PRODUCTION PLANNING WITH RAW MATERIAL SHELF-LIFE CONSIDERATIONS BY MIXED INTEGER PROGRAMMING

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ABSTRACT

PRODUCTION PLANNING WITH RAW MATERIAL SHELF-LIFE CONSIDERATIONS BY MIXED INTEGER PROGRAMMING

Andrés F. Acevedo

Besides being a widely studied area in operations research and in industrial and management science, production planning is considered one of the most fundamental elements in manufacturing systems. Due to the nature of the features involved in production planning problems, Mixed Integer Programming (MIP) is commonly used for optimization in this area. Also, the flexibility of MIP allows addressing specific problem characteristics and assumptions. This thesis tackles a multi-item multi-level capacitated production planning problem by MIP with a particular feature found in certain industries: raw material shelf-life. Manufacturing systems such as food, chemicals, composite materials and related industries, utilize components that are subject to limited shelf-life and must be disposed if they reach the end of it. Two MIP model formulations are proposed here: one without raw material shelf-life requirements as a basis of comparison, and one integrating raw material shelf-life. The models are flexible enough to be applied and validated for multiple problem instances with different variations and for an Automotive Industry case study. IBM® ILOG® CPLEX® Optimization Studio is used to achieve optimality. Results are analyzed and discussed in depth and future research topics are proposed.

Keywords: *production planning, mixed integer linear programming, shelf life, composites manufacturing.*

DEDICATION

This thesis is dedicated to my parents, Cristina Ojeda y Carlos Acevedo, for their indispensable complicity and support.

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1 INTRODUCTION

Production planning is one of the most relevant fields of study in operations research and management science. The range of decisions that are made based on production planning models and systems can be associated with virtually all the variables involved in a manufacturing process, sometimes including supply chain. In general terms, variables and parameters regarding quality, quantity, timing, sequences, and other features of raw materials procurement, production and distribution are addressed by production planning.

Mixed Integer Linear Programming (MILP) is often used to solve production planning problems, taking advantage of its flexibility to involve specific aspects and variables for each case. This study focuses on manufacturing processes with perishable raw materials or components. The perishability feature is addressed here using the concept of *shelf-life*, which is defined as the maximum length of time a component can be stored under specified conditions and remain suitable for use, consumption or for its intended function. Unlike other models of inventory control and production planning involving deteriorating inventory with lose of functionality depending on storage time, we consider raw materials fully functional until the end of its shelf-life. In addition to this feature, other relevant variables, parameters and assumptions such as ordering batch size and lead time are taken into account to analyze the impact of such considerations in problem results.

In order to make a more practical research contribution, the study is specifically contextualized in composites manufacturing field and related industries. In such systems, components shelf-life requirements are of particular relevance.

The thesis is organized as follows: We present a review of recent research contributions relevant to the topics of interest in Chapter 2. We first discuss production planning in general, followed by applications of Mixed Integer Linear Programming optimization, then considering the perishability characteristics, and finally contextualizing the study in composites manufacturing. In Chapter 3, we propose different variants for the studied problem assuming relevant features, variables, constraints and parameters. Subsequently, the mathematical formulation is applied to different problem instances and to an Automotive Industry case study in Chapter 4. Finally, Chapter 5 summarizes the main research conclusions and suggests future research and aspects to consider.

1.1 Scope and Delimitation of Thesis

This thesis focuses specifically on Production Planning using Mixed Integer Linear Programming optimization models considering shelf-life of components (raw materials). The specific problem addressed in this study refers to a multi-item multi-level production planning model, under the assumption that the end-products are made with perishable materials, i.e. they have limited shelf-life. Once defined, the mathematical formulation is applied to solve different hypothetical instances with different assumptions and a case study from the Automotive Industry using IBM® ILOG® CPLEX® Optimization Studio Version 12.4. Results are analyzed and discussed in depth.

1.2 Research Objectives and Contribution

The objectives and contribution of this research work are described as follows:

- To carry out a review of recent and relevant scientific contributions on topics related to production planning, mixed integer linear programming, shelf-life considerations and composites manufacturing. To analyze different approaches and methodologies in order to sufficiently substantiate the research contribution.

- To propose a Mixed Integer Linear Programming formulation that includes fundamental and specific variables, parameters and constraints to solve the production planning problem under consideration. Moreover, this formulation is intended to be flexible and general to be applied to different manufacturing systems. The main aspect of the study is to consider components or raw material shelf-life.

- To validate and analyze the efficiency and relevance of the proposed optimization model by applying it to solve different instances of the addressed problem with different assumptions. We also apply it to a case study based on information from industry sources.

- To present analysis and in-depth discussion on the performance and the important aspects of the mathematical model and implementation.

The above objectives are tackled throughout the thesis, keeping a logical order, but not strictly linear, i.e. it is likely that part of an objective is addressed in more than one section or chapter of the document.

1.3 Limitations of Thesis

The limitations of this research are mainly in two aspects: (1) in the set of assumptions made about the parameters, variables and/or constraints used to make a more specific model so that it can focus on its unique features. That is, some important considerations may have been overlooked or not taken into account. (2) In the proposition of more sophisticated and efficient methodologies for solving considerably large size instances of the production planning problem under consideration. However, these limitations can be addressed in our future in this area.

2 LITERATURE REVIEW

2.1 Production Planning

As proposed by Wolsey and Pochet (2006), production planning can be viewed as planning of the acquisition of resources and raw materials (components), as well as planning of the production activities required to transform materials into finished products. All of the above, meeting customer demand in the most efficient or economical way possible, i.e. minimizing total costs.

Typically, solving production planning problems involve making decisions regarding the size of production lots, or production levels, for each of the time periods in a planning horizon. Additionally, these problem solutions may also include decisions on the quantities of raw materials (components) to purchase, order or process, inventory levels for finished products and components, production sequence, and other variables related to these aspects.

In production planning, we usually consider material flow and inventory balance equations in time-indexed models using a relative coarse discretization of time, such as years, quarters, months or weeks (Kallrath, 2005). Linear Programming (LP), Mixed Integer Linear Programming (MILP), and Mixed Integer Non-Linear Programming (MINLP) models are often appropriate and successful for solving these problems with a clear quantitative objective function: net profit, contribution margin, cost, total sales, total production, etc (Kallrath, 2005).

2.1.1 History and Evolution of Production Planning Models

Harris and Wilson EOQ Models

The beginning of the study and development of production planning and production scheduling models dates back to 1913, with the Economic Order Quantity (EOQ) model proposed by F. W. Harris. The purpose of the EOQ model is to determine the order quantity that minimizes the total inventory holding costs and the ordering costs. Expanding Harris' contributions, R. H. Wilson developed the statistical re-order point model in 1934 with the objective of preventing components from running out of stock, introducing the notion of *safety stock*.

In the 1940s, Wilson combined his technique with that of Harris' EOQ and it became referred to as the Wilson EOQ Technique, or the Wilson Formula. These models became the main inventory control technique for almost 30 years (Adam and Sammon, 2004).

Wagner and Whitin Dynamic Lot-Sizing and MRP Models

Over a decade later, another crucial contribution was made by H. Wagner and T. Whitin. They introduced the Dynamic Lot-Sizing model in 1958 as a generalized version of the EOQ model, considering the demand as time-varying. Subsequently, "the introduction of Materials Requirement Planning (MRP) systems in the 1970s was a major step forward in the standardization and control of production planning systems" (Wolsey and Pochet, 2006). While MRP is primarily focused on planning and scheduling of materials, subsequent formulations called Manufacturing Resource Planning (MRP II) began to cover all aspects of manufacturing processes, including demand planning, sales and operations planning (S&OP), master production schedule (MPS), bill of materials (BOM) and inventory control, among others.

Advanced Planning and Scheduling and Enterprise Resource Planning Systems

During the 1980s and 1990s decades, the intentions of integrating MRP and MRP II transversally in supply chain and manufacturing facilities led to what is now knows as Advanced Planning and Scheduling (APS) and Enterprise Resource Planning (ERP). Thus, APS systems provide long, mid and short-term planning of the supply chain, including aspects of procurement, production, distribution, and sales (Newmann et al., 2002). Furthermore, ERP systems not only focus on planning and scheduling of internal resources, they strive to plan and schedule supplier resources as well (Chen, 2001). Additionally, ERP systems also include technology aspects, such as friendly graphical user interfaces, relational databases, use of fourth-generation language, and computer-aided software engineering tools (Adam and Sammon, 2004).

However, according to Wolsey and Pochet (2006), "MRP and its successors are not sufficient for the efficient planning of the factory or enterprise. Much criticism was leveled at the inability of such systems to deal effectively with lead times and capacity constraints. Even in APS and ERP systems, the planning modules are still seen as unusable, or unable to handle the complexity of the underlying capacitated planning problems".

2.1.2 Production Planning by Mixed Integer Linear Programming

When we intend to apply production planning in a more sophisticated way for complex manufacturing systems, it is usual to find such applications made through Mixed Integer Linear Programming (MILP) models. This is due to the nature of the decision variables for some features involved in such problems, e.g., set-up costs and times, start-up costs and times, machine assignment decisions, ordering costs and times, and so on. These costs and times are fixed per batch and are not proportional to the batch size. Therefore, binary or integer variables are required to model them (Wolsey and Pochet, 2006).

Recent and relevant contributions on the development of these production planning models by Mixed Integer Linear Programming are presented below.

Orçun et al. (2001) developed a continuous time model for production planning and scheduling applicable to batch processing plants. Initially, the proposed model is a Mixed Integer Nonlinear Program (MINLP), and it is then reformulated as a MILP using linearization techniques. The model aims at maximizing the net profit obtained from the batch production, and it is subject to restrictions relating batch assignment, operation and equipment setup time limitations, and scheduling periods. A multi-product batch paint processing plant is considered for a real case implementation to show the effectiveness of the model.

Timpe (2002) presents a combined Mixed Integer Linear Programming / Constraint Programming (MILP/CP) model for production planning in the chemical process industry with the objective function of minimizing setup, stock holding and backlogging costs.

The MILP model is a standard capacitated dynamic lot-sizing problem and involves material balance, machine usage production setups and inventory bounds constraints. The model is developed and programmed in C++, using Dash's XPRESS-MP library functions.

Floudas and Lin (2005) made a review of the progress of MILP approaches for short-term scheduling systems. The models presented are classified by the representation of time: discrete and continuous, and some approaches to accelerate the solution process are also shown. They analyze more specific decision variables used to assign tasks to units (binary), and the amount of materials produced, consumed and available (continuous), in specified time intervals. Thus, inventory balance equations, similar to the ones showing how to add shelf-life considerations in Kallrath (2005), are presented for both discrete and continuous time models.

Chen and Ji (2007) presented an Advanced Planning and Scheduling (APS) problem modeled by MILP. The model considers capacity constraints, operation sequences, lead times, due dates and multi-level product structures (Bill of Materials). Chen and Ji addressed the MILP problem of finding the optimal schedule for the orders by composing the objective function in two main parts: first, the production idle time is to be minimized (equivalent to maximizing machine utilization), and second, orders must be completed as close to their due date as possible (minimizing tardiness and earliness penalties). The model considers precedence constraints of items, which means subassemblies and components should be completed before processing final products. The model is illustrated by a four level structure product example and solved using CPLEX. Optimal numerical results are shown and graphically presented in a Gantt chart.

Moreno and Montagna (2009) proposed a MILP model to simultaneously optimize production planning and design decisions applied to multiproduct batch production plants over a multi-period scenario. The model involves deterministic seasonal variations of costs, prices, demands and supplies. The objective of the model is to maximize the net present value of the profit (sales, investment, inventories, waste disposal and resources costs). The model calculates the plant structure and allocation of intermediate storage tanks, unit sizes, inventory levels of both product and raw materials and purchases. They present two problem examples to illustrate the key features of the formulation approach, as well as its versatility and usefulness.

As it can be see, production planning using Mixed Integer Linear Programming is an area that has been worked extensively, considering various aspects, systems, and perspectives. However, there still seems to be a long way to go in this topic, not only in formulation but also on the efficient solution of such models.

2.2 Production Planning for Perishable Products

Regardless of the formulation technique, production planning models are applied to multiple types of manufacturing systems. Due to the formulation flexibility of these models, variables and constraints are adjusted to requirements and specifics of each problem. Specifically, industries such as food and chemicals have manufacturing systems involving products and raw materials that have the characteristic of being perishable, meaning that, once they are produced, after a certain time, expire, deteriorate or cease to be completely useful and should be discarded or diminish its commercial value.

This characteristic of perishable products can be reflected in other aspects even beyond the physical conditions of the product (deterioration or depletion). Sarker and Xu (2003) considered the productive or marketable life of a product in a competitive emerging market as a form of perishability. “For example, though it is not a deteriorating item, a personal computer’s (PC) marketable life is completely dominated by the other emerging competitive items in the market. Hence, even though a relatively old PC is sparingly usable, this product has a very short shelf-life after which it is not saleable” (Sarker and Xu, 2003). In this sense, the concept of shelf-life can be defined as the time period during which a product can be stored without loss of function for which it was designed, or without loss of its usability.

To present recent and relevant contributions in the area of production planning models considering this special feature of perishability or shelf-life, we can begin with Kallrath (2002), who made an overview of some of the most encountered production planning and scheduling problems in the chemical process industry and their specific characteristics. He took into account and distinguished three classes of production systems: continuous, batch and semi-batch production. Among the several aspects needed for undertaking such problems, Kallrath refers to the possible limitations on the shelf-life time of products. Thus, it is specified that product-aging time should be traced, giving way to the application of constraints such as: “maximum shelf-life time, disposal costs for time expired products, and the setting of selling prices as a function of product life”.

Newmann et al. (2002) introduced a Mixed Integer Nonlinear Programming model for an Advanced Planning System (APS) in the context of batch production for process industries. The model is reduced to a Mixed Binary Linear Program of moderate size, and includes constraints referring to perishability of products, where production tasks are assigned to consuming tasks so that no perishable product is kept in stock at any time, i.e. the amount produced by a batch must equal the amount consumed in following tasks without delay. The proposed model was applied for a chemical industry production plant and solved by a branch-and-bound algorithm in C under MS-Visual C++ 6.0.

Entrup et al. (2005) developed three Mixed Integer Linear Programming (MILP) models that incorporate shelf-life limitations for final products in planning and scheduling for an industrial case study of stirred yoghurt production. The models presented focus on the flavoring and packaging steps of the yoghurt production process.

Considering a shelf-life-dependent pricing component, M. Entrup et al. (2005) included the shelf-life aspect in their models’ objective function, which aims at maximizing the contribution margin. Numerical investigation was carried out to assess the suitability of the models for specific planning problems. Computations were performed using ILOG's OPL Studio 3.6.1 as a modeling environment and its incorporated standard optimization software CPLEX 8.1.

Kallrath (2005) presented a compilation of Mixed Integer Optimization (including MILP) for solving planning and design problems. One of the special features in planning in the process industry where Kallrath's work delves more specifically is the case of limited shelf-life for products. According to Kallrath, in these cases, such limitations on the shelf-life require controlled records to trace time stamps of products. For this, a variable disposal cost is associated with products that have exceeded its shelf-life, are no longer useful and need to be discarded. From the above, inventory balance equations are presented and show how to add the shelf-life aspect for products.

Corominas et al. (2007) proposed two MILP models to solve production, working hours and holiday weeks for human resources in a multi-product process with perishable products. Both models have the same objective function: maximizing the profit (income minus costs due to production, product elimination, lost demand, and inventory, among others), introducing a unit cost of eliminating product that have reached its shelf-life and must be discarded. A computational experiment was conducted to evaluate the model efficiency and was solved using ILOG CPLEX 8.1.

Wang et al. (2009) presented a binary integer programming model for operations planning involving product traceability, production batch size, inventory levels, product shelf-life, and other aspects in perishable food production. They modeled two different scenarios: one with two-level bill of materials (raw materials and finished products), and one with three-level bill of materials (adding components). Shelf-life is considered to be the period between manufacture and retail purchase of a product during which the product is of satisfactory quality or saleable condition, and it is calculated by deducting the product storage time from the product life. To incorporate the shelf-life factor, a temporary price discount is applied quantifying product deterioration cost. Wang et al. note that the model is applicable not only in perishable food manufacturing contexts, but in a wider area of batch production and assembly processing. A case study with numerical simulation is implemented using Microsoft Excel. Sensitivity analyses were conducted to illustrate the proposed work.

Although we present only a portion of the available literature, most studies on the subject focus on production planning models considering shelf-life of finished-products. However, when studying systems like the ones in the composite materials manufacturing industry, it is very common to find this feature of perishability in the components (raw materials), and not so prominent in finished-products.

The following section presents available research related to shelf-life of components and finished-products mainly from the field of inventory control theory and modeling. Subsequently we introduce further the components shelf-life considerations in industries related to composites manufacturing.

2.3 Inventory Theory and Models for Perishable Products

Nahmias (1982) presented a review of existing literature related to perishable inventory theory. In this case, perishability is divided into two classes: fixed lifetime and random lifetime. The first one refers to cases in which shelf-life is known a priori and is independent of any other parameters in the system. This category also separates the cases depending on demand: deterministic and stochastic. While the second class is related to an exponential decay of shelf-life and includes cases in which it behaves randomly with a specific probability distribution.

Raafat (1991) conducted a study of the available literature in mathematical modeling of inventory systems for deteriorating (decaying) items. The reviewed models consider the decay or deterioration processes as: "any process that prevents an item from being used for its intended original use". Raafat distinguishes between cases in which all items in inventory become obsolete simultaneously at the end of their planning horizon, and those where the items deteriorate throughout it.

Goyal and Giri (2001) extended the work in Raafat (1991). They presented a review of the advances of deteriorating inventory literature since early 1990s. Deteriorating items are referred to as those (a) having a maximum usable lifetime (perishable products) and/or (b) those having no shelf-life at all (decaying products). Subsequently, based on shelf-life characteristics, the inventory models reviewed are classified into models for inventory with (i) fixed lifetime, (ii) random lifetime, and (iii) decays corresponding to the proportional inventory decrease in terms of its utility or physical quantity. According to the authors, in case (i), if a product remains unused up to its life-time, it is considered to be outdated and must be disposed; and in case (ii), the products' lifetime cannot be determined in advance while in stock. Finally, principal features of the models are specified and discussed.

