Two Secondary School Mathematics Teachers' Use of Technology through the Lens of Instrumentation Theory

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Abstract

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The primary goal of our research was to gain a sense of the technology that secondary school mathematics teachers in Montreal, Québec are currently using, how they are using it, and some of the reasons why they use the technology that they do, in the ways that they do. The secondary goal was to test the effectiveness of our approach in obtaining this information. The approach consisted of interviewing two local secondary school mathematics teachers. The interview questions prodded at what Instrumentation Theory suggests to be some of the fundamental aspects of one's interactions with technology; the 'artifact' (the particular technology), the subject (the user of that technology), the 'task' that the subject tries to complete with the artifact, the 'instrumented techniques' that they employ to complete the task (which reveal some of their 'schemes of use'), and the process through which the subject and the artifact interact and 'shape' each other, called 'an instrumental genesis'. The teachers' responses to the interview questions revealed that, although they both used most of the same technology (with a few exceptions), significant differences existed between the ways that they used some of them, why certain technologies were used, and why others were not. The two teachers also differed in their views on the value of their instrumented techniques. These findings are discussed in light of the literature review, demonstrating some of the effectiveness of our approach. We believe that our approach was useful as it allowed us to elicit detailed descriptions of these two teachers' uses

of technology and because it facilitated the analysis of the data (as the questions were based on the same theoretical framework that was then used to analyze the teachers' responses). We conclude with some suggestions for future research. One of the suggestions addresses ways in which our approach could be improved to give researchers who might use it in the future more informative responses.

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Chapter 1: Introduction

In March of 2008, National Council of Teachers of Mathematics (NCTM) released a statement on what they proposed should be "the role of technology in the teaching and learning of mathematics" (Appendix C). The statement included strong claims such as: "technology is an essential tool for learning mathematics in the 21st century"; "effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics"; "calculators and other technological tools, such as computer algebra systems, interactive geometry software, applets, spreadsheets, and interactive presentation devices, are vital components of a high-quality mathematics education"; "all schools and mathematics programs should provide students and teachers with access to instructional technology, including appropriate calculators, computers with mathematical software, Internet connectivity, handheld data-collection devices, and sensing probes". Suggesting that technology should have such an important place in mathematics education, the NCTM cautioned the teachers, however, that,

> The use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills. In a balanced mathematics program, the strategic use of technology enhances mathematics teaching and learning. Teachers must be knowledgeable decision makers in determining when and how their students can use technology most effectively.

Some of the NCTM statements were formulated in a conditional form (which I find more appropriate), such as, "when technology is **used strategically**, it can provide access to mathematics for all students", or "**in a well-articulated mathematics program**, students can use these tools for computation, construction, and representation as they explore problems". Although the current state of knowledge about the effects of technology on the teaching and learning of mathematics does include some awareness of the conditions under which technology can be beneficial for mathematics teaching and learning, the above conditional statements of the NCTM don't seem very useful. The points that they reflect are that the usefulness of technology in mathematics education depends not only on the technology used, but on how it is used, and that "the ways in which teachers adopt technology" depends on "what counts as good teaching in their institutional culture" (Goos & Bennison, 2007). The NCTM is not, however, clear on what is meant by "a well-articulated" or "balanced mathematics program", or "the strategic use of technology".

The statement left me with the impression that a break-through had occurred; that teaching with technology has been widely accepted as ultimately better than teaching without it, and that teachers who were not making regular use of it were disadvantaging their students. This had me think of the secondary school mathematics teachers that I had once worked with, of varying academic and professional backgrounds. Thinking of their various levels of comfort with using technology, knowledge about how to do so to some pedagogical advantage, and beliefs about the usefulness of teaching with technology, I wondered what effect on these teachers' morale, practice, and their students' learning would be, should they take to heart the NCTM statement and immediately start implementing a large variety of sophisticated technology into their practice. My concern was that the NCTM statement did not mention how, perhaps, the pace at which teachers integrate technology into their practice should depend on their current experience with, comfort in using, and knowledge about technology.

This concern inspired me to seek out examples of teachers' current uses of technology, the reasons behind their choices of which technology to use and which not, to complete specific tasks, in specific ways. Rabardel and Vérillon's (1995) theory of processes of learning to work with all kinds of technological artifacts ("Instrumentation Theory") inspired me to seek information not only about the tasks that teachers were completing with technology, but also in the *techniques* they employed to complete those tasks.

The primary goal of my research thus became to gain a sense of the technology that secondary school mathematics teachers in Montreal, Québec are currently using, how they are using it, and some of the reasons why they use the technology that they do, in the ways that they do. To reach this goal, I asked several secondary school teachers from this area to agree to a sequence of detailed semi-structured interviews on their practices in using technology. I was somewhat acquainted with those teachers and chose to contact them because they appeared to represent some variety in the use of technology in mathematics teaching. Two teachers agreed. These two teachers appeared to present widely different practices and views, which would allow me to see more sharply the characteristics of each.

The secondary goal of the research was to test the effectiveness of the interview design in obtaining this information. The interview was structured based on the framework of Instrumentation Theory. The interview questions aimed at identifying what Instrumentation Theory suggests to be some of the fundamental dimensions of one's interactions with technology:

- The 'artifact' (the particular technology);
- The subject (the user of that technology);
- The 'task' that the subject tries to complete with the artifact;

- The 'instrumented techniques' that they employ to complete the task (which reveal some of their 'schemes of use').

The teachers' responses to the interview questions revealed that, although they both used most of the same types of artifacts (with a few exceptions), significant differences existed between the tasks they used them for, the techniques they employed and how what they did in class was affected by the technology, and affected their use of the technology. The two teachers also differed in their views on the value of their instrumented techniques.

Designing the interview questions with the Instrumentation Theory in mind facilitated the analysis of the transcripts of the interviews, since the aim of the analysis was to answer the research questions and the research questions were inspired by the same theory.

This thesis is composed of eight chapters. The first is the present introductory chapter. The second presents a review of some of the literature on secondary school mathematics teachers' uses of technology, the factors that discourage them from using it and the opportunities that many researchers believe can be afforded by teaching with technology. The third chapter gives a brief overview of the main aspects of Instrumentation Theory. The fourth chapter presents the methodology of the research by explaining how Instrumentation Theory was used to structure the interview questions and analyze the results in view of the goals of the research. The choice of the semi-structured interview method is also justified.

Chapters 5 and 6 present the results of the interviews. Chapter 5 is organized along the artifacts that each teacher used in the past, were using at the time of the interviews, and those that they planned to use in the future. Each artifact is then discussed in term of the other dimensions of the Instrumentation Theory (tasks, techniques, ways in which the subject and the technology shape each other). Each teachers' reason(s) for using or hesitating to use each type of technology in certain ways are also discussed, as are their views on the epistemic and pragmatic values of their uses of technology. Chapter six uses the accounts of individual interviews presented in Chapter 5 to compare the two teachers' uses of technology and views about the use of technology in mathematics teaching in general. Although some readers might be interested in the details presented in Chapter 5, others might not be, and can skip to Chapter 6.

In Chapter 7, the results are discussed in light of the literature review. In Chapter 8, some suggestions are made for future research.

Chapter 2: Literature Review

As mentioned above, the primary goal of my research was to get an idea of the technology that secondary school mathematics teachers are actually using, the tasks they seek to complete with it, how they are using technology to complete those tasks, why they are using this technology in those ways, and why they hesitate to use certain technology in certain ways. The literature reviewed in this chapter will be that which attempted to answer such questions in the past. One exception was made, however. Rather than review literature on the reasons why teachers use technology, I will review literature on the opportunities that researchers claim can be afforded by teaching with technology. I do so mainly for two reasons. For one, the 2008 NCTM statement encouraged the use of technology by alluding to some of the latter research. I therefore believed that presenting it might help the reader understand why the NCTM is so adamant about the importance of the use of technology in secondary school mathematics education. Second, I do so because reviewing this literature allowed me to compare these opportunities with teachers' reasons for hesitating to use technology and to conjecture why, even if teachers were aware of these opportunities and believed that they could be afforded by technology, many might still hesitate to use it. It became clear after our interviews that teachers who are currently using technology compare the opportunities they believe can be afforded by certain technology with **their** reasons for hesitating to use it, relative to each task **they** consider using technology for. Last, although I did not ask teachers whether they believed that these opportunities could be afforded by the (or their) use of technology, I was curious to see whether some of these opportunities would be mentioned when the teachers discussed their reasons for using technology.

The literature on secondary school mathematics teachers' reasons for hesitating to use technology about integrating technology into their practice, on the opportunities that researchers believe can be afforded by teachers integrating technology into their practice, and the types of technology used is quite extensive. There has been much less work done, however, on how teachers actually use technology in their practice, and the tasks that they seek to complete with it. Further, the literature that does exist on the latter differs considerably in how authors describe the ways in which teachers use technology. Examples of this will be given below.

As I will obviously not be able to review all of this work, I decided, in terms of the work on teachers' uses of technology, only to review some of the work published after 2004. I made this decision based on my conjecture that, with new technology being created seemingly every month, teachers' practice years ago would likely no longer be representative of their current use. In terms of the teachers' reasons for hesitating to use technology, a handful of papers (Norton et al., 2000; Forgasz, 2006; Thomas, 2006) will be synthesized. In terms of the opportunity that researchers believe can be afforded by technology, this review will focus on pedagogical opportunities and those related to assessment (Pierce & Stacey, 2010; Bokhove & Drijvers, 2010).

Section 2.1 will give examples of the types of technology that different researchers have found secondary school mathematics teachers and their students to be using. Section 2.2 will describe the ways that they used the technology (the tasks they completed with it, and the instrumented techniques they employed to do so). Section 2.3 will list some of the opportunities that researchers claim can be afforded by teaching with technology. Section 2.4 will list several reasons why teachers hesitate to use technology.

2.1 Types of technology

According to researchers' findings (Hennessy, Ruthven, & Brindley, 2005; Goos & Bennison, 2007; Peirce & Ball, 2010; Stoilescu, 2011), teachers use a variety of hardware, software, internet resources and classroom settings when working with technology in their mathematics classrooms. Hardware use ranges from handheld calculators (four-function, scientific, graphing, with computer algebra systems), through desktop computers, laptops, projectors (for handheld calculators, for computers, and video projectors), interactive whiteboards, CDs accompanying a textbook to classroom clickers (for classroom clickers, see, e.g., Wit, 2003). Teachers use software such as CAS (computer algebra systems), dynamic geometry software, dynamic data software, LOGO, data-loggers, word processors, spreadsheets, presentation software and PDF converters. Internet resources mentioned as used by teachers include animated and interactive graphs or figures such as Applets (for Applets, see, e.g., Heath, 2002); internet revision sites; lesson materials from dedicated websites such as "Gizmos" (ExploreLearning. Cambium Learning Group, 2012), and Video clips. One of the teachers mentioned in (Stoilescu, 2011) maintained a personal website and planned to construct or join a teachers' community website ("Wiki"). Some teachers worked only in regular classrooms, some combined regular classroom and computer lab sessions, and some used the computer lab for all their classes.

For the reader's benefit, I made a list of many of the types of technology that have attracted the attention of researchers. I include this list in Appendix C. The list may not be exhaustive, but it could be a useful resource for researchers who are new to the domain, as it will give them a sense of the range and the amount of literature that exists. I also include references to at least one paper on the use of each type of technology in mathematics education. Last, I include a ‘map’ of how this research ‘fits together’ (i.e. it explains, visually, which types of technology are examples of other, more general types of technology). This map is included to help researchers associate the literature referred to; for example, to understand that Computer Algebra Systems (CAS), Dynamic Geometry Software (DGS), Spreadsheets, and Microworlds are examples of Mathematical Analysis Software (MAS).

2.2 How technology is used

The descriptions available in the literature of how technology is used are described in very different ways by each author. Also, although all of the papers that I have read on how teachers use technology described (in some way) the tasks that technology was used to complete, only one (Goos & Bennison, 2007) described the ways in which the teachers used technology to complete those tasks.

Hennessy et al. (2005) distinguished between tasks that use technology to support existing practice from tasks where technology substantially changes the existing practice. The first type of tasks include those where students learn the same material but with a more experimental or investigative approach, and have access to more information resources. The second, at least in their research, included “employing dynamic visual representations to improve student access to difficult underlying mathematical and scientific concepts and using simulations for interaction with otherwise dangerous or complex phenomena” (ibid.).

A different division of tasks is proposed by Pierce and Ball (2010). They distinguished between teachers’ tasks and students’ tasks, and then divided students’ tasks into tasks for “doing mathematics” and tasks for “learning mathematics”. In tasks for “doing mathematics” by students, software is used to “perform harder, more complicated calculations quickly and correctly”. These tasks were the most commonly used among the teachers surveyed by these researchers. Technology was rarely purposefully used to support "deeper learning" of mathematical ideas.

Thomas (2006) also described tasks that teachers had completed with technology. To be able to obtain this information, he sent surveys to 336 secondary schools in New Zealand. 193 of the 336 schools and a total of 465 teachers responded. He categorized tasks as "teachers using the computer for skill development", teachers using computers for "demonstrations", teachers allowing their students "free use of the computer", teachers accommodating "student-centred investigation and problem solving", and students' use of computers for "programming". In terms of Pierce and Ball's (2010) categorization of tasks, Thomas (2006) learnt of teachers' using technology towards completing teachers' tasks (using it for demonstration and to develop their students' skills) and of students using technology towards "learning mathematics" (using technology freely, investigating and problem solving, and programming). He had not learnt of students using technology for "doing mathematics".

Goos and Bennison (2007) described two activities. According to Pierce and Ball's (2010) categorization, one involved students' tasks for learning mathematics and the other – teachers' tasks and students' tasks for learning. In one of the examples, students had to do systematic observations of the flight of a projectile which they could control using two variables, tabulate the data, and construct a mathematical model of the relationship between the distance covered and the values of one of the two variables. The students used desktop computers, animated and interactive figures, and graphing calculators to complete these tasks. The description of the way technology was used was given in some detail.

For example, students worked on a task that asked them to investigate projectile motion as a practical application of quadratic functions. They viewed a computer simulation in which the Sesame Street character Gonzo was shot from a cannon towards a bucket of water some distance away (http://www.funny-games.biz/flying-gonzo.html). The simulation allowed students to vary the angle of projection and the cannon "voltage" (i.e., muzzle velocity) and observe the effects on the distance Gonzo travelled as they "aimed" him at the bucket of water. They were to use their graphics calculators to tabulate and plot data that would allow them to find a mathematical model for the relationship between this distance and the muzzle velocity. Algebraic methods were then to be used to determine the best cannon settings for Gonzo to hit a target at a given distance.

The reader of the paper is given a link to the game that was used in the task, so that she or he can play it and thus imagine what techniques the students could invent to solve the task.

2.3 A list of opportunities that researchers claim can be afforded by teaching with technology

The following list of opportunities has been categorized into 'opportunities related to enrichment of the mathematical content', 'opportunities related to more efficient learning, particularly algebra', 'opportunities related to more balanced power and authority relations in classroom interactions', and 'opportunities related to more efficient assessment'.

Opportunities related to enrichment of the mathematical content

According to Pierce and Stacey (2010), technology can help teachers:

- "Exploit contrast of ideal & machine mathematics". This can be achieved when "teachers deliberately use 'unexpected' error messages, format of expressions, graphical displays as catalyst for rich mathematical discussion";

- "Re-balance emphasis on skills, concepts, applications". This can be achieved when "teachers adjust goals: spend less time on routine skills; more time on concepts and applications. Increase emphasis on mathematical thinking";
- "Build metacognition and overview". This can be achieved when "teachers give overview as introduction or summation: link concepts through manipulation of symbolic expressions and use of multiple representations";
- "Use real data". This can be achieved when teachers and their students "work on real problems involving calculations that, done by hand, are error prone and time consuming";
- "Explore regularity and variation". This can be achieved when teachers "strategically vary computations", "search for patterns", "observe effect of parameters", or "use general forms";
- "Simulate real situations". This can be achieved when Teachers and their students "use dynamic diagrams, drag and collect data for analysis" or "use technology generated statistical data sets";
- "Link representations". This can be achieved when teachers use technology to "move fluidly between geometric, numeric, graphic and symbolic representations."

Opportunities related to more efficient learning, particularly algebra

Technology is deemed to help students:

- Learn pencil-and-paper skills. For example, allowing students to use their graphing calculators during class to check the rough shape of the graphs that they draw (with pencil and paper), provides them with feedback on their understanding of, and their ability to use, pencil-and-paper techniques for graphing functions; (Pierce & Stacey, 2010)

- Make sense of symbolic expressions; (Zorn, 2002)
- Develop algebraic skills (Bokhove & Drijvers, 2010)

Opportunities related to more balanced power and authority relations in classroom interactions

Technology can:

- Change the dynamics of social interactions in the classroom, when teachers do not impose a strict agenda, encourage group work and sharing of students' learning in discussions with their peers; (Pierce & Stacey, 2010)
- Changes the role of the teacher to one of a facilitator and allow their students to use technology to support the construction of their knowledge. (Pierce & Stacey, 2010)

Opportunities related to more efficient assessment

Bokhove and Drijvers (2010) suggest that certain technology can facilitate the creation of formative assessment ("assessment for learning") as they:

- Can provide several assessment modes (e.g., practice, test);
- Can cater for several types of feedback (e.g., conceptual, procedural, corrective);
- Have a review mode showing what the student has done wrong or right;
- Support more than one language;
- Can take the mastery and profile of the student into account and serve up appropriate questions.
    - They also suggest that certain technology can help with the preparation and correction of summative assessments ("assessment of learning") as they:
- Have the ability to randomize algebra assignments;
- Have the ability to combine questions into larger units to enable multi-component tasks;

- Have authoring functions that enable teachers to add or modify content (e.g., questions, texts, links, graphs, feedback);
- Can allow for the use of several question types (e.g., multiple choice, open);
- Can be programmed to electronically assess and grade students' answers;
- Store the answers and the solution processes of the students.

2.4 A list of teachers' reasons for hesitating to use technology (at all or to complete certain tasks)

The following list of teachers' reasons for hesitating to use technology has been categorized into, 'concerns about possible impoverishment of mathematical content', 'concerns about possibly less efficient learning', 'concerns about disturbing the balance of power and of authority relations in the classroom', 'concerns related to time management', and 'concerns about some teachers' the lack of confidence in using technology for teaching'.

Concerns about possible impoverishment of mathematical content

- One teacher in Norton et al. (2000) was concerned that, when students use computers, they "don't get the opportunity to work it out for themselves" because "they've got a gadget to do it for them". She believed, however, that "going through algorithms is very important since everything is based upon those algorithms". She believed that mathematics is "based upon laws and theorems and algorithms" and that "pressing a button and the computer giving the answer doesn't help students' understanding of what mathematics is all about".
- Another teacher was concerned that "computers were not useful in explaining mathematical concepts to students or in illustrating examples of mathematics

procedures." He believed that the usefulness of computers was more in their ability to facilitate "number crunching", but that "teachers explanation and student practice were most effective for teaching mathematics." (Norton et al. 2000)

Concerns about possibly less efficient learning

- Thomas (2006) noticed that some teachers believe that "sometimes some students rely too heavily on [technology] without really understanding basic concepts and unable to calculate by hand." (Thomas, 2006)
- Norton et al. (2000) described how one teacher who participated in their study believed that computers could be used to illustrate concepts and speed-up computations, but was concerned that computers were not "as efficient as [teacher] explanations (time wise)" (Norton et al. 2000). She saw this as an important consideration, mentioning that teachers' time to prepare their students for their evaluations was limited. She therefore only used (or had her students use) technology "after students had learnt the concepts through traditional instruction", and only if "time permitted".
- The concern that computers "do not prepare students for assessment" (Norton et al. 2000), or at least not as well as their traditional teaching practices did, was fairly common in the literature. The truth or falsity of this statement likely depends on how the student is being assessed, and could have been voiced by a teacher who tested his/her students in a pencil-and-paper environment.

Concerns about disturbing the balance of power and of authority relations in the classroom

Forgasz (2006) found that many teachers were concerned about student discipline, attitudes, behaviours, and changes to their role as teachers.

