

Deflection Analysis of
Prestressed Concrete Segmental
Cable Stayed Bridges

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ABSTRACT

Deflection Analysis Of Prestressed
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Analytical studies are carried out on precast prestressed concrete segmental cable stayed bridges. The prototype bridge is analysed for both erection and service stages. The method of predicting initial camber during erection and calculation of the required pre-tensioning forces in the cable stays to produce this camber are discussed. A numerical design example and a computer program are also presented.

The results obtained for the cable stayed bridge are compared to a structure without cable stays. These results are presented in a table.

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NOTATIONS

a	Length of cantilever during erection stage.
A	Area of cross section of girder.
A_1, A_2, \dots, A_i	Area of cross section of temporary cable stays.
A_r	Area of cross section, permanent cable stays.
A_T	Area of cross section of tower.
b	Length of each segment.
C_a, C_c	Flexibilities of cable stays at 'A' and 'C'.
e	Eccentricity of prestressing tendons
e_n	Eccentricity of group-1 tendons
E_2, e_{3a}, e_{3b}	Eccentricities of group-2, group-3a and, group-3b prestressing tendons respectively.
e_A, e_o	Eccentricity of group-2 tendons at the outer and inner ends respectively.
E, E_T	Young's modulus of elasticity of girder and tower materials respectively.
E_e	Equivalent modulus of elasticity of cable stay.
$f_{aa}, f_{ab}, \dots, f_{nn}$	Flexibilities of the simple beam 'AG'.
f_T	Flexibility, of the tower.
f'_c	Cylindrical strength of concrete in compression.
F_1, F_2, \dots, F_n	Prestressing forces in the first, second, ..., n^{th} tendons.

-x-

$F_t, \Sigma F_n$	Total prestressing force at the support due to group-1 cantilever prestress.
$\Sigma F_2, \Sigma F_{3a}, \Sigma F_{3b}$	Total prestressing force due to group-2, group-3a, and group-3b tendons respectively.
H_k	Horizontal component of tension force in cable stay.
H_T	Height of the tower.
I	Moment of inertia of girder.
k	Number of temporary cable stays.
l	Length of main span.
l'	Length of flanking span.
L	Total length of the bridge.
L_1, L_2, \dots, L_k	Length of auxiliary cable stays.
L_r	Length of permanent cable stays.
M_1, M_2, \dots, M_n	Prestressing moment at the supports for the cantilever beam when the successive segments are erected.
M_x	Moment at section 'x'.
M_r	Maximum bending moment in the girder.
$(M_r)_B$	Bending moment at 'B' for the primary system.
M_r^i	Bending moment at 'r' due to unit load at 'i'.
n	Number of segments in the cantilever span.
n_1, n_2, \dots, n_k	Number of segments up to the respective cable points.

n_x	Number of segments up to any section distant 'x' from the free end.
q	Total load/unit length acting on the girder.
$R'_{B,F}$	Secondary reaction at intermediate supports.
r	Number of segments up to point of permanent cable stay.
s	Total length of horizontal projection of group-2 tendons.
$2s_1$	Length of straight portion of group-3a tendons in the main span.
s_2	Horizontal projection of bent down portion of group-3a tendons in the main span.
$2s_3$	Straight portion of group-2 tendons in flanking span.
s_4	Horizontal projection of bent down portion of group-2 tendons in flanking span.
t	Time after casting (days).
t_o	Age of concrete (days).
T	Cable stay tension.
T_i	Initial tension in cable stays.
T_f	Final tension in cable stays.
V_1, \dots, V_k	Vertical component of tension force, in cable stays.
w	Uniformly distributed load due to self weight.

W	Concentrated load due to construction equipment.
w_L	Uniformly distributed live load due to lane loading.
w_c	Concentrated load due to lane loading..
w_e	Weight per unit length of cable stay.
x	Distance from a given section.
x_b, x_f	Support reactions at 'B' and 'F'.
x_1, x_2, \dots, x_k	Pretensioning forces in the auxiliary cable stays.
x_i	Required cable force to reduce M_r to $C_o M_r$.
x_a, x_c	Vertical components of cable stay force at 'A' and 'C'.
x_r	Pretensioning force in permanent cable stays.
$x_{r\phi}$	Modified tension in cable stays to account for creep.
$\alpha_1, \dots, \alpha_k$	Angle of inclination of auxiliary cable stays.
α_r	Angle of inclination of permanent cable stays.
β_{c1}	Factor accounting for relative humidity of ambient medium and composition of concrete.
β_{c2}	Factor depending on theoretical thickness of structural element.
$\beta_{d(t-t_o)}$	Factor variable from zero to infinity indicating variation of ϕ_d with time.

- $\beta_f(t) - \beta_f(t_0)$ Factor variable from zero infinity indicating variation of ϕ_f with time.
- δ_{el} Elastic deflection of girder.
- $\delta'_a, \dots, \delta'_g$ Deflection of girder at 'A', 'B', ..., 'G' respectively due to action of total loads on the structure.
- $(\delta'_b)_{\text{conc. live load.}}$ Deflection at 'B' due to concentrated live load.
- $(\delta'_c)_{\text{conc. live load.}}$ Deflection at 'C' due to concentrated live load.
- $(\delta'_b)_{\text{u.d.l.}}$ Deflection at 'B' due to uniformly distributed lane loading.
- $(\delta'_c)_{\text{u.d.l.}}$ Deflection at 'C' due to uniformly distributed lane loading.
- $(\Delta_A)_p, \dots, (\Delta_G)_p$ Deflection at 'A', 'B', ..., 'G' due to primary prestressing forces in girder.
- $(\Delta_{B,F})_p$ Deflection at 'B' or 'F' due to primary prestressing forces.
- $\Delta_{\text{conc. load}}$ Deflection of the free end of the cantilever due to the concentrated moving load of the equipment.
- $\Delta_{c.p.}, (\Delta_D)_{c.p.}$ Deflection of the free end of cantilever due to prestressing forces.
- $\Delta_{c.p. \text{ up to } x}$ Deflection of the free end of cantilever due to prestressing up to a point distant 'x' from the fixed end.

$(\Delta_D)_P$	Deflection at 'D' due to total primary prestress.
$(\Delta_D)_1, (\Delta_D)_2$	Deflection at 'D' due to group-1 and group-2 prestressing tendons.
$(\Delta_D)_{3a}, (\Delta_D)_{3b}$	Deflection at 'D' due to group-3a and group-3b prestressing tendons respectively.
$\Delta_{g1}, \dots, \Delta_{gk}$	Deflection of cable stay points due to axial shortening of the girder.
Δ_{gr}	Deflection of permanent cable stay point due to axial shortening of girder.
Δ_{total}	Total deflection at the free end of the cantilever.
Δ_T	Deflection of girder due to shortening of the tower.
Δ_x	Required camber and deflection at any section distant 'x' from the fixed end.
$\Delta_{x1}, \dots, \Delta_{xk}$	Deflection at the respective cable points due tension forces in the stays.
Δ_{ll}	Deflection at 'B' or 'F' due to unit loads applied at 'B' and 'F'.
$\Delta L_{g1}, \dots, \Delta L_{gk}$	Axial shortening of girder due to cable stay forces.
$\Delta L_1, \dots, \Delta L_k$	Extension of cable stays due to tension.
ΔL_T	Axial shortening of the tower.
$\Delta L_{x1}, \dots, \Delta L_{xk}$	Deflection of girder due to extension of cable stays.

Δ'_a, Δ'_c	Deflection due to rotation of tower at 'A' and 'C'.
Δ''_a, Δ''_c	Deflection due to elastic stretch of cable stays, during service stage.
ϕ	Area of bending moment diagram due to prestressing.
ϕ_d	Recoverable creep or delayed plasticity.
$\phi_{d\infty}$	Delayed plasticity at infinity.
ϕ_f	Irrecoverable creep or flow.
$\phi_{f\infty}$	Flow at infinity.
ϕ_n	Rotation of towers.
ϕ_t	Creep factor.
$\phi(t, t_0)$	Creep factor at time 't' for a concrete specimen loaded at time 't ₀ '.
ψ	Shape function for a cantilever beam.

CHAPTER-1

INTRODUCTION

In recent years, cable stayed bridges have found wide application throughout the world, especially in European countries.

The concept of supporting the stiffening girders or beams by cables was known long ago but the type of cables and the material with which they were made were very crude. The cables were made out of bamboo sticks and chains. Also the construction techniques were crude.

The renaissance of cable stayed bridges in modern bridge engineering can be traced back to the post war years in Germany. Due to shortage in construction materials, German and other European engineers started investigating ways of obtaining optimum structural performance.

The modern renaissance of cable stayed bridges is considered to have begun in 1955 with the construction of Stromsund bridge in Sweden. The development of cable stayed structures was mainly due to progress in the methods of structural analysis and use of Electronic Computers for exact solutions.

Although most of the cable stayed bridges have been of steel construction with orthotropic decks, some with pre-stressed and precast concrete box girders have also been constructed. The precast, prestressed concrete structures possess better damping capacity and ability to resist vibrations.

The first concrete cable stayed structure constructed was the Tempul Aqueduct, crossing Guadelete River in Spain. However, the first concrete cable stayed bridge on American continent was Morandi's Lake Maracaibo Bridge constructed in 1962 in Venezuela.

In general the development of cable stayed bridges can be attributed to the following factors¹.

- 1) Development of methods of structural analysis of statically indeterminate structures and use of electronic computers.
- 2) Development of post-tensioned, segmental, double cantilever type of construction in case of concrete bridges and orthotropic decks in case of steel structures.
- 3) Experience from previously built bridges
- 4) Development of methods of analysing bridges through model studies.

The significance of cable stayed bridges lies in the utilisation of high strength cables which act as supports for the bridge girder, replacing the piers, thus enabling the girder to span longer distances. A cable stayed bridge is structurally very stiff with very little deformation of its members.

Until recently, segmental cantilever construction without cable stays was considered to be economical in the range of 200-300 feet spans. But with the application of cable stays to prestressed, segmental cantilever construction, the span range has been increased to 500-1600 feet.

So far most of the technical work described in the literature deals with cable stayed bridges with orthotropic steel decks. A method of linear analysis suitable for computer use was proposed by Smith^{2,3} in 1967. The assumption of linear behaviour means that deflection is proportional to the loads acting on the structure. These articles describe the behaviour of single and double plane cable stayed bridges under the action of loads. Troitsky and Lazar⁴ described an experimental procedure to determine the tension forces required in the cable stays in order to reduce the bending moments and displacements of stiffening girders and towers in cable stayed bridges. A detailed analysis and application of reduction method

(Transfer matrix method) to cable stayed bridges was presented by Tang⁵. The effect of axial forces due to tension in cable stays have been included in the analysis. Analysis of cable stayed bridges using stiffness method has been described by Lazar¹¹. This article describes application of stiffness method of analysis taking into account the non linear behaviour. The non linearity in cable stayed bridges is due to large displacements, bending moment-axial force interaction and catenary action of cables. The stiffness matrix for the system is assembled on basis of the deformed geometry of the structure. Each subsequent step 'i' uses data in the previous step i-1. The iteration continues until the last displacement vector obtained is a fraction of the total displacement.

This study is concerned with prestressed, post-tensioned concrete girders erected by segmental cantilever method. Analytical studies are carried out in order to develop methods of predicting initial camber required and deflections in bridge structures erected by segmental cantilever method. The camber can be provided at different stages of construction by using auxiliary and permanent cable stays. The camber can also be provided by adjusting the tension force in cable stays after the structure is completed.

The initial camber needed at each stage of

erection, prediction of final camber to compensate for dead load deflection and tension forces required to produce the necessary camber are to be analysed.

The analysis has to be carried out both in the cantilever erection stage and during the final service stage when the system is continuous.

For a preliminary study, of a given geometric layout, a reasonable prepreliminary design may be obtained by considering that the dead load amounts to approximately 75% to 90% of the total load in long span bridges and that girders are cambered to compensate for dead load deflection.

The main advantage of using precast segmental construction is that the segments can be cast in the casting yard and transported to site for erection, thus reducing creep in the concrete and eliminating elastic shortening and thus shrinkage in the concrete.

It is possible to extend the competitive span range of such structures to dimensions that had previously been considered impossible.

CHAPTER-2

DESCRIPTION OF STRUCTURAL SYSTEM

2.1 PROTOTYPE STRUCTURE :

Theoretical investigations were carried out on the post tensioned segmental bridge structure during the erection and service stages. Comparable investigation was carried out on systems with and without cable-stays, for the same depth of the main girder. The deflections and pre-tensioning forces in the prototype structure are analysed. A typical structure with cable stays on which the study is to be carried out is shown in figure 2.1. The system has a constant depth throughout its entire span. A typical cross section of the concrete girder with prestressing tendons is shown in figure 2.2.

2.2 ARRANGEMENT OF POST TENSIONING TENDONS:

The prestressing tendons required to hold the segments in position during erection, and to provide continuity between the two cantilevers to make the structure continuous can be grouped as follows^{6,7}:

Group 1: Cantilever post tensioning tendons in top slab.

Group 2: Tail span continuity post-tensioning tendons in the web.

- Group 3: a) Centre span continuity post-tensioning tendons in bottom slab and webs.
- b) Centre span continuity post-tensioning tendons located in the top slab.

The different types of arrangement of prestressing tendons in the main girder may be grouped into three common types.

Arrangement I : Straight tendons bent down as shown in Figure 2.3.

Arrangement II : Parabolic tendon profile as shown in Figure 2.4.

Arrangement III : System with straight tendons as shown in Figure 2.5.

2.3 METHOD OF CONSTRUCTION:

The widely used construction method^{7,8} is to assemble precast segments on falsework or the segmental cantilever erection. The great advantage of the cantilever method lies in the fact that the complete erection may be accomplished without any falsework and hence without traffic interruption.

The cable is hinged at connections to girder and towers. Therefore the cable can take up only axial forces, but not the bending moments. The cables stays are assumed to be always under tension.

$$L = 850'$$

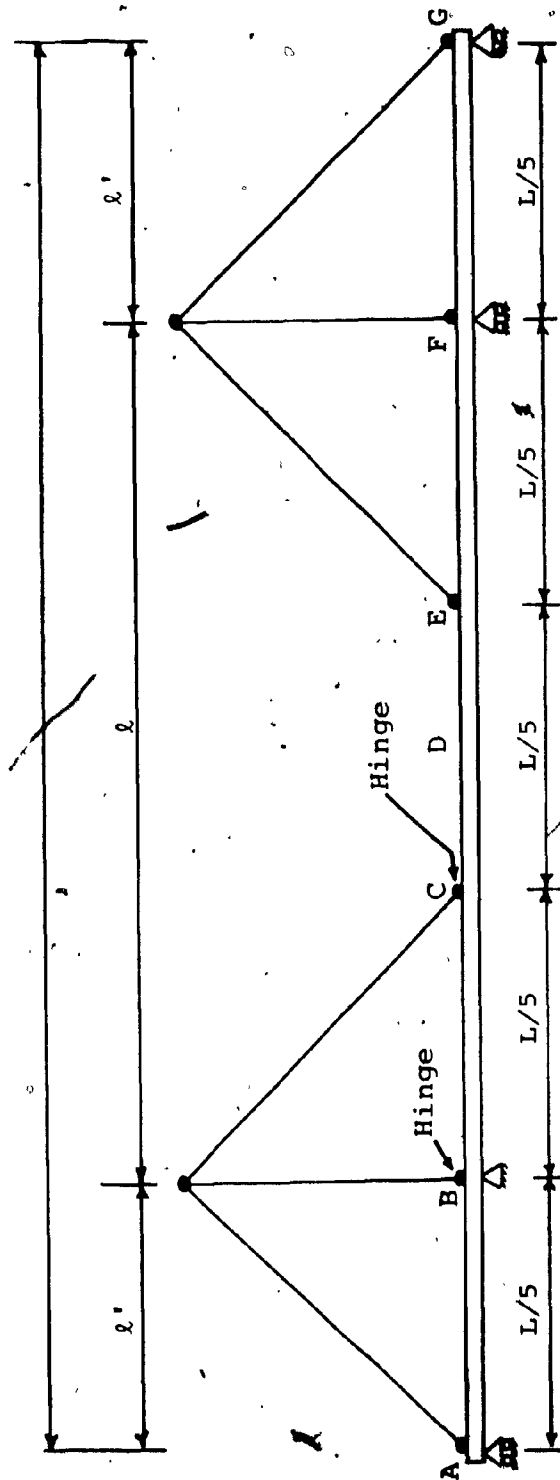
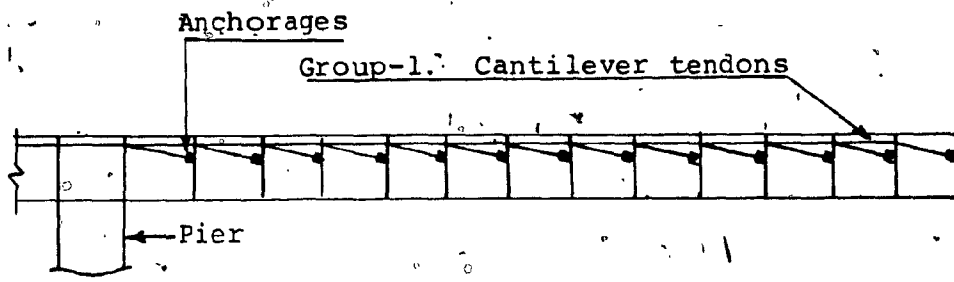
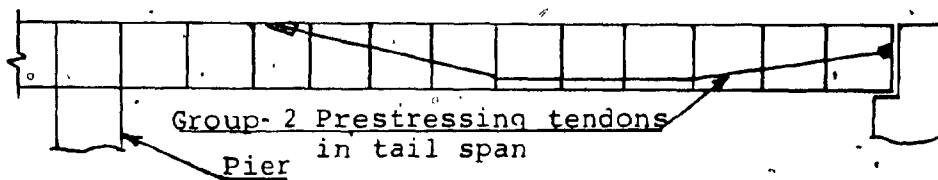


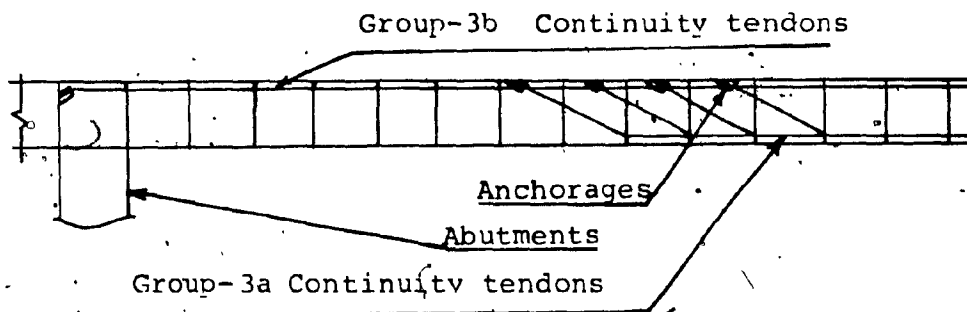
Fig.2.1 Elevation of Cable - Stayed Bridge.



a) Cantilever tendon layout

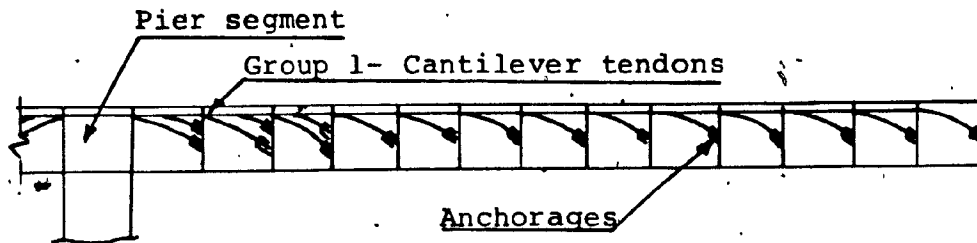


b) Prestressing tendons in the flanking span

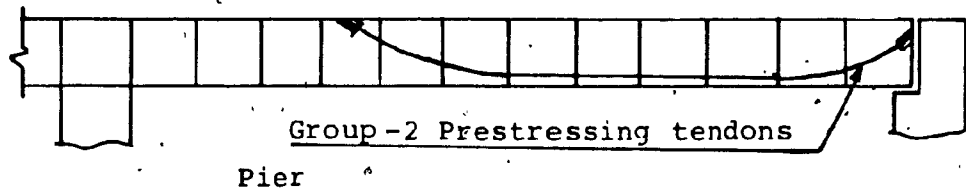


c) Main span continuity tendons

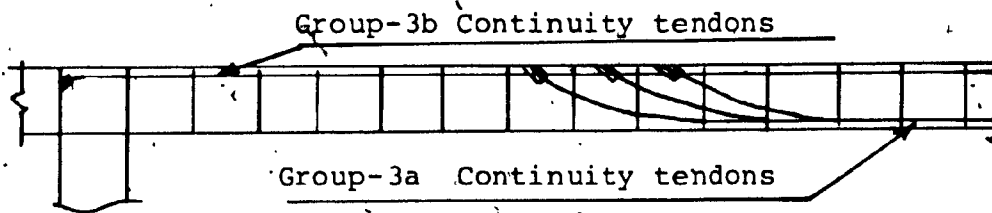
Fig. 2.3 Layout of post tensioning tendons in segmental cantilever construction
Tendon arrangement I.



a) Cantilever tendon layout

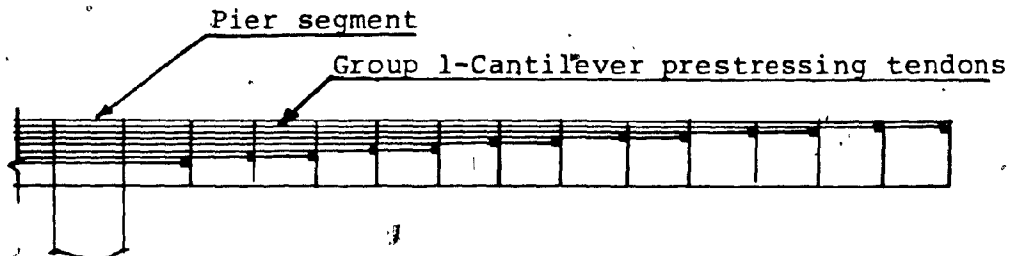


b) Prestressing tendons in flanking span

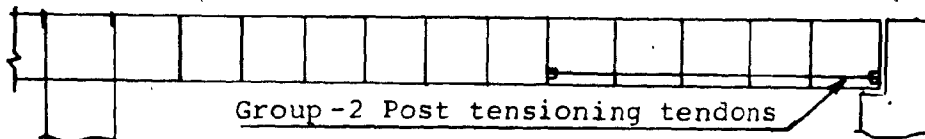


c) Main span continuity tendons

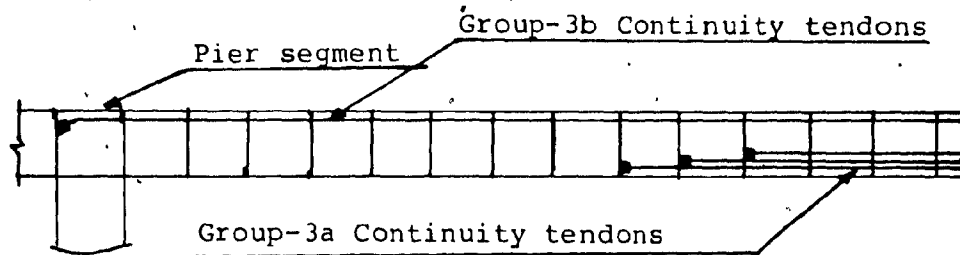
Fig 2.4 Tendon Arrangement II
i.e. parabolic profile.



a) Cantilever tendon layout



b) Prestressing tendons in the flanking span



c) Main span continuity tendon layout

Fig. 2.5 Tendon Arrangement III
System with straight tendons

Note: All tendons are located at the same level.
They are shown at different levels for
clarity.

