Feynman Rules and Static Quantities of the Charged Vector Bosons in a Left-Right Supersymmetric Extension of the Standard Model

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ABSTRACT

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The Standard Model is reviewed and its extension is proposed to incorporate left-right symmetry as well as supersymmetry. Model principles are developed to build a gauge theory with supersymmetry and the minimal supersymmetric extension of the Standard Model (MSSM) is briefly discussed. The left-right supersymmetric extension of the Standard Model is introduced and the symmetry-breaking pattern is described. The physical vector bosons, the charginos and neutralinos, and the Higgs particles are identified. The Lagrangian terms for the interactions of the vector bosons A, Z_L, Z_R, W_L, W_R amongst themselves as well as all corresponding Feynman rules are written down. The interaction Lagrangian terms with three fields are obtained at least one of which is the Photon A or a neutral vector boson Z_L or Z_R or a charged vector boson W_L or W_R. Feynamn rules are written down for interactions with three fields at least one of which is the Photon A, or a charged vector boson W_L or W_R. The Feynman rules are used in order to calculate the anomalous magnetic moments and quadrupole moments of the charged vector bosons W_L or W_R of the left-right supersymmetric extension of the Standard Model.

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INTRODUCTION

While much has been said about the success of the Standard Model in the description of the elementary particles that our world is made of and of the interactions between them, the predominant wisdom in the field is that the Standard Model is not yet the theory of everything. This is not surprising in principle, as the fundamental underlying principle of renormalizability already implies that the Standard Model is only meant to be applicable within a certain energy range. We learn from the Standard Model that at energies below approximately 300 GeV matter particles interact via the electro-weak interactions, which are unified, and via the strong interaction, which stands in a separate position. But the Standard Model teaches us little about what happens at energies higher than 300 GeV and the gravitational force is ignored entirely. The principal goal in particle physics is to formulate, if possible, a theory that unifies all fundamental forces of nature under a single framework. From this point of view, the Standard Model is unsatisfactory, even if it has been able to stand the test of time in the accessible energy regime.

There are a wide variety of models attempting to go beyond the Standard Model. Some of them already began in the mid-seventies when the Standard Model was relatively young and substantial parts of its experimental verification were yet to come. In 1974-75 Pati, Salam, and Mohapatra introduced the concept of left-right symmetry amongst matter fermions and gauge bosons that mediate the forces between them. The goal was originally to provide a mechanism for parity violation as well as a natural framework in which neutrinos could be allowed to be massive.

At approximately the same time Georgi and Glashow proposed the grand unification model SU(5) where all fundamental forces of the Standard Model are united at high energies as one force with a single coupling constant. Moving towards lower energies, the known Standard Model regime is created due to a mechanism that breaks the original SU(5) symmetry. Minimal SU(5) appears to be ruled out today because of its contradiction with the estimated proton lifetime. However the idea of grand unification is still appealing and it might be realized in the context of some extension of the Standard Model.

In 1973 Wess and Zumino introduced a new fundamental concept that is known as supersymmetry. Supersymmetry transforms fermions and bosons into one another and, in so doing, inevitably predicts new particles, because bosons and fermions must be introduced as pairs of particles of equal mass. In nature this is evidently not the case, as the superpartner of, say, the electron, would have had to be seen in some experiment, if both particles had the same mass. It follows that supersymmetry must be broken so as to lift the mass degeneracy and to render the new particles relatively heavy and hard to detect. Moreover, the gauge group symmetries must be broken in such a way as to arrive at the low energy limit, as we know it from Standard Model physics. Despite the absence of conclusive experimental evidence, supersymmetry has been very fascinating for numerous reasons. One of the advantages of model building with supersymmetry is that radiative corrections to the masses of scalar particles are canceled in a relatively unproblematic way due to corresponding corrections from superpartners. Furthermore, local supersymmetry necessarily includes gravity and therefore it is perhaps an appropriate perspective for the unification of all fundamental forces.

We cannot say today which model or which combination of models will ultimately form the one that explains 'everything'. However, in the context of this work we believe that at least supersymmetry must be involved, what ever the theory of everything might otherwise look like. We also adopt left-right symmetry to build a left-right supersymmetric extension of the Standard Model. Both symmetries, left-right symmetry and supersymmetry, predict new particles. In particular, left-right symmetry introduces a 'right-handed' gauge force. In the physical spectrum it materializes in the form of the bosons Z_μ^R and $W_\mu^{R\pm}$, while the Standard Model Z and W bosons are henceforth denoted as Z_μ^L and $W_\mu^{L\pm}$. On the other hand, supersymmetry introduces all of the superpartners.

In order to prove that supersymmetry exists, one must either directly observe the new particles or else measure the effects that they have on other observable quantities of the model. While we have to wait for the next generation of accelerator physics to go into operation, we can make theoretical predictions. In this work the subject is to calculate the anomalous magnetic moments and the quadrupole moments of the charged gauge bosons at one-loop level. The results depend in general on the particle content of the model. Experimental verification of these results would constitute a signal of supersymmetry.

In Chapter 1 we begin with a review of the Standard Model and introduce the idea of left-right symmetry. Chapter 2 develops the principles to build a gauge theory with supersymmetry and the minimal supersymmetric extension of the Standard Model is presented. Chapter 3 introduces the left-right supersymmetry and describes the symmetry- breaking pattern. In Chapters 4, 5, and 6 we develop the Feynman rules that are needed to calculate the anomalous magnetic and quadrupole moments of the charged gauge bosons. We finish with a conclusion.

Chapter 1: The Standard Model

1.1 The Particle Content of the Standard Model

In a Standard Model [1] world matter consists of elementary fermions, three generations (or flavors) of leptons and three generations of quarks. These matter fermions interact through the strong, weak, and electromagnetic interactions that are mediated by the exchange of virtual spin-one bosons. The leptons do not participate in the strong interactions, the quarks do. In a group-theoretical framework, the fermions transform under gauge groups and the requirement of local invariance under the gauge groups necessitates, or explains, the existence of the spin-one gauge bosons. There are gauge bosons for each group and their number equals the number of generators of the group. The gauge group of the Standard Model is the product $SU(3)_c xSU(2)_L xU(1)_Y$. Here $SU(3)_c$ is the strong interaction group and the combination $SU(2)_L xU(1)_Y$ represents the electro-weak interactions. Hence, there are eight gauge bosons for $SU(3)_c$, the gluons G^a_μ (a=1...8), three gauge bosons for $SU(2)_L$, W^a_μ (a=1, 2, 3), and one for $U(1)_Y$, B_μ. Moreover, there is an elementary scalar particle, the famous Higgs field, which originates from an $SU(2)_L$ doublet. This doublet is postulated in order to achieve the symmetry breaking that will be discussed below.

We first introduce the notation for fermions. Quarks and electrons may be represented by Dirac spinors and the neutrino may be represented by a Majorana spinor. The symbols ν_m, ε_m, u_m, and d_m denote the four-component spinors of neutrinos, electrons, up-type, and down-type quarks respectively.

$$\varepsilon_m = \begin{pmatrix} \varepsilon_{Lm} \\ \varepsilon_{Rm} \end{pmatrix}, \; u_m = \begin{pmatrix} u_{Lm} \\ u_{Rm} \end{pmatrix}, \; d_m = \begin{pmatrix} d_{Lm} \\ d_{Rm} \end{pmatrix}, \; \nu_m = \begin{pmatrix} \nu_{Lm} \\ i\sigma_2 \nu^*_{Lm} \end{pmatrix} \tag{1.1.1}$$

Here m is a generation index. In order of increasing mass:

ν_m denotes $\nu_1 = \nu_e$, $\nu_2 = \nu_\mu$, $\nu_3 = \nu_\tau$
ε_m denotes ε_1 ='e', ε_2 ='μ', ε_3 ='τ'
q_{um} denotes q_{u1} ='u', q_{u2} ='c', q_{u3} ='t'
q_{dm} denotes q_{d1} ='d', q_{d2} ='s', q_{d3} ='b'

The left-handed components of these Dirac spinors and the upper component of the neutrino are assigned to the group $SU(2)_L$ as doublets while the right-handed components of the electron and the quarks are singlets under $SU(2)_L$. This is so chosen, of course, in order to achieve a model that is in agreement with experimental observations, but at the same time this choice of gauge groups opens the door to a certain criticism of the Standard Model. We will return to this at the end of this chapter. The $SU(2)_L$ fermion-doublets and singlets are:

$$L_m = \begin{pmatrix} \gamma_L \nu_m \\ \gamma_L \varepsilon_m \end{pmatrix}, \; E_m = \gamma_R \varepsilon_m, \; Q_m = \begin{pmatrix} \gamma_L u_m \\ \gamma_L d_m \end{pmatrix}, \; U_m = \gamma_R u_m, \; D_m = \gamma_R d_m \tag{1.1.2}$$

The scalar $SU(2)_L$ doublet and its charge-conjugate in $SU(2)_L$-space are written as:

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}, \; \Phi^c \equiv i\tau_2 \Phi^* = \begin{pmatrix} \Phi^{0^*} \\ -\Phi^{+^*} \end{pmatrix} \tag{1.1.3}$$

The scalar doublet is needed to break the group symmetry of the model. This will be motivated and illustrated below.

The transformation properties of particles of the model under $SU(3)_c x SU(2)_L x U(1)_Y$ are summarized in Table 1.1:

L_m	$\left(\underline{1}\ \ \underline{2}\ \ -\frac{1}{2}\right)$	G^a_μ	$\left(\underline{8}\ \ \underline{1}\ \ 0\right)$
ε_{R_m}	$\left(\underline{1}\ \ \underline{1}\ \ -1\right)$	W^a_μ	$\left(\underline{1}\ \ \underline{3}\ \ 0\right)$
Q_m	$\left(\underline{3}\ \ \underline{2}\ \ \frac{1}{6}\right)$	B_μ	$\left(\underline{1}\ \ \underline{1}\ \ 0\right)$
u_{R_m}	$\left(\underline{3}\ \ \underline{1}\ \ \frac{2}{3}\right)$		
d_{R_m}	$\left(\underline{3}\ \ \underline{1}\ \ -\frac{1}{3}\right)$		
Φ	$\left(\underline{1}\ \ \underline{2}\ \ \frac{1}{2}\right)$		

Table 1.1: Particle content of the Standard Model - The numbers in brackets represent the quantum numbers of the respective particles under the corresponding group representations.

The convention regarding the quantum numbers in this chapter is the following: the coupling to the $U(1)_Y$-gauge boson in the covariant derivatives stands for $-iYg_YB_\mu$ and the Gell-Mann-Nishima [2] formula takes the form:

$$Q = t^3_L + Y \tag{1.1.4}$$

Here Q is a particle's relative electric charge and Y its hypercharge as specified in Table 1.1. Furthermore $t^3_L = \pm 1/2$ for any up/down component of a SU(2) doublet respectively, and $t^3_L = 0$ for any SU(2) singlet of Table 1.1.

1.2 The Lagrange Density of the Standard Model

The Lagrange density may now be written down in three parts: $\pounds_{fg}$ gives the kinetic energy terms for fermions and gauge bosons and their self-interactions, $\pounds_{Higgs}$ gives the kinetic and potential energies of the scalar doublet, and $\pounds_{Yukawa}$ gives the couplings of the scalar doublet to fermions.

$$\pounds_{SM} \equiv \pounds_{fg} + \pounds_{Higgs} + \pounds_{Yukawa} \tag{1.2.1}$$

$$\begin{aligned}
\pounds_{fg} &= -\frac{1}{4}G^{a}_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^{a}_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&- \frac{\Theta_1}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}\, B^{\mu\nu}B^{\lambda\rho} - \frac{\Theta_2}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}\, W^{a\mu\nu}W^{a\lambda\rho} - \frac{\Theta_3}{16\pi^2}\epsilon_{\mu\nu\lambda\rho}\, G^{a\mu\nu}G^{a\lambda\rho} \\
&+ \overline{L}_m i\gamma^{\mu}\left(\partial_\mu - \frac{i}{2}g_L W^{a}_{\mu}\tau^{a} + \frac{i}{2}g_Y B_\mu\right)L_m + \overline{E}_m i\gamma^{\mu}\left(\partial_\mu + ig_Y B_\mu\right)E_m \\
&+ \overline{Q}_m i\gamma^{\mu}\left(\partial_\mu - \frac{i}{2}g_S G^{a}_{\mu}\lambda^{a} - \frac{i}{2}g_L W^{a}_{\mu}\tau^{a} - \frac{i}{6}g_Y B_\mu\right)Q_m \\
&+ \overline{U}_m i\gamma^{\mu}\left(\partial_\mu - \frac{i}{2}g_S G^{a}_{\mu}\lambda^{a} - \frac{2i}{3}g_Y B_\mu\right)U_m + \overline{D}_m i\gamma^{\mu}\left(\partial_\mu - \frac{i}{2}g_S G^{a}_{\mu}\lambda^{a} + \frac{i}{3}g_Y B_\mu\right)D_m
\end{aligned} \tag{1.2.2}$$

$$\pounds_{Higgs} = \left|\left(\partial_\mu - \frac{i}{2}g_L W^{a}_{\mu}\tau^{a} - \frac{i}{2}g_Y B_\mu\right)\Phi\right|^2 - \lambda\left(\Phi^{\dagger}\Phi - \frac{\mu^2}{2\lambda}\right)^2 \tag{1.2.3}$$

$$\pounds_{Yukawa} = -f_{mn}\overline{L}_m E_n \Phi - h_{mn}\overline{Q}_m D_n \Phi - g_{mn}\overline{Q}_m U_n \Phi^{c} + \text{c.c.} \tag{1.2.4}$$

The field strength tensors in equation (1.2.2) are:

$$\begin{aligned}
G^{a}_{\mu\nu} &= \partial_\mu G^{a}_{\nu} - \partial_\nu G^{a}_{\mu} + g_S f^{abc} G^{b}_{\mu} G^{c}_{\nu} \\
W^{a}_{\mu\nu} &= \partial_\mu W^{a}_{\nu} - \partial_\nu W^{a}_{\mu} + g_L \epsilon^{abc}\, W^{b}_{\mu} W^{c}_{\nu} \\
B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu
\end{aligned} \tag{1.2.5}$$

Here f^{abc} and ϵ^{abc} are the structure constants of $SU(3)_c$ and $SU(2)_L$, respectively, while $U(1)_Y$ is an Abelian group.

The terms proportional to Θ_1, Θ_2, Θ_3 have been included for completeness. While the first two of them appear to be unimportant, the one proportional to Θ_3 gives rise to the so-called strong CP-problem.

The Lagrange density part (1.2.2) is thus far invariant under $SU(3)_c x SU(2)_L x U(1)_Y$; however, as a consequence of this symmetry, all fermions and gauge bosons are massless. In order to generate masses, the gauge group symmetry must be broken. The symmetry breaking scheme is $SU(3)_c x SU(2)_L x U(1)_Y \rightarrow SU(3)_c x U(1)_{em}$. This means that the theory is no longer invariant under $SU(2)_L x U(1)_Y$ but it is still invariant under gauge transformations of the electromagnetic force that we observe in nature. Electromagnetism is represented by the Abelian group $U(1)_{em}$. In this way two symmetries are unified into one. However, the symmetry breaking does not affect the strong interactions that are represented by the group $SU(3)_c$. For this reason the Standard Model is often said to achieve only a partial unification of its three fundamental forces.

The Higgs doublet (1.1.3) is introduced into the theory in order to achieve this breakdown. Once the Higgs doublet is called into existence, it gives rise to the Lagrange densities (1.2.3) and (1.2.4). The former is responsible for gauge boson masses and the latter for fermion masses. In particular, the gauge bosons W_μ^a and B_μ of $SU(2)_L$ and $U(1)_Y$ respectively form linear combinations to become the massive vector bosons $W_\mu^\pm$, Z_μ and the massless photon A_μ.

The symmetry breaking will be discussed in more detail shortly. But first, there is another ingredient in the Standard Model theory. This is the requirement of renormalizability and it is applied to gauge theories in general.

1.3 Renormalizability

Physical theories generally come with an implicit minimum distance or maximum cut-off energy, Λ. For example, QED as the theory of the electron and photon is physically correct only up to an energy of twice the mass of the lightest particle heavier than the electron, $\Lambda = 2m_\mu$, which is twice the muon mass. At energies higher than this, muons can be pair-produced in the final state even if they were not present in the initial state. Then QED becomes the theory of the electron, the muon, and the photon. In a quantum field theory the cut-off energy Λ must be carefully specified, because all states can contribute to any given process in the form of virtual particles and calculations therefore depend explicitly on the cut-off scale Λ. A theory is called renormalizable, if Λ enters into physical predictions only through a finite number of parameters (such as the masses and charges of the particles of the theory). Once the incalculable parameters are taken from experiment, the theory is able to make definite predictions without detailed knowledge of the physics at energies of the order of Λ.

Dimensional analysis provides a practical way to check the renormalizability of a given Lagrange density. In units $\hbar = c = 1$, the dimension of the action integral $S = \int d^4x \pounds(x)$ in powers of mass has to be zero, [S]=0. Since the dimension of the volume element is $[d^4x] = -4$, the dimension of a renormalizable Lagrange density must be always $[\pounds(x)] = 4$. Inspection of canonical mass terms such as $\frac{1}{2}m^2B^\mu B_\mu$, $-\frac{1}{2}m^2HH$, $-m\overline{\Psi}\Psi$ shows that, the dimensions of vector, scalar and fermion fields are $[B^\mu] = 1$, $[H] = 1$, $[\Psi] = \frac{3}{2}$, respectively. Furthermore, in a renormalizable theory the dimensions of coupling constants are non-negative. In the Standard Model the couplings have dimensions $[g_S] = [g_L] = [g_B] = 0$.

Given the particle content of the Standard Model, the Lagrange density $£_{SM} = £_{fg} + £_{Higgs} + £_{Yukawa}$ is the most general one that respects the symmetry $SU(3)_c xSU(2)_L xU(1)_Y$ and that is renormalizable. The renormalizability of the Standard Model was proven by 't Hoft in 1971 [3].

1.4 Symmetry Breaking

As mentioned before, the gauge group symmetry $SU(3)_c xSU(2)_L xU(1)_Y$ needs to be broken so as to generate the masses of the known particles. Here is how this works. The Higgs doublet is expanded about its vacuum expectation value v:

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 + i\Phi_2) \\ \frac{1}{\sqrt{2}}(\Phi_3 + i\Phi_4) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v + H(x)) \end{pmatrix} \tag{1.4.1}$$

It may be shown that this form can always be reached from any arbitrary field configuration via a gauge transformation, and the particular form (1.4.1) constitutes the unitary gauge. The three degrees of freedom that are seemingly lost reappear in the form of masses for three vector bosons as will be shown below.

The vacuum expectation value, in short v.e.v., is given a value of approx. v=246 GeV. It is determined by minimizing the scalar potential

$$V_{Higgs} = \lambda\left(\Phi^\dagger\Phi - \frac{\mu^2}{2\lambda}\right)^2 \tag{1.4.2}$$

Its minimum is found at:

$$\Phi^{\dagger}\Phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2} \tag{1.4.3}$$

The constants λ and μ^2 are real (unitarity), and furthermore, λ is positive (stability) and μ^2 is positive (so that the ground state is not invariant with respect to $SU(2)_L xU(1)_Y$).

The scalar ground state configuration

$$\langle\Phi\rangle \equiv \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix} \tag{1.4.4}$$

breaks $SU(2)_L xU(1)_Y$, but it remains invariant under a symmetry whose generator is found to be $t_L^3 + Y$, which is identified with the electric charge Q. Thus $SU(2)_L xU(1)_Y$ is broken to $U(1)_{em}$.

The general procedure now is to insert

$$\Phi \rightarrow \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\left(v + H(x)\right) \end{pmatrix} \tag{1.4.5}$$

into $\pounds_{SM}$ of equation (1.2.1) in order to identify the fields that are mass eigenstates of the broken theory and the values of their masses. In the process the entire Lagrange density is rewritten in terms of these mass eigenstates, as their interaction terms determine the physical processes that are observed in experiments.

The subsequent parts of this chapter define the mass spectrum and some of the interaction terms in the electro-weak part of the theory.

1.5 The Higgs Mass

The Higgs mass is directly produced in the Higgs-potential V_{Higgs} of (1.4.2). It is:

$$M^2_{Higgs} = 2\lambda v^2 = 2\mu^2 \tag{1.5.1}$$

The Higgs mass is estimated to be 95,3 GeV < M_{Higgs} [4].

1.6 The Gauge Boson Masses

The kinetic part of $\pounds_{Higgs}$ gives:

$$\left|\left(-\frac{i}{2}g_L W^a_\mu \tau^a - \frac{i}{2}g_Y B_\mu\right)\langle\Phi\rangle\right|^2 = \left(\frac{g_L v}{2}\right)^2 W^+_\mu W^{-\mu} + \frac{1}{2}\left(\frac{g_L v}{2}\right)^2 \frac{1}{\cos^2\theta_W} Z_\mu Z^\mu + 0 \bullet A_\mu A^\mu \tag{1.6.1}$$

The Lagrange density piece (1.6.1) becomes mass diagonal by means of an orthogonal transformation that removes mixed terms in the fields. Any orthogonal 2x2 matrix may be parameterized by one variable; here it is called the mixing angle θ_W.

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \tag{1.6.2}$$

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L{}^2 + g_Y{}^2}}, \quad \sin\theta_W = \frac{g_Y}{\sqrt{g_L{}^2 + g_Y{}^2}} \tag{1.6.3}$$

Furthermore the following linear combinations of fields are introduced that diagonalize the generator of electric charge:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right) \tag{1.6.4}$$

The masses are from (1.6.1):

$$M_W = \frac{g_L v}{2},\ M_Z = \frac{g_L v}{2}\frac{1}{\cos\theta_W} \tag{1.6.5}$$

The masses have been measured at M_W=80,419 GeV, M_Z=91,1882 GeV. The photon A_μ is massless.

One important relation that depends on details of the symmetry breaking is the mass ratio

$$\frac{M_W}{M_Z} = \cos\theta_W \tag{1.6.6}$$

1.7 The Fermion Masses

The fermion masses are generated in the Yukawa potential:

$$\begin{aligned}
&-f_{mn}\overline{L}_m E_n\langle\Phi\rangle - h_{mn}\overline{Q}_m D_n\langle\Phi\rangle - g_{mn}\overline{Q}_m U_n\left\langle\Phi^c\right\rangle + \text{c.c.} \\
&= -\frac{v}{\sqrt{2}} f_{mn}\overline{\gamma_L\varepsilon_m}\gamma_R\varepsilon_n - \frac{v}{\sqrt{2}} h_{mn}\overline{\gamma_L d_m}\gamma_R d_n - \frac{v}{\sqrt{2}} g_{mn}\overline{\gamma_L u_m}\gamma_R u_n + \text{c.c.} \\
&= (*)
\end{aligned} \tag{1.7.1}$$

This is made diagonal by means of the unitary transformations

$$\varepsilon_{Lm} = U^{(\varepsilon)}_{mn}\varepsilon'_{Ln},\ \varepsilon_{Rm} = U^{(\varepsilon)*}_{mn}\varepsilon'_{Rn}$$

$$u_{Lm} = U^{(u)}_{mn}u'_{Ln},\ u_{Rm} = U^{(\varepsilon)*}_{mn}u'_{Rn}$$

$$d_{Lm} = U^{(d)}_{mn}d'_{Ln},\ d_{Rm} = U^{(\varepsilon)*}_{mn}d'_{Rn} \tag{1.7.2}$$

The matrices have the properties

$$U^{(\varepsilon)}_{km}f_{mn}U^{(\varepsilon)*}_{nj} = f_k\delta_{kj},\ U^{(u)}_{km}f_{mn}U^{(u)*}_{nj} = g_k\delta_{kj},\ U^{(d)}_{km}f_{mn}U^{(d)*}_{nj} = h_k\delta_{kj} \tag{1.7.3}$$

such that the parameters f_k, g_k, h_k are all real. Substituting these field redefinitions into (1.7.1) and writing the result in terms of the Dirac spinors (1.1.1) gives:

$$(*) = -\frac{v}{\sqrt{2}} f_m \bar{\varepsilon}_m \varepsilon_m - \frac{v}{\sqrt{2}} h_m \bar{d}_m d_m - \frac{v}{\sqrt{2}} g_m \bar{u}_m u_m \tag{1.7.4}$$

Hence, the fermion masses are

$$M_{\varepsilon_m} = \frac{v}{\sqrt{2}} f_m, \; M_{d_m} = \frac{v}{\sqrt{2}} h_m, \; M_{u_m} = \frac{v}{\sqrt{2}} g_m \tag{1.7.5}$$

$M_{\nu_e} < 3$ eV	$M_e \approx 0{,}510$ MeV	$M_u \approx 5$ MeV	$M_d \approx 7$ MeV
$M_{\nu_\mu} < 0{,}19$ MeV	$M_\mu \approx 105{,}658$ MeV	$M_c \approx 1{,}3$ GeV	$M_s \approx 0{,}2$ GeV
$M_{\nu_\tau} < 18{,}2$ MeV	$M_\tau \approx 1777{,}03$ MeV	$M_t \approx 175$ GeV	$M_b \approx 4{,}5$ GeV

Table 1.2: Fermion masses in the Standard Model

In the absence of right-handed neutrinos, the Yukawa-potential $\pounds_{Yukawa}$ of (1.2.4) bears one term less for leptons than it does for quarks, and upon inserting the Higgs v.e.v. (1.4.4) the neutrinos drop out altogether, so that only electrons and quarks obtain masses. This is as expected, but one interesting point about it is that it is necessary to make different field redefinitions for up-type and down-type quarks, while we retain the liberty to redefine neutrinos under the same matrix as electrons:

$$\nu_{Lm} = U^{(\nu)}_{mn} \nu'_{Ln} \equiv U^{(\varepsilon)}_{mn} \nu'_{Ln} \tag{1.7.6}$$

As a consequence the matrix product in the following expression between neutrinos and electrons becomes a unit matrix due to the unitarity of the matrices of (1.7.2):

$$\begin{aligned} &\overline{\nu_{L\,m}} \gamma^\mu \varepsilon_{Lm} = U^{(\nu)*}_{mk} U^{(\varepsilon)}_{mj} \overline{\nu'_{L\,k}} \gamma^\mu \varepsilon'_{Lj} = U^{(\varepsilon)*}_{mk} U^{(\varepsilon)}_{mj} \overline{\nu'_{L\,k}} \gamma^\mu \varepsilon'_{Lj} = \delta_{kj} \overline{\nu'_{L\,k}} \gamma^\mu \varepsilon'_{Lj} \\ &\rightarrow \overline{\nu_{L\,m}} \gamma^\mu \varepsilon_{Lm} = \bar{\nu}_m \gamma^\mu \gamma_L \varepsilon_m \end{aligned} \tag{1.7.7}$$

Here in the end the summation indices are renamed, primes are dropped, and the spinors (1.1.1) are employed. On the other hand, the corresponding expression for up-type and down-type quarks does not simplify to unity, although experimentally it turns out to be relatively close:

$$\overline{u_{L\,m}}\gamma^{\mu}d_{Lm} = U^{(u)\,*}_{mk}U^{(d)}_{mj}\overline{u'_{L\,k}}\gamma^{\mu}d'_{L\,j} = \left(U^{(u)\dagger}U^{(d)}\right)_{kj}\overline{u'_{L\,k}}\gamma^{\mu}d'_{L\,j}$$

$$\rightarrow V_{mn}\overline{u_{L\,m}}\gamma^{\mu}d_{L\,n} = V_{mn}\bar{u}_m\gamma^{\mu}\gamma_L d_n \qquad (1.7.8)$$

The matrix

$$V_{mn} = \left(U^{(u)\dagger}U^{(d)}\right)_{mn} \qquad (1.7.9)$$

is called the Cabbibo-Kobayashi-Maskawa matrix, in short CKM-matrix, and it plays a role in the charged current interactions in the following section.

1.8 Electro-Weak Interactions: QED, Charged Currents, Neutral Currents

Here we examine the interactions resulting from the covariant energy terms from the Lagrange density $\pounds_{fg}$ for quarks and leptons:

$$\bar{L}_m i\gamma^{\mu}\left(-\frac{i}{2}g_L W^a_{\mu}\tau^a + \frac{i}{2}g_Y B_{\mu}\right)L_m + \overline{\varepsilon_{R\,m}}i\gamma^{\mu}\left(ig_Y B_{\mu}\right)\varepsilon_{Rm}$$
$$+\bar{Q}_m i\gamma^{\mu}\left(-\frac{i}{2}g_L W^a_{\mu}\tau^a - \frac{i}{6}g_Y B_{\mu}\right)Q_m + \overline{u_{R\,m}}i\gamma^{\mu}\left(-\frac{2i}{3}g_Y B_{\mu}\right)u_{Rm} + \overline{d_{R\,m}}i\gamma^{\mu}\left(\frac{i}{3}g_Y B_{\mu}\right)d_{Rm}$$

$$(1.8.1)$$

If the physical vector bosons (1.6.2) and the physical fermions (1.7.2) are inserted into (1.8.1), and if finally the result is cast in terms of the spinors (1.1.1), the following three types of interactions are found.

1.8.1 Electromagnetic Interactions

Electromagnetic interactions of quarks and leptons may be summarized conveniently as:

$$\pounds_{em} = eA_{\mu}\sum_{f_m} Q_{f_m}\bar{f}_m\gamma^{\mu}f_m \qquad (1.8.1.1)$$

The electric charge e is given by

$$e = g_L \sin\theta_W = g_Y \cos\theta_W \tag{1.8.1.2}$$

The summation is over the charged spinors of (1.1.1), i.e. $f_m : \varepsilon_m,\ u_m,\ d_m$.

The photon couples to all charged fermions in proportion to their respective charges. Hence, QED is reproduced exactly from the Lagrange density of the Standard Model. More precisely, it is a consequence of the electromagnetic symmetry of the scalar ground state configuration.

1.8.2 Charged-Current Interactions

The charged-current interactions of quarks and leptons can be written as:

$$£_{cc} = \frac{g_L}{\sqrt{2}} W_\mu^+ \left(\bar{\nu}_m \gamma^\mu \gamma_L \varepsilon_m + V_{mn} \bar{u}_m \gamma^\mu \gamma_L d_n\right) + \frac{g_L}{\sqrt{2}} W_\mu^- \left(\bar{\varepsilon}_m \gamma^\mu \gamma_L \nu_m + \left(V^\dagger\right)_{mn} \bar{d}_m \gamma^\mu \gamma_L u_n\right) \tag{1.8.2.1}$$

The weak charged currents are the only ones that change the fermion flavor. If such interactions did not exist. the lightest fermion of any species would be absolutely stable, as flavor would be conserved. As a consequence, charged current interactions are responsible for the majority of observed particle decays in the Standard Model.

The charged interactions of quarks break P and C, because they involve only left-handed spinor components, and they also break CP (or T), however in a very detailed way. CP-violation occurs, if it is impossible to make the CKM-matrix purely real by any redefinition of fields. It may be shown that, in the presence of three quark generations there is one complex phase in the CKM-matrix and CP-violation occurs. But if there were only two quark generations, the CKM-matrix would be able to become real and CP-violation would be impossible in this sector.

The charged interactions of leptons break P and C, but conserve CP (or T), because there is no CKM-matrix for leptons. This is related to the absence of a right-handed neutrino, as was argued in section 1.7. As a result, there are separate lepton number conservation laws for each lepton generation and leptons participate in the charged current interactions with equal strength, which is called weak universality. Weak universality is indeed experimentally observed, which speaks either for the masslessness of the neutrino or for the difficulty to detect the mass, because it must be very small. If a right-handed neutrino is included in the Standard Model framework, or alternatively, if the model is fully extended into a left-right symmetric version, the lepton sector also acquires a CKM-matrix, much like the quarks but with one exception. The electrically neutral neutrino may be either a Majorana or a Dirac particle, while the electrically charged electrons and quarks are carriers of an additive conserved quantum number and therefore must be Dirac particles. If the Neutrino is a Majorana particle, it may be shown that two generations of leptons are sufficient for CP-violation.

1.8.3 Neutral-Current Interactions

The neutral-current interactions of quarks and leptons can be summarized as:

$$\pounds_{nc} = \frac{eZ_\mu}{\cos\theta_W \sin\theta_W} \sum_{f_m} \bar{f}_m \gamma^\mu \left(t_L^3 \gamma_L - Q_{f_m} \sin^2\theta_W \right) f_m \tag{1.8.3.1}$$

The summation is over the spinors of (1.1.1), i.e. $f_m : \nu_m,\ \varepsilon_m,\ u_m,\ d_m$. The neutral interactions of fermions do not change the fermion flavor. These interactions break P and

C, because they involve only left-handed spinor components, but they conserve CP (or T).

1.9 Beyond the Standard Model

The Standard Model has been very successful in describing the low-energy world to which we presently have experimental access. And yet it is generally believed that it cannot be the final word in particle physics. Apart from the unsatisfied desire to unify all fundamental forces in nature, there are a number of concrete arguments of why the Standard Model needs at least to be extended to a more complete theory. The following sections list the arguments that point to the relevance of a left-right supersymmetric extension.

1.9.1 Left-Right Symmetry

The first portion of the critique of the Standard Model is on the fermion content of the model and on the asymmetric way in which the fermions are assigned to SU(2). We present a left-right symmetric version and put it in contrast to the Standard Model.

The basic idea is that right-handed fermion components transform as doublets under a right-handed group $SU(2)_R$ just as left-handed fermion components transform under $SU(2)_L$. This concept requires a few new fields. There must be right-handed neutrinos, so that quarks and leptons can be all assigned to doublets:

$$L_{L_m} = \begin{pmatrix} \gamma_L \nu_m \\ \gamma_L \varepsilon_m \end{pmatrix},\ L_{R_m} = \begin{pmatrix} \gamma_R \nu_m \\ \gamma_R \varepsilon_m \end{pmatrix},\ Q_{L_m} = \begin{pmatrix} \gamma_L u_m \\ \gamma_L d_m \end{pmatrix},\ Q_{R_m} = \begin{pmatrix} \gamma_R u_m \\ \gamma_R d_m \end{pmatrix} \tag{1.9.1.1}$$

If right-handed neutrinos are included in addition to the left-handed ones, it is possible to generate neutrino masses through Yukawa interactions as outlined in section 1.7. Left-right symmetry suggests itself as a more natural framework to consider massive neutrinos. Furthermore, let the gauge bosons with respect to $SU(2)_L$ be denoted from now on as W_μ^{La} and let a new set of gauge bosons W_μ^{Ra} be introduced for $SU(2)_R$. Omitting QCD, the kinetic Lagrange density (1.2.2) becomes:

$$\begin{aligned}
\pounds_{fg} &= -\frac{1}{4}W_{\mu\nu}^{La}W^{La\mu\nu} - \frac{1}{4}W_{\mu\nu}^{Ra}W^{Ra\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&+ \overline{L_L}_m i\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_L W_\mu^{La}\tau^a + \frac{i}{2}g_Y B_\mu\right)L_{Lm} + \overline{L_R}_m i\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_R W_\mu^{Ra}\tau^a + \frac{i}{2}g_Y B_\mu\right)L_{Rm} \\
&+ \overline{Q_L}_m i\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_L W_\mu^{La}\tau^a - \frac{i}{6}g_Y B_\mu\right)Q_{Lm} + \overline{Q_R}_m i\gamma^\mu\left(\partial_\mu - \frac{i}{2}g_R W_\mu^{Ra}\tau^a - \frac{i}{6}g_Y B_\mu\right)Q_{Rm}
\end{aligned}$$

(1.9.1.2)

In the unbroken theory, left-right symmetry implies that the Lagrange density remains unchanged under an interchange of the labels 'L' and 'R'. This form of extending the Standard Model is interesting for many reasons.

