ANALYSIS AND SYNTHESIS OF AN RC MULTIPLE-LOOP FEEDBACK NETWORK WITH FEED-FORWARD USING AN OPERATIONAL AMPLIFIER

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ABSTRACT

For the multiple loop feedback network with feedforward elements constrained by an operational amplifier, rules for writing the A and C parameters have been formulated.

A realization for a second order voltage transfer function using a low pass type structure has been given. The same can be used to obtain a high pass realization by the RC : CR transformation. The sensitivity of such a realization has been studied for a single parameter.

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CHAPTER 1

INTRODUCTION

1.1 GENERALITIES

With the advent of microminituarization and integrated circuits and also due to the almost ideal nature of the resistance and capacitance much effort has been directed towards the realization of active filters using these elements. The active RC filters as compared to the passive RC filters can realize poles located outside the negative real axis of the s-plane and hence can obtain higher Q factors. [1]

Many realizations have been formulated using RC elements and an operational amplifier as the active element. Ideally, the operational amplifier is an infinite gain d.c. amplifier with infinite input impedance, zero output impedance, whose output voltage is of opposite polarity to that of one of the inputs and is of the same polarity to that of the other input terminal. In practice, however, the operational amplifier is a non ideal device, characterized by a frequency dependent voltage gain. It can be realized with an open-loop d.c. gain of 10^4 or more, an input impedance greater than 100 k ohms and an output impedance less than 100 ohms. In addition, there exists a limitation regarding the maximum amplitude of the input and output signals in order that the amplifier be in the linear range.

For all analysis and synthesis, such an operational amplifier is assumed to be an ideal device, since the above limitations do not affect the overall characteristics of the circuit to any appreciable extent over the frequency range of operation.

The main problems associated with any active network are "stability" and "sensitivity". A passive network is inherently stable, as the poles are always contained in the left-half of the s-plane, irrespective of the changes in the elemental values. But, an active network can become unstable with variations in the elemental values.

Sensitivity is another problem which requires close scrutiny. Variations in elemental values give rise to the displacement of poles and zeroes from their nominal positions, thereby altering the characteristics considerably. Therefore, the designer must ensure that the sensitivity shall not be high. This is the reason why any higher order transfer function is realized by a cascade connection of an appropriate number of filters, each of them realizing a second order function.

1.2 SCOPE OF THIS STUDY

One of the circuits used both in active filters and analog computers is the multiple-loop feedback network constrained by an operational amplifier. [2],[3],[4]

The object of this study is to analyze and realize the network shown in figure 1.1 when the amplifier gain K is finite and infinite. (This network was considered by Sewell [2] in another context.)

It will be noted that if $Y''_2, Y''_4, \ldots Y''_{2N}$ are made zero the multiple-loop feedback network mentioned in references [2], [3] and [4] results. The inclusion of these admittances constitutes feedforward and hence the network is known as a multiloop feedback system with feedforward constrained by an operational amplifier.

The study consists of analyzing the network of figure 1.1 to obtain the A and C parameters and also to realize voltage transfer functions having poles and zeroes in the left half s-plane. In addition the sensitivity and stability properties of such a realization are examined.

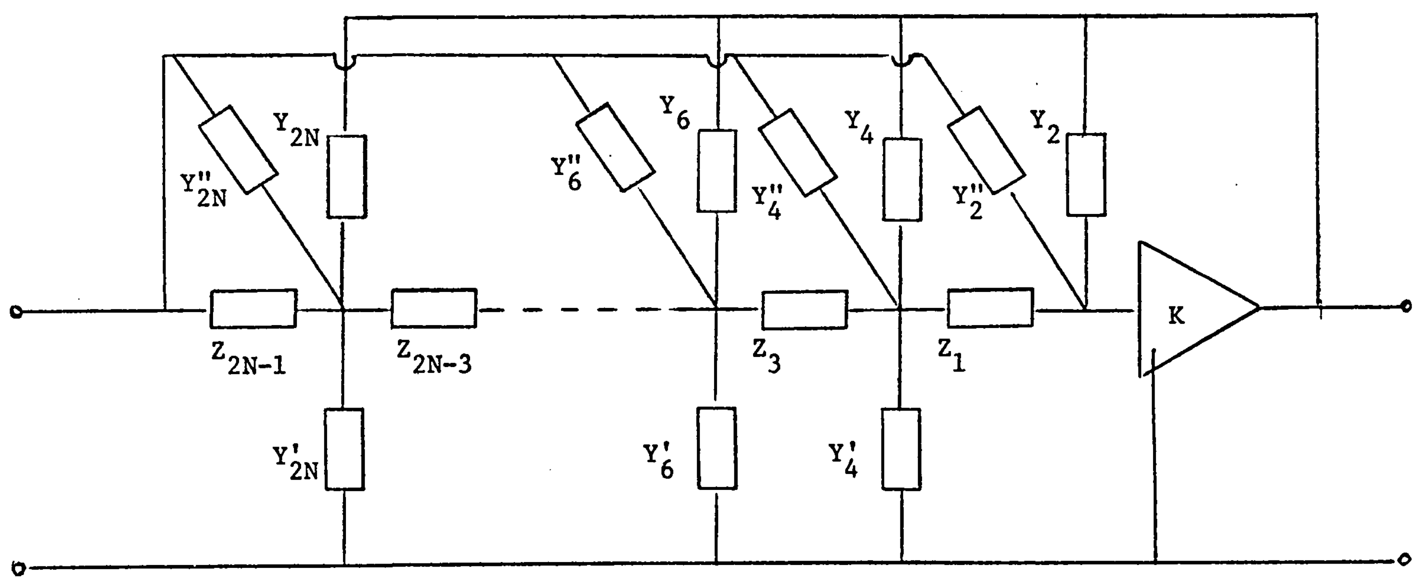

FIGURE 1.1 Multiple loop feedback network with feedforward constrained by an operational amplifier.

CHAPTER 2

ANALYSIS OF THE NETWORK

2.1 INTRODUCTION

In this chapter, we shall analyze the basic network to get the A and C parameters. Also, the rules of writing the A and C parameters are formulated.

2.2 THE A-PARAMETER

Referring to the figure 1.1, the indefinite admittance matrix [5] will be as shown on page 6.

