes

LST architecture design objective is to achieve enormous transmission rates over MIMO channels through maximizing spatial multiplexing gain. The LST receiver is a simple receiver that employs a sub-optimal multi-user detection algorithm for signal detection and treats detection and decoding separately. The computational complexity of the LST receiver increases linearly with the number of employed antennas, making LST quite suitable for high data rate applications.

A block diagram of a general LST transmitter is shown in Fig. 2.2. As illustrated, data stream is first multiplexed into sub-streams, called layers. Channel coding, modulation and space-time mapping of data in each layer are treated as separate blocks. Data in each layer is

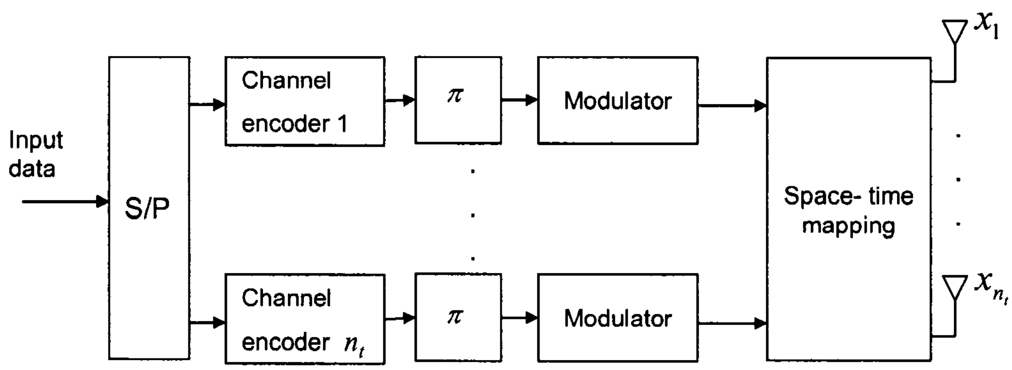



**Fig. 2.2: General LST transmitter**

channel encoded, interleaved and modulated independently from other layers. Modulated symbols from different layers are then mapped to transmit antennas according to the space-time mapping scheme employed. In VBLAST, for each layer, modulated symbols are transmitted from a certain transmit antenna. In DBLAST, mapping of symbols from different layers to transmit antennas is cycled periodically. A DBLAST transmission matrix for $n_t$=4 is illustrated in Fig. 2.3.

$$\begin{pmatrix} x_1^1 & x_1^2 & x_1^3 & x_1^4 & x_5^1 \ldots\ldots \\ 0 & x_2^1 & x_2^2 & x_2^3 & x_2^4 \ldots\ldots \\ 0 & 0 & x_3^1 & x_3^2 & x_3^3 \ldots\ldots \\ 0 & 0 & 0 & x_4^1 & x_4^2 \ldots\ldots \end{pmatrix}$$

**Fig. 2.3: Transmission matrix of DBLAST**

Modulated symbols from the four layers are respectively $x_1^1$ $x_2^1$ $x_3^1$ $x_4^1$..., $x_1^2$ $x_2^2$ $x_3^2$ $x_4^2$....,$x_1^3$ $x_2^3$ $x_3^3$....., and $x_1^4$ $x_2^4$..., where $x_k^i$ represents the *k*th symbol from the *i*th layer. As shown, symbols from each layer occupy a diagonal in the transmission matrix. Thus, transmission of each layer is shared by all the $n_t$ transmit antennas. As such, all layers are equally exposed to all fading profiles of channel paths between the transmitter and the receiver. This offers increased spatial multiplexing gain compared to VBLAST.

TLST goes a step further beyond D-BLAST. Modulated symbols from layers are spatially interleaved in place. Zeros padded in DBLAST transmission matrix that stand for unutilized transmit antennas at specific times in every transmission frame are eliminated, thus resulting in improved spectral efficiency.

In the LST receiver, detection and decoding of the received signal vector are carried out separately. A sub-optimal multi-user detection algorithm, such as the *ordered successive interference cancellation and nulling* proposed for VBLAST [16], or the *minimum mean square error* (MMSE) [22, 23], with successive interference cancellation is employed at the receiver front to separate the received modulated symbols into their respective layers. In each layer, decisions at the detector output are used to cancel the contribution of the layer in the received signal, prior to detection of subsequent layers. After detection, data in each layer is demodulated, de-interleaved and decoded independently from other layers.

It is clear that LST architecture is sub-optimal since it is based on the concept of layering [2]. However, LST attains the lower bound on capacity of an $n_t \mathrm{x} n_r$ MIMO channel, given by [5]

$$C = \sum_{i=(n_t-(n_r-1))}^{n_t} \log_2(1+\chi_{2i}^2) \tag{2.4}$$

where $\chi_{2i}^2$ is a Chi-square distributed random variable with $2i$ degrees of freedom.

The enormous capacity offered and the linear increase in implementation complexity with respect to the number of employed antennas make the scheme quite suitable for high data rate applications. However, LST receiver suffers a potential drawback. Layers face different levels of interference according to their order of detection, (i.e.), firstly detected layers suffer greater interference from layers not yet detected. As detection of each layer depends on the successful detection of previously detected layers, errors in detection propagate through layers. This causes deterioration of overall error performance. It is argued in [5] that the employment of powerful error correcting codes and/or maximum SNR detection could be possible cures to mitigate this problem. However, the employment of powerful error correcting codes does not eliminate the problem, and implies a loss in spectral efficiency for a little improvement in performance. As well, results in [5] show that the improvement in performance of the LST architecture with maximum SNR detection is marginal.

## 2.3 Multi-Level Coding (MLC)

MLC can be viewed as a realization of the chain rule for mutual information [19]. MLC was first introduced as a technique for achieving high spectral efficiency over SISO channels. The high spectral efficiency and moderate complexity made MLC an attractive candidate for extension to communication over MIMO channels, which was first introduced in [17]. For a system that employs $n_t$ transmit antennas and $M$-ary modulation, the MIMO channel is decomposed into $l = n_t \, . \log_2(M)$ parallel channels. A number of $l$ bits are sent over the $l$ parallel channels in every channel use. According to the chain rule of mutual information, the mutual information between the transmitter and the receiver can be expressed as

$$I(\mathbf{b};\mathbf{y}) =$$

$$I(b_0;\mathbf{y} \mid \mathbf{H}) + I(b_1;\mathbf{y} \mid \mathbf{H}, b_0) + ..... + I(b_i;\mathbf{y} \mid \mathbf{H}, b_0,...,b_{i-1}) + ..... + I(b_l;\mathbf{y} \mid \mathbf{H}, b_0,...,b_{l-1}) \tag{2.5}$$

where $\mathbf{b}=[b_0, b_1,....,b_{l-1}]$ is the $l$-bits vector, mapped to $n_t$ modulated symbols which are transmitted over the $n_t$ transmit antennas in every channel use, and $\mathbf{y}$ is the received signal vector. An MLC transmitter is illustrated in Fig.2.4.

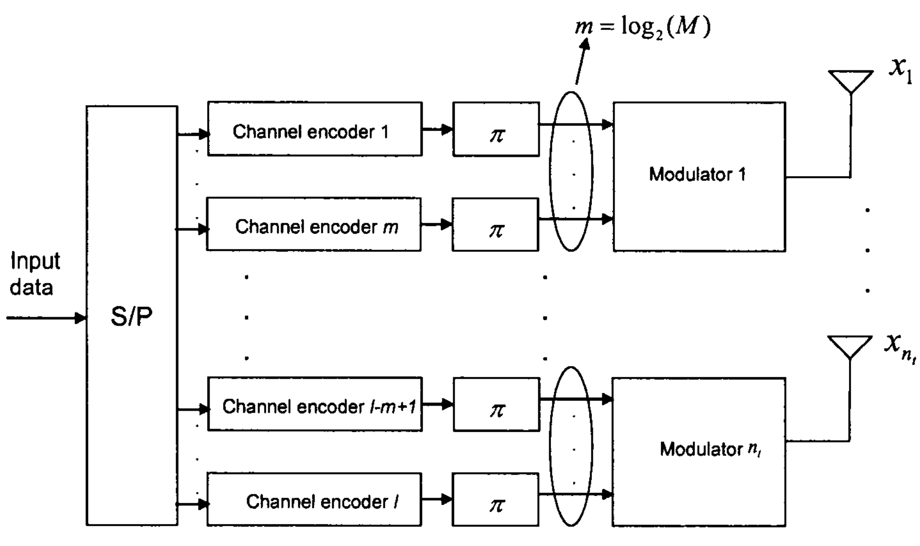



**Fig. 2.4: MLC transmitter**

As shown, data is first divided into $l$ levels, where data in each level is channel encoded and randomly interleaved independently from data in other levels. Afterwards, the $l$ levels are grouped into $n_t$ groups, each group containing $m$ levels. For every channel use, output bits in each group are mapped to a modulated symbol. The produced $n_t$ modulated symbols are transmitted simultaneously over the channel.

Multi-stage decoding (MSD) technique is used in the MLC receiver. A block diagram of an MLC receiver is illustrated in Fig. 2.5.

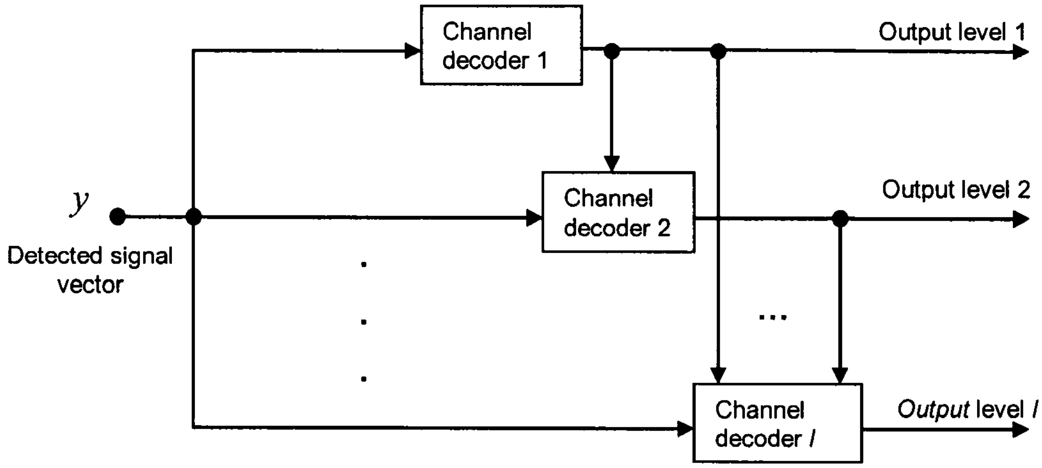



**Fig. 2.5: A multi-stage decoder for MLC**

A sub-optimal multi-user detection algorithm is employed at the receiver front for detection. Upon detection and demodulation of received symbols, bits in each level are de-interleaved and decoded independently from other levels. For better detection, decoding decisions information at the decoder output in each level are re-interleaved and forwarded to the detector in the next level to help cancel the decoded level contribution in the received signal. This operation continues successively until decoding of all levels is accomplished. Apparently, detection in each level relies on decoding decisions in previously decoded levels which are assumed with high probability to be true. This effectively applies the mutual information chain rule principle in decoding, and exploits the redundancy in error correcting codes to improve detection.

It was shown in [17] that MLC achieves the ergodic capacity of the channel if and only if the coding rate in each level is chosen to be equal to its corresponding sub-channel capacity. That is, for level $i$, coding rate $r_i$ must be chosen such that

$$r_i = c_i \quad \forall i \in \{0,1,\ldots,l-1\} \tag{2.6}$$

where $c_i$ is the $i$th sub-channel capacity, given by

$$c_i = I(b_i; \mathbf{y} \mid \mathbf{H}, b_0, b_1, \ldots\ldots, b_{i-1}) \tag{2.7}$$

However, the use of $n_t.m$ channel encoders at the transmitter increases implementation complexity and introduces considerable decoding delay. This puts a limit on the number of employed antennas and/or constellation size thus making the scheme hard to implement in high data rate applications. Furthermore, capacities of the sub-channels depend on the channel state. The possible fall of capacities of sub-channels below optimized transmission rates due to variations in channel state, coupled with the inter-level dependence in decoding at the receiver could cause the overall performance to rapidly deteriorate. This makes the system inflexible especially for communication over slow fading channels, in which certain levels might be trapped in deep fades over a large number of consecutive symbols.

In the MSD receiver, firstly detected levels have poorer error performances due to heavy interference from levels not yet detected, which results in error propagation. Exploitation of redundancy in error correcting codes in interference cancellation does not eliminate this problem. It was suggested in [17] that soft decoding information could be used in interference cancellation as to reduce detection errors and error propagation. Another proposed solution was the feedback of decoding decisions in higher levels to firstly detected levels, and repetition of the whole MSD process in an iterative manner, which eventually increases complexity of the MSD receiver to approach that of the turbo receiver.

## 2.4 Bit-Interleaved Coded Modulation (BICM)

The near optimum performance of BICM over SISO channels motivated its application for transmission over MIMO channels, which was firstly introduced in [20]. BICM follows a simple approach that has been proven to achieve high capacities. A block diagram of a BICM transmitter is shown in Fig. 2.5.

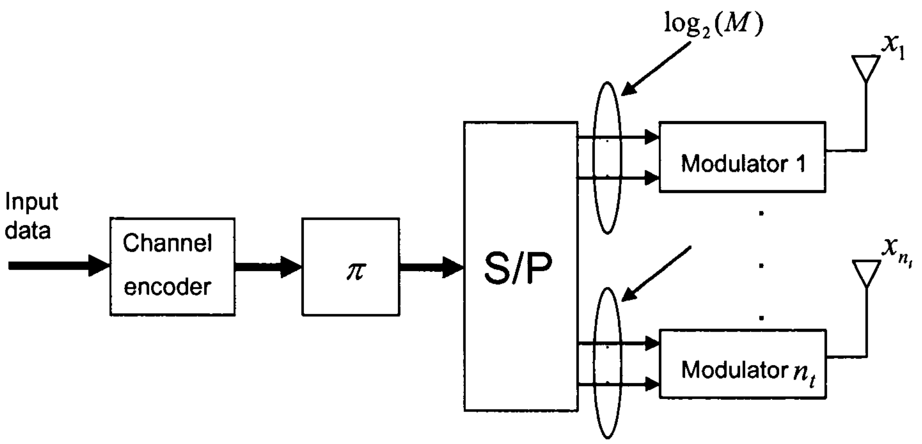



**Fig.2.6: BICM transmitter**

As shown, the input data stream is encoded using a single channel code. Encoded data are then interleaved and multiplexed into $n_t$ sub streams. Data sub-streams are modulated and transmitted simultaneously over the channel. From an information theoretic point of view, BICM neglects the dependencies between bit levels in mutual information between the transmitter and the receiver. Thus the mutual information between the transmitter and the receiver in BICM reduces to

$$I(\mathbf{b};\mathbf{y}) = \sum_{i=1}^{n_t} I(b_i;\mathbf{y} \mid \mathbf{H}). \tag{2.8}$$

This implies equal rate assignment which justifies the use of a single code. The employment of encoding prior to data multiplexing increases multiplexing gain and simplifies implementation. The BICM receiver is an iterative joint detection and decoding turbo receiver. Detection and decoding of data are carried out jointly using a turbo receiver. Decoding decision information are fed back to the detector to help better detection of the received signal. This operation is repeated further in an iterative fashion. It was shown in [20] that BICM with Gray labeling achieves near optimum performance over the two-input two-output (2x2) MIMO channel. However, as number of antennas increases, the gap between BICM performance and optimum performance increases. Also, the scheme is sensitive to the employed signal labeling technique. In the BICM turbo receiver, computational complexity increases exponentially with the number of employed antennas. Results in [24] suggest that BICM is a convenient alternative for MLC only for the case of the 2x2 MIMO channel.

