ANALOG/HYBRID SIMULATION OF NON-DARCY FLOW IN ROCKFILL STRUCTURES

Peter Ludwig Kotiuga

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ABSTRACT

ANALOG/HYBRID SIMULATION OF NON-DARCY FLOW IN ROCKFILL STRUCTURES

Peter Ludwig Kotiuga

This thesis presents a one-dimensional analog/hybrid model for non-Darcy flow in rockfill structures. The model is used to determine the family of phreatic surfaces within the rockfill due to an external impact wave; the rockfill is bounded by an impervious core at the downstream end.

The impact wave is classified as "slow drop" or "fast drop" depending respectively on whether or not the fall rate of the external wave is slower or faster than the maximum seepage velocity. The movement of the outcrop point was determined for the more complex "fast drop" case for one wave period using five techniques: MIMIC, numerical, analytic, approximate, and analog. The inputs were taken from earlier experimental data (reported by Nasser [35]) for four rock types.

The outcrop point movement represents a boundary condition for the main model. The main model is based on the continuity and momentum equations for long, shallow water waves. These equations form a system of quasi-linear hyperbolic partial differential equations; the method of
characteristics is applied to render the equations stable from an error propagation point of view. The time variable is discretized to produce ordinary differential equations in the spatial variable only. These are solved iteratively to satisfy the zero normal velocity boundary condition at the core.

Comparisons with experimental results are given where applicable and readily available.
ACKNOWLEDGEMENTS

His path led through the sea
His way through the mighty waters
Though His footprints were not seen.

— Psalm 77:19

I dedicate this work to Him who comforted me in my darkest hours, who flooded my mind with love and hope and peace when fear tried to grip me, who gave me a new life and the highest principles to live by, who helps me love all people; I dedicate this to the Person and my Lord Jesus Christ.

Dr. M. S. Nasser is no ordinary person either. It has been the greatest honour to work with, study under, and share with a man of wisdom and understanding, who upholds high principles and commands the deepest respect of those who truly know him. His suggestions, moral support, his concern and thoughtfulness, his time, are immensely appreciated. His confidence in and patience with me have hopefully not gone unrewarded.

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Extra special thanks are due to my family, especially my mother, who cared and encouraged and understood and prayed. They have waited for this day.

Last but certainly not least, I dedicate this work to the young woman who has captured my heart and soul and has often helped soothe a troubled mind. Grace, I love you! Thanks for sharing.

September 28, 1979
Montreal, Quebec

Peter Ludwig Kotiuga
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CHAPTER 1
INTRODUCTION

1.1 DEFINITION OF THE PROBLEM

As a wave attacks a rockfill structure, a maximum amplitude referred to as the maximum run-up and a minimum water level referred to as the limit of rush-down will be encountered on the interface; these two levels constitute the impact wave height. Part of the incident wave's kinetic energy is consumed in the uprush, part is reflected on the rock face and the remaining energy is transmitted into the rockfill. Within the rockfill the flow is usually in the non-Darcy regime and further dissipation of the wave energy occurs due to high damping effects.

The model used is based on a rockfill embankment with an impermeable core (Fig. 1). By the time the wave reaches the core it will have lost most of its energy so that the transmitted wave height at the core will be significantly smaller than that of the impact wave. Simulation of the successive phreatic line profiles would provide a better understanding of interior wave motion.

1.2 OBJECTIVES

The main objective of this thesis is to develop an analog/hybrid model to simulate the phreatic line profiles within a porous breakwater.
Fig. 1.1 Definition of Problem

- Mean water level
- Impervious core
- Maximum run-up
- Envelope of phreatic profiles
- Limit of rush-down
- Rockfill
or a rockfill dam. A Continuous-Space Discrete-Time technique is utilized to generate a family of phreatic lines for given boundary conditions.

In determining the oscillating phreatic lines within the structure, the movement of the outcrop point relative to an assumed sinusoidal impact wave is utilized as an entrance boundary condition. The vertical rate of fluctuation of the impact wave is classified as either "slow" or "fast" dropping. A slow drop case is one in which the oscillations of the external wave and the outcrop point are synonymous. A fast drop case is one in which a surface of seepage occurs at the rockfill interface because the maximum fall velocity of the internal wave is less than that of the external wave.

This study emphasizes the digital and analog analysis of the more complex fast drop case boundary condition and the subsequent generation of the internal phreatic surfaces for a complete wave period.

1.3 Motivation

Very few attempts have been made using hybrid simulation for specific engineering field problems governed by hyperbolic equations. Vichnevetsky has published two excellent papers: one dealing with a river flooding problem and another simulating electrical-transmission lines. Civil engineers have shown very little interest in hybrid programming and hence there exists a challenge to fill that vacuum. Due to lack of exposure to hybrid computing by most engineers, the thesis will attempt to present some of the frustrations likely to be encountered in solving problems by this method. Since the method is solved from a civil engineering point of view, many complexities of the system will not be emphasized.
The importance of this study is manifested in its practical application. Attainment of the stated objective could lead to a more rational approach for estimating the freeboard on the face and core of a dam, thereby resulting in better control over the maximum allowable reservoir level. Further, the height of such shore protection structures as gabion walls could more effectively be established. Tracing the unsteady water surface profiles is of value in determining the pressure distribution within the embankment.

1.4 **Approach in General**

Analog computers can only integrate one independent variable and since two independent variables exist (space and time), one of them must be discretized. If the time is chosen to be continuous, then the method is known as Discrete-Space Continuous-Time (DSCT) or the Parallel method. This method requires too many components if solved only on the analog computer or a special multiplexer if solved on the hybrid computer. The method chosen is the Continuous-Space Discrete-Time (CSDT) or the Serial method. There is an economy of components but error propagation becomes a problem. The two system equations are modified into characteristic equations. These resulting ordinary differential equations have characteristics of opposite sign but are integrated in the same direction. The development of the boundary conditions plays a major role in the accuracy of the solutions. A separate study of the movement of the outcrop point was made to provide the wave entrance conditions. Analytic, numeric, approximate, and analog
results are compared in the study. The continuous output derived from the plotters yields a family of curves showing the unsteady water surface profiles. These profiles are compared with the results obtained by Nasser and McCorquodale [33, 35].
CHAPTER 2

LITERATURE REVIEW

Non-Darcy flow problems have been treated by a few analytical or numerical techniques [19, 33, 35].

2.1 Scope

Nasser [35] recommended that the analog simulation of long water waves introduced by Henry [23] might be adapted to solving the differential equations governing wave-induced unsteady non-Darcy flow. This chapter reviews prominent contributions to the wave phenomenon of non-linear flow in rockfill structures and the hybrid implementation of hyperbolic partial differential equations (PDE). The governing equations are introduced after a brief description of the nature of flow in porous media. Different methods of solution of non-Darcy flow are then reviewed. A summary of the various hybrid techniques developed for hyperbolic PDE's is presented, the reasons justifying the choice of the method employed in this work are emphasized.

2.2 Flow Regimes in Porous Media

In 1856 Darcy first proposed his familiar law which expresses the relationship between the 'macroscopic' velocity, \( q \), and the hydraulic
gradient, \(i\), for linear (or laminar) flow in a porous medium, viz.

\[ q = ia \]  

(2.1)

where 'a' is a resistance coefficient.

It was not until 1901 that Forchheimer [16] published a paper indicating that Darcy flow does not universally characterize flow in porous media. He proposed the following equation which incorporates both linear and non-linear flow behaviour [1].

\[ i = aq + bq^2 \]  

(2.2)

where 'b' is a non-Darcy resistance coefficient.

Other expressions for flow resistance in non-Darcy flow have also been proposed [11, 15, 59, 31, 60]. In equation 2.2, a large \(a/b\) ratio suggests a predominantly laminar flow whereas a very low ratio suggests turbulent flow. Transitional flow would contain various intermediate values of \(a/b\). There is another region of non-Darcy flow where extremely low flows exist. Kovacs [29] represents this microseepage regime by:

\[ i = i_0 + aq \]  

(2.3)

where \(i_0\) is a threshold gradient. The various possible regimes of flow in porous media can be represented graphically after Kovacs as in Figure 2.1.

As the flow increases in the transitional regime, the viscous effects decrease and hence the stable streamlines in the pores through which the flow propogates, start to break down and the inertial effects become predominant. In the case of rockfill structures such as break-
Fig. 2.1 Regimes of Flow in Porous Media
waters, the interstitial spaces tend to be quite large, thereby furnishing greater propensity for inertial than viscous effects.

For the present study, both the coefficients $a$ and $b$ are essential inputs. The data available on different experimental values is very limited: the values determined by Nasser [35] were chosen for convenience. Nasser used a steady flow permeameter to establish the Darcy and non-Darcy coefficients, $a$ and $b$. A least squares technique was used to correlate $1/q$ and $q$ in Equation (2.2) and deduce the intercept $a$ and the slope $b$ of the line of best fit. Nasser also pooled his data with that of Lane, McCorquodale, Dudgeon and Ng. Empirical or semi-empirical expressions for $a$ and $b$ are generally considered constants for a particular medium, fluid and flow regime.

2.2.1 State of the Art on Non-Darcy Flow

A comprehensive description of the studies performed on Darcy and non-Darcy flows can be found in references [33, 34, 35]. The present analysis has been mainly influenced by the work of Dracos, McCorquodale, and Nasser.

Dracos [10] presented a study of linear flow in an infinite rectangular sand medium. He uses the classical method of characteristics to solve the Darcy flow equations:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial n}{\partial x} + \frac{g}{K} n = 0 \quad \cdots \quad (2.4)
\]

\[
\frac{\partial n}{\partial t} + u \frac{\partial n}{\partial x} + (h_0 + \eta) \frac{\partial u}{\partial x} = 0 \quad \cdots \quad (2.5)
\]

where $m$ = porosity

$g$ = acceleration due to gravity
\[ u = \text{horizontal pore velocity} \]
\[ \eta = \text{perturbation height with respect to the mean water level} \]
\[ h_0 = \text{mean water level} \]
\[ K = \text{conductivity for Darcy flow.} \]

His solution is verified by experiments performed in a Hele-Shaw apparatus. He presents the analysis of the movement of the outcrop point, i.e., the point at which the phreatic line intersects the boundary of the sand medium. Dracos shows that the outcrop point movement may undergo different phases depending on whether the maximum fall rate of the phreatic line within the rockfill structure is faster or slower than the fall rate of the free water level. Nasser [35] extended Dracos' analysis to the more complex non-Darcy flow regimes. Appendices A and B give the detailed analysis and solution of the initial value problem. Five solution methods are investigated and compared with special emphasis on the general analog solution. Computer programs and plots illustrate the types of possible initial conditions.

McCorquodale [33] uses a finite element model to study the problem of wave propagation through a rectangular rockfill embankment with an impervious core. He represents the tailwater piezometric boundary condition by a periodic function and uses triangular elements in discretizing his solution domain. After each time increment, the position of the free surface is calculated from the known previous value and the surface particle velocity during that increment.

Nasser [35] uses the method of characteristics coupled with a finite difference technique to treat wave motion in porous structures.
More recently, Hannoura and McCorquodale [19] combine the finite element and method of characteristics to determine the phreatic line and internal pressure distribution corresponding to maximum run-up. The results provide useful information for studying the stability of seawalls under both static and dynamic loading conditions. They indicate that about 60% less CPU time is required in comparison to the implementation of a purely finite element method.

The use of analog/hybrid techniques to solve this problem is missing from the literature. Some hyperbolic wave-type systems have been solved on the hybrid computer but none for porous media problems.

### 2.2.2 The Nature of the System Equations

Henry [23] presents the digital applications of a method devised originally to facilitate the analog and hybrid simulation of hyperbolic PDE's. The method was originally used to discretize the space variable and integrate with respect to time. He considers the problem of tidal motions in rivers and estuaries and the response of coastal inlets to tsunamis, both of which are long wave phenomena governed by hyperbolic equations. He introduces the continuity and general dynamic equilibrium equations respectively as:

\[
\begin{align*}
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + h \frac{\partial u}{\partial x} &= q \quad \ldots \ldots \ldots \ldots (2.6)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial h}{\partial x} &= g(S_0 - S_f) \epsilon - \frac{qu}{h} \quad \ldots \ldots \ldots (2.7)
\end{align*}
\]
where \( h \) = water level
\( u \) = horizontal velocity
\( q \) = lateral inflow
\( S_0 \) = channel slope
\( S_f \) = friction slope.

Nasser [35] introduces a porosity term and a representative resistance term in the momentum equation. His equations in a matrix form are:

\[
\begin{bmatrix}
\frac{h_0 + \eta}{m} & 0 & u/m & 1 \\
u/m & 1 & gm & 0 \\
dx & dt & 0 & 0 \\
0 & 0 & dx & dt
\end{bmatrix}
\begin{bmatrix}
\frac{du}{dx} \\
\frac{du}{dt} \\
\frac{d\eta}{dx} \\
\frac{d\eta}{dt}
\end{bmatrix} = \begin{bmatrix}
0 \\
-gmF_\eta u \\
du \\
d\eta
\end{bmatrix}
\]

(2.8)

where \( F = a + bq \)

This is a hyperbolic system of equations; setting the 4 x 4 determinant of the matrix of coefficients to zero yields the positive and negative characteristic equations:
\[ \alpha = \frac{dx}{dt} = \frac{u}{m} + \sqrt{g(h_0 + n)} \quad \ldots \quad (2.9) \]
\[ \beta = \frac{dx}{dt} = \frac{u}{m} - \sqrt{g(h_0 + n)} \quad \ldots \quad (2.10) \]

Nasser then determines the equations of motion along the characteristic directions, \( \alpha \) and \( \beta \), respectively as:

\[ \frac{d}{dt} \left( \frac{u}{m} + 2 \sqrt{g(h_0 + n)} \right) = -gF_u \quad \text{on} \quad \alpha \quad \ldots \quad (2.11) \]
\[ \frac{d}{dt} \left( \frac{u}{m} - 2 \sqrt{g(h_0 + n)} \right) = -gF_u \quad \text{on} \quad \beta \quad \ldots \quad (2.12) \]

He utilizes a finite difference approach to reduce the time consumption on a digital computer.

Nasser's basic equations will also be used in this study with the appropriate manipulation for a hybrid procedure.

2.3 Hybrid Methods for Hyperbolic PDEs

In the 1950s when computers were beginning to develop, the power and relative ease of programming of analog computers in simulation studies presented a substantial advantage over digital computers. Analog computers were known for their speed, their parallel nature of operation, their man-machine interaction, the continuous output produced and their ability to integrate, a feature of prime importance in differential equations [4].

Throughout the 1950s and 1960s, great strides were made in reducing digital hardware and increasing the speed, accuracy and memory
capabilities of the digital computers. The theoretical aspects of numerical solutions of differential equations were developed so rapidly that the digital computer was accepted as the general-purpose research tool. More time and money was invested into the digital market so that the analog simulations became obsolete to most researchers. There are virtually only a handful of analog computer producing companies today compared with the many in existence in the 1950s.

Hybrid computers were produced in the early 1960s basically out of economic motivation. Analog methods tended to require large amounts of hardware while numerical techniques were often too time consuming. Hybrid computers combine the advantages of the analog computer, i.e., speed and integration, with those of the digital computer, i.e., accuracy and memory capabilities. It must be remembered that the analog computer can integrate with respect only to one independent variable. Therefore, all the hybrid techniques for treating PDEs involve the reduction of the PDEs into a system of ordinary differential equations (ODEs). These techniques will be described shortly.

Vichnevetsky [54] suggests two reasons for preferring a hybrid solution of PDEs to the numerical solution:

a. the PDEs are only a part of a larger system, where some other parts are represented by ODEs and the use of an analog/hybrid system has been dictated by over-all economic requirements;

b. the numerical implementation of the PDE, more or less by itself, is prohibitively long and expensive.

In these cases, hybrid computers have been shown to have the potential to be several hundreds of times faster and cheaper (per solution) than
digital computers.

It is emphasized that the primary concern in this study is not the economic justification of the use of a particular method but rather the exploration of the potential for utilizing a technique not commonly adopted today in civil engineering practice. More detail regarding the advantages and disadvantages of analog computers as well as a description of their major components can be found in Appendix C.

2.3.1 Methods of Lines \([54, 62]\).

Methods of lines are those where PDEs are approximated by ODEs along a certain uni-directional path of integration or 'line' within the space-time domain. The path of integration may be along one of the following directions (considering the space-time domain):

a. parallel to the time axis
b. parallel to the space axis
c. along a characteristic direction.

The first method is the classical analog discrete-space continuous-time (DSCT) method. It is also referred to as the Parallel method since the whole system is analyzed with increasing time. Its hybrid nature is mainly associated with the multiplexing of components.

The second method is commonly referred to as the CSDT or serial method. This method, which is truly hybrid in nature, will be discussed later in greater detail.

The method of characteristics (MOC) has not seen much direct application in analog/hybrid computation. However, the theory associated with the characteristics and their properties in hybrid problems
has been an important tool in the analysis and development of other methods of lines (mainly for CSDT methods).

Schuchmann [46] presents a general paper covering all aspects of analog, hybrid and digital simulation of PDEs. The PDE must be reduced to ODEs since only one continuous variable can be treated by the analog computer. Independent variables can be classified as continuous (C), discretized (D), or transformed (T).

McCann [32] observes that two independent variables with the above three classifications result in nine combinations as shown in Table 2.1.

Since only one variable can be represented continuously, the first combination is no longer valid. The transformation of variables (T) by Laplace or Fourier series was not considered in the present problem since the method did not get support from any of the simulations of practical problems in the literature. A DSDT (discrete-space discrete time) framework would be handled more easily on a digital computer. This leaves the two main methods already mentioned: DSCT and CSDT.

Schuchmann [46] concludes that the DSCT method seems too coarse for an application to flow and wave-type systems and that the hybrid CSDT method seems to be the best for real time simulations of systems containing flow terms, i.e., hyperbolic equations.

2.3.2 DSCT (Parallel) Method [62].

This is the traditional technique used for solving PDEs. The equations are reduced to a set of ODEs in time by using a suitable numerical approximation for the space derivative and the resulting set of equations are solved in parallel.
### Table 2.1
#### Classification of 2-D Problems

<table>
<thead>
<tr>
<th>Combination</th>
<th>Space Variable</th>
<th>Time Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>T</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>C</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td>6</td>
<td>D</td>
<td>T</td>
</tr>
<tr>
<td>7</td>
<td>T</td>
<td>C</td>
</tr>
<tr>
<td>8</td>
<td>T</td>
<td>D</td>
</tr>
<tr>
<td>9</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>

*C = continuous  
D = discrete  
T = transformed*
In principle this method offers the fastest solution yielding
time histories of the system behaviour at selected points in the domain.
The resulting equations are normally stable and the boundary conditions
are rapidly solved on the analog computer. The main difficulty arises
in the amount of analog equipment needed. Adequate accuracy requires
that a sufficient number of spatial grid points be provided, each of
which must be represented by a set of differential equations. Even for
simple problems, the equipment requirements can become excessive making
the method suitable only to the largest analog installations.

Two important contributions are worth noting; Henry [23] refers
to a paper by Hsu and Howe [24] in deriving his numerical sectioning pro-
cedure. Hsu and Howe replaced the all-parallel analog solution with a
series-parallel combination. Instead of using many analog components,
by their multiplexed DSCT method, they employ a relaxation technique.
Their method involves integrating a subset of the continuous time finite
difference differential equations for the whole period of time and then
iterating through the different subsets to satisfy the coupling solution.
This method is a special form of cell multiplexing; it retains most of
the advantages of the DSCT method except that the speed of solving the
problem is reduced due to the iterative process. Hsu and Howe use the
iterative DSCT method to solve various diffusion-type equations. Due to
the lack of a suitable hybrid computer system, their solution is simulated
on a digital computer. Since their work is a preliminary investigation
with many unanswered questions regarding implementation of field problems,
it was not seriously considered as a method of solution.
Wright and Mutti [62] utilized a multiplexed DSCT method to model the hyperbolic nature of unsteady fluid flow systems. Their problem dealt with systems subjected to time-varying flow of compressible fluids (quasi-linear hyperbolic PDEs). Because of difficulties encountered in the breakdown of solutions at sonic flow or stagnation points (i.e. fluid velocity is zero), they concluded that the CSDT method is unsatisfactory for unsteady compressible fluid flow problems. The general consensus is to try the CSDT method first and then switch to other methods if it is not applicable.

2.3.3 Method of Characteristics

In the method of characteristics, at every point in the space-time (x-t) plane, there are two characteristic directions in which the PDEs reduce to equations involving total differentials only. In these directions, the equations are not complicated by the presence of partial derivatives in other directions. This method is especially suited for the analyses of hyperbolic PDEs since the two characteristic directions are real and of opposite sign (whereas for elliptic equations, there are two imaginary directions and parabolic equations yield two real and equal directions) [48].

The standard hybrid MOC solution involves integration along one family of characteristics with finite differencing along the other family [62]. A subcomputer block of computing elements is required to integrate along one characteristic and hence N blocks are necessary to handle N characteristics. In addition to the minimum number of blocks needed to maintain numerical accuracy, the number of computing elements per subcomputer block can become large when the characteristics are functions of
the solutions. Therefore, the MOC as a hybrid technique reassembles the DSCT method in that it requires a large number of analog components.

Paul and Ahmed [43] developed a continuous-characteristics approximation technique that slightly differs from the finite difference-differential method described. In their procedure, the characteristics of the equation are used to scan and give data on the initial line, thereby allowing the boundary conditions of the problem to be introduced.

Although their subcomputer blocks contain more complicated set-ups than the standard method, they note that there is a saving in the use of fewer blocks and claim that their method is an improvement upon similar digital techniques because of the increased speed of integration and the continuous nature of integration along one of the characteristics. Paul et al present worked examples which deal with supersonic and compressible flows. The CSDT method is not recommended for solving such problems as was noted earlier.

McAvoys [31] uses a modification of an error analysis developed by Bosgra and Buis [5] which results in a perfect frequency response relative to the exact solution. He concludes that the MOC can handle problems where discontinuities may arise in the solution. The drawback of McAvoys method is that it does not use the full potential of the hybrid computer, i.e., the integration process. In his somewhat unusual hybrid implementation, the digital computer is used for function storage and playback while the analog computer solves a system of algebraic equations and determines whether a storage point has been reached. It is only hybrid in the sense that the algebra is performed in parallel.
A mechanical distributed parameter system with a non-linear force was studied by Fugita et al [17] using the theory of characteristics and a finite difference approximation technique. He comments that his method is very useful to such practical problems: his method is not outlined in the paper but in essence it resembles the MOC finite difference differential equations. Upon determining the characteristic lines and making some substitutions, the second order system is reduced to simultaneous first order PDEs. These are then reduced to simultaneous difference equations by using certain transformations and finite difference approximations. Repetitive corrections are then carried out on the non-linear friction force term. This method is not generally applicable as Fugita suggests.