Neutrinos probably have a tiny mass after all. In particular, the masses of left-handed neutrinos are expected to be relatively small and the masses of right-handed neutrinos are expected to be relatively big. The mechanism that achieves this is called a seesaw mechanism. Using Higgs fields in triplet representations, the left- and right-handed neutrinos become Majorana-particles with masses that are very different in magnitude [5]. If neutrinos do have masses, several problems in cosmology can receive a plausible solution. One of these problems is the solar neutrino puzzle, which is a discrepancy between the observed neutrino flux from the sun and the value that is predicted by the Standard Model of the sun. Massive neutrinos would also contribute to the missing mass in the universe.

In the Standard Model there is no apparent mechanism to cause parity violation. Instead parity is broken by assuming that left-handed and right-handed fermion components transform differently with respect to SU(2). By contrast, in a left-right symmetric model, parity is originally conserved and may by broken in the low-energy regime due to a suitable mechanism [6].

In the Standard Model the role of the $U(1)_Y$ group is to explain the existence of the electromagnetic field, however the interpretation of the associated hypercharge quantum number is obscure. By contrast, in the left-right version of the model the associated quantum number is B-L (baryon number minus lepton number), which is the only anomaly-free quantum number that had been left ungauged. Hence, the symmetry group of the electro-weak sector is called $SU(2)_L x SU(2)_R x U(1)_{B-L}$.

One of the puzzles of the Standard Model is why the CKM-matrix is so close to being a unit matrix and why CP violation is so small. In the left-right symmetric model at least this smallness can be explained by relating it to the suppression of V+A-currents [7] and the strength of CP-violation is found to be proportional to the small mass ratio M_W^L/M_W^R.

1.9.2 Super-Symmetry

A problem that befalls all renormalizable models with fundamental scalar particles has to do with radiative corrections to scalar masses. The scalar masses receive additive higher-order contributions, which are quadratically divergent. By contrast, fermion masses are only logarithmically divergent, since they receive multiplicative renormalizations, and the divergences can be eliminated with the aid of chiral symmetries. But for scalar masses there are no symmetries to control the divergence. Instead, the parameters of the model must conspire in some 'unnatural' way so as to cancel the scalar divergences. This is not impossible but problematic for the following reason. If the cut-off scale of the renormalizable model is of the order of Λ, so is approximately the bare mass of the scalar field that is responsible for symmetry breaking. With a view to a grand unification scenario, we may estimate $\Lambda \geq 10^{15}$ GeV. On the other hand, the renormalized mass of the scalar particle is of the order of $M_{Higgs} \approx 10^{2}$ GeV, for the masses of the W and Z bosons to be of the correct magnitude. The existence of two so widely separated mass scales in nature is referred to as the gauge hierarchy problem (GHP). As a result, the size of the scalar mass renormalization is proportional to a factor of $(M_{Higgs}/\Lambda)^{2} \approx 10^{-26}$. Not only is this factor amazingly small, but in addition the renormalized mass value must be readjusted or fine-tuned to every order in perturbation theory, which is felt to be rather unnatural.

There are two classes of models that attempt to overcome the naturalness or fine-tuning problem. One class of models presumes that there are in fact no fundamental scalars. Instead, the appearing scalars have a deeper substructure consisting of new sets of fundamental fermions that are connected through forces similar to QCD. These models are known by the name of Technicolor. The other class of models is known as supersymmetry. Supersymmetry postulates the existence of partner-particles for each 'known' particle in such a way that the pairs of partners each consist of one particle of integer spin and one of half-integer spin and equal mass, and such that these partners are transformed into one another through supersymmetry transformations. As a consequence, perturbation theory also produces pairs of loops, which can cancel one another. To every bosonic loop that contributes to the renormalization of a scalar mass there is a corresponding fermionic loop of opposite sign that cancels this contribution. The cancellation of such partner-loops occurs to every order in perturbation theory, rendering fine tuning unnecessary. This point is discussed further in the next chapter.

1.9.3 Grand Unification?

If the ultimate goal is to unify all fundamental forces of nature in a simple framework, one may wonder about the group structure of the model, given that we generally believe in gauge theories. In the unbroken theory, the Standard Model is based on a product of gauge groups $SU(3)_c x SU(2)_L x U(1)_Y$. To each group there corresponds a coupling constant of arbitrary value. Perhaps a more satisfactory theory would be able to begin at high energies with a single gauge group along with a single coupling constant and get broken down at lower energies to resemble the Standard Model scenario. Georgi and Glashow proposed a model with such properties in 1974 [8]. In their model the product $SU(3)_c x SU(2)_L x U(1)_Y$ is embedded into one grand unification group SU(5). Then the group SU(5) undergoes two stages of symmetry breaking. The first stage takes place at about 10^{15} GeV and breaks in the manner SU(5) -> $SU(3)_c x SU(2)_L x U(1)_Y$; the second stage takes place at about 10^2 GeV and breaks as $SU(3)_c x SU(2)_L x U(1)_Y$ -> $U(1)_{em}$. However, minimal SU(5) is ruled out as the ultimate grand unification scheme. The model requires new, as yet unobserved super-heavy gauge bosons that are associated with the first stage of symmetry breaking and allow processes transforming leptons into quarks. Such interactions contribute to the proton decay but the predicted proton lifetime is in contradiction with present observations. Moreover, the unification of the three coupling constants into one unique numerical value at about 10^{16} GeV also does not work exactly. Nevertheless, the principal idea of starting with a single grand group continues to inspire great enthusiasm. In fact, minimal SU(5) is merely a grand unification scheme for a Standard Model world. It may be conjectured that nature is really based on a left-right-symmetric extension of the Standard Model, in which case the grand unification group would be SO(10), which is not ruled out. However, both SU(5) and SO(10) grand unifications achieve symmetry breaking, like the Standard Model, by means of fundamental scalar fields and therefore these models are all plagued with the same difficulty of quadratically divergent mass corrections. In this work, we do not wish to investigate grand unification in greater detail, as we are mainly interested in a phenomenologically viable theory in the low-energy regime. But from our perspective several points are noteworthy. Firstly, we probably ought to anticipate that nature contains the large grand unification mass scale of approx. 10^{16} GeV in some way. Secondly, if grand unification is a true concept, supersymmetry once again ought to play a role in it in order to successfully deal with the divergent mass corrections. Thirdly, in a supersymmetric grand unification scenario it is found that the three gauge couplings of $SU(3)_c \times SU(2)_L \times U(1)_Y$ do actually meet at one point at about 10^{16} GeV [1].

Chapter 2: Gauge Theories and Supersymmetry

It has already been stated that the particles in supersymmetric field theories come in pairs of bosons and fermions that are transformed into one another. The name symmetry implies that there must be generators to perform such transformations. We begin with a simple Lagrange density, introduce the generators of supersymmetry and then go on to build a realistic gauge theory.

2.1 A Basic Supersymmetric Field Theory

The obvious way to illustrate symmetry between bosons and fermions is to write down a Lagrange density with both types of fields along with the transformation rules ([9], [10]).

$$£=\frac{1}{2}\partial_\mu A\partial^\mu A+\frac{1}{2}\partial_\mu B\partial^\mu B+\frac{1}{4}\overline{\Psi}i\gamma^\mu\bar{\partial}_\mu\Psi+\frac{1}{2}F^2+\frac{1}{2}G^2 \tag{2.1.1}$$

This is a simplified version of the Wess-Zumino model. Here A and B are two neutral scalar fields and Ψ is a Majorana spinor. The fields A and B may be combined in the usual fashion into one charged scalar field, which is the superpartner of the Majorana fermion Ψ. The fields F and G are auxiliary scalar fields. They have no kinetic energy terms and they are not meant to interact with any of the other fields of the theory. Furthermore, their dimensions in powers of mass are [F]=[G]=-2 as opposed to the canonical dimensions of the 'physical' scalars [A]=[B]=-1. But their presence is required so that the bosonic and fermionic degrees of freedom (four real components of a Majorana spinor) match exactly. It may now be shown that the action

$$S=\int\left(\frac{1}{2}\partial_\mu A\partial^\mu A+\frac{1}{2}\partial_\mu B\partial^\mu B+\frac{1}{4}\bar{\Psi}i\gamma^\mu\bar{\partial}_\mu\Psi+\frac{1}{2}F^2+\frac{1}{2}G^2\right)d^4x \tag{2.1.2}$$

is invariant under the following set of global supersymmetry transformations.

$$\begin{aligned}
\delta A &= \bar{\varepsilon}\Psi\\
\delta B &= i\bar{\varepsilon}\gamma^5\Psi\\
\delta F &= i\bar{\varepsilon}\gamma^\rho\partial_\rho\Psi\\
\delta G &= -\bar{\varepsilon}\gamma^5\gamma^\rho\partial_\rho\Psi\\
\delta\Psi &= -i\gamma^\rho\varepsilon\delta_\rho A+\gamma^\rho\gamma^5\varepsilon\partial_\rho B-\varepsilon F-i\gamma^5\varepsilon G\\
\delta\bar{\Psi} &= -i\gamma^\rho\bar{\varepsilon}\partial_\rho A+\bar{\varepsilon}\gamma^5\gamma^\rho\partial_\rho B-\bar{\varepsilon}F-i\bar{\varepsilon}\gamma^5 G
\end{aligned} \tag{2.1.3}$$

Here ε is a constant Majorana spinor playing the role of a parameter of the transformation. The transformation is a global one, since the parameter ε is constant. It is thus possible to construct a theory in which bosons and fermions transform into one another. But what is the nature of the transformation rules (2.1.3)? This is the subject of the next section.

2.2 The Generators of Supersymmetry - The Supercharges

The generators of supersymmetry are operators that convert bosonic into fermionic states and vice versa and commute with the Hamiltonian. Such operators are therefore themselves fermionic in character. Since the generators commute with the Hamiltonian, they correspond to conserved quantities, which motivates the name supercharge in place of generator. If the supercharges carry spin 1/2, they may be represented by two-component spinors Q_α and, correspondingly, $\bar{Q}_{\dot{\beta}}$. The anti-commutator $\{Q_\alpha,\bar{Q}_{\dot{\beta}}\}$ is a non-vanishing 2x2-matrix that transforms as (½,½) under Lorentz transformations. A Lorentz-covariant expression is

$$\left\{Q_\alpha, \overline{Q}_{\dot{\beta}}\right\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{2.2.1}$$

where the right-hand side also transforms as (½,½). However, a theorem by Coleman and Mandula [11] states that, if a quantum field theory has a second conserved vector quantity in addition to the energy-momentum four-vector, the S-matrix equals one and no scattering is allowed. Thus the only choice for P_μ is the total energy-momentum. Furthermore, the Coleman-Mandula theorem also rules out any higher-spin conservation laws. Hence, it follows also that spin 1/2 is the only choice for the supercharges and not 3/2 or higher.

From a group-theoretical point of view, equation (2.2.1) implies that the generators of supersymmetry by themselves do not form a group; instead they form an extension of the Poincaré-group (the Lorentz-group with its rotations and boosts plus translations in four-dimensional space-time), because they are related to P_μ, which in turn is a generator of the Poincaré-group. It would appear that the Poincaré-group was not the whole story. In fact, the algebra of the super-Poincaré-group is a set of relations involving both commutators and anti-commutators as shown below in equations (2.2.2) and (2.2.3). Equation (2.2.2) gives the relations of the generators of the Poincaré-group in the absence of supersymmetry.

$$\begin{aligned} \left[J_{\mu\nu}, J_{\rho\sigma}\right] &= i\left(\eta_{\nu\rho}J_{\mu\sigma} - \eta_{\mu\rho}J_{\nu\sigma} + \eta_{\mu\sigma}J_{\nu\rho} - \eta_{\nu\sigma}J_{\mu\rho}\right) \\ \left[P_\mu, J_{\rho\sigma}\right] &= i\left(\eta_{\mu\rho}P_\sigma - \eta_{\mu\sigma}P_\rho\right) \\ \left[P_\mu, P_\nu\right] &= 0 \end{aligned} \tag{2.2.2}$$

$$
\begin{aligned}
&\left\{Q_\alpha, Q_\beta\right\} = 0 = \left\{\overline{Q}^{\dot{\alpha}}, \overline{Q}^{\dot{\beta}}\right\} \\
&\left\{Q_\alpha, \overline{Q}_{\dot{\beta}}\right\} = 2\sigma^{\mu}_{\alpha\dot{\beta}} P_\mu \\
&\left\{Q^\alpha, \overline{Q}^{\dot{\beta}}\right\} = 2\overline{\sigma}^{\mu\dot{\beta}\alpha} P_\mu \\
&\left[Q_\alpha, P_\mu\right] = 0 \\
&\left[J_{\mu\nu}, Q_\alpha\right] = -i\left(\sigma_{\mu\nu}\right)_\alpha^{\beta} Q_\beta \\
&\left[J_{\mu\nu}, \overline{Q}_{\dot{\alpha}}\right] = -i\left(\overline{\sigma}_{\mu\nu}\right)_{\dot{\alpha}}^{\dot{\beta}} Q_{\dot{\beta}}
\end{aligned}
\tag{2.2.3}
$$

In order to fully understand how the Q-operators are generators, it would be necessary to introduce the notion of super-space. Super-space consists of eight coordinates; four of these coordinates are the space-time coordinates introduced in special relativity and the other four are fermionic partner-coordinates, realized by means of anti-commuting Grassmann-variables. Formally, $z^m = \left(x^\mu, \theta^\alpha, \overline{\theta}_{\dot{\alpha}}\right)$ would be an eight-vector in super-space. It is then possible to explicitly construct Q_α as a differential operator in super-space and relate it to $\partial\theta_\alpha$, similarly to the four-momentum generator-operator, which is $P_\mu = -i\partial x_\mu$. Thus supersymmetry transformations may be seen as translations in super-space. Furthermore, particles are represented by super-fields that fall into multiplets in super-space. In this way the concepts 'boson' and 'fermion' are unified as different aspects of the same entity. This 'unification' along with the generalized group structure (2.2.2) explains why it is felt that supersymmetry is an elegant theory. However, we do not pursue this interesting theoretical concept of super-space further, because we are in a position to discuss the appeal of supersymmetry in particle physics without it.

2.3 Building a Gauge Theory with Supersymmetry

Having argued that supersymmetry is beautiful it is now time to explain why it is useful. The subsequent sections present two arguments. Supersymmetry is useful because it helps greatly in the cancellation of quadratic divergences of scalar fields and because it appears to be connected with gravity in a natural way. In fact, gravity comes to our aid indirectly in justifying the way in which we chose to break symmetry.

2.3.1 Cancellation of Quadratic Divergences

Once the particles are introduced as boson-fermion pairs, the Feynman diagrams that represent the particle interactions can also be related to one another like partners. It has already been mentioned in Section 1.9.2 that diagrams can cancel in this way. A general proof would consider a complete set of Feynman graphs that occur in a given model. To give an example that illustrates the relevant graphs in question, one may add to the model of Section 2.1 interaction terms of the form [12]:

$$\pounds = -\frac{1}{2}g^2 AAAA - g\overline{\Psi}\Psi A \tag{2.3.1.1}$$

Here g is a coupling constant and A and Ψ are the scalar and fermion fields introduced in Section 2.1. These interaction terms give rise to mass corrections from graphs, which at one-loop level have the structure indicated in Fig.2.1.

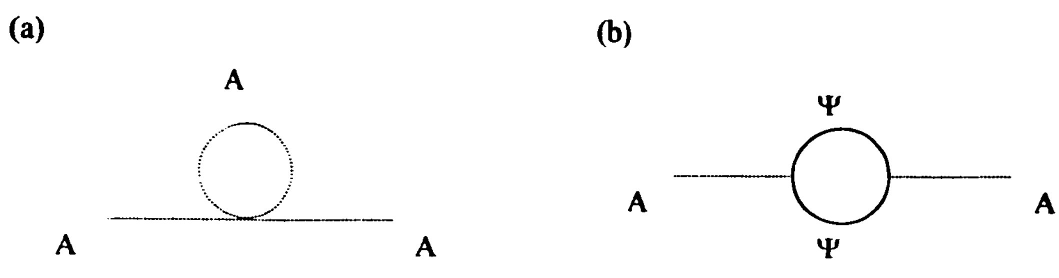


Figure 2.1: Cancellation of quadratic divergences in supersymmetry

The graphs (a) and (b) of Fig. 2.1 correspond to the first and second terms of equation (2.3.1.1), respectively. While both interaction terms have the same sign, the two loops (a) and (b) have opposite signs, since (b) involves a closed fermion loop. The sum of both graphs contributes to the squared scalar mass an amount δm^2, which is found to be:

$$\delta m^2 \approx \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\left(\frac{1}{k^2+m_s^2}-\frac{1}{k^2}\right) = -\frac{m_s^2 \ln\left(\Lambda^2/m^2\right)}{16\pi^2} \tag{2.3.1.1}$$

The mass m_s^2 is a supersymmetry breaking mass. Unbroken supersymmetry implies $m_s^2 = 0$ in which case $\delta m^2 = 0$. If supersymmetry is broken, then $m_s^2 \neq 0$ and consequently $\delta m^2 \neq 0$. However δm^2 is only logarithmically sensitive to the cutoff Λ and can be rendered finite.

Theories with unbroken supersymmetry are automatically free of quadratic divergences. The cancellation ofquadratically divergent terms demonstrated above works to all orders of perturbation theory and no fine-tuning is required. If supersymmetry is broken, quadratic divergences still cancel to give a total result that is finite. Furthermore, it is possible to introduce terms that break supersymmetry explicitly and still keep the theory finite and, in particular, free of quadratic divergences [13]. Terms with this property are called soft breaking terms. More about them will be said below in the discussion of symmetry breaking.

2.3.1 The Role of Gravity

Gravity by itself is outside the scope of this work. However, supersymmetry points to it so clearly and gravity also has its implications on particle physics. Therefore a few remarks are in order. From one perspective, the presence of gravity is already formulated within the algebra of supercharges. From another perspective, it is impossible to build a viable particle theory, if gravity is entirely ignored. This is because of problems in the process of symmetry breaking. The next two paragraphs investigate both perspectives in turn.

2.3.2.1 Supersymmetry and Gravity

The toy model of paragraph 2.1 only investigates global transformations. It is natural to wonder what happens if the transformations are allowed to become local, i.e. functions of space-time. The result is that there is a connection between supersymmetry and gravity. This may be demonstrated here by stating a few known results, the detailed calculations of which are found in the literature [13].

If the transformations (2.1.3) are generalized for $\varepsilon = \varepsilon(x)$, it is necessary to include a term of the form:

$$\Psi^{\mu} \rightarrow \Psi^{\mu} + \kappa^{-1}\partial^{\mu}\varepsilon \qquad (2.3.2.1.1)$$

Here Ψ^{μ} is a spinorial vector. The dimension of the constant κ is M^{-1}. A universal coupling constant associated with local supersymmetry could therefore be $\kappa = \sqrt{8\pi G_N}$. G_N is Newton's constant and κ is the gravitational constant of general relativity.

The anti-commutator of two successive supersymmetry transformations, applied to a field of a given theory, is found to be:

$$\{\delta(\varepsilon_1(x)), \delta(\varepsilon_2(x))\}X = -2i\overline{\varepsilon_2}(x)\gamma^{\mu}\varepsilon_1(x)\partial_{\mu}X \qquad (2.3.2.1.2)$$

This is in fact nothing else but equation (2.2.1) and it may be verified for our toy model with a constant ε and $X = A, B, \Psi, F, G$. The right-hand side is a translation and it now depends explicitly on the space-time point x^{μ}. This is exactly the type of coordinate transformation that gives rise to General Relativity.

The variation of the action integral depends on the energy-momentum tensor and the only way to make the variation vanish, so that the action is invariant, is to allow the metric tensor to be non-constant, namely:

$$\delta g_{\mu\nu} = \overline{\Psi}_{\mu}\gamma_{\nu}\varepsilon \tag{2.3.2.1.3}$$

This is how matter fields are coupled to gravity. In so doing, one principal problem comes from the fact that General Relativity is not a renormalizable theory. Lagrange densities may be obtained but the dimension counting of Section 1.3 will be violated. However, it is possible to impose the condition that the only nonrenormalizable terms are those that depend on the gravitational constant, and that the theory becomes renormalizable in the flat limit in which the gravitational constant goes to zero and space-time becomes the Minkowski space. Imposing this condition is feasible because General Relativity is locally flat over sufficiently small fragments of the space-time manifold.

Finally, local symmetries require the introduction of new fields that are called gauge fields. This is essentially how the electromagnetic field is introduced in QED when the symmetry $U(1)_{em}$ is made local. In the same fashion, in local supersymmetry it becomes necessary to postulate the existence of a (Majorana) fermion partner of 3/2, which plays the role of a gauge field of gravity. This fermion is called the gravitino, because its bosonic superpartner is of spin 2 and is easily identified with the graviton. General relativity is not originally a gauge theory. It is based on the principle of equivalence instead. But it is interesting that local supersymmetry might reconcile both principles.

2.3.2.2 Symmetry Breaking and Gravity: Spontaneous versus Explicit Breaking

In the unbroken theory the supersymmetric particle content is made of boson-fermion partners of equal masses. This equality of masses is evidently not a property of nature, because if it were, superpartners of the known particles surely would have been observed. The gauge group symmetry must be broken in order to generate masses and supersymmetry must be broken so that the masses of superpartners are no longer equal, unless breaking supersymmetry can achieve both. We know from the Standard Model how to implement a spontaneous breaking of the gauge symmetry. Ideally one would wish to carry out the breakdown of supersymmetry in a spontaneous way as well. Two possible mechanisms are known. One mechanism is the O'Raifertaigh mechanism [14] and another is the Fayet-Illiopoulos [15] mechanism. They are called F-type and D-type mechanisms, respectively, because the scalar potential consists of an F-part and a D-part, as will be shown below. Supersymmetry may be broken by either one or by a combination of both mechanisms. While this is an interesting topic to study in its own right, we shall implement broken supersymmetry in a different way. The point is that *spontaneously broken global supersymmetry* does not lead to an acceptable quantum theory of realistic particles for several reasons.

1) It is possible to derive mass formulas that show that the boson masses cannot be heavier than the fermion masses, which is clearly in contradiction with established observations [16].

2) There are still light scalar particles that should have been observed. In particular, one of the scalar quarks is always lighter than the lightest known quark.

3) It may be shown from (2.2.1) that:

$$E_{vac} \equiv \langle 0|H|0\rangle = \langle 0|P^0|0\rangle = |Q_\alpha|0\rangle|^2 \geq 0 \qquad (2.3.2.2.1)$$

Since unbroken global supersymmetry implies $Q_\alpha|0\rangle = 0$, it follows that $E_{vac} = 0$ if symmetry is conserved, whereas broken global supersymmetry requires $E_{vac} > 0$. But this is problematic because it is shown to imply a non-vanishing cosmological constant. However, in super-gravity the symmetry breakdown can be achieved at vanishing vacuum energy [13].

Once it is established that global supersymmetry is insufficient to allow for a realistic symmetry-breaking scenario, it may be concluded that it is in the nature of things to make the theory local and in so doing to include gravity. On one hand this is a wonderful opportunity, on the other hand it is also an extremely difficult and as yet unresolved task. However, it may be proven that, it is possible to begin with a local supersymmetry, which includes gravity, and to ask how the low-energy limit of the theory would look like. The answer is that the low-energy limit can be described as a global supersymmetry with soft breaking terms [13]. There are three classes of soft breaking terms:

1) Gaugino mass terms $m\lambda\lambda$ or $m\lambda^a\lambda^a$ (Abelian and non-Abelian)

2) Scalar mass terms for superpartners of 'known' particles of the form $\mu^2\tilde{\varepsilon}^*\tilde{\varepsilon}$

3) Tri-linear scalar interactions, for example $\tilde{Q}^\dagger H_d \tilde{d}_R$ (all particles being scalars)

This way of breaking supersymmetry is also used in our left-right supersymmetric model.

2.4 The Minimal Supersymmetric Standard Model (MSSM)

The remaining part of this chapter presents a first realistic model that incorporates the principles of model building that have been outlined so far. The MSSM ([12], [13], [17]) is a suitable opportunity to demonstrate how a supersymmetric gauge theory works, because the field redefinition problems have explicit solutions (if there is only one generation of quarks and leptons). By contrast, the left-right symmetric model of the next chapter requires the diagonalization of matrices that are 3x3-dimensional and even larger and must be solved numerically.

2.4.1 Two-Component Notation

Following supersymmetry conventions, we use from now on 2-spinors instead of the traditional 4-spinors of Chapter 1. One motivation for 2-spinors is that, apart from leptons and quarks it is by no means clear from the beginning what the 4-spinors are going to be in the low-energy limit. The mixing of fields, which takes place as vacuum expectation values are inserted, is more practically described with 2-spinors. The 2-component notation is explained in detail in Appendix 2. However, it is possible to read this chapter and the following ones without much effort, once a few simple definitions are made. Let the symbol ψ denote a 4-spinor of a fermion and γ_μ the Dirac matrices.

$$\psi \equiv \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix}, \ \bar{\psi} \equiv \begin{pmatrix} \eta & \bar{\xi} \end{pmatrix}, \ \gamma_\mu \equiv \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \tag{2.4.1.1}$$

It follows from (2.4.1.1) that:

$$\bar{\psi}_1 \gamma_\mu \psi_2 = \bar{\xi}_1 \bar{\sigma}^\mu \xi_2 + \eta_1 \sigma^\mu \bar{\eta}_2 \tag{2.4.1.2}$$

This establishes the connection between 4-component language, left side of (2.4.1.2), and 2-component language, right side of (2.4.1.2). The reader is asked mainly to accept that some variables are naturally barred according to (2.4.1.1). These are the 2-spinor that are lower components of a 4-spinor as well as the 2x2 block-matrix to the lower left of a Dirac matrix. The bar over a 2-component object represents complex conjugation and it is not to be confused with a bar over a 4-component spinor, which stands for $\overline{\psi} = \psi^\dagger\gamma^0$.

In accordance with (2.4.1.1), the 4-spinors of quarks and leptons are naturally written as:

$$\varepsilon = \begin{pmatrix} \varepsilon_L \\ \overline{\varepsilon_R} \end{pmatrix},\ u = \begin{pmatrix} u_L \\ \overline{u_R} \end{pmatrix},\ d = \begin{pmatrix} d_L \\ \overline{d_R} \end{pmatrix},\ \nu = \begin{pmatrix} \nu_L \\ \overline{\nu_L} \end{pmatrix} \tag{2.4.1.3}$$

The conventions (2.4.1.1) suggest the point of view that the equations (1.1.1) always meant to be (2.4.1.3). As a consequence however, in this new notation ε_L and $\overline{\varepsilon_R}$ mean to create equal electric charge, while ε_L and ε_R create opposite charge.

2.4.2 The Minimal Supersymmetric Standard Model - The Particle Content

Comparing Table 2.1 with Table 1.1, the particles of the MSSM are the ones of the Standard Model plus the corresponding superpartners. However, besides that the MSSM requires the use of two scalar doublets plus partners and there are siglet partner fields ψ_x and $\widetilde{\psi}_x$. There are three reasons why there is second Higgs doublet.

1) The scalar doublet Φ_d resembles the scalar doublet of the Standard Model. However, the Yukawa-interactions of the Standard Model are not all permitted. Supersymmetry forbids the interaction $Q^\dagger \overline{u_R} \Phi_d^c$, which occurs in the Standard Model equation (1.2.4).

Bosons	Fermions	$SU(3)_c x SU(2)_L x U(1)_Y$
$\widetilde{L}_L = \begin{pmatrix} \widetilde{\nu}_L \\ \widetilde{\varepsilon}_L \end{pmatrix}$	$L_L = \begin{pmatrix} \nu_L \\ \varepsilon_L \end{pmatrix}$	$\left(\underline{1} \;\; \underline{2} \;\; -\frac{1}{2}\right)$
$\widetilde{\varepsilon}_R$	$\bar{\varepsilon}_R$	$(\underline{1} \;\; \underline{1} \;\; -1)$
$\widetilde{Q}_L = \begin{pmatrix} \widetilde{u}_L \\ \widetilde{d}_L \end{pmatrix}$	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\left(\underline{3} \;\; \underline{2} \;\; \frac{1}{6}\right)$
$\widetilde{u}_R$	$\bar{u}_R$	$\left(\underline{3} \;\; \underline{1} \;\; \frac{2}{3}\right)$
$\widetilde{d}_R$	$\bar{d}_R$	$\left(\underline{3} \;\; \underline{1} \;\; -\frac{1}{3}\right)$
$\widetilde{\psi}_x$	ψ_x	$(\underline{1} \;\; \underline{1} \;\; 0)$
W_μ^a	λ^a	$(\underline{1} \;\; \underline{3} \;\; 0)$
B_μ	λ_Y	$(\underline{1} \;\; \underline{1} \;\; 0)$
$\Phi_u = \begin{pmatrix} \phi_u^0 \\ \phi_u^- \end{pmatrix}$	$\widetilde{\Phi}_u = \begin{pmatrix} \widetilde{\phi}_u^0 \\ \widetilde{\phi}_u^- \end{pmatrix}$	$\left(\underline{1} \;\; \underline{2} \;\; -\frac{1}{2}\right)$
$\Phi_d = \begin{pmatrix} \phi_d^+ \\ \phi_d^0 \end{pmatrix}$	$\widetilde{\Phi}_d = \begin{pmatrix} \widetilde{\phi}_d^+ \\ \widetilde{\phi}_d^0 \end{pmatrix}$	$\left(\underline{1} \;\; \underline{2} \;\; \frac{1}{2}\right)$

Table 2.1: The Particle Content of the Minimal Supersymmetric Standard Model

Generation indices are suppressed. The numbers in brackets represent the transformation properties of the respective particle multiplets under their gauge groups. The $SU(3)_c$ quantum numbers are noted, however gluons and their interactions will be ignored.

This is demonstrated properly in super-space. A product of three super-fields is invariant under supersymmetry, if all three super-fields are of the same chirality. However, while $Q^\dagger$ and $\bar{u}_R$ are left-handed super-fields, Φ_d^c is a right-handed super-field. Since Yukawa-interactions of charge conjugates of Higgs doublets are ruled out, it is impossible to generate masses for up-type quarks with Φ_d alone.

2) It is necessary to have a second higgsino doublet $\tilde{\Phi}_u$ along with $\tilde{\Phi}_d$ with opposite U(1) quantum number in order to cancel anomalies amongst fermions.

3) Three massless higgsino-fermions (Higgs superpartners) are needed in order to generate the masses for the fermion-partners of the physical gauge bosons $W^\pm$ and Z. One higgsino doublet would only provide two fermions as candidates for absorption.

The siglet partners ψ_x and $\tilde{\psi}_x$ are required in order to construct an unbroken supersymmetric theory with a unique tree level ground state that breaks SU(2)xU(1) to U(1) [12]. However they are relatively unimportant in the phenomenology of the MSSM and they disappear automatically in the left-right supersymmetric extension of the model. The singlets ψ_x and $\tilde{\psi}_x$ are included here for completeness.

