UNDER UNCERTAINTY TRUST ESTIMATION IN MULTI-VALUED SETTINGS

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Abstract

Under Uncertainty Trust Estimation in Multi-Valued Settings

Sina Honari

Social networking sites have developed considerably during the past couple of years. However, few websites exploit the potentials of combining the social networking sites with online markets. This, in turn, would help users to distinguish and engage into interaction with other unknown, yet trustworthy, users in the market. In this thesis, we develop a model to estimate the trust of unknown agents in a multi-agent system where agents engage into business-oriented interactions with each other. The proposed trust model estimates the degree of trustworthiness of an unknown target agent through the information acquired from a group of advisor agents, who had direct interactions with the target agent. This problem is addressed when: (1) the trust of both advisor and target agents is subject to some uncertainty; (2) the advisor agents are self-interested and provide misleading accounts of their past experiences with the target agents; and (3) the outcome of each interaction between the agents is multi-valued.

We use possibility distributions to model trust with respect to its uncertainties thanks to its potential capability of modeling uncertainty arisen from both variability and ignorance. Moreover, we propose trust estimation models to approximate the degree of trustworthiness of an unknown target agent in the two following problems: (1) in the first problem, the advisor agents are assumed to be unknown and have an unknown level of trustworthiness; and (2) in the second problem, however, some interactions are carried out with the advisor agents and their trust distributions are modeled. In addition, a certainty metric is proposed in the possibilistic domain, measuring the confidence of an agent in the reports of its advisors which considers the consistency in the advisors' reported information and the advisors' degree of trustworthiness. Finally, we validate the proposed

approaches through extensive experiments in various settings.

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Chapter 1

Introduction

1.1 Context of Research

Social networking sites have become the preferred venue for social interactions. Despite the fact that social networks are ubiquitous on the Internet, only few websites exploit the potential of combining user communities and online marketplaces. The reason is that users do not know which other users to trust, which makes them suspicious of engaging in online business, in particular if many unknown other parties are involved. This situation, however, can be alleviated by developing trust metrics such that an agent can assess and identify trustworthy agents. An agent, indeed, can acquire information about unknown agents of interest (target agents) through other agents in the network (which we refer to as advisors) who have already interacted with one of the target agents. Estimation of the degree of trustworthiness of a target agent, in turn, would help an agent to decide whether or not to engage in an interaction with that target agent. This problem gets more challenging when: (1) the advisors are self-interested and may not report honestly about target agents; and (2) the degree of trustworthiness of both advisor and target agents is subject to uncertainty. The uncertainty, considered in this thesis, is driven from lack of adequate information on advisor and target agents' trust and variability in their degree of trustworthiness. The variability in an agent's trust has the consequence of the agent's trustworthy behavior in some interactions and its untrustworthy behavior in other interactions such that the prediction of the agent's degree of trustworthiness becomes challenging for future interactions.

1.2 Problem Statement

In this thesis, we develop a trust model for estimating the trust of a target agent, who is unknown, through the information acquired from a group of advisor agents who had direct experience with the target agent. Figure 1 illustrates the multi-agent system studied in this thesis. As shown in this figure, an evaluator agent (namely a^{S}) wants to decide whether or not to interact with an unknown target agent (namely a^{D}). On this purpose, it acquires information from its advisors in set A (set A consists of agents a_1 to a_n) who have already interacted with agent a^{D}. Through the acquired information from the agents in set A, agent a^{S} models the trust of agent a^{D}. Consequently, a^{S} can have a prior estimation on the degree of trustworthiness of agent a^{D} before engaging in any interactions with it.

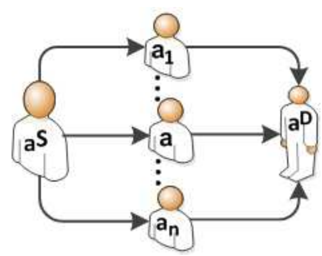


Figure 1: Multi-agent System

The trust model developed in this thesis, which measure the degree of trustworthiness of the target agent a^{D}, is in the following context:

- There is uncertainty in the degree of trustworthiness of both advisor and target agents. The uncertainty is risen from: (1) ignorance about advisor and target agents; and (2) variability in the degree of trustworthiness of both advisor and target agents. Ignorance is due to insufficient

[1]Without loss of generality, in our model, agents a^{S} and a^{D} are not in the set A. However, in a more general approach each agent may ask a group of agents about an agent unknown to him. Moreover, we assume the agents in the set A are willing to share information with a^{S} about the target agent a^{D}.

information about advisor and target agents. Variability, on the other hand, is due to diversity in degree of trustworthiness of the advisor and target agents. Indeed, variability in trust of an agent means that the agent's degree of trustworthiness may change from one interaction to another, which would make its degree of trustworthiness hard to predict for future interactions.

- The advisor agents are self-interested and may not report honestly to the evaluator agent a^{S} about their past experience with the target agent a^{D}.
- The domain of trust is multi-valued. After each interaction between every pair of connected agents in Figure 1, say agent α and agent β, α rates the trustworthiness of β by giving it a rating chosen out of a discrete set of trust values. For example, if the domain of trust values has five elements, agent α can rate β by giving it a value from one to five.

In this context, we address two different problems:

1. In the first problem, we assume the advisor agents are unknown to agent a^{S} and a^{S} acquires information from its advisors on the trust of agent a^{D}. Then, agent a^{S} models the trust distribution of a^{D} through information acquired from its advisors which have unknown degree of trustworthiness. In this problem, it is not clear to what extent a piece of information given by an advisor in A can be relied upon.
2. In the second problem, we assume that agent a^{S} has interacted with the advisor agents in set A. Therefore, agent a^{S} models the trust of the advisor agents in A through usage of the empirical values a^{S} has acquired in its direct interactions with the agents in A. Later, when a^{S} wants to measure the trust of the target agent a^{D}, it considers the degree of trustworthiness of the advisor agents. This, in turn, would help a^{S} figure out to what extent the information received from each agent in A about the target agent a^{D} is reliable.

1.3 Contribution of the Thesis

The contributions of this thesis are manifold. Here we provide an overview of them:

- The first contribution of this thesis is deployment of possibility theory to address the uncertainty in the aforementioned problem context. As discussed later, possibility is a strong tool for representing uncertainties arisen from insufficient knowledge and also variability. It also provides strong tools (e.g., fusion rules) for aggregating information acquired from different sources.
- The main contributions of this thesis is modeling the trust distribution of a target agent through information acquired from the advisor agents. A target agent's trust is estimated in two different problems: (1) when the advisor agents are unknown; and (2) when the advisor agents are known. In the latter problem, two different approaches are proposed:
  1. In the first approach, a methodology is proposed to merge possibility distributions representing the trust of agents at successive levels in a multi-agent system. In other words, we merge the trust distributions of an agent in its advisors with the trust distribution of the advisor agents in a target agent. More specifically, we merge the following possibility distributions:
     (a) The possibility distributions of agent a^{S}'s trust in its advisors.
     (b) The possibility distributions of the advisor agents's trust in the target agent a^{D}.

     These two possibility distributions represent the trust of different entities. While the former represents the trust distribution of the advisor agents, the latter demonstrates the trust distribution of the target agent a^{D}.
  2. In the second approach, a single trust value, representing the trust of an agent in its advisors is measured which is estimated from the direct experience of an agent with its advisors. This value, then, is employed in the fusion rules to measure the trust of a target agent.
- The third contribution of this thesis, is measuring a certainty value over the information acquired by agent a^{S} from its advisors. The information received by agent a^{S} from each one of its advisors is a possibility distribution representing the trust of that advisor in the target

agent a^{D}. The certainty metric developed here is aimed to represent the confidence of agent a^{S} in the quality of information received from its advisors. The certainty metric considers both the inconsistency among the advisors' reported possibility distributions and the degree of trustworthiness of the advisors. The certainty value differs from trust in the sense that trust represents the degree of reliability of an agent as measured or estimated by another agent while the certainty value represents the degree of reliability of the information acquired from the entire advisor agents which considers both the consistency in the information and the degree of trustworthiness of the advisor agents.

- Finally, we develop two metrics to evaluate the estimated possibility distribution of a target agent. The first metric is the information level of a possibility distribution which measures the degree of information provided in a possibility distribution. The second metric measures the approximated error of the estimated possibility distribution of agent a^{D}. On this purpose, this metric measures the difference between the true and estimated possibility distributions of the target agent a^{D}.

1.4 Thesis Outline

The thesis is organized as follows. In Chapter 2, the background information is provided. It explains concepts of trust, agent, multi-agent systems, uncertainty and possibility theory.

Chapter 3 makes a comprehensive review of trust and reputation models developed both with or without consideration of uncertainty.

Chapter 4 briefly covers the technical tools developed by other researchers which are used in our model. This chapter provides an overview of fusion rules, probability to possibility transformation technique and the process of inferring a possibility distribution from empirical data.

Chapter 5 contains the main contribution of the thesis. In this chapter, our proposed methodologies for modeling the trust distribution of a target agent is discussed. The estimation of a target agent's trust is developed when: (1) the advisor agents are unknown to agent a^{S}; and (2) the advisor agents are known to agent a^s. Finally, a certainty metric measuring the confidence of agent a^s in the quality of information acquired from its advisor is developed and analyzed in this chapter.

Chapter 6 contains the simulation results evaluating our proposed tools in comprehensive experiments. The experiments aim to validate our proposed approaches in different context settings. This chapter first introduces two evaluation metrics and then presents the experimental results.

Finally, Chapter 7 contains the conclusions of the thesis and discusses our future works.

Chapter 2

Background

This section provides the basic definitions of the core concepts used in the thesis. The definitions of the following terms are provided: trust, agent, multi-agent systems, uncertainty and possibility theory.

2.1 Trust

"trust is a term with many meanings" [1]. It has been given various definitions in different domains [2, 3]. Trust has been defined and used in economy [4, 5], sociology [6, 7], philosophy [8], psychology [9, 10], management [11, 12], political [13] and cognitive sciences [14]. For example, in sociology and economics, trust is typically defined as follows:

- **Sociology.** Trust is "a social relationship in which principals–for whatever reason or state of mind–invest resources, authority, or responsibility in another to act on their behalf for some uncertain future return" [6]. Trust exists [7] in a social system where the members act according to some norms and are secure in their expected futures through presence of other members in the society. Therein, trust is analyzed in a social context asking how the existence/betrayal of trust would benefit/harm the individuals in the society.

- **Economy.** Trust is the degree of confidence that the other party of an exchange will not take advantage of one's susceptible vulnerabilities [4]. In economy, trust is the key for successful exchanges where trust is considered as the degree of risk that the trusting party is willing to take with respect to the exchange partner. As described by Barney and Hansen "An exchange partner is trustworthy when it is worthy of the trust of others. An exchange party worthy of the trust is one which will not exploit others' exchange vulnerabilities ... Trust is an attribute of a relationship between exchange parties, trustworthiness is an attribute of individual exchange partners" [5].

In a general sense, trust is the degree of belief that a future phenomenon will be experienced as it is expected. For instance, we go to work trusting that the transportation system is reliable enough to get us to work which is driven by our expectation of the transportation system. Another example can be the banking system: we put our money in a bank trusting that it does not steal our money nor it would go bankrupt and we can get our money back whenever we wish. As described in [15] "trust is strongly linked to confidence in something, be it the person to be trusted, the environment, or whatever it is that the desirable outcome is contingent upon". Such general definitions of trust are applicable to a wide range of domains. Some authors, instead, define trust by addressing the interactions between the entities, especially in a society. As described in [16], trust is "the expectation or the belief that a party will act benignly and cooperatively with the trusting party". Thereupon, trust is defined as a relation between the two entities involved in it. These entities are referred to as the trustor and trustee. More specifically,

- **Trustor.** The entity who trusts the other party by taking the risk of being better off if the trust is being honored or worse-off if the trust is being betrayed.
- **Trustee.** The entity whom is being trusted. The trustee can either honor the trust by acting honestly or betray the trust by acting opportunistically.

  Considering such relations between a trustor and a trustee, trust can be defined as "the extent to which the trustor is willing to take the risk of trust being abused by the trustee" [17].

Some researchers modeled trust [18, 19] as the degree of belief that the trustee supports the trustor's plan. In these models, trust is a triplet consisting of belief, disbelief and uncertainty where trust, distrust and uncertainty arises when:

- **Trust:** belief (in a good outcome) is high, disbelief (or belief in a bad outcome) is low, and uncertainty is low. In this case, there is a high belief that trust will be honored by the trustee.
- **Distrust:** belief is low, disbelief is high, and uncertainty is low. In this case, there is a high disbelief that the trustee will honor trust.
- **Lack of both trust and distrust:** belief is low, disbelief is low, and uncertainty is high. In such a case, the trustor does not know whether the trustee is trustworthiness or not and to what extent the trust will be honored or abused.

In such a model, lack of trust is interpreted either as distrust in the trustee or uncertainty about the trustee. Moreover, increase of either trust or distrust in the trustee would increase the certainty on the trustee's degree of trustworthiness meaning that the trustor has more confidence on how trustworthy the trustee is. Regarding such view, trust is considered as "confident positive expectations regarding another's conduct" [20] and distrust as "confident negative expectations regarding another's conduct" where increase of either trust or distrust accumulates certainty on trustee's conduct.

In this thesis, we define trust as a relation between a trustor and a trustee which is the extent to which the trustee honors trust and acts cooperatively and benevolently with the trustor.

2.2 Agent

The same as trust, there are different definitions for an agent. "Agents do things, they act: that is why they are called agents" [21]. This is a very primitive definition of an agent which can categorize even many trivial applications or even products in the category of agents. For example, even a very basic Java code can be considered as an agent regarding this definition. Recent literature provide more comprehensive definition of the agents and describe them based on their expected features.

As defined in [22] “an agent is a computer system that is *situated* in some environment, and is capable of *autonomous action* in this environment in order to meet its delegated objectives”. In this definition, the emphasis is made on situatedness and autonomy as the main features of an agent. Another definition is provided in [23] where Wooldridge and Jennings describe an agent as an entity whose actions are autonomous and rational, where these two features carry the following meanings:

- **Autonomy.** Autonomy means that an agent’s actions are independent of direct human (or other) intervention or influence.
- **Rationality.** Rationality is mainly concerned with maximization of an agent’s rewards or performance considering some ‘valuation function’. Reward is the immediate feedback received after completing an action which is commonly in the form of a number. Valuation function specifies how to maximize an agent’s rewards in the long run. In other words, considering future actions and rewards of an agent, valuation function indicates what is good for the agent in the foreseeable future.

Although autonomy and rationality are the expected characteristics of an agent, these features are yet weak criteria for being an agent. As exemplified in [23], even a transistor can be called an agent which is consistent with this definition.

Wooldridge [24] [page 32] distinguishes between an agent and an intelligent agent arguing the latter is capable of flexible autonomous actions in order to meet its objectives, where flexibility implies the three following notions:

- **Reactivity.** The agent is capable of perceiving the environment and respond to it in a timely manner to satisfy its own goals.
- **Pro-activeness.** Intelligent agents make the goal-oriented initiatives in order to fulfill their own objectives.
- **Social ability.** Intelligent agents have the capability of interacting with other agents to fulfill their own objectives.

In a more generalized and comprehensive definition [25], an agent is an entity subject to beliefs, desires, goals, etc. In such a definition an agent is an intentional system [26] with human characteristics such as knowledge, desires, beliefs, intentions, commitments and goals. These characteristics are categorized [23] as information attitudes and pro-attitudes of an agent:

- **Information Attitude.** Consists of what an agent knows about the world it belongs to. Knowledge and beliefs are in this category.
- **Pro-Attitudes.** Consists of what drives an agent towards its actions. Desires, intentions, commitments and goals are all part of this category.

In this thesis, we consider the above definition where an agent has human characteristics like knowledge, desires, intentions and goals. We assume the agents in our model are goal oriented and the manipulation of information by them is a consequence of their intentional behavior which satisfies their overall goal.

2.3 Multi-Agent Systems

Multi-Agent Systems (MAS) are distributed systems composed of several independent agents where each agent either cooperates with other agents (to reach a shared objective or contribute to goal of other agents) or only pursues its own objectives. D'Inverno and Luck [27] define MAS as "typically distributed systems in which several distinct components, each of which is an independent problem-solving agent come together to form some coherent whole". While some features such as "having a distributed system" and "contributing to the goal of the system" are emphasized in this definition, MAS are categorized differently based on the goal of the agents and their degree of cooperation. Moreover, communication and exchange of information with other agents is a key property of MAS. Each agent communicates and exchanges with other agents in an environment or society in order to accomplish its own goals or contribute to the goals of the system. With respect to the environment in MAS, the following properties are usually assumed [24]:

1. Multi-agent environments should provide an infrastructure facilitating communication and

interaction among agents.

2. Multi-agent environments are typically open and have no central designer.
3. Multi-agent environments contain autonomous agents which can be either cooperative or self-interested.

In such systems, the question is how these agents relate to one another and what the entire system does fulfill? Considering the cooperation level, MAS are categorized into two types [28], independent and cooperative:

- **Independent MAS.** A multi-agent system is independent when each agent pursues its own agenda [29]. It such systems, agents are self-interested perusing their own goals where they tend to compete against each other instead of cooperating.
- **Cooperative MAS.** A multi-agent system is cooperative when collaboration with other agents in the system is compatible with agent's objectives [28]. In such systems each agent satisfies either or both of the following conditions:
  1. The agents pursue a common goal.
  2. Each agent performs actions which in addition to its own goals satisfy the goals of other agents.

Compositional MAS [30] are a specific form of cooperative MAS, in which each agent fulfills a specific task such that the composition of the tasks carried out by the entire agents accomplishes the overall goal of system.

In this thesis, we consider an independent multi-agent system where the agents are self-interested and pursue their own objectives. Note that throughout the exchange of information between the agents, an agent can help other agents' achieve their goals when such cooperation is in conformance to the agent's objectives. This may put our multi-agent system on the border of independent and cooperative MAS; however, we consider our system to be more on the independent side than on the cooperative one.

2.4 Uncertainty

As for trust, there are different definitions and interpretations of uncertainty [31]. The differences in the definition and understanding of uncertainty are due to different points of view of objectivists (who consider that uncertainty is driven from the system under study) and subjectivists (who consider that uncertainty is associated with the degree of belief of the observer or its knowledge). Dubois considers two types of uncertainty [32]:

- **Variability.** This type of uncertainty is “subject to intrinsic variability” [32]. In this case, the uncertainty is due to the fluxing behavior of the system under study which makes it indeterminate for accurate measurement. An example can be the weather temperature in a city like Montreal. This type of uncertainty is objective arising from the stochastic behavior of the system under study, which is not driven from the judgment or view of the observer [33].
- **Ignorance.** The type of uncertainty is “totally deterministic but anyway ill-known” [32]. In this case, uncertainty arises from lack of information about the system under study. An example can be a number of ants in a complex colony which is deterministic but unknown. This sort of uncertainty is subjective [33] driven from lack of an individual or an entity’s knowledge of the system.

Uncertainty may also emerge from a combination of variability and ignorance where there is insufficient knowledge on a quantity that has a fluxing and non-deterministic nature. This case usually arises from situations when the observations are poor due to lack of proper measurements and there is alteration in the system under study.

The variability and ignorance should be addressed differently [32]. While ignorance can be addressed by more study, variability arises from the nature of the system under study which is independent of our knowledge about the system. However, ignorance may remain in cases when additional information cannot be acquired either due to lack of access to the system or the difficulty of its measurement (e.g., the number of ants in a complex ant colony).

The uncertainty that we address in this thesis is a combination of both ignorance and variability. We will discuss it in more details in the following chapters where we provide more details on our research.

Probability theory is too normative to consider all aspects of uncertainty according to [34]. The uncertainty driven from variability can be modeled by using a unique probability distribution [32,33]; however, the uncertainty driven from lack of knowledge cannot be modeled by a probability distribution. In this type of uncertainty, the probability of events is unclear, except maybe their lower and upper bounds. The uncertainty driven from lack of knowledge can be represented by either interval analysis or possibility theory [34]. Interval analysis is not a single probability distribution; rather it is a region within which the true probability distribution most probably lies.

We used possibility theory as it is capable of addressing both types of uncertainty (the uncertainty driven from variability and the uncertainty driven from ignorance) [31]. In addition, as mentioned in [32], it is the simplest theory for addressing incomplete information (ignorance). These are the main factors guiding us to use the possibility theory for addressing uncertainty in this thesis. We elaborate the advantage of possibility theory in more details in the following chapters.

2.5 Possibility Theory

Possibility theory is one of the current theories for addressing uncertainty. It was first introduced by Zadeh [35] as a graded semantics for representing linguistic terms such as "today will possibly rain". This representation had close links to fuzzy sets where the possibility of an element was introduced as the degree of membership of that element in a set. Zadeh considers probability and possibility as the tools that give human the "ability to reason in approximate terms".

Possibility theory has been developed further by Dubois and Prade [32,36] where the relation between probability and possibility theories have been discussed and elaborated. Possibility theory is one of the current uncertainty theories devoted to handle incomplete information. As stated by Dubois and Prade [32], possibility theory is a "simple approach to reasoning with imprecise probabilities". It is powerful enough to represent different sorts of information such as numbers, intervals, consonant (nested) random sets and linguistic information. There are four meanings for the term 'possibility' [32]:

- **Feasibility**: Here, the term 'possible' refers to the easiness of fulfillment of a task, or satisfying the constraints of a problem. For example, the expression of "solving this problem is possible" implies the feasibility of solving the problem.
- **Plausibility**: In this case, the term 'possible' indicates the likelihood of an event to occur. It is expressed in sentences such as "it is possible that the bus comes on time" which indicates the likelihood of the arrival of bus on time.
- **Consistency with the available information**: In this case, the term 'possible' is used when a proposition does not contradict with available information. It is implied in expressions such as "it is possible that the cat has opened the door". This meaning of possibility is logical, which is an "all-or-nothing" [32] version of plausibility.
- **Deontic**: In this case, the term 'possible' means allowed or permitted by the law. For example, "it is possible to park my car here".

Considering the logical view of possibility, it measures the degree of plausibility of an event. The dual of possibility is necessity [36], which measures the degree of certainty of an event. While the possibility value measures the degree of plausibility of an event, the necessity value measures the degree of certainty over an event. This leads to presentation of possibility theory commonly in form of possibility and necessity values.

Suppose some quantity x ranges on a set U in which the elements in U are mutually exclusive, meaning that only one element in the domain of U is the true value. The possibility value of each element $u \in U$ indicates the degree of plausibility that element u is the true value of x. The possibility distribution [32], defined over a set U, gives each element $u \in U$ a value in $[0, 1]$, which indicates its possibility value. The possibility value of element u is represented by $\pi(u)$. Having $\pi(u) = 0$ means that event u is impossible while having $\pi(u) = 1$ means that element u is usual, normal, unsurprising. This is a much weaker representation of the degree of plausibility of element u compared to a probability distribution. An element having a possibility value between 0 and 1 ($0 < \pi(u) < 1$) is interpreted as being partially possible and partially impossible. Unlike probability distribution in which the total sum of the probabilities of elements in the set U should sum up to one, there is no upper bound considering the sum of possibility of the elements of U. Each element $u \in U$ can indeed have a maximum possibility of one. However, the normalization in the possibility domain is satisfied by having at least one element in U that has a possibility of one. This is due to the fact that at least one element in the domain is the true value which should be totally possible. Otherwise, if all elements in U have a possibility less than one the meaning is logically at odds implying that all of them are partially impossible. More formally, a possibility distribution defined over a set U should satisfy:

$$0 \leq \Pi(u) \leq 1 \qquad \forall u \in U \tag{1}$$

$$\exists u \in U : \; \Pi(u) = 1. \tag{2}$$

Finally, the possibility distribution defined over a set U is represented as:

$$\Pi : U \to [0,1] \qquad \text{where} \max_{u \in U} \Pi(u) = 1 \tag{3}$$

In this chapter, we provided a brief description of the concepts used in this thesis in order to familiarize the reader with the overall context of the thesis. In the next chapter, we concentrate more on the details of our research and we review the researches closely related to our own.

Chapter 3

Literature Review

In this chapter, we review relevant literature in the domains of trust, reputation, and uncertainty, which are related to our model.

3.1 Trust and Reputation Models Without Uncertainty

First, we review the works accomplished in trust and reputation models that do not address uncertainty. Considerable research has been accomplished in multi-agent systems representing models of trust and reputation, a detailed overview of which is provided in [37]. In reputation models, an aggregation of opinions of agents towards an individual agent is publicly maintained. In trust models, on the other hand, the focus is mainly on the private trust measurement by an agent towards other agents in the system. In this section, we review some of the trust and reputation models which are close to our own.

3.1.1 Sporas and Histos

In *Sporas* [38,39], the reputation of a user can be evaluated and rated by other users in the system. *Sporas* provides a mechanism for capturing the changes in the behavior of each user over time. In this model, the reputation of a user, say u_i, giving a rating to another user, say u_j, is considered such that the extent of influence of user u_i on the reputation update of user u_j is based on u_i's reputation. The higher the u_i's reputation, the higher its influence on the update of u_j's reputation. In [38, 39] the authors also introduce the *Histos* reputation model where a user u_i acquires information from other users in the web of trust about the reputation of a target user u_j. On this purpose, the most recent paths from u_i to u_j are found [1] and the reputation of u_j is measured based on the ratings[2] and reputation of the agents in the selected paths from u_i to u_j. However, in both *Sporas* and *Histos* models, it is implicitly assumed that an agent's reputation is a fixed unknown value at each given time. Consequently, it does not address the variability in an agent's reputation.