Chang and Chou (2008) proposed inventory models for perishable products in the aerospace industry. The authors note that in this industry, "perishable products are raw chemical materials used on the airplane, or the raw materials for the manufacture of compound materials". An assumption of the study is that the age of arriving inventory units is zero, i.e. they arrive fresh and shelf-life begins to deduct. In addition, as in most of the related work, units that have not been used before its expiration date are discarded and are applied an outdate cost. Based on the above, Chang and Chou proposed a model with four different policy options: the first one ignores the possibility of negotiation between the supplier and the customer; model 2 includes the supplier and considers return policies; the third one joins the customer considering discounts; and the last one involves all of the above. A real aerospace enterprise was taken as an example to verify the validity of the model.

2.4 Composites Manufacturing

Although the feature of perishability of raw materials is a consideration in manufacturing systems in many different industries, the main focus of this study is in composites manufacturing and related industries. The reason for this is the great importance of composite materials for industries such as aerospace, automotive, motorsports, marine, among others.

According to Advani and Sozer (2001), in engineering, the definition of composite materials can be narrowed down to “a combination of two or more distinct materials into one with the intent of suppressing undesirable constituent properties in favor of the desirable ones”. There are mainly three types of composites: polymer matrix composites, metal matrix composites, and ceramic matrix composites.

Polymer matrix composites are constituted by two individual components: polymer resin and fibers. “The role of the polymer resin, which is also called the matrix phase of the composite, is primarily to bind the fibers together, give a nice surface appearance, and provide overall durability” (Advani and Sozer, 2011).

One of the composites processing aspects that make the manufacturing stage to be of special importance is that not only the part of the desire shape is made, but also the materials with specific properties are manufactured throughout the multiple processing phases. In addition, it is during the manufacturing process that the matrix material and the fiber reinforcement are combined and consolidated to form the composite.

2.4.1 Shelf-Life Considerations in Composites Manufacturing

As shown in Figure 1, depending on the materials to be made and the manufacturing methods, the polymer matrix can be either a thermoset or a thermoplastic material. In both categories, we can observe the presence of a crucial and relevant material: prepregs.

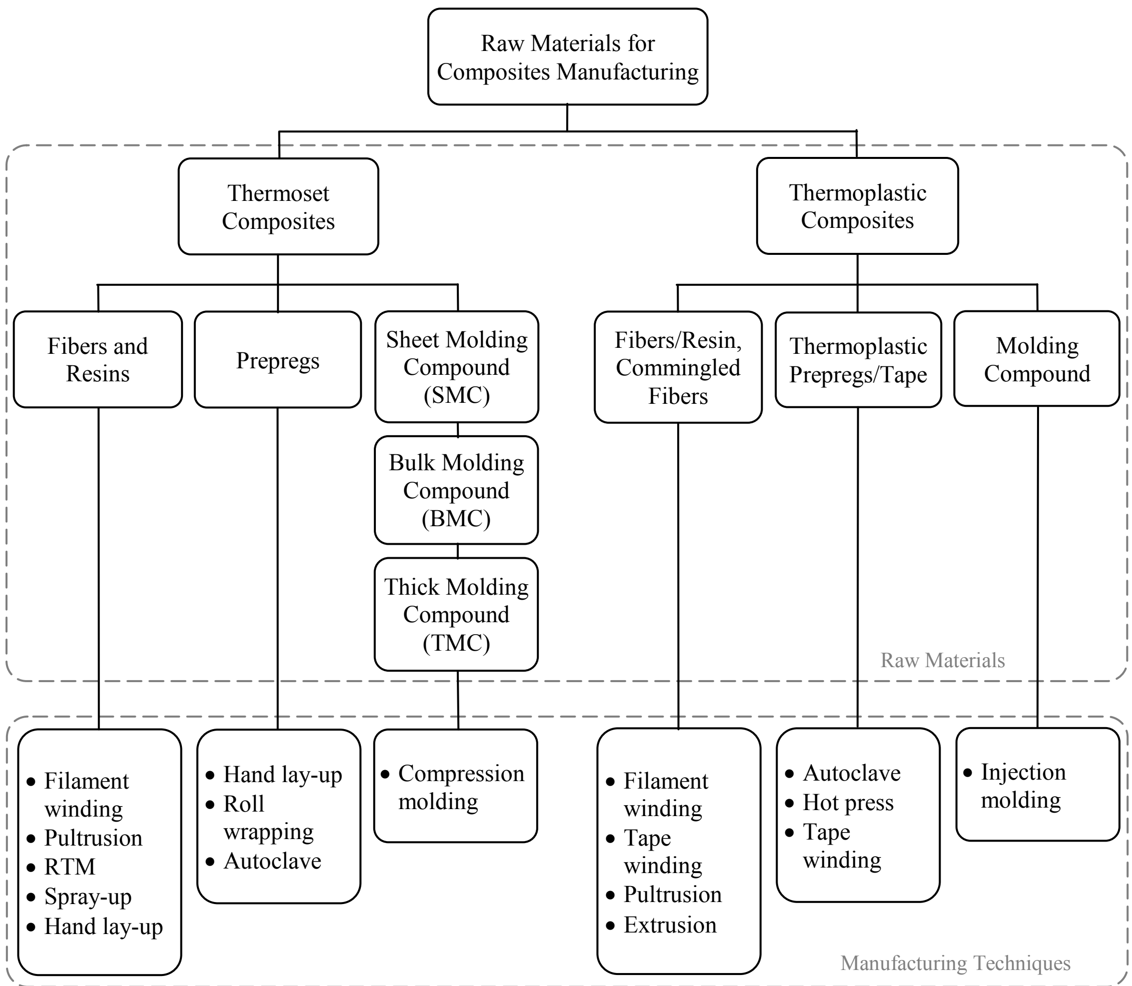


Figure 1 Classification of Raw Materials for Composites Manufacturing (Mazumdar, 2001)

A prepreg is a combination of fiber reinforcement material preimpregnated with a resin matrix. These materials and other materials consisting of mixed resin and hardener will gradually cure, even at low temperatures (Strong, 2007). For this reason, these materials are subject to a maximum storage time, shelf-life. To have longer shelf-lives, these materials may be stored at low temperatures. However, "even when kept refrigerated, the manufacturer usually sets a maximum shelf-life after which the material is assumed to have become too hard (cured) to use" (Strong, 2007). The material that has reached its maximum shelf-life is discarded from the manufacturing process.

2.4.2 Production Planning in Composites Manufacturing

Mazumdar (2001) describes the production planning procedures in composites manufacturing summarizing the following stages: (i) establishing total time needed for procurement of raw materials, inspection of raw materials, storage, manufacturing operations, delays, quality control, packaging, and shipping; (ii) calculating manufacturing equipment capacities, raw material storage, lay-up area, and more; (iii) preparing Bill of Materials and identifying methods for procuring all the materials and parts needed; (iv) establishing the list of all major activities and sub-activities (tasks); and (v) estimating manufacturing lead times and preparing schedules.

From a systematic point of view, Zhongyi et al. (2011) proposed the production planning and scheduling module of a Manufacturing Execution System (MES) for composite component manufacturing in an aerospace enterprise. Figure 2 shows the production planning and scheduling structure and workflow in composite component manufacturing.

The authors mentioned that many optimal algorithms such as simulated annealing, genetic algorithms, tabu search and neural networks are developed to solve the problem of meeting products demands in this context. They proposed an improved genetic algorithm and develop it adopting three-layer structure based on Web technology and browser/server (B/S) architecture. The developing languages are ASP.NET and C# and is implemented and applied in the composite component manufacturing workshop of an aerospace enterprise.

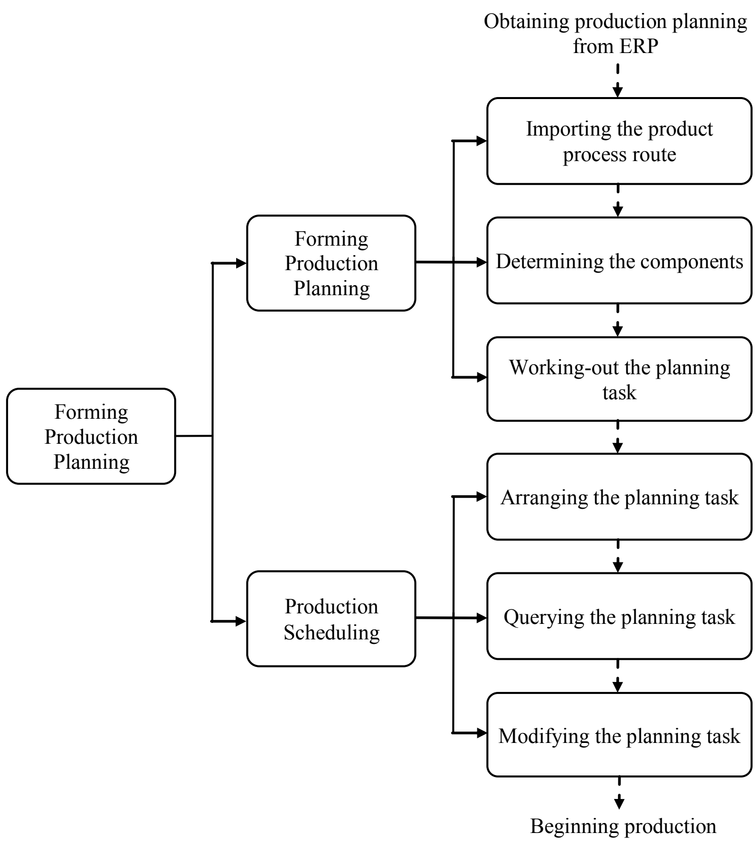


Figure 2 Production Planning and Scheduling in the Composites Manufacturing (Zhongyi et al., 2011)

From this literature review, in the next chapter we introduce the production planning problem to study and the model formulation to solve it.

3 Problem Definition and Mathematical Model

The current chapter introduces the structure of the production planning problem studied, the assumptions made, and the mathematical formulation proposed to solve it. We present two different models: the first one, called "Basic Variant", does not consider shelf-life, and it is introduced as comparison point for the main modeling. While the second one, called "Core Variant", does consider the raw material shelf-life requirement. These formulations will be applied to several problem instances using different considerations that affect the most significant problem variables and the process to achieve optimality.

While each model and problem instance will be described in detail in the following sections, they can be briefly summarized as in Table 3.1. The table shows the different considerations for each instance in which the two model formulations are applied.

Table 3.1 Model Variants, Problem Instances and Considerations

	Considerations		
	Component Shelf-Life	**Order Batch Size**	**Order Lead Time**
Model Basic Variant			
Problem Instance B1	×	×	×
Problem Instance B2	×	✓	×
Problem Instance B3	×	✓	✓
Model Core Variant			
Problem Instance C1	✓	×	×
Problem Instance C2	✓	✓	×
Problem Instance C3	✓	✓	✓

3.1 Problem Definition

The objective of the proposed core formulation is to solve a multi-item multi-level production planning problem, where the end-products are made with components (raw materials) that have shelf-life restrictions, i.e. they can be stored or kept in stock only for a limited time.

More specifically, the addressed problem considers:

- the acquisition of the components (raw materials): when and how much to order,
- the production activities to meet customer demand: when and how much to produce (transforming or assembling the components into end-products),
- the components and end-products inventory control: how much to keep on inventory during each period of the planning horizon,
- the disposal of components that have reached their shelf-life and are not suitable for use or consumption: when and how many units of component to be discarded.

These planning decisions are subject to production and ordering capacities, and to the objective of incurring the lowest possible cost. Costs are related to components and their purchase, production, holding of components and end-products in inventory, and finally, for this specific case, to components disposal.

3.1.1 Shelf-Life Consideration

The core aspect of the problem under study is the perishability condition of the components considered as raw materials for production. These components are those that, for various reasons, will expire after certain date, or can only be used for a determined period of time. The term “shelf-life” refers to the maximum length of time a component can be stored under specified conditions and remain suitable for use, consumption or for its intended function.

The relevance of the above lies in the need to track the age of components with specific time-stamps for each of them. Individual inventory control is required to properly handle the ordering/receiving of materials, their remaining shelf-life, their consumption and the subsequent disposal.

As described by Kallrath (2005), most of the data associated with inventories have to be duplicated for problems involving shelf-life, regarding additional shelf-life index. Besides the amount of inventoy kept in stock, we also need to know when the material has been ordered or received. In order to track the inventory of components that must be discarded when they expire, it is required to keep specific records of the period in which the components were received. Thus, they are later totalized as inventory to be unsuitable for use after expiration date.

If a component reaches the end of its shelf-life and expires, it will have to be discarded. This will cause additional costs: besides the cost of acquiring the component and holding it in stock, it may need to be transported to a certain disposal site, and may incur a treatment cost.

Hence, we present a mathematical model formulation that takes this factor into account by adding variables, indexes and constraints for these individual time-stamps. Also, a disposal cost per unit of discarded component is applied.

3.2 Assumptions and Notation

In this research, we consider that a manufacturer needs to develop its production plan involving a set of *N* types of end-products, using a set of *J* types of components, over a planning horizon of T periods. The company seeks at simultaneously optimizing production and component orders, as well as the inventory levels, consumption and disposal of components to minimize operating costs.

To structure the problem and the mathematical formulation, we make use of the following assumptions and notation:

- **Demand is deterministic with no back-orders:** there is no uncertainty about the quantity or timing of demand. For each end-product type $i \in N$, and each period $t \in T$, we assume that there is a forecasted demand $d_{i,t}$ that needs to be filled on time, i.e. no back-orders are considered.

- **Production is instantaneous and immediate:** we define $x_{i,t}$, $i \in N$ and $t \in T$, as the units of end-product type *i* to be produce in period *t*, and assume the entire lot is produced simultaneously. We also assume that demand for each period is to be filled with production in the same period, plus available on-hand inventory from the previous period $(t-1)$, i.e. there is no time lag between production and availability to satisfy demand.

- **End-Product Inventory:** available on-hand inventory is defined by $I_{i,t}$, $i \in N$, $t \in T$, as the units of end-product type *i* kept in stock at the end of period *t*.

- **Bill of Materials:** the dependent relationship between components and end-products is modeled through the definition of the product structure (BOM). For every component type $j \in J$ and every end-product type $i \in N$, we define $b_{j,i}$ as the amount of component type *j* needed to produce a unit of end-product type *i*.

- **Component Orders:** we consider three different problem variations relating component ordering. In first instance, for the case under the assumption that the manufacturer may order the exact amount of components required for production with immediate receipt, i.e. there is no determined component order batch size or order lead time, let $Q_{j,t}$, $j \in J$, $t \in T$, be defined as the units of component type *j* to order in period *t*. Then, let also $Q_{j,t}$, be the number of batches of component *j* to order in period *t* for the case where the manufacturer is subject to order components in batches of size S_j. We further assume that the manufacturer is also subject to ordering lead times l_j. In this case, we additionally define $q_{j,t}$ as the number of batches of component *j* scheduled to be received in period *t* as a parameter. Thus, $q_{j,t}$ is determined in advance and applies to the initial periods in which the orders $Q_{j,t}$ are not yet being received $(t \leq l_j)$. For this case, it is also important to clarify that the orders received in period *t* are those made in period $t - l_j$).

- **Component Inventory and Disposal:** as mentioned in Section 3.1.1, for problems involving shelf-life, besides the amount of inventory kept in stock, we also need to know when the material was received. Therefore, the component inventory variable $v_{j,t,r}$, for every $j \in J$, $t \in T$ and $r \in T$, is defined as the amount of component type *j* kept in stock at the end of period *t* that was received in period *r*. We assume the planning horizon starts with no available component inventory, i.e., $v_{j,0} = 0$, for problems with no order lead time considerations. For problems considering order lead times, we assume an initial inventory $v_{j,0} \geq 0$ that has been received in period *t* = 0, which means that, all inventory at the beginning of the planning horizon will have a age of 1 period, and therefore, a remaining shelf-life $a_j - 1$. All units of components kept on inventory and that have reached the end of their shelf-life will not be considered useful. For this, we define z_j as the units of component $j \in J$ that have been discarded. In addition, for problems that do not consider component shelf-life, the inventory variable is simply $v_{j,t}$, i.e, it does not need to have the *r* index,

- **Component Consumption:** a key auxiliary variable for a problem involving shelf-life considerations for components is the one referring to component consumption. To have an inventory control that allows us to track the antiquity of items, we define $e_{j,r,t}$ as the units of component type *j* that were received in period *r* and are consumed in period *t*, $j \in J$, $r \in T$, $t \in T$. This variable is not relevant for the model formulation that does not consider component shelf-life.

- **Shelf-Life:** let a_j be the maximum number of t periods that each component type $j \in J$ can be stored before it is considered unsuitable for use or consumption and must be discarded. It is assumed that the shelf-life of each component begins to be deducted from the moment it is received by the manufacturer, i.e. the age of a component with shelf-life a_j received in a given period, at the end of the same period, is 1; meaning that its remaining shelf-life is $a_j - 1$.

- **Order Batch Size and Order Lead Time:** For problem variants considering order batch size, for each component type $j \in J$, we define S_j as the order batch size. And for variants considering order lead time, for each component type $j \in J$, we define l_j as the order lead time: number of periods it takes an order to be received from the moment it is ordered.

- **Capacity:** We assume that both, production and orders have capacity constraints. For every end-product type $i \in N$, we define its capacity K_i as the maximum units of end-product type i to produce in any period t. And for every component type $j \in J$, for problem variants where there is no determined component order batch size, we define its capacity L_j as the maximum units of component type j to order in any period t. For problem variants where the manufacturer is subject to order components in batches, L_j is defined as the maximum number of batches of component type j to order in any period t.

- **End-Product Unit Costs and Component Costs:** for each end-product type $i \in N$, we define p_i as the cost of producing one unit of end-product type i in any period (not including component costs). And for each component type $j \in J$, for problem variants where there is no determined component order batch size, we define a cost c_j per unit of component type j ordered in any period. For the case where the manufacturer is subject to order components in batches, c_j is defined as per batch of component type j ordered in any period.

