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Cassirer's Contribution Towards a Philosophical Understanding of Einstein's Theory of Relativity

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ABSTRACT

CASSIRER'S CONTRIBUTION TOWARDS A PHILOSOPHICAL UNDERSTANDING OF EINSTEIN'S THEORY OF RELATIVITY

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This thesis deals with the relationship between philosophy and modern science. Ernst Cassirer's philosophy is an important step forward in the twentieth century because his philosophy offers the hope that scientific knowledge can yet be integrated into a philosophical framework. In Part I of this paper an attempt is made to understand and assess Cassirer's philosophy as it relates to scientific knowledge. Cassirer's views regarding philosophy itself, physics, mathematics, epistemology, and metaphysics are studied in order to grasp the essential connectedness of the different branches of knowledge within the mainstream of Cassirer's thought. His epistemological conviction that all knowledge is knowledge of relations and his functional approach to metaphysics are of particular interest.

In Part II some basic concepts from Einstein's theory of relativity are examined. Part II is intended to fit together with Part I as an example of Cassirer's method in operation. The philosophical implications of

the theory of relativity have not yet been fully explored and nor does philosophy fully understand its own relation to modern science. The analysis of Einstein's innovative use of such physical concepts as space, time, matter and energy is an attempt to show that Cassirer's philosophy provides an interpretive matrix within which scientific knowledge can be situated. The new meaning Einstein gave to the concepts of space and time within the theory of relativity shows that the type of progress Cassirer envisioned as a possibility for epistemology in general has actually been made within the limited sphere of the epistemology of physics. Furthermore, a parallelism can be shown to exist between Cassirer's metaphysics of substance and function and the modern physical treatment of matter and energy.

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Introduction

If one were to compare a few standard histories of western philosophy and look through their tables of contents, the chances are that one would immediately be struck by the fact that the Milesian philosophers, Thales, Anaximander and Anaximenes, are almost invariably listed first, as the progenitors of western philosophy. The 'physicists', as some authors style them, were the first to inquire into the nature and substance of the universe and the first to reason along rational and scientific lines. The mythological world-view, as it had evolved in the writings of Hesiod and Homer, was unable to satisfy the intellectual curiosity of these men, and consequently, the first stirrings of a hunger for rational explanation and ordered thought were felt in the Greek world. Needless to say, a great deal has transpired in the history of philosophy in the interval between Thales' initial cryptical assertion that 'all is water' and Einstein's formulation of the "General Theory of Relativity"; but what has to be noticed and even felt (as a doctor feels a patient's pulse) is that the history of philosophy is a history of ideas. Moreover, the history of ideas is also the history of the impact those ideas have had upon the civilizations which gave birth to them and the lives of the men who pursued them.

We should also be aware of the close relationship that has obtained between philosophy and physics since their twin inception. The phenomenon of the philosopher-physicist who gives new vision to the men of his and future eras has repeated itself over and over again from Aristotle to Bacon to Galileo to Newton to Einstein, to name but a few of the luminaries in science's pantheon of great thinkers. Philosophy informs and is informed by physics at every stage of its development and the dialectical **tension** between the two disciplines gives rise to the possibility of a new synthesis of human knowledge, new solutions for old problems and new problems with the old solutions. Seen in this light, what we are witnessing is a continuous and protracted evolution of thought. The objective of this study is to enter into this evolutionary process and to view modern physics from the perspective of modern philosophy. There is little doubt in this writer's mind that physics and philosophy are still in search of a deeper understanding of each other and of themselves.

For obvious reasons, Einstein's theory of relativity has been chosen as a focal point of this study. The theory of relativity automatically suggests itself as an excellent place to begin looking at modern physics because its formulation was probably the outstanding intellectual feat of twentieth century physics and yet it has become sufficiently old by now for the initial shock to have worn off and for the scientific community to have begun evaluating its import. The choice of Ernst Cassirer as the representative philosopher of the twentieth century was perhaps not so obvious. While the average person on the street is quite likely to have heard of Albert Einstein (though few could say much about his theory), it is the rare person who will be familiar with the name Ernst Cassirer and rarer still would be the person who is conversant with his philosophy. Nevertheless, one of the main theses being communicated here is that Cassirer's relative obscurity is unmerited and that if more attention were paid to Cassirer's work, contemporary philosophy might not perceive itself to be in such a quagmire.

The above-mentioned thesis underlies the whole thrust of this study and is an implicit part of the entire project herein undertaken. The thesis explicity advanced is that Cassirer's philosophy provides an interpretive matrix within which the philosophical implications of Einstein's theory of relativity can be explored and charted. Ernst Cassirer was born in 1874, Albert Einstein was born in 1879; both understood the nature of the problems threatening to engulf nineteenth century science and both opened the door to a broader twentieth century view of these problems. Einstein published his "Special Theory of Relativity" in 1905, Cassirer published <u>The Problem of Knowledge</u> in 1907 and <u>Substance and Function</u> in

1910. To the latter was added _Einstein's Theory of Relativity_ in 1921, when Cassirer became aware of what Einstein had accomplished in physics. It is significant that Cassirer could so easily graft _Einstein's Theory of Relativity_ onto _Substance and Function_. His thought flows evenly from one book to the other with no break or discontinuity and it is obvious that he anticipates Einstein's conceptual breakthrough even in _The Problem of Knowledge_. The important point is this: Einstein's theory of relativity did not come as a shock to Cassirer's philosophy as it did to that of most of his contemporaries; the theory of relativity was an expected consequence of the direction in which Cassirer perceived scientific thought to be moving. Accordingly, no apology needs to be made to those not familiar with the name Ernst Cassirer - they ought to be!

For purposes of systematic coherence, this study has been divided into two parts. The first part is an attempt to come to grips with Cassirer's philosophy and to understand Cassirer's view of scientific knowledge. It deals consecutively with the disciplines of philosophy, physics, mathematics, epistemology and metaphysics. It is hoped that by surveying these branches of science both individually and in their interrelation, a foundation can be constructed from which to view the theory of relativity. The second part applies Cassirer's philosophy to a discussion of some aspects of the real content of the theory of relativity: the concepts of space and time, and those of energy and matter. It should be understood in advance that the primary emphasis of this thesis is intended to be an assessment of Cassirer's philosophy and that the consideration of Einstein's theory of relativity is undertaken with this objective in mind. What has been sought is a broad overview of the philosophical implications of the theory of relativity and an analysis of the epistemological and metaphysical import of its conclusions; but this has been sought as a means of understanding Cassirer more fully. For readers in search of a more systematic presentation of Cassirer's philosophy, David R. Lipton's book, Ernst Cassirer: The Dilemma of a Liberal Intellectual in Germany 1914-33 and Paul Arthur Schillp's biography are recommended. Also, John M. Krois has just produced a new book entitled, Cassirer, Symbolic Forms and History (New Haven: Yale Univ. Press, 1987) which, although not available at present, is expected soon.

The first five chapters represent, as it were, the convergence of five parallel lines of thought. The diversity of the disciplines of knowledge has to be acknowledged, yet it is still possible to assert, not inconsistently, that knowledge is also an interdisciplinary whole. The increased mass of empirical data available has resulted in a pressing demand for specialization within twentieth century science, but in direct proportion to the extent to which knowledge becomes specialized and technical, a counter-demand for unification asserts itself. The danger that our knowledge may become fragmented and disconnected from the wholeness of reality impels us to step back and to listen carefully and impartially to what each of the disciplines is saying. It is particularly important for philosophy to be aware of this need and respond to it. Therefore, in assessing Cassirer's philosophy, we will be specially interested in his view of the connectedness of all knowledge and in the way he integrates the physical perspective with that of philosophy.

The remaining two chapters, which comprise Part II, are intended to be a foray into the labyrinth-like complexity of modern physics itself. Although we enter the world of physics as philosophers, the vast majority of those of us who have not felt called upon to pursue the study of physics beyond the basic high school curriculum are apt to experience feelings of perturbation and inadequacy when confronted by the strange symbolisms and mysterious formulae of theoretical physics. But, what we fail to bear in mind is that the whole of modern physics is nothing more than a hypothetical explanation of physical intuitions native to all of us. Obviously, an Einstein, a Planck or a Bohr devotes a great deal more reflection and scrutiny to his physical intuitions than most people do, but there is no a priori reason why their intuitions should be any more correct than anyone else's. The intellectual task at hand is to follow another person's reasoning and to see where his ideas lead him. Questions have to be asked, mists surrounding concepts have to be dispelled and a mental effort has to be made to grasp what is not always perfectly lucid. Kierkegaard, in his Concluding Unscientific Postscript, relates the story of Jacobi who, having failed to fathom Lessing's meaning when he referred to the 'leap of faith', said to Lessing: "Next time you jump, take me with you." Modern physics is a bit like that; sometimes it is difficult to follow the conceptual leaps being made, especially when those 'leaps' are of the quantum variety. Nevertheless, even the attempt is thought to be worthwhile.

As a unity, this study stands or falls not so much on the narrow point of whether it has anything meaningful to add to the discussion of the role of the theory of relativity in modern physics, but on the broader basis of whether Einstein's theory of relativity has stimulated fresh insights in philosophy and whether Cassirer's philosophy, in particular, is a suitable philosophy in the context of the modern state of scientific knowledge. The two principal theses of Cassirer's that are stressed throughout are: (1) To know is to relate; and (2) What we know is function, not substance. The fact that Einstein's basic epistemological stance reduces to approximately the same two theses greatly facilitates the exchange of ideas between physics and philosophy and between their respective spokesmen, Einstein and Cassirer.

Part I: Ernst Cassirer and Scientific Knowledge

Chapter One

Philosophy and Science

The relationship between philosophy and science has always been an intimate one. It is well known that what we now term 'physics' was once called 'natural philosophy' and, in truth, all that is now subsumed under the category of scientific knowledge was once considered to be a part, or subset, of the broader category of philosophical wisdom. In the days of Aristotle, empirical facts about topics ranging from the observed motion of the planets to human physiology were lumped together with speculations concerning the teleological organization of the universe, and all branches of inquiry were integrated into a philosophical framework. Unfortunately, or perhaps fortunately, this situation no longer persists. It is unfortunate because, to a large degree, the twentieth century has lost the notion of the unity of all knowledge and has devoted itself to the investigation of the parts of knowledge to the exclusion of the whole. It is fortunate because the process of separating the factual from the fanciful and the speculative has allowed man to make a wide and free use of his reasoning powers, to discard unfounded assumptions, and to eliminate errors in thought.

At present, science is deemed distinct from philosophy in that the scientific method yields results that are useful, interesting, and, very often, verifiable; whereas, the philosophical method produces controversy, argument and widespread uncertainty. Modern universities routinely classify philosophy under the general heading 'Arts' and whilst a residual respect for the work done by philosophers still remains, the great majority of people appear to regard the 'Sciences' as the true repositories of meaningful knowledge about the world. However, in a world which has brought itself to the brink of catastrophes such as nuclear war, it appears timely to re-examine the relationship between philosophy and science and to attempt once more to integrate scientific knowledge into a philosophical framework. It is not a question of failing to see the forest for the trees, or indeed, of failing to see the trees for the forest; what needs to be done now is to see both the forest and the trees, the general and the particular, and to understand this relationship in the most thorough possible way.

The philosophy of Ernst Cassirer offers us exactly this possibility. Cassirer was both well-acquainted with modern developments in the various sciences and, at the same time, a philosopher of the old school, a man who believed in the unity of knowledge. Of course, the rapid pace at which scientific research is advancing in the twentieth century implies that Cassirer, who died in 1945, may already be out of date; but this is not necessarily so, for there is a definite sense in which good philosophy is never out of date and, furthermore, Cassirer did not have to live in the 1980's to be aware of the general trend contemporary science was taking - he had the foresight which characterizes great thinkers.

The obvious method of exploring Cassirer's ideas concerning the nature of scientific thought and the purpose of the philosophical enterprise is to proceed immediately to an examination of his works. However, it is also true that Cassirer's idea of philosophy can as readily be inferred from his method of doing philosophy as from the inner logic of the arguments he presents. William Curtis Swabey, who translated <u>Substance and Function</u>, writes that Cassirer "...speaks rather as a scholar writing in a well-stocked library," and that "nature is for him something known only indirectly, primarily through the books of scientists."[1] Another close source, Charles W. Hendel, testifies as follows in his Introduction to <u>The Philosophy of Symbolic Forms</u> (Volume Three):

> Cassirer had a deep-lying theory of his own to formulate which had to do not only with knowledge of the physical world but more comprehensively with "the structure and articulation of a theoretical world-view."[2]

Swabey tells us that Cassirer's knowledge was a mediated knowledge, that the reality Cassirer interpreted was a second-hand reality already distorted and over-simplified by the scientists who describe it. This is an accurate criticism in some ways, but not necessarily a damaging criticism. After all, to give Cassirer his due, the task of completing a survey of all of the various sciences was enough of a Herculean labour to have undertaken in the first place; thus, practically speaking, Cassirer is required to refer to the opinions of experts rather than attempt to do all his own original research. In fact, viewed from a sympathetic perspective, the knowledge to be found in scientific treatises and other such scholarly works represents not a distortion of reality but its distilled essence. Furthermore, aside from the age-old problem of the unreliability of the knowledge which comes to us through our senses, there is another more fundamental problem. Given the complexity and detail of first-hand empirical knowledge of reality, synthesis and synopsis is the only possible manner of proceeding: therefore, the many instances have to be compressed into the single rule and the many rules have to be consolidated into the single principle wherever possible.

Hendel's comment, on the other hand, is of a different nature, displaying a concern more for questions pertaining to motivation than methodology. Hendel quite rightly points out that, in effect, Cassirer marshalls a huge mass of information, derived from an incredibly wide variety of different sources, solely in order to orchestrate a concert whose score he has composed himself. From the pre-scientific mathematical mysticism of Pythagoras to the ultra-scientific relativistic physics of Einstein, each empirical fact, each new theory, each expansion of human knowledge is a part of the central theme Cassirer is seeking to communicate. His philosophy is the symphonic expression of an idea. Hendel recognizes this in Cassirer's philosophy and he points it out to us. Accordingly, if the question becomes, "Is this a valid way to do philosophy?", then the answer must categorically be answered in the affirmative.

What did Mozart do when he wrote a symphony? He translated music he felt in his soul into music that people could hear and appreciate. He arranged the tubas and the cellos, the trumpets and the clarinets, and all of the other instruments in such a manner that they complemented and criticized one another, boomed loudly and whispered softly, and ultimately, gathered in harmony to express a central theme. Cassirer does exactly the same thing in philosophy. He begins with fundamental insights into the nature and structure of knowledge that are intuitive and perhaps even inchoate, and he puts them into language others can understand and appreciate. He discusses physics and psychology, mathematics and mythology, everything and anything; and, in the end, he organizes all the parts of knowledge into a whole and he lets his theme express itself. Thus, in An Essay on Man, we read Cassirer's summary of his own philosophical conclusion:

> In language, in religion, in art, in science, man can do no more than to build up his own universe - a symbolic universe that enables him to understand and interpret, to articulate and organize, to synthesize and universalize his human experience.[3]

Far from being critical of this type of artistry, philosophy welcomes and even demands it. The failure to support one's contentions with either facts drawn from experience or history, or logical reasons, or both, is regarded on all sides as a serious philosophical defect. Conversely, the greater the weight of empirical evidence supporting one's position and the sounder the arguments adduced, the greater is the respect accorded to the philosophy. It cannot be a serious criticism of Cassirer to say that he interprets the information at his disposal in such a way as to fit his own underlying theory because this is precisely what is expected of a philosopher. If, however, bad faith is alleged and proven, then we are talking about a completely different thing and the criticism is indeed very serious. Intentional misinterpretation has nothing to do with philosophy. However, Cassirer's real genius consists in the fact that he never displays questionable academic conscience by hiding inconvenient facts or omitting to give proper weight to inferences that clash with his own theories. What Cassirer does do is to demonstrate a marvellous abitity to discover the connection between seemingly disjointed pieces of knowledge and to sift the essential from the peripherally important with uncanny acumen. At first, this may seem somewhat disconcerting, but with patience and time, a deeper insight into Cassirer's philosophy develops and as Walter M. Solmitz writes in his article, "Cassirer on Galileo: An Example of Cassirer's Way of Thought":

> If one is familiar with Cassirer's thought, one is so used to the fact that Cassirer can agree and disagree at the same time that he no longer wonders about it.[4]

But this is not bad faith or intellectual dishonesty, nor is it the incompetence of a shallow thinker who picks and chooses theories eclectically with no thought for any systematic understanding that may connect them. In Solmitz' view, this trait of Cassirer's is a sure indication of the depth and breadth of his thought. Thus, we read:

> A statement by Cassirer, valid in itself, must yet be seen within its dialectical context. Furthermore, in addition to the horizontal dialectics, there is a kind of vertical dialectics: a historical statement by Cassirer has a systematic significance at the same time. The fact that Cassirer could "revive" and "re-present" Galileo on the contemporary intellectual scene is obviously due to the special form of Cassirer's systematic interest in Galileo.[5]

