POPULARIZATION OF MATHEMATICS AS INTERCULTURAL COMMUNICATION–AN EXPLORATORY STUDY

Klara Kelecsenyi

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ABSTRACT

Popularization of mathematics as intercultural communication – An exploratory study

Klara Kelecsenyi, Ph.D.
Concordia University, 2009

Popularization of mathematics seems to have gained importance in the past decades. Besides the increasing number of popular books and lectures, there are national and international initiatives, usually supported by mathematical societies, to popularize mathematics. Despite this apparent attention towards it, studying popularization has not become an object of research; little is known about how popularizers choose the mathematical content of popularization, what means they use to communicate it, and how their audiences interpret popularized mathematics.

This thesis presents a framework for studying popularization of mathematics and intends to investigate various questions related to the phenomenon, such as:

- What are the institutional characteristics of popularization?
- What are the characteristics of the mathematical content chosen to be popularized?
- What are the means used by popularizers to communicate mathematical ideas?
- Who are the popularizers and what do they think about popularization?
- Who are the audience members of a popularization event?

- How do audience members interpret popularization?

The thesis presents methodological challenges of studying popularization and suggests some ideas on the methods that might be appropriate for further studies. Thus it intends to offer a first step for developing suitable means for studying popularization of mathematics.

ACKNOWLEDGMENTS

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I also wish to thank the lecturers and audience members who kindly agreed to participate in my study, and were willing to offer their time and share their views and comments with me.

Finally, I would like to thank my family and friends for their constant support and encouragement during the years of my studies.

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CHAPTER 1

INTRODUCTION

> *Sadly, the level of abstraction in mathematics, and the consequent need for notation that can cope with that abstraction, means that many, perhaps most, parts of mathematics will remain forever hidden from the non-mathematician; and even the more accessible parts – the parts described in books like this one – may at best be dimly perceived, with much of their inner beauty locked away from view. Still, that does not excuse those of us who do seem to have been blessed with an ability to appreciate that inner beauty from trying to communicate to others some sense of what it is we experience – some sense of the simplicity, the precision, the purity, and the elegance that give the patterns of mathematics their aesthetic value.* (Devlin, 2002b: 8-9)

In the last two decades, there seems to have been an increase in the activity of popularization of mathematics and a growing interest in it within the mathematical community. Besides the many popular books on mathematics published and displayed on bookstore shelves, magazines targeting high school students, public lectures and math fairs organized by schools and universities, there have been large scale initiatives at the national or international level.

Year 2000 was declared the World Mathematical Year[1], aiming, in particular, at making modern research in mathematics more visible to "the man in the street". That year, a series of large and colorful posters with messages about mathematics were on display in the Montreal Metro stations. Year 2008 was a similar event on the national

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[1] For additional information about the program see the official site of WMY2000: http://wmy2000.math.jussieu.fr/ (Downloaded: July 5, 2009)

level in Germany: Jahr der Mathematik[2]. There is now an annual set of activities, called The Math Awareness Month[3], targeting mostly the North American audience.

There are, thus, many popularization of mathematics projects, but, most of the time, these projects are not evaluated (Ernest, 1996:785).

Popularization of mathematics is often considered as a possible remedy for the problems that mathematics has with its public image, attitudes towards mathematics, appreciation of mathematics in the society at large, and low enrollment in mathematics and mathematics related topics (science, engineering) at the university level (Landsman, 2008). According to my knowledge, however, there has been very little systematic research on whether these expectations are realistic or not. I am aware of one empirical study that seemed to address these questions in the particular case of the effects of Square One TV on children's attitudes and constructs of mathematics (Debold, Hall, Fisch, Bennett, & Solan, 1990).

Little is also known about how popularizers choose what to present to the general public and decide how to do it, and even less about their audiences' impressions, reactions, opinions and understanding of popularized mathematics. These were the questions that I was the most interested in.

There could be several reasons for the dearth of research on popularization of mathematics. One of them is that the object of study, "popularization of mathematics" is

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[2]The following site contains additional information about the event http://www.jahr-der-mathematik.de/ (Downloaded: July 5, 2009)

[3] The annual event is organized by The American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics, for further information see http://www.mathaware.org/index.html (Downloaded: July 5, 2009)

not a well defined area. There does not seem to be a consensus, even within the mathematical community, on what counts as popularization and what does not. It is also not clear which academic discipline it belongs to: mathematics, mathematics education, or science communication. It cannot be a discipline in itself; that much is rather obvious. In fact, some mathematics educators are adamant about putting mathematics education research in the same "bag" as popularization of mathematics, as it is done in the International Congresses of Mathematicians:

> In the International Congress of Mathematicians, one of the parallel sessions was devoted to 'Teaching and popularization of mathematics' (1998) and 'Mathematics Education and popularization' (2002). Some researchers in didactics of mathematics were invited as official lecturers.... However, why is teaching paired with popularization? Is it a way of emphasizing once again that, for mathematicians, teaching mathematics is similar to popularizing mathematics? Whichever is the mathematicians' opinion, we must insist that they are related, yet different, things. Popularization might be related only to some aspects of the research in didactics of mathematics, like students' motivation and the reorganization of a field of knowledge around a few fundamental ideas. But teaching is much more (and very different) than popularization.... (Bartolini Bussi & Bazzini, 2003)

I became aware of another reason for the paucity of research on popularization of mathematics the hard way, by stumbling on methodological challenges in trying to conduct such research myself. Methods used in research on mathematics teaching and learning in school contexts do not apply, because students in those contexts usually do not have a choice between attending or not attending, paying attention or not, unless they do not care about the formal consequences of their behavior. Popularization is a free activity chosen according to the participants' wishes, and generally, there is no assessment following the event. If the researcher, however, wants to investigate the audience members' reactions or interpretations of a popular event, he or she must recruit some of them to talk or write about the event. This may feel like a test to some participants, and turn the popularization event into a teaching event. Therefore, it is not popularization that we would be studying but something different.

Moreover, since there can be no norms about the interpretation of the mathematical contents of a popular event – participants are free to attend to whatever they wish – no measure of understanding can be established, and evaluation of the event becomes problematic. There seems to be some awareness of the methodological difficulties in studying popularization in the mathematical and mathematics education community (Holton, Muller, Oikkonen, Valenzuela & Zizhao, 2009: 12).

This thesis presents the results of my struggles (and sometimes the story of these struggles) to study popularization of mathematics nevertheless. I present a framework for studying popularization that I finally came up with in Chapter 2, and its application to analyzing, in detail, three popular lectures in the remaining chapters. I observed two of them, and conducted myself the third one. I interviewed several participants after the observed talks, and asked the participants of my talk to fill out a questionnaire. This experience allowed me to realize the methodological challenges of studying popularization and get some insight into what could be the methods that are more appropriate. I consider this thesis as a first step for developing suitable means for studying popularization of mathematics.

Chapter 2 addresses the question, what is popularization. Popularization of mathematics is described by ten characteristics that lead to questions about popularization which structure the rest of the thesis. These questions are:

What are the institutional characteristics of popularization? (Chapter 3)

What are the characteristics of the mathematical content chosen to be popularized? (Chapter 4)

How can mathematics be popularized? What are the means used by popularizers? (Chapter 5)

Who are the popularizers and what do they think about popularization? (Chapter 6)

Who are the audience members of a popularization event? (Chapter 7)

How do audience members interpret popularization? (Chapter 8)

Chapters 3 to 6 are organized into two parts. In the first part of the chapters, I discuss the main question of the chapter in general terms, looking at various examples of popularization. In the second part, I address the question specifically in the case of the two popular lectures that I observed. I analyze each lecture individually, and then look at similarities and differences between them. Chapters 7 and 8 are devoted entirely to studying these two lectures' audiences' perceptions of the lectures.

Chapter 9 is devoted to the lecture I designed and conducted myself with two audiences, one composed of Hungarian secondary school students, and the other – of Canadian college students and teachers. The questions that I posed to analyze popularization provide the structure of this chapter. The thesis ends with Chapter 10 which contains conclusions.

CHAPTER 2

BUILDING A FRAMEWORK FOR STUDYING POPULARIZATION OF MATHEMATICS: POPULARIZATION AS INTERCULTURAL COMMUNICATION

2.1 INTRODUCTION

In this chapter, I present a general framework for studying popularization of mathematics, in the form of questions to be asked about this activity. The starting point was a reflection on the possible implications of the metaphor of popularization of mathematics as "mathematical tourism", inspired by the title, *The Mathematical Tourist* by Ivars Peterson (1998). In this metaphor, popularizers give "guided tours" of mathematical landscape and culture to visitors from "foreign" cultures (Fig. 2.1). From this point of view, popularization of mathematics becomes a kind of *intercultural communication.* Exploiting the implications of the tourist metaphor leads to distinguishing ten important aspects of popularization of mathematics. These particular aspects of popularization of mathematics lead to a structured set of questions to be asked about a popularization of mathematics event by a researcher.

2.2 THE TOURIST METAPHOR AND ITS IMPLICATIONS

Let us formulate the "tourist metaphor" for popularization of mathematics as follows: *Popularization of mathematics consists in communicating selected parts of mathematical culture to groups of "tourists" from other cultures, in the aim of improving their appreciation of the mathematical culture.*

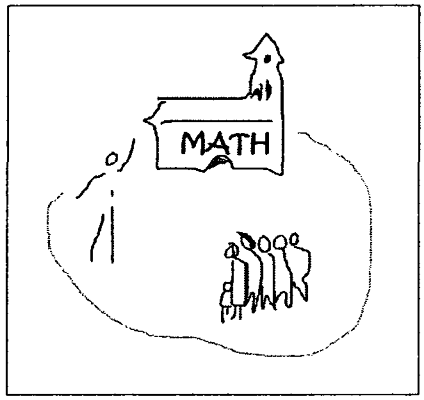

Figure 2.1. The tourist metaphor of popularization of mathematics

Let us exploit the implications of this metaphor.

2.2.1 Popularization of mathematics is organized action

One mathematician talking about mathematics with a non-mathematician in a chance encounter (e.g. during a flight or train journey) is not a popularization event, just as walking in historic parts of a foreign city or a scenic mountainous area is not necessarily tourism. Tourism is organized by tourist agencies; it is an institutionalized activity. Likewise, popularization events are planned, organized actions aimed at larger groups of people, although the level of institutionalization of popularization of mathematics is certainly much lower than in tourism. A tourist agency must organize the trip, ensure the means of transportation, hire the guides, etc. The organizers of a popularization event must ensure there will be a speaker or an animator who will serve as a "guide" for mathematical culture in the event; that the theme or object of activity is sufficiently attractive; that there will be a physical space (e.g. a room) for the meeting or activity; that the event is properly advertised, etc.

Organization of a tourist tour and organization of a school "field trip" may have a lot in common, but participation in these two types of events is different in ways similar to the differences between popularization of mathematics and teaching mathematics. Several aspects of popularization discussed below will highlight these differences.

2.2.2 Admission to a popularization of mathematics event is non-selective

Like tourists visiting a culture, participants in a popularization of mathematics event are not selected; access is open to all. In particular, admission is not based on candidates' age or achievement on examinations, as in the case of courses in mathematics taken for credit in an educational institution. Accessibility to the wider public was mentioned as one of the characteristics of popularization of mathematics in Howson and Kahane (1990: 5-6), and Ernest (1996: 786).

2.2.3 Participation in a popularization of mathematics event is not compulsory

There is no obligation to engage in tourism. There is no obligation to participate in a popularization of mathematics event, either, and this aspect, like the previous one, is part of the existing characterizations of popularization of mathematics (Howson & Kahane, 1990: 5-6; Ernest, 1996:786). The non-compulsory character of popularization of mathematics distinguishes it from teaching and learning mathematics in a diploma awarding educational institution, where participation is obligatory by contract, both for the teachers and for the students.

2.2.4 Popularization of mathematics attempts to make mathematics appear attractive to the visitors

Travel agencies are trying to make tourist sites as attractive as possible. So do announcements of popularization of mathematics events. This is yet another difference between popularization and institutionalized teaching of mathematics, where making mathematics engaging and fun is optional. If some teachers do it, it is on their own initiative. There is no penalty if they do not. Students of boring teachers, on the other hand, can be punished if they leave the classroom for that reason. However, if a popularization of mathematics event is boring, the audience leaves the site and the popularizer has to close shop, maybe forever. One can be an unpopular popularizer only once.

2.2.5 Mathematics is a culture

Our fundamental metaphor of popularization of mathematics as stated above assumes that the tourists are visiting "a part of mathematical culture". This requires an explanation: in what sense is mathematics a culture?

I will use the definition proposed by the anthropologist Clifford Geertz, where culture refers to

> an historically transmitted pattern of meanings embodied in symbols, a system of inherited conceptions expressed in symbolic forms by means of which men communicate, perpetuate, and develop their knowledge about and attitudes towards life (Geertz, 1973: 89).

Mathematics seems to satisfy this definition. It is, indeed, "an historically transmitted pattern of meanings", with its own (rather unique) symbolic formalism and value system. The value system not only internally distinguishes between elegant and ugly proofs, meaningful and pointless mathematics or fruitful and sterile approaches (see, e.g. Anglin, 1997), but also more or less directly influences many decisions we make in life (at least those that rely on some measure or measurement).

Mathematics as a culture consists of various subcultures. It includes, but is not limited to the different domains of mathematical research usually represented in sectional addresses of the International Congresses of Mathematicians[4], such as Logic and Foundations, Algebra, Geometry, Number Theory, Analysis, Topology, Probability and Statistics or Applications of Mathematics in the Sciences. These domains share many aspects of the mathematical culture but there are subtle differences in symbolism (e.g. algebraists and topologists may use slightly different notations and graphic representations), and values (e.g. a topologist may value visual representations more than an algebraist).

The list of sections at the ICM 2006 in Madrid did not contain such titles as Actuarial Mathematics or Financial Mathematics, although in some Mathematics and Statistics departments at universities these areas attract many (if not the majority of students), and scholarship in these domains is already quite well developed[5]. This suggests the developing and changing nature of the mathematical culture.

Besides the various domains of research mathematics, which altogether constitute "mathematicians' mathematics", it can be argued that school mathematics (Civil, 2002), as well as everyday mathematics used by various ethnic communities (including

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[4] See, for example, the sections of ICM 2006, Madrid, available at http://www.icm2006.org/scientificprogram/scientificsections/ (Downloaded on May 16, 2008)
[5] See, for example, the web page of Interdisciplinary Master of Science Degree in Financial Mathematics at Stanford University (http://finmath.stanford.edu/)

aboriginal groups, see Saxe, 1990) and in different occupations (including children selling candy in the streets, see Nunes, Schliemann & Carraher, 1993) also constitute subcultures of the mathematical culture in the broad sense.

2.2.6 The visited mathematical culture must be somewhat "exotic" for the audience to warrant organizing a popularization of mathematics event to show it

One cannot be a tourist in one's own familiar surroundings. The tourist wants to be surprised, amazed; tourism is fuelled by the curiosity of the unknown and the appeal of the "exotic". Similarly, popularization applies only to themes that are new, strange or surprising to the audience. Thus, a popularization of mathematics event may consist in a mathematician specializing in one domain presenting – in an attractive way – this domain to mathematicians specializing in another domain. However, a mathematician presenting his or her results to colleagues working in the same domain, no matter how attractive or exciting the presentation, would not qualify as a popularization of mathematics event.

The mathematical subculture visited in a popularization of mathematics event does not have to be part of the "mathematicians' mathematics". Puzzles and other kinds of "recreational mathematics" are not part of mathematicians' mathematics, yet they are often the site of the popularization of mathematics "tourism", visited by lay people and mathematicians alike. Also "ethnic mathematics", developed in various oral aboriginal traditions can be an attractive subject of a popularization of mathematics event, besides being the object of study in the vast area of research at the intersection of anthropology and mathematics education called "ethnomathematics" (Ascher & D'Ambrosio, 1994).

"Exotic" culture is also "foreign" to the visitor. "Foreignness", however, has certain negative connotations that "exotic" has not. In fact, in mathematics education research, the foreignness of the school mathematical knowledge is largely treated as a *problem*, causing students' difficulties and sometimes their alienation from school culture in general. Students' difficulties in school mathematics have been explained by the cultural clash or conflict between the familiar everyday mathematics and the foreign school mathematics (e.g., Lave, 1988; Nunes, Schliemann and Carraher, 1993; Bishop, 1994; Civil, 2002). Also, the analogy between learning mathematics and learning a foreign language has been drawn and thoroughly exploited both for explaining what is at stake in learning mathematics and for proposing ways of facilitating the process (Pimm, 1987; Prediger, 2004).

2.2.7 Popularization of mathematics involves communication between two cultures, but not encultururation in *or* acculturation to *a foreign culture*

Just as tourism is distinct from immigration, popularization of mathematics does not aim at enculturation or acculturation of the visitors into the mathematical culture or one of its subcultures. These anthropological concepts have been adapted to study phenomena of mathematics education by Bishop (1988; 1994; 2002). Bishop described mathematics learning as a process of *enculturation* whereby students are gradually integrated into mathematical culture. Ideally, this process is supposed to be similar to children's integration into their home culture. Less ideally, however, the process can become one of acculturation (Bishop, 2002), which refers to imposed modification of one culture by another (as in the context of colonization). In the case of acculturation, one culture is often endowed with institutional power, and changes are a one-way process where the more influential culture dominates.

This institutional power is lacking in the case of popularization of mathematics. In fact, a popularizer has neither the time nor the authority either to enculturate or to acculturate his or her audience. Both enculturation and acculturation aim at living (and functioning) in (a new) culture, as when students have to live (or survive) and function in a school environment. Popularization of mathematics only proposes a brief guided tour of a mathematical culture. This tourist position of the audience is entirely different from that of students of mathematics who find themselves in a position of "immigrants" landing on a foreign soil.

This aspect of popularization of mathematics has its positive side: it is free from the oppressive aspects of enculturation and acculturation. There is a downside, however, as well: a tourist learns considerably less about the visited culture than an immigrant does, and sometimes the knowledge acquired can be quite biased.

It is more appropriate to think of popularization of mathematics as communication between two cultures without the intention of one of the interlocutors to "naturalize" the other, although the wish to exert some influence is certainly there. This aspect will be discussed separately later, under the title of "Popularization of mathematics has a political agenda". First, we will look at the fact that, just as there are different models of communication between the host culture and the visitors in tourism, there are different models of communication with the audience in popularization.

2.2.8 There are different models of communication between popularizers and the audience in popularization of mathematics

There are different forms of tourism. The most common form is the silent group of people following an incessantly talking guide in Old City streets or museum halls. But there are also forms of tourism where exchanges between the tourists and the host culture are more symmetric. Similarly, in popularization of mathematics one can observe different forms of communication between the popularizer and the audience. On the one hand, there are televised lectures that can only be watched; on the other – there are interactive workshops or displays of mathematical puzzles that visitors can touch and play with (as in, e.g. the *Cité des Sciences et de l'Industrie* in Paris La Vilette, France).

To identify, describe and name these different forms of communication, it was useful to look at general models of communication in communication theory (Shepherd, St. John, & Striphas, 2006) and models identified in theory of science communication (e.g., Gross, 1994; Logan, 2001; Weigold, 2001; Lewenstein, 2006).

One way of classifying these models can be to look at the direction of the information flow between the interlocutors and distinguish between models where the flow is uni-directional and models where the flow is bi-directional (Figure 2.2).

I will look at examples of both kinds of models and how they have been or could be applied to communication in the context of popularization of science and mathematics.

2.2.8.1 Uni-directional communication: the dissemination model

In communication theory, the dissemination model applies especially to mass communication (Peters, 2006). It applies to situations where a message is broadcast many times as, for example, in repeated lectures, or lectures broadcast on television or via the internet, theatre plays, museums, etc. In this type of communication, the intentions of the sender of the message and the representations of the message are well known, but little is known about the receiver's interpretation of the message: the sender receives little or no feedback on his or her presentation.

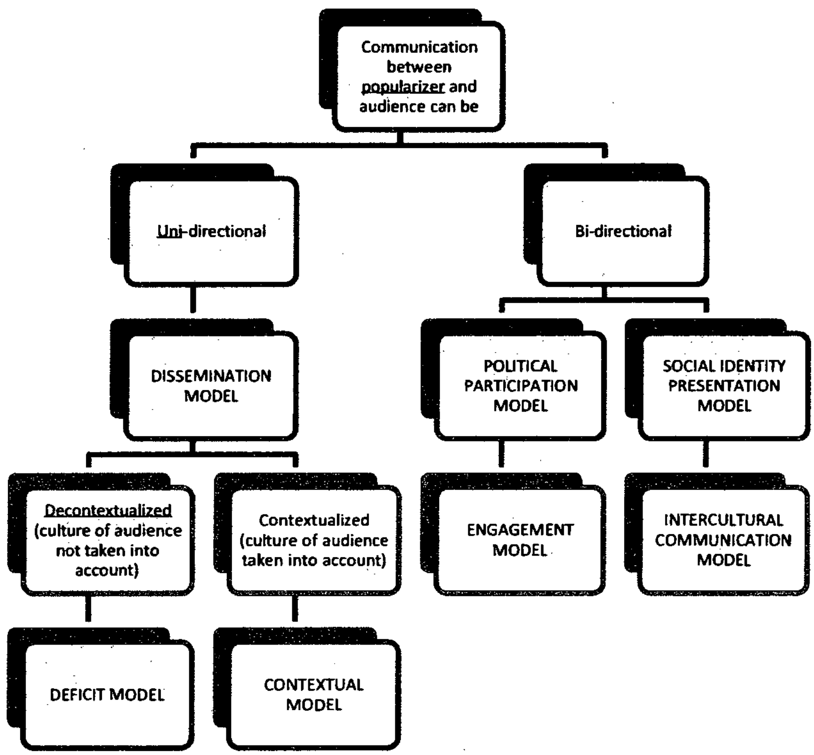

Figure 2.2. Different models of communication in Communication Theory and their existing applications in theory of Science Communication. The "Intercultural communication" model of popularization is my addition.

Some forms of tourism follow the *dissemination model* of communication. This is what often happens during a guided tour in a famous town, church, museum, etc. The guide simply tells the tourists what they *should* know about a particular monument, piece of art, etc, and is not interested in listening to the tourists' impressions, stories and accounts of their knowledge of the visited site. The tourists sign up for a guide speaking a particular language they can understand and that is about all the contextual information normally taken into account in this communication. In the case of a special audience (for example for people with the same profession, e.g. architects) the tour guide might adapt the message to the special group by focusing on specific details close to the group's interest.

The dissemination model of communication has been extensively used and studied in science communication (e.g. Gross, 1994; Logan, 2001; Weigold, 2001). It was seen as representing a communication situation where popularizers, acting as interpreters for scientists, disseminate scientific information to the general public, thus filling in a "deficit" in the public's knowledge (whence the "deficit model" of science communication, see Figure 2.2). The popularizer's task was to "translate" scientific information into a language and form that general public could understand. This model, however, was generally criticized for not taking into account the contextual information about the audience. This was considered to be the reason why popularization did not lead to a better understanding of science among the general public (Gross, 1994; Logan, 2001).

In its modified version, also known as the *contextual model* of science communication (Lewenstein, 2006), dissemination-type communication would vary according to the expected participants' background such as their previous experience with science (level of education) or belonging to an ethnic minority. The new model, however, did not bring about the expected results. Surveys did not show any significant improvement in the level of public understanding of science as a result of participating in popularization events following this model (e.g. Clark & Illman, 2001). There are no analogous results for popularization of mathematics.

2.2.8.2 Bi-directional models of communication

Recent models of communication assume some symmetry in the contributions of the interlocutors. This is certainly the case of the "political participation" and presentation of "social identity" models, which have their counterparts in studies of science communication and in mathematics education.

2.2.8.2.1 Communication as political participation

From the point of view of the "political participation" model (Kelshaw, 2006), all communication is interaction (and therefore two-way) and all communication is political since politics is a universal condition influencing all kinds of social interactions. Even refusing to communicate is participation since, in fact, it provides a very definite feedback to the other participants of the act; moreover, it conveys a political statement.

The view of communication as political participation (or more generally participation in social interactions) corresponds to another major model of science communication, namely the engagement-based model[6] (Lewenstein, 2006). Science popularization events following this model would engage the audience in doing science through workshops, and in decision making through public debates.

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[6]In fact, Lewenstein (2006) distinguished between two engagement-based political participation models. The first model focused on lay expertise by acknowledging and asking for lay experience about science (e.g. radioactivity). The second model tried to involve the public in science through participation, and even in decision making through political debates about science.

Active participation of the audience highlighted in this model has been sometimes seen as a remedy for problems of general public's lack of interest in science. Also, in popularization of mathematics, starting from the common belief that "mathematics is not a spectator sport", some events would be organized to encourage the audience's engagement. Math Fairs, games, workshops, or logical puzzles are examples of popularization activities that followed this model. Although popularization of mathematics does not always follow this model, engagement of the audience in doing mathematics has become part of the characterization of popularization of mathematics by Howson and Kahane (1990: 5-6) and Ernest (1996: 786). In science communication, the engagement model has been criticized for reaching only small audiences and for the fact that debates would often drift away from scientific matters towards political issues (Lewenstein, 2006).

2.2.8.2.2 Communication as presentation of social identities: focus on intercultural communication

The model of communication as conveying a social group identity (Harwood, 2006) focuses on intergroup communication rather than on interpersonal communication. It is concerned with how social identity influences communication, and, conversely, how communication shapes social identity. Social identity of a particular person is determined by the different groups (professional, ethnic, etc.) the person belongs to (or does/does not want to belong to). This way, the intergroup communication can be considered as intercultural communication, encompassing international communication, as well as communication between different professions, or between a profession and lay people relative to this profession, as could be the case in a popular lecture about mathematicians' mathematics to general public. In this model, the lecturer represents not his or her individual self but mathematicians as a professional group.

From the point of view of intercultural communication, tourism involves communication between tourists' culture and the hosts' culture. This rough picture has been refined by Holliday, Hyde and Kullmann (2004) (see Figure 2.3). The refined model distinguishes between tourists' home culture and "tourist culture", as well as between hosts' home culture and hosts' culture of dealing with tourists.

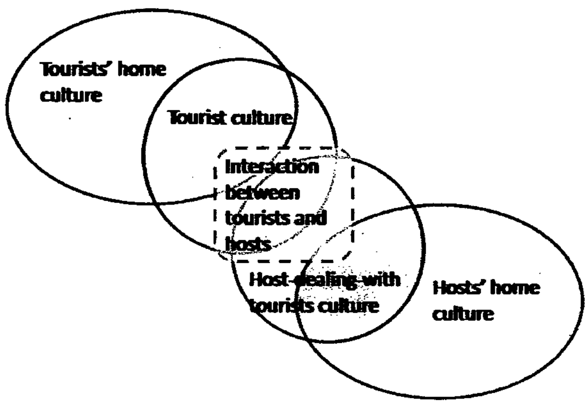

Figure 2.3. Relations among tourists' and hosts' cultures

(based on Holliday et al., 2004: 27)

The model of tourism proposed by Holliday et al. (2004) draws attention to the fact that the image the tourists get of the foreign culture is not identical with that of the members of the culture. The image shown to tourists is rather the image intended to "sell" the culture to outsiders. The souvenir shops sell "folk art" objects which are generally far from those that have actually been used in the given culture, even in a distant past. Similarly, the mathematics presented in popular lectures can be very far from the mathematics done by researchers. It is mathematics specially made for popularization.

2.2.9 Popularization of mathematics has a political agenda

The tourist industry is not a-political. In fact, as mentioned before, some communication theorists claim that no social interaction is a-political (Kelshaw, 2006). Interlocutors want to win each other over to their cause or point of view. They may try to achieve this aim by force (physical or legal). But they may also use "seduction", which is certainly the case of tourism. Tourist agencies have no means to force anybody to visit a country. But they want the tourists to come back and to encourage others to visit. Therefore, they "put their best foot forward", showing the beauty of the country, not the slums, the misery of unemployment, or the drudgery of everyday work in a factory or office. There is an element of propaganda in every message intended for tourists.

The same can be said of popularization of science or mathematics. "To improve the image of mathematics" has even been inscribed in the "definition" of popularization of mathematics proposed in Howson and Kahane (1990: 5-6) and Ernest (1996: 786). The very reason for organizing science or mathematics popularization events is often a real or perceived decrease of public support for financing research and development in these areas or for the decreasing weight mathematics and science are given in compulsory education. The support can be "moral" only, but, when we speak of "public funding support", it is measurable in the portion of the budget that a government spends on funding scientific research and development. This money comes from taxes and concerns all citizens and businesses. The approach to popularization of science where public support is the main concern has been called the "persuasion model" (Clark & Illman, 2001: 9).

Activity in popularization of mathematics or science is likely to increase in the wake of journalistic accounts of reports about falling enrollment in science and mathematics at the university, or shortage of mathematics and science teachers. There is some belief in the impact popularization may have on people's career choices, even if the reports themselves point to systemic factors of the falling enrollments and teacher shortages that cannot be solved by persuasion alone. However, here is in a quote from a 2007 report for the Australian government, where such factors are pointed out and no appeal to improvement of public image of science or mathematics through popularization is made:

> [W]hile most science occupations are not in short supply, there is a recognised shortage of engineers and of secondary school teachers in science and mathematics. The shortage of engineers is partly self-correcting as it has elicited a rapid growth in salaries for both graduate and experienced engineers, encouraging entry into the profession. In the case of science and mathematics teachers, shortages have instead been accommodated by using teachers without adequate skills in these areas. This may adversely affect student performance and engagement and decrease future university enrolments in the sciences. In teaching, price signals have not been able to respond to shortages *due to the inflexible pay levels and structures*. This should be subject to reform.
>
> *Job satisfaction amongst scientists appears to be falling*, with potential consequences for productivity and future recruitment. This morale problem reflects scientists' concerns about *poor career pathways*, *excessive use of short-term contract employment* and a burgeoning *non-research workload*. Many of the issues are best addressed by negotiation and agreement between employers and employees. However, *job satisfaction can also be increased through: longer-term funding certainty; carefully designed performance assessment processes that reward higher performing institutions, research teams and individuals; a level of academic freedom consistent with the strategic interests of the employing institution; and the minimisation of non-research workloads.*[7] (my emphasis)

The myth of the influence of popularization on the public image of mathematics or science may have its source in a traditional model of communication proposed by

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[7] "Public Support for Science and Innovation." Australian Government. Productivity Commission Research Report. Overview. Downloaded February 18, 2009 from http://www.pc.gov.au/__data/assets/pdf_file/0014/37121/scienceoverview.pdf

Newcomb in the 1950s (Newcomb, 1953). According to this model, the goal of communication is to reach an equilibrium regarding the participants' view of their social environment. This model – which appears to assume symmetry in the influence of the communicating sides – has been somewhat politicized in the metaphor of communication as "social influence", studied by Boster (2006). This model treats communication essentially as a way of changing beliefs, attitudes or behavior: "social influence is a result of all communication" (ibid, p.183); "communication necessarily impacts beliefs" (ibid, p.185). Newcomb's model suggests that, if two-way exchanges are allowed, this influence can be mutual for the communicating parties and that it depends on the participants' cultural values (or "cultural lenses"). In the context of popularization of mathematics, this influence would depend on participants' (both the audience's and the popularizer's) values related to mathematics, or mathematical culture.

The political aspects of communication have been extensively investigated in mathematics education, from the perspective of ethnomathematics and broader cultural and sociological points of view. I have already mentioned some of the conclusions drawn by mathematics educators working from these perspectives (Bishop, 1994).

2.2.10 Popularization of mathematics faces several important challenges: problems of communicability and translation

Tourists may fear they will be lost in the foreign culture; that they will not understand the local people and will not be able to make themselves understood; that they will do something contrary to the local custom and will get into unpleasant situations. Organized tourism is reassuring because it minimizes direct contacts between the tourists and the local people; there are specially appointed mediators (travel agents, guides, interpreters, etc.) who take care of communication between the tourists and the local culture.

The situation is very similar in popularization. Popularizers act as mediators between the mathematical culture and the audience. Participants in a popularization of mathematics event are not given a research paper to read or listen to; rarely if ever are they invited to do original research. The popularizer must choose what to communicate to the audience and how. The popularizer is thus faced with issues of "communicability" and problems of "translation".

2.2.10.1 Communicability

Some communication theorists consider communicability to be their main object of study. For some of them, the problem is to identify conditions, under which communication is possible, and those under which it fails (Chang, 2006). For others, communication is bound to failure; it is fundamentally impossible (St. John, 2006). According to St. John, the available sign systems are not able to communicate our thoughts, feelings, etc., and we can only imagine how the others receive and interpret what we intend to communicate. Accepting this theory would lead, of course, to abandoning all efforts of popularizing mathematics, since mathematics is abstract ideas that cannot be communicated by pointing to objects or using everyday language. Even if we do not take this radical position, we have at least to admit that communicating mathematics is an extremely difficult task. This means that the issue of communicability must be treated in all seriousness.

In popularization, we have to do with intercultural communication, where communicability is rendered more difficult by the fact that messages are filtered by the different cultural identities, background knowledge, values, sensitivities and dialects, i.e. the "cultural lenses" of the interlocutors. If a message is filtered by such cultural factors, then what parts of the message are accessible? Some popularizers appear to believe that a possible solution lies in a judicious choice of the subject or theme for a popularization event. This could explain the frequency of themes such as puzzles and games (some mathematical puzzles, such as Sudoku, have become part of everyday culture); number theory (as natural numbers are part of everyday culture as well); geometry (because it can be represented in a visual manner), or chaos theory, which, although mathematically quite complicated, can be described using impressive yet simple examples (the famous "butterfly effect") and colorful dynamical pictures[8]. An interesting question would be to investigate what happens with other areas of mathematics. Are they neglected because they are not fashionable, not interesting enough, or because they address topics that are not communicable this way? Can we identify some characteristics (topics, types of presentation, etc.) of popularization, which necessarily lead to failure? Indeed, there are certain areas of mathematics (e.g. ring theory or cohomology theory) that are considered impossible to popularize, even by well-known popularizers such as Ian Stewart or Keith Devlin (see Kruglinski, 2004).

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[8] See e.g., the applet illustrating the Lorenz attractor at http://to-campos.planetclix.pt/fractal/lorenz_eng.html (downloaded February 18, 2009).

2.2.10.2 Translation problems

Besides the choice of the topic to present in a popularization event, there is the question of its "translation" into a language more accessible to the audience. Translation is so closely related to communication that some communication scientists do not distinguish between them. For example, Striphas (2006) argues that every communication can be considered as translation; when speaking to someone we are, in fact, translating our ideas or feelings into signs (words, sentences, images, sound, etc.). Speaking about linguistic translations literally, he distinguishes between interlingual and intralingual translations by referring to the process of communication across and within sign systems, respectively. The translator's task is to be aware of communication problems caused by translation. In particular, he mentions interlingual problems such as structural-grammatical differences (when a certain grammatical situation, for example a verb tense, does not have an equivalent in the other language), and vocabulary differences (when a particular word or phrase does not exist in one language, or has a different meaning or connotation in different languages). Striphas also points out that intralingual translation problems can be explained similarly to the interlingual ones (e.g. differences in meaning of a word for people with different backgrounds).

The perspective of communication as translation has led to quite extensive research on the teaching and learning of mathematics. I have already mentioned the work of Pimm (1987) where the starting point was the analogy between learning mathematics and learning a foreign language. Prediger's (2004) research also starts from this assumption but it is based on a broader framework of intercultural perspectives, which also goes beyond the notions of cultural conflict, enculturation and acculturation envisaged by Bishop (1994). Prediger argues that instead of considering the "vertical" view of integrating students into the mathematical culture, it is worth taking a "horizontal" view, which acknowledges the existence of simultaneous cultures, and using them to enrich mathematics teaching and learning. Obstacles to learning mathematics may be explained not so much by cultural conflicts, as by the fact that in learning mathematics one has to learn a whole new vocabulary that describes a world that is in some ways totally different from the one experienced in everyday and out-of-school life (Prediger, 2004: 379-380). There may be no conflict, but just an encounter with a different culture. In this process, the mathematics teacher should be aware of the difficulties of the students and help them translate the new cultural (linguistic and non-linguistic) elements into a language they can understand. Prediger saw four sources of difficulties in this translation. One was the already mentioned aspect of *mathematics as a foreign language.* Second source was the problem of *intercultural misunderstandings.* These refer to linguistic and non-linguistic situations in an intercultural setting that could cause discrepancies in the interpretation of a certain situation by different participants (such as conflicting values, customs, etc.). Third, she mentions the *effects of overlapping,* referring to the fact that, although mathematical language and everyday experiences often overlap, simply transferring the everyday usage of the language to the mathematical situation can interfere with the mathematical meaning. Fourth, an affective component of intercultural issues is *foreignness as an experience.* Many students experience mathematics as foreign and, in some cases, this uncomfortable feeling can lead to fear and anxiety.

Although Prediger's ideas capture some important intercultural aspects of learning mathematics, not all her ideas can be applied to popularizing mathematics. In the case of school mathematics, students are expected to function in an everyday life context in the culture of mathematics. In other words, students have to live in a foreign culture and learn to speak the foreign language. Participation in a popularization event does not require as much. Neither is foreignness an unpleasant experience in popularization, as mentioned before. The fact remains, however, that mathematics is a foreign language also for the popularization event audience. The popularizer is not given the time necessary to teach the audience this foreign language; he or she must translate the mathematical language into a language that the audience can understand. This is very difficult because this translation has to overcome the other two obstacles identified by Prediger: the intercultural misunderstandings and the effects of overlapping. A popular presentation of mathematics must minimize the use of the technical aspects of mathematics such as formal definitions, equations and other expressions. Mathematics is, largely, a written language, and uses symbolic forms that have no counterparts in everyday language (one cannot even read them literally, as they are written). Therefore, abstract mathematical ideas must be conveyed using everyday language to build analogies and illustrative examples; this inevitably leads to the above-mentioned problems of intercultural misunderstandings and effects of overlapping.

In this section, I have identified ten essential aspects of popularization of mathematics. I will now combine these aspects into a framework for studying popularization of mathematics.

2.3 A FRAMEWORK FOR STUDYING POPULARIZATION OF MATHEMATICS

From the perspective of the tourist metaphor, popularization of mathematics can be seen as organized action whereby mathematicians volunteer to engage a willing audience in a communication about mathematics or a particular mathematical topic. The aim of the action is to increase the audience's understanding and/or appreciation of mathematics or the particular topic. The communication style and/or the mathematical theme or activity must be sufficiently attractive and novel for the audience to maintain its willingness to participate. Communication in popularization of mathematics is "intercultural": the popularizer represents a mathematical culture; the audience – either a different mathematical subculture or a culture that has little to do with mathematics. The popularizer and the audience view the object of communication and understand the language of communication through their respective "cultural lenses". This implies that the means of communication must be adjusted to overcome the usual challenges of intercultural communication: problems of communicability, and the risk that some essential aspects of mathematics might be lost in translation.

Schematically, we can represent a popularization of mathematics event as in Figure 2.4.

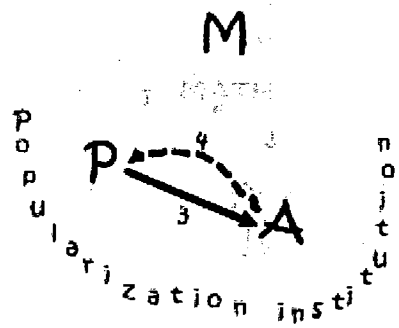

Figure 2.4. Schema of a popularization event, containing elements such as the popularizer (P), audience (A), and the mathematical culture. Popularizers and audience members see the mathematical culture through cultural lenses. The communication between the participants is dominated by the popularizer (3) with some optional feedback from the audience members(4).

The schema highlights the following aspects of popularization. The event is organized within an institution. There is a popularizer (P). There is an audience (A). P engages A in communication (3) about an aspect of mathematical culture (M). P obtains some feedback from A (4) but communication is engaged from P's initiative. P views M through cultural lenses (1). A views M through possibly quite different cultural lenses (2). The success of communication depends on how much of what P tells A about M can be grasped through A's cultural lenses.

In studying popularization of mathematics, the researcher can ask questions about each element of this general situation:

1. What are the *institutions* that organize popularization of mathematics? What are the different models of the organization?
2. (P)[9] Who are the *popularizers*? Why do they volunteer to engage in this activity? What are their objectives? What do they expect the audience to understand?
3. (1) How do *popularizers* view mathematical culture?
4. (M) What criteria do *popularizers* use to select parts of mathematical culture for popularization?
5. (3) What are the means that *popularizers* use to overcome the challenges of intercultural communication with respect to mathematics?
6. (A) Who are the members of the *audience*? Why do they participate in the popularization event? What do they expect from the activity?
7. (2) How do members of the *audience* view mathematical culture in general and the part of the mathematical culture they are being shown?
8. (4) How do members of the *audience* react to the event? What do they think they have understood from the communication?

In my research, I have been seeking answers to all these questions in relation to a selection of popularization of mathematics events and activities (mainly popular lectures and books). I will report on my findings in the consecutive chapters, although not always in the order of the questions listed above, and I will sometimes deal with several questions in one chapter.

In Chapter 3, I will focus on Question 1, related to the popularization institutions (I label this chapter as *INSTITUTIONS*).

Chapter 4 will be devoted to the content of popularization or Question 4 (*WHAT* is being popularized).

In Chapter 5, I will talk about the means used in popularization (*HOW*

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[9] Symbols in brackets before the questions refer to codes of the elements in the schema in Figure 2.4.

mathematics is popularized). This chapter corresponds to Question 5.

Chapter 6 corresponds to Questions 2 and 3. Here, I will investigate who the popularizers are and what are their "cultural lenses", or how they see mathematics and its popularization (*WHO* are the *POPULARIZERS*).

Chapters 7 and 8 correspond to Questions 6, 7 and 8, and are related to the audience.

Chapter 7 (Questions 6 and 7) looks at the cultural lenses of the audience (*WHO* are the *AUDIENCE*), on the example of nine members of audiences of two particular popular lectures, that I have attended, analyzed and used throughout the thesis as two constant examples of popularization events.

Chapter 8 (Question 8) looks at the audience members' reactions to the lectures (*AUDIENCE's REACTIONS*).

Chapter 9 presents my own experience in designing and conducting a popular lecture in mathematics, and analyzes it based on all 8 questions of the framework.

CHAPTER 3

INSTITUTIONAL ENVIRONMENTS OF POPULARIZATION OF MATHEMATICS

3.1 INTRODUCTION

This chapter looks at the institutional environments of popularization of mathematics. I will start by explaining what I mean by "institution". Using the adopted definition of institution, I will then reflect on the extent to which popularization of mathematics is institutionalized. A comparison between institutional environments of popularization of mathematics and those of popularization of science will lead to observing the relative weakness of the former compared to the latter. I will then look in more detail at institutional environments of one kind of popular activity, namely public lectures on mathematics. I will base my analysis on empirical data from two popular lectures that I attended, and whose organizers, lecturers and members of the audience I have interviewed.

3.2 A NOTION OF INSTITUTION

Institutional theory aims to reveal the underlying aspects of certain social structures. As such, it embraces theoretical and empirical studies of institutions from the point of view of sociology, political science, economics and philosophy, and takes many different approaches, such as rational choice institutionalism, normative institutionalism, or historical institutionalism (Peters, 1999). These approaches differ mainly in how they treat the relationship between the individual and the social. For example, rational choice theory stresses the fact that individuals act autonomously and choose actions that would maximize their personal utility based on rational assumptions.

However, whatever role is attributed to the individual in an institutional theory, it is always taken for granted that "an institution transcends individuals to involve groups of individuals in some sort of patterned interactions that are predictable based upon specified relationships among the actors" (Peters, 1999: 18). Moreover, in all theories reviewed, Peters was able to identify four commonly accepted characteristics of an institution (1999: 18) which I list below and use as a "definition" of institution here:

1) An institution is a structural feature of a society (or polity); the structure can be formal (legal) or informal (as in a network of organizations).
2) An institution has some stability over time.
3) An institution constrains the individual behavior of its members through rules and norms.
4) Members of an institution share certain values and goals and give common meaning to the basic actions of the institution.

(based on a summary from Peters, 1999: 18, proposed by Sierpinska, Bobos & Knipping, 2008)

The third characteristic above mentions "rules" and "norms". A distinction between these two types of constraints has been drawn by Ostrom (2005). Norms can be represented by statements such as, "Participants in such and such positions and in such and such situations will be expected to behave so and so". Note that the statement does not say what will happen if a participant decides not to behave in the expected way. A rule differs from a norm by the addition of a sanction for not behaving in the expected way: "Participants in such and such positions and in such and such situations will behave so and so *or else*…". Norms characterize any repetitive activity in a culture. There must be rules for the activity to become institutionalized.

For example, if a few people get together on some sunny afternoons to play tennis, this social activity will not count as an institution, even if it shares some of the above defining features of it (e.g. the meetings have some stability over time, the people most probably all like playing tennis). If the same group of people, however, decides to start a club that will – for a fixed fee – provide its members with a permanent tennis court to play, a trainer, and will organize classes or even competitions for the members, then the occasional and informal activity is well on its way of becoming institutionalized. Soon there will be a president, a treasurer, membership lists and cards, and statutes spelling out all the rights and obligations of the members and the executive committee. The club members will be united in their devotion to playing tennis and teaching this sport to others, but the statutes of the club will also constrain the members by setting some entrance criteria, such as a minimum age, or requirements on physical condition, as well as rules institution-members should obey (e.g. paying a regular membership fee). Sanctions will apply if a participant violates the rules and regulations of the club.

National and provincial mathematical societies (e.g. *American Mathematical Society, Canadian Mathematical Society, Association Mathématique du Québec, Bolyai János Matematikai Társulat, Polskie Towarzystwo Matematyczne*) are institutions according to the above definition. There are formal structures of the society, usually legalized at the state level. Many have a long history, preceding the establishment, in 1920, of an international organization representing all mathematicians in the world, namely the *International Mathematical Union.* Their functioning is regulated by a formal document ("Bylaws", "Statutes") where amendments can be introduced only through a formal vote. In applying to become a member of the society, a candidate declares his or her agreement with the basic aims, values and regulations of the society. Thus all four conditions of an institution are satisfied.

Mathematical societies exemplify institutions that organize or, at least, promote popularization of mathematics on a national level. For example, the *American Mathematical Society* (AMS), a formal organization established in the late 19th century (1888), is devoted to "promote mathematical research and its uses, strengthen mathematical education, and foster awareness and appreciation of mathematics and its connections to other disciplines and to everyday life[10]". It is the "foster awareness and appreciation of mathematics" part of this statement that points to popularization activities. This aim of the society is materially realized, among others, through its contribution to an award for such activities: the JPBM Communications Award[11].

Moreover, AMS has included popularization of mathematics into its formal structures. There is a special office devoted to improving the public image and public understanding of mathematics: the AMS Public Awareness office. Its main responsibility is to improve "public relations" between the mathematical community and the general public. It "works with the media, scientific societies, institutes, universities, and museums to promote awareness of mathematics and to publicize meetings, events, prizes, and AMS activities."[12] It sponsors magazines on mathematics, and provides information for the general public about mathematics-related books, films, art-works websites, etc. It also

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[10] http://www.ams.org/ams/about.html Downloaded: July 10, 2008

[11] The Joint Policy Board for Mathematics Communications Award was established in 1988, "to reward and encourage communicators who, on a sustained basis, bring mathematical ideas and information to non-mathematical audiences" (downloaded July 30, 2008, from http://www.ams.org/prizes/jpbm-comm-award.html).

[12] Information about the AMS Public Awareness Office http://www.ams.org/ams/about.html Downloaded: July 10, 2008

gives contact information about mathematicians that can be useful for journalists. The goals and values of the AMS Public Awareness Office include improving public relations, and are declared as follows:

> The goal of public awareness is more than just making the layperson understand (or love?) mathematics. It's making people realize that mathematics is a field of research, just like physics, chemistry, or biology. It's helping other scientists to realize this as well. It's providing mathematicians with material that allows them to better explain to non-mathematicians what mathematicians do. It's giving everyone, mathematicians and non-mathematicians alike, a pride in mathematical accomplishments. And it's promoting the Society's accomplishments, both to the mathematical community and to the world beyond[13]. (Ewing, 2002: 5)

Popularization of mathematics is not the sole activity of national societies of mathematics; they could probably survive by serving only the internal needs of their own members' community. The question is, therefore, if popularization of mathematics would survive on its own, outside of the national societies. This is the topic of the next section.

