Motivating Adult Students Taking a Basic Algebra Course in a University Setting

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Abstract

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Understanding the motivation of students learning mathematics and using this understanding to strengthen motivation can improve mathematics instruction for students, especially students who may dislike math. Motivation is modeled as arising from an interaction of needs, goals and dimensions of the self, resulting in behavior that is regulated by feedback and external factors.

The study uses a Conjecture-Driven design in the teaching situation of MATH 200 – Fundamental Concepts of Algebra which is a required course for many students. That they have to take a basic algebra course at the university level is indicative of some previous difficulties with mathematics which in turn can be linked to negative affect towards math. The conjecture was that motivation would be lacking among this group of students but that a class that is taught from a motivational standpoint would result in better attitudes towards math. Based on an a priori profile of motivational characteristics, *the hypothetical student*, the course was taught with the aim of improving motivation. Observations, course evaluations, a questionnaire and a survey were used to: (1) create a profile of observed motivational characteristics, *the realistic student,* and (2) to describe the effect of the course on student motivation.

It was found that a classroom that addressed students' needs for autonomy, competence and relatedness, promoted an understanding of why a procedure was used (rather than just how to apply the procedure), and that at all times respected the dignity of students, was motivational. In this classroom, the students reported improved affect towards mathematics across the dimensions of emotions, attitudes, beliefs and values.

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My thanks also go to the students in MATH 200, in the summer session of 2011. It was a pleasure to teach and interact with this very fine group of individuals.

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Chapter 1

Introduction

1.1 Motivation for this Research

There are many people who confess that they do not particularly like mathematics. These people may use basic math skills on a daily basis, yet if questioned they admit that the math they learned in school was mostly useless and largely incomprehensible. This group may also disclose that they were not very good at math either. I have had the occasion to encounter some people who held these views. When I returned to university to study mathematics after an absence of more than 20 years, many people were incredulous that I would consider mathematics as a chosen discipline. N.C. Miller (2000) aptly describes this phenomenon in the subtitle of her doctoral thesis, "Perceptions of Motivation in Developmental Mathematics Students: I Would Rather Drill My Own Teeth."

Moreover, success in mathematics is highly valued by parents, educators and potential employers. A skill in mathematics is often equated with the potential to succeed in postsecondary studies and the workforce. While this may be a view that is held by many, it is not my contention. My belief is that there are many pathways to success, and many brilliant people who might or might not have an aptitude for mathematics. Nevertheless, there is a great deal of pressure that is placed on students to succeed in mathematics.

It is not surprising that people who have difficulties with mathematics as it is taught in high school or college (CEGEP in the province of Quebec) fail to develop a taste for further mathematics studies. Furthermore, it is reasonable to expect that people who dislike mathematics might encounter difficulties. The twin questions of *why* individuals dislike mathematics and *how* they may be helped to learn are likely to be quite complicated but it may be possible to derive some insight from the study of motivation.

1.2 The Study of Motivation

The study of motivation is concerned with the motives of individuals; with the reasons why they act the way that they do. Inherent in this field of study is also the idea that if one can access peoples' motives then one can predict their behavior and attempt to direct it in a particular way. Motivation becomes of interest in the field of education as soon as there are teachers who want their students to learn; perhaps more than the students want to learn for themselves. Teachers may attempt to motivate students using a variety of strategies that have been recommended in published articles (for example, Diette & Howe, 2003; Menon, 2004) or promoted on internet sources (National Council of Teachers of Mathematics, 2012). While these efforts may be well-intentioned, the adopted strategies may reflect more of what might seem motivating to the teacher than what is actually motivating for the students. Designing effective motivational strategies involves an understanding of what motivates students to learn. Furthermore, motivating strategies that might be effective for high school students learning algebra may not be applicable to adult students in a university algebra course. Galligan & Taylor (2008) review some of the research that has been conducted among this population of students (termed the "bridging mathematics community"). The authors conclude that this research domain is "under-theorized" and does not focus on effective teaching practices.

In general principles, motivation follows interest. People are generally strongly motivated to participate in activities that they like or enjoy (Reeve, 2005). Given that a large segment of the general population may not like mathematics, an algebra classroom of adults could very well contain many individuals who are weakly motivated to study. Strengthening the motivation of students is one way to improve mathematics instruction for this group of students.

1.3 Preparatory Courses

Concordia University offers preparatory courses (MATH 200 – MATH 209) that cover pre-university level material. Students may be required to take one or more of these courses as a condition of their acceptance into the program of their choice or mature students may be advised to take them if their mathematics knowledge is not current. The first of these is MATH 200 – Fundamental Concepts of Algebra. In the summer of 2011, I had the opportunity to teach this course. One of my first considerations was the population of students that would be in the class.

The topics in MATH 200 included: an introduction to real numbers, arithmetical operations, algebraic expressions, solving algebraic equations, rational expressions and the final topic was an introduction to radicals. The entire content of the course is covered in the high school curriculum in Quebec, suggesting that the population of students would include individuals who had experienced difficulties (or failure) with this material in the past. There might also be students who were seeking to improve their record to meet the entrance requirements for a particular program and students who needed a review of basic concepts. It was my concern that the majority of these students might not like mathematics very much at all and that their motivation for the course would be weak. Naturally, some students could be taking the course as an elective but these students would be much more likely to be interested in mathematics and hence more strongly motivated. My intention as an instructor was to deliver a course that would improve the motivation of the students.

My other concern was that the students in the class would be adults, fully cognizant of the fact that this course covered high school material that they had not yet mastered. Remedial courses often repeat the same material at a slower pace and many students still do not succeed (Miller, 2000). When someone is speaking in a language that is foreign to me, I do not understand any better if they speak slower (or louder). What I felt was needed was a different approach but one that respected the students as adults. Sierpinska (2006) observed that preparatory courses focus primarily on techniques of arriving at a solution rather than offer any theoretical justification and this can be frustrating for students. My aim was to develop the students' abilities by consistently showing how to apply the techniques but also why the techniques were appropriate. In short, I wanted the students to understand.

1.4 Goals of this Research

The above defines some of my goals as an instructor. My goals as a researcher were to investigate and describe the motivation of the students in this class.

The goals of the research that is the object of this thesis then are threefold:

1. To develop a model (concept map) of motivation that can be used to describe the motivations of students in a basic algebra class in a university setting.
2. To use the model of motivation to develop a profile of the students who would be taking such a course (the *hypothetical student* model) and to refine the profile based on information derived from the course (the *realistic student* model).
3. To design effective motivational teaching strategies for this group.

As both instructor and researcher, I had multiple goals. As an instructor, I taught the course to the best of my abilities, attempting to motivate the students as much as possible. The students' learning was my primary consideration and any decisions or interventions that I made were in what I thought were the students' best interests. As a researcher, I documented those decisions and interventions. Attempting to play both roles is a situation that can introduce bias but it can be kept to a minimum if the researcher/instructor is mindful of the distinction (Tabach, 2006).

1.5 Structure of the Thesis

Motivation has been studied extensively in the field of psychology but it is still a matter of some debate and there are many theories that contribute. A single theory of motivation has not been developed (Reeve, 2005). In Chapter 2 of this thesis, I discuss some of the theories of motivation as they are pertinent to a discussion of the motivation of students in a mathematics classroom. A concept map for motivation is presented that includes the relevant aspects of the theories that are discussed. This map is not intended to be a unified theory but it serves to illustrate the components of motivation and how they interact.

The research that is the subject of this thesis is the experiment of teaching MATH200. It is situated in the dynamic situation of a classroom over the length of a summer semester. It is not a controlled experiment in which a hypothesis is tested; it is a "Conjecture-Driven" research design, as described by Confrey & Lachance (2000). In this design, a conjecture is developed that is based on an ideological stance and theoretical considerations. The conjecture is improved and elaborated upon over the course of the experiment. In Chapter 3, the design is described in more detail and the conjecture of this thesis is developed. The conjecture that was adopted is: "Motivation is lacking in a beginning algebra class but if a class can be taught in a way that is motivating for students who dislike mathematics, it will result in better attitudes towards mathematics." In order to make use of this conjecture, I developed the construct of the hypothetical student (HS) – a compilation of the motivational characteristics that a typical student, whose experiences in mathematics had been negative, would potentially exhibit. In Chapter 3, the assumptions that were made about the HS are described.

Chapter 4 describes the research procedures that were used. The course is described in detail and there is a brief introduction to the population of the students in the class. The theme and some examples of the content of the first lecture are presented as these illustrate the steps that were taken to address the motivation of the HS at the outset of the course. The measures that addressed the needs of the HS in the remainder of the course are presented and Chapter 4 closes with a description of the instruments that were used to describe the motivation of the students. These are: observations from class, e-mail communications with students, marks, two questionnaires that were completed during the term and a survey that was circulated to students (who gave consent to participate) after completion of the course.

Observations and results of the instruments used to describe motivation are presented in Chapter 5. Detailed results of the survey can be found in Appendix D of this document.

The model of the HS is refined in Chapter 6. The aim of this chapter is to improve upon the description of the HS based on the data that was collected and to describe the effect of the course on these characteristics. The assumptions that were made for the HS are presented with a summary of the pertinent evidence. Where warranted by the evidence, modifications are made to the assumptions. A compilation of the revised characteristics represent a more realistic model of motivational characteristics and hence is called the realistic student (RS). The effect of the course on motivation is considered with each characteristic as it is appropriate. A detailed presentation of the evidence appears in Appendix E.

In Chapter 7, the limitations of the research are described and the results are discussed in the context of the literature. Recommendations and conclusions are presented.

Chapter 2

Theoretical Framework

2.1 Overview of the Study of Motivation

The study of motivation has roots that can be traced back to ancient Greek civilization. Until the time of the European renaissance, two types of motivation were recognized. The 'will' represented active, good, rational motives and 'bodily desires' were primitive impulsive and biological impulses. As theories of evolution gained acceptance, the study of motivation focused on a genetic concept of 'instinct' which could explain a great deal of unlearned behavior in animals but also in humans (Reeve, 2005). R.S. Woodsworth (as cited in Reeve, 2005) introduced the concept of drive in 1918 to replace instinct. Sigmund Freud, in 1915 and Clark Hull, in 1943 (as cited in Reeve, 2005) both developed drive theories which proposed that behavior was energized by the needs of the organism.

As psychology and especially cognitive psychology emerged and developed as a field of study, motivation research rejected the passive view that humans are controlled only by instinct or drive and began to adopt a more active portrayal of humans. Several perspectives evolved within the field of psychology to explain and predict behavior. Reeve (2005) lists seven main perspectives: behavioral, physiological, cognitive, social-cognitive, evolutionary, humanistic and psychoanalytical. Each perspective views motives as arising from different sources. For example, a behaviorist views motives as emerging from incentives and rewards. Many theories now exist to explain and predict behavior and Reeve refers to these as "mini-theories." The following are examples of mini-theories and a parenthetical supporting reference (as cited in Reeve, 2005) is given for each: Flow theory (Csikszentmihalyi, 1975), an attributional theory of achievement motivation (Weiner, 1972), achievement motivation theory (Atkinson, 1964) and self-efficacy theory (Bandura, 1977).

Each perspective has many supporters and detractors, as do the mini-theories, and a single unified theory of motivation does not exist. The emergence of a wealth of mini-theories though allowed the study of motivation to be applied to a wide range of socially relevant problems as each theory seeks to explain aspects of human behavior. These theories overlap and sometimes disagree with each other and the definitions are not always consistent. In this chapter, I will attempt to describe the underlying concepts that are generally accepted in the field of motivation, highlighting the aspects of this field that are pertinent to a university mathematics classroom. The perspective that I will use is largely cognitive but will also incorporate aspects drawn from behavioral, socio-cognitive and humanistic perspectives where needed or helpful.

2.2 Definitions

The terms *motive*, *motivate* and *motivation* are used frequently in everyday contexts. The noun motive is generally used to mean the reason(s) why someone does something. To motivate someone is to make them willing to act. Motivation is a word commonly used to describe how much willingness or enthusiasm a person has for a particular task.

Merriam-Webster's definition of motive is, "Something (as a need or desire) that causes a person to act." To motivate someone is to "provide with a motive", and motivation is "the act or process of motivating" or "the condition of being motivated" (Merriam-Webster Inc. , 2010).

As a psychological term, Reeve (2005) defines motive as, "An internal process that energizes and directs behavior" (p. 6) and motivation as, "Those processes that give behavior its energy and direction" (p. 39). Hannula (2006) defines motivation as a "potential to direct behavior..." (p. 166).

A motive can be a reason, a need or a process. The word then has both a static connotation as in a particular reason for an act and a more fluid connotation as a changing process that causes a person to act. For the purposes of this research, I will use the term motive in the everyday sense. It is the reason(s) for a particular behavior. The term *need* will be used according to Reeve's (2005) definition, "Needs are conditions within the individual that are essential and necessary for the maintenance of life and for the nurturance of growth and well-being" (p. 6).

The term motivation will be used to capture the more dynamic aspects of the topic. It is a collection of motives but also the potential and process by which the motives are translated into action. With this viewpoint, the relative strength of a person's motivation can be described. A person who is vigorously and energetically performing a task can be described as strongly motivated whereas another person who gives a lackluster performance would be weakly motivated. This person may have strong motives to act but may be experiencing some difficulty or blockage with translating these reasons into action.

An additional definition is required when considering the complexity of motives. At any point in time, a person may have a complex range of motives and some of these may be conflicting (Hannula, 2006). A person may have a good reason to study but also want to watch a TV program, eat supper and connect with family and friends. The relative strengths of these motives may also be in a continuous state of change, even over a short period of time. For this, Reeve (2005) uses the term *motivational state* (p.17). If this person is actually studying then it is because at this time the motivational state for studying dominates other motives. Ten minutes later, when the phone rings, the individual's motivational state changes and the motive for answering the call dominates. Motivating someone then involves changing their motivational state and increasing the likelihood that it will reoccur in the future.

Intrinsic motivation is defined by Reeve (2005) as, "The innate propensity to engage one's interests and to exercise one's capacities and in doing so, seek out and master optimal challenges" (p. 134). *Extrinsic motivation* is, "An environmentally created reason to initiate or persist in an action," (p. 134).

Since motivation is about the reasons for a course of action, the topic cannot be separated from cognitions or emotions. Cognitions are mental events and encompass beliefs, expectations and self-concept. Emotions are defined by Reeve (2005) as, "Short-lived subjective-physiological-functional-expressive phenomena that orchestrate how we react adaptively to the important events in our lives" (p. 38). Cognitions are what we think, emotions are how we feel.

2.3 Assessing Motivation

In order to describe motivation, one needs to address the aspects of quality and quantity. Motivation varies from person to person and from time to time in both aspects. In terms of quality, two people engaging in the same activity may do so for completely different reasons. A person's strength of motivation is also a quantity that is difficult to measure. A person who is studying in the face of many possible distractions can be said to have stronger or more motivation that a person who studies because there is little else to do.

It is not possible to directly observe or measure motivation. Instead, motivation can be inferred from behavior. Since behavior exists within an environment, it is necessary to sufficiently describe the aspects of the environment that might have bearing on the behavior. It may be difficult to summon the motivation to study in a coffee shop with friends. The conditions that lead up to the situation – antecedents – are also important to describe motivation. After failing to solve 3 algebra problems, the motivation to continue study may be greatly decreased or extinguished completely.

Aspects of behavior can provide information about motivation. The choice of action, the effort expended, facial expressions and bodily gestures during the behavior are all aspects that can be observed. Latency (time from exposure to response initiation), persistence (time from initiation to cessation) and probability of response (the number of times the behavior occurs) are all aspects that can not only be observed but under some circumstances measured. Physiological changes (temperature, blood pressure etc.) can also be monitored (Reeve, 2005). Although facial expressions and bodily gestures play a significant role in interpersonal communication in a classroom setting, the class was not videotaped so there is no information available from these. Similarly, physiological changes were not monitored.

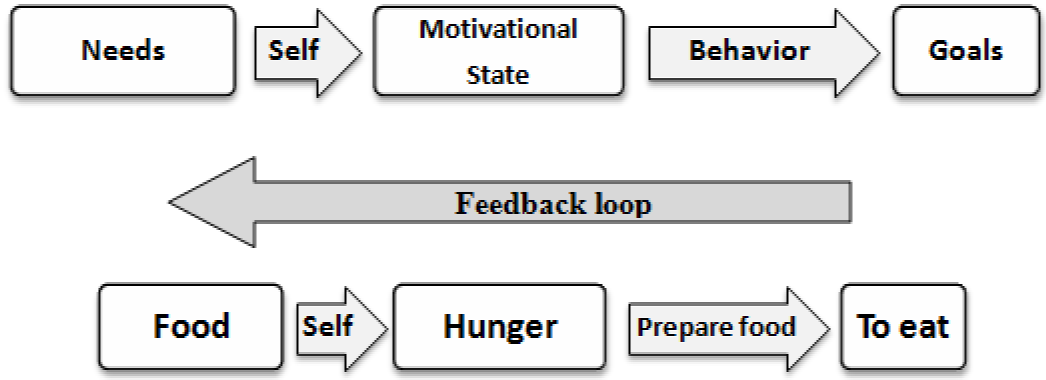
It is also possible to obtain information about motivation by simply asking. Questionnaires are a tool that can be used for this purpose but there are two cautions that must be considered before relying too heavily on this method. Firstly, people are often reluctant to report their true motives and secondly, people may not be aware of all the sources of motivation for their behavior (Reeve, 2005). However, questionnaires can be used to confirm or refute inferences from other data.

2.4 Concept Map

The study of what motivation *is* leads inevitably to the idea that what we do can influence others – the idea that one can motivate people. To accomplish this objective it is necessary to understand how motivation works.

People are complex organisms and no two individuals are alike. However, all individuals have needs that must be met in order to live and grow. Motives arise from these needs and direct behavior towards goals. (A goal is simply defined as what one wants or hopes to achieve.) A simple model is shown in Figure 2.1.

Figure 2.1 A basic model for motivation



The wide arrows in the figure are intended to indicate that the relationship can be one-to-one (as in out of one need arises one motive) but it can also be one-to-many, many-to-one or many-to-many. The first arrow, "self" indicates that although individuals may share needs, the ways that this provokes the motivational state is very much dependent upon the individual. The self in this model is a compilation of everything an individual is born with (genetic and physiological makeup) and all of their past experiences. The arrow marked "behavior" also indicates a choice which is affected not only by the self but also by the environment. In this example, the environmental factor would be the availability of food. If there is no food in the house, a feedback loop is initiated causing a revision of motives and behavior until the goal is either achieved or abandoned.

The model is simple but it will be developed in the following sections to describe the complex motivation of students who are taking a beginning course in algebra in a university setting.

2.5 Needs

An *organismic* approach to motivation, according to Deci & Ryan (as cited in Reeve, 2005), is a model that describes human needs in 3 types: Physiological needs, psychological needs and social needs. This model, with one minor change will be used.

2.5.1 Physiological Needs

These are, as expected, the biological needs of an individual. The body needs food, water, warmth etc. When a state of deprivation exists, these needs dominate the motivational state; when satisfied, they fade into the background of consciousness until they reoccur. These needs are regulated biochemically and neurologically within the body (Reeve, 2005). Boekaerts (as cited in Hannula, 2006) asserts that physiological needs can interfere with learning in a classroom. In a university classroom, it will be assumed that the students have the means and abilities to satisfy their needs. These needs may surface during a 3 hour long class but a university student should be able to appropriately manage them. Satisfying these needs over the semester however, may occupy a significant portion of the students' time and may divert attention away from their studies, as would be the case if the student had a summer job.

2.5.2 Psychological Needs

These needs are inherent in all individuals (to varying degrees) and represent the strivings of the individual. They provide the energy to participate in activities that are not directly connected to a person's survival and provide the motivation to learn, grow and develop (Reeve, 2005). They are the needs for *autonomy*, *competence* and *relatedness*.

Autonomy. Autonomy is a person's need to feel that they control their own actions and that they choose to act ("I want to study" instead of "the teacher assigned these problems"). Environments can be controlling or autonomy-supportive. In a controlling environment, a person is told what to think, feel and do implying motivation that is external. In an autonomy-supportive environment, people are able to direct their own behavior. Hence motivation is internal, stemming from interests and preferences. A great deal of evidence has been gathered that autonomy-supportive environments are beneficial (Reeve, 2005). In short, people learn better, perform better and feel better when the environment addresses a need for autonomy. A student-centered classroom where students learn in groups is supportive of their need for autonomy (Hannula, 2006).

It is not always possible to provide an autonomy-supportive environment in a university classroom. The MATH 200 curriculum is fixed and a detailed course outline, designed by the course coordinator, governs the workload, putting strong limitations on what an instructor can do. It is not possible to allow students to pursue their own interests at their own pace but some elements of an autonomy-supportive nature can be incorporated. For example, the instructor can listen carefully to the students, allow them time to talk, respond to their questions and acknowledge their perspective. The students can sometimes be given options as to what they want to do (more problems, take a break, etc.) by means of polling the class. The instructor can take the time to encourage effort and praise progress towards a goal. Lastly, and of tremendous importance, the instructor can provide a rationale for studying mathematic in general or for the specific topic at hand. When an individual is provided with reasons for an act and the reasons make sense, it facilitates their internalization, defined as, “the process through which an individual transforms a formerly externally prescribed value into an internally endorsed one” (Reeve, 2005, p. 123).

Competence. A psychological need for competence is the need to feel able to cope with the environment. A need for competence provides the motivation to seek out and master the challenges that the environment provides (Reeve, 2005). Challenge must match ability. Csikszentmihalyi & Nakamura (as cited in Reeve, 2005), established that, "Flow is a state of concentration that involves a holistic absorption in an activity. Flow is such a pleasurable experience that the person often repeats the activity with the hope of experiencing flow again and again" (p.115). Flow happens when a person's ability is matched to the amount of challenge and the result is that the activity is enjoyed. If a task is too challenging (for a person's skill level) it threatens a person's competence and the individual becomes worried or anxious, prompting avoidance behaviors. If a task is too easy it results in boredom and poor performance because it neglects the individual's need for competence (Reeve, 2005). Thus, there is an optimal amount of challenge for an individual that permits the experience of flow. It is in flow that the best learning happens. The worst combination is low skill and low challenge; a situation that produces apathy. In a beginning algebra class, the students most likely have low skills so it is of utmost importance that the challenge of the tasks be carefully chosen.

Vygotsky, (1978) describes a *zone of proximal development* when referring to the development of children, "It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). This is precisely the zone that is required in order to ensure optimal challenge for students in a classroom. It is the situation in which they are *almost* capable of arriving at a solution independently. The task is not too easy because they *are* able to arrive at a solution with some assistance and for exactly the same reason the task is not too challenging. It is an optimally challenging situation.

The need for competence begins to be satisfied when the individual receives feedback about their performance. Feedback can be obtained from: the task itself (obtaining a correct answer), a comparison to one's past performance, a comparison to others or obtained from external sources (a grade on a midterm). Feedback can provide a sense of success or failure. If the challenge is optimal then both are equally likely (Reeve, 2005). It is generally accepted that it is from our failures that we learn the most but success is certainly better. The environment then must encourage success but be tolerant of and even value failure. An explanation of why a student got a wrong answer provides better quality feedback than just a numerical score.

Structure is also of importance when considering feedback. Expectations must be clearly explained and the criteria for success must be modeled, otherwise feedback is independent of the task. It is not sufficient to show how to complete an algebraic problem; it must also be shown 'what a good answer looks like', i.e. the answer that will receive full marks on a test.

It is from optimal challenge that people learn new skills and improve existing ones. Progress is monitored by feedback. With improved skills, greater challenges can be undertaken. In any classroom however, there are students for whom the challenge is too great. It is these students that are at risk of failure. Remedial measures are necessary that do not assault these students' need for competence.

Relatedness. Relatedness is the third of the psychological needs. It is a need to form positive relationships. Reeve (2005) defines two types of relationships. The first are exchange relationships in which people do business together and the second are communal relationships in which people monitor and keep track of each other's needs without regard for gain. In a classroom situation, a relationship is formed between the students and the instructor. In a typical university class of 50 or more students, this relationship is typically of the exchange type. The opportunity does exist however to create, not a communal relationship, but one that is somewhere between the two types. An instructor that takes the time to learn students' names, communicates in a positive manner, offers feedback appropriately and supports the students' needs and interests is establishing a more positive relationship.

It is in positive relationships that internalization occurs. It is more likely in a supportive relationship that a student will adopt the values and prescriptions of the instructor. For this to be of benefit to learning, the values and prescriptions must be appropriate and the student must also see their value, meaning and usefulness. For example, if the instructor explains the value of practicing on a regular basis, it is the nature of the relationship that determines (in part) whether or not the student will accept this as good advice and adjust their behavior accordingly.

In summary, when people's needs for autonomy, competence and relatedness are being satisfied, they show high levels of engagement in an activity. According to Wellborn (as cited in Reeve, 2005), this is characterized by high levels of attention, effort, persistence, verbal participation and positive emotions. This is what a classroom of motivated students would look like.

2.5.3 Social Needs

These needs originate from a person's experiences and exist as part of an individual's personality. They include the needs for achievement, affiliation or intimacy and power. Atkinson, Heyns & Veroff (as cited in Reeve, 2005) defined *affiliation* and *intimacy* as the needs of an individual to establish and maintain an affective relationship with another. Winter (as cited in Reeve, 2005), described *power* as the need to establish, maintain and expand control. Neither of these needs play a significant role in the motivation of students in a university classroom in any way that can be described or measured here.

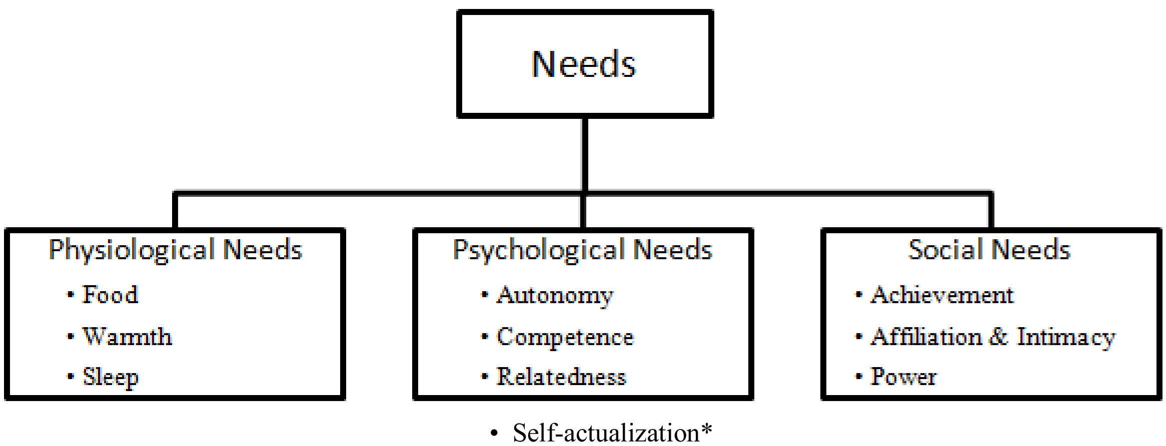
An achievement situation is one in which there is a standard of excellence provided and a university course certainly meets this criterion. Reeve, (2005) citing several studies, found that individuals who have a high need to achieve, relative to a standard of excellence, react with approach emotions (hope, pride etc.) and approach behaviors (choose difficult versions of a task, engage in the task quickly, show more effort, and persist more) (p.168), as opposed to avoidance emotions and behaviors. Reeve characterizes high need achievers as individuals who have perceptions of high ability and high expectations for success (relating to self-efficacy beliefs). These individuals respond to difficulties with renewed effort (mastery orientation), remain optimistic even when confronted by failure (explanatory style) and place a high value on success (values) (p. 169). Self-efficacy, orientation, explanatory style and values are dimensions of the self that will be described in Section 2.8, below. Thus, it is the dimensions of the self that are used to determine the extent of this need for an individual.

The extent to which an achievement situation is regarded as a competition also effects different people in different ways. It encourages approach behaviors in high need achievers but avoidance behaviors in people who have a strong fear of failure (Reeve, 2005). Displaying students' marks would add an aspect of competition to the achievement situation of a university class.

This model of human needs is summarized in Figure 2.2. Before leaving the topic, I would like to add one other aspect to the palette of needs.

In 1943, A. H. Maslow described a hierarchy of human needs that became a very popular model (as cited in Reeve, 2005, p. 412). Although Maslow's theories are very intuitive, little evidence has been produced that confirms that human needs exist in a hierarchy (Reeve, 2005). Maslow placed the need for *self-actualization* at the top of his pyramid and identified it as a growth need. He described this as a tendency that guides the individual towards a genetically determined potential and it motivates the individual to want to undertake new and challenging experiences (as cited in Reeve, 2005). A person taking a mathematics course later in life because they want to realize their potential is acting out of self-actualizing (or self-determining) tendencies. I would like to include this tendency in the model because I feel that it represents a need that does not fit easily into any of the other categories of the organismic approach.

Figure 2.2 Types of needs in an organismic approach



Adapted from, *Understanding Motivation and Emotion*, 4th ed., Johnmarshal Reeve, 2005, (p. 103). *The item, Self-actualization was added by this author of this thesis.

2.6 Goals

All people set goals. A goal is what a person wants to accomplish; what they are striving for. They can be long-term (a career in business), intermediate (a B average this semester) or short-term (complete 10 algebra problems this evening). A number of short-term goals may be linked together in a series or a web[1], progressing towards a longer-term goal. Obtaining a university degree is a web of many intermediate goals and these include the goals of passing all of the

[1] In a web, a collection of short term goals may all be required to attain a longer-term goal but they need not necessarily be obtained in a specific order.

courses required for the degree. Passing an algebra course can be seen as a step in this path but it too is made up of a set of shorter-term goals. Within a series or web of goals, a particular goal can be inhibitory, (Hannula, 2006) as could be the case for a student whose long-term goal was a degree in English Literature who *had* to take a course in mathematics.

An individual may be just starting on a path towards a long-term goal or attaining this goal may be imminent. *Goal proximity* refers to how close an individual is to attaining their goals. A goal that is too distant may not provide much motivation. For uninteresting tasks, long-term goal may provide extrinsic motivation but setting short-term goals can create opportunities for positive feedback, satisfying a need for competence that enhances intrinsic motivation. Setting a goal can provide the motivation that is not inherent in the task itself (Reeve, 2005).

Goals may be set internally by an individual or they may come from external sources. If externally set (as a goal set by an instructor), then the amount of motivation that the goal generates is dependent upon the extent to which the individual accepts the goal. Reeve (2005) identifies the following:

> Four factors determine whether an externally set goal will be accepted or rejected:
> - The perceived difficulty of the task
> - The amount of participation of the individual in the goal-setting process
> - The credibility of the person assigning the goal
> - Extrinsic incentives (p. 211)

Setting a goal creates an ideal state (what the person wants to be) that is different from the present state (what the person is now) and it is from this incongruity (dissonance) that the motivation to act is formed (Reeve, 2005). Once motivated, the individual forms a plan to accomplish the goal. Different people may have the same goal but go about accomplishing it in very different ways. Students are individuals with differing learning styles, abilities and perceptions of the effectiveness of any particular strategy. In an algebra class of 50 students there would be many different plans formed, some adaptive and some mal-adaptive.

As an individual implements their plan, they receive feedback about their progress. Depending on the nature of this feedback, the individual may revise either their goal or their plan, or it may change their motivation (self-regulation).

Some general principles about goals:

- People who set goals outperform people who do not
- Performance increases as the goal difficulty increases
- Effort increases with increased goal difficulty
- Goal attainment brings satisfaction and can lead to setting more difficult goals
- Failure brings dissatisfaction and can lead to abandonment of the goal
- Goal proximity effects persistence – a goal that is too far away provides little motivation
- Focusing on the goal adversely affects performance. Focusing on implementation strategies improves performance (Reeve, 2005, pp. 204-219)

In an achievement situation (for example, a university course setting), an integrated model of achievement motivation (Elliot, as cited in Reeve, 2005) can be used to describe how a need for achievement effects the motivation of students. In this model, the need for achievement is reflected by the types of goals that an individual adopts. These are; mastery goals, performance-approach and performance-avoidance goals. Hannula, (2006) refers to performance-avoidance goals as "ego-defensive" goals (p. 165).

If an individual adopts *mastery goals*, he or she is seeking to develop or improve competence. A typical mastery goal would be, "I want to improve my understanding of fractions." Mastery goals focus the attention on developing competence and a goal is achieved when the person is making progress.

Performance-approach goals are adopted when an individual wants to demonstrate their competence relative to that of others. A performance-approach goal would be, "I want to get an A" (assuming that not everyone in the class will also get an A).

A *performance-avoidance goal* would be, "I do not want to fail this course." People adopt performance-avoidance goals out of fear or anxiety. The fear of failure interferes with performance, persistence and emotionality and manifests as poor coping strategies for stressful situations. Mastery and performance-approach goals are both productive goal orientations but performance-avoidance goals undermine achievement and positive outcomes (Reeve, 2005). Using a multiple goals perspective (in which students goals were described as a blending of all three types to varying degrees), Luo, Paris, Hogan and Luo (2011), found that performance-avoidance goals interfered with learning even when the students also adopted mastery and performance-approach goals.

2.7 Types of motivation

Needs and goals supply the necessary information to describe a person's behavior in sufficient detail to establish that they indeed have some motivation for a particular task or plan. In order to change or improve someone's motivation, it is necessary to understand the different types of motivation that exist and the effect that different interventions can have on different types of motivation.