3.1.2 Regret

Regret is another reputation model presented in [40] which describes different dimensions of reputation including "individual dimension", "social dimension", and "ontological dimension".

Individual dimension is measured through consideration of previous direct interactions with an agent in which the time recency of each interaction is addressed. Considering the social dimension, it is assumed that each agent belongs to a social group. In order to measure the reputation of an agent j in group G_j by agent i in group G_i the following aspects of reputation are measured:

- The reputation of agent j measured by agent i in direct interactions between them.
- The reputation of the agents of group G_j as directly experienced and measured by agent i.
- The reputation of agent j as measured by the agents of group G_i in mutual direct interactions with agent j.
- The reputation of the agents of group G_j as measured by the members of group G_i.

These four aspects of reputation in the social context are combined in order to measure the

[1] A path from user u_1 to user u_n is a sequence of users $(u_1, u_2, \ldots, u_n)$ such that considering each pair of users (u_i, u_{i+1}), user u_i had direct experience with user u_{i+1}. Here, a path is the same as a path in a tree. If we denote the nodes of a tree as the users and create an edge between every pair of users who had direct experience, a path is a sequence of vertices where each vertex is connected directly by an edge to the next vertex. In *Sporas*, the time of the most recent interaction between every pair of agents is considered. Consequently if there are several paths from user u_1 to u_n, the path is selected in which the agents in the path have made more recent interactions compared to the other paths.

[2] A rating is a value given by a trustor agent to a trustee agent in an interaction between the trustor and trustee agents which indicates the degree of satisfaction of the trustor agent in that interaction. For example, in a one to five rating scale, one and five indicate the lowest and the highest satisfaction levels, respectively.

reputation of agent j by agent i.

In ontological dimension, different aspects of reputation that are important for a model (e.g. delivery date, product quality) are considered. In general, in the *Regret* model, the focal emphasis for measuring a target agent's reputation is put on the target agent's social status and also on the aspects of reputation important for a given model (time, quality, etc). However, this model does not address the possibility of receiving erroneous information by agents whose opinions are considered on the target agent.

3.1.3 Multi-Dimensional Trust

Griffiths [41] presents a multi-dimensional trust containing dimensions such as success, cost, timelines and quality where each dimension is updated over time based on the contract outcomes. The focus in this work is on the possible criteria that is required to build a trust model. In addition, the trust measured here is based on direct interactions. How the trust and reputation of an agent can be propagated and measured in a network of agents is not addressed.

3.1.4 FIRE

FIRE [16] measures the trust of a target agent considering "interaction trust", "role-based trust", "witness reputation" and "certified reputation". Here is a short description of these aspects:

- **Interaction trust.** Previous history of interactions with the target agent is considered by the evaluator agent.
- **Role-based trust.** This type of trust is domain specific and is based on the role of the target agent or the relationship between the target agent and the estimator agent. For example, an agent may trust any agent belonging to a group or certified by an organization.
- **Witness reputation.** The agents who had direct interactions with the target agent share their experience with the evaluator agent.

- **Certified reputation.** This aspect of trust is based on the references that the target agent introduces to the evaluator agent in order to certify its behavior.

These four trust components are integrated to measure the trust of a target agent. The *FIRE* model is based on two assumptions:

- **Assumption 1.** Agents are willing to share their experiences with others agents.
- **Assumption 2.** Agents are truthful when exchanging information with each other.

The second assumption is much stronger compared to the first one which is less likely in real world scenarios. In addition, although the underlying trust of an agent is assumed to have a normal distribution, the estimated trust is a single value instead of a distribution. A distribution contributes more information compared to a single value. Moreover, a single value cannot represent the uncertainty associated with the occurrence of each element of the trust domain.

3.1.5 Maintenance Based Trust

A maintenance based trust (MBT) is introduced in [42] where an online learning methodology is proposed to: (1) estimate the trustworthiness of the target agents; and (2) update and evaluate the degree of trustworthiness of the advisor agents. The MDB model considers two issues:

1. Online learning of a target agent's trust: In this aspect, both direct and indirect interactions with a target agent is considered to measure its agent's degree of trustworthiness. While the direct interactions happen between the evaluator and the target agents, the indirect interactions either are based on the information acquired from the network of trustee, known by the evaluator agent, or from the referees provided by the target agent. This aspect of trust considers both the number of interactions and the time recency of the interactions occurred between the agents.
2. Online learning of an advisor agent's trust: When the evaluator agent interacts with the target agent and observes its true value, the evaluator agent updates the degree of trustworthiness of the advisor agents (network of trustees and referees) based on their provided feedback.

To this end, an optimization algorithm for trust adjustment is proposed where the trust of honest and dishonest advisors are increased and decrease respectively with respect to an error threshold. In addition to this optimization part, a maintenance process is proposed to measure the overall trust trend of an advisor agent based on its current trust values measured by the optimization part. The maintenance process is aimed at measuring a unified and consistent trust value of an advisor agent based on its recent provided ratings. Consequently, this model distinguishes the trustworthy advisors and relies more on the information received from them for more accurate future evaluations.

All models in this section do not address uncertainty. Uncertainty is one of the aspects that our trust model aims to address.

3.2 Uncertainty in Trust and Reputation Models

In all of the trust and reputation literature presented in Section 3.1 the uncertainty in the trust of an agent is not modeled. We now review the works which address uncertainty.

3.2.1 Multi-Dimensional Trust for Heterogeneous Contracts

Reece *et al.* [43] present a multi-dimensional trust model in which the reputation reports of advisors are combined to measure multi-dimensional contracts. Each contract dimension corresponds to a service (e.g., video, audio, data service, etc.) and has a binary value (successful or unsuccessful). The contract dimensions are subject to correlated failure (e.g., due to shared resources or infrastructure) where the failure of one dimension may lead to failure of another. In this model, each advisor agent has observed a subset of dimensions and the main focus is to combine the advisors' reported heterogeneous subsets to model all dimensions of a contract. On this purpose, Kalman filter is applied for data fusion to combine heterogeneous contract observations. Moreover, the probability of having each service successfully delivered is estimated. Later, the uncertainty and correlations between these probabilities are measured in a covariance matrix.

This model is mainly concentrated on fusing information received from advisor agents who had direct observations over a subset of services (incomplete information) to derive a complete picture on all trust dimensions of a target agent. In this reputation model, when the reports of other agents are provided, the notion of having manipulated information provided by malicious agents is ignored, which makes the model less realistic in open environments.

3.2.2 Referral System

Yu and Singh [44] introduce belief functions measuring the probability of trust, distrust and uncertainty of an agent's quality of service. The rating given to each interaction between agents is a member of a discrete set of values in the interval of $[0, 1]$. In this model, the reported belief functions of the witness agents is merged in order to get the probability of trust, distrust and uncertainty of the target agent. The uncertainty measured in this work is equal to the frequency of the interaction results in which the agent's performance is neither highly trustworthy nor highly untrustworthy which can be inferred as lack of both trust and distrust in the target agent. However, the uncertainty that we capture in our model is due to the variability in the target agent's degree of trustworthiness and lack of adequate information about the target agent. We do not consider uncertainty as lack of trust or distrust, but the variability in the degree of trustworthiness of the agent and insufficient knowledge on it. This, in turn, makes the target agent's degree of trustworthiness hard to predict for future interactions. In addition, in [44] when the witness reports are gathered, the possibility of having malicious agents providing falsified reports is not considered.

3.2.3 Interval-Based Approach

Ben-Naim and Prade [45] present a trust model based on interval-based approach. In this model, the ratings given on a trustee are real numbers in $[0, 1]$. These numbers are then summarized in an interval to represent the past behavior of the trustee. The derived interval demonstrates a region within $[0, 1]$ in which the ratings were mostly concentrated. However, it does not consider the manipulation of information and assumes the information acquired on the trustee agent is reliable.

3.2.4 Subjective Logic Approach

In [19], Jøsang provides a probabilistic computational model measuring belief, disbelief and uncertainty out of binary interactions (positive or negative). In this approach, belief, disbelief and uncertainty sums up to one. Moreover, uncertainty represents imperfect knowledge (ignorance) whereas belief and disbelief represent certainty on the system under study. Let r and s be the number of positive and negative events, then belief (b), disbelief (d) and uncertainty (u) are measured as:

$$b = \frac{r}{r+s+1}; \qquad d = \frac{s}{r+s+1}; \qquad u = \frac{1}{r+s+1}. \tag{4}$$

Jøsang also elaborates how belief, disbelief and uncertainty is propagated by agents. If agent A's opinion on agent B is represented by $\omega_B^A = \{b_B^A, d_B^A, u_B^A\}$ and B's opinion on a proposition p is represented as $\omega_p^B = \{b_p^B, d_p^B, u_p^B\}$, then agent A's opinion about proposition p is measured as:

$$b_p^{AB} = b_B^A b_p^B; \qquad d_p^{AB} = b_B^A d_p^B; \qquad u_p^{AB} = d_B^A + u_B^A + b_B^A u_p^B. \tag{5}$$

In this approach, the belief of agent A in proposition p (b_p^{AB}) is a multiplication of agent A's belief in B (b_B^A) with agent B's belief in proposition p (b_p^B). Moreover, Agent A's disbelief in proposition p (d_p^{AB}) is a multiplication of agent A's belief in B (b_B^A) with agent B's disbelief in p (b_p^B). As can be observed, the belief and disbelief of agent A in proposition p is based on its degree of belief in the recommender agent B. Uncertainty of agent A in proposition p is derived from agent A's disbelief and uncertainty in agent B plus the multiplication of b_B^A and b_p^B. This model has a number of shortcomings. As mentioned in [46] "it must be assumed that the agents in the chain do not change their behavior". In other words, this model does not consider the variability in an agent's behavior. Despite the fact that agent A has an opinion on agent B's trust, which in turn impacts agent A's opinion model on p, there is no assumption of manipulation of information by agent B. Indeed, agent B reports its opinion model on agent p without modification. Consequently, it is not clear how the opinion model of proposition p should be measured in the presence of manipulation of information. In addition, this model is based on binary domain of events (each interaction is either positive or negative) which is a restriction case of more generalized scenarios where each interaction is multi-valued.

As explained in [18], this model correctly increases certainty by increasing the confirmatory evidence (for a fixed ratio of positive to negative interactions, higher number of evidence, or equivalently interactions, increases certainty); however, this model mistakenly increases certainty in the presence of conflicting evidence. For example, if all of the interactions are positive or all of them are negative the same certainty value is measured as in the case where the evidence is equally split among the positive and negative interactions. In the latter case, the certainty should be less since the target entity's trustworthiness is more unclear since it demonstrates neither absolute trustworthiness nor absolute untrustworthiness.

3.2.5 Evidence-Based Trust

Wang and Singh [18] provide another probabilistic computational model measuring belief, disbelief and uncertainty from binary interactions (either positive or negative). Similar to the proposed model of Jøsang in [19], Wang and Singh [18] measure a single value for each one of belief, disbelief and uncertainty in the interval of $[0, 1]$ such that the sum of them equals one. The model formulates a probability density function measuring the probability of having a positive experience with the target agent. Then, the mean absolute deviation of the difference between this distribution and the uniform distribution is measured to derive the certainty value. The certainty value measured in this approach considers the following two characteristics.

- **Effect of Evidence.** Certainty increases by increase in evidence. In other words, given the same ratio of positive to negative events, higher evidence (higher number of provided events) would increase certainty.

- **Effect of Conflict.** Certainty decreases by increase in the degree of conflict. More specifically, given a fixed number of evidence (fixed number of positive and negative events), the more the number of positive and negative events become equal, the more conflict exists in the evidence, and therefore the less would be the certainty. For example, consider a scenario where Bob deals with Alain 4 times or receives fully trustworthy reports on Alain from 4 witnesses. The evidence would be between 0 to 4 positive experiences. The certainty is higher if the evidence is 0 (all experiences are negative) or 4 (all experiences are positive) compared to the cases where the number of positive events are among these 2 extreme ends (the evidence contains both positive and negative experiences). This is due to the fact that in the former cases, Alain's trustworthiness is more predictable (completely trustworthy or completely untrustworthy) and therefore the certainty about the degree of trustworthiness that would be experienced in a future interaction would be higher. The model of Jøsang [19] yields the same certainty value of 0.8 for all these cases, which does not consider the conflict in the information.

Both models described in [19] and [18] are designed for binary domains of events in which each interaction is either positive or negative. This, in turn, is a limited case of a more generalized multi-valued domains where each interaction can take a value chosen out of a discrete set of events. The work presented in [18] can represent the deviation in the degree of trustworthiness of an agent through the probability-certainty density function (PCDF). However, it cannot demonstrate the uncertainty risen from lack of information. This is a shortcoming of probabilistic models compared to possibilistic models. In addition, the evidence-based trust measured here is based on the information provided by trustworthy sources. It is not clear how, in presence of inaccurate reports provided by other agents, trust can be measured.

3.2.6 A Personalized Approach

A personalized approach is presented in [47] where the reputation of an advisor is modeled by a buyer agent based on the accuracy of its provided ratings for commonly rated sellers. In this model, the reputation of an advisor is measured from the previous ratings that both the buyer and advisor agents have given to the commonly rated sellers. Indeed, it is assumed that both advisor and buyer agents have rated a group of sellers where each rating is binary. Then, the ratings given to each seller are partitioned into time windows. If the rating provided by the advisor agent on a seller agent at a given time window matches the one of the buyer, then it is considered as a positive rating experience by the advisor. Otherwise, it is considered as a negative rating experience. Finally, based on the ratio of the positive rating experiences to the total number of experiences the trustworthiness of the advisor is measured. This constitutes the buyer's private reputation of the advisor. In this model, if a buyer agent does not have adequate experience with an advisor, in addition to the private reputation, it considers the public reputation of the advisor. The public reputation of an advisor represents the public's opinion on that advisor. The trustworthiness of the advisor is measured by combining the weighted private and public reputations where the weights of these reputations are based on the degree of reliability of the private reputation.

This work also measures the buyer's private reputation of a seller, based on the buyer's own experience with the seller. If the buyer's experience with the seller is not adequate, it can rely on the ratings provided by the advisor agents in which case the public reputation of the seller is considered. Finally, the trustworthiness of the seller is measured by combining its private and public reputations. This model considers the manipulation of information (unfair ratings). In addition, it addresses the changes in an agent's (seller or advisor) trustworthiness by giving each interaction a time-related weight. However, it does not model the uncertainty in an agent's behavior due to lack of information. Moreover, in this model, each event is binary (either positive or negative) which is more restrictive compared to multi-valued events.

3.2.7 Travos

Teacy *et al.* present *Travos* [48], which considers both manipulation of information and uncertainty. *Travos* measures the trust and reputation of the agents in the presence of inaccurate information sources. Moreover, the result of each inter-agent interaction is binary (successful or unsuccessful), which is based upon the underlying probability that an agent fulfills its obligations. The binary domain of events, in turn, have facilitated the usage of beta probability density function to constitute the probability of having a successful interaction with the target agent in the future. The expected value of the beta distribution is considered as the trust of the target agent. Then, a confidence is measured for this trust value such that it lies within an acceptable margin of error. If the confidence value, which is measured in the direct experience, is low *Travos* seeks information from witness agents. After receiving information from witness agents, *Travos* interacts with the target agent and compares its observation with the received information from the witness agents. Thereupon, it measures the probability that the report of a witness agent supports the true probability of the target agent. If the reports of a witness are biased, *Travos* gives it less influence for future trust measurement of a target agent.

While this model has a strong probabilistic approach and covers many issues, it is yet restricted to binary domain of events where each interaction is either successful or unsuccessful. Our work generalizes this aspect by considering a multi-valued domain. Moreover, we use possibility theory which is a flexible and strong tool to address uncertainty and at the same time it is applicable to multi-valued domains.

3.2.8 BLADE

BLADE [49] is a Bayesian Reputation model which addresses subjectivity and deception of the advisor agents. The subjectivity considers the specific view of an advisor which may differ from the buyer's view. For instance, if a buyer evaluates the quality of certain product as high, the advisor's view about the product may be normal. Another example can be the delivery date of a product.

In a buyer's view a product delivered up to one day after the deadline can still be on time while an advisor may consider it as a delay. Deception, on the other hand, arises from the advisor's intentional falsification of information which does not comply with its real opinion.

Although subjectivity and deception are different concepts, they are addressed together. In order to address subjectivity and deception, BLADE uses a Bayesian Network (BN) to learn seller features and the advisor evaluation functions. The Bayesian learning approach deals with deception and subjectivity without resorting to heuristics or filtering untrustworthy advisors. In this approach, a buyer interacts with a seller over a number of interactions and, meanwhile, receives information from advisors about that seller. Using the acquired information, the buyer models the seller's features (e.g., time, condition of the goods). The buyer also models the advisor evaluation functions through the information acquired from the advisor on a seller's features. In addition, a dynamic Bayesian network is proposed to adapt to the advisor and seller's behavioral changes over time.

In BLADE, each feature of a seller can take a finite number of discrete values. For example, the product quality feature can take values of highly-satisfactory, satisfactory, normal and poor. In addition, in this model, a seller's behavior is not necessarily deterministic and can change from one interaction to another. Indeed, a seller's behavior is drawn from a multinomial distribution. Consequently, the uncertainty arising from variability is considered in BLADE. However, the uncertainty due to ignorance is not modeled in BLADE. This is a shortcoming of probabilistic models which cannot demonstrate the uncertainty arising from ignorance.

In this chapter, we reviewed trust models that address uncertainty and the ones that do not. We provided a brief overview of each model and mentioned the shortcoming of each model compared to the model that we are going to propose in this thesis. We consider, *Travos* and BLADE as the closest models to our own. *Travos* is restricted to binary events while BLADE considers a multi-valued ratings. Both approaches consider uncertainty and both employ probability theory to model uncertainty. Their proposed uncertainty handles variability. However, the probabilistic models have a shortcoming in addressing uncertainty driven from ignorance. In our proposed model, we use possibility theory since it is capable of addressing both types of uncertainty (ignorance and variability).

Chapter 4

Technical Tools

In this chapter, we describe the technical tools that we employ in our model. These tools have been developed by other researchers and we use them in order to build our trust estimation model in a possibilistic domain. On this purpose, we review the literature that provides these technical tools and we further present a brief overview of each tool.

4.1 Fusion Rules

In possibility theory, fusion rules aggregate information from different sources. In other words, fusion rules facilitate combining different observations made on an entity (for example observation of n sensors monitoring a variable) when the information may be conflicting, imprecise and uncertain. Fusion rules are represented with a function $F : [0,1]^n \rightarrow [0,1]$ in which the information taken from n sources are aggregated to derive a single output. Both the inputs and output of this function are within the interval $[0,1]$, which makes them suitable for addressing trust since trust is usually represented in the same interval. In our multi-agent platform the fusion rules are used to aggregate the possibility distributions acquired from the agents in the set A. Each acquired possibility distribution represents the trust distribution of the target agent a^{D} as measured and reported by an agent $a \in A$. Consequently, the fusion rules facilitate merging the information reported on the target agent a^{D} in order to derive a unified distribution on a^{D}. In this section, we elaborate on the fusion rules applied in our model.

4.1.1 Classical Fusion Rules

Intersection and Union [50] are among the most widely used rules in possibility domain. These rules are equivalent to the t-norm and t-conorm functions in fuzzy theory [51]. Moreover, they assume the information is coming from sources of unknown dependence or non-interaction. In other words, it is not clear whether the sources have exchanged information about the system under study before reporting the information or not. The intersection or conjunctive rule is used when the sources are considered reliable. The output of this fusion rule represents the information that all sources agree upon. In the context of our trust model, let τ be a trust rating from the set T of all ratings and $\Pi_i(\tau)$ be the possibility of the trust rating τ received from source i (note that possibility is a function $\Pi : T \rightarrow [0, 1]$ as defined in Section 2.5), then the intersection rule on possibility distributions received from n sources is represented as:

$$\Pi_{\bigcap}(\tau) = \min_{i \in \{1,\ldots,n\}} \Pi_i(\tau) \qquad \forall \tau \in T. \tag{6}$$

The intersection rule considers the minimum possibility value for each trust rating τ.

After applying the intersection rule, the subsequent possibility distribution should be normalized. This is due to the fact that the resulting possibility distribution may not satisfy the following two conditions, which should be met by each possibility distribution to be considered normalized:

- The possibility value of every trust rating $\tau \in T$ should fall into $[0, 1]$.
- The possibility value of at least one trust rating in T should be equal to 1. In other words, $\exists \tau \in T : \Pi(\tau) = 1$

Let $\widetilde{\Pi}(\tau),\ \tau \in T$ represent a non-normalized possibility distribution. Either of the following formulas [51] generates a normalized possibility distribution $\overline{\Pi}(\tau)$:

$$\text{normalization 1: } \overline{\Pi}(\tau) = \widetilde{\Pi}(\tau)/h, \tag{7}$$

$$\text{normalization 2: } \overline{\Pi}(\tau) = \widetilde{\Pi}(\tau) + 1 - h, \tag{8}$$

$$\text{where } h = \max_{\tau \in T} \widetilde{\Pi}(\tau). \tag{9}$$

Normalization 1 is not defined in the case of absolute contradiction when the value of h is equal to 0. The value of h measured in (9) indicates the degree of contradiction among the sources. If h equals 1, the sources have agreement while if h equals 0, then the sources have absolute disagreement.

Union [50] or disjunctive rule is another fusion rule which is among the most applied ones. It is utilized when there is a belief that some information sources are trustworthy, but it is not clear which sources are trustworthy and which sources are not [52]. The union rule is represented as:

$$\Pi_{\bigcup}(\tau) = \max_{i \in \{1,\ldots,n\}} \Pi_i(\tau) \qquad \forall \tau \in T. \tag{10}$$

Union rule considers the maximum possibility value for each trust rating τ. Unlike the intersection rule, the union rule reflects all of the information provided by all of the sources even if they are contradictory. If only one source provides a piece of information that the other sources do not agree upon, that piece is included in the possibility distribution resulting from the union rule. Consequently, the final possibility distribution may converge to the uniform distribution, if each piece of information is recommended by at least one agent. The closer the consequent distribution is to the uniform distribution, the more ignorance exists on the target entity. This is due to the fact that the uniform distribution contributes no information as all of the trust ratings in T have the same possibility of 1. In the context of possibility distributions, a uniform distribution is termed as "complete ignorance" [53]. The union rule does not require a normalization process, assuming the distributions are normalized before applying the fusion rule. This is due to the fact that each possibility distribution reported by source i, $(1 \leq i \leq n)$ satisfies $\exists \tau \in$T: $\Pi(\tau) = 1$. Therefore, this value is maintained in the subsequent distribution of the union rule which would maintain its normalization.

The intersection and the union rules lead to two extreme points of a spectrum. In the former only the information that all sources agree upon is maintained, while in the latter all pieces of information are reflected. In order to take a midway approach, the mean of the data can be considered. The **mean** rule [54] is constructed as follows:

$$\Pi_\mu(\tau) = (1/n) \sum_{i \in \{1,\ldots,n\}} \Pi_i(\tau) \qquad \forall \tau \in T. \tag{11}$$

By applying the mean rule, the final possibility distribution is an average of the possibility distributions provided by all of the n sources. Consequently, all of the sources have the same influence on the result of the mean fusion rule. After applying the mean rule, normalization is needed. This is due to the fact that the normalization of the possibility distributions of the n sources are not necessarily maintained in the derived possibility distribution after applying the mean rule.

Representation of the Fusion Rules in Our Multi-Agent Platform: In our model, the sources of information are the agents of a set A and the information received from each agent is a possibility distribution on a target agent's trust. Assume an agent, say a^{S}, receives a possibility distribution from each agent $a \in A$ and let $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ be the possibility distribution reported by agent a indicating its trust in a target agent, say a^{D}. Then, the intersection fusion rule applied on these possibility distributions is represented as:

$$\textbf{intersection } [50]: \qquad \Pi^{\bigcap}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = \min_{a \in A} \Pi_{a \to a^{\mathrm{D}}}(\tau) \qquad \forall \tau \in T \tag{12}$$

where $\Pi^{\bigcap}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ represents the possibility distribution of agent a^{S}'s trust in agent a^{D} as measured by the intersection rule.

Similarly, union and mean rules can be measured in our model. Having the same set of distributions $(\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T, a \in A)$, received from the agents in A, the union and mean rules are represented respectively as follows:

$$\textbf{union} \quad [50]: \qquad \Pi^{\bigcup}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = \max_{a \in A} \Pi_{a \to a^{\mathrm{D}}}(\tau) \qquad \forall \tau \in T. \tag{13}$$

$$\textbf{mean} \quad [54]: \qquad \Pi^{\mu}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = (1/|A|) \sum_{a \in A} \Pi_{a \to a^{\mathrm{D}}}(\tau) \qquad \forall \tau \in T. \tag{14}$$

In (13), $\Pi^{\bigcup}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ represents the trust distribution of agent a^{S} in a^{D} as measured by the union rule. Similarly, $\Pi^{\mu}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ is the trust distribution of agent a^{S} in a^{D} derived from the mean rule which is represented in (14).