Genlina and Cleret [18] propose a method of solving hyperbolic equations using quasi-linear hybrid methods. They use McAvoy's characteristic method to yield the boundary conditions of the problem. They then use a second method of integration using the boundary conditions stored by the MOC solution to yield the total solution. The second method of resolution incorporates various speeds of integration and track/store devices. The method is not clearly presented.

In summary, the MOC hybrid technique can be used in problems where discontinuities may arise or in scanning the domain of a problem to determine the boundary conditions. This method is not generally pursued due to its limited applications and the quantity of analog components required. Thus, the most viable and truly hybrid technique remains to be discussed.
2.3.4 CSDT (Serial) Method

Since its introduction [26] in 1961, the serial method has steadily been growing in popularity to the point that today it is the standard hybrid approach of computation. The method consists of reducing the PDEs to ODEs in space by a suitable numerical representation of the time derivative. This reduces to a boundary value problem which must be solved at each time step of the time-marching integration process [52].

CSDT methods are inherently dependent on the capability of store and playback functions. This severely taxes the hybrid interface hardware and the high speed of the analog computer is only possible to the degree allowed by the time for information to be read in and out of the digital computer. Therefore, this method is usually reserved for complex PDEs.

This method has the following advantages over the classical DSCT method [20]:

1. It allows for higher resolution and accuracy without a proportional increase in equipment and as such is important in dealing with non-linearities in equations.

2. Greater use is made of the high-speed computational capabilities of the analog computer.

3. The integration interval of the continuous space variable can be controlled so that problems with moving boundaries are easily solved.

The major set-back with CSDT methods is the problem of error propagation. Typical hybrid errors result from truncation of results and inaccuracies in analog equipment. These errors can be controlled to a certain degree and do not drastically change the results. The CSDT
method has the disadvantage that the system equations are inherently unstable, i.e., the errors will accumulate rapidly and become substantial. Various techniques have been developed to overcome this drawback.

The target-shooting method consists mainly of iteratively correcting guessed values. It suffers from serious error instability properties and is not frequently used.

Calza-Bini et al [52] used a superposition technique whereby the solution of the homogeneous equation is added to the computed solution so that the boundary condition is satisfied. The error instability is reduced because there is no iteration but the problem still exists.

Witsenhausen [52] expressed the solution as a functional or definite integral of the space variable with the use of Green's functions.

Vichnevetsky's paper [51] introduced the Decomposition Method in 1968 and was instrumental in establishing the CSDT method as the standard hybrid approach. His method decomposes an unstable problem of the second order into problems of the first order that can be made stable by the proper choice of the spatial direction of integration.

The method can be best illustrated using a linear coefficient parabolic equation, viz.

\[ \frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} \tag{2.13} \]

Discretizing the time yields

\[ \frac{d^2 u^{j+1}}{dx^2} - \frac{u^{j+1}}{k\Delta t} = - \frac{u^j}{k\Delta t} \tag{2.14} \]
The operator of this equation,

\[ L = \frac{d^2}{dx^2} - \frac{1}{K\Delta t} \quad \cdots \quad (2.15) \]

may be decomposed into the product:

\[ L = \left( \frac{d}{dx} - a_F \right) \left( \frac{d}{dx} - a_B \right) = L_F \cdot L_B \quad \cdots \quad (2.16) \]

where \( a_F = \frac{-1}{\sqrt{K\Delta t}} \); \( a_B = \frac{1}{\sqrt{K\Delta t}} \).

A particular solution of the system may be obtained by solving two computationally stable initial value problems:

\[ L_F y_1 = \frac{dy_1}{dx} - a_F y_1 = -\frac{u^i}{K\Delta t} \quad \cdots \quad (2.17) \]

integrated in the forward direction \( (x = 0 \text{ to } x_{\text{max}}) \) and

\[ L_B y_2 = \frac{dy_2}{dx} - a_B y_2 = y_1 \quad \cdots \quad (2.18) \]

integrated in the backward direction \( (x = x_{\text{max}} \text{ to } 0) \) or

\[ \frac{dy_2}{d(-x)} + a_B y_2 = -y_1(x^i) \quad \cdots \quad (2.19) \]

The same analog cell can be used twice to represent these equations as can be seen in Figure 2.2.

Vichnevetsky also introduced an interpolated form of Equation 2.13 to reduce the time-skew effect which results in a truncation error of order \( \Delta t \).
Fig. 2.2  Typical Analog Cell for a Parabolic System
\[
\frac{d^2 u^{j+1}}{dx^2} - \frac{u^{j+1}}{\theta \Delta t} = - \frac{u^j}{\theta \Delta t} \left( \frac{1 - \theta}{\theta} \right) \frac{d^2 u^j}{dx^2} \quad \cdots (2.20)
\]

where \( \theta \) is the interpolation factor,

which is reduced to

\[
\frac{d^2 u^{j+1}}{dx^2} - \frac{u^{j+1}}{\theta \Delta t} = - \frac{u^j}{\theta \Delta t} \quad \cdots \cdots \cdots \quad (2.21)
\]

\( u \) satisfies the recurrence relation

\[
u^j = \frac{u^j}{\theta} + \frac{1}{6} \left( u^{j+1} - u^j \right) \quad \cdots \cdots \cdots \quad (2.22)
\]

A detailed analysis of this process would show that a necessary and sufficient condition for time-marching stability is \( 1/2 < \theta < 1 \).

Later in 1968, Hara and Karplus [20] introduced their CSDT version for solving PDEs with stable equations. A control function is digitally generated and is imposed as a forcing function upon the analog circuit. A steepest descent optimization routine is then used to minimize the errors of the control forcing function. They feel that the high speed computational capabilities of the hybrid computer can offset the more complicated fractional optimization problem and hence more hardware, introduced by their method.

The introduction of novel computing serial schemes reached a peak in 1970. Vichnevetsky and Tomalesky [58] solved a time dependent river pollution problem which was basically an extension of Vichnevetsky's previous work. In order to reduce the computer time consumption, they introduced a technique by which truncation errors induced by taking larger grid-time steps could be corrected for in a semi-exact fashion.

In 1969, Vichnevetsky [53] introduced a different method applicable only to quasi-linear hyperbolic problems. He presented a second
order hyperbolic system, viz:

\[ \frac{\partial u}{\partial t} = f(u) \frac{\partial u}{\partial x} + g(x,t) \]  \hspace{1cm} (2.23)

where \( u \) is a two dimensional vector of dependent variables, \( f \) is a square matrix of functions of \( u \) and \( g(x,t) \) is the external or forcing function, thus,

\[
\begin{bmatrix}
  u_1(x,t) \\
  u_2(x,t)
\end{bmatrix}
\quad ; \quad
f(u) = \begin{bmatrix}
  f_{11}(u) & f_{12}(u) \\
  f_{21}(u) & f_{22}(u)
\end{bmatrix}
\quad ; \quad
q = \begin{bmatrix}
  g_1(x,t) \\
  g_2(x,t)
\end{bmatrix}
\]  \hspace{1cm} (2.24)

The system can be transformed to its characteristic form:

\[ \frac{\partial W}{\partial t} = \lambda I \frac{\partial W}{\partial x} + G(x,t) \]  \hspace{1cm} (2.25)

where \( W \) is the vector of transformed dependent variables

\( I \) is the identity matrix

\( \lambda \) is the vector of eigenvalues

\( G \) is the resulting forcing functions, i.e.,

\[
W = \begin{bmatrix}
  W_1(u_1, u_2) \\
  W_2(u_1, u_2)
\end{bmatrix}
\quad \lambda = \begin{bmatrix}
  \lambda_1(W_1, W_2) \\
  \lambda_2(W_1, W_2)
\end{bmatrix}
\]  \hspace{1cm} (2.26)

Upon discretizing the time variable in Equation 2.25, the resulting boundary value problem reduces to two initial value ODEs with stable error propagation properties. The equations are decoupled by a time interlaced integrated scheme which can be seen in the computer block diagram in Figure 2.3.
Fig. 2.3 — Computer Block Diagram of Interlaced Integration Scheme.

\[ w_{j+1}^i = 2w_{j+1}^i - 2w_{j-1}^i \]

\[ \frac{d w_{j+1}^i}{d(x)} = \frac{w_{j+1}^i - w_j^i}{\Delta x} \]

\[ w_j^i = w_j^i(x, t_j) \]

Boundary Condition in \( x = X_{\text{max}} \)

Boundary Condition in \( x = 0 \)
Vichnevetsky uses a river flooding problem to illustrate his method. This method cannot indiscriminately be applied to the present study despite the similarity in the problem definitions. This is due to the fact that Vichnevetsky's flood wave has no downstream constraint, i.e., the wave is allowed to freely dampen out along its course whereas the solution domain in the present study is constrained by the core which not only inhibits further flow but also introduces wave reflection.

Vichnevetsky's method permits the decoupling of the discretized characteristic equations because the wave disturbances propagate in different directions. His initial value equations are simpler than the two point boundary value problem at hand. An iterative procedure was found necessary to evaluate the downstream condition in this study.

André [2] suggested two methods similar to Vichnevetsky's. In the first method, given first order PDEs, a similarity transformation is carried out whereby the equations to be solved at each time period on the analog are totally decoupled and hence solved simultaneously. In the second method, he applies the method of characteristics (MOC) to quasilinear hyperbolic equations. He uses increments in both time and space dimensions and claims that accuracy is not sacrificed at discontinuities.

Nelson [38] presented a preliminary study into the use of invariant bedding. This technique is essentially a method of converting a two-point boundary value problem for a linear system of ODEs to an equivalent initial value problem for an associated non-linear system of ODEs. A progress report on a digital solution appeared later in the year [39]. They felt it was a highly effective method for solving some types of PDEs.
Silvey and Barker [47] presented a decomposition-iterative method that uses one trial run and two correcting runs at each time step to satisfy the boundary conditions to within 0.1 per cent. They felt their method was more accurate than Vichnevetsky's decomposition-superposition method but at the expense of computing time.

Bosgra and Buis [5,6,7,8] introduced a directional difference method which was generally applicable to sets of hyperbolic equations. They found the decomposition-iterative method very impractical and searched therefore for a better method of solution. Assuming a sufficiently smooth function, they chose a point lying on a straight line joining two solution points for which the mean value theorem was valid. This method involved taking both time and spatial increments (as did André) and could be considered more general than Vichnevetsky's scheme. By choosing the proper ratio of space-to-time grid sizes, stability could be ensured for the finite difference-differential equations. Bosgra et al present error analysis in both the physical and numerical sense: the suitability of the method as a tool for the development of new classes of methods, etc. is discussed in subsequent papers. A complex multi-point hybrid CSDT difference approximation is introduced that eliminates the problem of inversion of the coefficient matrix as well as the suitability problem.

In 1971, Nelson and Altom [40] published their hybrid solution using their method of invariant imbedding. For the parabolic problem considered, they found their technique slightly less efficient (efficiency being measured by the number of operations) than the decomposition-superposition method but considerably more efficient than the decomposition-iteration method. They feel their method is at least competitive with
other stable serial methods for some problems.

Appolovicova and Babirad [3] in 1973 solved a general hyperbolic system by a method of distribution. The resulting equations and formulations closely parallel the decomposition-superposition method. The difference lies in their definition of two optional and arbitrary functions in the solution that must be chosen in such a way that the final solution must fulfill the initial and boundary conditions. This method does not appear generally applicable to this study.

In 1974, Vichnevetsky and Tomalesky [59] presented a computer algorithm for treating hyperbolic PDEs with special application to the simulation of electrical transmission lines. Truncation errors cause a spurious diffusion term to be introduced into the problem. This paper, similar to the one in 1969, introduces a time-centered approximation which results in a new recurrence algorithm. Incremental values are transferred across the interface to alleviate the effects of round-off errors.

O'Brien and Edge [42] in 1975 extended Vichnevetsky's work to yield a hybrid solution of the water quality in an estuary. Their graphical results were very instructive.

Stockton [49] solved the wave equation in isotropic and anisotropic media by employing MOC. He obtains satisfactory results for the isotropic case by using a predictor/corrector estimator for the unknown boundary conditions. For the anisotropic case, he implements a method whereby a solution is obtained from two computationally independent solutions. He indicates that the hybrid procedure is significantly superior to the digital procedure with respect to execution time.
In 1976, Janac [25] presented CSDT methods to solve PDEs in several space dimensions. The schemes are unconditionally stable with no truncation error in relation to the space variables. He makes use of the 'weak approximation' theory to approximate two-dimensional problems (in space) by two one-dimensional problems. He then uses a Crank-Nicolson scheme [48] to develop the recursive equations. He solves the boundary value problems for a system of ODEs (these can be solved in parallel). He claims that a very fine space discretization is possible without accumulating larger errors and without the need for more accurate computations.

In summary, the CSDT method is the most versatile, truly hybrid technique but has a problem of computational instability. Numerous techniques have been developed to overcome this problem, some highly specialized, others very general in application. In particular, the work of Vichnevetsky and O'Brien et al have influenced the present study.
CHAPTER 3

THEORETICAL DEVELOPMENTS

This chapter will describe the development and modification of the non-Darcy flow equations to suit the CSDT method of analysis. The theory presented considers a rectangular embankment; however, sloping configurations would basically differ in the geometric treatment.

3.1 Governing Equations

The derivation of the basic continuity and momentum equations for unsteady flow can be found in numerous references [9, 22, 35].

Fig. 3.1 shows an impact wave with a mean water level, $h_0$, and a maximum amplitude, $A_0$, attacking a rockfill structure; the resultant interior flow has a horizontal velocity, $u$. Within the porous media, the wave gradually loses its kinetic energy due to friction; the height of the phreatic surface with respect to the mean water level is termed $\eta$, so that the actual depth, $y$, of the phreatic surface is

$$y = h_0 + \eta$$ \hspace{1cm} (3.1)

Since $h_0$ is constant, the differential height of the phreatic surface is

$$dy = d\eta$$ \hspace{1cm} (3.2)
The porosity, \( m \), is the ratio of the volume of the voids to the total volume of the medium and can therefore be taken as a measure of the actual cross-sectional area through which the water can flow. Hence, the actual pore velocity, \( V \), is

\[
V = u/m
\]

(3.3)

3.1.1 Continuity Equation

Henderson [22] gives the continuity equation for a long shallow water wave (Fig. 3.2), i.e.

\[
\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + D \frac{\partial V}{\partial x} = 0
\]

(3.4)

where:

\( D = \) hydraulic depth = \( y \) for rectangular channels
\( x = \) spatial horizontal direction
\( t = \) time variable.

Substituting Equations 3.1, 3.2, and 3.3 into the above equation yields the continuity equation for wave motion in rockfill:

\[
\frac{\partial n}{\partial t} + \left( \frac{u}{m} \right) \frac{\partial n}{\partial x} + \left( h_0 + n \right) \frac{\partial (u/m)}{\partial x} = 0
\]

(3.5)

or the equation given by Nasser [35], viz.

\[
\frac{\partial n}{\partial t} + \left( \frac{u}{m} \right) \frac{\partial u}{\partial x} + \left( h_0 + n \right) \frac{\partial u}{\partial x} = 0
\]

(3.6)

To simplify the analysis, it is helpful to introduce the celerity, \( c \), being the velocity of a wave relative to the velocity of flow. For small gravity waves, the celerity is defined as:
Fig. 3.2 Continuity of Unsteady Flow in an Open Channel.
\[ c = \sqrt{g(h_0 + \eta)} \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.7) \\

where \( g \) = acceleration due to gravity 
\[ = 980.6 \text{ cm/sec}^2 \]

or
\[ c^2 = g(h_0 + \eta) \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.8) 

Taking the differential of both sides of the equation results in:
\[ 2c \frac{dc}{dt} = d(g(h_0 + \eta)) = g \frac{d\eta}{dt} \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.9) 

or
\[ d\eta = \frac{2c}{g} \, dc \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.10) 

Upon substitution of equations 3.7 through 3.10 into equation 3.6, the continuity equation reduces to the form:
\[ \frac{\partial c}{\partial t} + \frac{c}{2m} \frac{\partial u}{\partial x} + \frac{u}{m} \frac{\partial c}{\partial x} = 0 \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.11) 

3.1.2 Equation of Motion

Assuming a prismatic channel, Henderson [22] gives the equation of motion for unsteady, open channel flow (Fig. 3.3) as:
\[ \frac{1}{g} \frac{\partial V}{\partial t} + \frac{\alpha V}{g} \frac{\partial V}{\partial x} + \frac{\partial V}{\partial x} = S_0 - S_f \] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.12) 

where \( \alpha \) is the energy coefficient (assumed to be unity)

\( S_0 \) is the channel bed slope

\( S_f \) is the energy line slope.
Fig. 3.3. Representation of Energy in Open Channel Flow.
Substituting equations 3.1, 3.2, and 3.3 into 3.12 and rearranging results in the following interior flow equation:

\[
\frac{\partial u}{\partial t} + \frac{u}{m} \frac{\partial u}{\partial x} + g_m \frac{\partial n}{\partial x} = g_m (S_o - S_f) \quad \cdots \cdots \quad (3.13)
\]

The right hand side resistance term must be a function of the porosity, gravity, hydraulic gradient i, particle properties and velocity. Among the various resistance formulae available, it has been determined [1, 34, 35] that the Forchheimer equation is the most convenient expression available, i.e.

\[
i = a q + b q^2 \quad \cdots \cdots \quad (2.2)
\]

Since the hydraulic gradient is dimensionless and suitably describes the energy loss, it will be used to describe the slope terms in equation (3.13), viz.

\[
\frac{\partial u}{\partial t} + \frac{u}{m} \frac{\partial u}{\partial x} + g_m \frac{\partial n}{\partial x} = -g_m i \quad \cdots \cdots \quad (3.14)
\]

The hydraulic gradient can also be expressed as:

\[
i = F u \quad \cdots \cdots \quad (3.15)
\]

where \(F = a + b|u|\).

The equation of motion can easily be expressed in terms of velocity and celerity:

\[
\frac{\partial u}{\partial t} + \frac{u}{m} \frac{\partial u}{\partial x} + 2cm \frac{\partial c}{\partial x} = -g_m F u \quad \cdots \cdots \quad (3.16)
\]
Both equations 3.11 and 3.16 are quasilinear partial differential equations which when coupled form a hyperbolic system. This suggests that Vichnevetsky's [53] special procedure for solving quasilinear hyperbolic PDE's is applicable to this study.

3.2 BOUNDARY CONDITIONS

A time dependent function that describes the movement of the outcrop point is used as the boundary condition at the water rockfill interface. The derivation and solution of this time dependent function is given in Appendix A. Five solution methods are presented. The discrete points comprising the outcrop point curve are stored and used as the input data for the boundary conditions described by the hyperbolic system.

At the impervious core, the boundary condition of no normal flux is stated, i.e. the horizontal velocity must be zero; thus

$$u(t,x_{\text{max}}) = 0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.17)$$

Because a boundary condition is specified at each end of the solution domain, the phenomenon is a boundary value problem and therefore an iterative procedure may be used to satisfy the downstream condition of zero velocity.
3.3 Initial Conditions

Originally the water is considered to be at rest with the reservoir. Therefore, the velocity is zero and there will be no perturbation. Therefore the celerity of the wave initially is given as:

\[ c(0,0) = \sqrt{gh_0} \] \hspace{1cm} (3.18)

3.4 Hybrid Formulation

There exists a wide range of hybrid techniques to solve the existing problem but due to constraints of analog hardware, efficiency, complexity, etc., the CSDT method was chosen. For the reasons mentioned in Chapter 2, Vichnevetsky's method of lines \[53\] was deemed best suited in analyzing the governing equations.

3.4.1 Characteristic Transformation

The conservation of mass (3.11) and momentum (3.16) equations can be written in matrix notation, viz.

\[
\frac{\partial}{\partial t} \begin{bmatrix} u \\ c \end{bmatrix} + \begin{bmatrix} u/m \\ c/2m \end{bmatrix} \cdot \begin{bmatrix} 2cm \\ u/m \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} u \\ c \end{bmatrix} = \begin{bmatrix} -gmFu \\ 0 \end{bmatrix} \] \hspace{1cm} (3.19)

The eigenvalues are determined by solving for:

\[
\det \begin{bmatrix} u/m - \lambda & 2cm \\ c/2m & u/m - \lambda \end{bmatrix} = \lambda^2 - 2\lambda \frac{u}{m} + \frac{u^2}{m^2} - c^2 = 0 \]

\hspace{1cm} \ldots \ldots (3.20)

or

\[
\lambda_{1,2} = \frac{u}{m} \pm c \] \hspace{1cm} (3.21)
The adjoint-eigenvectors, \( w^1 \) and \( w^2 \) are defined as:

\[
\begin{align*}
  cw_1^1 + \frac{c}{2m} w_2^1 &= 0 \\
  -cw_1^2 + \frac{c}{2m} w_2^2 &= 0
\end{align*}
\]

which yield

\[
\begin{align*}
  w^1 &= \begin{bmatrix} 1 \\ 2m \end{bmatrix} \\
  w^2 &= \begin{bmatrix} 1 \\ -2m \end{bmatrix}
\end{align*}
\]

These two real matrices are linearly independent vectors. By multiplying \( 3.19 \) by the transpose of \( w^1 \) and \( w^2 \), two equivalent equations are obtained but in a new set of dependent variables. This yields the characteristic form of the equations. For example, multiplying the transpose of \( w^1 \) by \( 3.19 \) yields:

\[
\begin{align*}
  \frac{3}{\delta t} u + \frac{u}{m} \frac{\delta u}{\delta x} + 2mc \frac{\delta c}{\delta x} + 2m \frac{\delta c}{\delta t} + c \frac{\delta u}{\delta x} + 2u \frac{\delta c}{\delta x} &= -gmFu
\end{align*}
\]

Rearranging the terms and performing a little algebra yields the integrable form:

\[
\frac{3}{\delta t} (u + 2mc) + (u/m + c) \frac{\delta u}{\delta x} (u + 2mc) = -gmFu
\]
A similar expression can be obtained by multiplying the transpose of \( \mathbf{w}^2 \) and rearranging the terms, viz.

\[
\frac{3}{3 t} (u - 2mc) + \frac{3}{3 x} (u - 2mc) = -\gamma m F u \quad \ldots \ldots \ldots \ldots \ldots \ldots (3.26)
\]

Equations 3.25 represents the forward propagation of a wave, \( W_1 = u - 2mc \), at a velocity of \( u/m + c \), whereas 3.26 represents a backward wave propagation, \( W_2 = u - 2mc \), at a wave velocity of \( u/m - c \).