Since the death of Hegel, and particularly since Marx's inversion of the Hegelian method, dialectics has been viewed with a great deal of suspicion and the once cherished dream of eventually achieving a "complete" systematic philosophy is now regarded as a hopelessly unrealistic illusion. Yet Cassirer presses resolutely onwards, undaunted by the enormity of the problems facing him but also cognizant of his own limitations and the limitations of knowledge. The logician, Kurt Gödel, has shown that it is impossible to prove consistency in a formal axiomatic system without resorting to methods from outside the system and, in physics, Werner Heisenberg's "Uncertainty Principle" implies that physical knowledge can never attain to anything more than a high degree of statistical probability. Thus, the very possibility of systematic philosophy has been seriously attacked and the very foundations of knowledge have been shaken; but the requirement of consistency and the need for a systematic understanding still remains even when the possibility of attaining such an understanding is restricted. In Cassirer's words:

> The era of the great constructive programs, in which philosophy might hope to systematize and organize all knowledge, is past and gone. But the demand for synthesis and synopsis for survey and comprehensive view, continues as before, and only by this sort of systematic review can a true historical understanding of the individual developments of knowledge be attained.[6]

In Cassirer's hands, an almost imperceptible change in the meaning of systematic philosophy results in a noticeable revival of the possibility of its existence. Just as Cassirer's Substance and Function seems to anticipate Einstein's theory of relativity, his unfailing intuitive feel for the direction the philosophy of the future must take also seems to have warned him to keep clear of the problems associated with Gödel's "Theorem of Undecidability". Cassirer's philosophy does not purport to be and is not systematic in the sense of "system" to which Gödel's theorem applies. Gödel proves that it is impossible to begin with a finite number of axioms and then successively deduce the truth or falsity of all statements that arise within the system. But Cassirer isn't seeking to reduce knowledge to the logical consequences of a few basic propositions; in fact, his objective is the exact opposite of this. In his own words:

> It appears as the task of a truly universal criticism of knowledge not to level this manifold, this wealth and variety of forms of knowledge and understanding of the world, and compress them into a purely abstract unity, but to leave them standing as such.[7]

It is true that within each science, each form of knowledge, concepts are compressed in order to allow for greater generality of application; but this compression has to be carefully distinguished from the kind of compression that obliterates differences and effaces individuality. Cassirer tells us that, "...with every transition to a new type of physical statement a double process is involved: an extensive enlargement of knowledge and a corresponding intensive compression."[8] Thus we see that the forms of knowledge grow ever more distinct as their characteristic concepts become more sharply defined, so that philosophy is constantly confronted with a greater mass of knowledge in need of systematic integration and with a wider disparity amongst perspectives within which this knowledge is interpreted. Accordingly, Gödel's theorem is merely the expression in logic of a practical assessment of the existing state of affairs which philosophy had already made. Heisenberg's "uncertainty" remains to be discussed, but this is really a question of the interpretation of quantum theory and beyond the scope of this study. For the present, however, it should be remarked that inability to measure with absolute precision is a vastly different thing from inability to know with certainty. The major problem facing philosophy is not really measurement, but rather the diversity of possible ways observed facts concerning the physical world can be interpreted. It is worth quoting the following passage from The Problem of Knowledge for Cassirer's analysis of the situation:

> Even within certain fundamental lines of philosophical thought that are closely related to one another and seem actually to spring from the same source, the different ways of formulating the problem of knowledge are to be noticed and felt at every step ... Each of these formulations of the question, connected with a different discipline, contains important and

> fertile problems. But everyone pretends to speak not only for his own department but for the whole of science, which he believes himself to represent and to embody in an exemplary fashion. Thus arise ever new discords and constantly sharper conflicts, and there is no tribunal that can compose these quarrels and assign to each party its respective rights.[9]

"There is no tribunal to compose these quarrels..." - It is the realization of the relativity of knowledge that prevents Cassirer's philosophy from ever becoming dogmatic. Modern philosophy, if it is to be 'modern' and adequate to the needs of twentieth century man, can no longer seek to act as judge in the trial of the competing knowledge claims. Far from implying that philosophy must forfeit its prerogative of forming an independent opinion, however, this means that philosophy's independence is a crucial prerequisite. Thus, for example, the theory of relativity is thought to express fundamental truths about the nature of the physical universe, but philosophy errs gravely when it takes these truths at face value and absolutizes them because the truths are only true within the restricted context physics gives them and it is always possible to formulate these truths in different symbols and different contexts. The era when philosophy felt called upon to make authoritative pronouncements is at an end. Instead, philosophy now looks beyond the disordered diversity of thought's initial encounter with reality in search of the principle of unity whose many faces and names have continually baffled and bewitched man from time immemorial. It is a basic postulate of Cassirer's thought that each of the aspects under which this principle appears are at last reconcilable in the same sense that the terms of a mathematical series are all comprehended by the function defining it. With reference to philosophy, this idea translates into the conclusion that:

> It is the task of systematic philosophy, which extends far beyond the theory of knowledge, to free the idea of the world from this one-sidedness. It has to grasp the whole system of symbolic forms, the application of which produces for us the concept of an ordered reality.[10]

It now becomes clear that the preliminary characterization of philosophy as the study of the fundamental principles and basic concepts of other branches of knowledge is not entirely accurate. While it is true that philosophy actively canvasses as many of the reputed sources of knowledge as can be discovered, it is misleading to assume that philosophy stops here. To "grasp the whole system of symbolic forms" and to apply this system, philosophy must contribute an extra something of its own. The kernel of the philosophical method is the conviction that rationality inheres in reality and is capable of being discovered. The human mind imposes order on the phenomena, but the philosopher never ceases to believe that order is also an objective presence in the very fabric and structure of reality. For Cassirer, the "principal and original question" is the question of "what philosophy is and what it is about,"[11] because philosophy constantly questions itself and criticizes every static model of a reality it knows to be dynamic. If we look to the essence and origin of consciousness itself, then Cassirer contends that "ignorabimus (we will never know) is the only answer that science can give."[12] Therefore, we begin, as Descartes does, with "I think," and we progressively realize not only that "I am," is a rational conclusion but also that phenomenal reality itself is systematically coherent and intelligible. The paradox of motion and change will always remain an anomalous feature of the system unless philosophy itself is represented as a dialectical process and the continual interplay of concepts is accepted as a methodological axiom. In Cassirer's words:

> The concept of philosophy shows itself again and again as a problem of philosophy, as a problem which itself never comes to rest, but which must always be undertaken anew in a continual dialectical movement of thought.[13]

Accordingly, we recognize that, strange as it may sound, there is such a thing as philosophy of philosophy. Philosophy, *qua* method of solving problems, encounters and merges with philosphy, *qua* problem. The double aspect under which philosophy constantly presents itself makes a dialectical reconciliation of these opposing tendencies a necessary prerequisite to all philosophical thought. In scientific thought, conclusions ensue from hypotheses; in logic, 'then' follows from 'if'; and in philosophy, principles order the data of perception; but in every case, the question of the validity of the method is suspended during the actual operation of the method. Later on, we judge by results and we compare the creation of our thought with the known empirical facts. What is unspoken, however, is the problematic nature of the empirical fact and what philosophy specifically refrains from doing is to enter into in-depth analysis of the criteria of its own validity. The positivism of the Vienna Circle is one attempt to confront this problem and Karl Popper's theory of falsification is another, but neither gets to the real core of the issue. Knowledge is and inevitably remains a problem - this is the first fact of cognition. Consequently, philosophy is only valid when it incorporates this fact into itself; but the only legitimate way around an undisguised and full-fledged skepticism is a philosophy of the possible and a speculative metaphysics.

In spite of all that Cassirer says about the unknowableness of substance, he still acknowledges that, "Empirical knowledge cannot avoid the concept of substance," but it is precisely because he qualifies his meaning that we can see how philosophy is yet possible: "...genuine philosophical progress is in understanding it (substance) and evaluating it as a concept."[14] At this point, an analogy can be drawn to physics where scientists had hoped to replace the imprecise concept of matter by that of energy. Here also, it was found that the substantial concept (i.e. matter) could not be entirely eliminated from the system of thought. Even Einstein, who dedicated himself to the possibility of elaborating a unified field theory, was forced to admit that the concept of energy was incapable of solving all of the problems involved. Thus, in the Evolution of Physics, we read:

> But we have not yet succeeded in formulating a pure field physics. For the present, we must still assume the existence of both: field and matter.[15]

The fact that philosophy remains a problem does not impede the activity of philosophy any more than Einstein was impeded from doing theoretical physics by the problematic nature of the concepts he worked with. Whether or not there is any justifiable reason for doing so, the truth or falsity of a theory will always be judged according to the pragmatic standard of its use-value. Einstein's theory is useful because it can predict and explain, and by the same token, Cassirer's philosophy is useful because it allows for the organization and integration of what we term 'human knowledge'.

It seems fitting in concluding this chapter to refer to the words of Cassirer's great mentor, Immanual Kant. It is hoped that by considering the several branches of knowledge independently and also in their interrelation, a basis can be established for the latter part of this essay where we will investigate specific features of Einsteinian physics and their relation to a broader theory of knowledge in general. Kant, working within a framework of physical knowledge that was less comprehensive than that of the present day, nevertheless understood a great deal about the principles by means of which human knowledge must be interpreted, and Kant's best insights into the nature of thought itself and the meaning of the philosophical enterprise will remain essentially unaffected by any new discoveries that may be made in the empirical realm. Thus, in the Critique of Pure Reason, Kant comes to the following conclusion about philosophy:

> We cannot learn philosophy, for where is it, who is in possession of it, and how shall we recognize it? We can only learn to philosophise, that is, to exercise the talent of reason in accordance with its universal principles, on certain actually existing attempts at philosophy, always, however, reserving the right of reason to investigate, to confirm, or reject these principles in their very esssence.[16]

Philosophy has to understand itself before it can begin to understand modern science. The reason why it is important to enter into Cassirer's intellectual world and to struggle with the difficult points of Cassirer's philosophy is that in Cassirer we have an example of a philosopher who is attempting "to exercise the talent of reason in accordance with its universal principles" and who is also fully conversant with the main features and corresponding epistemological problems of modern science. Philosophy often results in controversy and argument precisely because philosophy is so pre-eminently critical and because it refuses to accept the supposedly certain conclusions of science at face value without further comment. It is necessary that the interplay between the two disciplines be furthered and that, in investigating the knowledge claims of modern science, we do so as philosophers who search for unity in diversity and broad vision rather than narrow focus.

Chapter Two

Physics as a Perspective and Method in the Natural Sciences

This is a study of some of the philosophical implications of Albert Einstein's famous "Theory of Relativity". At the outset, it is important to realize that there are many sciences and that "relativity" is not new to philosophy, within which the whole science of physics is itself relativized as one possible perspective among many. Philosophy surveys the various branches of human knowledge from a position of independence only when it refuses to accord to any particular science a dignity greater than that which the particular science merits; and thus, in spite of the high esteem in which the empirical facts and well-verified laws of physics have traditionally been held, philosophy still retains the prerogative to look beyond physics and to interpret the knowledge claims of physics within the broader framework of human knowledge in general. For this reason, the philosophy of Ernst Cassirer seems particularly apt as a vantage point from which to view the other sciences since Cassirer's immense erudition and his deep familiarity with all of the various sciences lends a singular credibility to his philosophical reasonings. This is neither an attempt to deny the ultimate veracity of the laws of physics nor a means of elevating philosophy to a position of prestige above the other fields of human knowledge; rather it is a simple recognition of the fact that each of the sciences has its own motive and its own characteristic way of organizing and synthesizing the empirical data available. As Cassirer writes:

> Even in "nature," the physical object will not coincide absolutely with the chemical object, nor the chemical with the biological - because physical, chemical, biological knowledge frame their questions each from its own standpoint and, in accordance with this standpoint, subject the phenomena to a special interpretation and formation.[1]

Physics, to be sure, has its own peculiar perspective and its own manner of applying the scientific method to its problems, and thus, it is evident that the interpretation placed on phenomena by the physicist bears the distinctive stamp fundamental to the outlook of the science of physics. Each of the various sciences is organized so that extraneous data can easily be separated from relevant information; indeed, the crucial defining feature of the particular science itself may very well turn out to be the criteria it uses in choosing between the "extraneous" and the "relevant". In physics, the theory of relativity is especially interesting because it inaugurates a revolutionary alteration in our basic conceptions of space and time. The intuitive ideas of space as a vast receptacle and of time as an enduring succession of moments are challenged by the new interpretation given to these concepts in Einsteinian physics; and coincident with the enlargement of these concepts in relativistic physics is a corresponding need for readjustment in the other sciences also, insofar as these concepts (i.e. space and time) are also fundamental for them. As Hans Reichenbach remarks in his article, "The Philosophical Significance of the Theory of Relativity":

> The analysis of knowledge has always been the basic issue of philosophy; and if knowledge in so fundamental a domain as that of space and time is subject to revision, the implications of such criticism will involve the whole of philosophy.[2]

Of course, we do not try to hide the fact that physics defines space and time self-consistently - that is, to suit its own purposes - but, at the same time, we should not fail to notice that a physical understanding of these concepts is necessarily at the base of any philosophy seeking to retain its relation to the world of empirical fact. A key element to the motive underlying the physical definition is the physicist's desire to provide the most cogent possible explanation for the observed "facts" which comprise the phenomenal world. Thus, to a large extent, the philosopher is bound to accept the physicist's appraisal of empirical reality; for it is undeniably true that physics not only explains, but explains well and in great detail, and furthermore, physics is able to predict the occurence of all sorts of phenomena with a trustworthiness that inspires great confidence. Nevertheless, Cassirer still maintains - and quite rightly so, as I shall argue - that the essential limitedness of physics' perspective (and of the perspectives of the other sciences also) should never be overlooked or disregarded. Physics defines space and time in the peculiar fashion that it does, because physics can only achieve an ordered, self-consistent view of reality by doing so. It is always a mistake, however, to absolutize either the physical definition or the physical world-view.

A second key element underlying the physicist's motive is the desirability of introducing the possibility of measurement into the hitherto chaotic mass of disjointed observations and perceptions that comprise phenomenal reality. Standards of comparison are needed in order to be able to relate events located in different frames of reference and to eventually be able to subsume such events under a single general law. Therefore, the physicist has to define in such a way as to allow for measurement and this involves a mathematization and an objectification of concepts so that, in physics, knowledge must make a steady progress away from distinctions of a merely qualitative nature and towards quantitative distinctions. It is unimportant that the standard of measurement may itself be arbitrarily determined, what matters is that such a standard be applied everywhere with the same logical rigour and that an undeviating application may lead to results capable of being formed into the general law. As Cassirer saw it, the theory of relativity was no startlingly new development in physics, but rather the completion of an evolutionary process whereby physics came to realize the epistemological consequences of its underlying motives. He writes:

> The general relativity of all places, times and measuring rods must be the last word of physics, because "relativization," the resolution of the natural object into pure relations of measurement constitutes the kernel of physical procedure, the fundamental cognitive function of physics.[3]

Now Einstein himself seems to have been quite conscious of his own motives for defining space-time in the special way that he did in his theory of relativity and we can see that he is in substantial agreement with Cassirer on this point. Thus, we find the following statement in Einstein's autobiographical notes, (Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, Volume VII):

> The system of concepts is a creation of man together with the rules of syntax, which constitute the structure of the conceptual systems. Although the conceptual systems are logically entirely arbitrary, they are bound by the aim to permit the most nearly possible certain (intuitive) and complete coordination with the totality of sense experiences; secondly, they aim at the greatest possible sparsity of their logically independent elements (basic concepts and axioms), i.e., undefined concepts and underived (postulated) propositions.[4]

The issue of what might be called "concept-economy", (i.e., "the greatest possible sparsity of their logically independent elements"), will be dealt with more fully at a later stage, but for the moment our attention is focused on the method of theoretical physics. The description of phenomena and the formulation of rules of order governing phenomenal reality are accomplished in the furtherance of a particular motive within the physical perspective and, accordingly, the reality that physics makes intelligible is a reality circumscribed by the operation of the fundamental axioms and definitions of physics. The same is true of chemistry, of biology and, as Cassirer argues in his Philosophy of Symbolic Forms, of all the different branches of human intellectual endeavor.