The assembly of precast segments is carried out in sequential balanced cantilevering from piers towards the span centre lines. Initially the "Hammer Head" is formed by erecting the first segment and attaching it to the pier to provide unbalanced moment capacity. The adjoining segments are then erected on both sides of the pier and post tensioned successively through the pier segment as shown in figure 2.6. Auxiliary and permanent cables are employed to reduce the moments and deflections in the girder.

The stage by stage erection and post tensioning of segments is continued until the cantilever span reaches the centre of main span. In this configuration the span is ready for closure. The steps involved in making the structure continuous are as follows:

- 1) Ensure that the displacements of the two cantilever ends are equal.
- 2) Cast a closure strip.
- 3) Post-tension through the closure strip to ensure continuity.

According to the usual prefabrication methods, a gap exists between the different elements which must be filled with mortar. If the same method was used for bridges erected by cantilevering, the whole advantage of prefabri-

cation would be lost due to waiting time between the erection and stressing of an element to allow for setting time of mortar.

To overcome this drawback "Thickless" joints called "conjugate" or "Match cast" joints are used which ensure a proper fit during erection. These joints are coated with bonding material such as the epoxy resins.

It is important that all the operations such as epoxy application, setting of segment in place and partial tensioning are completed during the pot life of the bonding agent.

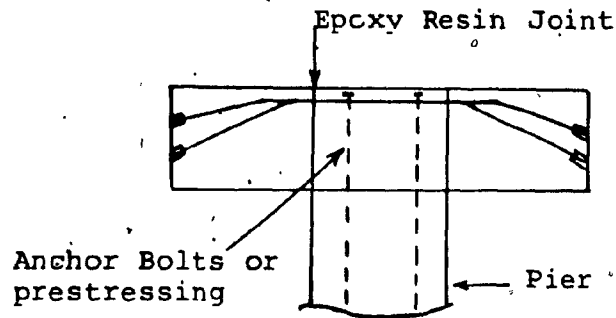
The recommended practice governing the application of bonding agents is specified by the Prestressed Concrete Institute⁶. The epoxy bonding material is supplied to the erection site in sealed containers, preproportioned in the proper reacting ratio. Surfaces to which epoxy material is to be applied should be free from oily materials. All the contaminants are removed by light sand blasting or high pressure water blasting with a minimum pressure of 5000 psi. The wet surface is dried before application of epoxy agent.

Instructions for storage, mixing and handling of the epoxy bonding agent are provided on the container. The epoxy material is applied to all surfaces to be joined within the first half of the gel time as shown on the container.

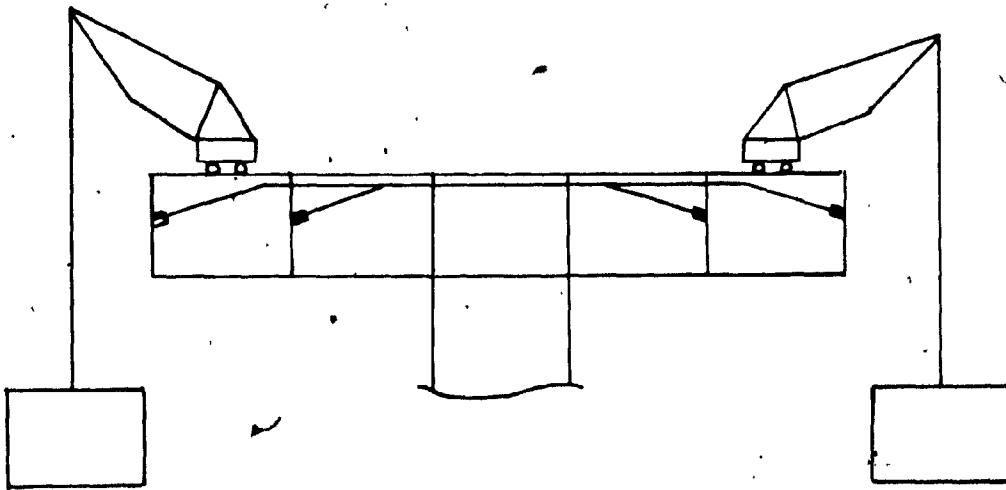
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The segments are joined within 45 minutes after application of the first epoxy material placed and a minimum average temporary prestress of 50 psi over the cross section is applied within 70 percent of open time of epoxy material.

The joints are checked immediately after erection to verify proper fit. Excess epoxy resin is removed.



a) Construction of Hammer Head

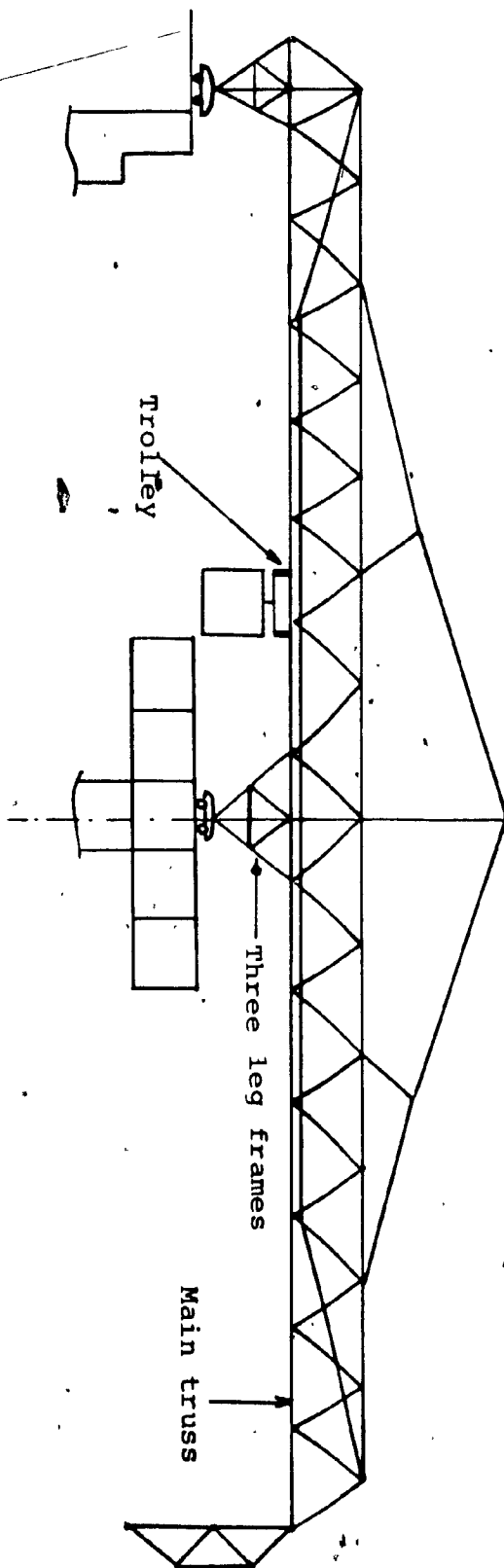


b) Segmental Cantilevering using Mobile Cranes.

Fig.2.6 Erection of Segments.

c) Segmental cantilever construction using launching gantry

Fig.2.6 Continued. Erection of Segments



CHAPTER-3

THEORETICAL ANALYSIS

3.1 BASIC CONCEPTS:

Theoretical analysis was carried out on the prototype bridge structure shown in figure 2.1 at different stages, namely :

- 1) Erection or cantilever stage
- 2) Service or continuous girder stage.

The analysis was also carried out separately for two structural systems namely:

- 1) Structure without cable stays and
- 2) Structure with cable stays.

Analysis during the cantilever stage is simple and can be carried out using basic principles of statics and mechanics of materials. During this stage the structure is statically determinate and represents a fixed cantilever beam.

After the structure is made continuous the analysis becomes more complicated both for systems with and without cable stays. In this stage the structure is eight times statically indeterminate.

The theoretical investigation is based on the assumption that the behaviour of structure is linear under the applied loads, the displacement under loads is small, and proportional to the applied loads. Linear superposition is valid for deflections caused by different load groups. The cable stays are hinged at connections to towers and girders and hence they can take up only axial forces and not bending moments. The cable stays are always under tension. In the first part, the bending moments, axial forces, shear forces and deflections for a system without cable stays are determined. In the second part, tension forces in cable stays required to reduce these displacements, the initial camber and final displacements are calculated.

3.2 ERECTION STAGE:

In the erection stage the system is treated as a double cantilever. The forces to be considered are the dead load of the girder, the weight of moving equipment and the prestressing forces in main girder. In cable stayed bridges the tension forces in the cable stays also have to be taken in to account.

3.2.1. SYSTEM WITHOUT CABLE STAYS:

Deflection in a system without cable stays can be calculated^{9,10} by superposition of dead load deflection, deflection due to moving equipment and deflection due to prestressing forces in the girder. These deflections may be calculated separately using principles such as virtual work or moment area method.

3.2.1.1. ERECTION MOMENTS AND DEFLECTION DUE TO

OWN WEIGHT:

During erection, the bending moments over the pier increase with the addition of each pair of segments⁶ as shown in figure 3.1. The increase of moment caused by addition of the seventh segment at the cantilever end is shown as an example by the shaded area in figure 3.1. These moments are resisted by the post-tensioning tendons provided in the top slab which may be anchored at the face of the segments or in buildouts inside the box sections.

The use of buildouts makes it possible to place the segments and stress the tendons in two separate operations, but tends to complicate the manufacturing of segments.

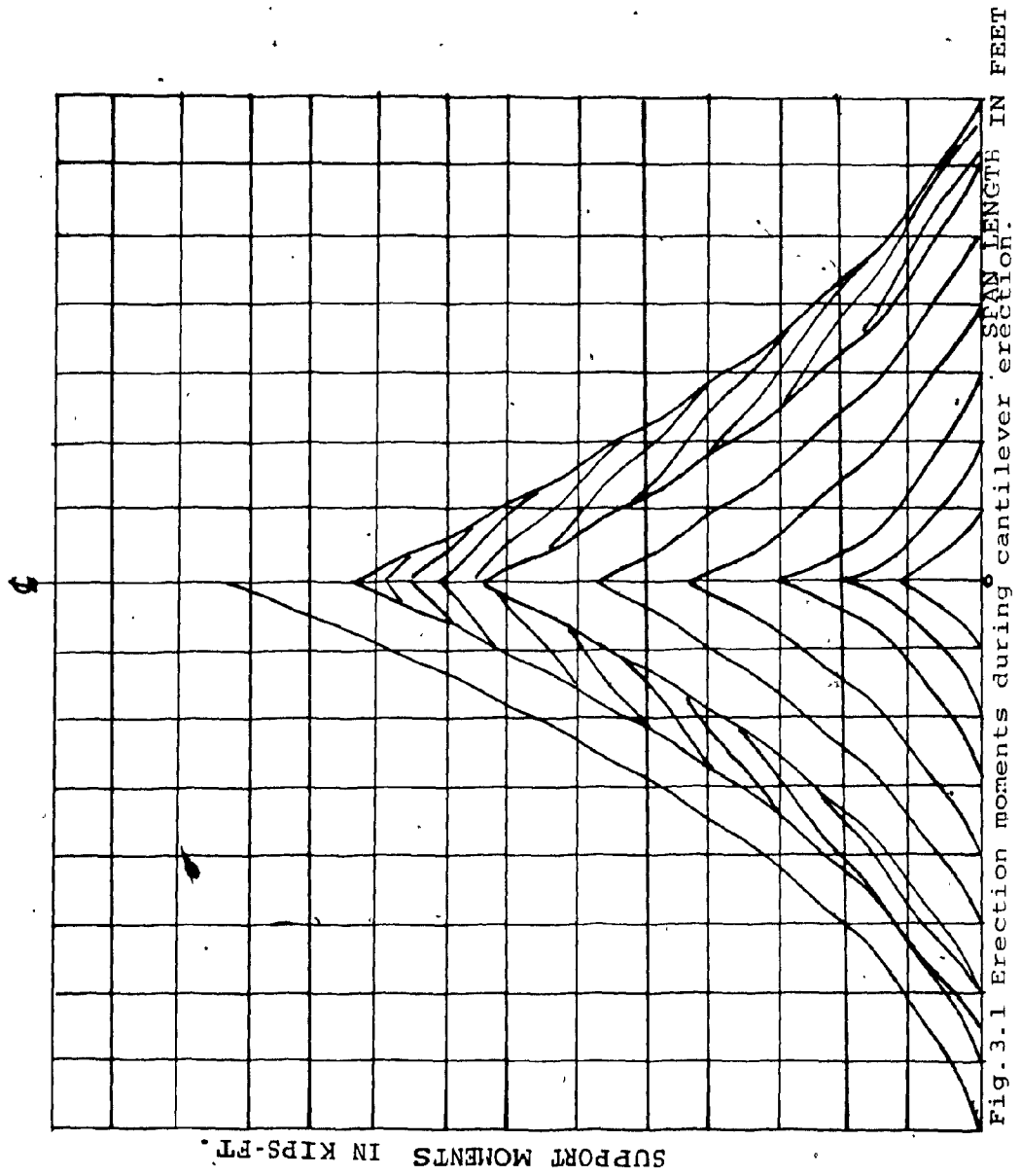


Fig. 3.1 Erection moments during cantilever erection.

If a section of constant section is considered, then the deflection at the free end of the cantilever due to its own weight is given by equation (3.1).

$$\Delta_D = \frac{wa^4}{8EI} \quad (3.1)$$

If $a=nb$, the deflection at the free end of the cantilever due to its own weight is given by equation (3.2).

$$\Delta_D = \frac{w(nb)^4}{8EI} \quad (3.2)$$

When the erection of the cantilever reaches full span length "a", the deflection at any intermediate point can be calculated using equation (3.3).

$$\Delta_x = \frac{w}{24EI} (x^4 - 4a^3x - 3a^4) \quad (3.3)$$

Where x = distance from free end of cantilever.

When the erection of the cantilever span has reached the point of cable stay attachment only, then the deflection at that point is :

$$\Delta_C = \frac{w(rb)^4}{8EI} \quad (3.4)$$

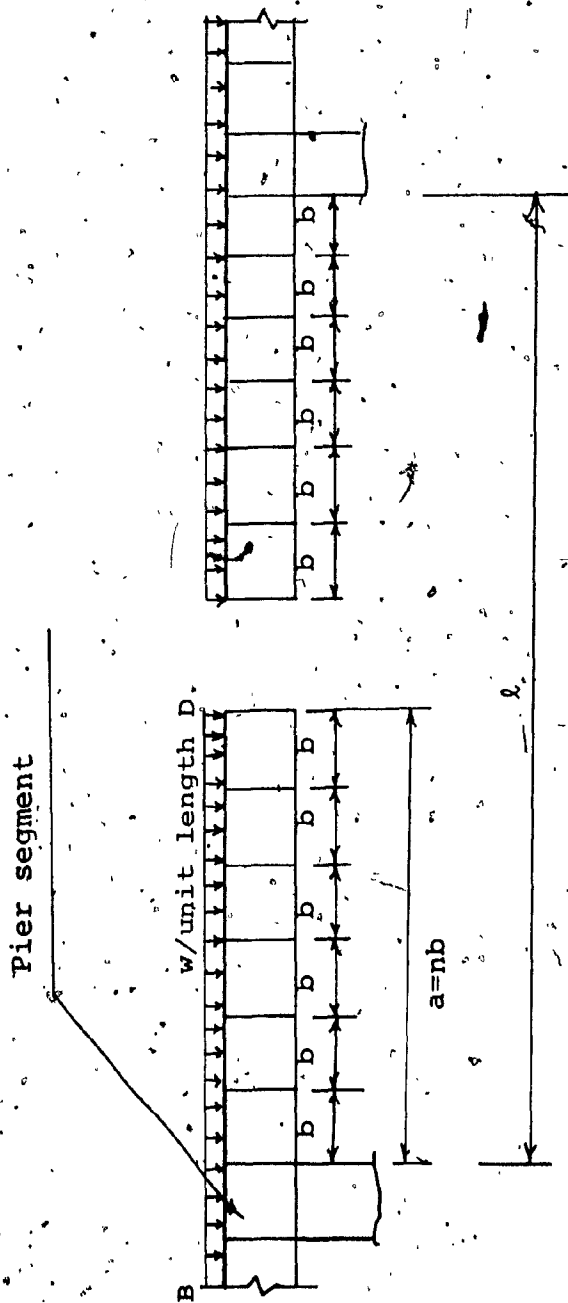


Fig.3.2 Cantilever spans loaded with own weight.

3.2.1.2: DEFLECTION OF CANTILEVER DUE TO MOVING

EQUIPMENT:

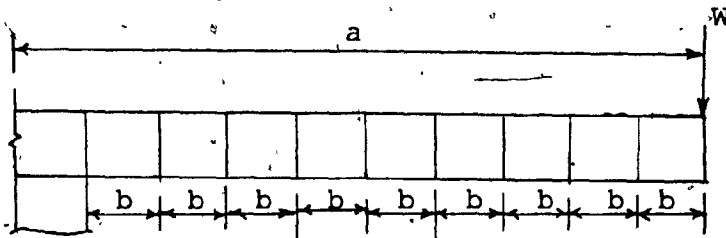


Fig. 3.3. Cantilever main span acted upon by load of construction equipment.

The maximum deflection in the cantilever occurs when the load is placed at the free end and can be calculated as follows:

$$\Delta_{\text{conc. load}} = \frac{Wa^3}{3EI} \quad (3.5)$$

At different stages of erection, the deflection at the free end is given by equation (3.6).

$$\Delta_{\text{conc. load}} = \frac{W(nb)^3}{3EI} \quad (3.6)$$

When the span length reaches the full cantilever span "a", the deflection at any point distant 'x' from the free end is given by equation (3.7).

$$\Delta_x = \frac{W}{6EI} (2a^3 - 3a^2x + x^3) \quad (3.7)$$

The deflection at the point of cable attachment 'C' when the erection reaches full span 'a' is given by

$$\begin{aligned}\Delta_C &= \frac{W}{6EI} [2a^3 - 3a^2(\frac{1}{3}a)^3] \\ &= \frac{W}{6EI} (2a^3 - a^3 + \frac{a^3}{27}) \\ &= \frac{Wa^3}{162EI} (28) = \frac{14Wa^3}{81EI} \quad (3.8)\end{aligned}$$

Deflection of the free end under a moving concentrated load, when the load is at any intermediate location is

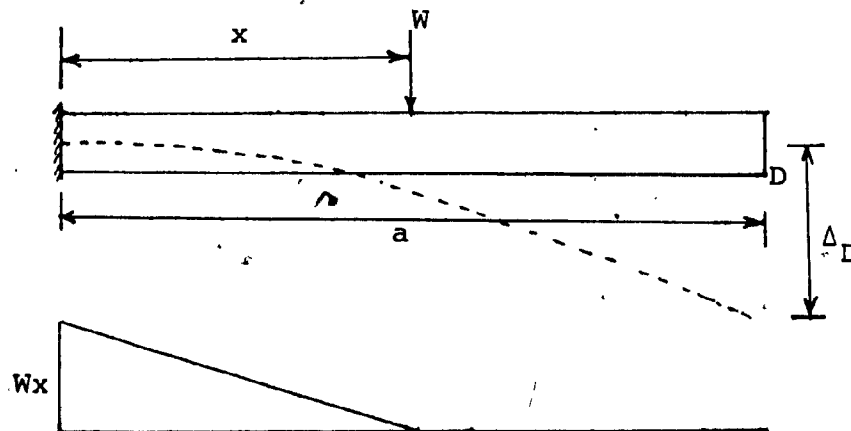


Fig.3.4 Bending moment Diagram due to concentrated load of equipment.

defined by equation (3.9),

$$\Delta_D = \left(\frac{1}{2} \cdot \frac{Wx \cdot x}{EI} \right) \left(a - \frac{1}{3}x \right) = \frac{Wx^2}{6EI} (3a - x) \quad (3.9)$$

where x = distance from the fixed end.

If we substitute $x=a$, we obtain the deflection at the free end when the load is placed at the point.

3.2.1.3. DEFLECTION DUE TO PRESTRESSING:

The arrangement of prestressing tendons considered is shown in figures 2.3 to 2.5. During the cantilever construction stage only group-1 cantilever tendons are provided. Each of the segments erected are post tensioned successively, either partially or in full.

Prestressing the tendons with a force 'F' placed with an eccentricity 'e' causes a moment 'M', where

$$M = Fe$$

The prestressing moments act in a direction opposite to the dead load moments. Hence the effect of prestressing is to reduce tensile stresses in concrete, to introduce negative deflection and thus to reduce the final bending moments in the structure.

If F_1, F_2, F_3 etc. are the successive prestressing forces applied to each of the segments, then

Prestressing force at the support after stressing each of the segments is

$$F_t = F_1 + F_2 + F_3 + \text{-----} + F_{n-1} + F_n = \sum_{i=1}^n F_i$$

Where,

F_1 = Prestressing force in first tendon

F_2 = Prestressing force in second tendon

F_n = Prestressing force in the n^{th} tendon

Total prestressing force at the support when the first segment is erected, $F_t = F_1$

Total prestressing force at the support when the second segment is erected, $F_t = F_1 + F_2$

Total prestressing force at the support when the n^{th} segment is erected is

$$F_t = F_1 + F_2 + \text{-----} + F_n = \sum_{i=1}^n F_i \quad (3.10)$$

The moment at support when the first segment is erected is $M_1 = F_1 e_1$

Moment at the support when second segment is erected is

$$M_2 = F_1 e_1 + F_2 e_2$$

Moment at the support when n^{th} segment is erected

$$M_n = F_1 e_1 + F_2 e_2 + \text{-----} + F_n e_n = \sum_{i=1}^n F_i e_i \quad (3.11)$$

In case when $F_1 = F_2 = F_n$ and all segments are of equal length, then the moment diagram will be a straight line as shown in fig.3.5.