- **Inventory Holding Costs:** end-product inventory holding cost per unit per period for each $i \in N$ is defined by h_i, and component inventory holding cost per unit per period for each $j \in J$ is defined by m_j.

- **Component Disposal Cost:** if a component is stored in inventory until the end of its shelf-life, expires, and is not suitable for use or consumption, a disposal cost f_j per unit of component type $j \in J$ is incurred.

- **Fixed Set-Up and Ordering Costs:** regardless of the amount of end-product units produced, a production run incurs a fixed set-up cost of A_i for each end-product type $i \in N$. Also, independently of the amount of components ordered, placing an order at a period incurs a fixed ordering cost of g_j for each component type $j \in J$. For these parameters, we define the binary variables $y_{i,t}$, $W_{j,t}$ and $w_{j,t}$, $i \in N$, $j \in J$ and $t \in T$, iqual to 1 if and only if $x_{i,t} > 0$, $Q_{j,t} > 0$ and $q_{j,t} > 0$, respectively.

3.2.1 Summary of Model Parameters and Variables

Initial Inventory and Scheduled Order Receipt:

$I_{i,0}$ Initial End-Product Inventory,

$v_{j,0}$ Initial Component Inventory,

$q_{j,t}$ Component Scheduled Order to Receive,

Demand, BOM and Shelf-Life:

$d_{i,t}$ End-Product Demand,

$b_{j,i}$ Bill of Materials,

a_j Component Shelf-Life,

Component Order Batch Size and Lead Time:

S_j Order Batch Size,

l_j Order Lead Time,

Costs and Capacity:

f_j Component Unit Disposal Cost,

c_j Component Cost,

p_i End-Product Unit Cost,

h_i End-Product Inventory Holding Cost,

m_j Component Inventory Holding Cost,

A_i Fixed Set-Up Cost,

g_j Fixed Ordering Cost,

K_i Production Capacity,

L_j Ordering Capacity,

Variables:

$x_{i,t}$	Production,
$I_{i,t}$	End-Product Inventory,
$Q_{j,t}$	Component Orders,
$v_{j,t,r}$	Component Inventory,
$e_{j,r,t}$	Component Consumption,
z_j	Component Disposal,
$y_{i,t}$	Fixed set-up binary variable,
$w_{j,t}$	Fixed scheduled ordering binary variable,
$W_{j,t}$	Fixed ordering binary variable.

3.3 MIP Optimization Model

This section presents the mathematical formulation to solve the discussed problem. The formulation is a Mixed Integer Linear Programming Optimization Model with the objective function to minimize the total costs.

Two model variants are considered to evaluate different aspects of the problem. Table 3.2 details the changes in each variant.

Table 3.2 Model Variants Definition

Model Variants	Definition
Basic Variant (No Component Shelf-Life)	
This is the most basic variant. It refers to a production planning model with no components shelf-life restrictions, and it is to be applied to problem instances considering: (1) no ordering batch sizes and no order lead times, (2) ordering batch size S_j and no order lead times, and (3) ordering batch size S_j and ordering lead time l_j.	
Core Variant (Component Shelf-Life)	
This is the main formulation considering the core aspect of the study: component shelf-life. As well as the Basic Variant, it is applied to problem instances considering: (1) no ordering batch sizes and no order lead times, (2) ordering batch size S_j and no order lead times, and (3) ordering batch size S_j and ordering lead time l_j.	

The variables and the parameters have certain modifications corresponding to each of these variants as specified in Section 3.2. The details of the mathematical formulation for each variant are presented below.

3.3.1 Basic Variant: No Component Shelf-Life Considerations

In this section, we present a Mixed Integer Linear Programming model for the basic production planning problem without component shelf-life limitations. The point of presenting this model is that, later in the stage where we analyze the implementation of the proposed core mathematical formulation, relevant comparison is made between the two scenarios: production planning with and without component shelf-life requirement.

The mathematical model for the Basic Variant, which can be considered an extension of the lot-sizing with capacities formulation by Wolsey & Pochet (2006), is as follows:

Objective Function:

$$\min \sum_{i=1}^{N}\sum_{t=1}^{T} p_i x_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j Q_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j q_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{i,t}$$
$$+ \sum_{j=1}^{J}\sum_{t=1}^{T} m_j v_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} A_i y_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j W_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j w_{j,t} \qquad (3.1)$$

Minimize Total Costs: The objective function (3.1) is to minimize:

- ✓ Cost of production: $\sum_{i\in N}\sum_{t\in T} p_i x_{i,t}$,
- ✓ Cost of components: $\sum_{j\in J}\sum_{t\in T} c_j Q_{j,t} + \sum_{j\in J}\sum_{t\in T} c_j q_{j,t}$, where $q_{j,t}$ are the orders of component *j* scheduled to be received in period *t*, which for problem instances with the ordering lead time l_j consideration will be $q_{j,t} = 0$. Also, the component cost parameter c_j is per component unit for problem instances with no ordering batch consideration, and per component batch for instances with ordering batch.

- ✓ End-product and component inventory costs: $\sum_{i\in N}\sum_{t\in T} h_i I_{i,t} + \sum_{j\in J}\sum_{t\in T} m_j v_{j,t}$, where the component inventory variable $v_{j,t}$ does not need to specify the period *r* in which the component was received,
- ✓ Set-up and ordering costs: $\sum_{i\in N}\sum_{t\in T} A_i y_{i,t} + \sum_{j\in J}\sum_{t\in T} g_j W_{j,t} + \sum_{j\in J}\sum_{t\in T} g_j w_{j,t}$.

Subject to:

$$I_{i,t} = I_{i,t-1} + x_{i,t} - d_{i,t} \quad \forall i \in N,\ \forall t \in T \tag{3.2}$$

End-Product Inventory Balance, Production, and Demand Fulfillment: constraint (3.2) imposes that end-product demand is filled in every period, either with production, inventory, or both. It also sets the end-products inventory level for each period *t*. Inventory level at the end of period *t* ($I_{i,t}$) must be equal to inventory of end-products at the end of previous period ($I_{i,t-1}$), plus the units of end-products to be produced ($x_{i,t}$), minus the demand ($d_{i,t}$), which can also be interpreted as end-product consumption.

$$v_{j,t} = v_{j,t-1} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ 1 < t \leq l_j \tag{3.3}$$

$$v_{j,t} = v_{j,t-1} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ t > l_j \tag{3.4}$$

Component Inventory Balance, Component Ordering, and BOM: constraints (3.3) and (3.4) ensure component orders to satisfy the requirements for production. They also define the component inventory level for each period *t*. Component inventory at the end of period *t* ($v_{j,t}$) must be equal to inventory at the end of previous period ($v_{i,t-1}$), plus units of component received in period *t*, either from scheduled orders ($q_{j,t}$) or from orders made within the planning horizon ($Q_{j,t}$), minus component consumption according to bill of materials ($b_{j,i}$) for end-products. Note that $Q_{j,t}$ only starts to be relevant when $t > l_j$, because no orders made within the planning horizon will be received before the order lead time l_j. For problem instances without ordering batch size consideration $S_j = 1$, and for problem instances without ordering lead time consideration $l_j = 0$.

$$x_{i,t} \leq K_i \qquad \forall i \in N, \ \forall t \in T, \tag{3.5}$$

Production Capacity: constraint (3.5) sets the limits on production capacity for each end-product type $i \in N$.

$$Q_{j,t} \leq L_j \qquad \forall j \in J, \ \forall t \in T \tag{3.6}$$

Ordering Capacity: constraint (3.6) sets the limits on ordering capacity for each component type $j \in J$. For the case of problem instances with no ordering batch size consideration, the ordering capacity parameter L_j is defined as the maximum units of component type $j \in T$ to order in any period t. For the case with ordering batch size consideration, L_j is defined as the maximum number of batches of component to order in any period t.

$$x_{i,t} \leq My_{i,t} \qquad \forall i \in N, \ \forall t \in T, \tag{3.7}$$

$$Q_{j,t} \leq MW_{j,t} \qquad \forall j \in J, \ \forall t \in T, \tag{3.8}$$

$$q_{j,t} \leq Mw_{j,t} \qquad \forall j \in J, \ \forall t \in T, \tag{3.9}$$

Fixed Set-Up and Ordering Cost Binary Variables: constraint (3.7) ensures that if $x_{i,t} > 0$, then the fixed set-up cost binary variable $y_{i,t} > 0$, and so necessarily $y_{i,t} = 1$.

Constraint (3.8) ensures that if $Q_{j,t} > 0$, then the fixed ordering cost binary variable $W_{j,t} > 0$, and so necessarily $W_{j,t} = 1$. And constraint (3.9) respectively does the same for $q_{j,t}$ and $w_{j,t}$ *(M is a large positive number)*.

Basic Variant Model Formulation Summary

Summarizing the above discussion, the mixed integer programming model formulation for the Basic Variant is given below:

$$\min \sum_{i=1}^{N}\sum_{t=1}^{T} p_i x_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j Q_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j q_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} m_j v_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} A_i y_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j W_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j w_{j,t} \quad (3.1)$$

Subject to:

$$I_{i,t} = I_{i,t-1} + x_{i,t} - d_{i,t} \quad \forall i \in N, \quad \forall t \in T \quad (3.2)$$

$$v_{j,t} = v_{j,t-1} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J, \ \forall t \in T, \ 1 < t \le l_j \quad (3.3)$$

$$v_{j,t} = v_{j,t-1} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J, \ \forall t \in T, \ t > l_j \quad (3.4)$$

$$x_{i,t} \le K_i \quad \forall i \in N, \ \forall t \in T, \quad (3.5)$$

$$Q_{j,t} \le L_j \quad \forall j \in J, \ \forall t \in T \quad (3.6)$$

$$x_{i,t} \le M y_{i,t} \quad \forall i \in N, \ \forall t \in T, \quad (3.7)$$

$$Q_{j,t} \leq MW_{j,t} \qquad \forall j \in J,\ \forall t \in T, \tag{3.8}$$

$$q_{j,t} \leq Mw_{j,t} \qquad \forall j \in J,\ \forall t \in T, \tag{3.9}$$

$$x_{i,t},\ I_{i,t},\ Q_{j,t},\ v_{j,t},\ z_j\ \geq 0 \quad \forall i, j, t$$

$$y_{i,t},\ W_{j,t},\ w_{j,t}\ \in\ \{0, 1\}$$

3.3.2 Core Variant: Component Shelf-Life Considerations

In this section we present in detail the core mathematical formulation to solve the production planning problem with raw material shelf life. It is a MILP optimization model specific enough to involve the most unique features of the problem, and sufficiently flexible and/or general to be applied to multiple problem instances with different considerations. These considerations relate to the variables and parameters that mostly affect the results of the problem: ordering batch size and ordering lead time.

The formulation for the Core Variant production planning problem is as follows:

Objective Function:

$$\begin{aligned} \min \sum_{i=1}^{N}\sum_{t=1}^{T} p_i x_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j Q_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j q_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{i,t} \\ + \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{r=0}^{T} m_j v_{j,t,r} + \sum_{j=1}^{J} f_j z_j \\ + \sum_{i=1}^{N}\sum_{t=1}^{T} A_i y_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j W_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j w_{j,t} \end{aligned} \tag{3.10}$$

Minimize Total Costs: The objective function (3.10) is to minimize the cost of production, cost of components, end-product and component inventory holding costs, component disposal cost ($\sum_{j \in J} f_j z_j$), and set-up and ordering cost. It is important to note that in this case, the component inventory variable $v_{j,t,r}$ includes the component receipt index r, to specify the period in which the component is received. In addition, for problem instances considering initial inventory of component $v_{j,0} \geq 0$ with an age of 1 period, index r will begin to be consider from period t = 0, i.e., we assume that the inventory of component at the beginning of the planning horizon has been received in period t = 0, and therefore has a shelf-life $a_j - 1$. Just as in the Basic Variant, the component cost parameter c_j is per unit of component for problem instances with no ordering batch consideration, and per component batch for instances with ordering batch.

Subject to:

$$I_{i,t} = I_{i,t-1} + x_{i,t} - d_{i,t} \quad \forall i \in N,\ \forall t \in T \tag{3.11}$$

End-Product Inventory Balance, Production, and Demand Fulfillment: constraint (3.11) imposes that end-product demand is filled in every period, either through production, inventory, or both. It also sets the inventory level for end-products for each period t. Inventory level at the end of period t ($I_{i,t}$) is equal to inventory of end-products at the end of previous period ($I_{i,t\text{-}1}$), plus the units of end-products to be produced ($x_{i,t}$), minus end-product consumption ($d_{i,t}$).

Component Inventory Balance, Component Ordering, and BOM: The following constraints (3.12), (3.13), (3.14), (3.15) and (3.16) ensure component orders to satisfy the requirements for production. They also define the component inventory level for each period *t*. Notice that, inventory level calculation varies depending on the period.

$$\sum_{r=0}^{t} v_{j,t,r} = v_{j,0} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ \forall r \in T,\ t = 1 \tag{3.12}$$

Equation (3.12) specifies the component inventory ($v_{j,t,r}$) at the end of the first period $t = 1$. In this case, the component inventory consists on the one coming from the initial inventory ($v_{j,0}$) i.e., interpreted as having been received in period $r = 0$, and the one cause by the receipt of scheduled orders in period $t = 1$ ($S_j q_{j,t}$). In cases where no ordering batch size is considered $S_j = 1$.

$$\sum_{r=0}^{t} v_{j,t,r} = \sum_{r=0}^{t-1} v_{j,t-1,r} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ 1 < t < a_j,\ t \leq l_j \tag{3.13}$$

Now in equation (3.13), when component inventory is being observed in periods that are lower than component shelf-life ($t < a_j$) and lower or equal to the order lead time $t \leq l_j$, it is still possible to have component in inventory from the one assumed to be received in period $r = 0$. Also, the component inventory contains the remaining units of scheduled orders that are received in period *t*. In cases where no ordering lead time is considered $l_j = 0$.

$$\sum_{r=t+1-a_j}^{t} v_{j,t,r} = \sum_{r=t+1-a_j}^{t-1} v_{j,t-1,r} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t,\ t \geq a_j,\ t \leq l_j \tag{3.14}$$

Equation (3.14) makes sure that when period $t \geq a_j$, only those components received starting from period $t + 1 - a_j$ can be hold in inventory. All units of inventory received before then would have had to be discarded. Additionally, it still maintains the assumption that $t \leq l_j$, which means that only scheduled orders $q_{j,t}$ would have been received in period t. It is important to remember that scheduled orders are only relevant when ordering lead time l_j is considered.

$$\sum_{r=0}^{t} v_{j,t,r} = \sum_{r=0}^{t-1} v_{j,t-1,r} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t, \;\; 1 < t < a_j, \;\; t > l_j \tag{3.15}$$

Constraint (3.15) still considers the time before expiration $t < a_j$ as in (3.13), but now it calculates the component inventory for periods greater than the order lead time ($t > l_j$). This means that component orders $Q_{j,t}$ made within the planning horizon in period $t - l_j$ would have been received.

$$\sum_{r=t+1-a_j}^{t} v_{j,t,r} = \sum_{r=t+1-a_j}^{t-1} v_{j,t-1,r} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t, \;\; t \geq a_j, \;\; t > l_j \tag{3.16}$$

Finally, constraint (3.16) calculates the component inventory for periods beyond component shelf-life and order lead times ($t \geq a_j$ and $t > l_j$), which implies that only those components received in periods starting from $t + 1 - a_j$ can be considered, because the ones received before then are already discarded. Furthermore, equation (3.16) considers the receipt of orders made within the planning horizon ($Q_{j,t}$) in period $t - l_j$.

$$\sum_{i=1}^{N} b_{j,i} x_{i,t} = \sum_{r=0}^{t} e_{j,r,t} \qquad \forall j \in J, \;\; \forall t \in T, \;\; t < a_j \tag{3.17}$$

$$\sum_{i=1}^{N} b_{j,i} x_{i,t} = \sum_{r=t+1-a_j}^{t} e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\quad a_j \leq t \tag{3.18}$$

Component Consumption: Constraints (3.17) and (3.18) define the component consumption variable $e_{j,r,t}$. Units of component j consumed for production in period t, ordered in different previous periods r and that have not yet exceeded its shelf life time a_j, must be equal to all component requirements in that same period t. Notice that, depending on the period t when it is being calculated, the components considered for consumption vary. Thus, if the consumption for component j is being calculated in a period t lower than the component's shelf-life a_j (3.17), then all components ordered starting from period $r = 0$ will be considered. However, if the consumption for component j is being calculated in a period t greater or equal to component's shelf-life (3.18), then only those components received starting at period $t + 1 - a_j$ will be considered, because all components received before then would have been discarded.

$$v_{j,t,r} = S_j q_{j,t} - e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\quad \forall r \in T,\ r = t,\ t \leq l_j \tag{3.19}$$

$$v_{j,t,r} = S_j Q_{j,t-l_j} - e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\quad \forall r \in T,\ r = t,\ t > l_j \tag{3.20}$$

$$v_{j,t,r} = v_{j,t-1,r} - e_{j,r,t} \quad \forall j,t,r,\ t - r < a_j,\ 1 \leq j \leq J,\ 1 \leq r \leq T \tag{3.21}$$

Component Individual Inventory: constraints (3.19), (3.20) and (3.21) represent conservation of components. Inventory of component j at the end of period t that was received in period r ($v_{j,t,r}$) is equal to, in (3.19) when $r = t$ and $t \leq l_j$, the units of component scheduled to be received in period t ($S_j q_{j,t}$), minus consumption of the same component in the same period ($e_{j,r,t}$). In (3.20) when $r = t$ and $t > l_j$, component inventory is equal to the units of component ordered in period $t - l_j$ ($S_j Q_{j,t-l_j}$), minus the consumption of the same component in the same period ($e_{j,r,t}$). And in (3.21) when $t - r < a_j$, to the inventory of component from previous period ($v_{i,t-1,r}$) received in period r, minus the component consumption at the same period ($e_{j,r,t}$). Notice that, if the component inventory is being observed in the same period r that it was received ($t = r$), then it is going to depend on the units received in that same period t ($Q_{j,t}$). In contrast, if the component inventory is being calculated on a period t different than the one r when it was received, it will depend of the inventory of the same component at the end of the previous period ($v_{j,t-1,r}$), as long as the storage time of the component is not greater than its shelf-life ($t - r < a_j$), otherwise, it would have had to be discarded.

