Discrete Event Control Kit

DECK 1.2012.10

User Manual

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Abstract

Discrete Event Control Kit (DECK) is a toolbox (a set of functions) written in the programming language of MATLAB [2] for the analysis and design of supervisory control systems based on discrete–event models. This software has been developed by Shahin Hashtrudi Zad and two of his graduate students, Shauheen Zahirazami and Farzam Boroomand. DECK is provided under the terms of the GNU General Public License, version 2, as published by the Free Software Foundation. The text of the license appears in Appendix B.

Contents

1 Introduction

Discrete Event Control Kit (DECK) is a toolbox (set of M–file functions) written in the programming language of MATLAB [2] for the analysis and design of supervisory control systems based on the Ramadge–Wonham (RW) theory of supervisory control of discrete–event systems (DES) [3]. The current version of DECK supports the case of supervision under full event observation. Future versions will extend the support to other cases. For information about supervisory control, the reader is referred to [5, 1].

DECK has been developed in the familiar environment of MATLAB as an educational tool for a graduate course at Concordia University on the supervisory control of discrete–event systems. Furthermore, it offers a set of functions that, along with the matrix and set operations of MATLAB, provide a convenient setup for implementing new algorithms and applying them to useful and interesting test cases.

The rest of this user manual is organized as follows. Section 2 explains how discrete–event models are built in DECK and provides an overview of the analysis and design functions that are available in DECK.The application of these functions to two examples is covered in Section 3. The second example is a well–known benchmark problem for which performance results (specifically, execution times) are provided. Section 4 contains the description of the functions. Appendix A contains a note on installation and Appendix B provides the text of the GNU General Public License, version 2.

2 Building Models and Solving Problems

In the RW supervisory control theory [3], it is assumed that the plant and the design specifications can be modeled as finite–state automata. A finite–state automaton is a five–tuple

$$G = (X, \Sigma, \eta, x_0, X_m)$$

where X is the finite state set, Σ the event set, x_0 the initial state, $X_m \subseteq X$ the set of marked states and $\eta : X \times \Sigma \to X$ is the transition (partial) function. An example is given in Fig. 1 with $X = \{1, 2, 3\}$, $\Sigma = \{11, 12\}$, $x_0 = 1$ and $X_m = \{2, 3\}$. The marked states are

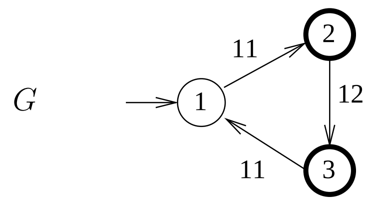


Figure 1: Automaton G.

shown with thick circles.

In DECK, an automaton is characterized by

1. `N`, the number of states,
2. `TL`, the list of transitions, and
3. `Xm`, the set of marked states.

`N` is a nonnegative ($\texttt{N} \geq 0$) integer. For an automaton with $\texttt{N} \geq 1$, the state set is $\{1, \ldots, \texttt{N}\}$, with 1 used for the initial state. For an empty automaton, $\texttt{N} = 0$.

The event labels are nonnegative integers [1]. Zero (0) is reserved for clock tick. In supervisory control, the event set is partitioned into controllable and uncontrollable event sets. The controllability status of an event depends on the extent and scope of a supervisor's control over the plant and hence on both the plant and the supervisor. This issue becomes important, for instance, in decentralized supervisory control. Therefore, in DECK when an automaton is defined, the controllable/uncontrollable status of events are not specified. This status is declared only when a function related to supervisory control (i.e., `supcon` and `controllable`) is used.

`TL` (transition list) is a matrix with three columns. Each row represents a transition in the form `[x1 e x2]`, where `x1`, `e` and `x2` are the source state, event and target state. The event set Σ is implicitly taken to be the event labels that appear in the second column of the transition list, `TL`.

[1] The default type for numerical variables in MATLAB is double precision floating point. To prevent roundoff errors in storing integer labels, the event labels should be no larger than 9×10^{15}.

`Xm` is a *row* vector containing the marked states.

The function `automaton` builds automaton objects. For example, the following statements build an automaton object `G` corresponding to the automaton G in Fig. 1.

```
N=3;
TL=[1 11 2; 2 12 3; 3 11 1];
Xm=[2 3];
G=automaton(N,TL,Xm);
```

Alternatively, the following compact form may be used.

```
G=automaton(3, [1 11 2; 2 12 3; 3 11 1], [2 3]);
```

The vector `Xm` (marked states) is optional and if omitted in the `automaton` statement, the default value of empty (`Xm=[]`) will be taken. Once the automaton object `G` is created, typing `G` returns a list of its properties, namely, the number of states, transition list and marked states (An automaton object does not have any methods).

```
G
```

```
G =

  automaton

  Properties:
     N: 3
    TL: [3x3 double]
    Xm: [2 3]

  Methods
```

Note that thoroughout this manual, MATLAB/DECK responses are shown in italics.