2.4.3 The MSSM Lagrange Density

$$\pounds_{MSSM} = \pounds_{gauge} + \pounds_{matter-fermion} + \pounds_{matter-boson} + \pounds_{\tilde{x}\tilde{g}x} + \pounds_{Yukawa} + \pounds_{Soft} - V \tag{2.4.3.1}$$

$$\pounds_{gauge} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu a} + i\bar{\lambda}^a \bar{\sigma}^\mu \left(\delta_{ac}\partial_\mu + g_L \in_{abc} W^b_\mu\right)\lambda^c - \frac{1}{4} B_{\mu\nu}B^{\mu\nu} + i\bar{\lambda}_Y \bar{\sigma}^\mu \partial_\mu \lambda_Y \tag{2.4.3.2}$$

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_L \in^{abc} W^b_\mu W^c_\nu \tag{2.4.3.3}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

The canonical form of a covariant derivative for gauginos in (2.4.3.2) involves the structure constants, as does the field strength tensor of the gauge boson partner in (2.4.3.3). Summation over repeated indices is usually implied. A rule to transform the structure constants into generators is given in Appendix 3.

$$\begin{aligned}
&£_{\text{matter-fermions}} = \\
&iL^{\dagger}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}+\frac{i}{2}g_{Y}B_{\mu}\right)L+\varepsilon_{R}i\sigma^{\mu}\left(\partial_{\mu}+ig_{Y}B_{\mu}\right)\bar{\varepsilon}_{R} \\
&+iQ^{\dagger}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}-\frac{i}{6}g_{Y}B_{\mu}\right)Q+u_{R}i\sigma^{\mu}\left(\partial_{\mu}-\frac{2i}{3}g_{Y}B_{\mu}\right)\bar{u}_{R}+d_{R}i\sigma^{\mu}\left(\partial_{\mu}+\frac{i}{3}g_{Y}B_{\mu}\right)\bar{d}_{R} \\
&+i\tilde{\Phi}_{u}^{\dagger}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}+\frac{i}{2}g_{Y}B_{\mu}\right)\tilde{\Phi}_{u}+i\tilde{\Phi}_{d}^{\dagger}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}-\frac{i}{2}g_{Y}B_{\mu}\right)\tilde{\Phi}_{d}+i\bar{\psi}_{x}\bar{\sigma}^{\mu}\partial_{\mu}\psi_{x}
\end{aligned} \tag{2.4.3.4}$$

$$\begin{aligned}
&£_{\text{matter-bosons}} = \\
&\left|\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}+\frac{i}{2}g_{Y}B_{\mu}\right)\tilde{L}\right|^{2}+\left|\left(\partial_{\mu}+ig_{Y}B_{\mu}\right)\tilde{\varepsilon}_{R}\right|^{2} \\
&+\left|\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}-\frac{i}{6}g_{Y}B_{\mu}\right)\tilde{Q}\right|^{2}+\left|\left(\partial_{\mu}-\frac{2i}{3}g_{Y}B_{\mu}\right)\tilde{u}_{R}\right|^{2}+\left|\left(\partial_{\mu}+\frac{i}{3}g_{Y}B_{\mu}\right)\tilde{d}_{R}\right|^{2} \\
&+\left|\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}+\frac{i}{2}g_{Y}B_{\mu}\right)\Phi_{u}\right|^{2}+\left|\left(\partial_{\mu}-\frac{i}{2}g_{L}W_{\mu}^{a}\tau^{a}-\frac{i}{2}g_{Y}B_{\mu}\right)\Phi_{d}\right|^{2} \\
&+\left|\partial_{\mu}\tilde{\psi}_{x}\right|^{2}
\end{aligned} \tag{2.4.3.5}$$

$$\begin{aligned}
£_{\tilde{x}\tilde{g}x} &= i\sqrt{2}\,\tilde{L}^{\dagger}\left(\frac{g_{L}}{2}\lambda^{a}\tau^{a}-\frac{g_{Y}}{2}\lambda_{Y}\right)L+i\sqrt{2}\,\tilde{\varepsilon}_{R}^{*}\left(-g_{Y}\lambda_{Y}\right)\varepsilon_{R} \\
&+i\sqrt{2}\,\tilde{Q}^{\dagger}\left(\frac{g_{L}}{2}\lambda^{a}\tau^{a}+\frac{g_{Y}}{6}\lambda_{Y}\right)Q+i\sqrt{2}\,\tilde{u}_{R}^{*}\left(\frac{2}{3}g_{Y}\lambda_{Y}\right)u_{R}+i\sqrt{2}\,\tilde{d}_{R}^{*}\left(-\frac{g_{Y}}{3}\lambda_{Y}\right)d_{R} \\
&+i\sqrt{2}\,\Phi_{u}^{\dagger}\left(\frac{g_{L}}{2}\lambda^{a}\tau^{a}-\frac{g_{Y}}{2}\lambda_{Y}\right)\tilde{\Phi}_{u}+i\sqrt{2}\,\Phi_{d}^{\dagger}\left(\frac{g_{L}}{2}\lambda^{a}\tau^{a}+\frac{g_{Y}}{2}\lambda_{Y}\right)\tilde{\Phi}_{d} \\
&+\text{h.c.}
\end{aligned} \tag{2.4.3.6}$$

$$\begin{aligned}
&\pounds_{\text{Yukawa}} = \\
&-h_d^L\left(L^\dagger \varepsilon_R \Phi_d + L^\dagger \tilde{\bar{\varepsilon}}_R \tilde{\Phi}_d + \tilde{L}^\dagger \bar{\varepsilon}_R \tilde{\Phi}_d\right) + \text{h.c.} \\
&-h_u^Q\left(Q^\dagger \bar{u}_R \Phi_u + Q^\dagger \tilde{\bar{u}}_R \tilde{\Phi}_u + \tilde{Q}^\dagger \bar{u}_R \tilde{\Phi}_u\right) + \text{h.c.} \\
&-h_d^Q\left(Q^\dagger \bar{d}_R \Phi_d + Q^\dagger \tilde{\bar{d}}_R \tilde{\Phi}_d + \tilde{Q}^\dagger \bar{d}_R \tilde{\Phi}_d\right) + \text{h.c.} \\
&-x\left(\tilde{\Phi}_d^\dagger \Phi_u^c \bar{\psi}_x + \tilde{\Phi}_d^\dagger \tilde{\Phi}_u^c \tilde{\psi}_x^* + \Phi_d^\dagger \tilde{\Phi}_u^c \bar{\psi}_x\right) + \text{h.c.}
\end{aligned} \tag{2.4.3.7}$$

$$V = \left|F_i^* F_i\right|^2 + \frac{1}{2} D_L^a D_L^a + \frac{1}{2} D_Y D_Y + V_{\text{soft}} \tag{2.4.3.8}$$

$$\begin{aligned}
&\left|F_i^* F_i\right|^2 = \\
&\left|h_u^Q \tilde{Q} \tilde{u}_R + x \tilde{\psi}_x \Phi_d\right|^2 + \left|h_d^Q \tilde{Q} \tilde{d}_R + h_d^L \tilde{L} \tilde{\varepsilon}_R + x \tilde{\psi}_x \Phi_u\right|^2 + \left|h_u^Q \Phi_u \tilde{u}_R + h_d^Q \Phi_d \tilde{d}_R\right|^2 \\
&+ \left|h_d^Q \Phi_d \tilde{\varepsilon}_R\right|^2 + \left|h_u^Q \tilde{Q}^\dagger \Phi_u^c\right|^2 + \left|h_d^Q \tilde{Q}^\dagger \Phi_d^c\right|^2 + \left|h_d^L \tilde{L}^\dagger \Phi_d^c\right|^2 + x^2 \left|\Phi_u^\dagger \Phi_d^c - \mu^2\right|^2
\end{aligned} \tag{2.4.3.9}$$

$$D_L^a = \frac{g_L}{2}\left(\tilde{L}_L^\dagger \tau^a \tilde{L}_L + \tilde{Q}_L^\dagger \tau^a \tilde{Q}_L + \Phi_u^\dagger \tau^a \Phi_u + \Phi_d^\dagger \tau^a \Phi_d\right) \tag{2.4.3.10}$$

$$D_Y = \frac{g_Y}{2}\left(-\frac{1}{2}\tilde{L}_L^\dagger \tilde{L}_L - \tilde{\varepsilon}_R^* \tilde{\varepsilon}_R + \frac{1}{6}\tilde{Q}_L^\dagger \tilde{Q}_L + \frac{2}{3}\tilde{u}_R^* \tilde{u}_R - \frac{1}{3}\tilde{d}_R^* \tilde{d}_R - \frac{1}{2}\Phi_u^\dagger \Phi_u + \frac{1}{2}\Phi_d^\dagger \Phi_d\right) \tag{2.4.3.11}$$

$$\begin{aligned}
&V_{\text{soft}} = \\
&\mu_u^Q h_u^Q \tilde{Q}^\dagger \Phi_u \tilde{u}_R + \text{h.c.} + \mu_d^Q h_d^Q \tilde{Q}^\dagger \Phi_d \tilde{d}_R + \text{h.c.} \\
&+ \mu_d^L h_d^L \tilde{L}^\dagger \Phi_d \tilde{\varepsilon}_R + \text{h.c.} + \mu_x x \tilde{\psi}_x \Phi_u^\dagger \Phi_d^c + \text{h.c.} \\
&+ \mu_Q^2 \tilde{Q}^\dagger \tilde{Q} + \mu_L^2 \tilde{L}^\dagger \tilde{L} + \mu_U^2 \tilde{u}_R^* \tilde{u}_R + \mu_D^2 \tilde{d}_R^* \tilde{d}_R + \mu_E^2 \tilde{\varepsilon}_R^* \tilde{\varepsilon}_R
\end{aligned} \tag{2.4.3.12}$$

$$\pounds_{\text{soft}} = m_L\left(\lambda^a \lambda^a + \bar{\lambda}^a \bar{\lambda}^a\right) + m_Y\left(\lambda_Y \lambda_Y + \bar{\lambda}_Y \bar{\lambda}_Y\right) \tag{2.4.3.13}$$

While this Lagrange density is very impressive in size, most terms are quite regular and common to any supersymmetric gauge theory. The main features are as follows.

1) The pieces $\pounds_{\text{gauge}}$, $\pounds_{\text{matter-fermion}}$, $\pounds_{\text{matter-boson}}$, and $\pounds_{\tilde{x}\tilde{g}x}$ are completely determined, once the particle content is specified. The covariant derivatives in the kinetic energy reflect the transformation properties of the fields under their gauge groups. In $\pounds_{\tilde{x}\tilde{g}x}$ scalar fields interact with their fermion partners via the appropriate gauginos. The

interactions again have the form of a covariant derivative with gauge bosons being replaced with gauginos.

2) The piece $\pounds_{\mathrm{Yukawa}}$ is in equally well determined from Standard Model experience, bearing in mind however that supersymmetry forbids the use of charge conjugate scalar doublets in combination with quarks and leptons. The notation in this chapter is chosen such as to make a comparison with the non-supersymmetric Standard Model transparent. There are other conventions in use but the Yukawa part of the Lagrangian is not the main focus in this work.

3) The scalar potential parts are not so completely forced, as there is a measure of arbitrariness with phenomenological parameters. However, the 'D-part' is an unmistakable function of group generators and hypercharge quantum numbers. The D-part is $\frac{1}{2}D_L^a D_L^a + \frac{1}{2}D_Y D_Y$ and it is derived from a vector super-space multiplet. Its counterpart is the 'F-part' $\left|F_i^* F_i\right|^2$, which is derived from a scalar super-multiplet. Within the scope of this work the F-part may be regarded as another series of renormalizable interactions that are permitted.

4) The pieces $\pounds_{\mathrm{soft}}$ and V_{soft} provide the soft breaking terms that are discussed in section 2.3.2.2. The combinations of different fields in V_{soft} are similar in form to the ones in $\pounds_{\mathrm{Yukawa}}$ and thus well determined.

2.4.4 Symmetry Breaking and Mass Eigenstates

Having implemented the breakdown of supersymmetry with explicit breaking terms, it remains to break $SU(2)_L xU(1)_Y$ to $U(1)_{em}$. To this end the scalar doublets Φ_u and Φ_d are to assume vacuum expectation values v_u and v_d respectively. Detailed calculations for the MSSM are available in the literature [12]. Below we merely list some of the physical fields and their interactions in order to comment on the hopes for experimental verification.

The vev's of the scalar doublets are in general:

$$\langle \Phi_u \rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} v_u \\ 0 \end{pmatrix},\ \langle \Phi_d \rangle \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_d \end{pmatrix} \qquad (2.4.4.1)$$

It is impossible to put both scalar doublets into a simple form like equation (1.4.5) at the same time. The number of scalars that are absorbed to generate boson masses is the same as in the Standard Model. As a result the MSSM predicts additional physical scalar particles. A suitable definition of variables is:

$$\Phi_u \to \Phi_u + \langle \Phi_u \rangle \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_u + H_u(x) + iz_u(x)\right) \\ \phi_u^-(x) \end{pmatrix} \qquad (2.4.4.2)$$

$$\Phi_d \to \Phi_d + \langle \Phi_d \rangle \equiv \begin{pmatrix} \phi_d^+(x) \\ \frac{1}{\sqrt{2}}\left(v_d + H_d(x) + iz_d(x)\right) \end{pmatrix} \qquad (2.4.4.3)$$

It is instructive to make a simplified model calculation taking $v_u = v_d$ and omitting at first the soft breaking terms. From the scalar potential one finds this Higgs spectrum:

$$H^+ = \frac{1}{\sqrt{2}}\left(\phi_d^+ + \phi_u^{-*}\right),\ H^- = \left(H^+\right)^* \qquad M_{H^\pm} = M_W \qquad (2.4.4.4)$$

$$H^0 = \frac{1}{\sqrt{2}}\left(H_d^0 - H_u^0\right) \qquad M_{H^0} = M_Z \qquad (2.4.4.5)$$

$$h_l^0 = \frac{1}{\sqrt{2}}\left(H_d^0 + H_u^0\right) \qquad M_{h_l^0} = M_H \qquad (2.4.4.6)$$

$$h_2^0 = \frac{1}{\sqrt{2}}\left(z_d^0 + z_u^0\right) \qquad M_{h_2^0} = M_H \tag{2.4.4.7}$$

$$h_3^0 = \sqrt{2}\,\mathrm{Re}(\widetilde{\psi}_x) \qquad M_{h_3^0} = M_H \tag{2.4.4.8}$$

$$h_4^0 = \sqrt{2}\,\mathrm{Im}(\widetilde{\psi}_x) \qquad M_{h_4^0} = M_H \tag{2.4.4.9}$$

The Goldstone bosons are:

$$G^+ = \frac{1}{\sqrt{2}}\left(\phi_d^+ - \phi_u^{-*}\right),\ G^- = \left(G^+\right)^* \tag{2.4.4.10}$$

$$G^0 = \frac{1}{\sqrt{2}}\left(z_d^0 - z_u^0\right) \tag{2.4.4.11}$$

The Goldstone bosons are massless and they are absorbed to give masses to gauge bosons, which form the massive fields $W_\mu^\pm$, Z_μ, and the massless photon A_μ. The vector boson spectrum is the same as in the Standard Model. All formulae of section 1.6 are still valid, except that one must make the change:

$$v \to \sqrt{v_u^2 + v_d^2} \tag{2.4.4.12}$$

The fermion partners of the scalar doublet components and the gauginos form 4-spinors of particles that are identified as the superpartners of $W_\mu^\pm$, Z_μ, and A_μ. Using the last line of (2.4.3.6) and the last line of (2.4.3.7) it is found that:

$$\widetilde{\omega}^+ = \begin{pmatrix} \widetilde{\phi}_d^+ \\ \overline{i\lambda^-} \end{pmatrix},\ \widetilde{\omega}^- = \begin{pmatrix} \widetilde{\phi}_u^- \\ \overline{i\lambda^+} \end{pmatrix} \qquad M_{\widetilde{\omega}^\pm} = M_W \tag{2.4.4.13}$$

$$\widetilde{\varsigma} = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(\widetilde{\phi}_u^0 - \widetilde{\phi}_d^0\right) \\ \overline{i\lambda_Z} \end{pmatrix} \qquad M_{\widetilde{\varsigma}} = M_Z \tag{2.4.4.14}$$

$$\widetilde{h} = \begin{pmatrix} \frac{-i}{\sqrt{2}}\left(\widetilde{\phi}_u^0 + \widetilde{\phi}_d^0\right) \\ i\overline{\psi}_x \end{pmatrix} \qquad M_{\widetilde{h}} = M_H \tag{2.4.4.15}$$

$$\widetilde{\gamma} = \begin{pmatrix} -i\lambda_\gamma \\ i\overline{\lambda}_\gamma \end{pmatrix} \qquad M_{\widetilde{\gamma}} = 0 \tag{2.4.4.16}$$

The gauginos in (2.4.4.13) are in analogy to (1.6.4):

$$\lambda^{\pm} = \frac{1}{\sqrt{2}}\left(\lambda^1 \mp i\lambda^2\right),\ \lambda^0 = \lambda^3 \tag{2.4.4.17}$$

The new gauginos in (2.4.4.14) and (2.4.4.16) are in analogy to (1.6.2):

$$\begin{pmatrix} \lambda_Z \\ \lambda_\gamma \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} \lambda^0 \\ \lambda_Y \end{pmatrix} \tag{2.4.4.18}$$

Hence there are multiplets of equal mass and equal number of fermionic and bosonic degrees of freedom:

$$\left(H^{\pm},\ \widetilde{\omega}^{\pm},\ W_\mu^{\pm}\right) \qquad M = M_W \tag{2.4.4.19}$$

$$\left(H^0,\ \widetilde{\varsigma},\ Z_\mu\right) \qquad M = M_Z \tag{2.4.4.20}$$

$$\left(h_1^0,\ h_2^0,\ h_3^0,\ h_4^0,\ \widetilde{h}\right) \qquad M = M_H \tag{2.4.4.21}$$

$$\left(\widetilde{\gamma},\ A_\mu\right) \qquad M = 0 \tag{2.4.4.22}$$

The presence of such mass multiplets means that supersymmetry is unbroken. If $£_{soft}$ is taken into consideration, supersymmetry must be broken, since $£_{soft}$ will interfere with all results of equ. (2.4.4.13) through (2.4.4.18). In general, higgsinos and gauginos are combined in a nontrivial way to form particles that are called charginos and neutralinos. These are denoted in 4-component language as $\widetilde{\chi}_k^{\pm}$ and $\widetilde{\chi}_k^0$ respectively. Calculations of chargino and neutralino interactions will be performed in detail as of the next chapter.

2.4.5 Interactions and Phenomenology

There is at present no conclusive experimental evidence of supersymmetry. The unification of the three different gauge couplings at a GUT-point of approximately 10^{16} GeV serves only as indirect evidence. If supersymmetry can induce the scale of electro-weak interactions, it is reasonable to hope for the discovery of supersymmetry effects in the accessible low-energy regime. Low energy will mean relatively soon energies of approximately 1 TeV. However, it is difficult to make very specific predictions and to claim that they are binding. The full realization of the minimal supersymmetric standard model, including QCD-related interactions and three generations of quarks and leptons, carries 124 independent parameters. It is found that the greatest part of the parameter space is unrealistic. Either this is a new naturalness problem, or constraints are imposed to make the model more naturally realistic. But then, theoretical assumptions can be made in a variety of ways and different assumptions may cause very different phenomenological predictions [4]. In any case, it is at least possible to calculate lower bounds for the masses of unobserved particles. The MSSM-related models predict lower-bound values starting at about 10^2 GeV, which is not at all out of reach. Another important prediction is that the scale of supersymmetry breaking must be not higher than approximately 1-10 TeV for a successful resolution of the naturalness problem that was discussed in Section 1.9.2. Hopefully it will be possible to probe this energy region relatively soon.

Another feature common to a large class of models is the existence of a 'lightest supersymmetric particle' (LSP), which follows from a new quantum number. If B-L is taken as conserved, supersymmetric models possess a multiplicative quantum number called R-parity. It is defined for a particle of spin S as:

$$R = (-1)^{3(B-L)+2S} \tag{2.4.5.1}$$

Equation (2.4.5.1) implies that the 'known' Standard Model particles are R-even while their superpartners are R-odd. If R-parity is conserved, supersymmetric particles are produced in pairs. In general they are unstable and quickly decay into lighter states, but the lightest particle that carries a conserved quantum number must be absolutely stable. A prime candidate in many models is the lightest neutralino. Neutralinos are also interesting, because they are, like neutrinos, candidates for dark matter in the universe [18]. Cosmological constraints suggest that the LSP is almost certainly electrically neutral and color-less [19]. For if dark matter contained a particle that is charged and absolutely stable, one would have to explain why the missing mass of the universe is not charged. And if dark matter contained a particle that is colored and absolutely stable it would be hard to explain the abundance of this colored particle as a free particle and why dark matter should exhibit a preference for a particular color.

In the context of this work, a possible interaction type with neutralinos in the context of this work is:

$$\pounds_{\tilde{\chi}^0\tilde{\chi}^0 Z} = \frac{eZ_\mu}{\cos\theta_W \sin\theta_W} \overline{\tilde{\chi}_j^0} \gamma^\mu \frac{1}{2}\left(\left(N^0\right)^L_{jk} \gamma_L + \left(N^0\right)^R_{jk} \gamma_R\right)\tilde{\chi}_k^0 \tag{2.4.5.2}$$

Here $\left(N^0\right)^L_{jk}$ and $\left(N^0\right)^R_{jk}$ are model-dependent matrix elements. There are 4 neutralinos $\tilde{\chi}_k^0$ in the MSSM with lower bound masses below 100 GeV. Being neutral and color-less, neutralinos interact weakly in ordinary matter and are expected to escape the detector, avoiding direct observation just as neutrinos do. Consequently, the experimental signature in R-parity conserving models would be an amount of missing energy.

The goal in this work is to make theoretical prediction on the anomalous magnetic moments and the quadrupole moments of gauge bosons where signals of supersymmetry may be expected. The magnetic and quadrupole moments have been calculated both in the Standard Model and in the MSSM. We shall make the calculations in the left-right supersymmetric model, which is introduced in the next Chapter.

Chapter 3: Left-Right Supersymmetric Extension of the Standard Model

3.1 The Particle Multiplets

GAUGE FIELDS		
Bosons	Fermions	$SU(2)_L x SU(2)_R x U(1)_{B-L}$
W_μ^{La}	$\Lambda_L = \begin{pmatrix} -\lambda_L^+ \\ \lambda_L^0 \\ \lambda_L^- \end{pmatrix}$	$(\underline{3}\ \ \underline{1}\ \ 0)$
W_μ^{Ra}	$\Lambda_R = \begin{pmatrix} -\lambda_R^+ \\ \lambda_R^0 \\ \lambda_R^- \end{pmatrix}$	$(\underline{1}\ \ \underline{3}\ \ 0)$
V_μ	λ_V	$(\underline{1}\ \ \underline{1}\ \ 0)$
MATTER FIELDS		
Bosons	Fermions	$SU(2)_L x SU(2)_R x U(1)_{B-L}$
$\tilde{L}_{Lm} = \begin{pmatrix} \tilde{\nu}_{Lm} \\ \tilde{\varepsilon}_{Lm} \end{pmatrix}$	$L_{Lm} = \begin{pmatrix} \nu_{Lm} \\ \varepsilon_{Lm} \end{pmatrix}$	$(\underline{2}\ \ \underline{1}\ \ -1)$
$\tilde{L}_{Rm} = \begin{pmatrix} \tilde{\nu}_{Rm} \\ \tilde{\varepsilon}_{Rm} \end{pmatrix}$	$L_{Rm} = \begin{pmatrix} \bar{\nu}_{Rm} \\ \bar{\varepsilon}_{Rm} \end{pmatrix}$	$(\underline{1}\ \ \underline{2}\ \ -1)$
$\tilde{Q}_{Lm} = \begin{pmatrix} \tilde{u}_{Lm} \\ \tilde{d}_{Lm} \end{pmatrix}$	$Q_{Lm} = \begin{pmatrix} u_{Lm} \\ d_{Lm} \end{pmatrix}$	$\left(\underline{2}\ \ \underline{1}\ \ \frac{1}{3}\right)$
$\tilde{Q}_{Rm} = \begin{pmatrix} \tilde{u}_{Rm} \\ \tilde{d}_{Rm} \end{pmatrix}$	$Q_{Rm} = \begin{pmatrix} \bar{u}_{Rm} \\ \bar{d}_{Rm} \end{pmatrix}$	$\left(\underline{1}\ \ \underline{2}\ \ \frac{1}{3}\right)$

HIGGS SECTOR		
Bosons	Fermions	$SU(2)_L x SU(2)_R x U(1)_{B-L}$
$H_L = \begin{pmatrix} -\Delta_L^{++} \\ \Delta_L^{+} \\ \Delta_L^{0} \end{pmatrix}$	$\tilde{H}_L = \begin{pmatrix} -\tilde{\Delta}_L^{++} \\ \tilde{\Delta}_L^{+} \\ \tilde{\Delta}_L^{0} \end{pmatrix}$	$(\underline{3}\ \ \underline{1}\ \ 2)$
$K_L = \begin{pmatrix} -\delta_L^{0} \\ \delta_L^{-} \\ \delta_L^{--} \end{pmatrix}$	$\tilde{K}_L = \begin{pmatrix} -\tilde{\delta}_L^{0} \\ \tilde{\delta}_L^{-} \\ \tilde{\delta}_L^{--} \end{pmatrix}$	$(\underline{3}\ \ \underline{1}\ \ -2)$
$H_R = \begin{pmatrix} -\Delta_R^{++} \\ \Delta_R^{+} \\ \Delta_R^{0} \end{pmatrix}$	$\tilde{H}_R = \begin{pmatrix} -\tilde{\Delta}_R^{++} \\ \tilde{\Delta}_R^{+} \\ \tilde{\Delta}_R^{0} \end{pmatrix}$	$(\underline{1}\ \ \underline{3}\ \ 2)$
$K_R = \begin{pmatrix} -\delta_R^{0} \\ \delta_R^{-} \\ \delta_R^{--} \end{pmatrix}$	$\tilde{K}_R = \begin{pmatrix} -\tilde{\delta}_R^{0} \\ \tilde{\delta}_R^{-} \\ \tilde{\delta}_R^{--} \end{pmatrix}$	$(\underline{1}\ \ \underline{3}\ \ -2)$
$F_{u,d}^{I} = \begin{pmatrix} \phi_{1u,d}^{0} \\ \phi_{2u,d}^{-} \end{pmatrix}$	$\tilde{F}_{u,d}^{I} = \begin{pmatrix} \tilde{\phi}_{1u,d}^{0} \\ \tilde{\phi}_{2u,d}^{-} \end{pmatrix}$	$(\underline{2}\ \ \underline{2}\ \ 0)$
$F_{u,d}^{II} = \begin{pmatrix} \phi_{1u,d}^{+} \\ \phi_{2u,d}^{0} \end{pmatrix}$	$\tilde{F}_{u,d}^{II} = \begin{pmatrix} \tilde{\phi}_{1u,d}^{+} \\ \tilde{\phi}_{2u,d}^{0} \end{pmatrix}$	$(\underline{2}\ \ \underline{2}\ \ 0)$

Table 3.1: The Particles of the Left-Right Supersymmetric Standard Model Extension
The numbers in brackets represent the group transformation properties of the particle multiplets

The full Lagrange density of the left-right supersymmetric model is shown in Ref. [20]. In this work only those Lagrangian parts will be invoked that are needed and the similarity with the MSSM will be stressed.

In the fermion sector of matter fields the model possesses a right-handed neutrino (per generation). In this way all leptons and quarks can be assigned in a symmetric way to

$SU(2)_L$ and $SU(2)_R$, as indicated in Chapter 1.9.1. The corresponding superpartners are now added also. The gluons and their superpartners as well their interactions with the quarks are omitted, since QCD interactions are outside the scope of this work. Particles that couple to the $U(1)_{B-L}$ boson have a covariant derivative term $-iY_{B-L}g_V B_\mu$ and the particle quantum numbers under $SU(2)_L x SU(2)_R x U(1)_{B-L}$ satisfy the Gell-Mann-Nishima relation in the form:

$$Q = t_L^3 + t_R^3 + \frac{Y_{B-L}}{2} \tag{3.1.1}$$

Here Q is a particle's relative electric charge and Y_{B-L} its hypercharge as specified in Table 3.1. Furthermore, $t_L^3 = \pm 1/2$ for any up/down component of a $SU(2)_L$ doublet respectively and $t_L^3 = +1, 0, -1$ for any up, middle, down component of a $SU(2)_L$ triplet respectively. $SU(2)_L$ singlets have $t_L^3 = 0$. The same may be said, if the label 'L' is replaced with the label 'R'.

In this work all SU(2) triplets are written in the form of column vectors. Appendix 3 gives an alternative notation. The four multiplets $F_{u,d}^{I}$ and $F_{u,d}^{II}$ are called bi-doublets, because they transform under both $SU(2)_L$ and $SU(2)_R$, as indicated in Table 3.1. There is also an alternative notation for the bi-doublets, which is shown in Appendix 4. The bi-doublet quantum numbers t_L^3 and t_R^3 are subject to a special rule that is also explained in Appendix 4.

The increase in the number of Higgs multiplets, as opposed to the MSSM, is due to the following reasons. The symmetry-breaking scheme of our left-right supersymmetric model is more complicated and requires more scalars to acquire vacuum expectation values. The particle content must initially reflect left-right symmetry, i.e. invariance under the interchange of the labels 'L' and 'R'. Finally, as pointed out in the MSSM, it must be possible to generate all up-type and down-type fermion masses and to cancel anomalies generated by Higgsinos. This is why $\widetilde{H}$-type Higgsinos have an anomaly partner of $\widetilde{K}$-type with opposite hypercharge.

While the Higgs bi-doublets are typical of left-right symmetry, Higgs triplets may possibly be replaced with doublets. However, triplets are preferred, because they are able to generate small Majorana-masses for ν_L and at the same time large Majorana masses for ν_R. By contrast, Higgs doublets generate Dirac masses that make it is more problematic to obtain a small ν_L mass [5].

3.2 Symmetry Breaking ([21], [22], [23], [24])

The first breakdown is that of supersymmetry at a scale of approximately $M_{SUSY} \approx 1\ \mathrm{TeV}$. It is implemented in the model from the start in the form of explicit soft breaking terms. As a result, gauginos, squarks and sleptons are the only particles that carry mass. The quarks and leptons as well as the gauge bosons are all still massless as a consequence of the invariance of the Lagrange density under $SU(2)_L x SU(2)_R x U(1)_{B-L}$. Furthermore, the model is initially invariant under parity. This means $g_L = g_R$, where g_L and g_R are the coupling constants corresponding to $SU(2)_L$ and $SU(2)_R$ respectively.

The gauge group symmetry must be broken spontaneously in order to generate masses for quarks, leptons and gauge bosons and in order to break parity. As in previous chapters, spontaneous symmetry breaking means that some Higgs multiplets receive vacuum expectation values. There are breakdowns at three different stages. At the first stage only the discrete symmetry of parity is broken.

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \xrightarrow{M_{Parity}} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad (3.2.1)$$

M_{Parity} is the scale at which the breaking occurs. The result is $g_L \neq g_R$, however, no gauge boson masses are generated. The second stage takes place at a scale M_{W_R} due to $\langle H_R \rangle \neq 0$. In this work we also take $\langle K_R \rangle \neq 0$.

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_{W_R}} SU(2)_L \times U(1)_Y \quad (3.2.2)$$

This generates masses for right-labeled gauge bosons. It is allowed to assume $M_{Parity} = M_{W_R}$. The particle world now possesses the same symmetries that the Standard Model takes as a starting point. The final breakdown stage is the analogue of the Standard Model breakdown and it takes place at the weak scale of M_{W_L}.

$$SU(2)_L \times U(1)_Y \xrightarrow{M_{W_L}} U(1)_{em} \quad (3.2.3)$$

This is achieved by taking $\langle F_u^I \rangle \neq 0$ and $\langle F_d^{II} \rangle \neq 0$ and possibly but not necessarily $\langle H_L \rangle \neq 0$ and $\langle K_L \rangle \neq 0$.

As the discussions of the Standard Model and the MSSM have shown, the physical fields, or mass eigenstates that occur in nature may arise as linear combinations of the original fields of the particle content. In particular, in left-right supersymmetry:

1) The photon and massive vector bosons arise from gauge bosons mixings.

2) Gauginos Λ and Higgsinos $\tilde{H}$, $\tilde{K}$, $\tilde{F}$ form charginos $\tilde{\chi}^{\pm}$ and neutralinos $\tilde{\chi}^0$.

3) The Higgs multiplets H, K, F form physical scalars $H^{\pm}$, pseudo-scalars A^0 and Goldstone bosons $G^{\pm}$ and G^0.

All of these mixings are considered in turn in the next three chapters.

Chapter 4: Vector-Boson Mass-Eigenstates and Interactions

The goal in the following three chapters 4, 5, and 6 is to obtain the interactions and Feynman rules of the photon and the massive vector bosons with all other physical fields of the theory. The photon is denoted as A_μ and the neutral and charged massive vector bosons as Z_μ^R, Z_μ^L, $W_\mu^{R\pm}$, $W_\mu^{L\pm}$. These fields are formed out of the gauge bosons that are specified in Table 3.1. Subsequently, in chapter 7 the Feynman rules are used to calculate properties of the charged vector bosons $W_\mu^{R\pm}$ and $W_\mu^{L\pm}$. This means that the focus in this work is on the interactions that involve A_μ, $W_\mu^{R\pm}$, and $W_\mu^{L\pm}$. However, the interactions with Z_μ^R and Z_μ^L also come out naturally and they are noted as well for future reference. This chapter begins with the interactions that vector bosons have amongst themselves.

4.1 Mixing of Gauge-Bosons into Mass-Eigenstate Vector-Bosons

Masses for right-labeled vector bosons are generated at the symmetry breaking stage of equation (3.2.2). To this end the following triplets assume vacuum expectation values.

$$H_R \to \langle H_R \rangle + H_R, \quad K_R \to \langle K_R \rangle + K_R$$

$$\langle H_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_{\Delta_R} \end{pmatrix}, \quad \langle K_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} -v_{\delta_R} \\ 0 \\ 0 \end{pmatrix} \tag{4.1.1}$$

(see also Table 3.1.) Only neutral fields may develop a v.e.v. in order to preserve electromagnetic gauge invariance when the symmetry breakdown is complete. It is sufficient to substitute $H_R \to \langle H_R \rangle$ and $K_R \to \langle K_R \rangle$ to identify the physical fields and their masses. Substituting $H_R \to \langle H_R \rangle + H_R$ and $K_R \to \langle K_R \rangle + K_R$ gives the full interaction Lagrangian. We also define:

$$v_R \equiv \sqrt{v_{\Delta_R}^2 + v_{\delta_R}^2} \tag{4.1.2}$$

Mass terms arise from the covariant kinetic energy of the corresponding scalar triplets.

$$\pounds_{\mathrm{kin}\,H_R,K_R} = \left|\left(\partial_\mu - ig_R\tau^a W_\mu^{Ra} - 2ig_V V_\mu\right)H_R\right|^2 + \left|\left(\partial_\mu - ig_R\tau^a W_\mu^{Ra} + 2ig_V V_\mu\right)K_R\right|^2 \tag{4.1.3}$$

Inserting $H_R \to \langle H_R \rangle$ and $K_R \to \langle K_R \rangle$ from (4.1.1) into (4.1.3) gives the mass terms:

$$\pounds_{\mathrm{Mass}\,W_R^\pm,Z_R,B} = \frac{1}{2} v_R{}^2 g_R{}^2 W_\mu^{R+} W^{R-\mu} + \frac{1}{2} v_R{}^2 \left(g_R{}^2 + 4g_V{}^2\right) Z_\mu^R Z^{R\mu} + 0 \bullet B_\mu B^\mu \tag{4.1.4}$$

In analogy to chapter 1.6, mixed terms in the fields have been removed due to an orthogonal transformation:

$$\begin{pmatrix} Z_\mu^R \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} W_\mu^{R0} \\ V_\mu \end{pmatrix} \tag{4.1.5}$$

$$\cos\varphi = \frac{g_R}{\sqrt{g_R{}^2 + 4g_V{}^2}} \equiv \frac{g'}{g_V}, \qquad \sin\varphi = \frac{2g_V}{\sqrt{g_R{}^2 + 4g_V{}^2}} \equiv 2\frac{g'}{g_R} \tag{4.1.6}$$

Furthermore, as usual:

$$W_\mu^{R\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^{R1} \mp iW_\mu^{R2}\right), \qquad W_\mu^{R3} = W_\mu^0 \tag{4.1.7}$$

The masses are from (4.1.4):

$$M_{W_R} = \frac{v_R g_R}{\sqrt{2}}, \qquad M_{Z_R} = v_R\sqrt{g_R{}^2 + 4g_V{}^2} = \frac{v_R g_R}{\cos\varphi},$$
$$M_B = 0 \tag{4.1.8}$$

The masses of $W_\mu^{R\pm}$ and Z_μ^R have a ratio of:

$$\frac{M_{W_R}}{M_{Z_R}} = \frac{\cos\varphi}{\sqrt{2}} \tag{4.1.9}$$

A comment on the factor of $1/\sqrt{2}$ in (4.1.9) will follow shortly.

A similar process is repeated at the symmetry breaking stage of equation (3.2.3) where the masses for left-labeled vector bosons are generated. The definition of vacuum expectation values at this stage is:

$$F_u^I \rightarrow \langle F_u^I \rangle + F_u^I, \qquad F_d^{II} \rightarrow \langle F_d^{II} \rangle + F_d^{II}$$

$$\langle F_u^I \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \kappa_u \\ 0 \end{pmatrix}, \qquad \langle F_d^{II} \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \kappa_d \end{pmatrix} \tag{4.1.10}$$

The remarks under (4.1.1) apply similarly to (4.1.10). The corresponding covariant kinetic energy of the scalar bi-doublets is:

$$£_{\text{kin } F_u^I, F_d^{II}} = \left| \left(\partial_\mu - \frac{i g_L}{2} \tau_\downarrow^a W_\mu^{La} - \frac{i g_R}{2} \tau^a W_\mu^{Ra} \right) F_u^I \right|^2 + \left| \left(\partial_\mu - \frac{i g_L}{2} \tau_\uparrow^a W_\mu^{La} - \frac{i g_R}{2} \tau^a W_\mu^{Ra} \right) F_d^{II} \right|^2 \tag{4.1.11}$$

Inserting the vacuum expectation values form (4.1.10) into (4.1.11) gives the mass terms:

$$\begin{aligned} £_{\text{Mass } W_L^\pm, Z_L, A} &= \frac{1}{4} g_L{}^2 \left(\kappa_u{}^2 + \kappa_d{}^2 \right) W_\mu^{L+} W^{L-\mu} \\ &+ \frac{1}{2}\frac{1}{4} \left(\kappa_u{}^2 + \kappa_d{}^2 \right)\left(g_L{}^2 + g_R{}^2 \sin^2\varphi \right) Z_\mu^L Z^{L\mu} + 0 \bullet A_\mu A^\mu \end{aligned} \tag{4.1.12}$$

This entails an orthogonal transformation that is the analogue of that in the Standard Model:

$$\begin{pmatrix} Z_\mu^L \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^{L0} \\ B_\mu \end{pmatrix} \tag{4.1.13}$$

$$\cos\theta_W = \frac{g_L}{\sqrt{g_L^2 + g_R^2 \sin^2\varphi}} = \frac{g_L}{\sqrt{g_L^2 + 4g'^2}}$$
$$\sin\theta_W = \frac{g_R \sin\varphi}{\sqrt{g_L^2 + g_R^2 \sin^2\varphi}} = \frac{2g'}{\sqrt{g_L^2 + 4g'^2}} \tag{4.1.14}$$

Two approximations have been made in the equations (4.1.12) through (4.1.14). Firstly, it is assumed that the field Z_μ^R decouples from the breakdown process at this stage. Secondly, in general there is also a CP-violating mixing between the fields $W_\mu^{R\pm}$ and $W_\mu^{L\pm}$ [25]. However it is very small and we neglect it. Our approximation is realized by assuming $v_R >> \kappa_u, \kappa_d >> v_L$ where v_L is the would-be vacuum expectation value of a neutral left-handed triplet field.