The original variables and the characteristics can now be expressed in terms of the new independent variables \( W_1 \) and \( W_2 \):

\[
u = \frac{W_1 + W_2}{2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.27)
\]

\[
c = \frac{W_1 - W_2}{4m} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.28)
\]

\[
\eta = \frac{1}{g} \left[ \frac{W_1 - W_2}{4m} \right]^2 - h_0 \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.29)
\]

\[
\lambda_1 = \frac{u}{m} + c = \frac{3W_1 + W_2}{4m} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.30)
\]

\[
\lambda_2 = \frac{u}{m} - c = \frac{W_1 + 3W_2}{4m} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.31)
\]

Equations 3.25 and 3.26 can be expressed in terms of the new independent variables, i.e.
\[
\frac{\partial W_1}{\partial t} + \frac{3W_1 + W_2}{4m} \frac{\partial W_1}{\partial x} = -\frac{gmF}{2} (W_1 + W_2) \quad \ldots \ldots \ldots \quad (3.32)
\]

\[
\frac{\partial W_2}{\partial t} + \frac{W_1 + 3W_2}{4m} \frac{\partial W_2}{\partial x} = -\frac{gmF}{2} (W_1 + W_2) \quad \ldots \ldots \ldots \quad (3.33)
\]

Vichnevetsky states that an equation of the form of 3.22 is computationally stable from the error propagation point of view when integrated in the forward direction, i.e. from \( x = 0 \) to \( x = x_{\text{max}} \). Equation 3.33 would then have to be integrated in the backward direction, i.e. from \( x = x_{\text{max}} \) to \( x = 0 \). In his example, the equations can be decoupled but in the present study such is not the case and therefore his serial interlaced integration is not applicable. The two characteristic equations must be solved simultaneously either by a matrix method or by an iterative procedure.

3.4.2 CSDT Formulation

Since the CSDT method is being applied, the time is discretized by intervals of \( \Delta t \). A typical time frame is expressed as:

\[
\int_{t^j} t^j = j\Delta t \quad \ldots \ldots \ldots \quad (3.34)
\]

where \( j = 0, 1, 2 \ldots \ldots \)

The waves are denoted as:

\[
W_1^j = W_1(x, t^j) \quad \ldots \ldots \ldots \quad (3.35)
\]

\[
W_2^j = W_2(x, t^j) \quad \ldots \ldots \ldots \quad (3.35)
\]
Introducing a backward finite difference scheme for the two time derivatives results in the following discretization:

\[
\frac{\partial W_{1,2}}{\partial t} \bigg|_{t^j} = \frac{W_{1,2}^j - W_{1,2}^{j-1}}{\Delta t}
\]

where subscripts \(1, 2\) indicate the applicability of equation 3.36 to \(W_1\) and \(W_2\).

Upon substitution of equation 3.36 into 3.32 and 3.33 and rearranging, the following equations are obtained:

\[
\frac{dW_1^j}{dx} + \frac{W_1^j}{\lambda_1 \Delta t} = \frac{1}{\lambda_1 \Delta t} W_1^{j-1} - \frac{gmF}{2\lambda_1} (W_1 + W_2)
\]

\[
\frac{dW_2^j}{dx} + \frac{W_2^j}{\lambda_2 \Delta t} = \frac{1}{\lambda_2 \Delta t} W_2^{j-1} - \frac{gmF}{2\lambda_2} (W_1 + W_2)
\]

At time \(t^j\), both equations 3.37 and 3.38 become ordinary differential equations with respect to the spatial direction \(x\).

3.4.3 Solution Method

Equations 3.37 and 3.38 state that the dependent variables are coupled with respect to wave velocities and the resistance term. This suggests that the equations must be solved simultaneously. This is achieved on a digital computer by utilizing a Runge-Kutta and/or a predictor-corrector method for a system of ODE's. On an analog computer, the two equations are patched up with the proper cross connections of the two variables and solved in a continuous fashion because only one independent.
variable exists. Both variables are updated from one time frame to another. The initial condition, ie. \( t = t^0 \), for the system in terms of \( W_1 \) and \( W_2 \) is:

\[
\begin{align*}
W_1(0) &= u(0) + 2mc(0) = 2m \sqrt{gh_0} \\
W_2(0) &= u(0) - 2mc(0) = -2m \sqrt{gh_0}
\end{align*}
\] (3.39)

Because of the time differencing, the system can only begin to be solved at \( t = t^1 \). At this second time frame, the celerity is known from the outcrop point movement but the macroscopic velocity is unknown, ie.

\[
\begin{align*}
W_1(t^1) &= u(t^1) + 2m \sqrt{gh_0 + \eta(t^1)} \\
W_2(t^1) &= u(t^1) - 2m \sqrt{gh_0 + \eta(t^1)}
\end{align*}
\] (3.40)

Since \( u(t^1) \) is not known, it must be given a trial value such that at the end of the integration of the two equations, 3.37 and 3.38, the downstream boundary condition is satisfied, viz.

\[
W_1(t_j, x_{max}) = -W_2(t_j, x_{max})
\] (3.41)

If the trial value does not satisfy equation 3.41, a new guess is required. On the digital computer, this is achieved by specifying the limits on the velocity, entering an initial guess and changing one limit depending on whether the velocity \( u \) (or \( (W_1 + W_2)/2 \)) is positive or negative, and thereby converging to the correct solution. This Bisector Search Method can yield a solution quickly if the initial guess is close enough (Fig. 3.4).
Fig. 3.4 A Possible Sequence of Events for the Bisector Search Method
Once a solution is obtained for a particular time frame, the variables are converted back to the original parameters to yield a plot of the variation of the water level with distance, i.e. the phreatic surface within the rockfill structure. The time advances one frame, using the reference values of $W_1$ and $W_2$ at the mean water level, and the procedure is continued until a family of phreatic surfaces is obtained for one complete period of the outcrop point movement.

This method was first programmed on a CDC 6600/Cyber digital computer to remove all the programming obstacles and then was implemented on the EAI 690 analog/hybrid computer.
CHAPTER 4

ANALYSIS

A number of methods were attempted to solve the characteristic equations of this study (Eqs. 3.32 & 3.33). Vichnevetsky [53] suggested that his serial-interlaced solution could easily be implemented using a standard Runge-Kutta 4th order subroutine. This would enable the programmer to obtain a fully digital solution from which proper scaling of the analog circuit could be established; thus, an accurate solution could be determined and used as a check against the hybrid solution. This technique proved helpful in this study since the hybrid program was not conveniently edited with the existing facilities. It was assumed that once the numerical program was correctly formulated, transferring the functioning algorithm to the hybrid computer would be relatively easy. Hence, it is imperative that the digital formulation be emphasized as a prelude to the hybrid solution; obtaining the proper algorithm presented the biggest challenge.

4.1 PROGRAMMING CONSIDERATIONS

Vichnevetsky's [53] serial-interlaced solution was attempted first to obtain some more information on the nature of the problem. It was quickly realized that a purely Runge-Kutta solution would be quite time
consuming because the method utilized four function evaluations for each value calculated. However, the method is a self-start one and can easily be programmed. Predictor/Corrector methods only required two function calls for each value calculated for the same order of accuracy. Thus, they were less time consuming but were not self-starting. Therefore, the numerical integration was performed utilizing a Runge-Kutta method to obtain the first four integration values and the solution was then propagated using an Adams-Moulton predictor/corrector method; the computer programs are given in Appendix D.

It was also found that one defining function statement could be used to model both characteristic equations.

Vichnevetsky's method did produce a propagating solution but it was noticed that: (a) the velocity term, $u$, was not satisfying the zero velocity criterion at the core; (b) the velocity generated was too high, in fact unrealistic; (c) the damping of the water-wave at the maximum run-up was minimal and in some cases the phreatic surface even sloped upwards (a physical impossibility). Further, the $W_2$ wave was not adequately accounted for.

In order to satisfy the downstream boundary condition, a term was introduced to force the velocity to zero at the core. This method was not acceptable even though it appeared to generate a plausible solution, because the system behavior was far more complicated than the forcing function indicated and there was instability in the equations that required very small space increments. To this end, it was decided to re-evaluate Vichnevetsky's work.
Vichnevetsky's claims were based on the definition of his problem: the routing of a flood wave. Since he had no downstream constraint, an input wave was allowed to dampen out to its steady state value sufficiently far downstream. His problem statement was consistent with the hydraulics of a flood disturbance generating wave fronts upstream and downstream ($W_2$ and $W_1$ waves respectively). He integrated the $W_1$ wave in the positive downstream direction and the $W_2$ wave in the negative upstream direction beginning with his input at his original axis as in Fig. 4.1. Hence, his $W_1$ and $W_2$ waves were also independent of each other which was not the case in the present study.

Because of the interdependence of the equations in this study, both the variables $W_1$ and $W_2$ were needed simultaneously to satisfy the downstream condition; this is a typical boundary value problem as opposed to Vichnevetsky's initial condition problem. Because both variables had to be solved simultaneously, a more elaborate integrating subroutine had to be devised. Instead of solving for each variable separately in a serial fashion, both variables had to be updated simultaneously. This called for a matrix solution method or a trial and error procedure. The latter method was used because of its convenient implementation in the present study. A typical slow drop case was used, for comparison purposes, in modelling all the problem features correctly. The range or the envelope of the phreatic lines for one reference slow drop case is given in Fig. 4.2.

Experimental findings [10, 35] indicated that the water wave continued to enter the breakwater even after the maximum run-up has been reached, i.e., the water wave velocity changes direction some time after the crest has been reached. This can be seen in Fig. 4.3 which shows the
Fig. 4.1 — Definition of Vichnevetsky's Flood Routing Problem.
Fig. 4.2 — Typical Envelope of Phreatic Lines for a Slow Drop Case.
Fig. 4.3 — Typical Velocity Variation at Outcrop Section for a Slow Drop Case.
variation of the sign and magnitude of the velocity at the outcrop section with respect to the input wave profile.

The definition of the velocity, \( u = (W_1 + W_2)/2 \), suggests that the \( W_1 \) and \( W_2 \) terms reach the same magnitude at the core where the resultant velocity is zero. Fig. 4.4 serves to illustrate how the \( W_1 \) and \( W_2 \) waves of representative time exposures A and B must have equal magnitude but opposite signs, i.e., \( W_1 = -W_2 \).

As noted previously, Vichnevetsky integrated his waves in their respective forward directions. When this same procedure was used in the present study, it was found that the velocity at the outcrop had to be zero to satisfy the downstream conditions. This was not acceptable. However, upon integrating the equations in the same positive downstream direction, realistic velocity distributions but unrealistic \( W_2 \) profiles resulted.

Based on the previous discussion, a qualitative description of the phenomenon can be formulated. The solution must satisfy the following conditions: (1) the velocity within the rockfill must be less than the maximum fall rate, \( V \); (2) the horizontal velocity must change direction at a time delay after the maximum run-up or minimum rush down; (3) the wave must necessarily dampen towards the core, and (4) zero velocity at the core must be maintained.

4.2 Time Discretization

Explicit numerical techniques for solving PDE are generally hampered by instability problems. Nasser [35] adopted the following stability criterion for his finite difference formulation, viz.
Fig. 4.4 - Typical $W_1$ and $W_2$ Profiles at Different Time Frames.
\[ \frac{\Delta t}{\Delta x} < \frac{1}{2} \left( \frac{u}{m} + c \right)^{-1} \]  

\[ \text{(4.1)} \]

where \( \Delta t \) and \( \Delta x \) are the respective time and spatial increments. This criterion was not applicable in this study.

The numerical integration necessitated small spatial increments but time increments were determined separately for the outcrop point movement. It was difficult to determine an upper limit on the size of \( \Delta t \). The value of \( \Delta t \) used, based on experimentation, ranged between .022 sec. and .114 sec., depending on the value of the wave period, the average \( \Delta t \) used was in the neighborhood of .05 sec. The value of \( \Delta t \) was kept relatively constant for any solution case to minimize velocity fluctuations. With finite difference techniques, the solution should continue for a few wave periods to ensure that the non-linearities are properly accounted for. The nonlinearities associated with the equations and downstream conditions were determined, in this study, by direct integration and iteration, respectively. The time effect was more difficult to analyze.

Originally, it was felt that a backward differencing scheme would logically account for the \( \partial W / \partial t \) term in the governing equation:

\[ \frac{\partial W}{\partial x} = - \frac{1}{\lambda} \left( \frac{\partial W}{\partial t} - gmfA \right) \]

\[ \text{(4.2)} \]

The main difficulty was associated with the proper differencing of the term:

\[ \frac{\partial W}{\partial t} = \frac{\Delta W}{\Delta t} \bigg|_{j} = \frac{W_{j}^{j} - W_{j}^{j-1}}{\Delta t} \]

\[ \text{(4.3)} \]

where \( j \) denotes the time period under study.

Since the first term, i.e., \( W_{j}^{j} \), would be the continuous variable, the problem shifted to the definition of the previous value, \( W_{j}^{j-1} \). This
term was investigated in numerous ways. Originally, \( W_{j-1} \) was taken as the previous initial value of \( W_{j} \) for a working solution. This yielded unreasonable answers; only the criteria of zero velocity at the core and reasonable velocities in the solution were realized.

The next step was to store the complete previous profile to better simulate the continuous nature of the problem. This did not improve the solution but did increase the memory storage requirements and time consumption.

It was then reasoned that since all systems when left alone, seek the stable state, then successive profiles should not be compared with each other but rather with the mean water level. This resulted in more reasonable profiles but there was no time lag of the change in the direction of the horizontal velocity. A sample output showing typical results for a half wave period using this method is given in Fig. 4.5.

The final solution utilized a term that needs special explanation. It is difficult to determine the point at which the horizontal velocity changes direction but it is known that at the time constant \( t_1 \) (see Appendix A), the water begins to leave the rockfill embankment.

It is assumed that this is the point at which the horizontal velocity will change. Similarly, the time constant \( t_C \) defines the point at which the external wave meets the outcrop point and both rise together. Thus, this is the point at which water will re-enter the rockfill, provided that \( t_C \) is greater than three quarters of the wave period.

Fig. 4.6 represents the region in which the water is leaving the rockfill at its maximum vertical velocity (not necessarily at a maximum horizontal velocity).
Fig. 4.5 — Sample Phreatic Profiles for a Half Wave Period with $W_{j-1} = h_0$. 

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- Theoretical Results
- Experimental Envelope
- Mean Water Level
Fig. 4.6 — Definition of the Region of Change in Horizontal Velocity.
Based on the assumption of change of velocity direction at times \( t_1 \) and \( t_C \), \( W_1 \) and \( W_2 \) can be determined, viz.

\[
W_1(t_1) = -W_2(t_1) = 2m \sqrt{gh(t_1)} \quad \ldots \ldots \ldots \quad (4.4)
\]

\[
W_1(t_C) = -W_2(t_C) = 2m \sqrt{gh(t_C)} \quad \ldots \ldots \ldots \quad (4.5)
\]

The initial conditions are also known, i.e.,

\[
W_1(0) = W_2(0) = 2m \sqrt{gh_0} \quad \ldots \ldots \ldots \quad (4.6)
\]

An assumption is made of a constant \( W_1^{j-1} \) along the length of the embankment at each time increment since its variation with length cannot be known in advance. According to the preliminary investigations attempting the storage of complete wave profiles, the difference in result did not seem to indicate any significant change. To simulate a change in the variation between the known values of \( W_1 \) and \( W_2 \), linear interpolations of the \( W_1^{j-1} \) term were used. These modifications were introduced into the hybrid model. The following three conditions pertain to the time discretization:

\[
W_1^{j-1} = (W_1(0) \cdot (t_1 - t) + W_1(t_1) \cdot t_1)/t_1 \quad \text{for} \ t < t_1 \quad \ldots \ldots \quad (4.7)
\]

\[
W_1^{j-1} = (W_1(t_1) \cdot (t_C - t) + W_1(t_C) \cdot (t - t_1))/(t_C - t_1) \quad \text{for} \ t_1 < t < t_C \quad \ldots \ldots \quad (4.8)
\]

\[
W_1^{j-1} = (W_1(t_C) \cdot (t_1 + T - t) + W_1(t_1) \cdot (t - t_C))/(t_1 + T - t_C) \quad \text{for} \ t_C < t < t_1 + T \quad \ldots \ldots \quad (4.9)
\]

A curve representing the variation of \( W_1^{j-1} \) for one wave length is given in Fig. 4.7. The value of \( t_C \) should not be below \( 3/4 \) \( T \).
Fig. 4.7 — Definition of the Variation of $W^{j-1}$ for One Complete Wave Period.
4.3 Analog/Hybrid Program

The hybrid program differs from the numerical program in that the analog circuit replaces the integration routine and special library subroutines must be called to activate the EAI 693 Interface Unit. References 12, 13 and 14 give all the explanations of the use of the hybrid FORTRAN software used.

The digital program reads in all the relevant parameters encountered in the program, makes any necessary calculations or checks, provides a means of printing out digital answers, and controls the operation of the analog computer. The analog computer, besides integrating the characteristic equations simultaneously, provides the means for obtaining the continuous plot of any of its variables for a given time increment.

4.3.1 Analog Circuitry

Fig. 4.8 shows the pot and amplifier components comprising the analog circuit used in this study. The circuit can be subdivided into five sections as illustrated in Fig. 4.9. Box B contains the A/D and D/A signals required; boxes A and E contain the circuits for \( W_1 \) and \( W_2 \) respectively; box D shows the components shared commonly in the characteristic equations by A and E; and box C illustrates some components required for plotting. Select variables are included at the output of certain amplifiers to acquaint the reader with the functions performed.

4.3.2 Analog Scaling

At the outset, amplitude scaling posed a problem due to the large values of \( W_1 \) and \( W_2 \) and their small summations. The resistance term also presented a challenge. In this study, the values of \( W_1, 2 \) never
greatly exceeded the value of the celerity which was directly related to the depth of flow. It was decided that a scale factor of 1/300 would suffice in dealing with the most extreme cases. This scale was thus introduced throughout the solution. Since the output at amplifier A05 was always small, its gains were increased to 10. Potentiometer P12 was then multiplied by 30 to yield a scaled value of $a + b |u|$ at the output of A11. Potentiometer P06 had to be divided by 500 to obtain a pot coefficient less than 1.0. Thus, the scaling at the output of P06 was 1/15,000. This necessitated P01 to be divided by 50 to yield a similar scaling. P00 had a value of 1/6 to maintain a consistent scale factor through integrator A00. Fig. 4.10 displays the scaling fractions at the appropriate locations in the circuit.

The problem did not need to be time scaled. It was decided to use a slow integration time to better analyze the problem and obtain adequate plots. The time of integration can be chosen according to the needs of the problem.

The output of the analog computer could be measured continuously by a strip chart recorder, or occasionally by an XY recorder. The output would have to be properly scaled to give accurate results. There are two sets of scaling in determining the output of a particular component: the scalings introduced by amplitude scaling and those introduced by the plotting machine itself. For instance, amplifier A38 yields a scaled fraction of the term $x$, the depth to be solved for. The circuit currently takes an average of the difference of $W_L$ and $W_{25}$. Thus, a reading off the digital voltmeter of A38 would have to be interpreted as:
Fig. 4.10 — Amplitude Scaling in the Analog Circuit
\[ y = \frac{c^2}{g} = \frac{1}{980.6} \left[ \text{output of A38} \right] \frac{1}{16} m^2 \cdot 4 \cdot (300)^2 \]

\[ = \frac{22.945}{m} \left[ \text{output of A38} \right] \] \hspace{1cm} (4.15)

The individual graph would have to be scaled to the sensitivity and speed of the chart recorder or pen on the XY recorder. The whole aspect of determining the scaling factors and interpreting output causes a great inconvenience. This also introduces greater chance for errors to be introduced into the solution.

The pot coefficients for this hybrid program are automatically determined in Subroutine Pots included in the computer listings of Appendix D.

4.4 Results

The analysis of the outcrop point movement, covered in greater detail in Appendices A and B, provided excellent results. The success of the analog solution in treating the full range of fast drop cases studied proved the worth of the method as a viable alternative to other more popular techniques. Comparison of the results indicate consistently excellent agreement among the various methods applied.

The analog solution nevertheless had an advantage over the digital solution in that the contributions of the various parameters of the damping equation (A.12) could be analyzed. In particular, an investigation carried out determined the roles of each of the two resistance coefficients \( a_1 \) and \( b_1 \). This was easily done by disconnecting the outputs of pots P03 and P02 representing \( a_1 \) and \( b_1 \) respectively (see Fig. B.1).Disconnecting
pot P03 had no apparent effect while disconnecting P02 completely altered the viscous damping curve. (A typical case is depicted in Fig. 4.11). This indicates that the non-linear damping term $b$ is more prominent in the solution and therefore the interior flow is truly non-Darcy. This behaviour could be determined from the digital solution but not in the same dramatic fashion. This is attributed to the fact that analog computers, once properly scaled, would be more flexible in studying changes in the parameters. The analog solution was more interactive and better simulated the physical nature of the outcrop point movement.

With the encouraging results of the outcrop point movement, a limited attempt was made to proceed to show how the hybrid analysis can be extended to simulate the interior wave motion. A digital formulation was then developed to furnish a basis for the hybrid simulation (see Appendix E). It is noted that the digital program in its present form can only treat embankments of short lengths.

The results of two typical cases are presented. Fig. 4.12 gives pertinent information about case H-2 (see Appendix E for details on the organization of the data), and Fig. 4.13 shows a plot of its Outcrop Point Movement. Fig. 4.14 illustrates the family of phreatic profiles obtained from one wave period. Similarly, Figs. 4.15 through 4.17 represent results for case K-2.