The number of states, transition list and marked states of an automaton object `G` can be accessed through `G.N`, `G.TL` and `G.Xm`. For example:

```
G.TL
```

```
ans =

     1    11     2
     2    12     3
     3    11     1
```

The function `automatonchk` can be used to verify that an automaton object conforms to the conventions of DECK for naming states and event labels. This function is useful for checking DECK models that are created for the first time or imported from an input file. It can also help with debugging new functions developed in the DECK environment.

After the desired automata are built, they can be analyzed and manipulated using the available functions in DECK. The `reach` function can be used to perform reachability analysis on the transition graphs of automata. The function `reachable` and `trim` return the reachable and trim subautomata while `complement` finds the complement of an automaton. The `selfloop` function adjoins selfloops to each state of its input automaton.

The functions `product` and `sync` perform the parallel product (also known as "meet") and the synchronous product of an arbitrary number of automata `G1`, ..., `Gn`. The `product` and `sync` functions can also find the parallel and synchrnous products of an array of automata. Furthermore, for every state of the resulting automaton, these functions return the information about the state of each of the constituent automata `G1`, ..., `Gn`.

Finally, the function `supcon` can be used to find the supremal controllable sublanguage and design minimally restrictive supervisor. The function `controllable` can be used to see if a language is controllable and in cases where controllability test fails, to obtain information about the circumstances under which the property has failed.

More details about each function is provided in Sec. 4 (Toolbox Functions). In the following section, two illustrative examples will be discussed.

3 Examples

3.1 Small Factory

This example has two parts. First it illustrates supervisor design using the Small Factory problem (Example 3.4.4, [5]). Next, it explains how it can be verified whether the plant (small factory) under the supervision of a given supervisor meets the design specifications.

The Small Factory (Fig. 2) consists of two machines, MACH1 and MACH2, and a buffer

Figure 2: Small Factory.

with a capacity of one. The automaton model of MACHi is depicted in Fig. 3. Starting

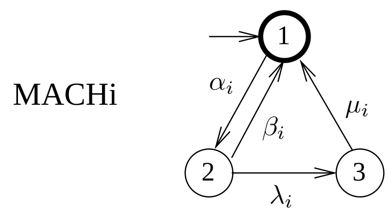


Figure 3: MACHi (i=1,2).

from the initial state I (idle), MACH1 takes a piece (α_1) from an (infinite) input bin and enters state W (work). Once the work of MACH1 is done, it deposits the workpiece in the buffer (β_1). If MACH1 breaks down (λ_1) and enters state D (down), the workpiece will be discarded but the machine can be repaired (μ_1). MACH2 takes workpieces from the buffer and puts the finished workpieces in an output bin.

The objective is to design a supervisor to have a nonblocking system (i.e. plant under supervision) that meets the following design specifications.

(S1) The buffer must not oveflow or underflow.

(S2) In case both machines fail, MACH2 must be repaired first.

The above two specifications can be captured using the following automata [5]. For the purpose of control, the events α_1, α_2, μ_1 and μ_2 are controllable and the rest of events are uncontrollable.

To design supervisor using DECK, we use the following codes for events (which are the same as those in [5]).

$\alpha_1 : 11 \quad \beta_1 : 10 \quad \lambda_1 : 12 \quad \mu_1 : 13$
$\alpha_2 : 21 \quad \beta_2 : 20 \quad \lambda_2 : 22 \quad \mu_2 : 23$

