Higher Partial Moment Uncertainty in Portfolio Allocation and Asset Pricing

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ABSTRACT

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This study considers the influence of relaxing the widely held assumption that investors operate according to monotonically declining marginal utility as proposed by Bernoulli in 1738. The analysis is conducted by examining the assumption change has on the performance of optimal portfolio theory, the accuracy of the capital asset pricing model (CAPM), and the magnitude of the equity risk premium puzzle. The data used includes the Ibbotson monthly frequency series used by Mehra and Prescott (1984) as well as the CRSP real return series for the SP500 companies that survived on the index through the 1990's. Although the study is preliminary, it suggests that investors do indeed behave in a manner unlike Bernoulli's solution. The suggested value function of Kahneman and Tversky (1979) appears to minimize the magnitude of the premium puzzle, produces a portfolio process that offers significantly positive ex ante return for risk borne and also allows for the theoretical existence of a two-beta CAPM that better predicts asset returns. Conclusions indicate that there are both empirical and theoretical failings with the Bernoulli solution and that further refinement and study of investor utility functions may result in derived models that are superior both in estimating positive investor behavior and in prescribing normative investor behavior.

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I would like to dedicate this thesis to:

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HIGHER PARTIAL MOMENT UNCERTAINTY IN PORTFOLIO ALLOCATION AND ASSET PRICING

1. INTRODUCTION

The Bernoulli solution to the St.Petersburg paradox has, for the last century and a half, laid the foundations for economic theory in the area of investor decision-making under risk. Despite the wide acceptance of the implied expected utility theory, there have been valid objections to the use of monotonically declining marginal utility models in describing individual investor behavior. This thesis attempts to show that replacing expected utility theory with prospect theory delivers asset pricing models and portfolio allocation processes that better reflect positive reality. To do this, the mean higher partial moment, or positive semi-variance, is considered in the derivation and application of the capital asset pricing model as well as for the optimal return-to-risk portfolio allocation. This study also considers the influence that this change of assumption has on the equity risk premium puzzle, and it explores the mathematical behavior of the value function in contrast to expected utility theory.

To ascertain whether a value function is a better proxy of positive reality, this thesis examines whether or not statistically significant excess returns can be earned by active investors who follow rules of portfolio allocation that diverge from those prescribed by the traditional normative mean-variance approach. To explore this possibility an alternative framework for building the portfolio optimization process is employed. The alternative framework is derived from a Prospect Theory class of value functions. Therefore, it is assumed for the purpose of the model that uncertainty, as measured by the positive semi-deviation above the expectation, is desirable. With regard to negative semi-deviations below the expectation, the model ascribes an aversion to this portfolio characteristic. This is similar to semi-variance optimization and the more widely used minimum variance approaches. The thesis also shows that by splitting risk into two kinds of uncertainty that investors may engage in limited risk taking behavior while simultaneously being risk averse. A two-dimensional expansion of both uncertainties in the small will also provide a theoretical derivation of a two Beta CAPM, which is empirically explored by other researchers. Finally, the alternative perspective on uncertainty is used to provide a preliminary study of the risk premium puzzle by exploring the soundness of this anomaly's presence.

This thesis finds that the use of prospect theory in place of expected utility theory does have some statistical advantages in improving return prediction. Specifically, the two-beta CAPM that is derived from a value function assumption has statistically superior adjusted coefficients of determination. The ex ante optimization process yields Sharpe ratios that are statistically greater than zero, which implies a reward for bearing risk of the conventional measure. Finally, the preliminary study of the equity risk premium puzzle seems to indicate that the magnitude of the puzzle is greatly diminished by the introduction of the value function into an asset-pricing framework.

The thesis is organized as follows. The next section provides a historical discussion of portfolio optimization, which deals with the historical models and assumptions that comprise the current paradigm of portfolio allocation. The third section reviews the relevant literature by addressing applications of the conventional framework, and provides the intuitive and empirical reasoning for a relaxation of some of the early assumptions. The fourth section develops the inductive models used herein to build an alternative set of portfolio allocation rules. The fifth section describes the methodology and data. The sixth and seventh sections present and discuss empirical analyses as to whether the alternative rule set has merit. Corollary analyses also are conducted in section eight to examine the support for this framework and also to better specify the parameters for the modeling process. The final section, section nine, reviews the major findings of this thesis and their implications for the implementation of portfolio optimization theory and practice.

2. EVOLUTION OF THE UNDERLYING THEORY

2.1 The Evolution

In 1738, Nicolas Bernoulli asks his cousin, Daniel Bernoulli, for assistance to find a solution to an intractable problem called the St.Petersburg paradox (Appendix A). Daniel Bernoulli (hereafter Bernoulli) provides a solution that resembles Cramer's 1728 (Sommer 1965) solution to the same problem. The seminal insight is that wealth is not valued by the average individual in a linear sense but rather as a function that depends on the utility that wealth provides.

Bernoulli's fundamental assertion is that marginal utility is a monotonically declining function of wealth bounded on the lower side. This solution does not solve all of the St. Petersburg Paradoxes as Menger (1936) later notes. Nevertheless, this solution to the paradox lays the foundation for another famous work, the *Theory of Games and Economic Behaviour*. In this book, Von Neumann and Morgenstern create a set of axioms, logic systems and game theories to provide the skeleton for the quantification of finance. This work is so compelling and well received that virtually all of the contemporary economists accept virtually every tenet and implied result within its covers.

Despite the major contributions of these previous works and subsequent refinements, one cannot ignore some of the evident failings of the assumptions made by these authors. Though the assumption of a monotonically declining marginal utility function solves certain problems it gives rise to a host of new concerns. One of these is that uncertainty, whether of unexpected losses or gains, is always undesirable. The implication is that an individual with a utility function as described by either Bernoulli or Von Neumann and Morgenstern will never enter into an actuarially negative gamble. That is, the monotonic concave function does not allow for investment in gambles that have a non-positive expected value. This is not only counterintuitive but also if one believes simulated gambles such as those conducted by Gordon, Paradis and Rorke (1972), a dissonance exists between economic theory and applied finance.

The assumption that a monotonic concave utility function is realistic allows for the development of a modern portfolio theory. Stone (1973) shows that all risk measures, such as VaR, standard/semi-standard deviation, or mean absolute deviation or MAD are all of the same generalized loss function. Probably due to mathematically simplicity, standard deviation is used most frequently and is the starting point of Markowitz's seminal piece (1952). Markowitz demonstrates that the combined standard deviation of a portfolio is less than a direct weighted linear combination of the components if any set S of assets has return distributions not wholly correlated with one another.[1] He

[1] Markowitz's formulation is restated mathematically in section 4.2.

demonstrates that this eliminates idiosyncratic risks, and allows for the selection of the optimal return/risk portfolio.

Using a Markowitz efficient frontier, Sharpe (1964) formulates a capital asset pricing model by demonstrating that all investors agree on the optimally efficient portfolio of risky assets,[2] and that all other portfolios on the efficient set are obtained by over- or underweighting this unique portfolio as per the Tobin (1958) separation theorem. This removes the risk preferences of individual investors from the determination of the efficient frontier, and re-introduces them when the investor moves away from holding this "market" portfolio by either lending or borrowing to intersect on their maximum utility curve. Unfortunately, the LSM CAPM[3] is plagued by weak estimation power that may itself result from poorly stated risk aversion and utility assumptions.

What Markowitz and Sharpe may not capture adequately is the positive reality that individuals seemingly express a taste for risk, even actuarially negative risk. Other researchers attempt to account for the observed conditional risk proclivity by developing a theory of gambling effect. Fishburn (1980) develops the axioms for one such theory. The essential argument of this approach is a function, $\phi(p)$, of the probability distribution of gamble that maintains *Uncertainty and Certainty Equivalence*. Under this theory, individuals have two distinct types of ordering processes, one that governs risk-laden decisions and another that copes with riskless decisions. One of the value function terms dependent upon the probability nature of risk accounts for the differential between risky and riskless opportunities. The value of each type of decision is derived by a different value function but may be equated for ordinal purposes with the use of an additional

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[2] By efficient we mean highest return per unit of risk absorbed

[3] Lintner (1965), Sharpe (1964), and Mossin (1966) each develop the equilibrium capital asset pricing model.

term. Diecidue et al. (2002) restate Fishburn's theory more succinctly and express this formulation as:

$$W(x) = W(P) - C(x, P) \tag{1}$$

W is the preference function of some risk theory such as utility, x is the value of a sure outcome, P is a risky lottery, and C(x,P) is the certainty preference or the additional term used to compare risky and risk-free situations. Although this model has two notable benefits it has a major flaw that limits its use in this study. The two advantages of this model are that it allows for insurance and gambling and the possibility of a saturation point. That is, gambling may be desirable up to a limit under this framework. The Diecidue et al. model is not used here, however, because it is either unintelligible in terms of calculus or no different than the simple formulation of Bernoulli as will be briefly shown below. Thus, the model used herein concentrates on an optimal combined asset that does allow for gambling, unlike the Markowitz framework, but does not have an explicit limit to the quantity of gambling beyond wealth constraints.

To see the failing of the Diecidue et al. framework one need only consider the limit of C(x,P). If the limit as P approaches unity converges to a finite non-zero value then a constant proclivity or aversion of risk exists in all its forms. However, this becomes untenable if we push P infinitesimally close to unity. While the constant remains, the risk for all purposes conceivable to the human threshold of perception is non-existent. Thus, in this case an individual should have a very definite preference between \$1 with certainty and \$1 with near absolute certainty.

Alternatively, C(x,P) may collapse to zero or may diverge. In the case where it collapses to zero as P approaches unity, one is left with the approach of a utility of expectation and expectation of utility. Bernoulli's quarter millennium old solution reappears. If however C(x,P) instead explodes as P approaches unity, two unintuitive results are possible. If the certainty preference is negative, then the individual will progressively prefer that gamble over certainty as the probability on the gamble increases. This individual will seek gambling over certainty, but will prefer gambles involving near certain outcomes. This may explain a predilection for gaming behavior but it cannot explain a rational market as no one wishes to hold the riskless securities under any environment. Alternatively, if the certainty increases then we have an individual who is less affected by larger risks. While this is similar to Thaler's (1980) observation that individuals exhibit decreasing marginal sensitivity to downside risk, it ultimately comes to the implausible conclusion that an individual will pay any certain amount to play a highly uncertain game rather than a slightly uncertain game.

Whatever C(x,P) converges or diverges to either makes little intuitive sense or offers no additional coping mechanism for risk proclivity over the original approach of Bernoulli. For this reason, the gambling effect framework is not discussed further in this thesis.

2.1 The Case for Change: Monotonic Utility

As early as Menger (1936), it was apparent that Bernoulli's solution to the St.Petersburg paradox, and by corollary the implied concave utility function, was suspect. Menger noted that the Bernoulli solution does not apply since some combination of payoffs could be found such that their expectations were infinite, and hence expected utility also would be infinite. That is, if one adjusts the rules of the St. Petersburg game, one can find other situations that result in an infinite price expectation from playing the game. Menger suggested that the utility function has to be bounded on the upper side to deal with this problem. The difficulty with Menger's solution is that it is as non-intuitive as the problem that gave rise to the paradox in the first place. The institution of an upper bound on the utility function implies that there is some value for which an incremental dollar change bears no increase in utility. Though one might not have much use for the next marginal trillion dollars once one already owns a hundred trillion, it is difficult to conceive of an individual that does not prefer an endowment of \$101 trillion versus \$100 trillion. Though Menger's criticism of the solution of Cramer is valid, his own solution is not a substantial improvement.

Despite the problems associated with Bernoulli's solution it did allow Von Neumann and Morgenstern (1943) to produce basic game theory and, more salient to this thesis, lay out a set of axioms to quantify economic utility. Von Neumann and Morgenstern show that numerical assignment of value is not necessarily contrary to observed reality and offer the essential rules of transitivity and addition necessary for risky proposition ranking. The authors demonstrate that a broad family of ordinal utility functions can be used in riskless situation ranking without excluding the use of cardinal ranking for risk situations. The authors do not develop empirical support for this claim but Friedman and Savage (1948) offer such evidence from surveys. Though the Friedman and Savage model of utility is appealing, as it allows for a coexistent proclivity for lotteries and seeking of insurance, it fails the test of rational behavior. This is illustrated by examining the solution proposed by Friedman and Savage as depicted in Figure 1.

Markowitz (1952) shows that the inflected utility functions proposed by Friedman and Savage suffer from the malaise of implying improbable investor behavior. In the case above, if C is the midpoint wealth endowment between points A and B, then two investors would be indifferent to playing a game in which they may rise to B or fall to A. This disagrees with usual expectations as one does not see average wealth individuals indifferent to taking on large symmetric gambles. Individuals near B, the "almost wealthy", would be eager to play games with high probability of a small gain and bearing the low probability of a massive loss. Such an individual would aggressively seek the opportunity to underwrite insurance even with the expectation of a loss. Therefore, Markowitz shows that a greater than order one utility curve limited to a single axis will result in improbable investor behavior at some points along the endowment.

Allais (1953) reduces the problems Markowitz describes by suggesting probability estimates are not objectively derived in decision-making. Allais demonstrates that Bernoulli's formulation may be normatively rational but does in no way reflect the intuitive behavior of the average individual. He proposes that humans view risk subjectively, based upon their degree of optimism or pessimism. An individual optimistic by nature would be inclined to overestimate the probability of a gain and therefore overvalue the opportunity to play an actuarially fair gamble. Counterpoise, the pessimist would undervalue such an opportunity by over weighting the probability of a loss. This allows for the benefits of the Friedman and Savage utility formulation, permitting both simultaneous risk aversion and risk loving, without the problem of improbable behavior.

Prospect theory, formulated by Kahneman and Tversky (1979), is a further development in the use of subjective probabilities that were proposed by Allais. The basic tenets of Prospect theory are that individuals view gains and losses not simply as mirror equivalents of each other (e.g., a gain is not a negative loss) but as entirely different sorts of experiences. The Prospect theory value function also differs from expected utility theory in that there is a context dependent reference point. Markowitz (1952) suggested this reference point in his criticism of the Friedman and Savage inflected utility function. Kahneman and Tversky conduct numerous surveys and find a general pattern that individuals suffer greater "aggravation" from a loss than foregoing an equal value opportunity. The implied utility curve from the data provides an empirical function that appears bent about a zero reference value, as is illustrated in Figure 2.

Thaler (1985) conducts additional surveys and finds much the same results, both within the general student population and amongst MBA students which have some economic education. Grether and Plott (1979) conduct tightly controlled studies with political science and economic students in an attempt to discredit earlier works by psychologists (Slovic and Lichtenstein 1971) that are at odds with preference theory. The authors suppose that poorly controlled test environments may cause the violation of the rules of stochastic transitivity, known as the preference reversal anomaly. Instead these authors find that they can not reject the theory that individuals assess risk according to a contextual reference point as would be consistent with a value function. Thaler (1991) also provides extensive literature on the subject but as it will not add substantial value to the development of this thesis, the reader is directed to Thaler's *Quasi-Rational Economics* (1991) for a more detailed discussion of this topic.

Both Allais' insight and Prospect theory itself rely upon the assumption that objective probabilities are replaced by subjective decision weights. This allows for events of extremely unlikely nature to be more heavily weighted in the decision process of the individual when considering risk. The proposed decision weight function offered by Kahneman and Tversky is of a form as is illustrated in Figure 3.

Although there are similarities between the work of Allais and Prospect theory, one notable difference is that the latter theory allows for the reference point to be shifted depending on the individuals' expectations. This permits additional 'irrational' behaviors to be explained. Examples include some special cases of the *Endowment effect*. The effect itself states than an individual will tend to ascribe a higher value to an object/prospect already in possession versus the value ascribed to the opportunity to gain that object/prospect. The tendency is to create seemingly irrational behavior as is illustrated in Appendix B. Although Allais' formulation could cope with the endowment effect, it could not contend with such issues as lost opportunities being treated as losses. A brief example to illustrate this point is an employee who suffers a substantial loss in utility when an upcoming bonus is to be taxed at a rate higher than originally anticipated. The endowment of the bonus was not yet in hand and so a tax increase should be interpreted as a foregone gain under Allais' model. However, Prospect theory allows that the reference point may have already 'priced' the bonus into the income and any reduction in that bonus will now be interpreted as a loss to an otherwise guaranteed endowment. In the aggregate this may contribute to the fact that salaries tend to be sticky on the upside as a loss of an expected raise can lead to significant reduction in employee morale.

2.2 Derived Theories of Monotonic Utility

While economists argue about the basis of decision making under risk, a large body of financial theory has been developed that implicitly assumes the debate is resolved. Two of the most widely used theories that rest upon the concept of monotonically declining marginal utility are the Mean Variance asset allocation procedure and the Capital Asset Pricing Model or CAPM. In both instances, there have been issues not only of application and intermittent empirical support for the theories but also concerns about the implied results if the theory were to hold. This section addresses some of the matters concerning the CAPM so as to lay the foundation for further analysis of the CAPM later in the thesis.

Studies by Fama and French (1992), for example, find that the CAPM has little capacity in estimating returns. This is especially true when other factors are controlled for such as firm size and book to market value ratios. Ross and Roll (1994) argue that the inefficacy is caused by the selection of an inefficient market portfolio resulting in other variables having explanatory power. This argument has validity but still assumes that the efficient portfolio can be found using a minimum variance approach. Other researchers have offered alternative reasons why the CAPM may not be effective in predicting returns (Kothari, Shanken, Sloan, 1995). Given that so many studies rest upon the implied risk premium from the CAPM, it is important to improve upon this model. Yamaguchi (1994) offers some advice on how to estimate the equity risk premium using downside probabilities in recognition of the aforementioned need but does not offer any theoretical validation for his suggested approach.

Other empirical work, such as that by Bhardwaj and Brooks (1993), find that a Dual Beta CAPM can provide substantially improved return estimates. Pettengill, Sundaram and Mathur (1995) also find that a one parameter Beta, conditioned on up and down markets, can enhance estimation accuracy. Some studies such as Howton and Peterson (1998) re-examine old anomalies like the January and size effects with a dual-beta model and find that the CAPM under this formulation re-absorbs much of the explanatory power of the variables used in the Fama and French studies (1992, 1996). Nevertheless, there remains some dispute as to whether a conditional single factor or two factor Beta model is more appropriate (Pettengill et al. 2002). The argument against the two-beta model is not particularly strong as it cites the adequacy of the single factor model in estimation as support for its adoption. Given the ease of developing a dual beta model once the monotonically declining marginal utility assumption is discarded, as shown in section 5.3, we tend to support the Bhardwaj and Brooks approach over that of Pettengill et al. In summary, there is a need for producing more reliable return estimates, and the dual beta model has empirically been shown to produce those estimates. In turn, the model can be developed theoretically once Bernoulli's arguable economic assumptions are replaced.

In review, the CAPM is an important development in financial theory with numerous studies relying on the concepts it puts forward. However, the empirical evidence seems to suggest that there may be some weakness with the theory in its present state. What should be considered though before dismissing the theory or holding the collection of accurate data at fault is that the broad ideas of CAPM could be valid but are undermined by risk metric assumptions. This thesis now considers the efforts of some researchers to handle the larger problems of risk specification before returning to a deeper analysis of the CAPM.

The issues with the CAPM also reflect upon the Risk Premium puzzle. Mehra and Prescott (1985) find that the magnitude of relative risk aversion required to generate the observed risk premium over the past decades would need to be a value in the order of ten to thirty. The implication is that the average U.S. investor has to be so averse to a gamble of losing half their wealth they would be willing to forfeit as much as forty-nine percent of that wealth to avoid the game. [4]

Recognizing this non-intuitive implication, various researchers attempt to reduce the magnitude of the equity premium. Constantinides (1990) uses a habit formation process to reduce the size of the equity premium puzzle. By relaxing Mehra and Prescotts' (1985) assumption of time separable utility, Constantinides finds that a subsistence consumption rate of about eighty percent of the most recent consumption can explain the equity premium observed by Mehra and Prescott. Brown, Goetzmann and Ross (1995) show how survival as well as mean reversion may increase the empirically observed premium. They find that for any series with a strictly positive survival probability, the unconditional premium will be smaller than the conditional premium. The interpretation is that a survivorship bias will in fact produce an upward bias in the reported equity premium, and therefore create the puzzle. Basak and Cuoco (1998) propose a pure exchange model with investing agents being restricted by barriers such as transaction or information costs. They find that with such restrictions on participants, an RRA of 1.3 can explain the historically observed equity premium.

[4] This is based on aggregate data.

Thaler and Siegel (1997) review additional efforts by the research community to reduce or eliminate the premium puzzle. They call attention to the Siegel study (1992) that extends the period of the analysis of Mehra and Prescott to determine if the premium puzzle is of the same magnitude in earlier history. Not only does the study find that the puzzle did exist prior to 1925 but that if one compares the volatility of fixed income and equities, the former exhibits mean aversion while the latter exhibits mean reversion. The result is that risk by the conventional metric measure is actually higher on longer horizons for fixed income than it is for equity, which only exacerbates the puzzle.

The summary of Thaler and Siegel also covers the work of Reitz (1988), who suggests that survivorship bias may be at the root of the puzzle. The argument is that historically observed risk measures are not illustrative of the expected risk ex ante for each period. Reitz suggests that the closing of indices, exchanges and rampant bankruptcies would account for risk of equity investing not evident in a simple standard deviation. Hirose and Tso (1995) point out the difficulty with the argument of Reitz. Hirose and Tso examine the performance of fixed income and equity investors during periods of financial turmoil. Concentrating on Japan and Germany post World War Two, the authors find that bondholders suffered a worse fate than equity holders. They conclude that the most extreme financial holocausts actually have more impact on bondholders by totally wiping them out due to hyperinflation or wealth confiscation.

Although numerous authors find means by which to reduce the size of the premium, there is still no general consensus on which solution is most plausible. This author interprets such a theoretical vacuum to indicate the need to re-examine the assumptions that imply the existence of the puzzle. The foundation of the equity premium puzzle, the single term absolute risk aversion measure developed by Pratt (1964), is itself reliant on Bernoulli's utility curve. In relaxing Bernoulli's assumption of monotonic risk aversion, already shown to be flawed both theoretically by Menger (1936) and empirically by Kahneman and Tversky (1979), the need for habit formation, agent restrictions or conditional survival may not be necessary to explain the puzzle for the puzzle may have disappeared. This thesis provides a development of an alternative uncertainty measure in the small, which mirrors the efforts of Pratt and offers an opportunity to analyze the premium in a way implied by Yamaguchi.

3. MODIFIED THEORIES AND INTUITION

3.1 Mean Lower Partial Moment and Higher Moment CAPM

Various authors attempt to modify either existing theories of the CAPM or portfolio theory or offer alternative means of approaching the problem of risk under uncertainty in hopes of mitigating the observed empirical anomalies. In one effort to stream-line the theory of risk, Stone (1973) formulates a three parameter risk measure showing variance, semi-variance, mean absolute variance and VaR all to be special cases of a larger 'loss function'. The definition he provides is given as:

$$L(W_0,k,A) = \int_{-\infty}^{A} |W - W_0|^k \, dF(W), \qquad k \geq 0 \tag{2}$$

W is future wealth, a stochastic variable, F(W) is the cumulative probability distribution for W, and W_0 is the reference level of wealth from which deviations are measured. k is the degree of impact small or large deviations from the reference level have upon the risk metric value. The interested reader is referred to the Stone article for the Lemmas equating the three-parameter loss function to the aforementioned risk metrics. Nevertheless, the equation in its analogous form to the metric suggested here is given by:

$$L(W_0,k,g,A,\alpha)=\int_{-\infty}^{A}|W-W_0|^k dF(W)-\alpha\int_{A}^{\infty}|W-W_0|^g dF(W),\ k,g\geq 0 \qquad (3)$$

The difference of this formulation from that given in Stone is that for almost all purposes A is equal to the reference wealth level, acting to discern losses from gains, and that the second term of the equation allows for the positive pricing of 'upside uncertainty'. In (3), α is a ratio that describes the degree to which upside and downside uncertainties are proportionally valued. An α greater than one indicates a relatively higher individual investor sensitivity to upside uncertainty than downside. The powers k and g determine the curvature of the expanded loss function with regard to downside and upside uncertainty, respectively. For k or g greater than one the individual investor treats the particular uncertainty in a convex fashion. If k, g and α are all equal to one, then the investor is effectively risk neutral.

Like the three parameter function, it can be shown that this function is equal to any of the risk metrics listed earlier. Furthermore, it has greater flexibility as it allows for the inclusion of a new risk metric that accounts for the higher partial moment as being desirable. This agrees with Prospect Theory, and so can sidestep the matter of a monotonically concave utility function.

In terms of portfolio theory development, Markowitz (1952) suggests optimizing according to semi-variance rather than variance but does not do so because of the then prevailing mathematical and computational difficulties of doing so. Hogan and Warren (1974) recognize this shortcoming and move towards developing a semi-variance CAPM.

