Computer Aided Synthesis of Optimal Multi-Speed Gear Drive Designs — Using Problem Reduction Strategy

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A Thesis

in

The Department

of

Mechanical Engineering

Presented in Partial Fulfillment of the Requirements
for the Degree of Doctor of Philosophy at
Concordia University
Montréal, Québec, Canada

August, 1987

ISBN 0-315-37078-5

ABSTRACT

Computer Aided Synthesis of Optimal Multi-Speed
Gear Drive Designs
Using Problem Reduction Strategy

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Concordia University, 1987

A method for the synthesis of optimal multi-speed gear drive designs is presented incorporating kinematic requirements, component strength requirements and location of eigenvalues. The synthesis is accomplished through a problem reduction optimization strategy which efficiently determines the optimum values for diameters and faces widths of all gears. Conventional, single composite and double composite arrangements can be analysed for all layout diagrams.

General kinematic equations to define the diameters of all gears in a parallel axis multi-speed gear drive are developed. These equations apply to single composites and double composites as well as the conventional arrangement. Equations for the diameters of gears for a particular kinematic arrangement are readily obtained from the general equations, given the layout diagrams and the types of arrangements of interest (i.e. conventional, single composite or double composite). Since the equations are based on the layout diagrams, which are mathematically formulated they are easily computerized.

A mesh optimization strategy to rate components is presented for standard spur gears which determines the optimum module of a mesh as a function of gear ratio and pinion speed for any specified set of operating conditions. This analysis is used to generate a minimum diameter matrix which is used during the arrangement optimization to

ensure that required component strengths are achieved along with kinematic requirements. The optimum diameter matrix can be calculated for both spur and helical gearing with standard or non-standard tooth geometry.

An eigenvalue analysis is performed on optimal arrangements to ensure that torsional natural frequencies do not occur near operating speeds. In the event that eigenvalues need to be modified, eigenvalue derivatives with respect to shaft stiffness elements are calculated and used to adjust the design.

A Problem Reduction optimization strategy is used to eliminate dependent variables and simplify constraints. This results in a very efficient formulation of the problem which allows optimization of as many arrangements as desired.

A four speed drive verification problem is used through-out the thesis to illustrate concepts and verify techniques. Case studies are presented and compared with other works for the 9 speed and 18 speed gear drives. Results of the case studies are shown to correlate with actual multi-speed machine tool gear drives.

ACKNOWLEDGEMENTS

The author would like to express his sincere appreciation to Dr. M.O.M. Osman and Dr. S. Sankar for their guidance and supervision.

Many thanks to Dr. M. Donato for long discussions, helpful suggestions and encouragement during the course of this work.

Thanks are due to the Concordia University Computer Center staff who provided excellent support. In particular the author thanks Steve Bush for his technical support.

Special thanks to my parents and my wife, Allison, for their patience and encouragement.

TABLE OF CONTENTS

		Page
ABSTRACT		iii
ACKNOWLEDGEMENTS		v
LIST OF ILLUSTRATONS		xii
LIST OF TABLES		xv
LIST OF SYMBOLS		xx
CHAPTER 1	INTRODUCTION	1
1.1	Objective	1
1.2	Background on Kinematic Analysis and Relevent Literature	2
1.3	Background on Gear Mesh Design and Optimization	14
	1.3.1 Rating the Power Capacity of a Gear Mesh	15
	1.3.2 Optimum Design of a Gear Mesh	18
1.4	Background on Dynamic Analysis	21
	1.4.1 Eigenvalues & Eigenvectors	21
	1.4.2 Eigenvalue Sensitivity Analysis	24
1.5	The Optimization Process	25
1.6	Overview	29
CHAPTER 2	DEVELOPMENT OF GOVERNING EQUATIONS FOR THE KINEMATIC ANALYSIS OF GEAR DRIVES	32
2.1	Introduction	32
2.2	Analysis of a Two Shaft Gear Train	37
2.3	Extension to the Multi-Shaft Arrangement	41

TABLE OF CONTENTS (Continued)

2.4 Layout Diagrams 43

2.5 Composite Arrangements 46

2.5.1 Double Composite Arrangements 46

2.5.2 Single Composite Arrangements 48

2.6 Four Speed Gear Drive - Verification Problem 48

2.6.1 Objective Function - Radial Dimensions 50

2.6.2 Minimum Diameter Matrix 51

2.6.3 Optimization Strategy 53

2.6.4 Discussion of Results 56

2.7 Constraints on the Lowest Output/Input Ratio 59

2.7.1 Case of Two Groups and a Double Composite . . 61

2.7.2 Case of More than Two Groups and a Double . . 61
Composite

2.7.3 Case of Two Groups and a Single Composite . . 62

2.7.4 Fours Speed Drive Results Constrained 63
Output/Input Ratio .

2.8 Summary 63

CHAPTER 3 GEAR MESH DESIGN & OPTIMIZATION 68

3.1 Introduction 68

3.2 Spur & Helical Gear Power Rating 69

3.2.1 Bending Strength/Tooth Breakage 69

3.2.2 Pitting Resistance 70

3.2.3 Scoring 71

TABLE OF CONTENTS (Continued)

3.3 Variables Which Affect the Power Rating of a Gear Mesh . . . 72

3.4 Mesh Optimization Strategy 75

3.4.1 Guidelines for Generating the Minimum Diameter Matrix 77

3.4.2 Calculation of the Minimum Diameter Matrix . . 79

3.5 Input Values of Design Variables for Verification Problem 80

3.5.1 Operating Conditions 80

3.5.2 Material Properties 80

3.5.3 Tooth Geometry 84

3.5.4 Gear Mesh Size Variables 84

3.6 Minimum Diameter Matrix by Mesh Optimization . . . 85

3.7 Four Speed Drive – Verification Problem 86

3.8 Summary 89

CHAPTER 4 DYNAMIC ANALYSIS 94

4.1 Introduction 94

4.2 Equations of Motion for Multi-Speed Gear Drives . . . 95

4.3 Calculations of Physical Parameters 100

4.3.1 Shaft Design 100

4.3.2 Inertia of Cylindrical Bodies 101

4.3.3 Torsional Stiffness of a Shaft 102

4.3.4 Stiffness of Other Components 102

4.3.5 Equivalent Stiffness and Inertia 102

TABLE OF CONTENTS (Continued)

4.4 Method of Calculating Natural Frequencies 103

4.4.1 The Jacobi Method 103

4.5 Eigenvalues and Eigenvectors - Four Speed Drive . . . 104
Verification Problem

4.5.1 Calculation of Gear Width 104

4.5.2 Inertia and Stiffness Matrices 106

4.5.3 Presentation and Discussion of Results 107

4.6 Derivatives of Eigenvalues 111

4.7 Eigenvalue Derivatives for the Four Speed Drive . . . 113
Verification Problem

4.8 Summary 117

CHAPTER 5 OPTIMIZATION STRATEGY 119

5.1 Introduction 119

5.2 Problem Reduction Optimization Strategy 119

5.3 Definition of Optimization Problem 121

5.3.1 Volume Objective Function 121

5.3.2 Operating Requirements 126

5.4 Background and Development of the Mathematical . . . 126
Model

5.4.1 Standard Approach to the Design of 127
Gear Drives

5.4.2 Highlights of the Conventional Design 130
Approach

5.4.3 Key Elements of the Optimization Process . . . 131

TABLE OF CONTENTS (Continued)

5.4.4 Operating Requirements Addressed by the Mathematical Model . . . 133

5.5 Optimization Techniques 136

5.5.1 Discrete Random Process 136

5.5.2 Patterned Move 136

5.5.3 Random Search 137

5.5.4 Univariate Technique 137

5.5.5 Direct Methods 137

5.6 Four Speed Results - Volume Objective Function . . . 138

5.7 Summary 139

CHAPTER 6 CASE STUDIES FOR A 9 SPEED AND AN 18 SPEED SPEED GEAR DRIVE 152

6.1 Introduction 152

6.2 Results and Discussion for the 9 Speed Gear Drive . . 152

6.3 Results and Discussion for the 18 Speed Gear Drive . . 155

6.4 Summary 156

CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK . . . 176

7.1 Recommendations for Future Work 179

REFERENCES 181

TABLE OF CONTENTS (Continued)

APPENDIX I — A COMPARISON OF MATHEMATICAL TECHNIQUES FOR THE KINEMATIC ANALYSIS OF MULTI-SPEED GEAR TRAINS . . . 192

APPENDIX II — OVERVIEW OF SOFTWARE USED FOR ANALYSIS OF MULTI-SPEED GEAR DRIVES 210

APPENDIX III — COMPUTER GENERATED RESULTS FOR THE OPTIMAL DESIGN OF MULTI-SPEED GEAR DRIVES 221

LIST OF ILLUSTRATIONS

FIGURE		Page
1.1	Conventional Arrangement for the 4 Speed Gear Drive using 3 Shafts	3
1.2	Layout Diagrams for the 4 Speed Drive Shown in Figure 1.1	4
1.3	Single Composite Arrangement. Shaded Gear on Center Shaft Acts as an Output Gear and as an Input Gear	6
1.4	Double Composite 8 Speed 10 Gear Arrangement	8
1.5	Double Composite 9 Speed 10 Gear Arrangement	9
1.6	Triple Composite 9 Speed 9 Gear Arrangement	10
1.7	Typical Strength Behavior of a Gear Mesh for Different Failure Modes Calculated from AGMA 218	19
2.1	Conventional 2 Shaft Arrangement for the 4 Speed Gear Drive	33
2.2	Conventional 3 Shaft Arrangement for the 4 Speed Gear Drive	34
2.3	Single Composite Arrangement for the 4 Speed Gear Drive	35
2.4	Double Composite Arrangement for the 4 Speed Gear Drive	36
2.5	Simple 2 Shaft Gear Drive Arrangement	38
2.6	Layout Diagram for Simple 2 Shaft Gear Drive	39
2.7	Combining 2 Groups to make a 3 Shaft Conventional Arrangement	42
2.8	Illustrating the Range Exponent Using Layout Diagrams for the 4 Speed Drive	44

LIST OF ILLUSTRATIONS (Continued)

FIGURE		Page
2.9	Conventional Arrangement for the Four Speed Gear Drive - Verification Problem	49
2.10	Variable Assignment for Radial Objective Function	54
2.11	Gear Drive Based on Optimization 4 from Table 2.4	60
2.12	Gear Drive Based on Optimization 1 from Table 2.5	66
2.13	Speed Diagram Based on Optimization 1 from Table 2.5	67
3.1	Variables Defining Cutter Geometry	74
3.2	Strategy to Generate Minimum Diameter Matrix	81
3.3	Center Distance as a Function of Gear Ratio and Pinion Speed for a Conventional and Composite Mesh. Data from Table C.10	87
3.4	Gear Drive Based on Optimization 1 Table 3.9 Ratio for a Conventional Mesh. Data from Table 3.8	91
3.5	Speed Diagram Based on Optimization 1 Table 3.9 Ratio for a Composite Mesh. Data from Table 3.9	92
4.1	Dynamic Parameters for the 4 Speed Verification Problem	96
4.2	Dynamic Model for 4 Speed Drive Verification Problem	97
4.3	Mode Shapes Associated with Lowest Frequency	110
4.4	Dynamic Model for Verification Problem with Mass Node 6 Isolated for Clarity	118
5.1	Problem Reduction Strategy	122
5.2	Definition of Volume Objective Function	123
5.3	Correction to Volume Objective Function for Composite Arrangements	124

LIST OF ILLUSTRATIONS (Continued)

FIGURE		Page
5.4	Effect of Face Width/Diameter on Volume Objective Function for Conventional Arrangements	142
5.5	Gear Train Based on Optimization 1 Table 5.2	145
5.6	Speed Diagram Based on Optimization 1 Table 5.2	146
6.1	Conventional Arrangement for the 9 Speed Gear Drive	158
6.2	Layout Diagrams for the 9 Speed Gear Drive (a) Crossed Arrangement (b) Open Arrangement	159
6.3	Gear Train Based on Optimization 20 Table 6.2	168
6.4	Speed Diagram Based on Optimization 20 Table 6.2	169
6.5	Gear Train Based on Optimization 6 Table 6.6	171
6.6	Speed Diagram Based on Optimization 6 Table 6.6	172
6.7	Section of Spindle Head for Mitsui-Seiki H5C	173
6.8	Main Drive for 18 Speed Gearbox by Rambaudi	174
A.1	Layout Diagram for 9 Speed Gear Drive Used by White [14], and Osman, Sankar, and Dukkipati [22]	193
A.2	Conventional Arrangement for 9 Speed Gear Drive. Double Composite Arrangement Under Consideration Occurs When $d_4 = d_9$ and $d_5 = d_7$	194
B.1	Flow Chart of Main Routine for Gear Drive Analysis	211
B.2	Flow Chart for Solution 1	213
B.3	Flow Chart for Solution 1A	214
B.4	Flow Chart for Solution 2	216
B.5	Flow Chart for Solution 3	217
B.6	Flow Chart for Solution 4	219

LIST OF TABLES

TABLE		Page
1.1	Possible Kinematic Arrangements as a Function of Output Speeds	11
2.1	Operating Requirements for the Four Speed Verification Problem	50
2.2	Diameter Matrix for Kinematic Optimization Conventional or Composite Input Gear	52
2.3	Kinematic Requirements for each Optimization for 4 Speed Arrangements	57
2.4	Optimization Results for the Verification Problem Four Speed Gear Drive	58
2.5	Optimization Results for the Verification Problem Four Speed Drive (S=0.30)	65
3.1	Gear Pair Design Variables - Operating Conditions/Requirements	73
3.2	Gear Pair Design Variables - Tooth Shape	75
3.3	Gear Pair Design Variables - Material Properties	76
3.4	Gear Pair Design Variables - Gear Size	76
3.5	Design Strategies to Minimize Mesh Center Distance	78
3.6	Value of the Mesh Input Variables Selected for the Verification Problem	82
3.7	Quality Categories used for Analysis	83
3.8	Sample Interpolation Using Averaged Cubic Relationship Between Optimum Gear Diameter (mm) and Power Capacity	88

LIST OF TABLES (Continued)

TABLE		Page
3.9	Optimization Summary for the 4 Speed Drive Verification Problem for the Radial Objective Function using Standard Operating Conditions	90
3.10	A Comparison of the Optimum Solutions Presented in Chapters 2 and 3	93
4.1	Optimum Arrangement with Initial Shaft Diameters	106
4.2	Ratio of Shaft Speed to Input Shaft Speed	107
4.3	Basic and Equivalent Inertia of Gears Referenced To Input Shaft Speed (kgm^2)	107
4.4	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #1	108
4.5	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #2	108
4.6	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #3	108
4.7	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #4	109
4.8	Initial Eigenvalues (Hz)	109
4.9	Stiffness Derivatives Associated with Eigenvalue 6 Speed 2	114
4.10	Modified Shaft Diameters (mm)	116
4.11	Modified Eigenvalues (Hz)	116
5.1	Summary of Optimum Convention Arrangements Face Width/Diameter Ratio = 0.1 - 1.0	141

TABLE		Page
5.2	Optimization Summary for the 4 Speed Drive Verification Problem Volume Objective Function	143
5.3	Optimum Arrangement with Shaft Diameters for Four Speed Verification Problem Volume Optimization	147
5.4	Ratio of Shaft Speed to Input Shaft Speed	147
5.5	Basic and Equivalent Interia (kgm^2) of Gears Referenced to Input Shaft Speed	148
5.6	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #1	148
5.7	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #2	149
5.8	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #3	149
5.9	Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #4	150
5.10	Eigenvalues (Hz)	150
5.11	A Comparison of Optimum Solutions	151
6.1	Kinematic Arrangements for the 9 Speed Gear Drive	160
6.2	Optimization Summary for the 9 Speed Gear Drive - Case Study	161
6.3	Design of Gearing for Optimum 9 Speed Drive	166
6.4	Error in Output Speeds for Optimum 9 Speed Arrangement	166
6.5	Eigenvalue Summary for the 9 Speed Gear Drive	167
6.6	Comparison of Results for the 9 Speed Drive with those in Literature	167

LIST OF TABLES (Continued)

TABLE		Page
6.7	Optimization Summary for the 18 Speed Gear Drive for Selected Arrangements	170
6.8	Design of Gearing for Optimum 18 Speed Drive	175
C.1	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.1	222
C.2	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.2	223
C.3	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.3	224
C.4	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.4	225
C.5	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.5	226
C.6	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.6	227
C.7	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.7	228
C.8	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.8	229
C.9	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.9	230
C.10	Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 1.0	231
C.11	Minimum Diameter Matrix (mm) for Hardened Pinion and Gear Materials with Face Width/Diameter Ratio of 0.8	232

LIST OF TABLES (Continued)

TABLE		Page
C.12	Matrix of Optimum Number of Teeth on Input Gear for a Face Width/Diameter Ratio of 0.5	233
C.13	Matrix of Optimum Number of Teeth on Input Gear for a Face Width/Diameter Ratio of 1.0	234
C.14	Matrix of Optimum Module for Face Width/Diameter Ratio of 0.5 for Standard Operating Conditions	235
C.15	Optimum Module on Input Gear (mm). Face Width/Diameter Ratio of 1.0	236
C.16	Eigenvalue Derivatives $\partial w/\partial M(i,i)$ [(krad/sec)/(kgm^2)]	237
C.17	Eigenvalue Derivatives $(\partial w/\partial k)$ [(krad/sec)/(MNm/rad)]	239
C.18	Optimum Arrangement with Initial Shaft Diameters	242
C.19	Ratio of Shaft speed to Input Shaft Speed	242
C.20	Basic and Equivalent Inertia of Gears Referenced to Input Shaft Speed (kgm^2)	243
C.21	Stiffness of Basic and Equivalent Couplings (MNm/rad)	244

LIST OF SYMBOLS

A	Mass matrix
A_i	Axial length of ith group (mm)
AR_i	Axial length of volume reduction associated with ith composite (mm)
$a_1, a_2, \ldots a_n$	Angular acceleration
a	Vector of angular accelerations
B	Stiffness matrix
C_f	Finish factor
C_H	Hardness factor
C_m	Load modification factor
C_p	Elastic coefficient
C_s	Size factor
C_{SF}	Service factor
C_T	Temperature factor
C_v	Dynamic factor
C_1	Constant based on step ratio and range exponent
C_2	Constant based on step ratio and range exponent
D_1	Diameter of first composite gear
D_2	Diameter of second composite gear
D_i	Depth of ith group
DR_i	Depth associated with volume reduction of ith composite
d	Component pitch diameter
d_b	Shaft diameter based on bending strength
d_s	Shaft diameter based on shear strength

$d_1, d_2 \ldots d_n$	Gear diameter
d_{in}	pitch diameter of input gear
dof	Basic kinematic degrees of freedom
d_{out}	pitch diameter of output gear
E	Youngs modulus.
$F, F_1, F_2, \ldots F_i$	Face width
F_n	New face width
F_o	Old face width
f_i	i^{th} factor of the number of speeds
G	Shear modulus
I	Geometry factor
I_s	Area inertia of shaft
$I_1, I_2 \ldots I_n$	Component inertia
J	Geometry factor
K	Stiffness matrix
K_m	Lead modification factor
K_s	Size factor
K_{SF}	Service factor
K_T	Temperature factor
K_v	Dynamic factor
$k, k_1, k_2, \ldots k_n$	Component stiffness
L	Shaft length
M	Inertia matrix
MG	Gear ratio
m	Module

LIST OF SYMBOLS (Continued)

N_1	Speed of input shaft
N_2	Speed of 2^{nd} shaft
N_L	Lowest speed of output shaft
n	Number of meshes in group
n_c	Number of composites
n_f	Number of factors
n_g	Number of gears in a group
n_s	Number of shafts in an arrangement
n_p	Pinion speed
P	Required power
P_{ac}	Pitting resistance power capability
P_{at}	Bending strength power capability
P_c	Calculated power capacity
P_d	Diametral pitch
p, p_1, p_j	Range exponent
p_j	Inertia or stiffness parameter
r	Material density
R_1	Radial dimension across a group
RR_1	Radial dimension across a group associated with the volume reduction for composite gears
S	Lowest output speed/input speed ratio
S_{ac}	Material fatigue strength
S_{at}	Material bending strength
S_b	Shaft allowable bending strength
S_s	Shaft allowable shear stress

LIST OF SYMBOLS (Continued)

SCN	Scoring number
T	Torque
T_{in}	Number of teeth on input gear
T_{out}	Number of teeth on output gear
V	Volume of drive without correction for composites
V_i	Volume of ith group in drive
VR_i	Volume reduction associated with the ith composite gear
V_t	Total volume of drive
W_t	Tangential Load
	Vector of gear diameter design variables
$X_1, X_2, \ldots X_i$	Lowest gear ratio of ith group
x_g	Profile shift coefficient
$x_1, x_2, \ldots x_i$	Angular displacement
x	Vector of angular displacements
$Y_i, Y_1, Y_3, \ldots Y_{odd}$	Diameter of smallest gear on input shaft in a group
$Y_o, Y_2, Y_4, \ldots Y_{even}$	Diameter of largest gear on output shaft in a group
y_{max}	maximum permissible shaft deflection
z	Number of speeds
α	Simplification constant
β	Simplification constant
ϕ	Step ratio
u	Density
n, n_i	Natural torsional frequency

CHAPTER 1

INTRODUCTION

1.1 Objective

Multi-speed gear drives are critical components in all metal cutting machine tools as well as being important sub-assemblies in many other mechanical systems. The design of a multi-speed gear drive is an involved process drawing on expertise from a number of engineering fields including kinematics, machine design, and dynamics. A kinematic analysis is required to ensure that the correct output speeds are obtained using an efficient gear arrangement. Principles of machine design are used to ensure that components, such as the gears, are capable of carrying the operating power for the life of drive. Dynamic analysis of the system is required to verify that operating speeds do not lie near system natural frequencies.

The analysis for each of these activities is substantial. In general, a kinematic analysis is performed first, followed by component design and dynamic analysis. By dealing with each analysis separately, the complexity of the problem is somewhat reduced. This approach is generally used in designing gear drives [1] and was recently used by Rao and Eslampour to synthesize designs for an objective function of minimum volume [2]. This series approach has been used because it allows for acceptable solutions to the problem in reasonable time frames.

A complete analysis for all possible combinations of design variables, which would determine the optimum, is not practical. Instead, for problems such as these, it is necessary to find strategies which search the design space efficiently. Systematic search strategies for complex problems [2-8] are under investigation. Substantial improvements have been achieved, when strategies are used which reduce the design space based on analysis of constraints.

This work studies the optimal design of parallel axis multi-speed gear drives. The focus is on providing a mathematical model and optimization strategy which integrate the significant aspects of key fields into one unified analysis. The goal is to synthesize optimal designs which correlate with drives in use.

1.2 Background on Kinematic Analysis and Relevent Literature

The output speeds of a multi-speed machine tool gear drive normally form a geometric progression which simplifies design and improves machine tool efficiency [9,10]. The ratio between speeds, known as the step ratio, is selected from one of a number of standard values depending on the speed range which must be covered and the number of speeds allowed. Shaw [11] critically examined the use of the geometric progression in machine tool applications. He found that manufacturing costs are minimized and that tool life and surface finish requirements are adequately met when a geometric progression of output speeds is used, as long as all speeds are equally likely to be required.

Based on the number of speeds a layout diagram [9] can be prepared. For the conventional gear drive, figure 1.1, the layout

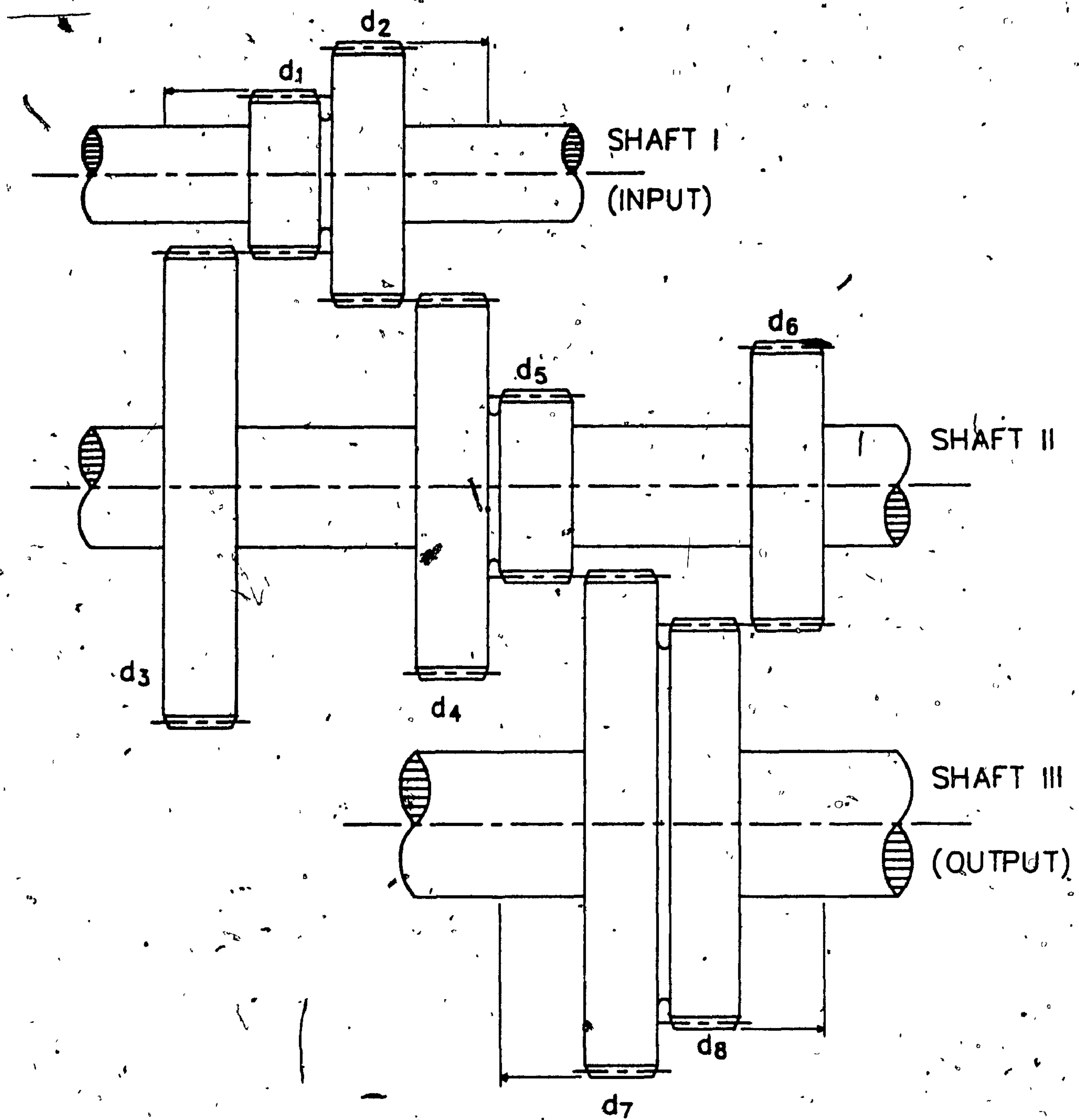
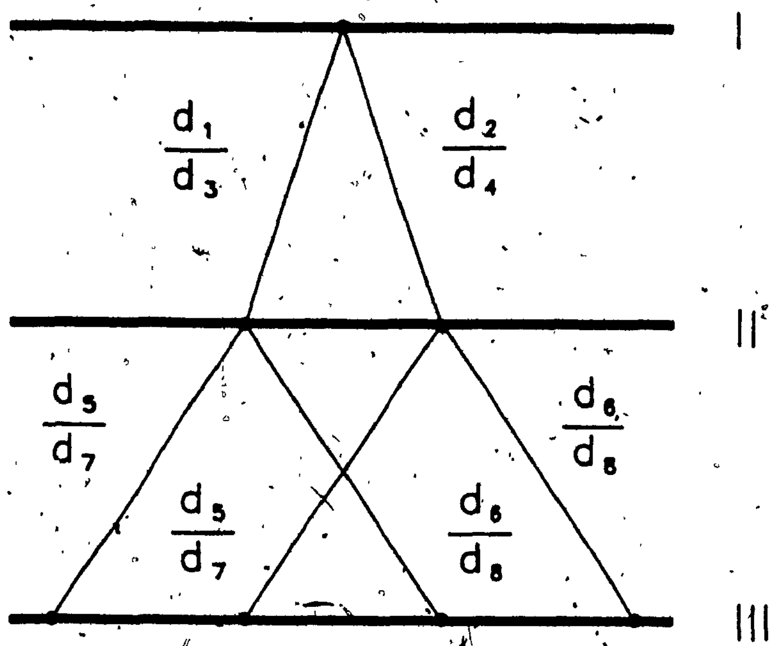


FIGURE 1.1

Conventional Arrangement for the 4 Speed
Gear Drive using 3 Shafts



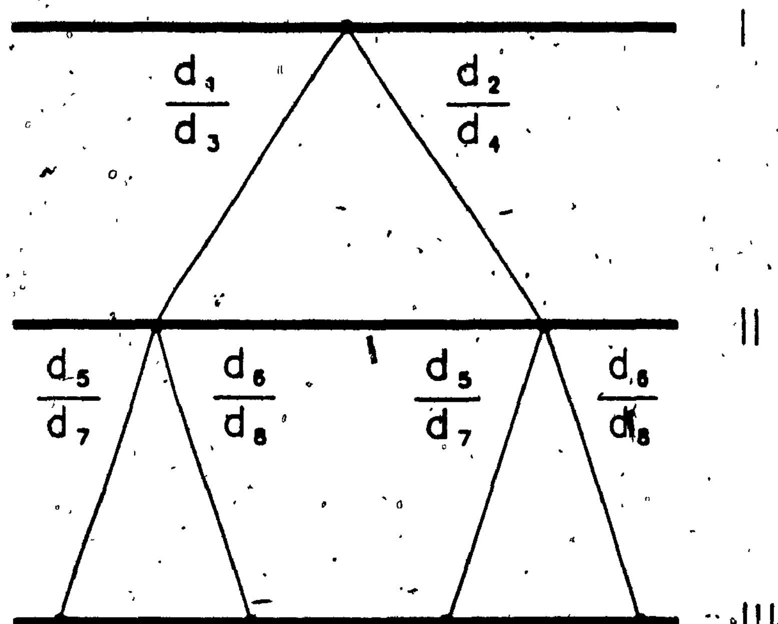


FIGURE 1.2

Layout Diagrams for the 4 Speed Drive Shown in Figure 1.1

diagram as shown in figure 1.2, identifies the number of shafts and the number of gear pairs (i.e. ratios) between each shaft. For a particular number of speeds, there may be many layout diagrams. The quantity depends on the number of speeds [9].

The number of gears used to satisfy a particular layout diagram depends on the arrangement selected. A conventional arrangement is defined as one which uses two gears for each gear mesh. In some cases, it is possible to use a driven gear from one mesh of a group as the driving gear in another mesh of the next group as shown in figure 1.3. This reduces the total number of gears in the gearbox compared to conventional arrangements.

Arrangements which utilize a gear on an intermediate shaft as both an input gear and an output gear are know as "composite" arrangements. If only one gear on the intermediate shaft has this property, the layout is termed a "single composite". If two gears on the same intermediate shaft have this property, the layout is termed a "double composite". Similarly, an arrangement having 3 gears on the same intermediate shaft is termed a "triple composite".

Composite gear arrangements have a reduced number of degrees of freedom and oblige the designer to develop mathematical models to find the gear diameters. Because these arrangements reduce the number of gears required to produce a given number of speeds they have received the attention of gear train designers.

Hall [12,13] was one of the first to use a mathematical procedure to determine the gear diameters of a multi-speed gear train arrangement with output speeds in a geometric progression. His work was directed at determining the number of teeth for each gear in composite gear drive

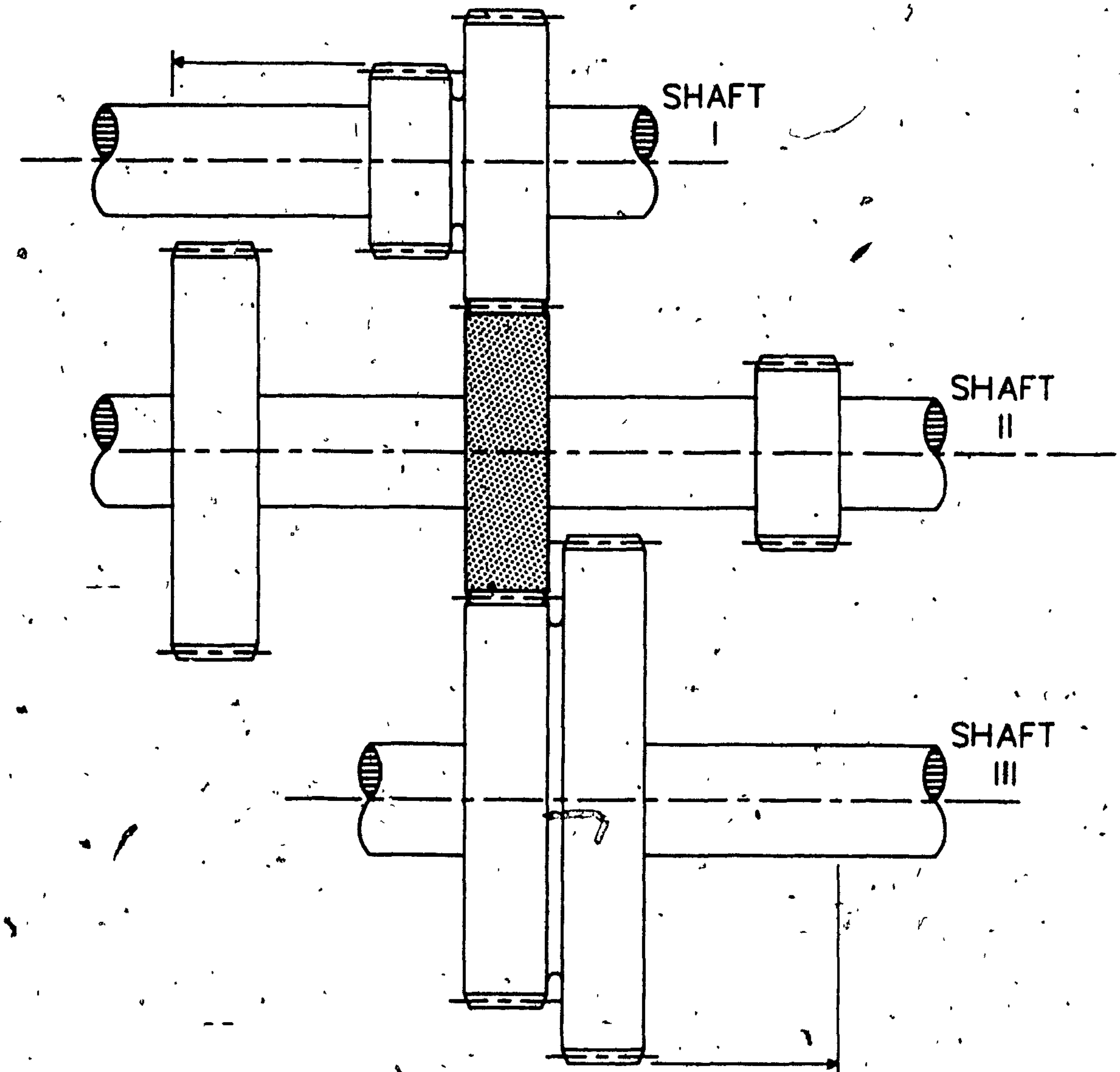


FIGURE 1.3

Single Composite Arrangement. Shaded Gear on Center Shaft Acts as an Output Gear and as an Input Gear.

arrangements. He examined three arrangements (figures 1.4 - 1.6) each with three shafts: The 8 speed 10 gear and the 9 speed 10 gear double composites and the 9 speed 9 gear triple composite. Hall analysed the above arrangements using an orderly procedure to obtain the number of teeth required on each gear. The analysis was based on the assumption that output speeds formed a geometric progression and that center distance between gears in a group was constant.

White [14] developed a rigorous mathematical method of analysis (see appendix A) in order to write analytic equations for the gear diameters of minimum gear (composite) arrangements. He applied this method to a number of 9 speed arrangements to obtain analytic equations for the diameters of the gears. Triple composite arrangements were shown to be incapable of providing output speeds in a geometric progression thus the 9 speed - 9 gear arrangement was rejected. Attention was next focused on the 9 speed - 10 gear double composite arrangement shown in figure 1.5. White developed algebraic equations for the diameter of each gear of the train in terms of the lowest output/input ratio, a reference gear, and the step ratio. As with Hall, the output speeds formed a geometric progression and the distance between the shafts was constant. This analysis applied to one of 38 kinematic possibilities, as shown in table 1.1, for the 9 speed 10 gear 3 shaft arrangement.

With analytic equations for the diameters of the gears, a natural step was to develop a diameter based objective function to minimize gear diameters. White and Sanger [15] presented results for the 4 speed double composite arrangement using the technique introduced by White. They also developed an analytic equation to find the optimum value of

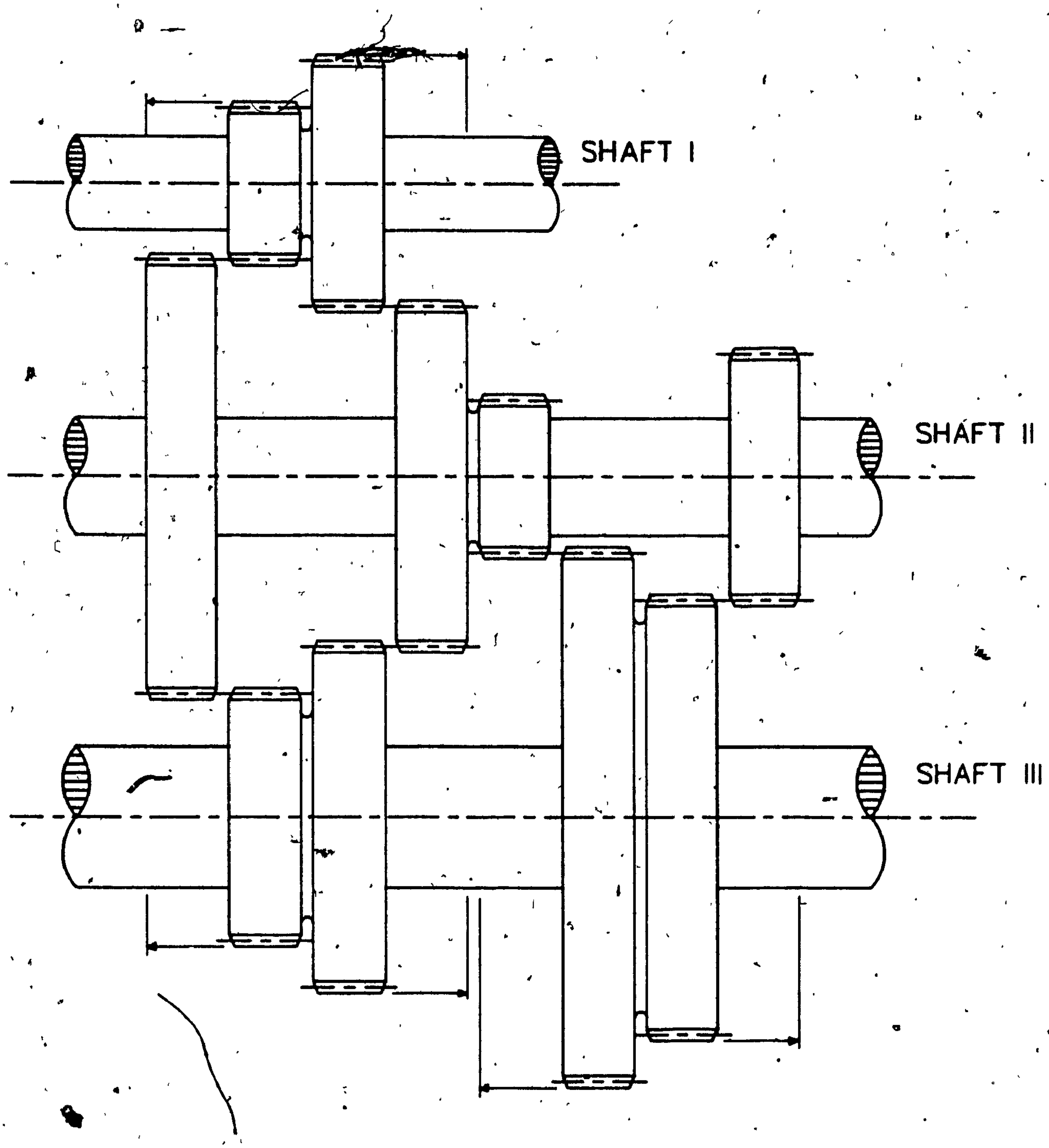


FIGURE 1.4

Double Composite 8 Speed 10 Gear Arrangement

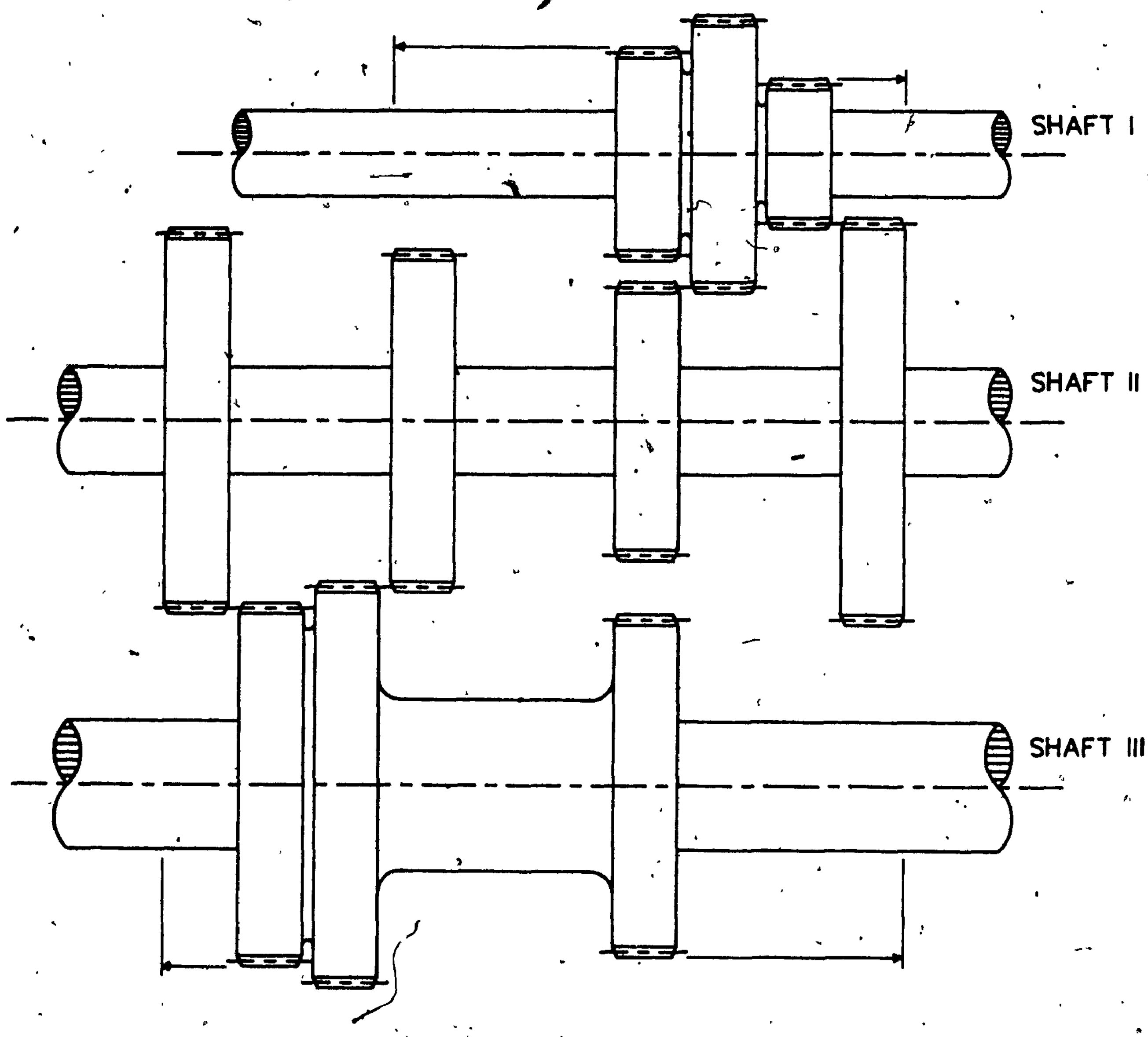


FIGURE 1.5

Double Composite 9 Speed 10 Gear Arrangement

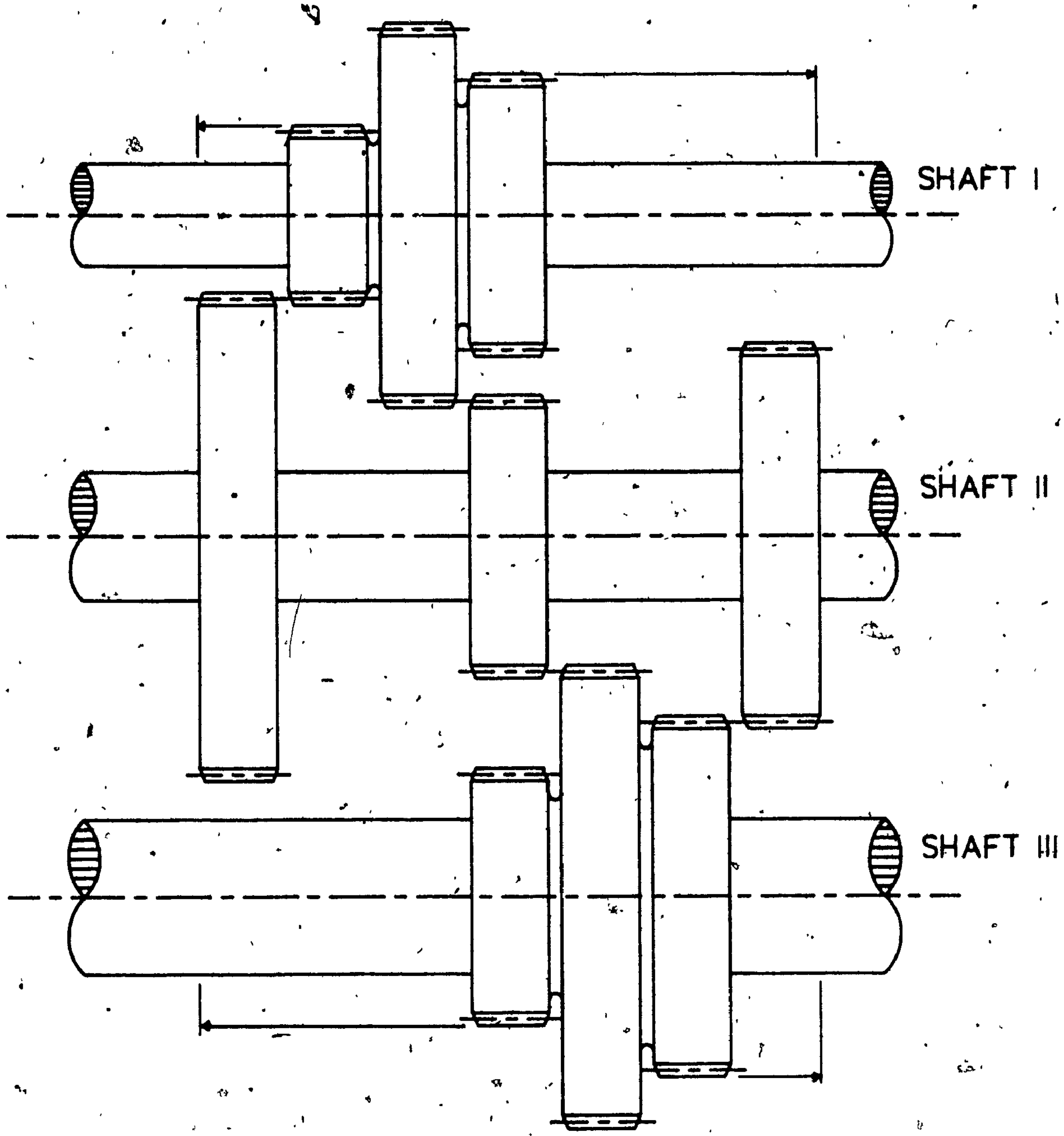


FIGURE 1.6

Triple Composite 9 Speed 9 Gear Arrangement

TABLE 1.1

Possible Kinematic Arrangements as a Function of Output Speeds

SPEEDS	FACTORS	RANGE EXP.	# OF COMPOSITES BY TYPE			TOTAL
			NONE	SINGLE	DOUBLE	
4	2-2	1-2	1	4	1	6
		2-1	1	4	1	6
						12
9	3-3	1-3	1	9	9	19
		3-1	1	9	9	19
						38
18	3-3-2	6-2-1	1	15	12	28
		2-6-1	1	15	12	28
		6-1-3	1	15	12	28
		1-6-3	1	15	12	28
		3-1-9	1	15	12	28
		1-3-9	1	15	12	28
	3-2-3	6-3-1	1	12	6	27
		3-9-1	1	12	6	27
		6-1-2	1	12	6	17
		1-9-3	1	12	6	17
		2-1-6	1	12	6	17
		1-3-6	1	12	6	17
	2-3-3	9-3-1	1	15	12	28
		3-6-1	1	15	12	28
		1-6-2	1	15	12	28
		9-1-3	1	15	12	28
		3-1-6	1	15	12	28
		1-2-6	1	15	12	28
						438

the ratio of lowest output speed/input speed as a function of the step ratio based on an objective of equating the diameters of the largest gears. White and Sanger [16,17] and Prasad, Dukkipati, and Osman [18] pursued various objective functions, such as equal diameters of the largest gears, minimum overall radial dimensions, and finding the number of gear teeth which resulted in the minimum resulting error in output speeds.

White and Sanger [19] studied all possible double composite arrangements for the 9 speed 3 shaft gear train, a task requiring considerable time and excellent algebraic skills, to determine which one was best. They considered 24 sets of equations for 9 speed 10 gear double composite arrangements which could produce a geometric progression of output speeds. A measure of the overall radial dimensions for each arrangement was obtained by summing the diameters of the gears in a line. Since analytic equations were available, as a function of the lowest output/input ratio and the step ratio for each of the gear diameters, a single analytic objective function was developed. The effect of the lowest output/input ratio on overall radial dimensions was studied and found to have an optimum value for a particular step ratio.

The method of White and Sanger had two serious limitations: (1) the analysis was extremely tedious and (2) the resulting equations applied to only a single specific arrangement. Sanger and White [20,21] developed a general theory for double composite gear trains in an effort to eliminate the need to analyze from scratch any change in the arrangement or layout diagram. This theory used a six gear double composite arrangement as the core and added extra gear pairs as required to obtain additional speeds. This method was mathematically complex requiring tables of index numbers and algebraically complicated expressions. It was further limited to double composite arrangements using 3 shafts.

Osman, Sankar and Dukkipati [22] used a multi-parameter optimization technique to obtain results identical to those obtained by White and Sanger [19] with much less analytic effort. The equations for the gear diameters were written with 3 variables, assuming 3 degrees of freedom instead of 1, thus reducing the algebraic manipulations required. It was then possible to write 2 additional constraints so that the resulting problem was similarly constrained (see appendix A). This converted the complex algebraic analysis problem to one of moderate algebraic analysis and a multi-parameter optimization subject to 2 equality constraints. The work required to obtain solutions to different objective functions was much reduced. Unfortunately the method still required significant amounts of mathematical development to obtain gear diameter equations valid only for a selected composite gear arrangement and accompanying layout diagram.

Although most previous work has been devoted to studying double composite arrangements, Murthy [23] used a mathematical procedure to minimize radial dimensions of a single composite gear drive. His technique did not develop equations for the diameters of each of the gears, however, his objective to minimize radial dimensions was similar to that of White and Sanger. He found that in particular cases, the single composite arrangement was radially as small as the conventional gear arrangement. His technique did not easily extend to double composite arrangements.

The utility of previous methods to define gear diameters in a drive was limited by the algebraic development required to generate the system of equations specific for each layout and arrangement. As shown in table 1.1, there are 12 possible kinematic arrangements for a drive with 4 speeds, 38 for one with 9 speeds and 438 for one with 18 speeds. A designer must be able, in a practical fashion, to evaluate as many of those arrangements as deemed necessary in order to determine the optimal one. Since there are many possible arrangements for a particular number of speeds, the designer must perform the mathematical analysis many times. Methods presented in the literature have not been practical for the investigation of the general problem.