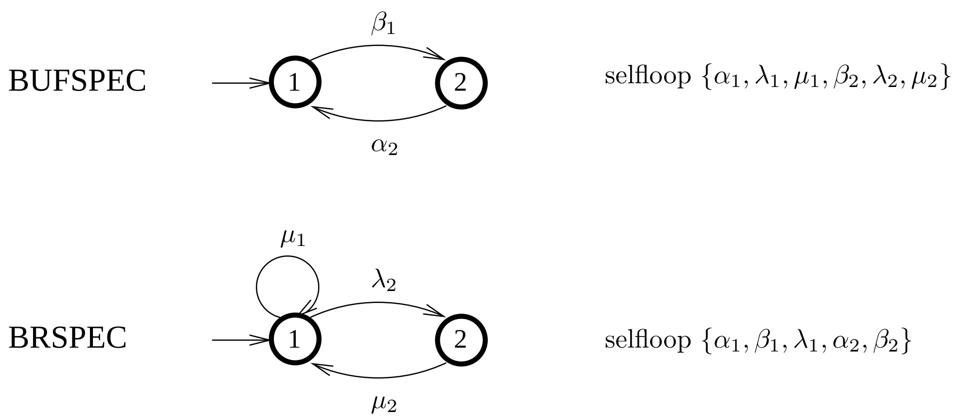


Figure 4: Small Factory: design specifications.

States I, W and D are named 1, 2 and 3. The following scripts design a minimally restrictive supervisor.

```
N1=3;
TL1=[1 11 2; 2 10 1; 2 12 3; 3 13 1];
Xm1=1;
MACH1=automaton(N1,TL1,Xm1);

N2=3;
TL2=[1 21 2; 2 20 1; 2 22 3; 3 23 1];
Xm2=1;
MACH2=automaton(N2,TL2,Xm2);

FACT=sync(MACH1,MACH2);

BUFSPEC=automaton(2, [1 10 2; 2 21 1], [1 2]);
BUFSPEC=selfloop(BUFSPEC,[11,13,14,20,22,23]);

BRSPEC=automaton(2, [1 13 1; 1 22 2; 2 23 1], [1 2]);
BRSPEC=selfloop(BRSPEC,[10,11,13,20,21]);

SPEC=product(BUFSPEC,BRSPEC);

Euc=[10 12 20 22];
FACTSUP=supcon(SPEC,FACT,Euc);
```

First two automata objects `MACH1` and `MACH2` are created. Next the plant model `FACT` is obtained using the `sync` procedure. After automata objects for BUFSPEC and BRSPEC are built, the automaton object `SPEC` is found using the `product` command. Finally the set of uncontrollable events are defined and the supervisor (in the form of an automaton FACTSUP) is obtained using `supcon`. `FACTSUP` has 12 states and 24 transitions.

```
FACTSUP

FACTSUP =

  automaton

  Properties:
     N: 12
    TL: [24x3 double]
    Xm: [1 3]

  Methods
```

The transition list and marked states can be examined as follows.

```
FACTSUP.TL

ans =

     1    11     2
     2    10     3
     2    12     4
     3    21     5
     4    13     1
     5    11     6
     5    20     1
     5    22     7
     6    10     9
     6    12     8
     6    20     2
     6    22    10
     7    11    10
     7    23     1
     8    13     5
     8    20     4
     8    22    11
     9    20     3
     9    22    12
    10    10    12
    10    12    11
```

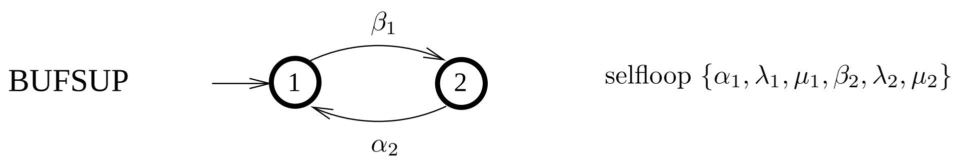


Figure 5: Small Factory: proposed supervisor BUFSUP.

```
    10    23    2
    11    23    4
    12    23    3



FACTSUP.Xm

ans =

     1     3
```

In the second part of this problem, let us consider Small Factory again with the objective of finding a nonblcking supervisor enforcing design specification S1 (regarding buffer overflow and underflow). First suppose the automaton in Fig. 5 has been proposed for supervisor. Note that this automaton is the same as the spec automaton BUFSPEC. The following script first builds the plant model which contains 9 states. This time, in addition to `FACT`, we have decided to get the information of the states of the components (`MACH1` and `MACH2`) in the 9×2 matrix FACT_States. Next the proposed suprevisor `BUFSUP` is built and the `controllable` procedure is used to determine if `BUFSUP` is admissible (i.e., the closed behavior L(BUFSUP) is controllable).

```
MACH1=automaton(3, [1 11 2; 2 10 1; 2 12 3; 3 13 1], 1);
MACH2=automaton(3, [1 21 2; 2 20 1; 2 22 3; 3 23 1], 1);
[FACT,FACT_States]=sync(MACH1,MACH2);

BUFSPEC=automaton(2, [1 10 2; 2 21 1], [1 2]);
BUFSPEC=selfloop(BUFSPEC,[11 12 13 20 22 23]);

BUFSUP=automaton(2, [1 10 2; 2 21 1],[1 2]);
BUFSUP=selfloop(BUFSUP,[11 12 13 20 22 23]);

Euc=[10 12 20 22];
[ic,S,E]=controllable(BUFSUP,FACT,Euc)
```

The results are returned in `ic`, `S`, `E`.