The rationalization is that it allows investors to 'exhibit conservatism toward losses' while remaining 'aggressive toward returns'. The optimal portfolio is found by the operation:

$$\min\left(\frac{SStd_{portfolio}}{E(R_{portfolio}) - Benchmark}\right) \tag{4}$$

$E(R_t)$ is the expected return of the portfolio, t is the period, and *Benchmark* is a minimum expected return for the portfolio. So to remain consistent with the Sharpe ratio used in the minimum variance approach, the *Benchmark* is taken herein as the risk free rate. The semi-standard deviation (SStd) is given by:

$$SStd_{portfolio} = \sqrt{\sum_{t=0}^{N} \min(0, R_t - E(R_t))} \tag{5}$$

Although this derivation is similar to the model provided in this thesis, two differences do exist. First, since this model relies on a single co-semi-variance, it has but one risk measure,[5] whereas the model developed later in the thesis incorporates a second uncertainty term. Secondly, this formulation assumes that the reference point or benchmark is the expected return from historical observations. The model described later is more general in that it may be set to this expectation. To match the behavior of Kahneman and Tversky's value function (1979), the reference point is set to zero to segregate losses from gains.

Numerous other studies also examine the suitability and reliability of Semi-Variance as the measure of risk. Mao (1970) compares the use of semi-variance to variance in

[5] The terms downside risk, downside semi-variance and LPM or lower partial moment are used synonymously from this point on.

capital budgeting decisions. The emphasis of this study is on normative project selection rules. However, Mao also queries the executives of three forestry companies as to their investment preferences to lend support for the use of semi-variance. He finds a near identical image of the Prospect Theory value function in all three cases. If one puts confidence in this study, then it seems that individuals consider risk, whether in the form of portfolio or project decisions, with the use of a value function similar to that described by Kahneman and Tversky.

Porter (1974) also gives theoretical justification to the use of semi-variance over variance for the purpose of asset pricing. Although Porter does not conduct an empirical study,[6] he shows that semi-variance decision criterion may be much more general in form and still yield stochastically dominant decisions. Specifically, he shows that for all distributions excluding those with identical mean and lower partial moments, the LPM method of decision-making yields the stochastically dominant result. Moreover, the theory suggests a natural preference for skewness, as is later supported by Harlow and Rao (1989). Efficient Variance measures by contrast can produce confused and conflicting results that are at odds with the more theoretically pleasing rules of stochastic dominance.

The semi-variance CAPM has not yet demonstrated itself as being definitely superior in Asset Pricing compared to the traditional CAPM. Jahankhani (1976) tests the efficacy of the two models with the use of portfolios formed from the CRSP files. The testing procedure is similar to that of Fama and MacBeth (1973) and uses portfolios to eliminate

[6] It should be noted that Ruszczynski and Vanderbei (2002) also demonstrate the same relation between semi-variance and stochastic dominance, and also offer a self-dual simplex method of determining an efficient frontier of non-stochastically dominated portfolios at a much improved computation time over conventional approaches.

biased errors proclaimed to be present in individual securities. Jahankhani finds that for both models there is a linear relation between the estimated betas and return, and that beta is the only contributor to return. However, both models fai! to find that the slope is equal to the expected risk premium or that the intercept passes through the risk free rate. Given the mystery of the Premium puzzle this is not a surprising discovery but does underscore the inability of EV or ES CAPM to robustly predict asset prices.

Nantell and Price (1979) and Natell, Price and Price (1982) find that the ES CAPM does have some power. The former authors develop a simple test of the work of Hogan and Warren and find that under the assumption of bivariate normal returns, equilibrium rates of return are unaffected by the selection of semi-variance or variance as the risk measure. In the latter study, the authors examine the consideration of non-normal returns and find that the ES CAPM is statistically distinguishable from the EV CAPM, and superior in predicting returns, so long as the market and security distributions are not strongly negatively skewed. However, there is a problem with this discovery. The degree of risk absorbed or beta is not always significant in determining returns for either model that weakens the argument that ES CAPM is efficient. Despite this, the ES CAPM does imply a *lower* risk free rate and *higher* risk premium than the traditional EV CAPM. The implication is that the ES CAPM would better fit the Premium Puzzle observations than the traditional CAPM and may be worthy of consideration when studying that anomaly.

Later studies by Harlow and Rao (1989) investigate more generally the Mean Lower Partial Moment CAPM (MLPM) and find that the traditionally derived CAPM is rejected as a well-specified model of asset returns. Interestingly, this study finds that the target rate of return that allows the LPM to best fit the data is not the risk free rate as would be analogous to EV CAPM but rather the implicit expected return derived from historical observations. This concurs with expectations if one develops a modified CAPM from the starting position of Prospect Theory. Individuals utilize a self-determined *reference point* for distinguishing gains and losses. While Rao and Harlow only consider losses with the use of the LPM, it is plausible that investors use the recent average historical return of risky securities as a benchmark for distinguishing a gain from a loss.

Rather than trying to find a family of interrelated risk functions or modifying the CAPM to better reflect positive reality, other researchers advocate the use of higher moments in risk analysis. Alderfer and Bierman (1970) suggest using the third moment and skewness (derived from the third moment) in addition to variance. They point out that Pruitt and Coombs (1967) find individual investor preference for skewness when variance is held constant. The authors conduct surveys of three groups; namely, doctoral students and professors, a group of cost-accounting students and the third group of corporate executives. In all three groups they find a willingness to suffer a lower expected mean and higher variance in exchange for a higher skewness. This response was somewhat muted in the group of executives, where they found that a small section of executives (14%) were willing to take gambles with a high chance of a small gain and a low chance of a large loss. Although the group was not representative of the norm, their behavior is in agreement with the proposed Friedman and Savage utility function (1948) that Markowitz criticized. This thesis is in agreement with Markowitz on this matter but does recognize that perhaps a small percentage of the population has a value function not well characterized by the norm. Regardless, Alderfer and Bierman report empirical evidence that variance and mean are not sufficient alone to make an accurate model of investor risk decisions.

Further study by Golec and Tamarkin (1998) seem to confer support to the conclusion of Alderfer and Bierman. Golec and Tamarkin examine the nature of risk taking at horse tracks where previous researchers have concluded that bettors are risk loving since their observed behavior includes a willingness to absorb greater variance with less expected returns. The authors note however that this does not necessarily imply that the bettors are in fact risk loving if one considers the risk taking choice with a higher moment model. The most obvious difficulty with assuming risk loving is that a risk lover would not bet on more than one horse. Yet, various forms of diversification such as "boxing" or "wheeling" are prevalent. The authors suggest that such bettors may in fact be risk averse in terms of variance but favor positive skewness.

In addition to Golec and Tamarkin (1998), Kane (1982) finds individual investor preference for the third moment of asset returns. There is still difficulty in utilizing these findings effectively in application. Lee, Moy and Lee (1996) test a three moment CAPM model proposed by Kraus and Litzenberger (1976). The model is expressed as:

$$E(R_i) - R_f = \left[\left(\frac{dE(w)}{d\sigma_w}\right)\sigma_m\right]\beta_i + \left[\left(\frac{dE(w)}{dm_w}\right)m_m\right]\gamma_i \qquad (6)$$

$E(R_i)$ is the expected return on asset i, R_f is the risk free rate, w denotes wealth and σ_w, m_w, σ_m, m_m are the second and third moments about end of period wealth and the second and third moments about mean market return, respectively. β_i and γ_i are the systematic and co-skewness risk pricing factors.

Testing a period from 1946 to 1985 from the CRSP files, Lee, Moy and Lee (1996) do find that the model statistically and significantly prices co-skewness and covariance risk. However, they also reveal that it does not describe expected returns well. This does not support the supposition of Klaus and Litzenberger that the risk of a security is simply the weighted average of co-skewness and covariance. Lee et al. conclude "the three-moment CAPM does not describe the pricing behavior of risky assets".

While the evidence suggests that skewness does play some role in pricing of assets, the theoretical formulation fails to properly describe the relation of skewness in that pricing process. We argue that skewness as a measure is in most instances positively correlated with the positive risk metric of the Prospect Theory curve. Perhaps the survey results confirming a preference for skewness under certain circumstances are in fact confirming a decision process similar to Prospect Theory. The fact that weighted co-skewness does not well describe the risk of a security may be because skewness is simply a proxy for another underlying measure of risk not yet fully articulated.

In the next section of this thesis, we provide an intuitive argument for change before giving a formulation of a modified method for risk measurement.

3.2 Consideration of Multifactor Model of Uncertainty

One might question why such metrics should be used rather than applying higher moments. When using a finite term Taylor approximation, higher moment models often result in ambiguous decisions under uncertainty. This is best illustrated by an old fable of the mathematician who believed an average by itself was an adequate description of a process and drowned in a stream with an average depth of two inches (Elton and Gruber, 1987, p.16). This story could be taken one step further, as illustrated in table 1.

If one takes our non-swimming mathematician who stands 6'1" tall, and asks which pool of uncertainty he would prefer to be thrown into even models including up to the third moment would state pool two because it has less average depth, less variance and identical skew. This example illustrates the failure of a higher moment model to provide a sensible decision. The example is also valuable in providing a concrete analog as to why individual investors should eventually become insensitive to marginal increases in downside uncertainty. The additional depth of the pool yields no negative impact to the swimmer. However, unlike 'going for broke', there is no externality on a third-party. The individual swimmer is simply attempting to select the highest probability of survival, and not attempting to maximize an expected outcome.

However, the purpose of this study is not to detail the failings of higher moment models. Instead, this initial purview was conducted only to demonstrate why higher moment models are not considered further in this thesis. The main objective of this work is to examine whether the assumption that individuals behave according to Prospect theory, in place of expected utility theory, improves the efficacy of derived asset pricing models in describing positive reality. It is then prudent that the two functions of individual preferences under uncertainty be contrasted.

As both expected utility theory and the value function of prospect theory have infinite sized families of parameter choices, this study just analyzes some of the most commonly used values and then generalizes to the broader set of functions. In particular, the square root power function is used when analyzing utility theory, while the parameters for the value function are the square root for losses and 0.4 for gains. These values are selected to fall between zero and one so as to create the convexity and concavity of losses and gains as per Thaler's (1980) value function. The power modifying the gains term is taken to be less than that for the losses term so that the individual, though possessing a love of upside uncertainty,[7] has a behavior that is slightly more dominated by the fear of losses. For purposes of contrasting these two functions, these parameter selections suggest that individuals are either risk averse (expected utility theory) or tend to the side of caution (prospect theory), though one could just as easily model risk loving or higher uncertainty proclivity.

For simplicity, the comparison assumes a binomial gamble with a high and a low state. The probability of the high and low states varies to illustrate the predilection for upside uncertainty when individuals behave according to the value function. The context dependent reference point of the value function is the probability weighted expectation. Any outcome that exceeds the expectation is assessed as a gain while any point below the expectation is assessed as a loss. In contrast, expected utility theory has a preset endowment, and perceives gambles as adjustments to the endowment.

The first example to contrast the difference in behavior between the two theories creates a scenario of a home worth $100,000 that faces the potential of flood damages that would reduce its value by $10,000. There are only two outcomes so that the probability of an outcome categorized as a loss is mutually exclusive to a gain, and combined they are exhaustive of all outcomes. The comparison is captured by the insurance premium that the hypothetical individual investor would be willing to pay so to avoid the gamble. If premium values are greater than zero this indicates a purchase of

[7] Upside uncertainty will also be referred to as MHPM (the mean higher partial moment).

insurance, whereas a negative value indicates that the individual would pay to take on a game of chance. This is obviously at odds with the classic tenets of rationality that state that an individual prefers a risk free gain over a risky one. Where the selected value function diverges is that it implies that individuals have some love of gambling itself. Under certain conditions, penchant for games of chance will be of sufficient magnitude to prompt investment in actuarially negative assets.

Such a form of individual decision-making under risk should not be dismissed outright since the function's implied behavior may be more consistent with reality. Games such as state lotteries, casino slot machines and even board games or games of sport all indicate that pleasure is derived by being put into specific uncertain situations. Under expected utility theory, one needs to argue that lottery tickets are somehow a consumption good to be enjoyed. This is consistent with the formulation of the certainty preference proposed by Diecidue et al. (2002) but, as was shown earlier, this model design suffers from numerous theoretical shortcomings. Thus, if individuals behave according to expected utility theory, why have bank machines not replaced lottery ticket dispensers. In the former case one can get a slip of paper with numbers on it, and by depositing a few dollars can enjoy a risk free gain equivalent to their savings account rate. In contrast, the lottery ticket is a slip of paper with numbers on it that assures no such gain, and moreover is extremely actuarially negative. The same argument can be raised of board games dependent on the roll of dice. It must be noted that such games do offer a chance to socialize so there may be utility in human interaction. If such were the case then why would such games exist on a computer? Playing such games on a computer offers no chance to socialize. If, alternatively, the utility of the game comes from winning the individual could presumably just cheat so to win constantly. Of course, the anecdotal retort to this later action is 'there is no sport in it'. Whether or not it is advisable, or rational in the classical sense, individuals do exhibit risk proclivity.

Figure 4 depicts the risk premium for the two specified functions as a probability of a gain. As can be seen from the curve of the function for the expected utility theory, an individual is willing to pay the highest premium when the probability of the high state is equal to the probability of a low state. The variance of the payoff is highest at this point. Under the formulation of the value function, the individual is initially unwilling to pay any insurance premium when the probability of a loss is near certain. As the likelihood of a loss decreases, the individual is initially willing to pay a larger premium to avoid the gamble and to accept a certain sized loss equivalent to the premium. However, at some point, the premium declines and eventually the individual is indifferent to insuring against the loss or not. At that point, they are not willing to pay a premium for insurance and are effectively risk neutral. Near a high chance of a gain, the individual now exhibits active upside uncertainty proclivity and actually enjoys the gamble although it is actuarially negative. In other words, such individuals are willing to pay to maintain the gamble and need to be compensated to take insurance.

It may be of interest to examine how an individual that behaves according to the value function may price a gamble that only has a positive or negative payoff. The St.Petersburg paradox is well suited to answer this question. Figure 5 depicts the prices that each of two types of investors would be willing to pay for the game. As was shown by Bernoulli, the expected utility function with the parameter of a square root reaches an asymptotic value of $1 when the flips of the coin approach infinity. Similarly, the value function also achieves an asymptote but at a higher price of $3.76. Both prices are of course dependent upon the parameters selected. An increase in the expected utility power term increases the finite price to a limit where if the power is equal or greater to one the price paid for the game explodes (the individual is risk neutral or risk loving). Similarly, if the power of the gains term in the value function increases to one or beyond, then the price paid also explodes. If the power on the loss term in the value function increases, the prices decline. If the power on the loss term in the value function decreases, the price increases to a limit determined by the power on the gains term. In either model, the St.Petersburg paradox can be resolved to a desired finite price simply by selecting the appropriate parameter set.

In exploring the two types of functions, the nature of insurance behavior and the price offered to play a purely positive outcome game have been considered. Since this thesis is interested in whether or not prospect theory can derive asset pricing models and allocation processes that better reflect positive reality, we have greater interest in how the use of the value function might affect the risky investment choices of individuals. Two assets are considered for this analysis. One is free of risk and pays an assured amount. The other asset pays the same amount as the risk free asset in the high state and pays nothing in the low state. If investors behave according to expected utility theory, then they should not be willing to invest any proportion of their portfolio into the risky asset except for points of assured gain. Doing otherwise would only serve to reduce expected return, and will always increase the overall risk of the portfolio. At the point that the risky asset is assured the high state, the risky asset is effectively identical to the risk free asset. At that point, the investor acting under utility theory is identically indifferent between assets one and two. In contrast, the individual behaving according to the value function prescribed above will invest some proportion of the portfolio at risk as the probability of the gain increases. While this reduces expected returns, it increases the value perceived by the investor. Figure 6 illustrates the optimal proportion put at risk as the probability of a high state varies. The amount at risk initially increases, is then subject to the constraint of no short sales, and then declines as the chance of the high state nears certainty. The decline in the amount put at risk no longer offers as much upside risk. Instead, the amount at risk approaches the characteristics of a risk free asset.

4. MODEL DEVELOPMENT

4.1 Continuous MacLaurin Approximation

Pratt's (1964) expansion of the risk premium in the small makes uses of an N-term Taylor Polynomial expansion. Using the identity that the expectation of the utility should equate, after some modifier, to the utility of the expectation, Pratt identifies the modifier known as the Risk Premium. This approach however implicitly assumes that the utility function is continuous about the point of initial wealth to a range of wealth plus, and minus, the Risk Premium π. This is expressed as:

$$E(U(w)) = U(E(w) - \pi) \tag{7}$$

For all concave functions π will be greater than zero. However, this approach is very sensitive to the assumption that utility functions are monotonically increasing in wealth. If this assumption is violated, the π can vary enormously simply by subtle adjustments in the distribution of w. For example, if one considers the suggested Value function by Thaler (1980), it is readily apparent that if the function is not symmetrical about axes x and y sequentially, that the Premia will vary considerably by tiny shifts in the nature of the gamble, as depicted in Figure 7.

Moreover, if the actual utility curve is discontinuous about any point on the range [$w-\pi$, $w+\pi$], then the Taylor Series expansion is not mathematically valid. There may very well exist a discontinuity precisely at x, the initial wealth. This approach suggests that individuals consider changes of utility when ranking outcomes rather than alternative states of utility.[8]

Even in the event that there are no discontinuities in the range, the accuracy of a Taylor approximation suffers when the number of terms used is small. Pratt (1964) assumes that three terms are sufficient to describe individual risk preferences. However, this runs contrary to recent findings by Golec and Tamarkin (1998) that higher moments such as distribution skewness do indeed matter to individual investors' preference choices. Even if a greater number of moments are taken for the approximation, difficulties may remain if the function is highly irregular about *w*.

The alternative framework conforms to Thaler's observations (1985) that gains and losses are viewed independently. Furthermore, the framework considers relative rather than absolute positioning. Instead of w being initial wealth, initial wealth change is defined at the point zero, consistent with the value function. Alternatively, the initial point might be expectations of a minimum return, such as a risk free rate, with returns above which are considered gains, while those below are considered losses. Therefore, the value function V(w) can be described as a composite of two lesser functions, the gain function, G(x) and the loss function, L(y). In these functions, x is defined as w>cutoff,

[8] Same contention as in Allais (1953).

normally zero although a risk free rate might be used, and y will be defined as w<cutoff. The cutoff is the reference point from which the individual ranks outcomes. The terms α_0 and α_1 are linear multipliers. If the power terms of G(x) and L(y) are equal to one, then the individual is effectively risk neutral. The alpha terms in such a case would be the rate of value acquired (lost) per marginal unit of gain (loss). If $\alpha_0 = \alpha_1$, then the individual would behave in this case identically to the risk neutral investor under expected utility theory. This value function is given as:

$$V(x,y) = \alpha_0 G(x) + \alpha_1 L(y) \tag{8}$$

The point at which the individual is now indifferent between the gamble and the certainty equivalent is no longer a function of π, but rather relies on two distinct 'premia'. Formally, the equations to be solved are now given by:

$$E(V(x,y)) = V(E(x,y)) \tag{9}$$

Using a Taylor approximation about zero in two dimensions, x (gains) and y (losses), the approximate polynomial P_n is given by:[9]

$$P_n(x,y) = \sum_{m=0}^{n}\sum_{j=0}^{m} \frac{1}{j!(m-j)!} D_1^j D_2^{m-j} V(a,b)(x-a)^j (y-b)^{m-j} \tag{10}$$

In (10), a and b are the initial points of y and x. However, as this is a value function, the initial values of gains and losses are by definition zero, or a=b=0.

D_1^j denotes the derivative of V(x,y) with respect to the first variable, gains, while D_2^{m-j} denotes the m^{th} -j^{th} derivative V(x,y) with respect to losses, the second variable.

[9] Note that the alpha terms are dropped from the derivation to avoid unnecessary clutter. As linear multipliers, they have no impact on the derivatives and so can be added as necessary once the derivation is complete. However, it is assumed that losses are unpleasant and so the multiplier in front of L(y) would in effect be -1. So long as the order of L(.) and G(.) are between 0 and 1, and the loss multiplier is -1, the MacLaurin expansion produces a curve that is convex for losses and concave for gains as per the value function.

However, as the variables x (gains) and y (losses) are wholly independent, the only time at which a term is non-zero is when j=0 for y not equal to zero, or when m-j=0 for x not equal to zero. As a result, the polynomial is simply the composite of two MacLaurin series of x and y. This allows for the simplification of the equality to the series of equations:

$$E(G(x)) + E(L(y)) = G(E(x) - \gamma) + L(E(y) - \varepsilon) \tag{11}$$

$$E(G(x)) = G(E(x) - \gamma) \tag{12}$$

$$E(L(y)) = L(E(y) - \varepsilon) \tag{13}$$

Now, introduce a random variable Z, which represents the gamble an individual may opt to take. The heuristic for the decision process is to take the gamble if:

$$E(G(x)) + E(L(y)) > G(E(x)) + L(E(y)) \tag{14}$$

Any stochastic variable may be decomposed into two other independent stochastic variables. Thus, Z is split into ρ and λ, where ρ represents the negative outcomes (i.e., those less than zero or the selected cutoff), and λ represents the positive outcomes or gains. That is,

$$\tilde{Z} = \tilde{\rho} + \tilde{\lambda} \tag{15}$$

Since the expected values of x and y are zero in the absence of a gamble, the expectation of the change is identical to the expectation of the gains or losses on the gamble, respectively. Equations 16 & 17 are now equal to:

$$E(G(\tilde{\lambda})) = G(E(\tilde{\lambda}) - \gamma) \tag{16}$$

$$E(L(\tilde{\rho})) = L(E(\tilde{\rho}) - \varepsilon) \tag{17}$$

Taking the separate MacLaurin approximations of equation (16) the L.H.S. is:

$$E(G(\tilde{\lambda})) = E\{G(0) + \tilde{\lambda}G'(0) + \frac{\tilde{\lambda}^2}{2!}G''(0) + ...\} \approx E\{G(0) + \tilde{\lambda}G'(0) + \frac{\tilde{\lambda}^2}{2!}G''(0)\} \tag{18}$$

$$\Rightarrow E\{G(0)\} + E(\tilde{\lambda})G'(0) + \frac{E(\tilde{\lambda}^2)}{2!}G''(0) \tag{19}$$

As E{G(0)}=0, then the L.H.S. reduces to:

$$E(\tilde{\lambda})G'(0) + \tfrac{1}{2}E(\tilde{\lambda}^2)G''(0) \tag{20}$$

Expanding the R.H.S. produces:

$$G(E(\tilde{\lambda}) - \gamma) = G(0) + G'(0)(E(\tilde{\lambda}) - \gamma) + ... \approx G(0) + G'(0)E(\tilde{\lambda}) - G'(0)\gamma \tag{21}$$

However, since G(0)=0, the R.H.S. reduces to:

$$G'(0)E(\tilde{\lambda}) - G'(0)\gamma \tag{22}$$

Equating the L.H.S. to the R.H.S. gives:

$$E(\tilde{\lambda})G'(0) + \tfrac{1}{2}E(\tilde{\lambda}^2)G''(0) = G'(0)E(\tilde{\lambda}) - G'(0)\gamma \tag{23}$$

Canceling terms and isolating γ yields:

$$\gamma = -\tfrac{1}{2}E(\tilde{\lambda}^2)\frac{G''(0)}{G'(0)} \tag{24}$$

Performing the expansion and isolation for ε gives:

$$\varepsilon = -\tfrac{1}{2}E(\tilde{\rho}^2)\frac{L''(0)}{L'(0)} \tag{25}$$

Rather than use the formalized terminology of Risk Aversion, the terminology Uncertainty Aversion is used to describe G''(0)/G'(0) and L''(0)/L'(0) as the gains are more readily described as uncertainty rather than as risk.

The author expects that the increase in value per unit of expected gains declines less quickly than the decreases in value per unit of expected losses. Stated differently, the next marginal unit of gain adds more value than the symmetric next marginal unit of losses subtracts from value. In the terminology of Pratt (1964) one might say that the absolute upside-uncertainty proclivity of an individual is anticipated to be less than the absolute downside-uncertainty aversion. Formally this may be written as:

$$G'(0) < L'(0) \tag{26}$$

$$\left|\frac{G''(0)}{G'(0)}\right| < \left|\frac{L''(0)}{L'(0)}\right| \tag{27}$$

When it is empirically fitted to observed data, the likely function V(x,y) is selected to be of the form $V(x,y)=\alpha_0(G(x)+\alpha_1 L(y)$. To fit Thalers' (1980) observations, the curve is of the log form such that:

$$V(x, y) = \alpha_0 \log(x) + \alpha_1 \log(|y|) \tag{28}$$

where $\alpha_0<\alpha_1$.

