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CHAPTER I

Introduction

In elementary particle physics there is not any well formulated dynamical theory because the exact potential is not known.[1] For this reason it is useful to study conservation laws and interactions of the elementary particle systems, using symmetry and dynamical groups. The study of elementary particles using group theory began after the introduction of the isospin quantum number by Heisenberg[1]. The three components of the isospin given by the Pauli matrices generate the algebra of the special unitary group SU(2). Under the SU(2) group all the hadrons are classified in isoplets (i.e. multiplets having the same isospin value). The SU(2) group gives the symmetry corresponding to the charge independancy of the strong forces. Later on a larger symmetry was considered among isoplets because of the relatively small difference in mass between them. As is discussed in chapter two, this gave rise to the necessity of the extension of the SU(2) group in a larger group, the SU(3) group, proposed by Gell-Mann[3] and Ne'eman[4]. The introduction of the charm quantum number resulted in a further extension of the maximum symmetry of the hadrons SU(3) to SU(4). This symmetry group provides a very good scheme of classification for the hadrons.

If a dynamical process is to be studied by means of group theory one must break the existing symmetry of the system in order to observe the evolution of the system during this process.

An algebraic mechanism of breaking the symmetry is the dynamical group. A dynamical group is equiped with the operators associated with a dynamical process. These operators together with the generators of the maximum symmetry group of a system generate the algebra of the dynamical group. In chapter three an example is given about the choice of a dynamical group.

The subject of this thesis is the calculation of cross-sections of meson-baryon reactions. The work done in this thesis is based on a calculation of cross-sections of two body strong interactions done by Barbari and Kalaman[21] using as symmetry group of the hadrons the SU(3) group and as dynamical group, the group SU(1,3). They obtained an equation for the cross-section (see chapter V) where some of the parameters were determined. In this thesis the same model was considered. Our calculation uses the larger symmetry group SU(4) and the corresponding dynamical group SU(1,4). It provides the same type of equation for the cross-section as the one of Barbari and Kalman. Using the larger symmetry it is possible to determine the important parameter of the model which specifies the irreducible representation of SU(1,4). A partial calculation of some other parameters is also possible. In chapter five our calculation are presented. An analytical example of our calculation of the equation for the cross-section is presented in appendix A.

Our calculation is based on the consideration of baryons as orthonormal vectors of a basis of an irreducible representation determined by the Gel'fand-Graev method and of mesons as the corresponding operators. The Gel'fand-Graev method of the determination of finite dimensional irreducible representations of the Lie algebra of the unitary special groups is described in chapter four. In chapter six we present our conclusions.

CHAPTER II

Unitary Symmetry

Physical entities are called symmetric, or we say that they exhibit a certain symmetry, if they remain invariant under a certain transformation. For instance, a sphere viewed from any point in space has the same appearance. Therefore we say that the sphere is symmetric under rotation. Symmetry and invariance may be dealt with using group theory. The application of group theory in physics is very important and helpful especially for systems of unknown Hamiltonians as is the case of elementary particles.

In this chapter the unitary symmetry of hadrons is discussed. In the first section we present the isospin symmetry in hadrons. Isospin symmetry is very similar to spatial symmetry. It expresses the invariance of the strong forces under an interchange of electric charge between almost identical particles. Sections two and three deal with the hypercharge and charm symmetries respectively. Finally, in section four we briefly discuss the quark model in the context of the symmetries of elementary particles.

1. ISOSPIN SYMMETRY

To introduce the isospin symmetry we will first give an analogous example of symmetry from a more familiar domain, atomic physics.

Consider an atomic level of angular momentum I, in the absence of an electromagnetic field. Any rotation of the spin angular momentum vector will result in the same atomic level without any change in the energy. Thus spatial symmetry results in the degeneracy of the energy. The Hamiltonian of this level is therefore invariant under the spin rotation. This invariance can be expressed in mathematical language by the commutator $[H,S] = 0$. The 3-dimensional rotational symmetry can be given by a symmetry group whose generators will be the three components of the spin vector given by the Pauli matrices. The commutation relations of the Hamiltonian and the spin, the spin matrices among themselves and the raising and lowering operators which we can construct from the Pauli matrices, give a Lie algebra which is the algebra of the special unitary group, SU(2).

If we apply an electromagnetic field $\vec{B}$ with direction, say along the $\vec{z}$ axis, the spin will align itself along the same direction. The E-M field breaks the existing symmetry. Any rotation of the spin vector will result in another sublevel with the same total angular momentum, but with different energy. There are (2S+1) such sublevels. The breaking down of the symmetry removes the degeneracy of the energy and allows us to distinguish the (2S+1) sublevels.

The proton and neutron have the same external properties. They are only distinguished by a small difference in mass which results from their different interactions (because of their charges) with the electromagnetic field. In the absence of this field the electric charge is not revealed and the two particles appear identical. We observe that in this case there is a symmetry similar to the above example. In the absence of the E-M field the two particles appear as a single one. If we perform a charge rotation we would obtain the same single particle as in the above example with the spin. In the presence of the E-M field the electric charge is revealed. Like the spin, it aligns itself along the E-M field and consequently the single particle we had before splits in two, the neutron and the proton. A unitary charge rotation will resolve either of these two particles. In other words in this case we have degeneracy in charge which is removed by the E-M field.

In 1932 Heisenberg[1] introduced the isospin operator, I, in order to formulate mathematically this symmetry. The isospin, or isotopic spin, operator is analogous to the ordinary spin operator but it performs unitary rotational operations in the charge space. As the ordinary spin distinguishes electrons with ½ and -½ spin so I_3 (the third component of isospin) distinguishes the two states of the single particle the nucleon, (proton and neutron). The Hamiltonian of the nucleon commutes with the isospin and we write $[H,I] = 0$, when the E-M field is not present. When it is present, H and I do not commute anymore but the new Hamiltonian $(H + H_{EM})$ does commute with the third component of the isospin. This means that an operation, done by I on a state of the nucleon, leaves this state unchanged. The components of the isospin are also given by the Pauli matrices. The commutation relations of the Hamiltonian and the isospin components are the same as the ones in the previous examples and therefore have the same algebra. Hence, the isospin symmetry can also be expressed by the SU(2) group.

In the isospin space, particles with the same physical properties (baryon number, spin, parity etc.) and very close mass are placed in isoplets. For example, the proton and neutron form an isodoublet, the nucleon. The three pions form an isotriplet. The different states of an isoplet are distinguished by the third component, I_3, of the isospin. The multiplicity of an isoplet is given by (2I + 1) where I is the isospin of the isoplet. All the isoplets can be represented in terms of the SU(2) algebra by its irreducible representations. The fundamental representation is the 2-dimensional irreducible representation. With the kronecker product of copies of this fundamental representation all the irreducible representations of SU(2) can be constructed. The symmetry of SU(2) or the isospin summetry is exact for strong interactions. If we include the E-M interaction, the symmetry is only approximate. The E-M field breaks the symmetry down just as it breaks the spin degeneracy in atoms, as we saw in the previous example. The breaking of symmetry in the case of SU(2) is not large since it results in a mass difference of the order of a few Mev for particles if their mass is roughly 1000 Mev.

2. SU(3) SYMMETRY

Mass differences between neighboring isoplets of the order of 100 Mev of baryons each with the same J^P, led to the consideration of the existence of another larger symmetry broken by some interaction as the isospin symmetry is broken by the Electromagnetic interaction. In analogy to the isospin symmetry, this symmetry, in the absence of such interaction, results in the degeneracy in mass of all the isoplets of hadrons with the same baryon number and J^P. These isoplets appear as a single multiplet as the proton and the neutron appear as a single particle, the nucleon, in the absence of an E-M field. (e.g. The four baryon isoplets, N, Σ, Λ, Ξ, considering this larger symmetry, appear as an octet with degenerate mass). In order to distinguish the isoplets of a multiplet, we need another quantum number, the hypercharge, Y. The hypercharge is conserved by the E-M and the strong interactions, but it is violated by the weak interactions. As I_3 distinguishes the proton from the neutron, so does the hypercharge distinguish the isoplets from each other.

In analogy with the isospin, it was thought that symmetry could be expressed by means of a Lie group. Many groups were proposed corresponding to different models of hadrons[2]. Finally the group SU(3), proposed by Gell-Man[3] and Ne'eman[4] independently, was accepted because its predictions were very close to the experimental situation. SU(3) has eight generators. Three of them are the isospin generators and the remaining five mix the isospin and hypercharge quantum numbers. The generators and their algebra can be found in Lichtenberg[5].

As in the case of isospin where we can represent all the isoplets by the irreducible representations of the SU(2) group, so also in SU(3) we can represent the multiplets by irreducible representations. These are obtained by the kronecker product of the copies of the fundamental 3-dimensional representation and its conjugate.

The SU(3) symmetry is presumed to be broken by medium-strong interactions. Their presence removes the degeneracy in mass and allows us to distinguish the different isoplets. The charge formula of a hadron is given in SU(3) by

$$Q = I_3 + \tfrac{1}{2}Y, \qquad (1)$$

with I_3 and Y expressed by the generators of SU(3).

The SU(3) symmetry group provides a very good scheme for the classification of the hadrons.

3. SU(4) SYMMETRY

In 1974, a new particle, the meson J/ψ, was discovered. This particle was very heavy compared with the old mesons and it had a very narrow width. There were two explanations for this peculiarity. The explanation that was accepted was that a new quantum number, charm, introduced previously by Bjorken and Glashow[6], was involved. The introduction of the charm quantum number makes SU(3) symmetry inadequate to give the symmetry of all hadrons. It must be extended just as SU(2) symmetry was extended by hypercharge, in SU(3). The charm quantum number, just like hypercharge, is conserved by the strong and electromagnetic interactions but it is violated by the weak interactions.

The Hamiltonian of a hadron can be written

$$H = H_O + H_{E-M} + H_{M-S} + H_W \qquad (2)$$

If we neglect all the three, electromagnetic, medium-strong and weak interactions the remaining part of the Hamiltonian H_O is invariant under a transformation belonging to a special unitary group of higher dimension SU(4). The SU(4) group has fifteen generators. Eight of them are the generators of SU(3) and the remaining seven mix the isospin and hypercharge with the new quantum number charm. The generators and their algebra can be found in Lichtenberg[5]. The SU(4) symmetry is very badly broken compared to the SU(2) and SU(3) symmetries. The mass splitting in SU(4) is of the order of 1000 Mev. The charge of a hadron in SU(4) is again given by the Gell-Mann-Nishijima formula extended to:

$$Q = I_3 + \tfrac{1}{2}Y + C \qquad (3)$$

4. QUARK MODEL

From our discussion of unitary symmetry we can point out three puzzling points:

The strong interactions are a property only of hadrons. Comparing SU(2), SU(3) and SU(4), we observe that, for higher symmetry we have greater splitting in mass. When SU(3) was accepted as the symmetry group of Y and I all the then known hadrons were placed in multiplets whose irreducible representations were not the fundamental 3-dimensional representation of SU(3).

If we assume that the hadrons are not elementary but they are made up by some fundamental particles ~~which~~ interact strongly it is easy to explain the mass difference by assuming that the different members of a multiplet are made up by different combinations of these fundamental particles. Gell-Mann[7] and Zweig[8] proposed three such fundamental particles, the quarks, that would fit in the fundamental 3-dimensional representation of SU(3). Those particles are the u, d and s (up, down and strange), quarks. The up and down quarks have almost the same mass and they are responsible for the isospin symmetry. The strange quark is more massive than the other two and it is responsible for the hypercharge symmetry. Later on, the charm quark, c, was introduced to complete the fundamental quartet in SU(4). The c quark is the most massive one.

The quarks have fractional charge, hypercharge and baryon number. Their baryon number is 1/3. A baryon (B = 1) is made up by three quarks and a meson (B = 0) by a quark-antiquark pair. In table 1. the quantum numbers of quarks are given and in table 2. the specific combinations of quarks that make up each hadron appear.

