Optimization of Machining Parameters in Multi-Pass Turning and Milling Operations

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Abstract

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The primary objective in machining operations is to produce products with low cost and high quality. Machining parameter optimization plays an important role in achieving this goal. Machining parameter optimization in multi-pass machining operations usually involves the optimal selection of cutting speed, feed rate, depth of cut and the number of passes. In this thesis, the parameter optimization problem for multi-pass machining operations is studied. Mathematical programming models for both multi-pass turning and face-milling operations with single-tool applications are developed based on the minimum production cost criterion. Maximum and minimum cutting speeds, feed rates and depths of cut, as well as tool life, surface roughness, cutting force and cutting power consumption are considered as constraints. Optimal values of machining parameters are found by two methods. One involves using integer programming and the other using nonlinear programming. When solving the optimization problem by the method using integer programming, two steps are adopted. The first step is to minimize the costs for individual finishing and roughing passes for various possible depths of cut. In the second step, an optimal combination of depths of cut for the finishing and roughing passes, the optimal number of passes and corresponding cutting speeds and feed rates, based on minimum total unit cost, are determined using an integer programming model. Examples are presented to illustrate the effectiveness of the optimization models and the solution methods developed. The effect of tool replacement time on the optimization results is evaluated. Certain conclusions related to the problem are made.

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Nomenclature

B	Width of workpiece in milling (mm)
C	Constant in tool life equations in turning
C_f	Constant in cutting force equations in milling
CI	Machine idle cost due to loading and unloading operations and idling tool motion ($/piece)
CM	Machining cost by actual cutting time ($/piece)
CR	Tool replacement cost ($/piece)
CT	Tool cost ($/piece)
C_v	Constant in tool life equations in milling
D_t	Diameter of milling cutter (mm)
D_w	Diameter of workpiece in turning (mm)
d_{ri}, d_s	Depths of cut per pass in roughing and finishing passes (mm) (i=1, 2...n)
$d_{r,\min}$, $d_{r,\max}$	Minimum and maximum recommended depths of cut in roughing (mm)
$d_{s,\min}$, $d_{s,\max}$	Minimum and maximum recommended depths of cut in finishing (mm)
d_t	Total depth of cut (mm)
$f_{\min}$, $f_{\max}$	Minimum and maximum allowable feed rates (mm/rev in turning or mm/tooth in milling)

f_{ri}, f_s	Feed rates in roughing and finishing passes (mm/rev in turning or mm/tooth in milling) (i=1, 2...n)
F, F_{max}	Cutting force and maximum allowable cutting force (N)
h_1, h_2	Constants relating to tool travel and approach/depart time (min/mm, min)
K_f	Constant in cutting force equations in milling
K_v	Constant in tool life equations in milling
k_0	Direct labor cost plus overhead ($/min)
k_t	Cost of cutting edge ($/edge)
k_1	Constant in cutting force and power equations in turning
L	Length of workpiece (mm)
L_t	Cutting travel length for each turning pass (mm)
L_{tr}, L_{ts}	Cutting travel lengths for roughing and finishing passes in milling (mm)
l	Constant exponent in tool life equations in milling
m_i	Number of divisions of recommended depth of cut range in i-th pass (i = 0, 1, 2...n)
n	Number of passes in roughing
P, P_{max}	Cutting power and maximum power of the machine tool (kW)
p_f, q_f	Constant exponents in cutting force equations in milling
p_v, q_v	Constant exponents in tool life equations in milling

R_a	Arithmetic average surface roughness (μm)
$R_{s.\max}$, $R_{r.\max}$	Surface roughness (arithmetic average) requirements in finishing and roughing (μm)
r_e	Cutter nose radius (mm)
s_f	Constant exponent in cutting force equations in milling
s_v	Constant exponent in tool life equations in milling
T_r, T_s, T	Tool lives in roughing and finishing, and general of tool life (min)
t_e	Tool change time (min/edge)
t_i	Idle tool motion time such as tool travel and tool approach/depart time (min)
t_l	Machine idle time due to loading and unloading workpiece and idle tool motions (min)
t_m	Actual machining time (min)
t_{mr}	Total roughing time (min)
t_{ms}	Finishing time (min)
t_p	Preparation time relating to loading and unloading of workpiece (min/piece)
UC	Unit production cost except material cost ($/piece)
UC_r	Unit cost for total roughing passes ($/piece)
UC_{ri}	Unit cost for roughing pass i ($/piece) ($i$=1, 2...$n$)
UC_s	Unit cost per finishing pass ($/piece)

$V_{\min}$, $V_{\max}$	Minimum and maximum allowable cutting speeds (m/min)
V_{ri} , V_s	Cutting speeds in roughing and finishing passes (m/min) (i =1, 2...n)
x_f , y_f	Constant exponents in cutting force equations in milling
x_v , y_v	Constant exponents in tool life equations in milling
Z	Number of cutter teeth in milling
α, β, γ	Constant exponents in tool life equations in turning
μ, ν	Constant exponents of feed and depth of cut in cutting force and power equations in turning
η	Efficiency of the machine tool

Subscripts

s	Corresponds to a finishing pass
r	Corresponds to roughing passes
i	Corresponds to i-th machining pass
j	Corresponds to j-th value of division of recommended range of depth of cut in i-th pass

Superscripts

opt	Corresponds to optimal values for the selected machining problem
*	Corresponds to optimal values in single roughing and finishing passes

Chapter 1

Introduction

1.1 Machining and Machining Economics

The machining process is a manufacturing process to shape metal parts by removing unwanted material. In the machining process of any part, one must follow given quality specifications such as surface finish, accuracy and surface integrity. At the same time, minimum production cost or machining time is desired. To satisfy these objectives, an optimal combination of machining parameters including cutting speed, feed rate, depth of cut and the number of passes needs to be determined by seeking optimal solutions of properly formulated mathematical models of the machining processes. These models are developed for optimal production cost, production time, production profit or other criteria, subject to various constraints from given quality specifications and machining conditions.

1.2 The Optimization of Machining Parameters

The selection of machining parameters is traditionally carried out by process planners or machinists based on their experiences and industrial handbooks. The parameters so determined are usually on the conservative side. Colding (1992) reported that 40% longer cutting times are used in the U.S. and Europe, when compared to optimal parameters.

The optimization of machining parameters usually consists of two steps: the first step is to formulate a mathematical optimization model based on certain economic criteria for machining conditions with various realistic constraints; the second step is to design a suitable solution procedure to seek the optimal or near optimal solutions. The optimization of machining parameters has been addressed using two basic approaches: single-pass machining operations and multi-pass machining operations. A single-pass operation removes the total desired depth of cut in just one pass. In practice, however, this rarely happens. Therefore, a multi-pass approach has to be considered for the determination of the machining parameters.

Consideration of machining parameter optimization began as early as 1907, when Taylor (1907) recognized the existence of an optimum cutting speed in single-pass turning operations. Research on machining parameter optimization has increased since the 1950's. It became more extensive with wide use of CNC machining. High initial investment and operation cost place a great demand to optimize the machining parameters for economic yield. With rapid development of computer technology, solution techniques for solving machining parameter models have been largely extended.

1.3 Optimization Criteria

When developing optimization models, objective functions are determined by optimization criteria. The following are the four criteria used in the optimization of machining parameters. Criterion employment depends upon the production objective.

1.3.1 The Minimum Production Cost Criterion

This criterion minimizes the production cost per piece, and coincides with the maximum profit criterion if the unit revenue is constant (Chua *et al.* 1991). The minimum cost per component criterion (along with maximum production rate criterion) was first proposed by Gilbert in his "Economics of Machining" in 1950. This criterion will lead to a low production rate and therefore is the criterion to be adopted when there is ample time for production. Minimum cost per piece criterion is so far the most frequently used optimization criterion, adopted by many researchers in both single and multiple pass machining analysis (Al-Ahmari 2001, Cakir and Gurarda 2000, Chen and Tsai 1996, Ermer and Kromodihardjo 1981, Ermer and Patel 1974, Gopalakrishnan and Al-Khayyal 1991, Gupta *et al.* 1995, Lambert and Walvekar 1978, Liang *et al.* 2001, Onwubolu and Kumalo 2001, Shin and Joo 1992, Shunmugam *et al.* 2000, Tolouei-Rad and Bidhendi 1997, Walvekar and Lambert 1970).

1.3.2 The Minimum Production Time or Maximum Production Rate Criterion

This criterion maximizes the amount of production in a unit time interval, sometimes in a specific time period. Therefore, it minimizes the production time per unit piece and usually gives a high cost per component. It is the criterion to be adopted when an increase in physical productivity or productive efficiency is desired, regardless of the production cost and/or profit. Some researchers (Armarego *et al.* 1994, Dereli *et al.* 2001, Prasad *et* *al.* 1997, Tolouei-Rad and Bidhendi 1997, Wang *et al.* 2002) used minimum production time as their optimization criterion.

1.3.3 The Maximum Profit Rate Criterion

This criterion involves maximizing the return on the operation in a given time interval. "The maximum profit" was first put forward by Okushima and Hitomi (Okushima and Hitomi 1964) in 1964 and has been named "the maximum profit rate" by Armarego and Russell (Armarego and Russell 1966) since 1966. It is the criterion to be recommended when there is insufficient capacity for a specific time interval. This criterion was adopted by Boothroyd and Rusek (Boothroyd and Rusek 1976).

1.3.4 Weighted Combination of Several Objective Functions

Agapiou (Agapiou 1992a, Agapiou 1992b) considered the optimization of machining parameters by using this criterion incorporating both production cost and production time. A constant multiplier is used to normalize the production cost and time criteria.

The first two approaches have received much more attention since the mid-1960s. The third approach is not commonly used due to lack of information and uncertainty during manufacturing. In this thesis, the minimum production cost criterion is adopted because economic consideration is concerned in most cases.

1.4 Research in the Thesis

In this thesis, mathematical programming models are proposed for optimal selection of machining parameters in multi-pass operations, both for turning and face milling. Two solution methods are developed to solve the models. The models are to minimize unit production cost. The constraints are maximum and minimum cutting speeds, feed rates, depths of cut as well as tool life, surface roughness, cutting force and cutting power. The first solution method, which involves using integer programming, consists of two steps. The first step is to minimize the costs for individual finishing and roughing passes for various fixed possible depths of cut and to obtain corresponding optimal cutting speeds and feed rates. In the second step, an optimal combination of depths of cut for the finishing and roughing passes, an optimal number of passes, the minimum total cost, and corresponding optimal cutting speeds and feed rates are determined using an integer programming model. The second solution method is a direct nonlinear programming approach.

Based on a careful literature review in Chapter 2, details of model formulation are presented in Chapter 3. Chapter 4 introduces the two solution methods. Example problems are given in Chapter 5 to illustrate the models and solution methods. Conclusions are drawn in Chapter 6.

Chapter 2

Literature Review

Consideration on machining parameter optimization begins with single-pass operations. As early as in1907, Taylor (Taylor 1907) recognized the existence of an optimum cutting speed for maximizing the material removal rate in single-pass turning operations. Gilbert (Gilbert 1950) used an analytical procedure to determine the cutting speed that minimizes the machining cost for a single-pass turning operation with fixed feed rate and depth of cut. Armarego and Brown (Armarego and Brown 1969) presented equations for determining optimal machining variables with the depth of cut fixed according to minimum production cost. Walvekar and Lambert (Walvekar and Lambert 1970) utilized geometric programming for the simultaneous determination of the optimal cutting speed and feed rate subject to certain practical constrains. Ermer and Kromodihardjo (Ermer and Kromodihardjo 1981) proposed an optimization model minimizing total machining cost for a single-pass turning operation. The cutting conditions for minimum cost are found by selecting the highest possible feed rate. Other research (Wu and Ermer 1966, Ermer and Wu 1967, Ermer and Morris 1969, Iwata *et al.* 1977, Hitomi 1989, Gopalakrishnan and Al-Khayyal 1991, Agapiou 1992a, Kilic *et al.* 1993, Armarego *et al.* 1993, Prasad *et al.* 1997, Wang *et al.* 2002) contributed to the optimization of machining parameters for single-pass operations.

In single-pass machining operations, the depth of cut and the number of passes are usually predetermined and eliminated from the decision variables of the optimization problem, leading to a simplification of the solution. The optimization problem for a single-pass operation can be represented in two-dimensional space, allowing for a graphical illustration of the problem formulation. The objective function is generally nonlinear and the various constraints could be linear or nonlinear. The single-pass approach can only be applied in cases where the total desired depth of cut could be removed in just one pass. But in practice, this rarely happens and a single-pass operation is also not always the most economical or the most productive, especially when considering practical constraints such as available horsepower, desired surface finish, minimum tool life and maximum permissible feed rate and cutting speed. It can be shown that two passes, or sometimes even three passes, can be less expensive or take less production time (Ermer and Kromodihardjo 1981). Therefore, a multi-pass approach has been in a dominant trend in determining machining parameters.

The subsequent studies explored multi-pass operations to determine the optimal machining parameters (Agapiou 1992a, Al-Ahmari 2001, Alberti and Perrone 1999, Arezzo *et al.* 2000, Armarego *et al.* 1994, Cakir and Gurarda 1998, Cakir and Gurarda 2000, Chua *et al.* 1991, Chen and Tsai 1996, Crookall and Venkataramani 1971, Dereli *et al.* 2001, Ermer and Kromodihardjo 1981, Gupta *et al.* 1994, Gupta *et al.* 1995, Hitomi 1989, Iwata *et al.* 1972, Iwata *et al.* 1977, Kals *et al.* 1978, Kee 1995, Kee 1996, Lambert and Walvekar 1978, Mesquita *et al.* 1995, Onwubolu and Kumalo 2001, Saravanan and Sachithanandam 2001, Shin and Joo 1992, Shunmugam *et al.* 2000, Sonmez *et al.* 1999, Tan and Creese 1995, Tolouei-Rad and Bidhendi 1997, Wang and Da *et al.* 2002, Yellowley 1983, Yellowley 1989). The machining economics problem has traditionally been solved using various optimization algorithms including geometric programming (Ermer 1971, Ermer and Kromodihardjo 1981, Gopalakrishnan and Al-Khayyal 1991, Petropoulos 1973), dynamic programming (Agapiou 1992b, Arezzo *et al.* 2000, Iwata *et al.* 1977, Lambert and Walvekar 1978, Shin and Joo 1992), linear programming (Ermer and Patel 1974), integer programming (Gupta *et al.* 1995), the sequential unconstrained minimization technique (Hati and Rao 1976) and circular direction search method (Cakir and Gurarda 2000). Previous studies did not consider all the cutting constraints because the numerous constraints complicate the machining optimization problem. The additional variables and number of passes makes the multi-pass problem NP-hard. Consequently, local search techniques have been recently applied to solve multi-pass machining optimization problems. Local search techniques include genetic algorithm (GA) approach (Dereli *et al.* 2001, Onwubolu and Kumalo 2001, Shunmugam *et al.* 2000, Wang and Da *et al.* 2002) and simulated annealing (SA) algorithm (Chen and Tsai 1996, Khan *et al.* 1997). In the following part of this chapter, a detailed review is given to the research on the multi-pass parameter optimization problem.

2.1 Review of the Research on Optimization Models

Hitomi (Hitomi 1979) proposed several objective functions for optimizing machining parameters. These models are based on the first three production criteria mentioned in Section 1.3. Other authors also discussed optimization model formulation in their publications (Armarego and Brown 1969, Boothroyd 1985, Shaw 1984, Stephenson and Agapiou 1997).

Shin and Joo (Shin and Joo 1992) presented a mathematical model for multi-pass turning operations with realistic machining constraints. They divided the cutting process into multi-pass rough cutting and finish cutting operations, and redefined machine idle time as a sum of constant and variable terms. When using a dynamic programming approach for the selection of depth of cut for individual passes, the final finishing pass is fixed to the minimum allowable depth of cut. The remaining depth of cut is divided into a number of roughing passes with equal size to obtain the minimum total cost.

Agapiou (Agapiou 1992a, Agapiou 1992b) proposed an objective function incorporating the minimum production cost and minimum production time criteria to determine the optimum machining conditions. The two criteria are prioritized through their weight coefficients. A constant multiplier is used to generate a weighted average of the objective functions. The optimum number of machining passes for a given total depth of cut is obtained by dynamic programming. The optimum cutting speed and feed rate for each pass are independently determined by the Nelder-Mead simplex search method.

Gupta *et al.* (Gupta *et al.* 1994) presented an optimization model based on the maximum profit criterion to determine the optimal machining parameters for multi-pass turning operations. Geometric programming combined with linear programming was used to solve the problem.

Cakir and Gurarda (Cakir and Gurarda 1998) presented optimization models based on the minimum production cost criterion for both multi-pass turning and milling operations.

The cut volume is divided into several sections. Each section was treated as a single-pass operation with constraints of maximum and minimum feed rate and cutting speeds, cutting power, tool life, deflection of workpiece, pre-load and surface roughness. Optimum values of machining parameters are found by using search methods.

Tolouei-Rad and Bidhendi (Tolouei-Rad and Bidhendi 1997) proposed mathematical models for both single-tool and multi-tool multi-pass milling operations for minimum unit cost, minimum unit time and maximum profit rate. They used the method of feasible directions to solve the problem.

In developing an optimization model, physical limitations on cutting conditions due to the characteristics of the machine-tool-workpiece system and quality requirements of the part should be identified from previous experience and taken into account as constraints. Commonly used constraints include (1) parameter constraints, (2) tool life constraints, (3) operation constraints, (4) roughing and finishing parameter relations. For given machining conditions, there exist certain ranges of values in selecting cutting speed, feed rate and depth of cut. Parameter constraints are usually expressed in terms of minimum and maximum values. Most publications in machining parameter optimization consider the relationship between tool life and machining parameters for given machining conditions. At the same time, tool life values are restricted to certain acceptable ranges. The actual tool replacement time is usually derived from economic production consideration and quality requirements of the machined part. In machining practice, limitations exist on the requirements of machined surface finish, cutting force and cutting power. These constraints were considered by many researchers. An important relationship between roughing and finishing parameters is that the total depth of cut should be equal to the depth of finishing cut plus the total depth of roughing cuts.

Besides the above commonly used constraints, some others were considered by several researchers. During the roughing pass, the depth of cut and feed rate are usually greater than those for the finishing pass and the speed is usually less than that for the finishing pass. Chen and Tsai (Chen and Tsai 1996) took this constraint into account in their research. In order to prevent chatter, adhesion and the formation of build-up edge, constraint on stable cutting region has been suggested by some researchers (Chen and Tsai 1996, Narang and Fischer 1993, Philipson and Ravindran 1979). Because the cutting capability of the cutter decreases and the cutter can no longer be used if temperature exceeds the reasonable limit, the constraint on the chip-tool interface temperature was used by Chen and Tsai (Chen and Tsai 1996).

2.2 Review of the Research on Optimization Techniques

There are mainly two types of methods to solve problems of machining economics: optimization algorithms and approximation algorithms. Optimization algorithms are capable of solving combinatorial optimization problems, yielding globally optimal solutions in a possibly prohibitive amount of computation time. Approximation algorithms can be divided into two categories: constructive methods and neighborhood or local search methods. Local search techniques currently used in solving these problems include simulated annealing (SA), genetic algorithms (GA), and Tabu search methods.