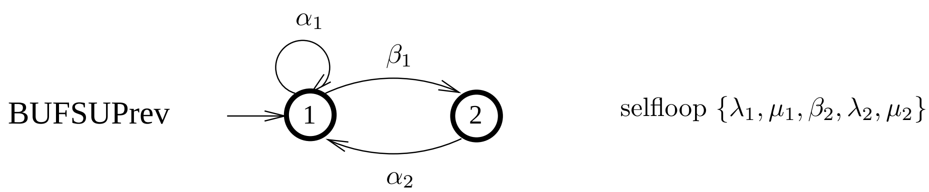


Figure 6: Small Factory: revised supervisor.

```
ic =

     0


S =

     2     2
     2     5
     2     8


E =

    [10]    [10]    [10]
```

It is observed that BUFSUP is not admissible. Uncontrollable event disablement would occur when the plant and supervisor are in states given by the rows of `S`. The corresponding disabled uncontrollable events are provided in `E`. For instance, one such disablement occurs when `FACT` and `BUFSPEC` are in states 2 and 2. To determine the states of `MACH1` and `MACH2` when `FACT` is in state 2, we use

```
FACT_States(2,:)

ans =

     2     1
```

Thus `MACH1` is in state 2 and `MACH2` in state 1. In other words, when the buffer is full, `MACH1` is in state 2 (W) and potentially ready to deposit another workpiece in the buffer, `BUPSUP` attempts to disable the uncontrollable event β_1 (event 10). In order to correct this issue, we remove the α_1 selfloop in state 2 of `BUFSUP` after the occurrence of β_1 (workpiece deposit in buffer) and effectively disable controllable event α_1 when the buffer is full. The resulting supervisor BUFSUPrev is shown in Fig. 6. We can see that this supervisor is addmissible.

```
[icrev,Srev,Erev]=controllable(BUFSUPrev,FACT,Euc)

icrev =

     1


Srev =

     []


Erev =

     {}
```

To verify that the plant under supervision is nonblocking and satisfies the design spec, first it must be built.

```
FACTuSUP=product(FACT,BUFSUPrev)

FACTuSUP =

  automaton

  Properties:
      N: 12
     TL: [25x3 double]
     Xm: [1 3]

  Methods
```

Next the automaton `FSco` is constructed which marks $L_m(\mathrm{FACT}) \cap [L_m(\mathrm{SPEC})]^{\mathrm{co}}$.

```
SPECco=complement(BUFSPEC);
FSco=product(FACTuSUP,SPECco)
```

We note that the marked state set of `FSco` is empty and hence, $L_m(\mathrm{FACT}) \subseteq L_m(\mathrm{SPEC})$.

```
FSco.Xm

ans =

   Empty matrix: 1-by-0
```

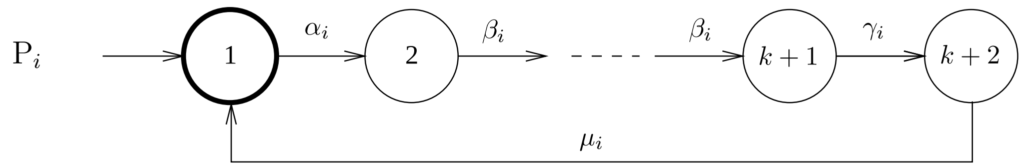


Figure 7: Dining Philosophers: Philosopher P_i.

Finally, to see if the (reachable) automaton `FACTuSUP` is nonblocking, the `trim` procedure is used.

```
FACTuSUPt=trim(FACTuSUP)

FACTuSUPt =

  automaton

  Properties:
     N: 12
    TL: [25x3 double]
    Xm: [1 3]

  Methods
```

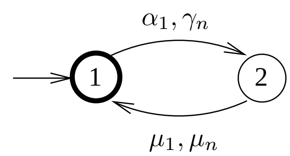
`FACTuSUP` has the same number of states as `FACTuSUPt` and hence it is nonblocking.

