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# Design Optimization and Performance Evaluation of a Gravity Check Valve for Gas Service

Enzo M. Cavazzoni

A Thesis

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# ABSTRACT.

## Design Optimization and Performance Evaluation of a Gravity Check Valve for Gas Service.

Enzo M. Cavazzoni

The objective of this thesis is to optimize the design parameters and to evaluate the performance of a wafer check valve, when used for gas service.

The wafer valve is a gravity swing check valve (Chap. 1), widely used in liquid lines. When used with gaseous fluids, the valve design optimization requires the minimization of a total cost function (Chap. 2), considering its energy efficiency and dynamic performance, and its mutual reaction with the piping system.

The valve energy efficiency is investigated (Chap. 3), using the concept of availability destruction. Formulae for flow torque, acting on the valve disc, are derived (Chap. 4) and used in a valve dynamic model (Chap. 5). A valve for a typical application (courtesy of RITEPRO Inc.) is designed using energy efficiency and dynamic performance concepts.

Basic formulae of unsteady, compressible flow in a pipe are developed and the characteristic of an ideal numerical model established (Chap. 6). The experimental testing of a piping model is rejected because it is too expensive (Chap. 7). As an alternative, a comparison with the solution of a test case with the Method of Characteristics is proposed. The continuous, unsteady, compressible flow through a pipe is then modelled in lumped elements (Chap. 8) and solved with three numerical techniques: Finite Volumes, Finite Elements and Finite Differences. The test case is solved and results favourably compared with the Method of Characteristics solution (Chap. 9). The valve closing is represented as a sudden closure (Chap. 10). A travelling shock is modelled, and the influence of delayed sudden closures on the shock strength is discussed.

Finally in Chapt. 11, the wafer check valve is evaluated as being suitable for gas service.

All concepts and computer programs, presented in this thesis have been successfully used to select valves for difficult applications. A list of the most significant cases (Courtesy of RITEPRO Inc.) is shown in Appendix 1.

## ACKNOWLEDGEMENT

The author wishes to thank his supervisors, Dr. J. Svoboda and Dr. M. McKinnon, for their support and valuable suggestions.

The late Dr. S. Katz is especially remembered for his encouragement. His presence is sadly missed.

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To

Anna, Cristiana, Marco, Patrizia

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## LIST OF SYMBOLS (1)

| | | |
|---|---|---|
| $A$ | = | Pipe Section area |
| $A_{cv}$ | = | Control Volume Availability |
| $A_i$ | = | Area of Pipe Section i |
| $A_1$ | = | Entering Pipe Diameter |
| $C_v^*$ | = | Coefficient of Discharge. Dimensionless |
| $C_v$ | = | Coefficient of Discharge |
| $C$ | = | Capacitance |
| $\bar{C}$ | = | Capacitance per Unit Length |
| $D$ | = | Pipe Diameter |
| $E_i$ | = | Control Volume Total Energy |
| $F$ | = | Wall Friction Term |
| $F_d$ | = | Net Thrust. Drag Force |
| $F_i$ | = | Elementary Net Thrust. Drag Force |
| $F_{cl}$ | = | Clapper Drag Force |
| $F_o$ | = | Drag Force of Seat Alone |
| $F_r$ | = | Drag Force Reverse. |
| $H_i$ | = | Elementary Drag Compressibility Coefficient |
| $H_{ri}$ | = | Elementary Drag Compressibility Coefficient. Reverse |
| $I$ | = | Availability Destruction |
| $I$ | = | Inductance |
| $\bar{I}$ | = | Inductance per Unit Length |
| $\bar{K}_i$ | = | Element Coeff. of Resistance. Ref. to Elem. Greater Sect. |
| $K_i$ | = | Coeff. of Resistance. Ref. to Pipe Section |
| $K_c$ | = | Coeff. of Resistance. Contraction |
| $K_{cl}$ | = | Coeff. of Resistance of Clapper Alone. Theoretical |
| $K_{clex}$ | = | Coeff. of Resistance of Clapper Alone. Experimental |

| | | |
|---|---|---|
| $K_E$ | = | Coeff. of Resistance. Expansion |
| $K_D$ | = | Coeff. of Resistance. Direct |
| $K_0$ | = | Equivalent Length |
| $K$ | = | Coefficient of Resistance |
| $K_r$ | = | Coefficient of Resistance. Reverse |
| $K_s$ | = | Coefficient of Resistance. Smooth Contraction |
| $L$ | = | Pipe Length |
| $M_x$ | = | Mach Number |
| $N_i$ | = | Shape Function |
| $R$ | = | Resistance |
| $\bar{R}$ | = | Resistance per Unit Length |
| $\bar{\bar{R}}$ | = | Specific Resistance per Unit Length |
| $R_1$ | = | Resistance Due to Density Space Variation |
| $R_2$ | = | Resistance Due to Density Time Variation |
| $R_3$ | = | Viscous Resistance |
| Rey | = | Reynold Number |
| $R_g$ | = | Gas Constant |
| $S_F$ | = | Equivalent Surface |
| $S_1$ | = | Equivalent Surface |
| $T$ | = | Temperature |
| $T_L$ | = | Entering Temperature |
| $T_2$ | = | Exiting Temperature |
| $T_{EN}$ | = | Environment Temperature |
| $T_w$ | = | Weight Torque |
| $T_F$ | = | Flow Torque |
| $T_j$ | = | Inertial Torque |

$T_s$ = Spring Torque

$T_c$ = Counterweight Inertial Torque

$T_{cw}$ = Counterweight Weight Torque

$T_D$ = Dashpot Torque

$T_{vw}$ = Clapper Weight Torque

$T_{Dw}$ = Damper Weight Torque

$T_{rF}$ = Flow Torque. Reverse

$V$ = Velocity

$V_{cL}$ = Clapper Velocity

$V_i$ = Velocity at Pipe Section i.

$V_o$ = Initial Velocity

$V_1$ = Entering Velocity

$V_2$ = Exiting Velocity

$Y$ = Net Expansion Factor

$Y_i$ = Elementary Net Expansion Factor

$Y_D$ = Net Expansion Factor. Direct

$Y_r$ = Net Expansion Factor. Reverse

$W_c$ = Counterweight Weight

$W$ = Shock Velocity Relative to Flow

$Z$ = Compressibility

AV = Blocks "Pressure Loss" and "Energy Degr."

GH = Block "Gas Hammer"

Ks = Spring Constant

OO = Blocks "Valve Open." and "Oscillation"

VS = Absolute Shock Velocity

$\$I$ = Investment Cost

$\$R$ = Recovery Cost

$\$O$ = Operating Cost

$\$M$ = Maintenance Cost

$\$II$ = Indirect Investment Cost

$\$IM$ = Indirect Maintenance Cost

(VS) = Valve Nominal Size

(VA) = Valve Opening Angle

(VO) = Valve Orifice Diameter

(VL) = Valve Stress Level

ETA = Equivalent Torque Arm

TAC = Torque Arm Coefficient

$a$ = Pseudo Sound Velocity

$a_i$ = Specific Availability

$a_{fi}$ = Entering Flow Availability

$a_{fe}$ = Exiting Flow Availability

$c$ = Local Sound Velocity

$c_1$ = Entering Sound Velocity

$c_2$ = Exiting Sound Velocity

$c_0$ = Initial Sound Velocity

$c_p$ = Constant Pressure Specific Heat

$c_v$ = Constant Volume Specific Heat

$d_0$ = Hinge Pin Displacement

$d_F$ = Equivalent Diameter

$d_{s1}$ = Equivalent Diameter

$d_1$ = Pipe Diameter

$d_2$ = Orifice Diameter

$d_i$ = Element First Diameter

$d_{i+1}$ = Element Second Diameter

$f$ = Friction Factor

$h$ = Specific Enthalpy

$h_i$ = Entering Enthalpy

$h_2$ = Exiting Enthalpy

$l_a$ = Hinge Length

$l_c$ = Counterweight Arm

$l_s$ = Spring Length

$l_{so}$ = Spring Free Length

$m$ = Mass Flow

$m_r$ = Relative Mass Flow

$m_c$ = Critical Mass Flow

$p$ = Pressure

$P_{EN}$ = Environment Pressure

$P_i$ = Entering Pressure

$P_1$ = Entering Pressure

$\Delta P_i$ = Elementary Pressure Drop

$P_o$ = Initial Pressure

$P_{ATM}$ = Atmospheric Pressure

$P_c$ = Critical Pressure

$q_x$ = Fluid Conductive Heat Flux

$q_w$ = Time Rate of Heat per Unit Length

$s$ = Entropy

$s_1$ = Entering Entropy

$s_2$ = Exiting Entropy

$t$ = Time

$t_0$ = Initial Time

$u$ = Specific Internal Energy

$v$ = Specific Volume

$x$ = Length Coordinate

$\alpha$ = Polytropic Coefficient

$\gamma$ = $c_p/c_v$

$\theta$ = Disc Angle

$\rho$ = Density

$\rho_i$ = Elementary Density

$\rho_1$ = Entering Density

$\rho_2$ = Exiting Density

$\rho_c$ = Critical Density

$\sigma$ = Entropy Production

NOTE (1)

Indexes may be combined.
Example:

$K_c$ Coefficient of Resistance. Contraction

$K_i$ Coeff. of Resistance. Ref. to Element. Greater Section

$K_{ic}$ Element Coeff. of Resistance. Contraction.

# CHAPTER 1
# INTRODUCTION

## 1.1 SUMMARY

Piping systems need protection from flow reversals. A check valve is a flow activated device for intercepting the fluid flow, when a reversal occurs.

Many types of check valves have been developed and are currently used. The gravity swing check valve is the most commonly used in industrial lines over 2" diameter.

Gas lines operate in a wide range of pipe sizes and fluid properties, but may be classified based on the ideal gas model.

A particular type of swing check, the wafer valve, has already been investigated, when used with liquids; the evaluation of its performance in gaseous lines is the main objective of this report.

## 1.2 CHECK VALVES IN GAS LINES

### 1.2.1 RATIONALE.

A piping system is a combination of piping components with the objective of carrying incompressible or compressible fluids to one or several users.

The fluid is brought into the system by pumps or compressors and its flow is guided through a desired path by sequencing strategically placed shut off valves.

An industrial piping system may be considered, for most of its operating time, to be in the steady state, as the oscillations of the flow parameters about their average values are of small amplitude. Transients, results of pump and compressor pulsations, piping shape and geometry may, nevertheless, trigger effects, like the spinning of discs and clappers. This can reduce the life of certain piping components. Opening or closing valves or compressors and accidents, such as pump malfunctions or pipe ruptures, create unsteady flow conditions and reversed flows. This may generate damage to pumps and compressors or spill system fluid into the environment.

The check valve is a flow activated device, which is designed to protect the system, by preventing flow reversals.

### 1.2.2 TYPES OF CHECK VALVES.

Some of the most commonly used check valves are classified here according to operating principle. Ball Check Valves (Fig. F 1.2.2.1) consist of a ball inserted into the flow to permit essentially unrestricted flow in the preferred direction. Flow in the reverse direction carries the ball to seat, immediately blocking the line. The valve may be spring or gravity loaded so that the minimum flow per operaton is controllable. The ball check valve is used for high viscosity liquids, in small size (max. 2") tranport lines and in hydraulic and pneumatic power systems.

A Piston Check Valve (Fig. F 1.2.2.2) consists of a piston similar to a globe valve disc, which, in normal operation, is kept suspended by the flow forces. The piston falls by gravity when the flow forces become insufficient or negative. This type may also be spring or gravity loaded. The piston check valve is widely used for liquids and gases

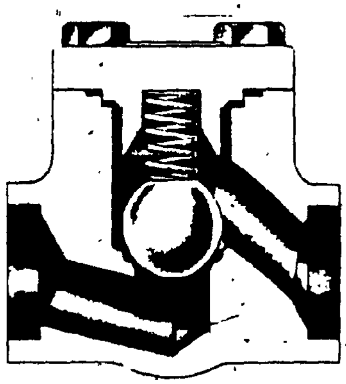


FIG. F 1.2.2.1 Ball Check.

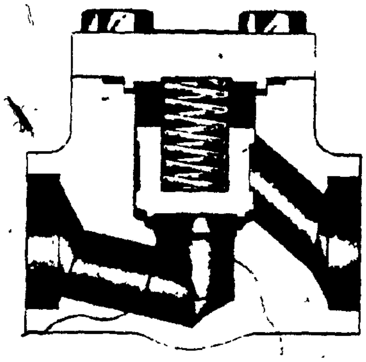


FIG. F 1.2.2.2 Piston Check.

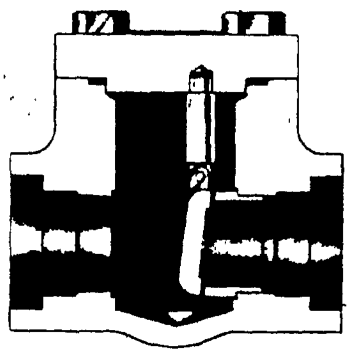


FIG. F 1.2.2.3 Swing Check.

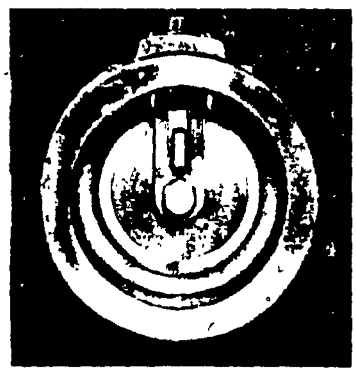


FIG. F 1.2.2.4 Wafer Check.

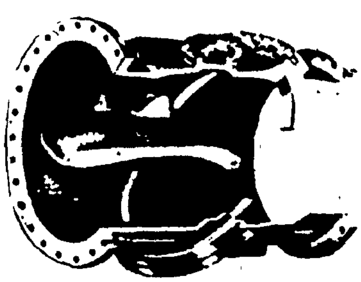


FIG. F 1.22.5 Tilting Disc.

FIG. F 1.22.6 Double Door.

and is particularly effective for high pressure steam service. High manufacturing costs and heavy pressure losses limit the use of piston check valves to power plant systems and to a maximum of 12" pipe diameter.

A Swing Check Valve (Fig. F 1.2.1.3) consists of a disc inserted into the line and rotating about a pivot shaft. In normal operation, the disc or clapper is kept open by the flow forces. The clapper "closes" by gravity when the fluid forces become insufficient. Counterweights, dashpots, or springs, may be added to control the disc closing characteristics. The swing check is the type most commonly used for liquid and gaseous pipelines, in a wide range of pipe dimensions (from 2" to over 48" diameter) and operating pressures. Subclasses of swing check valves are:

: Conventional bore or reduced orifice swing check valves for power lines,

: Full bore swing check valves for pipe lines,

: Wafer swing check valves (Fig. F 1.22.4), which are essentially conventional bore types, with reduced length and weight and favourable economic and installation characteristics.

The disc of the swing check valve can reach dimensions over 40" diameter and weights over 2 tons, where counterweights become necessary to open the valve in normal operating conditions and dampers are required to reduce the dynamic forces. When counterweights and dashpots are added, torsional stresses are induced on the pivot shaft and the closing time is increased. Therefore, disc weight and diameter represent an upper limit for an effective application of swing check valves.

The Tilting Disc Swing Check Valve (Fig. F 1.2.2.5) is a successfull attempt to reduce the regular swing check dynamic forces without worsening the closing characteristics and increasing the energy dissipation through the valve. The disc, still used as a closing device, rotates about a pivot shaft which is positioned inside the valve orifice. Its shape approximates, as much as possible, an airfoil and its gravity center is close to the shaft axis, which is positioned to let the disc open by rotation inside the conically shaped seat. Low opening forces and quick closing times are the main characteristics of the tilting disc check valves, but these are achieved by keeping the disc and shaft inside the fluid stream. Nevertheless, the pressure losses are not much higher then those of a standard swing check valve of the same size.

A relatively new development of the tilting disc check valve is the "Tricenter" valve, which is an interesting attempt to use the same design (not the same unit) as shut-off and check valve.

Another attempt to reduce the amount of flow required to open the valve is made with the "Door" Swing Check Valve, essentially a swing check valve with vertical pivot shaft, and with the "Double Door" Check Valve (Fig. F 1.2.2.6), which uses as closing devices two semidiscs, rotating about a vertical pivot shaft, relying heavily on two springs for correct closing movement.

### 1.2.3 GAS LINES.

Excluding pneumatic power systems, industrial gas lines may be classified as:

: Pipelines for long distance transportation of gaseous fuels,

: Blowing systems, where low pressure and high velocity gases (mostly air) are transported for short distances,

: Process systems where gaseous process fluids are transported to various stations,

: Utility systems, where industrial fluids are transported to using stations,

: Fuel systems, where gaseous fuels are transported to burning units,

: Steam systems, divided in:

: Low pressure heating systems,

: Medium pressure service systems,

: High pressure power systems.

From a different point of view, another classification may be made, using the Ideal Gas Property Relation:

$$p*v = Z*R*T$$

The following categories may be defined:

i) Gases with such pressure-density characteristics that the Ideal Gas Relation:

$$p*v = R*T$$

may be considered valid.

ii) Gases with such pressure-density values that the Gas Property Relation:

$$p*v = Z*R*T$$

can be applied providing Z is properly assumed according to the line parameters.

iii) Saturated steam, when water steam mixtures must be considered.

The first category is typical of flows in low pressure air blowing lines and in medium pressure high temperature steam and process lines. Typical of the second category are natural gas pipelines and moderately superheated steam lines. The third category is typical of flows in steam service systems.

Unless otherwise specified, only the (i) category of fluids will be taken into consideration.

## 1.3 THE WAFER CHECK VALVE

### 1.3.1 DESCRIPTION.

As already mentioned, the Wafer Check Valve (Fig. F 1.3.1.1) is a swing check valve with a gravity activated clapper. An additional spring system reduces the clapper closing time.

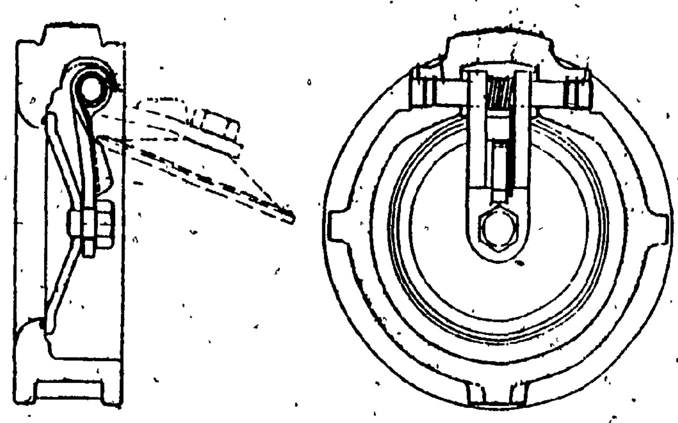


FIG. F 1.3.1.1 Typical Wafer Check Valve.