$$z_j = \sum_{t=a_j}^{T} v_{j,t,t+1-a_j} \quad \forall j \in J \tag{3.22}$$

Component Disposal: constraint (3.22) consolidates the units of component to be discarded because they exceeded their shelf-life.

$$x_{i,t} \le K_i \quad \forall i \in N,\ \forall t \in T \tag{3.23}$$

Production Capacity: constraint (3.23) sets the limits on production capacity for each end-product type $i \in N$.

$$Q_{j,t} \le L_j \quad \forall j \in J,\ \forall t \in T \tag{3.24}$$

Ordering Capacity: constraint (3.24) sets the limits on ordering capacity for each component type $j \in J$.

$$x_{i,t} \leq My_{i,t} \quad \forall i \in N,\ \forall t \in T \tag{3.25}$$

$$Q_{j,t} \leq MW_{j,t} \quad \forall j \in J,\ \forall t \in T \tag{3.26}$$

$$q_{j,t} \leq Mw_{j,t} \qquad \forall j \in J,\ \forall t \in T, \tag{3.27}$$

Fixed Set-Up and Ordering Cost Binary Variables: constraint (3.25) ensures that if $x_{i,t} > 0$, then the fixed set-up cost binary variable $y_{i,t} > 0$, and so necessarily $y_{i,t} = 1$. Constraint (3.26) ensures that if $Q_{j,t} > 0$, then the fixed ordering cost binary variable $W_{j,t} > 0$, and so necessarily $W_{j,t} = 1$. And constraint (3.27) does so respectively for $q_{j,t}$ and $w_{j,t}$ *(M is a large positive number).*

Core Variant Model Formulation Summary

Summarizing the above discussion, we have that, for *Variant C*, the mixed integer programming model formulation is as follows:

$$\begin{aligned}\min \sum_{i=1}^{N}\sum_{t=1}^{T} p_i x_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j Q_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} c_j q_{j,t} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{i,t} \\ + \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{r=0}^{T} m_j v_{j,t,r} + \sum_{j=1}^{J} f_j z_j \\ + \sum_{i=1}^{N}\sum_{t=1}^{T} A_i y_{i,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j W_{j,t} + \sum_{j=1}^{J}\sum_{t=1}^{T} g_j w_{j,t}\end{aligned} \tag{3.10}$$

Subject to:

$$I_{i,t} = I_{i,t-1} + x_{i,t} - d_{i,t} \quad \forall i \in N, \qquad \forall t \in T \tag{3.11}$$

$$\sum_{r=0}^{t} v_{j,t,r} = v_{j,0} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ \forall r \in T,\ t = 1 \tag{3.12}$$

$$\sum_{r=0}^{t} v_{j,t,r} = \sum_{r=0}^{t-1} v_{j,t-1,r} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j \in J,\ \forall t \in T,\ 1 < t < a_j,\ t \leq l_j \tag{3.13}$$

$$\sum_{r=t+1-a_j}^{t} v_{j,t,r} = \sum_{r=t+1-a_j}^{t-1} v_{j,t-1,r} + S_j q_{j,t} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t,\ t \geq a_j,\ t \leq l_j \tag{3.14}$$

$$\sum_{r=0}^{t} v_{j,t,r} = \sum_{r=0}^{t-1} v_{j,t-1,r} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t,\ 1 < t < a_j,\ t > l_j \tag{3.15}$$

$$\sum_{r=t+1-a_j}^{t} v_{j,t,r} = \sum_{r=t+1-a_j}^{t-1} v_{j,t-1,r} + S_j Q_{j,t-l_j} - \sum_{i=1}^{N} b_{j,i} x_{i,t} \quad \forall j, t,\ t \geq a_j,\ t > l_j \tag{3.16}$$

$$\sum_{i=1}^{N} b_{j,i} x_{i,t} = \sum_{r=0}^{t} e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\ 1 \leq t < a_j, \tag{3.17}$$

$$\sum_{i=1}^{N} b_{j,i} x_{i,t} = \sum_{r=t+1-a_j}^{t} e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\ \ a_j \leq t < T \tag{3.18}$$

$$v_{j,t,r} = S_j q_{j,t} - e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\ \ \forall r \in T,\ r = t,\ t \leq l_j \tag{3.19}$$

$$v_{j,t,r} = S_j Q_{j,t-l_j} - e_{j,r,t} \qquad \forall j \in J,\ \forall t \in T,\ \ \forall r \in T,\ r = t,\ t > l_j \tag{3.20}$$

$$v_{j,t,r} = v_{j,t-1,r} - e_{j,r,t} \quad \forall j \in J,\ \forall t \in T,\ \forall r \in T,\ r < t,\ t - r < a_j \tag{3.21}$$

$$z_j = \sum_{t=a_j-1}^{T} v_{j,t,t+1-a_j} \quad \forall j \in J \tag{3.22}$$

$$x_{i,t} \leq K_i \quad \forall i \in N,\ \forall t \in T \tag{3.23}$$

$$Q_{j,t} \leq L_j \quad \forall j \in J,\ \forall t \in T \tag{3.24}$$

$$x_{i,t} \leq My_{i,t} \quad \forall i \in N,\ \forall t \in T \tag{3.25}$$

$$Q_{j,t} \leq MW_{j,t} \quad \forall j \in J,\ \forall t \in T \tag{3.26}$$

$$q_{j,t} \leq Mw_{j,t} \qquad \forall j \in J,\ \forall t \in T, \tag{3.27}$$

$$x_{i,t},\ I_{i,t},\ Q_{j,t},\ v_{j,t,r},\ e_{j,r,t},\ z_j\ \geq 0 \ \ \forall i,j,t,r$$

$$y_{i,t},\ W_{j,t},\ w_{j,t}\ \in\ \{0,1\}$$

Refer to Appendix 1. IBM® ILOG® CPLEX® OPL Model Source File to view the model formulations written in IBM® ILOG® CPLEX® OPL format.

3.4 Model Solution Methodology

In order to validate and evaluate the relevance of the optimization model presented above, we apply it to different instances of the production planning problem. Initially, we assume hypothetical values for the parameters and test the model using IBM® ILOG® CPLEX® Optimization Studio Version 12.4.

Subsequently, we present a production planning problem applied to the automotive industry, although still mostly hypothetical, the parameter values are based on actual data collected from the composite manufacturing industry and from the automotive industry.

The results of applying the model are analyzed and discussed in depth in Chapter 4. We analyze the behavior of each of the model variants and instances of the problem, discussing specific aspects related to processing times, number of constraints and variables used to solve it, optimality gaps, as well as outcomes related to the most unique variables of the model: components shelf-life and their disposal.

3.4.1 IBM® ILOG® CPLEX® Optimization Studio

The IBM® ILOG® CPLEX® Optimization Studio is an optimization software package that solves linear, mixed integer linear, quadratic, mixed integer quadratic and constraint programming formulations with CPLEX® Optimizers.

CPLEX® was originally developed by Robert E. Bixby, current research professor of management in Rice University and who is a noted authority on the theory and practice of optimization (Rice University, 2008). CPLEX® was offered commercially starting in 1988 by CPLEX Optimization Inc., which was acquired by ILOG® in 1997. ILOG® was later acquired by IBM® in January 2009, for approximately $340 million USD (IBM, 2009). CPLEX® uses the programming language called OPL (Optimization Programming Language) to model the mathematical formulation. The specific individual formulations for each variation of the mathematical model written in OPL format can be found in Appendix 1. IBM® ILOG® CPLEX® OPL Model Source File.

CPLEX® Mixed Integer Optimizer employs a branch-and-cut technique that takes advantage of innovative strategies to provide high-performance solutions for the hardest mixed integer programs. CPLEX® can solve mixed integer linear, mixed integer quadratic and mixed integer quadratically constrained problems. CPLEX® Mixed Integer Optimizers include the CPLEX® presolve algorithm, sophisticated cutting-plane strategies and feasibility heuristics (IBM Corporation, 2010).

4 NUMERICAL EXAMPLE, RESULTS AND ANALYSIS

In this chapter we illustrate, validate and evaluate in depth the previously proposed model formulation. We present different numerical example instances of the production planning problem with component shelf-life considerations. These problems are solved by the mixed integer linear programming optimization model proposed in Chapter 3.

Although the example instances are hypothetical, we also present a case in which the values are based on data and information collected from real sources of composite materials manufacturing.

In order to have a better understanding of the performance of the mathematical formulation, multiple instances for the same numerical example problem are solved and analyzed. The different instances vary in size, i.e. number of time periods in the planning horizon, and number of types of components. They also vary with respect to lower or higher costs, shorter or longer shelf-life times, and smaller of bigger order batch sizes. Each of the mathematical model variations is applied to solve the example problem instances using IBM® ILOG® CPLEX® Optimization Studio Version 12.4 on a computer with a 2.00 GHz Intel® Core™ 2 Duo processor, 4.00 GB installed memory (RAM), and a 32-bit operating system. The results are presented, analyzed and discussed.

4.1 Example Problem Instances

The features of the different problem instances are presented in Table 3.1 in Chapter 3 to specify which formulation is applied. In this section, Table 4.1 displays them again focusing on the different considerations related to component shelf-life, ordering bath size, and ordering lead time. Additionally, Table 4.1 also specifies the model variant to be applied for each instance depending on its assumptions.

Table 4.1 Different Example Problem Instances by Considerations

Problem Instances					
No Order Batch Size		Order Batch Size			
No Order Lead Time		No Order Lead Time		Order Lead Time	
B1	*C2*	*B2*	*C2*	*B3*	*C3*
No Shelf-Life	Shelf-Life	No Shelf-Life	Shelf-Life	No Shelf-Life	Shelf-Life
Basic Variant	Core Variant	Basic Variant	Core Variant	Basic Variant	Core Variant

In addition to the above classification, the problem instances to develop also differ in their parameters and size. For this, we divide them into two groups: ALPHA and BETA.

ALPHA instances have:

- $J = 2$ types of components,
- Lower inventory costs,
- Shorter component shelf-life,
- Shorter ordering lead times (when applicable),
- Lower fixed set-up costs,
- Lower disposal costs,

- Smaller ordering batch size (when applicable).

BETA instances have:

- $J = 3$ types of components,
- Higher inventory costs,
- Longer component shelf-life,
- Longer ordering lead times (when applicable),
- Higher fixed set-up costs,
- Higher disposal costs,
- Bigger ordering batch size (when applicable).

All problem instances are applied using $N = 2$ types of end-products, and four different planning horizons, $T = 6$, $T = 8$, $T = 10$ and $T = 12$.

All parameter values for each of the problem instances are presented below.

4.1.1 Demand, Bill of Materials and Shelf-Life

For our numerical problem examples, we assume that a manufacturer company requires planning its production to meet the end-products demand. Table 4.2 shows the values of the end-products demand parameter $d_{i,t}$ for each period t in the planning horizon T, for each of the different instances proposed.

Table 4.2 Demand Parameter for Each Problem Instance

$t \in T$	1	2	3	4	5	6	7	8	9	10	11	12
***T* = 6**												
***i* = 1**	100	90	100	120	100	120	-	-	-	-	-	-
***i* = 2**	70	80	120	110	90	90	-	-	-	-	-	-
***T* = 8**												
***i* = 1**	100	90	100	120	100	120	100	90	-	-	-	-
***i* = 2**	70	80	120	110	90	90	100	100	-	-	-	-
***T* = 10**												
***i* = 1**	100	90	100	120	100	120	100	90	110	130	-	-
***i* = 2**	70	80	120	110	90	90	100	100	110	110	-	-
***T* = 12**												
***i* = 1**	100	90	100	120	100	120	100	90	110	130	100	90
***i* = 2**	70	80	120	110	90	90	100	100	110	110	90	80

Having determined the end-product demand for the planning problem, the next step is to establish the product structure, i.e. Bill of Materials ($b_{j,i}$), to know the required amount of components j that constitute each end-product type i. Table 4.3 presents these amounts, as well as the component shelf-life a_j for each problem instance.

It is important to emphasize that the component shelf-life and their inventory holding costs are directly related. Depending on the context in which the problem is addressed, it is to be expected that storage conditions influence the time that the components can be stored. Thus, for longer shelf-lives, we usually have to incur higher inventory (storage) costs and vice versa. This direct relation between shelf-life and component inventory holding costs is taken into account in defining of the example problem instances.

Table 4.3 Bill Of Materials and Shelf-Life for Each Problem Instance

	Bill Of Materials $b_{j,i}$		**Component Shelf-Life a_j**
	i **= 1**	i **= 2**	
ALPHA (J = 2, shorter shelf-life)			
j **= 1**	5	2	2
j **= 2**	3	4	3
BETA (J = 3, longer shelf-life)			
j **= 1**	5	2	3
j **= 2**	3	4	4
j **= 3**	2	2	5

4.1.2 Capacities, Component Order Batch Size and Order Lead Time

Table 4.4 shows the production capacity K_i for each end-product type $i \in N$, the ordering capacity L_j, order batch sizes S_j, and order lead times l_j for each component type $j \in J$. All of the above for each example problem instance.

Depending on the problem instance to be solved, these parameters are used differently. For example, the capacity parameter L_j is used in terms of component units for instances that do not consider order batch sizes, and it is used in terms of batches for problems with order batch size considerations. The same applies to the orders lead time l_j, it will only be used when applying model instances *B3* and *C3*.

Table 4.4 Production and Ordering Capacity, and Order Batch Size

Production Capacity K_i		Ordering Capacity L_j		Order Batch Size S_j*	Order Lead Time S_j*
ALPHA (lower batch size, shorter lead times, and $J = 2$)					
$i = 1$	210	$j = 1$	6,400 (8 batches)	800	2
$i = 2$	160	$j = 2$	4,900 (7 batches)	700	2
BETA (higher batch size, longer lead times, and $J = 3$)					
$i = 1$	210	$j = 1$	9,600 (8 batches)	1200	2
$i = 2$	160	$j = 2$	8,400 (7 batches)	1200	3
		$j = 3$	6,000 (6 batches)	1000	4

* Only used when applying model Variants *Basic 2*, *Basic 3*, *Variant B*, and *Variant C*.

4.1.3 Costs

The purpose of the application of mathematical model to a practical production planning problem is to optimize the results, i.e. to minimizing total operating costs.

Table 4.5 shows all the costs associated with end-products production and inventory.

Table 4.5 End-Product Unit, Inventory Holding and Fixed Set-Up Costs

	End-Product Unit Cost p_i (\$)	Inventory Holding Cost h_i (\$)	Fixed Set-Up Cost A_i (\$)
ALPHA (lower set-up costs)			
$i = 1$	60	16	3,000
$i = 2$	70	18	3,500
BETA (higher set-up costs)			
$i = 1$	60	16	6,000
$i = 2$	70	18	5,500

Table 4.6 shows all costs associated with components, component inventory and disposal, and component ordering.

Table 4.6 Component, Inventory Holding, Disposal and Fixed Ordering Costs

	Component Cost c_j (\$)*	Inventory Holding Cost m_j (\$)	Disposal Cost f_j (\$)	Fixed Ordering Cost g_j (\$)
ALPHA (lower component inventory costs, and lower disposal costs)				
$j = 1$	3.125 (2,500 / batch)	2	7	1,000
$j = 2$	2.286 (1,600 / batch)	3	8	1,500
BETA (higher component inventory costs, and higher disposal costs)				
$j = 1$	2.08 (2,500 / batch)	3	9	1,000
$j = 2$	1.33 (1,600 / batch)	4	10	1,500
$j = 3$	2.00 (2,000 / batch)	5	11	1,200

** Component cost c_j per batch is only used when applying model variants with order batch size consideration.

We assume the planning horizon starts with no available end-product inventories ($I_{i,0} = 0$) for model variants with no order lead time considerations, and with $I_{i,0} \geq 0$ for model variants considering order lead time

4.2 Results and Analysis

The following are the results of the different model formulations to address each of the above introduced instances. We compare the behavior of each model variant for each instance in terms of the number of constraints, variables (binary, general integer and continuous), average running time and number of nodes and iterations required to achieve the optimal solution.

Additionally, we compare the formulations in terms of their optimality gap between the objective value and the linear relaxation, as well as the most relevant variable associated cost: component disposal cost.

4.2.1 Model Comparison for ALPHA Problem Instances

All instances in the ALPHA category have the same parameter levels: lower inventory costs, shorter shelf-life, shorter order lead times, lower set-up costs, smaller order batch size, $J = 2$, and vary in the number of periods T.

Table 4.7 presents the computational results for the ALPHA problem instances and shows obvious variations in the majority of indicators depending on the formulation and the considerations included in the problem. For example, problem instances considering No Component Shelf-Life ($B1$, $B2$ and $B3$) have a smaller number of constraints and variables, but not necessarily the optimal solution is achieved with fewer nodes, iterations or running time. In fact, for ALPHA instances with T = 6, in terms of time, all formulations require similar computational times.

Table 4.7 Model Comparison / ALPHA Problem Instances (T = 6)

ALPHA T **= 6,** N **= 2,** J **= 2**	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_j**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	96	338	96	338	120	434
Variables	73	161	73	161	85	188
Binary	24	24	24	24	36	36
Integer	48	134	48	134	48	149
Continuous	1	3	1	3	1	3
Non-zero coefficients	164	321	164	321	172	341
Avg. Time (hrs:min:sec:cs)	00:00:02:21	00:00:02:44	00:00:03:76	00:00:03:98	00:00:02:32	00:00:04:61
# of Nodes	15	51	4,607	3,908	457	615
# of Iterations	115	205	152,520	19,905	2,111	3,163
Linear Relaxation	$128,296	$130,168	$128,295	$128,295	$130,760	$130,760
Objective Value	$141,965	$141,965	$152,520	$153,124	$149,235	$150,128
Optimality Gap	0.0096	0.0083	0.158	0.162	0.124	0.129
Disposal Cost ($\sum_{j\in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	$2,182	No Disposal	$3,490

Optimality Gap: As to the optimality gap, When comparing each of the formulations with the same considerations in pairs, that is *B1* with *C1*, *B2* with *C2*, and *B3* with *C3*, there is not a significant variation in the optimality gap values. However, when transversely comparing between them, the optimality gap will be lower for variants with No Order Batch Size, and higher for instances *B2* and *C2*

No disposal cost for instance *C1*: One of the most important aspects to note when comparing the different model applications is presented in instance *C1*. Even considering component shelf-life requirements, it is not expected to incur disposal costs, since problem instance *C1* includes the No Order Batch Size assumption. The manufacturer can always order the exact amount of components required for production. Therefore, the optimal solution will not incur components disposal.