Some types of motivation have already been alluded to above. According to Ames & Archer (as cited in Reeve, 2005), if a person is adopting a mastery goal then the motivation can be inferred to be the motivation to learn which is different from the motivation to perform. Elliot asserts that the motivation to approach success is not the same as the motivation to avoid failure (as cited in Reeve, 2005). Of these types, the motivations to learn and to approach success will have the better outcomes.

Proponents of self-determination theory assert that there are 3 types of motivation; amotivation (the absence of motivation), extrinsic motivation (4 types) and intrinsic motivation (Reeve, 2005). For the purposes of this research, it will suffice to categorize motivation as simply intrinsic or extrinsic.

Intrinsic motivation is defined above (Section 2.2) and it is when a person's actions are guided by their interests. An intrinsically motivated person is satisfying their psychological needs for competence and autonomy. It occurs when, for example, a person studies mathematics because they like mathematics. Popularization activities seek to increase an interest in mathematics.

A *popularization of mathematics activity* is defined by Kelescenyi (2009) as:

> [It is an] organized action whereby mathematicians volunteer to engage a willing audience in a communication about mathematics or a particular mathematical topic. The aim of the action is to increase the audience's understanding and/or appreciation of mathematics or the particular topic. (p. 28)

Appealing mathematical activities, for which participation is voluntary, meet the definition of a popularization of mathematics activity and have the potential to increase a person's intrinsic motivation for further mathematics involvement.

When an activity is inherently dull or uninteresting to a person then the motivation to participate in the activity must come from external sources. Extrinsic motivation comes from environmental incentives and consequences. Consequently, the motivation is not for the activity itself but it is a desire for the attractive consequences of the activity (Reeve, 2005). How much of a person's motivation is intrinsic or extrinsic is difficult to determine from observations of behavior alone. When I think of my own motives, there are often aspects of both to varying degrees. I study mathematics because it appeals to me but, in a course, I also care about grades.

From a behavioral perspective, extrinsic motivation works via operant conditioning. *Operant conditioning* is a term that refers to how an individual learns to operate in their environment. Incentives precede behavior and constitute what a person expects will be the consequences of their actions. They affect the probability that the behavior will occur. Responses to situational cues depend on past experiences with similar cues. Consequences follow behavior and increase or decrease the persistence of behavior and the probability that it will reoccur (Reeve, 2005).

Reinforcers are factors that will increase the chances of a behavior occurring and they follow a response (Reeve, 2005). Positive reinforcers are rewards that increase the likelihood of a desired behavior. Good grades, approval and awards are examples. Negative reinforcers are stimuli that increase the likelihood of the behavior that removes the stimulus. An annoying alarm clock is a negative reinforcer because it increases the behavior of getting out of bed to turn off the alarm. Negative reinforcers prompt escape and avoidance behaviors – to remove or avoid the aversive stimulus. A deadline could be considered a negative reinforcer because it prompts the behavior of working at a task to avoid the consequences of not completing the task on time.

Punishers are stimuli that decrease the future probability of an undesired behavior. Criticism and bad grades punish the behavior of not studying effectively. (This is not the same as feedback!) Punishers come in two types; positive and negative. Positive punishers remove a positive consequence (response cost), negative punishers administer an undesirable consequence.

If a situational cue of “show your work” is given on a test, and the response is an incorrect answer but some work is shown, then part marks for the question would be a positive reinforcer for attempting the question. If the response is a correct answer with no work shown, then part-marks would be a punisher for the attempt.[2]

Punishers usually result in immediate compliance but at a cost. They often generate negative emotions of anger, fear and resentment and they can cause harm to the relationship involved. (Reeve, 2005)

People attempt to motivate the weakly-motivated by the application of extrinsic motivators, i.e. incentives and rewards. There are hidden costs. Counter-intuitively, an expected, tangible reward will decrease a person's intrinsic motivation for a particular activity. This is the phenomenon that occurs when someone enters a profession that they enjoy but find that once they start to be paid, their enjoyment lessens. It happens because expected rewards decrease a person's perception of autonomy and undermine the intrinsic motivation that a person may have (Reeve, 2005). Expected and tangible rewards interfere with learning and creativity because they shift the goal of the activity away from learning in favor of obtaining the reward. Extrinsic rewards can be used if there is truly no other motivation for the activity. In this situation, there is nothing to lose. Several researchers (as cited in Reeve, 2005, p.145) have concluded that unexpected, verbal or other intangible rewards do not undermine intrinsic motivation.

A behaviorist perspective explains much but people are thinking and feeling organisms. Before a person responds to a situation the information about the situation is subject to cognitive processing and it is washed in a sea of emotions.

Ryan concluded that external events work best to increase motivation if they are administered in an informational rather than a controlling manner (as cited in Reeve, 2005), and praise is most effective when it is specific rather than general. "Good job you did what was

[2] If part marks is viewed as taking away some of the marks, then this is a positive punisher. If part marks is viewed as getting a poor grade, then this is a negative punisher.

expected of you," is more controlling and less effective than, "good job, I like the way you solved that problem."

External events cannot create intrinsic motivation but they can be used to support and facilitate any intrinsic motivation a person may have. Intrinsic motivation is worth promoting because it leads to: greater persistence, more creativity, better learning, optimal functioning and well-being (Reeve, 2005).

In order to motivate the un-motivated then, it is necessary to acknowledge and validate their viewpoint that the task is indeed boring, explain why it is important (autonomy), offer optimal challenge and interest (competence), then apply incentives and consequences as needed in an informational manner.

2.8 The Self

The self in the model of motivation presented above is probably the most difficult to describe. It is a compilation of everything an individual is born with (genetic endowment) and everything that they have experienced. Psychologists have developed many theories in an attempt to understand the self but it is still a topic of much debate. In this section, some of the dimensions of the self will be described as they apply to understanding motivation.

A person's past experiences are the essence of learning how to deal effectively with the environment, given the tools that they were born with. Abilities, even ability in mathematics, are the sum total of experience and genetics and they are developable. Bandura (as cited in Anderman, 2010), asserts that motivation to engage in an activity is "intricately tied to prior knowledge and experiences..." (p.55). A person who has experiences of failure in mathematics would most likely approach an algebra class differently than someone whose previous efforts had been successful. Furthermore, people are not always aware of the motivational basis of their behavior. Some of the motives, desires, moods and emotions that regulate human behavior are subconscious (Hannula, 2006).

Some dimensions of the self are described in the following sub-sections. These are: implicit theories, self-efficacy, orientation, explanatory style, ego-defensiveness, hope, self-esteem and affect.

Implicit theories. *Entity theorists* tend to believe that personal qualities are fixed whereas *incremental theorists* believe that they are malleable (Reeve, 2005). Being a 'math person' would be an example of a personal quality. Incremental theorists adopt mastery goals and see effort as a tool to greater achievement because they can change an aspect of the self. An entity theorist adopts performance goals and expending a lot of effort means that their abilities must be low. High effort then attacks their need for competence so they adopt self-protecting strategies and may decrease effort or withdraw from the task completely (Reeve, 2005). A person who believes that being good at mathematics is a quality that one is born with and that it cannot change will see poor performance as confirmation that they are indeed "not very good at math." If however, a person believes that this quality can be changed, a poor performance will be translated into renewed and increased effort.

Self-efficacy. In order for behavior to be energetic and goal-directed (i.e. motivated), participants must have fairly high *self-efficacy* expectations. This is an individual's belief that he or she can accomplish the task. Self-efficacy is defined by Bandura as: "one's judgment of how well one will cope with a situation given the skills one possesses and the circumstances that one faces," (as cited in Reeve, 2005, p. 228). The components of a high self-efficacy belief are: perceptions of high ability, high expectations of success and a belief that what one does will produce the desired outcome.

People are willing to expend more effort at a task and persist more in the face of difficulties if they have a strong self-efficacy belief. Empowerment is the mechanism by which a person's self-efficacy beliefs can be improved (Reeve, 2005).

Orientation. A *mastery or helpless orientation* refers to how a person responds to failure. With a mastery orientation, the person remains task-focused and maintains their mastery goals even when confronted with dismal failure. A person with a helpless orientation may abandon the task believing the situation to be out of their control and they may try to deal with their negative emotions and doubt by changing the task or the rules (Reeve, 2005). A request for a higher grade would be an example of a student attempting to change the rules.

Learned helplessness occurs when people believe that their behavior will not change the outcome. An individual may feel, "No matter how hard I study, I don't get any better at this." Everyone is susceptible to learned helplessness because it occurs when the environment is not controllable or predictable or it is perceived that way. In a study conducted by Diener & Dweck in 1978 (as cited in Reeve, 2005), people were exposed to either solvable or unsolvable problems. Both groups were then asked to solve some moderately difficult problems. The group that was first exposed to solvable problems significantly out-performed the group that had attempted the unsolvable problems (p. 243). Learned helplessness may be observed in an algebra class if the students' prior experiences have been ones in which mathematics was incomprehensible.

Explanatory style. When people attribute their poor performance to factors that are external, and controllable, they adopt an *optimistic explanatory style*. ("I failed because the textbook is awful.") A *pessimistic explanatory style* is a tendency to attribute failure to internal and uncontrollable factors. ("I failed because I'm not good at math.") An optimistic style may be a somewhat distorted view of reality but it is much more adaptive than a pessimistic style (Reeve, 2005).

Ego-defensiveness. The ego is a concept that is taken from a psychoanalytic perspective. Freud, in 1923 (as cited in Reeve, 2005) proposed that the ego, or personality, filled an adaptive role needed to function in society. Psychodynamic theory asserts that the ego is a process and that it is in a constant state of vulnerability (Reeve, 2005). Entering an algebra class, especially if one's previous experiences with mathematics were negative, is a situation that can certainly be perceived as threatening to one's self or ego. When a threat is perceived, negative emotions are experienced. Individuals react with defensive mechanisms and when sufficient threat is perceived, avoidance behaviors result. Care must be taken to minimize the threat to ego in order to allow motivation to flourish. Preserving the dignity of individuals is a matter of respect.

Hope. All students enter a course with some hope of positive outcomes. It is of great benefit to maintain and encourage hope because "hope alone leads to better performance," (Reeve, 2005, p. 255). In this research, it will be assumed that all the students in a university algebra course are somewhat hopeful.

Self-esteem. This is a measure of how well a person feels about himself or herself. Increasing a person's self-esteem does not improve their performance but it has been shown that increasing a person's abilities has a positive effect on their self-esteem. (Seligman, quoted in Azar, 1994; Helmke & van Aken, 1995, as cited in Reeve, 2005, pg. 261). Self-esteem then, is a dimension of the self but it is an outcome; if improved motivation leads to better performance, then self-esteem can be increased.

Affect. Affect is the observable self. Its manifestations reflect the inner self (DeBellis & Goldin, 2006). McLeod (as cited in Hannula, 2006) identified three components of affect:

emotions, attitudes and beliefs. DeBellis and Goldin (2006) also suggest that a fourth component – values – should be included. Whether or not a person likes mathematics, is a question that can be best answered by the affect that the idea of mathematics generates.

The basic *emotions* of; fear, anger, disgust, sadness, joy, and interest are emotions that are experienced by all. These emotions motivate behavior in expected ways and represent the coping strategies of an individual (Reeve, 2005). Interest in mathematics provides a reason (motive) to participate in mathematics activities. The joy of fulfilling this motive ensures that the participation will likely reoccur.

Attitudes are orientations or predispositions. An attitude can be an orientation towards sets of emotional feelings (DeBellis & Goldin, 2006), or toward patterns of behavior (Zan, Brown, Evans, & Hannula, 2006). An attitude towards mathematics then (for the purposes of this research) is the set of feelings and cognitions that the idea of math generates. DeBellis & Goldin (2006) see attitudes as moderately stable thus allowing for the possibility of lasting change.

Beliefs are essentially what one holds to be true. DeBellis & Goldin (2006) state that beliefs are, "highly stable, highly cognitive and highly structured…" (p. 135). Hence, a belief that mathematics is completely useless would be resistant to change. Since cognition is involved more than emotion, changing a person's belief would have to address the premises of the belief in a very rational manner.

Closely related to ethics and morals, the ideas that an individual has a commitment to, are his or her *values.* They regulate long-term choices and short-term priorities and may be highly structured (DeBellis & Goldin, 2006). Values are what one holds to be important and people tend to act in ways that are consistent with their values. A person's values are involved when they choose not to cheat on an exam, not out of fear of the consequences but because it is not "right." An individual values mathematics if they see it as being important in the real world or their own personal situation.

DeBellis & Goldin (2006) see these four components of affect as interconnected and interacting dynamically. They also introduce the concepts of "mathematical intimacy" and "mathematical integrity" (p. 137). Mathematical intimacy is an emotional relationship to mathematics that involves feelings of warmth, excitement, amusement and affection (among others). This is the concept of 'ownership' of mathematics. Mathematical integrity is a "psychological posture" that describes a quality of commitment to truth and understanding in mathematics (DeBellis & Goldin, 2006). Although these are both interesting concepts, worthy of further exploration as they relate to students' motivation, they are not qualities that can be described or measured in this research.

2.9 Self-Regulation and Feedback

Self-regulation is the process by which individuals manage their behavior (and motivation) through the control systems of attention, meta-cognition, emotion, volition control and others. (Boekaerts and Niemivirta, as cited in Hannula, 2006) For the purposes of this research, it is how affect is involved in the changing behaviors of students over time. Feedback is the mechanism by which individuals determine how to modify their behavior. Almost as soon as an individual begins to act, they start to receive feedback about how they are doing so far.

Types of feedback have been noted above and can be received from the following sources: the task itself, external sources, and/or comparisons of one's performance to past performance or to the performance of others.

One important type of feedback is the feeling that an understanding has been obtained. What constitutes an understanding has been a subject of much philosophical debate. Sierpinska

(1994) envisions understanding as a lattice of acts and that it is achieved by overcoming obstacles:

> Processes of understanding are seen as lattices of acts of understanding linked by various reasonings (explanations, validations) and a (relatively) 'good' understanding of a given mathematical situation (concept, theory, problem) is said to be achieved if the process of understanding contained a certain number of especially significant acts, namely acts of overcoming obstacles specific to that mathematical situation. (p. XIV)

It is generally accepted that when someone experiences an act of understanding, it is accompanied with positive emotions – the feelings of relief, joy, pride and/or satisfaction that stem from overcoming obstacles. "I understand" becomes synonymous with "I finally get it!", "That makes sense", "I see" or "it fits". Kim & Hodges (2011) call this the "discovery experience." In a study that examined the motivation of developmental mathematics students by means of student learning journals, it was found that understanding was the single most important factor cited by students that increased their motivation (Miller, 2000).

Any feedback signifies that the environment has changed. Consciously or subconsciously, the person reacts to the change. It can have a very small effect that may not be observable or it may have a significant impact on behavior. It can cause a revision of goals or the setting of new goals. It can change the quantity or quality of motivation for the chosen behavior or plan. This is the dynamic nature of motivation. It can change from minute-to-minute or day-to-day. Certainly over a course in mathematics, motivation can be improved.

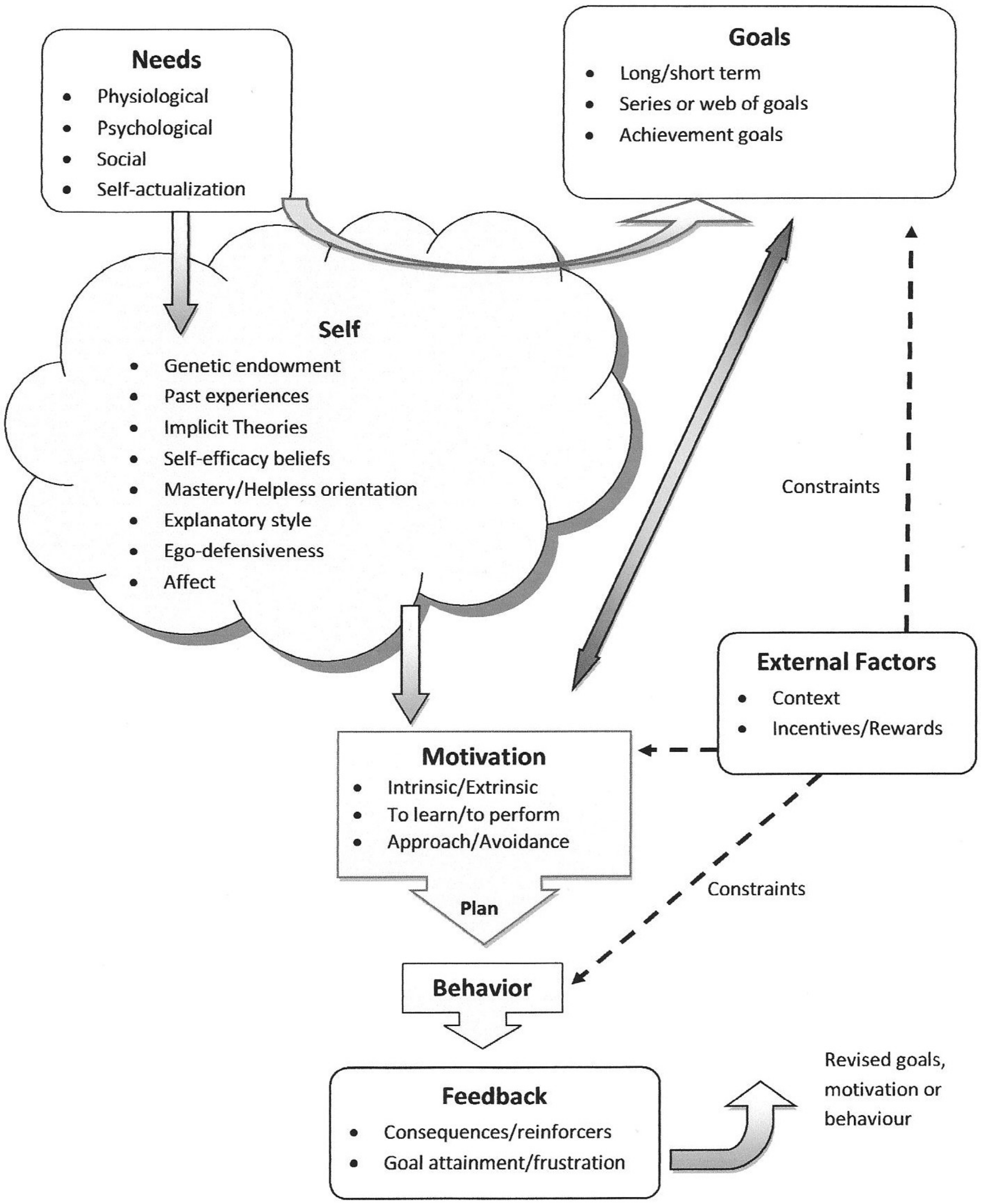
2.10 Elaboration of the concept map

The model of motivation (Figure 2.1, Section 2.4) can now be elaborated upon to provide a more complete concept map of motivation that will be used for this thesis. This map is shown in Figure 2.3.

The map now illustrates the following points and connections:

- Everyone shares the same kinds of needs
- People set goals to fulfill their individual needs and the goals that they set are based on the self
- Needs, filtered through a complex 'self', give rise to motivation and this can be the motivation to set additional goals
- Setting goals also provides motivation
- A motivated person creates a plan, selects behaviors
- Behavior involves implementation of a plan(s)
- Feedback follows behavior and provokes revisions of goals, motivation and behavior
- External factors can set constraints on the goals that people set or their behavior
- External incentives can influence motivation

Figure 2.3 Concept Map of Motivation



2.11 Afterword

Motivation has been the object of considerable study in the field of psychology. In this chapter, I have drawn from the body of literature, the theoretical considerations that I consider to be of importance in the study of motivation in a mathematics classroom especially as it relates to understanding the motivations of students taking an algebra course at the university level. The theory described in this chapter will be used to develop a profile of the students who would be taking MATH 200, a profile that I will call the "hypothetical student" (HS). If motivation can be described for this student, who will stand as representative for the class, then it should serve to highlight opportunities to improve upon the motivation within the class.

Chapter 3
Research Methodology

3.1 A Teaching Experiment

Schoenfeld (2000) offers two goals of research into mathematics education: "To understand the nature of mathematical thinking, teaching and learning;" and "To use such understandings to improve mathematics instruction" (p. 641). Mathematics education addresses these goals using theory and models derived from theory.

Sustained learning of mathematics requires motivation. Understanding what motivates people to learn mathematics is part of understanding the nature of mathematical learning. This understanding may then be used to teach so as to strengthen students' motivation. These were the goals of the research that is the object of this thesis: to understand the role of motivation in mathematical learning and to improve mathematics instruction for a group of students based on this understanding. The theories of motivation described in Chapter 2 and the model that is shown in Figure 2.3 form the framework underlying my understanding of motivation in mathematics learning.

According to Schoenfeld (2000), a 'good' theory or model is one that is able to describe a phenomenon and its elements. It can explain how things work in the situation and predict what will happen under certain circumstances. A theory or model can be tested in a controlled environment, as is the case of the interpretation of the problem-solving episode of 'Frank", described in detail by six articles in *Educational Studies in Mathematics, Special Issue* (Zan, Brown, Evans, & Hannula, 2006) or it can be applied in a classroom situation. The advantage of the controlled environment is the ability to limit the number of confounding variables but the advantage of using a classroom situation is that the teaching practices that evolve are in a form that can more easily be implemented by teachers (Confrey & Lachance, 2000). This research was based on a teaching experiment in a classroom situation.

3.2 Conjecture-Driven Teaching Experiment

The teaching experiment used a "Conjecture-Driven" research design, as described by Confrey & Lachance (2000). In this design, a conjecture is developed based on an ideological stance and situated in a theory. It is dynamic and evolves over the course of the experiment. A conjecture has two significant dimensions; mathematical content and pedagogy. The end result is an elaborated conjecture which can be subjected to review for quality, coherence and fidelity (Confrey & Lachance, 2000).

3.2.1 Ideological Stance

An ideological stance is a set of beliefs and values on the part of the researcher(s) so it is appropriate to consider my beliefs and values as they pertain to the design of the research. Confrey & Lachance (2000) cite 'equity' as the cornerstone of their ideology. They believe that mathematics education should provide equal opportunities for all students to participate regardless of gender, ethnicity or socio-economic group and I wholeheartedly agree.

Furthermore, I believe that our mathematics education system fails for a lot of students. I see evidence for this in the large number of adults who say that they dislike mathematics. The reasons why people do not like mathematics are probably as varied as the individuals themselves but our system could do better.

I believe that mathematics education could do more to inspire students rather than serving as a "gate-keeper" that shuts down many a career pathway for students. Increasingly today, careers do require that their entrants have extensive abilities in mathematics but the selection process begins in grade 7! We separate students into ability streams at a very early age that limit their career choices. Students who are placed in the enriched-stream program have the most choice but they are faced with a very demanding curriculum and if they fail to meet the performance standards, they are placed in a less demanding program. There are few avenues to move from a regular-stream or mid-stream mathematics program to an enriched-stream program. Our system sets external performance goals for students that assault their need for competence and neglect their need for autonomy – it is not surprising that we graduate students with little intrinsic motivation for mathematics.

I value life-long learning and believe in second chances. I value effort and achievement; especially achievement in the face of difficulties. I do not think that mathematics should be viewed as some kind of test of intelligence. I value mathematics for its own sake and see its value in a practical sense every day.

I believe that a dislike of mathematics leads to poor performance which in turn leads to disliking it more but I also believe that this is a cycle that can be reversed. I maintain that motivation is the mechanism by which 'dislike' becomes 'like' (or at least 'tolerate') mathematics and that promoting understanding in mathematics is one very good way to achieve this. I believe that good teaching addresses the motivation of students. In a class of students who are majoring in mathematics, there is probably intrinsic motivation to spare but in a beginning algebra class that students have to take as a required course for their discipline or as prerequisite, it is most probably lacking.

3.2.2 The Conjecture

Having established my ideological stance I can now elaborate on the conjecture that is central to my research. In the context of mathematics education, a conjecture is an idea that guides research but is not a hypothesis to be tested. The aim of the research is not to prove or disprove the conjecture but to improve upon it. There must be a mathematical content dimension and a pedagogical one and in addition, the conjecture must be situated in a theory (Confrey & Lachance, 2000). Here I will have to diverge from Confrey and Lachance somewhat in that the content of the course was fixed by the course outline and I was not at liberty to change in any significant way anything that appeared on the outline. The method of assessment was also governed by the outline (2 midterm tests and a final exam). I was encouraged to stay close to the approach used in the textbook and I did. Students at university expect a mathematics class to be a lecture format, with very little participation or involvement on their part and I did deliver lectures. How I taught the course (teaching style) was left up to me and I chose to teach the course with the hope of motivating students throughout.

The conjecture is firmly grounded in the theory of motivation. Stated simply, my conjecture is: "Motivation is lacking in a beginning algebra class but a class can be taught in a way that is motivating for students who dislike mathematics and this will result in better attitudes towards the subject."

3.3 The Hypothetical Student

In order to visualize and make use of the general conjecture, I developed a construct that I will call the hypothetical student (HS). The HS is a compilation of all my assumptions, guesses, prejudices and ideas about the students that would be taking a beginning algebra class at the university level. The HS was not a rigid construct; as central to the conjecture, it was allowed to evolve during the course. It was not an exclusive construct either; I recognized at all times that the class was a diverse collection of individuals. This was my first attempt at teaching such a course and I had no previous experience upon which to draw, therefore my construct may at times seem naïve to more experienced teachers. It is to this fictitious student that I addressed my first lecture.

The entire curriculum of MATH 200 is covered in a regular-stream mathematics program in Quebec so the underlying assumption that I made is that if a student was taking this course it was because in some way, mathematics had been an area of difficulty. Skills might have been poorly acquired or forgotten. I assumed that if the students were registered in this course it was either because of their program requirements or because they had been advised to do so. I did not neglect the possibility that some students were taking the course for interest but if this was the case, then interest alone would supply generous motivation.

In the following sections, I describe the assumptions that were made about the general characteristics, needs and goals of the HS as well as assumptions about the self and the motivation of the student. No assumptions will be made about the social needs for affiliation (intimacy) or power as these are individualistic traits and not pertinent to this discussion.

The dimensions of the self that are considered are: implicit theories, self-efficacy, orientation, explanatory style, ego-defensiveness and affect (emotions, attitudes, beliefs and values). The dimensions of hope and self-esteem are not included. It was assumed that hope would not be a dimension that would distinguish the HS from any other population of students in that all students in all classes would be somewhat hopeful. Self-esteem is an outcome of a multitude of life experiences thus, a successful result in this course could very well have an impact on an individual's self-esteem but there are too many other factors (outside of this course) that must be considered for this effect to be described in any meaningful way here.

3.3.1 General Characteristics

These are the assumptions that were made about the population of students that would be taking a beginning algebra class at the university level. No assumptions were made about gender (the HS could be male or female), ethnicity, or nationality. It was assumed that the HS is proficient in English.

Age. The HS is a young adult who is approximately 25 years of age.

Previous curriculum. The student has (at least) had access to the curriculum that is taught in the regular-stream mathematics program in Quebec high schools or an equivalent.

Student status. The HS has attended university for at least one year and is familiar with university procedures (registration, course websites etc.). The HS could be a full-time or a part-time student.

Major. The HS is majoring in a discipline other than mathematics. MATH 200 is a required course for this student.

Background. Mathematics has been an area of difficulty. The student probably did not receive very good marks in high school and may have failed a course in this subject. It is at least a couple of years since the HS took a mathematics course.

Mathematical prerequisites. The HS can perform arithmetical calculations but may rely heavily on a calculator. The student has been exposed to negative numbers and fractions in school. He or she was once taught how to manipulate these but the skills have not been practiced in a long time. The HS probably remembers more about the rules for manipulating integers and fractions than the underlying principles. The student has been taught algebra before but came away from this instruction with a poor understanding of how to use algebra to solve problems. The HS has little memory of or knowledge of geometry. Because mathematics is taught in

Quebec in discrete 'chunks' (fractions is a section, geometry is a section), the student probably has had difficulty integrating their mathematics knowledge.

3.3.2 Needs

The assumptions that are made about the needs of the HS are described in this section. The strength of a particular need is described as *low*, *moderate* or *high*. It is very difficult to be precise about such quantities, but since needs direct behavior it will be assumed that a high (or strong) need would be attended to before a need that is moderate or low.

Physiological needs. The student is a young adult, fully capable of managing their own physiological needs but with a busy life.

Psychological needs/autonomy. This is a moderate need for the student. As a young adult, this student is beginning to assert independence in his or her personal life. At university, the HS is starting to have a degree of autonomy in academic life. The student can, to some extent, choose courses and make choices about how or when to study. Mathematics courses in the past however, have probably not been autonomy-supportive – offering little choice in what to learn (or think), what to do or how to do it.

Psychological needs/competence. The HS has a strong need for competence but the student may not have felt very competent at all in mathematics courses before. If this is a required course for the HS, the student needs to be able to demonstrate competence in this course in order to continue with their chosen program. With low skills, optimal challenge is of paramount importance for this student.

Psychological needs/relatedness. No assumptions will be made about the needs of the HS to form positive relationships outside of the class. Any attempt to investigate the social life of the students would be unnecessarily intrusive. If the student lacks confidence in their mathematical abilities then the student may seek out a support network in the class. The student may prefer to study with friends. Of interest to the study of motivation is also the relationship that is formed between the student and the instructor. If the HS has experienced difficulties in the past, the student might have felt that it was because of the relationship with the instructor; the HS may have blamed teachers for previous poor performance in mathematics. The student will seek out positive relationships (this is more than a low need) but it is not likely that the student will see relationships as crucial to success or failure in mathematics (this is less than a high need). It will be assumed that the HS has a moderate need for relatedness.

Social needs/achievement. The HS is probably a high need achiever in his or her area of specialty. In mathematics, the student is a low need achiever when confronted by a standard of excellence. Using the characteristics established above for achievement needs (Section 2.5.3), the student would show perceptions of low ability and low or moderate expectations of success. The student may exhibit a helpless orientation (show little persistence when difficulties arise) and a pessimistic explanatory style (attribute their difficulties to internal and uncontrollable factors). The HS may not value success in mathematics to any great extent because the student has been able to manage effectively so far without strong skills in mathematics. Competition would encourage avoidance behavior for this student.

Self-actualization. This is a low need for the HS at this point in his or her life because the student is probably preoccupied with attending to other, more pressing needs but there may be students in the class who are acting to satisfy this need. A growth need represents an opportunity to change attitudes. Individuals who study mathematics for its own sake are satisfying a need for self-actualization.

3.3.3 Goals

Long-term goals. The HS has set a long-term career goal. Although society and family pressures may have been an influence, this goal is largely one that the student has chosen. A university degree is a necessary step on the path towards this career so it is also a long-term goal of the student but it is one that is to a much greater extent, set externally.

Intermediate goals. Passing MATH 200 is a necessary step towards a degree for the HS. The goal has been set externally (by their degree requirements or an academic advisor) and the HS has had very little participation in the setting of this goal. The task is difficult and attaining the goal carries no material reward, but it is set by a credible authority (the university) so it is likely that the goal has been accepted by the student but quite reluctantly.

Short-term goals. At the outset of the course, short term goals would be passing the midterms and the final exam. These are externally set goals, laid out by the course outline and for the same reasons as above; they have been reluctantly accepted by the student.

Goal proximity. The degree and career goals are too distant to provide much motivation for the student. It will be assumed that the HS finds mathematics uninteresting so it is from short term goals that the student will need to derive the motivation for the course.

Achievement goals. Using a multiple goals perspective, it will be assumed that what the student hopes to achieve in the course are predominantly performance goals (that demonstrate competence) and more of a performance-avoidance nature (fear of failure) than performance-approach. The HS would set few mastery goals (that develop competence).

3.3.4 The Self

Implicit theories. Prior experiences in which hard work did not result in good grades may have left the HS believing that mathematical abilities are more fixed than changeable. Thus, the HS is more of an entity theorist than an incremental theorist.

Self-efficacy. If mathematics has been an area of difficulty in the past, the HS may have more than a little doubt about his or her ability to succeed in this course. This dimension is low for the HS.

Mastery/helpless orientation. The HS has probably encountered difficulties or even failure in mathematics before and reacted by avoiding mathematics wherever possible (a helpless orientation). The student may exhibit features of learned helplessness, believing that they cannot control the outcome. Registering for MATH 200 suggests that the student has overcome some of this aversion to all things mathematical.

Explanatory style. The HS attributes poor performance to any number of internal or external factors. The student is eager to explain away poor performance. The student may *say* that their performance was due to external factors but they may *feel* that it was their own fault.

Ego-defensiveness. The HS is vulnerable; more vulnerable than most students entering most classes. The student knows that the material in the course has been taught in high school and that they have either insufficiently mastered or forgotten much of the content. The student who perceives this situation as threatening will be reluctant to ask questions in class or to seek help.

Affect/Emotions. Mathematics has previously aroused predominantly negative emotions for the HS.

Affect/Attitudes. The HS does not particularly like mathematics.

Affect/Beliefs. Previous experience may have led the HS to believe that mathematics is difficult, that algebra is abstract and that only the very gifted can succeed at mathematics. (The student may believe that being a "math-person" is a quality that they do not possess.)

Affect/Values. The HS has managed so far with poor skills in mathematics and does not see mathematics as practical or useful in the real world. The student may believe that one can solve everyday problems without having to resort to using algebra. The student may value academic success but not necessarily success in mathematics.