In an epistemological sense, the theory of relativity itself lends support to the thesis that no particular concept of reality belonging to any of the sciences uniquely ever coincides with reality absolutely; and Cassirer finds evidence for this proposition not only in his scientific studies but also in his researches into language, art and myth. Accordingly, Cassirer seems well-justified in arguing, quite consistently with his neo-Kantian epistemological stance, that space and time are pure forms of intuition. The particular construction given to space and time by physics is remarkable for the breadth of its explicative power and for the mathematical simplicity of its expression, but it remains a construction derived from a particular motive and a particular perspective. Philosophy, however, strives to find the unifying principle capable of comprehending a variety of different perspectives. As Cassirer writes:

> What time and space truly are in the philosophical sense would be determined if we succeeded in surveying completely this wealth of nuances of intellectual meaning and in assuring ourselves of the underlying formal law under which they stand and which they obey. The theory of relativity cannot claim to bring this philosophical problem to its solution; for, by its development and scientific tendency from the beginning, it is limited to a definite particular motive of space and time. As a physical theory it merely develops the meaning that space and time possess in the system of our empirical and physical measurements.[5]

The example of the meaning given to the concepts of space and time within physics is only one example of the way in which the science of physics orients itself. It would be unnecessarily tedious to spend our time discussing other such examples when the point being made is sufficiently clear: Not only in physics but also in the other natural sciences and, indeed, in every field into which human minds enquire, the manner in which the fundamental concepts are defined and applied characterizes the nature of the enquiry itself. Physics is that science which most nearly succeeds in mathematizing its concepts so that the entire system of physical reality can be resolved into pure relations of number and the basic laws can be understood in the form of mathematical equations; but chemistry exhibits this trait also (witness the periodic table), and even in biology we notice a progression from the purely classificatory schemata of an Aristotle or a

Linnaeus towards the highly mathematical techniques employed by modern biologists in such fields as genetics. The question is not whether biology should be based on physics or physics on biology or whether the method of any one science should be a paradigm for all of the other sciences[6]. The real question is: what truths, what knowledge of reality, does each of the individual sciences yield, and in what way can we synthesize the disparate pieces of knowledge produced severally by the sciences into a harmonious understanding of reality as one?

Clearly, the natural sciences are those to which the mathematical concept of serial order according to a determined rule is most easily wed, but just as it is wrong to hypostasize physics as a paradigm for the other natural sciences, it is also a mistake to overestimate the value of the natural sciences vis-a-vis other forms of human knowledge. Cassirer emphasizes over and over again that, "It is the task of systematic philosophy ... to grasp the whole system of symbolic forms..."[7] and that, "We must always direct our gaze to the whole if we are to reach an understanding of particulars and individuals. For it is just this irreducible coherence of whole and part that constitutes the essence of organic nature."[8] Accordingly, if we are ready to accept the general meaning of Cassirer's assertion that, "there is no more a chemical definition of life than there is a static and physical one. Life is activity and in a sense only a functional definition is possible."[9], then we should also be willing to accept the logical consequence of this mode of thought, viz., that a principle underlying all of the many forms of knowledge must be expressive of a reality of which mathematical order is only one aspect.

Now the natural sciences, and physics in particular, aim not so much at an exact determination of the nature of reality as at a truth consistent with the empirically known physical facts; a truth arrived at by a process of successive approximation, rather than a deduced truth of which we can be absolutely certain. As Bertrand Russell writes in his ABC of Relativity:

> Science does not aim at establishing immutable truths and eternal dogmas: it aims to approach the truth by successive approximation, without claiming that at any stage final and complete accuracy has been achieved.[10]

Obviously, this fundamental tendency is entirely consistent with Cassirer's and with the general neo-Kantian position. The Kantian thing-in-itself is unknowable; but beyond the realm of the noumenal is the phenomenal world and the possibility of knowledge emerges when analysis of phenomenal reality is in terms of the behaviour and properties of the 'things' perceived. Thus, it is the causal law given in the definition of the concept, rather than the thing itself with which physics concerns itself. In the words of William H. Werkmeister, ("Cassirer's

Advance Beyond Neo-Kantianism", The Philosophy of Ernst Cassirer, The Library of Living Philosophers, Volume VI):

> Plato, therefore, was right in the opinion of the neo-Kantians when he saw the task of science in an infinite process of determining the indeterminate.[11]

Insofar as the concept is interpreted metaphysically as the idealized expression of the thing as substance, there can, of course, be no hope of scientific exactitude. However, Cassirer has another way of interpreting the concept which involves a shift away from the metaphysical notion of substance and towards a non-metaphysical idea of functional unity. We will be discussing the twin ideas of substance and function in much greater detail in Chapter Five, but for our present purposes it is important to note that one great advantage of the functional interpretation of the concept is that it does allow for the possibility of scientific law-like precision. The functional concept is a man-made creation and because it is a human invention it can be forced to obey its master's will. Thus, Einstein can say that, "Science is the attempt to make the chaotic diversity of our sense experience correspond to a logically uniform system of thought,"[12] precisely because the definite manner in which he arranges his concepts produces the "logically uniform system of thought" which he needs as his starting point.

Naturally, for the scientist, the original choice of concepts is therefore a vitally important part of the task.

One might even venture to say that the choice of concepts is crucial since no system can ever hope to be more secure than the axiomatic base upon which it is built. Thus, it is interesting to note that a sort of direct proportionality exists between the degree to which the concepts are abstract and their utility as building blocks for a systematic understanding of reality. Einstein, himself, remarks that:

> The theory of relativity is a fine example of the fundamental character of the modern development of theoretical science. The initial hypotheses become steadily more abstract and remote from experience.[13]

Cassirer, too, is quite cognizant of this tendency and, furthermore, he has an interpretation for it that is perfectly consistent with the rest of his epistemology. In The Philosophy of Symbolic Forms, we read:

> Science seeks the unambiguousness of the concept. But in raising this demand from its very beginnings, natural science made a still sharper break with the world of common experience. This break separated from the world of the scientific object not only the world of words but also that of immediate perception. In order to penetrate to the sphere of these objects, in order to apprehend nature in its objective reality and objective determinacy, thought had to leave sensation and sensory intuition behind it, as well as names.[14]

The mention of Cassirer's epistemology brings this chapter to a close, as this is the subject matter of the fourth chapter. Before moving on, however, it seems worthwhile to briefly summarize the most important theses advanced so far. The theses to be recapitulated are six: (1) Physics in particular (and the other sciences, in general) views reality from its own particular perspective and interprets reality according to its own, self-consistent, canons of interpretation; (2) Philosophy's role consists not in adopting the perspective proper to any particular science but rather in surveying the whole complex of different perspectives with an eye to discovering the unifying principle or rule of order which relates them all to each other in a cogent and consistent fashion; (3) There is a definite sense in which Einstein's theory of relativity can be viewed as a final stage in the evolution of physics and as the end product of a method which stresses the explanation of the measurable; (4) The concepts with which scientists work are man-made, and what is more, they are made with a particular motive in mind; (5) A necessary inexactitude must prevail in physics, and in all the sciences, so long as these concepts are metaphysically interpreted as being expressive of substance, but the functional interpretation suggested by Cassirer allows for a real possibility of precision; and (6) The natural epistemological progress of the sciences is towards increasingly abstract concepts whose origin is pure thought, rather than sensation.

Chapter Three

Mathematics and Logic

It is obvious that an analysis of Einstein's theory of relativity must inevitably come to grips with the subject of mathematics. This is particularly true when we are discussing the theory of relativity with reference to Cassirer's philosophy. Although neither Einstein nor Cassirer were mathematicians as such, both had a great appreciation for the elegance and power of mathematical thought and both sought to integrate the results achieved independently by mathematicians into their own theories. Einstein could probably not have achieved the formulation of the theory of relativity had it not been for the mathematics behind the Lorentz-transformation and for the advances suggested by Minkowski in the field of tensor-calculus; by the same token, Cassirer is much indebted to Pointcaré for his denial of geometrical empiricism and to Heinrich Hertz for his theory of symbols in science. Evidently, in the hands of thinkers of this stature, mathematics is capable of becoming a very powerful tool. Moreover, the essence of the mathematical method consists in understanding and evaluating concepts in terms of relations and number. Thus, what makes mathematics so powerful is that it allows for certainty within a tightly defined knowledge situation where we are concerned not with the empirically 'true', but rather with the logically possible. As Cassirer writes in Substance and Function, mathematics' ability to lend numerical comprehensibility to the phenomena of experience is a key component of rationality itself:

> If there were no number, nothing could be understood in things, either in themselves or in their relations to each other. The Pythagorean doctrine remains unchanged in its real import through all the changes in philosophical thought. The claim to grasp the substance of things has gradually been withdrawn; but at the same time insight has been deepened and clarified that in number is rooted the substance of rational knowledge.[1]

Not only does Cassirer assert that number is 'rooted in the substance of rational knowledge', but he goes even further when, in An Essay on Man, one of his last works, he states that, "Science no longer speaks the language of common sense-experience; it speaks the Pythagorean language. The pure symbolism of number supersedes and obliterates the symbolism of common speech."[2] We have only to pick up and read a modern physics textbook to realize the truth of this dictum. For the uninitiated, a cloud of mathematical formulae obscures any possible meaning the scientist may be seeking to communicate and the density of the thought contained in these thrifty formulae can only be unravelled by a person well-versed in the 'Pythagorean language'. However, much as we may regret the trend in modern science towards an esoteric language that only an exclusive few can understand, we also have to acknowledge its usefulness and even its necessity. It simply is not possible to accurately describe the events that comprise physical reality in the ambiguous terms of ordinary speech. Extremely precise and dense scientific concepts have to be employed by the modern scientist who wishes to express himself clearly and, as a consequence, there is a need to resort to the pure symbolism of mathematics.

Not only does the language of science become increasingly technical and difficult for the layman to understand, but the experiences described also become less and less comprehensible in relation to our sense experience. A 6-dimensional mathematical space, for example, has no experiential analogue; yet it may be the most cogent way of understanding a set of physical relations. Cassirer explains this growing rift between mathematical concepts and experience by tracing the process of concept formation backwards to its origin. Right at the beginning of <u>Substance and Function</u> we find the following passage:

> Mathematical concepts which arise through genetic [sic - probably 'generic'] definition, through the intellectual establishment of a constructive connection, are different from empirical concepts, which aim merely to be copies of certain factual characteristics of the given reality of things. While in the latter case, the multiplicity of things is given in and for itself and is only drawn together for the sake of an abbreviated verbal or intellectual expression, in the former case we have first to create the multiplicity which is the object of consideration, by producing from a simple act of construction (Setzung), by progressive synthesis, a systematic connection of thought-constructions (Denkgebilden).[3]

Undoubtedly, arithmetic stands as the foremost example of Cassirer's thesis. In arithmetic, thought's capacity for the creation of the multiplicity displays itself most clearly. The whole set of natural numbers is given by the simple rule of serial ordering which makes each new number exactly one greater than its immediate predecessor, the set of integers introduces the notions of the zero and of sign, and the set of rational numbers can be derived in its entirety from another rule of a slightly more complex character. Thus, numbers are the first 'systematic connection of thought-constructions', the first particulars to be generated from a humanly created universal. But no scholastic "problem of universals" appears on the horizon, no metaphysical and epistemological difficulties have to be contended with, because the universal is a principle governing thought rather than a substantial 'thing'. As Cassirer writes:

> Here no insuperable gap can arise between the "universal" and the "particular", because the universal itself has no other meaning and purpose than to represent and to render possible the connection and order of the particular. If we regard the particular as a serial number and the universal as a serial principle, it is at once clear that the two moments, without going over into each other and in any way being confused, still refer throughout in their function to each other.[4]

The dispute between Cassirer and Russell as to the logical priority of ordinal and cardinal numbers takes on new meaning in this context. While Russell insists that number has an essentially empirical origin and that we only know number by reference to classes of objects which exemplify number, Cassirer argues that we first create the numbers and then apply them to the classes of empirical objects that perception reveals to us. A fundamental difference in approach can be detected when the question is examined in greater depth. It is not only the difference between an epistemology which holds that thought **creates** its concepts and a contrary epistemology holding that thought **discovers** the concepts, but it is also a difference as to the objective meaningfulness of the concepts. In his article, "Cassirer Versus Russell", Smart explains:

> In any case, so Cassirer maintains, logical priority pertains not to cardinal number, as Russell argues, but to ordinals. Again, whereas Russell holds that mathematical propositions are analytic and tautological, Cassirer just as stoutly maintains that they are synthetic and hence objectively meaningful. It follows that deduction is not, for Cassirer, as it is for Russell, a simple analytic unfolding of what is already given in the premisses, but is a geuinely progressive, synthetic and constructive process.[5]

Oddly enough, Russell, who theorizes that numbers have an empirical origin in classes, also claims that mathematical concepts have no objective meaning; but Cassirer, who makes number a pure creation of thought, regards the propositions of mathematics as objectively meaningful. This apparent reversal of positions is explicable when we realize that, for Cassirer, objectivity enters into thought in the form of fixed rules according to which thought must proceed; whereas, for Russell, objectivity is achieved only when thought severs all connection with the phenomenal world and the rules governing it. It is essential to be aware of the distinction being made here. In conformity with the generally accepted Positivistic outlook of his time, Russell endorses a correspondence theory of truth along with the necessary corollary that objectivity consists in reducing subjective input to zero so that reality can be known **as it is.** On the other hand, Cassirer, working within a Kantian framework and concentrating not on the actual, but rather on the **realm of possible experience,** considers the objective to be a rule or principle which governs experience and gives order to it. Thus, for Cassirer, the objective meaningfulness of mathematical propositions is derived not so much from what they say about the world of experience as from the logical necessity which characterizes them.

Cassirer's view, as we have seen, is quite consistent with his epistemology and is, futhermore, reinforced by the conclusions Einstein comes to in his theory of relativity. The same cannot be said of Russell. The theory of relativity explicitly rejects the view that objectivity can be achieved by attaining an absolute frame of reference and it implicitly supports the idea that objectivity consists in defining even the most arbitrarily chosen concepts according to a definite scheme. Noting

Russell's difficulty, Smart also writes:

> Precisely in this connection Cassirer acutely points out that Russell can only regard it as a fortunate coincidence, which might just as well have been otherwise, and for which therefore no satisfactory explanation in terms of epistemology can be advanced, that mathematical concepts, though inherently void of objective meaning, experimental reference, or truth, are nevertheless indispensable to the solution of problems in the physical sciences.[6]

Now, number, and arithmetic in general, constitutes one aspect of mathematics, but a second area to be explored is geometry, and this pertains even more closely to the philosophical implications of the theory of relativity. Kant, as is well known, held that time and space were 'pure forms of intuition'. Cassirer also adopts this view, though with modifications. Whereas Kant had based the concept of number on the intuition of time, Cassirer reverses his position, holding number to be more basic in that, "time... presupposes number, but number does not, conversely, presuppose time."[7] And whereas Kant had worked within the limited conception of space represented in Euclid's geometry, Cassirer is flexible enough to acknowledge that one's view of space depends on the geometry one has chosen to work in. Nevertheless, in spite of the differences in his approach, Cassirer still remains faithful to the central Kantian idea in that he seeks to achieve a "truly logical and strictly deductive construction of the science of space," and to "restore intuition to its full scope and independence."[8]

The connection between time and space reaches its fullest development in the Einsteinian conception of a 4-dimensional space-time manifold. Corresponding to this connection, there must also be an equally close connection between arithmetic and geometry if epistemology is to keep pace with physics. But this problem does not faze Cassirer who is able to look back in history even beyond Descartes and his co-ordinate geometry to the very origins of mathematical thought in search of corroborating evidence for the validity of his epistemological stance. Time and space, arithmetic and geometry, have always been products of the human mind's synthesizing activity and accordingly, Cassirer writes:

> Since the time of the Pythagoreans there has been an intimate association, not to say an indissoluble correlation, between the theory of numbers and the theory of extension: between arithmetic and geometry ... If this tie were broken, if the origin of number were sought in the mind and the origin of space "outside" it, then the former unity of mathematics would be gone and it would comprise entirely different classes of objects and go back to different sources of knowledge.

What has already been stated with reference to arithmetic also applies to geometry. Just as it is possible to work within a variety of different number systems, it is also possible to employ alternate geometries depending upon the purpose one has in mind. Cassirer notes that space, viewed as the 'form of possible coexistences', is a basic assumption of all the different geometries, but that each geometry characterizes itself as peculiar by virtue of the unique metric that it adopts. The different modes of interpreting spatial relations may initially seem completely arbitrary, yet when the geometries are compared with each other, a distinct pattern of thought emerges. As Cassirer puts it, "...the various geometries do not exist side by side promiscuously and without relationship; they develop one from the other in accordance with a rigourously determined principle."[10] Thus, the question of which geometry is the 'true geometry' loses its relevance and our attention shifts to the pragmatic question of which geometry is best suited to our purposes.

As we have already seen, the search for concepts of ever-increasing generality of application (which is an inherent element in the scientific method), results in a corresponding emptying of the experiential import contained in the concepts. Accordingly, even the intuitive simplicity of Euclidean space has to be sacrificed in order to achieve greater precision of scientific expression. It is for this reason that Cassirer writes as follows:

> Pure Euclidean space stands, as is now seen, not closer to the demands of empirical and physical knowledge than the non-Euclidean manifolds but rather more removed. For precisely because it represents the logically simplest form of spatial construction it is not wholly adequate to the complexity of content and material determinateness of the empirical.[11]

The motive underlying the choice of a particular geometry is now recognized as its usefulness in accurately describing the phenomenal reality that we perceive. Physical thought was emancipated by the breakthrough in mathematics that de-absolutized Euclidean geometry so that new theories could be found to fit the facts rather than always having to squeeze the facts into the old theory. Newton's great achievement consisted in his ability to formulate laws of nature consistent both with Euclidean geometry and the observed facts; but Einstein makes a conceptual leap beyond Newton when he dares to assume a non-Euclidean geometry and, as a consequence, Einstein is able to explain more empirical facts. In this context, Victor F. Lenzen ("Einstein's Theory of Knowledge", Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, Volume VII) remarks:

> Newton's law of gravitation is marked out from other conceivable laws of force through a success in prediction. By contrast, Einstein's law of gravitation was found by seeking the simplest co-variant law for space-time with a Riemann metric.[12]

The extent to which geometry always retains an association with empiricism remains an undiscusssed topic so far. The axioms of geometry do not, as it were, materialize in the mind of the geometer from thin air; they are suggested by, and reflect, experience of the phenomenal world. The very term 'geometry' connotes the idea that we are here concerned with 'earth measurement'.