The wide open clapper angle is usually about 60 deg., and in this position the flow forces must be greater than the combined effect of clapper weight and spring load.

The valve cage is set in line between two flanges welded to the main pipe, and its length is that required to contain seat, clapper and hinge. Therefore, the clapper moves inside the main pipe, and a reduced seat diameter is required to allow valve opening.

As already mentioned, the reduced dimensions of the valve cage permit significant cost savings in comparison with swing check valves of the same diameter. Furthermore, the low weight makes the valve attractive when the line total weight is a factor.

The reduced seat diameter is responsible for the greater pressure losses of the wafer type, when compared to the full bore (API 6D) swing check valves. Like all the gravity swing check valves, this type requires considerable mass flow into the line to reach the maximum opening angle. Hence, with gaseous fluid, complete valve opening is not always possible.

Dashpots, counterweights, soft seats and all the other possible additions to a check valve may be installed on the wafer check.

Typical wafer check valves are manufactured in cast iron, cast steel, cast alloys, fabricated steel, and from steel and alloy bars.

1.3.2 REVIEW OF PREVIOUS WORK.

Investigation of check valve dynamics and check valve related pressure surges have been conducted by POOL and PORWIT, CARLTON and POOL (1,2). It was found that the water hammer could be predicted, knowing the basic design of a check valve.

KRANE and CHO (3) examined the design parameters affecting pressure drop, flow reversal and pressure surge. In their experimental results the pressure loss was expressed as a function of disc angle. A method of calculating fluid torque on a moving disc was described. It was shown that the fluid torque was mainly dependent on the pressure drop across the valve.

URAM (4) presented an analysis and interpretation of the valve disc dynamics and steam hammer effects that allow a rapid estimate of the disc impact speed. Simple relationships are used to represent the nature of the valve pressure drop during closure.

WEAVER, ADUBI and KOUWEN (5) studied the flow induced vibrations of a check valve with a spring damper to prevent slamming. As the discharge-displacement characteristics of the valve are dependent on its geometry, modifications of this geometry were examined and one found which eliminated the vibrations.

Design outlines and rules for an optimum design of the wafer check valve have been estabilished by SVOBODA, KATZ and FITCH (6). Analytic and experimental work was done to minimize the pressure loss (14) for liquid flow; the basic concepts and methodology may be expected to be valid also for gaseous media.

LEE (7) investigated the valve physical system with and without the addition of counterweights and dampers, then made and tested a numerical simulation model.

Experimental work has been done by HONG (13) to determine flow forces at different opening angles. Its extension to compressible fluids seems, at least in principle, possible.

HONG and SVOBODA (8) presented the computer model of a swing check valve suitable for use in lumped parameter piping network simulation. The valve is viewed as a variable resistor with a resistance being the function of the disc angle. The fluid is considered incompressible. This model, not yet confirmed with an experimental analysis, does not apply to gaseous fluids.

# CHAPTER 2

# CONCEPTS OF OPTIMIZATION

## 2.1 SUMMARY

A design procedure involves several steps, including identification of design requirements and the formulation of alternative design configurations. The optimal design will be the configuration which satisfies performance requirements and minimizes total costs. It can be stated that the optimal design of a check valve depends on both operating conditions and piping system layout.

A Valve Cost Function (9) may be obtained as the sum of annualized present weighted costs, using the concepts of Weighted Costs, Present Worth Factor and Capital Recovery Factor. Direct and Indirect Capital, Operating and Maintenance costs contribute to the Valve Cost Function.

An engineering analysis is required to define the acceptable valve configurations and the amount of the Valve Cost Function factors. A study of the valve efficiency, dynamic behaviour and life and safety are required to ensure its acceptability from all view points.

Cost Function Factors and Independent Variables of the valve system are defined and the determination of the relationships between factors and variables is established as the objective of the design procedure. Conceptual Minimization of Valve Cost Function and a Design Optimization Procedure are proposed.

## 2.2 DESIGN PARAMETERS

### 2.2.1 DESIGN REQUIREMENTS.

An ideal check valve should satisfy several, sometimes contradictory requirements.

The main purpose of a check valve is to protect upstream equipment from backflows. A sudden valve closure fullfills this requirement, however, it can result in seat damages.

In steady operation, the pressure losses through the valve should be kept as low as possible. The major portion of the pressure loss occurs in the clapper zone ($\approx$80%) (6) and depends heavily on the clapper opening angle. The clapper is kept open by flow forces (lift and drag) which depend on the opening angle. Insufficient flow forces will allow the clapper to oscillate in a semi-open position. With the system low damping factor, the oscillations continue for several cycles and, when their fequency is close to the system's natural frequency, the disc opening angle can reach its minimum and maximum allowable values. As the order of magnitude of a medium size valve natural frequency is 1 cycle/sec., the number of disc impacts on the valve seat and cage can exceed the fatigue limit in a relatively short time and mechanical failures can occur in a few months of operating life.

In gas lines the pipe size may reach 72". The dynamic forces of a swinging clapper may smash the seat as well as the clapper contact surfaces. Dashpots slow down the clapper movement but increase torsional stresses in the critical clapper pivot shaft.

### 2.2.2 DESIGN SOLUTIONS.

To cope with these contradictory requirements several design alternatives were explored.

The addition of springs and counterweights modifies the dynamic behaviour of the valve system. The use of springs decreases the closing time, but requires greater flow forces to keep the valve open, while the use of counterweights allows modifications of the closing and opening characteristics, but, acting on the system inertia, increases torsional stresses in the clapper shaft.

A minimum flow velocity is required to lift the clapper to full open and stable position (10). Sizing a check valve for gas to the minimum velocity requirement often leads to small size valves or to reduced clapper maximum opening angles. Smaller valves and reduced opening angle mean reduced or similar capital expenditures but greater operating costs.

The stabilization of fluttering valves can also be achieved with a dashpot arrangement, but, again, the shaft torsion stresses must be taken into consideration.

## 2.3 ECONOMIC OPTIMIZATION

### 2.3.1 THE VALVE COST FUNCTION.

The swing check valve, as part of the complete piping system, may be represented by a cost function. The cost function includes all direct and indirect costs generated by the presence of the valve in the piping system. When these costs are calculated with reference to a sufficient number of Compatible Valves, the design optimization consists of choosing the Compatible Valve with the lowest value of Cost Function at given Operating Conditions. A Compatible Valve is a valve of the same type and pipe size as the basic valve under consideration and with operating characteristics fullfilling the piping system main requirement, i.e. able to intercept, with acceptable efficiency, line backflows. Compatible Valves differ in design solutions and in energy and dynamic performance.

The Cost Function of a device, particularly that of a wafer check valve is the sum of:

$I Investment Costs, including standard valve purchase price, additions and modifications purchase price, installation costs;

$M Maintenance Costs, including preventitive and break-down valve maintenance costs and break-down operating costs;

$O Operating Costs, including energy degradation and energy consumption (if any) costs;

$R Recovery Costs, the residual value at the valve life end (negative);

$II Indirect Investment Costs, the additional investment cost to be sustained in the piping system, with reference to the standard valve.

$IM Indirect Maintenance Costs, the additional maintenance and repair costs anticipated in the pipe system compared to a system with standard valves.

Some of these costs are incurred at the time of valve installation, others are distributed over the system operating life. Some of the future costs WILL be incurred, some COULD be incurred. Some future costs may be constant, while others vary year to year. All these costs, to be properly introduced into the cost function, must be weighted and annualized.

Weighted Costs are costs that could be incurred, weighted with their statistic probability of occurrence (W). Their unweighted magnitude may be found by engineering and cost analysis. Their occurrence statistic probability may often be found only with good engineering judgment.

Annualized (Levelized) Costs (9) are required to assign the same cost figure to each year of the asset (valve) life. The sum of the annual costs is the total cost associated with the asset (valve) over its life, taking into account the time value of money.

Before designing an annualization procedure and building the cost function, some additional definitions are required.

Present Worth Factor (PWF). Given an original sum of money \$P, at an annual interest rate of \$i, the value, \$Y, after n annual interest periods is:

$$\$Y = \$P*(1+\$i)^{n} \qquad (2.3.1.1)$$

Conversely, a future sum \$Y has a present worth \$P:

$$\$P = \frac{\$Y}{(1+\$i)^{n}} \qquad (2.3.1.2)$$

The present worth of a given future sum \$Y diminishes by a factor $(1+\$i)$ for each year in the future.

The Present Worth Factor PWF($i,n) is;

$$PWF(\$i,n) = (1+\$i)^{-n} \quad (2.3.1.3)$$

Capital Recovery Factor (CRF). Given n amounts ($\$Y_1$, $\$Y_2$, ...., $\$Y_n$), where $\$Y_m$ is the amount to be paid at the end of year m (m=1, ..., n), the Present Worth of $Y_m$ is:

$$\frac{Y_m}{(1+\$i)^m}$$

and the Present Worth of the total amount to be paid in the period is:

$$\$P = \sum_{m=1}^{n} \frac{\$Y_m}{(1+\$i)^m} \quad (2.3.1.4)$$

For the particular case of equal annual amount, the total Present Worth becomes:

$$\$P = \$Y*\sum_{m=1}^{n}(1+\$i)^{-m}$$

or:

$$\$P = \frac{\$Y*(1-(1+\$i)^{-n})}{\$i} \quad (2.3.1.5)$$

The term:

$$\frac{\$i}{(1-(1+\$i)^{-n})} \qquad (2.3.1.6)$$

is called Capital Recovery Factor (CRF).

The Annualized Cost over n years of a cost $E, incurred at the year m, is given by:

$$\$Y = \$E*(PWF)*(CRF) \qquad (2.3.1.7)$$

Cost Function is the sum of all the positive and negative weighted and annualized costs:

$$C(\$/year) = \quad C *(PWF) *(CRF) \qquad (2.3.1.8)$$

2.3.2 ENGINEERING ANALYSIS.

Each factor of each Valve Cost Function must be determined as quantitative (dollar) value and as dependent variable of certain design parameters. Therefore, a complete engineering analysis of each Compatible Valve, as dynamic structure interacting with a piping system must be pursued.

The engineering analysis of a check valve requires

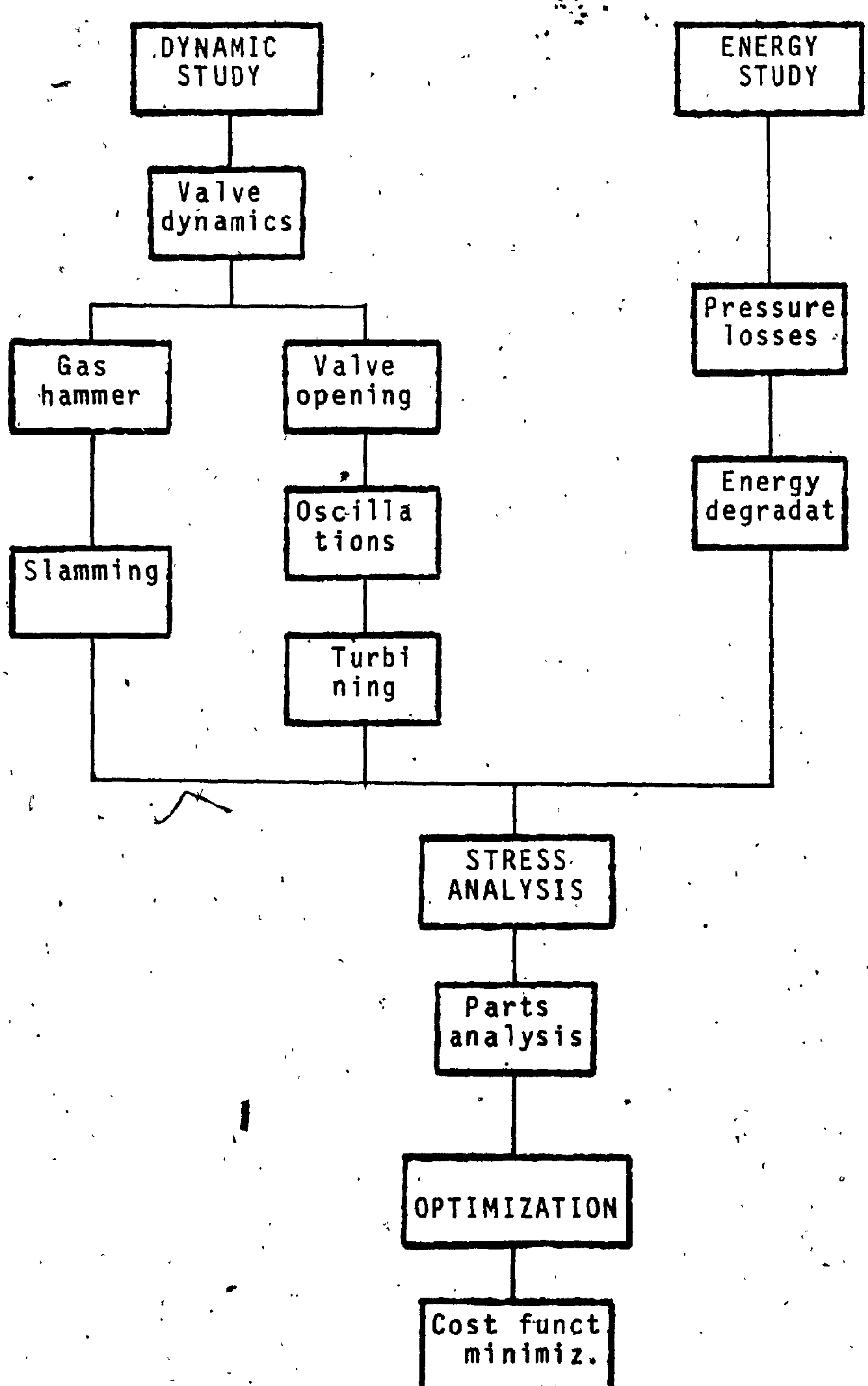



FIG. F 2.3.2.1 Design Optimization.

(Fig. F 2.3.2.1):

i) a study of the valve energy efficiency,

ii) a study of the valve dynamic behaviour,

iii) a valve components stress analysis.

The study of the valve energy efficiency determines:

: the valve pressure losses under the full range of operating conditions and for all possible clapper opening positions.

: the amount of energy degradation due to pressure losses.

The study of the valve dynamics will determine the valve response to the line steady and unsteady operating conditions and alert on possible abnormal behaviours of both valve and piping system. In fact, certain combinations of valve dynamic characteristics, piping system configurations and operating conditions may yield:

: the clapper partial opening and its oscillation with possible mechanical failure.

: the clapper spinning (turbining) with its possible detachment.

Furthermore, the combination of valve dynamic characteristics, piping system configuration and emergency conditions determine:

: the amount of pressure surge (gas hammer) downstream of the valve,

: the valve closing time and disc impact on the valve seat.

The stress analysis of the valve, in addition to being a requirement for a reliable design, will help to determine the valve life expectancy.

2.3.3 COST FUNCTION MINIMIZATION.

The piping system layout and/or operating conditions may influence the value of a Valve Cost Function. Therefore, a Valve Cost Function can assume a particular dollar value for each possible combination of piping system layout and operating conditions:

THE OPTIMAL DESIGN OF A CHECK VALVE DEPENDS ON BOTH OPERATING CONDITIONS AND PIPING SYSTEM LAYOUT.

The Cost Function Factors may be classified as:

i) Costs "internal" to valve (IC):

: Investment costs ($I)

: Recovery costs ($R)

: Operating costs ($O)

: Maintenance costs ($M)

ii) Costs "external" to valve (EC):

: Indirect investment costs ($II)

: Indirect maintenance costs ($IM)

For given operating conditions and piping system layout, the Cost Function major independent variables are:

: Valve nominal size (VS)

: Valve dynamic regulators (VR)

: Valve opening angle (VA)

: Valve orifice diameter (VO)

: Valve stress level (VL)

The Valve Cost Function can now be written:

$$
\begin{aligned}
C\ (\$/year) = {} & \$I*(PWF)_{\$I}*(CRF)_{\$I}+\$M*W_{\$M}*(PWF)_{\$M}*(CRF)_{\$M} \\
& +\$O*(PWF)_{\$O}*(CRF)_{\$O}+\$R*(PWF)_{\$R}*(CRF)_{\$R}+ \\
& +\$II*(PWF)_{\$II}*(CRF)_{\$II}+\$IM*(PWF)_{\$IM}*(CRF)_{\$IM}
\end{aligned}
$$

Table T 2.3.3.1 shows the independent and dependent variables. The relation between independent (valve factors) and dependent (cost items) variables will be obtained (Chap. 11) as the logical sum of the partial relations of each analysis block (Fig. F 2.3.2.1). This is are the subject of the following topics.

The Project Cost Function is the domain of all the Compatible Valve Cost Functions, for a specified piping system layout and given operating conditions (Fig. F 2.3.3.2). Its minimum is the Design Optimal Solution.

If a set of operating conditions is given, a Project Cost Function for each operating condition must be obtained

TABLE T 2.3.3.1 Independent and Dependent Variables.

| | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|
| $I : Investment costs | | | | | |
| $R : Recovery costs | | | | | |
| $O : Operating costs | | | | | |
| $M : Maintenance costs | | | | | |
| | | | | | |
| $II: Ind. Invest. costs | | | | | |
| $IM: Ind. Maint. costs | | | | | |

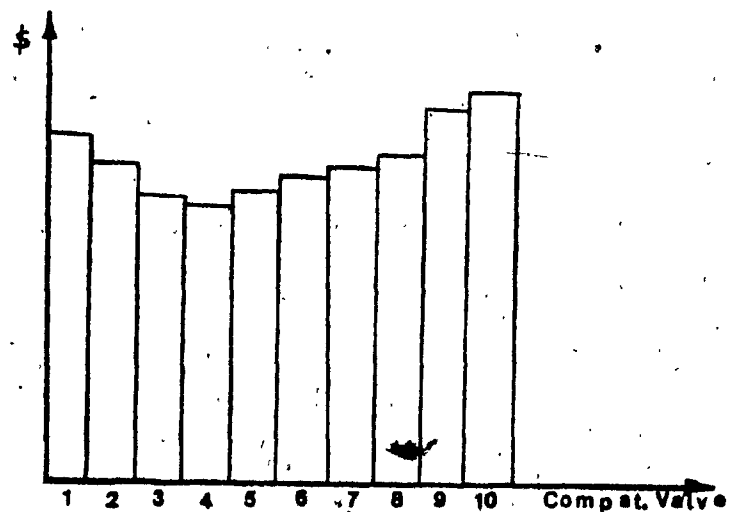



FIG. F 2.3.3.2 Project Cost Function.

and the Design Optimal Solution obtained as the minimum of all minimums.