3.2 Dining Philosophers

In this example, an extension of the problem of Dining Philosophers (which was proposed in WODES 2008 as a benchmark problem for supervisory control software tools) is examined. In this problem, n philosophers $P_1, \ldots, P_n$ ($n \geq 2$) are seated around a table and n forks $F_1, \ldots, F_n$ placed on the table in the following order: $F_1, P_1, F_2, P_2, \ldots, F_n, P_n$. The automaton modeling philosopher P_i is shown in Fig. 7. Philosopher P_i takes the fork on his left F_i (event α_i) and executes $k-1$ ($k \geq 1$) intermediate events β_i till it reaches state $k+1$. Then it takes the fork on his right (that is F_{i+1} when $1 \leq i \leq n-1$, and F_1 when $i = n$.) and enters eating state $k+2$ (event γ_i). Finally, P_i returns the forks (event μ_i) and goes to its initial (idle) state. The automata for forks are shown in Fig. 8. The events "philosopher takes the left fork" (α_i) are assumed uncontrollable when i is even. The rest of events are controllable. The objective is to design a maximally permissive nonblocking supervisor.

The script for solving the problem is provided at the end of this section. Note that the automata for philosophers and forks are put together to form an array of automata `Gs`. The commands `tic` and `toc` measure the execution time of `supcon`. The execution time, along

F_1



F_i

$(2 \leq i \leq n)$

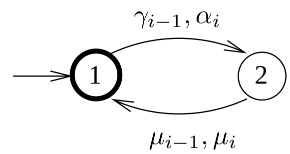


Figure 8: Dining Philosophers: Fork F_i.

with the number of states of the supervisor, is provided for various values of n and k in Table 1.

Remark: Note that the philosopher and fork automata are placed in array `Gs` in the order $F_1, P_1, \ldots, F_n, P_n$. The order affects the execution time (and space requirement) of the `sync` operation (despite the fact that the order of arguments in `sync` function does not change the final result – except perhaps for the name of the states). For instance, for $n = 10$ and $k = 1$, `sync(P1,...,Pn,F1,...,Fn)` takes 24 times longer to run compared with `sync(F1,P1,...,Fn,Pn)`. For an explanation, refer to the description of the `sync` function in Sec. 4.

$k \setminus n$	2	3	4	5	6	7	8	9	10	11	12	
1	<0.1	<0.1	<0.1	<0.1	0.2	0.5	1.7	5.8	28.1	185	1476	T (sec)
	3	12	30	78	190	470	1138	2770	6694	16206	39138	N
2	<0.1	<0.1	<0.1	0.3	1.4	7.9	89.8	1240				T (sec)
	4	24	83	321	1082	3855	12863	44172				N
4	<0.1	<0.1	0.4	3.5	69	2795						T (sec)
	6	60	327	2175	11602	69785						N
8	<0.1	0.3	4.8	325								T (sec)
	10	180	1703	21267								N
16	<0.1	2	313									T (sec)
	18	612	10695									N

Table 1: Dining Philosophers: Execution time of `supcon`, T (sec), and the number of states of the resulting automaton, N, for various values of philosopher number, n, and intermediate states, k. Execution times are measured on a PC with Intel Core i5-2500, 3.10 GHz, 8GB RAM, running 64-bit Windows 7 and MATLAB R2011b (64-bit). Execution times longer than one hour have not been measured.

Script for solving the problem of Dining Philosophers

```
%
% Solves the "Dining Philosophers" benchmark problem of WODES 2008
%
%

display('Dining Philosophers')

% Number of philosophers (and forks)  (n>=2)
n=9;
fprintf('Number of philosophers (n): %d\n', n)

% Number of intermediate states  (k>=1)
k=1;
fprintf('Number of intermediate states (k): %d\n', k)

%
% Plant model (G)
%
display('Building Plant model (G)')

%
% Event coding
%
%  For philosopher Pi:
%    alpha(i)=10*i+1   beta(i)=10*i+2   gamma(i)=10*i+3   mu(i)=10*i+4
%
alpha=10*(1:n)+1;
beta=10*(1:n)+2;
gamma=10*(1:n)+3;
mu=10*(1:n)+4;

%
% Gs is an array of the automata of Philosophers and Forks
% in the order F1, P1, F2, P2, ..., Fn, Pn.
for i=1:2*n
  Gs{i}=automaton(0,[],[]);
end
%
% Philosophers
%
for i=1:n
```

```
  TLi=[1 alpha(i) 2];
  for j=2:k
    TLi=[TLi; j beta(i) j+1];
  end
  TLi=[TLi; k+1 gamma(i) k+2; k+2 mu(i) 1];
  Gs{2*i}=automaton(k+2, TLi, 1);
end

%
% Forks
%
% The first fork
Gs{1}=automaton(2, [1 alpha(1) 2; 1 gamma(n) 2; 2 mu(1) 1; 2 mu(n) 1], 1);
%
% The other forks
for i=2:n
  Gs{2*i-1}=automaton(2, [1 gamma(i-1) 2; 1 alpha(i) 2;  ...
                          2 mu(i-1) 1; 2 mu(i) 1], 1);
end

%
G=sync(Gs);

%
% Spec (H)
%
display('Building Spec model (H)')

% Event set
if k==1
  E=[alpha gamma mu];
else
  E=[alpha beta gamma mu];
end

H=selfloop(automaton(1,[],1),E);

% Uncontrollable events
Euc=alpha(2:2:n);

%
display('Supcon')
```

```
tic
K=supcon(H,G,Euc);
toc
%
%
% End of code
```