The kronecker product of the fundamental representations of quarks gives us all the possible representations of hadrons. For example one can obtain the mesons representations in SU(3) by taking the kronecker product of the 3-dimensional representation by the 3-dimensional one, giving the antiquark triplet. e.g.

$$3 \times \bar{3} = 1 \oplus 8$$

That means, that mesons can form a singlet or an octet in SU(3).

The quarks have not been observed experimentally. Although, there is indirect experimental evidence which supports the theory of the quark model[9]. It is also believed that two more quarks exist, the t (top) and b (bottom) quarks[10].

We have seen that the similarities in symmetry between the well-known Zeeman effect and nucleons helped to introduce the isospin concept and the SU(2) symmetry. A generalization of this symmetry results in the SU(3) and the SU(4) groups as the symmetry groups of hadrons. The unitary symmetry proves itself useful from the point of view of a classification scheme. Using the symmetry group it was possible to predict the existence of other particles which were confirmed experimentally later on.

CHAPTER III

Dynamical Groups

If physical states exhibit some symmetry, they are undistinguishable. Thus, for example, consider states which are symmetric under SU(3) transformations. In the absence of the medium strong interactions all the isoplets of hadrons with the same baryon number and J^P are degenerate in mass and cannot be distinguished. To distinguish different states, the existing symmetry must break down, so the degeneracy will be removed. An algebraic mechanism for breaking the symmetry is the dynamical group. The symmetry breaking is due to the fact that some of the generators of the dynamical group do not commute with the Hamiltonian of a given system, therefore they are not symmetry operators.

In the following, in the first section, we give an example concerning the choice of a dynamical group in the case of the well known hydrogen atom. In section two we give the maximum symmetry group and the dynamical group for charmed hadrons.

1. HYDROGEN ATOM

The Hamiltonian of the hydrogen atom is

$$H = \frac{-\hbar^2}{2\mu}\Delta - \frac{k}{r} \tag{4}$$

with $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$, $k = Ze^2$ and $r = (x_1^2+x_2^2+x_3^2)^{\frac{1}{2}}$

Aside from the energy, there are two constants of the motion, the angular momentum

$$\vec{L} = \vec{r} \times \vec{p} \tag{5}$$

and the Runge-Lenz vector

$$\vec{M} = \frac{1}{2\mu}\left[\vec{p} \times \vec{L} - \vec{L} \times \vec{p}\right] - \frac{k\vec{r}}{r} \qquad (6)$$

Their corresponding operators commute with the Hamiltonian of the atom and therefore generate the symmetry group of the atom. Since there are no other operators[11] which depend on the position coordinates that commute with the Hamiltonian, the angular momentum and the Runge-Lenz operators form the maximum symmetry group of the atom. The commutation relations of the L and M are given in Bander and Itzykson[12] for the bound states. These commutation relations correspond to the algebra of the orthogonal group O(4). For the scattering states they correspond to the algebra of the pseudo-orthogonal group O(1,3).

In both groups, the expression for the energy can be obtained simply by the commutation relations without solving any Schrödinger equation.

We will show how the energy can be obtained in the case of O(4).

The commutator of M gives

$$\left[M_i, M_j\right] = \frac{-2i\hbar}{\mu} E\, \epsilon_{ijk}\, L_k \qquad (7)$$

To express this commutator exactly as in O(4) the M_i can be written as $M_i = \left(\frac{\mu}{-2E}\right)^{-\frac{1}{2}} \tilde{M}_i \qquad (8)$

The energy in terms of the L and $\tilde{M}$'s magnitudes, is given by

$$\frac{-\mu k^2}{2(\tilde{M}^2 + L^2 + \hbar^2)} = E \qquad (9)$$

In addition the two Casimir operators $C_1 = \left[\frac{1}{2}(L + \tilde{M})\right]^2$, $C_2 = \left[\frac{1}{2}(L - \tilde{M})\right]^2$ of the two O(3) subgroups contained in

O_4 $O_3 \otimes O_3$ have same eigenvalue because

$$\left[\tfrac{1}{2}(L+\tilde{M})\right]^2 = \left[\tfrac{1}{2}(L-\tilde{M})\right]^2$$

$$\tfrac{1}{2}\left[(L+\tilde{M})\right]^2 = \tfrac{1}{4}\left[L^2+\tilde{M}^2 + L.\tilde{M} + \tilde{M}.L\right]$$

$$\tfrac{1}{2}\left[(L-\tilde{M})\right]^2 = \tfrac{1}{4}\left[L^2+\tilde{M}^2 - L.\tilde{M} - \tilde{M}.L\right] \quad (10)$$

but $\tilde{M} \perp L$ in the motion of the H atom and therefore

$L . \tilde{M} = \tilde{M} . L = 0$

As usual the eigenvalue of the Casimir operators of the O_3 subgroup can be set equal to $\hbar^2\, j(j+1)$. For the moment it is not clear if j can take integer or half integer values. However we may assume that 2j is an integer.

From equations (10) we can then write

$$\tilde{M}^2 + L^2 + k^2 = 4\left[\tfrac{1}{2}(L+\tilde{M})\right]^2 + \hbar^2 =$$

$$\hbar^2\left[4j\,(j+1)+1\right] = \hbar^2\,(2j+1)^2 \quad (11)$$

On the other hand eq. (9) can be written as

$$\frac{2E}{\mu}(\tilde{M}^2 + L^2 + \hbar^2) = -k^2 \quad (12)$$

From (11) and (12) $\frac{2E}{\mu}\hbar^2(2j+1)^2 = -k^2$ and therefore

$$\bar{E} = -\frac{k^2\mu}{2\hbar^2}\left(\frac{1}{(2j-1)}\right) \quad (13)$$

Identifying 2j + 1 with the n principal quantum number we get

$$E = -\frac{k^2\mu}{2\hbar^2}\,\frac{1}{n^2} \quad (14)$$

which is the energy of the levels in the Coulomb potential. n can take every positive integer value and therefore j can take positive integer or half integer values. For every $n = 2j+1$ there are 2j possible values for ℓ. If we want to distinguish the levels labeled by different ℓ we must remove this degeneracy. This can be obtained by using a dynamical group. This group must have as generators the generators of O(4) and in addition, operators that will shift the ℓ quantum number, so they will lift the degeneracy. This group generated by such operators is the O(1,4) pseudo-orthogonal group[12]. Furthermore, if we would like to calculate transition rates between states, we must introduce the magnetic dipole operator. With the introduction of this operator the O(1,4) must be extended. Barut and Kleinert[13] have shown that the resulting dynamical group is the group O(2,4). With this latter group transition rates can be calculated without resorting to the Schrödinger equation for the Hydrogen atom.

2. DYNAMICAL GROUP OF CHARMED HADRONS

In this thesis the cross sections of meson-baryon reactions are calculated. The SU(4) symmetry group is considered to be the maximum symmetry group for the charmed hadrons. But this group, as in the previous example does not provide means to calculate results from dynamical processes. Kalman[14] has shown that SU(1,4) can be taken to be the dynamical group for calculations of properties of charmed hadrons. His method is based on the Kariyan-Sudarshan strong coupling model. Consider a Hamiltonian of the form

$$H = Ho^{part} + Ho^{field} + H' \qquad (15)$$

where Ho^{part} is the Hamiltonian of the free particle, Ho^{field} is the Hamiltonian of the free field and H' is the interacting part of the Hamiltonian. Ho^{part} and Ho^{field} are invariant under SU(4) symmetry.

Then H' can be expressed in the form:

$$H' = \sum_{i \leq j} A_{ij} \; M_{ij} \qquad (16)$$

where the M_{ij}'s correspond to mesons operators and A_{ij} are scalars in SU(4).

In the frame of the quark model the interacting part of the Hamiltonian is due to the interactions of the quarks in the presence of a field. Therefore we can express the M_{ij}'s by the means of quarks. Since mesons are bound states of a quark-antiquark pair the M_{ij}'s can be expressed as

$$M_{ij} = C_{ij} \, q_i \, \bar{q}_j \qquad i,j = 1, 2, 3, 4 \qquad (17)$$

where q's are quark operators and C_{ij} coefficients depending on the structure of the mesons.

The algebra of SU(4) is given by:

$$\left[A_{ij} , A_{km} \right] = \delta_{im} \, A_{kj} - \delta_{jk} \, A_{im} \qquad (18)$$

$$\sum_{i=1}^{4} A_{ii} = 0 \qquad (19)$$

Taking the commutator of A_{ij} generators of SU(4) and q_i operators we get:

$$\left[A_{ij}, q_k \right] = \delta_{ik} \; q_i \qquad (20)$$

$$\left[A_{ij}, \bar{q}_k \right] = -\delta_{jk} \; \bar{q}_i \qquad (21)$$

All these operators correspond to physical entities. The operators A_{ij}, q_k must be closed to form a Dynamical group. It is undesirable to add a large number of additional operators in making this closure since they will not correspond to physical entities. For this reason we consider the smallest possible closure of the algebra of q's and A's which is

$$[\, q_i \, , \bar{q}_j \,] = \theta \, (\delta_{ij} A_{55} - A_{ji})$$

This condition gives three possibilities.

For $\theta = +1$ we obtain the Lie algebra of SU(5).

For $\theta = -1$ we obtain the one of SU(1,4).

For $\theta = 0$ we obtain the one of $T_{10} \otimes SU(4)$.

The last possibility is ruled out because it does not predict transitions from one state to the other. Furthermore, the group SU(5) has only integer parameter representations. Since scattering states correspond to a continuous spectrum of energy the non-compact SU(1,4) is the appropriate dynamical group.

It has been shown in the case of the hydrogen atom that by means of the dynamical group we can break the symmetry allowing the observation of different states and also the calculation of transition probabilities. Similarly, for scattering of hadrons, the appropriate group has been shown to be SU(1,4).

CHAPTER IV

Representation of SU(1,4)

Generally, it is not the group itself but the Lie algebra of the Lie group which is of interest. A representation of the Lie algebra in a Hilbert space is given by a set of operators, defined in this space which have the same commutation relations as the abstract algebra.

As we saw in the previous chapter the maximum symmetry group for the bound states of the hydrogen atom is the group O(4). All the degenerate levels, of the atom, in energy constitute the basis vectors of an irreducible representation of the Lie algebra of group O(4). Similarly, in general, quantum mechanical states which are degenerate under a unitary transformation may be identified with the basis vectors of an irreducible representation of the Lie algebra of their symmetry group. In this chapter we present the Gel'fand-Graev[15] method of the determination of finite dimensional irreducible representations of the Lie algebra of the unimodular unitary groups. We first consider the compact groups in particular, the group SU(4) and following that, the non-compact group, in particular the group SU(1,4).