The moment at any section distant 'x' from the free end is given by

$$M_x = \sum F_n e_n \cdot \frac{x}{a} \quad (3.12)$$

To find the deflection at the free end we can used the moment area method.

$$\begin{aligned} \Delta_D &= \int_0^a \frac{M_x}{EI} (x) dx = \int_0^a \frac{\sum F_n e_n}{EI} \left(\frac{x}{a} \right) x dx \\ &= \frac{\sum F_n e_n}{EI \cdot a} \left[\frac{(x^3)}{3} \right]_0^a = \frac{\sum F_n e_n \cdot a^2}{3EI} \end{aligned}$$

$$(\Delta_D)_{C.P} = \frac{\sum F_n e_n}{3EI} (nb)^2 \quad (3.13)$$

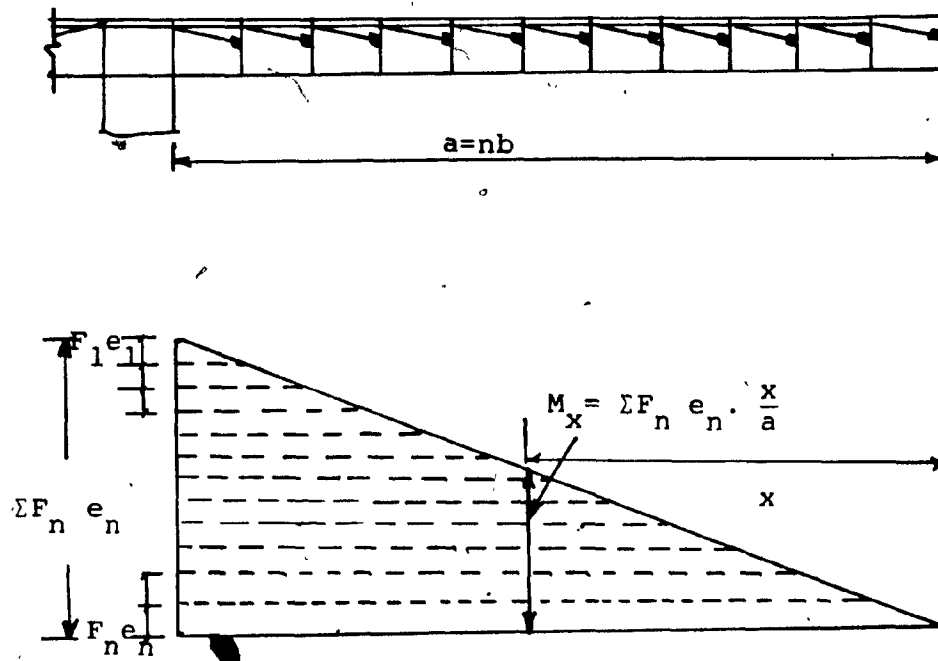


Fig.3.5 Bending Moment Diagram due to Tendon arrangement I.

The final deflection at the free end of the cantilever can be found as a sum of

$$\begin{aligned}\Delta_{\text{total}} &= \Delta_{\text{D.L}} + \Delta_{\text{Conc}} + \Delta_{\text{C.P}} \\ &= \frac{w(nb)^4}{8EI} + \frac{w(nb)^3}{3EI} - \frac{\sum F_n e_n (nb)^2}{3EI}\end{aligned}\quad (3.14)$$

For large spans, it is desirable to introduce parabolic prestress to reduce the shear force in the girder. In case of parabolic pre-stressing, the moment over the supporting piers due to prestressing of successive segments increases gradually as shown in figure 3.6. Moment at any section along the girder is assumed to vary parabolically by proper arrangement of tendon profile or varying the strength of individual tendons.

Let the ordinate of moment diagram at any section be given by equation (3.15).

$$y = k x^2 \quad (3.15)$$

$$\text{at } x = a, \quad y = \sum F_n e_n$$

$$\sum F_n e_n = k a^2, \quad k = \frac{\sum F_n e_n}{a^2} \quad (3.16)$$

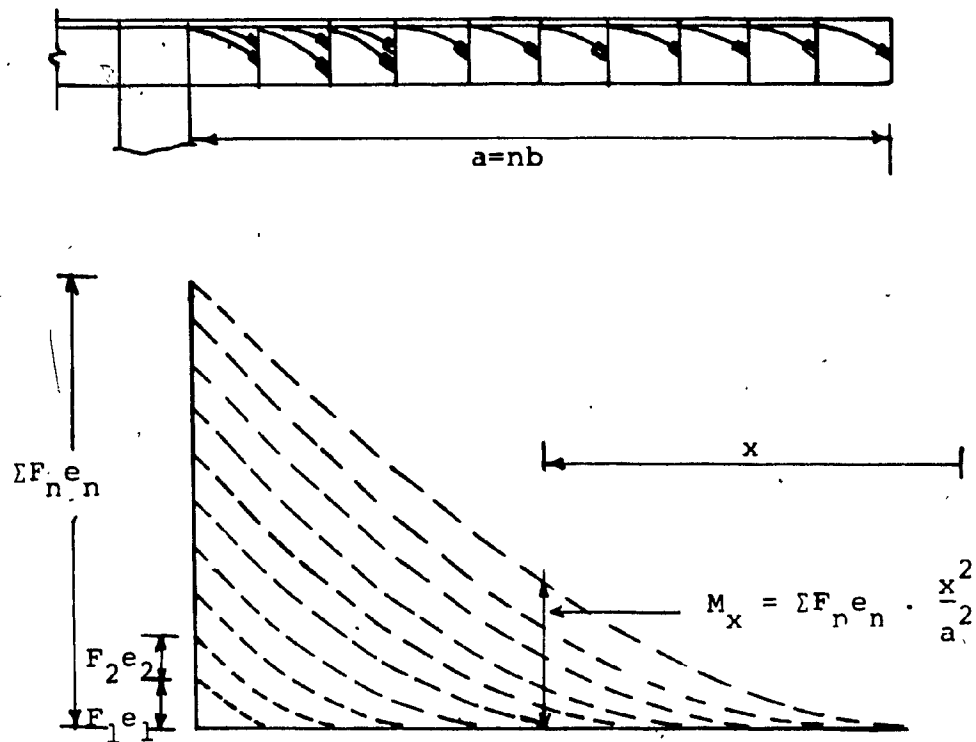


Fig. 3.6. Bending moment diagram due to Tendon arrangement II.

$$y = \frac{\sum F_n e_n}{a^2} x^2$$

$$M_x = \frac{\sum F_n \cdot e_n}{a^2} x^2 \quad (3.17)$$

The deflection at the free end of the cantilever due to prestressing can be found by using moment area method as follows:

$$(\Delta_D)_{C.P} = \int_0^a \frac{M_x}{EI} (x \, dx) = \int_0^a \frac{\sum F_n e_n}{EI} \left(\frac{x^2}{2}\right) (x) \, dx \quad (3.18)$$

$$= \frac{\sum F_n e_n}{a^2 EI} \left(\frac{a^4}{4}\right)$$

$$(\Delta_D)_{C.P} = \frac{a^2 \sum F_n e_n}{4EI} \quad (3.19)$$

The deflection due to prestress during different stages of erection can be calculated using

$$(\Delta_D)_{C.P} = \frac{\sum F_n e_n}{4EI} \cdot (nb)^2 \quad (3.20)$$

For example, when the span length reaches the point of cable attachment 'C', the deflection at that point can be calculated as follows:

$$(\Delta_C)_{C.P} = \frac{\sum F_r e_r}{4EI} (rb)^2 \quad (3.21)$$

$(\Delta_C)_{C.P}$ = Deflection at 'C' due to cantilever
prestress when erection has reached point 'C'.

ΣF_r = Total prestressing force at support for 'r'
segments

e_r = Eccentricity of prestressing tendons

$$\begin{aligned} \Delta_{total} &= \Delta_{D.L} + \Delta_{Conc} + \Delta_{C.P} \\ &= \frac{w(nb)^4}{8EI} + \frac{W(nb)^3}{3EI} - \frac{\Sigma F_n e_n}{4EI} (nb)^2 \end{aligned} \quad (3.22)$$

For a system with straight tendon profile,
moment over the support is given by equation (3.11).

The deflection at the free end is calculated
as follows:

$$\begin{aligned} (\Delta_D)_{C.P} &= [F_1 e_1 \cdot b(a - \frac{b}{2}) + F_2 e_2 \cdot 2b(a - 2b/2) \\ &\quad + \dots + F_n e_n \cdot nb(a - \frac{nb}{2})] \frac{1}{EI} \end{aligned} \quad (3.23)$$

$$(\Delta_D)_{C.P} = \frac{\Sigma_{i=1}^n F_i e_i}{EI} \cdot nb \left(a - \frac{nb}{2} \right) \quad (3.24)$$

$$\Delta_{total} = \Delta_{D.L} + \Delta_{CONC} + \Delta_{C.P}$$

$$\Delta_{total} = \frac{w(nb)^4}{8EI} + \frac{W(nb)^3}{3EI} - \frac{\Sigma F_n e_n \cdot nb(a - nb/2)}{EI} \quad (3.25)$$

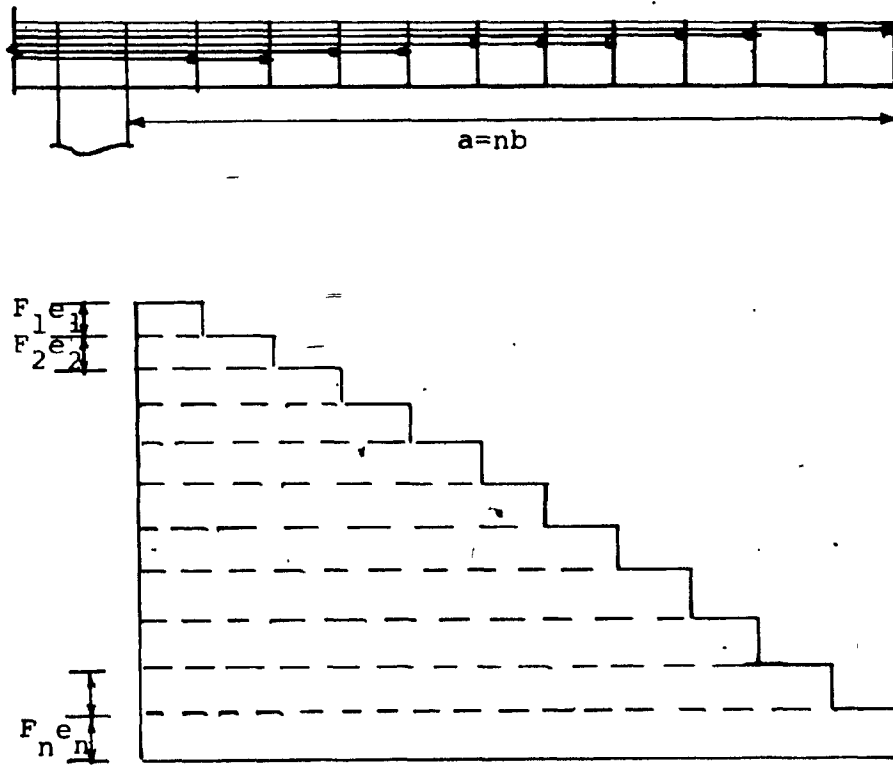


Fig. 3.7 Bending Moment Diagram due to Tendon arrangement III

Note : It is assumed that in practice all tendons are located at the same level. Only for picture clarity each tendon is represented by separate lines.

3.2.2 DEFLECTION CONTROL USING TEMPORARY

CABLE STAYS:

With increasing spans during segmental construction by the cantilever method, deformations of the main girder may reach considerable proportions. Therefore for large spans, auxiliary cable stays are used to reduce these deformations within allowable limits.

Consider the first temporary cable installed at a distance $(n_1 b)$ from the supporting pier. The forces acting on the system is shown in figure 3.8. The vertical component of cable force ' V_1 ' should always be less than the reaction at B.

The total dead load deflection at the point of cable stay attachment, when the cantilever span is complete can be found by assuming a suitable shape function.

The shape function¹⁹ for a cantilever beam is given by equation (3.26).

$$\psi(x) = 1 - \cos \frac{n_x \pi b}{2a} \quad (3.26)$$

The deflection at any section distant ' x ' from the fixed end is given by equation (3.27).

$$\Delta_x = \psi(x) \cdot \Delta_{\text{total}} \quad (3.27)$$

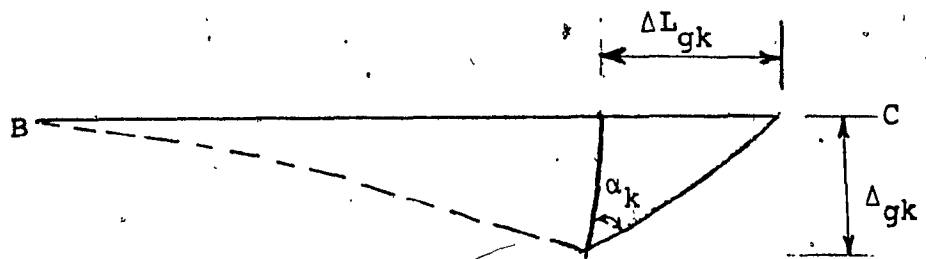
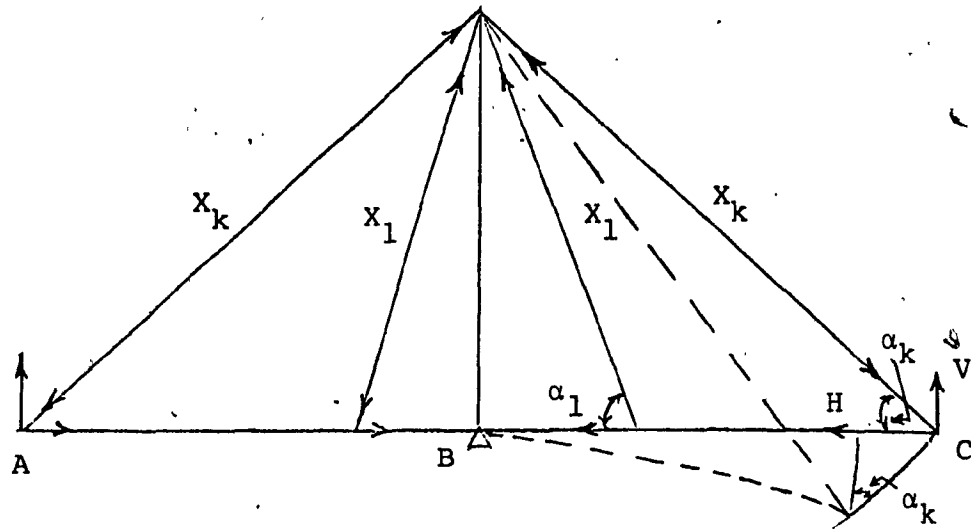


Fig.3.8 Forces due to cable stay tension acting on the system.

The deflection of the girder at a distance $(n_1 b)$ from the fixed end is given by equation (3.28).

$$\Delta_{n_1} = \left(1 - \cos \frac{n_1 \pi b}{2a}\right) \left[\frac{w(nb)^4}{8EI} + \frac{W(nb)^3}{3EI} - \Delta_{C.P} \right] \quad (3.28)$$

The required initial camber can be provided using auxiliary cable stays at intermediate locations during erection. The tension forces in these cable can be adjusted at any time, if necessary.

The vertical component of tension force in the first auxiliary cable stay is

$$V_1 = X_1 \sin \alpha_1 \quad (3.29)$$

The horizontal component of tension force in the first cable stay is

$$H_1 = X_1 \cos \alpha_1 \quad (3.30)$$

The horizontal component of the cable stay force causes elastic shortening of the girder. Due to this shortening the cable stay and the girder move down to the new position as shown in figure 3.8. Since these deformations are small, the arcs along which they deform are approximated as straight lines.

The shortening of the girder due to tension force X_1 in the first cable stay is

$$\Delta L_{g1} = \frac{H_1(n_1 b)}{AE} = \frac{X_1 \cos \alpha_1 (n_1 b)}{AE} \quad (3.31)$$

The deflection due to this elastic shortening is calculated from figure 3.8.

$$\tan \alpha_1 = \frac{\Delta L_{g1}}{\Delta_{g1}} \quad (3.32)$$

$$\begin{aligned} \Delta_{g1} &= \frac{\Delta L_{g1}}{\tan \alpha_1} = \frac{X_1 (n_1 b) \cos \alpha_1 \cot \alpha_1}{AE} \\ &= \frac{V_1 (n_1 b)}{3EI} \cot^2 \alpha_1 \end{aligned} \quad (3.33)$$

The upward deflection due to tension force in the first temporary cable stay is

$$\Delta_{X_1} = \frac{V_1 (n_1 b)^3}{3EI} = \frac{X_1 \sin \alpha_1 (n_1 b)^3}{3EI} \quad (3.34)$$

The cable stays stretch elastically due to the tension force. The deflection due to elongation of cable stays is calculated from figure 3.9.

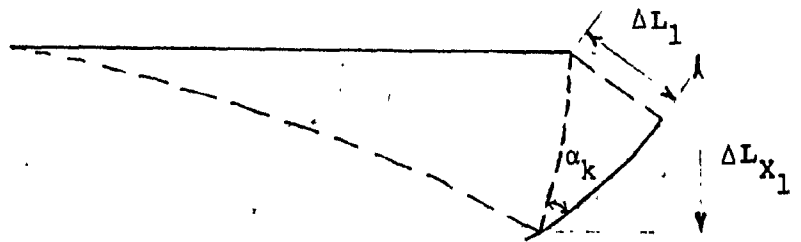
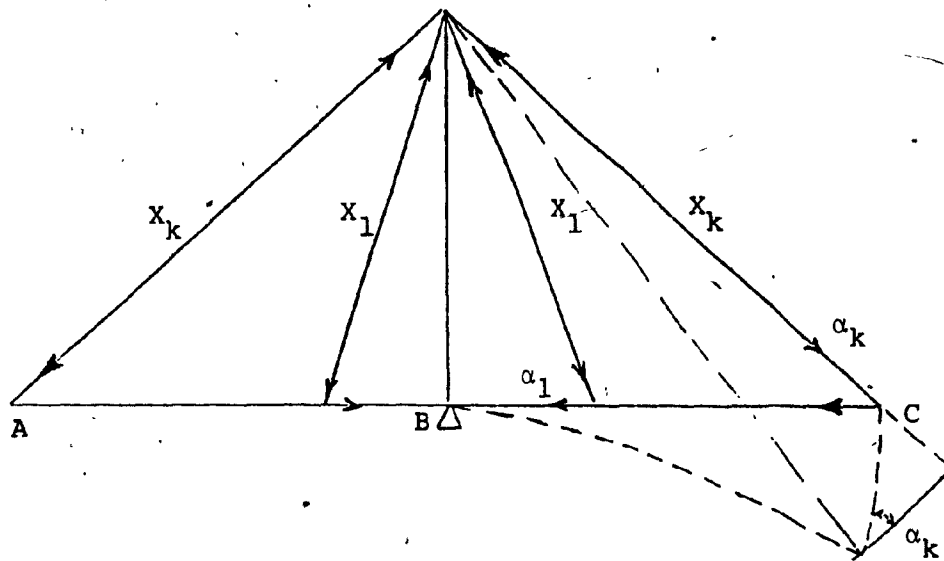


Fig.3.9 Deflection due to elongation of cable stay.

Extension of the cable stay due to 'X₁' is

$$\Delta L_1 = \frac{X_1 L_1}{A_1 E_e} \quad (3.35)$$

Deflection of the girder at the cable stay point is

$$\Delta L_{X_1} = \frac{X_1 L_1}{A_1 E_e \sin \alpha_1} \quad (3.36)$$

Axial shortening of the tower is

$$\Delta L_T = \frac{2X_1 \sin \alpha_1 (H_T)}{A_T E_T} \quad (3.37)$$

Deflection of girder at the cable point due to shortening of the tower is

$$\Delta_T = \frac{2X_1 \sin \alpha_1 (H_T)}{A_T E_T} \quad (3.38)$$

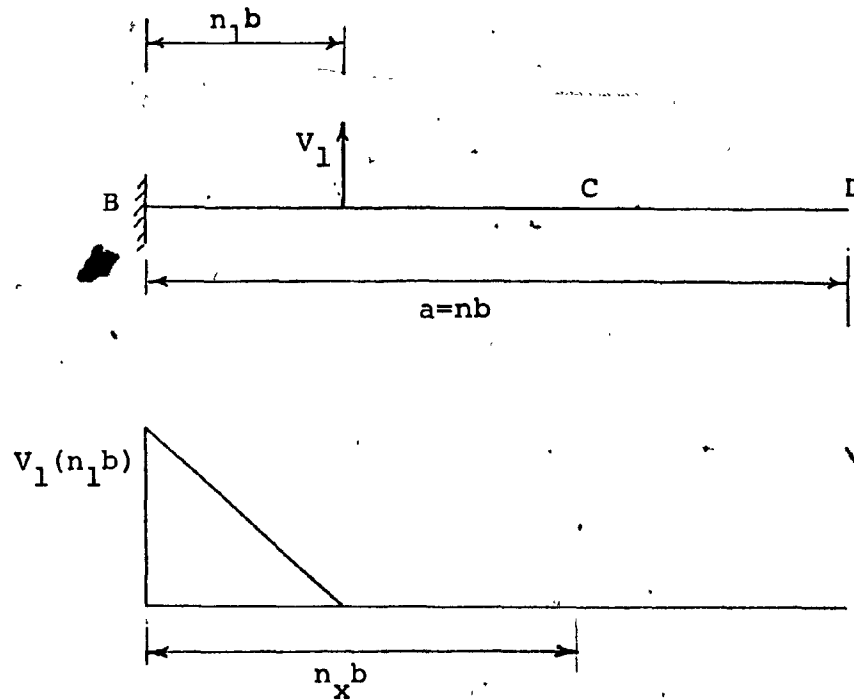


Fig.3.10 Bending moment diagram due to cable stay force.

