An Efficient Optimization Scheme for WDM/TDM PON Network Planning

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Abstract

With the growing popularity of bandwidth demanding services such as HDTV, VoD, and video conferencing applications, there is an increasing demand on broadband access. To meet this demand, the access networks are evolving from the traditional DSL (xDSL more recently) and cable techniques to a new generation of fiber-based access techniques. While EPONs and GPONs have been the most studied passive optical access networks (PONs), WDM-PON is now clearly seen as the next generation trend with an hybrid set of switching equipment.

We propose here an original optimization scheme for the deployment of greenfield PON networks where we minimize the overall deployment cost. Given the geographical location of ONUs and their incoming/outgoing traffic demands, the newly proposed scheme optimizes the placement of splitters/AWGs in a PON and the link dimensioning in order to provision the overall demand.

The optimization scheme proceeds in three phases. In the first phase, we generate several potential equipment hierarchies, where each equipment hierarchy is associated with an ONU partition such that a switching equipment is associated with each cluster, each ONU belongs to a single cluster, and the splitting ratio of the equipment corresponds to the number of ONUs in the cluster. In the second phase, for each equipment hierarchy, we make use of a column generation (CG) mathematical model to select the type and location of the switching equipment that leads to the minimum cost multi-stage equipment topology which accommodates all the traffic demand. The third phase selects the best hierarchy among all the generated and dimensioned

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hierarchies.

The optimization model encompasses the particular cases where all switching equipment are either splitters and AWGs, and outputs the location of the switching equipment together with the dimensioning of the PON network. We performed numerical experiments on various data sets in order to evaluate the performance of the optimization model, and to analyze the type of equipment hierarchies which are generated depending on the traffic and the location of the ONUs.

Keywords: PON Networks, equipment location, network provisioning, equipment selection.

1. Introduction

The Internet has enjoyed rapid growth in users and applications since the early 1990s, and while the driving factors have evolved from emails to web browsing to a nowadays continued surge in mobile-ready devices such as tablets, smart phones, and widespread mobile video content consumption, Internet goes on enjoying a steady growth. As an ultimate broadband access solution for future Internet, passive optical networks (PONs) bring many advantages such as cost-effectiveness, energy savings, service transparency, and signal security over other last/first-mile technologies.

The basic architecture of a PON consists of an optical line terminal (OLT) at the CO (Central Office), a number of optical network units (ONUs), one or multiple passive remote terminals (RTs) splitting optical power from one fiber into multiple fibers and vice versa, placed in between the OLT and the ONUs. The ONUs are located either at end user premises resulting in FTTPC/FTTH/FTTB (Fiber-to-the-PC/Home/Building) solutions or at the curb site in case of a FTTC (Fiber-to-the-Curb) architecture, see Figure 1 for an illustration.

In a traditional PON, the high capacity of an optical fiber is mingled with the low installation cost of a passive infrastructure by running a single fiber from the OLT to the RT which feeds individual short length fibers to all the ONUs in a neighborhood. As the common feeder fiber part of a PON is shared by all the ONUs, an appropriate channel access mechanism is required in the upstream direction in order to multiplex the traffic streams generated by the ONUs onto the common fiber in a collision-free way [1]. PONs are usually built following either time sharing principle known as time division

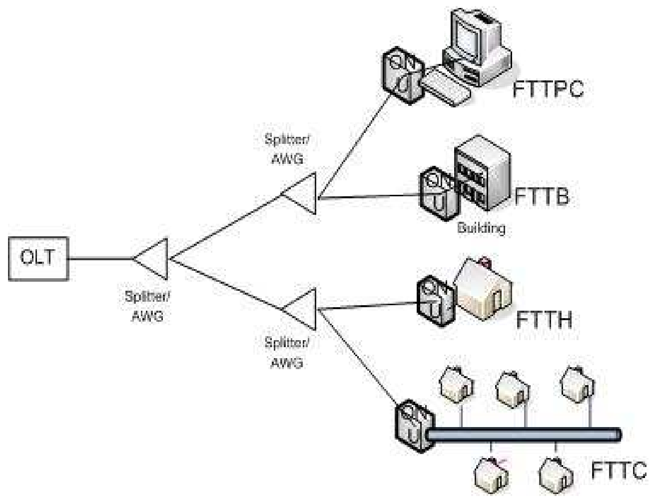


Figure 1: PON Architecture

multiplexed PON (TDM PON) or spectrum sharing principle recognized as wavelength division multiplexed PON (WDM PON) [2].

In a TDM-PON, a single wavelength channel is used along the downstream direction for broadcasting the same signal from the OLT to all ONUs by utilizing a passive optical power splitter or a cascade of passive splitters as the RT and another dedicated channel is used along the upstream direction for multiplexing signals from different ONUs in the time domain toward the OLT.

Although TDM PONs provide higher bandwidth than copper wire based access technologies, it is not anymore enough in view of the still increasing bandwidth demands, especially with applications such as video on demand or online gaming. Indeed, TDM-PON architectures are bandwidth limited as a single wavelength is shared among the PON set of users, resulting in a reduction of the average bandwidth per user to a few tens of megabits per second [3]. The end users' demand for more bandwidth can be satisfactorily mitigated by employing WDM-PON technology without drastically changing the fiber infrastructure. WDM-PONs support multiple wavelengths in either or both upstream and downstream directions by using a passive WDM coupler/arrayed waveguide grating (AWG) router as the RT.

A simple WDM PON architecture requires expensive WDM components such as dedicated transceiver per user at the OLT and optical source at each

ONU. To reduce the cost of WDM-PON technology, several technologies have been proposed and demonstrated by both academia and industry. There are mainly two approaches [4]. The first one is the remodulation method of the downlink signal at each ONU such as using saturated semiconductor optical amplifier (SOA) [5] [6], injection-locked Fabry-Perot laser diode (F-P LD) [7], mutually injected F-P LD [8]. The second approach is the controlling of the wavelength source in the CO rather than in the ONUs by using additional devices in CO or each ONU for the up-link wavelength source such as employing spectrum sliced light-emitting diode (LED) [9], spectrum sliced LED with cyclic AWG [10], spectrum-sliced amplified spontaneous emission(ASE) of erbium-doped fiber amplifier(EDFA) [11], ASE injection locked F-P LD [12], and the wavelength-seeded reflective SOA(RSOA) [13].

More recently, hybrid WDM/TDM PON networks (also called TWDM-PON networks) have been proposed in order to take into account the advantages of power splitting in TDM-PON and wavelength routing in WDM-PON, see, e.g., [14, 15].

In this paper, we investigate the network dimensioning and the placement of switching equipment in a hybrid WDM/TDM PON network in which, in order to increase the number of subscribers, we both consider splitters and AWGs. The paper is organized as follows. In Section 2, we review the recent studies related to a PON network planning and the placement of switching equipment in a PON network. Our proposed optimization process is described in Section 3. Section 4 describes the first phase of the optimization process, i.e., the clustering algorithm. Phase II, which relies on the solution of a column generation model for selecting the location of the equipment and performing the network dimensioning, is presented in Section 5. The solution of the column generation model is described in Section 6. Section 7 discusses the numerical experiments which have been conducted on various data sets. Conclusions and future work are drawn in the last section.

2. Related Work (Network Planning and Placement of Equipment in PONs)

Some of the studies on the placement of switching equipment in PONs have exploited the resemblance with the location/allocation (L/A) problem for the planning of logistic systems. Indeed, the L/A problem consists of optimally locating a number of service facilities among a finite number of demand points and simultaneously assigning each demand point to be served by the closest service facilities. While there are definitively some resemblance, there are also some differences such as the attenuation constraints which depend on the type of switching equipment and which limit the reach of the PON networks.

Another general comment is that most studies only consider the use of splitters, or of AWGs, but none of them do consider a mix of both switching equipment based on the characteristics of the traffic (e.g., mix of unicast and multicast requests) and on the location of the ONUs, as we do in the optimization process that is proposed in this paper. We specify below, for each reference, the assumptions and limitations of the switching equipment selection.