4.1.2 Fusion Rules Involving Trust of the Sources

In Section 4.1.1, we reviewed three commonly employed fusion rules, which do not take into account the trust of the information sources. However, in realistic settings, entities are distinguished based on their degree of trustworthiness. Thereupon, the information provided by these sources should be taken into consideration based on their honesty and accuracy. Through such consideration of the degree of trustworthiness of the information sources (which in our model corresponds to the advisor agents), the subsequent possibility distribution measured for the target agent a^{D} would be enhanced. In this approach, the influence of each information source depends on its degree of trustworthiness. In our multi-agent platform, the information sources are the advisor agents in the set A. Indeed, each advisor agent $a \in A$ reports a possibility distribution about the trust of the target agent a^{D}. The fusion rules introduced in this section facilitate aggregating the possibility distributions acquired from the agents in A with respect to the degree of trustworthiness of the advisor agents. The derived possibility distribution from the fusion rules, represents the trust distribution of the target agent a^{D}. In this section, we review the most commonly used fusion rules involving the trust of the information sources.

In our model, agent a^{S}, receives a possibility distribution $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ from each agent $a \in A$ representing the trust distribution of agent a in the target agent a^{D}. Moreover, let $\tau_{a^{\mathrm{S}} \to a} \in [0, 1]$ denote a scalar value of agent a^{S}'s trust in agent a. By scalar, we mean that $\tau_{a^{\mathrm{S}} \to a}$ can only hold a single value but not a set of values (e.g., a distribution). In this section, we review the fusion rules that consider the trust of agent a^{S} in each agent $a \in A$. The subsequent possibility distribution denoted by $\Pi^{X}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T \quad (X \in \{T, Y, DP\})^1$, which is derived from a fusion rule, represents a^{S}'s trust in a^{D}.

We explore three fusion rules, among the most commonly used. The first one is the trade-off rule [54], which builds a weighted mean of the possibility distributions:

$$\textbf{trade-off } [54]: \qquad \Pi^{T}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = \sum_{a \in A} \omega_a \times \Pi_{a \to a^{\mathrm{D}}}(\tau) \qquad \forall \tau \in T \tag{15}$$
$$\text{where } \omega_a = \tau_{a^{\mathrm{S}} \to a} / \sum_{a \in A} \tau_{a^{\mathrm{S}} \to a}$$

$\Pi^{T}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ indicates the possibility distribution of a^{S}'s trust in a^{D} measured through the trade-off rule. Note that the trade-off rule considers all of the possibility distributions reported by the agents in A. However, the degree of influence of the possibility distribution of $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ on final distribution of $\Pi^{T}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ is weighted by the normalized trust of agent a^{S} in each agent a , which is ω_a.

The next two fusion rules belong to a family of rules which first modify the possibility distribution of $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ based on the trust value associated with it, $\tau_{a^{\mathrm{S}} \to a}$, and then apply the intersection rule on them. Therein, if $\tau_{a^{\mathrm{S}} \to a} = 1$, $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ remains unchanged, meaning that agent a^{S}'s full trust in a results in total acceptance of the possibility distribution $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ reported by a. The closer $\tau_{a^{\mathrm{S}} \to a}$ gets to zero, the less trustworthy is agent a and therefore the less reliable is its report. When the trust in agent a decreases, its reported distribution of $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ moves towards the uniform distribution. In the context of possibility distributions, a uniform distribution provides no information as all trust values in domain T are considered equally possible. It is acknowledged as "complete ignorance" [53]. Consequently, when $\tau_{a^{\mathrm{S}} \to a}$ decreases, the possibility

[1] T, Y and DP respectively stand for Trade-off, Yager and Dubois-Prade fusion rules which are introduced later in this section.

distribution $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ provided by an agent a moves more towards a uniform distribution, and thereupon its information level is decreased. Consequently, its reported information gets more neglected. In this context, we selected two fusion rules which we refer to as Trust Modified (TM) rules.

$$\textbf{Yager } [55]: \qquad \Pi^{Y}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = \min_{a \in A} \left[\tau_{a^{\mathrm{S}} \to a} \times \Pi_{a \to a^{\mathrm{D}}} + 1 - \tau_{a^{\mathrm{S}} \to a} \right], \qquad \forall \tau \in T. \tag{16}$$

$$\textbf{Dubois and Prade } [56]: \qquad \Pi^{DP}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) = \min_{a \in A} \left[\max(\Pi_{a \to a^{\mathrm{D}}}, 1 - \tau_{a^{\mathrm{S}} \to a}) \right], \qquad \forall \tau \in T. \tag{17}$$

In Yager's fusion rule, the possibility of each trust value of τ moves towards a uniform distribution as much as $(1 - \tau_{a^{\mathrm{S}} \to a})$, which is the extent to which the agent a is not trusted. In Dubois and Prade's fusion rule, when an agent's trust declines, the max operator would more likely select $1 - \tau_{a^{\mathrm{S}} \to a}$ and, hence, the information in $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ reported by a is ignored. The higher the value of $1 - \tau_{a^{\mathrm{S}} \to a}$, the more the distribution of $\Pi_{a \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ converges towards a uniform one.

The possibility distribution reported by each agent $a \in A$ is first transformed by using a TM rule. Then, an intersection of these possibility distributions is taken. In (16) and (17), the intersection corresponds to the min operator.

Let $\Pi^{\mathrm{TM}}_{a \in a^{\mathrm{D}}}(\tau), \forall \tau \in T$ represent either $\Pi^{DP}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau)$ or $\Pi^{Y}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau)$, which indicates the modified distribution of agent a's report by virtue of a TM rule. The possibility distribution $\Pi^{\mathrm{TM}}_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau), \forall \tau \in T$ is then normalized to represent the possibility distribution of agent a^{S}'s trust in a^{D}. As mentioned above in (16) and (17), the intersection is represented by the min operator. However, in general, in this family of fusion rules, the reported possibility distributions of agents in the set A are first modified. The modification keeps the reported distribution of a trustworthy agent intact and makes the reported distribution of a complete distrusted agent converge to a uniform distribution. Then, an intersection of the modified distributions is considered. Since the reported distributions of more trustworthy agents are less modified, they have a higher degree of influence on the consequent possibility distribution derived from the intersection process.

We selected the above fusion rules based on their common usage in the literature and our assumption of unknown dependence of the sources meaning that it is not known whether the sources have exchanged information or not. We considered both extreme fusion rules (e.g., union and intersection) and moderate fusion rule (e.g., the mean rule). A description of the suitability criteria of some of the fusion rules is presented in [52]. The selection of more specific fusion rules for our model will be accomplished in future work.

4.2 Probability to Possibility Transformation

In this section, we present the transformation procedure of a probability distribution to a possibility distribution. This is of interest in our model when we transform the underlying (true) probability distribution of a target agent's trust to a possibility distribution in order to compare it with the estimated possibility distribution of the target agent's trust. The estimated possibility distribution of the target agent a^{D} is the distribution measured through the information acquired from the advisor agents. This is discussed in more details in Section 6.1.2. We use the transformation rules of Dubois *et al.* in [57, 58]. We provide a brief review of the transformations rules of [57, 58] based on the descriptions presented in [59]. In this section, we first elaborate the ordering relations and then discuss the transformation rules.

4.2.1 Ordering Relations

A binary relation on set U is a subset of the Cartesian product U^2. Let u, v and w be elements of set U where uRv indicates that u is in relation with v. A relation R is:

- transitive if $\forall (u, v, w) \in U^3, uRv$ and $vRw \Rightarrow uRw$;
- antisymmetric if $\forall (u, v) \in U^2, uRv$ and $vRu \Rightarrow u = v$;
- irreflexive if $\forall (u, v) \in U^2, uRv \Rightarrow u \neq v$;
- complete if $\forall (u, v) \in U^2, u \neq v \Rightarrow uRv$ or vRu.

An antisymmetric and transitive relation is considered a partial order relation. An irreflexive partial order is said to be strict. A transitive, antisymmetric and complete relation is a linear order relation. An irreflexive linear order relation is said to be a strict linear order relation. A linear relation ℓ is compatible with a partial ordered relation ρ if and only if $\rho \subseteq \ell$. In this case, ℓ is noted as a linear extension of ρ.

4.2.2 Dubois and Prade's Transformation

Consider a discrete domain $\Omega = \{\omega_1, \omega_2, \ldots, \omega_k\}$ containing k elements where P denotes the unknown probability distribution of a random variable X on the set Ω in which $p_i = P(\omega_i)$.

Zadeh [35] first mentioned the relation between probability and possibility by noting that *what is probable should be possible*. Later, Dubios and Prade [34, 60] presented this statement in the following form:

$$P(A) \leq \pi(A) \quad \forall A \subseteq \Omega \tag{18}$$

where $P(A)$ and $\pi(A)$ are the probability and possibility of a subset A in the domain Ω. Therefore, π dominates P. Dubois and Prade [57,58] added the following strong order preservation constraint:

$$p_i < p_j \Leftrightarrow \pi_i < \pi_j \quad \forall i, j \in \{1, \ldots, k\}. \tag{19}$$

where $p_i = P(\omega_i)$ and $\pi_i = \pi(\omega_i)$ are respectively the probability and possibility of element $\omega_i \in \Omega$. We now look for the most specific possibility distribution satisfying constraints (18) and (19). Note that the possibility distribution of π is more specific than π' having satisfied:

$$\pi(\omega_i) \leq \pi'(\omega_i) \quad \forall i \in \{1, \ldots, k\}. \tag{20}$$

Having $p_i \neq p_j,\ \forall i \neq j$, let ℓ be a strict linear order on $\Omega = \{\omega_1, \omega_2, \ldots, \omega_k\}$ such that:

$$\omega_i <_\ell \omega_j \Leftrightarrow p_i < p_j. \tag{21}$$

This means that if ω_i and ω_j are ordered by ℓ, then the corresponding probability of $P(\omega_i)$ is smaller than the one of $P(\omega_j)$. Let σ be a permutation on the indices $\{1, \ldots, k\}$ to satisfy the strict linear order ℓ such that $p_{\sigma(1)} < p_{\sigma(2)} < \cdots < p_{\sigma(k)}$. It satisfies the following:

$$\omega_{\sigma(i)} <_\ell \omega_{\sigma(j)} \Leftrightarrow p_{\sigma(i)} < p_{\sigma(j)}. \tag{22}$$

Example 1. Assume the elements of set Ω have the following probability values:

$p_1 = 0.25,\ p_2 = 0.3,\ p_3 = 0.35,$ and $p_4 = 0.1$. Here, we have $\omega_4 <_\ell \omega_1 <_\ell \omega_2 <_\ell \omega_3$ which gives $p_4 < p_1 < p_2 < p_3$. By applying the permutation σ on the indices of $\{1, 2, 3, 4\}$, the values of $\sigma(1) = 4, \sigma(2) = 1, \sigma(3) = 2, \sigma(4) = 3$ are derived. In this case, $\omega_{\sigma(1)} <_\ell \omega_{\sigma(2)} <_\ell \omega_{\sigma(3)} <_\ell \omega_{\sigma(4)}$ which indicates $p_{\sigma(1)} < p_{\sigma(2)} < p_{\sigma(3)} < p_{\sigma(4)}$.

The permutation has a function of $\sigma(x) = y,\ \ x, y \in \{1, \ldots, k\}$ where $\omega_{\sigma(x)}$ is the x'th element in the ordered list ℓ and has an index y in the set of Ω. In example 1, $\sigma(2) = 1$ means that the second element in ℓ has index 1 in Ω. In order to know the ranking of an element ω_i in the ordered list ℓ, we need to get the inverse function σ. The permutation function of σ is bijective and the inverse of permutation function $\sigma^{-1}(y) = x$ gives the rank of each $\omega_y \in \Omega$ in the ascending ordered list ℓ which is x. In other words, element with index y in the unordered set of $\{p_1, p_2, \ldots, p_y, \ldots, p_k\}$ has a ranking of x in the ascending ordered set of $\{p_{\sigma(1)}, p_{\sigma(2)}, \ldots, p_{\sigma(x)}, \ldots, p_{\sigma(k)}\}$.

Example 2. Assuming the elements of Ω have the probability values of $p_1 = 0.25,\ p_2 = 0.3,\ p_3 = 0.4,\ p_4 = 0.05$, p_2 has rank 3 in the ordered list of $p_{\sigma(1)} = 0.05,\ p_{\sigma(2)} = 0.25,\ p_{\sigma(3)} = 0.3,\ p_{\sigma(4)} = 0.4$ where we have $\sigma^{-1}(1) = 2,\ \sigma^{-1}(2) = 3,\ \sigma^{-1}(3) = 4,\ \sigma^{-1}(4) = 1$.

Considering the inverse permutation σ^{-1}, the most specific possibility distribution satisfying constraints (18) and (19) can be constructed. Thereupon, Dubois and Prade's transformation of probability to possibility is represented as follows:

$$\pi_i = \sum_{\{j|\sigma^{-1}(j)\leq\sigma^{-1}(i)\}} p_j. \tag{23}$$

where $\sigma^{-1}(j) \leq \sigma^{-1}(i)$ means that element $\omega_j \in \Omega$ is ranked less than ω_i in the ascending ordered set of $(p_{\sigma(1)}, p_{\sigma(2)}, \ldots, p_{\sigma(k)})$ and therefore the probability of $P(\omega_j)$ is smaller than $P(\omega_i)$.

Example 3. Assume the elements of set Ω have the probabilities provided in Example 1. Applying (23) yields the following possibility values:

$$\pi_1 = p_1 + p_4 = 0.25 + 0.1 = 0.35,$$

$$\pi_2 = p_2 + p_1 + p_4 = 0.3 + 0.25 + 0.1 = 0.65,$$

$$\pi_3 = p_3 + p_2 + p_1 + p_4 = 0.35 + 0.3 + 0.25 + 0.1 = 1,$$

$$\pi_4 = p_4 = 0.1.$$

Equation (23) can be used only when all probability values in P are pairwise different are identical). If at least two probabilities in P are the same, then there is no strict linear order on Ω. Instead, there is a partial order ρ on Ω. This partial order can be represented by a set of compatible linear orders $\Lambda(\rho) = \{\ell_l, l = 1, \ldots, L\}$. Each linear order ℓ_l of $\Lambda(\rho)$ can be associated with a permutation σ_l on Ω such that:

$$\omega_{\sigma_l(i)} <_{\ell_l} \omega_{\sigma_l(j)} \Leftrightarrow p_{\sigma_l(i)} \leq p_{\sigma_l(j)}. \tag{24}$$

In this case, the most specific possibility distribution on the probability set of $\{p_1, p_2, \ldots, p_k\}$ is obtained by taking the maximum of all possible permutations in $\Lambda(\rho)$:

$$\pi_i = \max_{l=1,\ldots,L} \sum_{\{j|\sigma_l^{-1}(j)\leq\sigma_l^{-1}(i)\}} p_j. \tag{25}$$

Example 4. Assume the probabilities of set P are: $p_1 = 0.3,\ p_2 = 0.3,\ p_3 = 0.35,$ and $p_4 = 0.05$. Then, there are two possible permutations: $\sigma_1(1) = 4,\ \sigma_1(2) = 1,\ \sigma_1(3) = 2,\ \sigma_1(4) = 3$ and

$\sigma_2(1) = 4,\ \sigma_2(2) = 2,\ \sigma_2(3) = 1,\ \sigma_2(4) = 3$. In permutation σ_1, we have $\sigma_1^{-1}(1) = 2, \sigma_1^{-1}(2) = 3, \sigma_1^{-1}(3) = 4, \sigma_1^{-1}(4) = 1$. In permutation σ_2, we have $\sigma_1^{-1}(1) = 3, \sigma_1^{-1}(2) = 2, \sigma_1^{-1}(3) = 4, \sigma_1^{-1}(4) = 1$. Applying (25) yields the following possibility values:

$$\pi_1 = max(p_1 + p_4, p_1 + p_2 + p_4) = max(0.35, 0.65) = 0.65,$$

$$\pi_2 = max(p_1 + p_2 + p_4, p_2 + p_4) = max(0.65, 0.35) = 0.65,$$

$$\pi_3 = p_4 + p_1 + p_2 + p_3 = 0.05 + 0.3 + 0.3 + 0.35 = 1,$$

$$\pi_4 = p_4 = 0.05.$$

In the above example, $p_1 = p_2$ resulted in $\pi_1 = \pi_2$ which should be satisfied in order to have strong order preservation.

4.3 Inferring a Possibility Distribution from Empirical Data

In this section, we provide a brief overview of the research accomplished in [59] which measures a possibility distribution out of empirical data. This measurement is used in the thesis, where a possibility distribution is constructed out of the interactions made with an agent in order to model the agent's possibility distribution of trust. This measurement is used for every pair of connected agents that have made direct interactions. Indeed, for every pair of connected agents, the trustor agent measures the possibility distribution of the trustee agent out of the empirical interactions it has made with the trustee agent. The measured possibility distribution represents the trust distribution of the trustee agent. The model described here is not the only model for measuring a possibility distribution. When there are very few measurements (which in our model is the number of interactions), the model presented in [61] can be used. However, the model presented in [59] can be used when the number of interaction is not necessarily few.

Suppose that a total of N samples over the space of $\Omega = \{\omega_1, \omega_2, \ldots, \omega_k\}$ is generated according to an underlying probability distribution P (in which $p_i = P(\omega_i)$) where the number of observation of the elements in the set Ω equals to $\mathbf{n} = (n_1, n_2, \ldots, n_k)$. The classical approach of measuring a probability distribution out of the observations of vector $\mathbf{n}$ is to get the frequency vector $\mathbf{f} = (f_1, f_2, \ldots, f_k)$ in which $f_i = n_i/N$ and then apply the Dubois and Prade transformation as given by equation (25). However, this approach does not consider the uncertainty driven from the sampling procedure. In order to consider such uncertainty, confidence interval on each element $\omega_i \in \{\omega_1, \omega_2, \ldots, \omega_k\}$ should be measured instead of its frequency.

A confidence interval on a parameter at a given level α contains the true value with a probability of $1 - \alpha$, where $\alpha \in [0, 1]$ is a small nonnegative value close to zero. In the approach proposed in [59], first the bounds (upper and lower bounds) of $p_i, i \in \{1, \ldots, k\}$ are estimated (by using the confidence intervals on multinomial proportions), and then the possibility values are measured from these intervals. The procedure satisfies that in at least $100(1-\alpha)\%$ of the cases the derived possibility distribution dominates the true probability. In other words:

$$P(\pi(A) \geq P(A) \;\; \forall A \subseteq \Omega) \geq 1 - \alpha \tag{26}$$

where $\pi(A)$ is the measured possibility value from the empirical data and $P(A)$ is the unknown but constant probability value of event A.

4.3.1 Measuring Confidence Intervals for Multinomial Proportions

Simultaneous confidence intervals are measured for multinomial proportions with a joint confidence level of $1-\alpha$. The goal is to find a confidence region $\mathbb{C}_n$ in the parameter space $\{p = (p_1, p_2, \ldots, p_k) \in [0;\ 1]^K \,|\, \sum_{i=1}^{K} p_i = 1\}$ which is the Cartesian product of K intervals $[p_1^-, p_1^+] \times \cdots \times [p_K^-, p_K^+]$ such that $P(p \in \mathbb{C}_n) \geq 1 - \alpha$. Note that p_i^- and p_i^+ respectively refer to the lower and upper bounds of the interval that the probability value p_i can fall into, where $i \in \{1, \ldots, k\}$.

The bounds of the confidence intervals are measured based on [62]. Let us consider:

$$A = \chi^2(1 - \alpha/K, 1) + N \tag{27}$$

where $\chi^2(1-\alpha/K, 1)$ is the quantile of order $1-\alpha/K$ of chi-square distribution with one degree of freedom and $N = \sum_{i=1}^{K} n_i$ is the total number of observations over all k elements in Ω. The bounds of the simultaneous confidence intervals are then measured as:

$$\left[p_i^-, p_i^+\right] = \left[\frac{B_i - \Delta_i^{1/2}}{2A}, \frac{B_i + \Delta_i^{1/2}}{2A}\right], \quad \forall i \in \{1, \ldots, k\} \tag{28}$$

where:

$$B_i = \chi^2(1-\alpha/K, 1) + 2n_i; \qquad \Delta_i = B_i^2 - 4AC_i; \qquad C_i = \frac{n_i^2}{N}. \tag{29}$$

Example 5. Assume the true (underlying) probability over set $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ is $p = (0.2, 0.35, 0.4, 0.05)$. Suppose 100 data sample is randomly generated without knowing the true probability distribution in this distribution. The randomly generated values have the frequencies of 18, 45, 35 and 2 for ω_1 to ω_4, respectively. Having $\alpha = 0.1$, the probability bounds measured for sample data is demonstrated in Table 1.

i	1	2	3	4
p_i^-	0.10	0.34	0.25	0
p_i^+	0.28	0.56	0.46	0.08

Table 1: Confidence Intervals in Example 5

4.3.2 Generating a Possibility Distribution from Confidence Intervals

After measuring the bounds of the confidence intervals, the problem of measuring a possibility distribution from empirical data can be reformulated as finding the most specific possibility distribution that dominates every probability distribution defined by $p_i \in \left[p_i^-, p_i^+\right], \quad \forall i \in \{1, \ldots, k\}$. The most specific possibility distribution for the above measured intervals can be measured through the probability to possibility transformation described in Section 4.2.

Let ρ indicate the partial order induced by the intervals $[p_i] = \left[p_i^-, p_i^+\right]$ such that

$$\omega_i <_\rho \omega_j \Leftrightarrow p_i^+ < p_j^- . \tag{30}$$

where there are a total of L compatible linear extensions of $\Lambda(\rho) = \{\ell_l, l = 1, \ldots, L\}$. Based on the descriptions of Section 4.2, each linear extension corresponds to one permutation. Therefore, there are a total of L permutations of $\{\sigma_l, l = 1, \ldots, L\}$.

For each possible permutation σ_l associated with each linear order in $\Lambda(\rho)$, and each element ω_i, the following linear program (LP) should be solved:

$$\pi_i^{\sigma_l} = \max_{\{j|\sigma_l^{-1}(j) \leq \sigma_l^{-1}(i)\}} p_j \ . \tag{31}$$

under the following constraints:

$$\sum_{i=1}^{K} p_k = 1, \tag{32}$$

$$p_k^- \leq p_k \leq p_k^+ \quad \forall k \in \{1, \ldots, K\}, \tag{33}$$

$$p_{\sigma_l(1)} \leq p_{\sigma_l(2)} \leq \cdots \leq p_{\sigma_l(K)}. \tag{34}$$

After considering all L permutations of $\{\sigma_l, l = 1, \ldots, L\}$ and solving the above LP for each class ω_i in each permutation of σ_l, the distribution dominating all the distributions of $\pi_i^{\sigma_l}$ should be taken:

$$\pi_i = \max_{l=1,L} \pi_i^{\sigma_l}, \quad \forall i \in \{1, \ldots, K\}. \tag{35}$$

If the confidence intervals used here are measured based on equation (28) with confidence interval of $1 - \alpha$, then the formula of (35) derives the most specific possibility distribution that dominates all of the compatible probability measured. Therefore, it satisfies property (26).

The above procedure explores all linear extensions of $\Lambda(\rho)$ and solves each linear program which becomes computationally complex when K grows (say $K \geq 10$). The authors in [59] provide a more simplified computational procedure to derive the possibility values measured in (35) while reducing its complexity. On this purpose, the following steps are taken:

- first, all linear extensions are grouped in separate subsets;
- then, the best solution in each subset is found;
- finally, it is shown the exploration of each solution in each subset in not necessary.

The possibility values derived in (35) are accurate. These steps just reduce the complexity of the values measured in (35). More elaboration on these steps are provided in [59].

In this chapter, we provided a brief overview of the technical tools developed by other researchers. We use these tools in our trust model (which is introduced in the next chapter) in order to construct a trust estimation model for our platform. The main purpose of using the technical tools described in this chapter is to develop a possibilistic trust model in the context that we have already introduced. The context includes: (1) addressing uncertainty arisen from both variability and ignorance; (2) considering a platform where agents are self-interested and manipulate the information before reporting it to other agents; and (3) The trust domain is multi-valued. In the next chapter, we elaborate our model.

Chapter 5

Under Uncertainty Trust Estimation

In social networks, estimation of the degree of trustworthiness of a target agent through the information acquired from a group of advisor agents, who had direct interactions with the target agent, is challenging. The estimation gets more difficult when, in addition, there is some uncertainty in both advisor and target agents' trust. In this chapter, we estimate the trust of the target agent when: (1) there is uncertainty arisen from both variability and ignorance in the degree of trustworthiness of advisor and target agents; (2) the advisor agents are self-interested and provide misleading accounts of their past experiences with the target agents; and (3) the outcome of each interaction between the agents that have direct interaction is multi-valued.

In this thesis, we study a model where a set $A = \{a_1, a_2, \ldots, a_n\}$ of n agents [1] (e.g., customers) have made a given number of interactions with a target agent (e.g., service provider), say agent a^{D}. A new agent, say agent a^{S}, gets information from A about the agent a^{D}. We consider the agents of the set A as the advisor agents of a^{S}. a^{D} is referred to as the target agent whose degree of trustworthiness is evaluated by agent a^{S}. Estimating the degree of trustworthiness of the target agent a^{D} helps a^{S}

[1] In our trust model, we assume that the set of A is nonzero.

decide whether or not to interact with a^{D}. Figure 2 illustrates the network of agents that we study in this thesis. For any pair of connected agents, an agent at the tail of an arrow is a trustor, i.e., trusts the other agent, and the agent at the head of the arrow is a trustee whom is trusted.