Deflection at any section ($n_x b > n_1 b$) due to ' V_1 ' is given by equation(3.39).

$$\begin{aligned} \Delta n_x &= \frac{1}{2} V_1 (n_1 b)^2 \left(\frac{2}{3} n_1 b + n_x b - n_1 b \right) \\ &= \frac{V_1}{3EI} (n_1 b)^2 \cdot \frac{1}{2} (3n_x - 1) \end{aligned} \quad (3.39)$$

The tension force ' X_1 ' is calculated by equating the deflection due to total loads at the final stage to the total deflection due to the tension force in cable stay.

$$X_1 \left[\frac{\sin \alpha_1 (n_1 b)^3}{3EI} - \frac{\cos \alpha_1 \cot \alpha_1 (n_1 b)}{AE} - \frac{L_1}{A_1 E_e \sin \alpha_1} - \frac{2 \sin \alpha_r (H_T)}{A_T E_T} \right] \\ = \left(1 - \cos \frac{n_1 \eta b}{2a} \right) \left(\frac{w(nb)^4}{8EI} + \frac{W(nb)^3}{3EI} - \Delta_{C.P} \right) \quad (3.40)$$

Let

$$B_1 = \left[\frac{\sin \alpha_1 (n_1 b)^3}{3EI} - \frac{\cos \alpha_1 \cot \alpha_1 (n_1 b)}{AE} - \frac{L_1}{A_1 E_e \sin \alpha_1} - \frac{2 \sin \alpha_r (H_T)}{A_T E_T} \right] \quad (3.41)$$

$$D_n = \frac{w(nb)^4}{8EI} + \frac{W(nb)^3}{3EI} - \Delta_{C.P} \quad (3.42)$$

$$X_1 B_1 = D_n \left(1 - \cos \frac{n_1 \eta b}{2a} \right) \\ X_1 = \frac{D_n}{B_1} \left(1 - \cos \frac{n_1 \eta b}{2a} \right) \quad (3.43)$$

Once the required tension force ' X_1 ' has been calculated, the initial camber to be provided can be calculated using equation (3.45).

$$\text{Let } D_x = \left[\frac{w(n_x b)^4}{8EI} + \frac{W(n_x b)^3}{3EI} - \Delta_{C.P} \text{ up to 'x'} \right] \quad (3.44)$$

$$\Delta_x = D_x \left(1 - \cos \frac{n_x b}{2a} \right) - X_1 B_1 \cdot \frac{1}{2} \left(\frac{3n_x}{n_1} - 1 \right) \quad (3.45)$$

Similarly, the deflection at the k^{th} auxiliary cable due to tension force in all the cable stays provided up to that point can be calculated as follows.

$$\begin{aligned} \Delta_{x_k} = & X_k \left[\frac{(n_k b)^3 \sin \alpha_k}{3EI} - \frac{\cos \alpha_k \cot \alpha_k (n_k b)}{AE} - \frac{L_k}{A_k E \sin \alpha_k} \right. \\ & - \frac{2 \sin \alpha_k (H_T)}{A_T E_T} \left. \right] + \sum_{i=2}^k X_{i-1} \left[\frac{(n_{i-1} b)^3 \sin \alpha_{i-1}}{3EI} \right. \\ & - \frac{\cos \alpha_{i-1} \cot \alpha_{i-1} (n_{i-1} b)}{AE} - \frac{L_{i-1}}{A_{i-1} E \sin \alpha_{i-1}} \\ & - \frac{2 \sin \alpha_{i-1} (H_T)}{A_T E_T} \left. \right] + \frac{1}{2} \left(\frac{3n_k}{n_{i-1}} - 1 \right) \\ = & X_k B_k + \sum_{i=2}^k (X_{i-1} B_{i-1}) \frac{1}{2} \left(\frac{3n_k}{n_{i-1}} - 1 \right) \quad (3.46) \end{aligned}$$

The pretensioning force in the k^{th} temporary cable is calculated as follows.

$$X_k B_k + \sum_{i=2}^k (X_{i-1} B_{i-1}) \frac{1}{2} \left(\frac{3n_k}{n_{i-1}} - 1 \right) = D_n \left(1 - \cos \frac{n_k \pi b}{2a} \right)$$

$$X_k = \frac{D_n \left(1 - \cos \frac{n_k \pi b}{2a} \right) - \sum_{i=2}^k (X_{i-1} B_{i-1}) \frac{1}{2} \left(\frac{3n_k}{n_{i-1}} - 1 \right)}{B_k} \quad (3.47)$$

The required initial camber at or after the k^{th} cable stay point can be calculated using equation (3.48).

$$\Delta_x = D_x \left(1 - \cos \frac{n_x \pi b}{2a} \right) - \sum_{i=1}^k (X_i B_i) \frac{1}{2} \left(\frac{3n_x}{n_i} - 1 \right) \quad (3.48)$$

Where

X_{i-1} = Required force in $(i-1)^{th}$ temporary cable stay.

n_{i-1} = Number of segments up to $(i-1)^{th}$ temporary cable stay point.

A_{i-1} = Area of cross section of $(i-1)^{th}$ temporary cable stay.

α_{i-1} = Angle of inclination of $(i-1)^{th}$ temporary cable stay with the horizontal.

The pretensioning forces in the cable stays required to produce the necessary initial camber is calculated using equations 3.47 and 3.48. The erection is continued further until the point of attachment of permanent cable stay is reached. At this stage, permanent cable stays have to be provided and all the temporary cables removed, so that the entire load is transferred to these cables.

In the equations derived above, the deflection due to cantilever prestressing should be substituted with equations (3.49) to (3.54).

I. Arrangement: straight tendons bent down

$$\Delta_{C,P} = \frac{\sum F_n e_n}{3EI} (nb)^2 \quad \text{and} \quad (3.49)$$

$$\Delta_{C.P. \text{ up to 'x'}} = \frac{\sum F_x e_x (n_x b)^2}{3EI} \quad (3.50)$$

II Arrangement: parabolic tendon profile

$$\Delta_{C.P.} = \frac{\sum F_n e_n (nb)^2}{4EI} \quad \text{and} \quad (3.51)$$

$$\Delta_{C.P. \text{ up to 'x'}} = \frac{\sum F_x e_x (n_x b)^2}{4EI} \quad (3.52)$$

III Arrangement: straight tendons

$$\Delta_{C.P.} = \frac{\sum F_n e_n}{EI} \cdot nb (a - nb/2) \quad \text{and} \quad (3.53)$$

$$\Delta_{C.P. \text{ up to 'x'}} = \frac{\sum F_x e_x}{EI} \cdot n_x b \left(a - \frac{n_x b}{2} \right) \quad (3.54)$$

3.2.3 DEFLECTION CONTROL USING PERMANENT

CABLE STAYS:

When the required span length has been erected, permanent cables stays are provided to transfer the loads to the tower. After the permanent cables are attached, all the temporary cables are removed and the entire load is now transferred only by the permanent cable stays.

Initially when the erection has reached point 'C' and permanent stays are installed, there will not be any negative moments in the girder. But as the erection is continued further, the girder is subjected to negative moments at the cable stay connections. Hence it is desirable to use a girder of constant section.

If the spans on either side of the tower are different, the dead load and hence the prestressing forces will be different. A hinge is provided at the base of the tower so that, the tower rotates about the hinge. But when the cantilever erection reaches the permanent cable points, the end 'A' of the flanking span is pinned down, thus eliminating the rotation of the tower.

$$\begin{aligned} (\Delta_c)_{\text{rotation of tower}} &= \phi_n \cdot BC = 0 \end{aligned} \quad (3.55)$$

Considering zero rotation of towers the expression for the required tension force in cable stay is given by equation (3.56).

$$X_r = \frac{D_n (1 - \cos \frac{r b}{2a})}{B_r} \quad (3.56)$$

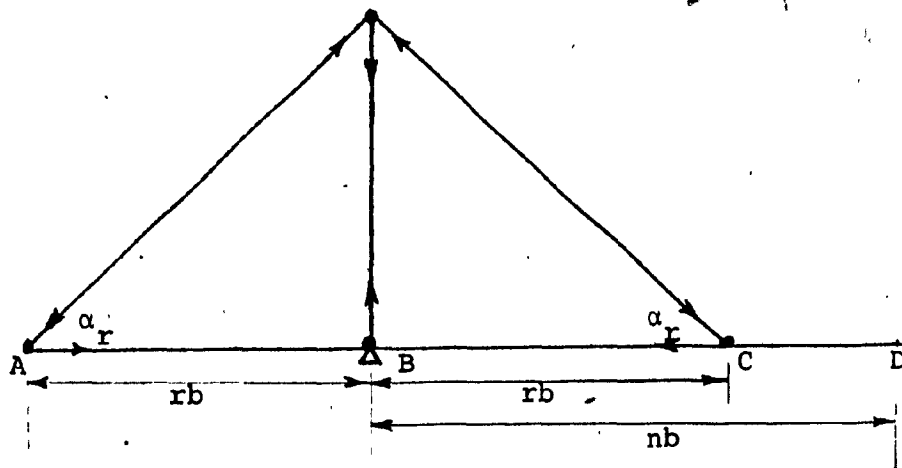


Fig. 3.11 Forces acting in a system with permanent cable stays.

$$B_r = \left[\frac{\sin \alpha_r (rb)^3}{3EI} - \frac{\cos \alpha_r \cot \alpha_r (rb)}{AE} - \frac{L_r}{A_r E_e \sin \alpha_r} - \frac{2 \sin \alpha_r (H_T)}{A_T E_T} \right] \quad (3.57)$$

and D_n is given by equation (3.42).

The subscript 'r' denotes the section and geometric properties for permanent cable stays. If a number of permanent cable stays are used then the pre-tensioning force can be calculated using equation (3.47).

The required camber to be provided at the cable point can be calculated using equation (3.56).

$$\Delta_x = D_x \left(1 - \cos \frac{n \pi b}{2a}\right) - (X_r B_r) \frac{1}{2} \left(\frac{3n}{r} - 1\right) \quad (3.58)$$

Where D_x and B_x are given by equations (3.44) and (3.57) respectively.

If a number of permanent cables are provided then the required initial camber can be calculated using equation (3.48).

3.3 EQUIVALENT MODULUS OF ELASTICITY:

The cable stays provided in bridge structures which are pre-tensioned, present a great problem because of their non linear behaviour^{1,17} and change in sag under load. Displacement of ends of the free hanging cable under an axial load depends not only on the cross sectional area, but to a certain extent of sag of the cable. One of the proposed methods for considering this non-linear behaviour is to substitute for the modulus of elasticity of the curved cable member, an equivalent modulus of elasticity

which is compatible with that of the curved member, considering material as well as geometric elongation.

The equivalent modulus of elasticity¹⁷ can be taken as equal to

$$E_e = \frac{E}{1 + \left[\frac{(w_e l)^2 AE}{12T^3} \right]} \quad (3.59)$$

Where, E_e = Equivalent cable modulus.

E = Modulus of cable material.

w_e = Weight per unit length of the cable.

A = Cross-sectional area of the cable.

T = Cable tension.

As seen from the Equation (3.59) the equivalent modulus of elasticity of the cable varies with change in cable tension, due to change in geometry of the cable.

Equivalent modulus of elasticity¹⁷ at the end of any load increment is

$$E_e = \frac{E}{1 + \left[\frac{(w_e l)^2 (T_i + T_f) AE}{24 T_i^2 T_f^2} \right]} \quad (3.60)$$

T_i = Initial cable tension.

T_f = Final cable tension.

In the above formula the material modulus of elasticity remains unaffected. Equation (3.60) can be used to determine an equivalent modulus of a particular load increment. It depends on evaluation of the final tension force, which in turn is evaluated from the previously calculated value E_e . Because of change in geometry during application of load, an iterative procedure must be employed so that final tension converges to a value within allowable limits.

Applying the concept of equivalent stiffness^{1,17}

$$(AE)_e = AE - \frac{AE}{1 + \frac{12T^3}{AE(w_e \ell)^2}} \quad (3.61)$$

$$(AE)_e = AE - \frac{\frac{AE}{24T_1^2 T_f^2}}{1 + \frac{AE(w_e \ell)^2}{24T_1^2 T_f^2} (T_i + T_f)} \quad (3.62)$$

3.4. ANALYSIS OF CONTINUOUS SYSTEM:

3.4.1. SYSTEM WITHOUT CABLE STAYS:

After completion of cantilever stage, continuity tendons are introduced in the top and bottom slabs and the closure segment is cast in place to make the system a continuous one. The prestressing continuity strands are now stressed to the required initial prestressing force and anchored in the top slab, thus providing continuity.

The moment in the cantilever structure will be modified by change in statical system due to coupling of the two cantilevers from adjacent piers in to a continuous structure. Subsequent to completion of stressing of strands and closure of joint, the influence of concrete creep modifies the continuity moments.

The total deformation during this stage includes:

1. Elastic and creep deformations due to the superimposed dead load.
2. Elastic and creep deformations due to prestressing forces from the cantilever and continuity tendons provided in stiffening girder.

3. Deformation of stiffening girder due to losses in the prestressing force in tendons.

The prototype bridge structure analysed consists of 3 spans as shown in figure 2.1. All the cable connections to the girder and tower are hinged, so that the cable stays are subjected to axial loads only. The connection between the tower and the girder are also hinged in order to eliminate bending moment in the tower.

If we consider the same structure, figure 2.1, without cable stays, the system will be twice statically indeterminate. The two unknown redundants are the interior support reactions. The deflection of this type of structure can be calculated using basic principles of mechanics of materials¹⁰. The deflection due to prestressing forces and dead load are superposed to obtain the resultant deflection for the structure.

A cable stayed structure shown in figure 2.1, is eight times statically indeterminate. The eight unknown redundants are :

Internally redundant cable stay forces	..	4
Externally redundant reactions at piers	..	2
Rotation of two towers	..	2
Total		<u>8</u>

For a symmetric structure, the number of redundants is halved in number, i.e. 4 times statically indeterminate. Mixed method of analysis developed by Stafford Smith^{2,3} was used for the analysis of the structure.

SOLUTION OF THE SYSTEM:

- 1) Remove the redundant forces to obtain a statically determinate system.
- 2) Write equilibrium equations for deflection at point of application of each redundant using principle of consistent deformations. The number of equations will be equal to the number of unknown redundants.

3.4.1.1. DEAD AND LIVE LOAD DEFLECTIONS FOR THE

PRIMARY SYSTEM:

Referring to figure 3.12.

Let w = Dead load of the structure
 w_L = Uniform live load due to lane loading
 q = Total uniformly distributed load = $w + w_L$
 w_c = Concentrated load to lane loading.

Displacement of points 'B' and 'C' due to uniformly distributed loading is :

$$(\delta'_b)_{U.D.L} = -\frac{q(l+2l')}{12EI} + \frac{ql'^4}{24EI} + \frac{q(l+2l')^3(l')}{24EI} \quad (3.63)$$

$$\begin{aligned} (\delta'_c)_{U.D.L} &= -\frac{q(l+2l')(2l')^3}{12EI} + \frac{q(2l')^4}{24EI} + \frac{q(l+2l')^3(2l')}{24EI} \\ &= -\frac{2q(l+2l')(l')^3}{3EI} + \frac{2ql'^4}{3EI} + \frac{q(l+2l')^3 \cdot l'}{12EI} \end{aligned} \quad (3.64)$$

The deflection due to concentrated live load at the cable point 'C' occurs under the load. When the load w_c is at 'C' ,

$$(\delta'_c)_{\text{Conc. live load}} = \frac{Pb}{6EIL} [(l^2 - b^2)x - x^3] \quad (3.65)$$

$$L = (l + 2l'), \quad b=l, \quad a=2l', \quad x=2l'$$

$$(\delta'_c)_{\text{conc. live load}} = \frac{w_c(l) [(l+2l')^2 - (l)^2] (2l') - (2l')^3}{6EIL(l+2l')}$$

$$= \frac{w_c(l) [(l^2 + 4ll' + 4l'^2 - l^2) (2l') - 8l'^3]}{6EI(l+2l')}$$

$$= \frac{w_c(l) (8ll'^2 + 8l'^3 - 8l'^3)}{6EI(l+2l')}$$

$$= \frac{4w_c(l^2 l'^2)}{3EI(l+2l')} \quad (3.66)$$

$$L = l + 2l', \quad b = l, \quad a = 2l', \quad x = l'$$

$$\begin{aligned} (\delta')_{b \text{ conc. live load.}} &= \frac{w_c(l) [(l+2l')^2 - l^2] (l') - (l')^3}{6EI(l+2l')} \end{aligned}$$

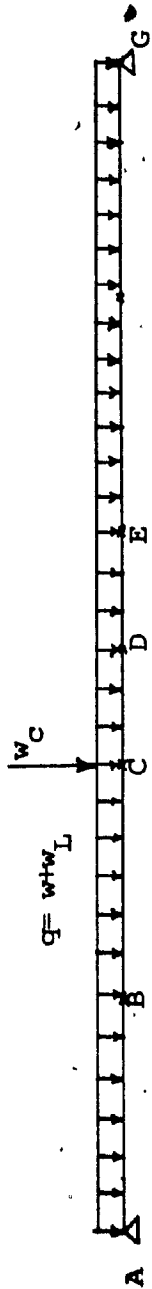
$$= \frac{w_c(l) [l^2 + 4ll' + 4l'^2 - l^2] l' - l'^3}{6EI(l+2l')}$$

$$= \frac{w_c(l) (4ll'^2 + 4l'^3 - l'^3)}{6EI(l+2l')}$$

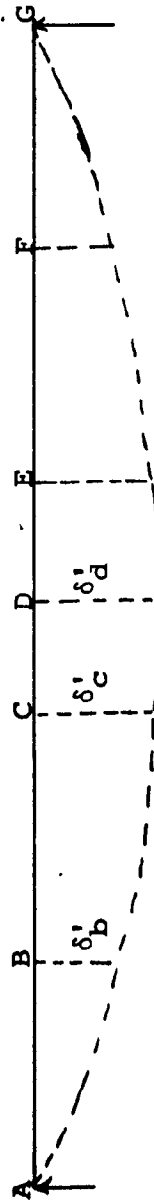
$$= \frac{w(l)}{c} \frac{(4ll'^2 + 3l'^3)}{6EI(l+2l')} \quad (3.67)$$

$$(\delta')_b = (\delta'_b)_{udl} + (\delta'_c)_{conc}$$

$$(\delta')_o = (\delta'_b)_{udl} + (\delta'_c)_{conc} \quad (3.68)$$



a) Load position for maximum deflection



b) Elastic curve under total load

Fig.3.12 Live load position and elastic curve.

3.4.1.2 DEFLECTION DUE TO PRIMARY PRESTRESSING MOMENTS:

DEFLECTION DUE TO GROUP 1. CANTILEVER PRESTRESS:

The deflection due to group-1 cantilever prestress remains the same as that for cantilever stage and only the deflection due to total load changes. The prestressing deflections are as follows:

Arrangement I

$$\Delta_D = \frac{\sum F_n e_n (nb)^2}{3EI} \quad (3.69)$$

Arrangement II

$$\Delta_D = \frac{\sum F_n e_n (nb)^2}{4EI} \quad (3.70)$$

Arrangement III

$$\Delta_D = \frac{\sum F_n e_n .nb.(a-nb/2)}{EI} \quad (3.71)$$

DEFLECTION DUE TO CONTINUITY POST TENSIONING TENDONS (GROUP 3b):

COMMON FOR TYPES 1, 11, 111 TENDON ARRANGEMENTS:

The bending moment diagram for group- 3b post-tensioning tendons is shown in figure 3.12.

The deflection at the centre of main span 'D' due this prestressing is

$$\Delta_D = \phi . \bar{x}$$

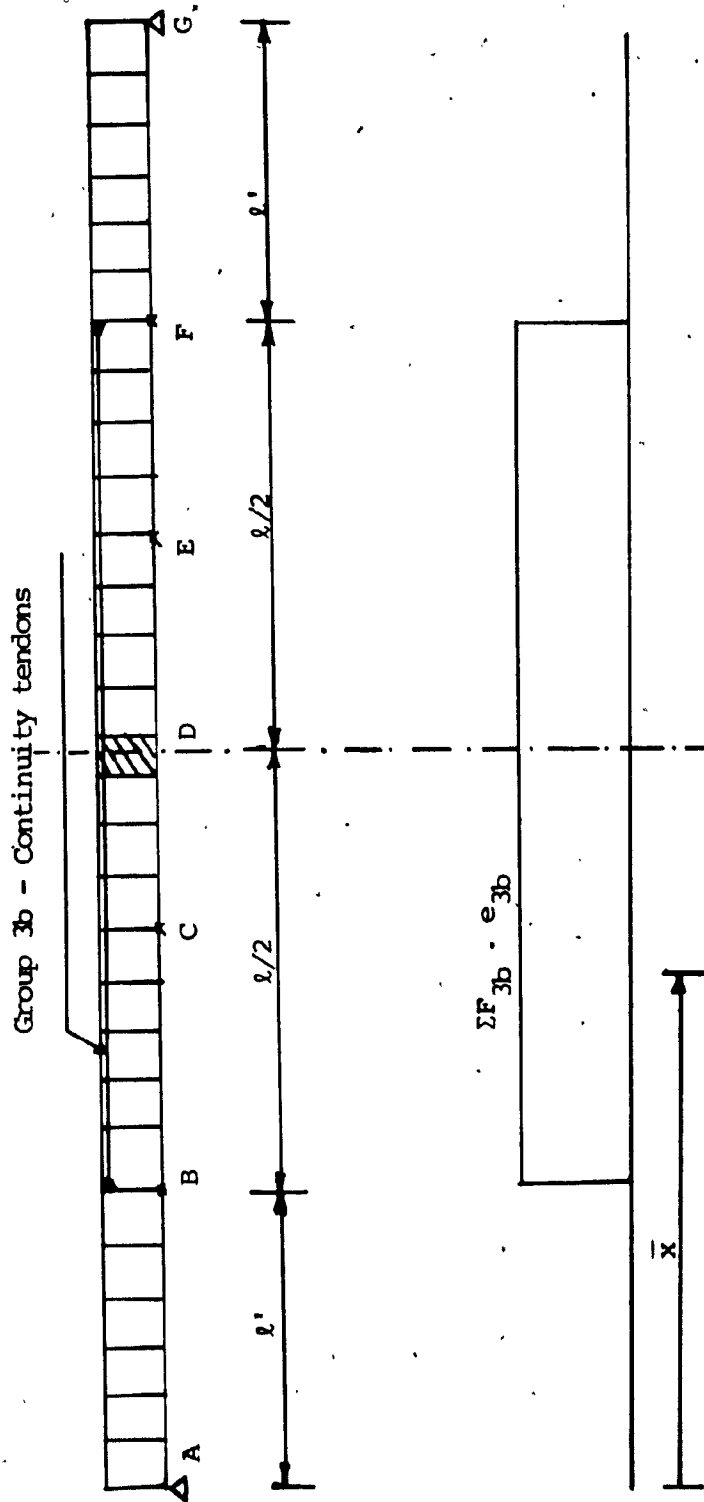


Fig. 3.13- Bending moment diagram for group-3b Prestress.

ΣF_{3b} = Total prestressing force in continuity tendons of group 3b.

e_{3b} = Eccentricity of group 3b. tendons.

$$\phi = \frac{\Sigma F_{3b} \cdot e_{3b} \cdot \frac{l}{2}}{E I}$$

$$\bar{x} = l' + \frac{l}{4}$$

$$\begin{aligned} \Delta_D &= \phi \cdot \bar{x} = \frac{F_{3b} e_{3b}}{EI} \cdot \left(l' + \frac{l}{4} \right) \frac{l}{2} \\ &= \frac{\Sigma F_{3b} e_{3b}}{8EI} \cdot l (l + 4l') \end{aligned} \quad (3.72)$$

DEFLECTION DUE TO CONTINUITY POST TENSIONING TENDONS IN
BOTTOM SLAB (GROUP- 3a):

Arrangement I:

The bending moment diagram is shown in figure 3.14.

Let

ΣF_{3a} = Total prestressing force in group-3a.
post tensioning tendons in the bottom slab.

e_{3a} = Eccentricity of group-3a tendons at 'D'.

e_0 = Eccentricity of group-3a tendons at anchorages.

$2S_1$ = Length of straight portion of tendons parallel to horizontal.

S_2 = Horizontal projection of the bent down portion of tendons, Figure 3.14.

