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DESIGN AND ANALYSIS OF AC HARMONIC FILTER  
NEAR A HVDC STATION  
Using  
OPTIMIZATION TECHNIQUES

Murali K. Komaragiri

A Thesis  
in  
The Department  
of  
Electrical and Computer Engineering

Presented in Partial Fulfillment of the Requirements  
for the Degree of Master of Engineering at  
Concordia University  
Montréal, Québec, Canada.

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## **ABSTRACT**

### **Design Of AC Harmonic Filter Near a HVDC Station Using Optimization Techniques**

**Murali K. Komaragiri**

This research work proposes a new method of designing an AC harmonic filter connected to an HVDC converter bus. The traditional method of designing an AC harmonic filter is based on a trial-and-error approach which is both time-consuming and tedious. The proposed method, however uses optimization techniques and replaces the trial-and-error approach with a faster iterative approach.

The principal objective of the present research is to propose a methodology for designing an optimal filter. This process involves expressing the filter cost equation as the objective function and expressing various filter design and performance specifications as constraint functions. The optimization technique is applied to design the filter, yielding an optimal filter. A test case showed that the filter designed with the proposed method is comparable to a filter designed with the traditional methods.

The proposed method has the capability to perform sensitivity analysis type of studies where the impact of one filter parameter on the overall cost of the total filter can be investigated.

Another feature of the optimization process is the ability to compute the worst harmonic voltage and a method to obtain this is given.

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## SYMBOLS USED

### Current Symbols

$I_a$	: Line Current
$I_1$	: Fundamental Current
$I_d$	: Direct Current
$I_r$	: Current Through Resistor
$I_L$	: Current Through Inductor
$I_C$	: Current Through Capacitor
$I_h$	: Harmonic Current
$I_f$	: Current Through Filter
$I_s$	: Current Through System
$ I_R $	: Absolute Value of Current Through Resistor
$I_{rms}$	: RMS Current
$IT$	: Harmonic Current Factor
$(I_L)_h$	: Inductor Current at Harmonic Loading

### Voltage Symbols

$V_s$	: Supply Voltage
$V_L$	: Line Voltage
$V_1$	: Fundamental Voltage
$V_h$	: Harmonic Voltage
$V_{Lh}$	: Worst Harmonic Voltage
$V_C$	: Voltage Across Capacitor
$V_L$	: Voltage Across Inductor
$V_r$	: Voltage Across Resistor.

### Impedance Symbols

$Z$	Impedance
$Z_s$	: System Impedance
$Z_f$	: Filter Impedance
$Z_t$	: Total Impedance
$Z_{st}$	: Single Tuned Filter Impedance
$Z_{hp}$	: High-Pass Filter Impedance
$Y_s$	: System Admittance
$Y_f$	: Filter Admittance
$Y_t$	: Total Admittance
$Y_{st}$	: Single Tuned Filter Admittance
$Y_{hp}$	: High-Pass Filter Admittance

### Basic Filter Design Symbols

$C_f$	: Filter Capacitor
$P$	: Active Power
$Q_p$	: Reactive Power
$Q_t$	: Total Reactive Power
$Q$	: Filter Quality Factor
$X_t$	: Transformer Reactance
$X_0$	: Reactance at Tuned frequency
$\delta$	: AC system frequency deviation
$h$	: Harmonic Order
$p$	: Pulse Number
$q$	: Integer Number
$R$	: Filter Resistor
$L$	: Filter Inductor

**C:** Filter Capacitor  
**M:** Damping Factor for the high-pass  
 **$\omega$ :** Angular Frequency  
 **$\omega_n$ :** Tuned Angular Frequency  
 **$X_0$ :** Reactance At Tuned Frequency  
 **$f_0$ :** Starting harmonic frequency for high-pass filter  
 **$F_r$ :** Resonant Frequency  
 **$F_1$ :** Frequency One  
 **$F_2$ :** Frequency Two  
 **$R_i$ :** Radius of the system harmonic impedance locus  
 **$\theta_m$ :** Maximum cut-off harmonic impedance angle  
 **$\theta_f$ :** De-tune angle of the filter

#### Performance Specifications Symbols

**$K_h$ :** Coupling Co-efficient  
 **$P_h$ :** Weighting factor of the harmonic order  
 **$m$ :** Highest harmonic order considered  
 **$X_c$ :** Reactance due to Capacitance at fundamental  
 **$X_l$ :** Reactance due to Inductance at fundamental  
 **$Q_h$ :** Reactance power at harmonic frequencies  
 **$Q_f$ :** Reactance power of the filter at fundamental frequency  
 **$Q_T$ :** Total reactance power

#### Filter Cost Symbols

**$U_k$ :** Constant cost component  
 **$U_L$ :** Inductor incremental cost

$U_c$ : Capacitor incremental cost  
 $S$ : Filter size  
 $SC_{fl}$ : Single tuned filter capacitor size at  
fundamental Loading  
 $SC_{hl}$ : Single tuned filter capacitor size at harmonic  
Loading  
 $SC_{tl}$ : Single tuned filter capacitor total size  
 $SP_c$ : Loss factor of the capacitor  
 $K_{cl}$ : Loss factor of the capacitor  
 $SI_{fl}$ : Single tuned filter Inductor size at fundamental  
loading  
 $SI_{hl}$ : Single tuned filter Inductor size at harmonic  
loading  
 $SI_{tl}$ : Single tuned filter Inductor total size  
 $SP_r$ : Power loss due to a resistor  
 $P_v$ : Present value factor  
 $P_{vl}$ : Present value cost energy loss  
 $U_u$ : Cost of energy loss per kilowatt-hour  
 $F_u$ : Filter utilization factor  
 $U_T$ : Total constant cost  
 $ST_{cost}$ : Single tuned filter total cost  
 $h_0$  : Ratio of the starting frequency of high-pass filter  
to the supply frequency  
 $HC_{fl}$ : High-pass filter capacitor size at fundamental  
Loading  
 $HC_{hl}$ : High-pass filter capacitor size at harmonic  
Loading

$HC_{T1}$ : High-pass filter capacitor total size  
 $HC_{p1}$ : Power loss due to the capacitor  
 $HI_{f1}$ : High-pass filter Inductor size at fundamental loading  
 $HI_{h1}$ : High-pass filter Inductor size at harmonic loading  
 $HI_{T1}$ : High-pass filter Inductor total size  
 $HLP_{f1}$ : High-pass filter Inductor power loss at fundamental loading  
 $HLP_{h1}$ : High-pass filter Inductor power loss at harmonic loading  
 $HLP_{T1}$ : High-pass filter Inductor total loss  
 $HR_f$ : High-pass filter power loss due to resistor at fundamental loading  
 $HR_h$ : High-pass filter power loss due to resistor at harmonic loading  
 $HR_T$ : High-pass filter Total power loss due to resistor  
 $(X_L)_h$ : Inductor reactance at harmonic loading  
 $R_L$ : Resistance in the Inductor of high-pass filter  
 $(R_L)_h$ : High-pass filter Inductor's resistor at harmonic frequencies  
 $(R_L)_h$ : High-pass filter Resistor at harmonic frequencies  
 $Q_L$ : High-pass filter Inductor Quality factor  
 $P_{vh}$ : High-pass filter present value factor  
 $HP_{cost}$ : High-pass filter total cost



### Optimization Symbols

$\mathbf{x}^k$ : Feasible trial point

$\bar{\mathbf{x}}$ : Optimal point

$\mathbf{x}^*$ : Local optimal point

$\hat{C}(\mathbf{x})$ : Constraint function

$\hat{A}(\mathbf{x})^T$ : Jacobian matrix

$\lambda$ : Lagrange multiplier

$L(\mathbf{x}, \lambda)$ : Lagrange function

$\phi$ : AC network admittance angle

$D$ : Maximum diameter of the AC impedance locus

$d$ : Minimum diameter of the AC impedance locus

## **LIST OF ABBREVIATIONS**

<b>THD</b>	<b>:</b>	<b>Total Harmonic Distortion</b>
<b>TIFF</b>	<b>:</b>	<b>Telephone Interference Factor</b>
<b>HVDC</b>	<b>:</b>	<b>High Voltage Direct Current</b>
<b>AC</b>	<b>:</b>	<b>Alternating Current</b>
<b>DC</b>	<b>:</b>	<b>Direct Current</b>
<b>(M) VAr</b>	<b>:</b>	<b>Reactive Power Unit</b>
<b>SVC</b>	<b>:</b>	<b>Static VAr Compensators</b>
<b>(M) W</b>	<b>:</b>	<b>Active Power Unit</b>
<b>RMS</b>	<b>:</b>	<b>Root Mean Square</b>
<b>RSS</b>	<b>:</b>	<b>Root Sum Square</b>

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**CHAPTER ONE**  
**1.0 INTRODUCTION**

**1.1 REVIEW**

**1.1.1 Defining Harmonic Filter**

**1.2 OBJECTIVE**

**1.3 OUTLINE OF THE RESEARCH**

## 1.1 REVIEW

AC harmonic filter design is an integral and important part of HVDC converter station design. The AC filter constitutes about 15% [3] of the total cost of the converter station. The AC harmonic filter at an HVDC converter station (Figure 1.0) is designed to control the converter generated level of AC harmonics which are injected into the AC system, in addition to providing reactive power for the converter.

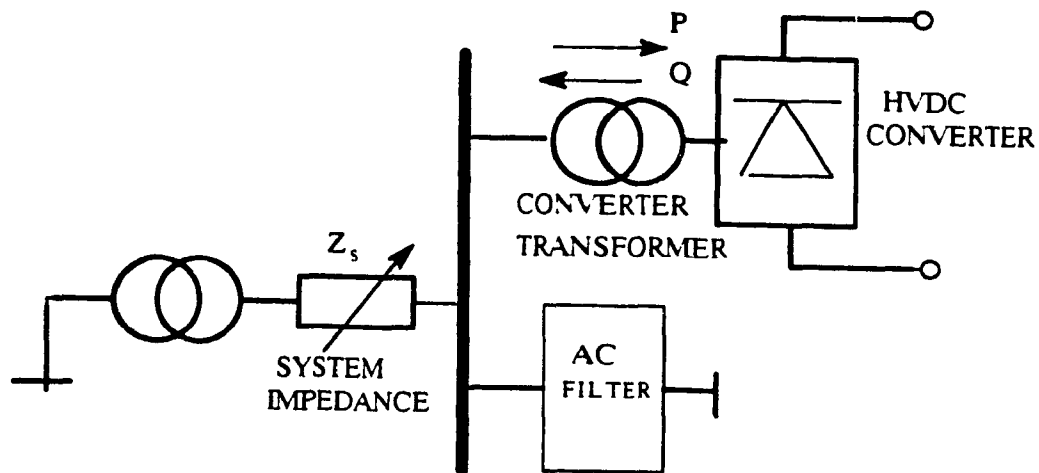


Figure 1.0 LAYOUT OF HVDC CONVERTER STATION

Traditionally, designing the filter is a tedious assignment. Trial-and-error approach is adopted in designing a filter; the approach constitutes designing a filter, then checking to see if this design meets the requirements such as harmonic voltage distortion, etc. The process is repeated till all functional requirements are

met. This method of computing gets complex as the number of filters and performance requirements increase.

Computerized techniques speeded up the design approach, however, the basic trial-and-error approach remained. Filter designers are looking for more sophisticated methods which will aid in the design of a filter and at the same time avoid the trail-and-error approach.

#### 1.1.1 Defining Harmonic Filter

The AC harmonic filter reduces the amplitude of one or more fixed frequency harmonic currents and voltages. It generally consists of one or more L-C tuned circuits. R-L-C circuits constitute a special type of damped harmonic filters. These filters, in addition to filtering the harmonic currents/voltages, can also reduce either overshoot or rate of rise of a transient current or voltage.

By definition, a filter  $F$  is a function of

$$F = f [ Z_s , X_t , I_h , C_f , P , Q_t ]$$

where

$Z_s$  = System impedance

$X_t$  = Converter transformer reactance

$I_h$  = Harmonic current at harmonic  $h$

$C_f$  = Filter capacitor

$Q_t$  = Total reactive power demanded of converter

$P$  = Active power transmitted by converter

## 1.2 OBJECTIVES

The objective of this research is to evaluate a methodology to design AC harmonic filters at a HVDC converter bus; this methodology should systematically compute the filter parameters while simultaneously satisfying the filter performance requirements.

The proposed method adopts an optimization technique to design the filter. All the parameters that constitute the filter design are formulated as the 'objective function'. Performance specifications are formulated as the 'constraint function'. Various optimisation techniques can be applied to solve for the 'objective function' subject to the 'constraint function'. By nature, this method is iterative and the optimisation process converges to the final design parameters of the filter.

The proposed method has a distinct advantage, in such that it avoids the trial-and-error approach. Furthermore, with computers the optimisation process can be fast and relatively easy. This method also aids an engineer to perform sensitivity type of studies, where the impact of



change in one parameter on the filter design can be studied. Such studies are very important for a design engineer and the proposed method has the capability to address such studies with ease.

### 1.3 OUTLINE OF THE THESIS

Ideally electrical power systems operate at a fixed frequency (typically at 50 or 60 Hz) and at a constant voltage level. The problem of voltage regulation and its control is the subject matter of considerable power system research and analysis. The problem of voltage waveform distortion is also of interest as is the emission of harmonics from various sources and devices.

Chapter Two deals with characteristic harmonics generated by an HVDC converter station. The general expressions for harmonics of a 6-pulse and 12-pulse converter are given. Various other sources which generate harmonics are also listed. The adverse effects on electrical equipment due to the injection of harmonic frequency voltages and currents are discussed.

The literature review of papers on design of AC harmonic filters in Chapter Three is classified into four sections: a) General filter design, b) Self-tuned filters, c) Filter performance estimation, and d) Specification of filters.

The chapter concludes with the introduction of a new method to design an AC harmonic filter.

Chapter Four deals with various aspects of the AC filter design at an HVDC converter bus. Various types of AC harmonic filters are discussed. Generalised impedance expressions for two specific type of filters, namely single-tuned and high-pass filter (of second order) are developed. Design considerations for a filter are broadly classified into two parts: a) Basic filter design and b) Filter performance specifications. Basic filter design requires consideration of current source, filter and system admittances and harmonic voltages, besides such quantities as filter Quality Factor ( $Q$ ) and AC system Frequency Deviation ( $\theta$ ). The performance specification requires consideration of Total Harmonic Distortion (THD), Telephone Interference Factor (TIFF) etc.

Chapter Five deals with the method of computing the cost function of the filter for single-tuned and high-pass filters. Each filter component's contribution to the filter cost is considered in detail. The energy losses attributed to each component are studied. The cost of the energy losses is expressed in terms of equivalent capital cost by use of present value factor (eq. 5.19). Each component's cost factor is added to produce a total cost expression for

each type of filter. The cost associated with the reactive power, which the filter generates at fundamental and harmonic frequencies are computed. This total cost is then expressed in a generalized form. The cost function developed in Chapter Five details the various parameters that contribute to the cost of the filter.

Chapter Six deals with the method of optimization of the cost of the filter. The basic principles of Optimization Techniques are briefly explained. The filter cost function in terms of the optimization format is given. Another important aspect in the optimization process is the computation of the worst case harmonic voltage. The procedure to compute this is given in detail. The chapter ends with a brief note on a commercial package used to optimize the filter cost equation.

## **CHAPTER TWO**

### **2.0 HARMONICS GENERATED BY HVDC CONVERTERS**

#### **2.1 INTRODUCTION**

#### **2.2 HARMONICS GENERATED BY CONVERTERS**

#### **2.3 HARMONIC SOURCES**

#### **2.4 EFFECTS OF HARMONICS ON ELECTRICAL APPARATUS**

##### **2.4.1 Additional Losses and Heating in Machines**

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##### **2.4.3 Interference With Ripple Control Systems**

##### **2.4.4 Interference With The Converter controls**

##### **2.4.5 Noise On Voice Frequency Telephone Lines**

##### **2.4.6 Apparatus Effected By Harmonics**

#### **2.5 NEED TO ELIMINATE THE HARMONICS.**

#### **2.6 SUMMARY**

## 2.1 INTRODUCTION

Electrical power systems operate at a fixed frequency (typically at 50 or 60 Hz) and at a constant voltage level. The emission of harmonics from various sources and devices is a matter of concern to power system engineers. The identification of these harmonic sources and development of effective methods of controlling the magnitude and the flow of these harmonics into the power system and other electrical devices are part of engineering design assignments. Various sources which generate harmonics are listed. The adverse effects on electrical equipment due to the injection of harmonic frequency voltages and currents are discussed.

This chapter deals in particular with harmonics generated by an HVDC converter station. The characteristic harmonics generated by the converter are briefly discussed. The general expression for the 6 and 12-pulse convertor harmonics are given.

## 2.2 HARMONICS GENERATED BY AN HVDC CONVERTER

The Converter acts as an harmonic current source on the AC side, and an harmonic voltage source on the DC side. A converter of pulse number "p" generates harmonics [8] principally of orders h.

$$h = pq \text{ (On DC Side)} \quad 2.1$$

$$h = pq \pm 1 \text{ (On AC Side)} \quad 2.2$$

where q is an integer.

In general the amplitude of the harmonics decreases with increasing harmonic order. For example, if the AC current at fundamental frequency is  $I_1$ , then the current at order  $h$  is  $I_1/h$ .

The harmonics generated are classified either as characteristic or non-characteristic harmonics. Characteristic harmonics are those defined in equations 2.1 and 2.2. Non-characteristic harmonics are those of other orders. The pulse number  $p$  of a converter is the number of non-simultaneous commutations/cycle of fundamental alternating voltage. The "order of a harmonic"  $h$  is the ratio of its frequency to the fundamental frequency. Detailed mathematical derivation of characteristic harmonics based on the pulse number is available in [8]. Based on the definition and assumptions in [8], the line current of a 6-pulse converter is

$$I_a = (2\sqrt{3}/\pi) \cdot I_d \left[ \cos(\theta) - 1/5 \cdot \cos(5\theta) + 1/7 \cdot \cos(7\theta) - 1/11 \cdot \cos(11\theta) + 1/13 \cdot \cos(13\theta) - 1/17 \cdot \cos(17\theta) + 1/19 \cdot \cos(19\theta) - \dots \right] \quad 2.3$$

This contains only harmonics of order  $6q \pm 1$ , where  $I_d$  is the value of the direct current. The crest value of the fundamental frequency current is

$$I_1 = (2\sqrt{3}/\pi) \cdot I_d = 1.103 I_d \quad 2.4$$

and its effective (RMS) value is

$$\begin{aligned}
 I_{rms} &= I_1/\sqrt{2} = (1.103/\sqrt{2}) \cdot I_d \\
 &= 0.780 I_d
 \end{aligned}
 \tag{2.5}$$

The effective value of the  $h^{th}$  harmonic is

$$I_h = I_1/h \tag{2.6}$$

Similarly the line current of a 12-pulse converter is

$$\begin{aligned}
 I_a = 1.103 I_d \left[ \cos(\theta) - 1/11 \cdot \cos(11\theta) + 1/13 \cdot \cos(13\theta) - \right. \\
 \left. 1/23 \cdot \cos(23\theta) + 1/25 \cdot \cos(25\theta) - \right]
 \end{aligned}
 \tag{2.7}$$

This contains only harmonics of order  $12q \pm 1$

### 2.3 HARMONIC SOURCES

Although the static converter is one of the major sources of harmonic generation, there are other sources within power systems which generate harmonics. These are:

- a) Non-linear loads using power electronic switches.
- b) Synchronous machines when suffering sharp variations in the load and air gap.
- c) Rectifiers, inverters, arc furnaces, solid state voltage controllers, frequency converters etc.
- d) Inter-connections of wind and solar power converters.

The potential use of direct energy conversion devices such as magneto-hydro dynamics may increase power harmonics. Most of the above sources mentioned, share the common

technology of solid state power switching devices.