1.3 Background on Gear Mesh Design and Optimization

The diameter and width of gears in a gear drive largely determine overall performance. These components must be optimized.

The power rating of a gear mesh is difficult to determine because of the large number of design variables, the wide range of operating conditions, and the inexplicit relationships between variables. Basic information on the geometry of gearing and basic equations have been presented by Dudley [24] and Buckingham [25]. Detailed design information is contained in design standards [26-30]. The American Gear Manufacturers Association (AGMA) design standards [26,29,30] form the basis of most North American designs. These standards are based on fundamental theories which are combined with design factors. Design factors are determined through a combination of theory and practice and are updated at intervals to incorporate improvements.

1.3.1 Rating the Power Capacity of a Gear Mesh

Gears used to transmit power must be sized to resist, for their design life, failure because of tooth breakage or surface fatigue pitting or scoring. Tooth breakage is a result of a single load which exceeds the yield strength or repeated loads which exceed the fatigue bending strength of the gear tooth material. Pitting of the gear tooth surface is a result of Hertzian contact stresses on the face of the gear tooth which exceed the material fatigue strength. Scoring of the gear tooth surface is caused by a breakdown of the lubricant film resulting in metal to metal contact.

Design methods for the avoidance of gear tooth breakage are based on the bending endurance limit of the gear material. The goal is to have sufficient strength to avoid a bending fatigue failure due to cracking at the tooth root fillet. Usually in these methods the gear tooth is analyzed as a cantilever beam with the addition of semi-empirical service and geometry factors. The bending strength is measured in terms of the tensile bending stress in a cantilever plate modified to include the effects of: (1) compressive stress at the tooth roots, (2) non-uniform moment distribution, (3) stress concentrations at the tooth root fillets, and (4) load sharing between adjacent teeth in contact.

Design methods to resist surface fatigue are based on the concept of a surface fatigue endurance limit. The Hertzian contact stress, which is proportional to the square root of the applied load, is estimated and then modified with service condition and geometry factors to become the stress number. The stress number is compared against the surface fatigue endurance limit, to ensure an adequate fatigue life.

Scoring is a wear phenomena which occurs as a result of a breakdown of the lubricant oil film, allowing metal to metal contact. The resulting welding and tearing apart of the tooth surfaces causes a rapid deterioration of the tooth profile. Scoring is most common with high speed gearing under heavy loads but is also evident with heavily loaded gears operated at low pitch line velocities. In most cases it may be controlled by selecting proper lubricant and/or by modifying the gear tooth profiles. The prime variables in determining oil film thickness are pitch line velocity, oil temperature and type of oil, although surface finish and materials may also be significant.

The equations to determine the power rating as suggested by design standard AGMA 218 [26] are cumbersome to evaluate. Most factors have been obtained as a combination of theoretical and empirical results accurate for specific operating conditions.

The dynamic factor is based on extensive studies over a period of time and is not yet determined for all conditions. Shimamura and Noguchi [31] used dynamic photoelastic tests to determine an analytical expression for dynamic factor. Houser and Seireg [32,33] performed extensive testing in a further effort to obtain analytic expressions for dynamic factors valid over a range of operating conditions. Tucker [34] suggested that the dynamic factor is additive rather than multiplicative and should be treated as such. Conrey and Seireg [35] presented a programming technique for the establishment of load distribution and optimal modification of a spur or helical gear mesh.

The complexity of the calculation to determine power rating for a specific gear pair using design standards resulted in a search for simplified rating expressions usually applicable to specific cases [36-

40]. The goal of such works was to arrive at analytic expressions for the power rating of a gear mesh. These provided approximate power ratings useful for the investigation of variable sensitivity but unable to correctly rate a gear mesh for a range of applications.

Power ratings for special cases, where one of the failure modes was clearly dominant, have also been published [41-47]. For specific applications, useful analytic methods are possible and have been demonstrated. However, the scope of application is distinctly limited and does not well serve the general problem. Invariably, the trend is towards the use of AGMA 218, and the more recent focus has been on the development of software to perform the necessary calculations in a practical fashion.

Cockerham and Waite [48] developed a computer aided procedure to design standard 20° pressure angle spur or helical gearing for strength with calculations based on the procedure outlined in BS436 [49]. They note that when a design is required for a specific power rating an iterative procedure is required. The ability to computerize the calculation of power rating was noted to be extremely useful.

With AGMA 218 the area of greatest difficulty was in the calculation of the geometry factor J. Normally, and still the basis of the design standard, the geometry factor J is determined using graphical methods. The standard presents tables of J values based on graphical methods for a limited number of standard tooth forms.

Analytic procedures for the determination of root geometry were presented by Mitchiner and Mabie [50] and Lopez and Wheway [51] based on the root profile of the tooth. These methods were applicable to both standard and non-standard tooth geometry. Earlier works by Kharalla

[52] and Gitchell [53] presented techniques to evaluate root geometry. Enrrichello [54] presented computer coding capable of calculating geometry factors. AGMA 218 provides a numerical technique to calculate the geometry factor which is developed from these earlier methods.

The ability to calculate the Lewis form factor using an analytical method suitable to the computer was very significant because it meant that it was no longer necessary to determine geometry factors by layout procedures. This made it practical to evaluate gear strengths for bending strength and pitting resistance using computer software.

The use of AGMA design standards are recommended, in the absence of more specific analytical and experimental results, to determine the design for a particular application.

1.3.2 Optimum Design of a Gear Mesh

Methods to optimize gear mesh performance have been known for many years. The goal, in general, is to balance the strength between the gear and the pinion. A secondary goal is to balance the bending and pitting resistance strengths with-in each gear. Figure 1.7 shows typical behavior of the four failure modes; (1) pinion bending strength, (2) pinion pitting resistance, (3) gear bending strength, and (4) gear pitting resistance as a function of tooth size.

Brass [55] and Bookmiller [56] provided design information to take advantage of non-standard gears. Brass suggested the use of modified pressure angles to simplify the design of gears for specific operating conditions. Bookmiller presented charts to design long/short addendum gears to avoid undercutting in the pinion or to balance strength. Oda and Shimatomi [57] presented results on the effect of

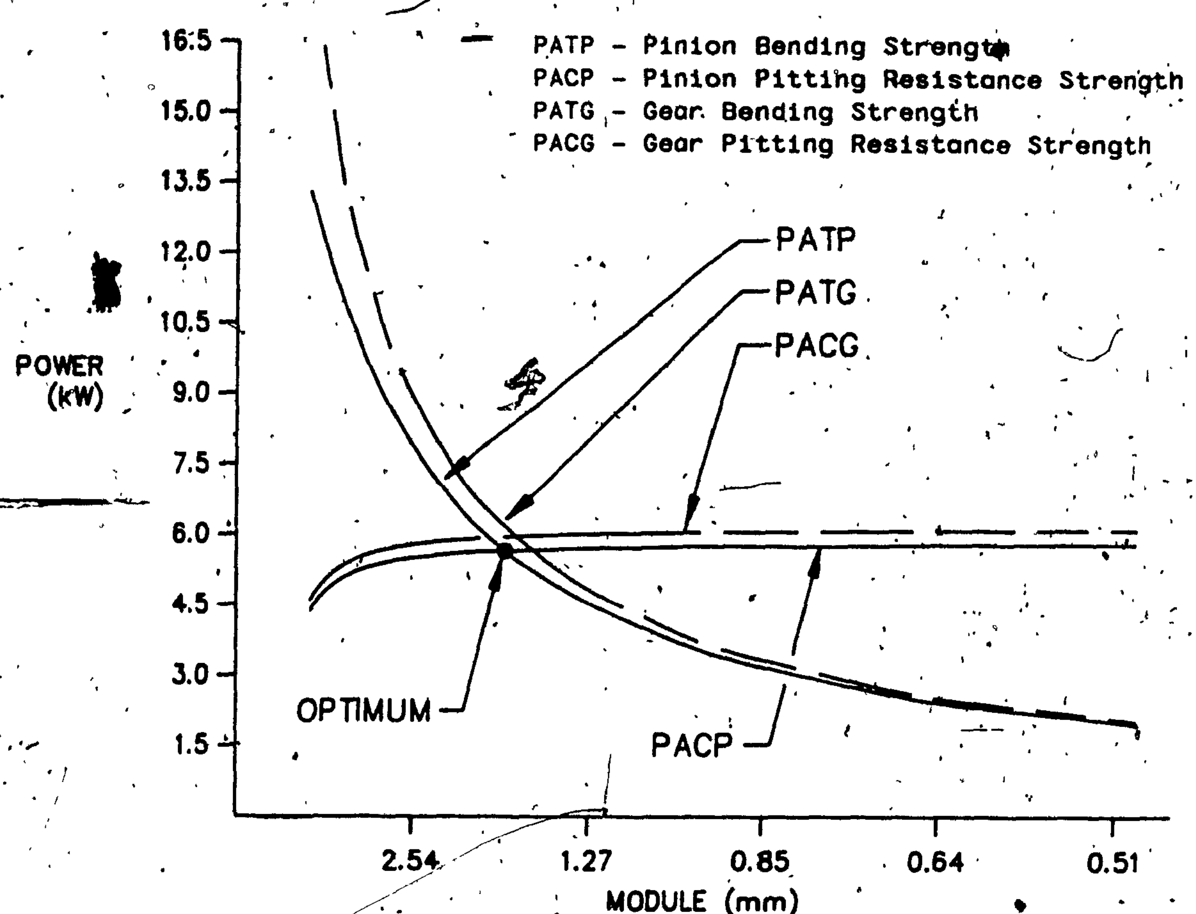


FIGURE 1.7

Typical Strength Behavior of a Gear Mesh
for Different Failure Modes Calculated from AGMA 218

addendum modifications for spur gears manufactured from both normalized steel gears and case hardened gears. They found that it was possible to raise the bending strength of case hardened gears by as much as 25% over standard gearing using addendum modifications. Oda, Tsubokura, and Namba [58] extended the study to gears with higher pressure angles. Li et al [59] studied the effect of large pressure angles on gear strength. They found that gears with high pressure angles (25°) provided greater strength and high anti-scoring capability for precision gearing. Cornell and Westervelt [60] reported on the use of high contact ratio gear teeth. Normal contact ratios for spur gears are between 1.2 and 1.6. This study examined dynamic tooth loads and stresses for gear pairs with contact ratios as high as 2.5. Further studies were recommended to determine design guidelines for high contact ratio gear teeth.

Estrin [61] looked closely at gear mesh parameters with the goal of optimizing tooth proportions. His analysis found outside radii and pressure angles to minimize stress or maximize contact ratio or maximize tooth tip thickness. Six constraints were developed to ensure that a quality mesh was obtained. Contact ratio, difference between base and limit diameters, compressive stress, maximum distance across outside diameters, tooth tip thickness and hob tip thickness were all constrained. Savage, Coy and Townsend [62] developed a design technique to determine the optimal tooth numbers of compact standard spur gear sets. They developed approximate expressions for the limiting pitting resistance stress, bending stress, scoring condition, and involute interference. These were plotted to provide a design space. With an objective function to minimize center distance, a number of equally suitable designs were apparent. Module was shown to have a significant effect on strength. Colbourne [63] estimated geometry factors based on data for standard tooth forms. These estimated geometry factors were used to provide a method of predicting the change in strength associated with addendum modifications. Hefeng et al [64] describe a FORTRAN computer program which can be used to investigate the root geometry of rack generated spur gears. The computer model was able to generate graphic data which allowed the visualization of the design.

Selection of the optimum kinematic arrangement for a multi-speed gear drive requires that an efficient method is available to calculate gear diameters - one which can be used as the basis of a gear mesh optimization.

1.4 Background on Dynamic Analysis

Torsional vibrations can cause serious problems in gear trains as a result of dynamic loads occurring in the components when natural frequencies lie near operating speeds. The low level of natural damping in torsional modes tends to aggravate this situation [65]. This requires that the natural frequencies be calculated during the design process to check their locations. In the event of a problem, it is necessary to modify the design in such a manner that the natural frequency in question is relocated.

1.4.1 Eigenvalues and Eigenvectors

The natural frequencies of mechanical systems have been found most commonly using the Raleigh Ritz method, matrix eigenvalue methods and transfer matrix methods [66]. Each of these methods of analysis is useful for specific applications.

The Raleigh Ritz method uses an estimation of the first mode shape to converge rapidly to the lowest or fundamental natural frequency. The approach equates the maximum kinetic energy to the maximum potential energy. Higher modes are found using orthogonality relationships to eliminate lower modes once they have been determined. This approach is sensitive to the accuracy of the matrix operations and cannot be relied upon to find higher frequencies correctly. McLay and Burroughs [67] developed a technique for sparse matrices based on the Raleigh Quotient. Since most engineering problems result in a sparse dynamic matrix, this method is useful for problems where the lowest frequency is required.

The transfer matrix method [68-75] breaks down a large system into subsystems with simple properties. The problem is formulated using the state vector, the point matrix and the field matrix [69]. The state vector is a column vector of the displacements and internal forces. The point matrix contains the dynamic properties of the subsystem while the field matrix contains elastic properties. Holzer developed a well known tabulation method used to find the eigenvalues of a mechanical system based on this approach. Improvements [71-75] were made to the basic approach by Holzer to handle closely located frequencies as well as branched systems.

Matrix eigenvalue methods find the roots of the dynamic matrix. The dynamic matrix is inverted or transformed using matrix algebra [76]. The dynamic matrix is of the order of n^2 and may become extremely large for complex systems. Efficient numerical transformations have been developed to take advantage of the symmetric sparse stiffness matrix common to gear drives [77,78]. These methods used orthogonal transformations to zero off-diagonal elements of the dynamic matrix. The objective is to zero those elements without creating new non-zero elements at other off-diagonal locations.

The three methods have frequently been used to find torsional natural frequencies in gear systems. Those using the Raleigh Ritz approach are effective for finding the first natural frequency and/or improving or accelerating the convergence of another method to the desired frequency. The transfer matrix approach is useful for finding a few frequencies within a given speed range. Matrix eigenvalue methods are most effective when all frequencies of the system are required. The transfer function and state-space method are useful when the actual response to an input is required.

The non-linear behavior of gear drives is well recognized. Methods of analysis are based on approximating the stiffness of the elements more accurately as well as accounting for backlash at the gear teeth and bearing clearances. The objective of these types of analyses is to better predict dynamic tooth loads, to determine stability of the overall system, and to more accurately determine natural frequencies. All methods [79-88] are computationally expensive.

General purpose programs for the dynamic analysis of mechanical systems are available. NASTRAN [89] is widely used for structural analysis and is quite capable of handling complex linear and non-linear systems using any or all of the solution methods previously presented. Wang [69] and Laschet and Troeder [90] have developed computer codes to analyse the general multi-speed gear drive problem. These codes are tailored towards the gear drive problem and have built-in models for the stiffness of each of the components reducing the time required for modelling.

Dynamic Analysis of complex mechanical systems is normally carried out at two or three specific stages of the design process, rather than at every design iteration. The reasons for this are that (1) most design iterations consist of small changes to the design variables which have a limited effect on overall system behavior and (2) the cost of dynamic analysis is generally large in comparison to the cost of the analysis of the objective function with respect to design variables.

1.4.2 Eigenvalue Sensitivity Analysis

When eigenvalues are found to be near operating speeds of the gear drive it is necessary to change the design of at least some of the components in the system. Derivatives of the eigenvalues with respect to mass or stiffness elements can be used to identify which elements are most profitable to re-design.

Lund [91] found the sensitivity to critical speeds of a rotor to changes in the design. Methods to find the derivatives of eigenvalues and eigenvectors have been presented by Fox and Kapoor [92] and Rogers [93] for symmetric matrices. These works were extended by Garg [94], and Rudisill [95]. Nelson [96] developed a method to find the eigenvector derivatives using eigenvalues and their associated right and left eigenvectors. This method was attractive for large systems where one of the methods to yield the first few eigenvalues and eigenvectors had been used. Doughty [97] applied these methods to simple torsional systems. Later, Doughty [98] presented an analysis which included the effect of damping.

Young and Shoup [99] used eigenvalue and eigenvector derivatives to improve the dynamic performance of a cam mechanism. This was an important step in utilizing the methods of dynamic analysis developed over the past two decades for the purpose of design. By knowing the influence of design variables on the natural frequencies of the system, a much more scientific approach to the re-design of the system was taken.

1.5 The Optimization Process

The computer aided design and optimization of gear-drives has received wide attention. Articles focus on both design aspects and optimization strategy. Both parts of the problem are important.

Designers were interested in exploiting the ability of a computer-aided approach to more fully investigate problems and in general to accelerate the design process. In examining more designs, they were in fact using a basic optimization technique. Dale [100] used computer graphics to boost the speed at which multi-speed gear drives could be designed. The computer performed routine calculations and presented graphic results relying on the designer to make decisions. He found that it was possible to design gear boxes four times faster at approximately 50% of design cost. Dil Pare [101], Selfridge and Riddle [102] and Orthwein [103] developed computer techniques to determine tooth numbers of compound gear trains which best achieve an overall ratio. Selfridge [104] investigated compound gear drives to determine gear ratios with minimum component inertia.

Optimization specialists have focused on the method and mathematics of the optimization process, selecting more complex problems as the methods and computational power of computers improved. Kowalski [105] studied a double reduction helical gear unit using Rosenbrock's algorithm for the optimization. Seireg and Conry [106] studied a double reduction unit with the goal of optimizing the gear units for surface durability. The objective was to maximize the "power to weight" ratio. The optimum solution was determined by the ratio of the first speed reduction to the total reduction in the case where the limiting constraint was allowable Hertzian stress (i.e. pitting resistance).

The weight optimization of a speed reducer, first formulated by Golinski [107], has been repeatedly studied. The attraction of the problem is its complexity as an optimization problem (7 variables bounded by 27 constraints) and its comparative simplicity when formulated as a realistic design problem. Golinski [108] used both gradient-type methods and random methods to solve this problem. With the Monte Carlo method he found that of 825,000 random searchings only 67 of them were contained in the permissible area. Lee [109] re-formulated the problem and improved on earlier results using heuristic combinatorial optimization techniques. Dotseris [110] solved the problem using heuristic and decomposition techniques further improving the results and again reducing execution time.

Rao [111] and Rao and Das [112] studied multi-speed gear trains by idealizing the gear train as a weakest-link kinematic chain, and considered all parameters affecting the design as random variables. Dhande and Gupta [113] performed a computer-aided interactive design of a multi-speed gearbox which made use of design standards for the components. Kamenetskaya [114] and Osyczka [115] have started to develop methods for the optimal design of multi-speed machine tool gear drives which combine mathematical optimization methods with proper design practice: Honggi et al [116] and Rao and Eslampour [2] have presented gear drive designs which present a strategy to generate multi-speed gear drives which satisfy component design and kinematic requirements.

There are many optimization techniques available for digital computers today. These methods include non-linear methods for constrained or unconstrained problems, with or without the use of gradient information [117-120]. These techniques address the needs of problems where the objective function and mathematical model have specific behavior.

The computer aided optimal synthesis of complex designs presents many problems. In a panel session entitled "Future Trends in Optimization" [121] a number of authors discussed problems and techniques for the optimization of mechanical systems. Wilde emphasized the potential benefit of global, non-iterative, analytical optimization techniques to provide insight into the design space and give confidence that solutions were global optima. Haug emphasized the value of design derivatives for interactive, computer aided design optimization of dynamic systems. Siddall emphasized the need to incorporate optimization into the design process. Ragsdell also emphasized the need for a "marriage of optimization and design" where "optimization brings discipline to the design activity". He also highlighted current problem areas such as discrete variables, multiple objectives, large problems, and identification of global solutions. Freudenstein noted the need for a multi-disciplinary approach to the optimum design of mechanical systems.

The use of understanding methods are generally accepted for large optimization problems which have discrete variables, multiple objectives, many design variables and are tightly constrained. Johnson's MOD [122] and Papalambros and Wilde's monotonicity analysis [123] both advocate this type of an approach.

Heuristic combinatorial methods, developed by Lin [124], have been used successfully to solve complex problems in mechanical engineering by Lee [109], Datseris [110], and Lee and Freudenstein [125,126]. The basic approach is to force the design variables to take on discrete values so that a finite number of possibilites are allowed. A heuristic algorithm is then developed to make variable assignments in such a manner that the value of the objective function is reduced and the design variables assume their optimum value.

Methods which analyse an optimization problem with the goal to eliminate constraints and reduce parameters appear to have great potential for complex optimization problems [3-8, 127-130]. These methods substantially improve optimization efficiency because they identify the feasible design space. This eliminates useless function evaluations.

Because design constraints have a significant impact on the selection of the kinematic arrangement, there is a definite need to combine these two analyses into a single comprehensive analysis using an integrated optimization strategy.

1.6 Overview

The large number of design variables and significant number of detailed calculations necessary for each design iteration have, in the past, made analysis of more than a few arrangements impractical. A more effective method is required to generate the system of equations defining the gear diameters in multi-speed gear drives. The method must address both conventional and single composite arrangements in addition to double composites. Further, there must be verification of component strength requirements and eigenvalue placement for the arrangement selected.

This work presents methods to evaluate as many gear arrangements as required, in sufficient detail to obtain a practical optimum solution to a given problem. The approach makes use of a method of kinematic analysis to provide the correct gear ratios, a method of gear design to ensure design life, and a dynamic analysis to ensure acceptable dynamic performance. A problem reduction optimization strategy is used that efficiently incorporate kinematic and component design constraints.

In chapter 2, equations to define the kinematic relationships in a general multi-speed gear train are developed for conventional, single composite and double composite arrangements. A degree of freedom analysis is performed to identify kinematic design variables and constraints for a 2 shaft arrangement. Results for the 2 shaft arrangement are generalized and expanded to handle multi-shaft arrangements. It is shown how the general equations are applied to specific layout diagrams to generate equations for a particular kinematic arrangement. The effect of a fixed lowest output speed/input speed ratio is considered and appropriate equations for the general case are developed. Results are presented for the 4 speed drive for conventional, single composite, and double composite arrangements based on an objective function of minimum radial dimensions.

In chapter 3, equations to rate the power capacity of a gear mesh are presented and discussed. The variables are identified which influence the design of a general gear mesh. These variables are grouped to facilitate understanding of their behavior and influences. Different strategies for the optimization of a gear mesh are studied to understand relative benefit/cost. An approach to determine the optimum diameter of a gear mesh is developed for standard tooth geometry and results are presented for sample operating conditions. A matrix of optimum diameters, called the minimum diameter matrix, is introduced to couple component strength requirements with the kinematic analysis. The 4 speed gear drive presented in chapter 2 is re-optimized with the additional constraint that gear diameters be sufficiently large to meet design life requirements. The effect of including design constraints in the optimization strategy is observed by comparing results of chapters 2 and 3.

In chapter 4, an analytical method to calculate and modify the eigenvalues of a multi-speed gear drive is presented. The form of the mass and stiffness matrices for the general multi-speed gear drive are developed using the four speed gear drive. The Jacobi matrix method of solution is shown to adequately extract eigenvalues and eigenvectors from the characteristic equation. A method to obtain derivatives of the eigenvalues with respect to stiffness and mass elements is applied to the case of multi-speed drives. Results are calculated and presented for the optimal four speed drive obtained in chapter 3.

In chapter 5, a problem reduction optimization strategy is introduced. The key concept used is that of equating the number of parameters in the optimization process to the number of degrees of freedom through reduction of constraints. Objective functions to define drive volume are developed for conventional, single composite and double composite arrangements. For the four speed gear drive subject to an objective function of minimum volume, there are 12 variables and 2 degrees of freedom. The reduction process is demonstrated in detail for this problem. Optimization techniques used in the analysis are described. Results for the 4 speed gear drive, optimized with the objective of minimum volume, are presented and compared with the results of chapters 2 through 4.

In chapter 6, case studies are made of two larger systems to demonstrate the theories presented in chapters 2 through 5. All 38 arrangements of the 9 speed 10 gear drive are optimized for an objective function of minimum volume. These are compared with results published in the literature. Results for the six best conventional arrangements for the 18 speed gear drive are presented. General trends in the analyic results are compared with production drives to corroborate the mathematical model and objective function.

In chapter 7 general conclusions about the study are presented along with an outlook for future work.

CHAPTER 2

DEVELOPMENT OF GOVERNING EQUATIONS FOR THE KINEMATIC ANALYSIS OF GEAR DRIVES

2.1 Introduction

There are many possible arrangements which will produce a specific number of speeds and selecting the best is not obvious. Figures 2.1 through 2.4 show that a four speed gear train, with output speeds forming a geometric progression, can be obtained using 2 shafts and 8 gears, 3 shafts and 8 gears, 3 shafts and 7 gears, or 3 shafts and 6 gears. Each arrangement has advantages and disadvantages. A practical method must be available to analyse all arrangements if the best is to be selected for a particular application. This chapter develops a new approach for the kinematic analysis of multi-speed gear drives.

Past efforts [12-22] have concentrated on developing diameter equations for the single and double composite gear train arrangements because the greatest savings in number of gears is possible. Equations are required for minimum gear arrangements because the degrees of freedom of the system are reduced and the diameters of gears in different groups become interdependent. When these equations are expanded to apply to other types of arrangements they become unnecessarily complicated. Equations for the conventional arrangement are most important since single and double composite arrangements are

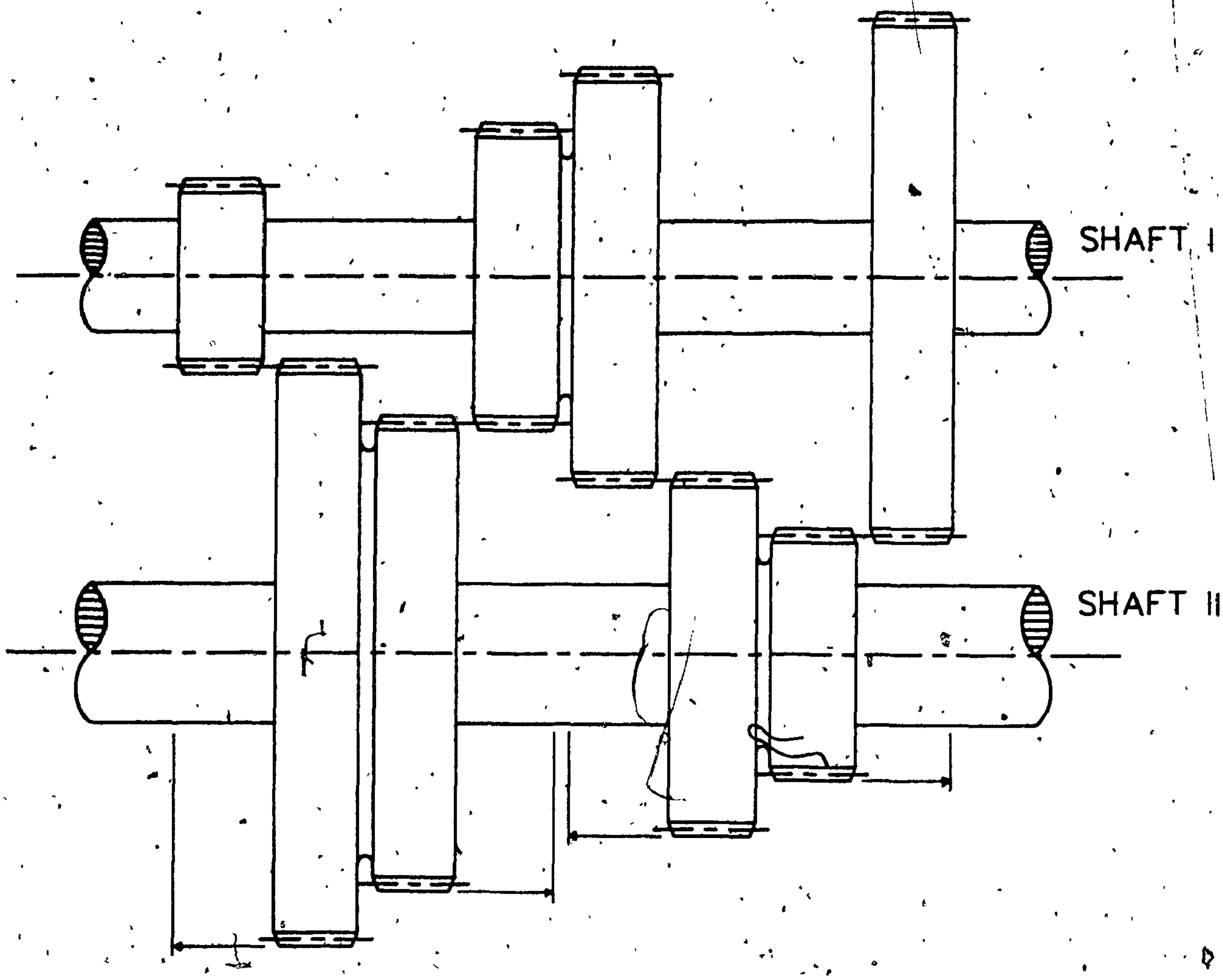


FIGURE 2.1

Conventional 2 Shaft Arrangement for the 4 Speed Gear Drive

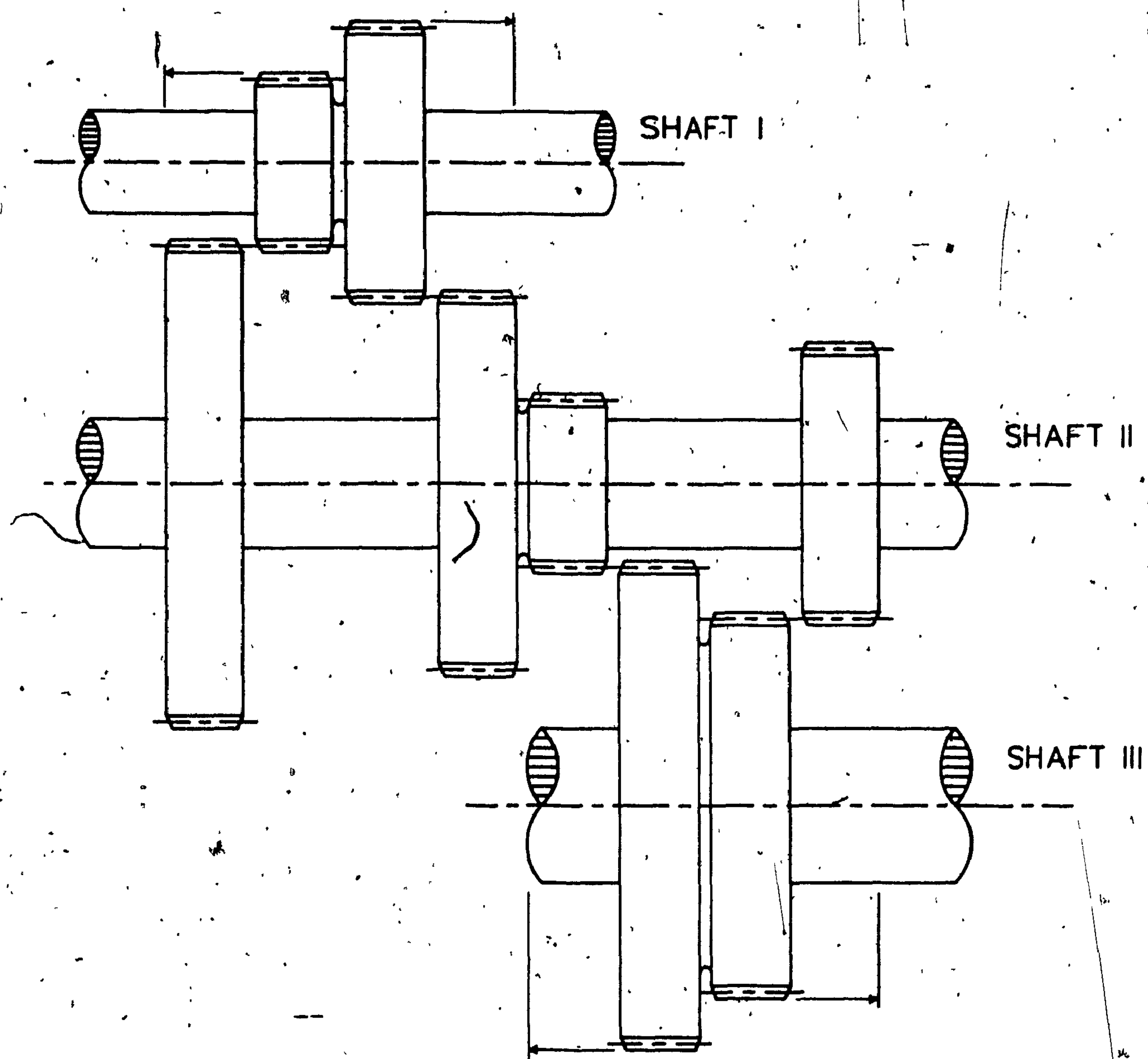


FIGURE 2.2

Conventional 3 Shaft Arrangement for the
4 Speed Gear Drive

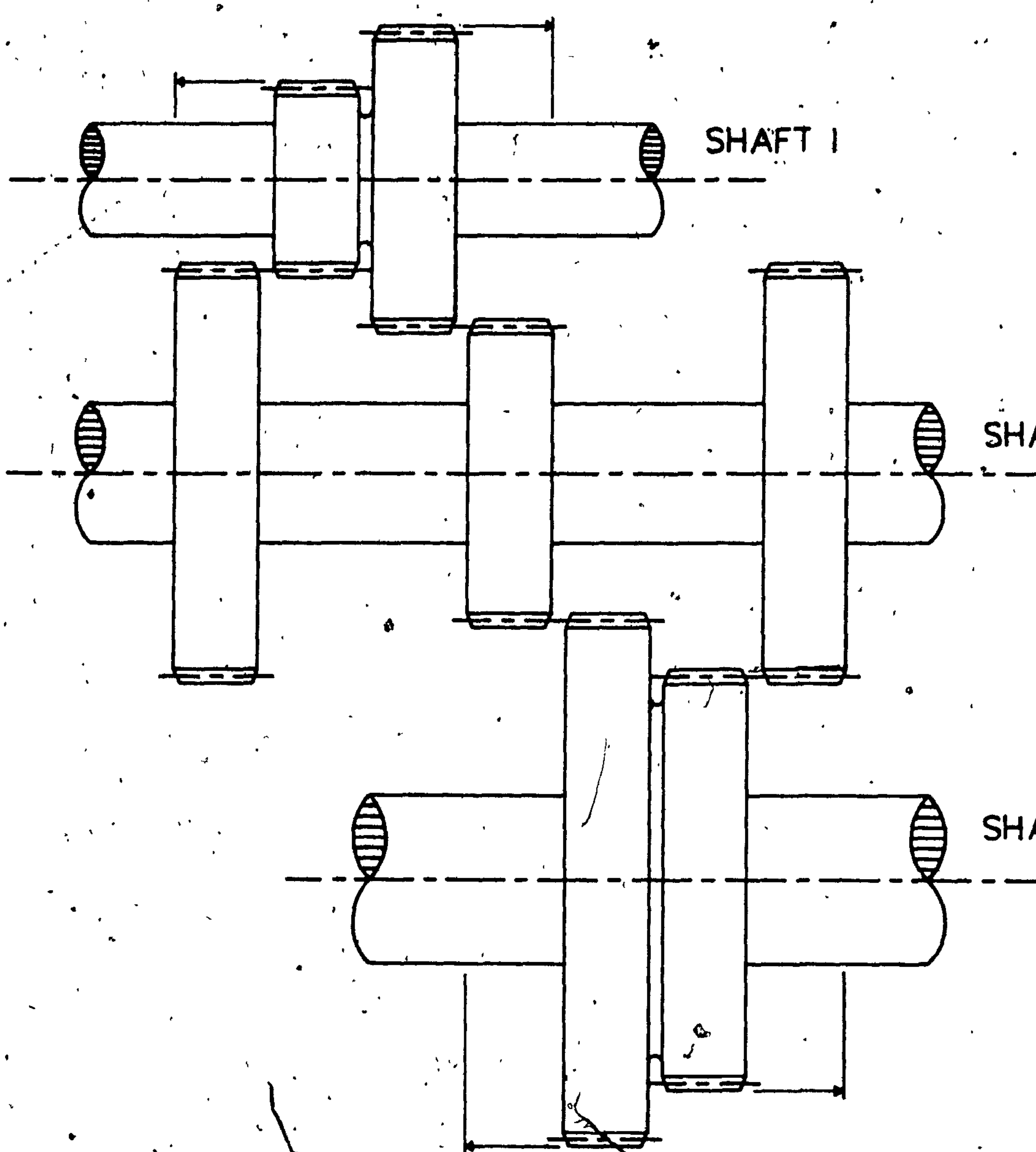


FIGURE 2.3

Single Composite Arrangement for the
4 Speed Gear Drive

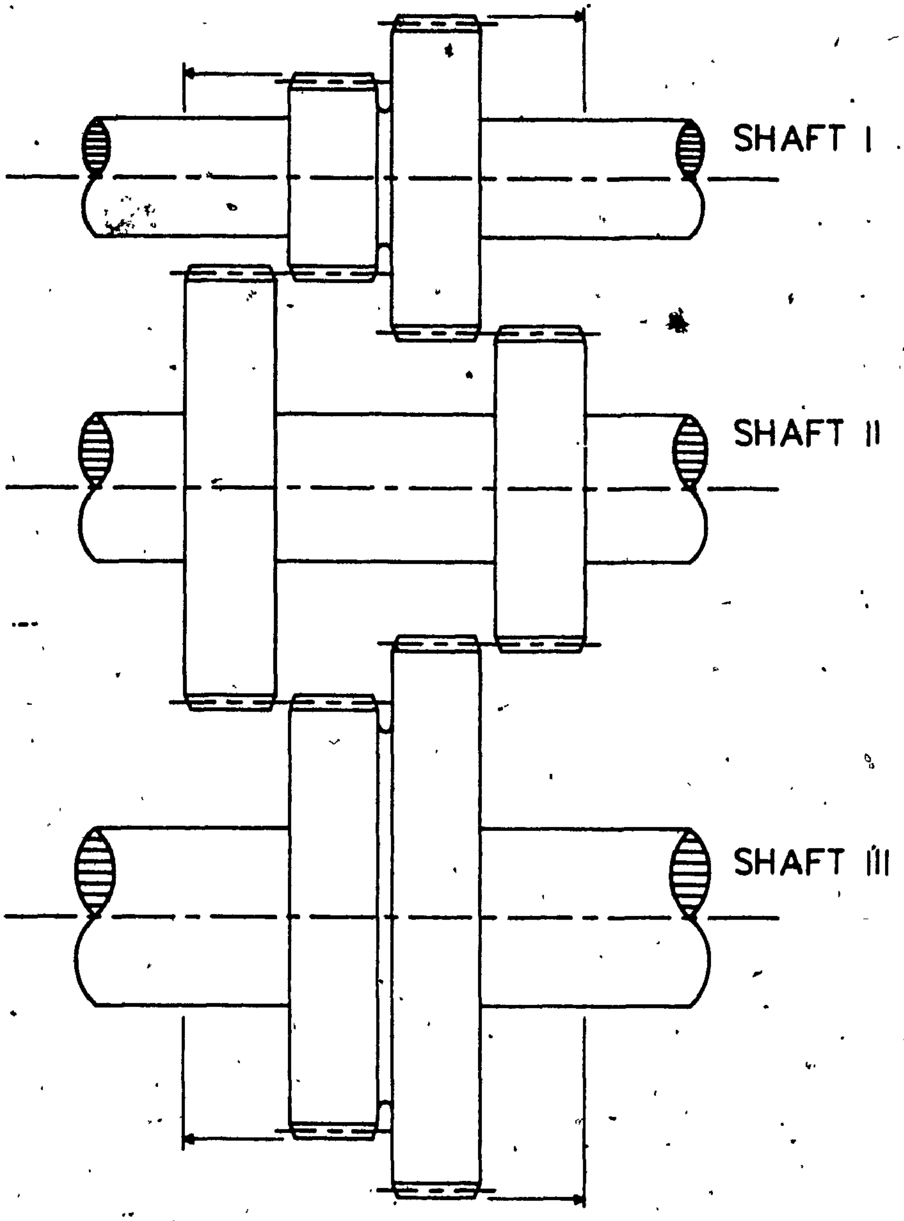


FIGURE 2.4

Double Composite Arrangement for the
4 Speed Gear Drive

special cases of the conventional arrangement. The initial focus will therefore be on developing equations for the conventional arrangement.

2.2 Analysis of a Two Shaft Gear Train

An analysis of a simple gear train will identify important basic properties of a group which can be used as the foundation of a general approach to equations for multi-speed gear drives.

The simplest case of a multi-speed gear train consists of 2 shafts with 2 or more gear pairs and is known as a group. One possible gear drive is shown in figure 2.5. The layout diagram is shown in figure 2.6.

The center distance between shafts is constant for all gears. Therefore the following equations must be satisfied.

$$d_1 + d_5 = d_2 + d_6 \qquad (2.1)$$

$$d_1 + d_5 = d_3 + d_7 \qquad (2.2)$$

$$d_1 + d_5 = d_4 + d_8 \qquad (2.3)$$

In addition, the output speeds are required to form a geometric progression with step ratio ϕ. The variable "S" defines the lowest output/input speed ratio.

$$S = d_1/d_5 \qquad (2.4)$$

$$S\phi = d_2/d_6 \qquad (2.5)$$

$$S\phi^2 = d_3/d_7 \qquad (2.6)$$

$$S\phi^3 = d_4/d_8 \qquad (2.7)$$

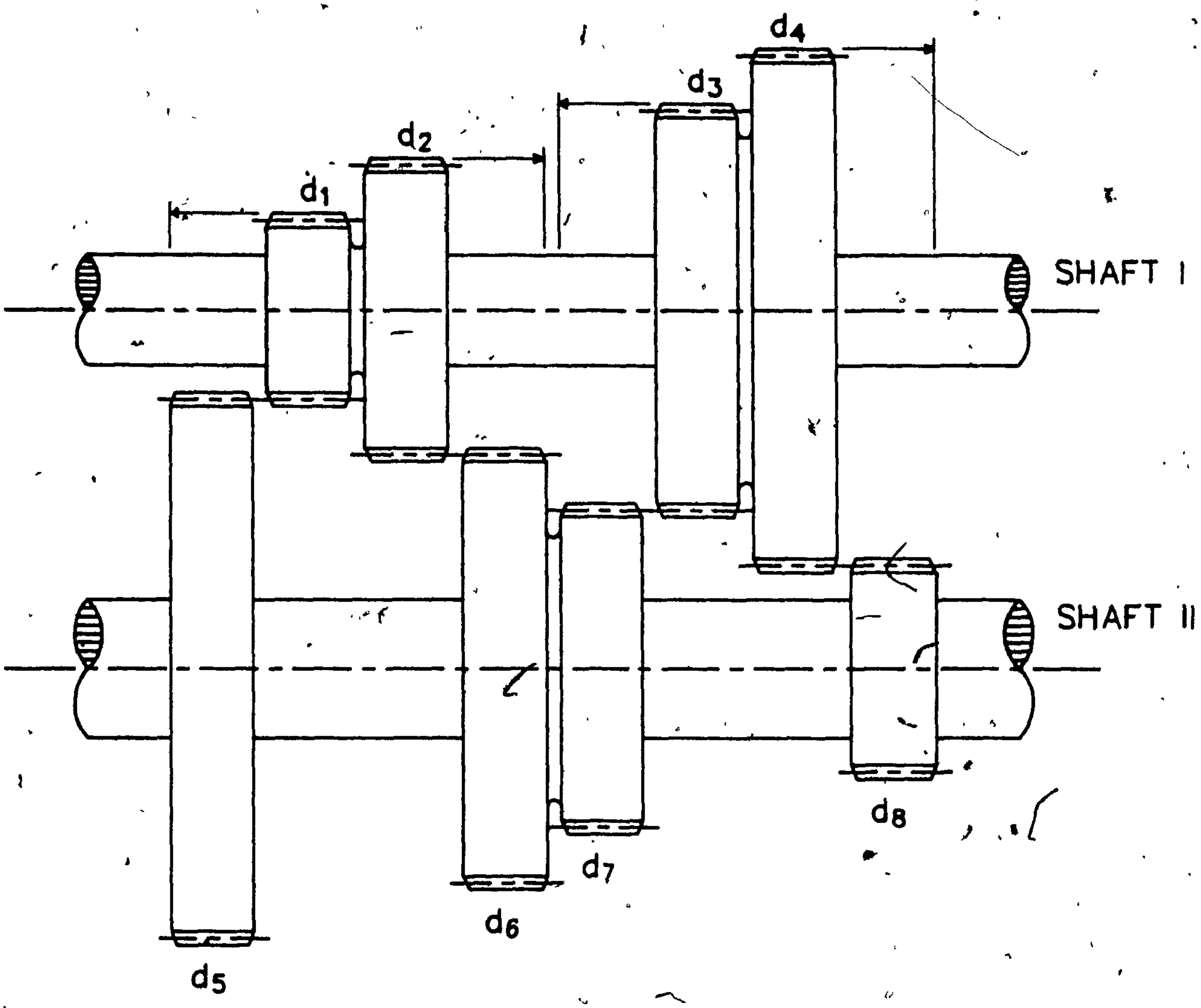


FIGURE 2.5

Simple 2 Shaft Gear Drive Arrangement

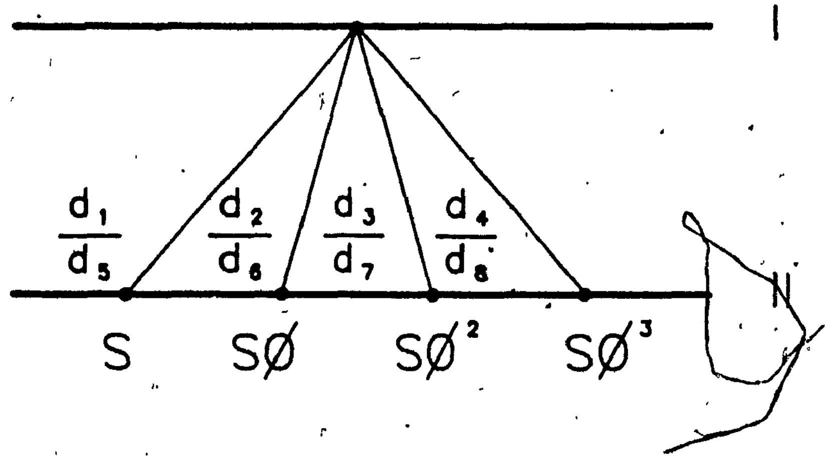


FIGURE 2.6

Layout Diagram for Simple 2 Shaft Gear Drive

Each two shaft arrangement can be shown to have two degrees of freedom. This arrangement has 9 unknowns (d_1 through d_8 and S). To solve for these 9 unknowns there are 7 equations (2.1) through (2.7). Therefore each 2 shaft arrangement has 2 degrees of freedom.

Selecting the diameters of the first gear on each shaft to be the independent variables for a group, it is a simple matter to solve for the diameters of the remaining gears. Designating d_1 (the first gear on the input shaft) as Y_1 and d_5 (the first gear on the output shaft) as Y_2 equations (2.1) through (2.7) can be re-written:

$$Y_1 + Y_2 = d_2 + d_6 \tag{2.8}$$

$$Y_1 + Y_2 = d_3 + d_7 \tag{2.9}$$

$$Y_1 + Y_2 = d_4 + d_8 \tag{2.10}$$

$$S = Y_1/Y_2 \tag{2.11}$$

$$S\phi = d_2/d_6 \tag{2.12}$$

$$S\phi^2 = d_3/d_7 \tag{2.13}$$

$$S\phi^3 = d_4/d_8 \tag{2.14}$$

The diameters of the remaining gears can be found as functions of Y_1, Y_2, and ϕ by re-arranging equations (2.8) through (2.14).

$$d_2 = \frac{Y_1\phi(Y_1 + Y_2)}{(Y_1\phi + Y_2)} \tag{2.15}$$

$$d_3 = \frac{Y_1\phi^2(Y_1 + Y_2)}{(Y_1\phi^2 + Y_2)} \tag{2.16}$$

$$d_4 = \frac{Y_1\phi^3(Y_1 + Y_2)}{(Y_1\phi^3 + Y_2)} \tag{2.17}$$

$$d_6 = \frac{Y_2(Y_1 + Y_2)}{(Y_1\phi + Y_2)} \tag{2.18}$$

$$d_7 = \frac{Y_2(Y_1 + Y_2)}{(Y_1\phi^2 + Y_2)} \tag{2.19}$$

$$d_8 = \frac{Y_2(Y_1 + Y_2)}{(Y_1\phi^3 + Y_2)} \tag{2.20}$$

Comparing equations for the diameters of the gears on the input shaft (i.e. d_1 to d_4) the general form for the diameter of the input gear is observed, where n is the number of meshes in the group.

$$d_j = \frac{Y_1\phi^{j-1}(Y_1 + Y_2)}{(Y_1\phi^{j-1} + Y_2)} \qquad j = 1,2,\ldots n \tag{2.21}$$

Comparing equations for the diameters of the gears on the output shaft (i.e. $d_5 - d_8$), the general form for the diameter of the output gear is observed.

$$d_{j+n} = \frac{Y_2(Y_1 + Y_2)}{(Y_1\phi^{j-1} + Y_2)} \qquad j = 1,2,\ldots n \tag{2.22}$$

A second group attached to the output shaft of the first group as shown in figure 2.7 results in a conventional 3 shaft arrangement and provides two additional degrees of freedom. Each group that is added and used in a conventional manner adds 2 degrees of freedom to the problem.

The total degrees of freedom for a conventional arrangement can be calculated when the number of shafts n_s are known.

$$dof = 2(n_s - 1) \tag{2.23}$$

2.3 Extension to the Multi-Shaft Arrangement

The equations developed for a 2 shaft arrangement can be applied to the general multi-speed gear train if the required arrangement is constructed using a combination of groups (figure 2.7). The multi-shaft arrangement can be isolated into smaller parts for the purpose of writing gear diameter equations. Equations (2.21) and (2.22) can be applied to each group taking into consideration the different step ratio between groups. The variables Y_1 and Y_2 are replaced by the more general values of Y_1 (variable associated with the first input gear of a group) and Y_o (variable associated with the first output gear of a group).

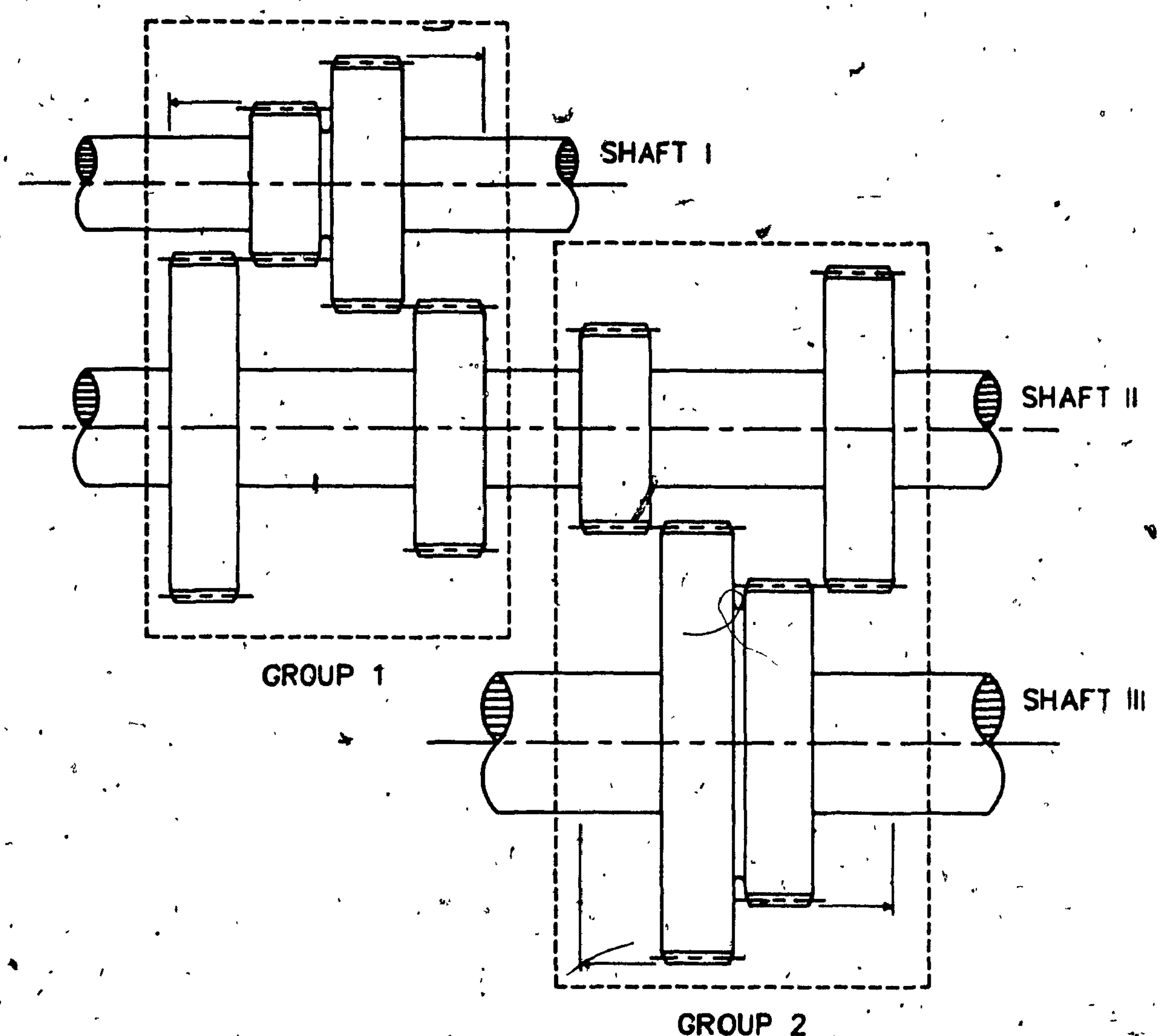


FIGURE 2.7

Combining 2 Groups to make a 3 Shaft Conventional Arrangement

In the two shaft arrangement used to develop the general form of the equations, the speeds increase on the output shaft by a constant factor of ϕ. With multi-shaft arrangements, intermediate shafts must have speed increases of ϕ^p where p is an integer factor known as the range exponent. The range exponent can easily be obtained from the layout diagram by graphical or analytical methods. The range exponent is illustrated in figure 2.8 for the four speed drive.