From this indefinite admittance matrix $\hat{Y}$, the A parameter is given by the equation

$$A = \text{Sign } (m-n) \text{ Sign } (i-j) \frac{\hat{Y}^{mn}_{mn}}{\hat{Y}^{mn}_{ij}}$$

where

$$\hat{Y}^{mn}_{ij} = (-1)^{m+n+i+j} \times \left[\begin{array}{l} \text{determinant of the submatrix} \\ \text{obtained by omitting the m row} \\ \text{and n row, the i column and the} \\ \text{j column of } \hat{Y} \end{array} \right]$$

$$\hat{Y}^{mn}_{mn} = (-1)^{m+n+m+n} \times \left[\begin{array}{l} \text{determinant of the submatrix} \\ \text{obtained by omitting the m row} \\ \text{and n row, the m column and the} \\ \text{n column of } \hat{Y} \end{array} \right]$$

As can be seen, the A parameter takes the form

$$A = - \frac{K\ F_2(Z,Y) - F_3(Z,Y)}{K\ F_1(Z,Y)} \qquad (2.1)$$

$$
\begin{bmatrix}
Y_{2N-1}+\sum_{i=1}^{N} Y''_{2i}, & -Y_{2N-1}-Y''_{2N} & , & -Y''_{2N-2} & , \cdots\cdots , & -Y''_2 , 0, & 0 & , 0 \\
-Y_{2N-1}-Y''_{2N}, & Y_{2N-1}+Y_{2N-3} & & -Y_{2N-3} & , \cdots\cdots , & \cdots 0, 0, & -Y_{2N} & , -Y'_{2N} \\
 & +Y_{2N}+Y'_{2N}+Y''_{2N}, & & & & & & \\
-Y''_{2N-2}, & -Y_{2N-3} & , & Y_{2N-3}+Y_{2N-5} & -Y_{2N-5}, & \cdots 0, 0, & -Y_{2N-2}, & -Y'_{2N-2} \\
\vdots & \vdots & & +Y_{2N-2}+Y'_{2N-2}+Y''_{2N-2}, & & & & \\
 & & & \vdots & & & & \\
0 , & 0 & , & \cdots\cdots\cdots\cdots & , \cdots\cdots\cdots & 0, 0, & 0 & , 0 \\
0 , & 0 & , & \cdots\cdots\cdots\cdots & , \cdots\cdots\cdots & K, 0, & -1 & , 1-K \\
0 , & -Y_{2N} & , & -Y_{2N-2} & , \cdots\cdots\cdots & 0, -1, & \sum_{i=1}^{N} Y_{2i}+1, & 0 \\
0 , & -Y'_{2N} & , & -Y'_{2N-2} & , \cdots\cdots\cdots & -K, 1, & 0 , & K-1 \\
 & & & & & & & +\sum_{i=1}^{N} Y'_{2i}
\end{bmatrix}
$$

This form is similar to the one found by other authors [6].

Furthermore it will be noted that the non-K multiplied portion of the numerator is nothing but the A parameter of a passive ladder network as shown on figure 2.1 and the rules for writing this parameter have already been given by other authors [7].

When the amplifier gain K is made infinite, the A parameter reduces to the form

$$A = -\frac{F_2\ (Z,Y)}{F_1\ (Z,Y)}$$

It is now essential to write the set of rules for $F_2(Z,Y)$ and $F_1(Z,Y)$. For this purpose Table 2.1 is set up. Following the rules set down by Ramachandran and Mehrotra [6] and also by Holt and Sewell [8] for the double ladder network without Y''_2, Y''_4,....Y''_{2N}, the A parameter for the network under consideration along with the topological interpretation can be written as follows:

2.2.1 DENOMINATOR TERMS F_1 (Z,Y)

All terms in the denominator contain K as a factor and consists of a unity factor 1, a set of two terms product of the type ZY, a set of four terms product of the type ZZYY, a set of six terms product of the type ZZZYYY, etc... The last term has

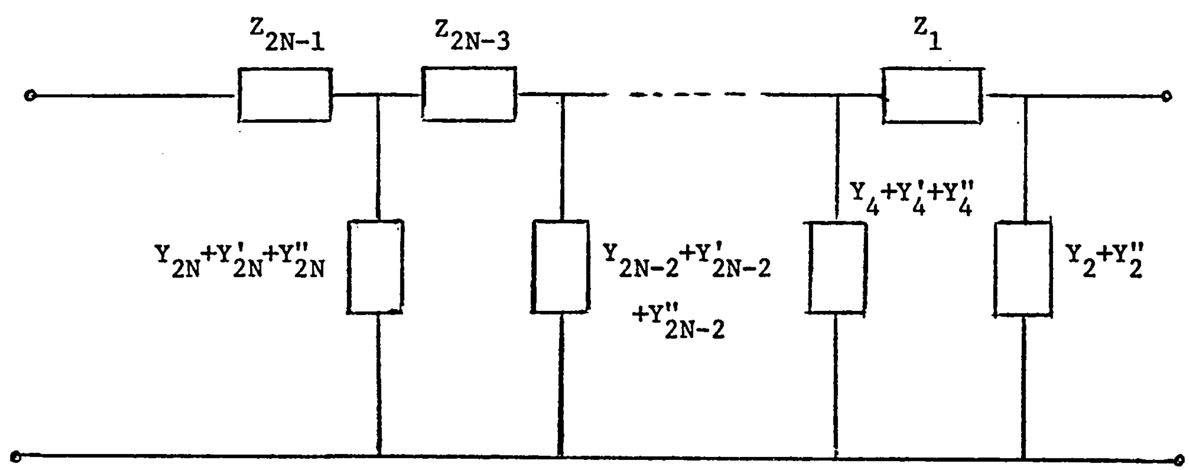

FIGURE 2.1 The passive network for the A-parameter

TABLE 2.1

ZY Table for writing the transfer function

Section	1	2	3	....	N-1	N
Series Element	Z_1	Z_3	Z_5			Z_{2N-1}
Upper Ladder Element	Y_2	Y_4	Y_6			Y_{2N}
Lower Ladder Element	Y'_2	Y'_4	Y'_6			Y'_{2N}
Feedforward element	Y''_2	Y''_4	Y''_6			Y''_{2N}

N factors in Z and N factors in Y.

The terms can be written by these rules.

Set of terms with one Z and one Y

Considering any one series impedance, the preceding and following* series impedances are short circuited. All ladder admittances Y_{2i} and Y'_{2i} are open-circuited. The product of the series impedance and the sum of the remaining admittances constitute a term in this set. The other terms are obtained by considering each series impedance once. The set is finally made up of the sum of all such terms.