#### **2.4 EFFECT OF HARMONICS ON ELECTRICAL APPARATUS**

The injection of harmonic frequency voltage and currents into power systems can produce many adverse effects [17] in equipment connected to or adjacent to that system. The most important of these effects are:

##### **2.4.1 Additional Losses And Heating In Machines & Capacitors**

The presence of harmonic currents in machines causes heating of the windings and consequently reduces the maximum torque available. Harmonic voltages at the terminals of power factor correction capacitors produce a disproportionate increase in the current in the capacitor which have to be derated to compensate for the increased di-electric loss and hence internal heating.

##### **2.4.2 Overvoltages Due To Resonance**

A small harmonic voltage introduced at one point in a system may result in a very much larger voltage appearing elsewhere in the system due to series resonance. For example, resonance between an intermediate transformer on a AC line and a power factor correction capacitor.

##### **2.4.3 Interference With Ripple Control Systems**

Some electric power utilites sell power at especially reduced rates for off-peak loads and control the hours of usage by transmitting audio frequency tones over the power



network to control contactors in series with customer's apparatus. Harmonics from converters may cause undesired operation of such equipment.

#### **2.4.4 Interference With The Converter Controls**

The addition of voltage harmonics to the fundamental frequency voltage wave may cause a shift in the voltage zero crossover points which, in some control systems are used as a reference for firing pulses for thyristors. The shift in zero crossover points causes unequally spaced firing of the valves which in turn generates non-characteristic harmonics. If the system impedance is high at one of these non-characteristic harmonics, the resulting distortion may seriously affect converter operation.

#### **2.4.5 Noise On Voice Frequency Telephone Lines**

The frequency range used in voice transmission is 200 to 3500 Hz and this range includes many harmonics of the fundamental power frequency. There is a great difference between the power levels of the telephone circuits and power lines and consequently when power lines are placed in close proximity to telephone lines, there is a great potential for inducing unwanted voltages and currents in the telephone lines.

#### **2.4.6 Apparatus Effected By Harmonics**

The following equipment/devices are effected by harmonics :

- Transformers
- Rotating machines
- Capacitor banks
- Switchgear
- Protective relays
- Metering devices
- Electronic equipment
- Lighting devices.

System studies are needed to define the combined affect of distributed sources of harmonics in an integrated power system.

## **2.5     NEED TO ELIMINATE THE HARMONICS**

The above sections dealt with the adverse effects of harmonics on electrical equipment in the power systems. These harmonics have to be eliminated or drastically reduced to minimise their effects on a wide range of power apparatus harmonic filters are commonly used to eliminate or reduce effects of these harmonics on the system and apparatus.

## **2.6 SUMMARY**

In this chapter a brief description of harmonics generated by an HVDC converter is provided. Various other harmonic sources with power systems are identified. Finally, The adverse effects of these harmonics on electrical apparatus and the need to eliminate them is discussed.

## **C H A P T E R   T H R E E**

### **3.0    STATE OF THE ART IN FILTER DESIGN TECHNIQUES**

#### **3.1    INTRODUCTION**

#### **3.2    LITERATURE SURVEY**

##### **3.2.1    General Filter Design**

##### **3.2.2    Self-Tuned Filters**

##### **3.2.3    Filter Performance Estimation**

##### **3.2.4    Specification Of Filters**

#### **3.3    PROPOSED NEW TECHNIQUE IN FILTER DESIGN**

#### **3.4    SUMMARY**

### **3.1 INTRODUCTION**

AC harmonic filter design is part of the converter station design. Although an exhaustive amount of literature is available on various aspects of the converter station design, a literature survey yielded only a few classical papers on design of AC harmonic filters. Most of these papers have treated harmonic filter design as a secondary item to generation of reactive power. A few papers have treated filter design in terms of meeting the performance requirements along with generation of reactive power. None of these papers, however, have stressed the need to use optimization techniques to design the filter.

### **3.2 LITERATURE SURVEY OUTLINE**

The classical papers reviewed on harmonic filter design are classified into four sections i.e. papers dealing with:

- 1) General Filter Design [1, 3, 14, 15],
- 2) Self-Tuned filters [2],
- 3) Filter Performance Estimation [5, 8],
- 4) Specification of a filter [16].

#### **3.2 1 General Filter Design**

Parker [1] has presented a set of design procedures for a unique filter configuration taking into account the types of filters, filter efficiency, reactive power compensation and point of connection. His underlying principle was to design a filter with reactive power compensation as the

governing criteria. This criteria determined the number of single-tuned filters and high-pass filters. A flow chart was given to achieve the filter design. Double-tuned filter design procedure was carried out solely using single-tuned filters with conversion to compound type being affected in the final design stage.

Filter efficiency was specified to be the limit of the total harmonic voltage appearing at the AC system terminals to a given fraction of the fundamental voltage under given system conditions. The reactive power generation was important since it played a vital role in the final form of the filter design. The author also talked about point of connection of filters, i.e. connection of filters to the primary side of converter transformer (i.e. directly to the AC system) or to a tertiary winding of the same transformer.

Brewer, Clarke and Gavrilovic [3] suggested that harmonic filters be considered in an overall system study as reactive power generating units besides providing filtering. The authors pointed out that reactive power consumed by a converter varies with load. At low load, the filter will overcompensate and the excess reactive power will be fed into the system. If the system cannot accept this excess, then the filter size has to be reduced. The authors also stressed that a minimum cost filter (section

6.1) normally supplies only part of the total reactive power required by the converter and that the capacitor amounts to about 60% of the cost of the filter

The characteristics of the AC system have a considerable influence on the design and cost of the harmonic filter. This was considered in terms of the harmonic voltage distortion appearing at the converter AC terminals. If the required distortion was particularly low, then the number of filter arms or the size of the arms may have to be increased. The authors have provided some theoretical performance criteria to aim for:

- 1) No single harmonic voltage should exceed 1% of nominal system voltage,
- 2) The arithmetic sum of the theoretical harmonic voltages from the 5<sup>th</sup> to 25<sup>th</sup> harmonic inclusive should not exceed 2.5 % of the nominal system voltage,
- 3) Frequency range of  $\pm 1\%$  from normal system frequency.

The impedance of the network to harmonic frequency currents influenced the cost of a filter from the point of view of both current magnification and harmonic voltage distortion. The authors concluded by stressing that the filter component ratings be selected for the most severe condition.

Mohamed and Swift [14] have investigated the transient performance of the interconnected AC/DC power system for filter design. Simulation studies done by them showed that

fixed-tuned filters are better than self-tuned filters from system performance point of view.

Morand, Erikson, and Morais [15] studied the impact of the busbar inductances on the AC filter performance. They considered the impedance of the connecting elements in filter design analysis. However, it was observed that the steady state currents and voltages on each filter component calculated with busbar inductances are not expected to be larger than if the busbar inductances were ignored. The authors felt that designing the AC filter taking into account the busbar inductances was at least time consuming and may prove to be unnecessary.

### 3.2.2 Self-Tuned Filters

Clark and Johanson-Brown [2] suggested a methodology for designing self-tuned filters. Self tuning was achieved by adjustment to the inductor (which is a variable), resulting in small frequency deviation  $\theta$  and hence higher filter quality  $Q$  and filter efficiency. This in turn resulted in lower  $R$  values and losses. The frequency deviation was monitored by means of a control circuit, and necessary control action initiated. They stated that a self-tuned filter will always have better performance than conventional harmonic filters, with respect to lower harmonic voltage. A filter arm can go off-tune due to component changes caused by (a) changes in the ambient

temperature, (b) the temperature rise due to current, and (c) due to changes in the system frequency. Initial mistuning must also be considered. As a filter goes off-tune, the impedance of the arm was given approximately by

$$Z = R(1 + j Q^2 \delta) \quad 3.1$$

where

$\delta$  is the per unit effective frequency detuning from nominal due to all causes, and

$Z$  and  $R$  are filter impedance and resistance respectively.

However, they added that, where de-tuning was small or where the required fundamental frequency reactive power of the filter arm was high, self-tuning may offer no cost advantage. For minimum harmonic voltage, the values of  $C$  and  $Q$  should be as high as possible. The capacitor accounts for about 60% of the total filter cost depending largely on the total VAR rating. A high value of  $C$  meant a large fundamental rating and high cost. As  $Q$  is increased the required capacitor VAR rating is also increased. To obtain reasonable ratings and also adequate filtering, a compromise between  $C$  and  $Q$  must be found.

Furthermore, they stated that, the economic advantages of using self-tuned versus fixed-tuned filters will vary with each application. The indication was that the higher the converter rating the more economic self-tuned filters



became. It should however, be noted that where both frequency and temperature variation are small or where high reactive power generation at fundamental frequency was required, then fixed-tuned filters may still be preferred.

### 3.2.3 Filter Performance Estimation

Brewer [5] adopted a simple method for estimating harmonic performance by using the impedance locus approach (section 4.4.3, Figs 4.2a and 4.2b) to describe the system harmonic impedance in terms of two parameters:

- 1)  $R_L$ , radius of the system harmonic impedance locus,  
and
- 2)  $\phi_m$ , maximum cut-off harmonic impedance angle.

Brewer observed that the value of  $R_L$  varies from tens of ohms in 11 kV networks to several hundred ohms in 400 kV networks. The impedance angle of the system was always above  $75^\circ$ . Brewer also suggested that  $\phi_f$  the detune angle of the filter, caused by drift in component values has a common value used by filter designers. The maximum detune angle of  $\phi_f = 45^\circ$  is obtained when  $2QS = 1$ .

where  $Q$  = quality factor of the filter arm and

$S$  = equivalent frequency deviation.

He also stated, that the maximum harmonic voltage distortion at the filter and the system terminals occurred when the combined admittance of the filter and the system was at a minimum, assuming the harmonic source was

essentially constant current. In this condition, the admittance of the filter was determined and the minimum total admittance  $Y_t$  was obtained.

If  $I_h$  is the harmonic current generated at the  $h^{\text{th}}$  harmonic, the corresponding harmonic voltage is

$$V_h = I_h / Y_t \quad 3.2$$

The current flowing into the filter is

$$\begin{aligned} I_f &= V_h Y_f \\ &= I_h [Y_f / Y_t] \end{aligned} \quad 3.3$$

The current flowing into the rest of the system including the other filter arms is

$$I_s = V_h Y_s = I_h Y_s / Y_t \quad 3.4$$

It is important to note that the ratio of  $(Y_f/Y_t)$  is greater than unity [8], and therefore the harmonic current into the filter is greater than that produced by generated harmonic current due to partial resonance. For this reason, filters are often rated for the worst condition of the system impedance angle of 90 degrees to ensure that they are adequately rated at maximum detune. Brewer's paper [5] provides an approximate guide to the performance of tuned harmonic filters.

#### **3.2.4 Specification Of Filters**

Fletcher and Clarke [16] are of the opinion that the lower order harmonics such as 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup> (which have large harmonic magnitudes) should have separately tuned filters. The 17<sup>th</sup> and higher harmonics can be absorbed by high-pass filters. In addition to the harmonics mentioned above, other harmonics are also produced by the converter and, although relatively small in magnitude, must be considered in filter design.

The effectiveness of the filters to limit the harmonics was given in terms of total harmonic voltage distortion (THD), telephone interference factor (TIF) on the AC bus at the converter station, and the IT factor, which was a measure of harmonic current flowing into the AC system. The AC system harmonic impedance must be considered in order to calculate the performance parameters.

The ratings of the components of the filter have to be adequate for all stages of converter station expansion upto one bipole, including the condition where any one valve group could be out of service. The choice of the components was based on meeting the performance requirements at the lowest total cost. The choice of the component values for the high-pass filter was usually based purely on performance requirements. This was because the harmonic

content was very low and hence the cost was not a governing criteria.

The authors state that a self-tuned filter has one advantage over a conventional filter; with a variable reactance capability, the possibility of resonance between the filter arm and the rest of the system can be minimized.

### 3.3 PROPOSED NEW TECHNIQUE IN FILTER DESIGN

The review of the technical literature provided information on various aspects of filter design i.e. the role of frequency de-tuning factor  $\delta$ , the role of the performance criteria (such as single harmonic voltage to be limited to 1% of the nominal system voltage) etc. However, the underlying principle in all these papers on filter design was to satisfy the reactive power requirements. In addition, the recommended procedure of designing the filter was based on a trial-and-error approach. There were two inherent weaknesses in the above procedure. Firstly, the reactive power compensation was the primary goal of the filter. In fact, an equally important goal of the filter was to reduce or eliminate the harmonic voltages and currents. Secondly, the recommended trial-and-error approach was both time consuming and tedious.

The proposed new technique addresses these two aspects of

filter design. It takes into account the design procedures of Parker [3], which includes among other things, filter efficiency, filter quality factor(Q) etc. It accepts the performance criteria of Brewer, Clarke, Gavrilovic [3], which requires among other things that the individual harmonic voltage be limited to 1% of the nominal system voltage. Brewer's [5] method for estimating harmonic performance by impedance locus method is part of the proposed method. It also takes Fletcher and Clarke's [16] proposition that the lower order harmonics such as 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> should have separately tuned filters. However, the proposed method treats reactive power compensation as one of the performance criteria along with TIF, THD, etc. The filters can be designed with or without the reactive power compensation as a performance criteria.

The proposed method attempts to design the filter while satisfying all the design and performance criteria. The method uses a mathematical optimization technique to achieve this. The procedure consists of expressing the filter cost equation as the objective function. The other design parameters (such as frequency deviation  $\delta$ , filter quality factor Q etc.), along with performance criteria (such as 1% limit on individual harmonic voltage, reactive power requirements etc.) are expressed as the constraint function. This procedure of expressing the filter cost equation as the objective function and performance criteria

as the constraint functions, addresses the first weakness of the traditional methods, pointed out in the literature review. That review summarised the reactive power compensation as the governing criteria of the filter design. The proposed method treats reactive power compensation as one of the performance criteria, but not the only one. Filter design has to satisfy other performance criteria along with reactive power compensation.

The proposed method also addresses the second concern of traditional filter design techniques i.e. trial-and-error approach. The proposed method, by design, is iterative. The final solution is reached only after all the performance criteria which are expressed as a set of constraint functions are completely satisfied. Hence the time consuming trial-and-error approach is replaced by a faster iterative approach.

However, since the method is iterative, the problem of convergence to a solution exists. In case studies, it is observed that the convergence problems occurred either due to 1) Improper data or 2) Wrong initial values. They can be corrected to overcome the convergence problem. However, it is pointed out that in any iterative method, convergence can be a problem. The proposed method has this inherent weakness.

### 3.4 SUMMARY

A general review of the literature on AC harmonic filters was presented in this section. The literature reviewed is categorised into four different sections based on specific aspects of filter design. Overall the literature survey showed that the criteria for supply of reactive power was the governing criteria for the filter design. The traditional method of filter design was based on a trial-and-error approach. The proposed method replaces the trial-and-error approach with an iterative approach and reactive power is treated as one but not the only performance criteria.

## C H A P T E R F O U R

### 4.0 AC HARMONIC FILTER DESIGN

#### 4.1 INTRODUCTION

#### 4.2 TYPES OF AC HARMONIC FILTERS

##### 4.2.1 Single Tuned Filter

##### 4.2.2 High Pass Filter

#### 4.3 DESIGN CONSIDERATIONS FOR A FILTER

#### 4.4 BASIC FILTER DESIGN

##### 4.4.1 Harmonic Source Current ( $I_h$ )

##### 4.4.2 Filter Admittance ( $Y_f$ )

###### 4.4.2.1 Quality Factor (Q)

###### 4.4.2.2 Filter De-Tuning ( $\theta$ )

##### 4.4.3 System Admittance ( $Y_s$ )

##### 4.4.4 Harmonic Voltage ( $V_h$ )

#### 4.5 PERFORMANCE SPECIFICATION

##### 4.5.1 Total Harmonic Distortion ( THD )

##### 4.5.2 Telephone Interference Factor ( TIF )

##### 4.5.3 Harmonic Current Factor ( IT )

##### 4.5.4 Reactive Power Requirement

##### 4.5.5 Reactive Power Generation

#### 4.6 SUMMARY



#### 4.1 INTRODUCTION

This chapter deals with design of various types of AC filter at an HVDC converter bus. Generalised impedance expressions for two specific type of filters, namely single-tuned and high-pass of second order, are developed. Design of any filter is classified into two parts:

a) Basic filter design: Basic filter design requires consideration of current source, filter and system admittance and harmonic voltages, besides such quantities as filter Quality factor ( $Q$ ) and AC system frequency deviation ( $\delta$ ).

b) Performance Specification: The performance specification requires consideration of such specification as Total Harmonic Distortion (THD), Telephone Interference Factor (TIF) etc. After the basic design is completed, the performance specifications are used to develop the final filter design.

#### 4.2 TYPES OF AC HARMONIC FILTERS

There are two categories of filters required by an HVDC converter 1) AC filters and 2) DC filters. This thesis is directed towards AC filters only, although similar techniques could be applied to the DC filter design also. The AC filters at a converter station may be classified according to the functions they perform. A filter may be tuned to a specific frequency (Single-tuned filter,

Fig.4.1a) or a filter may be designed to cover a broad range of frequencies (High-pass filter, Fig.4.1b).

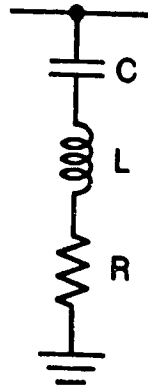


Fig.4.1a, Single-tuned filter

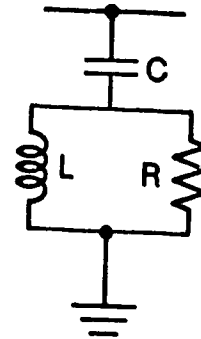


Fig.4.1b, High-Pass filter

Tuned-filters comprise of single, double or even triple-tuned filters. A single-tuned filter is responsive to one single frequency. Two single-tuned filters may be combined to form a double-tuned filter which is responsive to two frequencies, etc.

Various types of high-pass filters exist, i.e.

- a) second order (Fig.4.2a),
- b) third order (Fig.4.2b),
- c) C-type (Fig.4.2c).

The discussion will be limited to second order type filters only. The other types of filters are left for future development.