3.3 To what extent is popularization of mathematics institutionalized?

An institution is, first of all, a group of people. So the first question we have to ask is: who are the popularizers of mathematics? If we only look at the recipients of the JPBM Communications Award, we find that less than one third are mathematicians by profession and not one of the recipients has been trained as a "professional popularizer of mathematics". There is no such profession. There are no departments of popularization of mathematics in colleges or at universities. Nor can one obtain a PhD in mathematics based on popularization products. Mathematicians in a department of mathematics are evaluated based on their activity in three domains: research, teaching and service for the

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[13] http://www.ams.org/ams/state-of-ams2002.pdf (Downloaded: July 10, 2008)

community. With the good will of the evaluating committee, popularization activity can be included in the "service for the community". Popular papers and books hardly ever count for as much as research papers in the evaluation. For a research mathematician, popularization activity is not academically rewarding: it only takes the precious time away from research. Ian Stewart, in his response upon receiving the JPBM Communications Award in 1999, expressed his gratitude for what he felt was still quite *exceptional*, namely the recognition, by fellow mathematicians, that "communicating mathematics to the public is now... a respectable activity for an academic rather than a feeble substitute for serious research"[14].

Moreover, there is too little interaction among popularizers of mathematics to develop rules and norms of behavior. They usually work individually, not in groups; they do not form associations. The AMS Public Awareness Office and the JPBM Communications Award do not bring popularizers to work together; they only identify the individuals and ask them to perform, or reward their performance. The *Mathematics Awareness Centre at Warwick* that Ian Stewart talks about in the above-mentioned response apparently has greater institutional ambitions since it aims at "*coordinating activities* in the Public Understanding of Science, with special emphasis on the mathematical sciences"[15] according to the description on its website.

It is interesting that even at the *Mathematical Awareness Centre* in the University of Warwick, popularization of mathematics is not the only activity, but it is broadened to popularization of science. Institutionalization of science popularization has a longer

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[14] http://www.ams.org/notices/199905/comm-jpbm.pdf (downloaded June 29, 2009)
[15] http://freespace.virgin.net/ianstewart.joat/macaw.html#macaw (viewed June 29, 2009)

tradition, and Stewart might have wanted to graft popularization of mathematics on this stronger "trunk" to make it grow better.

When studying Journalism at the university, one can specialize in Science Journalism. Science popularization is also supported by the existence of academic research that takes it as its object. There are graduate programs in "science communication" at some universities. The results of this research can be communicated in international conferences[16], and published in specialized regular scientific journals such as *Public Understanding of Science, Science Communication,* or *Journal of Science Communication.* The journals are devoted to following an interdisciplinary approach by covering "all aspects of the inter-relationships between science (including technology and medicine) and the public" (from policy statement of *Public Understanding of Science*) and publishing the results of "international scholarly exploration of three broad but interrelated topics: Communication within research communities - Communication of scientific and technical information to the public - Science and Technology communications policy [with regard to] social science, engineering, medical knowledge, as well as the physical and natural sciences" (*Science Communication*). These journals also try to define the meaning of "science communication" in today's society and to identify the sociological and epistemological issues related to the popularization of science (*Journal of Science Communication*). The journals accept papers on:

- surveys of public understanding and attitudes towards science and technology
- perceptions of science

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[16] Information about the conferences http://www.scienceinpublic.com/scienceinmelbourne2007/ (Downloaded: July 12, 2008)

- popular representations of science, images and representations of science and technology
- science and the media
- public communication of and discourses on science and technology
- understanding public communication of science and technology

The university programs and courses are hosted by different areas, such as Science, Educational Studies or Social Studies of Science and Communication Studies. For admission into a graduate program in science communication, an undergraduate degree in science is certainly an asset but may not be absolutely necessary. Some general characteristics of the different university courses and programs, as well as the state of the research domain were summarized, for example, by Turney (1994), and a more recent account was given by Mulder, Longnecker, and Davis (2008). Besides the actual course offerings of universities (e.g. University of Bath[17]), some information about science communication courses can also be found in Littmann's (2005) overview. Although some of these programs have science modules, they offer courses mainly in science communication, in history and philosophy of science, and practical communication courses such as interviewing techniques, or news and feature writing. The emphasis is rather on communication and on cultural aspects of science than on the scientific content[18].

In addition to international journals and graduate programs, there exist national associations of science writers (journalists, public relation officers, researchers), such as the *Canadian Science Writers' Association*, the *Association of British Science Writers*, or

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[17] Further information about the program can be found on the following page of the University of Bath: http://www.bath.ac.uk/prospectus/postgrad/psychology/progs/comm.shtml (Downloaded, June 25, 2009)
[18] See http://www.bath.ac.uk/psychology/srhc/content.html for further details (Downloaded, June 25, 2009)

the *National Association of Science Writers* (NASW) in the U.S.A.. NASW was established in 1934 in New York; with more than 2400 members, it is, by far, the largest national organization of science writers worldwide today. The association is devoted to "fostering the dissemination of accurate information regarding science through all media normally devoted to informing the public." Its members are freelancers and employees of major magazines and broadcasting media as well as public information officers. Membership requires five samples of work produced for a lay audience in the past five years, along with the sponsorship of two members of the association.[19]

Although these international journals and associations devoted to science communication include mathematics, mathematics as a discipline on its own is usually underrepresented compared to other scientific fields, such as biology or physics. For example, in the journal *Public Understanding of Science*, so far (Summer 2009) only one paper has addressed public understanding of mathematics (von Roten, 2006).

Popularization of mathematics has no (at least to my knowledge) specialized international journals or associations devoted to the study and improvement of this activity (this does not include popular magazines such as the Russian language journal Kvant). This may change in the future because this activity has known increased interest, in recent times, among mathematicians and mathematics educators. One sign of this tendency is that issues related to popularization of mathematics started to be addressed in international conferences. In the next section, I will describe this development, first in the International Congresses of Mathematicians (ICMs), and then in congresses and

[19] The information is posted on the home of NASW, available: http://www.nasw.org/ (Downloaded: July 10, 2008)

conferences for mathematics educators.

3.4 Position of popularization of mathematics in international conferences

3.4.1 Popularization of mathematics in ICMs

The coverage of a topic in international conferences can be a measure of its status in the mathematical community. The oldest and probably the most prestigious international mathematical conference is the *International Congress of Mathematicians* (ICM) held every four years in different countries. The ICMs now have a regular section on "Mathematics Education and Popularization of Mathematics", with information about its work included in the Proceedings on a par with sections on Algebra, Number Theory or Probability and Statistics. It has not always been so; the areas of mathematics education and popularization of mathematics used to have, for some time, special symposia organized on the site of the congress, but information about the work of these symposia were not included in the ICM Proceedings (Howson, 1984; Letho, 1998).

In the ICMs, there has always been at least one section devoted to something else than a strict domain of mathematics (such as Algebra or Number Theory). In the early ICMs, there was one such section, titled, in ICM 1897 – *History and Bibliography*; in ICM 1900 – *Bibliography and History, Teaching and Methods*; in ICM 1904 – *Pedagogy*. This was usually the last section to be mentioned in the list of sections, and it could well be called "Other matters" or "Miscellaneous". Here, I will call it, the "Last Section". Around this time, many talks in the Last Section addressed issues related with the construction of an internationally unified mathematical terminology and with the identification of important mathematical problems to work on. This, in fact, served the process of institutionalization of mathematical research at the international level. For the international cooperation, the construction of a unified language was, indeed, essential, as was a consensus on what is important to investigate. On the latter issue, 1900 Hilbert's talk "Sur les problèmes futurs des Mathématiques" delivered in the Last Section, had an enormous impact of the development of mathematics in the 20th century.

In the following congresses, with the demand for clarifying the role and foundations of mathematics, philosophy was more and more emphasized, as can be seen from the titles of the Last Section in the ICMs between 1908 and 1928: 1908 – *Philosophical, Historical and Didactic Questions*; 1912 – *Philosophy and History. Didactics*; 1920 – *Philosophical, Historical and Didactic Questions*; 1924 – *History, Philosophy, Didactics*; 1928 – *Philosophy and History of Mathematics*. Gradually, however, together with foundations of mathematics (and, later, logic) and history, philosophy was transferred to another section. In 1936, philosophy became part of Section 7: *Logic, Philosophy and History*, leaving *Pedagogy* alone in the last section, which, in ICMs 1932 and 1936, was titled, simply, "Pedagogy".

So far, "popularization" had not appeared in the above-mentioned titles of the Last Section. However, some lectures addressed topics that we would now consider as popular. For example, in 1912, Zermelo delivered a talk titled, *Ueber eine Anwendung der Mengenlehre auf die Theorie des Schachspiels* (On an application of set theory to chess). Also Loher's (1937) talk on *Goethes Stellung zur Mathematik* (Goethe's attitude toward mathematics) would fit well into a public talk on mathematics. Besides these talks, which only addressed a popular topic, in 1920, Hatzidakis addressed directly the issue of popularization, and reflected on its importance in his talk *Systematische*

Rekreationsmathematik in den mittleren Schulen (Systematic recreational mathematics in middle schools).

After WWII, the ICM's Last Section seemed quite stable by embracing history and education. In Cambridge (1950) and Amsterdam (1954) the role of mathematics in society seemed an important issue. Wilder's (1950) talk on *The cultural basis of mathematics* was one of the invited lectures and Kurepa's (1957) presentation of the results of the ICMI study on *The Role of Mathematics and Mathematicians at the Present Time* was also published in full length in the Proceedings.

In the aftermath of WWII, mathematics as a teaching subject has grown in importance (Garrett & Davis Jr., 2003). However, by the 1990's, after the disappointment with the radicalism of the "New Math" reforms in the 1960s and 70s, and the subsequent, not necessarily convincing, attempts to return to moderation in school mathematics, the importance of mathematics in the eyes of the general public was no longer obvious. Mathematicians have become more and more aware of the problems with the public image of mathematics, and felt the need for action. Thus, from 1994 on, ICM addressed also questions related to popularization of mathematics. That year, the Last Section was split into two: *Teaching and Popularization of Mathematics*; and *History of Mathematics*. The title of the former section remained unchanged in 1998, but, in 2002 and 2006, it became *Mathematics Education and Popularization of Mathematics*.

At the ICM in Beijing in 2002, the description of the section on *Mathematics Education and Popularization* was very long and detailed. The part on Popularization was as follows:

> Popularization: Broadly accessible expositions of significant mathematical concepts and developments. Narrative or dramatic accounts of important mathematical events. High quality and creative mathematical journalism. Connections with section 19 [History of mathematics].

This quote reflects the ICM's Program Committee's collective understanding of what counts as "popularization of mathematics". At the ICM in Madrid in 2006, the description was much abbreviated and contained only the name, "popularization of mathematics", and it is the same for the ICM 2010.

This review shows that, in the history of ICMs, Popularization of Mathematics was never assigned a section of its own. Does this mean that, for the International Mathematical Union, "communicating mathematics" does not have the status of "an acceptable activity for an academic", as Ian Stewart had hoped?

There is some hope, however, in the fact that, at the more recent ICMs, talks in the section on mathematics education and popularization included not only a "popular talk" on mathematics, but also a discussion of issues related to popularization and a reflection on the activity of popularizing. Such were the presentations by the project director of Square One TV, Joel Schneider (1995; *Issues for the Popularization of Mathematics*); the organizer of the World Mathematical Year 2000, Vagn Lundsgaard Hansen (2002; *Popularizing Mathematics: From Eight to Infinity*); or Ian Stewart (2006; *Mathematics, the media, and the public*). These speakers shared their experience and their views on popularization, and discussed topics especially appropriate for popularization or worthy of popularization. The ICM in Madrid in 2006 also held a panel discussion on the problem of popularization by investigating the question: *Should mathematicians care about communicating to broad audiences? Theory and Practice.*

Based on the above historical remarks, it seems that, in the last century, popularization of mathematics evolved from an occasional activity done by a few to a phenomenon whose problems and consequences generate discussions within the international mathematical community. This community acknowledges the need for and the difficulties of popularization. The academic status of popularization of mathematics, however, remains undefined. There is no tendency towards establishing popularization as an area of academic *mathematical* activity.

In the next section, I move on to speak about the place of popularization of mathematics in the community of mathematics educators.

3.4.2 Popularization of mathematics in conferences on mathematics education

Many mathematics educators – especially those whose professional activities (research, teaching or teacher education) focus on post-elementary mathematics education – like to maintain strong institutional links with mathematicians. This is why the largest congress of mathematics education (ICME) is organized by the International Commission on Mathematics Instruction (ICMI), which is an official commission of the International Mathematical Union. Every four years, ICMI conducts a "study" devoted to a particularly important issue in mathematics education. The study is made available in a comprehensive publication representing expert views and research on the issue[20]. Popularization of mathematics featured as the subject of one of the ICMI studies (Howson & Kahane, 1990). It has not been, however, very prominent in the ICMEs. Apart from a Topic Group on *Mathematics for All* (ICME-5); *Video, Film; Mathematical Games and Recreation* (ICME-6) and a discussion group on the *Public Understanding of*

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[20] http://www.mathunion.org/icmi/ICMIstudies_org.html (Downloaded: July 27, 2008)

Mathematics (ICME-10), only a few lectures addressed related topics. Among those, Jin Akiyama (2004) gave a talk on *Mathematics for mass media.* The two major conferences on research in mathematics education, Psychology of Mathematics Education (PME) and its North-American chapter (PME-NA) seem not to consider popularization of mathematics as an area for research. I could find only a few presentations somewhat related to popularization (e.g., Beisiegel, 2006; Evans, 2004).

Mathematicians thus appear to be more interested in the topic than mathematics educators, but neither of the two communities view popularization of mathematics as an object of research. It is only invoked sometimes as a means to address certain cultural issues, such as lack of appreciation of mathematics by the society at large and its poor "public image". As such, it is more likely to be addressed in round table discussions than in plenary talks. For example, the European Congress of Mathematics has had several round table discussions in relation to popularization (*Public Image of Mathematics* at ECM1 and ECM2, and *Raising Public awareness of Mathematics* at ECM3).

Based on these facts, we can quite confidently state that institutionalization of popularization of mathematics is considerably weaker than that of popularization of science, especially in terms of the level of formal and stable organizational structures. The activity of popularization of mathematics does seem, however, to be constrained by certain rules that I will try to identify in the next section.

3.5 THE "RULES OF THE GAME" IN POPULARIZATION OF MATHEMATICS

The rules of science communication can be gleaned from official documents in related university programs or professional associations, or workshops and guidebooks[21] (Malavoy, 1999). The guidebooks give advice regarding the style (e.g. straightforward, humorous) and language (avoid jargon and formulas), etc. Malavoy mentions three general principles of popularization:

> *Popularization is not teaching.* Good popular science texts are much more than simply didactic texts. The challenge here is not only to explain well the aspects of a scientific research but also to generate interest in the reader. Never forget that the latter cannot be won ahead. A popularizer should be read or listened to with pleasure. A challenge worth trying!
>
> *Popularization is not mystifying science.* Popularization should not advertise science and limit its image to a success story. Don't hide the pitfalls, the problems a scientist should face, and so provide a more human image of science. Popularization is telling a story, sharing an adventure of science and also of the scientists involved.
>
> *Popularization encourages critical thinking.* A good popularizer emphasizes the "side effects" of the presented research whether they are social, cultural, economic, politic or environmental. It is especially important that the lectures should generate questions (and not answer those that were not asked). (Malavoy, 1999: 7, my translation)

I am not aware of the existence, in AMS, of similar guidelines that could serve as a source of rules for popularization of mathematics. However, perhaps some rules can be derived already from the fact that participation in popular mathematical activities is optional; it is not part of compulsory education. If an activity is not pleasurable, it will be abandoned. Popularization of mathematics events usually have to compete against many other activities on offer. Because there isn't a set assignment in the end of these events, the audience can take away whatever they want from these events. Moreover, the audience in any single event is varied with respect to age and background education. A

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[21] Short guides are also posted on related websites, e.g. http://www.scidev.net/en/practical-guides/ (Downloaded: July 12, 2008)

popularizer must take all these facts into account.[22]

Explicit rules, as far as I know, exist for a borderline case of popularization, namely mathematicians writing expository papers about advanced mathematics (especially recent developments) for other mathematicians who may not be experts in the given subject. It is debatable whether these activities can be considered as popularization at all. If we interpret popularization as intercultural communication, however, then this kind of activity, presenting a subculture to people who might belong to the broader culture but not necessarily coming from the same subculture, can also qualify as popularization.

This special type of popularization is typical of mathematical magazines that claim to target a wide audience of mathematicians and sometimes also students. This is the case, for example, of the *Notices of the American Mathematical Society*, *The American Mathematical Monthly*, *Focus*, *Math Horizons*, *The Mathematical Intelligencer*, etc. They all contain "lighter" articles on historical and cultural topics. For example, the profile description of *The Mathematical Intelligencer* and its Instructions for Authors mention several desirable features of the papers published there that could well function as rules of "popular writing" since papers that do not satisfy the conditions are not accepted:

> This journal publishes articles about mathematics, mathematicians, and the history and **culture** of mathematics. It presents expository articles on all kinds of mathematics and **interdisciplinary** trends, and articles that portray the **diversity** of mathematical communities and mathematical thought.
>
> Not only does *The Mathematical Intelligencer* **inform** a **broad audience of**

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[22] The rules were also identified by Godot (2005) as those characteristics of a didactic contract in a situation of popularization that distinguish this activity from teaching.

> **mathematicians and the wider intellectual community**, it also **entertains.** Throughout the journal, **humor, puzzles, poetry, fiction, and art** can be found. The journal also features information on emergent **mathematical communities** around the world, new **interdisciplinary trends**, and relations between **mathematics and other areas of culture.**
>
> Instructions to Authors: The Mathematical Intelligencer welcomes submissions from within and without the international mathematical community. Articles may feature **new results, surveys of recent work** in a particular field, **profiles of mathematicians past or present**, and so on: the scope is wide (see below).
>
> We welcome **controversy**; this is an international forum for issues on which mathematicians disagree. But whatever their subject, all articles should be written in a **relaxed, engaging style**, and should be **accessible to the entire community, irrespective of specialty**. Articles are peer-reviewed.
>
> Authors need not feel confined to non-fiction: we will consider **humor, poetry, fiction, and art forms not yet invented.** (Policy statements of *The Mathematical Intelligencer*, my emphasis)

Based on the policy statements of *The Mathematical Intelligencer* and the other journals mentioned above, it is possible to surmise the following desirable features of "popular writing" for "[all] mathematicians and the wider intellectual community" (which might be applicable in a wider context as well):

1. Intended for a broad audience, i.e. for people who are not necessarily expert in the given subject.
2. Intended to satisfy widespread interests.
3. Puts mathematics in an interdisciplinary context and shows it in the light of a broader, historical or cultural perspective.
4. Stimulating, challenging, well motivated, thought-provoking (for example, through addressing controversial issues).
5. Informal; clear explanation of the essential idea of an argument is more important than logical rigor and technical detail.

6. The novelty of the mathematical results is not so important; the paper can contain an interesting and clear exposition of old results; in any case, clarity of exposition is more important than the originality of the results;
7. Entertaining (e.g., through using various literary genres such as fiction, or visual art forms, humor, puzzles).

The above features function like formal rules in an institutionalized activity in the sense that, if they are not satisfied, the audience will quit, and popularization without an audience cannot exist.

So far, I was searching for rules for popularizers. Audience members are also participants in popularization; are there any rules for them? The only rule seems to be, "no rules for the audience members". The audience must be free to come and go; otherwise, it is not popularization. Popularizers, however, bear the responsibility to make them come and stay, rather than go. Therefore, they offer advice or soothing remarks of the kind, "don't be afraid if you don't understand everything". For example, in the Preface to his 2004 book, Penrose gives such direct advice to his readers:

> Do not be afraid to skip equations (I do this frequently myself) and, if you wish, whole chapters or parts of chapters, when they begin to get a mite too turgid! There is a great variety in the difficulty and technicality of the material, and something elsewhere may be more to your liking. You may choose merely to dip and browse. (Penrose, 2004: xix)

In the next section, I will describe the particular institutional environments of the two popular lectures that I use in the thesis as constant examples. The title of the first lecture was *Medial Representations: Mathematics, Algorithms and Applications*. The title of the second - *The mathematics of Escher's "Print Gallery"*. I will refer to the first lecture as the "Medial representation" lecture, and to the second as the "Escher" lecture.

3. 6 THE INSTITUTIONAL ENVIRONMENTS OF THE "MEDIAL REPRESENTATION" LECTURE AND THE "ESCHER" LECTURE

Information presented in this section is based on my interviews with the organizers of the lectures.

The lectures were organized in two Canadian research institutions as part of popular lecture series, not restricted to mathematics. The declared goals of the series of lectures were, in both cases, to offer non-specialists some insight into important recent research. The mathematics-related lectures were thus intended for non-mathematicians.

3.6.1 The institutional environment of the "Medial representation" lecture

The first lecture I attended was part of a lecture series organized by the Science and Engineering departments of a leading English Canadian University and supported by the Royal Society of Canada. The goal of the lecture series, as I was informed, was to enhance communication among scientists representing different disciplines and between scientists and the general public. There were normally eight lectures per year. In the last four years, there were three mathematics-related presentations compared to an average of seven in physics, biology or medicine. The lectures were announced on the university's web page and in a university newspaper. Lecturers were selected and invited based on what the organizers knew about them. Having given a popular talk before was not a prerequisite.

The make-up of the audience changed depending on the topic (with an average of 30-50 people), but usually the majority were the university's professors and students.

The talks took place on the university campus and lasted an hour. After the lectures, there was usually a small reception. The reception gave the audience an additional opportunity to talk with the lecturer and ask questions they did not have a chance to ask during the short question period after the talk.

The title of the actual presentation was "Medial Representations: Mathematics, Algorithms and Applications" given by a professor from the Department of Computer Science. The professor was a representative of an interdisciplinary research group of the university working on computer vision. The audience was varied in terms of age and previous knowledge, but most belonged to the university community (faculty and students).

This was supposed to be a popular lecture and therefore participation in it should have been optional and not a compulsory task followed by an assessment. It turned out, however, that at least one of the audience members – who agreed to be interviewed by me after the lecture – was there as a student on the assignment of attending and writing a report on a popular lecture in mathematics. The assignment was compulsory for a course she was taking in the same university. Thus, some university professors may force their students to attend popular lectures, violating the spirit of popularization.

In a way, my interviews with audience members also interfered with the popular character of the lecture. I had approached some of the audience members before the lecture, asking them if they would grant me an interview after the lecture about what they have understood from it. Those who agreed may have felt obliged to stay in the lecture and pay attention even if they did not enjoy it and would have gladly left the room much earlier without this promise. Indeed, the lecture did cause more frustration and shock than pleasure for some of the participants I have interviewed.

3.6.2 The institutional environment of the "Escher" lecture

The second lecture was organized by a Canadian research institute, and, similarly to the previous talk, it was also part of a lecture series, but the lectures were all devoted to mathematics. The series was aimed at communicating the "power" and the "beauty" of recent mathematical research to the general public. The lecturers were asked to present the ideas in a language accessible to non-mathematicians. The organizers were looking for speakers with a special gift for communicating mathematics in an "exciting" way. Having previous experience in popularization (talks, books or articles) thus was a prerequisite. The organizers were also trying to attract speakers knowledgeable about modern applications of mathematics (cryptography, quantum computing, chaos in meteorology or financial systems, brain imagery, biotechnology, etc).

There were three to four lectures per year. Topics of the lectures in the past few years included mathematical thinking, logic, dynamical systems, geometry, biographies of famous mathematicians, and mathematics in art.

The lectures were advertised in daily newspapers. Those planning to attend were advised to register before the lecture, to give the organizers an idea about the required room size. On the average, 150-200 people participated in the events. Emails advertising the lectures were also circulated among faculty members and students of the mathematics departments of universities in the neighborhood of the research institution.

The lectures were held in a big auditorium of a university, and lasted an hour. Similarly to the previously presented lecture series, time was reserved for a short question period after the presentations. There was also a reception after the talk, and the lecturer had to be prepared for additional questions in less formal circumstances.

The title of the talk was, "The mathematics of Escher's *Print Gallery*". Its abstract promised to use computer animations to explain the mathematics of one of Escher's most elusive pictures and fill the "mysterious" white spot left by Escher in the middle.

There were about 200 people in the audience and I interviewed only a few of them. Although one of the interviewees was advised by one of her professors to attend the talk, the attendance itself was not part of the course assignment. Thus none of the interviewees were attending the talk on a course assignment, but, of course, they were obliged to stay in the talk because of the promise they made to me. Unlike in the first lecture, however, none of them felt like leaving the talk in the middle. They all enjoyed it immensely.

3.6.3 General remarks about the institutional contexts of the lecture series

For both lectures, a research institution supported the event, invited the speakers, advertised the lecture, and provided a venue as well as a *seal* of the scientific community. The choice of the means of dissemination of the information about the talks already served as a filter for the audience: English speakers, internet users, the particular newspaper readers. Members of the academic community had easier access to the information because it arrived in their e-mail. The lecturers were constrained by the time (1 hour) allotted for the talk, the language of the presentation (English) which needed not be their mother tongue, the request to formulate the ideas in a language accessible to non-mathematicians. At the same time, the lecture would have to be interesting for mathematicians as well, since they were likely to come. These factors already constrained somewhat the aspects of mathematics that the lecturers could communicate. On the other hand, there were no strict rules imposed on the lecturer regarding how to choose the topic, the title, what technical aids to use, how to prepare the presentation, how to deliver it, how to answer the questions of the audience, whether and what kind of additional resources to suggest to the audience for further information, etc. Similarly, the behavior of the public (willing to come, listen to the talk and paying attention to it, asking questions, talk to the lecturer after the presentation or later) was not constrained by strict rules, except those that generally apply in similar social situations, like how one should behave in a lecture.

No systematic evaluation was used in the case of the lectures. The lecturers got feedback from the audience's reaction, in the form of questions during the talk or in the reception after the talk and could change the presentation (for the next time) accordingly. The organizers' feedback came from their own perceptions about the lecture and about the audience. Based on this perception they could invite other lecturers (e.g. choosing lecturers working of areas which seemingly attracted more people), or change the means of disseminating the information about the lecture.

Referring to the models of communication described in Chapter 2, both talks seem to fall into the uni-directional, contextualized dissemination model, since the lecturers seemed to take into account the cultures represented in the audience (they knew there will be both mathematicians and non-mathematicians), but there was very little interaction between the lecturers and the audience.

3.7 CONCLUSIONS

Popularization of mathematics has become an activity that increasingly provokes reflection within the mathematical community. However, popularization of mathematics has a considerably lower level of institutionalization than science popularization. Except for a few individuals (freelance writers, moviemakers), popularization of mathematics relies mainly on the mathematical community (mathematicians, educators) affiliated with research and educational institutions and on governmental funding. In science (including medical sciences), the funding can be provided by industrial organizations that are financially interested in promoting (or even organizing and evaluating) popularization activities. Educational and research institutions, on the other hand, usually do not reward popularization the way they do research or teaching. Popularization activity must be fuelled, therefore, by reasons other than material: a hobby, an interest for popular topics, or a particular talent that pushes one to creative action. Popularization of mathematics is more an art than a profession. It has its "masters" and its "sponsors", but, as yet, no "schools". The masters seem to have developed some techniques, but there is little or no language to speak about these techniques and teach them to others. This is what weak institutionalization of popularization of mathematics means.

CHAPTER 4

THE POPULARIZED MATHEMATICS

4.1 INTRODUCTION

The question posed in this chapter is: what part of the mathematical culture popularizers choose to "show" the audience? Can some criteria be identified?

Recent reviews of popular works on mathematics mention that there have been a large number of publications in this area over the last twenty-odd years, but also underline the commonality of their contents. For example, in a review of Higgins' book (2002), Morics (2003) states that many books try to give an overview of all branches of mathematics, with some topics – such as Fermat's Last Theorem or Ramanujan's story – inevitably mentioned:

> Within the last fifteen years, many books have been published which attempt to discuss mathematics in a manner which non-mathematicians can understand. While other disciplines have been pursuing this line of public relations for some time, mathematicians have only recently expended the effort to place their field of study in contexts, which appeal to the general reader. Since this effort is so new, it's not surprising that few of these efforts have stood out from their kin. *The books of this genre have begun to resemble each other, with much of their content overlapping from book to book.* As more people take their turn at adding to the growing list of mathematics books for the mainstream reader, we should expect these books to be more focused, eschewing yet another mention of *Fermat – Wiles* or *Ramanujan* and the *taxicab* in favor of a more detailed description of a smaller region of the field of mathematics. Science books have been doing this for a long time.... [P]ractically every popular mathematics book I have read tries to *cover every branch of mathematics.*" (Morics, 2003, the emphasis is mine).

Morics lauds Higgins' book for not trying to "cover every branch of mathematics" and focusing on geometry instead. The selection of geometric topics, however, is not much different from those chosen by other authors (e.g., Pythagorean theorem; spherical geometry; rotational symmetry and its order; the geometry of crystals; geometric models of planetary motion). The originality, according to Morics, lies in the author's attempt to use visual arguments whenever possible.

Higginson (2006) also feels compelled to express his impression of a sharp increase in the production of popular books on mathematics in the last years.

> 'Popular' treatments of mathematical ideas are not new. Authors like W.W. Sawyer, Lancelot Hogben, W.W. Rouse Ball and Martin Gardner, for instance, wrote prolifically for large and appreciative audiences through the previous century.... But, for whatever reasons, the former trickle of publications has become a torrent. In the period of a few months in late 2000 and early 2001 there were two books published on the topic of zero (Kaplan, 2001; Seife, 2000). More recently, an even shorter time frame saw the publication of three substantial books on the Riemann conjecture (Derbyshire, 2003; du Sautoy, 2003; Sabbagh, 2002). Some other recent texts have focused on a particular branch of mathematics. For instance, Barabasi (2002) [networks], Beltrami (1999) [chance and order] and Havil (2003) [Euler's constant]. (I have more than thirty other titles in this category on my shelves alone.) (Higginson, 2006: 136-137)

Writing later than Morics, Higginson's examples suggest that at least some of the recent popularization books attempt to focus on a smaller number of related ideas, rather than to "cover all branches of mathematics". While the common topics of popularization such as number, Riemann hypothesis, Euler's constant, or processes of proof appear to be still very present, Higginson notices a strong emphasis of many books on the aesthetic aspects of mathematics.

> A surprisingly large number of authors choose to emphasize the artistic, aesthetic and spiritual connections of their publications in the titles they give the books. See, for instance, *The Artful Universe* (Barrow, 1995), *The Universe and the Teacup: The Mathematics of Truth and Beauty* (Cole, 1998) or *It Must be Beautiful: Great Equations of Modern Science* (Farmelo, 2002) – again, there are over thirty more I could mention, all with this characteristic. (Higginson, 2006: 137)

Higginson also notes an increase of interest in the human aspects of mathematics through books about mathematicians (e.g., Nasar, 1998, about J.F. Nash; Hoffmann, 1999, about Paul Erdős) as well as theatre plays and feature films where the main characters are mathematicians (e.g. *Good Will Hunting*, *Pi* or *Proof*).

Reading popular books gives one the impression that some themes are indeed quite popular. Topics such as chaos or cryptography are certainly frequent. Books also often address core mathematical concepts, such as number, infinity, parallel lines, etc. Moreover, what they choose to say about them and how they say it, is similar from book to book.

In investigating popularization of mathematics, it seemed a very natural question to try to pin down the content of popularization of mathematics, and break it into some sensible categories. I thought it would be an easy task, based on my impression of there being only a few recurrent themes. A closer look at the themes, however, suggests a more complex picture. Over the period of the last 10-15 years, the content has been changing and there are various conjectures about the direction of these changes. Singh (2005), for example, perceives a trend towards a growing importance of what he calls the *narrative non-fiction* in popularization of both science and mathematics:

> Traditionally popular science writers have put the emphasis on explanation, concentrating on conveying to the reader an understanding of scientific concepts.... However, the last five years have witnessed the burgeoning of a new type of science writing, so-called *narrative non-fiction*, in which the emphasis is not solely on the explanation of science. Instead, the author also writes about the scientists, their motives, adversities and triumphs. All of this is framed within an overarching narrative. These books explain science, but they also tell the tale of scientific discovery or have a biographical thread. [Narrative non-fiction is different from] fiction based on scientific or mathematical themes. In these books the story is naturally more important than any explanation of scientific concepts but they do explain what drives scientists, describing the culture and atmosphere of scientific research. Recently there have been several fictional books about mathematics namely *Uncle Petros and Goldbach's Conjecture* by Apostolos Doxiadis and *The Parrot's Theorem* by Denis Guedj. Arguably the trend towards narrative non-fiction began with Dava Sobel's *Longitude*, a description of the invention of the marine chronometer, which also tells the story of its inventor John Harrison, who had to battle with the establishment in order to get his breakthrough recognized and adopted. Subsequently, many other books have been categorized as narrative non-fiction, including my own books, *Fermat's Last Theorem* and *The Code Book*. (Singh, 2005: 183)

Recent examples of popularization suggest that, besides the narrative non-fiction trend mentioned by Singh above, there is also much interest in modern applications of mathematics and statistics and in the various abuses of mathematics and statistics in the media. Books on great ideas in mathematics including old and new mathematical results are still being published (e.g. Devlin, 2002a; Szpiro, 2007), but more attention seems to be paid to applications including not only mathematics applied to physics and cosmology (e.g. Penrose, 2004; Szpiro, 2003), but also to biology, understanding nature, information technology (e.g. Casti, 2000; Ball, 2003; Higgins, 2007; Ratzan, 2004), and the use of mathematics in everyday life (e.g. Herzog, 2007; Zev, Segal & Levy, 2009).

In my research, I was trying to somehow "define", "characterize" or "categorize" the mathematical content of popularization. I was asking, is it possible to add something to the observations about the content of popularization of mathematics made by the authors quoted above? This chapter presents the results of my efforts in sections 4.2 and 4.3. One of these results is that I found the categorization of the content of popularization a rather impossible task. I will explain why. Then, I tried to at least identify a pattern in recurrent topics of popular books. Part of this pattern was the absence of proofs for most of the results presented in popular books. Yet, proofs are often considered to be the heart of mathematics. I noticed, however, that two proofs were recurrent in the books: the infinity of primes, and the irrationality of the square root of two. I tried to understand the authors' reasons for including these proofs in their books. What values did they intend to communicate with these proof? I write about my findings related to irrationality of the square root of two in section 4.3. Finally, in section 4.4, I deal with the most modest task about the content of popularization: I describe the mathematical content of the "Medial representation" and "Escher" lectures. The chapter ends, in section 4.5, with a tentative list of characteristics that it is desirable for a mathematical topic to possess if it is to be chosen for popularization.

4.2 Attempts at categorizing the mathematical content of popularization

My first approach to studying the content of popularization of mathematics was to categorize it somehow. I was looking at popular books, journals, and magazines, trying to find some pattern in their content. The results, however, were not very promising. I thought that the difficulty comes from the fact that the category of "popularization of mathematics" is not clearly defined and contains too great a variety of publications. I started searching for some reliable source for deciding whether something can be considered as popularization of mathematics or not, but did not find any explicit definition.

What I found, however, were some implicit hints on what counts, for the mathematical community, as *good popularization*. The American Mathematical Society has a special award, called The Joint Policy Board for Mathematics (JPBM) Communications Award. This Award was established, in 1988, "to reward and encourage communicators who, on a sustained basis, bring mathematical ideas and information to nonmathematical audiences"[23]. Until 2009, nineteen awards have been given. The list includes some very famous names, such as Martin Gardner, Ian Stewart, Keith Devlin and Roger Penrose. Some of the reasons for giving the award were:

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[23] Information about this award, together with the list of winners, is posted on the website of the American Mathematical Society: http://www.ams.org/prizes/jpbm-comm-award.html (Viewed: July 25, 2009).

- "... for communicating the beauty and fascination of mathematics and the passion of those who pursue it"
- "... for increasing the public's understanding of mathematical concepts"
- "... for making a consistent effort to reach out to a wider audience"
- "... for being an erudite spokesman for mathematics, communicating its charm and excitement to thousands of people from all walks of life"
- "... for artful and accessible essays and lectures elucidating the mathematical concepts"
- "... for the discovery of Penrose tilings, which have captured the public's imagination"

These and other justifications confirmed my conviction that the winners of the award were practicing popularization of mathematics in the sense of the characteristics highlighted in Chapter 2. The awards were not given to "great teachers" or skilled textbook writers. The award winners were producing books, articles or works of art (e.g. sculptures) for anybody willing to stop by and read or look. They were the "spokespersons for mathematics": trying to win the public appreciation of mathematics by making it attractive (beautiful, fascinating, charming, exciting, capturing the public's imagination). They were successful in overcoming, to some extent, the challenges of the intercultural communication: their works were believed to "increase public understanding of mathematical concepts", and make mathematics more "accessible" and clear.

Therefore, looking at the JPBM Communications Award winners' work could give some idea of what parts of the mathematical culture popularizers choose to show to nonmathematical audiences. I first tried to find a categorization of popular works based on how the award winners' publications were classified according to different classification systems used by mathematicians, mathematics educators and librarians. The attempt simply showed that, indeed, there seem to be some recurring topics (such as those mentioned in section 4.1), but also provided evidence that topics addressed by popularizers are often unclassifiable according to the existing catalogue systems. Moreover, it turned out that what counts as popularization changes from community to community. The notion of "popular work in mathematics" may be different for a librarian and a mathematician.

After these unsuccessful trials, I have given up looking for a general way of categorizing the content of popular mathematical activities. Instead, I decided to look at a sample of books written by the winners of the JPBM Communications Award in some detail. The result of this experience is described below.

As noted by Morics (2003), some books are *monographs* of a particular domain or problem of mathematics, and others take the reader on a mathematical tour displaying various more or less disconnected *snapshots* of mathematical culture. There seems to be more books of the latter kind than of the former. Perhaps the most spectacular example of the "monograph" approach is Penrose's (2004) book, which aims at no less than giving the reader *A Complete Guide to the Laws of the Universe*. Examples of the "snapshots" approach abound in Peterson's books. In *The Mathematical Tourist* (Peterson, 1988), the subtitle promises to show the reader *Snapshots of Modern Mathematics* (Peterson, 1988). In another book, the author invites his readers to *A Mathematical Mystery Cruise* (Peterson, 1990). I was expecting the choice of topics to be more varied in the monographs, but at least in the examples I saw, the same themes appeared to come up again and again.

Numbers and logic always seem to come to mind when people think about mathematics. Thus it is not surprising that these two themes appear frequently in the popular literature. In relation to numbers, authors often speak about the extension of the notion of number from whole numbers, used in the context of counting or ordering, to integers (positive and negative), to rational, to real and, sometimes, to complex numbers. The notion of irrational number receives much attention, often with a story of their discovery by the Pythagoreans, and a proof of the irrationality of the square root of two (but not the square root of three, for example). Numbers such as π, e, and i are introduced in a similar vein, with their history or just historical anecdotes, explanations of their mathematical meaning and significance in the progress of mathematical theories. Presentation of natural numbers is often accompanied by the discussion of factoring them into prime numbers and applications in cryptography. The history of classical problems such as the number of prime numbers, or of perfect numbers appears to be necessary in a chapter devoted to number theory. Looking for patterns in sequences of natural numbers such as those in the Fibonacci sequence is a popular topic, as well.

The theme of logic may be presented in the context of the classical logic puzzles, (as in the books by Martin Gardner) or in the context of foundational problems of mathematics such as Gödel's incompleteness theorem or issues related to infinite sets, which border on the philosophy of mathematics.

In Euclidean geometry, the most popular topics seem to be, besides the Pythagoras theorem and its various proofs (especially the visual ones), the Kepler's and Poincaré conjectures. Discussion of the parallel postulate, together with the history of mathematicians' attempts at proving it, inevitably leads to the invention of non-Euclidean geometries and the transformation of the philosophy of mathematics with which this invention was associated. Personal stories of Bolyai, Lobatchevsky and Gauss are usually told in this connection. In particular, hyperbolic geometry – and its artistic representations in Escher's works – has received special attention (see, e.g. Penrose, 2004, section 2.4). Geometry also serves as a vehicle to convey the fundamental concepts of group theory. These classical geometric subjects, however, seem to give way, in more recent times, to such new forms of geometric thinking in mathematics as those found in topology or knot theory.

The traditional ideas of calculus, although still present in popular literature, appear to lose fame to the more "hot" topics such as dynamical systems, chaos and fractals. Advances in technology have allowed producing beautiful color plates and animations to represent these ideas. Chance and probability are also gaining in popularity; their applications involving money such as card games or lottery may be quite appealing to the general audience. The growing importance of networks provides also a place for graph theory in popularization of mathematics.

Across all the above-mentioned "must-sees" in popularization of mathematics, the mathematician is concerned with testing his or her conjectures using proofs, often based on processing of formal expressions. Such proofs distinguish mathematics from other domains of scholarly knowledge. Proofs, however, do not appear to belong to the "must-sees" for the "average" popularizer. Detailed proofs generally require the reader or the listener to be familiar with a specialized mathematical language, and this is not normally assumed about the so-called general audience. If proofs are an essential part of the mathematical culture, then how, without proofs, can one give the audience an appropriate picture of this culture? There are certainly a few possibilities to overcome this paradoxical situation.

Probably the most common solution is to include only a few proofs – chosen as "paradigmatic examples" but still easy to follow – to give an idea of the essence of a mathematical proof.

Another approach is to justify the particular result one is focusing on, but without the technical details: offering only a narrative sketch of the main idea of the proof. This was the option chosen by Singh in his book on Fermat's Last Theorem (Singh, 1997).

Yet another solution is to convey the idea of mathematical proof on an artificial example, specially constructed for this purpose. For example, Stewart would explain what it means to prove in mathematics by showing that reaching the word DOCK by transforming the word SHIP one letter at a time will necessarily include a word containing two vowels (Stewart, 2006: 72).

It is also possible to omit a proof of a theorem in the main body of the book, but include it in an appendix (e.g. Singh, 1997).

Penrose – certainly not an "average" popularizer – would simply not give up proofs or formal processing in his *The road to reality. A complete guide to the laws of the universe.*

> The reader will find that in this book I have not shied away from presenting mathematical formulae, despite dire warnings of severe reduction in readership that this will entail. I have thought seriously about this question, and have come to the conclusion that what I have to say cannot reasonably be conveyed without a certain amount of mathematical notation and the exploration of genuine mathematical concepts. (Penrose, 2004: xv)

Penrose proposed that his book can be read at four levels (ibid., xv-xvi). The first three levels are for readers who are not already familiar with the subjects approached in the book. The first level is for the reader who has trouble with any kind of mathematical notation (including operations on fractions). This reader can just read the text and skip the formulas. There is still, Penrose claims, something to be learned from the text alone. The second level is for the reader who is ready to read the mathematical formulas but is not inclined to verify them. The third level is for readers who not only are willing to read the formulas but also are interested in understanding why they are true. For this level of reading, Penrose has provided, in the footnotes, exercises of three levels of difficulty. Finally, the fourth level is for the expert, who is already familiar with the mathematics. This reader will not waste his or her time in doing the exercises, but might be interested in the author's unconventional point of view on various modern theories such as the big bang theory or black holes.

This is an interesting approach to writing popular books, but it risks to highly increase their volume – Penrose's book is over a thousand pages – and make their writing a much more complex enterprise. Very few popularizers want to devote eight years to writing a single book.

In the next section, I will share some observations about the proof of the irrationality of the square root of two in a few popular books.

4.3 INVESTIGATING ONE RECURRENT TOPIC IN POPULARIZATION: THE PROOF OF THE IRRATIONALITY OF THE SQUARE ROOT OF TWO

Two proofs are frequent in popular works in mathematics: those of irrationality of $\sqrt{2}$ and of the infinity of primes. What could be the reasons of the popularity of these proofs?

To gain some insight into this question, I looked at a sample of publications written by the JPBM Communications Award winners and tried to find the reasons of choosing to include, in particular, a proof of the irrationality of the square root of 2. I picked only four books but even this small (and rather opportunistic) sample suggests that there could many reasons for the choice and presentation of the proof.[24]

4.3.1 Proof of irrationality of $\sqrt{2}$ in Constance Reid's "From zero to infinity"

The first edition of the book had a chapter devoted to each of the natural numbers from 0 to 9. In one of the revised editions, a chapter on the number *e* was added. This was the first chapter with an irrational number in its title. It is in this chapter that the proof of irrationality of the square root of 2 is mentioned. The main idea of the proof is given, namely that 2 cannot be expressed as a square of the ratio of two whole numbers, but detailed argumentation is missing. The context indicates that the purpose of mentioning the proof was to motivate the existence of irrationals and to gain some idea about their characteristics (e.g. the characteristics of their decimal representation). Thus, Reid used the proof for introducing irrational numbers as mathematical objects. With its rich connections to various mathematical objects, $\sqrt{2}$ indeed serves as a perfect illustration of a meaningful mathematical object. The main argument for the existence of $\sqrt{2}$ in this chapter is not so much in the above mentioned proof, as in its geometric interpretation as a measure of the diagonal of the unit square. Although other irrationals could have been

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[24] The books I looked at were the following:
Reid, C. (1992). *From zero to infinity. What makes numbers interesting.* MAA Spectrum
Stewart, I. (2006). *Letters to a young mathematician.* New York: Basic Books
Penrose, R. (2004). *The Road to Reality. A Complete Guide to the Laws of the Universe.* London: Jonathan Cape
Davis, P. J. and Hersh, R. (1981). *The Mathematical Experience.* Boston: Birkhauser

used (e.g. $\sqrt{3}$), the square root of two gave Reid an opportunity to speak about Pythagoreans to endow the notion of irrationality with a dramatic flavor[25], and also explain the restricted character of the Greek concept of number, which did not include all irrationals.

4.3.2 Proof of irrationality of $\sqrt{2}$ in Ian Stewart's "Letters to a young mathematician"

The purpose of mentioning the proof in Stewart's book is different from that in Reid's. Stewart is using the proof in the context of giving advice on how to read mathematical texts. The book is meant to give an insight into the mathematician's work and life. Although one of the chapters has the word "proof" in the title, and the chapter titled "mathematical stories" is also devoted to the notion of proof, Stewart decided to mention the proof of irrationality of square root of 2 in the chapter on how to study mathematics. An efficient way of reading mathematical texts plays an important part in studying mathematics. Reading the proof of irrationality of square root of two serves as an illustration of the more general process of reading mathematical texts. In particular, the author suggests that if one is stuck in reading a mathematical text, it is often a good technique to keep on reading, in the hope that subsequent explanations will shed light on the encountered difficulties and the problem will be solved. The choice of this particular proof in this context seems rather arbitrary. Other examples might easily serve the same purpose. For example, reading the definition of linear independence, saying that vectors $v_1, \ldots, v_n$ are linearly independent if $a_1v_1 + \ldots + a_nv_n = \theta$ implies that $a_1 = \cdots = a_n = 0$, certainly better represents the kind of mathematical text that undergraduate

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[25] This is an example of the narrative non-fiction mentioned in the Introduction to this chapter.

students have trouble reading and interpreting. Stewart's book, however, is not addressed to undergraduate students, although it speaks about studying mathematics at the university, doing a PhD in mathematics and eventually becoming a mathematician. The book's intended readership are mainly secondary school students who are about to make a decision what to study at the university. This audience would not understand examples from university mathematics; proof of irrationality of $\sqrt{2}$ is accessible to them.

4.3.3 Proof of irrationality of $\sqrt{2}$ in Phillip J. Davis and Reuben Hersh, "The Mathematical Experience"

The Mathematical Experience is a book that tries to describe how mathematicians feel and think. The authors have described their task as "to explain to nonprofessionals just what these people are doing, what they say they are doing, and why the rest of the world should support them at it" (Davis & Hersh, 1981: xii). The book aims to give a general impression about mathematics through various questions related to history, philosophy, and content of mathematics. The number $\sqrt{2}$ is discussed in two chapters. In one of them (Algorithmic vs. Dialectic Mathematics), the problem of the existence of the number is posed and two proofs of the existence of a solution to the equation $x^2 = 2$ are given, one "algorithmic", the other – "dialectic". The distinction between these two approaches is the main idea and the proofs merely illustrate it. Proofs of irrationality of the number appear in the chapter titled "Comparative Aesthetics". Again, there are two proofs and one is argued to represent "a higher degree of aesthetic delight", in the authors' words. The not-so-delightful proof (Proof I) is longer, contains detailed algebraic argumentation, deriving the contradiction in a step-by-step fashion. The one considered aesthetic (Proof

II) is shorter, contains no algebraic manipulations, and omits details, but clearly states the main idea of the proof, or "the heart of the matter", as the authors say.