Finally, a Design Optimization procedure can be outlined:

i) Selection of the Compatible Valves.

ii) Analysis, for each Compatible Valve, of each block of the three main design steps:

: Dynamic Study

: Energy Study

: Stress Analysis

iii) Definition, for each block, of a relation between dependent variables ($I, $R, $O, $M, $II, $Im) and independent variables (VS, VR, VA, VO, VL) (Partial Cost Functions).

iv) Definition of the Valve Cost Function for each Compatible Valve:

$$VCF = \sum PCF$$

v) Selection of the Compatible Valve with minimum value of Valve Cost Function (Design Optimal Solution).

## 2.4 CONCLUSIONS

The basic concepts of Engineering Economics provide a common sense approach to optimizing a valve design by forcing the designer to determine the cost of all the design factors. Although in most cases imperfect, the anticipated cost is in fact the only effective basis by which to compare possible alternatives.

In many instances, some of the design blocks of Fig. F 2.3.2.1 do not require investigation, as their influence on the cost function is negligible.

If high velocity and density flows keep the clapper well open, the "oscillation" effect can be ignored. Low pressure pipelines are somewhat less affected by gas hammer. In this case gas hammer and consequent indirect investment and indirect maintenance costs may be neglected.

In the next sections the various design analysis blocks will be examined. The blocks:

: Turbining

: Stress Analysis

will not be covered, assuming an adequate mechanical design.

The design analysis block:

: Slamming

will be examined only indirectly, thus ignoring possible, and for large valves important, damage of the seat-clapper contact surfaces.

Experimental tests are not economically feasible to support some of the block analyses.

PART 1

THE CHECK VALVE SYSTEM

# CHAPTER 3

# ENERGY STUDY

## 3.1 SUMMARY

In this chapter the blocks "Pressure losses" and "Energy Degradation" (Fig. F 2.3.2.1) will be investigated.

In the last decade, with sky rocketing oil prices, tremendous emphasis was put on energy conservation in all aspects of our society. An effective and systematic method, based on first and second principles of thermodynamics, was developed for the performance analysis of energy systems. This method (9) is called "availability analysis" or "exergy analysis".

In this section the basic concepts of "availability", "flow availability" and "availability destruction" are summarized and used to implement an analysis of the energy performance of a wafer check valve, which is considered a perfectly insulated system.

A fluid dynamic analysis of the wafer check valve (6) has been made for liquids. This analysis is proven to be in full agreement with the equivalent length of pipe approach (10) in which flow losses through a valve are assumed to be similar to those that would result from a fixed length of pipe.

The wafer valve characteristic pipe length is calculated and procedures established for computing the pressure losses and the availability destruction, in Kwh/year and in $/year, for given operating conditions and compressible fluids.

A computer program, COMPR, is then presented, to calculate the energy efficiency of a check valve. Results for various sizes and operating conditions are shown.

Through the analysis of some typical cases, the importance of "availability destruction" as a design parameter is demonstrated, and shown to be a more comprehensive approach than simple pressure drop considerations.

## 3.2 AVAILABILITY AND PRESSURE LOSSES

### 3.2.1 ENERGY DEGRADATION.

It is common practice to consider compressible fluid flow through a valve as an adiabatic process. This assumption is quite justified for fluid temperatures close to the environment temperature, otherwise a good insulation is required.

Under these assumptions, the flow through a valve becomes a closed system of flow through a control volume without work and heat exchange with the environment (Fig. F 3.2.1.1) (9)

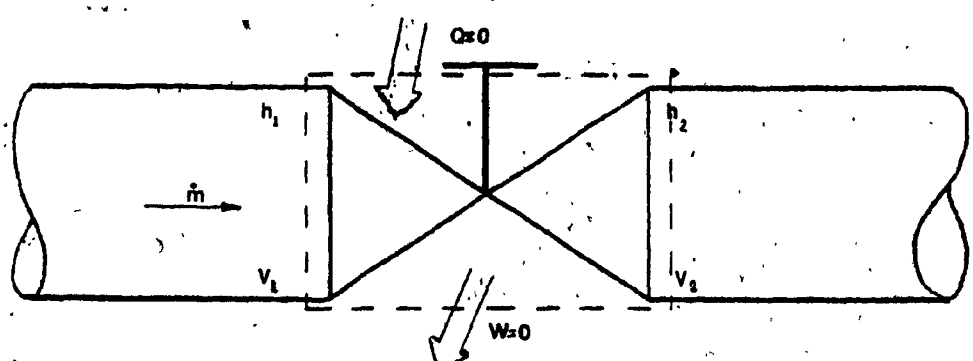



FIG. F 3.2.1.1 Valve Control Volume.

The Energy Equation, applied to the valve control volume for a steady flow, gives:

$$\frac{dE}{dt} = 0 = \dot{m}*(h_1 + \frac{V_1^2}{2}) - \dot{m}*(h_2 + \frac{V_2^2}{2}) \quad (3.2.1.1)$$

Therefore, no energy losses are experienced for an adiabatic flow through a valve. What is lost (9) through the valve is not a certain QUANTITY, but another energy aspect: its POTENTIAL or QUALITY. The energy quality is measured by a property called AVAILABILITY.

**AVAILABILITY** is the maximum work that can be extracted from the combined system of control mass and environment as the control mass passes from a given state to the dead state.

Some definitions are required:

**ENVIRONMENT** is that part of the system ( the valve) surrounding, the intensive properties (p, v, h, s, etc.) of which do not change significantly as a result of the process under consideration.

**RESTRICTED DEAD STATE** is the state in which a control mass is in thermal and mechanical equilibrium with the environment.

THERMOMECHANICAL FLOW AVAILABILITY at the valve entering and exiting side is defined as:

$$a_{fi} = a_i + (p_i - p_{EN})*v_i \quad (3.2.1.2)$$

Applying the Control Volume Availability Equation (9) to the steady flow through a valve, it may be written for an adiabatic process:

$$\frac{d\ A_{cv}}{dt} = 0 = \dot{m}*(a_{fi} - a_{fe}) - \dot{I} \quad (3.2.1.3)$$

where the term $\dot{I}$ accounts for destruction of availability within the valve, due to internal irreversibilities. The $\dot{I}$ dimensions are joule/s, since $\dot{I}$ is a rate of energy and represents the loss of work per second, extractable from the fluid under consideration, due to energy degradation through the valve.

The steady flow, control volume, availability equation becomes:

$$\dot{I} = \dot{m}*(a_{fi} - a_{fe})$$

$$\dot{I} = \dot{m}*((h_1 - h_2) - T_{EN}*(s_1 - s_2) + (\frac{V_1^2 - V_2^2}{2})) \quad (3.2.1.4)$$

### 3.2.2 PRESSURE LOSSES THROUGH A VALVE.

For an ideal gas and an adiabatic flow and the control volume of Fig. 3.2.1.1 it is:

| | | | |
|---|---|---|---|
| $\dot{m} = \rho_i * V_i * A_i$ (i = 1,2) | Continuity | (3.2.2.1 ) |
| $h_1 - h_2 = -(\frac{V_2^2 - V_1^2}{2})$ | Energy | (3.2.2.2 ) |
| $p = \rho * R_g * T$ | State | (3.2.2.3 ) |
| $dh_i = c_p * dT$ | Enthalpy | (3.2.2.4 ) |
| $s_2 - s_1 = \int_1^2 \frac{1}{T} dh - \int_1^2 \frac{1}{\rho * T} dp$ | Entropy | (3.2.2.5 ) |
| $p_1 - p_2 - F_v = \rho_2 * V_2^2 - \rho_1 * V_1^2$ | Momentum | (3.2.2.6 ) |

The term $F_v$ represents the pressure loss due to irreversibilities.

For an incompressible viscous flow through a straight pipe the momentum equation reduces to:

$$\Delta p = \int_1^2 -f * \rho * \frac{V^2}{2*D} * dx \qquad (3.2.2.7)$$

which is the well known Darcy equation. For a compressible viscous flow through a straight pipe the momentum equation bec??es:

$$\frac{dp}{dx} = -\frac{d}{dx}(\rho * V^2) - f * \rho * \frac{V^2}{2*D} \qquad (3.2.2.8)$$

KEENAN and NEUMANN (11) measured friction coefficients for turbulent flows in smooth pipes over a range of Mach Numbers between zero and nearly unity. For a fully developed profile, no significant effect of Mach Number was observed, i.e. the relation between friction coefficient and pipe Reynold Number, for subsonic flows, was found to agree with the well known Karman-Nikuradse formula for incompressible flows:

$$\frac{1}{\sqrt{4*f}} = -0.8+2*\log_{10}(Re*\sqrt{4*f}) \qquad (3.2.2.9\ )$$

Therefore it may be assumed that the Moody diagram is valid also for compressible fluids.

The numerical integration of the momentum equation (3.2.2.8 ) gives:

$$p_1-p_2 = \varrho_2*V_2^2-\varrho_1*V_1^2+f*L*\left(\frac{\varrho_1*V_1^2}{4*D}+\frac{\varrho_2*V_2^2}{4*D}\right) \qquad (3.2.2.10.a)$$

$$p_1-p_2 = \frac{f*L*\dot{m}^2}{2*D*\varrho_1*A^2}*\left(\frac{2*\varrho_1*D}{\varrho_2*f*L}-\frac{2*D}{f*L}+\frac{1}{2}+\frac{\varrho_1}{2*\varrho_2}\right) \qquad (3.2.2.10.b)$$

$$p_1-p_2 = \frac{f*L}{2*D}*\frac{\dot{m}^2}{\varrho_1*A^2}*\frac{1}{Y^2} \qquad (3.2.2.10)$$

with:

$$\frac{1}{Y^2} = \left(\frac{2}{K}*\left(\frac{\varrho_1}{\varrho_2}-1\right)+\frac{1}{2}*\left(1+\frac{\varrho_1}{\varrho_2}\right)\right) \qquad (3.2.2.11)$$

and:

$$K = \frac{f*L}{D} \qquad (3.2.2.12)$$

Using the (3.2.2.10, 3.2.2.11, 3.2.2.12) the Darcy formula for straight pipe, and compressible fluids, becomes:

$$\dot{m} = \frac{A*Y*\sqrt{2*\Delta p*Q_1}}{\sqrt{K}} \qquad (3.2.2.13)$$

Y is called "net expansion factor" (12) and may be calculated solving the continuity, momentum and energy equations.

Experiments have shown (10) that the pressure loss due to valves and fittings is proportional to a constant power of the flow velocity. For all practical purposes, in the turbulent range, the exponent may be assumed to be 2. The pressure loss through a valve or fitting may be expressed with the same type of equation used for straight pipe with the addition of a coefficient K, representing the pressure losses through the valve. Assuming L as length of a pipe of inside diameter D and f a yet undefined friction factor, the Darcy formula for a valve becomes:

$$\dot{m} = \frac{A*Y*\sqrt{2*\Delta p*q_1}}{\sqrt{K}} \quad (3.2.2.14)$$

with:

K = valve Coefficient of Resistance,

or:

$$\dot{m} = C_v^{*}*A*Y*\sqrt{2*\Delta p*q_1} \quad (3.2.2.15)$$

assuming:

$$K = \frac{1}{C_v^{*2}} \quad (3.2.2.16)$$

and:

$$\dot{m} = C_v^{*}A*Y*\sqrt{2*\Delta p*q_1} \quad (3.2.2.17)$$

with:

$$C_v = C_v^{*}*\sqrt{\frac{2*A^2}{q_1}} \quad (3.2.2.18)$$

The Coefficient of Resistance (10) K represents an equivalent length, in pipe diameter of straight pipe, that will cause the same pressure drop as the valve under the same flow conditions multiplied by a factor f:

$$K = f \cdot \frac{L}{D} = f \cdot K_o \qquad (3.2.2.18)$$

Experiments have shown (10) that, with an acceptable approximation, a typical value of L/D exists for each type of valve of a given diameter.

The Coefficient of Resistance, K, would theoretically be a constant for all sizes of a given design or line of valves and fittings if all sizes were geometrically similar. The geometrical similarity is never, or very seldom, achieved.

The K coefficient for a number of lines of valves and fittings have been plotted against size (10) and it has been found that the Coefficient of Resistance, for a given line of valves or fittings, tends to vary with size as does the friction factor, f, for straight clean commercial steel pipe, and that the equivalent length L/D tends toward a constant for the various sizes of a given line of valves or fittings.

Typical values of Equivalent Length for various types of valves are shown in table T 3.2.2.1.

TABLE T 3.2.2.1 Typical values of Ko

| TYPE OF VALVE | Ko |
|---|---|
| Gate | 8 |
| Globe Vertical | 340 |
| Globe Inclined | 55 |
| Swing Check | 50 |
| Check Tilting Disc | 90** |
| ** sizes 10"-14" | |

## 3.3 AVAILABILITY DESTRUCTION

### 3.3.1 COEFFICIENT OF RESISTANCE OF A WAFER VALVE.

SVOBODA and KATZ (6) studied the pressure losses through a wafer valve, using water as fluid, and found excellent correspondence on values obtained with an original model approach. The check valve is modelled as two orifices in series and a cumulative coefficients of discharge is calculated. For angles over 30 deg. and an incompressible fluid, it is: (Fig. F 4.21.1):

$$C_V = \frac{C_{V1} * C_{V2}}{(C_{V1} + C_{V2})^{1/2}} \qquad (3.3.1.1)$$

with:

$$C^*_{v1} = \frac{Cv_1}{A_1 * \sqrt{\frac{2}{\rho_1}}} = \frac{(d_2/d_1)^2}{(K_1 + ((d_2/d_1) - 1))^{1/2}} \quad (3.3.1.2)$$

$$C^*_{v2} = \frac{Cv_2}{A_1 * \sqrt{\frac{2}{\rho_1}}} = \frac{Fo}{(K_3 + (Fo-1)^2)^{1/2}} \quad (3.3.1.3)$$

$$Fo = 1-(d_3/d_1) * \cos\theta - 4*Ao/(\pi * d_1^2) \quad (3.3.1.4)$$

where Ao is the projected area of clapper and clapper arm. Its value can be calculated as:

$$\begin{aligned} Ao = & \; x_o((d_1/2)^2 - x_o^2) + (d_1/2)^2 * \sin^{-1}(2*x/d_1) \\ & - (x_o((d_3/2)^2 - x_o^2)^{\frac{1}{2}} + (d_3/2)^2 * \sin^{-1}(2*x_o/d_3)) * \cos\theta \\ & - 2*l_a * x_o(1-\cos\theta) + 2*d_o*x_o*\sin\theta \end{aligned} \quad (3.3.1.5)$$

where $x_o$ and $l_a$ are the half width and total length of the clapper arm and $d_o$ the disc shaft displacement with respect to the seat plane.

For an incompressible fluid and using the (3.2.2.17) it is:

$$C^*_V = \frac{1}{\sqrt{K_1 + K_2 + K_3 + K_4}} = \frac{1}{\sqrt{f*K_{o1} + f*K_{o2} + f*K_{o3} + f*K_{o4}}} \quad (3.3.1.6)$$

$$C^*_V = \frac{1}{\sqrt{K}} \quad (3.3.1.7)$$

with:

$$K_1 = \frac{0.04}{(d_2/d_1)^4} \qquad (4.2.1.4)$$

$$K_2 = \frac{(1-(d_2/d_3))^2}{(d_2/d_1)^4} \qquad (3.3.1.8)$$

$$K_3 = \frac{(1-d_3/d_1)^2 * \sqrt{\sin(90-\theta)}}{(d_3/d_1)^4} \qquad (4.2.1.8)$$

$$K_4 = \frac{(1-Fo)^2}{Fo^2} \qquad (3.3.1.9)$$

and:

$$K = K_1+K_2+K_3+K_4 \qquad (3.3.1.10)$$

$$Ko = Ko_1+Ko_2+Ko_3+Ko_4 \qquad (3.3.1.11)$$

with K and Ko Coefficient of Resistance and Equivalent Length of the valve.

Values of Equivalent Length for wafer check valves, calculated with the Svoboda-Katz procedure for various sizes and clapper opening angles, are shown in Table T 3.3.1.1.

TABLE T 3.3.1.1 Values of Ko

| NOM. SIZE | CLAPPER ANGLE | | | | |
|---|---|---|---|---|---|
| | 30 | 45 | 60 | MAX. | 0 |
| 2" | 1455 | 496 | 163 | 84 | 70 |
| 4" | 1452 | 471 | 144 | 72 | 69 |
| 6" | 1018 | 384 | 131 | 66 | 70 |
| 8" | 748 | 313 | 119 | 82 | 66 |
| 10" | 960 | 366 | 125 | 67 | 69 |
| 12" | 752 | 316 | 119 | 87 | 65 |
| 18" | 797 | 348 | 144 | 111 | 60 |
| 36" | 942 | 435 | 176 | 130 | 65 |

The design of sizes 36" and 18" is not optimized according the Svoboda-Katz rules.

For a wide open valve, the Characteristic Equivalent Length of a wafer check valve can be approximated as:

$$Ko = 76 \qquad (3.3.1.12)$$

when the optimization procedure is applied, while significant discrepancies result for the non optimized sizes.

The Coefficient of Resistance of a wafer check valve, for clapper angles greater then 30 deg., may be predicted using the quadratic relationship:

$$a \cdot K_o^2 + b \cdot \theta \cdot K_o + \theta^2 + c = 0 \qquad (3.3.1.13)$$

Values of coefficients a, b, c are shown in Table T 3.3.1.2.

TABLE T 3.3.1.2 Quadratic Coefficients.

| SIZE | a | b | c |
|---|---|---|---|
| 2" | -0.0010 | 0.1436 | -4981 |
| 4" | -0.0011 | 0.1432 | -4817 |
| 6" | -0.0015 | 0.1887 | -5060 |
| 8" | -0.0021 | 0.2523 | -5376 |
| 10" | -0.0017 | 0.1970 | -5053 |
| 12" | -0.0020 | 0.2454 | -5316 |
| 18" | -0.0018 | 0.2491 | -5710 |
| 36" | -0.0007 | 0.1877 | -5565 |

The Characteristic Equivalent Length may be used to compare the wafer check valve with other types of check valves (Table T 3.3.1.3), remembering that the Coefficient of Resistance, for a given mass flow, is directly proportional to the pressure drop.

TABLE T 3.3.1.3 Ko for Various Types of Check Valves. Small Sizes.

| TYPE OF VALVE | Ko | Ko |
|---|---|---|
| Wafer check | 76 | - |
| Swing check full bore | 50 | -34% |
| Tilting disc ($\alpha$=5 deg.) | 40 | -47% |
| Tilting disc ($\alpha$=15 deg.) | 120 | +58% |
| ($\alpha$ = angle of attack) | | |

When a compressible fluid is taken into consideration, the pressure drop for each elementary contraction or expansion becomes (see also Chapt. 4):

$$\Delta p_i = \overline{K} * \frac{\dot{m}^2}{2*A_{ii}^2 *Y_i^2 *\varrho_{ii}} \qquad (3.3.1.14)$$

with:

$A_{ii}$ = Greater section area of contraction (expansion) i.

$Y_i$ = Net Expansion Factor of contraction (expansion) i.