1. GROUP SU(n)

Let G^n be the group of nonsingular complex matrices of n^{th} order and L its algebra. Suppose that a basis of the algebra L is constituted by matrices $e_{k\ell}$ $k,\ell = 1,2,\ldots,n$ where their elements satisfy the condition.

$$(e_{k\ell})_{mn} = \delta_{km}\,\delta_{\ell n}$$

The commutation relations of $e_{k\ell}$ matrices are

$$\begin{aligned} [e_{ik}, e_{k\ell}] &= e_{i\ell} \quad \text{for } i \neq \ell \\ [e_{ik}, e_{ki}] &= e_{ii} - e_{kk} , \\ [e_{ik}, e_{\ell m}] &= 0 \quad \text{if } k \neq \ell \text{ and } i \neq m \end{aligned} \qquad (22)$$

The group of linear operators by which this algebra is represented in the Hilbert space must have the same commutation relations with the matrices $e_{k\ell}$. Therefore if $E_{k\ell}$ are the linear operators, we can write

$$\begin{aligned} [E_{ik}, E_{k\ell}] &= E_{i\ell} \quad \text{for } i \neq \ell \\ [E_{ik}, E_{ki}] &= E_{ii} - E_{kk} \\ [E_{ik}, E_{\ell m}] &= 0 \quad \text{if } i \neq m \text{ and } k \neq \ell \end{aligned} \qquad (23)$$

Considering three successive in order operators E_{kk}, $E_{k,k-1}$, $E_{k-1,k}$, we will give the commutation relations that they must satisfy.

$$\begin{aligned} [E_{ii}, E_{kk}] &= 0 \\ [E_{ii}, E_{k,k-1}] = [E_{ii}, E_{k-1,k}] &= 0 \text{ for } i < k-1 \\ [E_{ii}, E_{i,i-1}] = [E_{i,i-1}, E_{i-1,i-1}] &= E_{i,i-1} \\ [E_{i-1,i-1}, E_{i-1,i}] = [E_{i-1,i}, E_{ii}] &= E_{i-1,i} \\ [E_{i,i-1}, E_{i-1,i}] &= [E_{ii} - E_{i-1,i-1}] \\ [E_{i,i-1}, E_{k-1,k}] &= 0 \quad \text{for } k \neq i \\ [E_{i,i-1}, E_{k,k-1}] = [E_{i-1,i}, E_{k-1,k}] &= 0 \quad \text{for } k \neq i \pm 1 \end{aligned} \qquad (24)$$

Every operator E_{kl} can be determined using the formulas

$$E_{k,k-p} = \left[E_{k,k-1}, E_{k-1,k-p} \right] \tag{25}$$

$$E_{k-p,k} = \left[E_{k-p,k-1}, E_{k-1,k} \right] ;$$

The orthonormal vector basis of an irreducible representation may be constituted by triangular arrays of the type:

$$m = \begin{pmatrix} m_{1n} & m_{2n} & ---- & m_{nn} \\ m_{1,n-1} & m_{2,n-1} & ---m_{n-1,n-1} & \\ & ---- & & \\ & ---- & & \\ & m_{11} & & \end{pmatrix} \tag{26}$$

called Gel'fand-Cetlin pattern. The n integers $m_{1n}, m_{2n}, \ldots m_{nn}$ specify the representation i.e. if it is a representation of SU(4) etc. All the integers m_{ij} of the triangular array satisfy the conditions[15]:

$$m_{ij} \geq m_{i,j-1} \geq m_{i+1} , j \tag{27}$$

The action of E_{kk}, $E_{k,k-1}$, $E_{k-1,k}$ operators on the m triangular array is defined as:

$$E_{kk}\, m = \left(r_k - r_{k-1} \right) m \tag{28}$$

with $r_k = m_{1k} + m_{2k} + \ldots + m_{kk}$, $k = 1, \ldots, n$ and $r_o = o$

$$E_{k,k-1}\, m = a^1_{k-1}\, m^1_{k-1} + \ldots + a^{k-1}_{k-1}\, m^{k-1}_{k-1} \tag{29}$$

where m^j_{k-1} is the array obtained from m by replacing the element $m_{j,k-1}$ by $m_{j,k-1}-1$. The $E_{k,k-1}$ is a lowering operator. It can lower one element of each row of m at the time, preserving always the relationship (27).

The coefficients a^j_{k-1} are given by:

$$a^j_{k-1} = \left[- \frac{\prod_{i=1}^{k}\{m_{ik} - m_{j,k-1} - i+j+1\} \prod_{i=1}^{k-2}\{m_{i,k-2} - m_{j,k-1} - i+j\}}{\prod_{i\neq j}^{k-1}\{m_{i,k-1} - m_{j,k-1} - i+j+1\}\{m_{i,k-1} - m_{j,k-1} - i+j\}} \right]^{1/2} \quad (30)$$

The $E_{k-1,k}$ is raising operator. The application of $E_{k-1,k}$ on the m pattern leads in to the change of m array as

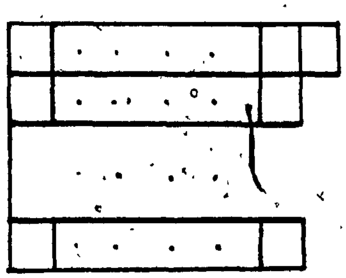
$$E_{k-1,k}\, m = b^1_{k-1}\, \hat{m}^1_{k-1} + \ldots + b^{k-1}_{k-1}\, \hat{m}^{k-1}_{k-1} \quad (31)$$

where this time $\hat{m}^j_{k-1}$ is the array obtained from m by replacing $m_{j,k-1}$ by $m_{j,k-1}+1$. The b^j_{k-1} are given by:

$$b^j_{k-1} = \left[- \frac{\prod_{i=1}^{k}\{m_{ik} - m_{j,k-1} - i+j\} \prod_{i=1}^{k-2}\{m_{i,k-2} - m_{j,k-1} - i+j-1\}}{\prod_{i\neq j}^{k-2}\{m_{i,k-1} - m_{j,k-1} - i+j\}\{m_{i,k-1} - m_{j,k-1} - i+j-1\}} \right]^{1/2} \quad (32)$$

Again the operator $E_{k-1,k}$ can raise one element of each row of the pattern m preserving the relation (27).

Each row of the Gel'fand-Cetlin pattern can be related to the well known Young tableau[17]. For the k^{th} row of the array (26) the Young's tableau has the arrangement



The top row as we said before is fixed, and specifies the representation which belong to let's say U(n). The next rows correspond each one to successive subgroups of U(n) (e.g. the row $m_{1,n-1}$ corresponds to U(n-1), $m_{1,n-2}$ to U(n-2), m_{11} to U(1)). The Young's tableau for a representation of SU(n) has the same form. But in the case of SU(n) we can subtract the number of boxes, common to each row, because they do not affect the dimensionality of the representation[18]. For example if $m_{1k} = 4$, $m_{2k} = 3$ and $m_{3k} = \ldots = m_{kk} = 2$ the representation having this Young's tableau has the same dimensionality as the representation with $m_{1k} = 2$, $m_{2k} = 1$, $m_{3k} = \ldots = m_{kk} = o$, where the rows with the same number of boxes have been removed. If we put $m_{1k} = m_{1k}$ the remaining rows can be written as $m_{2k} = m_{1k} -1$, $m_{3k} = \ldots = m_{kk} = m_{1k} - 2$. According to this example we can write the 20-dimensional representation, with mixed symmetry, of SU(4) in terms of the parameter m_{14} i.e. $m_{14} = m_{14}$, $m_{24} = m_{14} - 1$, $m_{34} = m_{44} = m_{14} - 2$. For non-charmed baryons, the second row of the array corresponding to the above representation of SU(4) is the 8-dimensional representation of SU(3) which from the Young's tableau corresponds to $m_{13} = m_{13}$, $m_{23} = m_{13} -1$, $m_{33} = m_{13} -2$. From the triangular inequality (eq.27) this corresponds to $m_{13} = m_{14}$, $m_{23} = m_{14} -1$, $m_{33} = m_{14} -2$. The Gel'fand-Cetlin pattern corresponding to this case is then

$$m = \begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 \\ & m_{14} & & m_{14}-1 & & m_{14}-2 & \\ & & m_{12} & & m_{22} & & \\ & & & m_{11} & & & \end{pmatrix} \quad (33)$$

where m_{12}, m_{22}, m_{11} satisfy the conditions (27). For different permissible values of m_{12}, m_{22}, m_{11} one obtains different members (states) of the 20_m -dimensional representation. (e.g. for $m_{12} = m_{14}$, $m_{22} = m_{14}-1$ and $m_{11} = m_{14}$ one obtains the proton).

As Jakimow and Kalman[19] have shown, the quantum numbers of a state can be obtained by linear combinations of the integers of each row. For the SU(4) group then, we have the following relationships:

Charm:	$C = m_{14} + m_{24} + m_{44} - (m_{13} + m_{23} + m_{33})$	
Hypercharge:	$Y = m_{12} + m_{22} - \frac{2}{3}(m_{13} + m_{23} + m_{33})$	
Isospin:	$I = \frac{1}{2}(m_{12} - m_{22})$	(34)
Third component of I:	$I_3 = m_{11} - \frac{1}{2}(m_{12} + m_{22})$	
Charge:	$Q = I_3 + \frac{1}{2}Y + \frac{2}{3}C$	

2. NON-COMPACT GROUP, SU(1,4)

For irreducible representations with continuous parameters[15], the elements m_{1n} and m_{nn}, change to $-\frac{1}{2}(n-1) + \sigma$ and $\frac{1}{2}(n-1) + \bar{\sigma}$ respectively. Here, σ is a complex number and $\bar{\sigma}$ is its conjugate. All other elements are integers and they satisfy the conditions (27) except the $m_{1,n-1}$ and $m_{n-1,h-1}$ elements. The operators E_{jk}, by which the algebra $L^{1,n-1}$ is represented in the Hilbert space, satisfy also commutation relations of eq. (25) and they operate in the same way on the array m as the operators of a compact group. Specificly the representation of SU(1,4) which contains the 20_m- dimensional representation of SU(4) and the 8-dimensional one of SU(3) is expressed by the array:

$$m = \begin{pmatrix} -2+\sigma & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & & 2+\bar{\sigma} \\ & m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\ & & m_{14} & & m_{14}-1 & & m_{14}-2 & & \\ & & & m_{12} & & m_{22} & & & \\ & & & & m_{11} & & & & \end{pmatrix} \qquad (35)$$

3. APPLICATION TO MESON-BARYON REACTION

As we noted in the previous chapter, the dynamical group of the hadrons, having as maximum symmetry group the SU(4) group, which permits a calculation of the cross-sections of meson-baryon reactions, is SU(1,4).

Following Kalman[20] the baryons can constitute the orthonormal basis of an irreducible representation of the algebra $L^{1,4}$ and the mesons expressed as a quark-antiquark pair will be the operators associated with the representation. The 20_m-dimensional representation of the uncharmed baryons is given by:

$$
m = \begin{pmatrix}
-2+\sigma & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & & 2+\bar{\sigma} \\
& m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\
& & m_{14} & & m_{14}-1 & & m_{14}-2 & & \\
& & & m_{12} & & m_{22} & & & \\
& & & & m_{11} & & & &
\end{pmatrix} \quad (36)
$$

For the charmed baryons the representation is obtained by changing the values of m_{13}, m_{23}, m_{33} in the third row. By giving appropriate values in m_{12}, m_{22}, m_{11} we obtain all the 20 baryons charmed and non-charmed.

The mesons, as we saw in the previous chapter are given by:

$$M_{ij} = C_i C_j \, q_i \bar{q}_j \quad (37)$$

The q_i, $\bar{q}_j$ quark operators are the operators of the Gel'fand-Cetlin basis. We identify them as follows:

$$
\begin{array}{ll}
E_{15} \longrightarrow u & E_{51} \longrightarrow \bar{u} \\
E_{25} \longrightarrow d & E_{52} \longrightarrow \bar{d} \\
E_{35} \longrightarrow s & E_{53} \longrightarrow \bar{s} \\
E_{45} \longrightarrow c & E_{54} \longrightarrow \bar{c}
\end{array} \quad (38)
$$

The L^n, $L^{1,n-1}$ algebras can be represented in the Hilbert space by a set of operators having the same commutation relations with the basis elements of this algebra. The orthonormal basis of an irreducible representation is given by the

Gel'fand-Cetlin pattern. This pattern is a very compact way of representing a state giving all the information about it. Thus considering SU(1,4) as a dynamical group we have identified the scattering states of charmed and non-charmed baryons with a particular set of representations of the Lie algebra of SU(1,4) described in terms of these patterns.

CHAPTER V

Calculation of Cross-sections

Barbari and Kalman[21] calculated the cross-sections of uncharmed meson-baryon reactions using as symmetry group the SU(3) group and as dynamical group the group SU(1,3). They obtained the following expression for the cross-section.

$$\sigma^{\frac{1}{2}} = C_{M_1} C_{M_2}\left[A(m_{13}) + B(m_{13})\gamma^2 + C(m_{13})\gamma^4\right] \quad (39)$$

They were able to evaluate some of the parameters of the above equation by means of a least squares fit.

It was thought that by using the SU(4) as symmetry group and the SU(1,4) as dynamical group it would be possible to evaluate the rest of the parameters.

In section one we briefly represent the work done by Barbari and Kalman and in section two we present our calculations and results.