Concerns related to time management

Teachers were concerned about the time required:

- To "cover the syllabus"; (Forgasz, 2006)
- To "plan computer-based mathematics learning"; (Forgasz, 2006)
- For students to develop "computer skills" and "mathematical skills"; (Forgasz, 2006)
- "By both students and teachers in order to become familiar with the technology". They were also concerned about the effort necessary to do so. (Thomas, 2006)

Concerns about some teachers' lack of confidence in using technology for teaching

Teachers claimed that they lacked the confidence needed to teach with technology (and, in particular, with computers), as they:

- Suffered from a "computer phobia"; (Forgasz, 2006)
- Lacked professional training in using computers in mathematics instruction (Forgasz, 2006):
    - o Lacked knowledge about appropriate ways to use computers to enhance mathematics learning; (Forgasz, 2006)
    - o Lacked "models of how to integrate technology into mathematics teaching". (Norton et al. 2000)

2.5 A theoretical debate between researchers and teachers

Teachers and researchers seem to disagree about whether using technology and/or allowing their students to use technology will enrich or impoverish mathematical content. They also seem to disagree over whether their students will learn more or less efficiently when technology is used. Further, because of their opposing views on these two points, I believe that teachers and researchers would likely disagree about whether teaching with technology is more effective and/or efficient than teaching without it, in preparing students for summative assessments. As mentioned above, whether researchers and teachers agree or disagree about this would likely depend (for one) on how the students are being assessed. For instance, if students were to be tested in pencil-and-paper environments, then teachers and researchers would likely disagree, as some researchers have claimed that "technology can be used to support traditional methods and existing curriculum" (Program Committee of ICMI-ALGEBRA Study, 2000); some teachers, however, do not seem convinced of this. Stacey and Chick (2000) argue, however, that using technology towards teaching these traditional goals does not allow for the full range of benefits of teaching with technology.

> The increased availability of computers and calculators will change what mathematics is useful as well as changing how mathematics is done. At the same time as challenging the content of what is taught, the technological revolution is also providing rich prospects for teaching and is offering students new paths to understanding. (Stacey & Chick, 2000)

I think that this difference of opinion about the potential for technology to improve mathematics education is highlighted best by an interesting insight by Pierce and Stacey (2010). They claim that the functional opportunities provided by technology ("to be able to carry out routine mathematical procedures (calculating, drawing, solving etc.) quickly and accurately"), which teachers seem aware of, "form a foundation for" pedagogical opportunities and for opportunities related to assessment. It is clear, however, that some teachers do not share this belief.

Although they might disagree over certain issues, teachers and researchers would likely agree about how, if technology is to be "used strategically" (as the NTM put it), the balance of power and authority relations in the classroom will change. Researchers might even encourage teachers to re-define their ideas of discipline and 'appropriate' behaviour as they change their roles from dictators to facilitators. In terms of teachers' concerns about students' attitudes, Souter (2002) claimed that "students in technology-enhanced classes… were more motivated, and had a more positive attitude than those in traditional algebra classrooms".

Many researchers and teachers would also likely agree that time and effort is required by students and teachers to become familiar with technology and, therefore, to be able to use it, and have their students use it, to their respective benefits. They would also likely agree that some of this time need be spent in professional development and other forms of training "on appropriate ways to use computers to enhance mathematics learning" (Forgasz, 2006). Researchers might argue that some of this time can be allowed for by delegating certain tasks related to preparing and correcting different types of assessment.

Norton et al. (2000) claim that "if reformers wish to encourage teachers to adopt the potential of computers, the teachers need to be convinced that the use of computers can make their tasks more effective and efficient than their current strategies". To be convincing, reformers might work to make teachers aware of Pierce and Stacey's (2010) insight, that the functional opportunities allow for pedagogical opportunities and opportunities related to assessment. Something that I am unsure about, however, even after reading of researchers' optimism about the potential usefulness of teaching with technology, is whether certain researchers believe that any teacher's practice would be improved by integrating technology as regular part of it and whether all teachers should be working towards using similar types of technology in similar ways. I know teachers who do not believe that all teachers will ever become so comfortable using technology that the benefits for their students will be guaranteed. I hope that our accounts of two teachers' uses of technology will encourage researchers and reformers to consider which technology they want teachers to be using and how they want them to be using that technology. I believe that this should differ from teacher to teacher.

Chapter 3: Theoretical Framework

In this section, I will introduce some of the fundamental ideas of Rabardel and Vérillon's (1995) instrumentation theory. These ideas shaped both the design of the interview questions and the analysis of the transcripts of the interviews.

In the 1980s, two ergonomists, Pierre Rabardel and Pierre Vérillon, started developing a theory of the cognitive processes that take place and are necessary for people to learn to use material objects (artifacts) as the instruments of their work. In the 1990s, the theory started to be known as "Instrumentation theory" and attracted the interest of several mathematics educators in France (Artigue, 1997; Guin & Trouche, 2002; Lagrange, 1999; Lagrange, 2000). The interest has now spread to other countries as well (e.g., Drijvers & Trouche, 2008; Thomas, Monaghan, & Pierce, 2004).

The theory offers "a theoretical point of view" on "the relationship between cognition and the artifactual nature of many of the objects on which it is brought to bear in everyday, work and school situations" (Rabardel and Vérillon, 1995, p.77). Mathematics educators adopted the theory to speak about the relationship between students' and teachers' cognition and the technology that used in teaching and learning mathematics.

The central notion of this theory is that of *instrument*. Vérillon (2000) defines 'an instrument' as "any object that a subject associates to his or her action in order to carry out a task" (p. 4). Before it is associated with the subject's action, however, the object is referred to as an 'artifact'. 'Artifact' is meant to refer broadly to all kinds of objects, material and symbolic (e.g., a technical drawing is a symbolic object). In this thesis, however, the term 'artifact' will be restricted to any type of information and communication technology (ICT).

Guin and Trouche (2002) describe an instrument as an entity composed of a material object and the cognitive schemes engaged in the execution of a type of task with the help of the material object. The schemes are sometimes called *utilization schemes* since they determine how a person organizes his or her behavior in dealing with tasks of a certain type, and are, therefore, a basis of the person's knowledge in the area (Trouche, 2004). Belonging to the subject, these utilization schemes are seen as "private", but also as "social", as actions are sometimes learnt from others (perhaps from reading an instruction manual, attending a professional development session, or from a teacher or fellow student) (Rabardel and Vérillon, 1995). The process through which the subject associates uses to the artifact is referred to as an *instrumental genesis*: the construction of the utilization schemes turns a material object into an "instrument" for accomplishing some work.

A utilization scheme, which is a cognitive entity, is not directly observable. Observing a person performing always the same sequence of actions in dealing with tasks of the same type suggests, however, that a utilization scheme is at work. We can say that the person has a *technique* for performing such tasks (Guin & Trouche, 2002). A technique can be understood as "the observable counterpart of the invisible mental scheme" (Drijvers, Kieran, & Mariotti, 2010).

Artigue (2002) explains how every technique has 'a pragmatic' and 'an epistemic value'. That is, techniques enable the subject to perform actions (pragmatic value), but can also be used to learn something (epistemic value). For example, Vérillon (2000) suggests how a climber might use an ice pick either to stabilize him/herself or to test the depth of a patch of snow.

The theory also suggests that an instrumental genesis affects both the user and the instrument. The process through which the user of an artifact decides what to do with it and how is called 'instrumentalization'. The process through which the artifact's features (the opportunities that it provides and the constraints that it imposes) 'shape' the subject's schemes and techniques is called 'instrumentation' (Drijvers et al., 2010). An instrumental genesis is often an ongoing, never-ending process of mutual interactions between instrumentalization and instrumentation: "when developing the first uses, a person pilots the artefact through existing schemes, then this primitive experience is the occasion of an adaptation of the schemes, and the better adapted schemes are a basis for developing new uses, and so on" (Lagrange, 1999). It has also been acknowledged that sometimes the user will 'shape' an artifact in ways that were "unplanned by the designer: modification of the task bar, creation of keyboard shortcuts, storage of game programs, automatic execution of some tasks (… methods and ways of solving particular classes of equations…)" (Trouche, 2004).

Researchers have recognized "the necessity (for a given institution – a teacher and her/his class, for example) of *external steering* of students' instrumental genesis" (Trouche, 2004), and have termed this phenomenon 'instrumental orchestration'. One example of this would be a teacher teaching his/her students how to complete a specific task by using specific techniques.

All of these ideas influenced the design of our research. I did not have time to speak in depth about each of them, however, with each teacher. I therefore prioritized questions that prodded at the artifacts that they were using, the tasks that they sought to complete with each artifact, the techniques that they employed to do so, the reasons why they used certain artifacts in certain ways (i.e. by employing certain techniques towards performing certain tasks), and the reasons why they hesitated to use certain technology to complete certain tasks. When time allowed, however, I also asked each teacher whether they had 'shaped' certain artifacts in any way (giving the example of whether they had created a program for the graphing calculator, to automate a task), whether features of the software influenced their uses, and about how they ‘orchestrated’ their students’ instrumental geneses (by asking about how they interacted with the students during while they were using certain technology; whether they would walk around, give hand-outs with specific instructions, etc.). Unfortunately, there was only time for very few of the latter types of questions. Out of them, I prioritized the third, as I quickly realized that describing instrumental geneses in detail would require a separate study.

A more detailed description of the questions asked and how they related to the ideas of instrumentation theory is described in sections 4.2.3 and 4.2.4.

Chapter 4: Methodology

This chapter is composed of two parts, sections 4.1 and 4.2. Originally, I was ambitious to have section 4.1 overview all of the research methodologies that I might have employed to learn about teachers' uses of technology and the factors that influence those uses, and to justify why, amongst these, we took the approach that we did. Learning of the large amount of literature available on research design, however, I began to feel that this task would not be accomplishable. Luckily, I found the following quotes by Alan H. Schoenfeld and Gerald A. Goldin, which I found both helpful and comforting:

> Given the broad spectrum of work, any attempt in a single chapter to deal with educational research methods must of necessity be selective rather than comprehensive. (Schoenfeld, 2002)
>
> A research design must be tailored to answering the questions asked. (Goldin, 2000)

With their advice in mind, I adjusted the goal and, in section 4.1, I will summarize two research methods that helped categorize the research design.

Section 4.2 will describe the data sources, the participants, the school at which they worked, and the research procedures.

4.1 A summary of two research approaches

Of the methodological categories with which I have become familiar to some extent, *Interpretive Research* and *Clinical Interviews* seemed best suited to situate the research and help answer the research questions with desired detail. Below, I provide a brief summary of each of these.

4.1.1 Interpretive Research

Tobin (2000) defines interpretive research as "an umbrella term used to describe studies that endeavour to understand a community in terms of actions and interactions of the participants, from their own perspectives". Borko, Whitcomb, and Byrnes (2008) explain that, to acquire this understanding, researchers study the participants' "voice and discourse". To "increase the credibility of what is learned", they use multiple data sources, including transcripts of oral text, videotaped lessons, photographs of "classroom events and phenomena", "sketches and diagrams from the field", "artifacts collected from the classroom or school" (such as samples of students work, lecture notes, etc.), and "field notes and analytic memoranda" based on their observations of the classroom or "the site of study (i.e. the field)" (Tobin, 2000). The analysis of this data is recursive, rather than linear; that is, previously analyzed data is re-analyzed in the light of analyses of subsequent data, and further collection of data is informed by previous analyses. Data analysis is conducted not only after all data is collected but also between interviews, classroom observations, and collections of artifacts. Researchers allow this analysis to direct their research (Borko, Whitcomb, & Byrnes, 2008). For example, answers to interview questions collected one day might influence the questions that the researchers ask on another, or encourage them to visit the classroom or to collect artifacts that they see as necessary to answer their research questions.

In interpretive research, researchers choose "participants serially and contingently", and "accordingly, binding decisions are not made beforehand about who will participate in a study or how many participants will be involved" (Tobin, 2000). If new participants are seen to be needed, the researcher should "choose someone who is most different from the first participant[s] selected and studied" (ibid.).

4.1.2 Task-Based Interviews

Goldin (2000) proposes that structured task-based interviews consist of at least one interviewer asking at least one participant to complete a prepared task (for example, to answer a set of questions or solve a given set of problems). Follow-up questions are often prepared as well, based on certain answers that the interviewer might expect (Hunting, 1997). Hunting (1997) suggests that both the initial and follow-up questions should be open-ended, encourage conversation, and "allow both student and interviewer to reflect on their respective thought processes". To insure that participants' thoughts and actions are freely discussed during the interviews, he also suggests that the interviewer should try to create "a relaxed atmosphere and establish a relationship of trust" (Hunting, 1997).

To keep track of the large amount of information that is elicited, interviews are often video- or audio-taped.

Goldin (2000) also explains how interviewers often prepare interview scripts to allow for other researchers to "employ similar methods, compare results, and confirm or contradict each other. These scripts should include the planned interview questions, descriptions and/or copies of the tasks given to the interviewees, and any other details (e.g., follow up questions, descriptions of the environment in which the participants were interviewed, etc.) that would facilitate their results and analysis to be compared to others. Further, to be able to compare and criticize research, researchers are encouraged to provide detailed descriptions of the theoretical frameworks that influence their analysis of their findings.

As with interpretive research, analysis of task-based interviews can also be recursive and heavily depends on interpretation of the participants' discourse (Hunting, 1997).

4.2 Methodology of the study

Interested in discovering high school teachers' uses of technology, our first major decision was to decide between a large scale study and a small scale one. After a summer of reading various papers on teachers' uses of technology (e.g., Thomas, 2006), it appeared to me that the trend had been to conduct large scale studies, based on survey methods, and whose data was collected through questionnaires that contained open and closed questions. Such surveys do tell us something about the scale of computer use and the kinds of applications used, but not about the way they are used (Hennessy et al., 2005, p.160). Although two of our three main research questions seem to have attracted some attention ('what technology are teachers using' and 'what tasks are they using them to accomplish'), our third question (that of constructing detailed accounts of the instrumented techniques that teachers were using to accomplish these tasks) seemed to have acquired little or no attention. To answer this third question, we thought it necessary to be able to interact with the teachers, to prod further into any potential superficial responses (e.g., "I use the graphing calculator"). We sought details such as which calculator keys students were expected to use in a given activity. We did so, again, because this had not seemed to be explored in past research, but also because there are often several ways to accomplish a given task with a given instrument; mentioning tasks alone and allowing broad descriptions of techniques seemed insufficient to accurately describe the reality of teachers' uses of technology.

To allow for such interactions, we decided to interview a small population of teachers. Therefore, our method of inquiry is primarily that of task-based interview research. Clement's (2000) reminder of the "open-endedness and qualitative and interpretive nature" (Clement, 2000, p. 548) of clinical interviews tempts me to categorize our study as interpretive research as well. Our methodology, however, did not bear all the characteristics of either approach. For example, participants in this study were not given the transcripts of the interviews to verify and suggest changes, although this is considered to be essential in interpretive research (Tobin, 2000, p.494-5). The most obvious way in which we did not adhere to the standards of task-based interviews, is that we did not propose the tasks! As the second of our three main research questions, we asked the teachers what tasks they completed with the different technologies that they had mentioned using in response to the first question. Once we learned of the tasks, however, we did ask them to describe in detail the specific instrumented techniques that they used to complete the tasks, and asked them to demonstrate these techniques whenever possible (which is more in line with traditional clinical interview research).

To describe how, in most respects, however, our research was well spoken for by the descriptions of interpretive and task-based interview methodologies, we will now describe in detail our data sources, participants, and research procedure of our study, and justify our decisions in selecting them.

4.2.1 The participants

In line with Tobin's (2000) description of interpretive research, we started with a small number of teachers, but were prepared to seek more participants in case there was not enough variation of practice among those first recruited. Having worked previously as a high school mathematics teacher myself, I wrote to the former department head of the mathematics department at the school where I was once employed (to whom we will henceforth refer as teacher T2) asking him to forward my invitation to participate in our study to any or all of the mathematics teachers. Out of the six mathematics teachers employed at the school at the time of the study (May, 2011), two volunteered to participate: T2 and one of the younger teachers (T1). T1 and T2 were very different in almost all relevant ways. That school year was T1's sixth year teaching and T2's thirteenth. Both had taught at the same school for their whole teaching careers and had used technology in some way every year in their teaching. T1 taught mainly the first two grades of the 5-years secondary school; T2 – mostly the last two grades, which included pre-calculus content. Both taught regular classes and enriched curriculum classes. T2 has also served as the mathematics department head for many years.

Teaching mathematics was T1's first career, and her only university degree was a Bachelor of Education. T2 had a degree in Mechanical Engineering, and worked "mostly in computer & control systems engineering" for nine years before getting his Bachelor in Education degree. This was his only other professional career. T2 explained, during the interviews, that because he "came to teaching from engineering", that he "had an inherently technological background to begin with". He further explained that he was "okay at developing in things like Basic and FORTRAN, and Java" and he had "some level of comfort with programming".

In the end, we did not ask teachers from other schools to participate. We wanted to compare teachers with the same resources available to them, to have our conclusions more clear, and found the two teachers that volunteered to be different enough to represent teachers near two ends of a spectrum of high school teachers' experience with and knowledge about technology.

4.2.2 Technological resources at the school

I chose to ask teachers from the school at which I used to work for two reasons. Firstly, I knew the teachers, and it was easy for me to "create a relaxed atmosphere and establish a relationship of trust" which is deemed important in clinical interviews (Hunting, 1997, p.154). The other reason was that I had a fair sense of the technological resources available to the teachers at that school, having worked there recently. I knew that the school supported the use of technology and had invested into constructing a new wing of the school that included four computer labs with thirty six computers each, data projectors and large screens. Some laptops had also been purchased for students' use, but there were not enough for everyone in a class for simultaneous use.

The technological environment in the school, therefore, was quite rich. The question was what technology teachers, who had those resources at their disposal, would actually use. This research began, in fact, with a conjecture that there is a large gap between certain researchers', teachers educators', and policy makers' call for using technology in mathematics teaching and the actual use of technology in teaching. If teachers who have easy access to technology do not use it to the extent recommended by researchers, then it seems that reluctance to the use of technology might be even more pronounced in less well equipped schools.

4.2.3 Research procedures

Four interviews were conducted with each teacher. Each interview lasted an hour to an hour and a half. The interview was semi-structured by eleven questions I had planned to ask. I allowed anywhere between a week and two and a half weeks between each interview with each teacher so that I had the time to listen to the audio recordings, and consider them in the planning the next interview. In particular, I listened for anything that I thought was unclear, what I might have wanted to learn more about, and kept track, making sure that I had asked each of the interview questions relative to each piece of technology named by the teacher in their response to the first question.

A list of the interview questions is included at the end of this chapter.

The questions were designed on the basis of our framework. In fact, our first questions, which remained our primary questions, were inspired by one sentence in Vérillon (2000) that "an instrument is any object that a subject associates with his or her action in order to carry out a task." Therefore, our primary interview questions became:

- What technology have you been using, or had used in the past, whether in the classroom or not, in some way towards your teaching? (Identifying the artefacts)
- What actions were you performing with them? (Identifying the techniques)
- What tasks were you trying to accomplish by doing so? (Identifying the tasks)

The exact oral wording of the questions varied from one interview to the next.

As described in Chapter 3, the theory also suggests that schemes behind the users' techniques are adapted from schemes that the users have constructed in relation to another similar object. Therefore, we also asked how teachers used to perform each task that they mentioned, before doing so with the mentioned technology. To investigate the schemes behind the techniques, we asked why they chose the mentioned technology to complete the mentioned task, and why they used it in the way that they mentioned. Due to time constraints, however, the investigation of schemes 'took a back seat', and we eventually concentrated on the above three questions, after having acquired superficial answers to most of the questions.

Time constraints were such an imposing constraint, in fact, that we looked to other methods of data collection to help us gain as many examples of the teachers' uses of technology as possible. In particular, after the second interview, I asked both teachers to email me examples of a Word file, PowerPoint file, videos that they had shown in class, and websites that they used with the students. I analyzed each of them before the next interview and prepared questions that I hoped would clarify the tasks that they were trying to accomplish with it.