$\varrho_{ii}$ = Entering density of contraction (expansion) i.

$\overline{K}_i$ = Coefficient of Resistance of element i, referred to the ELEMENT greater section area

or:

$$\Delta p_i = K_i * \frac{\dot{m}^2}{2*A_i^2 *Y_i^2 *\varrho_{ii}} \qquad (3.3.1.15)$$

with:

$$K_i = \frac{A_i^2}{A_{ii}^2} *\overline{K}_i \qquad (3.3.1.16)$$

and:

$K_i$ = Coefficient of Resistance of element i , referred to the PIPE section area.

The total pressure drop becomes:

$$\Delta_t p = \sum_{1i}^{4} \Delta p_i = \frac{\dot{m}^2}{2*A_1^2*q_2} \sum_{1i}^{4} \left( \frac{K_i}{\frac{q_{2i}}{q_2} * Y_i^2} \right) \quad (3.3.1.17)$$

A similar relationship may be derived with reference to the downstream flow parameters:

$$\Delta p_i = \frac{K_i * \dot{m}^2}{2*A_1^2 * Y_{ri}^2 * q_{2i}} \quad (3.3.1.18)$$

$$\Delta p = \frac{\dot{m}^2}{2*A_1^2*q_2} \sum_{1i}^{4} \left( \frac{K_i}{\frac{q_{2i}}{q_2} * Y_{ri}^2} \right) \quad (3.3.1.19)$$

with

$$\frac{1}{Y_{ri}^2} = \left( \frac{2}{K_i} * \left(1 - \frac{q_{2i}}{q_{1i}}\right) + \frac{1}{2} * \left(1 + \frac{q_{2i}}{q_{1i}}\right) \right) \quad (3.3.1.20)$$

3.3.2 EVALUATION OF AVAILABILITY DESTRUCTION.

Taking into consideration the limited valve body surface, compared to its equivalent length, as well as effective valve insulation at high temperature, an adiabatic process can be assumed. Therefore, with reference to (3.2.1.4) and using (3.2.2.2), the availability destruction, due to the flow through the valve, is:

$$\dot{I} = \dot{m} * T_{EN} * (s_1 - s_2) \qquad (3.3.2.1)$$

Accepting the ideal gas model, the entropy equation (3.2.2.5) becomes:

$$s_2 - s_1 = c_p * \ln(\frac{T_2}{T_1}) - R_g * \ln(\frac{p_2}{p_1})$$

$$s_2 - s_1 = R_g * ((\frac{\gamma}{\gamma - 1}) * \ln(\frac{T_2}{T_1}) - \ln(\frac{p_2}{p_1}))$$

$$s_2 - s_1 = R_g * \ln((\frac{p_1 - \Delta p}{p_1})^{\frac{1}{\gamma - 1}} * (\frac{\rho_2}{\rho_1})^{\frac{\gamma}{\gamma - 1}}) \qquad (3.3.2.2)$$

The energy equation of an adiabatic, steady, flow through a valve is:

$$\frac{\gamma}{(\gamma - 1)} * \frac{p_1}{\rho_1} + \frac{\dot{m}^2}{2 * \rho_1^2 * A_1^2} = \frac{\gamma}{(\gamma - 1)} * \frac{p_2}{\rho_2} + \frac{\dot{m}^2}{2 * \rho_2^2 * A_1^2} \qquad (3.3.2.3)$$

or:

$$\rho_2^2 * ((\frac{p_1}{\rho_1} + (\frac{\gamma - 1}{\gamma}) * \frac{\dot{m}^2}{2 * \rho_1^2 * A_1^2})) - \rho_2 * (p_2) - ((\frac{\gamma - 1}{2\gamma}) * \frac{\dot{m}^2}{A_1^2}) = 0$$

which gives:

$$\rho_2 = \frac{p_2 + \sqrt{p_2^2 + 4 * (\frac{p_1}{\rho_1} + (\frac{\gamma - 1}{2 * \gamma}) * \frac{\dot{m}^2}{\rho_1^2 * A_1^2}) * ((\frac{\gamma - 1}{2 * \gamma}) * \frac{\dot{m}^2}{A_1^2})}}{2 * (\frac{p_1}{\rho_1} + (\frac{\gamma - 1}{2 * \gamma}) * \frac{\dot{m}^2}{\rho_1^2 * A_1^2})} \qquad (3.3.2.4)$$

with:

$$p_2 = p_1 - \Delta p$$

Hence:

$$\rho_2 = \rho_2(p_1, \rho_1, \Delta p, \dot{m}, A_1) \qquad (3.3.2.5)$$

$$s_2 - s_1 = \Delta s = \Delta s(p_1, \rho_1, \rho_2, R_g) \qquad (3.3.2.6)$$

and:

$$\dot{I} = \dot{m} * T_{EN} * \Delta s(p_1, \rho_1, \rho_2, R_g) \quad \text{joule/s} \qquad (3.3.2.7)$$

or:

$$\dot{I} = \frac{(24*365)}{1000} * \varepsilon * \dot{m} * T_{EN} * \Delta s \quad \text{KWh/year} \qquad (3.3.2.8)$$

with

$$\varepsilon = \frac{\text{Standard Yearly Operating Hours}}{\text{Calendar Hours}}$$

### 3.3.3 COMPUTER PROGRAM "COMPR".

The FORTRAN coded Program COMPR (Appendix 2) allow calculation of the energy efficiency of a valve, given its geometric parameters and operating conditions. Program COMPR configuration is:

Input:

: Valve size.

: Operating pressure, density and flow velocity.

: Gas constant.

: Valve geometric parameters D1, D2, DCL, X, DIS, ALA, TETAF (Fig. F 3.3.3.1).

: Environment temperature.

: Cost per KWh.

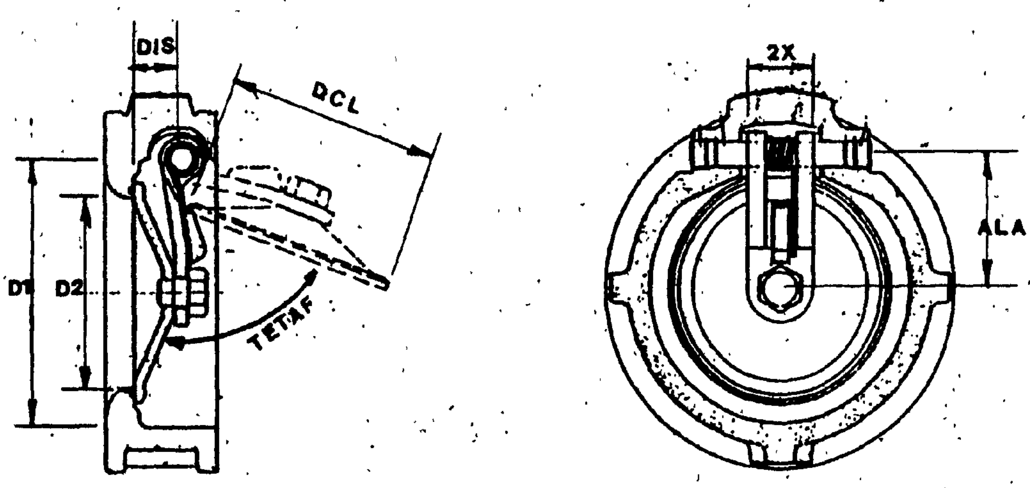



FIG. F 3.3.3.1 Valve Geometric Parameters.

Output:

: At disc angle 30°, 45°, 60° and full open:

: Coefficient of Resistance.

: Pressure drop.

: Availability destruction in Kwh/year.

: Availability destruction in $/year.

The quadratic formulae are of the form:

$$X^2+a*\theta*X+b*\theta^2+c = 0 \qquad (3.3.3.1)$$

with:

$\theta$ = clapper angle

X = unknown parameter

Structure:

The program structure is shown in Fig. F 3.3.3.2.

Results for valve sizes 10", 18", 36" and three typical operating conditions are shown in Tables T 3.3.3.1, 2, 3 at the end of this chapter, and the availability variation with flow velocity is plotted in Diagrams D 3.3.3.1, 2, 3 and with pressure in Diagrams D 3.3.3.4, 5, 6.