1. CROSS-SECTIONS USING SU(1,3)

The reactions considered by Barbari and Kalman[21] are of the type:

$$M_1 + B_1 \longrightarrow M_2 + B_2 , \quad (40)$$

where M_1, M_2 are meson operators belonging to the SU(3) meson octet and B_1, B_2 are baryon states belonging to the SU(3) baryon octet. The baryons are expressed by the Gel'fand-Cetlin patterns discussed in the previous chapter and the mesons by the non-charmed operators of eq. (38).

The expression for the cross-section is given by:

$$\sigma_{tot} = \left| \sum_{\substack{\text{int.}\\ \text{states}}} \langle B_2 \,|\, M_{op2} \,|\, B_{int} \rangle \langle B_{int} \,|\, M_{op1} \,|\, B_1 \rangle \right|^2 \qquad (41)$$

The intermediate states are obtained by operating with the meson operators on the Gel'fand patterns corresponding to baryon states. The initial and final states of baryons are as already noted members of an SU(3) octet. The intermediate states may be members either of SU(3) octet or decuplet or 27-plet. After the summation on the intermediate states eq. (41) takes the form of the eq. (39) stated in the introduction. In eq. (39) the C_{M_1}, C_{M_2} are coefficients of the meson operators depending on the mesons' structure. The A, B and C are polynomials of m_{13}, of fourth, second and zero order respectively. The parameter γ^2 is identified with the lab momentum given to the meson.

$\gamma^2 = \left(\frac{P_{LAB}}{P_o}\right)^{-n}$, where P_o is a constant having the momentum dimensions and n is a positive constant. From a least squares fit to the experimental data[22] Barbari and Kalman[21] obtained a satisfactory value of n ($n = \frac{1}{4}$). Following them, let us now set

$$A' = A\,(C_{M_1}\,C_{M_2})$$
$$B' = B\,(C_{M_1}\,C_{M_2}) / P_o^{-\frac{1}{4}} \qquad (42)$$
$$C' = C\,(C_{M_1}\,C_{M_2}) / P_o^{-\frac{1}{2}}$$

Substituting in eq. (39) the above values we get:

$$\sigma^{\frac{1}{2}} = A' + B'\,P_{LAB}^{-\frac{1}{4}} + C'\,P_{LAB}^{-\frac{1}{2}} \qquad (43)$$

For $n = \frac{1}{4}$ it was observed that $A^2 - \frac{B^2}{4AC} \sim 0$. Using this relationship the quadratic in P_{LAB} can be written:

$$\sigma^{\frac{1}{4}} = \left\{P_{LAB}^{-\frac{1}{4}} + \frac{B}{2C}\right\}^2 \times C_{M_1}\, C_{M_2}\, P_o^{-\frac{1}{4}} \quad \text{or also}$$

$$\sigma = \left\{E \left\{\frac{P_{LAB}}{P_o}\right\}^{-\frac{1}{4}} - D\right\}^4 \qquad (44)$$

where $E = C^{\frac{1}{2}} \times C_{M_1}\, C_{M_2}$ and $D = A^{\frac{1}{2}} \times C_{M_1}\, C_{M_2}$

Comparing equation (44) with Morrison's[23] formula

$$\sigma \sim \left\{\frac{P_{LAB}}{P_o}\right\}^{-n}$$

we see that they are in agreement if $E \left\{\frac{P_{LAB}}{P_o}\right\}^{-\frac{1}{4}} > D$ and the momentum is not too large. To fit high momentum values, D must be zero, this is because for high momentum values the number of channels increases and the total cross-section would approach infinity if the cross-section of the individual channels remains finite. It is impossible, however, to set D = O because D is function only of m_{13}. However, for the region of momentum values for which experimental data exists, equation (44) seems to be valid. The model is not, however, entirely satisfactory as the authors were unable to obtain value of m_{13} and thus could only make use of parameters A', B', C' derived from the least squares fit.

2. CROSS-SECTIONS USING SU(1,4)

In the hope of deriving the value of all parameters in the model, in this thesis we will consider the extension of the symmetry group to the charm group SU(4) as noted in Chapter III. The corresponding dynamical group is SU(1,4).

The reactions are taken to be of the type of eq. (40). The expression of the cross-sections is the same as in eq. (41). The baryons are expressed by the Gel'fand-Cetlin patterns and the mesons by the operators of eq. (38). The baryons of the initial and final states are considered to be members of the 20_M - dimensional representation. In table 3. the Gel'fand patterns corresponding to the baryons members of the 20_M - plet are given.

The expression of the cross-section in eq.(41) is obtained by summing over the intermediate states obtained in the process $M_1 B_1 \rightarrow$ inter. states $\rightarrow M_2 B_2$. As we noted in chapter III the meson operators correspond to a 15-dimensional representation and the baryons to a 20_M-dimensional representation of SU(4). To obtain the representations corresponding to these intermediate states we take the kronecker product of the SU(4) 15-dimensional representation for mesons with the 20_M-dimensional one for baryons.

$$15 \times 20_M = 2 \times 20_M + 20_S + 36 + 140 + 60 + 4$$

From all these representations we are interested in the 20_S because it is known[24] to contribute a great deal to the scattering cross-sections and also in the 20_M since it is the representation of the initial and final states and is clearly also of importance to the scattering cross-sections. The top row of the Gel'fand pattern must specify such a representation of SU(1,4) which contains the 20_M and 20_S representations. According to condition (27) we have the following permissible inequalities for the integers of the first row.

$$m_{14} \geq m_{25} \geq m_{14}-1$$

$$m_{14}-1 \geq m_{35} \geq m_{14}-1$$

$$m_{14}-2 \geq m_{45} \geq m_{14}-2$$

Therefore the first row can be written in four ways.

1)	$-2+\sigma$	m_{14}	$m_{14}-1$	$m_{14}-2$	$2+\bar{\sigma}$
2)	$-2+\sigma$	m_{14}	$m_{14}-2$	$m_{14}-2$	$2+\bar{\sigma}$
3)	$-2+\sigma$	$m_{14}-1$	$m_{14}-1$	$m_{14}-2$	$2+\bar{\sigma}$
4)	$-2+\sigma$	$m_{14}-1$	$m_{14}-2$	$m_{14}-2$	$2+\bar{\sigma}$

For case 4) operating on the Gel'fand pattern with meson operators yields intermediate states. The second rows of the pattern corresponding to the intermediate states and the corresponding SU(4) representations that these belong to, are as follows:

m_{14}	$m_{14}-1$	$m_{14}-2$	$m_{14}-2$	:	20_M
$m_{14}+1$	$m_{14}-1$	$m_{14}-2$	$m_{14}-2$	:	140
m_{14}	m_{14}	$m_{14}-2$	$m_{14}-3$	:	60

Similarly for the case 2) we obtain:

m_{14}	$m_{14}-2$	$m_{14}-2$	$m_{14}-2$	:	20_M
$m_{14}+1$	$m_{14}-2$	$m_{14}-2$	$m_{14}-2$	:	20_S
$m_{14}+1$	$m_{14}-1$	$m_{14}-2$	$m_{14}-3$	:	140
m_{14}	m_{14}	$m_{14}-2$	$m_{14}-2$	:	$\overline{20}_S$

For the case 3)

m_{14} $m_{14}-1$ $m_{14}-2$ $m_{14}-2$: 20_M

$m_{14}+1$ $m_{14}-1$ $m_{14}-2$ $m_{14}-3$: 140

$m_{14}-1$ $m_{14}-1$ $m_{14}-1$ $m_{14}-2$: 4

For the case 4)

m_{14} $m_{14}-1$ $m_{14}-2$ $m_{14}-2$: 20_M

$m_{14}+1$ $m_{14}-1$ $m_{14}-2$ $m_{14}-3$: 140

$m_{14}+1$ $m_{14}-2$ $m_{14}-2$ $m_{14}-2$: 20_S

Clearly cases 1) and 3) must be rejected since they do not contain the 20_S. Case 2) must be rejected because it contains the $\overline{20}_S$ and therefore contains antibaryons, $\bar{B}$. Such a situation where $M_1 B_1 \rightarrow \bar{B} \rightarrow M_2 B_2$ is clearly impossible. Therefore case 4) is the Lie algebra of SU(1,4) to be used to consider two body exclusive strong interactions of mesons and baryons. In our representation of baryons, for simplicity we have considered only the imaginary part (identified with $i\gamma$) of the complex number σ.

The way that mesons operate on baryon patterns is given in chapter IV by eq.(29 or 31) and the coefficients corresponding to each intermediate state by eq.(30 or 32). For the reaction $\pi^- p \rightarrow K^0 \Lambda$ the application of the operators and the resulting coefficients is described in detail in appendix A. For this particular reaction after the evaluation of the coefficients and the summation on the intermediate states equation (41) takes the form:

$$\sigma = C_{\pi^-}^2 \, C_{K^0}^2 \Big[0.0724\, m_{14}^4 - 0.3185\, m_{14}^3 + 0.4287\, m_{14}^2 - 0.472\, m_{14} + 2.6952 + \gamma^2 (0.1447\, m_{14}^2 - 0.3185 m_{14} + 0.9633) + 0.0724\, \gamma^4 \Big]^2 \quad (45)$$

The first polynomial is identified with the parameter A

$$0.0724\ m_{14}^4 - 0.3185\ m_{14}^3 + 0.4287\ m_{14}^2 - 0.472\ m_{14} + 2.6952 \equiv A$$

The second polynomial with B

$$0.1447\ m_{14}^2 - 0.3185\ m_{14} + 0.9633 \equiv B$$

The constant multiplying the parameter γ^4 is identified with C

$$0.0724 \equiv C$$

Following Barbari and Kalman[21] we identified the parameter γ^2 with the laboratory momentum as follows:

$$\gamma^2 = \left\{\frac{P_{LAB}}{P_o}\right\}^{-\frac{1}{2}} \qquad (46)$$

Equation (45) can then be rewritten:

$$\sigma = C^2_{\pi^-}\ C^2_{K^o} \left[A + B \left\{\frac{P_{LAB}}{P_o}\right\}^{-\frac{1}{2}} + C \left\{\frac{P_{LAB}}{P_o}\right\}^{-\frac{1}{2}} \right]^2 \qquad (47)$$

Barbari and Kalman[21] used $\frac{4AC}{B^2} \simeq 1$. But the least squares fit results given in Barbari's dissertation[22] we can see that there is a better agreement with the experimental values of cross-sections if $\frac{4AC}{B^2} \in [.8, .9]$. Taking the ratio $\frac{4AC}{B^2}$ in this region we were able to obtain a value of m_{14} which satisfies most of the reactions considered. Some of the values of the ratio $\frac{4AC}{B^2}$ and of the parameter m_{14} are given in table (4) for all the reactions considered. It can be seen that the value $m_{14} = 9$ is suitable for most of the reactions.

Using this value of m_{14} we can calculate the values of the parameters A and B. Table (5) gives the values of A, B and C for each reaction. Hence in equation (47) three parameters remain unevaluated, the C_{π^-} C_{K^0} and P_0. An attempt to evaluate in general the C_M's and P_0 was made by taking the ratios $\frac{A'}{A}$, $\frac{B'}{B}$ and $\frac{C'}{C}$ where A', B', C' are:

$$A' = A\, C_{M_1}\, C_{M_2}$$

$$B' = B\,(C_{M_1}\, C_{M_2}/P_o^{-\frac{1}{4}})$$

$$C' = C\,(C_{M_1}\, C_{M_2}/P_o^{-\frac{1}{2}})$$

and are evaluated by the least squares fit in Barbari-Kalman[21] for each reaction. The A, B and C are given in the above table for each reaction. Since mesons are made up by a quark-antiquark pair it was thought to express each C_M coefficient by a product of coefficients depending on the quarks that they form the considered meson (e.g. π^- meson is formed by a d and $\bar{u}$ quarks. Therefore C_{π^-} can be written as C_d X C_u considering that $C_q = C_{\bar{q}}$.