Hunting's (1997) reflection on his early research experiences, "of attempting to include more tasks than can reasonably be administered in a given time" (Hunting, 1997, p.151), speaks very well to our experience. When it became apparent that I could not ask all questions about each piece of technology, I therefore prioritized certain questions (as explained in Chapter 3), out of those that I had planned to ask. More specifically, I prioritized questions about which artifacts the teachers were using, and how they were using them (i.e. the tasks they were performing with each artifact and the techniques that they employed to do so). Next, I prioritized questions on why each teacher was using each artifact, why they were using it in certain ways, and why they hesitated to use certain technology to complete certain tasks. Because of this, I was only left with very little time to ask about whether they 'shaped' the artifacts that they used in any way and whether their uses were influenced by the features of the artifact.

Also, as Hunting (1997) suggests, our questions were open-ended, as we wanted to "maximize opportunity for discussion or dialogue".

A copy of the set of interview questions that we had planned to ask and notes on them that I had with me, both reminding me what questions to ask and in what order to ask them, is provided below. I include this as a rough interview script for researchers who might want to use a similar approach to that which we did or simply for those who would like more information on it.

4.2.4 Interview questions with notes that I had added for myself: A first stage in the development of my interview script

Clarify my intentions behind this research and that I am "not a salesman!"

The goal of the first interview is to obtain information about:

- What technology teachers use?
- What tasks they use it to accomplish, and how they use it?

- Why they use the technology that they do, in the ways that they do, to complete these tasks?

To achieve this goal, a discussion will be held, revolving around the following 11 questions:

- How many years have you been teaching?
- For how many of those years have you been using some form of technology (e.g., Excel, any type of calculators, SMART boards, etc.), whether during your lessons, in preparation for them, or in any way related to your profession?
- What technology are you using? (Explain that this includes all technology that they use, inside and outside of the classroom.)

Make note of each of the pieces of technology mentioned, and ask the following question relative to each.

- How and why did you come into contact with this technology?

  Potential follow-up questions:
- Did you discover it or was it suggested to you by someone else?
- If the former, were you looking for a specific type of technology (to serve a specific purpose) or were you just seeing what was available to you?
- What inspired you to look?
- Did you start searching because you were suggested (for example, by the school curriculum or someone at your school) to ‘use technology’ without any specificity as to which?
- What task(s) are you using this technology to accomplish:

- Student tasks (e.g., finding the coordinates of the vertex of a quadratic function, given its algebraic representation of the form $f(x) = ax^2+bx+c$)?
- Teacher tasks (e.g., corrections, organizing and computing grades, etc.)?

- Why do you use this technology to accomplish this (these) task(s)?

Listen for, or ask about, answers relating this technology to other objects used in the past to accomplish this task.

- How do you use, or encourage your students to use, this technology in order to complete these tasks (remind them to please be as specific as possible)?

Make sure to give them time. Chase their answers. Offer an example of these techniques (when they are used and what they are).

- Why do you use this technology in this (these) way(s)?

Listen for, but do not force them to come up with, ways in which the features of the technology shape their uses.

- Did this task only appear when you started using this technology? (Repeat what the task was).

a. If so, what motivated you (the features of the technology, suggestions from others, etc.) to take on this task?

- What do you perceive as the benefits of having done so?

b. If not, how (and using what object(s)) did you complete this task before you started using this technology to do so?

- Did the task change in any way when you started using this technology?

- Have you altered this technology in any way (for example, created an automatic execution for finding the vertex of a quadratic function in response to the user entering the algebraic representation of the function in general form)? If so, what did you (or did you hope to) accomplish by doing so?

Chapter 5: Interviews with two secondary school teachers

In this chapter, I present some results of our research. The chapter is split into two parts. The first (section 5.1) presents the two teachers' responses to the planned interview questions. The second (section 5.2) presents their views on the pragmatic and epistemic value of their instrumented techniques that became evident through the teachers' responses to the planned interview questions and also through their reaction to certain unplanned follow up questions or in impromptu conversations. These views are included as they help explain the teachers' choices and actions.

5.1 The two teachers' uses of technology

This section gives an account of the teachers' responses to the planned interview questions; first, the teacher T1's responses (section 5.1.1), and then – the teacher T2's responses (section 5.1.2). For each teacher, I list the artifacts (i.e. the pieces of technology) that he or she were using at the time of the interviews, those that they had used in the past but were no longer using, and those that they planned to use in the future but had not used yet. I then describe the tasks that each teacher or their students had completed with each artifact as well as the techniques that they employed to complete those tasks. Last, for both teachers, I will present some of the reasons why they were using, had used, or planned to use each artifact and some of the reasons why they hesitated to use that artifact (at all or to complete certain tasks).

I distinguish between tasks and techniques by classifying as a task the part of an activity that would be delegated to an artifact (e.g., 'graph a function', 'create a table of values', 'present information', etc.), and, as technique, the way in which the student or teacher would use the technology to complete the tasks (e.g., 'pressing buttons', 'sliding sliders', 'navigating through menus and submenus', 'using the utilities or programs of their graphing calculators', etc.).

When the teachers first showed their students how to complete a given task with a given artifact, they demonstrated a technique for how to do so. After this point, however, the students were not forced to use that technique when faced with that task, even if they chose to use the same technology as their teacher had. For example, both teachers allowed their secondary 4 students to use scientific or graphing calculators in class and during their exams. They demonstrated techniques for creating a table of values corresponding to a function, given an algebraic representation of it. If a student had a model of calculator different from their teacher's, however, he or she would have had to adapt the demonstrated technique to their artifact. Further, students were free to use other techniques on the same or different type of calculator; they were not forced to use the one demonstrated by the teacher.

5.1.1. T1's uses of technology

This section is divided into two major subsections; 5.1.1.1, 'the technology that T1 had used in the past, but was no longer using', and 5.1.1.2, 'the technology that T1 was using at the time of the interview'.

5.1.1.1 The technology that T1 had used in the past, but was no longer using

Computer labs equipped with internet access

Task

In years prior to that during which these interviews were held, T1 brought her students to the computer labs to have them play computer games and to use Microsoft Excel.

Technique

She would meet her students in their regular classroom and bring them to the computer lab. The computer labs each had enough computers for each student, and they worked individually. The particular ways in which the computer games and Excel were used will be described later, in the descriptions of T1's uses of each type of software.

Why they were used

She brought her students to the labs because she wanted each of her students to be able to play the games or use Excel, simultaneously. The only alternative would have been to have them use the computer available in their regular classroom, one at a time.

Why she hesitated to use it (at all or to complete certain tasks)

T1 explained that she stopped going to the computer labs because "it just wasn't worth the time of literally moving them from one room to another and then getting them to work on the computer, only to find that it doesn't work, or they can't open what they need to open". She explained how the computers "were horrible", that they would crash (i.e., shutdown without warning or command from the student or teacher), and that students would sometimes have trouble logging into their accounts (for example, they would forget their login information or, even when the correct information was entered, the computer might take a long time to process it).

5.1.1.2 The technology that T1 was using at the time of the interview

Four function calculators

It was required by the school that students in secondary 1 purchase four-function calculators before the start of the school year. Neither scientific nor graphing calculators were allowed to be used (in the classroom or during their summative assessments). In fact, secondary 1 students were not even allowed to use their four-function calculators (in class or during evaluations), except to check their pencil-and-paper calculations, until they had covered the topics planned for the first two of their three semesters. During the first two semesters, they were to learn how to add, subtract, multiply, divide, and compare rational numbers (using inequality symbols), as well as take powers and find integer factors of given integers, reduce fractions, convert from one metric unit to another, write large numbers in scientific notation, and find the mean, median, mode of a small set of data, understand what is meant by percent and apply the concept. According to the course outline, this would usually take them two of their three semesters to learn, but some years, this was all that they would have time to cover. If four-function calculators were used to check pencil-and-paper solutions, their pencil and paper work would have to be shown. In other words, for example, it was seen as insufficient to compare two fractions by finding their decimal representations with a calculator.

Tasks

The four-function calculator was only used to add, subtract, multiply, and divide rational numbers (both integers and non-integer rational numbers expressed as decimals), and to evaluate exponential expressions (e.g., to represent 3^3 as 27).

Techniques

The students' techniques consisted only of pressing, in an appropriate order, the buttons on the calculator (buttons for entering single digit numbers, buttons to add, subtract, multiply, and divide two rational numbers expressed as decimal, and a button to enter a decimal point). The order in which buttons were pressed could vary depending on the calculator a student was using and on the student's preferred uses of this artifact. Some students may plan the calculation of the value of a complex expression entirely on the calculator; others may prefer to calculate the values of parts of an expression separately, note the values on paper, and use those in subsequent calculations on the calculator or on paper.

Why it was used

T1 claimed to have had her students use the four-function calculators mainly to "speed up" their completing the tasks mentioned above.

It was clear, however, that she did not believe that her students' use of these calculators improved their understanding of mathematics. She explained,

> [Some teachers] are talking about introducing them earlier, just because we're finding that there are so many errors that are happening, because they don't have basic number sense, although I don't know if we're gonna really help them if we do introduce calculators earlier, it's just, the test results would probably be stronger. (T1)

Non-graphing scientific calculators

It was required by the school that students in secondary 2 purchase a scientific calculator before the start of the school year. Graphing calculators were allowed (in their classroom or during their summative assessments) and four-function calculators would not have sufficed to complete all of the tasks that they are asked to complete (e.g., evaluating the square root of a given rational number).

Tasks:

Student tasks

Students used these calculators to add, subtract, multiply, and divide rational numbers (both integers and non-integer rational numbers expressed as either decimals or fractions), to evaluate exponential expressions, and to evaluate square roots of given rational numbers (e.g., in order to find the radius of a circle given its area). They could delegate these tasks to the calculator when they were asked to "display and interpret graphs and tables of values", "interpret, construct and manipulate algebraic expressions", "solve algebraic equations and word problems", "apply concepts of ratio and rate to proportional situations", "interpret formulas and solve problems pertaining to circles", "interpret formulas and solve problems pertaining to polygons, as well use nets and formulas to find the areas of solids", "apply the concept of percent in solving everyday situations", and "calculate the probability of outcomes and events" (quoting from the learning outcomes described on the students' course outline).

Teacher tasks

T1 used a scientific calculator to complete the same tasks just mentioned (binary operations on, and square roots of, rational numbers, etc.) when solving problems in class or when solving problems relevant to the course at home, but also used it to double-check her students' grades after she had calculated them with Excel.

Techniques

As with the four-function calculators, T1 and her students' techniques consisted of pushing buttons on the calculator. More specifically, she and her students used the number buttons, the buttons for addition, subtraction, multiplication and addition, the button to find the square of a number ($\boxed{x^2}$), buttons for turning rational numbers represented with decimals into their representation as fractions, buttons to add fractions and receive the result as a fraction, and a button to compute square roots ($\boxed{\sqrt{\ }}$). If the students asked, she would also explain what the other buttons for taking powers did ($\boxed{x^3}$ and $\boxed{x^y}$).

Why it was used

She mentioned how the main reason for her using, and having her students use, scientific calculators was to "speed up" the numeric computation described above. In particular, she mentioned how the calculator was needed for her and the students to find the square root of certain numbers.

She also allowed students to use these calculators to boost their confidence.

> They seem to feel a lot more comfortable if there is a calculator in front of them, they can just start punching things in. They may not always know what they're doing, but it's a security blanket in a lot of ways. (T1)

The fact that some might not have understood what they were doing did not, however, deter her from allowing her students use these calculators.

Graphing calculators (TI-82, TI-83, TI-83 Plus, or TI-84 Plus)

It was required by the school that students in secondary 4 purchase a graphing calculator before the start of the school year. Some did not do so however, or lost their calculator part-way through the year. T1 therefore allowed her students to use scientific calculators, instead, during any evaluation or class activity. For each of the tasks that she taught her secondary 4 students to complete with their graphing calculator, she also taught alternate methods for how to do so with their scientific calculator, pencil, and paper.

Tasks

Both T1 and her students used their graphing calculators to:

- Graph functions, knowing their algebraic representations (i.e. the algebraic representations were either given or obtained from a table of values or the details of a word problem). T1 would sometimes have them graph more than one function in the same plane;
- Create a table of values, knowing an algebraic representation of a function (again, the algebraic representation was either given or found by the student somehow).

Although she did not have her students use graphing calculators to do so that year, in years prior, she also had her students use them to:

- Calculate the mean and median of a single-variable distribution;
- Graph a box-and-whisker plot, given come data (whether in the form a list or in a frequency table).

Techniques

T1 taught her students how to complete the above-mentioned tasks by pressing buttons on the calculator and navigating through the submenus, entering data when required by a given "built-in utility" (her words) or applet of the calculator. Examples of data that the T1 and her students enter are algebraic representations of different functions in order to graph them; the numerical values of parameters in different algebraic representations of a quadratic function to find x-intercepts of the graph of the function; side lengths and angles measures to calculate other side lengths or angle measures; and the coordinates of two points to calculate the coordinates of the midpoint of the segment joining those entered). In particular, the buttons that they pressed were [+], [-], [x], [÷], [.], [(-)], [SIN], [COS], [TAN], [LOG], [LN], [ENTER], [$\sqrt{\ }$], [x^2], [x^y], [a b/c], [MATH], [Y=], [GRAPH], [WINDOW], [2^{nd}], [ALPHA], and arrow buttons. In years prior, she has also used some of the utilities accessed by pressing the [STAT] button to calculate means and medians and to graph box-and-whisker plots.

Why it was used

As with the other two types of calculators, she used the graphing calculator with her secondary 4 students "to speed things up": all of the tasks could be completed without the calculator and students were taught how to. She claimed to have mainly been using it because it was a ‘required material’ for all students in senior high school (i.e. secondary 3, 4, and 5).

Why she hesitated to use it (at all or to complete certain tasks)

T1 explained how, that year, she had used the graphing calculator less, and did not teach her students about as many of its utilities, because she felt less comfortable using it than she had in years past.

She explained how, closer to when she had started using it, she downloaded the user manual (for her TI-83) and had planned to read through the whole document. As she put it, she believed, “it was a good idea for [her] to know the calculator really well”, or at least that she should be as familiar with its features (utilities, applets, etc.) as her students were. She explained that she did not read the whole document, however, as she felt that she didn’t need to know most of it; that much of it was “too advanced” for her students.

It became clear, however, that more than being influenced by what it would offer, she had stopped reading about its uses because her students were not motivated to use it, except for in very simple ways (to graph functions or create tables of values). Speaking about the statistics utilities that it had, she mentioned:

> In the past I had gone through a little bit, but I just found that it made things more complicated, because half the class wouldn’t want to use their graphing calculator anyway, so, they’re doing everything by hand. (T1)

She mentioned how one reason why her students hadn’t wanted to use the graphing calculators was because they found them difficult to use.

> [There are] just too many buttons to push and they can’t remember exactly where things are located. (T1)

These statements revealed a part of her personality. It was clear (and became clearer throughout the interviews) that she was very concerned about her students' attitudes towards mathematic and mathematics education; to the extent that it strongly influenced the ways in which she used technology (e.g., she would hesitate to use it in ways that her students found too difficult).

Microsoft Excel

That year, T1 used Excel to perform certain tasks, but did not have her students use it at all. In years prior, however, she did have her students use Excel, and used it herself to perform a wider variety of tasks.

Tasks:

Student tasks

In years prior, T1 had her secondary 1 students use Excel to:

- Create two-column frequency tables, given a list of data;
- Create bar graphs and broken-line graphs; given two-column frequency tables.

Teacher tasks

- In years prior, T1 had used Excel to log her students' marks and calculate weighted averages of them.
- Both in years prior and that year, she had used it to create two-column frequency tables, bar graphs, broken line graphs, pie charts, and scatter plots to include in certain tests that she would design.

Techniques

- T1 and her students typed data into cells (using their keyboards) to create frequency tables.
- She and her students used the cursor to highlight cells in the tables that they created to call up and select options from different types of menus (pop-up menus and drop-down menus) to create the different types of graphs listed above.
- T1 described how she would give her students "detailed handouts" with suggested techniques for completing the above mentioned 'student tasks' with Excel. She did not mind, however, if her students used different techniques.
- To calculate weighted averages of her students' marks, T1 used Excel's "Sum" utility (found in the drop-down menu labelled "AutoSum" on the software's user interface).

Why it was used

T1 taught her students how to create graphs and frequency tables in Excel because, at the time when she did, it was mandatory that all secondary 1 students at the school prepare a presentation for a local science fair, which the school expected them to create with a computer (to have the presentations look "professional"). T1 and the other secondary 1 teacher noticed how some students might be interested in presenting information that they had discussed in class (e.g., with pie charts) and perceived "an opportunity to do something interdisciplinary". T1 used Excel to create certain tests as she saw this as "a professional way" to do so, as well. She explained how she had started using Excel to log marks and calculate weighted averages of them when the school stopped paying for MarkBook (a 'class management program'). When T1 started teaching at the school, MarkBook was installed on all of the computers. It was what most of the teachers (except T2) used to log their marks and to calculate weighted averages of them.

They would calculate these averages either to discuss a students' progress with their parents, or to submit term marks for the students' report cards (the system that the term marks were then entered into, COBA, will be described later). One year, the school stopped paying for MarkBook, however, and that year T1 began using Excel to complete these tasks.

She claimed to have chosen Excel as Microsoft Office was already installed on all of the computers at school and on her laptop and because she was interested in improving her skills with it. After one year however, she stopped using Excel and began using MarkBook again, despite having to pay thirty dollars every year for a license.

Why she hesitated to use it (at all or to complete certain tasks)

T1 and the other secondary 1 teacher stopped having their students use Excel when the school stopped making it mandatory for each student to participate in the science fair; they no longer saw it as beneficial to the students to bring them to the computer labs to use it.

T1 explained how she stopped using Excel for her marks and switched back to MarkBook because she was "just not familiar with all the features in Excel", whereas she was very comfortable using MarkBook. She felt that she could do more with MarkBook and that completing the marking tasks with it was easier than doing so with Excel.

Microsoft Word

Tasks

- T1 created all her tests in Word. She would copy and paste images that she would find on the internet, as well as graphs and two-column frequency tables, bar graphs, broken line graphs, pie charts, and scatter plots that she would create in Excel, into some of these Word files.

- She would also use Word to present information in class (using a multimedia projector), such as prepared solutions to worksheets that she had given students to work on.

Techniques

To create tests, T1 used, beside the keyboard characters, the Word's "Insert Equation" and "Insert Symbol" tools from the "Insert" menu. The Insert menu and the copy and paste functions were used to insert graphical material from other files or the internet.

Why it was used

She mentioned that she used Word to create her tests because she grew up using it and because she felt that she could "easily manoeuvre" in it.

She also mentioned how she used it to create tests and to present information in class because she had a copy of Microsoft Office at school and on her personal laptop.

She would also spontaneously open a Word file during a lesson to type in definitions, steps of solving a problem, or other types of information, if there was glare from the window on the marker board or if the markers were out of ink.

Microsoft PowerPoint

T1 used Microsoft PowerPoint very little in the secondary 1 and 2 grades, but used it almost every class in secondary 4.

Task

In secondary 4, T1 would present all of the content of her lesson that she anticipated using (e.g., definitions, graphs of functions, tables of values and other images, questions, solutions to questions, etc.) with PowerPoint; the use of blackboard, marker board, and Word was reserved for spontaneous actions (such as answering a question that would arise during class).

Techniques

When preparing her presentations, T1 would use scrollbars to navigate through the slides. She used the copy and paste functions and the Insert menu to include images from the internet; graphs, charts, and tables that she would create in Excel. She would use the Drawing Tools to insert geometric shapes from the *Shapes* palette. The Animations menu was used to make her presentations more dynamic. For example, she would frequently program a hint or a solution to appear only later in displaying a slide with the statement of a question, at her command, by using the Entrance palette (e.g., Fly in, or Fade). She also used Emphasis Effects, for example, to make it possible for her to change the color of a shape during the presentation. She made the shapes move by programming the Motion Path. This way, she could superimpose one triangle onto a congruent one (filled with a different color) to demonstrate their congruence.

When presenting in class, she would start a "Slide Show" by clicking on the Slide Show button on the user interface and move through the slides using the arrow buttons on her keyboard.

Why it was used

Primarily, she used PowerPoint to make her lessons more "entertaining" for her students and because she found presenting information this way interesting.