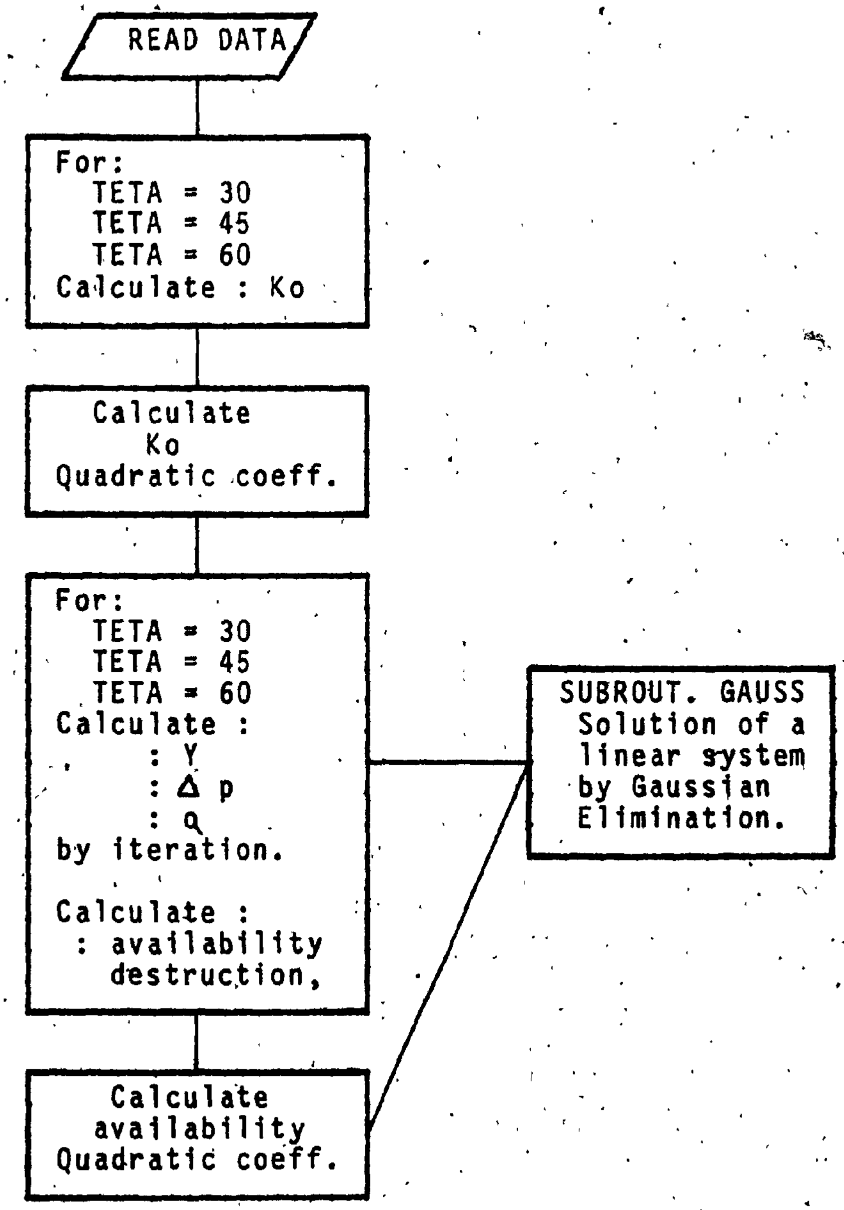



FIG. F 3.3.3.2 Program COMPR

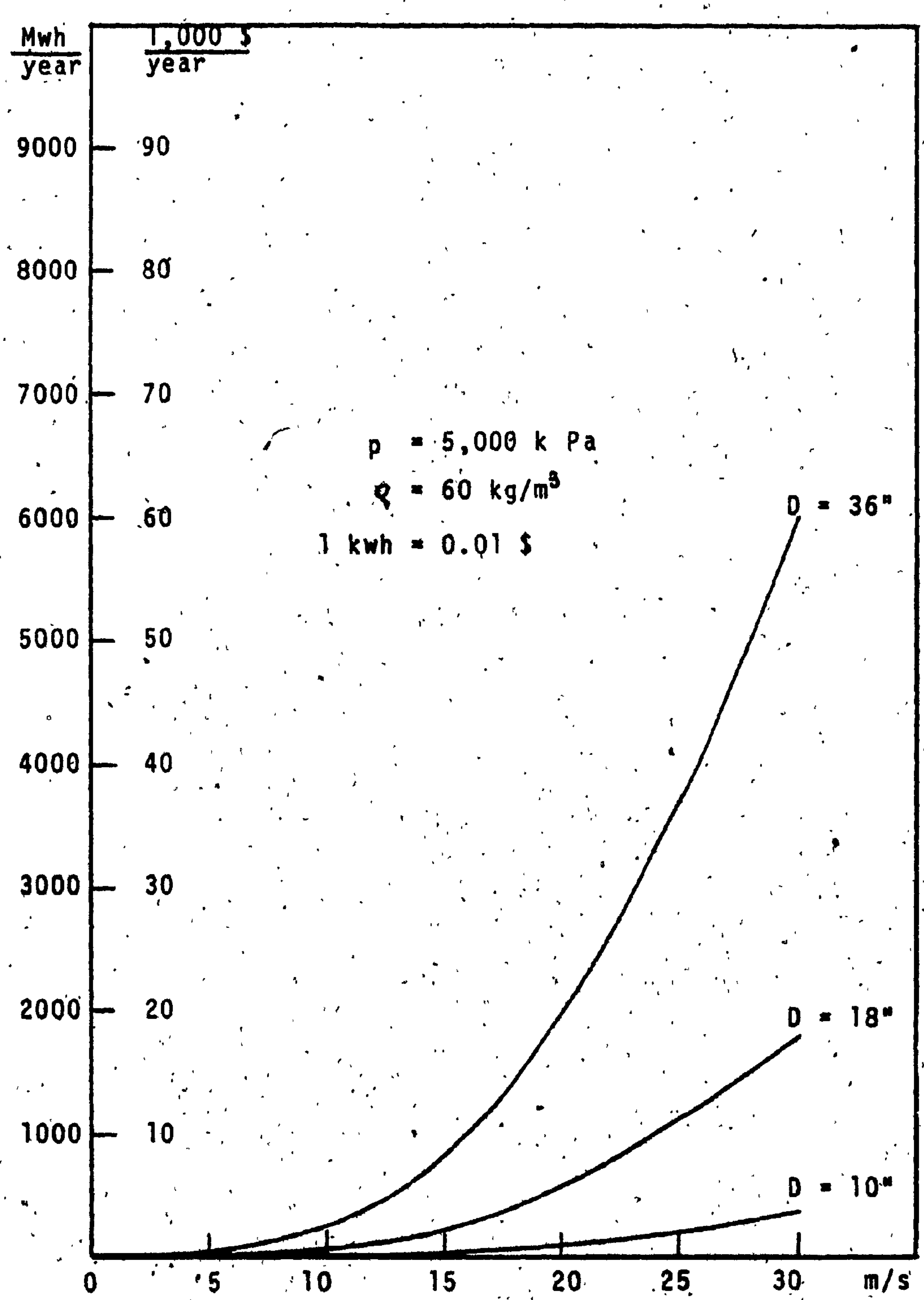



Diagram D 3.3.3.1. Availability Destruction vs. Air Velocity

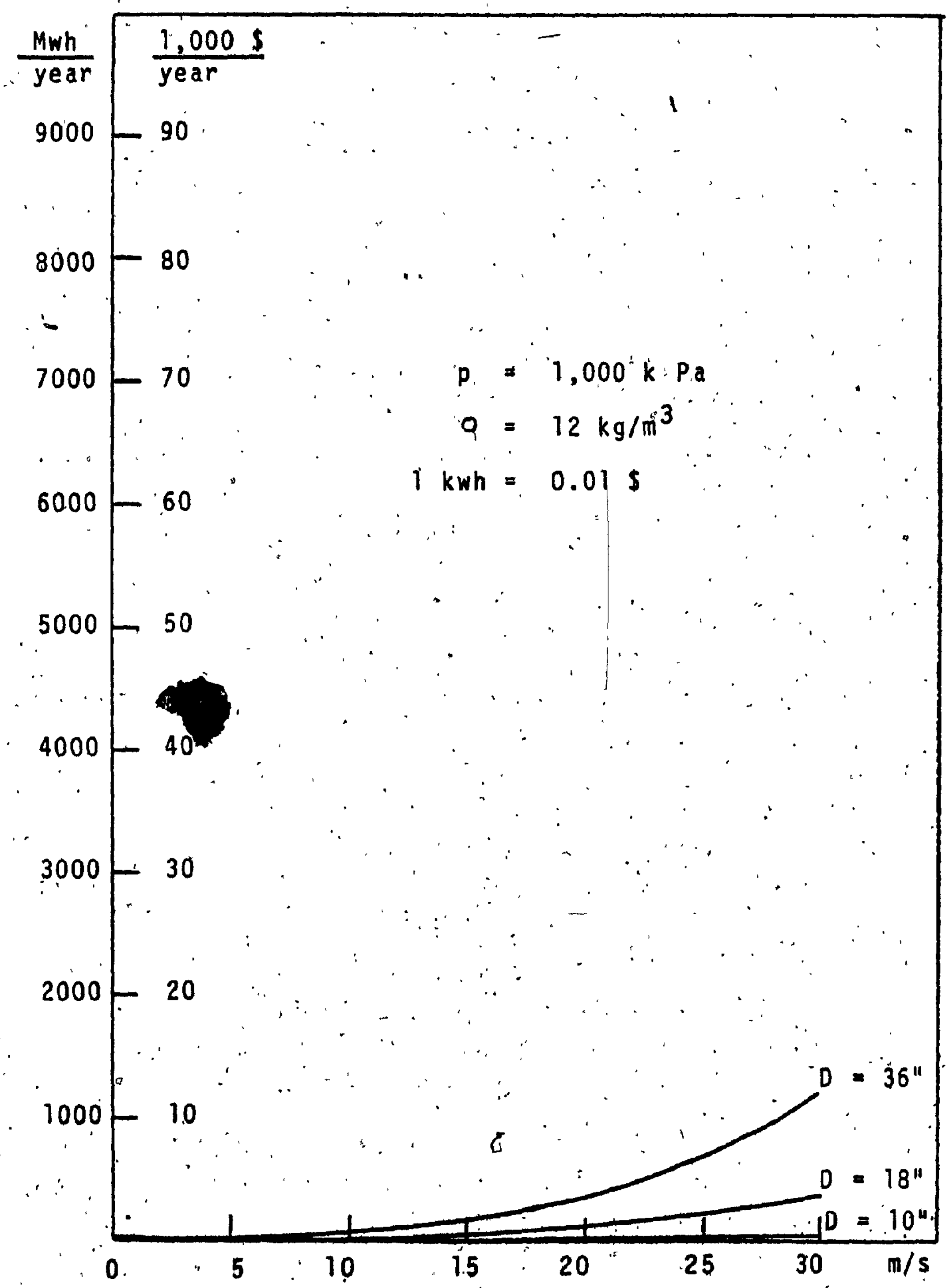



Diagram D 3.3.3.2 Availability Destruction vs. Air Velocity

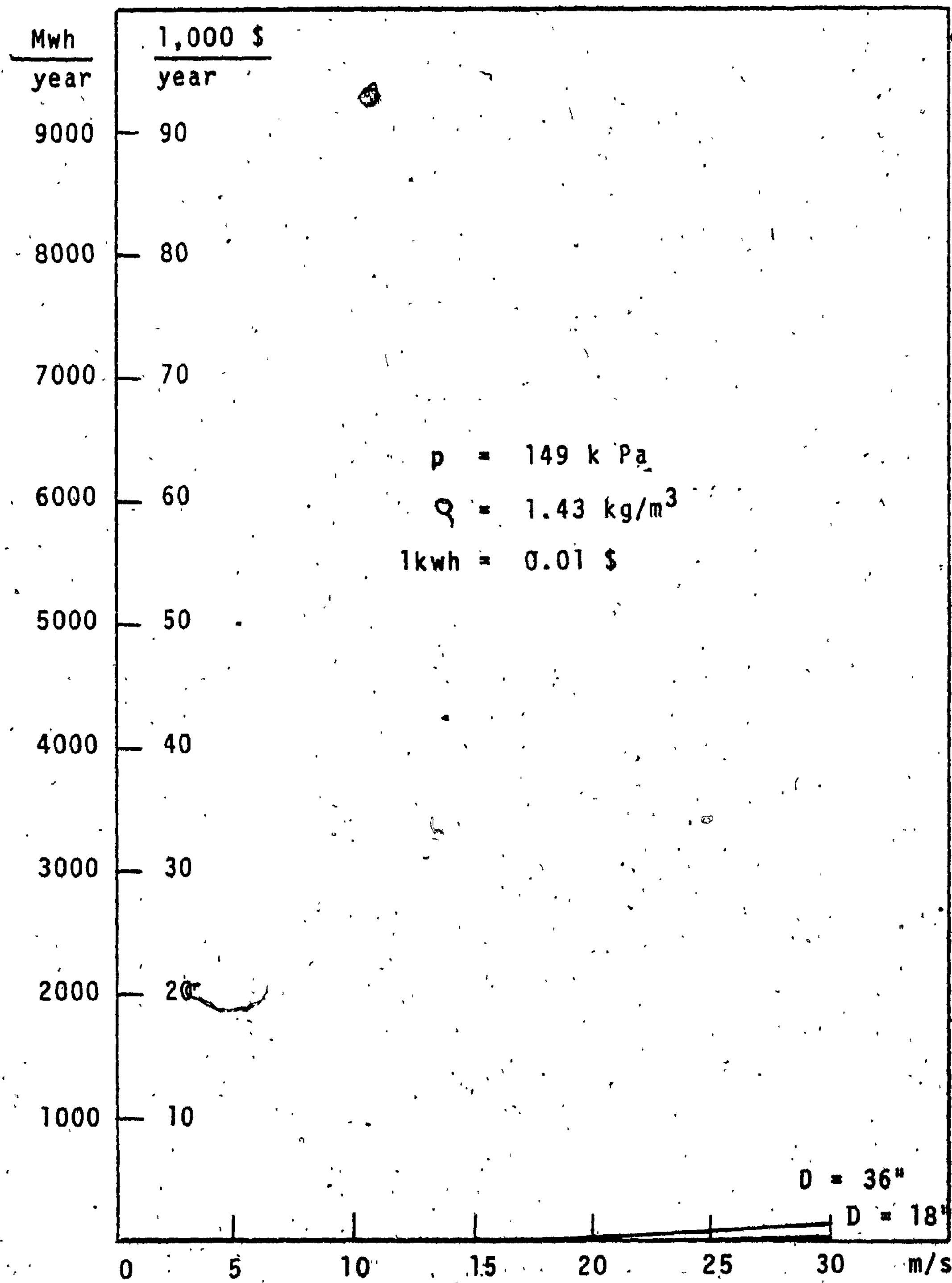



Diagram D 3.3.3.3 Availability Destruction vs. Air Velocity

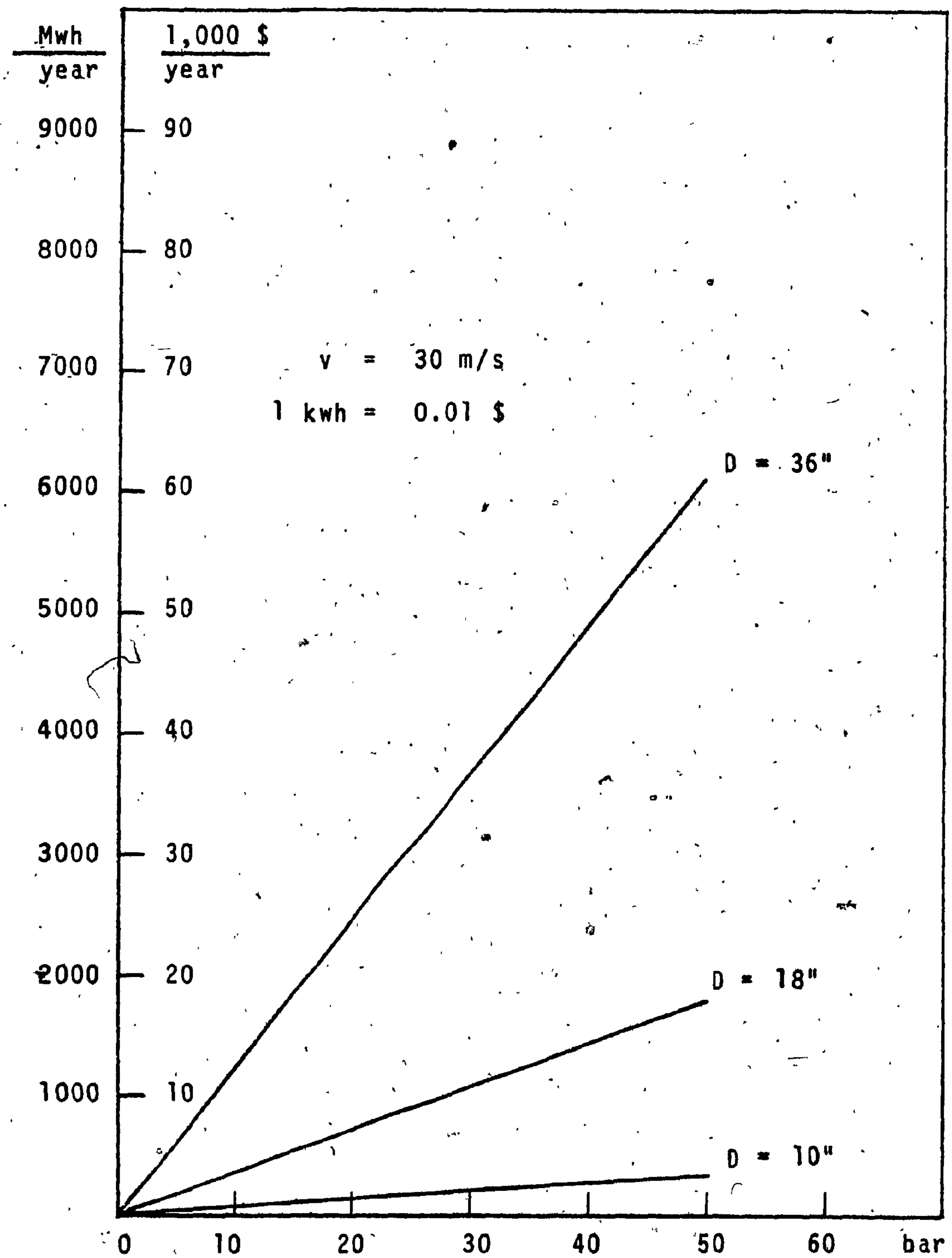



Diagram D 3.3.3.4 Availability Destruction vs. Air Pressure.

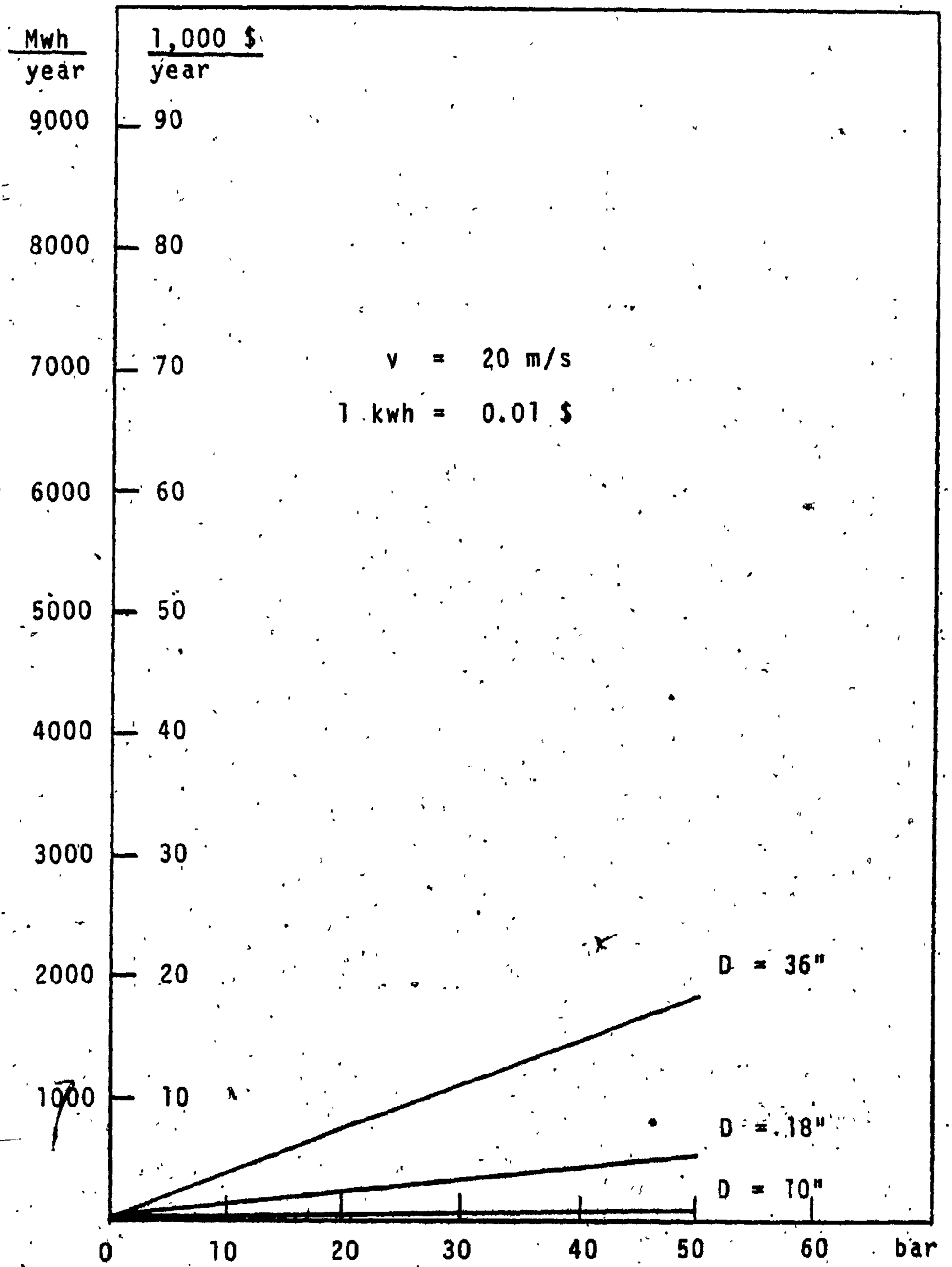



Diagram D 3.3.3.5 Availability Destruction vs. Air Pressure.

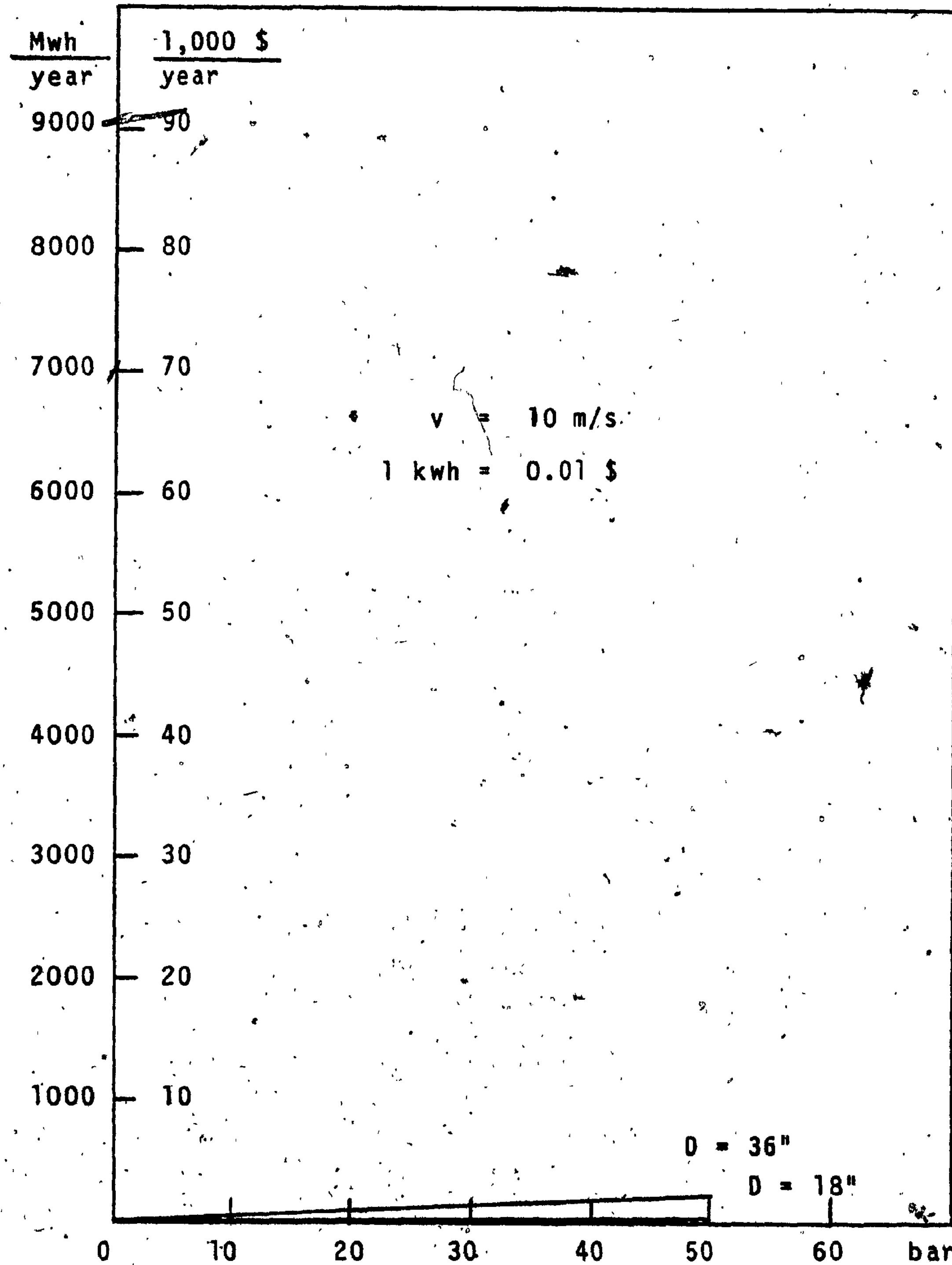



Diagram D 3.3.3.6 Availability Destruction vs. Air Pressure

### 3.3.4 PARTIAL COST FUNCTION.

The dollar form of the Availability Destruction Equation:

$$I^2+a*\theta*I+b*\theta^2+c = 0 \qquad (3.4.1.1)$$

is also the valve Partial Cost Function of the block Energy Degradation. The independent dependent variables relationship is shown in Table T 3.3.4.1.

TABLE T 3.3.4.1 Energy Degradation Partial Cost Function.

| | | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|---|
| $I | Investment Costs | X | | | | |
| $R | Recovery Costs | X | | | | |
| $O | Operating Costs | X | | X | X | |
| $M | Maintenance Costs | | | | | |
| $II | Indirect Investment Costs | | | | | |
| $IM | Indirect Maintenance Costs | | | | | |

The function domain is:

Opening angle θ : 30 - 70. deg.

Valve size : 2" - 12"

The valve size domain can be extended to any dimension, if proper values of Ko are known. The validity of the

Orifice Sequence Model for large valves is likely, but not supported by experimental tests (See Chapter 4).

## 3.4 CONCLUSIONS

Installing a check valve in a piping system is like buying insurance against damage to expensive piping components for unwanted flow reversals. The availability destruction represents the yearly cost, after investments, of this protection.

High pressure and high velocity fluids flowing through a check valve, may waste "good energy" at an impressive rate, especially for large diameter piping. Assuming complete (theoretical) line utilization and an indicative cost of:

0.01 $/KWh

with air at:

| | | |
|---|---|---|
| V = | 30 m/s | (100 ft/s) |
| p = | 5000 KPa | (750 psia) |

some typical figures of availability destruction are:

| | | | | |
|---|---|---|---|---|
| 36" dia. | 5,000,000 | KWh/year | or | 60,000 $/year |
| 18" dia. | 1,800,000 | KWh/year | or | 18,000 $/year |

| | | | | | | |
|---|---|---|---|---|---|---|
| 10" | dia. | 320,000 | KWh/year | or | 3,200 | $/year |

Low pressure, ambient temperature and high velocity fluids (air blowers) still have significant availability destruction, but at a much less impressive rate. For:

V = 30 m/s ( 100 ft/s)

p = 149 kpa ( 22 psia)

some typical figures of availability destruction are:

| | | | | | | |
|---|---|---|---|---|---|---|
| 36" | dia. | 116,000 | KWh/year | or | 1,160 | $/year |
| 18" | dia. | 34,000 | KWh/year | or | 340 | $/year |
| 10" | dia. | 6,000 | KWh/year | or | 60 | $/year |

In the selection of check valves for high pressure and high velocity pipelines a low coefficient of resistance is a primary requirement. For high pressure service, reducing the opening angle of a 36" check valve from 65 to 60 deg. will cost:

2,000,000 KWh/year or 20,000 $/year

For low pressure, high velocity lines the availability destruction is less important. Low pressure flows often do not generate enough force (Chapters 4 and 5) to keep the valve fully open. The quest for valve dynamic stability may yield a drastic reduction of the clapper maximum opening angle. For an 18" valve and low pressure service, reducing the maximum opening angle from 60 to 45 deg. will increase the operating costs of:

49,000 KWh/year or 490 \$/year

bringing the total cost of energy degradation to:

83,000 KWh/year or 830 \$/year

In conclusion, while the availability destruction is a most important parameter for evaluating a check valve for high pressure and high velocity operating conditions, its weight is lower for less severe operating conditions.