The ratio $\frac{A'}{A}$ is not used in the calculation of C_M's even though it provides a straight forward calculation of them. Theoretically A' should be zero, as it is noted in the first section of this chapter, to preserve finiteness of the cross-section. Equation (39) obtained by Barbari and Kalman[21] for values of the laboratory momentum (P_{LAB}) corresponding to available data is in agreement with Morrison's formula, $\sigma \sim \left\{\frac{P_{LAB}}{P_o}\right\}^{-n}$. Since Morrison's formula does not contain a parameter like A', it is clear that Barbari's[22] least squares fit is not sensitive to the parameter A'. For these reasons the calculation of C_M's is done using the ratios $\frac{B'}{B}$ and $\frac{C'}{C}$.

We give an example of our calculation of C's using the three following reactions:

$K^- n \rightarrow \pi^o \Sigma^-$ $\qquad K^- p \rightarrow \Xi^- K^+$

$\pi^- p \rightarrow \Lambda K^o$

For the $K^- n \rightarrow \pi^o \Sigma^-$

$$\frac{A'}{A} = 0.00425 = C_s C_u \left[C_u^{\ 2} - C_d^{\ 2} \right]$$

$$\frac{B'}{B} = .3792 = (C_s)\ (C_u) \left[C_u^2 - C^2_d \right] P_o^{\frac{1}{2}}$$

$$\frac{C'}{C} = 41.3141 = (C_s)\ (C_u) \left[C_u^2 - C_d^2 \right] P_o^{\frac{1}{2}}$$

$$\left\{\frac{B'}{B}\right\}^2 \div \left\{\frac{C'}{C}\right\} = \left[C_u^2 - C_d^2 \right] X\ (C_s)\ (C_u) = 0.00348$$

Similarly for the remaining reactions, we have:

$\pi^- p \rightarrow \Lambda K^o$

$$C_s C_u C_d^{\ 2} = 0.00466$$

$K^- p \rightarrow \Xi^- K^+$

$$C_u^{\ 2}\ C_s^{\ 2} = 0.004696$$

From the reactions $K^- n \rightarrow \pi^o \Sigma^-$ and $\pi^- p \rightarrow \Lambda K^o$, if we divide the product of the coefficients of each other, we obtain

$$\left[\frac{C_u^{\ 2}}{C_d^2} - 1 \right] = 0.7466 \Rightarrow C_u \doteq 1.32 C_d \qquad (48)$$

Also, from the reactions $K^- p \rightarrow \Xi^- K^+$ and $K^- n \rightarrow \Sigma^- \pi^o$, we obtain,

$$C_u^2 - C_d^2 \doteq 0.0508 \qquad (49).$$

From (48) and (49), we obtain

$$C_u = 0.3447$$
$$C_d = 0.2608.$$

Attempts were made to evaluate C_s for all the reactions. Results for those reactions with a large amount of experimental data are given below.

$K^- p \longrightarrow nK^0$	$C_s = 0.1854$
$K^- p \longrightarrow \pi^- \Sigma^+$	$C_s = 0.0795$
$K^- p \longrightarrow K^+ \Xi^-$	$C_s = 0.1988$
$K^- n \longrightarrow \pi^0 \Sigma^-$	$C_s = 0.1988$
$K^- n \longrightarrow \pi^- \Sigma^0$	$C_s = 0.0455$
$\pi^- p \longrightarrow \Lambda K^0$	$C_s = 0.1988$
$\pi^- p \longrightarrow K^+ \Sigma^-$	$C_s = 0.0826$

The reason that in the reactions $K^- p \rightarrow K^+ \Xi^-$, $K^- n \rightarrow \pi^0 \Sigma^-$ and $\pi^- p \rightarrow \Lambda K^0$ we obtain the same value of C_s is that these three reactions were used to calculate C_u and C_d and therefore the values of C_u and C_d satisfy them exactly. Among the other reactions, the $K^- p \rightarrow nK^0$ is consistent with the others with only 6.7% difference. The $K^- p \rightarrow \pi^- \Sigma^+$ and $\pi^- p \rightarrow K^+ \Sigma^-$ are consistent with each other but not with the rest of the presented reactions. The reaction $K^- n \rightarrow \pi^- \Sigma^0$ is not consistent with any. Re-examining the least squares fit in Barbari's[22] dissertation, we see that it does not include all the experimental points.

In figure 5.1 we give a semilog plot of versus P_{LAB} for the reaction $\pi^- p \longrightarrow K^0 \Lambda$. From equation (39) it is expected that in the plot appears a series of straight lines. This is because eq. (39) contains different powers of the variable P_{LAB}. The data indeed does seem to have the appearence as indicated by the two lines drawn on the graph. The model we use for the calculation of the cross-sections does not provide any means for the calculation of the cross-section near the threshold point. Therefore the first region of the graph is not taken into account. The third region contains the points used by Barbari[23] in his least squares fit. The results of the least squares fit are indicated by the curve made by the points which are circled. The points of region 2. are not included in Barbari's least squares fit. It can be easily seen from Barbari's curve extended to region 2. that the equation used for the cross-section (eq. 39), indeed roughly gives the behaviour of the reaction even in this region. However if the points of region 2. were included, one could obtain a better fit. This might be the reason for the apparent inconsistency in the above values of C_s.

The parameter P_o was also calculated for all the reactions. The interval of P_o's values is [94.95, 214,92] in $^{Mev}/_{C}$ units. An attempt was made to find a value of P_o which satisfies most of the reactions but there was no satisfactory result.

Calculations about charmed reactions were also made. But in absence of experimental data it is not possible to calculate the product of the coefficients depending on the quarks. To evaluate the parameters A, B, and C, we used m_{14} = 9 since the charmed baryons are in the same representation as the uncharmed. In appendix B. we give the final expressions of the cross-sections for charmed and uncharmed reactions.

The calculation by Barbari and Kalman[21] for cross-sections of meson-baryon reactions using the dynamical group SU(1,3) was extended using the dynamical group SU(1,4). As it is noted in the introduction, it was thought that using a larger group it could be possible to evaluate more parameters of the model presented by Barbari and Kalman[21]. Indeed it was possible to determine the parameter m_{14} which along with P_{LAB} defines the representation of the dynamical group.

It was tried also to calculate the coefficients C_{N} of the meson operators which are associated with the meson structure. In this calculation there were some inconsistencies. Re-examining the least squares fit of Barbari's[22] dissertation, it was seen that not all the experimental points are included. This might be a reason for the above inconsistency.

CHAPTER VI

Conclusions

For the exclusive cross-sections of two body strong interactions there is not any theoretically derived equation. The only existing equation is an empirical formula called Morrison's formula.

Barbari and Kalman sought to derive an equation for such cross-sections using the dynamical group method. This method provides an explicit model for the exclusive cross-sections of the two body strong interactions. Using the Gel'fand-Cetlin basis it is possible to find all the intermediate states of a reaction. Barbari and Kalman considered SU(3) as the maximum symmetry group for the hadrons, and SU(1,3) as the dynamical group for the process of meson-baryon interactions. The equation that they obtained is

$$\sigma^{\frac{1}{2}} = C_{M_1} C_{M_2} \left[A(m_{13}) + B(m_{13}) \gamma^2 + C \gamma^4 \right]$$

They identified the parameter γ^2 with $(P_{LAB} / P_0)^{-n}$. The parameters n and

$$A' = A\, C_{M_1} C_{M_2}$$

$$B' = B\, C_{M_1} C_{M_2} / P_0^{-\frac{1}{4}}$$

$$C' = C\, C_{M_1} C_{M_2} / P_0^{-\frac{1}{2}} ,$$

are evaluated by a least squares fit. The value of n was found tó be ½.

The above equation for cross-sections is in agreement with Morrison's formula, given in the previous chapter, for the range of momentum for which there is experimental data.

If all parameters of the above equation were evaluated, one could obtain the cross-section of any reaction. However four parameters remained undetermined; m_{13}, C_{M_1}, C_{M_2} and P_0.

It was thought that considering a larger symmetry one would be able to determine the remaining parameters. In this thesis, the larger group , SU(1,4), is taken to be the dynamical group for the process. As maximum symmetry group for the hadrons, SU(4) is considered.

The calculation of the cross-sections using SU(1,4) resulted in the same type of equation as the one of Barbari and Kalman, as expected. The only difference is that the terms A and B are polynomials of m_{14} which is the discrete parameter of the irreducible representation of the group SU(1,4). Using the larger dynamical group SU(1,4), it was possible indeed, to determine the parameter m_{14}.

The irreducible representation of SU(1,4) that was used for the hadrons, is specified by two parameters; a discrete parameter m_{14} and a continuous parameter γ. γ^2 was already identified by Barbari and Kalman to be proportional to P_{LAB}. With the determination of m_{14}, the representation is now known.

Besides m_{14}, the evaluation of the parameters C_{M_1}, C_{M_2}

and P_0, was also attempted. In the calculation of the C_M's, some inconsistencies appeared. Re-examining the least squares fit that we used, it was seen that not all the points of the experimental data were included. This might be a reason for the apparent inconsistency. As far as the P_0 parameter is concerned, it was not possible to find a unique value which would fit satisfactorily all the reactions considered. If one would be able to obtain good values for the C_M's, then it would also be possible for one to calculate P_0 as P_0 can be obtained from the ratio $C_{M_1} C_{M_2}$ /(B'/B).

The model provided by the dynamical group is totally determined since the two parameters of the irreducible representation used, γ and m_{14}, are determined. The evaluation of the parameters C_M and P_0, is a matter of fitting. It would be of interest if one would use another least squares fit including all the points of the experimental data, to recalculate the parameters C_M and P_0. This also would be a test for the model used. If one finds satisfactory values for these parameters one could also test the model by making predictions about cross-sections already evaluated experimentally. The equation obtained by the dynamical group method contains relatively few parameters compared with the number of reactions on which we can apply this equation to the cross-section. It is therefore very easy to test the model once we have consistent values for the parameters C_M and P_0. If the model is succesful then we have a theoretically derived equation for the cross-section of two body strong interactions.

Appendix A.

Reaction $\pi^- p \longrightarrow K^o \Lambda$

Operating with the π^- meson operator $(C_\pi - I_{25} I_{51})$ on the Gel'fand pattern of the proton

$$\left(\begin{array}{ccccccccc} -2+i\gamma & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & & 2-i\gamma \\ & m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\ & & m_{14} & & m_{14}-1 & & m_{14}-2 & & \\ & & & m_{14} & & m_{14}-1 & & & \\ & & & & m_{14} & & & & \end{array}\right) \qquad (A.1)$$

yields the following intermediate states. (The top row is unchanged and hence we omit it).

$$\left(\begin{array}{ccccccc} m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\ & m_{14}-1 & & m_{14}-1 & & m_{14}-3 & \\ & & m_{14} & & m_{14}-1 & & \\ & & & m_{14}-1 & & & \end{array}\right) \equiv |B\rangle$$

$$\left(\begin{array}{ccccccc} m_{14}-1 & & m_{14}-2 & & m_{14}-2 & & m_{14}-2 \\ & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\ & & m_{14}-1 & & m_{14}-2 & & \\ & & & m_{14}-1 & & & \end{array}\right) \equiv |C\rangle \qquad (A.2)$$

$$\left(\begin{array}{ccccccc} m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\ & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\ & & m_{14}-1 & & m_{14}-2 & & \\ & & & m_{14}-1 & & & \end{array}\right) \equiv |D\rangle$$

$$
\begin{pmatrix}
m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\
& m_{14}-1 & & m_{14}-1 & & m_{14}-3 & \\
& & m_{14}-1 & & m_{14}-2 & & \\
& & & m_{14}-1 & & &
\end{pmatrix} \equiv |\Xi\rangle
$$

$$
\begin{pmatrix}
m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 \\
& m_{14} & & m_{14}-1 & & m_{14}-2 & \\
& & m_{14} & & m_{14}-1 & & \\
& & & m_{14}-1 & & &
\end{pmatrix} \equiv |\eta\rangle \qquad (A.2)
$$

$$
\begin{pmatrix}
m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\
& m_{14} & & m_{14}-1 & & m_{14}-2 & \\
& & m_{14} & & m_{14}-1 & & \\
& & & m_{14}-1 & & &
\end{pmatrix} \equiv |\Lambda\rangle
$$