Of course, the human mind is creative in its manner of structuring experience, but the raw data with which it begins is experience itself. Lenzen points out that, "The axiomatic construction of Euclidean geometry has an empirical foundation, and Einstein declares that forgetfulness of this fact was responsible for the fatal error that Euclidean geometry is a necessity of thought which is prior to all experience."[13] Cassirer, too, is entirely in agreement on this point. For him, "The axioms of geometry ... must be regarded as expressing fundamental experiences of such a general sort that their empirical character is apt to be forgotten."[14]

Having recognized the empirical character of geometrical axioms, however, it is also necessary to qualify our view and to be aware of the extent to which geometry is non-empirical. The whole system of axioms, taken together, constitutes a distinct science with a distinct motive. The non-Euclidean geometries were first developed when mathematicians began to speculate as to the consequences of eliminating the axiom of parallels, and in this historical development the true nature of geometrical thought reveals itself more closely. Considered as a paradigm for systematic thought in general, geometry is the search for the greatest possible degree of internal consistency compatible with a concurrent ability to make positive assertions with regard to the subject matter. In

Einstein's words, "...the grand aim of all science ... is to cover the greatest possible number of empirical facts by logical deduction from the smallest number of hypotheses or axioms."[15] Thus, the axioms and even the geometry are chosen by the scientist, rather than being strictly dictated by experience. Geometry is itself only one way of organizing reality into a comprehensive whole and it is only once this fact is clearly recognized and understood that the scientist is capable of moving freely amongst the various geometries and the various interpretations of physical reality corresponding to each of them. Thus, Cassirer writes:

> No geometry is absolutely derived from experience or given by it, but it can be so chosen that it will be best suited to certain problems set by experience and will solve them in the simplest way. The theory of relativity has shown that even physics in its own internal progress may lead to problems that require reference to a geometry other than Euclidean.[16]

In a sense, geometry is a system of mathematical thought erected without any regard for considerations of utility. It is a pure exposition of the relations existing between its fundamental axioms and it extrapolates to pure thought constructions that experience could never suggest. Unlike the physicist, the mathematician is not concerned that his theory may be contradicted by observable facts and, unlike the philosopher, the mathematician has no worry about the effects the latest toys that he makes himself to play with have on a general understanding of knowledge. The beauty of the mathematical concept is that it is totally hypothetical - its being is simply not an issue. Whether or not there are any instances of it (or even approximations) in objective reality, the equilateral triangle is an easy thought construction to conceive of; but the important thing is that this idea (and other similar conceptions) is created according to a specific rule of thought once the basic concepts of magnitude and number are given. To the mathematician, the triangle itself is the interesting thing and the exploration of its properties becomes a fascinating exercise; but it is the rule of thought which is of permanent interest to the philosopher and the triangle is merely a particularly clear illustration of the operation of this rule. In the triangle, the concepts of magnitude and number are united in the process of measurement and once thought has created an angle of 60 degrees, it is a trivial matter to create a second and a third.

Due perhaps to the clarity with which mathematical concepts are defined, the study of mathematics offers many rich insights into the nature of thought itself. Not only does mathematics teach us a great deal about the art of defining, but reflection on the method of mathematics allows us a vision of thought in activity and makes possible the discernment of the laws governing thought. Cassirer obviously feels that philosophy has much to learn from mathematics for he tells us that the goal of the philosophical method is "to conceive all the objects with the same strictness of systematic connection as the system of numbers."[17] However, mathematical thought is far from being philosophy's only source of insight and accordingly, Casssirer qualifies the last statement as follows:

> The constructive concepts (Konstruktionsgriffe) of mathematics may be fruitful and indispensable in their narrow field; but they seem to lack an essential element for serving as an example for the whole circle of logical problems, as typical of the properties of the concept in general. For however much logic limits itself to the "formal," its connection with the problem of being is never broken.[18]

This brings us to the topics of epistemology and metaphysics. We know that each of the intellectual disciplines has its own characteristic perspective and its own method of interpreting knowledge and we shall see that the fundamental ideas underlying each science, when analyzed in terms of their functional unity, are generally nothing more than the pure expressions of relations. Mathematics, being the most highly formalized mode of thought known to man, is best equipped to deal with these pure expressions of relations and to resolve its concepts into fixed laws; but mathematics is still only one among many different sciences and, in any case, there is some danger that in effacing the individuality of a thing by reducing it to an unknown "x" in an equation, mathematics overlooks essential aspects of the thing's reality. The only way to give a complete account of reality is to accept the mathematical perspective for what it is and to augment it by viewing the object from other perspectives also. Reality is not comprised of sizeless mass-points, but rather of full-bodied things having irregular but definite shapes as well as colours, tactile qualities and odours. Thus, in writing about Cassirer's conception of the task of philosophy, Carl H. Hamburg comments:

> It follows that Cassirer could not consider as adequate any philosophical analysis of space, time, cause, number, etc., unless beside mathematical and physical spaces, it also attempted to account for the expressive and intuitional spaces of common sense, art, myth, and religion.[19]

Philosophy, however, in asserting its independence from mathematical thought, must always refrain from actually coming into conflict with the conclusions arrived at by mathematics in its own sphere. As Cassirer puts it, "...philosophy cannot and dare not attack or reject what mathematics has to teach about pure form or about the logical structure of space but must build on it as a sure foundation."[20] Thus, philosophy bases its claim to independence not so much upon the novelty of its concepts as upon the thoroughness of the method it espouses. Not only does philosophy refuse to allow mathematics to dictate its answers, but neither does it accept the answers of physics, biology or any other discipline without subjecting them to criticism and mediating them in the wider context of knowledge in general. No one would think of denying that the truths of mathematics are true within their limited context, but philosophy is asserting that we have to go beyond limited contexts if we are seeking truths that apply to a reality bigger than ourselves or the thought systems we create.

Chapter Four

Cassirer's Response to Science in his Epistemology

The analysis of the theory of relativity would necessarily be incomplete if no attempt were made to establish the connection between the specialized knowledge conceived of by relativistic physics and the philosophical theory of knowledge in general. An obvious precondition to the attainment of a proper understanding of Einstein's theory of relativity is, of course, the construction of an epistemological base in relation to which the knowledge claims being advanced can be interpreted. The soundness of this view is recognized not only by philosophers and logicians, but also by scientists themselves. Epistemology both influences and is influenced by developments in all of the sciences and therefore it is no surprise to find Einstein commenting as follows:

> The reciprocal relationship of epistemology and science is of a noteworthy kind. They are dependent upon each other. Epistemology without contact with science becomes an empty scheme. Science without epistemology is - insofar as it is thinkable at all - primitive and muddled.[1]

Now, as has been stressed throughout, the creation of concepts is always accomplished with a particular motive in mind. The physicist has the twofold objective of being able to account for the multiplicity of observed phenomena and to unite the diverse elements of phenomenal reality under a single system of laws; while, at the same time, remaining completely consistent with his initial premises and avoiding any internal contradictions amongst his concepts. Accordingly, epistemological analysis of physical theory has to be aware of both the perspective within which the physicist is operating and, especially, of the internal dialectic obtaining between the concepts of physics. Cassirer suggests that in order to satisfy these criteria two things are necessary: (1) The metaphysical interpretation of the concept as substantial has to be eschewed in favour of a functional approach; and (2) Epistemology must never lose sight of its own role in relation to the methodology which characterizes the science of physics. This idea is undoubtedly best expressed in Cassirer's own words:

> Thus physics knows its fundamental concepts never as logical "things in themselves," but only in their reciprocal combination; it must, however, be open to epistemology to analyze this product into its particular factors. It thus cannot admit the proposition that the meaning of a concept is identical with its concrete application, but it will conversely insist that this meaning must already be established before any application can be made.[2]

In physics, the concepts become increasingly abstract as the physicist attempts to formulate laws of ever-increasing generality and to extend the domain of physical knowledge farther and farther beyond that of immediate perception. The parallel development of this trend in epistemolgy exhibits itself in the analysis of concepts derived to an ever-greater degree from pure thought rather than sensory intuition. Thus, epistemology receives a tremendous impetus from theoretical physics insofar as the progress of physics is a steady movement towards objectivity. The Platonic distaste for the type of knowledge to be gathered via the untrustworthy medium of the sense organs finds its modern day expression in a physics that seeks to formulate frame-independent laws and which relativizes the totality of all frame-dependent observations derived from a single observer's experience. At the level of epistemology this implies that theoretical physics begins by concentrating on the internal relations of order determined by the manner in which the concepts are selected to the exclusion of any real concern regarding the concrete application of such concepts. Pure epistemology, in Cassirer's view, is 'pure' precisely because it relinquishes its attachment to the phenomenal uncertainty of the world of empirical fact in order to discover the logical and mathematical certainty of concepts ordered with respect to each other in a systematic fashion. In The Philosophy of Symbolic Forms, he writes:

> As long as we remain within the sphere of pure epistemology and concern ourselves with the presuppositions and validity of the basic scientific concepts, the world of sensory intuition and perception is defined only with a view to precisely these concepts and is evaluated as a phase preliminary to them.[3]

There is a danger, however, implicit in the type of idealistic philosophy which immerses itself so completely in the contemplation of concepts that it loses touch with the immediate reality of the factually present. Critical Idealism always needs to remain in contact and to dialogue with the very reality it criticizes; otherwise criticism itself becomes just so much empty speech - the "flatulence" of the nominalists. In assessing Cassirer's allegiance to the neo-Kantian school of Critical Idealism, William H. Werkmeister points out that, "Only one condition is indispensable. The content of cognition must be grounded in a unitary origin of thought,"[4] and Cassirer himself, while rejecting the 'copy' theory of knowledge, still recognizes that it is vital that the connection between thought and reality be maintained: "For thought and reality ought not merely to correspond to each other in some sense but must permeate each other."[5]

Einstein, who was also a well-known opponent of the 'copy' theory of knowledge, felt that somehow the fundamental hypotheses of physics ought to be "... suggested by the world of experience itself,"[6] but was at a loss to say exactly how this process of suggestion worked. A possible solution to this problem may be found in Cassirer's philosophy of symbolic forms, wherein he circumnavigates the dilemmas associated with the traditional way of framing the question by regarding concepts, "... no longer as passive images of something given but as symbols created by the intellect itself."[7] Again, it must be emphasized that, for Cassirer, the intellect is an active agent, freely inventing and modifying the concepts necessary for the production of an ordered and systematic reality not inconsistent with the reality known empirically through sensation. The symbols employed by the intellect in the representation of passively experienced reality are chosen for their heuristic value and their capacity for compressing a variety of differing experiences into a single form as well as for the fidelity with which they express the phenomenal reality they purportedly represent. In his introductory note to The Philosophy of Symbolic Forms, Charles W. Hendel provides insight into Cassirer's meaning when he makes the following remark:

> Cassirer is ready to include science as one of the "artificial" symbolisms along with language, art and myth. Thus he found Heinrich Hertz' theory of symbols in science a point of departure for the exposition of his own general theory in the present book. While there are basic forms such as space, time, and the categories of understanding, there are special constructive forms for each science and every theoretical construction within science. The meanings of concepts depend upon the whole structure of the scientific system in which they are conceived.[8]

When we once realize, with Cassirer, that each science achieves theoretical unity only by combining a manifold of diverse phenomena according to its own "special understanding and formulation of the concept of reality,"[9] then we are finally in a position to begin the task of epistemological analysis. There is no such thing as a concept in vacuo. Every concept is given meaning by the structure of the scientific system surrounding it. The mind creates symbols in order to simplify and give unity to experience, and the very essence of rational thought is rooted in the possibility of ordering these symbols conceptually into symbolic forms. The symbolic form is not itself a symbol, but rather a rule governing the manner in which different symbols can be arranged in relation to each other. Physics, as a particular science, thus constitutes an extensive symbolic form because, having created a multitude of symbols such as the mass-point, the rigid body, and the path of least resistance, physics proceeds to ennunciate the set of laws whose formulation is the result of the physicist's ceaseless striving to attain systematic unity.

It would be one thing if the thesis that Cassirer is setting forth were merely the child of his philosophical daydreams or the speculative ramblings of an overly fertile imagination, but it is quite another thing when we realize that these epistemological assertions regarding the creative activity of the human intellect are based as much, if not more, on solid empirical evidence as theory. As Carl H. Hamburg comments in his article, "Cassirer's Conception of Philosophy":

> The thesis, accordingly, that the mind (Bewusstein, Geist) is symbolically active in the construction of all its universes of perception and discourse is not suggested as a discovery to be made by or grounded upon specifically philosophical arguments. Instead of presupposing insights different from and requiring cognitive powers or techniques superior to those accessible to empirical science, the thesis is developed as issuing from an impartial reading of the scientific evidence in all branches of investigation.[10]

Of course, we know that, in general outline, Cassirer's position coincides not only with the observed psychological facts concerning perception and concept formation, but also with the epistemology of Immanuel Kant. It was Kant who first suggested the view of the mind as an active synthesizer of concepts and this idea was retained by the neo-Kantians, but Cassirer's true original contribution is his decision to emphasize the functional nature of the concepts created by the mind in his analysis of the process of knowing.

When we speak about the functional nature of the concepts that the mind creates, we are referring not merely to the laws governing thought itself, but also to the internal law which confers systematic unity upon the concepts. In his book, <u>Ernst Cassirer: Scientific Knowledge and the Concept of Man</u>, Seymour W. Itzkoff tells us that, "Cassirer was insistent that the meaning of scientific concepts could only be derived by an examination of the laws which ordered and interrelated their several parts."[11] In other words, it is the entire system of scientific concepts in their mutual dependency that comprises the symbolic form which is the epistemological basis of a particular science. Thus, with regard to physics for example, it is epistemology's role to investigate and delineate the fundamental concepts involved in the formation of a model such as Einstein's relativity theory. And it is only after epistemology has completed the arduous task of repeating this investigation for all the sciences, taking each one as an internally consistent whole, that philosophy is able to contemplate the even greater challenge of surveying the entire system in its interrelatedness.

In his other book, entitled simply, Ernst Cassirer, Itzkoff summarizes Cassirer's fundamental epistemological stance in one brief sentence: "The search for structure is inherent in thought."[12] Now, according to Cassirer, the mode typically employed by thought in the search for structure is symbolism. Cassirer even goes so far as to say that the ancient characterization of man as animal rationale ought to be modified to animal symbolicum; and in his An Essay on Man, we read:

> In language, in religion, in art, in science, man can do no more than to build up his own universe - a symbolic universe that enables him to understand and interpret, to articulate and organize, to synthesize and universalize his human experience.[13]

Epistemology, therefore, could perhaps best be defined as the analysis of the building blocks used by man in the construction of his "symbolic universe"; but with this qualification - we never grasp the full meaning of any particular symbol by itself, but always in its relation to the other symbols and also the symbolic form of which it is a part. As was mentioned earlier[14], neither the whole nor the parts are fully comprehensible in isolation.