## 2.5 Other Multi-Level Coding Based Schemes

There have been a number of schemes based on the MLC principle. These schemes are aimed to attain capacities close to that offered by MLC but with more flexibility and ease of implementation. *Hybrid coded modulation* (HCM) [24] is a variant of MLC that reduces the complexity and delay of the scheme. In HCM, the chain rule of mutual information is applied on a per-antenna basis. Consequently, the number of levels, thus number of required channel codes is reduced to $n_t$. This architecture has an interesting resemblance to the VBLAST scheme, except for the fact that the coding rates in levels in HCM are optimized to be equal to capacities of the corresponding sub-channels. It is obvious that HCM induces a certain loss in capacity when compared to MLC. However, results in [24] show that capacity loss for HCM with Gray labeling when compared to MLC is negligible, and that the performance of HCM with Gray labeling can approach the theoretical limits within 1-1.5 dBs. However, HCM performance is sensitive to signal labeling technique used. As well, transmitter complexity increases with increasing $n_t$.

*Multi-level coding with spatial multiplexing* (MLC-SM) [25] offers flexibility with respect to signal labelling techniques and number of transmit antennas $n_t$. In MLC-SM, the chain rule of mutual information is applied on a per-bit address level, (i.e), number of levels is equal to $\log_2(M)$, when $M$-ary modulation is used. Data is divided into $m$ levels, where data in each level are encoded and interleaved separately. After that, bit levels enter the modulation stage. For every channel use, output bits of all $m$ levels are grouped in tuples of $m$-bits each, where the $i$th bit in each tuple is taken from the inter-leaver output in level $i$. Then, every group of $n_t$ consecutive tuples are mapped to $n_t$ modulated symbols, which are then transmitted simultaneously from the $n_t$ transmit antennas. It can be observed that according to this structure, all levels in MLC-SM share equal access to all $n_t$ transmit antennas. Therefore, MLC-SM achieves high spatial multiplexing gain. Moreover, MLC-SM implementation complexity is decoupled from the number of employed antennas, making the scheme more suitable for high data rate applications.

Results in [25] show that, for non-iterative receivers, MLC-SM achieves higher ergodic capacities when compared with BICM and HCM and, unlike HCM and BICM, is not sensitive to the signal labeling technique used. Similar results were obtained for transmission over quasi-static channels, this is due to the spatial multiplexing gain offered by the scheme, and highly valued over quasi-static channels with scarce temporal diversity. However, results show that the gap between performance of the non-iterative receiver and the theoretical limits over quasi-static channels widens. This raises the need for iterative approaches for further improvements in performance.

# Chapter 3

## Improved LST Architecture for Fast Fading Channels

In this chapter, we present an improved LST architecture that employs multi-stage decoding at the receiver, combined with unequal allocation of transmission power among layers at the transmitter. Power allocation is optimized to guarantee equal ergodic capacities among layers. This, as opposed to rate optimization in MLC, provides flexibility and ease of implementation. As well, the use of MSD effectively applies the MLC principle and improves detection in the LST scheme, relaxing the need for complex iterative approaches.

The chapter is organized as follows: First, we begin with a description of the proposed architecture. Then, we derive a theorem for the unequal power allocation that achieves equal ergodic capacities among layers in the LST scheme with MSD decoding for the asymptotic case. Afterwards, we investigate the loss in ergodic capacity in the proposed scheme and derive a closed form representation of the scheme asymptotic capacity loss. Finally, we present simulation results that compare performances of the proposed scheme and other transmission schemes of interest over flat fast Rayleigh fading channels.

## 3.1 System Structure and Description

The proven high spectral efficiency of LST scheme coupled with its moderate implementation complexity has motivated its use in high data rate transmission over MIMO channels. LST maximizes multiplexing gain through spatial interleaving of layers. For practical complexity, sub-optimal receivers that perform separate detection and decoding are often used. As detection and decoding in LST are treated separately, conventional LST receivers do not make use of the redundancy in channel codes to reduce detection errors. Due to this, significant performance degradation can be observed as compared to other schemes that achieve high data rate transmission, like the near optimum BICM combined with a turbo receiver. However, BICM is often undesirable for high data rate applications due to its high computational complexity.

We propose the employment of MSD in LST receivers as a better way to improve LST performance apart from complex iterative approaches. MSD offers a compromise between detection improvement and implementation complexity. MSD was originally proposed for the decoding of MLC schemes. Interestingly, an LST transmitter can be regarded as an MLC scheme with coding performed on a per-antenna level (HCM), which nearly achieves channel capacity. However, rate optimization in HCM requires use of $n_t$ different channel encoders with fine granularity at the transmitter. Furthermore, coding rates are sensitive to SNR. These, coupled with variations in channel SNR makes rate optimization difficult in practice. As an alternative to rate optimization, we propose unequal transmit power allocation in layers. The unequal power allocation is optimized to achieve equal ergodic capacities among layers. This simplifies implementation and improves performance when multi-user detection methods are used at the front end of the receiver. As shown later in the chapter, the proposed scheme significantly outperforms conventional LST without increasing implementation complexity.

Figs. 3.1 and 3.2 are block diagrams of the transmitter and receiver structures in the proposed scheme, respectively. In Fig. 3.1, the transmitter is the same as the LST transmitter except for the unequal power allocation among layers, (i.e.), $p_i \neq p_j$ for $i \neq j$, $i, j \in \{1, \ldots, n_t\}$. Unequal power allocation is optimized to guarantee equal capacities among layers. Generally, transmission power is assigned to layers according to their order of detection. That is, higher power is assigned to early detected layers that suffer higher levels of interference from layers not yet detected. As a general rule, $p_i > p_j$ for $i < j$. Equal capacities among layers raise the possibility of using the same channel code in all layers, thus resulting in simplified code design and easy parallel implementation. Moreover, the proposed power allocation optimizes overall error performance as it reduces the gap in error performance between layers.

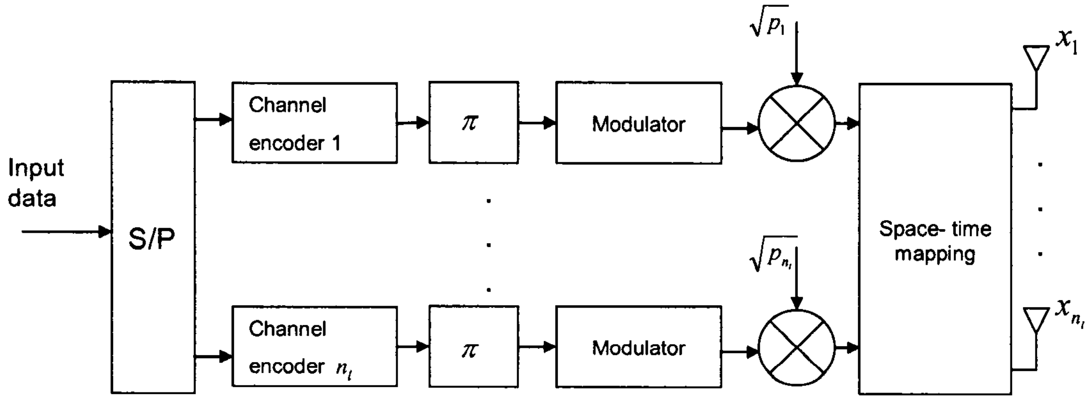



**Fig. 3.1: Block diagram of the transmitter in the proposed architecture**

As long as the fading channel is sufficiently fast, the proposed transmitter design is applicable to all LST mapping schemes. In practice, however, TLST is preferable as it offers higher multiplexing gain and bandwidth efficiency than other schemes.

Fig. 3.2 is the block diagram of the proposed MSD receiver. For convenience, it is assumed that $n_t \leq n_r$, so that simple linear multi-user detection algorithms can be used at the receiver front end. For the case when $n_t > n_r$, a number of layers equal to $n_r$ need to be generated. In fact, there is a little incentive to use more than $n_r$ data layers since the maximum multiplexing gain of an $n_t n_r$ channel is $\min(n_t, n_r)$.

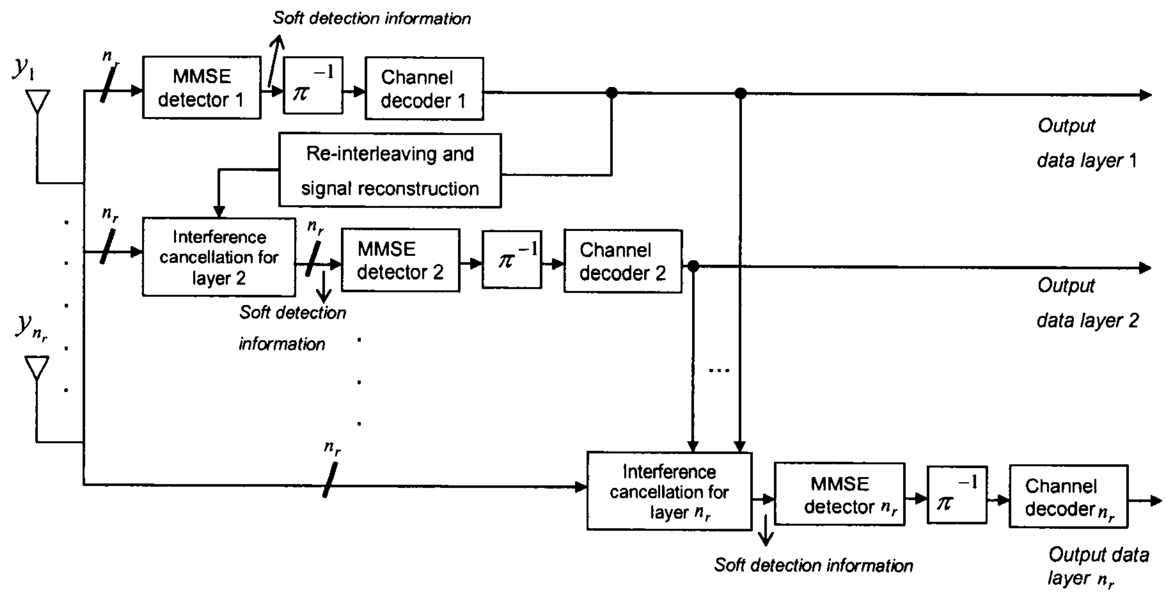



**Fig. 3.2: Block diagram of the receiver in the proposed architecture**

In Fig. 3.2, the MMSE detector is first employed to provide soft estimates of the symbols from layer 1. The soft estimates at the detector output in layer 1 are then de-interleaved and used in decoding by a soft Viterbi decoder. Afterwards, the decoded layer 1 data are re-interleaved and modulated to construct its contribution in the received signal, which is then cancelled from the received signal $\mathbf{y}$ by the interference canceller in layer 2. The resulting signal is then used for the detection of layer 2. This process continues successively until all the $n_t$ layers are decoded.

The proposed receiver exploits the inherent redundancy in channel codes to improve LST detection. Furthermore, in addition to simplified transmitter and receiver design, the proposed power optimization when combined with MSD offers higher protection against interference to firstly detected layers. This has a direct influence in improving detection and decreases error propagation through successively detected layers. Thus, as opposed to rate optimization, the proposed power optimization improves overall error performance while eliminating the need for iterative MSD which is unsuitable for high data rate and delay sensitive applications.

In the next section, we derive a theorem for the power allocation required to ensure equal channel capacities among layers in the proposed architecture.

## 3.2 Derivation of the Unequal Power Allocation in the Proposed LST Architecture

In this section, we derive the power allocation required to achieve equal ergodic capacities among layers in the proposed scheme for a system with $n_t$ transmit and $n_r$ receive antennas. It is assumed that $n_t \le n_r$. It is also assumed that channel state information is perfectly known at the receiver but unknown at the transmitter.

In referral to (2.1) in Chapter 1, the system model equation is rewritten for convenience as

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{1/2}\mathbf{x} + \mathbf{n} \tag{3.1}$$

where $\mathbf{P} = diag[p_1,..p_i,...p_{n_t}]$, with $p_i$ the transmit power assigned to layer *i*.

Since the error rate in a practical communication system has an arbitrarily small value, it is fair to assume ideal detection and decoding for each layer. As such, for the detection of layer *i* in the proposed MSD receiver, contribution of layers *1* to *i-1* is perfectly cancelled from the received signal before detection. Thus, the input to the detector in layer *i*, denoted by $\mathbf{y}_i$, is

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{P}_i^{1/2} \mathbf{x}_i + \mathbf{n} \tag{3.2}$$

where $\mathbf{H}_i$ is an $n_r \times (n_t - i + 1)$ matrix consisting of the last $n_t - i + 1$ columns of **H**, $\mathbf{x}$=$[x_i x_{i+1} ... x_{n_t}]$, and $\mathbf{P}_i = diag[p_i, p_{i+1},..., p_{n_t}]$. Apparently, $\mathbf{y}_1 = \mathbf{y}$. Given this, it follows that the capacity of layer *i* in nats can be expressed as

$$\begin{aligned} C_i &= E(\log(1 + p_i \mathbf{h}_i^H (\mathbf{H}_{i+1}{}^H \mathbf{P}_{i+1} \mathbf{H}_{i+1} + \sigma_n^2 \mathbf{I}_{n_r})^{-1} \mathbf{h}_i) \\ &= E \log(a_i) \end{aligned} \tag{3.3}$$

where the expectation is taken with respect to the channel matrix $\mathbf{H}$. By definition of $\mathbf{H}_i$, $\mathbf{H}_{n_t+1}$ is a zero matrix. The term $p_i \mathbf{h}_i^H \mathbf{h}_i$ stands for the power of the received symbol $y_i$, while the term $(\mathbf{H}_{i+1} \mathbf{P}_{i+1} \mathbf{H}_{i+1}^H + \sigma_n^2 \mathbf{I}_{i+1})$ represents the total noise present at the detector input in layer *i*, which is comprised of interference from layers *i+1*.... $n_t$ not yet detected, and the channel Gaussian noise.

Noting that $a_i$-1 is the output SNR of the MMSE detector in layer *i*, it is related to the Cholesky decomposition of the auto-correlation matrix of the channel [26-27]. Specifically, the autocorrelation matrix is given by

$$\hat{\mathbf{R}} = \hat{\mathbf{P}}^{1/2} \hat{\mathbf{H}}^H \hat{\mathbf{H}} \hat{\mathbf{P}}^{1/2} \tag{3.4}$$

where $\hat{\mathbf{H}}$ is the matrix obtained by left-right flipping **H,** (i.e.), $\hat{\mathbf{H}} = [\mathbf{h}_{n_t}\mathbf{h}_{n_t-1}...\mathbf{h}_1]$, and similarly, $\hat{\mathbf{P}} = [P_{n_t}P_{n_t-1}...P_1]$. Let $\mathbf{L}^H\mathbf{D}\mathbf{L}$ be the Cholesky decomposition of $\mathbf{R}_r/\sigma_n^2 + \mathbf{I}$, where $\mathbf{L}$ is a lower triangular matrix with unit diagonal entries and $\mathbf{D}$ is a diagonal matrix with positive diagonal entries. It can be checked out that

$$a_i = d_{n_t-i+1} \tag{3.5}$$

where $d_i$ is the $i$th diagonal entry of $\mathbf{D}$. To this end, the optimal power allocation that ensures equal capacities among layers can be written as the solution for the following equation

$$C_1 = C_2 = ... = C_i = ... = C_{n_t} \tag{3.6}$$

with

$$\sum_{i=1}^{n_t} p_i = P \tag{3.7}$$

where $P$ is the total transmit power and $C_i$ is given by (3.3). In general, a closed form solution of the above equation is difficult to find. Numerical Monte-Carlo methods can be used to find optimum power allocation at any given SNR. Below, we derive the asymptotic power allocation in closed form.