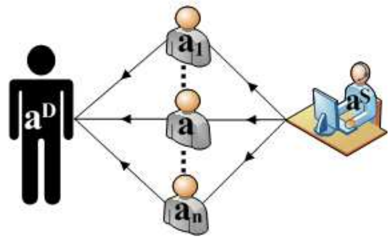


Figure 2: Network of Agents

We consider two different problems in this thesis:

- In the first problem, we assume that agent a^{D} and the agents in set A are unknown to a^{S}. Agent a^{S} receives information from each agent $a \in A$ on the degree of trustworthiness of a^{D}. In this problem, a^{S} cannot distinguish the degree of trustworthiness of each advisor agent and therefore the information received from each agent a is subject to an unknown degree of reliability. By using the acquired information from the agents in A, a^{S} estimates the degree of trustworthiness of the target agent a^{D}. We call the approach used to address this problem the unknown agents approach.
- In the second problem, we assume that a^{S} relatively knows the agents of set A and has accomplished a number of interactions with them. Therein, a^{S} can distinguish the agents in A based on their degree of trustworthiness and can differentiate the information received from them based on their trustworthiness. Finally, a^{S} predicts the trust distribution of a^{D} considering both the information received from the agents in A and their trustworthiness. We propose two approaches for this problem. In the first approach, we merge two successive sets of possibility distributions, namely the possibility distributions of agent a^{S}'s trust in the advisor agents and the possibility distributions of the advisor agents' trust in agent a^{D}. We call this approach the successive merging approach. In the second approach, a single value representing the trust of agent a^{S} in each advisor is measured and then the fusion rules are used to merged

the possibility distributions reported by the advisor agents on the trust of the agent a^{D}. We name this approach the single trust approach. These two approaches are described in more details later in this chapter. Finally, in this problem, the degree of certainty of a^{S} on the information acquired form set A of agents is also measured.

Figure 2 shows the network of agents that we study in both problems. The link between the agents of set A and agent a^{D} demonstrates the flow of interactions among every pair of connected agents $(a \in A, a^{\mathrm{D}})$. However, the arrows connecting a^{S} to each agent $a \in A$ have different meanings in the above problems. In the first one, they are considered as transfer of information from the trustee agent a to the trustor agent a^{S}. The transferred information is the trust distribution of the target agent a^{D} as reported by the agent a. In the second problem, on the other hand, each link indicates that interactions have been carried out among every pair of trustor and trustee agents $(a \in A, a^{\mathrm{D}})$, while at the same time it implies the transfer of information between the pair, the same as in the first problem.

In this chapter, in order to alleviate the notations, we simplify the notation of a possibility distribution $\Pi_X(\tau), \forall \tau \in T$ to Π_X where $(X \in \{a^{\mathrm{S}} \to a, a \to a^{\mathrm{D}}, a^{\mathrm{S}} \to a^{\mathrm{D}}\})$ and we indicate the possibility value of a trust rating τ by $\Pi_X(\tau)$. Note that in this chapter whatever we borrow from the chapter of technical tools (chapter 4) is developed by other researchers and we reuse them in our trust model. The rest of the materials in this chapter are contributions of this thesis.

The rest of this chapter is structured as follows. First, we detail the platform of our multi-agent system in Section 5.1. Then, in Section 5.2, we propose our model for the trust estimation of the target agent in both scenarios described above, which are described in Sections 5.2.1 and 5.2.2, respectively. Finally, in Section 5.3, we explain our prosed model for measuring certainty over the information acquired from the set A of agents.

5.1 Multi-Agent Platform

In the rest of this section, we present the components that build the multi-agent environment and the motivation behind each choice. We first discuss the set of trust values (Section 5.1.1). Then, we talk about the agent's internal trust distribution (Section 5.1.2) and the interactions among the agents (Section 5.1.3). Later, we describe the formation of the possibility distribution of an agent's trust (Section 5.1.4) out of its empirical interactions and the potential algorithms for the manipulation of information by the agents in A before reporting it to agent a^{s} (Section 5.1.5). Finally, the game scenario in this thesis is discussed (Section 5.1.6).

5.1.1 Trust Values

Service providers ask customers to provide their feedback on the received services commonly in form of a rating selected from a multi-valued set. A multi-valued set is a set which has more than one element yet the number of elements are finite. The selected rating indicates a customer's degree of satisfaction or, in other words, its degree of trust in the provider's service. Since the majority of user surveys are carried out through multi-valued sets (e.g., e-bay, amazon, IMDB), in which users can select a rating out of a set, we use a multi-valued set for our trust domain. We define a multi-valued set of trust ratings denoted by T, with $\underline{\tau}$ being the lowest, $\overline{\tau}$ being the highest and $|T|$ representing the number of trust ratings[1]. All trust ratings are within $[0, 1]$ and they can take any value in this range. However, if the trust ratings are distributed in equal intervals, the ith trust rating will be equal to: $(i-1)/(|T|-1)$ for $i = 1, 2, \ldots, |T|$. For example, if $|T| = 5$, then the set of trust ratings is $\{0, 0.25, 0.5, 0.75, 1\}$.

5.1.2 Internal Probability Distribution of an Agent's Trust

In our multi-agent platform, each agent is associated with an internal probability distribution of trust, which is only known to the agent itself. This allows modeling a rather specific and yet

[1] In our trust model, we assume that the set of T is nonzero.

not deterministic degree of trustworthiness in that agent which is subject to some uncertainty. The uncertainty is due to the variability in the degree of trustworthiness of the agent which is a consequent of having nonzero values for each element in the probability distribution. In this distribution, each trust rating τ is given a probability of occurrence. This means that at each iteration, the degree of trustworthiness of the agent may be driven from any nonzero trust rating value in the domain. However, the chance of occurrence of each trust rating is proportional to it's probability. In order to model a distribution, given its minimum, maximum, peak, degree of skewness and peakness, we use a form of beta distribution called modified Pert distribution [63]. It can be replaced by any distribution that provides the above mentioned parameters. Well known distributions, e.g., normal distribution, are not employed as they do not allow positive or negative skewness of the distribution. In a modified Pert distribution, the peak of the distribution, which is denoted by τ_a^{PEAK}, has the highest probability of occurrence. This means that while the predominant behavior of the agent is driven by τ_a^{PEAK} and the trust ratings next to it, there is a small probability that the agent does not follow its dominant behavior. Figure 3 demonstrates an example of the internal trust distribution of an agent. The closer the peak of the internal distribution to $\overline{\tau}$, the more trustworthy the agent is and vice-versa.

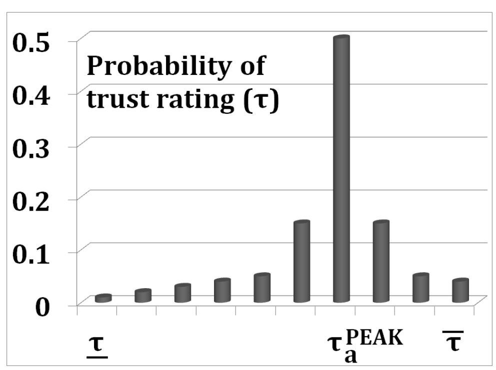


Figure 3: Internal Probability Distribution of Agent a

5.1.3 Interaction between Agents

When an agent rates another agent's degree of trustworthiness, its rating depends not only on that agent's degree of trustworthiness but also on the rater's personal point of view. In this thesis, we just model an agent's degree of trustworthiness. In each interaction, a trustor agent, say α, requests a service from a trustee agent, say β. Agent β should provide a service in correspondence with its degree of trustworthiness, which is represented by its internal trust distribution. On this purpose, a random value from the domain of T of agent β is generated by using its internal probability distribution of trust. The peak of the internal trust distribution, τ_a^{PEAK}, has the highest probability of selection while other trust ratings in T have a relatively smaller probability to be chosen. This will produce a mostly specific and yet not deterministic value. Agent β reports the generated value to α which α considers as the degree of trustworthiness of β in that interaction.[1]

5.1.4 Building Possibility Distribution of Trust

Upon completion of a number of interactions between a trustor agent, α, and a trustee agent, β, agent α can model the internal trust distribution of β, by usage of the values received from β throughout their interactions. We use possibility distributions in order to model α's trust with respect to the uncertainty associated with: (1) the variability in the degree of trustworthiness of β; and (2) the lack of adequate information on agent β's trust (ignorance) due to insufficient number of interactions among α and β. As described in Chapter 2 (Section 2.5), possibility distributions can represent the degree of possibility of each element in the domain, which in our model corresponds to trust rating $\tau \in T$. A possibility distribution is defined as: $\Pi : T \to [0,1]$ with $\max\limits_{\tau \in T} \Pi(\tau) = 1$. The measured possibility distribution represents the trust distribution of agent β as perceived and measured by agent α.

We apply the approach described in Section 4.3 to measure a possibility distribution from empirical data given the desired confidence level. In this approach, first simultaneous confidence intervals

[1] In our trust model, if two agents are assumed to have interactions, their number of interactions is nonzero.

for all trust ratings in the domain are measured by usage of the empirical data (which in our model are derived from interaction among agents). Then, the possibility of each trust rating τ considering the confidence intervals of all trust ratings in T is found.

5.1.5 Manipulation of the Possibility Distributions

An agent, say a^{S}, needs to acquire information about the degree of trustworthiness of agent a^{D} unknown to him. On this purpose, it acquires information from its advisors like a who have already interacted with a^{D}. Each agent $a \in A$ is not necessarily truthful for reasons of self-interest, therefore it may manipulate its possibility distribution of trust in a^{D} before reporting it to a^{S}. The degree of manipulation of information by an advisor a is based on its internal probability distribution of trust. More specifically, if the internal trust distributions of agents a and a' indicate that a's degree of trustworthiness is lower than a', then the reported possibility distribution of a is more prone to error than a'. Algorithms I and II are examples of the manipulation algorithms.

Manipulation Algorithm I

1: **for** $\tau \in T$ **do**

2: $\tau' \leftarrow$ random trust rating value from T, according to agent a's internal trust distribution

3: $\mathrm{error}_\tau = 1 - \tau'$

4: $\Pi_{a \rightarrow a^{\mathrm{D}}}(\tau) = \widehat{\Pi}_{a \rightarrow a^{\mathrm{D}}}(\tau) + \mathrm{error}_\tau$

5: **end for**

$\widehat{\Pi}_{a \rightarrow a^{\mathrm{D}}}$ is the possibility distribution of a's trust in a^{D} measured through their interactions and $\Pi_{a \rightarrow a^{\mathrm{D}}}$ is the manipulated possibility distributions.

In algorithm I, for each trust rating $\tau \in T$ a random trust value, τ', is generated following the internal trust distribution of a. For highly trustworthy agents, the randomly generated value τ' is close to $\overline{\tau}$ and the subsequent error (error_τ) is close to 0. Therefore, the manipulation of $\widehat{\Pi}_{a \to a^{\text{D}}}(\tau)$ is insignificant. On the other hand, for highly untrustworthy agents, the value of τ' is close to $\underline{\tau}$ and consequently the derived error, error_τ, is close to 1. In such a case, the possibility value $\widehat{\Pi}_{a \to a^{\text{D}}}(\tau)$ is considerably modified causing noticeable change in the original values.

After measuring the distribution $\Pi_{a \to a^{\text{D}}}$, it is normalized and then reported to a^{S}. We use the normalization described in Chapter 4, Section 4.1.1.

Manipulation Algorithm II

---

for $\tau \in T$ **do**

  $\tau' \leftarrow$ random trust rating value from T, according to agent a's internal trust distribution

  $\max\_\text{error}_\tau = 1 - \tau'$

  $\text{error}_\tau =$ random value in $[0, \max\_\text{error}_\tau]$

  $\Pi_{a \to a^{\text{D}}}(\tau) = \widehat{\Pi}_{a \to a^{\text{D}}}(\tau) + \text{error}_\tau$

end for

As for Algorithm I, in Algorithm II the distribution of $\Pi_{a \to a^{\text{D}}}$ is normalized before being reported to a^{S}. In Algorithm I, the trust rating τ_a^{PEAK} and the trust values next to it have a high probability of being selected. Therefore, the error added to $\widehat{\Pi}_{a \to a^{\text{D}}}$ may be neglected when the distribution is normalized. However, in Algorithm II, an additional random selection value is added which is selected uniformly from $[0, \max\_\text{error}_\tau]$ to curb the effect of normalization. More specifically, if an agent is highly untrustworthy, the random trust value τ' is close to $\underline{\tau}$ and thereupon the error value $\max\_\text{error}_\tau$ is close to 1. This causes the uniformly generated value from $[0, \max\_\text{error}_\tau]$ to be considerably random and unpredictable, which makes the derived possibility distribution highly erroneous after normalization. On the other hand, if an agent is highly trustworthy, the error value of $\max\_\text{error}_\tau$ is close to $\overline{\tau}$ and the random value generated from $[0, \max\_\text{error}_\tau]$ would be even smaller, making the error of the final possibility distribution insignificant. While incorporating some random processes, both algorithms manipulate the possibility distribution based on the agent's degree of trustworthiness causing the scale of manipulation of information by more trustworthy agents less than untrustworthy agents and vice-versa. However, the second algorithm acts more randomly. We design these algorithms to observe the extent of dependency of the derived results with respect to the manipulation algorithms employed. In other words, we want to figure out the influence of the manipulation algorithms on the accuracy of the target agent's estimated trust.

5.1.6 Game Scenario

In this thesis, we study a model arising in social networks where each agent a in a set $A = \{a_1, a_2, \ldots, a_n\}$ of n agents (agent a^{S}'s advisors) makes a number of interactions with the target agent a^{D}. In each interaction between an agent $a \in A$ and the target agent a^{D}, a trust rating $\tau \in T$ is given to a^{D} (as explained in Section 5.1.1). A number of interactions is carried out in the same style between every pair of connected agents $(a \in A, a^{\text{D}})$ (as explained in Section 5.1.3). When a number of interactions is completed between these pairs of agents, by usage of the empirical data derived throughout their interactions, a possibility distribution representing the trust of each agent $a \in A$ in a^{D} is measured (as described in Section 5.1.4). Each agent in A, in turn, measures an independent possibility distribution of trust through its own interactions with a^{D}. The measured possibility distribution indicates the trust of a in a^{D}. When a^{S} wants to evaluate the level of trustworthiness of a^{D}, (which is unknown to a), it acquires information from its advisors, the agents in A, on the possibility distributions of a^{D}'s trust. Agents in A are not necessarily truthful. Therefore, through usage of the manipulation algorithms (Section 5.1.5), each agent $a \in A$ manipulates its measured possibility distributions $\widehat{\Pi}_{a \to a^{\mathrm{D}}}(\tau)$, $(\forall a \in A)$ according to its degree of trustworthiness and reports the manipulated distributions to a^{S}. Agent a^{S} uses the reported distributions $\Pi_{a \to a^{\mathrm{D}}}(\tau)$ by each agent $a \in A$ in order to estimate the possibility distribution of a^{D}'s trust.

We study two problems in this thesis:

- In the first problem, the agents of the set A are unknown to a^{S} and their degree of trustworthiness is unknown to a^{S}. In this case, a^{S} cannot distinguish the reliability of the information received from each agent in A. This problem is discussed in Section 5.2.1.
- In the second problem, a^{S} makes a number of interactions with each agent $a \in A$. The interactions carried out here are the same as the interactions between every pair $(a \in A, a^{\mathrm{D}})$ as described above. After completing several interactions between every pair $(a^{\mathrm{S}}, a \in A)$, a possibility distribution representing trust of a^{S} in each agent $a \in A$ is measured out of the empirical data acquired in their interactions. The instructions described in Section 5.1.4 are employed for inferring such possibility distributions (the same as the possibility distributions built by the agents of A on a^{D}'s trust). The derived possibility distribution represents trust of a^{S} in $a \in A$. In this problem, a^{S} measures the trust distribution of a^{D} out of (1) the trust distributions reported by the agents of the set A on a^{D}'s trust, and (2) the degree of trustworthiness of the agents in the set A as measured by a^{S} through direct interactions. This problem is discussed in Section 5.2.2.

5.2 Estimating a Target Agent's Distribution of Trust

As described in the previous section, we first explore trust estimation of a^{D} when agents $a \in A$ are unknown to a^{S} (Section 5.2.1). Later, in Section 5.2.2, we measure trust distribution of a^{D} when the agents of set A are known to a^{S}.

5.2.1 Trust Estimation Through Unknown Agents

In this approach, we explore the estimation of a target agent's trust out of the distributions received from a set of unknown agents. The agents of set A are assumed to be unknown here. We employ the fusion rules described in Section 4.1.1 (Chapter 4) to aggregate the possibility distributions reported by the agents in A, representing their trust in a^{D}. Through usage of these fusion rules, the possibility distribution of a^{D}'s trust can be estimated. In fact, since the agents in A are unknown to a^{S}, their reported distributions are not treated differently based on their degree of trustworthiness. However, the fusion rules presented in Section 4.1.1 treat these distributions differently. The intersection rule only considers the pieces of information that all of the agents agree upon while the union rule takes into account all pieces of information received from all the agents in A. The trade-off rule, on the other hand, has a midway approach giving each reported distribution equal influence on the final distribution of a^{D}'s trust. We explore the accuracy of the results obtained through these fusion rules in the next chapter and observe the results generated from each rule. Therein, the results are evaluated in correspondence with: (1) the degree of the trustworthiness of the agents in set A (although it is unknown to a^{S}); (2) the number of interactions carried out between every pair of connected agents as shown in Figure 2; and (3) the number of agents in set A.

5.2.2 Trust Estimation Through Known Agents

In this approach, we consider the fusion rules that we reviewed in Section 4.1.2. These fusion rules consider the trust of the advisor agents while merging the possibility distributions received from them. Based on these fusion rules, we propose two different approaches. The first approach is based upon merging possibility distributions at different levels of a multi-agent network while the second approach employs directly the fusion rules of Section 4.1.2. More specifically:

- In the first approach, we propose a new methodology for merging the following two sets of possibility distributions: (1) The possibility distributions of a^{S}'s trust in agents of A; and (2) the possibility distributions representing trust of A's agents in the target agent a^{D}. These two sets of possibility distributions represent trust distributions of different agents. The first set represents trust in A's agents (as measured by a^{S}) while the second set represents the trust in a^{D} (as measured and reported by the agents of A).

- In the second approach, we measure a single value of trust representing a^{S}'s trust in each agent a of A. Then, we use the fusion rules presented in Section 4.1.2 for estimating the possibility distribution of a^{D}'s trust.

Note that except the fusion rules which are developed by other researchers (as described in Chapter 4), the rest of the techniques, methodologies and equations developed in this section (for both approaches) are contributions of this thesis.

A Trust Estimation Through Merging Successive Possibility Distributions

In this section, we propose a methodology for merging the possibility distribution of $\Pi_{a^{\mathrm{S}}\to a}$ (representing the trust of a^{S} in its advisors) with the possibility distribution of $\Pi_{a\to a^{\mathrm{D}}}$ (representing the trust of A's agents in a^{D}). These two possibility distributions are associated with the trust of entities at successive levels in a multi-agent system and hence giving it such a name. The former set of distributions represent the trust of agents directly connected to a^{S} while the latter distributions represents the trust of a^{D} which is indirectly connected to a^{S} by its advisors. We first discuss possible ways of merging two possibility distributions at such different levels of a multi-agent system, and then, we explore possible ways of estimating the possibility distribution of a^{D}.

- **How Successive Possibility Distributions Can Be Merged?** In order to perform such a merging, we need to know how the distribution $\Pi_{a\to a^{\mathrm{D}}}$ changes, depending on the characteristics of the possibility distribution $\Pi_{a^{\mathrm{S}}\to a}$. We distinguish the following cases for a proper merging of the successive possibility distributions.

Particular Case: Consider a scenario where $\exists! \tau'$, $\underline{\tau} \leq \tau' \leq \overline{\tau}$ and $\Pi_{a^{\mathrm{S}}\to a} = \begin{cases} 1, \ \tau = \tau' \\ 0, \text{ otherwise} \end{cases}$, i.e., only one trust value is possible in domain T and the possibility of all other trust values is equal to 0. Then, trust of a^{S} in agent a can be associated with a single value $\tau_{a^{\mathrm{S}} \to a} = \tau'$ and the fusion rules described in Section 4.1.2 can be applied to get the possibility distribution $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$.

Considering the TM fusion rules (DP and Yager), for each agent a, first the possibility distribution $\Pi_{a \to a^{\mathrm{D}}}$ is transformed based on the trust value $\tau_{a^{\mathrm{S}} \to a} = \tau'$ as discussed in Section 4.1.2. Then, an intersection of the transformed possibility distribution is taken and the resulting distribution is normalized to get the possibility distribution $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$.

General Case: For each agent a, we have a subset of trust ratings, which we refer to as T_a^{Pos}, such that:

$$1)\quad T_a^{\mathrm{Pos}} \subset T, \tag{36}$$

$$2)\quad \text{If } \Pi_{a^{\mathrm{S}} \to a}(\tau) > 0, \text{ then } \tau \in T_a^{\mathrm{Pos}}, \tag{37}$$

$$3)\quad \text{If } \Pi_{a^{\mathrm{S}} \to a}(\tau) = 0, \text{ then } \tau \in \{T - T_a^{\mathrm{Pos}}\}. \tag{38}$$

Each trust rating value in T_a^{Pos} is possible. This means that the trust of a^{S} in a can possibly take any value in T_a^{Pos} and consequently any trust rating $\tau \in T_a^{\mathrm{Pos}}$ can be possibly associated with $\tau_{a^{\mathrm{S}} \to a}$. However, the higher the value $\Pi_{a^{\mathrm{S}} \to a}(\tau)$, the higher the likelihood of occurrence of trust rating $\tau \in T_a^{\mathrm{Pos}}$. We use the possibility distribution $\Pi_{a^{\mathrm{S}} \to a}$ to get the relative chance of happening of each trust rating in T_a^{Pos}. In this approach, we give each trust rating τ, a Possibility Weight (PW) equal to:

$$PW(\tau) = \Pi_{a^{\mathrm{S}} \to a}(\tau) / \sum_{\tau' \in T_a^{\mathrm{Pos}}} \Pi_{a^{\mathrm{S}} \to a(\tau')}. \tag{39}$$

Higher value of $PW(\tau)$ implies more occurrences chance of the τ value. Hence, any trust rating $\tau \in T_a^{\mathrm{Pos}}$ is possible to be observed with a weight of $PW(\tau)$ and merged with $\Pi_{a \to a^{\mathrm{D}}}$ using one of the fusion rules.

Considering the General Case, there are a total of $|A| = n$ agents and each agent a has a total of $|T_a^{\mathrm{Pos}}|$ possible trust values. For a possible estimation of $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$, we need to choose one trust rating of $\tau \in T_a^{\text{Pos}}$ for each agent $a \in A$. Having $|A| = n$ agents and a total of $|T_a^{\text{Pos}}|$ possible trust ratings for each agent $a \in A$, we can generate a total of $\prod_{a \in A} |T_a^{\text{Pos}}| = K$ possible ways of getting the final possibility of $\Pi_{a^{\text{S}} \to a^{\text{D}}}$. This means that any distribution out of K distributions is possible. However, they are not equally likely to happen. If agent a^{S} chooses trust rating $\tau_1 \in T_{a_1}^{\text{Pos}}$ from agent a_1, $\tau_2 \in T_{a_2}^{\text{Pos}}$ from agent a_2, and finally τ_n for agent $a_n \in T_{a_n}^{\text{Pos}}$, then the possibility distribution of $\Pi_{a^{\text{S}} \to a^{\text{D}}}$ derived from these trust ratings has an Occurrence Probability(OP) of $\prod_{i=1}^{n} PW(\tau_i)$.

For every agent a, we have: $\sum_{\tau \in T_a^{\text{Pos}}} PW(\tau) = 1$, then considering all agents we have:

$$\sum_{\tau_1 \in T_{a_1}^{\text{Pos}}} \sum_{\tau_2 \in T_{a_2}^{\text{Pos}}} \ldots \sum_{\tau_n \in T_{a_n}^{\text{Pos}}} PW(\tau_1) \times PW(\tau_2) \times \ldots \times PW(\tau_n) = 1. \quad (40)$$

As can be observed above, the PW is normalized in such a way that, for every set of trust ratings $\{\tau_1, \tau_2, \ldots, \tau_n\}$ (where $\tau_i \in T_{a_i}^{\text{Pos}}$), the corresponding OP of this set can be measured through multiplication of PW of the trust ratings in the set, namely $PW(\tau_1) \times PW(\tau_2) \times \ldots \times PW(\tau_n)$.