The general nature of the phreatic profiles confirms the trends observed by Nasser [35]. Fig. 4.18 shows the variation of velocity at the outcrop section for one period. Sample output of the wave profile is given in Table 4.1. It can be seen that the velocity never exceeds the maximum fall rate.
Fig. 4.1 — Influence of Coefficients $a_1$ and $b_1$ on the Outcrop Point Movement.
### CASE H-2

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<tr>
<td>A = 0.0170</td>
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<td>B = 0.0330</td>
<td>B(_{\text{PRIME}}) = 7.6433</td>
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<td>TSR = 1.0151</td>
</tr>
<tr>
<td>POROSITY = 0.4660</td>
<td>TSR(_{1}) = 1.6149</td>
</tr>
<tr>
<td>GRAVITY = 980.6000</td>
<td>HYD. COND. (K) = 5.2533</td>
</tr>
<tr>
<td>SLOPE (RAD) = 1.5708</td>
<td>FREQUENCY = 2.3890</td>
</tr>
<tr>
<td>AMPLITUDE (AO) = 6.0000</td>
<td>TANGENT PT. = 41.9407</td>
</tr>
<tr>
<td>DEPTH (H) = 38.0000</td>
<td>MAX. DEVIATION = 1.3974</td>
</tr>
</tbody>
</table>

\[ Y(0) = 35.4567 \quad VEL(0) = -10.8092 \quad ACCEL(0) = 0.0168 \]

\[ W = 0.7541 \quad DEL = 701.7189 \quad PK (\%) = 11.6452 \]

---

### SECOND ORDER DIFFERENTIAL DAMPING

FOR \( H = 0.00800 \) AND \( N = 7 \)

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<tr>
<th>TIME</th>
<th>NUM</th>
<th>ANAL. APPROX</th>
<th>YDOT</th>
<th>ANAL. APPROX</th>
<th>% DIFF</th>
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Fig. 4.72 Perinent Data for Case H-2
Fig. 4.13 — Outcrop Point Curve for Case H-2.

CASE H-2
M = 0.4850
a = 0.0170
θ = 0.0330
THETA = 90.0000 (DEGREES)
Fig. 4.14 — Envelope of Phreatic Profiles for Case H-2.
CASE K-2

GIVEN

\[ A = 0.0100 \]
\[ B = 0.0210 \]
\[ PERIOD = 2.6200 \]
\[ POROSITY = 0.3770 \]
\[ GRAVITY = 980.6000 \]
\[ SLOPE (RAD) = 1.5708 \]
\[ AMPLITUDE (A0) = 13.0000 \]
\[ DEPTH (H) = 30.0000 \]

CALCULATED

\[ \text{APRIME} = 3.6969 \]
\[ \text{BPRIME} = 3.9268 \]
\[ \text{TSR} = 0.9865 \]
\[ \text{TSR1} = 2.7135 \]
\[ \text{HYD. COND. (K)} = 6.6667 \]
\[ \text{FREQUENCY} = 2.3982 \]
\[ \text{TANGENT PT.} = 40.7065 \]
\[ \text{MAX. DEVIATION} = 7.1420 \]

\[ Y(0) = 26.4356 \]
\[ \text{VEL}(0) = -17.6835 \]
\[ \text{ACCEL}(0) = 0.054 \]

\[ W = -5672 \]
\[ \text{DELT} = 137.3000 \]
\[ \text{PK (%) = 27.4693} \]

-----------------------------------------------------------------------------------

\[ \text{POT SETTING} \]
\[ \text{KD} = 1/(45.) \]
\[ \text{K1} = 1/(25.) \]
\[ \text{RETA} = 10. \]

-----------------------------------------------------------------------------------

SECOND ORDER DIFFERENTIAL DAMPING

\[ \text{FOR H = 0.0900 AND N = 6} \]

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Fig. 4.15 Pertinent Data for Case K-2
Fig. 4.16 — Outcrop Point Curve for Case K-2.
Fig. 4.17 — Envelope of Phreatic Profiles for Case K-2.
Fig. 4.18 — Velocity Variation at Outcrop Section for Case K-2.
Table 4.1  Sample Output of Wave Profile (Case K-2)

### STEP NO. 9 REQUIRED 10 ITERATIONS

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<th>(C)</th>
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<th>(W_2)</th>
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<th>(W_2)</th>
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The validity of the digital formulation was tested by comparing the points on the maximum and minimum phreatic envelopes with those determined experimentally by Nasser [35]. It can be seen from Table 4.2 and Fig. 4.14 that there is excellent agreement for the maximum envelope whereas the values of the minimum envelope are consistently lower than the experimental results.

Tables 4.3 and Fig. 4.17 reflect the same discrepancies for case K-2. A closer agreement with the experimental results can be obtained if an asymmetric wave form is used to describe the impact wave.

It is believed that the digital formulation can be best improved to treat wider embankments by resorting to a non-dimensionalized form of the governing equations. Further, non-dimensionlization of the equations would lead to a more versatile program. A possible non-dimensional form is given in the next section.

It was not possible to test the hybrid program for interior wave profiles because of hardware malfunction.

4.5 The Proposed Non-Dimensional Form

The original differential equation can be written as:

\[ \frac{dW_{1,2}}{dx} = \frac{m}{\sqrt{\phi W_{1,2} + \frac{A}{2}}} - \frac{\Delta W_{1,2}}{A \Delta b} \left[ \frac{b}{\sqrt{a + b \left( \frac{W_{1,2} + W_{2}}{2} \right) \left( \frac{W_{1,2} + W_{2}}{2} \right)}} \right] \]

The variables, \( W_1 \) and \( W_2 \), have the units of a velocity. Thus, the best term to serve as a reference would be the celerity corresponding to the mean water level, i.e.
Table 4.2: Comparison of Results for Case H-2

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<tr>
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<td>Maximum (cm)</td>
<td>Minimum (cm)</td>
<td>Maximum (cm)</td>
</tr>
<tr>
<td>0.0</td>
<td>44.00</td>
<td>32.06</td>
<td>44.00</td>
</tr>
<tr>
<td>2.5</td>
<td>43.07</td>
<td>32.06</td>
<td>43.30</td>
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<tr>
<td>4.5</td>
<td>42.70</td>
<td>32.06</td>
<td>43.00</td>
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<tr>
<td>7.0</td>
<td>42.66</td>
<td>32.06</td>
<td>43.00</td>
</tr>
</tbody>
</table>

Table 4.3: Comparison of Results for Case K-2

<table>
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<tr>
<th>Distance (cm)</th>
<th>Theoretical Depth</th>
<th></th>
<th>Experimental Depth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Maximum (cm)</td>
<td>Minimum (cm)</td>
<td>Maximum (cm)</td>
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<tr>
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<td>43.00</td>
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<tr>
<td>2.5</td>
<td>47.20</td>
<td>19.90</td>
<td>43.00</td>
</tr>
<tr>
<td>8.5</td>
<td>40.66</td>
<td>19.99</td>
<td>40.80</td>
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<td>16.5</td>
<td>40.53</td>
<td>19.99</td>
<td>40.70</td>
</tr>
</tbody>
</table>
\[ W_{1*} = \frac{W_{1*}}{g h_0} \] ... (4.11)

The spatial and time variables are best described in relation to the length of embankment, \( L \), and wave period, \( T \), respectively. Therefore,

\[ x^* = \frac{x}{L} \] ... (4.12)

\[ t^* = \frac{\Delta t}{T} \] ... (4.13)

The two remaining terms are \( g m a \) and \( g m b \), with units of \((\text{time})^{-1}\) and \((\text{length})^{-1}\) respectively, suggesting the last two dimensionless groups to be \( g m a T \) and \( g m b L \). Thus, the dimensionless governing characteristic equation for \( W_1 \) is:

\[
\frac{dW_1^*}{dx^*} = \frac{-m}{75 W_1^*} \frac{\Delta W_1^*}{\Delta t^*} + \left( g m a T + g m b L \right) \frac{1}{2} \left( \frac{W_1^* + W_2^*}{2} \right) \left( \frac{W_1^* + W_2^*}{2} \right)
\] ... (4.14)

A similar expression for \( W_2 \) can be obtained by interchanging \( W_1^* \) and \( W_2^* \).

4.6 GENERAL REMARKS

Chapters 1 and 2 summarize the reasons for attempting the method of solution used in this study, but this section will concentrate on specific frustrations civil engineers might encounter in pursuing analog/hybrid simulation for the analysis of their problems. Some of the setbacks are localized while others are of a universal nature.

A disadvantage of the hybrid computer is that the accuracy of the solution is governed by the least accurate electronic component.
Further, a malfunction in one component can affect the entire solution. An apparent error in the time delay subroutine call from the digital computer prevented the reporting of any specific analog/hybrid results for the simulation of the interior wave. The hyperbolic nature of the equations plus the interactive procedure required prevented the manual implementation of the program on the analog computer only. This reinforced the need for hybrid simulation for the present study.

Literature dealing with analog/hybrid simulation has made few inroads into making engineers aware of its potential. Most of the books written date back to the late 50's and early 60's, suggesting no substantial growth of the hybrid simulation field. Most hybrid techniques have only been developed within the last decade so that good references are hard to come by.

At the same time, there are few people who are actually using hybrid computers to simulate PDEs and therefore there is little or no exchange of ideas or feedback among the users. Further, equipment is seldom updated and hence technological advances in hardware have very little impact on the market.

Civil engineers would have to become more familiarized with basic electronics just to develop a finer sense of the limitations and the sensitivity of the analog component hardware.

A crucial feature of analog simulation is the scaling of the circuit to ensure that no components overload. This often requires a digital solution to determine the range of variables and hence the scale factors. The circuit must then be scaled at every component. This becomes difficult as the complexity of the differential equation increases.
Since the circuits have to be physically patched on a patch panel, the program has to be checked before using. This is done by a static check test. This necessitates storing analog circuits which presents an inconvenience as far as other users are concerned.

The facilities at Concordia have rather unsophisticated methods of loading the digital programs required for the hybrid simulation. Creating or editing files are not conveniently done.

There are many questions as to the future of hybrid computation. Rubin et al [45] present a very optimistic view; they mention the development of a hybrid programming language called SIMTRAN whose objective is to lower the fixed cost of program preparation, debugging and checking. This would considerably lower the break-even point in relation to digital computers (see Fig. 4.19). SIMTRAN would enable the hybrid user to enter his source program in a remote batch terminal and have all programming and check calculations done automatically.

Rubin et al foresee the features of next generation of hybrid computers to include automatic patching, high language programmability and local and remote terminals. This would decrease the variable cost of simulation to produce even more efficient computers. Far down the road, they expect hybrid computers to contain smaller circuitry, have lower costs, more integrators and faster speeds (Fig. 4.20).
Fig. 4.19 — Break-Even Diagram Between Hybrid and Digital Computers.

Fig. 4.20 — Cost Comparisons Between Hybrid and Digital Computers.
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

Based on this study, the following conclusions are drawn:

1. A combination of a Runge-Kutta and Adams-Moulton predictor/corrector method is successfully implemented in the numerical simulation of the phreatic profiles. It is, nevertheless, not a replacement for the more economic and efficient hybrid program.

2. An analog/hybrid simulation of the analysis of wave motion in rockfill embankments is a viable alternative to the Finite Element and Finite Difference Methods. The adjustable integration rate associated with analog simulation represents a definite advantage over numerical integration.

3. The various methods used to calculate the Outcrop Point Movement are successful. The solutions obtained can automatically be plotted on a strip chart recorder, an XY plotter, a Tektronix graphics terminal or Decuwriter terminal.
4. Experimentation with trial cases confirm Nasser's [35] observations of increasing wave transmission to the impervious core with increasing conductivity and decreasing wave steepness and embankment width.

5. The findings of this study indicate that the maximum water level at the core falls significantly below the maximum run-up level at the outcrop section. This could lead to a more economical design than the current practice.

5.2 RECOMMENDATIONS

Possible extensions of this work are summarized in the following recommendations:

1. The analog/hybrid technique used ought to be refined to improve the simulation. A possible approach is to resort to a backward differencing of second order accuracy. It is also worthwhile to investigate other hybrid techniques such as the hybrid method of characteristics or the scheme devised by André [2].

2. There is a definite need for a review acquainting Civil Engineers with the advantages of available hybrid techniques. Such a review should focus on the classification of the techniques in terms of the minimum size of the hybrid system required, the type of hardware needed, the expected time of computation and the simplicity of implementation.

3. The analysis of the Outcrop Point Movement should be adopted to simulate the exit conditions of a continuous breakwater. The introduction
of the vertical velocity at the outcrop section in the hybrid simulation would lead to a definite improvement.

For the class of nonlinear waves considered in this study, a better approximation of the seepage face could be achieved if the analysis of the outcrop point movement is based on an asymmetric external fluctuation of the input wave. This would also improve the simulation of the interior wave.

4. A challenging undertaking would be an attempt to apply an analog technique to layered dams with various geometries and porosities.

5. For a purely digital solution, it would be worthwhile to consider a Burlisch-Stoer integration method. It features from 6\textsuperscript{th} to 12\textsuperscript{th} order accuracy, variable step size, three possible error criteria, and is very stable for highly nonlinear problems.
APPENDIX A

OUTCROP POINT MOVEMENT
APPENDIX A
OUTCROP POINT MOVEMENT

The wave induced unsteady non-Darcy flow under consideration in this study requires the outcrop point movement relative to the external wave as a boundary condition (Fig. A.1).

When a water wave attacks a porous structure, the wave reaches a maximum run-up position on the face of the structure and recedes to a minimum rush-down level; these maximum and minimum levels constitute the impact wave height. Experiments done by Nasser [35] show that a sine wave can be used to represent the impact water wave. Assuming a sine function to describe the external water level fluctuation, then:

\[ Y_f = h_0 + A_0 \sin(\omega t) \quad \text{(a.1)} \]

where \( Y_f \) = elevation of the free water level at the interface with respect to the channel bottom

\( h_0 \) = mean water level

\( A_0 \) = amplitude of the water wave

\( \omega \) = frequency of oscillation

\( t \) = time

Dracos [10] states that "as long as water flows into a porous body, the phreatic line and the free water level meet at the same point"
on the boundary. When, however, the water exits from the porous body, a seepage surface may be interposed between the free water level and the phreatic line: a seepage surface will not occur unless the internal fall rate is less than the fall rate of the external water level. The vertical velocity of fluctuation, \( V_f \), is the derivative of equation (a.1) or:

\[
V_f = A_0 \cos(\omega t) \quad \quad \quad \quad \quad \text{(a.2)}
\]

And hence the maximum fall rate, \( V_{f_{\text{max}}} \), is

\[
V_{f_{\text{max}}} = \omega A_0 \quad \quad \quad \quad \quad \text{(a.3)}
\]

The maximum internal fall velocity, \( V \), is given by Dracos as:

\[
V = \frac{k}{m} \sin^2 \theta \quad \quad \quad \quad \quad \text{(a.4)}
\]

where \( m \) = porosity
\( k \) = non-Darcy hydraulic conductivity
\( \theta \) = angle of inclination of the structure face with the horizontal
\( k = 1/(a + bm|V|) \quad \quad \quad \quad \quad \text{(a.5)}\)

The value of \( V \) can be calculated directly from (a.4) and (a.5) to be:

\[
V = \frac{1}{2} \left[ \frac{-a}{b m} + \left( \frac{a}{b m} \right)^2 + \frac{4 \sin^2 \theta}{m^2 b} \right] \quad \quad \quad \quad \quad \text{(a.6)}
\]

The drop rate of the free water level can be classified as 'slow' if:

\[
V_f < V = \frac{-k}{m} \sin^2 \theta \quad \quad \quad \quad \quad \text{(a.7a)}
\]

or 'fast' if:
V_f > V = -k \frac{\sin^2 \theta}{m} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (a.7b)

A slow drop case implies that the outcrop point movement coincides with the free water level.

Comparing equations (a.7) and (a.2) and introducing the dimensionless parameter

\[ W = \frac{k}{\omega A_0 m} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (a.8) \]

yields:

\[ \cos(\omega t)/W < \sin^2 \theta \quad \text{for slow drop} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (a.9a) \]

\[ \cos(\omega t)/W > \sin^2 \theta \quad \text{for fast drop} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (a.9b) \]

Equating the terms yields the time, \( t_1 \), during which internal and external movements coincide:

\[ t_1 = \frac{1}{\omega} \cos^{-1}(-W \sin^2 \theta) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (a.10) \]

This now provides some information in determining the outcrop point movement (see Fig. A.2).

1. For \( W \sin^2 \theta > 1 \), \( t_1 \) cannot exist or else it is equal to \( T/2 \), where \( T \) is the wave period. This condition defines a slow drop case.

2. For \( W \sin^2 \theta < 1 \), a number of cases arise which must be studied separately.

A. For \( t < t_1 \), the water is entering the porous media and therefore the outcrop point movement is identical to the movement of the free water level or region I in Fig. A.2.
B. For \( t_1 < t < t_2 \), the outcrop point falls at the maximum internal velocity \( V \) as represented by the slope at time \( t_2 \) in region II. Point \( t_1 \) (or \( T - t_2 \)) is the time at which there is a maximum deviation, \( \Delta Y_p \), between the outcrop point and the free water level. The equation representing this region is:

\[
Y = h_0 + A_0 \sin(\omega t_1) + V(t-t_1)
\]  
(a.11)

C. Region III for \( t_1 < t < t_2 \)

At \( t = t_2 \), the outcrop point will have reached its maximum deviation, \( \Delta Y_p \), from the free water level curve and hence the two curves will start to converge. The non-linear viscous damping effect is described by a second order differential equation:

\[
\ddot{Y}_c + (a_1 + b_1|\dot{Y}_c|)\dot{Y}_c + \delta Y_c = \delta f(t)
\]  
(a.12)

where

\[
a_1 = gma
\]

\[
b_1 = gm^2b
\]

\[
\delta = g \sin^2\theta / \Delta Y_p
\]

\[
\Delta Y_p = 2 A_0 \sin(\omega t_1) - V(t_2-t_1)
\]

\[
f(t) = h_0 + A_0 \sin(\omega t)
\]

\((\cdot)\) refers to the time derivative

The required initial conditions for this equation are:

\[
\dot{Y}_c(t_2) = V = \frac{k}{m} \sin^2\theta
\]  
(a.13)

\[
Y_c(t_2) = h_0 + A_0 \sin(\omega t_2) + V(t_2-t_1)
\]  
(a.14)
This region is the most difficult to analyze. Five solution methods were investigated and will be covered in depth.

D. Region IV is a repetition of Region I where the water is once again entering the porous media and hence the outcrop point rises with the free water level.

SOLUTION OF VISCOUS NON-LINEAR DAMPING EQUATION (REGION III)

Three parameters were used to analyze and describe the nature of the fast drop cases particularly in the region of non-linear viscous damping. These parameters are:

1. \( \omega \), from equation (a.8), is a dimensionless parameter that varies between 0 (zero) and \( 1/\sin^2 \theta \) for fast drop cases. The lower the value of \( \omega \), the greater the damping. As \( \omega \) increases, the case approaches the slow drop state.

2. \( \delta \), given in equation (a.9) and possessing the units \( \text{sec}^{-2} \), is of prime importance in the analog solution because it is a potent setting. In the cases studied, its value varied between 4 and 111,000. Low values of \( \delta \) represent a highly damped system whereas high values, i.e., \( \delta > 800 \), resemble slow drop cases.

3. A practical, dimensionless percentage was developed to indicate quantitatively how much the fast drop case differed from the slow drop case. This number is given as:

\[
P_k = \frac{\Delta V_p}{2A_0} \times 100
\]  

(a.15)
Pk varied between 0 (zero) and 85% in the cases studied. A Pk value of zero is identical to the slow drop case. High values indicated substantial deviations from the slow drop case and therefore represented highly damped cases.

Five methods were used to get solutions in region III: MIMIC, analytic, numerical, approximation and analog. Each of these methods provided a solution for the region of viscous damping.

1. MIMIC

MIMIC is a parallel simulation language that numerically solves differential equations. Its advantage lies in the ease of programming and convenient plots of the solution. A major disadvantage of the Mimic package available at Concordia University is the inaccuracy and unreliability of the solutions. Large errors were introduced into the solution when the MIMIC program consistently determined an initial acceleration of 500 cm/sec² at time t₂ instead of a zero acceleration. For low values of δ, the general form of the solution curve was obtained but at high δ values, the solution broke down. With the existing software, it was not possible to obtain one continuous curve for the four regions described.

2. Numerical

Since the MIMIC simulation package was inaccurate, a Runge-Kutta fourth order method was used. An adequately small step was required (Δh < .01) to obtain any solution. A program was developed to piece together the total outcrop point curve and plot the results. The results were very good.
3. **Analytic Solution**

Dracos gave an analytic solution in terms of an external cosine water wave but because of an error in choosing a proper dimensionless number \( \chi \), his solution was wrong. The \( \chi \) term chosen for this study yields results in close agreement with the numerical solution but exhibits large errors for cases with low \( \delta \) values in sloping rockfaces. The general computer program developed gives a comparison of the analytic and numerical solutions.

The solution subject to the given initial conditions of the differential equations in region III is:

\[
Y = \frac{D-C \lambda_2}{\lambda_1-\lambda_2} e^{\lambda_1(t-t_2)} - \frac{D-C \lambda_1}{\lambda_1-\lambda_2} e^{\lambda_2(t-t_2)} + h_0 \\
+ A \sin \omega t - B \cos \omega t 
\]  

(a.16)

where

\[
\lambda_1 = -\frac{gm}{2K} \left(1 - \sqrt{\chi} \right) 
\]  

(a.17)

\[
\lambda_2 = -\frac{gm}{2K} \left(1 + \sqrt{\chi} \right) 
\]  

(a.18)

\[
\chi = 1 - \frac{4K}{gm} \sin^2 \theta \frac{\Delta Y}{H_0} 
\]  

(a.19)

\[
TKG = \frac{g \mu \omega}{K} 
\]  

(a.20)

\[
\omega = \frac{2\pi}{T} 
\]  

(a.21)

\[
\delta = \frac{g \sin^2 \theta}{\Delta Y} p 
\]  

(a.22)
\[ A = A_0 \frac{\delta(\delta - \omega^2)}{TKG^2 + (\delta - \omega^2)^2} \quad \quad \quad \quad \quad \quad \quad (a.23) \]

\[ B = A \frac{TKG}{(\delta - \omega^2)} \quad \quad \quad \quad \quad \quad \quad (a.24) \]

\[ C = (A_0 + A)\sin \omega t_1 + B\cos \omega t_1 + V(t_2 - t_1) \quad \quad \quad \quad \quad \quad \quad (a.25) \]

\[ D = -\omega A\cos \omega t_1 + \omega B\sin \omega t_1 + V \quad \quad \quad \quad \quad \quad \quad (a.26) \]

\( \chi \) is only a little smaller than unity so that \( \lambda_1 \) becomes small and \( \lambda_2 \) becomes a large negative number. The large negative number \( \lambda_2 \) causes the second term of the solution to tend to zero very fast and it can safely be disregarded. Therefore, the outcrop point movement in the viscous damping region is given by the modified equation:

\[ Y = \frac{D-C}{\lambda_1 - \lambda_2} e^{\lambda_1(t-t_1)} + A\sin \omega t_1 - B\cos \omega t_1 \quad \quad \quad \quad \quad (a.27) \]

When this function equals the external sine function \( (Y_C = h_0 + A_0 \sin \omega t) \), then the two curves rise again as the water will be entering the porous body from the reservoir.