I don't want to do the same thing all the time. I need to have fun myself somehow. So if I was always just, like, using the same acetates or something like that, I'd get bored. (T1)

T1 used PowerPoint to superimpose one triangle onto another that was congruent to it because she claimed that it was "easier [for the students to understand] than just saying or putting on the board: 'here's a triangle and it's identical to this one'… you can really see that everything is the same, because the side lengths are identical and the angles are the same as well."

She also used PowerPoint because doing so allowed her to email the presentation to her students; she did so after every class. This way, even students who were not in class that day had information about the contents of the lesson.

Why she hesitated to use it (at all or to complete certain tasks)

She admitted that she "would love to" teach algebra with PowerPoint, but that she felt restricted from doing so by her "abilities" in using PowerPoint. She therefore tended to use the blackboard or the marker board.

> If I had really strong skills in manipulating things in PowerPoint, I would love to do it in PowerPoint, but it's just easier… with the whiteboards to use colour to show how things move, and to literally move them myself. (T1)

She also mentioned how it was more difficult to respond to students' questions using PowerPoint than when she would use a marker board or blackboard.

> With PowerPoint, it would be a little bit harder because they [referring to her students] usually have a lot of little questions. They want to know what could happen in a slightly different scenario. So, the board makes it easier. (T1)

MarkBook (Asylum Inc.)

Tasks

T1 used MarkBook (http://www.asyluminc.com/markbook.html) a class management program, to log her students' marks, calculate weighted averages of them, and graph the grade distribution of each of her classes. In years prior, she had also used it to create seating arrangements.

Techniques

For each course she was teaching, she would start by using the New Class command to create an empty MarkBook file, which she would then name and save to a location on her C: drive. She would create a new file for each class, each term. Next, Markbook would ask her to enter each student's name, their homeroom, whether they were male or female, and a student number. She would label her students 1, 2, 3, and use the Add Name command to add each to a list. Once the students' information was loaded, Markbook would ask her to name each mark set that she wanted to create within this file and to give a brief description of it. She would usually create one mark set corresponding to each competency that she was evaluating the students' on that term.

Whenever she would open a file, she could select which class's marks, or which student's marks (which she would select after selecting their class) she wanted to modify. She could use the New Entry command to add a new evaluation or the Multiple Entry command to enter marks for several evaluations simultaneously. As she would keep a paper copy of her grades, she mentioned how, if she "was good", she would enter the marks soon after each evaluation, but that, more often, she only entered her marks at the end of each term, so that she could use MarkBook to calculate her students final marks for each competency. For each evaluation that she created, she had to enter the date of the evaluation, how much the test was 'out of', and the weight (as a percentage) that it should have against the other assessments (for example, she could have one evaluation worth 40% of the students' marks for that competency, that term, and another worth 60%). She would also have to name each evaluation. When she wanted to enter the marks then, she would only have to enter the mark that each student received (as it was assumed that they were every students; score for that evaluation was 'out of' the same amount). MarkBook would then calculate the percentages.

For each class she also used the Class Report function that ranked her students to see who was struggling and who was excelling. She admitted that this was only useful if she inputted marks throughout a given term, soon after she had marked each evaluation.

Why it was used

She used MarkBook because it saved her time over completing her teacher's administrative tasks with pencil, paper, and calculator, and because she found that it required less effort that any other way of completing these tasks (using any other software or pencil and paper). In fact, she mentioned how it was "like a nice, easy version of Excel".

COBA (a Management Software)

At the time of the interviews, the school's administration required all teachers to use COBA[1] to enter their grades, enter attendance at the beginning of each class, and check e-mails that they would receive from parents or students and messages that the school's administration posts for the parents and/or the parents and/or the students. Every parent, student, administrator, and teacher had an account.

Tasks

T1 used COBA to:

- Submit grades and comments for report cards;
- Enter attendance;

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[1] http://www.coba.net/secteurs/anglais/elementary_secondary/elementary_secondary.html

- Check her colleague’s teaching schedules, when she needed to arrange to meet one or more of them;
- Receive messages from, and send messages to, students and parents;
- Retrieve documents that the school’s administration had posted on COBA for the teachers and/or parents and/or students;
- Consult a posted calendar containing other teachers’ planned test dates. This calendar was updated daily by an administrator. All teachers were required to check the calendar before scheduling a test to make sure that no more than three tests would be scheduled for a given day and to inform the administrator when they planned to give a test;
- Print out a copy of each of her classes’ students’ names. She did so to keep a paper copy of her students’ test and assignment grades that she would later enter into MarkBook to calculate her final marks, before submitting those into COBA.

Techniques

She would open the Gradebook menu to enter her marks and comments for report cards (the students names were already loaded for her). She would open the Attendance feature (under the Class List menu) to enter her students’ attendance each class and to print lists of her students names (before she had entered any attendance information) to keep a paper copy of her students grades. She would open the Mail menu to send and receive messages. When sending a new message, she would select receivers from an available drop-down list of names of all of the students in the school (sorted by their ‘homeroom’), their parents, the faculty and the staff (which were located under submenus with those titles). She could select one or multiple receivers without needing to repeatedly re-open the list. She would open the Personnel menu to check her colleagues' teaching schedules, by searching their name. She would also open the Public Documents menu to retrieve documents posted by the administration.

Why it was used

It was required by the school that she use it to check messages sent to her by parents, students, and colleagues, enter attendance and end of term grades, and keep track of her students upcoming tests dates when schedule hers.

Why she hesitated to use it (at all or to complete certain tasks)

T1 mentioned that she only used COBA because she had to, and did not use it to complete tasks that she was not required to with it.

T1 claimed not to use COBA's mail feature because she had Outlook and did not want to use both for the same task. She also hesitated to use it because the COBA mail feature did not allow her to attach a document to a message. She described, "it just seems like it's a program that has less features than a typical e-mail program would have." She would encourage her students not to contact her through COBA and to email her instead. She only used COBA's mail feature to read and respond to messages sent to her by parents of her students though COBA.

Microsoft Outlook

If any information needs to be communicated to the parents and some (or all) of the faculty, it will be posted on COBA. If the administration only needs to communicate any information to her or a group of teachers however, it will be emailed to her or them. All teachers are, for this reason and others, required by the school to read emails sent to them from her colleagues and from parents sent to their Outlook addresses. She would sometimes respond to these with a phone call, but would more often reply via email.

Task

T1 encouraged her students to e-mail her with any questions they had (with respect to the course material). As mentioned above, she used it to read and respond to any messages sent to her by faculty or staff of the school. T1 emailed her secondary 4 students any Word files or PowerPoint presentations that were used or created in class, before their next class together.

Techniques

She used the Compose, Reply and Save sent mail commands.

Why it was used

T1 used Outlook because she was required to. She had mentioned that most time, when she wanted to initiate contact with parents, she would call them. That is, she would usually only use COBA or her email to reply to message that she received.

Videos

T1 showed two types of videos in her classes. To all of her classes (secondary 1, 2, and 4), she showed videos that were related to mathematics topics that they had learnt or would learn about that year. With her ‘enriched’ secondary 2 students, she also showed videos containing riddles that were often not related to the content of the course. All of the videos were short (less than five minutes long).

Tasks

T1 showed the videos mainly to entertain her students. She would also use them to introduce her students to definitions of mathematical objects that would be, or had been, discussed in class (e.g., vertex, rays, etc.). She used the riddles to encourage her students to pay more attention to details and, as she put it, “to get them to think outside the box”, as the details of the gritty riddles were often needed for their solution. She also used these riddles with the students in her advanced group to “to slow them down a little bit” (speaking of the pace at which they cover the curriculum).

Techniques

She found the videos using an internet search engine (usually Google) with a name of a mathematical topic and “math games” as keywords (e.g., by typing “geometry math games” or “algebra math games”). To find the riddles, she would always go to the same YouTube site that she was subscribed to (http://www.youtube.com/user/CoolRiddles?ob=5).

She would play the video in class by going directly to the website where she found it, using the classroom computer and LCD projector. She would include the link to the webpage in her PowerPoint presentation so that she could easily access it. She would always play it from start to finish without interruption.

Why it was used

T1 used videos to entertain her students’ and to capture and keep their attention because she realized that, due to how frequently they had math class with her (six days out of a nine-day cycle), “after a while, it gets kind of boring”.

She also mentioned how, as much as she showed them to entertain her students, she too found many of them “very neat”, “fascinating”, and “interesting”.

Last, she used riddles to “slow down” her enriched secondary 2 class. She mentioned how, in past years, the enriched group covered more topics than the regular stream of students[2], but that a few years prior to the interviews, the mathematics department “decided that it wasn't really fair to the students who could potentially be getting 90s on a secondary 2 test, but their marks were dropped because they were being given more difficult assignments and tests”. She explained how, as a result of this decision, the department decided not to "test them on this advanced material," and that she therefore had to find other ways to take up class time.

She also used riddles to encourage her students to pay attention to details, because she had noticed that they often missed important details when they tried to solve word problems on the summative assessments. Although she mentioned that she had not noticed a significant change in their attention to details (after I asked her whether she had), she claimed that she used them nonetheless because her students "absolutely love[d] them, they beg[ged] to do them, and they want[ed] to be the person [to] answer."

Computer Games

The selected games would involve mathematical ideas that had been or would be discussed that year in class. One example of a game that she would have her students play is: http://education.jlab.org/topquarkgame/index.html

Tasks

T1 had her student play computer games to have them practice solving problems on specific topics, after she had introduced some basic ideas related to that topic.

Techniques

To play the games students would have to enter information (numbers and/or words) and click on buttons using a cursor. For example, when playing the game cited above, students would enter coordinates where they would like a point to be plotted, click on points of a given graph corresponding to coordinates that were given by the computer, and type in their name.

In past years (as described above), T1 would take her students to a computer lab so that they could each play a game, simultaneously. She stopped doing so however and now had one or two students at a time play the games on the classroom computer. Sometimes she would project the computer screen and have the rest of the class discuss solutions with those playing. Other times, she would have the students who were not at the computer work on math problems that she would assign. In either case, she would have the students take turns playing the game at the computer.

Why it was used

She used the games because they offer "immediate feedback" to the student about whether their answers to the questions were correct or not. She also used them because, as she put it, they offer "a different way of learning, rather than me just standing at the board"; similar to why she showed her students videos (again, the theme of making math fun came up).

Computers in classrooms, with internet access, connected to multimedia projectors

Tasks

T1 used the classroom computers (as mentioned earlier) to play and project videos, to have her students play math games, to run COBA on so she can enter attendance, and to present all of her PowerPoint presentations and Word files.

Techniques

She would control the presentations and input data into COBA using the keyboard and mouse.

Why it was used

T1 had started using the classroom computers to run COBA because she had to. She started using them to have the students play games because she did not want to bring students to the lab anymore. She used them to present PowerPoint presentations and Word files, she claimed, in part, because of the option of doing so was made possible by the availability of the classroom projector and LCD projector.

5.1.2 T2's uses of technology

This section is divided into three major subsections; 5.1.1.1, 'the technology that T1 had used in the past, but was no longer using', 5.1.1.2, 'the technology that T1 was using at the time of the interview', and 5.1.2.3, 'technology that T2 planned to use in the future, but had not used yet'.

5.1.2.1 The technology that T2 had used in the past, but was no longer using

Computer labs equipped with internet access

Tasks

In years prior to the year of these interviews, T2 had brought his secondary 4 students to the computer lab to play with interactive applets on representations of quadratic functions.

Techniques

As T1 had, T2 would meet his students in their regular classroom and bring them to the computer lab. He would project instructions for the activity on a large screen at the front of the room. He would have the students work individually and would walk around to answer questions.

Why it was used

He brought his students to the lab to have them learn about the effects of parameters in the 'standard' form of a quadratic function ($f(x) = a(x-h)^2 + k$), and the 'general' form ($f(x) = ax^2 + bx + c$) on their graphs, because he believed that "the students are gonna learn better if they play with it themselves".

Why he hesitated to use it (at all or to complete certain tasks)

T2 mentioned how he hesitated to bring his students to the computer labs because the "performance" of the computers was sometimes "slow", because it was sometimes difficult to book time in the labs (despite the school having four computer labs), and because of the issues with the "reliability of the hardware platform" and the network. The latter issues also a reason why students were not tested in the computer labs. Another reason was that this is not how they would be evaluated at the college level.

> If anyone asks us why don't you test in the computer lab, why don't you do dynamic, interactive tests? Every teacher will say, that's not the way they do it at college or university, and we have an obligation to prepare students for the way they are being tested. To a great extent, that's true, at this point. The other point being that, most teachers don't have the training, we don't have the physical resources to do that. If I were to arrange to do the sketchpad-based, or a gizmos based test, which are available, a dynamic, interactive test, I cannot be sure that the hardware is going to be robust enough, or the network will be available enough to make sure that I have 70 minutes of clear, good computation. Because it's happened to me where I've tried to project something very, very simple, very low load on desktop and it just freezes, and I can't do that to them during intense [work]. (T2)

By the 'robustness of the software' T2 was referring to the "network bandwidth and CPU/RAM capacity of the individual workstations".

Explore Learning's Gizmos

As he was not using this artifact that year, we decided to leave the discussion of the tasks he had used it to complete and how he used it to do these things until the end. However, time ran short, and we did not have enough time to discuss either task or technique in much detail. He did mention however, that he had used different Gizmos (ExploreLearning. Cambium Learning Group, 2012), to teach about probability and supplied me with some examples of these[2].

Why it was used

He enjoyed the fact that the website offered "its own multiple choice tests with instant feedback" and contained "teacher guides and student guides."

He mentioned that "they're certainly very, very good at providing an infrastructure for support of the teachers and students, which you won't find with many other software." He explained that "they have a very well maintained and documented site; everything is there and I know how to find it instantaneously."

He also mentioned that the students enjoyed using them and that he believed that they helped his students develop their understanding of probability.

The kids liked it. Many of them, I think, improved their, sort of their core understanding of probabilities, especially successive events, dependent events. That can be a little tricky for some of them. (T2)

Why the teacher hesitated to use it (at all or to complete certain tasks)

He mentioned how he had not used Gizmos very much, because the Gizmos that existed at the time did not fit well with the curriculum. He found that since he could only use them sporadically, their use seemed "like a gimmick".

> If they had more Gizmos that suited the high school curriculum in Québec, then you'd be using it on maybe a monthly basis, it wouldn't seem like a

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[2] He had used all of the applets on this page: http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=268.

gimmick to the students, the students would become familiar with it. You'd get to a level of efficiency, after a few repeated uses. (T2)

He also found that, even of those that did fit with the curriculum, none were exactly as he would have liked them to be (in terms of their user-interfaces and in terms of how he could use them to teach).

I found myself so often saying, 'oh I wish it could do this' and, 'I wish it could do that' and, 'oh, I don't need this slider, but I wish it had that'. (T1)

As will be explained later, he therefore stopped using Gizmos at one point; in part because he was able to find free interactive applets on the internet that did exactly what he wanted.

The Geometer's Sketchpad

T2 was not using this software at the time of the interviews, but he had used it in the past.

Tasks

When he taught secondary 2, several years prior, he and his students would use The Geometer's Sketchpad while working through the Geometry section of their course. In particular, he had them use the software to "verify properties" such as whether the sum of the angles of a triangle is always 180 degrees, whether the median lines in a triangle always intersect at a single point, and whether the diagonals of a parallelogram will intersect, in Euclidean geometry.

Techniques

T2 would have the students create triangles, have the software "indicate measures of angles, and segments, etc., or locations of points of intersection", "alter a figure by mouse-dragging, translating, vertices or sides", and observe what stays invariant under those changes.

Why it was used

He mentioned how he liked The Geometer's Sketchpad because he noticed that it allowed him to cater to different styles of learning.

> You have so many different modes of learning. For some, a logical justification works, for others - a graphical one. For others, it's gotta be something dynamic, [and] that's where Sketchpad and GeoGebra [a software that he had not yet used with his students, but which he planned to use in the future] are wonderful. That's where their real strength is. It's that I can grab a point and I can drag it, and all the elements that are associated, whether they are points or surfaces, drag with it, is very powerful for those students who learn that way. And others don't like to see things move, because too many concepts open up in their head. So they need to see them in a different way, a logical way, or what have you, and so the ability to use multiple tools allows you to address multiple forms of intelligence. (T2)

He also mentioned how he had his students use it to verify properties of triangles because he found it useful in convincing them about them.

> For mathematicians, it's obviously not the same thing, but it is equivalent for where these students are at in terms of their academic maturity. It is equivalent. This is as good, if not better than a proof. (T2)

Why he hesitated to use it (at all or to complete certain tasks)

He told me how he had started using Geometers' Sketchpad while he was teaching secondary 2. He said that some of the other secondary 2 teachers didn't, however, which "caused some difficulties within the level in terms of evaluation, in terms of activities, and so forth." He mentioned how this stopped the school from buying licenses (and him from using it) because "the idea was to get the entire grade level, all the teachers, to use it, and then to continue using it in secondary three, and secondary four so that the students would develop a familiarity, a comfort level with it." He mentioned how this would have been important because "it has a very, very particular user interface, so it's not something that is intuitive for a students, there's a lot of little things that you have to think about." He had believed that "the user interface had such a steep learning curve that unless everyone got on board with it, and agreed to scale up what the students knew within sketchpad, in terms of commands and user interface, it wouldn't work in the higher-level grades." He said that, in the end, "there just wasn't the acceptance from the other teachers. Some did and some didn't."

He admitted that the software's constraints could inspire some students to ask questions and learn, but that such learning would be more appropriate for older students.

> Geometer's Sketchpad's has very peculiar ways of drawing things, so you can ask yourself, well, why can't I draw it like this, and you learn something, but that's fairly high level. That's not middle school. (T2)

5.1.2.2. The technology that T2 was using at the time of the interview

That year, graphing calculators and interactive applets were the only artifacts that T2 planned to have his students use. He used a variety of technologies, however, for his teaching tasks. We describe the technologies in detail below. The descriptions are more detailed than those of the technologies he used in the past, because our primary interest in this research was in the technologies that teachers were using at the time of the interviews.

Graphing calculators

As mentioned above, the school required all secondary 4 students (and therefore that T2's secondary 4 students) to purchase graphing calculators before the start of that school year. However, some of T2's students did not do so. Therefore, T2 also allowed students to use scientific calculators during any evaluation or class activities in its place. He also taught his students alternate methods, using a scientific calculator and pen and paper techniques, to complete all of the tasks listed below. He mentioned how the students were allowed to write the techniques on a "memory aid" that they could bring to every assessment.

The required graphing calculators were programmable and T2 made a point to refer to these calculators as "graphing and programmable calculators", noting that "the graphing takes you one direction… and the programming takes you another". That year, he was using a T1-84 Plus, and although it was permissible for his students to do so as well, he mentioned that he believed that most of them were using TI-82s or TI-83s.

Tasks

Similar to T1 and her secondary 4 students, both T2 and his secondary 4 and secondary 5 students used their graphing calculators to:

- Create a table of values from a given function rule;
- Graph functions knowing their algebraic representations (as with T1, the algebraic representations are given in the question or found somehow). Also, similar to T1, he would sometimes have his students graph more than one function in the same plane. He differed from her, however, in that he would have them give the curves different 'weights' (or line thicknesses), which T1 did not.

Moreover, T2 and his students used their graphing calculators to:

- Find properties of a function (its zeros, x- and y-intercepts, maximum or minimum values, if such exist, etc.), knowing one of its algebraic representation;
- Find the measure of an angle or a side length of a (right or scalene) triangle;
- Plot data from a given one- or two-variable distribution;

- "Match" the plotted data with a linear regression line, if the distribution involves two variables;
- Determine how well a suggested type of regression line "matches" given data;
- Plot the regression line onto a graph containing the data points previously plotted to visualize how well it fits the data;
- Find the standard deviation and mean of a one-variable distribution;
- Find the z-score of a given value from given data that the students are told follows a normal distribution;
- Play with the effect of different parameters in the 'standard' algebraic representations of square root, rational, exponential, logarithmic, and trigonometric functions (e.g., $f(x) = a \cdot \sin\big(b(x-h)\big) + k$) and of the standard and general forms of the algebraic representation of quadratic functions (mentioned above) on their graphs;
- Test whether "there are some values of the independent variable for which [a given] function does not work" (for example, to determine that –1 could not belong to the domain a real-valued function, whose rule of correspondence could be represented algebraically by $f(x) = \sqrt{x}$);
- Find the binomial factors of second degree polynomials, whose coefficients are real numbers (if factors with, real number coefficients, exist);
- Find the coordinates of a point that divides a given line segment in a given ratio;
- Find the distance between two points in the Cartesian plane;
- Find the point of intersection of two intersecting lines, given an algebraic representation for each;

- Chose different line widths (‘weights’) for each of the graphs of different function, if they graph more than one function of the same type (for example, two or more different quadratic functions) simultaneously.