To extend the above, assume that, at any period t, we order a number of component type j ($Q_{j,t}$) large enough to generate an inventory that, at the end of period $t + a_j$, has to be discarded; because the component orders are not subject to a specific batch size, there is always the possibility of ordering less component, even the minimum needed for production, and thus, only incurring inventory costs and not disposal.

Table 4.8 Model Comparison / ALPHA Problem Instances (T = 8)

ALPHA T **= 8,** N **= 2,** J **= 2**	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_j**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	128	546	128	546	160	674
Variables	97	247	97	247	113	282
Binary	32	32	32	32	48	48
Integer	64	212	64	212	64	231
Continuous	1	3	1	3	1	3
Non-zero coefficients	220	437	220	437	232	461
Avg. Time (hrs:min:sec:cs)	00:00:02:40	00:00:03:86	00:00:47:64	00:00:21:17	00:00:04:16	00:00:06:13
# of Nodes	57	38	301,868	68,933	6,864	10,847
# of Iterations	296	266	1,304,423	376,967	33,623	65,645
Linear Relaxation	\$168,766	\$170,961	\$168,764	\$168,764	\$171,230	\$171,230
Objective Value	\$188,476	\$188,476	\$204,752	\$204,540	\$199,411	\$200,838
Optimality Gap	0.104	0.093	0.175	0.175	0.141	0.147
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	\$3,346	No Disposal	\$2,546

Higher total costs when no component shelf-life: It is important to emphasize that, in situations (such as the one shown in Table 4.8 occurring between instance *B2* and *C2*), it is possible for the total costs to be higher when there are no component shelf-life considerations.

This is because, due to orders batch size, depending on the number of periods that are being planned and the relations between inventory and disposal costs, we may have to hold inventories for components and/or end-products for several periods and in amounts such that the costs are higher than those occurred if the components were discarded.

The aforementioned phenomenon can be explained more specifically as follows: first, assume for instance *C2* (with component shelf-life), at any period *t*, we have to dispose $v_{j,t,t+1-a_j}$ units of component type *j*, incurring in inventory holding and disposal cost of $(m_j + f_j) \times v_{j,t,t+1-a_j}$. Now, for variant *B2* (no component shelf-life), those same units of component type *j* will not be discarded but kept in inventory at the end of the same period *t* ($v_{j,t}$) with an inventory holding cost of $m_j \times v_{j,t}$. In the event that, for the following period (*t* + 1), it is required to order a number $Q_{j,t+1}$ of component batches S_j, but not a sufficient component consumption is conducted, then it is feasible that the inventory in *t* + 1 for variant *B2* ($v_{j,t+1}$) is higher enough than the one for *C2* ($v_{j,t+1,t+1}$) so that the total cost for the two periods is greater in *B2* $(m_j \times v_j) + (m_j \times v_{j,t+1}) >$ $[(m_j + f_j) \times v_{j,t,t+1-a_j}] + (m_j \times v_{j,t+1,t+1})$.

It is also crucial to mention that this phenomenon is due mainly to two assumptions that we are doing in the problem. Firstly, we are considering the **same component inventory**

costs for both cases: with and without shelf-life. Normally it is to be expected that in the case in which the components have shelf-life, since they must be stored under special conditions, these costs would be higher than when there is no shelf-life consideration, which would make it less likely for the phenomenon to occur. Secondly, we are interpreting the **inventory (of both components and end-products) solely as a cost**, which makes the mere fact of holding inventory and adversely affect for the objective function. This is contrasted with the fact that **inventories can also be considered as assets**. Having available inventory can be interpreted as an opportunity to sell it and receive profits (or recoup investments). If we were to consider such inventories as assets, the phenomenon mentioned above would not occur (or would not have the logic that does in this case).

Longer running times for some variants: Table 4.8, Table 4.9 and Table 4.10 show a significant increase in processing times for problem instances *B2* and *C2*, which are the ones considering ordering batch size and no ordering lead time. A significant increase in the number of nodes and iterations used by CPLEX® to solve the problem is also observed. This occurs in all instances of the problem example for the same variants. In contrast to the above, the problems that do consider ordering lead time (*B3* and *C3*) not only take less time to reach the optimal solution, but they also use fewer nodes and iterations.

Table 4.9 Model Comparison / ALPHA Problem Instances ($T = 10$)

ALPHA $T = 10, N = 2, J = 2$	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_j**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	160	802	160	802	200	962
Variables	121	349	121	349	141	392
Binary	40	40	40	40	60	60
Integer	80	306	80	306	80	329
Continuous	1	3	1	3	1	3
Non-zero coefficients	276	553	276	553	292	581
Avg. Time (hrs:min:sec:cs)	00:00:03:15	00:00:03:15	00:07:04:82	00:09:25:96	00:00:12:40	00:00:19:06
# of Nodes	429	224	1,773,651	1,944,153	44,154	46,593
# of Iterations	2,201	1,218	10,126,595	11,309,316	225,504	274,983
Linear Relaxation	$216,336	$219,234	$216,334	$216,334	$218,799	$218,799
Objective Value	$244,418	$244,418	$262,724	$263,694	$257,282	$256,500
Optimality Gap	0.115	0.103	0.176	0.179	0.149	0.147
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	$3,878	No Disposal	$3,570

Through all the results, it can be evidenced that problem instances *C1* and *C2* (both involving components shelf-life) always use the same number of constraints and variables to reach optimality, although differing in the order batch size consideration. This is because the component orders variable $Q_{j,t}$, although being handled under different definitions, in both cases it only depends on the number of component types $j \in J$ and in the number of periods $t \in T$. The remaining variables are all used in the same way in both model formulations, so there is no reason to generate changes in the amounts of variables used.

Table 4.10 Model Comparison / ALPHA Problem Instances (T = 12)

ALPHA T **= 12,** N **= 2,** J **= 2**	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_i**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	192	1,106	192	1,106	240	1,298
Variables	145	467	145	467	169	518
Binary	48	48	48	48	72	72
Integer	96	416	96	416	96	443
Continuous	1	3	1	3	1	3
Non-zero coefficients	332	669	332	669	352	701
Avg. Time (hrs:min:sec:cs)	00:00:03:12	00:00:04:32	00:07:14:80	00:07:14:16	00:01:07:48	00:01:16:87
# of Nodes	519	375	1,626,039	1,334,919	311,327	226,528
# of Iterations	2,109	2,473	9,754,558	8,353,792	1,785,494	1,639,288
Linear Relaxation	$253,542	$256,513	$253,539	$253,539	$256,005	$256,005
Objective Value	$286,667	$286,667	$303,078	$305,004	$301,784	$302,434
Optimality Gap	0.115	0.105	0.163	0.169	0.152	0.153
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	$1,990	No Disposal	$2,230

4.2.2 Model Comparison for BETA Problem Instances

In this section we present the results of applying the model formulations to each of the BETA instances. BETA instances maintain the same parameter levels among them: Higher inventory costs, longer shelf-life, longer order lead times, higher set-up costs, higher disposal costs, bigger order batch size, and $J = 3$, varying only the size of the planning horizon T.

We can see that all discussions presented in the preceding section on the application of the model to ALPHA instances still apply for BETA instances.

Table 4.11 Model Comparison / BETA Problem Instances ($T = 6$)

BETA $T = 6, N = 2, J = 3$	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_j**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	120	483	120	483	156	585
Variables	91	237	91	237	109	282
Binary	30	30	30	30	48	48
Integer	60	203	60	203	60	233
Continuous	1	4	1	4	1	1
Non-zero coefficients	211	526	211	526	220	569
Avg. Time (hrs:min:sec:cs)	00:00:03:05	00:00:03:99	00:00:03:79	00:00:06:04	00:00:03:10	00:00:03:41
# of Nodes	29	19	3,902	4,089	24	60
# of Iterations	164	128	22,139	32,370	196	497
Linear Relaxation	\$142,063	\$145,270	\$144,681	\$144,681	\$181,206	\$181,386
Objective Value	\$160,374	\$160,374	\$188,210	\$188,210	\$191,596	\$191,710
Optimality Gap	0.114	0.094	0.231	0.231	0.054	0.054
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	No Disposal	No Disposal	\$2,380

No disposal in variants with shelf-life and batch size considerations: as shown in Table 4.11, although instance *C2* assumes component shelf-life, it is perfectly possible to have no disposal of components when having this consideration. In this case, the value of the objective function is always going to be the same as that of the corresponding basic instance (which in this case is *B2*). When this situation occurs, the optimality gap is also the same for both variants.

One of the most crucial parameters for this problem of production planning is the component order batch size S_j. Depending on the relationship between the order batch size and the requirements of components, in a case applied to a manufacturing process, the flexibility of the producer will be critically affected by this. High order batch sizes could generate high levels of component disposal f_j, because the producer is forced to purchase large quantities of component that probably does not need to use. However, if end-products inventory holding costs h_i are not considerably high and with enough production capacity K_j, having large amounts of components will allow the manufacturer to produce end-products for inventory and not to have to dispose materials.

Table 4.12 Model Comparison / BETA Problem Instances ($T = 8$)

BETA $T = 8, N = 2, J = 3$	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_i**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	160	787	160	787	208	907
Variables	121	369	121	369	145	426
Binary	40	40	40	40	64	64
Integer	80	325	80	325	80	361
Continuous	1	4	1	4	1	1
Non-zero coefficients	283	742	283	742	298	791
Avg. Time (hrs:min:sec:cs)	00:00:03:08	00:00:04:11	00:00:05:65	00:00:11:08	00:00:02:99	00:00:04:83
# of Nodes	79	51	9,325	14,697	1,584	2,019
# of Iterations	439	399	66,293	131,777	9,579	15,172
Linear Relaxation	\$186,498	\$189,994	\$189,125	\$189,125	\$225,254	\$225,794
Objective Value	\$212,925	\$212,925	\$248,820	\$249,752	\$258,322	\$256,690
Optimality Gap	0.124	0.107	0.234	0.243	0.128	0.120
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	\$1,190	No Disposal	\$3,030

Observing problem instances *B2* and *C2* in Table 4.12 and Table 4.13, we can see the significant increase in processing time mentioned above when passing from $T = 8$ to $T = 10$, as well as in the number of nodes and iterations used by CPLEX® to reach the optimal solution. However, these increases are not accompanied by a significant difference in the optimality gap between the linear relaxation and the objective value.

In fact, the optimality gap presents variations in all corresponding model formulations through all the problem instances.

Table 4.13 Model Comparison / BETA Problem Instances ($T = 10$)

BETA $T = 10, N = 2, J = 3$	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_j**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	200	1163	200	1,163	260	1,293
Variables	151	525	151	525	181	594
Binary	50	50	50	50	80	80
Integer	100	471	100	741	100	513
Continuous	1	4	1	4	1	1
Non-zero coefficients	355	958	355	958	376	1,013
Avg. Time (hrs:min:sec:cs)	00:00:03:14	00:00:04:57	00:00:52:55	00:01:51:44	00:00:10:21	00:00:19:57
# of Nodes	773	431	182,715	194,818	25,338	24,266
# of Iterations	4,031	2,496	1,386,785	1,803,414	190,219	249,308
Linear Relaxation	\$238,737	\$243,178	\$241,375	\$241,375	\$277,505	\$278,045
Objective Value	\$278,504	\$278,504	\$319,644	\$318,404	\$333,290	\$332,142
Optimality Gap	0.142	0.127	0.245	0.242	0.167	0.163
Disposal Cost ($\sum_{j\in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	\$1,000	No Disposal	\$3,026

Another aspect to consider is that it is also feasible, through longer planning horizons, that the inventory costs of components and end-product is higher when low amounts of components are disposed. In this situation, the “forced” disposal of components (due to order batch sizes) may not be considered a negative limitation, but on the contrary, a mechanism to prevent high total inventory holding costs. This under the assumption of inventories only as costs, and not as assets.

Table 4.14 Model Comparison / BETA Problem Instances (T = 12)

Instance: 4-BETA T **= 12,** N **= 2,** J **= 3**	**No Order Batch Size No Order Lead Time**		**Order Batch Size (S_j)**			
			No Order Lead Time		**Order Lead Time l_i**	
	B1	***C1***	***B2***	***C2***	***B3***	***C3***
	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j	No Component Shelf-Life	Component Shelf-Life a_j
# of Constraints	240	1,611	240	1,611	312	1743
Variables	181	705	181	705	217	786
Binary	60	60	60	60	96	96
Integer	120	641	120	641	120	689
Continuous	1	4	1	4	1	1
Non-zero coefficients	427	1,174	427	1,174	454	1235
Avg. Time (hrs:min:sec:cs)	00:00:03:52	00:00:05:10	00:06:11:46	00:09:04:36	00:01:03:86	00:03:15:23
# of Nodes	920	812	1,069,380	968,941	187,800	278,615
# of Iterations	5,022	5,118	9,031,701	9,913,765	139,199	3,202,670
Linear Relaxation	$279,597	$283,900	$282,243	$282,243	$318,372	$318,913
Objective Value	$326,229	$326,229	$367,512	$368,412	$393,882	$394,269
Optimality Gap	0.143	0.129	0.232	0.234	0.192	0.191
Disposal Cost ($\sum_{j \in J} f_j z_j$)	No Disposal	No Disposal	No Disposal	$500	No Disposal	$5,130

The following section presents a comparison of the problem instances considering the component shelf-life requirement where the core model formulation variant is applied.

4.2.3 Comparison of Core Model Variants for all Instances

In Table 4.15, we now present the previous results comparing all instances solved with the core model variant at once. The purpose of this mode of comparison is to more clearly visualize the differences in the application of the core model variant to solve each of the different problem instances and get a deeper view of its performance.

Table 4.15 Core Model Comparison for all Problem Instances

	# of Const.	Variables				Non-zero coeff.	Average Time (seg)	# of Nodes	# of Iterations	Optimality Gap
		Total	Binary	Integer	Cont.					
Instance *C1* (No Order Batch Size, No Order Lead Time)										
ALPHA ($J = 2$)										
$T = 6$	338	161	24	134	3	321	2.44	51	205	0.0083
$T = 8$	546	247	32	212	3	437	3.86	38	266	0.093
$T = 10$	802	349	40	306	3	553	3.15	224	1,218	0.103
$T = 12$	1,106	467	48	416	3	669	4.32	375	2,473	0.105
BETA ($J = 3$)										
$T = 6$	483	237	30	203	4	526	3.99	19	128	0.094
$T = 8$	787	369	40	325	4	742	4.11	51	99	0.107
$T = 1$	1,163	525	50	471	4	958	4.57	431	2,496	0.127
$T = 12$	1,611	705	60	641	4	1,174	5.10	812	5,118	0.129
Instance *C2* (Order Batch Size S_j, No Order Lead Time)										
ALPHA ($J = 2$)										
$T = 6$	338	161	24	134	3	321	3.98	3,908	19,905	0.162
$T = 8$	546	247	32	212	3	437	21.17	68,933	376,967	0.175
$T = 10$	802	349	40	306	3	553	565.96	1,944,153	11,309,316	0.179
$T = 12$	1,106	467	48	416	3	669	434.16	1,334,919	8,353,792	0.169
BETA ($J = 3$)										
$T = 6$	483	237	30	203	4	526	6.04	4,089	32,370	0.231
$T = 8$	787	369	40	325	4	742	11.08	14,697	131,777	0.243
$T = 10$	1,163	525	50	741	4	958	111.44	194,818	1,803,414	0.242
$T = 12$	1,611	705	60	641	4	1,174	544.36	968,941	9,913,765	0.234
Instance *C3* (Order Batch Size S_j, Order Lead Time l_j)										
ALPHA ($J = 2$)										
$T = 6$	434	188	36	149	3	341	4.61	615	3,163	0.129
$T = 8$	674	282	48	231	3	461	6.13	10,847	65,645	0.147
$T = 10$	962	392	60	329	3	581	19.06	46,593	274,983	0.147
$T = 12$	1,298	518	72	443	3	701	76.87	226,528	1,639,288	0.153
BETA ($J = 3$)										
$T = 6$	585	282	48	233	1	569	3.41	60	497	0.054
$T = 8$	907	426	64	361	1	791	4.83	2,019	15,172	0.120
$T = 10$	1,293	594	80	513	1	1,013	19.57	24,266	249,308	0.163
$T = 12$	1,743	786	96	689	1	1,235	195.23	278,615	3,202,670	0.191

The tables in the previous section are intended to present the results of the problem instances solution and compare each of the mathematical formulations with its respective basic counterpart and with different key assumptions. In this section, the Table 4.15 presents a comparison of the core model applied to different problem sizes in terms of number of T periods in the planning horizon and J types of components. Most of the conclusions drawn above can also be observed here, but we display more clearly the difference in performance when changing the number of periods or components. For example, it clearly shows the significant increase in computational time to solve the problem when moving from 8 to 10 and to 12 time periods for all instances. However, as also mentioned above, we do not observe significant changes in the optimality gap.

In order to further analyze the application of the core model formulation, and to contextualize it in an industry where raw material shelf-life considerations are relevant, the following section presents a case study based on actual information from composites manufacturing and applied to an example in the automotive industry.

4.3 Case Study (Automotive Industry)

Before stating the case study example, it is important to contextualize its field (industry) of application. As described by Mazumdar (2001), composite materials have come to be considered the “material of choice” for over a decade in various applications in the automotive industry, either to make exotic sports cars, passenger cars, or small, medium or heavy trucks.

Manufacturers have the opportunity to meet the requirements of cost, appearance and performance, replacing metal parts with lightweight composite parts, by using materials such as customized glass fiber reinforced plastic (GFRP), and carbon fiber reinforced plastic (CFRP) prepregs. Surface finish body panels, impact structures, interior and exterior structural features, and aesthetic-pleasing cosmetic parts are some key elements in which these materials are used.