Li and Shen [16] investigate the problem of network planning for PON deployment. They decompose the problem into two subproblems: (1) the allocation subproblem with a clustering of ONUs in order to determine the ONUs to be connected to the same splitter, (2) the location subproblem to determine the optimal number and locations of the splitters. The authors propose two heuristics to solve this complex optimization problem. The first one is an extension of the benchmark sectoring algorithm in which the given parameters are a set of ONUs distributed in a full-circle and a maximum split ratio, S_r, for the splitters to be deployed. The circle is sliced into multiple sectors with each sector having S_r ONUs, except for the last one which may have less than S_r ONUs. The second heuristic is the so-called Recursive Allocation and Location Algorithm (RALA), derived from the classical L/A Cooper's algorithm [17]. RALA is designed to find a set of splitters so that each splitter connects to a maximum number of ONUs while satisfying the maximum split ratio, the maximal transmission distance, and the maximum differential distance in a PON network. RALA solutions output the number and location of the required splitters, as well as the arrangement of all the ONUs into clusters. The authors carried out simulations to measure and compare the cost per user for three planning schemes namely, benchmark sectoring, RALA, and RALA incorporated with a MILP (Mixed Integer Linear Program). Their results show that the pure RALA scheme reduces the PON deployment cost by 50%-70% compared to the sectoring scheme. For a medium-size design, the RALA MILP scheme further reduces the corresponding cost by about 10%. Note that the authors do not integrate the attenuation constraints in their algorithms, and did not investigate the compromise between one level networks with maximal signal splitting and two or more levels with reduced signal splitting.

Lee *et al.* [18] also examine the deployment of PONs throughout the location-allocation problem of splitters. They formulate the single splitting problem (SSP) and the distributed splitting problem (DSP) in which SSP includes single-level splitters and DSP multi-level splitters. For both problems, they use a mixed integer programming (MIP) modelling to determine the optimal placement of splitters. In order to solve the MIP models, they provide a tight representation by using the reformulation-linearization technique (RLT), and develop a column generation model taking advantage of polyhedral characteristics of the problems. The authors present preliminary computational results for both SSP and DSP models where the lower bound is obtained by linear programming (LP) relaxation and disaggregation analysis. They compute upper bounds by using the CPLEX Integer Linear Programming (ILP) solver (branch-and-bound method) on the restricted master problem defined by the set of columns. The optimality gaps (difference between lower and upper bounds) are quite large (up to 81%), so it is quite difficult to assess the quality of their solutions. Later, Kim *et al.* [19] proposed a relaxation of the objective function proposed in [18] and, with the help of valid inequalities and a local search heuristic, they reduce the optimality gap between the solutions of their LP and ILP formulations, and therefore obtained a better estimation of the quality of their solutions.

Hajduczenia *et al.* [20] investigate a multi-constrained optimization problem for automated PON deployment. They propose a model that takes into account several issues such as power budget, splitter location, existing network resources (trenches, aerial lines), and obstacles (both traversable: roads, greenfield areas and non-traversable: houses, industrial zones) for selecting the subscribers to be assigned to a given passive star coupler (PSC), assuming the location of the PSCs are given.A clustering algorithm is applied to find a grouping of the subscribers. Experiments are conducted on artificial maps where the ONUs are scattered. Their initial results show that their automated PON deployment tool achieves lower network cost compared to the hand made cost computed by an experienced network planner. Note that their model is for establishing a set of PONs, for a given set of PSCs (with their locations already set) and a given set of subscribers, and is therefore not limited to the deployment of a single PON.

Mitcsenkov *et al.* [21] propose a heuristic solution to address TDM PON topology planning, minimizing deployment cost along with operational aspects. They also propose an ILP to serve as a reference for smaller cases so that the performance of their heuristic can be compared with the optimal solution obtained by their ILP. It corresponds to a traffic flow problem where all customers are covered by a flow such that the splitters split an incoming flow to a set of outbound flows by the actual split ratio. The authors claim that the solutions obtained by their proposed heuristic are within the 10-20% of their computed ILP. Due to TDM technology, multiple splitters along with multiple feeder fibers are used which results in an overall increased deployment cost. The topology supports only single stage splitting architecture. It neither supports a multi stage architecture nor it utilizes AWG in the PON topology. The ILP does not optimize the location of the splitting nodes, it only connects the customers with the given splitting nodes. In the formulation of the ILP, the distance between the CO and the customers are not taken into account which is required to take into account the attenuation constraints.

Zhang and Ansari [22] present a heuristic scheme to minimize the cost of AWGs and of the optical cables in deploying a WDM PON. While optimizing the trade-off between the AWG cost and optical fiber cable cost, they decompose the network planning problem into the following subproblems:(i) determine the subscribers connected to each AWG exploiting tree-partitioning algorithms, (ii) decide geometric locations of AWGs, (iii) determine the cascaded AWG architecture by proposing a recursive partition-combination based algorithm. No information is given on the performance and the efficiency of the proposed heuristic in terms of solution accuracies.

Li and Shen [23] formulate a mathematical optimization model to minimize the deployment cost of a single-stage architecture based PON. Their proposed optimization model is non-linear. Moreover, the authors assume that the cost factor of a splitter has a linear relationship with the number of output ports of the corresponding splitter (which is not true in practice). As their proposed model is not tractable in practice, experiments are conducted with the heuristic proposed in [16].

Khan and Ahmed [24] reformulate the PON layout design problem as a theoretical graph problem. They explore several graph techniques and propose an algorithm for designing a PON layout. They compare the results of their proposed approach with those of randomized layouts. But no information is provided on the effectiveness of the proposed approach with respect to realistic PONs.

As a summary of all the studies reviewed in this section, we note that none of the previously published heuristics and ILP formulations considers the traffic unicast/multicast flows of individual ONUs for the placement of equipment in PON. There is no study investigating the placement of both splitters and AWGs in a given WDM PON network. In the solution process that will be proposed in the subsequent sections, we aim to find the optimum location of splitters and AWGs in a WDM PON network according to the traffic demand and the location of a set of ONUs, while taking care of the attenuation constraints.

3. PON Deployment: Problem Statement and Optimization Process

3.1. Problem Statement

Our goal is to determine the topology of a hybrid WDM/TDM PON network with the objective of minimizing the overall network deployment cost. It consists of the initial infrastructure installation and the maintenance cost. Installation cost comprises the price of the equipment (OLT, ONUs, splitters and AWGs), of the optical fiber cable and the cost for trenching and laying fibers. The cost of an equipment depends on the number of available outlet ports. Note that there is no maintenance cost for the switching equipment as it is a passive one. The optimization model excludes the installation and maintenance cost of the OLT and the ONU as these are fixed and unavoidable costs.

Placing splitters/AWGs close to the OLT will increase the fiber cost significantly as separate fibers are required to connect each ONU to the splitter. On the contrary, locating the splitters/AWGs toward the proximity of ONUs will reduce the fiber cost but it will increase the number and the cascading of required switching equipment. We propose an optimization model which determines the optimal locations of splitters/AWGs such that the distance between an ONU and its corresponding splitter/AWG is minimized while satisfying the PON network design constraints, in particular the maximum allowed signal power loss (attenuation) at each ONU.

The input of our problem includes the location of the OLT and the ONUs along with the requested (unicast/multicast) upstream/downstream traffic demand matrix of each ONU. It implies that the ONU cost is a fixed cost, and therefore with not be considered in our network deployment cost function. The input data of the mathematical model also includes a potential set of equipment locations together with their distance matrices between any pair of potential locations. Note that those distances are not necessarily the shortest distance between the two locations, and do take into account the logistic obstacles for trenching and layering the optical fibers, as well as the available ducts to host the optical fibers.

We consider that all ONUs are capable of transmitting and receiving single or multiple wavelengths. Indeed, ONUs are commonly associated with residential FTTH services. However, in addition to FTTH, there are a number of other network and service applications where FTTx becomes a strategic imperative, e.g., FTTB (Building/Business) or FTTC (Curb/Cabinet). Consequently, the traffic of an ONU is not necessarily limited to one wavelength. Should it be an issue, an easy way to go around the difficulty of one ONU with several wavelengths is to replace it by a cluster of ONUs, where each ONU is limited to one wavelength. Other technological solutions might be soon available.