Set of terms with two Z and two Y

Considering any two series impedances, Z_{2i-1} and Z_{2k-1} where $k > i$, all other series impedances are short circuited. All following ladder admittances Y_{2i}, Y_{2i-2}, etc... and Y'_{2i}, Y'_{2i-2}, etc... are open circuited.

The set of terms is made up of the sum of all such terms, All Z_{2i-1} Z_{2k-1} are multiplied by $(k > i)$:

---

* If Z_{2i-1} is considered, Z_{2i+1} is termed the preceding series impedance and Z_{2i-3} is termed the following series impedance. In addition, Y_{2i}, Y_{2i-2}, etc... are termed the following admittances and Y_{2i+2}, etc... are termed the preceding admittances.

(i) Y''_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's plus Y''_ℓ 's

$2i < \ell \leq 2k$, ℓ is even

Set of terms with three Z and three Y

Considering any three series impedances, Z_{2i-1}, Z_{2k-1} and Z_{2p-1} where $i < k < p$, all other series impedances are short circuited. All following ladder admittances Y_{2i}, Y_{2i-2}, etc..., Y'_{2i}, Y'_{2i-2}, etc... are open circuited.

The set of terms is made up of the sum of all such terms,

All Z_{2i-1} Z_{2k-1} Z_{2p-1} are multiplied by $(i < k < p)$:

(i) Y''_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's plus Y''_ℓ 's ,

$2i < \ell \leq 2k$, ℓ is even

(iii) the sum of Y_m 's plus Y'_m 's plus Y''_m 's ,

$m \geq 2p$, m is even

Rules for sets of four Z and four Y, five Z and five Y, etc... can also be written by expanding on the method given above for N sections of the network represented in Figure 1.1.

2.2.2 NUMERATOR TERMS $F_2(Z,Y)$

This portion of the numerator is caused by the active device and consists of a set of two terms product of the type ZY, a set of four terms product of the type ZZYY, a set of six terms product of the type ZZZYYY, etc..., a last set of N factors in Z and N factors in Y, all multiplied by the amplifier gain K. The terms can be written by these rules.

Set of terms with one Z and one Y

Considering any one series impedance the preceding and following series impedances are short circuited. All lower ladder and feedforward admittances are open circuited i.e. Y', Y". The product of the series impedance and the sum of the remaining admittances constitute a term in this set. The other terms are obtained by considering each series impedance once. This set is finally made up of the sum of all such terms.

Set of terms with two Z and two Y

Considering any two series impedances, Z_{2i-1} and Z_{2k-1} where k > i, all other series impedances are short circuited. All following feedforward admittances and lower ladder admittances, Y''_{2i}, Y'_{2i}, Y''_{2i-2}, Y'_{2i-2}, etc... are open circuited.

This set of terms is made up of the sum of all such terms,

All $Z_{2i-1}\ Z_{2k-1}$ are multiplied by (k > i):

(i) Y_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ's plus Y'_ℓ 's plus Y''_ℓ 's

$2i < \ell \leq 2k$, ℓ is even

Set of terms with three Z and three Y

Considering any three series impedances, Z_{2i-1}, Z_{2k-1} and Z_{2p-1} where $i < k < p$, all other series impedances are short circuited. All following feedforward admittances are open circuited i.e. Y''_{2i}, Y''_{2i-2} etc...

This set of terms is made up of the sum of all such terms,

All $Z_{2i-1}\ Z_{2k-1}\ Z_{2p-1}$ are multiplied by $(i < k < p)$:

(i) Y_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ's plus Y'_ℓ 's plus Y''_ℓ 's ,

$2i < \ell \leq 2k$, ℓ is even

(iii) the sum of Y_m 's plus Y'_m 's plus Y''_m 's $(m \geq 2p)$ m is even

Rules for sets of four Z and four Y, five Z and five Y, etc... can also be written by expanding on the method given above for N sections of the network represented by figure 1.1.

2.3 THE C PARAMETER

Knowing the A parameter for the network of figure 1.1, the C parameter is obtained once the driving point impedance has been determined using the method described in [5].

The C parameter takes the form

$$C = -\frac{KF_{c2}(Z,Y) - F_{c3}(Z,Y)}{K\ F_1(Z,Y)} \qquad (2.2)$$

Using the table 2.1, the set of rules for $F_{c2}(Z,Y)$ and $F_{c3}(Z,Y)$ i.e. the numerator of C, will now be written along with the topological interpretation.

The rules for the denominator have already been given since the denominator of A and C parameters are the same.

2.3.1 NUMERATOR TERMS $F_{c2}(Z,Y)$

This consists of a set of terms of Y 's only, a set of three terms product of the type ZYY, a set of five terms product of the type ZZYYY, a set of seven terms product of the type ZZZYYYY, etc... the last term containing (N-1) factors in Z and N factors in Y. All factors are multiplied by the amplifier gain K.

These terms can be written by the following rules,

Set of terms with Y only

Considering any one upper ladder admittance, Y_{2i}, the series impedances are short circuited. All lower ladder admittances Y'_{2i}, are open circuited. The sum of the upper ladder admittances, Y_{2i}, Y_{2i+2}, etc... Y_{2N} constitutes the set of terms specified.

Set of terms with one Z and two Y

Considering any one series, impedance Z_{2i-1}, the preceding and following series impedances are short circuited. All following lower ladder admittances, Y'_{2i}, Y'_{2i-2}, etc... are open circuited.

The set of terms is made up of the sum of the following subsets:

(a) All Z_{2i-1} are multiplied by:

(i) Y_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ's plus Y'_ℓ 's , $\ell > 2i$, ℓ is even

(b) All Z_{2i-1} are multiplied by:

(i) Y''_j 's , $j \leq 2i$, j is even

(ii) the sum of Y_ℓ's , ℓ from 2 through 2N , ℓ is even

<u>Set of terms with two Z and three Y</u>

Considering any two series impedances, Z_{2i-1} and Z_{2k-1} where $k > i$, all other series impedances are short circuited. All following lower ladder admittances Y'_{2i}, Y'_{2i-2}, etc... are open circuited.