3.3.5 Motivation

Intrinsic/extrinsic. The HS has very little intrinsic motivation for mathematics in general. Mathematics is not an interest. The student is extrinsically motivated – largely by grades. The student may have received the positive reinforcer of good grades for hard work in other courses but hard work in mathematics has probably not "paid off" much in the past for this student.

3.4 Implications for teaching

The construct of the HS suggests opportunities for improving the motivation of this student. Creating an autonomy-supportive classroom which delivers optimal challenge and establishes a supportive student/teacher relationship could provide motivation by addressing the psychological needs of this student for autonomy, competence and relatedness. The assumption that the HS is a low need achiever (social needs achievement) suggests avoiding elements of competition. Encouraging an interest in mathematics might activate a need for self-actualization even if this is a low priority.

Encouraging an understanding of the concepts of algebra could alleviate some of the HS's fear of failing (performance-avoidance goals) and allow the student to experience positive feedback (emotions). The student could then perceive a change in his or her mathematical abilities (incremental theorist) and might feel increased self-efficacy. If the HS can achieve a better understanding of algebra in this course then the student can attribute previous poor performance to external factors (optimistic explanatory style).

Assuming that the HS is in a position of vulnerability (ego-defensiveness) implies that at all times the dignity of this student *should* be respected. Otherwise, the student might be reluctant to ask questions or seek help. The assumption of a helpless orientation for the HS suggests establishing a failure tolerant environment in the classroom to encourage persistence.

Theory suggests that authentic positive experiences in mathematics can lead to more positive affect and hence greater intrinsic motivation. Reeve (2005) asserts that when an individual acts from intrinsic motivation, better learning can result.

These considerations guided the teaching style that I adopted and my interactions with the students.

Chapter 4
Research Procedures

4.1 MATH 200 – Fundamental Concepts of Algebra

Concordia University offers preparatory courses in mathematics (MATH 200-209) that allow students who do not already have the mathematics requirements for their degree program to obtain them at university. The students in these courses perhaps did not take mathematics at the college level (CEGEP in Quebec) or are interested in improving their record for entry into another program. Mature students might be counseled to take one or more of these courses if it has been a long time since their last mathematics course.

MATH 200 is the first (most basic) of the preparatory courses. A student cannot receive credit for this course if they have taken any other mathematics course at Concordia. A student who is taking MATH 200 and aspires to enter the prestigious John Molson School of Business must take not only this course but also: MATH 201 or 206 (Elementary Functions or Algebra and Functions), MATH 208 (Fundamental Mathematics I) and MATH 209 (Fundamental Mathematics II). A student hoping for a career in Science, Engineering or Computer Science would take this course (MATH 200) and would also need 4 or 5 other courses depending on the specific requirements of the program (Concordia University, 2011-2012). A path exists towards these fields but it is a long and difficult route if the student is starting at MATH 200.

The course was given in the first summer session of 2011, from May 4 to June 20. The final exam was held on June 27. Lectures were held twice a week, on Mondays and Wednesdays from 6:30 to 9:00 p.m. with a 10-15 minute break scheduled for midway through the lecture. A total of 13 lectures were given over a period of 8 weeks (May 23 was a statutory holiday). The pace of a summer course is almost double that of a course offered in the fall or winter semesters, i.e. the entire course must be learned in a much shorter period of time.

Lectures were held in a windowless, basement classroom. The lecture hall was wide and had long tiered desks for students. A podium was provided for the instructor and the room was equipped with a computer (not used), an overhead projector (used once), a screen and a wide blackboard. Lectures were delivered from the podium and the blackboard was used extensively.

The course outline is reprinted in Appendix A and it was available to all students electronically at the mathematics department website.

The text used for the course was, *Elementary Algebra,* 5th edition and a detailed solutions manual was also available. The course covered chapters 1 to 9.3 excluding section 4.5 (graphs of linear inequalities) and section 8.5 (systems of linear inequalities) (Larson & Nolting, 2010). The material included: properties and operations of real numbers, an introduction to algebraic expressions and equations, graphing, polynomials, factoring polynomials, manipulating rational expressions (i.e. “simplifying”) and solving equations, systems of 2 linear equations and an introduction to roots and radicals.

Assessment. There were two midterm tests and a final exam. The grade for the course was the higher mark of two options. In the first option, the final exam was worth 100%. In the second option, the final exam was worth 60% and the midterm tests were worth 40%. The first midterm test was held on May 25, during lecture 6 and covered material from lectures 1-5. Calculators were not permitted for the first test. The second test was held during lecture 11 on June 13 and covered material from lectures 6-9. Midterms were approximately 1 hour and 15 minutes. There were bonus questions on both the midterms but not on the exam. The final examination was held on June 27, from 7:00-10:00 p.m. and covered material from the whole course. Calculators, with a sticker indicating approval by the department, were allowed on the second midterm test and the final exam.

Assignments. The course outline lists "suggested problems" for each section of the text. These are the odd numbered exercises for which there are answers in the back of the book. No assignments were handed in, so there were no marks given for completing the problems. On the one hand, this addresses a need for autonomy on the part of the hypothetical student (HS); the student may choose exactly how many problems to attempt. On the other hand, the HS has set more performance goals than mastery goals and there is no performance incentive (no marks) for attempting the problems unless the student sees the assignments as a path towards a goal of a good grade on the tests. There is also no opportunity for external feedback about progress.

The course website. Concordia University uses a course management system (a "Moodle" interface). Each instructor is provided with a course template which can be modified to suit the instructor's needs. When activated, the students can access the Moodle from their Concordia portal. I used this website: as a calendar (the sections from the text and the suggested problems were indicated for each lecture), to make additional resources available and to communicate with the students. The Moodle website also has an interface in which a quiz can be created and graded electronically. This quiz feature was used to prepare 3 *online quizzes* which were billed as review questions for the tests.

4.2 The Students

At the outset of a course, the number of students fluctuates as students register, attend a class and then perhaps choose to drop the course. Initially, 62 students were registered and more than 60 students attended the first class. By the end of the course, 54 students were still registered. Of these, 8 students did not write the final exam.

Not all students had a declared concentration of studies but of those that did, only one student was registered in a program that could be considered mathematics-related (finance) and one other student was registered in a science program (biology). The declared concentrations of the rest of the students were mainly in the social sciences and arts and included: Sociology, English literature, philosophy, anthropology, psychology and music.

4.3 Teaching Style and the First Lecture

"*You never get a second chance to make a first impression.*" (Will Rogers)

The course content and method of assessment was fixed and I taught the course content using the approach from the textbook. The only way to effect motivation at all was by the style of teaching. Based on theories of motivation, and the assumption that intrinsic motivation was lacking in the class, I endeavored to create a classroom that was somewhat autonomy-supportive and presented optimal challenge to a majority of the students. I attempted to create a relationship that was supportive of their needs and formal enough that the students would respect me as an authority. Wherever possible, I cultivated an interest in mathematics and promoted mastery (understanding) of the concepts as a path to better performance.

The first lecture was of paramount importance to me; it had to set the tone of the course. I wanted to establish a relationship with the students, engage their interest and calm some of their fears. As a novice teacher, I was nervous myself but highly motivated. I taught the first class to the HS with the expectation that my assumptions about this student would be challenged.

I introduced myself to the students and told the students that I would answer to my first name. I shared with the students that I had returned to studies in mathematics after a long absence and that I understood some of the difficulties that could arise if they were in the same situation (empathy). It is standard practice to provide administrative particulars in the first class and I explained these in detail because for some students this was their first mathematics class at Concordia. I negotiated office hours that were acceptable to all students (autonomy) and I encouraged students to contact me by e-mail if they had short mathematical questions or to visit during office hours if they needed a more detailed explanation. The students were given the contact information for the MathHelp Center and the Counseling and Development department; both of these are excellent support services that Concordia University provides. I also told the students that they could find copies of past exams at the Digital Store and I gave them its location.

The students were given a rough outline of how the lectures would be structured. I told the students that I would set aside time at the beginning of class for any questions they had about the material that had been covered and I encouraged them to ask questions during the lecture. Before each lecture, I wrote a mini-review of the material that had been covered in the previous class on the blackboard along with an outline of the topics to be covered that day.

The suggested problems were optional and I was concerned that the students would not be highly motivated to do very many of the problems. In the introductory part of the lecture, I attempted to access the performance goals of the HS by extolling the benefits of practice in mathematics. Up to this point, I had no relationship with the students so I was relying on their perception of me as an authority but I explained that the class tests and the final exam were a bit of a race and that practice was the key to being able to write the tests fluidly. I stressed that this would result in better performance. (I returned to this idea before the first class test when I had established more of a relationship with the students.)

With the administration details out of the way, I introduced the topic of "algebra" and explained that we would be talking about numbers, symbols, sets and operations. As the word algebra is probably one that evokes negative emotions for the HS, I offered some information about the history of algebra and that it developed out of a need to solve problems (rationale).

The material for the first lecture was taken from the first 3 sections of Chapter 1 (Real Numbers) in the textbook. These are: "Section 1.1, Real Numbers: Order and Absolute Value"; "Section 1.2, Adding and Subtracting Integers"; and "Section 1.3, Multiplying and Dividing Integers" (Larson & Nolting, 2010, pp. 1-31). All of this material is typically covered in the high school curriculum by the end of grade 7. Presenting this material to a group of adults potentially creates a situation that is ego-threatening ("I know all this, does the teacher think that I am dumb?"). Measures were definitely needed to preserve the dignity of the students in the class.

What I adopted was an attitude – a posture that I hoped would communicate to the students the message that: They already knew this material but that it was in the course outline so I had to cover it and that it was important because I was going to show them the concepts that we were going to use when we started working with algebraic notation. This posture would satisfy a need for competence and address performance goals for the HS. It laid the groundwork for the idea that performance goals could be achieved through understanding and at the same time it allowed me to present the material in an ego-preserving way for the students who really did not know the basics. The following are two examples of this posture.

1) When I introduced what Larson & Nolting (2010) term "Classifying Real Numbers" (which simply involves being able to place a particular number in one or more of the subsets of the real numbers), I mentioned a study that I had read that had indicated that pre-service mathematics teachers were not always able to do this (Fischbein, 1994). A perception of understanding the concept then provides the HS with some positive feedback: "I get this, and they didn't so maybe I'm not so bad after all."

2) I wanted to present an example of the standard multiplication algorithm and I did but alongside the familiar algorithm, I explained how this was an example of applying the distributive property of multiplication over addition. I stressed that they knew how to do the algorithm but what they were really doing was decomposing the number as they multiplied. Thus if, 3×24 is desired, the computation can be written as $3(4 + 20)$. I referred back to this when I presented an algebraic expression of the form $a(bx + c)$.

In this and the next few lectures I made a list on the side of the board of notations that I was using and their meanings. One of the difficulties that students have in mathematics (in my experience) is that letters and symbols often serve multiple purposes. For example, in the expression $5 - (-3)$, the first minus is identified as an operation and the phrase "add the opposite of" is used in the text. The text indicates that the second minus sign means that the quantity is "negative" 3 (Larson & Nolting, 2010). The same symbol used with different meanings introduces ambiguity. Thus, $(-x)$ can be read as "negative x" or as "add the opposite of x." I used and explained the expressions in the text because the students needed an understanding of these to be able to consult the text but I also promised that I would be clear about the meaning of the symbols that I used.

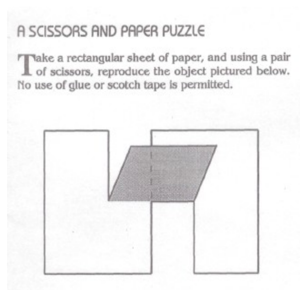
4.4 Fostering an Interest in Mathematics

Popularization of mathematics activities that are reputed to increase interest in mathematics include voluntary activities that place mathematics in a historical context (Kelecsenyi, 2009). Although the students had not volunteered for a history lesson, I introduced the real numbers from a historical perspective.

In the second half of the first lecture, and after I had described prime and composite numbers, I gave the class an activity – the Sieve of Eratosthenes. I explained how to proceed and handed out sheets of the first 100 numbers in a table.

At the end of the class I handed out a puzzle. (Figure 4.1) I told the students that it was strictly voluntary but that I thought they might enjoy it. I said that it was a visual math type puzzle and that they could probably solve it much quicker than I did. As such, the puzzle meets Kelecsenyi's (2009) definition of a popularization activity and if an inkling of interest in mathematics existed, it might have appeal to the HS.

Figure 4.1 Example of a puzzle



Source: *After Math, Puzzles and Brainteasers*, by Ed Barbeau, 1995 (p. 1). Also available at: http://boingboing.net/2011/11/30/fun-paper-puzzle.html

I offered a set of two puzzles to the students in the third lecture and left a pile by the door. One other puzzle was intentionally photocopied to the back of a sheet of problems that I used in class (Barbeau, 1995).

After the first midterm, a graduate student seeking volunteers polled the class to solicit participation in a mathematics activity that was billed as a "fun math experience." Volunteers for this activity would be students who had an interest in mathematics.

4.4 Introducing Fractions and Algebra: Promoting an Understanding

It is my experience that students who say that they don't like algebra also report that they do not like fractions. When confronted with a calculation involving fractions, the HS resorts to using a calculator and more often than not, converts the fractions to decimal numbers (a type of avoidance). The introduction of fractions was presented using the same posture outlined above. I acknowledged to the class that a lot of people don't like fractions. I indicated that we needed an understanding of fractions because although the calculator can add $\frac{7}{16}+\frac{2}{3}$, it has difficulty with $\frac{1}{x}+\frac{3x}{2}$ (rationale).

Going back to the very beginning of the topic, I drew part-to-whole representations of fractions, as circles (pizzas), rectangles (chocolate bars) and collections of objects. I used these representations to show basic operations on fractions, especially the need for a common denominator when adding fractions with unlike denominators. Fractions were represented on a number line and also as a ratio. The line separating numerator from denominator was identified as also meaning "divide by" and students were shown how to convert improper fractions to mixed numbers by dividing. A fraction as an operator was presented when multiplying a whole number by a fraction. These are the 5 representations (part-to-whole, ratio, operator, quotient and measure) that have been identified as sub-constructs that are needed for a robust understanding of the concept of fractions (Behr, Halel, Post and Silver, 1983 as cited in Verschaffel, 2006). In short, I did my very best to give the students the tools needed to reach a point where they could say; "I understand fractions much better now."

I wanted the students to be able to say the same about algebra, so the topic was introduced in a way that hopefully would address some of the difficulties that have been identified in the body of literature on teaching algebra. Rojano (2002) describes some of the difficulties that students experience in transferring from arithemetic to algebra. In arithemetic, the symbols $"+, -, \times, \div$ $and\ ="$ mean "do this to get that", as in $3 + 2 = 5$. In algebra the operation must be suspended, as in the expression $(3x + 5)$. The symbol "=" represents an equivalence in algebra, rather than an invitation to calculate. Letters can represent constants (π), unknowns (the hunt for the elusive x) or variables (elements of a set). The number 32 means 30 plus 2, whereas $3a$ denotes 3 times $"a"$ (Rojano, 2002) where $"a"$ represents a variable ranging over some set of numbers. Sutherland and Rojano (1993) note the difficulties that students experience in operating on the unknown. Assuming that the students had already had difficulty with this and other concepts in algebra, I attempted to ease the transition from numbers to symbols.

Algebra was introduced using very concrete examples in which each element of an algebraic expression or equation was explained. It was connected to arithemetic (previous knowledge) by showing how the properties of real numbers were applicable to algebraic symbols. Emphasis was placed on why the rules of algebra worked, rather than just how to use the rules. The concept of variable was introduced when evaluating an expression for several values in a table. I made distinction between simplifying an expression (= as operator) and solving an equation (= as equivalence). A balance analogy was used to introduce solving linear equations and I showed the students how the equation x=5 can be transformed to a more complicated equation and that solving an equation was the reverse of the process. Inevitably, I was asked if one could simply shift a term to the other side and change the sign and I used this as an opportunity to show what they had learned was true and worked but it worked *because* they were adding a term to both sides of an equivalence.

Bednarz, Kieran & Lee (1996), in *Approaches to Algebra: Perspectives for Research and Teaching*, (Chapter 1) present four approaches that have been used to introduce algebra. These are: An approach using generalizations (patterns), a problem-solving approach, a perspective of algebra as modeling and a functional approach. These are elaborated upon in the subsequent chapters of the book (Bednarz, Kieran, & Lee, 1996). I introduced algebra as a tool that developed out of a need to solve problems and showed in problems how a concrete situation could be modeled using algebra. I incorporated elements of a functional approach when discussing systems of 2 linear equations but did not use patterns extensively. Some geometrical examples were used and I showed examples of a "geometric model of a polynomial product" (Larson & Nolting, 2010, p. 299) explain multiplication of binomials.[3]   Elements of the different approaches were incorporated but the bulk of the material as presented in the text book is about how to solve an equation or simplify an expression.

4.5 Autonomy-Supportive Measures

Students were offered some choices. Office hours were negotiated, the timing of the second midterm was rearranged and I frequently asked the class if they wanted more examples or to continue with the material. One of the students asked if there was going to be an online quiz to prepare for the exam and I responded by polling the class. I agreed to offer a third quiz only after an overwhelming majority of students raised their hands in favor.

Students' questions were fielded with respect and with the attitude that if a student had not understood, it was because I had not explained effectively. I listened carefully for clues as to what was causing the difficulty and then, rather than repeat myself, I supplied additional information or a slightly different approach. I answered individual questions during the break and for 15-20 minutes after class. Occasionally, if a student was experiencing specific difficulties and having trouble communicating these to me, I asked the student to speak with me

[3] This model resembles "algebra tiles" (Leitze & Kitt, 2000).

after class. Mathematical questions posed in e-mails were responded to promptly, praising effort and with additional information as required.

I acknowledged that mathematics was probably not their favorite subject and I did provide a rationale whenever I saw the opportunity to do so.

Some autonomy is possible in choosing a mathematical approach to an algebraic problem and I showed options. I also explained the difference between an elegant solution to a mathematical problem and one that works. Students were encouraged to choose an approach that they were comfortable with but only if it made sense mathematically. On the tests, they were required to use an algebraic method and to show their work.

4.6 Competence Measures

Providing optimal challenge for a group of 50 individuals is difficult because the skill level of the students can vary greatly. The lecture itself must provide sufficient challenge to engage the students without either boring them or leaving them completely bewildered. It is advantageous to present the material in the Zone of Proximal Development (Vygotsky, 1978).

To introduce a new concept, I activated competence by linking the new concept to previous knowledge. For example, factoring was introduced by showing that it is the reverse of using the distributive property to expand an expression. Examples were given that progressed from simple to more complicated ones.

The goal of the lectures was to augment the skills of the students so that the suggested problem sets would provide optimal challenge. Hopefully, a student would experience ‘flow’ when doing the problems and receive the positive feedback of getting the same answer as the one in the back of the text. According to the model (Figure 2.3), feedback can significantly affect motivation. However, feedback will only be a factor if a student actually *does* the suggested problems and I had no way of knowing whether or not they were. I assumed that the behavior of doing the problems for the HS would have long latency and short persistence. In order to offer an alternative method of practice, I created the first online quiz in the Moodle course website and activated it one week prior to the first midterm. I hoped that a computer assisted activity would be more appealing to students than solving problems from the textbook.

4.6.1 Online Quizzes

The first quiz contained 23 questions, most with multiple parts, and each part was worth 1 mark. The quiz was graded out of 40 possible marks. The students could attempt the quiz as many times as they wanted to, and after each attempt the system would indicate which questions were correct or partially correct. The questions were very similar to the questions in the book and reflected the topics that they would need for the first midterm.

After the students had completed the first midterm, I circulated a questionnaire and asked for feedback about the online quiz. The questionnaire was completed anonymously and the information was used to decide whether or not to create more quizzes and if so, how to improve them. This questionnaire is reproduced in Appendix B.

Based in part on the students' responses to the questionnaire, a second quiz was prepared and activated one week prior to the second midterm. The second quiz had 20 questions and only a few of these had multiple parts. Each question was worth one mark and the students had five attempts. The questions on this quiz were slightly more difficult than what would be expected on the midterm.

A third quiz was requested to prepare for the final exam. This quiz had 23 questions for a total of 25 possible marks. Three attempts were allowed and again, the difficulty was a little more than would be required for the final exam. For all of these, in the introductory notes, it was stated that the quizzes would not count towards their grade for the course and that the students would have to show their work on the tests.

4.7 Relationships

A relationship that is supportive is developed in small steps and happens over the entire length of the course. At the beginning of the course, students asked quite a few procedural questions in class and by e-mail. Each question was answered as if it was the first time I had been asked. I provided the details or directions on where to find the information requested.

Some of the elements that contribute to a supportive relationship are more subtle. I smiled, paid attention to anyone who asked a question, made eye-contact and despite being slightly nervous, I strived to appear relaxed in front of the class. I truly believe that mathematics can be fun and I attempted to convey this to the class. As classroom discipline was never a problem, I encouraged students to call out bits of information or answers and every response was acknowledged enthusiastically.

I did not take myself (or mathematics) too seriously in front of the students. The night of the Stanley Cup final game there were less than 30 students in the class. I chatted with some of the students and admitted my utter ignorance of anything to do with hockey. If I drew a line that was noticeably crooked, I asked them to pretend that it was actually straight. I occasionally made light of the wording of problems (some problems are truly artificial).

Mathematics has a very formal language but it may be difficult to feel 'warm' towards something that is very formal and stuffy. I presented the formal language to the extent that it was used in the textbook but I often used much less formal descriptors and terms. For example, the text uses the term "complex fractions" to represent the division of two rational expressions. I told the students that the term 'complex numbers' usually had a different meaning and that I preferred the term “complicated fractions”. The students adopted this term and I heard it used more than once.

This level of informality might not be appropriate in an upper-level mathematics course but it was adopted here to develop a relationship in which the students’ affect towards mathematics could potentially be improved.

4.8 Feedback

If the challenge is optimal for students, then feedback can be a sense of accomplishment or one of failure. Attempts were made to create a classroom environment that was tolerant of failure. This is not to say that mathematics is in any way flexible about what constitutes a wrong answer, it is not. Questions that indicated a failure to understand on the part of the student were treated as opportunities to address the underlying reasons for a lack of understanding. I made mistakes on the board, some were intentional but most were accidental and when the students corrected me, I often asked for an explanation.

The only other opportunity for me to provide feedback to the students was on the midterms. Structure was provided so that this feedback was meaningful. Expectations were outlined before the test. Students were told that they needed to show their work and, if they used a letter, that I needed to know what quantity it represented. I indicated that a calculation error would result in a small deduction in marks but that an algebraic mistake would be more serious. A detailed solution set for a problem sheet was posted on the course website.

Marks provide the most significant feedback to the students with performance goals. Students were given part marks for partially correct responses as a reinforcer for the attempt. I also gave part marks if the student had copied a question incorrectly but then solved his or her question correctly. There was a bonus question on each of the midterms offering the incentive of a mark greater than 100%. I did not write comments next to the students' grades (good job or excellent etc.) because these are vague and do little to motivate students. I did write comments on their solutions indicating what part was correct and how to proceed next. These are external informational reinforcers and according to theory they are more effective.

Students' marks were not posted on the course website and when returning midterms, I was careful to cover an individual's mark so as not to introduce any elements of competition. In an achievement situation, it is only those with a mastery orientation that would be motivated by a comparison of marks.

Students who received very low grades on the first midterm were sent an encouraging e-mail that reiterated the measures they could use to seek help.

4.9 Instruments to Describe Motivation

At the end of the last class, I circulated a consent form. The consent form asked for permission to use data gathered during the course that could include: observations, course productions, marks, questionnaires, course evaluations and correspondence. The students were also asked for permission to be contacted to complete a survey. These forms were sealed in an envelope and students were informed that they would not be opened until after I had corrected final exams and submitted marks so that no bias could be introduced or perceived. A total of 37 students completed the consent form, of these, 31 agreed to both the use of data from the course and to be contacted to complete a questionnaire. Six students agreed to the use of their data but not to participate in the survey.

The online quiz questionnaire was completed anonymously and there is no way to be sure that all the students who completed this form also signed a consent form. Global results for the questionnaire will be used but comments will be summarized.

Course evaluations were completed anonymously by the students in class during lecture 9. Again, there may be students who completed course evaluations but did not give consent, so individual comments will be summarized. Global results for the questions will be used to indicate students' attitudes as a group, midway through the course. A reproduction of the course evaluation form can be found in Appendix B.

An online free utility (LimeSurvey), was chosen to prepare the survey. The questions were designed to elicit information about needs, goals, strength of motivation and choice of path (behavior) towards a goal. The students were asked about affect towards mathematics both before and after the course. There were no mathematics questions.

This was a mathematics related activity that students were asked to do without any incentive so motivation to complete the survey was assumed to be low. The number of questions was kept to a minimum but even so there were a total of 73 base questions and 27 questions that were conditional on an answer to a previous question. The survey was easy to complete. The instructions were given on an introduction page and students had to fill in the information required in a text box or click on an appropriate answer for multiple choice questions. Many of the questions were given in the form of a statement and the student was given the options to: "Agree," "Disagree' or remain "Neutral." If the student selected the neutral option, a text box appeared and the student was asked to explain why they neither agreed nor disagreed with the statement. Most of the multiple choice questions were mandatory (the student could not navigate to the next page until an answer had been provided) but the text questions were not. Additional boxes for comments were provided for some questions. The survey was arranged in six parts. A reproduction of the questions, as they appeared on the survey is provided in Appendix C.

Chapter 5
Observations and Results

5.1 Observations from Class

The class was not video-taped and during class, my focus was teaching so what is presented here are some very general observations.

The number of students attending class was always greater than 30. Attendance was not taken but before each class I performed a rough 'head-count'. The majority of students arrived on time and a handful arrived within the first 10 minutes of the lecture. No students left the class during the lecture but it is possible that some students did not return after the break.

Students appeared to be listening and writing during class. The lectures were interrupted by a cell phone only a few times. If students were talking to each other, it was never intrusive enough for me to comment upon.

Questions from the class, especially during the first few lectures were mostly of a procedural nature. Students asked questions about the mathematics department and the course. These were questions about, for example, the date of the final exam, approved calculators and the grading scheme for the course.

There were a few hesitant, mathematical questions in the first few lectures but the students gained confidence and by the second half of the course, students would interrupt the lecture if they had a question.

Questions of a mathematical nature included requests for additional information or explanation. Students asked for more examples (often for more 'word problems') and they sometimes asked me to explain what I had written on the board. There were also questions about choices for solving a mathematical problem; students asked if they could use a different approach or technique. Some of these questions seemed to be questioning the validity of a particular approach ("Will I get the right answer if I do it this way?") but the majority of these types of questions seemed to be about what would constitute an acceptable solution ("Will I get full marks if I do it this way?").

The students were very concerned about the criteria that would be used to evaluate their performance. There were several questions about the format of the tests, the material that would be covered and the types of answers that would result in a 'good' mark on the tests.

There were also a few questions that seemed to indicate an interest in material that was beyond the scope of the course. One student asked why division by zero was not allowed and one other student asked about the ratio of the areas of rectangles if a scale change had been applied to the sides.

5.2 Encouraging an Interest in Mathematics

After the first class, during which there had been a presentation of some of the history of mathematics in general and algebra in particular, one student asked whether the "history stuff" would be on the test. There was a relieved sigh from some of the students when I said, "No." Mid-way through the course, one student did mention that the historical information was interesting and this student also mentioned an interest in pursuing mathematics further.

The Sieve of Eratosthenes was presented as an activity for the students in the first class. A few students did start to work on the sieve but I was getting quite a number of blank stares from the rest. One student asked me if they should start now. Clearly, an activity mid-class was not expected and not welcomed either.

A total of 4 puzzles were offered to students during the first few lectures in an attempt to generate interest among the students. No student ever mentioned the puzzles or asked for a solution, either in class or in any communications outside of class.

The graduate student who polled the class looking for volunteers for a 'fun' math activity was not contacted by any of the students in the class.

5.3 Marks for the Class

Midterm 1. There were 49 students who wrote the 1st midterm. The class average was 78%, and 5 students received a mark less than 50%.

Midterm 2. There were 47 students who wrote the 2nd midterm. The class average was 72% and 11 students received a mark less than 50%.

Final Exam. There were 46 students who wrote the final exam. The average of these marks was 66.5% and 9 received a mark of less than 50%.

The e-mail reiterating the possible alternatives for seeking help with the course was sent to a total of 6 students in the class who received low or failing grades on the first midterm. Two students in the class acknowledged the e-mail and both of these responses indicated that the situation was under control. Three of the 6 students were in the consent group (the 37 students who signed the consent form) but only one of these students replied to the e-mail. The student indicated that he had been ill and had not had the time to study.

5.4 Online Quizzes

Online quiz 1. There were a total of 104 attempts (including partial attempts) that were recorded for the first online quiz. There were 35 students who attempted the quiz at least once.

Online quiz 2. There were 32 students who used the quiz for a total of 65 attempts.

Online quiz 3. There were 36 students who used the quiz for a total of 51 attempts.

5.5 Online Quiz Questionnaire

This questionnaire circulated after the students had written the first midterm test. Students would, by then, have an idea whether or not they had studied sufficiently and effectively for the test. Students were told that they did not need to complete the questionnaire if they had not used the quiz. Of the 49 students who wrote the midterm, 35 questionnaires were completed and of these, 8 students reported that they did not use the quiz at all. The purpose of the questionnaire was to gather feedback about the quiz in order to decide whether or not to create more quizzes and if so, how to improve them. The results are summarized below in Table 5.1. Results for questions 2-9 are tabulated for the 27 students who used the quiz. Students were asked for feedback (comments) but since the students had not yet given consent for inclusion in this research, the comments will not be reproduced here. Aggregate results will be mentioned as it affected the design of subsequent quizzes.

All of the students who completed the questionnaire found the quiz helpful in some way and it provided optimal challenge for all but 6 of the students (question 3). Since the combination of low challenge and low abilities is one that can produce apathy in students and there were 5 students who reported that the quiz was too easy, the following quizzes contained questions that were a little harder.

Table 5.1 Results for the online quiz questionnaire

	A lot	Somewhat	Not at all
1. I used the quiz	17	10	8
2. The quiz was helpful	17	10	0
	Too hard	**Just right**	**Too easy**
3. The questions were...	1	21	5
	Too long	**Just right**	**Too short**
4. The quiz was...	2	20	5
	Easy to use		**Difficult to use**
5. The Moodle system was	23		4
	A lot	**Some**	**None**
6. The amount of other studying that I did for the midterm	16	7	4
	More	**About the same**	**Less**
7. Because of the online quiz, I studied	12	13	2
	Yes		**No**
8. I would use an online quiz to prepare for the next midterm	26		1
9. I would use an online quiz to do homework problems	23		4

There were 4 students who reported that they did no other studying for the midterm. These results were broken down further (Table 5.2). My intention was *not* to reduce the amount of studying!

Table 5.2 Individual student responses

	1st Student	2nd Student	3rd Student	4th Student
Q1. I used the quiz	A lot	A lot	A lot	A lot
Q2. The quiz was helpful	A lot	A lot	Somewhat	A lot
Q7. Because of the online quiz, I studied...	More	More	About the same	About the same

The 4 students all reported that this was “A lot” of studying and no student reported that they studied less because of the quiz. The quiz did not represent a reduction in the amount of studying; rather it replaced studying by another path for these 4 students.

There were 2 students who reported that they studied less because of the online quiz. Both these students also reported that they used the quiz “A little” and they did “Some” other studying for the midterm. They may have studied less but neither student indicated (in their comments) that they felt that they did not study enough.

There were 25 students who offered comments. Eight of these were positive and one negative. Ten students indicated that they would have preferred to have the correct answers displayed after an attempt. The correct responses were not displayed by the system because the students were given unlimited attempts at the quiz. Subsequent quizzes had fewer attempts allowed so that a student could exhaust their attempts and then see the correct responses. Some students remarked that the questions with multiple parts would only record the number of correct parts and not which parts were correct so the next quizzes had less questions of this type.

5.6 Course Evaluations

Course evaluations (on paper) were conducted on June 8 in class and were completed by 33 students. The results for the multiple choice questions are summarized in Table 5.3.

Most students reported an “average” level of knowledge and interest in the course material before taking the course. All other dimensions about the course and the instructor were ‘above average’ or ‘well above average’ (as compared to a bank of data that is maintained by the university), indicating that at this point in the course, the students’ expectations were being met or exceeded. Responses to questions 8 through 14 seem to indicate that a positive relationship was indeed developing between students and instructor.