It is difficult to say with precision exactly what Cassirer meant by the term "symbolic form". After all, his philosophy of symbolic forms is contained in a three volume work, each volume of which exceeds three hundred pages in its English translation. Fortunately, however, this task is made easier by the fact that we are here attempting to focus on the philosophical implications of the theory of relativity. Looked at in the context of one particular scientific theory only, the symbolic form is the meta-theory constituted by the set of basic axioms or undefined concepts created in order to orient the secondary and tertiary concepts and to give a systematic character to the science as a whole. The theory of relativity happens to be particularly in tune with Cassirer's basic epistemological outlook because it represents a new plateau in objectification. Einstein's relativistic physics passes beyond the initial stage of a "naive realism" which concerns itself with things-in-themselves to a more profound functional understanding which concentrates on the relations obtaining between things rather than the things themselves. In <u>Determinism and Indeterminism in Modern Physics</u>, Cassirer writes:

> Every rightly framed hypothesis in keeping with its factual meaning sets up a law concerning phenomena more general than what has till then been directly observed; it is an attempt to ascend to an orderliness even more general and comprehensive.[15]

The Einsteinian hypothesis of space-time as a four-dimensional continuum allowed for a greater objectivity in physics in that the notion of an absolute frame of reference was discarded as unnecessary (and even factually misleading, as the Michelson-Morley ether experiment seems to show). The same empirical knowledge that we already had could be deduced from a more general assumption and other physical facts such as the effect of gravity on light rays could also be accounted for. Thus, for Cassirer, the theory of relativity is paradigmatic in that it compresses the concepts of physics while at the same time expanding our knowledge of the physical universe[16]. What Einstein has accomplished is the creation of a newer and more powerful symbolic form and, as a consequence, a new stage of objectification is reached. From the epistemological point of view, this means that the system of relations obtaining between physics' basic concepts has been simplified - simplified in order to approximate reality more closely. As Felix Kaufman comments in his article, "Cassirer's Theory of Knowledge", the resulting epistemological progress is not at the cost of lost fidelity to reality as it is known empirically:

> Each stage of objectification is represented by a specific system of linguistic symbols. But this fact must not be interpreted as a creation of concepts (meanings) by words, as radical nominalists would have it.[17]

Einstein's own expressed epistemological convictions indicate that he also is in complete agreement with Cassirer on this point. Even if reality can only be defined functionally and never in substantial terms, the object of a scientific theory is to explain empirical reality, as we know it through observation, ever more fully. The new generalization is a useless one if it cannot accomplish this. Thus, in The Evolution of Physics from Early Concepts to Relativity and Quanta, Einstein and Infeld write:

> A method of generalization is not uniquely determined, for there are usually numerous ways of carrying it out. One requirement, however, must be rigourously satisfied: any generalized concept must reduce to the original one when the original conditions are fulfilled.[18]

Great care should be taken, however, in distinguishing between the notion of the truth of a scientific theory within which Cassirer and Einstein are operating and the type of 'correspondence' theory of truth advocated by Bertrand Russell and others. Mere consistency with the body of established knowledge, though important, is not sufficient; Cassirer requires that the additional criterion of systematic coherence must also be met[19]. Not only must a theory be able to reconcile itself to the facts of empirical reality, but it must also demonstrate internal consistency of concepts and, furthermore, the very principles of empirical procedure which ultimately determine the meaning and the nature of the 'facts' must also participate in the overall function of the theory.

The discussion of the symbiotic relationship between theory is really too broad to be treated properly here. Initially, it may seem that the distinction between theory and fact is clear and unproblematic; but upon closer scrutiny, we become aware that a major problem does exist. At one point in his Philosophy of Symbolic Forms, Cassirer remarks that the poet, Goethe, has written that all fact is in itself theory[20]. This is the key to understanding the problem. What is a fact? Is a fact a statement about reality that can be verified? A statement which every man acknowledge to be true if he knew the fact upon which it was based? If we say that a fact is the former, we have to ask what verification is and we have to conclude that verification is only possible subjectively because the thing-in-itself cannot be known objectively. Thus, there can be no objectivity of scientific truth. If we say that a fact is a statement which is absolutely true, we will be forced to admit that no statement about measurement can possibly be a fact since all measurements are performed in particular reference frames and no particular reference frame can be absolute.

Admittedly, it is a hypothesis of relativity theory that no reference frame is absolute, but this is the central hypothesis of the Special Theory of Relativity.

What has to be remembered is that we are dealing with a physical theory aimed at explaining motion, and the theory does so by making all facts only relatively true. Thus, the theory explains and even postulates the facts, and the facts justify the theory. If it is objected that this reasoning is circular, then the **apparent problem** of theory and fact presents itself; but the scientist points out that the problem is only apparent because he is advocating his theory **as a theory**, and skeptics are invited to observe their own facts and to agree or disagree with the theory's conclusions as they please. The **real problem** of theory and fact, however, is not dismissed so easily. While we would not wish to argue that there is, metaphysically, no substantiality to the objects of our experience, we would also be disinclined to deny that, within a physical theory, the substantiality of the objects of experience is irrelevant because the theory explains appearance. Thus, we have the paradox of a theory about reality which explains appearance: modern physics!

Interesting as the problem of theory and fact may be, and interesting as the closely connected problem of measurement may also be, they must be put aside for the time being while we discuss the epistemological antecedents to these problems. Cassirer maintains that thought arrives at its fundamental (and therefore, most universal) concepts without being limited by experience and that it is only after the work of its essential epistemological formation has been completed that a scientific theory resorts to positivism and the empirical verification of its conclusions. In contradistinction to Russell, however, he also maintains that these fundamental concepts are man-made symbols rather than independently existing archetypes. As Harold R. Smart puts it, ("Cassirer's Theory of Mathematical Concepts"):

> According to Russell thought merely discovers the subsisting essences of this ideal, trans-empirical realm; whereas according to Cassirer thought actively creates these universals, thus generating its own world out of its internal resources.[21]

The mind's active role in creating concepts capable of imposing epistemological order on phenomenal reality has already been stressed. More importantly, the shift with regard to the focus on function has also been emphasized, as Smart recognizes when he summarizes Cassirer's basic epistemological thesis in five words: "To know is to relate."[22] In my view, this is the key to understanding Cassirer. The cornerstone of not only his epistemology, but also his whole philosophy, is what I will call his 'metaphysics'. Cassirer always insists on uncovering the functional relationships implicit in thought and, consequently, his metaphysics is a metaphysics of function and not a traditional metaphysics of substance. Epistemology always roots itself in a metaphysics of some kind, but as we shall see, Cassirer's metaphysics is quite unique.

Chapter Five

Substance and Function: Cassirer's Metaphysics

In the light of the well-known Kantian antipathy for the discipline of metaphysics and of Cassirer's strong affiliation with the Marburg school of neo-Kantians, it may seem somewhat odd to meet with a chapter heading entitled, "Cassirer's Metaphysics"; nevertheless, it is hoped that, aside from arousing the reader's curiosity, there may also be some intimations of a truth worth communicating in the title. Kant's objection to metaphysics was based on the view that it involved human reason attempting to surpass its inherent limits, for Kant felt that the metaphysical thing-in-itself would forever be an enigma to human understanding. Now, to say that Cassirer has a metaphysics is not to assert that Cassirer disagreed with Kant on this point; rather we should argue that, since Kant, the conception of what metaphysics can be has changed. Kant sought to understand Plato better than Plato had understood himself, and in their turn, the neo-Kantians sought to understand Kant better than he had understood himself. Thus, Cassirer agrees with Kant insofar as the thing-in-itself is to be interpreted as substance, but he points the way towards overcoming Kant's antimetaphysical bias when he begins to interpret the thing-in-itself functionally as a symbol and as a part in a system of relations that has unity as a whole.

The movement away from the metaphysical mode of thinking which Kant began culminated in the Logical Positivism propounded by members of the Vienna Circle such as Rudolf Carnap. Later on, the British philosopher, A.J. Ayer took such an extreme position that his book, Language, Truth and Logic, actually serves as a sort of 'reductio ad absurdum' for the antimetaphysical tendency. For, when Ayer makes statements such as, "... all metaphysical assertions are nonsensical,"[1] we become aware that this position is itself nonsensical and we begin to wonder where the flaw in such reasoning lies. In fact, the back-to-metaphysics rallying cry was sounded well before Ayer wrote his book. In the 1920's, Martin Heidegger was actively proclaiming a metaphysical reinterpretation of Kant and in 1938, Etienne Gilson wrote that, "all philosophical knowledge ultimately depends on metaphysics,"[2] while Einstein spoke of a "fear of metaphysics" that "has come to be a malady of contemporary empiricist philosophizing."[3] If metaphysics is interpreted as the set of all statements asserting unarguable propositions such as, "... the Absolute enters into, but is not capable of, evolution and progress,"[4] then there does indeed seem to be some validity to Ayer's contention, but nothing prevents us from taking a broader view of what metaphysics can be. Thus, Cassirer's metaphysics is a metaphysics of function and far from being nonsensical or beyond criticism, his metaphysical assertions are both intelligible and analyzable. In Ernst Cassirer: Scientific Knowledge and the Concept of Man, Itzkoff writes:

> Instead of asking questions of substance or reality, as for instance regarding space and time, the true critical analysis merely asks the function of the various elements of the knowledge situation.[5]

The metaphysical 'object', once regarded as the given starting point from which scientific enquiry proceeds, is now defined as a thing by the same law which defines all other objects as things also. At the beginning, what the object 'really is' is irrelevant because we are only interested in the way it relates to other objects. As Cassirer says, "... the object is not treated as a given fact but as a problem; it serves as the goal of knowledge not as its starting point."[6] Thus, metaphysics being what Aristotle termed 'the science of first principles', the individual thing in its individuality is not its true subject matter and Cassirer is correct in turning metaphysics away from the thing itself and towards the principle exemplified by the thing. This means that we study what is general about the thing rather than what is unique about it, and furthermore, we study the thing never in isolation, but always in the context of its relations with other things. The principle is therefore extracted wholly from the manifold of things and the only sense in which the principle can be said to be derived from the particular thing is that the existence of the particular is a sine qua non for the existence of the manifold. Metaphysics has to start somewhere, but as soon as it does begin it seeks to negate the arbitrariness of its starting point through the process of abstraction. Cassirer's functional approach to metaphysics is well illustrated by the following quotation:

> But precisely to the extent that the concept is freed of all thing-like being, its peculiar functional character is revealed. Fixed properties are replaced by universal rules that permit us to survey a total series of possible determinations at a single glance. This transformation, this change into a new form of logical being, constitutes the real positive achievement of abstraction.[7] (emphasis added)

In attempting to develop the implications of the type of metaphysics found in Cassirer's philosophy, it is illustrative to contrast his ideas with those of Russell, who is apparently still mired in the metaphysics of substance and attribute. In Human Knowledge: Its Scope and Limits, Russell's dissatisfaction with his own metaphysics is made manifest by the following remark:

> The notion of substance as a peg on which to hang predicates is repugnant, but the theory that we have been considering cannot avoid its objectionable features.[8]

To be sure, Russell still seeks other solutions - without, however, altering the basic framework of his metaphysical thought; consequently, his theory of 'compresence' leaves much to be desired. Interestingly enough, both Russell and Cassirer undertook to write major expositions of Leibniz's philosophy at formative times during their respective academic careers; but whereas Cassirer's subsequent philosophy reflects Leibniz's thought to a great extent, Russell can only say:

> Our age has penetrated more deeply into the nature of things than any earlier age, and it would be false modesty to overestimate what can still be learned from the metaphysicians of the seventeenth, eighteenth and nineteenth centuries.

The above statement coupled with Russell's tentative metaphysics contrasts sharply with Cassirer's deep respect for the thinkers of the past and his elaborate and systematic metaphysics of function. In Leibniz, Cassirer finds exactly the basis he needs for the task of constructing a post-Kantian metaphysics*. Thus, he substitutes a thoroughly modern, and at the same time quite ancient, notion of **substance as a principle of order** for the pre-Socratic conception of substance as that which underlies all material things. A perceptible evolution in thought can be traced from Thales' idea that "all is water" to Anaximander's concept of the <u>apeiron</u>, the indefinite; but when Anaximenes returns to the original view by arguing that air is the primary substance he falls back into the fundamental metaphysical fallacy of trying to identify the indefinite. The only way out is Leibniz's

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*Although Cassirer would probably never have referred to his own philosophy in these terms, there is no reason to think that he would disagree with the general point being made here - especially in view of the fact that the term 'metaphysics' is being used in a very special sense.

way - define substance in terms of what it does rather than what it is. This should not be taken to imply that being is necessarily incomprehensible, rather it means that the way of achieving an understanding of being is through analysis of it in its full complexity and essential orderliness. Cassirer explains:

> Hence substance as fundamental force is likened by Leibniz to the "universal member" of certain algebraic series, the member that specifies the rule according to which the series progresses from one element to the next, while the "derivative force" corresponds to the particular members of the series.[10]

Note, however, the fundamental metaphysical assumption at work here; viz., that orderliness is intrinsic to reality. Laplace, following Leibniz, elevated the postulate of intrinsic order into the most full-blown determinism yet created by human thought. In order for Cassirer to avoid the pitfall of Laplacian determinism or its complement, Aristotelian teleogy, he has to be extremely cautious in defining order. On one hand, the crux of his epistemology is the thesis that the human mind creates and imposes order upon phenomenal reality, and on the other hand, the historical evolution of life forms reveals an order of an altogether different character. The humanly created concept of order is objective - that is, in reality itself - only to the extent that man is capable of approximating true objectivity in his most abstract thought; but the order made manifest by the history of species is a subjective order of structure, a progressive realization of a purpose. This second type of order is also an order created by man, but it differs from the first kind of order in that it is a lived experience rather than an intellectual activity. Cassirer invokes the authority of Immanuel Kant to explain this idea more fully:

> The Critique of Judgement aims to prove that there is no antimony whatsoever between these two forms of order in knowledge. They cannot contradict each other because they relate to problems in distinct fields that must be carefully kept apart. Causality has to do with knowledge of the objective temporal succession of events, the order in change, whereas the concept of purpose has to do with the structure of those empirical objects that are called living organisms.[11]

As long as the notion of causality is applied purely in the domain of knowledge, determinateness is only a principle governing the arrangement of concepts. As long as purpose is not attributed to non-sentient reality, teleology has nothing to do with the science of physics. Only by keeping both of these provisos in mind is it possible to continue our analysis of the philosophical implications of the theory of relativity. Consider, for instance, the vast amount of metaphysics that lies behind Einstein's compact formula: $E=mc^2$. Here, an equivalence is established between the total energy and the total mass of a physical system. The new hypothesis suggested by Einstein effects a revolution in metaphysical thought because the functional concept of energy replaces the substantial concept of matter as the key feature of the physical object. As the role assigned to matter diminishes within the metaphysical picture, a number of associated metaphysical problems also diminish in importance. Thus, we notice that, on all sides, support can be found for Cassirer's metaphysical position. Einstein writes:

> It therefore appears unavoidable that physical reality must be described in terms of continuous functions in space. The material point, therefore, can hardly be conceived anymore as the basic concept of the theory.[12]

and Russell also says:

> It is energy, not matter, that is fundamental in physics. We do not define energy; we merely discover laws as to the changes in its distribution.[13]

The metaphysics of function which Cassirer espouses is completely consistent with his epistemology. The determinate character of scientific thought is a logical consequence of the finite number of ways in which the basic concepts of science can be permuted and combined. At the same time, each separate combination of concepts entails a specific manner of relating the concepts to each other and to the system as a whole. Since each system of concepts is a creation of the human intellect, there is no need to invoke the teleological notion of purpose and no need to represent the system as a copy of reality 'as it really is'. Thus, matter may or may not exist, but the question of substantial being has lost its relevance for a metaphysics concerned with function. The fact that material things can be viewed as pockets of highly concentrated energy makes a metaphysical description of reality into a mathematical problem rather than a series of dogmatic assertions. Energy is itself defined as the ability to do work and work is defined in terms of energy transfer; but the circularity here is not vicious because the problem of being has been recognized as insoluable from the outset. Once the focus is shifted from what energy **is** to what energy **does**, we are free to enter into the real metaphysical problem of the analysis of the conditions under which work (energy transfer) is possible.

Now work has one meaning for the physicist and quite another for the artist and still another for the sociologist because all of them interpret work with regard to their own disciplines. From the point of view of philosophy, however, work is not fully known by any of the individual disciplines because, as Cassirer writes, "... a philosophical critique seeks a rule governing the concrete diversity of the functions of cognition."[14] Just as energy, qua substance, is undefinable, work, interpreted as function from a multiplicity of different perspectives, is only definable with reference to the perspective from which it is viewed. In order to achieve a unified understanding, therefore, it is necessary to be aware of the motive according to which each perspective operates and to find the abstract principle underlying each of its concrete applications. Thus, each branch of knowledge assumes a particular metaphysics of function peculiar to itself, while philosophy probes in search of a functional unity generalizable to all. Cassirer is well worth quoting on this point:

> True, the unity of knowledge can no longer be made certain and secure by referring knowledge in all its forms to a "simple" common object which is related to all these forms as the transcendent prototype to the empirical copies. But instead, a new task arises: to gather the various branches of science with their diverse methodologies - with all their recognized specificity and interdependence - into one system, whose separate parts precisely through their necessary diversity will complement and further one another. This postulate of a purely functional unity replaces the concept of a unity of substance and origin, which lay at the core of the ancient concept of being.[15]

Having pointed out that Cassirer's metaphysics does not deal directly with the question of what energy is, it is also important to mention that he still does address himself to the fundamental metaphysical problem - being. The reader should bear in mind, however, that the answer Cassirer gives to the question of being is an answer wholly determined by his postulate of a purely functional unity. Thus, it only makes sense to talk about the being of energy when we remember that energy is known only through its self-manifestation as work. Nevertheless, energy is not a pure concept created by human minds, it is reality unfolding as it actually does unfold. Cassirer writes:

> In energy we grasp the real because it is effective. Here no mere symbol comes between us and the physical thing; here we are no longer in the realm of mere thought, but in the realm of being.[16]

In other words, the real thing is energy, acccording to Cassirer. The process of abstraction culminates not so much in the creation of the concept of energy, but in the discovery that energy is, "... a being that fulfills all the conditions of true and independent existence, since it is indestructible and eternal."[17]

We should not, however, misunderstand Cassirer's meaning here, for this point is a vital one. Energy is no concrete thing, having a 'whatness' or 'quiddity'; it is the exact opposite. Energy is the metaphysical first principle by means of which reality can be ordered; but this statement can only be comprehended when we remember that it is a metaphysics of function, not of substance, with which we are dealing. Thus, Cassirer's metaphysics can never claim to be complete because the totality of energy's functions is not yet known. Having passed beyond the substantial viewpoint which makes the object the mere sum of its properties, we need to know not only how it is that these properties are unified in the thing, but also all of the relations that obtain between the properties comprising the thing and between the thing and other things in order to understand energy fully. Plainly, this is impossible, but nevertheless it is a goal for human knowledge to work towards. Thus, Cassirer writes:

> Energy is able to institute an order among the totality of phenomena, because it itself is on the same plane with no one of them; because, lacking all concrete existence, energy only expresses a pure relation of mutual dependency.[18]

The similarity between Cassirer's functional metaphysics and the mathematical notion of a function is no accident. A simple mathematical function F(x) sets out a unique rule by which the members of a definite series can be determined. The function is not itself a member of the series; it is the rule governing a set of ordered pairs or, to express it another way, the simplest possible symbolic representation of the order intrinsic to the series. In the same way, the description of physical reality in terms of energy functions is also the simplest possible symbolic representation of the order intrinsic to reality. Once the rule given in the function is established, the characterization of the individual things (the members of the series) is a matter of manipulation of mathematical symbols. Any value we choose can be assumed for the unknown individual member "x" and the function will give us a corresponding value F(x) by reference to which the particular "x" chosen can be compared and contrasted with an infinity of other possible "x's".

