DESIGN AND IMPLEMENTATION OF A
100kHz GYRATOR BAND-PASS FILTER

by

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ABSTRACT

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In this report, the design and implementation of high-Q, bandpass filters using gyrators is reviewed. Chapter 1 describes passive band-pass filter synthesis, since this is the basis of the gyrator filter design procedure. Chapter 2 described the various imperfections of operational amplifiers. Chapter 3 describes the theoretical analysis of gyrator behaviour, and finally, Chapter 4 describes the implementation of a specific band-pass filter using gyrators instead of inductors. The results show that with adequate design precautions, gyrators can be used to design band-pass filters centered at 100KHz.
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INTRODUCTION

The unsuitability of inductors for the microelectronic realization of filters has led to the development of active RC circuits which simulate inductors. Over the period of the past twenty years, many attempts have been made to design and construct inductorless high quality filters. One of these achievements was to replace each inductor in a conventional LC ladder filter by a circuit that simulates an inductor, i.e. a capacitively-loaded gyrator.

The gyrator is an element that until few years ago had an available, if incomplete theory, but (at least at frequencies below the microwave range) only inadequate realizations. In the last years, however, several good realizations have appeared in literature, so that the gyrator now becomes available as a practical active element.

The major drawback for gyrators' performance is the effect of operational amplifier imperfections, which can be described by parasitic elements in the equivalent circuits. Because of these parasitic elements, the performance of the active filters may deteriorate considerably. Suitable steps for increasing the useful frequency range of active filters and for avoiding deterioration of its transfer performance are:

a) Optimal design of the nonideal gyrator
b) Incorporation of the parasitic elements into the LC structure
CHAPTER 1
PASSIVE LC BAND-PASS FILTERS

1.1 Introduction

In theory, coupled LC filters have the lowest sensitivity to component variation. These doubly terminated reactive two-port produce a frequency response by reflecting power back to the source in the stopbands, while in the pass band, power transfer is maximum at the zero loss frequencies determined by the filter's transfer function.

In reality, a change in any component of the filter can only cause a reduction in the load power, down from the maximum. This power loss at each of the zero loss frequencies increases while at other frequencies a small loss exists because of the ripple caused by reflected power. At any point within the passband, then, the change in loss have a well behaved and slowly varying characteristic that follows the changes of any component (inductor or capacitor) in the network.

The synthesis of these networks has been studied extensively by many pioneers in this field using mathematical models, which approximate a certain transfer function to a permissible error tolerance. The results of their work are tabulated in mathematical tables to facilitate the solution of certain filter requirements.

The most commonly used filter approximations are the Butterworth, Chebyshev, and elliptic approximations. In this report the Chebyshev approximation is used.
c) Compensation of the parasitic elements (i.e. parasitic capacitance).

In the present report the Antoniou type gyrator was investigated. This gyrator circuit has been incorporated into a band-pass filter and satisfactory results were obtained, after optimization, for a center frequency of 100KHz.
1.2 Specifications of the Filter

The required band-pass filter should have a center frequency of 100KHz, bandwidth of 20KHz, (passband edges \( f_1 = 90\text{KHz}, \ f_2 = 110\text{KHz} \)) with 0.1db band-pass ripple, and transmission loss, at frequencies \( f_A = 140\text{KHz} \), and \( f_B = 160\text{KHz} \), equal to or more that 40 db.

With the above specifications for the filter's performance, we first derive a prototype lowpass LC ladder network which meets the specifications. Then we transform the low-pass prototype to a band-pass filter, and substitute each individual inductance with a gyrator, terminated by a capacitance to simulate the exact value of this inductance.

This technique is superior to other techniques used, because we get filters with very low sensitivities, provided that gyrators used are optimized for low sensitivity.

From Ref. [1] we see that a third order low-pass prototype filter (Chebyshev) will meet the requirements. The prototype low-pass filter is given by Ref. [1] as shown in Fig. 1.1. The cutoff frequency of this filter occurs at \( \Omega = 1 \).
Chebyshev low-pass prototype, 0.1 db ripple

Variation of $\Omega$ with respect to $\omega$

Elements transformation from low-pass to band-pass
Alternatively for $n=1$, the corresponding cut-off frequency is given by:

$$
\omega_2 = \frac{B}{2} + \sqrt{\frac{B^2}{4} + \omega_0^2} \quad (1.6)
$$

Also from equation 1.3, we see that the region $-1 < n < 1$ corresponds to:

$$
\omega_1 < \omega < \omega_2
$$

and

$$
-\omega_2 < \omega < -\omega_1 \quad (1.7)
$$

Therefore, for $\omega > 0$, the passband corresponds to

$$
\omega_1 < \omega < \omega_2 \quad (1.8)
$$

and the region outside corresponds to the stopband. The bandpass of the filter is given by:

$$
B = \omega_2 - \omega_1 \quad (1.9)
$$

Also we have

$$
\omega_0^2 = \omega_1 \omega_2 \quad (1.10)
$$

That is, the "centre frequency" $\omega_0$ is the geometric mean of the lower and upper pass-band frequencies. Thus the transformation yields a filter having geometric symmetry [2]. The elements $L, C$, in the lowpass filter are transformed to series or parallel resonance circuits in the band-pass filter as shown in Fig. 1.2(b).

Applying the above transformation to the low-pass prototype filter Fig. 1.1, the required band-pass filter shown in
1.3 Low-pass To Band-pass Transformation

The band-pass filter structure is derived by transforming the basic normalized low-pass filter. The transformation applied on the normalized low-pass filter is

$$ S_n = \frac{S^2 + \omega_0^2}{SB} = \frac{\omega_0}{B} \left( \frac{S}{\omega_0} + \frac{\omega_0}{S} \right) \quad (1.1) $$

whose cut-off frequencies are $\omega = \pm 1$

Is $S = j\omega$ we get

$$ S_n = j\omega = \frac{-\omega^2 + \omega_0^2}{j\omega B} \quad (1.2) $$

or

$$ \eta = \frac{\omega^2 - \omega_0^2}{\omega B} \quad (1.3) $$

The variation of $\eta$ with respect to $\omega$ is as shown in Fig. 1.2(a).

For $\eta = -1$

we get

$$ \omega^2 + B\omega - \omega_0^2 = 0 \quad (1.4) $$

or

$$ \omega = \frac{-B \pm \sqrt{B^2 + 4\omega_0^2}}{2} \quad (1.4a) $$

and for the region $\omega > 0$, the cut-off frequency $\omega_1$ is given by:

$$ \omega_1 = -\frac{B}{2} + \sqrt{\frac{B^2}{4} + \omega_0^2} \quad (1.5) $$
Fig. 1.3 is derived. Impedance scaling can now yield the filter of Fig. 1.4 where \( R = R_s = R_L = 10K \).