The range exponent p is used to modify the equations to account for the output speeds of the intermediate shafts which have speed increases in multiples of ϕ. To include p and ϕ in the previous equations must be replaced by ϕ^p giving the final form of the gear diameter equations for a group as:

$$d_j = \frac{Y_1\phi^{p(j-1)}(Y_1 + Y_o)}{(Y_1\phi^{p(j-1)} + Y_o)} \qquad j=1,2,\ldots n \tag{2.24}$$

$$d_{j+n} = \frac{Y_o(Y_1 + Y_o)}{(Y_1\phi^{p(j-1)} + Y_o)} \qquad j=1,2,\ldots,n \tag{2.25}$$

Note that the values of Y_1, Y_o, p and n change for each group and for different layout diagrams.

Given the layout diagram one can directly write the equations for the diameters of the gears relating to the conventional arrangement by repeatedly using equations (2.24) and (2.25) for each group.

2.4 Layout Diagrams

Layout diagrams are used to graphically define the different combinations of gear ratios which can be used to obtain a given number

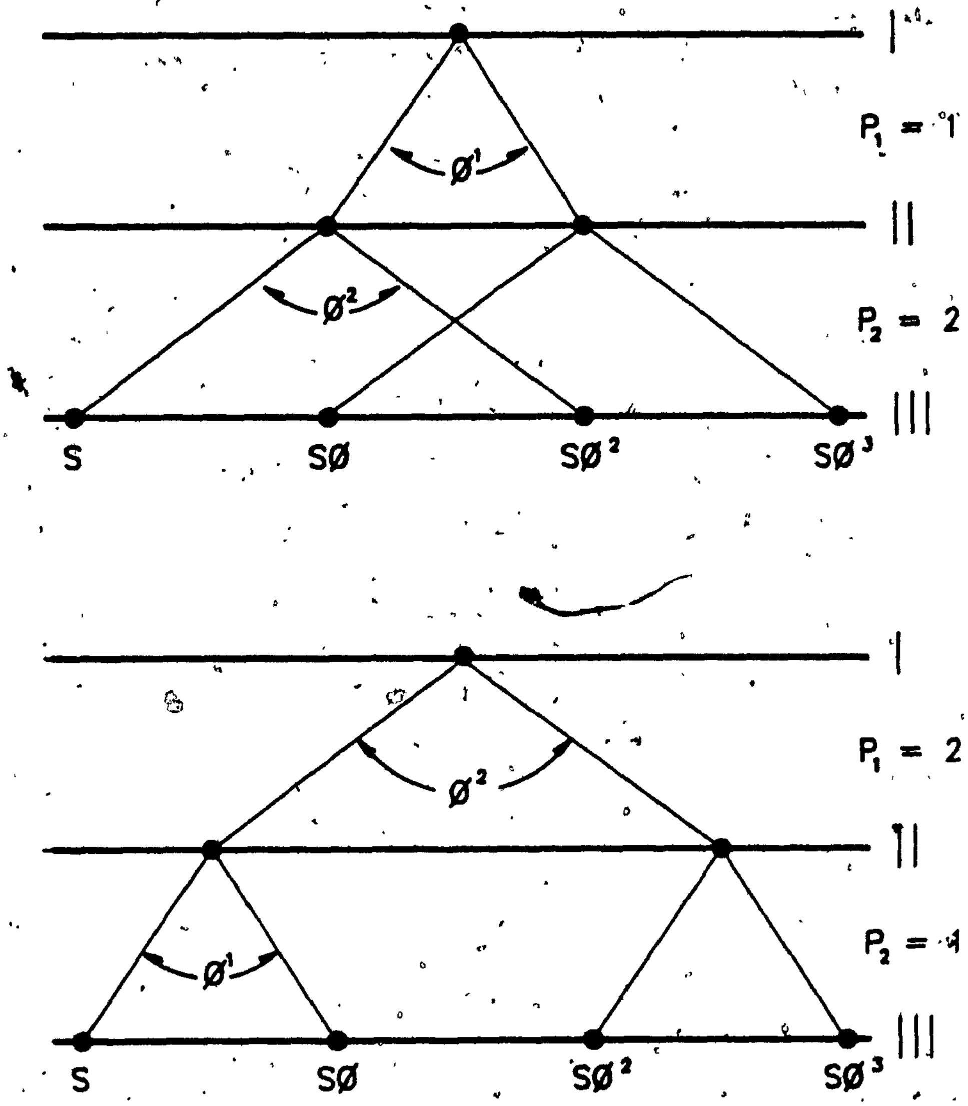


FIGURE 2.8

Illustrating the Range Exponent Using Layout Diagrams for the 4 Speed Drive of speeds. The speeds are plotted horizontally on a logarithmic scale and the shafts are shown as horizontal parallel lines at equal distances from each other. The speed values, appearing on the logarithmic scale are equally spaced since the output speeds form a geometric progression. The transmission ratios at each mesh are indicated by lines between the horizontal lines representing the shafts. If the distance between the horizontal lines for the shafts is constant, then the slope of the mesh lines are an indication of the relative gear ratios. Mesh lines with a negative slope are speed increasing, those with a positive slope are speed reducing.

Layout diagrams are re-named speed diagrams once the actual operating speeds and gear transmission ratios have been determined and applied to a particular layout diagram. To effectively study different arrangements for multi-speed gear trains it is desirable to be able to generate the various layout diagrams automatically based on specific mathematical rules. The following rules serve this purpose.

(a) The number of speeds produced must match the number of speeds required. The number of speeds produced is equal to the product of the number of meshes in each group. Typically, there are from 1 to 3 gear pair meshes per group.

$$z = \prod_{i=1}^{n_s-1} f_i \qquad (2.26)$$

(b) The ratios of the range exponents must be integer and unique for all groups. Identical range exponents produce two paths to the same speed, with the result that the overall range has gaps.

$$p_i \neq p_j \qquad i=1,2,\ldots n_f \text{ and } j=1,2,\ldots n_f \qquad (2.27)$$

(c) The overall speed range must be correct. For a given number of speeds and a given speed ratio, a specific speed range must be covered.

$$\sum (r_i-1)p_i = z-1 \qquad i=1,2,\ldots n_f \qquad (2.28)$$

(d) The range exponents must be selected in such a manner that no speed is obtained by more than one set of gear ratios. The product of the number of speeds in a group multiplied by the range exponent must be divisible by a common factor for all groups.

2.5 Composite Arrangements

The system of mathematical equations defining the conventional arrangement can be adopted for use with composite arrangements. Equality constraints are developed which force the diameter of an output gear in one group to be the same as an input gear in the next group.

2.5.1 Double Composite Arrangements

For a double composite arrangement, the diameters of two input gears in a group are made equal to the diameters of 2 output gears in an adjacent group respectively. For a 3 shaft arrangement with two groups the variables associated with the 4 degrees of freedom would be Y_1, Y_2, Y_3, and Y_4. Since two equality constraints are required to define a double composite, the variables associated with group 2 (Y_3 and Y_4) are dependent and can be solved for explicitly using the following method.

Two output gears in group 1 are used in the composite. The diameters of these gears, D_1 and D_2, can be calculated using variables Y_1 and Y_2 which are known. These diameters can be equated with the equations for the composite input gears of the following group.

$$D_1 = \frac{Y_3C_1(Y_3 + Y_4)}{(Y_3C_1 + Y_4)} \tag{2.29}$$

and

$$D_2 = \frac{Y_3C_2(Y_3 + Y_4)}{(Y_3C_2 + Y_4)} \tag{2.30}$$

For a specific layout and composite, the terms with ø are known and are constant. They have been represented by the constants C_1 and C_2. Both equations (2.29) and (2.30) can be re-arranged to solve for Y_4.

$$Y_4 = \frac{Y_3C_1(Y_3 - D_1)}{(D_1 - Y_3C_1)} \tag{2.31}$$

or

$$Y_4 = \frac{Y_3C_2(Y_3 - D_2)}{(D_2 - Y_3C_2)} \tag{2.32}$$

The variable Y_3 can be found by equating the right side of equations (2.31) and (2.32) and re-arranging to obtain:

$$Y_3 = \frac{D_1D_2(C_2 - C_1)}{D_1C_2(1-C_1) - D_2C_1(1-C_2)} \tag{2.33}$$

Equation (2.33) is solved first to find variable Y_3 which can then be used in either equation (2.31) or (2.32) to find variable Y_4.

If there are more than 2 groups, the composite may reside in any two adjacent groups. The variables which are removed from the parameter list are always from the group closest to the output of the drive.

This method is extremely effective for use in an optimization problem because it can be handled by simply removing 2 variables from the parameter list. This has the direct beneficial effect of reducing the number of parameters to be optimized, as well as the indirect value of not having to solve a multi-parameter optimization problem subject to equality constraints.

2.5.2 Single Composite Arrangements

In the event of a single composite arrangement, only one equality constraint must be developed. In this case Y_3 is used as a parameter and equation (2.31) or (2.32) is used directly to solve for Y_4.

2.6 Four Speed Gear Drive - Verification Problem

The theory of chapter 2 provides equations to duplicate the kinematic optimizations for minimum radial dimensions performed by White and Sangar [14] and Osman, Sankar and Dukkipati [22], but for many kinematic arrangements.

The four speed gear drive using 3 shafts shown in figure 2.9 will be analysed in a progressive fashion to demonstrate the validity of the theory in chapters 2 through 5 and provide a practical illustration. The kinematic requirements of the drive, in future referred to as the verification problem, are given in table 2.1. The objective function

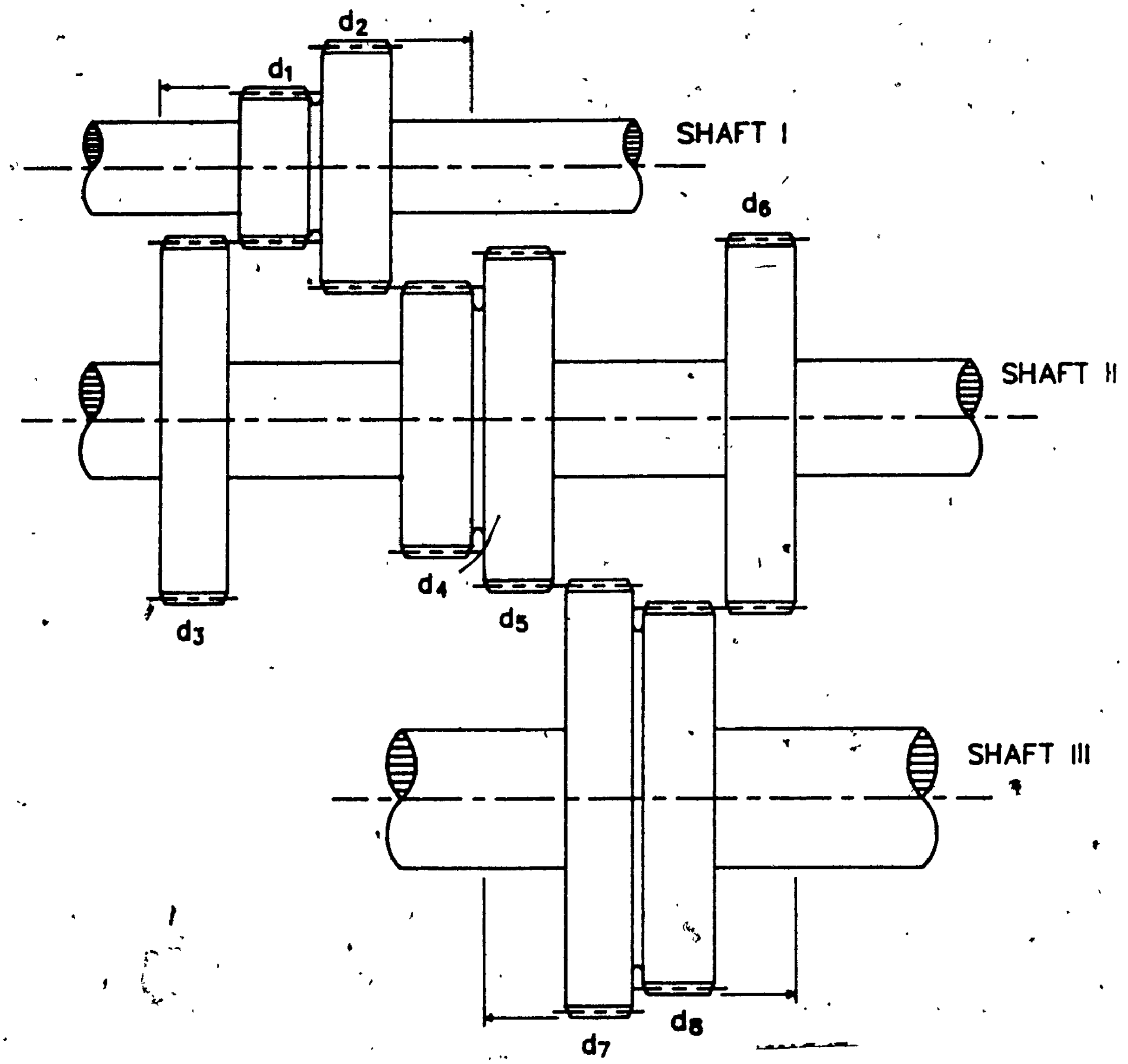


FIGURE 2.9

Conventional Arrangement for the Four Speed Gear Drive - Verification Problem and optimization strategy are presented briefly. A more detailed explanation, with justification of approach, is made in chapter 5.

2.6.1 Objective Function - Radial Dimensions

In this chapter the values of variables Y_1, Y_2, Y_3 and Y_4 which minimize the overall radial dimensions are required for all 3 shaft 4 speed arrangements. This is a common objective [14,15,22,23] and adequately demonstrates the kinematic equations.

TABLE 2.1

Operating Requirements for the Four Speed Verification Problem

NUMBER OF SPEEDS	4
STEP RATIO	1.590
LOWEST OUTPUT SPEED (RPM)	30
INPUT SHAFT SPEED (RPM)	100

The radial dimension was calculated as the sum of the center distances between shafts plus one half the diameter of the largest gear on the first input shaft and last output shaft. For the 4 speed verification problem (chapter 2) the objective function can be written in terms of the gear diameters as follows. Prior to evaluation of the objective function the diameters are calculated using variables Y_1 through Y_4 and equations (2.35) through (2.42).

$$\underset{f(Y_1,Y_2,Y_3,Y_4)}{\text{minimize}} \left[d_2 + \frac{d_4 + d_5}{2} + d_7 \right] \tag{2.34}$$

$$d_1 = Y_1 \tag{2.35}$$

$$d_2 = \frac{Y_1 * p1(Y_1 + Y_2)}{(Y_1 * p1 + Y_2)} \tag{2.36}$$

$$d_3 = Y_2 \tag{2.37}$$

$$d_4 = \frac{Y_2(Y_1 + Y_2)}{(Y_1 * p1 + Y_2)} \tag{2.38}$$

$$d_5 = Y_3 \tag{2.39}$$

$$d_6 = \frac{Y_3 * p2(Y_3 + Y_4)}{(Y_3 * p2 + Y_4)} \tag{2.40}$$

$$d_7 = Y_4 \tag{2.41}$$

$$d_8 = \frac{Y_4(Y_3 + Y_4)}{(Y_3 * p2 + Y_4)} \tag{2.42}$$

p1 = range exponent for group 1

p2 = range exponent for group 2

During the optimization process the diameters of the gears were normalized to ensure that the smallest gear had a diameter value of one. This was accomplished with the use of a matrix of minimum allowable input gear diameters called the minimum diameter matrix.

2.6.2 <u>Minimum Diameter Matrix</u>

A table of minimum diameters is used to provide the optimization routine with the minimum allowable diameter of the input gear of each mesh. The minimum diameter is taken to be a function of the speed reduction ratio, the input gear shaft speed, and composite type. This assumption will be shown to be valid in chapter 3.

With the requirement on diameter being to ensure that the smallest gear is greater than 1, the diameter matrix is simple to obtain. For speed reduction ratios, the entry in the diameter matrix is 1 since the output gear will always have a diameter greater than 1. For speed increasing ratios, the entry in the diameter matrix is equal to the inverse of the gear ratio since the output gear is required to have a minimum value of 1. For a kinematic optimization which neglects component strength, input shaft speed and composite gears have no effect on the minimum diameter. The complete matrix is presented in table 2.2.

TABLE 2.2

Diameter Matrix for Kinematic Optimization.
Conventional or Composite Input Gear

INPUT SPEED	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
50	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
100	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
200	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
300	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
500	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
750	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
1000	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
2000	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00
4000	10.00	5.00	2.00	1.00	1.00	1.00	1.00	1.00

2.6.3 Optimization Strategy

The variables (Y_1 through Y_4) need to be determined to evaluate the objective function. Variables Y_1 and Y_3 are selected from the minimum diameter matrix. Variables Y_2 and Y_4 are calculated from the optimization parameters X_1 and X_2 which represent the lowest reduction ratio of each group. In the event that the diameter of one of the gears in a group is less than 1, then the appropriate variables for the group, (Y_1,Y_2) for group 1 or (Y_3,Y_4) for group 2, are multiplied by the inverse of the smallest gear in the group. The ratios, X_1 and X_2, are used as optimization parameters, rather than Y_2 and Y_4, because they are dimensionless and cover a predictible range. The variable assignments associated with the optimization process are shown in figure 2.10 and explained in further detail below.

(a) Conventional Arrangement

Parameters X_1 and X_2, which represent the speed reduction ratios of group 1 and group 2 respectively, are passed to the routine evaluating the objective function. Given that the lowest input speed is N_L, then the speed of shaft 1 (the input shaft to group 1) can be calculated as:

$$N_1 = \frac{N_L}{X_1 + X_2} \tag{2.43}$$

N_1 = speed of 1st input shaft

N_2 = speed of 2nd input shaft

N_L = speed of output shaft

Similarly the speed of the second shaft can be calculated as:

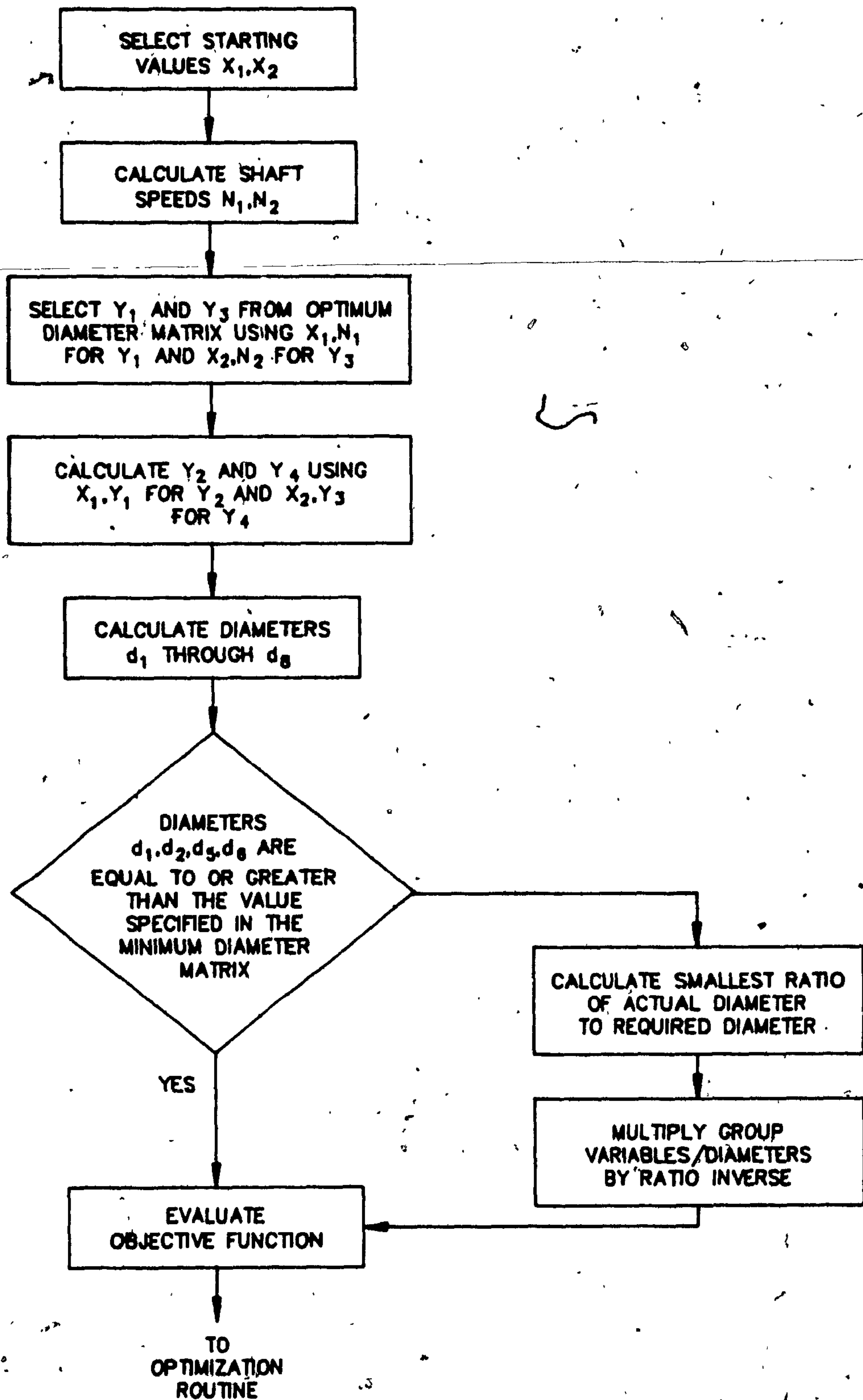


FIGURE 2.10

Variable Assignment for Radial Objective Function

$$N_2 = \frac{N_L}{X_2} \tag{2.44}$$

Given N_1 and X_1, the minimum diameter Y_1 for group 1 can be obtained by interpolation from the minimum diameter matrix. Next Y_3 can be be obtained using N_2 and X_2. Variables Y_2 and Y_4 can then be calculated.

$$Y_2 = Y_1X_1 \tag{2.45}$$

and

$$Y_4 = Y_3X_2 \tag{2.46}$$

Variables Y_1 through Y_4 completely define the kinematic arrangement and allow the calculation of all diameters in the drive.

A final check is made to ensure that the diameter of the input gear in each mesh is equal to or larger than the diameter in the diameter matrix. If a diameter is too small, the variables associated with the group in question are ratioed up to the value required to meet the minimum diameter requirements.

(b) Single Composite

The procedure for the single composite is similar to that for the conventional arrangement except that variable Y_3 is calculated directly using a modified form of equation (2.31). Diameter D_1 represents the diameter of the composite gear and can be calculated using variables Y_1 and Y_2 as shown in section 2.5.

$$X_2 = \frac{Y_4}{Y_3} \tag{2.47}$$

Substituting equation (2.47) into (2.31):

$$Y_3 = \frac{D_1(C_1 + X_2)}{C_1(1+ X_2)} \tag{2.48}$$

This diameter is then compared with the diameter specified in the minimum diameter matrix. The larger of the two diameters is used.

(c) Double Composite

For a double composite arrangement both Y_3 and Y_4 are calculated directly using the diameters D_1 and D_2 calculated from variables Y_1 and Y_2 as presented in section 2.5.

2.6.4 Discussion of Results

The definition of the different kinematic arrangements are shown in table 2.3. Results for the 4 speed gear drive are presented in table 2.4 for a step ratio of 1.59. Optimum gear diameters are presented for 1 conventional arrangement, 4 single composite arrangements and 1 double composite arrangement for each of 2 layouts. The gears are labeled in accordance with the conventional arrangement for the 4 speed gear drive shown in figure 2.5.

A number of simple checks can be made to verify the correctness of the equations.

(a) The center distance for each group must be a constant.

(b) The output speeds must form a geometric progression with lowest output/input speed ratio equal to S.

(c) the diameters of gears identified as composites in table 2.3 must be equal.

TABLE 2.3

Kinematic Requirements for each Optimization for 4 Speed Arrangements

OPT #	FACTORS #1	FACTORS #2	RANGE EXPONENT #1	RANGE EXPONENT #2	COMPOSITE ARRANGEMENTS	
1	2	2	1	2	-	-
2	2	2	1	2	$d_3 = d_5$	-
3	2	2	1	2	$d_3 = d_6$	-
4	2	2	1	2	$d_4 = d_5$	-
5	2	2	1	2	$d_4 = d_6$	-
6	2	2	1	2	$d_3 = d_6$	$d_4 = d_5$
7	2	2	2	1	-	-
8	2	2	2	1	$d_3 = d_5$	-
9	2	2	2	1	$d_3 = d_6$	-
10	2	2	2	1	$d_4 = d_5$	-
11	2	2	2	1	$d_4 = d_6$	-
12	2	2	2	1	$d_3 = d_6$	$d_4 = d_5$

A number of simple checks can also be made to verify the correctness of the optimization process.

(a) The value of the objective function for the conventional arrangement must be equal to the minimum because the diameters assumed can always be equivalent to a single or double composite arrangement (the number of gears does not affect the objective funtion).

(b) A diameter equivalent inverted arrangement should be found for each of the optimizations since the layouts are reciprocal.

(c) Since diameters 4 and 5 of conventional arrangement 1 are equal, optimization 4 which is a composite arrangement

TABLE 2.4

Optimization Results for the Verificaiton Problem
Four Speed Gear Drive

		OPTIMIZATION NUMBER					
		1	2	3	4	5	6
DIAMETER	1	1.000	1.000	1.000	1.000	1.000	4.723
	2	1.261	1.261	1.295	1.261	1.322	5.613
	3	1.261	1.261	1.590	1.261	1.912	3.520
	4	1.000	1.000	1.295	1.000	1.590	2.631
	5	1.000	1.261	1.000	1.000	1.000	2.631
	6	1.590	1.921	1.590	1.590	1.590	3.520
	7	1.590	1.660	1.590	1.590	1.590	1.889
	8	1.000	1.000	1.000	1.000	1.000	1.000
RATIOS	S	0.499	0.603	0.396	0.499	0.329	1.868
	1	0.793	0.793	0.629	0.793	0.523	1.342
	2	1.261	1.261	1.000	1.261	0.831	2.133
	3	0.629	0.760	0.629	0.629	0.629	1.393
	4	1.590	1.921	1.590	1.590	1.590	3.520
F		3.851	4.051	4.033	3.851	4.207	10.133

		OPTIMIZATION NUMBER					
		7	8	9	10	11	12
DIAMETER	1	1.000	1.000	1.000	1.000	1.000	1.000
	2	1.590	1.590	1.590	1.590	1.660	1.889
	3	1.590	1.590	1.590	1.590	1.921	3.520
	4	1.000	1.000	1.000	1.000	1.261	2.631
	5	1.000	1.590	1.295	1.000	1.000	2.631
	6	1.261	1.912	1.590	1.261	1.261	3.520
	7	1.261	1.322	1.295	1.261	1.261	5.613
	8	1.000	1.000	1.000	1.000	1.000	4.724
RATIOS	S	0.499	0.756	0.629	0.499	0.413	0.133
	1	0.629	0.629	0.629	0.629	0.521	0.284
	2	1.590	1.590	1.590	1.590	1.316	0.718
	3	0.793	1.203	1.000	0.793	0.793	0.469
	4	1.261	1.912	1.590	1.261	1.261	0.745
F		3.851	4.207	4.033	3.851	4.051	10.133

using these gears should find the equivalent design point.

In all cases these checks are seen to be true to 3 decimal place accuracy.

Optimizations 1, 4, 7 and 10 all provide solutions with equivalent values of the objective function. Optimizations 1 and 4 have a close ratio in the first group as compared to optimizations 7 and 10 which have the wide ratio in the first group. Optimization 4 and 10 are both single composites.

Based on this analysis, arrangements 4 and 10 would be selected over conventional arrangements 1 and 7 because of the fewer number of gears and the potentially smaller width. Analytically, arrangements 4 and 10 are equivalent. The gear drive of optimization 4 is illustrated in figure 2.11.

It is interesting to note that both double composite arrangements are 2.6 times larger than the optimum solution.

2.7 Constraints on the Lowest Output/Input Ratio

Frequently, the input shaft speed and the output shaft speeds are fixed within narrow limits because of operating requirements. This condition results in a further equality constraint of the form:

$$S = \prod_{j=1}^{n_g} Y_{2j-1}/Y_{2j} = \text{constant} \qquad (2.49)$$

A constraint on the lowest output/input ratio reduces the degrees of freedom of the drive by one and requires special consideration for

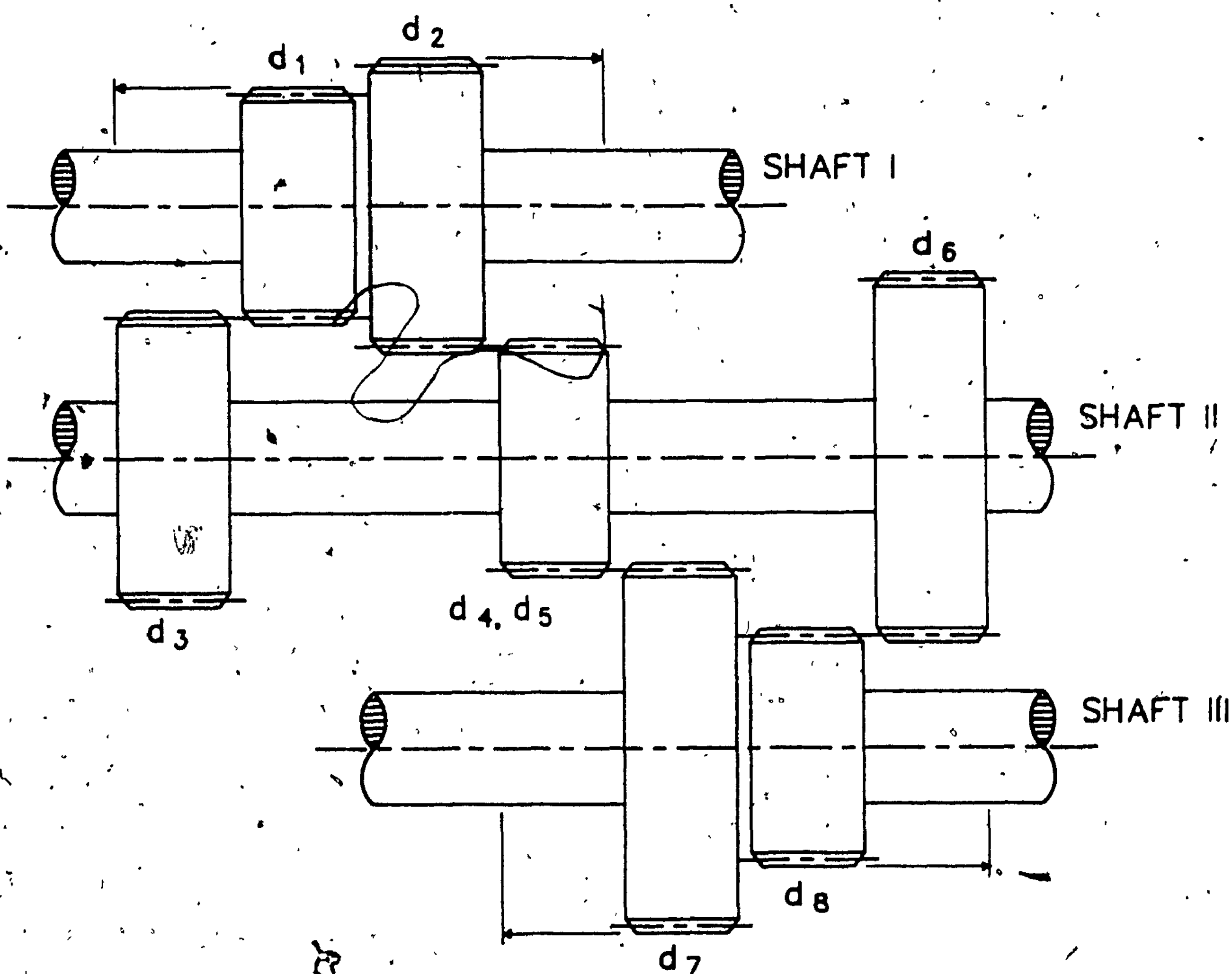


FIGURE 2.11

Gear Drive Based on Optimization 4 from Table 2.4

single and double composite arrangements in order to avoid less efficient optimization techniques.

2.7.1 Case of Two Groups and a Double Composite

If the number of groups is two, then a double composite arrangement requires Y_1 and Y_2 to be known in order to calculate diameters d_1 and d_2 and Y_3 and Y_4 are solved for using equations (2.31) and (2.33). Since Y_1 is used as a reference gear, only Y_2 remains available for use in satisfying this constraint. Although a single unique solution is mathematically possible, it most frequently results in one of the diameters taking on a negative value. Problems with 2 groups, a double composite arrangement, and a required value of S are considered over constrained and are not analysed.

2.7.2 Case of More than Two Groups and a Double Composite

If the number of groups is greater than two and the composite is not part of the last group, then variable Y_{2ng} is solved for directly using the following equation after the variables associated with the composite have been calculated.

$$Y_{2ng} = \prod_{j=1}^{n_g} \frac{Y_{2j-1}}{Y_{2j}} \frac{Y_{2ng-1}}{S} \tag{2.50}$$

Analysis for the remaining portions of the drive are carried out as presented in sections 2.5 and 2.6.

If the composite arrangement is part of the last group, then Y_2 is solved for using the equation:

$$Y_2 = \prod_{j=1}^{n_g} \frac{Y_{2j-1}}{Y_{2j}} \frac{Y_1}{S} \tag{2.51}$$

2.7.3 Case of Two Groups and a Single Composite

With two groups and requirements for a single composite arrangement special equations are used to find Y_3 and Y_4. The equation for the lowest output/input ratio is given as:

$$S = \frac{Y_1 Y_3}{Y_2 Y_4} \tag{2.52}$$

This equation is re-arranged to solve for Y_3 which is then used to replace that variable in equation (2.31). The result is the following two equations which can be used to solve for Y_3 and Y_4.

$$Y_4 = \frac{d_1 Y_1 (SC_1 Y_2 + Y_1)}{SC_1 Y_2 (SY_2 + Y_1)} \tag{2.53}$$

and

$$Y_3 = \frac{SY_2 Y_4}{Y_1} \tag{2.54}$$

The resulting system of equations ensure one single composite (any of those possible for a specific arrangement) and one equality constraint on the lowest input/output ratio.

2.7.4 Four Speed Drive Results for Constrained Output/Input Ratio

Results are presented for the four speed drive in table 2.5. Operating conditions are identical to those for the results presented in table 2.4 however the lowest output/input speed ratio is constrained to a value of 0.3 (i.e. the requirement specified in table 2.1 for an input speed of 100 RPM is satisfied).

The optimization strategy is altered slightly from that presented in section 2.6 since the number of parameters is reduced. Because of the equality constraint, there are no degrees of freedom and hence no solutions for the double composite arrangements.

The best arrangement was optimization 1 with an objective function value of 4.306. The requirement for this particular lowest step ratio resulted in an increase in radial dimensions of 11.8% when compared with the global optimum. If the value of S selected (0.3) had been further away from the optimum value of S (.499) the increase could have been larger. The gear drive is shown in figure 2.12. The speed diagram is shown in figure 2.13.

2.8 Summary

In this chapter, basic methods to generate equations for the diameter of each gear in a multi-speed gear drive are presented. The method can deal with conventional, single composite, double composite and restricted output/input ratio arrangements in a straight forward and efficient manner. These equations result in gear diameters which satisfy all kinematic requirements. The concept of optimum diameter matrix for use in relating the kinematic requirements to physical size of components is introduced. This process will be further developed in chapter 3 to provide a link between the kinematic analysis and gear power capacity. Results are presented for the 4 speed gear drive which verify the mathematical correctness and utility of the equations developed.

TABLE 2.5

Optimization Results for the Verification Problem
Four Speed Drive (S = 0.30)

		OPTIMIZATION NUMBER					
		1	2	3	4	5	6
DIAMETER	1	1.000	1.000	1,000	1.000	1.000	NO SOLUTION
	2	1.335	1.261	1.303	1.261	1.325	
	3	2.096	1.261	1.673	1.261	1.942	
	4	1.761	1.000	1.371	1.000	1.618	
	5	1.000	1.261	1.000	1.000	1.000	
	6	1.590	2.246	1.673	1.781	1.618	
	7	1.590	3.333	1.992	2.644	1.716	
	8	1.000	2.349	1.319	1.863	1.098	
RATIOS	S	0.300	0.300	0.300	0.300	0.300	
	1	0.477	0.793	0.598	0.793	0.515	
	2	0.758	1.261	0.950	1.261	0.819	
	3	0.629	0.378	0.502	0.378	0.583	
	4	1.590	0.956	1.269	0.956	1.473	
F		4.306	5.725	4.480	4.905	4.350	

		OPTIMIZATION NUMBER					
		7	8	9	10	11	12
DIAMETER	1	1.000	1.000	1.000	1.000	1,000	NO SOLUTION
	2	1.744	1.590	1.590	1.590	1.670	
	3	2.398	1.590	1.590	1.590	1.974	
	4	1.654	1.000	1.000	1.000	1.304	
	5	1.000	1.590	1.190	1.000	1.000	
	6	1.275	2.123	1.590	1.335	1.304	
	7	1.390	3.333	2.496	2.096	1.689	
	8	1.115	2.800	2.097	1.761	1.385	
RATIOS	S	0.300	0.300	0.300	0.300	0.300	
	1	0.417	0.629	0.629	0.629	0.507	
	2	1.054	1.590	1.590	1.590	1.281	
	3	0.719	0.477	0.477	0.477	0.592	
	4	1.144	0.758	0.758	0.758	0.941	
F		4.461	6.219	5.181	4.687	4.511	

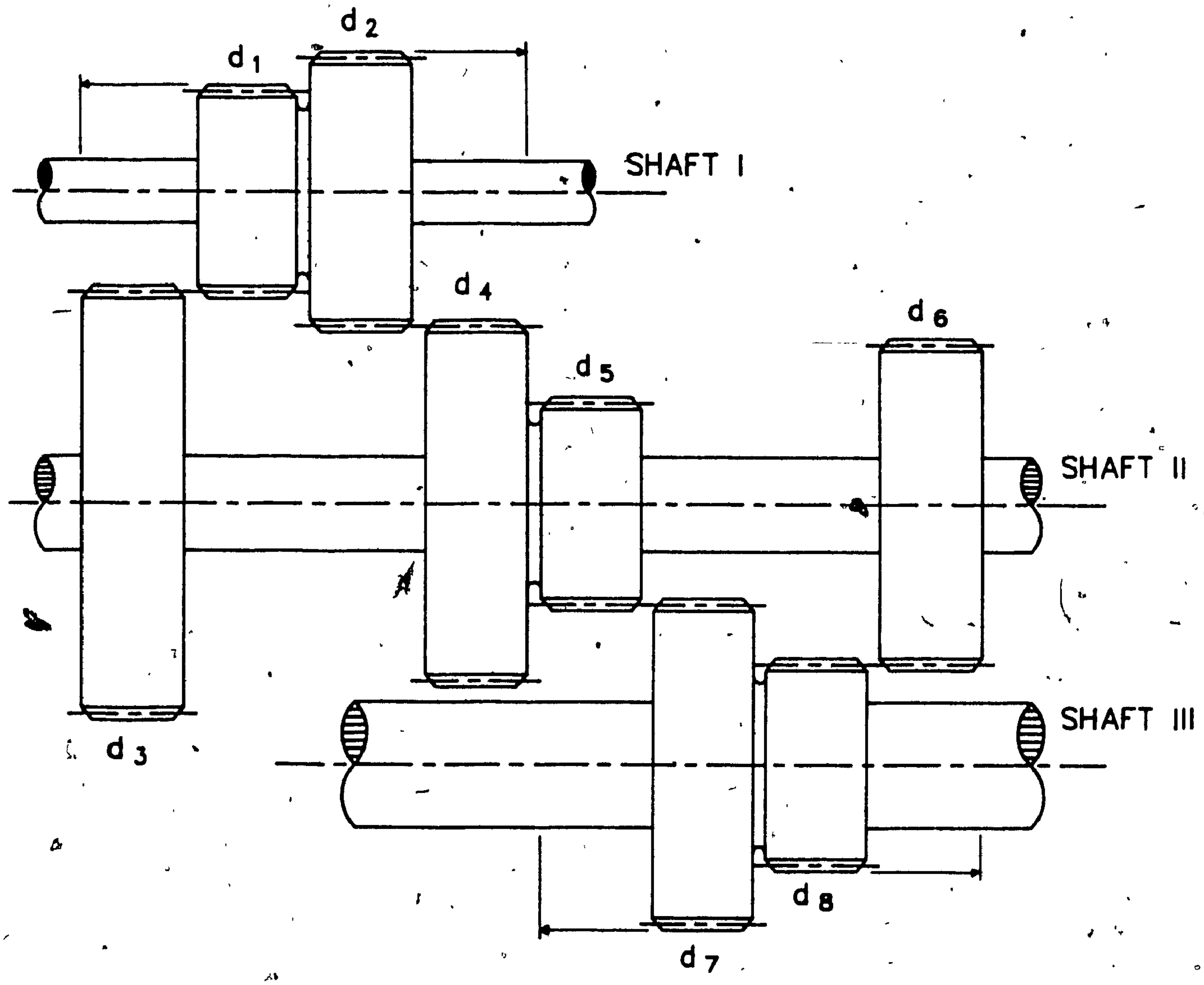


FIGURE 2.12

Gear Drive Based on Optimization 1 from Table 2.5

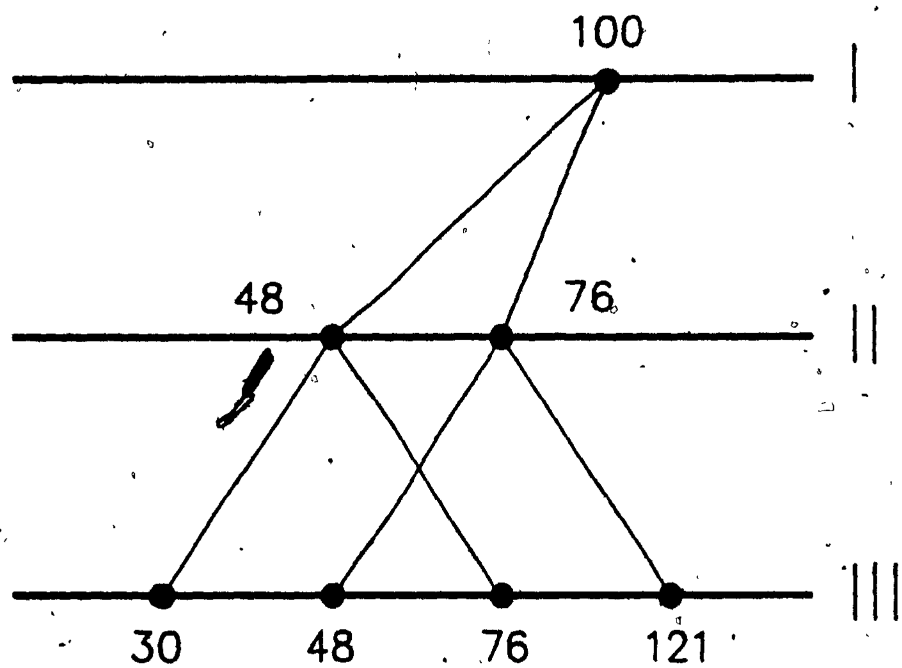


FIGURE 2.13

Speed Diagram based on Optimization 1 from Table 2.5

CHAPTER 3

GEAR MESH DESIGN AND OPTIMIZATION

3.1 Introduction

In chapter 2, optimum arrangements for the four speed drive were determined based on an objective function of minimum radial dimensions without regard for component power capacities. In practice, these solutions could not be implemented efficiently. The size of gears on shafts running at slow speeds have the same size as gears on shafts running at high speeds (i.e. $d_1 = d_5 = 1.000$ from table 2.3) despite the fact that they are carrying significantly different torques. In a real drive, if face widths are to be reasonable, the input gear diameters must change because they are a function of gear ratio and input gear speed.

Required is a method to incorporate the diameter constraints imposed by component design on the arrangement optimization. The arrangement optimization determines diameters of specific gears through the use of the minimum diameter matrix. Therefore it is possible to incorporate component design requirements through this matrix.

In the following sections, the optimization of a gear mesh consisting of a pinion and gear is presented. The goal is to determine the values of the design variables which minimize center distance for given operating conditions and power requirements. A matrix of these optimum values is then generated as a function of gear ratio and input gear speed for use in the arrangement optimization.

3.2 Spur and Helical Gear Power Rating

The power capacity of each gear in a drive can be calculated for bending strength and pitting resistance. The power capacity is calculated in accordance with AGMA 218, the North American standard for rating the pitting resistance and bending strength of spur and helical involute gear teeth.

3.2.1 Bending Strength/Tooth Breakage

From AGMA 218 the bending strength power capacity (i.e. the power which can be transmitted for the design life of the gear drive without causing root fillet cracking or failure) can be calculated using the following equation:

$$P_{at} = \frac{n_p d K_v \; F m \; J \; s_{at}}{1.91 \times 10^7 \; K_{SF} \; K_s K_m \; K_T} \tag{3.1}$$

P_{at} - Bending strength power capacity

d - Pitch diameter

n_p - Pinion speed

K_v - Dynamic factor

F - Face width

J - Geometry factor

s_{at} - Material bending strength

K_{SF} - Service factor

m - Module

K_s - Size factor

K_m - Lead modification factor

K_T - Temperature factor

A check must also be made to ensure that the yield strength will not be exceeded when the number of fatigue cycles is low. The calculation for bending strength must be carried out for both the input gear and the output gear in a mesh. The allowable power capacity is equal to the lower of the values obtained.

3.2.2 Pitting Resistance

From AGMA 218 the surface fatigue power capacity for the avoidance of surface fatigue (i.e. the power that can be transmitted for the design life of the gear drive without destructive pitting of the gear teeth occurring) can be calculated using the following equation.

$$P_{ac} = \frac{n_p F \, I C_v}{1.91 \times 10^7 \, C_{SF} C_s C_m C_f} \left[\frac{d S_{ac} C_H}{C_p \quad C_T} \right]^2 \tag{3.2}$$

P_{ac} - Pitting resistance power capacity

n_p - Pinion speed

F - Face width

I - Geometry factor

C_v - Dynamic factor

d - Pitch diameter

Sac - Material fatigue strength

C_H - Hardness factor

C_{SF} - Service factor

C_s - Size factor

C_m - Lead modification factor

Cf - Finish factor

Cp - Elastic coefficient

C_T - Temperature factor

Equation (3.2) must be applied to both the input gear and the output gear in a mesh. The allowable power capacity is equal to the lower of the values obtained.

The analysis of load and stress modifying factors is similar to that for bending strength and, as a result, many of those factors have identical values.

3.2.3 Scoring

Gearing operates in three regimes of lubrication [131]. In regime I, oil film breakdown is caused by high pressure squeezing the film out from between the gear teeth resulting in asperity contact. Pitch line velocity is low, providing a boundary lubrication environment where there is constantly some asperity contact. In regime II, good lubrication normally occurs since pitch line velocities are sufficient to develop a proper lubricant film without excessive heat being generated. In regime III, oil film breakdown is a result of high gear blank temperature or overload. Since the pitch line velocity is high, even short contact of the tooth profile can produce significant heat which causes a reduction in the oil film and rapid failure.

For the range of analysis under consideration, scoring is not expected to be the active constraint. As a precaution, the probability of scoring can be assessed by calculating the scoring number as suggested by Dudley [24].

$$SCN = \frac{W_t^{3/4} \, n_p^{1/2}}{F \, P_d^{1/4}} \tag{3.3}$$

A scoring number of less than 10,000 indicates that the design is adequate, provided gear blank temperature does not exceed 150°C and pitch line velocity is greater than 1.0 m/sec (197 ft/min). Higher gear blank temperatures or lower pitch line velocities require special consideration for lubrication to ensure adequate performance. When the design is marginal, extreme pressure lubricants may be used. These lubricants react chemically at locations of high pressure and temperature, forming a protective coating with low shear strength [132].

3.3 Variables Which Affect the Power Rating of a Gear Mesh

Tables 3.1 though 3.4 provide lists of variables which affect the power capacity of a gear mesh. The lists are long and the relationships between variables are complex. A number of the design variables are strongly dependent and care must be taken to ensure a consistent set when making changes.

The list of variables are grouped as follows; (1) variables which define operating conditions, (2) variables which relate to material properties, (3) variables which define tooth geometry, and (4) variables which determine tooth size and gear diameters.

Operating condition variables are determined by the application to which the gear mesh is to be used. They are normally based on user requirements and are thus constant for a particular design. They are not allowed to be changed during the optimization process.

TABLE 3.1

Gear Pair Design Variables -
Operating Conditions/Requirements

VARIABLES	DESCRIPTION OF VARIABLE
NY	Life in Years 24 hr Service
NF	Failures/1000 Parts
SO	Characteristic of Source
LO	Characteristic of Load
QV	Quality of Gearing
LM	Lead Modification
FP	Tooth Surface Finish
LSHAR	Load Sharing
ST	Critical Service
CN	Mesh Centerline to Bearings
CO	Type of Composite
NP	Input Speed
MG	Gear Ratio
POW	Required Power Capacity

Material properties are defined by the materials selected for the pinion and gear and the precision of the drive. As the quality and cost of the gear materials increase, the center distance of the mesh is reduced. In addition to reducing mesh size, selection of materials can be used to minimize costs. Pinions or gears in a mesh which have reserve capacity can have less expensive materials substituted with no impact on the remainder of the drive.