The automotive industry has been chosen to be the base for the numerical case study example, due to the importance that the composite materials represent for it. Below, information and actual data from the automotive and composites manufacturing industries are presented for later use in defining the case.

4.3.1 Composite Materials Used in the Automotive Industry

According to the British composite materials manufacturer Amber Composites Ltd. (which produces composites for high performance lightweight structures), the most typical applications of GFRP and CFRP prepregs and other composites in the automotive industry are (Amber Composites Ltd., 2012):

Tooling: high-performance tool making from tooling board, release agents, sealers, vacuum bags and other consumables to high performance tooling prepreg,

Structural parts: prepregs, honeycombs and adhesive films work together seamlessly to manufacture structural parts such as high-performance chassis,

Interiors: aesthetic and structural parts such as dashboards, door panels, and trim,

Cosmetic bodywork: rapid laminating epoxy resin systems reduce laminating times and enable first-class surfaces.

For example, spanish manufacturer GTA Motor's super sports car 'Spano' is manufactured using surfacing films for tooling, body panel systems for the bodywork, a combination of high visual quality cosmetic prepregs for the interior, exterior cosmetic trim panels and epoxy prepreg systems. For high impact resistance in certain areas of the car, GTA Motor is using resin systems on hybrid aramid/carbon reinforcement (Advanced Composites Group Ltd., 2012). Most of these materials mentioned above are composites whose shelf life must be considered when using them for production.

Shelf-Life Consideration

Shelf-life of composite materials depends on the type and compound formulation. Furthermore, depending on storage conditions, the shelf-life may vary dramatically. Typically, to achieve maximum shelf-life times, materials need to be stored at refrigeration temperatures (18°C / 0°F). At ambient temperatures (20°C / 68°F), shelf-life times can be reduced to only a few days.

Following is a series of examples of some composite materials used in the automotive industry where shelf-life is a consideration depending on the conditions under which they are stored. Specifically, Table 4.16 and Table 4.17 below show component and tooling prepregs, and other composites manufactured by Amber Composites Ltd. and Advanced Composites Group Ltd., two major companies in the composites manufacturing industry. The tables contain: the original manufacturer specification (OM), main characteristics of the material, some typical applications and information related to storage and shelf-life.

Table 4.16 Prepreg Systems Developed by Amber Composites Ltd. (Amber Composites Ltd., 2012)

Low-Temperature Prepregs				
Low-temperature curing component prepreg incorporates epoxy resin systems with a range of viscosities, pre-impregnated into high-performance fibers such as carbon, glass and kevlar. They are designed for rapid manufacturing from low cost tooling. Do not require autoclaves.				
OM Spec.	**Characteristics**	**Typical Applications**	**Shelf-Life**	
			At ambient temp. (20°C / 68°F)	**Refrigerated at (-18°C / 0°F)**
E644	Very good surface finish under autoclave or vac-bag. Fast curing at elevated temperatures.	Motorcycle exhausts, motor racing bodywork, marine applications, medical applications.	7 days	Up to 12 months
E650	Excellent surface finish with autoclave or vac-bag. Improved handleability and good mechanical properties.	Motor racing bodywork, leisure industry, medical, commercial, automotive, wind turbines.	5 days	Up to 12 months
8020	Designed for excellent surface finish with reduced layup times.	Automotive & Marine.	30 days	Up to 12 months

Cyanate Ester Prepregs				
Suited for motorsport, automotive, aerospace and industrial applications where high temperature performance is paramount. Resin systems can be pre-impregnated into carbon, glass and aramid fibers.				
OM Spec.	**Characteristics**	**Typical Applications**	**Shelf-Life**	
			At ambient temp. (20°C / 68°F)	**Refrigerated at (-18°C / 0°F)**
C640	Very high end-use temperature with flexible low to medium cure.	Motor sport exhaust systems and high temperature applications.	4 days	Up to 6 months
C740	Very high end-use temp. Excellent handleability and surface finish.	Aerospace and motor sport exhaust systems and engine covers.	1 month	Up to 12 months
Out-Of-Autoclave Prepregs				
With out-of-autoclave systems, components can be laminated quickly and more efficiently in a non-autoclave environment. Benefits include: excellent surface finishes, more flexibility in manufacturing, less expensive tooling and energy savings.				
8020 Rapi-ply Rapi-core Surface Film	Multiple layer product that significantly reduces lay-up times compared with traditional prepreg.	Structural components, automotive, marine, wind energy and many other applications.	1 month	Up to 12 months
Fire Retardant Prepregs				
For quick component lamination and more efficiency in a non-autoclave environment. Benefits include: excellent surface finishes, more flexibility in manufacturing, less expensive tooling and energy savings.				
8020-FR	Flexible low to medium cure schedules. Excellent drape and good adhesive properties.	Motor sport exhaust systems and high temperature applications.	1 month	Up to 12 months
E721-FR	Excellent adhesive properties suitable for sandwich construction.	Motor racing, marine, aircraft interiors, rail, automotive.	1 month	Up to 12 months

Tooling Prepregs				
Excellent surface finish, ease of handling and longevity of tool life. Up to 200°C (392°F) end-use temperatures. Tools can be produced using a range of materials and methods.				
OM Spec.	**Characteristics**	**Typical Applications**	**Shelf-Life**	
			At ambient temp. (20°C / 68°F)	**Refrigerated at (-18°C / 0°F)**
HX42	Longest out life, best surface finish at high temperature.	Large aerospace and automotive tooling.	5 days	Up to 12 months
HX50	Excellent surface finish.	Fast curing automotive tooling.	60 hours	Up to 6 months
HX70	Shorter autoclave curing time and very low curing temp.	Very fast curing automotive tooling.	30 hours	Up to 6 months
HX90N	Lower coefficient of thermal expansion.	High precision aerospace and automotive tooling.	30 hours	Up to 6 months

As can be seen in the above table, the maximum shelf-life of the presented examples of composite materials may vary from 6 to 12 months, always depending on storage conditions. Additionally, when these materials are kept at ambient temperatures, their shelf-life (also called out-life in these cases) may be significantly reduced to 5 days.

Table 4.17 gives further examples of composites with shelf-life considerations produced by Advanced Composites Group Ltd. ACG Ltd. is part of part of Umeco Composites Structural Materials (UCSM), and is a leading manufacturer of advanced composite carbon and glass fiber (CFRP and GFRP), reinforced plastic pre-impregnated materials (prepregs), custom formulated for components, and structural and tooling applications in a diverse range of industries (including the automotive industry) (Advanced Composites Group Ltd., 2012).

Table 4.17 Prepreg systems Developed by Advanced Composites Group Ltd. (Advanced Composites Group Ltd., 2012)

Structural Epoxy Prepreg Resin Systems				
Low Temperature Molding (LTM)				
OM Spec.	**Characteristics**	**Typical Applications**	**Shelf-Life**	
			At ambient temp. (20°C / 68°F)	**Refrigerated at (-18°C / 0°F)**
LTM®23	40°C (104°F) initial cure. Toughened. Honeycomb bondable.	Prototypes. Low cost structural parts. Offshore structure repair.	2 to 3 days	6 to 12 months
LTM®26	Toughened LTM structural system. Wet service temp. 90°C (194°F). Excellent resin clarity and UV resistance.	Light aircraft. UAVs. Automotive prototypes. Flame retarded and low smoke variants.	4 to 6 days	6 to 12 months
LTM®40	Toughened LTM structural system. Wet service temperature 130°C (266°F).	Aircraft prototypes. UAV's. Automotive engine parts.	3 to 15 days	6 to 10 months
LTM®45	Toughened LTM structural system. Wet service temperature 130°C (266°F). Autoclave quality from vacuum bag processing.	Aircraft prototypes. Structural aircraft components. Automotive engine parts.	5 to 6 days	6 to 10 months
Medium Temperature Molding (MTM)				
MTM®28	Good handling. Variants to meet specific handling requirements. Excellent damage tolerance. Honeycomb bondable.	Widely used structural material for commercial applications.	1 month	Up to 12 months
MTM®29FR/SFR	Toughened LTM structural system. Wet service temperature 130°C (266°F).	Rail, marine and transport structural parts.	1 month	Up to 12 months
MTM®44-1	Out-of-Autoclave processing. Low density 1.18g/cc.	Primary aircraft and automotive structures.	21 days	Up to 12 months

Medium Temperature Molding (MTM)				
OM Spec.	Characteristics	Typical Applications	Shelf-Life	
			At ambient temp. (20°C / 68°F)	Refrigerated at (-18°C / 0°F)
MTM®45-1/FR	Out-of-Autoclave processing. 90°C (194°F) initial cure for prototypes. Conventional 180°C (356°F) cure for production. Low density 1.18g/cc.	Primary aircraft and automotive structures for prototypes and series production.	21 days	Up to 6 months
MTM®48	Good tack and drape. Good environmental resistance. Honeycomb bondable.	Honeycomb sandwich structures and secondary structures.	1 month	Up to 12 months
MTM®49	High temperature resistance and wet service. Good balance of temperature resistance and toughness.	Automotive and motorsport structures, e.g. chassis, wings and roll hoops.	1 to 2 months	Up to 12 months
MTM®57	Low cost, aesthetically clear resin, offering good UV resistance.	Component manufacture. Cosmetic interior and exterior automotive panels.	1 month	Up to 12 months
MTM®58B/FRB	Excellent tack and drape. Good hot/wet performance.	Automotive components.	2 months	12 to 18 months
MTM®71	Good tack and handling.	Motorsport chassis and side intrusion structures.	1 month	Up to 12 months
MTM®249	Good dry property retention up to 180°C (356°F). Low density 1.18g/cvc. High toughness.	Motorsport chassis and impact structures at high service temperatures.	21 days	12 to 18 months

High Temperature Molding (HTM)				
OM Spec.	Characteristics	Typical Applications	Shelf-Life	
			At ambient temp. (20°C / 68°F)	Refrigerated at (-18°C / 0°F)
HTM®60	High temp. resistance. Honeycomb bondable.	High temp. automotive structural and bodywork applications.	28 days	Up to 12 months

Variable Temperature Moulding (VTM)				
OM Spec.	**Characteristics**	**Typical Applications**	**Shelf-Life**	
			At ambient temp. (20°C / 68°F)	**Refrigerated at (-18°C / 0°F)**
VTM®243FRB VTM®244FRB VTM®260	Fabric reinforcement weights up to 300g/m^2, 700g/m^2, and more. Excellent vacuum-bag consolidation for low void laminates. Good dry property retention.	Large Out-of-Autoclave structures for automotive, marine and other industries.	1 month	Up to 18 months
Body Panels Systems				
VTF242 FRB	Pit free finish under vacuum consolidation. Stable under environmental conditioning. Excellent drape and handling. Flame retarded.	Automotive body panels. Rapid prototype construction. Models.	7 days	12 to 18 months
VTS®243 FR	Rapid build-up of part thickness. Excellent drape and handling. Stable under environmental conditioning. Flame retarded.	Automotive body panels. Rapid prototype construction. Models.	30 days	Up to 12 months
Tooling Prepregs				
LTM®10 LTM®12 LTM®16	Low temp. cure epoxide based resin systems which may be post-cured to produce high temperature (200°C/392°F) composite tools.	Automotive and other industries tooling.	3 to 7 days	6 to 12 months
HTM®512	Bismaleimide (BMI) systems specifically formulated for the manufacture composite tooling operating at high temperatures.	Automotive and other industries tooling.	21 days	Up to 6 months

The above items shown in Table 4.16 and Table 4.17 are just some of the many composite materials used in the automotive industry (among others industries). All these materials, as specified in the tables, are limited to storage conditions and a given shelf-life time depending on such conditions.

If the material is stored at room temperature, the term “out-life” is more appropriate to refer to the expiration period. For moderately long periods of shelf-life time, the material can be stored in a refrigerator at approximately 5°C / 40°F. To obtain the maximum shelf-life time, the material must be stored frozen at 18°C / 0°F. Additionally, several suppliers recommend to allow prepreg to reach room temperature before opening its packing following removal from cold storage.

Pricing and Other Specifications

Multiple variables influence the final prices of materials for manufacturers. The price ranges can vary significantly depending on the type of composite material, the specifications required by the manufacturer for each product, the quantity and frequency of orders, the supplier, the time of the year (according to demand and product availability), etc.

Additionally, depending on material specifications, market prices are established on various units of measure. For example, prices for epoxy prepreg systems may be specified by the area ($ per square foot: $/ft^2, $/sq-ft, or $ per square meters: $/m^2), but some suppliers may also manage prices by the weight ($/lb or $/kg), or even some just by custom units (e.g. rolls, sheets, cut forms, etc.).

For this specific case study, the measurement units that are used to specify component prices, requirements and other parameters are square foot (ft^2), as done by Mazumdar (2001) when describing production planning and manufacturing instructions for composite materials.

Although it is complex to establish pricing standards for such materials, it seems proportionate to say, based on various sources of the composites materials manufacturing industry, that market prices of thermoplastic and thermoset prepreg systems for manufacturers can vary around the range of US$1.5 to US$4 per square foot. It should be noted that some suppliers may have higher prices, and even that seasonal variations may cause significant increase in prices. The above range is used to approximate the prices of components for the automotive case study of matter.

4.3.2 Demand/Sales and Production in the Automotive Industry

Demand/Sales Behavior

The demand is one of the fundamental elements of any production planning model, as it establishes patterns related to the objective function and system constraints. In order to develop a consistent case study with realistic elements, the automotive industry demand/sales behavior is briefly analyzed below, to then make approximations applied to the case definition.

According to the National Automobile Dealers Association (NADA), which represents nearly 16,000 new car and truck dealers, with 32,500 franchises internationally, in its

2010 State of the Industry Report, sales of new cars fell dramatically in 2010 compared to previous year, as the recession deepened in the first half of the year. On the other hand, demand of used cars increased rapidly, even faster than supply, causing a significant shortage. This, mixed with other factors, led to an increase in new car's sales in the second half of the year.

In order to get a better idea about the behavior and the proportionality of the demand in the automotive industry, Table 4.18 records the monthly sales data of an important portion of that market over the past three years.

Table 4.18 New-Car / New-Truck Dealer Total Monthly Sales (million units) (National Automobile Dealers Association, NADA, 2012)

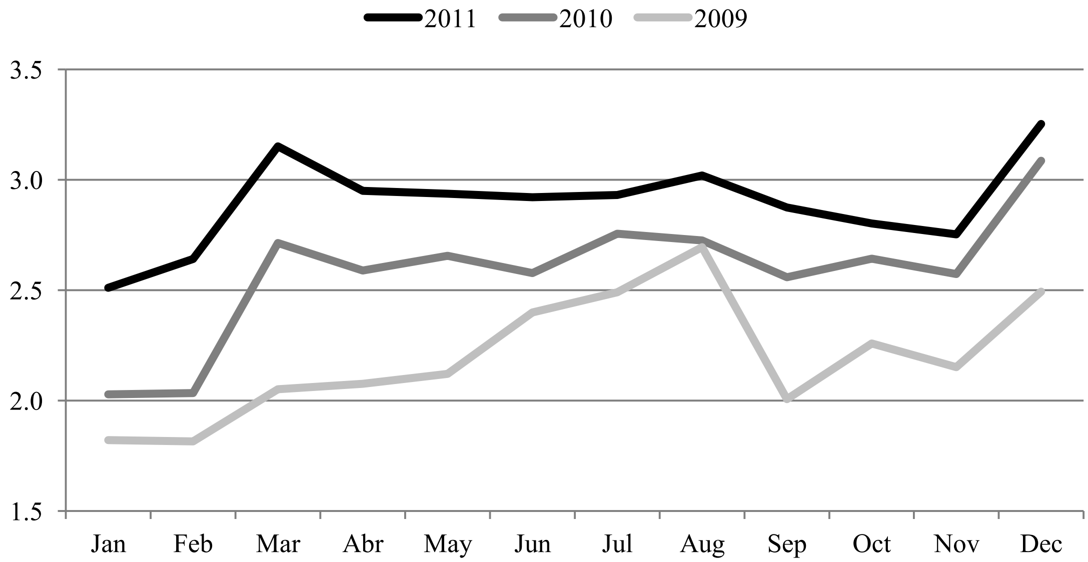
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
2011	2.51	2.64	3.15	2.95	2.94	2.92	2.93	3.02	2.87	2.80	2.75	3.25	**34.74**
2010	2.03	2.03	2.71	2.59	2.66	2.58	2.76	2.73	2.56	2.64	2.57	3.09	**30.94**
2009	1.82	1.81	2.05	2.08	2.12	2.40	2.49	2.70	2.01	2.26	2.15	2.49	**26.38**
Total	**6.36**	**6.49**	**7.92**	**7.62**	**7.71**	**7.90**	**8.18**	**8.44**	**7.44**	**7.70**	**7.48**	**8.83**	**92.07**

The above data is later used to approximate the demand parameter for the case study.

Next, Figure 3 graphically illustrates the behavior of the sales data and evidence the presence of an increasing yearly trend. Observing the trend in sales, it is also clear that there are season patterns throughout the year.