Certainly, other proofs could be used to illustrate the aesthetic component of mathematics. For example, a collection of such results can be found in Aigner & Ziegler (2003). Undoubtedly, the brevity and elementary character of the proofs of irrationality of $\sqrt{2}$ were good reasons why this particular theorem was chosen in a book addressed to general audience. Interestingly, however, the only places in the book where irrational numbers are mentioned are those where the number $\sqrt{2}$ is mentioned. This number thus appears to be regarded by the authors as *the* paradigmatic example of irrational number.

4.3.4 Proof of irrationality of $\sqrt{2}$ in Roger Penrose's "The Road to Reality. A Complete Guide to the Laws of the Universe"

Penrose's approach to writing a popular book is certainly quite different than any of the authors' mentioned above. His goals are quite ambitious as he offers a "complete guide to the laws of the universe". It is not a tour of a few interesting mathematical concepts and results chosen for communicating certain general characteristics of the mathematical culture. The author's ambitious goal requires appropriate mathematical tools and techniques, and Penrose spared no effort in acquainting the reader with them. Penrose aimed at giving a complete, self-contained guide. With the choice of topics, he intended to support his main goals; each mathematical tool in the book is there for a purpose, and they are all interconnected. The choice of topics seems to be guided by their contribution towards a coherent presentation rather than by offering an insight into the world of mathematics and mathematicians. The proof that $\sqrt{2}$ is irrational is included because

Penrose heavily relies on the special characteristics of the set of real numbers in the book, thus its completeness had to be addressed at some point at the beginning.

The proof is different from those in the previously mentioned books. Penrose uses the method of infinite descent. The proof starts by assuming that there is a solution to the equation $(\frac{a}{b})^2 = 2$, or, equivalently, $a^2 = 2b^2$, where a and b are positive integers. This looks almost like the classical proof evoked by the previous three authors, except that no assumption is made about the fraction $\frac{a}{b}$ being reduced to lowest terms. This assumption is essential in obtaining the contradiction in the classical proof. Penrose doesn't need this assumption, however, since he uses the infinite descent method and obtains a contradiction with the fact that every strictly descending sequence of positive integers is finite. The argument goes as follows. The last equation, $a^2 = 2b^2$, implies not only that a is even and so a^2 is divisible by 4, and therefore b is even, but also that $a^2 > b^2 > 0$. Since b can be written as $b = 2c$ for some positive integer c, we have $b^2 = 4c^2$, whence $b^2 > c^2 > 0$. Continuing this process, we obtain an infinite sequence of positive integers $a, b, c, d, \ldots$ with $a^2 > b^2 > c^2 > d^2 > \cdots > 0$. However, a decreasing sequence of positive integers must come to an end, which contradicts the above conclusion that the obtained sequence is infinite.

Penrose explains his choice of proof by the fact that the claim that it is possible to reduce any fraction to lowest terms is not obvious and requires a proof (which can be based on Euclid's algorithm), while the method of infinite descent is more elementary, closer to the axioms (in particular the axiom of well ordering). Another advantage that Penrose sees in his proof is the possibility it offers to highlight an essential difference between real numbers and natural numbers: it is not true that a strictly descending sequence of real numbers must end. He shows where his argument of the non existence of square root of two in positive integers breaks down when "integer" is replaced by "real".

Penrose takes the time to enumerate the properties of numbers that were used in his proof to illustrate the mathematicians' tendency to be suspicious about the "obvious" and always trying to "identify the precise assumptions that go into a proof" (Penrose, 2004: 53), since this may save them from unjustified generalizations.

As in the other books, the proof is also used as an occasion to speak about the Ancient Greek restricted concept of number and the breakthrough that the discovery of the impossibility to represent the square root of two as a quotient of integers has brought about.

4.3.5 Conjectures about reasons for including the proof of irrationality of $\sqrt{2}$ in a popular book

The examples above suggest that the proof is rarely used to convince the reader solely about the irrationality of this particular number. If irrationality is in the focus at all (Reid, Penrose), then the purpose is to show that integers and quotients of integers are not enough to solve even simple equations or to measure such familiar segments as the diagonal of a square, and there is a need to extend the notion of number. The proof that $\sqrt{3}$ is irrational might serve the same purpose, but apparently not that well. The square root of 2 has become the paradigmatic example of irrational number at school, and this notion is perpetuated through popularization.

As the examples above show, however, the proof can serve other purposes as well. It seems to have a great potential to connect with and illustrate various aspects of the culture of mathematics: its history (Reid, Penrose); its objects and their various representations (Reid, Penrose); its special notion of truth and methods of justification (Davis & Hersh, Penrose); its aesthetic values (Davis & Hersh); its utilitarian values (Penrose), and skills necessary for its study (Stewart). I elaborate on these purposes below.

History. The story of the discovery, by Pythagoreans, of the impossibility to represent the ratio of the diagonal of a square to its diagonal as a ratio of two integers leads far in various directions. It provides an opportunity for introducing elements of the Pythagorean philosophy and religion. It also gives space for discussing the Pythagorean view of number and thus provides an immediate link to the *Elements*, which might be a starting point for talking not only about many mathematical ideas but also about epistemological issues regarding the conceptualization of the notion of number. Human aspects of mathematical history can also be touched upon in the context of the proof, by telling the legend of the Pythagorean who lost his life as a result of the discovery of irrationality of square root of two.

Objects and their representations. The proof asserts that a mathematical object, whose existence was not recognized before, becomes a valid entity. This may contradict previous expectations, but the logical argument claims its right for existence. This argument, however, is not the only "evidence" for the existence of the square root of two. Usually, the popularizer will talk about the construction of this number as the length of the diagonal of a unit square, which satisfies many people's conception of number as measure. The situation is more difficult if one wants to convince the audience that it makes sense to say that i is a "number". Therefore, the square root of two is a more convincing illustration of the process of extending the notion of number in mathematics. Square root of 2 is, moreover, a special number and as such it already exemplifies *number* as an important mathematical object. It has different immediate representations (analytic, numerical, geometric), and this allows popularizers to make excursions into various areas of mathematical culture. It provides immediate links to important mathematical concepts, such as infinity, limits, completeness, unpredictable decimal representations, etc. It also gives an opportunity for talking about other special irrational numbers and for looking at their properties and significance in mathematics. The distinction between constructible and non-constructible numbers as well as between algebraic and transcendental numbers can be drawn.

Methodology. There are several ways to prove the irrationality of the square root of 2 and comparing them gives the popularizer the opportunity to discuss important methodological issues. The fact that the theorem states an impossibility makes it natural to use a proof by contradiction, which is almost a trademark of the mathematical culture. The method of proof by contradiction is not intuitive for many people, because it is based on the law of excluded middle, which does not apply in most everyday situations. Illustrating this method on the example of irrationality of the square root of two can be a good choice since the proof is quite short and straightforward and thus technical complications do not obscure the main idea of the proof.

Aesthetic Values. The proof of the irrationality of square root of 2 is often cited as an example of an elegant mathematical proof. Short and based on not more than mathematical facts known from school, the proof is considered accessible to general audience (Betts, 2005). The examples of books presented above show, moreover, that there are several approaches to proving the fact and that their comparison can also lead to the discussion of aesthetic values in mathematics.

Utilitarian values. The proof offers an opportunity to discuss the usefulness of irrational numbers in the applications of mathematics. Extending the notion of number from integers and ratios of integers (to which this notion was restricted in Greek mathematics) to real numbers certainly made calculations easier to manage, especially when using decimal representations. For example, the use of real numbers was necessary for the development of calculus and therefore also for the theory of motion in physics. Numerical methods for approximating square roots opened the way towards other techniques used in modern numerical mathematics.

Study skills. As we have seen in Stewart's book, the proof of irrationality of the square root of 2, can be used as a context for talking about study skills to secondary school students.

I will not give an account here of my investigation into the purposes of including the proof that there are infinitely many primes in popular books, but I found that they are similar in kind to those mentioned above for the square root of two.

4.4 The Mathematical Content of Two Popular Lectures

In the following, I will describe the mathematical content of the two lectures that I am using as one constant example about which I am asking all the questions posed about popularization in the thesis. For the description of the mathematical content, I used not only my notes from the lectures but also information about this content in the scholarly literature to which the lecturers referred. I will first present the content of the first lecture, then that of the second lecture and finally I will briefly compare the contents of the two lectures at the end of the section.

4.4.1 The mathematical content of the first lecture: Medial representation

The title of the lecture was, *Medial Representations: Mathematics, Algorithms and Applications*. The lecture aimed at presenting the mathematics behind storing, retrieving and processing computer images. The main concept was that of the "medial axis" of a shape, as opposed to its "boundary representation". The medial axis of a shape can be likened to the "skeleton" of an object, while the boundary representation is its "contour", as shown in Figure 4.1. "Skeleton" is a technical term in the domain of computer imagery (Yushkevich, 2003) but, in the lecture, it was first used as a metaphor.

The theory proposes that if an object can be described by a set of "maximal inside balls", which are spheres (or circles in the case of 2D objects) that are tangent to (at least) two sections of the boundary, then the "skeleton" of the object (also called its "medial axis") can be described by a pair of data (x,r), where x is the center of the maximal inside ball and r is its radius at the given point (Yushkevich, 2003). Knowing the data for a sufficient number of inside points allows reconstructing the boundary of the object.

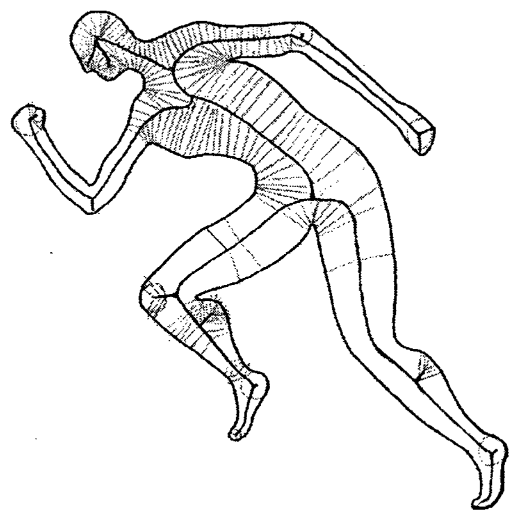

Figure 4.1. The "skeleton" of a man

Information about x gives the "medial curve". For a given skeleton-point (x, r), using the radius, r and the local behavior of the medial curve we can reconstruct vectors pointing from x to the corresponding boundary points. These vectors are likened to the paddles of a rowboat touching the bank of a narrow canal[26]. We can determine the direction of the boundary shape by calculating the unit tangent vector of the medial curve, whose angle with the vector pointing to the boundary is given by arccos $(-\frac{dr}{ds})$, where s is the arc length along the medial curve.

This way, a close correspondence obtains between medial representation and

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[26] A picture illustrating the analogy can be found for example in Yushkevich (2003: 24). Available: http://midag.cs.unc.edu/pubs/phd-thesis/PYushkevich03.pdf (Downloaded: July 7, 2009).

boundary representation. Moreover, medial representation can capture some special aspects of the object. The method of medial representations has been also used to understand features of human vision (Blum, 1967; Yushkevich, 2003).

Some scholarly publications on the subject focus on finding the medial curve, referring to it as Blum's Medial locus. There exist several definitions or descriptions of this concept in the literature. Yushkevich (2003) gives a short overview of these. Besides the one just mentioned, the set of the centers of the inner balls is often described using the grass fire analogy. The object is imagined as a patch of grass that catches fire along its boundary. The skeleton of the patch consists of points where the fire fronts hit each other. The movement of the fire fronts can be described by a partial differential equation of a motion with a constant speed in the direction normal to the boundary: $\frac{\delta C(t,p)}{\delta t} = -\alpha N(p)$, where *C(t, p)* denotes the fire front parameterized by *p* at time *t*, *N(p)* is the unit outward normal to the fire front and α is a constant whose sign depends on the direction of the propagation.

Although there is a differential equation that describes the motion, the "brute force" method of determining the desired points by solving the partial differential equation raises serious difficulties. An attempt at solving the problem was suggested in Siddiqi, Bouix, Tannenbaum, and Zucker (1999, 2002) and Dimitrov, Damon, and Siddiqi (2003). The proposed method was based on the behavior of the gradient vector field of the distance from the boundary function. This vector field changes exactly at the points of this skeleton, as if these points functioned as charges. Using this analogy, the problem can be reduced to a well-known question in physics, namely to detect the place of the charged particles of an electromagnetic field.

Using the gradient vector field of the distance from the boundary function we can compute the average outward flux. To visualize the situation better, color coding is sometimes used. Around the singularities, the flux has large negative values while the smooth regions close to the boundary have zero flux. To indicate the singularities inside the objects, the points can be colored according to the value of the flux. Thus coloring could change around the singularities; the closer one gets to the singularities the brighter (or darker) the colors are. In the literature (e.g., Dimitrov et al., 2003; Pizer, Siddiqi, Szekely, Damon & Zucker, 2003; Siddiqi et al., 2002), this idea is often illustrated by applying the method to the image of a panther[27].

The method based on Blum's medial loci was presented as an alternative representation in data processing in Stolpner and Siddiqi (2006) with an emphasis on medical applications, such as brain imagery (Bouix, Siddiqi & Tannenbaum, 2005; Yushkevich, 2003, etc.) or endoscopy (Bouix, Pruessner, Collins & Siddiqi, 2005). Researchers tested the efficiency of the method by applying it to sorting shapes. Using graph matching techniques based on eigenvalues of the adjacency matrix, researchers identified about twenty different groups of objects. The method was able to capture characteristics such as having many additional parts as in the case of an octopus or a human shape (Zhang, Siddiqi, Macrini, Shokoufandeh & Dickinson, 2005).

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[27] A picture similar to the one used in the lecture is shown, for example, in Pizer et al. (2003: 165) [Available: http://www.cim.mcgill.ca/~shape/publications/ijcv03.pdf, Downloaded: July 7, 2009] where the singularities are colored black while the points closer to the boundary are marked using grey colors. This way, the skeleton is outlined by the dark lines in the middle. (The additional white lines outside of the panther are noises coming from the fact that the distance function is defined also for points outside of the object but the medial loci were defined only for points inside the image. The lines are not part of the medial axis using the definitions of inner circles, or the non-linear differential equation.)

4.4.2 The mathematical content of the second lecture: the mathematics of Escher's "Print Gallery" lithograph

Escher's drawings have attracted much interest amongst mathematicians, who investigated how geometric transformations, such as symmetry (Chung, Chan & Wang, 1998), and non-Euclidean geometries, particularly hyperbolic geometry (Adcock, Jones, Reiter & Vislocky, 2000), inspired his artwork. Although the artist himself was not trained as a mathematician, Bool (1982) argues that he had strong connections with mathematicians, was inspired by Pólya and corresponded with Coxeter and Penrose.

The April 2003 issue of the *Notices of the American Mathematical Society* published two articles on the mathematics of Escher's drawings. They were a contribution to the 2003 Mathematical Awareness Month, as an illustration of the theme "Mathematics and Art". In the first of these articles, the authors, Bart de Smit and Hendrik Lenstra (2003) sketched "The mathematical structure of Escher's *Print Gallery*". "Print Gallery" refers to Escher's 1959 lithograph, titled "Prententententoonstelling", which represents, in a rather unobvious way, a man standing in an art gallery looking at a picture showing himself standing in an art gallery looking at a picture of himself standing in an art gallery, etc. The infinite repetition in the picture, however, is not as obvious as in the picture on the *Droste* cocoa box, where a nurse carries a tray with a *Droste* cocoa box with the picture of herself carrying a tray, etc. In the case of the cocoa box, the consecutive pictures seem to be obtained by iterating a scaling mapping, which preserves ratios of lengths and produces an object similar to the original one. In the *Print Gallery*, the iterated mapping not only shrinks, but also rotates and distorts objects considerably. The picture does not show what the mapping is exactly; only the start of the iteration is shown and further steps are hidden within a white spot in the centre of the picture, containing only Escher's signature. De Smit and Lenstra successfully reconstructed the mapping, and were able to fill in the white hole, based on Escher's original sketches for the picture, where he used transformations of grids. It turned out that, to obtain a picture like the *Print Gallery*, one needs a rotation and an exponential map on the complex plane, a special *conformal* mapping. A group at the University of Leiden (The Netherlands) developed an animated explanation of this reconstruction, using simple grid examples and videos. It can be viewed on a special website devoted to the topic[28], titled, "Escher and the Droste effect".

One tends to think of popularization as a "mere" transposition of material from existing mathematics to a product that could be appealing to a mathematical tourist. Escher's pictures have been used in popular works in mathematics to illustrate various geometric transformations and the idea of tiling, or how mathematics can be successfully used in art (Ernst, 1976; Hofstadter, 1979). De Smit's and Lenstra's paper is, however, a not very common example of the "inverse transposition": from popular work to scholarly mathematics. In fact, however, we have here an instance of the process of going "from popular work to scholarly work and back": the scholarly work of de Smit and Lenstra has been popularized in the form of a website devoted to it and the many lectures of which the one I am describing here is an example, and generated publications in scholarly journals (e.g. Leys, 2007; Carphin & Rousseau, 2009).

[28] http://escherdroste.math.leidenuniv.nl (Viewed: July 25, 2009)

4.4.3 Comparison of the mathematical content of the two lectures

The topics of both lectures certainly provided a wide scope for presenting different features of the mathematical culture, and a rich set of mathematical objects (e.g. distance, iteration, etc.). Both lectures also created links between mathematics and other fields (medicine or information retrieval in the first lecture, art in the second), thus making it possible to communicate mathematical ideas through "translation" into other languages, perhaps closer to the audience's cultures. With these extra-mathematical references, however, the two lectures appealed to different values: the first – to utilitarian values, the second – to aesthetic values. Apart from these differences, the first lecture contained much more technical details than the second.

4.5 CONCLUSIONS

The choice of a particular topic for a mathematical tour seems to be guided by the popularizers' intention to show as much of the mathematical culture as possible in a brief time and space. Thus a topic is more likely to be chosen if,

- it bears some important core characteristics of the mathematical culture, i.e. it exemplifies some fundamental mathematical concept or method;
- is deeply rooted in the culture with a rich network of links to other elements of the culture;
- is easy to show to an outsider, requires little advanced knowledge of mathematical concepts and familiarity with specialized mathematical symbolism;
- contains a deep and surprising idea;

- exemplifies the more elegant and alluring aspects of the mathematical culture;
- is likely to resonate with the audience's emotions by being linked with universal human concerns represented in historical facts or anecdotes;
- has links to other cultures, preferably those represented in the audience, and therefore provides the possibility of translating its meaning or significance using extra-mathematical associations.

The above list is by no means exhaustive and it does not imply that a mathematical topic addressed by a popular activity should have all these characteristics. It simply provides a summary of the characteristics of mathematical questions that are often used in popularization.

CHAPTER 5

ANALYZING THE MEANS THAT POPULARIZERS USE TO COMMUNICATE THEIR MESSAGE

5.1 INTRODUCTION

This chapter focuses on *how* popularizers communicate their message; what means do they use? There have been, to my knowledge, no systematic studies of this question in relation to popular works in mathematics, but I have found such research in the domain of popularization of science. Discourse analyses of popular works in science suggest that these works are not merely simplified versions of scholarly publications but represent a different *genre* of scientific writing (Nwogu, 1991; Myers, 2003; Calsamiglia, 2003; Calsamiglia & van Dijk, 2004):

> [A] popularization article is not a simplified version of the research article, but a discursive reconstruction of scientific knowledge to an audience other than the academic one (de Oliveira & Pagano, 2006: 628).

How do we distinguish the popular mathematics genre from the mathematics textbook genre or the scholarly writing genre? This is not an easy question. Let us look at the following brief texts. Without knowing their sources, can one tell, which ones belong to a popular work in mathematics, and which ones – to a mathematics textbook?

1) "... the essence of [linear transformations] is that these are transformations that 'preserve' the vector operations of addition and scalar multiplication."[29]
2) "A linear transformation preserves the vector-space structure of the space on which it acts."[30]

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[29] Poole, D. (2006). *Linear Algebra. A Modern Introduction. Second Edition.* Thomson. Brooks/Cole (p. 211)

[30] Penrose, R. (2004). *The Road to Reality. A Complete Guide to the Laws of the Universe.* London: Jonathan Cape (p. 255)

3) "A transformation $T: R^n \to R^m$ is called a linear transformation if
   1. $T(\boldsymbol{u} + \boldsymbol{v}) = T(\boldsymbol{u}) + T(\boldsymbol{v})$ for all $\boldsymbol{u}$ and $\boldsymbol{v}$ in R^n and
   2. $T(c\boldsymbol{v}) = cT(\boldsymbol{v})$ for all $\boldsymbol{v}$ in R^n and all scalars c."[31]
4) "The concept of linearity is pervasive in linear algebra. When a mapping has the property of linearity, matters that are otherwise difficult can become simple. For this reason, *linearity* has become a broad area of study in applied mathematics, containing many theories, facts and formulas – probably far beyond what any one person can comprehend!"[32]
5) "In the general study of groups, there is a particular class of symmetry groups that have been found to play a central role. These are the groups of symmetries of vector spaces. The symmetries of a vector space are expressed by the linear transformations preserving the vector-space structure."[33]
6) "Geometrically, a linear transformation is one that preserves the 'straightness' of lines and the notion of 'parallel' lines, keeping the origin O fixed."[34]
7) "A linear transformation maps one line segment into another."[35]

Texts 1) and 2) are very similar, yet the first one is taken from a textbook and the second from a popular work. Text 3) raises no doubts about its origin in a textbook, although it might appear in a popular work as well. The difference would be in the context: in a textbook, it could appear without comments such as 1), 2) or 4), while leaving it without such comments would not be acceptable in a popular work. The difference would also be in the intended rapport of the reader to a text like 2): in a popular work, text 2) would be optional; in a textbook, the reader may skip the 'superfluous introductions' such as 4) but must read text 2). Texts 6) and 7) are similar, yet the first one is in a popular work, and the second – in a textbook. The difference is, again, not in these texts themselves, but in the contexts in which they appear. Text 6) is part of an explanation of the general notion of linear transformation: part of the author's

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[31] Poole, ibid.
[32] Cheney, W. & Kincaid, D. (2009). *Linear Algebra. Theory and Applications*. Jones and Bartlett (p. 170).
[33] Penrose, ibid. (p. 254)
[34] Penrose, ibid. (p. 255-6)
[35] Cheney & Kincaid, ibid., p. 181.

attempt to communicate, "what linear transformations look like" (Penrose, ibid.). Text 7) is a theorem and is followed by a proof. Texts 4) and 5) are both the first paragraphs of sections on linear transformations; 4) – in an undergraduate textbook in Linear Algebra, 5) – in a popular work. No author of an undergraduate text in Linear Algebra intended for Canadian universities would dare to precede the notion of linear transformation by the notion of group of transformations. A popular work author, on the other hand, may derive a more elementary notion from a more advanced notion because the reader is not expected to have learned the more advanced notion to the point of being able to apply it as part of a technique in solving problems.

These examples show that sentence-by-sentence discourse analysis of popular works in mathematics is not enough to identify the characteristics of this genre and that a more global, structural view is needed.

Seeking a framework to organize an analysis of popular texts or lectures from the point of view of the means mathematics popularizers use to convey their message, I turned to several sources:

- Analyses of popular texts in scientific fields, such as medicine (Nwogu, 1991), biology and nanotechnology (Miller, 1998; Beacco, Claudel, Doury, Petit & Reboul-Touré, 2002; Knudsen, 2003; Calsamiglia & van Dijk, 2004), physics (Leane, 2007), or general science (de Oliveira & Pagano, 2006);
- A framework for analyzing the discourse of mathematical textbooks proposed by Richard & Sierpinska (2004), based on a combination of two functional models of language: Jakobson's model of communication (Jakobson, 1963), and Duval's model of mathematical language (Duval, 1995). I will refer to this framework as

the *Duval-Jakobson framework.*

Frameworks used in analyzing popular science were not sufficient for studying the popularization of mathematics genre, because they were usually restricted to looking at textual (Myers, 2003) and visual (Miller, 1998) aspects, ignoring symbolic notation, which plays a central role in mathematics. Only some general assumptions about the means used in science popularization could apply also to popularization of mathematics. For example, Calsamiglia & van Dijk (2004) recommended that analyses of popular texts should take into account not only the internal features of the texts or lectures, but also their social/institutional and cognitive contexts. With regard to the cognitive context, the authors were offering distinctions between different kinds of knowledge (e.g. episodic knowledge, abstract knowledge) and strategies of knowledge management to be used as analytic tools for analyzing popularization. I have devoted a special chapter in this thesis to the institutional context of popularization. The cognitive aspects will receive more attention in the chapter on the audience's reactions to lectures.

The present chapter will be devoted to the internal structure of popular texts or talks. For this aspect of popularization (in relation to science popularization) Calsamiglia & van Dijk (ibid., pp. 372-373) were offering such analytic categories of description as, *introduction*, *explanation*, *clarification*, and filling the knowledge-gap through *paraphrasing*. They classified explanations into *denomination*, *definition/description*, *exemplification*, *generalization* and *analogies* (comparisons and metaphors). These very general categories are not sufficient to capture the specific characteristics of a popular text or lecture in mathematics.

This is why I sought some inspiration in the second of the above-mentioned frameworks. I present this framework in the next section (Section 5.2). In Section 5.3, I show how I interpret the proposed categories in the context of popularization of mathematics. In the remaining part of the chapter I will apply the framework to analyze the mathematical discourse in the case of written (Section 5.4) and oral (Section 5.5) communication of mathematical ideas. In section 5.4, I will compare the discourse in a research paper and in a more popular text, while in Section 5.5 I will present and compare the discursive means of communication in the two popular lectures I use as a constant example in the thesis.

5.2 THE DUVAL-JAKOBSON FRAMEWORK FOR STUDYING MATHEMATICAL TEXTS

Richard and Sierpinska (2004) proposed a framework for analyzing mathematical texts that combines Duval's framework designed for this purpose (Duval, 1995) and Jakobson's model of communication (Jakobson, 1963). Jakobson's model was added because Duval's was not sufficiently detailed with regard to the communicative functions of language.

Duval distinguished

- discursive,
- meta-discursive, and
- non-discursive uses of language.

The *discursive function* refers to four uses of language:

- referential function (identifying and naming objects);
- apophantic function (making statements about these objects);
- discursive elaboration function (linking those statements into a coherent whole),

and

- discursive reflectivity function (signaling the value, the mode, or the status of the statements).

The *meta-discursive functions* include

- communication,
- objectivation, and
- processing.

Communication aims at establishing a rapport between the interlocutors, maintaining and controlling it, and clarifying the meaning of messages. The aim of objectivation, on the other hand, for the individual engaged in it, is to represent his or her ideas or solutions to problems in a certain symbolic system to depersonalize them and make them more accessible for analysis and tests of consistency. This is what a mathematician does in writing a proof of a theorem he or she has discovered: the aim is not to communicate to others the cognitive process of discovery but to turn its result into an object – whence the term 'objectivation' – to be studied from the point of view of mathematical rules and meanings. Processing refers to transformations of expressions using conventions and rules of manipulation such as those found in grammar or algebra.

The term *non-discursive functions* refers to such uses of language as

- editorial organization,
- synoptic function, and
- graphical representations.

Structuring a text using chapters and sections marked by headings of different levels are ways of using language for the purposes of editorial organization.

Numbering sections in a hierarchical way (e.g. 1, 1.1, 1.2, 2, 2.1, etc.) serves the synoptic function; it facilitates the grasp of the overall organization of the whole text.

Graphical representations such as diagrams, charts, icons, graphical symbols that abound in mathematical texts do not produce a coherent discourse on their own, but they are meaningful in their function of representing objects and ideas discussed in the accompanying text.

Duval was not interested in the study of the function of communication; he focused more on objectivation. Jakobson, on the contrary, was interested in modeling communication, and he proposed several categories of the functions of language used in *communication*, namely,

- poetic (making the message more pleasant to the listener or reader),
- conative (producing a change in the receiver's attitudes or actions) ,
- phatic (maintaining contact with the receiver), and
- metalinguistic (making statements about the form of one's utterance)

functions.

In the next section, I will explain in more detail how I understand the communication functions, and all other functions of language in the context of popularization of mathematics. Before I go into that, however, let me clarify what I mean by "language".

In Duval's work, "language" has precise technical meaning. In fact, he distinguishes between the meanings of the French words *langue* and *langage*. The former refers to semiotic systems that make possible the realization of all the four discursive functions. Natural languages such as English or French are "langues".

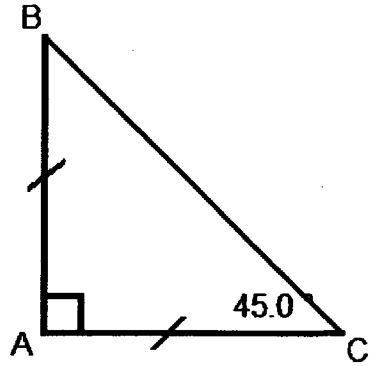

Fig. 5.1 Diagrams used in geometry do not constitute a "langue"

Diagrams used in geometry can be used to refer (e.g. the diagram in Fig. 5.1 refers to a triangle); to make statements (the diagram shows for example that the angle BAC is a right angle), and to signal the status of these statements (the diagram suggests that the statement that the triangle is right-angled is true). It is not possible, however, to decide based on the diagram whether the statement that the angle BAC is a right angle is an assumption or has been deduced from other statements (proved or assumed as true) such as that angle BCA measures 45 degrees and the triangle is isosceles. Diagrams, on their own, do not allow us to "elaborate" or link statements into a coherent whole. Therefore, they do not constitute a "langue" (Richard & Sierpinska, 2004). "Langage", on the other hand, encompasses all sign systems that make it possible to refer and to mean, including not only the "langues" but also such convention-based means of expression as diagrams, drawings, pictures, font color schemes, text bordering and highlighting, sounds and gestures. It is in this larger sense of *langage* that I use the English word "language" here.

5.3 INTERPRETATION OF THE DUVAL-JAKOBSON FRAMEWORK IN THE CONTEXT OF POPULARIZATION OF MATHEMATICS

In this section, I will interpret the meta-discursive, discursive and non-discursive functions of language of the Duval-Jakobson framework in the context of popularization of mathematics.

5.3.1 The meta-discursive functions

I will discuss, in turn, the functions of communication, objectivation and processing in popularization of mathematics.

5.3.1.1 Communication

Jakobson distinguished the poetic, the conative, the phatic and the meta-linguistic functions of language in communication. How do popularizers use these functions to convey their message?

5.3.1.1.1 The poetic function

The *poetic* function refers to the use of means that make a talk or a text pleasant to listen to or read: an appropriate length and rhythm of the sentences; a melodious or harmonious way of linking the words; the aesthetic value of visual illustrations. These aspects are taken into account in advice given to science popularizers; it is recommended that they avoid long words and sentences for addressing the general audience (Malavoy, 1999). Malavoy also stressed the importance, in popularization, of using varied punctuation and vocabulary, and enlivening the text or talk with metaphors, word play and verbal puns. These are also instances of the poetic functions of language in communication. Such means, used in the title, are likely to attract an audience, and maintain the audience's attention if used during a talk or within a text. In popularization of mathematics, word play is quite common, although some authors (e.g. Roger Penrose, Constance Reid) use it less often than others (particularly Ivars Peterson). Peterson's "The Mathematical Tourist" is full of word play: the chapter on number theory is titled "Prime pursuits" and one of its subtitles is, "Breaking up is hard to do" (the section is about factoring).

Keeping up with the poetic principle in popularization of mathematics is a challenge whenever formulas and symbolic manipulations come into play. Should they be presented at all, and if yes, how? Clearly, embedding mathematical formulas "inline" within the text may be considered as violating the poetic principle, because they inevitably interrupt the flow of reading the text. Replacing formulas by narrative explanations or mixed representations (Pimm, 1987), or inserting formulas in the "display" mode (in a separate paragraph or in the margin) might reduce these interruptions.

5.3.1.1.2 The conative function

The *conative* function refers to expressions of the communicator's intentions regarding the effects of his or her message on the audience (e.g. to provoke an attitude, a feeling towards a thing or a person, or cause the audience to do certain things). Popularizers of mathematics do not have to engage the audience in mathematical activity – as teachers are obliged to do – but they want the audience to develop at least an interest in mathematics and a positive attitude towards it. How do they do it? Using lively metaphors, emphasizing the importance and excitement related to the discovery of a result, relating anecdotes about private lives of mathematicians and particularly their "endearing foibles", or making the story of a mathematical discovery read like a thriller. A good example of the last method is the first paragraph of the section "Breaking up is hard to do" in Peterson's "The Mathematical Tourist":

> Several decades ago, an interest in factoring was the mark of a mathematical eccentric. A small, unheralded group of mathematicians worked quietly, prying open large composite numbers to unlock their prime secrets. They reveled in the pure delight of calculation and in the immense pleasure of devising elegant algorithms to do their work. Like many ardent hunters, they even kept lists of 'wanted' and 'most wanted' targets. (Peterson, 1988: 43)

Let us note, in the quote above, the expression, "the *pure delight* of calculation". "Pure delight" is not a very common qualifier of "calculation" in everyday conversations. Much more often, we hear and speak about "*tedious* calculations" or about the "*drudgery* of calculations". Obviously, Peterson was trying to break the association, in his readers' minds, of calculation with boredom and hard work, and replace it with "delight", the excitement of "unlocking secrets" or "ardent hunting".

While conative function plays an important role in teaching, teaching involves a different kind of mathematical communication than popularization and the two differ mostly in their conative aspects. Popularizers want their audiences to change their images or attitudes towards mathematics, but rarely intend to induce actions. Contrary to students of mathematics in compulsory education, participants in popularization are not forced to do anything by institutional rules. Moreover, if some activities are included in a popularization event, they are normally quite different from those in a regular mathematics course. It is hard to imagine, for example, that an instructor of a calculus course would assign the same set of problems as Penrose (2004) proposed to his readers in Chapter 6 of *The Road to Reality*. Besides the fact that Penrose's problems generally asked for knowledge not explicitly taught or even mentioned in the previous chapters in the book, the problems usually asked for proofs and not for a straightforward application of a computational technique, as would be the case for most exercises in a calculus course.

5.3.1.1.3 The phatic function

Staying in touch with the audience or the readers is very important in popularization. Otherwise, they might just walk out or leave reading. Unlike taking a course, participation in popularization is optional. Therefore, engaging audience in active participation is recommended in guidelines for popularizers. This was even the central point in the so-called "engagement model of science communication", presented in Chapter 2.

Popularizers use a variety of means not to lose their audience. They try to create a bond with the participants. For example, they seek feedback by using humor. If the audience is laughing, it is already a sign that they are with the speaker, and that they have understood something. They may also address the audience directly, ask questions, invite participants to ask questions, offer opinions or share their experience with mathematics. This technique is used not only in talks but also in written work. For example, Stewart's "Letters to a young mathematician"[36] has been written as a collection of the author's responses to questions in letters from "Meg", who, at the beginning of the correspondence, is a finishing secondary school student considering, but not being sure

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[36] Stewart, I. (2006). *Letters to a Young Mathematician*. New York: Basic Books.

about, studying mathematics at the university.

Another way of creating a bond with the audience is by saying, "I am like you". For example – "Like you, I am also not a mathematician, and found mathematics difficult, but I was able to understand something about mathematics, and if I could understand, so can you". This is the technique used by Peterson:

> Years ago, as a University of Toronto undergraduate majoring in physics and chemistry (with a heavy dose of mathematics on the side), there were times when I felt I had dropped down a rabbit hole into a bewildering land. More than once, I remember sitting in cavernous lecture halls, surrounded by dozing or fidgeting classmates, trying to figure out what was going on. The lecturer would be scrawling equation after equation across the blackboard and speaking in a puzzling language that sounded like English but somehow wasn't. Although I could pin a meaning to practically every word, I didn't seem to understand anything he said. I was as lost as an accidental tourist wandering in a very foreign country. (Peterson, 1988: xv).

The above quote is the first paragraph of the *Preface* to Peterson's book. Creating a bond with the readers was the first thing this author wanted to ensure.

5.3.1.1.4 The metalinguistic function

This function refers to saying something and then reflecting on what one has said, for example, by paraphrasing it, saying it in different words to make it simpler, clearer (e.g. "What this theory really means is …"). Reformulation is a meta-linguistic technique frequently applied in scholarly texts. Finding a different, simpler representation of a mathematical concept can even be an important result in mathematics. This is certainly the case of matrix representations of linear transformations. In the section on linear transformations in *The Road to Reality* (section 13.3), Penrose puts a lot of stress on paraphrasing the statement, "A linear transformation preserves the vector-space structure of the space on which it acts". The seven pages of this section contain at least sixteen different ways of describing linear transformations. There are structural-algebraic descriptions (preservation of the vector-space structure), geometric descriptions (preservation of straightness), and analytic descriptions ("each new coordinate is expressed as a linear combination of the original ones, i.e. by a separate expression like $\alpha x + \beta y + \gamma z$, where $\alpha, \beta, and\, \gamma$ are constant numbers", Penrose, 2004: 256). There are descriptions using index notation, matrices, diagrammatic notation, and others. In a Linear Algebra textbook, not all of these reformulations would appear and they would certainly not appear in a single section. Having so many of them in a popular book is a sign of the use of the meta-linguistic function of language in a communicative intention.

The meta-linguistic function has received a lot of attention in studies of science popularization: the role of reformulation in expository texts including popular science was studied, in particular, in Ciapuscio (1997; 2003), and Bach (2001a; 2001b).

5.3.1.2 Objectivation

In Sierpinska's words (2005: 222), "objectivation is the use of language in the aim of obtaining some control over one's activity and one's experience, whether physical or mental. Objectivation organizes and reorganizes one's activity and experience and makes it the object of conscious evaluation and decision." Thus, its primary concern is not communication, which is the main aim of popularization. In fact, the main difference between scholarly mathematical texts and popular works in mathematics lies in the different meta-discursive functions they intend to fulfill. This difference is clear when we compare a scholarly and a popular paper on the same mathematical topic. An example is provided later in this chapter, in the comparison of two papers, a research journal publication by Rubinstein & Sarnak (1994) in *Experimental Mathematics*, and a paper intended for the broader audience of *The American Mathematical Monthly*, by Granville & Martin (2006), on the same result in Number Theory, related to prime numbers. The former was presenting a new mathematical result, and it is clear from the analysis of its discourse that the authors wrote it primarily to organize and verify this result. The authors of the *Monthly* article are trying to explain this result – which is no longer new – to a broader audience. There is an emphasis on motivating examples, illustrations and reformulations.

5.3.1.3 Processing

Formalized processing of expressions written using an operational symbolism plays an important role in modern mathematics. It was not always so, however, as the history of pre-Viète mathematics shows. There are also differences in emphasis on processing even across mathematical domains. Moreover, mathematical domains differ in the formal notational systems they use. Therefore, whenever a speaker is invited to give a plenary talk to a larger audience, even if all members of the audience are mathematicians, he or she is asked to avoid the "technicalities" (which includes formal processing of expressions) and use a convincing narrative argument in natural language or visual representations to convey the main ideas. In popularization of mathematics, the seriousness of abiding by this recommendation ranges from complete avoidance of processing (as in, e.g. Stewart's *Letters to a Young Mathematician*) to unabashed use of it (as in Penrose's *The Road to Reality*). Penrose appears to have had to fight with reviewers and publishers for his right to use mathematical formalism, but he remained unconvinced by their arguments, because he did not believe that what he had to say could be conveyed without a minimum of mathematical notations:

> The reader will find that in this book I have not shied away from presenting mathematical formulae, despite dire warnings of the severe reduction in readership that this will entail. I have thought seriously about this question and have come to the conclusion that what I have to say cannot reasonably be conveyed without a certain amount of mathematical notation and the exploration of the genuine mathematical concepts. The understanding that we have of the principles that actually underlie the behavior of our physical world indeed depends upon some appreciation of mathematics. (Penrose, 2004: xv)

The use of processing in Penrose's book has apparently not deterred the readers, if we judge by the high number of reviews of the book on Amazon.com (165) and the even higher number of people who bothered to evaluate these reviews. The reviews, while praising the scope and depth of the contents of the book, warn the readers, however, that the book is "not for the faint of heart" and that "this is a very, very heavyweight book for non-mathematicians". The reviewer who was writing the quoted statements, in fact, contrasts this book with what he calls the "more 'pop' science" books:

> The first half of this extremely challenging book takes the reader through huge swathes of mathematical territory – hyperbolic geometry, complex numbers, complex calculus, Riemann surfaces, n-manifolds and many more topics are covered. These chapters don't just convey a general impression of each subject in layman's English, but make heavy use of formulae and mathematical notation, effectively letting the math do the talking where a more 'pop' science book would be breaking out the strained analogies. Although Penrose takes care to provide the reader with all groundwork necessary to understanding these subjects, this is still fundamentally difficult and unintuitive stuff and non-mathematicians will find that each page requires heavy concentration; skipping or skimming any part of these chapters renders later chapters unintelligible. Still, careful reading reaps huge rewards – the ideas these chapters cover are deep and beautiful. (Reviewer "MikeF"; review written in 2004; excerpt downloaded from Amazon.com website June 20, 2009).

This reviewer suggests that heavy use of processing in a text may effectively make it unfit for the "popular works in mathematics" category.

5.3.2 The discursive functions

In this section, I will look at the four discursive functions of language – referential, apophantic, discursive elaboration and discursive reflectivity – in relation to popularization of mathematics.

5.3.2.1 The referential function

Language used in the referential function is meant to identify an object and constrain the freedom of its interpretation. Tools used to fulfill this function include denomination (naming the object), and description using analogies and metaphors (Calsamiglia & van Dijk, 2004). The role of metaphors in the referential function of language is extensively studied by linguists and cognitive scientists (e.g. Lakoff & Johnson, 1980), sometimes focusing on mathematical metaphors (Lakoff & Núñez, 2000).

The use of metaphors was emphasized in learning (Muscari, 1988; Duit, 1991; Ortony, 1993; Brown, 2003) popularization of science (Knudsen, 2003), as well as in mathematics education (e.g., Pimm, 1981, 1987, 1991; Sierpinska, 1994: 92-111; 122-3; Presmeg, 1998; Lakoff and Núñez, 2000). The use of metaphors by mathematicians was addressed also in some studies (Sfard, 1994; Burton, 1998; Manin, 2007). Metaphors became an area of investigation also from the point of view of education in general. Some authors (e.g. Tourangeau & Sternberg, 1982; see also Leino and Drakenberg, 1993) consider metaphors as a way of relating new knowledge to old knowledge; the metaphor correlates concepts of two different domains to which they belong. In the light of intercultural communication, the domains can be interpreted in terms of cultures (or subcultures). Thus, metaphors are constructs that make it possible to relate objects that belong to the same or to different cultures. However, the relation is made not only between the objects themselves, but it also affects their "environments", images closely related to the objects in the given culture. Thus, a metaphor might be interpreted differently according to different cultural lenses.

In linguistics (e.g., Richards, 1936), metaphor is a figure of speech where one thing or idea (called the *tenor*) is described with words normally used for something else (called the *vehicle* of the metaphor). Similarities between the tenor and the vehicle are called the *ground* of the metaphor, while differences between them are referred to as the *tension*. For example, if the statement, "medial axis of a shape is its skeleton" is treated as a metaphor to explain what the notion of "medial axis" means to people who are not familiar with the technical vocabulary of image processing but are familiar with the anatomical notion of skeleton, then "medial axis" is the tenor and "skeleton" – the vehicle. This metaphor was used in one of the lectures I will be analyzing later on in the chapter.

In scholarly mathematical literature, the referential function is fulfilled mostly by definitions. These definitions are not easy to understand for non-mathematicians because they usually use already defined technical terms and specialized notations. Therefore, popularizers have to use other means to name and describe mathematical concepts. Analogies with the familiar are common, but mathematicians are more curious about the counter-intuitive than the intuitive and so *contrasting* newly discovered (or invented) mathematical objects with familiar ones is often more useful. This is the technique that Peterson used in introducing the concept of fractal dimension. After having introduced the word "fractal" and derived some of its meaning from its Latin etymology, he proceeded to describe the contrasts between fractals and the familiar school geometry objects such as spheres, triangles and lines:

> Fractals turn out to have some surprising features, especially in contrast to such geometric shapes as spheres, triangles and lines. In the world of classical geometry, objects have a dimension expressed by a whole number.... Fractal curves can wiggle so much that they fall into the gap between two standard dimensions. Indeed, they can have a dimension anywhere between one and two, depending on how much they meander.... (Peterson, 1988: 119)

In analyzing the "Medial representation" and "Escher" lectures, I have noticed yet another technique of referring and meaning, that I will call the "hook technique". In each lecture, there was some kind of central symbol intended to represent (signify, refer to) the main idea the lecturer wanted to convey in the lecture. In the first lecture, it was the picture of the panther. In the second – the picture on the Droste cocoa box. This central symbol was in each case visually attractive, vivid, and therefore likely to attract people's interest and attention. In this sense, the symbol was a "hook"[37] for the audience: it attracted their attention. Once, however, the lecturer had the audience's attention, he could explain the meaning of the symbol, and refer to it in the rest of the talk by a single word associated with this symbol ("panther"; "Droste"). The way I want to use the word "hook" to refer to this technique could be explained through the metaphor of "fishing": the lecturer is "fishing" for the audience's attention and understanding of his lecture; he or she prepares a "hook" to get this attention and direct it to the intended meaning. The lecturer may have a "central hook" to convey the main idea of the talk, and some additional hooks to convey the meanings of some details. This technique seems to be rather specific to lectures. I will refer to this technique in the rest of the thesis. In particular, I will use this notion in analyzing the audience members' reactions to the

[37] According to Longman's dictionary, one of the meanings of "hook" as a noun is "something that is attractive and gets people's interest and attention". An example of the use of the word in this sense is, "You need a bit of a hook to get people to come to the theatre".

lectures and, in this context, I will be saying that the audience members "get hooked" set by the lecturer (or not).

5.3.2.2 The apophantic function

In scholarly mathematical texts, the apophantic function (saying something about objects named using the referential function) is most seriously played in the formulation of theorems. In popularization of mathematics, the definition-theorem-proof style of exposition is not acceptable and it is not practiced. If stated at all, theorems are conveyed informally, as "facts", and they are rephrased in many ways, as well as represented using pictures, diagrams, animations and only rarely – formulae. For example, Stewart (1996) states not only the theorem but also its consequences as follows:

> To see the way in which topological insight can illuminate a wide variety of apparently unrelated things, consider one of its minor triumphs, which says (in suitably refined language) that you can't comb a hairy ball to make it smooth everywhere. Here are some direct consequences:
>
> (1) At any moment, there is at least one point on the Earth's surface where the horizontal wind speed is zero. (If not, comb along the wind.)
>
> (2) A spherical magnetic bottle, as used in 1950s fusion reactor experiments, must leak. (It leaks where lines of magnetic force run perpendicular to the surface of the bottle, i.e. where the horizontal component is zero.)
>
> (3) Dogs, bears, cheetahs, and mice must have a parting in their fur. (Self-explanatory.)
>
> (4) Every polynomial equation has a complex root. (Technical proof based on the fact that the complex numbers, plus a point at infinity, form a sphere.)
>
> (5) A chemical gradient on the surface of a cell must have a critical point (where there is no local 'downhill' direction). (Stewart, 1996: 213)

It is important to note that diagrams are frequently used in scholarly papers as well. However, their role is different in the scholarly and popular works. In relation to popularization of science, Miller (1998) noted that graphical representations that are integral part of scientific research papers – as part of empirical evidence, for example, or as elements of formal diagrammatic notation – may appear only as loose illustrations in popularization of science. The role of graphical representations will be discussed in detail as part of the non-discursive functions of language.

5.3.2.3 The discursive elaboration function

Proofs are examples of the application of the discursive elaboration function in scholarly mathematical texts. One could perhaps risk saying that they are the only part of a scholarly paper where the exercise of the discursive elaboration function is "compulsory". Longer "discourses" in such papers are usually proofs. Not so in popular texts. Popularizers are advised to organize their texts or lectures in a special way (Malavoy, 1999). Proofs are rare in popular works in mathematics, yet these works are full of long discourses. Popularizers must be able to produce mathematical discourses that are not proofs. They must have a literary or oratory talent on top of a good understanding of mathematics to communicate it to non-mathematical audience.