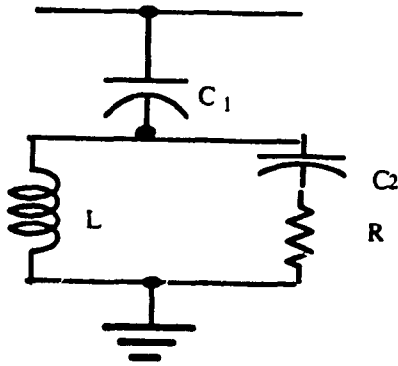


Fig.4.2a  
Second Order

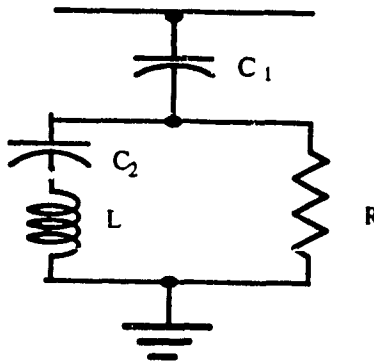


Fig.4.2b  
C- Type

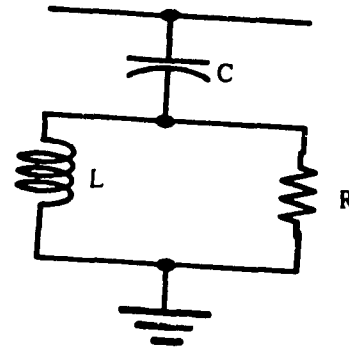


Fig.4.2c  
Third Order

#### 4.2.1 Single Tuned Filter

A single-tuned filter is a series RLC circuit (Figure 4.1a)

The impedance of such a filter is given by

$$Z_{st} = R + j ( \omega.L - 1/\omega.C ) \quad 4.1$$

At the resonant frequency  $f_n$ , the impedance has a low resistance  $R$ . Its passband (PB) is bounded by frequencies  $f_1$  and  $f_2$  at which  $| Z_f | = \sqrt{2}.R$ . At these frequencies the net reactance equals the resistance, and the impedance angle is  $\pm 45^\circ$ . The impedance versus frequency plot is shown in Fig 4.3

Fig.4.3 Single-Tuned Filter Impedance vs. Frequency

Now at tuned frequency angular frequency (rad/sec) is expressed as

$$\omega_n = 1/ \sqrt{L \cdot C}. \quad 4.2$$

and  $\delta$ , the deviation (per unit) of frequency from the tuned frequency as

$$\delta \cong (\omega - \omega_n) / \omega_n \quad 4.3$$

and  $X_0$  reactance of inductor or capacitor in ohms as

$$\begin{aligned} X_0 &= \omega_n \cdot L \\ &= 1/ \omega_n \cdot C \\ &= \sqrt{L/C} \end{aligned} \quad 4.4$$

The quality factor of the filter or sharpness of tuned filter can be expressed as

$$Q = X_0 / R \quad 4.5$$

Rewriting eq.4.3 yeilds

$$\omega = \omega_n ( 1 + \delta ) \quad 4.6$$

Rewriting eq.4.4 and substituting for  $X_0$  from eq.4.5 yeilds an expression for Capacitor C.

$$\begin{aligned} C &= 1/(\omega_n \cdot X_0) \\ &= 1/(\omega_n \cdot R \cdot Q) \end{aligned} \quad 4.7$$

Similarly Inductor L can be expressed as

$$\begin{aligned} L &= X_0 / \omega_n \\ &= R \cdot Q / \omega_n \end{aligned} \quad 4.8$$

Substituting eq.4.6, eq.4.7 and eq.4.8 in the equation 4.1 yeilds

$$Z_{st} = R \left[ 1 + j Q \delta \cdot \left( \frac{2 + \delta}{1 + \delta} \right) \right] \quad 4.9$$

If  $\delta$  is too small i.e.  $\delta \ll 1$  then

$$Z_{st} = R ( 1 + j.2.\delta. Q ).$$

4.10

#### 4.2.2 High Pass Filter

The high-pass damped filter (Fig. 4.1b) has capacitive reactance at fundamental frequency, and low, predominantly resistive, impedance over a wide band of higher frequencies (Fig.4.4).

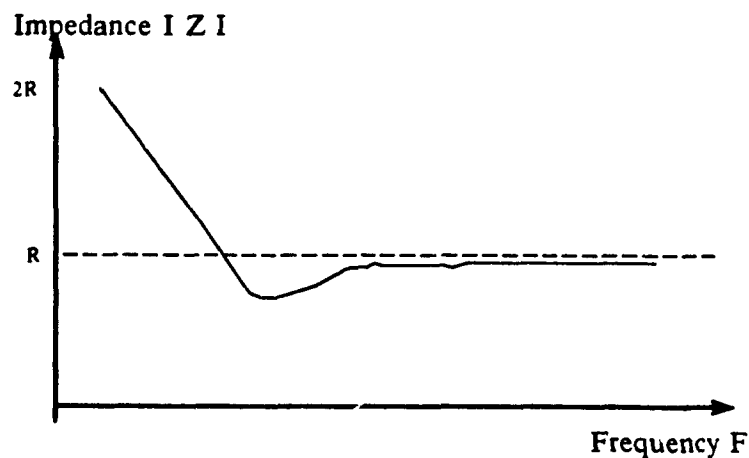


Figure 4.4 High-Pass Filter Impedance vs. Frequency.

The advantages of using a high-pass filter are:

- a) Its performance and loading is less sensitive to temperature variation, frequency deviation, component manufacturing tolerances, loss of capacitor elements etc.
- b) It provides a low impedance for a wide spectrum of harmonics without the need for subdivision of parallel branches with increased switching and maintenance problems, and
- c) The use of tuned filters often results in parallel resonance between the filter and the system admittances at

a harmonic order below the lowest tuned filter frequency or in between tuned frequencies. In such cases the use of one or more damped filters is a more acceptable alternative.

The main disadvantages of a high-pass filter are:

- a) To achieve a similar level of filtering performance, the damped filter needs to be designed for higher fundamental VA ratings, though in most cases a good performance can be obtained within the limits required for power factor correction.
- b) The losses in the resistor and reactor are generally higher.

When designing a damped filter, the  $Q$  is chosen to give the best characteristics over the required frequency band and there is no optimal  $Q$  as with tuned filters. The behaviour of damped filters has been described by Ainsworth [9] with the help of two parameters; these are

$$f_0 = 1/2\pi CR \quad 4.12$$

$$M = L/R^2C \quad 4.13$$

where  $f_0 = N * \text{fundamental frequency}$  ( $N$  is the starting frequency of the high-Pass filter.)

$M$  = damping factor, normally varies between 0.5 and 2

Rewriting equations 4.12 and 4.13 in terms of  $R$  and  $L$  yeilds

$$R = \frac{1}{2 \cdot \pi \cdot f \cdot C} \quad 4.14$$

$$L = M \cdot R^2 \cdot C \quad 4.15$$

The high pass filter impedance is

$$Z_{hp} = (1/j \cdot \omega \cdot C) + [j \cdot R \cdot \omega \cdot L / (R + j \cdot \omega \cdot L)] \quad 4.16$$

substituting for R and L from equation 4.14 and 4.15 respectively results in

$$Z_{hp} = \frac{1}{j \cdot \omega \cdot C} + \frac{j \cdot \omega \cdot M}{2\pi f_0 C (2\pi f_0 + j\omega M)} \quad 4.17$$

### 4.3 DESIGN CONSIDERATIONS FOR A FILTER

The primary objective of a filter is to reduce (or eliminate) the amplitude of the harmonic currents and voltages. In addition, it should also provide all or part of the reactive power consumed by the converter, and meet other performance specifications, such as TIF, IT, etc. Theoretically, a filter can be designed for a very high quality factor (Q) i.e. very sharply tuned to a specific frequency. However, a slight deviation of the AC system frequency will de-tune the filter, resulting in increased harmonics into the AC system.

Another consideration in the filter design is the AC system impedance. AC system network topology changes due to various loading conditions. Any change in the network topology, brings changes in the AC network impedance. AC harmonic filter design depends on the AC network impedance. Data of the AC network impedance expressed in terms of network impedance radius  $R_L$  and network impedance phase angle  $\pm \phi_m$  (section 4.4.3) are taken into consideration to design an AC harmonic filter.

AC system frequency deviation ( $\theta$ ), which is built into the impedance expression (eq.4.9) is another requirement to be considered for the filter design. Single-tuned filter, which is tuned to a specific frequency is sensitive to AC system frequency deviation ( $\theta$ ). For a high-pass filter, starting frequency  $f_0$ , which is some times referred to as high-pass tuning frequency, along with the damping factor  $M$  are to be considered for filter design (eq.4.17). Limits on harmonic voltages and currents flowing into the AC system are also considered as requirements for filter design.

Practical filter design is carried out in two parts, a) Basic filter design and b) Design for meeting performance specifications.

#### **4.4 BASIC FILTER DESIGN**

In the basic filter design, four design parameters require



specific consideration:

- 1) Harmonic Current  $I_h$
- 2) Filter admittance  $Y_f$
- 3) System admittance  $Y_s$
- 4) Harmonic voltage  $V_h$

#### 4.4.1 Harmonic Current $I_h$

The harmonic current generated by a converter depends on its pulse number and operating firing angle, as explained in section 2.3 i.e. if a 12-pulse converter is used, then the AC current is comprised of harmonics of order  $12q \pm 1$ , where  $q$  is an integer. Similarly the converter firing angle value determines the magnitudes of the harmonic currents [8] Normally, harmonic current orders upto 50 are considered for the filter design. This assumes that the current magnitudes of higher order harmonics are very small and can be ignored.

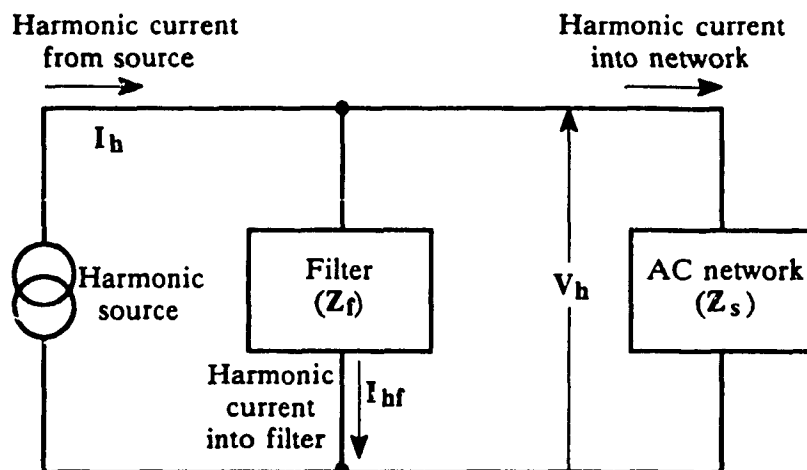


Figure 4.5. Current Source

#### 4.4.2. Filter Admittance $Y_f$

Filter admittance is a function of the filter parameters namely R, L and C (section 4.2.1.). The parameters that affect the filter admittance are: a) Quality factor Q of the filter or sharpness of tuning of the filter, and b) Frequency de-tuning  $\delta$ . The filter parameters R, L and C are expressed in terms of Q,  $\delta$  and  $\omega$ , such that the final filter admittance will be a function of these terms.

##### 4.4.2.1. Quality Factor ( Q )

The quality factor of a filter determines the sharpness of tuning and filters may either be high Q or low Q type. The high Q filter is sharply tuned to one of the lower harmonic frequencies (eg. 5<sup>th</sup>) and its typical Q value is between 30 and 90. The low Q filter, typically has a Q value in the region of 0.5 - 5, and also has a low impedance over a wide range of frequencies. The high-pass filter is of this type. Normally, Q is expressed as

$$Q = X_0 / R$$

where  $X_0$  is the reactance of the filter at the tuned frequency and R is the filter resistance.

##### 4.4.2.2 Filter de-tuning ( $\delta$ )

The filter de-tuning from the nominal tuned frequency is represented by a factor  $\delta$ . In practice, a filter is not always tuned exactly to the harmonic frequency that it is

intended to eliminate. De-tuning is also caused by:

- 1) Variation in the filter capacitance C and inductance L. Of these two, the capacitance changes more because of ageing and changes in the temperature and self-heating.
- 2) Variation in the power system frequency causing harmonic frequency to change proportionally.
- 3) Initial off-tuning caused by manufacturing tolerances and finite size of tuning steps.

A change of L or C of 2% [8] causes the same detuning as a change of system frequency of 1%. The overall de-tuning in per-unit of the nominal tuned frequency is

$$\delta \cong (\omega - \omega_n) / \omega_n \quad 4.18$$

where  $\omega_n$  is the tuned angular frequency (rad/sec).

The total de-tuning  $\delta$  or equivalent frequency deviation is given as [8]

$$\delta \cong \delta F/F_n + 1/2 [\delta L/L_r + \delta C/C_n] \quad 4.19$$

#### 4.4.3 System Admittance $Y_s$

The reactive components of system impedance and of the impedance of the resonant filters can give rise to parallel resonance [6] that tend to amplify harmonic voltage and current in the filters and the system. Consequently a knowledge of the network impedance at the various harmonic frequencies is one of the important factors in the rating of the components of resonant filters as well as in

calculating filter performance. Figures 4.6 and 4.7 shows plots of the system impedances for various frequencies for two different topologies. These figures illustrate the dynamic nature of the AC system impedance.

The maximum amplification of the harmonic current in the filter for large frequency and temperature variations depends on both the system impedance and the Q factor. A high Q filter suffers from a greater risk of amplification but, on the other hand, smaller losses and smaller harmonic voltages when the filter is correctly tuned [8].

The methods of computing the system impedance [6] are fairly involved. Figure 4.6 and 4.7 illustrate system

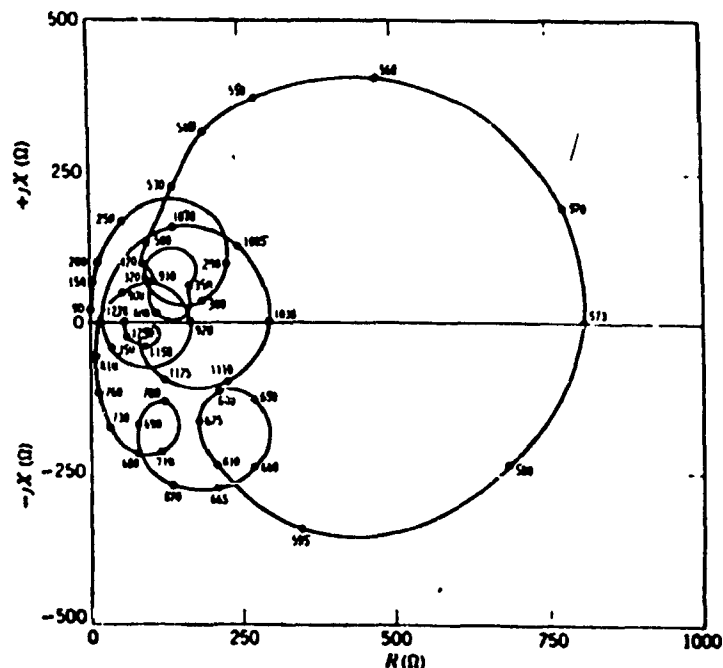


Figure 4.6 Calculated Impedance of 220 kV Network at Minimum Load with One Line Disconnected



$R_L$  - AC system operating radius which circumscribe all possible load considerations of the region.

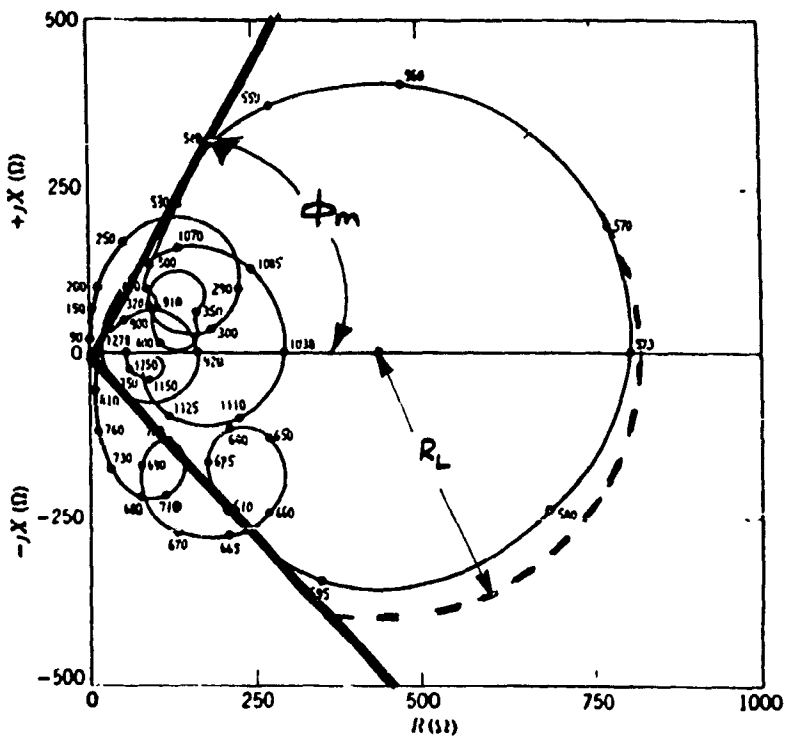
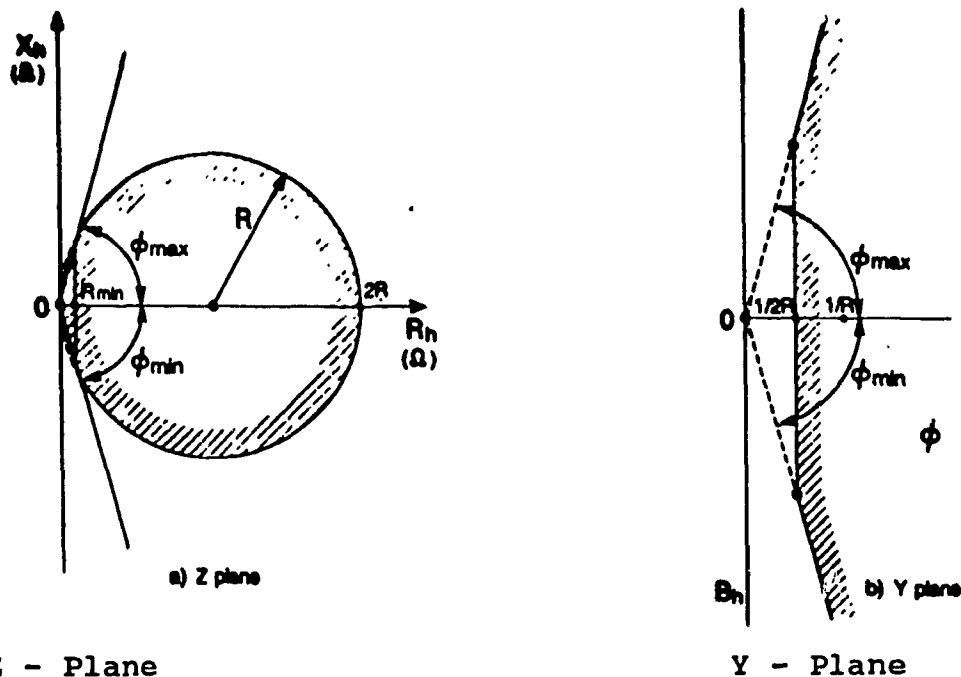


Figure 4.8    Circumscribing a Network Plot

The maximum phase angle  $\pm \phi_m$  of the system impedance is limited to a value below 90 degrees and generally decreases with increasing frequency. Further restrictions to the circumscribing locus can be applied when the limits of individual harmonic impedance are known.

Figure 4.8 can be simplified further and redrawn using only  $\phi_m$  and  $R_L$  to define system admittance as shown in Fig.4.9.



Z - Plane

Y - Plane

Figure 4.9 Circumscribed System Impedance/Admittance

#### 4.4.4. Harmonic Voltage $V_h$

Practical filter design includes minimising harmful telephone interference (commonly expressed in terms of harmonic current or harmonic voltage or both). In order to obtain the maximum voltage distortion it is required to compute the minimum total equivalent admittance ( $Y_s + Y_f$ ) of both filter and system (Fig.4.5) at each frequency. Once harmonic voltage is computed, the basic design of the filter is complete. The harmonic voltage,  $V_h$  at the filter busbar is

$$V_h = I_h / Y_t \quad 4.20$$

where  $I_h$  is source harmonic current and

$$Y_t \text{ is } (Y_s + Y_f)$$

To minimise the harmonic voltage it is necessary to increase the overall admittance of the filter in parallel

with the AC system.

All the parameters mentioned above such as source harmonic current  $I_h$ , filter admittance  $Y_f$ , system admittance  $Y_s$  etc. form part of the basic filter design.

#### 4.5 FILTER DESIGN TO MEET PERFORMANCE SPECIFICATION

The basic filter design does not satisfy all the system performance requirements such as

- 1) Total Harmonic Distortion THD,
- 2) Telephone Interference Factor TIFF,
- 3) Harmonic Current Factor IT and
- 4) Reactive Power Requirements.

The final filter design is a product of these additional requirements, hitherto not considered as part of the basic filter design. These performance specifications are discussed in the following sections.