ϕ = Area of bending moment diagram for group '3a' prestressing for one half of symmetric structure.

$$\begin{aligned} \phi &= \frac{1}{2} \frac{\sum F_{3a} (e_{3a} + e_0) (s_2)}{EI} + \frac{\sum F_{3a} (e_{3a} + e_0) (s_1)}{EI} \\ &\quad - \frac{\sum F_{3a} e_0 (s_1 + s_2)}{EI} \\ &= \frac{1}{2} \frac{\sum F_{3a} \cdot e_{3a} (s_2)}{EI} - \frac{1}{2} \frac{\sum F_{3a} \cdot e_0 (s_2)}{EI} + \frac{\sum F_{3a} e_{3a} (s_1)}{EI} \\ &= \frac{\sum F_{3a} e_{3a} (s_2 + 2s_1)}{2EI} - \frac{\sum F_{3a} \cdot e_0 (s_2)}{2EI} \quad (3.73) \end{aligned}$$

$$\bar{x}_1 = \ell' + \frac{\ell}{2} - s_1 - \frac{s_2}{3} = \frac{1}{6} (6\ell' + 3\ell - 6s_1 - 2s_2)$$

$$\bar{x}_2 = \ell' + \frac{\ell}{2} - \frac{s_1}{2} = \frac{1}{2} (2\ell' + \ell - s_1)$$

$$\bar{x}_3 = \ell' + \frac{\ell}{2} - \frac{s_1 + s_2}{2} = \frac{1}{2} (2\ell' + \ell - s_1 - s_2)$$

$$\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3}{3}$$

$$= \frac{1}{18} (6\ell' + 3\ell - 6s_1 - 3s_2 + 6\ell' + 3\ell - 3s_1 + 6\ell' + 3\ell - 3s_1 - 3s_2)$$

$$= \frac{1}{18} (18\ell' + 9\ell - 12s_1 - 5s_2) \quad (3.74)$$

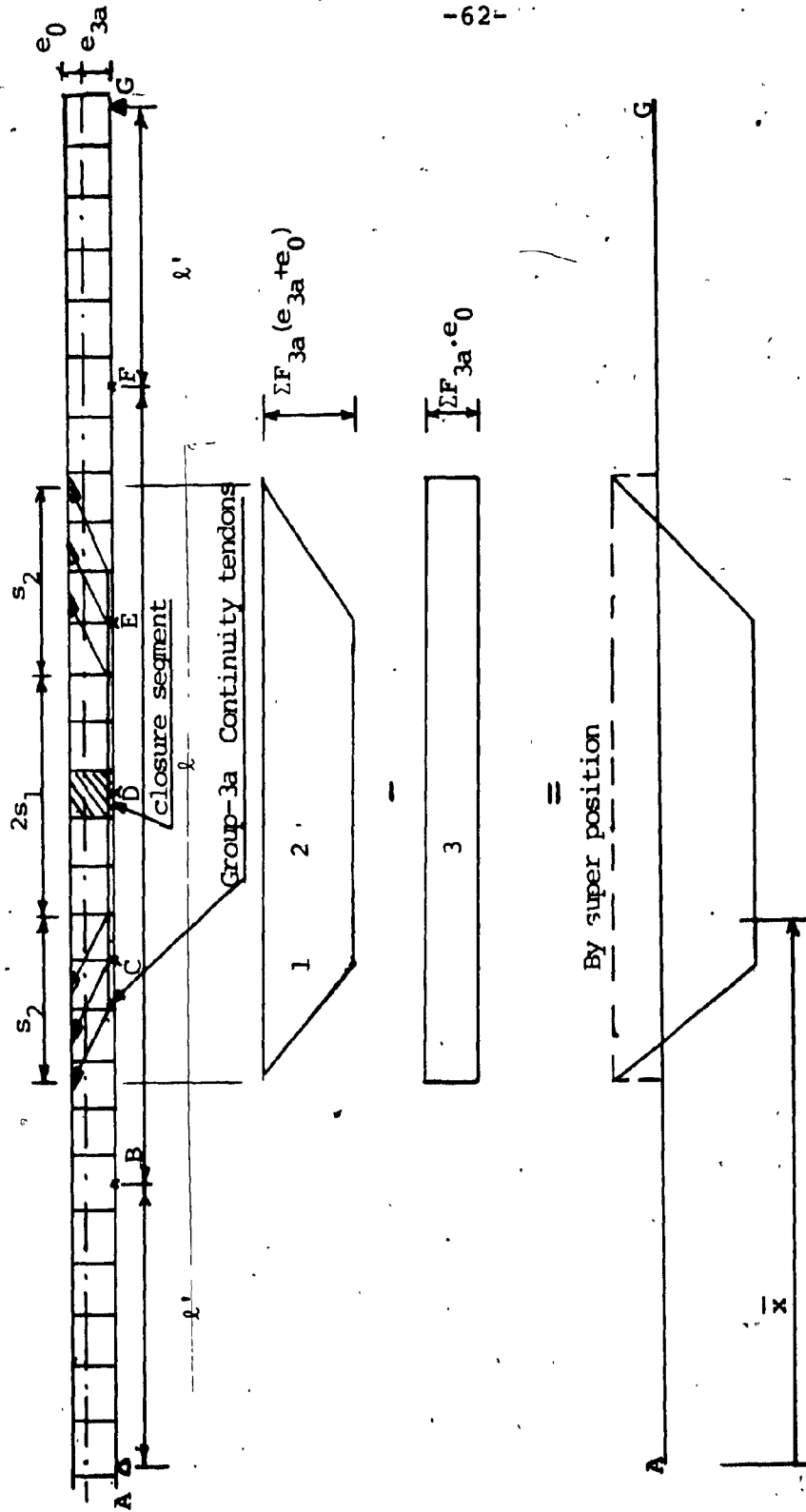


Fig. 3.1.4-Bending moment diagram for group-3a prestress.

$$\begin{aligned}
 (\Delta_{D3a}) &= \phi \cdot \bar{x} \\
 &= \frac{1}{36EI} [\Sigma F_{3a} e_{3a} (s_2 + 2s_1) - \Sigma F_{3a} e_0 \cdot s_2] (18l' + 9l - 12s_1 + 5s_2) \\
 &\quad (3.75)
 \end{aligned}$$

This deflection is calculated by superposition of the bending moment diagrams shown in fig. 3.14.

DEFLECTION DUE TO CONTINUITY POST TENSIONING TENDONS IN BOTTOM

SLAB (GROUP-3a):

Arrangement II:

The bending moment diagram due to this group of tendons is shown in figure 3.15.

$2s_1$ = Length of horizontal projection between end points of group-3a tendons for type II arrangement.

ΣF_{3a} = Total prestressing force at centre of main span 'D', due to group-3a continuity tendons.

e_{3a} = Eccentricity of group-3a tendons at 'D'.

e_0 = Eccentricity of group-3a tendons at the anchorages.

ϕ = Area of bending moment diagram for symmetric half of the structure.

$$\begin{aligned}
 \phi &= \frac{2}{3} \frac{\sum F_{3a} (e_{3a} + e_0)}{EI} (s_1) - \frac{\sum F_{3a} \cdot e_0}{EI} (s_1) \\
 &= \frac{2}{3} \frac{\sum F_{3a} \cdot e_{3a}}{EI} (s_1) - \frac{1}{3} \frac{\sum F_{3a} \cdot e_0}{EI} (s_1) \\
 &= \frac{s_1}{3EI} (2\sum F_{3a} \cdot e_{3a} - \sum F_{3a} \cdot e_0) \quad (3.76)
 \end{aligned}$$

$$\bar{x}_1 = \ell' + \frac{\ell}{2} - \frac{3}{8} s_1 = \frac{1}{8} (8\ell' + 4\ell - 3s_1)$$

$$\bar{x}_2 = \ell' + \frac{\ell}{2} - \frac{s_1}{2} = \frac{1}{2} (2\ell' + \ell - s_1)$$

$$\bar{x} = \bar{x}_1 + \bar{x}_2 = \frac{1}{2} (\ell' + \frac{\ell}{2} - \frac{3}{8} s_1) + \frac{1}{2} (\ell' + \frac{\ell}{2} - \frac{s_1}{2}) = \frac{1}{16} (16\ell' + 8\ell - 7s_1) \quad (3.77)$$

$$(\Delta_D)_{3a} = \phi \cdot \bar{x} = \frac{s_1}{48EI} (2\sum F_{3a} \cdot e_{3a} - \sum F_{3a} \cdot e_0) (16\ell' + 8\ell - 7s_1) \quad (3.78)$$

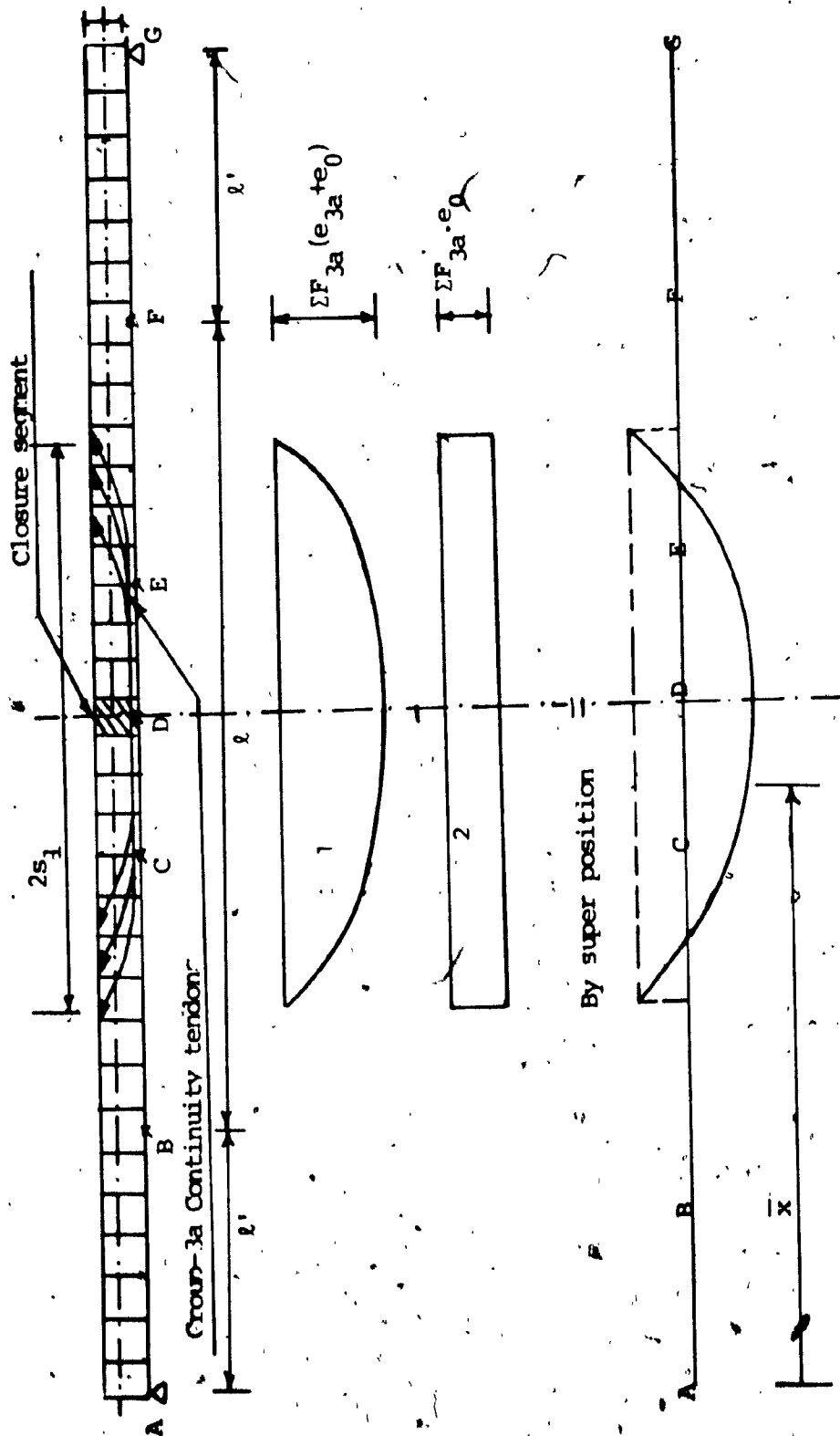


Fig. 3.15-Bending moment diagram for group 3a tendons.

DEFLECTION AT CENTRE OF MAIN SPAN "D" DUE TO

PRESTRESSING TENDONS (GROUP-3a):

TENDON Arrangement III:

The bending moment diagram for this type of tendon arrangement is shown in figure 3.16.

ΣF_{3a} = Total prestressing force at 'D' due to group-3a prestressing tendons, type III arrangement.

n_{3a} = Number of segments in one half of main span through which group- 3a prestressing tendons pass through.

$$(\Delta_D) = \phi_1 \bar{x}_1 + \phi_2 \bar{x}_2 + \dots + \phi_n \bar{x}_n = \Sigma \phi_n \cdot \bar{x}_n$$

$$\begin{aligned} &= [(F_1)_{3a} \cdot b \left(\ell' + \frac{\ell}{2} - \frac{b}{2} \right) + (F_2)_{3a} \cdot 2b \left(\ell' + \frac{\ell}{2} - \frac{2b}{2} \right) + \dots \\ &\quad + (F_n)_{3a} (n_{3a} b) \left(\ell' + \frac{\ell}{2} - \frac{n_{3a} \cdot b}{2} \right) \frac{e_{3a}}{EI}] \\ &= \sum_1^{n_{3a}} \frac{(F_n)_{3a} (n_{3a} b)}{EI} \left(\ell' + \frac{\ell}{2} - \frac{n_{3a} b}{2} \right) \quad (3.79) \end{aligned}$$

b = Length of each segment.

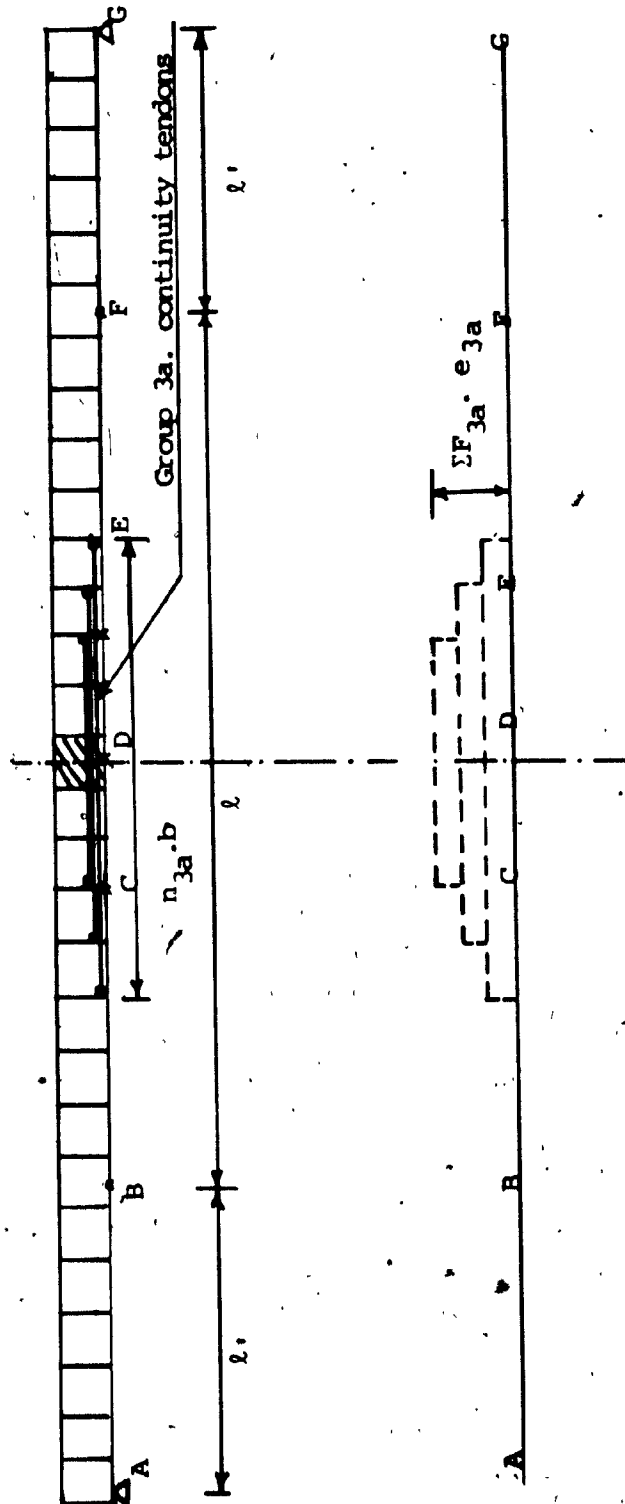


Fig. 3.16- Bending moment diagram for group-3a tendons.

DEFLECTION AT 'D' DUE TO CONTINUITY PRESTRESSING TENDONS IN
FLANKING SPAN (GROUP-2):

The maximum moment due to self weight of girder in flanking span occurs at a distance " $\frac{3}{8} \ell$ " from the end 'A'. as shown in figure 3.17. So the prestressing cables should be arranged symmetrically on either sides about this section.

TENDON ARRANGEMENT I :

$2s_3$ = Straight portion of tendons in the flanking span.

s_4, s_5 = Horizontal projection of the bent down portion of tendons as shown in figure 3.17.

s = Total length of horizontal projection of group-2 tendons.

e_0 = Eccentricity of right end of group-2 tendons.

e_A = Eccentricity of group-2 tendons at the end 'A' and 'G'.

$$\begin{aligned}
 &= \frac{\Sigma F_2 f_1 (2s_3 + s)}{2EI} - \frac{\Sigma F_2 \cdot e_A (s)}{EI} - \frac{1}{2} \frac{\Sigma F_2 (e_0 - e_A) (s)}{EI} \\
 &= \frac{\Sigma F_2 f_1 (2s_3)}{2EI} + \frac{\Sigma F_2 f_1 (s)}{2EI} - \frac{\Sigma F_2 e_A (s)}{EI} - \frac{1}{2} \frac{\Sigma F_2 e_0 (s)}{EI} + \frac{1}{2} \frac{\Sigma F_2 e_A (s)}{EI} \\
 &= \frac{\Sigma F_2 \cdot f_1 \cdot s_3}{EI} + \frac{\Sigma F_2 \cdot s (f_1 - e_A - e_0)}{2EI} \quad (3.80)
 \end{aligned}$$

Where

$$f_1 = \left(e_2 + \frac{e_A + e_0}{2} \right) = \text{Ordinate at centre of group 2 tendons} \\ \text{figure 3.17.}$$

$$\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3}{3} = \frac{1}{3} \left(\frac{s}{2} + \frac{s}{2} + \frac{2}{3}s \right) = \frac{5s}{9} \quad (3.81)$$

$$(\Delta_D)_2 = \left[\frac{\Sigma F_2 f_1 s_3}{EI} + \frac{\Sigma F_2 s}{2EI} (f_1 - e_A - e_0) \right] \frac{5s}{9} \quad (3.82)$$

By substituting $e_A = 0$ in the above expression, we obtain the deflection at 'D' when eccentricity at 'A' is zero.

DEFLECTION AT 'D' DUE TO PRESTRESS (GROUP-2) TENDONS):

TENDON ARRANGEMENT II:

The bending moment diagram is shown in figure 3.18.

Let $2s_3$ = Length of horizontal projection of group-2 tendons.

f_1 = Cable ordinate at " $\frac{3}{8}l$ " from 'A' as shown in figure 3.20.

$$f_1 = e_2 + \frac{e_A + e_0}{2}$$

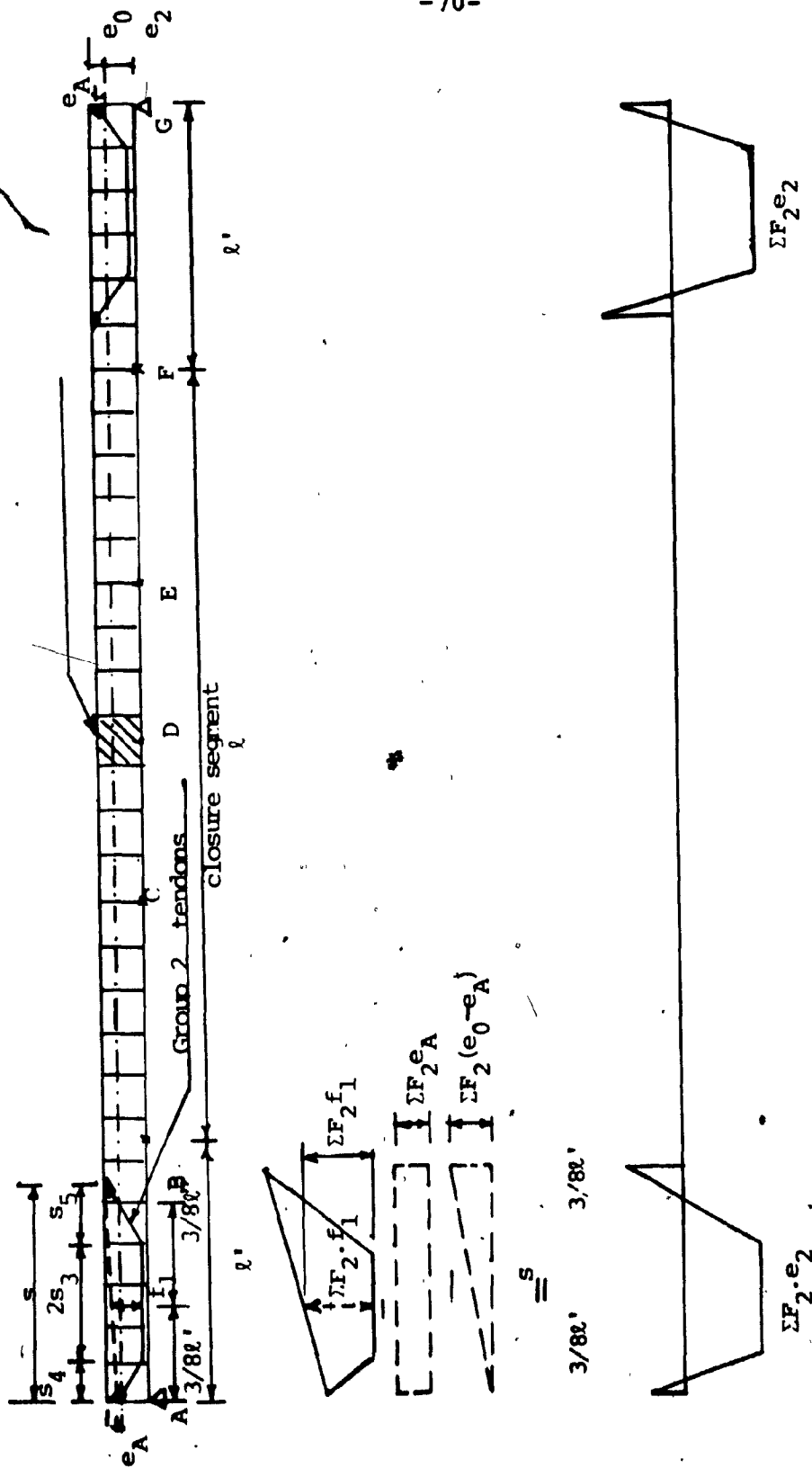


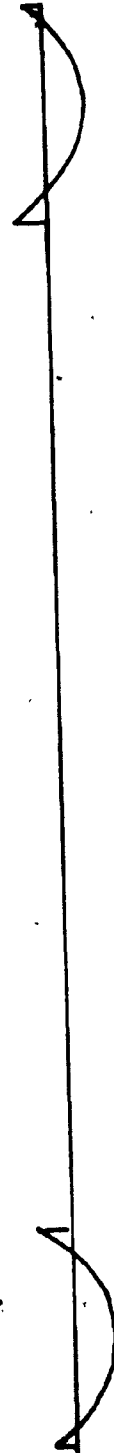
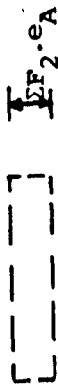
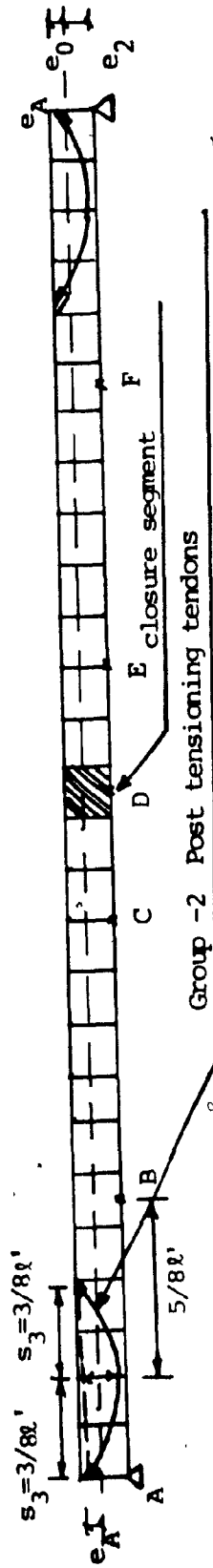
Fig. 3.17- Bending moment diagram due to group-2 post tensioning.

$$\begin{aligned}
 \phi &= \frac{2}{3} \frac{\Sigma F_2 f_1 (2s_3)}{EI} - \frac{\Sigma F_2 e_A (2s_3)}{EI} - \frac{\Sigma F_2 (e_0 - e_A) (2s_3)}{2EI} \\
 &= \frac{4}{3} \frac{\Sigma F_2 f_1 s_3}{EI} - \frac{\Sigma F_2 e_A s_3}{EI} - \frac{\Sigma F_2 e_0 s_3}{EI} \\
 &= \frac{\Sigma F_2 s_3 [4f_1 - 3e_A - 3e_0]}{3EI} \quad (3.83)
 \end{aligned}$$

$$\bar{x} = \frac{1}{3} (\bar{x}_1 + \bar{x}_2 + \bar{x}_3) = \frac{1}{3} (s_3 + s_3 + \frac{4}{3} s_3) = \frac{10s_3}{9} \quad (3.84)$$

$$(\Delta_D)_2 = \frac{10 \Sigma F_2 s_3^2 [4f_1 - 3e_A - 3e_0]}{27EI} \quad (3.85)$$

We obtain the deflections at 'D' for by substituting the values of prestressing force and the cable eccentricities.