4.2 Discrete Model Formulation

Application of this framework to portfolio theory requires that the model is designed to process discrete data. To do such, we begin with the expected dispersion of a sum of stochastic variables. For the sum of stochastic variables,

$$s = x_1 + x_2 + ... + x_n \tag{29}$$

and the expectation of the sum is given by:

$$E(s) = E(x_1) + E(x_2) + ... + E(x_n) \tag{30}$$

or identically as:

$$E(s) = a_1 + a_2 + ... + a_n \tag{31}$$

where a_i is the expectation of x_i. The expectation dispersion or variance of the sum may be expressed as the expectation of

$$[x_1 + x_2 + ... + x_n - (a_1 + a_2 + ... + a_n)]^2 \tag{32}$$

which, in turn, simplifies to:

$$\sum_{i=1}^{n}(x_i - a_i)^2 + 2\sum_{i,j}(x_i - a_i)(x_j - a_j) \tag{33}$$

This instantly recognized equation, lacking only weights, is used for variance minimization in the Markowitz portfolio theory.

However, this may also be expressed as the product of the matrices. The variance of the portfolio, assuming equal weights for the moment, can be expressed as AA^T for A^T the transpose matrix of A, where A is given as:

$$A = \begin{bmatrix} x_{11} - a_1 & x_{12} - a_2 & \dots & x_{1n} - a_n \\ x_{21} - a_1 & & & \\ \dots & & & \\ x_{n1} - a_1 & & & \end{bmatrix} \tag{34}$$

Where a_i is the mean of the n^{th} column returns. This value may be changed later in the bivariate risk model to either zero, or some other selected cutoff, such as a risk free rate.

A+B is equivalent to adding the corresponding entries of matrix A to matrix B provided A and B are of equal dimensions and (B+C)A=AB+AC. Matrix A may be decomposed into two 'halves', a positive and a negative half, such that:

$$A_+ + A_- = A$$

where $A_+ = \begin{bmatrix} \max(x_{11} - a_1, 0) & \max(x_{12} - a_2, 0) & \dots & \max(x_{1b} - a_n, 0) \\ \max(x_{21} - a_1, 0) & & & \\ \dots & & & \\ \max(x_{n1} - a_1, 0) & & & \end{bmatrix}$ and (35)

$$\text{where } A_- = \begin{bmatrix} \min(x_{11}-a_1,0) & \min(x_{12}-a_2,0) & \ldots & \min(x_{1b}-a_n,0) \\ \min(x_{21}-a_1,0) & & & \\ \ldots & & & \\ \min(x_{n1}-a_1,0) & & & \end{bmatrix} \quad (36)$$

Knowing that, by the *Left* and *Right distributive laws*, the following hold:

A(B+C)=AB+AC

(B+C)A=BA+CA

for matrices A, B, C and D, the product of matrix A and its transpose may be expressed as:

$$AA^T = (A_+ + A_-)(A_+^T + A_-^T) = A_+A_+^T + A_+A_-^T + A_-A_+^T + A_+A_-^T \quad (37)$$

Under a univariate risk framework, the minimum variance portfolio is identified by adjusting the proportional weights invested, $w_1, w_2, \ldots w_n$ of the n term weight vector so as to minimize the value $(AW)^TAW$. However, this implicitly assumes that all forms of risk are disliked to an equal degree as they are additive along the same axis (standard deviation).

Instead, the bivariate risk approach formulates the problem as:

$$\{(A_+ + A_-)W\}^T(A_+ + A_-)W = (A_+W)^T(A_+ + A_-)W + (A_-W)^T(A_+ + A_-)W \quad (38)$$

$$= (A_+W)^TA_+W + (A_-W)^TA_+W + (A_+W)^TA_-W + (A_-W)^TA_-W \quad (39)$$

However, rather than simply minimizing this sum, the new approach advocates that $\alpha_0[A_+W)^TA_+W]+\alpha_1[(A_-W)^TA_-W]$ be minimized subject to the values of α_0 and α_1. The value of each of the first and last terms of equation 39 are in fact the weighted square deviations from a selected cutoff (currently mean, but can be some other value), which are the expected square of the lower and higher stochastic variables, e.g. $E(\rho^2)$ and $E(\lambda^2)$.

These are the identical risk metrics as given by the polynomial expansion approach in equations 24 and 25.

5. DATA SETS

The data used in the empirical analysis are drawn from three sources, the CRSP files, Reuters Kobra and the Board of Governors Federal Reserve Website.[10] The data cover the period from January 1, 1990 to December 31, 2001 on a monthly frequency for a total of 144 months. The CRSP total returns sequence is used for each individual security, and the SP500 total return sequence from CRSP is used as the market proxy. The three-month U.S. treasury-bill data are taken from the Federal Reserve site, and are used as the risk free rate. The identification of the constituent firms of the SP100 and SP500 Indices utilitizes Reuters Kobra. Securities without complete information for the studied period are dropped, resulting in 85 of the SP100 and 377 of the SP500 firms being used in subsequent analysis.

6. OPTIMIZATION

The minimum variance portfolio allocation strategy is compared to the ES, maximum return, static equal-weighted and alternative portfolio strategies. For purposes of analysis, the minimum variance and ES strategies are in fact identifying the optimal Sharpe ratio portfolios for the respective risk measures (standard deviation and lower semi-standard deviation). The Equal-weighted (EQ) approach is used as a benchmark for the other strategies, as they should statistically exceed the risk adjusted returns of the EQ model given that they would require dynamic allocation. Such a strategy would result in

[10] URL address is www.federalreserve.gov.

both transaction costs and management expenses. The EQ model assigns a weight of approximately 1.176% to each of the 85 securities used in the analysis. These 85 securities are members of the SP100 index as of February 1, 2003 with complete data from January 1, 1990 to December 31, 2001 in the CRSP files. The SP100 is analyzed rather than the SP500 given the enormity of the computations for the latter. Given the square geometric increase in complexity of the optimization by adding one variable to the covariance/variance matrix, the use of the 377 qualifying securities of the SP500 would take about twenty times the computations to converge for the period analyzed. The current analysis with 85 firms requires just short of thirty computer hours on a 1.6 Ghz P4 processors to compute. For the broader index, such a comparison would require nearly a month to converge all sets with current desktop technology. The processing time for such an allocation could be substantially reduced to the order of a 12^{th} , if a single strategy was compared. Thus, it is feasible for reasonably large real applications.

Mathematically each of the dynamic strategies may be expressed, in any period t, as:

$$\text{Minimum Variance } Max\left(\frac{\sum_{i=1}^{85} w_i(R_i - R_f)}{\sum_{i=1}^{85} w_i^2\sigma_i^2 + \sum_{\substack{i=1 \\ i\neq j}}^{85}\sum_{j=1}^{85} w_i w_j Cov(r_i, r_j)}\right) \tag{40}$$

$$\text{ES, Minimum lower semi-variance } Max\left(\frac{\sum_{i=1}^{85} w_i(R_i - R_f)}{\sum_{i=1}^{85} w_i^2\rho_{i-}^2 + \sum_{\substack{i=1 \\ i\neq j}}^{85}\sum_{j=1}^{85} w_i w_j Cov_-(r_i, r_j)}\right) \tag{41}$$

Where ρ_{i-} is the negative semi-deviation of security *i* and $Cov_-(r_i,r_j)$ is the negative co-semivariance of securities i and j.

$$\text{Alternative strategy, } Max\left(\frac{\sum_{i=1}^{85} w_i(R_i)}{\sum_{i=1}^{85} w_i^2 \rho_{i-}^{2(\alpha)} - \beta \sum_{i=1}^{85} w_i^2 \lambda_{i+}^{2(\gamma)}}\right) \quad (42)$$

Where λ_{i+} is the positive semi-deviation, ρ_{i-} is the negative semi-deviation, α and γ are power multipliers for the degree of investor sensitivity to the lower and upper risk, and β is the linear weight preference of up risk to down risk. Higher and lower partial risk moment covariances are unconstrained in the expression as they reduce both enjoyed risk and disliked risk. When β is reduced to zero, this method produces results that are very similar to ES theory. A notable difference is that a zero point rather than the period t risk free rate is used as a benchmark. As explained earlier, this better reflects the demarcation of gains and losses about a zero endowment change implicit in the Prospect theory.

Given the use of extreme negative and positive outlying values in the alternative strategy, a geometric information decay rate is introduced in the data so as to put greater emphasis on later period data for the optimization estimation. Failure to do so would expose the alternative strategy to a much higher sensitivity to extreme values than the other strategies, and would likely produce results not illustrative of the method. The geometric weighting process used was $w_t = (0.96987)^T$, where T is the age of the observation in months. This reduces the oldest or 60 month back observation to 15% of the most recent observation.

In assessing the alternative strategy, ten different sets of possible parameter combinations are considered due to the time required to compute each strategy. An interesting issue for future research is whether or not better parameter estimates could improve the results reported herein. The parameter combinations that are analyzed are reported in table 2.

Each of the strategies are considered from both an ex ante and ex post perspective. Ex ante results are generated by using ex post, optimized weight vectors that are available in month t to generate returns that are realized in period $t+1$.

Of the 144 months of data available for this purpose, 60 months of data are used to generate the covariance/variance matrices. This allows for 84 periods of returns for each strategy. The program executes the analysis in a scrolling loop, so that the co-variances update by incorporating either time t data or time $t\text{-}1$ data to generate the optimal weight vectors for ex post and ex ante results, respectively. The optimal weight vectors all share identical constraints. This ensures complete investing and prevents short-sales, that is:

$$\sum_{i=1}^{85} w_i = 1$$

$$w_i \geq 0 \qquad (43)\ \&\ (44)$$

For computational tractability, it is necessary to split the optimization effort over multiple (5) computers. Each machine independently computes so that the weight vectors are not carried from one machine to the next. As a result, there are five fewer period observations for each strategy in the ex ante analysis than in the ex post analysis. This does not adversely affect the power of the difference of means tests.

Once the results are generated they are analyzed using a Tukey-Kramer difference of means within SAS. The ex post Model has 1176 observations and 14 levels, while the ex ante Model has 1106 observations with the same number of levels. More specifically, eight Tukey-Kramer difference of means tests are conducted; namely, two for each model on an ex post basis, two for each model on an ex ante basis, and one in each set for risk (as measured by standard deviation), return, compound return,[11] and Sharpe ratio as measured by the excess return over standard deviation.

Although the risk metric used herein is rather arbitrary as risk is effectively different for each optimization process, the alternative strategies are known to outperform the MV process as they are optimized to achieve this very result. The question then remains whether they can statistically improve returns over MV theory while not substantially increasing risk. If this is achievable, then the model would imply a multi-period dominance of the alternative strategies over MV. An alternative approach is to use second-order stochastic dominance to rank as suggested by Porter (1974). However, as noted by Ruszczynski and Vanderbei (2002), the computational power required to approach this problem using conventional methods for one hundred assets is not tractable. The use of a Simplex procedure to greatly reduce the computational effort and time was examined herein but this process is well beyond the scope of this initial study. The difference of means of the Sharpe ratios, as the measure of performance, by strategy are summarized in tables 4 and 8.

The optimization analysis does not support the unquestioned dominance of the minimum variance portfolio theory over other investment functions. As expected, the minimum variance portfolio allocation process does increase return and reduce risk over a naively equally diversified portfolio on an ex post basis, as shown in tables 5 and 6. However, these gains are not significant at the 95% level of confidence for the period

[11] Compound returns are assessed by taking the non-overlapping three-month geometric average of the monthly arithmetic averages. This reduces the number of data points available for the compound return difference of means test but does not so adversely affect power as to inhibit the drawing of conclusions as evident in the summary table 3.

studied. When both risk and return are considered on an ex post basis using a Sharpe ratio, the minimum variance allocation is generally superior to the other strategies but not at a 95% level of confidence. When assessed from an ex ante perspective, the slight advantage of minimum variance is reversed, and the alternative strategies and ES theory demonstrate dominance in returns. Although the difference in means is not significant between the strategies, the alternative strategies do possess strictly positive Sharpe ratios at a 95% confidence level whereas the MV process does not.

For all the ex post analyses, the R^2-values range from 66.4% to 87.0%, and all the models are significant at well below the 0.1% level of confidence. With regard to the ex ante analyses, only the difference of means in risks exhibit comparable power. The analysis of compound return is significant at the 10% level of confidence as the strategy alternatives account for about 5.4% of the variation in geometric returns. The ex ante Sharpe ratios are not statistically different from one another based on model selection, and the difference of means are not significant with a p-value of 0.23. It is not apparent that the minimum variance theory can maintain a statistically superior return-to-risk ratio over any other competing theory excluding the maximum return strategy. This last strategy consistently has a statistically lower Sharpe ratio than MV on an ex ante basis (see Table 8).

One explanation for the statistically lower Sharpe ratio of the maximum return strategy may be that the market exhibits mean reversion. Summers (1986) demonstrates that prevalent econometric techniques for measuring EMH introduce a bias that makes rejection highly unlikely. Poterba and Summers (1988) find that stock returns exhibit positive serial correlation over the short term and negative autocorrelation over the long term. Similarly, Fama and French (1988) also find long-term negative autocorrelation in stock returns. An analysis of EMH is beyond the scope of this study but intuitively one might expect to find that the maximum return strategy tends to find the point in a return sequence just before positive autocorrelation becomes negative. In turn, the maximum return strategy, which selects period t's highest return, would be exposed to a mean-reverting loss in period $t+1$. The result is that the maximum return strategy ex ante would tend to have very poor Sharpe ratios, as observed in this analysis. It is noted that without the advantage of prescient information, the MV theory can not be said to be the optimal choice for investors as the alternative strategies tend to reward risk better. Nevertheless, one may conclude that MV is superior to maximum return ex ante.

Despite the MV theory having less risk[12] than all the other strategies ex ante, one might infer that this strategy is in fact inferior to the alternatives examined herein. Only the MV and the maximum return strategies have return and compound returns that are not statistically greater than zero on an ex ante basis. As well, these two strategies have the lowest Sharpe ratios, which questions if an investor is being adequately compensated for the risk that is borne. In contrast, the ES and alternative processes provide returns that are significantly non-zero at the 95% confidence level for both arithmetic and geometric returns. The Sharpe ratios also are positively correlated with the increase in the taste for HMPM (β and γ) risk. That is, as the investor function is optimized so as to place emphasis on maximizing higher mean partial moment returns, the ex ante Sharpe ratio tends to increase. These strategies do expose the investor to greater fluctuations of returns but those returns are significantly positive. They are positive even to the point

[12] When risk is measured by the standard deviation.

where the increase in return more than compensates for the increase in risk, so as to improve the Sharpe ratio.

Examining the correlations between strategies for risk, returns and Sharpe ratios, we observe that MV generally has highly correlated returns with the other strategies but exhibits entirely uncorrelated changes in risk as measured by the standard deviation. This persists both in the ex ante and ex post frameworks. Select alternative strategy parameter sets and the maximum return strategy also are highly uncorrelated with MV in terms of compound returns and single period returns. Thus, these strategies appear to differ from one another quite substantially in how they achieve returns. This is consistent with the model design where MV explicitly attempts to identify a "safe-course" of growth whereas the alternative models attempt a path that is more volatile and more likely to experience intermittent periods of high growth.

These differences call into question what investors truly perceive as being risk. Under Bernoulli's assumption, risk is simply a mathematical tendency to vary from a given mean. However, in reality investors may not be so concerned with such variance depending on which side of the mean they are on. Although skewness incorporating asset-pricing models, such as those tested by Simkowitz and Beedles (1978), can account for some of the individual investor preference for this moment, the model is arranged so that mathematical terms are in fact at odds with each other. While the higher variance partial moment grows, the monotonic nature of the utility curve implies growing investor dissatisfaction while the simultaneous increase in skewness suggests an increase in investor satisfaction. The alternative optimization process allows investors to exhibit distinct preferences for partial mean variances. The evidence reported herein seems to confirm that such models, of which the ES may be viewed as a subset, do not tend to underperform the MV approach on an ex ante basis. On an ex post basis, the MV model has an evident advantage but in real applications security prices are not deterministic.

The summary conclusion is that the alternative models at least offer a significantly positive return for bearing risk in a simulation of real life investment possibilities. Therefore, these models need to be more closely examined, hopefully identifying and using the parameter set that best models aggregate investor behavior. Further study with appropriately calibrated parameters may illustrate yet higher Sharpe ratios, and possibly increase the significance of the ex ante advantage of non-monotonic utility derived investment functions over the MV approach.

7. THE TWO-BETA CAPM

Extending from equation 26 one can develop a theoretical foundation for the two parameter CAPM as formulated by Howton and Peterson (1998). The expression given previously was:

$$= \underbrace{(A_+W)^T A_+W}_{\substack{positive\,/\,positive \\ \text{cov}\,ariance}} + \underbrace{(A_-W)^T A_+W}_{\substack{negative\,/\,positive \\ \text{cov}\,ariance}} + \underbrace{(A_+W)^T A_-W}_{\substack{positive\,/\,negative \\ \text{cov}\,ariance}} + \underbrace{(A_-W)^T A_-W}_{\substack{negative\,/\,negative \\ \text{cov}\,ariance}} \quad (45)$$

Each of the four terms in this equation are measures of different forms of covariance. One obtains a four factor co-varying model by splitting the *n* period by *2* series matrix A, where the first column series is the excess return sequence of any given security in question and the second column series is the sequence of the market excess returns over the appropriate risk free rate. The first term would represent the security's excess returns to vary positively with the positive market premium. The fourth term would represent the security's excess returns covariance with a negative market premium. The two intermediary terms would represent the security's tendency to react negatively during a positive market and to react positively during a negative market, respectively. However, these intermediary terms are aggregated since they can not be viewed ex ante without estimating both the future direction of the market but also the relation of the security to the market movement in that given period. In a form analogous to the LSM CAPM, the model appears initially as:

$$\begin{aligned} R_{it} &= R_{ft} + \beta_{+sec/+mkt}(\max(R_{mt} - R_{ft},0)) + \beta_{-sec/+mkt}(\max(R_{mt} - R_{ft},0))\ldots \\ &\ldots + \beta_{+sec/-mkt}(\min(R_{mt} - R_{ft},0)) + \beta_{-sec/-mkt}(\min(R_{mt} - R_{ft},0)) + e_{it} \end{aligned} \tag{46}$$

Merging by λ_+ and λ_-, this equation simplifies to:

$$R_{it} = R_{ft} + \underbrace{(\beta_{+sec/+mkt} + \beta_{-sec/+mkt})}_{\beta_{+mkt}}\underbrace{(\max(R_{mt} - R_{ft},0))}_{\lambda_+} + \underbrace{(\beta_{+sec/-mkt} + \beta_{-sec/-mkt})}_{\beta_{-mkt}}\underbrace{(\min(R_{mt} - R_{ft},0))}_{\lambda_-} + e_{it} \tag{47}$$

$$R_{it} = R_{ft} + \beta_{+mkt}\lambda_+ + \beta_{-mkt}\lambda_- + e_{it} \tag{48}$$

To test the validity of this model, both the single parameter and dual-parameter one year CAPM are estimated for each of the 377 securities of the SP500 that had complete CRSP data for the 132 months over the period from January 1991 to December 2001. This generates 49,764 separate regressions for each of the two models from which statistical analyses are conducted. The regressions are conducted in SAS with a macro loop, and involve the excess security returns against the excess market returns with a constrained intercept of zero.

Two comparisons are made relative to the single parameter CAPM to assess the suitability of the dual beta model. The first is simply an examination of the descriptive statistics of the R^2, adjusted R^2 and F-test values of the two models.

The predictive performance of each model now is assessed using two methods. The first method examines the CAR or cumulative abnormal residuals of the two models over the 132-month period. To accommodate periods in which the Adjusted R^2 is negative, the CAR is given by:

$$CAR = \sum_{t=1}^{N} \underbrace{\max\left(\frac{AdjR^2{}_{t-1}}{\left|AdjR^2{}_{t-1}\right|},0\right)}_{Validity\ \ Coefficient} AR_t \tag{49}$$

The CAR methodology is used to identify what a practitioner may experience as model error in real application. Since no practitioner would knowingly make the error of applying a model with a negative adjusted R^2, estimates generated from such a model are dropped by calibrating the *validity coefficient* to zero in such instances.

The second method used to assess predictive power is a Model Weighted CAR as given by:

$$MWCAR = \sum_{t=1}^{N} \underbrace{\max\left(\frac{AdjR^2{}_{t-1}}{AvgR^2},0\right)}_{Confidence\ \ Coefficient} AR_t \tag{50}$$

The purpose of the *confidence coefficient* in this method is to more heavily weight those abnormal residuals where more accuracy was expected than those where less accuracy was anticipated. This reflects the application of a model where the practitioner would not put great confidence in a return estimate generated by a statistically weak model. Therefore, it should better illustrate which of the two CAPM processes best meet expected accuracy.

In both models, the abnormal returns are given by:

$$AR_t = \frac{\sum_{i=1}^{S} Actual_i - Estimate_i}{S} \tag{51}$$

In this equation, S is the number of firms (377) in the sample. The parameters selected for the 'Betas' are those that are generated in the previous period. That is, the beta estimates are generated ex ante such that the parameter estimates available to predict period t returns are those identified in period $t-1$. This realistically captures the information that a practitioner would have available at the time of a decision. The market premium however is the time t realized premium. It also is of interest to test the validity of applying the various forms of the CAPM when perfect information about market return realizations is assumed.

Both a 12- and 60-month model is tested. Unlike the 12-month model, the 60-month model is more precise statistically, but it allows for a less rapid change in the parameter estimates. The results are summarized in tables 12 and 13. Contrary to the conclusions of Nantell, Price and Price (1982) or Howton and Petterson (1998), no material advantage or disadvantage is identified for the use of the two-parameter CAPM. This is the case for either the best mathematical fit or predictive accuracy for both the 12- and 60-month CAPM.

Out of a possible 49,764 regressions for each 12-month model, 35,880 and 35,421 regressions for the one-parameter and two-parameter CAPM, respectively, have an adjusted R^2-value greater than zero. The average adjusted R^2 of 18.56% for the two-parameter CAPM is nearly identical to that (18.06%) for the one-parameter model. The one-parameter CAPM displays a 66.3% accuracy rate for predicting directional shifts in security returns given the known market premium for that period. The accuracy rate for the two-parameter model was slightly inferior at 64.25%.

Not surprisingly, the 60-month versions of the two models have a much greater number of non-negative adjusted R^2-values out of a possible 31,668 regressions for each model. The specific numbers are 30,573 and 30,573 for the single- and two-parameter models, respectively. As is the case for the 12–month versions, the adjusted R^2-value is higher for the two-parameter model for the 60-month versions of the two models, and the one-parameter model is slightly better at predicting security direction (65.947% versus 65.611% for the two-parameter model).

The cumulative abnormal returns or CARs for both window versions of the two models are reported in tables 14 and 15. Although the one-parameter CAPM has cumulative abnormal residuals or CARs that are fourteen and ten times larger than the CAR of the two-beta CAPM for the 12 and 60-month windows, respectively, none of these CAR are significantly different from zero. Thus, no estimation bias is evident for any model or window version. For the model-weighted CAR the one-beta CAPM has residuals that are 1.64 and 1.54 times as large as the two-beta CAPM for 12 and 60-month windows, respectively.

Non-positive adjusted R^2-values are then eliminated. Based on a Tukey-Kramer test of the difference of means, the two-parameter CAPM is slightly better than the one-parameter CAPM in adjusted R^2 for both window lengths. In all four cases, the null hypothesis that the means of the two measures are equal is rejected at greater than 99.99% confidence on a combined sample of 99,528 parameter estimates for the 12 month and 81,432 parameter estimates for the 60 month windows. Although the difference in means is small, the high level of confidence stems from the large size and consistency of the sample.

In summary, the case for the use of the two-beta CAPM in place of the one-beta model is neither rejected nor accepted considering the data and methodology used in its evaluation. Both models consistently produce only modest statistical support, though neither appears to have any upward or downward bias in ex ante return prediction. Therefore, the support for the use of the mean higher partial moment development of the CAPM is inconclusive. Where data is sufficiently abundant and degrees of freedom are of little concern, there seems to be a tiny advantage in using the two-beta CAPM. Where data availability is of greater concern, the greater degrees of freedom and the more parsimonious single factor CAPM is likely to be preferred.

8. RISK PREMIUM PUZZLE

An initial analysis of the equity premium puzzle using the new framework is now conducted. The data is taken from Ibbotson and spans the period of February 1927 to December 2002. The particular series used are the same as in the Mehra and Prescott seminal piece; namely, a monthly deflated real return series of the S&P500 as well as the monthly deflated real returns series of the US one month treasury bill. The objective of the analysis is to find parameter estimates of the alternative utility function that best describe observed returns for this period.