Assume the Cholesky decomposition of $R_r / \sigma_n^2$, given by

$$R_r / \sigma_n^2 = \hat{\mathbf{L}}\hat{\mathbf{D}}\hat{\mathbf{L}}^H \tag{3.8}$$

let $\hat{d}_i$ be the $i$th diagonal element of $\hat{\mathbf{D}}$, where $\hat{\mathbf{D}}$ relates to the Cholesky decomposition of $\hat{\mathbf{R}}/\sigma_n^2$ as $\mathbf{R}/\sigma_n^2 = \hat{\mathbf{L}}\hat{\mathbf{D}}\hat{\mathbf{L}}^H$, we have

$$\lim_{\frac{p}{\sigma_n^2}\to\infty} \frac{(1+\hat{d}_i)}{d_i} = 1 \tag{3.9}$$

hence, when SNR goes to infinity, we need only to ensure that $E\log(1+\hat{d}_i) = E\log(1+\hat{d}_j)$ for any *i, j* subject to the total energy constraint given in (3.7). Since $\hat{\mathbf{H}}^H\hat{\mathbf{H}}$ is a complex Wishart matrix, the *i*th diagonal element of its Cholesky decomposition is known to be Chi-square distributed with $2(n_r - i + 1)$ degrees of freedom, specifically, $2\hat{d}_i\sigma_n^2 / p_{n_t - i+1}$ [28-29]. It follows from (3.3), (3.5) and (3.9) that

$$\lim_{p/\sigma_n^2\to\infty} C_i = \lim_{p/\sigma_n^2\to\infty} E(\log(1 + d_{n_t - i+1}))$$

$$= \lim_{p/\sigma_n^2\to\infty} \int_0^\infty f(z, 2(n_r - n_t + i))\log(1 + \frac{p_i}{2\sigma_n^2} z)dz \tag{3.10}$$

where $f(z,2i) = \dfrac{z^{i-1}.e^{-z/2}}{2^i\Gamma(i)}$ is the probability density function of a $\chi_{2i}^2$ Chi-square random variable.

Define $I_k = E\log(1+\rho z)$ with $z \sim \chi_{2k}^2$ for any positive integer *k,* and $\rho = p_i/2\sigma_n^2$, That is

$$I_k = \frac{1}{2^k\Gamma(k)} \int_0^\infty \log(1+\rho z) z^{k-1} e^{\frac{-z}{2}} dz \tag{3.11}$$

using integration by parts, we have

$$I_k = \frac{k-1}{2^{k-1}\Gamma(k-1)} \int_0^\infty \log(1+\rho z) z^{k-2} e^{\frac{-z}{2}} dz + \frac{2^{1-k}}{\Gamma(k)} \int_0^\infty \frac{\rho z^{k-1}}{1+\rho z} e^{\frac{-z}{2}} dz \tag{3.12}$$

let $T_k = 2^{1-k} \int_0^\infty e^{\frac{-z}{2}} \frac{\rho z^{k-1}}{1+\rho z} dz$, as $\Gamma(k) = (k-1)\Gamma(k-1)$, then

$$I_k = I_{k-1} + \frac{T_k}{\Gamma(k)}, \quad k > 1 \tag{3.13}$$

using integration by parts again, we can write

$$T_k = \Gamma(k-1) - (2\rho)^{-1} T_{k-1}, \quad k > 1 \tag{3.14}$$

accordingly, it can be verified that

$$\begin{aligned} I_1 = T_1 = \int_0^\infty e^{-z/2} . \frac{\rho}{1+\rho z} dz \\ = e^{\frac{1}{2p_1}} \int_{\frac{1}{2p_1}}^\infty (e^{-x}/x) dx \\ = -e^{\frac{1}{2p_1}} \varepsilon(-1/2\rho) \end{aligned} \tag{3.15}$$

where the exponential integration function $\varepsilon(x)$ is given by [30]

$$\begin{aligned} \varepsilon(x) = \int_{\frac{1}{2p_1}}^\infty (e^{-x}/x) dx \\ = \gamma + \log(-x) + \sum_{i=1}^\infty \frac{x^i}{i.i!} \end{aligned} \tag{3.16}$$

where $\gamma$ is the Euler's constant [31]. As $p >> 1$, $T_1 = \log(2p) - \gamma$ [30]. Using recursive relations in (3.13), (3.14), $I_k$ for $k$>1 can be found as

$$I_k = \sum_{m=1}^{k} \frac{1}{\gamma(m)} . \sum_{n=2}^{m} \left(\frac{-1}{2\rho}\right)^{m-n} . \Gamma(n-1) + \sum_{m=1}^{k} \frac{1}{\gamma(m)} . \left(\frac{-1}{2\rho}\right)^{m-1} T_1 \tag{3.17}$$

substituting (3.15) in (3.17), as $\rho \to \infty$, we have

$$\lim_{p \to \infty} I_k = \log(2\rho) - \gamma + \sum_{m=1}^{k-1} \frac{1}{m} \tag{3.18}$$

It follows from (3.10), (3.11) and (3.18) that

$$\lim_{\rho\to\infty} C_i = \log(\frac{p_i}{\sigma^2}) - \gamma + \sum_{m=1}^{n_r-n_t+i-1} \frac{1}{m} \tag{3.19}$$

consequently, we have the following theorem

***Theorem*** *Consider a layered communication scheme using an MSD receiver over a Rayleigh fading channel with $n_t$ transmit and $n_r$ receive antennas with $n_r \geq n_t$. As SNR goes to infinity, the layers have equal ergodic capacities if*

$$\alpha_i = \frac{p_i}{p_1} = \exp\left( - \sum_{m=n_r-n_t+1}^{n_r-n_t+i-1} \frac{1}{m} \right) \quad : i>1. \tag{3.20}$$

The proposed power allocation can be found for any SNR. However, providing a closed form formula for all SNR is rather difficult. Tables (3.1), (3.2), and (3.3) show the optimized power allocation over a range of SNR values as well as in the asymptotic case for 2x2, 3x3 and 4x4 MIMO channels, respectively. The coefficients for the specified range of SNR were found using Monte-Carlo simulation based on the aforementioned discussion for the specified channel SNR. The allocated power is normalized by the total power *P*, that is, allocated power is expressed as a percentage of unity.

| SNR (dB) / $p_i$ | 0 | 5 | 10 | 15 | 20 | 25 | Asymptotic case |
|---|---|---|---|---|---|---|---|
| $p_1$ | 0.660 | 0.708 | 0.725 | 0.729 | 0.730 | 0.730 | 0.731 |
| $p_2$ | 0.340 | 0.292 | 0.275 | 0.271 | 0.270 | 0.270 | 0.269 |

**Table 3.1: Proposed transmission power allocation for a 2x2 LST scheme for different channel SNR values and the asymptotic case**

| SNR (dB) / $p_i$ | 0 | 5 | 10 | 15 | 20 | 25 | Asymptotic case |
|---|---|---|---|---|---|---|---|
| $p_1$ | 0.466 | 0.506 | 0.536 | 0.553 | 0.563 | 0.567 | 0.665 |
| $p_2$ | 0.312 | 0.298 | 0.285 | 0.277 | 0.272 | 0.269 | 0.245 |
| $p_3$ | 0.221 | 0.195 | 0.179 | 0.170 | 0.166 | 0.164 | 0.090 |

**Table 3.2: Proposed transmission power allocation for a 3x3 LST scheme for different channel SNR values and the asymptotic case**

| SNR (dB) / $p_i$ | 0 | 5 | 10 | 15 | 20 | 25 | Asymptotic case |
|---|---|---|---|---|---|---|---|
| $p_1$ | 0.387 | 0.432 | 0.466 | 0.489 | 0.502 | 0.508 | 0.644 |
| $p_2$ | 0.272 | 0.266 | 0.258 | 0.250 | 0.245 | 0.243 | 0.237 |
| $p_3$ | 0.193 | 0.175 | 0.161 | 0.152 | 0.148 | 0.146 | 0.087 |
| $p_4$ | 0.147 | 0.128 | 0.115 | 0.108 | 0.105 | 0.104 | 0.032 |

**Table 3.3: Proposed transmission power allocation for a 4x4 LST scheme for different channel SNR values and the asymptotic case**

Fig. 3.3 represents power ratio $\alpha_2$ in schemes with two transmit antennas and two, three and four receive antennas.

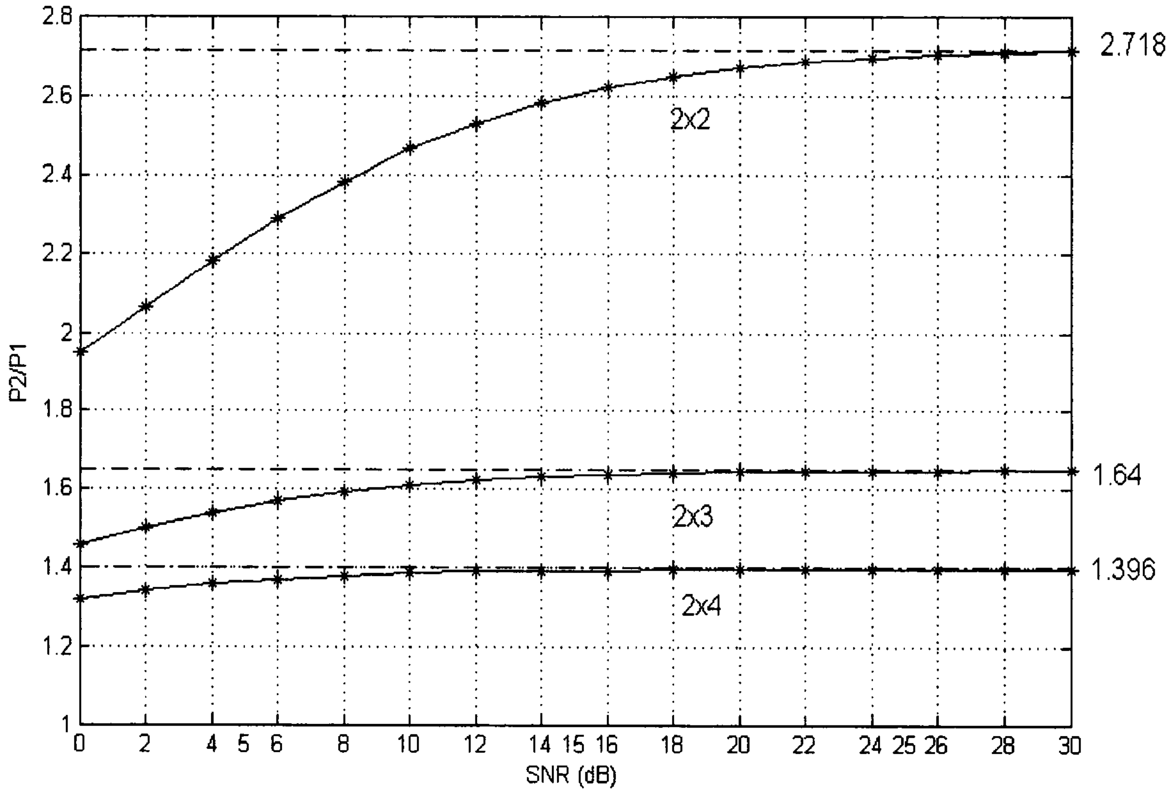



**Fig. 3.3: Ratio of the allocated transmit power among layers in the proposed architecture for 2x2, 2x3 and 2x4 schemes**

As can be seen, $\alpha_2$ converges to the asymptotic value at approximately 27, 23 and 15 dBs for $n_r = 2$, 3 and 4, respectively. It is clear that allocated power converges faster with increasing number of receive antennas. We also note that $\alpha_2$ decreases as $n_r$ increases. This can be deduced from (3.22) and can be explained as follows: For $n_t$ fixed, each additional receive antenna adds two degrees of freedom for $\chi^2_{2i}$, the Chi-square distributed random variable that stands for the SNR in layer $i$, for $\forall i \in \{1,...,n_t\}$; (i.e.), $\alpha_2$ in a 2x3 scheme is equivalent to $\alpha_3 / \alpha_2$ in a 3x3 scheme. It can be easily verified that, for a fixed number of transmit antennas, as $n_r \to \infty$, $\alpha_i$ approaches unity for all $i$.

It is also observed in Fig. 3.3 that deviation of power ratio $\alpha_2$ from the asymptotic ratio is limited, especially over mid-range SNR. This suggests the possibility of fixed power allocation over a wide range of channel SNR. Apparently, when compared to rate optimization, the proposed power optimization is less vulnerable to variations in channel SNR.

## 3.3 Ergodic Capacity Loss for Power Allocation

It is well known that the ergodic capacity of MIMO channel can be achieved when equal transmit power is assigned among transmit antennas [1]. Hence, it is interesting to investigate the loss in ergodic capacity of the proposed scheme. Assuming Gaussian distribution of channel input, the ergodic channel capacity with equal power allocation is given by

$$\hat{C} = E\log(\det(\mathbf{I} + \frac{P}{n_t\sigma_n^2}\mathbf{H}\mathbf{H}^H)) \tag{3.21}$$

by the information chain rule [19], the channel capacity can also be written as

$$\hat{C} = \sum_{i=1}^{n_t}\hat{C}_i \tag{3.22}$$

where $\hat{C}_i$ is the capacity of the $i$th layer, when data of layers 1 – *(i-1)* are perfectly cancelled from received signal, and $p_i = \frac{P}{n_t}$ for any $i$. From (3.19), as $\frac{p_i}{\sigma_n^2} \to \infty$, the capacity of a layer $i$ is independent from transmit power of other layers. That is, at high SNR, $\hat{C}_i$ is only a function of $p_i$, denoted as $C_i(p_i)$. By (3.20), $\hat{C}_i = -\log\alpha_i + \hat{C}_1$, and (3.22) can be written as

$$\lim_{P/\sigma_n^2 \to \infty} \hat{C} = n_t . C_1(P/n_t) - \sum_{i=2}^{n_t} \log(\alpha_i) \tag{3.23}$$

The achievable capacity of the proposed scheme is

$$C = \sum_{i=1}^{n_t} C_i(p_i) = n_t C_1(p_1) \tag{3.24}$$

where $p_1$ can be obtained from (3.20) as $p_1 = \dfrac{P}{(1+\sum_{i=2}^{n_t} \alpha_i)}$. Comparing (3.23) and (3.24), asymptotic capacity loss of the proposed scheme is given by

$$\lim_{p/\sigma_n^2 \to \infty} \hat{C} - C = n_t \log\left(1 + \sum_{i=2}^{n_t} \alpha_i\right) - n_t . \log(n_t) - \sum_{i=2}^{n_t} \log(\alpha_i) \tag{3.25}$$

From the above equation, the capacity loss is a constant as transmit power approaches infinity. Hence, the proposed scheme maintains full multiplexing gain.

Ergodic capacities for equal and unequal power allocation over a 2x2 fast fading MIMO channel are compared in Fig. 3.4.