Lambert and Walvekar (Lambert and Walvekar 1978) used geometric programming to determine optimal machining parameters and minimum production cost for two-pass turning operations subject to constraints of cutting force, power and surface finish.

Gopalakrishnan and Al-Khayyal (Gopalakrishnan and Al-Khayyal 1991) also used geometric programming to determine cutting speed and feed rate of turning operations to minimize total machining cost. Geometric programming is used as the basic methodology, and the solution approach for the selection of machine parameters is based on an analysis of the complementary slackness conditions and realistic machining conditions.

Prasad *et al.* (Prasad *et al.* 1997) combined geometric and linear programming techniques to determine machining parameters for turning operations. Tolerance and workpiece rigidity constraints, among others, were considered for multi-pass turning operations. Geometric programming has been considered for the determination of machining parameters by many other researchers (Brown 1962, Ermer 1971, Walvekar and Lambert 1970, Petropoulos 1973, Lambert and Walvekar 1978, Ermer and Kromodihardjo 1981, Jha 1990, Gopalakrishnan and Al-Khayyal 1991, Sonmez *et al.* 1999).

Gupta *et al.* (Gupta *et al.* 1995) considered the optimization of machining parameters in constrained multi-pass turning operations. They used two steps for solving the problem. The first step is the minimization of cost for roughing and finishing passes for various fixed depths of cut. In the second step, an optimal combination of depths of cut for each passes, the optimal number of passes and minimum total cost were determined by an integer programming model.

Al-Ahmari (Al-Ahmari 2001) presented a nonlinear programming model for the optimization of machining parameters and subdivisions of depth of cut in multi-pass turning operations based on the minimum production cost criterion with practical machining constraints.

Arezoo *et al.* (Arezoo *et al.* 2000) developed an expert system to select cutting tools and conditions of turning operations using Prolog. The system can select tool holder, insert and cutting conditions such as cutting speed, feed rate and depth of cut. Dynamic programming was used to optimize cutting conditions. Subdivisions of depth of cut were determined based on a fixed amount of stock.

Crookall and Venkataramani (Crookall and Venkataramani 1971) presented a computer analysis of parameter optimization for multi-pass turning operations, mainly for slender workpieces. They used cycle time as the optimizing criterion. The approach is essentially a computer simulation method with minimum possible generalization. The objective function contours can be drawn together with constraint functions so that the user can visually inspect the contours and evaluate the optimum point graphically. This provides an understanding of the whole problem and allows visualization of the effects of the constraints.

Among local search techniques, genetic algorithms (GA) were used by several researchers to solve parameter optimization problems. Onwubolu and Kumalo (Onwubolu and Kumalo 2001) proposed an optimization technique based on genetic algorithms to determine cutting parameters in multi-pass turning operations. The optimum machining parameters were determined by minimizing unit production cost subject to 20 practical machining constraints. They used a technique that converted crisp and non-crisp variables into binary information that were operated by the genetic operators. They compared their results with the results of other researchers (Gupta *et al.* 1995, Chen and Tsai 1996, Alberti and Perrone 1999).

Wang and Da *et al.* (Wang and Da *et al.* 2002) also used genetic algorithms to select optimal cutting parameters and cutting tools in multi-pass turning operations. The objective function includes the contributing effects of five major machining performance measures in all passes of the operation. The user can control the optimization process by configuring weighting factors for different machining performance measures.

Dereli *et al.* (Dereli *et al.* 2001) developed an optimization system for cutting parameters of prismatic parts based on genetic algorithms. It can be used as a standalone system or as an integrated module of a process planning system. The proposed optimization system suggested significant improvements in machining cost and timesaving over the handbook values.

Shunmugam *et al.* (2000) used genetic algorithms to determine machining parameters including number of passes, depth of cut in each pass, cutting speed and feed rate for multi-pass milling operations. A software for optimal allocation of total stock and minimization of total production cost was coded in C++. Computations were carried out for total stock removal of 8 mm. The total production cost for this optimal combination was lower than that calculated from handbook values.

Chen and Tsai (Chen and Tsai 1996) combined pattern search technique and simulated annealing algorithm to solve the multi-pass turning optimization problem for minimum unit production cost. They used the pattern search technique to generate a seed solution as input to the simulated annealing algorithm. Simulated annealing was used to find better solutions and moves towards the global optimal solution. They analyzed the machining parameters in roughing and finishing operations simultaneously rather than being subdivided into two separate sets.

Alberti and Perrone (1999) used genetic algorithms to solve a fuzzy probabilistic optimization model to determine cutting parameters. The fuzzy probabilistic approach can formalize the uncertainty affecting the constraints (probabilistic formulation) and data (fuzzy probabilistic formulation). They concluded that the application of the fuzzy probabilistic formulation and genetic algorithms seems to fit very well to machining economics problems.

From the literature review, one can see that much research has been done on machining parameter optimization, mainly for turning operations. With increasing applications of milling in the metal machining industry, there is a need to develop optimization models for milling operations. And developing an effective, simple and reliable solution approach is always a necessity in machining parameter optimization. In this thesis, mathematical programming models are proposed for optimal selection of machining parameters in both multi-pass turning and face-milling operations and solution methods are developed to solve these models. Solutions of the models are to minimize unit production cost. Limits on cutting speed, feed rate, depth of cut, tool life, surface roughness, cutting force and cutting power are constraints of the models. Two methods, one using integer programming and the other using nonlinear programming, are proposed to find optimal values of machining parameters. In the next Chapter, details of model formulation are presented.

Chapter 3

Model Development

Machining parameter optimization models are mathematical models formulated for realistic machining processes. These models have objective functions based on certain economic criteria and subject to various practical constrains from machining conditions and quality specifications. The formulation of process models requires the knowledge of mathematical equations to represent the relations of economical and physical parameters for the machining process and the knowledge on the whole machine-tool-workpiece system.

In this chapter, machining models, based on the minimum production cost criterion, are proposed for both multi-pass turning and face-milling operations in single-tool applications. The total depth of cut to be removed, in one finishing pass and *n* roughing passes, is cut by the same tool. Multi-pass machining operations are governed by complex machining conditions. Some of the formulations of these conditions presented in this thesis can be found in the existing literature. Others are developed in this chapter.

3.1 Objective Functions

In the optimization of machining parameters, objective functions are mathematical formulations governed by certain production criteria. They are the basis on which machining parameters are optimized.

If material cost is not considered, unit production cost UC ($/piece) can be expressed by (Armarego and Brown 1969)

$$UC = CM + CI + CR + CT \tag{3.1}$$

where CM ($/piece), CI ($/piece), CR ($/piece) and CT ($/piece) are actual machining cost, machine idle cost, tool replacement cost and tool cost, respectively. The expression in Eq (3.1) has been widely accepted by many researchers in this field.

Each cost term in Eq (3.1) is analyzed as follows.

3.1.1 Machining Cost per Unit Piece—*CM*

Machining cost CM is based on actual machining time t_m (min) and labor cost per unit time, k_0 ($/min), including overhead. It can be written as (Shaw 1984)

$$CM = k_0 t_m$$

By dividing the cutting process into one finish pass and n rough passes, machining time t_m can be expressed by (Shin and Joo 1992)

$$t_m = t_{ms} + t_{mr}$$

where t_{ms} (min) and t_{mr} (min) are finishing time and total roughing time, respectively.

Following Hitomi (1979), finish cutting time t_{ms} can be represented by

For turning: $$t_{ms} = \frac{\pi D_w L_t}{1000 V_s f_s}$$

For face milling: $$t_{ms} = \frac{\pi D_t L_{ts}}{1000 V_s f_s Z}$$

where D_w (mm) is the diameter of the workpiece in turning; L_t (mm) is cutting travel length for each turning passes; V_s (m/min) and f_s (mm/rev in turning or mm/tooth in milling) are cutting speed and feed rate for finishing operations, respectively. D_t (mm) is the diameter of the milling cutter; L_{ts} (mm) is cutting travel length for the finish milling pass; Z is the number of teeth of the milling cutter.

Rough machining time t_{mr} can be obtained by summing the cutting time for all roughing passes required.

For turning: $$t_{mr} = \sum_{i=1}^{n} \frac{\pi D_w L_t}{1000 V_{ri} f_{ri}}$$

For face milling: $$t_{mr} = \sum_{i=1}^{n} \frac{\pi D_t L_{tr}}{1000 V_{ri} f_{ri} Z}$$

where n is the number of roughing passes; V_{ri} (m/min) and f_{ri} (mm/rev in turning or mm/tooth in milling) are, respectively, cutting speed and feed rate for the i-th roughing pass; L_{tr} (mm) is cutting travel length for each rough milling passes.

Therefore, machining time t_m can be expressed by

For turning: $$t_m = \frac{\pi D_w L_t}{1000 V_s f_s} + \sum_{i=1}^{n} \frac{\pi D_w L_t}{1000 V_{ri} f_{ri}}$$

For face milling: $$t_m = \frac{\pi D_t L_{ts}}{1000 V_s f_s Z} + \sum_{i=1}^{n} \frac{\pi D_t L_{tr}}{1000 V_{ri} f_{ri} Z}$$

L_t can be calculated by

$$L_t = L + 3$$

where L (mm) is the length of the workpiece in turning and 3 (mm) is the recommended extra travel length of the cutter at the ends of each turning passes.

Following the method given in Nefedov and Osipov (1987), L_{tr} can be calculated by

$$L_{tr} = L + 0.5(D_t - \sqrt{D_t^2 - B^2}) + 3$$

where L (mm) and B (mm) are the length and width of the workpiece in milling, respectively. $0.5(D_t - \sqrt{D_t^2 - B^2})$ is the approach distance for symmetrical rough milling. 3 (mm) is recommended as the extra travel length of the cutter at the end of each rough milling pass. L_{ts} can be calculated by

$$L_{ts} = L + D_t + 3$$

where the length of D_t is used to completely clear the whole workpiece length in the finish milling pass. 3 (mm) is recommended as the extra travel length of the cutter at the end of the finish milling pass.

Therefore, machining cost CM can be calculated by

For turning:
$$CM = k_0 \left[\frac{\pi D_w L_t}{1000 V_s f_s} + \sum_{i=1}^{n} \frac{\pi D_w L_t}{1000 V_{ri} f_{ri}} \right] \quad (3.2)$$

For face milling:
$$CM = k_0 \left[\frac{\pi D_t L_{ts}}{1000 V_s f_s Z} + \sum_{i=1}^{n} \frac{\pi D_t L_{tr}}{1000 V_{ri} f_{ri} Z} \right] \quad (3.3)$$

3.1.2 Machine Idle Cost per Unit Piece—*CI*

The machine idle cost CI is defined (Shin and Joo 1992) as

$$CI = k_0 t_I$$

Machine idling time t_I (min) can be further divided into a constant term t_p due to workpiece loading and unloading operations and a variable term t_i due to tool idle motion:

$$t_I = t_p + t_i$$

where t_p (min/piece) is preparation time for loading and unloading a workpiece.

The idle tool motion time t_i (min) can be expressed by dividing tool motion into n rough passes and one finish pass as given below:

For turning: $t_i = n(h_1 L_t + h_2) + (h_1 L_t + h_2)$

For face milling: $t_i = n(h_1 L_{tr} + h_2) + (h_1 L_{ts} + h_2)$

where h_1 (min/mm) and h_2 (min) are constants related to tool travel and approach/depart time.

Therefore, machine idle cost CI can be expressed by

For turning: $$CI = k_0[t_p + n(h_1 L_t + h_2) + (h_1 L_t + h_2)$$ (3.4)

For face milling: $$CI = k_0[t_p + n(h_1 L_{tr} + h_2) + (h_1 L_{ts} + h_2)$$ (3.5)

3.1.3 Tool Replacement Cost per Unit Piece—*CR*

Following Shaw (1984), tool replacement cost CR can be written as

For turning: $$CR = k_0 t_e \frac{t_m}{T}$$ (3.6)

For face milling: $$CR = k_0 t_e Z \frac{t_m}{T}$$ (3.7)

where t_e (min/edge) is tool change time; T (min) is tool life.

3.1.4 Tool Cost per Unit Piece—*CT*

Following Shaw (1984), the tool cost CT can be given by

For turning: $$CT = k_t \frac{t_m}{T}$$ (3.8)

For face milling: $$CT = k_t Z \frac{t_m}{T}$$ (3.9)

where k_t ($/edge) is tool cost.

By substituting Eqs (3.2), (3.4), (3.6), (3.8) and (3.3), (3.5), (3.7), (3.9) into equation (3.1), respectively, the objective function to minimize the total unit production cost can be written as

$$\text{Minimize: } UC = CM + CI + CR + CT = UC_s + \sum_{i=1}^{n} UC_{ri} + k_0 t_p \tag{3.10}$$

where

For turning:

$$UC_s = \left(k_0 + \frac{k_t}{T_s} + \frac{k_0 t_e}{T_s} \right) \frac{\pi D_w L_t}{1000 V_s f_s} + k_0 (h_1 L_t + h_2) \tag{3.11}$$

$$UC_{ri} = \left(k_0 + \frac{k_t}{T_{ri}} + \frac{k_0 t_e}{T_{ri}} \right) \frac{\pi D_w L_t}{1000 V_{ri} f_{ri}} + k_0 (h_1 L_t + h_2) \tag{3.12}$$

For face milling:

$$UC_s = \left(k_0 + \frac{k_t Z}{T_s} + \frac{k_0 Z t_e}{T_s} \right) \frac{\pi D_t L_{ts}}{1000 V_s f_s Z} + k_0 (h_1 L_{ts} + h_2) \tag{3.13}$$

$$UC_{ri} = \left(k_0 + \frac{k_t Z}{T_{ri}} + \frac{k_0 Z t_e}{T_{ri}} \right) \frac{\pi D_t L_{tr}}{1000 V_{ri} f_{ri} Z} + k_0 (h_1 L_{tr} + h_2) \tag{3.14}$$

3.2 Constraints

In the optimization of machining parameters, physical limitations on cutting conditions due to the characteristics of the machine-tool-workpiece system should be identified from previous experience and taken into account as constraints in the optimization process.

When formulating the models in this thesis, it was considered: (1) parameter constraints, (2) tool life constraints, (3) surface finish constraints, (4) cutting force constraints, (5) cutting power constraints, and (6) roughing and finishing parameter relations. Some constraints, such as those on cutting speed, feed rate, etc., are simple lower and upper boundary, while others must be computed from empirical equations.

3.2.1 Parameter Constraints

Let V_{min} and V_{max} (m/min) be minimum and maximum allowable cutting speeds, respectively; f_{min} and f_{max} (mm/rev in turning or mm/tooth in milling) minimum and maximum allowable feed rates, respectively; $d_{s,min}$ and $d_{s,max}$ (mm) minimum and maximum recommended depths of cut in the finishing pass, respectively; $d_{r,min}$ and $d_{r,max}$ (mm) minimum and maximum recommended depths of cut in the roughing passes, respectively. Let V_s (m/min), f_s (mm/rev in turning or mm/tooth in milling) and d_s (mm) be cutting speed, feed rate and depth of cut in the finishing pass, respectively; V_{ri} (m/min), f_{ri} (mm/rev in turning or mm/tooth in milling) and d_{ri} (mm) cutting speed, feed rate and depth of cut in the roughing passes, respectively. For given machining conditions, there exist reasonable ranges of parameter values one can choose for the operation. They can be expressed in terms of minimum and maximum values.

For the finishing pass, parameter constraints are:

$$V_{min} \leq V_s \leq V_{max} \qquad (3.15)$$

$$f_{\min} \leq f_s \leq f_{\max} \tag{3.16}$$

$$d_{s,\min} \leq d_s \leq d_{s,\max} \tag{3.17}$$

For the roughing passes, the lower and upper limits are:

$$V_{\min} \leq V_{ri} \leq V_{\max} \tag{3.18}$$

$$f_{\min} \leq f_{ri} \leq f_{\max} \tag{3.19}$$

$$d_{r,\min} \leq d_{ri} \leq d_{r,\max} \tag{3.20}$$

The cutting speed has a greater effect on tool life than either the depth of cut or feed rate. The influence of these three machining parameters on tool life is expressed in tool life constraints discussed in Section 3.2.2.

3.2.2 Tool Life Constraints

Following Armarego and Brown (1969), tool life in turning operations can be expressed by

For the finish turning pass:

$$V_s T_s^{\alpha} f_s^{\beta} d_s^{\gamma} = C \tag{3.21}$$

where α, β, γ and C are constants; T_s is tool life in finishing (mm).

For the rough turning passes:

$$V_{ri} T_r^{\alpha} f_{ri}^{\beta} d_{ri}^{\gamma} = C \tag{3.22}$$

where T_r is tool life in roughing (mm).

Using Eq (3.15), Eq (3.21) can be revised as

$$\frac{C}{V_{\max} T_s^{\alpha}} \leq f_s^{\beta} d_s^{\gamma} \leq \frac{C}{V_{\min} T_s^{\alpha}} \tag{3.23}$$

Using Eq (3.18), Eq (3.22) can be revised as

$$\frac{C}{V_{\max} T_r^{\alpha}} \leq f_{ri}^{\beta} d_{ri}^{\gamma} \leq \frac{C}{V_{\min} T_r^{\alpha}} \tag{3.24}$$

As discussed in Nefedov and Osipov (1987), tool life in face milling can be expressed, in the finish face-milling pass, by

$$T_s^l = \frac{C_v K_v D_t^{q_v}}{V_s d_s^{x_v} f_s^{y_v} B^{s_v} Z^{p_v}} \tag{3.25}$$

For the rough face-milling passes:

$$T_r^l = \frac{C_v K_v D_t^{q_v}}{V_{ri} d_{ri}^{x_v} f_{ri}^{y_v} B^{s_v} Z^{p_v}} \tag{3.26}$$

where C_v and K_v are constants; l, p_v, q_v, s_v, x_v and y_v are constant exponents.

Using Eq (3.15), Eq (3.25) can be revised as

$$\frac{C_v K_v D_t^{q_v}}{T_s^l V_{\max} B^{s_v} Z^{p_v}} \quad f_s^{y_v} d_s^{x_v} \quad \frac{C_v K_v D_t^{q_v}}{T_s^l V_{\min} B^{s_v} Z^{p_v}} \tag{3.27}$$

Using Eq (3.18), Eq (3.26) can be revised as

$$\frac{C_v K_v D_t^{q_v}}{T_r^l V_{\max} B^{s_v} Z^{p_v}} \quad f_{ri}^{y_v} d_{ri}^{x_v} \quad \frac{C_v K_v D_t^{q_v}}{T_r^l V_{\min} B^{s_v} Z^{p_v}} \tag{3.28}$$

In single-tool applications, tool life is usually difficult to determine. In practice, tool replacement time is determined from economic considerations and quality requirements of the machined part. This also makes tool management much easier. In developing the models, one assumes that the tool life is identical in roughing and finishing operations and requires the same tool replacement time.