The intuition is that monotonic utility is blind to the difference between higher and lower partial variance. If investors have a greater disdain for lower partial variance than upper partial variance, as is suggested, then whichever category of asset has the greater downside variance will need to also have the larger expected return in equilibrium. If equities exhibit greater downside fluctuations than treasuries, and if investors are more sensitive to downside fluctuations, then investors will naturally demand a relatively higher rate of return to be compensated for this unpleasant nature of the variance of equities.

The method of least squares is used to identify the parameters. The fitted model is of the form:

$$F = \min \sum \left(Y_i - (\alpha_1 VLPM_i^{\beta_1} + \alpha_2 VHPM_i^{\beta_2})\right)^2 \qquad (52)$$

In equation (52), VLPM and VHPM are the historical lower partial moment variance and higher partial moment variance, and Y_i is the expected return of the security.[13] The terms α_1, α_2 and β_1, β_2 are the linear and power sensitivities to each kind of risk metric, respectively. As VLPM and VHPM are measured from a reference point of zero to distinguish losses from gains, they are identically equal to $E(\rho^2)$ and $E(\lambda^2)$, as expressed in equations (16) and (17).

To solve for the solution analytically, one derives equation (52) according to the variables α_1, α_2, β_1 and β_2, sets the First Order Conditions of each expression to zero, and then solves for each variable. The derivatives are of the form:

$$\frac{\partial F}{\partial \alpha_1} = \sum \; -2 \bullet \left(Y_i - (\alpha_1 VLPM_i^{\beta_1} + \alpha_2 VHPM_i^{\beta_2})\right) \bullet VLPM_i^{\beta_1} = 0 \qquad (53)$$

$$\frac{\partial F}{\partial \alpha_2} = \sum \; -2 \bullet \left(Y_i - (\alpha_1 VLPM_i^{\beta_1} + \alpha_2 VHPM_i^{\beta_2})\right) \bullet VHPM_i^{\beta_2} = 0 \qquad (54)$$

$$\frac{\partial F}{\partial \beta_1} = \sum \; -2 \bullet \left(Y_i - (\alpha_1 VLPM_i^{\beta_1} + \alpha_2 VHPM_i^{\beta_2})\right) \bullet \alpha_1 \bullet VHPM_i^{\beta_1} \bullet \log(VHPM_i) = 0 \qquad (55)$$

$$\frac{\partial F}{\partial \beta_2} = \sum \; -2 \bullet \left(Y_i - (\alpha_1 VLPM_i^{\beta_1} + \alpha_2 VHPM_i^{\beta_2})\right) \bullet \alpha_2 \bullet VLPM_i^{\beta_2} \bullet \log(VLPM_i) = 0 \qquad (56)$$

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[13] It is assumed that the best approximation of the future partial variance of a given security is the historically observed partial variances.

Although the analytical solutions potentially allow for tests of the estimated parameter confidence, the isolation of the terms has proven to be of a mathematical complexity that exceeds the scope of this work. As this section is only a preliminary fit of the data to a potential set of parameters, a numerical optimization following a gradient approach is used instead.

To produce the sequence of expected returns two methods are considered. The first is an average geometric return of the given asset for the prior T periods where T is from *t-n-1* to *t-1*. N is either 12 or 60, representing either the one year or five year average geometric return. The second method uses an arithmetic average with a decay rate where the weight of each observation is 1.01^{j-n}. N is either 13 or 61, and j is the period that has elapsed since the observation. In the case of the expectation being formed based on one year of historical data, the most recent observed return has a weight that is 1.137 times the oldest observation. In the case of the five-year expectation, the most recent observation is 1.816 times the weight of the oldest observation. In either case, the partial variance expectations are for the matching time frame. To illustrate, the one-year partial variances are used to generate estimates of the one year expectation for both the geometric and decayed arithmetic averages.

In numerically identifying the parameter estimates, the reader should be made aware that numerous local minima were identified. To best reflect the alternative optimization model and conventional monotonic utility, the initial vector for optimization was set so that all parameters are equal to unity. This start point differs from conventional measures of variance as the covariance terms are not present, but represents an upper limit that portfolio variance could attain provided all assets were wholly uncorrelated. An area for future research would include better specification of the parameters, hopefully through an analytic method or a numerical method with imposed theoretical boundary values for the parameters.

The current analysis results are summarized in table 20. The polynomial of best fit describes between 2.7% and 10.6% of the variation of the expected returns depending upon which model period (one year or five year) and which model type (geometric or arithmetic decay) is selected. Uniformly, each model ascribes a lower linear multiplier and lower power multiplier to the VHPM than to the VLPM.

Although the expectation is that the α_1 term should in fact be negative (implying a preference for VHPM), the small positive value is not inexplicable. A possible reason is that investors do indeed dislike upper moment risk as is implied by Bernoulli's assumption but that upper moment risk is much less disliked than lower moment contrary to the conjecture of Bernoulli.[14]

The value of β_2 for all the models is greater than unity, as expected. Values above one, given that all variance measures are in units such that their value is less than one, imply a steady desensitization to a specific form of risk. This occurs although the investor would always retain some preference to procure or rid oneself of one additional unit of risk, as determined by the sign of the alpha coefficient for each risk measure. β_1 similarly implies a steady rate of desensitization since its value is universally between zero and one. This suggests a concave, rather than a convex shape. The shape of the fitted polynomial precisely matches the general characteristics of the Value function, a concave response to gains and a convex response to losses.

[14] As noted before, the monotonic utility proposed by Bernoulli implies a symmetric dislike of risk regardless of which type of partial moment is being examined.

Using the best-fit parameter estimates (where R^2 is 10.6%), one can identify how an investor should respond to the gamble of 50% chance of 50% wealth gain versus a 50% chance of a 50% wealth loss. Recall, this same gamble with the parameter estimates of Mehra and Prescott (1984) suggest that an investor would divulge themselves of 49.9% of their wealth to avoid such a game. However, with the fitted polynomial, an investor would be willing to part with far less of their initial endowment to avoid such a game, in this case 5.945%. Such a value is evidently more plausible although it appears to be somewhat low. To examine the validity of such protests, a much more refined study of the polynomial, hopefully with an analytic solution, should be undertaken in future work.

9. CONCLUSIONS

The objective of this thesis is to question the validity of Bernoulli's 1738 solution to the St.Petersburg Paradox. The conceptualization of investors as risk averse, risk loving or risk neutral, as determined by the concavity (convexity/linearity) of an internal monotonic utility function, is parsimonious but is inconsistent with the findings of positive finance. A breadth of financial innovation eminates from this quintessential foundation, including the Pratt (1964) measurement of the risk aversion in the small, Markowitz's (1952) seminal insight into optimal portfolio allocation, and the Sharpe, Lintner and Moss CAPM. It is not the contention of this thesis to rebuke these developments but to offer a potential modification that might allow for a closer joining of prescriptive and descriptive finance, particularly in the area of asset allocation and pricing.

This thesis is not the first to note the weakness with Bernoulli's assumption. Allais (1953), Savage (1948) and Markowitz (1952) all noted the underlying problem. Attempts such as Stone's (1973) generalized loss function have directly attempted to streamline the theory to rectify the problem. Other theoretical models have been developed, including those by Constantinides (1990) and Basak and Cuoco (1998) to help explain anomalies, such as the size of the equity premium. In an effort to increase explanatory power, Pettengill, Sundaram and Mathur (1995) and Howton and Peterson (1998) also perform empirical adjustments to CAP models, which are descended from the monotonic utility function. Despite these efforts to reinforce the use of monotonic utility, other researchers such as Mao (1970) and Kahneman and Tversky (1979) persistently observe a value function behavior in empirical studies and not monotonic utility.

The work herein examined the theoretical development of a two-factor value function in both a continuous framework through a MacLaurin series expansion as well as through a discrete matrix expression. The analysis of the optimization reveals that the alternative allocation strategy has a slight advantage over the minimum variance approach on an ex ante basis. Moreover, the risk borne by investors utilizing the alternative strategy or ES (semi-variance) is rewarded with increased returns at the 95% confidence level. The analysis undertaken herein can not make any such claim for MV (minimum variance) theory. The discrete expression of the two-factor value function model naturally leads to a two-beta CAPM, as proposed empirically by Howton and Peterson (1998), among others. An examination of the efficacy of the two-beta model versus the single parameter model reveals a tiny, though strongly significant, advantage of the former over the latter. Finally, a preliminary study of creating a polynomial of best fit to the Ibbotson data used by Mehra and Prescott (1984) reveals parameter estimates that greatly reduce the magnitude of the equity premium puzzle.

In all the aforementioned cases, the strength of the analyses need further improvement. Furthermore, the theoretical models would benefit from refinement. Nevertheless, each analysis independently confirms that there may be an advantage to modeling individual investor preferences according to a Kahneman and Tversky value function in place of a Bernoulli monotonic utility function. While such a preliminary study can not possibly warrant an all out adoption of the value function for modeling individual investor behavior, it is reasonable on the basis of the research reported herein to further explore in future research just such a possibility.

APPENDICES

Appendix A: St.Petersburg – Original and Dual-risk formulations

Peter tosses a coin and continues to do so until it should land "heads" when it comes to the ground. He agrees to give Paul one ducat if he gets "heads" on the very first throw, two ducats if he gets it on the second, four if on the third, eight if on the fourth, and so on, so that with each additional throw the number of ducats he must pay is doubled. Suppose we seek to determine the value of Paul's expectation. (Sommer 1965, p.31)

The mathematical value of the series is:

$$E(x) = \sum_{0}^{S} pr(x_i)x_i = \sum_{0}^{\infty} \frac{1}{2^n} 2^n = 1 + 1 + 1 \ldots = \infty$$

S represents the number of states, and the number is without limit in this particular scenario. $pr(x_i)$ is the probability of state i and x_i is the payoff in that state. The mathematical expectation is obviously infinite, yet no sane individual would pay everything they have, and more, to obtain a chance to play this game.

Bernoulli offers the concept of the utility of wealth, such that a person will value a dollar based upon what it can do for their standard of living rather than valuing the dollar for itself. He also goes further to suggest that the utility function is monotonic and convex. Formally that is:

$$\frac{dU(x)}{dx} > 0 \quad \& \quad \frac{d^2U(x)}{dx^2} < 0$$

One such function that meets this requirement is $U(x) = x^r$ for $0<r<1$. Bernoulli suggests $r = 0.5$ (square root) as an example, giving the solution to the paradox as:

$$E(U(x)) = \sum_{0}^{S} pr(x_i)U(x_i) = \sum_{0}^{\infty} \frac{1}{2^n} U(2^n) = \sum_{0}^{\infty} \frac{1}{2^n} 2^{n/2} = \sum_{0}^{\infty} \frac{1}{2^{n/2}} = 1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + \ldots = \frac{\sqrt{2}}{\sqrt{2}-1}$$

This geometric series converges to a finite point and implies that anyone with the function as described above would never pay more than $11.66 to play this game.[15]

Under the two dimensional framework, the solution is very similar although there may exist two pure answers that can be linearly combined to create a set of possible solutions. Unlike Bernoulli there is not one utility function but rather a gain function and a loss function. These functions will also be monotonic but will have the following characteristics:

$$\frac{dL(x_-)}{dx} > 0 \qquad \frac{d^2L(x_-)}{dx^2} > 0$$

$$\&$$

$$\frac{dG(x_+)}{dx} > 0 \qquad \frac{d^2G(x_+)}{dx^2} < 0$$

The combined function has the form:

$$\begin{aligned}
V(x_+, x_-) &= \sum_{i=0}^{N} pr(x_{-i})L(x_{-i}) + \alpha \sum_{j=0}^{M} pr(x_{+j})G(x_{+j}) \\
&= \sum_{i=0}^{N} pr(x_{-i})(x_{-i})^Q + \alpha \sum_{j=0}^{M} pr(x_{+j})(x_{+j})^P = 0 + \alpha \sum_{j=0}^{M} \frac{1}{2^M} 2^{PM} \\
&= \alpha \sum_{j=0}^{M} 2^P = \frac{\alpha}{1-2^{-P}}
\end{aligned}$$

Under this approach, Paul will either pay Peter (an assured loss and less desirable than the loss of an assured gain):

$$L(x_-)^Q = \frac{\alpha}{1-2^{-P}}, x_- = \left(\frac{\alpha}{1-2^{-P}}\right)^{1/Q}$$

---

[15] $U(x) = \frac{\sqrt{2}}{\sqrt{2}-1}, x = \left(\frac{\sqrt{2}}{\sqrt{2}-1}\right)^2 = \frac{2}{3-2\sqrt{2}} \cong 11.6568$

or would be willing to trade an assured gain of:

$$G(x_+)^P = \frac{\alpha}{1-2^{-P}}, x_+ = \left(\frac{\alpha}{1-2^{-P}} \div \alpha\right)^{1/P} = \left(\frac{1}{1-2^{-P}}\right)^{1/P}$$

In the case of a=1, P=0.5 and Q=0.8, an individual is indifferent between the gamble and receiving an assured gift of $11.66, or trading this endowed chance to gamble for this amount. However, if an individual is not yet in possession of the gamble, they may be willing to pay no more than $4.64.

This formulation can account for the endowment effect noted in the economic literature. While it is not explored in this thesis, one might posit that the differential between P and Q could account for expectations of information asymmetry. If an individual perceives that the endowment of a positive expectation game offers some control, then they will put a premium on the sale of that game. The sense of control may also help to discount the amount that is paid to avoid a negative game, given that the player falsely assumes that the outcome is more favorable than suggested by the odds. When not endowed with control the potential player will assume the seller (buyer) of the gamble has superior information, and therefore will apply a discount (premium) to hedge oneself from buying a lemon (selling a gem).

Appendix B: Sample of 'Irrational' behavior

Source Kahneman and Tversky (1979): Empirically identified examples of irrational and improbable behavior through survey decisions.

Problem 1: Choose between

A: 2,500 with probability 0.33 2,400 with probability 0.66 0 with probability 0.01	B: 2,400 with certainty.
N=72 [18% A]	[82% B]*

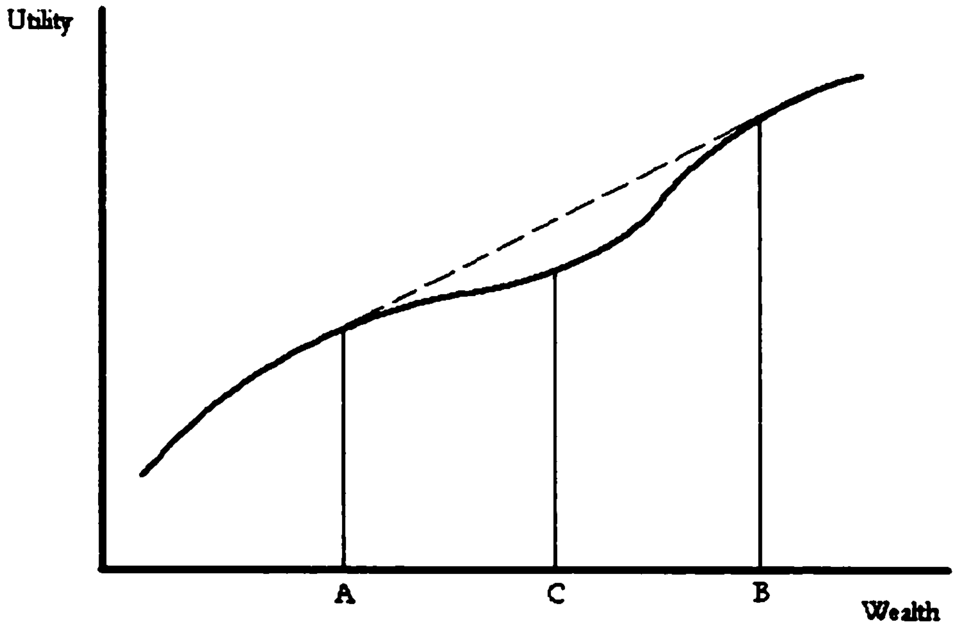
Problem 2: Choose between

C: 2,500 with probability 0.33 0 with probability 0.67	D: 2,400 with probability 0.34 0 with probability 0.66
N=72 [83% C]*	[17% D]

The data show that 92 per cent of the subjects chose B in Problem 1, and 83 per cent of the subjects chose C in Problem 2. Each of these preferences is significant at the .01 level, as denoted by the asterisk. Moreover, the analysis of individual patterns of choice indicates that a majority of respondents (61 per cent) made the modal choice in both problems. This pattern of preferences violates expected utility theory in the manner originally described by Allais. (Source Kahneman and Tversky 1979)

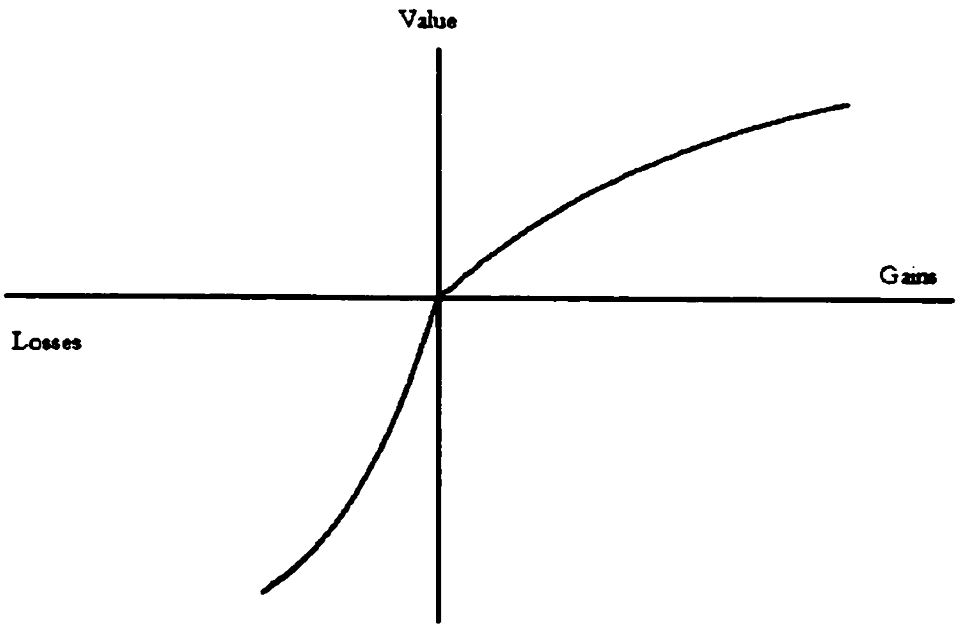
FIGURES

Figure 1.



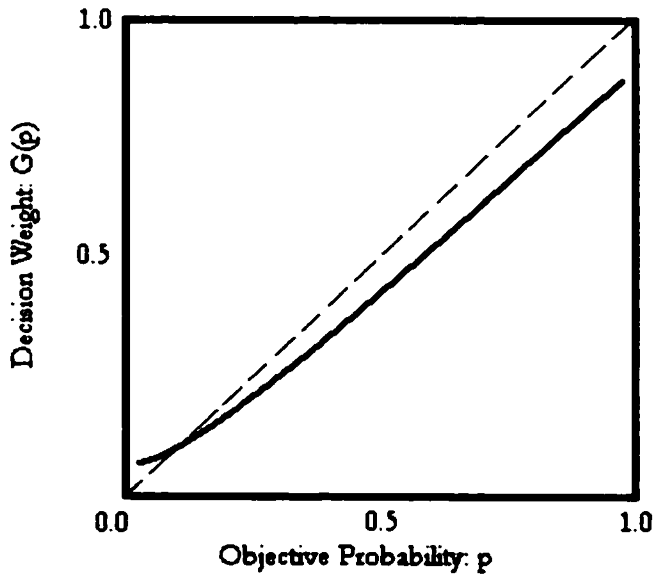
Source Friedman & Savage (1948): Hypothetical degree three utility curve exhibiting both concave and convex regions of risk preference. Allows for both gambling and insurance behavior but is shown by Markowitz (1952) to describe irrational and improbable investor behavior.

Figure 2.



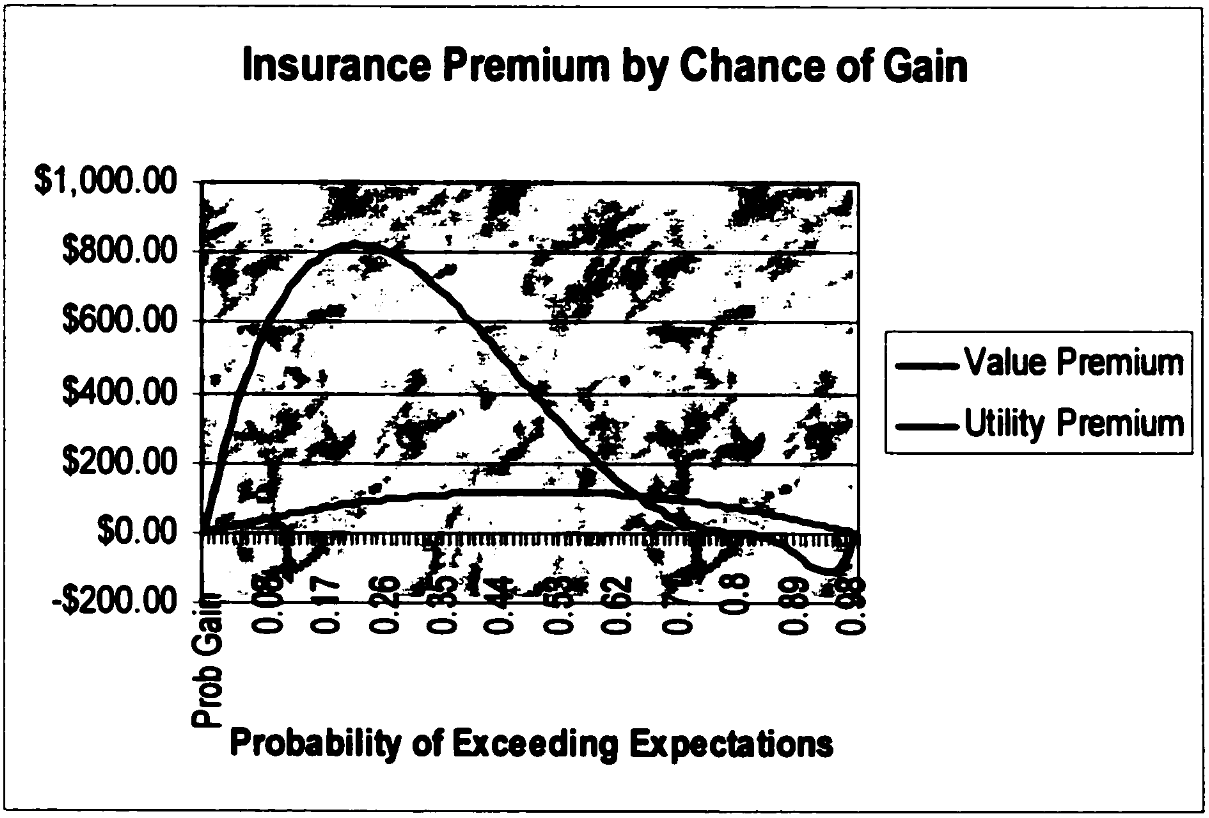
Source Thaler (1980): Plot of a potential value function. This value function differs from monotonic utility in the use of a reference point and non-symmetrical responses to either a foregone gain or a realized loss.

Figure 3.