On the other hand operating with the K^o meson operator $(C_{K^o}\, I_{25} I_{53})$ on the Gel'fand pattern of the lamda particle

$$
\begin{pmatrix}
-2-i\gamma & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & & 2-i\gamma \\
& m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 & \\
& & m_{14} & & m_{14}-1 & & m_{14}-2 & & \\
& & & m_{14}-1 & & m_{14}-1 & & & \\
& & & & m_{14}-1 & & & &
\end{pmatrix}
$$

yields the following intermediate states (omiting again the first row).

$$
\begin{pmatrix}
m_{14} & & m_{14}-1 & & m_{14}-2 & & m_{14}-2 \\
& m_{14} & & m_{14}-1 & & m_{14}-2 & \\
& & m_{14} & & m_{14}-1 & & \\
& & & m_{14}-1 & & &
\end{pmatrix} \equiv |\eta\rangle
$$

$$\begin{pmatrix} m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\ & m_{14} & & m_{14}-1 & & m_{14}-2 & \\ & & m_{14} & & m_{14}-1 & & \\ & & & m_{14}-1 & & & \end{pmatrix} \equiv |A\rangle \qquad (A.3)$$

$$\begin{pmatrix} m_{14}-1 & & m_{14}-1 & & m_{14}-2 & & m_{14}-3 \\ & m_{14}-1 & & m_{14}-1 & & m_{14}-3 & \\ & & m_{14} & & m_{14}-1 & & \\ & & & m_{14}-1 & & & \end{pmatrix} \equiv |B\rangle$$

The common intermediate states of $\pi^- p$ and $K^0 \Lambda$ are the $|n\rangle$, $|A\rangle$, $|B\rangle$, and are the ones that contribute in the expression of the cross-section given by equation (41). The meson operators are a combination of raising and lowering operator (e.g. $\pi^- = C_{\pi^-} I_{25} I_{51}$) where C_{π^-} is aconstant associated to the meson's structure. I_{25} is a raising operator that raises one element of the fourth third and the second rows. The I_{51} is a lowering operator and it lowers one element of the fourth till the first row included.If for example I_{25} raises the first element of each of the three rows and I_{51} lowers the second element of each of the four rows the resulting state is written $m^{1222}_{1234}\, \hat{m}^{111}_{234}$ .The m represents the lowered state and $\hat{m}$ the raised state. The subscripts 1,2,3,4 represent the rows and the superscripts the changed elements of each row. Analogously the coefficients corresponding to the operators are written $a_{1234}^{1222}\, b_{234}^{111}$. If both operators I_{51} and I_{25} operate on the first element of each row then the resulted state would be represented by m_1^1 which gives the net action of these operators.

Explicitly m_{1234}^{1222} corresponds to changes of the following elements:

$$m_{11} \longrightarrow m_{11}-1$$
$$m_{22} \longrightarrow m_{22}-1$$
$$m_{23} \longrightarrow m_{23}-1$$
$$m_{24} \longrightarrow m_{24}-1$$

analogously $\hat{m}_{234}^{111}$

$$m_{12} \longrightarrow m_{12}+1$$
$$m_{13} \longrightarrow m_{13}+1$$
$$m_{14} \longrightarrow m_{14}+1$$

Thus $\pi^- p = C_{\pi^-} I_{25} I51 |p\rangle$ =

$$\left[\begin{pmatrix} b_{234}^{222}\end{pmatrix}\begin{pmatrix} a_{1234}^{1222}\end{pmatrix} + \begin{pmatrix} b_{234}^{134}\end{pmatrix}\begin{pmatrix} a_{1234}^{1134}\end{pmatrix} + \begin{pmatrix} b_{234}^{224}\end{pmatrix}\begin{pmatrix} a_{1234}^{1224}\end{pmatrix} + \begin{pmatrix} b_{234}^{234}\end{pmatrix}\begin{pmatrix} a_{1234}^{1234}\end{pmatrix}\right]\left| m_{11}\right\rangle$$

$$+ \left[\begin{pmatrix} b_{234}^{131}\end{pmatrix}\begin{pmatrix} a_{1234}^{1134}\end{pmatrix} + \begin{pmatrix} b_{234}^{221}\end{pmatrix}\begin{pmatrix} a_{1234}^{1224}\end{pmatrix} + \begin{pmatrix} b_{234}^{231}\end{pmatrix}\begin{pmatrix} a_{1234}^{1234}\end{pmatrix}\right]\left| m_{14\, m_4}^{14-1}\right\rangle$$

$$+ \left[\begin{pmatrix} b_{234}^{111}\end{pmatrix}\begin{pmatrix} a_{1234}^{1134}\end{pmatrix} + \begin{pmatrix} b_{234}^{211}\end{pmatrix}\begin{pmatrix} a_{1234}^{1234}\end{pmatrix}\right]\left| m_{134}^{134}\, \hat{m}_{34}^{11}\right\rangle .$$

From eq (A.1) and (A.3) we note that the state $|n\rangle$ corresponds to the pattern m_{11} in eq. (A.4) and thus the coefficient of $|n\rangle$ is

$$\begin{pmatrix} b_{234}^{222}\end{pmatrix}\begin{pmatrix} a_{1234}^{1222}\end{pmatrix} + \begin{pmatrix} b_{234}^{134}\end{pmatrix}\begin{pmatrix} a_{1234}^{1134}\end{pmatrix} + \begin{pmatrix} b_{234}^{224}\end{pmatrix}\begin{pmatrix} a_{1234}^{1224}\end{pmatrix} + \begin{pmatrix} b_{234}^{234}\end{pmatrix}\begin{pmatrix} a_{1234}^{1234}\end{pmatrix}$$

Similarly that corresponding to $|A\rangle$ is

$$\left(b^{131}_{234}\right)\left(a^{1134}_{1234}\right) + \left(b^{221}_{234}\right)\left(a^{1224}_{1234}\right) + \left(b^{231}_{234}\right)\left(a^{1234}_{1234}\right)$$

That corresponding to $|B\rangle$ is

$$\left(b^{111}_{234}\right)\left(a^{1134}_{1234}\right) + \left(b^{221}_{234}\right)\left(a^{1234}_{1234}\right)$$

For $K^0 \Lambda$ the coefficients corresponding to state $|n\rangle$ is

$$\left(b^{111}_{234}\right)\left(a^{11}_{34}\right) + \left(b^{114}_{234}\right)\left(a^{14}_{34}\right) + \left(b^{134}_{234}\right)\left(a^{34}_{34}\right)$$

That corresponding to state $|A\rangle$ is

$$\left(b^{111}_{234}\right)\left(a^{14}_{34}\right) + \left(b^{131}_{234}\right)\left(a^{34}_{34}\right)$$

and that corresponding to state $|B\rangle$ is

$$\left(b^{111}_{234}\right)\left(a^{34}_{34}\right)$$

Evaluating these coefficients using equations (30 and 32) we obtain:

For $\pi^- |p\rangle$

State $|n\rangle$

$$\left(b^{222}_{234}\right)\left(a^{1222}_{1234}\right) = .1054 \left[(1-m_{14})^2+\gamma^2\right]$$

$$\left(b^{134}_{234}\right)\left(a^{1134}_{1234}\right) = .0208 \left[(4-m_{14})^2+\gamma^2\right]$$

$$\left(b^{224}_{234}\right)\left(a^{1224}_{1234}\right) = .0139 \left[(4-m_{14})^2+\gamma^2\right]$$

$$\left(b^{234}_{234}\right)\left(a^{1234}_{1234}\right) = .0417\left[(4-m_{14})^2+\gamma^2\right]$$

State $|A\rangle$

$$\left(b^{131}_{234}\right)\left(a^{1134}_{1234}\right) = .00352\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

$$\left(b^{221}_{234}\right)\left(a^{1224}_{1234}\right) = .01174\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

$$\left(b^{231}_{234}\right)\left(a^{1234}_{1234}\right) = .00704\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

State $|B\rangle$

$$\left(b^{111}_{234}\right)\left(a^{1134}_{1234}\right) = .069\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

$$\left(b^{211}_{234}\right)\left(a^{1234}_{1234}\right) = .01626\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

Similarly for $K^0\Lambda$

State $|n\rangle$

$$\left(b^{111}_{234}\right)\left(a^{11}_{34}\right) = .2449\left[(1+m_{14})^2+\gamma^2\right]$$

$$\left(b^{114}_{234}\right)\left(a^{14}_{34}\right) = .01021\left[(4-m_{14})^2+\gamma^2\right]$$

$$\left(b^{134}_{234}\right)\left(a^{34}_{34}\right) = .05103\left[(4-m_{14})^2+\gamma^2\right]$$

State $|A\rangle$

$$\left(b^{111}_{234}\right)\left(a^{14}_{34}\right) = .04313\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

$$\left(b^{131}_{234}\right)\left(a^{34}_{34}\right) = .00863\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

State $|B\rangle$

$$\left(b^{111}_{234}\right)\left(a^{34}_{34}\right) = .169\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{\frac{1}{2}}$$

Summing up for $\pi^- p\rangle$ we obtain.

$$.0764\left[(4-m_{14})^2+\gamma^2\right]+.1054\left[(1-m_{14})^2+\gamma^2\right]+.1076\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{1/2}$$

Similarly for $K^0\Lambda$

$$.0612\left[(4-m_{14})^2+\gamma^2\right]+.245\left[(1+m_{14})^2+\gamma^2\right]+.2208\left[(4-m_{14})^2+\gamma^2\right]^{\frac{1}{2}}\left[(2+m_{14})^2+\gamma^2\right]^{1/2}$$

The equation (41) after taking the scalar product becomes

$$\sigma^{\frac{1}{2}}=C_{\pi}-C_{K^0}\Big[.0047\left[(4-m_{14})^2+\gamma^2\right]^2+.0019\left[(4-m_{14})^2+\gamma^2\right]\left[(1+m_{14})^2+\gamma^2\right]$$
$$+.00645\left[(4-m_{14})^2+\gamma^2\right]\left[(1-m_{14})^2+\gamma^2\right]+.0258\left[(1+m_{14})^2+\gamma^2\right]\left[(1-m_{14})^2+\gamma^2\right]$$
$$+.0237\left[(4-m_{14})^2+\gamma^2\right]\left[(2+m_{14})^2+\gamma^2\right].$$

Expanding the brackets the above equation takes the form

$$\sigma^{\frac{1}{2}}=C_{\pi^-}C_{K^0}\Big[.0724m_{14}^4-.3185m_{14}^3+.4287m_{14}^2-.472m_{14}+2.6952$$
$$+\gamma^2(.1447m_{14}^2-.3185m_{14}+.9633)+.0724\gamma^4\Big]$$

Appendix B

The final expression of the cross-section given by eq.47 are given below. A detailed representation of how one arrives at this expression is given in the previous appendix. The parameter γ^2 is identified with $\left(\frac{P_{LAB}}{P_o}\right)-1$

The coefficients $C_{M_1}C_{M_2}$ can be expressed in terms of the quarks coefficients in the following way :

$$C_{\pi^-} = C_{\pi^+} = C_u C_d \qquad C_{\pi^o} = \frac{1}{\sqrt{2}}\left(C_u^2 - C_d^2 \right)$$

$$C_{\bar{K}^o} = C_{K^o} = C_s C_d \qquad C_{K^+} = C_{K^-} = C_u C_s$$

$$C_{D^+} = C_{D^-} = C_c C_d \qquad C_{D^o} = C_{\bar{D}^o} = C_c C_u$$

$$C_{F^+} = C_{F^-} = C_c C_s$$

Reaction $K^- p \longrightarrow K^+ \Xi^-$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{K^+} \left[0.1305 m_{14}^4 - 0.2715 m_{14}^3 + 0.6271 m_{14}^2 - 0.0575 m_{14} + 2.6663 + \gamma^2 (0.261 m_{14}^2 - 0.2715 m_{14} + 1.2511) + 0.1305 \gamma^4 \right]$$