The optimization model that is proposed can accommodate fixed or tunable ONU transceivers. Looking at the technology state of the art today, tunable transceivers will be possibly seeded by the OLT (e.g., RSOA based). In such a case, only one wavelength can reach the same ONU on the same fiber. As a consequence, for ONUs with traffic amounting to more than the transport capacity of a wavelength, they will need to be subdivided (e.g., according to the application or up/downstream traffic). Note that our model can be easily modified as to accommodate such a technology constraint, see the end of Section 5.2.

The output of the model corresponds to the definition of the topology and includes the selection and location of splitters/AWGs (with the cascading architecture of the PON network) by allocating each equipment to a group of ONUs in the WDM/TDM PON such that the bandwidth demand of each ONU is satisfied. It also includes the dimensioning of each link, i.e, how many wavelengths are required on each link in order to satisfy the demand.

3.2. Optimization Process

We propose the LAPON (Location/Allocation PON) algorithm which is a three-phase algorithm according to the process scheme depicted in Figure 2. The first phase, detailed in Section 4, consists in generating several potential equipment hierarchies, where an equipment hierarchy is defined by the physical cascading architecture of a PON network: it includes the clustering of the ONUS and the number of levels/stages of switching equipment (but not yet the choice of the switching equipment), see Figure 3 for an example of such a hierarchy. Therein, the hierarchy is a two-stage hierarchy. However, the type and geographical location of the passive equipment are not yet determined.

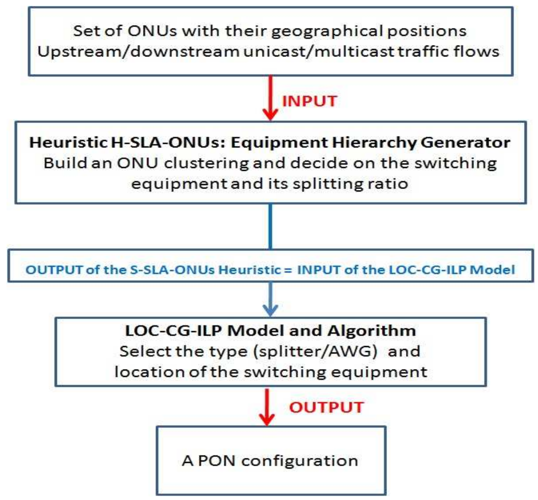


Figure 2: LAPON Solution Scheme

The second phase, detailed in Section 5.2, consists in selecting for each potential hierarchy the best type and location of its passive equipment in terms of the minimum network deployment cost and finally the best hierarchy is generated in the third step.

4. Phase I: Equipment Hierarchies and Clustering Heuristic

In order to generate equipment hierarchies, we use a clustering algorithm, called H-SLA-ONUs. It relies on the classical Single-Linkage Algorithm (SLA) [25] for the clustering of the ONUs: the idea is to successively merge clusters until all ONUs have been merged into a single remaining cluster. In each step of the H-SLA-ONUs algorithm, we get a new partition with a smaller number of clusters after the merging of the two closest clusters. Each partition leads to an equipment hierarchy by the process described below.

For each partition, the number of clusters defines the splitting ratio of the first level equipment, whereas the number of ONUs in a cluster C_j defines the splitting ratio of the equipment of the corresponding cluster, i.e., the splitting ratio of the second level equipment. Some cluster re-organization is performed in order to reconcile the cardinality of the clusters with the standard splitting values, as described in Algorithm 1, see below. We illustrated in Figure 4 the cardinality adjustments of algorithm H-SLA-ONUs

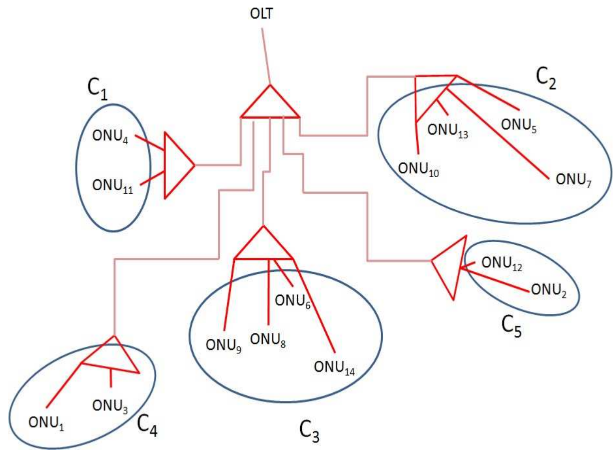
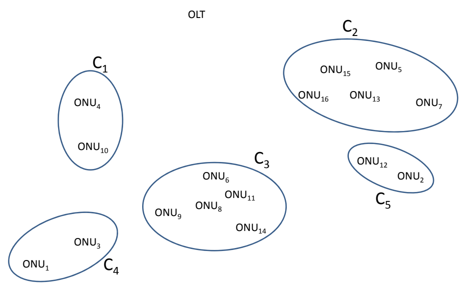


Figure 3: An Equipment Hierarchy

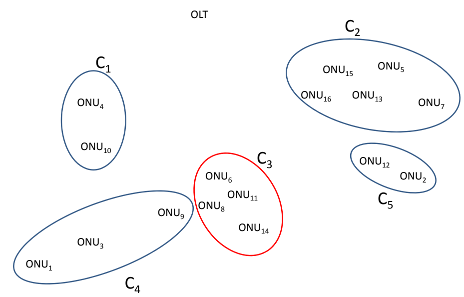
for a given clustering. Assume that clusters are ordered as follows: C_3, C_2, C_1, C_4, C_5. As the cardinality of C_3 is 5, it is rounded down to the closest available splitting ratio, i.e., 4. Consequently, we extract the ONU of C_3 which is the closest one to another cluster not yet considered, i.e., ONU_9 and we move it to C_4. Next, similarly, we move ONU_7 from C_2 to C_5. Finally, we round off the cardinalities of clusters C_5 and C_4 to 4. We are now done as all cardinalities matches standard splitting ratios.

5. Phase II: Optimization Model for Selecting the Location of the Passive Equipment

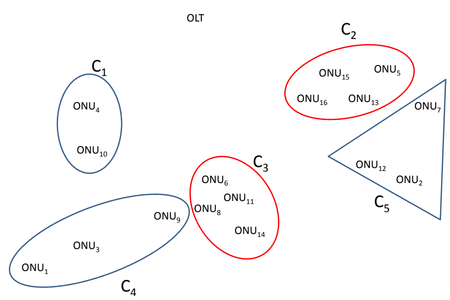
We propose the LOC-CG-ILP algorithm which determines which switching equipment and where to locate it within a given hierarchy, as generated by the H-SLA-ONUs heuristic. Indeed, several potential equipment hierarchies will be generated by the H-SLA-ONUs heuristic. Once the best switching equipment and the best location has been found by the LOC-CG-ILP algorithm for each potential hierarchy, the most economical equipment hierarchy will be selected. The LOC-CG-ILP algorithm relies on a large scale optimization model that is described in Section 5.2 after setting the notations in Section 5.1. Its solution uses column generation techniques and requires the solution of a so-called pricing problem for generating augmenting location configurations, see Section 6.2.



(a) Initial partition



(b) First modified partition



(c) Second modified partition

Figure 4: ONU partitioning

5.1. Notations

5.1.1. Hierarchy Parameters

Let $V = \{\text{OLT}\} \cup V^{\text{ONU}}$ be the set of nodes where $V^{\text{ONU}} = \{\text{ONU}_1, \text{ONU}_2, \ldots, \text{ONU}_n\}$.

For a given hierarchy, G is the set of ONU groups in a given equipment hierarchy, i.e., g_0 the cluster of the ONUs associated with the first level equipment and g any of the second level clusters, which is connecting a given subset of ONUs with the same switching equipment. We will denote by $|g|$ the splitting ratio of the switching equipment of cluster g. Let $G^\star$ be the set $G \setminus \{g_0\}$. In order to identify the membership of an ONU to a particular group, we use the parameter $\delta_{\text{ONU},g}$: It is equal to 1 if ONU belongs to group g in equipment hierarchy, and 0 otherwise.