TABLE T 3.3.3.1 Energy Efficiency. Low Pressure Line.

**PROGRAM COMPR : VALVE ENERGY EFFICIENCY**

| | | |
|---|---|---|
| Operating pressure | 149 | KPa. |
| Operating density | 1.43 | Kg/m3 |
| Operating velocity | 30 | m/s |
| Gas | air | |
| Environment temp. | 20 | C |
| Cost of Mwh | 10 | $/Mwh |

| VALVE SIZE : 10" | U | 30° | 45° | 60° | 69° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 940 | 367 | 125 | 67 |
| Pressure drop | kPa | 8.17 | 3.18 | 1.09 | 0.58 |
| Availab. destruction | MWh/y | 89 | 34 | 12 | 6 |
| Availab. destruction | $/y | 894 | 342 | 116 | 62 |

Friction factor 0.0135

X = L/D $X^2 - 0.001555 * X * \theta + 0.1957 * \theta^2 - 5045 = 0$

X = MWh/Y $X^2 - 0.185100 * X * \theta + 2.0918 * \theta^2 - 5031 = 0$

X = $/y $X^2 - 0.001850 * X * \theta + 0.2092 * \theta^2 - 5031 = 0$

| VALVE SIZE : 18" | U | 30° | 45° | 60° | 60° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 777 | 348 | 144 | 144 |
| Pressure drop | kPa | 6.00 | 2.68 | 1.11 | 1.11 |
| Availab. destruction | Mwh/y | 188 | 83 | 34 | 34 |
| Availab. destruction | $/y | 1877 | 830 | 341 | 341 |

Friction factor 0.0120

X = L/D $X^2 - 0.001556 * X * \theta + 0.2456 * \theta^2 - 5684 = 0$

X = MWh/Y $X^2 - 0.292100 * X * \theta + 1.0306 * \theta^2 - 5673 = 0$

X = $/y $X^2 - 0.000292 * X * \theta + 0.1031 * \theta^2 - 5673 = 0$

| VALVE SIZE : 36" | U | 30° | 45° | 60° | 65° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 932 | 435 | 177 | 131 |
| Pressure drop | kPa | 6.59 | 3.08 | 1.25 | 0.92 |
| Availab. destruction | Mwh/y | 842 | 388 | 156 | 116 |
| Availab. destruction | $/y | 8415 | 3881 | 1564 | 1158 |

Friction factor 0.011

X = L/D $X^2 - 0.0006355 X * \theta + 0.1862 * \theta^2 - 5550 = 0$

X = MWh/Y $X^2 - 0.000896 * X * \theta + 0.2088 * \theta^2 - 5538 = 0$

X = $/y $X^2 - 0.000009 * X * \theta + 0.0209 * \theta^2 - 5538 = 0$

TABLE T 3.3.3.2 Energy Efficiency. Medium Pressure Line.

**PROGRAM COMPR : VALVE ENERGY EFFICIENCY**

| | | | |
|---|---|---|---|
| Operating pressure | : | 1000 | KPa. |
| Operating density | : | 12 | Kg/m3 |
| Operating velocity | : | 30 | m/s |
| Gas | : | air | |
| Environment temp. | : | 20 | C |
| Cost of Mwh | : | 10 | $/Mwh |

| VALVE SIZE : 10" | U | 30° | 45° | 60° | 69° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 940 | 367 | 125 | 67 |
| Pressure drop | kPa | 68.58 | 26,71 | 9.11 | 4.85 |
| Availab. destruction | MWh/y | 943 | 359 | 12.2 | 65 |
| Availab. destruction | $/y | 9430 | 3594 | 1216 | 645 |

Friction factor, 0.0135

$X = L/D \quad X^2 - 0.001555 * X * \theta + 0.1957 * \theta^2 - 5045 = 0$

$X = Mwh/Y \quad X^2 - 0.001696 * X * \theta + 0.1992 * \theta^2 - 5028 = 0$

$X = \$/y \quad X^2 - 0.000017 * X * \theta + 0.0199 * \theta^2 - 5028 = 0$

| VALVE SIZE : 18" | U | 30° | 45° | 60° | 60° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 777 | 348 | 144 | 144 |
| Pressure drop | kPa | 50.36 | 22.54 | 9.29 | 9.29 |
| Availab. destruction | Mwh/y | 1975 | 870 | 357 | 357 |
| Availab. destruction | $/y | 19750 | 8705 | 3570 | 3570 |

Friction factor, 0.0120

$X = L/D \quad X^2 - 0.001556 * X * \theta + 0.2456 * \theta^2 - 5684 = 0$

$X = MWh/Y \quad X^2 - 0.000270 * X * \theta + 0.0983 * \theta^2 - 5670 = 0$

$X = \$/y \quad X^2 - 0.000003 * X * \theta + 0.0098 * \theta^2 - 5670 = 0$

| VALVE SIZE : 36" | U | 30° | 45° | 60° | 65° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 932 | 435 | 177 | 131 |
| Pressure drop | kPa | 55.30 | 25.81 | 10.46 | 7.75 |
| Availab. destruction | MWh/y | 8860 | 4073 | 1639 | 1213 |
| Availab. destruction | $/y | 88608 | 40730 | 16.386 | 12126 |

Friction factor, 0.011

$X = L/D \quad X^2 - 0.0006355 X * \theta + 0.1862 * \theta^2 - 5550 = 0$

$X = Mwh/Y \quad X^2 - 0.000008 * X * \theta + 0.0199 * \theta^2 - 5534 = 0$

$X = \$/y \quad X^2 - 8.35 * 10 * X * \theta + 0.0020 * \theta^2 - 5534 = 0$

TABLE T 3.3.3.3 Energy Efficiency. High Pressure Line.

**PROGRAM COMPR : VALVE ENERGY EFFICIENCY**

| | | |
|---|---|---|
| Operating pressure | 5000 | KPa. |
| Operating density | 60 | Kg/m3 |
| Operating velocity | 30 | m/s |
| Gas | air | |
| Environment temp. | 20 | C |
| Cost of Mwh | 10 | $/Mwh |

| VALVE SIZE : 10" | U | 30° | 45° | 60° | 69° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 940 | 367 | 125 | 67 |
| Pressure drop | kPa | 342 | 133 | 46 | 024 |
| Availab. destruction | MWh/y | 4715 | 1797 | 607 | 323 |
| Availab. destruction | $/y | 47750 | 17921 | 6078 | 3225 |

Friction factor 0.0135

X = L/D $X^2 - 0.001555*X*\theta + 0.1967*\theta^2 - 5045 = 0$

X = Mwh/Y $X^2 - 0.000068*X*\theta + 0.3984*\theta^2 - 5028 = 0$

X = $/y $X^2 - 6.78*10 *X*\theta + 0.0040*\theta^2 - 5028 = 0$

| VALVE SIZE : 18" | U | 30° | 45° | 60° | 60° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 777 | 348 | 144 | 144 |
| Pressure drop | kPa | 252 | 113 | 46 | 35 |
| Availab. destruction | MWh/y | 3875 | 4353 | 1785 | 1785 |
| Availab. destruction | $/y | 98750 | 43528 | 17848 | 17848 |

Friction factor 0.0120

X = L/D $X^2 - 0.001556*X*\theta + 0.2456*\theta^2 - 5684 = 0$

X = Mwh/Y $X^2 - 0.000108*X*\theta + 0.0197*\theta^2 - 5670 = 0$

X = $/y $X^2 - 1.08*10 *X*\theta + 0.0020*\theta^2 - 5670 = 0$

| VALVE SIZE : 36" | U | 30° | 45° | 60° | 65° |
|---|---|---|---|---|---|
| Equiv. length | L/D | 932 | 435 | 177 | 131 |
| Pressure drop | kPa | 276 | 129 | 52 | 39 |
| Availab. destruction | MWh/y | 44300 | 20365 | 8193 | 6063 |
| Availab. destruction | $/y | 443006 | 203650 | 81930 | 60632 |

Friction factor 0.011

X = L/D $X^2 - 0.0006355X*\theta + 0.1862*\theta^2 - 5550 = 0$

X = Mwh/Y $X^2 - 3.34*10 *X*\theta + 0.0040*\theta^2 - 5534 = 0$

X = $/y $X^2 - 3.34*10 *X*\theta + 0.0004*\theta^2 - 5534 = 0$

## CHAPTER 4

## VALVE DYNAMICS

### 4.1 SUMMARY

In this chapter the theoretical background for the analysis of blocks "Valve Opening" and "Oscillations" of Fig. F 2.3.2.1 is developed.

Pressure drop and flow forces are of primary importance in determining valve dynamic behaviour.

An Orifice Sequence Model is developed and matched to experimental results, in order to predict the pressure drop through a valve as a function of clapper angle, for both direct and reverse flows. Coefficients of Resistance for valve and for clapper alone are developed.

The clapper flow torque is investigated and some experimental results are used to determine a relationship between drag forces and total torque. An Equivalent Torque Arm and a Torque Arm Coefficient are introduced to determine the clapper torque which depends on Coefficient of Resistance and flow conditions.

The valve dynamic system is represented by a second order, non linear, differential equation. Additional elements such as counterweights and springs are examined and introduced, as additional terms, into the general differential equation. Finally, a simple and effective dashpot is presented.

## 4.2 DYNAMIC FORCES

### 4.2.1 FORWARD PRESSURE DROP.

Pressure drop and mass flow through an elementary contraction or expansion are related by the Darcy Formula (3.3.1.14).

A wafer check valve (Fig. F4.2.1.1 (a)) may be modelled as a sequence of orifices and the valve Coefficient of Resistance assumed as combination of the Coefficients of Resistance of the elementary contractions and expansions.

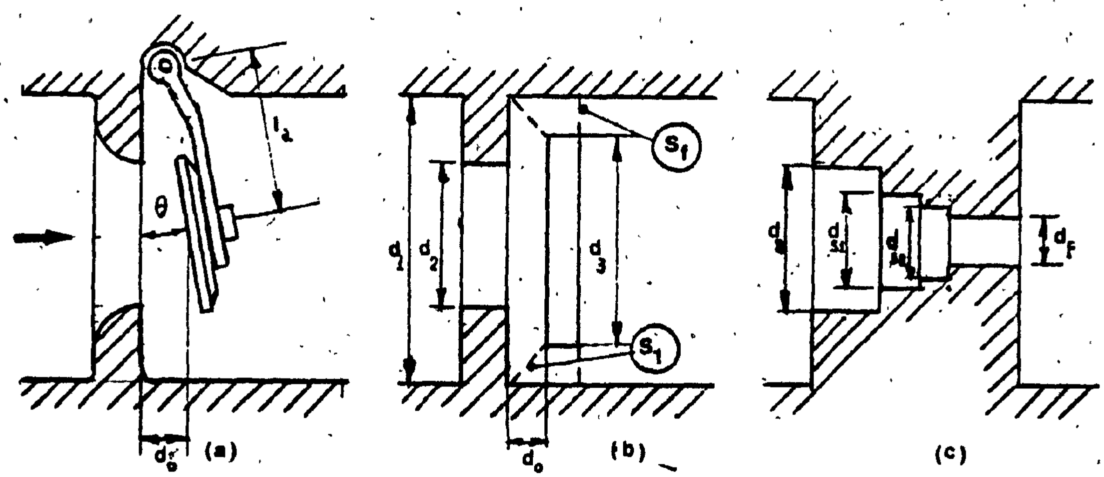



FIG. F 4.2.1.1 Orifice Sequence Model

(a) Valve geometry

(b) Clapper idealization

(c) Sequence of orifices

The valve's internal geometry (Fig. F 4.2.1.1 (b)) may be idealized as an orifice, the seat area, and a circular disc, the vertical projection of clapper and hinge (the clapper arm) area. Four characteristic surfaces can be identified (Fig. F 4.2.1.1 (c)) and their area assumed to be the surface area of four orifices in sequence.

Using Fig. F 4.2.1.1 notations, it is:

$\Psi$ = 0.7 (form coeff.)

$S_F = \pi * F * d_1^2$

$S_1 = \pi * d_2 * d_o$

$d_o$ = la* $\Psi$ *tgθ

$d_F = \sqrt{F * d_1^2}$

$d_{s1} = \sqrt{4 * S_1 / \pi}$

$d_3 = \sqrt{d_1^2 - d_F^2}$

The elementary Coefficients of Resistance are (10):

$$\overline{K}c = \frac{((1-(d_{i+1}/d_i)^2)*\sqrt{\sin(90-\theta)}}{(d_{i+1}/d_i)^4} \quad \text{(contraction)} \quad (4.2.1.1)$$

$$\overline{K}e = \frac{(1-(d_i/d_{i+1})^2)^2}{(d_i/d_{i+1})^4} \quad \text{(expansion)} \quad (4.2.1.2)$$

$$\overline{K}s = \frac{0.004}{(d_{i+1}/d_i)^4} \quad \text{(smooth contr.)} \quad (4.2.1.3)$$

The total pressure drop through a check valve, considered a sequence of contraction and expansions, is:

-----------

* The Coefficient of Resistance for a pipe contraction is empirically defined by Crane Co. as:

$$K = 0.5(1-(d_{i+1}/d_i))*\sqrt{\sin(\gamma/2)}/(d_{i+1}/d_i)$$

with:

$\gamma$ = angle of total contraction

The Coefficient of Resistance used for clapper contraction, allows for an irregular shape and is in good agreement with experimental results for a 4" valve and incompressible fluids. Its generalization to compressible fluids and bigger valves should be confirmed by experimentation.

$$\Delta p = \sum_{1}^{5}{}_{i}\Delta p_{i} = \frac{\dot{m}^2}{2*A_1^2*q_1} * \sum_{1}^{5}{}_{i}\left(\frac{K_i}{\frac{q_i}{q_1}*Y_i^2}\right) \quad (3.3.1.16)$$

with the Coefficients of Resistance for each elementary contraction and expansion:

$$K_1dc = \frac{0.04}{(d_2/d_1)^4} \quad (4.2.1.4)$$

$$K_2de = \frac{(1-(d_2/ds_1)^2)^2*(d_1/ds_1)^4}{(d_2/ds_1)^4} \quad (4.2.1.5)$$

$$K_2dc = \frac{0.5*(1-(ds_1/d_2)^2)*(d_1/d_2)^4}{(ds_1/d_2)^4} \quad (4.2.1.6)$$

$$K_3dc = \frac{((1+(d_F/ds_1)^2)*\sqrt{\sin(90-\theta)})*(d_1/ds_1)}{(d_F/ds_1)^4} \quad (4.2.1.7)$$

$$K_4de = \frac{(1-(d_F/d_1)^2)^2}{(d_F/d_1)^4} \quad (4.2.1.8)$$

and:

$$\frac{d_i}{d_j} < 1 \qquad d_{s1} \leqslant d_1$$

The comparison with experimental results, for water ($Y_i=1$) and a 4" valve, suggests a validity of the Orifice Sequence Model for:

$$15^\circ < \theta < 70^\circ$$

For clapper angles under 15 deg. the Coefficient of

Resistance may be considered as one entity and calculated using:

$$K = \frac{15*K(15)}{\theta\ (deg.)} \qquad (4.2.1.9)$$

Table T 4.2.1.1 shows a comparison of model and experimental results :

TABLE T 4.2.1.1 Forward Coefficient of Resistance. Comparison of Theoretical and Experimental Results for 4" Valve.

| | CLAPPER ANGLES (DEG.) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 30 | 45. | 60 | 69 |
| K dc | | | .0972 | .0972 | .0972 | .0972 | .0972 |
| K de | | | | .0846 | .3127 | .3127 | .3127 |
| K cc | | 2.6671 | 1.0659 | | | | |
| K dc | | 17.00 | 16.52 | 10.55 | 4.34 | 1.25 | 0.53 |
| K de | | 25.65 | 20.42 | 10.87 | 3.33 | 0.85 | 0.32 |
| Kd | 67.5 | 45.41 | 38.10 | 21.61 | 7.933 | 2.501 | 1.268 |
| Kcl | 67.5 | 45.00 | 37.70 | 21.20 | 7.523 | 2.091 | 0.859 |
| Kclex | 59 | 42 | 33 | 23.7 | 7.93 | 2.19 | 0.86 |
| ΔKcl | +8.5 | +3.0 | +4.7 | -2.5 | -0.41 | -0.10 | -0.00 |
| ΔKcl% | +14.4 | +7.2 | +14.2 | -10.6 | -5.1 | -4.5 | -0.2 |

## 4.2.2 FORWARD FLOW TORQUE.

With the pipeline in normal operating conditions, flow forces act on the clapper surface, keeping it in the open position. These flow forces are a combination of lift and drag effects. The magnitude and point of application of lift forces depend on the aerodynamic characteristics of clapper and hinge and on the angle of attack $\alpha$ ($\alpha$ = 90-0). The lift force increases with the opening angle (12) and reaches a maximum for opening angles greater than 70 deg.. Magnitude, direction and point of application of lift forces depend on the pressure distribution on the upper and lower surfaces of the clapper-hinge systems. Their true values could be predicted with a refined finite elements analysis combined with wind tunnel experiments.

The magnitude of the drag forces may be determined knowing the valve's total pressure losses. The control volume of Fig. F 4.2.2.1 encloses a pipe element of length L, representing an elementary contraction or expansion, of pressure drop:

$$\Delta p = p_i - p_{i+1}$$

The impulse theorem, applied to the control volume, gives:

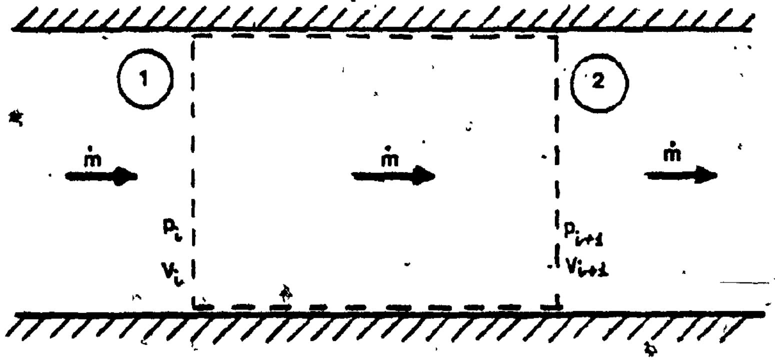



FIG. F 4.22.1 Control Volume.

$$Fd+p_1*A-p_2*A = \rho_2*A*V_2^2-\rho_1*A*V_1^2 \qquad (4.2.2.1)$$

with:

Fd = net thrust produced by the stream between section 1 and section 2.

or for the contraction (expansion) i, using (3.2.2.10), (3.2.2.10.a), (3.2.2.10.b) and (3.3.1.12):

$$Fd_i = A*\bar{K}_i*(\rho_i*\frac{V_i^2}{2}+\rho_{i+1}*\frac{V_{i+1}^2}{2})$$

and, remembering (3.2.2.11):

$$Fd_i = \frac{\dot{m}^2 * Ki}{2*A_i*\rho_i} * \left(\frac{1}{Y_i^2} - \frac{2}{Ki} * \left(\frac{\rho_i - \rho_{i+1}}{\rho_i}\right)\right) * \left(\frac{\rho_i}{\rho_i}\right) \qquad (4.2.2.3)$$

$$Fd = Ki*Hi^2 \frac{\dot{m}^2}{2*A_i*\rho_i} = Ki*Hi^2*\frac{\dot{m}^2}{2*A_i*\rho_i}*\left(\frac{\rho_i}{\rho_i}\right) \qquad (4.2.2.4)$$

with:

$$Hi^2 = \frac{1}{Y_i^2} - \frac{2}{Ki}*\frac{(\rho_i - \rho_{i+1})}{\rho_i} \qquad (4.2.2.5)$$

$H$ may be called "Drag Compressibility Coefficient" and is equal to unity for an incompressible fluid.