The masses at this stage are from (4.1.12):

$$M_{W_L} = \frac{1}{2}\sqrt{\kappa_u^2 + \kappa_d^2}\, g_L,$$
$$M_{Z_L} = \frac{1}{2}\sqrt{\kappa_u^2 + \kappa_d^2}\sqrt{g_L^2 + g_R^2 \sin^2\varphi} \tag{4.1.15}$$
$$= \frac{1}{2}\sqrt{\kappa_u^2 + \kappa_d^2}\sqrt{g_L^2 + 4g'^2} = \frac{1}{2}\sqrt{\kappa_u^2 + \kappa_d^2}\,\frac{g_L}{\cos\theta_W},$$
$$M_A = 0$$

The masses of $W_\mu^{L\pm}$ and Z_μ^L have a ratio of:

$$\frac{M_{W_L}}{M_{Z_L}} = \cos\theta_W \tag{4.1.16}$$

Comparing the mass ratios (4.1.9) and (4.1.16), it is found that they differ formally by a factor $1/\sqrt{2}$. This is due to the fact that the fields Z_μ^R and $W_\mu^{R\pm}$ receive mass from a Higgs field that is a member of a SU(2)-triplet, whereas the fields Z_μ^L and $W_\mu^{L\pm}$ receive their masses from SU(2) bi-doublets.

In summary, the model has undergone two stages of symmetry breaking and the fields that are specified in the particle content in the Table 3.1 have formed physical fields in the following way.

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow{M_{W_R}} SU(2)_L \times U(1)_Y$$

$$\begin{aligned} &W_\mu^{R3} = W_\mu^{R0},\ V_\mu \to Z_\mu^R,\ B_\mu \\ &W_\mu^{R1},\ W_\mu^{R2} \to W_\mu^{R\pm} \end{aligned} \tag{4.1.17}$$

$$SU(2)_L \times U(1)_Y \xrightarrow{M_{W_L}} U(1)_{em}$$

$$\begin{aligned} &W_\mu^{L3} = W_\mu^{L0},\ B_\mu \to Z_\mu^L,\ A_\mu \\ &W_\mu^{L1},\ W_\mu^{L2} \to W_\mu^{L\pm} \end{aligned} \tag{4.1.18}$$

The field B_μ corresponds to the $U(1)_Y$ gauge boson of the Standard Model in the unbroken theory. The coupling constant pertaining to this field is called g' here and it is defined from g_V and g_R as:

$$2g' = g_R \sin\varphi = \frac{2 g_V g_R}{\sqrt{{g_R}^2 + 4{g_V}^2}} \tag{4.1.19}$$

However, in our model B_μ is an intermediate field in the sense that it occurs in between two mixing stages. Since B_μ only appears at an intermediate level, it is practical to eliminate it so to get a relation between the initial fields W_μ^{L0}, W_μ^{R0}, V_μ on one hand and the final fields Z_μ^L, A_μ, Z_μ^R on the other.

$$\begin{pmatrix} W_\mu^{L0} \\ W_\mu^{R0} \\ V_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\varphi\sin\theta_W & \sin\varphi\cos\theta_W & \cos\varphi \\ -\cos\varphi\sin\theta_W & \cos\varphi\cos\theta_W & -\sin\varphi \end{pmatrix} \begin{pmatrix} Z_\mu^L \\ A_\mu \\ Z_\mu^R \end{pmatrix} \tag{4.1.20}$$

This transformation is orthogonal as are separately the two transformations at the two mixing-stages (4.1.5) and (4.1.13), and the inverse transformation is given by the transposed matrix. The matrix in (4.1.20) represents also the most general form to parameterize the transformation. In fact, the columns of the matrix are the unit basis vectors in three-dimensional space in spherical polar coordinates. However, approximations may be hidden in the particular definitions of the mixing angles.

The identities below are worth noting in view of their importance in virtually every practical calculation:

$$\frac{\sqrt{2}}{v_R} M_{W_R} = \frac{e}{\cos\theta_W \sin\varphi} = g_R , \qquad \frac{\cos^2\varphi}{v_R} M_{Z_R} = \frac{e}{\cos\theta_W \tan\varphi} \tag{4.1.21}$$

$$\frac{2}{\sqrt{\kappa_u{}^2 + \kappa_d{}^2}} M_{W_L} = \frac{e}{\sin\theta_W} = g_L , \qquad \frac{2}{\sqrt{\kappa_u{}^2 + \kappa_d{}^2}} M_{Z_L} = \frac{e}{\cos\theta_W \sin\theta_W} \tag{4.1.22}$$

$$g_R = \frac{e}{\sin\varphi\cos\theta_W}, \; g_V = \frac{1}{2}\frac{e}{\cos\varphi\cos\theta_W}, \; g_L = \frac{e}{\sin\theta_W}, \; g' = \frac{1}{2}\frac{e}{\cos\theta_W} \tag{4.1.23}$$

A consistency check

Schematically the number of parameters e, θ_W, φ equals the number of coupling constants g_L, g_R, g_V, since g' is more or less an auxiliary variable. However, some models assume $g_L = g_R$, in which case θ_W and φ are no longer independent. They are then related as follows:

$$\cos\varphi = \frac{\sqrt{\cos(2\theta_W)}}{\cos\theta_W}, \; \sin\varphi = \tan\theta_W , \; g_L = g_R \tag{4.1.24}$$

In that case the mixing of physical fields out of the initial ones assumes this shape:

$$\begin{pmatrix} Z_\mu^L \\ A_\mu \\ Z_\mu^R \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \tan\theta_W & -\tan\theta_W \sqrt{\cos(2\theta_W)} \\ \sin\theta_W & \sin\theta_W & \sqrt{\cos(2\theta_W)} \\ 0 & \dfrac{\sqrt{\cos(2\theta_W)}}{\cos\theta_W} & -\tan\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^{L0} \\ W_\mu^{R0} \\ V_\mu \end{pmatrix} \tag{4.1.25}$$

This set of equations is identical with the one given by Mohapatra and Senjanovic [5]. However, in this work we generally permit $g_L \neq g_R$.

4.2 Interactions of Vector Bosons

The interactions of the vector bosons amongst themselves arise from the kinetic energy terms for gauge bosons of the model.

$$\pounds_{\text{kin-gauge bosons}} = -\frac{1}{4} W_{\mu\nu}^{La} W^{L\mu\nu a} - \frac{1}{4} W_{\mu\nu}^{Ra} W^{R\mu\nu a} - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} \tag{4.2.1}$$

A useful identity is:

$$\begin{aligned} -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu a} &= -\frac{1}{2} W_{\mu\nu}^+ W^{-\mu\nu} - \frac{1}{4} W_{\mu\nu}^0 W^{0\mu\nu} + ig\left(W_{\mu\nu}^+ W^{-\mu} W^{0\nu} - W_{\mu\nu}^- W^{+\mu} W^{0\nu} + W_{\mu\nu}^0 W^{+\mu} W^{-\nu}\right) \\ &- \frac{1}{2} g^2 \left(W_\mu^+ W^{-\mu} W_\nu^+ W^{-\nu} - W_\mu^+ W^{+\mu} W_\nu^- W^{-\nu}\right) - g^2 \left(W_\mu^+ W^{-\mu} W_\nu^0 W^{0\nu} - W_\mu^+ W^{0\mu} W_\nu^- W^{0\nu}\right) \end{aligned} \tag{4.2.2}$$

On the left side of the identity (4.2.2) we have

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g \in^{abc} W_\mu^b W_\nu^c \tag{4.2.3}$$

On the right side of equation (4.2.2) all field strength tensors are just the linearized curls

$$W_{\mu\nu}^{\pm 0} = \partial_\mu W_\nu^{\pm 0} - \partial_\nu W_\mu^{\pm 0}, \quad W_\mu^{\pm} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \tag{4.2.4}$$

The identity (4.2.2) is specific to SU(2), because it relies on properties that are specific to the structure constants of SU(2), for example $\epsilon^{abc}\epsilon^{ade} = \delta^{bd}\delta^{ce} - \delta^{be}\delta^{cd}$.

The set of equations (4.1.20) is to be inserted into (4.2.1) while taking advantage of (4.2.2). The result is pure kinetic terms, trilinear and quadrilinear interactions.

Pure kinetic terms

$$\pounds_{\text{pure kinetic}}$$
$$= -\frac{1}{2}W^{L+}_{\mu\nu}W^{L-\mu\nu} - \frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}Z^{L}_{\mu\nu}Z^{L\mu\nu} - \frac{1}{2}W^{R+}_{\mu\nu}W^{R-\mu\nu} - \frac{1}{4}Z^{R}_{\mu\nu}Z^{R\mu\nu} \tag{4.2.5}$$

Interactions with three fields

$$\pounds_{W_L W_L Z_L} = ig_L \cos\theta_W \left(W^{L+}_{\mu\nu}W^{L-\mu}Z^{L\nu} - W^{L-}_{\mu\nu}W^{L+\mu}Z^{L\nu} + Z^{L}_{\mu\nu}W^{L+\mu}W^{L-\nu}\right) \tag{4.2.6}$$

$$\pounds_{W_L W_L A} = ig_L \sin\theta_W \left(W^{L+}_{\mu\nu}W^{L-\mu}A^{\nu} - W^{L-}_{\mu\nu}W^{L+\mu}A^{\nu} + A_{\mu\nu}W^{L+\mu}W^{L-\nu}\right) \tag{4.2.7}$$

$$\pounds_{W_R W_R Z_L} = -ig_R \sin\theta_W \sin\varphi \left(W^{R+}_{\mu\nu}W^{R-\mu}Z^{L\nu} - W^{R-}_{\mu\nu}W^{R+\mu}Z^{L\nu} + Z^{L}_{\mu\nu}W^{R+\mu}W^{R-\nu}\right) \tag{4.2.8}$$

$$\pounds_{W_R W_R Z_R} = ig_R \cos\varphi \left(W^{R+}_{\mu\nu}W^{R-\mu}Z^{R\nu} - W^{R-}_{\mu\nu}W^{R+\mu}Z^{R\nu} + Z^{R}_{\mu\nu}W^{R+\mu}W^{R-\nu}\right) \tag{4.2.9}$$

$$\pounds_{W_R W_R A} = ig_R \cos\theta_W \sin\varphi \left(W^{R+}_{\mu\nu}W^{R-\mu}A^{\nu} - W^{R-}_{\mu\nu}W^{R+\mu}A^{\nu} + A_{\mu\nu}W^{R+\mu}W^{R-\nu}\right) \tag{4.2.10}$$

Interactions with four fields

$$\pounds_{W_L W_L W_L W_L} = -\frac{1}{2}g_L{}^2\left(\left(W^{L+}_{\mu}W^{L-\mu}\right)^2 - W^{L+}_{\mu}W^{L+\mu}W^{L-}_{\nu}W^{L-\nu}\right) \tag{4.2.11}$$

$$\pounds_{W_R W_R W_R W_R} = -\frac{1}{2}g_R{}^2\left(\left(W^{R+}_{\mu}W^{R-\mu}\right)^2 - W^{R+}_{\mu}W^{R+\mu}W^{R-}_{\nu}W^{R-\nu}\right) \tag{4.2.12}$$

$$\pounds_{W_L W_L Z_L Z_L} = -g_L{}^2\cos^2\theta_W\left(W^{L+}_{\mu}W^{L-\mu}Z^{L}_{\nu}Z^{L\nu} - W^{L+}_{\mu}Z^{L\mu}W^{L-}_{\nu}Z^{L\nu}\right) \tag{4.2.13}$$

$$\pounds_{W_R W_R Z_L Z_L} = -g_R{}^2\sin^2\theta_W\sin^2\varphi\left(W^{R+}_{\mu}W^{R-\mu}Z^{L}_{\nu}Z^{L\nu} - W^{R+}_{\mu}Z^{L\mu}W^{R-}_{\nu}Z^{L\nu}\right) \tag{4.2.14}$$

$$\pounds_{W_R W_R Z_R Z_R} = -g_R{}^2\cos^2\varphi\left(W^{R+}_{\mu}W^{R-\mu}Z^{R}_{\nu}Z^{R\nu} - W^{R+}_{\mu}Z^{R\mu}W^{R-}_{\nu}Z^{R\nu}\right) \tag{4.2.15}$$

$$\pounds_{W_L W_L AA} = -g_L{}^2 \sin^2\theta_W \left(W_\mu^{L+} W^{L-\mu} A_\nu A^\nu - W_\mu^{L+} A^\mu W_\nu^{L-} A^\nu\right) \tag{4.2.16}$$

$$\pounds_{W_R W_R AA} = -g_R{}^2 \cos^2\theta_W \sin^2\varphi \left(W_\mu^{R+} W^{R-\mu} A_\nu A^\nu - W_\mu^{R+} A^\mu W_\nu^{R-} A^\nu\right) \tag{4.2.17}$$

$$\pounds_{W_L W_L Z_L A} = \tag{4.2.18}$$
$$-g_L{}^2 \sin\theta_W \cos\theta_W \left(2W_\mu^{L+} W^{L-\mu} Z_\nu^L A^\nu - W_\mu^{L+} Z^{L\mu} W_\nu^{L-} A^\nu - W_\mu^{L+} A^\mu W_\nu^{L-} Z^{L\nu}\right)$$

$$\pounds_{W_R W_R Z_R A} = \tag{4.2.19}$$
$$-g_R{}^2 \cos\theta_W \sin\varphi \cos\varphi \left(2W_\mu^{R+} W^{R-\mu} Z_\nu^R A^\nu - W_\mu^{R+} Z^{R\mu} W_\nu^{R-} A^\nu - W_\mu^{R+} A^\mu W_\nu^{R-} Z^{R\nu}\right)$$

$$\pounds_{W_R W_R Z_L A} = \tag{4.2.20}$$
$$g_R{}^2 \sin\theta_W \cos\theta_W \sin^2\varphi \left(2W_\mu^{R+} W^{R-\mu} Z_\nu^L A^\nu - W_\mu^{R+} Z^{L\mu} W_\nu^{R-} A^\nu - W_\mu^{R+} A^\mu W_\nu^{R-} Z^{L\nu}\right)$$

$$\pounds_{W_R W_R Z_L Z_R} = \tag{4.2.21}$$
$$g_R{}^2 \sin\theta_W \sin\varphi \cos\varphi \left(W_\mu^{R+} W^{R-\mu} Z_\nu^L Z^{R\nu} - W_\mu^{R+} Z^{L\mu} W_\nu^{R-} Z^{R\nu} - W_\mu^{R+} Z^{R\mu} W_\nu^{R-} Z^{L\nu}\right)$$

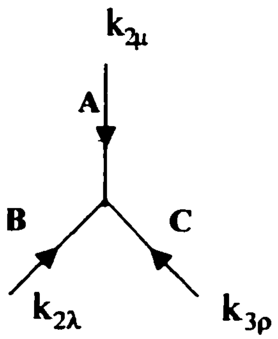
The corresponding Standard Model parts are reproduced within these interaction terms.

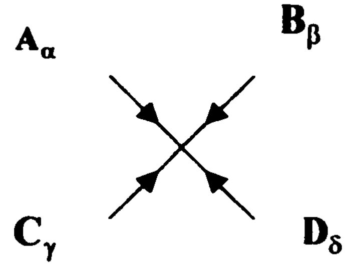
$$\begin{aligned}\pounds_{\text{Standard Model}} &= -\frac{1}{2} W_{\mu\nu}^{L+} W^{L-\mu\nu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu}^L Z^{L\mu\nu} \\ &+ \pounds_{W_L W_L Z_L} + \pounds_{W_L W_L A} + \pounds_{W_L W_L W_L W_L} + \pounds_{W_L W_L Z_L Z_L} + \pounds_{W_L W_L AA} + \pounds_{W_L W_L Z_L A}\end{aligned} \tag{4.2.22}$$

Interactions between $W_\mu^{L\pm}$ and Z_μ^R, such as $\pounds_{W_L W_L Z_R}$, $\pounds_{W_L W_L Z_R Z_R}$, $\pounds_{W_L W_L Z_R A}$, $\pounds_{W_L W_L Z_L Z_R}$, do not occur. This is a consequence of the fact that one matrix element in (4.1.20) is zero, which in turn reflects the way in which the gauge group symmetry is broken.

4.3 Feynman Rules for Vector-Bosons

Three-Field and Four-Field Interactions





$$F_{\mu\lambda\rho}(k_1,k_2,k_3) \equiv (k_1-k_2)_\rho \eta_{\mu\lambda} + (k_2-k_3)_\mu \eta_{\lambda\rho} + (k_3-k_1)_\lambda \eta_{\mu\rho} \qquad (4.3.1)$$

$$S_{\alpha\beta,\gamma\delta} \equiv 2\eta_{\alpha\beta}\eta_{\gamma\delta} - \eta_{\alpha\gamma}\eta_{\beta\delta} - \eta_{\alpha\delta}\eta_{\beta\gamma} \qquad (4.3.2)$$

A	**B**	**C**	**Feynman Rule**	
Z_L	W_L^+	W_L^-	$-ig_L\cos\theta_W\, F_{\mu\lambda\rho}(k_1,k_2,k_3)$	(4.3.3)
A	W_L^+	W_L^-	$-ie\,F_{\mu\lambda\rho}(k_1,k_2,k_3)$	(4.3.4)
Z_L	W_R^+	W_R^-	$ie\tan\theta_W\, F_{\mu\lambda\rho}(k_1,k_2,k_3)$	(4.3.5)
Z_R	W_R^+	W_R^-	$-ig_R\cos\varphi\, F_{\mu\lambda\rho}(k_1,k_2,k_3)$	(4.3.6)
A	W_R^+	W_R^-	$-ie\,F_{\mu\lambda\rho}(k_1,k_2,k_3)$	(4.3.7)

A_α	B_β	C_γ	D_δ	**Feynman Rule**	
W_L^+	W_L^+	W_L^-	W_L^-	$i{g_L}^2 S_{\alpha\beta,\gamma\delta}$	(4.3.8)
W_L^+	W_L^-	Z_L	Z_L	$-{g_L}^2\cos^2\theta_W\, S_{\alpha\beta,\gamma\delta}$	(4.3.9)
W_L^+	W_L^-	A	A	$-ie^2 S_{\alpha\beta,\gamma\delta}$	(4.3.10)
W_L^+	W_L^-	Z_L	A	$-ieg_L\cos\theta_W\, S_{\alpha\beta,\gamma\delta}$	(4.3.11)
W_R^+	W_R^+	W_R^-	W_R^-	$i{g_R}^2 S_{\alpha\beta,\gamma\delta}$	(4.3.12)
W_R^+	W_R^-	Z_R	Z_R	$-{g_R}^2\cos^2\varphi\, S_{\alpha\beta,\gamma\delta}$	(4.3.13)
W_R^+	W_R^-	A	A	$-ie^2 S_{\alpha\beta,\gamma\delta}$	(4.3.14)
W_R^+	W_R^-	Z_R	A	$-ieg_R\cos\varphi\, S_{\alpha\beta,\gamma\delta}$	(4.3.15)
W_R^+	W_R^-	Z_L	Z_L	$-e^2\tan^2\theta_W\, S_{\alpha\beta,\gamma\delta}$	(4.3.16)
W_R^+	W_R^-	Z_L	A	$ie^2\tan\theta_W\, S_{\alpha\beta,\gamma\delta}$	(4.3.17)
W_R^+	W_R^-	Z_L	Z_R	$eg_R\tan\theta_W\cos\varphi\, S_{\alpha\beta,\gamma\delta}$	(4.3.18)

Chapter 5: Fermion Interactions with Vector Bosons

5.1 Mixing of Gauginos and Higgsinos into Charginos and Neutralinos

As the Lagrange density is expanded about the vacuum expectation values (4.1.1) and (4.1.10), the gauginos and higgsinos of the model form linear combination to become massive physical fields. This occurs in two different places of the Lagrange density: in interactions of the form particle-sparticle-gaugino (analogues of equation (2.4.3.6) of the MSSM) and in some of the soft breaking terms. In our model the terms that are responsible for the formation of charginos and neutralinos are the following ones.

$$
\begin{aligned}
& i\sqrt{2}\begin{pmatrix}-\Delta_R^{++*} & \Delta_R^{+*} & \Delta_R^{0*}\end{pmatrix}\left(g_R\begin{pmatrix}\lambda_R^0 & -\lambda_R^+ & 0\\ \lambda_R^- & 0 & -\lambda_R^+\\ 0 & \lambda_R^- & -\lambda_R^0\end{pmatrix}+2g_V\lambda_V\underline{1}\right)\begin{pmatrix}-\tilde{\Delta}_R^{++}\\ \tilde{\Delta}_R^{+}\\ \tilde{\Delta}_R^{0}\end{pmatrix}+\text{h.c.}\\
& +i\sqrt{2}\begin{pmatrix}-\delta_R^{0*} & \delta_R^{-*} & \delta_R^{--*}\end{pmatrix}\left(g_R\begin{pmatrix}\lambda_R^0 & -\lambda_R^+ & 0\\ \lambda_R^- & 0 & -\lambda_R^+\\ 0 & \lambda_R^- & -\lambda_R^0\end{pmatrix}-2g_V\lambda_V\underline{1}\right)\begin{pmatrix}-\tilde{\delta}_R^{0}\\ \tilde{\delta}_R^{-}\\ \tilde{\delta}_R^{--}\end{pmatrix}+\text{h.c.}\\
& +i\sqrt{2}\begin{pmatrix}\phi_{1u}^{0*} & \phi_{2u}^{-*}\end{pmatrix}\left(\frac{g_L}{2}\begin{pmatrix}-\lambda_L^0 & \sqrt{2}\lambda_L^+\\ \sqrt{2}\lambda_L^- & -\lambda_L^0\end{pmatrix}+\frac{g_R}{2}\begin{pmatrix}\lambda_R^0 & \sqrt{2}\lambda_R^+\\ \sqrt{2}\lambda_R^- & -\lambda_R^0\end{pmatrix}\right)\begin{pmatrix}\tilde{\phi}_{1u}^{0}\\ \tilde{\phi}_{2u}^{-}\end{pmatrix}+\text{h.c.}\\
& +i\sqrt{2}\begin{pmatrix}\phi_{1d}^{+*} & \phi_{2d}^{0*}\end{pmatrix}\left(\frac{g_L}{2}\begin{pmatrix}\lambda_L^0 & \sqrt{2}\lambda_L^+\\ \sqrt{2}\lambda_L^- & \lambda_L^0\end{pmatrix}+\frac{g_R}{2}\begin{pmatrix}\lambda_R^0 & \sqrt{2}\lambda_R^+\\ \sqrt{2}\lambda_R^- & -\lambda_R^0\end{pmatrix}\right)\begin{pmatrix}\tilde{\phi}_{1d}^{+}\\ \tilde{\phi}_{2d}^{0}\end{pmatrix}+\text{h.c.}\\
& +\mu_1\left(\begin{pmatrix}\tilde{\phi}_{1u}^{+} & \tilde{\phi}_{2u}^{0}\end{pmatrix}i\tau_2\begin{pmatrix}\tilde{\phi}_{1d}^{0}\\ \tilde{\phi}_{2d}^{-}\end{pmatrix}-\begin{pmatrix}\tilde{\phi}_{1u}^{0} & \tilde{\phi}_{2u}^{-}\end{pmatrix}i\tau_2\begin{pmatrix}\tilde{\phi}_{1d}^{+}\\ \tilde{\phi}_{2d}^{0}\end{pmatrix}\right)+\text{h.c.}\\
& +\frac{1}{2}m_L\lambda_L^i\lambda_L^i+\frac{1}{2}m_R\lambda_R^i\lambda_R^i+\frac{1}{2}m_V\lambda_V\lambda_V+\text{h.c.}\\
& +\mu_2\left(\tilde{\Delta}_L^{++}\tilde{\delta}_L^{--}+\tilde{\Delta}_L^{+}\tilde{\delta}_L^{-}+\tilde{\Delta}_L^{0}\tilde{\delta}_L^{0}\right)+\text{h.c.}+\mu_3\left(\tilde{\Delta}_R^{++}\tilde{\delta}_R^{--}+\tilde{\Delta}_R^{+}\tilde{\delta}_R^{-}+\tilde{\Delta}_R^{0}\tilde{\delta}_R^{0}\right)+\text{h.c.}
\end{aligned}
\tag{5.1.1}
$$

The left-handed delta-fermions in (5.1.1.) do not participate in any mixing, as the corresponding vacuum expectation values are zero by assumption. However, they are included here, because they are also massive Higgsinos due to μ_2.

The matrices that need to be diagonalized in order to obtain canonical mass terms are too big to permit explicit solutions. The diagonalization problems require numerical solutions. In this case it is advantageous to insert the complete set of vev's (4.1.1) and (4.1.10) into (5.1.1) and to perform both breakdowns (3.2.2) and (3.2.3) simultaneously. Inserting the vev's produces two Lagrange densities, one for charginos and one for neutralinos:

$$\pounds_{f-mix} \equiv \pounds_{ch\,arg\,ino} + \pounds_{neutralino} \tag{5.1.2}$$

The subsequent sections deal with $\pounds_{ch\,arg\,ino}$ and $\pounds_{neutralino}$ in turn. The mixing will be described in terms of 2-spinors and then 4-spinors will be formed.

5.1.1 Chargino Mixing

$$\begin{aligned}
\pounds_{ch\,arg\,ino} = {} & \\
& i v_{\Delta_R} g_R \widetilde{\Delta}_R^{+} \lambda_R^{-} + i v_{\delta_R} g_R \widetilde{\delta}_R^{-} \lambda_R^{+} + \frac{i\kappa_u}{\sqrt{2}} \left(g_L \lambda_L^{+} + g_R \lambda_R^{+} \right) \widetilde{\phi}_{2u}^{-} + \frac{i\kappa_d}{\sqrt{2}} \left(g_L \lambda_L^{-} + g_R \lambda_R^{-} \right) \widetilde{\phi}_{1d}^{+} \\
& + \mu_1 \widetilde{\phi}_{1u}^{+} \widetilde{\phi}_{2d}^{-} + \mu_1 \widetilde{\phi}_{2u}^{-} \widetilde{\phi}_{1d}^{+} + m_L \lambda_L^{+} \lambda_L^{-} + m_R \lambda_R^{+} \lambda_R^{-} + \mu_3 \widetilde{\Delta}_R^{+} \widetilde{\delta}_R^{-} + \mu_3 \widetilde{\Delta}_R^{++} \widetilde{\delta}_R^{--} + \text{h.c.}
\end{aligned} \tag{5.1.1.1}$$

To write this Lagrange density in a mass-diagonal form, we need to define the following chargino states.

$$\begin{aligned}
\psi^{+^T} &\equiv \begin{pmatrix} -i\lambda_L^{+} & -i\lambda_R^{+} & \widetilde{\phi}_{1u}^{+} & \widetilde{\phi}_{1d}^{+} & \widetilde{\Delta}_R^{+} \end{pmatrix} \\
\psi^{-^T} &\equiv \begin{pmatrix} -i\lambda_L^{-} & -i\lambda_R^{-} & \widetilde{\phi}_{2u}^{-} & \widetilde{\phi}_{2d}^{-} & \widetilde{\delta}_R^{-} \end{pmatrix}
\end{aligned} \tag{5.1.1.2}$$

Then:

$$X \equiv \begin{pmatrix} m_L & 0 & 0 & \frac{g_L \kappa_d}{\sqrt{2}} & 0 \\ 0 & m_R & 0 & \frac{g_R \kappa_d}{\sqrt{2}} & g_R v_{\Delta_R} \\ \frac{g_L \kappa_u}{\sqrt{2}} & \frac{g_R \kappa_u}{\sqrt{2}} & 0 & -\mu_1 & 0 \\ 0 & 0 & -\mu_1 & 0 & 0 \\ 0 & g_R v_{\delta_R} & 0 & 0 & -\mu_3 \end{pmatrix} \tag{5.1.1.3}$$

With these variables the Lagrange density takes the form:

$$\pounds_{chargino} = -\frac{1}{2}\begin{pmatrix} \psi^{+T} & \psi^{-T} \end{pmatrix}\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c. = -\frac{1}{2}\psi^{+T} X^T \psi^- - \frac{1}{2}\psi^{-T} X \psi^+ + h.c. \tag{5.1.1.4}$$

A non-symmetric matrix X can be diagonalized with two unitary matrices U and V such that $U^* X V^{-1} = M_D$ where M_D is a diagonal matrix with real, non-negative entries, which may be called M_i. Then with a little calculation it may be shown that:

$$\pounds_{chargino} = -M_D \chi^{-T} \chi^{+} - M_D \overline{\chi^{-T}}\, \overline{\chi^{+}} = -\sum_{i=1}^{5} M_i \overline{\tilde{\chi}_i^{+}} \tilde{\chi}_i^{+} \tag{5.1.1.5}$$

In equation (5.1.1.5) χ^+ and χ^- are 'vectors' with 5 'components' like ψ^+ and ψ^- whose entries are, in turn, 2-component spinors.

$$\begin{aligned} \chi^{+T} &\equiv \begin{pmatrix} \chi_1^+ & \chi_2^+ & \chi_3^+ & \chi_4^+ & \chi_5^+ \end{pmatrix} \\ \chi^{-T} &\equiv \begin{pmatrix} \chi_1^- & \chi_2^- & \chi_3^- & \chi_4^- & \chi_5^- \end{pmatrix} \end{aligned} \tag{5.1.1.6}$$

And these 'components' are, in terms of the diagonalizing matrices U and V:

$$\begin{aligned} \chi_i^+ &= \sum_{j=1}^{5} V_{ij} \psi_j^+ \\ \chi_i^- &= \sum_{j=1}^{5} U_{ij} \psi_j^- \end{aligned} \tag{5.1.1.7}$$

On the other hand, the $\tilde{\chi}_i$ in (5.1.1.5) are 4-component Dirac spinors, defined by:

$$\widetilde{\chi}_i^+ = \begin{pmatrix} \chi_i^+ \\ \chi_i^- \end{pmatrix} \tag{5.1.1.8}$$

The charginos are thus defined in terms of the undetermined elements of the matrices U and V of (5.1.1.7).

5.1.2 Neutralino Mixing

$$\begin{aligned}
\pounds_{\text{neutralino}} = \\
& i v_{\Delta_R}\left(-g_R\lambda_R^0 + 2g_V\lambda_V\right)\widetilde{\Delta}_R^0 + i v_{\delta_R}\left(g_R\lambda_R^0 - 2g_V\lambda_V\right)\widetilde{\delta}_R^0 + \frac{i\kappa_u}{2}\left(-g_L\lambda_L^0 + g_R\lambda_R^0\right)\widetilde{\phi}_{1u}^0 \\
& + \frac{i\kappa_d}{2}\left(g_L\lambda_L^0 - g_R\lambda_R^0\right)\widetilde{\phi}_{2d}^0 + \frac{1}{2}m_L\lambda_L^0\lambda_L^0 + \frac{1}{2}m_R\lambda_R^0\lambda_R^0 + \frac{1}{2}m_V\lambda_V^0\lambda_V^0 \\
& - \mu_1\widetilde{\phi}_{1u}^0\widetilde{\phi}_{2d}^0 - \mu_1\widetilde{\phi}_{1d}^0\widetilde{\phi}_{2u}^0 + \mu_3\widetilde{\Delta}_R^0\widetilde{\delta}_R^0 + \text{h.c.}
\end{aligned} \tag{5.1.2.1}$$

This part is also expressed in vector-matrix-form, if we define:

$$\Omega^{0^T} \equiv \left(-i\lambda_L^0 \quad -i\lambda_R^0 \quad -i\lambda_V \quad \widetilde{\phi}_{1u}^0 \quad \widetilde{\phi}_{2d}^0 \quad \widetilde{\Delta}_R^0 \quad \widetilde{\delta}_R^0 \quad \widetilde{\phi}_{1d}^0 \quad \widetilde{\phi}_{2u}^0\right) \tag{5.1.2.2}$$

$$Z \equiv \begin{pmatrix}
m_L & 0 & 0 & -\frac{g_L\kappa_u}{2} & \frac{g_L\kappa_d}{2} & 0 & 0 & 0 & 0 \\
0 & m_R & 0 & \frac{g_R\kappa_u}{2} & -\frac{g_R\kappa_d}{2} & -g_R v_{\Delta_R} & -g_R v_{\delta_R} & 0 & 0 \\
0 & 0 & m_V & 0 & 0 & 2g_V v_{\Delta_R} & 2g_V v_{\delta_R} & 0 & 0 \\
-\frac{g_L\kappa_u}{2} & \frac{g_R\kappa_u}{2} & 0 & 0 & \mu_1 & 0 & 0 & 0 & 0 \\
\frac{g_L\kappa_d}{2} & -\frac{g_R\kappa_d}{2} & 0 & \mu_1 & 0 & 0 & 0 & 0 & 0 \\
0 & -g_R v_{\Delta_R} & 2g_V v_{\Delta_R} & 0 & 0 & 0 & -\mu_3 & 0 & 0 \\
0 & -g_R v_{\delta_R} & 2g_V v_{\delta_R} & 0 & 0 & -\mu_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_1 & 0
\end{pmatrix} \tag{5.1.2.3}$$

$$\pounds_{\text{neutralino}} = -\frac{1}{2}\Omega^{0^T} Z \Omega^0 + \text{h.c.} \tag{5.1.2.4}$$

Let M be the matrix that makes Z diagonal and m_i be the nine eigenvalues of Z, in other words $MZM^T = \text{diag}(m_i)$.

This gives rise to a new 'vector' whose entries are 2-component spinors

$$\chi^{0^T} \equiv \begin{pmatrix} \chi_1^0 & \chi_2^0 & \chi_3^0 & \chi_4^0 & \chi_5^0 & \chi_6^0 & \chi_7^0 & \chi_8^0 & \chi_9^0 \end{pmatrix}$$

$$\chi_i^0 = \sum_{j=1}^{4} M_{ij}\Omega_j^0 \tag{5.1.2.5}$$

In addition, there are the 4-component Majorana-spinors

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \overline{\chi_i^0} \end{pmatrix} \tag{5.1.2.6}$$

And the Lagrange density becomes

$$\pounds_{\text{neutralino}} = -\frac{1}{2}\sum_{j=1}^{9} m_i \chi_i^0 \chi_i^0 + \text{h.c.} = -\frac{1}{2}\sum_{i=1}^{9} m_i \overline{\tilde{\chi}_i^0} \tilde{\chi}_i^0 \tag{5.1.2.7}$$

5.2 Interactions between Vector Bosons and Fermions

The purpose of this section is to work out the interactions between the vector bosons $W_\mu^{L\pm}$, $W_\mu^{R\pm}$, Z_μ^L, Z_μ^R, A^μ and the fermions of the theory. The fermions are quarks, leptons, charginos, and neutralinos.