4. Approximate Solution

Nasser [35] found that a drop rate of about one half of the maximum fall velocity, \( V \), would give very close values to those obtained by his finite difference solution of the damped equation.

In this study, 147 fast drop cases were analyzed to compare the actual drop rate to the maximum velocity. Since the numerical and analog results were in close agreement, the approximate slopes were calculated
from Fig. A.3 as:

\[
\text{linear drop rate} = \frac{y_2 - y_3}{t_2 - t_3} \quad \ldots \quad (a.28)
\]

This was then compared to the maximum drop rate to determine the ratio needed:

\[
\text{Ratio} = \frac{y_2 - y_3}{t_2 - t_3} \times \frac{t_1 - t_2}{y_1 - y_2} \quad \ldots \quad (a.29)
\]

Based on the 147 cases studied, the following information was revealed:

mean \quad = \quad 0.7567
std dev. \quad = \quad 2.60
highest ratio \quad = \quad .836
lowest ratio \quad = \quad .676

Therefore, an approximate solution would be to use a drop rate of 75 per cent of the maximum fall velocity rather than the half value reported by Nasser.

5. Analog Solution

The analog solution most truly represents the system in program preparation and during computer operation. Essential to the proper calculation in this simulation was the digital logic, amplitude scaling, time scaling and the proper system diagram.

The versatility of the analog computer, once properly programmed (the programming procedures are dealt with in detail in Appendix B), lies in the ease of studying the whole system or parts of it separately. A preliminary study was made to measure the effect of the linear Darcy coeffi-
Fig. A.3. Illustration of the Approximate Solution
cient, \( a \), in equation (a.12). In most cases \( a \) was of very little significance in the solution whereas the non-Darcy coefficient, \( b \), was indispensable. This supports the assumption that the flow is turbulent in the fast drop cases (i.e. \( i = aq + bq^2 \)). The more weight that is put on coefficient \( b \), the more turbulent the flow will become.

Because of its parallel nature, the analog computer produces continuous solutions which can be monitored on a strip chart recorder, oscilloscope or on an X-Y plotter. X-Y plots proved to be very helpful since the outcrop point curve could be superimposed on the sine wave which provided an excellent basis for comparison. These plots also monitored the difference in solution with or without certain variables such as \( a \) or \( b \). These plots were only used for final comparisons and results. Intermediate results were obtained on the strip chart recorder.

After proper scaling, a vast range of cases were studied, from sloping dams with low \( \delta \) values of 4 to vertical breakwaters with values of \( \delta \) as high as 111,000. Figures A.4 to A.11 show the difference between the outcrop point movement and the input sine wave for varying values of \( \delta \). High values of \( \delta \) approach the slow drop case. The author believes that since deviations of 5% are not uncommon in the field, simulating fast drop cases with very high values of \( \delta \) might be very impractical. It is suggested that a cut-off factor be introduced so that any transition drop cases (fast drop cases sufficiently close to slow drop cases) can be treated in a special manner. For \( P_k \) less than 5%, the differences between \( Y_C \) and \( Y_f \) curves are negligible as can be seen in Figures A.12 to A.15.

The main disadvantage of the continuous or analog solution is that it is scaled and hence its accuracy at any point is only as good as
CASE 0-16,

M = 0.3500
R = 0.0100
θ = 0.0210
THETA = 28.5850 (DEGREES)

Note: Solid lines represent the outcrop point movement while the dashed lines represent the external waves in Fig. A.4 through A.15.

Fig. A.4. Outcrop Point Movement for δ = 9.22
Fig. A.5. Outcrop Point Movement for $\delta = 30.4$
Fig. A.8. Outcrop Point Movement for $\delta=143$. 

CASE F-6

$M = 0.4800$
$R = 0.0170$
$B = 0.0330$
$\theta = 90.0000 \text{ (degrees)}$

DEPTH (CM.)

TIME (SEC.)

0.00 0.40 0.80 1.20 1.60 2.00 2.40 2.80 3.20

20.00 24.00 32.00
CASE H-9

\( R = 0.4860 \)
\( A = 0.0170 \)
\( B = 0.0930 \)
\( \theta = 90.0000 \) (DEGREES)

Fig. A.11. Outcrop Point Movement for \( \delta = 1221 \).
CASE B-4

$\eta = 0.4900$
$A = 0.0040$
$B = 0.0050$

$\Theta = 90.0000$ (DEGREES)

Fig. A.14. Outcrop Point Movement for $P_k = 1.85\%$
Fig. A.15. Outcrop Point Movement for $P_k = 3.5\%$
the accuracy of the components and the method of interpreting a point. Nevertheless, the solution can usually be read accurately to four decimal places. This is more than adequate for comparison with numerical and analytical solutions.

6. Comparison of Results

Meaningful comparison of the methods is best based on the point where the $Y_C$ curve meets the $Y_F$ curve. The analog solution yields one distinct point of intersection because of the continuous nature of the solution whereas the numerical and analytical methods provide solutions at distinct points which may or may not be the points of intersection. The step sizes for the digital methods were determined such that about 51 points would represent the total outcrop point curve (this provided a convenient graphical plot on an 8 1/2 by 11 sheet of paper). For example, fast drop cases with periods of 5.0 and 1.7 seconds would have print-out step sizes of .1 and .035 seconds respectively. Therefore, most of the discrepancies between the analog and digital techniques can be attributed to this factor of uncertainty of the digital intersection point. The Runge-Kutta method used has some inherent errors due to approximation but these are of the order of $\Delta h^3$. The actual step size $\Delta h$ was always less than .01 so that these errors are negligible. The analog errors can generally be attributed to the accuracy of the components (.1 - 1%) and in the proper scaling of the recording device. A 2-4% error may be expected.

A brief comparison of seven cases studied shows the mean, standard deviation and the correlation coefficient for the intersection points by the analog, analytic and numeric techniques. The correlation coefficient suggests
a good correlation between the three methods even though some cases showed a wide variance (Table A.1).

In review, the analog yielded the best graphical solutions (i.e. provided the superimposed curves) whereas the numerical solution provided discrete points. The analytic solution would be most useful if one part of the curve needed to be studied more closely. The approximate solution would yield a good first approximation of the differential damped portion of the curve. Each method contributes worthwhile information to the study of the outcrop point movement.
### Table A.1
Comparison of Analog, Analytic and Numeric Analysis for Point $t_c$

<table>
<thead>
<tr>
<th>Case Studied</th>
<th>Period (T)</th>
<th>Method</th>
<th>Coordinates</th>
<th>Mean and Std. Dev.</th>
<th>Correlation Coefficient (R)</th>
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<td>Time ($^\circ$C)</td>
<td>Depth (y)</td>
<td>Time (t)</td>
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<td>B-1</td>
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<td>Analog</td>
<td>1.338</td>
<td>34.970</td>
<td>1.339</td>
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<tr>
<td></td>
<td></td>
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<td>1.330</td>
<td>32.925</td>
<td>.010</td>
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<td></td>
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<td>34.676</td>
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<td>C-1</td>
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<td>3.150</td>
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<td>O-4</td>
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<td>48.130</td>
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<td></td>
<td>Analytic</td>
<td>3.313</td>
<td>39.821</td>
<td>.760</td>
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<td></td>
<td></td>
<td>Numeric</td>
<td>3.439</td>
<td>47.907</td>
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<td>D-6</td>
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<td>2.910</td>
<td>15.300</td>
<td>2.921</td>
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<td></td>
<td></td>
<td>Analytic</td>
<td>2.921</td>
<td>15.312</td>
<td>.015</td>
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<tr>
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<td></td>
<td>Numeric</td>
<td>2.9385</td>
<td>15.366</td>
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<tr>
<td>D-7</td>
<td>5.5</td>
<td>Analog</td>
<td>4.400</td>
<td>4.500</td>
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<td></td>
<td></td>
<td>Analytic</td>
<td>4.425</td>
<td>4.050</td>
<td>1.455</td>
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<tr>
<td></td>
<td></td>
<td>Numeric</td>
<td>4.180</td>
<td>4.036</td>
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</tr>
</tbody>
</table>
APPENDIX B

ANALOG SOLUTION OF OUTCROP POINT MOVEMENT
APPENDIX B

ANALOG SOLUTION OF OUTCROP POINT MOVEMENT

Programming the outcrop point problem on the analog computer required special circuit diagrams, digital logic, scaling and various other steps. This appendix recapitulates the basic steps involved in setting up the problem.

A. SYSTEM EQUATIONS AND DIAGRAMS

Three basic equations comprise the generation of the outcrop point curve: (the parameters were previously defined)

\[ Y_f = h_o + A_o \sin(\omega t) \]  \hspace{1cm} (b.1)

\[ Y_1 = h_o + A_o \sin(\omega t_1) + V(t-t_1) \]  \hspace{1cm} (b.2)

\[ Y_c = (a_1+b_1|Y_c|)Y_c + \delta Y_c = \delta f(t) \]  \hspace{1cm} (b.3)

Figure B.1 shows the complete analog diagram which incorporates the above equations. To obtain one continuous curve, digital logic was required to operate the two track/store devices and three comparators. The first equation was simulated by the components in box A (Fig. B.2). At the first time constant, \( t_1 \), the value of \( Y_f \) was stored and the multiplying circuit (box B) took over to simulate equation (b.2). At the second time constant, \( t_2 \), the value of \( Y_1 \) was stored and the second-order differential
Fig. B.1 Analog Circuit Diagram for Outcrop Point Movement
Fig. B.2 Components of the Outcrop Point Equations.
damping circuit (box C) began operating. A comparator was used to compare
the signals out of amplifiers A65 and A30 so that when the value of $Y_f$
was greater than $Y_c$, the first equation took over once again (Fig. B.3).
This represented the rising water wave.

B. DIGITAL LOGIC

Figure B.3 shows the basic logic diagram to control the operation
of the computer. Two additional comparators were needed to ensure switching
at the proper time constants. The logic hardware to operate the amplifiers
(i.e., A00, A30, A35) and the track/stores (i.e., T31, T41), has two holes:
one for initial conditions (IC) and the other for operate (OP). When the OP
lead is high, the amplifier goes into the operate mode. This is essential
for the differential damping circuit where integrators A00 and A30 should
not operate before the second time constant $t_1$.

C. AMPLITUDE SCALING

Analog computers have a range of 10 volts reference (sometimes
100 volts) but the variables are scaled to values in machine units (1 machine
unit = reference voltage). The original differential equation was modified
after introducing the appropriate scale factors to yield:

$$
\ddot{Y}_c = -\frac{1}{K_1} \left( a_1 + b_1 \right) + K_1 \dot{Y}_c - \frac{\delta}{K_0} \left( K_0 Y_c - K_0 Y_f \right)
$$

(b.4)

where

$$K_1 = (1 \text{ machine unit})/\dot{Y}_{c_{\text{max}}}
$$

$$K_0 = (1 \text{ machine unit})/Y_{c_{\text{max}}}
$$

$K_0$ and $K_1$ were determined automatically within the general pro-
gram developed after it was noted that $\dot{Y}_{c_{\text{max}}}$ = V and $Y_{c_{\text{max}}}$ is always less
Fig. B.3 Digital Logic Diagram
than $h_0 + A_0$. To be on the safe side, the values of $K_1$ and $K_0$ were determined respectively as:

$$K_1 = \frac{1}{\text{INT}((V+7.5)/5*5)} \quad \text{(b.5)}$$

$$K_0 = \frac{1}{\text{INT}((h_0 + A_0 + 5)/5*5)} \quad \text{(b.6)}$$

(INT is the integer value of the number).

D. **TIME SCALING**

The generalized form of a linear second order differential equation (similar to the non-linear equation (b.3)) is given as:

$$\ddot{y} + 2\alpha \omega_n \dot{y} + \omega_n^2 y = \omega_n^2 f(t) \quad \text{(b.7)}$$

where

$\alpha =$ damping ratio

$\omega_n =$ undamped natural frequency

$$\omega = \omega_n \sqrt{1-\alpha^2} = \text{damped natural frequency of oscillation}$$

Equations with $\omega_n$ in the range $0.1 < \omega_n < 10$ usually have solution rates suitable for recording on strip chart and X-Y recorders. For systems outside of this range, a time scale factor, $\beta$, should be chosen such that $0.1 < (\omega_n/\beta) < 10$.

The integration rate is proportional to the gain of the integrator. Therefore, time scaling is simply a gain-adjusting operation for all the integrators by the same amount $(1/\beta)$. When $\beta = 1$, computer time and real time are one and the same; when $\beta$ is smaller than unity, the solution rate is speeded up; when $\beta > 1$ the solution rate is slowed down.

Comparing equations (b.3) and (b.4), it can be seen that

$$\omega_n = \frac{1}{\sqrt{\beta}}$$

and therefore the following time scaling constants were used:
\[0 < \delta < 81 \quad \beta = 1\]
\[81 < \delta < 4000 \quad \beta = 10\]
\[\delta > 4000 \quad \beta = 100\]
\[
\text{It is to be noted that time scaling was only required for the second order differential damping segment of the system. The frequency of the rest of the system was dictated by the frequency term,} \omega, \text{in the forcing function} \quad Y_f = h_0 A_0 \sin(\omega t). \text{No time scaling was necessary here since the frequency was always less than 10 cps.}
\]

E. ADDITIONAL SCALING

Reasonable potentiometer settings are in the range of 0.05 to 0.95. Two pot settings created additional problems in that their values were always much greater than unity. The first pot in question, P36, had a value of \(\delta \frac{K_1}{\beta K_0}\) which varied enormously since \(\delta\) varied from 4 to 111,000. Also pot P03 proved troublesome with a setting of \(b_1/K\) which yielded values ranging from 20 to 115. To circumvent these problems, these pots were reduced to settings less than unity and then the appropriate gains on amplifiers A00 and A30 were adjusted. Integrators have gains of 1 and 10 but in the digital logic, there is space provided for pins which when inserted into the F slot, increases the gain by 10, into the MS slot increases the gain by 1000, and into both slots increases the gain by 10,000. This was very important in the scaling of the circuit. Table B.1 shows examples of scaling for a wide range of \(\delta\) values.
### Table B.1
Samples of Analog Scaling of P36 and P02

<table>
<thead>
<tr>
<th>Case Studied</th>
<th>g</th>
<th>δ</th>
<th>Pot Settings</th>
<th>Gains From</th>
<th>Pino On</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>P36</td>
<td>P02</td>
<td>P36</td>
</tr>
<tr>
<td>B-1</td>
<td>1</td>
<td>37</td>
<td>70</td>
<td>41</td>
<td>10</td>
</tr>
<tr>
<td>J-5</td>
<td>1</td>
<td>75</td>
<td>165</td>
<td>72</td>
<td>-</td>
</tr>
<tr>
<td>D-6</td>
<td>10</td>
<td>158</td>
<td>32</td>
<td>68</td>
<td>-</td>
</tr>
<tr>
<td>F-5</td>
<td>10</td>
<td>426</td>
<td>.99</td>
<td>112</td>
<td>-</td>
</tr>
<tr>
<td>C-1</td>
<td>10</td>
<td>1075</td>
<td>151</td>
<td>69</td>
<td>10</td>
</tr>
<tr>
<td>H-10</td>
<td>10</td>
<td>1222</td>
<td>180</td>
<td>115</td>
<td>-</td>
</tr>
</tbody>
</table>
F. STATIC CHECK

Before running any problem on an analog computer, a static check was performed to see that the analog components functioned up to par. Basically, this involved putting arbitrary values on the pots and following the circuit to check what theoretical outputs could be expected at any amplifiers. These calculated values were then checked against the outputs measured on the analog computer while in the static test mode. The programmer could thereby determine whether certain components were faulty or locate an error in patching. Table B.2 and Fig. 4 show the calculations of a sample static check.

The beauty of the analog computer becomes evident once the circuit is in operation. The programmer can tap the solution at any point in the system, freeze the system and note the values at all points in the system, and modify the parameters to search for an optimal solution. Although the solution is restricted to a scaled four-digit accuracy, that is sufficient for most preliminary engineering analysis or design.
### TABLE B-2
SAMPLE STATIC CHECK

<table>
<thead>
<tr>
<th>Amplifier</th>
<th>Function</th>
<th>Calculated Values</th>
<th>Measured Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Derivative</td>
<td>Amplifier</td>
</tr>
<tr>
<td>A00</td>
<td>integ</td>
<td>.3400</td>
<td>.5000</td>
</tr>
<tr>
<td>A01</td>
<td>inv</td>
<td>-.2000</td>
<td>-.5000</td>
</tr>
<tr>
<td>A02</td>
<td>inv</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A05</td>
<td>integ</td>
<td>5.000</td>
<td>.0000</td>
</tr>
<tr>
<td>A06</td>
<td>sum</td>
<td>-.2500</td>
<td></td>
</tr>
<tr>
<td>A07</td>
<td>integ</td>
<td>.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>A10</td>
<td>sum</td>
<td>.3000</td>
<td></td>
</tr>
<tr>
<td>A30</td>
<td>integ</td>
<td>.3000</td>
<td>-.2500</td>
</tr>
<tr>
<td>A31</td>
<td>inv</td>
<td></td>
<td>.0000</td>
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<td>A32</td>
<td>inv</td>
<td></td>
<td>.0000</td>
</tr>
<tr>
<td>A33</td>
<td>mult</td>
<td>-.1250</td>
<td></td>
</tr>
<tr>
<td>A34</td>
<td>inv</td>
<td>-.1000</td>
<td>.2500</td>
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<td>integ</td>
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<td>A37</td>
<td>inv</td>
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<td>.0000</td>
</tr>
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<td>mult</td>
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<td>.0000</td>
</tr>
<tr>
<td>A41</td>
<td>sum</td>
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<td>.0000</td>
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<td>A60</td>
<td>integ</td>
<td>.0000</td>
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</tr>
<tr>
<td>A64</td>
<td>inv</td>
<td>-.3000</td>
<td></td>
</tr>
</tbody>
</table>

integ = integration
inv = inversion
mult = multiplication
sum = summation
Fig. B.4 Static Check Circuit Diagram
APPENDIX C

A NOTE ON ANALOG AND ANALOG/HYBRID COMPUTERS [27, 28, 44]
APPENDIX C

A NOTE ON ANALOG AND ANALOG/HYBRID COMPUTERS [27, 28, 44]

Electronic differential analyzers (EDA) commonly referred to as analog computers comprise only a small segment of the vast analog simulation field. Most people are familiar with scale models and their value in estimating real conditions. Special purpose mathematical models are often confined to research where the governing equations of one physical system are used to simulate another system. Electronic analog computers are classified as general purpose because of their more inherent mathematical nature (Fig. C.1).

Analog computers, as referred to in this thesis, implement mathematical relations between physical variables with electronic computing elements that operate simultaneously. Each element can perform a mathematical operation such as addition, subtraction, multiplication, division, and integration with respect to time. The solutions are measured voltages at any point of interest within a circuit. A circuit or program is established by interconnecting various computing elements with wires called "patch cords".

Analog computers exhibit many features that render it suitable for simulation. There is a close correspondence between the real and the simulated problem not only in the solution but in the solution technique.
Fig.C1.a Computation by Analogy

Fig.C1.b EAI 690 Hybrid Computer System
The components of the system operate in parallel and thus the answers are immediately available. Corrections and adjustments to parameters are easily made, thereby enhancing the flexibility of man-machine interaction. The integrating capability and the continuous nature of output are important advantages for analog computers.

Despite its advantages, the analog computer has lost ground to the digital computer because it is not as accurate (limited to .01% for each elementary operation), it is deficient in memory storage and it lacks the capability to perform logic operations. Hybrid computers have combined the advantages of both computers, i.e. accuracy, memory, programmability, speed, flexibility, and decision making capability. They have sophisticated interface systems that not only transfer analog and digital signals but allow the digital program to control the analog system. Perfunctory manual operations are executed automatically, such as setting pots. The digital program could also control successive simulation runs with recorded results, optimizing parameters during the runs, etc.

The hybrid computer is an invaluable simulation tool. Once the basic components are understood, they are not too difficult to program. The rest of this Appendix describes the basic analog and hybrid components found on the EAI 690 which was used in the present study.

References 4, 12, 13, 14, 21 and 61 can be consulted to get further descriptions of the components described below.
1. Potentiometers (Pots) (Fig. C.2)

Potentiometers are used to multiply a given voltage by a constant between zero and unity. They consist of a resistance with one end grounded and a sliding contact. They have a resolution of 0.01 per cent and can thus be set to four decimal digits. Potentiometers can be either hand-set or servo-set, i.e. set by a servo-mechanism within the analog computer. It is recommended maintaining pot values between 0.25 and 0.80.

2. Operational Amplifiers (Fig. C.3)

The operational amplifier is the basic functional unit in an electronic analog computer. The fundamental equation governing the output of an operational amplifier is:

$$V_o = -Z_f \frac{V_i}{Z_i}$$

where:

- $Z_i$ = input impedance
- $Z_f$ = feedback impedance
- $V_i$ = input voltage
- $V_o$ = output voltage

By varying the input and feedback impedances, the output is forced to behave in some desired way. These operational amplifiers find application in inverters, summers, integrators, high-gain amplifiers and track/store units. They will automatically invert the sign of an input voltage,
Fig. C.2 Potentiometer Symbol

Fig. C.3. Operational Amplifier Circuit

Fig. C.4. Inverter Symbol

Fig. C.5. Summer Symbol
3. **Inverters (Fig. C.4)**

Inverters change the sign of the incoming voltage(s) and can thus be used for subtraction, i.e., a voltage is inverted and then added to other voltages.

4. **Summers (Fig. C.5)**

Summers will algebraically sum the incoming voltages and invert the sign. Most summers have up to six inputs with gains of 1 and 10. These components are controlled to give four-figure accuracy.

5. **High Gain Amplifiers (Fig. C.6)**

These are basically summers with their feedback resistances removed. They are only used when external feedback is provided. The output voltage is such that the sum of the input voltages must be equal to zero.

6. **Integrators (Fig. C.7)**

This component differs from a summer in that its feedback component is a capacitor instead of a resistor. It will integrate a sum of voltages and invert the sign. It is, therefore, of immense value in solving differential equations. The EAI 680 is equipped with four different feedback capacitor values for every integrator. This is useful in solving problems with a wide range of time constants, for high-speed repetitive and iterative operation, etc. The integrator can be charged to an initial voltage to represent the initial conditions of the differential equation.
7. **Multipliers (Fig. C.8)**

The two most common methods of multiplying voltages in general-purpose analog computers are servo-multiplication and quarter-square,biased-diode multiplication.