4 Toolbox Functions

4.1 Automaton

Create an automaton model ('automaton' object) for use by Discrete Event Control Kit (DECK).

Syntax

`G=automaton(N,TL)`
`G=automaton(N,TL,Xm)`

Inputs

`N`	Number of states
`TL`	Transition list
`Xm`	Marked states (row vector)

Outputs

`G` Output automaton

Description

The class 'automaton' has the following properties:

`N` Number of states.

The states must be named using the following convention: the state set must be {`1`, ..., `N`}, with `1` used for the initial state.

`TL` Transition list.

`TL` is an m-by-3 matrix where m is the number of transitions. Each row of `TL` represents a transition in the form `[x1, sigma, x2]`, with `x1` and `x2` being the source and destination states, and `sigma`, the corresponding event.

`Xm` Marked states (row vector).

`G=automaton(N,TL,Xm)` returns an automaton model (object) `G` with state set {`1`, ..., `N`}, transition list `TL` and set of marked states `Xm`.

`G=automaton(N,TL)` returns an automaton `G` with `Xm=[]`.

4.2 Automatonchk

Verify the validity of an automaton object.

Syntax

```
flag1=automatonchk(G)
[flag1,flag2]=automatonchk(G)
```

Inputs

`G` Input automaton

Outputs

`flag1` Automaton validity flag (part 1)
`flag2` Automaton validity flag (part 2)

Description

`[flag1,flag2]=automatonchk(G)` verifies the validity of the automaton object `G` and returns the result in `flag1`. In cases where the automaton is not valid, the invalid property is identified in `flag2`. The various cases are explained in the following table.

`flag1`	`flag2`	Description
-3	1	`G.N` has wrong size (is not a scalar)
-3	2	`G.TL` has wrong size
-3	3	`G.Xm` has wrong size
-2	1	`G.N` is not an integer
-2	2	`G.TL` contains entry that is not an integer
-2	3	`G.Xm` contains entry that is not an integer
-1	1	`G.N` is negative
-1	2	`G.TL` has out–of–range entry
-1	3	`G.Xm` has out–of–range entry (The valid range for states is 1,...,`G.N`, and for events, nonnegative integers.)
0	2	`G.TL` has repeated rows
0	3	`G.Xm` has repeated entries
1	0	Automaton is valid

`Automatonchk` examines the above list of cases from the top. Once a case is identified, the function returns with the corresponding flags.

4.3 Complement

Complement of a deterministic automaton.

Syntax

`Gco=complement(G)`
`Gco=complement(G,Ea)`

Inputs

`G`	Input deterministic automaton
`Ea`	List of events (vector)

Outputs

`Gco`	Output deterministic automaton

Description

Let E denote the event set of the input automaton `G`.

`Gco=complement(G)` returns an automaton `Gco` with

$$L_m(G_{co}) = E^* - L_m(G), \qquad L(G_{co}) = E^*$$

where $L_m()$ and $L()$ denote marked behavior (marked language) and closed behavior (generated language), and E^* is the Kleene closure of E.

`Gco=complement(G,Ea)` returns an automaton `Gco` with

$$L_m(G_{co}) = E_e^* - L_m(G), \qquad L(G_{co}) = E_e^*$$

where $E_e = E \cup E_a$. The event set of the input automaton, E, and E_a must be disjoint.

4.4 Controllable

Determine if a language is controllable.