This set of terms is made up of the sum of subsets listed below:

(a) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y_j 's , j from 2 to 2N, j is even

(ii) Y''_ℓ 's , $\ell < 2i + 1$, ℓ is even

(iii) Y''_m 's , $2i + 1 < m \leq k$, m is even

(b) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y_j 's , $j \leq 2i$, j is even

(ii) Y''_ℓ 's , ℓ from 2 to 2N

(iii) the sum of Y_m 's plus Y'_m 's , $2i+1<m\leq 2k$, m is even

(c) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y_j 's , j from 2 to 2N

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's plus Y''_ℓ 's ,
$\ell > j$, ℓ is even

(iii) the sum of Y_m 's plus Y'_m 's plus $\alpha Y''_m$'s,
$m > \ell$, m is even
$\alpha=1, k=N$
$\alpha=0, k<N$

(d) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) $\alpha Y''_j$'s, $j \leq 2i$, j is even
$\alpha=1, k<N$
$\alpha=0, k=N$

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's, $\ell > j$, ℓ is even

(iii) Y_m 's, $m > \ell$, m is even

<u>Set of terms with three Z and four Y</u>

Considering any three series impedances, Z_{2i-1}, Z_{2k-1}, Z_{2p-1} where $i < k < p$ all other series impedances are short circuited. All following lower ladder admittances Y'_{2i}, Y'_{2i-2}, etc... are open circuited. This set of terms is made up of the sum of subsets listed below:

(b) All $Z_{2i-1}\ Z_{2k-1}$ are multiplied by $(i < k)$:

(i) Y_j 's , $j \leq 2i$, j is even

(ii) Y''_ℓ 's , ℓ from 2 to 2N

(iii) the sum of Y_m 's plus Y'_m 's , $2i+1<m\leq 2k$, m is even

(c) All $Z_{2i-1}\ Z_{2k-1}$ are multiplied by $(i < k)$:

(i) Y_j 's , j from 2 to 2N

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's plus Y''_ℓ 's ,

$\ell > j$, ℓ is even

(iii) the sum of Y_m 's plus Y'_m 's plus $\alpha Y''_m$'s,

$m > \ell$, m is even
$\alpha=1, k=N$
$\alpha=0, k<N$

(d) All $Z_{2i-1}\ Z_{2k-1}$ are multiplied by $(i < k)$:

(i) $\alpha Y''_j$'s, $j \leq 2i$, j is even
$\alpha=1, k<N$
$\alpha=0, k=N$

(ii) the sum of Y_ℓ 's plus Y'_ℓ 's, $\ell > j$, ℓ is even

(iii) Y_m 's, $m > \ell$, m is even

Set of terms with three Z and four Y

Considering any three series impedances, Z_{2i-1}, Z_{2k-1}, Z_{2p-1} where $i < k < p$ all other series impedances are short circuited. All following lower ladder admittances Y'_{2i}, Y'_{2i-2}, etc... are open circuited. This set of terms is made up of the sum of subsets listed below:

(a) All Z_{2i-1} Z_{2k-1} Z_{2p-1} are multiplied by $(i < k < p)$:

(i) Y_j 's , j from 2 to 2N

(ii) Y''_ℓ , $\ell < 2i+1$, ℓ is even

(iii) Y''_m , $2i+1 < m \leq 2k$, m is even

(iv) Y''_q , $q \geq 2p$, q is even

(b) All Z_{2i-1} Z_{2k-1} Z_{2p-1} are multiplied by $(i < k < p)$:

(i) Y_j , $j \leq 2i$, j is even

(ii) Y''_σ , where $\sigma \geq j$, σ is even

(iii) Y''_ℓ , where $\ell > \sigma$, ℓ is even

(iv) the sum of Y_m 's plus Y'_m 's when $m \neq \ell$ and $\sigma = j$, m is even

(c) All Z_{2i-1} Z_{2k-1} Z_{2p-1} are multiplied by $(i < k < p)$:

(i) Y''_j 's, $j \leq 2i$, j is even

(ii) Y''_σ 's , $\sigma > j$ to 2N , σ is even

(iii) Y_σ 's , $\sigma > j$ to 2N , σ is even

(iv) the sum of Y_m 's plus Y'_m 's, $m \neq \sigma$ but greater than j m is even

(d) All Z_{2i-1} Z_{2k-1} Z_{2p-1} are multiplied by $(i < k < p)$:

(i) Y_j , j = from 2 to 2i , j is even

(ii) Y''_ℓ, ℓ = from 2 to 2N, ℓ is even

(iii) The sum of Y_m 's plus Y'_m 's , $m > 2i+1$, m is even

(iv) The sum of Y_q 's plus Y'_q 's , $q > 2k+1$, q is even

Rules for sets of terms of four Z and five Y, five Z and six Y, etc... can also be written with a similar type of topological interpretation.

2.3.2 NUMERATOR TERMS $F_{c3}(Z,Y)$

These terms consist of sets of terms similar to $F_{c2}(Z,Y)$ with the exception that the factors formed are not multiplied by the amplifier gain K. From the indefinite admittance matrix shown on page 5 it can be seen that the $F_{c3}(Z,Y)$ is the C parameter of the network shown on figure 2.2. These terms can now be written by these rules.

Set of terms with Y only

Considering N sections of the network all series impedances Z_{2i-1} are short circuited, hence the feedforward admittances Y''_{2i}, Y''_{2i+2}, etc... are also short circuited. The sum of all lower ladder admittances i.e. Y'_{2i}, Y'_{2i+2}, etc... and upper ladder admittances ie. Y_{2i}, Y_{2i+2}, etc... constitute the set of terms specified.

Set of terms with one Z and two Y

Considering any one series impedance Z_{2i-1}, the preceding and following series impedances are short circuited. This set of terms is made up of subsets of such terms.

(a) All Z_{2i-1} are multiplied by:

(i) The sum of Y_j 's plus Y'_j 's , $j \leq 2i$, j is even

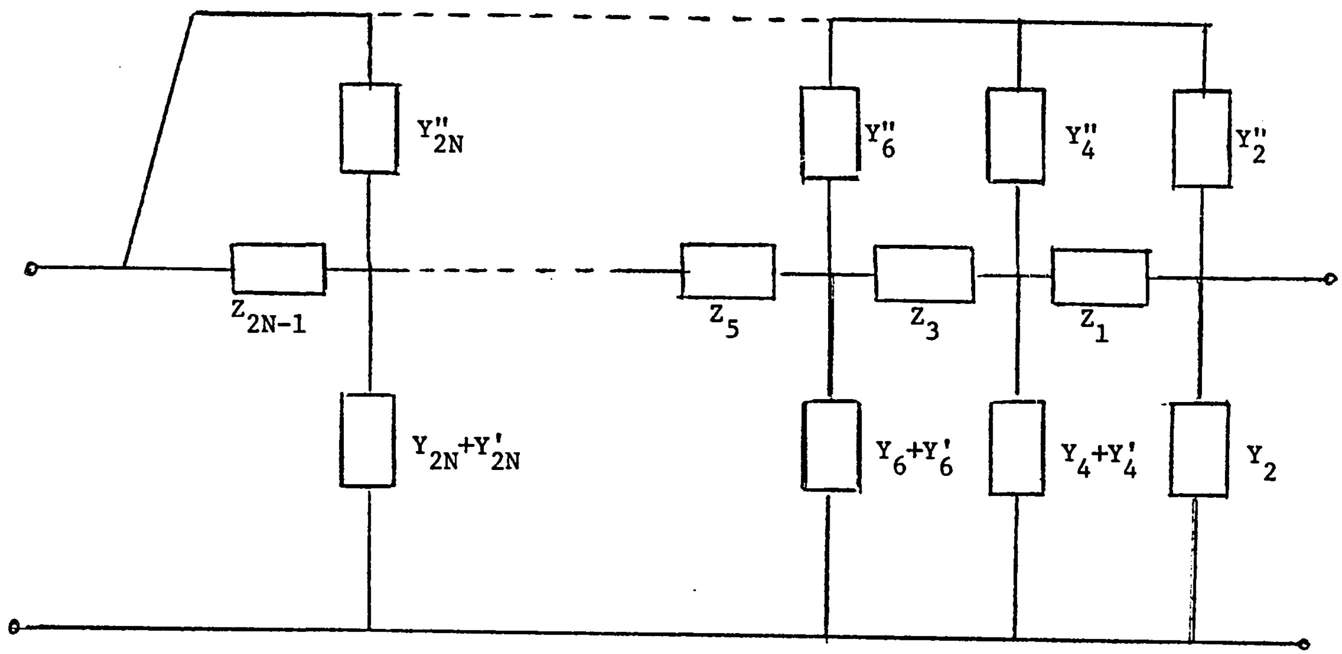

FIGURE 2.2 The passive network for the C-Parameter

(ii) The sum of Y_ℓ 's plus Y'_ℓ 's , $\ell > 2i$, ℓ is even

(b) All Z_{2i-1} are multiplied by:

(i) Y''_j 's, $j \leq 2i$, j is even

(ii) The sum of Y_ℓ 's plus Y'_ℓ 's, ℓ=from 2 to 2N, ℓ is even

Set of terms with two Z and three Y

Considering any two series impedances Z_{2i-1} and Z_{2k-1} where $k > i$ all other series impedances are short-circuited.

This set of terms is made up of subsets of terms described below.

(a) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y_j 's , $j < N$, j is even

(ii) Y''_ℓ 's , $\ell < 2i+1$, ℓ is even

(iii) Y''_m 's , $2i+1 < m \leq 2k$, m is even

(b) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y''_j 's , $j \leq 2i$, j is even

(ii) Y''_ℓ 's , $2i+1 < \ell \leq 2k$, ℓ is even

(iii) The sum of Y_m 's plus Y'_m 's , $m > 2i+1$, m is even

(c) All Z_{2i-1} Z_{2k-1} are multiplied by $(i < k)$:

(i) Y_j , $j < N$, j is even

(ii) Y''_ℓ , $\ell \leq 2k$, ℓ is even

(iii) The sum of Y_m 's plus Y'_m 's,

where $2i+1 < m$

but $m \neq 2N$ when ($k < N$ and $\ell = j$)

m is even

(d) All $Z_{2i-1}\, Z_{2k-1}$ are multiplied by ($i < k$):

(i) αY_j 's , $j > 2i-1$ $\alpha=0, k=N$
$\alpha=1, k<N$, j is even

(ii) The sum of Y_ℓ 's plus Y'_ℓ 's , $\ell > j$, ℓ is even

(iii) The sum of Y_m 's plus Y'_m 's , $m > \ell$, m is even

(e) All $Z_{2i-1}\, Z_{2k-1}$ are multiplied by ($i < k$):

(i) $\alpha Y''_j$, $j \leq 2k$, j is even
$\alpha=1, k=i+1$
$\alpha=0, k \neq i+1$

(ii) The sum of Y_ℓ 's plus Y'_ℓ 's , $2i-1<\ell<2k-1$

(iii) The sum of Y_m 's plus Y'_m 's , $m > \ell$, m is even

Rules for sets of terms of three Z and four Y, four Z and five Y, five Z and six Y, etc... can also be written in similar fashion.

2.4 SUMMARY

In this chapter, the rules for writing the A and C parameters of the network shown on figure 1.1 are given, with the aid of topological interpretation. The denominators for both the parameters are the same, and the numerators are bilinear functions. They consist of two parts, one multiplied by the amplifier gain K and the other corresponding to an appropriate passive network.

CHAPTER 3

REALIZATION OF THE NETWORK

3.1 INTRODUCTION

In this chapter the properties of the A and C parameters will be studied first.

With a knowledge of these an attempt will be made to realize the voltage transfer function $T_V(s) = \frac{1}{A}$. In order to minimize sensitivity, only a second order transfer function realization will be considered. The single parameter sensitivities of the Q factor and ω_o factor will be determined.

3.2 PROPERTIES OF THE A-PARAMETER

Considering the two section network only the A-parameter will be:

$$A = -\frac{\left[\begin{array}{l} K[Z_1Y_2+Z_3(Y_2+Y_4)+Z_1Z_3Y_2(Y_4+Y_4'+Y_4'')] \\ -[1+Z_1(Y_2+Y_2'')+Z_3(Y_2+Y_2''+Y_4+Y_4'+Y_4'') \\ +Z_1Z_3(Y_2+Y_2'')(Y_4+Y_4'+Y_4'')] \end{array}\right]}{K[1+Z_1Y_2''+Z_3(Y_2''+Y_4'')+Z_1Z_3Y_2''(Y_4+Y_4'+Y_4'')]} \qquad (3.1)$$

(i) If K is made negative we see that all the terms in the numerator and the denominator become positive.

(ii) By selecting the series impedances to be resistances, any one shunt admittance to be either a parallel combination of a capacitance and a resistance or a capacitance alone, the rest of the admittances being capacitances only, it can be seen that the poles and the zeroes lie only in the left half of the s-plane provided that K remains finite.