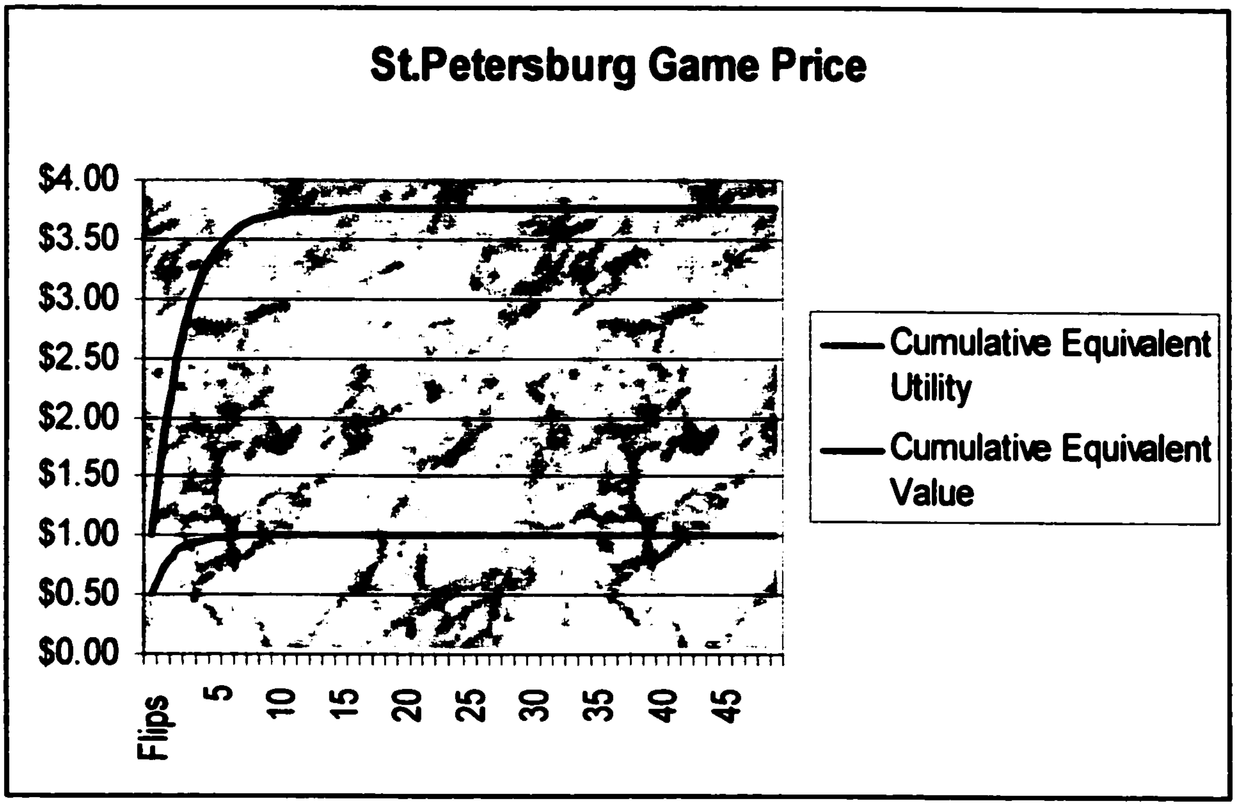
Source Allais (1953): Depiction of subjective interpretation of probability. Small probabilities are subjectively over-weighted in the decision while large probabilities are subjectively under-weighted.

Figure 4.



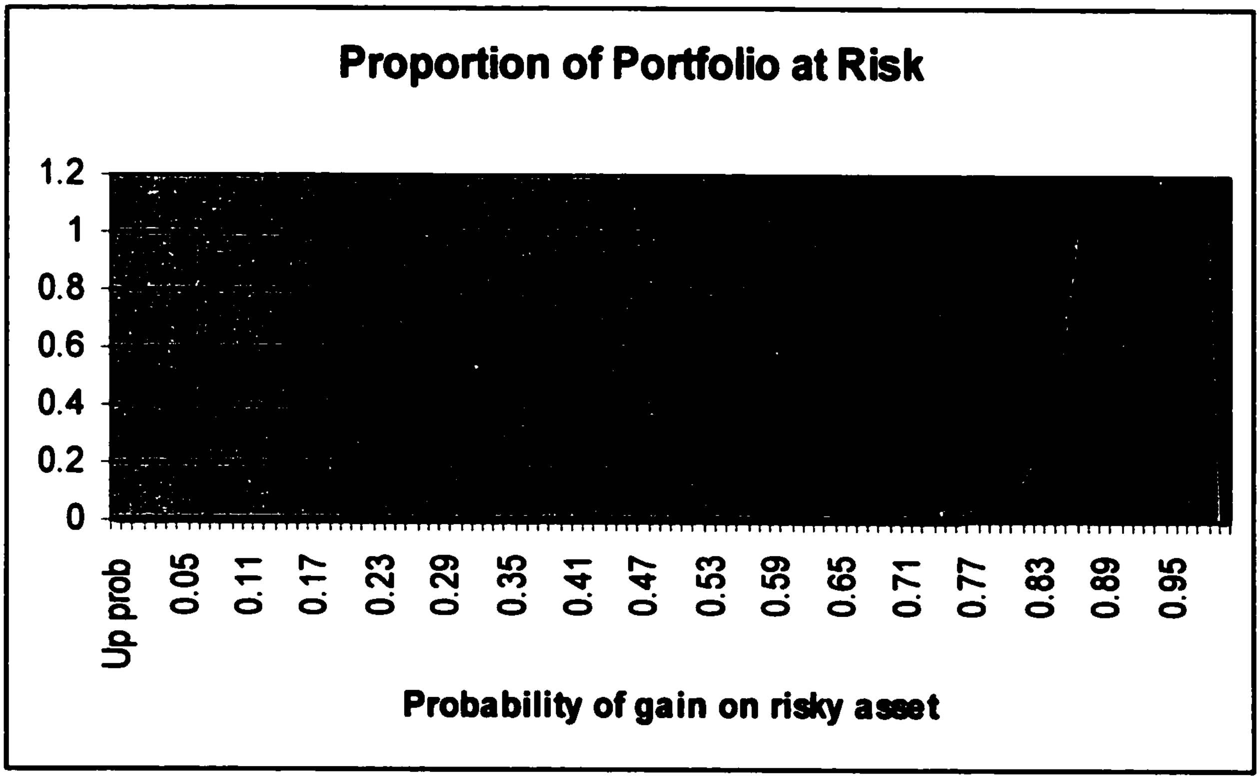
Insurance premium paid on $100,000 home facing potential flood damages of $10,000 according to uncertainty function. Utility premium is decided according to function $U(x) = (x)^{1/2}$, and determines the $100,000 home as the initial endowment. The value premium is determined by $V(x) = -(x_-)^{0.5}+(x_+)^{0.4}$, and uses the expectation (variable according to probability of gain) as the reference point. For example at a probability of 10%, the expected value of the home is $91,000 (or 0.9*$90K+0.1*$100K). Outcomes exceeding this value are treated as gains, while outcomes inferior to this value are assessed as a loss.

Figure 5.



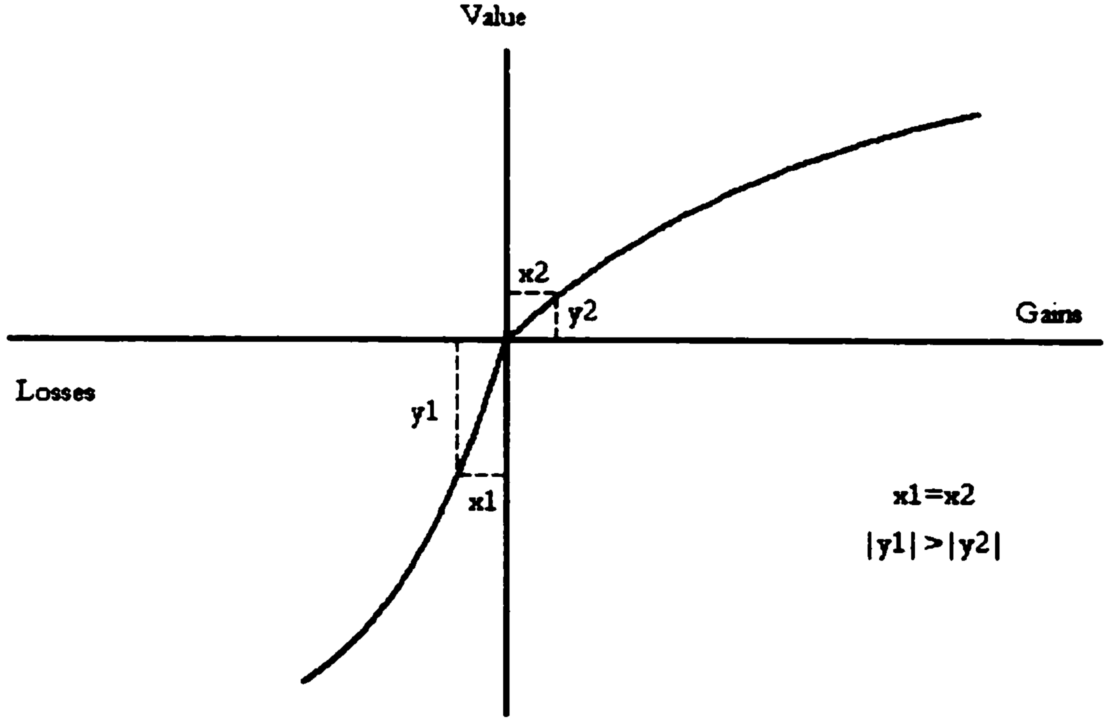
Dollar value of the St.Petersburg game by decision under uncertainty function. Cumulative equivalent utility depicts dollar price paid by individual under expected utility theory assuming $U(x) = (x)^{1/4}$. Cumulative equivalent value depicts dollar price paid by individual under prospect theory assuming $V(x) = -(x_{-})^{0.5}+(x_{+})^{0.4}$.

Figure 6.



Optimal proportion of portfolio put at risk according to the value function $V(x) = -(x_-)^{0.5}+(x_+)^{0.4}$. Two-asset case, one risk free yielding \$10,000 in both high and low states. The risky asset yields either \$0 or \$10,000. The high state is the probability of gain on the risky asset. Any investment in the risky asset will reduce expected return while incurring an increase in risk. Therefore, expected utility theory will find 0 optimal at all but 100% likelihood of risky return. In this case, expected utility theory will be indifferent. Short sales are prohibited, but proportion in risky asset would have a finite crest at exceedingly high values.

Figure 7.



Source Thaler (1985): Depiction of Prospect Theory value function with equal magnitude of marginal shifts in initial endowment but non-symmetric reduction/elevation in value (utility). Applying this particular function, a foregone gain does not distress an investor as much as a realized loss of equal size.

TABLES

Table 1.

Comparison of choice of two pool depths by a non-swimmer. Classic Minimum variance, or even third moment decision processes will advocate pool 2, although pool 1 results in the higher chance of survival.

State (pr)	*Pool 1 Depth*	*Pool 2 Depth*
1 (1/3)	30'	10.1'
2 (1/3)	2'	10'
3 (1/3)	2'	10'
Mean		
	11.33333'	10.03333'
Variance	261.3333	0.003333
Skew	-1.73205	-1.73205

Table 2.

Selected parameters for alternative optimization processes. α is the power raising MLPM variance, and β is the negative coefficient multiplying MHPM, which is in turn raised by γ.

Strategy	α	β	γ
1	1.00	0.00	0.00
2	1.00	1.00	0.25
3	1.00	0.25	0.25
4	1.00	0.10	0.25
5	1.00	1.00	0.50
6	1.00	0.25	0.50
7	1.00	0.10	0.50
8	1.00	1.00	1.00
9	1.00	0.25	1.00
10	1.00	0.10	1.00

Table 3.

Tukey-Kramer difference of means test on levels (strategies) compared according to ex ante and ex post Return, Risk, Compound Return (3 month) and Sharpe ratio.

Item and Method	*F-test (p-value)*	R^2	*Model DF*	*#Obs*
Return ex post	244.11 (<0.0001)	0.731974		
Risk (Std.Dev) ex post	177.15 (<0.0001)	0.664636		1176
Sharpe ex post	263.20 (<0.0001)	0.746490		
Compound Return ex post	194.86 (<0.0001)	0.870154		378
Return ex ante	1.37 (0.1685)	0.016016	13	
Risk (Std.Dev) ex ante	183.32 (<0.0001)	0.685768		1106
Sharpe Ratio ex ante	1.26 (0.2312)	0.014779		
Compound Return ex ante	1.57 (0.0927)	0.054988		364

Table. 4

Tukey-Kramer difference of means of Sharpe ratios compared by strategies under ex post assumption.

Ex Post Sharpe i	j	Means	90% Confidence Limits LSMean(i)	-LSMean(j)	95% Confidence Limits LSMean(i)	-LSMean(j)	Sig. Diff @ 95%
EQ	α:1 β:0.25 γ:0.5	1.529144	-0.55451	3.612802	-0.71173	3.77002	
EQ	α:1 β:0.1 γ:0.5	1.285341	-0.79832	3.368999	-0.95554	3.526217	
EQ	α:1 β:1 γ:1	0.041201	-2.04246	2.124858	-2.19968	2.282077	
EQ	α:1 β:0.25 γ:1	-0.02849	-2.11214	2.055171	-2.26936	2.21239	
EQ	α:1 β:0.1 γ:1	-0.04231	-2.12596	2.041353	-2.28318	2.198571	
EQ	MV	-0.38426	-2.46791	1.699403	-2.62513	1.856621	
EQ	ES	-0.00011	-2.08377	2.083547	-2.24099	2.240765	
EQ	MaxRet	-27.8354	-29.919	-25.7517	-30.0763	-25.594512	Yes
EQ	α:1 β:0 γ:0	-0.05149	-2.13515	2.032165	-2.29237	2.189383	
EQ	α:1 β:1 γ:0.25	1.414855	-0.6688	3.498513	-0.82602	3.655731	
EQ	α:1 β:0.25 γ:0.25	1.420539	-0.66312	3.504197	-0.82034	3.661415	
EQ	α: 1 β:0.1 γ:0.25	1.52252	-0.56114	3.606178	-0.71836	3.763396	
EQ	α:1 β:1 γ:0.5	1.59696	-0.4867	3.680618	-0.64392	3.837836	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	-0.2438	-2.32746	1.839855	-2.48468	1.997073	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	-1.48794	-3.5716	0.595715	-3.72882	0.752933	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	-1.55763	-3.64129	0.526027	-3.79851	0.683246	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-1.57145	-3.65511	0.512209	-3.81233	0.669428	
α:1 β:0.25 γ:0.5	MV	-1.9134	-3.99706	0.170259	-4.15428	0.327478	
α:1 β:0.25 γ:0.5	ES	-1.52926	-3.61291	0.554403	-3.77013	0.711622	
α:1 β:0.25 γ:0.5	MaxRet	-29.3645	-31.4482	-27.2809	-31.6054	-27.123656	Yes
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	-1.58064	-3.66429	0.503021	-3.82151	0.66024	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	-0.11429	-2.19795	1.969369	-2.35517	2.126587	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	-0.10861	-2.19226	1.975053	-2.34948	2.132272	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	-0.00662	-2.09028	2.077034	-2.2475	2.234252	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	0.067816	-2.01584	2.151474	-2.17306	2.308693	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	-1.24414	-3.3278	0.839518	-3.48502	0.996736	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	-1.31383	-3.39749	0.76983	-3.5547	0.927049	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-1.32765	-3.4113	0.756012	-3.56852	0.913231	
α:1 β:0.1 γ:0.5	MV	-1.6696	-3.75325	0.414062	-3.91047	0.57128	
α:1 β:0.1 γ:0.5	ES	-1.28545	-3.36911	0.798206	-3.52633	0.955424	
α:1 β:0.1 γ:0.5	MaxRet	-29.1207	-31.2044	-27.0371	-31.3616	-26.879853	Yes
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	-1.33683	-3.42049	0.746824	-3.57771	0.904042	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	0.129514	-1.95414	2.213172	-2.11136	2.37039	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	0.135198	-1.94846	2.218856	-2.10568	2.376074	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	0.237179	-1.84648	2.320837	-2.0037	2.478055	

Table 4. Continued.

α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	0.311619	-1.77204	2.395277	-1.92926	2.552496	
α:1 β:1 γ:1	α:1 β:0.25 γ:1	-0.06969	-2.15335	2.01397	-2.31056	2.171189	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.08351	-2.16716	2.000152	-2.32438	2.157371	
α:1 β:1 γ:1	MV	-0.42546	-2.50911	1.658202	-2.66633	1.815421	
α:1 β:1 γ:1	ES	-0.04131	-2.12497	2.042346	-2.28219	2.199565	
α:1 β:1 γ:1	MaxRet	-27.8766	-29.9602	-25.7929	-30.1175	-25.635713	Yes
α:1 β:1 γ:1	α:1 β:0 γ:0	-0.09269	-2.17635	1.990964	-2.33357	2.148183	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	1.373654	-0.71	3.457312	-0.86722	3.61453	
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	1.379338	-0.70432	3.462996	-0.86154	3.620215	
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	1.481319	-0.60234	3.564977	-0.75956	3.722195	
α:1 β:1 γ:1	α:1 β:1 γ:0.5	1.555759	-0.5279	3.639417	-0.68512	3.796636	
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.01382	-2.09748	2.06984	-2.2547	2.227058	
α:1 β:0.25 γ:1	MV	-0.35577	-2.43943	1.727889	-2.59665	1.885108	
α:1 β:0.25 γ:1	ES	0.028376	-2.05528	2.112033	-2.2125	2.269252	
α:1 β:0.25 γ:1	MaxRet	-27.8069	-29.8906	-25.7232	-30.0478	-25.566025	Yes
α:1 β:0.25 γ:1	α:1 β:0 γ:0	-0.02301	-2.10666	2.060651	-2.26388	2.21787	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	1.443341	-0.64032	3.526999	-0.79754	3.684218	
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	1.449026	-0.63463	3.532683	-0.79185	3.689902	
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	1.551006	-0.53265	3.634664	-0.68987	3.791883	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	1.625447	-0.45821	3.709105	-0.61543	3.866323	
α:1 β:0.1 γ:1	MV	-0.34195	-2.42561	1.741708	-2.58283	1.898926	
α:1 β:0.1 γ:1	ES	0.042194	-2.04146	2.125852	-2.19868	2.28307	
α:1 β:0.1 γ:1	MaxRet	-27.7931	-29.8767	-25.7094	-30.034	-25.552207	Yes
α:1 β:0.1 γ:1	α:1 β:0 γ:0	-0.00919	-2.09285	2.07447	-2.25006	2.231688	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	1.45716	-0.6265	3.540817	-0.78372	3.698036	
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	1.462844	-0.62081	3.546502	-0.77803	3.70372	
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	1.564825	-0.51883	3.648482	-0.67605	3.805701	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	1.639265	-0.44439	3.722923	-0.60161	3.880141	
MV	ES	0.384144	-1.69951	2.467802	-1.85673	2.62502	
MV	MaxRet	-27.4511	-29.5348	-25.3675	-29.692	-25.210257	Yes
MV	α:1 β:0 γ:0	0.332762	-1.7509	2.41642	-1.90811	2.573638	
MV	α:1 β:1 γ:0.25	1.79911	-0.28455	3.882768	-0.44177	4.039986	
MV	α:1 β:0.25 γ:0.25	1.804794	-0.27886	3.888452	-0.43608	4.04567	
MV	α: 1 β:0.1 γ:0.25	1.906775	-0.17688	3.990433	-0.3341	4.147651	
MV	α:1 β:1 γ:0.5	1.981215	-0.10244	4.064873	-0.25966	4.222091	
ES	MaxRet	-27.8353	-29.9189	-25.7516	-30.0762	-25.594401	Yes
ES	α:1 β:0 γ:0	-0.05138	-2.13504	2.032276	-2.29226	2.189494	
ES	α:1 β:1 γ:0.25	1.414966	-0.66869	3.498624	-0.82591	3.655842	
ES	α:1 β:0.25 γ:0.25	1.42065	-0.66301	3.504308	-0.82023	3.661526	
ES	α: 1 β:0.1 γ:0.25	1.522631	-0.56103	3.606289	-0.71825	3.763507	
ES	α:1 β:1 γ:0.5	1.597071	-0.48659	3.680729	-0.64381	3.837947	
MaxRet	α:1 β:0 γ:0	27.7839	25.70024	29.86755	25.54302	30.024771	Yes

Table 4. Continued.

MaxRet	α:1 β:1 γ:0.25	29.25024	27.16659	31.3339	27.00937	31.491119	Yes
MaxRet	α:1 β:0.25 γ:0.25	29.25593	27.17227	31.33959	27.01505	31.496804	Yes
MaxRet	α: 1 β:0.1 γ:0.25	29.35791	27.27425	31.44157	27.11703	31.598784	Yes
MaxRet	α:1 β:1 γ:0.5	29.43235	27.34869	31.51601	27.19147	31.673225	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.25	1.466348	-0.61731	3.550006	-0.77453	3.707224	
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	1.472032	-0.61163	3.55569	-0.76884	3.712908	
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	1.574013	-0.50965	3.657671	-0.66686	3.814889	
α:1 β:0 γ:0	α:1 β:1 γ:0.5	1.648453	-0.43521	3.732111	-0.59242	3.889329	
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	0.005684	-2.07797	2.089342	-2.23519	2.246561	
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	0.107665	-1.97599	2.191323	-2.13321	2.348541	
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	0.182105	-1.90155	2.265763	-2.05877	2.422982	
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	0.101981	-1.98168	2.185638	-2.1389	2.342857	
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	0.176421	-1.90724	2.260079	-2.06446	2.417297	
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	0.07444	-2.00922	2.158098	-2.16644	2.315317	

Table 5.

Tukey-Kramer Difference of means of arithmetic average return compared by strategies under ex post assumption.

Ex Post Return			90% Confidence Limits		95% Confidence Limits		Sig. Diff
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	0.058102	0.033956	0.082249	0.032134	0.084071	Yes
EQ	α:1 β:0.1 γ:0.5	0.047934	0.023788	0.072081	0.021966	0.073903	Yes
EQ	α:1 β:1 γ:1	0.001559	-0.02259	0.025706	-0.02441	0.027528	
EQ	α:1 β:0.25 γ:1	-0.00133	-0.02547	0.02282	-0.0273	0.024642	
EQ	α:1 β:0.1 γ:1	-0.0019	-0.02605	0.022246	-0.02787	0.024068	
EQ	**MV**	-0.00414	-0.02829	0.020003	-0.03011	0.021825	
EQ	**ES**	-5.1E-06	-0.02415	0.024141	-0.02597	0.025963	
EQ	**MaxRet**	-0.26314	-0.28729	-0.23899	-0.28911	-0.237173	Yes
EQ	α:1 β:0 γ:0	-0.00229	-0.02643	0.021861	-0.02825	0.023683	
EQ	α:1 β:1 γ:0.25	0.081092	0.056946	0.105239	0.055124	0.107061	Yes
EQ	α:1 β:0.25 γ:0.25	0.071906	0.04776	0.096053	0.045938	0.097875	Yes
EQ	α: 1 β:0.1 γ:0.25	0.069682	0.045536	0.093828	0.043714	0.09565	Yes
EQ	α:1 β:1 γ:0.5	0.065257	0.04111	0.089403	0.039288	0.091225	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	-0.01017	-0.03431	0.013979	-0.03614	0.015801	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	-0.05654	-0.08069	-0.0324	-0.08251	-0.030575	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	-0.05943	-0.08358	-0.03528	-0.0854	-0.03346	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-0.06	-0.08415	-0.03586	-0.08597	-0.034034	Yes
α:1 β:0.25 γ:0.5	**MV**	-0.06225	-0.08639	-0.0381	-0.08821	-0.036277	Yes
α:1 β:0.25 γ:0.5	**ES**	-0.05811	-0.08225	-0.03396	-0.08408	-0.032139	Yes
α:1 β:0.25 γ:0.5	**MaxRet**	-0.32124	-0.34539	-0.2971	-0.34721	-0.295275	Yes
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	-0.06039	-0.08453	-0.03624	-0.08636	-0.034419	Yes
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	0.02299	-0.00116	0.047136	-0.00298	0.048958	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	0.013804	-0.01034	0.03795	-0.01216	0.039772	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	0.01158	-0.01257	0.035726	-0.01439	0.037548	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	0.007154	-0.01699	0.031301	-0.01881	0.033123	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	-0.04638	-0.07052	-0.02223	-0.07234	-0.020407	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	-0.04926	-0.07341	-0.02511	-0.07523	-0.023292	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-0.04984	-0.07398	-0.02569	-0.0758	-0.023866	Yes
α:1 β:0.1 γ:0.5	**MV**	-0.05208	-0.07622	-0.02793	-0.07805	-0.026109	Yes
α:1 β:0.1 γ:0.5	**ES**	-0.04794	-0.07209	-0.02379	-0.07391	-0.021971	Yes
α:1 β:0.1 γ:0.5	**MaxRet**	-0.31108	-0.33522	-0.28693	-0.33704	-0.285107	Yes
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	-0.05022	-0.07437	-0.02607	-0.07619	-0.024251	Yes
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	0.033158	0.009011	0.057304	0.007189	0.059126	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	0.023972	-0.00018	0.048118	-0.002	0.04994	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	0.021748	-0.0024	0.045894	-0.00422	0.047716	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	0.017322	-0.00682	0.041468	-0.00865	0.04329	