The interactions of the fermions with the vector bosons arise from the fermions' kinetic part of the Lagrange density. In our model the fermion kinetic part is:

$$
\begin{aligned}
&\pounds_{\mathrm{kin.fermion}} = \\
&iL^{\dagger}_{Lm}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a W^{La}_{\mu}+ig_V V_{\mu}\right)L_{Lm}+iL^{\dagger}_{Rm}\sigma^{\mu}\left(\partial_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}+ig_V V_{\mu}\right)L_{Rm} \\
&+iQ^{\dagger}_{Lm}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a W^{La}_{\mu}-\frac{ig_V}{3}V_{\mu}\right)Q_{Lm}+iQ^{\dagger}_{Rm}\sigma^{\mu}\left(\partial_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}-\frac{ig_V}{3}V_{\mu}\right)Q_{Rm} \\
&+i\widetilde{H}^{\dagger}_{L}\bar{\sigma}^{\mu}\left(\partial_{\mu}-ig_L\tau^a W^{La}_{\mu}-2ig_V V_{\mu}\right)\widetilde{H}_{L}+i\widetilde{H}^{\dagger}_{R}\bar{\sigma}^{\mu}\left(\partial_{\mu}-ig_R\tau^a W^{Ra}_{\mu}-2ig_V V_{\mu}\right)\widetilde{H}_{R} \\
&+i\widetilde{K}^{\dagger}_{L}\bar{\sigma}^{\mu}\left(\partial_{\mu}-ig_L\tau^a W^{La}_{\mu}+2ig_V V_{\mu}\right)\widetilde{K}_{L}+i\widetilde{K}^{\dagger}_{R}\bar{\sigma}^{\mu}\left(\partial_{\mu}-ig_R\tau^a W^{Ra}_{\mu}+2ig_V V_{\mu}\right)\widetilde{K}_{R} \\
&+i\Lambda^{\dagger\,a}_{L}\bar{\sigma}^{\mu}\left(\delta_{ac}\partial_{\mu}+g_L\in_{abc}W^{Lb}_{\mu}\right)\Lambda_{L}{}^{c}+i\Lambda^{\dagger\,a}_{R}\bar{\sigma}^{\mu}\left(\delta_{ac}\partial_{\mu}+g_R\in_{abc}W^{Rb}_{\mu}\right)\Lambda_{R}{}^{c}+i\bar{\lambda}_V\bar{\sigma}^{\mu}\partial_{\mu}\lambda_V \\
&+i\widetilde{F}^{I\dagger}_{u}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a_{\downarrow}W^{La}_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}\right)\widetilde{F}^{I}_{u}+i\widetilde{F}^{II\dagger}_{u}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a_{\uparrow}W^{La}_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}\right)\widetilde{F}^{II}_{u} \\
&+i\widetilde{F}^{I\dagger}_{d}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a_{\downarrow}W^{La}_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}\right)\widetilde{F}^{I}_{d}+i\widetilde{F}^{II\dagger}_{d}\bar{\sigma}^{\mu}\left(\partial_{\mu}-\frac{ig_L}{2}\tau^a_{\uparrow}W^{La}_{\mu}-\frac{ig_R}{2}\tau^a W^{Ra}_{\mu}\right)\widetilde{F}^{II}_{d}
\end{aligned}
\tag{5.2.1}
$$

The next step is to insert the set of vectors (4.1.20), the charginos (5.1.1.7) and the neutralinos (5.1.2.5) into (5.2.1). Furthermore, the result will be formulated in terms of 4-spinors. The 4-spinors of the theory are:

$$
\begin{aligned}
&\nu_m=\begin{pmatrix}\nu_{Lm}\\ \hline \nu_{Rm}\end{pmatrix},\ \varepsilon_m=\begin{pmatrix}\varepsilon_{Lm}\\ \hline \varepsilon_{Rm}\end{pmatrix},\ u_m=\begin{pmatrix}u_{Lm}\\ \hline u_{Rm}\end{pmatrix},\ d_m=\begin{pmatrix}d_{Lm}\\ \hline d_{Rm}\end{pmatrix}, \\
&\widetilde{D}^{++}_{L}=\begin{pmatrix}\widetilde{\Delta}^{++}_{L}\\ \hline \widetilde{\delta}^{--}_{L}\end{pmatrix},\ \widetilde{D}^{+}_{L}=\begin{pmatrix}\widetilde{\Delta}^{+}_{L}\\ \hline \widetilde{\delta}^{-}_{L}\end{pmatrix},\ \widetilde{D}^{0}_{L}=\begin{pmatrix}\widetilde{\Delta}^{0}_{L}\\ \hline \widetilde{\delta}^{0}_{L}\end{pmatrix},\ \widetilde{D}^{++}_{R}=\begin{pmatrix}\widetilde{\Delta}^{++}_{R}\\ \hline \widetilde{\delta}^{--}_{R}\end{pmatrix}, \\
&\widetilde{\chi}^{+}_{i}=\begin{pmatrix}\chi^{+}_{i}\\ \hline \chi^{-}_{i}\end{pmatrix}\ \widetilde{\chi}^{0}_{i}=\begin{pmatrix}\chi^{0}_{i}\\ \hline \chi^{0}_{i}\end{pmatrix}
\end{aligned}
\tag{5.2.2}
$$

There are several interesting details to the calculation that is required at this point. However, there is only enough room to summarize the results below. Repeated indices shall imply summation. Chargino indices take the values 1 through 5 and neutralino indices take 1 through 9. Furthermore, generation indices for quarks and leptons take the values 1 through 3. Since neutrinos are possibly massive, there will be two CKM-matrices in interactions with $W_\mu^{L\pm}$ and $W_\mu^{R\pm}$, one for quarks, (X_{mn}), and one for leptons, (Y_{mn}). (The CKM-matrices arise due to field re-definitions in Yukawa-interactions; compare the general conclusions in Sections 1.7 and 1.8.2).

Interactions with two fermions and the photon

$$\pounds_{\bar{f}fA} = eA_\mu \bullet \Big(-\overline{\varepsilon_m}\gamma^\mu\varepsilon_m + \frac{2}{3}\overline{u_m}\gamma^\mu u_m - \frac{1}{3}\overline{d_m}\gamma^\mu d_m + \overline{\tilde{\chi}_j^+}\gamma^\mu\tilde{\chi}_j^+$$
$$+\overline{\tilde{D}_L^+}\gamma^\mu\tilde{D}_L^+ + 2\overline{\tilde{D}_L^{++}}\gamma^\mu\tilde{D}_L^{++} + 2\overline{\tilde{D}_R^{++}}\gamma^\mu\tilde{D}_R^{++}\Big) \tag{5.2.3}$$

Interactions between Z_μ^L and two fermions

$$\pounds_{\bar{f}fZ_L} = \frac{eZ_\mu^L}{\cos\theta_W \sin\theta_W} \bullet \Bigg(\overline{\nu_m}\gamma^\mu\left(\frac{1}{2}\gamma_L\right)\nu_m + \overline{\varepsilon_m}\gamma^\mu\left(-\frac{1}{2}\gamma_L + \sin^2\theta_W\right)\varepsilon_m$$
$$+\overline{u_m}\gamma^\mu\left(\frac{1}{2}\gamma_L - \frac{2}{3}\sin^2\theta_W\right)u_m + \overline{d_m}\gamma^\mu\left(-\frac{1}{2}\gamma_L + \frac{1}{3}\sin^2\theta_W\right)d_m$$
$$+\left(1-2\sin^2\theta_W\right)\overline{\tilde{D}_L^{++}}\gamma^\mu\tilde{D}_L^{++} - \sin^2\theta_W\,\overline{\tilde{D}_L^+}\gamma^\mu\tilde{D}_L^+ - \overline{\tilde{D}_L^0}\gamma^\mu\tilde{D}_L^0 - 2\sin^2\theta_W\,\overline{\tilde{D}_R^{++}}\gamma^\mu\tilde{D}_R^{++}$$
$$+\overline{\tilde{\chi}_j^+}\gamma^\mu\left(\left(Z_L^+\right)_{jk}^L\gamma_L + \left(Z_L^+\right)_{jk}^R\gamma_R\right)\tilde{\chi}_k^+ + \overline{\tilde{\chi}_j^0}\gamma^\mu\frac{1}{2}\left(\left(Z_L^0\right)_{jk}^L\gamma_L + \left(Z_L^0\right)_{jk}^R\gamma_R\right)\tilde{\chi}_k^0\Bigg) \tag{5.2.4}$$

$$\begin{aligned}
\left(Z_L^+\right)_{jk}^L &\equiv V_{k1}^*V_{j1} + \frac{1}{2}V_{k3}^*V_{j3} + \frac{1}{2}V_{k4}^*V_{j4} - \sin^2\theta_W\,\delta_{jk}\\
\left(Z_L^+\right)_{jk}^R &\equiv U_{j1}^*U_{k1} + \frac{1}{2}U_{j3}^*U_{k3} + \frac{1}{2}U_{j4}^*U_{k4} - \sin^2\theta_W\,\delta_{jk}\\
\left(Z_L^0\right)_{jk}^L &\equiv -\frac{1}{2}M_{k4}^*M_{j4} + \frac{1}{2}M_{k5}^*M_{j5} - \frac{1}{2}M_{k8}^*M_{j8} + \frac{1}{2}M_{k9}^*M_{j9}\\
\left(Z_L^0\right)_{jk}^R &= -\left(Z_L^0\right)_{jk}^{L*}
\end{aligned} \tag{5.2.5}$$

Interactions between Z_μ^R *and two fermions*

$$
\begin{aligned}
\pounds_{\bar{f}fZ_R} = & \frac{eZ_\mu^R}{\cos\theta_W \tan\varphi} \bullet \left(\overline{\nu_m}\gamma^\mu \left(\frac{1}{2}\gamma_R + \frac{1}{2}\tan^2\varphi \right)\nu_m + \overline{\varepsilon_m}\gamma^\mu \left(-\frac{1}{2}\gamma_R + \frac{1}{2}\tan^2\varphi \right)\varepsilon_m \right. \\
& + \overline{u_m}\gamma^\mu \left(\frac{1}{2}\gamma_R - \frac{1}{6}\tan^2\varphi \right) u_m + \overline{d_m}\gamma^\mu \left(-\frac{1}{2}\gamma_R - \frac{1}{6}\tan^2\varphi \right) d_m \\
& - \tan^2\varphi\, \overline{\tilde{D}_L^{++}}\gamma^\mu \tilde{D}_L^{++} - \tan^2\varphi\, \overline{\tilde{D}_L^{+}}\gamma^\mu \tilde{D}_L^{+} - \tan^2\varphi\, \overline{\tilde{D}_L^{0}}\gamma^\mu \tilde{D}_L^{0} + \left(1 - \tan^2\varphi\right) \overline{\tilde{D}_R^{++}}\gamma^\mu \tilde{D}_R^{++} \\
& \left. + \overline{\tilde{\chi}_j^+}\gamma^\mu \left(\left(Z_R^+\right)_{jk}^L \gamma_L + \left(Z_R^+\right)_{jk}^R \gamma_R \right)\tilde{\chi}_k^+ + \overline{\tilde{\chi}_j^0}\gamma^\mu \frac{1}{2}\left(\left(Z_R^0\right)_{jk}^L \gamma_L + \left(Z_R^0\right)_{jk}^R \gamma_R \right)\tilde{\chi}_k^0 \right)
\end{aligned}
\tag{5.2.6}
$$

$$
\begin{aligned}
\left(Z_R^+\right)_{jk}^L & \equiv V_{k2}^* V_{j2} + \frac{1}{2}V_{k3}^* V_{j3} + \frac{1}{2}V_{k4}^* V_{j4} - \tan^2\varphi\, V_{k5}^* V_{j5} \\
\left(Z_R^+\right)_{jk}^R & \equiv U_{j2}^* U_{k2} + \frac{1}{2}U_{j3}^* U_{k3} + \frac{1}{2}U_{j4}^* U_{k4} - \tan^2\varphi\, U_{j5}^* U_{k5} \\
\left(Z_R^0\right)_{jk}^L & \equiv \frac{1}{2}M_{k4}^* M_{j4} - \frac{1}{2}M_{k5}^* M_{j5} + \frac{1}{2}M_{k8}^* M_{j8} - \frac{1}{2}M_{k9}^* M_{j9} \\
& \quad + \left(-1 - \tan^2\varphi\right) M_{k6}^* M_{j6} - \left(-1 - \tan^2\varphi\right) M_{7j} M_{7k}^* \\
\left(Z_R^0\right)_{jk}^R & = -\left(Z_R^0\right)_{jk}^{L*}
\end{aligned}
\tag{5.2.7}
$$

Interactions between $W_\mu^{L\pm}$ *and* $W_\mu^{R\pm}$ *and two fermions*

$$
\begin{aligned}
& \pounds_{\bar{f}fW_L, \bar{f}fW_R} = \\
& \frac{g_L}{\sqrt{2}} W_\mu^{L+} \left(Y_{mn}\, \overline{\nu_m}\gamma^\mu \gamma_L \varepsilon_n + X_{mn}\, \overline{u_m}\gamma^\mu \gamma_L d_n \right) + \text{h.c.} \\
& + \frac{g_R}{\sqrt{2}} W_\mu^{R+} \left(Y_{mn}^*\, \overline{\nu_m}\gamma^\mu \gamma_R \varepsilon_n + X_{mn}^*\, \overline{u_m}\gamma^\mu \gamma_R d_n \right) + \text{h.c.} \\
& + g_L W_\mu^{L+} \left(-\overline{\tilde{D}_L^{++}}\gamma^\mu \tilde{D}_L^{+} + \overline{\tilde{D}_L^{+}}\gamma^\mu \tilde{D}_L^{0} \right) + \text{h.c.} - g_R W_\mu^{R+} \overline{\tilde{D}_R^{++}}\gamma^\mu \left(V_{k5}^* \gamma_L + U_{k5}\gamma_R \right)\tilde{\chi}_k^+ + \text{h.c.} \\
& + g_L W_\mu^{L+} \overline{\tilde{\chi}_j^+}\gamma^\mu \left(L_{jk}^L \gamma_L + L_{jk}^R \gamma_R \right)\tilde{\chi}_k^0 + \text{h.c.} + g_R W_\mu^{R+} \overline{\tilde{\chi}_j^+}\gamma^\mu \left(R_{jk}^L \gamma_L + R_{jk}^R \gamma_R \right)\tilde{\chi}_k^0 + \text{h.c.}
\end{aligned}
\tag{5.2.8}
$$

$$L^L_{jk} \equiv -M^*_{k1}V_{j1} + \frac{1}{\sqrt{2}}M^*_{k5}V_{j4} + \frac{1}{\sqrt{2}}M^*_{k9}V_{j3}$$

$$L^R_{jk} \equiv -U^*_{j1}M_{k1} - \frac{1}{\sqrt{2}}U^*_{j3}M_{k4} - \frac{1}{\sqrt{2}}U^*_{j4}M_{k8}$$

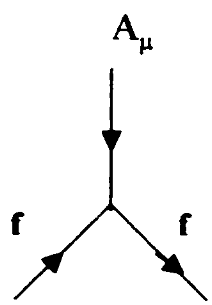
$$R^L_{jk} \equiv -M^*_{k2}V_{j2} + \frac{1}{\sqrt{2}}M^*_{k5}V_{j4} + M^*_{k6}V_{j5} + \frac{1}{\sqrt{2}}M^*_{k9}V_{j3}$$

$$R^R_{jk} \equiv -U^*_{j2}M_{k2} - \frac{1}{\sqrt{2}}U^*_{j3}M_{k4} + U^*_{j5}M_{k7} - \frac{1}{\sqrt{2}}U^*_{j4}M_{k8} \tag{5.2.9}$$

5.3 Feynman Rules for Vector Bosons and Fermions

The list of Feynman Rules contains interactions between two fermions and one vector boson. The vector boson is the photon A or $W^\pm_L$ or $W^\pm_R$.

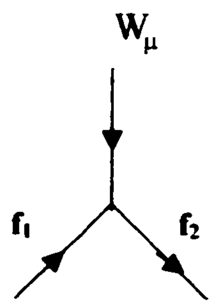
Feynman-Rules for Photon Interactions



f	Feynman Rule	
ε_n	$ie\gamma_\mu$	(5.3.1)
u_n	$-ie\frac{2}{3}\gamma_\mu$	(5.3.2)
d_n	$ie\frac{1}{3}\gamma_\mu$	(5.3.3)
$\tilde{\chi}^+_j$	$-ie\gamma_\mu$	(5.3.4)

$\widetilde{D}_L^{+}$	$-ie\gamma_\mu$	(5.3.5)
$\widetilde{D}_L^{++}$	$-2ie\gamma_\mu$	(5.3.6)
$\widetilde{D}_R^{++}$	$-2ie\gamma_\mu$	(5.3.7)

Feynman-Rules for Interactions with $W_\mu^{L\pm}$ *and* $W_\mu^{R\pm}$



f_1	f_2	W_μ	**Feynman Rule**	
ε_n	ν_m	W_μ^{L+}	$-i\frac{g_L}{\sqrt{2}}Y_{mn}\gamma^\mu\gamma_L$	(5.3.8)
ν_n	ε_m	W_μ^{L-}	$-i\frac{g_L}{\sqrt{2}}Y^*_{nm}\gamma^\mu\gamma_L$	(5.3.9)
d_n	u_m	W_μ^{L+}	$-i\frac{g_L}{\sqrt{2}}X_{mn}\gamma^\mu\gamma_L$	(5.3.10)
u_n	d_m	W_μ^{L-}	$-i\frac{g_L}{\sqrt{2}}X^*_{nm}\gamma^\mu\gamma_L$	(5.3.11)
ε_n	ν_m	W_μ^{R+}	$-i\frac{g_R}{\sqrt{2}}Y^*_{mn}\gamma^\mu\gamma_R$	(5.3.12)
ν_n	ε_m	W_μ^{R-}	$-i\frac{g_R}{\sqrt{2}}Y_{nm}\gamma^\mu\gamma_R$	(5.3.13)

d_n	u_m	W_μ^{R+}	$-i\frac{g_R}{\sqrt{2}}X_{mn}^*\gamma^\mu\gamma_R$	(5.3.14)
u_n	d_m	W_μ^{R-}	$-i\frac{g_R}{\sqrt{2}}X_{nm}\gamma^\mu\gamma_R$	(5.3.15)
$\widetilde{D}_L^+$	$\widetilde{D}_L^{++}$	W_μ^{L+}	$ig_L\gamma^\mu$	(5.3.16)
$\widetilde{D}_L^{++}$	$\widetilde{D}_L^+$	W_μ^{L-}	$ig_L\gamma^\mu$	(5.3.17)
$\widetilde{D}_L^0$	$\widetilde{D}_L^+$	W_μ^{L+}	$-ig_L\gamma^\mu$	(5.3.18)
$\widetilde{D}_L^+$	$\widetilde{D}_L^0$	W_μ^{L-}	$-ig_L\gamma^\mu$	(5.3.19)
$\widetilde{\chi}_k^+$	$\widetilde{D}_R^{++}$	W_μ^{R+}	$ig_R\gamma^\mu\left(V_{k5}^*\gamma_L + U_{k5}\gamma_R\right)$	(5.3.20)
$\widetilde{D}_R^{++}$	$\widetilde{\chi}_k^+$	W_μ^{R-}	$ig_R\gamma^\mu\left(V_{k5}\gamma_L + U_{k5}^*\gamma_R\right)$	(5.3.21)
$\widetilde{\chi}_k^0$	$\widetilde{\chi}_j^+$	W_μ^{L+}	$-ig_L\gamma^\mu\left(L_{jk}^L\gamma_L + L_{jk}^R\gamma_R\right)$	(5.3.22)
$\widetilde{\chi}_k^+$	$\widetilde{\chi}_j^0$	W_μ^{L-}	$-ig_L\gamma^\mu\left(L_{kj}^{L*}\gamma_L + L_{kj}^{R*}\gamma_R\right)$	(5.3.23)
$\widetilde{\chi}_k^0$	$\widetilde{\chi}_j^+$	W_μ^{R+}	$-ig_R\gamma^\mu\left(R_{jk}^L\gamma_L + R_{jk}^R\gamma_R\right)$	(5.3.24)
$\widetilde{\chi}_k^+$	$\widetilde{\chi}_j^0$	W_μ^{R-}	$-ig_R\gamma^\mu\left(R_{kj}^{L*}\gamma_L + R_{kj}^{R*}\gamma_R\right)$	(5.3.25)

Chapter 6: Scalar Mass Eigensates and Interactions with Vector Bosons

This chapter provides the last set of Feynamn rules of interest to this work. These are the Feynman rules for vector bosons and scalar particles, which are Higgs, sleptons, and squarks. The task is similar to that of the previous chapter: finding the physical Higgs particles that are formed as symmetry breaking takes place and inserting them into the Lagrange density along with (4.1.20). However, this chapter is organized differently than the previous one. In Section 6.1 we shall work out the consequences of inserting (4.1.20) alone, because this leads to a useful intermediate result. Section 6.2 presents the physical Higgs fields and Section 6.3 gives the scalar-vector interactions in the finished form. Finally, Section 6.4 lists the Feynman rules.

6.1 Interactions of Scalar Bi-Doublets and Triplets with Vector Bosons

The interactions between vector bosons and scalar particles arise from the covariant kinetic energy of the gauge bosons that are specified in Table 3.1. In our left-right supersymmetric model the relevant part of the Lagrange density is:

$$\begin{aligned}
£_{\mathrm{kin,scalar}} &= \left|\left(\partial_\mu - \frac{ig_L}{2}\tau^a W_\mu^{La} + ig_V V_\mu\right)\widetilde{L}_{Lm}\right|^2 + \left|\left(\partial_\mu - \frac{ig_R}{2}\tau^a W_\mu^{Ra} + ig_V V_\mu\right)\widetilde{L}_{Rm}\right|^2 \\
&+ \left|\left(\partial_\mu - \frac{ig_L}{2}\tau^a W_\mu^{La} - \frac{ig_V}{3} V_\mu\right)\widetilde{Q}_{Lm}\right|^2 + \left|\left(\partial_\mu - \frac{ig_R}{2}\tau^a W_\mu^{Ra} - \frac{ig_V}{3} V_\mu\right)\widetilde{Q}_{Rm}\right|^2 \\
&+ \left|\left(\partial_\mu - ig_L\tau^a W_\mu^{La} - 2ig_V V_\mu\right)H_L\right|^2 + \left|\left(\partial_\mu - ig_R\tau^a W_\mu^{Ra} - 2ig_V V_\mu\right)H_R\right|^2 \\
&+ \left|\left(\partial_\mu - ig_L\tau^a W_\mu^{La} + 2ig_V V_\mu\right)K_L\right|^2 + \left|\left(\partial_\mu - ig_R\tau^a W_\mu^{Ra} + 2ig_V V_\mu\right)K_R\right|^2 \\
&+ \left|\left(\partial_\mu - \frac{ig_L}{2}\tau_\downarrow^a W_\mu^{La} - \frac{ig_R}{2}\tau^a W_\mu^{Ra}\right)F_u^{I}\right|^2 + \left|\left(\partial_\mu - \frac{ig_L}{2}\tau_\uparrow^a W_\mu^{La} - \frac{ig_R}{2}\tau^a W_\mu^{Ra}\right)F_u^{II}\right|^2 \\
&+ \left|\left(\partial_\mu - \frac{ig_L}{2}\tau_\downarrow^a W_\mu^{La} - \frac{ig_R}{2}\tau^a W_\mu^{Ra}\right)F_d^{I}\right|^2 + \left|\left(\partial_\mu - \frac{ig_L}{2}\tau_\uparrow^a W_\mu^{La} - \frac{ig_R}{2}\tau^a W_\mu^{Ra}\right)F_d^{II}\right|^2
\end{aligned} \tag{6.1.1}$$

Next the set of equations (4.1.20) that relates the gauge bosons to physical vector bosons is inserted. The results can be expressed conveniently in the form of seven summation rules. The advantage of these rules is that the choice of vacuum expectation values of our model versus another is left optional. In fact, the choice of the Higgs sector altogether is left optional. If the Higgs sector were different, only the summation over the specified particles would have to be altered in a self-explanatory way. Here are the results from inserting (4.1.20) into (6.1.1).

1) Two identical scalars and one neutral vector

$$£_{xxA,xxZ_L,xxZ_R} \equiv \sum_x i\,x^* \overleftrightarrow{\partial}^\mu x$$

$$\bullet\left(Q_x e A_\mu + \frac{eZ_\mu^L}{\cos\theta_W \sin\theta_W}\left(t_L^{3x} - Q_x \sin^2\theta_W\right) + \frac{eZ_\mu^R}{\cos\theta_W \tan\varphi}\left(t_R^{3x} - \frac{Y_x}{2}\tan^2\varphi\right)\right) \tag{6.1.2}$$

The sum is over all possible scalar fields:

$\widetilde{\nu}_{Lm}$, $\widetilde{\varepsilon}_{Lm}$, $\widetilde{\nu}_{Rm}$, $\widetilde{\varepsilon}_{Rm}$, $\widetilde{u}_{Lm}$, $\widetilde{d}_{Lm}$, $\widetilde{u}_{Rm}$, $\widetilde{d}_{Rm}$,

$$-\Delta_L^{++},\ \Delta_L^{+},\ \Delta_L^{0},\ -\Delta_R^{++},\ \Delta_R^{+},\ \Delta_R^{0},\ -\delta_L^{0},\ \delta_L^{-},\ \delta_L^{--},\ -\delta_R^{0},\ \delta_R^{-},\ \delta_R^{--},$$

$$\phi_{1u}^{0},\ \phi_{2u}^{-},\ \phi_{1u}^{+},\ \phi_{2u}^{0},\ \phi_{1d}^{0},\ \phi_{2d}^{-},\ \phi_{1d}^{+},\ \phi_{2d}^{0}$$

$$\overleftrightarrow{\partial}_\mu \equiv \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$$

2) *Two scalars x and y of different flavor and one charged vector*

$$\begin{aligned}
&\pounds_{x_L y_L W_L^{\pm}, x_R y_R W_R^{\pm}} \equiv \\
&\sum_{x_L, y_L}\left(i g_L \sqrt{T_L^{xy}}\, x_L^{*} \overleftrightarrow{\partial}^{\mu} y_L W_\mu^{L+} + i g_L \sqrt{T_L^{xy}}\, y_L^{*} \overleftrightarrow{\partial}^{\mu} x_L W_\mu^{L-} \right) \\
&+ \sum_{x_R, y_R}\left(i g_R \sqrt{T_R^{xy}}\, x_R^{*} \overleftrightarrow{\partial}^{\mu} y_R W_\mu^{R+} + i g_R \sqrt{T_R^{xy}}\, y_R^{*} \overleftrightarrow{\partial}^{\mu} x_R W_\mu^{R-} \right)
\end{aligned} \tag{6.1.3}$$

The summations are over the following pairs

(x_L, y_L):

$$(\tilde{\nu}_{Lm}, \tilde{\varepsilon}_{Lm}),\ (\tilde{u}_{Lm}, \tilde{d}_{Lm}),\ (-\Delta_L^{++}, \Delta_L^{+}),\ (\Delta_L^{+}, \Delta_L^{0}),\ (-\delta_L^{0}, \delta_L^{-}),\ (\delta_L^{-}, \delta_L^{--}),$$

$$(\phi_{1u}^{0}, \phi_{2u}^{-}),\ (\phi_{1u}^{+}, \phi_{2u}^{0}),\ (\phi_{1d}^{0}, \phi_{2d}^{-}),\ (\phi_{1d}^{+}, \phi_{2d}^{0})$$

(x_R, y_R):

$$(\tilde{\nu}_{Rm}, \tilde{\varepsilon}_{Rm}),\ (\tilde{u}_{Rm}, \tilde{d}_{Rm}),\ (-\Delta_R^{++}, \Delta_R^{+}),\ (\Delta_R^{+}, \Delta_R^{0}),\ (-\delta_R^{0}, \delta_R^{-}),\ (\delta_R^{-}, \delta_R^{--}),$$

$$(\phi_{1u}^{0}, \phi_{2u}^{-}),\ (\phi_{1u}^{+}, \phi_{2u}^{0}),\ (\phi_{1d}^{0}, \phi_{2d}^{-}),\ (\phi_{1d}^{+}, \phi_{2d}^{0})$$

Moreover, the following replacements have to be made for squarks and sleptons:

$$\begin{aligned}
&\tilde{u}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{d}_{Lm} \to \tilde{X}_{mn} \tilde{u}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{d}_{Ln},\quad \tilde{d}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{u}_{Lm} \to (\tilde{X}^{\dagger})_{mn} \tilde{d}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{u}_{Ln} \\
&\tilde{u}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{d}_{Rm} \to \tilde{X}_{mn}^{*} \tilde{u}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{d}_{Rn},\quad \tilde{d}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{u}_{Rm} \to (\tilde{X}^{T})_{mn} \tilde{d}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{u}_{Rn} \\
&\tilde{n}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{e}_{Lm} \to \tilde{Y}_{mn} \tilde{n}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{e}_{Ln},\quad \tilde{e}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{n}_{Lm} \to (\tilde{Y}^{\dagger})_{mn} \tilde{e}_{Lm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{n}_{Ln} \\
&\tilde{n}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{e}_{Rm} \to \tilde{Y}_{mn}^{*} \tilde{n}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{e}_{Rn},\quad \tilde{e}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{n}_{Rm} \to (\tilde{Y}^{T})_{mn} \tilde{e}_{Rm}^{*} \overleftrightarrow{\partial}^{\mu} \tilde{n}_{Rn}
\end{aligned} \tag{6.1.4}$$

$T_L^{xy}\left(T_L^{xy}+1\right)$ is the eigenvalue of $\vec{T}_L^{\,2}$, in other words the quantum number of the angular momentum operator in $SU_L(2)$-space with respect to the multiplet of which x_L and y_L are members (and similarly for 'R'). Charged vector bosons that mediate flavor changing interactions clearly recognize the fields they interact with by the dimension of their SU(2) multiplet. By contrast, t_L^{3x} is the eigenvalue of the third component of the angular momentum operator in $SU_L(2)$-space with respect to the field x (and similarly for 'R'). For a field x that exists in the three-dimensional representation of $SU_L(2)$: $T_L^x = 1$ and $t_L^{3x} = \pm 1, 0$, while in two dimensions: $T_L^x = \frac{1}{2}$ and $t_L^{3x} = \pm\frac{1}{2}$ (and similarly for 'R').

3) Two identical scalars, two neutral vectors

$$\pounds_{xxAA,xxZ_LZ_L,xxZ_RZ_R,xxAZ_L,xxAZ_R,xxZ_LZ_R} \equiv \sum_x x^{*}x$$
$$\bullet\left(Q_x eA_\mu + \frac{eZ_\mu^L}{\cos\theta_W \sin\theta_W}\left(t_L^{3x} - Q_x \sin^2\theta_W\right) + \frac{eZ_\mu^R}{\cos\theta_W \tan\varphi}\left(t_R^{3x} - \frac{Y_x}{2}\tan^2\varphi\right)\right)^2 \qquad (6.1.5)$$

The summation is the same as under (6.1.2).

4) Two scalars of different flavor, one neutral vector, one charged vector

$$\pounds_{x_Lx_LAW_L^\pm,x_Rx_RAW_R^\pm,x_Lx_LZ_LW_L^\pm,x_Rx_RZ_LW_R^\pm,x_Lx_LZ_RW_L^\pm,x_Rx_RZ_RW_R^\pm} \equiv$$
$$\sum_{x_L,y_L}\left(g_L\sqrt{T_L^{xy}}\,x_L^{*}y_L W_\mu^{L+} + g_L\sqrt{T_L^{xy}}\,y_L^{*}x_L W_\mu^{L-}\right)\bullet\left(\left(Q_x+Q_y\right)eA_\mu\right.$$
$$\left.+\frac{eZ_\mu^L}{\cos\theta_W \sin\theta_W}\left(t_L^{3x}+t_L^{3y}-\left(Q_x+Q_y\right)\sin^2\theta_W\right)+\frac{eZ_\mu^R}{\cos\theta_W \tan\varphi}\left(t_R^{3x}+{}_R^{3y}-\frac{Y_x+Y_y}{2}\tan^2\varphi\right)\right)$$
$$+\sum_{x_R,y_R}\left(g_R\sqrt{T_R^{xy}}\,x_R^{*}y_R W_\mu^{R+} + g_R\sqrt{T_R^{xy}}\,y_R^{*}x_R W_\mu^{R-}\right)\bullet\left(\left(Q_x+Q_y\right)eA_\mu\right.$$
$$\left.+\frac{eZ_\mu^L}{\cos\theta_W \sin\theta_W}\left(t_L^{3x}+t_L^{3y}-\left(Q_x+Q_y\right)\sin^2\theta_W\right)+\frac{eZ_\mu^R}{\cos\theta_W \tan\varphi}\left(t_R^{3x}+{}_R^{3y}-\frac{Y_x+Y_y}{2}\tan^2\varphi\right)\right)$$

(6.1.6)

The summations in (6.1.6) are the same as in (6.1.3).

While the phi-Higgs scalars interact separately with Z_R, $W_L^{\pm}$ and $W_R^{\pm}$ (see (6.1.2), (6.1.3)), it urns out that they do not interact with the combinations $Z_R W_L^{\pm}$ and $Z_R W_R^{\pm}$.

This is so because the combination of quantum numbers $t_R^x + t_R^y - \frac{1}{2}(Y_x + Y_y)\tan^2\varphi$ is zero, if x and y are two phi-Higgs of the same SU(2)-(bi)-doublet. In that case $t_R^x = \frac{1}{2} = -t_R^y$ while Y_{B-L} is generally zero for phi-Higgs fields. It follows that phi-Higgs fields cannot change flavor through interactions with Z_R.

5) Two identical scalars and the combinations $W_L^+ W_L^-$, $W_R^+ W_R^-$

$$\pounds_{x_L x_L W_L^+ W_L^-, x_R x_R W_R^+ W_R^-} \equiv$$
$$\sum_{x_L} g_L^2 \left(T_L^x (T_L^x + 1) - (t_L^{3x})^2 \right) x_L^* x_L W_\mu^{L+} W^{L-\mu} + \sum_{x_R} g_R^2 \left(T_R^x (T_R^x + 1) - (t_R^{3x})^2 \right) x_R^* x_R W_\mu^{R+} W^{R-\mu} \tag{6.1.7}$$

The sums run over all possible scalars, not forgetting that the bi-doublet fields must be considered in the left-sum as well as in the right-sum.

6) Two identical phi-scalars and the combinations $W_L^+ W_R^-$, $W_R^+ W_L^-$

Because the phi-Higgs bi-doublet scalars transform under both $SU_L(2)$ and $SU_R(2)$, they do not 'see' the gauge bosons differently.

$$\pounds_{\phi\phi W_L^\pm W_R^\pm} \equiv \sum_{x=\phi_u,\phi_d} g_L g_R \sqrt{T_L^x (T_L^x + 1) - (t_L^{3x})^2} \sqrt{T_R^x (T_R^x + 1) - (t_R^{3x})^2}\, x^* x \left(W_\mu^{L+} W^{R-\mu} + W_\mu^{R+} W^{L-\mu} \right) \tag{6.1.8}$$

The sum is over phi-Higgs fields only.

7) Two triplet scalars of different flavor and combinations of two identical vectors

Triplet delta-Higgs can change the flavor such that the electric charge between the delta-fields changes by two units. As a consequence both of the participating gauge bosons must have the same electric charge.

$$
\begin{aligned}
&\pounds_{\Delta\Delta W^+W^+,\Delta\Delta W^-W^-} \equiv \\
&\sum_{x_L,y_L}\left(g_L^2 T_L^{xy} x_L^* y_L W_\mu^{L+} W_\mu^{L+} + g_L^2 T_L^{xy} y_L^* x_L W_\mu^{L-} W_\mu^{L-}\right) \\
&+ \sum_{x_R,y_R}\left(g_R^2 T_R^{xy} x_R^* y_R W_\mu^{R+} W_\mu^{R+} + g_R^2 T_R^{xy} y_R^* x_R W_\mu^{R-} W_\mu^{R-}\right)
\end{aligned}
\tag{6.1.9}
$$

Summation over

$(x_L, y_L): \left(-\Delta_L^{++}, \Delta_L^{0}\right), \left(-\delta_L^{0}, \delta_L^{--}\right); (x_R, y_R): \left(-\Delta_R^{++}, \Delta_R^{0}\right), \left(-\delta_R^{0}, \delta_R^{--}\right)$

Equations (6.1.2) through (6.1.9) complete our task of inserting the physical vector bosons (4.1.20) into the Lagrange density (6.1.1).