Table 5.3 Summary of multiple choice responses to course evaluation questions

	Very Good	Good	Fair	Poor	Very Poor	Overall Assessment
1. Overall, this course has been...	16	11	5	1	0	Above average
2. Overall, the instructor has been...	16	12	3	2	0	Above average
3. Overall, my learning has been...	13	12	6	2	0	Above average
	Strongly Agree	Agree	Neither	Disagree	Strongly Disagree	
4. Course outline and syllabus are clear, complete and well explained.	17	11	3	2	0	Well above average
5. Course materials, textbook or readings are useful or relevant.	18	8	5	1	1	Well above average
6. I have found this course intellectually challenging and stimulating.	10	17	4	0	2	Above average
7. The course met the objectives as stated in the course outline.	16	13	3	0	1	Above average
8. Instructor demonstrates a comprehensive knowledge of the subject matter.	17	13	2	1	0	Well above average
9. The instructor's explanations are clear.	17	10	3	3	0	Well above average
10. The instructor provides feedback in the form of exams and/or assignment grading.	14	13	2	2	2	Above average
11. Students are encouraged to ask questions.	26	5	1	1	0	Well above average
12. Students are encouraged to share their ideas and knowledge.	21	9	2	1	0	Well above average
13. The instructor is approachable.	26	6	0	1	0	Well above average
	Well above average	Above average	Average	Below Average	Well Below Average	
14. Overall, my involvement in this course has been...	7	9	14	1	0	Above average
	Very High	High	Average	Low	Very Low	
15. What was your level of knowledge of the subject prior to taking this course?	2	5	18	5	3	Average
16. What was your level of interest in the subject prior to taking this course?	5	8	13	5	0	Average
	Rq*	Ele	Mi	Ge	Fit	
17. What was your reason for taking the course?	14	3	7	6	3	NA

Note. Rq=Required for Major or Specialization, Ele=Elective for Major or Specialization, Mi=Elective or Minor, Ge=General Interest, Fit=Fit into schedule.

Questions 18 and 19 allowed for the students to enter comments about the facilities and the course. Although the course evaluations were completed anonymously, at the time that they were administered, the consent form had not yet been circulated. As it is not possible to be sure that every comment was from a student who also signed a consent form, the text of these comments will not be included here. These comments will be summarized.

Question 18 asks the students about the facilities for the course (e.g. air, seating, temperature, class size, etc.). There were 24 responses and of these 14 students were satisfied with or liked the facilities. There were 10 students who reported some difficulty and these included; broken or noisy seating, obstruction of the view of the blackboard, a lack of windows and insufficient maintenance of the seating or blackboard. Three students found the room cold but there were three other students who commented that the temperature was fine.

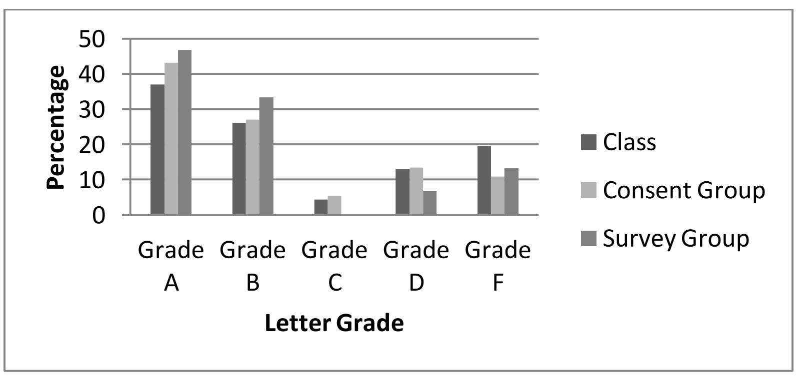
Question 19 asked: "What suggestions or comments do you want to give to your instructor about this course?" There were a total of 17 responses but 4 of these were equivalent to "I have no suggestions or comments." There were 5 comments that could be considered favorable towards the instructor and/or the course. In these, the instructor was described as "great", "approachable", "effective", "caring" and "informed". The course was described as; "well structured, "interesting" and "fun". One student found the course "difficult" and another found it "intensive". One student was "inspired" to continue in mathematics. One student liked the historical background and found the course "different" than other math courses. One comment was quite negative and referred to mistakes on the board, called for more practice on the part of the instructor and indicated that the instructor was "lazy" for not always writing "full answers" on the board. Suggestions for improving the course included; more examples shown in class, more online quizzes, more “organization” and “interest”, a better outline, more questions (other than the textbook) and that the correct answer be shown on the online quizzes.

5.7 The Groups of Students

All of the 37 students, who signed a consent form, wrote both midterms and the final exam. Of these, 31 agreed to be contacted for the survey and 15 students returned completed surveys.

The distribution of final marks for the class (the 46 students who wrote the final exam), the consent group (the 37 students who signed the consent form) and the survey group (SG, the 15 students who completed the survey) are compared in the chart below (Figure 5.1). Each bar on the chart represents the percentage of students in the group who received the corresponding letter grade.

Figure 5.1 Comparison of grades for the groups of students



The average of the marks for the 3 groups on the two midterms and the final exam are compared in Table 5.4.

Table 5.4 Summary of average marks for each of the tests

	Class %	Consent Group %	Survey Group %
Midterm 1	77.1	79.6	81.1
Midterm 2	70.5	75.5	76.3
Final Exam	66.5	71.4	71.6

The consent form was circulated on the last day of class so the consent group (37 students) only includes students that attended on that day. It is not a randomly chosen sample; it does however contain representatives that did very well and representatives that failed the tests but it is skewed. Students who received an A or a B are over-represented and students who received low marks are under-represented. These students can also be considered to be slightly more highly motivated (they attended class) than the class as a whole. The survey group is also potentially even more motivated – these students took the time to participate in an activity related to mathematics without any external incentive to do so.

Each student in the consent group has been assigned a number from S1 to S37 with sufficient randomness to ensure anonymity. Comments that reflect motivation will be drawn from e-mail communications with these students.

5.8 Results from the Survey

The results from the survey will be the primary vehicle used to develop a better construct of the hypothetical student (as established in Section 3.3). Data has been collated and a summary of the responses to all of the questions can be found in Appendix D. An explanation of the treatment of the data is also given in Appendix D.

For Part V of the survey, Your Opinions about Mathematics, students were asked to either "Agree," "Disagree" or remain "Neutral" for a statement that reflected a dimension of affect towards mathematics that they held before the course. Responses were assigned a value of +1 if it reflected a positive attitude towards mathematics, a value of -1 if it reflected a negative attitude and a value of 0 was given to a response of 'Neutral'. The sum of these values for each question is reported as *Net Affect*. Each of these questions had a pair in which the students were asked if this attitude had changed after the course. Again a positive change was given a value of +1, a negative change, a value of -1 and a value of 0 was given if there was no change. The sum of these values for each question about affect after the course is reported as *Net Change*. For example, Q501 asks the students to agree or disagree with the statement, "Before the course, I liked math." Eight of the students agreed and 6 disagreed, one student entered a response of 'Neutral'. Net Affect for this question $(8 - 6 = 2)$ is reported as (+2). For Q501a, there were 8 students who liked mathematics *more* after the course, 2 students who liked mathematics *less* and 5 students who indicated no change. Net Change for this question $(8 - 2 = 6)$ is reported as (+6). A summary of the values for Net Affect and Net Change for Q501-Q510 is presented in Table 5.5.

Table 5.5 Summary of dimensions of affect and change in affect

	Net Affect	Net Change
Q501: I like math	+2	+6
Q502: Algebra was too abstract	+2	+2
Q503: Math was difficult for me	+5	+5
Q504: I understood what was presented in class	+6	+7
Q505: I felt math was for some people but not for me	+6	+3
Q506: Math is a waste of time unless it is geared towards a career	+11	+2
Q507: I was not good at writing math tests	+1	+5
Q508: Math sometimes made me feel stupid	+1	+3
Q509: Math sometimes was fun	+4	+5
Q510: I thought I could succeed in math	+8	+7
Total	**+46**	**+45**

For all of these 10 questions, the dimension of affect reported *before* the course was more positive than negative among the survey group of students. The change in attitude for every question was also positive.

Chapter 6
Analysis

The profile of the hypothetical student (HS) was based on my assumptions about the students that would be taking MATH 200 and the inferences that could be drawn from theory about these characteristics. This was the student who I expected to teach in the first lecture. The evidence that was gathered during and after the course makes it possible to improve upon the description of the motivational characteristics of the HS. In this section, I will re-examine these characteristics and modify them where the evidence (from the survey, course evaluations, online quiz questionnaire and classroom observations) permits a better description of the motivation of the HS. Consequently, a compilation of these characteristics represents a profile of a typical student who was present at the start of the course. As this model is more realistic, it will be called the *realistic student* (RS). The RS profile is structured in the same way as the HS but the characteristics are now supported by the evidence that was gathered wherever possible. As for the HS, it does not represent any particular student in the class. However, it does model a typical student in my section of MATH 200. An instructor who would like to improve students' motivation in a similar course could start with this RS model as his or her initial HS model but would have to be prepared to adjust the model to fit the actual characteristics of another group of students.

The assumptions that were made about the HS will be presented here along with a short summary of the evidence that pertains to each characteristic. If the evidence suggests modifications to these assumptions or if additional descriptive information is available, then these are incorporated in the portrayal of the motivation of the RS. As the RS represents the students that were present in the class at the outset of the course, the impact that the course had on the students can also be considered. Where evidence from the course is available, the evolution of the RS is also described for the motivational characteristics. A discussion of motivational characteristics for a typical student leads to an examination of these characteristics for individuals. At the end of this chapter, some individual cases are considered.

A more detailed summary of the evidence can be found in Appendix E. The survey was completed by 15 students in the survey group (SG) and results for the survey are presented in Appendix D. Results for the online quiz questionnaire (completed by 27 students) and results for the course evaluations (33 students) were presented in Section 5.5 and Section 5.6, respectively.

6.1 General Characteristics of the RS

In this section the general or personal characteristics of the HS will be presented. A compilation of the evidence that supports the conclusions can be found in Section E-1 of Appendix E.

Age. The HS was assumed to be approximately 25 years of age but almost half of the students in the SG were over 25, indicating that the RS was slightly *older*.

Previous Curriculum. It was assumed that the HS had had access to a regular-stream mathematics curriculum as it is taught in Quebec or an equivalent. All of the students in the SG had taken regular-stream mathematics but some students had taken (and not necessarily succeeded at) mathematics courses that are more challenging than regular-stream mathematics either in high school or at the college level. The RS then had taken *at least* regular-stream mathematics.

Student Status. The initial assumption was that the HS had attended university for at least one year, either part-time or full-time and was familiar with university procedures. Almost all of the students in the SG had attended university for one year or more and about half of the SG students were attending to studies on a full-time basis. This assumption is supported by evidence from the survey thus it applies to the RS as it is written.

Major. It was assumed that the HS was majoring in a discipline other than mathematics and that MATH 200 was a required course for this student.

None of the students in the class were majoring in math but some of the programs of concentration that were reported have mathematics requirements in addition to MATH 200. Many students were taking the course because they 'had to' but 8 of the 15 students in the SG and 36% of the students who completed course evaluations chose (or might have chosen) to take this course. There is evidence here that the SG is potentially skewed towards individuals who were taking the course as an elective.

This establishes the conclusion that the RS did not major in mathematics but it might be important in the student's field. MATH 200 was *probably* a required course for the RS.

Background. It was assumed that mathematics had been an area of difficulty for the HS. The student probably did not receive very good marks in high school and might have failed a mathematics course. It was also assumed that it had been at least a couple of years since the HS had taken a mathematics course.

A (small) majority of SG students reported that their previous marks in mathematics were usually less than 74%, there were 6 students who had previously failed a mathematics course and only 3 students could describe their math background as "good."

The RS had experienced difficulties in mathematics and furthermore, was *aware* that his or her background was not strong.

Mathematical prerequisites. It was assumed that the HS had very few mathematical prerequisites but that this was not the first time that algebra had been presented to the student. Although a range of abilities was observed, there were more than a few students who experienced initial difficulties manipulating integers and fractions arithmetically. Students asked for examples of the long division algorithm and expressed difficulty when dividing two fractions. Some students were applying remembered 'rules of algebra' (for example, when adding a term to one side of an equality while subtracting it from the other side).

The conclusion was drawn that the RS would benefit from a *review of arithmetical operations with integers and fractions.*

Implications for teaching. The RS differs from the HS across these characteristics only slightly. The RS was somewhat older and had weaker skills in mathematics than expected. A review of arithmetical operations was presented at the outset of the course. It was presented in an ego-preserving manner so that the students who felt that their background was weak could benefit without feeling threatened.

6.2 Needs of the RS

In this section, the assumptions that were made about the physiological needs, the psychological needs (for autonomy, competence and relatedness) and self-actualization needs of the HS are presented along with a summary of the evidence that contributes to the more realistic model (RS). A discussion of how well the course was able to address the needs of the RS follows the characterization. The evidence is presented in detail in Section E-2 of Appendix E. The social need for achievement cannot be fully examined here because some of the criteria for this characteristic will be examined in later sections. A conclusion about achievement needs will be made in Section 6.5.

6.2.1 Physiological Needs of the RS

It was assumed that the HS was capable of managing physiological needs but that the student had a busy life.

A busy life can limit the amount of time that is available for study but it can also be used to rationalize poor performance. The evidence indicates that the RS had many commitments besides MATH 200. Additionally, approximately 30% of the students felt sufficiently inconvenienced by the facilities to enter a negative comment (on course evaluations) about the facilities for the course, which is suggestive that the HS was somewhat sensitive to the physical surroundings. The modification can be made then that the RS has a busy life and is capable of managing physiological needs *but may be easily distracted*.

Addressing physiological needs. Only one student in the SG indicated that it was difficult to pay attention in class indicating that for the most part, these needs *were* being managed by the RS during the course.

6.2.2 Psychological Needs/Autonomy

Autonomy was assumed to be a moderate need for the HS and this assumption can be maintained in the model of the RS.

Three examples are presented in Appendix E (Section E-2.2) of students who were beginning to investigate academic autonomy in mathematics. The students were asking whether or not they could choose an approach when solving a mathematical problem. They had to ask because they were not yet sufficiently confident in the theory to be sure that different approaches were equally valid and they also had to be sure that their approach would receive full marks. Sierpinska (2006), refers to this phenomenon as learning the rules of an “institutional game,” and argues that mathematics courses that stress procedural approaches to solving problems without offering a theoretical context can be frustrating for students. The examples illustrate that the RS was beginning to explore ways to achieve autonomy in mathematics.

Addressing a need for autonomy. An autonomy-supportive classroom is one in which the students are offered choices. Moreover, it is one in which the students feel that their questions and perspective will be acknowledged. If a rationale is provided in an autonomy-supportive class, it is more likely that the values that it relates to will be internalized by the students.

Within the restrictions imposed by the curriculum and the course outline, I gave the students some choices. Although initially hesitant about asking questions in class the students gained confidence. Some students however, preferred to ask questions during the break and after the class as well as in e-mail communications. On course evaluations, a large majority of the students reported that they felt encouraged to ask questions and to share their ideas and knowledge.

A rationale was provided consistently for studying mathematics, supporting the value that mathematics is useful. Although most of the SG students felt that mathematics was useful before the course there was a Net Change of +2 in this value by the end of the course.

When students asked about options for a mathematical solution, I showed why different approaches were valid. Whenever possible, I showed the students how to verify their own solutions giving them the ability to decide if they were correct or not.

For the RS then, the course was as autonomy-supportive as possible and there is some evidence that the need for autonomy was being addressed.

6.2.3 Psychological Needs/Competence

It was assumed that the HS had a strong need for competence and that mathematics was not an area in which the student had previously felt competent.

While there was evidence that the students had experienced difficulties in mathematics that would not have allowed a feeling of competence, almost half of the SG students reported receiving good marks in previous mathematics courses.

A majority of students in the SG *wanted* to help others in mathematics which indicates a need to be competent enough to help. The SG students were not interested in displaying their competence to family or friends, but there were students who indicated that they thought mathematics would help them in other courses or their chosen career. These students certainly needed to develop their competence in this course. Interestingly, 76% of the students who completed the course evaluations felt that their previous mathematics knowledge was average or above average. If these students were in fact *aware* that mathematics had been an area of difficulty, and still felt that their level of knowledge was average, then they were expressing a need to feel competent in comparison to others.

The conclusion is drawn then that the RS had a strong need for competence *in comparison to others* and *may not* have felt competence in mathematics courses in the past.

Addressing a need for competence. The course addresses the needs of the students for competence if the challenge that it provides is optimal. It is when the challenge is optimal that students can have the pleasurable experience of flow (Section 2.5.2). Too much challenge can lead to anxiety and too little challenge can result in boredom.

The online quizzes were offered to the students to provide optimal challenge and the evidence suggests that they did for a majority of students. Students also found optimal challenge when doing the suggested problems from the text book. Only one student thought that the problems were "very hard" (this student received an F for the course) and another thought they were "boring" (this student received an A).

Some students described the course material as "difficult." Finding the course material difficult is perfectly acceptable during the course, otherwise there is no challenge at all. Being able to resolve difficulties is an indication that the student is in the Zone of Proximal Development (Vygotsky, 1978). One student wrote in a succession of e-mail communications, "I understand your explanation...but I still don't quite get..." and "I just answered my own question, I understand that one now...but I have another question." Challenge is about right for this student.

When a situation is optimally challenging, individuals become involved and begin to enjoy the activity. On course evaluations, a majority of students reported that they found the course "intellectually challenging and stimulating" and only one student reported "below average" involvement in this course.

With only a few students at the extremes of finding the course too hard or too easy, it can be concluded that optimal challenge was achieved for the RS. It was established however, that the RS had a need to feel competence *compared to others*. As I did not anticipate this need (in the characterization of the HS), there is no way to determine whether or not this need was addressed.

6.2.4 Psychological Needs/Relatedness

It was assumed that this was a moderate need for the HS and that the HS might seek out a support network in the class. The student might have blamed teachers for previous poor performance in mathematics.

Slightly less than half of the SG students liked to study with friends and these students would potentially seek positive relationships with peers. Almost all of the students in the SG thought that some of their previous math teachers had been "good" and some students entered comments that indicated that "good" or "bad" teachers were responsible for their success or failure. There is insufficient evidence to conclude that the RS was blaming poor performance predominantly on poor teaching. There were however, a total of 6 students who felt that the quality of teaching was a factor in their success or failure. This result is sufficient to conclude that the RS *may* feel that performance in mathematics is influenced by the nature of the relationship between student and instructor. Relationships, with peers and with the instructor, were important to the RS.

Addressing a need for relatedness. The instruments used to investigate motivation did not address peer relationships. During the course, I attempted to establish a positive relationship with the students; one in which the students regarded me as an authority but that was also supportive and friendly without being overly familiar. In a positive relationship, students can accept and adopt a value if they view it as good advice.

The students always addressed me formally (as Professor, Mrs. or Miss.), sought (and were provided with) my support for administrative details, and they did regard me as an authority. One student sent me a link to a video that made light of the complicated wording that is typical of algebra ‘word problems’ but the student was also concerned that I might find the language somewhat offensive. Two students asked about a specific question in the text that required the students to calculate $9 - (-2)$ but the answer reported in the solutions manual was 7. Both students assumed that the fault was their own and were not able, initially, to entertain the possibility that the manual was wrong. They needed another ‘authority’ – the instructor – to override the authority of the manual.

On course evaluations, the overall assessment of the instructor was “above average” or “well above average” for all of the questions. The comments described the instructor as “effective,” “informed,” “caring,” and there was one negative comment.

This was precisely the relationship that I had hoped to establish. It was positive and neither too formal, nor too familiar. Theory predicts that within this kind of relationship, advice that is given by an instructor can be accepted by the students. I consistently advised the students that "practicing math" was the key to improving their grades on the tests. There were five comments on the survey from students who thought that "practice" (or doing the problems) was a good way to study.

6.2.5 Social Needs/Achievement

The assumption was made that the HS was a low need achiever. Several criteria need to be evaluated before a conclusion can be drawn about this characteristic. The criteria of mastery or helpless orientation, explanatory style and valuing of success in mathematics will be discussed in subsequent sections. Perceptions of ability and expectations of success are presented here.

It was assumed that the HS would show perceptions of low ability and low or moderate expectations of success. These would be the characteristics of a low need achiever.

It was established above (Section 6.1), that the RS was aware that his or her mathematics background was not strong. Several comments from students also support the conclusion that the RS did have perceptions of low ability. However, even though a majority of SG students were nervous about passing MATH 200, a majority of students also expected to receive high marks. A comparison of individual responses to survey questions (Table D.17, Appendix D) allows a characterization of individuals as *worried*, *unsure*, *confident* or *hopeful*. A majority (9 students) could be considered either confident or hopeful.

The conclusion was drawn that the RS did have perceptions of low ability but high expectations of success.

6.2.6 Self-actualization

This was assumed to be a low need for the HS. Students who are acting to satisfy a self-actualizing need would be seeking to improve themselves, i.e. they would be interested in learning for its own sake. This self-actualizing tendency can be a powerful motivator in that it may provide the motivation needed for low-interest activities.

A majority (11) of students in the SG indicated that they were pursuing a dream of obtaining a university degree and this can certainly provide motivation for taking MATH 200 if it is a requirement for the degree. A self-actualizing tendency *for mathematics* would be observable as an interest in mathematical activities that were not directly related to passing the course. Attempts were made to encourage an interest in mathematics by introducing puzzles & historical content but for the most part these were unsuccessful. Thus, the need for self-actualization may *indirectly* be a factor in the motivation of the RS. The student may be attending university (in part) in order to realize his or her potential and this coincidentally places the student in MATH 200. The RS may have had a greater need for self-actualization than was assumed for the HS but the student was not acting to realize his or her potential in the field of mathematics.

Addressing self-actualization needs. Since the popularization of mathematics activities did not serve to generate an interest, they were discontinued after a few attempts.

6.3 Goals of the RS

Physiological, psychological, social and self-actualization needs (above) supply motivation for behavior that satisfies one or many needs. One of these behaviors is goal setting. Goals, in turn, supply motivation for the behaviors that will result in goal-attainment. Students who set goals outperform students who do not (Reeve, 2005) but the types of achievement goals (performance or mastery) determine the quality of motivation (intrinsic or extrinsic). The evidence that relates to goals can be found in Section E-3 of Appendix E.

6.3.1 Long-term, Intermediate and Short-term Goals

It was assumed that the HS had set a long-term career goal and the goal of obtaining a university degree. This was found to be the case for a majority of the SG students. Some students had set graduate degree goals. The intermediate goal of passing MATH 200 and the short-term goals of passing the tests had also been adopted (as expected) by the students. The assumptions made for the HS can be adopted for the RS as well.

6.3.2 Goal Proximity

It was assumed that the long term career and academic goals of the HS would be too distant to provide much motivation for the HS. The evidence supports this assumption in the characterization of the RS (Section E-3.4). Thus, most of the motivation derived from goals for the RS would be from short-term and intermediate goals.

6.3.3 Achievement Goals

It was assumed that the HS had set predominantly performance goals and that these were of a performance-avoidance nature. It was also assumed that the HS had set few mastery goals.

Briefly, a student who sets mastery goals is seeking to develop competence and one who sets performance goals is seeking to display competence. In a classroom situation, one way to demonstrate competence is via grades. A student who is working for grades because of a fear of failing is adopting performance-avoidance goals whereas one who is seeking a better grade than peers is adopting performance-approach goals.

The students in MATH 200 were certainly very concerned about grades. In Part II of the survey, students were asked to rank reasons why they worked hard at mathematics and were offered choices of achievement goals. More students (6) ranked the performance-avoidance goal ("because I don't want to fail") first than any other option. Five students chose to rank one of the performance-approach options first and four students selected the mastery goal ("to understand more about math") as their first choice. Notably, of the 6 students who ranked the performance-avoidance goal first, 4 students received an A as a final grade (2/3 of this group) and the other 2 students received a B. Among the remaining 9 students who chose another goal first, only 3 students (1/3 of this group) achieved an A.

The RS then, had set multiple goals and *more* performance goals than mastery goals. The student had set performance-approach *and* performance-avoidance goals.

Addressing Achievement Goals. During the course, performance goals were acknowledged but mastery goals were not ignored. Students were given structure and hints as to what material they would be tested on, i.e. an idea of how to get a good grade which directly addresses the students' performance goals of demonstrating their competence on the tests. However, the underlying theme of my presentation was that, "if you can understand the principles, then you will perform better with less effort." This too accesses the performance goals of the students but by encouraging an understanding which is precisely what a student with some mastery goals would seek. There were 6 students who indicated on the survey that they felt they were "better at writing math tests" after this course (and only one student who felt worse) and there were 9 students who reported that they understood more in this class than in previous mathematics courses (and 2 students said they understood less).

6.3.4 Goal Attainment/Frustration

Goal attainment or frustration is a type of feedback that, in part, determines the motivation of students at the end of the course. Thirteen of the 15 students in the SG achieved the intermediate goal of passing MATH 200. These 13 students are one step closer to their long-term academic goals which allows for a sense of accomplishment. There were 12 SG students who reported that they achieved at least some of their goals for the course and 3 students who reported that they did not achieve their goals. Two of these students (S1 & S13) received a failing final grade but one student (S23) received a D+ for a final grade. Goal frustration for these 3 students may lead to decreased motivation, revised behavior (taking the course again) or revised goals (a change in their long-term goals). The situation of these 3 students will be discussed in the following sections and in Section 6.9.

6.4 The Self for the RS

This section will consider the motivational characteristics that relate to the self. These are: implicit theories, self-efficacy, mastery/helpless orientation, explanatory style, ego-defensiveness and affect.

One could reasonably expect to find some variation in any group of individuals among dimensions of the self. What is sought here is information about these traits that could be of importance to a description of the motivation of the RS. A detailed compilation of the evidence for these characteristics can be found in Section E-4 of Appendix E. The RS is characterized for each dimension then the impact of the course (for each dimension) is discussed.

6.4.1 Implicit Theories

It was assumed that the HS was more of an entity theorist than an incremental theorist.

This quality of the self relates to whether an individual feels that mathematics ability is a quality that can be improved upon (incremental theorist) or whether it is fixed (entity theorist). An incremental theorist tends to adopt mastery goals and believes that hard work should produce good results. A disappointing result, for an incremental theorist, means that they should work harder. An entity theorist adopts performance goals and sees disappointing results as confirmation that they are not very good at mathematics.

As established in Section 6.3.3 above, the RS had set multiple goals but more of these were performance goals than mastery goals as would be the situation of an entity theorist. However, there was also evidence of an incremental viewpoint. The students in the SG felt that hard work in mathematics should produce good results and a majority of students disagreed with the statement, "math is for some people but not for me." The evidence suggests that both viewpoints were present for the RS but neither was predominant.

Change in implicit theories. During the course, if the students had the experience in which their hard work produced good results *and* they held an incremental viewpoint, then this course should have allowed them to feel that their abilities in mathematics had improved. There were a total of 7 students who reported hard work, achievement of their goals and who also felt more strongly that they could succeed in mathematics after the course. These 7 students can be said to hold an incremental viewpoint after the course. One student (S25) who wrote, "I felt that I was very bad at math," *before* the course also wrote, "If I had spent half as much time in high school doing the homework as I did in this class, I would have done much better in Math!" This student's viewpoint has changed from an entity view to a much more incremental viewpoint.

If the students worked hard, and achieved their goals but held an entity viewpoint then they would not feel more strongly that they could succeed in mathematics after this course. This was the case for 2 students in the SG.

There were 3 other students in the SG who did not study 'a lot' but who did achieve their goals and all 3 students received an A or an A+ as a final mark. These students have not yet been challenged, so no inference is possible about their viewpoint.

One student (S1), who experienced goal frustration, also studied 'very little' but the student wrote, "…I know that I'll do well if I apply myself." The students' incremental viewpoint remains unchanged by this course because he did not work hard.

The remaining 2 students are S13 and S23, the other students who experienced goal frustration. Both students reported studying 'almost every day' for 'more than 2 hours' each time. They both visited the MathHelp center and sought the help of a tutor. This defines a situation in which hard work did not produce good results; a situation which can lead the students to believe that they are not very good at mathematics and one that reinforces an entity viewpoint.

For 8 students then, this course promoted an incremental viewpoint, and for 4 students the situation suggests entity view. A determination was not possible for the remaining 3 students.

6.4.2 Self-efficacy

Since mathematics has been an area of difficulty for the HS, it was assumed that the student would have more than a little doubt about his or her ability to succeed in the course. Self-efficacy was assumed to be low for the HS.

Most of the students in the SG felt that they could accomplish the task of passing the course, had high outcome expectations of success and believed that their efforts would have the desired effect. However, the students also exhibited perceptions of low ability accompanied by nervousness about passing the course. The modification can be made that the RS had a moderate self-efficacy belief.

Change in self-efficacy. With improved abilities and more incremental viewpoints, it is possible that this quality could have increased for the RS because of the course. If students were nervous about passing because their last mathematics course was more than 2 years ago, then an experience of success in this course should reduce their anxiety. A majority of the students in the SG did feel more strongly that they could succeed in mathematics after taking the course.

6.4.3 Mastery/Helpless Orientation

It was assumed that the HS (because of prior difficulties in mathematics) would have adopted a helpless orientation and might even have exhibited features of learned helplessness.

An orientation is determined based on how a student responds to difficulties or failure. Persistence at a task is indicative of a mastery orientation. A student who abandons the task is adopting a helpless orientation. Students who feel incapable of improving no matter what they do are exhibiting learned helplessness.

Some students in the SG reported that they persisted when they were 'stuck' on a math problem but two students indicated that they did not. This can be taken to indicate that both mastery and helpless orientations were present among the students at the beginning of the course. The only conclusion that can be drawn then is that the RS may exhibit either of these orientations.

Change in mastery/helpless orientation. Many of the students who used the online quizzes made more than one attempt but for the most part, students did not persist until they had either exhausted their attempts or obtained a perfect score. There was some persistence, indicative of a mastery orientation. Ten students entered comments on the online quiz questionnaire indicating that they wanted to have the correct answers posted after an attempt. These students are indicating an unwillingness to persist at the problems that they had got wrong.

When students who adopt a helpless orientation are faced with potential failure, they may attempt to change the task or the rules (Reeve, 2005). There were 2 individuals in the SG that did receive a failing final grade for this course and both these students (S1 & S13) contacted me before the exam. S1 wanted to delay the exam and S13 wanted to have the final grade entered as 'incomplete'. Even so, these students did not give up and they both wrote the final exam. S1 maintained a positive outlook (a mastery orientation) and offered the comment, "I have registered to take the course again in the fall and am excited for it."

S13's comment was, "I have tried continuously in this course and despite my efforts I was unable to obtain a passing grade." S13 worked very hard in this course and sought help. It is entirely possible that this course was responsible for the development of some learned helplessness which can happen when the situation is perceived as being uncontrollable.

For the majority of the students in the SG however, this dimension should show improvement after the course as they did indeed work hard and experienced the reinforcement of goal attainment.

6.4.4 Explanatory Style

It was assumed that the HS would exhibit a mixture of both optimistic and pessimistic styles.

An individual adopts an optimistic explanatory style if they attribute their difficulties to external and controllable factors and a pessimistic style if the factors are internal and uncontrollable. In general, an internal factor is related to the individual's character or behavior and an external factor is related to the environment. However, an external factor ("I did poorly because the teacher wasn't very good") could be perceived as being somewhat internal by an individual ("I made the wrong choice of teacher"). Similarly, it is whether or not a factor is perceived as being controllable (or not) that is important in a determination of explanatory style.[4]

The students in the SG attributed their previous difficulties in mathematics to a variety of internal and external factors that could be considered controllable or uncontrollable. No clear consensus could be established across this dimension of the self for the RS.

Change in explanatory style. Since most of the students in the SG reported that they had achieved at least some of their goals, this characteristic can only be described for the 3 students who did not achieve their goals. The remarks of S13 and S23 indicate that they thought they would have been able to perform better if they had taken the course during a full semester. Time, as an external factor that can be perceived as controllable (the students could opt to take the course during a full semester) is indicative of an optimistic explanatory style. S1 did not work very much during the course and cited "personal circumstances" as the reason. This student had a positive outlook (mastery orientation), even after a failure because he had attributed his difficulties to external factors and hence can be said to have an optimistic explanatory style.

6.4.5 Ego-defensiveness

It was assumed that the HS was in a position of vulnerability.

The students in the SG had experienced difficulties and even failure in mathematics prior to this course. For most of the SG students it had been more than 2 years since their last mathematics course and a majority of the students were nervous about passing. Six students felt

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[4] Other possibilities exist but in general, the more external and controllable a factor is perceived to be, the more optimistic the explanatory style. For example, a factor that is internal and controllable indicates a more optimistic style than one that is internal and uncontrollable.

that math had made them feel "stupid" at some point in time. An algebra class, under these circumstances, could certainly be perceived to be threatening.

It is important to maintain the description of vulnerability for the RS because to do otherwise could create a situation that might provoke avoidance of mathematical activities. An instructor who is aware that the students are vulnerable can avoid diminishing their motivation.

Addressing ego-defensiveness. Some students asked questions during class but there were several students who were more comfortable asking questions during the break or after the lectures. Questions were encouraged but no student was ever singled out and asked to provide an answer. If a student asked a question and seemed very confused then I asked the student to see me after class but I explained that it was because I could not understand what they were asking. I promoted a relaxed atmosphere during the lectures that was at all times respectful of the students. Mid-way through the course, the students certainly seemed more comfortable about asking questions in class.

6.4.6 Affect for the RS

The dimensions of affect that will be considered in this section are: Emotions, attitudes, beliefs and values. Evidence for these is drawn primarily from Part V of the survey. The students were asked to agree or disagree with 10 statements. Some statements can be interpreted in a context of more than one dimension of affect. For example, the statement that, "math is useless" is a belief (a student may believe that mathematics is useless) but it also serves to place a value on mathematics (something that is useless has no value). A belief of this nature about mathematics may arouse emotions (frustration, anger) which become part of an attitude that is held towards mathematics. The emotions that a belief generates cannot be predicted reliably as individuals may show considerable variability in their interpretation of a particular belief.