3.2.3 Surface Finish Constraints

Surface finish is one of the most important factors in a finishing operation because it directly affects the machining quality. Surface finish is generally affected by various parameters, but only feed and nose radius are considered here for simplification because they have the most dominant effects on surface finish. Following Boothroyd (1975), if micro millimeter is used as the dimensions of R_a, surface finish constraint on a finishing operation can be expressed by

$$R_a = \frac{32.1 f_s^2}{r_e} \quad R_{s,\max}$$

It can be rewritten as

$$f_s \quad \sqrt{r_e R_{s,\max} / 32.1} \tag{3.29}$$

where r_e , R_a and $R_{s,\max}$ are cutter nose radius (mm), arithmetic average surface roughness (μm) and the surface roughness (arithmetic average) requirement in finishing (μm), respectively.

The same inequality is used for roughing operations:

$$R_a = \frac{32.1 f_{ri}^2}{r_e} \quad R_{r,\max}$$

or

$$f_{ri} \quad \sqrt{r_e R_{r,\max} / 32.1} \tag{3.30}$$

where $R_{r,\max}$ is the surface roughness (arithmetic average) requirement in roughing (μm).

3.2.4 Cutting Force Constraints

Cutting force constraint is placed to limit the deflection of the workpiece, holding device and cutting tool to prevent chatter. If the effect of cutting speed is not considered, the cutting force constraints can be expressed by

For the finish turning pass:

$$F = k_1 f_s^{\mu} d_s^{\nu} \leq F_{\max} \tag{3.31}$$

where k_1 , μ, ν are constant exponents; F , $F_{\max}$ are cutting force and maximum allowable cutting force (N), respectively.

For the rough turning passes:

$$F = k_1 f_{ri}^{\mu} d_{ri}^{\nu} \le F_{max} \qquad (3.32)$$

For the finishing pass in face milling (Nefedov and Osipov 1987):

$$F = \frac{C_f K_f B^{s_f} Z^{p_f} d_s^{x_f} f_s^{y_f}}{D_t^{q_f}} \le F_{max} \qquad (3.33)$$

where C_f, K_f are constants; p_f, q_f, s_f, x_f, y_f are constant exponents.

For the roughing passes in face milling (Nefedov and Osipov 1987):

$$F = \frac{C_f K_f B^{s_f} Z^{p_f} d_{ri}^{x_f} f_{ri}^{y_f}}{D_t^{q_f}} \le F_{max} \qquad (3.34)$$

3.2.5 Cutting Power Constraints

Cutting power consumption should not exceed available power of the machine tool. It can be derived by multiplying cutting force and cutting speed.

For the finish turning pass, using Eq (3.31), cutting power can be expressed by

$$P = \frac{FV_s}{60000\eta} = \frac{k_1 f_s^{\mu} d_s^{\nu} V_s}{60000\eta} \le P_{max} \qquad (3.35)$$

where η is the efficiency of the machine tool; P and P_{max} are cutting power (kW) and the maximum power of the machine tool (kW), respectively.

For the rough turning passes, using Eq (3.32), cutting power can be expressed by

$$P = \frac{FV_{ri}}{60000\eta} = \frac{k_1 f_{ri}^{\mu} d_{ri}^{\nu} V_{ri}}{60000\eta} \le P_{max} \tag{3.36}$$

Similarly, cutting power for face-milling operations can be derived from Eqs (3.33) and (3.34) as

For the finish face-milling pass:

$$P = \frac{FV_s}{60000\eta} = \frac{C_f K_f B^{s_f} Z^{p_f} V_s d_s^{x_f} f_s^{y_f}}{60000\eta D_t^{q_f}} \le P_{max} \tag{3.37}$$

For the rough face-milling passes:

$$P = \frac{FV_{ri}}{60000\eta} = \frac{C_f K_f B^{s_f} Z^{p_f} V_{ri} d_{ri}^{x_f} f_{ri}^{y_f}}{60000\eta D_t^{q_f}} \le P_{max} \tag{3.38}$$

3.2.6 Roughing and Finishing Parameter Relations

The total depth of cut, d_t, should be equal to the depth of finish cut, d_s, adding the total depth of rough cuts, $\sum_{i=1}^{n} d_{ri}$. It can be expressed by

$$d_t = d_s + \sum_{i=1}^{n} d_{ri} \tag{3.39}$$

Chapter 4

Solution Methods

The primary objectives in solving the machining parameter optimization problems are reliability, accuracy of results and efficiency of computation. The selection of a suitable solution approach for the optimization problem depends on the problem itself. The form and complexity of the objective function and the constraints influence the solution procedure. The solution approaches have characteristics that affect their efficiency and accuracy. The multi-pass machining problem studied in this thesis has four decision variables. The number of passes and depth of cut for each pass can be determined by integer programming or dynamic programming. The optimal cutting speed and feed rate for each pass can be determined by a single-pass optimization method.

In this chapter, two solution methods to the multi-pass machining parameter optimization problem are proposed. One uses integer programming and the other uses nonlinear programming. The solution method using integer programming presented in Sections 4.1 and 4.2 consists two steps. The first step is to minimize the costs for individual finishing and roughing passes for various possible depths of cut to obtain corresponding optimal cutting speeds and feed rates. In the second step, an optimal combination of depths of cut for the finishing and roughing passes, an optimal number of passes, the minimum total cost and corresponding cutting speeds and feed rates are determined by an integer programming model. The solution procedure using a direct nonlinear mathematical programming model is introduced in Section 4.3.

4.1 Determining Optimal Costs for Individual Passes

In this section, the process to determine the optimal costs for the individual finishing and roughing passes considering various possible depths of cut in the recommended range is discussed. As it is well known, the production cost of a single pass operation decreases with the increase of the feed rate (Shin and Joo 1992). The maximum possible feed rate satisfying all of the constraints for each given possible depth of cut needs to be determined first when solving the problem. To do this, one needs to rewrite some of the constraint equations presented in Chapter 3.

4.1.1 Rewriting Constraints

(1) Turning operations

Eqs (3.23) and (3.31) can be rewritten as follows:

$$\left(\frac{C}{T_s^{\alpha} V_{\max} d_s^{\gamma}}\right)^{1/\beta} \quad f_s \quad \left(\frac{C}{T_s^{\alpha} V_{\min} d_s^{\gamma}}\right)^{1/\beta} \tag{4.1}$$

$$f_s \leq \left(\frac{F_{\max}}{k_1 d_s^{\nu}}\right)^{1/\mu} \tag{4.2}$$

Eqs (3.16) and (3.29) can be combined into

$$f_{\min} \leq f_s \leq \min\left(f_{\max}, \sqrt{r_e R_{s,\max} / 32.1}\right) \tag{4.3}$$

Therefore, for the finish turning pass, the objective function given by Eq (3.11) is minimized under Eqs (3.17), (3.35), (4.1), (4.2) and (4.3).

Similarly, Eqs (3.24) and (3.32) can be rewritten as follows:

$$\left(\frac{C}{T_r^{\alpha} V_{\max} d_{ri}^{\gamma}}\right)^{1/\beta} \quad f_{ri} \quad \left(\frac{C}{T_r^{\alpha} V_{\min} d_{ri}^{\gamma}}\right)^{1/\beta} \tag{4.4}$$

$$f_{ri} \leq \left(\frac{F_{\max}}{k_1 d_{ri}^{\nu}}\right)^{1/\mu} \tag{4.5}$$

Eqs (3.19) and (3.30) can be combined into

$$f_{\min} \leq f_{ri} \leq \min\left(f_{\max}, \sqrt{r_e R_{r.\max} / 32.1}\right) \tag{4.6}$$

Therefore, for the rough turning passes, the objective function given by Eq (3.12) is minimized under Eqs (3.20), (3.36), (4.4), (4.5) and (4.6).

Eq (3.39) considers the total available material stock to be removed in turning.

(2) Milling operations

Eqs (3.27) and (3.33) can be rewritten as follows:

$$\left(\frac{C_v K_v D_t^{q_v}}{T_s^{l} V_{\max} B^{s_v} Z^{p_v} d_s^{x_v}}\right)^{1/y_v} \quad f_s \quad \left(\frac{C_v K_v D_t^{q_v}}{T_s^{l} V_{\min} B^{s_v} Z^{p_v} d_s^{x_v}}\right)^{1/y_v} \tag{4.7}$$

$$f_s \leq \left(\frac{F_{max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f} d_s^{x_f}} \right)^{1/y_f} \qquad (4.8)$$

Therefore, for the finish milling pass, the objective function given by Eq (3.13) is minimized under Eqs (3.17), (3.37), (4.3), (4.7) and (4.8).

Similarly, Eqs (3.28) and (3.34) can be rewritten as follows:

$$\left(\frac{C_v K_v D_t^{q_v}}{T_r^l V_{max} B^{s_v} Z^{p_v} d_{ri}^{x_v}} \right)^{1/y_v} \quad f_{ri} \quad \left(\frac{C_v K_v D_t^{q_v}}{T_r^l V_{min} B^{s_v} Z^{p_v} d_{ri}^{x_v}} \right)^{1/y_v} \qquad (4.9)$$

$$f_{ri} \leq \left(\frac{F_{max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f} d_{ri}^{x_f}} \right)^{1/y_f} \qquad (4.10)$$

Therefore, for the rough milling passes, the objective function given by Eq (3.14) is minimized under Eqs (3.20), (3.38), (4.6), (4.9) and (4.10).

Eq (3.39) is also used to consider the total available depth of stock removal in milling.

4.1.2 Determination Procedure

To determine the costs for various possible depths of cut for the finishing pass and to get corresponding optimal cutting speeds and feed rates, we first identify a series of possible depths of cut. If the recommended range of depth of cut is divided into equal steps, the *j*-th value of the series, d_{sj}, can be calculated by

$$d_{sj} = d_{s,\min} + \frac{j(d_{s,\max} - d_{s,\min})}{m_0} \tag{4.11}$$

where $d_{s,\min}$ and $d_{s,\max}$ are, respectively, minimum and maximum recommended depths of cut in finishing, j = 0, 1, 2... m_0 and m_0 is a suitable integer. In practice, m_0 is normally determined based on machining experience. In solving the problem presented in this thesis, m_0 is determined based on accuracy of the results and computational considerations.

The step-by-step procedure to determine the optimal costs and corresponding cutting speeds and feed rates for various possible depths of cut for the finish face-milling pass is given below.

Step 1:

For a given possible depth of cut d_{sj} satisfying Eq (3.17), the optimal feed value f_{sj}^{*} is determined by the following model:

Maximize: f_{sj}

Subject to:

$$\left(\frac{C_v K_v D_t^{q_v}}{T_s^l V_{\max} B^{s_v} Z^{p_v} d_{sj}^{x_v}}\right)^{1/y_v} \quad f_{sj} \quad \left(\frac{C_v K_v D_t^{q_v}}{T_s^l V_{\min} B^{s_v} Z^{p_v} d_{sj}^{x_v}}\right)^{1/y_v} \tag{4.12}$$

$$f_{sj} \le \left(\frac{F_{max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f} d_{sj}^{x_f}} \right)^{1/y_f} \quad (4.13)$$

$$f_{min} \le f_{sj} \le \min\left(f_{max}, \sqrt{r_e R_{f.max} / 32.1} \right) \quad (4.14)$$

Step 2:

Using f_{sj}^*, T_s and d_{sj}, the optimal cutting speed V_{sj}^* is calculated by

$$V_{sj} = \frac{C_v K_v D_t^{q_v}}{T_s^l B^{s_v} Z^{p_v} f_{sj}^{y_v} d_{sj}^{x_v}} \quad (4.15)$$

Eq (4.15) is derived from Eq (3.25). Then, machining power is checked by Eq (3.37) using V_{sj}^*, f_{sj}^* and d_{sj}. If the calculated power exceeds the maximum power available, V_{sj}^* is recalculated by

$$V_{sj} = \frac{60000 P_{max} \eta D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f} f_s^{y_f} d_s^{x_f}} \quad (4.16)$$

Eq (4.16) is derived from Eq (3.37).

Step 3:

For the given d_{sj}, calculate minimum cost UC_{sj}^* by Eq (3.11) using V_{sj}^* and f_{sj}^*.

Step 4:

Repeat steps 1 to 3 for each d_{sj} to calculate UC_{sj}^* and corresponding V_{sj}^* and f_{sj}^*.

Similarly, to determine the costs for various possible depths of cut for the roughing passes, we first identify a series of possible depths of cut. Define d_{rij} as

$$d_{rij} = d_{r,\min} + \frac{j(d_{r,\max} - d_{r,\min})}{m_i} \tag{4.17}$$

where $d_{r,\min}$ and $d_{r,\max}$ are, respectively, minimum and maximum recommended depths of cut in roughing, $j = 0, 1, 2 \ldots m_i$, $i = 1, 2 \ldots n$, m_i are integers determined in a similar way as for m_0 in the finishing pass.

For the roughing passes, a computation procedure similar to that given in steps 1 to 4 can be applied to find optimal V_{rij}^* and f_{rij}^* and corresponding minimum cost UC_{rij}^* for each given possible depth of cut d_{rij}.

The resulting costs UC_{sj}^* and UC_{rij}^* can be tabulated along with optimal speeds V_{sj}^* and V_{rij}^* as well as optimal feed rates f_{sj}^* and f_{rij}^*, for the finishing pass and roughing passes, respectively.

The above procedure can be applied to turning to find a minimum cost and corresponding cutting speeds and feed rates for each given possible depth of cut for individual finishing and roughing passes.

4.2 Determining Depths of Cut by Integer Programming

With the optimal cutting speeds and feed rates for various fixed possible depths of cut for individual finishing and roughing passes obtained using the computation procedure in

Section 4.1, we need to determine the optimal depths of cut and optimal number of passes to minimize the total unit cost. A binary integer programming model is used to find optimal solutions to this problem.

As we discussed, there are n roughing passes and one finishing pass in the entire machining operation. The binary decision variables in the integer programming model are then defined as

$$X_{ij} = \begin{cases} 1, \text{if } d_{ij} \text{ value is selected in pass } i \\ 0, \text{otherwise} \end{cases}$$

With so defined decision variables X_{ij} and calculated cost coefficients, the integer programming model to minimize the total unit production cost can be written as

$$\text{Minimize: } UC = UC_s + \sum_{i=1}^{n} UC_{ri} + k_0 t_p$$

$$= \sum_{j=0}^{m_0} UC_{sj}^{*} X_{0j} + \sum_{i=1}^{n} \sum_{j=0}^{m_i} UC_{rij}^{*} X_{ij} + k_0 t_p \tag{4.18}$$

Subject to:

$$\sum_{j=0}^{m_0} X_{0j} = 1 \tag{4.19}$$

$$\sum_{j=0}^{m_i} X_{ij} \leq 1 \text{ for } i = 1, 2 \ldots n \tag{4.20}$$

$$\sum_{i=0}^{n} \sum_{j=0}^{m_i} d_{ij} X_{ij} = d_t \tag{4.21}$$

The objective function, Eq (4.18), is to minimize the total unit production cost. Eq (4.19) implies that one and only one d_{0j} should be selected for the finishing pass. Eq (4.20) enforces that only one d_{ij} is selected if a roughing pass is selected. Eq (4.21) implies that the sum of the individual depths of cut is equal to the total depth of cut.

The integer programming model can be solved by available optimization software. In this research, the model was coded by LINGO (Schrage 1991). The example problems presented in the next section were solved by LINGO on IBM-PC compatible platforms.

4.3 Solving the Problem by Nonlinear Programming

The mathematical models presented in this thesis for the optimization of machining parameters are multi-variable, nonlinear models. They can be formulated as nonlinear programming models and solved by LINGO. Results from the integer programming models and the nonlinear programming models can be compared to verify the solution approaches presented in this thesis.

For the turning parameter optimization problem, the nonlinear programming model can be described as

$$\text{Minimize: } UC = UC_s + \sum_{i=1}^{n} UC_{ri} + k_0 t_p \tag{4.22}$$

where

$$UC_s = \left(k_0 + \frac{k_t}{T_s} + \frac{k_0 t_e}{T_s} \right) \frac{\pi D_w L_t}{1000 V_s f_s} + k_0 (h_1 L_t + h_2) \tag{4.23}$$

$$UC_{ri} = \left(k_0 + \frac{k_t}{T_{ri}} + \frac{k_0 t_e}{T_{ri}} \right) \frac{\pi D_w L_t}{1000 V_{ri} f_{ri}} + k_0 (h_1 L_t + h_2) \tag{4.24}$$

Subject to:

$$V_{\min} \leq V_s \leq V_{\max} \tag{4.25}$$

$$V_{\min} \leq V_{ri} \leq V_{\max} \tag{4.26}$$

$$d_{s,\min} \leq d_s \leq d_{s,\max} \tag{4.27}$$

$$d_{r,\min} \leq d_{ri} \leq d_{r,\max} \tag{4.28}$$

$$V_s f_s^{\beta} d_s^{\gamma} \leq \frac{C}{T_s^{\alpha}} \tag{4.29}$$

$$V_{ri} f_{ri}^{\beta} d_{ri}^{\gamma} \leq \frac{C}{T_r^{\alpha}} \tag{4.30}$$

$$f_{\min} \leq f_s \leq \min \left(f_{\max}, \sqrt{r_e R_{s,\max} / 32.1} \right) \tag{4.31}$$

$$f_{\min} \leq f_{ri} \leq \min \left(f_{\max}, \sqrt{r_e R_{r,\max} / 32.1} \right) \tag{4.32}$$

$$f_s^{\mu} d_s^{\nu} \leq \frac{F_{\max}}{k_1} \tag{4.33}$$

$$f_{ri}^{\mu} d_{ri}^{\nu} \leq \frac{F_{\max}}{k_1} \tag{4.34}$$

$$V_s f_s^{\mu} d_s^{\nu} \leq \frac{60000 \eta P_{\max}}{k_1} \tag{4.35}$$

$$V_{ri} f_{ri}^{\mu} d_{ri}^{\nu} \leq \frac{60000\eta P_{\max}}{k_1} \quad (4.36)$$

$$d_s + \sum_{i=1}^{n} d_{ri} = d_t \quad (4.37)$$

Decision variables are V_s, f_s, d_s, V_{ri}, f_{ri}, d_{ri} and n.