A provisioned hierarchy is described by its switching equipment at each

Algorithm 1 H-SLA-ONUs

Apply the Single-Linkage Algorithm for a given number of clusters, say M, while forbidding the generation of clusters with more than the maximal allowed splitting ratio.
Order the clusters in the decreasing order of their cardinality
for all each cluster C in that order **do**
  Let card(C) be the cardinality of C
  Round off card(C) to the closest standard splitting ratio value
  if it corresponds to a rounding down **then**
    Extract from C the ONU which is the closest to another cluster which is, either smaller than C, or larger than C but with room for an additional ONU
    Repeat the operation until the number of ONUs in C is equal to the rounded down cardinality value
  end if
end for

level by the following parameters: $a_{g_0,k} = 1$ if there is an equipment with $k \in K = \{2, 4, 8, 16, 32, 64\}$ output ports at the first level, below the OLT, leading group g_0, and 0 otherwise. Similarly, the equipment selected at the second level is described by the parameter $a_{g,k}$, for $g \in G^\star$.

5.1.2. Location Parameters

A set P of discrete locations, indexed by p, such that: $P = \{P_{\text{OLT}}\} \cup P_{\text{ONU}} \cup P_{\text{EQ}}$, where the following locations are assumed to be known: *(i)* p_{OLT}, the OLT location, *(ii)* $P_{\text{ONU}} = \{p_{\text{ONU}_1}, p_{\text{ONU}_2}, \dots, p_{\text{ONU}_n}\}$, the ONU locations, and *(iii)* P_{EQ} the set of potential locations for switching equipment. As all these locations are known, it is easy to determine their pairwise distances $d_{pp'}$.

5.1.3. Cost Parameters

We denote by COST_{S}^k / $\text{COST}_{\text{AWG}}^k$ the cost of a splitter/AWG with $k \in K$ output ports. Let COST_{FT} be the cost of the fiber and of the trenching per kilometre. Our optimization model excludes the cost of the OLT and the ONUs as we have assumed that the OLT and the ONUs have fixed costs, independent of the location of the switching equipment.

5.1.4. Traffic Parameters

We assume the traffic to be described by a traffic matrix $T = (T_{sD})$ such that T_{sD} is the amount of bandwidth (traffic flow) to be carried out from node v_s to each node $v_d \in D \subseteq V^{\text{ONU}}$. Let $\mathcal{D}$ be the overall set of multicast destination sets. We distinguish:

- Upstream traffic: it is made of unicast traffic flows, each flow from one ONU to the OLT, denoted by $T_{\text{ONU,OLT}}$,
- Downstream traffic: it is made of unicast or multicast traffic flows, each flow from the OLT to a subset of ONUs, denoted by $T_{\text{OLT},D}$ where $D \in \mathcal{D}$.

5.2. Optimization Model

5.2.1. Location Configurations

Before setting the optimization model, we need to introduce the concept of location configurations. A configuration c corresponds to the bandwidth demands that can be routed on a given wavelength, on a given equipment hierarchy where either an AWG or a splitter has been set at some of the intermediate nodes. A location configuration can be either upstream or downstream. As one configuration means one wavelength, it is not possible to have upstream as well as downstream traffic on a single wavelength. We denote the overall set of configurations by $\mathcal{C}$ such that $\mathcal{C} = \mathcal{C}^{\text{UL}} \cup \mathcal{C}^{\text{DL}}$, where $\mathcal{C}^{\text{UL}}$ (resp. $\mathcal{C}^{\text{DL}}$) is the set of uplink (resp. downlink) configurations. Let COST_c be the cost of configuration c. For a given equipment hierarchy, a configuration $c \in \mathcal{C}$ is characterized by:

- $t^c_{sD} \in [0, 1]$ is the amount of bandwidth carried out by configuration c for $(v_s, \{v_d : v_d \in D\})$. Amount of bandwidth is normalized with respect to the transport capacity of a wavelength. The parameter t^c_{sD} can be of two types:

  $t^c_{\text{OLT}D} \in [0, 1]$ for downstream and

  $t^c_{s\text{OLT}} \in [0, 1]$ for upstream.

- $a\_a^c_{p,g,k} = 1$ if an AWG with $k \in K$ output ports is set at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster $g \in G$ in configuration c, 0 otherwise.
- $a\_s^c_{p,g,k} = 1$ if a splitter with $k \in K$ output ports is set at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster $g \in G$ in configuration c, 0 otherwise.

- $\alpha_d^c = 1$ if destination d is served by configuration c and 0 otherwise where $d \in D$.

5.2.2. Variables

- $z_c \in \{0, 1\}$ is a decision variable such that $z_c = 1$ if configuration c is selected, and 0 otherwise.

- $y\_s_{p,g,k} \in \{0, 1\}$ is a decision variable such that $y\_s_{p,g,k} = 1$ if a splitter with $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster g in the selected configurations (they must all concur for the switching equipment), and 0 otherwise.

- $y\_a_{p,g,k} \in \{0, 1\}$ is a decision variable such that $y\_a_{p,g,k} = 1$ if an AWG $k \in K$ output ports is placed at location $p \in P_{\text{EQ}}$ serving the ONUs of cluster g in the selected configurations (they must all concur for the switching equipment), and 0 otherwise.

- $y_{p,p',g,k} \in \{0, 1\}$ is a decision variable introduced for linearization purposes (see below), such that $y_{p,p'g} = 1$ if p (resp. p') are selected for the location of a switching equipment with $k \in K$ output ports in group $g \in G^\star$ (resp. g_0), where $p, p' \in P_{\text{EQ}}$ and 0 otherwise.

5.2.3. Objective

As mentioned before, the objective corresponds to the deployment cost of a given equipment hierarchy where the locations of its passive equipment are determined as to minimize the cost while satisfying the technological and traffic constraints. It is formally defined as follows:

$$\text{COST}(y) = \text{COST}^{\text{LINK}}(y) + \text{COST}^{\text{EQ}}(y) \tag{1}$$

where

$$\text{COST}^{\text{LINK}}(y) = \text{COST}_{\text{FT}} \sum_{i=1}^{3} \text{COST}_i^{\text{LINK}}(y) \tag{2}$$

$$\text{COST}_1^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} d_{\text{OLT},p}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k}) \tag{3}$$

$$\text{COST}_2^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} \sum_{g \in G^\star} \sum_{k \in K} \bullet$$

$$d_{pp'}(y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}) \tag{4}$$

$$\text{COST}_3^{\text{LINK}}(y) = \sum_{g \in G^\star} \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} \sum_{\text{ONU} \in P_{\text{ONU}}:\delta_{\text{ONU},g}=1} d_{p\text{ONU}}(y\_s_{p,g,k} + y\_a_{p,g,k}) \tag{5}$$

$$\text{COST}^{\text{EQ}} = \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} (\text{COST}_{\text{S}}^k \, y\_s_{p,g,k} + \text{COST}_{\text{AWG}}^k \, y\_a_{p,g,k}) \tag{6}$$

where $\text{COST}_1^{\text{LINK}}(y)$ (resp. $\text{COST}_2^{\text{LINK}}(y)$, resp. $\text{COST}_3^{\text{LINK}}(y)$) are the fiber deployment costs associated with the first level, i.e., from the OLT to the first passive equipment of g_0 (resp. from the passive equipment of g_0 to the passive equipment of the groups $g \in G^\star$, resp. from the passive equipment of the groups $g \in G^\star$ to their ONUs), and COST^{EQ} the cost of the selected passive equipment.