By super position

Fig. 3.18-Bending moment diagram for group-2 post tensioning

DEFLECTION AT THE CENTRE OF MAIN SPAN 'D' DUE TO

PRESTRESSING STRANDS (GROUP-2):

TENDON ARRANGEMENT III STRAIGHT TENDONS:

The bending moment diagram due to this group of prestressing tendons is shown figure 3.19.

$2s_3$ = Length of group-2 prestressing tendons.

$$= 2\left(\frac{3}{8} l'\right) = \frac{3}{4} l'$$

$$\Delta_D = \frac{\Sigma F_2 e_2 (2s_1)}{EI} \frac{3}{8} l'$$

Substituting

$$2s_1 = \frac{3}{4} l', \text{ we get}$$

$$\Delta_D = \frac{\Sigma F_2 e_2}{EI} \left(\frac{3}{4} l'\right) \left(\frac{3}{8} l'\right)$$

$$= \frac{9 \Sigma F_2 e_2 l'^2}{32 EI} \quad (3.86)$$

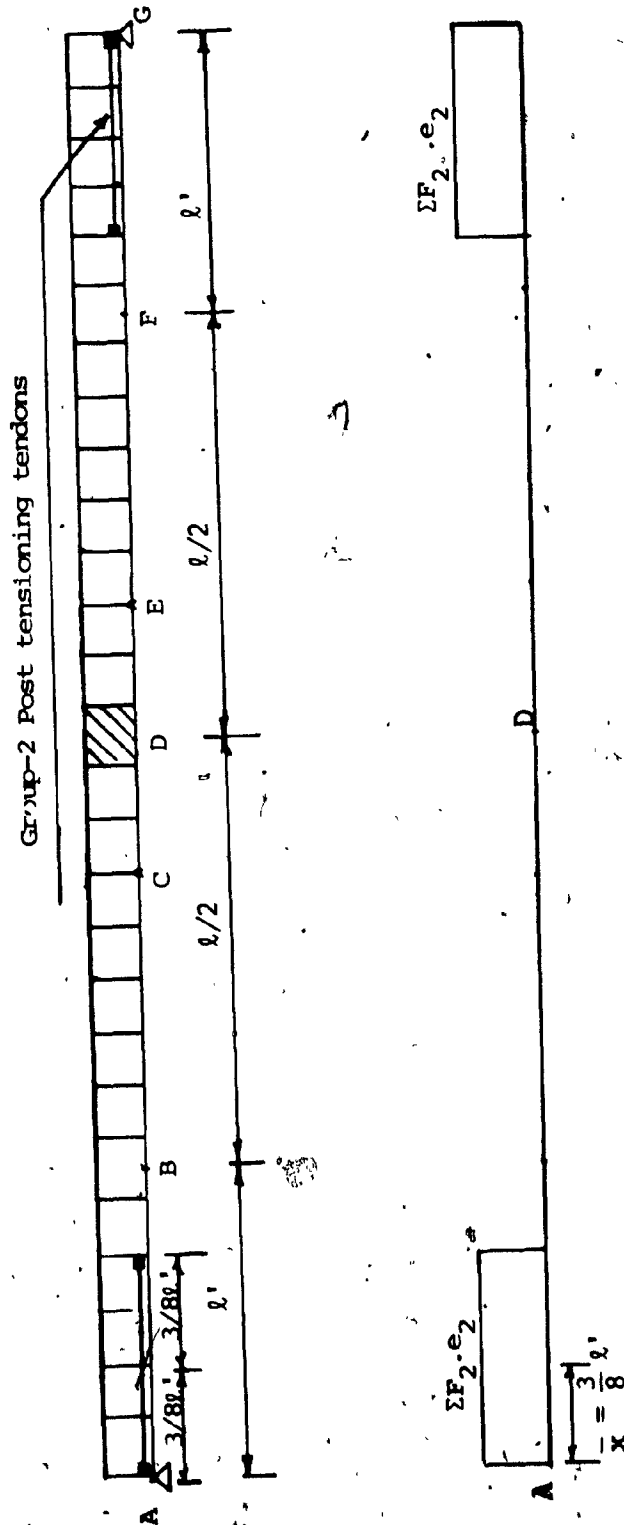


Fig. 3.19 - Bending moment diagram due to group-2 Prestress.

Total deflection due to primary prestressing

Type of Arrangement	Group - 1. Cantilever tendons.	Group - 2. tendons in flanking span.	Group-3a Continuity tendons in main span.	Group-3b tendons.
I	$\frac{\Sigma F_n e_n (nb)^2}{3EI}$ +	$\left[\frac{\Sigma F_2 f_1 s_3}{EI} + \frac{\Sigma F_2 s (f_1 - e_A - e_0)}{2EI} \right] \frac{(5s)}{9}$ +	$\frac{1}{36EI} [\Sigma F_{3a} e_{3a} (s_2 + 2s_1) - \Sigma F_{3a} e_0 s_2] - (18\ell' + 9\ell - 12s_1 + 5s_2)$	$\frac{\Sigma F_{3b} e_{3b} (\ell + 4\ell')}{8EI}$
II	$\frac{\Sigma F_n e_n (nb)^2}{4EI}$ +	$\frac{10\Sigma F_2 s_3^2 [4f_1 - 3e_A - 3e_0]}{27 EI}$ +	$\frac{s_1}{48EI} (2\Sigma F_{3a} e_{3a} - \Sigma F_{3a} e_0) - (16\ell' + 8\ell - 7s_1)$	$\frac{\Sigma F_{3b} e_{3b} \ell (\ell + 4\ell')}{8EI}$
III	$\frac{\Sigma F_n e_n nb (a-nb/2)}{EI}$ +	$\frac{9\Sigma F_2 e_2 \ell'^2}{32 EI}$ +	$\Sigma (F_n)_{3a} (n_{3a} b) (\ell' + \frac{\ell}{2} - \frac{n_{3a} b}{2}) - \frac{n_{3a} b}{2}$	$\frac{\Sigma F_{3b} e_{3b} \ell (\ell + 4\ell')}{8 EI}$
				(3.87)

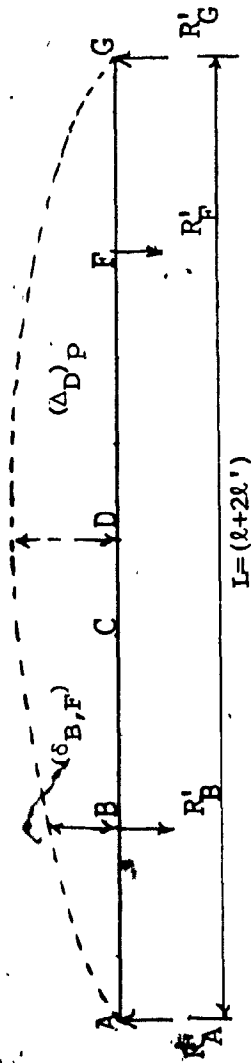
3.4.1.3 SECONDARY MOMENTS:

The primary prestressing forces cause deformation of the structure. In statically determinate structures, the line of thrust resulting from prestressing force is coinciding with the cable profile and the deformations take place freely without inducing any change in bearing reactions. On the other hand, in statically indeterminate structures¹⁵, the line of thrust does not coincide with the c.g.s. line, because redundant reactions are introduced at intermediate supports by deformations due to primary prestressing. These redundant reactions are also called parasitic and they produce a secondary moment.

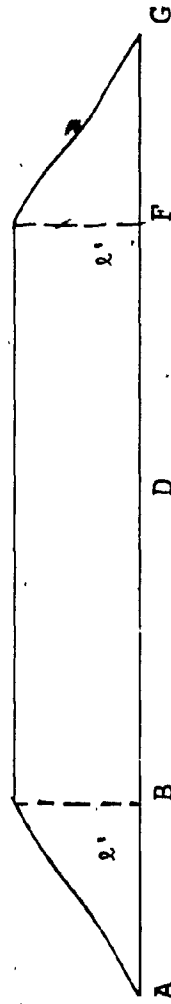
Considering the continuous beam with the geometric layout of tendons discussed in Chapter 2, secondary moments are produced at intermediate supports by the reactions induced due to primary prestressing.

Consider the basic system with simple supports at 'A' & 'G' as shown in figure 3-22, with the support reactions at 'B' and 'F' to be the redundant forces. So now for the simply supported beam 'AG' let the total deflection at the centre of the span due to primary prestressing force be represented by $(\Delta_D)_p$.

For the primary system, the deflected shape of the beam due to primary prestress can be assumed to follow a sine curve,



a) Elastic curve due to prestressing



b) Bending moment diagram for unit load at 'B' and 'F'.

Fig. 3.20- Elastic curve due to prestress and bending moment due to

unit load at 'B' and 'F'.

which may be represented by

$$y = z_{\max} \cdot \sin \frac{\pi x}{L}$$

where, x = Any section at a distance ' x ' from 'A'.

y = Ordinate of deflection at section ' x '.

z_{\max} = Maximum deflection at centre of span.

Here ,

$$z_{\max} = (\Delta_D)_P, \quad L = l + l', \quad x = l'$$

Therefore, the deflection at points 'B' and 'F', the intermediate support points is given by

$$(\Delta_{B,F})_P = (\Delta_D)_P \sin \frac{\pi l'}{L+2l'} \quad (3.88)$$

The reactions " $R'_{B,F}$ " induced at these intermediate support points can be given by the relation

$$(\Delta_{B,F})_P = (\Delta_{11}) (R'_{B,F}) \quad (3.89)$$

Where, Δ_{11} = Deflection at 'B' or 'F' due to unit loads applied at 'B' and 'F'.

$R'_{B,F}$ = Secondary reactions at 'B' or 'F'.

The deflections at intermediate support points 'B' and 'F' due to unit loads placed at 'B' and 'F' is given by

$$(\Delta_{B,F}) = \frac{Pa^2}{6EI} (3L - 4a) \quad (3.90)$$

Here, $L = L + 2\ell'$, $p=1$, $a=\ell'$

Therefore,

$$\begin{aligned}\Delta_{11} &= \frac{L^2}{6EI} [3(\ell + 2\ell') - 4\ell'] \\ &= \frac{\ell^2}{6EI} (3\ell + 2\ell')\end{aligned}\quad (3.91)$$

Where,

Δ_{11} = Deflections at 'B' or 'F' due to unit loads acting at 'B' and 'F'.

Now

$$(\Delta_B)_p = \Delta_{11} \cdot R'_B \quad (3.92)$$

$$(\Delta_F)_p = \Delta_{11} \cdot R'_F \quad (3.93)$$

Where,

$(\Delta_B)_p$ = Deflection at 'B' due to total primary prestress.

$(\Delta_F)_p$ = Deflection at 'F' due to total primary prestress.

$$\text{or } R'_B = \frac{(\Delta_B)_p}{\Delta_{11}} \quad (3.94)$$

$$R'_F = \frac{(\Delta_F)_p}{\Delta_{11}} \quad (3.95)$$

The secondary reactions and moments developed at the intermediate support points will be such that the total deflections at these points will be zero.

Hence, the resultant deflection at the centre of main span 'D' is given by :

$$(\Delta_D)_{\text{final}} = (\Delta_D)_p \left(1 - \frac{\sin \frac{\pi l'}{l+2l'}}{2} \right) \quad (3.96)$$

The resultant reactions at intermediate supports can be determined using principle of consistent deformation. Considering the intermediate support reactions as redundants, we get,

$$\delta'_b - (\Delta_B)_p = x_b f_{bb} + x_e f_{be} \quad (3.97)$$

Since $x_b = x_e$, by symmetry

$$x_b = \frac{\delta'_b - (\Delta_B)_p}{f_{bb} + f_{be}} \quad (3.98)$$

Where δ'_b = Deflection at 'B' for primary system due to dead and live loads.

f_{bb} = Deflection at 'B' due to unit load placed at 'B'.

f_{bf} = Deflection at 'B' due to unit load placed at 'F'.

After the support reactions are determined the deflection at any point can be easily calculated.

3.4.2 STRUCTURE WITH CABLE STAYS:

3.4.2.1 METHOD-1: DETERMINATION OF REQUIRED TENSION

FORCES IN CABLE STAYS BY MIXED METHOD OF ANALYSIS:

A cable stayed structure is many times statically indeterminate. The degree of redundancy is equal to the number of unknowns. As seen in Art. 3.4.1.3 the cable stayed structure shown in figure 2.1 is eight times statically indeterminate. By considering the symmetric nature of the structure the degree of redundancy is reduced to 4.

Let X_r = Tension force in the cables

X_a, X_c = The vertical component of cable force at 'A' and 'C'.

$$X_a = (X_r)_a \cdot \sin \alpha_r \quad (3.99)$$

$$X_c = (X_r)_c \cdot \sin \alpha_r \quad (3.100)$$

The basic equation for the continuous system can be formulated using principle of consistent deformations.

$$\delta'_a - f_{aa} X_a - f_{ab} X_b - f_{ac} X_c - f_{ae} X_e - f_{af} X_f - f_{ag} X_g = \Delta_a \quad (3.101)$$

$$\delta'_b - f_{ba} X_a - f_{bb} X_b - f_{bc} X_c - f_{be} X_e - f_{bf} X_f - f_{bg} X_g = \Delta_b \quad (3.102)$$

$$\delta'_c - f_{ca} X_a - f_{cb} X_b - f_{cc} X_c - f_{ce} X_e - f_{cf} X_f - f_{cg} X_g = \Delta_c \quad (3.103)$$

Since the structure is symmetrical, the equations for the other half will be exactly similar.

Similarly, the equations for the other half are:

$$\delta'_g = f_{gg}X_g - f_{gf}X_f - f_{ge}X_e - f_{gc}X_c - f_{gb}X_b - f_{ga}X_a = \Delta_g \quad (3.104)$$

$$\delta'_f = f_{fg}X_g - f_{ff}X_f - f_{fe}X_e - f_{fc}X_c - f_{fb}X_b - f_{fa}X_a = \Delta_f \quad (3.105)$$

$$\delta'_e = f_{eg}X_g - f_{ef}X_f - f_{ee}X_e - f_{ec}X_c - f_{eb}X_b - f_{ea}X_a = \Delta_e \quad (3.106)$$

These equations account only for total loads on the structure and they can be modified to include the effect of prestressing

$$[\delta'_a - (\Delta_A)_p] - (f_{aa} + f_{ag})X_a - (f_{ac} + f_{ae})X_c - (f_{ab} + f_{af})X_b = \Delta_a \quad (3.107)$$

$$[\delta'_b - (\Delta_B)_p] - (f_{ba} + f_{bg})X_a - (f_{bc} + f_{be})X_c - (f_{bb} + f_{bf})X_b = \Delta_b \quad (3.108)$$

$$[\delta'_c - (\Delta_C)_p] - (f_{ca} + f_{cg})X_a - (f_{cc} + f_{ce})X_c - (f_{cb} + f_{cf})X_b = \Delta_c \quad (3.109)$$

Where

δ'_a = Deflection at 'A' due to total load on the girder.

δ'_b, δ'_c = Deflections at 'B', 'C' ----- 'G', due to total loads on the girder.

$(\Delta_A)_p, (\Delta_B)_p$ = Deflections at 'A', 'B', ----- 'G', respectively due to total prestressing forces in girder.

f_{aa}, f_{bb} = Flexibilities of beam due to unit loads placed at these points.

f_{ab}, f_{ac} = Flexibilities of the beam, where the first subscript denotes the point at which flexibilities are required and the second subscript represents the point of application of unit load.

X_a, X_c = Vertical components of cable forces.

Increase in deflection due to rotation of tower at 'B' is

$$\Delta'_a = \phi'_n \cdot BA, \quad \Delta'_c = \phi'_n \cdot BC \quad (3.110)$$

Similarly, deflection due to elastic stretch of cables is given by

$$\Delta''_a = C_a X_a, \quad \Delta''_c = C_c X_c \quad (3.111)$$

$$\text{Where, } C_a = C_c = \frac{L_r}{A_c E \sin^2 \alpha_r} \quad (3.112)$$

$$\text{Elastic shortening of tower} = (X_a + X_c) f_T \quad (3.113)$$

Deflection due to axial shortening of the girder is given by equation (3.114).

$$\Delta_{gr} = \frac{X_c (BC)}{AE} \cot^2 \alpha_r = X_c \cdot f_{bc}^g \quad (3.114)$$

Where f_{bc}^g = Flexibility of girder due to axial loads.

A stability equation must be formulated; due to rotation of the tower about its hinge.

$$-X_a \cdot BA + X_c \cdot BC = 0 \quad (3.115)$$

$$-X_g \cdot FG + X_e \cdot FE = 0 \quad (3.116)$$

For a structure with cable anchored at the supports,

$$f_{aa} = f_{ag} = f_{ac} = f_{ae} = f_{ab} = f_{af} = 0$$

$$f_{ab} = f_{bg} = f_{ca} = f_{cg} = 0 \quad (3.117)$$

Substituting equations (3.110) to (3.117) in equations (3.107) to (3.109), we obtain

$$(C_a + f_T) X_a + (0) X_b + (f_T) X_c - \phi_n \cdot BA = 0 \quad (3.118)$$

$$(0) X_a + (f_{bb} + f_{bf}) X_b + (f_{bc} + f_{be}) X_c = [\delta'_b - (\Delta_B)_p] \quad (3.119)$$

$$(f_T) X_a + (f_{cb} + f_{cf}) X_b + (f_{cc} + f_{ce} + f_T + f_{bc}^g + C_c) X_c + \phi_n \cdot BC = [\delta'_c - (\Delta_C)_p] \quad (3.120)$$

$$-X_a \cdot BA + X_c \cdot BC = 0 \quad (3.121)$$

These equations are represented in a matrix

form as shown in equation (3.122).

$$\begin{bmatrix} C_a + f_T & 0 & f_T & -BA \\ 0 & (f_{bb} + f_{bf}) & (f_{bc} + f_{be}) & 0 \\ f_T & (f_{cb} + f_{cf}) & (f_{cc} + f_{ce} + f_{bc}^g + f_T + C_c) & BC \\ -BA & 0 & BC & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ \phi_n \end{bmatrix} = \begin{bmatrix} 0 \\ \delta_c' - (\Delta_B)_p \\ \delta_c' - (\Delta_c)_p \\ 0 \end{bmatrix} \quad (3.122)$$

EXPRESSION FOR FLEXIBILITIES OF BEAM :

The expression ¹⁰ for flexibility of beam due to different load positions can be calculated as follows:

$$f_{aa} = f_{ba} = f_{ca} = f_{da} = f_{ea} = f_{ra} = f_{ga} = 0 \quad (3.123)$$

The deflection at 'B' due to unit load placed at 'B' is

$$\Delta_{AB} = \frac{pb}{6EIL} (x^3 - (L^2 - b^2)x), \quad 0 \leq x \leq a \quad (3.124)$$

Here , $p = 1$, $b = l + l'$, $a = l'$, $L = l + 2l'$, $x = l'$

$$f_{bb} = \frac{(l + l')}{6EI(l + 2l')} [\{ (l + 2l')^2 - (l + l')^2 \} l' - l'^3]$$

$$f_{bb} = \frac{(\ell + \ell') \ell'}{6EI(\ell + 2\ell')} [\ell^2 + 4\ell\ell' + 4\ell'^2 - \ell^2 - 2\ell\ell' - \ell'^2 - \ell'^2]$$

$$f_{bb} = \frac{\ell'(\ell + \ell')}{6EI(\ell + 2\ell')} (2\ell'^2 + 2\ell\ell')$$

Unit load placed at 'B':

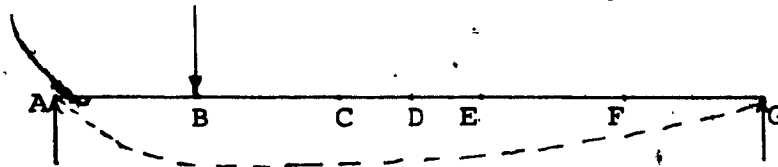


Fig. 3.2 1. Load position for beam flexibility.