What are these long discourses about in popular talks or writing if they are not or cannot be proofs? I have addressed this question in a previous chapter. Here, let me just stress again that, in contrast to the scholar's concern with consistency and truth, particular to objectivation, the popularizer must try to convey meanings and values of the mathematical culture to an audience of strangers to this culture. The question, for the audience, is not why something is true, but what is its historical significance and its meaning in present day applications of mathematics.

5.3.2.4 The discursive reflectivity function

In using this function, popularizers express their emotions and attitudes towards the subject of their talk or writing. This function can be exercised by means of directly telling the audience how the author thinks and feels about the subject, but also by indirectly signaling the value (the significance or importance), the mode (affirmative, interrogative or imperative), the status (whether it is a truth or a conjecture) of what is being said or written. In particular, exclamations such as "Oh!", "Phew!", "Arghh!" are used to express emotions such as surprise, relief and anger, respectively. Ways of conveying the status of truth or conjecture probably do not differ between scholarly and popular works, but scholarly work is certainly much more restrained in expressing emotions. Authors of mathematical textbooks are also constrained in their freedom to present their opinions and feelings. Penrose (2004), on the other hand, devotes a lot of space in his book to presenting his original philosophical views about mathematics and physics and claims that mathematicians and physicists, who are already familiar with the theories, will be interested mainly in this layer of the book.

In science communication studies, the author's sharing his or her reflections on the truth status of a statement (to what extent a statement can be true) is often referred to as *hedging*. Varttala (1998, 2001) found that hedging is extensively present in both scientific and popular texts on science, but there is a difference in the purpose for which this reflection is offered. In scholarly papers, hedging is intended to assure precision; in popularization, it is used for valuation purposes, since the audience is usually not in the position of being able to judge the truth-value of the popularizer's statements.

5.3.3 The non-discursive functions

In this section, I will briefly look at aspects such as editorial organization, synoptic means and graphical representations in popular works in mathematics.

5.3.3.1 Editorial organization

Editorial organization uses a variety of means. Structuring a text by titles and subtitles (verbal means); using nonverbal means such as borders, shading and color; distinguishing the significance of some elements using footnotes, different fonts, etc.; organizing information in tables or diagrams, are some of them. In popularization, editorial organization has an important role to play not only in structuring a text but also in capturing attention and maintaining interest. The synoptic function facilitates the grasp of the whole, and graphical representations assist in both the editorial organization and in clarifying the meaning.

It seems reasonable to assume that, in a popular text, sections should not be too long, the text on a page not too dense (the text should be broken into short paragraphs and illustrated with visual matter) and titles must be clearly marked and captivating. Interrupting the flow of reading by reference to previously introduced definitions and literature sources – common and, in fact, required in scholarly texts – should be used very sparingly in a popular text.

A look at some popular texts in mathematics reveals, however, that these apparently reasonable rules are not always respected. Peterson's *The Mathematical Tourist* is perhaps the closest to this editorial ideal. Paulos' *A Mathematician Reads the Newspaper* has short chapters, but the text is pretty dense and there are not many graphical illustrations. In Penrose's *The Road to Reality*, the text has been cut into 34 chapters, each with five to eighteen sections, but the book has over a thousand pages. There are, indeed, many illustrations, but the text on many pages (sometimes in a row) is extremely dense. References are used, but only in the form of endnotes, without interrupting the flow of the text.

5.3.3.2 Synoptic function

The function helps to grasp the overall concept of the text by certain global organizational method (like numbering the sections, etc.). Since popularization is often organized in a linear way (Miller, 1998), and the texts are usually not intended as a later reference (unlike research monographs, encyclopedias, etc.), the role of synoptic function is generally not significant. The reader, however, needs some "road posts" to keep track of where he is in a book or paper, and authors use language in the synoptic function to assist the reader in that. If the book is a collection of texts that can be read in whatever order, a table of contents with the titles of the texts, and starting each text on a new page is enough. The texts do not have to be numbered in the table. This is the case, for example, of popular books consisting of a collection of letters (Stewart, 2006), or dialogues (Rényi, 1967). Peterson's *The Mathematical Tourist* could also be regarded as a collection of separate articles on different domains of mathematics, but the author decided to use such synoptic means as a "map of the mathematical landscape" with names of imaginary countries such as "Algebria" or "Topologia" to help the reader orient him or herself in the mathematical tour. The synoptic means in Penrose's (2004) book are no different from those normally used in a textbook or a scholarly monograph. Helping the reader in grasping the structure of this more than a thousand-page long monograph certainly required some careful organization. Penrose not only used detailed hierarchical numbering of the chapters, sections, subsections, etc., but also used cross-referencing between the sections and chapters:

> My hope is that the extensive cross-referencing may sufficiently illuminate unfamiliar notions, so it should be possible to track down needed concepts and notation by turning back to earlier unread sections for clarification. (Penrose, 2004: xix)

References to particular research papers are generally omitted in popular books although some authors do it in the form of endnotes (Penrose, 2004; Stewart, 1996). The detailed bibliographic information is included not necessarily for reference but for further reading on the particular topic even with additional information on how to locate the cited sources.

5.3.3.3 Graphical representations

The function refers to the use of graphs, icons, symbols, animations, etc. in the text. As already mentioned for the apophantic function, graphical representations appear generally as illustrations in popular science texts, contrary to scholarly publications where graphical representations are integral parts of the text. In mathematics, this does not seem to hold as a general rule. Apart from examples where pictures are put as appealing illustrations as it is done, for example, in the color plates in Peterson (1988), they might be inserted into the text and referred to, as well (e.g. Reid, 1992: 157). While in many cases the representations are similar to those used in research papers and teaching (e.g. graphs of functions, drawings of geometric figures or topological objects) one can find also unconventional representations, such as the diagrammatic notation used by Penrose

(2004: 258-262).

In this section, I presented a framework for analyzing the discursive, meta-discursive and non-discursive ways of using language in popularization of mathematics. I will now apply this framework to two sets of data: two popular lectures and two texts on the same mathematical topic, a scholarly one and a popular one.

5.4. COMPARING THE MATHEMATICAL DISCOURSE IN A RESEARCH PAPER AND IN AN EXPOSITORY ARTICLE

In this section, I will apply the framework presented in the previous section for comparing two papers written about the same topic but using a different genre. One is a research paper and the other a popular text.

5.4.1 Choice of the papers

Finding two texts on the same topic but written with different purposes was not easy. I was looking for recent papers but the majority of the popular literature (similarly to regular textbooks and mathematics courses) contains older results. On the other hand, publications addressing new research in mathematics are either too short (short communications in newspapers announcing the results within a few columns, such as recent news in the New York times about symmetry of simple Lie groups[38], or about the gömböc[39], a convex three-dimensional shape with one stable and one unstable equilibrium points, etc.) or too long (books written about the entire background of a

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[38] http://www.nytimes.com/2007/03/20/science/20math.html?scp=1&sq=mathematics+symmetry&st=nyt
[39] http://query.nytimes.com/gst/fullpage.html?res=9B05EFDA1730F93AA35751C1A9619C8B63&scp=1&sq=mathematics+gomboc&st=nyt (Downloaded: July 24, 2009).

theorem or even a whole mathematical area such as Fermat's Last Theorem (Singh, 1997) or the Poincaré conjecture, O'Shea, 2007), and often not associated with a single research paper, which makes the comparison difficult. Moreover, choosing one particular paper from the variety of popularization literature necessarily restricts the observable features. The criteria I used for choosing the texts for the comparison were the following:

- The popular paper should exemplify intercultural communication by addressing an audience different from those of readers of research publication in the given area and so accessible to a non-specialist audience and displaying important features of the mathematical culture.
- The popular paper should contain a recontextualization of what is already contained in the research paper including a detailed description of the results.
- The popular paper should be long enough for such a comparison, but both the research paper, and the popular paper are still reasonably short (the length of an average scholarly paper).
- The popular paper should contain different representations of mathematical language (textual, symbolic, graphic).

Finally, I chose two papers in number theory. One of them, Chebychev's Bias written by M. Rubinstein and P. Sarnak is a recent research paper published in *Experimental Mathematics* in 1994. The other one, Prime Number Races written by A. Granville and G. Martin appeared in *The Mathematical Monthly* in 2006. The latter paper won the Lester R. Ford Award[40] recognizing "articles of expository excellence" among

[40]http://www.maa.org/awards/ford.html (Downloaded: July 22, 2008)

papers published in *The Mathematical Monthly* and in the *Mathematics Magazine*. Based on the Monthly's policy statement the latter one should be appropriate for undergraduates (sometimes even for high-school students), and read by a wide range of mathematicians. Its policy statement claims that:

> The MONTHLY publishes articles, as well as notes and other features, about mathematics and the profession. Its readers span a broad spectrum of mathematical interests, and include professional mathematicians as well as students of mathematics at all collegiate levels. Authors are invited to submit articles and notes that bring interesting mathematical ideas to a wide audience of MONTHLY readers.
>
> The MONTHLY's readers expect a high standard of exposition; they expect articles to inform, stimulate, challenge, enlighten, and even entertain. MONTHLY articles are meant to be read, enjoyed, and discussed, rather than just archived. Articles may be expositions of old or new results, historical or biographical essays, speculations or definitive treatments, broad developments, or explorations of a single application. Novelty and generality are far less important than clarity of exposition and broad appeal. Appropriate figures, diagrams, and photographs are encouraged.
>
> Notes are short, sharply focused, and possibly informal. They are often gems that provide a new proof of an old theorem, a novel presentation of a familiar theme, or a lively discussion of a single issue.[41]

According to the policy statement of *The Mathematical Monthly*, articles published in the journal belong to a special type of popularization, namely popularization among mathematicians. They are supposed to satisfy the criteria of didactic transmission proposed by Moirand (1992) (cited in Beacco et al., 2002: 278) and the phenomenon of the blurred boundaries between popularization and professional literature coming from the fact that scientists themselves constitute a considerable audience of science popularization literature (Myers, 2003: 268).

5.4.2 Some general remarks about the two papers to be compared

The two papers share quite a few characteristics, besides, of course, the fact that both are

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[41] http://www.maa.org/pubs/monthly.html (Downloaded: July 22, 2008)

devoted to the same mathematical problem. Both articles display the historical background of the problem and provide further generalizations. Thus, both present mathematics as a discipline in continuous development and not a dead subject. Both contain symbolic and visual representations. Neither contains problems for the reader to solve.

The way the two papers use language in the metadiscursive, discursive and non-discursive functions is, however, quite different. It will briefly summarize the differences regarding each of the three groups of functions in the next sections. I will pay special attention to differences regarding the communicative function. I will refer to the papers by the initials of the last names of their authors: [RS] for *Chebychev's Bias* and [GM] for *Prime Number Races*.

5.4.3 Comparing the role of metadiscursive functions of language in the two papers

The goals of the two papers are different. While the primary aim of the research article is objectivation, the popular text is intended to communicate the existing result. This difference is visible if we compare the functions of communication and objectivation in the two articles. Although some communicative functions are present also in [RS] they are not made explicit. The use of language in the phatic function is virtually absent from [RS], and the conative function is applied only in a limited way (mainly to induce an action). The communicative function will be discussed in detail in a separate section. Algebraic processing, although present in both papers, plays much more significant role in [RS] where algebraic manipulations give the backbone of the paper, whereas in [GM] they are considerably less frequent and generally followed by narrative explanations as well.

5.4.4 Comparing the role of non-discursive functions of language in the two papers

The organizational features in [RS] support a compact presentation of the topic in itself, while [GM] emphasizes the overall role of the result in mathematics. The relative space occupied by various sections shows this idea clearly: while in [RS] 15 pages of the 20 page-long paper are devoted to the main result and only 5 pages for introduction and generalizations, in [GM] it is the opposite; the main result is presented in only five pages, while the other 26 contain the introduction and generalizations (16 and 10 pages respectively). Besides the difference in the length of the sections, the transition between them is more clear-cut in [RS]. In [RS], the section titles (listed also at the beginning of the paper) refer to the content of a given section explicitly and objectively. Titles of sections in [GM] are more informal (e.g. "So what do we know about the count of primes up to x anyway?") and the transition from one section to another is rather smooth. While the topics of the subsections in [RS] seem more traditional (introduction, results, examples, generalizations), [GM] is organized around the metaphor of prime number races.

While in [RS] there are only two tables to illustrate the corresponding numerical values, [GM] contains eleven tables with the purpose of convincing the reader about the long-term behavior of certain numerical sequences and giving a feeling for the magnitude of the numbers to be dealt with. In [GM], the most important formulae are highlighted using borders. Unlike the general characteristics of popular science texts (Miller, 1998), [GM] contains several footnotes referring to historical information, personal data or references. The frequent use of these footnotes can be explained by the fact that the paper targeted a varied audience and in this way the authors could give additional information for interested readers.

Both papers contained different forms of representations (symbolic, graphical, etc.) which were integral parts of the texts. However, in [RS] there are no pictures, or graphs illustrating the actual content. The graphs and tables contained in the text show the results of the numerical computation; they are not meant to facilitate understanding. The role of graphs and histograms in [GM] is to illustrate the actual concept and foster understanding. They are always followed by explanation and interpretation of the given representation. In the use of graphical representations, [GM] does not follow the traditions in science popularization (Miller, 1998), where visual representations are integral parts of research papers but usually serve only as a loose illustration in popular articles.

The *Monthly* article was considerably less structured, assuming that the readers will read the text linearly. On the other hand, [RS] was heavily cross-referenced, with careful numbering of theorems, formulae and sections. The theorems were often mentioned only in the introduction and not necessarily stated again in the corresponding section, which made the later reference easier but did not facilitate reading.

5.4.5 Comparing the role of discursive functions of language in the two papers

As popularization is meant to "translate" elements of the culture (or subculture) to outsiders, it is not surprising that there is a considerable difference between the uses of discursive functions in the research paper and in the popular text. The referential function used for identifying and naming objects appears mostly in terms of strict definitions in [RS] often clearly indicated beforehand such as “define” or “introduce”. To have a common understanding of the terms used in the research paper, the authors clarify explicitly the objects and notations they intend to use. Hedging or the constraining of the interpretation of a given definition or theorem was done by careful listing of the assumptions, usually without giving any rationale for their necessity. In [GM] the authors use a much broader variety of means for naming objects. The main object, the idea of prime races, is introduced through an elaborated metaphor, a “hook”. Various cases exemplify the phenomenon by serving seemingly different didactical purposes (e.g. mod 4 – historical, mod 3 – simplicity, having only two contestants, mod 10 – corresponds to last digits, *Li(x)* vs. $\pi(x)$ – symmetry), which led to the general formulation of the problem. Definitions were avoided and replaced by mixed notations (e.g. “ #{primes $4n + 1 \leq x$}”); the exact definitions were put in footnotes (p. 1) or defined through analogy (e.g. the Dirichlet L-functions are “defined” only “as relatives of the Riemann zeta-function”, p. 15). The constraints on the definitions were often justified within the text or in the footnotes.

Although [RS] consists of complete sentences containing subject and verb, and the punctuation conform to standard grammatical rules, the sentences often contain mathematical formulae whose length can be up to half a page. In the case of [GM], despite the inserted mathematical formulation, the text is closer to the natural language. The formulae are less numerous, and they are usually shorter. [GM] contains a greater variety of representations, such as formulae, tables, graphs and histograms.

In [RS], the function of discursive elaboration is used mainly in proofs. The particular mathematical objects of study are identified in the introduction by definitions along with their characteristics stated in the form of theorems and the rest of the paper is devoted to proving these theorems. In [GM], on the other hand, the essential content of the paper refers to the implications of the race metaphor and the historical development of the problem elaborated in a narrative form.

Emotive and evaluative functions are rather weak in [RS]. The genre of scientific writing does not provide much scope for revealing personal feelings, and evaluations refer only to restricted objects and are meant to signal their relative importance within the paper (e.g. "it is a crucial point", "this is the key to the proof"). The authors do not evaluate the significance of particular results in mathematics. On the other hand, emotional factors are quite strong in [GM]. This takes the form, for example, of personification of mathematical expressions (e.g. "goes to infinity with great dignity") and expressions of the authors' feelings and judgment toward the subject throughout the paper. The importance of a particular result is often made explicit (e.g. "perhaps the most prominent open problem in mathematics"), and the evaluation refers to the global importance of the statement and signals its value within the mathematical culture. Although both papers contain proofs, the question of rigor and the role of proofs are considerably different in the two articles. In [RS] each statement is proved rigorously, while in [GM] presenting an argument which is convincing is more important than giving a rigorous proof.

5.4.6 Some additional remarks on the use of communicative functions of language in the two papers

The authors of [GM] put a great emphasis on introducing the metaphor of the races. They elaborated on this metaphor using extensively different communicative functions of language (poetic, conative, phatic). While the poetic function in [RS] is restricted to the aesthetic component in the mathematical content, such as searching for a symmetric treatment and presentation, in [GM] it appears in a variety of ways. While in [RS] natural language often serves only to bind the formulas, the text in [GM] itself reads more smoothly and can be considered as a narrative with some inserted formulae, presented regularly through mixed representations (Pimm, 1987) in a form close to the spoken language (e.g. using the #{definition of the set} notation). In [RS], the conative function is used to provoke an action (such as reproducing the missing calculations or look for reference); in [GM] its main function is to induce an attitude. Mathematical objects are presented, in [GM], as an integral part of human culture (e.g. "primes have music in them") and the results are signaled by positive attributes ("extraordinary claim", "amazing result", etc.). The mathematical results in this paper are personified by means of historical information about the mathematicians involved or giving some insight into the authors' personal life and interests at the end of the paper. Even the mathematical content is often approached through conative aspects, for example, listing the numbers in a multiple exponential form makes one feel how big the actual numbers are. Although not always as a form of an explicit command, the authors encourage the reader to perform investigations, such as examining characteristics of numerical results presented in the tables or look for patterns among them (e.g. "Do you see a pattern?"). Nevertheless, these investigations do not require lengthy computations as in [RS].

The use of phatic function of language for addressing the reader and maintaining the communication is rather apparent in [GM]. The reader is often addressed explicitly by "you" form. The style is informal and the text contains a lot of questions not only about the material (e.g. "Do you see a pattern?") but also about the personal thoughts and feelings of the reader as well, (e.g. "If so, you are right to be skeptical", or "You might have grown bored"). On first sight it seems that in [RS] the authors do not use the phatic function to keep contact with the reader (e.g. there are hardly any questions in the text and the verbs are usually in an impersonal form). In fact, however, the reader has to think and work together with the authors to be able to follow the text. In this sense, the connection between the authors and the reader is much stronger than in [GM].

Unlike [RS], the emotive-expressive component in [GM] is dominant throughout the paper. The reader's activity to understand the text is hidden in [RS] while in [GM] it is more apparent. Sáenz Ludlow (2006) states that the "acts of writing [of mathematical papers] are concomitant with acts of reading, listening, interpreting, thinking, and speaking" (p.234). Similarly to writing, reading of [RS] is also a complex process.

While the use of metalinguistic function is not common in [RS], explaining the meaning of terms or reasoning frequently happens in [GM]. The theorems are often restated in a "less mathematical" form introduced by terms such as "may be paraphrased", "in other words", etc. For example, "restate it in entirely elementary language" clarifies the meaning and makes it visible also through the reader's cultural lenses.

Based on the above analysis it can be stated that the written popular discourse bears some special characteristics and is considerably different from research publications. The main goal is communication (in fact intercultural communication) in the first, and objectivation in the second. In the following section, I will investigate how these special elements of the popular discourse were implemented in the oral discourse of the popular lectures I attended.

5.5. Comparing two popular lectures from the point of view of the means used to convey the message

In this section, I will look at the discursive and other means used in the "Medial representation" and "Escher" lectures. I will attempt to identify the main differences between the popular discourses involved in the lectures.

When I asked for permission to audiotape the lectures, the organizers offered to provide me with their own recording instead. The organizers of the first lecture failed, however, to fulfill their promise. During the second lecture, there was a technical glitch and I was able to obtain only a partial recording. Thus, instead of a detailed discursive analysis of the two talks, I was forced to take into account only the more global aspects of the lectures.

5.5.1 The "Medial representation" lecture

I will first describe how the speaker communicated the topic, focusing on the means he used and then I will analyze the lecture in terms of the Duval-Jakobson framework.

5.5.1.1 Description of the first lecture

The lecturer started by motivating the importance of image analysis, illustrating it by the problem of matching objects viewed from different perspectives. He showed a set of contours: three hands in various positions, a hammer and four views of a walking horse (similar to the figure in Siddiqi et al., 2002: 7). The categorization of these shapes would be an easy task for a 6-year-old, but, the lecturer said, it is a highly non-trivial problem for computers. Organizing and classifying two- and three-dimensional objects is very difficult, since, due to the fact that these objects undergo various transformations, or are seen from various perspectives, the actual images of a particular object can be considerably different. The idea that the lecturer was trying to convey was that the task of recognizing and distinguishing objects could be facilitated if the objects were represented by simpler drawings: abstracting from their "flesh" and taking into account only their "skeletons". He drew an analogy with drawings that young children make of people, representing their hands by thin "sticks". He pointed to the curves drawn inside the pictures of the hands, the hammer and the horses shown in the figure mentioned above.

The first time the lecturer used the word "skeleton", it was a metaphor; its vehicle was the anatomical sense of the word. In the course of the lecture, however, the lecturer was progressively narrowing down (or "hedging") the meaning of the word "skeleton". Already the curves inside the pictures of the hands, hammer and horses were indicating that he did not mean the skeleton in the anatomical sense. Ultimately, the meaning of this word was conveyed by the impressive colorful and dynamic figure of the panther

(Dimitrov et al., 2003)[42], shown at the peak of the lecture, as the "central hook", representing a method of obtaining the skeleton – in this technical sense – of a shape. This method was intended as the main point of the lecture. The lecturer wanted the audience to associate the word "skeleton" with the process of skeletonization[43] represented in the panther figure and, during the lecture, often referred to it, just using the word "panther" (e.g. "as we did it for the panther").

Analogies with other familiar objects or situations and their visual representations were used throughout the lecture. In particular, methods of finding the skeleton of a shape were described using both the grass fire analogy and as a set of centers of inscribed balls. The local medial geometry was illustrated by the rowboat analogy. These were the other, less central, "hooks" of the lecture. The lecturer did not have to invent these analogies and pictures for the purposes of the popular lecture; they were already standard in scholarly literature.

To demonstrate the technique for computing medial loci, the lecturer used not only the animated panther figure with appropriate coloring, and other visualizations, but also presented the mathematical background of the problem, including the formalization of the average outward flux method and the corresponding differential equation, and sketched the algorithms written to determine the medial loci. These were his hooks intended for the more expert members of the audience.

Applications of the method, such as medical applications in brain imagery, or interpreting camera images of blood vessels or intestines taken from the inside, and the

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[42] The color picture is available online at http://www.cim.mcgill.ca/~pdimit/cvpr03-poster.pdf (Downloaded: June 27, 2009)

[43] "Skeletonization" is another technical term in the domain, but it was not used in the lecture.

more playful application to finding the "skeleton" of a picture of the Venus of Milo sculpture displayed at Louvre, were other examples of the lecturer's hook technique.

The empirical evaluation of medial representations by retrieving 3D models was also presented during the talk, as an illustration of the scientific method of hypothesis testing.

5.5.1.2 Analysis of the first lecture

As mentioned in the description above, the lecturer's main discursive means for conveying his message were metaphors and analogies, and also technical terminology. The main concept was that of a shape's skeleton, and this technical word was first used as a metaphor, based on the similarity of its graphical representations with skeletons in the anatomical sense. The similarity is in the idea of a "central line in the middle". This is a rather loose resemblance, because the anatomical skeleton might be located differently, it consists of more bones, etc. Moreover, the technique applied to determine the skeleton of the panther produced additional points located outside of the shape, which is not conceivable for the anatomical skeleton. The anatomical skeleton can be obtained by means of an X-ray of an actual live animal. The technique presented in the lecture, however, was applied to an iconic representation of the animal, a very simplified drawing of a panther. The image of the panther was chosen because the shape was complicated enough to illustrate how the technique could be applied in complex situations and therefore it was representative of the general case. Examples of skeletons of simple objects such as circles would not be able to play this role.

"Skeleton" was only one among the many technical terms used in the lecture. It was the only one that could also function as a metaphor accessible to lay audience. Other terms, like "medial axis", "Blum's loci", "Blum's medial axis", were very remote from everyday experience, yet played an essential role in the lecture.

The lecturer used a variety of means to convey the meaning of the concepts: natural language, differential equations, static diagrams as well as animations and color. The shape of the panther was animated to shrink to the skeleton and the change of colors indicated the distance.

The lecturer succeeded in presenting a range of cultural values in the talk. His talk, however, had few features of "a tour for mathematical tourists". It was no different from a talk in a more general mathematical conference (such as a conference of a mathematical society). Expecting to have different professionals among the audience members (mathematicians, computer scientists, psychologists, medical workers), the lecture was intended to "sell" the results to people with various backgrounds, showing them the possibility of applications of the results in their domains. From this point of view, we could say that the lecturer engaged in multicultural communication. The focus, however, was on specific research results and the more general aspects of mathematical culture received no explicit attention at all. The lecturer adjusted his talk almost exclusively to the cultural lenses of other professional scientists possibly working in a field where the presented results could be applicable. Moreover, the translation provided by the lecturer referred only to limited professional cultures, particularly mathematics, computer science, and medicine. The content and the organization of the presented material were very similar to that in research publications.

The communicational functions of the message generally focused on the explicit desire to convince the audience of the usefulness and applicability of the research results. The speaker presented the results as a scholar, and the function of objectivation dominated over the communicative function. The poetic function was practically absent from the lecture. The slides and the verbal presentation in the lecture were intended to conform to the professional rather than to the aesthetic standards. The phatic function was generally not emphasized: the lecturer rarely if at all made eye contact with the audience, and did not seek feedback through questions or humorous remarks. With regard to the conative function, the lecturer was trying, using logical reasoning and examples, to convince the participants about the scientific and mathematical truth and usefulness of the results, but did not seem concerned about changing their attitudes towards mathematics. It was as if he took for granted that those who came to his lecture already had a positive attitude towards mathematics.

The title of the talk, *Medial Representations: Mathematics, Algorithms and Applications*, referred to the general organization of the lecture into three sections, presenting the mathematical background, the corresponding algorithms and the possible applications. The organization of the lecture was generally in accordance with the organization of research publications. Graphical representations were applied extensively in the presentation for introducing the concepts, justifying the statements, illustrating the applications and for convincing the audience about the advantages of the presented technique compared to previous ideas. The illustrations shown in the talk were identical to those used in research publications, although they seemed to be intended to serve as convincing visual explanations, not necessarily as a way for presenting the exact results.

5.5.2 The "Escher" lecture

This section is organized similarly to the one on the first lecture.

5.5.2.1 Description of the second lecture

Although the mathematical content of the presentation was based mainly on the paper by de Smit and Lenstra (2003), the organization of the lecture was considerably different from that of the paper. The lecturer first introduced some information about the artist's life as well as about the history of the creation of the lithograph.

The "central hook" of the lecture was the picture on the *Droste* cocoa box, with the term "Droste effect". It was meant to convey the main idea of infinite iteration of a mapping. In this case, the mapping is quite simple: scaling. The Droste effect was then shown on the picture of a ship on the sea with, on the board, a swimming pool with a toy ship with a swimming pool floating in the middle, etc. This was one of the additional hooks in the lecture. Gradually, the lecturer was showing examples of more complicated mappings and the effects of their iteration. The first complication was to combine scaling with a rotation; this was shown on the example of the picture on the circular box of the "La vache qui rit" cheese spread. The cow on the box has earrings that are boxes of the "La vache qui rit" cheese spread. Finally, the lecturer arrived at explaining the method used by mathematicians to discover the mapping used in the Print Gallery picture.

He presented some of the sketches Escher did in preparation for the picture, and emphasized the surprising mathematical sophistication of the graphics in spite of Escher's lack of formal training in the subject. The mathematical formalization used for the construction of the conformal grid was not avoided in the lecture, and was presented in a similar way as in de Smit and Lenstra (2003). It was, however, only very briefly "flashed" in the lecture, with a focus on the numerical values in the formulas, such as the scaling factor. The fact that the mapping was defined over the complex plane was briefly mentioned. Much more time was spent on the visual representation of the conformal map in the particular case of the lithograph. The lecturer referred the audience to the website[44] "Escher and the Droste effect" for additional details.

The lecture focused mostly on the effects of various mappings when applied to concrete grids and pictures. The effects were shown in a dynamic way, using animations, like on the *Escher and the Droste effect* website, and described using metaphorical language. In particular, it was shown in detail how the mappings that lead to pictures like those on the *Droste* cocoa box must be changed to obtain mappings leading to a picture like the *Print Gallery*. Several examples were given of both the "Droste effect" and the "Print Gallery effect".

In addition to the animated presentation of the method used for filling the hole in the *Print Gallery* picture, the technique (pull-back – scaling – push forward) was applied to other pictures. In particular, at the end of the lecture, a visualization of one of Van-Gogh's paintings distorted by the Riemann zeta-function was shown.

5.5.2.2 Analysis of the second lecture

Similarly to the lecture on medial representations, the message of the Escher lecture was communicated through visual means. Metaphors and dynamic visualization rather than definitions, theorems and proofs were the main vehicles for discursive functions. The

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[44] http://escherdroste.math.leidenuniv.nl (Viewed: July 25, 2009)

idea of conformal mapping was conveyed by means of visual representations of the effects of concrete examples of such mappings. Iteration of such mappings was conveyed by the familiar action of repetition, using animations. "Distortion of a picture or grid" was a metaphor for "mapping"; "repetition" – a metaphor for "iteration". Transformations of pictures where straight lines turned into curves but angles between them were preserved – a metaphor for "conformal mapping". The meanings of "straight line" and "curve" were suggested by pointing to elements of pictures and the everyday meaning of these words.

The aim of the lecture was not just to present the results of a mathematical modeling work. Using this one particular example of mathematical work, the lecturer wanted to guide the "mathematical tourist" around some of the mathematical culture and landscape. Certainly deductive reasoning and formal processing are important landmarks in the mathematical culture, but they were not a big part of this "guided tour". The lecturer stressed the culture of curiosity in solving mysteries of all kinds, even those created by an artist's genius.

The communicative functions, especially phatic (maintaining contact with the audience) and conative (influencing people's thinking), were dominant throughout the whole lecture. The poetic function was exercised through the display of pieces of art and animations. The lecturer frequently used humor; emphasized personal aspects of both the artists and the mathematicians working on the project. He suggested a visit of the website of the project. All these are examples of the use of these above-mentioned communicative functions. The results of the work of objectivation and processing were in the article in the *Notices*; in the talk, language was used mainly in its communicative function.

Although the title of the lecture – *The Mathematics of Escher's Print Gallery* – resonated with the title of the *Notices* article of de Smit and Lenstra (2003), the organization of the lecture differed greatly from that of the paper. Aside from the difference in space devoted to mathematical formalism and processing in the lecture and the paper, there was also a difference in the way each was introduced. Both announced the main idea to be presented, but the lecture introduced it using metaphors, pictures and animations, stressing the informal and ludic aspects of the idea. Graphical representations were so much an integral part of the lecture that it would be completely incomprehensible based on an audio-recording alone. There were some graphics in the paper as well, but they were less numerous and played a different role.

5.5.3 Comparing the two lectures

The main difference between the means the lecturers used to convey their messages seems to lie in the nature of their communication. The "Medial representation" lecture, which addressed a special application of mathematics (mathematical imaging), a well-defined and technically demanding problem, resembled a talk for non-specialists in the particular area, but still representing the same overall profession as the speaker. It was like a "tour" of an unfamiliar "subculture" for people belonging to a common culture. The lecturer appeared to assume that the audience is familiar with the general features of the overall culture, and thus saw no need to give a general picture of that culture; he focused on the subculture's specialized ideas. Even the editorial organization of the talk mirrored that of a scholarly paper: Motivation through a possible application, introduction of the main concept and methods, verification, applications.

The "Escher" lecture dealt with connections between mathematics and art along with a humanistic perspective, since the talk focused on the work of one artist, M. C. Escher. This lecture appeared to address an audience foreign to the presented culture altogether, and offered a general "guided tour" of it.

Both lecturers made an extensive use of metaphors. These were, however, different kinds of metaphors. The metaphors in the first lecture were "dead metaphors" in the sense that they were already practically lexicalized in the standard technical vocabulary and set of paradigmatic graphical illustrations of scholarly papers in the domain (Dimitrov et al, 2003; Pizer et al, 2003; Siddiqi et al., 1999; Yushkevich, 2003, etc.). The metaphors in the "Escher" lecture, such as "infinite repetition", "shrink", "distort", "twist" were ad-hoc, live metaphors, invented to convey the idea of iterations of various mappings to a lay audience. The graphical representations supporting these metaphors were also not the standard ones used in scholarly literature, but invented, if not for the purpose of this particular talk, then for the website aimed at popularizing the discovery of the mapping behind the *Print Gallery* lithograph.

According to the special characteristics of popularization, the mathematical content involved in popularization should differ from research mathematics and also from school mathematics. In particular, it is intended to present mathematical culture to people with diverse cultural backgrounds. The mathematical discourse in the lecture on medial representation was in many aspects similar, or even identical to that of research publications. Its organization followed that of conference presentations, its metaphors were identical to those used in scholarly papers and the use of language did not rely heavily on communicative aspects. Thus, it can be concluded that the lecture on medial representation did not fulfill the special characteristics of a popular discourse. The lecture on Escher's Print Gallery seemed more appropriate to provide a guided mathematical tour in terms of the means used by the lecturer.

5.6 Conclusions

As popularization is a special genre of scientific communication it appears to bear some special characteristics such as a greater emphasis on communicative functions of the language, the role played by different metaphors, the reformulation of statements and arguments, etc. The mathematical discourse differs from that used in research or teaching. The first and foremost aim of popularization is to communicate, while the aim of scholarly publications is to objectivate personal knowledge. Teaching, although designed for mathematical communication, differs especially in its conative aspects and organizational features of mathematics. According to Verret (1975: 145-174) knowledge intended for communication through institutionalized teaching, must be

1) Explicit
2) Impersonal
3) Divisible into small chunks that can be associated with learning practices and exercises
4) Programmable into teaching, study and assessment periods
5) Testable

Popularization does not have to abide by these norms. For example, Penrose's book (2004), *The Road to Reality*, fails in all five aspects. It asks the readers to solve problems often based on mathematical techniques that were not explicitly presented before in the book, and that it may not be even possible to transpose into an explicit object of teaching. The main interest of this book for experts in mathematics and physics are Penrose's personal views on various philosophical issues, such as whether mathematics is discovered or invented. The text is an indivisible whole if the reader's goal is to understand how the universe works. Cutting it into "lessons" followed by tests would defeat this purpose; it would result in fragmented knowledge, a collection of unrelated concepts.

CHAPTER 6

POPULARIZERS AND THEIR VIEWS OF MATHEMATICS AND POPULARIZATION

6.1 INTRODUCTION

This chapter deals with questions about popularizers. Who are the popularizers? What do they think about the goals, content and means of popularization of mathematics?

In my research, I have sought answers to these questions mainly by directly interviewing nine mathematicians and mathematics educators who engaged in different forms of popularization activity[45]. The sample was opportunistic; I interviewed popularizers that I could reach, by attending their popular lectures, or participating in conferences and other meetings at which they were present. The popularizers I interviewed engaged in different forms of popularization activity, such as giving lectures for the general audience, writing popular articles and books, developing websites, or organizing open houses and math fairs.

In section 6.2 of this chapter, I present a "family portrait" of the interviewed popularizers. In the fourth section, I present the individual portraits of two of the popularizers, whose lectures I attended and have been using to illustrate various aspects of popularization in the chapters of this thesis.

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[45] For ethical reasons, the names of the interviewed popularizers will not be revealed in this thesis.

6.2 A "FAMILY PORTRAIT" OF POPULARIZERS BASED ON NINE INTERVIEWS

In this section, I will speak about the popularizers' backgrounds and professions, and their views on the goals of popularization and criteria of its success.

6.2.1 Backgrounds and professions

The interviewed popularizers represented a variety of national and professional backgrounds: six North-Americans, three Europeans, pure and applied mathematicians, and mathematics educators. The sample did not include journalists, writers, filmmakers or artists, but people representing these professions are also sometimes highly valued – by mathematicians – for their work as popularizers of mathematics. This follows from the list of the winners of the JPBM Communications Award[46]. It is not necessary to be a mathematician or even to have an advanced academic degree in mathematics to receive the award. Of the 21 people who received the award between 1988 (the first award) and 2009, thirteen were non-mathematicians. Among these, eight (8) worked as journalists, freelance writers for journals and television programs or were writers of books and theatre plays: Carl Bialik, award received in 2008; Sylvia Nasar, 2000; Constance Reid, 1998; Gina Kolata, 1996; High Whitemore, 1990; Ivars Peterson, 1991 and James Gleick, 1988. Three (3) were filmmakers, television producers and directors: George Csicsery, 2009; John Lynch, 1999 (he received the award together with Simon Singh) and Joel Schneider, 1993. Two were artists (sculptors): Halaman and Claire Ferguson, 2002 (common award). Two other recipients had PhD degrees in mathematics or physics, but did not continue in research: Barry Cipra, 2005, and Simon Singh, 1995. The remaining

[46] http://www.ams.org/prizes/jpbm-comm-award.html (Downloaded: July 6, 2009)

six recipients have been active mathematicians at the time of receiving the award: Steven H. Strogatz, 2007, dynamics of synchrony; Roger Penrose, 2006, mathematics and mathematical physics; Robert Osserman, 2004, geometric problems (Riemann surfaces, minimal surfaces, etc.); Keith J. Devlin, 2001, applications of mathematics in linguistics and design of information systems; Ian Stewart, 1999, nonlinear dynamics and applications in, among others, fluid dynamics and mathematical biology, and lastly, Philip J. Davis, 1997, numerical analysis. It is noticeable that the work of most of the recipients in this last category was close to applications of mathematics.

Interestingly, however, the interviewed popularizers (especially research mathematicians) emphasized that research mathematicians should play a crucial role in popularization; stories about mathematicians or mathematical activities cannot replace direct contact with a live and active research mathematician.

One of the interviewees believed that popularization becomes increasingly a "specialization": some people "specialize" in this activity.

> [T]here are books now, and there are people specializing in it, [John] Allen Paulos[47] specializes in popularizing mathematics, Keith Devlin[48] is certainly a big figure in the US. And you start to see more and more popular books in mathematics. [SM]

He saw that more and more people engage in popularization and that "there is a market for it", but still believed that there were not enough popularizers and that "there should be much more".

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[47] John Allen Paulos is a research mathematician who has worked in the domains of logic and probability, and also a prolific writer of popular books on mathematics, e.g., *Innumeracy – Mathematical Illiteracy and its Consequences* (1988) or *A Mathematician Reads the Newspaper* (1995).

[48] Keith J. Devlin is also a research mathematician; his name already appeared in this chapter as one of the JPBM Communications Award recipients. His popular works in mathematics include, e.g., *Life by the Numbers* (1998) and *The Language of Mathematics: Making the Invisible Visible* (2002).

6.2.2 Views on the goals of popularization of mathematics

All interviewees believed in the need of popularization of mathematics, because mathematics is disliked and misunderstood by the general public and this results in dwindling enrollments in mathematics, science and engineering at the university, which is bad for the survival of mathematicians. More specifically, the interviewees mentioned such goals as improving

- public image of mathematics and mathematicians;
- public understanding of mathematics; and
- public attitudes towards mathematics and mathematicians.

I will give some more details about the interviewees' opinions about these goals in the next subsections.

6.2.2.1 Improving the public image of mathematics and mathematicians

The interviewees seemed to believe that the general public views mathematics as "something that's been left in books written hundreds of years ago and done by white European men" [MG]; that it is "dull and mechanical like book-keeping or something, just add numbers, you have to be precise and add them all". They wanted popularization to convince people that mathematics is a live and interesting domain of research; that it has recently been progressing "at an amazing rate", with the proofs of Fermat's Last Theorem and Poincaré's Conjecture, yet there is a "vast mathematical universe that is still waiting to be discovered" [EL].

Popularizers wanted also to present mathematicians as interesting people and their work as exciting. This motive can be also found in introductions to popular books about mathematics. For example, Constance Reid, in *From Zero to Infinity*, explains that she wanted to impart to the general audience some of the excitement of mathematical work and discovery that she witnessed as a sister and sister-in-law of mathematicians (Julia Bowman Robinson and Raphael Mitchel Robinson). In his Preface to *The Mathematical Tourist*, Ivars Peterson speaks of his difficulties in understanding advanced mathematics, feeling in mathematical lectures as an "accidental tourist lost in a foreign land"; he wrote the book to prove to himself and others that visiting the "foreign land" of mathematics can be a pleasant and exciting adventure.

Another image of mathematics that popularizers wanted to change was the view that mathematics is mainly a teaching subject and tool of academic selection, but otherwise not very useful. They wanted to show the public that "if science progresses, it's mainly, it's a lot because of mathematics; that mathematics is not only the universal language of science but it's also a basis of all science; people would not do science if they didn't have the mathematical models and so forth." [SM]. Some stressed that mathematical culture is strongly connected to other cultures. For example, mathematics was compared to music, as an "organic part of culture" [WS]. They also mentioned the need to show the public that mathematics pervades their daily lives, but that it is also abused in the media, leading to misconceptions about mathematics.

6.2.2.2 Improving the public understanding of mathematics

For some popularizers, an important goal was to fight the widespread innumeracy, expressed in the common misconceptions about numbers, randomness and statistical data. John Allen Paulos' work was mentioned in this context by one of the interviewed popularizers. Another referred to the same problem, proposing that an important goal of popularization is to get people to think critically about mathematics.

> ... to get people to think, think critically about it. Not to criticize but to think critically, you know, to ask questions. Like the research with breast cancer, right? Well, everybody is saying, well, hormone replacement, hormone replacement, but then one looked back and looked at the studies where hormone replacement seemed to have a good influence on women; well they found that those women were the healthiest women to start with. So it wasn't the hormone replacement, it was just that they were healthy to begin with, and so like getting people to think about that; what else could explain it. I think that for me that's the most important in statistics. That correlation is not the same as causation. Just because the price of alcohol goes up and teaching salaries go up, doesn't mean they are related; these are different things. You know what I mean? Get people to think about that. You know, just because two things are increasing at the same time, it doesn't mean that one is causing the other. [MF]

6.2.2.3 Improving the public attitude towards mathematics

One of the popularizers believed that thanks to movies such as *Good Will Hunting* or *A Beautiful Mind*, "people seem to be bit more sympathetic for mathematicians and their careers". [SM] Another was saying that popularizers should make people "feel attached [to mathematics] in some way, enjoy the fact that it surrounds them, so that they get to have a feeling that it belongs to them in some way." [LA]

To achieve this goal, popularizers should "try to show that mathematics is fun and useful" [SM], "interesting" and "fascinating" [EL], contrary to their previous negative experiences with school mathematics. This view is often found in popular books in mathematics; even in Penrose's "complete guide to the laws of the universe":

> Moreover, I hope that I may persuade my reader that, despite what she or he may have previously perceived, mathematics can be fun. (Penrose, 2004: 21)

For one of the interviewees, a research mathematician, a popularization act consisted, at least in part, in showing the public, using himself as an example, what it means to be a mathematician, to think like a mathematician by presenting himself as a warm person, passionate about his mathematical work [EL]. Thus popularization could "help people not normally involved in mathematics" to "like mathematicians" and "see that mathematics is something that can be interesting, engaging and something that's supposed to be experienced" [LB]. This, as it was hoped, could shift the attitudes of the "many [who] left school with a negative experience of mathematics".

Popularizers who were not research mathematicians themselves, also insisted on conveying the human and emotional side of mathematical culture in popularization through, for example, the biographical details of lives of famous mathematicians, or the dramatic stories of mathematical discoveries.

6.2.3 Popularizers' views on the conditions of success of popularization

According to the interviewed popularizers, the success of popularization depends on certain characteristics of popularizers and what they present and how.

Popularizers should, first of all, have a deep understanding of the mathematical area they want to present. They should also be fascinated by the subject and enjoy working on it. Last but not least, they should be able to represent abstract mathematical ideas in a visual form.

Regarding the mathematical content of popularization, it should be mathematically serious and important yet relevant for non-mathematicians, applicable, not too technical or narrowly specialized, and intellectually stimulating not only for mathematicians. Moreover, it should be possible to represent it in a lively way, visually, if possible. The presentation should be able to capture the audience's interest, for example, by relating it to daily life or topics with which people are confronted in the news. It should also stir critical thinking among the audience.

Popularizers mentioned some themes that are particularly suitable for popularization such as problems in applied mathematics; examples of mathematical modeling; geometry and measurement; topics associated with the calculation of risk particularly in the context of games of chance; logical problems; knot theory; the mathematics of soap bubbles, etc. Some popularizers, however, believed that any mathematical content could be good for popularization:

> If the speaker's skilled and inspired, I think you can make interesting stories in very many different mathematical areas. I don't think it is limited to, you know, applied areas or number theory.... I would say if you are a mathematician and you work on something and do enjoy it, you find that it is fun, there must be a way to convey that joy to express what is fascinating about the subject. If you think it is fascinating, you know, I think you can explain to other people why it is fascinating. [EL]
>
> Almost everything is [a] good [topic in mathematics to popularize]. I mean if you really sit down and think about it thoroughly, you will realize that in just about any concern in our daily, modern life, there is some mathematics you can explain and you can...relate to their daily life. You just have to make the exercise.... General public can [feel attached to] to mathematics if they can relate it to their daily lives. What happens in their daily lives? They use cell phones, they use television... they listen to the news, and they need to understand what graphics is all about, what statistics is all about, or probability.... The general concerns they have about [genetically modified organisms]... nuclear physics, whatever. You know, some science related aspects, which they need to maybe have an opinion on, or to be able to take a decision on. Surveys, for instance, is another example. And all these things they can appreciate. And perhaps realize that there is some very important science behind that and very important mathematics. So I do this equation between their daily preoccupation and the real science. [SM]

Even some ordinary school mathematics topics, generally considered as boring such as addition of fractions or factorization of polynomials, can be turned into an exciting activity, as one of the interviewees said, based on his own experience.

The condition of avoiding technicalities (including mathematical symbolism and proofs) in popularization was discussed by the popularizers as one of the hardest to satisfy. The price of giving up the technical aspects of mathematics in popularization results, according to one of the interviewees, in not showing "the real thing":

> [I]t's not the real thing. You got to go away from the real thing.... But you have to compromise somehow. You have to make a decision between real science and popular science. And I know for real scientists it's difficult to do that. To give up the rigor to just give the general lines and so forth. I know, it's not real science. But if you want to reach out to the people you have to go down a few steps. And you can do that while still being honest, scientifically honest. [SM]

Some popularizers were sharing their ways of avoiding the technicalities but still conveying a picture of mathematics that is not too far from "the real thing":

> But so I remember one article I wrote about the four color theorem, which is an easy question to understand, but hard to answer. But there are some interesting things that you can say about it that most people can understand and you can do some mathematics. So that the four colors problem takes place on a plane and you can have other surfaces you can ask about coloring, like for example the surface of a torus. And there the problem has been solved for a long time and it's a very nice formula and it's easier. It's much easier mathematics. It's an easy problem compared to the four color problem. So that's a good example, I think. [LA]

It was interesting to see that popularizers themselves had an ambivalent relation with the symbolic aspects of mathematics. While they stressed the importance of using, in popularization, alternative means of communicating mathematical ideas instead of the traditional mathematical formalism used by mathematicians, one of them mentioned that it were, in fact, the symbolic aspects and rigor that attracted him to mathematics, in the first place.

> When I was in 5th grade in primary school, I had this nun who was teaching mathematics and she was writing all these equations on the board and it seemed so interesting, so fun, that she really gave me a passion.... What I didn't like so much about physics was how they always approximated the results. And they were not so rigorous. While in mathematics you have to be rigorous if you want go anywhere. So that attracted me more than physics. [SM]

Another interesting view was that, although symbolism certainly has a great power in capturing the structure of mathematical objects, overemphasizing symbolism can be a cause of communication problems even between mathematicians working in different areas.