The results for Part V of the survey are reported as Net Affect (before the course) and Net Change (after the course).[5] For all of the questions in Part V, the value for Net Affect before the course was positive and the value for Net Change was also positive.

Affect/Emotions. It was assumed that mathematics had aroused predominantly negative emotions for the HS but the emotions that were reported by the SG students were not uniformly negative. It was concluded that mathematics had aroused some negative emotions for the RS.

Affect/Attitudes. The assumption was made that the HS did not particularly like mathematics. This could be changed to the statement that the RS did not especially dislike mathematics.

Affect/Beliefs. The initial characterization of the HS was that the student believed that mathematics was difficult, that algebra was abstract and that only the very gifted could succeed at mathematics. This was not found to be the case. The RS believed that mathematics was attainable and not overly difficult or abstract. The RS felt that he or she could succeed in mathematics.

Affect/Values. The initial assumption was made that the HS did not see mathematics as useful in the real world and hence, the student did not necessarily value success in mathematics. This assumption was also not supported by the evidence and the conclusion was made that the RS did see mathematics as useful and valued success in mathematics.

Change in affect for the RS. Theory predicts that if the students' needs are being addressed then there can be a positive change in emotions, attitudes, beliefs and values.

[5] These quantities are introduced and examples of the calculations appear in Section 5.8. Essentially, Net Affect is the number of responses that were positive minus the number of responses that were negative for a question about affect before the course. Net Change is the number of responses indicating a positive change minus the number of responses indicating a negative change.

The Net Change values for all of the questions about affect were positive and the total over the ten questions was +45. This indicates that the course had a positive impact on the affect of the RS. Individual responses however, were not uniformly positive. There were 9 students who reported a positive, *individual Net Change*[6] and 2 students who reported no change overall. The individual cases of the 4 students who reported a negative change will be presented at the end of this chapter in Section 6.9.

6.5 Conclusion for Social Needs/Achievement

As established above, the RS showed perceptions of low ability but had at least moderate expectations of success (Section 6.2.5). The student showed some signs of persistence when confronted by difficulties (Section 6.4.3) and exhibited a mixture of optimistic and pessimistic explanatory styles (Section 6.4.4). The RS valued success in mathematics (Section 6.4.6, Values). The RS cannot be considered a low need achiever across this dimension but the results were not uniformly positive for a majority of SG students either. The RS was a moderate-need achiever.

Addressing achievement needs. This dimension is being addressed if the abilities of the students are being developed during the course. A course that is failure tolerant is one that encourages the students to attribute their difficulties to external and controllable sources (encourages an optimistic explanatory style) and encourages persistence when difficulties are encountered (a mastery orientation).

Developing students' abilities is the goal of most courses at university and this course was no exception. Overall, the students' abilities in algebra developed considerably during the course. Polynomial division, for example, was introduced as a concept that was "hard" (acknowledging

[6] This value is the number of positive responses minus the number of negative responses for an individual student over the 10 questions.

difficulties) but it was compared to the long division algorithm (connected to previous knowledge) and the students were told that it would be on the test (performance goals). There were two students who had asked for an explanation of the long-division algorithm at the beginning of the course. These two students and many others were able to divide polynomials on the second midterm test and the final exam.

There were two notable areas of difficulty (word problems and cancelling terms) that were still being observed by the end of the course. Students said that they found word problems confusing and asked for more examples but the problems on the final exam were quite well done overall. Cancelling terms inappropriately was a consistent difficulty and despite several different attempts at explaining, this difficulty was noted on the final exam.

In order for a classroom to be perceived as failure tolerant, someone has to make mistakes. I did make calculation errors on the board and these were usually spotted by the students. Occasionally I asked for a volunteer to use a calculator to verify my arithmetic. I made some algebraic "errors" but never when the concept was being explained; these were made only if the concept had been previously established. These also were noted by the students and I asked them to explain what the problem was. One student noted this and remarked on course evaluations, "...she does a lot of mistakes..." but for most of the students, this did not impact their perception of me as 'knowledgeable' (question 8, Table 5.3). One student commented in an e-mail that was sent after the final exam but before the marks were posted, "I might have made some stupid mistakes but that is part of mathematics."

If students made mistakes or indicated a failure to understand, I reacted as if it was a failure on my part to explain fully. This allows them to attribute their lack of understanding to an external source – me.

6.6 Motivation of the RS

It was assumed that the HS had very little interest for mathematics and hence very little intrinsic motivation. The HS was assumed to be largely motivated extrinsically – by grades. It was found that the RS was not overly interested in mathematics but that the student 'liked' mathematics more than was originally assumed (Section 6.4.6). The student had set more performance goals than mastery goals (Section 6.3.3) and the motivation from these is extrinsic (grades). Thus, the RS may have had some intrinsic motivation but, for the most part, was motivated extrinsically.

6.6.1 Behavior of the RS

The behavior of the RS can approximate the strength of motivation that was present.

The students in the SG reported studying in expected ways. The students attended class, did the suggested problems from the text book, read explanations from the text, used the online quizzes and they attempted the practice exams. Also 12 of the 15 students in the survey group indicated that they did extra problems from the text, even if they weren't listed on the course outline. This last result suggests strong motivation on the part of the RS because there were many problems listed on the outline.

A student can be said to be strongly motivated if they are energetically pursuing an activity. Strongly motivated behavior shows short latency (time from exposure to response), high probability of response (the number of times the behavior occurs) and long persistence (time from initiation to cessation). A measure of latency could not be determined. A high probability of response was observed in the responses to the survey questions in Part IV. The students reported studying either frequently (more than 3 times a week) or for a long time (more than 2 hours) on a regular basis. Four students reported that they studied more than 30 hours for the three tests. Persistence was observed in the number of students who attempted the online quizzes more than once. More important perhaps, is the result that, whatever amount of studying they actually did, 9 of the students thought it was "a lot." I would tend to agree; there is evidence that the students in the SG did quite a lot of work which would be indicative of strong motivation.

The amount of studying is not an absolute indicator of the strength of motivation, many other factors intrude. The amount of studying that an individual *needs* to do is dependent upon their abilities, the outcome that they hope to achieve and the effectiveness of their study habits (and perhaps other factors as well). Two individuals of differing abilities may be equally motivated to get a high mark but an individual with strong skills may not need to study for as long as someone with a weaker skill set.

6.6.2 Self-Regulation and Feedback

This is the last element of the concept map of motivation to be considered. Feedback provides the information needed to regulate behavior. Feedback can be inherent in the task, by comparison to previous performance, by comparison to others or it may come from external sources.

Feedback, inherent to the task, is immediate when the students were attempting problems from the text book; they could check their solution against the answer in the back of the book. The online quizzes provided this feedback but not immediately, the students had to exit the quiz before they could see which questions had been answered correctly. (Other types of computer-assisted learning programs offer immediate feedback for each question.)

Introducing elements of competition were avoided during the course but some students may have sought feedback by comparing their results to others.

Feedback from external sources was primarily the students' grades and the comments that I wrote on their midterms. These were as informational and un-controlling as possible.

Feedback can lead to revised goals, motivation or behavior and this may very well have been the case but the survey did not address this point directly. The encouraging e-mail that was sent to students who had low marks on the first midterm was acknowledged by 2 of the students in the class and both students indicated that the situation was under control but it cannot be determined what changes the students made to their behavior.

Students also receive feedback when they compare their performance to previous results. A comparison of students previous results with their results in this course (Table D.17) shows that 7 students received the feedback of obtaining a better mark this time.

The emotions that accompany an understanding are also feedback to the student. A majority of students (on the survey and course evaluations) indicated that they were receiving this feedback.

6.6.3 Motivation derived from MATH 200

Some of the students will take other mathematics courses and a few will re-take MATH 200. (One student indicated that she was already enrolled in MATH 201 at the time the survey was completed.) If the students experienced a change in their affect towards mathematics in this course it is possible that they will have greater motivation for their next course. The majority of the students in the SG had the experience of success in this course. Feeling responsible for one's successes allows the emotion of pride in one's accomplishments and encourages an individual to seek out greater challenges. Feeling that the cause of success was due to other factors generates no pride and sets up the expectation that success will only happen again if the conditions are similar. The survey asked the students to list what they found was most helpful about the course.

There were 12 comments recorded for this question and students cited multiple factors that contributed to their success. The instructor was mentioned 8 times, the textbook (suggested problems, explanations or the solutions manual) was cited 7 times. Four students explicitly mentioned “explanations” (from the instructor, the text or a tutor in the MathHelp center) that were helpful in this course. If an explanation is successful, it provokes an internal understanding or a perceived understanding. For students who might not have understood algebra in the past, this is a significant achievement. The instructor’s explanations are external but the understanding that the students developed was their own. One course in which they did develop an understanding may not be enough for the students to have the confidence that in another course, with another instructor (and another text etc.) they will also be able to succeed. Many iterations of success may be needed but this course hopefully provided one instance of success that the students could ‘own’.

6.7 Individual Cases

Overall, a large, positive change in affect was observed in the responses to the survey but as noted above, (Section 6.4.6) not all students reported a positive change. Individual responses are summarized in Table 6.1. Individuals whose net change in affect was negative are highlighted. A comparison of responses to selected questions can be found in Tables D.16 and D.17.

There were 9 students who reported a net positive change in affect and another 2 students who reported no net change. Two students, S1 and S13 both received a failing grade for this course but S1 reported a large positive Net Change and S13 reported a negative Net Change. This discrepancy and an analysis of the motivation of the 4 students who reported a negative Net Change is presented here.

Table 6.1 Individual results for dimensions of affect

ID	Final Grade	Net Affect (Before MATH 200)	Net Change
S1	F	+8	+10
S2	A	-2	+8
S6	A+	-2	+3
S7	B+	+8	-1
S8	A-	+4	-5
S11	A	+4	0
S13	F	-4	-6
S16	A	+8	+10
S17	A+	+8	+6
S18	B-	+9	0
S23	D+	0	-1
S25	B+	-5	+7
S27	B+	-5	+5
S28	B+	+6	+6
S37	A-	+9	+3
Total		**46**	**+45**

Comparison of S1 and S13. Both S1 and S13 failed this course but S1 reported a high positive Net Affect (+8) before the course and a Net Change in affect of (+10). S13 had a negative Net Affect of (-4) and reported a negative Net Change of (-6). There are important differences between these two students that can account for this discrepancy.

S1 had not previously failed a mathematics course and reported receiving good marks. S13 had failed a mathematics course and reported usually receiving marks that were lower. The mathematics background of S13 includes more negative experiences (than S1) which accounts for a more negative Net Affect before this course.

S1 was taking this course as an elective and had set a mastery goal. MATH 200 was a prerequisite for S13 and the student's goals were of a performance nature. Even though neither student attained the performance goal of passing the course, S1 entered a comment, "I'm more familiar with terms and the fundamental concepts," indicating that the student had a greater perception of understanding and had achieved some mastery goals.

S1 worked "very little" for this course and cited external factors as the cause. This student did not try and fail; he did not try. This course did not impact S1's concept of himself to any great extent. His self-efficacy belief has not been tested. S1 offered the comment, "I had a slight fear of math because I've been away from it for so long..." and also wrote, "I like it (math) more because I know what to expect and know that I'll do well if I apply myself." This student can like mathematics more after the experience of the course because it is less "scary" and despite his failing grade, he is *more* confident about being able to succeed.

S13 however, worked very hard, sought help and still did not pass. This student is in a much more difficult position because it calls into question S13's self-efficacy belief – he may now believe that he is not capable of passing the course. This student may also have developed some learned helplessness with respect to mathematics. Since *this course* (or by inference, mathematics in general) is the cause of some discomfort for S13, it is not surprising that the student reported a much more negative affect after his experience.

Students reporting a negative change in affect. The 4 students reporting a negative Net Change are: S13, S23, S7 and S8. The case of S13 is described above.

The case of S23. This student received a D+ as a grade for the course, had a Net Affect of (0) and reported a small but negative Net Change of (-1). S23 worked very hard for this course but still experienced goal frustration. The student commented, "I was expecting a better grade due to the commitment I put into this class." The course may have reinforced an entity viewpoint and fostered some learned helplessness because hard work did not produce good results. Additionally, S23 had previously failed MATH 200 and reported previous mathematics marks were usually less than 60%. Even so, this student had set a goal of obtaining a mark of 75-100%. S23 had high (and possibly unrealistic) expectations and hence was disappointed when these proved to be unattainable.

The case of S7. S7 did not have unrealistic expectations, and achieved the goals that had been set. The student had a highly positive Net Affect towards mathematics before the course (+8) yet felt that after the course algebra was more abstract and mathematics was less useful but felt more strongly about being able to succeed in mathematics. This student also sent me an e-mail after the exam thanking me for my "patience" and "excellent explanations" which made it possible for this student to, "complete all the assignment (exam) without missing any (questions)." S7 had a feeling of success after the exam. After the marks were posted this student sent an additional e-mail and asked, "I got a B+ on the exam, but what does this mean... 80, 85, 90???" This student seems to have set higher expectations for the final grade *after* leaving the exam and may have found that a result of B+ was somewhat disappointing.

The case of S8. This student has a positive Net Affect of (+4) and a final grade of A- but a Net Change in affect of (-5). S8 had not previously failed a mathematics course, expected a high mark and achieved this goal. There are no comments recorded for this student for Part VI of the survey (Additional information) and there are no indications as to why this student felt more negatively towards mathematics after the course.

6.8 Summary

Almost all of the characteristics that were revised or reworded in the characterization of the RS were made in a direction that is either neutral or that positively impacts motivation. The RS was a moderate-need achiever and had a moderate need for self-actualization. The student had set some mastery goals, there was evidence of an incremental viewpoint and some signs of persistence when confronted by difficulties (mastery orientation). The RS had a moderate self-efficacy belief and an overall positive affect towards mathematics. All of these factors suggest stronger motivation for the RS than was assumed for the HS.

This course was to some extent autonomy-supportive, delivered optimal challenge for many students and the relationship that was established between the students and the instructor was a positive one. Achievement goals were addressed using the theme that better performance results from a better understanding of the principles of mathematics. The popularization of mathematics activities did not serve to encourage an interest in mathematics but many students felt that they had achieved their goals in this course.

There is evidence for positive change in the students' abilities, and their perceptions of their abilities (implicit theories). The conditions were satisfied that could provide a positive change in self-efficacy and orientation (mastery rather than helpless). A change in explanatory style could not be assessed for the RS because it relates only to attributions for difficulties. There is evidence for a large change in affect for the RS even though individual results were not uniformly positive.

Chapter 7

Discussion, Recommendations and Conclusions

The goals of this research were to describe, investigate and improve upon the motivation of students who would be taking a beginning algebra class in a university setting. My conjecture was that motivation is lacking in a beginning algebra class but that a class can be taught in a way that is motivating for students who dislike mathematics and this will result in better attitudes towards mathematics.

7.1 Limitations of the Methodology

The Conjecture-driven research design (Confrey & Lachance, 2000) that was used here, offers the advantage of situating mathematics education research in a dynamic classroom. It allows for reacting to the situation on an ongoing basis. I was able to fulfill my goals as an instructor by reacting to the needs of the students *as they arose*. The major drawback to this design is exactly the dynamic nature of a classroom. The changing situation introduces variables that limit the conclusions that can be drawn. In my attempts to create a motivating classroom, I sacrificed the ability to be absolutely sure precisely what was motivating to the students.

7.1.1 Timing

In order to investigate my conjecture, I made assumptions about the students who would be taking the course and the construct of the hypothetical student (HS) is a compilation of these assumptions. Based on this representation, I planned actions and structured the lectures in a way that I expected to be motivating. During the course, I adapted my presentation and actions according to the feedback that I received from the students. For example, I had only planned to create online quizzes for the two midterm tests. The online quiz for the final exam was only created because the students asked for me to do so.

It was not until *after* the course that I collected the data (from the survey) that allowed me to re-examine my original assumptions. This was the data that was used to refine the motivational characteristics of the HS to the more realistic profile of the realistic student (RS). The students were not polled to participate in this research study until the end of the last class. A more linear approach would have been to assess initial motivation using an instrument (perhaps a questionnaire) administered at the beginning of the course and then to re-assess at the end of the course allowing for a comparison of responses. I did not choose this approach because I did not want the students to feel that they were "experimental subjects" as they might have altered their behavior during the course. By doing so, the interactions and e-mail communications that I had with the students were authentic – they were the natural interactions between students and an instructor. This was achieved however, at the cost of reliable data from the outset of the course.

In the survey, students were asked to describe their motivation at the beginning of the course. At all times during the course I was attempting to motivate students and Reeve (2005), asserts that motivation is not entirely a conscious process. The experience of the course may have subtly influenced the students' descriptions of their initial motivation.

The timing of the data collection also meant that whereas I was planning interventions for the HS, it was the RS who was actually present in the class. My portrayal of the HS was pessimistic (motivationally) drawing inferences from the assumption that the student had experienced difficulties in mathematics prior to this course. The RS was found to be quite strongly motivated to study despite having had negative experiences in mathematics. Thus, some of my interventions might have been unnecessary.

7.1.2 Groups of Students

There were 46 students who wrote the final exam, 37 students who agreed to participate in this study and 15 students who completed the survey (survey group). The composition of the survey group is skewed, in that individuals who did well in the course are over-represented (Chapter 5, Figure 5.1 and Table 5.4). It was also found that this group was skewed towards individuals taking MATH 200 as an elective (Section 6.1). Participation in the survey was voluntary. These three factors (good marks, choosing to take the course and choosing to respond to the survey) are all indicative of stronger and better motivation among the individuals in the survey group than perhaps was present in the class population. Even though approximately a third of the students in the class completed the survey, this might represent the most motivated students in the class.

The survey group is a small sample (15 students) and not randomly chosen. Moreover, the survey was not anonymous. Students were asked to enter their name on the survey so that I could coordinate their responses with any communications made during the course and with their grades. There might have been more responses to an anonymous survey and the students might have felt more inclined to enter negative responses or comments.

7.1.3 A Qualitative Study

This study is qualitative and not meant to be in any way quantitative. The survey questions give no information about the quantity of a dimension. The students were asked if they liked mathematics, not how much they liked mathematics. Whereas eight students agreed that they did like mathematics, I'm sure that not all of these students would read a mathematics book for pleasure or attend a mathematics seminar out of an interest. It is entirely possible that agreeing with the statement represents liking mathematics a very little bit for all of these students.

The survey questions were only intended to provide a general direction of the dimensions of affect and subsequently whether or not there had been a change in this affect after completion of the course. Even so, a more extensive questionnaire or individual interviews would have the potential to produce a much more complete picture.

This study is descriptive in nature and it is descriptive only of the students in this specific class. It allows for valuable insight *in this instance* but there is no comparison to any other group of students. It is my contention that the motivational characteristics of the students should be considered *before* attempting to devise motivational interventions. A comparison to another population of students would have been ineffectual but the construct of the HS, based on theoretical considerations, served this purpose admirably. The description of the RS is a more accurate portrayal of the students that were in this class and it is this profile that could be used to design motivational strategies for adult students in a basic algebra course.

The profile of the RS could certainly be refined further by the inclusion of more data from other MATH 200 classes and by gathering data from a greater proportion of students in each class. A more controlled study could then be conducted to determine the effectiveness of differing motivational interventions.

7.1.4 Difficulties in the Characterization of the RS

Characterization of needs. The needs of the HS were described as low, moderate or high, as an assumption based on theoretical considerations. In the characterization of the RS, there was little evidence that could be derived from the instruments to support or refute the *strength* of needs. All individuals have the psychological needs for autonomy, competence and relatedness, (Reeve, 2005) but this was not a comparative study, therefore no inference is possible about the strength of the RS's needs, relative to any other group of students. The descriptors serve simply to suggest the relative priority that the RS would assign to these needs. The needs for autonomy and relatedness were assumed to be moderate (for the HS) and these descriptions were maintained in the RS model. A need for competence was assumed to be high (strong) and was also maintained. A need for self-actualization was initially characterized as low and revised as moderate. The implication here is that competence needs would have priority for the RS and the other needs would have lesser (but equal) importance. This is a simplistic model and it is based, in part, on assumptions. Further investigation is needed to substantiate (or modify) this implication.

Dimensions of the self. Difficulties were also encountered when attempting to describe the dimensions of the self for the RS.

The evidence for the dimension of implicit theories (entity or incremental) was drawn implicitly (from the result that the students had set performance goals) and more directly (from the question "math is for some people but not for me"). This dimension is positive, motivationally, if the student believes that they are "good" at mathematics or if they believe that they can improve. It negatively impacts motivation if a student feels that they are truly and irrevocably "not very good at math." Frustration and negative experiences in mathematics reinforce a negative belief but it may be partially subconscious. Students in an algebra class, who are attempting to improve their abilities, might be unwilling to admit a fear that they might not be able to do so. In a situation where ego or the appearance of competence is at stake, most people would answer the direct question, "Do you think you can improve at math?" with a positive response even if they were not sure. Hence, direct questioning might yield an unduly positive result.

The survey did not question the students about persistence in the face of difficulties before the course for a determination of the dimension of mastery or helpless orientation. This was an oversight on my part. Ego-defensiveness was not specifically addressed by the survey either and this dimension was inferred from the experiences of the students.

No clear consensus could be established for the explanatory style of the RS, either before or during the course, because the students attributed their difficulties to a variety of internal and external factors that could be perceived as either controllable or uncontrollable. The survey did not provide information about the students' *perceptions* of locus and controllability. Explanatory style only takes into account the factors that students feel are responsible for their difficulties and many students were successful in this course. Explanatory style is one part of a much larger theory in psychology, called Attributional Theory (Weiner, 1985) which does consider attributing factors for both success and failure.[7] Moreover, in a review of research in the area of motivation in mathematics education, Middleton and Spanias (1999) describe studies that indicate gender differences in attributional style that are established and stable by the time students enter university. The most recent of the cited studies[8] was published in 1990 and the situation of gender differences in mathematics education may have changed since publication.

Thus, it may be worthwhile to investigate students' attributions for success as well as for failure but it may also be the case that this dimension is one that shows too much individual variability to be characterized for a typical student in the class.

[7] Central to this theory is the idea that a 'cause' may have three properties: locus, stability and controllability. Locus (either internal or external) and controllability are as defined for explanatory style. Stability is the extent to which the factor can change or the outcome can be predicted (ability is generally perceived as being more stable than the amount of effort that is exerted). As with explanatory style, it is perceptions of locus, stability and controllability that matter and a spectrum of possibilities exist.

[8] The authors cite: Meyer and Fennema (1985); Kloosterman (1988); Amit (1988) and Fennema, Peterson, Carpenter & Lubinski (1990).

Survey instruments exist that have been used to investigate motivational characteristics. For example, Yates (2002 & 2009) investigated students' explanatory style with a 48 point questionnaire and used a "Student Behavior Checklist" with 24 questions to assess mastery orientation and learned helplessness in primary mathematics classes in Australia. These could be adapted for adults but, as written, they were too long to be used in this research.

7.2 Theoretical Considerations

The model that was described in Chapter 2 incorporated some of the aspects of the multiple theories of motivation and no major inconsistencies were found in the theory. The results however, were not all as was expected. One notable result was that 7 of the students in the survey group reported previous marks in mathematics that were usually better than 75%. Self-reporting of grades is not always accurate and the tendency is to self-report higher grades than were actually attained (Kuncel, Crede, & Thomas, 2005). On course evaluations, a majority of students reported their previous level of knowledge as average (or above average). There is a psychological tendency, termed *illusory superiority* and colloquially referred to as the "Lake Wobegon[9] effect," which identifies a reluctance of individuals to self-report personal abilities as below average.

7.2.1 Characterization of the RS

The initial characterization of the HS was pessimistic. Since the HS was never assumed to be representative of any particular student, a pessimistic outlook allowed for consideration of the most weakly motivated student without ignoring the more strongly motivated. It was found that despite negative experiences in mathematics, the RS could be characterized much more

[9] Lake Wobegon is a fictional town, popularized by Garrison Keillor in a radio broadcast, "A Prairie Home Companion." It is a town in which all the children are smarter than average. For more information: http://prairiehome.publicradio.org/

optimistically with respect to motivation (Section 6.8). Motivation did not appear to be lacking for the RS (as conjectured); the student was quite strongly motivated to study. The major differences that were documented for the RS are: moderate self-efficacy, strong motivation from performance goals (grades) and better initial affect towards mathematics.

Self-efficacy. The RS was quite hopeful of a positive outcome in this course. Two thirds of the survey group students expected to receive high marks in this course. While pessimism is associated with lower achievement in mathematics at an early age (Yates, 2002), hope or optimism supports persistence (Reeve, 2005).

Motivation from performance goals. It is well established in the theory of motivation that mastery and performance-approach goals both contribute to motivation but performance-avoidance goals undermine achievement and positive outcomes (Reeve, 2005), and interfere with learning (Luo, Paris, Hogan, & Luo, 2011). This result was not reproduced here. The RS was motivated by grades (either the expectation of receiving good grades or a fear of failing). However, the group of students who admitted to a fear of failing all did well in this course (Section 6.3.3). This particular sample was exceedingly small (6 students) so the result is insufficient to contradict the established theory. Overwhelming fear can be debilitating and can paralyze an individual but a little anxiety may prompt an individual to act even when the activity is uninteresting. It is possible that these 6 students felt a small amount of fear and used the emotion productively.

Affect. It was assumed that because of previous negative experiences in mathematics, the HS would present a negative affect towards mathematics. I fully expected that almost all of the students would report that they did not like mathematics at all. Interestingly, this result was not established by the survey responses. There were more students who reported positive affect towards mathematics than negative. Some students who had previously failed a mathematics course reported that they 'liked math' before the course. Kim and Hodges (2011) also found very little negative emotions reported among students taking an online introductory mathematics course. Some possible explanations of this result follow.

1) The result for positive affect is drawn from the 15 students in the survey group. As noted above (Section 7.1.2) the students in the survey group might represent the most motivated third of the class. These were the students who were interested enough to complete the survey and it is entirely possible that these students accurately self-reported but that this does not represent the affective nature of the class as a whole.

2) It is also possible that even these students reported more or even less positive affect than was actually present. The students were asked to describe how they felt *before* the course but they were not asked until *after* the course had been completed. While attitudes, beliefs and values are moderately stable, (DeBellis & Goldin, 2006) emotions can be fleeting. DeBellis and Goldin (2006) see these sub-domains of affect as interacting dynamically and Reeve (2005) acknowledges that people may be reluctant to accurately report or not aware of all the sources of their motivation. Over the period of the summer semester, the students' perceptions of how they did feel at the beginning of the course could have changed.

3) Another possible interpretation of the positive affect reported by the students in the survey group involves the concept of meta-affect as described by DeBellis & Goldin, (2006). According to the authors, meta-affect is *how* an individual feels and thinks about *what* they are feeling. It is the mechanism by which individuals monitor their affect and it can manifest as a "tower" of emotions. DeBellis & Goldin use an example of feeling guilty about one's anger.

By the end of the course, a positive student/instructor relationship had been established and the students were aware that I had initiated the survey. The comments that the students wrote were impersonal; they wrote “the professor…” or “the teacher…” and did not address me specifically but they knew that I would be reading their comments and I had presented myself as someone who does like mathematics. It is within the realm of possibility that students with neutral or slightly negative affect towards mathematics felt a little uncomfortable about their affect and were reluctant to report negative feelings within a positive relationship. In other words, the students could have reported what they thought or felt that I wanted to hear about what they felt. The tower of meta-affect might not even have been formulated consciously but might have existed as a subconscious tendency that manifested in more positive results overall. Thus the students may have accurately reported what they believed they felt about mathematics.

If the result for initial affect of the RS is potentially inflated, the result for change in affect that was recorded after the course might be inflated also. The students were still being asked to recall their emotions, beliefs, attitudes and values before the course but they were now comparing this to the state that existed for them at the time that they were completing the survey. Hopefully, this position was easier for the student to access and report accurately.

Intrinsic motivation. Neutral or positive affect can certainly generate motivation. Positive affect though, implies more; it implies that the students would be *intrinsically* motivated (Reeve, 2005). While the students did report a lot of studying, there was an almost complete lack of interest in the popularization of mathematics activities that were presented. The students did not seem at all interested in the puzzles, were worried that the history of mathematics would be on the test and there was not a single student who volunteered to participate in the “fun” activity that was offered by the graduate student who polled the class. There seems to be a discrepancy between the reported positive affect of the students and the lack of evidence of intrinsic motivation.

Popularization of mathematics activities are (by definition) voluntary (Kelecsenyi, 2009). They can cultivate an interest in mathematics but only if an individual agrees to participate. If a student decides to participate, there must be a 'spark' of interest present but it does not necessarily follow that declining to participate indicates a complete lack of interest – there could be many other factors. It could have been the case that the student with a busy life might not have had time for extra, non-credit activities or that the specific activities might not have been appealing to the student.

If the RS can stand as a representative for adult students in bridging mathematics courses, then it is the description of this student that explains why it is difficult to recruit students to participate in research studies from this population. The student is motivated by grades and has little interest in activities that are not directly related to performance.[10] If the students in this course had experienced a positive change in affect towards mathematics then it is possible that a little intrinsic motivation had been encouraged. While it was not presented as a result per-se, there were 15 students who completed my survey of the 31 students who were asked to participate. The survey was an activity that is related to mathematics for which there were no extrinsic rewards offered. These students found the time and the motivation to participate.

7.3 Examples of Motivational Teaching

The literature on motivation in mathematics education is extensive and much has been written about attempts to motivate or encourage an interest in mathematics. With the best of intentions, teachers and researchers attempt to motivate students and report a positive impact on

[10] Considering the performance goals of the RS, participation in research studies might be increased if a small amount of extra credit was offered to students.

motivation. There are two shortcomings inherent in these attempts. The first is described by Middleton and Spanias (1999) who write:

> …teachers seem to have little background knowledge pertaining to how students view mathematics activities from a motivational perspective. The teachers' own personal constructs of what makes mathematics intrinsically motivating play a pivotal role in determining the types of activities they choose or design for their classrooms. (pg. 76)

Thus these attempts often ignore the perspective of the students that are being "motivated." The second shortcoming is that the positive results might be misleading. In a classroom, especially at the high school level, there are individuals who *are* intrinsically motivated for mathematics. Motivational strategies may very well have appeal for students who are already equipped with some intrinsic motivation but these students, arguably, have little need for motivational intervention. It is the students who have little or no interest in mathematics that have the greater need for motivational support. Strategies designed to increase intrinsic motivation may not reach this latter group at all. These are essentially attempts to motivate the *already motivated*.

For example, Menon (2004) in an article retrieved from the NCTM website, (National Council of Teachers of Mathematics, 2012), suggests introducing algebra to high school populations through the use of algebraic generalizations incorporated in puzzles. In the first of the puzzles (a constant-sum grid), the students are asked to follow 9 steps of instructions and then to notice patterns amid the numbers. The process ultimately leads to an algebraic generalization. While I find the puzzle approach fascinating, I believe that it would be more appropriately situated in a math-club environment, among the already intrinsically motivated. The RS would find little appeal in this approach. The students in my class were unimpressed by the simpler Sieve of Eratosthenes and had not yet reached a level of comfort with arithmetical calculations that would have allowed them to notice patterns, i.e. it is not optimally challenging. The "interesting" result that after following each of the steps, the students all arrive at the same number (regardless of the initial number chosen) would offer little of a practical nature for students who were exerting their efforts towards passing the course (performance goals). The author suggests that this method will "empower students to create new puzzles or games of their own" (pg. 31). An algebra class at the high school level would include students who might be inspired to do so but there would be considerably fewer students in a basic algebra class at the university level; this approach does little to foster motivation where none already exists.

As a second example, retrieved also from NCTM (National Council of Teachers of Mathematics, 2012), Diette and Howe (2003) describe activities that have been used in a "first-year-experience course" in college algebra. One of these is to, "Write a research report on the career of a contemporary mathematician or a mathematician from an underrepresented group." Another is, "[Investigate] what kinds of careers are available to graduates who have a substantial mathematics background" (pg. 278-279). The activities were offered for extra credit or as part of the course grade. At the end of the course, students were asked how much their motivation for mathematics had increased because of the activities and other components of instruction (using a questionnaire). The authors found that activities that were directly related to careers were the most popular.

These activities do address weakly motivated students by encouraging a valuing of mathematics and the authors appear to have considered the population of students – students enrolled in a basic algebra course at the university level (which is similar to the population considered in this research). The authors assume that their students deny the value of mathematics because it involves hard work (my students worked very hard) and assume that their students need to convince themselves that mathematics is useful (this value was found to be already established in this study). Researching career opportunities in mathematics (or careers that require "substantial" mathematics) is only motivating if the career is one that the student sees as attainable. Finding out that an actuary makes a large salary may only be motivating to students who want to be, or can see themselves actually becoming, an actuary. My finding was that students had already established their career and degree goals and my observation was that they already knew exactly how much mathematics they would have to take to attain their chosen degree. The students in my class might have done these activities if grades were attached and probably exceedingly well; the activities (e.g. writing a research paper) address their competence in other fields. Researching a contemporary mathematician though, might be motivating to an instructor of mathematics but it is unlikely to be motivating to a student who is afraid they might fail the course (performance-avoidance goals). This student is preoccupied with learning the material sufficiently well to pass the tests and has little time or motivation for activities that do not immediately relate to these goals.