Servo-mechanisms are closed loop control devices that control in accordance with input signal loads that are coupled to the output. They are basically following devices that must respond quickly and precisely to changes in inputs. A servomultiplier is an electromechanical servo which provides products of analog variables.

The EAI 680 uses quarter-square multipliers. The name stems from the identity:

\[
xy = \frac{1}{4} [(x+y)^2 - (x-y)^2] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (C.2)
\]

The quarter-square multiplier contains two squaring networks: one to generate the first term on the right hand side of the identity and the other to generate the second term. This requires four input signals (\(x, y\)) to satisfy all possible outputs. The output signal may or may not be inverted depending on the arrangement of the input signals.

8. **Track/Store Device (Fig. C.9)**

This device provides some memory capability to the analog computer. It is controlled by digital logic in such a fashion that it behaves as a regular summer while in the tracking mode and holds the output constant in the store mode. Arbitrary initial conditions can be inputted.
Fig. C.6. High Gain Amplifier Symbol

\[ V_1 + V_2 + V_3 = 0 \]

Fig. C.7. Integrator Symbol

\[ V \rightarrow V_{IC} \rightarrow V_{IC} \rightarrow V_{OUT} \]

Fig. C.8. Multiplier Symbol

\[ x \times x = xy \]

Fig. C.9. Track/Store Symbol

\[ V \rightarrow -V \]

Fig. C.10. Limiter Symbol
9. Limiters (Fig. C.10)

This device basically sets a range in which the output voltage can exist. It is helpful in ensuring a circuit does not overload or in finding absolute values of voltages.

**DIGITAL LOGIC ELEMENTS**

1. Comparator (Fig. C.11)

The comparator compares the sum of two analog voltages and produces a high digital output signal (binary 1) when their sum is positive and a low signal (binary 0) when their sum is negative. In the outcrop problem, comparators were used to control the timing sequence and yield a continuous curve.

2. Digital Inverter (Fig. C.12)

This component produces the binary complement of the input digital signal, i.e., when the input is high, the output is low and vice versa.

3. And Gate (Fig. C.13)

The And gate performs the fundamental logic multiplication operation. Both inputs must be high for the output to be high. Either the direct or the inverted output may be selected.

4. Digital-Analog Relay Switch (Fig. C.14)

This provides for the logical switching of two analog inputs. A high digital signal closes the switch and a low signal opens the switch. The logic control input is terminated on the digital patch panel.
Fig. C.11. Comparator Symbol

Fig. C.12. Digital Inverter Symbol

Fig. C.13. And Gate Symbol

Fig. C.14. Relay Switch Symbol

(Note: N denotes address)
INTERFACE COMPONENTS  (Fig. C.15)

1. Senselines
   Senselines are flip-flops controlled by digital logic signals on the analog console that can be interrogated by the digital computer.

2. Control Lines
   Control lines represent flip-flops located on the analog-logic panel that can be set and reset by the digital computer.

3. Analog/Digital (A/D) Converters
   Analog/Digital converters are devices which translate pulses having an amplitude proportional to an analog voltage into the binary-coded equivalent of this amplitude.

4. Digital/Analog (D/A) Converters
   Digital/Analog (D/A) Converters are devices which accept input data in binary form and generate an analog voltage proportional to the binary number.

5. Interrupts
   Interrupts serve almost the same purpose as the sense-lines. When the input generated is high, an interrupt is set at the leading edge. Once triggered, the EAI 640 branches to whatever location is specified by the pointer stored in the interrupt trap cell. The users program is executed.
if $X = S$, then a sense line

$= C$, then a control line

$= I$, then an interrupt line

---

**ADC**

Analog/Digital Converter

**DAC**

Digital/Analog Converter

---

Fig. C.15. Interface Symbols
APPENDIX D

COMPUTER PROGRAMS
APPENDIX D

COMPUTER PROGRAMS

Four programs were developed for this study. The first one is a Mimic simulation of the Outcrop Point Movement; however, since certain features of the Mimic package did not work, the results were not reliable and one continuous output curve could not be obtained. The variables used are consistent with those of the study except that $P$ is utilized as the period of oscillation. Sample inputs are given at the end of the listing and include the seven essential parameters, $a, b, m, \theta, P, A_o, h_o$. A partial sample output is included.

The second program, "PROGRAM WAVE", was developed as a substitute for the Mimic program. It is interactive in nature and has various options available to the programmer. The program first determines whether a case is slow or fast drop. If the latter case applies, the program prints out all the pertinent constants and parameters necessary for the analysis of the outcrop point movement, otherwise the program stops. These calculations are determined in Subroutine "Fast". Included in the subroutine is an option to printout the pot settings needed for the analog simulation and the appropriate scale factors. The potentiometer addresses and its corresponding values are printed out.

Subroutine "Differ" contains a Runge-Kutta 4th order technique to solve the second order differential damping equation (region III in Figure A.2, Appendix A) numerically. Only the step size and the printout
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Subroutine "Differ" contains a Runge-Kutta 4th order technique to solve the second order differential damping equation (region III in Figure A.2, Appendix A) numerically. Only the step size and the printout
frequency have to be specified.

Subroutine "Analyt" contains the analytic and approximate solutions also corresponding to Region III of the Outcrop Point Movement. A printout of the comparison of the numerical, analytic and approximate solution is optional. The analytic solution is not very accurate for certain sloping cases but is much more accurate than Dracos' solution as given in paper [10].

Subroutine "Fitcurv" joins the various Regions of the Outcrop Point Movement to yield one complete period of oscillation. The printout of the numerical solution is optional. Two different plotting routines are available for the program, one to yield a digital plot that will fill an 8 1/2 x 11 sheet of paper and another that will yield a Calcomp plot on a Tektronix terminal. Figures A.4 through A.15 were obtained using the latter method.

Subroutine "POTS" determines the pot settings needed for the analog circuit of the main analog/hybrid program. Sample outputs of this program illustrating its various options are included at the end.

The third program, "PROGRAM ODE", can be used as a subroutine and added to the previous program. This program has two variations that will yield the solution of the family of phreatic profiles within the rockfill for one complete wave period. It uses the outcrop point movement determined in the previous program as the boundary conditions. The program uses a Runge-Kutta 4th order method to calculate the first four values of the integration whereupon an Adams-Moulton Predictor/Corrector Method is utilized to continue the solution. This method is adopted to reduce the number of function calls to the derivative function DERFN. The program uses a bisector search method to converge to the solution. Tape 5 is used
as input, Tape 6 yields the solution profiles at each time increment. Tape 7 contains the values of the gradient of the $W_1$ and $W_2$ curves at all times and space locations and Tape 8 stores the intermediate profile calculations for each iteration. An option is included to manually guess the profile solution.

The other variation of the program is a non-dimensionalized form which would overcome the limitations of units, and decrease the problem of determining adequate numerical integration intervals. At present, the program is not fully operational. The need for this program emerges from the fact that the proper scaling factors for the analog program are not known in advance and hence they must be determined. The numerical integration consumes too much computer time to be viable as a solution technique (ranging from 51 secs. for 100 steps of a unit embankment to about 160 secs. for 700 integration steps). This is where the economy of the analog/hybrid computer solution becomes apparent.

The fourth program is the hybrid program that will yield the phreatic line profiles. The program is interactive in that it asks the programmer to read in the proper data. It controls the operation mode of the analog computer and contains options for the amount of printout the programmer feels is necessary. Once the program yields a solution, the computer pauses and gives the operator an opportunity to plot the solution on an x-y plotter.

The analog solution can be obtained without the aid of the digital control but the method would be awkward and very time consuming. This is due to the fact that the solution for any particular time increment is determined in an iterative manner.
MINIC SIMULATION OF THE OUTCROP POINT MOVEMENT

CONSTANTS AND PARAMETERS AND THEIR DEFINITIONS

\[ \text{PAR}(A,B,M,\Theta) \]
\[ \text{PAR}(P,H,\alpha,\omega) \]

* A = Darcy Coefficient
* B = Non-Darcy Coefficient
* \( \psi \) = Porosity
* \( \Theta \) = Angle of Inclination of Rockfill
* P = Period of Oscillation
* \( \alpha \) = Mean Water Level
* \( \omega \) = Amplitude of Impact Wave

IMPORTANT QUANTITIES NEEDED IN THE SYSTEM EQUATIONS

\[ G = 980.6 \]
\[ \pi = 3.14159 \]
\[ \text{AS} = A/(A+M) \]
\[ \text{ST} = \sin(\Theta) \]
\[ \text{DS} = \sin((A+M+\alpha,A,\Theta)/(\pi \times \omega)) \]

** MAXIMUM INTERNAL VELOCITY (V)
\[ V = (-D\lambda_{A+M}/2) \]

** COEFFICIENT OF PERMEABILITY RELATING TO V (K)
\[ K = 1/((A+M+\alpha,\Theta)/(\pi \times \omega)) \]

** FREQUENCY OF THE WAVE (\( \gamma \))
\[ \gamma = 2.*\pi/\omega \]

** MAXIMUM DEVIATION BETWEEN EXTERNAL AND INTERNAL WAVES
\[ \text{DELYP} = 2.*A*\sin(\Theta)*\gamma*(\text{TSR}-\text{TSR}) \]

OTHER IMPORTANT QUANTITIES

\[ \text{AI} = G*\pi \]
\[ \text{BI} = \pi*H*\alpha \]
\[ N = K*V/(2.*\pi*\pi*\omega) \]
\[ \text{DEL} = \text{ST}_*/\text{DELYP} \]

IMPORTANT TIME CONSTANTS (TSR, \( \text{TSR} \))

\[ \text{TSR} = \text{AC}*(-\pi*\pi*\pi)*\omega \]
\[ \text{TSR} = \text{P} - \text{TSR} \]

EQUATIONS DEFINING THE SYSTEM

\[ YF = 10*A*\sin(\omega*\text{TSR}) \]
\[ YF1 = 10*A*\sin(\omega*\text{TSR}) + V*(\text{TSR}-\text{TSR}) \]
\[ YF2 = 10*A*\sin(\omega*(\text{TSR} + \text{TSR})) \]

NOTIFY

\[ \text{YC} = 10*A*\sin(\omega*(\text{TSR} + \text{TSR}) + V*(\text{TSR}) \]
\[ \text{DAMP} = (-A*\pi*\alpha,\psi,\omega)*\text{YF} + \text{DEL}*\text{YF1} + \text{DEL}*\text{YF2} \]
\[ \text{YCI} = \text{INIT(DAMP)} \]
\[ \text{YCI} = \text{INIT(DAMP)} \]

\[ \text{YCI} = \text{INIT(DAMP)} \]
LOGIC TO PIECE TOGETHER THE OUTCROP POINT CURVE

*IFLAG =FSW(T-TSR+.05,TRUE,TRUE,FALSE,FALSE)
*NIFLG =NOT(IFLAG)
*JFLAG YC =YF
*JFLG =FSW(T-TSR1,FALSE,TRUE,TRUE)
*NJFLG =NOT(JFLAG)
*NJFLG =AND(NIFLG,NJFLG)
*JFLAG JOIN =YC1-YF
*JFLG KFLAG =FSW(JOIN,TRUE,FALSE,FALSE)
*KFLAG YC =YF
*NKFLG =NOT(KFLAG)
*NKFLG =AND(JFLAG, NKFLG)
*NOTJK YC =YC1

FIN(T,4.5)

NTRAX =.01
NTWIN =.005
NT = 1

OUTPUT
INR(T,YF,YC,YC1,NAMP,NCYCI)
OUT(T,YF,YC,YC1,NAMP,NCYCI)
PLOT(T,YF,YC,YC1,NAMP,NCYCI)
TTPM MOVEMENT OF OUTCROP POINT
TTY(EXTERNAL SINE WAVE AND OUTCROP)
TTX(TIME-SECONDS)
END

* SAMPLE INPUT DATA

<table>
<thead>
<tr>
<th>Time</th>
<th>YF</th>
<th>YC1</th>
<th>NCYCI</th>
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<td>.015</td>
<td>.43</td>
<td>1.5709</td>
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<tr>
<td>2.6</td>
<td>.22</td>
<td>1.14</td>
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<tr>
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<tr>
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<td>.443</td>
<td>1.5709</td>
</tr>
<tr>
<td>5.1</td>
<td>.19</td>
<td>10.5</td>
<td></td>
</tr>
</tbody>
</table>
PROGRAM WAVE(INPUT,OUTPUT,TAPE6=OUTPUT,TAPE14)
COMMON /BL1/M,N,ALPHA,Theta,DELPHI,K,1,9,16/H,N/BL7/XL
COMMON /BL3/Q0,AD,TSR1,TSR2,OMEGA,WANT(5),DEL,T,K0
COMMON /BL5/APRIME,BPRIME,Y1,CASE
DIMENSION X(300),Y(300)
REAL M,K,K2,K1,CASE

INPUT THE NECESSARY DATA (INTERACTIVELY)

1 PRINT 2
   READ 3,CASE
   READ H,A,B,Theta,T,H0,A0
   IF (H.EQ.1) STOP
   IF (Theta.NE.90) GO TO 5
   PRINT 4
   READ *XL
   5 THETA=THETA*3.14159/180.
   WRITE (14,10) CASE,M,A,B,XL,H0
   TRY=T/50
   PRINT 6,TRY
   READ *H,N
   PRINT 7
   READ *,WANT(1),WANT(2),WANT(3),WANT(4),WANT(5)
   CALL CAY
   CALL SERIAL(X,Y)
   GO TO 1

2 FORMAT(111),10X,*ENTER IN FREE FORMAT AND IN ORDER, EACH OF*,
   *11X,*THE FOLLOWING QUANTITIES*,/15X,*1. A TITLE*
   ** FOR THE CASE UNDER STUDY*,/19X,*MUST BE ENTERED ON A*,
   ** SEPARATE LINE*,/15X,*2. POROSITY (IF M=1, THE*
   ** PROGRAM STOPS)*,/15X,*3. A (DARCY COEFFICIENT)*,/15X,
   **4. Q (NON-DARCY COEFFICIENT)*,/15X,*5. SLOPE OF THE DAM*
   ** INCLINE (IN DEGREES)*,/15X,*6. PEIOD OF OSCILLATION*,/15X,
   **7. MEAN WATER LEVEL*,/15X,*8. WAVE AMPLITUDE*)
3 FORMAT (140)
4 FORMAT (/11X,*ENTER THE LENGTH OF ROCKFILL*)
5 FORMAT (/11X,*FOR THE NUMERICAL SOLUTION, ENTER THE STEP SIZE*
   ** (H<0.1),/11X,*AND THE FREQUENCY OF PRINTOUT (INTEGER) SO THAT*
   ** THEIR*,/11X,*PRODUCT IS *,F7.5)
6 FORMAT (/11X,*ENTER 1 FOR EACH OF THE FEATURES DESIRED *
   ** OTHERWISE ENTER 0 */,15X,*A. POT SETTNGS*,/15X,
   **8. COMPARISON OF NUMERICAL AND ANALYTICAL RESULTS*,/15X,
   **C. ANALOG SCALING OF OUTPUT*,/15X,
   **D. NUMERICAL OUTFLOW POINT MOVEMENT*,/15X,
   **E. PLOT OF OUTFLOW POINT MOVEMENT*,/15X,
   **11X,*A SAMPLE INPUT IS *1,0,1,1,0*)
7 FORMAT (3X,A10,F8.3,2F8.4,F8.1,F8.1)
END
SUBROUTINE FAST
COMMON /BL1/M, G/BL2/A, B/BL4/THETA, DELYP, K, KL
COMMON /BL3/AD, TSR1, TSR, W, OMEGA, WANT5, DELT, K0
COMMON /BL5/APRIME, BPRIME, YC1, CASE
REAL M, K0, K, KL, CASE

CALCULATION OF FAST DROP PARAMETERS

G = 980.6
ABM = A / (B*M)
V = - (ABM * (A*B)**2 + 4 * M**2 * SIN(THETA)**2) / (M**2 * B)**5 / 2.
K = 1 / (A + 9 * M**2 * ABS(V))
W = K**2 / (6.2539 * M**4)
IF (ABS(W ** SIN(THETA))**2 > GT, 1) GO TO 20
D0 = INT(ABS(V) ** 4 + 2.5) / 5
D0 = D0 ** INT(H0 + 0.5) / 5
IF (K0 .LT. K1) KG = K1
K1 = 1 / KG
TSR = T / (6.2539 * M**2 * SIN(THETA)**2)
APRIME = G * M**2
BPRIME = G * M**2
OMEGA = 2 * 3.141597 / T
TSR1 = TSR
DELYP = 2 * A**2 * M**2 * SIN(OMEGA * TSR) * V ** (TSR1 - TSR)
YFC1 = YF1 * V ** (TSR1 - TSR)
YFC1 = M * A**2 * SIN(OMEGA * TSR)
YC1 = YF1 + V ** (TSR1 - TSR)
YC1 = M * A**2 * SIN(OMEGA * TSR)
ACCEL = (APRIME + BPRIME * ABS(V)) * V - DEL * YC1 + DEL * YFC1
PK = DELYP / 20

PRINTOUT OF RELEVANT FAST DROP PARAMETERS

PRINT 8, CASE, A, APRIME, B, BPRIME, T, TSR, TSR1, G,
     K, THETA, OMEGA, AD, YF1, H0, DELYP
PRINT 23, YC1, V, ACCEL
PRINT 24, W, DEL, PK

IF (WANT5 .LT. EQ, 6) RETURN

DETERMINING THE POT SETTINGS AND SCALE FACTORS

BETA = 1
IF (DEL .GT. 0.1) BETA = 10
IF (DEL .GT. 2.0) BETA = 100
P00 = 1 / BETA * K0 / K1
P01 = K1 / ABS(V)
P02 = APRIME / K1
P03 = APRIME
P05 = OMEGA
P07 = K3 / 40
P08 = K3 / ABS(V)
P10=K0*H0
P31=1/BETA
P35=1
P36=1/BETA*K1/K0*DEL
P62=1
P64=TSR/10
P94=TSR1/10

C PRINTOUT OF SCALED POF SETTINGS

WRITE(6,200) Q0,Q1,BETA,P30,P01,P02,P03,P05,Q07,P07,P08,P10,P31,
P35,P36,P62,P64,P94.
RETURN

20 PRINT 21

8 FORMAT(11H1,/,T37,A10,/,T20,*G I V E N*,T51,*CALCULATED*,/)
+T20,*T11=*,T51=*,T37=*
+T20,*T91=*,T51=*,T37=*
+APRIME =*T61,F8.4,,T12,*P01 =*T30,F8.4,T44
+BPRIME =*T51,F8.4,,T12,*PERIOD =*T30,F8.4,T44
+TSR =*T61,F8.4,,T12,*POROSITY =*T30,F8.4,T44
+TSR1 =*T61,F8.4,,T12,*GRAVITY =*T30,F8.4,T44
+HYD. COND. (K) =*T61,F8.4,,T12,*SLOPE (RAD) =*T30,F8.4,T44
+FREQUENCY =*T61,F8.4,,T12,*AMPLITUDE (A0) =*T30,F8.4,T44
+TANGENT PT. =*T61,F8.4,,T12,*DEPTH (H) =*T30,F8.4,T44
+MAX. DEVIATION =*T61,F8.4,/

C FORMAT(11X,*SORRY, THIS IS NOT A FAST DROP CASE!*,,/,,/,,11X,
+G9.1H*)

23 FORMAT(11X,*Y(0) = *,F8.4,2X,*VEL(0) = *,F8.4,2X,
+*ACCEL(0) = *,F10.4,/

24 FORMAT(T12,*W = *,F8.4,2X,*DEL = *,F11.4,2X,
+*PK (T) = *,F10.4,/,11X,57(1H*))

200 FORMAT(/,T22,*POT*,T30,*SETTING*,T45,*K0 = 1/(*,F3.0,1*)/,,
+T22,*---*,T30,*-----*,T45,*K1 = 1/(*,F3.0,1*)/,,
+T22,*BETA = *,F4.0,,T22,*P01*,T27,*F10.4,/,,
+T22,*P02*,T27,*F10.4,,T22,*P01*,T27,*F10.4,/,,
+T22,*P05*,T27,*F10.4,,T22,*Q07*,T27,*F10.4,/,,
+T22,*P07*,T27,*F10.4,,T22,*P06*,T27,*F10.4,/,,
+T22,*P10*,T27,*F10.4,/,,
+T22,*P35*,T27,*F10.4,,T22,*P35*,T27,*F10.4,/,,
+T22,*P36*,T27,*F10.4,,T22,*P62*,T27,*F10.4,/,,
+T22,*P94*,T27,*F10.4,,T22,*P94*,T27,*F10.4,/,,,11X,57(1H*)

RETURN

END
SUBROUTINE DIFFER(X,YDOT)

SUBROUTINE TO CALCULATE THE NUMERICAL SOLUTION
BY A RUNGE-KUTTA-4 METHOD AND TO COMPARE IT TO
AN APPROXIMATE ANALYTIC SOLUTION

COMMON /BL3/H0,A0,XIN,TSR,PIN,H,WANT(5),DEL,T,QQ
COMMON /BL5/A0,YIN,CASE/BL6/H,N
DIMENSION X(300),Y(300),YDOT(300),YC(300)
REAL K1,K2,K3,K4,L1,L2,L3,L4,K,N,CASE

ACCEL(P,X,Y)=-(A+R*ABS(P))*P+DEL*(H0+40*SIN(10*X)-Y)

IF (WANT(3).EQ.0) Q1=QQ=1
IF (WANT(2).EQ.0) GOTO 115
PRINT 110,H,N

NUMERICAL FRAMEWORK FOR SECOND ORDER DIFFERENTIAL EQUATION

115 X(1)=XX=X0=XIN
Y(1)=YY=Y0=YIN
YDOT(1)=PP=P0=PIN
KOUNT=0
I=N

125 KOUNT=KOUNT+1
K1=H*PP
L1=H*ACCEL(P,XX,YY)
XX=XX+L1/2
YY=YY+K1/2
P =P0+L1/2
K2=H*PP
L2=H*ACCEL(P,XX,YY)
YY=YY+K2/2
P =P0+L2/2
K3=H*PP
L3=H*ACCEL(P,XX,YY)
XX=XX+L3/2
YY=YY+K3
P =P0+L3
K4=H*PP
L4=H*ACCEL(P,XX,YY)
K1=XX+2*K2+2*K3+K4/6
L1=XX+2*L2+2*L3+L4/6
XX=XX+L1/2
YY=YY+Y0+K1
P =P0+P0+L1
IF (KOUNT.LT.1) GO TO 125
J=KOUNT/N+1
X(J)=XX
Y(J)=YY
YOOT(I,J) = P
IF (X,GE,(1,1*T)) GO TO 150
I = I + N
GOTO 125
150 CALL ANALYTIC(X, Y, YOOT, J)
110 FORMAT (* SECOND ORDER DIFFERENTIAL GATING*/,
+ T57, * ANAL *, T53, * APPROX */)
RETURN
END