From Fig. 1.4 the design values of the filter are:

\[
C_1 = \frac{C_n}{R_B} = \frac{1.032}{10^4 \times 20 \times 10^3} = 5.160 \text{pF} \quad (1.11)
\]

\[
L_1 = \frac{BR}{C_n w_0} = \frac{20 \times 10^3 \times 10^4}{1.032 (2\pi \times 100 \times 10^3)^2} = 490.897 \mu \text{H} \quad (1.12)
\]

\[
L_2 = \frac{L_n R}{B} = \frac{1.147 \times 10^4}{20 \times 10^3} = 573.5 \text{mH} \quad (1.13)
\]

\[
C_2 = \frac{1}{L_n R w_0} = \frac{20 \times 10^3}{1.147 \times 10^4 (2\pi \times 100 \times 10^3)^2} = 4.4168 \text{pF} \quad (1.14)
\]

\[
L_3 = \frac{BR}{C_n w_0} = \frac{20 \times 10^3 \times 10^4}{1.032 (2\pi \times 100 \times 10^3)^2} = 490.897 \mu \text{H} \quad (1.15)
\]

\[
C_3 = \frac{C_n}{R_B} = \frac{1.032}{10^4 \times 20 \times 10^3} = 5.160 \text{pF} \quad (1.16)
\]

1.4 **Ideal Transformations**

From the design values of the band-pass filter we see that some elements are too small, and consequently not realizable. One possible way to overcome this disadvantage is the use of ideal transformers. From Ref. 3, "The ideal transformer is a device which has zero leakage inductance, and infinite primary and secondary inductances, but so proportioned that their ratio is finite number". The voltage-current relationships are shown in Fig. 1.5.
Fig. 1.3
Frequency transformation of band-pass filter derived from low-pass prototype

Fig. 1.4
Impedance scaling of band-pass filter
Fig. 1.5

Ideal Transformer

Equivalent Circuits of Transformer
Fig. 1.7
Symmetrical band-pass filter

Fig. 1.8
Band-pass filter after modification

R = R_s = 10 K  L_2 = 11.854 mH  C_2 = 2.584 pF  L_5 = 11.854 mH
C_1 = 5.160 pF  L_1 = L_3 = .5112 mH  L_4 = L_6 = .5112 mH  C_3 = 5.160 pF

Fig. 1.9
Band-pass filter with actual values
From Fig. 1.5 we see that the volt-amperes in primary and secondary are equal, but the voltage to current ratios differ.

Since \( e_2/i_2 = n^2 e_1/i_1 \), and inductance in the secondary is like having an inductance \( 1/n^2 \) times as large placed in the primary, as far as the primary is concerned. This is valid also for resistance \( R \) and capacitance \( C \). Equivalent circuits for a transformer are shown in Fig. 1.6.

Modifying Fig. 1.4 as shown in Fig. 1.7 we get the determinant \( \Delta \) for nodes 1, 2, 3, 4 as follows.

\[
\begin{vmatrix}
SC_1 & 1 & 2 \frac{1}{SL_1} & -2 \frac{1}{SL_2} & 0 & 0 \\
& -2 \frac{1}{SL_2} & \frac{2}{SL_2} + SC_2 & -SC_2 & 0 \\
0 & -SC_2 & SC_2 + 2 \frac{1}{SL_2} & -2 \frac{1}{SL_2} \\
0 & 0 & -2 \frac{1}{SL_2} & SC_3 + \frac{1}{SL_3} + 2 \frac{1}{SL_2}
\end{vmatrix}
\]
The network of Fig. 1.7 has been made symmetric by splitting the series inductance \( L_2 \) into two equal inductances. From matrix theory we know that if we multiply all the elements of a row or column by a real positive number \( n \), then the determinant \( \Delta \) is multiplied by \( n \), i.e., \( n \Delta_{11}, n \Delta_{1k}, n \Delta_{kk}, \ldots \) etc. But since the input, output and transfer immitances are the ratios of two determinants, they do not change because the factor \( n \) cancels. In other words, if the first and last rows and columns are unchanged, the terminal conditions are unchanged, despite the manipulations of the other rows and columns.

For the above determinant \( \Delta \) the second and third rows and columns can be manipulated without changing the terminal conditions. Our aim is to develop a symmetrical network which will make easier the realization of the network from the components point of view. This is done by multiplying the second and third rows and columns by \( n \). The resultant determinant is:

\[
\begin{vmatrix}
\frac{SC_1}{1} + \frac{1}{SL_1} + \frac{2}{SL_2} & -\frac{2n}{SL_2} & 0 & 0 \\
-\frac{2n}{SL_2} & \frac{2n^2}{SL_2} + \frac{S_n}{S_n^2} & -\frac{Sn^2}{S_n} & 0 \\
0 & -\frac{2n}{S_n} & \frac{S_n^2}{S_n} + \frac{2n^2}{SL_2} & -\frac{2n}{SL_2} \\
0 & 0 & -\frac{2n}{SL_2} & \frac{SC_3}{1} + \frac{1}{S_1} + \frac{2}{S_2}
\end{vmatrix}
\]
With the information given by Figs. 1.5 and 1.6 and the small modification of Fig. 1.4 as shown in Fig. 1.7, we get the equivalent circuit of Fig. 1.8. In Fig. 1.8 we managed to shorten the gap between elements and to make the band-pass filter realizable. One should note that the source and load impedances are not affected [22].

Since transformers are not desirable elements, especially in active filters, they are replaced by equivalent circuits [3]. Each transformer can be replaced by, either an inductive or capacitive circuit. In Fig. 1.8 transformers are replaced by inductive circuits, and we see that the circuit has reduced element-value disparities.

To get equal values for shunt inductances we set

\[
\frac{L_1L_2}{2L_1(1-n)+L_2} = \frac{L_2}{2n(n-1)}
\]

or

\[2n(n-1)L_1L_2 = 2L_2L_1(1-n)+L_2^2\]

or

\[2n^2L_1L_2 - 2nL_1L_2 = 2L_1L_2 - 2L_1L_2^2 + L_2^2\]

or

\[2n^2L_1L_2 = 2L_1L_2 + L_2^2\] (1.17)

\[\eta^2 = \frac{2L_1L_2 + L_2^2}{2L_1L_2} = 1 + \frac{L_2}{2L_1}\] (1.18)
\[ n = \sqrt{1 + \frac{L_2}{2L_1}} = 24.189558 \quad (1.19) \]

Therefore
\[ L_1 = L_3 = L_4 = L_6 = \frac{L_2}{2n(n-1)} = 0.511278 \text{ mH} \quad (1.20) \]
\[ L_2 = L_5 = \frac{L_2}{2n} = 11.854288 \text{ mH} \quad (1.21) \]
\[ C_2 = n^2 C_2 = 2,584.98 \text{ pF} \quad (1.22) \]
\[ C_1 = C_3 = 5,160.00 \text{ pF} \quad (1.23) \]

The passive band-pass filter with actual values is shown in Fig. 1.9.

In the following chapters the simulation of the inductors by means of gyrators will be studied.
CHAPTER 2

INDUCTANCE SIMULATION

2.1 Introduction

An ideal inductor satisfies the first-order, linear differential equation \[ V = L \frac{dI}{dt} \], where \( L \) is the inductance in Henrys.

Therefore any two-terminal device that satisfies the above equation, can replace the inductor. Now if we compare the above equation with the equation for a capacitor, i.e. \( (I_c = \frac{CV}{dt}) \) we see that the roles played by voltage and current are reversed \([8]\), and to simulate an inductance with a capacitor we must transform the current \( I \) to a voltage \( V_L \) and the voltage \( V_C \) to a current \( I_L \). This is possible because both elements (inductor, capacitor) are storage elements.

The two-port device that can be used to perform this conversion is a "lossless" device and is referred to as a gyrator, as defined by Telegem \([9]\).

2.2 Properties of Active Networks

ACTIVITY AND PASSIVITY: A network is said to be passive if, for all admissible pairs \((v,i)\) (see Fig. 2.1) and all real \( t > -\infty \)

\[
\int_{-\infty}^{t} V(\tau)i(\tau)d\tau \geq 0
\]

(2.1)

assuming that at \( t = -\infty \), the network was relaxed and that

\[ V_1 \]
\[ I_i \]

\[ \text{NETWORK} \]

\[ \text{Fig. 2.1} \]

Sign conventions for voltages and currents
the admissible signal pair in the above equation is consistent with this. This equation states that the total energy delivered to the network can never be negative for passive networks. Activity, then, can be defined as the opposite of passivity. That is, a network which is not passive is active.

STABILITY: Stability can be expressed by two very useful definitions for active filters as follows [12]:

(A) A given network is said to be marginally stable or simply stable if all voltages and currents remain bounded when all independent generators are set to zero. Otherwise it is unstable. (B) A given network is said to be strictly stable if all voltages and currents tend to zero as \( t \to -\infty \), if they are initially finite and all independent generators are then set to zero.