T2 also created programs (for which he wrote the code either directly on the calculator or in BASIC, on a computer, which he would then upload).

Techniques

T2 and his students completed these tasks by navigating through the menus and submenus, entering data (such as an algebraic representation of a given function, the numerical values of parameters from such a representation, real numbers into a table, side lengths and angles measures, coordinates of points) when required by a “built-in utility” (his words) or applet of the calculator, or by a program that the students had downloaded onto their graphing calculator. The graphing calculator utility that gives the coordinates of an x-intercept of a given function (called ROOT or ZERO, accessed through the menu displayed after pressing the 2nd button then the TRACE button) also requires the user to move a cursor along the graph of the function, to the left and then to the right of the x-intercept, which he/she does using the arrow buttons on the calculator. Also, T2 and his students used these arrow buttons to choose the line weights for the graphs of the functions that they would plot.

T2 did not allow his students to use any programs, applets, or utilities of their graphing calculators on their first test on almost any topic (the course topics are briefly described on the course outlines). He explained that during “any subsequent test and exam, they [his students] may use any calculator-based software of their choosing”, provided that they write the name of the program/utility/applet and the “inputs” that they entered. He informed me that “the only exception to this [was] correlations and regressions in statistics in which case they may use the calculator on the first test."

Why it was used

T2 mentioned several reasons why he used and had his students use graphing calculators:

- He had decided that his students, that year, should focus on only using one type of technology (with one exception, the one class during which the students used an interactive applet) because he had assessed them as being very "weak" (both in mathematics and other subjects). He chose the graphing calculator to be this one technology because the school had a rule in place that it would be the only type of electronic technology that secondary 3, 4, and 5 students would be allowed to use during their evaluations (although he also allowed his students to use scientific calculators if they had lost their graphing calculators or if they were broken);
- T2 mentioned how features of the software allowed him to discuss mathematical ideas that were otherwise difficult to motivate (e.g., scale);
- He mentioned how when teaching statistics, or about the effects different parameters in algebraic representations of function on the function's graph, or about the restrictions of certain algebraic expressions (e.g., that $\sqrt{x}$ is not a real number for $x < 0$) he would use and would have his students use graphing calculators "on a very, very regular basis" and that he felt, without them, his "teaching of that topic wouldn't be as effective";
- He mentioned how, in life outside of school, "nobody" uses pencil and paper to find the equation of a regression line, and therefore, why should they. He also mentioned how, if they were to work with pencil and paper only, because of time restrictions and the calculation errors that would likely be committed, they would have to work with very

small samples, but that this would not demonstrate much of the usefulness of statistics (describing trends in larger populations).

> Imagine doing this by hand, using a Meyer line, or a median-median line, how many calculation mistakes there would be. And who actually, in real life calculates by hand the regression line? Nobody does. It's all done by Excel or whatever program they have. So we'll teach them what's the principle of a regression line, but, [we allow them to use their] calculator [to find the equation of the regression line]…. If I say, ok, just so we can do it by hand, I'm only gonna give you three values. That's not statistics. Right? I mean there's already a theoretical objection to that. Well ok, five values? It's still not good. Ten values? Ok that's gonna take them too long. So really this is where the calculator, I think, is valid as a teaching tool. (T2);

- The above quotes also point to the speed and accuracy that he understood could be afforded by allowing his students to use graphing calculators (which motivated him to allow his students to use them in certain situations);
- Programs/applets/utilities also improved the speed and accuracy of his and his students' computations;
- He mentioned how he had his students use graphing calculators so that they could delegate basic procedures to it and focus on "higher level skills".

> Automating the very simple processes… which for some of these students could be a fairly laborious and careless-error-ridden procedure, allows them on focus on establishing the procedure, and validating it, and not worrying about the computation. (T2);

- He mentioned how he allowed them to use programs on their graphing calculators during evaluation to complete certain subsidiary tasks another because he did not want their difficulties with topics covered earlier in the year to impede them from learning about and working with newer ideas. He also claimed that if students couldn't solve a problem because they weren't strong enough with the material from past chapters, then "either we shouldn't be giving them problems like that, or we should be helping them, by scaffolding them with this type of technology."

> Once you've passed or failed that chapter test, once we're past that as a class, you may now use the programs. Because you've shown me that you can or cannot do it. And I'm not going to double reward you, or double penalize you, for something that you've already shown me.
>
> You cannot allow their weaknesses or strength from the previous chapter interfere with the next chapter. (T2)

He had his students write down the name of the program/utility/applet that they used and the inputs they entered, during evaluations, because he wanted to evaluate their solution methods/processes. Similar to how he would ask his students to show their work when working with pencil and paper, so that he could determine where they went wrong when they did, he asked them to write the program/utility/applets and their inputs so that he could determine whether they used "the wrong analysis tool" or whether "they made a careless mistake where they didn't enter data properly, or hit the wrong key". He also wanted to be able to determine whether "they know how to translate the problem into a format that is useable by [a] formula".

Why he hesitated to use it (at all or to complete certain tasks)

He mentioned that he hesitated to use graphing calculators because:

- They were not easy to use.

There are menus and there are submenus, and there are programs and there are apps, and they're not the same things.

> [When graphing functions] I have to go and enter my function, I have to make sure my window is set properly, I have to make sure my axes are scaled properly.
>
> If you're trying to teach students what are the effects of varying parameters on a function with a graphing calculator, you have to make sure that they all vary the same parameter, that they don't make a sign mistake, that they don't confuse the negative with a minus and so forth (T1);

- They were not allowed during evaluations at the college level, and he felt an obligation as a senior high school teacher to prepare his students for assessment at the college level. Specifically, it had him disallow them from using certain programs during certain evaluations (as describe earlier).

> My philosophy is basically to have them prove that they can, or cannot (as the case may be) perform the algebraic manipulations by hand in at least one instance. Once they develop a little more academic maturity (i.e. once they're in college) the ability to deal with abstract manipulations increases dramatically; therefore I do not feel I'm selling them short on algebraic practice.
>
> My goal is to best prepare my students for success on exams at the college level, they have to know how to do multiple choice, they have to know how to do their calculations by hand, they have to have a certain set of formulas memorized, they're not necessarily gonna have a memory aid or a formula sheet. All of these things, we've chosen to implement or follow, based on our knowledge of the college system. (T2);

- They were subject to technical difficulties; "your memory might become over-loaded, your program might crash";

- To be able to use it to its full potential, he believed a student must have a strong understanding of some basic mathematics (which he did not believe that they all did).

> The graphing calculator and a lot of the apps that are available for it and a lot of the utilities it uses really only take off, I find, have a benefit for the students, after they have a foundation for some of the mathematical basics, things like operations with fractions, factoring, some basic sense of two dimensional geometry, the real basics, before you get to functions, for example. (T2)

He also mentioned how he hesitated to show his students' certain features of the calculator because they lay in conflict with some of the educational goals of the course.

> I don't like using [the zoom feature on the graphing calculator], because when the students use it, they don't have to choose a scale. The calculator will zoom in on an interesting, or important feature... but the problem there is that the students do no thinking. The idea of having them choose their window for their graph is key... So I don't tell them anything at all about zoom. And in fact I try to introduce situations where the zoom is deceptive. (T2)

He hesitated to use the graphing calculator for "exploratory learning" that year because (as mentioned above) he believed his class was "very mathematically weak." He had used them in such ways when he taught mathematically "stronger" groups of students.

Interactive applets

T2 would find the applets on the internet[3].

They were all open-source and could be run on a desktop computer (as opposed to those only to be used on cell phones or calculators), such as the computers installed in the school's

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[3] http://members.shaw.ca/ron.blond/TLE/QR.PARABOLA.APPLET/index.html

computer labs and in regular classrooms. They consisted only of a graphical and an algebraic representation of a function and sliders to change the value of each parameter one at a time. T2 often referred to these applets as "Java applets" because a majority of the ones that he used were programmed in Java. He mentioned that the software he was using belonged to the larger category of "interactive applets" that could be "executed locally".

Tasks

T2 used applets to guide his students towards an understanding of the effects of parameters in the standard and general forms of the quadratic function and the standard form of the exponential function.

Techniques

In the computer lab, he would have his students play with the sliders and observe the effect of changing the value of each parameter on the graph of the associated function displayed on their computer screen. That year, since he had assessed most of his students as mathematically weak, relative to the stream hat they were in, he would either give them a handout with instruction or give verbal instructions, to move a specific slider and say (for example) "ok, now slide the "a" slider back and forth - what do you notice?" In years prior, when he had stronger groups, he would not direct their "play", but still asked them to conclude what the effect of each parameter was on the shape of the graph.

As mentioned above, he would only bring his secondary 4 students to the computer lab, once per year, to have them "play" with an applet on quadratic functions (the first function they discuss in secondary 5), but would also use an applet, in class, to demonstrate the effects of the parameters in the "standard" algebraic representation of exponential functions ($f(x) = ac^{b(x-h)} + k$). When he used applets to demonstrate these effects, he would ask his students to predict the effect of changing the value of each parameter, based on what they had learned about quadratic functions, before he moved each slider.

> In an ideal situation, we would both use it for the quadratic, then the next function I teach, which I guess in sec[ondary grade] 4 would be exponential, I would show it with an applet, but they wouldn't play with it, we wouldn't have time. I'd say, okay, remember what we did with the quadratic? Do you see how it's the same? I'd have them predict. I'd say, 'okay I'm going to slide my slider over this way, which way should the graph go?' (T2)

At home, he would program applets himself, producing a source code for them. He mentioned that he had used an applet that he had programmed in NetLogo in one of the science courses that he taught. He did not publish any of his work, however, because of time and effort required to document it and bring the code to a state he would be proud of.

Why it was used

T2 mentioned several reasons why he used and had his students use interactive applets:

- They allowed him to represent the concepts of quadratic functions and exponential functions graphically and algebraically, simultaneously, which he believed helped his students understand the connection between representations of a function and the above-mentioned underlying concepts.

  > Students can get fixated on a particular presentation of a concept to their detriment. It is better for them to view multiple representations of a concept in order to see 'beyond' the representation. (T2);

- The applets that he and his students used were easy for the students to use, and therefore students could focus on the "instructional goal" rather than on how to use the software to complete attain that goal. He mentioned how this was important because students "really have a lot of difficulty with user interfaces".

> If you're running it locally [i.e. if you download it from the website and run it from a hard drive], then all you have to worry about is sliding sliders, and pushing buttons, and that's it.
>
> The student doesn't have to worry about manipulating the interface, except for specifically what is the instructional goal. (T2);

- They were effective at convincing students about the effect of parameters in algebraic representations of functions on their graph, as they could experiment and notice "regularity" (Pierce & Stacey, 2010) in the effects. He believed that convincing them, rather than having them just memorize these effects was important.

> At some point [students] need to be convinced, they need to see it over and over again, at least a couple of times, so that they can buy into it. They say 'yeah I get it, the 'a' parameter, the multiplicative parameter, is always going to do this because it's always in the same place, it always has the same mathematical effect on the function rule'. So at one point, [they] won't need to see it anymore. (T2);

- The applets that he used were free (as opposed to propriety applet, such as Gizmos);
- Comparing them to the graphing calculator, he mentioned how he liked using them because and that "you don't have to worry about the window". He mentioned how, to him, this was an important difference, because he taught the senior grades, in which having students choose a correct scale was only part of many larger tasks. As will be explained below, he admitted that if he could not download an applet, there could be issues with the website from which he would have to access it. This was not as serious of

an issue as the constraints of the graphing calculator (to him); however, as he would most often download them to avoid these complications;

- He mentioned that, because there were so many free applets available on the internet, that if you looked for long enough, you could "find pretty much exactly what you want";
- He explained that he took the students to work individually with applets in the computer labs because he believed they would "learn better if they play with it themselves", as opposed to him using it for demonstration. He only did this once per year though because he believed that the more often they went, the likely they were to run into "hardware difficulties" or "network difficulties". He would also, therefore, use one in class for demonstration. He explained how he did so because "projecting them, and using them as part of my lecture… that's fail safe".

Why he hesitated to use it (at all or to complete certain tasks)

He explained how he did not use, or have his students use, applets more often because of time constraints.

> [The amount that I use them is] going to vary by course, by grade level, by student. I think what it really comes down to is how much time do I have, and the answer is very little. (T2)

He mentioned how, in comparison to proprietary software, websites offering free applets were not often well maintained or organized, and that it could therefore take a long time to find the applet that you would like to use.

Again, comparing open-source to proprietary applets, he mentioned how open-source applets were not always accessible because of issues with the websites that hosted them, and that he therefore did not like to rely too much on them.

> Basically, in my experience, applets of any kind on freely-accessible websites (i.e. not like Gizmos where you pay a subscription fee) are not entirely reliable. The website could be down for maintenance, or have very high traffic (therefore slow/no response), or the owner/author could have made modifications to the applet so that it no longer had the exact features that are required. Therefore I am hesitant to be overly reliant on specific applets on specific websites. (T2)

He also mentioned how he didn't want his students to use applets on every type of function that they studied because he didn't "want them to become dependent on it."

Microsoft Excel

Tasks

Similar to how T1 had in past years, T2 was using Excel to log his students' marks and calculate weighted averages of them. With his secondary 4 students, he also used it to calculate the standard deviation of a given class's on a test, on an exam, and over a term.

Techniques

To do so, he used all of the same techniques as T1 did; he typed data into cells, used the cursor to highlight cells, selected options from different types of menus (pop-up menus and drop-down menus), and used Excel's "Sum" utility. He also used the "Average" utility (also found in the "AutoSum" menu).

Why it was used

He created spreadsheets of students' marks so that he could send them to parents who requested them or who he thought should see them, and so that he could submit the students' term averages to COBA at the end of each term.

He claimed to use Excel, rather than any other software, to complete these tasks, because "there's a lot of statistical functionality there that you're just not going to get anywhere else." Also, comparing it specifically to COBA, he mentioned how "COBA will not allow us to weight our evaluations", because doing so was not in line with the educational reform that was in place at the time.

Microsoft Word

Tasks

T2 would use Microsoft Word or Microsoft PowerPoint to present most of the content of his lessons (e.g., definitions, graphs of functions, tables of values, and other images, questions, solutions to questions). The software did not eliminate the use of the blackboard or marker board, which served a different purpose.

> To me, the blackboard and the whiteboard now have a very specific role, where it's a very long, complicated, maybe intricate mathematical procedure where I really want them to see how I cancel variables, and make my substitutions, and so, because it is a very physical thing, I exaggerate the motions and I try to get them to learn, almost kinaesthetically, that way, and it forms a very, very small part of my teaching. (T2)

Thus, he would use the blackboard to present a new computation that would be more than just trivial.

He also used Word to create all his tests.

Techniques

He mentioned how, in terms of the content of his presentations, "the text is short, a few lines per page". The text would include mathematical definitions, mathematical properties of the objects under discussion, and/or a diagram. He would go through the material, explaining different points, stressing the important ideas, etc.

In terms of creating tests, he mentioned how, “to create a graph of a function I do one of two things; one, if the graph required is ‘not to scale’, that is to say, the question involves general ideas but no explicit calculations or computations, then I use the draw tools in MS-Word” to create shapes “that closely approximate polynomial and rational curves” and click on and drag corners of this image to enlarge or shrink it. “Otherwise, if necessary for calculations, I can paste coordinate values onto the graph created by the previous method using the ‘text box’ tool”. Any text involved in his test, including algebra, was either typed-in using a keyboard or inserted using Word’s “Insert Equation” and “Insert Symbol” tools from the “Insert” menu.

Why it was used

T2 mentioned several reasons why he used Word:

- He explained that he would often create presentations in Word before PowerPoint because it was quicker for him to do so;
- He explained that “the text on the projected Word document is really to help keep me organized and to give me time to erase the blackboard, walk around the class etc. while they take notes.” He added that he did this because “there’s no need for them to see me writing out questions”, or “a pre-formulated figure”, or “a formula that they already know”;
- He also mentioned how preparing certain material in advance allowed him to save time. He did not have to spend as much time writing on the board and could spend that time interacting with students.

Why he hesitated to use it

T2 believed that certain things need to be done by hand, for example, geometric constructions, or solving a non-trivial equation involving multiple steps. To understand, he explained, the students needed to see the process of solving the equation.

> They need to see me. I need to justify that. But then I think there's a physical component, that's going to help it sink it, if they don't understand why the sign change happens, at least see me do different things on the board will help, you know, them internalize it, whether they understand it or not…. A lot of them are physical learners…. Just today, I had my weak TSO class. They were trying to solve a z-score problem, a standard score problem, and they were having trouble working backwards, and it's an (add – 'in terms of') basic algebraic task, with proportions, and so forth, so I say, 'so here's your class average, your mean, and I'm gonna take it over here [he gestures, as if carrying the term] to the other side of the equals sign,' and I made that physical move, and I saw that a lot of them, who had gotten it wrong, go, 'that's right! When you go to the other side of the equal sign you change the sign'. I can't yet do that on the screen. (T2)

He also gave a few examples of ideas in statistics that he believed were better conveyed by having his students work with pencil and paper than with their graphing calculators.

> One thing we teach them to do by hand, and I still do this, is to use the box method for getting the correlation coefficient... to help them understand what an outlier is... just to give them… and also for them to get a sense of the numerical value of the correlation coefficient.
>
> Also when you're given a scatter plot with no scale, you can't enter the data because you have no idea what the values of the points are, but you just know that they're there, and that they've been plotted accurately. Well then you can do that by hand. (T2)

Microsoft PowerPoint

Tasks

As mentioned above, T2 used either Word or PowerPoint to present all of the information from his lessons that do not involve "very long, complicated, maybe intricate mathematical procedure", and worked towards having all of his presentations on PowerPoint.

Techniques

Similarly to T1, when creating a PowerPoint, T2 navigated through PowerPoint's menus and submenus, clicked on buttons on the user interface, dragged corners of images to enlarge, stretch or shrink them, typed-in information where needed, and changed slides by clicking and dragging the scrollbar to navigate through the slides. He also animated slides, also by navigating through menus and submenus, keying in data, and clicking on buttons, when necessary. One example of an animation that he would include was one that would sequentially add the "vertex, axis of symmetry, congruent (perpendicular) distances between axis of symmetry and zeroes, etc." of parabola, to its graph, that he would have appear with the given slide, or to sequentially animate in "procedures for solving equations".

Also, as T1 did, to start the slide show in class, he clicked on a button on the user interface and moved through the slides throughout the lesson using the arrow buttons on the keyboards.

Why it was used

T2 mentioned several reasons why he used PowerPoint (many being similar to his reasons for using Word):

- Although he preferred to create presentations with Word originally (because it was quicker for him, as explained above), when time permitted, he would recreate the same lesson in PowerPoint. Given more time still, he would continue to work on these PowerPoint presentations and add animations. He worked to have all of his presentations in PowerPoint because this way of presenting information was more interesting for the students than his use of Word or the blackboard, because the content of the lectures could be illustrated with dynamic visual matter: drawings, diagrams, colors and animations;
- Scaled drawings and graphs could be more accurate and neat than drawn on the board;
- PowerPoint saved time: Time saved from having to write on the board allowed the teacher more time facing the class, interacting with students, "checking on them, making sure they're not miscopying, making sure that they keep up";
- Keeping students engaged.

T2 explained the last point by saying that "within a given classroom… there are varying ability levels", and by projecting his slides and having the students take notes, "potentially everyone is addressed, because I can give the stronger students the reign to solve the problem completely on their own, and yet I have guided practice on the screen. And finally, I'm walking around and helping the struggling students."