Syntax

```
isctrb=controllable(K,G,Euc)
[isctrb,States]=controllable(K,G,Euc)
[isctrb,States,Events]=controllable(K,G,Euc)
```

Inputs

`K`	Automaton representing the test language
`G`	Plant Automaton
`Euc`	Uncontrollable events (vector)

Outputs

`isctrb`	Test result
`States`	List of states where disablement of uncontrollable events occurs
`Events`	List of disabled uncontrollable events (cell array)

Description

Let $L_m()$ and $L()$ denote marked behavior (marked language) and closed behavior (generated language). `isctrb=controllable(K,G,Euc)` returns `isctrb=1` if and only if $L(K)$ is controllable with respect to $L(G)$ and E_{uc}. Otherwise, it returns `isctrb=0`. When K is trim, `controllable(K,G,Euc)` returns `1` if and only if $L_m(K)$ is controllable with respect to $L(G)$ and E_{uc}.

`[isctrb,States]=controllable(K,G,Euc)` returns a two-column matrix `States`. Each row of `States`, `[xK,xG]`, is a state of `product(K,G)` where disablement of uncontrollable events (if any) occurs, i.e., the test fails. If $L(K)$ is controllable (`isctrb=1`), then `States=[]`.

`[isctrb,States,Events]=controllable(K,G,Euc)` returns the list of disabled uncontrollable events (if any) in the cell array Events. The i-th cell of `Events`, `Events{i}`, is a row vector containing the uncontrollable events disabled in `[xKi,xGi]` (the i-th row of `States`).

4.5 Product

Product of automata.

Syntax

```
G=product(G1,...,Gn)
[G,States]=product(G1,...,Gn)

G=product(Ga)
[G,States]=product(Ga)
```

Inputs

`Gi`	Input automaton i (i=1, ..., n)
`Ga`	Cell array containing input automata

Outputs

`G`	Output automaton
`States`	State set of output automaton

Description

`G=product(G1,...,Gn)` returns the product of `G1`, ..., `Gn` ($n >= 2$). If $L_m()$ and $L()$ denote marked behavior (marked language) and closed behavior (generated language), then

$$\begin{aligned} L_m(G) &= Lm(G_1) \cap \ldots \cap L_m(G_n) \\ L(G) &= L(G_1) \cap \ldots \cap L(G_n). \end{aligned}$$

`[G,States]=product(G1,...,Gn)` returns an $N \times n$ matrix `States` where N is the number of states of `G`. Let `[xi1 ... xin]` be the i-th row of `States`. Then `xi1`, ..., `xin` are the states of `G1`, ..., `Gn` when `G` is in state i.

`PRODUCT` can be used with arrays of automata. Let `Ga` denote a cell array containing automata `Ga1`, ..., `Gan` ($n >= 2$). `product(Ga)` returns the product of `Ga1`, ..., `Gan`.

4.6 Reach

Find the reachable states of transition graph.

Syntax

```
Xr=reach(TL,S)
```

Inputs

`TL`	Transition list
`S`	Source states (vector)

Outputs

`Xr`	States reachable from `S` (row vector)

Description

`Xr=reach(TL,S)` returns the states of the (automaton) transition graph that are reachable from the set of source states `S` using the breadth–first–search algorithm. The reachable states appear in `Xr` in the order they are discovered in the breadth–first search.

4.7 Reachable

Find reachable subautomaton.

Syntax

```
Gr=reachable(G)
[Gr,Xr]=reachable(G)
```

Inputs

`G` Input automaton

Outputs

`Gr` Reachable subautomaton
`Xr` Reachable states of `G` (row vector)

Description

`Gr=reachable(G)` returns the subautomaton of `G` that is reachable (from the initial state of `G`). The states of `Gr` are renamed in the order they are discovered in a breadth–first search.

`[Gr,Xr]=reachable(G)` returns the reachable states of `G` in `Xr`.

4.8 Selfloop

Add selfloops to automaton.

Syntax

`Gs=selfloop(G,Es)`

Inputs

`G`	Input automaton
`Es`	List of events (vector)

Outputs

`Gs`	Output automaton

Description

Adds selfloop transitions `[x, e, x]` to the transition list of the input automaton `G`, for all states `x` of `G` and all events `e` in the event list `Es`. The event set of `G` and `Es` must be disjoint.

4.9 Supcon

Supremal Controllable Sublanguage.