For the purpose of this dissertation only one case will be considered where Z_1 and Z_3 are resistances and the rest are capacitances.**

(iii) By open circuiting the feedforward elements Y''_2, Y''_4.... etc... only the multiple feedback network is obtained whose low pass realization has already been formulated [4].

3.3 PROPERTIES OF THE C-PARAMETER

Even though this parameter is not directly used in the realization, the properties are given here for the sake of completeness.

For the two section network the parameter will be:

** Many variations are possible but only this one is considered.

$$C = -\frac{\begin{bmatrix} K[Y_2+Y_4+Z_1Y_2(Y_2''+Y_4+Y_4')+Z_1Y_2''Y_4+Z_3Y_2(Y_2''+Y_4'') \\ +Z_3Y_4(Y_2''+Y_4'')+Z_1Z_3Y_2Y_2''(Y_4+Y_4'+Y_4'') \\ +Z_1Z_3Y_4Y_2''Y_4''] \\ -[Y_2+Y_4+Y_4'+Z_1(Y_2+Y_2'')(Y_4+Y_4')+Z_1Y_2Y_2'' \\ +Z_3Y_2''(Y_4+Y_4')+Z_3Y_4''(Y_2+Y_4+Y_4')+Z_3Y_2Y_2'' \\ +Z_1Z_3Y_2Y_2''(Y_4+Y_4'+Y_4'')+Z_1Z_3Y_4Y_4''(Y_2+Y_2'') \\ +Z_1Z_3Y_4Y_4'(Y_2+Y_2'')] \end{bmatrix}}{K[1+Z_1Y_2''+Z_3(Y_2''+Y_4'')+Z_1Z_3Y_2''(Y_4+Y_4'+Y_4'')]} \qquad (3.2)$$

With the same selection of the different components made earlier it can be seen that:

(i) The numerator and the denominator coefficients are all positive.

(ii) The poles and the zeroes lie in the left half s-plane. The poles of A and C are the same whereas the zeroes of C are not the same as those of A.

3.4 REALIZATIONS

As mentioned earlier only one case will be considered and the realization is carried out by the coefficient matching technique [5].

All series impedances are resistances and all upper

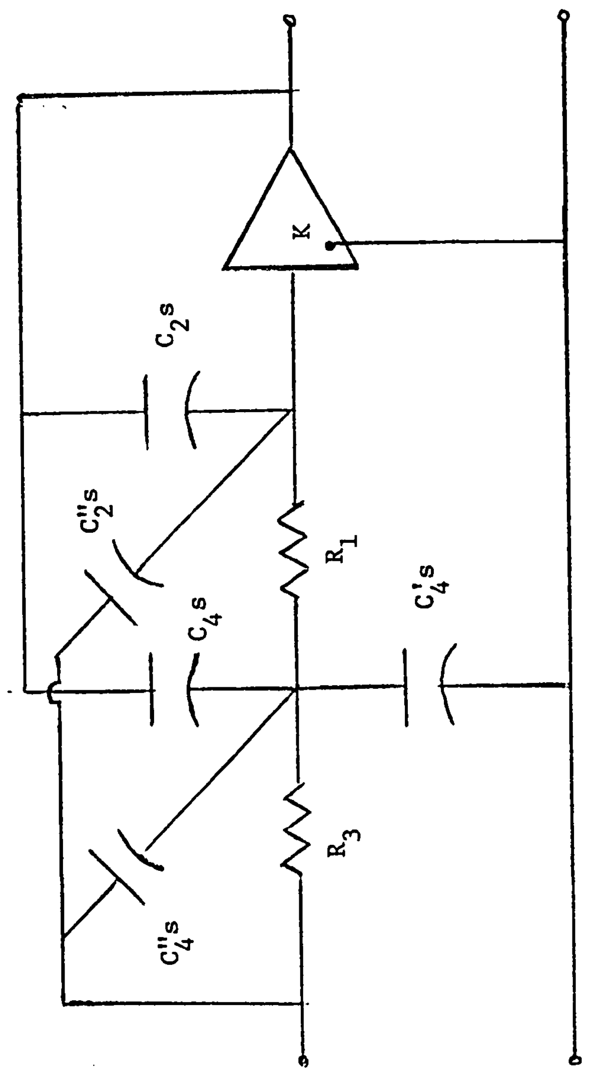

FIGURE 3.1 Proposed active realization

ladder, lower ladder and feedforward admittances are capacitances. Then the network will be as shown in figure 3.1.

From equation (3.1), the voltage transfer function $T_V(s)$ becomes,

$$T_V(s) = -\frac{K[R_1R_3C_2''(C_4+C_4'+C_4'')s^2+R_3(C_2''+C_4'')s+R_1C_2''s+1]}{\left[\begin{array}{l}K[R_1R_3C_2(C_4+C_4'+C_4'')s^2+R_3(C_2+C_4)s+R_1C_2s] \\ -[R_1R_3(C_2+C_2'')(C_4+C_4'+C_4'')s^2+R_3(C_2+C_2''+C_4+C_4'+C_4'')s \\ +R_1(C_2+C_2'')s+1]\end{array}\right]} \quad (3.3)$$

which is of the form

$$T_V(s) = \gamma \cdot \frac{b_2s^2 + b_1s + b_o}{a_2s^2 + a_1s + a_o} \quad (3.4)$$

whence equating like coefficients and letting $R_1 = \alpha R_3$

$$b_2 = \alpha R_3^2\ C_2''\ (C_4 + C_4' + C_4'') \quad (3.5)$$

$$b_1 = (1 + \alpha)\ R_3\ C_2'' + R_3\ C_4'' \quad (3.6)$$

$$a_2 = \alpha R_3^2\ (C_4 + C_4' + C_4'')\ \ \{C_2'' + C_2\ (1 - K)\} \quad (3.7)$$

$$a_1 = R_3[(C_4 + C_4' + C_4'') + (1 + \alpha)(C_2 + C_2'') - C_2K\ (1 + \alpha) - C_4K] \quad (3.8)$$

$$b_o = a_o = 1$$

Since $T_V(s)$ is always realizable within a multiplicative constant it is assumed that $T_V(s)$ is given in the form

$$T_V(s) = \frac{b_2 s^2 + b_1 s + 1}{a_2 s^2 + a_1 s + 1}$$

and without any loss of generalities it is further assumed

$$a_2 \geq b_2$$

and

$$a_1 \geq b_1$$

It is to be noted that $a_2 = b_2$ when K is unity. As can be seen we have four equations and seven unknowns and hence the solution does not appear to be unique.