"Animations" in the first reason above refers not only to animated figures, but also to presenting the text (steps of a procedure, for example) sequentially, so as not to overwhelm students with too much information at once. This simulated live writing on the board.

He would animate his slides because "students get confused and intimidated when there's too much info on a slide" and because "when you write on a black/whiteboard, the info gets added sequentially and students can see the connections and/or process; animations in

PowerPoint are the same idea." He preferred to animate the steps to the procedures that he would teach rather than have each step on a new slide because he wanted students to be able to see the full procedure together once the procedure was complete.

Why he hesitated to use it (at all or to complete certain tasks)

T2 also hesitated to use PowerPoint because (as mentioned above) he believed that some things needed to be done by hand. He explained how, with PowerPoint animations, "there is a small loss of flexibility to go back and forth and respond to student requests", and that because of this, for example, "if a student says, 'I didn't get it, can I see that again,' or just one part of it that I haven't really foreseen, in the PowerPoint animation, I have to do it on the blackboard."

T2 did not post his PowerPoint slides on the internet for all his students after class. He wanted them to learn to take notes which he considered to be a valuable skill in college and university studies, and he believed also that one learns through taking notes. He kept the slides and other documents prepared for his lessons, however, and made them available for students who could not come to school on a particular day.

COBA Management Software

Tasks

Most of T2's uses of COBA were very similar to T1's. In fact, T2 used COBA to complete all of the tasks that T1 did except for, for creating paper copies of his grades by printing out lists of his students' names. He would not use the mail feature of COBA very often to communicate with parents; however, as he admitted that "the vast majority" of his electronic communication with the parents of his students was via Outlook.

Unlike T1, T2 would extract the email addresses of the parents of his students from COBA (when needed) to email them from Outlook. He would also post "extra practice material", “fully worked out” solutions to tests that the students had written, and “instructions on how to use their graphing calculators towards what they learned about statistics”.

Techniques

How T2 used COBA could be described in the same way as T1’s techniques.

Why it was used

As mentioned earlier, it was required by the school that he use it to check messages sent to him from parents and colleagues, enter attendance, and end of term grades. He enjoyed using it to post the solutions to his evaluations because it released him from having to “redo every single example” in class, as he could simply post the solutions and it “would be up to them to check it.”

Although he did not use it often, he was interested in COBA’s “automated mailing lists” because (in his words), “I can communicate electronically with a parent instantaneously, I don’t have to go looking up their email address”, which he would have to do if he wanted to email them from Outlook, or call them. He also thought that another benefit of this feature was that “you can’t get junk mail in COBA, because nobody had your email address”, as opposed to email where “you run into junk mail problems, and of course if you've got personal student information that's on it also, that's a sensitivity issue”.

Why he hesitated to use it (at all or to complete certain tasks)

In response to this question, T2 mentioned two aspects of COBA: that it’s email system for communication with parents did not allow him to attach documents to messages, and that the system was very slow. T2 would use the attachments feature of his personal email to send the parents a scan of their child’s test, or class notes and homework assignments that a student might have missed. The slowness of the COBA system was caused by the fact that it was on the same server as the internet access for students. Students would use internet intensively during lunch time or one hour after school. Teachers would often like to use this time for entering grades, but access was slowed down by the simultaneous use of the server by the students.

T2 explained that the people at COBA had denied all Québec teachers access to certain of COBA's functionalities because of the education reform that was taking place in Québec preschools, elementary schools, and high schools, at the time of the interviews. For instance, the function, of the tool that allowed teachers to keep track of their students' marks, that allowed teachers to calculate weighted averages of their grades had been shut off. This was shut off because, under the reform, teachers' were not to weight their students' assessments. They were to 'eyeball' what the students' term marks should be for each competency, based on their performance on individual tests. Québec reform documents encourage them to "exercise judgement"[4] when determining their students grades. T2 still weighted his grades, however, and claimed that this was his way of using his "judgement". Since he could not do this with COBA, he did not use that tool in COBA, and instead used Excel to keep track of his students' grades and only entered the final grades, each term, for each of the three competencies on which they were evaluated.

Microsoft Outlook

Tasks

T2 would use his email to send messages to, and receive messages from, parents, administrators, and professional organizations. He mentioned how most of the messages that he would send to the parents of one of his students contain "either a copy of their daughter's test

[4] http://www.mels.gouv.qc.ca/lancement/Renouveau_ped/452771.pdf

that [he] scanned, so they can see the original", "an attachment for homework that the students have missed, or notes that the students have missed", or "an Excel spreadsheet of [a students'] marks over the last term".

Techniques

He used his email in the same ways that T1 did.

Why it was used

As mentioned above, T2 used Outlook because COBA would not allow him to attach a file to a message to any message, which is what the majority of the emails that he would want to send would contain. For example, he mentions how, "if a student is absent, I would like to send them the notes, but I can only do that in COBA by posting them to the whole class. So I end up e-mailing." He does this knowing that "if I e-mail and she e-mails it to everybody else, I haven't prevented anything," but that "at some point, the e-mail chain will stop, and that, that will be before the whole class gets the notes."

He also preferred Outlook to COBA for exchanging messages because "you can't give out your COBA email... it's on intra-web that exists only between parents, students, the administration, and the teachers", and "I need an outside e-mail, because I need to communicate with professional organizations".

T2 used his email as his primary source of contact with parents. He only called if he felt that he had to.

Computers in classrooms, with internet access and writeable SMART technologies' screens, connected to multimedia projectors

Tasks

As T1 did, T2 used the classroom computers to play videos, to run COBA on so he can enter attendance, and to present all of his PowerPoint presentations and Word files needed for his lessons. As mentioned above, he used the SMART technology to write extra information on Word files or PowerPoint slides.

Techniques

He would control the presentations and input data into COBA using the keyboard.

He mentioned how "sometimes, when they have a homework question in their textbook, I'll still have it up here [projected onto the partially rolled up projector screen above the blackboard]".

Why it was used

He mentioned how he liked using the SMART technology in unison with the Word documents and PowerPoint presentations (around which he would structure his lessons), because it allowed him a lot of the flexibility that the chalkboard did, but also afforded him time by allowing him to prepare some of the material in advance. For example, he mentioned that he could "pop-up a document that already [had] a nice equilateral triangle", and have it appear in a Cartesian plane containing a grid. In this case, he would draw a median line with the stylus, knowing that it would be "reasonably accurate" and that his students could see him "manually" doing something (which, as we described above, he thought was important).

He also liked working with the projector screen pulled part way up and projecting only a small bit of information above the blackboard. Because of the height at which the screen had been installed, relative to the top of the blackboard, he explained how he could have a small bit of text displayed without covering any of the blackboard, and therefore that he gained "more real-estate to work with; but a different kind, where I can be functioning in different phases or different speeds". Because of this set-up, he was able to work on the entire blackboard and "constantly refer back to what's there, or scroll forward a little bit." He believed this way of working was "powerful."

Why he hesitated to use it (at all or to complete certain tasks)

The school had a second building built a couple of years prior to the interviews. In the new building, classrooms and computer labs were equipped with computers with writeable screens, but they were made by a different company than those in the classrooms in the old building. T2 expressed how he hesitated to use the stylus in the old building because they were slightly different than the ones in the old building, which he had become used to.

> It's a real problem, because I'm really comfortable with the one in the new building, in the labs. I started off with that. For a year, I checked out all of the intricacies, I got really good, and this one [we were in the old building at the time of the interviews], I haven't had the time to sit down and go into all the details. I'm really awkward with this one, and I make mistakes. And so because of that, because of my awkwardness, I tend not to use it as much as I could. (T2)

I found it interesting how even he, who was comfortable with many artifacts and looked to use new ones in the future, was stunted by this. This was much like T1's decision to use MarkBook rather than Excel, or his to use Outlook as his primary means of communication with parents rather than COBA. Of course, the software that they compared are different in more ways than one, but all of these examples speak to how, once someone is able to comfortably complete a task with one technology, they are unlikely to use a different one to complete the same task unless they see some larger advantage (for instance, if T1 thought that learning Excel could allow her to do more things that she would like to do with her grades than MarkBook).

5.1.2.3. Technology that T2 planned to use in the future, but had not used yet

GeoGebra

At the time of the interviews, T2 had not yet tried using GeoGebra at the school, but had planned to use it the following year. He had not yet used it with his students. In particular, he had planned to develop a sequence of units, using pre-programmed functions of the software, and ones that he would program himself, to cover the whole curriculum except, perhaps, for statistics.

Why he was planning to use it

He mentioned how he was having difficulty teaching the chapter on algebraic expressions with the graphing calculator, and was optimistic that he could use GeoGebra to do this. He was, however, comfortable teaching algebra on the blackboard, so one might interpret this ‘optimism’ as a sign that he might have wanted the option to teach everything with technology one day, or that he saw something that could be afforded by it, that he could not do on the blackboard or any of his current tools.

He liked GeoGebra because, while it could do everything that The Geometer’s Sketchpad could do (and more), it was open source. Therefore, he explained, he could have it at home and at school and he would not “have to go through the administration” to be able to do so. Students could also download the software for free from the internet, but he could save it on CDs for students whose parents had an issue with downloading things from the internet. This way, he could use the software not only for work in class but he could also assign homework to be done on GeoGebra. In fact, he found that work on GeoGebra in class could be problematic because of issues with the availability of the computer labs and the slowness of the network connection.

It is important to stress that the last comment was coming from a teacher working in a well-equipped school, where there were more technological resources available than in many other schools. As mentioned before, the school had recently invested roughly two million dollars into building a new wing to the school with up-to-date technology, but T2 was still considerably worried about the costs of any new software that the school might purchase. This enforces the point that, even in schools with larger budgets, cost is always an issue (i.e., no one's budget is unlimited, and if the school could find cheaper software that fit their needs as well or better, then they would likely integrate it instead).

Mathematica

T2 was not using this software at the time of the interviews either, but he thought about using it in the future. He had even asked the school to purchase a license for Mathematica, but was not sure, at the time of the interview, if his request would be granted. We therefore did not prioritize discussing his uses of this artifact and, in the end, did not have time to discuss in detail what he would use it for or how. He mentioned how he had taken a demo version of the software "for a few test drives" at home.

What it would be used for

He mentioned how "what's nice about it is the cross-curricular work that you can do, where you can take mathematical concepts and bring them to the physics class, for example. At the high school level, physics is very, very mathematical, yet, sometimes it's very difficult to establish that connection. Things like Mathematica can really help with that."

Why he would hesitate to use it (at all or to complete certain tasks)

He admitted that "things like Mathematica and Maple" were very hard for first time users to work with. He added, however, that "if there's a module already designed that has a nice interface, it's not a problem, but if you're trying to put something together yourself, you have to learn their programming language, which we don't have time for".

5.2 The teachers' views on the pragmatic and epistemic value of their instrumented techniques

As the interviews progressed, it became clear that one of the main differences between T1's and T2's views on their uses of technology was that T1 did not seem to see any potential in her or her students uses of technology for improving their understanding of mathematics, whereas T2 certainly did.

This conclusion can be supported by the teacher's statements. For example, T1 said that that although allowing students to use calculators (simple or graphic) on tests could improve the test results or make certain procedures easier, it would probably not contribute to improving their number sense or understanding of graphs. She believed that any improvement in her students' understanding was caused by their pencil and paper work (at home or in the classroom), group discussions that she would encourage, and her (PowerPoint, Word-based, chalk board, or whiteboard) presentations. In fact, it seemed that T1 saw her use of technology, in general, as superfluous to helping them understand the mathematics that they were being taught.

When I asked her whether she believed that teaching without her videos, computer games, PowerPoint, or any of the technology that she was or her students were using, would cause the students to do worse on summative assessments, she confidently replied “no”. She also admitted that the main reasons for her using technology and for her having her students use technology in the classroom were to entertain her students and to “speed things up”. She did not seem to recognize, as Pierce and Stacey (2010) did, that the ‘functional opportunities’ afforded by technology might have afforded some ‘pedagogical opportunities’.

T2, on the other hand, seemed to see both a pragmatic and an epistemic value of his instrumented techniques.

> For me, technology has two possible uses in the classroom. Number one, it’s things like student utilities, like factoring, or the quadratic program, that speed up and simplify the task for them. But it’s not really part of the learning. And then there are other uses of technology that are part of the learning, and they don’t necessarily speed things up. (T2)

As an example of uses of technology that ‘speed things up’, he referred to having his students use programs (for instance, the ‘quadratic program’ that factors a second degree polynomial, given its coefficients and inputs). He mentioned, however, that these programs “can really get in the way of understanding, because they automate for you.”

As an example of uses of technology that improve students’ understanding, he mentioned how he was planning to use GeoGebra to create a dynamic rectangle whose side lengths would be represented by algebraic expressions, that could be enlarged and shrunk by dragging the sides of the rectangle, and would modify the expressions representing its dimensions instantaneously. Also, the area would be represented as the product of the side lengths, and would change with the side lengths. He conjectured that, as a consequence of employing this technique, “they’ll see how the... algebraic expressions change and how the factors relate to that situation.” He believed that this activity, in particular, could be useful as “they don't understand what factoring is. They realize it breaks it up in two, but what do these things mean, what does x-2 mean? You know? What's the physical meaning? And I think that's where something dynamic and colorful might help."

I conjecture that the difference in the value that they associate to their instrumented techniques is important to consider when thinking about how they might use technology in the future and how they might feel about 'optimistic' statements about the use of technology in mathematics education, such as those given by the NCTM.

Chapter 6: Synthesis of the results of the interviews

If we compare T1's and T2's uses of technology, we find an example of two secondary school mathematics teachers who had access to the same technology and technological support (as they worked at the same school), differed only slightly in the types of technology that they were using that year, but used the technology to complete (and have their students complete) very different tasks. Teachers differed also by the techniques employed to complete the tasks and the reasons for using the technology the way they did. Some of these differences might be explained by their beliefs about the value of their instrumented techniques. They might also be explained by the opportunities that each teacher believed could be afforded by teaching with technology (we will discuss these opportunities in Chapter 7, in light of the literature review).

I did expect some of these differences, since the teachers taught different grades and had different levels of experience in teaching with technology. I did not expect, however, so much similarity in the types of technology that they were using. I thought that the interviews with the teachers would reveal their use of and preferences for very different sets of technology, and that, therefore, interviews with two teachers only would reveal the existence of a wide range of teachers' practices relative to technology use.

Still, information collected on the two teachers' practices during the year of the interviews is sufficient to suggest that we do away with blaming the lack of access to technology for teachers' not using it or not using it in ways advocated by certain researchers or software developers.

This chapter spells out succinctly the similarities and differences between the two teachers' practices of using technology in mathematics teaching. The chapter is divided into five sections. The first will compare the technological artifacts that each teacher used, the second -

the tasks for which they used the technology, the third - the techniques for completing the tasks, the fourth – the teachers' reasons for using the technology that they did, in the ways that they did, and the fifth – the reasons why they hesitated to use certain pieces of technology.

6.1 Artifacts

T2 seemed to have had experience with a wider variety of technological artifacts than T1, some of which were not as user-friendly as those that T1 had used. At the time of the interviews, he was exploring, at home, the possibilities of *Mathematica*, without even being sure this software would ever be purchased and implemented in the school. T1 did not report engaging in such explorations. Yet, in terms of the technology that they were using in the year of the interview, T1 used a larger variety of technological devices than T2. Both teachers used graphing calculators, COBA, and Microsoft Outlook that were prescribed by the school, and both opted to use the Microsoft Office suite of Excel, Word, and PowerPoint. They both ran PowerPoint and Word from their classroom computers and projected their presentations using the available multimedia projectors. Neither T2 nor T1 forced students to purchase the graphing calculators and allowed them to use scientific calculators. The only other type of technology that T2 was using that year was interactive applets. T1, on the other hand, also used MarkBook, showed her students videos, and had her students use four-function calculators and play computer games.

6.2 Tasks

The teachers used Microsoft Word, PowerPoint, and Outlook for the same or similar tasks. There were differences, however, in tasks performed with other types of technology.

Some of these differences might be understood as consequences of their having taught different grades and different streams of the grade that they both taught (secondary 4). That being said, looking at T1's course outline, I can see many places in which she could have had, but had not, her students use their graphing calculators, for example, to perform tasks that T2 had his students delegate to it (e.g., finding x- and y-intercepts of a graph or finding the measure of an angle or a side length in triangles using appropriate programs related to the sine law or cosine law). In fact, it was in their use and their students' use of graphing calculators that we noticed the most significant differences in the tasks that they used technology to complete. T2 and his students completed a much wider range of tasks with them than T1's students did.

In terms of their use of COBA, both teachers' completed many of the same tasks with it, but T2 mentioned using it to post solutions to tests and instructions on how to use their graphing calculators, which T1 did not. She would use class time to review solutions to all of the questions that were on her tests, while T2 would only go over, in class, those problems that he believed his students had the most difficulty with.

T1 used videos to "slow down" her enriched secondary 2 class or fill the surplus of time she had with that class. T2 did not speak of doing this; in fact, he spoke of how he often struggled to get through the whole curriculum.

T1 had her students play computer games to have them practice solving problems individually. T2 would present problems to be worked on in class on hand-outs on Word or PowerPoint. These two approaches might have different effects on the social dynamic of the classroom during certain problem-solving sessions.

In the future, if things went according to his plan and he was able to integrate GeoGebra to the extent that he wanted (to teach each section of the secondary 4 course), then the tasks that they sought to complete with technology would likely differ further (assuming that she did not adopt it to the same extent that he did).

6.3 Techniques

The two teachers' instrumented techniques with PowerPoint, COBA, and Microsoft Excel were identical (except that T2 used the Average utility, while T1 (when asked) mentioned that she did not). To "orchestrate" his students techniques (whether when they were using their graphing calculators or not), however, T2 would walk around his class while projecting his PowerPoint presentations, whereas T1 would remain (for the most part) at the front of her classrooms. Most of the time, she would even ask her students to pose their questions aloud so that the whole class could hear them. She would only seldom go to a student's desk to speak to them about a question that they had individually.

In terms of their use of graphing calculators, their techniques were similar, although T2 allowed his students to use wider range of utilities and applets, and allowed them to use programs that he programmed himself.

In Word, to create tests, T1 would copy and paste images of graphs that she had either found on the internet or had generated using Excel, while T2 created his graphs using the drawing tools and text boxes available in Word.

In terms of their use of the classroom computers, T1 allowed her students to come up to play computer games, whereas T2 did not (although he could have allowed them to play with the applets in class).

Because T2 had used The Geometer's Sketchpad, GeoGebra, and Mathematica, he had also employed, and had his students employ, a wide variety of techniques that T1 had not employed. For instance, because how one creates geometric shapes in The Geometer's Sketchpad or GeoGebra is unlike how one would do so with PowerPoint, Word, or any software that T1 used to do this (outside of the fact that objects has to be selected from palettes available in menus). It was clear that he recognized these differences when we spoke about the user interfaces of different sophisticated technologies:

> How you handle things on the graphing calculator is very, very different than how you handle things on Geometer's Sketchpad, is very, very different than Maple [though he had not used this software before], is very, very, different than Mathematica. (T2)

6.4 Why they used certain technology

Both teachers used particular technologies because these technologies:

- Helped to pace the class. For example, T1 used riddles to keep her advanced grade 8 group busy. T2 used PowerPoint because it allowed his "to be functioning in different phases or different speeds", by projecting slides while working on the board or walking around to work individually with students, catering to the speeds at which different students in his class learnt;
- Improved the speed and accuracy with which students work. For example, T1 had her students use their four-function calculators to perform basic arithmetic operations. She also mentioned how she used MarkBook because she found it to be the fastest and easiest way to keep track of and tabulate her students' marks. T2 had his students use programs on their graphing calculators for subsidiary tasks during class work and summative assessments and projected images of geometric figures because of the speed and accuracy that these technologies provided him and his students;
- Were mandatory to use (e.g., COBA and Outlook);
- Made one or more tasks easier to complete than other technology or than completing the task(s) without technology. For instance, T1 was very comfortable using Word to prepare her test, much more than she was with any other software (and, though I did not ask, I

suppose she did not want to write her tests in pencil or pen). T2 had his students use applets because they were very easy to use;

- Facilitated interdisciplinary work. T1's colleague had noticed how the students could use Excel to complete some of their mathematical tasks and help to prepare them for their science fair. T2 had mentioned the cross-curricular work that could be done with Mathematica;
- Provided instant feedback. T1 mentioned how certain games that she had her students play provided this. T2 mentioned Gizmos;
- Made lessons more entertaining and/or interesting for their students. For instance, both teachers mentioned how PowerPoint had this effect;
- Clearly demonstrated concepts. T1 mentioned how PowerPoint helped her students distinguish between one triangle and another, congruent to it, and inscribed in it. T2 mentioned graphing calculators helped students see the how well certain regression lines fit the data that they were employed to;
- Offered paperless resources (which T2 posted on COBA and T1 emailed to her students).