Figure 3 New-Car / New-Truck Dealer Total Monthly Sales (million units)

(National Automobile Dealers Association, NADA, 2012)



Demand Forecasting With Seasonal Adjustment

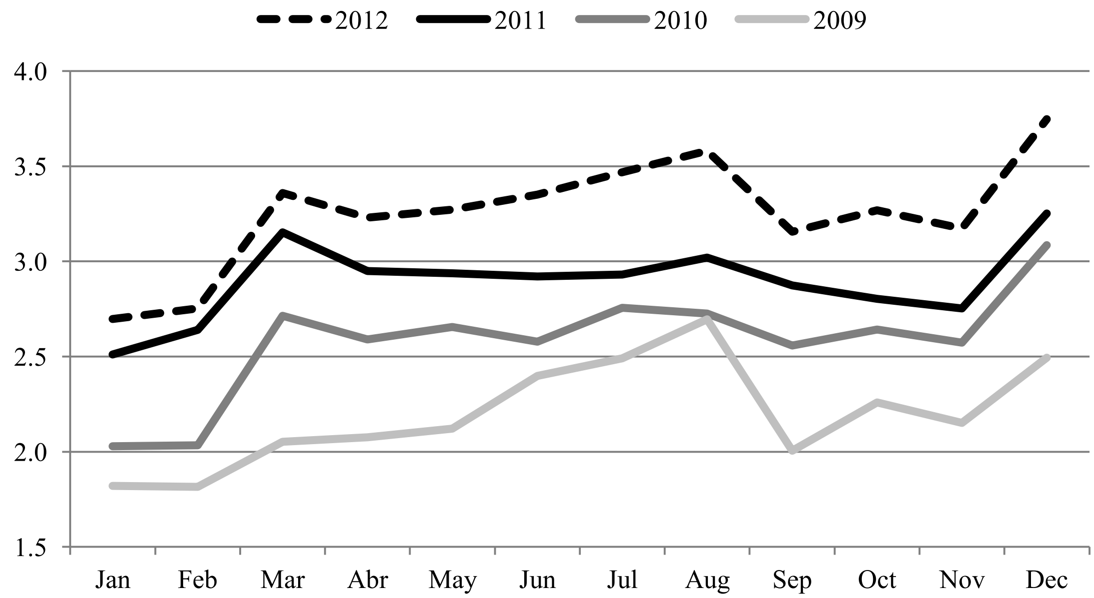
Using the historical data on sales for 2009, 2010 and 2011 presented above, and applying a simple forecasting model with seasonal adjustment, demand for 2012 is estimated to later use as a tool in formulating the case study.

Table 4.19 New-Car / New-Truck 2012 Demand Forecast (million units)

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**	**Total**
2012 *(Forecast)*	2.7	2.75	3.36	3.23	3.27	3.35	3.47	3.58	3.16	3.27	3.17	3.75	**39.05**

Figure 4 graphically shows the forecasted demand for 2012, along with the sales curves for 2009, 2010 and 2011. As it can be seen by comparing the estimated curve for 2012 with the past actual data, the estimate is relatively appropriate, reflecting both: seasonal variations and the overall annual trend.

Figure 4 New-Car / New-Truck 2012 Demand Forecast (million units)



Subsequently, when stating the case study problem, this estimate will be used to approximate the demand parameter (d_i). Additionally, a car manufacturing company with an annual production between 3 and 4% of the total world car production is to be assumed. That is, an aggregate of approximately 1,367,000 units for the forecasted year.

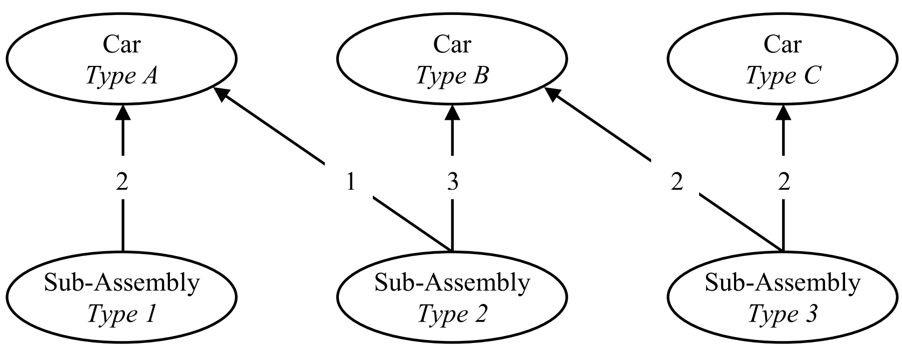
4.3.3 Case Study Definition

In the previous section of this chapter, the case study was contextualized in its field (industry) of application. Relevant information about the automotive and composites manufacturing industries was presented and analyzed to be used now as a reference in the definition of the case.

An automotive manufacturer company is regarded to implement and analyze the production planning model. A planning horizon of 12 periods (in this case months) is considered, in which the company (medium-size) will have a total production between 3 and 4% of the total world production of cars. Total production refers to 3 different types of cars: *Type A*, *Type B* and *Type C*.

Among all the sub-assemblies required to manufacture the different types of cars, only 3 sub-assemblies are considered as relevant for their content of composite materials with limited shelf-life for the purpose of the present case study. Figure 5 shows the bill of materials of sub-assemblies for the different car types.

Figure 5 Sub-Assembly Bill of Materials



As shown in Figure 5, Sub-Assembly Type 1 is only used to make Type A cars in quantities of 2 units per car; Sub-Assembly Type 2 is used in Type A and B cars in quantities of 1 and 3 units per car, respectively; and Sub-Assembly Type 3 is used in car types B and C in quantities of 2 units per car. The sub-assemblies are interpreted as the end-products ($i \in N$) for the implementation of the mathematical model.

Demand Parameter

To determine the demand parameter ($d_{i,t}$), the previously presented forecast demand for 2012 (Table 4.9) is taken into account. Thus, the company will sell (and so produce) a total of 1,367,000 cars: 40% of *Type A*, 35% of *Type B*, and 25% of *Type C*.

Table 4.20 shows the car demand for each period *t* in the planning horizon *T*. Finally, and more importantly, Table 4.20 shows the demand for sub-assemblies, which is the definitive demand parameter ($d_{i,t}$) for the case study.

Table 4.20 End-Product and Sub-Assembly Demand (thousand units)

$t \in T$	1	2	3	4	5	6	7	8	9	10	11	12	Total
Car Demand	95	96	118	113	114	117	121	125	111	114	111	131	**1,367**
Car Demand per Type													
Type A (40%)	38	39	47	45	46	47	49	50	44	46	44	53	**547**
Type B (35%)	33	34	41	40	40	41	43	44	39	40	39	46	**478**
Type C (25%)	24	24	29	28	29	29	30	31	28	29	28	33	**342**
Sub-Assembly Demand ($d_{i,t}$)													
Sub-Assembly 1 ($d_{1,t}$)	76	78	94	90	92	94	98	100	88	92	88	106	**1,096**
Sub-Assembly 2 ($d_{2,t}$)	137	141	170	165	166	170	178	182	161	166	161	191	**1,988**
Sub-Assembly 3 ($d_{3,t}$)	114	116	140	136	138	140	146	150	134	138	134	158	**1,644**

Component Bill of Materials and Shelf-Life Parameters

Among the various components to be considered as raw materials for the sub-assemblies, different types of composites materials will be taken into account. We assume these materials as components of the sub-assemblies to produce and that are stored under different conditions.

With the information covered previously on the applications and specifications of various component prepregs systems, tooling prepregs systems, and other composites with shelf-life considerations, we make the assumptions shown in Table 4.21 about the requirements ($b_{j,i}$) and the shelf-life (a_j) of different epoxy prepreg resins types (components $j \in J$) for each sub-assembly type $i \in N$. Since these materials are assumed to be stored under different conditions (none reaches its maximum shelf-life), for purposes of the case study, we consider shelf-lives between 3 and 10 months.

Table 4.21 Component Bill of Materials and Shelf-Life

	Component Bill of Materials ($b_{j,i}$)			**Component Shelf-Life (a_j)**
	Sub-Assembly Type 1	**Sub-Assembly Type 2**	**Sub-Assembly Type 3**	
Component Type 1	145 ft^2	200 ft^2	100 ft^2	6 months
Component Type 2	80 ft^2	144 ft^2	-	4 months
Component Type 3	24 ft^2	-	24 ft^2	10 months
Component Type 4	42 ft^2	60 ft^2	55 ft^2	3 months

Remaining Parameters

The remaining parameters needed to formulate the case study and the optimization model for each of the sub-assembly types are shown in Table 4.22.

Table 4.22 Sub-Assemblies Remaining Parameters

	Sub-Assembly Type 1	**Sub-Assembly Type 2**	**Sub-Assembly Type 3**
Production Capacity (K_i)	400 units	600 units	600 units
Unit Cost (p_i)	\$ 120	\$ 100	\$ 250
Inventory Holding Cost (h_i)	\$ 60	\$ 50	\$ 80
Fixed Set-Up Cost (A_i)	\$ 600	\$ 800	\$ 900
Initial Inventory ($I_{i,0}$)	0 units	0 units	0 units

Table 4.23 shows the remaining parameters for each of the component types.

Table 4.23 Components Remaining Parameters

	Component Type 1	**Component Type 2**	**Component Type 3**	**Component Type 4**
Order Batch Size (S_j)	5,000 ft^2	5,000 ft^2	10,000 ft^2	10,000 ft^2
Ordering Capacity (L_j)	60 batches	40 batches	3 batches	9 batches
Batch Cost (c_j)	\$ 7,500	\$ 20,000	\$ 35,000	\$ 25,000
Inventory Holding Cost (m_j)	\$ 4.0	\$ 5.0	\$ 5.0	\$ 4.0
Unit Disposal Cost (f_j)	\$ 6.0	\$ 8.0	\$ 8.0	\$ 6.0
Fixed Ordering Cost (g_j)	\$ 400	\$ 500	\$ 700	\$ 500
Order Lead Time (l_j)	3 periods	3 periods	2 periods	4 periods
Initial Inventory ($v_{j,0}$)	30,000 ft^2	20,000 ft^2	5,000 ft^2	0 ft^2

Scheduled Order Receipt ($q_{j,t}$)												
$t \in T$	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Component Type 1 ($q_{1,t}$)	20	0	10	-	-	-	-	-	-	-	-	-
Component Type 2 ($q_{2,t}$)	4	5	5	-	-	-	-	-	-	-	-	-
Component Type 3 ($q_{3,t}$)	0	2	-	-	-	-	-	-	-	-	-	-
Component Type 4 ($q_{4,t}$)	6	0	5	0	-	-	-	-	-	-	-	-

4.3.4 Case Study Results and Analysis

To solve the case study defined above, we apply the Mixed Integer Linear Programming Optimization core model formulation variant introduced in Chapter 3, Section 3.3.2, using IBM® ILOG® CPLEX® Optimization Studio Version 12.4, on a computer with a 2.00 GHz Intel® Core™ 2 Duo processor, 4.00 GB installed memory (RAM), and a 32-bit operating system. The following results were obtained.

Statistics and Optimality

- Number of constraints: 2,392
- Number of variables: 1,107
    - Binary: 132
    - Integer: 970
    - Continuous: 5
- Non-zero coefficients: 1,998
- Running Time: 27 min., 9.46 sec.
- Number of nodes: 3,380,218
- Number of iterations: 17,667,870
- Linear Relaxation: $ 4,836,566
- Objective value: $ 5,114,672
- Optimality Gap: 0.0544

It is important to note the significant increase in running time, number of nodes and iterations used by CPLEX® for the solution of the case study compared to the solution of the different problem instances previously presented in Section 4.2. With only an increase from $N = 2$ to $N = 3$, and from $j = 3$ to $j = 4$, the complexity of the solution process increases considerably. However, a significant impact is not shown on the optimality gap.

Solution for Variables Related to Sub-Assembly Production and Inventory

The case study solution values for sub-assembly production variables $x_{i,t}$ and $y_{i,t}$, and sub-assembly inventory variable $I_{i,t}$, are shown in Table 4.24.

Table 4.24 Sub-Assembly Production and Inventory Variables

$t \in T$	1	2	3	4	5	6	7	8	9	10	11	12	Total	Total Costs
Sub-Assembly Production ($x_{i,t}$)														
Type 1 ($x_{1,t}$)	77	235	0	204	125	0	141	13	125	0	189	0	**1,109**	**$ 133,080**
Type 2 ($x_{2,t}$)	235	43	173	512	0	0	165	305	139	173	69	174	**1,988**	**$ 198,800**
Type 3 ($x_{3,t}$)	131	160	154	80	269	0	265	21	291	0	142	152	**1,665**	**$ 416,250**
Sub-Assembly Production Set-Up ($y_{i,t}$)														
Type 1 ($y_{1,t}$)	1	1	0	1	1	0	1	1	1	0	1	0	**8**	**$ 4,800**
Type 2 ($y_{2,t}$)	1	1	1	1	0	0	1	1	1	1	1	1	**10**	**$ 8,000**
Type 3 ($y_{3,t}$)	1	1	1	1	1	0	1	1	1	0	1	1	**10**	**$ 9,000**
Sub-Assembly Inventory ($I_{i,t}$)														
Type 1 ($I_{1,t}$)	1	158	64	178	211	117	160	73	110	18	119	13	**1,222**	**$ 73,320**
Type 2 ($I_{2,t}$)	98	0	3	350	184	14	1	124	102	109	17	0	**1,002**	**$ 50,100**
Type 3 ($I_{3,t}$)	17	61	75	19	150	10	129	0	157	19	27	21	**685**	**$ 54,800**

As it can be seen in the table above, in this case production is carried out in almost all periods: for Sub-Assembly Type 1, production was carried out in 8 of the 12 periods, and for Sub-Assemblies Type 2 and 3, in 10. As also shown in Table 4.24, total production exceeds demand for the sub-assemblies 1 and 3, leaving inventory of 13 and 21 units respectively at the end of the planning horizon. The table also shows the totaled costs associated with each of the variables.

Solution for Variables Related to Component Ordering and Inventory

Below, Table 4.25 shows the values of the variables that determine the amounts and timing of orders for components, as well as those used to calculate the ordering costs.

Table 4.25 Component Ordering Variables

$t \in T$	1	2	3	4	5	6	7	8	9	10	11	12	Total	Total Costs
Component Ordering ($Q_{j,t}$)														
Type 1 ($Q_{1,t}$)	28	9	0	16	13	15	7	11	10	0	0	0	**109**	**$ 817,500**
Type 2 ($Q_{2,t}$)	18	2	0	7	9	6	5	5	5	0	0	0	**57**	**$ 1,140,000**
Type 3 ($Q_{3,t}$)	0	0	1	0	1	0	1	0	1	1	0	0	**5**	**$ 175,000**
Type 4 ($Q_{4,t}$)	2	0	3	2	3	1	2	2	0	0	0	0	**15**	**$ 375,000**
Component Scheduled Ordering Placement ($w_{j,t}$)														
Type 1 ($w_{1,t}$)	1	0	1										**2**	**$ 800**
Type 2 ($w_{2,t}$)	1	1	1										**3**	**$ 1,500**
Type 3 ($w_{3,t}$)	0	1											**1**	**$ 700**
Type 4 ($w_{4,t}$)	1	0	1	0									**2**	**$ 1,000**
Component Ordering Placement ($W_{j,t}$)														
Type 1 ($W_{1,t}$)	1	1	0	1	1	1	1	1	1				**8**	**$ 3,200**
Type 2 ($W_{2,t}$)	1	1	0	1	1	1	1	1	1				**8**	**$ 4,000**
Type 3 ($W_{3,t}$)	0	0	1	0	1	0	1	0	1	1			**5**	**$ 3,500**
Type 4 ($W_{4,t}$)	1	0	1	1	1	1	1	1					**7**	**$ 3,500**

The shaded areas determine the periods in which the calculation of the variable is not relevant. For example, in the case of the scheduled ordering placement variable ($w_{j,t}$), it is only relevant starting at the beginning of the planning horizon and until the period t equal to the lead time l_j of each component. After that time, we will be receiving orders corresponding to those made during the course of planning $Q_{j,t}$, which corresponds to the order placement variable $W_{j,t}$. Now, with respect to this variable $W_{j,t}$, one can observe that it is balanced with the above, since the shaded area corresponding the first one, equals the unshaded of the second. It is pertinent to clarify that if the two variables are transposed in the first periods, it is because during these periods we are receiving the scheduled orders $q_{j,t}$ and placing orders $Q_{j,t}$ at the same time. Table 4.25 also shows the total costs associated with each of the ordering variables.

Finally, Table 4.26 shows one of the most unique and relevant variables of the model: component inventory ($v_{j,t,r}$). It should be remembered that this variable is directly related to the auxiliary variable $e_{j,r,t}$ (component consumption) and at the same time determines the component inventory levels at the end of each time period t, the period r in which that component was received, and therefore also contains the component units that must be discarded because they reached their shelf-life limit.

The shaded area on the left side of Table 4.26 corresponds to combinations of t and r periods in which the variable does not exist. E.g., if a component batch is received in the period r = 2, there is no inventory of that component at end of period t = 1.

The shaded area on the right side of the table corresponds to the area in which the calculation of the variable is not relevant because the existing component inventory (if any) would have already expired and been discarded. E.g., in the case of component type 1 that is received in the period r = 1, because its shelf-life is a_1 = 6, the calculation of the variable is relevant only until t = 6. Moreover, the variable $v_{j,t,r}$ also contains the units of component j to be discarded. E.g., in the case of component type 2 received in period r = 2, since its shelf-life is a_2 = 4, the 8 units in inventory at end of period t = 5 should be discarded. The same applies to the 55 units of component type 1 received at r = 0 at end of period t = 5, and to the 1,230 units of component type 4 (a_4 = 3) received at r = 1 at the end of t = 3. For the case of component type 3, it does not present component disposal.