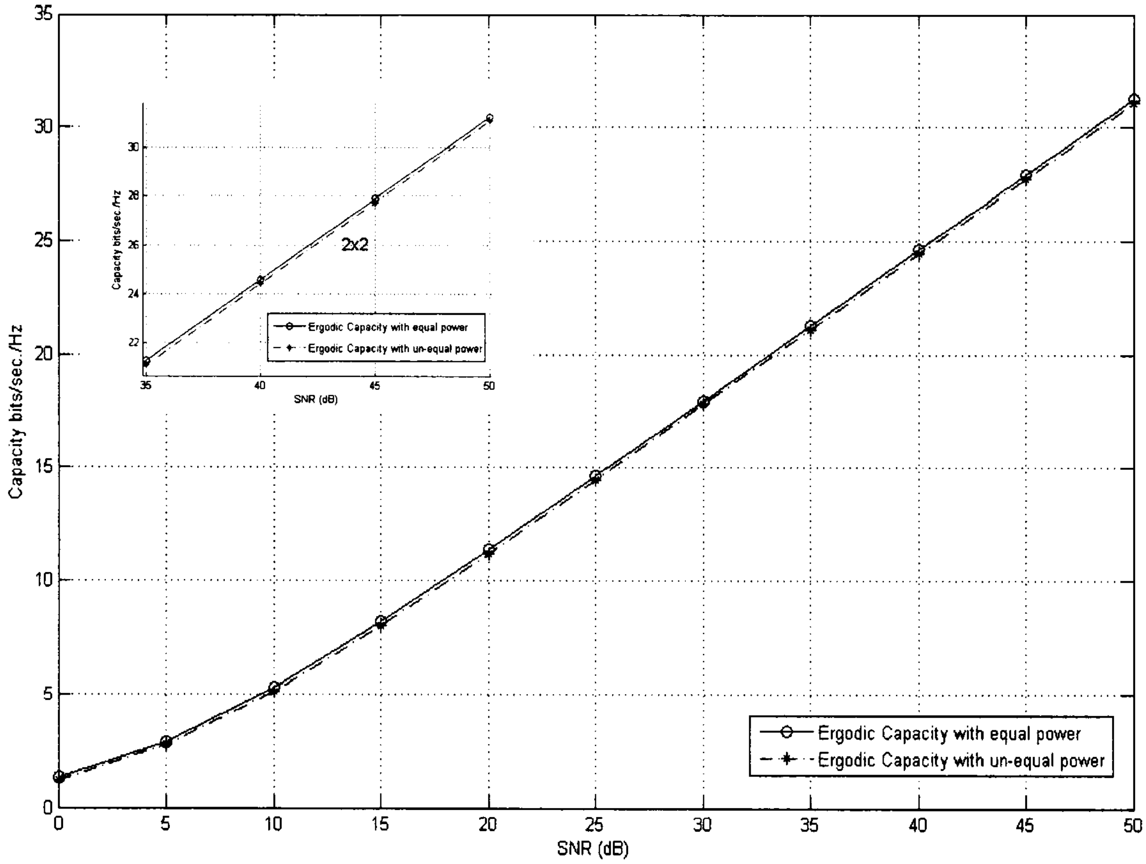
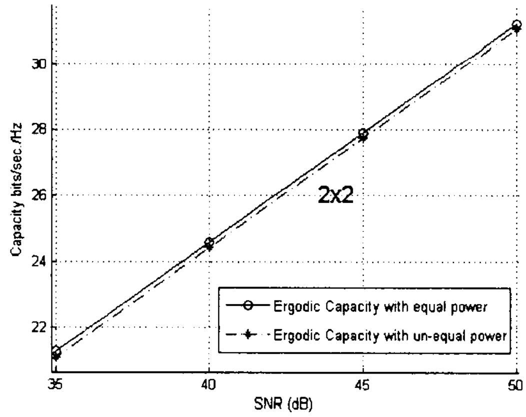


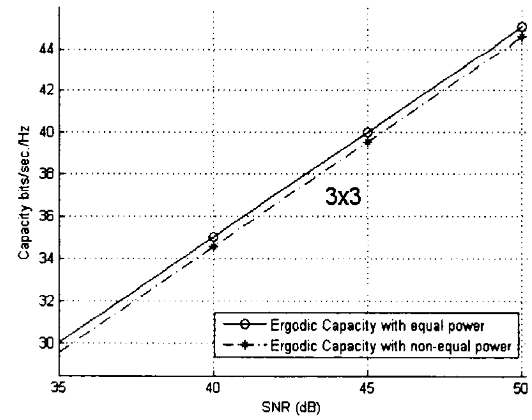

**Fig. 3.4: Ergodic Capacity with equal, and the proposed unequal power allocation for a 2x2 fast Rayleigh fading MIMO channel**

It is clear that loss in ergodic capacity due to unequal power allocation is marginal. Furthermore, it is observed that, as suggested in (3.28), loss in capacity reaches a constant value at high SNR.

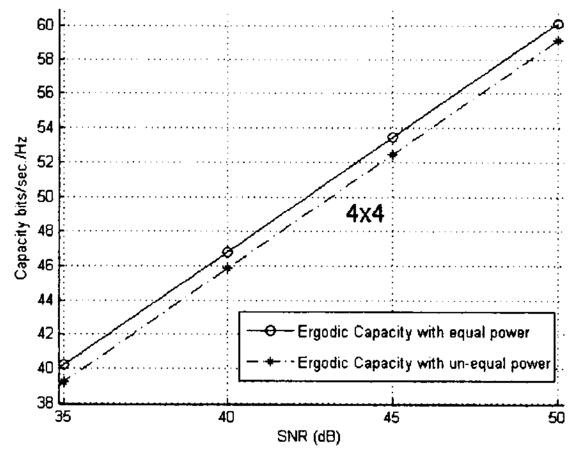
In Fig. 3.5, comparisons of ergodic capacities for equal and unequal power allocation at high SNR for 2x2, 3x3 and 4x4 MIMO channels are provided.




(a)




(b)




(c)

**Fig. 3.5: Ergodic capacity for equal and unequal power allocation at high SNR for: (a) 2x2 (b) 3x3 (c) 4x4 fast fading MIMO channels**

It is observed that loss in capacity is constant at high SNR. More specifically, capacity losses due to unequal power allocation are about 0.346, 0.86 and 1.5 (bits/sec./Hz) for 2x2, 3x3 and 4x4 schemes, which in terms of power loss are interpreted as 0.52, 0.86 and 1.13 dBs, respectively. However, capacity loss is still marginal and accounts for a small percentage of the total ergodic capacity available.

## 3.4 Simulation Results

In this section, error performance of the proposed LST scheme with MSD and unequal allocation of transmission power over flat fast Rayleigh fading MIMO channels is presented in comparison to performances of other schemes, namely, BICM with turbo receiver, VBLAST with ordered successive interference cancellation, and LST with MSD and equal transmission power allocation.

In all schemes, the channel code used is rate ½ convolutional code with constraint length equal to 5. The MATLAB function for random interleaving, 'randintrlv' [37] was employed for bit interleaving. The constellation used is 4-PSK with Gray labeling and unit symbol energy. The size of information data frame is 120 bits. The fast fading channel is assumed to change from channel use to channel use. As the channel is assumed extremely fast, results of simulated LST architectures are equally valid for VBLAST, DBLAST and TLST.

The detector utilized in all schemes is a soft MMSE detector with number of tabs equal to $n_t$. In the implemented VBLAST receiver, received symbols in layers are orderly detected according to their SNR. Soft decision information at the detector output is used in interference cancellation, as well as in soft decoding of information bits, which is carried out using a soft Viterbi decoder. In BICM, the receiver is a joint detection and decoding iterative turbo receiver. A soft input soft output BCJR [31] decoder is used in decoding. Soft decision information for decoded symbols is a by-product of the implemented BCJR, and is fed back to the detector to aid better detection. The process can be repeated for a number of iterations.

Figs. 3.6, 3.8 and 3.9 present the BER performances for transmission over 2x2, 2x3 and 4x4 MIMO channels, respectively; Fig.3.7 compares BER performances of layers in both cases of equal, and optimized un-equal power assignment for an LST scheme with two layers. Fig. 3.10 compares the BER performances of the proposed scheme and 4 iteration BICM over a 2x2 flat fast Rayleigh fading channel when the channel code is a convolutional code with rate ½ and constraint length equal to 3.In all figures, BER performances are plotted against $\frac{E_b}{N_0}$, where $E_b$ is the information bit energy, and $N_0$ is the PSD of Gaussian noise.

Fig. 3.6 presents the BER performances of the proposed scheme and schemes described earlier, over a 2x2 fast Rayleigh fading channel. It can be observed that the proposed scheme significantly outperforms conventional LST by 3.1 dBs at BER=$10^{-5}$. As well, the proposed scheme outperforms single iteration BICM by ~1dB and approaches 4-iteration BICM within 0.2 dB at BER=$10^{-5}$. The advantage of unequal power allocation is quite clear. Unequal power allocation improves performance by about 1.8 dB at $10^{-5}$ BER compared to the same architecture with equal power allocation. This can be viewed as a direct consequence of higher immunity against interference and error propagation achieved by the proposed scheme.

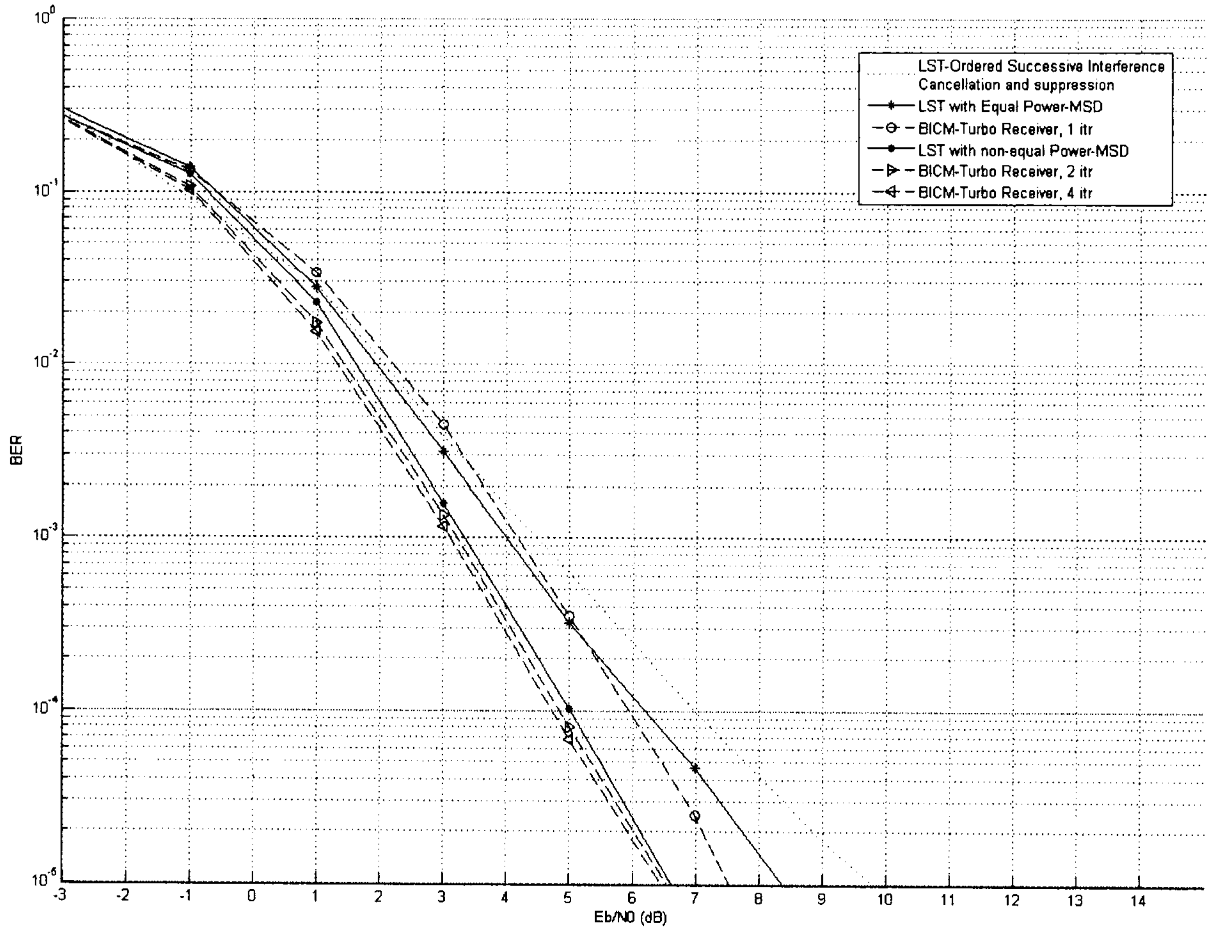



**Fig. 3.6:BER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over 2x2 fast Rayleigh fading channel**

It is interesting to investigate the effect of the proposed unequal power allocation on the gap in error performance of layers in the proposed architecture. Fig.3.7 illustrates this effect for a 2x2 scheme.

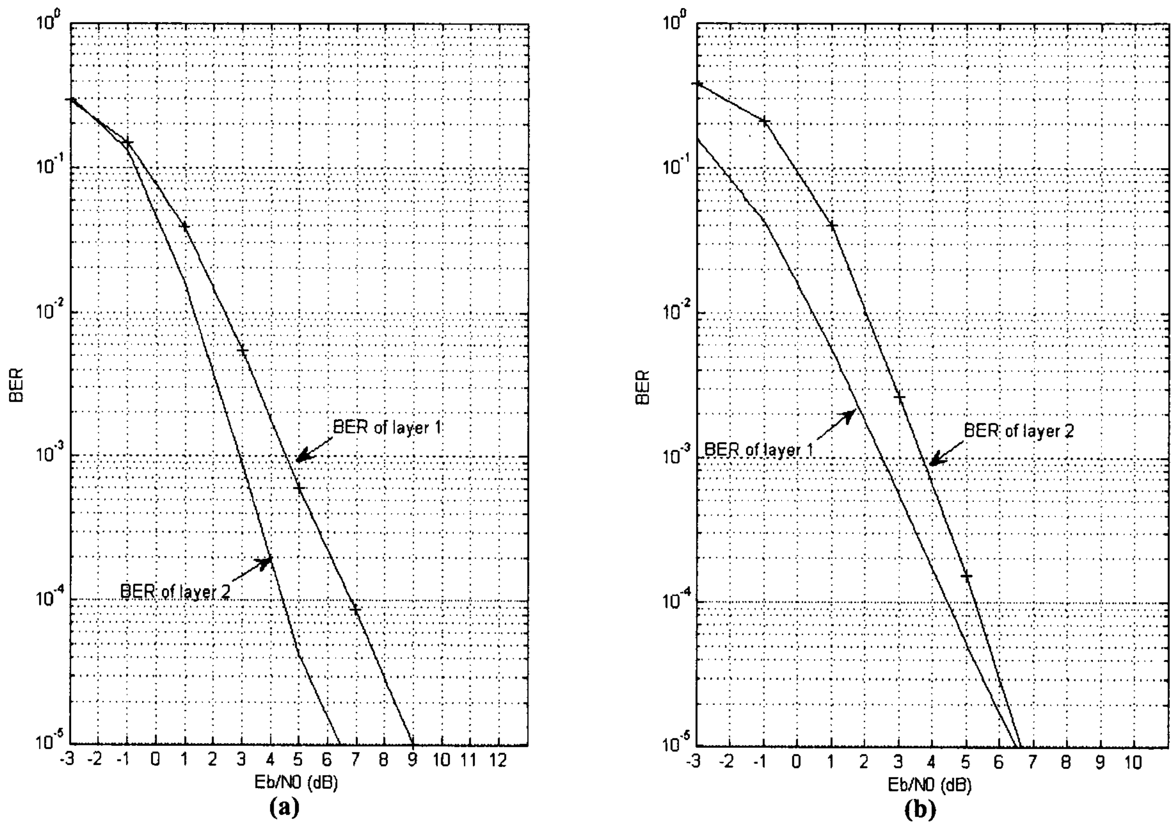



**Fig.3.7: Comparison of BER performances of layers 1 & 2, in a 2x2 LST scheme with MSD decoding, for: (a) equal allocation of transmit power (b) the proposed unequal allocation of transmit power**

From Fig. 3.7 (a), layer 2 outperforms layer 1 in error performance, and the gap between the two increases as $\frac{E_b}{N_0}$ increases reaching ~2.5 dBs at BER=$10^{-5}$. This is expected as layer 1 suffers interference from layer 2. In Fig.3.7 (b), the effect of unequal power allocation is obvious. At low $\frac{E_b}{N_0}$, or, equivalently, high noise power, layer 1 outperforms layer 2 as its assigned higher power. However, the gap in error performance between the two layers shrinks with increasing $\frac{E_b}{N_0}$ , and performances of layers 1 and 2 converge to achieve a $10^{-5}$ BER at 6.6 and 6.7 dBs, respectively. This highlights the effect of the proposed unequal power allocation in optimizing error performance among layers.