Cassirer's metaphysics of function goes hand in hand with his epistemology as expressed in The Philosophy of Symbolic Forms. The mathematical notion of serial ordering provides a paradigm of connectedness of concepts and logical simplicity for all other branches of knowledge to strive to emulate; but as the fundamental symbolisms of other disciplines become increasingly rich in experiential content, the determinateness which characterizes the mathematical function becomes increasingly difficult to achieve. Nevertheless, Cassirer still stresses that a functional approach should be taken in the analysis of the different symbolic forms and he demands precision amd internal coherence even in the analysis of the concepts of art and mythology. The realm of possible experience is indeed variegated and complex, but fruitful insights stand to be gained from the study of the metaphysical backdrops implied or presupposed in each and all of the fields of human knowledge.

There are two interrelated aspects of the problem of knowledge: on one hand, experience and sensation are prerequisites for knowing; and, on the other hand, the very act of intellectualizing lived experience, which is the essential core of the process of knowing, shapes and limits possible experience. The question of the 'truth' of what is known is a function of the certainty with which it is known, but experience can never yield certainty and knowledge is in a constant process of reformulation whilst it remains in dialogue with experience. But truth is not the correct metaphysical question to be asking, what we really need to inquire about is the function of the various elements of the knowledge situation and the role of the symbolic form under consideration. The symbols that man creates are his way of compressing experience in order to extract its essence and yet the symbols are suggested by experience itself and are not completely free inventions or pure fictions. A whole system of definitions has to be evolved in order to allow for the possibility of knowledge, but the substantial underpinnings of this system continually evade all analysis. Thus, the sphere of knowledge is limited by the symbols in which human beings are capable of expressing themselves.

Perhaps this explanation will be seen to be more cogent if we consider as an example the set of basic concepts that made it possible for Einstein to achieve a higher thought synthesis in physics. Obviously, the full implications of Einstein's theory of relativity are too broad a topic for a study such as this, and in any case, it is not necessary to plunge too deeply into physics when the goal is not mastery of modern physics, but philosophical reflection on Einstein's method. Thus, it must be possible to explore Cassirer's theory of concept-formation in science and to examine some of Einstein's basic concepts without becoming overly technical. The best way to appreciate the true depth and breadth of Einstein's thought is, of course, to enter into his world and to confront the problems that he himself faced, but the true purpose of this endeavor is to test Cassirer's ideas and to assess the viability of his philosophical program. Thus, it is important to remember that we are not so much interested in the theory of relativity's impact on modern physics as in its implications for philosophy. The second part of this essay constitutes an attempt to analyze the basic concepts of space, time, matter and energy. The full import of these concepts is not yet fully understood in physics and yet they are tailored to the specifically physical problem. The task of philosophy, therefore, is that much more difficult still because, in attempting to integrate the physical perspective into a broader view, philosophy has to consider the problem from many different angles. It is, however, to be hoped that the broader view which philosophy takes and the synopsis that philosophy achieves may enable philosophy to participate meaningfully in a dialogue with science and to contribute new insights and ideas.

PART II: BASIC CONCEPTS OF EINSTEIN'S THEORY

Chapter Six

Space, Time and Relativity

In Part I of this paper we dealt with Cassirer's interpretation of scientific knowledge and we saw that, in his view, the task confronting philosophy is to achieve synthesis and synopsis and to situate scientific knowledge within the wider framework of human knowledge in general. Such a project requires a painstaking analysis of the manner in which each science structures its concepts. The specific knowledge claims advanced have to be considered both with regard to their systematic coherence and with regard to their objective meaningfulness. Furthermore, philosophy has a role to play in surveying the knowledge to be derived from a variety of different perspectives so that the necessary allowance can be made for the bias inherent in each science's peculiar methodological orientation. Einstein's theory of relativity serves as an excellent illustration of Cassirer's basic epistemological thesis because Einstein's new approach to the concepts of space and time within physics implies a fundamental change in the methodology of physics. The new meaning Einstein ascribed to the 'facts' of physical reality harmonizes with a philosophical view which emphasizes the functional nature of scientific concepts and which sees knowing itself as a process of relating concepts according to the fixed rules governing reason's legitimate sphere of operation. Thus, by exploring the uses to which these concepts are put within physics, we gain a deeper understanding not only of their general scientific meaning (which philosophy must necessarily presuppose), but also of the inner workings of the method of physics.

In order to understand the significance of the theory of relativity properly it is necessary to go back to the pre-Einsteinian view of space so that by becoming familiar with the earlier view and its hidden presuppositions, we may fairly advance to the study of the modern view. Space, like time, is one of those concepts which everyone grasps initially with very little difficulty, but which becomes progressively more elusive the more we think about it. St. Augustine expressed this general idea most clearly when he said of time, "If no one asks me I know what it is. If I wish to explain it to him who asks me, I do not know."[1] Fortunately, however, the method of physics allows for provisional or operational definitions of concepts which would be otherwise difficult to articulate. It is nevertheless true that a great deal of metaphysical or, at any rate, philosophical, thought lies behind these definitions, but the advantage of this method of proceeding is that it allows us to bypass preliminary considerations of the truth or falsity of our axioms so that we can get on with the task of understanding physical reality. As Hans Reichenbach remarks:

> The logical basis of the theory of relativity is the discovery that many statements which were regarded as capable of demonstrable truth or falsity, are mere definitions.[2]

Prior to the twentieth century, the dominant theory of space and time was that upon which the edifice of Newtonian physics had been built. Newton was well aware of the relativity of different kinds of motion and, accordingly, he felt that space had to be defined as an absolute frame of reference in order to organize and make sense of the various motions he observed. With space at rest, each motion could be related to space and, by reference to space, to every other motion. Thus, a train moving at sixty miles per hour in one direction and a man running at ten miles per hour in the opposite direction could be related to each other because the velocities of both were in reference to an assumedly immobile space. Space itself had a status intermediate between existence and non-existence, but the metaphysical problem of space didn't deter Newton from producing a physics of exceptional explanatory potential. What was important was the concept of space as an absolute frame of reference. In his book, <u>The Existence of Space and Time</u>, Ian Hinckfuss comments:

> If we were willing to countenance the existence of space and yet deny that space was an object, we could allow that absolute motion was motion with respect to space. Since nothing can move with respect to itself, space would itself be absolutely at rest. This was Newton's position with respect to space.[3]

Newton's system, however, broke down as new scientific discoveries in the late nineteenth and early twentieth centuries forced physicists to re-examine Newton's basic assumptions. In fact, Leibniz had objected to the theory of absolute space on philosophical grounds at the very beginning, but support for Leibniz' position really grew when the weight of empirical evidence altered the balance of the scales against Newton. Consider the following remark by Leibniz:

> As for my opinion, I have said more than once, that I hold space to be something merely relative, as time is; that I hold it to be an order of co-existences, as time is an order of successions.[4]

In view of the failure of all attempts to detect the ether (which could have provided an alternative to the postulate of absolute space) and of a growing body of results suggesting that Newtonian physics was inadequate in dealing with very high velocities and the motion of subatomic particles, it was inevitable that a new conceptualization of space should be adopted. As Cassirer had foreseen in his philosophy, scientific thought moved away from the idea of space as a substantial entity and scientists began to take a purely functional approach. Heinrich Hertz had already taken the first step in another context when he equated Maxwell's electromagnetic theory of light to the system of Maxwell's differential equations[5], and Einstein was to take the second step when he deprived space of all objective content. Interpreted in substantial terms, the concept of space is void and meaningless; the whole meaning of 'space' is now exhausted by the set of functional relations encompassing it. As Leibniz wrote in the seventeenth century, "space denotes, in terms of possibility, an order of things which exist at the same time, considered as existing together,"[6] or as Einstein puts it in the twentieth century:

> According to general relativity, the concept of space detached from any physical content does not exist. The physical reality of space is represented by a field whose components are continuous functions of four independent variables - the coordinates of space and time.[7]

The mathematization of space is completely in accord with Cassirer's general program. In Einstein's hands, space-time is nothing other than what the theory of relativity needs it to be in order for the theory to remain consistent. Newtonian physics is not thereby rendered obsolete - it is only the hypothesis of absolute space which is eliminated - but Einstein's genius consists in showing that Newtonian physics does not need this hypothesis. Properly understood, Occam's rule of economy of concepts means that where an explanation requiring fewer assumptions (articles of faith) is as satisfactory as another explanation requiring additional assumptions, then the former ought to be preferred - there is no need to multiply hypothetical entities. Accordingly, if there is no need to assume that some ideal inertial system is absolutely at rest (i.e., that absolute space exist), then why make such an assumption? The substantial question of what space is (or may be) is thereby avoided and the only really meaningful issue is the question of function. Einstein's 'space' enables him to relate physical things and events to each other in such a way as to generate a coherent and systematic physical theory. Beyond the relational meaning of space necessary to the internal consistency of the system Einstein speculates no further; or, if he does, he recognizes it as speculation. Thus Hinckfuss writes:

> Relational theories of space are not so much theories of space per se, but rather affirmations of a belief that a program which has as its aim the reduction of statements concerning space could be successful.[8]

When one stops to think about it, the theory of relativity is really not saying anything all that astounding or radically new. Although an observer who habitually gazed heavenwards on starry nights might easily come to the conclusion that the stars move with respect to 'space', it is not difficult to convince most people that, in the sublunary sphere at least, one's perception of motion depends very much on the way one is situated. As Russell comments in his ABC of Relativity:

> But such things as differences in perspective, or differences of apparent size, due to difference of distance, are obviously not attributable to the object; they belong solely to the point of view of the spectator. Common sense eliminates these in judging of objects; Physics has to carry the same process much further, but the principle is the same.[9]

In the pre-Copernican era, when it was thought that the earth was fixed in the center of the universe, it was reasonable to assume that a stationary observer on the stationary earth could make a completely objective assessment of motion he perceived; but, now that it is known that not only the earth but all bodies in the universe are in motion, it would be unreasonable indeed to assume that perfect objectivity was possible. The next best thing, then, is the theory of relativity. We recognize the arbitrariness of every particular point of view and we seek to reconcile all differences with the reflection that a uniform system of measurement will yield results capable of being transformed in such a way as to mean the same thing in any of the reference frames available. There is no need to assume absolute space in order to make the results measured in one reference frame intelligible in another reference frame; we now possess the mathematical techniques necessary to make such an assumption superfluous. In William H. Werkmeister's words:

> The old idea of a unitary time and a unitary space has been abandoned, but its place has been taken by a one-to-one correlation of space-time values in empirically different systems.[10]

Here, we pause to draw a distinction between the "Special" and "General" theories of relativity. The "Special Theory" dealt with non-accelerating frames of reference only, but the "General Theory" goes one step further in also accounting for the possibility of accelerating reference frames. The most important thing that the Special Theory must do is to articulate laws of nature which are invariant regardless of all particular non-accelerating reference frames. The Lorentz transformation makes it possible to synthesize the meaning of data gathered in different reference frames, but the laws governing the arrangement of this data must be the same in all reference frames. As Russell writes:

> The laws of phenomena should be the same whether the phenomena are described as they appear to one observer or as they appear to another. This single principle is the generating motive of the whole theory of relativity.[11]

Essentially, Einstein does not formulate new laws of motion; he merely reformulates those of Newton within the new context of a four-dimensional space-time continuum. At first glance, this may not appear to be a very significant change, but when we reflect that it took Einstein eleven years to set out the "General Theory of Relativity" after he first published the "Special Theory" in 1905, we realize that the battle to retain and reformulate Newton's laws was not easily won.

The merger of the concepts of space and time into Einsteinian 'space-time' was challenging enough on a mathematical level but it was even more difficult on a philosophical level. Whilst it isn't very difficult conceptually to think in terms of 'world-lines' composed of 'events' that can be described mathematically as points in a four-dimensional co-ordinate system, it does require a more disciplined intellectual approach to move from a physics based on the idea of time as an independent variable to a physics which incorporates time into the relational structure of space. In relativistic physics, the space-time manifold actually assumes an active role and what is important about this shift in scientific thinking is that it allows for a radically new idea of motion whereby space-time is participant and not spectator. On an epistemological level, this corresponds to an equally radical conceptual shift because it becomes apparent that only a dynamic understanding of reality as a systematic unity can now be considered philosophically adequate.

In his book, The Philosophical Impact of Contemporary Physics, Milic Capek stresses that the **dynamization of space** should not be misconstrued as the **spatialization of time**[12]. What Einstein has accomplished is to interpret the mathematics of a four-dimensional space-time in such a way as to harmonize with the facts of everyday, empirical, physical reality. Space has been dynamized in the sense that it is now regarded as participating in motion, but although the passage of time will be measured differently according to the velocities of the reference frames in which the measurements are taken, time has not been spatialized. Thus, the paradoxes associated with time travel remain a subject for playful philosophical speculation and science fiction stories.

Einstein's 'space-time' is a purely conceptual ordering of events. It is meaningless to speak of moving either forwards or backwards in time because space-time is nothing other than the manner in which the universe is currently ordered. At any given instant we can conceptually 'stop the universe' to look more closely at the precise distribution of matter/energy momentarily constituting space-time, but the static model is now recognized as a pure thought-construction. The material bodies that comprise the physical universe are represented conceptually in a complex system of relations and space-time is the whole system conceived of as an irreducible unity. Note the congruity between this view and that expressed by Cassirer in Substance and Function:

> Space is by no means a static vessel and container into which ready-made "things" are poured; it is rather a sum of ideal functions, which complement and determine one another to form a unified result.[13]

It is evident that the above-stated view of space-time accords so well with Cassirer's epistemological outlook because it explicitly assumes a relational view of scientific knowledge, but what interests Capek, however, is the physical implications of the 'dynamization of space'. Gravity is no longer attributed only to the bodies occupying space, gravity has become a property of space itself. Conceived of as a field, space-time is 'curved' in regions of high gravitational force, and given the tendency of light (and all bodies, in general) to follow the path of least resistance (which coincides with the geodesic in this case), space becomes dynamic in that it is the cause of motion. Whether space, or more aptly 'space-time', is the ultimate cause of motion, or the final cause as Aristotle would have it, is another question entirely, but space is certainly an effective cause. Objects which are moving in space follow the precise paths they take through a sort of universal cosmic laziness; gravity is merely another way of saying that it is easier to bend with the curvature of space than to fight against it.

What was formerly explained as a consequence of the gravitational attraction of neighbouring bodies is now conceived of as the natural result of space-time's local topography. Thus, as far as empiricism is concerned, gravity (therefore space-time) is a cause of motion; as far as knowledge is concerned, space-time is the rule governing motion. In the mathematics of space-time, a motion is simply a process whereby a physical object endures for an interval. The interval can be 'time-like', 'space-like' or 'light-like'. A 'time-like' interval is one in which the time part of the interval dominates over the space part, a 'space-like' interval is one where the space part dominates over the time part, and a 'light-like' interval is one in which the time part equals the space part[14]. In all cases, however, physics considers motion purely as a change in space-time co-ordinates. Thus, Cassirer sums up the situation quite succinctly when he writes:

> Motion in the universal scientific sense, is nothing but a relation into which space and time enter.[15]

Along with his insightful analysis of the dynamization of space, Capek also makes remarks concerning the so-called 'relativity of simultaneity'. For, the theory of relativity introduces not only the notion of the relativity of reference frames, but also the relativity of simultaneity. As Capek says, "...absolute space and absolute simultaneity imply each other; the denial of one implies the denial of the other."[16] Whereas it had formerly been taken for granted that two events which an observer perceives at the same time must also have occurred simultaneously, the theory of relativity now casts doubt on this common-sense inference. Perhaps the time taken for the knowledge of the occurrence of the events to reach our perceiving apparatus is different. Thus, if a star should explode at a distance of exactly ten light years away from us and we were to perceive the explosion at the exact time that the bell in the square chimed for midnight, we would be mistaken in inferring that the two events occured simultaneously because in actual fact they are separated by a time interval of ten years. To a differently stationed observer, the two events would not appear to be simultaneous at all, and the same is true of any two events occurring in different regions: some observers will say that the events are simultaneous, some will say that **A** happened before **B**, and some will say that **B** happened before **A**. Not only is simultaneity relativized, but succession (though not causation) is also made to depend on the observer's frame of reference.