Tooth geometry is a function of the cutter used to generate the tooth. Figure 3.1 defines the different variables associated with the cutter. The pinion and gear may be cut using different cutters, hence these variables are defined for both. Changes in the cutter geometry are normally effected to balance power capacity or to reduce undercutting or to increase contact ratio. Changes which result in non-standard tooth geometry must be done carefully because of the potential

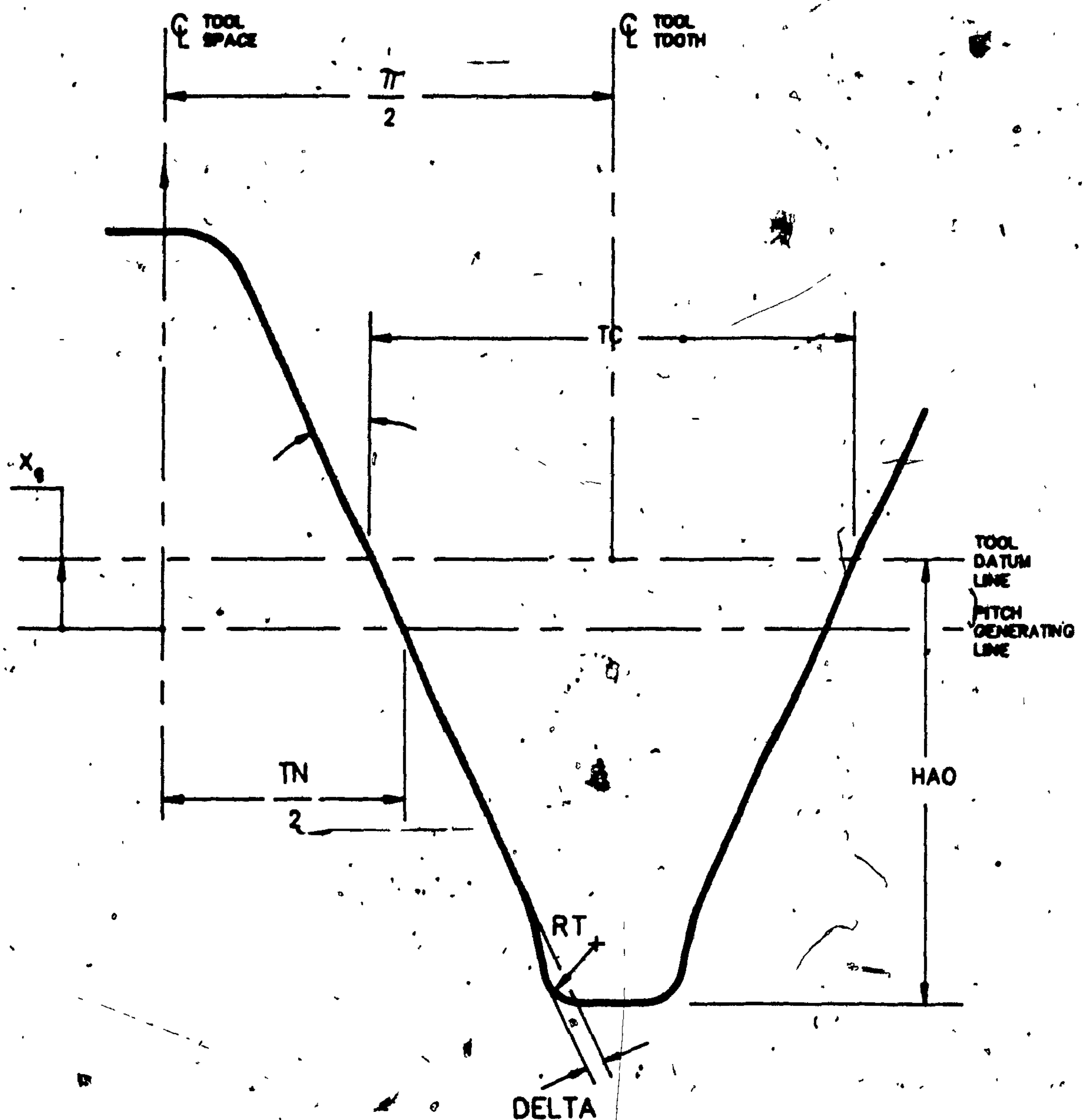


FIGURE 3.1

Variables Defining Cutter Geometry

TABLE 3.2

Gear Pair Design Variables - Tooth Shape

VARIABLES	DESCRIPTION OF VARIABLE
PHIC	Cutter Normal Tooth Thickness
CHIS	Helix Angle
CTYPEP	Pinion Cutter Type
CTYPEG	Gear Cutter Type
DCP	Pinion Cutter Diameter
DCG	Gear Cutter Diameter
AP	Pinion Addendum
AG	Gear Addendum
BP	Pinion Dedendum
BG	Gear Dedendum
TNP	Pinion Normal Tooth Thickness
TNG	Gear Normal Tooth Thickness
RTP	Pinion Cutter Tip Radius
RTG	Gear Cutter Tip Radius
DELTAP	Pinion Cutter Protuberance
DELTAG	Gear Cutter Protuberance
HAOP	Pinion Cutter Addendum
HAOG	Gear Cutter Dedendum
TCP	Pinion Cutter Tooth Thickness
TCG	Gear Cutter Tooth Thickness

for significantly different performance characteristics and increased manufacturing costs.

3.4 Mesh Optimization Strategy

Figure 1.7 shows a typical curve of power capacity versus module for each of 4 failure modes. Those modes are pinion bending strength, gear bending strength, pinion pitting resistance strength and gear pitting resistance strength. Generally the point of maximum strength occurs when the pinion bending strength and pinion fatigue resistance match. This, however, is not true for all operating conditions and it is necessary to evaluate all 4 ratings to ensure the minimum is found.

TABLE 3.3

Gear Pair Design Variables
Material Properties

VARIABLES	DESCRIPTION OF VARIABLE
MATP	Pinion Material Name
MATG	Gear Material Name
HTNP	Type of Heat Treatment of Pinion
HTNG	Type of Heat Treatment of Gear
HBP	Brinnell Hardness of Pinion
HBG	Brinnell Hardness of Gear
SATLP	Pinion Fatigue Strength - Low Value
SATHP	Pinion Fatigue Strength - High Value
SATLG	Gear Fatigue Strength - Low Value
SATHP	Gear Fatigue Strength - High Value
SACLP	Pinion Pitting Strength - Low Value
SACHP	Pinion Pitting Strength - High Value
SACLG	Gear Pitting Strength - Low Value
SACHP	Gear Pitting Strength - High Value
SAYP	Pinion Yield Strength
SAYG	Gear Yield Strength
UP	Pinion Poisson's Ratio
UG	Gear Poisson's Ratio
EP	Youngs Modulus - Pinion
EG	Youngs Modulus - Gear

TABLE 3.4

Gear Pair Design Variables
Gear Size

VARIABLES	DESCRIPTION OF VARIABLE.
NTP	Number of Pinion Teeth
NTG	Number of Gear Teeth
PND	Module (Normal Diametral Pitch)
LAMDA	Face Width to Diameter Ratio

For the gear ratios and speeds under consideration, scoring is not normally a problem, and although this failure mode is checked, the optimization is directed towards bending strength and pitting resistance.

In an optimum design, the difference between the strength of the pinion and the gear can be reduced to zero. Techniques such as addendum and dedendum modification and the use of exotic materials are commonly used. These modifications add cost and have limited impact on the overall design. As the gear diameter is reduced for a constant set of operating conditions, cost to manufacture and assemble rise exponentially. A tradeoff is thus required for each application to determine the extent to which optimization is justified. Table 3.5 pnovides a list of actions which can be taken to reduce gear diameters with associated advantages/disadvantages.

3.4.1 Guidelines for Generating the Minimum Diameter Matrix

A strategy for mesh optimization will now be developed using the variables for mesh design identified in tables 3.1 through 3.4 and the possible optimization strategies given in table 3.5.

Variables from table 3.1 represent given operating conditions which should not be altered by the mesh optimization. If the results of the arrangement optimization are unacceptable, it may be necessary to reconsider the defined operating conditions.

Materials with increased ultimate strength and/or increased shear strength produce stronger teeth and allow smaller diameter gears resulting in a smaller drive. In general, the best materials which can

TABLE 3.5

Design Strategies to Minimize Mesh Center Distance

#	ACTION	EFFECT	ADVANTAGE	DISADVANTAGE
1	Optimize module	Improves balance between bending strength and pitting resistance	Low cost when standard values of module are used	Cannot provide a totally balanced design between all failure modes
2	Helical gearing in place of spur gears	Results in greater contact ratio and larger apparent face width improving performance and reducing size	Smaller more compact gearing Smoother operation	Does not improve balance between failure modes Significantly higher cost
3	Addendum/dedendum modification of the tooth form	Increases pinion strength at expense of gear strength	Smaller more compact gearing Avoids undercutting of pinion	Significantly higher cost
4	Select special materials	Increases bending strength and/or pitting resistance strength of pinion or gear	Helps balance strength between pinion and gear	Slight increase in analysis costs

be justified in terms of cost for a specific application, should be selected for the generation of the minimum diameter matrix.

The tooth form variables, table 3.3, affect the relationship between the different failure modes. Standard tooth forms represent a conscious compromise between the different failure modes and will normally provide a reasonably balanced design over a wide range of operating conditions. For top performance, it is necessary to optimize tooth form to an application. This analysis, however, substantially increases the computational cost of the minimum diameter matrix.

The size of a gear mesh is determined by the number of teeth on the pinion and gear, the module (diametral pitch), and the face width. The module also defines the size of the gear teeth. Typically, bending strength of a gear increase with tooth size while pitting resistance strength decreases. The point where these two curves cross represents a balanced design when scoring is not the critical design criteria. These variables will be determined by mesh optimization.

3.4.2 Calculation of the Minimum Diameter Matrix

The determination of the design variables which specify a gear mesh for optimal performance are computationally expensive. The purpose of the minimum diameter matrix is to yield solutions to this problem for a discrete number of operating conditions. These discrete points can be interpolated to provide a reasonable approximation over a range of speeds and speed reduction ratios for use in the arrangement optimization.

The diameter matrix is calculated using an optimization process which determines the optimum number of teeth on the input gear and the optimum module for specific speed reduction ratios and input gear speeds. The power capacity is calculated based on a starting diameter and module. The diameter of the input gear in the mesh is re-calculated assuming that the power varies as the cube of the diameter. When the change is sufficiently small, normally requiring 3 or 4 steps for 3 decimal place accuracy, the module is optimized. The optimization process limits the module to discrete steps. The process is repeated until no reduction in size is possible and error in desired power capacity is sufficiently small. A flow chart of the process is shown in figure 3.2.

3.5 Input Values of Design Variables for Verification Problem.

This section selects values for the design variables for the calculation of the verification problem and explains the rational behind the selections.

3.5.1 Operating Conditions

Many of the variables associated with operating conditions are arbitrary with respect to the optimization process. Random selection of these variables would allow the calculations to be performed, but the solution would be of little interest. Therefore, the operating conditions of table 3.6 are specified. These are appropriate to those required for a precision multi-speed drive used in a milling machine.

3.5.2 Material Properties

Steel A5 is selected for both the pinion and gear for all gear pairs. This is an high quality steel resulting in reasonably compact

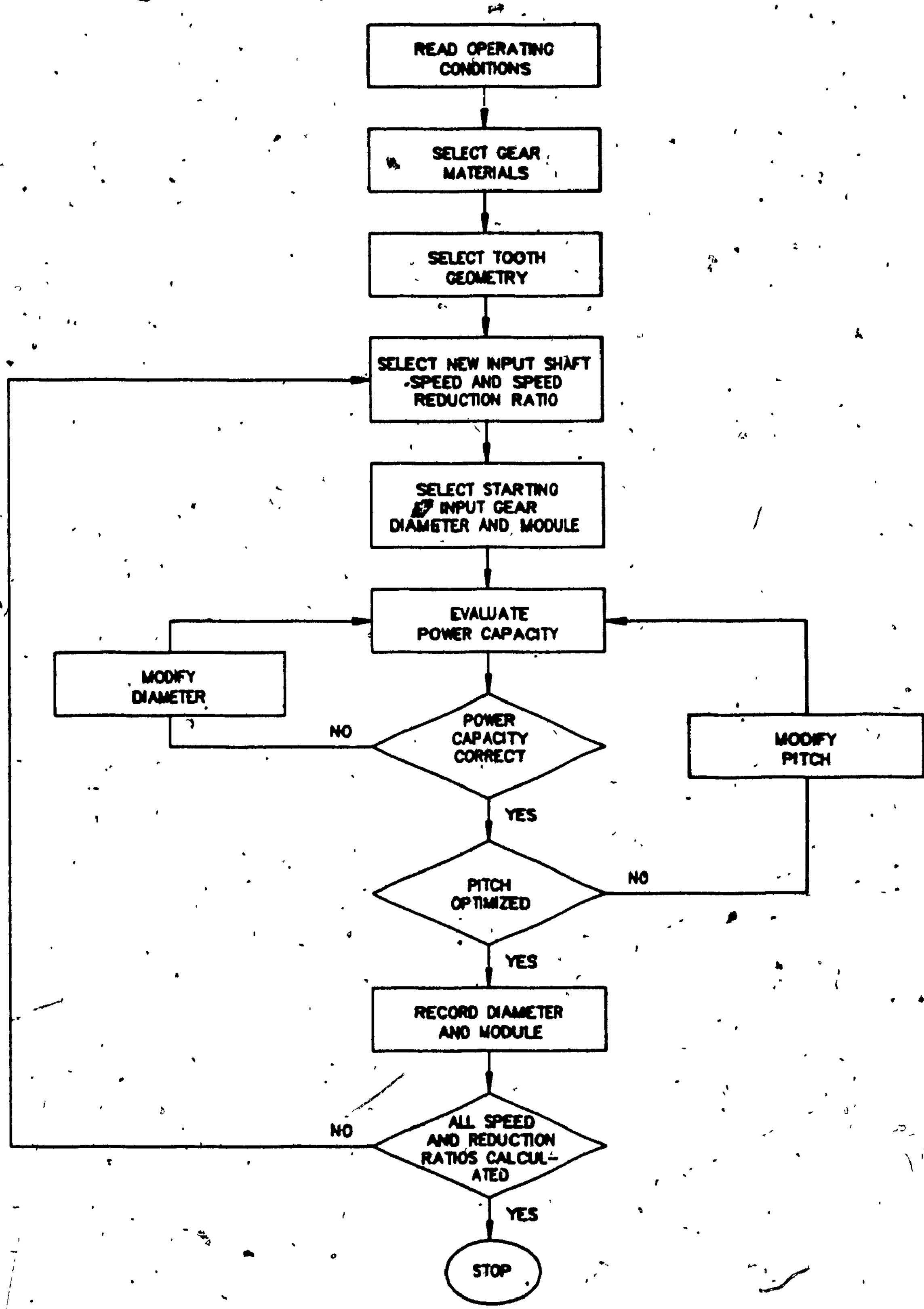


FIGURE 3.2

Strategy to Generate Minimum Diameter Matrix

TABLE 3.6

Value of the Mesh Input Variables
Selected for the Verification Problem

INPUT DESCRIPTION	INPUT NAME	INPUT VALUE	AFFECTS VARIABLES	COMMENTS
1) Title				Informative
2) Category	CA	3	QV, LM, FP LSHAR, SACP SATP, SACG SATG	Precision Enclosed Gearing. See Table 3.7.
3) Year of Operation	NY	10	NY	Continuous Service
4) Failures in 10000	NF	10	NF	Continuous Service
5) Character of Source	SO	1	SO	Uniform Load
6) Character of Load	LO	2	LO	Moderate Shock
7) Gear Mounting	CN	2	CN	Off Centerline
8) Type of Operation	ST	1	ST	Commercial Application
9) Composite	CO	1 2 3	CO	Conventional Mesh Composite Input Composite Output
10) Pressure Angle	PHIC	20	PHIC	Standard
11) Helix Angle	CHIS	0	CHIS	Spur Gears
12) Tooth Geometry		2	CTYPEP, CTYPEG, DCP, DCG, AP, AG, BP, BG, TNP, TNG RTP, RTG, DELTAP, TCP, DELTAG, TCG, HAOP, HAOG	Standard Tooth Form

TABLE 3.6 (Continued)

13) Face Width/ Diameter	LAMDA	1	Optimized	Minimize Diameter
14) Input Gear Material	MATP	5	Variable	Steel A5
15) Output Gear Material	MATG	5	Variable	Steel A5
16) Input Speed	NP		Input	Input
17) Pinion Speed	NTP		Optimized	
18) Gear Teeth	NTG		Calculated	From Input Gear Ratio
19) Module	PND		Optimized	
20) Required Power Capacity			Constant	Operating Conditions

TABLE 3.7

Quality Categories Used for Analysis

	1	2	3	4
QV	6	8	10	12
LM	1	2	2	2
FP	64	64	32	16
LSHAR	0	1	1	1
SACP	LOW	LOW	AVERAGE*	HIGH*
SATP	LOW	LOW	AVERAGE	HIGH
SACG	LOW	LOW	AVERAGE	HIGH
SATG	LOW	LOW	AVERAGE	HIGH

* AGMA publishes a range of values for material properties. It is assumed that applications in category 3 or 4 verify the material properties to ensure the safe use of average or high values

designs. To attain greater strength special heat treatments are required. The effect of higher strength material will be demonstrated in chapter 6.

3.5.3 Tooth Geometry

For multi-speed gear drives, spur or helical gearing is commonly used. Spur gears are used for commercial applications with low power and low pitch line velocities. Helical gears are used for precision applications with high power requirements and/or high pitch line velocities. Spur gears are used for the verification problem presented. This is consistent with the design of most current machine tool drives.

Standard tooth geometry is assumed throughout. Standard geometry produces reasonably balanced designs over the normal ranges of pinion speeds and gear reduction ratios. Standard tooth proportions also result in significantly lower production cost since standard tooling can be used. Non-standard tooth geometry is only appropriate when designing for specific, well defined, applications where a clear problem exists.

3.5.4 Gear Mesh Size Variables

The face width is a parameter available for optimization, however the face width/diameter ratio is limited to the range of 0.1 to 1.0. A face width/diameter ratio of 1.0 is used for optimization where the radial objective function is used, as the greater the face width to diameter ratio, the smaller the diameter required to carry a given power. Values greater than 1.0 are possible but require special care during assembly.

The pinion speeds and the speed reduction ratios used for the minimum diameter matrix are selected based on the expected range of values encountered during the arrangement optimization. The speed reduction ratios were selected to be .1, .2, .5, 1, 2, 3, 4, 5. The input gear speeds were selected to be 25, 50, 100, 200, 300, 500, 750, 1000, 2000, 4000 RPM. These values are somewhat arbitrary and are selected to provide a reasonable distribution over the design space.

3.6 Minimum Diameter Matrix by Mesh Optimization

Results of the optimization for minimum diameter based on standard operating conditions defined in section 3.5 are presented in tables C.1 through C.15. Data was calculated for a conventional mesh and a composite input gear mesh. Table C.10 contains optimum diameters for a face width/diameter ratio of 1.0. Tables C.13 and C 15 contain number of teeth and module for a face with/diameter ratio of 1.0 respectively. The case of a composite on the output gear is not required during the arrangement optimization because a composite output gear in one group is equivalent to a composite input gear in the next group and is thus correctly sized.

The composite pinion has a marked effect on the diameters for speed reduction ratios of 1.0 or more. For speed increasing ratios, the output gear is smallest and tends to drive the diameter of the mesh. This can be easily observed in the results.

The minimum diameter matrix is used to determine the diameter of the input gear, given the speed reduction ratio and input gear speed. Interpolation is used to obtain the input gear diameter for conditions other than those in the table. For speed reduction ratios the correct diameter is obtained using a linear interpolation of the mesh center distance. The center distances are calculated from the minimum diameter matrix as required. A plot of center distances vs speed reduction ratio is shown in figures 3.3 for a range of speeds using conventional mesh data from table C.10. The graph is sufficiently linear to justify the proposed approach.

The speed is assumed to be proportional to the cube of the diameter when calculating gear diameters for a gear mesh with input speeds other than those tabulated. The new diameter is calculated using the 2 speeds closest to the required speed. An average of the calculated values is used. Table 3.8 demonstrates that the assumption of the cubic relationship between input gear diameters and speed is valid.

3.7 Four Speed Drive - Verification Problem

In chapter 2, the four speed drive was optimized for minimum radial dimensions subject to kinematic requirements and the constraint that the smallest diameter have a value equal to 1.0. As noted, the drive could not be implemented as presented because of gear strength considerations.

The problem is re-analysed with the requirement that all gears have sufficient strength to meet the operating requirements of table 3.5. This is accomplished by using the diameter matrix of table C.10 as input data. The objective function, to minimize radial dimensions, as well as the optimization logic are the same as in chapter 2. The optimization of radial dimensions, table 3.9, using the minimum diameter matrices of chapter 3 yields a single optimum arrangement. Optimization

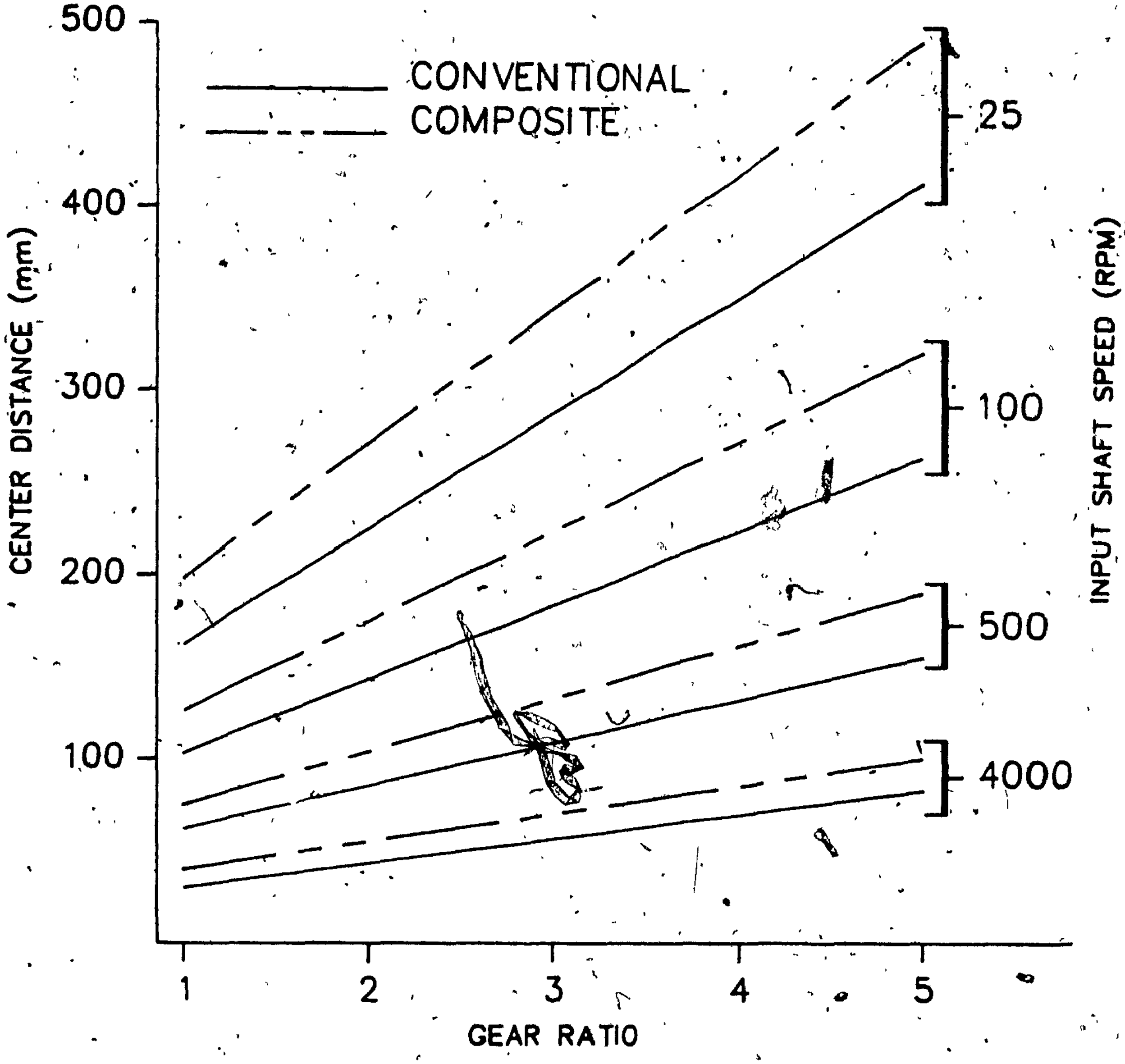


FIGURE 3.3

Center Distance as a Function of Gear Ratio and Pinion Speed for a Conventional and Composite Mesh. Data from Table C.10.

TABLE 3.8

Sample Interpolation Using Averaged Cubic Relationship Between Optimum Gear Diameter (mm) and Power Capacity

SPEED	SRR = 0.1			SRR = 1.0			SRR = 5.0		
	ACT.	EST.	%ERROR	ACT.	EST.	%ERROR	ACT.	EST.	%ERROR
25*	640	648	1.3	162	163	0.6	138	139	0.7
50	514	514	0.0	129	129	0.0	110	110	0.0
100	413	414	0.2	103	104	1.0	88	88	0.0
200	333	332	0.3	83	83	0.0	71	70	1.4
300	294	294	0.0	73	73	0.0	62	61	1.6
500	251	251	0.0	62	62	0.0	53	54	1.9
750	222	221	0.5	55	55	0.0	46	47	2.2
1000	203	205	1.0	50	50	0.0	42	43	2.4
2000	165	165	0.0	40	40	0.0	34	34	0.0
4000*	134	131	2.2	30	32	3.0	28	27	3.6
AVE.			0.6			0.5			1.4

* Interpolation from 1 direction only.

1, with a value of 463 mm, provides the minimum value of the objective function. This gear drive and speed diagram are shown in figures 3.4 and 3.5 respectively.

The difference between the results of the optimizations of chapter 2 and 3 can be seen by comparing relative diameters as shown in table 3.10. The results for chapter 2 show the minimum diameter in group 1 and group 2 to have a value of 1.000. For the result from chapter 3, the minimum diameter for group 1 is 1.000, however the minimum diameter for group 2 is greater. The diameter of the first input gear in group 2 is approximately 22% larger than group 1. The relationship between optimum diameters and power is approximately cubic. Since power is proportional to speed, the relationship between diameters of the smallest input gear from group 1 to group 2 should be approximately equal to the cube root of the shaft 1 to shaft 2 speed ratio. The speed ratio is 1.743 and the cube root is equal to 1.204. Hence the diameter in group 2 is expected to be approximqtely 20% larger, provided the face width/diameter ratio remains constant. This is in close agreement with the actual result.

Comparing speed diagrams, the inclusion of gear strength requirements has moved the input speed to higher values. This occurs because the size of the gears becomes smaller as the speed increases. A trade-off takes place between the reduction in diameter associated with increasing speed and the increase in diameter associated with the change in gear ratio required to implement this. The gears on the output shaft are largest and have the most effect on the objective function, therefore, these are optimized to have reciprocal gear ratios which minimize radial dimensions.

3.8 Summary

This chapter has examined the design variables of a gear mesh in order to determine a method to minimize center distance. The design is based on AGMA 218 which ensures that bending strength and pitting resistance are correctly evaluated. It was shown that an optimum module and input gear diameter for a mesh can be calculated as a function of input gear speed and speed reduction ratio. This analysis was used to build a minimum diameter matrix for use in the synthesis of optimal gear drive designs. The re-analysis of the 4 speed drive to include gear design requirements resulted in a new design when optimized using the identical radial objective function. This reflects the influence of components strength on the selection of the kinematic arrangement.

TABLE 3.9

Optimization Summary for the 4 Speed Drive Verification Problem for the Radial Objective Function using Standard Operating Conditions

		OPTIMIZATION NUMBER					
		1	2	3	4	5	6
DIAMETER	1	104.0	122.3	89.2	98.9	77.9	388.7
	2	136.1	155.2	119.3	129.9	107.2	487.6
	3	181.3	162.9	190.2	178.4	216.3	468.8
	4	149.2	130.0	160.0	147.4	187.0	369.9
	5	127.0	162.9	115.7	147.4	112.0	369.9
	6	198.8	248.8	190.2	236.2	187.0	468.8
	7	188.4	217.1	212.9	242.7	221.0	198.4
	8	116.6	131.2	138.4	153.9	146.0	99.5
RATIOS	S	0.387	0.563	0.255	0.337	0.183	1.545
	1	0.574	0.750	0.469	0.554	0.360	0.829
	2	0.912	1.193	0.745	0.881	0.573	1.318
	3	0.674	0.750	0.543	0.607	0.507	1.865
	4	1.704	1.897	1.374	1.535	1.281	4.714
F (mm)		462.7	518.7	470.0	520.0	477.7	1055.9

		OPTIMIZATION NUMBER					
		7	8	9	10	11	12
DIAMETER	1	106.3	122.3	111.3	92.1	85.0	66.5
	2	169.2	186.7	173.0	163.1	151.5	121.3
	3	169.9	162.9	160.2	237.3	225.4	196.9
	4	106.9	98.4	98.5	166.3	158.9	142.1
	5	123.7	162.9	123.8	166.3	122.5	142.1
	6	159.9	206.7	160.2	210.0	158.9	196.9
	7	194.2	217.1	196.1	212.2	197.4	426.2
	8	157.9	173.3	159.7	168.6	161.0	371.4
RATIOS	S	0.399	0.563	0.438	0.304	0.234	0.113
	1	0.626	0.750	0.695	0.388	0.377	0.338
	2	1.583	1.897	1.756	0.981	0.953	0.853
	3	0.637	0.750	0.631	0.783	0.621	0.333
	4	1.013	1.193	1.003	1.246	0.987	0.530
F (mm)		478.7	534.5	480.3	541.6	489.5	689.6

Diameters (mm)

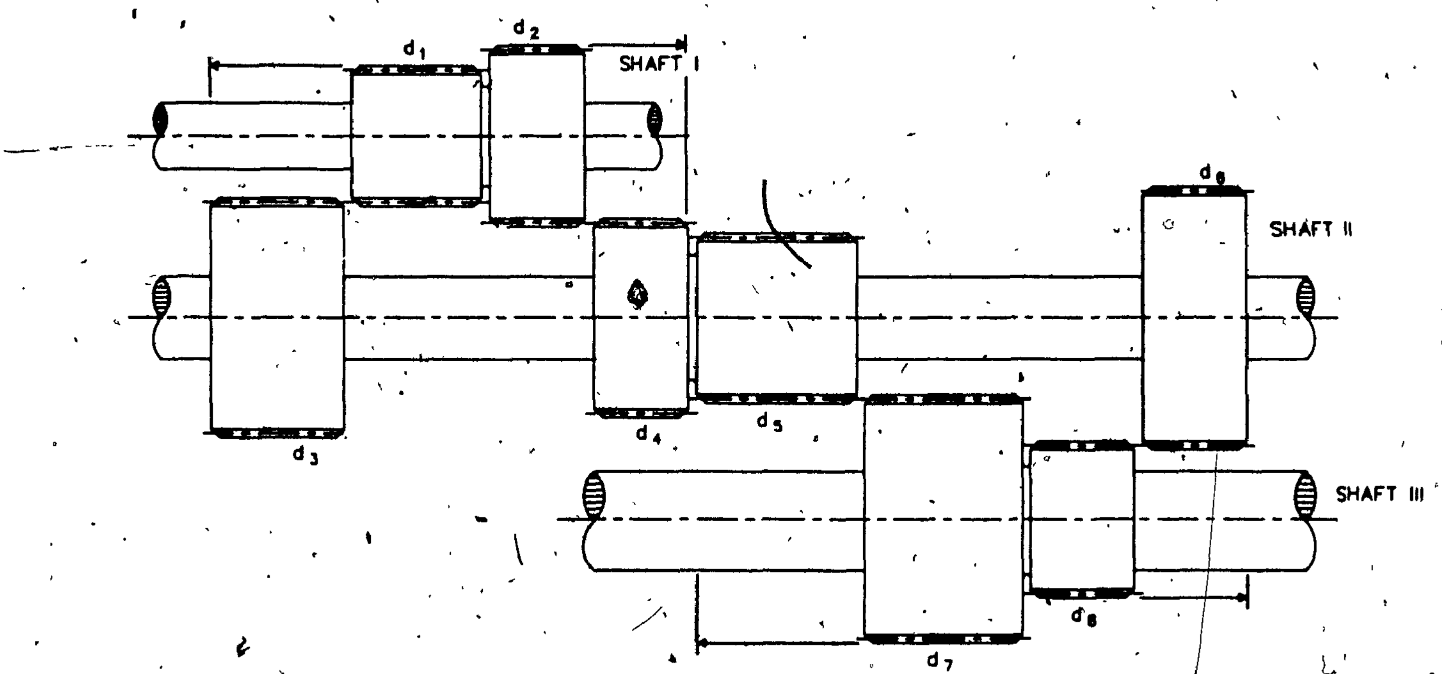


FIGURE 3.4

Gear Drive Based on Optimization 1 Table 3.9

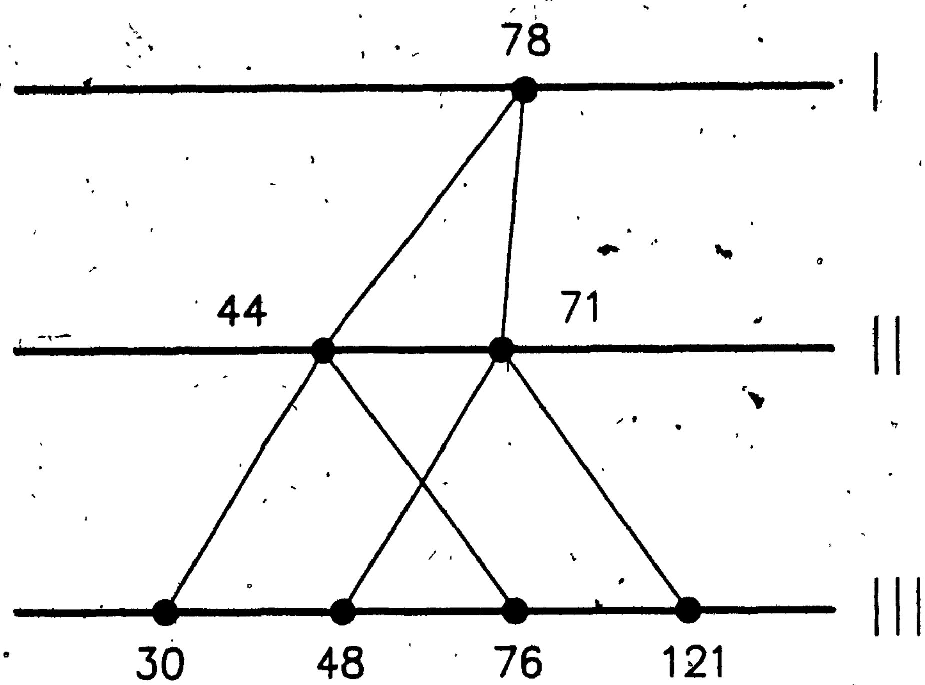


FIGURE 3.5

Speed Diagram Based on Optimization 1 Table 3.9

TABLE 3.10

A Comparison of the Optimum Solutions
Presented in Chapters 2 and 3

DIAMETER	CHAPTER 2	CHAPTER 3 (mm)	CHAPTER 3 RELATIVE
1	1.000	104.0	1.000
2	1.261	136.1	1.309
3	1.261	181.3	1.743
4	1.000	149.2	1.435
5	1.000	127.0	1.221
6	1.590	198.0	1.912
7	1.590	188.4	1.812
8	1.000	116.6	1.121
S	0.499	0.387	0.387
F	3.851	462.7	4.449

CHAPTER 4

DYNAMIC ANALYSIS

4.1 Introduction

In a multi-speed gear drive, each speed represents a different dynamic system and the natural frequencies of each must be verified as acceptable. To obtain a change in output speed, at least one gear mesh in the power path must be altered. These changes are normally accomplished with the use of a sliding block on a splined shaft. In all cases, the dynamic matrix is altered with the result that a new set of eigenvalues and eigenvectors exists which must be considered.

The most accurate method of mechanical analysis is through the use of finite elements. Components of the system are broken into small elements which have defined mass and stiffness properties. The stiffness of individual elements are combined into an overall system stiffness matrix using specified boundary conditions for each element. These accurate elements allow the calculation of detailed stress information, eigenvalues, and eigenvectors. Unfortunately, these models are time consuming to develop and expensive to run. NASTRAN [52] is one of a number of large commercially available finite element analyzers. It is rare that this type of analyser is used to determine system eigenvectors and eigenvalues in a repetitive manner because of the extensive computer times required.

Alternatively, mechanical components and assemblies can be modelled as lumped mass/inertia systems for the purpose of mechanical analysis. Equations of motion are formulated for each mass/inertia node, with up to six degrees of freedom per node. The mass/inertia nodes can be coupled by stiffness and damping elements. The resulting set of equations can be solved to provide system eigenvalues and eigenvectors.

When modeled in this fashion, it is assumed that the elastic components have no mass and that mass/inertia elements are absolutely rigid. The results of experiments have shown that many mechanical systems, including torsional gear drives, can be analysed with reasonable accuracy using these assumptions [52].

Torsional modes tend to be most critical for geared systems. Although both lateral and axial vibrations have been demonstrated [76], shafting in machine tool gear drives tends to be very stiff in order to limit deflections. As a result, lateral and axial vibrations occur at higher frequencies. For speed reducing drives, these modes can be neglected and emphasis placed on determining the torsional eigenvalues and eigenvectors of the system.

4.2 Equations of Motion for Multi-Speed Gear Drives

Equations of motion for the multi-speed gear drive are developed based on the group shown in figure 4.1. For the general case each mesh was separated by an elastic coupling. An elastic coupling was also shown between each of the gears in a mesh. The equations of motion, for the speed shown, can be written as:

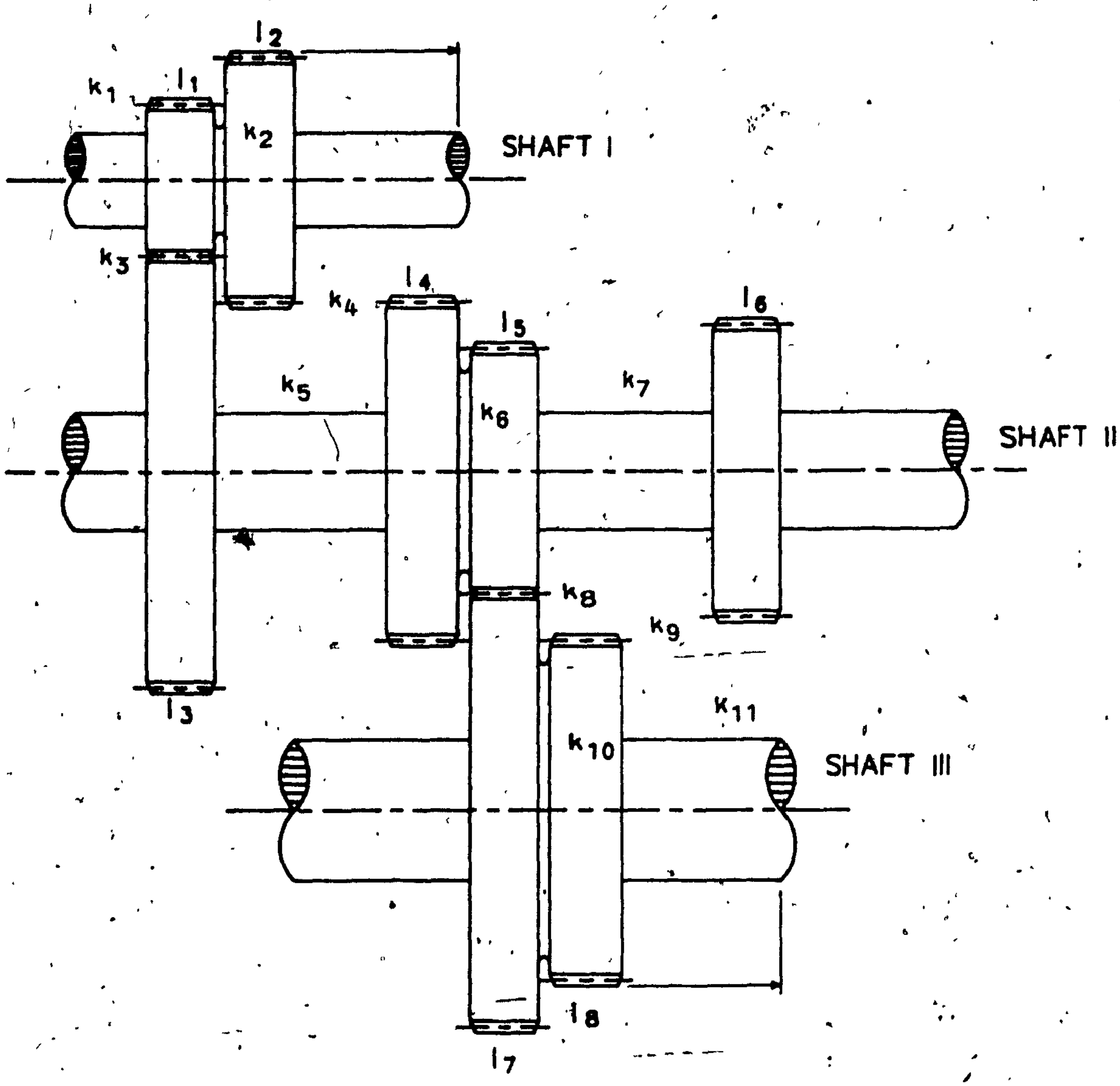


FIGURE 4.1

Dynamic Parameters for the 4 Speed Drive Verification Problem

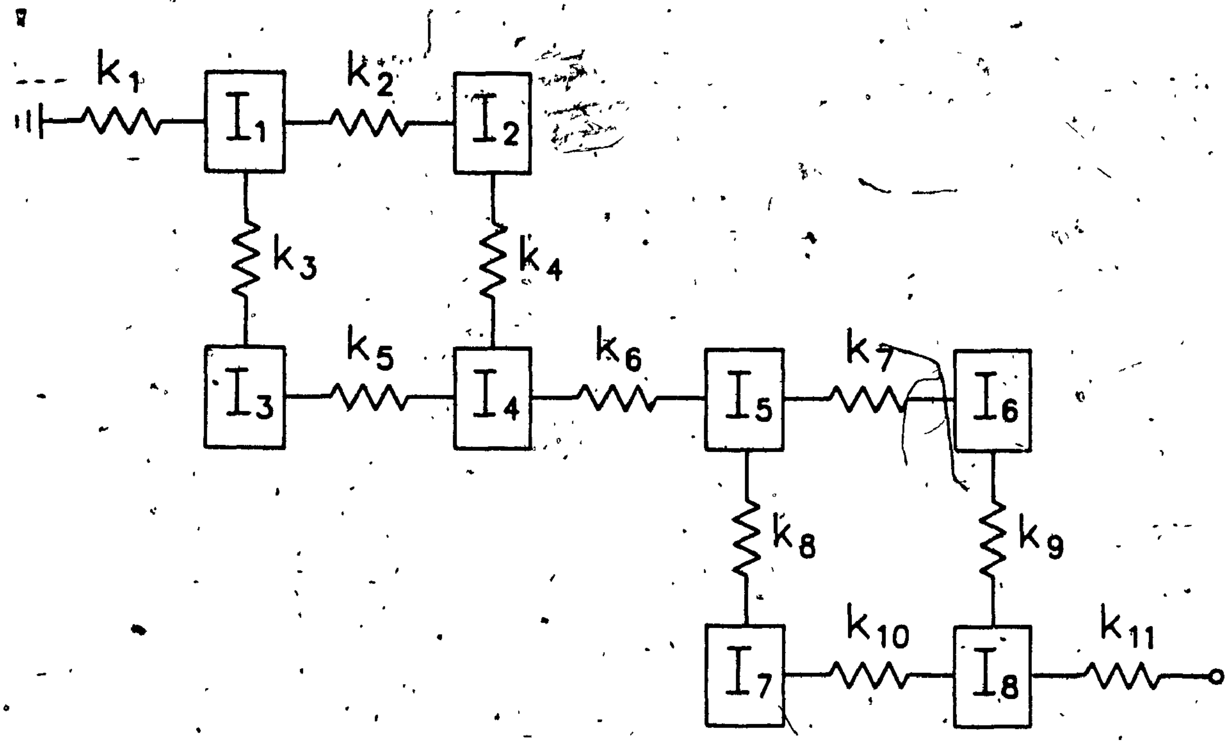


FIGURE 4.2

Dynamic Model for 4 Speed Drive Verification Problem

$$I_1a_1 + k_1x_1 + k_2(x_1 - x_2) + k_3(x_1 - x_3) = 0 \tag{4.1}$$

$$k_2(x_1 - x_2) = I_2a_2 + k_4(x_2 - x_4) \tag{4.2}$$

$$k_3(x_1 - x_3) = I_3a_3 + k_5(x_3 - x_4) \tag{4.3}$$

$$k_4(x_2 - x_4) + k_5(x_3 - x_4) = I_4a_4 + k_6(x_4 - x_5) \tag{4.4}$$

$$k_6(x_4 - x_5) = I_5a_5 + k_7(x_5 - x_6) + k_8(x_5 - x_7) \tag{4.5}$$

$$k_7(x_5 - x_6) = I_6a_6 + k_9(x_6 - x_8) \tag{4.6}$$

$$k_8(x_5 - x_7) = I_7a_7 + k_{10}(x_7 - x_8) \tag{4.7}$$

$$k_9(x_6 - x_8) + k_{10}(x_7 - x_8) = I_8a_8 + k_{11}x_8 \tag{4.8}$$

The equivalent mass and stiffness matrices for these equations are:

$$\begin{bmatrix} I_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_8 \end{bmatrix} \qquad (4.9)$$

The stiffness matrix in its general form is as follows:

$$\begin{bmatrix} k_1+k_2+k_3 & -k_2 & -k_3 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & k_2+k_4 & 0 & -k_4 & 0 & 0 & 0 & 0 \\ -k_3 & 0 & k_3+k_5 & -k_5 & 0 & 0 & 0 & 0 \\ 0 & -k_4 & -k_5 & k_4+k_5+k_6 & -k_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_6 & k_6+k_7+k_8 & -k_7 & -k_8 & 0 \\ 0 & 0 & 0 & 0 & -k_7 & k_7+k_9 & 0 & -k_9 \\ 0 & 0 & 0 & 0 & -k_8 & 0 & k_8+k_{10} & -k_{10} \\ 0 & 0 & 0 & 0 & 0 & -k_9 & -k_{10} & k_9+k_{10}+k_{11} \end{bmatrix} \qquad (4.10)$$

Couplings K_3, K_4, k_8 and K_9 represent the stiffness of the gear keys, webs, and teeth. When a mesh is disengaged, the associated stiffness value is zero. Stiffness elements of zero do not introduce singularities in the calculations, because each inertia element remains connected by a least one stiffness.

Element stiffness between 2 and 3 orders of magnitude larger than the shaft stiffnesses can be treated as infinitely stiff. When this happens, the row and column associated with that entry are added into an adjacent row and column respectively. This effectively combines two inertias into one and reduces the number of modes by one. The mode removed is a high frequency mode and the effect on the other modes is negligible. This reduction is useful to reduce matrix size and to avoid computational irregularities.

The equations of motion of the lumped parameter system can be represented by the following matrix equations:

$$Ma + Kx = 0 \tag{4.11}$$

Capital letters are used to designate matrices. Bold lower case letters are used to designate vectors. The mass matrix M is a diagonal matrix, the stiffness matrix K is a sparse symmetric square matrix, a is a column vector of angular accelerations and x is a column vector of angular displacements. Assuming harmonic motion, equation (4.11) can be rewritten as follows where Ω is the circular frequency.

$$-\Omega^2 Mx + Kx = 0 \tag{4.12}$$

Using equation (4.12) the following determinate is written which is known as the characteristic equation and whose roots are the eigenvalues of the problem.

$$\left| K - \Omega^2 M \right| = 0 \tag{4.13}$$

The natural frequencies of the system are equal to the square root of the eigenvalues. The mode shape x_i corresponding to the i^{th} eigenvalue is called the i^{th} eigenvector.

4.3 Calculations of Physical Parameters

The calculation of the eigenvalues and eigenvectors of the system requires the design of shafts which affect system stiffness and the calculation of component mass/inertia.

4.3.1 Shaft Design

Minimum diameters for the shafts are calculated in accordance with AGMA 360 [21] and consider both torsional and bending strength. AGMA 360 notes that it may be necessary to exceed shaft sizes obtained based on stress considerations in order to limit shaft deflections.

The diameter required to withstand bending stress and shear stress can be calculated using the following two equations respectively.

$$d_b = \left[\frac{32M}{\pi S_b}\right]^{1/3} \qquad (4.14)$$

$$d_s = \left[\frac{16T}{\pi S_s}\right]^{1/3} \qquad (4.15)$$

An approximation for the deflection of the shaft can be made by assuming a simply supported shaft with a load at its center. From Shigley [133]:

$$y_{max} = \frac{W_t L^3}{48EI_s} \qquad (4.16)$$

where for a circular shaft,

$$I_s = \frac{\pi d^4}{64} \qquad (4.17)$$

Re-arranging equation (4.17) gives:

$$d_s = \frac{64 W_t L^3}{48 \pi E y_{max}} \qquad (4.18)$$

The allowable deflection, y_{max} depends on the value of the quality factor. For machine tools the quality factor is normally between 8 and 10 and would require maximum deflections to be on the order of 0.025 mm (0.001").

4.3.2 Inertia of Cylindrical Bodies

The inertia of gears can be approximated using the equation for a cylinder spinning about it's central axis.

$$I_n = \frac{\mu \pi d_n^4 F_n}{32} \qquad n = 1, 2, \ldots, n_g \qquad (4.19)$$

μ = Density

d_n = Diameter

F_n = Width

4.3.3. Torsional Stiffness of a Shaft

The shaft stiffness of a shaft of circular cross section is given by:

$$k = \frac{\pi d^4 G}{32L} \tag{4.20}$$

d = Shaft diameter

L = Shaft length

G = Shear modulus

4.3.4 Stiffness of Other Components

For close approximations to the system eigenvalues it has been shown to be necessary to include the stiffness of components such as keyways, gear ribs, gear webs, gear teeth, and clutches.

In this analysis, the purpose of calculating natural frequencies and mode shapes is to ensure that the optimum design does not have serious vibration problems. An approximation of the eigenvalues is therefore satisfactory. The stiffness of components such as keys, gear teeth, gear webs, clutches, etc. has therefore been neglected.

4.3.5 Equivalent Stiffness and Inertia

The component stiffnesses and inertias must all be referred to a common speed before use in the mass and stiffness matrices. An equivalent system, one which retains the same kinetic and potential energy, can be obtained by multiplying all stiffnesses and inertias by the ratio n^2, where n is the ratio of the speed of the component under consideration as compared to a reference shaft [52]. The reference shaft is taken to be the input shaft for this study since is operates at the same RPM for all output speeds.

4.4 Method of Calculating Natural Frequencies

The requirement of this analysis is to ensure that the natural frequencies do not occur at any of the operating speeds. To do this it is necessary to calculate all eigenvalues which lie in or near the speed range of the drive or one of its components.

This rules out the use of techniques based on the Raleigh quotient which approximate the systems first natural frequency. Transfer matrices, which are similar to finite element methods, are also ruled out because the increased modelling effort and analysis time cannot be warranted.

This leaves matrix methods of analysis. These methods convert the mass and stiffness matrices into eigenvalue and eigenvector matrices using matrix transformations. The Jacobi method is a matrix eigenvalue method which yields all of the natural frequencies simultaneously. The method is described in more detail in the following section.

4.4.1 The Jacobi Method

The Jacobi method was selected for use in finding the eigenvalues and eigenvectors of the system. This method is based on simple orthogonal transformations which convert the dynamic matrix into the eigenvalues and, if identical operations are perform on an identity matrix, an identity matrix into the system eigenvectors.

Although not the most efficient technique available to obtain the eigenvalues, it is extremely stable and generates eigenvectors with little additional work. It is suitable for number of inertias < 40 where computer time is not paramount. Details for the Jacobi method can be found in reference [77].

In order to use the the Jacobi method it is necessary to convert the eigenvalue problem from the form:

$$Ax = \Omega Bx \tag{4.21}$$

to the form:

$$Hx = \Omega x \tag{4.22}$$

It is desirable to perform the transformation from equation (4.21) to (4.22) in such a manner that H remains symmetric. This transformation can be done using a Choleski decomposition [77] on matrices A or B. The requirement for a successful decomposition is that the matrix be positive definite. If the inertia matrix B is used, this requirement is always satisfied.

4.5 Eigenvalues and Eigenvectors - Four Speed Drive Verification Problem

The arrangement obtained in chapter 3 can be used to demonstrate the calculation of eigenvalues and eigenvectors and eigenvalue derivatives. To calculate physical properties of the components, widths as well as diameters are required for the gears.

4.5.1 Calculation of Gear Width

In chapter 3, the diameters of the gears in the four speed drive were calculated for an objective of minimum radial dimensions. The diameters selected have sufficient power capacity to carry the load with a maximum face width to diameter ratio of one. Some of the gear diameters are determined because of kinematic requirements and as a result have excessive capacity. This capacity can be reduced by reducing the face width/diameter ratio in proportion to the excess power rating. This optimization is performed prior to the calculation of eigenvalues to provide gear width information which is optimal with respect to the solution provided by chapter 3.