SUBROUTINE ANALYTIC(X, Y, YOOT, J)
COMMON /BLI/H, GOL3/H0, AO, TSR1, TSR, Y, WANT(5), DEL, T, QO
COMMON /H, THETA, DELYP, K, D1
DIMENSION X(1300), Y(1300), YOOT(1300)
REAL K, M
C
APPROXIMATE ANALYTICAL SOLUTION

C
TX = X/WM
TKG = 5/TK
C "I =1-(4*K*SIN(THETA)**2*DELPY)/((G*M*A0*T)
XLAMDA1 = G*M*(1-(CHI**5))/2*CHI
XLAMDA2 = G*M*(1+CHI**5)/2*CHI
COEFF1 = A0*DEL*(DEL-M**2)/(1KG**2*KDEL-M**2*272)
COEFF2 = COEFF1*TK/DEL*(DEL-M**2)
COEFF3 = COEFF2*(SIN(M*TSR) + COS(M*TSR) + V*(TSR1-TSR)
COEFF4 = COEFF3*COS(M*TSR) + H*COEFF3*SIN(M*TSR) + V
PART1 = COEFF3*COEFF4*XLAMDA2/(XLAMDA1-XLAMDA2)
PART2 = COEFF3*COEFF4*XLAMDA1/(XLAMDA1-XLAMDA2)
IF WANT(2), EQ, 0) GOTO 176
00 170 I = 1, J
TIME = X(J)
PART2 = PART1*EXP(XLAMDA1*(TIME-TSR1))
PART3 = PART2*EXP(XLAMDA2*(TIME-TSR1))
PART4 = H*COEFF4*SIN(M*TIME) - COEFF3*COS(M*TIME)
C
ZG = PART2 - PART3 + PART4
C
DIFF = (Y(J) - ZG)*100/Y(J)
SINE = -AO*2*SIN(W*TIME)
C
APPROXIMATE SOLUTION

C
APPROX = 0.5 + AO*SINH(TSR) + V*(TSR1-TSR)*0.75*V*(TIME-TSR1)
DIFF = (APPROX - Y(J))*100/Y(J)
PRINT 175, Y(J), Y(J), QO, ZG, Q0, APPROX*QQ, YOOI(1), Q1, DIFF, DIF
IF (SINE, GT, Y(J)) GOTO 176
170 CONTINUE
176 CALL FIGCURV(X, Y, YOOT, M)
175 FORMAT (12, F6.4, 41X, F8.4, 21X, F6.2)
RETURN
END
SUBROUTINE FITCURV(X,Y,YC)

SUBROUTINE TO FIT TOGETHER THE VARIOUS COMPONENTS
OF THE OUTCROP CURVE TO GET ONE CONTINUOUS CURVE

COMMON /AL2/H0, AO, TSR1, TSR, Y, K0, WANT(5), COL, T, KO/BL6/H, N

COMMON /BL6/H

DIMENSION X(300), Y(300), YC(350), TIME(350)

REAL K0

DO 23 I=1, 350

23 YC(I)=0.

TIME(I)=0.

YC(I)=H0

H=H*H

DT=H

I=L=1

YT1=H0+AO*SIN(W*TSR)

YT2=Y(I)

THE RISING PART OF THE OUTCROP POINT

38 I=I+1

TIME(I)=TIME(I)+H*(I-1)

THE MAXIMUM FALL RATE OF THE OYICROP POINT

IF(TIME(I)-TSR) GOTO 48

YC(I)=H0+AO*SIN(W*TIME(I))

GOTO 38

48 YC(I)=H0+AO*SIN(W*TSR)+W*(TIME(I)-TSR)

I=I+1

TIME(I)=TIME(I)+H*(I-1)

IF(TIME(I)-TSR) GOTO 48

GOTO 48

THE NON-LINEAR DIFFERENTIAL VISCOS DAMPING

64 TIME(I)=X(I)

YC(I)=YT(I)

YSIN=H0+AO*SIN(W*TIME(I))

IF((YC(I))-.YSINLE.0.) GO TO 74

I=I+1

L=L+1

GOTO 60

THE OUTCROP POINT RISES AGAIN WITH THE EXTERNAL Wave

74 YC=TIME(I)

YC(H0+AO*SIN(W*TYC))

IF((YC.GT.T)) GOTO 87

75 YC(I)=H0+AO*SIN(W*TIME(I))
I=I+1  
TIME(I)=TIME(I)+H*(I-1)  
IF(TIME(I).GT.T) GOTO 90  
GOTO 75  
87 II=I  
YC(I)=MO+AO*SIN(W*TIME(I))  
L=II-INT(I/H)-1  
GOTO 93  
90 II=I-1  
L=1  
93 PRINT 95,TSR,YT1,TSR1,YT2,YTC,YTC  
WRITE(14,1(5),TSR,YT1,TSR1,YT2,YTC,YTC  
IF(WANT(4).EQ.0) GOTO 97  
PRINT 96  
97 DO 100 J=L,II  
M=J-L+1  
X(M)=TIME(M)=TIME(J)  
YF=(MO+AO*SIN(W*TIME(J)))*K0  
YC(M)=YC(J)/K0  
IF(WANT(6).EQ.0) GOTO 100  
PRINT 99,M,TIME(M),YF,YC(M)  
CONTINUE  
CALL "**TS(0)  
CC 122 T=2,H  
102 WRITE(14,1(1)) TIME(I)-TIME(I-1),YC(I)  
IF(WANT(5).EQ.0) RETURN  
CALL PLOT(Y,YC,TIME,M,55,0)  
95 FORMAT(/,T28,*T1 = *,F8.4,T44,*Y(T1) = *,F6.4,/  
*T28,*T2 = *,F6.4,T44,*Y(T2) = *,F6.4,/  
*T28,*TC = *,F6.4,T44,*Y(TC) = *,F6.4)  
96 FORMAT(1H1/,T21,*T1,T32,*TIME*,T44,*SINE WAVE*,  
*T60,*OUTCROP*,//)  
99 FORMAT(19X,12,3(5X,F10.4))  
101 FORMAT(16X,2(F10.4,5X))  
105 FORMAT(16X,T5,*T1 = *,F8.4,T25,*Y(T1) = *,F8.4,/  
*T5,*T2 = *,F6.4,T25,*Y(T2) = *,F6.4,/  
*T5,*TC = *,F8.4,T25,*Y(TC) = *,F8.4)  
RETURN  
END
SUBROUTINE PLOT(ITY, Y, Z, N, XSCALE, IPRINT)

PROGRAM TO PLOT A SET OF X-VERSUS-Y VALUES.
X IS THE SYMBOL USED TO MARK THE POINT OF INTERSECTION
Y IS THE DEPENDENT VARIABLE ARRAY
Z IS THE INDEPENDENT VARIABLE ARRAY
N IS THE NUMBER OF DATA POINTS IN EACH ARRAY
XSCALE IS THE NUMBER OF COLUMNS USED TO PRINT THE X SCALE
IPRINT EQUALS 1 IF THE SCALED DATA POINTS ARE TO BE PRINTED OUT

DIMENSION Y(N), I(N), ITY(300), IY(300), IX(300)
REAL X, BLANK, DOT, MARK
INTEGER XSCALE, XSCAL
DATA BLANK, DOT, MARK / ' ', ' ', ' ' /

CREATE TEMPORARY STORAGE FOR ARRAYS (A=TY AND B=TX GENERALLY)

DO 100 I=1,N
   TX(I)=TY(I)
100   TX(I)=2*IX(I)

FIND THE MINIMUM AND MAXIMUM ELEMENTS IN EACH ARRAY

XMIN=YMENN=10.0
XMAX=YMENN=-10.0

DO 500 I=1,N
   IF(TY(I).GT.XMAX) XMAX=TY(I)
   IF(TY(I).LT.XMIN) XMIN=TY(I)
500 CONTINUE

PRINT 997, XMIN, XMAX, XMIN, YMAX

SCALE ELEMENTS IN X ARRAY IF NEGATIVE OR LESS THAN ONE

DO 601 I=1,N
   TX(I)=TX(I)-XMIN+1.0
   XMAX=XMAX-XMIN+1.0
601   XMIN=1.0

SCALE ELEMENTS IN Y ARRAY IF NEGATIVE OR LESS THAN ONE

DO 602 I=1,N
   TY(I)=TY(I)-XMIN+1.0
   YMAX=YMAX-XMIN+1.0
602   YMIN=1.0

FORM THE SCALING RATIOS FOR X AND Y AXES

RANGE=YMAX-YMIN
RANGE2=XSCALE-1
RANGE=XMAX-XMIN
PRINT 998, RANGE, RANGE2, RANGE

SCALE THE ELEMENTS OF THE ARRAYS
DO 700 I=1,N
TY(I)=1.0*(TY(I)-YMIN)*RANGEX/RANGEY

700 TY(I)=10.0*(TY(I)-YMIN)*70./RANGEY

C PLOT THE SCALED DATA ARRAYS
C CREATE AND PRINT THE Y AXIS AS DOTS

DO 398 I=1,9
LX(I)=BLANK
DO 400 I=1:83
WRITE(6,401)D(LX(I),I=1,83)

C INITIALIZE PRINT VECTOR

DO 402 J=1,83
LX(J)=BLANK
LX(1)=D00
XSCALE=XSCALE+1
II=I
DO 800 I=1,XSCALE.
LL=I
KS=0
J=TX(II)
IF(JJ.NE.I)GO TO 601
KS=1
KK=TY(II)
LX(KK)=MARK
ITY(LL)=KK
II=II+1
MM=TX(II)
IF(MM.EQ.I)GO TO 802

601 WRITE(6,401)D(LX(J),J=1,83)
IF(KS.EQ.0)GO TO 805
LL=LL+1
DO 804 NN=1,LL1
ITY1=ITY(NN)
804 WRITE(6,402)D,ITY1
805 KS=0
800 CONTINUE

IF(IPRINT.NE.1)GO TO 900
DO 901 I=1,N
WRITE(6,902)D,ITY(I)

900 CONTINUE

401 FORMAT(1H,83A1)
902 FORMAT(15,2F15.7)
907 FORMAT(1H1,9X,**XMIN=**,F9.4,**XMAX=**,F9.4,**YMIN=**,F9.4,**YMAX=**,F9.4)
908 FORMAT(1H,1X,**RANGEX=**,F13.4,**RANGE2=**,F13.4,**RANGEY=**,F12.4)
RETURN
END
SUBROUTINE POTS(DT)
THIS SUBROUTINE WILL GIVE THE PROPER POT SETTINGS
FOR THE HYBRID PROGRAM DEVELOPED

COMMON /BL1/H,0/L2A,0/BL7/XL
REAL H

P00=P31=1/6.
P01=P6=1/(50*DT)
P02=P32=.75
P03=P33=M
P06=P38=1.9612*M
P10=A
P12=3*9
P30=P35=25
FACTOR=1.
IF(XL.GE.10.) FACTOR=1.
P60=1./FACTOR
P65=XL/(10.*FACTOR)
WRITE(14,5) P00,P01,P30,P31,P02,P32,P33,P06,P35,
*P09,P36,P10,P60,P12,P65
5 FORMAT(15,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4,*F8.4)
RETURN
END
PROGRAM ODE (INPUT, OUTPUT, TAPES, TAPE7, TAPE8)
SOLUTION TO FIRST ORDER DIFFERENTIAL EQUATION BY RUNGE-KUTTA
DIMENSION WM2(4), WM1(4), RESULT(701,6), TIME(52), DEPTH(52)
REAL M

DEFINE DERIVATIVE FUNCTION

\[ \text{DERFN}(Y,Z,YM1) = \frac{-1}{(1.75*Y+2.5*Z)*(M*(Y-YM1)/DT+5*G*(A+B)+ABS(Y+Z)/2)} \]

READ IN PARAMETERS
ISTEP = 1
PRINT 202

READ IN TAPES
READ (5,*) M, A, B, XL, H0, PTA, BTB, U
PRINT *, 20H INPUT T1, H1, T2, H2, H
READ *, T1, H1, T2, H2
do 15 J = 1, 52
15 READ (5,*) TIME(J), DEPTH(J)
IMAX = 701
H = XL/(IMAX-1)
I10 = (IMAX-1)/10
I1 = IMAX-1
G = 980.6
FAC = 16*M*M*G
WRITE (7, 210)

DEFINE THE CONDITIONS AT INITIAL TIME

W1C = 2*M*(G*H0)**.5
W11 = 2*M*(G*H1)**.5
W12 = 2*M*(G*H2)**.5
TYME = C.
GO TO 2

1 PTA = U + 1*.5
PTB = U - 1*.5

MOVE ON TO THE NEXT TIME INCREMENT

2 PRINT 203
DT = TIME(ISTEP)
TYME = TYME + DT
IF (TYME.GT.T) STOP
IF (TYME.LE.T1) W10 = (W10*(T1-TYME)+W11*TYME)/T1
IF (TYME.GT.T1 .AND. TYME.LE.T2) W10 = (W11*(T2-TYME)+W12*(TYME-T1))/T2-T1
IF (TYME.GT.T2) W10 = (W12*(T+T1-TYME)+W11*(TYME-T2))/(T+T1-T2)
W20 = W10
IF (DT.LE.0) STOP
G = (G*DEPTH(ISTEP))**.5
ICOUNT = 1
GO TO 4
C DEFINE THE LIMITS AND THE INITIAL VALUE OF THE VELOCITY U
100 I=1,IMAX,1
13 PRINT 204,RESULTS(I,1),(RESULTS(I,J),J=2,6)
PRINT 204
READ *,PTA,Ptb,U
IF(PTA.EQ.100) STOP
ICOUNT=1
4 X0=0
I=1
C COMPUTE FOUR VALUES BY RUNGE-KUTTA AND PRINT OUT

WM1(I)=W1+2*H*C
WM2(I)=W2+2*H*C
CALL STORE(RESULTS,X0,W1,W2,FAC,M,I,IMAX)
DO 10 I=2,4
FK1=DERFN(W1,W2,W10)
FK2=DERFN(W2+5*FK11,W2+5*FK12,W10)
FK3=DERFN(W2+5*FK21,W2+5*FK22,W10)
FK4=DERFN(W2+5*FK31,W2+5*FK32,W10)
FK1=DERFN(W1+FK11,W2+FK12,W10)
FK2=DERFN(W1+FK21,W2+FK22,W10)
FK3=DERFN(W1+FK31,W2+FK32,W10)
FK4=DERFN(W1+FK41,W2+FK42,W10)
X0=X0+H
WM1(I)=W1+FK11+2.*FK21+2.*FK31+FK41/6.
WM2(I)=W2+FK12+2.*FK22+2.*FK32+FK42/6
W1=WM1(I)
W2=WM2(I)
CALL STORE(RESULTS,X0,W1,W2,FAC,M,I,IMAX)
10 CONTINUE
FY=DERFN(W1(1),W2(1),W10)
F2=DERFN(W1(2),W2(2),W10)
F3=DERFN(W1(3),W2(3),W10)
F4=DERFN(W1(4),W2(4),W10)
FW=DERFN(W1(2),W1(1),W2(1),W10)
FW=DERFN(W1(3),W2(2),W1(2),W10)
FW=DERFN(W1(4),W2(3),W1(3),W10)
FW=DERFN(W2(4),W1(4),W1(1),W10)
WRITE(7,211) ISTEP,ICOUNT,W1,W2,F2,F3,F4,F5,F6,F7,F8,F9,F10
I=4
C ADAMS-MOULTON PREDICTOR-CORRECTOR METHOD

30 I=I+1
PW1=W1*(55*F4-53*F3+37*F2-9*F1)/24
PW2=W2*(55*F9-59*F8+37*F7-9*F6)/24
F5=DERFN(PW1,PW2,W10)
F0=DERFN(W1+PW1,W2+PW2,W20)
GW1=W1*(9*F5+19*F4-5*F3+F2)/24
GW2=W2*(9*F9+19*F8-5*F7+F6)/24
W1=GW1+19*(PW1-GW1)/270
W2=GW2+19*(PW2-GW2)/270
X0=X0*H
F1=F2
F2=F3
F3=F4
F4=OFRFN(W1,W2,W10)
F6=F7
F7=F8
F8=F9
F9=OFRFN(W2,W1,W20)
IF(I/I10*10.E0.I) WRITE(7,212) I,F4,F9
CALL STORE(RESULTS,X0,H1,W2,FAC,M,I,IMAX)
IF(I.LT.IMAX) GOTO 30

C BISECTOR SEARCH FOR THE CORRECT VELOCITY FOR
C THE BOUNDARY VALUES PROBLEM

R=(W1+W2)/2
IF(ABS(R).LE.01) GOTO 40
IF(R) 25,25,32

C LOWER LIMIT IS ADJUSTED UPWARDS
--------------------
25 PTA=U
       GO TO 35
C UPPER LIMIT IS ADJUSTED DOWNWARDS
--------------------
32 PTA=U
C THE NEW VELOCITY IS COMPUTE?
--------------------
35 U=(PTA+PTB)/2
ICOUNT=ICOUNT+1
DO 37 1=1,IMAX,110
37 WRITE(1,201) (RESULTS(I,J),J=1,6)
IF(ICOUNT.LE.10) GO TO 4
PRINT 47
GO TO 3

C SUCCESS!! YOU HAVE CONVERGED TO A SOLUTION.
C PREPARE TO MOVE ON TO THE NEXT VALUE
40 WRITE(6,42) ISTEP,ICOUNT
WRITE(6,60)
DO 44 1=1,IMAX,110
WRITE(6,214) RESULTS(I,1),RESULTS(I,2)
44 WRITE(6,20C) RESULTS(I,1),RESULTS(I,J),J=2,6
ISTEP=ISTEP+1
GO TO 1
FORMAT STATEMENTS

42 FORMAT(* STEP NO.*, *I4,* REQUIRED *,I3,* ITERATIONS*/)
47 FORMAT(* SOMETHING IS WRONG, YOU ARE NOT CONVERGING*,
** FAST ENOUGH,*/* TRY READING IN A NEW VALUE OF U*/)
60 FORMAT (7X,*X*,12X,*Y*,14X,*U*,13X,*C*,12X,*W1*,12X,*W2*,/)
200 FORMAT (F10.4,3X,F14.7,4(3X,F10.4))
201 FORMAT(6(2X,G12.3))
202 FORMAT(* ENTER POROSITY(H), COEFFICIENTS(A,B), LENGTH(XL),*,
** MEAN WATER LEVEL(HD), UPPER AND LOWER LIMITS OF VELOCITY*,
** AND THE INITIAL GUESS FOR THE VELOCITY(U)*)
203 FORMAT(* ENTER THE INCREMENT AND THE NEW INITIAL HEIGHT*/)
204 FORMAT(* ENTER NEW UPPER AND LOWER LIMITS*,
** AND THE ACTUAL VELOCITY*/)
210 FORMAT(1H1.5X,* THIS TAPE GIVES THE VALUES OF THE DERIVATIVE *,
**/6X,* AT EACH STEP OF THE INTEGRATION. THIS VALUE FOR THE*,
**/6X,* ANALOG/HYBRID SIMULATION./5X,60(1H-1))
211 FORMAT(5X,*STEP NO.*, I4,4X,* ITERATION NO.*, I4, /*
+15X,*1*,5X,2G14.7/15X,*2*,5X,2G14.7/15X,*3*,5X,
+2G14.7/15X,*4*,5X,2G14.7)
212 FORMAT(13X,I3,5X,2G14.7)
214 FORMAT(10X,F5.2,3X,F10.4)
215 FORMAT(*-------- STEP NO. *, I4,*-------- ITERATION NO. *,
+I4,*--------*)
END

SUBROUTINE STORE(RESULTS,X0, W1, W2, FAC, PORE, NUM, IMAX)
DIMENSION RESULTS(IMAX,6)
RESULTS(NUM,1)=X0
RESULTS(NUM,2)=(W1-W2)**2/FAC
RESULTS(NUM,3)=(W1+W2)/2
RESULTS(NUM,4)=(W1-W2)/(4*PORE)
RESULTS(NUM,5)=W1
RESULTS(NUM,6)=W2
RETURN
END
DEVELOPED BY PETER KOTIUGA WITH THE HELP OF DAVID HARGREAVES.

THIS PROGRAM CALCULATES THE PHREATIC SURFACES OF THE WAVE
MOTION WITHIN ROCKFILL STRUCTURES USING A CSOT HYBRID
APPROACH. THE CONTINUITY AND MOMENTUM EQUATIONS HAVE BEEN
TRANSFORMED TO THEIR CHARACTERISTIC FORM AND ARE INTEGRATED
SIMULTANEOUSLY. BECAUSE IT IS A BOUNDARY-VALUE PROBLEM,
AN ITERATIVE BISECTOR SEARCH METHOD HAS BEEN IMPLEMENTED
TO YIELD A SOLUTION AT EACH TIME FRAME. AN OPTION TO PLOT
THE PHREATIC SURFACE AT EACH TIME FRAME IS GIVEN AND THEN
THE VARIABLES ARE UPDATED. THE SOLUTION IS OBTAINED FOR
ONE COMPLETE WAVE PERIOD.

COMMON T1,H1,T2,H2,EPS,SCALE
REAL M

SELECT ANALOG CONSOLE
CALL OSMTIN(IER,683)
CALL QSCI(IER)

PLACE ANALOG IN POT COEFFICIENT MODE
CALL OSPC(IER)
CALL OSQLL(1,.FALSE.,IER)

ENTER THE PARAMETERS

10 FORMAT(62H ENTER THE PERIOD, H.W.L., POROSITY, EPSILON,
+50H ENTER THE SCALE FACTOR (ABOUT 300) */
ACCEPT 15,T,H,EPS,SCALE
TYPE 15,T,H,EPS,SCALE
15 FORMAT(2F5.2,2F6.4,F6.1)

TYPE 20
20 FORMAT(62H ENTER T1,H1,T2,H2 (4F10.4) */
ACCEPT 25,T1,H1,T2,H2
1 TYPE 25,T1,H1,T2,H2
25 FORMAT(4F10.4)
G=980.6

CALL PHREA(G,T1,H0,M)
STOP
END
SUBROUTINE TO SOLVE FOR THE PHREATIC PROFILES

SUBROUTINE PHREA(G,T,H0,H1,H2,EPS,SCALE)
DIMENSION WIN(4),WOUT(2)
COMMON T1,H1,T2,H2,EPS,SCALE
LOGICAL SENSW
REAL M

HO=2.*M*SORT(G*H0)/SCALE
H1=2.*M*SORT(G*H1)/SCALE
H2=2.*M*SORT(G*H2)/SCALE
TIME=0.