$$f_{bb} = \frac{\ell'^2(\ell' + 2\ell)(\ell + \ell')}{6EI(\ell + 2\ell')} \quad (3.125)$$

$$\Delta_{BG} = \frac{Pa}{6EIL} (x^3 - 3x^2L + (2L^2 + a^2)x - a^2L) \quad (3.126)$$

$$f_{bc} = f_{cb} \quad \text{Here, } a = \ell', \quad x = 2\ell', \quad L = \ell + 2\ell', \quad p = 1$$

$$f_{bc} = f_{cb} = \frac{l'}{6EI(l+2l')} [8l'^3 - 12l'^2(l+2l') + \{2(l+2l')^2 + l'^2\}(2l') - (l')^2(l+2l')]]$$

$$= \frac{l'^2}{6EI(l+2l')} [8l'^2 - 12ll' - 24l'^2 + 4l'^2 + 16ll' + 16l'^2 + 2l'^2 - 2ll' - 2l'^2]$$

$$= \frac{l'^3(4l' + 3l)}{6EI(l+2l')} \quad (3.127)$$

$f_{be} = f_{eb}$, Here $a=l'$, $x=l'$, $L=l+2l'$,

$$f_{be} = f_{eb} = \frac{l'}{6EI(l+2l')} [l^3 - 3l^2(l+2l') + \{2(l+2l')^2 + l'^2\}(l) - l'^2(l+2l')]]$$

$$= \frac{l'}{6EI(l+2l')} [l^3 - 3l^3 - 6l'l^2 + 2l^3 + 8l^2l' + 8l'^2l + l'^2l - l'^2l - 2l'^3]$$

$$= \frac{l'}{6EI(l+2l')} (2l'l^2 + 8l'^2l - 2l'^3)$$

$$= \frac{l'^2}{3EI(l+2l')} [l^2 + 4ll' - l'^2] \quad (3.128)$$

$f_{bf} = f_{fb}$, $a=l'$, $x=l+l'$, $L=l+2l'$

$$\begin{aligned}
 f_{bf} = f_{fb} &= \frac{l' \cdot [(l+l')^3 - 3(l+l')^2(l+2l') + \{ 2(l+2l') + l'^2 \} \cdot (l+l') - l'^2(l+2l')]}{6EI(l+2l')} \\
 &= \frac{l' [l^3 + 3l^2l' + 2ll'^2 - l'^3 - 3l^3 - 6l^2l' - 3ll'^2 - 6l^2l' - 12ll'^2 - 6l'^3 + 2l^3 + 8l^2l' + 9ll'^2 + 2l^2l' + 8ll'^2 + 9l'^3]}{6EI(l+2l')} \\
 &= \frac{l'^2(2l'^2 + l'^2 + 4ll')}{6EI(l+2l')} \quad (3.129)
 \end{aligned}$$

FLEXIBILITIES DUE TO UNIT LOAD AT 'C':

$$a = 2l', \quad x = 2l', \quad p = 1, \quad L = l + 2l',$$

$$\begin{aligned}
 f_{cc} &= \frac{2l'}{6EI(l+2l')} [8l'^3 - 3(2l')^2(l+2l') + 2(l+2l')^2(2l')^2(2l') - (2l')(l+2l')] \\
 &= \frac{l'}{3EI(l+2l')} [8l'^3 - 12ll'^2 - 24l'^3 + (2l^2 + 8ll' + 8l'^2)(2l') + 8l'^3 - 4ll'^2 - 8l'^3] \\
 &= \frac{4l'^2l'^2}{3EI(l+2l')} \quad (3.130)
 \end{aligned}$$

$$a = l', \quad x = l, \quad p = 1, \quad L = l + 2l',$$

$$f_{ec} = f_{ce} = \frac{2l'}{6EI(l+2l')} [l^3 - 3l^2(l+2l') + \{2(l+2l')^2 + (2l')^2\}(l) - (2l')^2(l+2l')]$$

$$= \frac{l'}{3EI(l+2l')} [l^3 - 3l^2 - 6l^2l' + 2l^3 + 8l^2l' + 8ll'^2 + 4ll'^2 - 4ll'^2 - 8l'^3]$$

$$= \frac{2l'^2(l^2 + 4ll' - 4l'^2)}{3EI(l+2l')} \quad (3.131)$$

$$a = 2l', \quad x = l + l', \quad p = 1, \quad L = l + 2l',$$

$$f_{cf} = f_{fc} = \frac{2l'}{6EI(l+2l')} [(l+l')^3 - 3(l+l')^2(l+2l') + \{2(l+2l')^2 + (2l')^2\}(l+l') - (2l')^2(l+2l')]$$

$$= \frac{l'}{3EI(l+2l')} [l^3 + 3l^2l' - 6l^2l' - 7l'^3 - 3l^3 - 6l^2l' - 3ll'^2 - 6l^2l' - 12ll'^2 - 6l'^3 + 2l^3 + 8l^2l' + 12ll'^2 + 2l^2l' + 8ll'^2 + 12l'^3]$$

$$= \frac{l'^2}{3EI(l+2l')} (l^2 + 4ll' - l'^2) \quad (3.132)$$

DEFLECTION AT CABLE POINT 'C' AND INTERMEDIATE
SUPPORTS DUE TO PRESTRESSING:

$(\Delta_D)_p$ = Deflection at centre of main span 'D' due to primary prestressing forces.

The deflection at the cable stay point due to primary prestressing force is

$$(\Delta_C)_p = (\Delta_D)_p \cdot \sin \frac{2 \ell'}{\ell + 2\ell'} \quad (3.133)$$

The deflection at the intermediate supports 'B' and 'F' can be determined in a similar way using

$$(\Delta_B)_p = -(\Delta_D)_p \cdot \sin \frac{\pi \ell'}{\ell + 2\ell'} \quad (3.134)$$

$$(\Delta_A)_p = 0 \quad (3.135)$$

3.4.2.2.METHOD-II: PRE-TENSIONING FORCES REQUIRED TO PRODUCE MAXIMUM MOMENT AT INTERMEDIATE SUPPORTS:

The maximum bending moment due to dead load occurs at intermediate supports. Since these intermediate supports⁴ are considered as redundants in the primary system, the bending moment at these points is given by the equation,

$$(M_r)_B = -\frac{w(l+2l')}{2}l' + \frac{wl'^2}{2} = \frac{-wl'}{2}(l+l') \quad (3.136)$$

The procedure to evaluate the pre-tensioning force in cables consists of reducing the bending moment ' M_r ' due to dead load in the stiffening girder to $C_0 M_r$.

C_0 = A reduction factor less than unity.

The basic equation for the bridge system is

$$M_r + M_r^1 x_1 + M_r^2 x_2 + \dots + M_r^n x_n = C_0 M_r \quad (3.137)$$

Where x_1, x_2, \dots, x_n = pre-tensioning forces in the cables. Since the pre-tensioning forces in all the cables are equal,

$$x_1 = x_2 = x_3 = x_4 = x_i \quad (3.138)$$

$$\text{Therefore, } M_r + M_r^a x_i = C_0 M_r \quad (3.139)$$

Or

$$M_R^a x_i = M_R (C_0 - 1) \quad (3.140)$$

$$x_i = \frac{M_R (C_0 - 1)}{M_R^a} \quad (3.141)$$

$$\text{Where, } M_R^1 + M_R^2 + M_R^3 + M_R^4 = M_R^a \quad (3.142)$$

M_R^1 = Bending moment at location 'r' due to unit load placed at 'i', &

x_i = Required cable stay force to reduce M_R to $C_0 M_R$.

The moment at intermediate support point 'B'.

for the simple beam 'AG' is given by

$$M_R^a = 2 \sin \alpha_r (\ell') - 1 \sin \alpha_r (\ell') = \ell' \sin \alpha_r \quad (3.143)$$

$$x_i = \frac{M_R (C_0 - 1)}{\ell' \sin \alpha_r} = \frac{w \ell' (\ell + \ell') (C_0 - 1)}{2 \ell' \sin \alpha_r}$$

$$x_i = \frac{w \ell' (\ell + \ell') (C_0 - 1)}{2 \ell' \sin \alpha_r} \quad (3.144)$$

If $\Sigma F_n e_n$ = Moment due to prestressing at 'B' then the expression is modified as follows:

$$x_i = \frac{[w \ell' (\ell + \ell') / 2 - \Sigma F_n e_n] (C_0 - 1)}{\ell' \sin \alpha_r}$$

$$x_i = \frac{[w \ell' (\ell + \ell') - 2 \Sigma F_n e_n] (C_0 - 1)}{2 \ell' \sin \alpha_r} \quad (3.145)$$

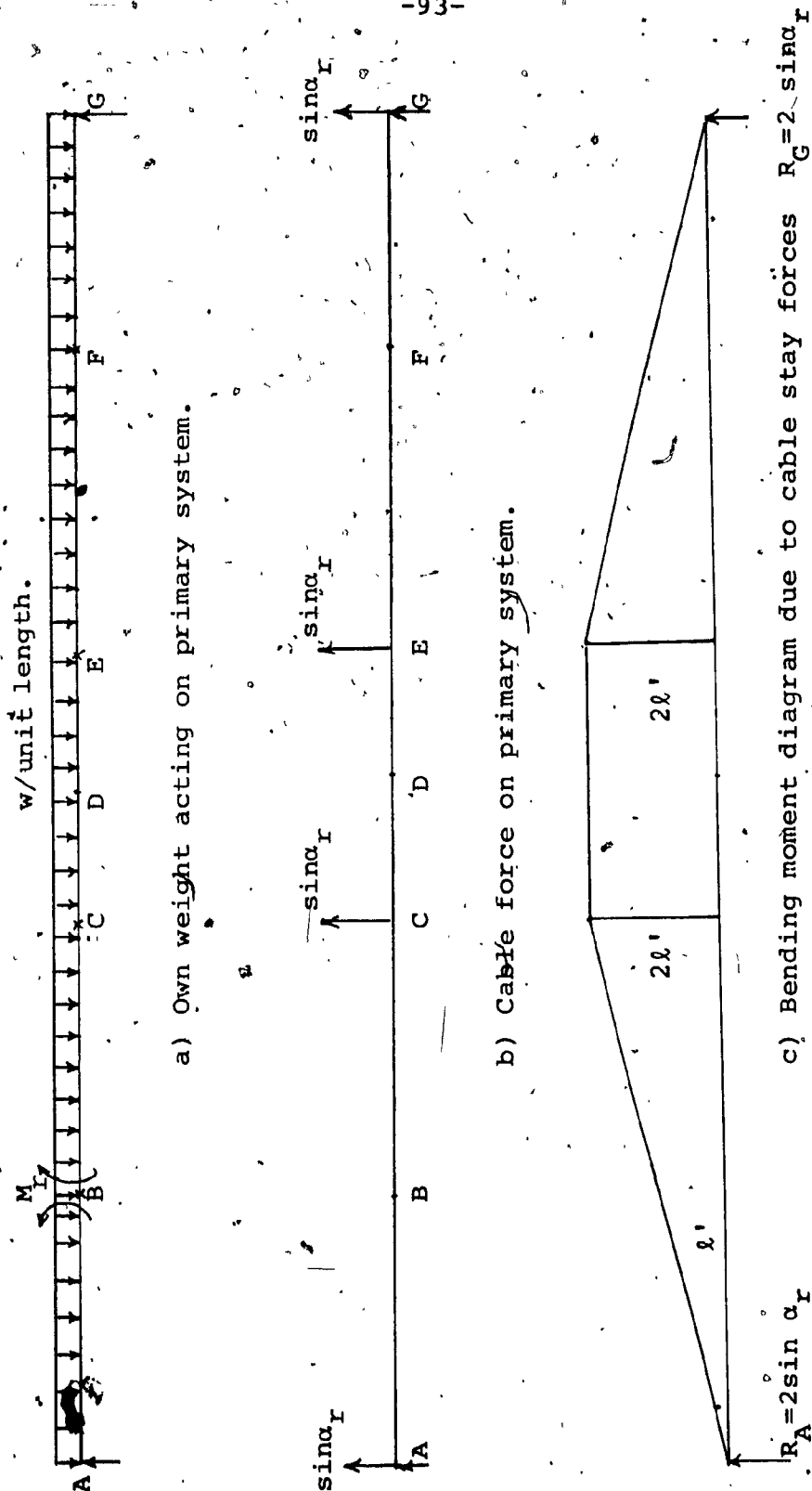


Fig. 3.22-Forces acting on primary system and bending moment diagram due to forces in cable stay.

CHAPTER-4

EFFECT OF CREEP IN PRECAST SEGMENTAL BRIDGES

4.1 CANTILEVER STAGE:

Creep deformation of concrete⁶ is that part of the inelastic deformation occurring with time under constrained load. The moment in the existing cantilever of precast segmental bridges will be modified by the creep. Creep deformations occur as a result of inelastic response of concrete to long term loadings, such as dead load, prestressing forces and the pre-tensioning forces in the cables stays. Restraint of creep deformations causes redistribution of moments.

The relation between creep and elastic deformations is linear. Their ratio is called the creep factor ϕ_t .

$$\epsilon_{cr} = \epsilon_e \cdot \phi_t \quad (4.1)$$

Where,

ϵ_{cr} = Creep strain

ϵ_e = Elastic strain

E = Modulus elasticity of concrete at 28 days

ϕ_t = Creep factor.

The modulus of elasticity of concrete can be calculated using the ACI Equation:

$$E = 33 w^{3/2} \sqrt{f'_c} \quad (4.2)$$

f'_c = Cylindrical strength of concrete in psi at time of erection.

w = Unit weight of concrete in lbs/cu.ft.

The modulus of elasticity may be assumed constant for precast segments after the concrete age reaches 28 days.

The creep factor, ϕ_t , can be considered to be the sum of recoverable creep, ϕ_d , and irrecoverable creep, ϕ_f .

$$\phi_t = \phi_d + \phi_f \quad (4.3)$$

Recoverable creep and irrecoverable creep are referred to as "Delayed plasticity" and flow" respectively. Both ϕ_d and ϕ_f are time dependent, but according to different relations.

Creep factor ϕ_t is given by,

$$\phi(t, t_0) = \phi_d \beta_d(t-t_0) + \phi_{f\infty} [\beta_f(t) - \beta_f(t_0)] \quad (4.4)$$

Where

$\phi(t, t_0)$ = Creep factor at time 't' for a concrete specimen loaded at time 't₀'.

$\phi_{d\infty}$ = Delayed elasticity at infinity.

$\beta_d(t-t_0)$ = Factor variable from zero to unity indicating the variation of ϕ_d with time.

$\phi_{f\infty}$ = "Flow" at infinity.

$\beta_f(t) - \beta_f(t_0)$ = Factor variable from zero to unity indicating the variation of ϕ_f with time.

t_0 = Age of concrete at loading (days).

t = Time after casting (days).

$\phi_{d\infty}$ = 0.4

The recoverable creep factor will have effect only for temporary loads acting on structure, such as those applied during construction.

The creep factor due to flow is given by

$$\phi_{f\infty} = \beta_{c1} \times \beta_{c2} \quad (4.5)$$

β_{c1} = Factor for accounting relative humidity of ambient medium and composition of concrete.

β_{c2} = Factor depending on the theoretical thickness of the structural element.

The deflection due to creep of concrete

$$= \delta_{el} \cdot \phi_t \quad (4.6)$$

δ_{el} = Elastic deflection of concrete.

$$\text{Total deflection} = \delta_{el} (1 + \phi_t) \quad (4.7)$$

The required tension force in the permanent cable stays will have to be modified accordingly as follows:

$$x_{r\phi} = x_r (1 + \phi_t) \quad (4.8)$$

The temporary or auxiliary cables contribute to the recoverable creep, whereas the dead load, prestressing forces, tension forces in cable stays contribute to irrecoverable creep deformations.

4.2 CONTINUOUS STAGE:

In the continuous stage the increase in angle of rotation is restrained. Hence the moment at the centre of the span has to be modified⁶ by the factor $(1 - e^{-\phi t})$. Accordingly, the deformations and the post tensioning forces required in the cable stays have to be modified by $(1 - e^{-\phi t})$.

$$x_{r\phi} = x_r(1-e^{-\phi t}) \quad (4.9)$$

In this analysis the effect of cable forces on the girder and corresponding creep of concrete has been considered.

CHAPTER-5

DESIGN CONSIDERATIONS AND NUMERICAL EXAMPLE

5.1. APPROXIMATE STRUCTURAL DESIGN PROCEDURE:

- 1) A preliminary set of sectional properties is assumed for all component members of the cable stayed bridge.
- 2) For these assumed sectional properties, stresses and displacements under the given load are determined, which should be within the limits specified (by The Task Committee On Cable Suspended Structures¹⁷).
- 3) A new set of sectional properties is chosen to satisfy the requirements of specifications.

Some of the sectional properties can be assumed as follows:

- 1) Height of tower in a cable stayed bridge =
 $(\frac{1}{6} \text{ to } \frac{1}{8})$ (main span).
2. The ratio of the depth of girder to main span length is a function of the maximum allowable ratio of live load deflection to main span. The economic depth-to-span ratio is in the range of 1/60 to 1/90.

- 3) Length of main span is 0.6 to 0.7 times the total length.
- 4) Allowable static deflection under loads $(\frac{1}{400} \text{ to } \frac{1}{500})$ span.
- 5) Optimum inclination of cable stays = 25° to 65° .

For preliminary design of cable system, the approximate maximum moments can be calculated using the following formulas.

For a system with 5 equal panels

$$M_{\max} = 0.007ql^2 \quad (5.1)$$

For a system with 7 equal panels

$$M_{\max} = 0.006ql^2 \quad (5.2)$$

Where q = dead load + live load.

5.2. DESIGN AND CHECK FOR THE CABLE STAYED STRUCTURE

SHOWN IN FIGURE 2.1:

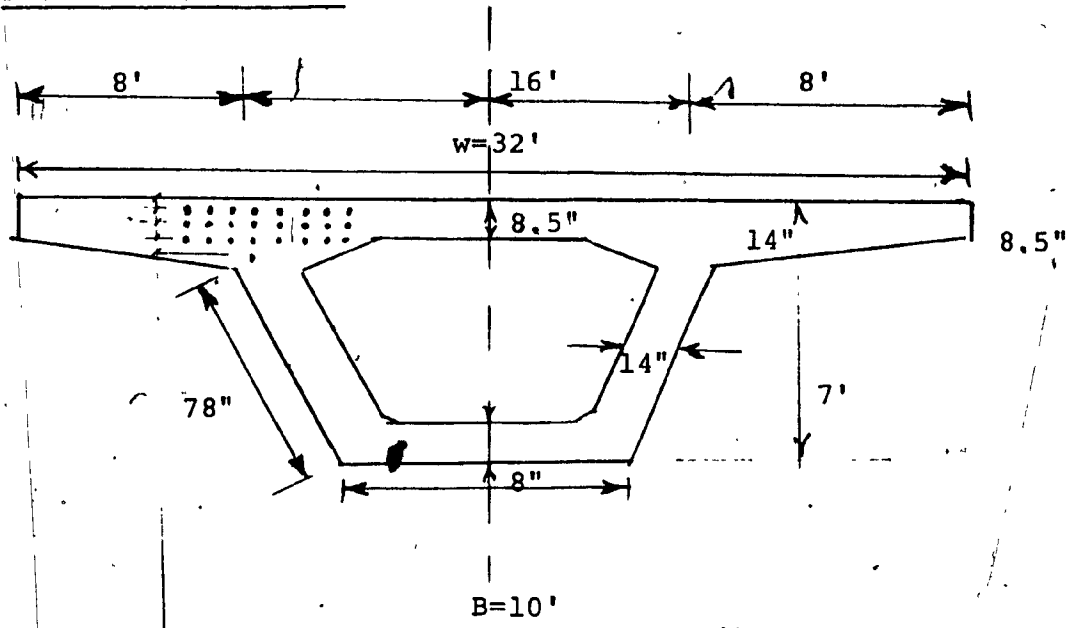


Figure 5.1

Referring to Figure 2.1 .

Length of main span 'L' = 510'.

Assume the dimensions of section as shown in figure 5.1.

Depth of girder = $\left(\frac{1}{60} \text{ to } \frac{1}{90}\right) (\text{span}) = \frac{1}{75} (\text{span})$.

$$= \frac{1}{75} (510) = 7 \text{ feet.}$$

$$(384 \times 8.5) 4.25 + \frac{(2 \times 5.5 \times 96)}{2} \left(8.5 + \frac{5.5}{3}\right)$$

$$+ \frac{(2 \times 5.5 \times 48)}{2} \left(8.5 + \frac{5.5}{3}\right) + (2 \times 14 \times 78) (49) + (120 \times 8) (80)$$

$$\bar{y} = \frac{(384 \times 8.5) + \frac{(2 \times 5.5 \times 96)}{2} + \frac{(2 \times 5.5 \times 48)}{2} + (2 \times 14 \times 78) + (120 \times 8)}{}$$

$$\bar{y} = \frac{13872 + 5456 + 2728 + 107016 + 69120}{3264 + 528 + 264 + 2184 + 864}$$

$$\bar{y}_t = 28" \quad \bar{y}_b = 56"$$

$$\text{Dead weight} = \frac{7104}{144} (150) = 7400 \text{ lbs/ft.}$$

$$w = 7.4 \text{ kips/ft.}$$

Let the concentrated load due to equipment

$$P = 25 \text{ kips.}$$

$$\begin{aligned} I_{xx} &= \frac{384 \times 8.5^3}{12} + (384 \times 8.5) (28 - 4.25)^2 \\ &+ 2 \left[\left(\frac{1}{12} \times 96 \times 5.5^3 \right) + \left(\frac{1}{2} \times 96 \times 5.5 \right) (28 - 8.5)^2 \right] \\ &+ 2 \left[\left(\frac{1}{12} \times 48 \times 5.5^3 \right) + \left(\frac{1}{2} \times 48 \times 5.5 \right) (28 - 8.5)^2 \right] \\ &+ 2 \left[\frac{14 \times 78^3}{12} \times \frac{2}{\sqrt{3}} + (14 \times 78) (56 - 35)^2 \right] \\ &+ \frac{108 \times 8^3}{12} + (108 \times 8) (80 - 28)^2 \end{aligned}$$

$$\begin{aligned} I_{xx} &= 19652 + 1841100 + 202103 + 101052 + 2258218 \\ &+ 4608 + 2336256 \end{aligned}$$

$$I_{xx} = 67630000 \text{ in}^4 = 327 \text{ ft}^4.$$

SECTION MODULUS:

$$z_b = \frac{I}{y_b} = \frac{6763000}{56} = 120768 \text{ in}^3 = 70 \text{ ft}^3.$$

$$z_t = \frac{I}{y_t} = \frac{6763000}{28} = 241536 \text{ in}^3 = 140 \text{ ft}^3.$$

1. CANTILIEVER STAGE:

Maximum moment due to (dead load of girder and concentrated load of equipment) is

$$\begin{aligned} M_{\max} &= \frac{7400(250)^2}{2} + 25000(250) \\ &= 23125 \times 10^4 + 625 \times 10^4 \\ &= 23750 \times 10^4 \text{ lb-ft.} \\ &= 237500 \text{ kips-ft.} \end{aligned}$$

Assuming a concrete strength of $f'_c = 6000 \text{ psi}$.