The optimization strategy is straight forward. The power capacity of each gear pair is calculated for the operating conditions determined by the kinematic analysis. The new width is calculated as follows:

$$F_n = F_o * \frac{P_c}{P} \tag{4.23}$$

F_n = new gear width

F_o = old gear width

P_c = calculated power capacity

P = required power capacity

Equation (4.23) is calculated repeatedly until the difference between P_c and P becomes acceptably small. This normally occurs in 3 or 4 iterations. This optimization for width guarantees that each mesh is optimum at the component level.

The optimum widths, as calculated using this approach for the results of chapter 3, are shown in table 4.1. Initial shaft diameters are also shown. These diameters are selected to satisfy strength and

TABLE 4.1

Optimum Arrangement with Initial Shaft Diameters

GEAR				SHAFT	
#	DIAM (mm)	LAMDA	WIDTH (mm)	#	DIAM (mm)
1	104.1	0.995	103.5	1	40.0
2	136.2	0.545	74.2	2	60.0
3	181.3		103.5	3	80.0
4	149.2		74.2		
5	127.0	0.992	126.0		
6	198.8	0.700	139.1		
7	188.4		126.0		
8	116.6		139.1		

deflection criteria and are increased, if necessary, to modify eigenvalues.

4.5.2 Inertia and Stiffness Matrices

Using the gear diameters and widths, the inertias of each mesh component can be calculated. The axial arrangement of the drive can also be deduced based on gear width information. Table 4.2 presents tabulated results showing the RPM of each shaft for each operating speed. This information allows for the calculation of equivalent inertias and stiffnesses of all components in the drive. The input shaft speed is selected to be the reference.

Table 4.3 provides basic and equivalent inertias for each of the gears in the drive. Tables 4.4 through 4.7 show basic and equivalent stiffnesses for each of the shafts at respective operating speeds.

TABLE 4.2

Ratio of Shaft Speed to Input Shaft Speed

SHAFT	SPEED			
	1	2	3	4
1	1.000	1.000	1.000	1.000
2	0.574	0.913	0.574	0.913
3	0.387	0.615	0.978	1.555

4.5.3 Presentation and Discussion of Results.

Eigenvalues for the 4 speed drive of chapter 3 are presented in table 4.8. The lowest natural frequency is 150 Hz and occurs for output speed 3. This is equivalent to an operating speed of 9000 RPM and hence is satisfactory. The mode shapes for the lowest frequency at each speed are shown in figure 4.3. It is clear that the modes for speeds 1 and 2 are associated with inertia 6.

TABLE 4.3

Basic and Equivalent Inertia of Gears Referenced To Input Shaft Speed (kgm^2)

		SPEED			
GEAR #	BASIC	#1	#2	#3	#4
1	0.0093	0.0093	0.0093	0.0093	0.0093
2	0.0196	0.0196	0.0196	0.0196	0.0196
3	0.0859	0.0283	0.0716	0.0283	0.0716
4	0.0283	0.0093	0.0235	0.0093	0.0235
5	0.0252	0.0083	0.0210	0.0083	0.0210
6	0.1669	0.0550	0.1390	0.0550	0.1390
7	0.1220	0.0183	0.0462	0.1168	0.2952
8	0.0198	0.0030	0.0075	0.0189	0.0479

TABLE 4.4

Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #1

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	60.0	0.347	0.347
2	1	2	4.0	5.199	5.199
5	3	4	186.6	0.564	0.186
6	4	5	6.0	17.545	5.781
7	5	6	278.3	0.378	0.125
10	7	8	8.0	41.588	6.227
11	8	9	398.3	0.835	0.125

TABLE 4.5

Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #2

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	246.6	0.084	0.084
2	1	2	4.0	5.199	5.199
5	3	4	186.6	0.564	0.470
6	4	5	6.0	17.545	14.613
7	5	6	278.3	0.378	0.315
10	7	8	8.0	41.588	15.742
11	8	9	398.3	0.835	0.316

TABLE 4.6

Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #3

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	60.0	0.347	0.347
2	1	2	4.0	5.199	5.199
5	3	4	186.6	0.564	0.186
6	4	5	6.0	17.545	5.781
7	5	6	278.3	0.378	0.125
10	7	8	8.0	41.588	39.797
11	8	9	120.0	2.773	2.653

TABLE 4.7

Stiffness (MNm/rad) of Basic and Equivalent Couplings - Speed #4

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	246.6	0.084	0.084
2	1	2	4.0	5.199	5.199
5	3	4	186.6	0.564	0.470
6	4	5	6.0	17.545	14.613
7	5	6	278.3	0.378	0.315
10	7	8	8.0	41.588	100.609
11	8	9	120.0	2.773	6.707

TABLE 4.8

Initial Eigenvalues (Hz)

EIGENVALUE #	SPEED			
	1	2	3	4
1	13874	4459	13873	5302
2	3118	11569	3119	11459
3	412	310	389	231
4	4424	2886	730	653
5	15450	10168	5815	3747
6	184	150	3838	2999
7	606	581	590	584
8	7785	7668	11847	9577

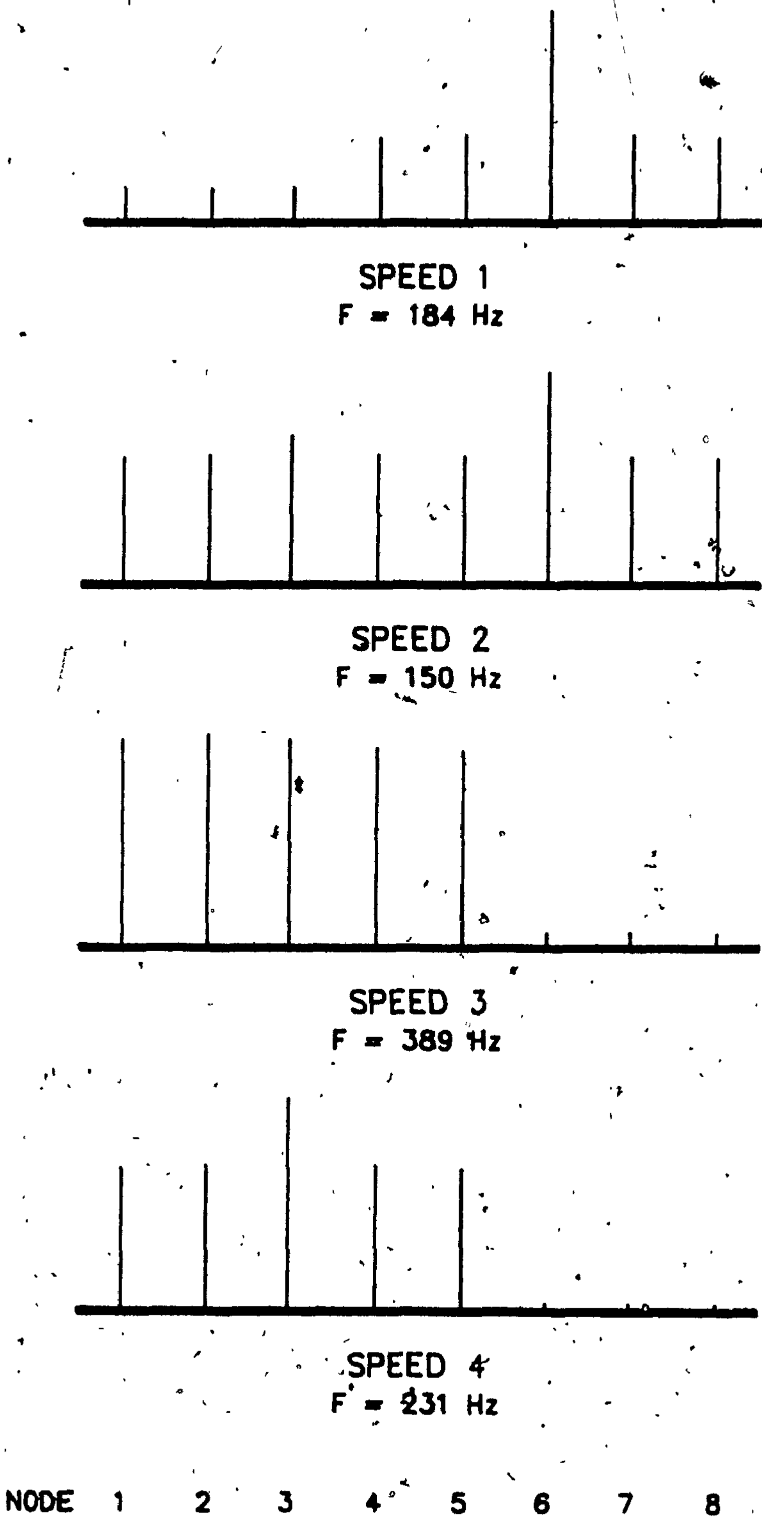


FIGURE 4.3

Mode Shapes Associated With Lowest Frequency

4.6 Derivatives of Eigenvalues

The modification of eigenvalues by trial and error can be a long process. The calculation of eigenvalue derivatives with respect to different mass and stiffness elements provides insight into the dynamic behavior of the system and quantitative information on the effect of design modifications.

Derivatives of eigenvalues and eigenvectors can be calculated using the Raleigh quotient, which is designed as the equation of motion of a particular mode multiplied by the transpose of the eigenvector of the same mode. The Raleigh quotient can be written as:

$$x_i^T (K - \Omega_i^2 M)\, x_i = 0 \qquad (4.24)$$

Following the method of Rogers [93] the Raleigh quotient is differentiated with respect to the j^{th} design parameter.

$$\frac{\partial}{\partial p_j}\left[(x_i^T (K - \Omega_i^2 M) x_i \right] = 0 \qquad (4.25)$$

$$\frac{\partial x_i^T}{\partial p_j} K\, x_i + x_i^T \frac{\partial K}{\partial p_j} x_i + x_i^T K \frac{\partial x_i}{\partial p_j} \qquad (4.26)$$

$$- \frac{\partial x_i^T}{\partial p_j} \Omega_i^2 M\, x_i - 2\, x_i^T\, \Omega_i \frac{\partial \Omega_i}{\partial p_j} M\, x_i$$

$$- x_i^T\, \Omega_i^2 \frac{\partial M}{\partial p_j} x_i - x_i^T\, \Omega_i^2 M \frac{\partial x_i}{\partial p_j}$$

Combining similar terms:

$$\frac{\partial x_i^T}{\partial p_j} (K - \Omega_i^2 M)\, x_i + x_i^T (K - \Omega_i^2 M) \frac{\partial x_i}{\partial p_j} \tag{4.27}$$

$$+\, x_i^T \frac{\partial K}{\partial p_j} x_i - 2\, x_i^T\, \Omega_i \frac{\partial \Omega_i}{\partial p_j} M\, x_i$$

$$-\, x_i^T\, \Omega_i^2 \frac{\partial M}{\partial p_j} x_i$$

The first two terms of equation 4.27 are zero since the term in brackets is set to zero to find the eigenvalues. Re-arranging the remaining terms to solve for $\partial \Omega_i / \partial p_j$ provides the following equation.

$$\frac{\partial \Omega_i}{\partial p_j} = \frac{x_i^T \dfrac{\partial K}{\partial p_j} x_i - x_i^T \Omega_i^2 \dfrac{\partial M}{\partial p_j} x_i}{2\, \Omega_i\, x_i^T M\, x_i} \tag{4.28}$$

The eigenvectors are mass orthogonal and can be normalized so that the term $x_i^T M x_i$ is 1 regardless of the eigenvector selected.

When the j^{th} parameter is a mass element, the term $\partial K / \partial m_j$ is zero since the derivative of any component stiffness with respect to a component mass is zero. The term $\partial M / \partial m_j$ is a matrix of zeros at all elements except for the matrix element $M(j,j)$ which becomes 1. For a mass parameter, equation (4.28) can be reduced to the following simple expression.

$$\frac{\partial \Omega_i}{\partial m_j} = - \frac{\Omega_i\, x(j)_i^2}{2} \tag{4.29}$$

When the j^{th} parameter is a stiffness element, the term $\partial M/\partial k_j$ is zero since the derivative of any component mass with respect to a component stiffness is zero. The term $\partial K/\partial k_j$ is zero at all locations except those that have a contribution of stiffness from the j_{th} parameter. Designating the inertias which are connected by the j_{th} stiffness as i1 and i2, then the non-zero stiffness elements are; K(i1,i1) = 1, K(i2,i2) = 1, K(i1,i2) = -1, and K(i2,i1) = -1. When these values are substituted into equation 4.18 the result is:

$$\frac{\partial \Omega_i}{\partial k_j} = \frac{(x(i1)_i - x(i2)_i)^2}{2\ \Omega_i} \qquad (4.30)$$

Equations (4.29) and (4.30) allow for the calculation of derivatives of the eigenvalues in a simple and efficient fashion with respect to both mass and stiffness elements.

4.7 Eigenvalue Derivatives for the Four Speed Drive Sample Problem

Although the lowest frequency of 150 hz calculated for the 4 speed drive in section 4.5 is not critical (since the operating speed at this frequency is only 48 RPM) the adjustment of eigenvalues can be demonstrated.

Assume that the goal is to modify this frequency from 150 Hz to 175 Hz. The frequency 150 Hz occurs as the sixth eigenvalue of speed 2. The eigenvectors associated with frequency 6, speed 2, are shown in figure 4.3. Derivatives with respect to mass/inertia elements are given in appendix C. It is undesirable to alter inertia elements because the change would alter gear diameters or widths which would affect the power rating of the gear mesh or the kinematic design of the drive. Derivatives with respect to the different stiffness elements are also given in appendix C. It is clear that inertia node 6 is the resonant element. The derivatives of frequency 6 with respect to different parameters have been extracted from appendix C and can be studied in table 4.9. For speed 2, this inertia is coupled to the system by stiffness element (5-6) only. On first reaction, one might expect this stiffness element to have maximum impact. However, stiffness element (0-1) is nearly twice as effective in modifying this frequency.

TABLE 4.9

Stiffness Derivatives Associated with Eigenvalue 6 Speed 2

Element	$\partial n / \partial k$ [(rad/s)/(MNm/rad)]
0-1	1.366
1-2	0.000
3-4	0.035
4-5	0.000
5-6	0.590
7-8	0.001
8-9	1.334

Because a number of stiffness elements affect eigenvalue 6 - speed 2, a number of iterations are required to achieve the objective of 175 hz. The first iteration will modify stiffness element (0-1) which has the greatest impact. The change in frequency required is:

$$\Delta\Omega = 2\pi\,(175-150) = 157 \text{ rad/s}$$

From table 4.9 - stiffness (0-1):

$$\frac{\partial\Omega}{\partial k_1} = 1366 \text{ (rad/s)/(MNm/rad)}$$

$$\Delta k = \frac{\partial k_1}{\partial\Omega}\Delta\Omega$$

$$\Delta k = (1/1366)\ 157 = 0.115 \text{ MNm/rad}$$

The new diameter of shaft 1 can be calculated as follows:

$$d_{new} = d_{old}\ (k^{new}/k^{old})^{1/4}$$

$$d_{new} = 40\ ((0.084 + 0.115)/0.084)^{1/4}$$

$$= 49.6 \text{ mm}$$

With this change in diameter implemented, the frequency under consideration moved from 150 Hz to 165 Hz. Checking eigenvalue derivatives it was found that $\partial\Omega/\partial k_1$ had changed to 914 (rad/s)/(MNm/rad). A second iteration resulted in a new diameter of 53.8 mm for shaft 1. This adjustment modified the frequency to 171 hz. A final adjustment was made to the system, however the change was to shaft 2 instead of shaft 1, because of the decreasing value of $\partial\Omega/\partial k_1$ Shaft 2 was modified in diameter from 60 mm to 61.3 mm with a resulting move in the natural frequency to 174 hz. This was considered sufficiently close to the design objective.

This sample calculation has demonstrated the utility of eigenvalue derivatives in modifying natural frequencies. The frequency selected was complex to move because the frequency was the result of a long stiffness path (figure 4.4). Although stiffness (0-1) was initially most sensitive, this changed by the second iteration. Eigenvalue derivatives made this change in sensitivity clear and simplified the process of making design modifications. The modifications were incorporated into the design and the resulting modified eigenvalues are presented in table 4.11.

TABLE 4.10

Modified Shaft Diameters (mm)

SHAFT NUMBER	OLD	NEW
1	40.0	53.8
2	60.0	61.3
3	80.0	80.0

TABLE 4.11

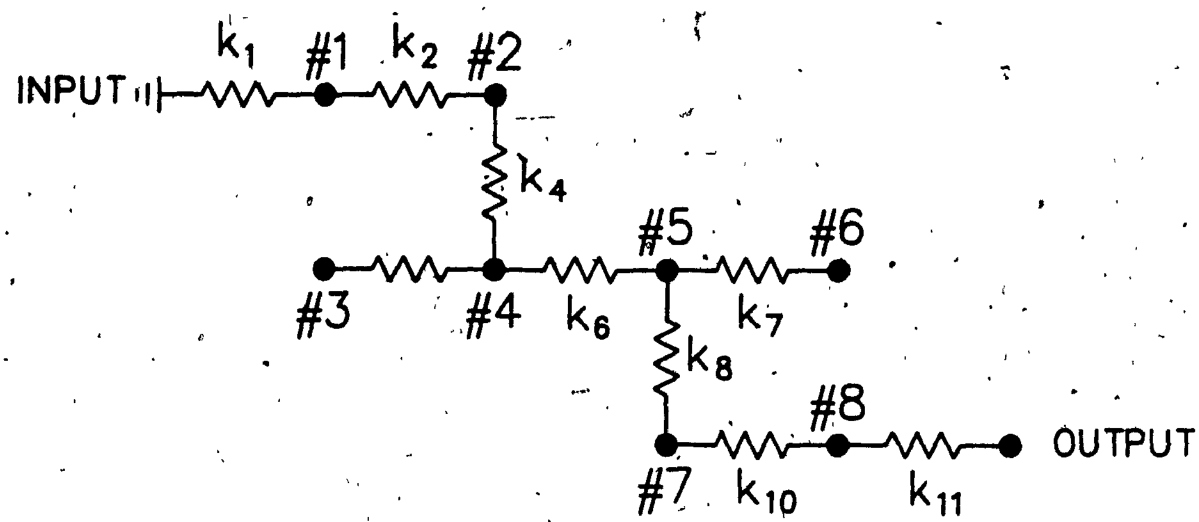
Modified Eigenvalues (Hz)

EIGENVALUE #	SPEED			
	1	2	3	4
1	14966	3162	14966	709
2	5233	12127	5232	12066
3	787	332	840	275
4	4598	7142	639	7487
5	15492	10321	6069	4761
6	198	174	3839	2999
7	535	622	561	59[illegible]
8	7796	7780	11847	9577

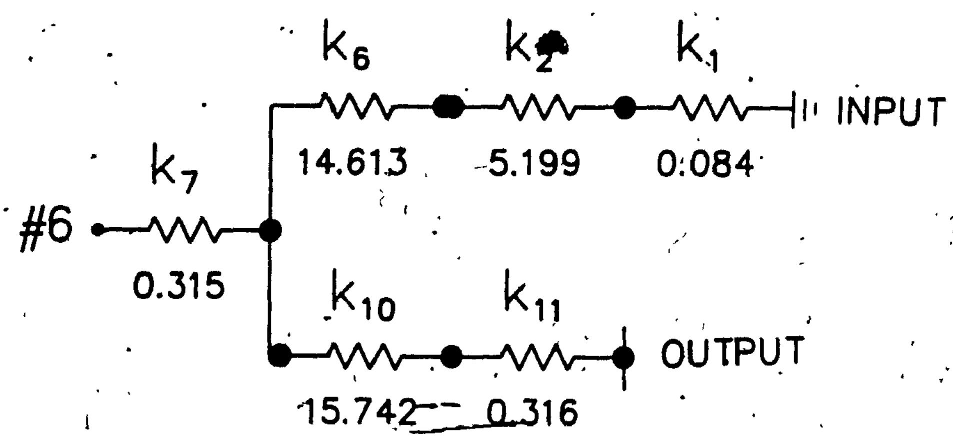
This ability to modify eigenvalues using stiffness elements is very appropriate to the analysis at hand. The method of modifying eigenvalues provides a feasible design approach which ensures that the optimum arrangement selected will not have to be discarded or invalidated because of dynamic problems.

4.8 Summary

This chapter has presented a mathematical model of multi-speed gear drives which can be used to evaluate the torsional natural frequencies for each operating speed. A strategy for modification of eigenvalues, based on adjustment of shaft diameters was presented. To assist in the selection of the shaft to be modified, a method to evaluate eigenvalue derivatives with respect to stiffness elements was also presented. Eigenvalues for the 4 speed verification problem were presented.



(a) Total Drive



(b) Rearranged with respect to node 6.

FIGURE 4.4

Dynamic Model for Verification Problem with
Mass Node 6 Isolated for Clarity.

CHAPTER 5

OPTIMIZATION STRATEGY

5.1 Introduction

The need for an optimization strategy for complex mechanical systems relates directly to the number of design variables and their interaction. The ability of all methods of non-linear optimization to move variables toward optimum values decreases rapidly with increased number of design variables and constraints. It is clear that for large problems, CPU limitations continue to exist.

The optimization of an engineering problem can be reduced to three primary tasks; (1) define an objective function and constraints which ensure that the operating requirements will be satisfied at the optimum point, (2) develop a mathematical model of the system with all significant physical variables represented and constrained as required, and (3) select or develop the optimization techniques which reliably find the minimum value of the objective function.

5.2 Problem Reduction Optimization Strategy

There is much written in the literature about the difficulties of realistic engineering problems which have discrete variables, multiple objectives, complex constraints, etc. [2,4,6,8,105-110,121-128]. The difficulties encountered in the optimal synthesis of designs for multi-speed gear drives are not unique. The following process, called problem reduction strategy, incorporates elements of Johnson's MOD [122], Papalambros and Wilde's monotonicity analysis [123] and Wilde's maximal activity principle [4]. The three main constituents of the solution method are: (a) determining how the different analyses interact, (b) formulating the problem such that optimization parameters cannot take on values which violate constraints, and (c) ensuring that the number of optimization parameters is equal to the degrees of freedom of the problem. The detailed process for these activites is as follows:

1. Review the conventional approach to producing designs for the problem at hand.
2. Note the strengths and weaknesses of the standard approach. Understand which activities are in parallel and which activities are in series and why. In particular, observe how the different analyses interact.
3. Identify all operating requirements and select an appropriate objective function.
4. Develop a mathematical model such that all parameters which affect the objective function within the desired accuracy bandwidth are included.
5. Assess whether the mathematical model can be solved directly, analytically, or using an available linear or non-linear optimization technique.
6 Simplify the mathematical model. Determine the number of actual degrees of freedom in the overall design process as compared to the number of parameters in the objective function. Reduce the number of parameters to the number of degrees of freedom.

7. Take advantage of known objective function behavior by customizing optimization routines.

Figure 5.1 presents problem reduction strategy in the form of a flow chart.

5.3 Definition of Optimization Problem

For engineering problems it is often difficult to define an objective function which is "best" for all applications. Volume, weight, radial size, axial size, etc are all valid objectives and have been studied in the literature. Of these, volume is most appropriate because it takes advantage of both gear diameter and face width information. Further it includes elements related to both size and weight. Therefore, volume of the drive envelope, on a per group basis, shall be the objective function.

5.3.1 Volume Objective Function

The volume function is calculated as the sum of the volume of each group with a volume correction made for composite arrangements. The basic volume calculated for a conventional arrangement is shown in figure 5.2. The correction (i.e. savings in volume) associated with a composite arrangement is illustrated in figure 5.3.

The diameters and widths of the gears are variables of the objective function. The following equations are used in the calculation of volume for the i^{th} group.

$$V_i = R_i * A_i * D_i \qquad (5.1)$$

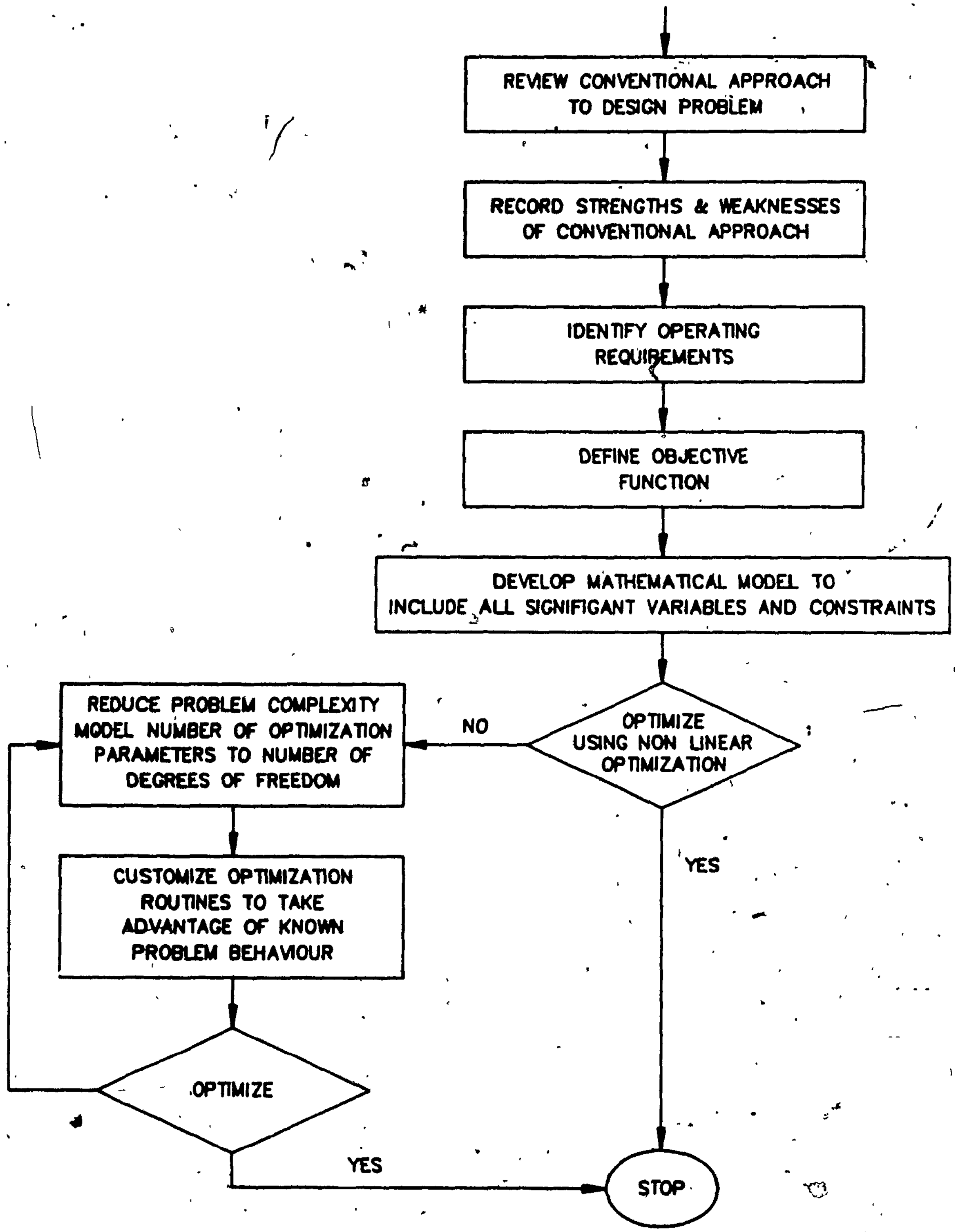


FIGURE 5.1

Problem Reduction Strategy

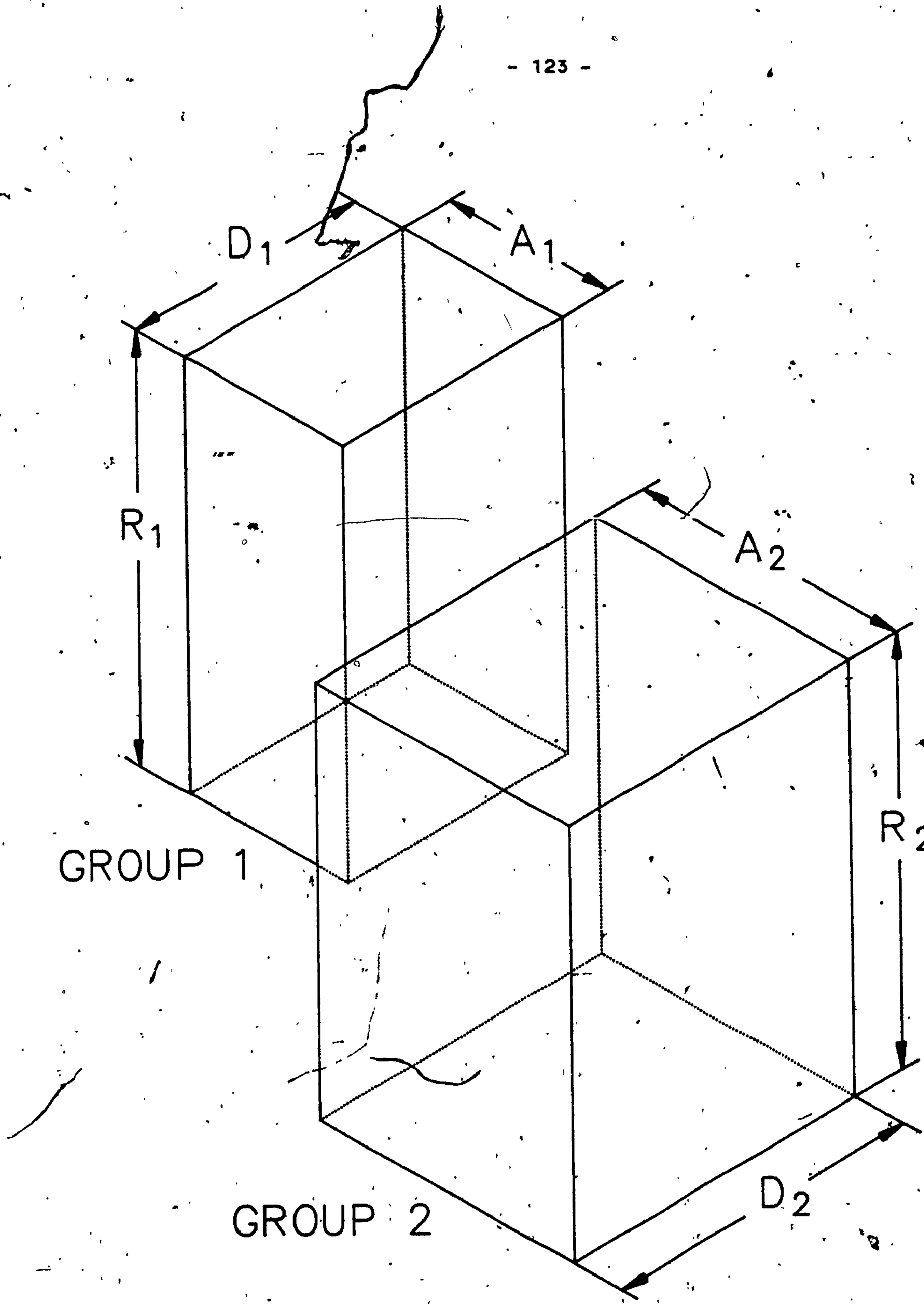


FIGURE 5.2

Definition of Volume Objective Function

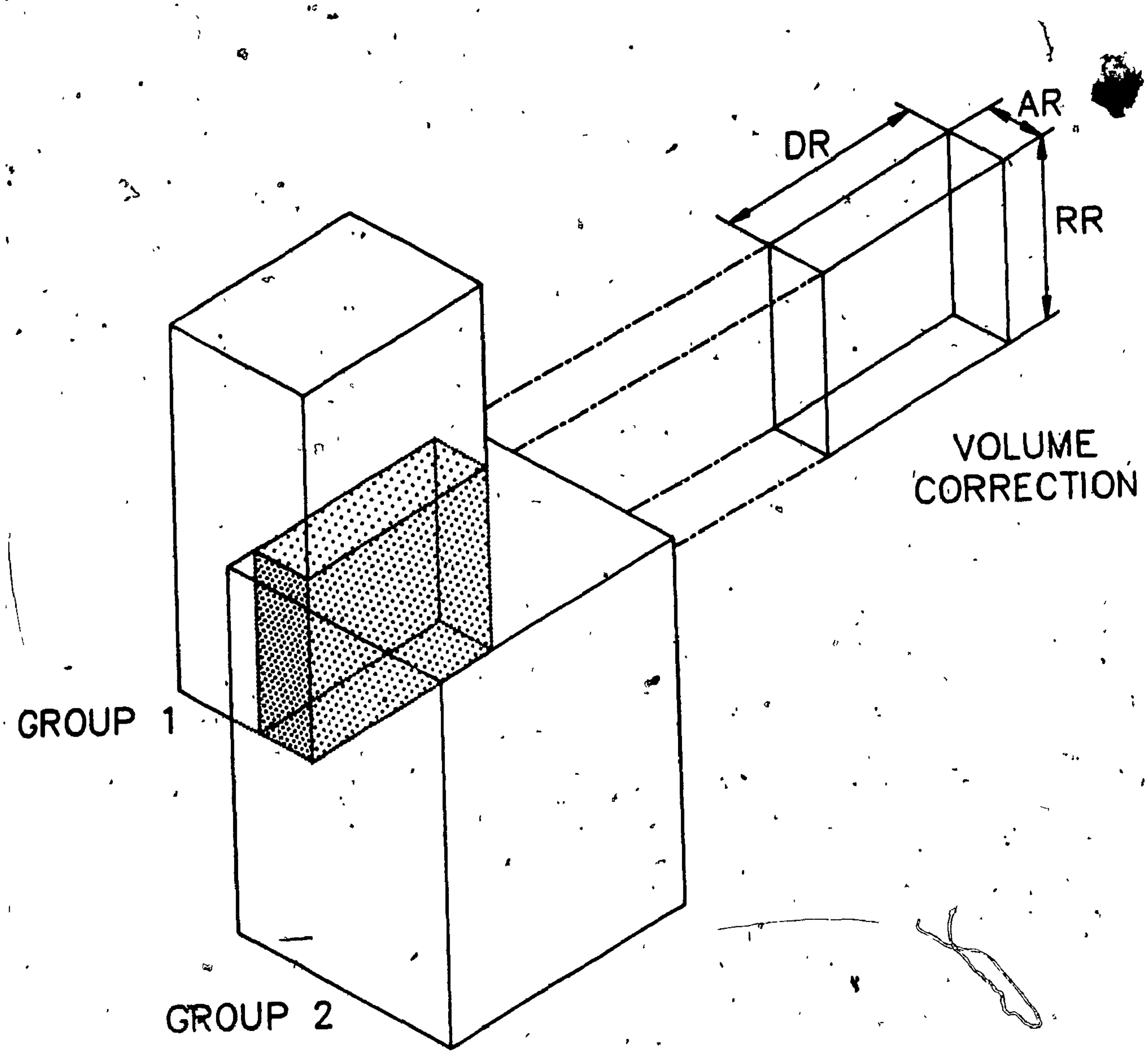


FIGURE 5.3

Correction to Volume Objective Function
for Composite Arrangements.

R_i = Sum of the center distances between shafts plus half the diameter of the largest gear on the input shaft and the output shaft

A_i = Twice the sum of the width of the gears on the input shaft.

D_i = Diameter of the largest gear in the group under consideration.

For the overall drive, the volume for the conventional arrangement is equal to the sum of the volumes of each group as calculated by equation (5.1).

$$V = \sum V_i \qquad i=1,2,3, \ldots n_s-1 \tag{5.2}$$

The correction in volume associated with each composite in a drive is calculated as:

$$VR_i = RR_i * AR_i * DR_i \tag{5.3}$$

RR_i = Half the diameter of the largest gear on the input side composite shaft plus half the diameter of the largest gear on the output side of the composite shaft.

AR_i = Twice the width of the composite gear under consideration

DR_i = The smaller one of the largest diameter in the two groups associated with the composite

This gives the total volume as calculated for the objective function as:

$$V_t = V - \sum VR_i \qquad i = 1,2,\ldots n_c \tag{5.4}$$

This volume calculation provides a representative estimate of the overall size of the drive.

5.3.2 Operating Requirements

In addition to the objective function, there are a number of operating requirements which must be satisfied. These are:

1. Output speeds must form a specified geometric progression starting from a given lowest speed.
2. Gear components must be rated equal to or greater than the drive design rating. At least one component must be equal to the design rating.
3. Face width to diameter ratio of the gearing must be between 0.1 and 1.0.
4. Natural frequencies of the system must not lie within 10 Hz of an operating speed.
5. The input speed may be specified.

These operating requirements further define the problem and have been mathematically formulated in chapters 2 to 4.

5.4 Background and Development of the Mathematical Model

The objective function developed in section 5.3 requires the diameters and face widths of each gear for its evaluation. For the 4 speed conventional arrangement, this turns out to be 12 variables. As will be shown, for the case when the input speed is fixed, the kinematic arrangement optimization has 1 degree of freedom. The remaining variables are constrained by equality constraints. The probability of finding a feasible solution for variables contained in the range of 1 to

10 would be on the order of $1/((10^4)^{11})$, assuming the constraints are satisfied to approximately 3 decimal places. This kind of hardship was demonstrated during the solution of the speed reducer [105-109].

5.4.1 Standard Approach to the Design of Gear Drives

Insight into the problem can be obtained by examining the traditional design methods particularly when they have been in use over an extended period of time. The design procedure for a gear drive follows a standard path where specific types of design and analysis are done at distinct stages. The following step by step procedure, representative of the normal design activities, is based on a paper by Tucker [1].

Step 1 - Identify Drive requirements. This step identifies the application of the drive and its associated design requirements. Typically this would include number and value of output speeds, power capacity, life, precision and environment. The results of the analysis are entirely dependent on the data provided in step 1. In a real design problem, step 1 is often re-visited if cost becomes excessive to meet requirements.

Step 2 - Select Gear Train Arrangement. There are many drive arrangments possible for a multi-speed gear drive. Most commonly, parallel axis arrangments are used. This limitation still leaves a wide range of possibilities as demonstrated by Koenigsburger [9] and Ackerkan [10]. Most of these arrangements will satisfy the kinematic requirements. Because it is expensive and time consuming to examine more than one arrangements, a selection is normally made after which this step is taken to be independent of the others. The best one is difficult to identify since it depends on many design variables.

Step 3 - Select Tooth Type. Spur or helical gearing is most commonly used for parallel axis drive systems. Spur gears are the simplest to design and manufacture and are used whenever practical. Helical gears can operate at higher pitch line velocities and with higher power to weight ratios than spur gears. The overlapping tooth action results in a smooth, more quiet operation. The tooth form is determined as the result of a trade-off between operating requirements and cost. Special tooth forms may be required for drives with complex arrangements. Given that the arrangement is of the parallel axis type, then the tooth type depends primarily on operating conditions.

Step 4 - Select Gear Materials - Gear materials affect the size and performance characteristics of the drive. Selection is a tradeoff between cost and performance. Given the tooth form, materials depend primarily on operating conditions.

Step 5 - Estimate Minimum Gear Diameters. An initial estimate of the gear diameters is made based on power capacity. It is normally assumed that the pitting resistance is the critical design constraint. This step introduces some of the more apparent difficulties associated with the design of gear drives. The diameters of the gears are a function of the drive kinematics. The resulting diameters may not be satisfactory and the designer is forced to return to step 1. A number of iterations are commonly expected.

Step 6 - Select Tooth Size and Proportions. This step is a refinement of the gear design. It results in small variations in gear diameters. The intent, however, is that the diameters are set after step 5 and are not changed because of these small variations. Tucker notes that the tooth size is a function of the allowable tooth bending stress, scoring resistance, tooth deflection, diametral pitch and addendum proportion. Standard tooth proportions result in a reasonably balanced design for most gearing. Special tooth proportions are used to minimize the size and balance the strength of pinions. Addendum modifications are also used to avoid undercut.

Step 7 - Specify Allowable Tolerances. Tolerances are determined based on pitch line velocity, accuracy requirements of the application, manufacturing limitations and cost considerations. Tolerancing of the components has little impact on the size of the diameters, provided the assumptions for quality are consistent with step 1. Changes in dimensions associated with tolerancing are samll and do not affect earlier steps.

Step 8 - Calculate Shaft Diameters. Shafts must support the gears between bearing centers with a minimum of bending. Their major impact is on the system dynamic properties, both torsional and lateral, which are very dependent on shaft stiffness. Since shaft lateral deflections are limited, machine tool drives are normally stiff with regard to lateral and axial vibration.

Step 9 - Perform Preliminary Dynamic Analysis. With principle dimensions of all major components available, a dynamic analysis is normally made to verify natural freqencies and loads in the drive. This stage may result in re-design which must be implemented as early as step 1. The step, however, is considered to be independent in the anticipation that design modifications will be small and confined to individual components.

Step 10 - Determine Gear Mounting. Steps 1 though 9 determine the size of each of the components. Kinematic performance, load capability and dynamic performance have been verified. This step includes design of the casing, bearings and packaging. This step is largely independent of earlier stages and is treated as such.

Step 11 - Lubrication Requirements. Cooling and heat dissipation are normally handled by the lubricant. With spur and helical gears, which have efficiencies of 98-99%, the losses are low and do not require special treatment until power and pitch line velocities become much higher than those reached in machine tools. It is important that the correct lubricant be selected to ensure that scoring is not a problem. This step is considered independent of earlier steps.

5.4.2 Highlights of the Conventional Design Approach

The following comments highlight the advantages and disadvantages of the standard procedure. The advantages were:

(1) the problem was partitioned along natural boundaries in order to achieve a solution.

(2) later stages were more nearly independent and are treated as such.

The disadvantages are:

(1) designer experience was extremely important in selecting an arrangement since it was not practical to evaluate more than 1 or 2,

(2) the design process was iterative, particularly for steps 1 through 5, since component size is a function of operating speeds and gear ratios (i.e kinematic arrangement).

(3) The inter-relationships in steps 1 through 5 were handled mostly by designer experience.

In the conventional approach the designer has partitioned the problem at sensible stages to minimize the impact of variable coupling between the different analyses required for the design. The main weakness of the conventional method was that few arrangements could be examined through the most important "beginning stages" of the design.

5.4.3 Key Elements of the Optimization Process

The initial steps of the design, up to the sizing of the gears, is very important and represents the area which was previously most difficult. The result using problem reduction strategy on this problem is a mathematical process which allows for the efficient synthesis of designs which satisty all key requirements for this critical initial stage in the design of a gear drive.

Consider the 4 speed conventional arrangement. The variables associated with the objective function are d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8, F_1, F_2, F_3, and F_4.

The kinematic analysis, presented in chapter 2, determines the relative diameters of all gears in the drive. It was shown that there are two degrees of freedom per group. These were ~~taken~~ to be the first input gear and the first output gear in each group. Thus, the diameters d_2, d_4, d_6 and d_8 are defined by kinematic requirements.

The sizes and face widths of the first input gear in each group should be as compact as possible and thus are defined using the minimum diameter matrix to ensure adequate power capability. Thus variables d_1, F_1, d_5, and F_3 are defined directly from the minimum diameter matrix.

The diameters of gears d_2 and d_6 are normally larger than the minimum specified in the minimum diameter matrix. Therefore, if the face width to diameter ratio is not reduced, these meshes will have excess power capacity. The widths of these two gears are therefore reduced, down to a minimum face width to diameter ratio of .1, until the required power rating is achieved. This step defines gear widths F_2 and F_4.

This leave two parameters to optimize for best arrangement which are d_2 and d_4. Since these diameters can have a wide range of values, the speed reduction ratios X_1 and X_2, which are dimensionless and have a normal range of .5 to 4, are used by the optimization routine.

If the input speed is fixed, then 1 further degree of freedom is removed. Parameter X_2 is eliminated from the optimization process and is calculated based on the relationships between X_1, the input shaft speed, and the lowest output speed.

Single composite arrangements do not reduce the degrees of freedom but constrain the diameter of the input gear in the second group, d_5 to a specific value greater than or equal to the value specified in the minimum diameter matrix.

Double composite arrangements reduce the degrees of freedom by one. Variable X_2 is dropped from the optimization parameter list and determined analytically. The 2 group, double composite arrangement with a fixed input speed is a special case which results in no degrees of freedom.

The dynamic behavior of the gear drive is affected by two types of parameters. These are inertias and stiffnesses. Changes to the inertias must be implemented by changes to the diameter or width of gears. Changes to stiffness elements can be implemented by changing the length or diameter of shafts. Changing gear diameters, gear widths or shaft lengths is undesirable because it modifies the arrangement optimization. This leaves shaft diameter which is well suited to this task because it has no impact on the objective function.

The diameter matrix provides a link between the kinematic optimization and the component design without introducing the many variables associated with the later. The interface is ideal because it decouples two time consuming analyses. The effect of different diameter matrices (i.e. extent to which diameter is optimized) can be examined without necessarily doing the analysis required to generate an accurate matrix. Parametric studies, such as the benefit to overall volume associated with a 25% reduction in input gear diameter, could be easily studied. If the benefit is sufficient, then an effort could be made to determine what modifications in mesh design are required to achieve the required diameter reduction.

5.4.4 Operating Requirements Addressed by the Mathematical Model

In addition to the objective function of minimum volume, there are required operating conditions and inputs which must be satisfied in order to resolve the problem. These operating requirements have been built into the mathematical model or are checked during evaluation of the objective function.

(1) Output Speeds Form a Geometric Progression. Standard values of geometric progresion are normally used. This is a basic requirement of the kinematic analysis. The step ratio was selected to be 1.59 for the analysis and is one of the recommended values.

(2) Diameter Relationships for Composite Arrangements. A composite arrangement requires a gear on an intermediate shaft to have the same diameter as a pinion on the same shaft. This relationship between diameters is maintained by the equations which relate gear diameter and kinematic requirements.

(3) Fixed Lowest Output/Input Ratio. Frequently the output speeds are fixed due to operating requirements and the input speeds are limited if syncronous motors are used. This results in an equality constraint on the lowest output/input ratio. The number of design variables is reduced by one and an equality constraint is solved to produce an exact result.

(4) Component strengths must be sufficient to satisfy operating requirements for the design life. An optimum face width/diameter ratio is selected for generation of the minimum diameter matrix. Some gears are forced to have diameters larger than the minimum in order to satisfy kinematic constraints. In those instances, the face width/diameter is reduced to optimize power capacity.

(5) Face width to diameter must be limited to ensure that the analysis for power capability is done in a valid range. Normal values range from .1 to 1.

(6) Contact ratio is a measure of the average number of teeth in contact at any time. It must clearly be greater than 1 and AGMA 360 [20] recommends this value to be greater than 1.4 for spur gears. A value between 1.2 and 1.4 is considered to be the minimum acceptable value. The greater the value of the contact ratio the smoother the mesh, the lower the dynamic loads, and the quieter the operation. The assumption of standard tooth form ensures this requirement.

(7) Standard module are normally used even for non-standard gearing. This makes it more likely that special tooling will not be required to manufacture the gear. The analysis ensures the use of standard module (whole numbers for diametral pitch).

(8) Tooth tip thickness must be sufficient to avoid pointed teeth. The tooth thickness is recommended to be larger than 0.300 at a pitch of 1. For power gearing it is also recommended that the tooth thickness of the gear be less than twice the tooth thickness of the pinion. Standard Tooth form ensures this requirement.

(9) Standard pressure angles to be selected. Standard pressure angles are normally specified for all gearing. The most common values are 20^{o}, 22.5^{o} and 25^{o}. A pressure angle of 20^{o} was selected for the analysis.

(10) Standard helix angles to be selected. The normal values are usually 15^{o}, 23^{o}, and 30^{o}. Values over 30^{o} usually require the use of double helicals. A helix angle of 0^{o} was selected for the analysis.

(11) Number of Design Variables greater than one. It is possible with three shafts, a constraint on the lowest output/input ratio and a double composite arrangement to have no degrees of freedom remaining and hence no optimization parameters. In this case the constraint on the Lowest output/input ratio has priority and double composite arrangements are disallowed.

(12) Diameter of the first pinion in each group minimized. The diameter of the first pinion in each group must have sufficient diameter and width to carry the power of the motor at its lowest operating speed. This diameter is critical for the correct operation of the gear train. A table of values is generated prior to the arrangement optimization,

which gives the minimum diameter of the first pinion in each group for the expected operating range. These diameters are calculated in a separate optimization process. They are calculated based on the defined operating conditions and recognized gear design criteria.

5.5 Optimization Techniques

The problem is partitioned such that a number of independent optimizations are performed. Better efficiency is achieved by the used of different optimization techniques for each analysis.

5.5.1 Discrete Random Process

A discrete random process was investigated for the generation of the minimum diameter matrices. The diameters of the gears are reasonably continuous functions and although there may be rigid constraints, if a move in a particular direction improves the objective function it can be expected that further moves in the same direction will also be beneficial. A directed discrete algorthm was developed and tested for generating the minimum diameter matrix. Computation time was approximately 10 times larger than for a more direct optimization which ignores the discrete behaviour of the variables. Difference between the matrices generated was not significant with regards to the arrangement optimization.

5.5.2 Patterned Search

A patterned search technique was used for the arrangement optimizations. The objective function was evaluated for small deltas for each of the variables. Based on the improvement in each direction, an overall vector was generated in the most favorable direction. It was found to be efficient and reliable in locating global optimums to high accuracy.

5.5.3 Random Search

A random search method was used to investigate the design space particularly for the arrangment optimization. The random optimization performed a number of useful functions.

(1) An inexpensive investigation of the design space was possible to observe objective function behavior.

(2) Single evaluations of a specific point were possible to assist in debugging and problem verification.

(3) Searchs of specific areas could be made to determine if the patterned move approach converged to the global minimum.

5.5.4 Univariate Technique

The univariate technique was used for the kinematic analysis for radial dimension optimization. This technique was found to be effective prior to the volume objective function which was found to have more complex behavior.

5.5.5 Direct Methods

Knowledge of the behaviour of the variables with respect to power capacity was used to optimize face width and diameter during particular stages of the design process. Diameter was found to vary exponetially from 2.8 to 3.1 with respect to power. Face width was found to vary approximately linearly with power. Direct methods when used are most efficient and converge to high accuracies.

5.6 Four Speed Results - Volume Objective Function

To determine the effect of face/width to diameter ratio on the arrangement optimization, conventional arrangements were optimized for a number of values. Minimum diameter matrices are given in tables C.1 through C.10 for face/width to diameter ratios of 0.1 through 1.0. The conventional arrangements were optimized for a variety of face widths. The results of this calculation are given in table 5.1 and plotted in figure 5.4. The optimum face width/diameter ratio of was found to be 0.5.

This optimum face width/diameter ratio was used for the optimization of all four speed arrangements. Tables C.12 and C.14 present tooth numbers and module for the optimum diameter matrix with a 0.5 face width/diameter ratio.

Table 5.2 presents optimum results for the 4 speed drive for all 12 arrangements for a face width/diameter ratio of 0.5. The best arrangement is optimization 1 which is conventional and has an objective function value of 0.0409 m^3. Other arrangements, however were found to be nearly as small. Optimization 3 has a function value of 0.0429 m^3 and optimization 7 has a function value of 0.0430 m^3. These are both within 6% of the optimum solution. Addition constraints on axial or radial dimension may make these slightly larger arrangements attractive. Figure 5.5 presents the gear train for optimization 1 table 5.2. Figure 5.6 presents the speed diagram for optimization 1 table 5.2.

The general behavior of the drive presented in chapter 5 is good. The face width/diameter ratio is equal to 0.5 for the first input gear in each group. The diameters are seen to increase as the input speed to the group decreases. The face widths of the larger input gears in a group have face width/diameter ratios of less than 0.5 in order to compensate for extra power capacity. The face width/diameter ratio of all gears in the drive are between 0.1 and, in this case 0.5.

Tables 5.3 through 5.9 present the basis of the dynamic analysis including component inertias and stiffnesses. Table 5.10 presents eigenvalues for the 4 speeds of the optimum arrangment. The lowest frequency is 191 hz. This is equivalent to an operating speed of 11500 RPM hence this frequency is satisfactory. The very high frequency is related to the verification problem rather than gear drives in general. The drive under consideration is operating at very low speeds and is very compact. This results in a very stiff design and accounts for the large difference between the highest operating speed and the first natural frequency.

The results of chapter 5 are compared to the results of chapters 2 and 3 in table 5.11. The optimization for volume results in another optimum design point. The input speeds move higher in order to reduce torque on the initial mesh. Also, the diameters of the gears for the volume optimization are larger in order to compensate for the smaller face widths.