Similarly, for the face-milling parameter optimization problem, the nonlinear programming model can be described as

$$\text{Minimize: } UC = UC_s + \sum_{i=1}^{n} UC_{ri} + k_0 t_p \quad (4.38)$$

where

$$UC_s = \left(k_0 + \frac{k_t Z}{T_s} + \frac{k_0 Z t_e}{T_s} \right) \frac{\pi D_t^{(1-q_v)} L_{ts} T_s^l B^{s_v} f_s^{(y_v-1)} d_s^{x_v}}{1000 C_v K_v Z^{(1-p_v)}} + k_0 (h_1 L_{ts} + h_2) \quad (4.39)$$

$$UC_{ri} = \left(k_0 + \frac{k_t Z}{T_r} + \frac{k_0 Z t_e}{T_r} \right) \frac{\pi D_t^{(1-q_v)} L_{tr} T_r^l B^{s_v} f_{ri}^{(y_v-1)} d_{ri}^{x_v}}{1000 C_v K_v Z^{(1-p_v)}} + k_0 (h_1 L_{tr} + h_2) \quad (4.40)$$

Subject to:

$$V_{\min} \leq V_s \leq V_{\max} \quad (4.41)$$

$$V_{\min} \leq V_{ri} \leq V_{\max} \quad (4.42)$$

$$d_{s,\min} \leq d_s \leq d_{s,\max} \quad (4.43)$$

$$d_{r,\min} \leq d_{ri} \leq d_{r,\max} \quad (4.44)$$

$$V_s f_s^{y_v} d_s^{x_v} \quad \frac{C_v K_v D_t^{q_v}}{T_s^l B^{s_v} Z^{p_v}} \tag{4.45}$$

$$V_s f_{ri}^{y_v} d_{ri}^{x_v} \quad \frac{C_v K_v D_t^{q_v}}{T_r^l B^{s_v} Z^{p_v}} \tag{4.46}$$

$$f_{\min} \le f_s \le \min\left(f_{\max}, \sqrt{r_e R_{s,\max} / 32.1}\right) \tag{4.47}$$

$$f_{\min} \le f_{ri} \le \min\left(f_{\max}, \sqrt{r_e R_{r,\max} / 32.1}\right) \tag{4.48}$$

$$f_s^{y_f} d_s^{x_f} \le \frac{F_{\max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f}} \tag{4.49}$$

$$f_{ri}^{y_f} d_{ri}^{x_f} \le \frac{F_{\max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f}} \tag{4.50}$$

$$V_s f_s^{y_f} d_s^{x_f} \le \frac{60000 \eta P_{\max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f}} \tag{4.51}$$

$$V_{ri} f_{ri}^{y_f} d_{ri}^{x_f} \le \frac{60000 \eta P_{\max} D_t^{q_f}}{C_f K_f B^{s_f} Z^{p_f}} \tag{4.52}$$

$$d_s + \sum_{i=1}^{n} d_{ri} = d_t \tag{4.53}$$

Decision variables are V_s, f_s, d_s, V_{ri}, f_{ri}, d_{ri} and n.

LINGO is a simple and effective tool for solving linear and nonlinear optimization problems.

Chapter 5

Example Problems

In this section, examples to determine optimal machining parameters for both multi-pass turning and face-milling operations are presented to test the optimization models and the solution methods developed in this thesis.

5.1 Turning Example

The multi-pass turning example given in Shin and Joo (1992) is considered in this thesis for the purpose of illustration. The example was used in Shin and Joo (1992) to explain a solution procedure to the turning optimization problem by dynamic programming. The same example was used in Gupta *et al.* (1995) to illustrate a solution approach using integer programming and in Al-Ahmari (2001) to illustrate a solution approach using nonlinear programming. Data related to cost, machining conditions and machining requirements are shown in Table 5.1.

5.1.1 Solving the Turning Example by Integer Programming

In solving the problem, we first followed the step-by-step procedure presented in Section 4.1 to compute the cost coefficients. Then we used the integer programming model to find the optimal solutions of the problem.

Table 5.1: Data for the given turning example

Given parameters or constants	Signs and units	Values
Length of the workpiece	L (mm)	300
Diameter of the workpiece	D_w (mm)	50
Nose radius of the cutter	r_ε (mm)	1.2
Direct labor cost plus overhead	k_0 (\$/min)	0.5
Cutting edge cost	k_t (\$/cutting edge)	2.5
Tool change time	t_e (min/cutting edge)	1.5
Preparation time	t_p (min/piece)	0.75
Tool return time	h_1 (min/mm)	7×10^{-4}
Tool advance/return time	h_2 (min)	0.3
Maximum cutting speed	$V_{\max}$ (m/min)	500
Minimum cutting speed	$V_{\min}$ (m/min)	5
Maximum feed rate	$f_{\max}$ (mm/rev)	0.9
Minimum feed rate	$f_{\min}$ (mm/rev)	0.1
Maximum depth of cut for finishing	$d_{s.\max}$ (mm)	2.0
Minimum depth of cut for finishing	$d_{s.\min}$ (mm)	0.5
Maximum depth of cut for roughing	$d_{r.\max}$ (mm)	4.0
Minimum depth of cut for roughing	$d_{r.\min}$ (mm)	1.0
Tool replacement time	T (min)	25
Surface roughness requirement for finishing	$R_{s.\max}$ (μm)	2.5
Surface roughness requirement for roughing	$R_{r.\max}$ (μm)	25
Maximum cutting force	$F_{\max}$ (N)	1960
Maximum cutting power	$P_{\max}$ (kW)	5
Machine tool efficiency	η	0.85
Constants and exponents in tool life equations	C=227, $\alpha = 0.2$, $\beta = 0.35$, $\gamma = 0.15$	
Constants and exponents in cutting force and power equations	k_1=1058, $\mu = 0.75$, $\nu = 0.95$	

For the recommended range of depth of cut from 0.5mm to 2.0mm for the finishing pass, a multiplication of 0.1mm was used for the generation of possible depths of cut. The division results are shown in Column 2 of Table 5.2. For the roughing passes, the range of cut is from 1.0mm to 4.0mm. It was divided into 30 segments, 0.1mm each, to generate possible depths of cut. The division results are shown in Column 2 of Table 5.3.

Following steps 1 to 4 described in Section 4.1, the costs for the finishing pass and roughing passes were minimized separately for each given possible depth of cut. For the finishing pass, f_{sj}^*, V_{sj}^* and UC_{sj}^* were calculated and entered in Table 5.2. For the roughing passes, f_{rij}^*, V_{rij}^* and UC_{rij}^* were calculated and entered in Table 5.3.

Table 5.2: Optimal parameters and costs for the finish turning pass (T = 25min)

No. j	d_{ij} (mm)	Single finishing pass		
		V_{sj}^* (m/min)	f_{sj}^* (mm/rev)	UC_{sj}^* ($/pass)
0	0.5	200.32	0.3057	0.7457
1	0.6	194.92	0.3057	0.7592
2	0.7	190.47	0.3057	0.7710
3	0.8	186.69	0.3057	0.7814
4	0.9	183.42	0.3057	0.7908
5	1.0	180.54	0.3057	0.7993
6	1.1	177.98	0.3057	0.8071
7	1.2	175.67	0.3057	0.8144
8	1.3	173.58	0.3057	0.8211
9	1.4	171.66	0.3057	0.8274
10	1.5	169.89	0.3057	0.8334
11	1.6	168.25	0.3057	0.8390
12	1.7	166.73	0.3057	0.8443
13	1.8	165.31	0.3057	0.8494
14	1.9	163.97	0.3057	0.8542
15	2.0	162.71	0.3057	0.8588

Table 5.3: Optimal parameters and costs for the rough turning passes ($T = 25$min)

No. j	d_{ij} (mm)	Single roughing pass		
		V^*_{rij} (m/min)	f^*_{rij} (mm/rev)	UC^*_{rij} ($/pass)
0	1.0	123.72	0.9	0.5253
1	1.1	121.97	0.9	0.5292
2	1.2	120.39	0.9	0.5328
3	1.3	118.95	0.9	0.5361
4	1.4	117.63	0.9	0.5393
5	1.5	116.42	0.9	0.5422
6	1.6	115.30	0.9	0.5450
7	1.7	114.26	0.9	0.5476
8	1.8	113.28	0.9	0.5502
9	1.9	112.37	0.9	0.5525
10	2.0	111.51	0.9	0.5548
11	2.1	111.19	0.8885	0.5596
12	2.2	112.72	0.8377	0.5736
13	2.3	114.20	0.7918	0.5877
14	2.4	115.63	0.7503	0.6017
15	2.5	117.03	0.7124	0.6157
16	2.6	118.38	0.6779	0.6297
17	2.7	119.70	0.6463	0.6437
18	2.8	120.98	0.6172	0.6576
19	2.9	122.23	0.5903	0.6716
20	3.0	123.46	0.5655	0.6855
21	3.1	124.65	0.5425	0.6995
22	3.2	125.81	0.5211	0.7134
23	3.3	126.96	0.5012	0.7273
24	3.4	128.07	0.4826	0.7412
25	3.5	129.17	0.4652	0.7550
26	3.6	130.05	0.4489	0.7697
27	3.7	130.05	0.4336	0.7878
28	3.8	130.05	0.4192	0.8061
29	3.9	130.05	0.4056	0.8245
30	4.0	130.05	0.3928	0.8430

Data shown in Table 5.2 and 5.3 were used to determine the optimal depths of cut by the integer programming model discussed in Section 4.2. The formulated integer programming model was solved by LINGO for d_t = 6.0, 7.0, 8.0, 9.0, 10.0 and 12.0mm. Computation results are shown in Table 5.4. LINGO code of the integer programming model for solving the turning example is given in Appendix B.1.

Table 5.4: Optimal solutions of the turning example

d_t (mm)	6.0	7.0	8.0	9.0	10.0	12.0
d_s^{opt} (mm)	2.0	2.0	2.0	2.0	2.0	2.0
d_{r1}^{opt} (mm)	4.0	2.9	3.5	3.5	4.0	3.5
d_{r2}^{opt} (mm)		2.1	2.5	3.5	4.0	3.5
d_{r3}^{opt} (mm)						3.0
f_s^{opt} (mm/rev)	0.3057	0.3057	0.3057	0.3057	0.3057	0.3057
f_{r1}^{opt} (mm/rev)	0.3623	0.5903	0.4652	0.4652	0.3623	0.4652
f_{r2}^{opt} (mm/rev)		0.8885	0.7124	0.4652	0.3623	0.4652
f_{r3}^{opt} (mm/rev)						0.5655
V_s^{opt} (m/min)	162.71	162.71	162.71	162.71	162.71	162.71
V_{r1}^{opt} (m/min)	138.18	122.23	129.17	129.17	138.18	129.17
V_{r2}^{opt} (m/min)		111.19	117.03	129.17	138.18	129.17
V_{r3}^{opt} (m/min)						123.46
n^{opt}	1	2	2	2	2	3
UC^{opt} ($/piece)	2.0768	2.4650	2.6045	2.7438	2.9198	3.4293

The results in the second column of Table 5.4 show that one roughing pass and one finishing pass are required when the total depth of cut is d_t = 6.0mm. The depths of cut are 4.0mm for the roughing pass and 2.0mm for the finishing pass, respectively. The total production cost is $2.0768/piece. According to the Machining Data Handbook (Machinability Data Center 1980), two roughing passes and one finishing pass are required for $d_t = 6.0$mm. The depths of cut are 4.0mm and 1.0mm for the roughing passes and 1.0mm for the finishing pass, respectively, with total production cost of \$2.9684/piece. The proposed optimization method reduces the unit production cost by 42.93% from the handbook's suggested cutting scheme. When the total cut is $d_t =$ 10.0mm, the optimal cutting scheme generated by our model is to have two roughing passes and one finish pass. The depths of cut are 4.0mm each for the two roughing passes and 2.0mm for the finishing pass, respectively. The total production cost is \$2.9198/piece. The Machining Data Handbook (Machinability Data Center 1980) recommends three roughing passes and one finishing pass when $d_t = 10$mm. The depths of cut of 4.0mm, 4.0mm and 1.0mm for the roughing passes and 1.0mm for the finishing pass are suggested. The total production cost will be \$3.8114/piece. The optimization method can reduce the unit production cost by 30.54%. The proposed optimization method also results in much lower unit production costs compared to the optimization solutions in Shin and Joo (1992) and Gupta *et al.* (1995).

The minimum unit production costs vary slightly when different values of tool life are adopted. In this thesis, we calculated the minimum unit production costs for different depth of cut ($d_t = 6.0$, 7.0, 8.0, 9.0, 10.0 and 12.0mm) corresponding to various tool replacement times ($T = 20$, 22, 25, 28, 30, 32, 35, 40, 45, 50 and 60min). The results are shown in Table 5.5 (The relative data are given in Table A.1 – Table A.20 in Appendix A). We also plotted the data at Column 6 in Table 5.5. The graph is shown in Figure 5.1. It shows that when $d_t = 10$mm, the effect of tool replacement time on the minimum unit production cost is negligible in the considered range from 20min to 60min.

Table 5.5: Minimum unit production costs in turning

d_t (mm) / T (min)	6.0	7.0	8.0	9.0	10.0	12.0
20	2.1106	2.4725	2.6133	2.7917	2.9839	3.4753
22	2.0947	2.4679	2.6081	2.7664	2.9542	3.4404
25	2.0768	2.4650	2.6045	2.7438	2.9198	3.4293
28	2.0640	2.4653	2.6045	2.7443	2.8940	3.4294
30	2.0598	2.4669	2.6060	2.7460	2.8849	3.4311
32	2.0619	2.4692	2.6085	2.7488	2.8880	3.4346
35	2.0658	2.4736	2.6134	2.7541	2.8938	3.4414
40	2.0740	2.5799	2.7211	2.7653	2.9060	3.5531
45	2.0835	2.4936	2.6354	2.7783	2.9201	3.4720
50	2.0935	2.5048	2.6479	2.7920	2.9350	3.4894
60	2.1144	2.5284	2.6740	2.8205	2.9660	3.5256

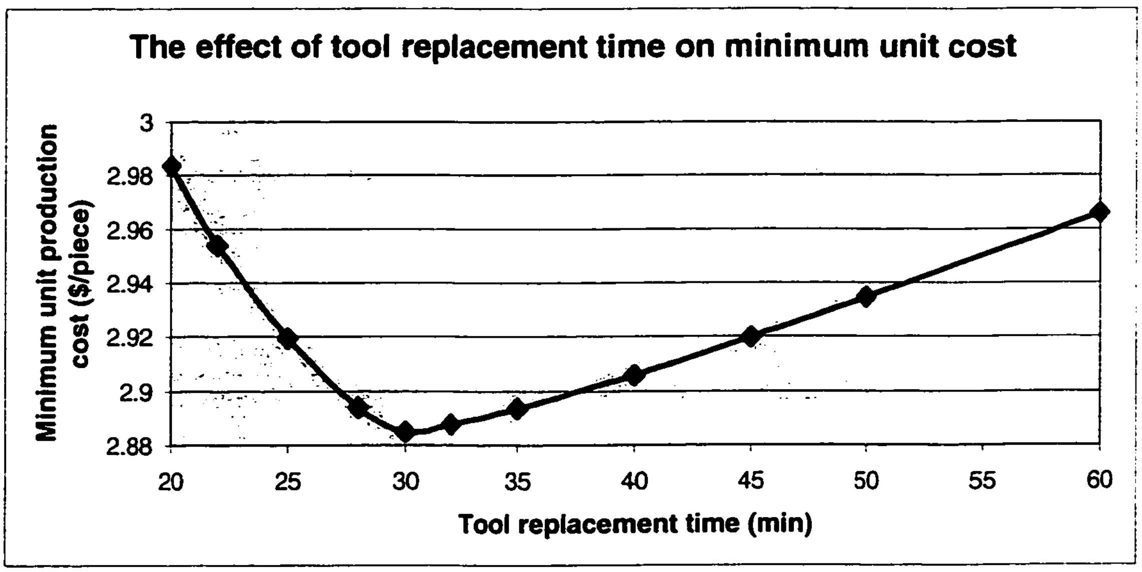

Figure 5.1: Tool replacement time and production cost in turning (d_t = 10mm)

5.1.2 Solving the Turning Example by Nonlinear Programming

The same turning example can also be solved by nonlinear programming. According to the data given in Table 5.1, UC_s and UC_{ri} were calculated by Eqs (4.23) and (4.24), respectively. After $k_0 t_p$ was calculated, the total unit production cost UC given by Eq (4.22) can be expressed by

$$UC = 29.985 V_s^{-1} f_s^{-1} + \sum_{i=1}^{n} (0.25605 + 29.985 V_{ri}^{-1} f_{ri}^{-1}) + 0.63105$$

We also calculated the bound values in Eqs (4.25) – (4.37), according to the data in Table 5.1. Then, the nonlinear programming model to minimize total unit production cost UC for the given turning example can be stated as

Minimize: $UC = 29.985 V_s^{-1} f_s^{-1} + \sum_{i=1}^{n} (0.25605 + 29.985 V_{ri}^{-1} f_{ri}^{-1}) + 0.63105$

Subject to:

$5 \le V_s \le 500$

$5 \le V_{ri} \le 500$

$0.5 \le d_s \le 2.0$

$1.0 \le d_{ri} \le 4.0$

$V_s f_s^{0.35} d_s^{0.15} \le 119.24$

$V_{ri} f_{ri}^{0.35} d_{ri}^{0.15} \le 119.24$

$0.1 \le f_s \le \min(0.9, 0.3057) = 0.3057$

$0.1 \le f_{ri} \le \min(0.9, 0.9667) = 0.9$

$f_s^{0.75} d_s^{0.95} \le 1.8519$

$f_{ri}^{0.75} d_{ri}^{0.95} \le 1.8519$

$V_s f_s^{0.75} d_s^{0.95} \le 240.83$

$V_{ri} f_{ri}^{0.75} d_{ri}^{0.95} \le 240.83$

$$d_s + \sum_{i=1}^{n} d_{ri} = d_t$$

The above nonlinear programming model was coded in LINGO and solved on IBM-PC compatible platforms for d_t = 6.0, 7.0, 8.0, 9.0, 10.0 and 12.0mm. LINGO code of the nonlinear programming model for solving the turning example is given in Appendix B.2. Optimal results on all instances were achieved in less than one second of computation. The results from the above nonlinear programming model are exactly the same as the results presented in Table 5.4 by solving the integer programming model. The proposed solution method results in lower unit production costs compared to the optimization solutions in Al-Ahmari (2001) when the same allowable range of depth of cut is adopted.

When using the nonlinear programming model to solve machining parameter optimization problems, we may obtain solutions with endless decimal places. Therefore, this method, after the solutions are rounded off properly, can be used to select machining parameters in NC/CNC machining with infinitely variable speeds and feed rates while the optimization method using integer programming can be used for machining parameter selection on conventional machine tools.

5.2 Face-Milling Example

The example problem of multi-pass face milling presented in this thesis is based on the face-milling example given in Nefedov and Osipov (1987). In this example, cemented carbide cutting tools are used to machine a gray cast iron workpiece (190HB). The same example was also used in Shunmugam *et al.* (2000). In that paper, a solution approach was developed to solve milling parameter problems using genetic algorithm. The problem data are given in Table 5.6.

5.2.1 Solving the Face-Milling Example by Integer Programming

Similar to solving the turning example problem by integer programming in Section 5.1.1, in solving this milling example problem, we first followed the step-by-step procedure presented in Section 4.1 to compute the cost coefficients. Then we used the integer programming model to find the optimal solutions of the problem.

For the recommended range of depth of cut from 0.5mm to 2.0mm for the finishing pass, a multiplication of 0.1mm was used for the generation of possible depths of cut. The division results are shown in Column 2 of Table 5.7. For the roughing passes, the range of cut is from 1.0mm to 4.0mm. It was divided into 30 segments, 0.1mm each, to generate possible depths of cut. The division results are shown in Column 2 of Table 5.8.