Finding a solution to the problem of causation is an essential prerequisite of any scientific theory. If event **A** is to be the cause of event **B**, then logic dictates that there must be a corresponding order of succession between the two events - i.e., event **A** must always occur before event **B** or, a 'time-like' interval must separate the events. In point of fact, this logic errs in not recognizing that cause and effect might also be simultaneous, but at any rate, the general point is that effect can never precede cause. Now, suppose that in the previously given example, the bell in the square is equipped with a special mechanism which will make it chime when the light from the exploding star hits it. Is it possible for the effect (i.e., the bell chiming) to ever precede the cause (i.e., the star exploding)? Is it possible for the two events to be perceived as simultaneous? The answers are: no, because the bell can only ring when the light strikes it and no known physical cause can travel faster than light; and no, because there will inevitably be a delay in the transmission of the impulse received from the light ray to the mechanism which rings the bell. An observer with ideal measuring instruments would infallibly note the occurrence of the cause before the effect. An uninstructed observer might have a distorted perception of cause and effect in the sense that he might fail to connect a cause ten light years away with an effect in his immediate neighbourhood, but this is another problem.

The relativity of simultaneity addresses a different question than cause and effect. Analyzed in terms of causation, every event has an 'absolute past' (its causes) and an 'absolute future' (its effects), but events which occur in different regions of the universe and are causally unconnected are a problem. Suppose the bell (without its photoelectric mechanism) chimes an hour earlier at 11:00 pm. An observer on earth will now think that the bell has chimed before the star exploded, but an observer one light year away from the earth and one light year closer to the star will think that the star has exploded long before the bell chimes. Whose perception will be correct? We can only say one or the other if we imagine an ideal reference frame as a standard of comparison, but this is precisely what the theory of relativity asks us not to do. What we must do is to say that both observers are correct within their own time frames and that a third observer stationed at the exact appropriate place would also be correct in thinking that the two events were simultaneous. Within the theory of relativity the analysis of simultaneity leads to the conclusion that it is relative and to the further conclusion that 'before' and 'after' are also relative in all cases where events are not causally connected (i.e., where the interval between the events is 'space-like' rather than 'time-like'). For all practical intents and purposes, however, all of these concepts make perfect sense to a fellow earth inhabitant because the differences between any two people's reference frames is so infinitesimally small relative to the size of the universe, that no Lorentz-transformation has to be applied in order to regularize our perceptions. Simultaneity being the most precise of the three concepts (before, after, at the same time), it is possible that earth observers stationed far enough apart might disagree over their interpretations of seemingly simultaneous events, but this is merely a clue which helped Einstein conceive of his theory, not a skeptical argument against the trustworthiness of our senses. The only real simultaneity is when cause coincides absolutely with effect, as when thought moves; and the only real 'before' and 'after' are the past and the future of the universe regarded from the ideal standpoint of history.

While we recognize the truth of Cassirer's assertion that "...there is no field of philosophy or theoretical knowledge into which the problem of space does not in some way enter and into which it is not interwoven in one way or another,"[17] we also have to bear in mind that the purpose of this discussion is to analyze the concepts of space and time as physical concepts. The questions of causality and historical knowledge are tangentially related to the present consideration but the principal focus of this study is Cassirer's interpretation of scientific knowledge and the nature of that knowledge itself. We have seen that the theory of space and time implied by Einstein's relativity is completely compatible with modern philosophy and there can be no doubt that Einstein's marvellous physical intuitions further epistemological analysis in all fields. It is primarily when we think on an interstellar or subatomic scale that the theory of relativity really applies, but history reveals that the greatest thinkers have been those who were able to turn their attention to the incalculably vast or the infinitesimally small. The consideration of the extremes is often the best way to understand the mean as Aristotle has demonstrated.

Chapter Seven

Matter and Energy (Substance and Function)

The consideration of space and time takes us directly into the next topic, which is the view of matter and energy set out by the theory of relativity. It is truly amazing to reflect that the choice of the geometry employed by the physicist can have such radical consequences for the physical theory evolved, but this is because it is so difficult to rid ourselves of the substantial concept of space as a thing. It cannot be stressed often enough that Einsteinian space-time is a pure system of functional relations and that it has no thing-like status of its own. The equation $E=mc^2$ follows as a theorem of Einstein's physics as surely as the Pythagorean theorem follows from simple Euclidean geometry*. Once the ramifications of a conception which makes space into a pure field of forces are fully worked out, we realize that not only space but also matter has been defined in terms of function. The link between matter and space is therefore on an epistemological level rather than anything else, for as Capek writes, referring to the

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*See From Pythagoras to Einstein, by K.O. Friedrichs for the mathematical proof of the derivation of Einstein's $E=mc^2$ from Pythagoras' $c^2 = a^2 + b^2$.

so-called 'curvature of space':

> It is wrong to say that matter "causes" the corresponding curvature in space, as is sometimes vaguely suggested, for there is no relation of causality between matter and a local warping of space. The relation is one of identity: matter and local curvature of space are one and the same reality.[1]

Just as space is the set of relations obtaining amongst the things 'in' it, matter is the way space behaves in a localized region. In other words, the very 'things' that comprise physical reality are, for the purposes of modern physics, nothing more than parameters or limiting factors whose role consists in connecting the mathematics of theoretical physics with our system of empirical measurements. As William Werkmeister observes:

> Mass, however, is an indispensable factor in all calculations through which the principles of pure mechanics are specifically applied to concrete physical situations. Mass ties the equations to physical "reality".[2]

Thought of in abstract terms, a mass is merely a coherent body of matter in space; but physics immediately conceives of mass as functional so that, from a physical perspective, we speak of inertial mass or gravitational mass. Prior to Einstein, these concepts were thought to be regulated by different laws, but in achieving a higher synthesis, the theory of relativity brought both inertial mass and gravitational mass under the same law.

There is no need to delve into the manner in which this synthesis was achieved at present; the more pressing concern is our exploration of the idea of mass as a physical quantification of matter. Mass is that physical concept which allows for the reduction of a material thing (a substance) into a functional relation (a specific number of units in a system that correlates units). The meaning of the units - be they kilograms, pounds or anything else - is determined by reference to an arbitrary standard; the number gives the thing its position in a mathematically ordered series. Thus, the unknowable and substantial thing-in-itself is related to the totality of other things through the determination of its mass. Of course, this is not the only way of relating the thing to other things, but for physics mass is extremely important since it translates into energy.

In fact, in physics matter has no other meaning than energy. Einstein remarks that, "The quantity of heat able to convert thirty thousand tons of water into steam would weigh about one gram."[3] Think what it means, therefore, for a man to consume a 16 oz (454g) steak in a single sitting. It is enough energy to hurl a boulder into orbit! Now, leaving aside the obvious point that the matter composing a steak is not plutonium and that the human digestive system is not a nuclear reactor, the general implication is clear enough: matter can be conceived of as enormous quantities of energy abiding in a relatively stable state. Furthermore, matter obeys the same laws as those obeyed by energy. The law of conservation of mass is made redundant by the law of conservation of energy. If the 'principle of equivalence' is correct (i.e., if gravitational mass is equivalent to inertial mass), then the mass of a body can always be described in terms of the gravitational field it generates or in terms of its reaction to the gravitational fields generated by other bodies. Even light, which was formerly regarded as 'pure' energy, is now subsumed under the category of things affected by gravity and, indeed, all forms of energy obey the universal law of gravitation. For, as Einstein and Infeld say, "Energy was regarded as weightless for so long simply because the mass which it represents is so small."[4] We should be careful to note, however, that weight (as opposed to mass) is not a property intrinsic to matter itself, but a relational concept which varies depending on the strength of the gravitational field the body happens to be occupying. Thus, while the rest mass of a photon is zero, it nevertheless has weight in the sense that it is subject to the influences of gravitational fields.

Mass is a fundamental concept in that it allows for the quantification of matter so that physical laws can be represented in mathematical form. Furthermore, mass can conceptually be converted into energy according to the formula $E=mc^2$, and this allows for even greater mathematical precision insofar as the field theory is superior to mechanical models in many respects. It might be objected that energy could equally well be conceived of as the substance of a field, and that, therefore, no metaphysical progress has been made. But this argument misses the point. Even if the field theory does not permit us to evade the metaphysical dilemma regarding the ultimate 'stuff' of which reality is composed, it does advance our enquiry by illuminating the fundamentally complementary nature of substance as energy-matter. The fact that an equivalence has now been established between matter and energy means that a functional metaphysics can now integrate a wider range of phenomena into a unified theory of being.

The transmutability of matter into energy does not appear to be reciprocal, however. This is what scientists refer to when they speak about the irreversibility of physical processes or the possible eventual entropy death of the universe. Alternate theories have been advanced in order to explain this irreversibilty, but none are very satisfactory philosophically. It seems evident from a historical standpoint that past events are irreversible, but whether all the matter in the universe will eventually be converted into energy and spent as work in attaining a universal heat equilibrium is another question. Two assumptions are at work here: Firstly, it is assumed that theory which holds that all matter is possibly convertible into energy is fully justified by empirical reality; and secondly, it is assumed that matter is not essentially different from energy in some respect (perhaps in respect of the fact that matter characteristically has form, perhaps in some other respect also). It is here that we have to remember that we are dealing with a physical theory designed with a particular motive and not with a metaphysical explanation of the nature of the universe couched in terms of the new wonder-substance, energy. On the one hand, Einstein and Infeld admit that the attempt to formulate a pure field physics has not yet been successful and that the concept of matter is still a necessary concept[5]; and on the other hand, Cassirer recognizes that substance is also a necessary concept for empirical knowledge[6] even though he drives towards a completely functional interpretation of substance in his metaphysics and his epistemology.

What is important is that, from a physical standpoint, energy behaves like matter[7]. We are not concerned with a static system of mass-points, we are dealing with a dynamic universe of 'events'. The energy field is a function of the electromagnetic and gravitational fields exerted by the matter composing physical reality. Events are motions, not with respect to the field, but **of the field** in the sense that the field function varies with the motion of the elements of the field. The metaphysical understanding of a succession of events (a "world-line"[8]) as matter enduring throughout a time interval (or as a being being) is not useful to physics insofar as it fails to eliminate the substantial notion of the thing. As Einstein writes, "Considered logically," the concepts of space, time and event are "free creations of the human mind."[9] The physical perspective is eminently pragmatic: A concept must be useful or else it is discarded. Accordingly, events are the behaviour of matter, or alternatively and because it is easier to describe in mathematical terms, the fluctuations of the energy field. Whether we describe the thing as a "string of events," as Russell does[10], or whether we speak of the energy density of the field in localized regions as Einstein does, we are adopting Cassirer's fundamental method in respect of the material thing - we are "understanding and evaluating it as a concept."[11]

The transmutability of matter into energy in physical theory is at an epistemological level because the physicist wishes to integrate both aspects of phenomenal reality into the same set of mathematical equations. The law governing the distribution of mass-points in space is replaced by a functional analysis of the distribution of energy in the field. That the analysis of reality in terms of energy functions should lead to more refined laws of even greater generality than Newton's laws is an essential prerequisite for the acceptance of the theory of relativity; but this condition is met, for the theory of relativity is a more generally applicable theory and, consequently, Newtonian physics becomes a limiting case. In the words of Einstein and Infeld, "any generalized concept must reduce to the original one when the original conditions are fulfilled."[12] With respect to the 'limiting case', Einstein writes:

> No fairer destiny could be allotted to any physical theory, than that it should of itself point out the way to the introduction of a more comprehensive theory in which it lives on as a limiting case.[13]

It is granted that a unified field theory has not yet been achieved and that not everything can be explained in terms of energy functions, but this does not mean that the theory is wrong; it is better to say that physical theory is incomplete. One explanation for theory's inadequacy is Niels Bohr's "complementarity", another is that physics awaits a new conceptual breakthrough; but in any case it is clear that Cassirer's basic idea is being realized: physical theory is striving to find laws capable of ordering observable physical reality on a conceptual level. As Cassirer writes:

> The concepts with which he (the physicist) operates, the concepts of space and time, of mass and force, of material point and energy, of the atom or the ether, are free "fictions". Cognition devises them in order to dominate the world of sensory experience and survey it as a world ordered by law.[14] (my insertion)

Bohr's "complementarity" deserves some explaining but we will only discuss it in a cursory manner at present since a full analysis of the topic would plunge us into the debate about quantum theory and the nature of light. Stated in an oversimplified form, the theory of complementarity amounts to an assertion that the explanation of phenomenal reality is only possible in dualistic terms and that a necessary inexactitude must always prevail in our description. Thus, the concept of matter is not made entirely redundant by that of energy, nor is the concept of substance made useless by function. The equation $E=mc^2$ does not purport to do this. It merely states that, for very special purposes quantities of matter can equally well be described as quantities of energy. Bohr's complementarity asserts that both descriptions are meaningful and this assertion is not in conflict with the Einsteinian attempt to formulate a theory which seeks to work within a unified perspective as much as possible. The relativity of all reference frames does not clash with the theory of the field because relativity is always the last word in the interpretation of 'events' within the field. Accordingly, it is recognized that no single perspective can ever be absolutized, but it is also clear that a general perspective can only be generated through the close analysis of individual perspectives.

Energy is a more useful concept than matter in that it can more readily be subsumed under a mathematical law. As Max Von Laue writes in his article, "On the Structure of Our Universe", (Albert Einstein: Philosopher-Scientist, The Library of Living Philosophers, Volume VII): "The concept of mass, formerly a basic concept of physics and a measure for the quantity of matter as such, is demoted to a secondary role."[15] Since energy is also, conceptually, a measure for the quantity of matter, it allows for the formulation of a more general mathematical law. The attainment of the more general mathematical law is the whole reason for the theory of relativity's success as a physical theory. We could equally describe energy in terms of matter, but this desciption would not allow us to escape the mechanistic interpretation of reality and to leave behind the concept of a force acting at a distance. There is no 'empty space' between bodies whose gravitational and electromagnetic fields are interacting. The medium of the transmission of the once mysterious 'force' is the field itself and the force is an energy function arrived at by assessing local disturbances or fluctuations of the field. It is admitted that the concepts of matter and energy are, in the final analysis, as mysterious as the concept of force, but the epistemological improvement consists in the increased parsimony of concepts and the recognition that these concepts are created because they are helpful and because they lend coherence to the scientific theory and not because they literally represent real things. 'Force' is no longer necessary because the description of reality in terms of energy functions makes the field (or perhaps more accurately, the fields - electromagnetic and gravitational) the explanation of motion.