5.7 Summary

This chapter has brought together the mathematical model of the gear drive developed in chapters 2 through 4 so that an overall optimization could be performed. The optimum arrangement determined meets the kinematic requirements specified, all gear components are rated for the required power, and the dynamic performance has been verified as satisfactory.

TABLE 5.1

Summary of Optimum Conventional Arrangements
Face Width/Diameter Ratio = 0.1 - 1.0

		FACE WIDTH TO DIAMETER RATIO						
	#	0.10	0.30	0.40	0.50	0.60	0.70	1.00
DIAMETER	1	206	142	129	108	102	106	92
	2	260	180	163	140	132	134	118
	3	261	180	163	173	162	139	128
	4	207	143	130	141	132	111	103
	5	201	138	126	118	110	105	94
	6	365	252	229	214	201	191	171
	7	578	403	369	340	324	311	281
	8	415	290	266	244	233	225	203
WIDTH	1	21	42	51	54	62	74	94
	2	18	38	46	40	46	64	79
	3	20	42	51	59	67	74	95
	4	11	22	27	32	34	39	47
RATIOS	S	0.275	0.271	0.269	0.217	0.214	0.257	0.241
	1	0.790	0.790	0.790	0.628	0.627	0.762	0.722
	2	1.256	1.256	1.256	0.998	0.998	1.211	1.148
	3	0.348	0.343	0.340	0.347	0.340	0.337	0.333
	4	0.879	0.868	0.861	0.876	0.861	0.852	0.843
F (m^3)		0.041	0.041	0.041	0.041	0.041	0.042	0.043

Diameter (mm), Width (mm)

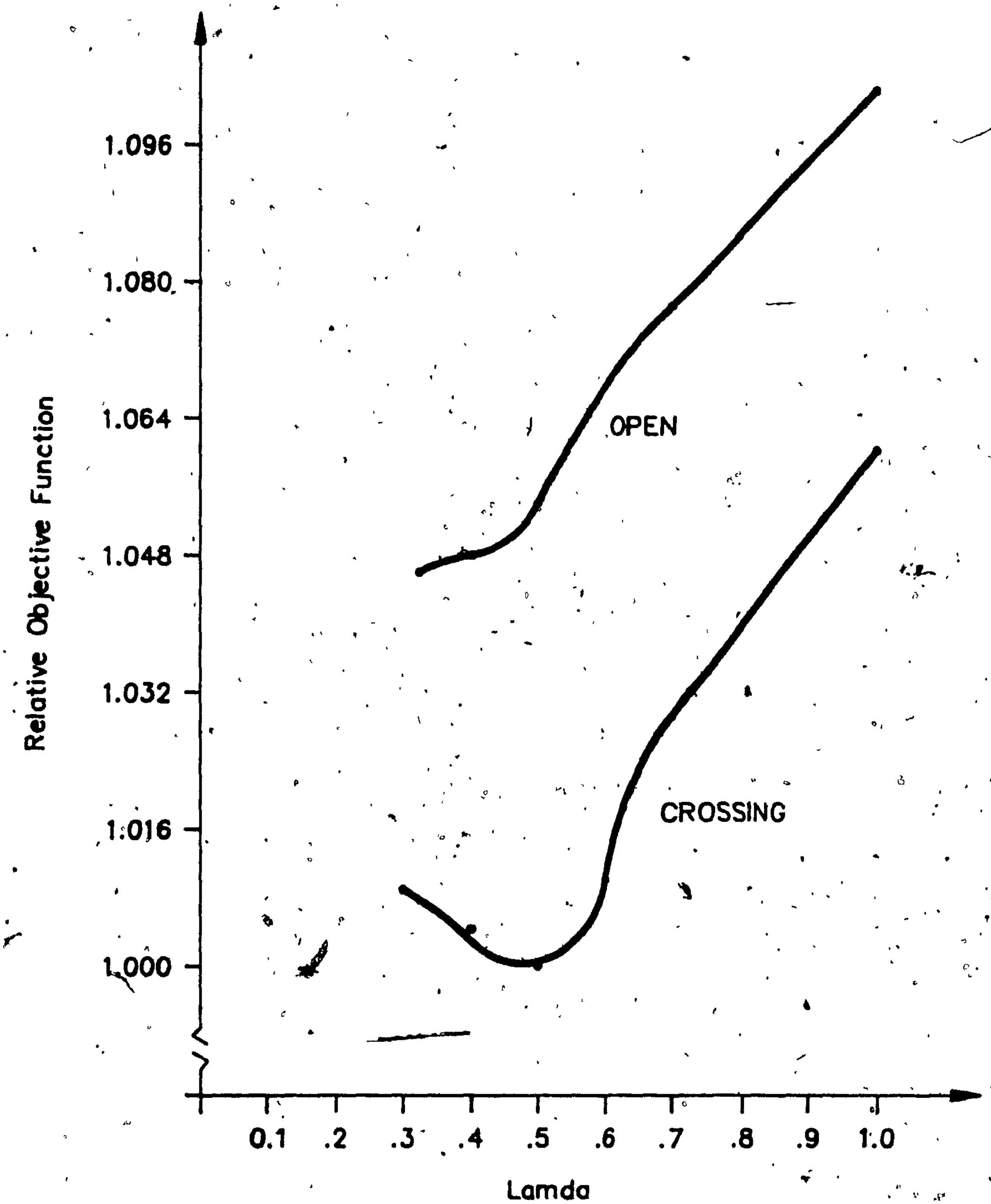


FIGURE 5.4

Effect of Face Width/Diameter on Volume Objective Function for Conventional Arrangements

TABLE 5.2

Optimization Summary for the 4 Speed Drive Verification Problem
Volume Objective Function

		OPTIMIZATION NUMBER					
		1	2	3	4	5	6
DIAMETER	1	108.5	129.0	97.8	127.6	101.6	472.4
	2	140.5	167.2	131.7	165.0	138.4	595.1
	3	172.9	205.7	220.8	199.9	257.2	590.5
	4	140.8	167.6	187.0	162.6	220.4	467.8
	5	117.7	205.7	126.0	162.6	125.4	467.8
	6	213.6	330.1	220.8	275.9	220.4	590.5
	7	339.6	340.6	309.1	344.7	311.0	245.0
	8	243.7	216.2	214.3	231.4	216.1	122.3
WIDTH	1	53.5	68.7	53.9	52.7	39.1	97.6
	2	40.3	46.0	35.7	80.7	53.5	84.9
	3	59.4	68.7	62.9	80.7	62.6	85.1
	4	31.6	31.4	53.9	36.5	53.5	97.6
RATIOS	S	0.217	0.379	0.180	0.301	0.159	1.528
	1	0.626	0.627	0.443	0.638	0.395	0.800
	2	0.998	0.998	0.704	1.015	0.628	1.272
	3	0.347	0.604	0.408	0.472	0.403	1.909
	4	0.876	1.526	1.031	1.192	1.020	4.827
F (m^3)		0.0409	0.0506	0.0429	0.0564	0.0464	0.2840

TABLE 5.2 (Continued)

		OPTIMIZATION NUMBER					
		7	8	9	10	11	12
D	1	110.1	127.9	116.3	100.0	90.0	82.3
I	2	175.1	204.3	184.9	173.3	160.1	150.2
A	3	175.5	207.8	184.9	232.9	236.3	243.7
M	4	110.4	131.3	116.3	159.6	166.2	175.9
E	5	120.0	207.8	136.8	159.6	121.3	175.9
T	6	164.9	270.4	184.9	214.4	166.2	243.7
E	7	330.8	344.2	321.1	353.2	325.9	528.3
R	8	285.9	281.6	273.0	298.4	280.9	460.5
W	1	55.0	67.3	70.2	50.0	44.2	41.3
I	2	38.5	41.8	42.5	79.4	69.4	46.0
D	3	60.3	67.3	55.9	79.4	60.9	46.0
T	4	43.7	34.3	70.2	40.6	69.4	41.3
H							
R	S	0.227	0.372	0.268	0.194	0.142	0.112
A							
T	1	0.627	0.615	0.629	0.429	0.381	0.338
I	2	1.586	1.556	1.590	1.086	0.963	0.854
O	3	0.363	0.604	0.426	0.452	0.372	0.333
S	4	0.577	0.960	0.677	0.719	0.592	0.529
F (m^3)		0.0430	0.0510	0.0482	0.0598	0.0526	0.0731

Diameter (mm), Width (mm)

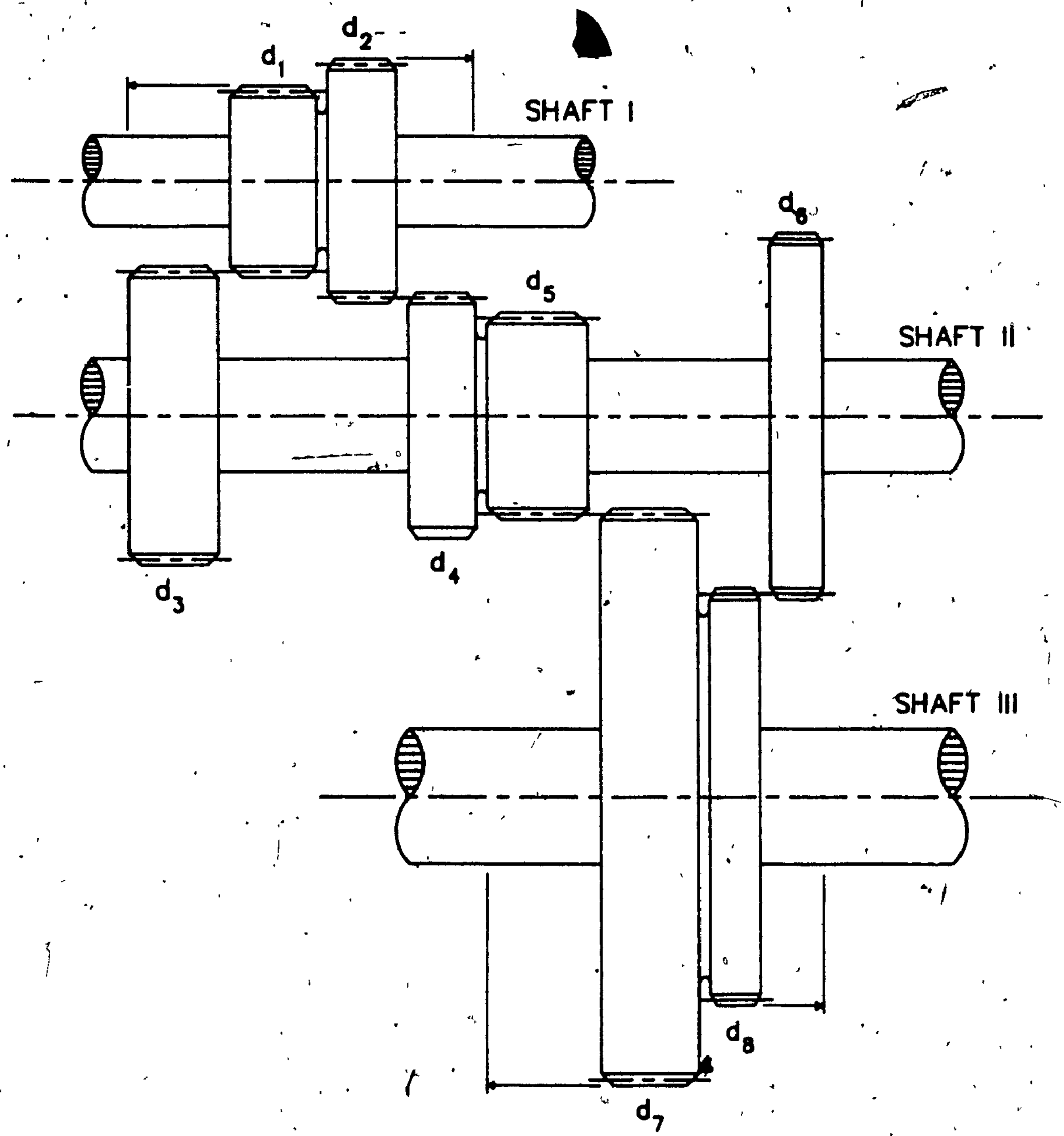


FIGURE 5.5

Gear Train Based on Optimization 1 Table 5.2

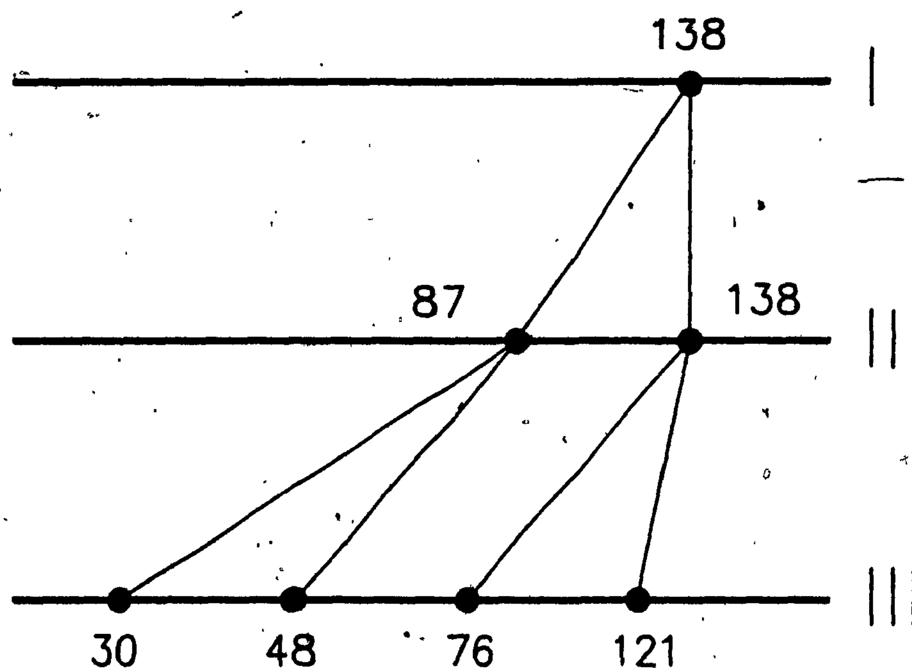


FIGURE 5.6

Speed Diagram Based on Optimization 1 Table 5.2

TABLE 5.3

Optimum Arrangement With Shaft Diameters for Four Speed Verification Problem Volume Optimization

GEAR				SHAFT	
#	DIAM (mm)	LAMDA	WIDTH (mm)	#	DIAM (mm)
1	108.5	0.493	53.5	1	40.0
2	140.5	0.287	40.3	2	60.0
3	172.9		53.5	3	80.0
4	140.8		40.3		
5	117.7	0.504	59.4		
6	213.6	0.148	31.7		
7	339.6		59.4		
8	243.7		31.7		

TABLE 5.4

Ratio of Shaft Speed to Input Shaft Speed

SHAFT	SPEED			
	1	2	3	4
1	1.000	1.000	1.000	1.000
2	0.627	0.998	0.627	0.998
3	0.218	0.346	0.550	0.874

TABLE 5.5

Basic and Equivalent Inertia (kgm^2) of Gears
Referenced to Input Shaft Speed

GEAR	SPEED				
#	BASIC	#1	#2	#3	#4
1	0.0057	0.0057	0.0057	0.0057	0.0057
2	0.0121	0.0121	0.0121	0.0121	0.0121
3	0.0367	0.0144	0.0365	0.0144	0.0365
4	0.0122	0.0048	0.0121	0.0048	0.0121
5	0.0088	0.0035	0.0087	0.0035	0.0087
6	0.0506	0.0199	0.0504	0.0199	0.0504
7	0.6070	0.0287	0.0726	0.1835	0.4640
8	0.0859	0.0041	0.0103	0.0260	0.0656

TABLE 5.6

Stiffness (MNm/rad) of Basic and Equivalent
Couplings Speed #1

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	60.0	0.347	0.347
2	1	2	4.0	5.199	5.199
5	3	4	98.5	1.068	0.421
6	4	5	6.0	17.545	6.907
7	5	6	95.6	1.101	0.434
10	7	8	8.0	41.588	1.967
11	8	9	215.6	1.543	0.073

TABLE 5.7

Stiffness (MNm/rad) of Basic and Equivalent Couplings Speed #2

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	158.5	0.131	0.131
2	1	2	4.0	5.199	5.199
5	3	4	98.5	1.068	1.063
6	4	5	6.0	17.545	17.462
7	5	6	95.6	1.101	1.096
10	7	8	8.0	41.588	4.974
11	8	9	215.6	1.543	0.185

TABLE 5.8

Stiffness (MNm/rad) of Basic and Equivalent Couplings Speed #3

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	60.0	0.347	0.347
2	1	2	4.0	5.199	5.199
5	3	4	98.5	1.068	0.421
6	4	5	6.0	17.545	6.907
7	5	6	95.6	1.101	0.434
10	7	8	8.0	41.588	12.575
11	8	9	120.0	2.773	0.838

TABLE 5.9

Stiffness (MNm/rad) of Basic and Equivalent Couplings Speed #4

#	END 1	END 2	LENGTH	BASIC	EQUIV
1	0	1	158.5	0.131	0.131
2	1	2	4.0	5.199	5.199
5	3	4	98.5	1.068	1.063
6	4	5	6.0	17.545	17.462
7	5	6	95.6	1.101	1.096
10	7	8	8.0	41.588	31.790
11	8	9	120.0	2.773	2.119

TABLE 5.10

Eigenvalues (Hz)

EIGENVALUE #	SPEED			
	1	2	3	4
1	18137	5845	18137	5076
2	4105	3332	4108	1375
3	1012	1048	641	571
4	6298	16087	1656	15534
5	21403	13926	9444	8260
6	752	783	10899	2853
7	293	191	312	292
8	3723	3963	2975	7181

TABLE 5.11

A Comparision of Optimum Solutions

DIAMETER	CHAPTER 2	CHAPTER 3	CHAPTER 5
1	1.000	1.000	1.000
2	1.261	1.309	1.295
3	1.261	1.743	1.594
4	1.000	1.435	1.298
5	1.000	1.221	1.085
6	1.590	1.912	1.969
7	1.590	1.812	3.130
8	1.000	1.121	2.246
WIDTH			
1	0.500*	0.995**	0.493
2	0.500	0.713	0.371
3	0.500	1.212	0.547
4	0.500	1.338	0.291

* Arbitrary

** Calculated to achieve required power rating.

CHAPTER 6

CASE STUDIES FOR A 9 SPEED AND AN 18 SPEED GEAR DRIVE

6.1 Introduction

This chapter will draw on the theory of chapters 2 through 5 to design and optimize 2 gear drives. Results for the 9 speed drive will be presented for all 38 possible arrangements. Results for the 18 speed gear drive will be presented for selected conventional arrangements.

6.2 Results and Discussion for the 9 Speed Gear Drive

The 9 speed conventional arrangement is shown in figure 6.1 The 2 layout diagrams for 9 speeds are shown in figure 6.2. There are 38 possible kinematic arrangements for this gear drive. These are identified in table 6.1. The minimum diameter matrices used for the arrangement optimization are for a face width/diameter ratio of 0.5 and are given in appendix C. This value of face width to diameter was found to minimize gear drive volume. Results for 28 of the 38 possible arrangements are presented in table 6.2. No practical solutions were found for the remaining 10 arrangements.

The smallest design was found to be optimization 20 of table 6.2 with a function value of 0.0650 m^3. Design values for each gear in this arrangement are shown in table 6.3. All meshes but 1 are rated at desired power capacity. Mesh 3 has a 16.2% over-capacity. This mesh is non-optimum because the face width/diameter ratio is equal to 0.1 and cannot be reduced further. Figure 6.3 illustrates the gear drive for optimization 20 table 6.2. Figure 6.4 gives the speed diagram for the same arrangement.

The smallest single composite arrangements were optimizations 23 and 24 both with objective function values of 0.0711 m^3. This is 9.4% larger than the conventional arrangement. The smallest double composite arrangement was optimization 16 with an objective function value of 0.0963 m^3. This is 48% larger than the conventional arrangement.

The implementation of actual numbers of teeth from theorectical diameters, results in some error. Table 6.4 presents tablulated results for % error comparing actual speed ratios with theorectical ratios. The largest error in an output speed is 1.62%. The average error is 0.70%. These errors are acceptable for machine tool applications, where other factors relating to the tool/workpiece interface have significantly more impact.

The results of the eigenvalue analysis are presented in brief in table 6.5. Information for the generation of the inertia and stiffness matrices is given in appendix C. The lowest frequency was found to be 28 Hz at speed 3. Since the highest shaft speed for this speed is 317 RPM or 5.3 Hz, this frequency is satisfactory. All other frequencies are located further away at an adequate distance.

By examining the conventional arrangement solutions, it is possible to determine which single composites are most likely to be competitive. Examining the optimum arrangement for the 9 speed gear drive, the best matches for diameter of gears on the intermediate shaft occur for $d_4 = d_9$, $d_5 = d_7$, and $d_5 = d_8$. Optimization 23 to 25 of table

6.2 confirm this expectation. Compared to the optimum arrangement, optimizations 23 and 24 are 9.4% larger and optimization 25 is 20.3% larger. All other single composites using this layout diagram are farther away from the optimum. It is thus possible to determine which composites are most likely to be beneficial.

The analysis of 9 speed drives can be compared with those published in the literature. Relative gear diameters are compared for 2 cases.

Case 1 compares the double composite arrangement published by White and Sanger [19] and Osman, Sankar, and Dukkipati [22] for a step ratio of 1.26 with results of this work. The input data are for the same double composite arrangement and layout diagram. The arrangements are different, but considering that different objective functions were used the trends are similar in both. This represents a case that works well for double composite arrangements. The objective function value, however, is 29.4% larger than for the conventional arrangement. This clearly demonstrates the disadvantage of double composites for most applications.

Case 2 compares the conventional arrangement presented by Rao [112] with this work. Face widths between the 2 methods shown somewhat similar behaviour, althought the basic face width/diameter ratios are different. The main difference lies in group 2 where the 1st input gear diameter is seen to be constant by Rao's solution yet increases in relation to speed for the results presented.

The 9 speed drive, as presented, has 2 problems. The largest speed increasing ratio is large with a value of 5.9 and the input shaft speed to the drive, with a value of 286 RPM, is very low. The principle cause of the problem is the large speed range required. With ϕ = 1.59 and 9 speeds the speed range is 40.8. To cover this range in two reductions, a gear ratio of 6.39 would be required. In practice this problem is resolved by using a greater number of shafts. The eighteen speed gear drive, presented next, has overcome these defficiencies.

6.3 Results and Discussion for the 18 Speed Gear Drive

The 18 speed gear drive has 18 conventional arrangements. Of these, the six with best results are presented in table 6.7. A face width/diameter ratio of 0.8 was used for the minimum diameter matrix as given in table C.8. The objective function values ranged from 0.0586 m^3 to 0.0599 m^3 which is a very narrow bandwidth. Arrangement 6 was selected as optimum. Although it has a slightly higher value of objective function, at 0.0588 m^3, the gears ratios are within desirable limits. The largest reduction ratio is 3.79 and the largest speed increasing ratio is 2.405. Figure 6.5 presents the arrangement using data from optimization 6, table 6.7. Figure 6.6 presents the layout diagram for the same arrangement. Details of each of the gears in the drive are presented in table 6.8.

The effect of high strength material was investaged for the 18 speed gear drive. A minimum diameter matrix, table C.11 was generated for case hardened steel. The diameters in the matrix are smaller than that for steel A5 as can be seen by comparing tables C.8 and C.11. This results in more compact drives. The arrangement, using case hardened steel was 59% of the volume of the drive presented in table 6.7. This demonstrates the effect of higher strength gearing. This analysis also demonstrates the ability of the optimization routine to do tradeoff studies.

The 18 speed gear drive shows all of the expected behavior in terms of component size that was present with the 9 speed drive. The two main advantages are that the speed ratios are acceptable and that the input speed was satisfactory with a value of 1440 RPM.

6.4 Summary

Figures 6.7 and 6.8 show actual drive trains. figure 6.7 shows a section of a head from an NC milling machine. Figure 6.8 shows an 18 speed drive for a precision milling machine. Both drives show some important common features.

1. The gears on the input shaft are smaller than the gears on the output shafts.
2. The face width of the gears vary to balance power capacity through-out the drive.
3 The module changes to increase the tooth size in relation to increasing torque.

The results of the 9 speed and 18 speed gear drive optimization highlighted 4 significant improvements over previous studies.

1. The diameters of the gears increased in relation to the decrease in operating speed. This was expected since torque increases as speed decreases and the face width/diameter ratio is maintained approximately equal for the smallest input gear in each group.
2. In table 6.3 and in table 6.8, the module is seen to increase as the input shaft speed decreases. This is

reasonable since larger teeth are required to carry the higher tangential load at lower input gear speeds.

3. The selected face width to diameter ratios, 0.5 in the case of the 9 speed and 0.8 in the case of the 18 speed, were satisfied within 2% for the first input gear in each group.

4. A constraint on the minimum face width/diameter ratio ensured a value greater than .1 for any pinion or gear in the drive.

For the 18 speed gear drive, the input speed and the gear ratios obtained were satisfactory.

None of the items above have previously been incorporated into a multi-speed gear drive optimization. All are fundamental to the synthesis of practical designs.

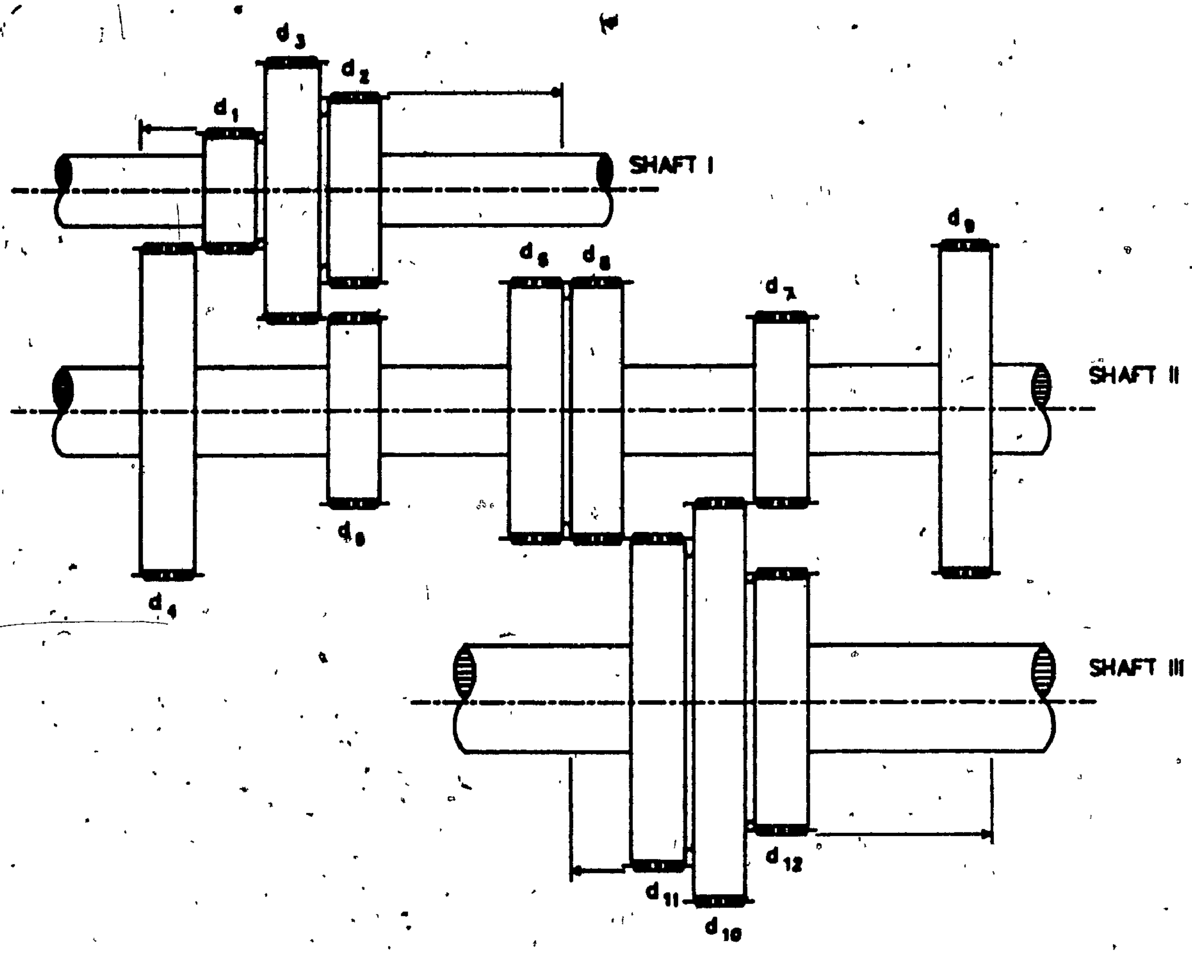


FIGURE 6.1

Conventional Arrangement for the 9 Speed Gear Drive

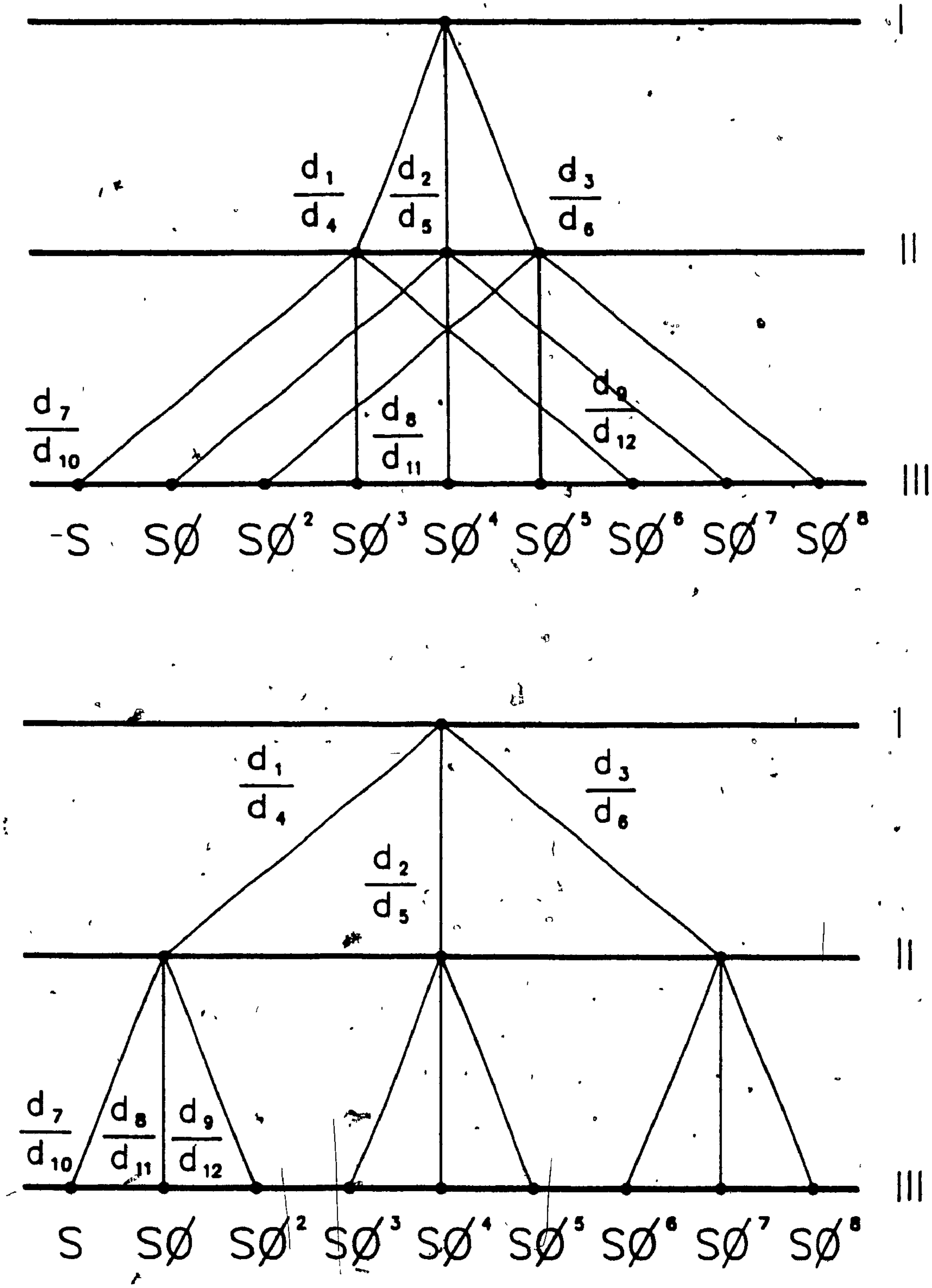


FIGURE 6.2

Layout Diagrams for the 9 Speed Gear Drive
(a) Crossed Arrangement
(b) Open Arrangement

TABLE 6.1

Kinematic Arrangements for the 9 Speed Gear Drive

OPT #	FACTORS		RANGE EXPONENT		COMPOSITE ARRANGEMENTS	
	#1	#2	#1	#2		
1	3	3	1	3	-	-
2	3	3	1	3	$d_4 = d_7$	-
3	3	3	1	3	$d_4 = d_8$	-
4	3	3	1	3	$d_4 = d_9$	-
5	3	3	1	3	$d_5 = d_7$	-
6	3	3	1	3	$d_5 = d_8$	-
7	3	3	1	3	$d_5 = d_9$	-
8	3	3	1	3	$d_6 = d_7$	-
9	3	3	1	3	$d_6 = d_8$	-
10	3	3	1	3	$d_6 = d_9$	-
11	3	3	1	3	$d_4 = d_8$	$d_5 = d_7$
12	3	3	1	3	$d_4 = d_8$	$d_6 = d_7$
13	3	3	1	3	$d_4 = d_9$	$d_5 = d_7$
14	3	3	1	3	$d_4 = d_9$	$d_5 = d_8$
15	3	3	1	3	$d_4 = d_9$	$d_6 = d_7$
16	3	3	1	3	$d_4 = d_9$	$d_6 = d_8$
17	3	3	1	3	$d_5 = d_8$	$d_6 = d_7$
18	3	3	1	3	$d_5 = d_9$	$d_6 = d_7$
19	3	3	1	3	$d_5 = d_9$	$d_6 = d_8$
20	3	3	3	1	-	-
21	3	3	3	1	$d_4 = d_7$	-
22	3	3	3	1	$d_4 = d_8$	-
23	3	3	3	1	$d_4 = d_9$	-
24	3	3	3	1	$d_5 = d_7$	-
25	3	3	3	1	$d_5 = d_8$	-
26	3	3	3	1	$d_5 = d_9$	-
27	3	3	3	1	$d_6 = d_7$	-
28	3	3	3	1	$d_6 = d_8$	-
29	3	3	3	1	$d_6 = d_9$	-
30	3	3	3	1	$d_4 = d_8$	$d_5 = d_7$
31	3	3	3	1	$d_4 = d_8$	$d_6 = d_7$
32	3	3	3	1	$d_4 = d_9$	$d_5 = d_7$
33	3	3	3	1	$d_4 = d_9$	$d_5 = d_8$
34	3	3	3	1	$d_4 = d_9$	$d_6 = d_7$
35	3	3	3	1	$d_4 = d_9$	$d_6 = d_8$
36	3	3	3	1	$d_5 = d_8$	$d_6 = d_7$
37	3	3	3	1	$d_5 = d_9$	$d_6 = d_7$
38	3	3	3	1	$d_5 = d_9$	$d_6 = d_8$

TABLE 6.2

Optimization Summary for the 9 Speed Gear Drive - Case Study

	#	1	2	3	4	5	6
		OPTIMIZATION NUMBER					
DIAMETER	1	106.0	127.8	83.6	60.1	113.6	88.7
	2	137.3	161.3	116.6	88.7	147.8	124.3
	3	168.6	193.0	154.9	126.4	182.2	166.4
	4	168.9	161.9	266.6	395.3	187.3	302.0
	5	137.6	128.4	233.7	366.8	153.2	266.3
	6	123.3	96.7	195.3	329.0	118.8	224.2
	7	114.5	161.9	118.4	113.0	153.2	117.9
	8	264.7	342.3	266.6	264.0	327.7	266.3
	9	393.0	473.7	387.1	395.3	457.3	387.8
	10	353.7	380.8	336.8	360.3	373.1	338.9
	11	203.4	200.3	188.6	209.3	198.6	190.5
	12	75.1	69.0	68.1	78.0	69.0	69.0
WIDTH	1	52.3	71.7	38.7	39.5	57.9	31.4
	2	38.7	50.8	25.8	36.7	76.4	38.0
	3	36.0	49.0	22.2	32.9	40.1	22.4
	4	58.2	71.7	59.7	57.5	76.4	59.4
	5	26.7	34.2	38.7	26.4	32.8	38.0
	6	39.3	47.4	38.7	39.5	45.7	38.8
RATIOS	S	0.203	0.336	0.110	0.048	0.249	0.102
	1	0.627	0.790	0.314	0.152	0.607	0.294
	2	0.998	1.256	0.499	0.242	0.965	0.467
	3	1.586	1.996	0.793	0.384	1.534	0.742
	4	0.324	0.425	0.352	0.314	0.410	0.348
	5	1.301	1.709	1.413	1.261	1.650	1.398
	6	5.229	6.868	5.682	5.069	6.632	5.620
F (m^3)		0.0724	0.1140	0.0737	0.0898	0.1088	0.0785

TABLE 6.2 - (Continued)

	OPTIMIZATION NUMBER						
	#	7	8	9	10	12	14
DIAMETER	1	57.7	97.8	67.4	55.7	583.8	464.8
	2	85.6	131.8	97.5	83.3	716.7	602.0
	3	123.2	168.8	135.6	120.9	836.6	739.1
	4	423.3	224.2	333.2	460.5	583.8	739.0
	5	395.4	190.1	303.1	433.0	450.8	601.9
	6	357.8	153.1	265.0	395.3	330.9	464.8
	7	113.0	153.1	115.2	113.0	330.9	344.8
	8	264.0	327.7	265.0	264.0	583.8	601.9
	9	395.4	457.4	391.8	395.3	720.8	739.0
	10	360.3	373.2	350.4	360.3	450.6	454.6
	11	209.3	198.7	200.6	209.3	450.6	197.4
	12	78.0	69.0	73.8	78.0	60.7	60.3
WIDTH	1	42.3	48.7	34.1	46.1	60.9	73.9
	2	39.5	35.3	30.3	43.3	71.7	60.2
	3	35.8	76.4	36.3	39.5	83.7	73.9
	4	57.5	76.4	58.5	57.5	83.7	45.5
	5	26.4	32.8	36.3	26.4	83.7	60.2
	6	39.5	45.7	39.2	39.5	72.1	73.9
RATIOS	S	0.043	0.179	0.067	0.038	0.734	0.477
	1	0.136	0.436	0.202	0.121	1.000	0.629
	2	0.217	0.693	0.322	0.192	1.590	1.000
	3	0.344	1.102	0.512	0.306	2.528	1.590
	4	0.314	0.410	0.329	0.314	0.734	0.759
	5	1.261	1.649	1.321	1.261	2.952	3.049
	6	5.069	6.630	5.310	5.069	11.867	12.255
F (m^3)		0.0984	0.1102	0.0838	0.1118	0.6305	0.5303

FIGURE 6.2 - (Continued)

	#	16	19	20	21	22	23
				OPTIMIZATION NUMBER			
DIAMETER	1	191.0	440.9	80.7	102.4	101.4	84.2
	2	254.7	576.6	182.0	230.6	228.3	189.8
	3	322.4	715.0	264.7	334.9	331.4	275.9
	4	395.4	764.7	230.8	291.6	288.4	240.6
	5	331.6	629.0	129.5	163.4	161.5	135.0
	6	264.0	490.5	46.8	56.8	58.3	48.8
	7	113.0	260.3	111.1	291.7	221.6	132.1
	8	264.0	490.5	155.5	377.8	288.4	182.7
	9	395.4	629.0	207.6	464.0	355.7	240.6
	10	360.4	433.5	369.7	464.8	367.1	388.1
	11	209.4	203.3	325.4	378.7	300.4	337.5
	12	78.0	64.8	273.3	292.5	233.0	279.6
WIDTH	1	39.5	76.5	40.7	50.9	50.1	42.1
	2	33.2	62.9	20.7	27.1	26.5	22.3
	3	34.9	71.5	26.5	33.5	33.1	27.6
	4	57.5	43.4	56.7	50.9	42.3	45.6
	5	34.9	71.5	33.3	37.9	50.1	36.5
	6	39.5	62.9	28.3	46.4	35.6	42.1
RATIOS	S	0.151	0.346	0.105	0.220	0.212	0.119
	1	0.483	0.577	0.350	0.351	0.352	0.350
	2	0.768	0.917	1.406	1.411	1.413	1.406
	3	1.221	1.458	5.651	5.670	5.682	5.651
	4	0.314	0.600	0.300	0.627	0.604	0.340
	5	1.261	2.413	0.478	0.998	0.960	0.541
	6	5.067	9.701	0.760	1.586	1.526	0.861
F (m^3)		0.0963	0.5115	0.0650	0.1311	0.0877	0.0711

TABLE 6.2 - (Continued)

	#	24	25	26	27	28	29
		OPTIMIZATION NUMBER					
DIAMETER	1	85.0	75.1	62.3	61.1	52.8	46.9
	2	195.8	180.4	165.7	177.0	157.6	147.8
	3	289.7	277.1	282.2	335.5	310.9	318.6
	4	259.3	261.7	305.4	415.9	405.8	469.1
	5	148.6	156.4	202.0	300.0	301.0	368.2
	6	54.7	59.8	85.5	141.5	147.8	197.4
	7	148.6	111.8	104.3	141.5	103.8	99.9
	8	203.2	156.4	148.3	195.6	147.8	143.4
	9	264.3	208.9	202.0	257.6	201.4	197.4
	10	390.5	372.0	417.2	415.5	420.6	446.4
	11	335.8	327.4	373.2	361.4	376.6	402.9
	12	274.7	274.9	319.6	299.4	322.9	348.9
WIDTH	1	42.5	37.2	31.0	41.6	40.6	46.9
	2	74.0	57.5	46.9	30.0	30.1	36.8
	3	29.0	27.7	28.2	71.5	57.1	43.0
	4	74.0	55.9	51.4	71.5	51.2	49.6
	5	36.2	57.5	37.3	36.1	57.4	40.3
	6	27.5	28.1	46.9	29.9	32.3	43.0
RATIO	S	0.125	0.086	0.051	0.050	0.032	0.022
	1	0.328	0.287	0.204	0.147	0.130	0.100
	2	1.318	1.154	0.821	0.590	0.523	0.402
	3	5.298	4.637	3.299	2.372	2.104	1.614
	4	0.381	0.300	0.250	0.340	0.247	0.224
	5	0.605	0.478	0.398	0.541	0.392	0.356
	6	0.962	0.760	0.632	0.861	0.624	0.566
F	(m^3)	0.0711	0.0906	0.0882	0.1234	0.1147	0.1353

TABLE 6.2 - (Continued)

	#	30	32	33	37
		OPTIMIZATION NUMBER			
D I A M E T E R	1	64.2	88.2	56.4	48.4
	2	194.4	220.7	172.9	143.8
	3	392.8	352.3	355.9	282.2
	4	529.3	350.8	491.3	365.7
	5	399.0	218.3	374.8	270.3
	6	200.6	86.7	191.8	131.9
	7	399.0	218.3	272.1	131.9
	8	529.3	284.2	374.8	192.4
	9	666.0	350.8	491.3	270.3
	10	785.8	363.5	766.7	729.9
	11	655.6	297.6	664.0	669.4
	12	518.8	231.0	547.5	591.5
W I D T H	1	65.6	45.1	54.7	36.6
	2	78.6	63.0	66.4	59.2
	3	39.3	35.2	35.6	73.0
	4	78.6	63.0	76.7	73.0
	5	65.6	32.3	66.4	66.9
	6	66.6	45.1	54.7	59.2
R A T I O	S	0.615	0.151	0.041	0.100
	1	0.121	0.252	0.115	0.132
	2	0.487	1.011	0.462	0.532
	3	1.959	4.063	1.855	2.139
	4	0.508	0.601	0.355	0.181
	5	0.807	0.955	0.564	0.287
	6	1.284	1.518	0.897	0.457
F (m^3)		0.4926	0.0970	0.3972	0.3053

. Diameter (mm), Width (mm)

TABLE 6.3

Design of Gearing for Optimum 9 Speed Drive

#	d_{in} (mm)	d_{out} (mm)	T_{in}	T_{out}	F (mm)	M (mm)	MG	N (RPM)	P_c (kW)
1	80.7	238.8	52	150	40.7	1.54	2.859	285	5.99
2	182.0	129.5	158	112	20.7	1.16	1.405	401	5.99
3	264.7	46.8	302	53	26.5	0.88	5.651	1611	6.96
4	111.1	369.7	44	146	56.7	2.54	3.328	100	5.99
5	155.4	325.4	64	135	33.2	2.42	2.093	100	5.99
6	207.6	273.3	102	134	28.3	2.03	1.316	100	5.99

TABLE 6.4

Error in Output Speeds for Optimum 9 Speed Arrangement

SPEED	RATIO 1	RATIO 2	CALCULATED	THEORETICAL	% ERROR
1	2.885	3.318	0.1045	0.1051	0.57
2	2.885	2.109	0.1644	0.1671	1.62
3	2.885	1.314	0.2638	0.2657	0.72
4	(1.410)	3.318	0.4251	0.4225	0.62
5	(1.410)	2.109	0.6688	0.6717	0.43
6	(1.410)	1.314	1.0734	1.0680	0.51
7	(5.698)	3.318	1.7173	1.6982	1.12
8	(5.698)	2.109	2.7018	2.7001	0.06
9	(5.698)	1.314	4.3364	4.2931	0.69
				AVERAGE	0.70

NOTE: Values in brackets indicate speed increasing ratios

TABLE 6.5

Eigenvalue Summary for the 9 Speed Gear Drive

Output Speed	Shaft Speed			Natural Frequency	
	1	2	3	Hz	RPM
1	286	100	30	223	13380
2	286	100	48	68	4080
3	286	100	76	28	1680
4	286	409	121	385	13160
5	286	409	192	141	8460
6	286	409	305	40	2400
7	286	612	405	400	24000
8	286	612	771	270	16200
9	286	1612	1225	62	3720

TABLE 6.6

Comparison of Results for the 9 Speed Drive With those in Literature

GEAR #	WHITE ET AL* OSMAN ET AL** ø=1.26	PRESENTED METHOD ø=1.26	RAO [112] ø=1.55	PRESENTED METHOD ø=1.55
1	1.000	1.000	1.080	1.000
2	1.589	1.538	1.440	1.349
3	2.248	2.104	1.840	1.740
4	2.847	2.328	2.600	2.684
5	2.260	1.790	2.240	2.336
6	1.600	1.224	1.840	1.944
7	2.260	1.790	1.000	1.340
8	2.554	2.055	2.360	3.063
9	2.847	2.328	3.720	4.678
10	2.848	2.976	3.720	4.460
11	2.554	2.711	2.360	2.738
12	2.260	2.438	1.000	1.123
1	-	0.492	0.191	0.502
2	-	0.683	0.103	0.386
3	-	0.293	0.138	0.295
4	-	0.683	0.536	0.684
5	-	0.378	0.144	0.309
6	-	0.492	0.198	0.468

* REF [19], ** REF [22]

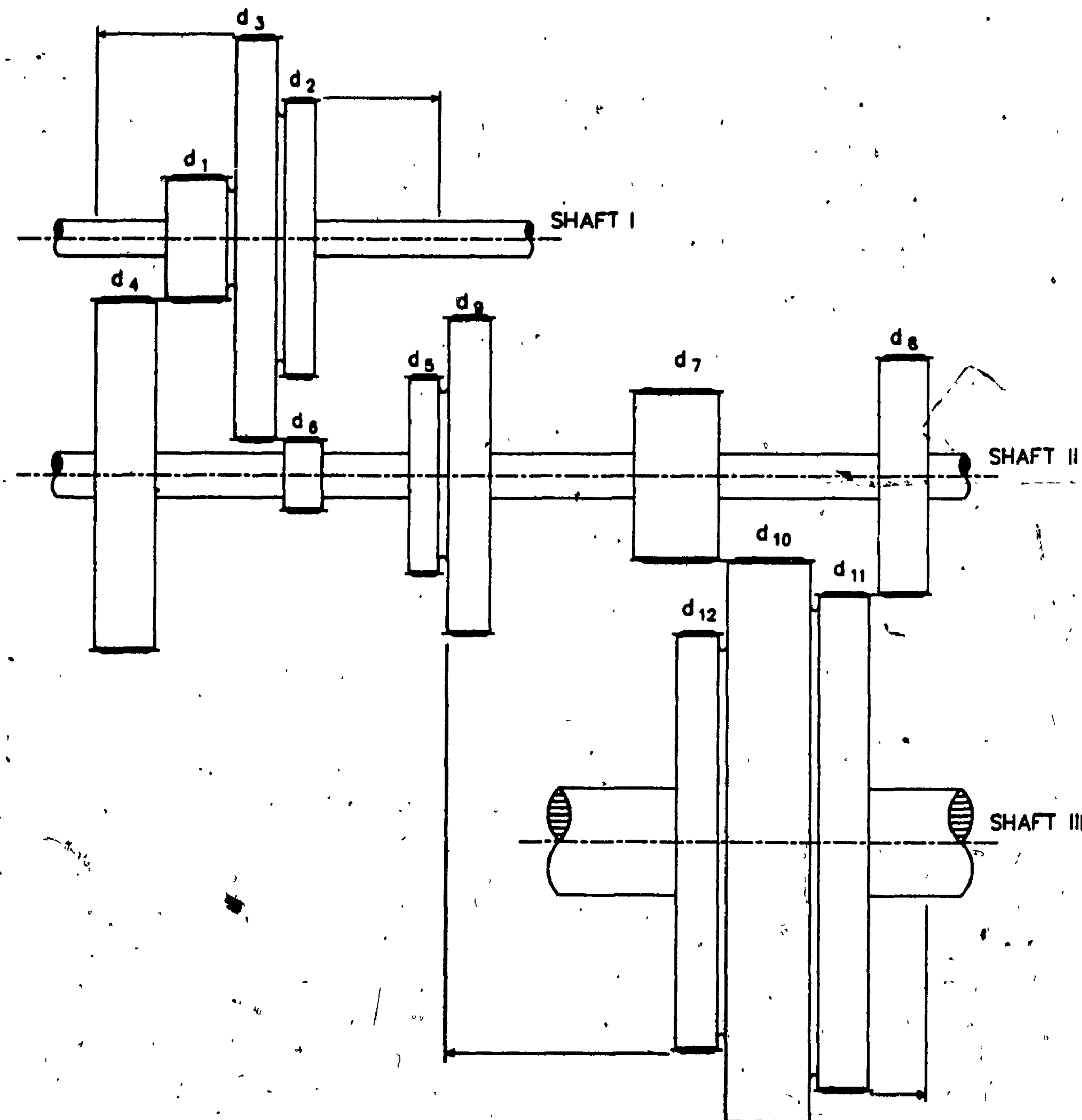


FIGURE 6.3

Gear Train Based on Optimization 20 Table 6.2

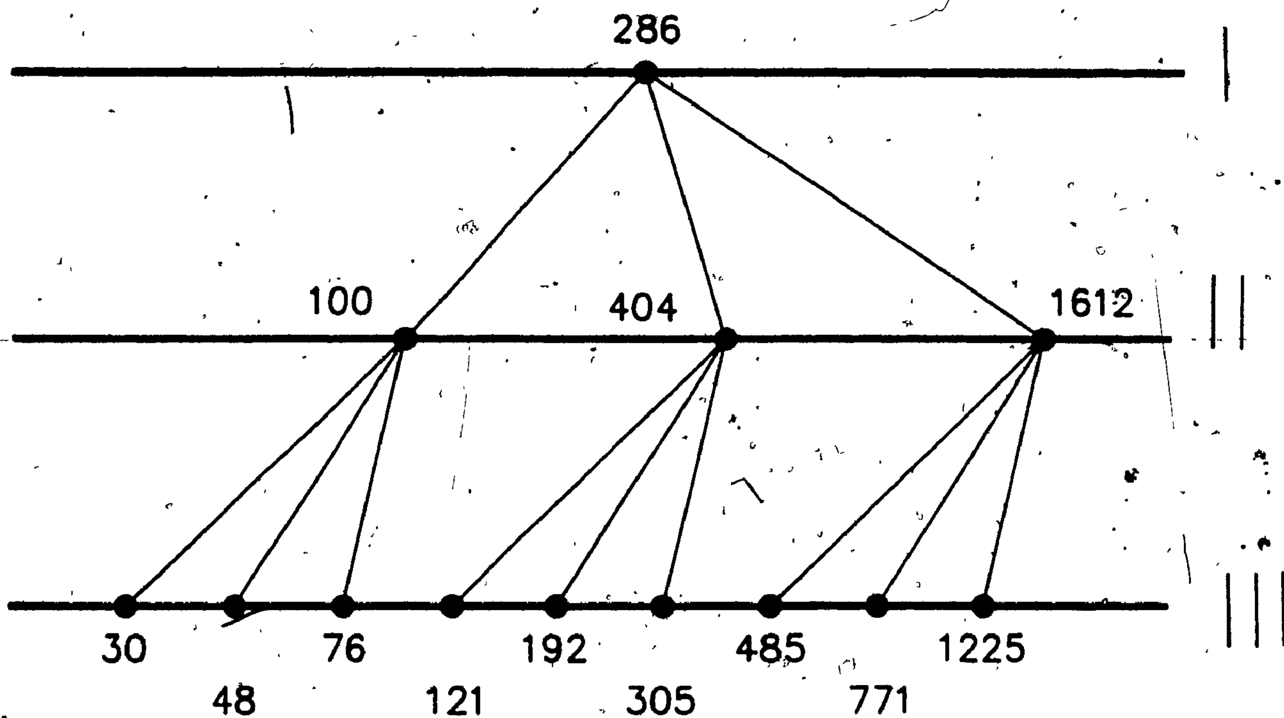


FIGURE 6.4

Speed Diagram Based on Optimization 20 Table 6.2

TABLE 6.7

Optimization Summary for the 18 Speed Gear Drive
for Selected Arrangements

	OPTIMIZATION NUMBER						
	#	1	2	3	4	5	6
DIAMETER	1	41.2	41.4	41.1	41.4	41.1	41.2
	2	101.8	58.2	102.8	303.3	68.5	49.3
	3	160.9	78.4	164.6	58.2	102.8	58.3
	4	158.4	149.0	164.8	149.0	164.8	155.9
	5	97.8	132.2	103.1	141.1	137.4	147.9
	6	38.6	112.0	41.3	132.2	103.1	138.9
	7	62.9	61.3	64.0	61.3	64.0	62.5
	8	88.9	153.3	76.3	153.3	76.3	103.5
	9	120.1	245.3	90.1	245.3	90.1	154.0
	10	235.9	245.3	230.0	245.3	230.0	238.3
	11	209.9	153.3	217.7	153.3	217.7	197.3
	12	178.7	61.3	203.9	61.3	203.9	146.7
	13	96.1	96.1	96.1	96.1	96.1	96.1
	14	114.2	114.2	156.1	156.1	293.7	293.7
	15	319.8	319.8	319.8	319.8	319.8	319.8
	16	301.7	301.7	259.8	259.8	122.1	122.1
WIDTH	1	32.9	33.1	32.9	33.1	32.9	32.9
	2	17.0	23.4	15.0	28.0	19.7	26.4
	3	16.1	19.7	16.5	23.4	15.0	23.3
	4	50.1	49.3	51.2	49.3	51.2	49.9
	5	34.3	21.2	42.6	21.2	42.6	33.2
	6	28.8	24.5	35.3	24.5	35.3	24.8
	7	78.6	78.6	78.6	78.6	78.6	78.6
	8	55.9	55.9	44.2	44.2	29.7	29.7
RATIOS	S	0.021	0.021	0.021	0.021	0.021	0.021
	1	0.260	0.278	0.249	0.278	0.249	0.264
	2	1.040	0.441	0.997	0.350	0.498	0.333
	3	4.163	0.699	3.990	0.441	0.997	0.420
	4	0.267	0.250	0.278	0.250	0.278	0.262
	5	0.523	1.000	0.351	1.000	0.351	0.525
	6	0.672	4.001	0.442	4.001	0.442	1.050
	7	0.300	0.300	0.300	0.300	0.300	0.300
	8	0.376	0.379	0.601	0.601	2.405	2.405
F	(m^3)	0.0596	0.0599	0.0589	0.0586	0.0592	0.0588