Table 5.6: Data for the given face-milling example

Given parameters or constants	Signs and units	Values
Length of the workpiece	L (mm)	240
Width of the workpiece	B (mm)	100
Diameter of the cutter	D (mm)	160
Tooth number of the cutter	Z	16
Nose radius of the cutter	r_e (mm)	1.0
Direct labor cost plus overhead	k_0 (\$/min)	0.5
Cutting edge cost	k_t (\$/cutting edge)	2.5
Tool change time	t_e (min/cutting edge)	1.5
Preparation time	t_p (min/piece)	0.75
Tool return time	h_1 (min/mm)	7×10^{-4}
Tool advance/return time	h_2 (min)	0.3
Maximum cutting speed	V_{max} (m/min)	300
Minimum cutting speed	V_{min} (m/min)	50
Maximum feed rate	f_{max} (mm/tooth)	0.6
Minimum feed rate	f_{min} (mm/tooth)	0.1
Maximum depth of cut for finishing	$d_{s,max}$ (mm)	0.5
Minimum depth of cut for finishing	$d_{s,min}$ (mm)	2.0
Maximum depth of cut for roughing	$d_{r,max}$ (mm)	4.0
Minimum depth of cut for roughing	$d_{r,min}$ (mm)	1.0
Tool replacement time	T (min)	240
Surface roughness requirement for finishing	$R_{s,max}$ (μm)	2.5
Surface roughness requirement for roughing	$R_{r,max}$ (μm)	25
Maximum cutting force	F_{max} (N)	8000
Maximum cutting power	P_{max} (kW)	10
Machine tool efficiency	η	0.8
Constants and exponents in tool life equations	$C_v = 445$, $l = 0.32$, $x_v = 0.15$, $y_v = 0.35$, $p_v = 0$, $q_v = 0.2$, $s_v = 0.2$, $K_v = 1.0$	
Constants and exponents in cutting force power equations	$C_f = 534.6$, $x_f = 0.9$, $y_f = 0.74$, $s_f = 1.0$, $p_f = 1.0$, $q_f = 1.0$, $K_f = 1.0$	

Following steps 1 to 4 described in Section 4.1, the costs for the finishing pass and roughing passes were minimized separately for each given possible depth of cut. For the finishing pass, f_{sj}^{*}, V_{sj}^{*} and UC_{sj}^{*} were calculated and entered in Table 5.7. For the roughing passes, f_{rij}^{*}, V_{rij}^{*} and UC_{rij}^{*} were calculated and entered in Table 5.8.

Data shown in Tables 5.7 and 5.8 were used to determine the optimal depths of cut by the integer programming model discussed in Section 4.2. The formulated integer programming model was solved by LINGO for d_t = 6.0, 7.0, 8.0, 9.0, 10.0, 12.0mm. Computation results are shown in Table 5.9. LINGO code of the integer programming model for solving the milling example is given in Appendix B.3.

Table 5.7: Optimal parameters and costs for the finish milling pass (T = 240min)

No. j	d_{ij} (mm)	Single finishing pass		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/tooth)	UC_{sj}^{*} ($/pass)
0	0.5	146.78	0.2791	0.5125
1	0.6	142.82	0.2791	0.5187
2	0.7	139.55	0.2791	0.5240
3	0.8	136.78	0.2791	0.5287
4	0.9	134.39	0.2791	0.5330
5	1.0	132.28	0.2791	0.5368
6	1.1	130.40	0.2791	0.5403
7	1.2	128.71	0.2791	0.5436
8	1.3	127.18	0.2791	0.5467
9	1.4	125.77	0.2791	0.5495
10	1.5	124.48	0.2791	0.5522
11	1.6	123.28	0.2791	0.5548
12	1.7	122.16	0.2791	0.5572
13	1.8	121.12	0.2791	0.5595
14	1.9	120.14	0.2791	0.5616
15	2.0	119.22	0.2791	0.5637

Table 5.8: Optimal parameters and costs for the rough milling passes ($T = 240$min)

No. j	d_{ij} (mm)	Single roughing pass		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	101.20	0.6	0.3378
1	1.1	99.760	0.6	0.3392
2	1.2	98.466	0.6	0.3405
3	1.3	97.291	0.6	0.3417
4	1.4	96.215	0.6	0.3428
5	1.5	91.019	0.6	0.3486
6	1.6	85.883	0.6	0.3550
7	1.7	81.322	0.6	0.3614
8	1.8	77.245	0.6	0.3678
9	1.9	73.576	0.6	0.3741
10	2.0	70.256	0.6	0.3804
11	2.1	67.238	0.6	0.3866
12	2.2	64.481	0.6	0.3928
13	2.3	61.952	0.6	0.3990
14	2.4	60.017	0.5947	0.4055
15	2.5	60.017	0.5659	0.4139
16	2.6	60.017	0.5395	0.4224
17	2.7	60.017	0.5153	0.4309
18	2.8	60.017	0.4930	0.4394
19	2.9	60.017	0.4724	0.4481
20	3.0	60.017	0.4534	0.4568
21	3.1	60.017	0.4356	0.4656
22	3.2	60.017	0.4191	0.4744
23	3.3	60.017	0.4037	0.4833
24	3.4	60.017	0.3893	0.4922
25	3.5	60.017	0.3758	0.5013
26	3.6	60.017	0.3632	0.5103
27	3.7	60.017	0.3513	0.5194
28	3.8	60.017	0.3401	0.5286
29	3.9	60.017	0.3295	0.5378
30	4.0	60.017	0.3195	0.5471

Table 5.9: Optimal solutions of the milling example

d_t (mm)	6.0	7.0	8.0	9.0	10.0	12.0
d_s^{opt} (mm)	2.0	2.0	2.0	2.0	2.0	2.0
d_{r1}^{opt} (mm)	4.0	2.5	3.0	3.6	4.0	3.4
d_{r2}^{opt} (mm)		2.5	3.0	3.4	4.0	3.3
d_{r3}^{opt} (mm)						3.3
f_s^{opt} (mm/tooth)	0.2791	0.2791	0.2791	0.2791	0.2791	0.2791
f_{r1}^{opt} (mm/tooth)	0.3195	0.5659	0.4534	0.3632	0.3195	0.3893
f_{r2}^{opt} (mm/tooth)		0.5659	0.4534	0.3893	0.3195	0.4037
f_{r3}^{opt} (mm/tooth)						0.4037
V_s^{opt} (m/min)	119.22	119.22	119.22	119.22	119.22	119.22
V_{r1}^{opt} (m/min)	60.017	60.017	60.017	60.017	60.017	60.017
V_{r2}^{opt} (m/min)		60.017	60.017	60.017	60.017	60.017
V_{r3}^{opt} (m/min)						60.017
n^{opt}	1	2	2	2	2	3
UC^{opt} ($/piece)	1.4858	1.7665	1.8523	1.9412	2.0329	2.3975

The results in the second column of Table 5.9 show that one roughing pass and one finishing pass are required when the total cut is $d_t = 6.0$mm. The depths of cut are 4.0mm for the roughing pass and 2.0mm for the finishing pass, respectively. The total production cost is $1.4858/piece. According to the Machining Data Handbook (Machinability Data Center 1980), two roughing passes and one finishing pass are required for $d_t = 6.0$mm. The depths of cut are 4.0 and 1.0mm for the roughing passes and 1.0mm for the finish pass, respectively, with total production cost of $1.8890/piece. The proposed optimization method reduces the unit production cost by 27.14% from the handbook's suggested cutting scheme. When the total cut is $d_t = 10.0$mm, the optimal cutting scheme generated by our model is to have two roughing passes and one finishing pass. The depths of cut are 4.0mm each for the two roughing passes and 2.0mm for the finishing pass, respectively. The total production cost is \$2.0329/piece. The Machining Data Handbook (Machinability Data Center 1980) recommends three roughing passes and one finishing pass when $d_t = 10$mm. The depths of cut of 4.0, 4.0 and 1.0mm for the roughing passes and 1.0mm for the finishing pass are suggested. The total production cost will be \$2.4361/piece. The optimization method can reduce the unit production cost by 19.83%. When $d_t = 8$mm, the proposed method generates two roughing passes of 3.0mm each and one finishing pass of 2.0mm. This gives the production cost of \$1.8523/piece as compared to \$2.0086/piece in Shunmugam *et al.* (2000). The solution method using integer programming proposed in this thesis is based on the fact that the production cost of a single pass operation decreases with the increase of the feed rate (Shin and Joo 1992). In Shin and Joo (1992), Gupta *et al.* (1995), Shunmugam *et al.* (2000) and Al-Ahmari (2001), feed rate was limited by power constraints. In another word, the power constraints were followed by decreasing the feed rate. In this thesis, however, we did not transfer power constraints into feed rate constraints. The power constraints were followed by decreasing the cutting speed instead of the feed rate in solving our models. Therefore, the proposed method yielded lower costs for each given possible depth of cut and, as a result, a lower total unit production cost was obtained. In this solution method, the largest possible feed rate is desirable in order to obtain a lower cost without reducing tool life and exceeding the power limitation by decreasing the cutting speed. This technique is particularly useful for roughing cuts, in which the maximum feed depends on the maximum force that the cutting edge and the machine tool are able to withstand. The largest possible feed rate is usually consistent with surface finish requirements in finishing.

The minimum production cost is related to tool replacement time. In this paper, we calculated the minimum unit production costs for different depth of cut (d_t = 6.0, 7.0, 8.0, 9.0, 10.0 and 12.0mm) corresponding to various tool replacement times (T = 200, 240, 360, 540, 720, 960, 1200, 1440 and 1680min). The results are shown in Table 5.10 (The relative data are given in Table A.21 – Table A.36 in Appendix A). We also plotted the data in Column 6 of Table 5.10. The graph is shown in Figure 5.2. It shows that when d_t = 10mm, the unit production cost takes the minimum value around T = 720min.

All computations were conducted on IBM-PC compatible platforms using LINGO optimization software. Optimal results on all instances were achieved in less than one second of computation.

Table 5.10: Minimum unit production costs in milling

d_t (mm) / T (min)	6.0	7.0	8.0	9.0	10.0	12.0
200	1.5102	1.7934	1.8842	1.9786	2.0758	2.4478
240	1.4858	1.7665	1.8523	1.9412	2.0329	2.3975
360	1.4615	1.7382	1.8154	1.8952	1.9778	2.3299
540	1.4559	1.7300	1.8014	1.8754	1.9516	2.2955
720	1.4610	1.7337	1.8021	1.8731	1.9465	2.2861
960	1.4723	1.7544	1.8104	1.8792	1.9500	2.2867
1200	1.4852	1.7867	1.8410	1.8955	1.9583	2.3099
1440	1.5070	1.8160	1.8728	1.9296	1.9860	2.3518
1680	1.5312	1.8612	1.9024	1.9611	2.0198	2.3910

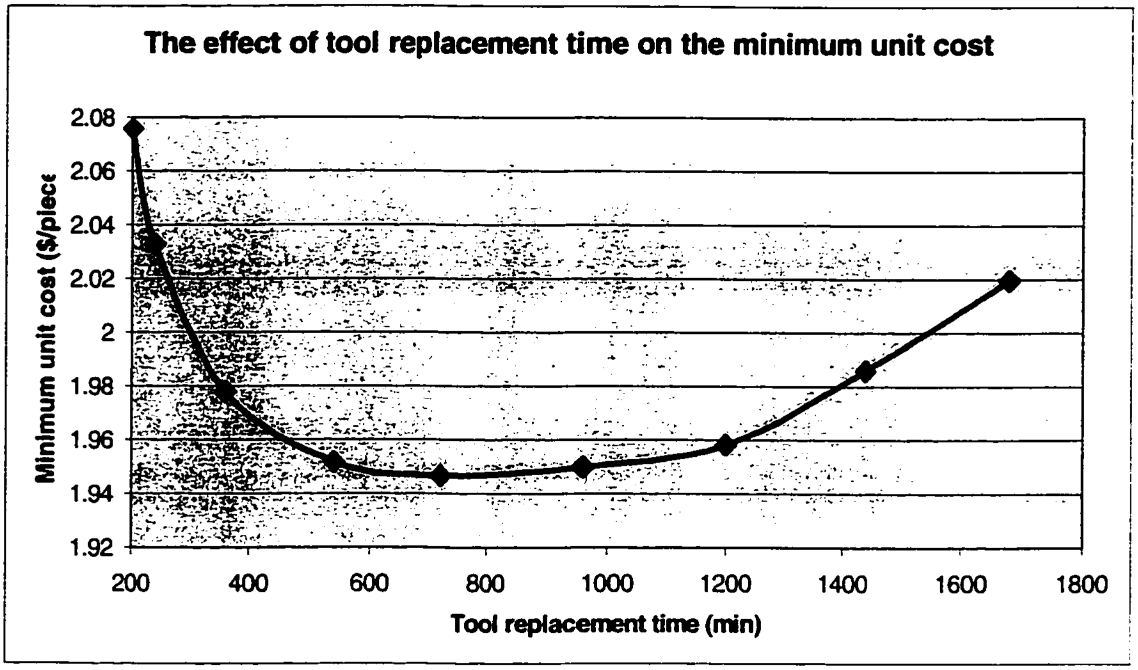

Figure 5.2: Tool replacement time and production cost in milling (d_t = 10mm)

5.2.2 Solving the Face-Milling Example by Nonlinear Programming

For the given milling example data in Table 5.6, UC_s and UC_{ri} were calculated from Eqs (4.39) and (4.40), respectively. After $k_0 t_p$ was calculated, total unit production cost UC in Eq (4.38) can be expressed by

$$UC = 9.07346 V_s^{-1} f_s^{-1} + \sum_{i=1}^{n} (0.24119 + 5.86623 V_{ri}^{-1} f_{ri}^{-1}) + 0.66606$$

We also calculated the bound values in Eqs (4.41) – (4.53), according to the data in Table 5.6. Then, the nonlinear programming model to minimize total unit production cost UC for the given milling example can be stated as

Minimize: $UC = 9.07346 V_s^{-1} f_s^{-1} + \sum_{i=1}^{n} (0.24119 + 5.86623 V_{ri}^{-1} f_{ri}^{-1}) + 0.66606$

Subject to:

$$50 \le V_s \le 300$$

$$50 \le V_{ri} \le 300$$

$$0.5 \le d_s \le 2.0$$

$$1.0 \le d_{ri} \le 4.0$$

$$V_s f_s^{0.35} d_s^{0.15} \le 84.6285$$

$$V_{ri} f_{ri}^{0.35} d_{ri}^{0.15} \le 84.6285$$

$$0.1 \le f_s \le \min(0.6, 0.2791) = 0.2791$$

$$0.1 \le f_{ri} \le \min(0.6, 0.8825) = 0.6$$

$$f_s^{0.74} d_s^{0.9} \le 1.4968$$

$$f_{ri}^{0.74} d_{ri}^{0.9} \le 1.4968$$

$$V_s f_s^{0.74} d_s^{0.9} \le 89.8349$$

$$V_{ri} f_{ri}^{0.74} d_{ri}^{0.9} \le 89.8349$$

$$d_s + \sum_{i=1}^{n} d_{ri} = d_t$$

The above nonlinear programming model was coded in LINGO and solved on IBM-PC compatible platforms for d_t = 6.0, 7.0, 8.0, 9.0, 10.0 and 12.0mm. LINGO code of the nonlinear programming model for solving the milling example is given in Appendix B.4. Optimal results on all instances were achieved in less than one second of computation. The results from the above nonlinear programming model are the same as the results presented in Table 5.10 by solving the integer programming model.

When using the nonlinear programming model to solve machining parameter optimization problems, we may obtain solutions with endless decimal places. Therefore, this method, after the solutions are rounded off properly, can be used to select machining parameters in NC/CNC machining with infinitely variable speeds and feed rates while the optimization method using integer programming can be used for machining parameter selection on conventional machine tools.

Chapter 6

Conclusions

6.1 Summary

In this thesis, the parameter optimization problem for multi-pass machining operations was studied. Based on an introduction to machining economics in Chapter 1 and a literature review of the research on machining parameter selection in Chapter 2, mathematical programming models were formulated in Chapter 3 for optimal selection of machining parameters in multi-pass operations, both for turning and face milling operations. Two solution methods were proposed in Chapter 4 to solve the models. Example problems were given in Chapter 5 to illustrate the models and solution methods developed in this thesis.

6.2 Contributions of the Thesis

In this thesis, mathematical programming models for both multi-pass turning and face-milling operations were developed. The optimization models are based on the minimum production cost criterion. Maximum and minimum cutting speeds, feed rates and depths of cut, as well as tool life, surface roughness, cutting force and cutting power are constraints of the models. Optimal values of machining parameters were found by two methods. One involves using integer programming and the other using nonlinear programming. The main optimization problems for multi-pass turning and face-milling operations have four variables governed by complex constraint functions. When solving the optimization problem by the integer programming method, the optimal cutting speeds and feed rates for each passes were determined using a single-pass optimization method. The number of passes and the depths of cut for each passes were determined by solving an integer programming model. The proposed procedures are effective, simple and reliable. Optimal solutions were obtained by solving the developed mathematical models within computational times of less than one second on widely available PC computers. The optimization methods proposed in this thesis generate feasible solutions and lower production costs compared to cutting schemes recommended by a machining data handbook. The research shows that tool replacement time affects the optimization results.

6.3 Future Research

With certain modifications, the proposed solution methods for solving the turning and face-milling parameter optimization problems can be used to solve similar problems for other machining operations such as drilling and grinding. This will be the topic for a future research in this area. Tool replacement time has an effect on the machining parameter optimization results. Further study on how to determine tool replacement time is of practical significance. There is also an interest in developing optimization models and solution techniques to solve parameter optimization problems in machining curved surfaces.