Table 5. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	-0.00289	-0.02703	0.021261	-0.02885	0.023083	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.00346	-0.02761	0.020687	-0.02943	0.022509	
α:1 β:1 γ:1	**MV**	-0.0057	-0.02985	0.018444	-0.03167	0.020266	
α:1 β:1 γ:1	**ES**	-0.00156	-0.02571	0.022582	-0.02753	0.024404	
α:1 β:1 γ:1	**MaxRet**	-0.2647	-0.28885	-0.24055	-0.29067	-0.238732	Yes
α:1 β:1 γ:1	α:1 β:0 γ:0	-0.00384	-0.02799	0.020302	-0.02981	0.022124	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	0.079533	0.055387	0.103679	0.053565	0.105501	Yes
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	0.070347	0.046201	0.094493	0.044379	0.096315	Yes
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	0.068123	0.043976	0.092269	0.042155	0.094091	Yes
α:1 β:1 γ:1	α:1 β:1 γ:0.5	0.063697	0.039551	0.087844	0.037729	0.089666	Yes
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.00057	-0.02472	0.023573	-0.02654	0.025394	
α:1 β:0.25 γ:1	**MV**	-0.00282	-0.02696	0.021329	-0.02879	0.023151	
α:1 β:0.25 γ:1	**ES**	0.001321	-0.02283	0.025468	-0.02465	0.027289	
α:1 β:0.25 γ:1	**MaxRet**	-0.26182	-0.28596	-0.23767	-0.28778	-0.235846	Yes
α:1 β:0.25 γ:1	α:1 β:0 γ:0	-0.00096	-0.02511	0.023188	-0.02693	0.025009	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	0.082419	0.058272	0.106565	0.05645	0.108387	Yes
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	0.073232	0.049086	0.097379	0.047264	0.099201	Yes
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	0.071008	0.046862	0.095155	0.04504	0.096977	Yes
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	0.066583	0.042436	0.090729	0.040615	0.092551	Yes
α:1 β:0.1 γ:1	**MV**	-0.00224	-0.02639	0.021903	-0.02821	0.023725	
α:1 β:0.1 γ:1	**ES**	0.001895	-0.02225	0.026041	-0.02407	0.027863	
α:1 β:0.1 γ:1	**MaxRet**	-0.26124	-0.28539	-0.23709	-0.28721	-0.235272	Yes
α:1 β:0.1 γ:1	α:1 β:0 γ:0	-0.00039	-0.02453	0.023761	-0.02635	0.025583	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	0.082992	0.058846	0.107139	0.057024	0.108961	Yes
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	0.073806	0.04966	0.097953	0.047838	0.099775	Yes
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	0.071582	0.047436	0.095729	0.045614	0.09755	Yes
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	0.067157	0.04301	0.091303	0.041188	0.093125	Yes
MV	**ES**	0.004138	-0.02001	0.028284	-0.02183	0.030106	
MV	**MaxRet**	-0.259	-0.28314	-0.23485	-0.28497	-0.233029	Yes
MV	α:1 β:0 γ:0	0.001858	-0.02229	0.026004	-0.02411	0.027826	
MV	α:1 β:1 γ:0.25	0.085235	0.061089	0.109382	0.059267	0.111204	Yes
MV	α:1 β:0.25 γ:0.25	0.076049	0.051903	0.100196	0.050081	0.102018	Yes
MV	α: 1 β:0.1 γ:0.25	0.073825	0.049679	0.097972	0.047857	0.099794	Yes
MV	α:1 β:1 γ:0.5	0.0694	0.045253	0.093546	0.043431	0.095368	Yes
ES	**MaxRet**	-0.26314	-0.28728	-0.23899	-0.2891	-0.237167	Yes
ES	α:1 β:0 γ:0	-0.00228	-0.02643	0.021866	-0.02825	0.023688	
ES	α:1 β:1 γ:0.25	0.081097	0.056951	0.105244	0.055129	0.107066	Yes
ES	α:1 β:0.25 γ:0.25	0.071911	0.047765	0.096058	0.045943	0.09788	Yes
ES	α: 1 β:0.1 γ:0.25	0.069687	0.045541	0.093834	0.043719	0.095655	Yes
ES	α:1 β:1 γ:0.5	0.065262	0.041115	0.089408	0.039293	0.09123	Yes
MaxRet	α:1 β:0 γ:0	0.260856	0.236709	0.285002	0.234887	0.286824	Yes

Table 5. Continued

MaxRet	α:1 β:1 γ:0.25	0.344233	0.320087	0.368379	0.318265	0.370201	Yes
MaxRet	α:1 β:0.25 γ:0.25	0.335047	0.310901	0.359193	0.309079	0.361015	Yes
MaxRet	α: 1 β:0.1 γ:0.25	0.332823	0.308676	0.356969	0.306855	0.358791	Yes
MaxRet	α:1 β:1 γ:0.5	0.328397	0.304251	0.352544	0.302429	0.354366	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.25	0.083377	0.059231	0.107524	0.057409	0.109346	Yes
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	0.074191	0.050045	0.098338	0.048223	0.10016	Yes
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	0.071967	0.047821	0.096114	0.045999	0.097935	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.5	0.067542	0.043395	0.091688	0.041573	0.09351	Yes
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	-0.00919	-0.03333	0.01496	-0.03515	0.016782	
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	-0.01141	-0.03556	0.012736	-0.03738	0.014558	
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	-0.01584	-0.03998	0.008311	-0.0418	0.010133	
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	-0.00222	-0.02637	0.021922	-0.02819	0.023744	
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	-0.00665	-0.0308	0.017497	-0.03262	0.019319	
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	-0.00443	-0.02857	0.019721	-0.03039	0.021543	

Table 6.

Tukey-Kramer difference of means of geometric average returned compared by strategies under ex post assumption.

Ex Post Compound			90% Confidence Limits		95% Confidence Limits		Sig. Diff
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	0.057512	0.030621	0.084402	0.028566	0.086458	Yes
EQ	α:1 β:0.1 γ:0.5	0.047348	0.020457	0.074239	0.018402	0.076294	Yes
EQ	α:1 β:1 γ:1	0.001392	-0.0255	0.028283	-0.02755	0.030338	
EQ	α:1 β:0.25 γ:1	-0.00144	-0.02833	0.025454	-0.03038	0.027509	
EQ	α:1 β:0.1 γ:1	-0.002	-0.02889	0.024888	-0.03095	0.026943	
EQ	**MV**	-0.00484	-0.03173	0.02205	-0.03379	0.024106	
EQ	**ES**	-4.7E-06	-0.0269	0.026886	-0.02895	0.028941	
EQ	**MaxRet**	-0.26044	-0.28733	-0.23355	-0.28939	-0.231496	Yes
EQ	α:1 β:0 γ:0	-0.00238	-0.02927	0.02451	-0.03133	0.026565	
EQ	α:1 β:1 γ:0.25	0.081499	0.054608	0.10839	0.052553	0.110445	Yes
EQ	α:1 β:0.25 γ:0.25	0.071773	0.044883	0.098664	0.042827	0.100719	Yes
EQ	α: 1 β:0.1 γ:0.25	0.069257	0.042366	0.096148	0.040311	0.098203	Yes
EQ	α:1 β:1 γ:0.5	0.064712	0.037821	0.091602	0.035766	0.093658	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	-0.01016	-0.03705	0.016727	-0.03911	0.018782	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	-0.05612	-0.08301	-0.02923	-0.08507	-0.027174	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	-0.05895	-0.08584	-0.03206	-0.08789	-0.030002	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-0.05952	-0.08641	-0.03262	-0.08846	-0.030569	Yes
α:1 β:0.25 γ:0.5	**MV**	-0.06235	-0.08924	-0.03546	-0.0913	-0.033406	Yes
α:1 β:0.25 γ:0.5	**ES**	-0.05752	-0.08441	-0.03063	-0.08646	-0.02857	Yes
α:1 β:0.25 γ:0.5	**MaxRet**	-0.31795	-0.34484	-0.29106	-0.3469	-0.289007	Yes
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	-0.05989	-0.08678	-0.033	-0.08884	-0.030947	Yes
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	0.023987	-0.0029	0.050878	-0.00496	0.052933	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	0.014262	-0.01263	0.041153	-0.01468	0.043208	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	0.011745	-0.01515	0.038636	-0.0172	0.040691	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	0.0072	-0.01969	0.034091	-0.02175	0.036146	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	-0.04596	-0.07285	-0.01907	-0.0749	-0.01701	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	-0.04879	-0.07568	-0.02189	-0.07773	-0.019839	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-0 04935	-0.07624	-0.02246	-0.0783	-0.020405	Yes
α:1 β:0.1 γ:0.5	**MV**	-0.05219	-0.07908	-0.0253	-0.08113	-0.023243	Yes
α:1 β:0.1 γ:0.5	**ES**	-0.04735	-0.07424	-0.02046	-0.0763	-0.018407	Yes
α:1 β:0.1 γ:0.5	**MaxRet**	-0.30779	-0.33468	-0.2809	-0.33674	-0.278844	Yes
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	-0.04973	-0.07662	-0.02284	-0.07868	-0.020783	Yes
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	0.034151	0.00726	0.061042	0.005205	0.063097	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	0.024425	-0.00247	0.051316	-0.00452	0.053371	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	0.021909	-0.00498	0.0488	-0.00704	0.050855	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	0.017364	-0.00953	0.044254	-0.01158	0.04631	

Table 6. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	-0.00283	-0.02972	0.024062	-0.03178	0.026117	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.0034	-0.03029	0.023495	-0.03234	0.025551	
α:1 β:1 γ:1	MV	-0.00623	-0.03312	0.020658	-0.03518	0.022713	
α:1 β:1 γ:1	ES	-0.0014	-0.02829	0.025494	-0.03034	0.027549	
α:1 β:1 γ:1	MaxRet	-0.26183	-0.28873	-0.23494	-0.29078	-0.232888	Yes
α:1 β:1 γ:1	α:1 β:0 γ:0	-0.00377	-0.03066	0.023118	-0.03272	0.025173	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	0.080107	0.053216	0.106998	0.051161	0.109053	Yes
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	0.070381	0.043491	0.097272	0.041435	0.099327	Yes
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	0.067865	0.040974	0.094756	0.038919	0.096811	Yes
α:1 β:1 γ:1	α:1 β:1 γ:0.5	0.06332	0.036429	0.09021	0.034374	0.092266	Yes
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.00057	-0.02746	0.026324	-0.02951	0.028379	
α:1 β:0.25 γ:1	MV	-0.0034	-0.0303	0.023487	-0.03235	0.025542	
α:1 β:0.25 γ:1	ES	0.001432	-0.02546	0.028323	-0.02751	0.030378	
α:1 β:0.25 γ:1	MaxRet	-0.25901	-0.2859	-0.23211	-0.28795	-0.230059	Yes
α:1 β:0.25 γ:1	α:1 β:0 γ:0	-0.00094	-0.02784	0.025946	-0.02989	0.028002	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	0.082936	0.056045	0.109826	0.05399	0.111882	Yes
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	0.07321	0.046319	0.100101	0.044264	0.102156	Yes
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	0.070694	0.043803	0.097584	0.041748	0.09964	Yes
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	0.066148	0.039257	0.093039	0.037202	0.095094	Yes
α:1 β:0.1 γ:1	MV	-0.00284	-0.02973	0.024053	-0.03178	0.026109	
α:1 β:0.1 γ:1	ES	0.001998	-0.02489	0.028889	-0.02695	0.030944	
α:1 β:0.1 γ:1	MaxRet	-0.25844	-0.28533	-0.23155	-0.28739	-0.229493	Yes
α:1 β:0.1 γ:1	α:1 β:0 γ:0	-0.00038	-0.02727	0.026513	-0.02932	0.028568	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	0.083502	0.056612	0.110393	0.054556	0.112448	Yes
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	0.073777	0.046886	0.100667	0.044831	0.102723	Yes
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	0.07126	0.044369	0.098151	0.042314	0.100206	Yes
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	0.066715	0.039824	0.093606	0.037769	0.095661	Yes
MV	ES	0.004836	-0.02206	0.031726	-0.02411	0.033782	
MV	MaxRet	-0.2556	-0.28249	-0.22871	-0.28455	-0.226655	Yes
MV	α:1 β:0 γ:0	0.002459	-0.02443	0.02935	-0.02649	0.031405	
MV	α:1 β:1 γ:0.25	0.08634	0.059449	0.11323	0.057394	0.115286	Yes
MV	α:1 β:0.25 γ:0.25	0.076614	0.049723	0.103505	0.047668	0.10556	Yes
MV	α: 1 β:0.1 γ:0.25	0.074097	0.047207	0.100988	0.045151	0.103043	Yes
MV	α:1 β:1 γ:0.5	0.069552	0.042661	0.096443	0.040606	0.098498	Yes
ES	MaxRet	-0.26044	-0.28733	-0.23355	-0.28938	-0.231491	Yes
ES	α:1 β:0 γ:0	-0.00238	-0.02927	0.024514	-0.03132	0.02657	
ES	α:1 β:1 γ:0.25	0.081504	0.054613	0.108395	0.052558	0.11045	Yes
ES	α:1 β:0.25 γ:0.25	0.071778	0.044887	0.098669	0.042832	0.100724	Yes
ES	α: 1 β:0.1 γ:0.25	0.069262	0.042371	0.096152	0.040316	0.098208	Yes
ES	α:1 β:1 γ:0.5	0.064716	0.037826	0.091607	0.03577	0.093662	Yes
MaxRet	α:1 β:0 γ:0	0.258061	0.23117	0.284951	0.229115	0.287007	Yes
MaxRet	α:1 β:1 γ:0.25	0.341941	0.31505	0.368832	0.312995	0.370887	Yes

Table 6. Continued.

MaxRet	α:1 β:0.25 γ:0.25	0.332215	0.305324	0.359106	0.303269	0.361161	Yes
MaxRet	α: 1 β:0.1 γ:0.25	0.329699	0.302808	0.356589	0.300753	0.358645	Yes
MaxRet	α:1 β:1 γ:0.5	0.325153	0.298263	0.352044	0.296207	0.354099	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.25	0.08388	0.056989	0.110771	0.054934	0.112826	Yes
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	0.074154	0.047264	0.101045	0.045208	0.1031	Yes
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	0.071638	0.044747	0.098529	0.042692	0.100584	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.5	0.067093	0.040202	0.093983	0.038147	0.096039	Yes
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	-0.00973	-0.03662	0.017165	-0.03867	0.01922	
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	-0.01224	-0.03913	0.014649	-0.04119	0.016704	
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	-0.01679	-0.04368	0.010103	-0.04573	0.012158	
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	-0.00252	-0.02941	0.024374	-0.03146	0.02643	
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	-0.00706	-0.03395	0.019829	-0.03601	0.021884	
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	-0.00455	-0.03144	0.022345	-0.03349	0.024401	

Table 7.

Tukey-Kramer difference of means of risk (metric standard deviation) compared by strategies under ex post assumption.

Ex Post Sharpe			90% Confidence Limits		95% Confidence Limits		Sig. Diff
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	1.529144	-0.55451	3.612802	-0.71173	3.77002	
EQ	α:1 β:0.1 γ:0.5	1.285341	-0.79832	3.368999	-0.95554	3.526217	
EQ	α:1 β:1 γ:1	0.041201	-2.04246	2.124858	-2.19968	2.282077	
EQ	α:1 β:0.25 γ:1	-0.02849	-2.11214	2.055171	-2.26936	2.21239	
EQ	α:1 β:0.1 γ:1	-0.04231	-2.12596	2.041353	-2.28318	2.198571	
EQ	**MV**	-0.38426	-2.46791	1.699403	-2.62513	1.856621	
EQ	**ES**	-0.00011	-2.08377	2.083547	-2.24099	2.240765	
EQ	**MaxRet**	-27.8354	-29.919	-25.7517	-30.0763	-25.594512	Yes
EQ	α:1 β:0 γ:0	-0.05149	-2.13515	2.032165	-2.29237	2.189383	
EQ	α:1 β:1 γ:0.25	1.414855	-0.6688	3.498513	-0.82602	3.655731	
EQ	α:1 β:0.25 γ:0.25	1.420539	-0.66312	3.504197	-0.82034	3.661415	
EQ	α: 1 β:0.1 γ:0.25	1.52252	-0.56114	3.606178	-0.71836	3.763396	
EQ	α:1 β:1 γ:0.5	1.59696	-0.4867	3.680618	-0.64392	3.837836	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	-0.2438	-2.32746	1.839855	-2.48468	1.997073	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	-1.48794	-3.5716	0.595715	-3.72882	0.752933	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	-1.55763	-3.64129	0.526027	-3.79851	0.683246	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-1.57145	-3.65511	0.512209	-3.81233	0.669428	
α:1 β:0.25 γ:0.5	**MV**	-1.9134	-3.99706	0.170259	-4.15428	0.327478	
α:1 β:0.25 γ:0.5	**ES**	-1.52926	-3.61291	0.554403	-3.77013	0.711622	
α:1 β:0.25 γ:0.5	**MaxRet**	-29.3645	-31.4482	-27.2809	-31.6054	-27.123656	Yes
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	-1.58064	-3.66429	0.503021	-3.82151	0.66024	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	-0.11429	-2.19795	1.969369	-2.35517	2.126587	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	-0.10861	-2.19226	1.975053	-2.34948	2.132272	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	-0.00662	-2.09028	2.077034	-2.2475	2.234252	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	0.067816	-2.01584	2.151474	-2.17306	2.308693	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	-1.24414	-3.3278	0.839518	-3.48502	0.996736	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	-1.31383	-3.39749	0.76983	-3.5547	0.927049	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-1.32765	-3.4113	0.756012	-3.56852	0.913231	
α:1 β:0.1 γ:0.5	**MV**	-1.6696	-3.75325	0.414062	-3.91047	0.57128	
α:1 β:0.1 γ:0.5	**ES**	-1.28545	-3.36911	0.798206	-3.52633	0.955424	
α:1 β:0.1 γ:0.5	**MaxRet**	-29.1207	-31.2044	-27.0371	-31.3616	-26.879853	Yes
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	-1.33683	-3.42049	0.746824	-3.57771	0.904042	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	0.129514	-1.95414	2.213172	-2.11136	2.37039	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	0.135198	-1.94846	2.218856	-2.10568	2.376074	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	0.237179	-1.84648	2.320837	-2.0037	2.478055	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	0.311619	-1.77204	2.395277	-1.92926	2.552496	

Table 7. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	-0.06969	-2.15335	2.01397	-2.31056	2.171189	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.08351	-2.16716	2.000152	-2.32438	2.157371	
α:1 β:1 γ:1	MV	-0.42546	-2.50911	1.658202	-2.66633	1.815421	
α:1 β:1 γ:1	ES	-0.04131	-2.12497	2.042346	-2.28219	2.199565	
α:1 β:1 γ:1	MaxRet	-27.8766	-29.9602	-25.7929	-30.1175	-25.635713	Yes
α:1 β:1 γ:1	α:1 β:0 γ:0	-0.09269	-2.17635	1.990964	-2.33357	2.148183	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	1.373654	-0.71	3.457312	-0.86722	3.61453	
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	1.379338	-0.70432	3.462996	-0.86154	3.620215	
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	1.481319	-0.60234	3.564977	-0.75956	3.722195	
α:1 β:1 γ:1	α:1 β:1 γ:0.5	1.555759	-0.5279	3.639417	-0.68512	3.796636	
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.01382	-2.09748	2.06984	-2.2547	2.227058	
α:1 β:0.25 γ:1	MV	-0.35577	-2.43943	1.727889	-2.59665	1.885108	
α:1 β:0.25 γ:1	ES	0.028376	-2.05528	2.112033	-2.2125	2.269252	
α:1 β:0.25 γ:1	MaxRet	-27.8069	-29.8906	-25.7232	-30.0478	-25.566025	Yes
α:1 β:0.25 γ:1	α:1 β:0 γ:0	-0.02301	-2.10666	2.060651	-2.26388	2.21787	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	1.443341	-0.64032	3.526999	-0.79754	3.684218	
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	1.449026	-0.63463	3.532683	-0.79185	3.689902	
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	1.551006	-0.53265	3.634664	-0.68987	3.791883	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	1.625447	-0.45821	3.709105	-0.61543	3.866323	
α:1 β:0.1 γ:1	MV	-0.34195	-2.42561	1.741708	-2.58283	1.898926	
α:1 β:0.1 γ:1	ES	0.042194	-2.04146	2.125852	-2.19868	2.28307	
α:1 β:0.1 γ:1	MaxRet	-27.7931	-29.8767	-25.7094	-30.034	-25.552207	Yes
α:1 β:0.1 γ:1	α:1 β:0 γ:0	-0.00919	-2.09285	2.07447	-2.25006	2.231688	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	1.45716	-0.6265	3.540817	-0.78372	3.698036	
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	1.462844	-0.62081	3.546502	-0.77803	3.70372	
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	1.564825	-0.51883	3.648482	-0.67605	3.805701	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	1.639265	-0.44439	3.722923	-0.60161	3.880141	
MV	ES	0.384144	-1.69951	2.467802	-1.85673	2.62502	
MV	MaxRet	-27.4511	-29.5348	-25.3675	-29.692	-25.210257	Yes
MV	α:1 β:0 γ:0	0.332762	-1.7509	2.41642	-1.90811	2.573638	
MV	α:1 β:1 γ:0.25	1.79911	-0.28455	3.882768	-0.44177	4.039986	
MV	α:1 β:0.25 γ:0.25	1.804794	-0.27886	3.888452	-0.43608	4.04567	
MV	α: 1 β:0.1 γ:0.25	1.906775	-0.17688	3.990433	-0.3341	4.147651	
MV	α:1 β:1 γ:0.5	1.981215	-0.10244	4.064873	-0.25966	4.222091	
ES	MaxRet	-27.8353	-29.9189	-25.7516	-30.0762	-25.594401	Yes
ES	α:1 β:0 γ:0	-0.05138	-2.13504	2.032276	-2.29226	2.189494	
ES	α:1 β:1 γ:0.25	1.414966	-0.66869	3.498624	-0.82591	3.655842	
ES	α:1 β:0.25 γ:0.25	1.42065	-0.66301	3.504308	-0.82023	3.661526	
ES	α: 1 β:0.1 γ:0.25	1.522631	-0.56103	3.606289	-0.71825	3.763507	
ES	α:1 β:1 γ:0.5	1.597071	-0.48659	3.680729	-0.64381	3.837947	
MaxRet	α:1 β:0 γ:0	27.7839	25.70024	29.86755	25.54302	30.024771	Yes

Table 7. Continued.

MaxRet	α:1 β:1 γ:0.25	29.25024	27.16659	31.3339	27.00937	31.491119	Yes
MaxRet	α:1 β:0.25 γ:0.25	29.25593	27.17227	31.33959	27.01505	31.496804	Yes
MaxRet	α: 1 β:0.1 γ:0.25	29.35791	27.27425	31.44157	27.11703	31.598784	Yes
MaxRet	α:1 β:1 γ:0.5	29.43235	27.34869	31.51601	27.19147	31.673225	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.25	1.466348	-0.61731	3.550006	-0.77453	3.707224	
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	1.472032	-0.61163	3.55569	-0.76884	3.712908	
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	1.574013	-0.50965	3.657671	-0.66686	3.814889	
α:1 β:0 γ:0	α:1 β:1 γ:0.5	1.648453	-0.43521	3.732111	-0.59242	3.889329	
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	0.005684	-2.07797	2.089342	-2.23519	2.246561	
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	0.107665	-1.97599	2.191323	-2.13321	2.348541	
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	0.182105	-1.90155	2.265763	-2.05877	2.422982	
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	0.101981	-1.98168	2.185638	-2.1389	2.342857	
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	0.176421	-1.90724	2.260079	-2.06446	2.417297	
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	0.07444	-2.00922	2.158098	-2.16644	2.315317	

Table 8.

Tukey-Kramer difference of means of Sharpe ratio compared by strategies under ex ante assumption.