> I think that there is a certain level of snobbery on the part of some mathematicians. They feel they have to be incredibly abstract in order to be doing [it] mathematically. So I think one of the worst things that has happened to mathematics in the 20th century is the development of the Bourbaki [volumes] which, I think, has been absolutely horrible. For example, if you look at a textbook in mechanics appeared in 1942, it is a perfectly straightforward textbook. It talks about the motion of bodies, planets, the solar system and the planets using a pretty straightforward language. And here is a modern textbook in mechanics from 1978[49]. When you look at it, you will not even realize that it has anything to do with the problems of mechanics it was developed from, which was the motion of planets. It's ridiculous, this desire of incredibly many people to make things as abstract and obscure as possible. And I think it's absolutely horrible. Because it's unnecessary, and it really makes [the reader] feel very confused. And it's also quite horrible in the sense that it's got to the point that a lot professional mathematicians can't talk [to each other] now. If you have an algebraist or an analyst, they can't go to a lecture or a seminar by a topologist and understand what's going on. And even mathematics departments usually have either seminars or colloquia. The colloquia are supposed to be a much more broadly based, and more intelligible [event], but often if you go to math department colloquia there are just the people [who] are the specialists. If it's an algebraic number theorist's talk, who goes? The algebraic number theorists. [MO]

I will now sketch the individual portraits of the lecturers of "Medial representation" and "Escher" talks.

6.3 Individual Portraits of Two Popularizers, with Their Lectures in the Background

The two popularizers were interviewed a few days after their popular lectures, and the interviews were strongly connected to the lectures. I will refer to the speaker in the "Medial representation" lecture by "ML", and to the speaker of the "Escher" lecture by "EL".

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[49] Referring to the book: Abraham, R. & Marsden, J. E. (1978). *Foundations of mechanics*. Reading, Mass.: Benjamin/Cummings Publ. Co.

6.3.1 The lecturers' backgrounds

ML was a professor in a university department of Computer Science, and member of a research group on *computer vision.* ML stressed that his background was in electrical engineering, not in mathematics, and that he only got in deeper contact with mathematics during his work in the area of computer vision, what he described as:

> ...a field interpreting and analyzing visual information. So it's to be able to do different tasks such as data perception or motion recognition. Computer vision is a quite unestablished field. It's a field that is sort of interdisciplinary; in some universities it is in Computer Science and in others it is in engineering, and in many of them it is applied mathematics, and certainly is related to human vision. So the problems require some expertise in mathematics and in signal processing. The problem of shape analyzing and shape understanding is rather the areas of computer vision. And that is what I am doing since my PhD years. So it's not like I trained in mathematics to do it, it's more the other way. [ML]

ML described his own learning of the necessary mathematical concepts as picking them up those through working on the applications. He thus experienced mathematics as an applied domain. For him, mathematics was a necessary and powerful tool to implement and justify various methods in engineering and computer science especially associated with computer vision.

> It's a fairly direct subject, it's not like you need esoteric advanced mathematics to get ahead. You pick up the pieces as you go along and at least you need to be aware of what you don't really understand. ... And it's the combination of mathematics and implementation of algorithms that allows us to do this. So in a sense it is certainly a transition between pure mathematics to applied mathematics that allows you to work with these models and compute descriptions of objects and then you can visualize those... That's, this is certainly a kind of subject where, if you have an interest in it and you understand a part of it, you can push deeper and pick up the tools as you need them.

Based on what ML said in the interview, he seemed to be familiar with learning mathematics through the particular problems he had to face in his research work. According to his field of study, these mathematical problems probably originated from a concrete (often visual) application of certain mathematical concepts usually coming from differential equations or differential geometry.

ML was quite new to popularization. Before the "Medial representation" lecture, he only gave one other talk to non-specialists in his area. The audience in this other talk consisted of mathematicians, but they were not familiar with computer vision.

The second lecturer, EL, on the other hand, was an experienced popularizer. He lectured in front of a variety of audiences including high school students and teachers and designed websites on popular topics such as the famous "hole" in Escher's lithograph "Prententoonstelling" (*Print Gallery*, 1956), the abc conjecture or the number π .

EL was trained as a mathematician and was still engaged in mathematical research at the time of the interview (Number Theory). He explained his decision of becoming a mathematician by his fascination with mathematics as a kid. He also mentioned a magazine for high school students, as well as Mathematical Olympiads as a source of inspiration in his high school years. In the interview, he was comparing mathematics to art, citing stories of mathematical intuition and stressing the fact that, in popularization, he wanted to show that "math is fun".

6.3.2 The choice of the topic of the lecture

6.3.2.1 ML

ML was invited to give a talk in a series of lectures to a broad audience on the "cutting-edge of science", but the organizers left the choice of the topic to his discretion. He decided to talk about his own research, because

- he knew the topic well and was enthusiastic about it;
- preparing a talk about one's own research for a "non-technical audience… forces

people to think about what they are working on… in a meaningful way" and is therefore useful for the researcher;

- he believed the subject to be both accessible and appealing to a broad audience, because it was directly connected with the everyday experience of human vision and could be understood at an intuitive, non-technical level;
- at the same time, the subject was sufficiently complex to satisfy the interests of three groups of people he expected to attend his lecture: general public, computer scientists, and mathematicians.

He said,

> I knew the general theme of the lecture series was to speak of a topic that could be interesting for a broad audience and was not necessarily too technical or too specialized. And this problem is one where the intuition is quite easy to capture, although the problems are quite challenging, technical. So I decided to talk about it because the aspects of the problems. There could be appealing for mathematicians, mainly for differential geometers but there are other aspects that could be appealing for anybody who is interested in shape perception, and mathematical biology and also for those who are interested in describing visual forms. So I thought I could give the talk at three different levels. One, it could be technical, one that should be more algorithmic for computer scientists and [one] which would illustrate the applications.

ML thus decided to "disseminate" knowledge about a certain area of research he was well familiar with.

6.3.2.2 EL

EL's goal was not so much to disseminate mathematical knowledge but to influence people's attitudes towards mathematics and images of mathematics. For EL, popularization was mainly a tool for recruiting high school students to study mathematics at universities: popularization as a possible remedy against the steady decline of the number of mathematics students. He stressed its great importance for the survival of academic mathematics. It was important to make the public (both the general public and the academic community) aware that mathematical research exists, and it is in fact an activity that people enjoy doing. Mathematicians, he said, should engage more often in popularization because he felt that "mathematics tends to be focused inward a lot" whereas it would be important to build public relations both outside and inside of the academic community.

When I asked EL about his goals in his lecture, he emphasized conveying a positive image of mathematics as fun and interesting, and dispelling misconceptions about mathematics.

> Well, the goal is to get more people interested in mathematics and to get people convinced that mathematics is a fun activity. [EL]

He wanted to show that, contrary to what people usually believe, intuition (and not only logical reasoning and computation) plays a very important role in mathematical research and that mathematics is not a finished science. He believed that the topic he chose was quite likely to achieve these goals.

6.3.3 Lecturers' expectations about the audience's understanding of the lectures

6.3.3.4 ML

ML assumed that anybody who would care to come to his lecture would be at least "interested in shape perception, mathematical biology and in describing visual forms". In addressing members of the *general public* group – which, in this case, were people who were neither mathematicians nor computer scientists, but who might be academics from other departments – ML assumed the following background:

- Having some previous experience with computer vision and human vision and image analysis, not more, however, than what can be gained through everyday experience.
- Being used to learning through examples, since he planned to introduce the main ideas through specific examples and metaphors, such as the metaphor of the panther.
- Being prepared to be presented with the inevitable technicalities that belong to the nature of the subject of his research, including mathematical formalism as well as empirical and theoretical testing of hypotheses. The lecturer expected that the audience will not be surprised by the steps generally used in this type of research such as posing the problem, formalizing it, developing an algorithm and testing it.

> And that you can think about what would be the ways to formalize this. What would be the necessarily mathematics. What assumptions would be made? Where would the techniques be successful and everything. ... Once you have algorithms developed, you can test and so continue to understand them. So if you chose the nature of the subject, I think the appeal of it would be actually that. If they understand the physical processes, they can visualize that in the sense of what the nature of the problem is. And they don't need sophisticated tools for understanding that. And furthermore you have a specific claim and hypothesis that you can test and demonstrate the experiments. They can understand [it]. So it's certainly what even quite a general audience can do. And then they could expect that there exist certain techniques that precisely answer the questions of flows. You can build it all through examples the mathematics that you use. [ML]

As for the group of *computer scientists*, ML was assuming these would be people capable of understanding computer implementations of the theory and would be, therefore, familiar with data structures and algorithms, as well as with some aspects of computational geometry (such as geometry of discrete structures, lattices, matrix, points, polygons and algorithms for specific processes).

ML elaborated most on his expectations about the group of *mathematicians* in his audience. He identified four subgroups of prerequisites that he considered necessary to understand certain aspects of the material presented:

- calculus and basic differential geometry to understand basic facts about smooth curves and surfaces;
- advanced calculus and advanced differential geometry to understand singularity theory;

- differential equations, partial differential equations to understand the process how curves move;
- differential topology, measure theory to understand the reverse process.

ML intended to communicate three different kinds of messages to the three groups of the audience.

To the general audience, he wanted to communicate the idea of medial locus as a way to compare two- and three-dimensional objects. He also wanted to show the general method of his research by indicating the specific technique (the mathematical idea on which an algorithm is constructed), and point to the potential applications.

> And to the general audience I wanted to get them a feeling that when you compare objects, often you do so in terms of the points. And these medial loci give me a way of thinking about parts of an object. That's mathematical, that's precise but it's also algorithmic. You can compute the parts, represent them and then apply them to for example to 3-D objects, to a trivial problem as a starting point.

To the computer scientists, ML intended to present how mathematics can be applied differently than in the previous methods used in the field of computer vision and pattern recognition. He also wanted to show the advantage of his method compared to other techniques and emphasize the importance of the theoretical basis in this kind of research.

> To the computer science oriented people I wanted to say that the intuition and the continuous mathematics is really useful because it provides new ways and new algorithms for computing these. And they were tempted [to use] much more direct algorithms, which basically failed. They haven't been generalize-worth in 3-dimensions. It's too much complexity, [nothing] is stable but fragile. [There are] a lot of heuristic assumptions you have to make.

For the mathematicians, the emphasis was on presenting a mathematically precise treatment of differential geometry along with highlighting the applications. More specifically, medial representations offer a nice way for connecting differential geometry, computational techniques and applications.

> For the mathematically sophisticated the main goal was to communicate the idea that there is really a new way of thinking about precisely differential geometry of curves and surfaces. And there is a rich way of doing all of that through this medial representation along with the details. They will develop a lot of aspects and interest of all about.

6.3.3.4 EL

EL did not expect the audience to come out from his lecture with some deep and precise understanding of the mathematical concepts, but he hoped that they would remember the main question of the talk and that the visual material and the animations would have made a lasting impression on them. He wanted them to remember that mathematicians had various ideas and methods for filling the hole in Escher's lithograph. He hoped they would be intrigued by the infinite repetition suggested in Escher's work and that they will enjoy the idea.

> I think what they'll all remember mostly is the visual material. I think the stuff I said and the whole talking around will not be remembered as much as just looking at the animations. ... I hope that is what they'll remember. So I hope what they remember is that there are these pictures and these mathematicians have a way to look at them that when you can do these amazing manipulations just by sort of a change of perspective. Yes. And I think the main message they will remember is that there was this print of Escher which had a hole in it and there were some mathematicians who figured out what goes into it. Maybe they will remember that there is this infinite repetition in it. But the main thing they should remember is that mathematics is fun. That's all.... So when we are asked why we do mathematics we usually say that because it makes us happy to do mathematics. So if you want to be a happy person like me, then you will also do mathematics. [EL]

Regarding EL's expectations about the audience, he said that he did not expect any previous knowledge about mathematics from the audience: "I think ... the best thing is to assume that they know no mathematics."

6.3.4 Popularizers' beliefs about effective means of communicating mathematics in their lectures

6.3.4.1 ML

In the interview, ML stressed that he planned to build his presentation on examples (mainly visual examples).

> ...a quite general audience can still understand principles by examples. Typically – visual examples. So if you can go through a specific example, describe the process for example in this context of this topic, the process is very simple. You have an object, the boundary is on fire. And the fire is eating its way to the inside of the object. Everybody can visualize that. And in the end they understand that finally the fire fronts hit each other and these are the regions we are interested in. So there is no mathematics really, that's all by example. [ML]

He cited the idea of curvature as a concept that could be understood this way:

> ...for example everybody knows bending of surfaces described in terms of a qualitative notion. The thing that precisely captures how much a surface is bending is the principal curvature and that's much more formal. Now from formal to abstract: a lot of the abstractions that people can understand in the world of mathematics as they can visualize it and they are familiar with this intuition. And once you are familiar with it, if they had the training and calculus and linear algebra of course, they can pick up enough of the basics to figure out, to understand what curvature is, what the principal curvature is. [ML]

He also stressed the use of animations and analogies in the presentation. For example, he intended to introduce some mathematical notions he used in the talk, such as the idea of divergence and flux. For this purpose, he used the picture of the panther, on which he also demonstrated the method he used to find the medial locus.

> ... the panther shape where I showed what this distance function is. Everybody can see that. If there is a move away from the boundary it gets brighter. That's intuitive. And I said let's compute the derivative; it looks like this flow. Everybody can visualize what a flow is. Even if they haven't understood [it] precisely. That intuition is simple. And then you sort of throw in this idea that you can compute the divergence of a vector field. People don't really know what that means, but they can imagine that there is a medium, and there is a flow and there are particles. And maybe the flow is pushing the medium. And intuitively divergence captures what's happening. So I think it's, it's useful to have

> the mathematics. They may not understand the notation but they could understand the description of the problem being solved. ... Because people remember the examples. [ML]

At the same time, expecting that computer scientists and mathematicians will be in the audience, ML also included a technical layer in his talk. It was as if he was a multilingual tour guide offering explanations in all languages spoken by the tourists in his group.

Based on the feedback ML received from the audience after the talk (the lecture was followed by a small reception where interested participants could talk to the lecturer), he considered it successful.

> I think it was at the right level. That's my sense, that's the feedback I got... [From] the questions I have received, I got the feeling that the audience really did understand the problem and what I wanted them to understand.

He described his experience during the reception as the following:

> They remember that thinking about a medial locus could address that problem. And certainly the biologist would be curious about the way to discover certain significant descriptive differences or similarities between shapes that would be useful for that particular example, analysis of fossils, or evolution of species, or such things. So they remember the aspects that we can compare objects by this representation. So I think they remember quite a bit. They remember the model and they remember some of its applications. They all understood the idea of lines through the center of some structure. So they remember the examples shown. And they understood that once you have this representation you can get back the boundary. That they understood. But mostly by the examples. They probably will not understand the detailed notation, or the process, or even the algorithms. But they might understand that there is a physical principle that explains it all. And once you read about it again maybe you'll remember it. Many of them admit that they didn't have the mathematics to dispute the theory, but that the examples were suggestive. [ML]

In response to the question about what he would change in the presentation, ML mentioned that time permitting, he would have talked a little more about the technical details such as the models used, and on graph matching for searching algorithms.

> Given a little more time, I would have explained more clearly... how these models were created, what their medial structure is, what graph matching is about. There is a huge

> field which is both mathematics and computer science and graph theory and subgraph isomorphism, related to the problem. I am certainly not an expert on that but it's a wide field and to describe the nature of that field also takes some time. So I wouldn't really attempt to change the content so much.

6.3.4.2 EL

EL devoted a substantial amount of time in the interview to discussing the problem of the level of mathematical rigor in popularization. He claimed that it is not necessary to give up rigor and proofs in popular talks, however, it has to be presented in an appropriate style.

> Well, I think [a popular lecture] can be very rigorous. But in a certain style, I think. So for instance, suppose [that in the audience, some] people are working in law. They have a notion of proof and we have a notion of proof. I think people can appreciate very strict logic. So I think you can do a proof in a general audience lecture. [EL]

Rigor, however, should be subordinated to the clarity of the presentation:

> Well, if you state that something is true but really there are technical conditions that need to be satisfied before it's true, yet you don't really need them [for what you want to say in your talk], then the whole lecture is better off by your omitting them. If this would disturb you too much as a mathematician you can say that something is 'usually true'. So that you compromise on the precision of the statement but, the usual way a mathematician would proceed just by stating these conditions, that is not the way for a general audience lecture. You don't want to bother people with complicated details because it will only detract from the main story.

EL was well aware that one has to be very careful in using mathematical formalism (and even the technical mathematical terminology) in popular talks. This was a major difficulty in popularization – a challenge – for him in preparing popular lectures. He had to find means to convey mathematical ideas otherwise. Similarly to ML, EL resorted to "translating" mathematical concepts into natural language and pictures, using analogies with familiar things and visual aids.

> So I think the main problem of communicating mathematics is that mathematics is a stacked science. In the first year you learn words that you use in the next year and you learn more words and then after five years or maybe 10 years you use all these words like they are normal words with the very few people who know what you are talking about. Only the people who have done the same thing in your program. So if we talk about complex numbers, well to me, it is like talking about sandwiches. But I lose, if I am talking in front of a general audience, 90% of the people. They don't know what I am talking about. And so it is very easy for mathematicians to talk over the heads of the audience. And to miss them. I think the challenge for us as mathematicians is to phrase things by analogies or by visual aids or by other means to phrase things using only normal words. And not using our own terminology. And this is difficult. And often mathematicians will say, this can't be done. If somebody doesn't know what a finite field is how can I talk to them? Well, I say, think about a bit more and find a way to talk about it without using "finite fields". [EL]

For these reasons, EL said, a popular talk requires a lot more preparation than teaching a class. Preparing the visual material and finding "substitutes" for the specific vocabulary takes time.

> ...popular talks for me, it's a lot more preparation than teaching a class. When I teach a class it happens much easier for me. Because I am a mathematician and I am doing mathematics with my students. But for a general audience, a lecture is often much more depending on visual materials that you have to prepare in advance. I have to think more about what it is that I want to say. When I teach, I use the mathematical vocabulary. I use all the terminology, which is so natural for me. And when I get a general audience lecture I have to avoid that. Or maybe use some terms but not in the way that are critical for the audience. So that is more work.

Another difference with teaching he mentioned was the motivation of the audience.

> If you are teaching something, if you are teaching a course for the students, they come to your class and they will try to pass the exam. And so you can make them work. And they are supposed to really learn the subject. So I think it is very different from giving a general audience lecture. Because in a general audience lecture you don't want to lose people. What you want to get, of course, is the inspiration. And if you teach, you want to actually communicate the real mathematics. And be precise, and really educate them. So I would say it is quite different. But, of course, if I want to..., it is important when you teach to inspire students as well. So, I mean a lot of ... these skills are the same, but the focus is very different.

Although EL has already given lectures on the same topics several times before, he said he constantly tries to improve the presentation often by adding more visual materials. EL learned from his experience with popularization to look at familiar things in mathematics with a fresh eye and learned to see their potential for discovery for someone who is a "tourist" in this culture:

> I think I have learned that, for me, as a mathematician, I find certain things fascinating but other things I am completely used to and they are so normal that they are not fascinating anymore. But, for a general audience, it is often these things that I find sort of trivial that are already a big discovery for them. And so it is often, I think, that we tend to undervalue our own mathematics. And we think that oh, that I have to tell more, or we have to make it more, but already just explaining something that for us is really very simple, it can be a big discovery for someone else. I think we have a tendency to make things too complicated before we think they are interesting. But that material can be interesting for someone else.

6.3.5 Comparison of the lecturers' views

I end this section with a comparative summary of ML's and EL's views of mathematics and popularization. Based on the interviews, the two lecturers expressed considerably different views of mathematics. For ML mathematics is a tool that offers a rich way for developing methods useful in science and engineering with a smooth transition between pure and applied mathematics. In contrast with ML's mostly utilitarian image of mathematics, EL considered mathematics as a creative art with rapid progress and with various open problems people enjoy doing.

Besides their different views on mathematics ML and EL considered popularization also very differently. ML interpreted the activity mainly in terms of the deficit model as its goal is spreading information, although he intended to give a presentation at three levels[50] depending on the audience members' background and expressed his expectations according to the different audience groups. Thus, he behaved

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[50] This approach is used also by other popularizers, for example Penrose (2004) also intended his book to be read at four levels from the general audience to the experts.

rather as a multilingual tour guide trying to provide the appropriate translation for the different groups in the audience. As means of the translation, he used analogies and images, computer implementations and symbolic representations. EL, on the other hand, wanted to invoke interest and show that mathematics is "fun" and inspiring. It seemed form the interview, that he interpreted popularization mostly in terms of the intercultural model, where he, as a tour guide offers a glimpse into the mathematical culture. Although the tour included both mathematicians and "outsiders" it seemed that he rather focused on the latter group. He provided a translation of mathematical ideas by means of animations which he continually improved over time. Unlike ML, he did not expect the audience members to remember any details of the presentation except probably the main question and the fact that mathematicians solved it.

ML was more satisfied with his performance than EL was, and did not see as many challenges in the task of popularization as EL did. Yet, as we will see in Chapters 7 and 8, some of ML's audience walked out with a rather negative attitude towards mathematics, while all interviewed audience members of EL's lecture found that it was "fun".

6.4 Conclusions

Popularizers as a collectivity possess a lot of experience and knowledge about the challenges of popularization and various means to overcome these challenges. It is a disappointing, however, that this knowledge remains, to a large extent, the individual popularizers' private knowledge and that there is no platform where this knowledge could be made explicit and transmitted to the novices in this special practice.

CHAPTER 7

ANALYSIS OF INTERVIEWS WITH MEMBERS OF THE AUDIENCE OF POPULAR LECTURES.

PART I. AUDIENCE MEMBERS' CULTURAL LENSES

7.1 INTRODUCTION

In this and the next chapter, I will analyze my interviews with audience members of the "Medial representation" and the "Escher" lectures. In this chapter, I will describe the audience members' "cultural lenses" (background, images of mathematics, previous experience with popularization and reasons for coming to the talk), that may have influenced their perception of the lecture. In the next (Chapter 8), I will focus on the audience members' perceptions of the talks (understanding, opinions).

In Section 7.2, I will describe the procedures of choosing the participants and administering the interviews. Section 7.3 will contain a description of the "Medial representation" audience members' cultural lenses; Section 7.4 will do the same for the "Escher" talk. In the final Section 7.5, I synthesize the results and discuss them in the light of the literature on people's images of mathematics.

7.2 PROCEDURES

7.2.1 Recruitment of audience members for the interviews

I engaged in recruitment of candidates for the interview before the lectures, as well as during the receptions after the lectures. Before the lectures, I would ask several of my (mainly non-mathematical) acquaintances and fellow graduate students if they would be interested in attending the lectures and be interviewed afterwards. Unfortunately, people were generally very reluctant to be interviewed, even if they were interested in attending the lectures. Many were discouraged from coming to the "Medial representation" lecture already after hearing its title. The late evening (after 8 pm) timing was a deterrent for attending the "Escher" lecture. Among those finally recruited prior to the lecture, some already held a degree in mathematics. Participants recruited before the lecture knew that I was going to interview them, but I did not tell them what questions I was going to ask. During the receptions held after the talks, I also tried to find participants to be interviewed by choosing audience members randomly. A few people agreed, and I contacted them later by email and phone, and finally a small number of them agreed to be interviewed. The sample may be not very representative especially that most of the non-mathematicians I approached refused to be interviewed.

Finally, I managed to recruit six audience members after the "Medial representation" (four were non-mathematicians) lecture and the same number of participants after the "Escher" talk (three were non-mathematicians). All but two of them were non-native English speakers. All were fluent in spoken English, however, and had no difficulty understanding the lecturer. Two interviewees were Hungarian and I interviewed them in this language which happens to be my mother tongue. For the purposes of quoting in the thesis, I translated some parts of these interviews into English. The interviews took place within a week after the lectures.

7.2.2 Interview questions and their rationale

The interview was based on questions about

1) the participants' background (especially their mathematical background),
2) their image and attitude toward mathematics,
3) their previous experience with popularization of mathematics and science,
4) their reasons for choosing to attend the talk,
5) their general opinion about the presentation, and
6) what they understood from it: I asked participants to summarize the talk and mention some details that they found engaging.

The questions were not necessarily asked in this order in the interview, which was conducted as an informal conversation.

I will give an account of participants' responses to questions 1 to 4 in this chapter and to questions 5 and 6 in the next.

Questions 5 and 6 have been the main motivation for my research on popularization of mathematics. How can we distinguish between good and bad popularization, if we don't know what effect it has on the audience – a general audience, that is, and not only the mathematicians on, say, the JPBM Communications Award committee? Why spend so much effort on preparing popular talks and writing popular books if one needs to have studied mathematics at the university to be able to understand them? What does someone without training in mathematics understand from the popular talks and books? How can we improve popularization if we don't even know what is wrong with it from the point of view of the audience?

Questions 1 to 4 were supposed to provide a context and perhaps some explanation to the interviewees' responses to Questions 5 and 6. If, as argued in Chapter 2, popularization can be considered as intercultural communication about mathematics, then it is important to know something about the audience members' "cultural lenses" since these will inevitably filter what they hear and see. I assumed that aspects of cultural lenses that are relevant in the context of popularization of mathematics are related to people's mathematical background, their images of mathematics and previous experience with popularization.

7.3 Cultural lenses of the "Medial representation" audience members

This section is devoted to describing the "Medial representation" audience members' cultural lenses. That is, I will present the interviewees' mathematical background, summarize their views on mathematics and on popularization and end with a synthesis of all these.

7.3.1 Interviewed audience members' backgrounds

After the lecture on medial representations, I interviewed six participants, of which four were non-mathematicians. In the following, I will describe in detail the profiles of only one of the two mathematicians (M1) and all four of the non-mathematicians. (I chose to present the profile of only one mathematician since the interviews with mathematicians were quite similar to each other from the point of the questions I address in the thesis.) Of the four non-mathematician audience members, two persons (G1 and G2) could be considered as representatives of the general public (in the sense given to this category by the lecturer, see Chapter 6), and two (C1, C2) had a computer science background.

All interviewees had post-secondary education; even those whom I classified as representatives of the "general public", namely G1 and G2.

G1 had a degree in psychology and took some courses in statistics during his studies. He had some superficial familiarity with mathematical applications and research through his wife who had a PhD in mathematical biology and was on a post-doctoral fellowship position in this area at the university where the "Medial representation" lecture was taking place. He described his knowledge of mathematics as very basic: "mathematics for me is just fundamentals; maybe basic equations but not too much".

G2 was an undergraduate student of biology, and, at the time of the interview, was at the earliest stage of post-secondary education amongst all the interviewees. Her last "pure mathematics" class was Calculus at the university four years earlier, but she also took some courses in mathematical applications to biology.

C1 and C2 had engineering degrees in computer science areas. C1 was working on her PhD and C2 already had a PhD. Both needed mathematics in their doctoral research, C1 more extensively than C2.

C1 had a Bachelor of Engineering degree, a Master's degree in software engineering, and was currently working on her PhD thesis in telecommunications engineering. She planned to use optimization methods in her research, combining mathematics, computer science and engineering and, to prepare to study linear programming more closely in the near future.

C2 studied computer systems engineering and completed her PhD in biomedical sciences. She mentioned that she used statistics and tools related to dynamical systems but she stressed that, outside of her field, she knew only the "basics" of mathematics.

M1 was on his way to become a research mathematician. At the time of the interview, he was working on a doctoral thesis in mathematical physics. He took courses in differential geometry and differential equations, and therefore belonged to both the group with a background in advanced calculus and advanced differential geometry and the group with a background in differential and partial differential equations, named by the lecturer (see Chapter 6). His choice of mathematics as an area of doctoral study was motivated mainly by his high school experience with the subject in his home country (not Canada). Mathematics was considered a key subject and being good at it meant a lot; it made those who were good at it "feel successful". He was selected to follow an enriched mathematics program during his secondary education, was successful in it, and became "stuck with mathematics"; his career choice was determined.

7.3.2 Interviewed audience members' images of mathematics

7.3.2.1 G1

G1, the psychologist, said he used to think of mathematics as being useful mainly as a mental exercise or challenge, its only applied area being statistics. Now, however, seeing what his wife was doing, and going to popular and professional lectures in mathematics with her, he was starting to view it more as a tool for science: construction of models, solving problems using these models in various domains.

> Before I attended these talks, I used to think about mathematics in terms of statistics or in terms of its use for exercise. But now, I think mathematics is a fundamental basis for research and for psychology.

More frequent contact with mathematicians also had an effect on his opinion about mathematicians and mathematical symbolism.

> Because I have learned that mathematics is a very fundamental researching tool, now I

> think mathematicians are more human, more interested in real life. Because before I used to watch mathematics as a science of symbols without [any] sense. But now I have learned that it makes sense to see symbols and concepts of the real life.

Operating with mathematical symbolism, for G1, became an exteriorization of a process of mathematical thinking, especially after he saw the movie *The Beautiful Mind*, based on a biography of John Nash:

> To see the mathematical signs in *The Beautiful Mind* and to see the guy drawing them was important to me because I understood that he has a main thing in his mind and he has a process in his mind running.

This idea was reinforced through his observation of his wife's work and attending with her the seminars in the biology department.

7.3.2.2 G2

G2 saw a sharp distinction between pure and applied mathematics. Applied mathematics is something closer to "everyday people". Doing pure mathematics, on the other hand, does not mean much for most people. It can be a very "satisfying" activity, but requires special abilities, and only "smart people", with "a lot of brain" can do it.

> So, I think, the more theoretical stuff; like, to me is just, wow, like, I can't handle it, like it's not just that I can't handle, it's like I could never do that, you know what I mean. If it's applied, I think, it is more... in touch, maybe? ... You have the people who create all these theorems and the people who take this theory and apply. And I guess I have more awe for the people who do this theory.
>
> You start algebra in school; you will never use it in your life. And you never use geometry in life unless you are an architect, and you never use calculus unless you are a physicist... Anyway, that's the thing, since we start algebra you're never going to use it again in your life unless you pursue a career in it. I think statistics is the one, you know, something that you can really need in your life.

Thus, G2 had a lot of admiration for mathematicians, people able to do the pure mathematics, but saw herself as unable to "do the theory".

7.3.2.3 C1

C1 stressed that she always loved mathematics. This subject attracted her most from the primary school on. Mathematics in her home country, however, she said, was taught as a "pure science". In primary school, she said she "loved the logical aspects of it because everything was making sense. There were no vague parts.... In the primary school, everything was very clearly defined and... two plus two is four, there is no discussion about it.... Logic and mathematics, I saw them as the same thing". She enjoyed abstracting patterns from relations among objects and formulating them in a mathematical language. In high school, however, she started questioning this "pure science" approach to mathematics.

> I wanted to do math and continue with math, but then math in my home country was pure science. The only application of it was to become a teacher and teach math again. So that part I found a bit obscure, it was not as lively because I believed that there must be some application of all this formalism and all these theories. But I couldn't see. Then..., in high school, we had a bit of linear programming, and that part, oh yeah, that was it, but it was still very limited. In that part, I could see that you can apply it in your daily life. It is something that applies. It is not just the logic, logic, all the pure science. Just to teach it again... to the next generation, and again, and again. And what to do with it? How do we apply it in life? So that part was a huge question mark in my head and nobody could answer that, or maybe I just didn't know how to ask it. But it was sort of my dilemma in math. Is it math for math's sake, or math for life. I couldn't see the application of it.

She appears to have finally found her way to applied mathematics by going into engineering at the university level, and now doing a PhD in telecommunications engineering.

7.3.2.4 C2

C2 did not say she "loved" mathematics, but accepted it as "a fundamental tool for science in general" (including social and human sciences). Like the previous audience members, she distinguished between pure and applied mathematics, but talked more about the difference between pure and applied *mathematicians*. She described pure mathematicians as those who understand the things that she doesn't, but who are also "cold" as people, in contrast to those working in applied fields whom she saw as more sympathetic toward non-mathematicians.

7.3.2.5 M1

In high school, for M1, mathematics consisted mainly in solving problems that, while challenging, were still school problems to be solved and stop there. They were not problems that would lead to the need to broaden one's knowledge of an area and further investigations. He only developed this attitude later, at the university, under the influence of his professors and engagement in research. He felt that the "problem solving view" of mathematics is not enough for a mathematical career he intended to pursue.

> [It is not enough] if someone stops at the point when he is able to solve all the problems correctly but it doesn't generate anything in him, like thinking about it further, like thinking 'How interesting, I would like to read more about this'.

To describe his present image of mathematics, he used the metaphor of exploring a land, which, for him, was still "largely undiscovered" and "beautiful".

> Mathematics is like a vast, largely undiscovered landscape: some parts of it you know by heart (you walk there every day), some parts you have seen from a birds-eye-view and admire its beauty from a distance.

He did not distinguish between "pure" and "applied" mathematics as sharply as G2 and C2. He just said that he was interested in knowing about useful applications of mathematics.

7.3.3 Interviewed audience members' images of popularization of mathematics

G1 and G2 had rather opposed motivations in attending popular talks in mathematics. G1 wanted to deepen his understanding of his own domain (psychology of scientific thinking). G2 wanted a distraction from her own domain and learn something different.

7.3.3.1 G1

As a psychologist, G1 was interested in cognitive processes relating thought and speech. Popular talks were, for him, an occasion to observe these processes, not in young children, but in mature scientists; not in cognitive development from birth to adolescence, but in the development of the scientific method. The scientific facts or results presented in the lecture did grab his attention to start with, but he was really attending to the cognitive processes engaged in scientific knowledge construction:

> I am interested in the way they express, the way they organize the information, which is a scientist's way. I always try to find out how they organize the information in their head, in their brain. I always attend these talks because I know the psychological approach to it. How people organize thoughts, how people raise the concepts to the order of speech,... the scientific method.

7.3.3.2 G2

G2 would go to lectures in domains other than her strict area of study because she enjoyed learning something new or different: "if you like learning, you like going to a lecture where something different is taught". She mentioned that she "wandered into" a physics talk before, but she has never attended talks related to mathematics prior to the "Medial representation" lecture. She came to this particular lecture not just by curiosity, however, but because, in one of her compulsory courses, she had an assignment to attend a mathematics or computer science lecture and write a report on it. She chose the "Medial representation" lecture because it fit her schedule. She took notes during the lecture, wrote a report and made it available to me.

7.3.3.3 C1

C1 came to the lecture upon my request and agreed to be interviewed after. During the interview, she mentioned that in her school years she regularly participated in mathematics competitions. She viewed popularization as a means to influence the image of mathematics not only among the general public but also among engineers. For her, this image (also in engineers) represents mathematics as a "septic, dry" domain with "a lot of logic", that is also "nasty" and "scary", and so difficult that "you have to kill yourself to understand" it. Showing applications of mathematics was a way, for her, to make it more meaningful, understandable and less scary. She said that seeing applications of mathematics was crucial in her career choice. Showing applications in popular talks must be properly done, however; it must be understandable. She was disappointed with this particular talk because she could not understand the mathematical content of the lecture although she expected it from herself.

7.3.3.4 C2

C2 did not have much experience in attending popular talks in mathematics; rather, she would read popular science sections is daily newspapers, and more specialized magazines (such as *Scientific American*). She mentioned that reading popular literature helped her in her research work. When she wants to look up something with which she is not yet familiar (especially in relation with medical issues), popular literature and websites are often her first sources of information about the topic. She goes on to more specialized professional literature only in a second step. C2 considered popularization as a very important tool in spreading scientific knowledge, and emphasizing its methods and its demand for objectivity. If people are more interested in science and understand it better then they are more willing to support it financially. They are also better prepared to distinguish between science and pseudo-science. She saw, however, a dilemma here: to popularize science, it must be simplified; but if it is overly simplified, then the difference between science and pseudo-science is harder to see.

7.3.3.5 M1

Previous experience of M1 with popularization included mainly reading books such as Simon Singh's book *Fermat's Last Theorem* (Singh, 1997), James Gleick's *Chaos: making a new science* (Gleick, 1987), and George Pólya's *How to solve it* (Pólya, 1973). The first two of these books were recognized by the JPBM Communications Award. To M1, these readings served as a motivation for his later studies about a particular subject. As for the way such books can be read by non-mathematicians, he proposed that the aim of popularization is to give a "general view" of the work of mathematicians, and convince the public that mathematics has many useful applications. The lecture he just attended did that and this is why he liked it.

7.3.4 Synthesis

All interviewed audience members stressed the applied aspects of mathematics and had a positive attitude towards these aspects.

For G1 and C2, mathematics was a tool for science. Mathematics without applications ("pure mathematics") appeared to them as senseless manipulation of symbols, for the sake of training one's own mind as a student, and the minds of others as a teacher. Those who practice this kind of mathematics appeared as distant, cold, and unsympathetic towards others.

The other three interviewees did not perceive pure mathematics as negatively as the previous two. G2 and C1, who distinguished rather sharply between "pure mathematics" and "applied mathematics", had a lot of appreciation for pure mathematics. G2 admired those who were able to do it (the "smart people"), although she did not think she could do it herself; C1 enjoyed studying it because she liked the precise and clear definitions and the logical connections that made everything make sense. Both, however, craved for applications; G2 – because this was what she could do; C1 – because this was what interested and intrigued her the most.

M1 did not distinguish between pure and applied mathematics. He did not even mention "pure mathematics" and only used the word "applications" a couple of times, when saying, for example, that showing applications of mathematics is useful in popularization. For him, the important distinction was between a problem solving approach to mathematics and an investigative approach to mathematics. The difference is between stopping after having solved a problem, and using a problem to pose further questions. It seems that one can practice either, in both pure and applied mathematics.

All but M1 had ambivalent feelings towards mathematics. For M1 mathematics was like a still "largely undiscovered landscape", and "beautiful". In other members of the audience, negative feelings were expressed mostly in connection with "pure math"; even C1, who enjoyed the "pure math" approach to mathematics in elementary school, eventually was disenchanted with it. Pure mathematics was associated mostly with negatively laden expressions such as "science of symbols without any sense"; "algebra, you are never going to use it in your life"; "obscure formalism"; "septic"; "dry"; "nasty"; "scary"; "cold"; "not sympathetic to non-mathematicians"; "[can be done only by] smart people... but I can't handle it"; "I could never do it"; "something I can't do"; "you have to kill yourself to understand it". It was only very rarely associated with positive feelings: "[thanks to] logic[al connections] everything was making sense"; "everything was clearly defined"; "[can be a] very satisfying activity". Applied mathematics gained praise for being "more human"; "more interested in real life"; "closer to everyday people".

Regarding images of popularization of mathematics, it was interesting to note that, in responding to questions about popularization of mathematics, G1 did not seem to make a difference between the departmental research seminars and popular talks. It was as if he treated both as popularization, because, in the departmental seminars, he was an outsider, like a tourist in a foreign land, observing the natives of a domain in their natural habitat.

Therefore, popularization is not a property of a talk, or a book or some other event, but a property of a relation between the speaker (author, animator) and the audience member (reader, participant).

This relation must involve "intercultural communication", whether there is a conscious effort on the part of the participants to make this communication more effective or not. Something to this effect is mentioned by Penrose (2004, *Preface*) when he compares the reading of his book at the level of skipping all mathematical formulas with his reading, as a child, chess magazines without understanding the technical notation describing the detailed moves, but attending to the stories of the exploits of the chess masters. These magazines were certainly not written in the intention of popularizing chess among people who have never played chess; they were written for active chess players. Yet, the rapport of Penrose with these magazines was a "popularization relation".

Interviewees' responses suggested several reasons for participating in popularization (reading books, attending talks):

- To understand one's own domain better
- To understand real life better
- To learn something about a domain totally different from one's own
- To get an initial basic information about a topic related to one's own work
- To find inspiration for further research in mathematics.

The suggested purposes of popularization of mathematics were:

- Popularization of mathematics amongst secondary school students helps them make career choices.
- Convince people that mathematics makes sense and is interesting by showing them a wide variety of applications of mathematics.
- Change negative images and attitudes towards mathematics amongst engineers.
- Spread scientific knowledge and distinguish it from pseudo-science.

7.4 Cultural lenses of the "Escher" audience members

I interviewed six people after the "Escher" lecture. Among these six people, two were non-mathematicians, one studied Political Science (interviewee PS) and the other – Microbiology (Mb). Another interviewee was an undergraduate student of physics (in a program with strong emphasis on mathematics). The other three were graduate students in a mathematics department. One of the three was in a master's level mathematics education program (ME), and another one was doing her master's degree in mathematical physics (MPh). I chose to describe in detail the cultural profiles of PS, Mb, ME and MPh. (I included only one student of mathematics as the interview with her represented the behavior of the other two students in mathematics or in a mathematics-related domain quite well.)

Contrary to the "Medial representation" lecture, the "Escher" lecturer did not classify his expected audience into "general public" and other groups. Therefore, I cannot treat the interviewees as representatives of the lecturer's groups. I can only propose my own categories. In an effort of having some analogy with the categories of interviewees in the previous lecture, I will consider PS and Mb as representing more or less the "general public" since they were non-mathematicians and worked in domains not related in any way to the theme of the lecture. As a mathematics educator, ME was already closer related to the theme of the lecture, since Escher pictures are being used in teaching as motivation for the introduction of geometric transformations. In this sense, she corresponds to C1 and C2 in the previous lecture. Finally, the mathematical physics student MPh corresponds to M1, since they both regarded themselves as mathematicians and worked in the same domain.

7.4.1 Interviewed audience members' backgrounds

When PS was in a social sciences and humanities program at College, she took classes in mathematics as electives. She has not taken mathematics courses at the university level.

Mb had a Bachelor of Science degree in microbiology, and worked in this area "to a certain point". At the time of the interview, he mentioned marketing as his current occupation.

EM was a graduate student in a master program in mathematics education but she already had an MSc degree in mathematics. She said that she chose to study mathematics at the university because she liked mathematics in school. She never planned to become a research mathematician, however, and was more interested in its teaching and in applications of mathematics in finance and actuarial science.

MPh was a graduate student in mathematical physics. She chose to study mathematics at the university partly because her family motivated her to do so (her father and brothers were engineers or mathematicians), and partly as a result of her positive experience in school.

7.4.2 Interviewed audience members' images of mathematics

7.4.2.1 PS

PS said that she liked mathematics and found it "interesting" and "fascinating". She stressed, however, that she found it interesting only when she could understand it. She was attracted to "the whole mystery of mathematics". Part of the mystery, for her, was the inborn "gift" for mathematics that some people appear to have while others do not.

> [Mathematicians are] people who have that capacity. It's a gift. It's a gift to be born with. People have it and they know they have it and use it. And once you have it you could use it or not. It's very interesting.

She would pay attention to news about mathematicians, published in newspapers. She was aware of a Fields medal being awarded to Grigori Y. Perelman in 2006, which made news a few weeks before the interview and was in newspapers all over the world not only because one of the long-standing mathematical conjectures (the Poincaré conjecture) was finally proved but also because Perelman refused to accept the award and did not come to the awarding ceremony. There was certainly an aura of mystery surrounding Perelman and PS must have been extrapolating his genius and eccentricity to all mathematicians at the time of the interview.

7.4.2.2 Mb

Mb said that he "always had a passion for mathematics when he was young" and he was in a group of young people with similar interests. Some of his "buddies" went on to study mathematics, but he decided to use mathematics as a tool in another science rather than "do research with it". For him, "mathematics is definitely a tool that every scientist must use", and it is also "a tool that you can use every day for everything". One doesn't have to be a mathematician, however, to be creative with mathematics; he said that he was "praised for being more creative" than his friends, the mathematicians. While he believed that "anyone can be a mathematician", he observed that mathematicians have certain "distinctive" characteristics in common: "interest for complicated things"; "interest for putting things in order"; "remember[ing] patterns that it would take somebody else more time [to remember]; "[ability] to simplify some concepts to people".

He was interested in the life and work of mathematicians (especially Arabic mathematicians), because "there should be a good story behind each of them" that could show that "they are not as geeks as you think". For Mb, mathematics (and science as well) was part of general culture, as he considered himself as an "eclectic" person by saying:

> I like to know about everything. Because this is odd. I can tell you something that I love mathematics and all that. And at the same time I love literature and philosophy. Highly. Some people think that if you like one you shouldn't like the other.

This was why he perceived himself as being similar to mathematicians since "[y]ou should go back to history. The biggest mathematicians became the biggest philosophers, actually".

7.4.2.3 ME

ME saw many different aspects of mathematics and did not have a clear dislike of any of them. Similarly to PS, ME stressed she generally liked the parts of mathematics that she could understand. She was not interested in becoming a mathematics researcher herself and was interested in applications, but this did not mean that she did not appreciate the value and interest of pure mathematical research. Her husband was on his way to becoming a research mathematician, her mother was a mathematics teacher, so mathematics and the different rapports one can have with it were very much part of her everyday life. She considered mathematics as an activity that first of all requires creativity.

7.4.2.4 MPh

MPh described mathematics as a language that helps to model and therefore understand real life situations. It seemed that the lecture provided a link between the mathematical theory she learned in classes and the applications to "real life situations" which turned out to be related to art in this case. She perceived that "related to the lecture, mathematics is useful" by saying:

> It was very nice [to see] actual things that we study in mathematics... So I found it very beautiful the use of functions we actually were studying in complex analysis. They can complete pictures, it's amazing... I couldn't imagine that they can be used like that. Sometimes, it seems so abstract that we would never really think something about that.

7.4.3 Interviewed audience members' images of popularization of mathematics

7.4.3.1 PS

As mentioned above, PS liked to study mathematics and was well-informed about mathematical "news", but she did not seek to know more about mathematics through attending popular talks or reading popular books in mathematics.

7.4.3.2 Mb

Mb said that he would go to popular lectures quite often: 6-10 popular talks per year. He seemed enthusiastic about the idea of popular talks by saying:

> I think the idea is wonderful. To have those kind of talks and because, you know, we call this general culture.

He also read popular books about mathematics and physics. He quoted, among others, the names of Hubert Reeves and Stephen Hawking. His reason for going to lectures and reading was to broaden his "general culture". This was also what he considered to be the purpose of popularization: spreading scientific information to the interested public.

7.4.3.3 ME

ME did not have much previous experience with popularization. When I explicitly asked her about lectures, magazines, books or competitions, she said that since she did not like reading books in general, she did not read popular works in mathematics, either. She did participate, however, in a few mathematics competitions because she appreciated the challenge. In each case, she would seek advice of her teachers, to tell her if they thought the challenge would not be too big for her.

Popular talks, for ME, were nothing more than "enrichment activities" within institutionalized mathematics education, attended by and intended for mostly teachers and students.

> I think it is mainly for people that already live in an academic environment... It is for those who already feel a need for a cultural, scientific, or whatever information. Those who want to know what research is all about. And not tired. Because these are educational-like programs.... You sit down and learn something. But most of the people are tired. They just finished work and don't want to learn any more or use their mind after a hard day.

She interpreted popularization rather broadly, without institutional constraints; for example, talking about one's mathematical work to family and friends who are not mathematicians also counted as popularization for her. Participation in enrichment programs and reading books other than textbooks in preparation for an examination were other examples, for her, of participation in popularization. ME came to the lecture because one of her professors recommended it as an additional source for a project she was working on. Thus the position from which she was looking at the popular talk was similar to that of G2 in the previously discussed lecture: popularization is used as an extended school task (e.g. used as enrichment) and puts the participant in the role of a student.

7.4.3.4 MPh

When asked about her previous experience with popularization, MPh said that she had none unless reading science fiction counts as such. Her attendance of the "Escher" lecture was rather an exception. She found information about the talk among the mathematical announcements, on a university web page containing mathematics related events aimed primarily at mathematicians and university students of mathematics. She chose the lecture because she was interested very much in things "that involve magical mathematical works", such as the pictures of Escher and Dali. She perceived this type of popular lecture as a way of changing the popular image of mathematics. She knew people thinking that the work of a mathematician consists in being good at sums, being fast at it, smart and thinking all the time, and that mathematics is finished and involves mainly calculations. The lecture provided a good way to challenge this view.

7.4.4 Synthesis

Generally, the interviewed audience members displayed a positive attitude towards mathematics. The attitude was nuanced in PS and ME who mentioned that they like the mathematics that they can understand. Both considered that mathematicians are special people. ME attributed it to the fact that doing mathematics requires creativity while PS considered it as an inborn gift, which one either has or has not.

Mb, on the other hand, believed that anybody can be a mathematician. Still, being a mathematician means to develop certain distinctive skills such as noticing and remembering patterns, and interest in complicated things that they are able to simplify.

Both Mb and MPh stressed the relationship between mathematics and real life. MPh saw mathematics as a language that helps to describe and model real life situations. Mb considered mathematics as a tool for science and "everything" because it simplifies things and helps to see a pattern. Besides this rather pragmatic view of mathematics, both stressed its connection to general culture, as MPh expressed her interest in "magical mathematical works" done by artists and Mb was attracted not only to mathematics but also to literature and philosophy, and stressed that, in history, mathematicians would often turn to philosophy to answer questions that mathematics could not solve.