In active filter theory another important definition for stability is the short-circuit and open-circuit stability. A network is said to be short-circuit stable if it is stable when the ports are short-circuited. Similarly, this applies for open-circuit stability.

SENSITIVITY: Network sensitivity is of particular concern with active filters, since most of them are more sensitive to element variations than are classical filters. A measure \( S \) of sensitivity of a network to a component value \( K \) must give an indication of the change in some performance
characteristic $P$ to variations of the component about its nominal value $K_o$.

Sensitivity measures may be divided broadly into two classes, namely microscopic and macroscopic. Microscopic measures take into account only infinitesimal variations of $K$ about its nominal value. If infinitesimal sensitivity measures may lead to incorrect results (microscopic) when elements with finite tolerances are used, then macroscopic measures should be used [12].

These properties apply if an active network (in our case filter) is well designed and optimized properly.

2.3 Operational Amplifiers

The operational amplifier is the most widely used active device of all linear circuits in production today. It is called "operational amplifier" because operations like inversion, summation, integration, etc., are readily performed when these amplifiers are used with RC networks. The most commonly used operational amplifiers are monolithic integrated circuits. These were in fact the first linear integrated circuits to be widely used. Circuit techniques used in integrated operational amplifiers are continually being modified and have many variants.

An ideal operational amplifier has two input ports. One is between the negative terminal and ground, and the other
Schematic presentation: The Ideal Operational Amplifier

(a) $I_1 = 0$
(b) $I_a = 0$

\[ V_a = 0 \quad V \rightarrow i \quad V = i = 0 \]

Fig. 2.2

 Equivalent circuit

(a) $V, i$ arbitrary
(b) $V, i$ arbitrary

Fig. 2.3

(a) Nullator
(b) Norator

Fig. 2.4

Nullor

Fig. 2.5

(a) Short circuit condition
(b) Open circuit condition

Fig. 2.6

(a) Nullor representation of the ideal op. amplifier
(b) Nullor representation of a transistor..
in different parts of an electrical network".

Fig. 2.5 shows an example of "electrical conditions".

It has been shown in the past that an ideal transistor is equivalent to a nullor. Also a nullor is equivalent to an infinite-gain voltage-controlled voltage source (V.C.V.S.) as shown in Fig. 2.6. Also it is equivalent to all the other types of infinite gain controlled sources.

The manipulation of nullators and norators simplifies the analysis and synthesis of active networks, and presents a better understanding of a given or new derived active network. These two elements exist only as paper elements and it is not possible to approximate them in practice. Therefore they must be used with care in analysis and synthesis of active networks.

2.4 Nonideal Operational Amplifiers

A practical operational amplifier differs from the ideal one shown in Fig. 2.2, in that gain and input impedance are finite, and that the output impedance is not zero. The equivalent circuit of a practical differential input operational amplifier is given in Fig. 2.27 [31].

The differential voltage gain $A$ of the amplifier is defined by

$$A = \frac{E_o}{E_i}$$

(2.2)
between the positive terminal and ground. It has only one output port, the voltage across which is shown as $V_o$ (Fig. 2.2).

When operating in the linear region, the operational amplifier has the following characteristics [11]:

1. It has infinite impedance at both input ports.
2. It has infinite gain.
3. It has zero output impedance.

The above three characteristics are expressed by the equivalent circuit shown in Fig. 2.2.

A better concept of operational amplifiers can be achieved by using nullors [13]. A nullor consists of two singular elements as shown in Fig. 2.4. These two singular elements are shown in Fig. 2.3 and were described by Carlin and Youla.

Fig. 2.3(a) represents a "pathological" one-port singular element for which the voltage across and the current through are simultaneously constrained to be zero. Such a one-port is called a nullator. Fig. 2.3(b) represents a "pathological" one port defined by the dual relations:

$$V = \text{arbitrary}, \quad i = \text{arbitrary}$$

and known as a norator. The values of the electrical variables for this singular element, are established by the rest of the circuit in which it is embedded [15]. In reality these two singular elements represent "electrical conditions.
Fig. 2.7
Non-ideal operational amplifier

Fig. 2.8
Amplitude $|A(j\omega)|$ and phase shift $\phi$ of the amplifier
Where,

\[ E_N \] - voltage at the non-inverting input
\[ E_I \] - voltage at the inverting input

The differential input voltage is defined as

\[ E_{id} = E_N - E_I \quad (2.3) \]

The differential input resistance \( R_{id} \) is generally in the range of \( 100 \text{K} \) to \( 10^6 \text{ M\( \Omega \}) \). In addition to this, there is common-mode resistance \( 2R_{cm} \) from each of the two inputs to ground. The value of \( R_{cm} \) is very high, generally ranging from \( 10 \) to \( 10^6 \text{ M\( \Omega \}) \). Since the common-mode input resistance \( R_{cm} \) is much larger than the differential input resistance, the effect of \( R_{cm} \) can be neglected for most practical applications. The output resistance \( R_o \) of the amplifier is low, generally ranging from a few ohms to \( 1 \text{ K\( \Omega \}) \).

Practical operational amplifier circuits may be classified in many ways, but perhaps the most useful is by the number of stages within the amplifier providing voltage gain. This system of classification is useful, because the voltage gain stages tend to contribute the dominant poles of the differential voltage gain function of the circuit, and hence dictate the amount and type of frequency compensation which must be used [35].

The finite but high open-loop gain is obtained by cascading several stages, and the gain of each stage starts falling
off at 6db/octave after a certain frequency. Thus the net roll-off of the total gain can be as high as 18db/octave. According to Bode's stability criterion, the rate of closure between the open-loop and the closed-loop response must be less than 12db/octave.

With a given operational amplifier it is usually not possible to apply arbitrary amounts of feedback to achieve a given closed-loop gain, unless the open-loop phase shift is corrected with additional circuitry. The correction procedure is called "frequency compensation" or "phase compensation", although the phase-shift is actually only partially compensated in a restricted band of frequencies. The frequency compensation can be achieved externally by alteration of the external feedback, or internally by the use of additional components in the amplifier circuit. Some of the methods most often used are called "lag compensation", "lead-lag compensation", "feed-forward compensation".

A reasonable approximation of the open-loop gain for a practical operational amplifier is given by

\[ A(s) = \frac{A_0}{\left(1 + \frac{s}{\omega_1}\right) \left(1 + \frac{s}{\omega_2}\right) \left(1 + \frac{s}{\omega_3}\right)} \]  \hspace{1cm} (2.4)

where \(0 < \omega_1 < \omega_2 < \omega_3\). The approximate (asymptotic) behaviour of \(\log |A(j\omega)|\) as a function of \(\log \omega\) is shown in Fig. 2.8 in the...
form of a Bode diagram. $|A(j\omega)|$ is constant for $\omega < \omega_1$. It falls off at 6db/octave between $\omega_1$ and $\omega_2$, at 12db/octave between $\omega_2$ and $\omega_3$, and at 18db/octave above $\omega_3$ [33].

The phase shift $\phi$ is shown as a function of $\log \omega$, and it is given approximately by $-45^\circ$, $-135^\circ$, and $-225^\circ$ at the corner frequencies $\omega_1$, $\omega_2$, and $\omega_3$, respectively, assuming that these frequencies are well separated. From Fig. 2.8 we derive a simple sufficient criterion for a good design for an operational amplifier. The open-loop gain should not fall off at more than 12db/octave at frequencies for which $|A(j\omega)| > 1$.