This point needs to be made clearly: The concept of force is not rendered obsolete by that of energy, but its role is transformed. The ghost which formerly inhabited the materialist machine now inhabits the universe of vibrations. The universe of vibrations is a more mathematically elegant reality than the machine, but its most basic presupposition is in common: something moves the parts of the universe. Both the mechanical viewpoint and the energeticist viewpoint reject at the outset the idea that motion could be an illusion. Things appear to move therefore they move. Intuition has to be trusted at least this far; but the reasoning physicist does not stop here. Things also have to be intuited - things <u>do exist</u> independently of their perceivers. The things may be intrinsically unknowable by their perceivers and there may be all sorts of illusions about the things caused by insensitivity in the perceiving apparatus being employed, but the things do exist. They are made of matter and they generate space in the form of a field of energy functions. Motion gives us time, and accordingly, physics works in space-time relying on its axiomatic metaphysical base. The whole explanation is very neat, but it deserves to be remembered that physics is a theory that the physicist creates and that the reality which physics is an interpretation of is a reality mediated through the senses. This is why Cassirer writes:

> The reality of the physicist stands over and against the reality of immediate perception as something through and through mediated; as a system, not of existing things or properties but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coordinations and dependencies of phenomena.[16]

The discussion of the concepts of energy and matter hits rock bottom when it becomes aware that they are intellectual symbols. The substantial question is left unanswered but the question of function becomes more pressing. What is happening? How does the universe function? The physicist speaks of matter, energy and motion. He cajoles reality into obeying his physical laws and he explains reality in terms of this same law. Naturally, reality does not always obey, nor is it always explainable. This is a problem for physics, but it is not an insurmountable one. The physicist recognizes the arbitrariness of his enterprise but he can also point to the statistical verification of his laws. Even the statistical falsification of a law is useful in that the old law's demise may pave the way to the new law's ascendance. The law is what is important in physics and it is a functional law. To deny that one is presupposing anything is pointless; what has to be done is to be clear about one's presuppositions. Substance is presupposed, function is the mathematical, philosophical and physical mode of knowing substance. To put it even more generally, function is how we know substance. Similarly, matter is presupposed and energy is how we know matter. It is significant that Cassirer quotes the German chemist Wilhelm Ostwald on this point. Cassirer obviously agrees that energy is the medium of our knowing in physics. Thus, in The Problem of Knowledge, we find Cassirer elaborating his underlying thesis with the help of the following quotation from Ostwald:

> What we hear originates in the work done on the eardrum and the middle ear by vibrations of the air. What we see is only radiant energy, which does chemical work on the retina that is perceived as light. When we touch a solid body we experience mechanical work performed during compression of our fingertips and, in suitable cases of the solid body itself ... From this standpoint the totality of nature appears as a series of spatially and temporarily changing energies, of which we obtain knowledge in proportion as they impinge upon the body and especially upon the sense organs fashioned for the reception of the appropriate energies.[17]

The parallelism between **matter and energy** and **substance and function** is very clear. On the one hand, matter and energy are the basic concepts needed by the physicist, and on the other hand, substance and function are the metaphysician's basic concepts. In each pair of concepts the latter is the explanation of the former; or to put it in other words, the latter is the law by which the former is deemed to be governed. The human mind creates the concepts in order to interpret reality. Sense perception, left to itself, could not order reality. For this task reason is required and intuition also. Matter, like space, is a pure form of intuition coming to us from sensation. Reason tells of the existence of energy although energy itself is never directly perceived, but always secondarily, through its effect - work. Assume that light is pure energy. Do we perceive light directly or do we perceive things because light causes us to see through the work it does on our eyes? It is obvious that the latter explanation only is compatible with empiricism; similarly, on an epistemological level, we notice that the concept of energy is the intellectual illumdnation of the concept of matter. That which was formerly regarded as inert is now conceived of as being in constant activity.

Energy is that which moves matter and motion is what is perceived. The supposed solid object which we do not perceive to be in motion is, according to our scientific and empirical knowledge, a mass of particles some of which are moving at extremely high velocities. The principle of conservation of energy and the other laws governing the behaviour of energy enable the physicist to have an understanding of the motion of matter which surpasses all intuition. Solidity can be defined in terms of conformity to certain laws - that is, functionally or relationally; but the intuitive notion of solidity in terms of unmoving substance cannot be subsumed under a law because it is a metaphysical basis upon which the functional law rests. What is known metaphysically is nothing, as Socrates first emphasized; but what is known in physics is that the law works and that it consistently explains phenomenal reality. In relation to a pillow, a desk is relatively solid. In relation to a desk, a granite block is also relatively solid. The absolutely solid object is unnecessary as a standard to which other solid objects can be compared because any standard at all can be chosen at the scientist's convenience. In fact, the absolutely solid object is an idealization extrapolated from the world of experience by taking the law defining solidity to its extreme limit. What is important epistemologically is not the absolute, but the law defining the absolute. Once the law is given, the absolute follows from it as a necessary consequence just as the whole range of intermediate values are corollary necessary consequences also. Lord Kelvin's absolute zero ($-273.16^{0}C$) for instance, the temperature of the ideal, perfectly rigid body whose parts know no motion, is an absolute arrived at by discovering the law capable of ordering the diverse phenomena of thermodynamics. Absolute zero is, in principle, unattainable yet modern research in the area of super-conductivity has unearthed experimental results of the greatest practical importance to designers of electromagnets and computer components. Thus, we see in this example how it is that the functional law allows for the possibility of passing beyond the narrow confines of sense perception so that physical knowledge can touch the very limits of what is knowable.

The material 'thing-in-itself' remains an enigma and the description of reality in terms of energy functions is never quite exact. However, this does not deter the physicist, nor the philosopher. The search for laws of ever-increasing generality continues in spite of the imprecise nature of the subject matter. What Cassirer hoped to accomplish in metaphysics has been accomplished by Einstein in physics. In both sciences, to know is to relate. In metaphysics, substance is known in terms of the functional relations we generate in order to understand it and in physics, matter is known by the behaviour of the energy field. The price physics has to pay for the mathematicization of the concept of matter is a permanent artificiality and a limitedness of perspective, but what physics achieves is a knowledge synthesis at an extremely high level. For all practical purposes then, physics can be said to fulfill its objective and the physical project is pronounced a success insofar as the field theory set out in Einstein's General Theory of relativity successfully explains phenomenal reality in greater depth and detail than has hitherto been possible. From the point of view of philosophy, however, the metaphysical project is not yet successful until other perspectives besides that of physics have been surveyed and related. Acccordingly, even in recognizing the importance of the theory of relativity, and in penetrating the meaning given to the concept of matter by Einsteinian physics, Cassirer still cautions us against the dangers of a facile metaphysics:

> Whether we characterize the ultimate being as "matter" or "life", "nature" or "history", there always results for us in the end confusion in our view of the world, because certain spiritual functions, that cooperate in its construction, are excluded and others are over-emphasized.[18]

Conclusion

This study began by reflecting on the relationship between philosophy and science. Whereas, in antiquity, scientific knowledge was thought to be subsumed under the broader category of philosophical wisdom, it is now widely held that modern science is quite distinct from philosophy. We are here concerned to show that such a separation need not exist. Thus, Cassirer's philosophy was selected as the focal point of this study because Cassirer believed in the fundamental unity of knowledge and because Cassirer exhibited a depth of familiarity with the many different branches of science that has become increasingly rare amongst philosophers. On one hand, philosophy has to struggle to understand itself and to define what philosophy is and, on the other hand, philosophy has to integrate scientific knowledge and empiricism into its own self-understanding. While recognizing that the era of the great constructive programs is over and gone, Cassirer nevertheless insists on the importance of philosophy's role in surveying, synthesizing and condensing the diverse knowledge claims of all of the various sciences. Philosophy has a very intimate relationship with physics in particular. Conclusions about the nature and structure of the physical universe were the first impetus philosophy received, and the history of philosophy is a continuous attempt to re-solve this problem. The theory of relativity speaks loud and clear: physics is embarking on a new program of self-understanding. Philosophy must try to keep pace and the new insights physics unearths will be food for philosophical thought also, just as philosophy's search for self-understanding will inevitably stimulate physics.

When we observe that physics is only one perspective among many, the theory of relativity is itself relativized. Physics openly acknowledges the objective of reducing the phenomenal world to a coherent law-abiding system of energy functions determined by strict laws of causality, but there are other perspectives from which phenomenal reality can be viewed also. Cassirer is willing to enter fully into the spirit of physics and to concern himself with the physical concept of causality, yet he is still able to reflect that other modes of understanding reality also have their own intrinsic validity. Thus, for example, consciousness remains an enigma for physics, and while it is possible to consider reality exclusively from the point of view of the interaction of forces and of the disposition of energy-matter in space-time, it is also important to be aware of the essential limitedness of the physical perspective even as we work within it.

Cassirer notes that what physics is really promoting is a particular concept of order. Teleology assumes that order is intrinsic to reality, but it is not this concept of order with which physics deals. Order, in physics, refers to the systematic interrelation of concepts which it is the task of the physical theory to provide. Expressed in their most crystalized form, these concepts give rise to the 'laws of nature' which are the heart of physics. Man defines nature and he orders it conceptually; in this respect, the theory of relativity is the most generally applicable physical theory yet advanced because it orders the greatest amount of available information. The more abstract the concept, the more it compresses scientific knowledge. The more scientific knowledge can be compressed, the greater the explanatory potential of the physical theory. Thus, we can see that Einstein's theory of relativity is an illustration of the broader philosophical thesis Cassirer is developing.

Kant asserted that, "... in every special doctrine of nature only so much science proper can be found as there is mathematics in it." Mathematics is the science of pure form: In number, we have the distilled essence of being and in geometry, concepts are manipulated freely as absolute idealizations. A point is indivisible and occupies no space; a unit is infinitely divisible and capable of generating all other numbers. Thus, mathematics allows for the formulation of principles and laws. The mathematical function can entertain as many variables as need be and still continue to perform its task of assigning to each element of the knowledge situation a specific place in a serial order. For Einstein, this is important because every observer can invent a co-ordinate system appropriate to his unique frame of reference yet the results of observation can be transposed from one co-ordinate system to another with the utmost ease. For Cassirer, mathematics is important because it is the means by which knowledge can be systematically organized and compressed. Cassirer's epistemological convictions are confirmed when the theory of relativity modifies geometry's claim to have an empirical foundation and announces that space is only bounded by the limits of what is conceptually possible. It is stressed that we must be aware that the theory of relativity is exactly what it purports to be - a theory. As Leibniz first pointed out, the discussion properly begins with the consideration of what is possible; and as Hume recognized, empiricism will tell us what is probable.

From an epistemological point of view, the theory of relativity is important because it has so much to contribute to the discusssion of concept formation. This study has assumed that Cassirer's, and by extension, Kant's, analysis of concept formation is correct: The mind actively creates the concepts it finds suitable for the task of ordering phenomenal reality. The thesis that our concepts correspond to reality, as it actually is, is explicitly rejected. Whether or not our concepts correspond to reality is, in principle, unknowable; what is more, the question is largely irrelevant. Ultimately, the question of correspondence may be vital, but for the practical purposes of science it is not the truth or falsity of our concepts that matters, but their utility.

Concepts are tools, they are used to create the possibility of a synthesis of knowledge. In epistemology, as in physics, it is important to strive for economy of concepts: The simplest ideas are the most profound. The symbolic forms in which our concepts are expressed are reminiscent of the Platonic Forms, the self-subsisting Ideas. But there is this difference: ideas are not more real than things in any metaphysical sense. Ideas are the reality the mind knows, the symbolic forms are communally shared ideas. It is because it is possible for us to create these symbolic forms that any communication of ideas is possible at all. As our language increases in sublety and nuance the symbolic forms become more and more powerful, so that a wealth of knowledge can be packed into a short formula such as : $E=mc^2$. Epistemology learns this lesson from Einstein.

The metaphysics of substance and function is the heart of Cassirer's philosophy. What is is substance, what is known is function. Metaphysics is impossible in the sense that being *per se* has no rational explanation; but metaphysics is a philosophical project when being is analyzed in terms of the meaning of being. This takes us back to the problem of causality. Causality has to be assumed because knowledge is not competent to go beyond that assumption except in a mystical mode. The meaning of being, however, can be interpreted scientifically when metaphysics is the inquiry into causal relations. The elements of the knowledge situation are the creations of the mind. The study of the function of the various elements and the subsuming of these elements under general laws is the work of the metaphysician. As the name suggests, metaphysics is prior to physics. It is prior in the same way that intuition is prior to rational thought. First there is sensation, then there is intuition, then there is concept formation and lastly, rational thought. The metaphysical fallacy of equating the intuition with the thing itself has always to be guarded against, but without intuition, the physicist's hands are tied.

Einstein's theory of relativity needs to be situated in a philosophical context. Since the time of Newton, science has been making very rapid progress but it is essential that philosophy should keep pace by continuing to interpret and integrate new knowledge claims. Thus, Cassirer's work proves itself to be invaluable, for he has well-reasoned answers to the questions raised and he is able to bring a historical perspective to bear on the problems encountered. Once the flaw in the thinking of Mach or Ostwald has been pointed out and once the innovation of the thought of Hertz or Pointcaré has been accentuated, the possibility of intellectual illumination becomes less remote and the material takes on new meaning. For Cassirer, a figure such as Galileo can even be an example both of what is good and what is bad about the method of physics and Cassirer gracefully gives credit where credit is due while exposing errors that are apt to lead into blind alleys. Philosophy is not judge in the court battle betwen competing knowledge claims, it is mediator in the dispute over division of labour.

Einstein, like Cassirer, learns from the mistakes of his predecessors and finds new ways of making competing theories compatible. Newtonian space was inadequate to the task of accounting for the diversity of observed phenomena. Thus, Einstein had to rid the concept of space of its last remnants of substantiality and reformulate the laws of physics within the context of a more generalized concept of space. Time, too, had to be integrated into the new conceptual framework and, consistent with Cassirer's proposal for a metaphysics of function, space-time was analyzed in terms of the dynamics of possible motion. The field theory was an attempt to provide a unified explanation of motion in space-time, but Einstein was not successful in bringing physical knowledge to so complete a synthesis. The elegance of his proposed solution has to impress even the most ardent critic of his epistemology, but the fact remains that too many discrepancies exist for anyone to put much faith in this particular course. Nevertheless, it is always possible that a missing piece in the conceptual jig-saw puzzle may be supplied by physicists of the future. Knowledge advances slowly; sometimes it is necessary to take a step backward in order to take two steps forward.

The theoretical equivalence of matter and energy is an outstanding example of the way in which Einstein is able to simplify and synthesize the basic concepts of physics. Closely allied to this synthesis is the further synthesis of the concepts of inertial mass and gravitational mass. Thus, it is perhaps true to say that Einstein's greatest accomplishment is in the epistemological domain. While the concept of matter is still a necessary and useful concept to the physicist, it is superseded in its application by the more mathematically workable concept of energy. The energy field is not only defined by a system of equations, it is actually constituted by these equations. Strict mathematical laws govern the fluctuations of the field and when physical reality is described solely in terms of energy functions, the rationalist ideal of a universe ordered by law appears within reach. But even on an epistemological level, determinism eludes us. The field theory is incomplete, but what is more disturbing is the fact that empiricism reports that anomalies and exceptions to physics' 'universal laws' crop up on all sides. The point Cassirer makes is that matter and energy are to physics what substance and function are to metaphysics. The former in each pair is the indispensable underpinning of the latter; but it is only the latter which is the proper subject of analysis because our knowledge is of relations and not of things.

It should be stressed that in assessing Cassirer's philosophy and his treatment of the problem of scientific knowledge, we do so from a historical perspective. Given that theoretical physics is necessarily a highly formalized and abstract system of thought, it is no surprise to find that Cassirer's philosophy is correspondingly difficult to grasp at points. Nevertheless, it is the task of a philosophical study to make things clear and to situate the physical theory within a broader understanding of knowledge in general. Insofar as Cassirer failed to do this and insofar as some issues may even have been confused rather than clarified, Cassirer's work is deserving of criticism. Obviously, the same is true of this study itself and points at which Cassirer's thought has been presented in an awkward manner are doubly regrettable in that we owe it to a philosopher to evaluate him on the basis of his own work rather than on the basis of what others have written about him.

The epistemological similarity between the thought of Cassirer and that of Einstein is so worthy of notice as to be interesting in itself. The reason why Einstein's theory of relativity has had such a powerful impact is that the fundamental conceptual alterations which characterize Einstein's theory are widely recognized and acknowledged as improvements to the hypothetical-theoretical structure of modern physics. This reflection lends credibility to the further thesis that Cassirer's philosophy ought to have an equally powerful impact. When we analyze knowledge in term of the functional relations implied by the manner in which our concepts have been selected, we discover that many fertile fields of research have become accessible. The mind's role in concept formation, is essentially an active and creative one; therefore, reality has to be reconstituted in order to make sense of the theoretical framework that we construct. The task of reconstituting reality according to a specific set of theoretical precepts is work in which both the physicist and the metaphysician must participate. While the physicist seeks to explain and to predict, the metaphysician is in a constant process of evaluating and re-evaluating the theoretical precepts upon which the physicist's reality is founded.

The importance of Cassirer's work derives from his realization that the process of checking our intuitions against the phenomenal reality that we experience is philosophically problematic. Not only are there a wide variety of different perspectives within which our intuitions can be meaningful, but the metaphysical thing-in-itself with which theory concerns itself is objectively unknowable. Cassirer's functional approach to metaphysics allows for the possibility that a theory about physical reality can remain coherent and useful even though it explains appearance rather than reality itself; and, on the other hand, an epistemology that stresses the relational character of our knowledge successfully sidesteps the problem of the inherent subjectivity of individual perspectives. Thus, Einstein's theory of relativity can be regarded as the physical theory logically consequential upon Cassirer's epistemological stance in that space and time are now defined as relational rather than absolute concepts. Furthermore, the theoretical equivalence of mass and energy implies that the substantial concept of matter can be treated functionally in terms of either the mechanics of mass-point interactions or of energy field fluctuations. The reader is encouraged to turn to the works of Cassirer himself for a more elaborate and fully developed treatment of these and other themes.

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3) Ian Hinckfuss, The Existence of Space and Time, (Oxford: Clarendon Press, 1975) p.24

4) H.G. Alexander (ed.), The Leibniz-Clark Correspondence, (Manchester Univ. Press, 1956) The Third Paper

5) Cassirer, The Problem of Knowledge, p.104

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9) Russell, ABC of Relativity, p.18

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13) Cassirer, The Philosophy of Symbolic Forms, Vol. I: Language, p.101

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16) Cassirer, <u>Substance and Function and Einstein's Theory of Relativity</u>, p.357

17) Cassirer, <u>The Problem of Knowledge</u>, p.100

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