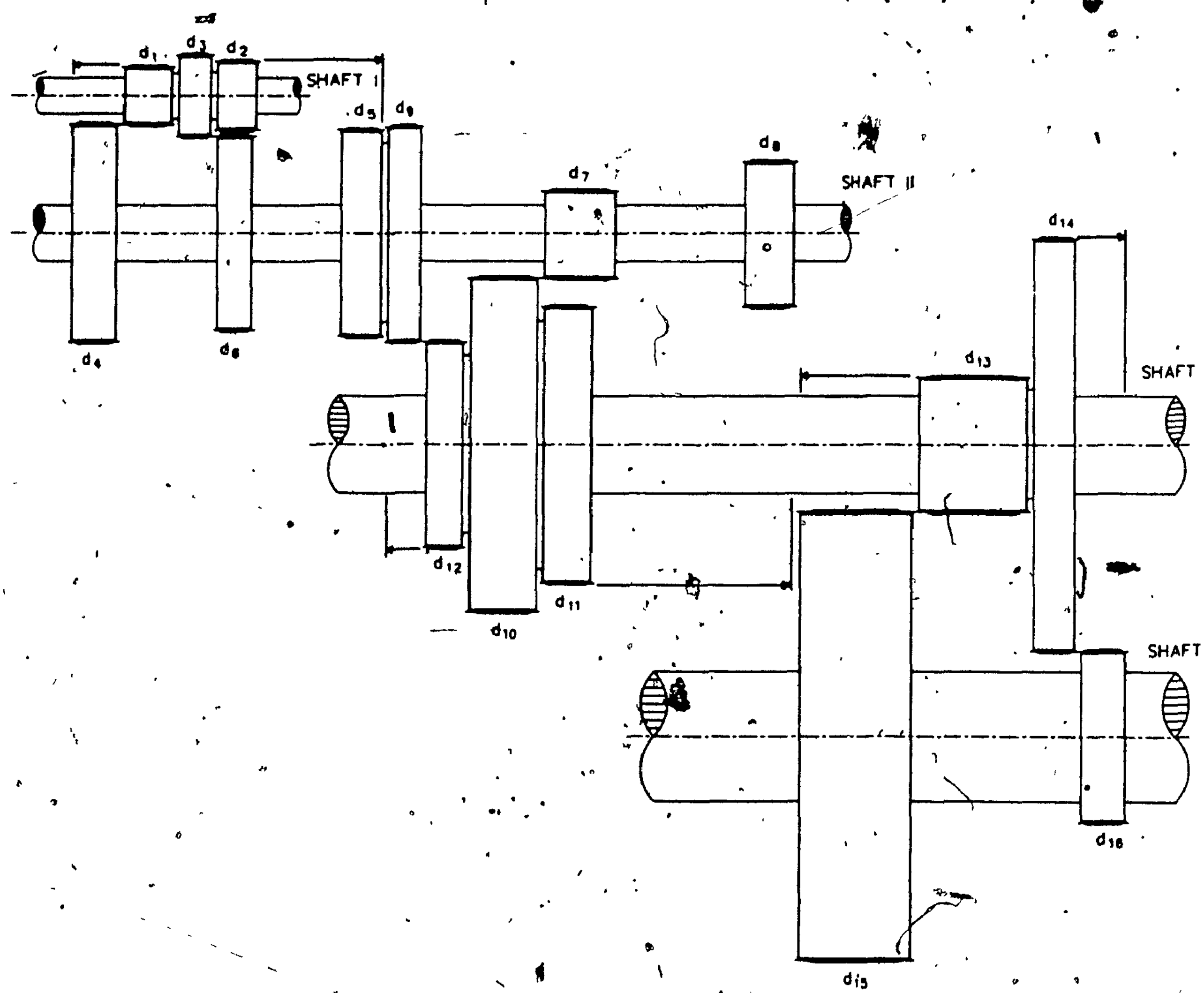


FIGURE 6.5

Gear Train Based on Optimization 6 Table 6.6

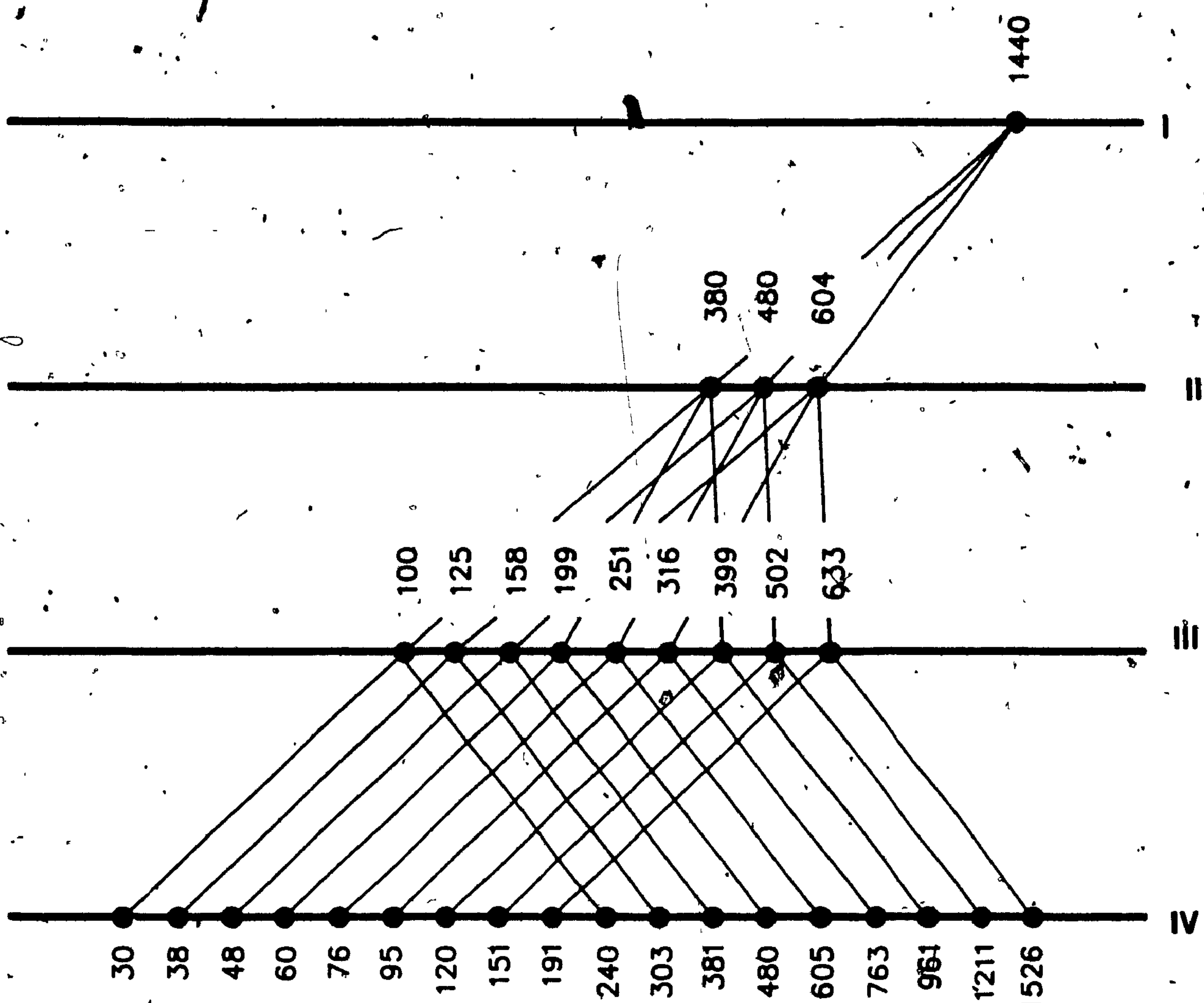


FIGURE 6.6

Speed Diagram Based on Optimization 6 Table 6.6

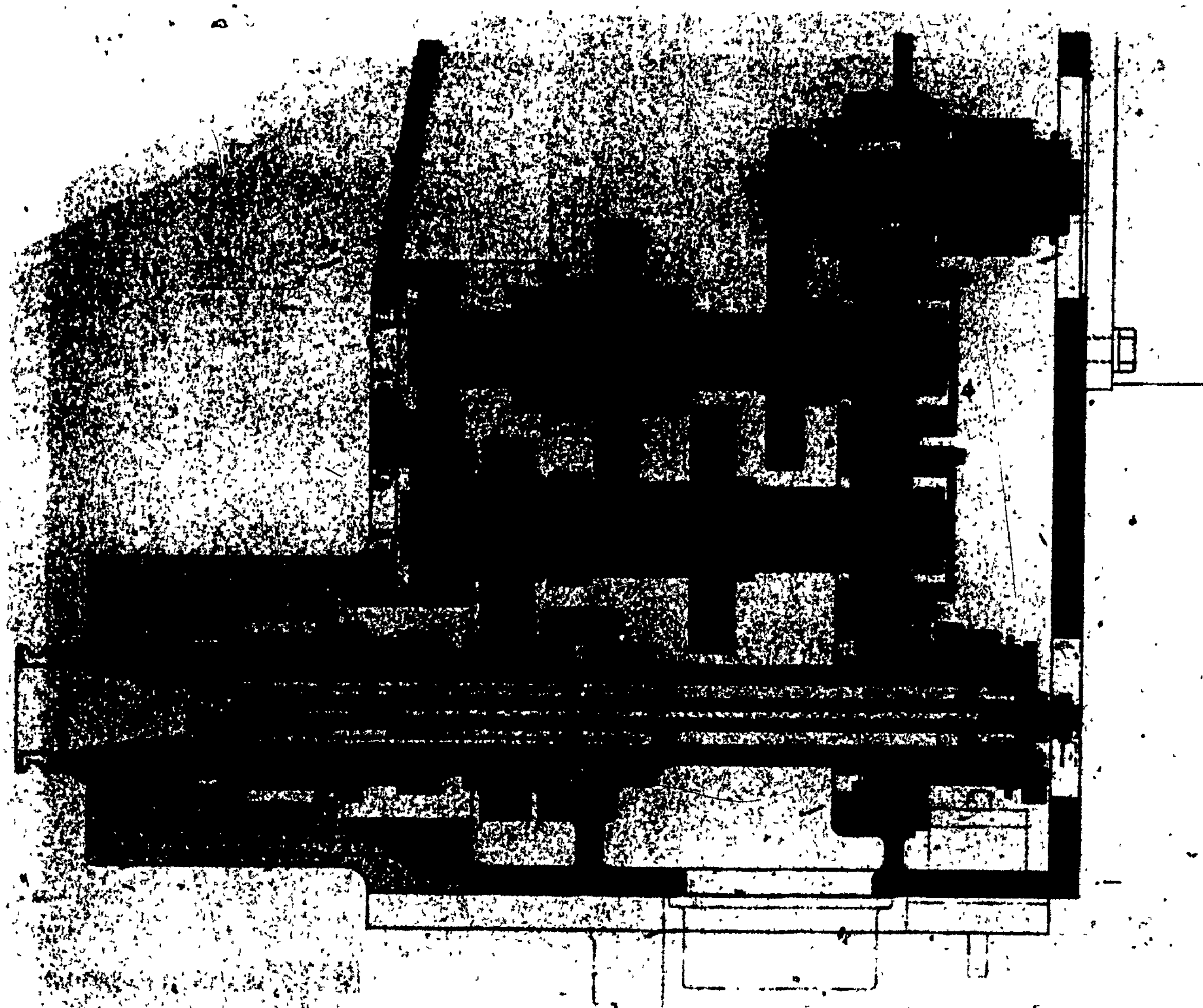


FIGURE 6.7

Section of Spindle Head for Mitsui-Seiki H5C

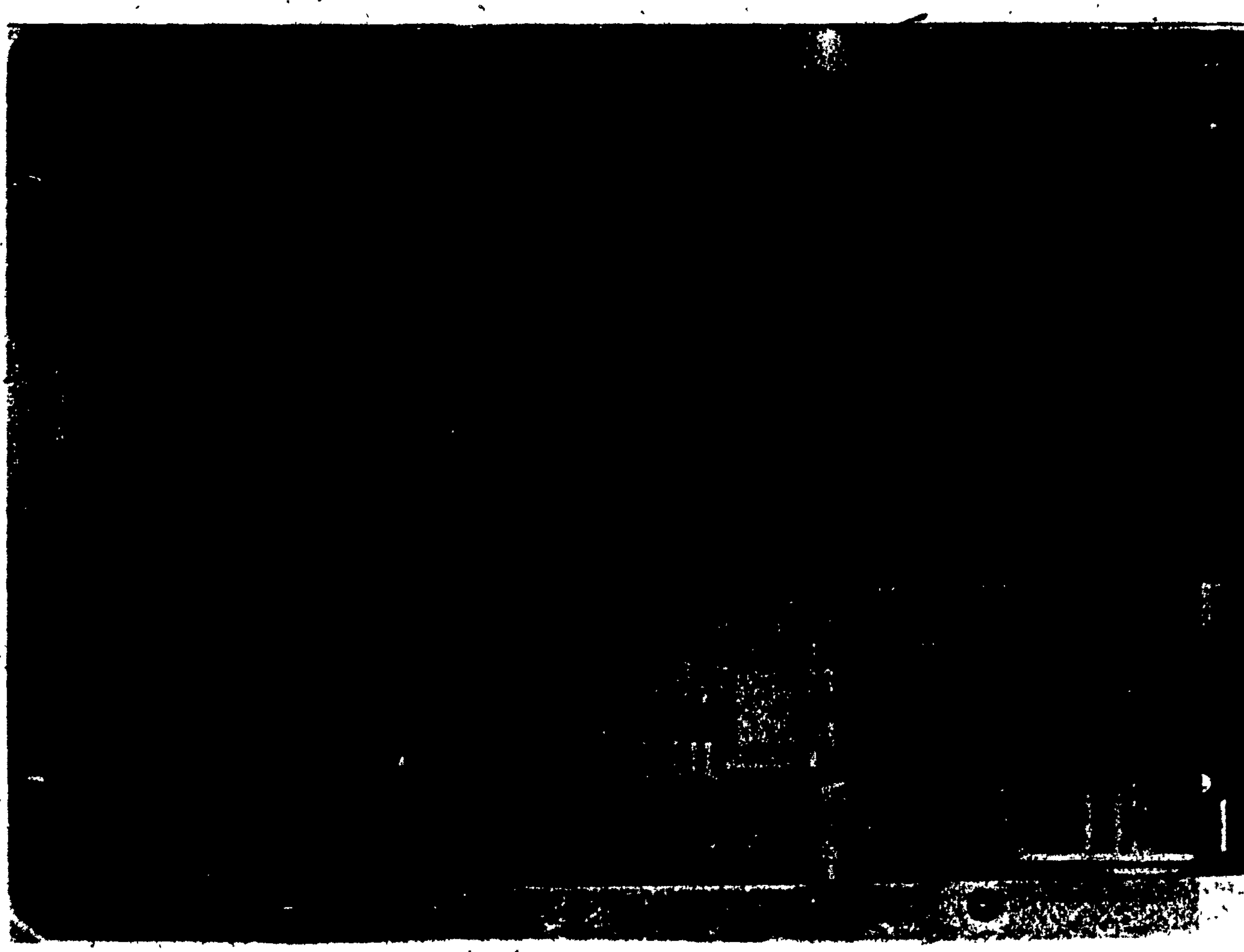


FIGURE 6.8

Main Drive for 18 Speed Gearbox by Rambaudi

TABLE 6-8

Design of Gearing for Optimum 18 Speed Drive

#	d_{in} (mm)	d_{out} (mm)	T_{in}	T_{out}	F (mm)	M (mm)	MG	N (RPM)	P (kW)
1	41.2	155.9	57	215	32.9	0.73	3.788	1440	5.59
2	49.3	147.9	68	204	26.4	0.73	3.003	1440	5.59
3	58.3	138.9	86	205	23.3	0.68	2.381	1440	5.59
4	62.5	238.3	54	206	49.9	1.16	3.819	380	5.59
5	103.5	197.3	110	210	33.2	0.94	1.905	380	5.99
6	154.0	146.7	185	176	24.8	0.83	1.050	400	5.99
7	96.1	319.8	44	145	78.6	2.21	3.333	100	5.99
8	293.7	122.1	208	87	29.7	1.41	2.405	241	5.99

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

The synthesis of optimal designs for multi-speed gear drives has been presented using new algorithms to incorporate both kinematic and design requirements into the optimization process. The method of analysis includes verification of acceptable location of torsional eigenvalues and provides for design modification without affecting the objective function value. A problem reduction strategy was demonstrated which successfully linked the requirements of the kinematic analysis with the requirements of component design without burdening either analysis. The reduction strategy forced the number of optimization parameters to equal the degrees of freedom of the system ensuring efficient accurate solutions. This efficiency made it practical to analyse the number of arrangements deemed necessary.

General equations were developed to define the kinematic relationships in a multi-speed gear drive for conventional, single composite and double composite arrangements. With these equations it was shown that all arrangements for a given number of speeds could be examined in detail.

The effect of a constraint to control the lowest output/input speed ratio on the kinematic equations was included in the analysis. Specific analytic equations were developed to remove one degree of freedom from the analysis for all types of arrangements to efficiently address this condition.

Variables which did not directly affect the objective function were segregrated from the main problem and handled separately. This was demonstrated in the generation of the minimum diameter matrices which were calculated independently of the arrangement optimizations. The use of the minimum diameter matrix effectively decoupled the cumbersome analysis of gear strength from the arrangement optimization by providing data in a simplified form.

Computer code was developed to rate external spur and helical gear pairs for bending strength and pitting resistance meeting all requirements of AGMA 218. This code was applicable to spur and helical gearing with standard or non-standard tooth geometry.

The diameter of the first pinion in each group was shown to be a function of shaft speed and gear ratio, hence important to the selection of the kinematic arrangement. This coupling between kinematic requirements and design constraints was demonstrated by comparison with previously published results.

An optimization strategy was presented to determine the module which minimized the size of a gear mesh for a given set of operating conditions. This analysis was used to generate matrices of minimum diameter, optimum module, and optimum number of teeth for different operating conditions.

Equations to define the dynamic characteristics of a multi-speed gear drive were developed. These equations were used to find approximations of the eigenvalues and eigenvectors for the optimal arrangement to verify that critical speeds were located away from operating speeds.

The use of eigenvalue derivatives to modify dangerous eigenvalues was presented. The stategy to modify shaft diameter was shown to be consistent with the arrangement optimization ensuring that the dynamic analysis would not modify the optimum design point.

The use of a combination of optimization techniques (discrete, random, patterned move, and univariate) were demonstrated to be most practical when attempting to solve problems with have a number of forms.

Direct methods were also shown to be efficient.

Problem reduction strategy was presented for use in obtaining optimal solutions to difficult problems. The method provides for the simplification of the optimization problem, dramatically improving efficiency, without the loss of important links between the different required analyses. The technique of problem reduction strategy was illustrated practically for gear drives.

The 4 speed gear drive was used in chapters 2 through 5 to illustrate the developement of the mathematical model. Key elements of the model defined kinematic performance, gear components strength requirements and location of torsional eigenvalues. These key elements were shown to generate results which correlate well with current design practice.

Results for the nine speed gear drive were presented and compared with previous published results. The sample problem demonstrated the ability of the mathematical model and optimization process to generate solutions meeting operating requirements. The kinematic requirements were satisfied, all gears were correctly rated for optimum power, and the dynamic performance of the drive was shown to be satisfactory.

Results were presented for the 18 speed gear drive. This drive was shown to meet all operating requirements as with the 9 speed drive. General characteristics of this drive were compared to existing designs. The similarities were clear with regards to the relative component size.

The ability of this analysis to be used for trade-off studies was demonstrated by the analysis of the 18 speed gear drive for different gear materials. It was shown that case hardened gears resulted in a reduction in drive size.

The characteristics of the drive were compared with actual drives and the similarities observed. The results of the volume optimizations correlated well with design practice in all cases.

Some of the results of this thesis have been published and presented at conferences [134-136].

7.1 Recommendations for Future Work

This work makes use of standard cutter geometries and does not attempt to optimize addendum/dedendum ratios to minimize radial dimensions further. A study on the effect this type of modification has on the minimum diameter matrix would be of value. The analysis would require additional development of the mathematical model for a mesh to ensure that designs could be implemented.

It is not obvious that the face/width to diameter ratio should be the same for each group. Providing a number of minimum diameter matrices, which cover a range of face/width to diameter ratios, would allow the optimization process to consider these additional parameters. It is probable that a substantial effort would have to be spent on the optimization routine to meet this objective reliably. The increased number of variables will make it more difficult to find global optima.

Results of the optimizations for conventional arrangements for each layout diagram were shown to provide information to identify which single and double composite arrangements have potentional. An analytic routine, which compares diameters of composite gears in the conventional arrangement, could automatically recommend specific composites for analysis rather than analysing all possible arrangements. This would be more efficient than the current practice of analysing all arrangements.

Expert system which "employ human knowledge to solve problems that ordinarily require human intelligence" [137] will make important contributions to engineering in the near future. For engineering problems, a large portion of the knowledge base will be analysis software such as presented in this work. Users will access the analysis program through an expert interface. The development of this interface, directed toward engineering applications, has much potential.

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APPENDIX A

A COMPARISON OF MATHEMATICAL TECHNIQUES FOR THE KINEMATIC ANALYSIS OF MULTI-SPEED GEAR TRAINS

The method of kinematic analysis presented in chapter 2 is compared to the techniques by White [14] and Osman, Sankar, and Dukkipati [22]. Equations for the diameters of the gears are derived for a particular double composite arrangement of the nine speed, three shaft gear box.

The layout diagram for the sample problem is shown in figure A.1. The conventional arrangement for the nine speed gear train is shown in figure A.2. The double composite arrangement which will be studied is the one which occurs when diameters $d_4 = d_9$ and $d_5 = d_7$. From figure A.1 the step ratio is seen to be ϕ.

A.1 Technique 1: Per White

White [14] was the first author to present an analytic technique for the analysis of multi-speed gear trains. His method can be summarized as follows:

a. Select a layout diagram,

b. Select a double composite arrangement,

c. Write equations to satisfy speed and mesh requirements as a function of the lowest output/input ratio, the step ratio, and a reference gear,

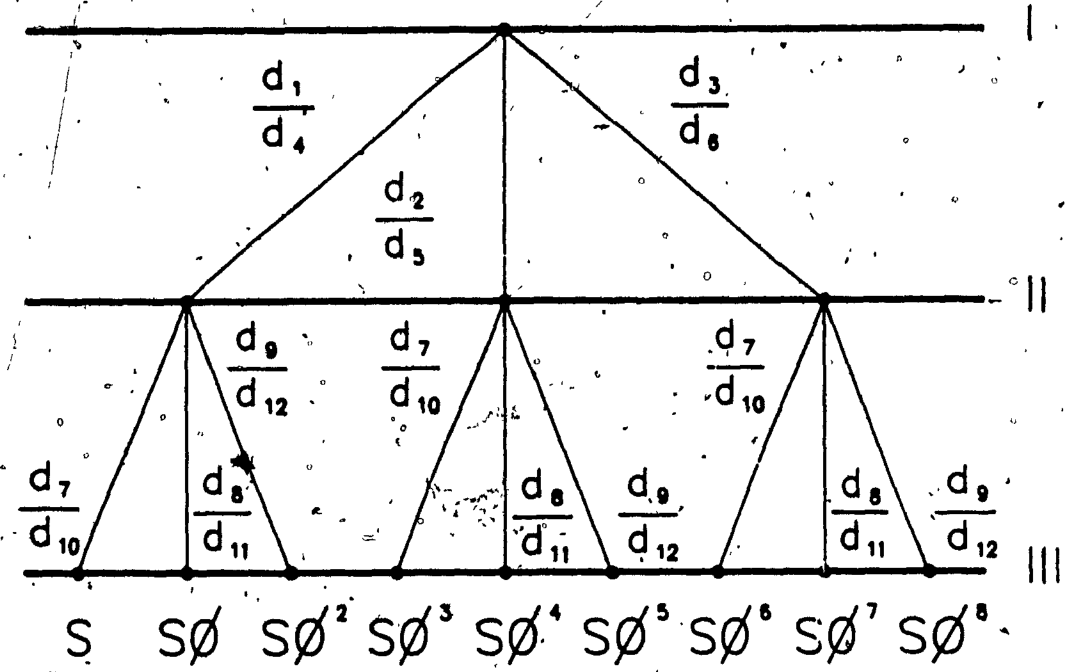

FIGURE A.1

Layout Diagram for 9 Speed Gear Drive Used by White [14] and Osman, Sankar, and Dukkipati [22]

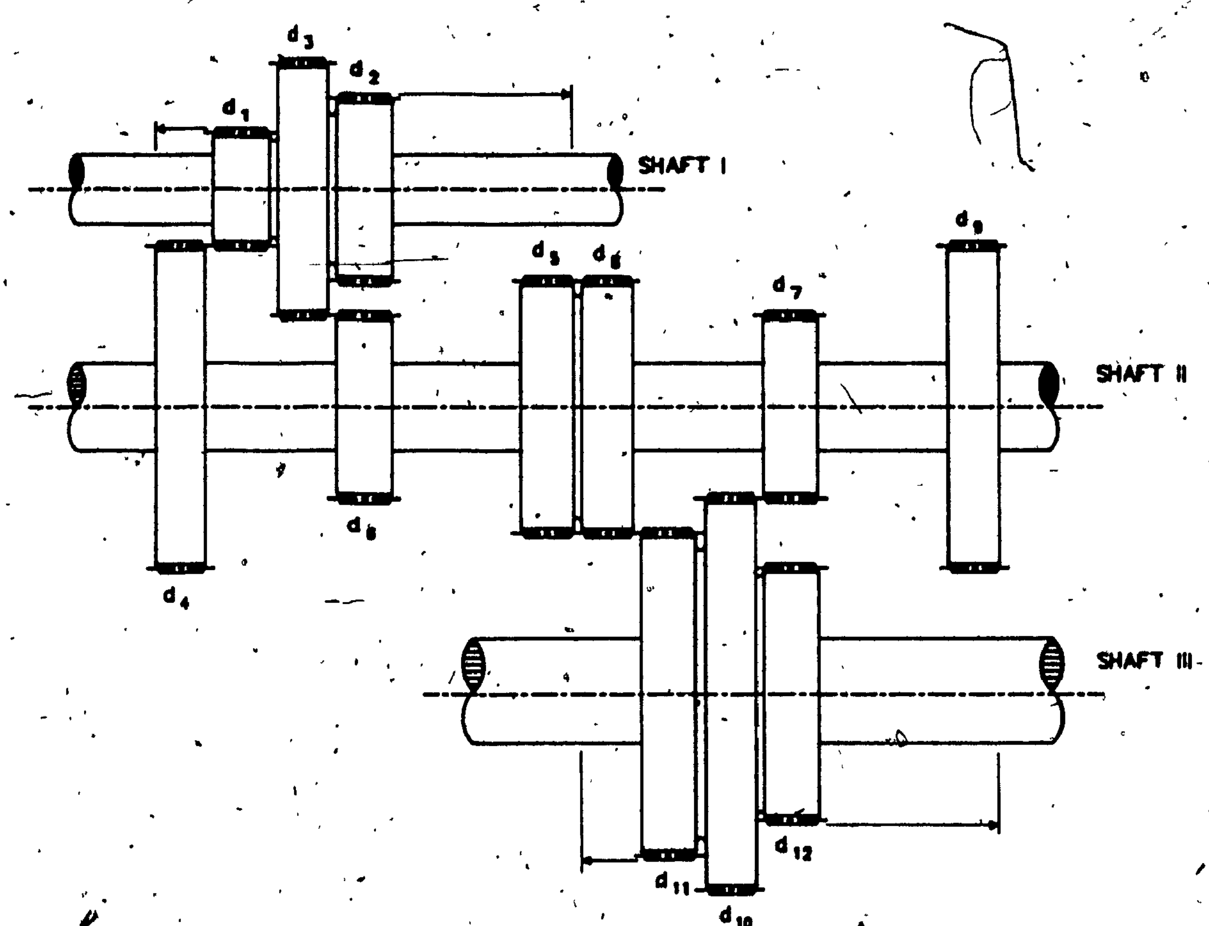

FIGURE A.2

Conventional Arrangement for 9 Speed Gear Drive. Double Composite Arrangement Under Consideration Occurs When $d_4 = d_9$ and $d_5 = d_7$

d. Re-arrange the equations to obtain analytic expressions for the diameters of each gear in the drive.

e. Evaluate diameters as a function of S and ø. Diameter d_1 is taken to be a reference gear and is assigned a value of 1.

The speed ratio, defined as the ratio of the output speed to the input speed, can be written as functions of the gear diameters.

$$S = \frac{d_1 d_7}{d_4 d_{10}} = \frac{d_1 d_5}{d_4 d_{10}} \tag{A.1}$$

The following speed relationships must be satisfied based on equation (A.1) and the requirement that the output speeds form a geometric progression.

$$S\phi = \frac{d_1 d_8}{d_4 d_{11}} \tag{A.2}$$

$$S\phi^2 = \frac{d_1 d_9}{d_4 d_{12}} = \frac{d_1 d_4}{d_4 d_{12}} = \frac{d_1}{d_{12}} \tag{A.3}$$

$$S\phi^3 = \frac{d_2 d_7}{d_5 d_{10}} = \frac{d_2 d_5}{d_5 d_{10}} = \frac{d_2}{d_{10}} \tag{A.4}$$

$$S\phi^4 = \frac{d_2 d_8}{d_5 d_{11}} \tag{A.5}$$

$$S\phi^5 = \frac{d_2 d_9}{d_5 d_{12}} = \frac{d_2 d_4}{d_5 d_{12}} \tag{A.6}$$

$$S\phi^6 = \frac{d_3 d_7}{d_6 d_{10}} = \frac{d_3 d_5}{d_6 d_{10}} \tag{A.7}$$

$$S\phi^7 = \frac{d_3 d_8}{d_6 d_{11}} \tag{A.8}$$

$$S\phi^8 = \frac{d_3 d_9}{d_6 d_{12}} = \frac{d_3 d_4}{d_6 d_{12}} \tag{A.9}$$

The mesh conditions, required since the center distance for each of the gear pairs on the same two shafts must be constant, can also be written in terms of the gear diameters.

$$d_1 + d_4 = d_2 + d_5 \tag{A.10}$$

$$d_1 + d_4 = d_3 + d_6 \tag{A.11}$$

$$d_7 + d_{10} = d_8 + d_{11} \text{ or } \quad d_5 + d_{10} = d_8 + d_{11} \tag{A.12}$$

$$d_7 + d_{10} = d_9 + d_{12} \text{ or } \quad d_5 + d_{10} = d_4 + d_{12} \tag{A.13}$$

The diameter "a" is taken to be a reference gear. Equation (A.3) can then be re-arranged to give the diameter of d_{12} as a function of S, ø, and d_1 only.

d_1 = reference diameter

$$d_{12} = \frac{d_1}{S\phi^2} \tag{A.14}$$

To find expressions for the remaining diameters requires manipulating equations (A.1) through (A.13). The diameters d_{10} and d_5 can be found using equations (A.10) and (A.13) and writing the diameter

d_2, d_4 and d_{12} as functions of S, ø, d_1, d_5 and d_{10}. With two equations and two unknowns, d_5 and d_{10} can be easily found.

Diameter d_{12} is already available as a function of the desired variables from equation (A.14). Diameter d_2 can be found by re-arranging equation (A.4). Diameter d_4 can be found by re-arranging equation (A.1).

$$d_2 = S\phi^3 d_{10} \tag{A.15}$$

$$d_4 = \frac{d_1 d_5}{S d_{10}} \tag{A.16}$$

Next equations (A.14) through (A.16) are substituted into equations (A.10) and (A.13) to give two equations for diameter d_5.

$$d_5^2 = \frac{S d_{10}(S d_{10}\phi^3 - d_1)}{(d_1 - S d_{10})} \tag{A.17}$$

and

$$d_5 = \frac{d_{10}d_1 - S d_{10}^2\phi^2}{S d_{10}\phi^2 - d_1\phi^2} \tag{A.18}$$

Combining equations (A.17) and (A.18):

$$d_{10} = \frac{d_1(1-S\phi^3)}{S\phi^2(1-S\phi^3)} \tag{A.19}$$

Substituting equation (A.19) into equation (A.18):

$$d_5 = \frac{d_1(1-S\phi^2)(\phi-1)}{[\phi^2(1-S\phi^3)-(1-S\phi^2)](1-S\phi^3)} \tag{A.20}$$

Diameter d_2 can be calculated by substituting equation (A.19) into equation (A.15).

$$d_2 = \frac{d_1 \phi (1 - S\phi^2)}{(1 - S\phi^3)} \tag{A.21}$$

Diameter d_4 can be calculated by substituting equations (A.19) and (A.20) into equation (A.16).

$$d_4 = \frac{d_1 \phi^2 (\phi - 1)}{\phi^2 (1 - S\phi^3) - (1 - S\phi^2)} \tag{A.22}$$

Diameters d_8 and d_{11} can be calculated using equations (A.2), (A.12) and previously calculated diameters.

$$d_{11} = \frac{d_1 d_9}{S\phi d_4} \tag{A.23}$$

Substituting (A.2) into (A.12) and substituting into the resulting equation the values of d_5 and d_{10} gives diameter d_8.

$$d_8 = \frac{d_1 \phi (1 - S\phi^2)(\phi + 1)}{[(1 - S\phi^2)(\phi^2 + \phi + 1) - \phi^2](1 - S\phi^2)(\phi^2 + 1) - \phi(\phi - 1)]} \tag{A.24}$$

Diameter d_{11} can be calculated by substituting equation (A.22) and (A.24) into equation (A.23).

$$d_{11} = \frac{d_1 (1 - S\phi^2)(\phi + 1)}{S\phi^2 [(1 - S\phi^2)(\phi^2 + 1) - \phi(\phi - 1)]} \tag{A.25}$$

In a similar fashion diameters d_3 and d_6 can be calculated using equation (A.7) and (A.11).

$$d_3 = \frac{S\phi^6 d_6 d_{10}}{d_5} \tag{A.26}$$

Substituting (A.26) and previously calculated diameters, diameter d_6 can be obtained.

$$d_6 = \frac{d_1(1-S\phi^2)(\phi^3-1)(\phi-1)}{[\phi^2(1-S\phi^3)-(1-S\phi^2)][\phi^2(1-S\phi^3)-(1-S\phi^2)]\phi^4+\phi-1]} \tag{A.27}$$

Using equation (A.26) and the diameters already calculated diameter d_3 is found.

$$d_3 = \frac{d_1\phi^4(\phi^3-1)(1-S\phi^2)}{[\phi^2(1-S\phi^3)-(1-S\phi^2)]\phi^4+\phi-1} \tag{A.28}$$

This gives an expression for each of the diameters of the gears as a function of S, ϕ, and the reference gear d_1. White performed a small additional amount of manipulation to put these equations in a final, more simplified form. Defining two new variables α and β as follows:

$$\alpha = (1-S\phi^2) \tag{A.29}$$

$$\beta = \phi^2(S+1)-(S\phi^2+1) \tag{A.30}$$

The diameters of the gears can be re-written as follows.

$$d_2 = \frac{d_1\alpha\phi}{1-\phi(1-\alpha)} \tag{A.31}$$

$$d_3 = \frac{d_1\alpha\phi^4(\phi^3-1)}{\phi^4\beta+\phi-1} \tag{A.32}$$

$$d_4' = \frac{d_1 \phi^2(\phi-1)}{\beta} \qquad \text{(A.33)}$$

$$d_5 = \frac{d_1 \alpha(\phi+1)}{\beta[1-\phi(1-\alpha)]} \qquad \text{(A.34)}$$

$$d_6 = \frac{d_1 \alpha(\phi^2-1)(\phi-1)}{(\beta\phi^4+\phi-1)} \qquad \text{(A.35)}$$

$$d_7 = d_5 \qquad \text{(A.36)}$$

$$d_8 = \frac{d_1 \alpha\phi(\phi+1)}{[\alpha(\phi^2+\phi+1)-\phi^2][\alpha(\phi^2+1)-\phi(\phi-1)]} \qquad \text{(A.37)}$$

$$d_9 = d_4 \qquad \text{(A.38)}$$

$$d_{10} = \frac{d_1 \alpha}{(1-\alpha)[1-\phi(1-\alpha)]} \qquad \text{(A.39)}$$

$$d_{11} = \frac{d_1 \alpha(\phi+1)}{(1-\alpha)[\alpha(\phi^2+1)-\phi(\phi-1)]} \qquad \text{(A.40)}$$

$$d_{12} = \frac{d_1}{(1-\alpha)} \qquad \text{(A.41)}$$

The method is clearly algebraically tedious. These equations apply to a specific layout diagram and composite arrangement. If there was interest in more than one arrangement, then a similar analysis is required. White and Sangar [12] examined all arrangements for the nine speed gear train. While they demonstrated that it is possible to study many arrangements it is obvious that the method has practical limitations.

A.2 Technique 2: Per Osman, Sankar and Dukkipati

Osman, Sankar and Dukkipati [22] applied numerical techniques to the analysis of the 9 speed double composite arrangement studied above. The method resulted in significantly less mathematical development to generate an equivalent system of equations.

The steps of this method can be listed as was done for White's method.

1. Select a layout diagram,
2. Select a double composite arrangement,
3. Write gear diameter equations to satisfy the speed and mesh requirement as a function of the lowest output/input ratio, the step ratio and a reference gear.
4. Select the gear diameters which determine the lowest output/input ratio S and arbitrarily designate them as independent variables Y_1, Y_2, Y_3, and Y_4. Y_1, the reference gear is set to 1. Re-arrange the equations to obtain analytic expression for the diameters of each gear as a function of Y_2, Y_3, Y_4, and ø, subject to 2 equality constraints.
5. Select values of X_2, X_3, and X_4 which satisfy the 2 equality constraints.
6. Evaluate diameters.

The technique follows the same approach as White's method however the equations are developed in a slightly different manner. Gear d_1 is once again considered the reference gear and equations (A.1) through (A.13) are used to produce a set of dimensionless equations.

$$S = \frac{(d_5/d_1)}{(d_4/d_1)(d_{10}/d_1)} \tag{A.42}$$

$$S\phi = \frac{(d_8/d_1)}{(d_4/d_1)(d_{11}/d_1)} \tag{A.43}$$

$$S\phi^2 = \frac{1}{(d_{12}/d_1)} \tag{A.44}$$

$$S\phi^3 = \frac{(d_2/d_1)}{(d_{10}/d_1)} \tag{A.45}$$

$$S\phi^4 = \frac{(d_2/d_1)(d_8/d_1)}{(d_5/d_1)(d_{11}/d_1)} \tag{A.46}$$

$$S\phi^5 = \frac{(d_2/d_1)(d_4/d_1)}{(d_5/d_1)(d_{12}/d_1)} \tag{A.47}$$

$$S\phi^6 = \frac{(d_3/d_1)(d_5/d_1)}{(d_6/d_1)(d_{10}/d_1)} \tag{A.48}$$

$$S\phi^7 = \frac{(d_3/d_1)(d_9/d_1)}{(d_6/d_1)(d_{11}/d_1)} \tag{A.49}$$

$$S\phi^8 = \frac{(d_3/d_1)(d_4/d_1)}{(d_6/d_1)(d_{12}/d_1)} \tag{A.50}$$

The mesh equations are similarly re-written.

$$1 + (d_4/d_1) = (d_2/d_1) + (d_5/d_1) \tag{A.51}$$

$$1 + (d_4/d_1) = (d_3/d_1) + (d_6/d_1) \qquad (A.52)$$

$$(d_5/d_1) + (d_{10}/d_1) = (d_8/d_1) + (d_{11}/d_1) \qquad (A.53)$$

$$(d_5/d_1) + (d_{10}/d_1) = (d_4/d_5) + (d_{12}/d_1) \qquad (A.54)$$

The lowest output/input ratio S is a function of (d_4/d_1), (d_5/d_1) and (d_{10}/d_1). Letting Y_2, Y_3 and Y_4 represent the normalized gear diameters respectively, equations (A.42) through (A.54) can be re-arranged to give the normalized diameters of each of the gears. The normalized diameters of d_2 and d_{12} can be found using equations (A.42), (A.44) and (A.45).

$$\frac{d_2}{d_1} = S\phi^3 Y_4 \qquad (A.55)$$

$$\frac{d_{12}}{d_1} = \frac{1}{S\phi^2} \qquad (A.56)$$

Diameters d_3 and d_6 can be found using equations (A.48) and (A.52). An expression for d_3 can be found by re-arranging (A.48).

$$\frac{d_3}{d_1} = \frac{S\phi^6 Y_4 d_6}{Y\phi_3 a} \qquad (A.57)$$

Substituting equation (A.57) in equation (A.48) and re-arranging gives the normalized diameter of d_6.

$$\frac{d_6}{d_1} = \frac{(1+Y_2)Y_3}{S\phi^6 Y_4 + Y_3} \qquad (A.58)$$

Substituting (A.58) into (A.57) gives the normalized diameter of d_3 as follows.

$$\frac{d_3}{d_1} = \frac{S\phi^6 Y_4(1+Y_2)}{(S\phi^6 Y_4+Y_3)} \tag{A.59}$$

Using equations (A.49) and (A.53) the normalized diameter d_8 and d_{11} can be found. Re-arranging equation (A.49) gives an expression for d_8.

$$\frac{d_8}{d_1} = \frac{\phi Y_3 d_{11}}{Y_4 d_1} \tag{A.60}$$

Substituting equation (A.60) into equation (A.53) and re-arranging gives an expression for the normalized diameter of d_{11}.

$$\frac{d_{11}}{d_1} = \frac{Y_4(Y_3+Y_4)}{(\phi Y_3+Y_4)} \tag{A.61}$$

Substituting equation (A.61) in equation (A.60) gives an expression for the normalized diameter of d_8.

$$\frac{d_9}{d_1} = \frac{\phi Y_3(Y_3+Y_4)}{(\phi Y_3+Y_4)} \tag{A.62}$$

Equations (A.51) and (A.54) have not been used to formulate the diameter equations. Substituting the equations for each of the diameters into the equations results in two equality constraints. Using equation (A.51):

$$1 + Y_2 = \frac{\phi^3 Y_3}{Y_2} + Y_3 \tag{A.63}$$

Using equation (A.54):

$$Y_3 + Y_4 = Y_2 + \frac{Y_2 Y_4}{Y_3 \phi^3} \tag{A.64}$$

The variable S is a function of Y_2, Y_3 and Y_4 and can be eliminated from equations (A.55) through (A.64). This gives equations for the diameters of the gears, with two equality constraints.

$$\frac{d_1}{d_1} = 1 \tag{A.65}$$

$$\frac{d_2}{d_1} = \frac{Y^3\phi^3}{Y_2} = \frac{\phi^3(1+Y_2)}{(\phi^3+Y_2)} \tag{A.66}$$

$$\frac{d_3}{d_1} = \frac{\phi^6(1+Y_2)}{(\phi^6+Y_2)} = \frac{\phi^6(1+Y_2)}{(\phi^6+Y_2)} \tag{A.67}$$

$$\frac{d_4}{d_1} = Y_2 = Y_2 \tag{A.68}$$

$$\frac{d_5}{d_1} = \frac{Y_2(1+Y_2)}{(\phi^3+Y_2)} = \frac{Y_2(1+Y_2)}{(\phi^3+Y_2)} \tag{A.69}$$

$$\frac{d_6}{d_1} = \frac{Y_2(1+Y_2)}{(\phi^6+Y_2)} = \frac{Y_2(1+Y_2)}{(\phi^6+Y_2)} \tag{A.70}$$

$$\frac{d_7}{d_1} = Y_3 = Y_3 \tag{A.71}$$

$$\frac{d_8}{d_1} = \frac{Y_3\phi(Y_3+Y_4)}{(Y_3\phi+Y_4)} = \frac{Y_3\phi(Y_3+Y_4)}{(Y_3\phi+Y_4)} \tag{A.72}$$

$$\frac{d_9}{d_1} = Y_3 = \frac{Y_3\phi^2(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \qquad (A.73)$$

$$\frac{d_{10}}{d_1} = Y_4 = Y_4 \qquad (A.74)$$

$$\frac{d_{11}}{d_1} = \frac{Y_4(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} = \frac{Y_4(Y_3+Y_4)}{(Y_3\phi+Y_4)} \qquad (A.75)$$

$$\frac{d_{12}}{d_1} = \frac{Y_2Y_4}{Y_3\phi^2} = \frac{Y_4(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \qquad (A.76)$$

The equality constraints become:

$$1 + Y_2 = \frac{Y_3}{Y_2}\phi + Y_3Y_3 = \frac{X_2(X1+Y_2)}{(\phi^3+Y_2)} \qquad (A.77)$$

$$Y_3 + Y_4 = Y_2 + \frac{Y_2Y_4}{\phi Y_3} Y_2 = \frac{Y_3\phi^2(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \qquad (A.78)$$

The method of Osman, Sankar, and Dukkipati requires far less mathematical development than White's method. The equations are still limited in that they apply only to one specific layout diagram and arrangement.

A.3 Technique 3: Per The Author

In chapter 2, a new technique was presented for developing the diameter equations based on the conventional arrangement. Because the equations are based on the conventional arrangement which is most general, they also apply to the single and double composite arrangements. The steps can be summarized as follows:

1. Select a layout diagram and note the number of speeds and range exponent for each group,
2. Write equations for the gears diameters corresponding to the conventional arrangement,
3. Equate diameters of composite gears to develop equality constraints corresponding to a particular arrangement,
4. Analytically evaluate the equality constraints for a consistent set of variables Y_1, Y_2, Y_3, and Y_4.
5. Evaluate gear diameters.

Using the layout diagram of figure A.1, the number of speeds for the first group is seen to be 3 and the range exponent is also 3 since the speeds increase by a factor of ϕ^3. For the second group, the number of speeds is again 3, however, the range exponent is 1.

$$d_1 = Y_1 \tag{A.79}$$

$$d_2 = \frac{Y_1\phi^3(Y_1+Y_2)}{(Y_1\phi^3+Y_2)} \tag{A.80}$$

$$d_3 = \frac{Y_1\phi^6(Y_1+Y_2)}{(Y_1\phi^6+Y_2)} \tag{A.81}$$

$$d_4 = Y_2 \tag{A.82}$$

$$d_5 = \frac{Y_2^2(Y_1+Y_2)}{(Y_1\phi^3+Y_2)} \tag{A.83}$$

$$d_6 = \frac{Y_2(Y_1+Y_2)}{(Y_1\phi^6+Y_2)} \tag{A.84}$$

$$d_7 = Y_3 \tag{A.85}$$

$$d_8 = \frac{Y_3\phi(Y_3+Y_4)}{(Y_3\phi+Y_4)} \tag{A.86}$$

$$d_9 = \frac{Y_3\phi^2(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \tag{A.87}$$

$$d_{10} = Y_4 \tag{A.88}$$

$$d_{11} = \frac{Y_3(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \tag{A.89}$$

$$d_{12} = \frac{Y_3(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \tag{A.90}$$

For the double composite arrangement under consideration the diameter of gear d_4 must equal the diameter of gear d_9. This results in the first equality constraint.

$$Y_2 = \frac{Y_3\phi^2(Y_3+Y_4)}{(Y_3\phi^2+Y_4)} \tag{A.91}$$

Also gear diameter d_5 must be equal to gear diameter d_7. This results in the second equality constraint.

$$\frac{Y_2(Y_1+Y_2)}{(Y_1\phi^3+Y_2)} = Y_3 \tag{A.92}$$

Equations (A.79) through (A.92) are equivalent to equations (A.65) to (A.78) if Y_1 is set equal to 1. The analysis of chapter 2 provides for an analytic procedure which is general in nature to solve for diameters Y_2, and Y_3, regardless of the double composite selected.

A.4 Summary

This final method offers two clear advantages. The equations for the gear diameters are written for the conventional arrangement and are also valid for all single and double composite arrangements. There is no need to develop a set of diameter equations for each arrangement. The second advantage is that the equations can be written by inspection of the layout diagram. Since the layout diagram is mathematically based, a computer based procedure can be written to generate the different arrangements possible.

APPENDIX B

OVERVIEW OF SOFTWARE USED FOR ANALYSIS OF MULTI-SPEED GEAR DRIVES

The analyses presented in chapters 2 through 6 are calculation intensive. A module FORTRAN program was developed specifically to perform these calculations. The following sections present a brief explanation of the analysis performed by each module.

B.1 Main Program

The program module was used for the selection of the problem to be analysed, selection of output to be written to disk, and optimization subroutines. Program calls to the objective functions were made from within different solutions. Figure B.1 shows a flow chart of the main program.