2.5 Other Imperfections of Nonideal Operational Amplifier

1. Slew rate: If an operational amplifier is over-driven by a large-signal pulse, or square wave having a fast enough rise time, the output does not follow the input immediately. Instead, it ramps or "slews" at some limiting rate determined by internal currents and capacitances. The magnitude of input voltage required to make the amplifier reach its maximum slew rate varies, depending on the type of input stage used [34]. As an example, in an operational amplifier with a slew rate of $0.5V/\mu$ sec, 60 $\mu$sec are required to change the output from $-15V$ to $+15V$. For a sinewave, the slew rate is defined as

$$\text{slew rate} = 2\pi f_p V_{\text{max}}$$

(2.5)
where $f_p$ is the maximum input frequency beyond which rated output cannot be obtained without significant distortion. $V_{\text{max}}$ is half the (rated) output swing of the operational amplifier.

2. Power bandwidth: Power bandwidth is defined as the maximum frequency at which full output swing (usually 10V peak) can be obtained without distortion. For a sinusoidal output voltage $V_o(t) = V_p \sin \omega t$, the slew rate required to reproduce the output is

$$\frac{dV_o}{dt} = \omega V_p \cos \omega t$$

(2.6)

This has a maximum when $\cos \omega t = 1$, giving

$$\frac{dV_o}{dt} \bigg|_{\text{max}} = \omega V_p$$

(2.7)

so the highest frequency that can be reproduced without slew limiting, $\omega_{\text{max}}$ (power bandwidth) is

$$\omega_{\text{max}} = \frac{1}{V_p} \frac{dV_o}{dt} \bigg|_{\text{max}}$$

(2.8)

Thus power bandwidth and slew rate are directly related by the inverse of the peak of the sinewave $V_p$.

3. Unity gain bandwidth: In order to ensure stability of a practical operational amplifier when used in a unity gain configuration, the compensation capacitor must be chosen
to provide a dominant pole. This dominant pole must produce a 6db/octave roll-off in gain so that unity gain is reached at some frequency $\omega_T$ lower than the next most important pole produced by the amplifier. Thus the value of $\omega_T$ and of compensation capacitance are determined by the frequency limitations of the devices in the amplifier. The unity gain bandwidth is inversely proportional to the duration of the slew rate. Thus a principal aim in high-speed operational amplifier design, is the achievement of high gain-bandwidth product with sufficient phase margin to give an optimally damped response.

The gain bandwidth product is determined by the location of the higher frequency poles, which contribute excess phase at frequencies at and above crossover [35]. These higher frequency poles are contributed by the fall-off of the gain of the various stages within the amplifier.

2.6 Minimization of Parasitic Effects

In order to achieve good high-frequency performance with operational amplifiers it is necessary to use them with large gain-bandwidth products, rather than the more usual high-gain operational amplifiers with gain-bandwidth products of the order of 1MHz to 10MHz. In fact, the very large dc gains of the common operational amplifiers are unnecessary in most active filter applications.
Other important factors are

a) Layout: a good layout can reduce stray capacitances and load impedances.

b) Technological improvements: technological improvements can be relied upon to continue to reduce many parasitic effects. Also, the effects of many parasitic components can be minimized by a correct choice of the impedance level in the network. Some parasitic effects can also be minimized by choice of dc operating currents and voltages.

A possible disadvantage in either case is that the dc power consumption may be increased. However, many parasitic components can be "designed in" or "swamped out". For instance, amplifier input resistances can usually be allowed for by slightly altering the design.
CHAPTER 3

GYRATORS

3.1 Introduction

An "ideal gyrator" is a two-port non-reciprocal network, which at either port, presents a driving-point impedance proportional to the terminating admittance connected across the other port, with a positive constant of proportionality [10]. This ideal gyrator is a lossless and unconditionally stable network and is described by the short-circuit admittance parameters [4]

\[ Y_{11} = Y_{22} = 0 \]  \hspace{1cm} (3.1)

\[ Y_{12} = -Y_{21} = G \]  \hspace{1cm} (3.2)

In practical gyrators instability and sensitivity problems can occur due to operational amplifier limitations. Therefore the design of a practical gyrator should be based on stability and sensitivity optimization. Direct transformation of capacitance into inductance is possible with gyrator. Therefore inductors simulated in this way can be used in the construction of very insensitive active filters [12]. Many gyrator designs and implementations can be found in the literature which exhibit good performance [16] to [20]. Gyrators can be realized by networks with resistors, capacitors and non-reciprocal components. The non-reciprocal components
This is the basic gyrator equation and can be made to show an inductive character in two ways. If $Z_1, Z_2, Z_3, Z_5$ are resistors and $Z_4$ is a capacitor in Fig. 3.1(b) the equation becomes

$$V_i = sLI_i$$  \hspace{1cm} (3.7)

where

$$L = \frac{R_1 R_3 R_5 C_4}{R_2}$$  \hspace{1cm} (3.8)

This will be referred to in this report as a type GA configuration. If, on the other hand, $Z_1, Z_3, Z_4, Z_5$ are resistors and $Z_2$ a capacitor as in Fig. 3.1(c) the equation becomes

$$V_i = sLI_i$$  \hspace{1cm} (3.9)

where

$$L = \frac{R_1 R_3 R_5 C_2}{R_4}$$  \hspace{1cm} (3.10)

This will henceforth be referred to as a type GB configuration. Whenever possible, all of the resistors are made equal for convenience, which gives $L=R^2C$ for either configuration. The term $R^2$ is often called the gyration constant. Consequently the circuit configurations shown in Fig. 3.1(b) and (c) convert a capacitance into an inductance.

### 3.3 Simulation of Multi-Inductor Networks

The circuit discussed above simulates only inductors having one end grounded, which are not suitable for many filter configurations. However, an interconnection of two gyrators
of practical interest with reference to present technology are the transistor and the operational amplifier [29]. As ideal transistors and operational amplifiers can be modelled by nullors, it has been proved in the past [5] that two nullors are necessary to realize a gyrator.

In this report, for the realization of the band-pass filter, the Antoniou GIC [5], [4] will be used and analyzed, because it has been shown to be superior to other known GIC realizations [6].

3.2 Ideal Analysis of Gyrators Using Operational Amplifiers

The circuit model developed by Antoniou is shown in Fig. 3.1 along with the definitions of the variables to be used in the analysis. In the ideal analysis the operational amplifiers are assumed to have infinite gain, infinite input impedance, zero output impedance, and zero offset. The voltages across the inputs of the amplifiers must be zero. Thus we have the equations

\[ I_1 z_1 + (I_1 - I_5) z_2 = 0 \]  \hspace{1cm} (3.3)

\[ (I_1 - I_5) z_3 + (I_1 - I_5 - I_4) z_4 = 0 \]  \hspace{1cm} (3.4)

Also

\[ V_i = I_1 z_1 + (I_1 - I_5)(z_2 + z_3) + (I_1 - I_5 - I_4)(z_4 + z_5) \]  \hspace{1cm} (3.5)

Combining these equations yields

\[ V_i = \left( \frac{z_1 z_3 z_5}{z_2 z_4} \right) I_i \]  \hspace{1cm} (3.6)
The Antoniou GIC basic

(b) Type GA

(c) Type GB

Fig. 3.1
Fig. 3.2

Simulation of multi-inductor networks
Substituting the value of $I_5$ to eqn. (3.12) we get

$$I_4 = -I_1 \left( \frac{Z_1 Z_3}{Z_2} \right) \left( \frac{Z_1}{Z_2 Z_4} + \frac{Z_1}{Z_2} \right)$$  \hspace{1cm} (3.14)

The gyrator voltages are given by

$$V_1 = (I_1 - I_5 - I_4) (Z_4 Z_5)$$  \hspace{1cm} (3.15)
$$V_2 = V_i - I_1 Z_1$$  \hspace{1cm} (3.16)

Using the two current equations, the above expressions can be simplified to

$$V_1 = V_i \left( 1 + \frac{Z_4}{Z_5} \right) \hspace{1cm} V_2 = V_i \left( 1 - \frac{Z_2 Z_4}{Z_3 Z_5} \right)$$  \hspace{1cm} (3.17)