In order to linearize the expression of (4), we introduce variables $y_{p,p',g,k}$ so that expression of $\text{COST}_2^{\text{LINK}}(y)$ becomes:

$$\text{COST}_2^{\text{LINK}}(y) = \sum_{p \in P_{\text{EQ}}} \sum_{p' \in P_{\text{EQ}}} \sum_{g \in G^\star} \sum_{k \in K} d_{pp'} \, y_{p,p',g,k} \tag{7}$$

with

$$y_{p,p',g,k} = (y\_s_{p,g_0,k} + y\_a_{p,g_0,k})(y\_s_{p',g,k} + y\_a_{p',g,k}),$$

together with the following additional constraints:

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} + y\_s_{p',g,k} + y\_a_{p',g,k} - 1 \leq y_{p,p',g,k} \tag{8}$$

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \geq y_{p,p',g,k} \tag{9}$$

$$y\_s_{p',g,k} + y\_a_{p',g,k} \geq y_{p,p',g,k} \tag{10}$$

for all $p \in P_{EQ}, p' \in P_{EQ}, g \in G^\star, k \in K$.

The linearization is valid under the assumption that

$$y\_s_{p,g_0,k} + y\_a_{p,g_0,k} \leq 1 \quad g \in G, p \in P_{EQ}, k \in K$$

which is fulfilled due to constraints (16) (to be described in the sequel)

5.2.4. Constraints

There are three sets of constraints which decompose into the equipment hierarchy constraints (Section 5.2.4), the equipment location constraints (Section 5.2.4), and the demand constraints (Section 5.2.4).

Equipment hierarchy constraints. The number of selected configurations generated around one equipment hierarchy is limited by the number of available wavelengths:

$$\sum_{c \in C} z_c \leq W. \tag{11}$$

The next set of constraints imply that only configurations associated with the selected equipment hierarchy can be themselves selected in the optimal solution. For all $p \in P_{\text{EQ}}, g \in G, k \in K$, we have:

$$\sum_{c \in C} a\_s^c_{p,g,k} \, z_c \geq y\_s_{p,g,k} \tag{12}$$

$$\sum_{c \in C} a\_a^c_{p,g,k} \, z_c \geq y\_a_{p,g,k} \tag{13}$$

$$\sum_{c \in C} a\_s^c_{p,g,k} \, z_c \leq W y\_s_{p,g,k} \tag{14}$$

$$\sum_{c \in C} a\_a^c_{p,g,k} \, z_c \leq W y\_a_{p,g,k}. \tag{15}$$

Equipment location constraints. All level 2 equipment must connect to the same equipment of level 1 (i.e., location of a 1st level equipment is same for all 2nd level equipment. The equipment of each group must be placed in a single location:

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) = 1 \qquad g \in G. \tag{16}$$

A given location cannot be selected more than once in a given hierarchy:

$$\sum_{g \in G} \sum_{k \in K} (y\_s_{p,g,k} + y\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{17}$$

Demand constraints. The upstream traffic will be granted if all its components are carried out.

$$\sum_{c \in \mathcal{C}^{\text{UL}}} t^c_{\text{ONU,OLT}} \, z_c \geq T_{\text{ONU,OLT}} \qquad \text{ONU} \in V^{\text{ONU}}. \tag{18}$$

The downstream traffic will be carried out only if every destination gets the signal and it is of two types:

$$\text{Unicast:} \quad \sum_{c \in \mathcal{C}^{\text{DL}}} t^c_{\text{OLT},d} \, z_c \geq T_{\text{OLT},d} \qquad d \in P_{\text{ONU}} \tag{19}$$

$$\text{Multicast:} \sum_{c \in \mathcal{C}^{\text{DL}}} \alpha^c_d \, t^c_{\text{OLT},D} \, z_c \geq T_{\text{OLT},D} \quad d \in D, D \in \mathcal{D}. \tag{20}$$

If we would like to restrict the routing of the traffic from/toward one ONU to a single wavelength, the following set of constraints can be added:

$$\sum_{c \in \mathcal{C}: t^c_{\text{ONU,OLT}} > 0 \text{ or } t^c_{\text{OLT},d} > 0 \text{ or } t^c_{\text{OLT},D} > 0} z_c \leq 1 \qquad \text{ONU} \in V^{\text{ONU}}. \tag{21}$$

6. Solution of the Model

6.1. Generalities

In order to solve the optimization model described in the previous section, we have two options: An off-line process in which all location configurations are pre-enumerated, or at least a subset of promising ones, or an on-line process in which location configurations are generated along with an iterative solution of the model. We choose the latter process relying on a column generation solution scheme, in which we start with a preliminary selection of a handful location configurations, and we add a new configuration only if it contributes to the improvement of the current solution of the linear relaxation of the model. For readers not familiar with column generation techniques, see, e.g., [26].

A column generation solution scheme corresponds to a decomposition made of a so-called master problem (here the optimization model described in the previous section) and a so-called pricing problem (PP), to be viewed as a configuration generator. Note that in practice, one works with a so-called Restricted Master Problem, as we only explicitly embed a subset of location configurations in the optimization model of Section 5.2. The PP guarantees the generation of an augmenting location configuration thanks to its particular objective, the so-called reduced cost, which has the following properties (again, readers not familiar with column generation techniques must refer to, e.g., [26]): if there exists a location configuration with a negative reduced cost, its addition to the restricted master problem (RMP) will lead to a new solution with a reduced deployment cost, otherwise, we can claim that we have reached the optimal solution of the linear relaxation of the master problem.

Once the linear relaxation of the Master Problem has been solved optimally by the column generation algorithm, one needs to derive an integer solution. Here, rather than developing a costly branch-and-cut algorithm (see, e.g., [27]), we solve the ILP model made of the columns generated in order to obtain the optimal linear programming solution. It is well known that it usually does not provide the optimal ILP solution, but, as will be seen in the numerical results section, in practice, it was enough in order to obtain satisfactory optimized solutions.

We next describe the pricing problem, first its set of variables (Section 6.2.1), next its objective (Section 6.2.2), and then its set of constraints (Section 6.2.3).

6.2. Pricing Problem

In order to alleviate the notations, although each pricing problem is associated with a given equipment hierarchy, and a given equipment location configuration (c), we will omit the c index if there is no confusion.

6.2.1. Variables

The variables of the pricing are the coefficients of the z_c variables in the master problem, i.e., the generic coefficients of a column vector associated with a z_c variable (see their definitions in Section 5.2.1). Therefore, the variables of the pricing problem are:

- $t_{sD,d} \in [0,1]$ (values of the traffic are normalized using the transport capacity of a wavelength)
- $a\_a_{p,g,k} \in \{0,1\}$
- $a\_s_{p,g,k} \in \{0,1\}$
- $\alpha_d \in \{0,1\}$ where $\alpha_d = 1$ if any ONU $\in P_{\text{ONU}}$ is associated with a configuration.

- $\beta_g \in \{0,1\}$ where $\beta_g = 1$ if any ONU of group $g \in G^\star$ is associated with a configuration.

6.2.2. Objective

The objective of the pricing problem is defined by the minimization of the reduced cost (see [26] if not familiar with linear programming concepts), which is expressed as follows for the upstream pricing problem:

$$\overline{\text{COST}}^{\text{UP}}(z) = -\sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{1_{p,g,k}} a\_s_{p,g,k} - \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{1_{p,g,k}} a\_a_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{2_{p,g,k}} a\_s_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{2_{p,g,k}} a\_a_{p,g,k} - \sum_{\text{ONU} \in P_{\text{ONU}}} u^t_{\text{ONU}} t_{\text{ONU,OLT}} \quad (22)$$

where $u^{\text{S}}_{1_{p,g}}$ and $u^{\text{AWG}}_{1_{p,g}}$ are the dual values associated with constraints (12-p, g) and (13-p, g) respectively, $u^{\text{S}}_{2_{p,g}}$ and $u^{\text{AWG}}_{2_{p,g}}$ are the dual values associated with constraints (14-p, g) and (15-p, g) respectively, and u^t_{ONU} is the dual value associated with constraint (18-ONU).