The results also show the increased immunity against interference in firstly detected layers.

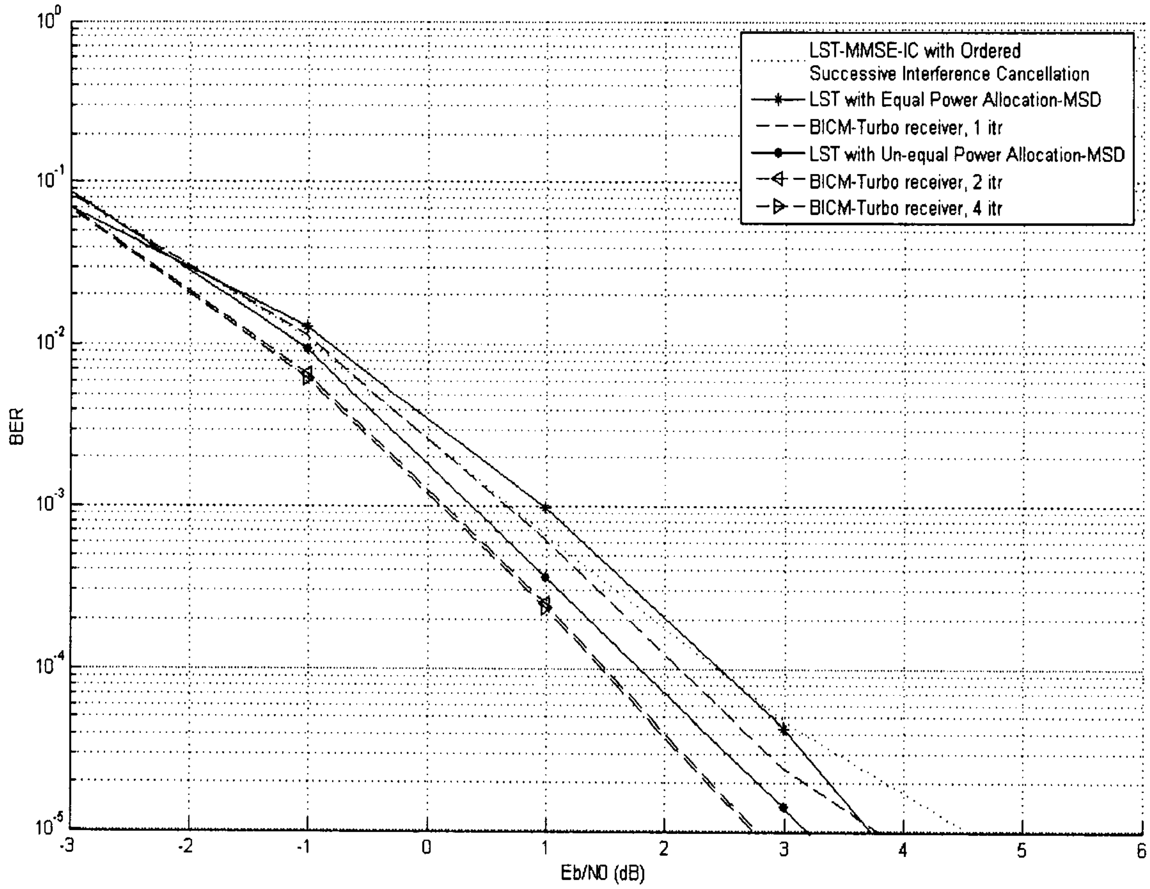



**Fig. 3.8:BER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over 2x3 fast Rayleigh fading channel**

Fig. 3.8 presents the BER performances of the proposed scheme and the other schemes of interest over a 2x3 MIMO channel. The proposed scheme outperforms conventional LST with ~1.2 dBs at BER=$10^{-5}$. However, the gap between the performance of BICM with 4 iterations and the proposed scheme widens to reach 0.5 dB. It is observed that schemes over the 2x3 channel significantly outperform schemes over the 2x2 channel. This is clearly due to the increased receive diversity in the 2x3 channel.

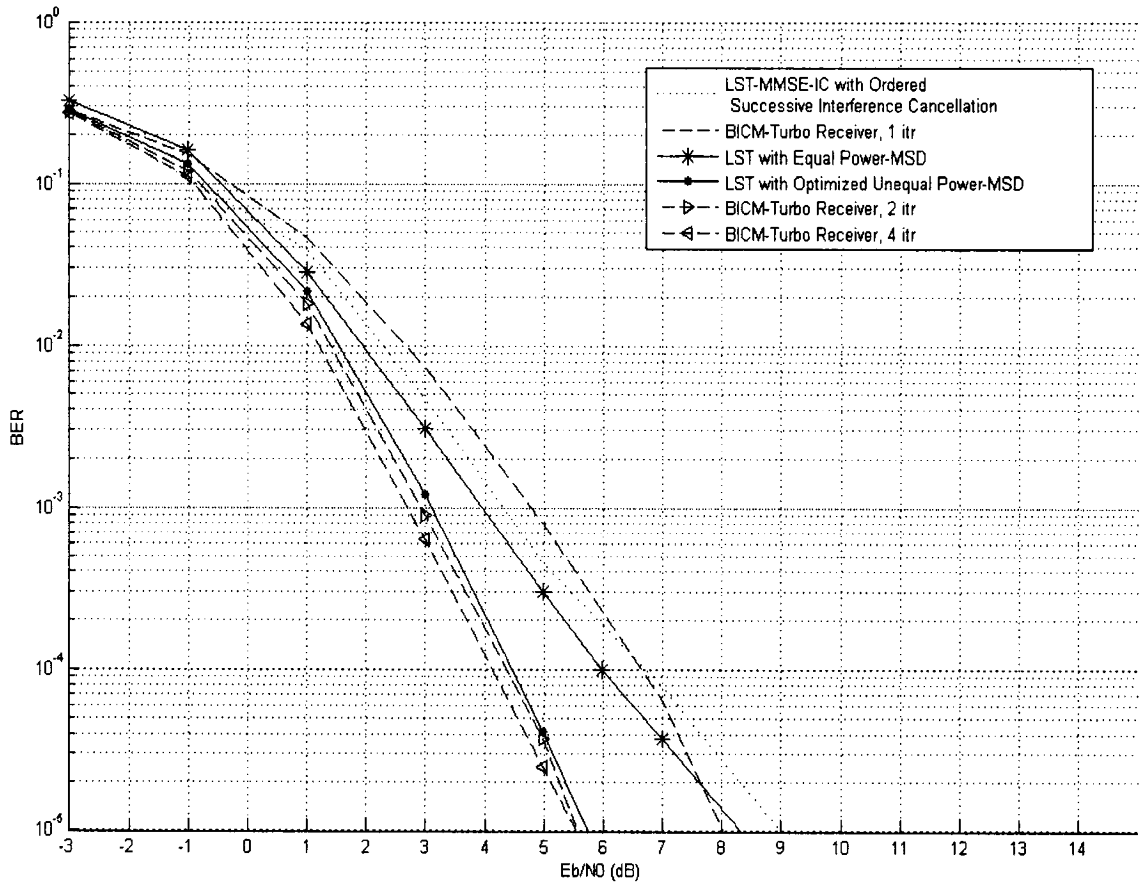



**Fig. 3.9: BER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over 4x4 Rayleigh fast fading channel**

BER performances of the proposed scheme and the other schemes of interest over a 4x4 channel are plotted in Fig. 3.9. Similarly, the proposed scheme outperforms conventional LST with~3.5 dBs and approaches 4-iteration BICM within 0.2 dB at BER= $10^{-5}$. It can be observed that schemes over the 4x4 channel have an advantage over schemes over 2x2 channels of about 1 dB. This is expected due to the higher spatial diversity provided by the 4x4 channel. However, it is observed that the proposed scheme and multiple iterations BICM achieve higher diversity gain than the other schemes. This illustrates the benefits of unequal power allocation proposed. As well, this highlights the need for more iterations in BICM. In comparison, the proposed scheme has the advantage of lesser delay and implementation complexity.

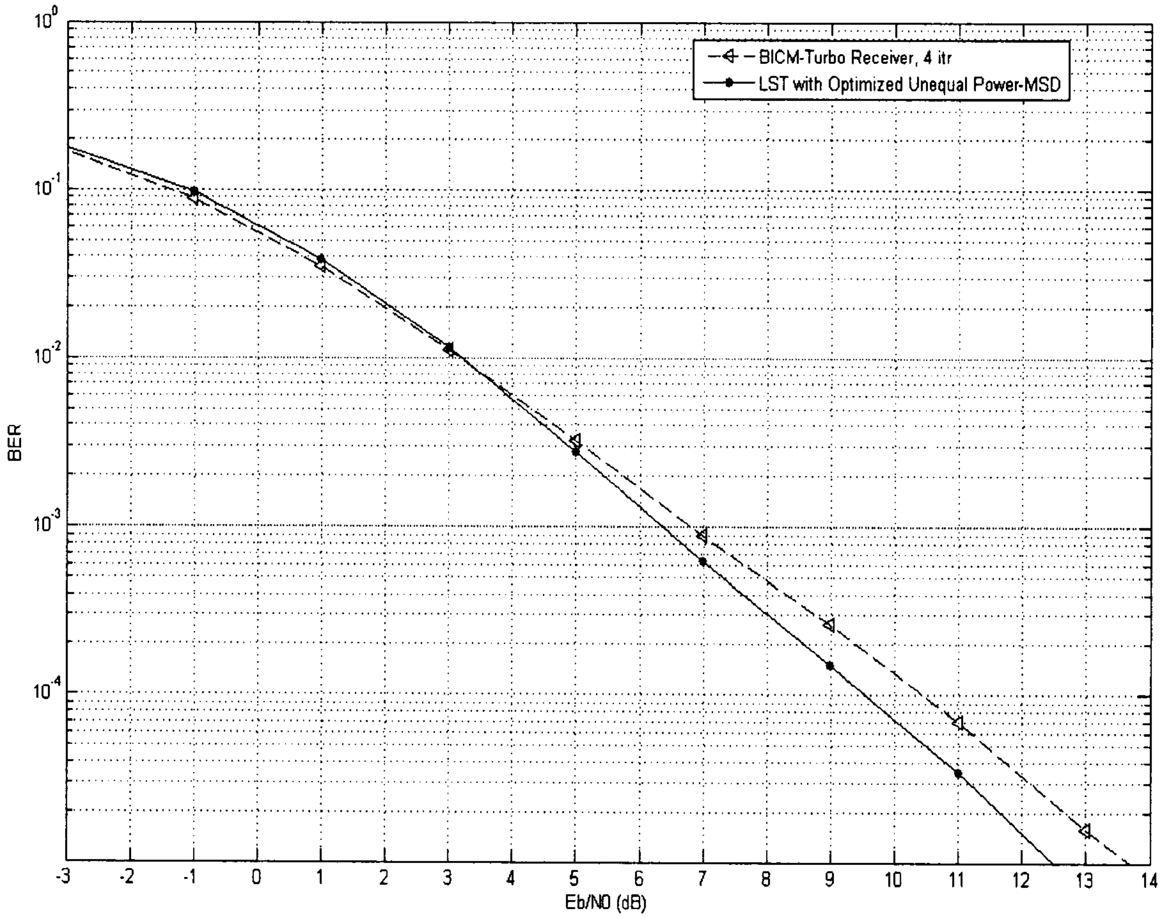



**Fig. 3.10: Comparison of BER performances over a flat fast Rayleigh fading channel between the proposed scheme and 4 iteration BICM for a convolutional channel code with rate ½ and constraint length equal to 3**

Fig. 3.10 presents the BER performances of the proposed scheme and 4 iteration BICM over a 2x2 fast Rayleigh fading channel when the channel code employed is a convolutional code with rate ½ and constraint length equal to 3. It is observed that the proposed scheme outperforms 4 iteration BICM; At BER= $10^{-5}$, the proposed scheme has an advantage of ~1 dB. This is expected as the performance of the BICM turbo receiver is more affected by the strength of the employed channel code. This shows that, for shorter constraint length convolutional codes, the proposed scheme provides improved performance when compared to BICM.

## 3.5 Conclusion

In this chapter, we presented an improved LST architecture that employs multi-stage decoding at the receiver, coupled with transmission power allocation optimized to achieve equal capacities among LST layers at the transmitter. A derivation and development of a theorem for the proposed power allocation for fast fading channels in the asymptotic case, as well as a closed form representation of asymptotic ergodic capacity loss due to the unequal power allocation were presented. It was shown that loss in ergodic capacity due to power allocation is marginal and converges to a constant at high SNR.

Simulation results show that the proposed scheme significantly outperforms conventional LST architecture, and approaches the near-optimum BICM scheme. Unlike BICM, the proposed scheme has a moderate implementation complexity that grows linearly with the number of antennas.

# Chapter 4

# Improved LST Architecture for Quasi-static Fading Channels

In this chapter, we extend the proposed architecture for transmission over quasi-static flat Rayleigh fading channels, where the ergodic capacity is no longer a measure of merits.

The chapter is organized as follows. First, we discuss the concept of outage capacity and its optimization in BLAST scheme. We then derive a theorem for power allocation that maximizes outage capacity given maximum outage probability. After that, we derive unequal power allocation required to achieve equal capacities among layers in the proposed architecture. We analyze the change in allocated power with different values of the maximum outage probability, and investigate the loss in outage capacity due to unequal power allocation. Finally, we present simulation results that compare performances of the proposed scheme and other transmission schemes of interest over flat quasi-static Rayleigh fading channels.

## 4.1 Outage Capacity

For quasi-static fading channels, the channel is assumed to be constant over a whole data frame but changes from frame to frame. The ergodic capacity of a quasi-static Rayleigh fading channel is known to be zero [2]. Regardless of the transmission rate, there exists a non-zero probability that the channel does not support transmission with an arbitrarily small probability of error. In other words, reliable transmission over a quasi-static Rayleigh fading channel 100% of time is not possible at any rate. In such a case, the ergodic capacity is no longer a useful measure. Instead, outage capacity is used in practice.

The concept of outage capacity deals with the supported transmission rates for a percentage of time, or, equivalently, a percentage of random channel realizations. A channel outage occurs if the channel realization does not support the employed transmission rate $r$. For a given $\varepsilon < 1$, the outage capacity, $C_\varepsilon$, is defined as the maximum transmission rate possible with the channel outage probability $P_{out} \leq \varepsilon$, that is

$$\begin{aligned} &C_\varepsilon = \max r \\ &P_{out} \leq \varepsilon . \end{aligned} \tag{4.1}$$

This means that reliable transmission at any rate within the capacity region defined by $C_\varepsilon$ is possible for a percentage $(1-\varepsilon)$ x100% or more of time.