##### 4.5.1. Total Harmonic Distortion THD

Limits are placed on the effective distortion of the AC voltage wave. This being a root-sum-square (R.S.S.) value of the distortion produced by all harmonics [17], it is defined as

$$THD = (1/V_1) \cdot \left[ \sum_{h=2}^m (V_h^2) \right]^{1/2} \quad 4.21$$



Typically this limit varies from 1% to 5%. The harmonics considered are all the characteristic harmonics from 2<sup>nd</sup> to the 50<sup>th</sup> harmonic.

#### 4.5.2. Telephone Interference Factor TIF

The telephone interference factor is a dimensionless value used to describe the influence of a power transmission line on a telephone line without taking into account detailed geometric aspects of coupling, and is given as

$$\text{TIF} = \frac{\left[ \sum_{h=2}^m (K_h P_h V_h)^2 \right]^{1/2}}{\left[ \sum_{h=2}^m (V_h)^2 \right]^{1/2}} \quad 4.22$$

where  $K_h$  is the coupling co-efficient and is equal to, by definition, to 5h.

$P_h$  is the weighting factor of the harmonic in question with a maximum value of 1 at 1000 Hz (called the C-message weighting factor)

m highest number of harmonic number considered.

#### 4.5.3. Harmonic Current Factor IT

The factor IT provides a similar measure of the influence of a power line on a telephone line. IT factor is a

root-sum-square combination of products of currents (in amperes) of various frequencies, each multiplied by the corresponding  $K_h$  and  $P_h$  factors i.e.

$$I_T = \left[ \sum_{h=2}^m (K_h P_h I_h)^2 \right]^{1/2} \quad 4.24$$

where  $I_h$  is the harmonic current flowing into the system. Typical values of  $I_T$  are between 10000 - 20000.

#### 4.5.4 Reactive Power Requirement

The major function of the filter is to absorb the harmonic currents generated from the converter (source) and therefore eliminate them from entering the power system. Since the filter is composed of reactive elements namely Capacitor  $C$  and Inductor  $L$ , they naturally generate reactive power. The converter consumes reactive power which can be supplied by any combination of four different sources:

- 1) Filter bank (including shunt capacitor),
- 2) Synchronous condensor,
- 3) Static VAR compensator, and
- 4) Power system.

In terms of reliability and economics of VAR supply with respect to a synchronous condensor and static VAR compensator (SVC), a synchronous condensor is preferred due

to its capability to supply/absorb VARs and increase the system short circuit ratio [6]. However, due to improved switching response and flexibility in terms of VAR generation, SVC's are becoming more acceptable to system engineers (i.e. Chateauguay project of Hydro Quebec). Filter banks are part of converter station design to filter out the harmonic voltages and currents. The filter bank also produces reactive power. Hence, a combination of filter banks, synchronous condensers and SVC's can be effectively used to meet the dynamic need for the reactive power of the converter station.

The converter demand for reactive power is a dynamic process. This demand changes with the converter firing angle. However, the filter bank generates a fixed amount of reactive power assuming that system voltage stays constant. There are three scenarios possible :

a) Converter VAR demand is fully met by the filter capability

In this case, there is no need for additional units such as synchronous condensers or SVC's to meet the converter demand. This case is highly idealised and is practically never realised due to the dynamic nature of the converter operation. In normal operation, reactive power generated by the filter bank partially meets the converter station demand.

b) Converter VAR demand is more than the filter capability. In this case, converter station demand for reactive power exceeds the generating capacity of the filter bank. The additional power is then supplied either by synchronous generator, SVC or the AC system.

c) Converter VAR demand is less than the filter capability. In this case the filter bank generates excess reactive power than required by the converter. This can occur when the converter is lightly loaded for some reason. When this happens, some of the filters in the filter banks are switched off in such a way that high harmonic currents do not enter the system.

The challenge to the filter designer is to match the VAR generation from the filters and shunt capacitor bank throughout the converter load range and at the same time meet the performance specification for filtering purposes. Extensive load flow type studies are carried out to match these performance criteria.

#### 4.5.5. Reactive Power Generation

At fundamental frequency, the filter banks are almost purely capacitive and the VAR generation of the bank is approximately equal to the VAR rating of the capacitors in the filter. However, due to the presence of other reactive elements (i.e. Inductors), a minor decrease in the VAR generation results. This is given as

$$Q_f = \frac{(V_L)^2}{(X_C - X_L)} \quad 4.24$$

where  $Q_f$  - Reactive power of the filter at fundamental frequency (MVA),

$V_L$  - AC line voltage (kV),

$X_C$  - reactance due to capacitance at fundamental,

$X_L$  - reactance due to inductance at fundamental.

Reactive power generation is not confined to fundamental frequency. Harmonic frequencies also contribute to the VAR generation. The reactive power at harmonic frequencies is given as

$$Q_h = \frac{(V_{1h})^2}{\sum_{h=2}^{50} (X_C(h) - X_L(h))} \quad 4.25$$

where  $Q_h$  - reactive power at harmonic frequencies.

$V_{1h}$  - Worst harmonic voltage.

$X_{Ch}$  - Harmonic reactance due to capacitor.

$X_{Lh}$  - Harmonic reactance due to inductor.

Total reactive power is the sum total of the reactive power generated at fundamental and harmonic frequencies, i.e.

$$Q_T = Q_f + Q_h$$

In high-pass filters, the VAR generation decreases below

the capacitor capability when harmonic components are considered. However, in case of single-tuned filters, VAR generation increases beyond the capacitor's capability.

#### **4.6 SUMMARY**

Two types of filters, single-tuned and high-pass filters have been discussed. The impedance expressions for both have been developed. Various design considerations for a filter have been discussed. Overall filter design is classified into two parts: a) Basic filter design and, b) Design for meeting performance specifications. Components which contribute to each part of the filter design have been dealt with and relevant expressions developed.

## C H A P T E R F I V E

### 5.0 F I L T E R C O S T

- 5.1 INTRODUCTION
- 5.2 COST CALCULATION METHODOLOGY
- 5.3 FILTER COMPONENT PROPERTIES
  - 5.3.1 Capacitor
  - 5.3.2 Inductor
  - 5.3.3 Resistor
- 5.4 FILTER COST
- 5.5 COST COMPUTATION FOR SINGLE TUNED FILTER
  - 5.5.1 Capacitor Cost
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- 5.6 COST COMPUTATION FOR HIGH PASS FILTER
  - 5.6.1 Capacitor Cost
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  - 5.6.3 Resistor Cost
  - 5.6.4 Total Cost
- 5.7 SUMMARY

## 5.1 INTRODUCTION

This chapter deals with the method of computing the cost of the filter for single-tuned and high-pass filters. Each component's contribution to the filter cost is considered. The energy losses attributed to these component are studied. The cost of the energy losses is expressed in terms of equivalent capital cost by use of "present value factor" (equation 5.19). Each component's cost factor is added to produce a total cost expression for each type filter. The cost associated with the reactive power, which the filter generates at fundamental and harmonic frequencies are computed. This total cost is then expressed in a generalised form.

## 5.2 COST CALCULATION METHODOLOGY

The cost of a filter varies with the size of the filter (Fig.5.0). The size of the filter (MVar) is defined as the reactive power that the filter supplies at fundamental and harmonic frequencies. A filter capacitor is subjected to currents and voltages of the fundamental power-system frequency and harmonic frequency (or frequencies). The rating of the capacitor (MVar) is the sum of the fundamental frequency reactive power and harmonic reactive power for which the filter is designed. The cost of the capacitor is assumed to be directly proportional to its rating. The size of the capacitor (MVar), is defined as the reactive power it supplies at fundamental and harmonic



frequencies.

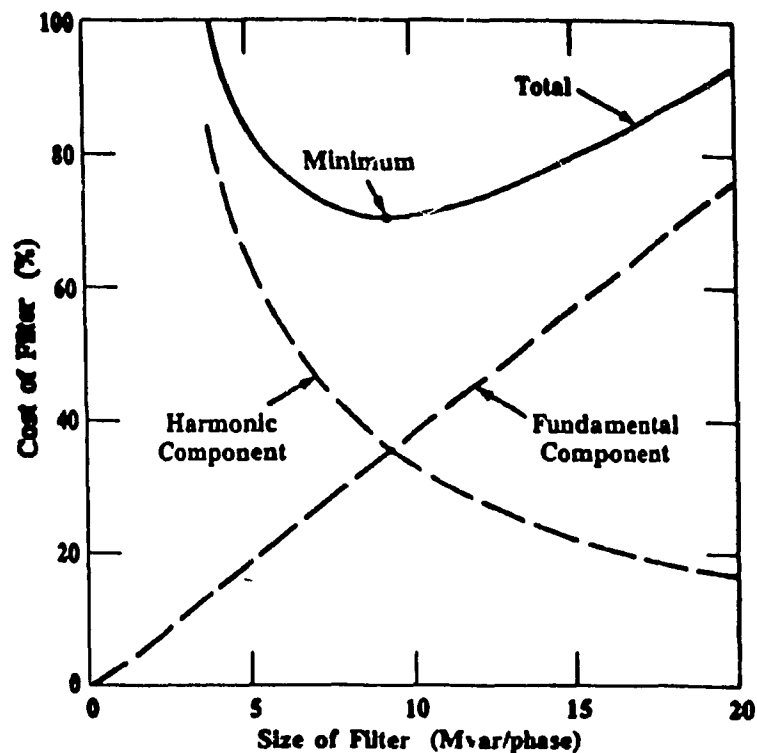


Figure 5.0 Cost of Filter versus Its Size

The fundamental frequency source is essentially a constant-voltage source, the harmonic source is a constant-current source [8]. Therefore, the fundamental frequency reactive power of the capacitor is directly proportional to its size, and the harmonic reactive power is inversely proportional to its size, as shown in Figure 5.0. Similarly the rating of the inductor depends on the sum of its fundamental and harmonic reactive power. The cost of the inductor is greatly affected by its insulation level; this component of cost should be relatively independent of filter size. The rating of the resistor (in

MW) is also proportional to the power loss at fundamental and harmonic frequencies.

### 5.3 FILTER COMPONENT PROPERTIES

The voltage and current ratings of the capacitors, inductors and resistors can be calculated based on information of fundamental and harmonic voltages at the relevant busbar. To prevent damage of these components, their ratings must be based on the most severe conditions expected i.e. the highest effective frequency deviation, highest fundamental voltage, harmonic currents from other sources and possible resonances between the filter and the AC system.

#### 5.3.1. Capacitor

The capacitor bank consists of standard capacitor units which are connected in series and/or in parallel for obtaining the desired overall voltage and MVA rating. The main property of the capacitor is its ability to supply reactive power. Power loss in the capacitor under normal operation is low.

Prolonged operation at moderate overvoltage or even brief operation at higher overvoltage must be avoided to prevent destructive ionization of the di-electric material. Capacitors obtain their high reactive power per unit volume by having low losses and operating at high voltage

stresses. The required reactive power rating of the capacitor is the sum of the reactive powers at each frequency to which it is subjected.

### 5.3.2 Inductor

Skin-effect and hysteresis losses are taken into account when designing an inductor for a filter unit. Skin-effect and hysteresis losses increase with frequency. The effect of the flux level in the iron i.e. de-tuning caused by magnetic non-linearity must be considered. This leads to use of low flux densities when using iron cores. Filter inductors are better designed with non-magnetic cores. Inductor rating depends mainly on the maximum R.M.S. current and on the insulation level required to withstand switching surges. Normally the R and L form the ground side of a tuned filter.

### 5.3.3 Resistor

Resistor value in the filter is inversely proportional to the quality factor of the filter. Losses in the filter are associated with the resistive elements of the filter. In the proposed method of filter design, the resistor is one of the optimisation variables. Hence, the losses due to a resistor (or resistive element) of the filter has to be varied to satisfy the objective function of the optimisation process. The resistor thus plays a vital role in the final design of the filter.

#### 5.4 FILTER COSTS

The cost of the filter is attributed to (1) suppression of harmonics and (2) supply of some reactive power. The cost of power losses due to resistive elements in the filter may be charged to the reactive power it supplies and/or to the filtering. In short the investment made in the filters is returned in terms of reactive power supply and suppression of harmonic voltages and currents. There is no logical basis for the division of cost between suppression of harmonics and generation of reactive power.

The following assumptions are made in the cost analysis of filter components:

- 1) The cost of the capacitor bank (which consists of a "matrix" of capacitor units, the unit incremental cost  $U_c$  varies with each type of capacitor units, each having a nominal rating at the prescribed operating voltage) is thus approximately constant up to the rating of the minimum matrix containing full units for the higher ratings. One or more units are added to each series group as required and a reasonable cost per MVAR or size can be computed. Although such factors would have to be included in the development of an accurate cost equation. The assumption here is that the capacitor's cost is proportional to its rating.
- 2) Although the cost of a filter inductor depends greatly on its method of construction (eg. oil insulated/cooled

units, natural air-cooled reactors of open construction, etc) their cost does not vary greatly for units of different rating. The cost approximation used in the analysis [10], therefore, is of the form

$$\text{Inductor cost} = U_k + U_L * (\text{total MVAR rating})$$

where  $U_k$  is the constant cost component and

$U_L$  is unit incremental cost of the inductor.

3) The power rating of the resistor necessary for Q-factor adjustment in each filter branch will doubtless affect the cost to some extent. The unit cost will be different due to the cooling media used (air cooled vs. oil cooled), but it is virtually independent of power rating.

## 5.5 COST COMPUTATION FOR SINGLE-TUNED FILTER

**BASIC ASSUMPTION** In a high Q circuit it is assumed that

$$V_C = V_L + V_S \quad 5.1a$$

where  $V_C$ ,  $V_L$  and  $V_S$  represents the capacitor, inductor and supply voltages respectively. The filter size S is given as

$$S = \frac{V_S^2}{(X_C - X_L)} \quad 5.1b$$

where  $X_C$  and  $X_L$  are reactance of capacitor and inductor respectively at fundamental frequency. However, when a filter is tuned to a harmonic frequency  $h$ , then

$$X_0 = h.X_L = X_C / h \quad 5.2a$$

$$X_L = X_C / h^2 \quad 5.2b$$

$$V_L = V_C / h^2 \quad 5.2c$$

Substituting 5.2a, 5.2b and 5.2c in 5.1b yields

$$S = \frac{V_s^2}{X_C - (X_C / h^2)} \quad 5.3$$

Therefore, on solution for  $X_C$ , we obtain

$$X_C = \frac{V_s^2}{S} \left( \frac{h^2}{h^2 - 1} \right) \quad 5.4$$

Now

$$V_s = V_C - V_L \quad (\text{from eq 5.1a})$$

Substituting from 5.2c for  $V_L$

$$V_s = V_C - V_C / h^2$$

which gives on solution

$$V_C = V_s \cdot \left[ \frac{h^2}{h^2 - 1} \right] \quad 5.5$$

Equation 5.4 and 5.5 expresses  $X_C$  and  $V_C$  in terms of supply voltage  $V_s$  and harmonic number  $h$ . The loading for each filter component at fundamental and harmonic frequencies are developed for cost calculation in the following sections.

### 5.5.1. Capacitor

At fundamental frequency loading, reactive power of the capacitor or its Size (  $SC_{fl}$  ) is given as

$$SC_{fl} = V_c^2 / X_c \quad 5.6a$$

Substituting for  $V_c$  from 5.5 and  $X_c$  from 5.4 results in

$$SC_{fl} = S \cdot \left[ \frac{h^2}{h^2 - 1} \right] \text{ MVar} \quad 5.6b$$

At harmonic frequency loading, reactive power of the capacitor or its Size (  $SC_{hl}$  ) is given as

$$SC_{hl} = I_h^2 \cdot (X_c / h) \quad 5.7a$$

Substituting for  $X_c$  from 5.4 yeilds

$$SC_{hl} = \frac{I_h^2 \cdot V_s^2}{S \cdot h} \cdot \left( \frac{h^2}{h^2 - 1} \right) \text{ MVar} \quad 5.7b$$

Total loading of the capacitor is

$$SC_{TL} = SC_{fl} + SC_{hl}$$

$$= S \cdot \left[ \frac{h^2}{h^2 - 1} \right] + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \cdot \left[ \frac{h^2}{h^2 - 1} \right]$$

$$SC_{TL} = \left[ \frac{h^2}{h^2 - 1} \right] \left\{ S + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \right\} \text{ MVar} \quad 5.8$$

Power loss in a capacitor at fundamental and harmonic loading is

$$SP_C = K_{C1} \cdot SC_{TL}$$

where  $K_{C1}$  is the loss factor of the capacitor which is expressed in (MW/MVA).

$$SP_C = K_{C1} \cdot \left[ \frac{h^2}{h^2 - 1} \right] \cdot \left\{ S + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \right\} \text{ MW} \quad 5.9$$

#### 5.5.2. Inductor

At fundamental frequency loading reactive power of the inductor  $SI_{f1}$  is expressed as

$$SI_{f1} = V_L^2 / X_L \quad 5.10a$$

substituting from 5.2c for  $V_L$  and from 5.2b for  $X_L$  in 5.10a yeilds

$$SI_{f1} = \frac{V_c^2 \cdot h^2}{(h^2)^2 \cdot X_c} \quad 5.10b$$

Substituting for  $X_c$  from 5.4 yeild

$$= \frac{V_c^2 \cdot S}{(h^2) \cdot V_s^2} \cdot \left[ \frac{h^2 - 1}{h^2} \right]$$

Substituting for  $V_c$  from 5.5 yeilds



$$= \frac{V_s^2 \cdot (h^2)^2 \cdot S}{(h^2 - 1)^2 \cdot (h)^2 \cdot V_s^2} \cdot \left[ \frac{h^2 - 1}{h^2} \right]$$

$$SC_{fl} = \frac{S}{h^2} \cdot \left[ \frac{h^2}{h^2 - 1} \right] \text{ MVar} \quad 5.10c$$

Harmonic frequency loading for the inductor is the same as for the capacitor since the reactances are equal at harmonic frequencies i.e.

$$SC_{hl} = SI_{hl}$$

$$SI_{hl} = \frac{I_h^2 \cdot V_s^2}{S \cdot h} \cdot \left[ \frac{h^2}{h^2 - 1} \right] \text{ MVar} \quad 5.11$$

Total loading of the inductor is given as

$$SI_{TL} = SI_{fl} + SI_{hl}$$

$$SI_{TL} = \frac{S}{h^2} \left[ \frac{h^2}{h^2 - 1} \right] + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \cdot \left[ \frac{h^2}{h^2 - 1} \right]$$

$$SI_{TL} = \left[ \frac{h^2}{h^2 - 1} \right] \cdot \left\{ \frac{S}{h^2} + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \right\} \text{ MVar} \quad 5.12$$

### 5.5.3 Resistor

The power loss in the resistor for fundamental and harmonic frequencies loading can be considered in stages as follows, rewriting eq.4.5 results in:

$$R = X_0 / Q \quad 5.13a$$

Substituting for  $X_0$  from 5.2a yeilds

$$R = X_c / (h.Q) \quad 5.13b$$

Fundamental current  $I_1$  is expressed as

$$I_1 = S / V_s \quad 5.14$$

Total power loss due to a resistor can be written as

$$SP_r = \left( I_1^2 + I_h^2 \right) \cdot R \quad 5.15$$

Substituting for  $R$  from 5.13b and for  $I_1$  from 5.14 yeilds

$$SP_r = \left( \frac{S^2}{V_s^2} + I_h^2 \right) \frac{X_c}{h.Q} \quad 5.16$$

Substituting for  $X_c$  from 5.4 results in

$$SP_r = \left( \frac{S^2}{V_s^2} + I_h^2 \right) \frac{V_s^2}{S} \left( \frac{h^2}{h^2 - 1} \right) \cdot \frac{1}{h.Q} \quad 5.17a$$

rewriting the above equation as

$$SP_r = \frac{S^2 V_s^2}{V_s^2 \cdot S \cdot h \cdot Q} \left( \frac{h^2}{h^2 - 1} \right) + I_h^2 \frac{V_s^2}{S \cdot h \cdot Q} \left( \frac{h^2}{h^2 - 1} \right) \quad 5.17b$$

$$SP_r = \left( \frac{h^2}{h^2 - 1} \right) \left[ \frac{S}{h.Q} + I_h^2 \frac{V_s^2}{S \cdot h \cdot Q} \right] \text{ MW} \quad 5.18$$