$K^- p \longrightarrow K^o \Xi^o$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{K^o} \left[0.1576 m_{14}^4 - 0.4878 m_{14}^3 + 1.6613 m_{14}^2 - 0.9339 m_{14} + 4.4449 + \gamma^2 (0.3153 m_{14}^2 - 0.4878 m_{14} + 1.7965) + 0.1576 \gamma^4 \right]$$

$K^- p \longrightarrow \pi^- \Lambda$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{\pi^-} \left[0.0791 m_{14}^4 - 0.3176 m_{14}^3 + 0.6022 m_{14}^2 - 0.8469 m_{14} + 2.3965 + \gamma^2 (0.1582 m_{14}^2 - 0.3176 m_{14} + 0.948) + 0.0791 \gamma^4 \right]$$

$K^- p \longrightarrow K^0 n$

$$\sigma^{\frac{1}{2}} = C_K - C_{K^0}\left[0.1434m_{14}^4 - 0.3226m_{14}^3 + 1.6527m_{14}^2 - 1.7802m_{14} + 4.614 + \gamma^2(0.2868m_{14}^2 - 0.3226m_{14} + 1.5827) + 0.1434\gamma^4\right]$$

$K^- p \longrightarrow \Sigma^+ \pi^-$

$$\sigma^{\frac{1}{2}} = C_K - C_{\pi^-}\left[0.1462m_{14}^4 - 0.4689m_{14}^3 + 1.5654m_{14}^2 - 2.1005m_{14} + 4.4471 + \gamma^2(0.2923m_{14}^2 - 0.4689m_{14} + 1.6131) + 0.1462\gamma^4\right]$$

$K^- p \longrightarrow \Sigma^- \pi^+$

$$\sigma^{\frac{1}{2}} = C_K - C_{\pi^+}\left[0.1024m_{14}^4 - 0.5125m_{14}^3 + 1.4042m_{14}^2 - 2.3805m_{14} + 4.0896 + \gamma^2(0.2049m_{14}^2 - 0.5125m_{14} + 1.3605) + 0.1024\gamma^4\right]$$

$\pi^- p \longrightarrow \pi^0 n$

$$\sigma^{\frac{1}{2}} = C_\pi - C_{\pi^0}\left[0.2678m_{14}^4 - 1.1829m_{14}^3 + 0.9832m_{14}^2 - 0.8638m_{14} + 12.7963 + \gamma^2(0.5356m_{14}^2 - 1.1829m_{14} + 4.1054) + 0.2678\gamma^4\right]$$

$\pi^- p \longrightarrow \Lambda K^0$

$$\sigma^{\frac{1}{2}} = C_\pi - C_{K^0}\left[0.0724m_{14}^4 - 0.3185m_{14}^3 + 0.4287m_{14}^2 - 0.472m_{14} + 2.6952 + \gamma^2(0.1447m_{14}^2 - 0.3185m_{14} + 0.9633) + 0.0724\gamma^4\right]$$

$\pi^- p \longrightarrow \Sigma^- K^+$

$$\sigma^{\frac{1}{2}} = C_\pi - C_{K^+}\left[0.077m_{14}^4 - 0.4264m_{14}^3 + 1.2691m_{14}^2 - 2.4837m_{14} + 3.8051 + \gamma^2(0.1541m_{14}^2 - 0.4264m_{14} + 1.1093) + 0.077\gamma^4\right]$$

$K^- n \longrightarrow \Sigma^- \pi^0$

$$\sigma^{\frac{1}{2}} = C_K - C_{\pi^0}\left[0.1122m_{14}^4 - 0.7019m_{14}^3 + 2.3065m_{14}^2 - 4.6769m_{14} + 6.3721 + \gamma^2(0.2245m_{14}^2 - 0.7019m_{14} + 1.7211) + 0.1122\gamma^4\right]$$

$K^- n \longrightarrow \Sigma^0 \pi^-$

$$\sigma^{\frac{1}{2}} = C_K - C_{\pi^-} \Big[0.0979m_{14}^4 - 0.5264m_{14}^3 + 1.4912m_{14}^2 - 2.7613m_{14} + 4.3056 + \gamma^2(0.1957m_{14}^2 - 0.5264m_{14} + 1.3441) + 0.0979\gamma^4\Big]$$

$K^- n \longrightarrow pK^0$

$$\sigma^{\frac{1}{2}} = C_{K^-} + C_{K^0} \Big[0.0317m_{14}^4 - 0.127m_{14}^3 - 0.3809m_{14}^2 + 1.0159m_{14} + 2.0317 + \gamma^2(0.0635m_{14}^2 - 0.127m_{14} + 0.6349) + 0.0317\gamma^4\Big]$$

$\pi^+ p \longrightarrow \Sigma^+ K^+$

$$\sigma^{\frac{1}{2}} = C_{\pi^+} C_{K^+} \Big[0.0794m_{14}^4 - 0.0952m_{14}^3 - 0.619m_{14}^2 + 1.2063m_{14} + 2.8571 + \gamma^2(0.1587m_{14}^2 - 0.0952m_{14} + 1.0317) + 0.0794\gamma^4\Big]$$

$K^- p \longrightarrow \Lambda \pi^0$

$$\sigma^{\frac{1}{2}} = C_K - C_{\pi^0} \Big[0.2715m_{14}^4 - 1.13m_{14}^3 + 2.2815m_{14}^2 - 2.8924m_{14} + 10.1535 + \gamma^2(0.543m_{14}^2 - 1.13m_{14} + 3.5926) + 0.2715\gamma^4\Big]$$

$K^- p \longrightarrow \bar{D}^0 \Xi^-_{+a}$

$$\sigma^{\frac{1}{2}} = C_K - C_{\bar{D}^0} \Big[0.0744m_{14}^4 - 0.1477m_{14}^3 + 0.4515m_{14}^2 - 0.0427m_{14} + 1.7379 + \gamma^2(0.1487m_{14}^2 - 0.1477m_{14} + 0.7625) + 0.0744\gamma^4\Big]$$

$\pi^- p \longrightarrow D^- \Sigma^0_+$

$$\sigma^{\frac{1}{2}} = C_\pi C_{D^-} \Big[0.066m_{14}^4 - 0.2529m_{14}^3 + 0.7446m_{14}^2 - 1.453m_{14} + 2.1319 + \gamma^2(0.132m_{14}^2 - 0.2529m_{14} + 0.747) + 0.066\gamma^4\Big]$$

$\pi^- p \longrightarrow D^- \Lambda_+$

$$\sigma^{\frac{1}{2}} = C - C_{D^-} \Big[0.0558m_{14}^4 - 0.2205m_{14}^3 + 0.3623m_{14}^2 - 0.3983m_{14} + 1.4386 + \gamma^2(0.1116m_{14}^2 - 0.2206m_{14} + 0.6196) + 0.0558\gamma^4\Big]$$

$\pi^- n \longrightarrow D^- \Sigma^-_+$

$$\sigma^{\frac{1}{2}} = C_{\pi^-} + C_{D^-} \Big[0.097m_{14}^4 - 0.351m_{14}^3 + 0.7248m_{14}^2 - 1.1181m_{14} - \\ + 2.5044 + \gamma^2(0.194m_{14}^2 - 0.351m_{14} + 1.0141) + 0.097\gamma^4 \Big]$$

$\pi^- n \longrightarrow \bar{D}^0 \Sigma^0_+$

$$\sigma^{\frac{1}{2}} = C_{\pi^-} - C_{\bar{D}^0} \Big[0.0823m_{14}^4 - 0.2507m_{14}^3 + 0.4265m_{14}^2 - 0.6659m_{14} - \\ + 2.1251 - \gamma^2(0.1646m_{14}^2 - 0.2507m_{14} + 0.7819) + 0.0823\gamma^4 \Big]$$

$\pi^+ n \longrightarrow \bar{D}^0 \Lambda_+$

$$\sigma^{\frac{1}{2}} = C_{\pi^+} + C_{\bar{D}^0} \Big[0.131m_{14}^4 - 0.0428m_{14}^3 + 0.7154m_{14}^2 + 0.5054m_{14} - \\ + 2.4262 + \gamma^2(0.2799m_{14}^2 - 0.0428m_{14} + 1.1664) - 0.131\gamma^4 \Big]$$

$K^- n \longrightarrow D^- \Xi^-_{+a}$

$$\sigma^{\frac{1}{2}} = C_{K^-} - C_{D^-} \Big[0.0706m_{14}^4 - 0.3097m_{14}^3 + 0.4727m_{14}^2 - 0.4621m_{14} - \\ + 2.1513 + \gamma^2(0.1412m_{14}^2 - 0.3097m_{14} + 0.8815) + 0.0706\gamma^4 \Big]$$

$K^- n \longrightarrow D^- \Xi^-_{+s}$

$$\sigma^{\frac{1}{2}} = C_{K^-} - C_{D^-} \Big[0.055m_{14}^4 - 0.2462m_{14}^3 + 0.715m_{14}^2 - 1.3829m_{14} - \\ + 2.1315 + \gamma^2(0.11m_{14}^2 - 0.2462m_{14} + 0.6939) + 0.055\gamma^4 \Big]$$

$K^- n \longrightarrow F^- \Sigma^-_+$

$$\sigma^{\frac{1}{2}} = C_{K^-} - C_{F^-} \Big[0.0565m_{14}^4 - 0.3484m_{14}^3 + 1.1386m_{14}^2 - 2.1244m_{14} - \\ + 2.5059 + \gamma^2(0.1129m_{14}^2 - 0.3484m_{14} + 0.769) + 0.0565\gamma^4 \Big]$$

$\pi^- p \longrightarrow \bar{D}^0 \Xi^-_{+s}$

$$\sigma^{\frac{1}{2}} = C_{\pi^-} - C_{\bar{D}^0} \Big[0.0688m_{14}^4 - 0.3643m_{14}^3 + 1.1901m_{14}^2 - 2.2108m_{14} - \\ + 2.4937 + \gamma^2(0.1377m_{14}^2 - 0.3643m_{14} + 0.828) + 0.0688\gamma^4 \Big]$$

$$K^- p \longrightarrow \bar{D}^0 \Xi^-_{+s}$$

$$\sigma^{\frac{1}{2}} = C_{\bar{K}} C_{\bar{D}^0}\Big[.0798 m_{14}^4 - .3568 m_{14}^3 + .8969 m_{14}^2 - 1.5284 m_{14} + 2.6477 + \gamma^2(.1597 m_{14}^2 - .3568 m_{14} + .9518) + .0798\gamma^4\Big]$$

$$K^- p \longrightarrow F^- \Sigma^0_+$$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{F^-}\Big[.0411 m_{14}^4 - .2432 m_{14}^3 + .8025 m_{14}^2 - 1.5115 m_{14} + 1.7696 + \gamma^2(.0823 m_{14}^2 - .2432 m_{14} + .9409) + .0411\gamma^4\Big]$$

$$K^- p \longrightarrow D^- \Xi^0_{+s}$$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{D^-}\Big[.0798 m_{14}^4 - .3542 m_{14}^3 + .8836 m_{14}^2 - 1.5288 m_{14} + 2.7135 + \gamma^2(.1595 m_{14}^2 - .3542 m_{14} + .9547) + .0798\gamma^4\Big]$$

$$K^- p \longrightarrow F^- \Lambda_+$$

$$\sigma^{\frac{1}{2}} = C_{K^-} C_{F^-}\Big[.0787 m_{14}^4 + .0117 m_{14}^3 + .4228 m_{14}^2 + .3966 m_{14} + 1.3647 + \gamma^2(.1575 m_{14}^2 + .0117 m_{14} + .6561) + .0787\gamma^4\Big]$$

Table 1.

Properties of quarks.

Quark label	B	Y	I	I_3	Q	S	C
u	1/3	1/3	1/2	1/3	2/3	0	0
d	1/3	1/3	1/2	-1/2	-1/3	0	0
s	1/3	-2/3	0	0	-1/3	-1	0
c	1/3	-2/3	0	0	2/3	0	1

Table 2.

Quark structure of hadrons.