The expression in (4.1) applies for the unconstrained optimum case. Below, we discuss the outage capacity of BLAST architecture.

### 4.1.1 Outage Capacity of BLAST with MSD

Successive interference cancellation in the BLAST architecture implies that the successful detection of a certain layer is undermined by the detection errors in previously detected layers. Consequently, for perfect interference cancellation, transmission rates in individual layers must not exceed the capacities in layers. This is true for all layers regardless of their order of detection, as far as reliable transmission at a given overall transmission rate is concerned. Thus, for transmission over quasi-static channels, the channel is considered in outage if, for any layer $i$, its capacity $C_i$ falls below the corresponding transmission rate $r_i$.

Under the assumption that channel inputs are Gaussian distributed, and that interference cancellation is perfect in all layers, capacity of a layer is solely determined by the output SIR value of the MMSE receiver [36]. Let $x_i$ stand for the SIR of layer $i$, the capacity $C_i$ is given by

$$C_i = \log(1+x_i) \qquad \text{for } i=1,\ldots,n_t \tag{4.2}$$

since $x_i$ depends on the random channel realization, it is a random variable, and so is $C_i$. For a certain layer transmission rate $r_i$ in bits, the required SIR value is $\zeta_i = 2^{r_i} - 1$. From the aforementioned discussion, a channel outage occurs if, for any layer $i$, the associated SIR falls below $\zeta_i$. Thus, the outage probability of BLAST with MSD is given by [33]

$$P_{out} = P(x_1 \le \zeta_1, \ldots, x_{n_t} \le \zeta_{n_t}) \tag{4.3}$$

for a given $\varepsilon$, the outage capacity in BLAST can be written as [32]

$$C_\varepsilon = \max(\sum_{i=1}^{n_t} \log(1+\zeta_i)$$
$$P_{out} \le \varepsilon . \quad (4.4)$$

That is, the outage capacity is obtained by selecting the combination of layer transmission rates that maximize the overall transmission rate under the constraint that the outage probability is less than $\varepsilon$.

From (4.3), the outage probability can be determined by the cumulative distribution function (CDF) of $x_i \forall i, i \in \{1,\dots,n_t\}$. As discussed in Chapter 3, $x_i$ relates to the Cholesky decomposition of the matrix $\mathbf{H}^H\mathbf{H}$ (3.4). At high SNR, $x_i$'s are independent Chi-square distributed random variables with $2(n_r - n_t + i)$ degrees of freedom. The complementary CDF of $x_i$ is given by [32]

$$CF_i(x_i) = e^{-(x_i/(p_i/\sigma_n^2))} \sum_{k=0}^{n_t - n_r + k - 1} \frac{(x_i/(p_i/\sigma_n^2))^k}{k!} \quad (4.5)$$

where $p_i$ and $\sigma_n^2$ are the transmit power of layer $i$ and Gaussian noise variance, respectively. Consequently, at high SNR, (4.3) can be rewritten as

$$\begin{aligned} P_{out} &= P(x_1 \le \zeta_1, \dots, x_{n_t} \le \zeta_{n_t}) \\ &= 1 - P(x_1 \ge \zeta_1, \dots, x_{n_t} \ge \zeta_{n_t}) \\ &= 1 - \prod_{i=1}^{n_t} CF_i(\zeta_i) \end{aligned} \quad (4.6)$$

consequently, outage capacity of BLAST can be put in the form [32]

$$C_\varepsilon^{BLAST} = \max_{\zeta_i : i \in [1,\dots,n_t]} (\sum_{i=1}^{n_t} \log(1+\zeta_i)$$

Subject to

$$\left( \prod_{i=1}^{n_t} CF_i(\zeta_i) \right) \ge 1 - \varepsilon \quad (4.7)$$

## 4.2 Power Allocation for the Maximum Outage Capacity

Before we derive the optimum unequal power allocation for the proposed architecture, it is interesting to find the power allocation that maximizes outage capacity in BLAST. At high SNR, the outage capacity can be written as the solution of the following optimization problem

$$\max_{p_i, z_i : i\in[1,\dots n_t]} \left(\sum_{i=1}^{n_t} \log(1+\rho_i z_i)\right) \tag{4.8}$$

subject to

$$\prod_{i=1}^{n_t} CF_i(\rho_i z_i) \geq 1-\varepsilon \tag{4.9}$$

and

$$\sum_{i=1}^{n_t} \rho_i = \frac{P}{\sigma_n^2} \tag{4.10}$$

where $\rho_i = \frac{p_i}{\sigma_n^2}$ and $z_i = \frac{x_i}{p_i}$. It can be shown that the above constraints (4.9) and (4.10) define a convex set [32]. Consequently, the above optimization problem has a unique global maximization solution [34], which can be solved for using the *Karush-Kuhn-Tucker* [33, 34] (KKT) theorem. For a function $f(\mathbf{x})$, the objective function of a set of variables **x**, with equality constraint(s) $g_i(\mathbf{x})$ and non-equality constraint(s) $h_j(\mathbf{x})$, the general Lagrangian is [34]

$$\Lambda(\mathbf{x}, \lambda, \mu) = f(\mathbf{x}) - \sum_k \lambda_k g_k(\mathbf{x}) - \sum_j \mu_j h_j(\mathbf{x}) \tag{4.11}$$

the KKT theorem states that, the set of variables that minimize/maximize the function $f(\mathbf{x})$ under equality and non-equality constraints can be found by solving

$$\nabla\Lambda(\mathbf{x},\lambda,\mu)=\nabla f(\mathbf{x})-\sum_k \lambda_k \nabla g_k(\mathbf{x})-\sum_j \mu_j \nabla h_j(\mathbf{x})=0 \tag{4.12}$$

subject to

$$\mu_j \le 0, \forall j \tag{4.13}$$

$$\sum_j \mu_j h_j(\mathbf{x})=0\,. \tag{4.14}$$

The condition in (4.14) states that, for any $j$, if $h_j(\mathbf{x})$ is not zero, then the associated Lagrange multiplier $\mu_j$ must be zero, which is referred to as *complementary slackness condition* [35].

Applying the KKT theorem to the constrained optimization problem (4.8-4.10), the BLAST outage capacity, $C_\varepsilon^{BLAST}$, a function of the set of variables $[z_1,..,z_{n_t},\rho_1,...,\rho_{n_t}]$ is the objective function. The linear constraint is given by

$$g(\mathbf{z},\rho)=\sum_{i=1}^{n_t}\rho_i-\frac{P}{\sigma_n^2}=0 \tag{4.15}$$

the non-linear constraints in the problem include the conditions that $z_i,\rho_i$ are greater than zeros ; and the outage probability constraint. Thus, we define

$$h_i=-z_i\le 0 \tag{4.16}$$

$$h_{n_t+i}=-\rho_i\le 0 \tag{4.17}$$

for $\forall i, i\in\{1,...,n_t\}$, and

$$h_{2n_t+1}=\prod_{i=1}^{n_t} CF_i(\rho_i z_i)\le\varepsilon \tag{4.18}$$

thus, the set of variables $[z_1,..,z_{n_t},\rho_1,...,\rho_{n_t}]$ that maximize the BLAST outage capacity $C_\varepsilon^{BLAST}$ can be found by solving

$$\nabla\Lambda(\mathbf{z},\rho,\lambda,\mu) = \nabla\left(\sum_{i=1}^{n_t}\log(1+\rho_i z_i)\right) - \lambda\nabla\left(\sum_{i=1}^{n_t}\rho_i - \frac{P}{\sigma_n^2}\right) - \sum_{j=1}^{2n_t+1}\mu_j \nabla h_j(\mathbf{z},\rho) = 0 \tag{4.19}$$

subject to

$$\mu_j \le 0, \forall j \tag{4.20}$$

$$\sum_j \mu_j h_j(\mathbf{z},\rho) = 0. \tag{4.21}$$

Solving (4.19-4.21), we get a system of homogeneous equations

$$\frac{\partial\Lambda}{\partial z_i} = \frac{1}{z_i} + \mu_i + \mu_{2n_t+1}\frac{\partial h_{2n_t+1}}{\partial z_i} = 0, \forall i \tag{4.22}$$

$$\frac{\partial\Lambda}{\partial p_i} = \frac{1}{p_i} - \lambda + \mu_{n_t+i} = 0, \forall i \tag{4.23}$$

$$\frac{\partial\Lambda}{\partial\lambda} = p_1 + \ldots + p_j + \ldots + p_{n_t} - \frac{P}{\sigma_n^2} = 0 \tag{4.24}$$

$$\frac{\partial\Lambda}{\partial\mu_i} = z_i = 0,\ or\!:\ \mu_i = 0, \forall i \tag{4.25}$$

$$\frac{\partial\Lambda}{\partial\mu_{n_t+i}} = \rho_i = 0,\ or\!:\ \mu_{n_t+i} = 0, \forall i \tag{4.26}$$

$$\frac{\partial\Lambda}{\partial\mu_{2n_t+1}} = P_{out} - \varepsilon = 0\ . \tag{4.27}$$

In (4.26, 4.27), if $z_i$, $\rho_i$ are zeros for all *i*, we get the outage capacity minimum. However, if transmission is active in all layers, $z_i$, $\rho_i$ are non-zeros while the Lagrange multipliers $[\mu_1 - \mu_{2n_t}]$ are considered zeros. Given this, solving (4.23) we get

$$\rho_i = \frac{1}{\lambda}, \forall i. \tag{4.28}$$

It follows from (4.24) that $\rho_i = \frac{P}{n_t \sigma_n^2}$ for all $i$. Given this, we present the following theorem.

***Theorem**: For a layered communication system over a quasi-static Rayleigh fading channel with $n_t$ transmit and $n_r$ receive antennas with $n_t \le n_r$, the maximum outage capacity for a given total transmission power $P$ when $\frac{P}{\sigma_n^2} >> 1$ is achieved by equal transmission power allocation among layers, regardless of the required outage probability ε.*

## 4.3 Proposed Power Allocation

In this section, we derive unequal power allocation under the constraint that all layers have the same rate for transmission over quasi-static channels. The derivation can be carried out using the same method in the previous section, but with an additional equality constraint that ensures equal rates among layers. For the purpose of clarity, we use a 2x2 MIMO channel as an example. To such case, (4.19-4.21) can be written as

$$\nabla\Lambda(\mathbf{z},\rho,\lambda,\mu) = \nabla\left(\sum_{i=1}^{2}\log(1+\rho_i z_i)\right) - (\sum_{k=1}^{2}\lambda_k \nabla g_k(\mathbf{z},\rho) - \sum_{j=1}^{5}\mu_j \nabla h_j(\mathbf{z},\rho) = 0 \tag{4.29}$$

Subject to

$$\mu_j \le 0, \forall j \tag{4.30}$$

$$\sum_j \mu_j h_j(\mathbf{z},\rho) = 0 \tag{4.31}$$

where

$$g_1(\mathbf{z},\rho)=\sum_{i=1}^{2}\rho_i-\frac{P}{\sigma_n^2}=0 \tag{4.32}$$

$$g_2(\mathbf{z},\rho)=\log(1+\rho_1 z_1)-\log(1+\rho_2 z_2)=0 \tag{4.33}$$

solving for power coefficients $\rho_1,\rho_2$ that maximize outage capacity, we get

$$\frac{\rho_1}{\rho_2}=\frac{1+\lambda_2}{1-\lambda_2} \tag{4.34}$$

where $\lambda_2$ is the Lagrange multiplier for $g_2(\mathbf{z},\rho)$. The solution depends on the given total power $P$ and maximum outage probability $\varepsilon$. Monte-Carlo simulations can be used to obtain unequal power allocation, for any given $P$ and $\varepsilon$.

The Monte-Carlo simulations were carried out using the MATLAB function 'fmincon' from the MATLAB optimization toolbox [37]. The 'fmincon' function finds the minimum/maximum points of a non-linear multivariable function, with a combination of linear and non-linear constraints. The 'fmincon' function requires the definition of the objective function, the linear and the non-linear constraints. For a given $n_t, n_r$, total transmission power $P$ and maximum outage probability $\varepsilon$, the objective function is given in (4.7). The equality constraints were defined as

$$\begin{aligned} g_1(\mathbf{z},\rho)&=\sum_{i=1}^{n_t}\rho_i-\frac{P}{\sigma_n^2}=0 \\ g_2(\mathbf{z},\rho)&=\log(1+\rho_i z_i)-\log(1+\rho_j z_j)=0, \forall i,j; i,j\in\{1,\dots,n_t\}. \end{aligned} \tag{4.35}$$

The non-equality constraints used in the simulation are those defined in (4.16-4.18).

Tables (4.1), (4.2) and (4.3) show the optimized unequal power allocation in layers normalized by total transmission power, for different values of maximum outage probability $\varepsilon$ in 2x2, 3x3 and 4x4 quasi-static channels. We note that the gap in allocated power between layers increases as ε is smaller.

| $p_i$ \ ε | **0.1** | **0.05** | **0.02** | **0.01** |
|---|---|---|---|---|
| $p_1$ | 0.707 | 0.747 | 0.796 | 0.829 |
| $p_2$ | 0.293 | 0.253 | 0.204 | 0.171 |

**Table 4.1: Optimized unequal power allocation for different values of ε over a 2x2 quasi-static MIMO channel**

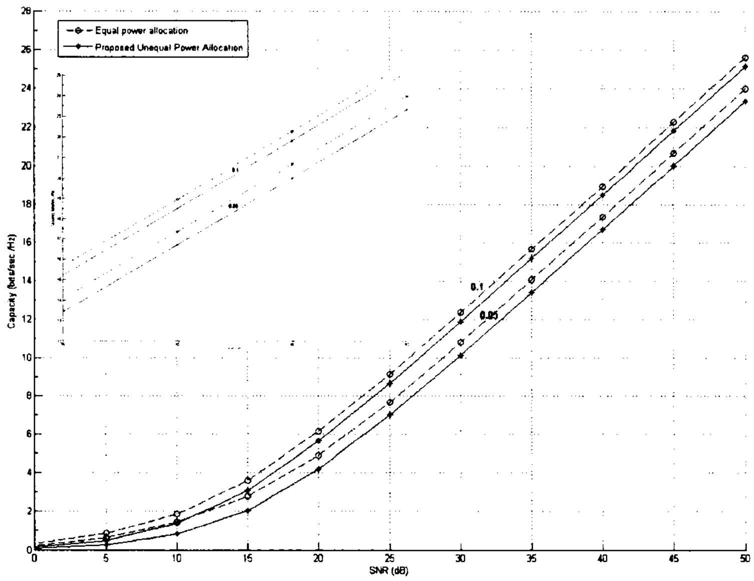
| $p_i$ \ ε | **0.1** | **0.05** | **0.02** | **0.01** |
|---|---|---|---|---|
| $p_1$ | 0.615 | 0.669 | 0.736 | 0.780 |
| $p_2$ | 0.251 | 0.223 | 0.186 | 0.159 |
| $p_3$ | 0.135 | 0.108 | 0.078 | 0.060 |

**Table 4.2: Optimized unequal power allocation for different values of ε over a 3x3 quasi-static MIMO channel**

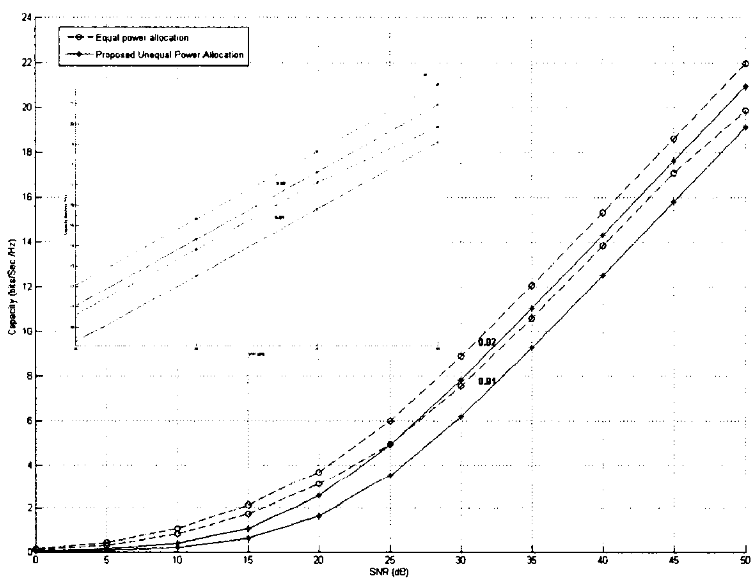
| $p_i$ \ ε | **0.1** | **0.05** | **0.02** | **0.01** |
|---|---|---|---|---|
| $p_1$ | 0.570 | 0.630 | 0.705 | 0.704 |
| $p_2$ | 0.227 | 0.210 | 0.181 | 0.183 |
| $p_3$ | 0.126 | 0.100 | 0.073 | 0.072 |
| $p_4$ | 0.077 | 0.060 | 0.041 | 0.041 |

**Table 4.3: Optimized unequal power allocation for different values of ε over a 4x4 quasi-static MIMO channel**

Since maximum outage capacity is achieved by equal power allocation among layers, it is interesting to investigate the loss in outage capacity due to power allocation in the proposed scheme. Maximum outage capacity for equal and unequal power allocation for different values of ε over a 2x2 quasi-static MIMO channel are illustrated in Fig. 4.1.