#### 5.5.4. Total Cost of Single Tuned Filter

The cost of energy losses is expressed in terms of

equivalent capital cost by use of a present value factor( $P_v$ ).

$$P_v = \frac{\left( (1 + I)^n - 1 \right)}{I (1 + I)^n} \quad 5.19$$

where  $I$  is the interest rate and " $n$ " is the estimated filter life. Thus the present value cost of energy loss  $P_{vL}$  is

$$P_{vL} = P_v \cdot U_u \cdot F_u \cdot (365) \cdot (24) \cdot (\text{total power loss}) \quad 5.20$$

where  $U_u$  is the cost of energy loss per kilowatt-hour and  $F_u$  is the filter utilization factor. The total cost of the filter can now be estimated in terms of the respective unit incremental costs (such as  $U_c$  for capacitor incremental cost per MVar) and power losses due to each component. These factors are added as follows,

$$ST_{\text{cost}} = U_T + \left( \text{unit costs} * \text{loading of each component} \right) + \left( \text{present value factor} * \text{total loss of energy} \right)$$

Substituting the respective loading and the respective losses results in the following expression,

$$ST_{\text{cost}} = U_T + \frac{h^2}{h^2 - 1} \left\{ \left[ U_c \left( s + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \right) + U_L \left( \frac{s}{h^2} + \frac{I_h^2 \cdot V_s^2}{S \cdot h} \right) \right] \right\}$$

$$+ 8760 P_v U_u F_u \left[ K_{cl} \left( s + \frac{I_h^2 \cdot V_s^2}{s \cdot h} \right) + \left( \frac{s}{h \cdot Q} + \frac{I_h^2 V_s^2}{s \cdot h \cdot Q} \right) \right] \Bigg\} \quad 5.21$$

where  $U_T$  is the total constant cost of the filter branch,  $U_C$  and  $U_L$  are unit incremental cost for capacitor and inductor respectively. Equation 5.21 can be expressed in a simplified form as

$$ST_{cost} = U_T + A \cdot S + B / S \quad 5.22$$

where

$$A = \frac{h^2}{h^2 - 1} \left[ U_C + \frac{U_L}{h^2} + 8760 P_v U_u F_u \left( K_{cl} + \frac{1}{h \cdot Q} \right) \right] \quad 5.23a$$

$$B = \left( \frac{h^2}{h^2 - 1} \right) \left( \frac{V_s^2 I_h^2}{h} \right) \left[ U_C + U_L + 8760 P_v U_u F_u \left( K_{cl} + \frac{1}{h \cdot Q} \right) \right] \quad 5.23b$$

## 5.6 COST COMPUTATION OF THE HIGH PASS FILTER

The cost estimation for the high-pass filter also proceeds in a similar manner as the case for the single-tuned filter. The basic assumption remains unchanged, except for equation 5.2 which is modified to

$$X_0 = X_L \cdot h_0 = X_C / h_0 \quad 5.24a$$

$$X_L = X_C / h_0^2$$

$$V_L = V_C / h_0^2$$

This modifies equation 5.4. to

$$S = \frac{V_s^2}{X_C} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.24b$$

and

$$X_C = \frac{V_s^2}{S} \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.24c$$

where  $h_0$  is the ratio of the tuned frequency to the supply frequency.

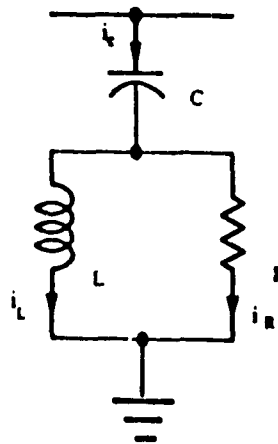


Figure 5.1 High Pass filter

#### 5.6.1. Capacitor

At fundamental frequency loading reactive power or the size  $HC_{f1}$  is obtained by substituting equation 5.6b and on replacing harmonic order  $h$  with  $h_0$ .

$$HC_{f1} = S \left( \frac{h_0^2}{h_0^2 - 1} \right) \text{ MVAR} \quad 5.25$$

At harmonic frequency loading, reactive power  $HC_{hl}$  is obtained by modifying equation 5.7b,

$$HC_{hl} = \frac{I_h^2 v_s^2}{h \cdot S} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.26$$

where  $I_h$  is the current for the respective harmonic  $h$ . Hence the total harmonic frequency for loading for the entire range of harmonics considered is given as

$$HC_{hl} = \frac{v_s^2}{S} \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \left[ \sum_{h=h_{\min}}^{h=h_{\max}} (I_h^2 / h) \right] \text{ MVar} \quad 5.27$$

Total loading of the high pass capacitor is the sum of the fundamental frequency loading and harmonic frequency loading i.e.

$$HC_{TL} = HC_{fL} + HC_{hL} \quad 5.28$$

Substituting for  $HC_{fL}$  and for  $HC_{hl}$  from 5.25 and 5.27 respectively yields

$$HC_{TL} = S \left( \frac{h_0^2}{h_0^2 - 1} \right) + \frac{v_s^2}{S} \left( \frac{h_0^2}{h_0^2 - 1} \right) \left[ \sum_{h=h_{\min}}^{h=h_{\max}} (I_h^2 / h) \right] \quad 5.29$$

Rewriting eq. 5.29 yields

$$HC_{TL} = \left( \frac{h_0^2}{h_0^2 - 1} \right) \left[ s + \frac{v_s^2}{s} \sum_{h=h_{\min}}^{h=h_{\max}} (I_h^2 / h) \right] \quad 5.30$$

The power loss due to the capacitor at fundamental and the entire harmonic range is

$$HC_{pL} = K_{cl} \cdot HC_{TL} \quad 5.31$$

Substituting for  $HC_{TL}$  from eq. 5.30 results in

$$HC_{pL} = K_{cl} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \left[ s + \frac{v_s^2}{s} \sum_{h=h_{\min}}^{h=h_{\max}} (I_h^2 / h) \right] \quad MW \quad 5.32$$

### 5.6.2. Inductor

At fundamental frequency loading reactive power or size of the inductor is  $HI_{f1}$  is i.e.

$$HI_{f1} = I_L^2 \cdot X_L \quad 5.33$$

The following basic assumptions are made to facilitate the computation, i.e.

$$R = Q X_0 = Q \cdot h_0 \cdot X_L \quad 5.34a$$

Thus if the high-pass filter is tuned to a frequency, say  $h_0 = 17^{th}$  and if  $Q = 1.0$ , Then

$$R = 17 X_L \quad 5.34b$$

Since

$$I_C = I_L + j I_r \quad 5.35a$$

It follows that, at fundamental frequency

$$I_C \approx I_L \quad 5.35b$$

At fundamental frequency loading,

$$I_L^2 \cdot X_L = I_C^2 \cdot X_c, \quad 5.36a$$

Substituting for  $X_L$  from 5.2b yeilds

$$I_C^2 \cdot X_L = I_C^2 \cdot X_c / h_0^2 \quad 5.36b$$

Rewriting 5.33 in light of 5.36b results in

$$HI_{fl} = I_C^2 \cdot X_c / h_0^2 \quad 5.37a$$

Substituting for  $X_c$  from 5.4 and for  $I_C$  yeilds

$$HI_{fl} = \left( \frac{S}{V_s} \right)^2 \cdot \frac{V_s^2}{S} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \frac{1}{h_0^2} \quad 5.37b$$

Rewriting 5.37b results in

$$HI_{fl} = \frac{S}{h_0^2} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.38$$

which is similar to equation 5.10d derived for single-tuned filter.

At harmonic frequency loading, reactive power of the inductor ( $HI_{hl}$ ) is given as

$$HI_{hl} = \left( I_L \right)_h^2 \cdot \left( X_L \right)_h \quad 5.39$$

Now



$$\left( I_L \right)_h = \frac{I_h \cdot R}{R + j h X_L} \quad 5.40$$

Substituting from 5.34 for R yeilds

$$\left( I_L \right)_h = \frac{I_h \cdot Q \cdot h_0 \cdot X_L}{Q \cdot h_0 \cdot X_L + j h X_L} \quad 5.41a$$

$$= \frac{I_h \cdot Q}{\left( Q + j \frac{h}{h_0} \right)} \quad 5.41b$$

$$\left| \left( I_L \right)_h \right| = \frac{I_h \cdot Q}{\sqrt{Q^2 + \left( \frac{h}{h_0} \right)^2}} \quad 5.41c$$

The inductive reactance at harmonic h is given as

$$\left( X_L \right)_h = \frac{X_0}{h_0} \cdot h \quad 5.42$$

subtituting  $X_0$  in terms of  $X_C$  from equation 5.24a yeilds

$$\left( X_L \right)_h = \frac{h}{h_0} \cdot \left( \frac{X_C}{h_0} \right) \quad 5.43a$$

Substituting for  $X_C$  from 5.24c results in

$$\left( X_L \right)_h = \frac{h \cdot V_s^2}{h_0^2 \cdot S} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.43b$$

Rewriting 5.39 by Substituting 5.41c and 5.43b for  $I_L$  and  $X_L$  respectively yeilds

$$\left( HI_{hl} \right)_h = \left\{ \frac{I_h \cdot Q}{\sqrt{Q^2 + \left( \frac{h}{h_0} \right)^2}} \right\}^2 \frac{h \cdot v_s^2}{h_0^2 \cdot S} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.44a$$

$$\left( HI_{hl} \right)_h = \frac{1}{S} Q^2 \cdot v_s^2 \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \left[ \frac{I_h^2 \cdot h}{Q^2 \cdot h_0^2 + h^2} \right] \text{ MVar} \quad 5.44b$$

Total harmonic loading over the defined range is

$$\left( HI_{hl} \right)_h = \frac{1}{S} Q^2 \cdot v_s^2 \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \sum_{h=h_{\min}}^{h=h_{\max}} \left[ \frac{I_h^2 \cdot h}{Q^2 \cdot h_0^2 + h^2} \right] \text{ MVar} \quad 5.45$$

Total loading of an inductor is the sum of fundamental loading and harmonic loading.

$$HI_{TL} = HI_{fl} + HI_{hl} \quad 5.46$$

Substituting for  $HI_{fl}$  and  $HI_{hl}$  from 5.38 and 5.45 yeilds

$$HI_{TL} = \frac{S}{h_0^2} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) + \frac{1}{S} Q^2 \cdot v_s^2 \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \sum_{h=h_{\min}}^{h=h_{\max}} \left[ \frac{I_h^2 \cdot h}{Q^2 \cdot h_0^2 + h^2} \right] \text{ MVar} \quad 5.47$$

The power loss of an inductor is obtained by using the inductor's series resistor,  $R_L$ , at fundamental frequency,

$$R_L = X_0 / Q_L \quad 5.48a$$

Substituting for  $X_0$  in terms of  $X_L$  results in

$$R_L = \left( \frac{h_0}{Q_L} \right) \cdot X_L \quad 5.48b$$

where

$Q_L$  is the quality factor of the inductor.

The power loss at fundamental frequency is

$$HLP_{f1} = I_1^2 \cdot R_L \quad 5.49$$

Substituting for  $R_L$  from 5.48b results in

$$HLP_{f1} = I_1^2 \cdot \left( \frac{h_0}{Q_L} \right) \cdot X_L \quad 5.50a$$

Expressing  $X_L$  in terms of  $X_C$  from 5.24a and 5.24c results in

$$HLP_{f1} = I_L^2 \cdot \left( \frac{h_0}{Q_L} \right) \cdot \frac{V_s^2}{S} \left( \frac{h_0^2}{h_0^2} \right) \frac{1}{h_0^2} \quad 5.50b$$

Substituting  $I_C \cong I_L = \frac{S}{V_s}$  in the above equation results in

$$HLP_{f1} = \frac{s^2}{v_s^2} \cdot \frac{h_0}{Q_L} \cdot \frac{v_s^2}{s} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \frac{1}{h_0^2} \quad 5.50c$$

$$HLP_{f1} = \frac{s}{h_0 Q_L} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.50d$$

At harmonic frequencies, the power loss is

$$HLP_{h1} = \left( I_L \right)_h^2 \cdot \left( R_L \right)_h \quad 5.51a$$

where

$$\left( R_L \right)_h = \left( \frac{h_0}{Q_L} \right) \cdot \left( X_L \right)_h \quad 5.51b$$

Substituting in 5.51a for  $I_L$  from 5.41c and for  $R_L$  from 5.51b

$$HLP_{h1} = \left[ \frac{I_h \cdot Q}{\sqrt{Q^2 + \left( \frac{h}{h_0} \right)^2}} \right]^2 \cdot \left( \frac{h_0}{Q_L} \right) \cdot \left( X_L \right)_h \quad 5.51c$$

substituting for  $X_L$  from 5.43b and expanding the square terms results in

$$HLP_{h1} = \frac{I_h^2 \cdot Q^2 \cdot h_0^2}{Q^2 \cdot h_0^2 + h^2} \cdot \left( \frac{h_0}{Q_L} \right) \cdot \frac{h}{h_0^2} \cdot \frac{v_s^2}{s} \left( \frac{h_0}{h_0^2 - 1} \right) \quad 5.52$$

Rewriting the above equation results in

$$HLP_{hl} = \frac{1}{S} Q^2 V_s^2 \left( \frac{h_0}{Q_L} \right) \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \left[ \frac{I_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right] \quad 5.53$$

Considering the entire range results in

$$HLP_{hl} = \frac{1}{S} Q^2 V_s^2 \left( \frac{h_0}{Q_L} \right) \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \sum_{h=h_{\min}}^{h=h_{\max}} \left[ \frac{I_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right] \quad 5.54$$

The total power loss due to an inductor is

$$HLP_{TL} = HLP_{fl} + HLP_{HL} \quad 5.55$$

Substituting for  $HLP_{fl}$  and  $HLP_{hl}$  from 5.50d and 5.55 respectively yeilds

$$HLP_{TL} = \frac{S}{h_0 Q_L} \left( \frac{h_0^2}{h_0^2 - 1} \right) + \frac{1}{S} Q^2 V_s^2 \left( \frac{h_0}{Q_L} \right) \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \sum_{h=h_{\min}}^{h=h_{\max}} \left[ \frac{I_h^2 h}{Q^2 h_0^2 + h^2} \right] \quad 5.56$$

### 5.6.3. Resistor

The power loss due to a shunt resistor R in a high pass

filter can be computed as follows,

$$R = Q \cdot X_0 \quad 5.57a$$

$$R = Q \cdot h_0 X_L \quad 5.57b$$

$$R = Q \cdot h_0 \cdot X_C / h_0^2 \quad 5.57c$$

Now

$$| I_R | = \frac{| I_L | \cdot X_L}{R} \quad 5.58a$$

$$= \frac{I_L \cdot X_L}{Q \cdot h_0 X_L}$$

$$= \frac{I_L}{Q \cdot h_0} \quad 5.58b$$

The power loss at fundamental is

$$HR_f = I_R^2 \cdot R \quad 5.59a$$

Substituting for  $I_R$  and  $R$  from 5.58b and 5.57c respectively

$$HR_f = \left( \frac{I_L}{Q \cdot h_0} \right)^2 \cdot Q \cdot h_0 X_L \quad 5.59b$$

$$= I_L^2 \cdot X_L \left( \frac{1}{Q \cdot h_0} \right) \quad 5.59c$$

Substituting  $X_L$  in terms of  $X_C$  from 5.24a and 5.24c yeilds

$$HR_f = \left( \frac{s^2}{v_s^2} \right) \cdot \left\{ \frac{v_s^2}{s} \left( \frac{h_0^2}{h_0^2 - 1} \right) \frac{1}{h_0^2} \right\} \frac{1}{Q \cdot h_0} \quad 5.60a$$

Rewriting the above equation

$$HR_f = \frac{s}{Q \cdot h_0^3} \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.60b$$

At harmonic frequencies the power loss is given as

$$HR_h = \left( I_R \right)_h^2 \cdot \left( R \right)_h \quad 5.61$$

where

$$\left| (I_R)_h \right| = \left| (I_L)_h \right| X_L / R \quad 5.62$$

Substituting for  $I_L$  from 5.41c and for  $R$  from 5.57b results in

$$\left| (I_R)_h \right| = \frac{I_h \cdot Q}{\sqrt{Q^2 + \left( \frac{h}{h_0} \right)^2}} \cdot \frac{X_L}{Q \cdot h_0 X_L} \quad 5.63$$

Similarly

$$\left( R \right)_h = Q \cdot h_0 \cdot \left( X_L \right)_h \quad 5.64a$$

Substituting for  $X_L$  from 5.43b yeilds

$$\left( R \right)_h = Q \cdot h_0 \cdot \frac{h \cdot V_s}{h_0^2 \cdot S} \cdot \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.64b$$

now rewriting the full expression for the  $HR_h$  by Substituting expression for  $I_R$  and  $R$  from 5.63 and 5.64b results in

$$HR_h = \left\{ \frac{I_h^2 Q^2 h_0^2}{Q^2 h_0^2 + h^2} \cdot \frac{1}{Q^2 h_0^2} \right\} \cdot \left\{ Q \cdot h_0 \frac{h V_s^2}{h_0^2 S} \left( \frac{h_0^2}{h_0^2 - 1} \right) \right\} \quad 5.65a$$

Simplifying the above equation results in

$$HR_h = \frac{1}{S} \frac{Q \cdot V_s^2}{h_0} \left( \frac{h_0^2}{h_0^2 - 1} \right) \cdot \left\{ \frac{I_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right\} \quad 5.65b$$

Power loss over the entire frequency range is given as

$$HR_h = \frac{1}{S} \frac{Q \cdot V_s^2}{h_0} \left( \frac{h_0^2}{h_0^2 - 1} \right) \sum_{h=h_{\min}}^{h=h_{\max}} \left\{ \frac{I_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right\} \text{ MW} \quad 5.66$$

Total power loss due to shunt resistor in the high pass filter due to fundamental frequency and the entire range of harmonic frequencies will be

$$HR_T = HR_f + HR_h \quad 5.67$$



Substituting for  $HR_f$  and  $HR_h$  from 5.60b and 5.66 yields

$$HR_T = \frac{s}{Q \cdot h_0^3} \left( \frac{h_0^2}{h_0^2 - 1} \right) + \frac{1}{s} \frac{Q \cdot v_s^2}{h_0} \left( \frac{h_0^2}{h_0^2 - 1} \right) \sum_{h=h_{\min}}^{h=h_{\max}} \left\{ \frac{J_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right\} \quad 5.68$$

#### 5.6.4 High Pass Filter Total Cost

The present value factor is similar to the single tuned filter

$$P_{vh} = P_v U_u F_u \cdot (365) \cdot (24) \cdot \left( \text{total power loss} \right) \quad 5.69a$$

$$P_{vh} = P_v U_u F_u \cdot 8760 \cdot HR_T \quad 5.69b$$

Total cost for the high pass filter is

$$HP_{\text{cost}} = U_T + ( \text{Unit costs} * \text{loading of the components} ) \\ + ( \text{present value factor} * \text{total loss in energy} )$$

$$\begin{aligned}
HP_{cost} = U_T + \frac{h_0^2}{h_0^2 - 1} & \left\{ \left[ U_C \left( s + \frac{v_s^2}{s} \sum_{h=h_{min}}^{h=h_{max}} (I_h^2) \right. \right. \right. \\
& + U_L \left( \frac{s}{h_0^2} + \frac{1}{s} Q^2 v_s^2 \sum_{h=h_{min}}^{h=h_{max}} \frac{I_h^2 \cdot h}{Q^2 \cdot h_0^2 + h} \right) \left. \right] \\
& + 8760 P_v U_u F_u \left[ K_{CL} \left( s + \frac{v_s^2}{s} \sum_{h=h_{min}}^{h=h_{max}} (I_h^2) \right. \right. \\
& + \left( \frac{s}{h_0 Q_L} + \frac{1}{s} Q^2 v_s^2 \left( \frac{h_0}{Q_L} \right) \cdot \sum_{h=h_{min}}^{h=h_{max}} \frac{I_h^2 h}{Q^2 h_0^2 + h} \right. \\
& \left. \left. \left. + \left( \frac{s}{Q \cdot h_0^3} + \frac{1}{s} \frac{Q \cdot v_s^2}{h_0} \sum_{h=h_{min}}^{h=h_{max}} \frac{I_h^2 \cdot h}{Q^2 h_0^2 + h^2} \right) \right] \right\} \quad 5.70
\end{aligned}$$

In short, equation 5.70 can be simply expressed as

$$HP_{cost} = U_T + A \cdot S + b / S \quad 5.71$$

where

$$A = \left\{ U_C + \frac{U_L}{h_0^2} + 8760 P_v U_u F_u \left( K_{CL} + \frac{1}{h_0 Q_L} + \frac{1}{Q h_0^3} \right) \right\} \left( \frac{h_0^2}{h_0^2 - 1} \right) \quad 5.72$$

$$B = \left( \frac{h_0^2}{h_0^2 - 1} \right) v_s^2 \sum_{h=h_{\min}}^{h=h_{\max}} \left( I_h^2 \right) \left\{ \frac{U_c}{h} + U_L \frac{Q^2 h}{Q^2 h_0^2 + h_0^2} + \right. \\ \left. 8760 P_v U_v F_v \left[ \frac{K_{cl}}{h} + \frac{Q^2 h_0}{Q_L} \left( \frac{h}{Q^2 h_0^2 + h^2} \right) + \frac{Q}{h_0} \left( \frac{h}{Q^2 h_0^2 + h^2} \right) \right] \right\}$$

5.73

## 5.7 SUMMARY

This chapter is devoted to developing the filter cost equation. After discussing the cost calculation methodology, the properties of filter components are explained. Single-tuned filter cost expressions have been developed. These expressions included each component (of the filter) loading both at fundamental and harmonic frequencies. Energy loss associated with each component is included in the above expression. Finally, present value factor is added to give the total cost expression for the single-tuned filter. The above process of developing the cost expression is repeated for high-pass filter. Cost expression for each type of filter is different.