6.2 Interlude:

Mixing of Scalar Bi-Doublets and Triplets

Here we formally define the mass eigenstates in the Higgs-sector. These states arise from the D-part and the soft breaking terms of the superpotential. As in the case of charginos and neutralinos, there are matrices to diagonalize numerically and results will be written in the form of undetermined matrix elements. The relevant parts of the potential may be taken to be most generally:

$$
V_{\text{sup erpotential}-\text{Higgs}} = \frac{1}{2} D_L^a D_L^a + \frac{1}{2} D_R^a D_R^a + \frac{1}{2} D^i D^i + V_{\text{soft}} \tag{6.2.1}
$$

$$D_L^a = \frac{g_L}{2} F_u^{I\dagger} \tau^a F_u^I + \frac{g_L}{2} F_u^{II\dagger} \tau^a F_u^{II} + \frac{g_L}{2} F_d^{I\dagger} \tau^a F_d^I + \frac{g_L}{2} F_d^{II\dagger} \tau^a F_d^{II}$$
$$+ g_L H_L{}^\dagger \tau^a H_L + g_L K_L{}^\dagger \tau^a K_L$$

$$D_R^a = \frac{g_R}{2} F_u^{I\dagger} \tau^a F_u^I + \frac{g_R}{2} F_u^{II\dagger} \tau^a F_u^{II} + \frac{g_R}{2} F_d^{I\dagger} \tau^a F_d^I + \frac{g_R}{2} F_d^{II\dagger} \tau^a F_d^{II}$$
$$+ g_R H_R{}^\dagger \tau^a H_R + g_R K_R{}^\dagger \tau^a K_R$$

$$D^l = \frac{g_V}{2}\left(2H_L{}^\dagger H_L + 2H_R{}^\dagger H_R - 2K_L{}^\dagger K_L - 2K_R{}^\dagger K_R\right)$$

$$V_{soft} = m_1^2 F_u^{I\dagger} F_u^I + m_1^2 F_u^{II\dagger} F_u^{II} + m_2^2 F_d^{I\dagger} F_d^I + m_2^2 F_d^{II\dagger} F_d^{II}$$
$$+ m_3^2 H_R{}^\dagger H_R + m_4^2 K_R{}^\dagger K_R + m_5^2 H_L{}^\dagger H_L + m_6^2 K_L{}^\dagger K_L$$
$$- m_{\phi,ud}^2\left(-F_u^I i\tau^2 F_d^{II} + F_u^{II} i\tau^2 F_d^I\right) - m_{\phi,ud}^2\left(-F_u^{I\dagger} i\tau^2 F_d^{II\dagger} + F_u^{II\dagger} i\tau^2 F_d^{I\dagger}\right)$$
$$- m_{R,\Delta\delta}^2\left(\Delta_R^{++}\delta_R^{--} + \Delta_R^{+}\delta_R^{-} + \Delta_R^{0}\delta_R^{0}\right) - m_{R,\Delta\delta}^2\left(\Delta_R^{++\dagger}\delta_R^{--\dagger} + \Delta_R^{+\dagger}\delta_R^{-\dagger} + \Delta_R^{0\dagger}\delta_R^{0\dagger}\right)$$
$$- m_{L,\Delta\delta}^2\left(\Delta_L^{++}\delta_L^{--} + \Delta_L^{+}\delta_L^{-} + \Delta_L^{0}\delta_L^{0}\right) - m_{L,\Delta\delta}^2\left(\Delta_L^{++\dagger}\delta_L^{--\dagger} + \Delta_L^{+\dagger}\delta_L^{-\dagger} + \Delta_L^{0\dagger}\delta_L^{0\dagger}\right) \quad (6.2.2)$$

The superpotential conventionally enters into the Lagrange density with an overall minus sign. We have to insert into (6.2.1):

$$\phi_{1u}^0 \to \frac{1}{\sqrt{2}}\left(\kappa_u + H_{1u}^0 + iz_{1u}^0\right) \qquad \Delta_R{}^0 \to \frac{1}{\sqrt{2}}\left(v_{\Delta_R} + H_{\Delta_R} + iz_{\Delta_R}\right)$$
$$\phi_{2d}^0 \to \frac{1}{\sqrt{2}}\left(\kappa_d + H_{2d}^0 + iz_{2d}^0\right) \qquad \delta_R{}^0 \to \frac{1}{\sqrt{2}}\left(v_{\delta_R} + H_{\delta_R} + iz_{\delta_R}\right)$$
$$\phi_{1d}^0 \to \frac{1}{\sqrt{2}}\left(H_{1d}^0 + iz_{1d}^0\right) \qquad \Delta_L{}^0 \to \frac{1}{\sqrt{2}}\left(H_{\Delta_L} + iz_{\Delta_L}\right)$$
$$\phi_{2u}^0 \to \frac{1}{\sqrt{2}}\left(H_{2u}^0 + iz_{2u}^0\right) \qquad \delta_L{}^0 \to \frac{1}{\sqrt{2}}\left(H_{\delta_L} + iz_{\delta_L}\right) \quad (6.2.3)$$

The neutral scalar fields in (6.2.3) are split into 'real' parts, which are scalars (CP-even), and 'imaginary' parts, which are pseudo-scalars (CP-odd). The scalar fields mix to form Higgs fields that are denoted with letters H and the pseudo-scalars mix to form fields that are denoted with letters A and suitable superscripts. The mixtures of charged scalars are again denoted with letters H and suitable superscripts. As in the mixing of charginos and neutralinos we need to define a few 'vectors'.

Doubly Charged Fields

$$x_R^{++T} \equiv \left(\Delta_R^{++} \quad \delta_R^{--*}\right),\ x_R^{--T} \equiv \left(\Delta_R^{++*} \quad \delta_R^{--}\right),$$
$$y_R^{\pm\pm T} \equiv \left(H_1^{\pm\pm} \quad H_2^{\pm\pm}\right) \tag{6.2.4}$$

$$x_L^{++T} \equiv \left(\Delta_L^{++} \quad \delta_L^{--*}\right),\ x_L^{--T} \equiv \left(\Delta_L^{++*} \quad \delta_L^{--}\right),$$
$$y_L^{\pm\pm T} \equiv \left(H_3^{\pm\pm} \quad H_4^{\pm\pm}\right) \tag{6.2.5}$$

Singly Charged Fields

$$x^{+T} \equiv \left(\Delta_L^{+} \quad \delta_L^{-*} \quad \phi_{2d}^{-*} \quad \phi_{1u}^{+} \quad \phi_{2u}^{-*} \quad \phi_{1d}^{+} \quad \Delta_R^{+} \quad \delta_R^{-*}\right),$$

$$x^{-T} \equiv \left(\Delta_L^{+*} \quad \delta_L^{-} \quad \phi_{2d}^{-} \quad \phi_{1u}^{+*} \quad \phi_{2u}^{-} \quad \phi_{1d}^{+*} \quad \Delta_R^{+*} \quad \delta_R^{-}\right),$$

$$y^{\pm T} \equiv \left(H_1^{\pm} \quad H_2^{\pm} \quad H_3^{\pm} \quad H_4^{\pm} \quad H_5^{\pm} \quad H_6^{\pm} \quad G_1^{\pm} \quad G_2^{\pm}\right) \tag{6.2.6}$$

Neutral Fields

$$x_s^{0T} \equiv \left(H_{\Delta_L} \quad H_{\delta_L} \quad H_{1d}^0 \quad H_{2u}^0 \quad H_{1u}^0 \quad H_{2d}^0 \quad H_{\Delta_R} \quad H_{\delta_R}\right),$$

$$y_s^{0T} \equiv \left(H_1^0 \quad H_2^0 \quad H_3^0 \quad H_4^0 \quad H_5^0 \quad H_6^0 \quad H_7^0 \quad H_8^0\right),$$

$$x_p^{0T} \equiv \left(z_{\Delta_L} \quad z_{\delta_L} \quad z_{1d}^0 \quad z_{2u}^0 \quad z_{1u}^0 \quad z_{2d}^0 \quad z_{\Delta_R} \quad z_{\delta_R}\right),$$

$$y_p^{0T} \equiv \left(A_1^0 \quad A_2^0 \quad A_3^0 \quad A_4^0 \quad A_5^0 \quad A_6^0 \quad G_1^0 \quad G_2^0\right) \tag{6.2.7}$$

The indices 's' and 'p' are to remind of 'scalar' and 'pseudo-scalar' respectively. There are two charged Goldstone bosons for the left-handed and the right-handed charged vector bosons and two neutral Goldstone bosons, similarly giving mass to the left and right-handed Z-bosons. The masses of the Goldstone bosons are theoretically zero. In order to make uncomplicated use of Einstein's summation convention we may also define

$$H_7^{\pm} \equiv G_1^{\pm},\ H_8^{\pm} \equiv G_2^{\pm},\ A_7^0 \equiv G_1^0,\ A_8^0 \equiv G_2^0 \tag{6.2.8}$$

Without loss of generality we may assume that the superpotential has been expanded and those pieces have been singled out that are quadratic in the fields and therefore responsible for mass terms. This part of the superpotential has this form:

$$\begin{aligned}\pounds_{\text{potential-Higgs Masses}} &= \sum_{i,j=1}^{2} x_{L\ i}^{--}\left(M_L^{\pm\pm}\right)_{ij} x_{L\ j}^{++} + \sum_{i,j=1}^{2} x_{R\ i}^{--}\left(M_R^{\pm\pm}\right)_{ij} x_{R\ j}^{++} \\ &+ \sum_{i,j=1}^{8} x_i^{-}\left(M^{\pm}\right)_{ij} x_j^{+} + \frac{1}{2}\sum_{i,j=1}^{8} x_{s_i}^{0}\left(M_s^0\right)_{ij} x_{s_j}^{0} + \frac{1}{2}\sum_{i,j=1}^{8} x_{p_i}^{0}\left(M_p^0\right)_{ij} x_{p_j}^{0}\end{aligned} \tag{6.2.9}$$

The matrices 'M' in this equation (6.2.9) have the dimensions of the corresponding 'vectors' and they are real symmetric. The matrices 'M' are diagonalized by new orthogonal matrices 'R' such that

$$\begin{aligned}&\left(R_R^{\pm\pm}\right)_{ij}\left(M_R^{\pm\pm}\right)_{jk}\left(R_R^{\pm\pm}\right)_{lk} = \mathrm{diag}\left(m_1^{\pm\pm}, m_2^{\pm\pm}\right), \\ &\left(R_L^{\pm\pm}\right)_{ij}\left(M_L^{\pm\pm}\right)_{jk}\left(R_L^{\pm\pm}\right)_{lk} = \mathrm{diag}\left(m_3^{\pm\pm}, m_4^{\pm\pm}\right), \\ &\left(R^{\pm}\right)_{ij}\left(M^{\pm}\right)_{jk}\left(R^{\pm}\right)_{lk} = \mathrm{diag}\left(m_1^{\pm},\ldots,m_6^{\pm},0,0\right), \\ &\left(R_s^{0}\right)_{ij}\left(M_s^{0}\right)_{jk}\left(R_s^{0}\right)_{lk} = \mathrm{diag}\left(m_{s\,1}^{0},\ldots,m_{s\,8}^{0}\right), \\ &\left(R_p^{0}\right)_{ij}\left(M_p^{0}\right)_{jk}\left(R_p^{0}\right)_{lk} = \mathrm{diag}\left(m_{p_1}^{0},\ldots,m_{p_6}^{0},0,0\right)\end{aligned} \tag{6.2.10}$$

Hence, the matrices 'R' of (6.2.10) can make the potential (6.2.9) mass-diagonal:

$$\pounds_{\text{potential}} = \sum_{i=1}^{2} m_{L\ i}^{++} y_{L\ i}^{--} y_{L\ i}^{++} + \sum_{i=1}^{2} m_{R\ i}^{\pm\pm} y_{R\ i}^{--} y_{R\ i}^{++} + \sum_{i=1}^{6} m_i^{\pm} y_i^{-} y_i^{+} + \frac{1}{2}\sum_{i=1}^{8} m_{s_i}^{0} y_{s_i}^{0} y_{s_i}^{0} + \frac{1}{2}\sum_{i=1}^{6} m_{p_i}^{0} y_{p_i}^{0} y_{p_i}^{0} \tag{6.2.11}$$

The new fields 'y' are related to the old fields 'x' through

$$\begin{aligned} &y_{R\ i}^{\pm\pm} = \left(R_R^{\pm\pm}\right)_{ij} x_{R\ j}^{\pm\pm},\ y_{L\ i}^{\pm\pm} = \left(R_L^{\pm\pm}\right)_{ij} x_{L\ j}^{\pm\pm},\ y_i^{\pm} = \left(R^{\pm}\right)_{ij} x_j^{\pm}, \\ &y_{s\,i}^{0} = \left(R_s^{0}\right)_{ij} x_{s\,j}^{0},\ y_{p_i}^{0} = \left(R_p^{0}\right)_{ij} x_{p_j}^{0} \end{aligned} \tag{6.2.12}$$

We do not attempt to obtain explicit expressions for the matrices 'M' and 'R', however numerical values for the matrices 'R' may be obtained from the literature [26], which is sufficient to make practical calculations with the corresponding Feynman rules.

6.3 Interactions of Higgs Fields with Vector Bosons

The symbols for our Higgs fields have been defined previously. The final step is to take the Lagrange density pieces (6.1.2) through (6.1.9) and insert first (6.2.3) and then (6.2.12). In so doing, we single out the interactions of three fields of which at least one is a vector boson and the others are Higgs or Goldstone bosons. To our knowledge interaction Lagrangians and Feynman rules like the ones in this part of the theory have not been published previously. The results are listed below.

$$\pounds_{H^{--}H^{++}A,H^{-}H^{+}A} = 2ieA_\mu \sum_{i=1}^{4} H_i^{--} \overleftrightarrow{\partial}^\mu H_i^{++} + ieA_\mu \sum_{i=1}^{8} H_i^{-} \overleftrightarrow{\partial}^\mu H_i^{+} \tag{6.3.13}$$

$$\pounds_{H^{-}H^{++}W_R} = -ig_R \sum_{k=1}^{8}\sum_{j=1}^{2} W_\mu^{R+} H_j^{--} \overleftrightarrow{\partial}^\mu H_k^{+} a_{1,jk} - ig_R \sum_{k=1}^{8}\sum_{j=1}^{2} W_\mu^{R-} H_k^{-} \overleftrightarrow{\partial}^\mu H_j^{++} a_{1,jk} \tag{6.3.14}$$

$$\pounds_{H^-H^{++}W_L} = -ig_L \sum_{k=1}^{8}\sum_{j=3}^{4} W_\mu^{L+} H_j^{--} \bar{\partial}^\mu H_k^+ a_{2,jk} - ig_L \sum_{k=1}^{8}\sum_{j=3}^{4} W_\mu^{L-} H_k^- \bar{\partial}^\mu H_j^{++} a_{2,jk} \tag{6.3.15}$$

$$\pounds_{H^0H^+W_R} = -ig_R \sum_{j,k=1}^{8} W_\mu^{R+} H_j^- \bar{\partial}^\mu H_k^0 a_{3,jk} - ig_R \sum_{j,k=1}^{8} W_\mu^{R-} H_k^0 \bar{\partial}^\mu H_j^+ a_{3,jk} \tag{6.3.16}$$

$$\pounds_{H^0H^+W_L} = -ig_L \sum_{j,k=1}^{8} W_\mu^{L+} H_j^- \bar{\partial}^\mu H_k^0 a_{4,jk} - ig_L \sum_{j,k=1}^{8} W_\mu^{L-} H_k^0 \bar{\partial}^\mu H_j^+ a_{4,jk} \tag{6.3.17}$$

$$\pounds_{A^0H^+W_R} = -g_R \sum_{j,k=1}^{8} W_\mu^{R+} H_j^- \bar{\partial}^\mu A_k^0 a_{5,jk} + g_R \sum_{j,k=1}^{8} W_\mu^{R-} A_k^0 \bar{\partial}^\mu H_j^+ a_{5,jk} \tag{6.3.18}$$

$$\pounds_{A^0H^+W_L} = -g_L \sum_{j,k=1}^{8} W_\mu^{L+} H_j^- \bar{\partial}^\mu A_k^0 a_{6,jk} + g_L \sum_{j,k=1}^{8} W_\mu^{L-} A_k^0 \bar{\partial}^\mu H_j^+ a_{6,jk} \tag{6.3.19}$$

$$\pounds_{H^0W_RW_R} = g_R^2 W_\mu^{R+} W^{R-\mu} \sum_{j=1}^{8} H_j^0 a_{7,j} \tag{6.3.20}$$

$$\pounds_{H^0W_LW_L} = g_L^2 W_\mu^{L+} W^{L-\mu} \sum_{j=1}^{8} H_j^0 a_{8,j} \tag{6.3.21}$$

$$\pounds_{H^0W_LW_R} = g_L g_R W_\mu^{L+} W^{R-\mu} \sum_{j=1}^{8} H_j^0 a_{8,j} + g_L g_R W_\mu^{R+} W^{L-\mu} \sum_{j=1}^{8} H_j^0 a_{8,j} \tag{6.3.22}$$

$$\pounds_{H^{++}W_RW_R} = -\frac{g_R^2}{\sqrt{2}} W_\mu^{R+} W^{R+\mu} \sum_{j=1}^{2} H_j^{--} a_{9,j} - \frac{g_R^2}{\sqrt{2}} W_\mu^{R-} W^{R-\mu} \sum_{j=1}^{2} H_j^{++} a_{9,j} \tag{6.3.23}$$

$$\pounds_{H^+AW_R} = eg_R A^\mu W_\mu^{R+} \sum_{j=1}^{8} H_j^- a_{10,j} + eg_R A^\mu W_\mu^{R-} \sum_{j=1}^{8} H_j^+ a_{10,j} \tag{6.3.24}$$

$$\pounds_{H^+AW_L} = eg_L A^\mu W_\mu^{L+} \sum_{j=1}^{8} H_j^- a_{11,j} + eg_L A^\mu W_\mu^{L-} \sum_{j=1}^{8} H_j^+ a_{11,j} \tag{6.3.25}$$

$$\pounds_{H^+Z_RW_R} = g_R^2 Z_R^\mu W_\mu^{R+} \sum_{j=1}^{8} H_j^- a_{12,j} + g_R^2 Z_R^\mu W_\mu^{R-} \sum_{j=1}^{8} H_j^+ a_{12,j} \tag{6.3.26}$$

$$\pounds_{H^+Z_LW_R} = g_L g_R Z_L^\mu W_\mu^{R+} \sum_{j=1}^{8} H_j^- a_{13,j} + g_L g_R Z_L^\mu W_\mu^{R-} \sum_{j=1}^{8} H_j^+ a_{13,j} \tag{6.3.27}$$

$$\pounds_{H^+Z_LW_L} = g_L^2 Z_L^\mu W_\mu^{L+} \sum_{j=1}^{8} H_j^- a_{14,j} + g_L^2 Z_L^\mu W_\mu^{L-} \sum_{j=1}^{8} H_j^+ a_{14,j} \tag{6.3.28}$$

The coefficients that appear in equations (6.3.13) through (6.3.28) are:

$$a_{1,jk} = \left(R_R^{\pm\pm}\right)_{j,1}\left(R^\pm\right)_{k,7} + \left(R_R^{\pm\pm}\right)_{j,2}\left(R^\pm\right)_{k,8} \tag{6.3.29}$$

$$a_{2,jk} = \left(R_L^{\pm\pm}\right)_{j,1}\left(R^\pm\right)_{k,1} + \left(R_L^{\pm\pm}\right)_{j,2}\left(R^\pm\right)_{k,2} \tag{6.3.30}$$

$$\begin{aligned} a_{3,jk} &= \frac{1}{2}\left(R^\pm\right)_{j,3}\left(R_s^0\right)_{k,3} - \frac{1}{2}\left(R^\pm\right)_{j,4}\left(R_s^0\right)_{k,4} \\ &+ \frac{1}{2}\left(R^\pm\right)_{j,5}\left(R_s^0\right)_{k,5} - \frac{1}{2}\left(R^\pm\right)_{j,6}\left(R_s^0\right)_{k,6} - \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,7}\left(R_s^0\right)_{k,7} - \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,8}\left(R_s^0\right)_{k,8} \end{aligned} \tag{6.3.31}$$

$$\begin{aligned} a_{4,jk} &= -\frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,1}\left(R_s^0\right)_{k,1} - \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,2}\left(R_s^0\right)_{k,2} \\ &+ \frac{1}{2}\left(R^\pm\right)_{j,3}\left(R_s^0\right)_{k,3} - \frac{1}{2}\left(R^\pm\right)_{j,4}\left(R_s^0\right)_{k,4} + \frac{1}{2}\left(R^\pm\right)_{j,5}\left(R_s^0\right)_{k,5} - \frac{1}{2}\left(R^\pm\right)_{j,6}\left(R_s^0\right)_{k,6} \end{aligned} \tag{6.3.32}$$

$$\begin{aligned} a_{5,jk} &= \frac{1}{2}\left(R^+\right)_{j,3}\left(R_p^0\right)_{k,3} + \frac{1}{2}\left(R^\pm\right)_{j,4}\left(R_p^0\right)_{k,4} \\ &+ \frac{1}{2}\left(R^\pm\right)_{j,5}\left(R_p^0\right)_{k,5} + \frac{1}{2}\left(R^\pm\right)_{j,6}\left(R_p^0\right)_{k,6} + \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,7}\left(R_p^0\right)_{k,7} - \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,8}\left(R_p^0\right)_{k,8} \end{aligned} \tag{6.3.33}$$

$$\begin{aligned} a_{6,jk} &= \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,1}\left(R_p^0\right)_{k,1} - \frac{1}{\sqrt{2}}\left(R^\pm\right)_{j,2}\left(R_p^0\right)_{k,2} \\ &+ \frac{1}{2}\left(R^\pm\right)_{j,3}\left(R_p^0\right)_{k,3} + \frac{1}{2}\left(R^\pm\right)_{j,4}\left(R_p^0\right)_{k,4} + \frac{1}{2}\left(R^\pm\right)_{j,5}\left(R_p^0\right)_{k,5} + \frac{1}{2}\left(R^\pm\right)_{j,6}\left(R_p^0\right)_{k,6} \end{aligned} \tag{6.3.34}$$

$$a_{7,j} = \frac{\kappa_u}{2}\left(R_s^0\right)_{j,5} + \frac{\kappa_d}{2}\left(R_s^0\right)_{j,6} + v_{\Delta_R}\left(R_s^0\right)_{j,7} + v_{\delta_R}\left(R_s^0\right)_{j,8} \tag{6.3.35}$$

$$a_{8,j} = \frac{\kappa_u}{2}\left(R_s^0\right)_{j,5} + \frac{\kappa_d}{2}\left(R_s^0\right)_{j,6} \tag{6.3.36}$$

$$a_{9,j} = v_{\Delta_R}\left(R_R^{\pm\pm}\right)_{j,1} + v_{\delta_R}\left(R_R^{\pm\pm}\right)_{j,2} \tag{6.3.37}$$

$$a_{10,j} = -\frac{\kappa_u}{2}\left(R^{\pm}\right)_{j,5} + \frac{\kappa_d}{2}\left(R^{\pm}\right)_{j,6} + \frac{v_{\Delta_R}}{\sqrt{2}}\left(R^{\pm}\right)_{j,7} + \frac{v_{\delta_R}}{\sqrt{2}}\left(R^{\pm}\right)_{j,8} \tag{6.3.38}$$

$$a_{11,j} = -\frac{\kappa_u}{2}\left(R^{\pm}\right)_{j,5} + \frac{\kappa_d}{2}\left(R^{\pm}\right)_{j,6} \tag{6.3.39}$$

$$a_{12,j} = -\frac{1}{\sqrt{2}}\frac{1+\sin^2\varphi}{\cos\varphi}\left(v_{\Delta_R}\left(R^{\pm}\right)_{j,7} + v_{\delta_R}\left(R^{\pm}\right)_{j,8}\right) \tag{6.3.40}$$

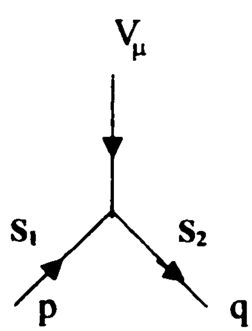
$$a_{13,j} = \cos\theta_W\left(-\frac{\kappa_u}{2}\left(R^{\pm}\right)_{j,5} + \frac{\kappa_d}{2}\left(R^{\pm}\right)_{j,6}\right) + \frac{\sin^2\theta_W}{\cos\theta_W}\left(-\frac{v_{\Delta_R}}{\sqrt{2}}\left(R^{\pm}\right)_{j,7} + \frac{v_{\delta_R}}{\sqrt{2}}\left(R^{\pm}\right)_{j,8}\right) \tag{6.3.41}$$

$$a_{14,j} = \cos\theta_W\left(-\frac{\kappa_u}{2}\left(R^{\pm}\right)_{j,5} + \frac{\kappa_d}{2}\left(R^{\pm}\right)_{j,6}\right) \tag{6.3.42}$$

6.4 Feynman Rules for Vector Bosons and Higgs Fields

The list of Feynman Rules contains the interactions between three fields at least one of which is the photon A or $W_L^{\pm}$ or $W_R^{\pm}$. More rules may be drawn from equations (6.1.2) through (6.1.9) according to circumstances.

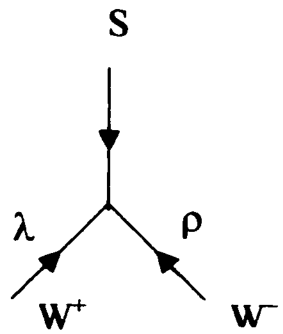
Feynman Rules for two Scalar Fields and one Vector Field



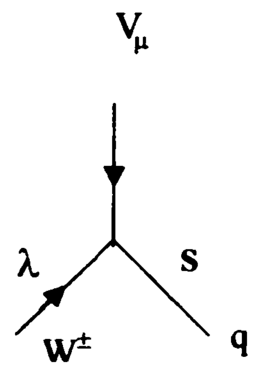
V_μ	S_1	S_2	**Feynman Rule**	
A	$\tilde{u}_{Ln}$	$\tilde{u}_{Ln}$	$-i\frac{2}{3}e(p+q)^\mu$	(6.4.1)
A	$\tilde{u}_{Rn}$	$\tilde{u}_{Rn}$	$-i\frac{2}{3}e(p+q)^\mu$	(6.4.2)
A	$\tilde{d}_{Ln}$	$\tilde{d}_{Ln}$	$i\frac{1}{3}e(p+q)^\mu$	(6.4.3)
A	$\tilde{d}_{Rn}$	$\tilde{d}_{Rn}$	$i\frac{1}{3}e(p+q)^\mu$	(6.4.4)
A	$\tilde{\varepsilon}_{Ln}$	$\tilde{\varepsilon}_{Ln}$	$ie(p+q)^\mu$	(6.4.5)
A	$\tilde{\varepsilon}_{Rn}$	$\tilde{\varepsilon}_{Rn}$	$ie(p+q)^\mu$	(6.4.6)
W_L^-	$\tilde{\nu}_{Ln}$	$\tilde{\varepsilon}_{Lm}$	$-i\frac{g_L}{\sqrt{2}}(p+q)^\mu\left(\tilde{Y}^\dagger\right)_{mn}$	(6.4.7)
W_L^+	$\tilde{\varepsilon}_{Ln}$	$\tilde{\nu}_{Lm}$	$-i\frac{g_L}{\sqrt{2}}(p+q)^\mu\tilde{Y}_{mn}$	(6.4.8)
W_R^-	$\tilde{\nu}_{Rn}$	$\tilde{\varepsilon}_{Rm}$	$-i\frac{g_R}{\sqrt{2}}(p+q)^\mu\left(\tilde{Y}^*\right)_{mn}$	(6.4.9)
W_R^+	$\tilde{\varepsilon}_{Rn}$	$\tilde{\nu}_{Rm}$	$-i\frac{g_R}{\sqrt{2}}(p+q)^\mu\left(\tilde{Y}^T\right)_{mn}$	(6.4.10)
W_L^-	$\tilde{u}_{Ln}$	$\tilde{d}_{Lm}$	$-i\frac{g_L}{\sqrt{2}}(p+q)^\mu\left(\tilde{X}^\dagger\right)_{mn}$	(6.4.11)
W_L^+	$\tilde{d}_{Ln}$	$\tilde{u}_{Lm}$	$-i\frac{g_L}{\sqrt{2}}(p+q)^\mu\tilde{X}_{mn}$	(6.4.12)
W_R^-	$\tilde{u}_{Rn}$	$\tilde{d}_{Rm}$	$-i\frac{g_R}{\sqrt{2}}(p+q)^\mu\left(\tilde{X}^*\right)_{mn}$	(6.4.13)

W_R^+	$\widetilde{d}_{R_n}$	$\widetilde{u}_{R_m}$	$-i\frac{g_R}{\sqrt{2}}(p+q)^\mu\left(\widetilde{X}^T\right)_{mn}$	(6.4.14)
A	H_i^{++}	H_i^{--}	$-2ie(p+q)^\mu, i\in\{1..4\}$	(6.4.15)
A	H_i^+	H_i^-	$-ie(p+q)^\mu, i\in\{1..8\}$	(6.4.16)
W_R^+	H_j^{--}	H_k^+	$ig_R a_{1,jk}(p+q)^\mu, j\in\{1,2\}, k\in\{1..8\}$	(6.4.17)
W_R^-	H_j^{++}	H_k^-	$ig_R a_{1,jk}(p+q)^\mu, j\in\{1,2\}, k\in\{1..8\}$	(6.4.18)
W_L^+	H_j^{--}	H_k^+	$ig_L a_{2,jk}(p+q)^\mu, j\in\{3,4\}, k\in\{1..8\}$	(6.4.19)
W_L^-	H_j^{++}	H_k^-	$ig_L a_{2,jk}(p+q)^\mu, j\in\{3,4\}, k\in\{1..8\}$	(6.4.20)
W_R^+	H_j^-	H_k^0	$ig_R a_{3,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.21)
W_R^-	H_j^+	H_k^0	$ig_R a_{3,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.22)
W_L^+	H_j^-	H_k^0	$ig_L a_{4,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.23)
W_L^-	H_j^+	H_k^0	$ig_L a_{4,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.24)
W_R^+	H_j^-	A_k^0	$g_R a_{5,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.25)
W_R^-	H_j^+	A_k^0	$-g_R a_{5,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.26)
W_L^+	H_j^-	A_k^0	$g_L a_{6,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.27)
W_L^-	H_j^+	A_k^0	$-g_L a_{6,jk}(p+q)^\mu, j,k\in\{1..8\}$	(6.4.28)

Feynman Rules for one Scalar Fields and two Vector Fields



S	W^+	W^-	Feynman Rule	
H_j^0	W_R^+	W_R^-	$ig_R^2 a_{7,j}\eta_{\lambda\rho}, j \in \{1..8\}$	(6.4.29)
H_j^0	W_L^+	W_L^-	$ig_L^2 a_{8,j}\eta_{\lambda\rho}, j \in \{1..8\}$	(6.4.30)
H_j^0	W_R^+	W_L^-	$ig_L g_R a_{8,j}\eta_{\lambda\rho}, j \in \{1..8\}$	(6.4.31)
H_j^0	W_L^+	W_R^-	$ig_L g_R a_{8,j}\eta_{\lambda\rho}, j \in \{1..8\}$	(6.4.32)
H_j^{--}	W_R^+	W_R^+	$-\sqrt{2}ig_R^2 a_{9,j}\eta_{\lambda\rho}, j \in \{1,2\}$	(6.4.33)
H_j^{++}	W_R^-	W_R^-	$-\sqrt{2}ig_R^2 a_{9,j}\eta_{\lambda\rho}, j \in \{1,2\}$	(6.4.34)



In the above diagram the charged vector boson, be it positive or negative, is always meant to be going into the vertex. The arrow of the corresponding charged scalar field S is omitted. It flows such that charge conservation is not violated.

V_μ	W^+	S	**Feynman Rule**	
A	W_R^+	H_j^-	$ieg_R a_{10,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.35)
A	W_R^-	H_j^+	$ieg_R a_{10,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.36)
A	W_L^+	H_j^-	$ieg_L a_{11,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.37)
A	W_L^-	H_j^+	$ieg_L a_{11,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.38)
Z_R	W_R^+	H_j^-	$ig_R^2 a_{12,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.39)
Z_R	W_R^-	H_j^+	$ig_R^2 a_{12,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.40)
Z_L	W_R^+	H_j^-	$ig_L g_R a_{13,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.41)
Z_L	W_R^-	H_j^+	$ig_L g_R a_{13,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.42)
Z_L	W_L^+	H_j^-	$ig_L^2 a_{14,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.43)
Z_L	W_L^-	H_j^+	$ig_L^2 a_{14,j}\eta_{\mu\lambda}, j \in \{1..8\}$	(6.4.44)

Chapter 7: Magnetic Moments and Quadrupole Moments of $W_L^{\pm}$ and $W_R^{\pm}$

7.1 Magnetic Moments and Quadrupole Moments in Left-Right Supersymmetry

The magnetic moments and quadrupole moments of $W_L^{\pm}$ and $W_R^{\pm}$ can be read off from the vertex function $\Gamma^{\mu\lambda\rho}$ calculated for interactions between the photon A and two W_L or the photon A and two W_R. The magnetic moment is called $\kappa-1$ and the quadrupole moment is ΔQ. The most general CP- and $U(1)_{em}$ invariant vertex, when all particles are taken on the mass shell, is:

$$
\begin{aligned}
\Gamma^{\mu\lambda\rho} &= ie\left(AP^{\mu\lambda\rho} + 2(\kappa-1)Q^{\mu\lambda\rho} + 4\frac{\Delta Q}{M_W{}^2}p^{\mu}q^{\lambda}q^{\rho}\right) \\
Q^{\mu\lambda\rho} &\equiv q^{\rho}\eta^{\mu\lambda} - q^{\lambda}\eta^{\mu\rho}, \quad P^{\mu\lambda\rho} \equiv 2p^{\mu}\eta^{\lambda\rho} + 4Q^{\mu\lambda\rho}
\end{aligned}
\tag{7.1.1}
$$

At tree-level the tensor structure of $\Gamma^{\mu\lambda\rho}$ is not complicated enough and one merely finds $A=1$, $\kappa-1=0$, and $\Delta Q=0$. In order to produce all the terms of equation (7.1.1) it is necessary to calculate the vertex function $\Gamma^{\mu\lambda\rho}$ at least to one-loop level. Figure 7.1 indicates the basic process that needs to be calculated. The particle masses and the momentum variables that correspond to the form of (7.1.1) are also shown. The particle that connects to the photon carries mass M and the particle that does not connect to the photon carries mass m. We use the mass ratios τ and σ respectively as well as the fine-structure constant α.

$$
\alpha = \frac{e^2}{4\pi}, \quad \tau = \frac{M}{M_W}, \quad \sigma = \frac{m}{M_W}
\tag{7.1.2}
$$

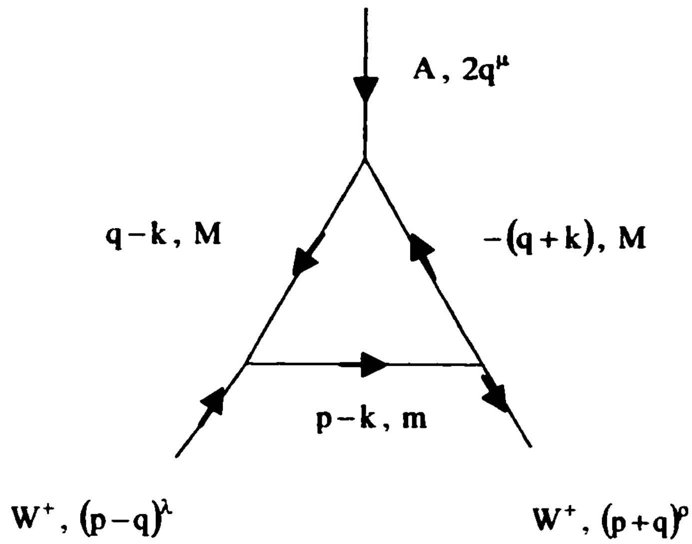


Figure 7.1: Vertex for the interactions AW_LW_L *or* AW_RW_R *at one-loop level. The W-bosons are strictly on the mass shell and the photon is taken in the limit* $q^{\mu} \to 0$.