Trust Event Coefficient: The $PW(\tau)$ value shows the relative possibility of τ compared to other values in T of an agent a. However, we still need to compare the possibility of a given trust rating τ, for an agent a, with other agents in A. If the possibility weights of two agents are equal, say 0.2 and 0.8 for trust ratings $\underline{\tau}$ and $\overline{\tau}$, and the number of interactions with the first agent is much higher than for the second agent, we need to give more credit to the first agent's reported distribution of $\Pi_{a \to a^{\text{D}}}$. However, the current model described above is unable of doing so. Therefore, we propose to use a Trust Event Coefficient for each trust value τ, denoted by $\text{TEC}(\tau)$, in order to consider the number of interactions, which satisfies:

$$1)\quad \text{If } m_\tau = 0,\ \ \text{TEC}(\tau) = 0. \tag{41}$$

$$2)\quad \text{If } \Pi_{a^{\mathrm{s}}\to a}(\tau) = 0,\ \ \text{TEC}(\tau) = 0. \tag{42}$$

$$3)\quad \text{If } m_\tau \geq m_{\tau'},\ \ \text{TEC}(\tau) \geq \text{TEC}(\tau'). \tag{43}$$

$$4)\quad \text{If } m_\tau = m_{\tau'} \text{ and } \Pi_{a^{\mathrm{s}}\to a}(\tau) \geq \Pi_{a^{\mathrm{s}}\to a}(\tau'), \text{TEC}(\tau) \geq \text{TEC}(\tau'). \tag{44}$$

where $\tau \in T_a^{\mathrm{Pos}}$, m_τ is the frequency of the trust rating τ in the interactions between agents a^{s} and a. Considering conditions 1) and 2) above, if the frequency of the trust rating τ or its corresponding possibility value is 0, then TEC is also zero. Condition 3) increases the value of TEC by increasing the frequency of the trust rating τ. As observed in Condition 4), if the frequency of two trust ratings, τ and τ' are equal, then the trust rating with higher possibility is given the priority. Comparing the frequency (in the number of interactions) and the possibility value $\Pi_{a^{\mathrm{s}}\to a}$, the priority is given first to the frequency, and then, to the possibility value $\Pi_{a^{\mathrm{s}}\to a}$ in order to avoid giving preference to the possibility values driven out of few interactions. The following formula is an example of a TEC function, which satisfies the above conditions.

$$\text{TEC}(\tau) = \begin{cases} 0,\ m_\tau = 0 \text{ or } \Pi_{a^{\mathrm{s}}\to a} = 0 \\ [1/(\gamma \times m_\tau)]^{(1/m_\tau)} + \dfrac{\Pi_{a^{\mathrm{s}}\to a}}{\chi}, \text{ otherwise} \end{cases} \tag{45}$$

where $\gamma > 1$ is the discount factor and $\chi \gg 1$. Higher values of γ impede the convergence of TEC(τ) to one and vice-versa. χ, which is a large value, insures that the influence of $\Pi_{a^{\mathrm{s}}\to a}$ on TEC(τ) remains trivial and is noticeable only when the number of interactions are equal. In (45), as m_τ grows, TEC(τ) converges to one. TEC(τ) can be utilized as a coefficient for trust rating τ when comparing different agents. Note that the General Case mentioned above gives the guidelines for merging successive possibility distributions and TEC feature is only used as an attribute when the number of interactions should be considered and can be ignored otherwise.

- **Possibility Distribution of a^S's Trust in a^D:** We propose two approaches for deriving the final possibility distribution $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ considering different available possible choices.

The first approach is to consider all K possibility distributions $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ and take their weighted mean by giving each $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ a weight equal to its Occurrence Probability (OP), measured by multiplying the possibility weights of the trust values, $PW(\tau_i)$, that are used to build $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$.

In the second approach, we only consider the trust ratings, $\tau \in T$ such that $\Pi_{a^{\mathrm{S}} \to a} = 1$. In other words, we only consider the trust ratings that have the highest weight of PW in the T_a^{Pos} set. Consequently, the $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ distributions derived from these trust values have the highest OP value which makes them the most expected distributions. We denote by μ_a the number of trust ratings, $\tau \in T_a^{\mathrm{Pos}}$ that satisfy $\Pi_{a^{\mathrm{S}} \to a} = 1$ for agent a. In this approach, we only select the trust ratings in μ_a for each agent a in A and build the possibility distributions $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ out of those trust ratings. After building $M = \prod\limits_{a \in A} \mu_a$ different possibility distributions $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$, we compute their average, since all of them have equal OP weight.

Proposition 1 *In both approaches, the conditions of the general case described in the previous section are satisfied.*

Proof: Proof can by simply done by replacing values.

Due to the computational burden of the first approach (which requires building K distributions of $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$), we used the second one in our experiments as it only requires building M distributions. The second approach has been simulated on a cluster using parallel programming in order to speed up the running time.

To conclude this section, we would like to comment on the motivation behind using possibility distribution rather than probability distributions. Indeed, if probability distributions were used instead of possibility distributions, a confidence interval should be considered in place of the single value of trust for each τ in T. Consequently, for representing the probability distribution of a^{S}'s trust in each agent $a \in A$, a confidence interval should be measured for each $\tau \in T$ to consider uncertainty. The same representation should be used for each agent $a \in A$'s trust in a^{D}. Now, in order to estimate the probability distribution of a^{D}'s trust with respect to its uncertainty, we need to find some tools for merging the confidence intervals of the probability distributions of a^{S}'s trust in A with the A's trust in a^{D}. To the best of our knowledge, only the works of [64, 65] address this issue. In [64], the frequency of each element in the domain, which is equivalent to the number of observances of each τ value in the interactions between agents a and a^{D}, is reported by agents in A to a^{S} and then, the probability intervals on the trust of a^{D} is built. The work [65] measures the confidence intervals of a^{D}'s trust out of several confidence intervals provided by agents in A. In both proposals, the manipulation of information by the agents in A is not considered and for building the confidence intervals of a^{D}, the trust of a^{S} in A is neglected. We employed possibility theory as it is capable of addressing both types of uncertainty (variability and ignorance). In addition, as mentioned in [32], possibility theory is the simplest theory for addressing incomplete information. Moreover, it offers flexible and straightforward tools for our trust model.

B Trust Estimation Through Single Trust Value of Advisor Agents

In this section, we measure a single value of trust, representing the trust of a^{S} in agent $a \in A$. Then, this value is considered as a trust weight of the possibility distributions $\Pi_{a \to a^{\text{D}}}$ (representing the trust of a in the target agent a^{D}). Finally, the fusion rules discussed in Section 4.1.2 are applied on the possibility distributions $\Pi_{a \to a^{\text{D}}}$, considering the single value of trust, to derive the possibility distribution of a^{S}'s trust in a^{D}, namely $\Pi_{a^{\text{S}} \to a^{\text{D}}}$. Here, we intend to use the traditional fusion rules (Section 4.1.2) more directly and compare the final trust distribution of $\Pi_{a^{\text{S}} \to a^{\text{D}}}$ with the distribution measured in Section 5.2.2.

- **Measuring Single Trust Values:** Here, we measure a single value of trust, $\tau_{a^{\text{S}} \to a}$, representing a^{S}'s trust in a. To this end, the empirical values obtained through interactions of a^{S}, as trustor, with a, as trustee, can be used. We introduce a weight of a^{S}'s trust in a measured from empirical interactions between these agents computed as follows:

$$E_{a^{\mathrm{S}}\to a} = \sum_{\tau \in T} \frac{n_\tau}{N} \times \tau, \qquad N > 0 \tag{46}$$

where n_τ is the frequency of the trust rating τ in the interactions between agents a^{S} and a such that $\sum_{\tau \in T} n_\tau = N$. The shortcoming of (46) is that it does not consider the number of interactions carried out between the two agents. In (46), if the number of observations of each trust rating for agent a is m times more than for agent a', (a and $a' \in A$), equation (46) cannot differentiate the E measured for agents a and a'. In order to consider the number of interactions, we propose an Interaction-Coefficient (IC) for the value of E such that the trust value of a^{S} in a is defined as $\tau_{a^{\mathrm{S}}\to a} = IC_{a^{\mathrm{S}}\to a} \times E_{a^{\mathrm{S}}\to a}$. Any formula measuring IC should satisfy the following conditions:

C1. If the number of the interactions N is low, the impact of $E_{a^{\mathrm{S}}\to a}$, on the value of $\tau_{a^{\mathrm{S}}\to a}$ should remain low.

C2. If $N \geq N'$, then $\tau_{a^{\mathrm{S}}\to a} \geq \tau_{a^{\mathrm{S}}\to a'}$.

C3. If N is high enough, $E_{a^{\mathrm{S}}\to a}$ should almost be equal to $\tau_{a^{\mathrm{S}}\to a}$.

Condition C1 avoids having high trust values of $\tau_{a^{\mathrm{S}}\to a}$ over few interactions. Comparing two agents a and a', if a's total number of interaction (N) is greater than a''s number of interactions (N'), then trust of a^{S} in a should be greater than a'. In condition C3, if the number of interactions is above a certain threshold, the value of IC would be ineffective on $\tau_{a^{\mathrm{S}}\to a}$. Therefore, only $E_{a^{\mathrm{S}}\to a}$ derives $\tau_{a^{\mathrm{S}}\to a}$. The following is an example of an IC formula which satisfies conditions C1 to C3:

$$IC_{a^{\mathrm{S}}\to a} = \left(\frac{1}{\gamma \times N}\right)^{\frac{1}{N}} \qquad \gamma > 1, N > 0 \tag{47}$$

where γ is a discount factor. Higher values of γ decreases the growth of IC.

- **Trust Estimation of Target Agent a^D:** The fusion rules described in Section 4.1.2 can be applied to the possibility distributions $\Pi_{a \to a^{\mathrm{D}}}$, $\forall a \in A$, by using the trust values $\tau_{a^{\mathrm{S}}\to a}$, $\forall a \in A$

measured in this section. Thereupon, an estimated possibility distribution of a^{S}'s trust in a^{D}, which is represented as $\Pi_{a^{\mathrm{S}} \rightarrow a^{\mathrm{D}}}$, $\forall \tau \in T$ can be measured. Despite the fact that each agent a measures the distribution $\Pi_{a \rightarrow a^{\mathrm{D}}}$ independent of other agents in A, we assume that a^{S} does not know whether the agents in A have exchanged information or not. Consequently, a^{S} cannot prefer any specific fusion rule in advance. In Chapter 6, the final possibility distribution $\Pi_{a^{\mathrm{S}} \rightarrow a^{\mathrm{D}}}$ measured here is compared with the distribution measured through merging successive distributions. This, in turn, would indicate which approach is more accurate.

The tools explained in Chapter 4 have made the possibility theory more adaptable with our model. However, we have developed additional tools, which were introduced in this chapter, in order to completely use the possibility theory for our trust model. As mentioned before, excluding the technical tools introduced in chapter 4 (which are developed by other researchers), the rest of materials in this chapter, (e.g., equations, formulas, methodologies and metrics) are all contributions of this thesis.

5.3 Measuring Certainty

In this section, we propose a new metric in order to figure out the extent to which a^{S} can rely on the information provided by the set A of agents. On this purpose, we measure a^{S}'s certainty over the possibility distributions $\Pi_{a \rightarrow a^{\mathrm{D}}}, \forall a \in A$ reported by the advisor agents on agent a^{D}'s trust. We call it the certainty metric and it measures the confidence that a^{S} has on the information acquired from its advisors with respect to their degree of trustworthiness. We consider two features to define the certainty metric:

(i) **Consistency:** Given a fixed number of agents in A, and a fixed set of trust values representing trust of a^{S} in the agents of A ($\tau_{a^{\mathrm{S}} \rightarrow a}$, $\forall a \in A$), the more the possibility distributions of $\Pi_{a \rightarrow a^{\mathrm{D}}}$ reported by the agents in A become similar, especially the ones reported by highly trusted agents, the higher the certainty over the distributions provided by members of A.

(ii) **Trust:** Given a fixed set of possibility distributions reported by the members of A to a^{S}

($\Pi_{a \to a^{\text{D}}}$, $\forall a \in A$) and a fixed number of agents in A, increase of a^{S}'s trust in an individual agent $a \in A$ increments the certainty, if the reported distribution $\Pi_{a \to a^{\text{D}}}$ is similar with other agents, especially the highly trusted ones. On the other hand, increase of a^{S}'s trust in an agent a decreases the certainty if its reported distribution is dissimilar to other agents (especially the highly trusted ones). The trust feature should also increase the certainty in the cases where, for a fixed number of agents, and a fixed set of reported distributions ($\Pi_{a \to a^{\text{D}}}$, for $a \in A$) the trust of a^{S} in all of the agents in the set A or the majority of them increases. This is due to the fact that the trust in the agents providing the same information is enhanced. For the evaluations made in this section, we use the single trust value measured in Section 5.2.2. This is due to the fact that in the current certainty model the trust of a^{S} in an agent $a \in A$ is considered as a single value, indicated by $\tau_{a^{\text{S}} \to a}$, instead of a distribution. Developing a certainty metric while considering the possibility distribution $\Pi_{a^{\text{S}} \to a}$ instead of a single value $\tau_{a^{\text{S}} \to a}$ is part of our future works. The certainty metric and all of the components developed in this section are also the contributions of this thesis.

5.3.1 Measuring Consistency

Consistency is expressed through measuring the degree of similarity among the reported possibility distributions of the advisors agents. On this purpose, we first measure the extent of dissimilarity among the possibility distributions provided by the agents in A, considering the trust of a^{S} in each agent $a \in A$, which we refer to as inconsistency. Then, we propose a normalization for the measured dissimilarity, which we denote by maximum possible inconsistency. Finally, we measure the consistency out of these two values, which is a normalized value and lies in $[0, 1]$.

A Inconsistency in Possibility Distributions

Given a fixed number of agents in A and a fixed set of trust values ($\tau_{a^{\text{S}} \to a}, \forall a \in A$) representing the trust of a^{S} in each agent a ($\forall a \in A$) , the more the reported possibility distributions $\Pi_{a \to a^{\text{D}}}$ are similar, the higher the consistency within the information provided by the agents in A and therefore the higher is the certainty. The certainty is especially higher when the reports of the highly trustworthy agents are similar as a^{S} relies more on their reports. In order to measure the inconsistency in the distributions provided by the agents of A, we measure the weighted average absolute deviation of every pair of possibility distributions $\Pi_{a\to a^{\text{D}}}(\tau)$ and $\Pi_{a'\to a^{\text{D}}}(\tau)$ reported by a and a' ($\forall\ a,\ a' \in A,\ a \neq a'$). The Inconsistency INC is defined as follows:

$$\text{INC} = \frac{1}{|T|} \sum_{(a,a')\in A^2, a\neq a'} \omega_{a^{\text{S}}\to a} \times \omega_{a^{\text{S}}\to a'} \times \sum_{\tau\in T} |\Pi_{a\to a^{\text{D}}}(\tau) - \Pi_{a'\to a^{\text{D}}}(\tau)| \quad (48)$$

where $\omega_{a^{\text{S}}\to a'} = (\tau_{a^{\text{S}}\to a'} / \sum_{a\in A} \tau_{a^{\text{S}}\to a})$ and $|T| > 0$. In (48), for every pair of distributions, $\Pi_{a\to a^{\text{D}}}(\tau)$ and $\Pi_{a'\to a^{\text{D}}}(\tau)$, we compare the difference in the possibility value over all trust ratings in the domain. The weight $\omega_{a^{\text{S}}\to a'}$ is equal to the ratio of the agent a''s trust to the trust of all agents in A. Usage of the weights in (48) causes the difference (absolute value) among the reports of more trusted agents to be further penalized as their reports are considered more important by agent a^{S}.

For illustration, consider an example with two highly trusted and two slightly trusted agents in A. If the reports of the two trusted agents are similar, the certainty is much higher compared to the case where the reports of these two agents are dissimilar. The influence of two slightly trusted agents would be trivial on the certainty as their reported distributions are not highly trusted.

B Maximum Possible Inconsistency

In order to normalize the INC value defined in Section 5.3.1.A, we need to evaluate the maximum possible inconsistency such that it provides an upper bound for the inconsistency value. The maximum inconsistency, denoted by $\text{INC}_{\max}$, arises from the situation where a^{S} has equal trust in the agents of A ($\tau_{a^{\text{S}}\to a} = \tau_{a^{\text{S}}\to a'},\ \forall a, a' \in A$) and these agents provide the most inconsistent possibility distributions, i.e., agents of A are divided into two subsets A_1 and A_2, such that the difference between the possibility distributions of the agents of A_1 and those of the agents of A_2 takes its largest possible value. It can be shown that this occurs when the possibility distributions reported by each subset are identical while the dissimilarity between the distributions reported by each agent $a_1 \in A_1$ and each agent $a_2 \in A_2$, is at its largest possible value, i.e., $|T|$. It means that, for each trust rating $\tau \in T$, we have: $|\Pi_{a_1 \to a^{\mathrm{D}}}(\tau) - \Pi_{a_2 \to a^{\mathrm{D}}}(\tau)| = 1$, for all $a_1 \in A_1$, and $a_2 \in A_2$ such that $\sum_{\tau \in T} |\Pi_{a_1 \to a^{\mathrm{D}}}(\tau) - \Pi_{a_2 \to a^{\mathrm{D}}}(\tau)| = |T|$. This is indeed the case when $\Pi_{a_1 \to a^{\mathrm{D}}}(\tau)$ is at one extreme end of the possibility value (either 0 or 1) and $\Pi_{a_2 \to a^{\mathrm{D}}}(\tau)$ is at the other extreme end ($1 - \Pi_{a_1 \to a^{\mathrm{D}}}(\tau)$ which is either 0 or 1).

In order to have the possibility distributions of $\Pi_{a_1 \to a^{\mathrm{D}}}(\tau)$ and $\Pi_{a_2 \to a^{\mathrm{D}}}(\tau)$ normalized, at least the possibility of one trust rating in T must be equal to 1, therefore $\sum_{\tau \in T} \Pi_{a_1 \to a^{\mathrm{D}}}(\tau)$ cannot be equal to 0 or $|T|$. For estimating $\textsc{inc}_{\max}$, we distinguish two cases:

Case 1. If $|A|$ is even, both A_1 and A_2 have the same cardinality, equal to $\frac{|T|}{2}$. Using the $\textsc{inc}$ expression of (48), we get: $\textsc{inc}_{\max} = 1/4$, where $\omega_{a^{\mathrm{S}} \to a} = \omega_{a^{\mathrm{S}} \to a'} = 1/n$.

Case 2. If $|A|$ is odd then one subset has $\frac{|A|+1}{2}$ members and the other subset has $\frac{|T|-1}{2}$ members. By replacing values in formula (48) we obtain: $\textsc{inc}_{\max} = 1/4 - \frac{1}{|A|^2}$. The normalized inconsistency in the distributions reported by the set A of agents is therefore equal to $\frac{\textsc{inc}}{\textsc{inc}_{\max}}$ and the consistency is equal to $1 - \frac{\textsc{inc}}{\textsc{inc}_{\max}}$.

5.3.2 Measuring Trust

Through the inconsistency formula proposed in Section 5.3.1.A , if we have a fixed number of agents in A and a fixed set of distributions $\Pi_{a \to a^{\mathrm{D}}}$, for all $a \in A$, increase or decrease of a^{S}'s trust in an individual agent a is reflected in the weight $\omega_{a^{\mathrm{S}} \to a}$, which influences the measured consistency value. Specifically, increasing the trust of an agent whose reported distribution is in conformance with the distributions reported by the mostly trusted agents would increase consistency and vice-versa. However, in a scenario where we have a fixed set of agents in A and a fixed set of distributions $\Pi_{a \to a^{\mathrm{D}}}$, if we, for example, double the trust values $\tau_{a^{\mathrm{S}} \to a} (\forall a \in A)$, our certainty in the information received by the set A should increase as we can rely more on those agents, but the consistency value measured in Section 5.3.1 does not distinguish such cases. In order to consider the overall trust of a^{S} in the set A of agents, we introduce a trust feature, T_f, which should satisfy:

$$(1)\ T_f(A) = 0, \qquad \text{if } \tau_{a^{\mathrm{s}} \to a} = 0, \quad \forall a \in A. \tag{49}$$

$$(2)\ T_f(A) = 1, \qquad \text{if } \tau_{a^{\mathrm{s}} \to a} = 1, \quad \forall a \in A. \tag{50}$$

$$(3)\ T_f(A) > T_f(A'), \qquad \text{if } \sum_{a \in A} (\tau_{a^{\mathrm{s}} \to a}) > \sum_{a' \in A'} (\tau_{a^{\mathrm{s}} \to a'}). \tag{51}$$

Conditions (1) and (2) satisfy the existence of T_f in $[0, 1]$ and condition (3) increases T_f by increasing a^{S}'s trust in the set A of agents. Set A' has the same number of agents as A while the trust of a^{s} in the agents of set A $(\tau_{a^{\mathrm{s}} \to a}, \forall a \in A)$ may differ from its trust in the agents of set A' $(\tau_{a^{\mathrm{s}} \to a'}, \forall a' \in A')$. The following function is a possible candidate for the trust feature:

$$T_f(A) = \left[\frac{1}{n} \sum_{a \in A} (t_{a^{\mathrm{s}} \to a}) \right]^k \tag{52}$$

Higher values of k slow down the growth of $T_f(A)$.

5.3.3 Certainty Function

The certainty over the information provided by the agents in A, is equal to: $C = (1 - \frac{\text{INC}}{\text{INC}_{\max}} \times \delta) \times T_f(A),\ 0 < \delta \leq 1$, where δ is the inconsistency coefficient. Therein, the consistency and trust features are combined together in such a way that the certainty function considers the issues mentioned in Sections 5.3.1 and 5.3.2. The inconsistency coefficient moderates the influence of inconsistency on the certainty. If $\delta = 1$ and $\text{INC} = \text{INC}_{\max}$, the certainty would be equal to zero no matter how many agents are supporting the most inconsistent situation. In order to make the certainty function less restrictive, δ should be less than one. Having set δ to such a value, even in the most inconsistent situation, the increase in the trust of the agents increases the certainty. We put our certainty metric to experiments to observe its behavior.

A Modifying Inconsistency with Fixed Trust

Let us assume the set of $\tau_{a^{\text{S}}\to a}$, for all $a \in A$, representing trust of a^{S} in the set A of agents, is fixed. We want to modify the possibility distributions $\Pi_{a\to a^{\text{D}}}$, $\forall a \in A$ to observe its effect on the certainty value. In order to only observe the effect of possibility distributions on certainty, we consider a scenario where we have four agents $A = \{a_1, a_2, a_3, a_4\}$ with an identical trust value $\tau_{a^{\text{S}}\to a} = 0.5$. Otherwise, if the trust values are not identical, the reported values of more trustworthy agents would have higher influence on the certainty metric. We consider the trust values equal in order to just observe the influence of the reported possibility values on the certainty metric. Without loss of generality, we assume $|T| = 2$ and $\Pi_{a\to a^{\text{D}}}(\overline{\tau}) = 1$, for all $a \in A$ under the assumption that the possibility distributions are normalized. We only modify the possibility value of $\Pi_{a\to a^{\text{D}}}(\underline{\tau})$ and measure the certainty for the changes made only on $\underline{\tau}$. We start with $\Pi_{a\to a^{\text{D}}}(\underline{\tau}) = 0, \forall a \in A$ and increase the value of $\Pi_{a_1\to a^{\text{D}}}(\underline{\tau})$ from 0 to 1 (0.25 each time), keeping the value of $\Pi_{a\to a^{\text{D}}}(\underline{\tau})$ for the other three agents fixed (equal to zero). Then, we keep increasing this value for agent a_2 in the same trend while $\Pi_{a_1\to a^{\text{D}}}(\underline{\tau}) = 1$ and $\Pi_{a_3\to a^{\text{D}}}(\underline{\tau}) = \Pi_{a_4\to a^{\text{D}}}(\underline{\tau}) = 0$. Later, we do the same for a_3 and a_4 increasing one at a time.

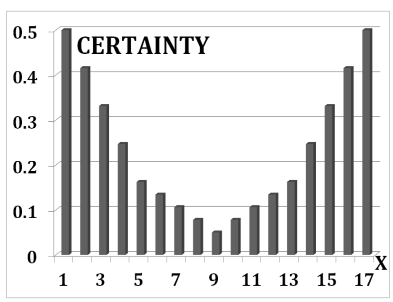


Figure 4: Modifying Inconsistency

X Axis	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
# $\Pi_{a_1\to a^{\text{D}}}(\underline{\tau})$	0	0.25	0.5	0.75	1	1	1	1	1	1	1	1	1	1	1	1	1
# $\Pi_{a_2\to a^{\text{D}}}(\underline{\tau})$	0	0	0	0	0	0.25	0.5	0.75	1	1	1	1	1	1	1	1	1
# $\Pi_{a_3\to a^{\text{D}}}(\underline{\tau})$	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1	1	1	1	1
# $\Pi_{a_4\to a^{\text{D}}}(\underline{\tau})$	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1

Table 2: The reported values of agents $a_i,\ i \in \{1, 2, 3, 4\}$ corresponding to the x values of Figure 4

Figure 4 demonstrates the results of this experiment. Table 2 demonstrates the reported possibility value of $\Pi_{a_i \to a^{\mathrm{D}}}(\underline{\tau})$ by each agent a_i, $i \in \{1, 2, 3, 4\}$ for x values in Figure 4. The certainty value shown in Figure 4 demonstrates a steady decrease from $x = 1$ to $x = 5$ (where $\Pi_{a_1 \to a^{\mathrm{D}}}(\underline{\tau})$ has increased from 0 to 1) and from $x = 6$ to $x = 9$ (where $\Pi_{a_2 \to a^{\mathrm{D}}}(\underline{\tau})$ has changed in the same trend). This is due to the fact that the reported values of a_1 and a_2 have deviated away from other two agents such that it makes a^{S} less certain about the true distribution of a^{D}. From $x = 1$ to $x = 9$, the more the reported distributions of a_1 and a_2 deviate away from the reported values of a_3 and a_4, the less certain a^{S} becomes about the true trust distribution of agent a^{D}. At $x = 9$ we have the lowest certainty as the agents are completely divided over the value $\Pi_{a \to a^{\mathrm{D}}}(\underline{\tau})$. from $x = 9$ to $x = 13$, where the report of a_3 converges towards the reports of a_1 and a_2 the certainty increases as the agents' reports are more consistent. The same trend happens for $x = 14$ to $x = 17$ when a_4's report converges towards other agents. at $x = 0$ and $x = 9$ we have the highest certainty values as the reported possibility values of all four agents are identical.