## C H A P T E R   S I X

### C O S T   O P T I M I S A T I O N

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#### 6.3 OPTIMISATION PRINCIPLE

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##### 6.3.3 Optimality

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###### 6.4.1.1 Constraint Qualification

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#### 6.5 FORMULATION FOR FILTER DESIGN

#### 6.6 MINOS 5.0

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#### 6.8 SUMMARY

## 6.1 INTRODUCTION

The cost equation developed in chapter five details the various parameters that contribute to the cost of the filter. One of the principle objectives of the present research is to propose a methodology for computing the optimum filter cost. This chapter deals with the method of optimisation of the cost of the filter. The basic principles of optimisation techniques are briefly explained. The formulation of the filter cost equation in terms of the optimisation format is given. Another important aspect in the optimisation process is the computation of the worst harmonic voltage. The procedure to compute the worst harmonic voltage is given in detail. The chapter ends with a brief note on a commercial package which was used to optimise the filter cost equation.

## 6.2 JUSTIFICATION FOR COST OPTIMISATION

The general expression for the filter cost (eq.5.71) is

$$\text{COST} = U_T + A.S + B/S \quad 6.1$$

The cost of the filter varies with the size of the filter (Fig.6.0). Where the filter cost is least, it is defined as a MINIMUM COST FILTER. This size of the filter may or may not give adequate filtering. A MINIMUM-FILTER is defined as one that adequately suppresses harmonics at the lowest possible cost, and supplies some reactive power.

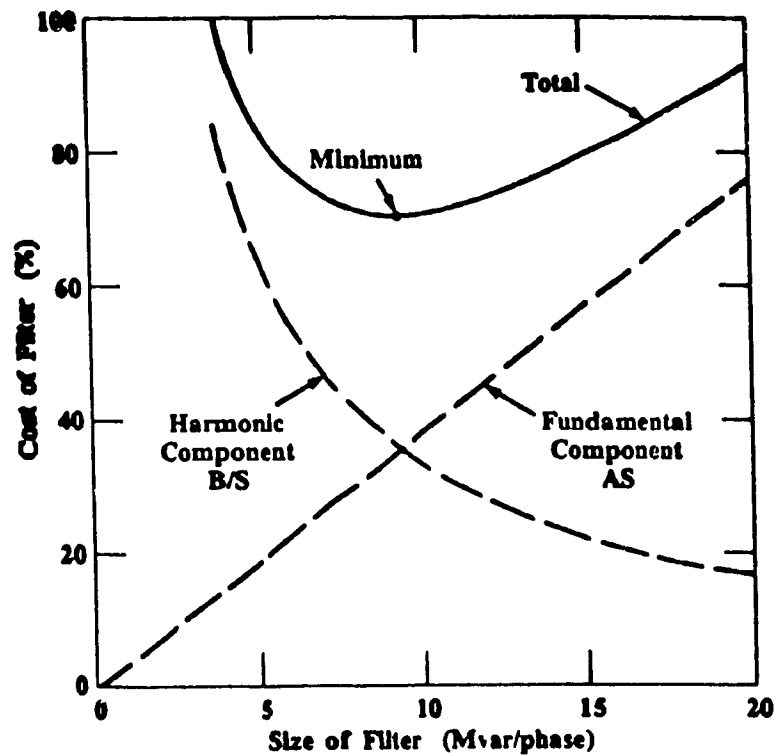


Figure 6.0 Cost Of Filter versus Its Size

Basic filter design (section 4.2.1) can also be termed as a minimum filter design, as per the above definition. The cost of the minimum filter is normally higher than the minimum-cost filter. In order to accomplish the best filter cost, which satisfies all the requirements, the process is quite complicated.

For example, for a single-tuned filter, the cost equation (equation 6.1) gives the minimum-cost filter. The components of this filter may result in a harmonic voltage ( $V_h$ , section 4.4.4), which may not be an acceptable value (say 1%). In order to satisfy the harmonic voltage specification, filter parameters have to be changed. New harmonic voltage value is computed, and then checked to see

if the 1% requirement is satisfied. If not, then the above process is repeated till the 1% requirement is satisfied. In case of another requirement, such as total harmonic distortion limit (THD) at 3%, the above process of trial-and-error has to be repeated till this requirement is also simultaneously satisfied. When such requirements are expanded to include others, such as TIF, IT, REACTIVE POWER, then the amount of work involved in the traditional methods of trial-and-error becomes enormous.

A better way of designing the filter is possible using optimisation methods. The filter design which has the 'best' cost while satisfying all the performance requirements is called the OPTIMUM-COST-FILTER and is defined as the filter, which satisfies all the performance requirements at the lowest possible cost.

Filter cost equation (eq.5.70) is a non-linear expression. Similarly total harmonic distortion THD (eq.4.21), telephone interference factor TIF (eq.4.22) etc. are also non-linear expressions. The optimum-cost filter can be achieved only when the performance requirements such as THD, TIF etc. are satisfied. Therefore, filter design problem can be formulated in a typical optimization format wherein the filter cost equation is considered as the objective function and performance requirements such as TIF THD etc. are considered as constraint functions. Once the

filter design problem is formulated in the above format, then optimization techniques can be applied. This concept of applying optimization techniques to filter design is new. Both processes i.e. optimization and filter design are well established, however unison between them was not explored earlier. A brief introduction to optimization principles is given below [11,18].

### 6.3 OPTIMISATION PRINCIPLE

Classical definition of anon-linear optimisation problem with non-linear constraints is to

Minimise the objective function

$$F( X_1, X_2, \dots, X_n ) \quad 6.2$$

subject to the constraint function

$$C_i( X_1, X_2, \dots, X_n ) \leq 0 \quad 6.3$$

for  $i = 1, 2, \dots, M$

where  $n$  is the number of variables and  $M$  is the number of constraints. Both  $F(X_1, X_2, \dots, X_n)$  and  $C_i(X_1, X_2, \dots, X_n)$  are real valued, nonlinear functions of  $n$  real variables.

Three basic terms in optimization are:

- 1) Feasible region
- 2) Convergence
- 3) Optimality



### 6.3.1 Feasible Region

The constraint function defines an area bounded by  $f(x)$  within which the solution is feasible (Fig.6.1). The algorithm for non-linear constraints is similar to that for linear constraints. Each iteration commences with a feasible trail point  $x^k$  and tests whether  $x^k$  is optimal. If  $f(x)$  can be increased further, a new trail point with a larger value of  $f(x)$  is found. However, difficulties may arise in determining the direction to move when the constraints are non-linear.

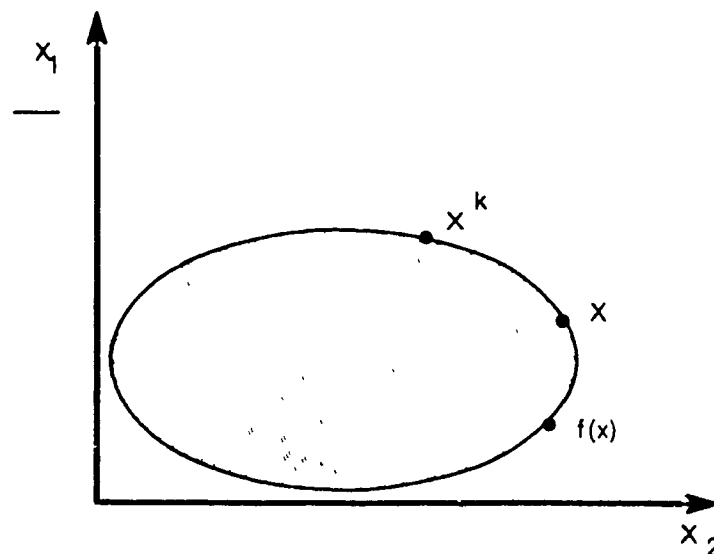


Fig.6.1 Feasible Region and Optimal Point  $\bar{x}$

### 6.3.2 Convergence

For convergence of the solution, it is necessary to consider only directions that point into the interior of the constraint region. When the trail solution is on a non-linear boundary there is always a direction of improvement unless the trail point is truly optimal.

Another approach is to use a grid of feasible points and approximate the constraint region by linearization among these points. In either approach, the successive values of  $f(x)$  converge to the optimal value  $F$  and the optimum solution is unique.

### 6.3.3 Optimality

Once the optimality conditions are satisfied, a point  $\bar{x}$  is said to be the optimal point and the respective  $F$  is the optimal solution.

## 6.4 OPTIMALITY CONDITIONS

This section reviews optimality conditions for the minimum of a non-linear function subject to non-linear constraints. Based on these conditions there are many numerical approaches for non-linear optimisation, but no general consensus as to which is the best. The optimality conditions which follow are often referred to as the Kuhn-Tucker conditions [11,13].

### 6.4.1. Conditions for a Minimum with Non-linear Equality Constraints

When all the constraints are equalities, the form of the problem becomes:

$$\begin{array}{ll} \text{Minimise} & F(x) \\ \text{subject to} & \hat{C}(x) = 0 \end{array} \qquad 6.4a$$

where  $x$  is the  $n \times 1$  vector of variables and  $\hat{C}$  is the  $t \times 1$  vector of equality constraints. A point  $x^*$  is a local minimum of problem of (6.4) if :

$$a) \hat{C}_i(x^*) = 0 \quad 6.4b$$

where  $i = 1, 2, \dots, t$  ( $x^*$  is feasible ).

$$b) F(x^*) \leq F(x) \text{ for all feasible } x \text{ in some neighbourhood of } x^* . \quad 6.4b$$

#### 6.4.1.1. Constraint Qualification

To verify whether a point  $x^*$  satisfies condition (b) above, it is necessary to characterise feasible perturbations from  $x^*$ , so that the behaviour of  $F(x)$  along such directions may be analyzed. In order to determine whether feasible perturbations exist, it is necessary to impose conditions on the constraint functions [13]. These conditions are commonly termed constraint qualifications. The constraint qualification with respect to the constraints of the non-linear equality problem holds at  $\bar{x}$  if [13]:

- i) The functions  $\hat{C}_i(x)$  are twice-continuously differentiable,
- ii)  $\hat{C}(x) = 0$ , and
- iii) Any non-zero vector  $p$  satisfying:

$$\hat{A}(\bar{x})^T p = 0 \quad 6.5$$

is tangent to a twice-differentiable arc emanating from  $\bar{x}$  along which all constraints are satisfied in some neighbourhood of  $\bar{x}$ . In the above, the matrix  $\hat{A}(x)^T$  is called the Jacobian matrix of the set of constraint

functions, i.e.

$$\hat{A}(x)^T = \frac{\partial \hat{C}(x)}{\partial x^T} \quad 6.6$$

Satisfaction of the constraint qualification allows a specification of necessary conditions for a point to be a solution of non-linear equality problem. If  $F(x)$  and  $\hat{C}(x)$  are continuously differentiable, and the constraint qualification holds at  $x^*$ , the following conditions are necessary for  $x^*$  to be a solution of Non-linear problem:

$$\begin{aligned} 1) \quad & \hat{C}(x^*) = 0 \\ 2) \quad & g(x^*) = \hat{A}(x^*) \lambda^* \end{aligned} \quad 6.7b$$

where  $g(x) = \partial F(x) / \partial x$

where the vector  $\lambda^*$  is called the vector of Lagrange multipliers, and represents the co-efficients in the expansion of  $g(x^*)$ . The condition (6.7b) is equivalent to the statement that  $x^*$  is a stationary point with respect to  $x$  of the Lagrange function defined by :

$$L(x, \lambda) = F(x) - \lambda^T \hat{C}(x) \quad 6.8$$

This characterisation of  $x^*$  is quite significant in the design of algorithms to solve Non-linear problem.

## 6.5 FORMULATION FOR FILTER DESIGN

The filter design problem can be formulated into the

classical definition of a non-linear optimisation. The filter cost equation (5.22) can be expressed as

Minimise  $F(x)$

Subject to  $H(x) \leq 0$

$L \leq x \leq U$

where

$x$  is a vector which will include the R,L,C parameters of the filters.

$F(x)$  is the objective function which is to be optimised i.e. filter cost (eq.6.1). Since the cost factor includes the losses, the resistor is one of the optimisation variables.

$H(x)$  are the constraint functions which represent the basic filter design parameters such as  $\delta$ ,  $Q$  and other performance criteria such as harmonic voltage TIF, IT, reactive power etc.

- frequency de-tuning  $\delta$  should not exceed, say 2%

- No single harmonic voltage ( $v_h$ ) should exceed 1% of the nominal system voltage.

- The arithmetic sum of the theoretical harmonic voltage (THD) from 2<sup>nd</sup> to 50<sup>th</sup> harmonic inclusive should not exceed say 3% of the nominal system voltage.

$L$  and  $U$  are the vectors of lower and upper limits of the state variables.

A general purpose non-linear optimisation package called

MINOS 5.0 [19 ] is used to achieve the stated objective. This package is very efficient at solving general non-linear optimisation problems with non-linear constraints.

#### 6.6 MINOS 5.0

MINOS solves a non-linear optimisation problem with non-linear constraints using the same optimality conditions as explained in section 6.4. MINOS solves the filter design problem by re-arranging the objective function (Cost equation) and the constraint function (Performance specifications, etc.) into the Lagrange format (equation 6.8). The solution to the problem lies in two stages:-

Stage one is called, major iteration. This involves solving the Lagrange equation to obtain a feasible point  $x^*$ . To verify, at the end of each iteration, if the new feasible point  $x^*$ , satisfies the objective function, MINOS performs a task called minor iteration, and this is stage two operation.

Stage two operation is similar to the "constraint qualification" (6.4.1.1), wherein it is necessary to characterise feasible perturbations from  $x^*$ , so that the behavior of  $F(x)$  along such directions can be analysed. If the constraint qualifications with respect to the constraints holds, then that feasible point is the optimal

point and the  $F(x)$  at that point is the optimal solution.

#### 6.6.2 Minos Operation

The general layout of MINOS operation is given in Figure 6.2 It consists of a file called MPS; this file stores all the user given data and specification, such as network impedance, harmonic voltage, etc. The main driver reads the user given information and stores in the MPS file. MINOS also requires two subroutines called Funobj and Funcon.

##### 6.6.2.1 Funobj

This subroutine computes the objective function to be optimised and its gradient. Gradients are evaluated by MINOS using a finite difference approach. Filter cost equation is evaluated in this subroutine.

##### 6.6.2.2 Funcon

This subroutine computes the non-linear constraint functions and their gradients. All the filter constraints, such as TIF, IT, Harmonic voltage etc. are computed in this subroutine.

The program flow chart is given in Figure 6.3

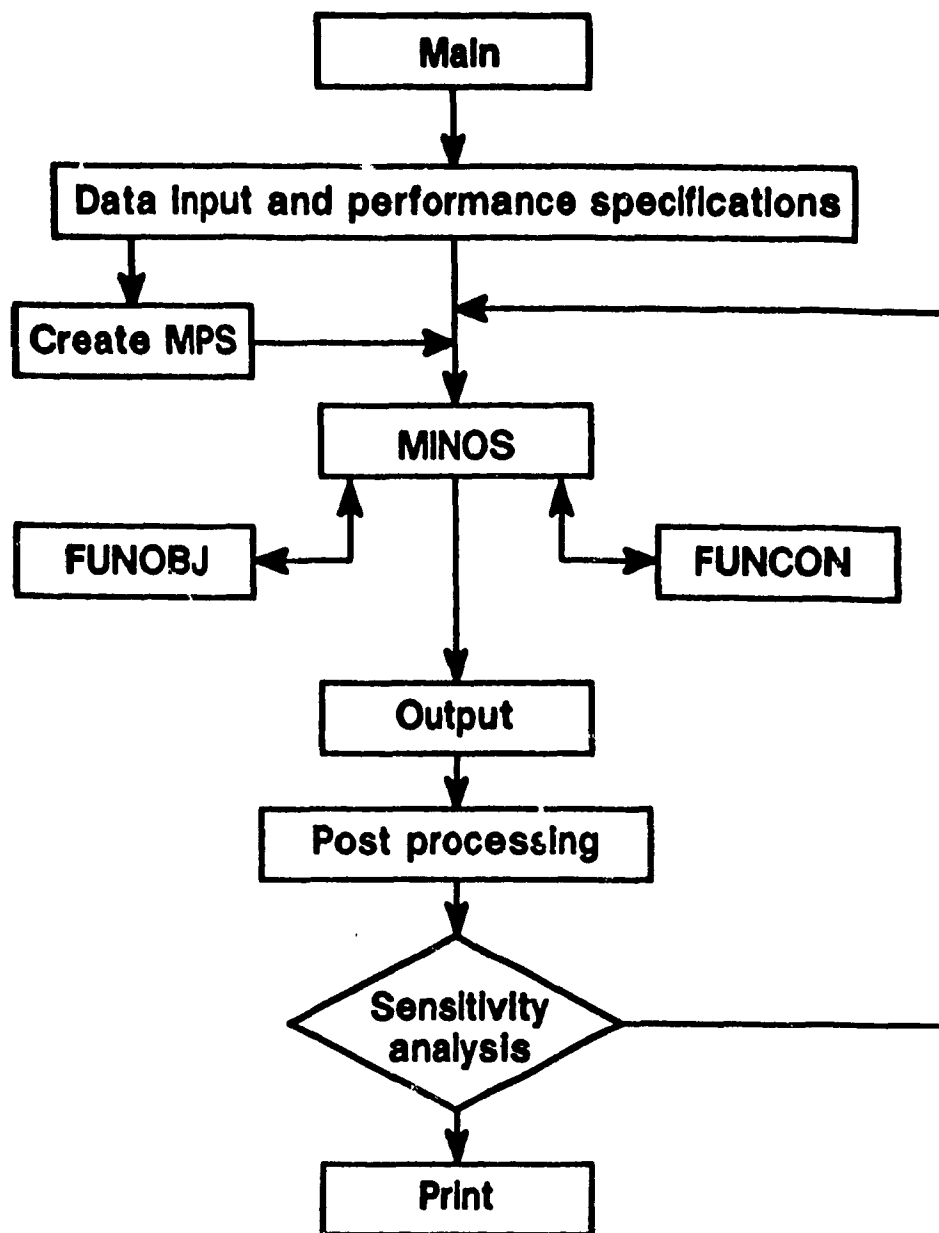


Figure 6.2

MINOS OPERATION



## 6.7 WORST HARMONIC VOLTAGE

Harmonic voltage on the bus is the result of (1) the converter current source, (2) Transformer saturation, and (3) any other switching devices in the AC system. All these sources produce harmonic currents. However, one of the principal harmonic current sources is the converter. Once the converter is designed the amount of harmonic voltage  $V_h$  on the bus is determined. Since there is a limit on the amount of harmonic voltage  $V_h$  an AC system will tolerate, one of the principle functions of the filter design is to keep the harmonic voltage  $V_h$  to an acceptable level. This is achieved in the following manner.