Compared to the first lecture, no sharp distinctions were made between pure and applied mathematics by the audience members. Only Mb mentioned "pure mathematics" once, when saying, "I don't say I am a pure mathematician but I have a high interest in that and I certainly have a high interest in literature". They did not stress the difference between these two "kinds" of mathematics. In fact, PS seemed indignant that mathematical work developed within a different domain is sometimes not recognized as mathematics. She gave the example of John Nash, who got a Nobel Prize in economics, not mathematics:

> It's not mathematics, it was economics. It's like the math was in economics. And for his theory he got a Nobel Prize in economics. Not in mathematics because there is no Nobel Prize in mathematics. It's well known. So then he got a Nobel Prize. But he did do a lot of mathematics also. But still his theory got the Nobel Prize in economics. It's not relevant for mathematics?

The other striking difference between the interviews after the "Medial representation" and the "Escher" lecture was the balance between positively and negatively laden emotional expressions about mathematics. Although some of the "Escher" lecture interviewees mentioned that school mathematics may make people scared, the negative expressions were not many. Mathematics was rather considered as "interesting", "fascinating", "amazing", "fun", and mathematicians were described not only as special people with a "gift" but also as "human beings who found something that is interesting".

Interviewees mentioned different reasons for participating in popularization, such as:

- to learn about art and design (thus to learn about a nonmathematical topic corresponding to the lecture);
- to be entertained, get distracted from classes;
- to learn about applications of abstract knowledge learned in mathematics courses;
- to use the acquired information for study;
- to access certain information not available from other sources;
- to learn new things;
- to hear about interesting things in mathematics (without theory).

They also suggested purposes of popularization such as follows:

- to educate people (mostly within the academia)
- to give a picture about culture in general;
- to give credit to and promote Arabic mathematicians;

- to show that mathematicians are "not as geek as you think";
- to show a good story about mathematicians;
- to show for the "people outside" that mathematics is not only about numbers;
- to entertain.

7.5 Discussion of the results

The above report of my interviews with audience member gives, of course, only a limited picture of their cultural lenses. My observations certainly cannot be generalized to popular lectures' audiences as such because the sample was very small and opportunistic. They may not even be considered as faithful representations of the cultural lenses of the interviewed people because their views may change in time and circumstances. A short conversation is often not enough to reveal the detailed aspects of these views. Thus drawing conclusions based on the interviews is certainly rather risky. There are, however, some lessons to be learned from these interviews. The striking difference between the emotional reactions of the non-mathematical audience members' after the two lectures cannot be just a coincidence. It is worth trying to understand what could have caused such reactions.

Mathematics, in general, provokes strong emotional reactions in people, and, perhaps because of its rather universally fundamental role in education (it is taught everywhere), everyone has some image of it (Furinghetti, 1993). The general opinion among mathematicians and mathematics educators is that the public image of mathematics is bad, and should be significantly improved (Howson & Kahane, 1990; Ernest, 1996, 2004; Fiori & Pellegrino, 1996; Lim, 1999). This opinion is supported by survey studies on images of mathematics, which, however, never addressed people as participants of popular events, but mainly as participants of formal education such as present or former students, or as teachers, mathematicians, or educators (e.g., Karsenty, 2004; Mura, 1993, 1995; Sterenberg, 2008; Wilson & Cooney, 2002). Ernest (1996), in giving a general overview of the public image of mathematics, noted that mathematics is often perceived as a "difficult, cold, abstract, theoretical, ultra-rational and largely masculine" domain, seen as an abstract academic subject completely remote from actual life and professions (ibid, p. 449).

Negative emotions towards mathematics were expressed more often by the interviewed audience members after the "Medial representation" lecture than after the "Escher lecture". These negative emotions, however, did not include the image of a domain "completely remote from actual life and professions". On the contrary, members of the audience were convinced that mathematics has many applications and the talk consolidated this view. All interviewed persons had positive and warm attitude towards the "applications" or "applied mathematics", which, in the talk, were conveyed by means of everyday language, vivid metaphors and dynamic visualizations involving familiar shapes. The negative emotions were associated with "pure mathematics", which, for the participants, appeared in the talk in the form of numerous technical terms, differential equations and graphs. These elements were completely out of reach for the audience and even the mathematicians did not fully understand how the differential equation was obtained, or what were, exactly, the experiments whose results were represented in the graphs. Some of the non-mathematicians were "shocked" or "distressed" by not being able to see the highly unobvious relations between the metaphors and pictures and the technical mathematical elements of the lecture. This may have led to this striking sharp distinction between "applied" and "pure mathematics" evoked by the interviewees, and the negative emotions towards the latter. In the Escher lecture the technical mathematical terms and symbols were not so many (or at least not fundamental for following the lecture) and more closely intertwined with everyday language, pictures of everyday objects and art in the presentation. This could explain why people were not separating mathematics into pure and applied.

On the other hand, this finding raises the question whether the distinction between pure and applied mathematics is widespread among the general public or not, and if yes, then do people generally have a better attitude towards applied than towards pure mathematics? It is also worth investigating if focusing on applications in mathematics teaching at school improves people's image of mathematics in general, or only of applied mathematics while pure mathematics continues to evoke negative feelings.

Some existing studies suggest that answers to these questions may depend on the age of the subjects. For example, in Lim's (1999) study of public images of mathematics in the UK, younger respondents' attitudes were significantly more negative, compared to middle aged participants'. The research showed also that young UK respondents have a much more utilitarian view of mathematics than middle aged ones (ibid, p.171). This may imply that applications of mathematics do not necessarily make mathematics more attractive or likeable to young people. Perhaps focusing on aesthetic and cultural values of mathematics rather than on their utility would be more appropriate for this age group, as suggested by certain authors (Ammari-Allahyari, 2006; Betts, 2005).

Aesthetic and cultural aspects of mathematics, whether “pure” or “applied”, may well appeal to any age group, as the audience’s reactions after the “Escher” talk seem to suggest. Nobody was making the distinction between pure and applied mathematics, and many positive emotions were expressed in relation to mathematics as such. Interviewees emphasized that mathematics is “fascinating”, “magical”, “requires creativity”, is part of general culture, etc. On the other hand, audience members found nothing “magical” or mysterious about the “Medial representation” topic; the talk was about useful research that could improve the accuracy of medical diagnosis based on pictures taken from inside a sick organ, a serious but not necessarily very aesthetic matter.

The “Medial representation” was, indeed, very serious and was taken as such by the audience. The interviewed members of the audience generally interpreted the goals of the lecture in terms of spreading information and good for recruitment purposes by showing interesting mathematical applications. The “Escher” lecture was much “lighter”, and the interviewees perceived it as entertainment.

Moreover, as I will show in the next chapter, in the “Medial representation” lecture, the interviewed non-mathematicians somehow expected to understand more of the technical mathematical aspects and were disappointed with themselves, and this may have caused them to express so many negative feelings about mathematics. In the “Escher” lecture, the non-mathematicians did not feel they “should have” understood the technical mathematics. They felt like outsiders, “tourists”, having fun in an exotic environment and so were not bothered by not understanding.

I can thus, so far, conjecture that, a lecture is more likely to induce positive feelings towards mathematics in the audience given the following conditions:

1) a closer connectedness between the mathematical notions and their visual representations,
2) stressing the cultural and aesthetic aspects of mathematics rather than serious applications,
3) making the encounter with mathematics entertaining, and
4) avoiding giving the non-mathematical audience the impression that they should know or understand the technical aspects of mathematics shown in the popular lecture,

I was trying to confirm this conjecture with the existing literature, but I have not found many relevant papers.

Somewhat relevant was Lim's (1999) paper. This research looked not only at people's images of mathematics, but also at the sources of major influence on these images. Besides mathematics teachers, parents, one's self, peers and the experience of learning mathematics in school, respondents also mentioned mass media. Some respondents said that their previous (negative) view about mathematics changed after watching mathematics-related television programs. It was not clear, however, what kinds of programs they were, and whether they satisfied the conditions proposed above or not. These conditions (at least the first three) were apparently satisfied in the Square One TV television programs, designed to popularize mathematics among high school students[51]. These programs were aligned with the school curriculum and teachers could schedule watching those programs at school as part of mathematics "enrichment activities". The influence of this program on students' images of mathematics was investigated by Debold et al. (1990). These authors claimed that viewers of the program (8- to 12-years old North-American children) referred more often to advanced mathematical content and problem solving in the interviews and in the written essays, but their views of mathematics remained focused mainly on arithmetic (similarly to non-viewers). This result suggests that just watching a popular event might enrich participants' views of mathematics but generally it will not change their already existing mathematical constructs. It is not known, however, if popularization has the same effect on adults.

Certainly, among my interviewees, G1's views of mathematics changed as a consequence of attending lectures on mathematics to include a larger scope of applications, and he also started seeing the significance of mathematical formalism for mathematical thinking.

G1 was, however, an adult, and a university educated person, not a child. This supports the conjecture that the influence of various forms of popularization on people's images of mathematics is age-sensitive. The effect might also depend on the type of activity (passive listening or engaging in some activity).

Another relevant paper was Hirano and Kawamura (2001), who argued that mathematical museums could have an effect on changing adults' negative images of

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[51] http://www.squareonetv.org/default.asp?ID=1 (viewed July 9, 2009)

mathematics and give an opportunity to understand mathematical principles experimentally. I am not aware, however, of any more systematic evaluation of the effect of popular activities on the public image of mathematics.

CHAPTER 8

ANALYSIS OF INTERVIEWS WITH MEMBERS OF THE AUDIENCE OF POPULAR LECTURES.

PART II. AUDIENCE MEMBERS' PERCEPTIONS OF THE LECTURES

8.1 INTRODUCTION

In this chapter, I look at how audience members understood the content of the lectures, how they assessed their understanding of it, what they thought about the lecturer's performance as popularizer of mathematics, and how all this compared with the lecturer's expectations and goals.

I derived this information from the interviewees' responses to my requests to summarize the main idea of the presentation; to mention the details they remembered and to describe what they found engaging or interesting in the talk and why. I also asked some specific questions about certain details of the talks. Based on the interviews, I tried to identify in what sense the reaction of the audience members met the lecturer's expectations and goals identified in the previous chapters.

At the end of the chapter, I will discuss the findings in the light of previous research.

8.2 INTERVIEW RESULTS. AUDIENCE MEMBERS' PERCEPTIONS OF THE "MEDIAL REPRESENTATION" LECTURE

In this section, I will describe the results of interviews with audience members in the

"Medial representation" lecture, regarding their understanding of the content of the lecture and what they liked or did not like about the lecture.

8.2.1 What was the talk about?

Among the interviewed audience members, only the mathematicians had no difficulty in summarizing the lecture.

8.2.1.1 G1

When I asked G1 to summarize the lecture, he could not immediately recall the subject of the talk. He appeared ashamed of not having paid more attention.

> G1: It's a very interesting question. (Pause). He was talking about (Pause) how images, (little pause) no, I can't remember any more, sorry. ... If I had known that I would be asked about that, I would have paid more attention. ... now I cannot remember any of it, the whole completely, even the title of the conference, I'm sorry. Now that I am thinking about the conference or the talk I cannot even remember its topic....

In the quote above, G1 said he would have paid more attention, had he known he would have to report on the talk. This suggests the rather significant difference between two research procedures: participants who know that they will be interviewed after the popular lecture attend to it differently than those who do not. Those who know may behave more like students than like regular popular talk audience members.

Memory came back to G1 after a few minutes. In his description, the talk was about the development of a mathematical method for analyzing complicated shapes; starting from analyzing simple shapes and gradually refining the method to analyzing more complicated ones.

> Maybe the goal was to present a tool or a way to go from a single concept or method (Pause) to see the things. I think that was the goal. The way maybe he started with very, very simple shapes and he then went to more and more complicated shapes. Now I can

> remember that. That he started with very simple, and went to complicated shapes and then went to those that are more difficult to distinguish. So he was trying to present a method or a tool, maybe mathematical tool (Pause) to find final ways to analyze complicated shapes.

He never mentioned the words "skeleton" or "panther", and never referred to medical applications. As it turned out later in the interview, it was not his goal to pay attention to the subject matter details of the lecture. For him, the lecturer was a cognitive subject that he observed as a psychologist interested in general cognitive mechanisms of advanced scientific and mathematical thinking (see Chapter 7).

8.2.1.2 G2

Similarly to G1, G2 also had problems recalling the main idea of the talk, even though she made notes during the lecture because she was there on an assignment for another course. The notes, however, contained bits of the terminology used by the lecturer, but not the basic idea or the research problem discussed. The first thing that she recalled was that the lecturer used an "analogy" but she needed her notes to recall what the analogy was about.

> I remember understanding that he used analogy at some point, as far as I remember. The analogy was the basic, well, I don't remember, I really need my notes. The thing is I took notes to prove that I was there. It is part of my work.

She referred to the analogy in her report, stating:

> Blum medial loci can be defined by fire coming inwards from the outline of the object. Where the fire quenches itself are the Blum Medial loci. This explanation was the only one of many (others were physics or computer science related) that my non-computer science trained brain even remotely grasped and understood....Blum morphologies are key in developing such algorithms, but either I did not understand the explained link between these and web repositories, or audience knowledge of it was assumed.

G2 had problems in seeing the connections among the various concepts presented in the lecture. She was, in a way, lost in the details. She remembered the grass-fire analogy, as shown above. Another detail she remembered was the image of the panther. In describing her understanding of the skeleton of the panther, she did not use the metaphor of "skeleton", however, but that of "valley".

> And the panther. Yeah, that's what made sense to me like if everything is coming into a center point, that's what made sense to me. Well, I think, there was an image, like a valley, wasn't there?

The valley metaphor that she constructed did capture some of the dynamic nature of the problem. It wasn't clear, however, if she made a link between the dynamic construction of the medial loci and the skeleton of the panther represented by the coloring. In fact, it seemed that this image did not help G2 in grasping the idea of the medial axis. A little later in the interview, she mentioned that the shape of the panther (which she called "puma" at this point) only confused her:

> See, the puma didn't make sense to me at all. Like, halfway through the lecture, when he was explaining all this divergence stuff, and then I didn't really understand. He said something like, 'everything is coming into the center', right? Or, something like 'everything is going into the center'. And then, it all collapsed. I didn't understand a word of what he was saying.

The metaphor of the valley suggests that, for G2, the colors of the animation indicated a depth in a three dimensional representation rather than a skeleton, which, although it can be obtained through a dynamic process, is still a static object.

8.2.1.3 C1

Like G1 and G2, C1 had difficulties in recalling the topic of the lecture. C1's difficulties could partly stem from the fact that she arrived late to the lecture and missed the introduction, where the main problem to be discussed in the talk was presented. She felt quite distressed about her difficulties in understanding right after the talk, to the point of losing confidence in her ability to apply mathematics in her own research. Her distress was so visible that, when she returned home that evening after the talk, her husband asked her, "what happened?".

> I went home. At home, my husband asked what happened, I said it was very bad. I didn't understand anything. It wasn't because of the lecture. The lecture was perfect. It was because of me, because of my expectations. I didn't understand anything. And it is terrible. I want to apply mathematics now and I, maybe I am wrong, maybe I can't. And my husband said: 'okay, explain it to me'. And I could remember exactly what that was. And that was the application. I think that is my interest. And that part can still trigger me, and I can build on it. If I can find some part of mathematics, which is applied, then I would be able to build on it. And of course now that you see I am trying to learn back about the linear programming, and try to apply it. It's again sort of going back to an old friend I once had. And also again, from the point of applications. But it was good, but it was good for me to know that there are a lot of things I don't know in mathematics, but there are still... the applications.

Thus, focusing on applications rather than on the mathematical theory helped C1 to calm down, and recall and summarize the lecture (for her husband, right after the lecture). She remained unsure, however, about her understanding of the lecture. In summarizing the lecture for me, she said that the talk was about finding a method of encoding information in a way that could help us to reconstruct even moving objects. She focused on the advantages of the method; its applicability for image processing of three-dimensional shapes and moving objects. She did not speak about the idea of skeleton.

> Based on what we have of the object, we can sketch the whole object. We can find the pattern, to build up from the limited amount of information that we can get from the dimension. That's what I understood. From the little information that we can have about an object, which is a dynamic object, a moving object, which is changing, so it is not a stable object, we can draw the object, we can have a good picture of the object and we can see they move. The way it is changing. That's the whole message I got. I don't know whether it was correct or not. ... I think the message somehow was clear. I don't know if the message that I got was correct, that's the thing, because since it had technical mathematical formulas and also it has this message, I said, okay, the message is, that you have application for this. But still I am not hundred percent sure that that was the message.

She implicitly referred to the idea of the points of the medial axis acting like attractors, illustrated by the image of the panther, by saying, "when they called back all the movements". Here again, she appeared to focus on the dynamic characteristics of the vector field rather than on the set of attractors.

8.2.1.4 C2

When C2 was asked to summarize the lecture, her first reaction was to say that she did "not remember anything about the specific mathematics he showed", and that she was "not very good at it".

> [T]he core of the mathematics, actually, when he explained like some functions or some manipulations with those functions that gives another thing related. I could not follow that.

In using dynamical systems in her work, she only "just know[s] the basics to apply", so when the "equations [are] different", she doesn't understand them that well.

For her, the main point of the lecture was to show a method of "manipulating the image, reduc[ing] the whole image to a few parameters, [so that] at least you can get its structure". This was, for her, analogous to children's drawings, where, say, arms or legs are represented by single lines, and the idea behind the method was to simulate, with an algorithm, the kind of cognitive processing that goes on in a child's mind.

> [R]educe the whole image to a few parameters, so at least you can get its structure. Suppose that the image is reduced to a few parameters. And the image is very, very close to how children reduce [images] to lines. The speaker thinks that the images can be reduced to lines just as children usually draw the lines at the beginning. He wonders if the computer system works like that, kind of reducing the information to single parameters. I found that very interesting.

> [T]hey work on the application of some mathematical concepts, tools, I mean, application of those things to some algorithms for computer. And analyze something, for me, an image. There were assumptions about, I mean, that they can parameterize the image in mathematics.

C2 referred to the picture of the panther as a tool to illustrate the technique used by the researchers, but she did not mention any more details about what exactly the lecturer wanted to illustrate with the panther image.

> I liked the example of the panther or tiger, I don't know which one. Maybe because it is something he used... because this was the analysis how to get results and then they are appealing things, tigers, perhaps because I like cats, maybe.

She did not appear to understand the relation between the mathematical symbolic representations of the skeleton (in the form of the differential equation) and the visual representation of the process of obtaining the skeleton for a concrete shape (that of a panther). She only said she found this picture appealing because a panther is a cat and she likes cats.

8.2.1.5 M1

For M1, the lecture was presenting a mathematical technique for constructing an algorithm for image processing and image retrieval and giving evidence for the application and effectiveness of the technique. He gave an illustration of this technique.

> And also for me, if I sit down in front of a computer and I would like to find a polka-dot puppy, and I like to find all polka-dot puppies in the world, then for me [this problem] could be deadly important. The problem, that if I give my picture to some kind of machine with the command of finding similar things to this, finding all possible images.

M1 did not mention any details, formal mathematical or other, of the talk. He said he did not understand all the mathematical details of the method, but was satisfied with "a feeling" for the problem and the connections among the ideas presented in the talk. Still, he was glad the lecturer did not completely omit the formulas in the talk.

> I wouldn't be able to write which thing was which, and of which object we have taken the vector field. But the ideas more or less came through, like what is going on there, why that surface is interesting. More or less. One could get a feeling how things are connected. It was certainly good for those who could decode the formulas, so for me, for instance it was pretty good that he talked about those. Again, the exact meaning cannot be grasped but the feeling that one has seen such thing and that this is a phenomenon of having seen something before. This is enough.

This notion of being satisfied by grasping the structure of connections among concepts without understanding the concepts themselves when listening to a lecture is probably not uncommon among mathematicians (see Sfard, 1994: 48). M1 said that not everything was clear for him, but he considered this quite natural; it did not disturb him in following the lecture.

> One learns to go on. It could be compared to a situation where you are reading something and the fact that you don't understand certain words disturbs you for a while. ... But after a while you can ignore the fact that you don't understand the thing in the middle and you can simply go on reading.

The technique of "going on" in reading to get a general idea of what a text is about, even if not every detail is clear, is well known in reading comprehension studies and the link to mathematics has been investigated by mathematics educators (see, e.g., Zack & Reid, 2003, 2004).

One of the few details M1 mentioned in his description of the lecture was the image of the panther. He did not remember the word "panther", however, and referred to the image as "leopard or jaguar". This image was meant, according to him, to convey the main idea of the talk in a visual manner.

> ... leopard or jaguar with the appropriate coloring. It would have been even better if the colors had been less blurred, but that conveyed the most important things.

He did not mention the "skeleton", however, and it seemed that, like C1, he also focused on the dynamics of the process rather than on its static outcome.

8.2.2 What is your opinion about the talk?

8.2.2.1 G1

G1's general opinion about the talk was very positive. He liked the lecture because he was interested in the topic of image processing from the point of view of cognitive psychology, and because the talk showed real life applications of mathematics.

> [I liked the lecture] because this guy was talking about – for me – all kinds of representations of images, and because I am interested in images, shapes and forms I have in my mind, in my brain. (...)
>
> When this guy tried to explain the concepts, the mathematical concepts applied to the real life, matching with the real life. (...)
>
> Well, when you try a test, trying to detect [visual] impairment in some people. For example, some people cannot detect some details in some figures, in some shapes. There are people who cannot see eyes, or cannot see ears, or they cannot see details like hair, and then the psychologist presents to the people different shapes in order to detect specifically where the problem is. That was very similar.

G1 mentioned that the mathematical symbolism shown in the lecture did not disturb him.

> When I go to any of these talks, I focus on the process of the topic and the parts that are especially meant to explain the ideas. More frequently, I wasn't deep into the details of the talks. I am going like, okay, this guy is talking about doing this.... I usually don't go deep into the details.

It seemed that he simply considered these more technical parts as additional information for specialists.

8.2.2.2 G2

G2's main impression of the lecture was the feeling of being "overwhelmed" and "taken aback" by the presentation, which was one of a series of science lectures she was forced to attend as part of the requirements for a course.

> This one, I actually found the most specialized among all of [the lectures I have attended before]. Maybe it's because I don't know anything about it, like I don't know things like algorithms and things like that, but I found this one the hardest to follow out of all of them. And that is just speaking about the material, not the presentation.
>
> But since I haven't seen any talk in computer science or any advanced talk in math, I was more taken aback. Like, usually, if I go to these really advanced things, somehow I follow along. [But in this lecture] I felt overwhelmed and I was just, like, wow.

In her report she made her feelings even clearer and described her experience as an "academic shock":

> [The lecture] was an academic shock for me. Never before had I been exposed to a lecture or topic where I would be.so completely lost. Therefore, the following summary is a highlight of points I think I grasped as the talk progressed, but do not swear to be accurate.
>
> [A]s I quickly realized I was going to have a tough time understanding. I glanced around and noticed a few people from my class whose facial expressions I was unable to read but also noted that the audience was predominantly male.

She did not try to blame her difficulties on the lecturer; she recognized that communicating cutting-edge scientific research is a hard thing to do, because it is formulated in a specialized "jargon" that is hard to avoid in speaking about it. She did not suggest eliminating the "jargon" from the lecture.

> Because [the lectures] should be cutting-edge but they should also be good for public and you can't, like it is hard to do, right? Like science is. All people use jargon.

She tended to blame her own lack of foundations in the subject of the talk for her difficulties of understanding.

> Well I think I have the foundation of biology, so I could built on it more. Whereas this one, I think, I lack the foundation. One thing I did relate with in this one though, is [when] he was talking about applying it to about the biological models like going through the hole and in the arteries, and that I could relate to, that I could understand.

In the above quotation, G2 was referring to applications of the presented method in medicine mentioned in the lecture.

8.2.2.3 C1

C1 was less indulgent for the lecturer than G2 to have used so much mathematical formalism and specialized terminology in a talk that was advertised as "open to the public". She was especially critical about referring to methods by the names of their inventors (Blum), or by some strange words ("medial axis") because they were empty names for her. She said this kind of discourse would be appropriate only in a regular course, where the method is thoroughly described and studied. Then the lecturer can refer to the method by a name. She noticed that, during the question period, some members of the audience were mentioning those "big words", and had "big questions", so the technical parts of the lecture were "probably very valuable" for them. As for herself, she couldn't "remember any of those names that this person used about what is this method or that method, because to me it is blablabla, it is nonsense because I have never heard that word before". She found that it was not appropriate to refer to concepts and methods by their technical names in a talk for "people who were not mathematicians and actually scared of math".

She felt diminished in her confidence about her own mathematical knowledge after the talk. She used to be successful in mathematics as a student, and she expected to understand more from the lecture. Now she felt "disappointed in herself". She used to be good at understanding and using symbolic notations; in fact, this is what she enjoyed most about mathematics. Yet, she could not follow the notation used by the lecturer.

> I was actually a bit disappointed in myself. Because there is (Pause) my love for math, although I have left it alone. Although I have this big love, I haven't paid much attention to it, I haven't been around it as much. But I thought I could still understand much more. ... I expected more from myself to understand. It was hard for me to follow. Especially when it was getting to the formulas. The thing that I loved for so long. To formulate the things. It was sort of strange in front of me, and I thought, oh my God, it's not my friend anymore. And that was a bit sad for me. But it was the expectation, the wrong expectation.

Despite her disappointment, she still perceived the goal of the lecture in terms of improving the image of mathematics, even to convince students to pursue a career in mathematics (or in related fields, such as engineering) which seems to contradict her opinion that the lecture was not appropriate for non-mathematicians. This is what she said about the goal of the lecture:

> I think the goal of the lecture, or maybe it is the way I wanted to see, was to ...make mathematics not scary, make it more popular. So it has usage. It has to be creative and it was. I think that was to introduce it more to the human beings. Among the engineers, they have all kinds of degrees but when it comes to a formula and mathematics, oh my god. Like if it is scary, nasty, difficult part. It is like septic, dry, not easy to understand. You have to have so much logic to kill yourself to understand. That is the image among the people. So if we could have used it, if we could have presented it better. It is the way that in my life, and in my career I choose. Because the lecture showed real applications, and [made it] not to be scared of.

She remarked that images and charts used in the lecture were helpful in grasping the general idea ("whole picture") of the topic.

> If you give an image, if you give a chart, if you give something ... intuitively, it is much easier to get a grasp. A grasp to remember. Because I am not going to be a mathematician by the lecture. I am not going to learn that method. But it's good to have the whole picture.

She mentioned some images used in the lecture (a chair in an upside down position, the panther) as helping in grasping "the whole picture".

> I am a visual person, so the image, and especially the image ... like of the chair, upside down, and it was especially the animal when they called it back all the movement. That gave me the whole picture, the message of the lecture. That was very useful.

She did not go into the details of what exactly in these images was helpful.

8.2.2.4 C2

Despite not having grasped the formal mathematical parts of the lecture, C2 still found the talk enjoyable, much more entertaining than C1 did. She liked it because the talk was related to engineering, and she was an engineer, too, and was familiar with some of the problems mentioned in the lecture. She found the main idea of the presented method (reduction of the number of parameters to obtain a simpler structure) very interesting. She also appreciated the visual representations in the talk.

She thought that the talk would be appropriate for an audience of mathematicians and students of mathematics but not for a "*general* general public".

8.2.2.5 M1

M1's general opinion about the lecture was positive. He said that he especially liked the main problem posed by the lecturer and the clarity and good organization of the presentation.

> I really liked the problem posed. He talked very clearly about the problem, why it is interesting and what is our goal with this. What would be the final goal toward which we are approaching. And he could certainly communicate this message. Practically he could tell what their research group is working on. And then, more or less successfully, he tried to sketch the mathematical background of this.

M1 said that he could connect the lecture to the things he was currently working on.

> I particularly liked also, and I actually paid attention to this part, that it seemed to me that a lot of things, like the type of figures he was working with, and the tools he applied could probably be useful in what I have to do now. So that attracted me. And what he did was really interesting. And then I got a bit scared when I saw the first slide with the algorithms because it brought back a memory of the horribly boring lectures [given by one of my former professors].

M1 also said that, since there was "mathematics" in the title, it was to be expected that there would be some mathematical symbolism in the lecture. For him, it showed that the research required serious work and indicated the depth of the subject. He remarked that although the technical part could be "frightening" for a non-mathematician, the lecturer still managed to "convey the most important things" using natural language and visualizations.

8.2.3 How close was the audience members' perception of the talk to the lecturer's expectations?

In this section I will compare the lecturer's expectations about the audience's cultural lenses and interpretation of the talk with what actually happened in the case of the interviewees. In particular, I give a comparative summary of the expectations and actual facts on the interviewed audience members' background, images of mathematics and popularization and their understanding of the lecture.

8.2.3.1 Background

The lecturer expected a diversity of backgrounds in his audience. This expectation was certainly satisfied, based on the sample I was able to meet and interview. They were not all computer scientists or mathematicians.

One category of audience members the lecturer anticipated was the "general public", of whom he did not expect to have more than an everyday experience with vision and image analysis, the ability of learning through examples, and some awareness of scientific methods such as hypothesis testing or using mathematical formalism. Anybody with secondary education would have satisfied these expectations. All participants that I have interviewed exceeded, however, these expectations, as they either already had a university degree or were on their way of getting one. Even G1 (psychologist) and G2 (biology student), who were neither computer scientists nor mathematicians, had, nevertheless, some professional knowledge related to the topic of the lecture (visual cognition, mathematical models in biology).

Regarding computer scientists, the lecturer expected them to be familiar with data structures, algorithms and computational geometry.

Based on their background, C1 and C2 were possibly familiar with data structures and algorithms that are parts of the core curriculum in computer science. However, they did not necessarily have previous experience with computational geometry. In spite of their computer science background, they stressed their lack of experience with mathematical formalism and they did not emphasize their previous experience with computer science when they talked about the lecture.

According to his background in mathematics, M1 could be classified as a member of the mathematicians belonging to the group with previous knowledge on differential equations and partial differential equations ('group c"). In a way he also exceeded the expectations of the lecturer by mentioning that he was working on a related topic.

8.2.3.2 Images of mathematics

While the lecturer considered the topic he presented (and was also working on) as a "transition between pure mathematics to applied mathematics", he did not succeed in showing this transition in the lecture since the audience members generally saw pure and applied mathematics as separate domains.

The lecturer considered the mathematical foundations of his research as "intuitive"; he said that the problem he was working on did not require any "esoteric advanced mathematics to get ahead". This could have been his own experience of learning the mathematics he used. Except for the mathematician, however, and the psychologist who did not even pay attention to the mathematical details, the audience members, found the technical mathematical aspects overwhelming. Even the mathematician did not use the word "intuitive" in relation to the differential equation displayed in the lecture.

8.2.3.3 Images of popularization of mathematics

In the interview, the lecturer said that he perceived popularization as a way of "making a technical subject non-technical" in a thought-provoking and inspiring way. A popular talk should be able to inspire the mathematicians in the audience to apply some of the presented idea in their own domains.

The interviewed audience members shared the lecturer's opinion that popularization should make the technical non-technical, but several perceived this particular lecture as too technical.

The lecturer's notion that popularization can be used for getting ideas to apply in one's own domain were apparently shared by some of the audience members (e.g. G1, M1), but it was only one among several possible purposes of popularization given by the interviewees. Different views of popularization could also affect how one interprets the lecture or processes the presented information. If an audience member wants to use a lecture as a source of inspiration then he or she only takes the part that is inspiring and feels satisfied. This is what happened in the case of G1, who related the presentation to his experience of the psychological aspects of image processing. G1 described his mathematical background as "only basics" and seemed not to expect to understand the technical part of the lecture. He did not perceive his lack of mathematical foundations as an obstacle to understanding the lecture; rather, he tried to find the part applicable in his profession and relate it to what he had heard. Instead of learning through examples (as the lecturer expected) he focused only on one example, namely, the first one presented by the speaker.

On the other hand, G2 interpreted popularization as a distraction from her regular classes. She sought something different from her own domain in attending popular talks. Therefore, she could not have been looking for something to inspire her in her own domain. Rather, she was trying to learn something completely new to her. This was an overly ambitious goal. The lecturer did not expect to teach anybody his domain of research in one hour. G2 was trying to achieve the impossible. Like a student, she paid attention to the details, took notes, got all the difficult terminology down, but ended up not grasping the main idea of the talk, even in the most general terms.

8.2.3.4 Understanding of the lecture

The lecturer expected the general audience to understand the following from his talk:

> As to the general audience, I wanted to get them a feeling that when you compare objects, often you do so in terms of the points. And these medial loci give me a way of thinking about parts of an object. That's mathematical, that's precise but it's also algorithmic. You can compute the parts, represent them and then apply them to, for example, to 3-D objects, to mention a trivial problem as a starting point.

He wanted people to understand that the problem could be represented by a mathematical model leading to an algorithm that could be implemented in an appropriate computer environment, that the technique could be applied in various situations and that the effectiveness of the model could be shown (at least according to a certain measure).

These expectations were not quite met, however, neither in the general public representatives, nor in the computer scientists, and only the mathematician came close to it. According to the interviewed audience members, the lecture was about

- A mathematical method for analyzing complicated shapes (G1)
- Reconstructing a whole object based on limited information about it, even if it is changing or moving (C1)
- Reducing the image of a whole object to a few parameters to get a grasp of its structure (C2) (Note: this process is inverse to the previous one)
- Presenting a mathematical technique for the construction of an algorithm for image processing and image retrieval and giving evidence for the application and effectiveness of the technique (M1)

None of these descriptions, alone, gives a sufficient idea of the content of the lecture; collectively, however, they are better, since each of the above statements describes some aspect of the lecture.

While G1 focused on the main problem of how to compare objects, he did not really make the connection with the method of medial representations and its technical realization presented in the talk. He attended to the fact that the problem captured the attention of applied mathematicians and computer scientists, because, for him, this used to be a problem of cognitive psychology. As mentioned above, G2 could not even grasp the main idea of the talk; she was lost in details.

The lecturer had additional expectations with regard to computer scientists:

> To the computer science oriented people, I wanted to say that the intuition and the continuous mathematics is really useful because it provides new ways and new algorithms for computing. And that one would be tempted to use much more direct algorithms, but that's basically failed. They have no generalization to three dimensions. It's too much complexity, nothing is stable but fragile. It's a lot of heuristic assumptions you have to make. So then the main point I wanted to convey was that, having even a single theorem that maps from the theory to applications is very useful....

Neither C1 nor C2, however, paid attention to the algorithmic part of the lecture, and only C2 mentioned that the research group represented by the lecturer was working on implementing methods of image analysis to "computer systems". Although C1 stressed that engineers and computer scientists need to appreciate the importance of mathematical theory and methods more, she did not make any *specific* comments on applications, in computer science or in the construction of search engines, of the results presented in the lecture. C1 and C2 understood the lecture more in the way expected of the general public; they referred to the connection of medial representations with reducing images to information coded by points (vectors) of the medial axis and using it for comparing images.

Finally, regarding mathematicians in the audience, the lecturer had the following message:

> For the mathematically sophisticated, the main goal was to communicate the idea that there is really a new way of thinking about, precisely, the differential geometry of curves and surfaces. And that there is a rich way of doing all of that through this medial representation along with the details. They will develop an interest about a lot of aspects.

None of the mathematicians I have interviewed met this expectation. Nobody saw the presented method as a precise mathematical way of thinking about curves and surfaces in general; rather, they perceived it as a method to be applied in medicine or in search engines. They all attended only to the aspects the lecturer expected from the general public. They noticed the mathematical formalizations, of course, but have not tried understanding them in detail. The difference between them and some of the other interviewed participants was that the mathematicians were not disturbed in the least by not understanding the mathematical details.

The lecturer thought that the main idea of the lecture could be grasped based on everyday experience regarding human vision, and some very general notion of scientific method. The interviewed members of the audience had much more knowledge than that, yet nobody had the impression of having understood the lecture fully. One person (G2) could not even say what the main idea of the lecture was, and the other four gave only very vague descriptions. Already recalling what the lecture was about was a problem for three people (G1, G2 and C1), and C2 said she did not remember the "specific mathematics he showed" because she did not understand it. The psychologist G1 did not remember the content details because he was not paying attention to them. The biology student G2 had a fragmented picture of the lecture, made of disconnected details. She remembered the panther image but "did not understand a word" from the verbal explanations accompanying it. One of the computer scientists, C1, said she had the feeling of not having understood anything and was extremely upset about it. The other, C2, said she did not understand the mathematics in the lecture. Only the mathematician (M1) did not complain about not remembering what the lecture was about in general or some of the details, even if he, too, did not remember the details and did not understand the mathematical models to the point of being able to reproduce them by himself.

The lecturer expected the audience to be prepared to see some mathematical formalism. He did not intend to intimidate anyone with this formalism. Yet, this is what happened in the case of some audience members, even in those who applied mathematics in their work.

What were the details that audience members remembered from the lecture (except for the mathematical formulas that scared some of them)? All except the psychologist (G1) mentioned the panther image. The psychologist paid no attention to details and only distinguished between "simple shapes" and "more complicated shapes".

The image of the panther seemed to be interpreted in various ways by the audience members. According to Presmeg (1992), experts' visual images function as "pattern imagery", where the concrete details of the image are disregarded and only pure relationships stay in memory. The interviewed audience members did not seem to behave as "experts", however. They focused on rather insignificant details of the picture, interpreting, for example, the colors as an illustration of depth and the skeleton as a "valley", or thinking about the panther as a cute cat, rather than as a non-trivial shape used to illustrate the technique of finding the skeleton. They also paid attention more to the dynamic process, visualized in the colored animation of the panther image, than to the outcome of the process in the form of the skeleton.

The lecturer assumed the ability of "learning from examples" in the audience, particularly in representatives of the "general public". The person who was perhaps the closest to his definition of general public, G2, was not, however, able to learn from examples. Examples were not enough for her to understand the main idea of the lecture. She could recall some elements of the examples given, but she could not understand what these examples were examples of, and her memory of the lecture remained very fragmented. The other "general public" representative, G1, mentioned only one example in the interview, namely the problem of categorizing images that the lecturer mentioned in his introduction as a motivation for his research.

8.3 INTERVIEW RESULTS, PART II: AUDIENCE MEMBERS' PERCEPTIONS OF THE "ESCHER" LECTURE.

In this section, I describe in detail the profiles of four audience members that I have interviewed. Two among them had a profession outside of mathematics, and two were students of mathematics. Similarly as in the previous section, I will describe the audience members' summaries of the main ideas of the lecture and their opinions about the talk. I will then compare them with the lecturer's expectations.

8.3.1 What was the talk about?

8.3.1.1 PS

For PS, the main idea of the talk was in "repeating forever" a picture "one inside the other". She thus focused on the idea of infinite iteration of a transformation as exemplified in the Droste-effect.

> He explained with the picture of the objects inside the design and the perfect equation between the angle (incomprehensible) the pi... And took examples like the cheese, 'La vache qui rit', or the cocoa box in Netherlands. And there were some designs repeating forever one inside the other. And then he explained how this could happen mathematically.

I had to ask her about the problem of filling in the hole in Escher's *Print Gallery*, to get her to talk about it. She had some idea about the method but could not recall the details. She indicated that for completing the picture mathematicians needed to "turn back the technique of production" and then "put it into perspective".

> KK: How did they fill the hole?
>
> E1: He did it by repeating the same technique, the technique of production and turned it back. Just repeated the same effect and filled the center.... So they could continue the perspective.

KK: So they finished the original one by drawing smaller and smaller pictures. Or did they do it in another way?

E1: They did it in another way. They filled it up and then they put the distance smaller and smaller and smaller. First they filled it up, I don't remember where and then ... they put some design on the picture, and the kind of motive and they finished the work with kind of something missing. And they just filled it all up. With the same idea as Escher.... They calculated mathematically... They drew the graphics and filled the hole by just completing the picture with the same idea. Same design and colors and anything. And they kind of finished it. And then they put it into perspective and it's about it. Then filled up the hole and then we could see the perspective.

What seems to have impressed her most was the Droste-effect introduced by the lecturer. She constructed her own metaphor for it, based on the image of "Russian dolls", idealized so that the repetition "never stops", and ever smaller and smaller but otherwise identical dolls are inserted into one another.

Because we had the cheese, we had the cocoa, we had everything in that prospect. I mean man doesn't realize how difficult it is to get that perfect effect. And it goes on forever because... it's, like, forever. And of thinking prospect of forever. So that was interesting. Like the Russian doll, you open one and another and one inside one inside one inside. And so it never stops. Well, the Russian doll has to stop but there you don't.

Even though the finite process represented by the actual Russian doll object captured the idea of iteration, PS still added that this should be changed to a situation where the iteration "never stops" and "goes like forever". This new example differed in various aspects from those chosen by the lecturer. It captured the idea of iteration but might not have served as an appropriate introduction for the conformal maps later.

8.3.1.2 Mb

When I asked Mb to summarize the talk, he listed some details of the presentation focusing on features that he found interesting in the talk.

OK, long day. Seriously I was about to sleep because I had a long, long day. And he started with this drawing. I looked at it, this is very interesting... Then he started talking about this Droste-effect, and all that. Yeah, I know this, I am sure. And then the "La vache qui rit" came, of course this is the Droste-effect with rotation of 90 degrees. OK.

> Then I knew things where he was going. And I just having this whole day in my body, and I just wanted to go sleep and that's it. It's only when he started to take this kind of Droste-effect and he explained the "La vache qui rit" and all that, and I said, oh my god, this is highly interesting. Now I see something that I didn't see at first when I looked at the picture. And I was all the more impressed also by the animation that was behind it. Because he took the basic drawing, and applied it. I don't remember this kind of mathematical effect... So he took the basic drawing, and he applied the same effect having this kind of point over there and boom, boom, boom. Can I say I would have never ever thought of this? So I said this is highly interesting. And my interest is high like that, to learn that even more. Take a picture and try to apply other things that you would do yourself or something like that. So that's exactly how I felt.

As for the mathematical formalism, Mb seemed to have ignored it and did not seem to be bothered with not understanding it.

Mb noted that he found it very interesting that it is possible to look at the "Print Gallery" from a different perspective. He expressed that he was amazed by the underlying mathematical structure.

> I probably have seen it in a book or in a magazine or anywhere else. And I would have looked at it, I said, wow, interesting, very interesting but I have never thought that there is a Droste-effect or anything like that. So I would have probably in a real life situation, without these kind of lectures attended, I would have looked at it for a couple of seconds, 15 seconds, would appreciate the drawing aspects of it, the creativity of the person, but I'd never have understood. I would not have been able to understand it myself what was the rationale behind the construction of this image. So this is what I have learned. How this image actually has an algorithm behind.

Regarding the main idea of filling the hole, Mb seemed to focus on the process. It was not clear, however, if he distinguished the back and forth maps for this process.

> Mb:...You have this man that is looking the basic one, without the effect. He is in this gallery and he is looking at this picture which represents the museum of the gallery where he is, and he can see himself looking the same gallery, basically. So he cut at that level, where this kind of Droste-effect is happening and you make this rotation. This is how I remember. So I remember ... you basically turn at the picture that is been created with this illusion. So it is how I remember. You cut and you turn. And you make two points that are similar in the two pictures together. So this is what I remember.
>
> KK: They said that there is a hole and they had to ask some people to complete the picture because it wasn't finished. How did they do this?
>
> Mb: When you do this kind of calculations, mathematical calculations and you use the perfect angle and the perfect ratio, I don't remember what the ratio was, you end up, I think it was the computer who did it. But anyway you can easily put everything back.

And I think Escher didn't have the perfect ... angle.

KK: The picture wasn't finished and they had to ask some designers to finish. Did they draw these things inside the picture?

Mb: I think they started from the basic picture again, and they applied the same concept from everything is back to normal. I don't remember actually.

Mb mentioned various details regarding the presentation especially in connection with the Droste-effect combined with transformations. He referred to it as repeating images revealed by zooming into the image, and distinguished two different kinds of concepts, the "basic Droste-effect" illustrated by the cocoa-box, and the Droste-effect with rotation exemplified in the "La vache qui rit" cheese box. He interpreted the iteration presented by the Droste picture in terms of repeating images where by zooming in the picture one can see "another image in the same one", but he never mentioned that the presented picture was meant to illustrate an "infinite repetition".

8.3.1.3 ME

When I asked ME to summarize the lecture, she described the process sequentially by focusing on as many details as possible, and by using expressions such as "well, it started with the cocoa", "and then", "after this" "wait, what kind of other pictures he had". She listed as many pictures as possible, a joke related to the picture as well, or circumstances that disturbed her like the following:

Because I had to pay attention for such a long time and some also asked questions and everything. Because I always tried to pay attention that for the cuts when will the window meet the window. But I should consider the other points too.

She described the main idea of the lecture as part of this sequential description as:

And then he asked the question that they had in connection with the Escher picture, how to fill the hole in the middle. This was very important and of course also that by what kind of mathematical formula or transformation we can push it.

Regarding the Droste-effect, ME only briefly mentioned the simple Droste-effect – "Well, it started with the cocoa and then he showed on a bunch of other pictures with this effect" – and then went on to give a detailed description of the process of forming the distorted Droste-pictures from general images:

> And then all kinds of animations how we can get back this Droste-picture from the doubly periodic one. And at the end, this is nice, how the Escher picture came from this. And this gave me really a lot. And of course I should have looked longer at that cut that he made, from the middle he made a cut and there he rotated it. ...And for the other he rotated everything... On the doubly periodic picture, he made two parallel cuts and he translated it. But I had to look at it for such a long time ... and then I always looked at when the windows meet for the cuts. But I should have paid attention to the other points too.

As seen from the excerpt above, her previous experience and expectations already shaped the way she looked at the lecture, focusing on information she identified as "useful" for her project. She interpreted the Droste-effect in terms of "always decreasing" and "self repeating images", and thought of it in the context of covering space on the complex plane. This interpretation did not appear in the interviews with PS and Mb.

8.3.1.4 MPh

MPh summarized the lecture by emphasizing the idea of the Droste-effect and the application of mathematical transformations to distort images. In the description, she stressed the fact that the picture should be "self connected", possibly referring to the fact that appropriate points of the image should be identified to produce a "nice" self-repeating picture.

> One of the main ideas was to show the Droste-effect. And its different variations. It was very fundamental. The boat, this can be one of the main ideas. The boat is kind of this cruise, the pool's inside and the replica of the pool's inside. So that was one thing and then show how to use mathematical functions for playing with images and make them do this kind of things and make this ... self connected and self contained in some way

and then how finally they just completed the Escher picture.

MPh described the idea of filling the hole by connecting it to the applied mathematical techniques and to their visual representations used in other situations in the talk as well:

> Well, first they drew new things which were missing from the original picture... And then they applied the cut and paste method that he showed with the functions. He played with the one of the snakes.
>
> ... they drew it outside, and they pasted it then and got inside in some sort of, how was it called, the Droste-effect? Because first you had those things, that's what I remember, the sketches. And they were not complete because there was this circle the middle. And then he did a grid in order to transform everything and make them to look self contained. Without the power point picture there was growing and it was the building the guy was in the picture inside.
>
> ... they did the same [as Escher] but they completed the part that was missing. So with the complete pictures they used those functions that they had for the snakes and the -2 snakes.

MPh described the Droste-effect by focusing on its property of self-repeating images that evoked, for her, the notion of fractal:

> The one about chocolate box with a nurse with the same box? It was like a fractal. So if you got inside the picture and went to the smaller box, you'd find the same picture, and if you got inside that...

Similarly to ME, MPh also interpreted the Droste effect in terms of "self repeating images"; however, she connected it to a new visual image, a fractal.

8.3.2 What is your opinion about the talk?

In this section, I will summarize the interviewees' opinions about the talk.

8.3.2.1 PS

PS interpreted the talk primarily as a positive, relaxing, "entertaining" experience.

> I found it was very entertaining. It was a little bit tough when he started putting it in a mathematical figure, but, of course, that was between mathematicians. So they got it. Everybody was laughing and having a good time. So I think it was okay.

Even if she sometimes felt as an "outsider", for not understanding the mathematical formulas, she was not particularly stressed by it. Rather, she felt as a tourist among natives who naturally speak their own language among themselves. This could have been, in fact, part of the fun. It was enough that the lecturer talked to the "foreigners" like her from time to time and gave simple examples and explanations that they could understand.

> It was that the speaker was very humorous, so I was very amazed to have simple example for people like me. To have an idea what Escher was seeing when you have to look at [his pictures]. Because when you get to the picture of Escher with this staircase and the persons then you understand better the idea of Escher.

When I asked her if she understood the lecture, she answered positively. She did not understand the formal part of the lecture ("mathematical equations" in her words) but she did not expect to understand it, in any case. She was satisfied with her understanding.