Table 4.26 Component Inventory Variables

$t \in T$	1	2	3	4	5	6	7	8	9	10	11	12	Total	
Component Type 1 ($v_{1,t,r}$)														
$r \in R$														
0	60	60	60	55	55								**290**	
1	58675	0	0	0	0	0							**58675**	
2		0	0	0	0	0	0						**0**	
3			0	0	0	0	0	0					**0**	
4				25	0	0	0	0	0				**25**	
5					0	0	0	0	0	0			**0**	
6						0	0	0	0	0	0		**0**	
7							55	40	40	40	40	0	**215**	
8								30	5	5	0	0	**40**	
9									0	0	0	0	**0**	
10										400	0	0	**400**	
11											0	0	**0**	
12												40	**40**	**Total Costs**
Total ($v_{1,t}$)	**58,735**	**60**	**60**	**80**	**55**	**0**	**55**	**70**	**45**	**445**	**40**	**40**	**59,685**	**$ 238,740**
Component Type 2 ($v_{2,t,r}$)														
$r \in R$														
0	0	0	0										**0**	
1	0	0	0	0									**0**	
2		8	8	8	8								**32**	
3			88	0	0	0							**88**	
4				40	40	40	0						**120**	
5					0	0	0	0					**0**	
6						0	0	0	0				**0**	
7							0	0	0	0			**0**	
8								40	0	0	0		**40**	
9									24	0	0	0	**24**	
10										112	56	0	**168**	
11											0	0	**0**	
12												0	**0**	**Total Costs**
Total ($v_{2,t}$)	**0**	**8**	**96**	**48**	**48**	**40**	**0**	**40**	**24**	**112**	**56**	**0**	**472**	**$ 2,360**
Component Type 3 ($v_{3,t,r}$)														
$r \in R$														
0	8	8	8	8	0	0	0	0	0				**32**	
1	0	0	0	0	0	0	0	0	0	0			**0**	
2		10520	6824	8	8	8	0	0	0	0	0		**17,368**	
3			0	0	0	0	0	0	0	0	0	0	**0**	
4				0	0	0	0	0	0	0	0	0	**0**	
5					552	552	0	0	0	0	0	0	**1,104**	
6						0	0	0	0	0	0	0	**0**	
7							816	0	0	0	0	0	**816**	
8								0	0	0	0	0	**0**	
9									16	16	16	0	**48**	
10										0	0	0	**0**	
11											2056	0	**2,056**	
12												8424	**8,424**	**Total Costs**
Total ($v_{3,t}$)	**8**	**10,528**	**6,832**	**16**	**560**	**560**	**816**	**0**	**16**	**16**	**2,072**	**8,424**	**29,848**	**149,240**
Component Type 4 ($v_{4,t,r}$)														
$r \in R$														
0	0	0											**0**	
1	35461	14211	1230										**50,902**	
2		0	0	0									**0**	
3			44131	443	0								**44,574**	
4				0	0	0							**0**	
5					398	398	0						**796**	
6						0	0	0					**0**	
7							1	0	0				**1**	
8								0	0	0			**0**	
9									405	0	0		**405**	
10										25	0	0	**25**	
11											137	0	**137**	
12												1337	**1337**	**Total Costs**
Total ($v_{4,t}$)	**35,461**	**14,211**	**45,361**	**443**	**398**	**398**	**1**	**0**	**405**	**25**	**137**	**1,337**	**98,177**	**392,708**

Table 4.26 also shows the total inventory at the end of each period t, and the total costs associated with each variable.

It does not seem entirely relevant to show the values for the auxiliary component consumption variable $e_{j,r,t}$, not only because it is extensive, but because it is an auxiliary variable used to calculate other of higher interest that are presented in the above tables.

Based on all the analysis and discussion above, and being consistent with the objectives set at the beginning of this study, in the next chapter we present the main conclusions and findings of the investigation. We also propose future research in this area.

5 CONCLUSIONS AND FUTURE RESEARCH

The following are the most conclusive aspects of this research thesis. We also propose future research topics in the area.

5.1 Conclusions

Regarding the first objective for this study, we conducted a review of recent research on topics related to production planning problems, MILP applied to solve them, and problems involving shelf-life or perishability requirements. From this review, we conclude that, although there is material related to cases in which production planning problems have perishable end-products, only some models of inventory control consider this feature for raw materials. Therefore, the importance and relevance of this study is the introduction of the shelf-life requirement feature for raw materials or components and applying MILP to solve the production planning problem.

The proposed optimization model tackles a multi-product multi-level production planning problem with the consideration of raw material shelf-life. In addition to the typical production planning variables, parameters and constraints (production and ordering variables, inventory variables, lead times, set-up and ordering costs, inventory holding costs, inventory balance equations, etc.), we introduce a number of features to address the raw material shelf-life requirement.

Raw material perishability is tackled by introducing a Shelf-Life variable as the maximum number of periods that a component can be stored before it is considered unsuitable for use or consumption and must be discarded. This variable is complemented by a component disposal cost parameter, applied to those units that are discarded when they reach their shelf-life. Additional time-stamps were implemented for the component inventory variables, specifying the period in which the component orders are received, so that we can keep independent track of the material inventory age. Finally, we formulate the equations that relate the ordering times, receipt times and disposal of components and also the use of auxiliary variables that control the consumption of materials in each period.

Two different formulation variants are presented to address the problem with different assumptions. The first variant called "Basic Variant" is introduced without considering shelf-life to compare with called "Core Variant". The core formulation involves the main feature of the study, raw material shelf-life. These formulations are applied to multiple problem instances differing in aspects of importance for the features in question. The presence or absence of ordering batch sizes and the presence or absence of ordering lead times are the assumptions that vary in the different problem instances and significantly impact results.

When it comes to implementing the model to different problem instances and to an Automotive Industry case study, we use IBM® ILOG® CPLEX® Optimization Studio to solve them. Analysis and discussion are carried out referring to important performance aspects such as: number of constraints, variables, time, nodes and iterations. Optimal solutions, optimality gaps and disposal cost values are also analyzed.

Additionally, a further observation on the behavior of the formulation and results is also held. One of the most important remarks is the one concerning the assumption of equal component inventory costs for cases with and without shelf-life; as well as the interpretation of inventory merely as costs and not as assets. These two observations are of crucial importance because they make a difference in the objective function values, in the general logic of the results, and in the model approximation to real cases.

Finally we conclude that the originally proposed research objectives have been successfully achieved and the proposed models are feasible for future applications, with relevant modifications or extensions.

5.2 Future Research

Future research of this study are directly related to the delimitations presented in the first chapter of this thesis.

In order to develop models with broader applicability in real industry cases, there is opportunity for formulations considering additional aspects such as:

- More directly involving the storage conditions of components with limited shelf-life, i.e. consider both the shelf-life variable, and the component inventory holding costs as a function of storage conditions,

- Assuming different component inventory holding costs for problems with the component shelf-life requirement that for those without it,

- Considering inventory holding (of both components and finished-products) as assets and not only as costs,
- Assuming random customer demand: uncertain demand modeled by probability distributions,
- Allowing backlog inventory with penalty costs,
- Considering inventory stock-out and related costs,
- Involving quality aspects of both components and end-products.

These are some of the possible considerations that would make the formulation more accurate for real world applications.

In addition, given that the proposed formulation presents a significant increase in time and iterations required to solve larger problems, there is an opportunity to use more efficient and sophisticated methodologies using heuristics or metaheuristics to locate optimal or near-optimal solutions of larger size problems.

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APPENDIX 1. IBM® ILOG® CPLEX® OPL MODEL SOURCE FILE

Appendix 1.A Basic Variant OPL Model Source File

```
/*  OPL Model Formulation */
/*  Basic Variant */
/*  No Shelf-Life */

/*SETS AND RANGES*/

 int tPeriods = ...;              /*Set of periods in planning horizon*/
 int iProducts = ...;             /*Set of end-product types*/
 int jComponents = ...;           /*Set of component types*/
 range T = 1..tPeriods;           /*Range of periods t*/
 range N = 1..iProducts;          /*Range of end-product types i*/
 range J = 1..jComponents;        /*Range of component types j*/

/*PARAMETERS*/

/*Demand, BOM, Lead Time and Shelf-Life*/
 int d[N][T] = ...;               /*End-Product Demand*/
 int b[J][N] = ...;               /*Bill of Materials*/
 int l[J] =  ...;                 /*Order Lead Time*/

/*Costs*/
 float c[J] = ...;                /*Component Cost (per unit or per batch)*/
 float p[N] = ...;                /*End-Product Unit Cost*/
 float h[N] = ...;                /*End-Product Inventory Holding Cost*/
 float m[J] = ...;                /*Component Inventory Holding Cost*/
 float A[N] = ...;                /*Fixed Set-Up Cost*/
 float g[J] = ...;                /*Fixed Ordering Cost*/

/*Component Order Batch Size*/
 int S[J] = ...;                  /*Order Batch Size*/

/*Capacity*/
 int K[N] = ...;                  /*Production Capacity*/
 int L[J] = ...;                  /*Ordering Capacity (number of units or batches)*/

/*Initial Inventory, scheduled order receipt*/
 int EPInv[N] = ...;              /*End-Product Initial Inventory*/
 int CInv[J] = ...;               /*Component Initial Inventory*/
 int q[J][T] = ...;               /*Component Scheduled Order Receipt*/

/*VARIABLES*/

dvar int+ x[N][T];                /*Production*/
 dvar int+ I[N][T];               /*End-Product Inventory*/
 dvar int+ Q[J][T];               /*Component Orders (number of units or batches)*/
 dvar int+ v[J][T];               /*Component Inventory*/
```

```
 dvar boolean y[N][T];          /*Fixed set-up cost binary variable*/
 dvar boolean w[J][T];          /*Fixed scheduled ordering cost binary variable*/
 dvar boolean W[J][T];          /*Fixed ordering cost binary variable*/
 int M = 100000000;             /*Big Number*/

/*OBJECTIVE FUNCTION*/

/*Minimize TotalCost*/

 minimize sum (t in T, i in N) p[i]*x[i][t]
             + sum (t in T, j in J) c[j]*q[j][t]
             + sum (t in T, j in J) c[j]*Q[j][t]
             + sum (t in T, i in N) h[i]*I[i][t]
             + sum (t in T, j in J) m[j]*v[j][t]
             + sum (t in T, i in N) A[i]*y[i][t]
             + sum (t in T, j in J) g[j]*w[j][t]
             + sum (t in T, j in J) g[j]*W[j][t];

/*CONSTRAINTS*/

 subject to {

/*(Basic3 II) End-Product Inventory Balance, Production, Demand Fulfillment*/
   forall (t in T, i in N)
     if (t == 1) {
       Basic3IIa: I[i][t] == EPInv[i] + x[i][t] - d[i][t];
 }
   forall (t in T, i in N)
     if (t > 1) {
       Basic3IIb: I[i][t] == I[i][t-1] + x[i][t] - d[i][t];
 }

/*(Basic3 IX and X) Component Inventory Balance, Component Ordering, BOM*/
   forall (t in T, j in J)
     if (t == 1) {
       Basic3IXa: v[j][t] == CInv[j] + S[j]*q[j][t] - sum (i in N) b[j][i]*x[i][t];
 }
    forall (t in T, j in J)
      if (t > 1 && t <= l[j])    {
        Basic3IXb: v[j][t] == v[j][t-1] + S[j]*q[j][t] - sum (i in N) b[j][i]*x[i][t];
 }
   forall (t in T, j in J)
     if (t > l[j]) {
       Basic3X: v[j][t] == v[j][t-1] + S[j]*Q[j][t-l[j]] - sum (i in N) b[j][i]*x[i][t];
 }

/*(Basic3 IV) Production Capacity*/
    forall (t in T, i in N)
      Basic3IV: x[i][t] <= K[i];

/*(Basic3 V) Ordering Capacity*/
   forall (t in T, j in J)
     Basic3V: Q[j][t] <= L[j];

/*(Basic3 VI) Fixed Set-Up Cost Binary Variable*/
   forall (t in T, i in N)
     Basic3VI: x[i][t] <= M*y[i][t];

/*(Basic 3 VII) Fixed Ordering Cost Binary Variable*/
   forall (t in T, j in J)
     Basic3VII: Q[j][t] <= M*W[j][t];
```

```
/*(Basic 3 XI) Fixed Scheduled Ordering Cost Binary Variable*/
   forall (t in T, j in J)
     Basic3XI: q[j][t] <= M*w[j][t];

 }
```

Appendix 1.B Core Variant OPL Model Source File

```
/*  OPL Model Formulation */
/*  Core Variant */

/*SETS AND RANGES*/

 int tPeriods = ...;              /*Set of periods in planning horizon*/
 int iProducts = ...;             /*Set of end-product types*/
 int jComponents = ...;           /*Set of component types*/
 range T = 1..tPeriods;           /*Range of periods t*/
 range R = 0..tPeriods;           /*Range of ordering periods r*/
 range N = 1..iProducts;          /*Range of end-product types i*/
 range J = 1..jComponents;        /*Range of component types j*/

/*PARAMETERS*/

/*Demand, BOM, Lead Time and Shelf-Life*/
 int d[N][T] = ...;               /*End-Product Demand*/
 int b[J][N] = ...;               /*Bill of Materials*/
 int l[J] =  ...;                 /*Order Lead Time*/
 int a[J] = ...;                  /*Component Shelf-Life*/

/*Costs*/
 float f[J] = ...;                /*Disposal Unit Cost*/
 float c[J] = ...;                /*Component Cost (per units or per batch)*/
 float p[N] = ...;                /*End-Product Unit Cost*/
 float h[N] = ...;                /*End-Product Inventory Holding Cost*/
 float m[J] = ...;                /*Component Inventory Holding Cost*/
 float A[N] = ...;                /*Fixed Set-Up Cost*/
 float g[J] = ...;                /*Fixed Ordering Cost*/

/*Component Order Batch Size*/
 int S[J] = ...;                  /*Order Batch Size*/

/*Capacity*/
 int K[N] = ...;                  /*Production Capacity*/
 int L[J] = ...;                  /*Ordering Capacity (number of units or batches)*/

/*Initial Inventory and Sheduled Order Receipt*/
 int EPInv[N] = ...;              /*End-Product Initial Inventory*/
 int CInv[J] = ...;               /*Component Initial Inventory*/
 int q[J][T] = ...;               /*Component Scheduled Order Receipt*/

/*VARIABLES*/
 dvar int+ x[N][T];               /*Production*/
 dvar int+ I[N][T];               /*End-Product Inventory*/
 dvar int+ Q[J][T];               /*Component Orders (number of component units or batches)*/
 dvar int+ v[J][T][R];            /*Component Inventory*/
 dvar int+ e[J][R][T];            /*Component Consumption*/
 dvar float+ z[J];                /*Component Disposal*/
 dvar boolean y[N][T];            /*Fixed set-up cost binary variable*/
 dvar boolean w[J][T];            /*Fixed scheduled ordering cost binary variable*/
 dvar boolean W[J][T];            /*Fixed ordering cost binary variable*/
 int M = 100000000;               /*Big Positive Number*/

/*OBJECTIVE FUNCTION*/
```

```
/*Minimize TotalCost*/

 minimize sum (t in T, i in N) p[i]*x[i][t]
              + sum (t in T, j in J) c[j]*q[j][t]
              + sum (t in T, j in J) c[j]*Q[j][t]
              + sum (t in T, i in N) h[i]*I[i][t]
              + sum (t in T, j in J, r in R) m[j]*v[j][t][r]
              + sum (j in J) f[j]*z[j]
              + sum (t in T, i in N) A[i]*y[i][t]
              + sum (t in T, j in J) g[j]*w[j][t]
              + sum (t in T, j in J) g[j]*W[j][t];

/*CONSTRAINTS*/

 subject to {

/*(C2) End-Product Inventory Balance, Production, Demand Fulfillment*/
   forall (t in T, i in N)
     if (t == 1) {
       C2a: I[i][t] == EPInv[i] + x[i][t] - d[i][t];
 }
   forall (t in T, i in N)
     if (t > 1) {
       C2b: I[i][t] == I[i][t-1] + x[i][t] - d[i][t];
 }

/*(C13) Component Inventory Balance, Component Ordering, Bill of Materials*/
   forall (t in T, j in J)
     if (t == 1) {
       C13a: v[j][t][0] + v[j][t][t] == CInv[j] + S[j]*q[j][t] - sum (i in N) b[j][i]*x[i][t];
 }
   forall (t in T, j in J)
     if (t > 1 && t <= l[j] && t < a[j]) {
       C13b: v[j][t][0] + sum (r in 1..t) v[j][t][r] == v[j][t-1][0] + sum (r in 1..t-1) v[j][t-1][r] + S[j]*q[j][t] - sum (i in N)
b[j][i]*x[i][t];
 }
    forall (t in T, j in J)
      if (t > 1 && t <= l[j] && t >= a[j]) {
        C13c: sum (r in t+1-a[j]..t) v[j][t][r] == sum (r in t+1-a[j]..t-1) v[j][t-1][r] + S[j]*q[j][t] - sum (i in N)
b[j][i]*x[i][t];
 }
   forall (t in T, j in J)
     if (t > l[j] && t < a[j]) {
       C13d: v[j][t][0] + sum (r in 1..t) v[j][t][r] == v[j][t-1][0] + sum (r in 1..t-1) v[j][t-1][r] + S[j]*Q[j][t-l[j]] - sum (i in
N) b[j][i]*x[i][t];
 }
   forall (t in T, j in J)
     if (t > l[j] && t >= a[j]) {
         C13e: sum (r in t+1-a[j]..t) v[j][t][r] == sum (r in t+1-a[j]..t-1) v[j][t-1][r] + S[j]*Q[j][t-l[j]] - sum (i in N)
b[j][i]*x[i][t];
 }

/*(C14) Component Consumption*/
    forall (t in T, j in J)
      if (t < a[j])    {
        C14a: sum (i in N) b[j][i]*x[i][t] == sum (r in 0..t) e[j][r][t];
 }
    forall (t in T, j in J)
      if (t >= a[j])    {
        C14b: sum (i in N) b[j][i]*x[i][t] == sum (r in t+1-a[j]..t) e[j][r][t];
 }
```

```
/*(C15) Component Individual Inventory*/
   forall (t in T, j in J)
     if (t == 1)    {
       C15: v[j][t][0] == CInv[j] - e[j][0][t];
 }
   forall (t in T, j in N, r in R)
     if (t == r && t <= l[j])    {
       C15a: v[j][t][r] == S[j]*q[j][t] - e[j][r][t];
 }
   forall (t in T, j in J, r in R)
     if (t == r && t > l[j])    {
       C15b: v[j][t][r] == S[j]*Q[j][t-l[j]] - e[j][r][t];
 }
    forall (t in T, j in J, r in R)
      if (r < t && t > 1 && t - r < a[j])    {
         C5b: v[j][t][r] == v[j][t-1][r] - e[j][r][t];
 }

/*(C6)Component Disposal*/
    forall (j in J)
      z[j] == sum (t in a[j]-1..tPeriods) v[j][t][t+1-a[j]];

/*(C7) Production Capacity*/
    forall (t in T, i in N)
      C7: x[i][t] <= K[i];

/*(C8) Ordering Capacity*/
   forall (t in T, j in J)
     C8: Q[j][t] <= L[j];

/*(C9) Fixed Set-Up*/
   forall (t in T, i in N)
     C9: x[i][t] <= M*y[i][t];

/*(C10) Fixed Scheduled Ordering*/
   forall (t in T, j in J)
     CXI: q[j][t] <= M*w[j][t];

/*(C11) Fixed Ordering*/
   forall (t in T, j in J)
     C11: Q[j][t] <= M*W[j][t];

 }
```