B Modifying Trust with Fixed Inconsistency

We consider a scenario where we keep the reported distributions $\Pi_{a \to a^{\mathrm{D}}}, \forall a \in A$ intact and we modify the trust of a^{S} in the agents of A to observe its influence on the certainty value. As in Section 5.3.3.A, we assume $|T| = 2$ and we consider $\Pi_{a \to a^{\mathrm{D}}}(\overline{\tau}) = 1, \forall a \in A$ in order to have the distributions normalized and just observe the certainty metric as measured over the trust rating $\underline{\tau}$. We consider a set of four agents, however, we associate $\Pi_{a \to a^{\mathrm{D}}}(\underline{\tau})$ to 1 for agents a_1 and a_2 and to 0 for agents a_3 and a_4. We intentionally set such values in order to observe the effect of trust modification in a case where the four agents have complete disagreement over the value of $\Pi_{a \to a^{\mathrm{D}}}(\underline{\tau})$. In such settings, we increase the trust of the agents in the same trend that we increased the possibility values of the agents in Section 5.3.3.A (one agent at a time). We start by $\tau_{a^{\mathrm{S}} \to a} = \epsilon$ (≈ 0), $\forall a \in A$ and increase the trust of a_1 by 0.25 each time while keeping the trust of other agents unchanged until $\tau_{a^{\mathrm{S}} \to a_1} = 1$. Then, we increment the trust of a_2 from $\tau_{a^{\mathrm{S}} \to a_2} = \epsilon$ to $\tau_{a^{\mathrm{S}} \to a_2} = 1$ while keeping the trust of the other three agents fixed ($\tau_{a^{\mathrm{S}} \to a_1} = 1$, $\tau_{a^{\mathrm{S}} \to a_3} = \tau_{a^{\mathrm{S}} \to a_4} = \epsilon$). Finally, we increase the trust of the agents a_3 and a_4 in the same trend, one at a time, keeping the trust values of the other three agents unchanged. Figure 5 illustrates the measured certainty values over these trust changes. Table 3 shows the trust values of the agents a_1 to a_4 for each x value of Figure 5.

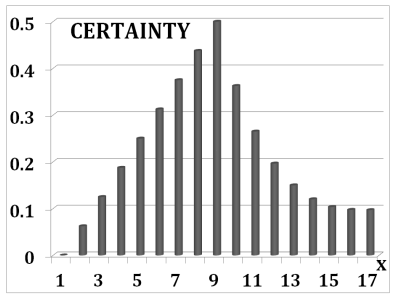


Figure 5: Modifying Trust

X Axis	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
# $\tau_{a^{\mathrm{S}}\to a_1}$	0	0.25	0.5	0.75	1	1	1	1	1	1	1	1	1	1	1	1	1
# $\tau_{a^{\mathrm{S}}\to a_2}$	0	0	0	0	0	0.25	0.5	0.75	1	1	1	1	1	1	1	1	1
# $\tau_{a^{\mathrm{S}}\to a_3}$	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1	1	1	1	1
# $\tau_{a^{\mathrm{S}}\to a_4}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0.5	0.75	1

Table 3: The trust values of $\tau_{a^{\mathrm{S}}\to a_i},\ i \in \{1, 2, 3, 4\}$ corresponding to the x values of Figure 5

Figure 5 illustrates that, from $x = 1$ to $x = 5$ as the trust value $\tau_{a^{\mathrm{S}}\to a_1}$ increases, a^{S} relies more on the reports of a_1 compared to the other agents, whose trust values of $\tau_{a^{\mathrm{S}}\to a},\ a \in \{a_2, a_3, a_4\}$ are very low, which results in the increase of the certainty value as a^{S} believes more that the possibility of 1 reported by a_1 is correct. Through increase of the trust of a^{S} in a_2 from $x = 6$ to $x = 9$ from 0 to 1, the certainty increases since a_2's reported value of $\Pi_{a_2\to a^{\mathrm{D}}}(\underline{\tau}) = 1$ is in conformance with the agent a_1's value. Consequently, increase in the trust of a_2 should increase certainty. At $x = 9$ we have the highest certainty, which is due to the fact that a^{S} has complete trust in a_1 and a_2 that have reported the same value of $\Pi_{a\to a^{\mathrm{D}}}(\underline{\tau}) = 1$ while a^{S}'s trust in a_3 and a_4 that provide a completely contradictory report (compared to the reports of a_1 and a_2) which equals to $\Pi_{a\to a^{\mathrm{D}}}(\underline{\tau}) = 0$ is trivial ($\tau_{a^{\mathrm{S}}\to a} \approx 0, a \in \{a_3, a_4\}$). When a^{S}'s trust in a_3 increases From $x = 10$ to $x = 13$, the certainty reduces since a_3's reported value contradicts with the reports of a_1 and a_2. The same scenario happens for a_4. At $x = 17$ the graph reaches a local minimum as the sets of $\{a_1, a_2\}$ and $\{a_3, a_4\}$ have total disagreement over the value of $\Pi_{a\to a^{\mathrm{D}}}(\underline{\tau})$, however the certainty at $x = 17$ is greater than

$x = 1$ since a^{s} has complete trust in the 4 agents compared to the case of $x = 1$.

In this chapter, we provided our main contribution, which is a possibilistic model for estimating the trust of a target agent thought the information acquired from the advisor agents. We considered both the problem that the advisor agents are unknown to a^{s} and the problem that they are known to a^{s}. Moreover, we proposed a certainty metric measuring the confidence of a^{s} in the information acquired from its advisors. Note that in this chapter whatever we borrowed from the chapter of technical tools (chapter 4) is developed by other researchers and we reuse them in our trust model. The rest of the materials introduced in this chapter are contributions of this thesis. This includes the approaches proposed to address the two problems described above, the methodologies proposed to solve the successive merging approach (including all of the equations introduced in this section), the techniques used to address the single trust approach (including all of the equations proposed to measure a single trust value in each advisor agent), the certainty metric proposed at the end of this chapter and finally the combination of all of these tools (both the ones proposed in this thesis and the ones described in chapter 4) to measure the trust of a target agent in the context described before which is: (1) there is uncertainty (arisen from variability and ignorance) in the advisor and target agent's trust; (2) the advisor agents may act selfishly and manipulate the information; and (3) each interaction between every pair of connected agents is multi-valued. These are the main contributions of this thesis, which were described in details in this chapter. In the next chapter, we provide the experimental results of our proposed approaches.

Chapter 6

Simulation Results and Analysis

In this chapter, we provide extensive numerical experiments to validate all proposed concepts. We use the same multi-agent network introduced in Chapter 5 and the game scenario described in Section 5.1.6, where a buyer agent, namely a^{S}, wants to estimate the degree of trustworthiness of a target agent, namely a^{D}, unknown to him. a^{S} receives a possibility distribution from each member a in the set A of agents where each agent a has already interacted with a^{D}. Figure 6 illustrates the aforementioned multi-agent network.

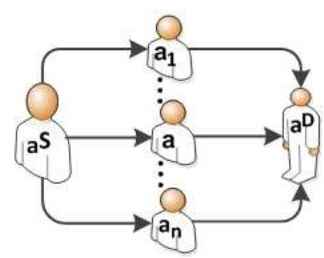


Figure 6: Network of Agents

We present an extensive experimental evaluation of the three different approaches described in Sections 5.2.1, 5.2.2.A, and 5.2.2.B. Specifically, the trust estimation approach made through unknown agents (described in Section 5.2.1) and the trust estimation approaches made through known agents (presented in Section 5.2.2) are experimentally investigated. We evaluate our proposed algorithms in these three approaches and perform evaluations in different experimental settings by changing: (1) the manipulation algorithms introduced in Section 5.1.5; (2) the number of agents in the set A; (3) the number of interactions between every pair of connected agents (as illustrated in Figure 6); and (4) the degree of trustworthiness of the agents in A. We intend to evaluate our proposed trust estimation approaches in different experimental settings in order to measure its performance in different scenarios. In addition, we measure the certainty over the possibility distributions acquired from the set A of agents (as described in Section 5.3) in every experiment.

In Section 6.1, we introduce two evaluation metrics for our experiments and then in Section 6.2 we present our experimental results. Finally, in Section 6.3 we discuss the conclusions of our simulation.

6.1 Evaluation Metrics

In order to evaluate the experimental results, we first introduce two evaluation metrics: (i) the information level of the estimated possibility distribution of a^{D}'s trust , and (ii) the approximated error of a^{D}'s estimated possibility distribution. The former metric measures the degree of provided information in the final possibility distribution of a^{D}'s trust. The latter metric, however, provides an approximation on the error of the estimated possibility distribution of a^{D}'s trust.

6.1.1 Metric I - Information Level of a Possibility Distribution

In the context of the possibility theory, the uniform distribution (53) contributes no information, as all of the trust ratings are equally possible and cannot be differentiated. The uniform distribution provides the state of "complete ignorance" [53].

$$\forall \tau \in T : \Pi(\tau) = 1, \tag{53}$$

The more a possibility distribution deviates from the uniform distribution, the more it contributes information. The following distribution provides the most informative possibility distribution which is referred to as "complete knowledge" [53]:

$$\exists!\ \tau \in T : \Pi(\tau) = 1 \text{ and } \Pi(\tau') = 0,\ \forall \tau' \neq \tau, \tag{54}$$

where only one trust value in T has a possibility greater than 0. We assign the information levels of 0 and 1 to the distributions of (53) and (54), respectively. In the general case, the information level (denoted by I) of a distribution Π having a total of $|T|$ trust ratings, is equal to:

$$I(\Pi) = \frac{1}{|T| - 1} \sum_{\tau \in T} (1 - \Pi(\tau)), \qquad |T| > 1. \tag{55}$$

Here, the distance of each possibility value of $\Pi(\tau)$ from the uniform distribution is measured first for all trust ratings of T. Then, it is normalized by $|T| - 1$, since at least one trust rating must be equal to 1 (property of a possibility distribution). We denote the information level of a possibility distribution by I in our experiments.

6.1.2 Metric II - Estimated Error of Target Possibility Distribution

In this Section, we measure the approximated error of the estimated possibility distribution of a^{D}'s trust, namely $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$. On this purpose, we measure the difference between the estimated and the true possibility distributions of a^{D}'s trust. In order to measure the true possibility distribution of a^{D}'s trust, its true probability distribution of trust (which is a^{D}'s internal probability distribution of trust as described in Section 5.1.2) should be transformed to a possibility distribution. Dubois *et al.* [58] provide a probability to possibility transformation procedure, which is described in Section 4.3. Through usage of this transformation procedure, the true possibility distribution of a^{D}'s trust can be measured and then compared with the estimated distribution $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$. Let $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ denote an estimated distribution, measured by one of the three estimation approaches (presented in Sections 5.2.1, 5.2.2.A, and 5.2.2.B) and let Π_F represent the true possibility distribution of a^{D}'s trust transformed from its internal probability distribution. The Estimated Error (EE) of distribution

$\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ is measured by taking the average of the absolute differences between the true and estimated possibility values over all trust ratings, $\tau \in T$. The EE metric is measured as:

$$EE(\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}) = \frac{1}{|T|} \sum_{\tau \in T} \left| \Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}(\tau) - \Pi_F(\tau) \right|. \quad (56)$$

We denote the estimated error of the possibility distribution $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$ by EE in our experiments.

6.2 Experimental Results

Here we report the experiments conducted to evaluate our three estimation approaches described in Sections 5.2.1, 5.2.2.A, and 5.2.2.B. We provide our experimental evaluations with respect to these three approaches.

In order to facilitate variation in the degree of trustworthiness of the agents in A, we divide the set A into three subsets. Each subset simulates a specific level of trustworthiness in the agents. The subsets are:

- A^{FT}: The subset of Fully Trustworthy agents where the peak of the internal probability distribution of trust(described in Section 5.1.2) is 1.
- A^{HT}: The subset of Half Trustworthy agents where the peak of the internal probability trust distribution is 0.5.
- A^{NT}: The subset of Not Trustworthy agents where the peak of the internal probability trust distribution is 0.

In each experiment, we start with $A = A^{\mathrm{NT}}$, where all of the agents in A are not trustworthy, and then at each step we gradually move a subset of the agents in A from A^{NT} to A^{HT} such that after a number of such steps, we reach the state of $A = A^{\mathrm{HT}}$ where all of the agents in A belong to the subset of half trustworthy agents (A^{HT}). Later, in the same trend, we move the agents in A from the subset of A^{HT} to A^{FT}, moving a subset of the agents in A from the set of half trustworthy agents (A^{HT}) to the set of fully trustworthy agents (A^{FT}) in each step such that we finally end up with $A = A^{\mathrm{FT}}$.

Through such transformation of the agents in A, we want to observe the influence of the degree of trustworthiness of the agents in A on the accuracy of the results we obtain. Indeed, the robustness of the estimated distribution $\Pi_{a^{S} \to a^{D}}$ is evaluated with respect to the nature of trustworthiness of the agents in A. We repeat this transition in different experimental settings by changing:

1. The number of agents in the set A;
2. The number of interactions between each pair of connected agents; and
3. The manipulation Algorithm I and II.

We intend to observe the influence of each one of these components on the final estimated distribution $\Pi_{a^{S} \to a^{D}}$. In all experiments, the number of trust rating events, $|T|$, is equal to 5 (a commonly used value in most surveys, e.g., ebay, IMDB and Amazon). Moreover, the possibility distributions constructed in our experiments (as described in Section 4.3) are measured with a confidence of $100(1-5)\%$. This , in turn, means that the measured possibility distribution dominates the true probability with 95% confidence, as presented in Equation (26) in Section 4.3.

6.2.1 Experiments Using the Manipulation Algorithm I

In the first set of experiments, the manipulation Algorithm I (described in Section 5.1.5) is used by the agents in A. We carry out four different experiments. We change the number of agents in A and the Number of Interactions (NoI) among every pair of connected agents (as illustrated in Figure 6) from one experiment to another. The following four experiments are carried out:

1. Experiment 1: $|A| = 30, NoI = 50$
2. Experiment 2: $|A| = 30, NoI = 20$
3. Experiment 3: $|A| = 10, NoI = 20$
4. Experiment 4: $|A| = 5, NoI = 10$

In the first experiment, the results are evaluated when both NoI and the number of agents are high enough. Then, through moving from the first to the second experiment, the number of agents is kept unchanged while NoI is reduced. Later, In the third experiment, NoI remains unchanged while the number of the agents is decreased. Finally, in the fourth experiment, both NoI and the number of agents are reduced. The last experiment aims to measure the performance of trust estimation approaches in low data presence (few number of interactions). In each one of these experiments, the set of agents in A is gradually transferred from A^{NT} to A^{HT} and later from A^{HT} to A^{FT} as explained before. Moreover, in each experiment, one trust estimation approach is made through unknown agents (described in Section 5.2.1) and two trust estimation approaches are done through known agents (presented in Section 5.2.2). Finally, the certainty of the information acquired by a^{S} is measured in each experiment (as described in Section 5.3). In this chapter, the elements of X axis are denoted by x. In the following sections, we present the results of these experiments.

A Experiment 1: $|A| = 30, NoI = 50$

In this experiment, we have 30 agents in A and a total of 50 interactions among every pair of connected agents (as illustrated in Figure 6). We start with $A = A^{\text{NT}}$ and at each step we transfer 10 agents from subset of (A^{NT}) to (A^{HT}). As shown in Table 4, each step is associated with a value of X axis. For each x value, the partition of A into $A^{\text{FT}} \cup A^{\text{NT}} \cup A^{\text{HT}}$ is presented in Table 4. At $x = 1$, we have $A = A^{\text{NT}}$ where all of the agents belong to A^{NT}. As x increases, the agents of A are redistributed from A^{NT} to A^{HT} such that at $x = 4$ we have $A = A^{\text{FT}}$. In the same trend, by increasing values of x, the members of A are redistributed from A^{HT} to A^{FT}. Finally, at $x = 7$, all the agents belong to A^{FT} such that $A = A^{\text{FT}}$.

X Axis	1	2	3	4	5	6	7
# $\|A^{\text{FT}}\|$	0	0	0	0	10	20	30
# $\|A^{\text{HT}}\|$	0	10	20	30	20	10	0
# $\|A^{\text{NT}}\|$	30	20	10	0	0	0	0

Table 4: Distribution of Agents in Figures 7 to 12 and Figures 19 to 24

For each distribution of the agents in A into $A^{\text{FT}} \cup A^{\text{NT}} \cup A^{\text{HT}}$ presented in Table 4, we carryout the experiments as discussed in Sections 5.2.1, 5.2.2.A, and 5.2.2.B. Indeed, as discussed in Section 5.2.1, we estimate the trust distribution of a^{D} for the fusion rules of intersection (Equation 6), union (Equation 13), and mean (Equation 14). In this approach, a^{S} does not know the agents of the set

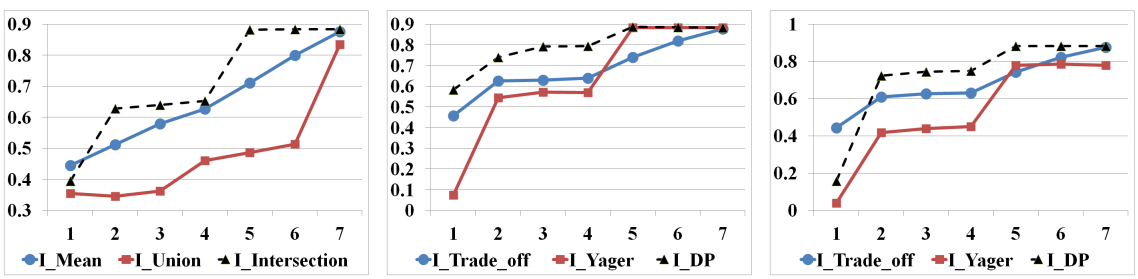
A. Later, based on the approach presented in Section 5.2.2.A, the following two trust distributions are successively merged: (1) the possibility distribution of a^{S}'s trust in agents of A, and (2) the trust possibility distributions of the set A in a^{D}. As described in Section 5.2.2.A, the fusion rules of trade-off (Equation 15), Yager (Equation 16) and DP (Equation 17) are used in this approach. Finally, based on the approach introduced in Section 5.2.2.B, a single trust value for each agent of set A is measured by a^{S} and this value is used in the fusion rules of trade-off (Equation 15), Yager (Equation 16) and DP (Equation 17) to estimate the trust distribution of a^{D}. The results presented in the figures of Section 6.2 are the average over 50 test runs measured for each fusion rule. In our experiments, we refer to the approach of Section 5.2.1 as "Unknown Agents", the approach of Section 5.2.2.A as "Successive Merging (SM)" and the approach of Section 5.2.2.B as "Single Trust (ST)".

Figures 6.7(a) and 6.8(a) illustrate the results carried out based on "Unknown Agents" approach. Figure 6.7(a) presents the Information level (I) (described in Section 6.1.1) of the possibility distribution of a^{D} measured through fusion rules of intersection (Equation 6), union (Equation 13), and mean (Equation 14). Figure 6.8(a) illustrates the Estimated Error (EE) of the the possibility distribution of a^{D} measured in the same experiment. In Figures 6.7(a) and 6.8(a), throughout the evolution of the agent distribution by increasing x values, we can observe the decrease of EE and the increase of I. This is a consequent of the fact that as the agents move from the subset of A^{NT} to A^{HT} and later to A^{HT} the agents become more trustworthy and therefore the information provided by the agents becomes less prone to error. Consequently, the estimation made through more accurate information enhances as the value of x increases in these figures. Comparing the fusion rules, the intersection rule outperforms the mean and union rules. This is a consequence of the fact that the intersection rule selects the information that all the sources agree upon. The results of the intersection rule deteriorate considerably at $x = 1$ as none of the sources are reliable.

Figures 6.7(b) and 6.8(b) show the experiment results carried out based on the "Successive Merging (SM)" approach of Section 5.2.2.A. Figure 6.7(b) illustrates the I metric as measured for the estimated possibility distribution of a^{D}'s trust through fusion rules of trade-off (Equation 15), Yager (Equation 16) and DP (Equation 17). Figure 6.8(b) demonstrates the EE of the the possibility distribution of a^{D} measured in the same experiment.

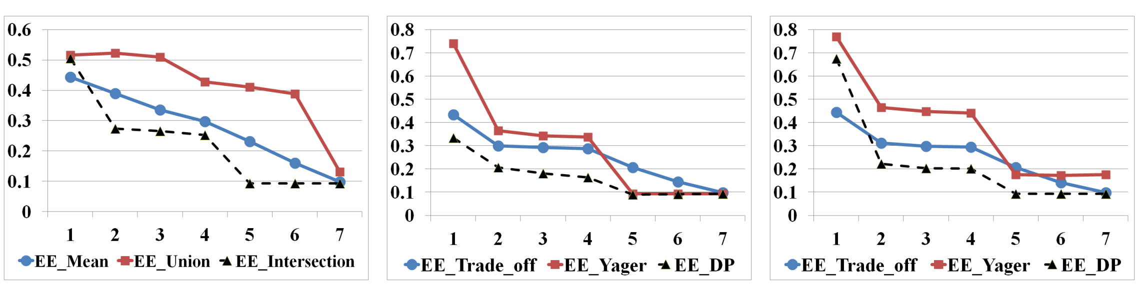
Figures 6.7(c) and 6.8(c) show respectively the I and EE metrics applied on the possibility distribution of a^{D}'s trust measured using the approach "Single Trust (ST) (Section 5.2.2.B) where the fusion rules of trade-off (Equation 15), Yager (Equation 16) and DP (Equation 17) are employed.

In Figures 6.7(b), 6.7(c), 6.8(b) and 6.8(c), by increasing the value of x, the results measured by metrics I and EE improve. Indeed by increasing x values, as the agents become more trustworthy, I increases and EE decreases. Comparing the fusion rules, DP outperforms fusion rules of Yager and trade-off in all Algorithm I and II's experiments which is due to the fact that the DP rule is more categoric in its ignorance of the agents who are not trustworthy compared to the two other fusion rules.



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 7: Experiment 1 - Algorithm I, $|A| = 30, NoI = 50$ - Metric I

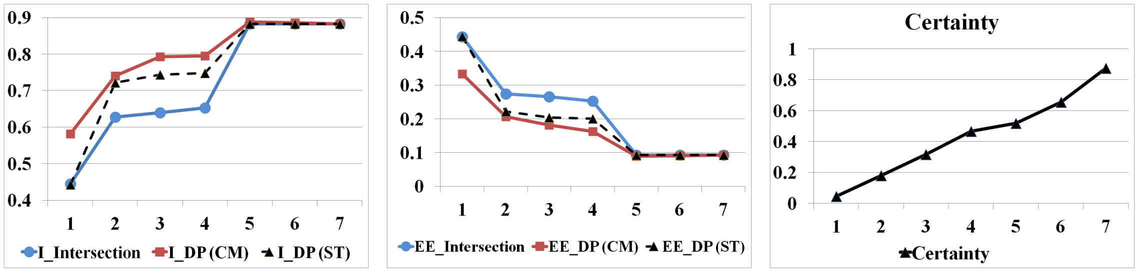


(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 8: Experiment 1 - Algorithm I, $|A| = 30, NoI = 50$ - Metric EE

In order the compare the results of the approaches presented in Sections 5.2.1, 5.2.2.A, and 5.2.2.B, we have selected the best result from each one of these three approaches. Indeed, for each value of x in each approach the best value is selected. For example, in Figure 6.7(a) at $x = 1$ the value of 0.45 from the mean rule is selected since it provides the highest I compared to intersection and union rules at $x = 1$. In Figure 6.7(a), at $x = 2$, the value of 0.63 belonging to the intersection fusion rule is selected since it contributes higher I compared to union and mean rules. Similarly, in Figure 6.7(b), at $x = 2$ the value of 0.74 from DP rule is selected since it's I value is higher than Yager and trade-off rules. In Figure 6.7(c), at $x = 2$ the value of 0.72 from DP is selected. In the same approach, for each x value in each subfigures a, b and c of figures 7 and 8, the best value is selected out of the three values. These selected values are shown in Figures 6.9(a) and Figure 6.9(b). Figures 6.9(a) illustrates the metric I while Figures 6.9(b) demonstrates the metric EE of these selected values. In Figures 6.9(a) and 6.9(b) "Unknown" stands for the selected value of the "Unknown Agents" approach (Section 5.2.1), SM stands for the "Successive Merging" approach (Section 5.2.2.A) and ST stands for the "Single Trust" approach (Section 5.2.2.B). The results shown in these figures illustrate that the SM and ST approaches outperform the "Unknown Agents"'s approach. This is due to the fact that these approaches also rely on the trust of the agents in the set A. However, the result of "Unknown Agents" is still satisfactory, which is mainly selected from the intersection rule, due to the nature of this rule that only takes information that all of the sources agree upon. Comparing SM and ST, SM performs better than ST. This is due to better trust modeling of the agents in the set A in the SM approach which builds a possibility distribution for each agent $a \in A$. However, in the ST approach a single trust value is measured for each agent a. Usage of a distribution allows a^{s} to model all of the possible trust ratings for each agent $a \in A$ which, in turn, would enhance its estimation results.