Syntax

`K=supcon(H,G,Euc)`

Inputs

`H`	Specification (deterministic) automaton
`G`	Plant (deterministic) automaton
`Euc`	Uncontrollable events (vector)

Outputs

`K`	Trim (deterministic) automaton marking supremal controllable sublangage

Description

Let $L_m()$ and $L()$ denote marked behavior (marked language) and closed behavior (generated language). `SUPCON` calculates the supremal sublanguage of $L_m(H) \cap L_m(G)$ that is controllable with respect to $L(G)$ and E_{uc}. The result is returned in the trim automaton `K` which marks the supremal controllable sublanguage. The calculations are based on the algorithm introduced in

> W.M. Wonham and P.J. Ramadge, "On the supremal controllable sublanguage of a given language," *SIAM J. Control and Optimization*, Vol. 25, No. 3, May 1987.

4.10 Sync

Synchronous product of automata.

Syntax

```
G=sync(G1,...,Gn)
[G,States]=sync(G1,...,Gn)
[G,States,Blocked_events]=sync(G1,...,Gn)

G=sync(Ga)
[G,States]=sync(Ga)
[G,States,Blocked_events]=sync(Ga)
```

Inputs

`Gi`	Input automaton i (i=1, ..., n)
`Ga`	Cell array containing input automata

Outputs

`G`	Output automaton
`States`	State set of output automaton
`Blocked_events`	Events blocked (absent) in output automaton (row vector)

Description

`G=sync(G1,G2)` returns the synchronous product of `G1` and `G2`. Let E_1 and E_2 be the event sets of G_1 and G_2. If $L_m()$ and $L()$ denote marked behavior (marked language) and closed behavior (generated language), then

$$\begin{aligned} L_m(G) &= L_m(G_1) \| L_m(G_2) \\ L(G) &= L(G_1) \| L(G_2) \end{aligned}$$

Here $L_1 \| L_2$ is the synchronous product of languages L_1 and L_2 defined according to

$$L_1 \| L_2 = P_1^{-1}(L_1) \cap P_2^{-1}(L_2)$$

where P_1 (resp. P_2) is the natural project of $(E_1 \cup E_2)^*$ onto E_1^* (resp. E_2^*).

`G=sync(G1,...,Gn)` returns the synchronous product of `G1`, ..., `Gn` ($n >= 2$).

`[G,States]=sync(G1,...,Gn)` returns the $N \times n$ matrix `States` where N is the number of states of `G`. Let `[xi1 ... xin]` be the i-th row of `States`. Then `xi1`, ..., `xin` are the states of `G1`, ..., `Gn` when `G` is in state i.

`[G,States,Blocked_events]=sync(G1,...,Gn)` returns the row vector `Blocked_events` containing the events that are in the transition list of at least one of the input automata (`Gi`) and absent in the transition list of the output automaton (`G`).

`SYNC` can be used with arrays of automata. Let `Ga` denote a cell array containing automata `Ga1`, ..., `Gan` ($n >= 2$). `sync(Ga)` returns the synchronous product of `Ga1`, ..., `Gan`.

Remark: The execution time (and space requirement) of the `sync` operation depends on the order of the input arguments `G1`, ..., `Gn`. This can be explained as follows. In DECK, `G=sync(G1,G2,G3,G4)`, for instance, is evaluated in the following steps. First `sync(G1,G2)` is calculated. Let us call the result `G12`. Next `G123=sync(G12,G3)` and finally `G=sync(G123,G4)` are found. If automata that have common events are listed next to eachother, then the intermediate automata resulting from successive application of `sync` (in our example, `G12, G123`) are likely to have fewer states because of the interactions among the neighboring automata on the list of input arguments. Therefore, this results in smaller intermediate automata (hence, less space requirement) and faster execution time. Coversely, if automata with no common events appear first on the list of `sync`, the intermediate automata in `sync` computations could become very large, resulting in prolonged execution times.

4.11 Trim

Find the reachable and coreachable subautomaton.

Syntax

```
Gt=trim(G)
[Gt,Xrc]=trim(G)
```

Inputs

`G`	Input automaton

Outputs

`Gt`	Trim subautomaton
`Xrc`	States of `G` that are reachable and coreachable (row vector)

Description

`Gt=trim(G)` returns the trim subautomaton of `G` (containing only those states of `G` that are both reachable and coreachable). The states of `Gt` are renamed in the order they are discovered in a breadth–first search.

`[Gt,Xrc]=trim(G)` returns the states of `G` that are reachable and coreachable in `Xrc`.

A Installation

Simply unzip the downloaded file. If you wish to keep your data files and scripts in a separate directory (folder) and run DECK from that directory, then from MATLAB menu bar, use “File” followed by “Set path” to add the directory of DECK files to MATLAB’s search path. Alternatively, this can be done using the command “path”.

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