The objective of the downstream pricing problem is expressed as follows:

$$\overline{\text{COST}}^{\text{DL}}(z) = -\sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{1_{p,g,k}} a\_s_{p,g,k} - \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{1_{p,g,k}} a\_a_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{S}}_{2_{p,g,k}} a\_s_{p,g,k} + \sum_{p \in P_{\text{EQ}}} \sum_{g \in G} \sum_{k \in K} u^{\text{AWG}}_{2_{p,g,k}} a\_a_{p,g,k} - \sum_{d \in P_{\text{ONU}}} u^t_{d,\text{UNI}}\, t_{\text{OLT},d} - \sum_{D \in \mathcal{D}} \alpha_d\, u^t_{d,\text{MULTI}}\, t_{\text{OLT},D} \quad (23)$$

where $u^{\text{S}}_{1_{p,g,k}}$ and $u^{\text{AWG}}_{1_{p,g,k}}$ are the dual values associated with constraints (12-p, g, k) and (13-p, g, k) respectively, $u^{\text{S}}_{2_{p,g,k}}$ and $u^{\text{AWG}}_{2_{p,g,k}}$ are the dual values associated with constraints (14-p, g, k) and (15-p, g, k) respectively, $u^t_{d,\text{UNI}}$ is the dual vector associated with constraint (19-d), and $u^t_{d,\text{MULTI}}$ is the dual vector associated with constraint (20-D).

The last term of the reduced cost is nonlinear, but we can easily linearized it: we can remove α_d in the above expression (23) of the reduced cost, and add the following constraint:

$$t_{\text{OLT},D} \leq \alpha_d \qquad d \in D, D \in \mathcal{D}, \quad (24)$$

as the values (i.e., the $t_{\text{OLT},D}$ values) of the traffic are normalized.

6.2.3. Constraints

Equipment Selection Constraints. For each cluster g, at most one splitter/AWG with $k = |g|$ output ports can be placed in a potential location. In other words, for each $g \in G, k \in K : a_{g,k} = 1$, we have:

$$\sum_{p \in P_{\text{EQ}}} a\_a_{p,g,k} \leq 1 \tag{25}$$

$$\sum_{p \in P_{\text{EQ}}} a\_s_{p,g,k} \leq 1 \tag{26}$$

$$\sum_{p \in P_{\text{EQ}}} (a\_s_{p,g,k} + a\_a_{p,g,k}) = 1 \tag{27}$$

In the case where an AWG has been selected at the first level of the equipment hierarchy in the configuration under construction, we only need to select one equipment in the 2nd level as each configuration is associated with a single wavelength. However, if a splitter has been selected in the first level, we need to select $k_2 = |G^\star|$ equipment in the 2nd level. Those constraints are the purpose of the following constraint:

$$\sum_{g \in G^\star} \sum_{p \in P_{\text{EQ}}} \sum_{k_1 \in K} (a\_s_{p,g,k_1} + a\_a_{p,g,k_1}) = \sum_{p \in P_{\text{EQ}}} \sum_{k_2 \in K} (k^2 \times a\_s_{p,g_0,k^2} + a\_a_{p,g_0,k_2}). \tag{28}$$

For each potential location, at most one equipment with a single splitting ratio can be placed.

$$\sum_{g \in G} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) \leq 1 \qquad p \in P_{\text{EQ}}. \tag{29}$$

For each cluster, at most one equipment with a single splitting ratio can be placed at a potential location.

$$\sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_s_{p,g,k} + a\_a_{p,g,k}) \leq 1 \qquad g \in G \tag{30}$$

Downstream Traffic Constraints. If the optimization model selects a splitter in the first level, the summation of traffic requests of all groups in the second level can be at most 1, in order not to exceed the transport capacity of a wavelength. If the selected switching equipment is an AWG in the first level, the individual traffic of each group can be at most 1. Similarly, if there is a splitter in the second level, the summation of traffic of all ONUs in the corresponding group can be at most 1 (and most likely even less than that in order to satisfy the capacity requirements at the upper level). Again, if there is an AWG in the second level, the individual traffic of each ONU can be at most 1. If there is a splitter in a group g, at most $|g|$ ONUs confined to that group can receive traffic in each configuration. But in case of an AWG, only one ONU can receive traffic in each configuration.

For downstream traffic, we need to take into account both unicast and multicast traffic requests.

Constraints for unicast traffic are as follows:

$$\sum_{d \in P_{\text{ONU}}} t_{\text{OLT},d} \leq 1 \qquad t_{\text{OLT},d} \leq \alpha_d, d \in P_{\text{ONU}}. \tag{31}$$

Constraints for multicast traffic are written as follows:

$$t_{\text{OLT},D} \leq \alpha_D \qquad \alpha_D \geq \alpha_d, d \in D, D \in \mathcal{D}. \tag{32}$$

Constraints for both unicast and multicast traffic are:

$$\sum_{d \in P_{\text{ONU}}:\delta_{d,g}=1} \alpha_d \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad g \in G^\star \tag{33}$$

$$\sum_{g \in G^\star} \beta_g \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{34}$$

$$\beta_g \geq \alpha_d \qquad g \in G^\star, d \in P_{\text{ONU}} : \delta_{d,g} = 1. \tag{35}$$

Upstream Traffic Constraints. The upstream traffic only consists of unicast requests.

$$\sum_{\text{ONU} \in P_{\text{ONU}}} t_{\text{ONU},\text{OLT}} \leq 1 \tag{36}$$

$$t_{\text{ONU},\text{OLT}} \leq \alpha_{\text{ONU}} \qquad \text{ONU} \in P_{\text{ONU}} \tag{37}$$

$$\sum_{\text{ONU} \in P_{\text{ONU}}:\delta_{\text{ONU},g}=1} \alpha_{\text{ONU}} \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g,k} + |g| \times a\_s_{p,g,k}) \qquad g \in G^\star \tag{38}$$

$$\sum_{g \in G^\star} \beta_g \leq \sum_{p \in P_{\text{EQ}}} \sum_{k \in K} (a\_a_{p,g_0,k} + |g_0| \times a\_s_{p,g_0,k}) \tag{39}$$

$$\beta_g \geq \alpha_{\text{ONU}} \qquad g \in G^\star, \text{ONU} \in P_{\text{ONU}} : \delta_{\text{ONU},g} = 1 \tag{40}$$

Attenuation Constraints. For each ONU, we have to make sure that the total signal loss is less than a given threshold value. In our numerical experiments, we use 20 dB, see, e.g., [28, 29] for detailed technical considerations. The total signal P is given by:

$$P_p = P_p^{\text{FIBER}} + P_p^{\text{THROUGH}} + P^{\text{INSERTION}} + P^{\text{MARGIN}} \tag{41}$$

where

P_p^{FIBER} is the signal loss on the fiber to reach the ONU located at p,

$P^{\text{INSERTION}}$ is the overall insertion loss (i.e., the ratio of the power received at the end of a line to the power transmitted into the line) for all the lines in the PON topology,

P^{MARGIN} is a power margin to ensure that the calculation of the total loss is within the power budget range, and

P_p^{THROUGH} is the loss provoked by going through the equipment towards the ONU located at p.

Note that the first two terms depend on the fiber lengths and on the type of switching equipment (see calculations below), while the last two losses have a constant value.