**(a)**




**(b)**

**Fig. 4.1: Outage capacity for equal and optimized unequal power allocation in a 2x2 quasi-static fading MIMO channel for: (a) ε= 0.1, 0.05 (b) ε= 0.02, 0.01.**

It is observed that the loss in outage capacity is tolerable and converges to a constant at high SNR. For ε= 0.1, 0.05, 0.02 and 0.01, the losses in outage capacity are ~ 0.5, 0.7, 1 and 1.4 (bits/sec./Hz) which in terms of power loss are interpreted as 0.75, 1, 1.5 and 2.1 dBs, respectively.

## 4.4 Simulation Results

In this section, error performance of the proposed LST scheme with MSD and unequal allocation of transmission power over flat quasi-static Rayleigh fading MIMO channels is presented in comparison to performances of other schemes, namely, BICM with turbo receiver, VBLAST with ordered successive interference cancellation, and LST with MSD and equal transmission power allocation.

In all schemes, the channel code used is rate ½ convolutional code with constraint length equal to 5. The MATLAB function for random interleaving, 'randintrlv' was employed for bit interleaving. The constellation used is 4-PSK with Gray labeling and unit symbol energy. The size of information data frame is 120 bits. The quasi-static fading channel is assumed to be constant over a whole information data frame but changes from frame to frame. TLST scheme is used in simulated LST architectures as it offers higher spatial multiplexing gain than other LST space-time mapping schemes.

The detector utilized in all schemes is a soft MMSE detector with number of tabs equal to $n_t$. In the implemented VBLAST receiver, received symbols in layers are orderly detected according to their SNR. Soft decision information at the detector output is used in interference cancellation, as well as in soft decoding of information bits, which is carried out using a soft Viterbi decoder. In BICM, the receiver is a joint detection and decoding iterative turbo receiver. A soft input soft output BCJR decoder is used in decoding. Soft decision information for decoded symbols are a by-product of the implemented BCJR, and are fed back to the detector to aid better detection. The process can be repeated for a number of iterations.

Figs. 4.2, 4.3 and 4.4 present the FER performances for transmission over 2x2, 2x3 and 4x4 MIMO channels, respectively. Fig. 4.5 compares the FER performances of the proposed scheme and 4 iteration BICM over a 2x2 flat quasi-static Rayleigh fading channel when the channel code is a convolutional code with rate ½ and constraint length equal to 3. In all figures, FER performances are plotted against $\frac{E_b}{N_0}$, where $E_b$ is the information bit energy and $N_0$ is the PSD of Gaussian noise.

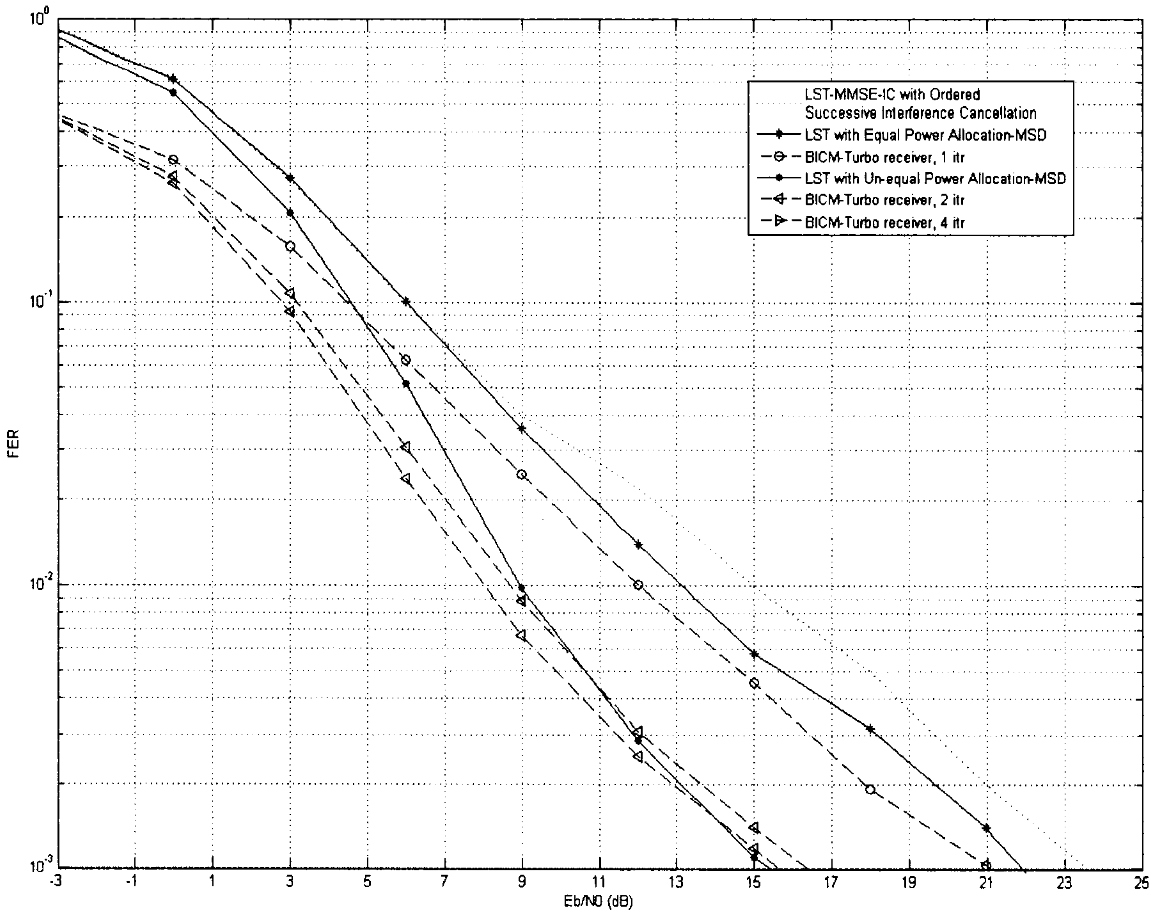



**Fig. 4.2: FER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over a 2x2 flat quasi-static Rayleigh fading channel**

Fig. 4.2 presents the FER performances of the proposed scheme and schemes described earlier, over a 2x2 flat quasi-static Rayleigh fading channel. It can be observed that the proposed scheme significantly outperforms conventional LST. As well, the proposed scheme outperforms single iteration BICM with ~5.5 dBs at FER=$10^{-3}$ and achieves nearly the same performance of 4-iteration BICM with ~15.7 dBs at FER=$10^{-3}$. The advantage of the proposed unequal power allocation is quite clear. The proposed unequal power allocation significantly improves performance with ~6 dBs at FER=$10^{-3}$.

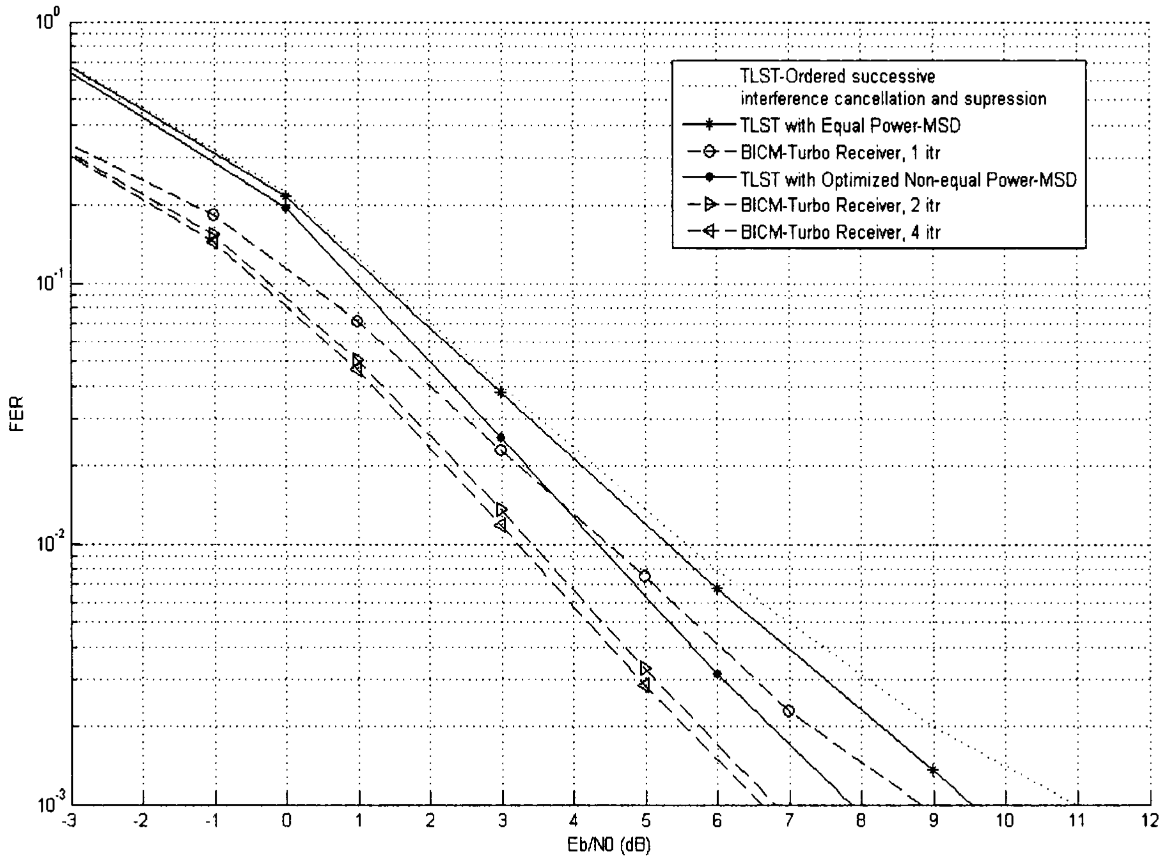



**Fig. 4.3: FER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over a 2x3 flat quasi-static Rayleigh fading channel**

FER performances of the proposed schemes and the other schemes over a 2x3 flat quasi-static Rayleigh fading channels are plotted for comparison in Fig. 4.3. The proposed scheme outperforms conventional LST with ~3 dBs at FER=$10^{-3}$. The gap in performance between the proposed scheme and 4-iteration BICM decreases with increasing $\frac{E_b}{N_0}$ and is ~1.1 dBs at FER=$10^{-3}$. The improvement due to power allocation is observed to reach ~1.5 dBs at FER=$10^{-3}$. The BICM and the proposed scheme achieve nearly the same diversity gain, while a loss in diversity gain is observed in conventional LST with increasing $\frac{E_b}{N_0}$. The schemes over the 2x3 channel significantly outperform the schemes over the 2x2 channel. This is clearly due to the increased receive diversity in the 2x3 channel.

FER performances for the proposed scheme and other schemes of interest over a 4x4 flat quasi-static Rayleigh fading channel are shown in Fig. 4.4.

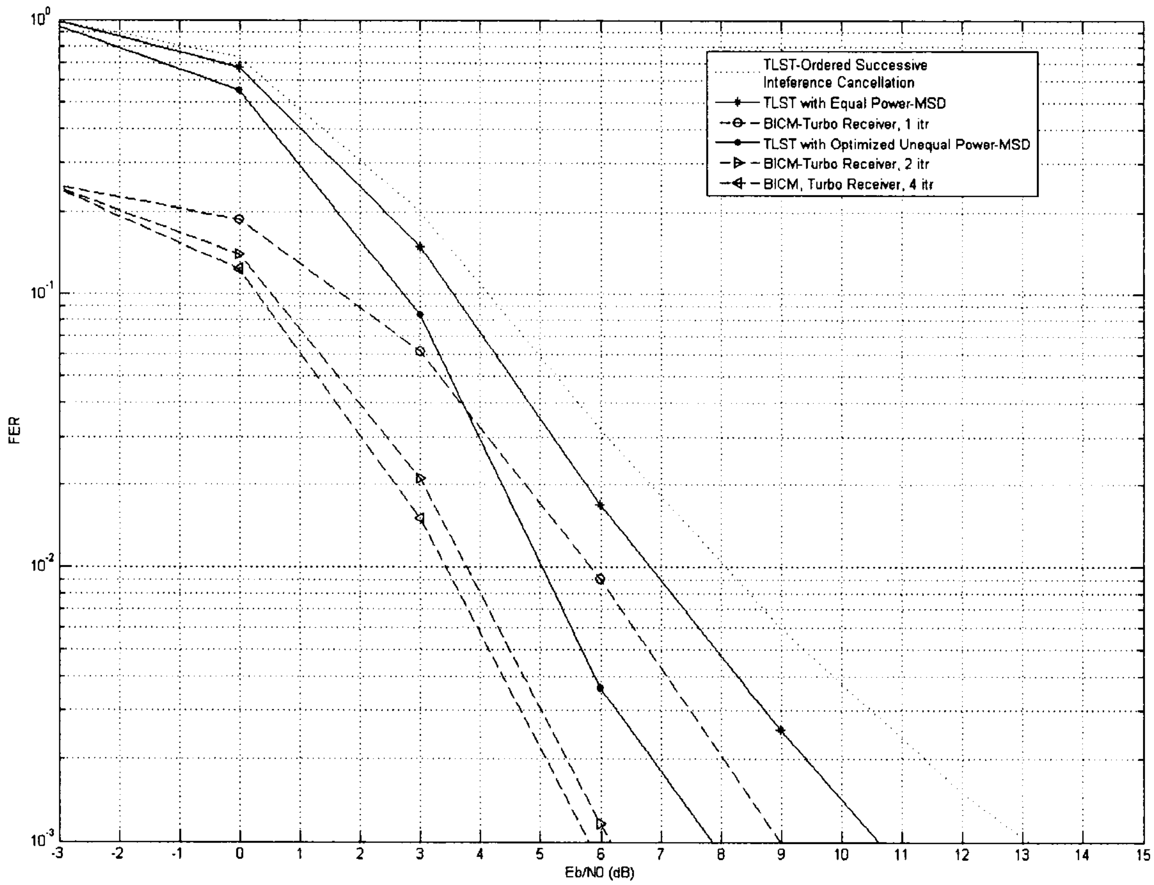



**Fig. 4.4: FER performance comparison of the proposed scheme, LST-MMSE IC, LST with MSD and equal power allocation, and BICM with multiple iterations over a 4x4 flat quasi-static Rayleigh fading channel**

As shown, the proposed scheme significantly outperforms conventional LST. The gap in performance between the proposed scheme and BICM shrinks with increasing $\frac{E_b}{N_0}$. At FER=$10^{-3}$, the proposed scheme outperforms single-iteration BICM in performance by ~1 dB but lags 2 and 4-iteration BICM by ~1.6, 1.8 dBs, respectively. The advantage of the proposed unequal power allocation is quite clear as it achieves a gain of 2.5 dBs over equal power allocation for the proposed scheme. It can be observed that the proposed scheme with equal and non-equal power allocation achieves the same diversity gain as BICM, while conventional LST lags the other schemes in terms of diversity gain at higher $\frac{E_b}{N_0}$. Comparing Figs. 4.4 and 4.2, the benefit of increased spatial diversity in the 4x4 channel over the 2x2 channel is clearly observed. This is of important significance in quasi-static fading channels which are inherently poor in temporal diversity.