Baryon	Quark structure
p	uud
n	ddu
Λ^{0}	uds
Σ^{+}	uus
Σ^{0}	uds
Σ^{-}	dds
Ξ^{0}	ssu
Ξ^{-}	ssd
Σ_c^{++}	uuc
Σ_c^{+}	udc
Σ_c^{0}	ddc
Ξ_c^{+}	usc
Ξ_c^{0}	dsc
Ω_c^{0}	ssc
Λ_c^{+}	udc
Ξ_c^{A+}	usc
Ξ_c^{A0}	dsc
Ξ_{cc}^{++}	ccu
Ξ_{cc}^{+}	ccd
Ω_{cc}^{+}	ccs

Table 2. (cont.)

Meson	Quark structure
K^+	$u\bar{s}$
K^0	$d\bar{s}$
π^+	$u\bar{d}$
π^0	$(1/2)^{\frac{1}{2}}(u\bar{u}-d\bar{d})$
π^-	$d\bar{u}$
η	$(1/6)^{\frac{1}{2}}(u\bar{u}-d\bar{d}-2s\bar{s})$
$\bar{K}^0$	$s\bar{d}$
K^-	$s\bar{u}$
F^+	$c\bar{s}$
D^+	$c\bar{d}$
D^0	$c\bar{u}$
$\bar{D}^0$	$u\bar{c}$
D^-	$d\bar{c}$
F^-	$s\bar{c}$

Table 3.

Gel'fand-Cetlin patterns for the $J^P = \frac{1}{2}^+$ baryons.
The first two rows corresponding to the SU(1,4) and SU(4) groups are omittedsince they are the same for all baryons.

$$\begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14} & & m_{14}-1 & \\ & & m_{14} & & \end{pmatrix} \equiv p \qquad \begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14} & & m_{14}-1 & \\ & & m_{14}-1 & & \end{pmatrix} \equiv n$$

$$\begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14}-1 & & m_{14}-1 & \\ & & m_{14}-1 & & \end{pmatrix} \equiv \Lambda \qquad \begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14} & & m_{14}-2 & \\ & & m_{14} & & \end{pmatrix} \equiv \Sigma^+$$

$$\begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14} & & m_{14}-2 & \\ & & m_{14}-1 & & \end{pmatrix} \equiv \Sigma^0 \qquad \begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14} & & m_{14}-2 & \\ & & m_{14}-2 & & \end{pmatrix} \equiv \Sigma^-$$

$$\begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14}-1 & & m_{14}-2 & \\ & & m_{14}-1 & & \end{pmatrix} \equiv \Xi^0 \qquad \begin{pmatrix} m_{14} & & m_{14}-1 & & m_{14}-2 \\ & m_{14}-1 & & m_{14}-2 & \\ & & m_{14}-2 & & \end{pmatrix} \equiv \Xi^-$$

$$\begin{pmatrix} m_{14} & m_{14}-2 & m_{14}-2 \\ & m_{14} & m_{14}-2 \\ & & m_{14}-2 \end{pmatrix} \equiv \Sigma^{-}_{+} \qquad \begin{pmatrix} m_{14} & m_{14}-2 & m_{14}-2 \\ & m_{14} & m_{14}-2 \\ & & m_{14} \end{pmatrix} \equiv \Sigma^{+}_{+}$$

$$\begin{pmatrix} m_{14} & m_{14}-2 & m_{14}-2 \\ & m_{14} & m_{14}-2 \\ & & m_{14}-1 \end{pmatrix} \equiv \Sigma^{0}_{+} \qquad \begin{pmatrix} m_{14}-1 & m_{14}-1 & m_{14}-2 \\ & m_{14}-1 & m_{14}-1 \\ & & m_{14}-1 \end{pmatrix} \equiv \Lambda^{0}_{+}$$

$$\begin{pmatrix} m_{14} & m_{14}-2 & m_{14}-2 \\ & m_{14}-1 & m_{14}-2 \\ & & m_{14}-2 \end{pmatrix} \equiv \Xi^{-}_{+s} \qquad \begin{pmatrix} m_{14} & m_{14}-2 & m_{14}-2 \\ & m_{14}-1 & m_{14}-2 \\ & & m_{14}-1 \end{pmatrix} \equiv \Xi^{0}_{+s}$$

$$\begin{pmatrix} m_{14}-1 & m_{14}-1 & m_{14}-2 \\ & m_{14}-1 & m_{14}-2 \\ & & m_{14}-2 \end{pmatrix} \equiv \Xi^{-}_{+a}$$

Table 4.

The variation of the ratio $4AC/B^2$ with m_{14} for non-charmed reactions.

Reaction	m_{14}	$\frac{4AC}{B^2}$
$K^- p \longrightarrow \Lambda \pi^0$	7	0.8000
	8	0.8429
	9	0.8743
	10	0.8977
	11	0.9153
$K^- p \longrightarrow nK^0$	6	0.9783
	7	0.9833
	8	0.9869
	9	0.9895
$K^- p \longrightarrow \Sigma^+ \pi^-$	6	0.9170
	7	0.9366
	8	0.9506
	9	0.9606
$K^- p \longrightarrow \Sigma^- \pi^+$	7	0.8487
	8	0.8839
	9	0.9088
$K^- p \longrightarrow \Xi^- K^+$	6	0.8690
	7	0.8968
	8	0.9175
	9	0.9329

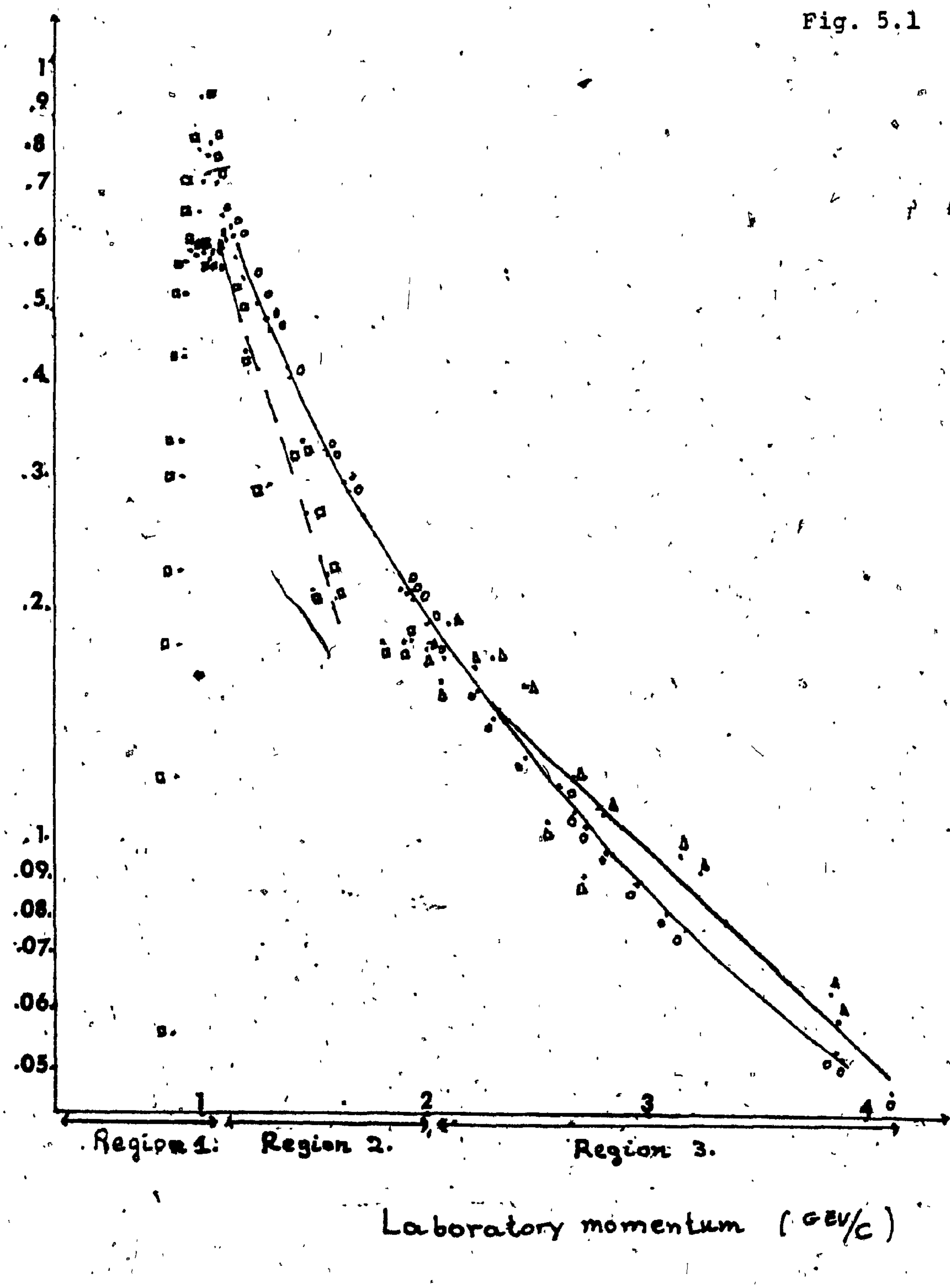
Reaction	m_{14}	$4AC/B^2$
$K^- p \rightarrow \Xi^0 K^0$	6	.9514
	7	.9576
	8	.9636
	9	.9693
$K^- n \rightarrow \Lambda \pi^-$	7	.4639
	8	.5196
	9	.5667
	20	.8030
$K^- n \rightarrow \Sigma^- \pi^0$	7	.8499
	8	.8880
	9	.9140
$K^- n \rightarrow \Sigma^0 \pi^-$	7	.8612
	8	.8945
	9	.9077
$K^+ n \rightarrow p K^0$	7	.3600
	8	.4756
	9	.5676
	10	.6400
	13	.8000
$\pi^+ p \rightarrow \Sigma^+ K^+$	8	.7274
	9	.7755
	10	.8126
$K^- p \rightarrow \Lambda \pi^0$	8	.8429
	9	.8743
	10	.8977

Reaction	m_{14}	$4AC/B^2$
$\pi^- p \longrightarrow \pi^0 n$	8	0.7173
	9	0.7728
	10	0.8145
$\pi^- p \longrightarrow \Lambda K^0$	8	0.7841
	9	0.8228
	10	0.8601
$\pi^- p \longrightarrow \Sigma^- K^+$	7	0.8357
	8	0.8757
	9	0.9035

Table 5.

Evaluation of the parameters A,B,C with $m_{14} = 9$ for non-charmed reactions.

Reactions	A	B	C
$K^- p \rightarrow \Lambda \pi^0$	772.309	25.05	0.1794
$K^- p \rightarrow n K^0$	828.119	21.91	0.1434
$K^- p \rightarrow \Sigma^+ \pi^-$	729.489	21.071	0.1462
$K^- p \rightarrow \Sigma^- \pi^+$	394.953	13.345	0.1024
$K^- p \rightarrow \Xi^- K^+$	711.179	19.947	0.1305
$K^- p \rightarrow \Xi^0 K^0$	809.209	22.943	0.1576
$K^- n \rightarrow \Lambda \pi^-$	331.149	10.039	0.0791
$K^- n \rightarrow \Sigma^- \pi^0$	375.855	13.587	0.1122
$K^- n \rightarrow \Sigma^0 \pi^-$	359.764	12.462	0.0979
$K^- n \rightarrow p K^0$	96.031	4.635	0.0317
$\pi^- p \rightarrow n \pi^0$	660.508	24.275	0.1694
$\pi^- p \rightarrow \Lambda K^0$	275.814	8.93	0.0724
$\pi^- p \rightarrow \Sigma^- K^+$	278.82	9.751	0.0770
$\pi^- p \rightarrow \Sigma^+ K^+$	414.855	13.032	0.0794



Plot of σ vs. lab. momentum for the reaction $\pi^- p \rightarrow K^0 \Lambda$. The points denoted by triangles are the experimental values used by Barbari to obtain a least squares fit to eq. (40). The points denoted by squares are not included in the fit. The small circles represent Barbari's fit to eq. (40). The dashed and continuous straight lines are eyeball fits to the experimental points in regions 2. and 3. respectively.

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11. Of course there are other operators which do not depend on the position coordinate that commute with the Hamil-