Ex Ante Sharpe			90% Confidence Limits		95% Confidence Limits		Sig. Diff.
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	-0.13406	-1.63029	1.362172	-1.74323	1.475113	
EQ	α:1 β:0.1 γ:0.5	-0.11102	-1.60725	1.385213	-1.72019	1.498153	
EQ	α:1 β:1 γ:1	-0.00536	-1.50159	1.490873	-1.61453	1.603814	
EQ	α:1 β:0.25 γ:1	0.000244	-1.49599	1.496476	-1.60893	1.609417	
EQ	α:1 β:0.1 γ:1	-0.21956	-1.71579	1.276674	-1.82873	1.389614	
EQ	**MV**	0.008135	-1.4881	1.504368	-1.60104	1.617308	
EQ	**ES**	7.26E-05	-1.49616	1.496305	-1.6091	1.609246	
EQ	**MaxRet**	1.348472	-0.14776	2.844705	-0.2607	2.957646	
EQ	α:1 β:0 γ:0	0.002105	-1.49413	1.498338	-1.60707	1.611278	
EQ	α:1 β:1 γ:0.25	0.072246	-1.42399	1.568479	-1.53693	1.681419	
EQ	α:1 β:0.25 γ:0.25	0.045158	-1.45107	1.541391	-1.56402	1.654331	
EQ	α: 1 β:0.1 γ:0.25	-0.01597	-1.5122	1.480261	-1.62514	1.593202	
EQ	α:1 β:1 γ:0.5	-0.15145	-1.64769	1.344779	-1.76063	1.457719	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	0.02304	-1.47319	1.519273	-1.58613	1.632213	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	0.128701	-1.36753	1.624933	-1.48047	1.737874	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	0.134304	-1.36193	1.630536	-1.47487	1.743477	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-0.0855	-1.58173	1.410734	-1.69467	1.523674	
α:1 β:0.25 γ:0.5	**MV**	0.142195	-1.35404	1.638428	-1.46698	1.751368	
α:1 β:0.25 γ:0.5	**ES**	0.134133	-1.3621	1.630365	-1.47504	1.743306	
α:1 β:0.25 γ:0.5	**MaxRet**	1.482533	-0.0137	2.978765	-0.12664	3.091706	
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	0.136165	-1.36007	1.632398	-1.47301	1.745338	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	0.206306	-1.28993	1.702539	-1.40287	1.815479	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	0.179218	-1.31701	1.675451	-1.42996	1.788391	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	0.118089	-1.37814	1.614321	-1.49108	1.727262	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	-0.01739	-1.51363	1.478839	-1.62657	1.591779	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	0.105661	-1.39057	1.601893	-1.50351	1.714834	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	0.111263	-1.38497	1.607496	-1.49791	1.720436	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-0.10854	-1.60477	1.387693	-1.71771	1.500634	
α:1 β:0.1 γ:0.5	**MV**	0.119155	-1.37708	1.615388	-1.49002	1.728328	
α:1 β:0.1 γ:0.5	**ES**	0.111092	-1.38514	1.607325	-1.49808	1.720265	
α:1 β:0.1 γ:0.5	**MaxRet**	1.459492	-0.03674	2.955725	-0.14968	3.068665	
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	0.113125	-1.38311	1.609357	-1.49605	1.722298	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	0.183266	-1.31297	1.679499	-1.42591	1.792439	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	0.156178	-1.34006	1.652411	-1.453	1.765351	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	0.095048	-1.40118	1.591281	-1.51413	1.704221	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	-0.04043	-1.53667	1.455799	-1.64961	1.568739	

Table 8. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	0.005603	-1.49063	1.501835	-1.60357	1.614776
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.2142	-1.71043	1.282033	-1.82337	1.394973
α:1 β:1 γ:1	MV	0.013495	-1.48274	1.509727	-1.59568	1.622668
α:1 β:1 γ:1	ES	0.005432	-1.4908	1.501664	-1.60374	1.614605
α:1 β:1 γ:1	MaxRet	1.353832	-0.1424	2.850064	-0.25534	2.963005
α:1 β:1 γ:1	α:1 β:0 γ:0	0.007464	-1.48877	1.503697	-1.60171	1.616637
α:1 β:1 γ:1	α:1 β:1 γ:0.25	0.077606	-1.41863	1.573838	-1.53157	1.686779
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	0.050517	-1.44572	1.54675	-1.55866	1.65969
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	-0.01061	-1.50685	1.48562	-1.61979	1.598561
α:1 β:1 γ:1	α:1 β:1 γ:0.5	-0.1461	-1.64233	1.350138	-1.75527	1.463078
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.2198	-1.71604	1.27643	-1.82898	1.38937
α:1 β:0.25 γ:1	MV	0.007892	-1.48834	1.504124	-1.60128	1.617065
α:1 β:0.25 γ:1	ES	-0.00017	-1.4964	1.496062	-1.60934	1.609002
α:1 β:0.25 γ:1	MaxRet	1.348229	-0.148	2.844461	-0.26094	2.957402
α:1 β:0.25 γ:1	α:1 β:0 γ:0	0.001862	-1.49437	1.498094	-1.60731	1.611035
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	0.072003	-1.42423	1.568235	-1.53717	1.681176
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	0.044915	-1.45132	1.541147	-1.56426	1.654088
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	-0.01622	-1.51245	1.480018	-1.62539	1.592958
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	-0.1517	-1.64793	1.344535	-1.76087	1.457476
α:1 β:0.1 γ:1	MV	0.227694	-1.26854	1.723927	-1.38148	1.836867
α:1 β:0.1 γ:1	ES	0.219632	-1.2766	1.715864	-1.38954	1.828805
α:1 β:0.1 γ:1	MaxRet	1.568032	0.071799	3.064264	-0.04114	3.177205
α:1 β:0.1 γ:1	α:1 β:0 γ:0	0.221664	-1.27457	1.717897	-1.38751	1.830837
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	0.291805	-1.20443	1.788038	-1.31737	1.900978
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	0.264717	-1.23152	1.76095	-1.34446	1.87389
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	0.203588	-1.29265	1.69982	-1.40559	1.812761
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	0.068105	-1.42813	1.564338	-1.54107	1.677278
MV	ES	-0.00806	-1.5043	1.48817	-1.61724	1.60111
MV	MaxRet	1.340337	-0.1559	2.83657	-0.26884	2.94951
MV	α:1 β:0 γ:0	-0.00603	-1.50226	1.490202	-1.6152	1.603143
MV	α:1 β:1 γ:0.25	0.064111	-1.43212	1.560344	-1.54506	1.673284
MV	α:1 β:0.25 γ:0.25	0.037023	-1.45921	1.533255	-1.57215	1.646196
MV	α: 1 β:0.1 γ:0.25	-0.02411	-1.52034	1.472126	-1.63328	1.585066
MV	α:1 β:1 γ:0.5	-0.15959	-1.65582	1.336643	-1.76876	1.449584
ES	MaxRet	1.3484	-0.14783	2.844632	-0.26077	2.957573
ES	α:1 β:0 γ:0	0.002033	-1.4942	1.498265	-1.60714	1.611206
ES	α:1 β:1 γ:0.25	0.072174	-1.42406	1.568406	-1.537	1.681347
ES	α:1 β:0.25 γ:0.25	0.045086	-1.45115	1.541318	-1.56409	1.654259
ES	α: 1 β:0.1 γ:0.25	-0.01604	-1.51228	1.480189	-1.62522	1.593129
ES	α:1 β:1 γ:0.5	-0.15153	-1.64776	1.344706	-1.7607	1.457647
MaxRet	α:1 β:0 γ:0	-1.34637	-2.8426	0.149865	-2.95554	0.262806
MaxRet	α:1 β:1 γ:0.25	-1.27623	-2.77246	0.220006	-2.8854	0.332947

Table 8. Continued.

MaxRet	α:1 β:0.25 γ:0.25	-1.30331	-2.79955	0.192918	-2.91249	0.305859
MaxRet	α: 1 β:0.1 γ:0.25	-1.36444	-2.86068	0.131789	-2.97362	0.244729
MaxRet	α:1 β:1 γ:0.5	-1.49993	-2.99616	-0.00369	-3.1091	0.109247
α:1 β:0 γ:0	α:1 β:1 γ:0.25	0.070141	-1.42609	1.566374	-1.53903	1.679314
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	0.043053	-1.45318	1.539286	-1.56612	1.652226
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	-0.01808	-1.51431	1.478156	-1.62725	1.591097
α:1 β:0 γ:0	α:1 β:1 γ:0.5	-0.15356	-1.64979	1.342674	-1.76273	1.455614
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	-0.02709	-1.52332	1.469144	-1.63626	1.582085
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	-0.08822	-1.58445	1.408015	-1.69739	1.520955
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	-0.2237	-1.71993	1.272532	-1.83287	1.385473
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	-0.06113	-1.55736	1.435103	-1.6703	1.548044
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	-0.19661	-1.69285	1.299621	-1.80579	1.412561
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	-0.13548	-1.63172	1.36075	-1.74466	1.473691

Table 9.

Tukey-Kramer difference of means of arithmetic average return compared by strategies under ex ante assumption.

Ex Ante Return			90% Confidence Limits		95% Confidence Limits		Sig. Diff.
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	-0.00668	-0.04034	0.026975	-0.04288	0.029515	
EQ	α:1 β:0.1 γ:0.5	-0.00453	-0.03819	0.029131	-0.04073	0.031672	
EQ	α:1 β:1 γ:1	-7.1E-05	-0.03373	0.033588	-0.03627	0.036128	
EQ	α:1 β:0.25 γ:1	0.000115	-0.03354	0.033774	-0.03609	0.036315	
EQ	α:1 β:0.1 γ:1	-0.0092	-0.04286	0.024462	-0.0454	0.027003	
EQ	**MV**	0.005809	-0.02785	0.039468	-0.03039	0.042009	
EQ	**ES**	5.32E-06	-0.03365	0.033664	-0.03619	0.036205	
EQ	**MaxRet**	0.022645	-0.01101	0.056304	-0.01355	0.058845	
EQ	α:1 β:0 γ:0	0.00019	-0.03347	0.033849	-0.03601	0.03639	
EQ	α:1 β:1 γ:0.25	-0.01248	-0.04614	0.021183	-0.04868	0.023724	
EQ	α:1 β:0.25 γ:0.25	-0.01047	-0.04412	0.023194	-0.04666	0.025735	
EQ	α: 1 β:0.1 γ:0.25	-0.00737	-0.04103	0.026292	-0.04357	0.028833	
EQ	α:1 β:1 γ:0.5	-0.0092	-0.04286	0.024462	-0.0454	0.027003	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	0.002156	-0.0315	0.035815	-0.03404	0.038356	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	0.006613	-0.02705	0.040272	-0.02959	0.042813	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	0.006799	-0.02686	0.040458	-0.0294	0.042999	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-0.00251	-0.03617	0.031146	-0.03871	0.033687	
α:1 β:0.25 γ:0.5	**MV**	0.012494	-0.02117	0.046153	-0.02371	0.048693	
α:1 β:0.25 γ:0.5	**ES**	0.00669	-0.02697	0.040349	-0.02951	0.042889	
α:1 β:0.25 γ:0.5	**MaxRet**	0.02933	-0.00433	0.062989	-0.00687	0.065529	
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	0.006875	-0.02678	0.040534	-0.02933	0.043074	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	-0.00579	-0.03945	0.027867	-0.04199	0.030408	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	-0.00378	-0.03744	0.029879	-0.03998	0.032419	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	-0.00068	-0.03434	0.032976	-0.03688	0.035517	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	-0.00251	-0.03617	0.031146	-0.03871	0.033687	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	0.004457	-0.0292	0.038116	-0.03174	0.040656	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	0.004643	-0.02902	0.038302	-0.03156	0.040842	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-0.00467	-0.03833	0.02899	-0.04087	0.031531	
α:1 β:0.1 γ:0.5	**MV**	0.010337	-0.02332	0.043996	-0.02586	0.046537	
α:1 β:0.1 γ:0.5	**ES**	0.004533	-0.02913	0.038192	-0.03167	0.040733	
α:1 β:0.1 γ:0.5	**MaxRet**	0.027173	-0.00649	0.060832	-0.00903	0.063373	
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	0.004718	-0.02894	0.038377	-0.03148	0.040918	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	-0.00795	-0.04161	0.025711	-0.04415	0.028252	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	-0.00594	-0.0396	0.027722	-0.04214	0.030263	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	-0.00284	-0.0365	0.03082	-0.03904	0.033361	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	-0.00467	-0.03833	0.02899	-0.04087	0.031531	

Table 9. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	0.000186	-0.03347	0.033845	-0.03601	0.036386
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.00913	-0.04279	0.024533	-0.04533	0.027074
α:1 β:1 γ:1	**MV**	0.00588	-0.02778	0.039539	-0.03032	0.04208
α:1 β:1 γ:1	**ES**	7.65E-05	-0.03358	0.033736	-0.03612	0.036276
α:1 β:1 γ:1	**MaxRet**	0.022717	-0.01094	0.056376	-0.01348	0.058916
α:1 β:1 γ:1	α:1 β:0 γ:0	0.000262	-0.0334	0.033921	-0.03594	0.036461
α:1 β:1 γ:1	α:1 β:1 γ:0.25	-0.01241	-0.04606	0.021254	-0.0486	0.023795
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	-0.01039	-0.04405	0.023266	-0.04659	0.025806
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	-0.0073	-0.04096	0.026363	-0.0435	0.028904
α:1 β:1 γ:1	α:1 β:1 γ:0.5	-0.00913	-0.04279	0.024533	-0.04533	0.027074
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.00931	-0.04297	0.024347	-0.04551	0.026888
α:1 β:0.25 γ:1	**MV**	0.005694	-0.02797	0.039353	-0.03051	0.041894
α:1 β:0.25 γ:1	**ES**	-0.00011	-0.03377	0.033549	-0.03631	0.03609
α:1 β:0.25 γ:1	**MaxRet**	0.022531	-0.01113	0.05619	-0.01367	0.05873
α:1 β:0.25 γ:1	α:1 β:0 γ:0	7.55E-05	-0.03358	0.033735	-0.03612	0.036275
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	-0.01259	-0.04625	0.021068	-0.04879	0.023609
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	-0.01058	-0.04424	0.023079	-0.04678	0.02562
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	-0.00748	-0.04114	0.026177	-0.04368	0.028718
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	-0.00931	-0.04297	0.024347	-0.04551	0.026888
α:1 β:0.1 γ:1	**MV**	0.015006	-0.01865	0.048665	-0.02119	0.051206
α:1 β:0.1 γ:1	**ES**	0.009202	-0.02446	0.042861	-0.027	0.045402
α:1 β:0.1 γ:1	**MaxRet**	0.031842	-0.00182	0.065501	-0.00436	0.068042
α:1 β:0.1 γ:1	α:1 β:0 γ:0	0.009387	-0.02427	0.043046	-0.02681	0.045587
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	-0.00328	-0.03694	0.03038	-0.03948	0.032921
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	-0.00127	-0.03493	0.032391	-0.03747	0.034932
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	0.00183	-0.03183	0.035489	-0.03437	0.03803
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	1.39E-17	-0.03366	0.033659	-0.0362	0.0362
MV	**ES**	-0.0058	-0.03946	0.027855	-0.042	0.030396
MV	**MaxRet**	0.016836	-0.01682	0.050495	-0.01936	0.053036
MV	α:1 β:0 γ:0	-0.00562	-0.03928	0.02804	-0.04182	0.030581
MV	α:1 β:1 γ:0.25	-0.01829	-0.05194	0.015374	-0.05449	0.017915
MV	α:1 β:0.25 γ:0.25	-0.01627	-0.04993	0.017385	-0.05247	0.019926
MV	α: 1 β:0.1 γ:0.25	-0.01318	-0.04684	0.020483	-0.04938	0.023023
MV	α:1 β:1 γ:0.5	-0.01501	-0.04867	0.018653	-0.05121	0.021194
ES	**MaxRet**	0.02264	-0.01102	0.056299	-0.01356	0.05884
ES	α:1 β:0 γ:0	0.000185	-0.03347	0.033844	-0.03602	0.036385
ES	α:1 β:1 γ:0.25	-0.01248	-0.04614	0.021178	-0.04868	0.023719
ES	α:1 β:0.25 γ:0.25	-0.01047	-0.04413	0.023189	-0.04667	0.02573
ES	α: 1 β:0.1 γ:0.25	-0.00737	-0.04103	0.026287	-0.04357	0.028827
ES	α:1 β:1 γ:0.5	-0.0092	-0.04286	0.024457	-0.0454	0.026997
MaxRet	α:1 β:0 γ:0	-0.02246	-0.05611	0.011204	-0.05866	0.013745
MaxRet	α:1 β:1 γ:0.25	-0.03512	-0.06878	-0.00146	-0.07132	0.001078

Table 9. Continued.

MaxRet	α:1 β:0.25 γ:0.25	-0.03311	-0.06677	0.000549	-0.06931	0.00309
MaxRet	α: 1 β:0.1 γ:0.25	-0.03001	-0.06367	0.003647	-0.06621	0.006187
MaxRet	α:1 β:1 γ:0.5	-0.03184	-0.0655	0.001817	-0.06804	0.004357
α:1 β:0 γ:0	α:1 β:1 γ:0.25	-0.01267	-0.04633	0.020993	-0.04887	0.023533
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	-0.01066	-0.04431	0.023004	-0.04686	0.025545
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	-0.00756	-0.04122	0.026102	-0.04376	0.028642
α:1 β:0 γ:0	α:1 β:1 γ:0.5	-0.00939	-0.04305	0.024272	-0.04559	0.026812
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	0.002011	-0.03165	0.03567	-0.03419	0.038211
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	0.005109	-0.02855	0.038768	-0.03109	0.041308
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	0.003279	-0.03038	0.036938	-0.03292	0.039479
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	0.003098	-0.03056	0.036757	-0.0331	0.039297
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	0.001268	-0.03239	0.034927	-0.03493	0.037467
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	-0.00183	-0.03549	0.031829	-0.03803	0.03437

Table. 10

Tukey-Kramer difference of means of geometric average return compared by strategies under ex ante assumption.

Ex Ante Compound			90% Confidence Limits		95% Confidence Limits		Sig. Diff.
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	-0.0066	-0.038	0.024808	-0.04041	0.027212	
EQ	α:1 β:0.1 γ:0.5	-0.00463	-0.03604	0.026775	-0.03844	0.029179	
EQ	α:1 β:1 γ:1	-0.00011	-0.03151	0.031301	-0.03392	0.033705	
EQ	α:1 β:0.25 γ:1	9.65E-05	-0.03131	0.031502	-0.03371	0.033906	
EQ	α:1 β:0.1 γ:1	-0.00883	-0.04023	0.02258	-0.04264	0.024984	
EQ	MV	0.005276	-0.02613	0.036682	-0.02853	0.039086	
EQ	ES	3.19E-06	-0.0314	0.031409	-0.03381	0.033813	
EQ	MaxRet	0.024605	-0.0068	0.056011	-0.00921	0.058415	
EQ	α:1 β:0 γ:0	0.000164	-0.03124	0.03157	-0.03365	0.033974	
EQ	α:1 β:1 γ:0.25	-0.00956	-0.04097	0.021843	-0.04337	0.024247	
EQ	α:1 β:0.25 γ:0.25	-0.00853	-0.03993	0.022879	-0.04234	0.025283	
EQ	α: 1 β:0.1 γ:0.25	-0.00641	-0.03781	0.024997	-0.04022	0.027401	
EQ	α:1 β:1 γ:0.5	-0.00883	-0.04023	0.02258	-0.04264	0.024984	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	0.001967	-0.02944	0.033373	-0.03184	0.035777	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	0.006493	-0.02491	0.037898	-0.02732	0.040302	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	0.006694	-0.02471	0.0381	-0.02712	0.040504	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	-0.00223	-0.03363	0.029178	-0.03604	0.031582	
α:1 β:0.25 γ:0.5	MV	0.011874	-0.01953	0.04328	-0.02194	0.045684	
α:1 β:0.25 γ:0.5	ES	0.006601	-0.02481	0.038007	-0.02721	0.040411	
α:1 β:0.25 γ:0.5	MaxRet	0.031203	-0.0002	0.062609	-0.00261	0.065013	
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	0.006762	-0.02464	0.038168	-0.02705	0.040572	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	-0.00297	-0.03437	0.028441	-0.03678	0.030845	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	-0.00193	-0.03334	0.029477	-0.03574	0.031881	
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	0.000189	-0.03122	0.031595	-0.03362	0.033999	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	-0.00223	-0.03363	0.029178	-0.03604	0.031582	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	0.004525	-0.02688	0.035931	-0.02928	0.038335	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	0.004727	-0.02668	0.036133	-0.02908	0.038537	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	-0.0042	-0.0356	0.027211	-0.03801	0.029615	
α:1 β:0.1 γ:0.5	MV	0.009907	-0.0215	0.041312	-0.0239	0.043716	
α:1 β:0.1 γ:0.5	ES	0.004633	-0.02677	0.036039	-0.02918	0.038443	
α:1 β:0.1 γ:0.5	MaxRet	0.029235	-0.00217	0.060641	-0.00457	0.063045	
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	0.004794	-0.02661	0.0362	-0.02902	0.038604	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	-0.00493	-0.03634	0.026473	-0.03874	0.028877	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	-0.0039	-0.0353	0.027509	-0.03771	0.029913	
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	-0.00178	-0.03318	0.029628	-0.03559	0.032032	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	-0.0042	-0.0356	0.027211	-0.03801	0.029615	

Table 10. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	0.000201	-0.0312	0.031607	-0.03361	0.034011	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.00872	-0.04013	0.022685	-0.04253	0.025089	
α:1 β:1 γ:1	**MV**	0.005381	-0.02602	0.036787	-0.02843	0.039191	
α:1 β:1 γ:1	**ES**	0.000108	-0.0313	0.031514	-0.0337	0.033918	
α:1 β:1 γ:1	**MaxRet**	0.02471	-0.0067	0.056116	-0.0091	0.05852	
α:1 β:1 γ:1	α:1 β:0 γ:0	0.000269	-0.03114	0.031675	-0.03354	0.034079	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	-0.00946	-0.04086	0.021948	-0.04327	0.024352	
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	-0.00842	-0.03983	0.022984	-0.04223	0.025388	
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	-0.0063	-0.03771	0.025102	-0.04011	0.027506	
α:1 β:1 γ:1	α:1 β:1 γ:0.5	-0.00872	-0.04013	0.022685	-0.04253	0.025089	
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-0.00892	-0.04033	0.022484	-0.04273	0.024888	
α:1 β:0.25 γ:1	**MV**	0.00518	-0.02623	0.036586	-0.02863	0.03899	
α:1 β:0.25 γ:1	**ES**	-9.3E-05	-0.0315	0.031312	-0.0339	0.033716	
α:1 β:0.25 γ:1	**MaxRet**	0.024509	-0.0069	0.055914	-0.0093	0.058318	
α:1 β:0.25 γ:1	α:1 β:0 γ:0	6.77E-05	-0.03134	0.031473	-0.03374	0.033877	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	-0.00966	-0.04107	0.021747	-0.04347	0.024151	
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	-0.00862	-0.04003	0.022783	-0.04243	0.025187	
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	-0.00651	-0.03791	0.024901	-0.04032	0.027305	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	-0.00892	-0.04033	0.022484	-0.04273	0.024888	
α:1 β:0.1 γ:1	**MV**	0.014102	-0.0173	0.045507	-0.01971	0.047911	
α:1 β:0.1 γ:1	**ES**	0.008828	-0.02258	0.040234	-0.02498	0.042638	
α:1 β:0.1 γ:1	**MaxRet**	0.03343	0.002025	0.064836	-0.00038	0.06724	
α:1 β:0.1 γ:1	α:1 β:0 γ:0	0.008989	-0.02242	0.040395	-0.02482	0.042799	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	-0.00074	-0.03214	0.030668	-0.03455	0.033072	
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	0.000299	-0.03111	0.031704	-0.03351	0.034108	
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	0.002417	-0.02899	0.033823	-0.03139	0.036227	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	1.91E-16	-0.03141	0.031406	-0.03381	0.03381	
MV	**ES**	-0.00527	-0.03668	0.026133	-0.03908	0.028537	
MV	**MaxRet**	0.019329	-0.01208	0.050734	-0.01448	0.053138	
MV	α:1 β:0 γ:0	-0.00511	-0.03652	0.026294	-0.03892	0.028698	
MV	α:1 β:1 γ:0.25	-0.01484	-0.04625	0.016567	-0.04865	0.018971	
MV	α:1 β:0.25 γ:0.25	-0.0138	-0.04521	0.017603	-0.04761	0.020007	
MV	α: 1 β:0.1 γ:0.25	-0.01169	-0.04309	0.019721	-0.04549	0.022125	
MV	α:1 β:1 γ:0.5	-0.0141	-0.04551	0.017304	-0.04791	0.019708	
ES	**MaxRet**	0.024602	-0.0068	0.056008	-0.00921	0.058412	
ES	α:1 β:0 γ:0	0.000161	-0.03125	0.031567	-0.03365	0.033971	
ES	α:1 β:1 γ:0.25	-0.00957	-0.04097	0.02184	-0.04338	0.024244	
ES	α:1 β:0.25 γ:0.25	-0.00853	-0.03994	0.022876	-0.04234	0.02528	
ES	α: 1 β:0.1 γ:0.25	-0.00641	-0.03782	0.024994	-0.04022	0.027398	
ES	α:1 β:1 γ:0.5	-0.00883	-0.04023	0.022577	-0.04264	0.024981	
MaxRet	α:1 β:0 γ:0	-0.02444	-0.05585	0.006965	-0.05825	0.009369	
MaxRet	α:1 β:1 γ:0.25	-0.03417	-0.06557	-0.00276	-0.06798	-0.00036	Yes

Table 10. Continued.

MaxRet	α:1 β:0.25 γ:0.25	-0.03313	-0.06454	-0.00173	-0.06694	0.000678
MaxRet	α: 1 β:0.1 γ:0.25	-0.03101	-0.06242	0.000392	-0.06482	0.002796
MaxRet	α:1 β:1 γ:0.5	-0.03343	-0.06484	-0.00203	-0.06724	0.000379
α:1 β:0 γ:0	α:1 β:1 γ:0.25	-0.00973	-0.04113	0.021679	-0.04354	0.024083
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	-0.00869	-0.0401	0.022715	-0.0425	0.025119
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	-0.00657	-0.03798	0.024833	-0.04038	0.027237
α:1 β:0 γ:0	α:1 β:1 γ:0.5	-0.00899	-0.0404	0.022416	-0.0428	0.02482
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	0.001036	-0.03037	0.032442	-0.03277	0.034846
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	0.003154	-0.02825	0.03456	-0.03066	0.036964
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	0.000737	-0.03067	0.032143	-0.03307	0.034547
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	0.002118	-0.02929	0.033524	-0.03169	0.035928
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	-0.0003	-0.0317	0.031107	-0.03411	0.033511
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	-0.00242	-0.03382	0.028989	-0.03623	0.031393

Table. 11

Tukey-Kramer difference of means of risk (metric standard deviation) compared by strategies under ex ante assumption.