The virtual particles of the internal lines depend on the particle-content of the model. All loops may be assigned to four distinct classes of problems indicated in Figure 7.2.

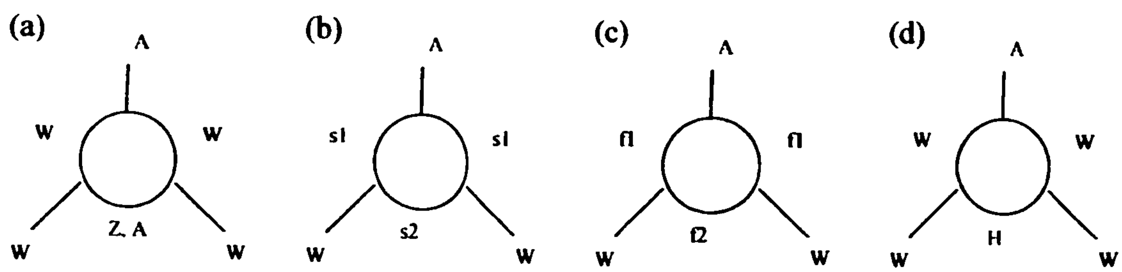


Figure 7.2: Classes of Loops; 's1' and 's2' are scalars, 'f1' and 'f2' are fermions, and H must be a Higgs-type scalar.

Tables 7.1 through 7.4 in the sections below list the results for all possible loops of our left-right symmetric model. The possible particle combinations for the internal lines are specified in the tables. The Feynman rules of the three previous chapters have been used. The calculations are made with the dimensional regularization technique ([1], [10], [27]) and in unitary gauge ([28], [29], [30]).

7.2 Virtual Supersymmetry

One crucial but outstanding test of the Standard Model is the observation of trilinear and quadrilinear gauge boson interactions. Such interactions have been discussed in Sections 4.2 and 4.3 and it is clear that they represent the non-Abelian structure of the Standard Model gauge forces (see the remark on top of p. 59). Experiments of the near future are aimed at the direct observation of three gauge boson couplings and it is here that we hope to observe signals of supersymmetry too. The interaction type shown in Fig. 7.1 is equally a trilinear interaction but with virtual particles plugged into the vertex. The contributions to the magnetic and quadrupole moments depend on the particle content of a given model, namely the combinations of particles that can be taken as virtual particles that contribute to the process. Thus it is hopefully possible to see effects from supersymmetry particles, even if the supersymmetry particles themselves are not observed as yet. One is justified to speak of virtual supersymmetry.

A recent calculation in the MSSM framework of the magnetic and quadrupole moments of the W-boson [31] ended with a conclusion that may be interpreted pessimistically as well as optimistically. Even if there are deviations from the Standard Model fit in the expected data output, they would be too small to be interpreted in favor of MSSM-supersymmetry. On the other hand, quantities that are hard to observe are the cleanest observables in favor of the interpretation of new physics, once something unexpected has been measured. If MSSM-supersymmetry is not what one may reliably expect, then we are doubly encouraged to hope that deviations from the Standard Model fit point to our left-right supersymmetric model, since its additional particles may produce more striking deviations from the Standard Model.

The following sections list the results of the calculations of the magnetic and quadrupole moments of the W-bosons of the model. The formulae have been compared with calculations that have been performed previously in the Standard Model [28], in a supersymmetric version of the Standard Model [29], [30], and in the MSSM [31]. While the particle content of our left-right supersymmetric model is richer, not all particles are new ones. Hence, several formulae have to be either identical or similar in form as previously. The result of the comparison is that the formulae in this work are reliable.

The next step to undertake will be a numerical evaluation of the results given in the following Sections 7.3 and 7.4. The current numerical value for κ can be taken from Ref. [4] and Ref. [33]. The full magnetic moment is given by $\mu_W = e(1+\kappa+\lambda)/2M_W$ and the electric quadrupole moment by $-e(\kappa-\lambda)/M_W^2$. In the Standard Model, at tree level the values of the variables are $\kappa = 1$ and $\lambda = 0$. Ref. [33] reports:

$-1.3 < \kappa < 3.2$ for $\lambda = 0$ and $-0.7 < \lambda < 0.7$ for $\kappa = 1$ in $p\bar{p} \to e\nu_e\gamma X$ and $\mu\nu_\mu\gamma X$ at $\sqrt{s} = 1.8$ TeV.

7.3 Magnetic Moments of $W_L^{\pm}$ and $W_R^{\pm}$

Table 7.1: *Contributions to the Magnetic Moment of* W_L
The External Fields (Fig. 7.1) are AW_LW_L

Particles in the loop	τ	σ	$\kappa-1$
$W_L\ W_L\ A$	1	0	$\frac{5}{3}\frac{\alpha}{\pi}$
$W_L\ W_L\ Z_L$	1	$\frac{M_{Z_L}}{M_{W_L}}$	$\frac{g_L{}^2}{16\pi^2}\left(\frac{20}{3}\frac{1}{\sigma^2}-\frac{5}{6}+\int_0^1 dz\frac{\frac{1}{2}z^4+5z^3-18z^2+16z-8}{\sigma^2(1-z)+z^2}\right)$
$\tilde{u}_{Ln}\ \tilde{u}_{Ln}\ \tilde{d}_{Lm}$	$\frac{M_{\tilde{u}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{d}_{Lm}}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^{*}_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{d}_{Ln}\ \tilde{d}_{Ln}\ \tilde{u}_{Lm}$	$\frac{M_{\tilde{d}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{u}_{Lm}}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^{*}_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{\varepsilon}_{Ln}\ \tilde{\varepsilon}_{Ln}\ \tilde{\nu}_{Lm}$	$\frac{M_{\tilde{\varepsilon}_{Ln}}}{M_{W_L}}$	$\frac{M_{\tilde{\nu}_{Lm}}}{M_{W_L}}$	$-\frac{g_L{}^2}{16\pi^2}(-1)\tilde{Y}^{*}_{mn}\tilde{Y}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$u_{Ln}\ u_{Ln}\ d_{Lm}$	$\frac{M_{u_n}}{M_{W_L}}$	$\frac{M_{d_m}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(\frac{2}{3}\right)X^{*}_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$d_{Ln}\ d_{Ln}\ u_{Lm}$	$\frac{M_{d_n}}{M_{W_L}}$	$\frac{M_{u_m}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(-\frac{1}{3}\right)X^{*}_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\varepsilon_{Ln}\ \varepsilon_{Ln}\ \nu_{Lm}$	$\frac{M_{\varepsilon_n}}{M_{W_L}}$	$\frac{M_{\nu_m}}{M_{W_L}}$	$-\frac{g_L{}^2}{16\pi^2}(-1)Y^{*}_{mn}Y_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{D}_L^{++}\ \tilde{D}_L^{++}\ \tilde{D}_L^{+}$	$\frac{M_{\tilde{D}_L^{++}}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^{+}}}{M_{W_L}}$	$-\frac{g_L{}^2}{8\pi^2}2\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau^2-1)}{\sigma^2(1-z)+z^2+z(\tau^2-1)}\right)$
$\tilde{D}_L^{+}\ \tilde{D}_L^{+}\ \tilde{D}_L^{++}$	$\frac{M_{\tilde{D}_L^{+}}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^{++}}}{M_{W_L}}$	$-\frac{g_L{}^2}{8\pi^2}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau^2-1)}{\sigma^2(1-z)+z^2+z(\tau^2-1)}\right)$
$\tilde{D}_L^{+}\ \tilde{D}_L^{+}\ \tilde{D}_L^{0}$	$\frac{M_{\tilde{D}_L^{+}}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^{0}}}{M_{W_L}}$	$-\frac{g_L{}^2}{8\pi^2}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau^2-1)}{\sigma^2(1-z)+z^2+z(\tau^2-1)}\right)$

Table 7.1 continued

Particles in the loop	τ	σ	$\kappa - 1$
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_L}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}\left(L_{kj}^{L*}L_{kj}^{L}+L_{kj}^{R*}L_{kj}^{R}\right) \bullet \left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $-\frac{g_L^2}{8\pi^2}\left(L_{kj}^{L*}L_{kj}^{R}+L_{kj}^{R*}L_{kj}^{L}\right) \bullet \left(\sigma_k\tau_j\int_0^1 dz\frac{2z^2-z}{\sigma^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$
$H_j^{++}\ H_j^{--}\ H_k^+$	$\frac{M_{H_j^{\pm\pm}}}{M_{W_L}}$	$\frac{M_{H_k^\pm}}{M_{W_L}}$	$-2\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{3,4\}, k\in\{1..6\}$
$H_j^+\ H_j^-\ H_k^{++}$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{H_k^{\pm\pm}}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{2,kj}a_{2,kj}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $k\in\{3,4\}, j\in\{1..6\}$
$H_j^+\ H_j^-\ H_k^0$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{H_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{4,kj}a_{4,kj}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\}, k\in\{1..8\}$
$H_j^+\ H_j^-\ A_k^0$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{A_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{6,kj}a_{6,kj}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\}, k\in\{1..6\}$
$W_L\ W_L\ H_j^0$	1	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^4}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3+4z^2}{\sigma_j^2(1-z)+z^2}\right)$ $j\in\{1..6\}$
$W_R\ W_R\ H_j^0$	$\frac{M_{W_R}}{M_{W_L}}$	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2}a_{8,j}a_{8,j}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3+4z^2}{\sigma_j^2(1-z)+z^2+z(\tau-1)}\right)$ $j\in\{1..6\}$

Table 7.2: ***Contributions to the Magnetic Moment of*** W_R
The External Fields (Fig. 7.1) are AW_RW_R

Particles in the loop	τ	σ	$\kappa-1$
$W_R\ W_R\ A$	1	0	$\frac{5}{3}\frac{\alpha}{\pi}$
$W_R\ W_R\ Z_L$	1	$\frac{M_{Z_L}}{M_{W_R}}$	$\frac{\sigma^2 e^2\tan^2\theta}{16\pi^2}\left(\frac{20}{3}\frac{1}{\sigma^2}-\frac{5}{6}+\int_0^1 dz\frac{\frac{1}{2}z^4+5z^3-18z^2+16z-8}{\sigma^2(1-z)+z^2}\right)$
$W_R\ W_R\ Z_R$	1	$\frac{M_{Z_R}}{M_{W_R}}$	$\frac{g_R{}^2}{8\pi^2}\left(\frac{20}{3}\frac{1}{\sigma^2}-\frac{5}{6}+\int_0^1 dz\frac{\frac{1}{2}z^4+5z^3-18z^2+16z-8}{\sigma^2(1-z)+z^2}\right)$
$\tilde{u}_{R_n}\ \tilde{u}_{R_n}\ \tilde{d}_{R_m}$	$\frac{M_{\tilde{u}_{R_n}}}{M_{W_R}}$	$\frac{M_{\tilde{d}_{R_m}}}{M_{W_R}}$	$-N_c\frac{g_R{}^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^{*}_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{d}_{R_n}\ \tilde{d}_{R_n}\ \tilde{u}_{R_m}$	$\frac{M_{\tilde{d}_{R_n}}}{M_{W_R}}$	$\frac{M_{\tilde{u}_{R_m}}}{M_{W_R}}$	$-N_c\frac{g_R{}^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^{*}_{mn}\tilde{X}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{\varepsilon}_{R_n}\ \tilde{\varepsilon}_{R_n}\ \tilde{\nu}_{R_m}$	$\frac{M_{\tilde{\varepsilon}_{R_n}}}{M_{W_R}}$	$\frac{M_{\tilde{\nu}_{R_m}}}{M_{W_R}}$	$-\frac{g_R{}^2}{16\pi^2}(-1)\tilde{Y}^{*}_{mn}\tilde{Y}_{mn}\left(\frac{1}{3}+\int_0^1 dz\frac{z^4-2z^3-z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$u_{R_n}\ u_{R_n}\ d_{R_m}$	$\frac{M_{u_n}}{M_{W_R}}$	$\frac{M_{d_m}}{M_{W_R}}$	$-N_c\frac{g_R{}^2}{16\pi^2}\left(\frac{2}{3}\right)X^{*}_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$d_{R_n}\ d_{R_n}\ u_{R_m}$	$\frac{M_{d_n}}{M_{W_R}}$	$\frac{M_{u_m}}{M_{W_R}}$	$-N_c\frac{g_R{}^2}{16\pi^2}\left(-\frac{1}{3}\right)X^{*}_{mn}X_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\varepsilon_{R_n}\ \varepsilon_{R_n}\ \nu_{R_m}$	$\frac{M_{\varepsilon_n}}{M_{W_R}}$	$\frac{M_{\nu_m}}{M_{W_R}}$	$-\frac{g_R{}^2}{16\pi^2}(-1)Y^{*}_{mn}Y_{mn}\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_n^2-1)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{D}_R^{++}\ \tilde{D}_R^{++}\ \tilde{\chi}_j^{+}$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^{+}}}{M_{W_R}}$	$-\frac{g_R{}^2}{8\pi^2}2\left(V^{*}_{k5}V_{k5}+U^{*}_{k5}U_{k5}\right)$ $\bullet\left(\frac{1}{6}-\int_0^1 dz\frac{z^4-z^3+z^2+z^2(\tau_k^2-1)}{\sigma^2(1-z)+z^2+z(\tau_k^2-1)}\right)$ $-\frac{g_R{}^2}{8\pi^2}2\left(V^{*}_{k5}U_{k5}+U^{*}_{k5}V_{k5}\right)$ $\bullet\left(\sigma\tau_k\int_0^1 dz\frac{2z^2-z}{\sigma^2(1-z)+z^2+z(\tau_k^2-1)}\right)$

Table 7.2 continued

Particles in the loop	τ	σ	$\kappa-1$
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{D}_R^{++}$	$\dfrac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\dfrac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$-\dfrac{g_R^2}{8\pi^2}\left(V_{k5}^*V_{k5}+U_{k5}^*U_{k5}\right)\bullet\left(\dfrac{1}{6}-\int_0^1 dz\dfrac{z^4-z^3+z^2+z^2\left(\tau_k^2-1\right)}{\sigma^2(1-z)+z^2+z\left(\tau_k^2-1\right)}\right)$ $-\dfrac{g_R^2}{8\pi^2}\left(V_{k5}^*V_{k5}+U_{k5}^*U_{k5}\right)\bullet\left(\sigma\tau_k\int_0^1 dz\dfrac{2z^2-z}{\sigma^2(1-z)+z^2+z\left(\tau_k^2-1\right)}\right)$
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{\chi}_k^0$	$\dfrac{M_{\tilde{\chi}_j^+}}{M_{W_R}}$	$\dfrac{M_{\tilde{\chi}_k^0}}{M_{W_R}}$	$-\dfrac{g_R^2}{8\pi^2}\left(R_{kj}^{L*}R_{kj}^{L}+R_{kj}^{R*}R_{kj}^{R}\right)\bullet\left(\dfrac{1}{6}-\int_0^1 dz\dfrac{z^4-z^3+z^2+z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $-\dfrac{g_R^2}{8\pi^2}\left(R_{kj}^{L*}R_{kj}^{R}+R_{kj}^{R*}R_{kj}^{L}\right)\bullet\left(\sigma_k\tau_j\int_0^1 dz\dfrac{2z^2-z}{\sigma^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$
$H_j^{++}\ H_j^{--}\ H_k^{+}$	$\dfrac{M_{H_j^{\pm\pm}}}{M_{W_R}}$	$\dfrac{M_{H_k^{\pm}}}{M_{W_R}}$	$-2\dfrac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\dfrac{1}{3}+\int_0^1 dz\dfrac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1,2\},k\in\{1..6\}$
$H_j^{+}\ H_j^{-}\ H_k^{++}$	$\dfrac{M_{H_j^{\pm}}}{M_{W_R}}$	$\dfrac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$-\dfrac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\dfrac{1}{3}+\int_0^1 dz\dfrac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $k\in\{1,2\},j\in\{1..6\}$
$H_j^{+}\ H_j^{-}\ H_k^{0}$	$\dfrac{M_{H_j^{\pm}}}{M_{W_R}}$	$\dfrac{M_{H_k^{0}}}{M_{W_R}}$	$-\dfrac{g_R^2}{8\pi^2}a_{3,kj}a_{3,kj}\left(\dfrac{1}{3}+\int_0^1 dz\dfrac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\},k\in\{1..8\}$
$H_j^{+}\ H_j^{-}\ A_k^{0}$	$\dfrac{M_{H_j^{\pm}}}{M_{W_R}}$	$\dfrac{M_{A_k^{0}}}{M_{W_R}}$	$-\dfrac{g_R^2}{8\pi^2}a_{5,kj}a_{5,kj}\left(\dfrac{1}{3}+\int_0^1 dz\dfrac{z^4-2z^3-z^2\left(\tau_j^2-1\right)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\},k\in\{1..6\}$

Table 7.2 continued

Particles in the loop	τ	σ	$\kappa - 1$
$W_L\ W_L\ H_j^0$	$\dfrac{M_{W_L}}{M_{W_R}}$	$\dfrac{M_{H_j^0}}{M_{W_R}}$	$\dfrac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2} a_{8,j} a_{8,j} \left(\dfrac{1}{3} + \int_0^1 dz \dfrac{z^4 - 2z^3 + 4z^2}{\sigma_j^2 (1-z) + z^2 + z(\tau - 1)} \right)$ $j \in \{1..6\}$
$W_R\ W_R\ H_j^0$	1	$\dfrac{M_{H_j^0}}{M_{W_R}}$	$\dfrac{g_R^4}{32\pi^2 M_{W_R}^2} a_{7,j} a_{7,j} \left(\dfrac{1}{3} + \int_0^1 dz \dfrac{z^4 - 2z^3 + 4z^2}{\sigma_j^2 (1-z) + z^2} \right)$ $j \in \{1..6\}$
$W_R\ W_R\ H_j^{++}$	1	$\dfrac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$\dfrac{g_R^4}{16\pi^2 M_{W_R}^2} a_{9,j} a_{9,j} \left(\dfrac{1}{3} + \int_0^1 dz \dfrac{z^4 - 2z^3 + 4z^2}{\sigma_j^2 (1-z) + z^2} \right)$ $j \in \{1,2\}$
$W_R\ W_R\ H_j^{--}$	1	$\dfrac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$\dfrac{g_R^4}{16\pi^2 M_{W_R}^2} a_{9,j} a_{9,j} \left(\dfrac{1}{3} + \int_0^1 dz \dfrac{z^4 - 2z^3 + 4z^2}{\sigma_j^2 (1-z) + z^2} \right)$ $j \in \{1,2\}$

7.4 Quadrupole Moments of $W_L^\pm$ and $W_R^\pm$

Table 7.3: *Contributions to the Quadrupole Moment of* W_L
The External Fields (Fig. 7.1) are $AW_L W_L$

Particles in the loop	τ	σ	ΔQ
$W_L\ W_L\ A$	1	0	$\frac{1}{9}\frac{\alpha}{\pi}$
$W_L\ W_L\ Z_L$	1	$\frac{M_{Z_L}}{M_{W_L}}$	$\frac{g_L{}^2}{16\pi^2}\frac{1}{3}\left(1+\frac{8}{\sigma^2}\right)\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$\tilde{u}_{Ln}\ \tilde{u}_{Ln}\ \tilde{d}_{Lm}$	$\frac{M_{\tilde{u}_{Lm}}}{M_{W_L}}$	$\frac{M_{\tilde{d}_{Lm}}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{d}_{Ln}\ \tilde{d}_{Ln}\ \tilde{u}_{Lm}$	$\frac{M_{\tilde{d}_{Lm}}}{M_{W_L}}$	$\frac{M_{\tilde{u}_{Lm}}}{M_{W_L}}$	$-N_c\frac{g_L{}^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{\varepsilon}_{Ln}\ \tilde{\varepsilon}_{Ln}\ \tilde{\nu}_{Lm}$	$\frac{M_{\tilde{\varepsilon}_{Lm}}}{M_{W_L}}$	$\frac{M_{\tilde{n}_{Lm}}}{M_{W_L}}$	$-\frac{g_L{}^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$u_{Ln}\ u_{Ln}\ d_{Lm}$	$\frac{M_{u_n}}{M_{W_L}}$	$\frac{M_{d_m}}{M_{W_L}}$	$N_c\frac{g_L{}^2}{16\pi^2}\left(\frac{2}{3}\right)X^*_{mn}X_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$d_{Ln}\ d_{Ln}\ u_{Lm}$	$\frac{M_{d_n}}{M_{W_L}}$	$\frac{M_{u_m}}{M_{W_L}}$	$N_c\frac{g_L{}^2}{16\pi^2}\left(-\frac{1}{3}\right)X^*_{mn}X_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\varepsilon_{Ln}\ \varepsilon_{Ln}\ \nu_{Lm}$	$\frac{M_{\varepsilon_n}}{M_{W_L}}$	$\frac{M_{\nu_m}}{M_{W_L}}$	$\frac{g_L{}^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z(\tau_n^2-1)}\right)$
$\tilde{D}_L^{++}\ \tilde{D}_L^{++}\ \tilde{D}_L^{+}$	$\frac{M_{\tilde{D}_L^{++}}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^{+}}}{M_{W_L}}$	$\frac{g_L{}^2}{2\pi^2}2\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2+z(\tau^2-1)}\right)$
$\tilde{D}_L^{+}\ \tilde{D}_L^{+}\ \tilde{D}_L^{++}$	$\frac{M_{\tilde{D}_L^{+}}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^{++}}}{M_{W_L}}$	$\frac{g_L{}^2}{2\pi^2}\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2+z(\tau^2-1)}\right)$

Table 7.3 continued

Particles in the loop	τ	σ	ΔQ
$\tilde{D}_L^+\ \tilde{D}_L^+\ \tilde{D}_L^0$	$\frac{M_{\tilde{D}_L^+}}{M_{W_L}}$	$\frac{M_{\tilde{D}_L^0}}{M_{W_L}}$	$\frac{g_L{}^2}{2\pi^2}\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2+z\left(\tau^2-1\right)}\right)$
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_L}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_L}}$	$\frac{g_L{}^2}{2\pi^2}\left(L_{kj}^{L*}L_{kj}^{L}+L_{kj}^{R*}L_{kj}^{R}\right) \bullet\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$
$H_j^{++}\ H_j^{--}\ H_k^+$	$\frac{M_{H_j^{\pm\pm}}}{M_{W_L}}$	$\frac{M_{H_k^\pm}}{M_{W_L}}$	$-2\frac{g_L^2}{8\pi^2}a_{2,kj}\,a_{2,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{3,4\}, k\in\{1..6\}$
$H_j^+\ H_j^-\ H_k^{++}$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{H_k^{\pm\pm}}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{2,kj}\,a_{2,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $k\in\{3,4\}, j\in\{1..6\}$
$H_j^+\ H_j^-\ H_k^0$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{H_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{4,kj}\,a_{4,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\}, k\in\{1..8\}$
$H_j^+\ H_j^-\ A_k^0$	$\frac{M_{H_j^\pm}}{M_{W_L}}$	$\frac{M_{A_k^0}}{M_{W_L}}$	$-\frac{g_L^2}{8\pi^2}a_{6,kj}\,a_{6,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\}, k\in\{1..6\}$
$W_L\ W_L\ H_j^0$	1	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^4}{32\pi^2 M_{W_L}^2}a_{8,j}\,a_{8,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}\right)$ $j\in\{1..6\}$
$W_R\ W_R\ H_j^0$	$\frac{M_{W_R}}{M_{W_L}}$	$\frac{M_{H_j^0}}{M_{W_L}}$	$\frac{g_L^2 g_R^2}{32\pi^2 M_{W_L}^2}a_{8,j}\,a_{8,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2+z(\tau-1)}\right)$ $j\in\{1..6\}$

Table 7.4: ***Contributions to the Quadrupole Moment of*** W_R
The External Fields (Fig. 7.1) are $AW_R W_R$

Particles in the loop	τ	σ	ΔQ
$W_R\ W_R\ A$	1	0	$\frac{1}{9}\frac{\alpha}{\pi}$
$W_R\ W_R\ Z_L$	1	$\frac{M_{Z_L}}{M_{W_R}}$	$\frac{e^2\tan^2\theta_W}{16\pi^2}\frac{8+\sigma^2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$W_R\ W_R\ Z_R$	1	$\frac{M_{Z_R}}{M_{W_R}}$	$\frac{{g_R}^2}{16\pi^2}\frac{2}{3}\left(1+\frac{8}{\sigma^2}\right)\int_0^1 dz\frac{z^3(1-z)}{\sigma^2(1-z)+z^2}$
$\tilde{u}_{R_n}\ \tilde{u}_{R_n}\ \tilde{d}_{R_m}$	$\frac{M_{\tilde{u}_{R_m}}}{M_{W_R}}$	$\frac{M_{\tilde{d}_{R_m}}}{M_{W_R}}$	$-N_c\frac{{g_R}^2}{16\pi^2}\left(\frac{2}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$\tilde{d}_{R_n}\ \tilde{d}_{R_n}\ \tilde{u}_{R_m}$	$\frac{M_{\tilde{d}_{R_m}}}{M_{W_R}}$	$\frac{M_{\tilde{u}_{R_m}}}{M_{W_R}}$	$-N_c\frac{{g_R}^2}{16\pi^2}\left(-\frac{1}{3}\right)\tilde{X}^*_{mn}\tilde{X}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$\tilde{\varepsilon}_{R_n}\ \tilde{\varepsilon}_{R_n}\ \tilde{\nu}_{R_m}$	$\frac{M_{\tilde{\varepsilon}_{R_m}}}{M_{W_R}}$	$\frac{M_{\tilde{\nu}_{R_m}}}{M_{W_R}}$	$-\frac{{g_R}^2}{16\pi^2}(-1)\tilde{Y}^*_{mn}\tilde{Y}_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$u_{R_n}\ u_{R_n}\ d_{R_m}$	$\frac{M_{u_n}}{M_{W_R}}$	$\frac{M_{d_m}}{M_{W_R}}$	$N_c\frac{{g_R}^2}{16\pi^2}\left(\frac{2}{3}\right)X^*_{mn}X_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$d_{R_n}\ d_{R_n}\ u_{R_m}$	$\frac{M_{d_n}}{M_{W_R}}$	$\frac{M_{u_m}}{M_{W_R}}$	$N_c\frac{{g_R}^2}{16\pi^2}\left(-\frac{1}{3}\right)X^*_{mn}X_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$\varepsilon_{R_n}\ \varepsilon_{R_n}\ \nu_{R_m}$	$\frac{M_{\varepsilon_n}}{M_{W_R}}$	$\frac{M_{\nu_m}}{M_{W_R}}$	$\frac{{g_R}^2}{16\pi^2}(-1)Y^*_{mn}Y_{mn}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_m^2(1-z)+z^2+z\left(\tau_n^2-1\right)}\right)$
$\tilde{D}_R^{++}\ \tilde{D}_R^{++}\ \tilde{\chi}_j^+$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\frac{{g_R}^2}{2\pi^2}2\left(V^*_{k5}U_{k5}+V^*_{k5}U_{k5}\right)\cdot\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau^2-1\right)}\right)$
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{D}_R^{++}$	$\frac{M_{\tilde{\chi}_k^+}}{M_{W_R}}$	$\frac{M_{\tilde{D}_R^{++}}}{M_{W_R}}$	$\frac{{g_R}^2}{2\pi^2}\left(V^*_{k5}V_{k5}+U^*_{k5}U_{k5}\right)\cdot\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau^2-1\right)}\right)$

Table 7.4 continued

Particles in the loop	τ	σ	ΔQ
$\tilde{\chi}_j^+\ \tilde{\chi}_j^+\ \tilde{\chi}_k^0$	$\frac{M_{\tilde{\chi}_j^+}}{M_{W_R}}$	$\frac{M_{\tilde{\chi}_k^0}}{M_{W_R}}$	$\frac{g_R{}^2}{2\pi^2}\left(R_{kj}^{L*}R_{kj}^{L}+R_{kj}^{R*}R_{kj}^{R}\right)\bullet\left(\frac{1}{6}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$
$H_j^{++}\ H_j^{--}\ H_k^{+}$	$\frac{M_{H_j^{\pm\pm}}}{M_{W_R}}$	$\frac{M_{H_k^{\pm}}}{M_{W_R}}$	$-2\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1,2\},k\in\{1..6\}$
$H_j^{+}\ H_j^{-}\ H_k^{++}$	$\frac{M_{H_j^{\pm}}}{M_{W_R}}$	$\frac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{1,kj}a_{1,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $k\in\{1,2\},j\in\{1..6\}$
$H_j^{+}\ H_j^{-}\ H_k^{0}$	$\frac{M_{H_j^{\pm}}}{M_{W_R}}$	$\frac{M_{H_k^{0}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{3,kj}a_{3,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\},k\in\{1..8\}$
$H_j^{+}\ H_j^{-}\ A_k^{0}$	$\frac{M_{H_j^{\pm}}}{M_{W_R}}$	$\frac{M_{A_k^{0}}}{M_{W_R}}$	$-\frac{g_R^2}{8\pi^2}a_{5,kj}a_{5,kj}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_k^2(1-z)+z^2+z\left(\tau_j^2-1\right)}\right)$ $j\in\{1..6\},k\in\{1..6\}$
$W_L\ W_L\ H_j^{0}$	$\frac{M_{W_L}}{M_{W_R}}$	$\frac{M_{H_j^{0}}}{M_{W_R}}$	$\frac{g_L^2g_R^2}{32\pi^2M_{W_L}^2}a_{8,j}a_{8,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2+z(\tau-1)}\right)$ $j\in\{1..6\}$
$W_R\ W_R\ H_j^{0}$	1	$\frac{M_{H_j^{0}}}{M_{W_R}}$	$\frac{g_R^4}{32\pi^2M_{W_R}^2}a_{7,j}a_{7,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}\right)$ $j\in\{1..6\}$
$W_R\ W_R\ H_j^{++}$	1	$\frac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$\frac{g_R^4}{16\pi^2M_{W_R}^2}a_{9,j}a_{9,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}\right)$ $j\in\{1,2\}$
$W_R\ W_R\ H_j^{--}$	1	$\frac{M_{H_k^{\pm\pm}}}{M_{W_R}}$	$\frac{g_R^4}{16\pi^2M_{W_R}^2}a_{9,j}a_{9,j}\left(\frac{2}{3}\int_0^1 dz\frac{z^3(1-z)}{\sigma_j^2(1-z)+z^2}\right)$ $j\in\{1,2\}$

7.5 Cancellation of Divergences

The triangle graphs whose internal lines are all vector bosons are the most difficult to calculate. In particular, the contributions to the magnetic moments of $W_L^{\pm}$ and $W_R^{\pm}$ become finite in a very detailed way.

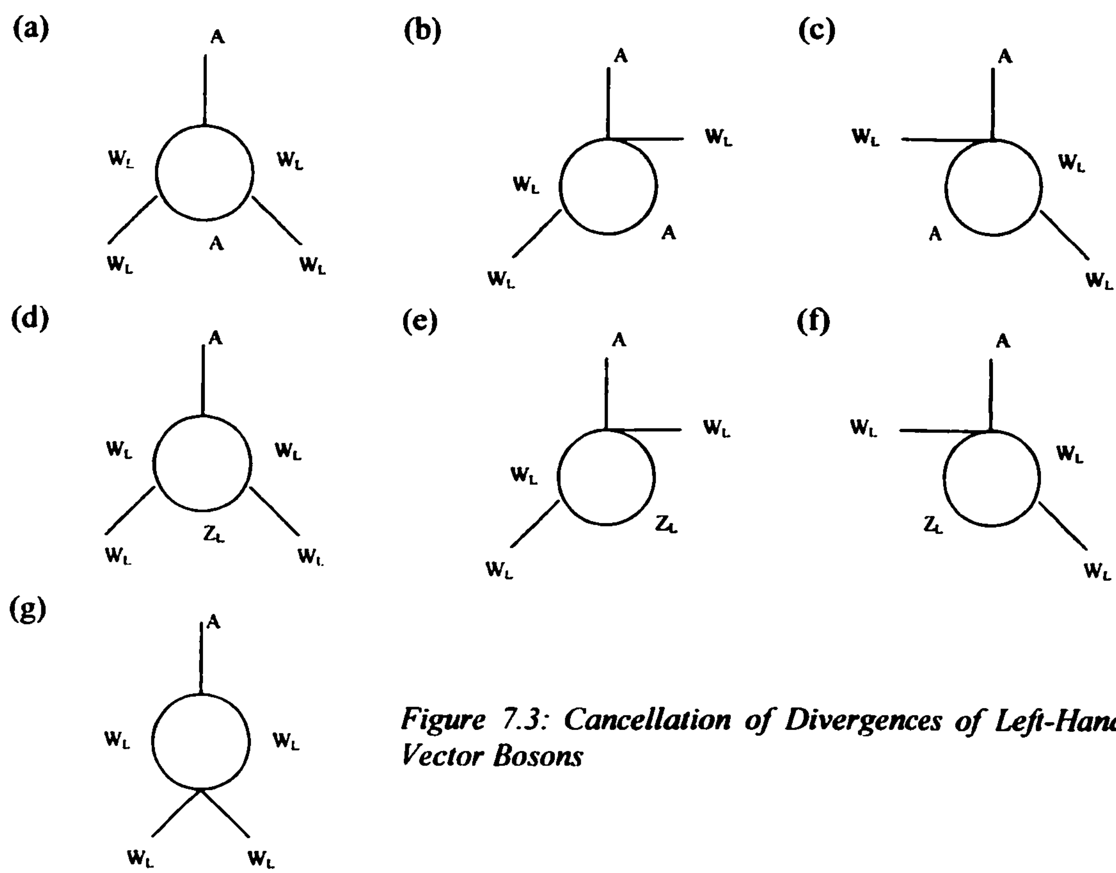


Figure 7.3: Cancellation of Divergences of Left-Handed Vector Bosons

In Fig. 7.3 the graphs (a) and (d) define the scope of the calculation as indicated earlier in Fig. 7.1. However, the remaining graphs (b), (c), (e), (f), (g) of Fig. 7.3 also contribute to the magnetic moment of $W_L^{\pm}$.

None of the graphs of Fig. 7.3 is finite, if taken individually. However, it has been known that the sum of all of them combined is finite. The graphs of Fig. 7.3 occur identically in the Standard Model and were calculated in 1972 [25]. Those results were confirmed in this work. It should be noted that the finiteness depends non-trivially on the mass ratio:

$$\frac{1}{\sigma} \mapsto \frac{M_{W_L}}{M_{Z_L}} = \cos\theta_W \tag{7.5.1}$$

Only the graphs (d), (e), and (f) in Fig. 7.3 depend on σ. If the form of the mass ratio is altered, one must check again how the cancellation of divergences overall works.

Now we turn to findings of our own in the right-handed sector. In Fig. 7.4 the graphs (a), (d), and (h) define the scope of the calculation. The graphs (a) through (g) of Fig. 7.4 all have analogues in Fig. 7.3 if the label 'R' is replaced with 'L'. However, in Fig. 7.4 the graphs (d), (e), and (f) now depend on the different mass ratio:

$$\frac{1}{\sigma} \mapsto \frac{M_{W_R}}{M_{Z_R}} = \frac{\cos\varphi}{\sqrt{2}} \tag{7.5.2}$$

Due to interference of the factor $1/\sqrt{2}$, not only are the graphs (a) through (g) of Fig. 7.4 individually divergent, but also their sum remains divergent.