Maximum allowable stresses in concrete are:

$$\begin{aligned} f_{cbf} &= 0.45 \times 6000 = 2700 \text{ psi} \\ f_{cbt} &= 0.6 \times 6000 = 3600 \text{ psi} \\ f_{ctf} &= 6 \times \sqrt{f'_c} = 464 \text{ psi.} \\ f_{ctt} &= 3 \times \sqrt{f'_c} = 232 \text{ psi.} \end{aligned}$$

Providing the prestressing tendons as shown in figures 5.1, $e = 20"$

Limiting the tensile stress in top fibres
 $= -464 \text{ psi.}$

$$\frac{P}{A} + \frac{P_e}{Z_t} - \frac{M_{d.l}}{Z_t} = -464$$

$$P \left(\frac{1}{7104} + \frac{20}{241536} \right) = \frac{23750 \times 10^4 \times 12}{241536} - 464$$

$$\text{Prestressing force } P = \frac{11799.50 - 464}{1.41 \times 10^{-4} + 8.28 \times 10^{-4}}$$

$$= \frac{11335.50 \times 10^4}{9.69} = 11698000 \text{ lbs.}$$

$$P = 1170 \text{ kips.}$$

Using 50 cables made of 7-0.6" diameter strands of 270 ksi'.

Maximum stressing forces of each strand = 58.1 kips.

Effective prestressing force = 34.86 kips.

Total Prestressing force provided = $50 \times 7 \times 34.86 = 12201 \text{ kips.}$

Each segment is stressed to 488 kips.

Therefore $F_1 = F_2 = F_3 = \dots = F_n = 488 \text{ kips.}$

$$\Sigma F_n = 12201 \text{ kips.}$$

RESULTS

TABLE-1.

Values of deflection at free end of girder during different stages of erection.

Girder depth. Span in ft.	Without cable stays.		With Cables	x_n in kips.
	L/75 Deflection in ft.	L/25 Deflection in ft.	L/75 Deflection in ft.	
10	0.0000	0.0000	0.0000	
20	0.0023	-0.0004	-0.0000	
30	0.0020	-0.0014	0.0010	
40	0.0072	-0.0029	0.0052	
50	0.0173	-0.0049	0.0154	
60	0.0424	-0.0070	0.0357	
70	0.0816	-0.0087	0.0711	
80	0.1433	-0.0094	0.1276	
90	0.2346	-0.0083	0.2121 -2.3140	3000
100	0.3350	-0.0045	-2.5450	
110	0.5395	0.0031	-2.7550	
120	0.7727	0.0160	-2.9350	
130	1.0742	0.0356	-3.0690	
140	1.4563	0.0635	-3.1470	
150	1.0322	0.1015	-3.1460	
160	2.5163	0.1516	-3.0630	
170	3.2235	0.2159	-2.8690 -5.9900	1670
180	4.0703	0.2967	-5.7000	
190	5.0734	0.3964	-5.2700	
200	6.2520	0.5178	-4.6800	
210	7.6246	0.6635	-3.7600	
220	9.2216	0.8366	-2.5000	

TABLE-1 Continued

230	11.0342	1.0400	-0.8400	404
240	13.1148	1.2777	1.2700 -2.3600	
250	14.4764	1.4423	0.0000	

2. CONTINUOUS STAGE:

Main span length $= l = 510'$

For a system with 5 equal panels, the approximate maximum bending moment is given by :

$$M_{\max} = 0.007 q l^2 \text{ (Influence lines}^1\text{)}$$

Dead load + Live load $= 7.4 + 0.640 = 8.04 \text{ kips/ft.}$

$$M_{\max} = 0.007 \times 8.04 \times (510)^2$$

$$= 14639 \text{ kips} - \text{ft.}$$

$$z_t = 140 \text{ ft}^3$$

$$z_t = 70 \text{ ft}^3$$

$$I = 327 \text{ ft}^4$$

Eccentricity of prestressing tendons $e = 20" = 1.67'$

Maximum allowable tensile stress $= 6 \sqrt{f'_c} = 464 \text{ psi}$

$$f_{cbt} = \frac{P}{A} + \frac{P e}{z_b} - \frac{M_{\max}}{z_b}$$

$$z_b = 120768 \text{ in}^3$$

$$-464 = \frac{P}{7104} + \frac{1.67 P}{120768} - \frac{14639 \times 10^3 \times 12}{120768}$$

$$P = \frac{-464 + 1415}{1.407 \times 10^{-4} + 0.138 \times 10^{-4}} = 6155340 \text{ lbs.}$$

$$= 6156 \text{ kips.}$$

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Using 22 cables each made of 7-0.6" strands

$0.7 f_{pu} A_{pu}$ for each strand = 40.7 kips.

Prestressing force provided = 6268 kips = 6268 kips.

The results are provided by the computer program.

6. CONCLUSIONS

The cable stayed bridge has numerous advantages over a structure without cable stays. A comparison can be made between the two types of structures by referring to the results presented for displacements and camber to be provided during erection, in Table 1.

- 1) For the same depth of the girder a cable stayed structure is structurally more rigid than the one without cables. As we can see from results presented in Table 1, deflection is almost nil, whereas in a structure without cables the deflections become very large with increasing spans.
- 2) A structure without cable stays requires 3 to 4 times the depth of girder in order to provide the same rigidity as that of a cable stayed bridge. This means, for a main span length of 510' a cable stayed bridge requires 5-7', whereas the one without cables requires 20-25' depth of girder.
- 3) The smaller sections and design moments reduce the prestressing forces required in the case of cable stayed structures thus eliminating the need for special anchorages.

- 4) In case of cable stayed bridges it possible to produce the desired initial camber in the girder by adjusting the pre-tensioning forces in the cable stays, so that the deflections are within allowable limits.

Upto a certain span length, the desired initial camber can also be obtained in a structure without cable stays by adjusting the prestressing forces in the girder. But for very large spans these prestressing forces become so large that it may be impossible to accommodate the strands in the girder sections even by providing buildouts.

- 5) For large spans we observe that any increase in prestressing forces in the girder has negligible effects in reducing the deflections in the stiffening girder. But in cable stayed bridges even a small increase in pre-tensioning forces in cable stays results in considerable improvement in reducing deflections.
- 6) Concrete cable stayed bridges become more economical over¹³ other structural types (suspension, arch-cantilever, cantilever) of structures with respect to the concrete volume required.

7. A structure without cable stays erected by double cantilever method required temporary supports, which can be eliminated in case of cable stayed bridges by using auxiliary cables.
8. Prestressed concrete girders with cable stays perform better than those without cable stays, since the horizontal components from the cable stays have the same effect as the prestressing in the girder. This in turn, adds to the rigidity of the structure.
9. Erection and handling of precast prestressed segments is easier than reinforced concrete segments, since in the latter case couplers, have to be used to join the reinforcements from adjacent segments.

```

PROGRAM ONE (INPUT,OUTPUT)
C   THIS PROGRAM CALCULATES DEFLECTION IN POST-
C   TENSIONED SEGMENTAL CANTILEVER BRIDGES
C   DURING ERECTION AT DIFFERENT STAGES
C   DEPTH OF THE STIFFENING GIRDER USED=L/75
C   EG=MODULUS OF ELASTICITY OF GIRDER
C   IG=MOMENT OF INERTIA OF GIRDER
C   W1=SELF WEIGHT OF GIRDER
C   W2=WEIGHT OF MOVING EQUIPMENT
C   B=SEGMENT LENTH
C   N=NUMBER OF SEGMENT
C   F1(1)= PRESTRESSING FORCE WHEN FIRST
C   SEGMENT IS ERECTED.
C   E1= ECCENTRICITY OF TENDON,
C   SIGMAF=TOTAL PRESTRESSING FORCE
C   D(I)= DEFLECTION AT DIFFERENT POINT
DIMENSION SIGMAF(25),D(25),F1(2)
REAL IG
INTEGER B
READ*,EG,IG,W1,W2,B,N,F1(1),E1
  SIGMAF(1)=F1(1)
  DO 5 I=2,N
    SIGMAF(I)=SIGMAF(I-1)+F1(1)
    D(I)=(((I*B)**2)/(EG*IG))* (W1*((I*B)
    **2)/8+(W2*I*B)/3-(SIGMAF(I)*E1)/4)
5    CONTINUE
  PRINT 10,EG,IG,W1,W2,B,F1(1),E1
10  FORMAT('1',25X,'EG=',F12.1,2X,'IG=',F6.1,
  $2X,'W1=',F5.2,2X,'W2=',F6.2,2X,'B=',
  $I3,/,25X,'F(1)=' ,F8.2,2X,'E1=' ,F4.2)
  PRINT 30
30  FORMAT('1',25X,'NO OF SEGMENTS',4X,
  $'TOTALTPRESTRESS',4X,'TOTAL DEFLECTION')
  DO 25 I=1,N
    PRINT 40,I,SIGMAF(I),D(I)
40  FORMAT(28X,I3,15X,F10.1,10X,F12.8,/)
25  CONTINUE
  STOP
  END

```

> ?

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EG= 676800.0 IG=4000.0 W1=15.00 W2= 25.00 B= 10

F(1)= 1332.00 EI=6.25

NO OF SEGMENTS	TOTALTPRESTRESS	TOTAL DEFLECTION
1	1332.0	0.00000000
2	2664.0	-.00047959
3	3996.0	-.00143160
4	5328.0	-.00295016
5	6660.0	-.00489628
6	7992.0	-.00696476
7	9324.0	-.00868417
8	10656.0	-.00941686
9	11988.0	-.00835896
10	13320.0	-.00454036
11	14652.0	.00317525
12	15984.0	.01609043
13	17316.0	.03567393
14	18648.0	.06356075
15	19980.0	.10155211
16	21312.0	.15161545
17	22644.0	.21588442

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18	23976.0	.29665891
19	25308.0	.39640503
20	26640.0	.51775512
21	27972.0	.66350773
22	29304.0	.83662763
23	30636.0	1.04024584
24	31968.0	1.27765957
25	33300.0	1.55233228

S.

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EG= 676800.0 IG= 327.0 W1= 7.40 W2= 25.00 B= 10

F(1)= 488.00 E1=1.67

NO OF SEGMENTS TOTALTPRESTRESS TOTAL DEFLECTION

1	488.0	0.00000000
2	976.0	.00023349
3	1464.0	.00191652
4	1952.0	.00721780
5	2440.0	.01932175
6	2928.0	.04241590
7	3416.0	.08169087
8	3904.0	.14334039
9	4392.0	.23456129
10	4880.0	.36355350
11	5368.0	.53952004
12	5856.0	.77266706
13	6344.0	1.07420378
14	6832.0	1.45634254
15	7320.0	1.93229878
16	7808.0	2.51629104
17	8296.0	3.22354096

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18	8784.0	4.07027328
19	9272.0	5.07371584
20	9760.0	6.25209958
21	10248.0	7.62465857
22	10736.0	9.21162993
23	11224.0	11.03425392
24	11712.0	13.11477390
25	12200.0	15.47643630

IS.

```
PROGRAM INVERT(INPUT,OUTPUT,TAPE 3=OUTPUT)
DOUBLE PRECISION A(20,20),B(20,20),F(20,20),
X(9),D(9)
```

THIS PROGRAM CALCULATES THE POST TENSIONING FORCES IN CABLE STAYS, REACTIONS AT INTERMEDIATE SUPPORTS AND ROTATION OF TOWERS.

L=LENGTH OF MAIN SPAN
L1=LENGTH OF FLANKING SPANS
SIGFN=TOTAL PRESTRESSING FORCE AT INTERMEDIATE SUPPORTS DUE TO GROUP 1 CANTILEVER TENDONS IN MAIN SPAN.
SIGFR=TOTAL PRESTRESSING FORCE IN THE TAIL SPAN PROVIDED OVER THE INTERMEDIATE SUPPORTS DUE TO GROUP 1. CANTILEVER TENDONS.
SIG3B=PRESTRESSING FORCE DUE TO GROUP 3B CONTINUITY TENDONS.
SIG3A=PRESTRESSING FORCE DUE TO GROUP 3A CONTINUITY TENDONS.
SIGF2=PRESTRESSING FORCE DUE TO GROUP 2 TENDONS
EN=ECCENTRICITY OF GROUP 1 CANTILEVER TENDONS IN MAIN SPAN.
ER=ECCENTRICITY OF GROUP 1 CANTILEVER TENDONS IN TAIL SPAN.
E3B=ECCENTRICITY OF GROUP 3B TENDONS
E3A=ECCENTRICITY OF GROUP 3A TENDONS
E2=ECCENTRICITY OF GROUP 2 TENDONS
2*S1=TOTAL HORIZONTAL PROJECTION OF GROUP 3A TENDONS.
EG=MODULUS OF ELASTICITY OF GIRDER
IG=MOMENT OF INERTIA OF GIRDER
Q=DEAD LOAD+LIVE LOAD ON THE GIRDER
LR=LENGTH OF PERMANENT CABLES PROVIDED
ALR=INCLINATION OF CABLE STAYS TO THE HORIZONTAL
AE = CABLE CROSS SECTIONAL AREA
EE=EQUIVALENT MODULUS OF ELASTICITY OF CABLE STAYS.
BA,BC=LENGTH OF GIRDER UP TO POINT OF CABLE CONNECTION.
HT=HEIGHT OF TOWER
AT=CROSS SECTIONAL AREA OF TOWER
ET=MODULUS OF ELASTICITY OF TOWER MATERIAL
FCC,FBB,FBC-----=FLEXIBILITIES OF THE GIRDER FOR THE PRIMARY SYSTEM.FIRST LETTER F REPRESENTS THE FLEXIBILITY , SECOND LETTER REPRESENTS THE POINT WHERE THE FLEXIBILITY IS REQUIRED, LAST LETTER REPRESENTS THE POINT WHERE THE UNIT LOAD IS APPLIED.
CC=ELASTIC STRETCH OF CABLE AT C
CA=ELASTIC STRETCH OF CABLE AT A
FT=FLEXIBILITY OF TOWER
DPD=DEFLECTION DUE TO PRIMARY PRESTRESS AT CENTRE OF MAIN SPAN D.

C
C
C
C
C
C
C
C

OFMAIN SPAN D.
DPC=DEFLECTION DUE TO PRIMARY PRESTRESS AT C
DPB=DEFLECTION DUE TO PRIMARY PRESTRESS AT B
DIB,DIC=DEAD LOAD +LIVE LOAD DEFLECTION AT B C
RESPECTIVELY (UNIFORM LOAD)
DIBC,DICC=DEFLECTION DUE TO CONCENTRATED LIVE LOAD
AT B,C RESPECTIVELY.

C
C

REAL L,L1,IG,LR
COMMON SIGFN,SIGFR,SIG3B,SIG3A,SIGF2,EN,ER,E3B
COMMON E3A,E2,S1,Q,WC,E0,EA,S3
READ*,SIGFN,SIGFR,SIG3B,SIG3A,SIGF2,EN,ER,E3B
READ*,E3A,E2,L,L1,S1,EG,IG,Q,LR,ALR,AE,EE
READ*,BA,BC,HT,AT,ET,WC,E0,EA,S3
TO CALCULATE THE FLEXIBILITY MATRIX
FOR THE CABLE STAYED STRUCTURE
CC=LR/(AE*EE*(SIN(ALR)**2))
CA=LR/(AE*EE*(SIN(ALR)**2))
FT=HT/(AT*ET)
FBB=(L1**2)*(L1+2*L)*(L+L1)/(6*EG*IG*(L+2*L1))
FBC=(L1**3)*(4*L1+3*L)/(6*EG*IG*(L+2*L1))
FCB=(L1**3)*(4*L1+3*L)/(6*EG*IG*(L+2*L1))
FCC=4*(L**2)*(L1**2)/(3*EG*IG*(L+2*L1))
FCE=2*(L1**2)*((L**2)+4*L*L1-4*(L1**2))
/ (3*EG*IG*(L+2*L1))
FCF=(L1**2)*((L**2)+4*L*L1-(L1**2))/
(3*EG*IG*(L+2*L1))
FEC=2*(L1**2)*((L**2)+4*L*L1-4*(L1**2))
/ (3*EG*IG*(L+2*L1))
FEB=(L1**2)*((L**2)+4*L*L1-(L1**2))/(3*EG*IG*
(L+2*L1))
FFB=(L1**2)*(2*(L1**2)+(L**2)+4*L*L1)/
(6*EG*IG*(L+2*L1))
FBE=(L1**2)*((L**2)+4*L*L1-(L1**2))/(3*EG*IG*
(L+2*L1))
FBF=(L1**2)*(2*(L1**2)+(L**2)+4*L*L1)/(6*EG*IG*
(L+2*L1))
F(1,1)=CA+FT
F(1,2)=0
F(1,3)=FT
F(1,4)=-BA
F(2,1)=0
F(2,2)=FBB+FBF
F(2,3)=FBC+FBE
F(2,4)=0
F(3,1)=FT
F(3,2)=FCB+FCF
F(3,3)=FCC+FCE+CC+FT
F(3,4)=BC
F(4,1)=-BA
F(4,2)=0
F(4,3)=BC
F(4,4)=0
F(1,5)=1

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F(1,5)=1
F(1,6)=0
F(1,7)=0
F(1,8)=0
F(2,5)=0
F(2,6)=1
F(2,7)=0
F(2,8)=0
F(3,5)=0
F(3,6)=0
F(3,7)=1
F(3,8)=0
F(4,5)=0
F(4,6)=0
F(4,7)=0
F(4,8)=1
DO 2 I=1,4
DO 1 J=1,8
A(I,J)=F(I,J)
1 CONTINUE
2 CONTINUE
WRITE(3,4)
4 FORMAT(1H1,6X,'FLEXIBILITY MATRIX A',//)
WRITE(3,5) ((A(I,J),J=1,4),I=1,4)
5 FORMAT(10X,4F12.6,///)
C TO FORMULATE THE INVERSE OF THE
C FLEXIBILITY MATRIX
DO 9 K=1,4
DO 6 J=1,8
6 A(K+4,J)=A(K,J)/A(K,K)
K1=K+1
K3=K+3
DO 8 I=K1,K3
B(I,K)=A(I,K)
DO 7 J=1,8
7 A(I,J)=A(I,J)-A(K+4,J)*B(I,K)
8 CONTINUE
9 CONTINUE
WRITE(3,21)
21 FORMAT(//,4X,' INVERSE OF MATRIX A ',//)
WRITE(3,5) ((A(I,J),J=5,8),I=1,8)
C TO CALCULATE THE POST TENSIONING FORCES
C IN CABLES, INTERMEDIATE REACTIONS AND
C ROTATION OF TOWERS
CALL PSTRESS(L,L1,IG,EG,DB,DC)
D(5)=0
D(6)=DB
D(7)=DC
D(8)=0
DO 40 K=1,4
X(K)=0
DO 3 J=5,8
X(K)=X(K)+A(K+4,J)*D(J)
3 CONTINUE

```

```

3      CONTINUE
40     CONTINUE
      WRITE(3,42)
42     FORMAT('1',6X,'VERT COMPONENT OF TENSION',1X,
$      'REACTION',1X,'ROTATION OF TOWERS',//)
      WRITE(3,43) (X(K),K=1,4)
43     FORMAT(5X,4(F14.7),//)
      XR=X(1)/SIN(3.142*45/180)
      WRITE(3,44)
44     FORMAT('1',20X,'POST TENSION FORCE')
      WRITE(3,45) XR
45     FORMAT(25X,F12.4)
      STOP
      END
      SUBROUTINE PSTRESS(L,L1,IG,EG,DB,DC)
      THIS PROGRAM IS TO CALCULATE THE DEFLECTION
      DUE TO PRESTRESSING FORCES DUE TO TOTAL
      LOADS ACTING ON THE STIFFENING GIRDER
      COMMON SIGFN,SIGFR,SIG3B,SIG3A,SIGF2,EN,ER,E3B
      COMMON E3A,E2,S1,Q,WC,E0,EA,S3
      REAL L,L1,IG
      C1=SIGFN*EN*((N*B)**2)/(3*EG*IG)
      C2=(SIG3B*E3B)*L*(L+4*L1)/(8*EG*IG)
      C3=S1*(2*SIG3A*E3A-SIG3A*E0)*(16*L1+8*L-7*S1)
      / (48*EG*IG)
      F=E2+(E0+EA)/2
      C4=10*SIGF2*(S3**2)*(4*F-3*EA-3*E0)/(27*EG*IG)
      DPD=-C1+C2-C3-C4
      DPC=DPD*SIN((2*3.142*L1)/(L+2*L1))
      DPB=DPD*SIN(3.142*L1/(L+2*L1))
      PRINT 50,DPD,DPB,DPC
50     FORMAT('1',12X,'DPD=',F10.4,5X,'DPB=',F10.4
$      ,5X,'DPC=',F10.4)
      C6=Q*(L+2*L1)*(L1**3)/(12*EG*IG)
      C7=Q*(L1**4)/(24*EG*IG)
      C8=Q*((L+2*L1)**3)*L1/(24*EG*IG)
      C9=2*Q*(L1**3)*(L+2*L1)/(3*EG*IG)
      C10=2*Q*(L1**4)/(3*EG*IG)
      C11=Q*L1*(L+2*L1)**3/(12*EG*IG)
      DIB=-C6+C7+C8
      DIC=-C9+C10+C11
      DICB=4*WC*(L**2)*(L1**2)/(6*EG*IG*(L+2*L1))
      DICC=WC*L*(4*L*(L1**2)+3*(L1**3))/(3*EG*IG
      (L+2*L1))
      DB=DIB+DPB+DICB
      DC=DIC+DPC+DICC
      PRINT 51,DIB,DIC,DICB,DICC
51     FORMAT('1',8X,'DIB=',F10.4,2X,'DIC=',F10.4,2X,
$      'DICB=',F6.4,2X,'DICC=',F6.4)
      PRINT 52,DB,DC
52     FORMAT('1',20X,'DB=',F14.4,5X,'DC=',F14.4)
      RETURN
      END

```

*53 LINES.

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1 FLEXIBILITY MATRIX A

.000431	0.000000	.000025	-170.000000
0.000000	.037739	.039219	0.000000
.000025	.039219	.104028	170.000000
-170.000000	0.000000	170.000000	0.000000

INVERSE OF MATRIX A

15.685722	-16.300848	15.685722	-.005840
-16.300848	43.438056	-16.300848	-.000044
15.685722	-16.300848	15.685722	.000042
-.005840	-.000044	.000042	-.000000

1. DPD= -3.1514 DPB= -1.8526 DPC= -2.9973
1 DIB= 146.6528 DIC= 235.1502 DICB= .2398 DICC= .5994
1 DB= 145.0400 DC= 232.7522
2 VERT COMPONENT OF TENSION REACTION, ROTATION OF TOWERS

1286.6119132 2506.1965473 1286.6119132 .0034525

1 POST TENSION FORCE
1819.3587

> ?

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