Introducing the Orifice Sequence Model, the drag force can be seen as the sum of five elementary components corresponding to the five contractions and expansions of figure F 4.2.1.1.

$$Fd = Fd_1 + Fd_2 + Fd_3 + Fd_4 + Fd_5 \qquad (4.2.2.6)$$

The drag force due to clapper alone can be obtained (13) by deducting from the total drag force the drag force due to the orifice alone:

$$Fcl = Fd - Fdo \qquad (4.2.2.7)$$

with:

Fcl = drag force due to clapper alone

Fdo = drag force due to seat alone.

HONG (13) obtained experimental values of clapper torque for all opening angles of a 4" valve. This experimental data has been used to determine an Equivalent Torque Arm (ETA) (Table T 4.2.2.1):

$$ETA = \frac{\text{Experimental Clapper Torque}}{Fcl} \qquad (4.2.2.8)$$

and a Torque Arm Coefficient (TAC):

$$TAC = \frac{ETA}{\text{hinge length}} \qquad (4.2.2.9)$$

TABLE T 4.2.2.1 Equivalent Torque Arm and Torque Arm Coefficient for a 4" Valve.

| | CLAPPER ANGLES (DEG.) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 30 | 45 | 60 | 69 |
| Q (m3/s) | .0015 | .0015 | .0015 | .0015 | .0288 | .0288 | .0288 |
| Exp. T (N*m) | 21.92 | 14.63 | 12.17 | 9.04 | 22.91 | 9.04 | 4.06 |
| Fdcl (N) | 541 | 360 | 302 | 170 | 365 | 102 | 42 |
| Hinge le. (m) | .0548 | .0548 | .0548 | .0548 | .0548 | .0548 | .0548 |
| ETA (m) | .0405 | .0406 | .0403 | .0532 | .0628 | .0886 | .0965 |
| TAC | .7391 | .7409 | .7354 | .9708 | 1.145 | 1.617 | 1.762 |

An acceptable algebraic expression of TAC is:

$0° < \theta < 20°$ TAC = 0,74 (4.2.2.10)

$20° < \theta < 70°$ TAC = 0.0209*θ+0.3220 (4.2.2.11)

Therefore, the clapper torque for forward flow can be obtained using:

$$Tfd = TAC*la*Fdc \qquad (4.2.2.12)$$

with:

la = hinge length

If TAC is assumed invariant with the valve size, the clapper torque can be predicted for any valve size, clapper position and flow condition. These assumptions are likely, but should be proved by experimentation.

The static clapper torque formula can also be used in dynamic conditions, introducing a Relative Mass Flow:

$$\dot{m}_r = \rho_l * A * (V_l - V_{cl} * \cos\theta) \qquad (4.2.2.13)$$

with the clapper closing velocity $V_{cl}$ calculated at its center.

### 4.2.3 REVERSE PRESSURE DROP.

Using the same notations of subparagraph 4.2.1 and Fig. F 4.2.1.1, the elementary Coefficients of Resistance for a reversed flow are:

$$K_1 re = \frac{(1-(d_2/d_1)^2)^2}{(d_2/d_1)^4} \qquad (4.2.3.1)$$

$$K_2 re = \frac{(1-(d_{s1}/d_2)^2)^2 *(d_1/d_2)^4}{(d_{s1}/d_2)^4} \qquad (4.2.3.2)$$

$$K_2 rc = \frac{0.5*(1-(d_2/d_{s1})^2)*(d_1/d_{s1})^4}{(d_2/d_{s1})^4} \qquad (4.2.3.3)$$

$$K_3 re = \frac{(1-(d_F/d_{s1})^2)^2*(d_{s1}/d_1)^4}{(d_F/d_{s1})^4} \qquad (4.2.3.3)$$

$$K_4 rc = \frac{1-(d_F/d_1)^2}{(d_F/d_1)^4} \qquad (4.2.3.4)$$

with:

$$\frac{d_i}{d_j} < 1$$

$$d_{si} < d_i$$

and the total pressure drop:

$$\Delta p = \sum_{i}^{5} \Delta p_i = \frac{\dot{m}^2}{2*A_1^2*\rho_1} * \sum_{i}^{5} \left( \frac{K_i r}{\frac{\rho_i}{\rho_1}*Y+1} \right) \qquad (4.2.3.5)$$

Comparison with experimental results for water

(Ydi = 1) and a 4" size valve suggests a validity of the Orifice Sequence Model, for reversed flow, for:

$$10^\circ < \theta < 70^\circ$$

For clapper angles under 10 deg., the Coefficient of Resistance may be calculated with:

$$K = \frac{10*K(10)}{\theta\ (deg.)} \qquad (4.2.3.6)$$

Table T 4.2.3.1 compares calculated and experimental results.

TABLE T 4.2.3.1 Reverse Coefficient of Resistance. Comparison of Theoretical and Experimental Results for 4" Valve.

| | CLAPPER ANGLES (DEG.) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 30 | 45 | 60 | 69 |
| K re | | | .3127 | .3127 | .3127 | .3127 | .3127 |
| K re | | 2.748 | .7661 | .0104 | | | |
| K rc | 8.1256 | | | | | | |
| K re | | 7.93 | 9.53 | 6.96 | 3.33 | 0.84 | 0.32 |
| K re | | 40.96 | 34.80 | 20.70 | 8.49 | 2.61 | 1.22 |
| Kr | 51.76 | 41.78 | 35.55 | 21.45 | 9.243 | 3.360 | 1.966 |
| Krcl | 51.01 | 40.95 | 34.80 | 20.70 | 8.492 | 2.612 | 1.218 |
| Krclex | 44 | 33 | 31 | 22.5 | 8.40 | 2.60 | 1.11 |
| ΔKrcl | +6.6 | +7.9 | +3.8 | -1.8 | +0.09 | +0.01 | +0.11 |
| ΔKrcl% | +14.8 | +23.9 | +12.0 | -8.0 | +1.1 | +.5 | +9.7 |

## 4.2.4 REVERSE FLOW TORQUE.

The net thrust, produced by the reversed stream on an elementary contraction or expansion, is:

$$Fri = \frac{\dot{m} * Kri}{2 * A_i * a_6} * \left(\frac{1}{Y^2 ri} - \frac{1}{Kri} * \left(\frac{a_{i+1} - a_i}{a_i}\right)\right) \qquad (4.2.4.1)$$

with:

$$Y^2 ri = \frac{1}{\left(\frac{2}{Kri} * \left(\frac{a_{i+1}}{a_i} - 1\right) + 0.5 * \left(1 + \frac{a_{i+1}}{a_i}\right)\right)} \qquad (4.2.4.2)$$

$$H^2 ri = \frac{1}{Y^2 ri} - \frac{1}{Kri} * \frac{(a_{i+1} - a_i)}{a_i} \qquad (4.2.4.3)$$

and, introducing the Orifice Sequence Model, the total thrust is:

$$Fr = Fr1 + Fr2 + Fr3 + Fr4 + Fr5 \qquad (4.2.4.4)$$

$$Fcl = Fr - Fro \qquad (4.2.4.5)$$

with Fro the reverse drag force of the seat alone.

An Equivalent Torque Arm (ETA) and a Torque Arm Coefficient (TAC) for reverse flow can be calculated (Table T 4.2.4.1) and an algebraic expression of TAC defined.

An acceptable algebraic expression of TAC is:

TABLE T 4.2.4.1 Equivalent Torque Arm and Torque Arm Coefficient for a 4" Valve.

| | CLAPPER ANGLES (DEG.) | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 15 | 20 | 30 | 45 | 60 | 69 |
| Q (m3/s) | .0015 | | .0015 | .0015 | .0288 | .0288 | .0288 |
| Exp. T (N*m) | 13.15 | | 9.5 | 7 | 21 | 12.2 | 8 |
| Frcl (N) | 409 | | 273 | 166 | 405 | 125 | 58 |
| Hinge le. (m) | .0548 | .0548 | .0548 | .0548 | .0548 | .0548 | .0548 |
| ETA (m) | .0330 | | .0348 | .0422 | .0519 | .0976 | .1379 |
| TAC | .6111 | | .6444 | .7851 | .9611 | 1.781 | 2.554 |

$$0^\circ < \theta < 20^\circ \qquad TAC = 0.65 \qquad (4.2.4.6)$$

$$20^\circ < \theta < 45^\circ \qquad TAC = 0.0117*\theta+0.4160 \qquad (4.2.4.7)$$

$$45^\circ < \theta < 70^\circ \qquad TAC = 0.0583*\theta-1.6820 \qquad (4.2.4.8)$$

and the clapper torque for static flow can be calculated with:

$$Tfr = TAC*la*Frc \qquad (4.2.4.9)$$

## 4.3 DYNAMIC MODEL

### 4.3.1 CIRCUIT REPRESENTATION.

While the simple clapper arm system is a pendulum oscillator, the addition of a counterweight, springs and a dashpot leads to the circuit representation shown in Fig. F 4.3.1.1, with:

| | | |
|---|---|---|
| $T$ = | torque | (through variable) |
| $\omega$ = | angular velocity | (across variable) |

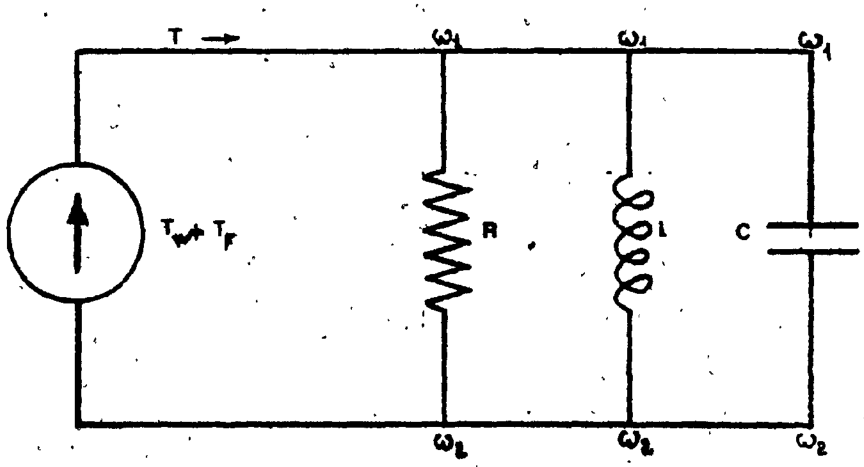



FIG. F 4.3.1.1 Circuit Representation

The Capacitor represents the total system inertia, clapper, hinge, counterweights and all other moving parts; in many applications only the inertia of clapper, hinge and counterweigts are taken into consideration.

The Inductance represents the springs effect and the elasticity of all mechanical components; only spring effect is usually taken into consideration.

The Resistor represents the damping effect of the clapper moving into the fluid and of dashpots. For gaseous fluids the disc damping effect may be neglected.

The torque source represents the combined effect of all weight and flow forces.

The generic second order differential equation of the valve physical system is (Fig. F 4.3.1.1):

$$C * \left(\frac{d\omega}{dt}\right) + \frac{1}{R} * + \int_0^t \frac{\omega}{I} * dt = T_W + T_F \qquad (4.3.1.1)$$

or:

$$C * \left(\frac{d^2\theta}{dt^2}\right) + \frac{1}{R_j} * \left(\frac{d\theta}{dt}\right) + \int_0^t \left(\frac{1}{I} * \left(\frac{d\theta}{dt}\right)\right) * dt = T_W + T_F \qquad (4.3.1.2)$$

with:

$\omega = \frac{d0}{dt}$

$T_W$ = combined weight torque

$T_F$ = flow torque

### 4.3.2 COUNTERWEIGHTS.

A counterweight is an auxiliary, external device, used to modify the static and dynamic characteristics of the clapper-hinge system. It may be used:

: to reduce the weight torque in open position,

: to modify the weight torque in closed position,

: to increase the system inertia.

The counterweight basic design rule is that the total weight torque must be on the clapper closing direction for all clapper positions.

The main disadvantages of a counterweight application are;

: a requirement of rigid connection between clapper and counterweight,

: the possibility of leakage where the rotating pivot shaft exits the valve body.

The counterweight dynamic equation is:

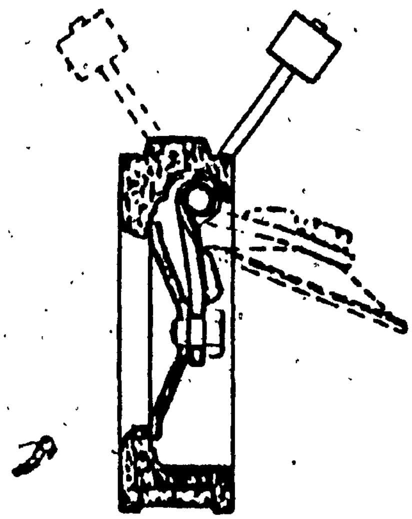


FIG. F 4.3.2.1 Counterweigt.

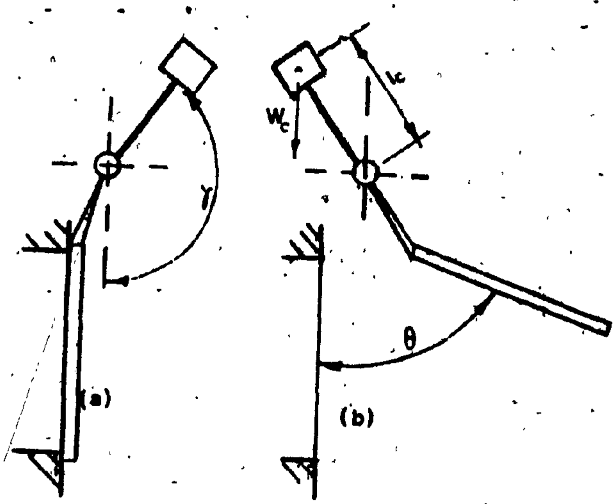



FIG. F 4.3.2.2 Kinematic.
a) valve closed
b) valve open

$$W_c * l_c^2 * \left( \frac{d^2\theta}{dt^2} \right) = W_c * l_c * \sin(\gamma + \theta) \qquad (4.3.2.1)$$

4.3.3 SPRINGS.

Adding springs (Fig. F 4.3.3) is the most common way of reducing the check valve closing time. Springs are also used to increase, or to generate, a closing torque, when the clapper is in near-closed position.

Springs, used in check valves, are usually of two types:

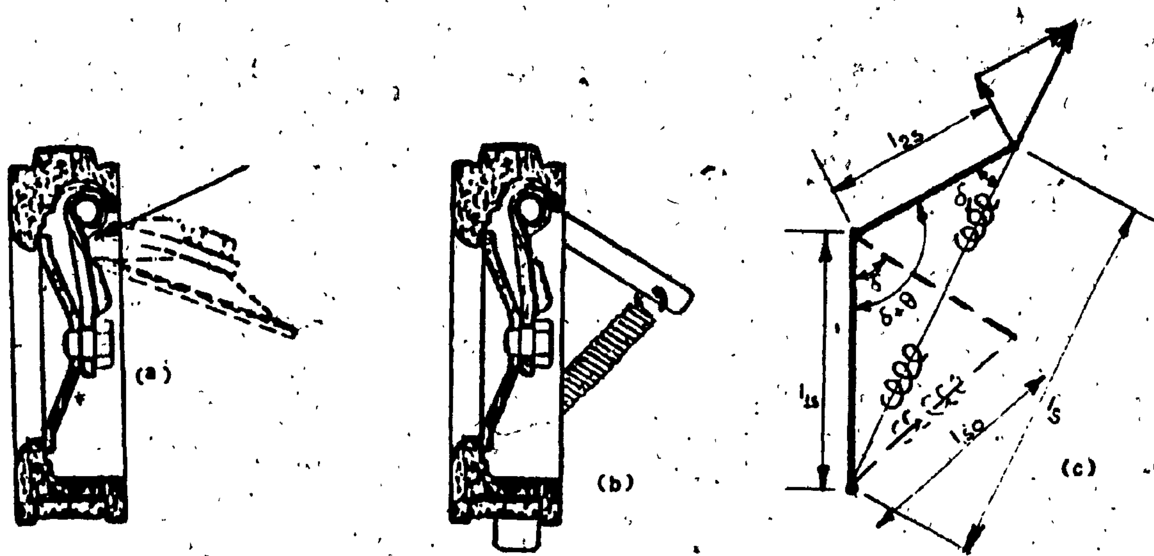



FIG F 4.3.3.1 Springs.

a) cylindrical helical bending spring.

b) cylindrical helical torsional spring.

c) kinematic of torsional spring.

: Cylindrical helical springs subject to bending.

: Cylindrical helical springs subject to torsion.

Helical springs subject to bending do not require rigid pivot connections, or external devices, yet cannot be changed without removing the clapper. The bending spring contribution to the valve differential equation is:

$$KS^*(\theta-\theta_0) = T_S \qquad (4.3.3.1)$$

with:

$KS$ = spring constant

$\theta_0$ = spring angular preload

$T_s$ = spring torque

Helical springs subject to torsion have all the counterweight disadvantages, but they can be easily inspected and changed. The torsional spring contribution to the valve differential equations (Fig. F 4.3.1) is:

$$KS*(l_s - l_{so})*l_{2s}*\sin\delta_1 = T_s \qquad (4.3.3.2)$$

with:

$$l_s = (l_{1s}^2 + l_{2s}^2 - 2*l_{1s}*l_{2s}*\cos(\theta+\delta))^{1/2}$$
$$\delta = \cos^{-1}\frac{(l_{2s}^2 + l_2^2 - l_{1s}^2)}{(2*l_{1s}*l_{2s})}$$

and

$KS$ = spring constant

$l_{so}$ = spring free length

4.3.4 DASHPOT.

A dashpot is a device used to damp the clapper system. In principle, the dashpot is a hydraulic, double effect, cylinder with upper and bottom chambers connected by a hydraulic circuit of variable Coefficient of Resistance.