Harmonic voltage on the bus is given as

$$V_h = I_h / Y_t \quad 6.4$$

where

$I_h$  = Harmonic current source

$Y_t = Y_s + Y_f$

$Y_s$  = System admittance

$Y_f$  = Filter admittance

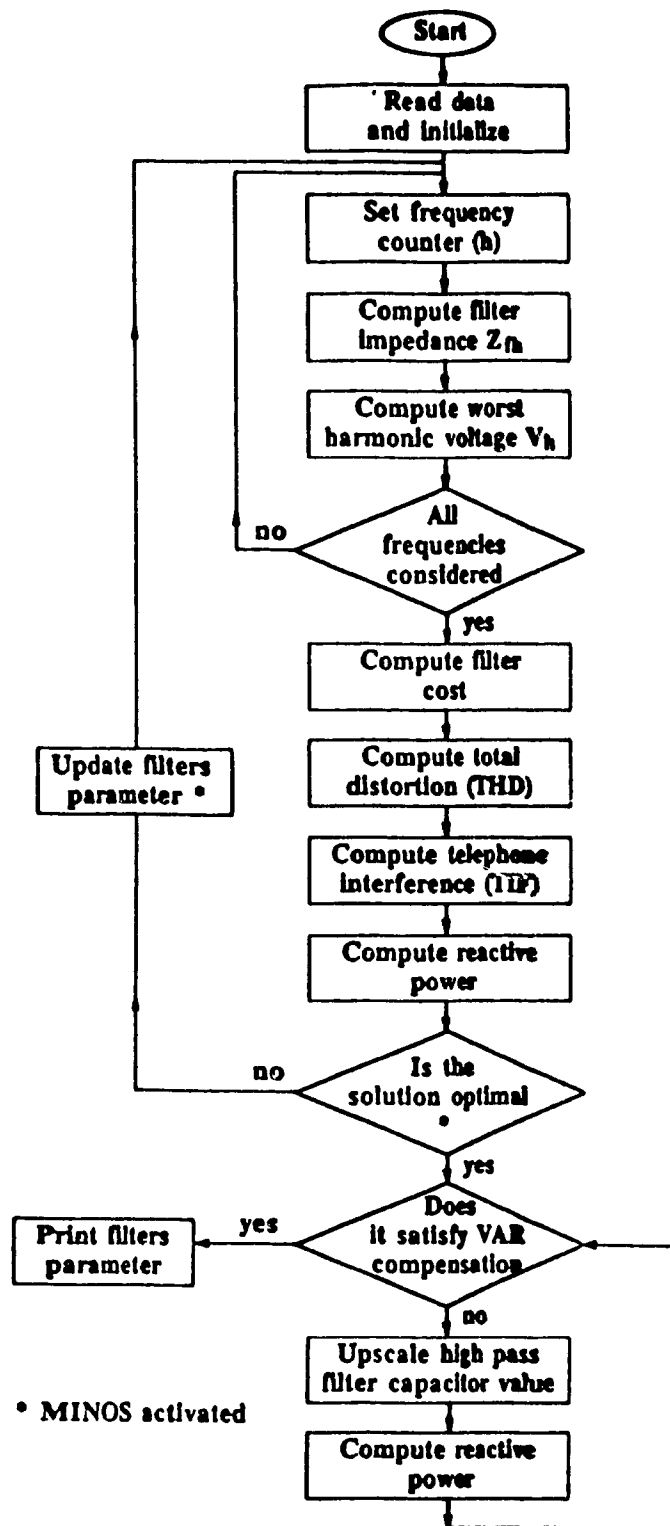


FIGURE 6.3

PROGRAM FLOW CHART

The variables that are not under the control of the filter designer are chosen pessimistically, which results in high harmonic voltages  $V_h$ . The variable that are under the designer's control are chosen optimally to give acceptable harmonic voltages  $V_h$ . The variables that the designer can choose are the filter parameters (i.e. Size of the filter etc.).

The AC network admittance angle  $\phi$  is defined as a value limited between  $\pm \phi_m$  ( where  $0 < \phi_m < 90^0$ ). It is shown [8] that the highest harmonic voltage  $V_h$  occurs when  $\phi = \phi_m$ . For a given AC network  $Y_s$  is fixed. Hence it is required to find and use the value that minimises total admittance  $Y_t$  which corresponds to maximising the harmonic voltage  $V_h$ . This value can come only from the filter admittance  $Y_f$ . It is shown in [8] that for a given filter admittance  $Y_f$ , the shortest vector for total admittance  $Y_t$  is perpendicular to the boundary (from  $Y_f$ ) and terminates on the boundary.

A modified version of Figure 4.9 (4.4.3) is redrawn as Figure 6.4. If the shaded area (Fig.6.4b) represents the AC system admittance, then any point in the unshaded area should represent the filter admittance [8], where D is the maximum diameter and d is the minimum diameter.

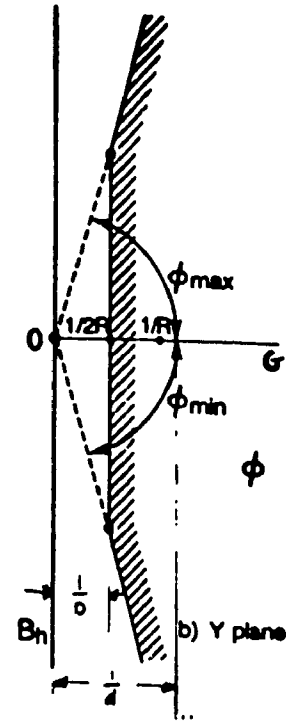
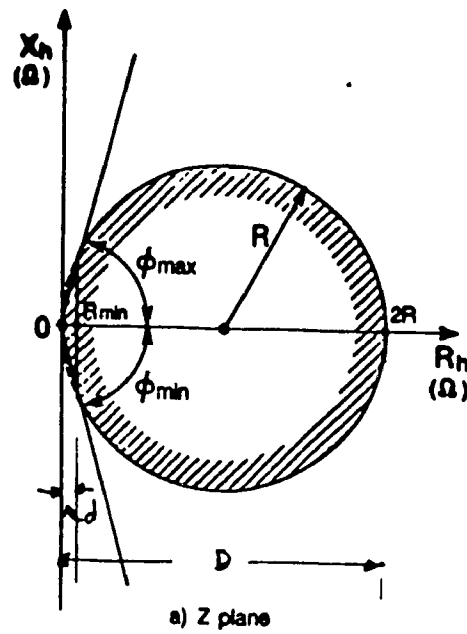


Fig. 6.4a AC Network impedance Fig.6.4b Admittance plane

Since

$$V_h = \frac{I_h}{Y_f + Y_s} \quad 6.6$$

the total admittance  $Y_t$  is to be minimised i.e.:

$$\text{Min } (Y_t) = Y_f + Y_s \quad 6.7$$

$$\text{Min } (Y_t - Y_f) = Y_s \quad 6.8$$

i.e. if  $Y_f$  is the filter admittance in the  $B \pm G$  plane, then  $-Y_f$  will be in  $-B \pm G$  plane.

#### 6.7.1 Locating Filter Admittance

At any given frequency the filter admittance lies at a point in the unshaded area of Fig. 6.4. In order to

facilitate the location of this point in this area, it is divided into nine zones, say Z1 to Z9 (Fig.6.5). All these zones are valid in the  $-G, \pm B$  plane. Admittance  $-Y_f$  can be placed or located in any one of these zones. Once the zone in which  $-Y_f$  is located is identified, then a perpendicular is dropped from the point to the boundary line of the shaded area. This length represents the shortest distance and hence the minimum total admittance  $Y_t$ .

The objective now is to establish a procedure for computing the minimum total admittance  $Y_t$ . Redrawing Fig.6.4b in the admittance plane and dividing it into nine different zones results in Fig.6.5. The co-ordinates  $(-x_f, -y_f)$  of  $-Y_f$  will be in any one of the ten zones i.e. outside the AC network operating zones and to the left of line MF.

#### 6.7.2 Definition Of Some Basic Terms

$$R' = 1 / D \quad 6.9a$$

$$r' = 1 / d \quad 6.9b$$

$$\phi_2 = -\phi_{\max} \quad 6.9c$$

$$\phi_1 = -\phi_{\min} \quad 6.9d$$

$$T = (r'/2, 0) \quad 6.9e$$

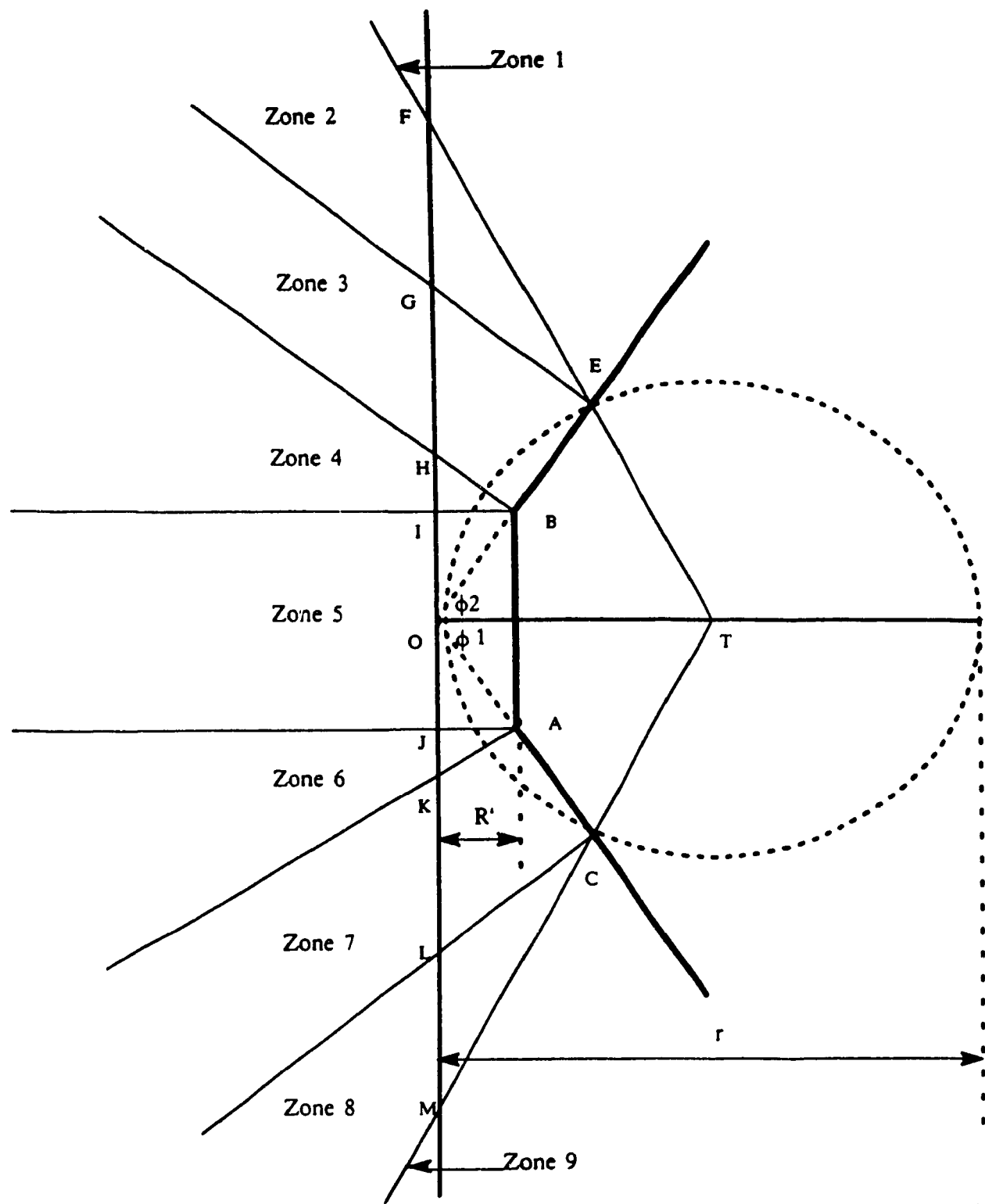


FIGURE 6.5 Determination Of Total Minimum Admittance

### 6.7.3 Definition Of Points

Similarly defining each point on the zones

$$B' = R', R' \tan(\phi_2) \quad 6.10$$

$$A' = R', R' \tan(\phi_1) \quad 6.11$$

$$I = 0, R' \tan(\phi_2) \quad 6.12$$

$$J = 0, R' \tan(\phi_1) \quad 6.13$$

$$H = 0, 2R' / \sin(2\phi_2) \quad 6.14$$

$$K = 0, 2R' / \sin(2\phi_1) \quad 6.15$$

$$E' = r' \cos^2(\phi_2), 1/2 r' \sin(2\phi_2) \quad 6.16$$

$$C' = r' \cos^2(\phi_1), 1/2 r' \sin(2\phi_1) \quad 6.17$$

$$G = 0, r' / \tan(\phi_2) \quad 6.18$$

$$L = 0, r' / \tan(\phi_1) \quad 6.19$$

$$F = 0, -r'/2 \tan(2\phi_2) \quad 6.20$$

$$M = 0, -r'/2 \tan(2\phi_1) \quad 6.21$$

### 6.7.4 Definition Of Lines

After defining the points, Slopes and lines can be defined.

Line TF'

Slope of the line TF' is given as

$$M_{tf} = \frac{1/2 r' \sin(2\phi_2)}{r' \cos^2(\phi) - r'/2} \quad 6.22a$$

$$= \frac{\sin(2\phi_2)}{2 \cos^2(\phi_2) - 1}$$

$$M_{tf} = \frac{\sin (2\phi_2)}{\cos (2\phi_2)}$$

$$M_{tf} = \tan(2\phi_2) \quad 6.22b$$

Line TF'

General line equation is  $Y = m.X + b$

if  $Y = 0$

$$\text{Then } X = -\frac{b}{m} = \frac{r'}{2}$$

$$b = -\frac{r'}{2} \cdot m$$

substituting for  $m$  from 6.22 yeilds

$$b = -r'/2 \tan(2\phi_2) \quad 6.23$$

Hence line TF can be written as

$$Y_{tf} = \tan(2\phi_2) X - r'/2 \tan(2\phi_2) \quad 6.24$$

Similarly other lines can be written as follows

Line TM

$$Y_{tm} = \tan(2\phi_1) X - r'/2 \tan(2\phi_1) \quad 6.25$$

Line E'G

$$Y_{e'g} = -\frac{1}{\tan(\phi_2)} \cdot X + \frac{r'}{\tan(\phi_2)} \quad 6.26$$

Line C'L

$$Y_{c'l} = -\frac{1}{\tan(\phi_1)} \cdot X + \frac{r'}{\tan(\phi_1)} \quad 6.27$$

Line B'H



$$Y_{b'h} = - \frac{1}{\tan(\phi_2)} \cdot X + \frac{R'}{\tan(\phi_2)} \quad 6.28$$

Line A'K

$$Y_{a'k} = - \frac{1}{\tan(\phi_1)} \cdot X + \frac{R'}{\tan(\phi_1)} \quad 6.29$$

Line B'I

$$Y_{b'i} = R' \tan(\phi_2) \quad 6.30$$

Line A'J

$$Y_{a'j} = R' \tan(\phi_1) \quad 6.31$$

#### 6.7.5 Zone Equations

The next stage is to develop expressions for any points in the respective zones as follows:

Zones 1 and 9

$$X_1 = r'/2 \left[ 1 + \sqrt{1 + \frac{1}{\left( \frac{Y_f}{x_f - r'/2} \right)^2}} \right] \quad 6.32$$

$$Y_1 = \frac{Y_f}{x_f - r'/2} \cdot \left( X_1 - r'/2 \right) \quad 6.33$$

Zones 2 and 8

$$X_{2/8} = r' \cos^2(\phi_2) \quad 6.34$$

$$Y_{2/8} = \frac{1}{2} r' \sin(2\phi_2) \quad 6.35$$

for zone eight replace  $\phi_2$  with  $\phi_1$

**Zones 3 and 7**

$$X_{3/7} = \left( Y_f + \frac{1}{\tan(\phi_2)} \cdot X_f \right) \cdot \frac{\sin(2\phi_2)}{2} \quad 6.36$$

$$Y_{3/7} = X_{3/7} \tan(\phi_2) \quad 6.37$$

for zone seven replace  $\phi_2$  with  $\phi_1$

**Zones 4 and 6**

$$X_{4/6} = R' \quad 6.38$$

$$Y_{4/6} = R' \tan(\phi_2) \quad 6.39$$

**Zones 5**

$$X_5 = R' \quad 6.40$$

$$Y_5 = Y_f \quad 6.41$$

Distance between  $- Y_f( X_f, Y_f )$  and the respective zone points (ex.  $X_1, Y_1$ ) can be expressed in the standard form as

$$\text{distance} = \sqrt{(X_f - X_1)^2 + (Y_f - Y_1)^2} \quad 6.42$$

Once the distances are computed then the minimum distance is selected. This value corresponds to the minimum total admittance of the system and filter combined. This value in turn provides the worst harmonic voltage.

#### 6.8 SUMMARY

Justification for cost optimisation is given. Optimisation principle is discussed, in brief. Basic terminology used is explained. Optimality conditions for a non-linear equation with non-linear equality and with non-linear inequality constraints are discussed. Equations for constraint qualifications are developed. Formulation of filter design into the classical definition of a non-linear optimisation, as discussed above, is given. MINOS and its operations are briefly discussed. Similarities of the MINOS operation with respect to the optimality conditions are stressed. A methodology is developed to derive an expression for worst harmonic voltage. This chapter provides the basis of how to optimise the filter design.

## **CHAPTER SEVEN**

### **7.0 EXAMPLES**

**7.1 Introduction**

**7.2 Example One**

**7.3 Example Two**

**7.4 Summary**

## 7.1 INTRODUCTION

The software written for the optimal filter design, based on the algorithms developed in the preceding chapters, is used to test and compare sample filter design problems. Two examples are considered. Example One deals with designing a separate 5<sup>th</sup> harmonic single-tuned filter. Example Two deals with a sample specification issued by a utility and the manufacturer's response to it. In both examples the results are compared and discussed.