To calculate the first two losses, we introduce the variables $x\_\text{ATT}_p^g$ to evaluate the total attenuation in order to reach the ONU of group g located at p, $p \in P_{\text{ONU}}$. Let us assume a loss of 0.2dB/km, and let ATT_k^{S} (resp. ATT^{AWG}) be the attenuation factor of the splitter S (resp. the AWG) heading group g, which depends on the number of outputs of S (resp. which is independent of the number of outputs of AWG). We get:

$$
\begin{aligned}
x\_\mathrm{ATT}_p^g = \sum_{k\in K}\sum_{p''\in P_{\mathrm{EQ}}} & (\mathrm{ATT}^{\mathrm{AWG}}\, a\_a_{p'',g_0,k} + \mathrm{ATT}_k^{\mathrm{S}}\, a\_s_{p'',g_0,k} \\
& + \sum_{k\in K}\sum_{p'\in P_{\mathrm{EQ}}} (\mathrm{ATT}^{\mathrm{AWG}}\, a\_a_{p',g,k} + \mathrm{ATT}_k^{\mathrm{S}}\, a\_s_{p',g,k}) \\
& + \sum_{k\in K}\sum_{p''\in P_{\mathrm{EQ}}} 0.2\, d_{\mathrm{OLT}p''}(a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k}) \\
+ \sum_{k\in K}\sum_{p''\in P_{\mathrm{EQ}}}\sum_{p'\in P_{\mathrm{EQ}}} & 0.2\, d_{p''p'}(a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k})(a\_a_{p',g,k} + a\_s_{p',g,k}) \\
+ \sum_{k\in K}\sum_{p'\in P_{\mathrm{EQ}}} & 0.2\, d_{p'p}(a\_a_{p',g,k} + a\_s_{p',g,k}) \qquad p\in P_{\mathrm{ONU}} : \delta_p^g = 1, g\in G^\star, \quad (42)
\end{aligned}
$$

where the first summation corresponds to the equipment attenuation at the first level (group g_0), the second summation corresponds to the equipment attenuation at the second level (group g), the third summation corresponds to the fiber attenuation between the OLT and g_0, the fourth (resp, the fifth) corresponds to the fiber attenuation between the first level and the second level (resp. between the second level and the ONU located at p).

The fourth summation in (42) contains non linear terms. In order to linearize it, we add a new variable

$$a_{p',p'',g,k} = (a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k})(a\_a_{p',g,k} + a\_s_{p',g,k}),$$

and the following constraints:

$$a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k} + a\_a_{p',g,k} + a\_s_{p',g,k} - 1 \geq a_{p',p'',g,k} \quad (43)$$

$$a\_a_{p'',g_0,k} + a\_s_{p'',g_0,k} \leq a_{p',p'',g,k} \quad (44)$$

$$a\_a_{p',g,k} + a\_s_{p',g,k} \leq a_{p',p'',g,k} \quad (45)$$

for all $p'', p' \in P_{\mathrm{EQ}}, g\in G^\star, k\in K$.

The last set of constraints expresses that the total loss for every ONU should not exceed 20 decibels:

$$x\_\mathrm{ATT}_p^g + P^{\mathrm{MARGIN}} + P^{\mathrm{INSERTION}} \leq 20dB \qquad p\in P_{\mathrm{ONU}}, g\in G^\star. \quad (46)$$

7. Numerical Results and Analysis

We implemented the optimization model of Section 5.2 within the Optimization Programming Language (OPL) platform and solved the linear and integer linear programs using the CPLEX package [30].

We conducted our experiments with four different scenarios (Scenarios 1, 2, 3, and 4) consisting of randomly generated Manhattan pattern geographic locations of 16, 32, 64 and 128 ONUs respectively. ONUs are generated in a 40×20 km^2 rectangular grid such that the OLT is located at the middle of the x-coordinate of the corresponding grid, i.e., at location (20,0). ONUs are located along several vertical lines so that each value of x-coordinate can accommodate several ONU locations.

Table 1 contains the values taken for the cost of the equipment [31], as well as the attenuation parameters, which depend on the number of output ports for the splitters, but not for the AWGs. For the costs related to optical fiber cables, we use the value of 7160$/km [31], assuming it includes the cost of trenching and laying the optical fiber cables. We randomly generated 15 potential locations for the placement of the passive equipment.

Table 1: Cost and Attenuation of Equipment

# output ports	Splitters		AWG	
	cost ($)	attenuation (dB)	cost ($)	attenuation (dB)
2	800	3	950	3
4	900	6	1,100	
8	1,100	9	1,400	
16	1,500	12	2,000	
32	2,300	15	3,200	
64	3,700	18	5,600	

We randomly generated the upstream unicast traffic flows within the range [0.05, 0.1] (recall that our traffic parameters are normalized using the wavelength transport capacities, see Section 5.1.4). Towards downstream direction, we randomly generated both unicast and multicast traffic flows within the range [0.1, 0.4]. We considered 15 multicast traffic requests destined for different groups of ONUs.

The first step of our LAPON scheme is to run the H-SLA-ONUs heuristic in order to generate three equipment hierarchies for each of the experimental scenarios, using the number of clusters as a parameter:

(i) Hierarchy 1 with all the ONUs of a scenario grouped into 2 clusters,

(ii) Hierarchy 2 with all the ONUs of a scenario grouped into 4 clusters,

(iii) Hierarchy 3 with all the ONUs of a scenario grouped into 8 clusters.

The next step of the LAPON scheme is to solve the column generation based optimization model in order to:

(i) select the type (splitter or AWG) and location of the passive equipment,

(ii) provision the traffic flows,

for each equipment hierarchy of each scenario. The last step of the LAPON scheme is then to select the best (minimum cost) equipment hierarchy. We now report on the numerical results, for various number of ONUs.

Table 2 shows a comparison of the PON 'greenfield' deployment costs for different hierarchies of Scenario 1 consisting of 16 ONUs. The type of the switching equipment, selected by the optimization model, is depicted in Figure 5 where the distribution of switching equipment is as follows:

Hierarchy 1. 1 splitter at the 1st level and 2 splitters at the 2nd level,

Hierarchy 2. 1 splitter at the 1st level and 4 splitters at the 2nd level,

Hierarchy 3. 1 splitter at the 1st level and 8 splitters at the 2nd level.

For Scenario 1, the minimum cost hierarchy is the Hierarchy 3 with splitters only, and 8 clusters, i.e., a 2 level hierarchy with eight switching equipment at the second level. The selection of the switching equipment is made based on the best choice taking into account the cost, the traffic flows (some unicast, some multicast) and the attenuation constraints. The optimality gap corresponds to:

$$\frac{\tilde{z}_{\text{ILP}} - z^\star_{\text{LP}}}{z^\star_{\text{LP}}},$$

where $z^\star_{\text{LP}}$ is a lower bound on the optimal value $z^\star_{\text{ILP}}$ (PON minimum cost) provided by the optimal value of the linear relaxation of the model (restricted master problem) described in Section 5, and $\tilde{z}_{\text{ILP}}$ is an upper bound on the optimal value $z^\star_{\text{ILP}}$ provided by the ILP solution of the ILP model associated with the last generated restricted master problem. As already observed by several authors for simpler ILP models, the optimality gaps are not very small, and vary from 0 to 8/11/12% in various case studies with 16/32/64 ONUs, therefore much smaller than those observed by, e.g., [18].

Table 2: Experimental Results for Scenario 1 (16 ONUs)

Hierarchy	Equip. Type	$z^\star_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Splitters	1,069,840	1,069,840	0
2	Splitters	1,308,437	1,422,180	8.7
3	Splitters	931,440	931,440	0

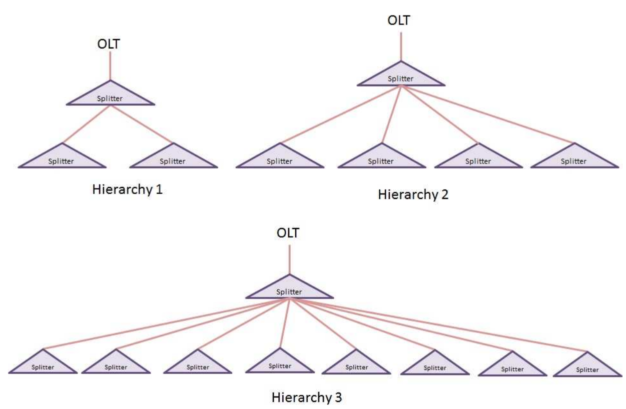


Figure 5: Type of equipment of Table 2

Table 3 shows a comparison of the PON deployment costs for different hierarchies of Scenario 2 consisting of 32 ONUs. The selection of switching equipment for all hierarchies is portrayed in Figure 6 which can be described as follows:

Hierarchy 1. 1 splitter at the 1st level and 2 splitters at the 2nd level,

Table 3: Experimental Results for Scenario 2 (32 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Splitters	2,158,960	2,158,960	0
2	Splitters	1,874,060	1,874,060	0
3	Mixed	2,151,249	2,408,000	11.9

Hierarchy 2. 1 splitter at the 1st level and 4 splitters at the 2nd level,

Hierarchy 3. 1 splitter at the 1st level and 5 splitters as well as 3 AWGs at the 2nd level.