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Appendices

Appendix A: Tables of Optimal Parameters and Costs

Table A.1: Optimal parameters and costs for the rough turning passes ($T = 20$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 20$min)		
		V_{rij}^{*} (m/min)	f_{rij}^{*} (mm/rev)	UC_{rij}^{*} ($/pass)
0	1.0	129.37	0.9	0.5269
1	1.1	127.53	0.9	0.5308
2	1.2	125.88	0.9	0.5344
3	1.3	124.38	0.9	0.5377
4	1.4	123.00	0.9	0.5409
5	1.5	121.74	0.9	0.5438
6	1.6	120.56	0.9	0.5466
7	1.7	119.47	0.9	0.5493
8	1.8	118.45	0.9	0.5518
9	1.9	117.50	0.9	0.5542
10	2.0	116.60	0.9	0.5565
11	2.1	116.27	0.8885	0.5613
12	2.2	117.86	0.8377	0.5754
13	2.3	119.41	0.7918	0.5895
14	2.4	120.91	0.7503	0.6036
15	2.5	122.37	0.7124	0.6177
16	2.6	123.78	0.6779	0.6318
17	2.7	125.16	0.6463	0.6459
18	2.8	126.50	0.6172	0.6599
19	2.9	127.81	0.5903	0.6739
20	3.0	129.09	0.5655	0.6880
21	3.1	130.05	0.5425	0.7030
22	3.2	130.05	0.5211	0.7213
23	3.3	130.05	0.5012	0.7398
24	3.4	130.05	0.4826	0.7584
25	3.5	130.05	0.4652	0.7772
26	3.6	130.05	0.4489	0.7962
27	3.7	130.05	0.4336	0.8152
28	3.8	130.05	0.4192	0.8344
29	3.9	130.05	0.4056	0.8538
30	4.0	130.05	0.3928	0.8733

Table A.2: Optimal parameters and costs for the finish turning pass ($T = 20$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 20$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/rev)	UC_{sj}^* (\$/pass)
0	0.5	209.47	0.3057	0.7485
1	0.6	203.82	0.3057	0.7621
2	0.7	199.16	0.3057	0.7739
3	0.8	195.21	0.3057	0.7844
4	0.9	191.79	0.3057	0.7938
5	1.0	188.78	0.3057	0.8024
6	1.1	186.10	0.3057	0.8103
7	1.2	183.69	0.3057	0.8176
8	1.3	181.50	0.3057	0.8243
9	1.4	179.49	0.3057	0.8307
10	1.5	177.64	0.3057	0.8367
11	1.6	175.93	0.3057	0.8423
12	1.7	174.34	0.3057	0.8477
13	1.8	172.85	0.3057	0.8528
14	1.9	171.45	0.3057	0.8576
15	2.0	170.14	0.3057	0.8623

Table A.3: Optimal parameters and costs for the finish turning pass ($T = 22$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 22$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/rev)	UC_{sj}^* (\$/pass)
0	0.5	205.51	0.3057	0.7467
1	0.6	199.97	0.3057	0.7603
2	0.7	195.40	0.3057	0.7721
3	0.8	191.52	0.3057	0.7826
4	0.9	188.17	0.3057	0.7920
5	1.0	185.22	0.3057	0.8005
6	1.1	182.59	0.3057	0.8083
7	1.2	180.22	0.3057	0.8156
8	1.3	178.07	0.3057	0.8224
9	1.4	176.10	0.3057	0.8287
10	1.5	174.29	0.3057	0.8346
11	1.6	172.61	0.3057	0.8403
12	1.7	171.05	0.3057	0.8456
13	1.8	169.59	0.3057	0.8507
14	1.9	168.22	0.3057	0.8555
15	2.0	166.93	0.3057	0.8602

Table A.4: Optimal parameters and costs for the rough turning passes (T = 22min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, T = 22min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/rev)	UC^*_{rij} (\$/pass)
0	1.0	126.928	0.9	0.5259
1	1.1	125.13	0.9	0.5298
2	1.2	123.50	0.9	0.5334
3	1.3	122.03	0.9	0.5368
4	1.4	120.68	0.9	0.5399
5	1.5	119.44	0.9	0.5428
6	1.6	118.29	0.9	0.5456
7	1.7	117.22	0.9	0.5483
8	1.8	116.22	0.9	0.5508
9	1.9	115.28	0.9	0.5532
10	2.0	114.39	0.9	0.5555
11	2.1	114.07	0.8885	0.5602
12	2.2	115.64	0.8377	0.5743
13	2.3	117.16	0.7918	0.5884
14	2.4	118.63	0.7503	0.6024
15	2.5	120.06	0.7124	0.6165
16	2.6	121.45	0.6779	0.6305
17	2.7	122.80	0.6463	0.6445
18	2.8	124.12	0.6172	0.6585
19	2.9	125.40	0.5903	0.6725
20	3.0	126.65	0.5655	0.6865
21	3.1	127.88	0.5425	0.7004
22	3.2	129.07	0.5211	0.7144
23	3.3	130.05	0.5012	0.7290
24	3.4	130.05	0.4826	0.7472
25	3.5	130.05	0.4652	0.7656
26	3.6	130.05	0.4489	0.7841
27	3.7	130.05	0.4336	0.8028
28	3.8	130.05	0.4192	0.8215
29	3.9	130.05	0.4056	0.8405
30	4.0	130.05	0.3928	0.8595

Table A.5: Optimal parameters and costs for the rough turning passes (T=28min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, T = 28min)		
		V_{rij}^{*} (m/min)	f_{rij}^{*} (mm/rev)	UC_{rij}^{*} (\$/pass)
0	1.0	120.95	0.9	0.5254
1	1.1	119.23	0.9	0.5293
2	1.2	117.69	0.9	0.5329
3	1.3	116.28	0.9	0.5362
4	1.4	115.00	0.9	0.5394
5	1.5	113.81	0.9	0.5423
6	1.6	112.72	0.9	0.5451
7	1.7	111.70	0.9	0.5477
8	1.8	110.74	0.9	0.5502
9	1.9	109.85	0.9	0.5526
10	2.0	109.01	0.9	0.5549
11	2.1	108.70	0.8885	0.5596
12	2.2	110.19	0.8377	0.5737
13	2.3	111.64	0.7918	0.5878
14	2.4	113.04	0.7503	0.6018
15	2.5	114.40	0.7124	0.6158
16	2.6	115.73	0.6779	0.6298
17	2.7	117.02	0.6463	0.6438
18	2.8	118.27	0.6172	0.6578
19	2.9	119.49	0.5903	0.6717
20	3.0	120.69	0.5655	0.6857
21	3.1	121.86	0.5425	0.6996
22	3.2	123.00	0.5211	0.7135
23	3.3	124.11	0.5012	0.7274
24	3.4	125.20	0.4826	0.7413
25	3.5	126.27	0.4652	0.7552
26	3.6	127.32	0.4489	0.7691
27	3.7	128.35	0.4336	0.7829
28	3.8	129.35	0.4192	0.7968
29	3.9	130.05	0.4056	0.8119
30	4.0	130.05	0.3928	0.8300

Table A.6: Optimal parameters and costs for the finish turning pass ($T = 28$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 28$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/rev)	UC_{sj}^* (\$/pass)
0	0.5	195.84	0.3057	0.7458
1	0.6	190.55	0.3057	0.7594
2	0.7	186.20	0.3057	0.7712
3	0.8	182.50	0.3057	0.7816
4	0.9	179.31	0.3057	0.7910
5	1.0	176.50	0.3057	0.7995
6	1.1	173.99	0.3057	0.8073
7	1.2	171.74	0.3057	0.8146
8	1.3	169.69	0.3057	0.8213
9	1.4	167.81	0.3057	0.8276
10	1.5	166.08	0.3057	0.8336
11	1.6	164.48	0.3057	0.8392
12	1.7	162.99	0.3057	0.8445
13	1.8	161.60	0.3057	0.8496
14	1.9	160.30	0.3057	0.8544
15	2.0	159.07	0.3057	0.8590

Table A.7: Optimal parameters and costs for the finish turning pass ($T = 30$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 30$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/rev)	UC_{sj}^* (\$/pass)
0	0.5	193.15	0.3057	0.7464
1	0.6	187.94	0.3057	0.7600
2	0.7	183.64	0.3057	0.7718
3	0.8	180.00	0.3057	0.7822
4	0.9	176.85	0.3057	0.7916
5	1.0	174.08	0.3057	0.8001
6	1.1	171.61	0.3057	0.8080
7	1.2	169.38	0.3057	0.8152
8	1.3	167.36	0.3057	0.8220
9	1.4	165.51	0.3057	0.8283
10	1.5	163.81	0.3057	0.8342
11	1.6	162.23	0.3057	0.8399
12	1.7	160.76	0.3057	0.8452
13	1.8	159.39	0.3057	0.8503
14	1.9	158.10	0.3057	0.8551
15	2.0	156.89	0.3057	0.8597

Table A.8: Optimal parameters and costs for the rough turning passes ($T = 30$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 30$min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/rev)	UC^*_{rij} ($/pass)
0	1.0	119.29	0.9	0.5257
1	1.1	117.60	0.9	0.5296
2	1.2	116.08	0.9	0.5332
3	1.3	114.69	0.9	0.5366
4	1.4	113.42	0.9	0.5397
5	1.5	112.25	0.9	0.5426
6	1.6	111.17	0.9	0.5454
7	1.7	110.17	0.9	0.5481
8	1.8	109.23	0.9	0.5506
9	1.9	108.34	0.9	0.5530
10	2.0	107.51	0.9	0.5553
11	2.1	107.21	0.8885	0.5600
12	2.2	108.68	0.8377	0.5741
13	2.3	110.11	0.7918	0.5881
14	2.4	111.49	0.7503	0.6022
15	2.5	112.84	0.7124	0.6162
16	2.6	114.14	0.6779	0.6302
17	2.7	115.41	0.6463	0.6442
18	2.8	116.65	0.6172	0.6582
19	2.9	117.86	0.5903	0.6722
20	3.0	119.03	0.5655	0.6862
21	3.1	120.19	0.5425	0.7001
22	3.2	121.31	0.5211	0.7140
23	3.3	122.41	0.5012	0.7280
24	3.4	123.49	0.4826	0.7419
25	3.5	124.54	0.4652	0.7558
26	3.6	125.57	0.4489	0.7697
27	3.7	126.59	0.4336	0.7836
28	3.8	127.58	0.4192	0.7974
29	3.9	128.56	0.4056	0.8113
30	4.0	129.52	0.3928	0.8251

Table A.9: Optimal parameters and costs for the rough turning passes (T=32min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, T = 32min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/rev)	UC^*_{rij} ($/pass)
0	1.0	117.76	0.9	0.5262
1	1.1	116.09	0.9	0.5301
2	1.2	114.59	0.9	0.5337
3	1.3	113.22	0.9	0.5370
4	1.4	111.97	0.9	0.5402
5	1.5	110.81	0.9	0.5431
6	1.6	109.75	0.9	0.5459
7	1.7	108.75	0.9	0.5486
8	1.8	107.83	0.9	0.5511
9	1.9	106.95	0.9	0.5535
10	2.0	106.13	0.9	0.5558
11	2.1	105.84	0.8885	0.5605
12	2.2	107.29	0.8377	0.5746
13	2.3	108.70	0.7918	0.5887
14	2.4	110.06	0.7503	0.6028
15	2.5	111.39	0.7124	0.6168
16	2.6	112.68	0.6779	0.6309
17	2.7	113.93	0.6463	0.6449
18	2.8	115.15	0.6172	0.6589
19	2.9	116.35	0.5903	0.6729
20	3.0	117.51	0.5655	0.6869
21	3.1	118.64	0.5425	0.7009
22	3.2	119.75	0.5211	0.7148
23	3.3	120.84	0.5012	0.7288
24	3.4	121.90	0.4826	0.7427
25	3.5	122.94	0.4652	0.7566
26	3.6	123.96	0.4489	0.7706
27	3.7	124.96	0.4336	0.7845
28	3.8	125.95	0.4192	0.7984
29	3.9	126.91	0.4056	0.8122
30	4.0	127.85	0.3928	0.8261

Table A.10: Optimal parameters and costs for the finish turning pass ($T = 32$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 32$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} (\$/pass)
0	0.5	190.67	0.3057	0.7472
1	0.6	185.53	0.3057	0.7609
2	0.7	181.29	0.3057	0.7727
3	0.8	177.69	0.3057	0.7831
4	0.9	174.58	0.3057	0.7925
5	1.0	171.85	0.3057	0.8011
6	1.1	169.41	0.3057	0.8089
7	1.2	167.21	0.3057	0.8162
8	1.3	165.21	0.3057	0.8229
9	1.4	163.39	0.3057	0.8293
10	1.5	161.71	0.3057	0.8352
11	1.6	160.15	0.3057	0.8409
12	1.7	158.70	0.3057	0.8462
13	1.8	157.34	0.3057	0.8513
14	1.9	156.07	0.3057	0.8561
15	2.0	154.88	0.3057	0.8608

Table A.11: Optimal parameters and costs for the finish turning pass ($T = 35$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 35$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} (\$/pass)
0	0.5	187.29	0.3057	0.7489
1	0.6	182.23	0.3057	0.7625
2	0.7	178.07	0.3057	0.7744
3	0.8	174.54	0.3057	0.7849
4	0.9	171.48	0.3057	0.7943
5	1.0	168.79	0.3057	0.8029
6	1.1	166.40	0.3057	0.8108
7	1.2	164.24	0.3057	0.8180
8	1.3	162.28	0.3057	0.8248
9	1.4	160.49	0.3057	0.8312
10	1.5	158.83	0.3057	0.8372
11	1.6	157.30	0.3057	0.8428
12	1.7	155.88	0.3057	0.8482
13	1.8	154.55	0.3057	0.8533
14	1.9	153.30	0.3057	0.8581
15	2.0	152.12	0.3057	0.8628

Table A.12: Optimal parameters and costs for the rough turning passes ($T = 35$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 35$min)		
		V_{rij}^* (m/min)	f_{rij}^* (mm/rev)	UC_{rij}^* (\$/pass)
0	1.0	115.67	0.9	0.5271
1	1.1	114.03	0.9	0.5310
2	1.2	112.55	0.9	0.5346
3	1.3	111.21	0.9	0.5380
4	1.4	109.98	0.9	0.5411
5	1.5	108.85	0.9	0.5441
6	1.6	107.80	0.9	0.5469
7	1.7	106.82	0.9	0.5496
8	1.8	105.91	0.9	0.5521
9	1.9	105.05	0.9	0.5545
10	2.0	104.25	0.9	0.5568
11	2.1	103.96	0.8885	0.5615
12	2.2	105.38	0.8377	0.5757
13	2.3	106.77	0.7918	0.5898
14	2.4	108.11	0.7503	0.6039
15	2.5	109.41	0.7124	0.6180
16	2.6	110.68	0.6779	0.6321
17	2.7	111.91	0.6463	0.6462
18	2.8	113.11	0.6172	0.6603
19	2.9	114.28	0.5903	0.6743
20	3.0	115.42	0.5655	0.6883
21	3.1	116.54	0.5425	0.7024
22	3.2	117.63	0.5211	0.7164
23	3.3	118.69	0.5012	0.7304
24	3.4	119.74	0.4826	0.7443
25	3.5	120.76	0.4652	0.7583
26	3.6	121.76	0.4489	0.7723
27	3.7	122.74	0.4336	0.7862
28	3.8	123.71	0.4192	0.8002
29	3.9	124.65	0.4056	0.8141
30	4.0	125.58	0.3928	0.8280

Table A.13: Optimal parameters and costs for the rough turning passes ($T = 40$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 40$min)		
		V_{rij}^{*} (m/min)	f_{rij}^{*} (mm/rev)	UC_{rij}^{*} ($/pass)
0	1.0	112.62	0.9	0.5290
1	1.1	111.02	0.9	0.5329
2	1.2	109.59	0.9	0.5365
3	1.3	108.28	0.9	0.5399
4	1.4	107.08	0.9	0.5431
5	1.5	105.98	0.9	0.5461
6	1.6	104.96	0.9	0.5489
7	1.7	104.01	0.9	0.5516
8	1.8	103.12	0.9	0.5541
9	1.9	102.29	0.9	0.5566
10	2.0	101.50	0.9	0.5589
11	2.1	101.22	0.8885	0.5637
12	2.2	102.61	0.8377	0.5779
13	2.3	103.95	0.7918	0.5922
14	2.4	105.26	0.7503	0.6064
15	2.5	106.53	0.7124	0.6206
16	2.6	107.76	0.6779	0.6347
17	2.7	108.96	0.6463	0.6489
18	2.8	110.13	0.6172	0.6631
19	2.9	111.27	0.5903	0.6772
20	3.0	112.38	0.5655	0.6913
21	3.1	113.47	0.5425	0.7055
22	3.2	114.53	0.5211	0.7196
23	3.3	115.57	0.5012	0.7337
24	3.4	116.58	0.4826	0.7477
25	3.5	117.58	0.4652	0.7618
26	3.6	118.55	0.4489	0.7759
27	3.7	119.51	0.4336	0.7899
28	3.8	120.45	0.4192	0.8040
29	3.9	121.37	0.4056	0.8180
30	4.0	122.27	0.3928	0.8320

Table A.14: Optimal parameters and costs for the finish turning pass ($T = 40$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 40$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} ($/pass)
0	0.5	182.35	0.3057	0.7523
1	0.6	177.43	0.3057	0.7661
2	0.7	173.38	0.3057	0.7780
3	0.8	169.94	0.3057	0.7886
4	0.9	166.96	0.3057	0.7980
5	1.0	164.34	0.3057	0.8067
6	1.1	162.01	0.3057	0.8146
7	1.2	159.91	0.3057	0.8219
8	1.3	158.00	0.3057	0.8288
9	1.4	156.26	0.3057	0.8352
10	1.5	154.65	0.3057	0.8412
11	1.6	153.16	0.3057	0.8469
12	1.7	151.77	0.3057	0.8523
13	1.8	150.48	0.3057	0.8574
14	1.9	149.26	0.3057	0.8623
15	2.0	148.12	0.3057	0.8670

Table A.15: Optimal parameters and costs for the finish turning pass ($T = 45$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 45$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} ($/pass)
0	0.5	178.11	0.3057	0.7562
1	0.6	173.30	0.3057	0.7701
2	0.7	169.34	0.3057	0.7821
3	0.8	165.98	0.3057	0.7928
4	0.9	163.08	0.3057	0.8023
5	1.0	160.52	0.3057	0.8111
6	1.1	158.24	0.3057	0.8190
7	1.2	156.19	0.3057	0.8264
8	1.3	154.32	0.3057	0.8333
9	1.4	152.62	0.3057	0.8398
10	1.5	151.05	0.3057	0.8459
11	1.6	149.59	0.3057	0.8516
12	1.7	148.24	0.3057	0.8570
13	1.8	146.97	0.3057	0.8622
14	1.9	145.78	0.3057	0.8671
15	2.0	144.67	0.3057	0.8719

Table A.16: Optimal parameters and costs for the rough turning passes (T=45min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 45$min)		
		V^{*}_{rij} (m/min)	f^{*}_{rij} (mm/rev)	UC^{*}_{rij} ($/pass)
0	1.0	110.00	0.9	0.5311
1	1.1	108.44	0.9	0.5351
2	1.2	107.03	0.9	0.5388
3	1.3	105.76	0.9	0.5422
4	1.4	104.59	0.9	0.5454
5	1.5	103.51	0.9	0.5484
6	1.6	102.51	0.9	0.5512
7	1.7	101.59	0.9	0.5539
8	1.8	100.72	0.9	0.5565
9	1.9	99.905	0.9	0.5590
10	2.0	99.139	0.9	0.5613
11	2.1	98.859	0.8885	0.5661
12	2.2	100.22	0.8377	0.5805
13	2.3	101.53	0.7918	0.5948
14	2.4	102.81	0.7503	0.6091
15	2.5	104.05	0.7124	0.6235
16	2.6	105.25	0.6779	0.6378
17	2.7	106.42	0.6463	0.6520
18	2.8	107.56	0.6172	0.6663
19	2.9	108.68	0.5903	0.6806
20	3.0	109.76	0.5655	0.6948
21	3.1	110.82	0.5425	0.7090
22	3.2	111.86	0.5211	0.7232
23	3.3	112.88	0.5012	0.7375
24	3.4	113.87	0.4826	0.7516
25	3.5	114.84	0.4652	0.7658
26	3.6	115.79	0.4489	0.7800
27	3.7	116.73	0.4336	0.7942
28	3.8	117.64	0.4192	0.8083
29	3.9	118.54	0.4056	0.8224
30	4.0	119.43	0.3928	0.8366