The amplifier currents can also be written in terms of the input voltage, as follows

$$I_4 = \frac{V_i}{Z_5} \left( 1 + \frac{Z_4}{Z_3} \right)$$  \hspace{1cm} (3.18)
$$I_5 = \frac{V_i}{Z_1 Z_3 Z_4} \left( 1 + \frac{Z_1}{Z_2} \right)$$  \hspace{1cm} (3.19)

Recalling that $\frac{V_i}{I_i} = j\omega L$ and assuming a type GA gyrator, these equations can be further rewritten as

$$V_1 = V_i \left( 1 + \frac{R_1 R_3}{j\omega LR_2} \right)$$  \hspace{1cm} (3.20)
$$V_2 = V_i \left( 1 - \frac{R_1}{j\omega L} \right)$$  \hspace{1cm} (3.21)
$$I_4 = \frac{V_i}{R_5} \left( 1 + \frac{R_1 R_5}{j\omega LR_2} \right)$$  \hspace{1cm} (3.22)
$$I_5 = \frac{V_i}{j\omega L} \left( 1 + \frac{R_1}{R_2} \right)$$  \hspace{1cm} (3.23)
can simulate more general inductor networks [21], as shown in Fig. 3.2. The resistive network interconnecting the two gyrators is exactly the same as the inductive network being simulated. However, in order for these circuits to perform correctly, the corresponding impedances $Z_1$, $Z_2$, $Z_3$, $Z_4$ of each of the two gyrators must be identical. Thus each inductor is proportional to the corresponding resistor in the interconnection network.

3.4 Voltage and Current Handling Capability of Gyrators

The maximum input voltages and currents which the gyrators can handle are determined by the passive element values and the voltage and current levels at which the operational amplifiers begin to saturate.

From the basic configuration of the gyrator we will determine the operational amplifier currents. The gyrator is assumed to be ideal in this analysis.

The two current equations were found as

\[ I_1 Z_1 + (I_1 I_5) Z_2 = 0 \]  \hspace{1cm} (3.11)

\[ (I_1 - I_5) Z_3 + (I_1 - I_5 - I_4) Z_4 = 0 \]  \hspace{1cm} (3.12)

From equation 3.11 we readily find $I_5$ as

\[ I_5 = I_1 (1 + \frac{Z_1}{Z_2}) \]  \hspace{1cm} (3.13)
does not occur in the proposed filter configurations and estimate, if possible, the size of the output offset voltages.

The basic gyrator configuration in Fig. 3.3 will be considered first. Modifying the basic equations from the ideal analysis to include offset voltage and bias current terms gives

\[(I_1+i_1)Z_1+(I_1+i_1-I_5)Z_2 = e_1 \]  
\[(I_1+i_1+i_2+i_3-I_5)Z_3+(I_1+i_1+i_2+i_3-I_5-I_4)Z_4 = -e_2 \]  
\[
V_i = (I_1+i_1)Z_1+(I_1+i_1-I_5)Z_2+(I_1+i_1+i_2+i_3-I_5)Z_3^+
+ (I_1+i_1+i_2+i_3-I_5-I_4)Z_4+(I_1+i_1+i_2+i_3+i_4-I_5-I_4)Z_5
\]

Solution of these equations gives

\[
V_i = I_i \left( \frac{Z_1Z_3Z_5}{Z_2Z_4} \right) + e_1 (1 - \frac{Z_3Z_5}{Z_2Z_4}) - e_2 (1 + \frac{Z_5}{Z_4}) + i_1 (Z_1+Z_2)x
x \left( \frac{Z_3Z_5}{Z_2Z_4} \right) - (i_1+i_2+i_3)(Z_3+Z_4)(\frac{Z_5}{Z_4}) + i_1+i_2+i_3+i_4 \right)Z_5
\]

where \(e_1, e_2\) are offset voltages and \(i_1, i_2, i_3, i_4\) are bias currents.

Clearly if a \(1/s\) term (step function) multiplied any of these constants, it would increase linearly and eventually saturate the gyrator. To illustrate the meaning of the equations, let all the gyrator resistors have a value \(R\) and the capacitor a value \(C\). Then, for type GA gyrator we have
Knowing the maximum voltages and currents of the operational amplifiers, one can calculate the maximum input voltages before the onset of distortion. A similar set of equations can be written for a type GB gyrator. In theory, one can determine optimum gyrator element values to allow a maximum input but, in practice, this could result in unrealistic element values.

3.5 Analysis of nonideal gyrators

DC considerations:

An ideal operational amplifier should respond only to the difference in value of the signals applied to the non-inverting and inverting terminals. Unfortunately, a practical operational amplifier has both input offset voltage and input bias currents and may have noticeable effects on the circuits in which they are used [15]. Their effect on circuit behaviour should be determined. The amplifier's DC gain is very large and will be assumed infinite. DC offset at the output of a bandpass filter is not a serious problem, since the output can be AC coupled, as long as the offset is not large enough to interfere with proper circuit operation. The problem is that when the power is turned on, several of the amplifier output voltages drift slowly toward saturation. The voltages increase linearly with time and the whole process takes 5-10 sec., suggesting that a very small bias current is charging a capacitor. We must ensure that such behaviour
\[ V_i = I_i(sCR^2) + e_1(l-SCR) - e_2(l+SCR) - (i_2+i_3)(2sCR^2) + (i_1+i_2+i_3+i_4)R \]  
(3.28)

and for type GB

\[ V_i = I_i(sCR^2) + e_1(l-SCR) - 2e_2 + i_1(R+SCR^2) - (i_1+i_2+i_3)2R + (i_1+i_2+i_3+i_4)R \]  
(3.29)

From the above equations we see that the two circuits are unconditionally stable as expected, and shown by [5].

At DC, \( s=0 \), and so we conclude that

Offset voltage (type GA) = \( e_1 - e_2 + (i_1+i_2+i_3+i_4)R \)  
(3.30)

Offset voltage (type GB) = \( e_1 - 2e_2 + (i_4-i_2-i_3)R \)  
(3.31)

For typical operational amplifiers, \( e_1 \) and \( e_2 \) are around 5mV and each of the bias currents are of the order of 100-200nA. From the above equations we conclude that the gyrator produces a steady offset voltage independent of the load. If that load is an inductor (or another gyrator) the offset voltage will produce a rising current, limited only by the inductor's internal resistance. Such a situation should be avoided.

3.6 AC Behaviour

The nonideal AC behaviour of a gyrator is determined by the parameters of the operational amplifier. In practice, the operational amplifier is a nonideal device. It is
Fig. 3.3

Gyrator dc offset analysis
\[ G_{p1} = \frac{1}{R_{p1}} + \frac{1}{A_o \omega R} \left( 2 + \frac{R_o}{R} \right) + \frac{R}{R_I (2R + 3R_o + A_o \omega_o R^2 C)} \]  
(3.33)

\[ G_{p2} = \left( \frac{1}{R_{N1}} + \frac{1}{R_{N2}} + \frac{2R_o}{R (2R + 3R_o + A_o \omega_o R^2 C)} \right) \]  
(3.34)

\[ G_{p3} = \frac{2 \omega^2}{A_o^2 \omega_o^2 R} \left( 1 + \frac{4R_o}{R} \right) - \frac{(4R_o R_I + 2R^2 R_o - 4R_o^2 R_I - 2R^3)}{R (2R + 3R_o + A_o \omega_o R^2 C)} \]  
(3.35)

\[ C_D = \frac{1}{A_o \omega R} \left[ 2 + \frac{R_o}{R} + \frac{1}{R R_I} \left( \frac{R_o (R_o R_I - R^2)}{2R + 3R_o + A_o \omega_o R^2 C} \right) \right] \]  
(3.36)

\[ r_{p4} = \frac{2R}{A_o} + \frac{3R_o}{A_o} + g_L R^2 \]  
(3.37)

\[ r_{p5} = \frac{2 \omega^2}{A_o^2 \omega_o^2} (R + 6R_o) \]  
(3.38)

\[ L_p = \frac{2R}{A_o \omega_o} + \frac{(3R_o)}{A_o \omega_o} \]  
(3.39)

and

\[ L_B = R^2 C \]  
(3.40)
Fig. 3.4

Model for nonideal operational amplifier.