Figures 6.9(c) demonstrates the certainty value measured in Experiment 1 (based on the explanations of Section 5.3). In this figure, by increasing x values, the agents in A become more trustworthy and their report becomes more consistent. This, in turn, increases the certainty value as a^{s} can rely more on the information provided by the members of A.



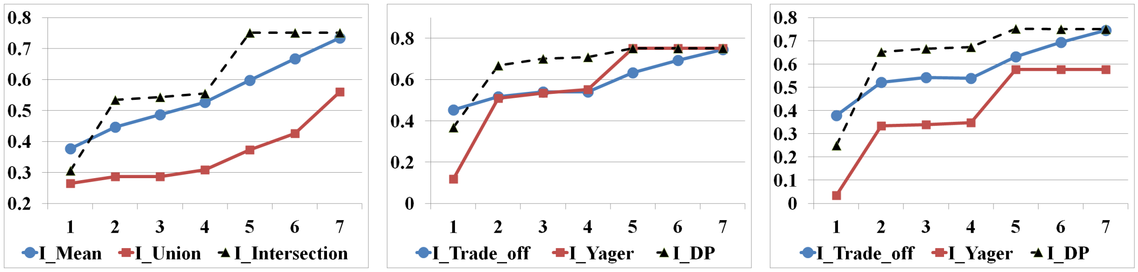
(a) Metric I (b) Metric EE (c) Certainty Value

Figure 9: Experiment 1 - Algorithm I, $|A| = 30, NoI = 50$ - Comparison of the Selected Results

B Experiment 2: $|A| = 30, NoI = 20$

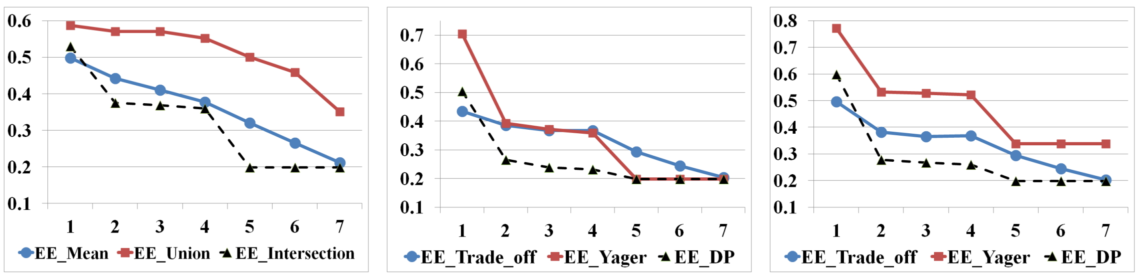
We perform the same experiment as Experiment 1 while reducing NoI from 50 to 30. The same approaches are used and the same experiments are carried out. Figures 6.10(a) and 6.11(a) demonstrate the I and EE metrics applied on the "Unknown Agents" approach. Figures 6.10(b) and 6.11(b) illustrate the approach of "Successive Merging (SM)" and Figures 6.10(c) and 6.11(c) show the results of the "Single Trust (ST)" approach. The results are almost the same as Experiment 1. However, the outcome of Experiment 2 has deteriorated compared to Experiment 1. Figure 10 shows lower values of I compared to Figure 7. In the same trend, Figure 11 demonstrates higher values of EE compared to Figure 8. This is due to the fact that reduction in the number of interactions between every pair of connected agents has reduced the accuracy of the possibility distributions measured by the trustor agent (in the pair of connected agents trustor agent is the agent who trust the other agent). Consequently, the estimation results have deteriorated since the estimation is made through lower amount of information.

As in Experiment 1, we choose the best result in each one of the three approaches of "Unknown Agents", "Successive Merging (SM)", and "Single Trust (ST)". Figures 6.12(a) and 6.12(b) demonstrate the selected values for the metrics of I and EE, respectively. Finally, the certainty over the results of Experiment 2 is shown in Figure 6.12(c).



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 10: Experiment 2 - Algorithm I, $|A| = 30, NoI = 20$ - Metric I



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 11: Experiment 2 - Algorithm I, $|A| = 30, NoI = 20$ - Metric EE

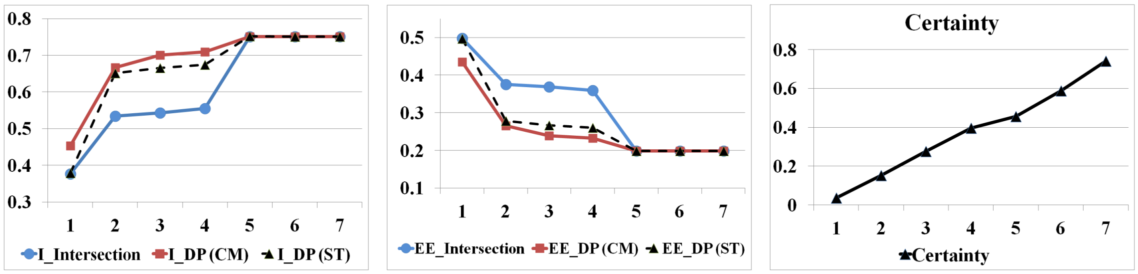
C Experiment 3: $|A| = 10, NoI = 20$

In this experiment, we set $|A|$ to 10 and NoI to 20. Compared with Experiment 2, we reduce the number of agents from 30 to 10 while keeping the NoI intact. We intend to observe the influence of the number of agents in A given a fixed manipulation algorithm and a fixd NoI. Since the number of agents is reduced to 10, the distribution of agents in A into $A^{\text{FT}} \cup A^{\text{NT}} \cup A^{\text{HT}}$ for each x value has changed. Tabel 5 shows the partition of A for each x value. At $x = 1$, all the agents belong to the set A^{NT}. From $x = 2$ to $x = 6$ we transfer two agents from A^{NT} to A^{HT} such that at $x = 6$ all the agents are in the set of A^{HT} and we have $A = A^{\text{HT}}$. From $x = 7$ to $x = 11$, agents are redistributed from A^{HT} to A^{FT}, two agents at a time. Finally, at $x = 11$, all the agents are in the set A^{FT}.

X Axis	1	2	3	4	5	6	7	8	9	10	11
# $\lvert A^{\text{FT}}\rvert$	0	0	0	0	0	0	2	4	6	8	10
# $\lvert A^{\text{HT}}\rvert$	0	2	4	6	8	10	8	6	4	2	0
# $\lvert A^{\text{NT}}\rvert$	10	8	6	4	2	0	0	0	0	0	0

Table 5: Distribution of Agents in Figures 13 to 15 and Figures 25 to 27

The values shown in Tabel 5 correspond to the experiments illustrated in Figures 13 to 15.



(a) Metric I (b) Metric EE (c) Certainty Value

Figure 12: Experiment 2 - Algorithm I, $|A| = 30, NoI = 20$ - Comparison of the Selected Results

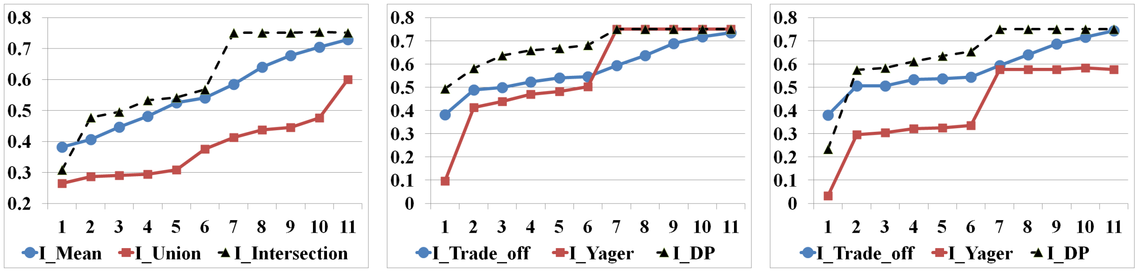
Figure 13 demonstrates the performance of our three estimation approaches ("Unknown Agents", "Successive Merging (SM)" and "Single Trust (ST)") as measured by metric I. Figure 14 illustrates the values for metric EE as measured for the these three estimation approaches.

Comparing Figures 6.13(a) and 6.14(a) with Figures 6.10(a) and 6.11(a), the graphs demonstrate almost the same results. Only, the results obtained by intersection rule have slightly deteriorated from $x = 2$ to the case where all of the agents are in the subset A^{HT}. This is illustrated in the decrease of I and increase of EE in Figures 6.13(a) and 6.14(a) for intersection rule from $x = 2$ to $x = 6$.

Comparing Figures 6.13(b) and 6.14(b) with Figures 6.10(b) and 6.11(b), the value of metrics I and EE for fusion rules of Yager and DP have moderately deteriorated from $x = 2$ to the case where all of the agents are in the subset A^{HT}. The results of trade-off rule, on the other hand, have remained almost unchanged.

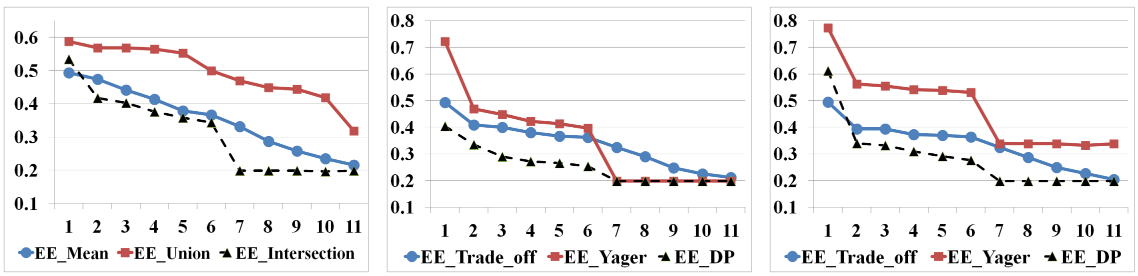
Finally, the DP rule in Figures 6.13(c) and 6.14(c) show moderate deterioration compared to the results shown in Figures 6.10(c) and 6.11(c) from $x = 2$ to the case where all of the agents are in the subset A^{HT}. This is demonstrated in the increase of the EE metric and decrease of the I metric.

Figures 6.15(a) and 6.15(b) show the selected best values of each one of the "Unknown Agents", "Successive Merging (SM)" and "Single Trust (ST)" approaches. Comparing the graphs of Figures 6.15(a) and 6.15(b) with their equivalents 6.12(a) and 6.12(b), the results are almost unchanged except for the values from $x = 2$ (where a subset of agents are in A^{HT}) to the case where all of the agents are in the subset A^{HT}. This is due to the fact that the intersection and DP rules, have slightly



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 13: Experiment 3 - Algorithm I, $|A| = 10, NoI = 20$ - Metric I



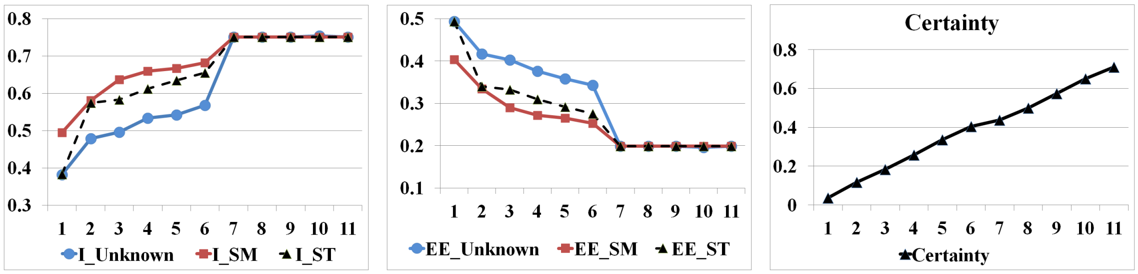
(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 14: Experiment 3 - Algorithm I, $|A| = 10, NoI = 20$ - Metric EE

deteriorated in this range in Figures 13 and 14 compared to Figures 10 and 11 and since these fusion rules (DP and intersection) are selected in the graphs of Figures 15 and 12, they demonstrate a slight modification comparing Figures 6.15(a) and 6.15(b) with 6.12(a) and 6.12(b).

Comparing the results of Experiments 2 and 3 we can conclude that reducing the number of agents has a slight influence on the final results. The influence of the number of agents is mainly observable in the cases where no subset of agents is fully trustworthy. In other words, in cases where the agents of A belong to the subsets A^{HT} and A^{NT}, the reduction in the number of agents reduces the accuracy of the results.

Figure 6.15(c) demonstrates the certainty value over the results of Experiment 3. Comparing Figures 6.12(c) and 6.15(c), the certainty value is almost unchanged. This is a consequent of the fact that the amount of information provided in Experiments 2 and 3 is almost unchanged and consequently the degree of inconsistency in the information received by a^{S} has not changed.



(a) Metric I (b) Metric EE (c) Certainty Value

Figure 15: Experiment 3 - Algorithm I, $|A| = 10, NoI = 20$ - Comparison of the Selected Results

D Experiment 4: $|A| = 5, NoI = 10$

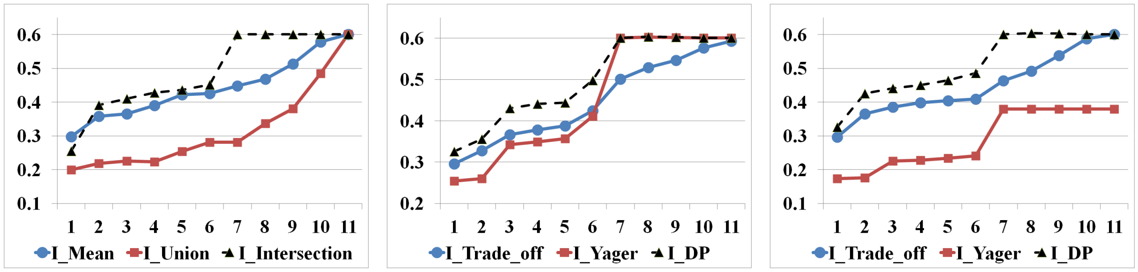
In this experiment, we reduce both values of $|A|$ and NoI to measure the performance of our estimation approaches when the amount of data is considerably low. Tabel 6 shows the distribution of A into A^{FT}, A^{NT} and A^{HT} for each value of x in this experiment. At $x = 1$ all the agents are in the set A^{NT}. From $x = 2$ to $x = 6$ one agent is transfered from A^{NT} to A^{HT} at each step. At $x = 6$ all the agents are in the subset A^{HT}. Later, from $x = 7$ to $x = 11$ the agents are redistributed from A^{HT} to A^{FT} in the same trend as before. Finally, at $x = 11$ all of the agents are in the subset A^{FT}.

X Axis	1	2	3	4	5	6	7	8	9	10	11
# $\|A^{\text{FT}}\|$	0	0	0	0	0	0	1	2	3	4	5
# $\|A^{\text{HT}}\|$	0	1	2	3	4	5	4	3	2	1	0
# $\|A^{\text{NT}}\|$	5	4	3	2	1	0	0	0	0	0	0

Table 6: Distribution of Agents in Figures 16 to 18 and Figures 28 to 30

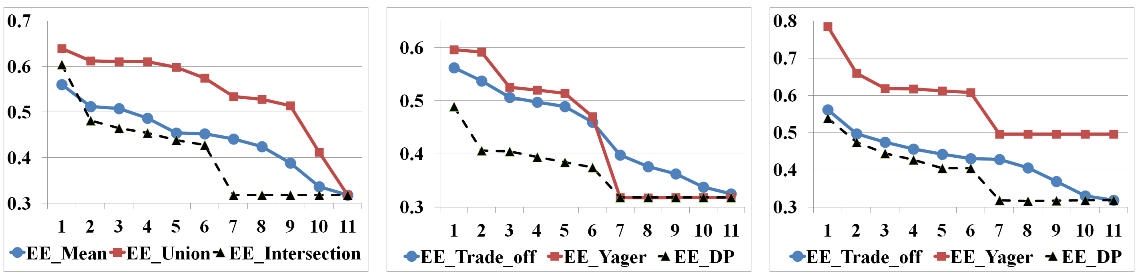
Tabel 6 corresponds to the experiments illustrated in Figures 16 to 18. These figures demonstrate the same measurements as Experiments 1 to 3. Figure 16 shows the outcome of our three approaches ("Unknown Agents", "Successive Merging (SM)" and "Single Trust (ST)") measured by Metric I, while Figure 17 shows the results using the Metric EE applied in these three approaches. Figures 6.18(a) and 6.18(b) illustrate the selected value of each one of these three approaches. Figure 6.18(c) , in turn, shows the certainty metric in Experiment 4.

Comparing the graphs of Experiment 4, with their equivalent of Experiment 2, the results have deteriorated. The metric I shows a decrease in Figure 16 compared to Figure 13 and the EE metric shows an increase in Figure 17 compared to Figure 14. The same result is observed while comparing



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 16: Experiment 4 - Algorithm I, $|A| = 5, NoI = 10$ - Metric I

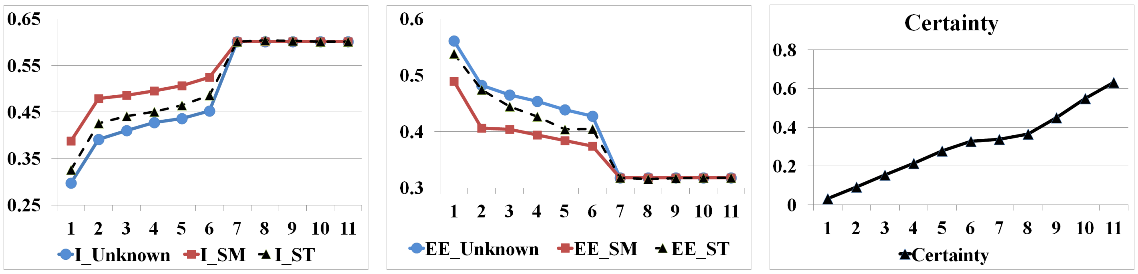


(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 17: Experiment 4 - Algorithm I, $|A| = 5, NoI = 10$ - Metric EE

the Figures 6.18(a) and 6.18(b) with Figures 6.15(a) and 6.15(b). This is due to decrease in the number of agents and NoI compared to Experiment 3. However, the influence of the reduction of NoI is more than the influence of the decrease in the number of agents in A. Reducing NoI reduces information exchanged in the interactions and this in turn deteriorates the possibility distributions built out of the few provided interactions. Consequently, the estimated distributions of a^{D} made through such few provided information is less accurate than Experiment 3. However, the results obtained here shows that the proposed approaches still provide satisfactory results in the cases of scarce data.

The certainty value demonstrated in Figure 6.18(c) is decreased compared to Figure 6.15(c) which is due to the reduction in the number of interactions among every pair of connected agents. This, in turn, decreases the consistency among the reported possibility distributions by the agents of the set A. Consequently, the measured values for certainty decreases.



(a) Metric I (b) Metric EE (c) Certainty Value

Figure 18: Experiment 4 - Algorithm I, $|A| = 5, NoI = 10$ - Comparison of the Selected Results

6.2.2 Experiments Using the Manipulation Algorithm II

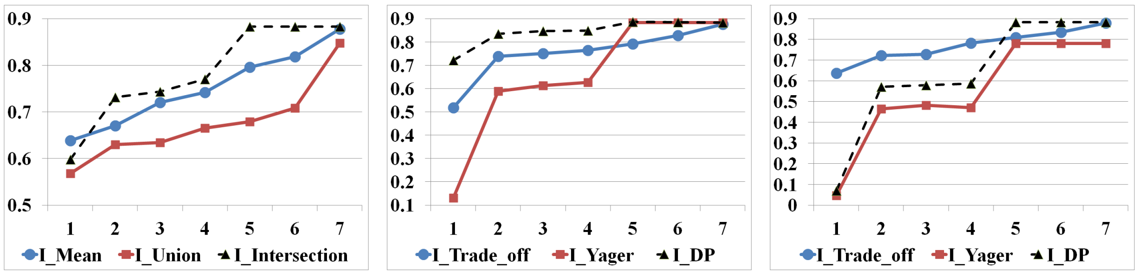
We repeat the same experiments with manipulation Algorithm II to observe the extent of influence of the manipulation algorithm chosen by the agents in the set A on the final distribution of $\Pi_{a^{\mathrm{S}} \to a^{\mathrm{D}}}$. Similar to the experiments of Section 6.2.1, the following experiments are carried out:

1. Experiment 5: $|A| = 30, NoI = 50$
2. Experiment 6: $|A| = 30, NoI = 20$
3. Experiment 7: $|A| = 10, NoI = 20$
4. Experiment 8: $|A| = 5, NoI = 10$

A Experiment 5: $|A| = 30,\ NoI = 50$

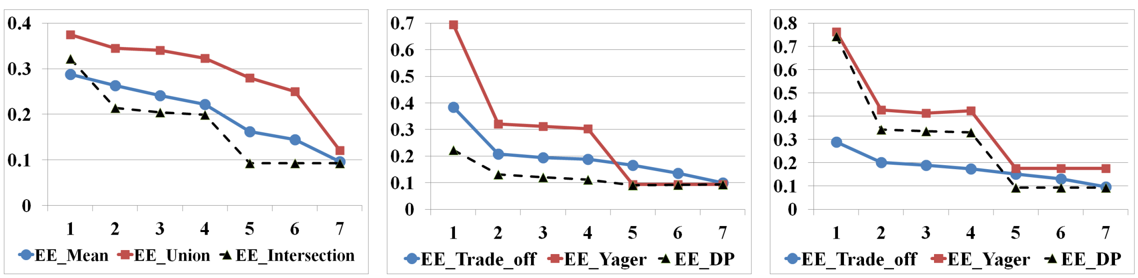
This experiment is the same as Experiment 1 (Section 6.2.1.A) only with the difference of using Algorithm II instead of Algorithm I. Figures 6.19(a) and 6.20(a) correspond to the results of the "Unknown Approach" as measured using the metrics I and EE, respectively. Figures 6.19(b) and 6.20(b) illustrate the results of the "Successive Merging (SM)" approach measured using the same metrics. Finally, Figures 6.19(c) and 6.20(c) show the results of the "Single Trust (ST)" approach. For each value of x, the distribution of the agents in A into the subsets A^{NT}, A^{HT} and A^{FT} is given in Table 4.

The graphs of Experiment 5 provide the same trends as Experiment 1. There is a noticeable difference comparing the results of Figures 6.19(c) and 6.20(c) with the graphs of Figures 6.7(c) and 6.8(c) where the trade-off rule outperforms the DP rule in the graphs of Figures 6.19(c) and 6.20(c) in the "Single Trust (ST)" approach. This shows that this fusion rule is more affective when the manipulation Algorithm II is being used. However, this observation is limited to the cases where none of the agents belong to the subset A^{FT}.



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 19: Experiment 5 - Algorithm II, $|A| = 30, NoI = 50$ - Metric I



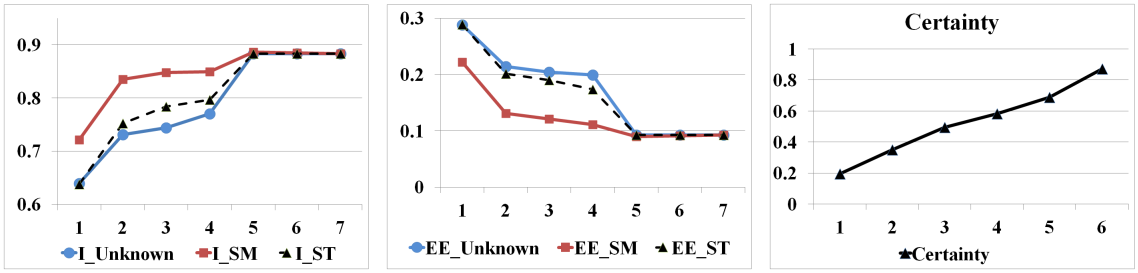
(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 20: Experiment 5 - Algorithm II, $|A| = 30, NoI = 50$ - Metric EE

As in previous experiments, we select the best results of each one of the three approaches of "Unknown Agents", "Successive Merging (SM)" and "Single Trust (ST)". Figures 6.21(a) and 6.21(b) demonstrate the selected values of these approaches as measured by the I and EE metrics, respectively. Finally, Figure 6.21(c) shows the certainty value in Experiment 5. The results observed here are the same as in Figures 9 where "Successive Merging (SM)" outperforms the other two approaches. The result of "Single Trust (ST)", in turn, is better than "Unknown Agents".