B.2 Solution 1

This module evaluates the power capacity of a gear pair for a wide range of operating conditions and design. The analysis is based on AGMA 218 and includes all relevent calculations to determine the power capacity of a pair of external spur or helical gears. Inputs to solution 1 are as described in detail in chaper 3. A flow chart for solution 1 is show in figure B.2. Solution 1A performs multiple calls

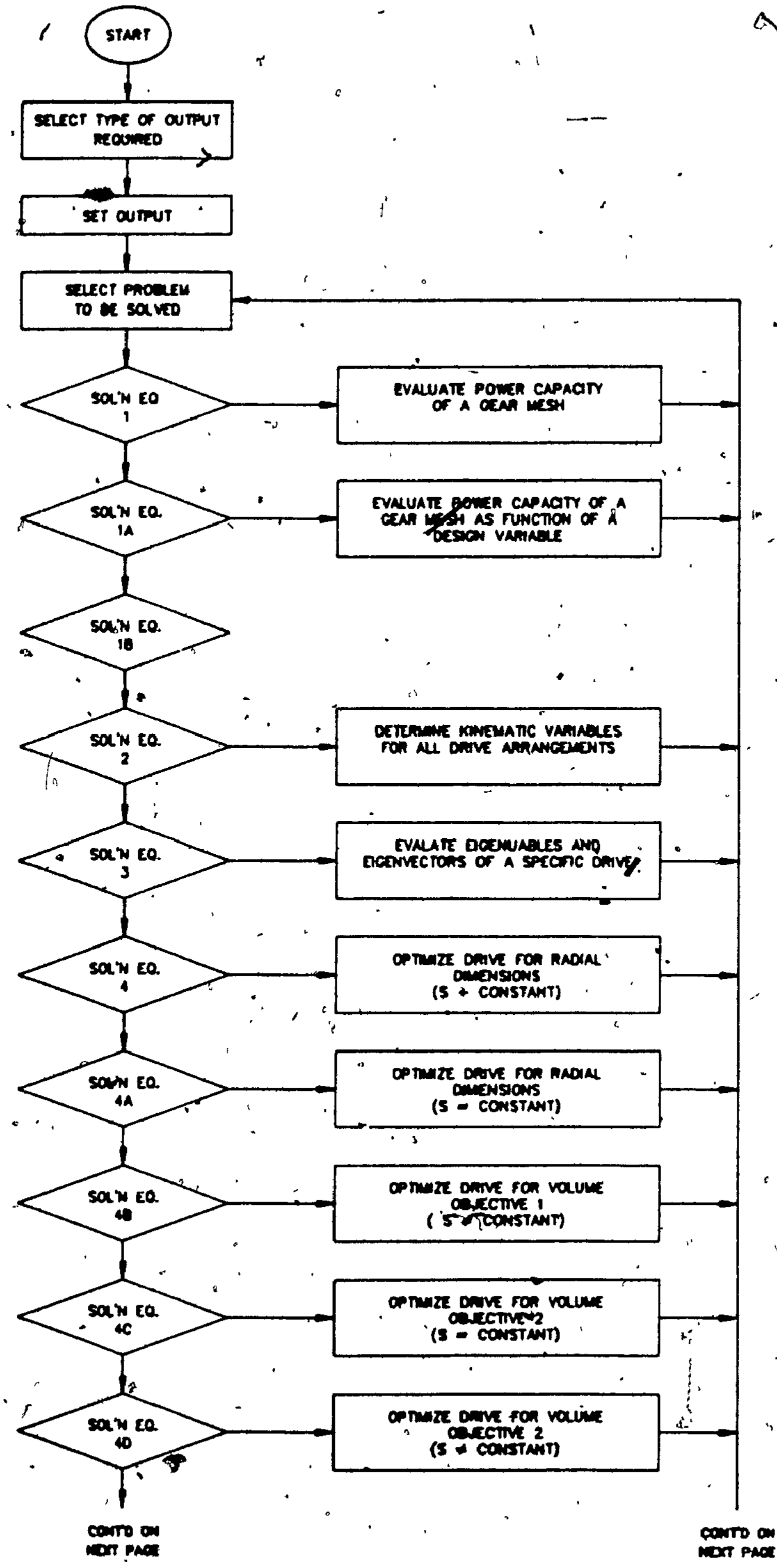

FIGURE B.1

Flow Chart of Main Routine for Gear Drive Analysis

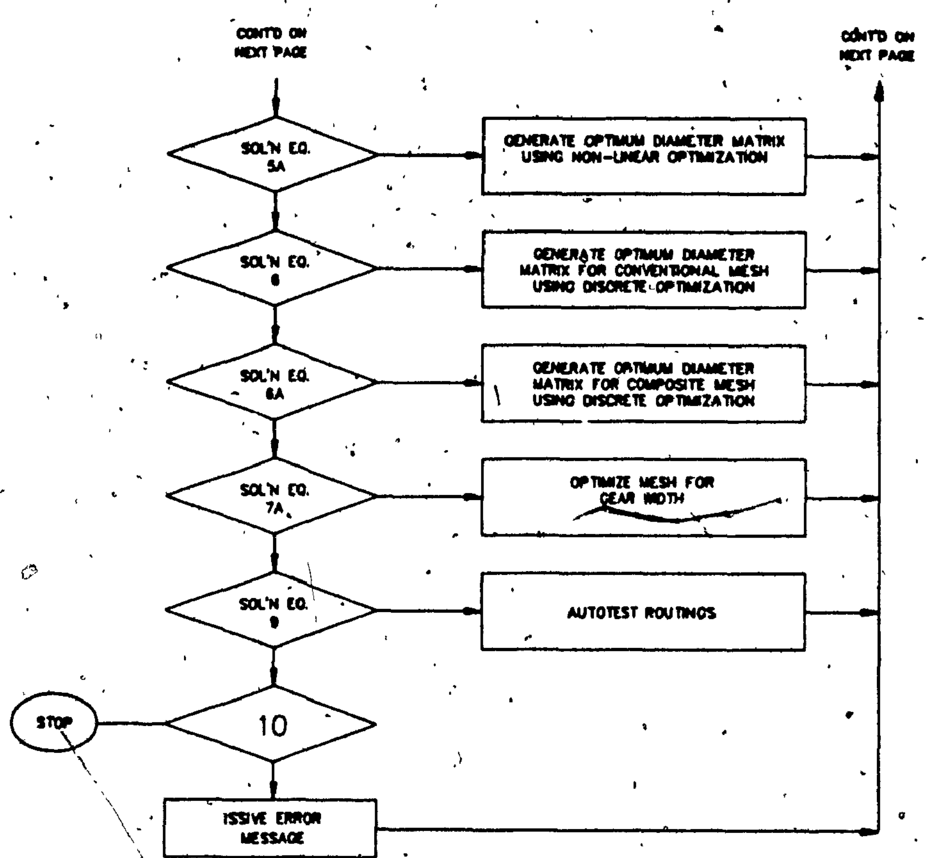

FIGURE B.1 (Continued)

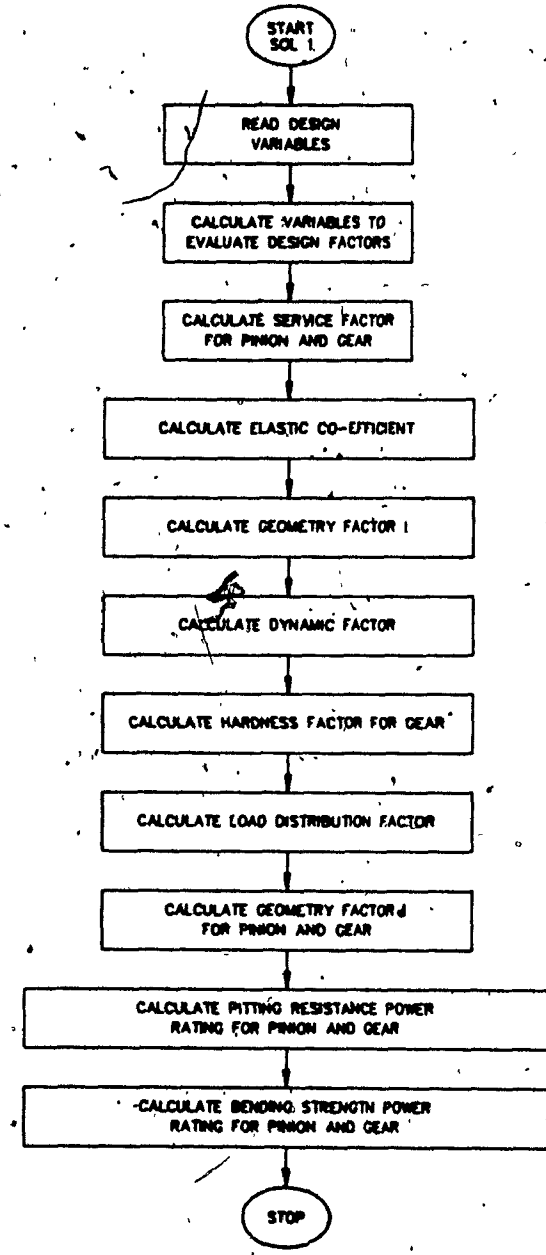

FIGURE B.2

Flow Chart for Solution 1

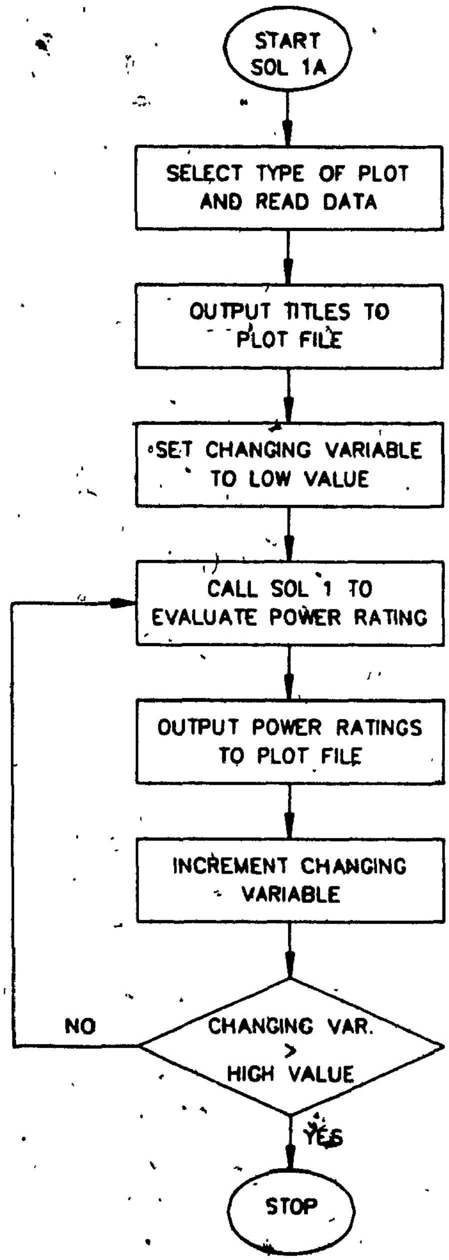

FIGURE B.3

Flow Chart for Solution 1A

B.3 Solution 2

This module is used to determine the different kinematic arrangements which are possible for a given number of speeds. The routine is capable of handling up to 6 shafts; conventional, single composite and double composite arrangements; and different layout diagrams.

Layout diagrams are generated based on the number of speeds desired. The program finds the factors of the number of speeds and their unique combinations. Once the factors and their combinations are found the allowable range exponents that are associated with a set of factors are evaluated. For each unique combination of factors there exists a set of single composite and double composite arrangements in addition to the conventional arrangement. All combinations of layout diagrams and composite arrangements are determined. A flow chart of solution 2 is shown in figure B.4.

B.4 Solution 3

This module evaluates the dynamic performance of the gear drive. Mass and stiffness properties of the individual components are required as input for each speed. Eigenvalues and eigenvalue derivatives are calculated. The mass and stiffness matrices are transformed into a symmetric dynamic matrix with equivalent eigenvalues. Eigenvalues and eigenvectors are calculated using the Jacobi method. Eigenvalue derivatives are calculated as shown in chapter 4. A flow chart of solution 3 is shown in figure B.5

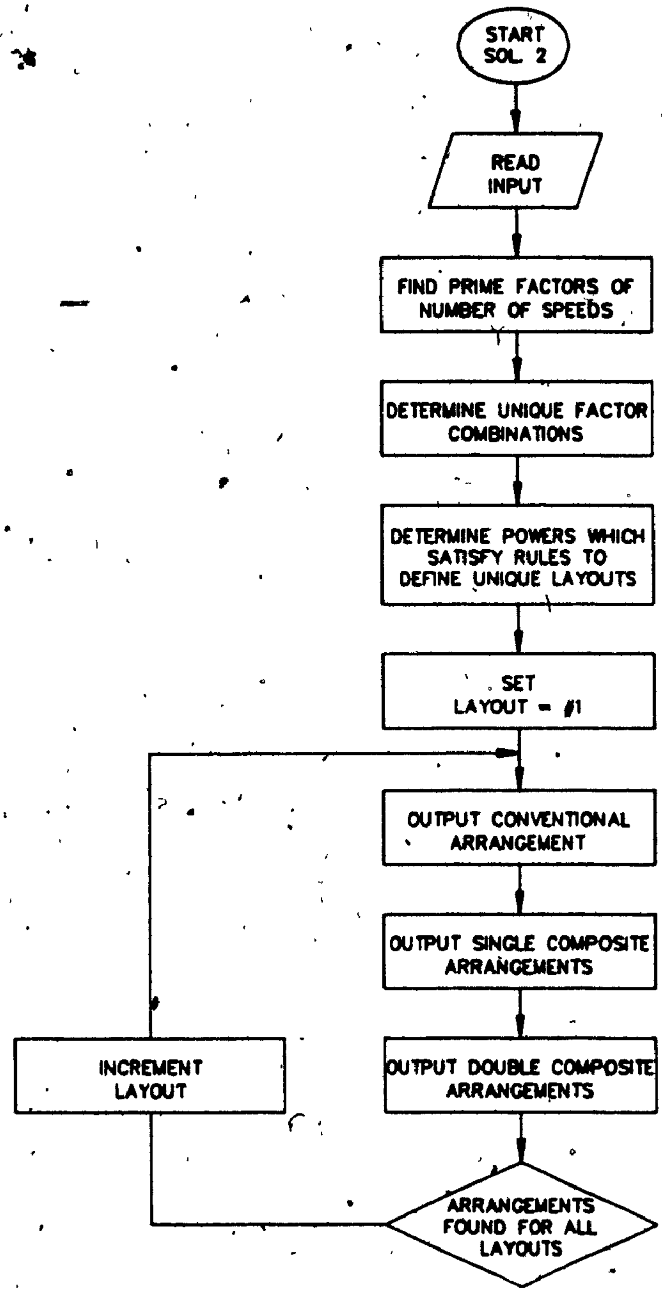

FIGURE B.4

Flow Chart for Solution 2

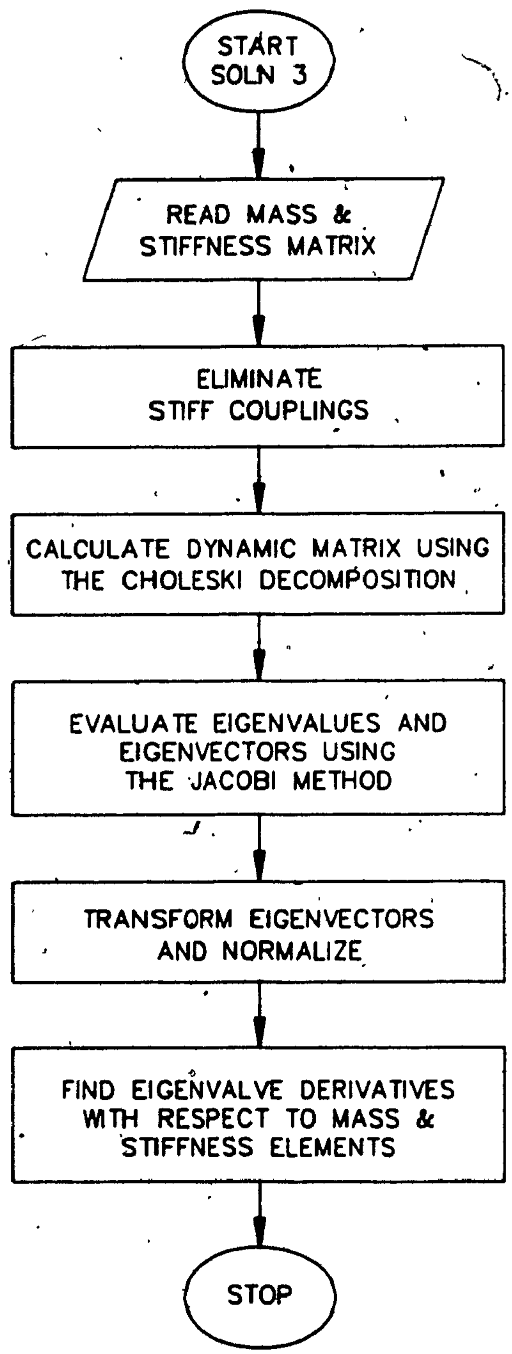

FIGURE B.5

Flow Chart for Solution 3

B.5 Solution 4

This module performs an arrangement optimization for an objective function of minimum radial dimensions or maximum stiffness (stiffness criteria based on speed ratios only). The diameters of all gears are calculated during the optimization process. The minimum diameter array is used to place constraints on the gear diameters for minimum size. A number of different optimization strategies can be used with this routine. The univariate method was normally most efficient for this analysis. A flow chart of solution 4 is shown in figure B.6

B.6 Solution 4a

This module performs an arrangement optimization similar to solution 4 except that the lowest output/input speed ratio is fixed. The exact value is calculated using the specified input shaft speed and lowest out speed.

B.7 Solution 4b

The module performs an arrangement optimization for an objective function of minimum volume. The diameters of all gears are calculated during the optimization process. The minimum diameter array is used to place constraints on the gear diameters for minimum size. As with solutions 4 and 4a, a number of optimization routines can be used. The random routine and the patterned move methods were most effective for this optimization.

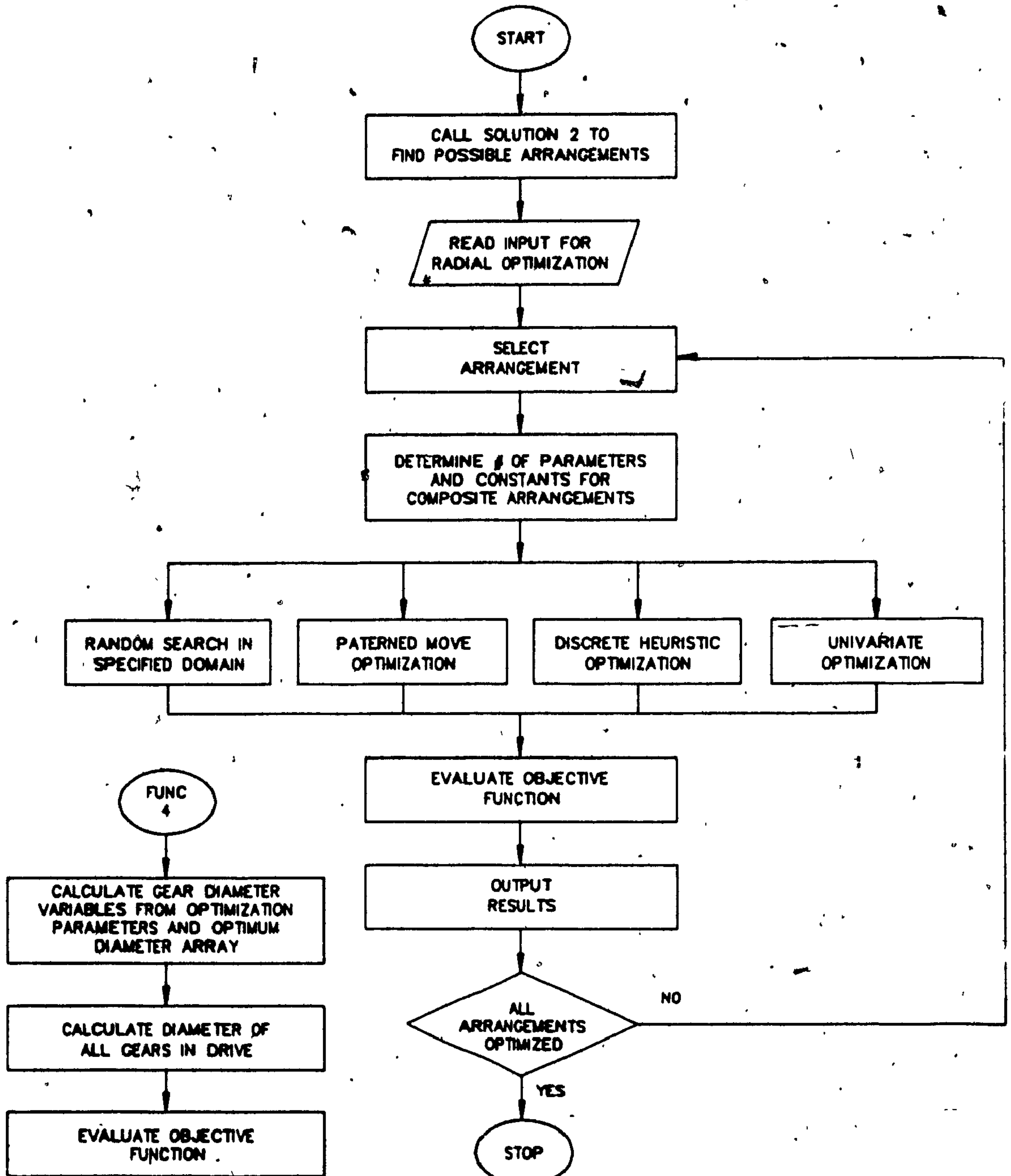

FIGURE B.6

Flow Chart for Solution 4

B.8 Solution 5a

This module calculates the minimum diameter matrix assuming the variables are continuous. The desired operating conditions, composite type, speed range, and speed reduction ratio range are required inputs. For each input speed and speed reduction ratio combindtion, the minimum input gear diameter is calculated. The optimum module and number of teeth are determined for each input gear. The actual power capacity is also calculated and printed for verification. A flow chart of solution 5a is shown in figure 3.2.

B.9 Solution 6

This module calculates the optimum diameter matrix using a discrete optimization technique. Results from solution 6 are approximately equivalent to those of solution 5a. Cost to obtain those solutions is significantly larger. The only advantage is that the variables can be limited to discrete values. This routine makes use of a discrete optimization technique.

B.10 Solution 7

This module optimizes the face width to diameter ratio of a mesh to ensure that the rated power capacity exactly matches the required power. For those meshes which represent minimum diameter input gears, the face width to diameter ratio will be very close to the value used to generate the minimum diameter matrix. Those gears with excess power capacity as a result of kinematic requirements are reduced in width to ensure all components are optimized to the power requirements.

APPENDIX C

COMPUTER GENERATED RESULTS FOR THE OPTIMAL DESIGN OF MULTI-SPEED GEAR DRIVES

C.1 Minimum Diameter Matrices for Standard Operating Conditions

TABLE C.1

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.1

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1359	868	493	336	307	296	290	287
50	1097	698	398	270	246	239	235	232
100	884	563	320	217	199	192	188	186
200	717	454	258	175	160	154	151	149
300	632	401	228	155	141	136	133	132
500	542	343	195	132	120	116	114	113
750	480	304	172	117	106	102	100	99
1000	440	279	157	107	97	94	92	91
2000	358	226	128	86	79	76	74	73
4000	292	184	104	70	64	62	60	60

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1374	868	496	408	372	359	351	347
50	1105	705	398	328	299	288	282	278
100	887	567	322	264	240	231	227	225
200	720	456	259	213	194	187	183	181
300	636	402	229	188	171	164	161	159
500	544	345	195	161	146	141	138	136
750	482	305	173	142	129	124	122	120
1000	443	280	158	130	118	114	111	110
2000	360	227	128	105	96	92	90	89
4000	294	185	104	85	78	75	73	72

TABLE C.2

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.2

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1077	687	393	268	244	237	233	230
50	866	553	315	215	196	189	186	183
100	701	445	253	173	157	152	149	147
200	565	359	204	139	127	122	120	118
300	500	317	180	122	112	108	106	104
500	428	271	154	105	95	92	90	89
750	379	240	136	92	84	81	80	79
1000	347	220	125	84	77	74	73	72
2000	282	178	101	68	62	60	59	58
4000	230	145	82	55	50	49	48	47

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1086	694	393	325	296	285	279	275
50	870	557	317	261	238	229	224	222
100	704	447	255	210	191	184	180	178
200	568	361	205	169	154	148	145	143
300	501	319	181	149	135	130	128	126
500	429	272	154	127	116	111	109	108
750	381	241	136	112	102	98	96	95
1000	349	221	125	103	93	90	88	87
2000	284	179	101	83	76	73	71	70
4000	231	146	82	67	61	59	58	57

TABLE C.3

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.3

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	939	599	343	234	214	206	202	200
50	756	483	276	188	172	165	163	161
100	610	388	221	151	138	133	130	129
200	493	313	178	121	111	107	105	103
300	436	276	157	107	97	94	92	91
500	373	236	134	91	83	80	79	78
750	329	209	118	80	73	71	69	69
1000	302	191	108	74	67	65	63	63
2000	246	155	88	60	54	52	51	51
4000	200	126	71	48	44	42	41	41

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	945	603	345	285	259	250	245	242
50	762	485	277	229	208	201	196	194
100	614	389	222	184	167	161	158	156
200	494	315	179	148	134	129	127	125
300	437	277	158	130	118	114	112	110
500	374	237	135	111	101	97	95	94
750	331	210	119	98	89	86	84	83
1000	304	192	109	90	81	78	77	76
2000	247	156	88	72	66	63	62	61
4000	201	127	71	59	53	51	50	50

TABLE C.4

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.4

RPM	SPEED REDUCTION RATIOS							
	CONVENTIONAL MESH							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	853	546	313	214	195	188	185	182
50	688	438	251	171	156	151	148	146
100	553	352	201	137	125	121	119	117
200	447	284	162	110	101	97	95	94
300	395	251	143	97	89	85	84	83
500	338	214	122	83	76	73	71	70
750	299	189	108	73	67	64	63	62
1000	274	173	98	67	61	59	58	57
2000	221	141	80	54	49	47	46	46
4000	182	114	65	44	40	38	38	37
	COMPOSITE INPUT GEAR							
RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	856	550	314	260	237	228	224	221
50	691	440	252	208	190	182	179	177
100	557	355	202	167	152	146	143	141
200	449	286	163	134	122	118	115	114
300	397	252	143	118	108	104	101	100
500	339	215	122	101	92	88	86	86
750	300	190	108	89	81	78	76	75
1000	275	174	99	81	74	71	70	69
2000	223	141	80	66	60	57	56	56
4000	182	115	65	53	48	47	46	45

TABLE C.5

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.5

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	793	506	290	199	181	176	172	170
50	638	407	233	159	145	140	138	136
100	513	327	187	127	117	112	110	109
200	415	264	150	103	94	90	88	87
300	366	233	133	90	82	79	78	77
500	313	199	113	77	70	68	66	65
750	277	175	100	68	62	60	58	58
1000	254	161	91	62	57	54	53	53
2000	206	130	74	50	46	44	43	43
4000	168	106	60	41	37	36	35	34

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	797	509	291	242	220	212	208	205
50	640	409	234	194	176	170	166	164
100	516	329	188	155	141	136	133	132
200	416	265	151	125	114	109	107	106
300	367	233	133	110	100	96	94	93
500	315	200	113	94	85	82	80	79
750	278	176	100	83	75	72	71	70
1000	255	161	92	76	69	66	65	64
2000	207	131	74	61	55	53	52	52
4000	168	106	60	49	45	43	42	42

TABLE C.6

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.6

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	749	479	275	188	172	165	163	161
50	601	384	220	151	137	132	130	129
100	485	309	177	121	110	106	104	103
200	391	249	142	97	88	85	83	83
300	345	219	125	85	78	75	73	72
500	295	187	107	73	66	64	63	62
750	261	165	94	64	58	56	55	54
1000	239	152	86	59	53	51	50	50
2000	194	123	70	47	43	41	41	40
4000	158	100	56	38	35	34	33	32

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	755	481	276	229	208	201	197	194
50	605	385	221	183	167	161	157	155
100	486	311	177	147	134	129	126	124
200	393	250	143	118	107	103	101	100
300	347	220	125	104	94	91	89	88
500	297	188	107	88	80	77	76	75
750	262	166	94	78	71	68	67	66
1000	240	152	86	71	65	62	61	60
2000	195	123	70	58	52	50	49	49
4000	159	100	57	47	42	41	40	39

TABLE C.7

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.7

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	713	457	262	180	164	158	155	154
50	574	367	210	144	131	126	124	122
100	462	294	168	115	105	101	99	98
200	118	237	135	92	84	81	80	79
300	329	209	119	81	74	71	70	69
500	281	178	102	69	63	61	60	59
750	248	157	90	61	56	54	52	52
1000	228	144	82	56	51	49	48	47
2000	185	117	66	45	41	39	39	38
4000	150	95	54	36	33	32	31	31

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	716	460	263	219	199	192	188	185
50	576	368	211	175	159	153	150	148
100	464	296	169	140	127	123	120	119
200	374	238	136	112	102	98	96	95
300	330	210	120	99	90	87	85	84
500	282	179	102	84	77	74	72	71
750	249	158	90	74	67	65	64	63
1000	229	145	82	68	62	59	58	57
2000	185	117	66	55	50	48	47	46
4000	151	95	54	44	40	39	38	37

TABLE C.8

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.8

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	686	438	252	173	158	152	149	147
50	550	351	202	138	126	121	119	117
100	442	283	162	111	101	97	95	94
200	357	227	130	89	81	78	76	76
300	315	200	114	78	71	69	67	66
500	269	171	97	66	61	58	57	57
750	238	151	86	59	53	51	50	50
1000	218	138	79	54	49	47	46	45
2000	177	112	64	43	39	38	37	37
4000	144	91	51	35	32	31	30	30

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	689	439	253	211	191	184	181	178
50	553	353	202	168	153	147	144	142
100	444	284	162	135	122	118	115	114
200	358	228	130	108	98	94	92	91
300	316	201	115	95	86	83	81	80
500	270	172	98	81	74	71	69	68
750	239	152	86	71	65	62	61	60
1000	219	139	79	65	59	57	56	55
2000	178	112	64	53	48	46	45	44
4000	144	91	52	43	39	37	36	36

TABLE C.9

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 0.9

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	659	423	243	167	152	147	144	142
50	530	339	194	133	122	117	115	114
100	427	272	156	107	97	94	92	91
200	344	219	125	86	78	75	74	73
300	303	193	110	75	69	66	65	64
500	259	165	94	64	58	56	55	54
750	229	145	83	56	51	50	49	48
1000	210	133	76	52	47	45	44	44
2000	170	108	61	42	38	37	36	35
4000	139	88	50	34	31	30	29	29

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	665	425	244	204	185	178	174	172
50	534	341	195	163	148	142	139	137
100	429	273	157	130	118	114	111	110
200	346	220	126	104	95	91	89	88
300	305	194	111	92	83	80	79	78
500	261	166	94	78	71	68	67	66
750	230	146	83	69	62	60	59	58
1000	211	134	76	63	57	55	54	53
2000	197	108	61	51	46	44	43	43
4000	139	88	50	41	37	36	35	35

TABLE C.10

Minimum Diameter Matrix (mm) for Standard Operating Conditions with Face Width/Diameter Ratio of 1.0

CONVENTIONAL ARRANGEMENT								
RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	639	410	235	161	147	142	139	137
50	514	328	188	129	117	113	111	110
100	413	263	151	103	94	90	89	87
200	332	212	121	82	75	72	71	70
300	293	186	106	72	66	64	62	62
500	250	159	91	62	56	54	53	52
750	221	140	80	54	49	48	47	46
1000	203	128	73	50	45	43	43	42
2000	164	104	59	40	36	35	34	34
4000	134	84	48	32	29	28	28	27

COMPOSITE PINION								
RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	642	411	236	197	179	172	169	166
50	515	329	189	157	143	137	135	133
100	415	264	151	126	114	110	107	106
200	334	213	121	101	91	88	86	85
300	295	187	107	88	80	77	76	75
500	252	160	91	75	68	66	64	63
750	222	141	80	66	60	58	57	56
1000	204	129	73	60	55	53	52	51
2000	165	104	59	49	44	42	42	41
4000	134	85	48	39	36	34	33	33

C.2 Minimum diameter Matrices for Case Hardened Material

TABLE C.11

Minimum Diameter Matrix (mm) for Hardened Pinion and Gear Materials with Face Width/Diameter Ratio of 0.8

CONVENTIONAL ARRANGEMENT								
RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	578	368	210	144	131	128	126	124
50	464	296	169	115	105	102	100	99
100	374	238	135	92	84	81	80	79
200	300	191	109	74	68	65	64	63
300	265	168	96	90	60	58	56	56
500	227	144	82	55	51	49	48	48
750	200	127	72	49	45	43	42	42
1000	183	116	66	45	41	39	39	38
2000	149	94	53	36	33	32	31	31
4000	121	76	43	29	27	26	25	25

COMPOSITE INPUT GEAR								
RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	583	371	211	175	160	154	151	149
50	467	299	169	140	127	123	121	119
100	376	239	136	112	102	98	97	95
200	302	193	109	90	82	79	77	76
300	267	169	96	79	72	69	68	67
500	228	145	82	67	61	59	58	57
750	201	127	72	59	54	52	51	50
1000	184	117	66	54	49	48	47	46
2000	149	95	53	44	40	38	38	37
4000	121	77	43	35	32	31	30	30

C.3 Optimum Number of Input Gear Teeth Matrix. Standard Operating Conditions.

TABLE C.12

Matrix of Optimum Number of Teeth on Input Gear for a Face Width/Diameter Ratio of 0.5

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	437	239	114	78	57	41	41	40
50	452	256	110	75	57	44	43	43
100	485	258	118	80	55	53	52	43
200	490	249	118	81	57	50	49	48
300	490	256	125	85	58	56	49	48
500	518	266	133	85	61	53	52	52
750	523	276	133	85	63	56	55	50
1000	520	278	137	88	67	56	55	50
2000	536	287	140	91	68	59	54	54
4000	542	292	141	93	70	62	58	54

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	377	200	92	90	69	67	65	65
50	403	193	92	107	83	67	65	65
100	406	207	89	110	78	75	63	62
200	426	209	95	118	80	77	67	67
300	434	221	94	112	87	76	74	66
500	421	220	98	118	77	77	70	69
750	438	222	102	117	89	80	72	71
1000	441	229	101	119	92	78	76	70
2000	456	237	105	125	92	84	78	73
4000	464	234	109	128	95	85	80	76

TABLE C.13

Matrix of Optimum Number of Teeth on Input Gear for a Face Width/Diameter Ratio of 1.0.

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	453	226	111	76	58	45	44	43
50	446	233	119	71	56	54	44	43
100	488	249	119	81	59	50	49	48
200	498	267	124	85	59	52	51	50
300	509	265	126	86	63	55	49	49
500	514	264	129	83	62	56	50	50
750	524	277	133	86	63	57	52	51
1000	528	274	133	87	64	59	54	53
2000	532	288	140	89	66	58	55	54
4000	549	293	140	92	70	61	57	54

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	404	195	93	109	71	68	67	66
50	406	208	89	112	79	65	64	63
100	393	209	95	109	81	69	68	67
200	421	218	96	111	87	76	68	67
300	418	222	101	119	89	73	72	71
500	436	227	101	119	87	78	71	70
750	438	223	101	121	90	78	76	71
1000	434	224	104	120	91	80	74	73
2000	456	231	108	124	95	81	76	75
4000	466	241	110	128	96	85	80	76

C.4 Optimum Module Matrix. Standard Operating Condtions

TABLE C.14

Matrix of Optimum Module for Face Width/Diameter Ratio of 0.5 for Standard Operating Conditions

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1.81	2.12	2.54	2.54	3.18	4.23	4.23	4.23
50	1.41	1.59	2.12	2.12	2.54	3.18	3.18	3.18
100	1.06	1.27	1.59	1.59	2.12	2.12	2.12	2.54
200	0.85	1.06	1.27	1.27	1.59	1.81	1.81	1.81
300	0.75	0.91	1.06	1.06	1.41	1.41	1.59	1.59
500	0.60	0.75	0.85	0.91	1.15	1.27	1.27	1.27
750	0.53	0.64	0.75	0.79	0.98	1.06	1.06	1.15
1000	0.49	0.58	0.67	0.71	0.85	0.98	0.98	1.06
2000	0.38	0.45	0.53	0.55	0.67	0.75	0.79	0.79
4000	0.31	0.36	0.42	0.44	0.53	0.58	0.60	0.64

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	2.12	2.54	3.18	2.54	3.18	3.18	3.18	3.18
50	1.59	2.12	2.54	1.81	2.12	2.54	2.54	2.54
100	1.27	1.59	2.12	1.41	1.80	1.81	2.12	2.12
200	0.98	1.27	1.59	1.06	1.41	1.41	1.59	1.59
300	0.85	1.06	1.41	0.98	1.15	1.27	1.27	1.41
500	0.75	0.91	1.15	0.79	0.98	1.06	1.15	1.15
750	0.64	0.79	0.98	0.71	0.85	0.91	0.98	0.98
1000	0.58	0.71	0.91	0.64	0.75	0.85	0.85	0.91
2000	0.45	0.55	0.76	0.49	0.60	0.64	0.67	0.71
4000	0.36	0.45	0.55	0.38	0.47	0.51	0.53	0.55

TABLE C.15

Optimum Module on Input Gear (mm). Face Width/ Diameter Ratio of 1.0

CONVENTIONAL MESH

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1.41	1.81	2.12	2.12	2.54	3.18	3.18	3.18
50	1.15	1.41	1.59	1.81	2.12	2.12	2.54	2.54
100	0.85	1.05	1.27	1.27	1.59	1.81	1.81	1.81
200	0.66	0.79	0.98	0.98	1.27	1.41	1.41	1.41
300	0.57	0.70	0.85	0.85	1.06	1.16	1.27	1.27
500	0.48	0.60	0.71	0.75	0.91	0.98	1.06	1.06
750	0.42	0.50	0.61	0.64	0.79	0.85	0.91	0.91
1000	0.38	0.47	0.55	0.58	0.71	0.75	0.79	0.79
2000	0.31	0.36	0.42	0.45	0.55	0.61	0.64	0.64
4000	0.24	0.28	0.34	0.35	0.42	0.47	0.49	0.51

COMPOSITE INPUT GEAR

RPM	SPEED REDUCTION RATIOS							
	0.10	0.20	0.50	1.00	2.00	3.00	4.00	5.00
25	1.59	2.12	2.54	1.81	2.54	2.54	2.54	2.54
50	1.27	1.59	2.12	1.41	1.81	2.12	2.12	2.12
100	1.06	1.27	1.59	1.16	1.41	1.59	1.59	1.59
200	0.79	0.97	1.27	0.91	1.06	1.16	1.27	1.27
300	0.71	0.84	1.06	0.75	0.91	1.06	1.06	1.06
500	0.57	0.70	0.91	0.64	0.79	0.85	0.91	0.91
750	0.50	0.63	0.79	0.55	0.67	0.75	0.75	0.79
1000	0.47	0.57	0.71	0.51	0.61	0.67	0.71	0.71
2000	0.36	0.45	0.55	0.40	0.47	0.53	0.55	0.55
4000	0.28	0.35	0.44	0.31	0.37	0.41	0.42	0.44

C.5 Chapter 4 - Eigenvalue Derivatives

TABLE C.16

Eigenvalue Derivatives $\partial \Omega / \partial M(i,i)$ (krad/s)/(kgm^2)

MASS NODE 1	SPEED			
EIGENVALUE #	1	2	3	4
1	-4672	-1501	-4671	-1785
2	-100	-26	-100	-27
3	-45	-2	-43	-5
4	0	-3977	-17	-91
5	0	-7	0	-3966
6	0	-1	0	0
7	-35	-38	-3	-6
8	0	-2	0	0
MASS NODE 2	SPEED			
EIGENVALUE #	1	2	3	4
1	-6	-231	-6	-1700
2	-500	-1854	-500	-1837
3	-48	-2	-45	-5
4	0	-721	-20	-89
5	0	-263	0	-2
6	0	-1	0	0
7	-39	-38	-3	-6
8	0	-18	0	0
MASS NODE 3	SPEED			
EIGENVALUE #	1	2	3	4
1	-430	0	-430	0
2	-162	0	-162	0
3	-46	-14	-43	-10
4	0	0	-17	-36
5	0	0	0	0
6	0	-2	0	0
7	-34	-35	-3	-5
8	0	0	0	0

TABLE C.16 - Continued

EIGENVALUE #	1	2	3	4
1	0	-260	0	-1022
2	0	-1691	0	-1539
3	-28	-2	-39	-5
4	-1493	-385	-246	-87
5	-29	-61	-1763	-29
6	-2	-1	0	0
7	-98	-37	-6	-6
8	-25	0	0	0

MASS NODE 5	SPEED			
EIGENVALUE #	1	2	3	4
1	0	0	0	-3116
2	0	-374	0	-37
3	-27	-2	-39	-5
4	-67	-221	-250	-87
5	-5848	-1522	-2201	-561
6	-2	-1	0	0
7	-103	-39	-7	-6
8	-196	-193	0	0

MASS NODE 6	SPEED			
EIGENVALUE #	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	-7	-5	0	0
4	0	0	-2	-5
5	0	0	0	0
6	-11	-3	-219	-67
7	-4	-2	-15	-6
8	0	0	-76	-8

MASS NODE 7	SPEED			
EIGENVALUE #	1	2	3	4
1	0	-27	0	0
2	0	-33	0	0
3	-27	-2	0	0
4	-158	-528	-2	-5
5	-1122	-341	0	0
6	-2	-1	-58	-15
7	-104	-40	-16	-6
8	-113	-23	-8	-7

TABLE C.16 - Continued

MASS NODE 8	SPEED			
EIGENVALUE #	1	2	3	4
1	0	-66	0	0
2	0	-15	0	0
3	-26	-2	0	0
4	-371	-708	-2	-5
5	-94	-401	0	0
6	-2	-1	-29	0
7	-102	-38	-15	-6
8	-8263	-3216	-1966	-629

TABLE C.17

Eigenvalue Derivatives ($\partial\eta/\partial k$) (krad/s)/(MNm/rad)

STIFFNESS (0-1)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.615	1.913	0.615	1.609
2	0.261	0.005	0.261	0.005
3	0.679	0.623	7.178	2.173
4	0.000	12.091	0.797	5.413
5	0.000	0.002	0.000	7.156
6	0.213	1.366	0.000	0.001
7	2.381	2.878	0.204	0.433
8	0.000	0.002	0.000	0.000
STIFFNESS (1-2)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.660	3.708	0.660	6.279
2	2.727	0.439	2.728	0.445
3	0.005	0.000	0.004	0.000
4	0.000	3.985	0.006	0.001
5	0.000	0.086	0.000	6.851
6	0.000	0.000	0.000	0.000
7	0.008	0.000	0.001	0.000
8	0.000	0.014	0.000	0.000

TABLE C.17 - Continued

STIFFNESS (3-4)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.057	0.337	0.057	0.932
2	0.400	0.321	0.437	0.298
3	0.311	1.202	0.015	0.493
4	1.967	1.220	18.575	13.905
5	0.003	0.015	1.332	0.053
6	0.449	0.035	0.000	0.000
7	17.129	10.905	1.266	1.611
8	0.010	0.000	0.000	0.000

STIFFNESS (4-5)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.000	0.354	0.000	6.944
2	0.001	0.692	0.000	0.397
3	0.002	0.002	0.001	0.001
4	2.838	3.615	0.001	0.000
5	0.711	0.239	5.920	0.607
6	0.000	0.000	0.000	0.000
7	0.005	0.001	0.000	0.000
8	0.150	0.095	0.000	0.000

STIFFNESS (5-6)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.000	0.000	0.000	2.822
2	0.000	0.071	0.001	0.007
3	9.320	3.481	5.442	1.909
4	0.087	0.680	14.095	7.803
5	0.621	0.373	1.656	1.030
6	2.723	0.590	0.379	0.185
7	10.031	4.241	0.124	0.000
8	0.082	0.083	0.014	0.200

STIFFNESS (7-8)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.000	0.010	0.000	0.000
2	0.000	0.018	0.000	0.000
3	0.001	0.000	0.000	0.000
4	0.058	0.040	0.000	0.001
5	0.198	0.363	0.000	0.000
6	0.000	0.001	0.290	0.046
7	0.001	0.001	0.002	0.001
8	4.311	1.629	0.403	0.212

TABLE C.17 - Continued

STIFFNESS (8-9)	SPEED			
EIGENVALUE #	1	2	3	4
1	0.000	0.084	0.000	0.000
2	0.000	0.003	0.000	0.000
3	3.931	0.547	0.038	0.006
4	0.480	2.152	0.097	0.267
5	0.010	0.098	0.000	0.000
6	1.276	1.334	0.050	0.000
7	7.000	2.887	1.065	0.426
8	3.453	1.386	0.355	0.174

C.6 Inertia and Stiffness Matrices for 9 Speed Drive

TABLE C.1[illegible]

Optimum Arrangement With Initial Shaft Diameters

GEAR				SHAFT	
#	DIAM (mm)	LAMDA	WIDTH (mm)	#	DIAM (mm)
1	80.7	0.504	40.7	1	30.0
2	182.0	0.114	20.8	2	40.0
3	264.7	0.100	26.5	3	50.0
4	230.8		40.7		
5	129.5		20.8		
6	46.8		26.5		
7	111.1	0.510	56.7		
8	155.5	0.214	33.3		
9	207.6	0.136	28.2		
10	369.7		56.7		
11	325.4		33.3		
12	273.0		28.2		

TABLE C.19

Ratio of Shaft Speed to Input Shaft Speed

	SPEED				
SHAFT	1	2	3	4	5
1	1.000	1.000	1.000	1.000	1.000
2	0.350	1.406	5.651	0.350	1.406
3	0.105	0.422	1.698	0.167	0.672

	SPEED				
SHAFT	6	7	8	9	
1	1.000	1.000	1.000	1.000	
2	5.651	0.350	1.406	5.651	
3	2.700	0.266	1.069	4.296	

TABLE C.20

Basic and Equivalent Inertia of Gears Referenced to Input Shaft Speed (kgm^2)

GEAR	SPEED				
#	BASIC	#1	#2	#3	#4
1	0.0013	0.0013	0.0013	0.0013	0.0013
2	0.0175	0.0175	0.0175	0.0175	0.0175
3	0.0999	0.0999	0.0999	0.0999	0.0999
4	0.0888	0.0109	0.1755	2.8358	0.0109
5	0.0045	0.0006	0.0089	0.1433	0.0006
6	0.0001	0.0000	0.0002	0.0031	0.0000
7	0.0066	0.0008	0.0131	0.2119	0.0008
8	0.0149	0.0018	0.0295	0.4770	0.0018
9	0.0403	0.0049	0.0797	1.2869	0.0049
10	0.8143	0.0090	0.1453	2.3475	0.0227
11	0.2868	0.0032	0.0512	0.8267	0.0080
12	0.1206	0.0013	0.0215	0.3477	0.0034
GEAR	**SPEED**				
#	**5**	**6**	**7**	**8**	**9**
1	0.0013	0.0013	0.0013	0.0013	0.0013
2	0.0175	0.0175	0.0175	0.0175	0.0175
3	0.0999	0.0999	0.0999	0.0999	0.0999
4	0.1755	2.8358	0.0109	0.1755	2.8358
5	0.0089	0.1433	0.0006	0.0089	0.1433
6	0.0002	0.0031	0.0000	0.0002	0.0031
7	0.0131	0.2119	0.0008	0.0131	0.2119
8	0.0295	0.4770	0.0018	0.0295	0.4770
9	0.0797	1.2869	0.0049	0.0797	1.2869
10	0.3673	5.9349	0.0576	0.9303	15.0317
11	0.1294	2.0900	0.0203	0.3276	5.2936
12	0.0544	0.8789	0.0085	0.1378	2.2262

TABLE C.21

Stiffness of Basic and Equivalent Couplings (MNm/rad)

SPEED 1

#	A	B	L	BASIC	EQUIV
1	0	2	159.4	0.320	0.3198
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	0.1320
8	6	4	67.2	0.759	0.0928
9	4	9	4.0	12.745	1.5588
10	9	7	84.9	0.601	0.0734
11	7	8	89.9	0.567	0.0693
15	12	10	5.0	10.196	0.1125
16	10	11	5.0	10.196	0.1125
17	11	13	164.9	0.309	0.0034

SPEED 2

#	A	B	L	BASIC	EQUIV
1	0	2	159.4	0.320	0.3198
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	0.1320
8	6	4	67.2	0.759	0.0928
9	4	9	4.0	12.745	1.5588
10	9	7	84.9	0.601	0.0734
11	7	8	89.9	0.567	0.0693
15	12	10	5.0	10.196	0.2846
16	10	11	5.0	10.196	0.2846
17	11	13	75.0	0.680	0.0189

TABLE C.21 (Continued)

SPEED 3

#	A	B	L	BASIC	EQUIV
1	0	2	159.4	0.320	0.3198
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	0.1320
8	6	4	67.2	0.759	0.0928
9	4	9	4.0	12.745	1.5588
10	9	7	84.9	0.601	0.0734
11	7	8	89.9	0.567	0.0693
15	12	10	5.0	10.196	0.7195
16	10	11	5.0	10.196	0.7195
17	11	13	249.8	0.204	0.0144

SPEED 4

#	A	B	L	BASIC	EQUIV
1	0	2	45.0	1.133	1.1328
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	2.1333
8	6	4	67.2	0.759	1.5001
9	4	9	4.0	12.745	25.1862
10	9	7	84.9	0.601	1.1868
11	7	8	89.9	0.567	1.1203
15	12	10	5.0	10.196	1.8190
16	10	11	5.0	10.196	1.8190
17	11	13	164.9	0.309	0.0551

SPEED 5

#	A	B	L	BASIC	EQUIV
1	0	2	45.0	1.133	1.1328
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	2.1333
8	6	4	67.2	0.759	1.5001
9	4	9	4.0	12.745	25.1862
10	9	7	84.9	0.601	1.1868
11	7	8	89.9	0.567	1.1203
15	12	10	5.0	10.196	4.5988
16	10	11	5.0	10.196	4.5988
17	11	13	75.0	0.680	0.3065

TABLE C.21 (Continued)

SPEED 6

#	A	B	L	BASIC	EQUIV
1	0	2	45.0	1.133	1.1328
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	2.1333
8	6	4	67.2	0.759	1.5001
9	4	9	4.0	12.745	25.1862
10	9	7	84.9	0.601	1.1868
11	7	8	89.9	0.567	1.1203
15	12	10	5.0	10.196	20.1489
16	10	11	5.0	10.196	20.1489
17	11	13	249.8	0.204	0.4032

SPEED 7

#	A	B	L	BASIC	EQUIV
1	0	2	92.2	0.553	0.5527
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	34.4710
8	6	4	67.2	0.759	24.2398
9	4	9	4.0	12.745	406.9552
10	9	7	84.9	0.601	19.1761
11	7	8	89.9	0.567	18.1020
15	12	10	5.0	10.196	29.3923
16	10	11	5.0	10.196	29.3923
17	11	13	164.9	0.309	0.8910

SPEED 8

#	A	B	L	BASIC	EQUIV
1	0	2	92.2	0.553	0.5527
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	34.4710
8	6	4	67.2	0.759	24.2398
9	4	9	4.0	12.745	406.9552
10	9	7	84.9	0.601	19.1761
11	7	8	89.9	0.567	18.1020
15	12	10	5.0	10.196	74.3079
16	10	11	5.0	10.196	74.3079
17	11	13	75.0	0.680	4.9538

TABLE C.21 (Continued)

SPEED 9					
#	A	B	L	BASIC	EQUIV
1	0	2	92.2	0.553	0.5527
2	2	3	3.0	16.993	16.9928
3	3	1	3.0	16.993	16.9928
7	5	6	47.2	1.080	34.4710
8	6	4	67.2	0.759	24.2398
9	4	9	4.0	12.745	406.9552
10	9	7	84.9	0.601	19.1761
11	7	8	89.9	0.567	18.1020
15	12	10	5.0	10.196	187.8558
16	10	11	5.0	10.196	187.8558
17	11	13	249.8	0.204	3.7599