Fig. 3.5

Model for the capacitively terminated gyrator.
and

\[ \frac{\Delta M(\omega)}{M(\omega)} = \int_{B}^{A(s)} ds \]

for the deviation due to a finite \( A_0 \) and \( B \), respectively.

With the above typical values we conclude that the gain-bandwidth product \( B \) is a very important parameter for frequencies above 50KHz, because it has greater effect than the dc open-loop gain.

The basic gyrator configuration (type GA) has been modelled by [24] for non-ideal operational amplifier, shown in Fig. 3.5 (equivalent circuit) and the following parasitic elements were introduced:

1. A constant series parasitic inductance \( L_p \).
2. A constant parallel parasitic capacitance \( C_p \) (analogous to the winding capacitance in an inductor).
3. A frequency-dependent parallel resistance \( 1/G_p3 \) which reduces as the frequency is increased.
4. A frequency-dependent series resistance \( r_p5 \) which increases as the frequency is increased (analogous to the winding resistance in an inductor which increases due to the skin effect).
5. For a nonzero \( R_o \), enhancement is introduced.

Also, the various values of the parasitic components were given.
characterized by a frequency-dependent voltage gain whose magnitude starts from a very high value at DC (usually in the range of 80 to 120db) and then monotonically decreases for higher frequencies. Likewise, the phase of the voltage gain is a monotonically decreasing function starting from 0° at dc.

The nonideal voltage transfer function $A(s)$ of the operational amplifier is usually approximated by the expression

$$A(s) = \frac{A_O \omega_c}{s + \omega_c} \quad (A_O \omega >> 1). \quad (3.32)$$

where $A_O$ is the dc open-loop gain of the operational amplifier, and $\omega_c$ is the cut off radian frequency.

Typical values given in [24] for the parameters of the model shown in Fig. 3.4 are

$A_O = 3,000 - 200,000, \quad \omega_c = 30-150 \times 10^3 \text{rads/sec}$

$p, R_N : 50-500M\Omega, \quad R_I : 50k\Omega - 2M\Omega, \quad R_o = 50-200\Omega$

Ref. [23] reports that for sufficiently small deviations $\Delta M(\omega)$ of the gain function (magnitude), the effect of finite $A_O$ and finite $B$ on $M(\omega)$ are given by

$$\frac{\Delta M(\omega)}{M(\omega)} = -\int \frac{A(s)}{A_O} ds$$
\[
Y_{IL} \quad {\text{Simplified low-frequency gyrator model}}
\]
\[
Y_{IH} \quad {\text{Simplified high-frequency gyrator model}}
\]

Fig. 3.6

Fig. 3.7

Simulated inductor given by [67].

Fig. 3.8

The Antoniou GIC used to realize a simulated inductance
In this analysis all resistors were assumed equal. From the above parameters, \( L_0 \) represents the nominal inductance, and the rest represent parasitic elements due to imperfections of the operational amplifiers and load capacitor. Also the magnitudes of these elements depend on frequency (i.e. gain bandwidth product of the amplifiers).

Therefore two simplified models were introduced [25], for better representation of the gyrator for low frequencies and high frequencies, as shown in Fig. 3.6.

The analysis of Ref. [25] is the most extensive one, compared to other reports up to date, because all the parasitic elements and non-ideal parameters, of an operational amplifier are taken into account.

A more simplified analysis is given by Ref. [67]. The GA type gyrator is analyzed and the following parasitic elements are reported.

\[
G_p = \frac{2}{A_o R_1} \left( 1 + \frac{\omega^2}{A_o \omega^2} \right) \tag{3.41}
\]

\[
C_p = 2 (A_o R_1)^{-1} \tag{3.42}
\]

\[
R_p = \frac{2 R_1}{A_o} \left( 1 + \frac{\omega^2}{A_o \omega^2} \right) \left( 1 - \frac{4 \omega^2}{A_o \omega^2 C_1 R_5} \right) \tag{3.43}
\]

\[
L = L_0 \left( 1 + \frac{2}{A_o \omega C_1 R_5} \right) \tag{3.44}
\]
This analysis assumes $R_2 = R_3 = R$, and

$$R_o \ll (R_1, R_2, R_3, R_5) \ll (R_D, R_C) \quad (3.45)$$

where

- $R_o$ is the output impedance of the operational amplifier.
- $R_D$ is the differential input impedance of the operational amplifier.
- $R_C$ is the common-mode impedance of the operational amplifier.

The presence of these parasitic elements raises design problems, which may be minimized by using as criterion the $Q$-factor for optimum design. The $Q$-factor of the simulated inductor with impedance $Z_L$ is defined by

$$Q_L = \frac{\text{Im} Z_L}{\text{Re} Z_L} \quad (3.46)$$

The design is optimized by maximizing the $Q$-factor. For a given frequency $f_k$, the $Q$-factor is maximum if

$$R_1 = \omega_L(1 + \frac{2f_k}{A_{r0}}) \quad (3.47)$$

$$C = \frac{1}{\omega_K R_s} \left( 1 - \frac{4f_k}{A_{r0}} \right) \quad (3.48)$$

For this critical frequency $f_k$, a suitable value is chosen at which a high $Q$-factor is of prime importance. This is the case where the tolerance sensitivity of the filter is greatest and therefore component tolerances have
the maximum effect on attenuation distortion. This occurs at the frequency at which the attenuation curve has the steepest slope and the group delay is a maximum [6].

3.7 Effects of Finite Operational Amplifier's Gain

Above 50KHz, the operational amplifier's gain can be approximated as

\[ A(s) = \frac{A_0 \omega_0}{s} \quad \text{provided that} \quad \omega < \omega_c \]  

(3.49)

The input admittance of the GA type gyrator is given by [6]

\[ \frac{Y_{I_1}}{V_1} = \frac{Y_{I_2}}{V_2} \frac{Y_{I_3}}{V_3} \frac{Y_{I_4}}{V_4} \left[ \frac{1 + \frac{1}{A(s)} \left( 1 + \frac{V_2}{4} I_2 \right)}{1 + \frac{1}{A(s)} \left( 1 + \frac{V_4}{4} I_2 \right)} \right] \]

(3.50)

From the above approximation we can ignore the second order terms of \( A(s) \), and the input admittance of the gyrator becomes
Substituting equation (3.55) to equation (3.52) and (3.53) we get

\[ 1 - 1 + Y_4 \frac{V_2}{I_2} - \frac{1}{V_4} \frac{I_2}{V_2} = 0 \]  \hspace{1cm} (3.56)

\[ 1 - 1 + Y_4 \frac{V_2}{I_2} - \frac{1}{V_4} \frac{I_2}{V_2} = 0 \]  \hspace{1cm} (3.57)

or

\[ Y_4 \frac{V_2}{I_2} = \frac{1}{V_4} \frac{I_2}{V_2} \]

which implies that

\[ \frac{I_2}{V_2} = Y_4 \]  \hspace{1cm} (3.58)

Eqn. (3.55) is easily satisfied by letting

\[ Y_2 = Y_3 = g \]  \hspace{1cm} (3.59)

where \( g \) is real.