From the above equations (3.5), (3.6), (3.7) and (3.8) these results are obtained choosing C_2'' and R_3 as arbitrary variables.

$$C_4'' = \frac{b_1}{R_3} - (1 + \alpha)\, C_2'' \tag{3.9}$$

$$C_2 = \frac{C_2''}{(1 - K)} \left(\frac{a_2 - b_2}{b_2} \right) \tag{3.10}$$

$$C_4 = \frac{b_2 - a_1 R_3\, \alpha C_2'' + (1+\alpha)\, C_2''^2 \frac{a_2}{b_2} \alpha R_3^2}{K\, \alpha R_3^2\, C_2''} \tag{3.11}$$

$$C_4' = \frac{b_2}{\alpha R_3^2 C_2''} \quad (1 - \frac{1}{K}) + \left(\frac{a_1}{KR_3} - \frac{b_1}{R_3}\right) + C_2''(1+\alpha)\left(1 - \frac{a_2}{b_2 K}\right) \tag{3.12}$$

If K is less than unity C_2 is always positive.

C_4'' will be positive if $C_2''(1+\alpha) \leq \frac{b_1}{R_3}$

$$\text{Let } C_2'' \; (1+\alpha) = \frac{\beta b_1}{R_3} \quad , \; \beta \leq 1 \tag{3.13}$$

Consider the numerator for C_4. If K is negative, we require that

$$\frac{a_1^2}{4a_2} \geq 1 + \frac{1}{\alpha}$$

which means $T_V(s)$ shall have negative real axis poles only, and hence this case is not considered.

If K lies between zero and unity, we require

$$\frac{a_1^2}{4a_2} - 1 \leq \frac{1}{\alpha}$$

and this is always satisfied irrespective of the value of α if the poles of $T_V(s)$ are complex. Hence, C_4 is always positive.

Now let us consider C_4' which can be arranged as

$$\left(\frac{a_1}{KR_3} - \frac{b_1}{R_3}\right) + \left\{\frac{b_2}{\alpha R_3^2 C_2''} \; (1 - \frac{1}{K}) + C_2''(1 + \alpha) \;\; (1 - \frac{a_2}{b_2} \; K)\right\}$$

It can be seen that the first term is positive, whereas the second term is negative and in order that C_4' be positive we require that

$$\alpha > \frac{b_2^2\,(1 - K)}{[\beta a_1 b_1 b_2 - b_2^2(1-K) - \beta^2 a_2 b_1^2 - b_1^2(1-\beta)\beta K b_2]} \tag{3.14}$$

This is always possible provided the denominator is not zero which requires that

$$K \neq \frac{b_2^2 - b_1\beta(a_2 b_1\beta + a_1 b_2)}{b_2^2 - b_1^2 b_2\beta(1 - \beta)} \tag{3.15}$$

Hence the procedure is to select a suitable β which could be unity, in which case $C_4'' = 0$, then select a value of K and hence α and therefore all other elements are chosen.

It has been proven earlier [4] that without feedforward the zeroes of $T_V(s)$ are contained only on the negative real axis. By the introduction of only one feedforward element the zeroes of $T_V(s)$ can be located anywhere in the left half of the s-plane.

3.5 Q AND ω_o SENSITIVITY OF THE VOLTAGE TRANSFER FUNCTION T_V

The characteristics of an active network may change for many reasons since the elements composing it are not ideal. The changes may be due to internal as well as external conditions.

A detrimental effect of these variations is the displacement of poles and zeroes of the network function. The displacement can be sufficiently large to render the circuit unstable in some cases. Hence, a study of the sensitivity of the active network is essential.

In this section the voltage transfer function T_V is studied for its Q-sensitivity and ω_o-sensitivity due to a variation of a single parameter.

Using the classical definition

$$S_x^f = \frac{df}{f} \cdot \frac{x}{dx}$$

the sensitivity of Q and ω_o are calculated for the network realized.

Since we know that

$$Q = \frac{\sqrt{a_2}}{a_1}$$

the Q-sensitivities are as follows:

$$S_{C_2}^Q = \frac{1}{2}\ \frac{C_2(1-K)}{\{C_2''+C_2(1-K)\}} - \frac{C_2(1+\alpha)(1-K)}{[(C_4'+C_4'')+\{(1+\alpha)C_2+C_4\}(1-K)+C_2''(1+\alpha)]} \qquad (3.16a)$$

$$S_{C_2''}^Q = \frac{1}{2}\ \frac{C_2''}{\{C_2''+C_2(1-K)\}} - \frac{C_2''(1+\alpha)}{[(C_4'+C_4'')+\{(1+\alpha)C_2+C_4\}(1-K)+C_2''(1+\alpha)]} \qquad (3.16b)$$

$$S^{Q}_{C_4} = \frac{1}{2}\,\frac{C_4}{(C_4+C_4'+C_4'')} - \frac{C_4(1-K)}{[(C_4'+C_4'')+\{(1+\alpha)C_2+C_4\}(1-K)+C_2''(1+\alpha)]} \tag{3.16c}$$

$$S^{Q}_{C_4'} = \frac{1}{2}\,\frac{C_4'}{(C_4+C_4'+C_4'')} - \frac{C_4'}{[(C_4'+C_4'')+\{(1+\alpha)C_2+C_4\}(1-K)+C_2''(1+\alpha)]} \tag{3.16d}$$

$$S^{Q}_{C_4''} = \frac{1}{2}\,\frac{C_4''}{(C_4+C_4'+C_4'')} - \frac{C_4''}{[(C_4'+C_4'')+\{(1+\alpha)C_2+C_4\}(1-K)+C_2''(1+\alpha)]} \tag{3.16e}$$

$$S^{Q}_{R_3} = 0 \tag{3.16f}$$

$$S^{Q}_{R_1} = 0 \tag{3.16g}$$

and S^{Q}_{K} because of the restrictive nature of K is derived here in detail as

$$S^{Q}_{K} = K\left[\frac{a_2'}{2a_2} - \frac{a_1'}{a_1}\right] \tag{3.17}$$

where $a_2' = \dfrac{da_2}{dK}$

$$= -\,C_2\,\alpha R_3^2\,(C_4 + C_4' + C_4'') = -\,\frac{a_2-b_2}{1-K} \tag{3.18a}$$

and $$a_1' = \frac{da_1}{dK}$$

$$= - R_3 \, [C_2(1+\alpha)+C_4] \tag{3.18b}$$

from which

$$S_K^Q = K \left[- \frac{a_2-b_2}{2(1-K)a_2} - \left\{ \frac{- \frac{a_1}{K} - \frac{b_2}{K\alpha R_3 C_2''} + \frac{C_2'' R_3 (1+\alpha)}{1-K} \left(1 - \frac{a_2}{b_2 K}\right)}{a_1} \right\} \right]$$