## 7.2 EXAMPLE ONE

An example of a filter design problem was taken from [8] pp 370-371:

*Find the minimum-cost fifth harmonic filter for a bipolar four-bridge 12-pulse converter rated 1.00 kA, 300 kV on the DC side. The filters are to be connected to the 235kV 60HZ three-phase line. The fifth-harmonic filter is to be designed for the operation of the converter with one bridge out of service. Assume the unit cost of capacitors to be \$3.59/kvar and that of inductors \$ 8.00/kvar, with 5<sup>th</sup> harmonic current of 70.2 A. The limiting network impedance angle may be taken as 75 degrees.*

A two-part comparison between the results obtained by traditional methods and the proposed approach is made in Table 1.

In part 1, the losses (i.e. running costs/year) of the filter are not considered. In column a (Table 1), the filter parameters designed by the traditional method are presented. A comparison of these results with those in column b shows that traditional method gives higher cost. The reason for this is that the traditional method optimises the filter cost with respect to one variable only, namely the filter capacitor.

TABLE 1. Comparison of results for Example 1.

<div style="display: flex; justify-content: space-around; align-items: center;"> <span>← Part 1 →</span> <span>← Part 2 →</span> </div>								
Column	a	b	c	d	e	f	g	h
Filter parameters	Approach of Ref.(8)	Proposed method						
		Energy losses ignored			Energy losses considered			
		U = 0	U = 0	U = 0	U = 2	U = 4.4	U = 8	U = 10
Installed cost (\$)	73684	69346	89632	85853	71525	75834	83664	89202
Running cost (\$ - )	-	-	-	-	21748	41696	65516	74658
Capacitor ( $\mu$ F)	1.746	1.862	2.72	1.746	1.754	1.750	1.822	1.893
Inductor (H)	0.1611	0.1511	0.10344	0.1611	0.1604	0.1607	0.1544	0.1486
Resistor ( $\Omega$ )	9.3	13.11	6.00	6.00	10.45	8.42	6.21	5.14
Quality factor (Q)	32.5	21.7	32.5	50.6	28.9	35.96	46.85	54.5
Size (Mvar)	12.116	12.92	18.87	12.115	12.17	12.14	12.64	13.14
Current (A)	88.5	78.1	88.5	106.41	84.83	91.90	102.7	110.0
Admittance angle (deg)	-52.5	-40.1	-52.5	-63.7	-49.2	-55.1	-61.9	-65.35

$$U = X.X \text{ cents / kWh}$$

The proposed method, on the other hand, optimises the filter cost as a function of the resistance and capacitor

of the filter. The program also has the ability to fix one (or multiple) parameters permitting the computation of other variables; this feature is useful in the evaluation of the performance characteristics of the filter and permits the filter designer to make critical trade-offs.

In part 2 (Table 1), the losses in the filter are considered and a comparison between the two methods is made. The running costs of the filters due to losses can be significant. The impact on filter costs of a parametric variation in the cost of energy loss, i.e.  $U = 2$  to 10 cents/kWh can be seen in column e to h respectively. Considering column g with  $U = 8$  cents/KWh, it can be seen that the installed cost of the filter is \$ 83664, which is lower than the cost found in columns c or d in Table 1. (i.e. \$ 89632 and \$ 85853 respectively).

### 7.3 EXAMPLE TWO

In this example, the filter design from a typical HVDC station is considered. The filter performance specification and system data given by a utility to a manufacturer to design the filter is used to derive an alternative filter. A comparison shows that the proposed technique can provide a useful check for the utility. The filter design by the proposed method was done in stages to highlight some features of the design process.

**Filter Performance Specification:**

Individual Harmonic Voltage = < 1%

Total Harmonic Voltage = < 3.6%

TIF requirement = 20.

The harmonic source currents are provided in Appendix 1.

**System Data:**

Network Impedance Angle = 87 deg.\*\*

Network Impedance Circle radius  $R' = 140 \text{ ohms}$ \*\*

Minimum Impedance  $R, \min' = 0.5 \text{ ohms}$ \*\*

Detuning factor = 2%

System Bus Voltage  $V = 120 \text{ kV}$

DC power transmitted = 1000 MW

Total filter reactive power = 168 MVar

\*\* valid for all the range of harmonics.

The filter designed by the manufacturer is shown in the Figure 7.1. It is comprised of tuned filters at the 11<sup>th</sup> and 13<sup>th</sup> harmonics, a high-pass filter tuned to 23<sup>rd</sup> harmonic, and a double tuned filter at 3<sup>rd</sup> and 5<sup>th</sup> harmonics. The component values are marked in the figure as well the filter Quality factors.

The filter design by the proposed method was done in different stages to highlight some features of the design



process:

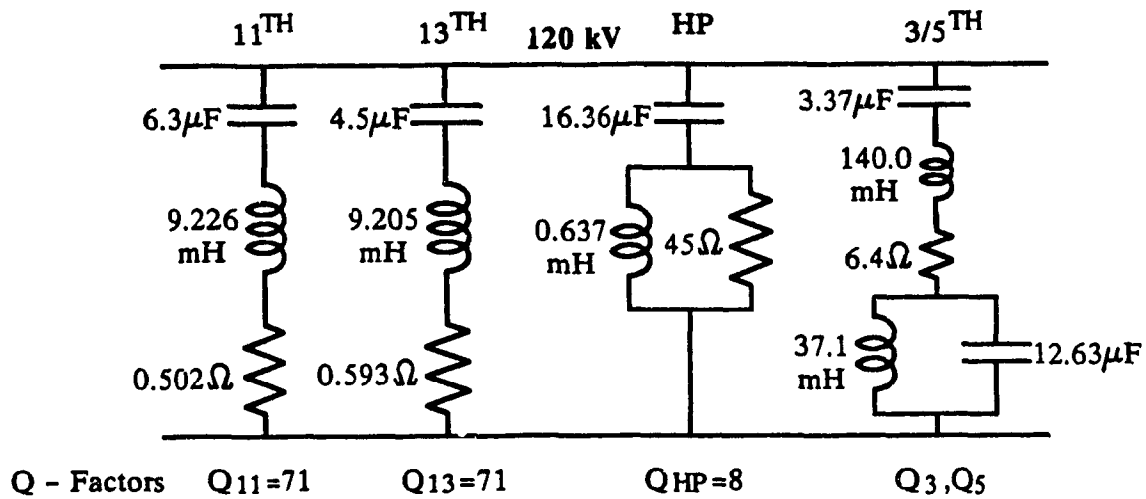


Figure 7.1: Typical filter designed by manufacturer.

**Stage 1:** As a first approximation, the cost of energy losses in the filter were ignored. A solution by the proposed method resulted in the design shown in Figure 7.2. The results show that filter parameters are comparable to the manufacturer derived filter except for the capacitor size in the high pass filter. The capacitor size for the high-pass filter is lower since the constraint for reactive power demanded by the converter has not yet been applied. The filter designed is therefore optimal for the requirements set by harmonic voltage, TIFV and THD only.

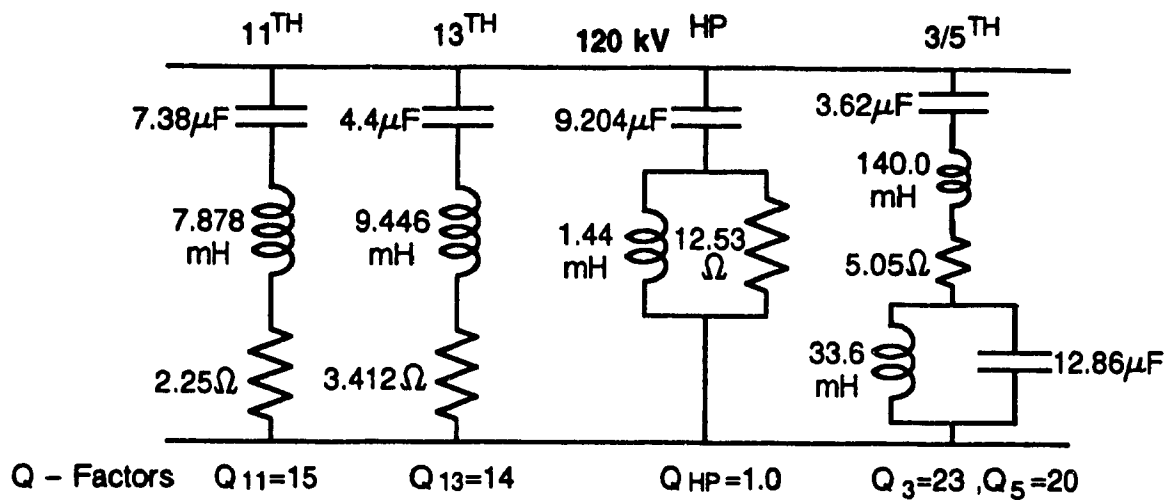


Figure 7.2 Filter designed by proposed method; without losses and reactive power consideration.

**Stage 2:** In the next step, the losses in the resistor are considered at a rate of 4.4 cents/KWh. This results in the filter design shown in Figure 7.3. Note that the Quality factors are higher than in Figure 7.2. (Stage 1, above).

**Stage 3:** Finally, the reactive power constraint is applied so as to impact only on the high-pass filter. This requirement increased the capacitor for the high-pass filter from  $9.204\mu\text{F}$  to  $14.32\mu\text{F}$ . Comparison of Figure 7.4 and Figure 7.1 shows that the design of the filter by the proposed method and the manufacturer is similar except for the discrepancy in Quality factors obtained by the two methods. This is reflected in terms of the differences in the resistor values. It is noted that the resistor

indicated

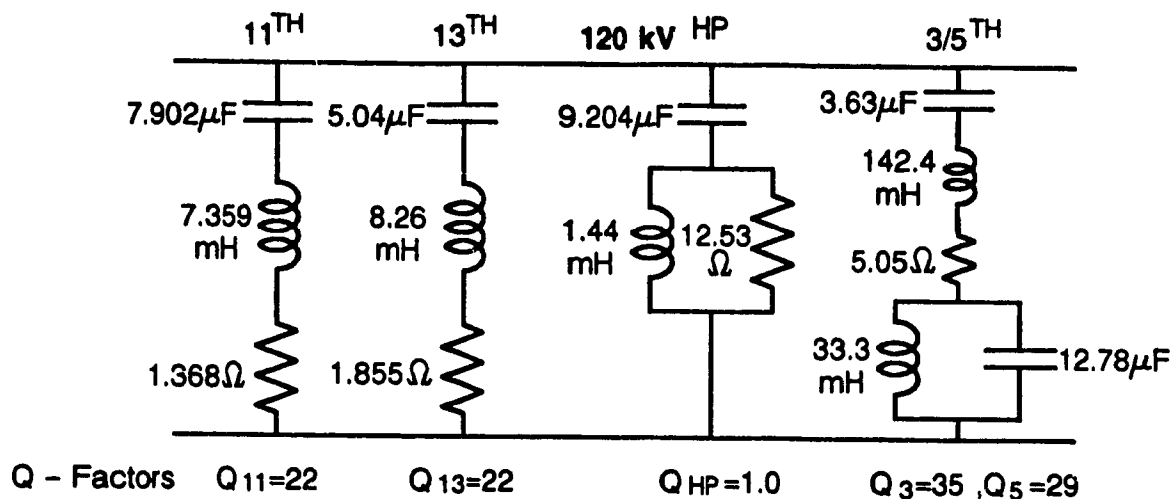


Figure 7.3: Filter designed by proposed method:  
with losses considered.

in the design of the manufacturer is the external resistor added to the filter; the internal resistance of the inductor has not been indicated and can only be estimated from the indicated value of the Q-factor of the inductor. It is presumed that the Q-factor indicated is that at fundamental frequency. On the other hand, the indicated resistor value in the proposed method assumes that the inductor is purely inductive and has no resistive component. The external resistor is therefore the sum of the added resistor plus the resistance of a "real" inductor.

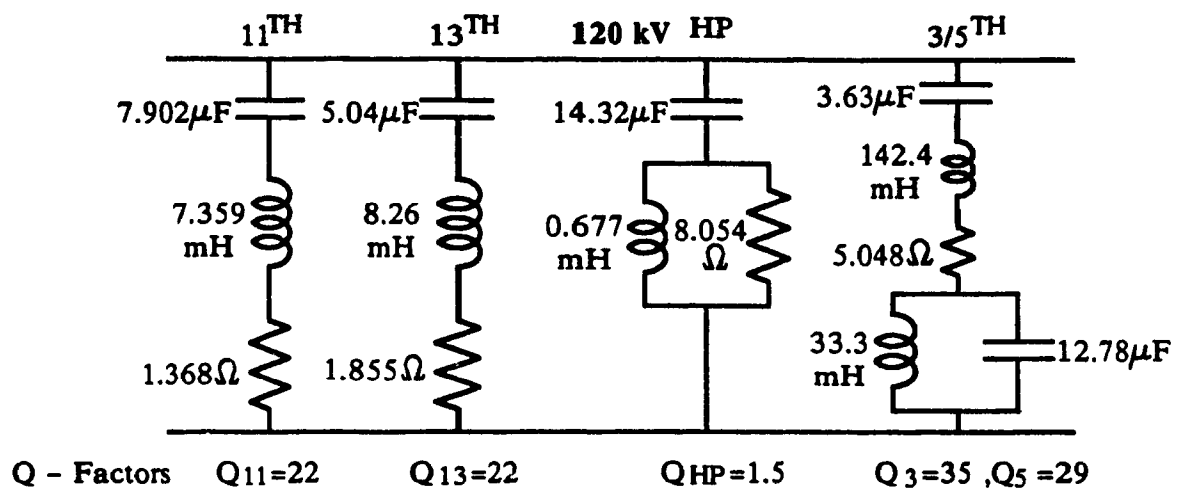


Figure 7.4: Filter designed by proposed method:  
reactive power considered.

It is logical, however, that filter Q-factors should be high to minimize the losses. It may be feasible that the Q-factors proposed by the manufacturer are selected to be as high as practically possible, and are probably limited due to manufacturing limitations.

In Figure 7.5 is shown a comparison of the impedance versus frequency characteristics of the filters designed in Figure 7.1 and 7.4. These comparative plots show the characteristics of the two filter designs. The major difference in the two characteristics is in the impedance of the high-pass filter. This is a result of the choice of the Q-factor. The proposed design (Figure 7.4) is deliberately chosen to have a low Q-factor.

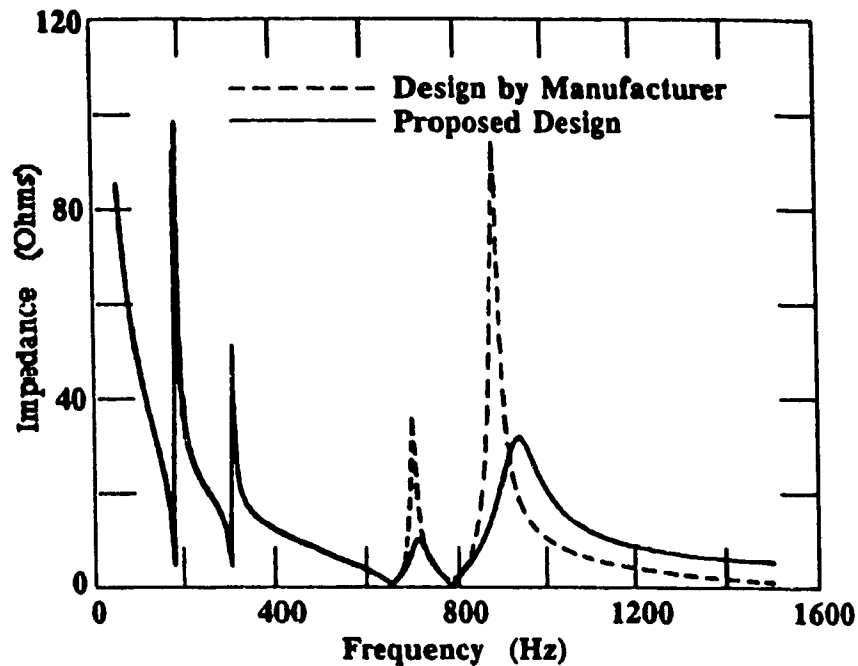


Figure 7.5 Impedance versus Frequency for filter design by proposed method versus manufacturer's method.

#### 7.4 SUMMARY

The proposed technique has been compared to a typical filter and has been shown to provide a similar solution. The comparison was done with two specific cases:

(1) A single-tuned filter at 5<sup>th</sup> harmonic frequency. Costs of this filter with energy loss and without energy loss are compared with the example.

(2) Design results of a filter bank near a HVDC converter bus, consisting of single-tuned, a high-pass and a double-tuned filters are compared. Furthermore, the ability to perform sensitivity studies by the proposed method was shown.

## CONCLUSIONS AND FUTURE DEVELOPMENT

### 8.1 CONCLUSIONS

A new technique to optimally design a filter has been presented. This technique has been compared to an existing filter and has been shown to provide a similar solution. The proposed technique is simple and direct optimization program to facilitate the work of the design engineer. Literature reviewed showed various methods for filter design, however these methods were based on a trial-and-error approach and were tedious to utilise. The application of optimization techniques to design filter is found to produce similar results with relative ease. Furthermore, the ability to perform sensitivity studies greatly aides the design process.

The proposed method addressed four major aspects of the AC harmonic filter design near a HVDC converter:

- 1) To keep the harmonic contents of voltages and currents at the converter bus to an acceptable level. This is achieved by filtering harmonic voltages at defined harmonic frequencies.
- 2) To generate required amount of reactive power to be consumed by the converter. This is achieved with the aid of reactive elements in the filter, namely the capacitor and the inductor, which generates the reactive power.
- 3) Capital cost of the filter is around 15% [3] of the

converter station costs. Optimizing the filter costs will help reduce the overall converter station costs. This is achieved by applying optimization techniques to the filter design process. This resulted in a filter, which satisfied all design and performance requirements and gave the optimum filter cost.

4) To keep the running/operating cost of the filter to a minimum level. This is achieved by including the running cost (expressed in terms of energy loss/kWh) into the filter cost expression.

The use of a general purpose non-linear optimization package has proven to be an efficient way of arriving at the required solution. The design procedure is interactive, permitting sensitivity analysis studies to be performed.

## **8.2 FUTURE DEVELOPMENT**

At present, the proposed method of applying optimization techniques to filter design is devised only for (1) single-tuned filters and (2) high-pass filters of second order. The proposed method is currently being applied to double-tuned filters. A full model of the double-tuned filter has yet to be developed. Extension of this work to cover high-pass filters of higher order, particularly that of third order and C-type has yet to be carried out. Design of DC filters using the proposed method needs to be explored.

AC harmonic impedance was defined with the help of  $R_L$  (AC harmonic impedance operating radius) and  $\phi_m$  (impedance angle) [4.4.4]. It can also be defined in an area enclosed by a polygon, wherein each point of the polygon is user-defined. The use of a polygon to define the AC harmonic impedance has not been explored.

The application of this technique to design filters at low power converter buses can also be explored.

The basic concept of applying the optimization techniques to filter design produced positive results. Hence this concept can now be applied to other types of filters.



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## APPENDIX I

The external look-up table can be read in by the program.  
In this case, both characteristic and non-characteristic harmonics can be considered.

**Table Two Harmonic Currents**

Harmonic number	Harmonic current (A)	Harmonic number	Harmonic current (A)
1	5560.0	26	0.6
2	2.6	27	3.9
3	26.2	28	0.7
4	4.0	29	2.1
5	18.3	30	0.5
6	3.3	31	1.1
7	14.3	32	0.3
8	2.4	33	2.0
9	12.6	34	0.2
10	1.1	35	25.7
11	240.1	36	0.2
12	0.5	37	22.3
13	163.4	38	0.4
14	0.5	39	2.0
15	2.3	40	0.6
16	0.3	41	1.9
17	1.2	42	0.6
18	0.4	43	1.7
19	2.7	44	0.5
20	0.7	45	1.3
21	4.6	46	0.3
22	0.5	47	12.6
23	58.7	48	0.2
24	0.4	49	13.1
25	48.4		