In practice, second-order effects which were ignored in Eqn. (3.51) will dominate if \( Y_2 = Y_3 = g \) [80]. However, Eqn. (3.58) is seldom satisfied, since \( \frac{I_2}{V_2} \) has, in general, components orthogonal to \( Y_4 \). Setting \( Y_2 = Y_3 \) in Eqn. (3.51) we get

\[ Y_1 = \frac{I_1}{V_1} = \frac{I_2}{V_2} \frac{Y_1}{Y_4} \left[ 1 + j \frac{2w}{\omega_c} (Y_4 \frac{V_2}{I_2} - \frac{1}{Y_4} \frac{I_2}{V_2}) \right] \]  \hspace{1cm} (3.60)

From the above equation one concludes that for best performance at the most critical frequency \( \omega_c \) (upper cut-off frequency of the band-pass filter) the quantity...
\[ Y_{IN} = \frac{I_1}{V_1} \approx \frac{I_2}{V_2} Y_2 Y_3 \left[ 1 + j \left( \frac{\omega}{\omega_c} \right) \left( 1 - \frac{Y_3}{Y_2} \cdot \frac{Y_4}{I_2} \right) \right] \]

\[ - \frac{Y_3}{Y_2 Y_4} \cdot \frac{I_2}{V_2} \cdot j \left( \frac{\omega}{\omega_c} \right) \left( \frac{Y_2}{Y_3} - 1 + \frac{Y_2 Y_4}{Y_3} \cdot \frac{V_2}{I_2} - \frac{1}{Y_4} \cdot \frac{I_2}{V_2} \right) \]

(3.51)

In this analysis the two amplifiers are assumed to be identical. From equation (3.51) we see that for ideal performance of the gyrator, the imaginary part should be equal to zero. Thus, the following equalities should hold

\[ 1 - \frac{Y_3}{Y_2} + Y_4 \cdot \frac{V_2}{I_2} - \frac{Y_3}{Y_2 Y_4} \cdot \frac{I_2}{V_2} = 0 \]

(3.52)

\[ \frac{Y_2}{Y_3} - 1 + \frac{Y_2 Y_4}{Y_3} \cdot \frac{V_2}{I_2} - \frac{1}{Y_4} \cdot \frac{I_2}{V_2} = 0 \]

(3.53)

Since in general \( \frac{V_2}{I_2} \) is a function of frequency and takes on a complex value, then \( 1 + \frac{Y_3}{Y_2} \) must be linearly independent of

\[ \frac{Y_4}{I_2} - \frac{Y_3}{Y_2 Y_4} \cdot \frac{I_2}{V_2} \]

(3.54)

Thus from equation (3.52) and (3.53) we conclude that it is necessary to have

\[ 1 - \frac{Y_3}{Y_2} = 0 \]

which implies

\[ Y_2 = Y_3 \]

(3.55)
Thus taking

\[ \omega_{cf} C_4 = \left| \begin{array}{c} I_2 \\ V_2 \end{array} \right| \quad \omega = \omega_{cf} \]  

(3.64)

minimizes \( M \). Then the corresponding value for the GA type gyrator transfer function is

\[ \frac{I_i}{V_i} = \frac{I_2}{V_2} \frac{Y_1}{Y_4} \left[ (1 - \frac{4\omega}{\omega_c}) \cos \theta \right] \]  

at \( \omega = \omega_{cf} \)

or

\[ \frac{I_1}{V_1} \bigg|_{\omega = \omega_{cf}} = \frac{Y_1 Y_5}{Y_4} (1 - \frac{4\omega}{\omega_c}) \]  

(3.66)

since \( \frac{I_2}{V_2} = Y_5 \)

Also

\[ Y_1 = \frac{1}{R_1}, \quad Y_5 = \frac{1}{R_5} \quad \text{and} \quad Y_4 = j\omega C_4 \text{ (for GA type)} \]  

(3.67)

At the frequency of interest the above equation becomes

\[ \frac{V_1}{I_1} \bigg|_{\omega = \omega_{cf}} = j\omega R_1 R_5 C_4 (1 + \frac{4\omega}{\omega_c}) \]  

(3.68)
\[ M = \left( Y_4 \frac{V_2}{I_2} - \frac{1}{Y_4} \frac{I_2}{V_2} \right) \]

should be as small as possible, because at this frequency, the group delay has its greatest value.

Letting
\[ \frac{V_2}{I_2} \bigg|_{\omega = \omega_{cf}} = Ae^{j\theta} \]

it follows that we require
\[ M = \frac{Y_4}{Ae^{j\theta}} - \frac{1}{Y_4} \frac{Ae^{j\theta}}{Ae^{j\theta}} \]

to be as small as possible.

The type of gyrator used for this filter is type GA which requires that \( Y_4 = j\omega C_4 \), where \( C_4 \) is real.

Therefore
\[ M = j\omega_{cf} C_4 A(s) e^{j\theta} + \frac{je^{-j\theta}}{\omega_{cf} C_4 A(s)} \]

which implies that
\[ M = j \left( \frac{1}{\omega_{cf} C_4 A(s)} + \omega_{cf} C_4 A(s) \cos \theta + \left[ \frac{1}{\omega_{cf} C_4 A(s)} - \omega_{cf} C_4 A(s) \right] \sin \theta \right) \]

Minimization of the above equation occurs when
\[ \omega_{cf} C_4 = \frac{1}{A(s)} \]

where the second term reduces to zero.
From the above equation we see that the input inductance has an infinite Q factor (since the real part of the equation is zero), but undergoes a deviation in value, and is given by

$$\frac{\Delta L}{L} \mid_{\omega = \omega_{cf}} = \frac{4\omega}{\omega C}$$

(3.69)

This equation implies that the deviation of the inductance value is strictly dependent on the gain-bandwidth product of the operational amplifier. The higher the gain-bandwidth product of the operational amplifiers the lower the deviation of the inductance. To minimize M (optimization of GA type of gyrator) we let

$$\omega_{cf} C_4 = \left| \frac{I_2}{N_2} \right|_{\omega = \omega_{cf}} = \frac{1}{R_5}$$

(3.70)

or

$$\omega_{cf} C_4 R_5 = 1$$

Thus eqn. (3.68) becomes

$$\frac{V_1}{I_1} \mid_{\omega = \omega_{cf}} = J R_1 \left( 1 + \frac{4\omega}{\omega C} \right)$$

(3.71)

This equation also suggests the use of $R_1$ to adjust or to trim the value of the input inductance, from the deviation due to finite gain-bandwidth product of the operational amplifiers.
CHAPTER 4

IMPLEMENTATION AND TUNABILITY OF BAND-PASS FILTER

4.1 Introduction

The value of a simulated inductance is determined by the value of five passive components in the ideal case. The inductance is typically described by an equation of the form: \( L = R_1 R_3 R_5 C_4 / R_2 \). Thus, the percentage errors of the components add up. If each component has a tolerance of \( \pm 1\% \), \( L \) has a maximum error of \( 5\% \). In practice, the component errors will not all have the same sign or magnitude, so the error in \( L \) will probably be considerably smaller. In any event, it is easy to adjust \( L \) by means of potentiometer, in place of \( R_1 \) as Eqn. (3.71) suggests. The tunability of the filter was relatively simple. Two different methods were used with the same results.

4.2 Band-Pass Filter Using Gyrators

The band-pass filter was implemented with the replacement of all inductors with multi-inductor network. The two \( \pi \) inductor sections were simulated with two gyrators, each one sharing one resistor to reduce the number of gyrators used. The four gyrators used were identical with optimized component values as follows.

\[
R_2 = R_3 = 4.99 k\Omega \pm 1\% \quad (4.1)
\]
for optimum performance. The second of these equations suggests that the lower the gain of the operational amplifier the higher the value of the capacitor used (for a certain frequency), and less influence of the parasitic capacitance of the gyrator.

A value of 3.6 nF was chosen for $C_4$. Then $R_5$ is determined as

$$R_5 = \frac{1}{\frac{1}{W_{C4}} = \frac{1}{2.7 \times 10^2 \times 3,500 \times 10^{-12}}} = 402 \Omega \quad (4.8)$$

The value of $R_5$ then determines the value of the sharing resistance, $R_6$ which is analogous to the ratio

$$\frac{L_1}{L_2} = \frac{R_5}{R_6} \quad (4.9)$$

From the above analogue the only unknown value is $R_6$.