8.3.2.2 Mb

Mb heard about the lecture from a friend who saw it advertised in the newspaper. He was interested because he was already familiar with Escher's drawings but was curious to see how they could be analyzed scientifically. He was not disappointed in his expectations and liked the lecture very much. It amazed him and stimulated his interest to the point of wanting to learn more about it and see how the method can be applied to create other similar pictures.

Asked if he felt he understood the lecture, he said, "Fully. Absolutely. Every single aspect of it." He also said that he found the flow of the lecture too slow at times, and the lecturer could have gone faster by referring the audience to additional information in the literature or on the web.

8.3.2.3 ME

ME mentioned that because of her project, she read about the presented material before the lecture. In fact, she used the lecture as an additional source of information for her project preparation. She stressed that she liked the talk especially because the displayed animations were unavailable in the printed material she had previously found. Besides her previous readings on the topic, she also connected the lecture to her personal experience: "I enjoyed the example with the cow, because, well, you know, sometimes I have the cheese for breakfast … ".

ME perceived that she understood the first part of the lecture which she connected also with affective components: "I certainly understood the first part. Because I also liked it and in general I usually like what I understand". ME considered the lack of formal mathematical knowledge as the source of difficulties for understanding other parts of the talk: "when he brought in the snakes, I probably did not understand it because I did not understand its mathematical foundation." It seemed, however, that in the end she was satisfied with her own understanding. From the interview, it seemed that she already had a clear idea what she intended to use from the lecture in her project: "... and I think I understood what I needed". This is again similar to the satisfaction of G2 who managed, after all, to write a report about the lecture for her class. Both G2 and ME, therefore, entertained a student's relation with the lectures, and not a popularization relation.

8.3.2.4 MPh

MPh found the lecture "really nice". She mentioned that it was particularly interesting for her because at the same time she had a class devoted to a related topic. She also liked the application of pure mathematics in the arts.

> It was very nice to see actual things that we study in mathematics. Something like that, you know. It's not only numbers, like this. Probably for the people outside. They know how to do this or that. It has nothing to do with arithmetic for instance. So I found it very beautiful the use of functions we actually were studying in complex analysis. They can complete pictures. It is amazing. So that's what I have learned. I couldn't imagine they can but used like that, sometimes, it seems I don't know, so abstract that we would never really thought something about that.

8.3.3 How close was the audience members' perception of the talk to the lecturer's expectations?

The section is organized similarly to that on the "Medial representation" lecture presented in Section 8.2.3.

8.3.3.1 Background

The lecturer expected that at least some people in the audience would have no previous knowledge of some of the mathematical concepts he was planning to use in the lecture (especially complex numbers). This is why he thought he should provide the audience with alternative means of understanding the idea, using visualizations of the transformations on examples of simpler pictures.

There were, indeed, some such people in the audience; PS was an obvious example. Mb had probably not taken a course on complex analysis before but he may have heard about complex numbers. The "general public" representatives, however, seemed to be outnumbered by mathematicians, and PS had the impression that the talk was mainly for mathematicians and among mathematicians.

8.3.3.2 Images of mathematics

The main goal of the lecturer was to convey to non-mathematicians an image of mathematics as an activity that is "fun" and "interesting". He also said that mathematics should be presented as an art, which is an inspired, prospering, informal and enjoyable activity. He wanted to convey the "surprise of discovery" experienced by mathematicians.

The interviews suggest that the lecturer did, indeed, succeed with communicating the above image to the audience. The interviewees mentioned that the lecture was "such a good fun" and "highly interesting", showing that mathematics is "fascinating" and "creative".

8.3.3.3 Images of popularization of mathematics

The lecturer viewed popularization mainly as a way to improve the public image of mathematics. The interviewees perceived the talk as an entertainment or an additional source of information. The former seemed to fit better with the lecturer's idea: one of the audience members noted: "if I want to learn mathematics I generally take a textbook". These audience members were more satisfied than those who sought learning some new mathematics.

8.3.3.4 Understanding of the lecture

Unlike the lecturer of the "Medial representation" talk, the lecturer of the second talk did not explicitly assume any previous knowledge (whether mathematical or non-mathematical) from the audience. He wanted the audience to remember, from the talk,

- The visual material (mainly the animations)
- The question posed and solved by mathematicians (How to fill the hole in Escher's drawing)
- The infinite repetition suggested in Escher's picture and also emphasized in the lecture

All interviewed audience members recalled (at least a part of) the visual material and seemed to hold vivid images of the pictures and animations presented in the talk. They not only remembered the images but all emphasized that they really enjoyed them.

The main question of the talk was interpreted by the interviewees as follows:

- Filled the hole by calculating mathematically (PS)
- Applying the Droste-effect to various pictures including Escher's drawing (Mb)
- Fill the hole in the middle of the picture by finding the formula of the appropriate mathematical transformation (ME)
- How to use mathematical functions to play with images and make them self-contained and in this way complete the Escher picture (MPh)

Thus, the second expectation of the lecturer was also mainly fulfilled as most of the interviewees recalled the question (Fill the hole in Escher's drawing) and mentioned that it was solved by a mathematical technique. The details of how these mathematical methods were applied, however, seemed to elude especially the lay public.

The infinite repetition in the Droste-effect was also clearly described by the audience members I have interviewed. What's more, they all distinguished between the different cases when the repetition involved only zooming and when it contained rotation as well. In fact, participants referred to the infinite repetition by reconstructing the image. The interpretations are summarized as follows:

- Russian dolls (but never stops)
- Repeating images when zooming inside
- Always decreasing self-repeating images
- A fractal

It seems that the lecturer's expectations regarding the audience's understanding were generally fulfilled. It must be stressed, however, that these expectations were not very ambitious. It seemed from the interviews that, although audience members enjoyed the colorful animations and the style of the presentation, they were not necessarily able to interpret the mathematical ideas in it. Thus, the glimpse it did provide was a rather superficial view of the mathematical culture.

Participants seemed to interpret the visual representations (the Droste picture and the picture on "La vache qui rit" cheese box) of the infinite repetition according to the lecturer's intentions. For example, in the case of the Droste picture they focused on the fact that the embedded pictures were identical to each other up to scaling; in some cases even the infinite nature of the iteration was emphasized.

8.4 Discussion and Conclusions

In this section, I will first discuss the differences in the two lectures audiences' perceptions of their understanding, seeking their possible causes. Then, I will discuss a possible more objective measure to assess the two lectures audiences' understanding of the lectures, inspired by Goffree's (1989) model of understanding lectures. Finally, I will question the lecturers' belief that visualization facilitates communication of abstract mathematical ideas and is particularly appropriate in popularization.

8.4.1 Differences in the audiences' perceptions of their understanding of the lectures

The interviewed audience members were considerably happier after the "Escher" lecture than after the "Medial representation" lecture. Some found the mathematics in the "Medial representation" lecture intimidating, while not understanding the more technical mathematical parts did not appear to bother the audience of the "Escher" lecture.

The "Medial representation" lecturer certainly underestimated the level of mathematical knowledge necessary to understand his lecture. He considered the mathematical foundations of the research he was presenting "intuitive" and basic and did not think it was necessary to "translate" this research into the language of a "guided tour" appropriate for popularization. The content and the organization of the talk were not different from the published research papers on the topic. The "Escher" lecture was, on the other hand, considerably different from the research paper on which it was based.

The "Medial representation" audience members' disappointment with their understanding cannot be blamed on the lecturer's lack of communication skills alone. Indeed, his relation with the topic of his lecture was more a "research relation" than a "popularization relation". In spite of this, however, some audience members were able to build their perception of the talk on a "popularization relation" and ended up being quite satisfied with their experience (G1, C2, M1). The worst experience was that of audience members who either expected to follow the lecture as well as if it were a research lecture in their own domain (C1), or had a "student's relation" with the lecture (G2). In the

"Escher" lecture, there was also a student with a "student's relation" with the lecture (ME), and, while she was not "shocked" by the lecture like G2, but unlike G2, EM did enjoy the lecture and was able to identify the main idea of the presentation.

It therefore seems a rather bad idea to behave as a student in a popularization event organized according to a dissemination model. It might work better in an engagement or intercultural communication model.

8.4.2 Assessment of audience members' understanding of the lectures

I have noted in the "Synthesis" sections above that the distance between the "Medial representation" lecturer's expectations about the audience's understanding of the lecture and the actual understanding was quite large, and certainly larger than in the "Escher" lecture. This was partly due to the very ambitious expectations of the "Medial representation" lecturer and very modest expectations of the "Escher" lecturer. Is there a way, however, to compare the two audiences' understandings of the lecture without reference to the expectations of the lecturers?

An attempt to construct a framework for this kind of independent assessment of understanding popular talks in science and mathematics was proposed by Goffree (1989). This author distinguished three kinds of understanding in this context, namely "receptive", "reproductive" and "productive" understanding. This is how he defined these categories:

> Receptive understanding: one has been able to follow the argumentation, but cannot retell anything thereof;
>
> Reproductive understanding: one can reconstruct the argumentation, but the knowledge acquired is insufficient for application in a new situation;
>
> Productive understanding: by applying the newly acquired knowledge, one can solve

problems and even extend one's knowledge. (Goffree, 1989: 53)

I have tried applying these categories to assess audience members' understanding of the lectures based on their responses to my request to summarize the talk. There were some difficulties, however.

The only category that applied rather easily was the reproductive understanding. Two "Medial representation" audience members (C2 and M1) and one 'Escher" audience member (Mb) could be classified as representing reproductive understanding.

"Receptive understanding" seemingly applied to those audience members who could not recall what the lecture was about when I asked them to summarize the talk. They recalled something, however, after a minute or two. I could not say, in anybody's case, if they could "follow the argument" or not.

The "productive understanding" category did not apply because the lectures were organized according to the dissemination model and the audience was not asked to do anything with the knowledge presented (solve problems, engage in a discussion, etc.). In fact, the first two of the above categories of understanding seem to be responses to a different question or task than the third one. Receptive and reproductive understandings are responses to the question, "Tell me about the lecture". Productive understanding is a response to, "Solve this problem". I did not ask such question in the interview. Perhaps some audience members did apply the knowledge learned in the lecture spontaneously, outside of the interview, but I did not seek evidence of this.

Goffree's categories also did not cover the kind of understanding found in audience members with a "student relation" to the talks (G2, ME) who remembered details (disconnected fragments) from the talks but did not see or did not attend to the conceptual relations among these details. This way of understanding the lecture could perhaps be called "fragmentary".

Goffree's categories also did not appear to capture the kind of understanding that I was observing in PS and MPh after the "Escher" lecture. These audience members' understanding could not be "productive" for reasons given above. It was also different from the other two kinds of understanding, because both PS and MPh have constructed original metaphors to represent the Droste-effect idea. Their understanding could perhaps be called "reconstructive understanding", referring to situations when a person reinterprets the scientific knowledge presented in popularization in original terms, based on his or her own knowledge and experience. PS's "reconstruction" was the idealized, never-ending Russian dolls metaphor for infinite repetition; she constructed another image, not presented in the lecture, yet represented quite well one of the essential ideas of the lecture. MPh's reconstruction took the form of the "fractal" metaphor. I observed the reconstructive understanding after the "Escher" lecture also in a mathematician different from MPh, whose interview I was not analyzing in detail in this thesis. The above mentioned other mathematician, also interviewed after the "Escher" lecture, constructed a more sophisticated explanation for the transformations of the kind involved in Escher's picture:

> ...take a roll film used for the old type of cameras, and we would need a film whose images are periodically rolled around a cylinder. And what I would do for visualizing it is that I would cover the cylinder with parallel rolls all of them containing the same image and the same picture would be repeated many times. And when you see the Droste picture it is like if you had looked into the cylinder somehow like "perspectively". And what happens if you cut it and deform it? Imagine that there is this cylinder and you just tear these strips because you say that they are rolled around it many times and you just pull the things along. And so these strips just go down and if you look into it now you will see that the picture like go into it in a spiral way.

Categorizing G1's understanding presented a special difficulty but also pointed to a very important methodological problem with Goffree's model. G1 had no memory whatsoever of anything beyond the first "hook" set by the lecturer, namely his introductory example (accompanied by pictures of hands, a hammer and horses) of the problem of recognizing and distinguishing between shapes. This was certainly not the "central hook" of the lecture. Yet, by way of catching on to this single hook, the lecture became meaningful for G1 as a psychologist. Moreover, in the lecture, G1 was busy observing the mathematical behavior of the lecture, not attending to the meaning of the mathematical concepts. It was not "receptive understanding" because one cannot say "he followed the argumentation" (because he didn't) and it was not true that he "couldn't retell anything" from the lecture (he remembered the first hook).

One major difference between "popularization relation" and "student relation" is that the effects of the latter can be measured by understanding what the lecturer intended to communicate ("catching on to the hook" set by the lecturer) and the former cannot. The "contract" in popularization relation does not bind the audience member to learn the mathematical content of the lecture. The audience member is free to choose what to attend to, what to learn. G1 chose to learn about the behavior of a mathematician in his natural habitat. It seems therefore that measuring the effectiveness of a popularization event by asking whether or not the audience "understood the lecture" in the sense of catching on to the hooks set by the lecturer does not make sense.

8.4.3 Visualization in mathematics

In the interviews, popularizers referred to visualization as effective means of conveying mathematical ideas especially to an audience not familiar with technical mathematical notations. Both lecturers strongly insisted on the use of visual images in their talks. In fact, they evoked the main ideas in their lectures by the names of the pictures intended to encapsulate these ideas: the image of the panther in the 'Medial representation" lecture and the Droste cocoa box picture in the 'Escher" lecture.

The "Medial representation" lecturer stressed visualization as a deliberately used means for conveying mathematical ideas for the general public:

> Because quite a general audience can still understand principles by examples. Typically *visual* examples. So if you can go through a specific example, describe the process for example in the context of this topic, the process is very simple. You have an object, the boundary is in fire. And the fire is eating the way in the inside of the object. Everybody can *visualize* that. And in the end they understand that finally the fire fronts hit each other and these are the regions we are interested in. So there is no mathematics really, that's all by example. And once they understand the example you can bring in the mathematics to explain the intuition. But of course that is true for this type of physical process that you were observing. It would be more of a challenge for abstract concepts of mathematics for a general audience.

Research results cast some doubt, however, on the lecturers' believes in the power of visualization in communication and understanding of mathematics (e.g. Arcavi, 2003; Sierpinska, 2004; Zimmermann & Cummigham, 1991). The following definition of "visualization" as used in mathematics education research and theory has been proposed:

> Visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings. (Arcavi, 2003: 217)

Researchers have acknowledged the importance that visualization may play in supporting learning and understanding of mathematics:

> Sophisticated mathematicians may claim to 'see' through symbolic forms, regardless of their complexity. For others, and certainly for mathematics students, visualization can have a powerful complementary role in... [at least] three aspects...: visualization as (a) support and illustration of essentially symbolic results (and possibly providing a proof in its own right)..., (b) a possible way of resolving conflict between (correct) symbolic solutions and (incorrect) intuitions,..., and (c) as a way to help us re-engage with and recover conceptual underpinnings which may be easily bypassed by formal solutions.... (Arcavi, 2003: 223-224)

Visualization in mathematics, however, is also found to be a non-trivial, complex cognitive process. It may facilitate understanding or producing analytical solutions, but the cognitive operations that are required for this to happen are themselves not easy or straightforward. A visual representation of a mathematical idea may be interpreted as a "picture" resembling the thing it is supposed to represent, and not, as usually intended, as a "symbol" of the thing. It is common among students to interpret the graph of the relation between time and height of a stone thrown vertically up as the trajectory of the stone. Popularizers expect their audiences to decode from the visualizations exactly what they have encoded into them. They may be deceiving themselves, if, as Arcavi says:

> 'We don't know what we see, we see what we know'. I was told that this sentence is attributed to Goethe. Its last part: "We see what we know" applies to many situations in which students do not necessarily see what we as teachers or researchers do. (Arcavi, 2003: 230).

Research literature is replete with examples of situations where students were attending to irrelevant details of visualizations they were shown and that were supposed to help them understand a complex mathematical idea (e.g. Presmeg, 1986; Sierpinska, Dreyfus & Hillel, 1999; Sierpinska, 2004). Examples of this phenomenon were observed also in my research. In the "Medial representation" lecture, some audience members did not pay attention to the skeleton in the dynamic coloring of the panther image, although the whole aim of the visualization was to show how the skeleton is obtained. In the "Escher" lecture, this phenomenon was not as salient; people seemed to attend to the relevant aspects at least in the Droste-picture.

In his paper, Arcavi stressed, moreover, that people's interpretation of a visual representation may depend on the context, also for experts:

> [W]hat we see is not only determined by the amount of previous knowledge which directs our eyes, but in many cases it is also determined by the context within which the observation is made. In different contexts, the 'same' visual objects may have different meanings, even for experts. (Arcavi, 2003: 232).

This phenomenon may be quite important to take into account in popularization. The audience filters the visualizations through their cultural lenses, bringing in interpretations of symbols from very different contexts than those in the lecture. This is bound to lead to misunderstandings.

The last important cognitive difficulty related to visualization that I will mention here is related to the need for multiple representations. It is not true that one can grasp a mathematical idea based on a visual representation alone. More, generally, understanding mathematical concepts requires conversions between at least two semiotic registers (Duval, 1995). This is said, in different terms, in Arcavi's paper as well:

> Another cognitive difficulty arises from the need to attain flexible and competent translation back and forth between visual and analytic representations of the same situation, which is at the core of understanding much of mathematics. Learning to understand and be competent in the handling of multiple representations can be a long-winded, context dependent, nonlinear and even tortuous process for students.... (Arcavi, 2003: 235)

Popularizers, however, expect the lay audience to "skip the equations" and understand the mathematical ideas from the visualizations alone. In view of the impossibility to perform visual-analytic translations, lay audience members may resort to translations of the visualization into a non-mathematical language and context they are more familiar with. This brings to mind the difference between using a single language dictionary, which gives definitions and examples of use of the words (like a Webster dictionary of English language), versus using a two language dictionary which translates the words from one language to another (e.g. an English-Hungarian dictionary). Translating something into a different representation in the mathematical language may help understanding, but only if the terms of the explanation are not completely foreign. Translation into another language may be as tricky as using an English-Hungarian dictionary: every English word has many different meanings, depending on the context of use, and if one does not know which context applies in the particular case, the dictionary is useless. In my research, the mathematicians in the audience were using the "single language", mathematical dictionary to interpret the visualizations (e.g. referring to the exponential map on the complex plane and its characteristics in interpreting the transformations of pictures presented by the lecturer to convey the method of reconstruction of the *Print Gallery* picture). The non-mathematicians, on the other hand, "translated" the visual images into a variety of other languages (e.g. psychology, in the case of G1). This helped them in making sense of the lectures to some extent, although both the audience and the lecturers were aware that something will inevitably be lost in translation.

The lecturers were trying to deal with this problem by using not only visualization and corresponding analytic representations but also other means (common in single language dictionaries as well) such as synonyms, explanations of the meaning in different terms, or examples of uses of a word in sentences. For example, in the "Medial representation" lecture, the lecturer treated the word "skeleton" as a synonym of medial axis, explained it using the grass fire analogy and the centers of inside balls, and used it in the examples of the skeleton of the panther. The differential equation (analytic representation) was not the only translation of the concept of medial representation that he gave. It seemed however, that audience members saw the various explanations as separate concepts rather than alternative explanations of a single concept. They, in fact, focused only on one representation, the panther, and interpreted it according to their own cultural lenses.

Explaining a concept by showing the uses of a word in different contexts seemed to be the approach taken in the "Escher" lecture. He showed the transformations underlying the construction of the Print Gallery picture in various examples of use and hoped that the audience will be able to derive their essential nature from them. It was not clear if this goal was achieved, since the lay audience members were not able to make their understanding of the transformations explicit.

Beside *cognitive* difficulties related to visualization, Arcavi (2003) also speaks, in his paper, of "cultural" and "sociological" difficulties that he explains in reference to institutionalized teaching of mathematics. Only the "cultural difficulties" may have some bearing in the context of popularization, and I will discuss them briefly here.

Cultural difficulties refer to the ambivalent perception of the legitimacy of visual proofs in mathematics. Students are often warned not to trust the relations visible in the drawing of a geometric figure when constructing a proof, because the drawing may represent a special case not assumed in the theorem they want to prove. A "visual proof" or "proof without words" may not be considered as acceptable proof in certain cultures or situations. In a popularization situation, however, a visual proof would be perfectly acceptable. We could say that visual proofs do not present cultural difficulties within the culture of popularization: this is where it is considered legitimate. This general acceptance of visual proofs in the popularization culture might, however, contribute to cultural difficulties outside of popularization; it could contribute to the belief that visual images somehow "devalue" mathematics, since popularizers (and also audience members) suggested that popularization is not necessarily about "communicat[ing] the *real* mathematics".

CHAPTER 9

DESIGNING A POPULAR LECTURE

9.1 INTRODUCTION

Besides observing popular lectures organized and conducted by others, I had the personal experience of preparing and giving a popular talk, and collecting information about the audience's reaction to it. I designed a 45-minute talk, and gave it several times, to different audiences, some in Hungary, and one in Canada. After each talk, I asked the audience to fill out a questionnaire, in the hope of obtaining some feedback on my talk.

In this chapter, I describe the various aspects of this experience, according to the intercultural framework presented in Chapter 2 of the thesis. I describe the institutional environments of the talks I had given, present the mathematical topic I chose and reasons for choosing it, and the various discursive and other means I used in the talk. I include a complete script of the talk. I then present some results about audience members' interpretations of the lecture, based on the questionnaire I distributed after the talk. I will end the chapter with a critical discussion of the research methodology I have used to obtain information about the audience members' reactions to the talk.

9.2 INSTITUTIONAL ENVIRONMENT OF THE LECTURE

I wanted to try my lecture with two audiences with similar academic backgrounds or orientations and previous experience with popularization but with different cultural backgrounds. I decided to give the lecture in Hungary and in Canada. In Hungary, I gave the talk in a secondary school. In Canada – in a "cegep" (Collège d'Enseignement Général et Professionnel).

Both the Canadian college and the Hungarian secondary school were academically oriented; the majority of their graduates would go on to study at the university. Students in the last two years of the Hungarian secondary school (grades 10 and 11) to whom I presented, and the college students were about the same age (16-19 y.o.). Both institutions have hosted popular talks in mathematics before. In Hungary, there is a tradition of schools hosting talks or special lessons given by university professors or graduate students. The purposes of the talks could be enrichment, or recruitment to study mathematics-related subjects at the university. In the Canadian college, there was a program of popular science lectures, regularly attended by students and teachers. There was a chance, therefore, that some students attending my talk would have had previous experience with popularization and would give me a more informed opinion about my talk.

The institutional constraints in Hungary forced me to give the talks during regular mathematics class periods. This had an important impact on the audience I ended up lecturing to, and on the possibility of conducting my research altogether. Had I given the talk after classes as I had planned to, those who came to the talk would have done so of their own free will and would have had a "popularization relation" with the talk. Since, however, I had to give the talk during regular class time, my audiences contained participants with relations other than popularization relation to my talk. Some would have never come to the talk, given the option. It was not obvious from the written responses who had what kind of relation with the talk and therefore the results were necessarily biased. I was trying to avert at least some of the effects of having the talks during regular classes by telling the students that their responses to the questionnaire would have no impact on their grades in high school. I also avoided giving the impression that the questionnaire is some kind of "test" of their understanding of the talk. The students knew from the beginning that I will ask them to fill out a questionnaire but they did not know what would be the connection between the questions and the presented material. This could help reduce the "student relation" effect but not necessarily induce "popularization relation". Some audience members could have, with the talk, the kind of relation that students often have with activities that take place in school and that they have to attend, but on which they are not evaluated ("time off" type of relation). The same relation could have been prevalent among the subjects in Debold et al.'s (1990) study, and therefore this study of the effects of the Square One TV popularization program on students' attitudes and constructs of mathematics was just as biased as mine.

Thus, in Hungary, I had an audience of students some of whom did not want to be there or were not interested at all. In Canada, all students who came to the talk did so without coercion, but only three students came! The rest of the audience were college mathematics or science teachers. There were thus 13 teachers and 3 students in the talk I gave in Canada. This was a problem for me because the talk I prepared and the questionnaire were meant for students. The teachers were also not particularly happy with filling out the questionnaire; this was usually not "part of the deal" in their attendance of the popular science talks. Because of the completely different audiences in the two settings (Hungary and Canada), I was not able to perform a systematic quantitative comparison of the responses to the questionnaires.

The above problems are clearly not just accidental; they are inherent to any project of studying the effects of popularization on the audience. Any audience coerced into attending a popular event will not necessarily have a popularization relation with it and therefore what we will be testing will not be the effects of this relation. If the audience has not been coerced, it will be random and not representing the population that we are interested in. If anything, this study has made me realize the tremendous methodological difficulties of research on popularization; I started understanding the possible causes of the paucity of this research.

Henceforth, I shall refer to participants in my talks in the Hungarian high school as "HS group" and to participants in the Canadian college as the "C group".

9.3 Choosing the Mathematical Content of the Lecture

In choosing the topic for my lecture, I had two main criteria: it has to represent the mathematical culture in an exemplary way, and it has to be attractive to 16-19 years old students. More precisely, I had in mind the following characteristics: the topic of the lecture should

1) bear some important core characteristics of the mathematical culture, i.e., it should be closely related to some fundamental mathematical concept or method;
2) have a rich network of links to other elements of the culture;
3) be easy to follow for a secondary school student, i.e., require only the mathematical knowledge that can be assumed in secondary school students, and possible to present without specialized symbolism;

4) contain an element of surprise: present some non-conventional, surprising mathematical results[52];
5) have an aesthetic value;
6) have a humanistic value;
7) have links with other cultures, for example, with art or with real life applications likely to attract students' attention;
8) be related to some recent results in mathematics, to show that mathematics is a live domain, presently making rapid progress and therefore attractive for studying at the university.

To satisfy condition 4), I was also looking for a less common theme, something students would not have heard about in previous popular talks.

Finally, I decided to talk about proofs in mathematics that involve ideas and techniques inspired by tangrams, namely the "dissect-and-rearrange proofs", in the context of the historical evolution of the problem of equidecomposition of figures, which involves also proofs by reduction to a previously demonstrated case. Tangrams are not uncommon in popularization and education, but they are often used for presenting elementary concepts to children (Bohning & Althouse, 1997; Irving & Bell, 2004; Yang, Li, & Lin, 2008). My plan was to use them for introducing more advanced mathematical ideas, such as

- the Bolyai-Gerwien theorem stating the equivalence between the property of being congruent by dissections and having the same area in the case of arbitrary

[52] This characteristic is inspired by the idea of presenting controversies to the public through popularization (Evans & Tsatsaroni, 2000).

polygons;

- the Banach-Tarski paradox, and
- the result of Laczkovich (1990) about squaring the circle.

This topic has the potential of exemplifying mathematical culture according to the eight characteristics mentioned before, for reasons I will outline below.

1) It speaks about fundamental mathematical concepts such as volume and area. As Stewart argues:

   > The Bolyai-Gerwien Theorem isn't just a recreational curiosity: it's important because it justifies one possible method for defining area. Start by defining the area of a square of the length of its side, so that for instance a 3 cm square has an area of 9 cm^2. Then the area of any other polygon is defined to be the area of the square into which it can be dissected. (Stewart, 1996: 172)

   It is also focused on proofs, which are one of the most important characteristics of the mathematical culture. It allowed me to include at least one proof (or the main idea of the proof) of a non-trivial mathematical theorem, namely the Bolyai-Gerwien Theorem.

2) The topic had direct links with elementary geometry, differential geometry, topology, measure theory.

3) The idea of dissect-and-rearrange proofs could be introduced through puzzles in a way that is easy to follow. The proof of the Bolyai-Gerwien Theorem was rather meant as an illustration of a certain proof methodology, namely that one can reduce a problem to a problem that has already been solved (such as reducing the problem of cutting polygons to the question of cutting triangles).

4) The Banach-Tarski paradox is certainly highly surprising.

5) Playing with mathematical puzzles might show that mathematics is "fun", an image popularizers often try to communicate.
6) Introducing the topic through puzzles gave me an opportunity to provide anecdotic and historical information about the tangram and other mathematical puzzles. It gave me a chance to talk a little bit about Chinese mathematics, but my choice of the topic was also influenced by the fact that I wanted to present results that would be related to Hungarian mathematicians, preferably to János Bolyai since his life was known by many of the students in the Hungarian school. In fact, János Bolyai was the patron of the school.
7) Displaying different mathematical games gave me a chance to connect mathematics with puzzles displayed in many formats (movable tables, postage stamps, etc.).
8) I was able to connect the results with recent mathematical research.

9.4 THE MEANS OF COMMUNICATION USED IN THE LECTURE

Knowing from research on science popularization that the dissemination model is highly criticized, I decided to follow the engagement model at least in a part of the event. I interrupted the talk at some points to let the participants to play with puzzles.

I first present the script of the talk (Section 9.4.1). In section 9.4.2, I will analyze the means I used in the talk in terms of the Duval-Jakobson framework.

9.4.1 The script of the talk

9.4.1.1 Part 1, organized according to the engagement model of popularization

I start the talk with the following introduction:

1. *I like puzzles, and I like playing with them. I have quite a few at home by now but I was in high school when I first saw a tangram. Have you heard about it?*
2. *It is a puzzle which consists of seven pieces only, a square, some triangles and a parallelogram.*
3. *To solve the puzzle means simply to form given shapes using all these seven pieces. Easy, isn't it? There are those 1000-2000-pieces puzzles (my cousin plans to buy a 5000-pieces puzzle!). Compared to those, tangrams are for kindergarten children.*
4. *To solve the puzzle means to form given shapes from the pieces. OK, but what are these shapes, really? Well, there are thousands of possibilities.*
5. *My favourites are paradoxical pairs of puzzles. What does it mean that they are paradoxical? The shapes vary only a little, as if it would be possible to solve one of them and then use just one more piece to find the other solution. However, we must always use exactly the same number of pieces. So although the shapes are similar, the arrangement of the pieces must usually be completely different.*

At this point, I show a few paradoxical puzzles from Slocum et al. (2003), and look at how the pictures are similar to each other and that solving the puzzles means arranging the pieces into the given shape.

6. *For example we can have one square or we can have two squares. Let's try to solve the first puzzle and form a square from the pieces.*

I encourage participants to form groups of 3-5 to work together on the puzzles. I distribute the tangrams among the groups. The groups should have enough time to solve their puzzle, and if they get stuck at some point, I help them. When the majority is finished, I show them the solutions. (See Figure 9.1)

Figure 9.1. Arranging tangram pieces to form a square

I then propose a more difficult task.

7. *Try to form two smaller squares from the seven pieces.*

When the majority is finished, I show the solution (Figure 9.2).

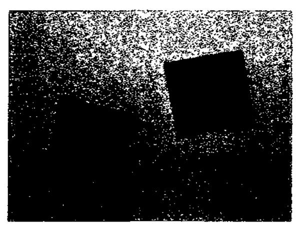

Figure 9.2. Arranging tangram pieces to form two small squares

Then, I show how a special arrangement of two puzzles leads to the proof of the Pythagorean Theorem in the case of an isosceles right triangle (Figure 9.3).

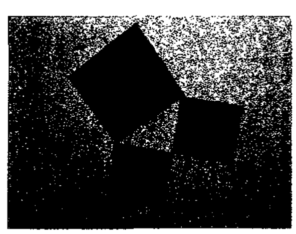

Figure 9.3. A "puzzle proof" of the Pythagorean Theorem for an isosceles right triangle

I continue:

9. *By forming these two squares we have actually proved the Pythagorean Theorem for an isosceles right triangle. This is only a special case of the theorem but considering that we were simply playing with a puzzle, it is not bad.*

At this point, I show some pictures of other rearrangement puzzles and tangrams of various forms (tangram tables, stamp collection, etc.) using the illustrations from Slocum et al. (2003).

10. *Rearrangement puzzles were popular throughout history, like the Stomachion in the ancient Greece and Rome, or a nineteen-piece puzzle in Japan.*
11. *In these puzzles the number and variety of the pieces were greater; for instance the Japanese puzzle contained even semicircles.*
12. *It is probably because of its simplicity that, unlike the Greek and Japanese puzzles, the tangram remained well known to this day. It is quite recent: it is only about 200 years old.*
13. *Its age is probably surprising but its origin is not.*
14. *We can find its roots in Chinese mathematics (for example, in Liu Hui's proof of the Pythagorean Theorem) and in similar Chinese puzzles[53]).*
15. *The first known tangram book was found at the beginning of the 19th century but the tangram craze quickly spread over North-America and Europe.*
16. *The tangram books became also more vivid at this time. Instead of the simple black and white puzzles they contained shapes of faces of famous people, or buildings. It also happened that the little elements were formed by pieces of furniture and so to solve a puzzle meant rearranging the furniture in a room. In Germany, for instance, children could collect the puzzles by buying some sweeties and getting them as a bonus, similarly as today.*
17. *We can also find tangrams today. I particularly like the Finnish stamp series, whose elements form the shape of tangram-pieces that can be arranged accordingly.*
18. *Since the method of rearranging the pieces of squares already worked for proving the Pythagorean Theorem in a special case, let's try to do it for a general right triangle. We will imitate Liu Hui's proof. For this let's first form a big square and then two small squares (possibly not of the same size).*

I distribute the puzzles similar to the tangrams to the participants and ask them to arrange the pieces to form one big square and two small squares.

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[53] The proof can be found in various places, for example Wagner (1985), Straffin (1998), or Wang (2009).

Figure 9.4. "Proving" the Pythagorean Theorem for an arbitrary triangle

When the majority is finished, I display the solution by using two sets of puzzles (Figures 9.4, 9.5).

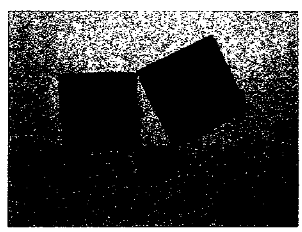


Figure 9.5. The Pythagorean Theorem for an arbitrary triangle

19. *If we arrange the pieces according to the picture, it becomes obvious that we have proved the Pythagorean Theorem for a general triangle.*
20. *Let's just stop and think a little about what we have done.*
21. *In fact the Pythagorean Theorem for us meant that we can cut two little squares into pieces and form a big square from the elements* (as shown in Figure 9.6)

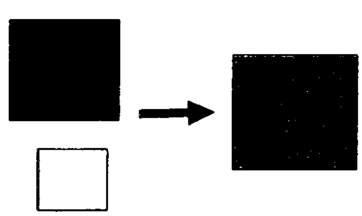


Figure 9.6. Two squares dissected into one big square

22. *We have seen that we can cut and rearrange two squares to a big square like the red and the yellow one into the green one.*
23. *What about shapes other than small squares? Can we get a square by cutting and rearranging a rectangle? Well, let's try.*

I distribute the puzzles to the participants and ask them to form first a rectangle and then a square from the pieces (as shown in Figure 9.7). At some point, I expected a question about how we can figure out the parameters of the final square and where we have to cut to get the desired result. I got such questions only after the talk in the Canadian college.

Figure 9.7. Dissecting a rectangle into a square

24. *What about triangles? Well, for an equilateral triangle it seems "easy", we need only three cuts (as shown in Figure 9.8), although quite surprising ones.*

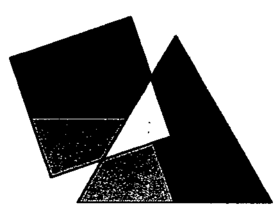


Figure 9.8. Converting an equilateral triangle into a square

25. *But what if we have a general triangle? It seems a little more complicated. If we have a triangle it is easy to get a rectangle...*

Participants are encouraged to rearrange the puzzle first into a triangle and then into a rectangle. (Figure 9.9)

Figure 9.9. Dissecting a triangle into a rectangle.

26. *... but not the square. How can we get a square instead of the rectangle?*
27. *Well, how do physicists and mathematicians boil water? They take a pot, fill with water, put on the stove, turn it on and wait till boiling. But what if the pot is already filled with water? The physicist takes the pot, puts it on the stove, turns it on and waits till it boils. What does a mathematician do? Well, he takes the pot, empties it, puts it back in its place. This way he has reduced the problem to the previous case and from now on he follows the original procedure: takes the pot, fills it with water, etc.*
28. *We can apply the same method for cutting the triangles.*
29. *From a triangle we can easily get a rectangle and from this rectangle we have already seen that it is possible to get a square.*
30. *We needed quite a few cuts but finally we got the square, which is what we wanted.(Figure 9.10)*

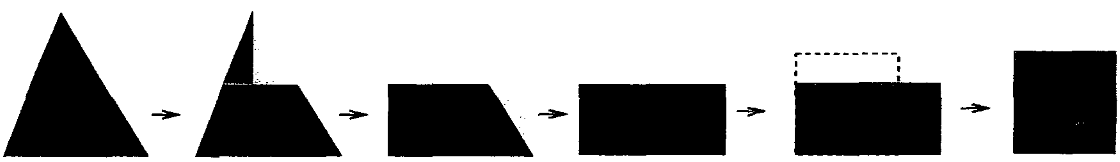


Figure 9.10. Dissecting a triangle into a square.

I was thinking of giving this task to the audience as a puzzle, but I found it too time consuming. I also felt that this approach would probably hide the main idea, so I finally divided it into two parts (See Figures 9.7, and 9.9). In fact, the proof is not complete; the rectangle has to have a certain height to be able to make the required cuts (if not, first we have to reach this height by appropriate cuts).

31. *Let me just summarize what we have done up till now.*
32. *We have cut and rearranged two small squares to make a big square, a rectangle to make a square, and a triangle to make a square.*
33. *Quite a lot, but all of them are special shapes. Can we do this for an*

arbitrary polygon?

34. *Let's use the method of boiling water again.*
35. *We can easily cut the polygon into triangles, from those triangles we form squares (as we did before) and from the small squares we get one big square (as in Figure 9.11)*

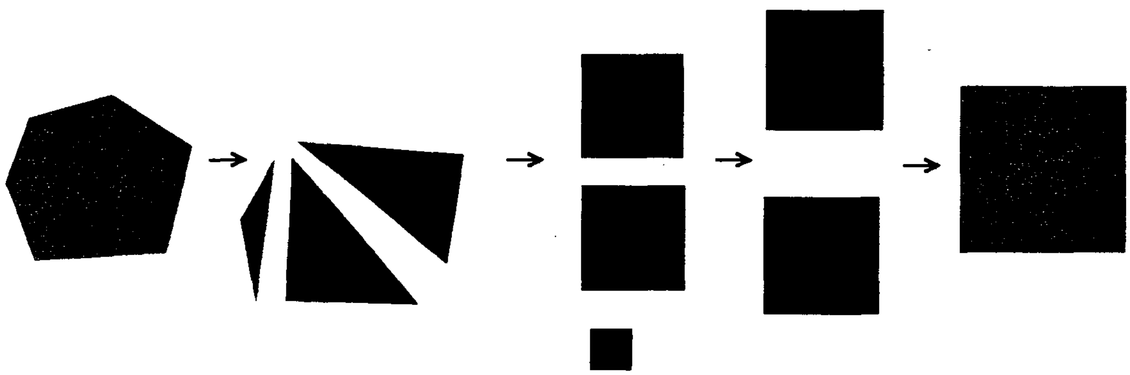


Figure 9.11. Dissecting arbitrary polygons into a square of the same area.

36. *We have seen so far that from an arbitrary polygon we can get a square by cutting it into pieces and then rearranging it as a puzzle.*
37. *Look at the two turquoise pictures [in Figure 9.11] closely. The two shapes have the same size.*
38. *It is not surprising since during this process the area remained the same.*

9.4.1.2 Part 2 of the talk, based on the dissemination model

In this part of the talk, I start discussing the Bolyai-Gerwien theorem.

39. *If we have one shape and we rearrange its pieces to another one then the areas always will be the same. Is this true backwards?*
40. *If we have two shapes of the same size, namely two polygons of the same area can we always cut one into pieces, rearrange the elements and get the other one?*
41. *The answer is yes. In fact this theorem has a name; it is known as the Bolyai-Gerwien theorem.*
42. *Here, "Bolyai" refers to the father of János, Farkas Bolyai.*

43. *The theorem states the following: Two polygons are congruent by dissection*[54] *if and only if they have the same area.*
44. *In particular, any polygon is congruent by dissection to a square with the same area.*
45. *This means that if we can get one shape from the other by cutting and rearranging the pieces by rotating and moving them, the area remains the same; and also, if we have two polygons of the same area, we can always cut one into pieces, rearrange the elements and get the other one.*
46. *It means that if we have two sheets of paper of different shapes but with the same area, we can always cut one into pieces and then reassemble it in the way that it will look the same as the other.*
47. *By using our previous method we can easily prove the theorem.*
48. *We can certainly cut both polygons and form squares from the pieces as we have seen before.*
49. *But let's just reverse one of the processes.*
50. *Let's cut the first polygon, say, a pentagon and rearrange it into a square.*

Here I refer to the pictures in Figures 9.12a and 9.12b.

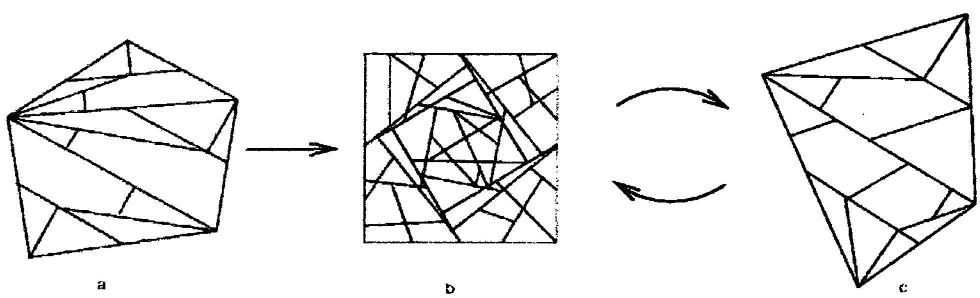


Figure 9.12. Illustration of the Bolyai-Gerwien Theorem

51. *And now let's cut the square and rearrange it to the other polygon, say a quadrilateral by simply reversing our previous process (we cut and reassemble the square to small squares, then to triangles, and then to the quadrilateral).*

[54] In other words, are equidecomposable with each other.

Referring to Figures 9.12b and 9.12c,

52. *We certainly need quite a lot of cuts even in this simple case (as can be seen in the picture) but we can surely do it.*
53. *Of course it usually doesn't give us the optimal solution, in certain cases only a few cuts are enough.*
54. *But the good thing about this method is that it works every time for all kinds of polygons.*
55. *Finding the good cuts is not easy, however.*
56. *You can find puzzles in magazines of this kind, like dissect a cross by four cuts into pieces and form a square from the elements. We might need thousands or millions of cuts, but we can do it.*
57. *From the Bolyai-Gerwien Theorem we have seen that a polygon can be cut and rearranged to another polygon with the same area.*
58. *But polygons are plane figures.*
59. *What about 3-dimensional figures? Is the same thing true in this case?*
60. *If two polyhedra have the same volume, is it possible to get one from the other by cutting it (by planes) into pieces and rearranging them?*
61. *A very famous mathematician, David Hilbert, asked the same question.*
62. *In 1900, Hilbert presented 23 problems. He intended those as a program for future mathematical research and so listed mathematical problems which seemed probably the most important. Hilbert's questions turned out to have strong impact on the 20^{th} century mathematics. Some of these problems still remain to be solved.*
63. *Solving such difficult mathematical problems seems to be a pretty good business, by the way; you can earn one million dollars by achieving this.*
64. *Our question was identical with the third problem of Hilbert, but it proved to be the easiest one.*
65. *Max Dehn found two tetrahedra with the same volume such that no matter how we cut one by plane cuts, it is not possible to form the other one from the pieces. In this way he proved that the 3-dimensional case is not as nice as the one in 2 dimensions, namely having the same volume does not guarantee that we can automatically cut and rearrange polyhedra to each other. What's this difference then?*
66. *In 3 dimensions we have to be careful with the angles formed by the cuts.*
67. *When we put them together they should fit, namely if we put them together they should form either the angle of the new polyhedron or become straight.*
68. *The monster-like complicated mathematical object which encodes this property of the angles is called Dehn-invariant.*

I had the formula of the Dehn-invariant on the slide but I wanted only to show it and explain it visually.

69. *If the Dehn-invariants of two polyhedra are equal then they are "scissors equivalent", namely one can be cut and rearranged to obtain the other.*

On the slides I had the necessary and sufficient conditions but I didn't want to go into details. However, I expected a question about it.

70. *It seems that the 3-dimensional case was more rigid concerning the rearrangement of polyhedra.*
71. *But what happens if we allow not only cuts by planes but also other kinds of cuts.*
72. *Imagine that up till this point we used only knives and from now on we are allowed to use more unusual tools to cut.*
73. *What happens then?*
74. *The result is shocking.*
75. *In fact, it is possible to dissect a ball into six pieces which can be reassembled by rigid motions to form two balls of the same size as the original.*
76. *This is called the Banach-Tarski paradox.*

I didn't want to spend much time on the paradox, simply state it as an interesting result.

77. *Practically, it means that, if you have, let's say, an orange, you can cut it into six pieces, put together the pieces and get two oranges of the same size.*
78. *In facts the cuts here are "pretty ugly".*

I didn't want to elaborate on it more. For the questions regarding the topic, I simply planned to say that it could be like "taking one point here or there", or to mention some "very strange" cuts (for example taking points with rational coordinates, which is not a good example here, since it is measurable, but already peculiar enough).

79. *Of course this cannot happen if you use a knife and make plane cuts.*
80. *I have to admit that you cannot really do this because the six pieces are very strange.*
81. *But theoretically it is possible.*
82. *In fact if we allow strange cuts we can get interesting results in two*

dimensions also.

83. *For example we can square the circle.*
84. *Squaring the circle was one of three classical problems of Greek mathematics which was proved to be impossible by Euclidean means. However, if we allow "strange" cuts, it is possible to cut the circle by a finite number of dissections ($\sim 10^{50}$having 50 zeros after the 1) and then reassemble the pieces to a square.*
85. *The cuts in this case are not very nice, either.*
86. *This is a quite recent result; Miklós Laczkovich proved it in 1988. So as you can see playing with mathematical puzzles can lead pretty far up till today's mathematics.*

I included it as an example for a quite recent result in mathematics but I did not want to go into any more details; just simply mention the fact.

After the talk, I distributed the questionnaires to the participants who were assured that their answers will by no means influence their grades in school.

9.4.2 Analyzing the means of communication in the talk according to the Duval-Jakobson framework

I summarize some aspects of the means I used in the lecture similarly to the talks presented in Chapter 5.

With regard to the meta-discursive functions, the emphasis was rather on communication than on objectivation and processing. Although I included proofs in the lecture they did not require formal processing. (It raises an immediate question about the accepted means of processing throughout history; while the proof by dissection was appropriate for Chinese mathematics it would not be for today's mathematicians.) The presentation also lacked the precise definitions and statements, thus it would not qualify for any kind of objectivation.

Regarding the communication functions, the workshop format of the first part of the talk made it possible to rely heavily on conative and phatic functions. I had the means to provoke an immediate action; in fact, students started to play with the puzzle before any explicit instructions. The rather small size of the groups (compared to a lecture given to hundreds of participants) made the interaction with the participants easier. In the second part of the talk, these functions were used much less. In this respect, the first part of the talk was an example for the engagement model of science communication presented in Chapter 2 (a similar approach was taken by Godot, 2005), while the second part followed the dissemination model, with a more uni-directional communication. In the second part, I also used the metalinguistic function quite often for formulating and rephrasing statements (as in the case of the Pythagorean Theorem or the Banach-Tarski paradox). As for the poetic function, I was consciously trying to make the presentation pleasant to the ear and to the eye. I tried to speak smoothly, using short and simple sentences. I never made a long speech without accompanying it with graphical representations that I also tried to make pleasing to the eye.

While the referential function was used often for identifying non-mathematical objects (such as different types of puzzles), it was not very transparent for defining mathematical terms. Similarly to the two lectures I analyzed in the previous chapters of this thesis, I also had a "central hook" to attract the audience's attention and convey the main idea of my talk. This central hook was represented in the tangram puzzle, which worked as a metaphor for the dissect-and-rearrange proofs throughout the lecture. The ground of the metaphor was the action one should make to solve the puzzles or to complete the proofs. However, the tenor and the vehicle certainly differed in many aspects that I generally did not make apparent in the presentation. Probably the main difference was between the methods involved; while for the puzzles a (small number of) cuts is made and the task is to find one (among the finitely many) solutions, in the case of theorems the question is about the *existence* of the cuts which generates infinitely many solutions.