Fig. 4.5 presents the FER performances of the proposed scheme and the 4 iteration BICM over a 2x2 quasi-static Rayleigh fading channel when the channel code employed is a convolutional code with rate ½ and constraint length equal to 3.

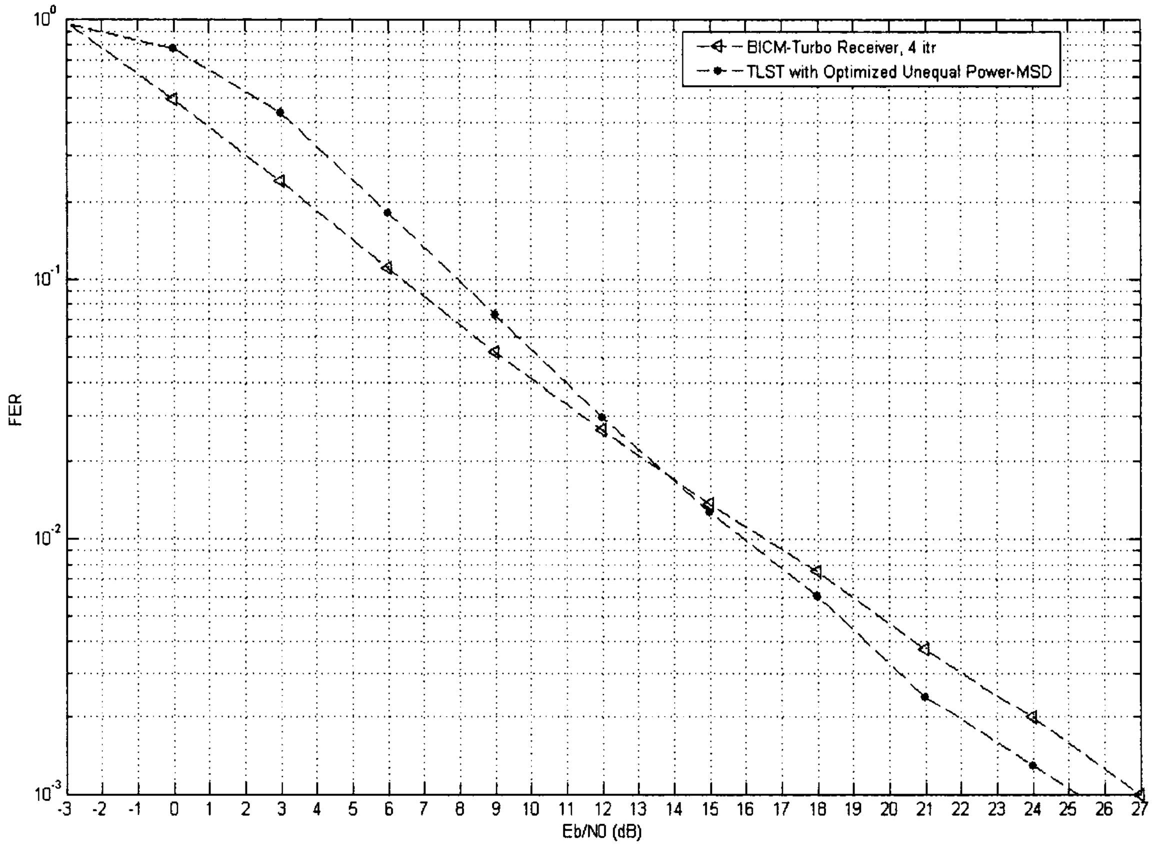



**Fig. 4.5: Comparison of FER performances over a flat quasi-static Rayleigh fading channel between the proposed scheme and 4 iteration BICM for a convolutional channel code with rate ½ and constraint length equal to 3**

The 4 iteration BICM outperforms the proposed scheme at lower values of $\frac{E_b}{N_0}$.

However, it is observed that the proposed scheme outperforms 4 iteration BICM as $\frac{E_b}{N_0}$ increases; At FER= $10^{-3}$, the proposed scheme has an advantage of ~1.9 dB. This is expected as the performance of the BICM turbo receiver is more affected by the strength of the employed channel code. This shows that, for shorter constraint length convolutional codes, the proposed scheme provides improved performance when compared to BICM.

## 4.5 Conclusion

In this chapter, we extended the architecture proposed in chapter 3 to transmission over quasi-static fading channels. The investigation of power allocation required to achieve equal capacities among layers was based on the discussion of outage capacity. We derived a theorem which proves that equal power allocation among layers in an LST scheme maximizes outage capacity for a given power and maximum outage probability in quasi-static channels.

Afterwards, we formulated the optimization problem of finding the power allocation required to achieve equal capacities among layers over quasi-static channels. By means of Monte-Carlo simulations, we presented the power allocation for different schemes and different values of maximum outage probability. We analyzed the loss in outage capacity due to unequal power allocation. It was shown that the loss in outage capacity is marginal and converges to a constant at high SNR.

Simulation results showed that the proposed scheme significantly outperforms conventional LST over quasi-static channels, and approaches the near optimum BICM, with certainly less complexity.

# Chapter 5

## Conclusions and Future Works

In this thesis, we presented a new LST architecture that employs multi-stage decoding combined with unequal transmit power allocation among layers. Multi-stage decoding is employed to exploit the redundancy in error correcting codes for detection improvement in LST with a moderate implementation complexity. The unequal power allocation is optimized to ensure equal capacities in layers. The architecture has been proposed for transmission over fast and quasi-static fading channels.

First, we proposed a new LST architecture with MSD receiver. The multi-stage decoding receiver uses soft detection information in decoding. Decoding decisions are then used in interference cancellation. The complexity of the receiver grows linearly with the number of employed antennas. The use of multi-stage decoding decreases error propagation through successively decoded layers in LST and improves error performance with a relatively similar level of implementation complexity when compared to conventional LST receivers.

The optimization of transmit power to achieve equal capacities among layers greatly simplifies code design and suggests easy parallel implementation. As well, it offers higher protection against interference for firstly detected layers in an LST scheme, which decreases error propagation. For transmission over fast fading channels, the proposed power allocation is optimized to yield equal ergodic capacities among layers. We derived and presented a theorem for finding power allocation in the proposed scheme over fast fading channels in the asymptotic case. The deviation of power allocation from the asymptotic value was shown to be limited over a wide range of channel SNR. This suggests the possibility of constant power allocation and shows the flexibility of the proposed power optimization as opposed to rate optimization in MLC. Simulation results showed that the loss in ergodic capacity due to unequal power allocation is marginal and converges to a constant at high SNR values. This shows that the multiplexing gain of the proposed scheme does not decrease with increasing SNR. We provided a closed form expression of the asymptotic capacity loss in the proposed scheme. Simulation results showed that the proposed scheme significantly outperforms conventional LST in error performance and approaches the near-optimum BICM scheme. Surely, as detection and decoding are carried out jointly in the BICM turbo receiver, the gap in performance depends on the strength of the employed error correcting code. However, the proposed scheme approaches BICM in performance with a much less implementation complexity and decoding delay. This makes the proposed scheme a good candidate for high data rate applications.

For transmission over quasi-static channels, we adopted outage capacity as a capacity index in design. We first investigated the power allocation that maximizes outage capacity for a given maximum value of outage probability. Using the KKT theorem, we proved that equal power allocation among layers achieves the maximum possible outage capacity under outage probability constraints in LST. Then, for the proposed architecture, the unequal power allocation was optimized as to maximize outage capacity with the assumption of equal rates among layers. We formulated the optimization problem and found the optimized unequal power for different numbers of employed antennas and with different values of maximum outage probability. Simulation results showed that the loss in outage capacity for a given maximum outage probability due to the proposed unequal power allocation is marginal, and converges to a constant at high SNR. Simulation results showed also that the proposed scheme outperforms conventional LST in error performance over quasi-static channels and approaches the BICM scheme.

The proposed architecture was shown to offer significant improvements in performance with moderate complexity for a negligible loss in capacity. The introduction of transmit power allocation as a design parameter was shown to offer a convenient trade-off between complexity and performance. Other possible uses of unequal power allocation are yet to be investigated. For instance, employment of the proposed power allocation in DBLAST over quasi-static channels is to be addressed. The employment of the proposed architecture in channel adaptive designs over quasi-static channels is also desirable. The advantages of the proposed architecture suggest the employment of unequal power allocation principle in other schemes in which multi-user detection algorithms are employed at the receiver. This implies an investigation of different power allocation design criteria for different schemes.

# References


[1] G.J. Foschini and M.J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311-335, 1998.

[2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585-595, Nov./ Dec. 1999.

[3] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, no. 8, pp. 1451-1458, Oct. 1998.

[4] D. Gesbert, M. Shafi, D. S. Shiu, P. Smith and A. Naguib, "From theory to practice: an overview of mimo space–time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, NO. 3, pp. 281-302, Apr. 2003

[5] G. J. Foschini, "Layered space-time architecture for wireless communication in fading environments when using multiple antennas," *Bell Labs Tech. J.,* vol. 1, pp. 41-59, Autumn 1996.

[6] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Transactions on Information Theory,* vol. 49, NO. 5, pp. 1073-1096, May 2003.

[7] S. Verdu, *Multiuser Detection,* Location: Cambridge University Press, 1998.

[8] V. Tarokh, N. Sechadri and A. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory,* vol. 44, no. 2, pp. 744-765, Mar. 1998.

[9] Z. Lie, G. B. Giannakis, S. Zhou and B. Muquet, "Space-time coding for broadband wireless communications," *Wirel. Commun. Mob. Comput.*, vol. 1, pp. 35-53, 2001.

[10] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory,* vol. 45, no. 5, pp. 1456-1467, Jul. 1999.

[11] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block coding for wireless communications: performance results," *IEEE J. Select. Areas Commun.,* vol. 17, no. 3, pp. 451-460, Mar. 1999.

[12] B. Vucetic and J. Yuan, *Space-Time Coding*, Location: John Wiley & Sons Ltd., 2003.

[13] W. Firmantoy, J. Yuanz, K. L. Loy, and B. Vuceticy, "Layered space-time coding: performance analysis and design criteria," *GLOBECOM '01. IEEE,* vol. 2, pp. 1083-1087, Nov. 2001.

[14] G. D. Golden, G. J. Foschini, R. A. Valenzuela and P. W. Wolniansky, " Detection algorithm and initial laboratory results using the V-BLAST space-time communication architecture," *Electronics Letters,* vol. 35, no. 1, pp. 14-15, Jan. 1999.

[15] H. El Gamal and A. R. Hammons, "The layered space-time architecture: a new perspective," *IEEE Trans. Inform. Theory,* vol. 47, pp. 2321-2334, Sept. 2001.

[16] G. D. Golden, G. J. Foschini, R. A. Valenzuela and P. W. Wolniansky, " Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE J. Select. Areas Commun.*, vol. 17, no. 3, pp. 1841-1852, Nov. 1999.

[17] U. Wachsmann, R. F. H. Fischer and J. B. Huber, "Multilevel codes: theoretical concepts and practical design rules," *IEEE Trans. Inf. Theory,* vol. 45, no. 5, pp. 1361-1391, Jul. 1999.

[18] H. Imai and S. Hirakawa, "A new multilevel coding method using error –correcting codes," *IEEE Trans. Inf. Theory,* vol. 23, no. 3, pp. 371-377, May 1977.

[19] Thomas M. Cover and Joy A. Thomas, *Elements of Information Theory,* Location: John Wiley & Sons Ltd., 1991.

[20] G. Caire, G. Taricoo and E. Biglieri, "Bit interleaved coded modulation," *IEEE Trans. Inf. Theory,* vol. 44, no. 3, pp. 927-946, May 1998.

[21] B.M.Hochwald and S.ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. on Communications*, pp.389-399, Mar. 2003.

[22] D. D. Falconer and G. J. Foschini. "Theory of minimum mean-square error QAM Systems employing decision feedback equalization," *Bell Labs Tech. J.,* pp. 1821-1849, Dec. 1973.

[23] J. Salz, "Optimum mean-square decision feedback equalization," *Proceedings of the IEEE,* vol. 67, no. 8, pp. 1143-1156, Aug. 1979.

[24] L. H. –J. Lampe, R. Schober, R. F. H. Fischer, "Multilevel coding for multiple antenna transmission," *IEEE Trans. On Wireless Communications,* pp. 203-208, Jan. 2004.

[25] M. Lamarca and H. Lou, "spectrally efficient mimo transmission schemes based on multilevel codes," *IEEE 6th Workshop on Signal Processing and Advances in Wireless Communications*, pp. 1063 – 1067, Jun. 2005.

[26] P. B. Rapajic, M. L. Honig, and G. Woodward, "Multi-user decision-feedback detection: performance bounds and adaptive algorithms," *Proc. IEEE International Symposium on Information Theory*, pp.34, Aug. 1998.

[27] Duel-Hallen, "A family of multi-user decision-feedback detectors for asynchronous code-division multiple-access channels," *IEEE Trans. Comm.*, pp. 421-434, Feb/Mar./Apr. 1995.

[28] R. Muirhead, *Aspects of Multivariate Statistical Theory*, Location: John Wiley & Sons, 1982.

[29] N. R. Goodman, "Statistical analysis based on a certain multivariate complex Gaussian distribution (an introduction)," *Ann. Math. Stat.*, vol. 34, no. 1, pp. 152-177, Mar. 1963.

[30] S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products,* Location: Academic Press, 1980.

[31] J. Proakis and Masoud Salehi, *Communication systems engineering,* Location: Prentice Hall, 2002.

[32] T. Guess, H. Zhang and T. Kotchiev, "The outage capacity of BLAST for MIMO channels," *IEEE International Conference on Communications,* vol. 4, pp. 2628 – 2632, May 2003.

[33] J. Borwein and A. Lewis, *Convex analysis and nonlinear optimization,* Location: Springer Publishing, 2006.

[34] A. Ruszczynski, *Nonlinear Optimization,* Location: Princeton University press, 2006.

[35] D. Klein, *Lagrange Multipliers without Permanent Scarring,* Location: http://www.cs.berkeley.edu/~klein/papers/lagrange-multipliers.pdf

[36] T. Guess and M. K. Varanasi, "Error exponents for maximum-likelihood and successive decoders for the Gaussian CDMA channel," *IEEE. Trans. Inform. Theory,* vol. 46, pp. 1683-1691, Jul. 2000.

[37] http://www.mathworks.com/