Ex Ante Risk			90% Confidence Limits		95% Confidence Limits		Sig. Diff.
i	j	Means	LSMean(i)	-LSMean(j)	LSMean(i)	-LSMean(j)	@ 95%
EQ	α:1 β:0.25 γ:0.5	-0.00149	-0.00851	0.005526	-0.00904	0.006056	
EQ	α:1 β:0.1 γ:0.5	6.84E-05	-0.00695	0.007085	-0.00748	0.007614	
EQ	α:1 β:1 γ:1	0.000223	-0.00679	0.007239	-0.00732	0.007769	
EQ	α:1 β:0.25 γ:1	0.000141	-0.00688	0.007157	-0.00741	0.007686	
EQ	α:1 β:0.1 γ:1	0.000123	-0.00689	0.007139	-0.00742	0.007669	
EQ	**MV**	0.0146	0.007584	0.021616	0.007054	0.022146	Yes
EQ	**ES**	0	-0.00702	0.007016	-0.00755	0.007546	
EQ	**MaxRet**	-0.07342	-0.08043	-0.0664	-0.08096	-0.06587	Yes
EQ	α:1 β:0 γ:0	0.000108	-0.00691	0.007124	-0.00744	0.007653	
EQ	α:1 β:1 γ:0.25	-0.03057	-0.03759	-0.02356	-0.03812	-0.02303	Yes
EQ	α:1 β:0.25 γ:0.25	-0.02185	-0.02887	-0.01484	-0.0294	-0.01431	Yes
EQ	α: 1 β:0.1 γ:0.25	-0.01413	-0.02114	-0.00711	-0.02167	-0.00658	Yes
EQ	α:1 β:1 γ:0.5	-0.00594	-0.01296	0.001073	-0.01349	0.001603	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:0.5	0.001558	-0.00546	0.008574	-0.00599	0.009104	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:1	0.001713	-0.0053	0.008729	-0.00583	0.009258	
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:1	0.00163	-0.00539	0.008647	-0.00592	0.009176	
α:1 β:0.25 γ:0.5	α:1 β:0.1 γ:1	0.001613	-0.0054	0.008629	-0.00593	0.009158	
α:1 β:0.25 γ:0.5	**MV**	0.01609	0.009074	0.023106	0.008544	0.023636	Yes
α:1 β:0.25 γ:0.5	**ES**	0.00149	-0.00553	0.008506	-0.00606	0.009036	
α:1 β:0.25 γ:0.5	**MaxRet**	-0.07193	-0.07894	-0.06491	-0.07947	-0.06438	Yes
α:1 β:0.25 γ:0.5	α:1 β:0 γ:0	0.001597	-0.00542	0.008614	-0.00595	0.009143	
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.25	-0.02908	-0.0361	-0.02207	-0.03663	-0.02154	Yes
α:1 β:0.25 γ:0.5	α:1 β:0.25 γ:0.25	-0.02036	-0.02738	-0.01335	-0.02791	-0.01282	Yes
α:1 β:0.25 γ:0.5	α: 1 β:0.1 γ:0.25	-0.01264	-0.01965	-0.00562	-0.02018	-0.00509	Yes
α:1 β:0.25 γ:0.5	α:1 β:1 γ:0.5	-0.00445	-0.01147	0.002563	-0.012	0.003093	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:1	0.000154	-0.00686	0.007171	-0.00739	0.0077	
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:1	7.22E-05	-0.00694	0.007088	-0.00747	0.007618	
α:1 β:0.1 γ:0.5	α:1 β:0.1 γ:1	5.44E-05	-0.00696	0.007071	-0.00749	0.0076	
α:1 β:0.1 γ:0.5	**MV**	0.014532	0.007515	0.021548	0.006986	0.022077	Yes
α:1 β:0.1 γ:0.5	**ES**	-6.8E-05	-0.00709	0.006948	-0.00761	0.007477	
α:1 β:0.1 γ:0.5	**MaxRet**	-0.07348	-0.0805	-0.06647	-0.08103	-0.06594	Yes
α:1 β:0.1 γ:0.5	α:1 β:0 γ:0	3.92E-05	-0.00698	0.007055	-0.00751	0.007585	
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.25	-0.03064	-0.03766	-0.02362	-0.03819	-0.0231	Yes
α:1 β:0.1 γ:0.5	α:1 β:0.25 γ:0.25	-0.02192	-0.02894	-0.0149	-0.02947	-0.01437	Yes
α:1 β:0.1 γ:0.5	α: 1 β:0.1 γ:0.25	-0.0142	-0.02121	-0.00718	-0.02174	-0.00665	Yes
α:1 β:0.1 γ:0.5	α:1 β:1 γ:0.5	-0.00601	-0.01303	0.001005	-0.01356	0.001534	

Table 11. Continued.

α:1 β:1 γ:1	α:1 β:0.25 γ:1	-8.2E-05	-0.0071	0.006934	-0.00763	0.007464	
α:1 β:1 γ:1	α:1 β:0.1 γ:1	-0.0001	-0.00712	0.006916	-0.00765	0.007446	
α:1 β:1 γ:1	**MV**	0.014377	0.007361	0.021393	0.006831	0.021923	Yes
α:1 β:1 γ:1	**ES**	-0.00022	-0.00724	0.006793	-0.00777	0.007323	
α:1 β:1 γ:1	**MaxRet**	-0.07364	-0.08065	-0.06662	-0.08118	-0.06609	Yes
α:1 β:1 γ:1	α:1 β:0 γ:0	-0.00012	-0.00713	0.006901	-0.00766	0.007431	
α:1 β:1 γ:1	α:1 β:1 γ:0.25	-0.0308	-0.03781	-0.02378	-0.03834	-0.02325	Yes
α:1 β:1 γ:1	α:1 β:0.25 γ:0.25	-0.02208	-0.02909	-0.01506	-0.02962	-0.01453	Yes
α:1 β:1 γ:1	α: 1 β:0.1 γ:0.25	-0.01435	-0.02137	-0.00733	-0.0219	-0.0068	Yes
α:1 β:1 γ:1	α:1 β:1 γ:0.5	-0.00617	-0.01318	0.00085	-0.01371	0.00138	
α:1 β:0.25 γ:1	α:1 β:0.1 γ:1	-1.8E-05	-0.00703	0.006998	-0.00756	0.007528	
α:1 β:0.25 γ:1	**MV**	0.014459	0.007443	0.021476	0.006914	0.022005	Yes
α:1 β:0.25 γ:1	**ES**	-0.00014	-0.00716	0.006876	-0.00769	0.007405	
α:1 β:0.25 γ:1	**MaxRet**	-0.07356	-0.08057	-0.06654	-0.0811	-0.06601	Yes
α:1 β:0.25 γ:1	α:1 β:0 γ:0	-3.3E-05	-0.00705	0.006983	-0.00758	0.007513	
α:1 β:0.25 γ:1	α:1 β:1 γ:0.25	-0.03071	-0.03773	-0.0237	-0.03826	-0.02317	Yes
α:1 β:0.25 γ:1	α:1 β:0.25 γ:0.25	-0.02199	-0.02901	-0.01498	-0.02954	-0.01445	Yes
α:1 β:0.25 γ:1	α: 1 β:0.1 γ:0.25	-0.01427	-0.02128	-0.00725	-0.02181	-0.00672	Yes
α:1 β:0.25 γ:1	α:1 β:1 γ:0.5	-0.00608	-0.0131	0.000933	-0.01363	0.001462	
α:1 β:0.1 γ:1	**MV**	0.014477	0.007461	0.021493	0.006931	0.022023	Yes
α:1 β:0.1 γ:1	**ES**	-0.00012	-0.00714	0.006893	-0.00767	0.007423	
α:1 β:0.1 γ:1	**MaxRet**	-0.07354	-0.08055	-0.06652	-0.08108	-0.06599	Yes
α:1 β:0.1 γ:1	α:1 β:0 γ:0	-1.5E-05	-0.00703	0.007001	-0.00756	0.007531	
α:1 β:0.1 γ:1	α:1 β:1 γ:0.25	-0.0307	-0.03771	-0.02368	-0.03824	-0.02315	Yes
α:1 β:0.1 γ:1	α:1 β:0.25 γ:0.25	-0.02198	-0.02899	-0.01496	-0.02952	-0.01443	Yes
α:1 β:0.1 γ:1	α: 1 β:0.1 γ:0.25	-0.01425	-0.02127	-0.00723	-0.0218	-0.0067	Yes
α:1 β:0.1 γ:1	α:1 β:1 γ:0.5	-0.00607	-0.01308	0.00095	-0.01361	0.00148	
MV	**ES**	-0.0146	-0.02162	-0.00758	-0.02215	-0.00705	Yes
MV	**MaxRet**	-0.08802	-0.09503	-0.081	-0.09556	-0.08047	Yes
MV	α:1 β:0 γ:0	-0.01449	-0.02151	-0.00748	-0.02204	-0.00695	Yes
MV	α:1 β:1 γ:0.25	-0.04517	-0.05219	-0.03816	-0.05272	-0.03763	Yes
MV	α:1 β:0.25 γ:0.25	-0.03645	-0.04347	-0.02944	-0.044	-0.02891	Yes
MV	α: 1 β:0.1 γ:0.25	-0.02873	-0.03574	-0.02171	-0.03627	-0.02118	Yes
MV	α:1 β:1 γ:0.5	-0.02054	-0.02756	-0.01353	-0.02809	-0.013	Yes
ES	**MaxRet**	-0.07342	-0.08043	-0.0664	-0.08096	-0.06587	Yes
ES	α:1 β:0 γ:0	0.000108	-0.00691	0.007124	-0.00744	0.007653	
ES	α:1 β:1 γ:0.25	-0.03057	-0.03759	-0.02356	-0.03812	-0.02303	Yes
ES	α:1 β:0.25 γ:0.25	-0.02185	-0.02887	-0.01484	-0.0294	-0.01431	Yes
ES	α: 1 β:0.1 γ:0.25	-0.01413	-0.02114	-0.00711	-0.02167	-0.00658	Yes
ES	α:1 β:1 γ:0.5	-0.00594	-0.01296	0.001073	-0.01349	0.001603	
MaxRet	α:1 β:0 γ:0	0.073523	0.066507	0.080539	0.065977	0.081069	Yes
MaxRet	α:1 β:1 γ:0.25	0.042843	0.035827	0.049859	0.035297	0.050389	Yes

Table 11. Continued.

MaxRet	α:1 β:0.25 γ:0.25	0.051563	0.044547	0.05858	0.044017	0.059109	Yes
MaxRet	α: 1 β:0.1 γ:0.25	0.059289	0.052272	0.066305	0.051743	0.066834	Yes
MaxRet	α:1 β:1 γ:0.5	0.067472	0.060456	0.074488	0.059926	0.075018	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.25	-0.03068	-0.0377	-0.02366	-0.03823	-0.02313	Yes
α:1 β:0 γ:0	α:1 β:0.25 γ:0.25	-0.02196	-0.02898	-0.01494	-0.02951	-0.01441	Yes
α:1 β:0 γ:0	α: 1 β:0.1 γ:0.25	-0.01423	-0.02125	-0.00722	-0.02178	-0.00669	Yes
α:1 β:0 γ:0	α:1 β:1 γ:0.5	-0.00605	-0.01307	0.000966	-0.0136	0.001495	
α:1 β:1 γ:0.25	α:1 β:0.25 γ:0.25	0.00872	0.001704	0.015736	0.001174	0.016266	Yes
α:1 β:1 γ:0.25	α: 1 β:0.1 γ:0.25	0.016446	0.009429	0.023462	0.0089	0.023991	Yes
α:1 β:1 γ:0.25	α:1 β:1 γ:0.5	0.024629	0.017613	0.031645	0.017083	0.032175	Yes
α:1 β:0.25 γ:0.25	α: 1 β:0.1 γ:0.25	0.007725	0.000709	0.014742	0.000179	0.015271	Yes
α:1 β:0.25 γ:0.25	α:1 β:1 γ:0.5	0.015909	0.008893	0.022925	0.008363	0.023455	Yes
α: 1 β:0.1 γ:0.25	α:1 β:1 γ:0.5	0.008184	0.001167	0.0152	0.000638	0.015729	Yes

Table 12.

Comparison of average R^2, adjusted R^2, and direction measures of accuracy for 12-month one-beta and two-beta CAPMs.

Model (12 month)	***Average R^2***	***R^2 Std.Dev.***	***Average Adjusted R^2***	***Adj. R^2 Std.Dev.***	***Direction Accuracy***	***Number of Regressions***
One-parameter CAPM	0.253482	0.210014	0.185617	0.22910	66.299%	35,880
Two-parameter CAPM	0.317184	0.208185	0.180621	0.249822	64.250%	35,421

Table 13.

Comparison of average R^2, adjusted R^2, and direction measures of accuracy for 60-month one-beta and two-beta CAPMs.

Model (60 month)	***Average R^2***	***R^2 Std.Dev.***	***Average Adjusted R^2***	***Adj. R^2 Std.Dev.***	***Direction Accuracy***	***Number of Regressions***
One-parameter CAPM	0.209924	0.128104	0.196533	0.130275	65.947%	30,573
Two-parameter CAPM	0.223475	0.127544	0.196699	0.1311942	65.611%	30,673

Table 14.

Comparison of cumulative abnormal returns (CAR) and model-weighted cumulative abnormal returns (MWCAR) for the 12-month one-parameter and two-parameter CAPMs.

Model (12 month)	***CAR***	***CAR Std.Dev.***	***MWCAR***	***MWCAR Std.Dev.***
One-Parameter CAPM	0.005467	0.094305	0.00418203	0.11027922
Two-Parameter CAPM	-0.00039	0.106867	-0.002544417	0.122664231

Table 15.

Comparison of cumulative abnormal returns (CAR) and model-weighted cumulative abnormal returns (MWCAR) for the 60-month one-parameter and two-parameter CAPMs.

Model (60 month)	***CAR***	***CAR Std.Dev.***	***MWCAR***	***MWCAR Std.Dev.***
One-parameter CAPM	0.004361	0.099433	0.002596	0.10794
Two-parameter CAPM	0.000441	0.100746	-0.00168	0.109637

Table 16.

Difference of means of R^2 and adjusted R^2 measures between the 12-month one-parameter and two-parameter CAPMs.

Measure (12 month)	***Difference of Means*** (one-parameter less two-parameter)	***95% Confidence Interval of difference***		***F-test of difference of means model***	***P-value***
R^2	-0.063703	-0.066301	-0.061140	2309.32	<0.0001
Adjusted R^2	-0.007406	-0.010077	-0.004735	29.54	<0.0001

Table 17.

Difference of means of R^2 and adjusted R^2 measures between the 60-month one-parameter and two-parameter CAPMs.

Measure (60 month)	***Difference of Means*** (one-parameter less two-parameter)	***95% Confidence Interval of difference***		***F-test of difference of means model***	***P-value***
R^2	-0.107261	-0.109815	-0.104706	6774.02	<0.0001
Adjusted R^2	-0.012430	-0.015085	-0.009775	84.19	<0.0001

Table 18.

Summary of Means of Return, Compound Returns, Risk (as proxied by the standard deviation) and Sharpe ratios for each strategy, based on the assumption of ex post optimization.

Strategy (ex post)	*Item*	*Mean*	*95% Confidence Limit*	
Equally Weighted	Return	0.017751	0.007031	0.028471
	Compound Return	0.017051	0.005130	0.028973
	Risk (Std.Dev.)	0.037963	0.035711	0.040215
	Sharpe	0.39778	-0.527250	1.322825
Minimum Variance	Return	0.021894	0.011174	0.032614
	Compound Return	0.021892	0.009970	0.033814
	Risk (Std.Dev.)	0.023063	0.020811	0.025315
	Sharpe	0.782042	-0.142995	1.707080
Minimum MLPM Variance	Return	0.017756	0.007036	0.28476
	Compound Return	0.17056	0.005134	0.028978
	Risk (Std.Dev.)	0.037963	0.035711	0.040215
	Sharpe	0.397898	-0.527139	1.322936
Maximum Return	Return	0.280892	0.270172	0.291611
	Compound Return	0.277493	0.265571	0.289415
	Risk (Std.Dev.)	0.108207	0.105164	0.111250
	Sharpe	28.233175	27.308137	29.158213
Alternative Strategy Parameters				
α:1 β:0 γ:0	Return	0.020036	0.009316	0.030756
	Compound Return	0.019432	0.007510	0.031354
	Risk (Std.Dev.)	0.037865	0.035613	0.040118
	Sharpe	0.449280	-0.475757	1.374318
α:1 β:1 γ:0.25	Return	-0.063341	-0.074061	-0.052622
	Compound Return	-0.064448	-0.076370	-0.052526
	Risk (Std.Dev.)	0.067043	0.064791	0.069295
	Sharpe	-1.017068	-1.942105	-0.092030
α:1 β:0.25 γ:0.25	Return	-0.054155	-0.064875	-0.043436
	Compound Return	-0.054722	-0.066644	-0.042800
	Risk (Std.Dev.)	0.058814	0.056562	0.061066
	Sharpe	-1.022752	-1.947789	-0.097714
α: 1 β:0.1 γ:0.25	Return	-0.051931	-0.062651	-0.041211
	Compound Return	-0.052206	-0.064128	-0.040284
	Risk (Std.Dev.)	0.051343	0.049091	0.053595
	Sharpe	-1.124732	-2.049770	-0.199695
α:1 β:1 γ:0.5	Return	-0.047506	-0.058225	-0.036786
	Compound Return	-0.047660	-0.059582	-0.035739
	Risk (Std.Dev.)	0.043560	0.041307	0.045812
	Sharpe	-1.199173	-2.124210	-0.274135
α: 1 β:0.25 γ:0.5	Return	-0.0406351	-0.051071	-0.029632
	Compound Return	-0.040460	-0.052382	-0.028539
	Risk (Std.Dev.)	0.039194	0.036942	0.041446
	Sharpe	-1.131357	-2.056394	-0.206319
α:1 β:0.1 γ:0.5	Return	-0.030184	-0.040903	-0.019464
	Compound Return	-0.030297	-0.042219	-0.018375
	Risk (Std.Dev.)	0.037732	0.035480	0.039984
	Sharpe	-0.887554	-1.812592	0.037483
α:1 β:1 γ:1	Return	0.016192	0.005472	0.026911
	Compound Return	0.015659	0.003737	0.27581
	Risk (Std.Dev.)	0.037823	0.035571	0.040075
	Sharpe	0.356587	-0.568451	1.281624
α:1 β:0.25 γ:1	Return	0.019077	0.008357	0.029797
	Compound Return	0.018488	0.006566	0.030410
	Risk (Std.Dev.)	0.037823	0.035571	0.040075
	Sharpe	0.426274	-0.498764	1.365130
α:1 β:0.1 γ:1	Return	0.019651	0.008931	0.030371
	Compound Return	0.019054	0.007133	0.030976
	Risk (Std.Dev.)	0.037848	0.035596	0.040100
	Sharpe	0.440092	-.484945	1.365130

Table 19.

Summary of Means of Return, Compound Return, Risk (as proxied by the standard deviation) and Sharpe ratios for each strategy, based on the assumption of ex ante optimization.

Strategy (ex ante)	*Item*	*Mean*	*95% Confidence Limit*	
Equally Weighted	Return	0.017495	0.002552	0.032437
	Compound Return	0.015883	0.001961	0.029804
	Risk (Std.Dev.)	0.037957	0.034842	0.041072
	Sharpe	0.399048	-0.265174	1.063271
Minimum Variance	Return	0.011685	-0.003257	0.026628
	Compound Return	0.010606	-0.003315	0.024528
	Risk (Std.Dev.)	0.023357	0.020242	0.026472
	Sharpe	0.390913	-0.273310	1.055135
Minimum MLPM Variance	Return	0.017489	0.002547	0.032431
	Compound Return	0.015880	0.001958	0.029801
	Risk (Std.Dev.)	0.037957	0.034842	0.041072
	Sharpe	0.398976	-0.265247	1.063198
Maximum Return	Return	-0.005151	-0.020093	0.009791
	Compound Return	-0.008722	-0.022644	0.005199
	Risk (Std.Dev.)	0.111372	0.108257	0.114487
	Sharpe	-0.949424	-1.613647	-0.285202
Alternative Strategy Parameters				
α:1 β:0 γ:0	Return	0.017304	0.002362	0.032246
	Compound Return	0.015719	0.001797	0.029640
	Risk (Std.Dev.)	0.037849	0.034735	0.040964
	Sharpe	0.396943	-0.267279	1.061166
α:1 β:1 γ:0.25	Return	0.029970	0.015028	0.044913
	Compound Return	0.025445	0.011524	0.039367
	Risk (Std.Dev.)	0.068529	0.065414	0.071644
	Sharpe	0.326802	-0.337421	0.991025
α:1 β:0.25 γ:0.25	Return	0.027959	0.013017	0.042901
	Compound Return	0.024409	0.010488	0.038331
	Risk (Std.Dev.)	0.059809	0.056694	0.062924
	Sharpe	0.353890	-0.310332	1.018113
α: 1 β:0.1 γ:0.25	Return	0.024862	0.009919	0.039804
	Compound Return	0.022291	0.008370	0.036212
	Risk (Std.Dev.)	0.052084	0.048969	0.055198
	Sharpe	0.415020	-0.249203	1.079242
α:1 β:1 γ:0.5	Return	0.026692	0.011749	0.041634
	Compound Return	0.024708	0.010787	0.038629
	Risk (Std.Dev.)	0.043900	0.040785	0.047015
	Sharpe	0.550502	0.113720	1.214725
α: 1 β:0.25 γ:0.5	Return	0.024179	0.009237	0.039121
	Compound Return	0.022480	0.008559	0.036402
	Risk (Std.Dev.)	0.039447	0.036332	0.042562
	Sharpe	0.533108	-0.131114	1.197331
α:1 β:0.1 γ:0.5	Return	0.022022	0.007080	0.036965
	Compound Return	0.020513	0.006592	0.034434
	Risk (Std.Dev.)	0.037889	0.034774	0.041003
	Sharpe	0.510068	-0.154155	1.174291
α:1 β:1 γ:1	Return	0.017566	0.002624	0.032508
	Compound Return	0.015988	0.002066	0.029909
	Risk (Std.Dev.)	0.037734	0.034619	0.040849
	Sharpe	0.404407	-0.259815	1.068630
α:1 β:0.25 γ:1	Return	0.017380	0.002437	0.032322
	Compound Return	0.015786	0.001865	0.029707
	Risk (Std.Dev.)	0.037816	0.034702	0.040931
	Sharpe	0.398805	-0.265418	1.063027
α:1 β:0.1 γ:1	Return	0.026692	0.011749	0.041634
	Compound Return	0.024708	0.010787	0.038629
	Risk (Std.Dev.)	0.037834	0.034719	0.040949
	Sharpe	0.618607	-0.45615	1.282830

Table 20.

Summary statistics of expected returns based on one year and five year geometric and decay-arithmetic models. Realized returns are stated as monthly average geometric or average arithmetic. Parameter estimates are derived from polynomials of best fit to the time period February 1927 to December 2002.

Parameter/Item	1 year Geometric model	1 year Decay Arithmetic model	5 year Geometric model	5 year Decay Arithmetic model
α_1	0.027418098	0.087534666	0.01620508	0.016205022
β_1	0.217837413	0.345433259	0.160629339	0.160629642
α_2	0.284157897	0.507072994	0.284157897	0.284157965
β_2	4.318426922	3.31207268	4.318426922	4.318425904
R^2	0.047492571	0.106048626	0.027296325	0.027296325
Average S&P VHPM	2.00465E-05		8.17855E-06	
Average S&P VLPM	3.93336E-06		1.81858E-06	
Average Treasury VHPM	6.51929E-11		5.55356E-11	
Average Treasury VLPM	1.1526E-09		6.82171E-10	
Average S&P expected monthly return	0.005894	0.007311	0.006436	0.006436
Average Treasury expected monthly return	0.000572	2.30423E-05	0.000524	0.000524
Realized S&P Average Monthly Geometric Returns	0.00556			
Realized S&P Average Monthly Arithmetic Returns	0.007175			
Realized Treasury Average Monthly Geometric Returns	0.00054			
Realized Treasury Average Monthly Arithmetic Returns	0.000554			

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