As a final example, Kim and Hodges (2011) designed and implemented an "emotion control treatment" for students taking an introductory, online, mathematics course. The authors found an increase in positive emotions, and hence motivation, among the students who received the treatment compared to a control group. Theoretically, this approach has merit in that it attempts to improve students' self-regulatory mechanisms. Their finding that students already equipped with positive emotions could experience an increase because of a treatment, suggests that even these students might have deficiencies in self-regulation, i.e. some students might be experiencing difficulties adequately processing their emotions. This may be the case but the effect of such a treatment on students (especially mature students) who might have fully intact, healthy and productive self-regulatory mechanisms is not noted.

7.4 Implications for Teaching Motivationally

In this research, I have attempted to avoid the pitfall of assuming that the students in my class would be motivated in the same way that I am motivated. To do so, I described the HS and this portrayal allowed me to consider students who were not *already motivated*. As the portrayal evolved during the course and was further refined in the description of the RS, I was able to address the *students'* needs for autonomy, competence and relatedness. Within a positive relationship, I promoted practice and understanding as a means to do well on the tests. With performance goals, this was of utmost importance to this group of students.

I had created the first online quiz because of my assumption that the HS would not be highly motivated to practice by doing the suggested problems in the text. Lucock (as cited in Middleton & Spanias, 1999) found that the motivation of students who did enjoy mathematics tended to plummet when they were confronted with routine, skills-related mathematical tasks. I had looked at the long list of problems and thought they would be boring for the students. What I had overlooked is that what might be boring for me was not necessarily the case for the students. Optimal challenge imparts interest in the activity as the students experience flow. The feedback of a sense of improvement then sustains the activity. Many students chose this method of studying and appeared to be well motivated to do the suggested problems.

The online quizzes were also optimally challenging, the students found them helpful and their confidence in their abilities increased. The quizzes more immediately addressed the students' performance goals (of doing well on the tests) because they were a review of the concepts that could appear on the tests. The quizzes offered more questions and more variety in the questions than the tests because, although the students were motivated largely by grades, *my* goal was that they become competent with the entire curriculum. The quizzes promoted persistence by offering multiple attempts and they provided the structure (my expectations of what they should be capable of) that is necessary for the feedback of their results to be effective. Although designed for the hypothetical student, the quizzes were well-suited for the actual students in this course.

The net result of the course was a positive change in the affect of the students towards mathematics. With a myriad of variables introduced for a group of diverse students, it is difficult to determine precisely the elements that lead to this change. It is sufficient for this study that the conditions were established that allowed for a positive change. To conclude that this course was responsible for generating an interest in mathematics though would be excessive. Middleton and Spanias (1999) assert that an interest in mathematics is a motivational attitude that is very stable over time. Furthermore, interest in mathematics is established at the elementary level and it generally decreases over time. To encourage an interest in mathematics among a group of adult students then is a formidable task. Interest does increase with perceptions of ability (Middleton & Spanias, 1999; Reeve, 2005) but many iterations of success are needed for this group of students to be able to entertain the possibility that mathematics is indeed interesting. This course did provide one instance of success for a majority of the students.

7.4 Recommendations

First and foremost, this study supports a recommendation that it is necessary to consider the population of students before attempting to motivate them and perhaps this should be a consideration before attempting to teach. Consideration of the population need not be as structured as it is here (in the constructs of the HS and the RS) but it is my contention that it is vital to designing motivational teaching strategies. Differing populations of students (e.g. business students, mathematics students) may present differing profiles and instruction can be tailored accordingly. Recommendations for teaching then are based on the profile of the realistic student.

Many of the students had set performance-avoidance goals and were ego-defensive. These students are motivated partly by fear so they are primarily concerned with developing the skills that they need to "survive" the course. These students need regular support. To develop confidence, mini online quizzes that were offered on a weekly basis would be appropriate. These should be optimally challenging and I would suggest unlimited attempts but with a small amount of credit offered for attaining a perfect score to encourage persistence. Weekly feedback about their improving abilities supports an incremental viewpoint.

Showing options for a solution to an algebraic problem is also important for students with performance goals as it allows the students to choose an approach that best fits with their developing understanding and it too supports confidence (Galligan & Taylor, 2008). Promoting an understanding (as a way to perform well on the tests) is important as this understanding is a precursor to being able to 'own' success in mathematics.

Students who have set mastery goals already have some intrinsic motivation for mathematics. It is this group that might be encouraged by activities that promote an interest in mathematics. Questions that highlight curiosities in mathematics are also appropriate for this group to avoid decreasing motivation. These could be offered as bonus questions or as an alternative to the weekly online quizzes, for the same amount of credit.

The above are teaching recommendations that were (or might have been) appropriate for this particular group of students. Students who have experienced difficulties in mathematics must be allowed the experience of developing their abilities, improved confidence and at least some success *before* a change in attitudes is even possible. If the environment also supports improved affect towards mathematics, *then* intrinsic motivation can begin to develop. Encouraging intrinsic motivation is worthwhile since many of these students will still need to take other mathematics courses and finding mathematics interesting will help them derive sufficient motivation when the course material is more difficult.

Since an interest in mathematics is developed early (Middleton & Spanias, 1999), addressing a lack of interest among the general population is perhaps an intervention that is best situated in a high school environment. Mathematics-appreciation courses or activities incorporated in the curriculum, have the potential to address a declining trend early enough to be beneficial. Mathematics-appreciation courses at the university level, open to the student body at large, have the more difficult task of reversing an already established trend but may also have merit.

7.5 Conclusions

The conjecture that was examined in this research was that motivation would be lacking in a basic algebra class at the university level but that a classroom could be structured that was motivating and that this would result in improved affect towards mathematics. Motivation was not found to be lacking; the students were quite strongly motivated to study. It was however, extrinsic in nature and related to performance goals. It was intrinsic motivation that was in short supply. The course was structured in a manner that was motivating to students and a positive change in affect towards mathematics was observed.

The interventions that I used were easy to implement. The teaching style that I adopted was one that supported autonomy, addressed competence and established a positive relationship. The posture that I adopted was one that preserved the dignity of the students. The only other intervention that I used was the online quizzes. None of these require any extensive preparation or special knowledge. I did strive to impart an understanding of algebra and, using the construct of the hypothetical student (HS), I considered the audience before starting to teach.

The realistic student (RS) profile better describes the motivational characteristics of this population of students. It remains to be determined: (1) if this set of characteristics can be further refined, (2) whether or not this profile distinguishes this population of students from other groups and (3) the specific teaching strategies or interventions that are effective for this group.

References

Anderman, E. (2010). Reflections on Wittrock's Generative Model of Learning: A Motivational Perspective. *Educational Psychologist* , 45(1), 55-60.

Barbeau, E. (1995). *After Math.* Toronto: Best Book Manufacturers Inc.

Bednarz, N., Kieran, C., & Lee, L. (1996). Chapter 1. Introduction. In *Approaches to Algebra: Perspectives for Research and Teaching* (pp. 3-12). Netherlands: Kluwer Academic Publishers.

Concordia University. (2011-2012). *Undergraduate Calendar.* Retrieved 2011, from http://registrar.concordia.ca/calendar/

Confrey, J., & Lachance, A. (2000). Chapter 10, Transformative Teaching Experiments Through Conjecture-Driven Research Design. In A. Kelly, & R. Lesh, *Handbook of Research Design in Mathematics and Science Education* (pp. 231-267). Mahwah, New Jersey: Lawrence Erlbaum Associates, Publishers.

DeBellis, V., & Goldin, G. (2006). Affect and Meta-Affect in Mathematical Problem Solving: A representational perspective. *Educational Studies in Mathematics, A PME Special Issue: Affect in Mathematics Education: Exploring Theoretical Frameworks* , 63; 131-147.

Diette, J., & Howe, M. (2003). Motivating students to study mathematics. *Mathematics Teacher* , 96(4), 278-280.

Fischbein, E. J. (1994). The irrational numbers and the corresponding eqistemological obstacles. *18th International Conference of the International Group for the Psychology of Mathematics Education Vol. 2.*, (pp. 352-367). Lisbon: PME.

Galligan, L., & Taylor, J. (2008). Adults Returning to Study Mathematics. In H. Forgasz, A. Barkatsas, A. Bishop, B. Clarke, S. Keast, W. Seah, et al., *(eds.) Research in*

Mathematics Education in Australasia 2004-2007 (pp. 99-118). Rotterdam: Sense Publishers.

Hannula, M. (2006). Motivation in Mathematics: Goals Reflected in Emotions. *Educational Studies in Mathematics, A PME Special Issue: Affect in Mathematics Education: Exploring Theoretical Frameworks* , 63; 165-178.

Kelecsenyi, K. (2009). *Popularization of Mathematics as Intercultural Communication - An Exploratory Study.* PhD Thesis, Montreal .

Kim, C., & Hodges, C. B. (2011). Effects of an emotion control treatment on academic emotions, motivation and achievement in an online mathematics course. *Instructional Science* , 40 (1), 173-192.

Kuncel, N., Crede, M., & Thomas, L. (2005). The validity of self-reported grade point averages, class ranks, and test scores: A meta-analysis and review of the literature. *Review of Educational Research* , 75(1), 63-82.

Larson, R., & Nolting, K. (2010). *Elementary Algebra 5th edition.* Belmont: Brooks/Cole.

Leitze, A. R., & Kitt, N. A. (2000). Using Homemade Algebra Tiles to Develop Algebra and Prealgebra Concepts. *Mathematics Teacher* , v93, n6, 462-466.

LimeSurvey. (n.d.). *LimeService*. Retrieved June 2011, from https://www.limeservice.com/

Luo, W., Paris, S., Hogan, D., & Luo, Z. (2011). Do performance goals promote learning? A pattern analysis of Singapore students' achievement goals. *Contemporary Educational Psychology* , 36; 165-176.

Menon, R. (2004). Motivating Activities that Lead to Algebra. *Mathematics Teacher* , 98(1), 26-31.

Merriam-Webster Inc. . (2010). *Merriam-Webster Dictionary*. Retrieved from http://www.merriam-webster.com/

Middleton, J. A., & Spanias, P. A. (1999). Motivation for Achievement in Mathematics: Findings, Generalizations, and Criticisms of the Research. *Journal for Research in Mathematics Education* , Vol. 30, No. 1 (Jan. 1999), pp.65-88.

Miller, N. (2000). *Perceptions of Motivation in Developmental Mathematics Students: I Would Rather Drill My Own Teeth.* Austin: Doctoral Dissertation, University of Texas.

National Council of Teachers of Mathematics. (2012). *Motivation Resources.* Retrieved January 31, 2012, from NCTM: http://www.nctm.org/resources/content.aspx?id=16479

Reeve, J. (2005). *Understanding Motivation and Emotion, 4th ed.* Hoboken: John Wiley & Sons, Inc.

Rojano, T. (2002). Mathematics Learning in the Junior Secondary School: Students'Access to Significant Mathematical Ideas. In L. English, *Handbook of International Research in Mathematics Education* (pp. 143-163). Mahwah: Lawrence Erlbaum Associates, Publishers.

Schoenfeld, A. (2000). Purposes and Methods of Research in Mathematics Education. *Notices of the American Mathematical Society* , Volume 47, #6; 641-649.

Sierpinska, A. (2006). Sources of students' frustration in bridging mathematics courses, Vol. 5. *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education*, (pp. 121-129). Prague, Czech Republic.

Sierpinska, A. (1994). *Understanding in Mathematics.* Bristol, PA: The Falmer Press.

Sutherland, R., & Rojano, T. (1993). A spreadsheet approach to solving algebra problems. *Journal of Mathematical Behavior* , 12, 353-383.

Tabach, M. (2006). Research and teaching--Can one person do both? A Case study. *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Vol.2* (pp. 233-240). Prague, Czech Republic: PME.

Verschaffel, L. G. (2006). Numerical Thinking. *Handbook of Research on the Psychology of Mathematics Education, Past, Present and Future* , 51-82.

Vygotsky, L. (1978). *Mind in Society.* London: Harvard University Press.

Weiner, B. (1985). An attributional theory of achievement motivation and emotion. *Psychological Review,* , 92, 548-573.

Yates, S. M. (2009). Teacher Identification of Student Learned Helplessness in Mathematics. *Mathematics Education Research Journal* , Vol. 21, No. 3, 86-106.

Yates, S. M. (2002). The Influence of Optimism and Pessimism on Student Achievement in Mathematics. *Mathematics Education Research Journal* , Vol. 14, No. 1, 4-15.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in Mathematics Education: An introduction. *Educational Studies in Mathematics, A PME Special Issue: Affect in Mathematics Education: Exploring Theoretical Frameworks* , 63; 113-121.

Appendix A: Course Outline

Department of Mathematics & Statistics
Concordia University

MATH 200
Fundamental Concepts of Algebra
Summer 2011

Instructor*: ______________________

Office/Tel No.: ______________________

Office Hours: ______________________

*Students should get the above information from their instructor during class time. The instructor is the person to contact should there be any questions about the course.

Course Examiner: Dr. C. Cummins

Text: *Elementary Algebra*, 5th Edition, Larson/Hostetler (Brooks Cole).

Credit: This is an introductory course in Algebra. Students with credits for any Concordia Math course will not receive credit for this course.

Math Help Centre: The Centre has been organized to help students in solving problems. A schedule of its operation and its location will be posted in the Department and on the math department web page: www.mathstat.concordia.ca.

Assignments: Are very important, even if they carry no credit and are not to be handed in. They indicate the level of difficulty of the problems which the student is expected to solve, so every effort should be made to do them.

Calculators: Only calculators approved by the Department are permitted in the class tests and final examination. The calculators are the **Sharp EL 531** and the **Casio FX 300MS**, available at the Concordia Bookstore.

Tests: Two one hour tests will be given during the course.
Test one: covers Lectures 1 to 5 inclusive.
PLEASE NOTE: For test 1 only, calculators are NOT permitted.
Test two: covers Lectures 6 to 9 inclusive.
Dates of these tests will be provided by your instructor.
PLEASE NOTE: It is the Department's policy that tests missed for any reason, ***including illness***, cannot be made up if the 100% final exam option is offered in the Final Grade (see below).

Final Exam: The final examination will be three hours long.
PLEASE NOTE: Students are responsible for finding out the date and time of the final exams once the schedule is posted by the Examinations Office. Any conflicts or problems with the scheduling of the final exam must be reported directly to the Examinations Office, ***not*** to your instructor. It is the Department's policy and the Examinations Office's policy ***that students are to be available until the end of the final exam period. Conflicts due to travel plans will not be accommodated.***

Final Grade: The final grade will be based on the higher of (a) or (b) below. A student who misses a test is on system (a).
a) The final examination (100%);
b) The weighted average of the final exam for 60%, with class tests for 40%.

Departmental website → http://www.mathstat.concordia.ca

Lectures	Sections	Assignments	
1	Chapter 1 Real Numbers	1.1 1.2 1.3	1, 3, 9, 13, 15, 35, 39, 43, 49, 51, 69, 71, 73 1-75 (odd numbers) 1-57 (odd numbers)
2		1.4 1.5	1-65 (odd numbers) 1-71 (odd numbers) review exercises (as many as possible)
3	Chapter 2 Fundamentals of Algebra	2.1 2.2 2.3 2.4	1-85 (odd numbers) 35-61 (odd numbers), 73-87 (odd numbers), 99-135 (odd numbers) 1-60 (odd numbers) 1-33 (odd numbers)
4	Chapter 3 Linear Equations	3.1 3.2 3.3	1-41 (odd numbers), 71-79 (odd numbers) 1-65 (odd numbers) 1-69 (odd numbers), 79-85 (odd numbers)
5		3.4 3.5 3.6	1-61 (odd numbers) 1-41 (odd numbers) 1-71 (odd numbers)
6	Chapter 4 Equations & Inequalities	4.1 4.2	1-25 (odd numbers), 35-39 (odd numbers) 51-57 (odd numbers) 1-15 (odd numbers), 25-35 (odd numbers)
		4.3 4.4	1-87 (odd numbers) 1-75 (odd numbers)
	Chapter 5 Exponents & Polynomials	5.1 5.2 5.3 5.4	1-147 (odd numbers) 31-65 (odd numbers), 99 & 101 1-99 (odd numbers) 1-51 (odd numbers)
7	Chapter 6 Factoring	6.1 6.2 6.3	1-79 (odd numbers), 101, 103 1-59(odd numbers) 1-71 (odd numbers)
8		6.4 6.5	1-59 (odd numbers) 67, 69 1-43 (odd numbers) 53-57 (odd numbers)
9	Chapter 7 Rational Expressions & Equations	7.1 7.2 7.3 7.4 7.5	41-89 (odd numbers) 1-81 (odd numbers) 15-79 (odd numbers) 1-35 (odd numbers) 1-63 (odd numbers)
10	Chapter 8 Systems of Linear Equations	8.1 8.2	1-25 (odd numbers) 11-45 (odd numbers)
11		8.3 8.4	5-61 (odd numbers) 1-57 (odd numbers) (omit 49)
12	Chapter 9 Roots & Radicals	9.1 9.2 9.3	1-57 (odd numbers) 1-103 (odd numbers) 1-119 (odd numbers)
13	**REVIEW**		

Appendix B: Questionnaires

B.1 Questionnaire for the Online Quiz - MATH 200-Summer session 2011

1) **I used the online quiz** a. A lot b. A little c. Not at all

2) **The quiz was helpful,** a. A lot b. Somewhat c. Not at all

3) **The questions were…** a. Too hard b. Just right c. Too easy

4) **The quiz was…** a. Too long b. Too short c. Just right

5) **The Moodle system was…** a. Easy to use b. Difficult (or frustrating) to use

6) **Studying habits (Choose the answer that applies to you)**

   a. I did a lot of other studying for the midterm

   b. This was the only studying that I did for the midterm

   c. I did some other studying for the midterm

   d. I did not use the online quiz for studying

7) **I probably studied ________ because of the online quiz.**

   a. More b. Less c. About the same

8) **I would use an online quiz to prepare for the next midterm** a. Yes b. No

9) **I would use an online quiz to do homework problems** a. Yes b. No

10) **If an online quiz was available, I would not need to do problems from the text book.**

    a. Yes b. No

<u>Comments or Suggestions</u>

____

B.2 Course Evaluation Form

Concordia University

PART-TIME FACULTY COURSE EVALUATION

Concordia University and the Concordia University Part-time Faculty Association (CUPFA) agree that the purpose of evaluating teaching is the improvement of instruction. Your instructor is participating in this effort by distributing a multipurpose questionnaire designed to gather your perspectives on several dimensions of the course you have taken: teaching, course materials and design, content, and learning. Results will be reported to your instructor and your Department Chair in the form of statistics, after final grades are reported. Any comments you write will be typed and given to your instructor only.

For each of the following statements and questions, please mark the response that most closely expresses your opinion; leave blank if you feel that no response applies.

Marking Instructions

- **Use black or blue pen or a HB pencil.**
- **Make dark marks that fill the circles completely.**
- **Do not use pens with ink that soaks through the paper.**
- **Make no stray marks.**

Incorrect Marks ⊘ ⊗ ◒ ⊙
Correct Mark ●

OVERALL RATINGS

	VERY GOOD	GOOD	FAIR	POOR	VERY POOR
1. Overall, this course has been...	①	②	③	④	⑤
2. Overall, the instructor has been...	①	②	③	④	⑤
3. Overall, my learning has been...	①	②	③	④	⑤

COURSE ORGANIZATION & CONTENT RATINGS

	STRONGLY AGREE	AGREE	NEITHER AGREE NOR DISAGREE	DISAGREE	STRONGLY DISAGREE
4. Course outline and syllabus are clear, complete and well explained.	①	②	③	④	⑤
5. Course materials, text book or readings are useful or relevant.	①	②	③	④	⑤
6. I have found this course intellectually challenging and stimulating.	①	②	③	④	⑤
7. The course met the objectives as stated in the course outline.	①	②	③	④	⑤

INSTRUCTOR RATINGS

	STRONGLY AGREE	AGREE	NEITHER AGREE NOR DISAGREE	DISAGREE	STRONGLY DISAGREE
8. Instructor demonstrates a comprehensive knowledge of the subject matter.	①	②	③	④	⑤
9. The instructor's explanations are clear.	①	②	③	④	⑤
10. The instructor provides feedback in the form of exams and/or assignment grading.	①	②	③	④	⑤
11. Students are encouraged to ask questions.	①	②	③	④	⑤
12. Students are encouraged to share their ideas and knowledge.	①	②	③	④	⑤
13. The instructor is approachable.	①	②	③	④	⑤

STUDENT SELF-RATINGS

	WELL ABOVE AVERAGE	ABOVE AVERAGE	AVERAGE	BELOW AVERAGE	WELL BELOW AVERAGE
14. Overall, my involvement in this course has been...	①	②	③	④	⑤

STUDENT SELF-RATINGS

	VERY HIGH	HIGH	AVERAGE	LOW	VERY LOW
15. What was your level of knowledge of the subject prior to taking this course?	①	②	③	④	⑤
16. What was your level of interest in the subject prior to taking this course?	①	②	③	④	⑤

	REQUIRED FOR MAJOR OR SPECIALIZATION	ELECTIVE FOR MAJOR OR SPECIALIZATION	ELECTIVE OR MINOR	GENERAL INTEREST	FIT INTO SCHEDULE
17. What was your reason for taking the course?	①	②	③	④	⑤

18. How do you find the facilities for this course, (e.g. air, seating, temperature, class size, etc.)?

19. What suggestions or comments do you want to give to your instructor about this course?

SUPPLEMENTAL QUESTIONS: Additional questions may be added by the Instructor, Department or Faculty.

20. ○○○○○ 21. ○○○○○ 22. ○○○○○ 23. ○○○○○ 24. ○○○○○

Appendix C: Survey Questions

A star indicates that a question is mandatory.

Part I. Personal Information

*** Enter your name:**

First Name: ____

Last Name: ____

***I am over 25 years of age**

○ Yes ○ No

Math 200 is the first math course I have taken since:

Enter the year:

____

I have been attending Concordia for:

Enter the number of years:

____

Only numbers may be entered in this field

My degree program (major) or concentration of study is:

____

***I am a full time student.**

○ Yes ○ No

***I had many commitments other than this course.**
Choose one of the following answers

○ Agree
○ Disagree
○ Neutral (Neither agree nor disagree)

*Going to university is a scary experience.
Choose one of the following answers

- Agree
- Disagree
- Neutral

Part II. Mathematics Background

*In high school I took:
Check any that apply

- Regular math
- Mid-stream math
- Enriched math
- Other:

*Regular math in Quebec would have been Math 416/516.
Mid-stream math was Math 426/526.
Enriched math was Math 436/536.
If you don't know, or did not take math in Quebec, check 'Other' and explain in the box provided.*

*Some of my math teachers in high school were good.
Choose one of the following answers

- Agree
- Disagree
- Neutral (Neither agree nor disagree)

*I took one or more math courses at CEGEP.

- Yes
- No

*My marks in math before I came to Concordia were usually:
Choose one of the following answers

- 75 - 100%
- 60 - 74%
- Less than 60%

I failed one or more math courses before I took this course.
Choose one of the following answers

- ○ Yes
- ○ No
- ◉ No answer

Please enter your comment here:

Enter a comment if you wish to explain. (Optional)

When I did well at math before taking Math 200 it was because:

When I had difficulties with math before taking Math 200 it was because:

***My math background before this course was:**
Choose one of the following answers

- ○ Good
- ○ Fair
- ○ Poor

Please enter your comment here:

Enter a comment if you wish to explain. (Optional)

***I learn best on my own.**
Choose one of the following answers

- ○ Agree
- ○ Disagree
- ○ Neutral

***I like to study with friends.**
Choose one of the following answers

- Agree
- Disagree
- Neutral

***I like to help other people to understand math.**
Choose one of the following answers

- Agree
- Disagree
- Neutral

*** I feel good about myself when I solve a math problem.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***List, in order of importance, the reasons that you study or work hard at math.**
Click on an item in the list on the left, starting with your highest ranking item, moving through to your lowest ranking item.

Your choices:

To please my family, friends or teachers
Because I don't want to fail
To understand more about mathematics
Because math will help me in other courses
Because math will help me get a better job

Your ranking:

1:
2:
3:
4:
5:

Click on the scissors next to each item on the right to remove the last entry in your ranked list

Additional comments about your math background.

Part III. Goals

My career goals are:

My academic goals are: (List any diplomas or degrees that you hope to obtain or that you will need to obtain in order to pursue your chosen career.)

***A university degree has always been a dream of mine.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***I took Math 200 as a(n):**

Choose one of the following answers

- Required course
- Prerequisite
- Elective
- Other:

***I would have taken this course for interest even if it was not required.**

Choose one of the following answers

- Yes
- No

This question only appears if "Required course' or 'Prerequisite' is chosen in the previous question.

I took this course because I wanted to:

? *Explain your reasons for taking this course and/or what you hoped to accomplish.*

***This is the first time I have registered for Math 200.**

○ Yes ○ No

Please indicate which situation applies to you.
Check any that apply

☐ I registered before and dropped the course.

☐ I completed the course and did not pass.

☐ I passed but I want to improve my mark.

? *Use the text boxes if you wish to explain. (Optional)*

This question only appears if 'No' is selected for the previous question.

***When I started this course I thought my mark would probably be:**
Choose one of the following answers

○ 75 - 100%
○ 50 - 74%
○ Less than 50%
○ I didn't think about what mark I would get.

***I was very nervous about passing this course when I started.**
Choose one of the following answers

○ Agree
○ Disagree
○ Neutral

***Overall, I achieved my goals for this course.**
Choose one of the following answers

○ I achieved or exceeded my goals
○ I achieved some of my goals
○ I did not achieve my goals

Please enter your comment here:

? Explain why you met or did not meet your goals in the box provided.

Additional comments about your goals:

Part IV. Study Habits

***I attended class:**
Choose one of the following answers

- Often
- Sometimes
- Rarely

Reasons for absence from class:

This question appears if the options 'Sometimes' or 'Rarely' are selected for the previous question.

***In class it was hard to pay attention.**
Choose one of the following answers

- Agree
- Disagree
- Neutral

Please explain why it was difficult to pay attention in class.

This question appears only if the option 'Agree' is selected for the previous question.

*Outside the class, I studied:
Choose one of the following answers

- A lot
- Somewhat
- Very little

* I studied on a regular basis:

- Yes
- No

*I studied regularily:
Choose one of the following answers

- Almost every day
- 3 - 5 times a week
- Once or twice a week

This question and the next appear only if the option 'Yes' is selected for the previous question.

*Each time I studied it was for approximately:
Choose one of the following answers

- More than 2 hours
- 1 - 2 hours
- Less than 1 hour

The approximate number of hours that I studied for the tests:

Only numbers may be entered in these fields

Approximate number of hours I studied for Midterm 1

Approximate number of hours I studied for Midterm 2

Approximate number of hours I studied for the final exam

? *If you did not study, enter zero.*

*I did suggested problems from the course outline:
Choose one of the following answers

- Most of the problems
- Some of the problems
- Very few of the problems

Doing the suggested problems from the text book was:
Check any that apply

- ☐ Boring
- ☐ Very hard
- ☐ A good way to learn
- ☐ A good way to practice skills
- ☐ Necessary to pass the course

Use the text boxes if you wish to make a comment. (Optional)

***I did extra problems from the text, even if they weren't on the course out**

◯ Yes ◯ No

***I read the explanations in the text book:**
Choose one of the following answers

- ◯ Often
- ◯ Sometimes
- ◯ Rarely

***The text book for this course was awful.**
Choose one of the following answers

- ◯ Agree
- ◯ Disagree
- ◯ Neutral

***I used one or more of the online quizzes on the moodle to study.**

◯ Yes ◯ No

The next 4 questions appear only if 'Yes' is selected for this question.

***Choose one or more of the answers that best describe the reasons why you did the online quizzes.**

Check any that apply

- ☐ To find out what would be on the tests
- ☐ To practice problems
- ☐ To check if I was ready for the tests
- ☐ To get feedback about my skills
- ☐ Other:

***The online quizzes boosted my confidence in my abilities.**
Choose one of the following answers

- Agree
- Disagree
- Neutral

***The online quizzes increased my anxiety about the tests.**
Choose one of the following answers

- Agree
- Disagree
- Neutral

Additional comments about the quizzes:

***I visited the MathHelp Center**
Choose one of the following answers

- More than once
- Once
- Never

The MathHelp Center was helpful to me.
Choose one of the following answers

- A lot
- Somewhat
- Not at all
- No answer

Please enter your comment here:

Please enter any comments you have about the MathHelp Center in the box provided. (Optional)

This question appears only if the options 'More than once' or 'Once' are selected for the previous question.

***I went to a tutor to help me through the course.**

Yes No

The tutor was helpful.
Choose one of the following answers

- A lot
- Somewhat
- Not at all
- No answer

Please enter your comment here:

Please enter any comments you have about the tutor in the box provided. (Optional)

This question appears only if the option 'Yes' is selected for the previous question.

***I attempted one or more of the practice exams.**

Yes No

Additional comments about your study habits. Were your study habits effective? Would you change your study habits if you took another math course?

Part V. Your Opinions about Mathematics

*** BEFORE the course, I liked math.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

*** AFTER the course, I like math:**

Choose one of the following answers

- MORE
- LESS
- About the same

* **BEFORE the course, Algebra was too abstract for me.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

* AFTER **the course, Algebra is:**

Choose one of the following answers

- MORE abstract
- LESS abstract
- About the same

* **BEFORE the course, math was difficult for me.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

* AFTER **the course, math is:**

Choose one of the following answers

- MORE difficult
- LESS difficult
- About the same

* **BEFORE the course, I usually understood what was presented in class.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***For this course, I understood what was presented in class:**

Choose one of the following answers

- MORE
- LESS
- About the same

* BEFORE this course I felt that: Math was for some people but not for n

Choose one of the following answers

- Agree
- Disagree
- Neutral

* **AFTER this course I feel that math is:**

Choose one of the following answers

- MORE for me
- LESS for me
- About the same

* **BEFORE this course I felt that: Math is a complete waste of time unless it i specifically geared towards your career.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

* **AFTER this course I feel that: Math is a complete waste of time unless it specifically geared towards your career.**

Choose one of the following answers

- Agree MORE with the statement
- Agree LESS with the statement
- Agree about the same

* **BEFORE this course, I was not good at writing math tests.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

* AFTER **the course, I am:**

Choose one of the following answers

- BETTER at writing math tests
- WORSE at writing math tests
- About the same

* **BEFORE this course, math sometimes made me feel stupid.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***AFTER the course, math makes me feel stupid:**

Choose one of the following answers

- MORE often
- LESS often
- About the same

* **BEFORE this course, math sometimes was fun.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***AFTER the course, I think that math is fun:**

Choose one of the following answers

- MORE often
- LESS often
- About the same

* **BEFORE this course, I thought that I could succeed in math.**

Choose one of the following answers

- Agree
- Disagree
- Neutral

***AFTER the course, I feel that I can succeed in math:**

Choose one of the following answers

- MORE strongly
- LESS strongly
- About the same

A text box for comments was provided with each question of the type 'AFTER the course...'

Part VI. Additional Information

The first 5 questions were displayed on the survey with long text boxes for replies.

What do you do when you get stuck on a math problem?

How do you convince yourself to study, even when you have better things to do?

List what was most helpful about this course:

List anything that would have helped you do better in this course

Any other comments or suggestions:

May I contact you again for additional information?

○ Yes ○ No ◉ No answer

Appendix D: Summary of Survey Responses

The survey was presented in five parts and each question was given a 3-digit question code. All survey questions are prefaced with the letter ‘Q’. The first digit of the code represents the number of the survey part and the remaining 2 digits represent the number of the question. Thus, the summary of responses for Q415 can be found in Part IV, and this is the 15th question in this part of the survey. If a letter follows a question code the question is dependent upon the response to the main question. Q304a only appeared for selected responses to Q304.

A star (*) next to a question indicates that the count has been revised. If a student selected ‘Neutral’ as a response and gave sufficient information in a comment to indicate their intentions, the count was adjusted. A plus (+) next to a question indicates that comments were recorded. Comments are reproduced with the original spelling and grammar.

The results for the following questions were interpreted so as to provide numerical data:

- Q103 asks the student to enter the year of the last math course they had taken. Results were tabulated for “more than 2 years” and “2 years or less”.
- Q 104 asks the students for the number of years they have been attending Concordia. Responses were tabulated for: “2 or more years” or “One year or less”.
- Q105 asks “My degree program (major) or concentration of study is:” Responses were sorted into the categories: “Math intensive major” or “Not math intensive major”.
- Q301 is: “My career goals are:” A count was made of the number of students who had set a long-term career goal.
- Q302 is: “My academic goals are:” Responses were sorted into the categories: “Graduate degree goal” or “Undergraduate degree goal”.

- Q404a and 404b, asks students, who reported that they studied on a regular basis (12 students), how often they studied and for how much each time. A rough chart was used to approximate the total number of studying hours per week.

For Part V, Your Opinions about Mathematics, the students were asked to either 'Agree' or 'Disagree' with a statement that reflected a dimension of affect towards mathematics that they held before the course. Responses were assigned a value of +1 if it reflected a positive attitude towards mathematics, a value of -1 if it reflected a negative attitude and a value of 0 was given to a response of 'Neutral'. The sum of these values is given as 'Net affect' – (meaning net dimension of affect) in Table D.13. Each of these questions had a pair in which the students were asked if this attitude had changed after the course. Again a positive change was given a value of +1, a negative change, a value of -1 and a value of 0 was given if there was no change. The sum of these values is given in Table D.13 as 'Net change'. A summary of 'Net Affect' before the course and 'Net change' are given in Table D.14.