T1 also mentioned that:

- Technology (such as four-function and scientific calculators) helped boost her students' confidence in learning and using mathematics;
- Videos helped keep her students' pay attention to detail;
- Introducing her students to technology (such as Excel) in class helped prepare them to use technology in their future careers;
- Technology (such as Word, Excel, and PowerPoint) helped her and her students present information in a "professional" way;

- Technology offered an alternative to the traditional marker board lecture (in case of glare).

T2 also mentioned that:

- Delegating certain student tasks to technology (such as their graphing calculators) allowed them to access “higher-level skills” faster;
- The constraints of certain technologies (such as graphing calculators) helped to motivate discussion about mathematical ideas that are otherwise hard to motivate (such as scale);
- Technology (such as The Geometer’s Sketchpad and GeoGebra) allowed him to cater to students’ different styles of learning (e.g., by representing ideas graphically and algebraically);
- Certain software (such as PowerPoint and Word) helped structure his lessons;
- Technology (such as Gizmos) offered ready-made tests (which saved him time and allowed his students more opportunities to evaluate their mathematical skills and understanding);
- He used technology to convince students of mathematical ideas before the students were at the level where they could understand formal mathematical proof;
- Technology allowed him to change the classroom’s social dynamic. For instance, he mentioned how PowerPoint allowed him to walk around and speak with individual students and groups of student privately, whereas he would have had to have used that time writing on the board with his back to the class if he did not have it;
- Technology could improve student understanding.

All in all, T2 stressed the epistemic value of using technology; it allowed him to keep a balance between procedural skills, understanding and problem solving.

6.5 Why they hesitated to use certain technology (at all or to complete certain tasks)

Both teachers mentioned several reasons for hesitating to use, or for not using, particular technologies.

- Learning the user interface would take too much effort compared to the advantages afforded by that technology. T1 hesitated to use her graphing calculator because her students found them very difficult to use, even when she gave them written instructions on how to do so. T2 mentioned how one of the reasons why The Geometer's Sketchpad was not accepted by most teachers because of its "very particular user interface".
- Teachers need to be comfortable in using technology to want to use it. T1 mentioned how she was not confident teaching with her graphing calculator and therefore that she did not use it (and did not teach her students to use theirs) to the extent that she was aware they could be. T2 mentioned how he hesitated to use the writeable screens in some classrooms, because he was not as confident in using them.
- Technology required a lot of their and their students' time. Both teachers mentioned how they had not animated many of their PowerPoint lectures, although they would both have liked to, mainly because of the time it would take to do so. Both teachers also mentioned how they hesitated, or did not, bring their students to computer labs because of how long it took to take them there, get them started, and to deal with technical difficulties (if any should occur). T2, however, believed that his students learned more from using applets by themselves than from his use of them for demonstration. T2 also believed that,

> one of the root problems with technology being implemented in, I think, education as a whole, not going where it's supposed to, is that, at least in

Québec… they haven't decreased the amount of topics in the curriculum" and therefore that there wasn't enough time for the students to become comfortable using certain types of technology (T2).

- Technology did not allow the as much "flexibility to go back and forth and respond to student requests" as teaching with the chalkboard or marker board did. Specifically, they both mentioned how this was the case with PowerPoint, but that they could remain more flexible (in this sense) when they used the blackboard or marker board in unison, and therefore that they did.
- Technical difficulties could arise. T1 mentioned that another reason for her not bringing her students to the computer labs was that she would often lose time with the computers malfunctioning. T2 had mentioned how the memory of a graphing calculator could overload and that its batteries could die. He also mentioned how, when he had a class right after lunch, when the server was busy with students accessing the internet on their break, it could be very slow for him to log onto COBA, as COBA did not have its own dedicated server. All of these issues can also be seen as time issues.
- If the technology that they were using was not freeware, cost was a factor. T1 mentioned how she had stopped using MarkBook originally because she would have had to pay for it herself. T2 mentioned how he was more likely to propose integrating GeoGebra over Mathematica the following year, because implementing Mathematica would be much more costly.
- Teachers need to perceive some advantage in using a particular piece to complete certain tasks. For example, after her secondary 1 students were no longer required to participate in the school science fair, T1 stopped teaching them to use Excel, which she had done mainly to help them produce more professional looking presentations. She saw no further

advantage to having them use it than the opportunity to work on something interdisciplinary. T2 hesitated to use the writeable screens in certain classrooms (powered by SMART technologies) because the writeable screens in other classrooms, which he had become accustomed to using, were powered by a different software (a product by Hewlett-Packard). He was otherwise keen to use new technology (he planned to try to implement Mathematica and/or GeoGebra over the following years). His unwillingness to learn to use this technology therefore seemed a result of his not perceiving any significant gain from using it, when he could simply schedule to be in the computer labs, which were equipped with the software that he was comfortable with (even though he admitted that they would sometimes be difficult to book).

T2 also offered other reasons for hesitating to use certain technology.

- Videos "can't answer questions".
- Use of a given technology should be regular, otherwise it can just seem like a "gimmick" to the students. Gizmos indeed appeared as a gimmick to students because it did not fit the Québec curriculum well and could only be used occasionally.
- When a piece of technology is adopted in a school, it should be adopted by all or most teachers. T2 mentioned how, with technology such as The Geometer's Sketchpad, with less user-friendly user interfaces, that if they were not used throughout the department issues of cost would come up, and parents might complain about some students having been allowed to (or forced to) use a given piece of technology that others were not.

6.6 Summary

As mentioned in section 5.2, one of the main differences between T1 and T2 was their views on the value of their instrumented techniques. T1 did not seem to see any potential in her or her students' uses of technology for improving their understanding of mathematics, whereas T2 certainly did. I conjecture that this is partly why T2 used, and had his students use, the graphing calculator in many ways that T1 did not.

Another difference that seemed to exist between the two teachers was that T2 seemed to spend more time thinking about how teachers' and students' uses of technology could or might affect students' understanding of mathematics than T1 did. For example, T1 had shown a four-minute YouTube video titled "The world without Pi", in which several objects that one would expect to be roughly cylindrically-shaped or spherically-shaped (e.g., car wheels, blueberries, soap bubbles, etc.) were not. When I asked her whether she believed that this might convey any false information to the students, such as that spherical objects exist only because of the number Pi, she replied, "I don't know; that's a good question." After prodding a bit further, it became clear that she was only showing the students videos to entertain them, and that she had not thought about the messages that they might convey, other than those which she intended them to. On the other hand, speaking to T2 about videos that he had shown his science class (as he did not show any videos in his math classes), he explained that choosing the right video and deciding how to use it requires a lot of thought.

> I've only got so much time for research, and these things take research; not just, 'I'm gonna show this because I need to show something for 15 minutes'. I need to figure out what is the instructional protocol that I'm going to wrap around this. (T2)

T1 put a lot of thought, however, into how she could make math more comfortable, and even fun, for her students to learn. She even seemed to prioritize this over trying to encourage their understanding of certain topics. For instance, speaking about her secondary 2 students, she mentioned how:

> They seem to feel a lot more comfortable if there is a calculator in front of them. They can just start punching things in. They may not always know what they're doing, but it's a security blanket in a lot of ways. (T1)

In terms of instrumentation theory, they seemed to differ in that T2 seemed to have instrumented the technology that he used more than T1 had. T2 seemed to think of the tasks that he wanted to complete (some of which were inspired by the technology) and would use technology to do so seemingly because it was instinctive for him to do so. His uses of technology seemed very natural. T1, on the other hand, seemed to try to find ways to use technology. She seemed to believe that the idea of using technology was interesting, but had mentioned how she struggled to use almost every type of technology that she did (except for the videos or the computer games).

In terms of the processes of instrumentation and instrumentalization, T1's teaching style hardly seemed changed by her uses of technology (which became clear when I would ask how she used to complete certain tasks). She seemed to fit her uses of technology to her traditional (mainly teacher-centered) teaching style. She also had not 'shaped' any of the technology that she used in any way. She would not even stop the videos that she would show to interject with an idea, comment, or question. T2, on the other hand, both shaped some of the technology that he used and allowed his uses of it to be 'shaped' by the technology. For example, T2 would create programs for the graphing calculator for his students to use. In terms of how his uses were shaped by certain technology, he explained, "I wouldn't teach statistics this way [for instance, by giving his students large sets of raw data to analyze] if I didn't have a graphing calculator". His style was and had been fairly traditional (teacher-centered) as well, but it seemed that his uses of technology were encouraging him to adopt certain approaches that were more student-centered (e.g., allowing his students to explore the effects of parameters of algebraic representations of functions on their graphs).

The value that they perceived of their instrumented techniques and the ways in which they shaped technology and were willing to be shaped by their interactions with technology, I believe, helps explain why they used technology in many different ways.

With respect to the 2008 NCTM statement, I believe that considering how different these two teachers were (in all of the ways described above), the suggestion that both use a variety of sophisticated technology "strategically" clearly does not ask the same of each of them. For one, the instrumental geneses that they would have to undergo with certain technology would be very different; they would require different amounts of time and different amounts of effort. They would also likely feel differently about having to use such sophisticated technology, especially if they felt that the pace at which they were being encouraged to integrate them was too quick.

Chapter 7: Discussion and Conclusions

In this chapter, I will discuss the results of my interviews with teachers T1 and T2 in light of the literature reviewed in Chapter 2.

The chapter will be structured in the same way as the literature review. In the first section, I will compare the types of technology (i.e. the artifacts) used by T1 and T2 and those mentioned in the literature. In the second section, I will similarly look at the tasks that the teachers' sought to complete with technology; in the third, the techniques employed by the teachers will be discussed; the fourth will focus on the teachers' reasons for hesitating to use technology, and the fifth – on their reasons for using the technology that they did, in the ways that they did.

7.1 Artifacts

When we compare T1's and T2's uses of technology to the 2008 NCTM statement (Appendix A), we find that T1 was not using computer algebra systems or interactive geometry software and neither T1 nor T2 was using handheld data-collection devices or sensing probes which were the types of technology that the NCTM had suggested them to use.

The new NCTM statement, appeared in 2011 (Appendix B), was not reviewed in Chapter 2, as I became aware of it only after I have completed the empirical part of my research. The new statement is similarly enthusiastic about the use of technology, but it appears more flexible and less intimidating by making the list of suggested technology less specific, and by justifying some of their claims by references to concrete sources. However, descriptions of the necessary conditions for having technology become useful remain vague (e.g., "strategic use" and “balanced mathematics program”). One of these conditions is “professional development”. This condition was mentioned by both T1 and T2.

The main issue that I have with the NCTM position is that it still seems to imply that teachers *can* use technology to improve their students’ mathematical skills and understanding, but that whether the students’ ability to do or understand mathematics is improved or not depends only on their teachers. It also seems to dismiss the debate on whether teaching with technology provides deeper understanding and stronger procedural skills than effective teachers in non-technological environments could help their students achieve. The debate is – to my knowledge – not settled, as evidenced by exchanges in mathematics education research journals (e.g., Wilson & Naiman, 2004; Ruthven, 2005), and more general education journals (e.g., Schneider, 2011). Also, there is no emphasis on how hard it is to use technology effectively, or of the time required to do so – issues that turned out to be very important for the interviewed teachers.

Although the NCTM position considers and refers to research, it does not refer to any of the large amount of work on teachers’ and researchers’ concerns about the use of technology when it is not used strategically (which may be how it is often used). The consensus amongst many researchers in mathematics education seems to be that there is potential for technology to improve students’ ability to “do and understand mathematics” (Norton et al., 2000), but that these benefits are not immediate. As Guin and Trouche (1999) point out, developing the comfort with technology necessary to realize these benefits is a “complex process”, both for the teacher and the student. More of this detail should be included in future statements.

In comparison to the technology mentioned in research articles reviewed (Hennessy et al., 2005; Goos and Bennison, 2007; Peirce and Ball, 2010; Stoilescu, 2011), T1 or T2 were also not using CAS calculators, dynamic data software, data loggers, PDF converters, laptops (in their classrooms), interactive whiteboards (though T2 did use a SMART technology writeable computer screen), CDs accompanying a textbook, or classroom clickers. Neither teacher had a personal website or belonged to a teachers' community website (a "Wiki").

As the teachers surveyed and interviewed for those studies had, however, T1 and T2 had used handheld calculators (four-function, scientific, and graphing – which T2 would sometimes attach to an overhead projector), desktop computers connected to multimedia projectors, word processors (Microsoft Word), spreadsheets (Microsoft Excel), presentation software (Microsoft PowerPoint), lesson materials from dedicated websites (Explore Learning's Gizmos), and Video clips. Both teachers were working in regular classrooms and had brought their students to work in computer labs. As mentioned above, T2 was also experimenting with dynamic geometry software (GeoGebra) and a computer algebra system (Mathematica), though neither he nor his students had used them in class. He and his students had also used interactive applets and he had used LOGO (although his students had not).

7.2 Tasks

In terms of Hennessy et al.'s (2005) categorization of tasks, it seemed that T1 used technology only to support her existing practice. Even her use of PowerPoint could be interpreted as a new way to present the same information that she would have in years prior (i.e. it was the technique that had changed, not the task). T2, on the other hand, used technology both to support his existing practice (in how he used Word and PowerPoint, for example), and to substantially change his practice. For instance, he would allow his students to use programs on their graphing calculators during certain evaluations to help prevent their difficulties with topics discussed earlier in the year from interfering with their understanding of the ideas introduced later in the year, and their ability to work with those ideas.

In terms of Pierce and Ball's (2010) categorization of tasks, we can differentiate between T1 and T2's uses of technology by noticing that T1 and her students used technology only "to do mathematics", while T2 and his students used technology both "to do mathematics" and to improve their understanding of it. This seemed consistent with their views on the value of their instrumented techniques. I did not find this categorization easy to use, however, as it assumes that "doing mathematics" cannot improve one's understanding of it (or, in other words, that teaching procedure skills cannot improve one's conceptual understanding (Rittle-Johnson & Alibali, 1999), which the interviewed teachers did not seem to endorse.

7.3 Techniques

The interviewed teachers' techniques do not compare well with those described in the literature review as I had only found one paper (Goos & Bennison, 2007) that described, with detail, how students and teachers used technology. In comparison with the example presented from (Goos and Bennison, 2007), T1's and T2's uses of technology differed mainly in that neither teacher had used two types of technology during the same activity. The students in (Goos and Bennison, 2007), however, played with a computer simulation and worked with data based on their play using their graphing calculators.

Our findings also did not compare well with those described in the literature review, as they did not allow me to compare the range of techniques that T1 and T2 employed with those completed by other teachers. In fact, I have still not found a paper that describes the range of techniques used by one or more teachers. This is not to imply that no such paper exists, as there is a large amount of literature that I have not read, but I am left with the impression that research describing the techniques that teachers employ when working with technology is not very common.

7.4 Reasons for hesitating to use technology

T1 and T2 shared many of the same concerns that the teachers spoken about in the literature review had. In particular, the interviewed teachers were concerned that,

- Students could become overly dependent on technology;
- Allowing the use of more sophisticated technology in class would not prepare the students for assessment where such technology would not be allowed (in our study, T2 mentioned assessment at the college level, where students would only have access to four-function or scientific calculators);
- Time needed to become comfortable with a user-interface, and to plan "computer based learning" may impinge upon the time necessary to cover the syllabus;
- Their abilities with, and confidence in using, a given piece of technology were sometimes insufficient (in part, because they lacked training);
- Access to computer labs was difficult;
- The cost of technology might be incommensurate with the benefits of using it.

Additionally, they mentioned being concerned about:

- Technical difficulties;
- Not being able to teach certain pencil-and-paper techniques with technology;
- Having to use a given piece of technology, rather than implementing technology that fits their needs; for example, one of the main reasons why T2 continued to use both Outlook and COBA (despite admitting that he would rather only use one or the other) was that he needed to "communicate with professional organizations";

- Whether or not the technology fits the curriculum; for example, T2 preferred using interactive applets rather than Explore Learning's Gizmos because the Gizmos available did not fit the Québec curriculum very well;
- Whether or not the technology fits the school; when speaking of implementing new technologies into a school, T2 mentioned that, "always the bottom line is, you got a look at what it's a gonna do for the school, because every technology is gonna have its proponents somewhere, and they all fit some kind of niche, but the question is, is it going to fit your organization well";
- How widely a given piece of technology will be adopted, when they are thinking of implementing it.

7.5 Reasons for using the technology

In explaining their reasons for using the technology that they did, in the ways that they did, the interviewed teachers also revealed some of the opportunities that they believed could be afforded by teaching with technology. Comparing these opportunities with the list of opportunities that researchers claim can be afforded by teaching with technology, presented in Chapter 2, we notice much less optimism on the part of the teachers (especially T1). It should be noted, however, that neither teacher was asked whether they believed that the opportunities listed in Chapter 2 could be provided by the (or their) use of technology. I compare some of the teachers' responses to the list of opportunities provided in Chapter 2, however, as I was curious to see whether some of these opportunities would be mentioned (naturally) when the teachers discussed their reasons for using technology.

T1 believed that teaching with technology could:

- Make teaching more interesting for her;

- Help prepare her students for their future careers (as she said, where they're "gonna have to figure out how to use different technologies");

T2 did not mention the above reasons. He believed that teaching with technology could:

- Allow his students to focus on "higher level skills" (or, in the words of Pierce and Stacey (2010), teaching with technology can allow teachers to "re-balance emphasis on skills, concepts, applications");
- Allow him to convince his students (who he claimed were not yet ready to discuss formal proof) about various mathematical properties;
- Facilitate generating multiple representations of certain mathematical notions, and therefore to "link representations" and "observe the effects of parameters", as it was put by Pierce & Stacey (2010);
- Help students understand "what it means to solve an equation", or, in the words of Zorn (2002), teaching with technology can "help students make sense of symbols and symbolic expressions".
- "Exploit contrast of ideal & machine mathematics". He would use the graphing calculator to motivate discussion about scale, as the default scale would not show certain features of certain graphs that other scales could.

Both teachers believed that teaching with technology could:

- Make the mathematics class more interesting for their students;
- Allow them to change their roles. When T1 showed her students videos (for instance) she would never stop the video to add something or comment on it and would always play it from start to finish; while it was playing, she was not a part of the class, in a sense. As

mentioned in Chapter 5, T2 had allowed his students to 'play freely' with the applets in the past; changing his role to one of a facilitator;

- Allow them and their students to complete several tasks much faster; for example, speaking of his use of the graphing calculator to analyze data with statistics, T2 mentioned, "We start analyzing all kinds of situations. Things we bring in from magazines, newspapers, and the web. And this allows us to get into the analysis really, really quickly, and not worry about the mechanics of mean deviation, standard deviation"; T1 mentioned how she had her secondary 1 and secondary 2 students use four-function calculators and scientific calculators, respectively, to "speed up" their numeric computations.

Unlike Pierce and Stacey (2010), however, neither T1 nor T2 mentioned anything about technology...

- helping students learn "pencil and paper skills";
- allowing students and teachers "to generate statistical data sets";
- allowing teachers to "overview [a topic] as introduction or summation";
- allowing teachers and students to "work on real problems involving calculation that, done by hand, are error prone and time consuming";
- allowing teachers to "change classroom social dynamics"; T2 mentioned how his use of PowerPoint allowed him to walk around, but not that it would encourage him to have his students work in groups or discuss more than they had prior to his use of the PowerPoint technology.

T1 and T2 also did not mention, as Zorn (2002) would have it, that teaching with technology can help with the preparation and correction of assessments (either formative or summative).