But then there is a new set (h), (i), (j) in Fig. 7.4 that does not have analogues in Fig. 7.3 with switched labels 'L' and 'R'. The absence of such analogues in Fig. 7.3 is a consequence of the fact that one of the matrix elements in the mixing matrix of physical fields is zero. We recall that:

$$\begin{pmatrix} W_\mu^{L0} \\ W_\mu^{R0} \\ V_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W & 0 \\ -\sin\varphi\sin\theta_W & \sin\varphi\cos\theta_W & \cos\varphi \\ -\cos\varphi\sin\theta_W & \cos\varphi\cos\theta_W & -\sin\varphi \end{pmatrix} \begin{pmatrix} Z_\mu^L \\ A_\mu \\ Z_\mu^R \end{pmatrix} \tag{7.5.3}$$

(See also the remarks at the end of Chapter 4.2.) The set (h), (i), (j) alone is again both individually and summarily divergent, but the combination of all graphs (a) through (j) of Fig 7.4 is found to be finite.

In summary, the cancellation of divergent terms has worked exactly. The exceptionally difficult graphs of Fig. 7.3, 7.4 serve as a grand consistency check for the form of the mixing matrix (7.5.3) and the two different mass ratios (7.5.1) and (7.5.2).

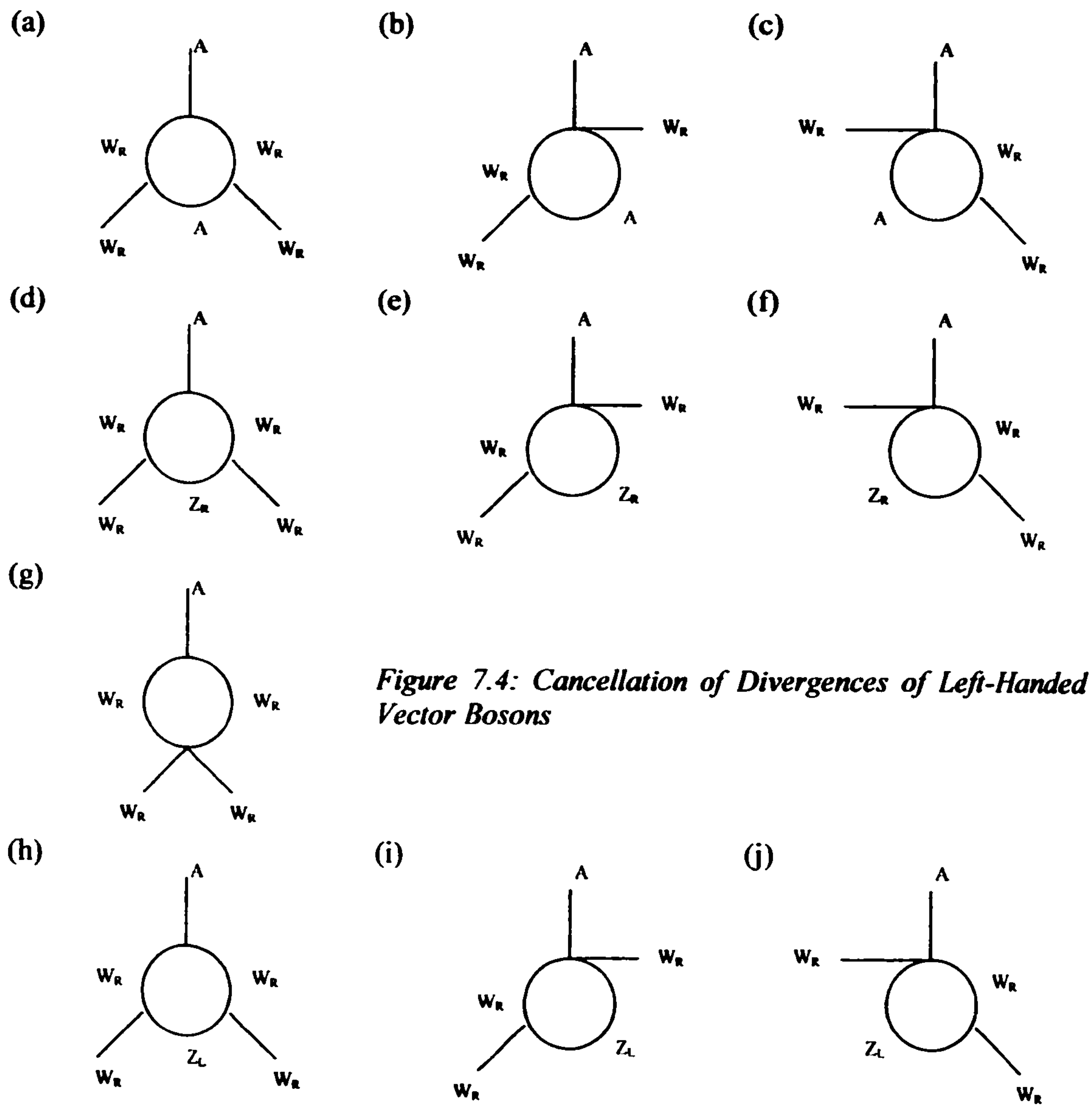


Figure 7.4: Cancellation of Divergences of Left-Handed Vector Bosons

CONCLUSION

The theoretical appeal of supersymmetry in building gauge models has been presented. Radiative corrections to scalar masses of scalar particles cancel and supergravity is shown to be capable of developing a globally supersymmetric low-energy limit plus soft supersymmetry-breaking terms.

A left-right supersymmetric extension of the Standard Model has been proposed. The symmetry-breaking pattern of the left-right symmetry has been described and the mixing of physical vector bosons from the gauge bosons of the particle content has been put in a neat form using two mixing angles that correspond to two different stages of breakdowns that break $SU(2)_L x SU(2)_R x U(1)_{B-L}$ to the electromagnetic symmetry $U(1)_{em}$. The right-handed vector bosons are shown to have a mass ratio that is different from the one of the Standard Model, because the corresponding scalar particle that receives a vacuum expectation value is a member of an SU(2)-triplet.

Rules have been formulated that summarize a large variety of vector boson interactions. The interactions with scalars and fermions are shown to be proportional to functions of quantum numbers of the involved particles and of the two mixing angles θ_W and ϕ. The summation rules in Section 6.1 for vector-scalar interactions may even be useful in models different from ours. For example, Ref. [21] raises the possibility of extending the Higgs sector in order to better describe parity violation. This work provides the tools that would be needed in order to easily modify our results accordingly, if it should be necessary.

The anomalous magnetic moments and the quadrupole moments of the charged vector bosons have been calculated in the framework of a left-right supersymmetric extension of the Standard Model. In particular, many results that are listed in Sections 7.3 and 7.4 have not been published previously. These are the results involving $W_\mu^{R\pm}$ as external particle and the results involving $W_\mu^{L\pm}$ as external particle in combination with virtual particles that are characteristic of left-right supersymmetry. Furthermore the finiteness of those loops with $W_\mu^{R\pm}$ as external particle has been demonstrated in detail in Section 7.5.

Relatively soon we expect relevant data to be available form measurements of three-gauge-boson couplings. While such measurements are interesting enough to the Standard Model in order to directly prove the non-Abelian structure of its gauge forces, they may be even more interesting to detecting signals of supersymmetry in the data. If reference [31] is right and small deviations from the Standard Model cannot be interpreted as MSSM-supersymmetry, we may hope that they can be interpreted as left-right supersymmetry instead. If this is true, it is going to be not only a signal of supersymmetry but also of left-right symmetry. A definite claim cannot be made at this point, since this work lacks a numerical evaluation of the theoretical results, for which there was not enough room. The numerical analysis is going to be the next logical step to take.

Another step can be taken in a different but related direction. The interaction Lagrangian of the neutral vector bosons have been worked out as well and the calculation of their static quantities suggests itself to follow up on this work. In this area one of the most recent works has been conducted once again in the context of MSSM-supersymmetry [32]. The present work on the other hand provides all the theoretical prerequisites to undertake the same adventure in the uncharted terrain of left-right supersymmetry.

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Appendix 1: Conventions

The Metric Tensor

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{A.1.1}$$

Spinors and the Projection Operators γ_L and γ_R

The upper components of a 4-spinor are taken to be 'left' and the lower ones to be 'right'.

$$\psi \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A.1.2}$$

The projection operators γ_L and γ_R are defined by:

$$\gamma_L\psi \equiv \begin{pmatrix} \psi_L \\ 0 \end{pmatrix},\ \gamma_R\psi \equiv \begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \tag{A.1.3}$$

The projection operators γ_L and γ_R satisfy a typical set of equations:

$$\begin{aligned} &\gamma_L^{\ 2} = \gamma_L \\ &\gamma_R^{\ 2} = \gamma_R \\ &\gamma_L + \gamma_R = 1 \\ &\gamma_L\gamma_R = \gamma_R\gamma_L = 0 \end{aligned} \tag{A.1.4}$$

The following equations are convention-independent and always valid:

$$\begin{aligned} &\gamma_L\gamma^\mu = \gamma^\mu\gamma_R,\ \gamma_R\gamma^\mu = \gamma^\mu\gamma_L \\ &\overline{\gamma_L\Psi} = \overline{\Psi}\gamma_R,\ \overline{\gamma_R\Psi} = \overline{\Psi}\gamma_L \end{aligned} \tag{A.1.5}$$

The Levi-Civita-Tensor and γ_5

There are (more than) two choices. Set number one:

$$\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3,\ \gamma_5 = \frac{i}{24} \in^{\mu\nu\lambda\rho} \gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\rho \tag{A.1.6}$$

$$\gamma_L = \frac{1}{2}(1+\gamma_5),\ \gamma_R = \frac{1}{2}(1-\gamma_5) \tag{A.1.7}$$

$$\in^{0123} \equiv 1 \tag{A.1.8}$$

Set number two:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \tag{A.1.9}$$

$$\gamma_L = \frac{1}{2}(1-\gamma_5),\ \gamma_R = \frac{1}{2}(1+\gamma_5) \tag{A.1.10}$$

$$\in_{0123} \equiv 1 \tag{A.1.11}$$

Even permutations of indices in the Levi-Civita Tensor preserve the sign, odd permutations change it and 'cyclic' happens to be odd.

The two sets (A.1.6, 7, 8) and (A.1.9, 10, 11) seem to be the ones that are most widely used. They both rely on the fundamental definitions (A.1.2) and (A.1.3) and they both have the consequence that $\mathrm{Tr}\left(\gamma_5\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\rho\right) = 4i \in^{\mu\nu\lambda\rho}$. In fact, the sign on the right-hand side of this trace theorem is convention dependent.

The two sets (A.1.6, 7, 8) and (A.1.9, 10, 11) are equivalent in the sense that they both agree with (A.1.2) and (A.1.3). If all formulae are expressed in terms of γ_L and γ_R rather than γ_5, as it is done in this work, then the formulae all have the same form in either set of conventions.

The Alternative Metric Tensor

For future reference it is helpful to note the alternative choice of the metric tensor, because it is as widely used as ours.

One may choose either $\eta^{\mu\nu} = \mathrm{diag}(1 \quad -1 \quad -1 \quad -1)$ or $\eta^{\mu\nu} = \mathrm{diag}(-1 \quad 1 \quad 1 \quad 1)$. However, the choice of the metric influences the algebra of Dirac matrices due to the Clifford equation: $\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}\underline{1}$. The Clifford algebra must be formally the same in either convention, i.e. without a change of sign on the right-hand side of the equation. To avoid the change of sign, a factor of minus i (it could as well be plus i) is defined into each Dirac matrix on the left-hand side. Table A1.1 lists various expressions in either metric (see next page).

$\eta^{\mu\nu} = \mathrm{diag}(1 \quad -1 \quad -1 \quad -1)$	$\eta^{\mu\nu} = \mathrm{diag}(-1 \quad 1 \quad 1 \quad 1)$
$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}\underline{1}$	$\gamma^{\mu}\gamma^{\nu} + \gamma^{\nu}\gamma^{\mu} = 2\eta^{\mu\nu}\underline{1}$
$x^{\mu}x_{\mu} = s^2$	$x^{\mu}x_{\mu} = -s^2$
$\overline{\psi} \equiv \psi^{\dagger}\gamma^0$	$\overline{\psi} \equiv i\psi^{\dagger}\gamma^0$
$\pounds_{\mathrm{Dirac}} = i\overline{\psi}\not{\partial}\psi - m\overline{\psi}\psi$	$\pounds_{\mathrm{Dirac}} = -\overline{\psi}\not{\partial}\psi - m\overline{\psi}\psi$
$(i\not{\partial} - m)\psi = 0$	$(\not{\partial} + m)\psi = 0$
$(\gamma^0)^2 = \underline{1}$, $(\gamma^k)^2 = -\underline{1}$, k=1, 2, 3	$(\gamma^0)^2 = -\underline{1}$, $(\gamma^k)^2 = \underline{1}$, k=1, 2, 3
$(\gamma^0)^{\dagger} = \gamma^0$, $(\gamma^k)^{\dagger} = -\gamma^k$, k=1, 2, 3	$(\gamma^0)^{\dagger} = -\gamma^0$, $(\gamma^k)^{\dagger} = \gamma^k$, k=1, 2, 3
$(\gamma^{\mu})^{\dagger} = \gamma^0\gamma^{\mu}\gamma^0$	$(\gamma^{\mu})^{\dagger} = \gamma^0\gamma^{\mu}\gamma^0$
$\gamma^{\mu}\gamma_5 + \gamma_5\gamma^{\mu} = 0$	$\gamma^{\mu}\gamma_5 + \gamma_5\gamma^{\mu} = 0$
$(\gamma_5)^{\dagger} = \gamma_5$	$(\gamma_5)^{\dagger} = \gamma_5$
$\gamma^0\gamma_5\gamma^0 = -\gamma_5$	$\gamma^0\gamma_5\gamma^0 = \gamma_5$
$\gamma^0\gamma_L\gamma^0 = \gamma_R$, $\gamma^0\gamma_R\gamma^0 = \gamma_L$	$\gamma^0\gamma_L\gamma^0 = -\gamma_R$, $\gamma^0\gamma_R\gamma^0 = -\gamma_L$
$(\overline{\Psi}_1\gamma^{\mu}\gamma_L\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma^{\mu}\gamma_L\Psi_1$	$(\overline{\Psi}_1\gamma^{\mu}\gamma_L\Psi_2)^{\dagger} = -\overline{\Psi}_2\gamma^{\mu}\gamma_L\Psi_1$
$(\overline{\Psi}_1\gamma^{\mu}\gamma_R\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma^{\mu}\gamma_R\Psi_1$	$(\overline{\Psi}_1\gamma^{\mu}\gamma_R\Psi_2)^{\dagger} = -\overline{\Psi}_2\gamma^{\mu}\gamma_R\Psi_1$
$(\overline{\Psi}_1\gamma_L\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma_R\Psi_1$	$(\overline{\Psi}_1\gamma_L\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma_R\Psi_1$
$(\overline{\Psi}_1\gamma_R\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma_L\Psi_1$	$(\overline{\Psi}_1\gamma_R\Psi_2)^{\dagger} = \overline{\Psi}_2\gamma_L\Psi_1$

Table A1.1: Comparison of Metric Tensor Conventions

Appendix 2: Two-Component Language - Dotted and Undotted Spinors

It is customary in supersymmetry to employ a notation that keeps the transformation properties of 2-spinors under Lorentz transformations transparent. The purpose of this chapter is to explain this notation ([10], [12]).

Transformation Properties

The Lorentz group SL(2,C) is equivalent to a product SU(2)xSU(2). It is possible to define 2-component spinors in such a way that they transform under either one of the two SU(2)-factors. Such spinors are called Weyl spinors and their quantum numbers are denoted as either (½ 0) or (0 ½). (½ 0) and (0 ½) are inequivalent representations of the Lorentz group. If the transformation matrix acting on a 2-component spinor in the representation (½ 0) is called M, the corresponding transformation matrix of (0 ½) is $\left(M^{-1}\right)^{\dagger}$. In this sense, if we write a 4-spinor as $\psi = \left(\psi_L, \psi_R\right)^T$, then ψ_L transforms under the matrix M and ψ_R transforms under the matrix $\left(M^{-1}\right)^{\dagger}$. We now introduce a notation that emphasizes the different transformation properties of 2-spinors.

Any four-component spinor is generally written in the form:

$$\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix} \tag{A2.1}$$

This notation is extremely precise, as it is loaded with information. The bar of $\bar{\eta}^{\dot{\alpha}}$ reflects the '†' of $\left(M^{-1}\right)^{\dagger}$, the raised index of $\bar{\eta}^{\dot{\alpha}}$ reflects the '-1' of $\left(M^{-1}\right)^{\dagger}$, and the dot on the index reflects the change from (½ 0) into (0 ½). Hence, one can always read the transformation properties of any given 2-spinor from the way it is written.

The bar over a 2-spinor is not to be confused with a bar over a 4-spinor, which is defined as $\overline{\psi} \equiv \psi^{\dagger}\gamma^{0}$. Instead, the bar over a 2-spinor stands for 'something like' complex conjugation, namely:

$$\overline{\chi}^{\dot{\alpha}} = (\chi^{\alpha})^{*}, \; \overline{\chi}_{\dot{\alpha}} = (\chi_{\alpha})^{*}. \tag{A2.2}$$

In fact, this is merely another way of stating the relation between (½ 0) and (0 ½). One can read from (A2.2) that M and $(M^{-1})^{\dagger}$ are transformation matrices of inequivalent representations, but that M* and $(M^{-1})^{\dagger}$ are transformation matrices of equivalent representations.

Spinor indices can be raised or lowered using a metric tensor:

$$\overline{\chi}_{\alpha} = \varepsilon_{\alpha\beta}\overline{\chi}^{\beta}, \; \overline{\chi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\overline{\chi}_{\dot{\beta}} \tag{A2.3}$$

The spinor metric is anti-symmetric and its components are:

$$\varepsilon^{\alpha\beta} = \varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -\varepsilon_{\alpha\beta} = -\varepsilon_{\dot{\alpha}\dot{\beta}} = i\sigma^{2} \tag{A2.4}$$

However this is also the charge conjugation matrix in SU(2) space. Hence re-inspecting the upper and lower components of (A.2.1) along with (A.2.3) shows that the representations (½ 0) and (0 ½) are linked by charge conjugation. The charge conjugate of ξ_{α} is

$$(\xi_{\alpha})^{c} = i\sigma^{2}(\xi_{\alpha})^{*} = \varepsilon^{\beta\alpha}(\xi_{\alpha})^{*} = (\xi^{\beta})^{*} = \overline{\xi}^{\dot{\beta}} \tag{A2.5}$$

Here too the notation indicates the change from (½ 0) into (0 ½) in all detail.

Definitions

Equation (A.2.1) defines the Dirac spinor ψ and the 'natural' positions of its indices. The term natural means here that the metric has not yet been used to raise or lower an index.

The following equations define a few more important quantities and the natural positions of their indices:

$$\overline{\psi} = \begin{pmatrix} \eta^{\alpha} & \bar{\xi}_{\dot{\alpha}} \end{pmatrix} \tag{A2.6}$$

$$\psi^{c} = \begin{pmatrix} \eta_{\alpha} \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} \tag{A2.7}$$

$$\gamma_{\mu} = \begin{pmatrix} 0 & \sigma_{\mu\alpha\dot{\beta}} \\ \bar{\sigma}_{\mu}^{\dot{\alpha}\beta} & 0 \end{pmatrix} \tag{A2.8}$$

There is also a relation for barred and unbarred components of Dirac matrices:

$$\bar{\sigma}^{\mu}_{\dot{\alpha}\beta} = \sigma^{\mu}_{\beta\dot{\alpha}} \tag{A2.9}$$

Majorana spinors satisfy $\psi^{c} = \psi$ and therefore take the form

$$\psi_{\text{Majorana}} = \begin{pmatrix} \chi_{\alpha} \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix} \tag{A2.10}$$

For practical calculations it is sometimes useful to take a Dirac spinor $\psi = \begin{pmatrix} \xi_{\alpha} \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}$ as a pair of Majorana spinors $a = \begin{pmatrix} \xi_{\alpha} \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix}$ and $b = \begin{pmatrix} \eta_{\alpha} \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}$ and to calculate with $\psi = \gamma_{L} a + \gamma_{R} b$ and $\psi^{c} = \gamma_{R} a + \gamma_{L} b$ using only the algebra of the projection operators γ_{L} and γ_{R}.

How to Avoid Indices

While the notation is now very precise, it is quite cumbersome to use in practical work. Here is a little taste. Raising and lowering indices with an anti-symmetric matrix has the property that

$$\eta^{\alpha}\xi_{\alpha} = -\eta_{\alpha}\xi^{\alpha} \tag{A2.11}$$

This is so, because:

$$\eta^{\alpha}\xi_{\alpha} = \varepsilon^{\alpha\beta}\varepsilon_{\alpha\gamma}\eta_{\beta}\xi^{\gamma} = -\varepsilon^{\beta\alpha}\varepsilon_{\alpha\gamma}\eta_{\beta}\xi^{\gamma} = -\varepsilon^{\beta}{}_{\gamma}\eta_{\beta}\xi^{\gamma} = -\eta_{\beta}\xi^{\beta} = -\eta_{\alpha}\xi^{\alpha} \tag{A2.12}$$

Moreover, *2-spinors that carry indices behave like anti-commuting variables*:

$$\eta^{\alpha}\xi_{\alpha} = -\xi_{\alpha}\eta^{\alpha} \tag{A2.13}$$

However, the indices can be largely ignored while calculating, if we define:

$$\eta\xi \equiv \eta^{\alpha}\xi_{\alpha} \tag{A2.14}$$

This defines the new index-less expression $\eta\xi$ as well as the natural positions of its indices, if it is desired to show them.

Then it follows from (A.2.11) and (A.2.13) that:

$$\eta^{\alpha}\xi_{\alpha} = -\eta_{\alpha}\xi^{\alpha} = \xi^{\alpha}\eta_{\alpha} \tag{A2.15}$$

Applying the definition (A.2.14) on the left and the right side of (A.2.15), it follows that:

$$\eta\xi = \xi\eta \tag{A2.16}$$

2-component spinors that do not carry indices behave like regular commuting variables.

In the same way it may be shown that:

$$\begin{aligned}
\eta\xi &\equiv \eta^{\alpha}\xi_{\alpha} = \xi\eta \\
\overline{\eta\xi} &\equiv \bar{\eta}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}} = \overline{\xi\eta} \\
\bar{\xi}_1\bar{\sigma}^{\mu}\xi_2 &\equiv \bar{\xi}_{1\dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}\xi_{2\beta} = -\xi_2\sigma^{\mu}\bar{\xi}_1 \\
\eta_1\sigma^{\mu}\bar{\eta}_2 &\equiv \eta_1{}^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\bar{\eta}_2{}^{\dot{\beta}} = -\bar{\eta}_2\bar{\sigma}^{\mu}\eta_1
\end{aligned} \tag{A2.17}$$

Equation (A.2.9) has been used in the third line and fourth line of (A2.17).

Note that $\bar{\varphi}\bar{\sigma}^{\mu}\chi = -\chi\sigma^{\mu}\bar{\varphi}$, but $\bar{\varphi}_i\bar{\sigma}^{\mu}D_{ij}\chi_j = -\chi_i\sigma^{\mu}\left(D^{T}\right)_{ij}\bar{\varphi}_j$, where D is a 2x2 matrix and the indices i, j are linear algebra indices.

In the present work the rules (A.2.17) generally allow us to make calculations completely without indices. The rules also allow us to put the indices back at any given time simply by observing their natural positions according to definition.

Simplified Definitions

For the purposes of this writing it is sufficient to use the simplified definitions

$$\psi = \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix},\ \bar{\psi} = \begin{pmatrix} \eta & \bar{\xi} \end{pmatrix},\ \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix} \tag{A2.18}$$

in order to convert from 2-component language into 4-component language and back, for example:

$$\begin{aligned} \bar{\psi}_1\gamma_\mu\psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2 + \eta_1\sigma^\mu\bar{\eta}_2 \\ \bar{\psi}_1\gamma_\mu\gamma_L\psi_2 &= \bar{\xi}_1\bar{\sigma}^\mu\xi_2 = -\xi_2\sigma^\mu\bar{\xi}_1 \\ \bar{\psi}_1\gamma_\mu\gamma_R\psi_2 &= \eta_1\sigma^\mu\bar{\eta}_2 = -\bar{\eta}_2\bar{\sigma}^\mu\eta_1 \end{aligned} \tag{A2.19}$$

The New Notation at Work in Physics

From Chapter 2 onward all 2-spinors are expressed as dotted and undotted Weyl 2-spinors. Indices are suppressed, taking advantage of (A.2.17). However many 2-spinors are still naturally barred, in fact a half of all of the 2-spinors are. For example, following the general definition (A.2.1), the leptons of the Standard Model are represented as:

$$\varepsilon = \begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix},\ \nu = \begin{pmatrix} \nu_L \\ \bar{\nu}_L \end{pmatrix} \tag{A2.20}$$

The natural positions of indices are implied as defined in (A.2.1). The convention is that the original notation (1.1.1) from Chapter One always meant to be this (A2.20). However, one must get accustomed to the fact that it is 'now' ε_L and $\bar{\varepsilon}_R$ (meaning $\varepsilon_{L\alpha}$ and $\overline{\varepsilon_R}^{\dot{\alpha}}$) that generate equal electric charge, while ε_L and ε_R (meaning $\varepsilon_{L\alpha}$ and $\varepsilon_{R\alpha}$) generate opposite charge. (Observe for example that $\bar{\varepsilon}\varepsilon = \varepsilon_R{}^{\alpha}\varepsilon_{L\alpha} + \overline{\varepsilon_{L}}_{\dot{\alpha}}\overline{\varepsilon_R}^{\dot{\alpha}} = \varepsilon_R\varepsilon_L + \overline{\varepsilon_L}\,\overline{\varepsilon_R}$.) There is nothing peculiar about this, it is simply how the notation works and everything is well defined.

In conclusion, here is an example. It is an enlightening exercise to write the kinetic energy of leptons of the Standard Model in 4-component as well as 2-component language.

Let all of the 4-spinor and 2-spinor variables below be defined by (A.2.20). In 4-component language $SU(2)_L$ doublets and a kinetic energy are given as:

$$L = \begin{pmatrix} \gamma_L \nu \\ \gamma_L \varepsilon \end{pmatrix},\ E = \gamma_R \varepsilon \qquad \text{(A2.21)}$$

$$\pounds_{Lep,kin} = \bar{L} i\gamma^{\mu} D^{L}_{\mu} L + \bar{E} i\gamma^{\mu} D^{R}_{\mu} E \qquad \text{(A2.22)}$$

The same in 2-component language is:

$$L_L = \begin{pmatrix} \nu_L \\ \varepsilon_L \end{pmatrix},\ \bar{\varepsilon}_R \qquad \text{(A2.23)}$$

$$\pounds_{Lep,kin} = iL^{\dagger}\bar{\sigma}^{\mu} D^{L}_{\mu} L + \varepsilon_R i\sigma^{\mu} D^{R}_{\mu} \bar{\varepsilon}_R \qquad \text{(A2.24)}$$

Here D^{L}_{μ} and D^{R}_{μ} are the covariant derivatives under $SU(2)_L$ of left- and right-handed spinors respectively.

Using equations (A2.18) through (A2.20) it is relatively straightforward to show that (A2.22) and (A2.24) are identically equal.

Appendix 3: Covariant Derivatives of SU(2)-Triplets

Consider a triplet of fields, for example gauginos that have a kinetic energy of the form:

$$£_{\text{triplet-kinetic}} = i \sum_{a,b,c=1}^{3} \bar{\lambda}^a \bar{\sigma}^\mu \left(\delta_{ac}\partial_\mu + g_L \in_{abc} W_\mu^b\right)\lambda^c \tag{A3.1}$$

One has to expand the sum (A.3.1) and substitute the $U(1)_{em}$ charge eigenstates:

$$\lambda^\pm = \frac{1}{\sqrt{2}}\left(\lambda^1 \mp i\lambda^2\right),\ W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right) \tag{A3.2}$$

It is more practical to handle generators instead of structure constants, because matrix multiplication is very straightforward. Matrices can be introduced in two different ways.

With Three-Dimensional Generators

$$T^1 \equiv \sqrt{\frac{1}{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ T^2 \equiv \sqrt{\frac{1}{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ T^3 \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{A3.3}$$

$$\Lambda \equiv \begin{pmatrix} -\lambda^+ \\ \lambda^0 \\ \lambda^- \end{pmatrix} \tag{A3.4}$$

The matrices (A3.3) are generators of 3-dimensional SU(2) and (A.3.4) is a spinor in 3-dimensional SU(2)-space. The minus sign in front of λ^+ plays a role when 4-spinors are formed. The connection with (A3.1) is:

$$i\bar{\lambda}^a \bar{\sigma}^\mu \left(\delta_{ac}\partial_\mu + g_L \in_{abc} W_\mu^b\right)\lambda^c = i\Lambda^\dagger \bar{\sigma}^\mu \left(\partial_\mu - i g_L T^a W_\mu^a\right)\Lambda \tag{A3.5}$$

The fields $-\lambda^+$, λ^0, λ^- behave as carrying the SU(2) quantum numbers +1, 0, -1 respectively, see for example the summation rules of Chapter 6.1.

With Two-Dimensional Generators

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A3.6}$$

$$\underline{\Lambda} \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\lambda^0 & \lambda^+ \\ \lambda^- & -\frac{1}{\sqrt{2}}\lambda^0 \end{pmatrix} \tag{A3.7}$$

The matrices $\frac{1}{2}\tau^a$, built from (A3.6), are generators of 2-dimensional SU(2).

The rule is in this case:

$$i\bar{\lambda}^a\bar{\sigma}^\mu\left(\delta_{ac}\partial_\mu + g_L \in_{abc} W_\mu^b\right)\lambda^c = i\mathrm{Tr}\left(\underline{\Lambda}^\dagger\bar{\sigma}^\mu\left(\partial_\mu - ig_L\tau^a W_\mu^a\right)\underline{\Lambda}\right) \tag{A3.8}$$

It is still true of course, that the fields $-\lambda^+$, λ^0, λ^- behave in calculations as carrying the SU(2) quantum numbers +1, 0, -1. All expressions given in (A.3.5) and (A.3.8) are identically equal.

The three-dimensional convention (A3.4) is the author's choice. The two-dimensional convention (A.3.7) is often used in the literature for triplet scalars that occur in left-right symmetric models. Triplet scalars are formally different than gauginos, because they carry nonzero $U(1)_{B-L}$-quantum numbers. As a result, the electric charges of triplet scalars are not (+, 0, -) but (++, +, 0) or (0, -, - -), which is a reflection of the Gell-Mann-Nishiyima formula (see Table 3.1 and equation (3.1.1)).

Appendix 4: Quantum Numbers of SU(2) Bi-Doublets

The quantum numbers t_L^3 and t_R^3 of the bi-doublets $F^I_{u,d}$ and $F^{II}_{u,d}$ of Table 3.1 are:

$$F^I_{u,d} = \begin{pmatrix} \phi^0_{1u,d} \\ \phi^-_{2u,d} \end{pmatrix} \mapsto \begin{pmatrix} t_L^3 = -\frac{1}{2} \\ t_L^3 = -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} t_R^3 = +\frac{1}{2} \\ t_R^3 = -\frac{1}{2} \end{pmatrix} \qquad F^{II}_{u,d} = \begin{pmatrix} \phi^+_{1u,d} \\ \phi^0_{2u,d} \end{pmatrix} \mapsto \begin{pmatrix} t_L^3 = +\frac{1}{2} \\ t_L^3 = +\frac{1}{2} \end{pmatrix}, \begin{pmatrix} t_R^3 = +\frac{1}{2} \\ t_R^3 = -\frac{1}{2} \end{pmatrix}$$

(A4.1)

The same applies to the superpartner bi-doublets.

W^R_μ recognizes the 'spin-up' and 'spin-down' states of bi-doublet Higgs fields as expected, but W^L_μ 'sees' the members of $F^I_{u,d}$ both as 'spin-down' and the members of $F^{II}_{u,d}$ both as 'spin-up'. There is a special rule for bi-doublets to take this into account. In addition to the usual Pauli matrices (A3.6) we need two more sets of matrices that differ in the definition of the third matrix, where the distinction between 'spin-up' and 'spin-down' is made.

$$\tau^1_\downarrow = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2_\downarrow = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3_\downarrow = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\tau^1_\uparrow = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^2_\uparrow = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^3_\uparrow = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \text{(A4.2)}$$

These 'Pauli-matrices' are used in the equations (4.1.11), (5.1.1), (5.2.1), and (6.1.1).

Another convention is found in the literature. The SU(2)-spinors $F^{I}_{u,d}$ and $F^{II}_{u,d}$ may be defined into one 2x2-matrix:

$$F_{u,d} \equiv \begin{pmatrix} \phi^{0}_{1u,d} & \phi^{+}_{1u,d} \\ \phi^{-}_{2u,d} & \phi^{0}_{2u,d} \end{pmatrix} \tag{A4.3}$$

But again the quantum numbers (A4.1) make it necessary to formulate a special covariant derivative rule. Distinguish first:

$$\mathrm{diag}(F_{u,d}) \equiv \begin{pmatrix} \phi^{0}_{1u,d} & 0 \\ 0 & \phi^{0}_{2u,d} \end{pmatrix} \qquad \mathrm{off}(F_{u,d}) \equiv \begin{pmatrix} 0 & \phi^{+}_{1u,d} \\ \phi^{-}_{2u,d} & 0 \end{pmatrix} \tag{A1.4}$$

Yet another set of 'Pauli-matrices' is needed:

$$\tau^{1}_{x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau^{2}_{x} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau^{3}_{x} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A1.5}$$

And then in this language:

$$D_{\mu}F_{u,d} \equiv \left(\partial_{\mu} - \frac{ig_{L}}{2}\tau^{a}W^{La}_{\mu} - \frac{ig_{R}}{2}\tau^{a}W^{Ra}_{\mu} \right) F_{u,d}$$

$$\equiv \left(\partial_{\mu} - \frac{ig_{L}}{2}\tau^{a}_{x}W^{La}_{\mu} - \frac{ig_{R}}{2}\tau^{a}W^{Ra}_{\mu} \right) \mathrm{diag}(F_{u,d}) + \left(\partial_{\mu} - \frac{ig_{L}}{2}\tau^{a}W^{La}_{\mu} - \frac{ig_{R}}{2}\tau^{a}W^{Ra}_{\mu} \right) \mathrm{off}(F_{u,d})$$

(A1.6)

Appendix 5: Integration of Feynman Parameters

Triangle loops have three propagator factors corresponding to three internal lines. The appropriate Feynman parameters rule is:

$$\frac{1}{abc} = 2\int_0^1 dx \int_0^{1-x} dy \frac{1}{\left(a(1-x-y)+bx+cy\right)^3} \tag{A5.1)}$$

This is nontrivial because the two integrations are interdependent. However, under some circumstances the denominator of (A5.1) depends only on the combination x + y. In that case the two integrations separate.

Let $f(x,y)$ be a function of x and y and $g(x+y)$ a function of x + y. Then it may be shown that:

$$\begin{aligned}\int_0^1 dx \int_0^{1-x} dy\, f(x,y)\, g(x+y) &= \int_0^1 dv \int_v^1 dz\, f(v,z-v)\, g(z) \\ &= \int_0^1 dz\, g(z) \int_0^z dv\, f(v,z-v)\end{aligned} \tag{A5.2)}$$

Example:

$$\begin{aligned}&\int_0^1 dx \int_0^{1-x} dy \frac{x^2+xy}{\left(K^2 - M^2\left(\sigma^2(1-x-y)+(x+y)^2+\left(\tau^2-1\right)(x+y)\right)\right)^3} \\ &= \int_0^1 dz \frac{1}{\left(K^2 - M^2\left(\sigma^2(1-z)+z^2+\left(\tau^2-1\right)z\right)\right)^3} \int_0^z dv\left(v^2+v(z-v)\right)\end{aligned} \tag{A5.3)}$$

The integration over the parameter v can be evaluated at once.