Part I: Personal Background

Table D.1 Questions 102 - 108

Question #	Text of the question	Response	Count
Q102	I am over 25 years of age	Yes: 7 No: 8	
Q103	Math 200 is the first math course I have taken since: Enter the year	More than 2 years 2 years or less No answer:	11 3 1
Q104	I have been attending Concordia for: Enter the number of years	2 or more years: 1 year or less: No answer:	7 7 1
Q105	+My degree program (major) or concentration of study is:	Math intensive: Not math intensive: No answer	2 12 1
+Civil Engineering and Finance were considered math intensive. Psychology, Applied Human Sciences, Philosophy, Graphic Applications, Political Science, Linguistics, Classics & Arts & Sciences were considered to be 'not math intensive'.			

Q106	I am a full time student	Yes: 8 No: 7	
Q107	*I had many commitments other than this course.	Agree: Disagree: Neutral:	11 3 1
*Comments with response 'Neutral' Comment 1: "equal commitments" – No inference is drawn from this comment. Comment 2: "I worked two jobs, have a family to support" – Represents many commitments and will be assumed to be 'Agree'.			
Q108	*Going to university is a scary experience	Agree: Disagree: Neutral:	2 12 1
*Comments with response 'Neutral' Comment 1: "it can be initially" – Agree Comment 2: "I do not want it to be a waisted experience my only fear" – Disagree Comment 3: "It depends on the individual, how prepared they are and how they manage their time, etc..." – No inference			

Part II: Mathematics Background

Table D.2 Q201 - Q205

Question #	Text of the question	Response	Count
Q201	*In high school I took:	Regular math only: Mid-stream math only: Enriched math only: All of the above	9 1 3 2
*Comment with 'Other': "IB math" – This is an enriched math stream.			
Q202	Some of my math teachers were good.	Agree: Disagree: Neutral:	13 2 0
Q203	I took one or more math courses at CEGEP.	Yes: 2 No: 13	
Q204	My marks in math before I came to Concordia were usually:	75 – 100% 60 – 74% Less than 60%	7 6 2
Q205	+I failed one or more math courses before I took this course.	Yes: 6 No: 8 No answer: 1	
+Comment with response 'Yes': "During high school, Math was my weakest subject."			

Table D.3 Q206 & Q207 (These are paired for comparison)

Q206: When I did well at math before taking Math 200 it was because:	Q207: When I had difficulties with math before taking Math 200 it was because:
prerequisites	prerequisites
only because of practice, and an approachable instructor	lack of practice, i know this might seem like an obvious and generic answer, but its the trut
I studied a lot.	The teacher wasn't good and was very strict and mean.
	I did not study enough
The course was at a slower pace and tutorial sessions allowed students to better grasp the concepts.	There was too little time and I had other obligations to fulfull making it difficult to manage my time efficiently.
Having a teacher or tutor being able to properly explain something, as well as practice.	Not being able to grasp the concepts due to lack of explanation or lack of attention of my behalf.
i had someone to study with me, or someone pushing me to study. I also had many tutors.	I had no motivation, i did not care to do well.
The teacher was good, My father helped me to study	Lack of motivation
	I find the processes very complicated.
was my first course at Concordia	i was taking higher courses before taking the beginners courses
Good teachers and study habits	Bad teacher, lack of personal motivation
Really did not do well in high school.	I did not spend the time required in order to succeed. I had failed some courses and felt that I was very bad at math. The feeling is still there!

Table D.4 Q208 - Q214

Question #	Text of the question	Response	Count
Q208	+My math background before this course was:	Good Fair Poor	3 9 3
+Comment with response 'Good': "I took math 536 in high school."			
Q209	*I learn best on my own.	Agree: Disagree: Neutral:	7 6 2
*Comments with response 'Neutral' (The purpose of this question was to evaluate autonomy.) Comment 1. "I find i leanr better attending every class to hear the classes input" – Agree Comment 2. "I need the teacher's explanations and such, however when studying I don't really prefer studying with others" – Agree Comment 3. "I prefer a mixed approached. I like to understand it on my own but like to study with others to compare and learn how they understand and see the problems aswell" – No inference			
Q210	*I like to study with friends	Agree: Disagree: Neutral:	7 8 0
*Comments with response 'Neutral' (The purpose of this question was to evaluate a need for relatedness.) Comment 1. "Completing practice examples on my own is beneficial, but at the same time it is nice to study as a group and to help others and be helped." - Agree Comment 2. "Only when the friends are committed to there studying and not fooling around" – Agree			

Q211	*I like to help other people to understand math.	Agree: Disagree: Neutral:	10 2 3
*Comments with response 'Neutral' Comment 1. "I don't really know enough about math to help others, but if I did I'm sure I'd like to help'" – Agree, this student wants to help. Comment 2. "I am not confident about my knowledge therefore not able to help others understand math" – No inference.			
Q212	I feel good about myself when I solve a math problem.	Agree: Disagree: Neutral:	13 0 2
Q213	List in order of importance the reasons that you study or work hard at math		

Response	Ranked 1st	Ranked 2nd	Ranked 3rd	Ranked 4th	Ranked 5th
To please family, friends or teachers (A1)	0	0	1	7	7
Because I don't want to fail (A2)	6	3	2	3	1
To understand more about mathematics (A3)	4	3	5	1	2
Because math will help me in other courses (A4)	2	8	1	2	2
Because math will help me get a better job (A5)	3	1	6	2	3

Q214	Additional comments about your math background.
Comment 1: "until grade 10 i had good math teachers, in grade 11 and 12 i was in community school and teaching wasn't that great, i had completed my school in (another country)"	

Part III: Goals

Table D.5 Q301 - Q304

Question #	Text of the question	Response	Count
Q301	My career goals are:	Long term goal: No answer:	11 4
Individual responses: I would like to work in management at a hotel; Law; Degree in arts and sciences; Own my own business; BA in psychology; To become a Transport or Civil Engineer; To own a company; Developmental Psychologist; Child Psychologist/Social Worker; Engineering; To be a hotel executive.			
Q302	My academic goals are: (List any diplomas or degrees that you hope or need to obtain.)	Graduate Degree: Undergraduate: No answer:	5 5 5
Graduate degree goals: Masters and PhD in Business Administration, Minor in marketing; Masters Degree in Educational Psychology; PhD; Bach. Civil Eng. & eventually a Masters in Urban Development; Bachelors , Masters in Engineering. Undergraduate degree goals: Degree in arts and sciences; Business Administration; BA in Psychology; I would like to go into John Molson in management; Bachelors degree. Neither: To have a diverse background.			
Q303	A university degree has always been a dream of mine.	Agree: Disagree: Neutral:	11 3 1

Q304	*I took Math 200 as a(n):	Required course: Prerequisite: Elective: Other:	1 7 6 1
*Comments with response 'Other' Comment 1. "to refresh my math studies" – No inference Comment 2. "Practice for further math classes" – Equivalent to taking the course as a prerequisite.			
Q304a	I would have taken this course for interest even if it was not required.	Yes: 2 No: 5 Not displayed: 8	

Table D.6 Q305

Question #	Text of the Question
Q305	I took this course because I wanted to:
1. "Refresh my knowledge of my high school math, so I could take harder math courses that are required to get into JMSB."	
2. "learn the concepts and understand the basis on mathematics and the importance of the course."	
3. "get a better grade"	
4. "i had a slight fear of math, because ive been away from it for so long, but i have always been very intrigued and fascinated by it as well" –	
5. "Take a course (Math) I hadn't taken in awhile"	
6. "I had not taken a math class in many years and needed to brush up on alot of the base notions in order to prepare for the harder stuff to come."	
7. "to refresh and have no difficulties in the next math courses"	

Table D.7 Q306 - Q308

Question #	Text of the Question	Response		Count
Q306	This is the first time I have registered for Math 200	Yes: 11 No: 4		
Q306a	Please indicate which situation applies to you.	Count		
	I registered before and dropped the course: I completed the course and did not pass: I passed but I want to improve my mark: All of the above:	0 1 2 1		

Q307	When I started this course I thought my mark would probably be:	75 – 100% 50 – 74% Less than 50% I didn't think about what mark I would get	9 5 0 1
Q308	*I was very nervous about passing this course when I started	Agree: Disagree: Neutral:	8 6 1
*Comment with 'Neutral': "Because I knew that I used to like math and that if I applied myself correctly I could do well. Of course it depends of other factors aswell that I was a bit unsure about" – Disagree, this student was not very nervous.			

Table D.8 Q309 & Q310

Question #	Text of the Question	Response	Count
Q309	Overall, I achieved my goals for this course	I achieved or exceeded goals: I achieved some of my goals: I did not achieve my goals:	8 4 3

Response for Q309	Comments for Q309 and Q310 – Additional comments about your goals
I did not achieve my goals	Q309: "i had certain responsibilities during the course i was not able to attend to, once i had the luxury of focusing on the material, it was a little too late" Q310: "whatever i practiced or inquired about with the teacher, i learned and had zero difficulty with, i didn't fail the course because of the material or the instructor, i failed because of my own personal situation; regardless, i have registered to take the course again in the fall 2011 and am excited for it"
I did not achieve my goals	Q309: "I have tried continuosly in this course and despite my efforts, I was unable to obtain a passing grade for the course."
I did not achieve my goals	Q309: "I was expecting a better grade due to the commitment I put into this class."
I achieved some of my goals	Q309: "I worked hard for a better grade than what i scored" Q310: "I want to complete a university degree, without one its hard to advance in the work environment"
I achieved or exceeded my goals	Q309: "If I had spent half as much time in HS doing the homework as I did in this class, I would have done much better in Math!"

Part IV: Study Habits

Table D.9 Q401 – Q404b

Question #	Text of the question	Response	Count
Q401	I attended class	Often:	15
Q402	[+]In class it was hard to pay attention.	Agree: Disagree: Neutral:	1 14 0
[+]Comment with 'Agree': "summer classes were two hours long, i find it hard to concentrate for that long"			
Q403	Outside the class, I studied	A lot: Somewhat: Very little:	9 4 2
Q404	I studied on a regular basis	Yes: 12 No: 3	
Q404a	I studied regularly	Almost every day: 3-5 times a week: Once or twice a week	3 5 4
Q404b	Each time I studied it was for approximately:	More than 2 hours: 1 – 2 hours: Less than 1 hour:	7 4 1

Responses for Q404, 404a & 404b converted to an approximate total amount of work using the chart in Table D.10. Results are summarized in Table D.11.

Table D.10 Conversion chart for Q404

	Less than 1 hr.	1-2 hrs.	More than 2 hrs.
6 to 7 days	+++	++++	+++++
3 to 5 days	++	+++	++++
1 to 2 days	+	++	+++

Table D.11 Estimate of total amount of work for Q404

	Not regularly	+	++	+++	++++	+++++
# of students	3	1	0	6	3	2

TableD.12 Q405 - Q416

Question #	Text of the question	Response	Count
Q405	The approximate number of hours that I studied for the tests: (Number of hours studying for the 2 midterms and the final exam combined)	0 – 10 hours 11-20 hours 21-30 hours more than 30 hours	4 6 1 4
Q406	I did suggested problems from the course outline	Most of the problems Some Very few	10 2 3
Q407	+Doing the suggested problems from the textbook was: (More than one response was possible)	Boring: Very hard: A good way to learn: A good way to practice: Necessary to pass:	1 1 10 12 8
+Comment with 'Necessary etc.': "maybe not totaly necessary but highly recommended"			
Q408	I did extra problems from the text, even if they weren't on the course outline.	Yes: 12 No: 3	
Q409	I read the explanations in the text book:	Often: Sometimes: Rarely:	9 3 3
Q410	*The text book for this course was awful.	Agree: Disagree: Neutral:	2 12 1
*Comment with 'Neutral': "It was okay. But not all the answers were provided for all the questions and some answers could have been explained further." – Disagree (Okay is not awful.)			
Q411	*I used one or more of the online quizzes on the Moodle to study	Yes: 14 No: 1	
*One student who responded 'No' did complete quiz 1 once and quiz 3 twice. The count has been revised but this student did not complete the remainder of the questions about the online quizzes.			
Q411a	Choose one or more of the answers that best describe the reasons why you did the online quizzes.		
	To find out what would be on the tests	Yes: 13	
	To practice problems	Yes: 13	
	To check if I was ready for the tests	Yes: 12	
	To get feedback about my skills	Yes: 12	
Q411b	*The online quizzes boosted my confidence in my abilities.	Agree: Disagree: Neutral:	10 3 0
*Comment with 'Neutral': "they were hard!" – Disagree			
Q411d	The online quizzes increased my anxiety about the tests.	Agree: Disagree: Neutral:	2 11 0
Q412	Additional comments about the quizzes:		
Comment 1: "I thought they were the perfect way to make sure whether or not I was ready for the test and had studied enough." Comment 2: "were great but i would prefer them hard copy, and not multiple choices" Comment 3: "Very good help" Comment 4: "They were a great tool!"			

Q413	I visited the MathHelp Center	More than once: Once: Never:	2 3 10
Q413a	The MathHelp Center was helpful to me.	A lot: Somewhat: Not at all: No answer:	3 1 0 1
Comment with response ‘Once’ for Q413 and ‘A lot’ for Q413a: “spoke to instructor briefly and she walked me through one or two problems”			
Q414	I went to a tutor to help me through the course.	Yes: 4 No: 11	
Q414a	The tutor was helpful.	A lot: Somewhat: Not at all: No answer:	1 2 0 1
Comment with response ‘Somewhat’: “6 hours of tutoring didnt help, once again, its about continous practice”			
Q415	I attempted one or more of the practice exams.	Yes: 13 No: 2	
Q416	Additional comments about your study habits. Were your study habits effective? Would you change your study habits if you took another math course?		
Comment 1: “I thought my study habits were pretty good. I did as many problems as I could before each exam to make sure I was ready.” Comment 2: “I think I did a lot of practice, which I consider enough in math.” Comment 3: “Practicing is the best way to study Math and i will never stop practicing for math” Comment 4: “I enjoyed doing the assigned homeworks and they realy helped to understand how certain things work, to keep the material fresh in my head and to boost my confidence in the know how of the subject matter” Comment 5: “I would study the same way that I did for this course. I found that with practicing the problems helped me understand where my strengths and weaknesses which helped my focus on the areas that I needed to work on.”			

Part V: Your Opinions about Mathematics

Table D.13 Q501 - Q510

+Q501: Before the course, I liked math.		Q501a : After the course, I like math.	
Agree: (positive) Disagree (negative) Neutral Net Affect:	8 6 1 (+2)	More (positive) Less (negative) About the same (zero) Net Change:	8 2 5 (+6)
+Comment 1, with 'Agree' and 'More': "i like it more because i know what to expect and know that ill do well if i apply myself" Comment 2, with 'Agree' and 'More': "I remembered how fun it could be" Comment 3, with 'Disagree' and 'More': "Thanks to the professor!"			
Q502: Before the course, algebra was too abstract for me.		Q502a: After the course, algebra is	
Agree: (negative) Disagree (positive) Neutral Net Affect:	6 8 1 (+2)	More abstract (negative) Less abstract (positive) About the same Net Change:	3 5 7 (+2)
Q503: Before the course, math was difficult for me.		Q503a: After the course, math is:	
Agree: (negative) Disagree (positive) Neutral Net Affect:	4 9 2 (+5)	More difficult (negative) Less difficult (positive) About the same Net Change	1 6 8 (+5)
Q504: Before the course, I usually understood what was presented in class.		Q504a: For this course, I usually understood what was presented in class.	
Agree: (positive) Disagree (negative) Neutral Net Affect:	10 4 1 (+6)	More (positive) Less (negative) About the same Net Change:	9 2 4 (+7)
Q505: Before this course, I felt that math was for some people but not for me.		Q505a: After this course, I feel that math is:	
Agree: (negative) Disagree (positive) Neutral Net Affect:	4 10 1 (+6)	More for me (positive) Less for me (negative) About the same Net Change:	5 2 8 (+3)

[+]Q506: Before this course, I felt that: Math is a complete waste of time unless it is specifically geared towards your career.		Q506a: After this course I feel that: Math is a complete waste of time unless it is specifically geared towards your career.	
Agree: (negative) Disagree (positive) Neutral Net Affect:	2 13 0 (+11)	Agree more & Disagree less (neg.) Agree less & Disagree more (pos.) About the same Net Change:	3 5 7 (+2)
[+]Comment 1, with 'Agree' and 'Agree less': "I understand the use of basic math and algebra is necessary for everyday life." Comment 2, with 'Disagree' and 'Disagree more': "Math is essential for everyday life"			
[+]Q507: Before this course, I was not good at writing math tests.		Q507a: After this course, I am:	
Agree: (negative) Disagree (positive) Neutral Net Affect:	6 7 2 (+1)	Better at writing math tests (pos.) Worse at writing math tests (neg.) About the same Net Change:	6 1 8 (+5)
[+]Comment with 'Agree' and 'About the same': "I tend to not be able to sit in a exam room for a long period of time and hold my concentration, whether the subject is Math or not is irrelevant."			
Q508: Before this course, math sometimes made me feel stupid.		Q508a: After this course, math makes me feel stupid:	
Agree: (negative) Disagree (positive) Neutral Net Affect:	6 7 2 (+1)	More often (negative) Less often (positive) About the same Net Change:	3 6 6 (+3)
[+]Q509: Before this course, math sometimes was fun.		Q509a: After the course, I think that math is fun:	
Agree: (positive) Disagree (negative) Neutral Net Affect:	7 3 5 (+4)	More often (positive) Less often (negative) About the same Net Change:	7 2 6 (+5)
[+]Comment with 'Agree' and 'More often': "im more familiar with terms and fundamental concepts"			
Q510: Before this course, I thought that I could succeed in math.		Q510a: After this course, I feel that I can succeed in math:	
Agree: (positive) Disagree (negative) Neutral Net Affect:	10 2 3 (+8)	More strongly (positive) Less strongly (negative) About the same Net Change:	9 2 4 (+7)

Table D.14 Summary of net affect and net changes in affect

Question	Net Affect (BEFORE MATH 200)	Net Change
Q501: I like math	2	+6
Q502: Algebra was too abstract	2	+2
Q503: Math was difficult for me	5	+5
Q504: I understood what was presented in class	6	+7
Q505: I felt math was for some people but not for me	6	+3
Q506: Math is a waste of time unless it is geared towards a career	11	+2
Q507: I was not good at writing math tests	1	+5
Q508: Math sometimes made me feel stupid	1	+3
Q509: Math sometimes was fun	4	+5
Q510: I thought I could succeed in math	8	+7
Total	**46**	**+45**

Part VI: Additional Information

Table D.15 Q601 - Q605

Q601	What do you do when you get stuck on a math problem?
"look at other problems that relate, read the chapter more thoroughly or ask the teacher, usually i just attempt it until i connect the dots" "I try to figure out as many ways as possible to figure it out until I get one that works." "Ask for help or try to figure out similar problems" "I try to remember the rules and way of attempting the problem, it usually backfires when I use the wrong formula or forget to add or complete certain steps, resulting in failure despite knowing what the question is asking." "Keep going and go back to it later, I often try many different approaches. If all else fails I ask my sister and we often work it out together." "ask for help in class, ask family for help or call tutor" "I take a break or move on to the next problem. Usually it means I'm having a math overload when I get stuck. When I come back to it I can usually get it. Otherwise I will ask for help." "I stop, leave the problem them and then try to work it out again." "Take a break or move to another problem" "keep at it or try a different approach ask a friend or a teacher for help understanding" "move on" "Review the text book explanations, review my notes from class, look at the answer key and work it backwards so that I can understand where I went wrong."	
Q602	How do you convince yourself to study, even when you have better things to do?
"i dont convince myself, my job is to study right now" "I usually watch tv as I study, it actually makes studying math enjoyable for me." "Just do it" "I focus on the future and force myself to study as I want to go into business and require this course in order to fulfill my career later on in life." "I study best as one of the first activities of the day. I feel it is when I am most focused." "i build motivation to do well so that once i see i am capable maybe math will get easier and continue to be less scary" "I convince myself with the though that I have to do well in my courses, which is "the better thing to do", anyway." "I just say to my self that to pass the course is my responsibility, so I work on it." "the deadline i have to meet and i don't like to study last minute" "I tell myself that practice makes perfect and that you can never practice enough..." "Because I am back at school late in life, I am very disciplined with my study habits. I regret not to have been the same 20 years ago!"	

Q603	List what was most helpful about this course:
“the instructor -very approachable, an absolute mandatory quality for a professor to have for an intro class” “the online quizzes, the suggested problems, teaching style” “Practice tests on Moodle” “The online quizzes and suggested problems” “The way the teacher was able to explain things incredibly clearly - which I realize now taking Math 201, is not an easy task. Also the book came with an answer book as well, and inside the answer book there were not only the answers but all the work step by step, which was a great way to learn and check where your mistakes were when practicing.” “it teaches you a good base for the future, its concepts that apply to many areas of life that used on a daily basis.” “The teacher was very good, she explained things very well, and she always replied to our questions through email or in office hours. She was very helpful! Also, I thought the book was really straight forward with most of its explanations.” “The teacher explanations were pretty clear and the tutors in the math centre too.” “Professor was great, suggested questions and practice test” “Very good teacher, good text book and good schedule” “GPA boost” “The way the professor explained the subject matter and the way the textbook took us step by step.”	
Q604	List anything that would have helped you do better in this course
“Having more time and being able to have class everyday for a shorter period of time rather than less often as it was too much material at one time.” “A tutor and more time to study for the final exam.” “if i would have had a full semester rather than a summer semester” “Maybe some more studying, more motivation.” “more time, since I took it during the summer and I felt I did not have enough time to do better.” “more explanation and more information on wording problems” “More time! As it was a summer intensive class, I found that the information was sometimes hart to assimilate...next time, I am not sure that I would take a math course in the summer session.”	
Q605	Any other comments or suggestions:
“Problem solving can be very tricky if a student doesn't understand correctly, i would suggest more problem solving as different problems has different wording”	

Table D.16 Summary of net affect and net change with selected questions

ID	Final Grade	Previous Failure? Q205	Expected Mark Q307	Term study time* Q404	Study time a lot? Q403	Achieved goals? Q309	Succeed more after Q510a	Net Affect (before) Part V	Net Change (after) Part V
S1	F	No	N/A	N/A	No	No	+	+8	+10
S2	A	No	50-74%	(+++)	Some	All	+	-2	+8
S6	A+	No	75-100%	(+)	Some	All	0	-2	+3
S7	B+	No	50-74%	(++++)	Yes	All	+	+8	-1
S8	A-	N/A	75-100%	(++++)	Yes	All	-	+4	-5
S11	A	No	75-100%	N/A	No	All	0	+4	0
S13	F	Yes	50-74%	(+++++)	Yes	No	-	-4	-6
S16	A	Yes	75-100%	(+++)	Yes	All	+	+8	+10
S17	A+	No	75-100%	N/A	Some	All	+	+8	+6
S18	B-	No	50-74%	(+++)	Some	Some	0	+9	0
S23	D+	Yes	75-100%	(+++++)	Yes	No	0	0	-1
S25	B+	Yes	50-74%	(++++)	Yes	All	+	-5	+7
S27	B+	No	75-100%	(+++)	Yes	Some	+	-5	+5
S28	B+	Yes	75-100%	(+++)	Yes	Some	+	+6	+6
S37	A-	Yes	75-100%	(+++)	Yes	Some	+	+9	+3
							Total	46	+45

*Term study time is an approximation of the amount of studying that a student reported (+<++<+++<++++<+++++). Tables E.10 & E.11 (above) show how this quantity is calculated.

Table D.17 Comparison of individual responses for selected questions

ID	Final Grade	Previous Marks Q204	Previous Failure? Q205	Nervous about passing? Q308	Expected Mark Q307	Expecting a mark that is…	Emotion
S1	F	75-100%	No	No	N/A		
S2	A	60-74%	No	Yes	50-74%	Same	Unsure
S6	A+	75-100%	No	No	75-100%	Same	Confident
S7	B+	75-100%	No	Yes	50-74%	Lower	Worry
S8	A-	75-100%	No answer	Yes	75-100%	Same	Unsure
S11	A	75-100%	No	Neutral	75-100%	Same	Confident
S13	F	60-74%	Yes	No	50-74%	Same	Confident
S16	A	60-74%	Yes	Neutral	75-100%	Higher	Hope
S17	A+	75-100%	No	Yes	75-100%	Same	Unsure
S18	B-	75-100%	No	Yes	50-74%	Lower	Worry
S23	D+	Less than 60%	Yes	No	75-100%	Higher	Hope
S25	B+	Less than 60%	Yes	Yes	50-74%	Higher	Hope
S27	B+	60-74%	No	Yes	75-100%	Higher	Hope
S28	B+	60-74%	Yes	Yes	75-100%	Higher	Hope
S37	A-	60-74%	Yes	No	75-100%	Higher	Hope

Appendix E: Interpretation of Data for the Characterization of the RS

The details of the evidence that was used to describe the motivational characteristics of the 'realistic student' (RS) are presented in this appendix.

Conclusions are made based predominantly on the survey results of the survey group (SG) of 15 students. This is a small sample and it is skewed towards individuals who did well in the course (Chapter 5, Section 5.7). Other sources include: classroom observations; course evaluations (33 students) and the online quiz questionnaire (27 students). Comments and observations, in particular, provide qualitative information that may be used to better describe the characteristics of the RS. The interpretations presented here are my own but they are disciplined and informed by the theory described in Chapter 2 and justified by empirical evidence to the extent that it was available.

Conclusions that are drawn from the results of the survey use the demarcation of half. If a majority of students in the SG respond in the same way then the information is incorporated into the characterization of the RS. In other cases, a result that is found for *some* of the students (typically more than 25% of the SG students) may be incorporated to better describe the characteristic for the RS. For example, six students admitted that they had previously failed a mathematics course (Q205). This is not a majority of the SG but it will be considered to be a large enough proportion to conclude that the RS *may* have failed a mathematics course.

A particular characteristic may be described as 'low', 'moderate' or 'high' (or strong). These were the descriptors that were used in the initial assumptions made about the HS and they were based on theoretical considerations. If a quality is described as 'low' it simply means that one could reasonably expect it to be higher elsewhere (in another group of students). These descriptions are used when referring to the strength of needs for the HS and will be treated as stable inasmuch as they will be revised (in the characterization of the RS) only if it can be established that a majority of students seem to exhibit the quality.

The motivational characteristics are presented in the same order here as was established for the characterization of the HS (Section 3.3) and the description of the RS (Chapter 6). For each characteristic, the initial assumption that was made about the HS is presented along with the pertinent evidence and the conclusion that was drawn for the RS. Evidence that pertains to the changes that were observed during the course is presented in the same order as established in Chapter 6.

Results for the survey are presented in Appendix D along with an explanation of the treatment of the data and the question codes. Results for the online quiz questionnaire appear in Section 5.5 and results for the course evaluations appear in Section 5.6.

E-1 General Characteristics

E-1.1 Age

Initial assumption. The HS was a young adult, approximately 25 years of age.

Evidence. Of the 15 students in the survey group, (SG) 7 were over 25 years of age (according to the responses to Q102).

Conclusion. The RS was slightly older than 25 years of age.

E-1.2 Previous Curriculum

Initial assumption. The HS had (at least) had access to the curriculum that is taught in the regular-stream mathematics program in Quebec high schools or an equivalent.

Evidence. All of the SG students had taken at least regular-stream mathematics in high school and for 9 students this was the only mathematics that they had taken (Q201). Two students had taken at least one mathematics course at CEGEP (Q203) but both these students also reported that they had failed one or more mathematics courses before taking MATH 200 (Q205). Some students had taken (but not necessarily succeeded at) mathematics courses that are more challenging than regular-stream mathematics.

Conclusion. The RS had taken (at least) regular-stream mathematics at high school.

E-1.3 Student Status

Initial assumption. The HS had attended university for at least one year, either part-time or full-time and was familiar with university procedures.

Evidence. There were 7 SG students who reported that they had been attending Concordia for two or more years (Q104). One student did not answer this question, one student entered zero and the other 6 students entered one year. Additionally, 8 of the students were attending university full-time and 7 were part-time students (Q106).

Conclusion. The RS had attended university for at least one year and might be attending university either full-time or part-time.

E-1.4 Major

Initial assumption. The HS was not majoring in mathematics. MATH 200 was a required course for this student.

Evidence. In Part I of the survey (Q105), the students were asked to enter their degree program. No student entered mathematics as their major and no student entered mathematics as a career goal or an academic goal (Q301 and Q302). This supports the conclusion that the RS does not major in mathematics.

Only two students entered a discipline (Civil Engineering and Finance) that I considered to be mathematics intensive. Some of the other programs that were entered in response to Q105 also have additional mathematics requirements (Appendix D, Table D.1). For example, psychology students must also take a statistics course. Hence, this characteristic (Major) better describes the RS if it includes the additional information that mathematics *might be important.*

Six students in the SG were taking MATH 200 as an elective (Q304) and of the 8 students taking the course as a prerequisite or as a required course, 2 students reported that they would have taken the course for interest (Q304a). Of the 33 students who completed the course evaluations, 6 students were taking the course for 'General Interest', another 3 students were taking the course as an elective and 3 additional students took the course because it fit in their schedule (Table 5.3). Eight of the students in the SG and 36% of the students who completed course evaluations chose (or might have chosen) to take this course. This establishes the conclusion that MATH 200 was *probably* a required course but it also provides evidence that the SG is potentially skewed in favor of individuals who have taken the course as an elective.

Conclusion. The RS did not major in mathematics *but mathematics might be important in the student's field*. MATH 200 was *probably* a required course for the RS.

E-1.5 Background

Initial assumption. Mathematics had been an area of difficulty for the HS. The student probably did not receive very good marks in high school and may have failed a math course. It was assumed that the HS had not taken a mathematics course for at least a couple of years.

Evidence. In Part II of the survey (Q204), the students were asked to indicate their marks in mathematics before taking MATH 200. There were 6 students who reported receiving marks that were between 60% and 74% and 2 students reported that their marks were usually lower than 60%. This is a majority of the SG students (albeit a small one) so there is not sufficient evidence to change the assumption that the HS probably did not receive very good marks. However, there were 7 students who did report marks that were usually better than 75% which suggests that the SG includes individuals who had positive experiences in mathematics prior to taking this course.[11]

Six of the students in the SG reported that they had failed one or more mathematics courses before taking MATH 200 (Q205) and 2 students had previously failed MATH 200 (Q306a). While this does not represent a majority of the students in the SG, it is sufficient to conclude that the RS *may* have failed a mathematics course.

For 11 of the students in the SG, it had been more than two years since they had taken a mathematics course (Q103). One response was 1980! This supports the conclusion that the mathematics background of the RS was not current.

Some comments were: "…math was my weakest subject" (Q205); "Really did not do well in high school" (Q206); and "I find the processes very complicated" (Q207). Failing a mathematics course can almost never be described as a positive experience. The comments and the number of students (6) who had failed a mathematics course are indications that the RS had experienced difficulties with mathematics in the past.

For Q208, the SG students were asked to describe their mathematics background as 'Good', 'Fair' or 'Poor'. There were only 3 students who chose the response 'Good' and these were all students whose marks in high school were 75-100%. Nine students chose 'Fair' and 3 students chose 'Poor' as a response to this question. There were 4 students who reported receiving high marks in high school (Q204) that could still not describe their mathematics background as being

[11] Students may self-report somewhat higher marks that were actually obtained (Kuncel, Crede, & Thomas, 2005).

better than ‘Fair’. It is for this reason that the additional statement that “The RS was aware that his or her background was not strong” was appended to the model of the RS for this characteristic.

Conclusion. Mathematics had been an area of difficulty for the RS. The student probably did not receive very good marks in high school and may have failed a math course. The mathematics background of the RS was not recent and the student was *aware* that his or her background was not strong.

E-1.6 Mathematical Prerequisites

Initial assumption. It was assumed that the HS had very few mathematical prerequisites but that this was not the first time that algebra had been presented to the student.

Evidence. I observed, in class and on the first midterm, quite a wide range of abilities among the students. One student asked for an example of long division in class and one student (S30) wrote in an e-mail, “I am having difficulty understanding how to do [the] long division algorithm when using decimals.” Another student (S37) wrote, “...I had forgotten how to divide two fractions...” Some students were comfortable with basic arithmetic operations but had trouble manipulating algebraic expressions. Some were applying remembered ‘rules’ of algebra (for example, adding a term to one side of an equation and subtracting it from the other side). All the students in the SG had taken at least regular-stream mathematics in high school (Q201) so the students had been taught the basic concepts of algebra before but they did need a review of arithmetical operations on integers and rational numbers.

Conclusion. The RS would benefit from a review of basic operations on integers and fractions.

E-2 Needs

E-2.1 Physiological Needs

Initial assumption: It was assumed that the HS was capable of managing physiological needs but had a busy life.

Evidence. Q107 asks the students to agree or disagree with the statement "I had many commitments other than this course". Eleven of the 15 SG students agreed with this statement, 3 disagreed and one student responded, "Neutral." Some comments were: "I had certain responsibilities during the course…" (Q309) and, "I worked two jobs, have a family to support," (Q107). In an e-mail communication, one student wrote that a personal situation had been "very time consuming…"

Question 18 on the course evaluations asked the students to comment on the facilities for the course. There were 24 responses to this question and 10 students reported some difficulty with the facilities that could impact their learning or ability to concentrate (obstructed view of the blackboard, noisy seating etc.). If 30% of the students felt sufficiently inconvenienced by the facilities to enter a comment, it is suggestive that the HS may be somewhat sensitive to the physical surroundings.