Therefore

$$R_6 = \frac{R_5 L_2}{L_1} = \frac{402 \times 11.874599 \times 10^{-3}}{0.512982 \times 10^{-3}} = 9,305.56 \Omega \quad (4.10)$$

$R_6 = 9.31 \text{K} \pm 1\%$ was used, as the closest standard value.

At the beginning the operational amplifier used was the Harris type HA4622, dual-in-line 14 pin package containing 4 operational amplifiers. The gain bandwidth product of this type is 70 MHz which was suitable for the band-pass filter implementation. The filter was implemented but it was unstable. The reason for instability was the operational
for Q enhancement with respect to amplifier bandwidth \([23]\). The Q factor has been derived and is given by Ref. \([80]\) as

\[
Q(\omega) = \frac{A_0}{2} \cdot \frac{1}{\omega RC + \frac{1}{\omega RC}}
\]  

(4.2)

In the above equation resistances \(R_1, R_2, R_3, R_5\) were assumed all equal, therefore \(L_0 = CR^2\).

Hence

\[
Q(\omega) = \frac{A_0}{2} \cdot \frac{1}{\omega L_0 \cdot \frac{R}{R} + \frac{1}{\omega L_0}}
\]  

(4.3)

This equation suggests that for a maximum Q at a given frequency \(\omega_c\)

\[
R = \omega_c L_0
\]  

(4.4)

This frequency in this band-pass filter is the upper cut-off frequency equal to 110KHz therefore the value of R is

\[
R = 2 \pi \times 110 \times 10^3 \times 0.512982 \times 10^{-3} = 354.54 \Omega
\]  

(4.5)

This value was used for resistor \(R_1\), and since with this resistor we control the value of the inductance, as equation \((3.71)\) suggests, then a potentiometer was used for trimming purposes.

Also from Eqn. \((3.63), (3.70)\) we have

\[
W_0 C_4 R_5 = 1
\]  

(4.6)

\[
W_0 C_4 = \frac{1}{A(s)}
\]  

(4.7)
Fig. 4.1
Band-pass filter (realized)
amplifier itself, because this type of operational amplifier, besides the excellent parameters, is unstable for a gain less than five. For the implementation of gyrator the operational amplifier must be stable for a gain of unity.

The RCA operational amplifier, CA3100T was then used. This amplifier has a gain bandwidth product of 38MHz, and unity gain stability. The filter is shown in Fig. 4.1 with actual values.

4.3 Tuning the Band-Pass Filter

Tuning of the filter was done as follows: through a 10MΩ resistor, we feed the signal to the parallel LC circuit, at center frequency of 100KHz, and we trim the 500Ω potentiometer to achieve resonance at the center frequency, as in Fig. 4.2.

Since all the shunt inductors have the same value, the adjustment for each gyrator is very easy and accordingly the band-pass filter is tuned without any other difficulty, by interconnecting the two sharing resistors,
Fig. 4.2

Tunning gyrator \( (G_1) \)
Another way of tuning this filter is by using Lissajous figures. Since this filter is designed symmetrical, the tuning can be done in two parts by disconnecting capacitor $C_2$ in Fig. 4.3.

Using point A as the output, the first gyrator is set by shorting point B to ground and adjusting $R_1$ to achieve resonance at the center frequency. This results into $0^\circ$ phase-shift relative to the source. Then we disconnect point B from ground, and tune $R_1$ of the second gyrator for $90^\circ$ phase-shift at the output. We follow the same procedure for the second part of the filter. Then by connecting back capacitor $C_2$, the filter is tuned with an accuracy better than $2^\circ$.

4.4 Parasitic Effects

The band-pass filter after tuning had a response far from ideal. The deviation from the ideal response was due to the parasitic elements introduced by Ref. [24] as $G_{p_3}, r_{p_5}, L_p,$ and $C_p$. Since the values of all, resistors, capacitor, and various parameters of the operational amplifier (for each gyrator) were established, the parasitic elements were calculated as follows.
\[ G_{p3} = \frac{R_2}{R_1} \left( \frac{1}{R_3\omega_0} + \frac{1}{R_2\omega_0} \right) = \frac{2}{R_1\omega_0} = 5.6497 \times 10^{-6} \text{Mho} \]

or

\[ \frac{1}{G_{p3}} = R_3 = 177 \Omega \]

\[ r_{p5} = \frac{2\omega^2}{A_0^2 \omega_0} (R + 6R_0) = \frac{R_1}{R_2} \left( \frac{R_2}{A_0} + \frac{R_3}{A_0} \right) = \frac{R_1 (R_2 + R_3)}{R_2 A_0} = 0.709 \text{m}\Omega \]  \hfill (4.11)

\[ L_p = \frac{R_1 (R_2 R_3)}{R_2 B} = 0.003 \mu \text{H} \]  \hfill (4.12)

\[ C_p = \frac{R_2}{R_1} \left( \frac{1}{R_3 B} + \frac{1}{R_2 B} \right) = \frac{2}{R_1 B} = 23 \text{pF} \]  \hfill (4.13)

From the above calculations we see that the only significant parasitic element is capacitance \( C_p \). This parasitic capacitance affects the quality factor \( Q_L \) of the simulated inductor. This quality factor is the ratio of the capacitor \( C_4 \) to the parasitic capacitance \( C_p \), Ref. [6].

\[ Q_L = \frac{C_4}{C_p} = \frac{3.600 \text{pF}}{23 \text{pF}} = 156.5 \]  \hfill (4.14)

Two different compensation techniques have been introduced for elimination (or reduction) of this parasitic capacitance. Ref. [25] suggests the use of a capacitor \( C_c \) with a value as

\[ C_c = \frac{2R + R_0}{A_0 \omega_0 R^2} \]  \hfill (4.15)
Fig. 4.4
Frequency response of active Band-pass filter
where \( R = R_2 = R_3 \)

Ref. [6] suggests the use of a resistor \( R_c \) with a value as

\[
R_c = R \frac{C_4}{C_p} \quad (4.16)
\]

where \( R = R_2 = R_3 \)

Both compensating elements are inserted between the common inverting input terminals of the operational amplifiers to ground.

Both methods were tested and in my opinion the use of a compensating variable resistor is better, because the tuning process is faster than using capacitor. At the frequency of 100KHz, the simulated inductance is also sensitive to stray capacitances, and a good layout is very important for best results.

With the use of compensating resistors the response of the band-pass filter improved significantly and the aim of this report was achieved.
CONCLUSIONS

The feasibility of implementing high-Q, active, band-pass filters with gyrators has been studied. The design is based on a suitably transformed passive filter network, by replacing the inductors by capacitively-terminated gyrators. With the use of gyrators instead of inductors, the filter preserves its insensitivity to component variations, and since the gyrator is an active circuit it does not add dissipative elements to the filter.

From the various types of gyrators that have appeared in the literature, the Antoniou type has been investigated and analysed. The main sources of errors are the imperfection of the operational amplifiers used due to limited gain-bandwidth product. From all the parasitic elements introduced, the parasitic capacitor is the most important one. Optimization of the gyrator consists of the elimination of this parasitic capacitance which is done by two different approaches. Both methods improved the performance of the gyrators, and a frequency response has been achieved for the band-pass filter which is close to the desirable one.
REFERENCES


[15]. Bhattacharyya, B.B., "Active Filter Design", Notes, Concordia University.


[40]. Ramakrishna, K., Soundarajan, K., Aatre, V.K., "Effect of Amplifier Imperfectins on Active Networks", IEEE


[52]. Saraga, W., Haigh, D.G., and Barker, R.G., "Micro-electronic Active-RC Channel Bandpass Filters in the
No. 10, October 1979, pp. 892-893.