Letting T, being the time constant, equal R_3C_2'' and rearranging, we get

$$S_K^Q = - K \left[\frac{a_2-b_2}{2(1-K)a_2} + \frac{1}{K} - \frac{b_2}{a_1 K\alpha T} + \frac{\beta b_1}{1-K} \left(1 - \frac{a_2}{b_2 K}\right) \right]$$

Let

$$\chi = \frac{a_2-b_2}{2(1-K)a_2} + \frac{1}{K} - \frac{\beta b_1}{1-K} \left(\frac{a_2}{b_2 K} - 1\right) \tag{3.19}$$

If $$\beta < \frac{\frac{a_2 - b_2}{2(1-K)a_2} + \frac{1}{K}}{\frac{b_1}{(1 - K)} \left(\frac{a_2}{b_2 K} - 1\right)} \tag{3.20}$$

it is seen that $\chi > 0$.

Hence,

$$S_K^Q = - K \left(\chi - \frac{b_2}{a_1 K\alpha T} \right)$$

which can be made zero by choosing

$$\chi = \frac{b_2}{a_1 K \alpha T} \tag{3.21}$$

From this consideration, β shall be

$$\min \left\{ \frac{\dfrac{a_2 - b_2}{2(1-K)a_2} + \dfrac{1}{K}}{\dfrac{b_1}{(1-K)}\left(\dfrac{a_2}{b_2 K} - 1\right)} \, , \, 1 \right\} \tag{3.22}$$

Also it can be observed that

$$|S_C^Q| < \frac{1}{2} \, ,$$

where C is any of the capacitances used.

Now we shall consider the sensitivity of ω_o. We know that

$$\omega_o = \sqrt{\frac{1}{a_2}}$$

Therefore the ω_o-sensitivities are as follows:

$$S_{C_2}^{\omega_o} = -\frac{1}{2} \quad \frac{C_2(1-K)}{\{C_2'' + C_2(1-K)\}} \tag{3.23a}$$

$$S_{C_2''}^{\omega_o} = \frac{1}{2} \quad \frac{C_2''}{\{C_2'' + C_2(1-K)\}} \tag{3.23b}$$

$$S_{C_4}^{\omega_o} = -\frac{1}{2} \frac{C_4}{(C_4+C_4'+C_4'')} \tag{3.23c}$$

$$S_{C_4'}^{\omega_o} = -\frac{1}{2} \frac{C_4'}{(C_4+C_4'+C_4'')} \tag{3.23d}$$

$$S_{C_4''}^{\omega_o} = -\frac{1}{2} \frac{C_4''}{(C_4+C_4'+C_4'')} \tag{3.23e}$$

$$S_{R_3}^{\omega_o} = -1 \tag{3.23f}$$

$$S_{R_1}^{\omega_o} = -1 \tag{3.23g}$$

$$\begin{aligned} S_K^{\omega_o} &= \frac{1}{2} \frac{KC_2}{\{C_2''+C_2(1-K)\}} \\ &= \frac{1}{2} \frac{K \dfrac{C_2''}{(1-K)} \left(\dfrac{a_2-b_2}{b_2}\right)}{C_2'' + C_2'' \left(\dfrac{a_2-b_2}{b_2}\right)} \\ &= \frac{K}{2(1-K)} \frac{a_2-b_2}{a_2} \end{aligned} \tag{3.23h}$$

If K is close to unity it is seen that $S_K^{\omega_o}$ tends to become high, hence, it is preferable to restrict K in the region zero to one-half. This sets a limitation on β which has to be chosen.

3.6 SUMMARY

If $T_V(s)$ is given in the form

$$\frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

it is seen that this is realizable by the network having two series resistances and either four or five capacitances.

In order to keep the sensitivity $S_K^{\omega_o}$ low, K is to be chosen between zero and one-half. The factor β shall be chosen to satisfy equation (3.15) and (3.22). Hence the other element values can be found and the network is realized.

This network possesses the property that the $S_K^Q = 0$ and all the other Q-sensitivities with respect to the capacitances are less than half while the Q-sensitivities with respect to the resistances are zero. In addition all ω_o-sensitivities have a magnitude which will be at most unity.

It is to be observed that by applying the RC : CR transformation the same transfer function is realizable by a similar network having two capacitances and either four or five resistances.

CHAPTER 4

CONCLUSIONS

A multiple loop feedback network with feedforward constrained by an operational amplifier has been analyzed. The A and C parameters were evolved and a set of rules have been framed to write them. The numerator of the A parameter is a bilinear function of the amplifier gain K. It is shown that the terms without K correspond to the passive part of the network shown on figure 2.1. The amplifier gain K is a multiplying factor of the denominator. Similarly the numerator of the C parameter is a bilinear function of the amplifier gain K and the terms without K correspond to the passive part of the network shown on figure 2.2. It is also shown that the denominator is the same as that of the A parameter.

Considering a low pass type network having resistances in the series arm and capacitances in the rest of the arms the properties of A and C parameters are investigated. It is found that all zeroes and poles lie in the left-half of the s-plane.

A realization using the coefficient matching technique has been worked out which will realize a second order voltage transfer function containing zeroes and poles anywhere in the left-half of the s-plane excluding imaginary axis poles. It is shown that all the elements are positive and they can be chosen such that the Q-sensitivity with respect to K is always zero and that the other Q-sensitivities are less than one-half. It is further shown that all ω_o-sensitivities have a magnitude which will be at most unity.

It is noted that, by introducing two additional elements to the multiple feedback network in the feedforward position, zeroes anywhere in the left-half of the s-plane (as compared to only negative real axis zeroes without them) can be realized and also the amplifier gain is restricted between the value zero and one-half. Hence the poles of the operational amplifier are not considered and it is shown that the circuit is insensitive.

By an examination of the expressions for the different elements and the sensitivities it is seen that the spread could be large. Therefore it is suggested for further investigation that operational amplifiers might be introduced in suitable arms as to increase Q and reduce the elemental spread.

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