I used apophantic function by a variety of means such as stating facts rephrased in different ways, pictures, diagrams, dynamic representations. The state of truth was not given a great importance in the lecture. I deliberately skipped constraints of theorems and details of proofs (although in some cases I expected questions about them). I generally felt that these details were not worth mentioning.

The organization of the first part of the talk was in accordance with those found in the popular paper and the lectures discussed in Chapter 5. The talk was generally organized around the metaphor of rearranging puzzles. Considerable time was devoted to introducing the problem with anecdotic aspects and puzzles, while the presentation of the actual mathematical results (and their justification) took much less time. The presentation was not structured formally (there was no slide with a "Plan" of the lecture presented at the beginning of the talk; the slides were not numbered; there were no cross-references, etc.). The graphical representations were certainly meant to play the most important part in conveying ideas. I relied heavily on pictures and dynamic images of puzzles. Instead of symbolic representations, I used colors for easier reference.

9.5 MY CULTURAL LENSES AND GOALS FOR THE TALK

Based on the interviews with popularizers, I identified two main views of mathematics that were usually intended to be communicated through popularization: *Mathematics is useful* (the utilitarian view of mathematics), and *Mathematics is beautiful and fun* (the aesthetic and enjoyable view of mathematics). I decided to convey mainly the aesthetic and enjoyable view of mathematics as expressed in the ingenuity of reasoning and imaginative proofs. I chose to present a proof method quite characteristic of the mathematical culture: a technique of reduction to a previously proved case. In my choice, I was also certainly influenced by the fact that the talk was planned as an extracurricular activity only for those students who were interested in mathematics and possibly planning to continue their studies at the university in this direction.

9.6 The Questionnaire

The questionnaire was administered right after the talk. The HS group had 45 minutes to complete it (one full classroom period). The C group was not given any time limits, but most were done after half an hour.

The questionnaire had two parts. The first part was intended to reveal the audience's cultural lenses; the second – their interpretations of the lecture. Each part had ten questions. The large number of questions was meant to make up for the limitations of the questionnaire method as compared to an interview, outlined above. By asking several questions on a similar topic, I was hoping to reveal more aspects of the participants' cultural lenses or their interpretations of the lecture.

For reasons that I will explain later, I was not able to obtain useful conclusions from a detailed quantitative analysis of the responses to Part I of the questionnaire alone. I took them into account, to some extent, in analyzing responses to Part II.

I reproduce the questionnaire below, without displaying the spaces left for answers in the original sheets given to the audience members. Together with the spaces, the questionnaire had three pages.

Part I. Audience members' cultural lenses

I.1 What is your attitude toward mathematics? (Like/Dislike/Neutral)Why?

I.2 Where do you plan to continue your studies? Do you expect to use mathematics in your future profession?

I.3 What is mathematics good for?

I.4 What do you think modern mathematics is about?

I.5 Complete the following sentence: Mathematics is ...

I.6 List a few mathematicians.

I.7 Complete the following sentence: Mathematicians are ...

I.8 Have you ever taken part in a mathematics competition?

I.9 Have you read popular books or attended popular lectures about mathematics? If yes, please give a brief description (for example, the title, or what where they about). What do you think about these books or lectures?

I.10 Are you interested in popular lectures? If yes, which topics are you interested in?

Part II. Audience members' interpretations of the lecture

II.1 What was the presentation about?

II.2 Have you understood the presentation?

1) No 2) A few things 3) Yes, essentially 4) Yes, completely

Which parts caused the most trouble?

II.3 What is the connection between the tangram and mathematics?

II.4 In what form have we used the Pythagorean Theorem for the cutting?

II.5 How could we cut and then rearrange

a) a triangle to obtain a rectangle

b) a rectangle to obtain a square

c) a polygon to obtain a square

II.6 State the.Bolyai-Gerwien theorem. Give an outline of the proof. Was it a correct proof?

II.7 What is the Dehn-invariant? What is it good for?

II.8 What does the Banach-Tarski paradox say?

II.9 What do you think about the cuts in the Banach-Tarski paradox by which it is possible to cut one ball into two?

II.10 Which parts of the presentation were the most unclear?

In the following section (9.7), I give an account of the information I could derive about participants' interpretations of the lecture, based, mainly, on their responses to Part II of the questionnaire. Unlike in the case of the "Medial representation" lecture and the "Escher lecture", I will not look at cultural lenses and understanding separately but will describe them together. In section 9.7.1, I describe my reasons for this different organization.

9.7 PARTICIPANTS' INTERPRETATIONS OF THE LECTURE

9.7.1 Introduction

My original aim was to assess participants' understanding based on their responses to questions intended to measure "reproductive understanding" in Goffree's sense (1989; see Chapter 8). Thus, I asked them the questions in Part II of the questionnaire. I expected to be able to classify the responses into categories that would be a refined version of the Goffree model.

Already the interviews with the "Medial representation" and "Escher" lecture audiences have shown, however, that Goffree's model of understanding a popular lecture may not be appropriate to describe audience members' interpretations of the lectures (see Chapter 8). Nevertheless, the interviews were able to reveal what people have found in the lectures that was useful or interesting for them even if it was not necessarily the memory and understanding of the mathematical facts, ideas and methods presented in the lectures. The interviews were loosely structured conversations, and people did not have to respond to a question directly, in one sentence; they could come back to a question if they wanted and whenever they wanted, or comment on a question rather than answer it. In the interviews, I could also ask additional questions, triggered by something the interviewee told me, and I could ask people to elaborate, if their answer was too short and not clear enough. All this was not possible with the methodology of questionnaires. I was not testing any premeditated hypothesis but exploring the different kinds of reactions and interpretations audiences may have, and therefore, in fact, I could not plan the appropriate questions to ask beforehand. I also had to decide how much space to leave on the questionnaire for the essay items, and this was a constraint on how much information the respondents could give. Another constraint was having to formulate an answer in writing; it is much more difficult to communicate in writing than orally, so people would write a lot less than they would be willing to talk about.

Moreover, talking about a lecture during an informal conversation with those who already agreed to be interviewed seems to be necessarily more revealing because audience members who consent to an interview "have something to say". Some of the interviewees indeed talked a lot and the interview for them was a way for making their opinions heard (as Mb) or for expressing their frustration (C1). An interview situation induces much less of a feeling of being tested, provides an opportunity for being flexible and for asking participants to clarify their answers.

In the institutional context of my lecture to the HS group, with 99 students in three classes, given during regular class hours, made it difficult for the audience to construe the lecture based on the popularization relation. Some audience members might have felt they were being tested. Some others may have treated the classes as "time off", only pretending that they participate in them (being physically but not mentally present).

Indeed, there were responses so laconic (e.g., the lecture was about "tangrams" or "geometry") or evasive ("I don't know"), or most questions not answered at all, that I was suspecting their authors to have the time off relation with my lecture. One participant even explicitly noted: "[Answering the questions] is not important unless I have to write a (school) test about it". The laconic or evasive answers could be, however, also a symptom of treating the questionnaire as a test and simply "not knowing the answer". It was difficult to tell which position these students were taking. There were, however, a number of thoughtful responses that seemed to reflect what the participants paid attention to during the lecture and what they took with them from it. These responses were mostly given by students who replied positively to the question I.10, indicating that they would be interested in participating in popularization in the future, and therefore were more likely to be in a popularization relation with the talk. In this section, I will therefore present, beside the reactions of the C group, the reactions of only those nineteen HS students who replied positively to question I.10. As in my accounts of audience members' reactions to the "Medial representation" and "Escher lectures", I will write mainly about the differences between the lecturer's (mine in this case) expectations regarding the message conveyed and what I hoped the audience will attend to, and the actual interpretations of my lecture, revealed in Part II of the questionnaire.

I have labelled the HS group members using symbols such as HS11a/1, HS11a/2, etc., or HS10/1, HS10/2, etc., where "11a" stands for grade 11, section "a" class, and "/1" identifies the student in this class. Some HS group members were from grade 10; then they were labelled with "HS10/" followed by their number in my database. The college teachers were labelled CT1,... ,CT13, and the college students – CS1, CS2 and CS3, but, in my analysis, I will not distinguish between teachers' and students' behaviour in the C group.

9.7.2 Results based on responses to the questionnaire

In this section, I will describe the audience members' interpretations of the lecture. The section will be organized into subsections corresponding to my criteria of choice of the topic for the talk, presented in section 9.3. The results for the HS group, as mentioned, refer only to the behavior of the 19 students who responded positively to question I.10, i.e. to those who said they are interested in participating in popularization of mathematics. The results for the C group take into account responses of all college participants.

9.7.2.1 Connections with mathematical culture

The talk addressed some important characteristics of the mathematical culture, such as dissect-and-rearrange proofs, proofs by reduction, and the historical evolution of the question of equidecomposition of figures by dissection.

About half of the audience members in each group were able to reproduce (usually as an answer to the question II.5c) an argument involving a dissect-and-rearrange proof or a proof by reduction (50% in the C group and 53% in the HS group). For example, respondents would state that it is possible to cut and then rearrange a polygon to obtain a square "by dividing it into triangles and then into square, etc." (CS3). Among them, only one person identified proof by reduction as the topic of the presentation, referring to it by "the previously solved problem idea" (CT12). Another participant wrote that the lecture "presented the tangram method" (HS11a/22). I did not use the term "tangram method" in my lecture. Therefore, I infer that this participant was not "reproducing" what he heard in the lecture but rather "reconstructing it". I am referring here to the notion of "reconstructive understanding" proposed in my discussion of Goffree's categories of understanding a popular lecture in Chapter 8, section 8.4.2.

There was a considerable difference in the level of detail of the presentation recalled by the C and HS group members in their answers, especially to question II.5. While the C participants gave at most very rough sketches of dissections in solutions, the HS participants' responses were often extremely detailed and usually very similar to those presented in the puzzles and on the slides. The difference can be seen in the examples in Figure 9.13.

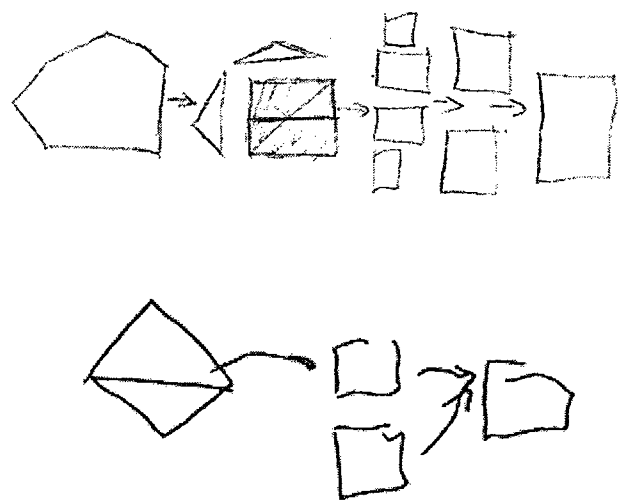


Figure 9.13. Examples of reponses to question II.5c.

The sketch at the top was given by HS10/13; the one at the bottom by CT12.

Responses to the questionnaire suggested that audience members perceived the presented techniques only in terms of the exemplary cases represented in the puzzles, and focused on the shapes of the parts of the puzzle rather than on the reconstruction of the cuts based on the properties of the shapes or the fact that dissections preserve angles. Thus, in their responses to question II.5a (How to cut and rearrange a triangle to obtain a rectangle) they usually provided a triangle similar to the one they played with in the puzzles (Figure 9.14), and showed how to rearrange pre-given pieces into a rectangle.

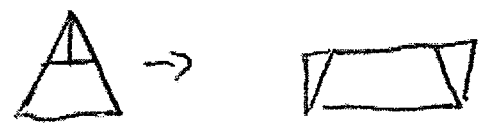


Figure 9.14. Answer to the question II.5a on cutting and rearranging a triangle into a rectangle (CS1)

In many cases, the methods represented in answers to II.5 were applicable only in special situations. For example, in question II.5a, the method shown could apply only in the case of an equilateral triangle (Figure 9.15). One participant proposed to cut a rectangle in two squares and then cut each of the squares into two triangles; in this special case of a rectangle decomposable into two squares, these triangles could be rearranged into a square (Figure 9.16a). Another one started with the same type of rectangle and then implied that one can use the Pythagoras theorem to make one square from two (Figure 9.16b). Yet another participant answered question II.5c on the example of a polygon made of four little squares (Figure 9.17).

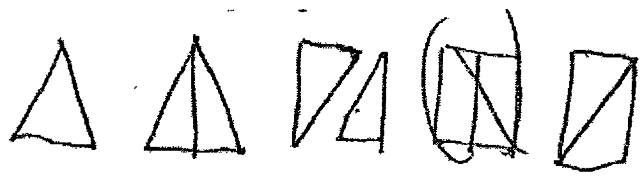


Figure 9.15. Answer to the question II5.a on the method of cutting and rearranging a triangle into a rectangle (HS11a/16)

(a) (b)

Figure 9.16. Answers to the question II.5b on the method of cutting and rearranging a rectangle into a square (a) (HS10/16); (b) (HS11b/8)

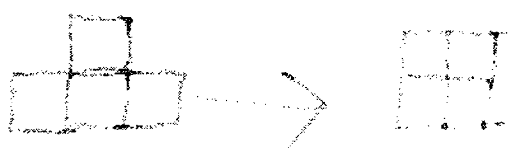
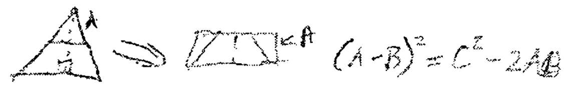


Figure 9.17. Answer to the question II.5c on the method of cutting and rearranging a polygon into a square (HS11b/10)

Participants were usually not curious to know how the cuts were obtained. Only one audience member (in the C group) asked how it is possible to construct the cuts in the puzzles and what should be taken account to do them. However, a few participants reflected upon the cuts and reconstructed them in the case of triangles (Figure 9.18).

(a)



(b)

Figure 9.18. Answers to the question II.5a on the method of cutting and rearranging a triangle into a rectangle (a) (HS11b/9), (b) (CT5)

Interestingly, the image of mathematics as a "science of proofs" was held in the C group among those who did use or refer to the technique of proof by reduction somewhere in their answers, whereas among the HS participants this view was not associated with the reproduction of the particular proof. This could be explained by the fact that, among the HS participants, the view of mathematics as a science of proofs was held mainly by those who had a negative attitude towards mathematics ("Dislike" in response to question I.1), whereas, in the C group, this view of mathematics was associated with positive attitudes. In both groups, the ability to reproduce proofs was higher among those who saw mathematics as a mental exercise or perceived it as "fun" or enjoyable activity.

Besides the general method of the proof, some participants noticed also other aspects of the mathematical culture. For example, historical aspects of mathematical culture were mentioned by some of the audience members; the topic of the presentation was described as "[t]he evolution of geometry and resulting problems" (CS1) or the response mentioned Chinese mathematics by associating it with the topic or with tangrams (HS11a/6, HS11a/17, HS11a/20). However, some participants over-generalized my historical remarks; for example, one participant perceived tangrams as "essentially the roots of mathematics" (CS2).

9.7.2.2 Links with additional elements of the mathematical culture

The topic was meant to provide a rich network of links to other elements of the mathematical culture. There were certainly a few mathematical objects I "put on display" in the talk, such as the Pythagorean Theorem, the preservation of area and volume and their problematic definition, etc. Among these, the Pythagorean Theorem attracted the most attention and was often identified as one of the main ideas in the presentation. The preservation of area and its role in dissections was also a recurrent theme in the responses. While I expected that some answers, especially to question II.3 (What is the connection between tangrams and geometry), would list geometric shapes and mention the Pythagoras theorem, I certainly did not intend to convey the impression that the Pythagoras theorem was the main topic of the talk.

In their responses, participants mentioned also mathematical ideas that I did not speak about in the talk nor expected the audience to make indirect associations with. Interestingly, this behavior appeared only in the C group, and was almost totally absent in responses of the HS group (with maybe a few exceptions). This difference of behavior could perhaps be explained by the fact that most of the HS audience appeared to attend the talk as students in a school period allotted to mathematics, while the C audience was in a popularization relation with it. The HS participants responded to the questionnaire as they would to a test: duly answering the questions to the best of their memory of the "lesson", but not offering anything extra, such as opinions, reflections, comments, because it is not the student's job to do such things. In the C group, respondents offered the following "loose associations" with the ideas presented in the talk:

- Complex numbers: CS3 wrote that the cuts in the Banach-Tarski paradox "probably involve imaginary and complex numbers", after stating earlier that he would be interested in popularization events related to "unreal numbers".
- Infinity: In response to question II.10 (what was most unclear?), CT2 posed the question, "Is 10^{50}finite?" referring to the number of cuts in the proof about "squaring the circle".
- Tiling: CT3 mentioned "tiling" as the connection between tangrams and mathematics.
- Axiom of Choice was mentioned by CT4 as a problematic idea in mathematics. The proof of the Banach-Tarski paradox does rely on the Axiom of Choice; however, I did not mention it in the talk.
- Algebraic identities: The formula $(A - B)^2 = C^2 - 2AB$ (CT5) was written in the space allotted for the question II.5a, related with cutting and rearranging a triangle to obtain a rectangle.
- Teaching-learning approaches: tangrams were seen by some participants as a good "entry point in teaching" (CT5) because it helps "asking the right question" (CT5), and it is good for "geometrical intuition" (CT8). A similar idea was expressed also by one HS participant who considered tangrams as a way for improving "geometric intuition" (HS11a/20).

9.8.2.3 Elements of surprise

The Banach-Tarski paradox is certainly a non-conventional, surprising mathematical result. This paradox, however, was not intended to be the most important idea in my talk, and I did not spend much time on it. Nonetheless, many participants took it as the most important or interesting idea of the talk. Several members of the C group came to me after the talk saying that they would be interested in hearing more about it. Responses to questions II.8-9 contained remarks such as "[the Banach-Tarski paradox] is fascinating, yet impossible in practice, so less fun" (CS1), "troubling and intriguing" (CT6), "pretty incredible" (CT4), "weird" (CT10) or "I don't believe it" (CT13). In fact, the paradox was mentioned in responses to various questions of the questionnaire by 81% of the C participants and by 68% of the HS participants; however, while 75% of C participants also reflected on it in some ways (expressed feeling or emotions, added visual representations or reformulated it), only 32% of the HS participants did something similar. An interesting feature was that, while, in Part II of the questionnaire, HS participants filled out mainly questions II.1-II.6, C participants answered mostly questions II.1-II.5, skipped II.6-7 and went directly to those on the Banach-Tarski paradox. Responses to the questions on the paradox included,

- connecting the paradox to real life: "It seems economical. I wish it could be applied in reality" (CT2);
- making a joke: "here go the orange multiplications" (CT6)
- considering its (in fact, not true) mathematical implications: "These is no $SO(3)$-invariant measure on R^3 " (CT7), or
- representing it by a picture (CT9, see Figure 9.19).

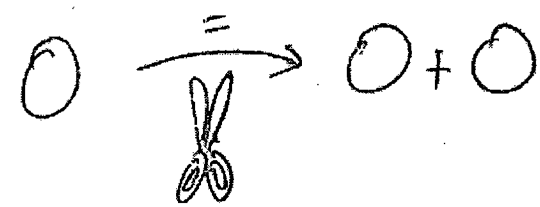


Figure 9.19. Visual image of the Banach-Tarski paradox (CT9)

Thus it seems that the Banach-Tarski paradox could be a good topic for popularization, at least in front of an audience of college teachers and students in popularization relation with the talk.

9.7.2.4 Aesthetic values

The presentation intended to convey aesthetic values of mathematics especially in the form of ingenuity and elegance of proofs and the fun of their representations through puzzles. Expressions such as "fascinating", "fun" (CS1), or that the talk was about "playfulness" (HS11b/9) of mathematics in responses to Part II of the questionnaire showed that this effort succeeded at least in the case of a few participants. However, answers where the expression of appreciation of the aesthetic value was explicit were not numerous: two HS participants described mathematics as an enjoyable activity; four C participants said that it is "fascinating" or "mysterious" and ten others in this group said that it is "fun".

9.7.2.5 Humanistic values

Although the talk mentioned names of mathematicians, their stories as discoverers or inventors of the presented mathematical results were not part of it. Thus, the humanistic aspects of mathematics were only very briefly and lightly touched upon in the lecture. This point seemed to be well understood by the participants. Very few respondents seemed to think that the talk was about mathematicians.

9.7.2.6 Links to other cultures

The topic provided links with other cultures, for example with the culture of games. Although many participants mentioned the tangram when they summarized the topic of the talk, it was not clear, from what they had written, to what extent they perceived it in terms of a game. A few audience members stated solving puzzles among the ideas associated with the presentation. One participant (in the C group) reacted to the questions by saying, "I was too busy playing with 'toys' to pay as much attention as I should have here".

9.7.2.7 Presenting mathematics as a living science

The talk referred to a recent result in mathematics, namely the result obtained by Laczkovich, to show that it is a live domain, presently making progress and therefore attractive for studying at the university. It did not seem from the responses to the questionnaire that this intention resonated with the participants in any way (although I did not ask any explicit question about it). The name of Laczkovich did not even appear in responses to question I.6.

9.7.3 Conclusions about participants' interpretations of the lecture

The main difference between the two groups of audience members was certainly how they interpreted the questionnaire. HS participants seemed to treat it mostly as a school task, while C group members saw it as an opportunity to express their opinions.

HS participants often gave very detailed answers, mentioning much irrelevant information. It was often not clear from their responses, how fragmented and procedural their idea about the lecture actually was. However, the large number of students who could reproduce the method of the proof in questions II.5 or 6, indicates that detailed descriptions did not necessarily mean fragmented understanding (such as in the case of G2 in Chapter 8), but often were simply a consequence of memorizing a variety of details, for which they are usually trained in schools. They attempted to reproduce solutions similar to those in the puzzles and on the slides, and generally did not engage in more reflection than they thought was necessary to answer the questionnaire.

The C participants, who, as mentioned, were much more in a popularization relation with the talk, provided considerably less detailed responses and omitted questions more often than the HS participants. They added comments or presented associations in some relation with the questions asked. Thus, reformulating the main difference between the two audience groups in the terms of "hooks", as in Chapter 8, HS participants generally noticed the hooks (probably also the connections between them) but generally did not "catch on" to them, while C participants generally caught on a hook, although not the one that I meant as the central hook (tangram, or the method of proof); most often they caught on to the Banach-Tarski paradox.

The questionnaires also showed that my expectations about the advantages of the engagement model as compared to the dissemination model were not met. The reasons why they weren't, however, seemed to be different in the two audience groups. I expected that participants will answer mostly questions about Part 1 of the talk, based on the engagement model, omitting the questions about the second part, based on the dissemination model, or giving some laconic answers. HS participants, however, often answered all questions, paying equal attention to all of them. This could be because they were forced into a student position and thus tried to recall as much information as possible. Even if they mentioned that it was hard to remember the names, they still responded similarly to the two parts of the lecture, giving detailed descriptions without extra associations. C participants, on the other hand, reacted mainly to the dissemination part of the talk. They were interested in the Banach-Tarski paradox, which provoked many spontaneous associations. Thus, the models themselves do not say much about popularization; other considerations should be taken into account as well.

I certainly learned a lot from the experience and would change the talk accordingly, had I to give it next time around. From the questionnaires, it seemed that, in a talk for teachers, it would be a good choice to focus more on the Banach-Tarski paradox.

It seemed that audience members were often paying more attention to the means (images and manipulatives) by which the ideas were presented than to a mathematically justified argumentation. Their responses contained a rather faithful representation of those means. However, I do not think that this should be changed. The aim of popularization is not to teach rigorous mathematics. It is fine if the audience has a loose idea of what the talk was about; if the lecture generates some reflection about mathematics or a particular concept in them, this should be treated as a bonus.

On the other hand, I was impressed by the rich network of related ideas audience members associated with the topic. I certainly experienced a feeling similar to the

"Escher" lecturer's (Chapter 6) impression that audiences are often fascinated by elements of the mathematical culture one would have never thought about. I also realized that I should have been more careful in choosing the central "hook", especially that the talk served also my research purposes. I will reflect on this problem in the Conclusions section.

9.8 Conclusions

The analysis presented in section 9.7.2 is a result of a long struggle to make sense of my audiences' responses to the questionnaire. I have categorized the responses in a variety of ways, and calculated the frequencies of the different categories in the groups of participants, taking into account the whole HS group as well as the subset of those students who declared, in question I.10, that they would be interested in participating in popularization. Somehow, however, these minute quantitative analyses failed to lead to a satisfactory picture of the audiences' general interpretations and impressions of the lecture and of the variations among the groups. It seemed not to make sense to separate the analyses into "cultural lenses" and "interpretations of the lecture" as I did for the individual profiles of audience members in Chapters 7 and 8.

Certainly, this quantitative work was useful for me because it made me aware of methodological issues related to studies of popularization and, at the same time, of specific characteristics of popularization.

My main problem was how we can design an appropriate talk and a method for analyzing it that would help us in investigating popularization. Is it possible at all?

As I already emphasized in the thesis, measuring understanding in popularization is practically impossible since by "testing" participants in any way we violate one of the main characteristics of popularization. Observing a class might cause students and teachers to behave differently than when they are alone, but "having no observers in the classroom" is not a defining characteristic of teaching. Audience members of popular lectures, on the other hand, will most probably not engage in certain behaviours if they know that they will be asked about the talk at the end. They will probably not let their minds wander away, or play freely and be lost in playing with toys while not paying attention to the speaker. Thus measuring is hard. Still, I had the impression that, through informal conversations, I was able to gain information about some audience members' interpretations of a popular lecture. I was not able to obtain such information in the case of my presentation, especially in the case of HS responses. It seemed that the questions I asked in the questionnaires were not the appropriate means of measuring understanding, even though I have asked the same type of questions in the interviews. I even asked my audience members to interpret the main metaphor of the lecture (question II.3) (which I did not do explicitly in the case of the interviews), and also to perform some operations (which again I did not do in the case of the interviews).

It still did not work as I had expected, and not because participants did not write anything. They took more time and effort than I expected to answer the questions. Still I was often not able to decide whether their understanding was fragmented (like G2), or they considered the lecture as "time off", or else their minds were wandering around some unexpected topic but they did not bother to write it down. I had serious difficulty especially if they referred to my "hook" and wrote that the presentation was about tangrams but did not give much clarification; thus I did not know whether they wrote it because they noticed the hook or because they caught on to it.

It is true that, in the interviews, I had to ask participants quite often to clarify their thoughts to me. I could not do this in the questionnaire. However, I also realized that interviews revealed that audience members often pay attention to details and aspects the lecturer did not think of when preparing the lecture.

The questionnaire was not able to detect whether or not audience members created new metaphors for themselves for the metaphor of the tangram I provided in the lecture. However, C participants seemed to reflect a lot on the Banach-Tarski paradox. It seems that tangram was simply not a good metaphor to provide. Apparently, some participants were more responsive to controversy than to playing with puzzles, or at least it gave them more scope for reflection. On the other hand, many participants mentioned tangrams, often by identifying the topic of the presentation with this metaphor. It was certainly among my goals to give an impression that mathematical theorems and their proofs can sometimes be interpreted in terms of puzzles: finding a proof is like solving a puzzle. In this sense they found the central metaphor. Can we then say that they understood the lecture? I do not think so. I was unable to judge whether they had any more to say about this metaphor. Was it because they just did not want to write it, or because they merely got that word without any connection to the rest of the talk? I created this obstacle by choosing a wrong metaphor. I used a hook labelled by an unknown word everyone remembered (just as in the case of the Droste-effect). In this way it seems that "hooks" are chosen by popularizers, in a way, to "be on the safe side". These hooks do not necessarily contain the main idea of the lecture (although they have a link to it), but they are easy to recognize and remember, and as such they are meant to give something to every one in the audience to bring home from the lecture. This seems reasonable if the task is not to use the talk to evaluate the audience's understanding (and, by definition, it is generally not the case). However, the hook could be a very bad choice for later assessment especially if one wants to try to identify how the hook was connected to the lecture based on a questionnaire. My tangram hook was too easy to identify without finding any connection with the rest of the material presented, and it was not very likely to provoke a conceptual reconstruction, as it happened in the case of PS and MPh after the "Escher" lecture.

Thus, when studying popularization in terms of hooks and metaphors, etc., it would be preferable to use in-depth interviews rather than questionnaires (where the interpretation of the metaphors can be discussed in detail); moreover, the central hooks for the talks should be constructed more carefully. Preferably, they should be easily identifiable but not obvious. They should be also sufficiently alluring to tempt the audience members to reconstruct them, possibly by connecting them to experience that they would have brought with them to the lecture, as it was in the case of the Droste-picture.

If one decides to present the metaphor by "saying it in different contexts" (Chapter 8), what these appropriate contexts could be? Is there a reasonable limit on the number of "sentences" provided? The speaker of the "Escher" lecture gave many such "sentences", most of them were appealing but they did not really facilitate understanding. After a few well-chosen examples, they simply became redundant.

I bring these "lessons", and others that I have learned through my research, together in the last chapter of the thesis, Chapter 10.

CHAPTER 10

CONCLUSIONS

10.1 INTRODUCTION

The theoretical framework for this chapter is based on the tourist metaphor and the ten implications of the metaphor presented in Chapter 2. I will look at these ten characteristics again in the light of my findings. At the end of the chapter, I summarize some methodological issues that it would be useful to take into account in further studies.

10.2 THE TOURIST METAPHOR AND ITS IMPLICATIONS - REVISITED

It seemed from the interviews that popularization is not a characteristic of an event or an activity, or even of the type of communication followed by the popularizer, but rather a feature of the *relation* of the participants with this communication. Consequently, a popularizer might be perceived as a teacher for some audience members, or a researcher for others depending on the circumstances, institutional constraints, the participants' cultural background, etc. Thus, the tourist metaphor should be modified accordingly. A business trip can be considered as tourism if the person interprets it so. If he or she considers the trip as tourism, he or she finishes the business-related part quickly and engages in tourism; then that person is a tourist. If a student perceives the field trip as tourism then he or she is a tourist as well. It makes more sense therefore to speak about "popularization relation" that a person holds with an event, or not.

10.2.1 Popularization of mathematics is organized action

In view of the above notion of *popularization relation*, it does not have much sense to constrain it with fixed institutional bounds. A person can wander around a city with a friend who volunteered to show the most important sites, if he or she feels like a tourist. However, non-institutionalized tourism is pretty difficult to deal with (e.g., organize, evaluate). Thus, it is useful to consider only activities that are institutionalized at least to some extent. In Chapter 3, we have seen that popularization of mathematics is seemingly on the way to institutionalization. However, its institutional constraints are much weaker than those of teaching or of research mathematics, but also weaker than those of popularization of science. It is questionable whether there is any need for stronger institutional constraints.

If popularization is about "selling mathematics to tourist" than one would expect some analogy between institutions for popularization and institutions that exist in some countries for promoting ethnic or national culture for the purposes of tourism. For example, CEPELIA is a Polish foundation for the protection, organization, development and popularization of the Polish folk and artistic handicraft, art, and artistic industry, both traditional and new[55]. CEPELIA stores are located in the "old town" parts of Polish cities and tourists visit them to buy souvenirs. The handicraft objects sold there are specially made for CEPELIA and while they resemble objects that used to be made by actual folk artists in the traditional Polish culture (e.g. tapestry, wooden sculptures, embroidered tablecloths, etc.), they are more or less stylized, polished, made more appealing for

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[55] http://www.cepelia.pl/english/fundacja.htm (Downloaded: July 24, 2009).

today's tourists, and they do not represent the full range of the traditional handicraft and art. Popularization of mathematics shares some aspects with institutions like CEPELIA: both try to protect a certain culture from extinction; their target public is tourists rather than experts (whether in art or in mathematics); they promote the culture by "stylizing" it and picking (*filtering in*) only some objects of the culture for display, thus creating an image of this culture which may be not completely accurate, but certainly appealing to tourists' tastes.

There is, however, a rather important difference between institutions such as CEPELIA and popularization of mathematics in the order of institutionalization. CEPELIA is strongly institutionalized, with a central administrative organ, legalized statutes, stable financial resources and human resources based on a large pool of permanent employees and contractual craftspeople. This guarantees a steady flow of products and activities. These products and activities often look as if they came from the same mould (and many of them literally are). Nothing of that is true of popularization of mathematics. I had tried to find some "common mould" or structure, or a stable set of categories in the content of popularization and how it is communicated, but, as I explained in Chapter 4, I wasn't able to find it. It seems that popularization of mathematics is more like fine arts than like folk arts and crafts: each event seems to be unique, and requires an artist's exceptional inspiration. Artists need sponsors and managers and there is a market for their products (some people collect popular books in mathematics) but this market is not made of the kind of tourists that would go and buy CEPELIA type of souvenirs after their visit to "Mathland". If participation in a popular event in mathematics is like tourism, it is an intellectually demanding type of tourism, both for the guides and for the tourists. For this reason, it cannot be strongly and centrally institutionalized. This kind of institution cannot survive and make plans based on a few authors' inspiration and volunteer organizers' free time.

On the other hand, students of fine arts learn some techniques (that they might later abandon deliberately). Isn't there a way to collect such techniques for popularizers and make the techniques available for them to learn? It appears, however, that popularizers tend to act in isolation without sharing their experiences with others. There is a certain similarity between knowledge inherent in popularization practices and knowledge inherent in teaching practices. The latter is also not made public, and not shared. Hiebert, Gallimore and Stigler (2002) have become aware of this problem and proposed the creation of a repository system for collecting information about teaching practices that thus could be then shared by a wider community. I am not sure if such collection of individual practices would be a good idea in the case of popularization.

10.2.2 Admission to a popularization of mathematics event is non-selective

The difficulty of communication involved in popularization comes mainly from the fact that the audiences' backgrounds might be very varied. They arrive from a variety of socio-economical situations, they represent a variety of professions, and their views are influenced by a variety of cultures. Popularizers, however, often do not have a chance to gain any information about the audiences' cultural backgrounds, and about their cultural lenses that will inevitably filter the communication. Based on my interviews with randomly chosen audience members of popular talks, it seemed that even if they did not like mathematics, they certainly did not have a decidedly negative attitude towards it. On the other hand, audience members and popularizers may interpret the situation of popularization in terms of a variety of different models, such as the dissemination model, the engagement model, etc. (SeeChapter 2). Depending on these interpretations, the popularizers' and audience members' expectations from the activity might be considerably different. Thus, it is hard to compare them. This property of a popular activity also complicates research, since neither the performance of the popularizer nor that of the audience members can be assessed in terms of predefined categories.

10.2.3 Participation in a popularization of mathematics event is not compulsory

Although the freedom of choosing the activity was among the defining characteristics of popularization, this key feature does not seem always to hold. People often do not participate of their own accord. They might simply accompany their friend or wife to a popular talk, but in some cases popularization might become a compulsory activity where the participation or even the understanding of the presentation is evaluated. In these situations the popularization relation may not hold and participants might interpret the popular event from the point of view of the student relation. Thus, they are not just strolling around some cultural sites by pleasure but want to learn about the culture. The situation, however, has not been designed for that. Their position might easily lead to failure and cause frustration instead of a positive experience with the foreign culture.

10.2.4 Popularization of mathematics attempts to make mathematics appear attractive to the visitors

Popularizers have to compete for the audience's attention; the audience is not forced by institutional rules to pay attention. We have seen that lecturers often use "hooks" to capture (and also to maintain) attention. What can be the good hooks that can be used both to capture and maintain attention, and to represent the mathematical culture well? A good hook should have a variety of connections. Thus, a good hook, through these connections, makes it possible for participants to build a structure around it and possibly reinterpret it according to their own cultural lenses. In fact, certain characteristics of a good hook could be gleaned from our conjectures about reasons for including a particular proof in a popular book on mathematics: insight into various aspects of mathematical culture, for example, its history, its objects (objects and representations, methods), its values (utilitarian, aesthetic), and its daily life (study skills). On the other hand, hooks are intended to capture audience's attention and thus they should operate with a variety of communicative functions identified in Chapter 5, such as poetic (hooks can come from a word play, or aesthetically pleasing images), conative (they are intended to induce feelings or action), phatic (might serve as a way for building a connection between the popularizer and the audience, for example, by humor), or metalinguistic (reformulating the meaning of the hook in different ways). Choosing the right hook and communicating accordingly certainly requires creativity, but identifying some general characteristics of "good hooks" might help popularizers to design lectures. However, the characteristics of these hooks depend largely on the purposes of the lecturer and his or her interpretation of a popular activity. The lecturer could use a hook as a mnemonic device, an object to touch and play with, or a metaphor that is likely to induce a conceptual reconstruction. Designing "good hooks" for the purposes of research poses another question, namely how to design hooks that make it possible for a researcher to investigate audience members' interpretations of the hooks.

10.2.5 Mathematics is a culture

Accepting that popularization involves communication about mathematical culture to foreigners immediately raises the question what part of the culture can be communicated this way. Sierpinska (1994: 161-162) suggested the use of the framework proposed by E.T. Hall (1959). Hall identifies three layers of culture, the formal, the informal and the technical. The formal level of culture is the level of traditions, rules, rituals, conventions, etc., i.e. elements of culture that are communicated explicitly with no justification. Members of the culture are strongly emotionally attached to it. The informal level is, on the other hand, implicit. It cannot be explicitly taught. It is learned by (consciously or unconsciously) imitating a model (by doing what everybody else is doing). The technical level of culture is learned by explicit, intentional, justified and matter-of-factual transmission of knowledge from one person to another. Admonition is not sufficient here. "Why should I do it this way" requires a serious answer and not just the "do as I told you" reaction. This is the level of explicit mathematics with well established formalism and proofs.

Functioning effectively in a culture requires the ability to function on all three levels of it. To be able to function in a *mathematical* culture effectively, its members need all three types of knowledge. Applied to mathematical culture, this theory led to the following interpretation of the levels:

> In a 'formal' or 'technical' way we can acquire certain knowledge about mathematics, we can learn algorithms, some methods of proof (mathematical induction, reductio ad absurdum, etc.), solving some 'typical' problems, ready and written parts of a theory. We can be passive users of mathematics. But it is only on the informal level, by working with mathematicians, through 'imitation and practice', as Pólya used to say, that we can learn to pose sensible questions, put up hypotheses, propose generalizations, synthetise concepts, explain and prove.

> 'Informal' knowledge and understanding are thus an indispensable support of any creative thinking in mathematics. On the other hand, however, this same knowledge and ways of understanding, as not fully conscious, and unquestioned, and drawn from experience in concrete situations, can guide our thinking in new situations in a way that will make the resolution impossible. (Sierpinska, 1994: 165)

Learning mathematics is like *acculturation* to mathematical culture, and therefore all three levels of the culture are supposed to be learned. Popularization, however, is not meant to acculturate its participants. Learning about a culture as an outsider necessarily deprives one of access to all levels, especially to the informal level, which is the least explicit. Informal learning takes place through imitation while already functioning in the culture; participation in popular events, on the other hand, rarely gives one a chance to function in the mathematical culture. In the case of mathematics, tourists usually do not have access to the technical level, either. Since mathematical tourists usually do not speak the symbolic language of mathematics, the technical level is generally available only in a very restricted format (e.g. in the case of certain recurring types of proofs listed in Chapter 4). The only remaining level of the culture, namely the formal one, is usually transmitted in emotionally charged situations. This is rarely the case, however, in popularization where participants often feel as outsiders and the short contact with the culture (and its representatives) do not really make it possible to build emotional ties such as usually involved in learning on the formal level. The above suggests that learning the mathematical culture through popularization is, in fact, impossible. Thus, participants who come to a popularization event with the goal of learning mathematics directly from the lecture will inevitably end up being frustrated and disappointed (unless the activity offers a substitute for learning, for example, entertainment).

10.2.6 The visited mathematical culture must be somewhat "exotic" for the audience to warrant organizing a popularization of mathematics event to show it

Interpreting popularization as a relation implies that "exotic" is not an objective characteristic of an event; rather, participants should perceive something exotic in the event. For example, G1, who had a popularization relation with the departmental seminars he attended, perceived the conventional mathematical symbolism as exotic. The idea that different audiences perceive different things as popularization came through in my efforts to categorize popular books. For example, while Stewart and Golubitsky (1992), *Fearful symmetry. Is God a geometer?*[56] is seen as popularization according to the subject classification used by mathematicians, it is categorized as part of metamathematics by mathematics educators, and does not even belong to mathematics (but to general science) in a library catalogue.

Thus Prediger's ideas about learning mathematics as an experience with a foreign culture (presented in detail in Chapter 2) and their interpretation in the case of popularization should be revisited as follows:

- <u>The language of mathematics is like a foreign language</u>: the experience with mathematical formalism and other special features of the mathematical language might not bother people in a popularization relation who think that it is completely normal if they do not understand it. At the same time, they seem to

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[56] Categorized according to the AMS Mathematics Subject Classification 2000 as 00A05 (00A08 00A69 92C15) which stands for General mathematics (Recreational mathematics; General applied mathematics; Developmental biology, pattern formation); according to The Mathematics Education Subject Classification it is categorized as E20 I90 G50 denoting Philosophy and mathematics; Miscellaneous topics in analysis; Transformation geometry; and according to the Library of Congress subject classification – as Q172.5, Symmetry (General Science).

feel as outsiders. For those, however, who are not in popularization relation, the experience of not understanding the local language might be painful and frustrating.

- Intercultural misunderstandings: the conflicts between different values held by the popularizers and by audience members might lead inevitably to intercultural misunderstandings (as in the case of the role of mathematical formalism in the "Medial representation" lecture). For people with popularization relation, these misunderstandings are not a problem, because they, in a way, expect it.
- Effects of overlapping: considering that interrelations with non-mathematical culture are much more common in popularization than in teaching, the effects of overlapping should be taken seriously in this case. While popularizers might become aware of these effects in the case of using natural language, they may be less alert in the case of visual interpretations which seem to be considered as one of the main communicational means. The use of images and metaphors are full of inherent possibilities for effects of overlapping (e.g. in the case of the interpretation of the image of the skeleton of the panther shown in the "Medial representation" lecture), especially if they are made attractive by colors and animations.
- Foreignness as an experience: while the experience should not be negative for the mathematical tourist, it could be upsetting in certain situations. It could well be that, for those who otherwise do not like mathematics, popularization could reinforce the feeling that mathematics is the privilege of the few and that they will never belong to this group. It might be the case especially for audience members

in student relation. For them, "foreignness" induces rather negative emotions and the "exotic" content and organization of the material might result in the feeling that they are unable to learn from popularization.

10.2.7 Popularization of mathematics involves communication between two cultures, but not enculturation in or acculturation to a foreign culture

In Chapter 5 we have seen that the mathematical discourse involved in popularization is in many ways similar to that of teaching or research, but it is considerably different from both of them. In research, the main function of the language is objectivation. In teaching, the conative functions are used differently. The consequence for a researcher is that learning and understanding cannot be interpreted the same way as in teaching. The existing research methods and results are not applicable to studying popularization. Reading a popular book or understanding a popular lecture requires different skills than studying from a book or a lecture.

10.2.8 There are different models of communication between popularizers and the audience in popularization of mathematics

It seemed that both popularizers and audience members have more or less definite ideas what they expect from a popularization event and they organize it or interpret it accordingly. While popularizers usually consider it as a way for improving public image and public understanding of and public attitude towards mathematics, audience members see popularization as a source of inspiration for life and work, a way of distraction and entertainment and a place for gaining information. While in general terms there does not seem to be a large discrepancy between the two kinds of expectations, these differences can make a big difference in the case of actual popularizers and particular audience members. Some "tourists" might just not get what they "expected". From a researcher's point of view, popularization events organized according to the different models of communication may result in considerably different types of understanding (if it makes even sense to talk about it, considering that on some cases audience members behave in a completely unpredictable manner as they have a freedom to do so).

10.2.9 Popularization of mathematics has a political agenda

Popularization of mathematics is often claimed as a way for improving public image and public attitude towards mathematics and also considered as a remedy against enrollment problems in mathematics departments (and also in university programs with high mathematical component). In fact there is no evidence for these claims and the questions seem more complicated than simply organizing popular events and hope that the problems will be solved. Certainly universities generally do not have much influence on global political, economic, and societal issues which seem to affect the situation. Thus mathematics department mainly try to change their teaching methods, perhaps build some connection with the job market, and organize outreach programs. However, it is very difficult to measure (if, at all, possible) what kind of impact they might have.

> … there would appear to be no way of evaluating how successful outreach programmes are generally. How many students take mathematics at university who would not have without an outreach programme? How many adults do not immediately scorn mathematics because of the experience they have had with an outreach programme? It is very difficult to know how to collect this or any other relevant data. (Holton et al., 2009: 12)

Public understanding of science surveys suggest that popularization might have a minor impact on public image and on "public appreciation" of science, and thus it can be assumed that the case is similar in mathematics. However, this impact seems rather insignificant compared to that of formal education (Miller, 2004: 288-290).

It is also widely believed that showing useful applications of mathematics will "do the job" and the general public will hold a more positive attitude toward mathematics and mathematicians if they learn about the utilitarian aspects of mathematics. It would be worth investigating whether these assumptions are (at all) valid, or hold for the whole body of mathematics (and mathematicians), or they imply only a potential attitude-shift (if any) in the case of applied mathematics and applied mathematicians.

10.2.10 Popularization of mathematics faces several important challenges: problems of communicability and translation

Previously in this chapter, I was referring to E.T. Hall's theory of culture which proposes that cultures (and so mathematical culture) are learned on three levels, namely formal, informal and technical levels. I was saying that learning about the mathematical culture at all three of these levels is not accessible for audience members of popular activities. Still, participants of popularization events usually "bring something home" from these events. A tourist may get an impression of a culture (which is of course not enough to live in it). In what ways can some parts of the culture be translated? Looking at the three cultural levels more closely, we see that there might be a substitution for learning at the appropriate levels that could give at least a glimpse of the culture through popularization. Thus translations can be provided at different levels as follows:

- Formal: the level of unquestioned values, rules, and traditions can certainly been shown, however, since these aspects are usually learned by trial and error in emotionally charged contexts, it should be substituted by other ways, for example by relying heavily on communicative functions, using impressive hooks, jokes, etc. For example, in the case of the skeleton of the panther, simply to introduce the skeleton without any explanation and make a joke on the missing bones, or choose a picture where the skeleton itself is easy to remember, like reconstructing a shape based on a skeleton on an actual child's drawing.

- Informal: learning a language in an informal way can be done by being confronted with different situations, thus learning words through the different contexts in which they are used. Presenting an application of a mathematical technique in different situations might give a certain access to the informal use of this technique by mathematicians. For example, showing skeletons of a variety of objects might have such an effect.
- Technical: the explicit transmission of knowledge along with its justification is usually avoided in popularization since the audience is not familiar with (and often afraid of) the mathematical symbolism which is often necessary for communicating and justifying mathematical ideas. Popularizers propose a variety of means for overcoming this obstacle. They may try to warn their audience ahead of time not to expect to understand everything, as Penrose (2004). They may also use different substitutes of mathematical symbolism, such as colors, analogies, images, etc., often referring to visualization as the ultimate solution for the problem. ML in the "Medial representation" lecture certainly believed that the analogies and images are sufficient to understand the main idea even if one could

not make sense of the differential equation. However, in Chapter 8, we have seen that visualization might cause misunderstandings especially if it is used as a substitute for mathematical explanations, because pictures do not constitute a "langue", and therefore do not allow access to all discursive functions. Hence, they are not appropriate to provide explanations that are necessary to communicate cultural aspects on the technical level.

As already mentioned a few times in the thesis translational difficulties arise mostly from the fact that cultural lenses of the popularizers and those of the audience members might be very different and popularizers generally do not have information about the type of audience they can expect. Thus it is not clear how the content intended to be communicated by the popularizer will be interpreted by the audience. The examples in the interviews and in the questionnaires showed that often people have very loose associations with the actual content presented, making unexpected connections which popularizers certainly do not expect to happen. Although the popularizers sometimes claimed that these events might serve as inspiration for the participants, they would certainly be quite surprised to learn about the interpretations. Thus translational problems, although important to be aware of, might not be such a serious issue, since they seem unavoidable anyway.

In the thesis I tried to investigate different aspects of popularization. My research brought to my awareness a variety of methodological difficulties which might explain why there is so little research on it. I am not saying, however, that research on popularization is not possible nor that engaging in popularization is a hopeless activity, but certainly, this thesis offers a few caveats about both.

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