READ IN IMPORTANT PARAMETERS

17 TYPE 20
20 FORMAT(60H ENTER THE NEW TIME INCREMENT AND DEPTH*,//)
ACCEP* 25,0T,YN
TYPE 25,0T,YN
25 FORMAT(2F6.4)
IF(YN.LE.0.) RETURN
TYPE 30
30 FORMAT(60H ENTER RANGE OF VELOCITIES AND INITIAL*,+2ND GUESS*,//)
ACCEP* 35,VELA,VELB,VEL
35 FORMAT(3F9.1)

DEFINING THE INITIAL VALUES

CEL=2.*M*SORT(G*YN)/SCALE
VELA=VELA/SCALE
VELB=VELB/SCALE
VEL=VEL/SCALE
WIN(1)=VEL+CEL
WIN(2)=VEL-CEL
IF(TIME.GT.T) RETURN
IF(TIME.LE.T1) WIN(3)=(H0*(T1-TIME)*TIME*W2)/T1
IF(TIME.GT.T1.AND.TIME.LE.T2) WIN(3)=(H1*(T2-TIME)*TIME*W2)/T1
IF(TIME.GT.T2) WIN(3)=(H2*(T1+T-TIME)*TIME*W1)/T1
WIN(4)=-WIN(3)

INTEGRATE ON THE ANALOG COMPUTER

40 CALL RUN(WIN,WOUT)

BISECTOR SEARCH METHOD TO SEEK SOLUTION

VELN=(WOUT(1)+WOUT(2))/2.
IF(SENSW(11)) GO TO 45
YN=(WIN(1)+WIN(2))/2.*SCALE/(4.*M**G)**2.
YOU=(WOUT(1)+WOUT(2))*SCALE/(4.*M**G)**2.
WIN=WIN(1)+WIN(2))/2.
CIN=WIN(11)-WIN(21)/4.*M
COUT=WOUT(11)-WOUT(21)/4.*M

45 EXIT
SUBROUTINE TO SOLVE FOR THE PHREATIC PROFILES

TYPE 70, TIME, YIN, UIN, CIN, YOUT, VELN, COUT

70 FORMAT (/12X, 7HTIME = F8.2, 12X, 5HY(0) = F8.4, 5X, 5HU(0) = F7.4,
+ 5X, 5HC(0) = F8.3, 12X, 5HY(1) = F8.4, 5X, 5HU(1) = F7.4, 5HC(1) =
+ F8.3)!

45 IF (ABS(VELN) .LE. EPS) GO TO 55
IF (VELN .LT. 0.1) VELB = VELN
IF (VELN .GT. 0.1) VELA = VELN
VEL = (VELA + VELB) / 2.
MIN(1) = VEL + CEL
MIN(2) = VEL - CEL
CALL QHODAR(MIN, 1, 2, IER)
GO TO 40

C PLOT THE CURVE FOR THE PRESENT TIME FRAME

55 PAUSE 01
CALL PUN(MIN, WOUT)
PAUSE 02

C GO TO THE NEXT TIME FRAME
TIME = TIME + DT
GO TO 17
END
PAGE 4  SUBROUTINE TO TRANSFER ANALOG AND DIGITAL SIGNALS

SUBROUTINE RUN(WIN, WOUT)
DIMENSION WIN(4), WOUT(2).
LOGICAL LVAL.

C SET THE DACS AND TRANSFER SIGNALS
CALL OW30AR(WIN, 1, IER)
CALL OW30TA.

C PLACE ANALOG IN IC MODE
CALL OSICIER)

C LOOP TO DETERMINE A PHREATIC SURFACE
CALL OSOLYR(500, IER)
CALL OSOPIER)
CALL ORSL(LVAL, 0, IER)

45 CALL ORSL(LVAL, 0, IER)
IF(IER, NOT, LVAL) GO TO 45

C READ THE A/D CHANNELS
CALL OR9ADR(WOUT, 1, 2, IER)
CALL OSICIER)
RETURN.
END
PROGRAM ODE (INPUT, OUTPUT, TAPE5, TAPE6, TAPE7, TAPE8)
C
SOLUTION TO FIRST ORDER DIFFERENTIAL EQUATION BY RUNGE-KUTTA
DIMENSION WH2(4), WH1(4), RESULTS(101, 6)
COMMON GH0, XL
REAL
C
DEFINE DERIVATIVE FUNCTION

FUNCTION DERF1(Y, Z, YM1) = -H/(1.75*Y+25*Z) * ((Y-YM1)/DT+5*(GMAT+.5*
+ GM8L*ABS(Y+Z)) * (Y+Z))
C
READ IN PARAMETERS
ISTEP=1
PRINT 202
READ (5, *) M, A, B, XL, H0, PTA, PTB, U
PRINT 201
READ *, TI, H1, T2, H2, T
PRINT *, WM : ENTER THE VALUE OF TOLERANCE ON VELOCITY
READ *, EPS
IMAX=101
I1C=(IMAX-1)/10
IM1=IMAX-1
H=1./(IMAX-1)
G=980.6
FAC=16*M*H*G
GH0=(G*H0)**.5
GMAT=G*M*A*T
GM8L=G*M*B*XL
PTA=PTA/GH0
PTB=PTB/GH0
U=U/GH0
WRITE (7, 210)
C
DEFINE THE CONDITIONS AT INITIAL TIME

W10=2*M
W11=12*M*(G*H1)**.5/GH0
W12=12*M*(G*H2)**.5/GH0
TIME=0
GO TO 2
1 PTA=U+1.5
PTB=U-1.5
C
READ IN THE INCREMENT AND THE NEW INITIAL HEIGHT

2 PRINT 203
READ (5, *) DT, W1
TIME=TIME+DT
DT=DT/T
IF (TIME.GT.TI) STOP
IF (TIME.LE.TI) W10=(W10*(TI-TIME)+W11*TIME)/TI
IF (TIME.GT.T1.AND.TIME.LE.T2) W10 = (W11*(T2-TIME)+W12*(TIME-T1)) / (T2-T1)
IF (TIME.GT.T2) W10 = (W12*(T1+T-TIME) + W11*(TIME-T21)) / (T1+T-T2)

X = W10
C = (G*M1)**0.5/GHO
ICOUNT = 1
PRINT *, XM10 = XM10.8H TIME = TIME
GO TO 4
3 DO 15 I=1, IMAX, IM1
13 PRINT 203, [RESULTS(I, J), (RESULTS(I, J)), J=2, 6]
C

DEFINE THE LIMITS AND THE INITIAL VALUE OF THE VELOCITY U

PRINT 204
READ *, PTA, PTB, U, OPT
IF (U.GT.60) STOP
PTA = PTA/GHO
PTB = PTB/GHO
U = U/GHO
ICOUNT = 1
4 X0 = 0
I = 1
C

COMPUTE FOUR VALUES BY RUNGE-KUTTA AND PRINT OUT

XM1(I) = XM1 = U**2*M*C
XM2(I) = XM2 = U**2*M*C
CALL STORE (RESULTS, XM1, XM2, FAC, M, I, IMAX)
DO 10 I = 2, 4

FK11 = DERFN(M1, XM1, XM2, XM10)
FK12 = DERFN(M2, XM2, XM10)
FK21 = DERFN(M1, XM1, XM2, XM10)
FK22 = DERFN(M2, XM2, XM10)
FK31 = DERFN(M1, XM1, XM2, XM10)
FK32 = DERFN(M2, XM2, XM10)
FK41 = DERFN(M1, XM1, XM2, XM10)
FK42 = DERFN(M2, XM2, XM10)
X0 = X**H
XM1(I) = XM1 + H*(FK11+2.*FK21+2.*FK31+FK41)/6.
XM2(I) = XM2 + H*(FK12+2.*FK22+2.*FK32+FK42)/6.
M1 = XM1(I)
M2 = XM2(I)
CALL STORE (RESULTS, XM1, XM2, FAC, M, I, IMAX)
10 CONTINUE
F1 = DERFN(XM1, XM2, XM1, XM2, XM10)
F2 = DERFN(XM1, XM2, XM1, XM2, XM10)
F3 = DERFN(XM1, XM2, XM1, XM2, XM10)
F4 = DERFN(XM1, XM2, XM1, XM2, XM10)
F5 = DERFN(XM1, XM2, XM1, XM2, XM10)
F6 = DERFN(XM1, XM2, XM1, XM2, XM10)
F7 = DERFN(XM1, XM2, XM1, XM2, XM10)
F8 = DERFN(XM1, XM2, XM1, XM2, XM10)
F9 = DERFN(XM1, XM2, XM1, XM2, XM10)
WRITE (7, 211) ISTEP, ICOUNT, F1, F6, F2, F7, F3, F8, F4, F9
I = 4

C ADAMS-MOUlTON PREDICTOR/CORRECTOR METHOD

30 I = I + 1
PM1 = W1 + H*(55*F4 - 89*F3 + 37*F2 - 9*F1)/24
PH2 = W2 + H*(55*F9 - 89*F8 + 37*F7 - 9*F6)/24
F5 = DERFN(PW1, PW2, W10)
F0 = DERFN(PW2, PW1, W20)
CH1 = W1 + H*(9*F5 + 19*F4 - 5*F3 + F2)/24
CH2 = W2 + H*(9*F0 + 19*F9 - 5*F8 + F7)/24
W1 = CH1 + 19*(PH1 - CH1)/270
W2 = CH2 + 19*(PH2 - CH2)/270
X0 = X0 + H
F1 = F2
F2 = F3
F3 = F4
F4 = DERFN(WL, M2, W10)
F6 = F7
F7 = F8
F8 = F9
F9 = DERFN(W2, M1, W20)
IF (I/I10*10.0.E0, 10) WRITE (7, 212) I, F4, F9
CALL STORE (RESULT, X0, M1, M2, X1, M1, X1, IMAX)
IF (I.LT. IMAX) GOTO 30

C BISECTOR SEARCH FOR THE CORRECT VELOCITY FOR
C THE BOUNDARY VALUES PROBLEM

R = (WL - W2)/2
IF (OPT.EQ.1) GO TO 38
IF (AM/0 R) LE EPS) GOTO 40
IF (R) .25, .25, 32

C LOWER LIMIT IS ADJUSTED UPWARDS

25 PTA = U
GO TO 35

C UPPER LIMIT IS ADJUSTED DOWNWARDS

32 PTA = U

C THE NEW VELOCITY IS COMPUTED

35 U = (PTA + PTB)/2
ICOUNT = ICOUNT + 1
GO TO 37 I = 1, IMAX, 10

37 WRITE (8, 1991) (RESULT(I, J), J = 1, 6)
IF (ICOUNT.LE.10) GO TO 4
PRINT 47
GO TO 3
38 PRINT 150, U*4GHO, R*4GHO
150 FORMAT(*VELOCITY GUESS =".G14.7," VELOCITY AT CORE =",".G14.7," INPUT 1 IF SATISFIED, 2 TO TRY AGAIN, 3 TO STOP*)
   ICOUNT=1
   READ *,DECIDE,
   IF(DECIDE.EQ.3) STOP
   IF(DECIDE.EQ.1) GO TO 60
   PRINT*, 25H INPUT THE NEW VALUE OF U
   READ *, U
   U=U*4GHO
   GO TO 4

SUCCESS!! YOU HAVE CONVERGED TO A SOLUTION
PREPARE TO MOVE ON TO THE NEXT VALUE
-----------------------------------------------

40 WRITE(6,42) ,ISTEP,ICOUNT
WRITE(6,60)
DO 44 I=1,IMAX,I10
WRITE(8,24) RESULTS(I,1),RESULTS(I,2)
44 WRITE(6,20) RESULTS(I,1),RESULTS(I,J),J=2,6
   ISTEP=ISTEP+1
GO TO 1

42 FORMAT(**, STEP NO.*.,I3,* REQUIRED »,I3,* ITERATIONS */)  
47 FORMAT(***, SOMETHING IS WRONG, YOU ARE NOT CONVERGING.*
   ** FAST ENOUGH.** TRY READING IN A NEW VALUE OF U.**
60 FORMAT(7X,*X*,12X,*Y*,14X,*U*,13X,*G*,12X,*W1*,12X,*W2*,14X)
199 FORMAT(62X,G11.4))
200 FORMAT(F10.4,3X,F14.7,4(3X,F10.4))
211 FORMAT(* ENTER THE VALUES OF TIME AND DEPTH AT THE/ * 
   ** TWO TIME CONSTANTS T1 AND TC FROM THE OUTCROP ANALYSIS/ * 
   ** AND THE PERIOD OF OSCILLATION, T/.**)
202 FORMAT(* ENTER POROSITY(H), COEFFICIENTS(A,9), LENGTH(XL),/ * 
   ** MEAN WATER LEVEL(H0), UPPER AND LOWER LIMITS OF VELOCITY, / * 
   ** AND THE INITIAL GUESS FOR THE VELOCITY(U)**/)
203 FORMAT(* ENTER THE INCORRECT AND THE NEW INITIAL HEIGHT/)  
204 FORMAT(* INPUT (A) UPPER LIMIT, (B) LOWER LIMIT.*
   ** (C) GUESSED VELOCITY, AND** (D) AN OPTION TO CONTROL* 
   ** THE UPDATING OF THE VELOCITY (IF YES, TYPE 1)*/)
210 FORMAT(1H1,5X,"THIS TAPE7 GIVES THE VALUES OF THE DERIVATIVE", 
   */5X,* AT EACH STEP OF THE INTEGRATION. THIS OF VALUE FOR THE*, 
   */5X,* ANALOG/HYBRID SIMULATION,*6X,60(1H-1)
211 FORMAT(5X,*STEP NO.*,14,4X,*ITERATION NO.*,14,7, 
   */15X,*1*,5X,2G14.7,15X,*2*,5X,2G14.7,15X,*3*,5X, 
   */2G14.7,15X,*4*,5X,2G14.7)
212 FORMAT(14X,I3,5X,2G14.7)
214 FORMAT(10X,F5.2,3X,F10.4)
215 FORMAT(* ----- STEP NO.*,I4,* ------- ITERATION NO.*, 
   */I4,* * * * * * * * *)
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DIMENSION RESULTS(IMAX,6)
COMMON GH0,XL
RESULTS(NUM,1)=X0*XL
RESULTS(NUM,2)=(W1-W2)**2/FAC*GH0**2
RESULTS(NUM,3)=(W1+W2)/2*GH0
RESULTS(NUM,4)=(W1-W2)/(4*PORE)*GH0
RESULTS(NUM,5)=W1*GH0
RESULTS(NUM,6)=W2*GH0
RETURN
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APPENDIX E

ORGANIZATION OF DATA
APPENDIX E

ORGANIZATION OF DATA

The experimental data used in this study, after Nasser (35), consists of four types of porous media: 4.4 cm, 1.7 cm, 0.7 cm crushed rock, and 1.6 cm rounded quartz. There were eight parameters of particular interest for each rock type, seven of which are listed in Table E.1, along with their respective ranges: a, the linear resistance coefficient; b, the Non-Darcy resistance coefficient; m, the porosity; T, the period of oscillation; L, the length of rockfill; h₀, the mean water level; A₀, the maximum wave amplitude; and Θ, the slope of the embankment.

There were 15 case groups among the four rock types. There was one sloping embankment case group for each of the four media; i.e., case groups L, M, N, O. These had a constant slope of 26.565° (i.e., 1:2). All the remaining cases were vertical embankments.

The data is broken down into individual cases identified by a letter designating the group followed by a number, e.g. Case C-3. Each case group refers to a particular rock type that has identical a, b, m, L and Θ values, as can be seen in Table E.2. Therefore only T, h₀ and A₀ vary within a group.

The case groups are divided into fast drop and slow drop cases to avoid confusion. Tables E.3 through E.9 show the pertinent variables
for the fast drop cases while Table E.10 summarizes the slow drop cases. The fast drop cases have three extra calculated parameters listed which are helpful in the analysis of the outcrop point movement: $W$ is the dimensionless parameter that determines whether a case is fast drop, its value being necessarily less than $1/\sin^2 \theta$; $\delta$ is introduced into one of the pot settings in the analog simulation and has a very wide range of values which creates scaling problems; $P_k$, another dimensionless number that indicates the percentage deviation of the fast drop case from the slow drop case.

To summarize, there were 224 cases studied, 165 fast drop cases and 59 slow drop. Of the 165 fast drop cases investigated, 92 had vertical embankments. Only 2 slow drop cases had sloped embankments.
### Table E.1

**Properties of Media and Range of Variables**

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<th>Parameter</th>
<th>4.40 cm Crushed Rock</th>
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## Table E.2

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*Q = Quartz*
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1.6 cm Quartz Rectangular Embankment

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APPENDIX F

PHOTOGRAPHS OF THE EAI 690 HYBRID COMPUTER
APPENDIX F

PHOTOGRAPHS OF THE EAI 690 HYBRID COMPUTER

This appendix is intended to familiarize the reader with the hardware used in this study, the EAI 690 Analog/Hybrid Computer. This system comprises a digital computer EAI 640 (Fig. F.1), and interfacing unit EAI 693 (Fig. F.1) and an analog computer EAI 680 (Fig. F.2).

The EAI 640 can read or write on magnetic tape or paper tape using the teletype printer and octal converters as controlling devices. The EAI 640 can be operated as a separate unit, can be linked with the CDC Cyber processors within the University or connected to the EAI 680 Analog Computer through the EAI 693 Interfacing Unit.

The Analog Computer (Fig. F.3) consists basically of an analog patch panel, digital logic and a control panel. The control panel sets the mode and speed of operation, sets pots, contains a four-digit accurate digital voltmeter and indicates the logic components that are in operation. The digital logic consists of And Gates, comparators, switches, controls over certain operational amplifiers, etc. (Fig. F.4).

The Analog Patch Panel (Fig. F.5) consists of 120 operational amplifiers that have the capability of performing all the basic mathematical operations including integration. The facility at Concordia at present has only about 55 operational amplifiers which limits the methods
of simulation that can be applied. A colour code helps to avoid confusion in patching circuits. Table F.1 summarizes the functions of the various colours.

<table>
<thead>
<tr>
<th>Colour</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>Input</td>
</tr>
<tr>
<td>Red</td>
<td>Output</td>
</tr>
<tr>
<td>Black</td>
<td>Ground</td>
</tr>
<tr>
<td>Yellow</td>
<td>Potentiometers</td>
</tr>
<tr>
<td>White</td>
<td>Amplifier Control Options</td>
</tr>
<tr>
<td>Brown (Striped)</td>
<td>Relay Switch</td>
</tr>
<tr>
<td>Red⁺</td>
<td>Positive Reference Voltage</td>
</tr>
<tr>
<td>Orange⁻</td>
<td>Negative Reference Voltage</td>
</tr>
</tbody>
</table>
Fig. F.1 — EAI 680 Digital Computer and EAI 683 Interface.

Fig. F.2 — EAI 680 Analog Computer with Peripheral Output Devices.
Fig. F.3 — EAI 680 Analog Console and Patch Panel.

Fig. F.4 — Typical Digital Logic Patching Tray.
Fig. F.5 — Typical Analog Patching Tray.
APPENDIX G

NOMENCLATURE

a, a₁ - resistance coefficients
a_B, a_F - constants
A - constant
A₀ - maximum amplitude

b, b₁ - non-Darcy resistance coefficients
B - constant

c - celerity
C - constant
CSDT - continuous-space discrete-time

D - hydraulic depth; constant
DSCT - discrete-space continuous time

EDA - electronic differential analyzer
F - non-Darcy friction term; fast second
F-MS - fast-milli-second
f(t) - time dependent function
f(u) - square matrix of functions

g - acceleration due to gravity
g(x,t),G - forcing functions

h - water level
h_0 - mean water level
Δh - numerical step size

i - hydraulic gradient
i_0 - threshold gradient
I - identity matrix
IC - initial condition mode

j - index of discretized time

k - non-Darcy hydraulic conductivity
K_1 - Darcy hydraulic conductivity
K_0,K - scaling factors

L - length of rockfill
L,L_B,L_F - operators on functions

MOC - Method of Characteristics
ODE - Ordinary Differential Equations
QP - Operate Mode
PDE - Partial Differential Equations
P_k - dimensionless parameter
q - macroscopic velocity; lateral inflow
R - correlation coefficient
S_f - friction or energy line slope
S_o - channel bed slope
t - time independent variable
t_1, t_2, t_C - time constants
Δt - time increment
T - period of oscillation
TKG - constant
u - horizontal pore velocity
V - maximum internal fall velocity
V - actual pore velocity
V_f - vertical velocity
V_{fmax} - maximum internal fall velocity
V_I - input voltage
V_O - output voltage
\( \mathbf{W} \) - vector of transformed dependent variables; dimensionless parameter

\( \mathbf{w}^1, \mathbf{w}^2 \) - adjoint eigenvectors

\( W_1, W_2 \) - characteristic independent variables

\( \mathbf{X} \) - space independent variable

\( X_{\text{max}} \) - maximum distance

\( y \) - depth of water

\( Y_C \) - elevation of outcrop point movement

\( Y_f \) - elevation of free water level

\( \Delta Y_p \) - maximum deviation between \( Y_C \) and \( Y_f \)

\( Z_1 \) - input impedance

\( Z_f \) - feedback impedance

\( \alpha \) - characteristic direction; energy coefficient; damping ratio

\( \beta \) - characteristic direction; time scale factor

\( \delta \) - constant

\( \eta \) - perturbation height wrt to mean water level

\( \theta \) - interpolation factor; angle of inclination of rockfill with the horizontal

\( \lambda \) - vector of eigenvalues

\( \chi \) - dimensionless number

\( \omega \) - frequency of oscillation

\( \omega_n \) - undamped natural frequency of oscillation
BIBLIOGRAPHY
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35. NASSER, M.S., "Theoretical and Experimental Analyses of Wave Motion in Rockfill Structures", A Thesis presented to the University of Windsor in Partial Fulfillment of the Requirements for the Ph.D. degree in Civil Engineering, Windsor, Canada, 1974.