We observe that Hierarchy 2 incurs minimum cost compared to other hierarchies in the case study of 32 ONUs.

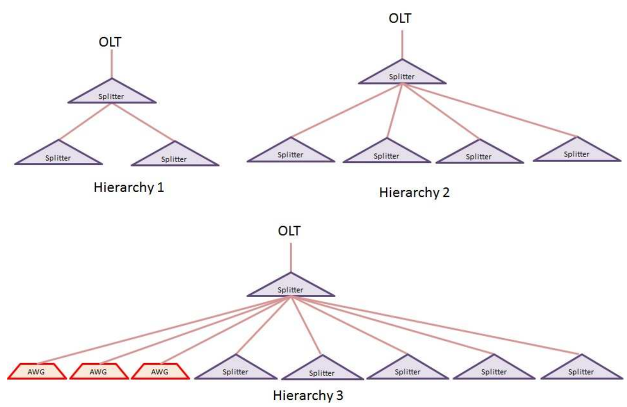


Figure 6: Type of equipment of Table 3

In Table 4, we conducted experiments with Scenario 3 with three similar hierarchies. For all hierarchies, the optimization model selects mixed-equipment PON architecture, as displayed in Figure 7, in which either a splitter or an AWG is assigned to each cluster. The distribution of switching equipment is described below:

Table 4: Experimental Results for Scenario 3 (64 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Mixed	4,281,720	4,281,720	0
2	Mixed	3,867,540	3,867,540	0
3	Mixed	3,895,640	4,344,100	12.8

Hierarchy 1. 1 splitter at the 1st level and 2 AWGs at the 2nd level,

Hierarchy 2. 1 splitter at the 1st level and 2 splitters along with 2 AWGs at the 2nd level,

Hierarchy 3. 1 splitter at the 1st level and 8 AWGs at the 2nd level.

We notice that Hierarchy 2 experiences minimum cost for the deployment of PON with the setting of 64 ONUs. It is obvious that if the optimization model could select splitters for all the clusters of a given hierarchy, the deployment cost would be the most economical one. However, there does not always exist a feasible passive equipment location/allocation with splitters only, due to the signal attenuation constraints. Indeed, in a splitter, the attenuation increases significantly with the increase of the number of output ports. However, the attenuation caused by an AWG is low and independent of the number of the output ports. While selecting the type of the equipment, the optimization model takes into account the attenuation constraint along with the bandwidth demand of each ONU and decides whether a splitter or an AWG will be assigned to a given cluster, according to the distance between the ONUs and the switching equipment.

Table 5 illustrates the deployment cost of a PON where all hierarchies of Scenario 4 are considered. The type of switching equipment selected in these hierarchies are shown in Figure 8 and described below:

Hierarchy 1. 1 splitter at the 1st level and 2 AWGs at the 2nd level,

Hierarchy 2. 1 splitter at the 1st level and 4 AWGs at the 2nd level,

Hierarchy 3. 1 splitter at the 1st level and 8 AWGs at the 2nd level.

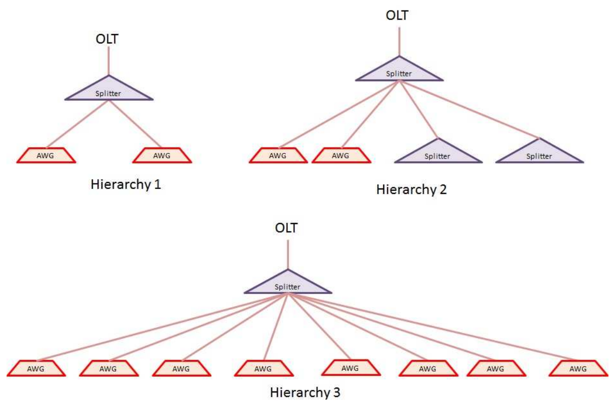


Figure 7: Type of equipment of Table 4

Table 5: Experimental Results for Scenario 4 (128 ONUs)

Hierarchy	Equip. Type	$z^{\star}_{\text{LP}}$	$\tilde{z}_{\text{ILP}}$	Optimality gap (%)
1	Mixed	7,952,440	7,952,440	0
2	Mixed	6,240,540	6,240,540	0
3	Mixed	6,042,096	6,100,740	0.97

Table 5 also reveals that the optimization model only assigns AWGs as the 2nd level equipment for all hierarchies. The reason behind it is that the selection of a splitter for a cluster can not generate any feasible solution due to the high power attenuation caused by the splitters. We perceive that Hierarchy 3 evolves as the best PON architecture for the Scenario 4 which consists of 128 ONUs.

In this paper, we experimented with four different scenarios consisting of different number of ONUs. For each scenario, we consider three types of hierarchies which are generated to investigate the impact of the number of output ports (i.e., split/AWG ratio) of the 1st and the 2nd level switching equipment while optimizing the overall deployment cost of each scenario. For example, in Hierarchy 1 of Scenario 3, 64 ONUs are grouped into 2 clusters,

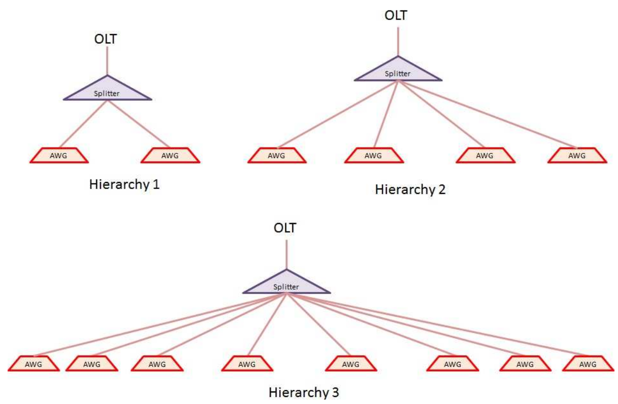


Figure 8: Type of equipment of Table 5

the number of output ports is 2 for the splitter of the 1st level and is 32 for both AWGs of the 2nd level; again in Hierarchy 2 of the same Scenario, the number of output ports is 4 for the splitter of the 1st level and is 32, 32, 2,4 for two splitters and two AWGs of the 2nd level respectively; finally in Hierarchy 3 of the same Scenario, the number of output ports is 8 for the splitter of the 1st level and is 32,8,32,2,2,4,4,2 for eight AWGs of the 2nd level. Similarly, for all scenarios, different values of split/AWG ratio are taken into account. By using different hierarchies, we are getting insight of the number of output ports of the switching equipment and obtaining the optimal values of the corresponding number. Our optimization model considers the following factors while deciding on the minimal cost PON hierarchy for each scenario: (i) type of traffic demand (unicast/multicast), (ii) split/AWG ratio of the equipment, (iii) cost of the equipment and the fiber, (iv) signal power loss caused by equipment and fiber. There is a trade off between the splitting ratio and the maximum allowable distance from the OLT to the ONUs. Increasing the split ratio will accommodate more ONUs to be served by the single equipment, but it will decrease the maximum acceptable distance from the OLT to the ONUs as the attenuation of a splitter depends on its number of output ports. In such a situation, AWGs can be deployed in the network as its attenuation is much less compared to a splitter and does not increase with the increase of its number of output ports. But AWGs are much expensive than splitters. However, the cost of a splitter or an AWG depends on its number of output ports. Our proposed optimization model takes into account all these aspects and selects different hierarchies for different scenarios as a economically feasible WDM PON architecture.

8. Conclusion

We have proposed an automated tool to find the provisioning of the unicast/multicast demand in a WDM/TDM PON network, while optimizing the location of the passive switching equipment assuming it can either be made of splitters or AWGs. As shown in the section on numerical results, the tool is quite powerful as data instances with up to 128 ONUs can be easily solved.

In the near future, we will look at expanding those tools in order to identify the best set of WDM/TDM PONs for a given geographical area, i.e., finding the optimal location of the switching equipment for each PON, as well as the provisioning of the unicast/multicast traffic.

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