Table A.17: Optimal parameters and costs for the rough turning passes ($T = 50$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 50$min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/rev)	UC^*_{rij} ($/pass)
0	1.0	107.71	0.9	0.5335
1	1.1	106.18	0.9	0.5375
2	1.2	104.80	0.9	0.5412
3	1.3	103.55	0.9	0.5446
4	1.4	102.41	0.9	0.5478
5	1.5	101.35	0.9	0.5509
6	1.6	100.38	0.9	0.5537
7	1.7	99.467	0.9	0.5564
8	1.8	98.618	0.9	0.5590
9	1.9	97.821	0.9	0.5615
10	2.0	97.072	0.9	0.5639
11	2.1	96.798	0.8885	0.5687
12	2.2	98.128	0.8377	0.5832
13	2.3	99.416	0.7918	0.5977
14	2.4	100.66	0.7503	0.6121
15	2.5	101.88	0.7124	0.6265
16	2.6	103.06	0.6779	0.6410
17	2.7	104.20	0.6463	0.6554
18	2.8	105.32	0.6172	0.6698
19	2.9	106.41	0.5903	0.6841
20	3.0	107.47	0.5655	0.6985
21	3.1	108.51	0.5425	0.7128
22	3.2	109.53	0.5211	0.7272
23	3.3	110.52	0.5012	0.7415
24	3.4	111.49	0.4826	0.7558
25	3.5	112.45	0.4652	0.7701
26	3.6	113.38	0.4489	0.7844
27	3.7	114.29	0.4336	0.7987
28	3.8	115.19	0.4192	0.8130
29	3.9	116.07	0.4056	0.8272
30	4.0	116.94	0.3928	0.8415

Table A.18: Optimal parameters and costs for the finish turning pass ($T = 50$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 50$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} ($/pass)
0	0.5	174.39	0.3057	0.7605
1	0.6	169.69	0.3057	0.7744
2	0.7	165.81	0.3057	0.7866
3	0.8	162.52	0.3057	0.7973
4	0.9	159.68	0.3057	0.8069
5	1.0	157.17	0.3057	0.8157
6	1.1	154.94	0.3057	0.8238
7	1.2	152.93	0.3057	0.8312
8	1.3	151.11	0.3057	0.8382
9	1.4	149.44	0.3057	0.8447
10	1.5	147.90	0.3057	0.8508
11	1.6	146.47	0.3057	0.8566
12	1.7	145.15	0.3057	0.8621
13	1.8	143.91	0.3057	0.8673
14	1.9	142.75	0.3057	0.8723
15	2.0	141.65	0.3057	0.8770

Table A.19: Optimal parameters and costs for the finish turning pass ($T = 60$min)

No. j	d_{ij} (mm)	Single finishing pass (Turning, $T = 60$min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/rev)	UC_{sj}^{*} ($/pass)
0	0.5	168.15	0.3057	0.7692
1	0.6	163.61	0.3057	0.7834
2	0.7	159.87	0.3057	0.7957
3	0.8	156.70	0.3057	0.8066
4	0.9	153.96	0.3057	0.8164
5	1.0	151.54	0.3057	0.8254
6	1.1	149.39	0.3057	0.8336
7	1.2	147.46	0.3057	0.8412
8	1.3	145.70	0.3057	0.8482
9	1.4	144.08	0.3057	0.8548
10	1.5	142.60	0.3057	0.8611
11	1.6	141.23	0.3057	0.8670
12	1.7	139.95	0.3057	0.8725
13	1.8	138.75	0.3057	0.8778
14	1.9	137.63	0.3057	0.8829
15	2.0	136.58	0.3057	0.8878

Table A.20: Optimal parameters and costs for the rough turning passes ($T = 60$min)

No. j	d_{ij} (mm)	Single roughing pass (Turning, $T = 60$min)		
		V^{*}_{rij} (m/min)	f^{*}_{rij} (mm/rev)	UC^{*}_{rij} (\$/pass)
0	1.0	103.85	0.9	0.5382
1	1.1	102.38	0.9	0.5423
2	1.2	101.05	0.9	0.5461
3	1.3	99.843	0.9	0.5496
4	1.4	98.740	0.9	0.5529
5	1.5	97.723	0.9	0.5559
6	1.6	96.782	0.9	0.5589
7	1.7	95.905	0.9	0.5616
8	1.8	95.087	0.9	0.5643
9	1.9	94.319	0.9	0.5668
10	2.0	93.596	0.9	0.5692
11	2.1	93.332	0.8885	0.5741
12	2.2	94.614	0.8377	0.5888
13	2.3	95.856	0.7918	0.6036
14	2.4	97.060	0.7503	0.6183
15	2.5	98.229	0.7124	0.6329
16	2.6	99.366	0.6779	0.6476
17	2.7	100.47	0.6463	0.6623
18	2.8	101.55	0.6172	0.6769
19	2.9	102.60	0.5903	0.6915
20	3.0	103.63	0.5655	0.7061
21	3.1	104.63	0.5425	0.7207
22	3.2	105.61	0.5211	0.7353
23	3.3	106.56	0.5012	0.7499
24	3.4	107.50	0.4826	0.7644
25	3.5	108.42	0.4652	0.7790
26	3.6	109.32	0.4489	0.7935
27	3.7	110.20	0.4336	0.8080
28	3.8	111.07	0.4192	0.8226
29	3.9	111.92	0.4056	0.8371
30	4.0	112.75	0.3928	0.8516

Table A.21: Optimal parameters and costs for the rough milling passes ($T = 200$min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, $T = 200$min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	107.28	0.6	0.3378
1	1.1	105.75	0.6	0.3392
2	1.2	104.38	0.6	0.3405
3	1.3	103.14	0.6	0.3417
4	1.4	96.850	0.6	0.3482
5	1.5	91.019	0.6	0.3551
6	1.6	85.883	0.6	0.3619
7	1.7	81.322	0.6	0.3687
8	1.8	77.245	0.6	0.3754
9	1.9	73.576	0.6	0.3821
10	2.0	70.256	0.6	0.3888
11	2.1	67.238	0.6	0.3954
12	2.2	64.481	0.6	0.4020
13	2.3	61.952	0.6	0.4086
14	2.4	60.017	0.5947	0.4155
15	2.5	60.017	0.5659	0.4244
16	2.6	60.017	0.5395	0.4333
17	2.7	60.017	0.5153	0.4423
18	2.8	60.017	0.4930	0.4514
19	2.9	60.017	0.4724	0.4606
20	3.0	60.017	0.4534	0.4698
21	3.1	60.017	0.4356	0.4791
22	3.2	60.017	0.4191	0.4885
23	3.3	60.017	0.4037	0.4979
24	3.4	60.017	0.3893	0.5074
25	3.5	60.017	0.3758	0.5170
26	3.6	60.017	0.3632	0.5266
27	3.7	60.017	0.3513	0.5363
28	3.8	60.017	0.3401	0.5460
29	3.9	60.017	0.3295	0.5558
30	4.0	60.017	0.3195	0.5656

Table A.22: Optimal parameters and costs for the finish milling pass ($T = 200$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 200$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	155.59	0.2791	0.5126
1	0.6	151.40	0.2791	0.5188
2	0.7	147.94	0.2791	0.5241
3	0.8	145.00	0.2791	0.5288
4	0.9	142.46	0.2791	0.5330
5	1.0	140.23	0.2791	0.5369
6	1.1	138.24	0.2791	0.5404
7	1.2	136.45	0.2791	0.5437
8	1.3	134.82	0.2791	0.5468
9	1.4	133.33	0.2791	0.5496
10	1.5	131.95	0.2791	0.5523
11	1.6	130.68	0.2791	0.5549
12	1.7	129.50	0.2791	0.5573
13	1.8	128.39	0.2791	0.5596
14	1.9	127.36	0.2791	0.5617
15	2.0	123.78	0.2791	0.5696

Table A.23: Optimal parameters and costs for the finish milling pass ($T = 360$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 360$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	128.92	0.2791	0.5178
1	0.6	125.44	0.2791	0.5241
2	0.7	122.57	0.2791	0.5296
3	0.8	120.14	0.2791	0.5344
4	0.9	118.04	0.2791	0.5387
5	1.0	116.19	0.2791	0.5427
6	1.1	114.54	0.2791	0.5463
7	1.2	113.05	0.2791	0.5496
8	1.3	111.70	0.2791	0.5528
9	1.4	110.47	0.2791	0.5557
10	1.5	109.33	0.2791	0.5584
11	1.6	108.28	0.2791	0.5610
12	1.7	107.30	0.2791	0.5635
13	1.8	106.38	0.2791	0.5659
14	1.9	105.52	0.2791	0.5681
15	2.0	104.71	0.2791	0.5702

Table A.24: Optimal parameters and costs for the rough milling passes ($T = 360$min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, $T = 360$min) V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	88.882	0.6	0.3401
1	1.1	87.620	0.6	0.3415
2	1.2	86.484	0.6	0.3428
3	1.3	85.452	0.6	0.3441
4	1.4	84.507	0.6	0.3452
5	1.5	83.637	0.6	0.3463
6	1.6	82.832	0.6	0.3473
7	1.7	81.322	0.6	0.3493
8	1.8	77.245	0.6	0.3550
9	1.9	73.576	0.6	0.3607
10	2.0	70.256	0.6	0.3663
11	2.1	67.238	0.6	0.3719
12	2.2	64.481	0.6	0.3775
13	2.3	61.952	0.6	0.3831
14	2.4	60.017	0.5947	0.3890
15	2.5	60.017	0.5659	0.3965
16	2.6	60.017	0.5395	0.4041
17	2.7	60.017	0.5153	0.4117
18	2.8	60.017	0.4930	0.4195
19	2.9	60.017	0.4724	0.4272
20	3.0	60.017	0.4534	0.4351
21	3.1	60.017	0.4356	0.4430
22	3.2	60.017	0.4191	0.4509
23	3.3	60.017	0.4037	0.4589
24	3.4	60.017	0.3893	0.4669
25	3.5	60.017	0.3758	0.4750
26	3.6	60.017	0.3632	0.4832
27	3.7	60.017	0.3513	0.4914
28	3.8	60.017	0.3401	0.4996
29	3.9	60.017	0.3295	0.5079
30	4.0	60.017	0.3195	0.5163

Table A.25: Optimal parameters and costs for the rough milling passes (T = 540min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, T = 540min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	78.067	0.6	0.3454
1	1.1	76.958	0.6	0.3469
2	1.2	75.961	0.6	0.3483
3	1.3	75.054	0.6	0.3496
4	1.4	74.224	0.6	0.3508
5	1.5	73.460	0.6	0.3519
6	1.6	72.752	0.6	0.3530
7	1.7	72.094	0.6	0.3540
8	1.8	71.478	0.6	0.3550
9	1.9	70.901	0.6	0.3559
10	2.0	70.256	0.6	0.3570
11	2.1	67.238	0.6	0.3622
12	2.2	64.481	0.6	0.3674
13	2.3	61.952	0.6	0.3725
14	2.4	60.017	0.5947	0.3779
15	2.5	60.017	0.5659	0.3849
16	2.6	60.017	0.5395	0.3919
17	2.7	60.017	0.5153	0.3990
18	2.8	60.017	0.4930	0.4061
19	2.9	60.017	0.4724	0.4133
20	3.0	60.017	0.4534	0.4206
21	3.1	60.017	0.4356	0.4279
22	3.2	60.017	0.4191	0.4352
23	3.3	60.017	0.4037	0.4426
24	3.4	60.017	0.3893	0.4501
25	3.5	60.017	0.3758	0.4576
26	3.6	60.017	0.3632	0.4651
27	3.7	60.017	0.3513	0.4727
28	3.8	60.017	0.3401	0.4803
29	3.9	60.017	0.3295	0.4880
30	4.0	60.017	0.3195	0.4957

Table A.26: Optimal parameters and costs for the finish milling pass ($T = 540$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 540$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	113.23	0.2791	0.5299
1	0.6	110.17	0.2791	0.5366
2	0.7	107.66	0.2791	0.5423
3	0.8	105.52	0.2791	0.5474
4	0.9	103.67	0.2791	0.5520
5	1.0	102.05	0.2791	0.5561
6	1.1	100.60	0.2791	0.5599
7	1.2	99.294	0.2791	0.5635
8	1.3	98.109	0.2791	0.5668
9	1.4	97.025	0.2791	0.5698
10	1.5	96.026	0.2791	0.5727
11	1.6	95.101	0.2791	0.5755
12	1.7	94.240	0.2791	0.5781
13	1.8	93.435	0.2791	0.5805
14	1.9	92.681	0.2791	0.5829
15	2.0	91.970	0.2791	0.5852

Table A.27: Optimal parameters and costs for the finish milling pass ($T = 720$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 720$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	103.27	0.2791	0.5424
1	0.6	100.48	0.2791	0.5494
2	0.7	98.188	0.2791	0.5554
3	0.8	96.241	0.2791	0.5608
4	0.9	94.555	0.2791	0.5656
5	1.0	93.073	0.2791	0.5699
6	1.1	91.751	0.2791	0.5740
7	1.2	90.562	0.2791	0.5777
8	1.3	89.481	0.2791	0.5811
9	1.4	88.492	0.2791	0.5844
10	1.5	87.581	0.2791	0.5874
11	1.6	86.737	0.2791	0.5903
12	1.7	85.952	0.2791	0.5930
13	1.8	85.218	0.2791	0.5957
14	1.9	84.530	0.2791	0.5981
15	2.0	83.882	0.2791	0.6005

Table A.28: Optimal parameters and costs for the rough milling passes ($T = 720$min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, $T = 720$min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} (\$/pass)
0	1.0	71.201	0.6	0.3508
1	1.1	70.190	0.6	0.3524
2	1.2	69.280	0.6	0.3539
3	1.3	68.453	0.6	0.3552
4	1.4	67.696	0.6	0.3565
5	1.5	66.999	0.6	0.3577
6	1.6	66.354	0.6	0.3588
7	1.7	65.753	0.6	0.3599
8	1.8	65.192	0.6	0.3609
9	1.9	64.665	0.6	0.3619
10	2.0	64.170	0.6	0.3628
11	2.1	63.702	0.6	0.3637
12	2.2	63.259	0.6	0.3646
13	2.3	61.952	0.6	0.3672
14	2.4	60.017	0.5947	0.3724
15	2.5	60.017	0.5659	0.3791
16	2.6	60.017	0.5395	0.3858
17	2.7	60.017	0.5153	0.3926
18	2.8	60.017	0.4930	0.3995
19	2.9	60.017	0.4724	0.4064
20	3.0	60.017	0.4534	0.4133
21	3.1	60.017	0.4356	0.4203
22	3.2	60.017	0.4191	0.4274
23	3.3	60.017	0.4037	0.4345
24	3.4	60.017	0.3893	0.4416
25	3.5	60.017	0.3758	0.4488
26	3.6	60.017	0.3632	0.4561
27	3.7	60.017	0.3513	0.4634
28	3.8	60.017	0.3401	0.4707
29	3.9	60.017	0.3295	0.4780
30	4.0	60.017	0.3195	0.4855

Table A.29: Optimal parameters and costs for the rough milling passes ($T = 960$min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, $T = 960$min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	64.939	0.6	0.3576
1	1.1	64.017	0.6	0.3593
2	1.2	63.187	0.6	0.3608
3	1.3	62.433	0.6	0.3623
4	1.4	61.743	0.6	0.3636
5	1.5	61.107	0.6	0.3649
6	1.6	60.518	0.6	0.3661
7	1.7	59.970	0.6	0.3673
8	1.8	59.458	0.6	0.3683
9	1.9	58.978	0.6	0.3694
10	2.0	58.526	0.6	0.3704
11	2.1	58.099	0.6	0.3713
12	2.2	57.695	0.6	0.3722
13	2.3	57.312	0.6	0.3731
14	2.4	57.124	0.5947	0.3747
15	2.5	57.771	0.5659	0.3799
16	2.6	58.399	0.5395	0.3852
17	2.7	59.010	0.5153	0.3904
18	2.8	59.604	0.4930	0.3956
19	2.9	60.017	0.4724	0.4012
20	3.0	60.017	0.4534	0.4079
21	3.1	60.017	0.4356	0.4147
22	3.2	60.017	0.4191	0.4215
23	3.3	60.017	0.4037	0.4284
24	3.4	60.017	0.3893	0.4353
25	3.5	60.017	0.3758	0.4423
26	3.6	60.017	0.3632	0.4493
27	3.7	60.017	0.3513	0.4563
28	3.8	60.017	0.3401	0.4634
29	3.9	60.017	0.3295	0.4706
30	4.0	60.017	0.3195	0.4777

Table A.30: Optimal parameters and costs for the finish milling pass ($T = 960$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 960$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	94.188	0.2791	0.5579
1	0.6	91.647	0.2791	0.5653
2	0.7	89.552	0.2791	0.5718
3	0.8	87.776	0.2791	0.5774
4	0.9	86.239	0.2791	0.5825
5	1.0	84.887	0.2791	0.5872
6	1.1	83.682	0.2791	0.5915
7	1.2	82.597	0.2791	0.5954
8	1.3	81.611	0.2791	0.5991
9	1.4	80.709	0.2791	0.6025
10	1.5	79.878	0.2791	0.6058
11	1.6	79.109	0.2791	0.6088
12	1.7	78.392	0.2791	0.6117
13	1.8	77.723	0.2791	0.6145
14	1.9	77.095	0.2791	0.6171
15	2.0	76.504	0.2791	0.6196

Table A.31: Optimal parameters and costs for the finish milling pass ($T = 1200$min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, $T = 1200$min)		
		V_{sj}^* (m/min)	f_{sj}^* (mm/tooth)	UC_{sj}^* (\$/pass)
0	0.5	87.697	0.2791	0.5721
1	0.6	85.331	0.2791	0.5799
2	0.7	83.381	0.2791	0.5866
3	0.8	81.727	0.2791	0.5926
4	0.9	80.296	0.2791	0.5980
5	1.0	79.037	0.2791	0.6029
6	1.1	77.915	0.2791	0.6074
7	1.2	76.905	0.2791	0.6115
8	1.3	75.987	0.2791	0.6154
9	1.4	75.147	0.2791	0.6190
10	1.5	74.373	0.2791	0.6224
11	1.6	73.657	0.2791	0.6257
12	1.7	72.990	0.2791	0.6287
13	1.8	72.367	0.2791	0.6316
14	1.9	71.782	0.2791	0.6344
15	2.0	71.232	0.2791	0.6371