B Experiment 6: $|A| = 30, NoI = 20$

We perform the same experiments as Experiment 5 while reducing NoI from 50 to 30. The same

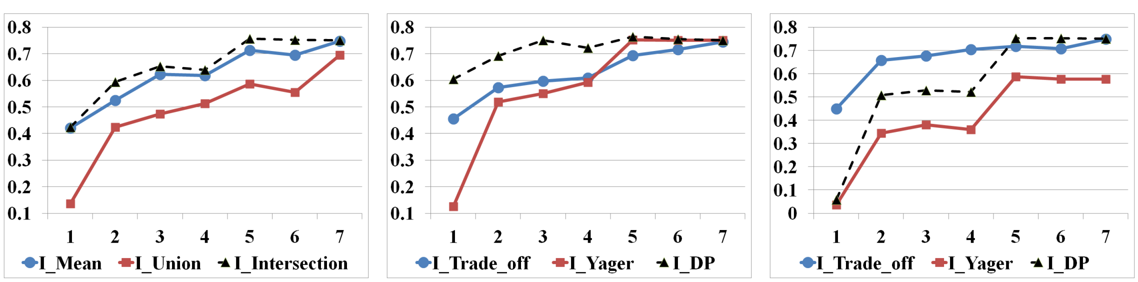


(a) Metric I (b) Metric EE (c) Certainty Value

Figure 21: Experiment 5 - Algorithm II, $|A| = 30, NoI = 50$ - Comparison of the Selected Results

measurements are carried out as in Experiment 5. Figures 6.22(a) and 6.23(a) demonstrate the I and EE metrics applied on the "Unknown Agents" approach. Figures 6.22(b) and 6.23(b) illustrate the approach of "Successive Merging (SM)" and Figures 6.22(c) and 6.23(c) show the results of the "Single Trust (ST)" approach. Figures 6.24(a) and 6.24(b) illustrate the selected values of these three approaches chosen from Figures 22 and 23.

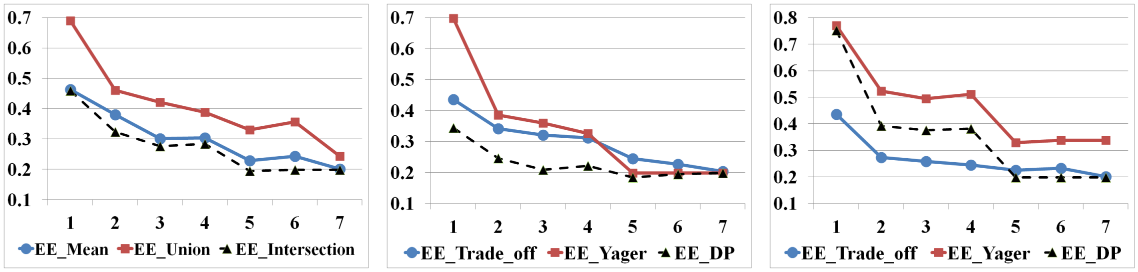
Comparing the results of Experiments 6 and 5, the outcome of Experiment 6 has deteriorated, meaning that the values of I and EE have decreased and increased, respectively. This outcome is expected as the number of interaction is reduced in Experiment 6 and therefore the acquired information is decreased compared to Experiment 5. This trend was observed when we compared the results of Experiments 1 and 2.



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

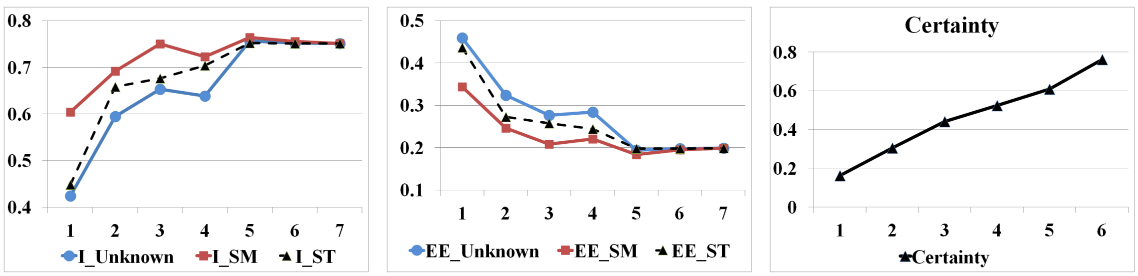
Figure 22: Experiment 6 - Algorithm II, $|A| = 30, NoI = 20$ - Metric I

Comparing Experiments 2 and 6, one noticeable difference is that the fusion rule of trade-off outperforms the DP rule in Figures 6.22(c) and 6.23(c) compared to 6.10(c) and 6.11(c). This trend was also observed in the results of Experiment 5 as compared with Experiment 1. Another noticeable



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 23: Experiment 6 - Algorithm II, $|A| = 30, NoI = 20$ - Metric EE



(a) Metric I (b) Metric EE (c) Certainty Value

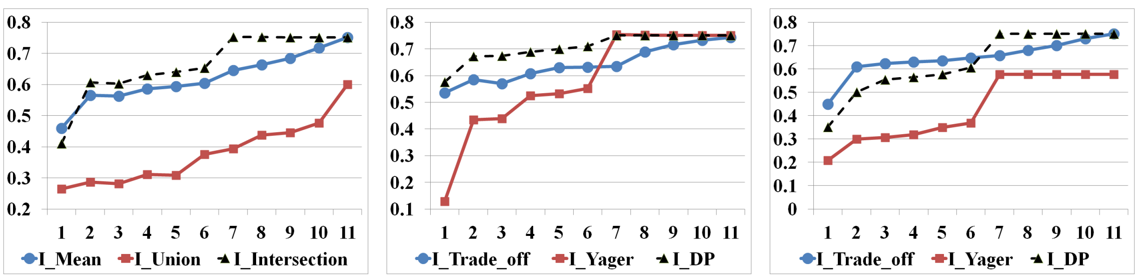
Figure 24: Experiment 6 - Algorithm II, $|A| = 30, NoI = 20$ - Comparison of the Selected Results

difference is higher degree of volatility in the results of Experiment 6 compared to Experiment 2. This means that the graphs of Experiment 6 are not monotonically changing. For example, in Figure 6.24(b) at $x = 4$, the value of EE has increased which was instead expected to decrease. Such volatility is driven from more random nature of Algorithm II compared to Algorithm I. Therefore, the results are not always following the monotonic patterns. However, the general trends are still observed as in Experiment 2, meaning that the graphs of the metric I increase by increasing x and the graphs of the metric EE decrease through higher values of x.

C Experiment 7: $|A| = 10, NoI = 20$

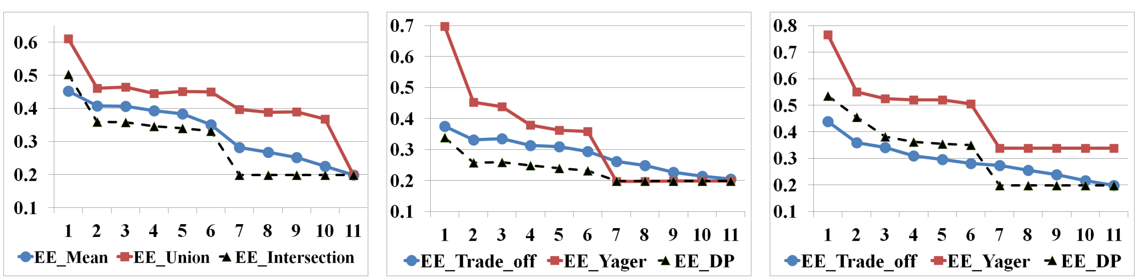
In this experiment, we reduce the number of agents from 30 to 10 compared to Experiment 6. Other values and settings are the same as in Experiment 6. Figure 25 illustrates the result of the metric I as measured for the three approaches of "Unknown Agents", "Successive Merging (SM)" and "Single Trust (ST)". Figure 26 illustrates the results of the metric EE as measured for the same three approaches. Finally, Figure 27 shows the selected best values from each one of these three approaches (Figures 6.27(a) and 6.27(b)) and the certainty metric as illustrated in 6.27(c). Note that the distribution of agents in A into the subsets A^{NT}, A^{HT} and A^{FT} corresponding to each value of x, is given in Table 5.

The results of Experiment 7 show slight deterioration compared to Experiment 6 from $x = 2$ (in which a subset of agents are in A^{HT}) to the case where all of the agents are in the subset A^{HT}. This demonstrates that when agents are not totally trustworthy, decreasing the number of agents would deteriorate the results (decrease of EE and increase of I).



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

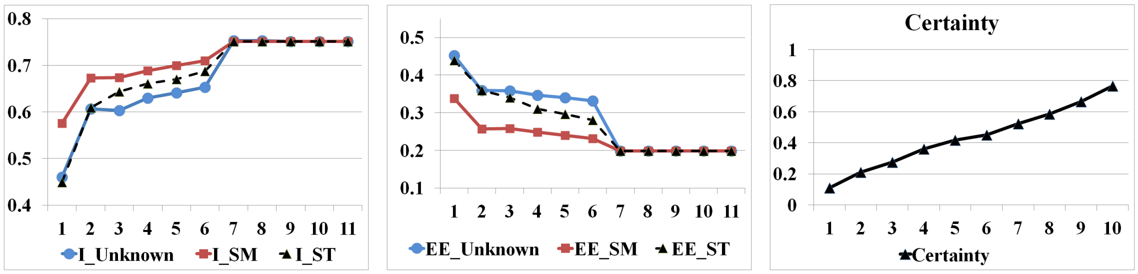
Figure 25: Experiment 7 - Algorithm II, $|A| = 10, NoI = 20$ - Metric I



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 26: Experiment 7 - Algorithm II, $|A| = 10, NoI = 20$ - Metric EE

Comparing results of Experiments 3 and 7, the graphs of Figure 7 demonstrate the same results as in Experiment 3. However, more volatility is observed in the results of Experiment 7 compared to Experiment 3. Indeed, as the value of x increases, the graphs of Experiment 3 are not monotonically changing. By increasing x values, the graphs of the metrics I and EE do not monotonically increase and decrease, respectively. This is due to higher randomness of Algorithm II compared to Algorithm



(a) Metric I (b) Metric EE (c) Certainty Value

Figure 27: Experiment 7 - Algorithm II, $|A| = 10, NoI = 20$ - Comparison of the Selected Results

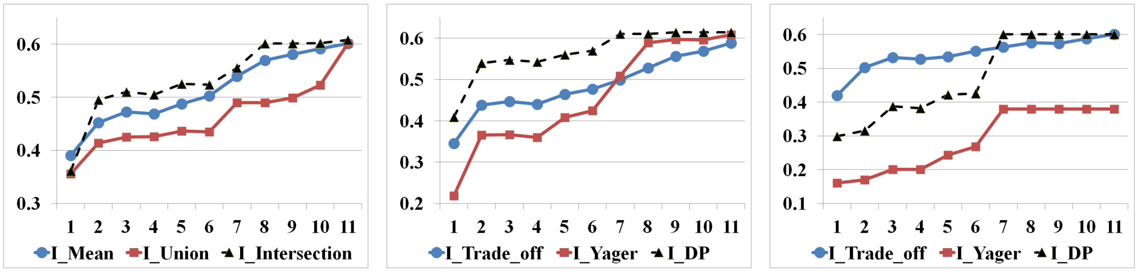
I. However, the overall trend in Experiment 7 is the same as Experiment 3.

D Experiment 8: $|A| = 5, NoI = 10$

In this experiment, we reduce the number of agents and NoI to 5 and 10 compared to Experiment 7 in order to measure the performance of our approaches while using manipulation Algorithm II in low data presence. The set of approaches are the same as before. Figure 28 demonstrates the results of the metric I, while Figure 29 illustrates the results of the metric EE applied on our three approaches. Figure 30 shows the selected values of each one of these three approaches (Figures 6.30(a) and 6.30(b)) and the certainty metric (Figure 6.30(c)). For each value of x, the distribution of agents in set A into the subsets A^{NT}, A^{HT} and A^{FT} is given in Table 6.

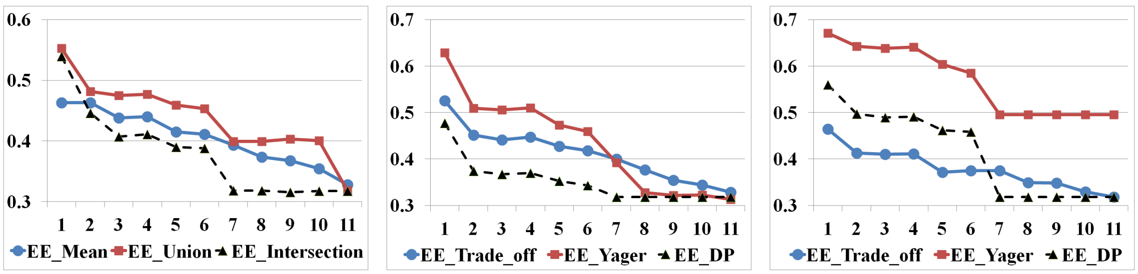
The results of Experiment 8 have deteriorated compared to Experiment 7. This is expected since the number of interactions and agents have reduced compared to Experiment 7. However, decreasing NoI has higher influence compared to reduction of agents. This is due to the fact that reduction of information reduces the quality of the measured possibility distributions. This, in turn, deteriorates the estimation results.

Comparing the results of Experiments 4 and 8, the graphs illustrate the same trends and the results are close. However, more fluctuation is observed in the graphs of Experiment 8 compared to Experiment 4. This phenomenon is also observed in previous experiments of manipulation Algorithm II. As explained before, the higher degree of volatility in the graphs of Algorithm II is a consequent



(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 28: Experiment 8 - Algorithm II, $|A| = 5, NoI = 10$ - Metric I



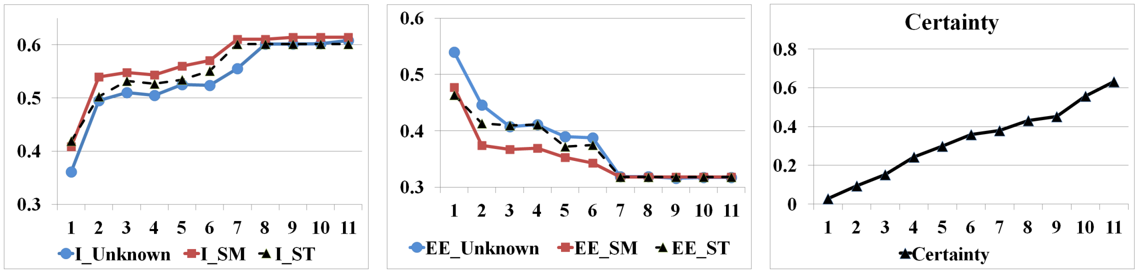
(a) Unknown Agents (b) Successive Merging (SM) (c) Single Trust (ST)

Figure 29: Experiment 8 - Algorithm II, $|A| = 5, NoI = 10$ - Metric EE

of higher randomness in the nature of Algorithm II compared to Algorithm I.

6.3 Conclusions of Simulations

In this chapter, we presented the simulation results of our trust model. Our trust model was aimed at estimating the trust distribution of a target agent through the information acquired from a set of advisor agents. The results are presented for the scenario that the advisor agents are unknown ("Unknown Agents") and the scenarios that the advisor agents are known ("Successive Merging (SM)" and "Single Trust (ST)"). The experiments were intended to validate the proposed methodologies introduced in Chapter 5. In general, the results validate our proposed approaches giving satisfactory results in the experiments. Moreover, the goal of separate experiments presented in this chapter was to evaluate the influence of: (1) the number of agents in set A which is $|A|$; (2) the number of interactions between each pair of connected agents (NoI); and (3) the manipulation Algorithm I and II. This led to eight different experiments which were demonstrated in this chapter.



(a) Metric I (b) Metric EE (c) Certainty Value

Figure 30: Experiment 8 - Algorithm II, $|A| = 5, NoI = 10$ - Comparison of the Selected Results

In each experiment, we evaluated the influence of the degree of the trustworthiness of the agents in A by gradually moving agents from the subset of A^{NT} to A^{HT} and later to A^{FT}.

Considering the simulation results of Section 6.2, by increasing the number of agents ($|A|$) and the number of interactions (NoI), the estimated possibility distribution of a^{D} improves. This improvement is observed in the results of manipulation algorithms I (starting from Experiment 4 and going back to Experiment 1) and manipulation algorithms II (starting from Experiment 8 and going back to Experiment 5) which is illustrated in the increment of the metric I and decrement of the metric EE. The influence of NoI, however, is more than $|A|$ since increasing the number of interaction would increase the information acquired on other agents in the multi-agent system which, in turn, improves the accuracy of the possibility distributions measured on the trust of other agents. Consequently, the results improve. This influence was observed when we compared the results of Experiments 1 and 2 (for manipulation Algorithm I) and Experiments 5 and 6 (for manipulation Algorithm II). However, the influence of $|A|$ is not to the same extent as NoI. This was illustrated when we compared the results of Experiments 2 and 3 (for manipulation Algorithm I) and Experiments 6 and 7 (for manipulation Algorithm II).

In Experiments 1 to 8, agent redistribution from the subset A^{NT} to A^{HT} and later from A^{HT} to A^{FT} improves the results. This improvement is observed through increment of the value of x where the metric I is increased and the metric EE is decreased. Through redistribution of the agents in A from A^{NT} to A^{HT} and later to A^{FT}, the advisor agents become more trustworthy and report more honestly. Therefore, the estimated trust distribution of a^{D} improves.

Comparing manipulation algorithms I and II, the graphs of manipulation Algorithm I (Experiments 1 to 4) are monotonically changing while the graphs of manipulation Algorithm II (Experiments 6 to 8), on the other hand, do not demonstrate such monotonic transformation. This influence is observed in the graphs of the metrics I and EE which is due to higher randomness in the nature of Algorithm II compared to Algorithm I. However, this influence is not observed in the graphs of Experiment 5 where NoI is 50. This indicates that when the number of interactions is high enough, the acquired information is adequate for correct measurement of a^{D}'s trust distribution. Consequently, having sufficient amount of information compensates for the random nature of manipulation Algorithm II.

The results of Experiments 4 and 8 aim to measure the performance of our proposed approaches when data is scarce. The results obtained in these two experiments are worse than other experiments. However, it shows that the proposed approaches still perform satisfactorily in low data presence.

Comparing the fusion rules of the "Unknown Agents" approach, intersection rule outperforms union and mean rules in experiments 1 to 8. This is due to the fact that the intersection rule only considers the information that all of the sources agree upon. Considering the results of the "Successive Merging (SM)" approach in experiments 1 to 8, DP rule outperforms Yager and trade-off rules. This is the consequent of the fact that DP fusion rule is more categoric in considering only the information acquired from the trustworthy sources. DP shows the same performance in the results of the "Single Trust (ST)" approach in experiments 1 to 4. However, in experiments of Algorithm II (Experiments 5 to 8), DP only dominates other fusion rules in the cases that a subset of the agents in A are in A^{FT} . When no agent is in the subset of A^{FT}, trade-off rule outperforms other fusion rules. This indicates that in cases where no agent is fully trustworthy, trade-off rule is more adaptable when manipulation Algorithm II is used and the "Single Trust (ST)" approach is applied.

Comparing the three approaches, the "Successive Merging (SM)" approach outperforms the "Single Trust (ST)" approach and the "Single Trust (ST)" approach, in turn, performs better than the "Unknown Agents" approach. This indicates that when the degree of the trustworthiness of the advisor agents is known, the estimation results improve. Although the "Successive Merging (SM)" approach yields the best results, there is a considerable computation burden for applying this approach. If the running time is important in an application, the "Single Trust (ST)" approach would be a better option, which keeps a balance between the performance and the running time.

The certainty metric, presented in Section 5.3, is aimed at measuring the confidence of a^{S} in the information acquired from its advisors. The certainty metric considers both the consistency in the information acquired from the agents in A and the trust of a^{S} in its advisors. Considering the certainty metric, increasing NoI enhances certainty. This trend was observed while comparing Experiments 1 and 2 (in manipulation Algorithm I) and Experiments 5 and 6 (in manipulation Algorithm II). Higher NoI contributes to more accurate possibility distributions built by agents in A on the trust distribution of a^{D}. Consequently, the consistency among the reported possibility distributions by the agents in A enhances which, in turn, increases certainty. Changing the number of agents in A does not influence certainty. This was observed while comparing Experiments 2 and 3 (in manipulation Algorithm I) and Experiments 6 and 7 (in manipulation Algorithm II) which is due to the fact that the quality of information has not changed and therefore the inconsistency among the reported distributions remains the same. Comparing the manipulation algorithms I and II, the certainty metric has unchanged while comparing experiment i $(1 \leq i \leq 4)$ with experiment $i + 4$. This is due to the fact that NoI has remained the same which keeps the inconsistency among the reported possibility distributions of agents in A unchanged. In all Experiments 1 to 8, through redistribution of agents from A^{NT} to A^{HT} and later to A^{FT} the certainty metric has improved. This is the consequent of the fact that when the agents in A are more trustworthy, their reported distribution become less manipulated and therefore the possibility distributions reported to a^{S} become more consistent. Consequently, the certainty increases by increase in the x values.

In this chapter, we provided extensive simulations in different experiments and we observed the influence of the multi-agent platform settings on the quality of the target agent's estimated distribution. The quality of the estimated distributions was evaluated through the the metrics I

and EE. Moreover, we presented the performance of our certainty metric and how it relates to the settings in the multi-agent platform. The simulations presented in this chapter were aimed at validating the approaches which were introduced in Chapter 5. In the next chapter, we present the conclusion and the future works.

Chapter 7

Conclusion and Future Work

7.1 Summary of the Thesis

In this thesis, we estimated the trust distribution of an unknown target agent through the acquired information from a set of advisor agents when: (1) there is uncertainty arising from variability and ignorance in the degree of trustworthiness of the advisor and target agents; (2) the advisor agents are self-interested and manipulate their information before reporting it; and (3) the trust domain is multivalued. We proposed the usage of the possibility distributions in such settings and considered three different approaches:

1. **Unknown agents approach.** We assumed the agents in the set A are unknown to a^{S}.
2. **Successive merging approach.** We assumed the agents in the set A are known to a^{S} and we merged the successive distributions of: (1) the possibility distributions of a^{S}'s trust in the agents of the set A; and (2) the possibility distributions of the set A's trust in a^{D}.
3. **Single trust approach.** We assumed the agents in the set A of advisors are known to a^{S} and we measured a single trust value, representing the trust of a^{S} in each agent $a \in A$. Then, we used these single trust values in the fusion rules of Yager, DP, and trade-off (introduced in Section 4.1.2) to estimate the possibility distribution of a^{S}'s trust in a^{D}.

Moreover, we measured a certainty metric over the possibility distributions reported by the agents in the set A in order to demonstrate the confidence of agent a^{S} in the information acquired from its advisors. For measuring the certainty metric we considered: (1) consistency in the reported possibility distributions by the agent in A; and (2) the trust of a^{S} in the agents of A. We measured the certainty metric in Section 5.3.3 and demonstrated the influence of modifying inconsistency and trust on the certainty value.

Finally, we presented extensive experiments in Section 6, where we first introduced two evaluation metrics of I and EE which respectively correspond to: (1) **I.** the information level of a possibility distribution; and (2) **EE.** the estimated error of the measured possibility distribution of a^{D}. Later, we validated our proposed approaches in extensive experiments and presented the results of our simulations by demonstrating the influence of the following experimental settings: (1) the employed manipulation algorithm; (2) the number of agents in A; (3) the number of interactions between every pair of connected agents (NoI); (4) the employed fusion rule; and (5) the degree of trustworthiness of the agents in A. The results demonstrated that the "successive merging" approach outperforms the "single trust" approach and the "single trust" approach, in turn, performs better than the "unknown agents" approach. In general, the trust estimation approaches and the certainty metric showed satisfactory results in the experiments.

7.2 Future Work

A possible extension to our model is consideration of prorogation of trust in multi-agent networks. In this thesis, we just considered trust estimation of a target agent who is connected to the evaluator agent through one intermediary agent. Indeed, each agent $a \in A$ connects a^{S} to a^{D}. This can be extended to the cases where the target agent can be reached through a set of connected intermediary agents. In such cases, the reputation of the target agent should be propagated more than once to reach the evaluator agent. Higher number of intermediary agents would probably decrease the accuracy of the information received by the evaluator agent and more robust approaches would be required to compensate for the manipulation of the information by the intermediary agents.

The possibility distribution measured in this thesis is based on the work of Masson and Denœux [59] which was explained briefly in Section 4.3. The possibility distribution is measured directly from empirical data. However, this measurement is done in a one shot approach, meaning that the possibility distribution is measured only once, which is after completion of the interactions. There is no mechanism for updating the possibility distribution when more data becomes available. Indeed, through the current mechanism the possibility distribution should be constructed entirely from scratch once more data is acquired. A methodology can be proposed for updating the possibility distribution measured in [59] in which the possibility distribution can be updated as more interaction is carried out over time and more data becomes available. This would, in turn, provide an online learning on the possibility distribution of an agent's trust.

Finally, we want to compare our trust model with the models of BLADE (introduced in Section 3.2.8) and Travos (introduced in Section 3.2.7) which are the closest trust models to our own. We became familiar with the BLADE model one month ago when I was almost at the end of writing my thesis, which was too late for an experimental comparison. The Travos model is proposed for binary domains which has a more restricted platform compared to our multi-valued platform. Both models of BLADE and Travos are probabilistic models which can only address uncertainty driven from variability. This is a shortcoming of probabilistic models compared to the possibilistic models which can address both types of uncertainty (namely variability and ignorance). Despite these differences, we want to compare our model with these two models as part of our future work.

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