Chapter 8: Suggestions for Future Research

Our study provides evidence that, even when teachers have access to the same technology and technical support, they might use technology in different ways towards completing different tasks. We also learned that their reasons for using certain technology in certain ways, and for hesitating to use certain technology to complete certain tasks, may differ. It is also clear that these differences (in the factors that influence their use of technology) might exist, not only between teachers, but also between a given teacher's reasons for using particular technology in the ways that they do. I believe that this is useful information, and that it should encourage researchers to:

1. Continue exploring the techniques that teachers and students employ when working with technology. More than understanding the tasks that they use technology to complete, I believe that understanding the techniques that they employ will allow us to understand the progress that teachers are making towards becoming "effective" users of technology.
2. Continue to ask about teachers uses of particular types of technology (e.g., asking those who use PowerPoint why they use PowerPoint in certain ways and asking those who don't, why they hesitate to). In some of the studies reported in Chapter 2, researchers had asked teachers about "the factors that encouraged and discouraged their use of computers for mathematics teaching" (Forgasz, 2006) or about "the advantages and disadvantages of using computers (technology) in mathematics" (Thomas, 2006). I believe that one issue with asking such general questions (about teachers' reasons for using or not using "technology" or "computers") is that when teachers think about their reasons for not using technology, they might be thinking of different types of technology than they do when they think about their reasons for using technology. For instance, Thomas (2006)

learned that the "time and effort needed by both students and teachers in order to become familiar with the technology" was a factor in how and whether teachers' used technology. Understanding teachers' uses of technology through instrumentation theory, however, we cannot say whether a given technology will be easy to use for any group of students or any teacher, without knowing which technology they are currently using (or have used in the past) and how they are (or were) using it.

To explore teachers' uses of technology and the factors that influence them, we must also continue to improve our methods for obtaining this information. Although I believe that conducting semi-structured interviews with a small group of teachers certainly supplied us with useful information, I do not believe that this should be the only approach taken. Large-scale studies, based on surveys that include both closed- and open-ended questions, analyzed both quantitatively and qualitatively, can also provide us with useful and complimentary information. Mixed methods approaches allow researchers to elicit detail and to remain flexible about what they want to learn, but also allow them to observe trends in larger populations. Further, the information obtained from semi-structured interviews can be used to shape the survey questions and the trends observed through statistical analysis of the survey data can be used to inspire better interview questions. For instance, having found that grade 10 and grade 11 teachers' (such as T2's) uses of technology might be influenced by how their students will be evaluated at the college level, we might survey a larger population that they belong to (such as grade 10 and 11 teachers in Montreal) to determine whether this is a factor that influences many teachers' uses of certain technology (for instance, graphing calculators) and how strong of an influence it has (by asking them to rank it against other common factors). In terms of the types of technologies used, if it was found that many secondary school teachers from a particular population were using a certain type of technology frequently, interviews could be conducted to discover examples of how they are being used (i.e. the techniques employed and the tasks that they are being used to complete). This could in turn encourage researchers to investigate how common certain uses of graphing calculators are, and the process could continue.

In terms of methodology, the time that it takes to publish findings is also an issue. Whether the results are reported in a journal article or a master's or doctoral thesis, they are often not read until several months (if not two or more years) after the study was conducted. With the seemingly ever-changing list of technology available to teachers, however, the types of technology that teachers are using and, therefore, the ways in which they are being used by teachers and students is likely also changing (although, perhaps, at a slower pace), and therefore, the uses reported have likely changed by the time a document is published. If we are to stay abreast of the realities of technology use, it seems important that our accounts be updated as continuously as possible. I would therefore suggest that perhaps setting up forums where teachers can blog about their uses of technology could be prove useful towards this goal. After reading some of these blogs, researchers could also (if permitted and if the blogger identifies her/himself) sit-in on those teachers' lessons and/or interview certain of their students, to confirm the uses described by the teachers.

It would also be interesting to learn of teachers' impressions of statements (such those released by the NCTM) on the appropriate or necessary role of technology in mathematics education; to ask whether they feel pressured (and why they feel pressured) to use technology in ways that they would not have of their own initiative, and what the effects this has on their uses of technology. I conjecture that those that feel pressured, might rush to integrate technology, and as a consequence, might use it in certain, less "strategic" ways, than the NCTM claims would be beneficial to their students.

In terms of the framework that we used, Drijvers at al. (2010) remind us that, when trying to understand the integration of technology into mathematics education, "instrumentation theory cannot of course be 'the complete solution to everything' and additional theoretical perspectives may be needed". Those who continue to investigate how technology is actually integrated into one's practice or how it could be (through theorizing about the affordances and constraints of using certain technologies in certain ways), should consider complimenting their use of instrumentation theory with other theoretical perspectives, as Drijvers et al. (2010) suggest, such as with the idea of semiotic mediation.

Nevertheless, I believe that the concepts of Instrumentation Theory were very useful in designing our research questions. First, because the theory encouraged us to look into teachers' techniques, which I now see as necessary to describe if we are to be able to assess whether uses of technology are beneficial or not. Second, because our framework encouraged us to look into the epistemic and pragmatic value that each teacher related to their and their students' instrumented techniques. As mentioned in section 6.6, I believe that understanding the value that a teacher relates to their uses of technology can help researchers understand why they use technology in certain ways and hesitated to use it in others.

Last, I suggest that researchers continuously criticize the goals of their and others' research on teachers' uses of technology. As our body of knowledge grows, certain questions will become more, and others less, relevant. For instance, large-scale studies that investigate the types of technology used by a given population, the percentage of that population that are using each, and the frequency with which each are being used, can inform professional developers of which types of technology they might want to provide training on. At some point, if it was known that all (or the vast majority of) teachers in a given population had limited their use of technology (for whatever reason) to the use of GeoGebra and one brand of course management software (such as COBA), for example, then the former questions need not be pursued until there is reason to believe that this reality has changed. To be sure that research questions are those that are most pressing and important to answer, it is important that research builds upon itself, and that researchers justify why they seek to answer the questions that they do.

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Appendix A: NCTM Statement 2008

Question: What is the role of technology in the teaching and learning of mathematics?

NCTM Position

Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology. Effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When technology is used strategically, it can provide access to mathematics for all students.

Calculators and other technological tools, such as computer algebra systems, interactive geometry software, applets, spreadsheets, and interactive presentation devices, are vital components of a high-quality mathematics education. With guidance from effective mathematics teachers, students at different levels can use these tools to support and extend mathematical reasoning and sense making, gain access to mathematical content and problem-solving contexts, and enhance computational fluency. In a well-articulated mathematics program, students can use these tools for computation, construction, and representation as they explore problems. The use of technology also contributes to mathematical reflection, problem identification, and decision making.

The use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills. In a balanced mathematics program, the strategic use of technology enhances mathematics teaching and learning. Teachers must be knowledgeable decision makers in determining when and how their students can use technology most effectively. All schools and mathematics programs should provide students and teachers with access to instructional technology, including appropriate calculators, computers with mathematical software, Internet connectivity, handheld data-collection devices, and sensing probes. Curricula and courses of study should incorporate instructional technology in learning outcomes, lesson plans, and assessments of students' progress.

Programs in teacher education and professional development must continually update practitioners' knowledge of technology and its classroom applications. Such programs should include the development of mathematics lessons that take advantage of technology-rich environments and the integration of technology in day-to-day instruction, instilling an appreciation for the power of technological tools and their potential impact on students' learning and use of mathematics. All teachers must remain open to learning new technologies, implementing them effectively in a coherent and balanced instructional program. These tools, including those used specifically for teaching and learning mathematics, not only complement mathematics teaching and learning but also prepare all students for their future lives, which technology will influence every day.

(March 2008)

Appendix B: NCTM Statement 2011

Question: What is the role of technology in the teaching and learning of mathematics?

NCTM Position

Technology is an essential tool for learning mathematics in the 21st century, and all schools must ensure that all their students have access to technology. Effective teachers maximize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When technology is used strategically, it can provide access to mathematics for all students.

Technological tools include those that are both content specific and content neutral. In mathematics education, content-specific technologies include computer algebra systems; dynamic geometry environments; interactive applets; handheld computation, data collection, and analysis devices; and computer-based applications. These technologies support students in exploring and identifying mathematical concepts and relationships. Content-neutral technologies include communication and collaboration tools and Web-based digital media, and these technologies increase students' access to information, ideas, and interactions that can support and enhance sense making, which is central to the process of taking ownership of knowledge. Findings from a number of studies have shown that the strategic use of technological tools can support both the learning of mathematical procedures and skills as well as the development of advanced mathematical proficiencies, such as problem solving, reasoning, and justifying (e.g., Gadanidis & Geiger, 2010; Kastberg & Leatham, 2005; Nelson, Christopher, & Mims, 2009; Pierce & Stacey, 2010; Roschelle, et al., 2009, 2010; Suh & Moyer, 2007).

In a balanced mathematics program, the strategic use of technology strengthens mathematics teaching and learning (Dick & Hollebrands, 2011). Simply having access to technology is not sufficient. The teacher and the curriculum play critical roles in mediating the use of technological tools (King-Sears, 2009; Roschelle, et al., 2010; Suh, 2010). Teachers and curriculum developers must be knowledgeable decision makers, skilled in determining when and how technology can enhance students' learning appropriately and effectively (ISTE, 2008). All schools and mathematics programs should provide students and teachers with access to instructional technology—including classroom hardware, handheld and lab-based devices with mathematical software and applications, and Web-based resources—together with adequate training to ensure its effective use.

Programs in teacher education and professional development must continually update practitioners' knowledge of technology and its application to support learning. This work with practitioners should include the development of mathematics lessons that take advantage of technology-rich environments and the integration of digital tools in daily instruction, instilling an appreciation for the power of technology and its potential impact on students' understanding and use of mathematics (Nelson, Christopher, & Mims, 2009; Pierce & Stacey, 2010). In addition to enriching students' experiences as learners of mathematics, use of these tools maximizes the possibilities afforded by students' increasing knowledge about and comfort with technology-driven means of communication vague (classroom hardware, handheld and lab-based devices with mathematical software and applications, and Web-based resources), and information retrieval (Gadanidis & Geiger, 2010; Project Tomorrow, 2011). (October, 2011)

Appendix C: Technology that attracted researchers' attention

In this Appendix, we will list some of the technology that has attracted the attention of researchers. References are included to papers on each type. We will also demonstrate how this research 'fits together' using a 'map'.

The broadest category of technology that we will consider is Information and Communication technology. This includes (but is not restricted to) computers, computer software, projectors, audio and visual recorders, Intranet, Internet, and Mathematical Analysis Software (MAS).

Mathematical Analysis Software includes:

- Algebra Tools (Kendal, Stacey, & Pierce, 2002)
- Geometry Tools (Laborde, 1992)
- Modeling software (Stratford, 1996)
- Microworlds (Sarama & Clements, 2002)
- Spreadsheets (Baker & Sugden, 2003)
- Cognitive Tutors (Bennane, 2010)
- Games (Klawe & Phillips, 1995)
- Programming Languages (Feurzeig, Papert, Bloom, Grant, & Solomon, 1969)

We now list in more detail the technologies that belong to the above large categories.

Algebra tools include:

- Computer Algebra Systems (CAS) (Zellers and Barzel, 2010)
- Graphing Programs (Jackson, Berger, & Edwards, , 1993)
- Multi-representational Software (Piliero, 1994)

- A computer-generated parallel axis representation (PAR) of function (Nachmias and Arcavi, 1990)

Computer Algebra Systems include:

- Maple (Judson, 1989; Aldis, Sidhu, and Joiner, 1999)
- Mathematica (Lefton and Steinhart, 1995; Park and Travers, 1996)
- MathCAD (Vlachos and Kehagias, 2000)
- Derive (Artigue, 1997)
- Symbolic/Complex Calculators (e.g., TI-92, TI-38G, TI-Nspire) (Lagrange, 1999)
- KETpic (Kaneko, Abe, Sekiguchi, Tadokoro, Fukazawa, Yamashita, & Takato, (2010).
- muMath (Heid, 1985)
- muPAD (Majewski, & Szabo, 2001)
- Scientific Notebook (Anderson, Bloom, Mueller, & Pedler, 2000)

Graphing programs include:

  - Graphing calculators (e.g., HP-48G and TI-89) (Girard, 2002)
  - Grapher (Schoenfeld, 1990)

Multi-representational software includes:

  - Function Probe (Confrey and Maloney, 2008)
  - The Function Analyzer (Yerushalmy, 1991)
  - The Function Explorer (Olsen, 1995)

PAR include:

  - Dynamaps (Heid, Zbiek, Blume, & Choate, 2004)

Geometry Tools include:

- Dynamic Geometry Software (DGS) (Jones, 2001)
- Static Construction (Fey, 1989)
- Turtle Geometry (Abelson & diSessa,1980)

  Dynamic Geometry Systems include:

  - Geogebra (Lavicza & Papp-Varga, 2010)
  - The Geometer's Sketchpad (Battista, 1998)
  - Cabri-Geometry (Noss, Hoyles, Healy, & Hölzl, 1994)
  - Cabri II (Sierpinska, Dreyfus, and Hillel, 1999)
  - GEOLOG (Holland, 2001)
  - Geometry Inventor (Arcavi & Hadas, 2000)

  Static construction software includes:

  - Geometric Supposer (Yerushalmy and Chazan, 1990; Chazan, 1993)

  Turtle geometry includes:

- Versions of LOGO

Modeling tools include:

- STELLA (Doerr, 1996)
- StarLogo (Resnick, 1994)
- The Algebra Sketchbook (Schwartz & Yerushalmy, 1995)

Microworlds include:

- SimCalc MathWorlds (Bowers & Doerr, 2001)
- TIMA microworlds (Olive, 2000)
- HOUSE (Hoyles &Noss, 1992)

Spreadsheets include:

- Excel (Drier, 2001)

Cognitive tutors include:

- Algebra Cognitive Tutors (Carnegie Learning, 2005)
- PUMP (Pittsburg Urban Mathematics Project) Algebra Tutor (PAT)" (Koedinger, Anderson, Hadley, & Mark, 1997)
- ACT: The Geometry Proof Tutor (Anderson, Corbett, Koedinger, & Pelletier, 1995)
- ANGLE (A New Geometry Learning Environment) (Koedinger and Anderson, 1993)

Programming languages include:

- LOGO (and other versions of it, LogoWriter (Harel, 1990))
- TurtleMath (Clements & Sarama, 1995)
- Microworlds (Olson, Kieren, & Ludwig, 1987)
- Boxer (diSessa & Abelson, 1986)
- BASIC (Cowell & Prosser, 1991)
- ISETL (Asiala, Brown, DeVries, Dubinsky, Matthews, & Thomas, 1996)

ICT that is not Mathematical Analysis Software includes:

- Web 2.0 technologies (e.g., blogging, Microcomputers wikis, etc.) (Dagien.e and Kurilovas, 2010)
- Microcomputers (Morris, 1983)
- Copycat (Davis, Hunting, & Pearn, 1993)

- SuperPaint (Aldus Corporation, 1994)
- Interactive Applets (programmed in Java) (Heath, 2002)
- TinkerPlots (Konold, 2002b)
- Fathom (Lock, 2002)
- Darts (a program on fractions) (Dughale & Kibbey, 1990)
- Interactive White Boards (IWBs) (DiGregorio and Sobel-Lojeski, 2010)
- PowerPoint (Wiwatanapataphee, Noinang, Wu, & Nuntadilok, 2010)
- Sliders (components of certain software) (Stryker, 2010)
- Calculator-Based Rangers (CBRs) (Kwon, 2002, internet - The Effect of Calculator-Based Ranger Activities on Students' Graphing Ability)
- Videos (Boyd & Rubin)
- Learning Management Software (Burrell & Maina, 2009)
- Course management software (e.g., Moodle and Blackboard Vista) (Dougiamas & Taylor, 2003).

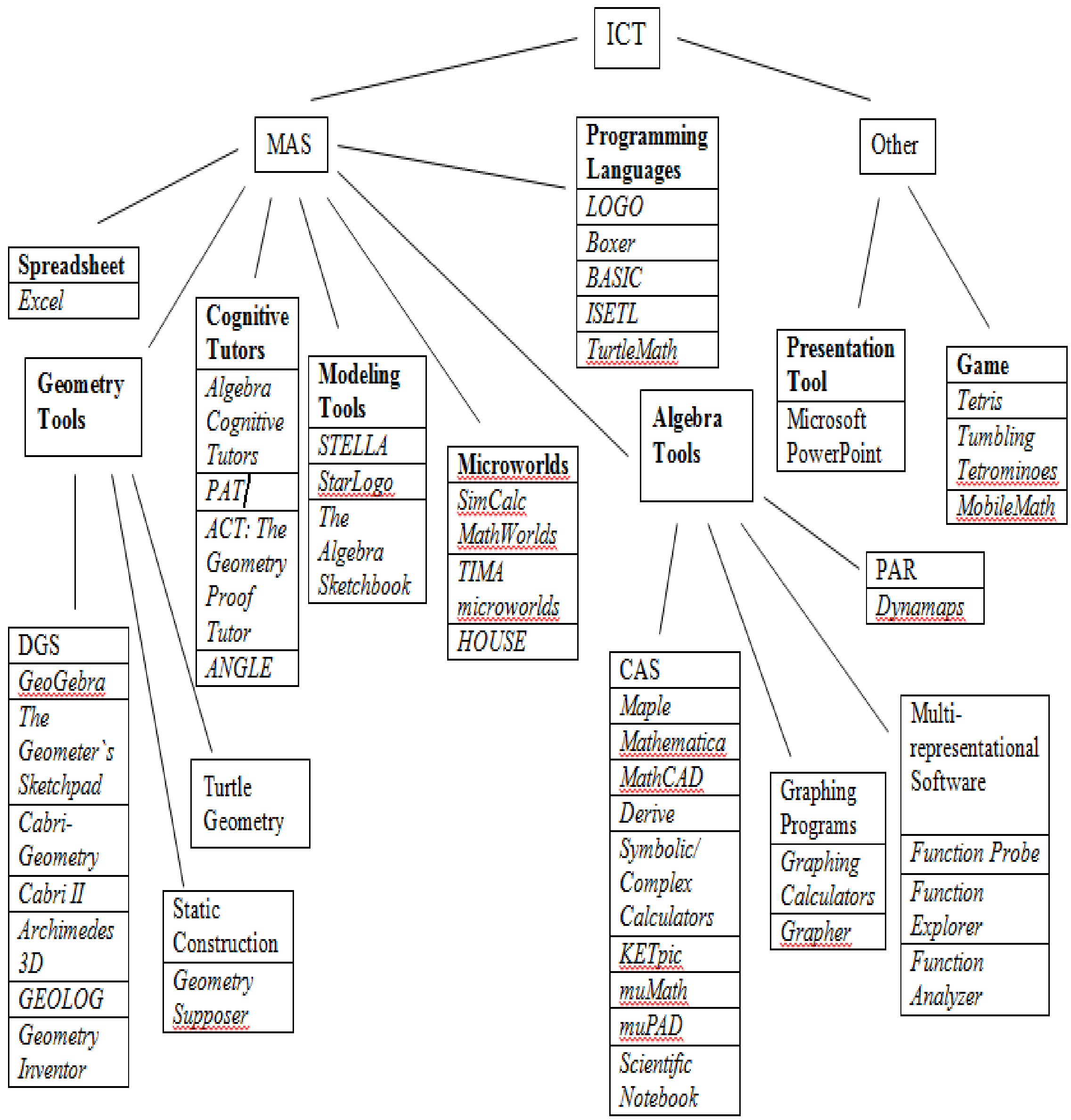
ICT is sometimes used in networks, such as:

- iCOPER best practice network (Dagien.e and Kurilovas, 2010)
- TI-Nspire Navigator networked system of handhelds (Clark-Wilson, 2010)

Games include:

  - Video games (e.g., Tetris) (Bright, Usnick, & Williams, 1992)
  - Computer games (e.g., Tumbling Tetrominoes) (Clements, Russell, Tierney, Battista, & Meredith, 1995)
  - Cell phone games (e.g., MobileMath) (Wijers, Jonker, and Drijvers, 2010)

Presented as a map



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