Table A.32: Optimal parameters and costs for the rough milling passes (T = 1200min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, T = 1200min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} (\$/pass)
0	1.0	60.463	0.6	0.3638
1	1.1	59.605	0.6	0.3656
2	1.2	58.832	0.6	0.3672
3	1.3	58.130	0.6	0.3687
4	1.4	57.488	0.6	0.3701
5	1.5	56.896	0.6	0.3715
6	1.6	56.348	0.6	0.3727
7	1.7	55.837	0.6	0.3739
8	1.8	55.361	0.6	0.3751
9	1.9	54.914	0.6	0.3762
10	2.0	54.493	0.6	0.3772
11	2.1	54.095	0.6	0.3782
12	2.2	53.719	0.6	0.3792
13	2.3	53.362	0.6	0.3801
14	2.4	53.188	0.5947	0.3818
15	2.5	53.789	0.5659	0.3873
16	2.6	54.374	0.5395	0.3928
17	2.7	54.943	0.5153	0.3983
18	2.8	55.496	0.4930	0.4037
19	2.9	56.036	0.4724	0.4092
20	3.0	56.562	0.4534	0.4146
21	3.1	57.076	0.4356	0.4201
22	3.2	57.577	0.4191	0.4255
23	3.3	58.068	0.4037	0.4309
24	3.4	58.548	0.3893	0.4363
25	3.5	59.018	0.3758	0.4417
26	3.6	59.478	0.3632	0.4471
27	3.7	59.929	0.3513	0.4525
28	3.8	60.017	0.3401	0.4591
29	3.9	60.017	0.3295	0.4661
30	4.0	60.017	0.3195	0.4731

Table A.33: Optimal parameters and costs for the rough milling passes ($T = 1440$min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, $T = 1440$min)		
		V^{*}_{rij} (m/min)	f^{*}_{rij} (mm/tooth)	UC^{*}_{rij} ($/pass)
0	1.0	57.037	0.6	0.3694
1	1.1	56.227	0.6	0.3713
2	1.2	55.498	0.6	0.3730
3	1.3	54.836	0.6	0.3746
4	1.4	54.230	0.6	0.3761
5	1.5	53.671	0.6	0.3775
6	1.6	53.154	0.6	0.3788
7	1.7	52.673	0.6	0.3800
8	1.8	52.223	0.6	0.3812
9	1.9	51.801	0.6	0.3824
10	2.0	51.404	0.6	0.3835
11	2.1	51.030	0.6	0.3845
12	2.2	50.675	0.6	0.3855
13	2.3	50.338	0.6	0.3865
14	2.4	50.173	0.5947	0.3883
15	2.5	50.741	0.5659	0.3940
16	2.6	51.293	0.5395	0.3998
17	2.7	51.829	0.5153	0.4055
18	2.8	52.351	0.4930	0.4112
19	2.9	52.860	0.4724	0.4169
20	3.0	53.357	0.4534	0.4226
21	3.1	53.841	0.4356	0.4283
22	3.2	54.314	0.4191	0.4340
23	3.3	54.777	0.4037	0.4396
24	3.4	55.230	0.3893	0.4453
25	3.5	55.673	0.3758	0.4509
26	3.6	56.107	0.3632	0.4565
27	3.7	56.532	0.3513	0.4622
28	3.8	56.949	0.3401	0.4678
29	3.9	57.359	0.3295	0.4734
30	4.0	57.760	0.3195	0.4790

Table A.34: Optimal parameters and costs for the finish milling pass (T = 1440min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, T = 1440min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/tooth)	UC_{sj}^{*} ($/pass)
0	0.5	82.727	0.2791	0.5850
1	0.6	80.495	0.2791	0.5932
2	0.7	78.655	0.2791	0.6002
3	0.8	77.095	0.2791	0.6065
4	0.9	75.745	0.2791	0.6121
5	1.0	74.558	0.2791	0.6172
6	1.1	73.499	0.2791	0.6219
7	1.2	72.546	0.2791	0.6263
8	1.3	71.680	0.2791	0.6303
9	1.4	70.888	0.2791	0.6341
10	1.5	70.158	0.2791	0.6377
11	1.6	69.482	0.2791	0.6411
12	1.7	68.853	0.2791	0.6443
13	1.8	68.265	0.2791	0.6473
14	1.9	67.714	0.2791	0.6502
15	2.0	67.195	0.2791	0.6530

Table A.35: Optimal parameters and costs for the finish milling pass (T = 1680min)

No. j	d_{ij} (mm)	Single finishing pass (Milling, T = 1680min)		
		V_{sj}^{*} (m/min)	f_{sj}^{*} (mm/tooth)	UC_{sj}^{*} ($/pass)
0	0.5	78.745	0.2791	0.5969
1	0.6	76.621	0.2791	0.6054
2	0.7	74.869	0.2791	0.6127
3	0.8	73.385	0.2791	0.6193
4	0.9	72.100	0.2791	0.6251
5	1.0	70.969	0.2791	0.6304
6	1.1	69.962	0.2791	0.6353
7	1.2	69.054	0.2791	0.6398
8	1.3	68.230	0.2791	0.6440
9	1.4	67.476	0.2791	0.6480
10	1.5	66.781	0.2791	0.6517
11	1.6	66.138	0.2791	0.6552
12	1.7	65.539	0.2791	0.6585
13	1.8	64.980	0.2791	0.6617
14	1.9	64.455	0.2791	0.6647
15	2.0	63.961	0.2791	0.6676

Table A.36: Optimal parameters and costs for the rough milling passes (T = 1680min)

No. j	d_{ij} (mm)	Single roughing pass (Milling, T = 1680min)		
		V^*_{rij} (m/min)	f^*_{rij} (mm/tooth)	UC^*_{rij} ($/pass)
0	1.0	54.292	0.6	0.3746
1	1.1	53.521	0.6	0.3765
2	1.2	52.827	0.6	0.3783
3	1.3	52.196	0.6	0.3800
4	1.4	51.619	0.6	0.3815
5	1.5	51.088	0.6	0.3830
6	1.6	50.596	0.6	0.3844
7	1.7	50.138	0.6	0.3857
8	1.8	50	0.5901	0.3885
9	1.9	50	0.5766	0.3919
10	2.0	50	0.5640	0.3953
11	2.1	50	0.5524	0.3986
12	2.2	50	0.5415	0.4017
13	2.3	50	0.5313	0.4048
14	2.4	50	0.5217	0.4078
15	2.5	50	0.5126	0.4108
16	2.6	50	0.5041	0.4136
17	2.7	50	0.4960	0.4164
18	2.8	50	0.4883	0.4192
19	2.9	50.316	0.4724	0.4240
20	3.0	50.788	0.4534	0.4299
21	3.1	51.250	0.4356	0.4359
22	3.2	51.700	0.4191	0.4418
23	3.3	52.141	0.4037	0.4476
24	3.4	52.571	0.3893	0.4535
25	3.5	52.993	0.3758	0.4594
26	3.6	53.406	0.3632	0.4653
27	3.7	53.811	0.3513	0.4711
28	3.8	54.208	0.3401	0.4769
29	3.9	54.598	0.3295	0.4828
30	4.0	54.980	0.3195	0.4886

Appendix B: LINGO Code for the Examples

B.1 LINGO Code of the Integer Programming Model for Turning

```
! Machining cost model of integer programming for mutipass turning operations:

SETS:
! Value of n;
! Value of m;
! Value of l;

i/1..4/;
j/1..16/;
k/1..31/;

! Cost:
COSTSET1(j): C1,X1,D1;
COSTSET2(i,k): C2,X2,D2;

ENDSETS

DATA:

! Values of dij:

D1=0.5,0.6,0.7,0.8,0.9,1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0;

D2=1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.
2,3.3,3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0;

! Values of Cij:

C1=0.7457, 0.7592, 0.771, 0.7814, 0.7908, 0.7993, 0.8071, 0.8144, 0.8211, 0.8274,
0.8334, 0.839, 0.8443, 0.8494, 0.8542, 0.8588;

C2=0.5253, 0.5292, 0.5328, 0.5361, 0.5393, 0.5422, 0.545, 0.5476, 0.5502, 0.5525,
0.5548, 0.5596, 0.5736, 0.5877, 0.6017, 0.6157, 0.6297, 0.6437, 0.6576, 0.6716, 0.6855,
0.6995, 0.7134, 0.7273, 0.7412, 0.755, 0.7697, 0.7878, 0.8061, 0.8245, 0.843,
```

```
0.5253, 0.5292, 0.5328, 0.5361, 0.5393, 0.5422, 0.545, 0.5476, 0.5502, 0.5525, 0.5548,
0.5596, 0.5736, 0.5877, 0.6017, 0.6157, 0.6297, 0.6437, 0.6576, 0.6716, 0.6855, 0.6995,
0.7134, 0.7273, 0.7412, 0.755, 0.7697, 0.7878, 0.8061, 0.8245, 0.843,
0.5253, 0.5292, 0.5328, 0.5361, 0.5393, 0.5422, 0.545, 0.5476, 0.5502, 0.5525, 0.5548,
0.5596, 0.5736, 0.5877, 0.6017, 0.6157, 0.6297, 0.6437, 0.6576, 0.6716, 0.6855, 0.6995,
0.7134, 0.7273, 0.7412, 0.755, 0.7697, 0.7878, 0.8061, 0.8245, 0.843,
0.5253, 0.5292, 0.5328, 0.5361, 0.5393, 0.5422, 0.545, 0.5476, 0.5502, 0.5525, 0.5548,
0.5596, 0.5736, 0.5877, 0.6017, 0.6157, 0.6297, 0.6437, 0.6576, 0.6716, 0.6855, 0.6995,
0.7134, 0.7273, 0.7412, 0.755, 0.7697, 0.7878, 0.8061, 0.8245, 0.843;

! Value of dt:

DT=6.0; ! DT=6.0,7.0,8.0,9.0,10.0,12.0;

ENDDATA

! Objective function to minimize unit production cost:

[OBJ] MIN=@SUM(COSTSET1(m):C1(m)*X1(m))+@SUM( COSTSET2(n,l):C2(n,l)*
X2(n,l))+0.375;

! Constraints:

! Finishing constraint:

! @FOR(i=0, j=0,1,2,...mi):@SUM (Xij)=1;
@SUM(costset1(m):X1(m))=1;

! Roughing constraint;

! @FOR(i=1,2,...n, j=0,1,2,...mi):@SUM (Xij)<=1;
@FOR(I(n):@FOR(K(l):@SUM (I(n):X2(n,l))<=1));

! Total depth of cut constraint:

! @FOR(i=0,1,2,...n, j=0,1,2,...mi):@SUM( dij*Xij)=dt;
@SUM(COSTSET1(m):D1(m)*X1(m))+@SUM(COSTSET2(n,l):D2(n,l)*X2(n,l))=DT;

! Xij are binary:

@FOR( J(m):@BIN(X1(m)));
@FOR( I(n):@FOR (K(l): @BIN(X2(n,l))));

END
```

B.2 LINGO Code of the Nonlinear Programming Model for Turning

```
! Machining cost model of nonlinear programming for mutipass turning operations:

! Data:
! N=10;

data:

n=10;

end data

sets:

roughpass/1..10/:dr,fr,vr,y;

endsets

! Min the total production cost of one finishing pass plus and n rough passes:
min=@sum(roughpass(i):0.25605*y(i)+29.985*y(i)*fr(i)^(-1)*vr(i)^(-1))+29.985*fs^(-
1)*vs^(-1)+0.63105;

! Constraints on the roughing passes:

! 1. Define binary variables:

@for(roughpass(i)|i#LE#N-1:y(i)-y(i+1)>=0);
@for(roughpass(i):@bin(y(i)));

! 2. Tool life constraint:

@for(roughpass(i):vr(i)*fr(i)^(0.35)*dr(i)^(0.15)<=119.24);

! 3. Minimum and maximum feed rate and surface finish constraint:

@for(roughpass(i):fr(i)>=0.1);
@for(roughpass(i):fr(i)<=0.9);

! 4. Cutting force constraint:

@for(roughpass(i):fr(i)^(0.75)*dr(i)^(0.95)<=1.8519);

! 5. Cutting power constraint:
```

```
@for(roughpass(i):vr(i)*fr(i)^(0.75)*dr(i)^(0.95)<=240.83);
! 6. Minimum and maximum depths of cut:

@for(roughpass(i):dr(i)<=4.0);
@for(roughpass(i):dr(i)>=1.0);

! 7. Minimum and maximum cutting speeds:

@for(roughpass(i):vr(i)<=500);
@for(roughpass(i):vr(i)>=5);

! 8. The total depth of cut should be equal to the depth of the finishing pass plus the total
depths of cut of roughing passes:

@sum(roughpass(i):dr(i)*y(i))+ds=6.0;!dt=6.0,7.0,8.0,9.0,10.0,12.0;

! Constraints on the finishing pass:

! 1. Tool life constraint:

vs*fs^(0.35)*ds^(0.15)<=119.24;

! 2. Minimum and maximum feed rate and surface finish constraint:

fs>=0.1;
fs<=0.3057;

! 3. Cutting force constraint:

fs^(0.75)*ds^(0.95)<=1.8519;

! 4. Cutting power constraint:

vs*fs^(0.75)*ds^(0.95)<=240.83;

! 5. Minimum and maximum depths of cut:

ds<=2.0;
ds>=0.5;

! 6. Minimum and maximum cutting speeds:

vs<=500;
vs>=5;

end
```

B.3 LINGO Code of the Integer Programming Model for Milling

```
! Machining cost model of integer programming for mutipass milling operations:

SETS:
! Value of n;
! Value of m;
! Value of l;

i/1..4/;
j/1..16/;
k/1..31/;

! Cost:
COSTSET1(j): C1,X1,D1;
COSTSET2(i,k): C2,X2,D2;

ENDSETS

DATA:

! Values of dij:

D1=0.5,0.6,0.7,0.8,0.9,1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0;

D2=1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.
2,3.3,3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0,
1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,1.9,2.0,2.1,2.2,2.3,2.4,2.5,2.6,2.7,2.8,2.9,3.0,3.1,3.2,3.3,
3.4,3.5,3.6,3.7,3.8,3.9,4.0;

! Values of Cij:

C1=0.5125, 0.5187, 0.524, 0.5287, 0.533, 0.5368, 0.5403, 0.5436, 0.5467, 0.5495,
0.5522, 0.5548, 0.5572, 0.5595, 0.5616, 0.5637;

C2=0.3378, 0.3392, 0.3405, 0.3417, 0.3428, 0.3486, 0.355, 0.3614, 0.3678, 0.3741,
0.3804, 0.3866, 0.3928, 0.399, 0.4055, 0.4139, 0.4224, 0.4309, 0.4394, 0.4481, 0.4568,
0.4656, 0.4744, 0.4833, 0.4922, 0.5013, 0.5103, 0.5194, 0.5286, 0.5378, 0.5471,
0.3378, 0.3392, 0.3405, 0.3417, 0.3428, 0.3486, 0.355, 0.3614, 0.3678, 0.3741, 0.3804,
0.3866, 0.3928, 0.399, 0.4055, 0.4139, 0.4224, 0.4309, 0.4394, 0.4481, 0.4568, 0.4656,
0.4744, 0.4833, 0.4922, 0.5013, 0.5103, 0.5194, 0.5286, 0.5378, 0.5471,
```

```
0.3378, 0.3392, 0.3405, 0.3417, 0.3428, 0.3486, 0.355, 0.3614, 0.3678, 0.3741, 0.3804,
0.3866, 0.3928, 0.399, 0.4055, 0.4139, 0.4224, 0.4309, 0.4394, 0.4481, 0.4568, 0.4656,
0.4744, 0.4833, 0.4922, 0.5013, 0.5103, 0.5194, 0.5286, 0.5378, 0.5471,
0.3378, 0.3392, 0.3405, 0.3417, 0.3428, 0.3486, 0.355, 0.3614, 0.3678, 0.3741, 0.3804,
0.3866, 0.3928, 0.399, 0.4055, 0.4139, 0.4224, 0.4309, 0.4394, 0.4481, 0.4568, 0.4656,
0.4744, 0.4833, 0.4922, 0.5013, 0.5103, 0.5194, 0.5286, 0.5378, 0.5471;

! Value of dt:

DT=6.0; ! DT=6.0,7.0,8.0,9.0,10.0,12.0;

ENDDATA

! Objective function to minimize unit production cost:

[OBJ] MIN=@SUM(COSTSET1(m):C1(m)*X1(m))+@SUM( COSTSET2(n,l):C2(n,l)*
X2(n,l))+0.375;

! Finishing constraint:

! @FOR(i=0,j=0,1,2,...mi):@SUM (Xij)=1;
@SUM(costset1(m):X1(m))=1;

! Roughing constraint:

! @FOR(i=1,2,...n,j=0,1,2,...mi):@SUM (Xij)<=1;
@FOR(I(n):@FOR(k(l):@ SUM(I(n):X2(n,l))<=1));

! Total depth of cut constraint:

! @FOR(i=0,1,2,...n,j=0,1,2,...mi):@SUM( dij*Xij)=dt;
@SUM(COSTSET1(m):D1(m)*X1(m))+@SUM(COSTSET2(n,l):D2(n,l)*X2(n,l))=DT;

! Xij are binary:

@FOR(J(m):@BIN(X1(m)));
@FOR( I(n):@FOR(K(l):@BIN(X2(n,l))));

END
```

B.4 LINGO Code of the Nonlinear Programming Model for Milling

```
! Machining cost model of nonlinear programming for mutipass milling operations:

! Data:
! N=10;

data:

n=10;

end data

sets:

roughpass/1..10/:dr,fr,vr,y;

endsets

! Min the total production cost of one finishing pass plus n roughing passes;
min=@sum(roughpass(i):0.24119*y(i)+5.86623*y(i)*fr(i)^(-1)*vr(i)^(-1))+9.07346*fs^
(-1)*vs^(-1)+0.66606;

! Constraints on the roughing passes:

! 1. Define binary variables:

@for(roughpass(i)|i#LE#N-1:y(i)-y(i+1)>=0);
@for(roughpass(i):@bin(y(i)));

! 2. Tool life constraint:

@for(roughpass(i):vr(i)*fr(i)^(0.35)*dr(i)^(0.15)<=84.6285);

! 3. Minimum and maximum feed rates and surface finish constraint:

@for(roughpass(i):fr(i)>=0.1);
@for(roughpass(i):fr(i)<=0.6);

! 4. Cutting force constraint:

@for(roughpass(i):fr(i)^(0.74)*dr(i)^(0.9)<=1.4968);

! 5. Cutting power constraint:
```

```
@for(roughpass(i):vr(i)*fr(i)^(0.74)*dr(i)^(0.9)<=89.8349);
! 6. Minimum and maximum depths of cut:

@for(roughpass(i):dr(i)<=4.0);
@for(roughpass(i):dr(i)>=1.0);

! 7. Minimum and maximum cutting speeds:

@for(roughpass(i):vr(i)<=300);
@for(roughpass(i):vr(i)>=50);

! 8. The total depth of cut should be equal to the depth of the finishing pass plus the total
depths of cut of the roughing passes:

@sum(roughpass(i):dr(i)*y(i))+ds=6.0;!dt=6.0,7.0,8.0,9.0,10.0,12.0;

!Constraints on the finishing pass:

! 1. Tool life constraint:

vs*fs^(0.35)*ds^(0.15)<=84.6285;

! 2. Minimum and maximum feed rates and surface finish constraint:

fs>=0.1;
fs<=0.2791;

! 3. Cutting force constraint:

fs^(0.74)*ds^(0.9)<=1.4968;

! 4. Cutting power constraint:

vs*fs^(0.74)*ds^(0.9)<=89.8349;

! 5. Minimum and maximum depths of cut:

ds<=2.0;
ds>=0.5;

! 6. Minimum and maximum cutting speeds:

vs<=300;
vs>=50;

end
```