Conclusion. The RS had a busy life and was capable of managing physiological needs *but might be easily distracted*.

Addressing physiological needs. For Q402, there was only one student who agreed with the statement, "In class it was hard to pay attention." This student also commented, "Summer classes were two hours long, I find it hard to concentrate for that long."

E-2.2 Psychological Needs/Autonomy

Initial assumption. Autonomy was assumed to be a moderate need for the HS.

Evidence. Q209 asked the students to agree or disagree with the statement, "I learn best on my own." There were 7 students who agreed with the statement, 6 students who disagreed and 2 students entered "Neutral." There was no evidence that the students had a strong need to learn autonomously.

One student (S14) frequently asked during class if it was possible to use shortcuts or to solve a problem in another way. This student had an excellent background in mathematics and had confidence in her abilities. Another student (S28) asked if he could use an intuitive solution because although he did not know how to write an algebraic solution, he said that he understood how to arrive at the correct answer. In an e-mail, S23 asked, "I get the correct process and correct answer but (if it is) a little bit different as it is in the book, (will I) get a full mark..." The student then showed two examples of collecting terms in a slightly different order. All three students are investigating their academic autonomy (choices) but they are being mindful of the requirements of the course. For all three of the examples presented above, I defined the course requirements, offered autonomy support and encouraged their questions. I explained that full marks would be given to any solution process that was mathematically sound (provided that the answer was correct also) and that on the tests the students would be required to show their work. I suggested to S28 that he could use his intuitive understanding with simple problems to generate an algebraic solution that would serve him when he was faced with more complicated situations. When S14 made suggestions in class, I used the opportunity to highlight the theory that allowed alternate approaches. I validated the approaches of S23 and praised her efforts because I wanted to encourage communication.

Conclusion. The RS had a moderate need for autonomy and was beginning to investigate autonomy in mathematics.

Addressing a need for autonomy. As in any university class, the students were free to choose their preferred method of studying and they did. In this class, the students chose to postpone the second midterm by one class and asked for a third online quiz to be prepared. Students chose whether or not to attempt the bonus question on the midterms and not all of the students who wrote the midterms had chosen to use the online quizzes. Students were free to choose an algebraic strategy on the tests, as long as it made sense mathematically. Within the restrictions imposed by the curriculum and the course outline, the students had choices.

From course evaluations (Table 5.3), all except 2 of the students agreed with the statement, "Students are encouraged to ask questions," (question 11) and all but 3 students, agreed with the statement, "Students are encouraged to share their ideas and knowledge." The overall assessment was that both these results were "Well above average" when compared to the bank of student course evaluations that is maintained by the university. It can be concluded that the students did feel that their ideas and concerns would be considered.

A rationale was consistently provided for studying mathematics and concrete examples were shown. The value that this rationale supports is that mathematics is useful to the students. For Q506 and Q506a there were 13 students who felt that mathematics was useful (outside of a career) before the course but even so, there was a net positive change (+2) in this belief after the course. Student comments were, for Q603, "...concepts that apply to many areas of life that (are) used on a daily basis," for Q506, "I understand the use of basic math and algebra is necessary for everyday life," and "Math is essential for everyday life." This value was already an internalized one for many of the students in the survey group and the change is not dramatic but it is in a positive direction.

E-2.3 Psychological Needs/Competence

Initial assumption. It was assumed that the HS had a strong need for competence and that mathematics had not been an area in which the student had previously felt competent.

Evidence. Only 3 students in the SG reported that their mathematics background was 'Good' (Q208) and 6 students had previously failed a mathematics course (Q205). There is evidence here that mathematics had been an area that did not allow a feeling of competence. However, 7 students in the SG reported receiving good marks in high school (Q204).

Ten students agreed with the statement, "I like to help other people to understand math," and 2 students disagreed (Q211). This question addresses a need for competence because in order to help others one must feel some confidence with the material. One student (S37) expressed this very succinctly in a comment for this question, "I don't really know enough about math to help others but if I did, I'm sure I'd like to help." Another student commented, "I am not confident about my knowledge therefore not able to help others understand math." It is the *wanting* to help that defines a need to feel competence and a majority of the students in the SG felt this need.

Q213 asks the students to rank, in order of importance, the reasons that they study or work hard at math. Responses for this question are reproduced in Table E.1.

Table E.1 Survey responses to Q213

Q213: List in order of importance the reasons that you study or work hard at math					
Response	Rank 1	Rank 2	Rank 3	Rank 4	Rank 5
To please family, friends or teachers (A1)	0	0	1	7	7
Because I don't want to fail (A2)	6	3	2	3	1
To understand more about math (A3)	4	3	5	1	2
Because math will help me in other courses (A4)	2	8	1	2	2
Because math will help me get a better job (A5)	3	1	6	2	3

The students in the SG were not interested in displaying their competence to others (A1), but students who work hard at math because they will need these skills in other courses (A4) or because math will help them in a career (A5) certainly need to develop their mathematical abilities in this course.

Q305 asked the students to finish the sentence, “I took this course because I wanted to…” There were seven responses and three of these were similar. They are: “refresh my knowledge of my high school math so I could take harder math courses that are required to get into the [School of Business],” “brush up on a lot of the base notions in order to prepare for the harder stuff to come,” and “to refresh and have no difficulties in the next math courses.” These students have a need to show their competence in a very practical sense.

For question 15 on course evaluations, 18 of the 33 students reported that their previous mathematics knowledge was ‘Average’. Seven students reported “high” or “very high” subject knowledge and 8 students reported “low” or “very low” prior knowledge (Table 5.3). If average can be taken to mean “just like everyone else,” then whatever the students do feel about their background, a majority want this to be the acceptable norm among whatever group the student is using for comparison. If the students are in fact *aware* that mathematics has been an area of difficulty, and still feel that their level of knowledge is ‘average’, then they are expressing a need to feel competent in comparison to others.

Conclusion. The RS had a strong need for competence *in comparison to others* and *may not* have felt competence in mathematics courses in the past.

Addressing a need for competence. For question 3 on the online quiz questionnaire, 21 of the 27 students who responded, felt that the difficulty level was “just right.” There were 5 students who thought it was “too easy” and one student found it “too hard.” For question 4, 20 students felt that the length of the quiz was “just right,” 5 students thought it was “too short” and for 2 students it was “too long.” For Q603 of the survey, there were 4 students who mentioned the online quizzes as an element that was helpful to them in the course. For Q411d, there were only 2 students who felt that the online quizzes increased their anxiety about writing the midterm tests and for Q411b, 10 students felt that the quizzes boosted their confidence in their abilities. Thus, the online quizzes were able to provide the right amount of challenge to the students without significantly impacting their anxiety.

There were comments from e-mail communications that indicated that some students were having difficulties with the material. These include: “I’m very confused...”; “I still don’t quite get...” and “I am having difficulty understanding...” Students’ comments on course evaluations included requests for more examples or more online quizzes and one student described the course as “difficult.” One student wrote in an e-mail, “I understand your explanation...but I still don’t quite get...” and “I just answered my own question, I understand that one now...but I have another question.” Challenge is about right for this student.

Only one student reported that mathematics was more difficult after the course (Q503a) and one student (on course evaluations) wrote that the instructor was “lazy” for not always providing full solutions on the board. For Q407, only one student indicated that doing the suggested problems was “boring” (this student received an A+ as a final grade) and one student indicated that they were “very hard” (this student received an F). Challenge was not optimal for these students.

From course evaluations (question 6), 27 students either agreed or strongly agreed with the statement, “I have found this course intellectually challenging and stimulating.” There were 2 students who strongly disagreed and 4 students who neither agreed nor disagreed. On question 14, students were asked to rate their level of involvement in the course. Sixteen students reported above average, 14 students reported average and one student reported below average involvement. For Q416, one student wrote, "I enjoyed doing the assigned homework," and on course evaluations, a student "enjoyed" the online quizzes. If the challenge is optimal, students enjoy the activity and become involved (flow) and there is sufficient evidence to conclude that this was the case for many of the students.

E-2.4 Psychological Needs/Relatedness

Initial Assumption. Relatedness was assumed to be a moderate need for the HS and that the HS would seek out a support network in the class. The student might have blamed teachers for previous poor performance in mathematics.

Evidence. There were 7 students who agreed with the statement "I like to study with friends" and there were 8 students who disagreed (Q210). One student commented, "…it is nice to…help others and be helped." There is no clear majority but the conclusion can be made that positive relationships in the class are somewhat important to the students.

For Q202, the students were asked to agree or disagree with the statement, "Some of my math teachers were good." Almost all of the SG students (13) agreed with the statement and 2 students disagreed. A majority of the students could have had positive relationships with previous mathematics teachers.

Questions Q206 and Q207 asked the students to give reasons why they had done well at mathematics and reasons for their difficulties respectively. There were ten comments entered for Q206 and of these 4 students cited "good" teachers as a reason for their success. There were 12 responses to Q207. One student wrote that the teacher was "bad" another wrote "The teacher wasn't good and was very strict and mean." There were other reasons given for both of these questions and these are listed in Appendix D, Table D.3. One other student offered a comment for Q214 (Additional comments about your mathematics background) and mentioned "good math teachers” until grade 10 and teaching that “wasn’t that great” in community school. If describing a teacher as ‘good’ can imply a positive relationship and ‘bad’ or ‘mean’ can imply a negative relationship then the RS could have experienced both but there is not enough evidence to conclude that the RS was to any great extent blaming poor performance on poor teaching. There are however, a total of 6 students who felt that the quality of teaching was a factor in their success or failure. This minority result is sufficient to conclude that the RS may feel that performance in mathematics is influenced by the nature of the relationship between student and instructor.

Conclusion. The RS had a moderate need for relatedness. Relationships with peers and with the instructor are important to the RS.

Addressing a need for relatedness. There were many requests in e-mails and in class for procedural details about the course and the mathematics department. The students might be familiar with the university but they are less familiar with the mathematics department and they sought this support from the instructor.

Two students (one in class and one in an e-mail) asked about a specific question in the text that required the students to calculate $9-(-2)$ but the answer reported in the solutions manual was 7. Both students assumed that the fault was their own and were not able, initially, to entertain the possibility that the manual was wrong. They needed another ‘authority’ – the instructor – to override the authority of the manual.

One student sent me a link to a video that made light of the complicated wording that is typical of algebra word problems but the student was also concerned that I might find the language somewhat offensive.

Even though I had indicated to the students that I would answer to my first name, I was only addressed this way by a couple of students. For the most part I was addressed as Professor, Mrs. or Miss along with my last name.

On course evaluations (Table 5.3), questions 2, 8, 9, 10, 11, 12 and 13 address the students' assessment of the instructor. For all of these questions the overall assessment was above average or well above average. No more than 6 students entered neutral or negative responses for any of the questions. The students thought the instructor was approachable, knowledgeable and provided clear explanations and feedback. In comments, the instructor was also described as "effective", "informed" and "caring" and there was one negative comment.

One piece of advice that I gave to the students was that practicing would allow them to become confident with the material and would improve their grades. For Q416 (additional comments about study habits), there were five responses – three that specifically indicated that "practice" was a good way to study, one cited "assigned homework" and one mentioned "[doing] problems." The students may have started the course with the belief that practice was important but there is evidence that this too was an internalized value by the end of the course.

E-2.5 Social Needs/Achievement

Initial Assumption. The assumption was made that the HS was a low need achiever. It was assumed that the student would show perceptions of low ability and have low or moderate expectations of success.

Evidence. As established above (Section E-1.5) the RS was aware that his or her mathematics background was not strong. One student commented in class, "I am not a strong student," another student made the comment, "I had failed some courses and felt that I was very bad at math," (Q207) and yet another wrote, "I find the processes very complicated," (Q207). There is sufficient evidence to conclude that the RS had perceptions of low ability.

In the SG, 8 students were very nervous about passing MATH 200 (Q308). Nine SG students were expecting a mark in the range of 75-100% for this course and 5 students expected a mark between 50% and 74%. No students expected a mark of less than 50% but one student checked the box for, "I didn't think about my mark," (Q307). Students had also entered the range of marks that they "usually" received before this course (Q204) so a comparison of individual responses to Q204, Q205, Q307 & Q308 are possible. This comparison is presented in Table D.17 of Appendix D.

- Four of the 8 students, who were very nervous about passing, expected high marks.
- There were 2 students who expected a lower mark than they were accustomed to receiving in the past and both these students were very nervous about passing. This situation is one in which the students can be considered to be *worried*.
- Three students expected marks that were in the same range as the marks they had received before but were still very nervous about passing. These students were *unsure*.
- Three students expected marks similar to what they had received in the past and were not very nervous about passing. These students were *confident*.
- There were 6 students who expected a better mark than what they had usually received in the past. These students were *hopeful*.
- Five of the hopeful students had previously failed a mathematics course and 4 of these students were expecting a mark of 75-100%. These 4 students were quite hopeful.

In all, 9 students were confident or hopeful about the outcome for this course.

Conclusion. The RS did have perceptions of low ability but high expectations of success.

E-2.6 Self-actualization

Initial assumption. This was assumed to be a low need for the HS.

Evidence. Q303 asks the students to agree or disagree with the statement, "A university degree has always been a dream of mine." Eleven students agreed, 3 disagreed and there was one response of "Neutral." This is certainly evidence that a majority of students are pursuing their dreams and the initial assumption that this quality was low for the HS can be revised to moderate with some confidence. This question was included in the survey because one mature student (S25) confided to me in class that she had always wanted to get a degree in psychology and she was returning to university in order to do so. In my opinion, this defines a high need for self-actualization that is present for S25 because her primary reason for attending university is self-improvement. Without additional evidence though, I am unwilling to conclude that this tendency is of paramount importance for the rest of the SG students, i.e. that this is a high need for the RS. Seven of the 11 students who agreed with the statement in Q303, also entered career goals for which their chosen degree program is a requirement. The growth need for self-actualization does exist for the students in the SG (it is a majority result) but it is also possible that students are fulfilling their dream of obtaining a university degree because it coincides with other priorities (such as career goals).

A self-actualizing tendency *for mathematics* would be observable as an interest in mathematical activities that were not directly related to passing the course. This would be an interest in learning mathematics for its own sake. Attempts were made to encourage an interest in mathematics by introducing puzzles & historical content. One student was "inspired", another was "intrigued" but for the most part these attempts were unsuccessful. Thus, the RS may have had a moderate need for self-actualization but it was not a need for self-actualization in the field of mathematics.

Conclusion. The RS may have a greater need for self-actualization than was assumed for the HS but the student is not acting to realize his or her potential in the field of mathematics.

E-3 Goals

E-3.1 Long-term Goals

Initial assumption. It was assumed that the HS had set a long-term career goal and a goal of obtaining a university degree.

Evidence. There were 11 students who entered a career goal in response to Q301. For Q302, students were asked to list any degrees or academic qualifications that would be required for their chosen career path. Five students entered a graduate degree goal and another 5 entered an undergraduate degree goal. One student offered the response, "To have a diverse background," and 4 students left this question blank. Actual responses appear in Table D.5 of Appendix D.

Conclusion. The RS had set long-term career and university degree goals.

E-3.2 Intermediate Goals

Initial assumption. The HS had set the intermediate goal of passing MATH 200.

Evidence. All of the students in the SG expected a mark greater than 50% (Q307).

Conclusion. The RS had set intermediate goals.

E-3.3 Short-term Goals

Initial assumption. The HS had set the short-term goals of passing the midterms and the final exam.

Evidence. All of the students who wrote the final exam also wrote both midterm tests.

Conclusion. The RS had set short-term goals.

E-3.4 Goal Proximity

Initial assumption. It was assumed that the HS was not close to realizing his or her degree and career goals.

Evidence. Seven of the students in the SG had been attending Concordia for two or more years (Q104). Individual survey responses show that 3 of these students had set a graduate degree goal (Q302), i.e. they were not close to obtaining their long-term goals. The remaining 4 of these students were all attending university part-time (Q106) so it is conceivable but unlikely that they are close to their degree goals. The other 7 students, who had been attending Concordia for one year or less (Q104), were most probably not close to obtaining a degree. Thus for a majority of the students in the SG, a university degree was still a distant goal.

Conclusion. The RS is not close to realizing long-term career or academic goals. Therefore, most of the motivation of the RS would be derived from short-term and intermediate goals.

E-3.5 Achievement Goals

Initial assumption. It was assumed that the HS had set predominantly performance goals (that demonstrate competence) and more of a performance-avoidance nature (fear of failure) than performance-approach goals (comparison to others). It was also assumed that the student would set few mastery goals (that develop competence).

Evidence. In Part II of the survey, Q213 addresses achievement goals and the results are tabulated in Table E-1, above. The question asked the students to rank, in order of importance, the reasons that they study or work hard at math. The first response (A1) is, “To please family, friends or teachers.” This is a typical performance-approach goal in that the student who chose this option would be interested in demonstrating their competence to others. No student ranked this option first or second.

The second option (A2) is, “Because I don’t want to fail.” This represents a performance-avoidance goal – the student wants to avoid the appearance of incompetence. A2 was ranked first by the most students (6) and only one student ranked this in last place. This suggests that performance-avoidance goals were prominent among the students of the SG.

The third response (A3) is, “To understand more about math.” This is a typical mastery goal in that the student is seeking to develop competence (abilities, understandings) in mathematics. Mastery goals are also well represented among the students of the SG (4 students ranked this option first) but there were more students who ranked the performance-avoidance goal first (6 students) than students who chose the mastery goal.

The last two options for this question (“Math will help me in other courses,” and “Math will help me get a better job.”) were considered in Section E-2.3, above. Students who choose these options are seeking to develop their competence but they will have to demonstrate their competence later. These two options reflect a need for competence which is more related to performance-approach goals than mastery goals.

Grades are one way to display competence; working to obtain a good mark is indicative of having set a performance-approach goal. Students asked about the class average for the midterms (and I did not supply this information). One student was taking the course, “to get a better grade,” (Q305), another student entered, “GPA boost,” when asked to list what was most helpful about the course (Q603). For Q309, one comment was, “I was expecting a better grade…” and another student remarked, “I worked hard for a better grade…” Nine students expected to receive a mark that was above 75% (Q307) – more than half of the students in the SG expected to score in the top third (approximately) of the class. Any student who needed this course to apply for another program would have to be concerned that their mark met the requirements for the program better than other applicants.

Conclusion. The RS had set all three types of achievement goals but more performance goals than mastery goals.

Addressing achievement goals. Students were given structure as to what constituted a good solution to an algebraic problem. Examples were shown in class and solved problems were posted on the course website.

There were 6 students who felt that they were "better at writing math tests" after this course (Net Change for Q507a was +5) and there were 9 students in the SG who reported that they understood more in this class than before (Net Change for Q504a was +7).

Of the 6 students who ranked the performance-avoidance goal (A2) first for Q213, 4 students received an A as a final grade and the other 2 students received a B. Among the remaining 9 students who chose another goal first, only 3 students received an A.

E-3.6 Goal Attainment/Frustration

Thirteen of the 15 students in the SG achieved the intermediate goal of passing MATH 200. For Q309 though, there were 3 students who reported that they did not achieve their goals. Two of these students (S1 & S13) received a failing final grade but one student (S23) received a D+ for a final grade.

E-4 The Self

E-4.1 Implicit Theories

Initial assumption. It was initially assumed that the HS was more of an entity theorist than an incremental theorist.

Evidence. As established in Section E-3.5, above, the RS had set multiple goals but more of these were performance goals than mastery goals as would be the situation of an entity theorist.

Q505 of the survey asked students to agree or disagree with the statement, "Before the course, I felt that math was for some people but not for me." Ten of the students in the SG disagreed with this statement, suggesting that most of the SG students felt that they were in some way a "math person."

Comment with Q308, "I knew…that if I applied myself correctly I could do well."

Comment with Q207, [When I had difficulties before it was because]: "lack of practice"; "I did not study enough"; "I had no motivation" and "lack of motivation."

Comments with Q206, [When I did well at math before it was because]: "I studied a lot"; "My father helped me to study"; "someone pushing me to study"; "only because of practice" and "…as well as practice."

The implication from these comments is that these students felt that more work would produce better results – an incremental viewpoint.

In response to Q207, one student (S25) wrote, "I felt that I was very bad at math." This student, clearly, could be described as an entity theorist at the beginning of the course.

Conclusion. Evidence was found for both viewpoints for the RS.

Change in implicit theories. A comparison of individual student responses appears in Table D-16 of Appendix D. There were a total of 7 students who reported hard work (Q404), achievement of their goals (Q309) and who also felt more strongly that they could succeed in mathematics after the course (Q510a). These 7 students can be said to hold an incremental viewpoint after the course. One student (S25) who wrote for Q207, "[I] felt that I was very bad at math," also wrote, "The feeling is still there!" but for Q309 entered, "If I had spent half as much time in high school doing the homework as I did in this class, I would have done much better in Math!" This student's viewpoint has changed from an entity view to a more incremental viewpoint.

There were 2 students in the SG who worked hard and achieved their goals but did not feel more strongly that they could succeed in mathematics after the course. The viewpoint of these 2 students is more probably an entity viewpoint.

There were 3 other students in the SG who did not study a lot (Q404) but who did achieve their goals and all 3 students received an A or an A+ as a final mark. For Q510a (feeling able to succeed in math), one student reported a positive change and the other 2 students reported no change at all. These students have not yet been challenged, so no inference is possible.

S1 (one of the students who experienced goal frustration) also studied 'very little' but for Q501a the student wrote, "…I know that I'll do well if I apply myself." The students' incremental viewpoint remains unchanged by this course because he did not work hard.

The remaining 2 students are S13 and S23, the other students who experienced goal frustration. Both students reported studying 'almost every day' for 'more than 2 hours' each time (Q404). They both visited the MathHelp center (Q413) and sought the help of a tutor (Q414). This defines a situation in which hard work did not produce good results; a situation which can lead the students to believe that they are not very good at mathematics and one that reinforces an entity viewpoint.

There are a total of 8 students who appear to hold an incremental viewpoint, 4 students who hold an entity view and a determination was not possible for the remaining 3 students.

E-4.2 Self-efficacy

Initial assumption. This was assumed to be low for the HS.

Evidence. It was concluded above (Section E-2.5), that the RS had perceptions of low ability, was nervous about passing this course but had high expectations of success. The RS also believed that hard work should produce good results (Section E-4.1). Ten SG students agreed with the statement, "Before the course, I thought that I could succeed in math," 2 disagreed and 3 students remained neutral (Q510).

Most of the students in the SG felt that they could accomplish the task of passing the course, had high outcome expectations of success and believed that their efforts would have the desired effect. This is evidence that the RS had more than a low self-efficacy belief. The student though, was aware that his or background in mathematics was weak and with perceptions of low ability accompanied by nervousness about passing the course, this dimension should not be considered high for the RS.

Conclusion. The RS had at least a moderate self-efficacy belief.

Change in self-efficacy. The Net Change for Q510a (feeling able to succeed in math) was +7. A majority of the students in the SG felt more strongly that they could succeed in mathematics after this course.

E-4.3 Mastery/Helpless Orientation

Initial assumption. It was assumed that the HS (because of prior difficulties in mathematics) had adopted a helpless orientation and might have exhibited features of learned helplessness.

Evidence. Q601 asked the students about their behavior when they were "stuck" on a mathematical problem. Ten students reported that they persisted or asked for help (mastery orientation). One student wrote, "Take a break or move on to another problem," another student simply wrote, "move on" (helpless orientation).

Conclusion. The only conclusion that can be drawn about this dimension at the outset of the course is that the RS did show some signs of persistence in the face of difficulties.

Change in mastery/helpless orientation. On the first online quiz, there were 104 recorded attempts that were made by 35 students. Despite having unlimited attempts for the first online quiz, no student in the class persisted until a perfect score had been recorded. On the 2nd and 3rd quizzes there were a limited number of attempts possible but only one student (on each quiz) exhausted their attempts. There was some persistence, indicative of a mastery orientation, but not to the extent of getting a perfect score.

From comments that were recorded on the online quiz questionnaire, 10 students (of the 27 students who completed the questionnaire) wanted to have the correct answers or solutions posted after an attempt. These students are indicating an unwillingness to persist at the problems that they had got wrong.

Two students wrote the two midterms but not the final exam. Both students had high marks on the midterms so they did not abandon the course because of difficulties with the material. All of the students in the consent group (37 students) wrote both midterms and the final exam. There was not a single student who abandoned the course, even if the student had very low marks on the midterms.

When students who adopt a helpless orientation are faced with potential failure, they may attempt to change the task or the rules (Reeve, 2005). There were 2 individuals in the SG that did receive a failing final grade for this course and both these students (S1 & S13) contacted me before the exam. S1 wanted to delay the exam and S13 wanted to have the final grade entered as 'incomplete'. Even so, these students did not give up and they both wrote the final exam. S1 maintained a positive outlook (a mastery orientation) and for Q310 made the comment, "I have registered to take the course again in the fall and am excited for it."

S13's comment for Q309 was, "I have tried continuously in this course and despite my efforts I was unable to obtain a passing grade." This comment was similar to a remark made by S23, "I was expecting a better grade due to the commitment I put into this class." (S23, along with S1 and S13 experienced goal-frustration but S23 passed the course.) Both S13 and S23 worked very hard in this course and sought help. It is entirely possible that their results in this course were responsible for the development of some learned helplessness.

E-4.4 Explanatory Style

Initial assumption. It was assumed that the HS would exhibit a mixture of both optimistic and pessimistic explanatory styles.

Evidence. The responses to Q207, which asks students to enter reasons why they had difficulties in mathematics before taking this course show both internal and external factors. External factors that were entered are: "The teacher wasn't good"; "prerequisites"; "I had other obligations to fulfill"; "lack of explanation"; "I was taking higher courses before taking the beginners courses"; and "bad teacher." Internal factors that were cited include: "lack of practice"; "I did not study enough"; "lack of attention"; "lack of motivation"; and "I find the processes very complicated." The extent to which these factors are seen as controllable may vary considerably from one individual to another.

Conclusion. The RS exhibited a mixture of explanatory styles.

Change in explanatory style. This characteristic can only be described for the 3 students (S1, S13 and S23) who did not achieve their goals. For Q604 (List anything that would have helped you do better in this course), S13 commented, "Having more time and being able to have class everyday for a shorter period of time rather than less often as it was too much material at one time," and S23 wrote, "time, since I took it during the summer and I felt I did not have enough time to do better." Time, as an external factor that can be perceived as controllable (the students could opt to take the course during a full semester) is indicative of an optimistic explanatory style. S1 did not work very much during the course and cited "personal circumstances" as the reason. This student had a positive outlook (mastery orientation), even after a failure because he had attributed his difficulties to external factors and hence can be said to have an optimistic explanatory style.

E-4.5 Ego-Defensiveness

Initial assumption. The initial assumption made about the HS was that the student was vulnerable.

Evidence. For Q508, 6 students in the SG agreed with the statement that math sometimes had made them feel stupid. As noted before, 6 students reported a previous failure in mathematics (Q205) and 8 students were nervous about passing this course (Q308). For 11 students in the SG, it has been more than two years since their last mathematics course. Entering an algebra class under these circumstances certainly could be perceived as threatening.

Conclusion. This assumption is maintained in the characterization of the RS.

Addressing ego-defensiveness. At the outset of the course, the students were somewhat reluctant to ask questions. Many students asked questions during the break and after class rather than during the lecture. Very few students visited during office hours. For Q413, there were 5 students who visited the MathHelp center and for Q414, 4 students reported that they had sought the help of a tutor. The conclusion can be made that some of the students sought help.

E-4.6 Affect

Evidence for dimensions of affect is drawn primarily from Part V of the survey. The students were asked to agree or disagree with 10 statements. The results for Part V of the survey (Q501-Q510) are reported as Net Affect. Results for questions about affect after the course are reported as Net Change. These quantities are explained in Appendix D. For all of the questions in Part V, the Net Affect before the course was positive and Net Change was also positive.

E-4.6.1 Affect/Emotions

Initial assumption. The assumption was made that mathematics had aroused predominantly negative emotions for the HS.

Evidence. For Q212, 13 students agreed with the statement that they felt "good about themselves when they solved a math problem." Eight students felt nervous about passing MATH 200 (Q308). Net Affect was +1 for Q508 [Math sometimes made me feel stupid], but there were still 6 students who had felt "stupid" because of mathematics. Seven students reported receiving high marks in high school mathematics courses (Q204) which can be responsible for positive emotions and 6 students reported a previous failure (Q205) which may have generated negative emotions.

Conclusion. There is no clear indication whether mathematics had aroused mostly positive or negative emotions for the RS but since they are not uniformly negative, the assumption will be stated in a more positive way: Mathematics had aroused *some* negative emotions for the RS.

E-4.6.2 Affect/Attitudes

Initial assumption. The assumption was made that the HS did not particularly like mathematics.

Evidence. Net Affect was +2 for Q501 [Before the course, I liked math]. Net Affect was +4 for Q509 [Before the course, math sometimes was fun].

On course evaluations, for question 16, students were asked what their level of interest was in the subject before taking the course (Table 5.3). The overall assessment of results for this question, when compared to a bank of data maintained by the university, was that the students' interest was average.

Conclusion. With positive net attitudes and an average level of interest, this assumption was also worded more positively. It was concluded that the RS did not especially *dislike* mathematics.

E-4.6.3 Affect/Beliefs

Initial assumption. The initial characterization of the HS was that the HS believed that mathematics was difficult, algebra was too abstract and that only the very gifted could succeed at mathematics. (The student may have believed that being a "math-person" is a quality that they did not possess.)

Evidence. The questions that relate to the beliefs of the students and the Net Affect recorded for the questions are summarized in Table E.2. All of the Net Affect values for these questions are positive. Net Affect for Q502 and Q507 are not large, meaning that several students in the SG do believe that algebra is abstract and that they were not "good" at writing mathematics tests. The RS believed quite strongly (Q510) that he or she could succeed in mathematics.

Table E.2 Summary of Net Affect for beliefs about mathematics

Question: Before the course...	Net Affect
Q502: Algebra was too abstract for me.	+2
Q503: Math was difficult for me.	+5
Q504: I usually understood what was presented in class.	+6
Q505: I felt that math was for some people but not for me.	+6
Q507: I was not good at writing math tests.	+1
Q510: I thought that I could succeed in math.	+8

Conclusion. The conclusion was made that the RS believed that mathematics was attainable and did not find it overly difficult or abstract. The RS felt that he or she could succeed in mathematics.

E-4.6.4 Affect/Values

Initial assumption. The HS did not see mathematics as useful in the real world and did not necessarily value success in mathematics.

Evidence. For Q506 [Math is a complete waste of time unless it is specifically geared towards your career], the Net Affect recorded was +11. For Q213 (Table 6.1), many of the students indicated that they worked hard because mathematics would help them in other courses or to get a better job.

Conclusion. The RS did see mathematics as useful and valued success in mathematics.

E-4.6.5 Change in Affect

All of the values for Net Change in the dimensions of affect were positive and the total of these was +45. Individual responses however, were not uniformly positive before the course and the net change was not positive for all students either.

There were however, 9 students who reported a net positive change in affect and another 2 students who reported no net change.

Note. There is no supplemental evidence for the conclusion about the social need for achievement of the RS. (The evidence is presented in Chapter 6.) This concludes the presentation of evidence for the characterization of the RS.

E-5 Motivation of the RS

In this section, the details of the evidence for the behavior and self-regulation of the RS during the course are presented.

E-5.1 Behavior of the RS

Evidence. The students in the SG reported studying in expected ways. All the SG students attended class (Q401), 12 students did at least some of the suggested problems from the text book (Q406), 12 students read explanations from the text (Q409), 14 students used the online quizzes at least once (Q411) and 13 students attempted the practice exams (Q415). Also for Q408, 12 students indicated that they did extra problems from the text, even if they weren't listed on the course outline.

A high probability of response was indicated by the following survey questions: Ten students did most of the problems from the text (Q406), 9 students read the explanations in the text book "often" (Q409), 12 students indicated that they studied on a regular basis (Q404) and 8 students studied more than 3 times a week (Q404a).

Persistence was observed in the number of attempts that students made for the online quizzes and for Q404b, 7 of the 12 students who studied regularly, studied for more than two hours each time they studied. There were 4 students who reported studying more than 30 hours for the tests (Q405). More important perhaps, is the result that, whatever amount of studying they actually did, 9 of the students thought it was a lot (Q403).

E-5.2 Self-regulation and Feedback

Evidence. There were 6 students in the consent group of 37 students who did not pass one or both of the midterms and of these, 2 students did receive a passing grade on the final but 4 did not. The encouraging e-mail that was sent to students who had low marks on the first midterm was acknowledged by 2 of the students in the class and both students indicated that the situation was under control but it cannot be determined what changes the students made to their behavior.

Students also receive feedback when they compare their performance to previous results. There were 6 students in the SG (Q507a) who felt they were better at writing mathematics tests after the course (8 students felt there had been no change and one student entered 'worse at writing tests'). A comparison of students previous results with their results in this course (Table D.17) shows that 7 students received the feedback of obtaining a better mark this time. The 2 students, whose previous marks were usually lower than 60%, both passed this course.

The emotions that accompany an understanding are also feedback to the student. For Q504a there were 9 students who felt that they understood more of what was presented in this class than in previous classes. On course evaluations, 25 of the 33 students felt that their learning in this course had been "good" or "very good."