An extensive dynamic analysis of high pressure dampers has been performed by LEE (7). A single effect, low pressure, damper (Fig. F 4.3.4.1) is discussed here.

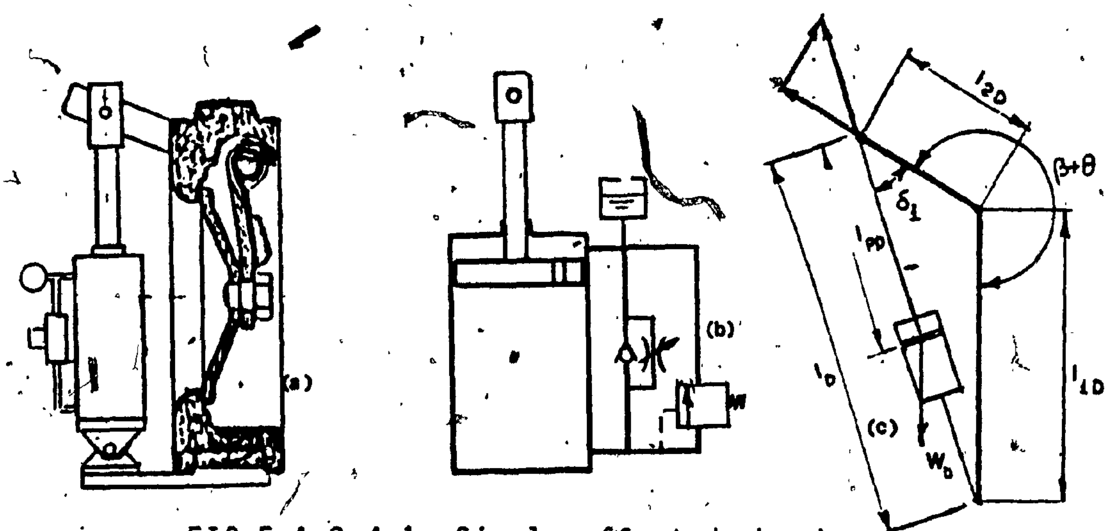



FIG F 4.3.4.1 Single effect dashpot.
a) General assembly.
b) Single effect circuit.
c) Kinematic.

In many low pressure gas lines a damper can be used to decrease the amplitude of the disc oscillations under reduced flow conditions.

While the valve opening time is relatively long, depending in most cases on the compressor start-up characteristic, the required valve closing time may be very short, depending not only on the compressor shut down characteristic, but also on the distance between valve and conceptual reservoir (manifold).

An attempt to slow the clapper, when it closes, may easily generate unacceptable torsion stresses in the valve pivot shaft. Therefore, it is always advisable to have the damping effect only on the disc opening direction. The single effect damper of Fig. F 4.3.4.1 is designed accordingly.

Neglecting all Coulomb friction forces and the low viscous forces generated when the valve opens, the dashpot differential equation is:

$$M_{cv}*\frac{dv}{dt}+B*v = F_D + F_{WD} \qquad (4.3.4.1.)$$

with:

$$B*v = F_d \quad \text{with} \quad B = 0 \text{ for } v < 0$$

$$F_d = p_l*A_{cY} - p_a*(A_{cY} - A_R)$$

$$p_l = p_a + \frac{Q^2*\rho_o*K_o}{2*A_o}$$

$$F_d = p_a*A_R + \frac{v^2*\rho_o*K_o}{2}*\left(\frac{A_{cY}}{A_o}\right)*A_{cY}$$

$$B = \frac{p_a*A_R}{v} + \frac{v*\rho_o*K_o}{2}*\left(\frac{A_{cY}}{A_o}\right)*A_{cY}$$

and:

$M_{cY}$ = cylinder mass

$B_l$ = cylinder viscous damping coefficient

$v$ = piston linear velocity

$F_d$ = damper reaction

$F_W$ = damper weight force component

$p_l$ = lower chamber pressure

$p_a$ = atmospheric pressure

$Q$ = oil flow rate through hydraulic connection

$A_{cY}$ = cylinder area

$A_o$ = hydraulic piping area

$A_R$ = rod section area

$K_o$ = hydraulic system Coefficient of Resistance

$\rho_o$ = oil density

The damper length is (Fig. F 4.3.4.1):

$$l_d = (l_{1d}^2 + l_{2d}^2 - 2*l_{1d}*l_{2d}*\cos(\beta+\theta))^{1/2}$$

and the piston velocity:

$$v = \frac{\omega * l_{2d} * \sin\delta_1 * \sin(\beta+\theta)}{|\sin(\beta+\theta)|}$$

with:

$$\delta_1 = \cos^{-1}\frac{(l_d^2 + l_{2d}^2 + l_{1d}^2)}{2*l_d*l_{2d}}$$

The damper torque is:

$$T_{D\omega} = F_d * l_{2d} * \sin\delta_1 \qquad (4.3.4.2)$$

and the weight torque:

$$T_{DW} = W_D*(l_{2d}*\sin(\beta+\theta) - l_{pd}*\sin(\beta+\theta+\delta_1)) \qquad (4.3.4.3)$$

The dashpot differential equation, in terms of torque, becomes:

$$M_{cy}*l_{2d}*\sin\delta_1*\frac{dv}{dt} - B*l_{2d}*\sin\delta_1*v = T_D + T_{DW} \qquad (4.3.4.4)$$

## 4.3.5 VALVE DIFFERENTIAL EQUATION.

The wafer check valve differential equation (4.3.1.1) may be rewritten as:

$$T_J + T_c + T_S + T_D = T_{VW} + T_{cW} + T_{DW} + T_F \qquad (4.3.5.1)$$

with:

$T_J$ = valve inertial torque
$T_C$ = ctwght. inertial torque
$T_S$ = spring torque
$T_D$ = dashpot torque
$T_{VW}$ = clapper weight torque
$T_{cW}$ = ctwght weight torque
$T_{DW}$ = damper weight torque
$T_F$ = flow torque

or:

$$T_J = J_V * \frac{d^2\theta}{dt^2} \qquad (4.5.5.2)$$

$$T_c = W_c * l_c * \frac{d^2\theta}{dt^2} \qquad (4.5.5.3)$$

$$T_S = KS*(\theta+\theta_o) \qquad (4.3.3.1)$$

$$T_{cW} = W_c * l_c * \sin(\gamma+\theta) \qquad (4.3.2.1)$$

$$T_{VW} = W_V l_a * \cos\theta \qquad (4.5.5.4)$$

$$T_F = (TAC) * l_a * F_{DC} \qquad (4.2.2.12)\ (4.2.4.8)$$

$$T_{DW} = W_D * (l_{ad} * \sin(\beta+\theta) - l_{pd} * \sin(\beta+\theta+\delta_1)) \qquad (4.3.4.11)$$

$$T_D = M_{cY} * l_{ad} * \sin\delta_1 * \frac{d^2\theta}{dt^2} + B * l_{ad} * \sin\delta_1 * \frac{d\theta}{dt} \qquad (4.3.4.10)$$

and:

$$\theta = \theta(t)$$

as only variable.

## 4.4 CONCLUSIONS

The valve dynamics can be represented with a set of ordinary differential equations, which can be solved (Chap. 5) using numerical techniques.

Many assumptions have been made, since the validity of the model was experimentaly verified for only one valve size and one fluid, water.

While the extension to compressible flows seems acceptable, at least for moderate pressure drops (14), the validity of the models for large sizes (over 12" diameter)is still to be proven.

Numerical results, obtained from model applications to large sizes (over 16"), should be considered more a qualitative than a quantitative description of facts.

# CHAPTER 5

# NUMERICAL MODEL AND APPLICATION

## 5.1 SUMMARY

In this chapter blocks "Valve Opening" and "Oscillations" of Fig. F 2.3.2.1 are investigated.

An experimental investigation of the valve dynamic behaviour would be very expensinve and, in the case of large valves, both difficult and time consuming. A numerical model is a valuable substitute. The complete numerical model consists of the combination of a valve dynamics model with a model of the piping system in unsteady flow conditions. While the piping system model will be investigated in Part 2, in this Chapter the valve numerical model is developed. The RUNGE KUTTA method is used to solve the second order differential equation, representing a general valve system operating under unsteady conditions. The resulting computer program, DYNA, is presented and discussed.

A field case (courtesy of RITEPRO Inc.) is used to develop a dynamic analysis procedure. A "Valve Specification Form" is proposed to rate a check valve through its energy efficiency and dynamic characteristics.

For the field case, three compatible solutions are defined and a cost supported decision made.

## 5.2 NUMERICAL MODEL

### 5.2.1 INTERACTION WITH THE PIPING.

The dynamic behaviour of a check valve is influenced by and interacts with the piping system flow parameters. As will be seen in Part 2, the unsteady flow through a piping system can be numerically described and the flow parameters known at successive time levels. At each time level, the flow parameters are known at any point of the piping system and in particular upstream and downstream of the check valve.

Approaching the numerical solution of the valve-piping combined system, a time level "n", where all the flow parameters are known and already influenced by the valve dynamic behaviour at time level "n-1" is considered. The valve dynamic behaviour and its Equivalent Length, at time level "n", can be numerically calculated and introduced into the piping model, to influence the flow parameters at time level "n+1".

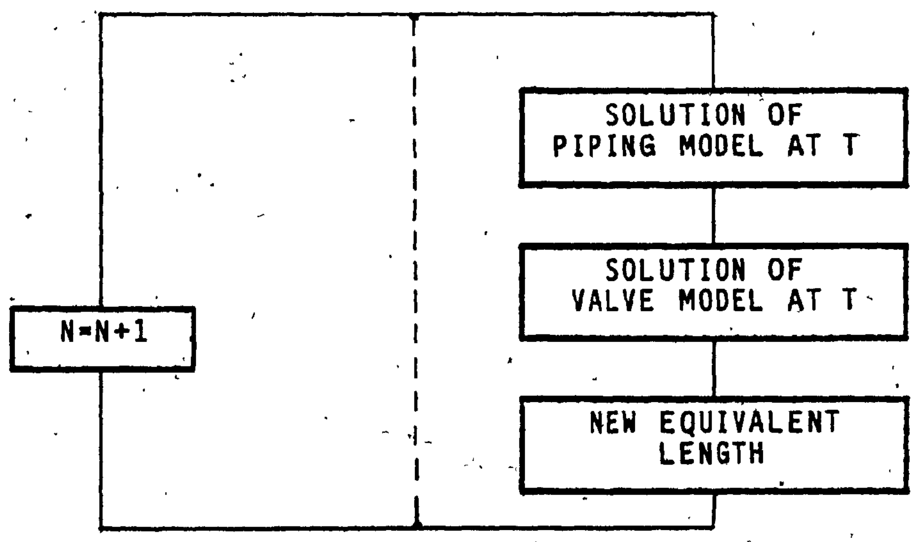



FIG. F 5.2.1.1 Valve Piping Interactive Solution.

Introducing the valve Equivalent Length into the pipe model means representing the valve with a pipe element. While unsteady flow conditions exist and are considered along all the piping system, this additional element is considered in steady state at each time level.

The Equivalent Length represents the pressure loss through the valve or the valve resistance, but does not represent the valve capacitance and inductance. Since both capacitance and inductance are affected by the valve effective length, neglecting their influence is an acceptable approximation.

Steady flow through the valve also means neglecting the time variation of the fluid density, or accounting for it in the Drag Compressibility Coefficient of subparagraph 4.2.2.

A rigorous iterative solution (Fig. F 5.21.1 dotted line) requires plugging the new eqivalent length into the piping model at time level "n" and iterating to convergence.

For small values of time increment between two consecutive time levels, the faster method (Fig. F 5.21.1 continuous line) can be used.

In order to compare different valve specifications, it is necessary to establish Standard Testing Conditions. The most logical choice, although unrealistic, is to ignore the influence of the valve on the piping, i.e. to assume the flow parameters independently of the clapper position and of valve and piping system interaction.

5.2.2 SONIC CONDITIONS.

In subparagraph 5.2.1 it has been seen that the pressure drop across a check valve can be represented by that of an Equivalent Length of pipe. Therefore, the check valve may be introduced into the piping system as an additional pipe segment of variable length. The only peculiarity of this pipe segment is that the time derivative terms of the Navier-Stoke equations will be neglected.

Along a pipe segment the three governing equations for a steady, unidimensional, adiabatic flow (Fanno flow) are (15):

$$\dot{m} = \text{const.} \qquad \text{(continuity)} \qquad (3.2.2.1\ )$$

$$p_1 - p_2 = \frac{K*\dot{m}^2}{2*\varrho_1*A_1*Y^2} \qquad \text{(momentum)} \qquad (3.2.2.10)$$

$$\frac{p}{\varrho}+\frac{(\gamma-1)}{2*\gamma}*\left(\frac{\dot{m}^2}{\varrho^2*A^2}\right) = \text{const.} \qquad \text{(energy)} \qquad (5.2.2.1\ )$$

An originally subsonic Fanno flow, with proper pipe exit pressure conditions, will increase in velocity until reaching sonic conditions. When a valve is taken in consideration, a large Equivalent Pipe Length, i.e. a substantial pressure drop, is required to reach sonic conditions. The Orifice Sequence Model (Chap. 4) indicates that the valve Coefficient of Resistance is the effect of a sequence of elementary contractions and expansions. A pipe contraction may result in development of local sonic conditions. Sonic conditions are most likely to occur in the case of smaller equivalent orifices. The global Coefficient of Resistance is affected not only by contractions but also by successive expansion. Therefore, a contraction sonic condition will most probably occur before reaching the Equivalent Length required for Fanno flow critical conditions.

When sonic conditions occur in an orifice, it is:

$$m = m_c \qquad (5.2.2.2)$$

$$\frac{p_1}{\rho_1} + \left(\frac{\gamma - 1}{2*\gamma}\right)*\left(\frac{\dot{m}_c^2}{\rho_1^2 * A_1^2}\right) = \frac{p_c}{\rho_c} + \left(\frac{\gamma - 1}{2*\gamma}\right)*\left(\frac{\dot{m}_c^2}{\rho_c^2 * A_c^2}\right) \qquad (5.2.2.3)$$

$$\frac{p_c}{\rho_c^{\gamma}} = \frac{p_s}{\rho_s^{\gamma}} \qquad (5.2.2.4)$$

with:

$A_c$ = minimum orifice area

$p_s$ = orifice upstream stagnation pressure

$\rho_s$ = orifice upstream stagnation density

5.2.3 THE RUNGE KUTTA NUMERICAL SOLUTION.

The solution of a second order ordinary differential equation, with known initial conditions, is obtainable with a fourth order RUNGE-KUTTA algorithm (16), reducing the second order equation to a system of two first order equations:

$$u' = \frac{d^2y}{dt^2} = f(t,y,\frac{dy}{dt}) = f(t,y,u)$$

$$y' = \frac{dy}{dt} = u$$

$$y^{(n)} = y^{(n-1)}+(AK11+2*AK21+2*AK31+AK41)/6 \qquad y_{t=0} = y_0$$

$$y'^{(n)} = y'^{(n-1)}+(ak12+2*AK22+2*AK32+AK42)/6 \qquad u_{t=0} = u_0 = \left(\frac{dy}{dt}\right)_{t=0}$$

$$AK11 = \Delta t*(y'^{(n-1)})$$

$$AK12 = \Delta t*(f(t^{(n-1)},y^{(n-1)},y'^{(n-1)}))$$

$$AK21 = \Delta t*(y'((t^{(n-1)}+0.5*\Delta t),(y^{(n-1)}+0.5*AK11),(y'^{(n-1)}+0.5*AK12)))$$

$$AK22 = \Delta t*(f((t^{(n-1)}+0.5*\Delta t),(y^{(n-1)}+0.5*AK11),(y'^{(n-1)}+0.5*AK12)))$$

$$AK31 = \Delta t*(y'((t^{(n-1)}+0.5*\Delta t),(y^{(n-1)}+0.5*AK21),(y'^{(n-1)}+0.5*AK22)))$$

$$AK32 = \Delta t*(f((t^{(n-1)}+0.5*\Delta t),(y^{(n-1)}+0.5*AK21),(y'^{(n-1)}+0.5*AK22)))$$

$$AK41 = \Delta t*(y'((t^{(n-1)}+\Delta t),(y^{(n-1)}+AK31),(y'^{(n-1)}+AK32)))$$

$$AK42 = \Delta t*(f((t^{(n-1)}+\Delta t),(y^{(n-1)}+AK31),(y'^{(n-1)}+AK32)))$$

The overall differential equation of a swing check valve may be written:

$$A*\frac{d^2\theta}{dt^2} = F(t,\theta,\omega)$$

where:

A = system total moment of inertia

F = angular velocity, opening angle, time, depending terms.

θ = clapper opening angle

ω = clapper angular velocity

The system total moment of inertia is the sum of the moments of inertia of all moving parts, clapper, counterweight, damper, etc.

The F terms are not represented by an algebraic equation but are a set of mixed (mathematic and logical) relations describing:

: Operating conditions

: Clapper Coefficient of Resistance

: Valve subsonic (sonic) conditions

: Flow torque

: Counterweight effect

: Spring effect

: Damper effect

: Valve geometric and dynamic parameters

The Runge Kutta fourth order algorithm can still be applied, assuming:

$$u = \omega \qquad y = \theta$$

and knowing:

$$\omega(t=0) = \omega_0 \qquad \theta(t=0) = \theta_0$$

The $AK_{ij}$ coefficients are COMPUTED solving all the mathematical and logic expressions, i.e. the numerical model, for:

$t^{(n-1)}$, $\theta^{(n-1)}$, $\omega^{(n-1)}$

$(t^{(n-1)}+0.5*\Delta t)$, $(\theta^{(n-1)}+0.5*AK11)$, $(\omega^{(n-1)}+0.5*AK12)$

$(t^{(n-1)}+0.5*\Delta t)$, $(\theta^{(n-1)}+0.5*AK12)$, $(\omega^{(n-1)}+0.5*AK22)$

$(t^{(n-1)}+\Delta t)$, $(\theta^{(n-1)}+AK31)$, $(\omega^{(n-1)}+AK32)$

## 5.3 THE COMPUTER PROGRAM "DYNA"

### 5.3.1 FLOW CHART.

A FORTRAN Program, DYNA, has been coded (Appendix 3) to solve the check valve dynamic model. The program is set to be used for valve evaluation under Standard Testing Conditions, but can be modified easily to accept any piping parameter and to be incorporated into a general piping system model. The program DYNA schematic flow chart is shown in Fig. F 5.3.1.1.

### 5.3.2 PROGRAM DESCRIPTION.

Input:

: Operating upstream pressure and density,

: Surrounding pressure and density,

: Unsteady operating parameters:

: Oscillation amplitude

: Oscillation angular velocity

: Valve geometry,

: counterweight geometry, weight and inertia,

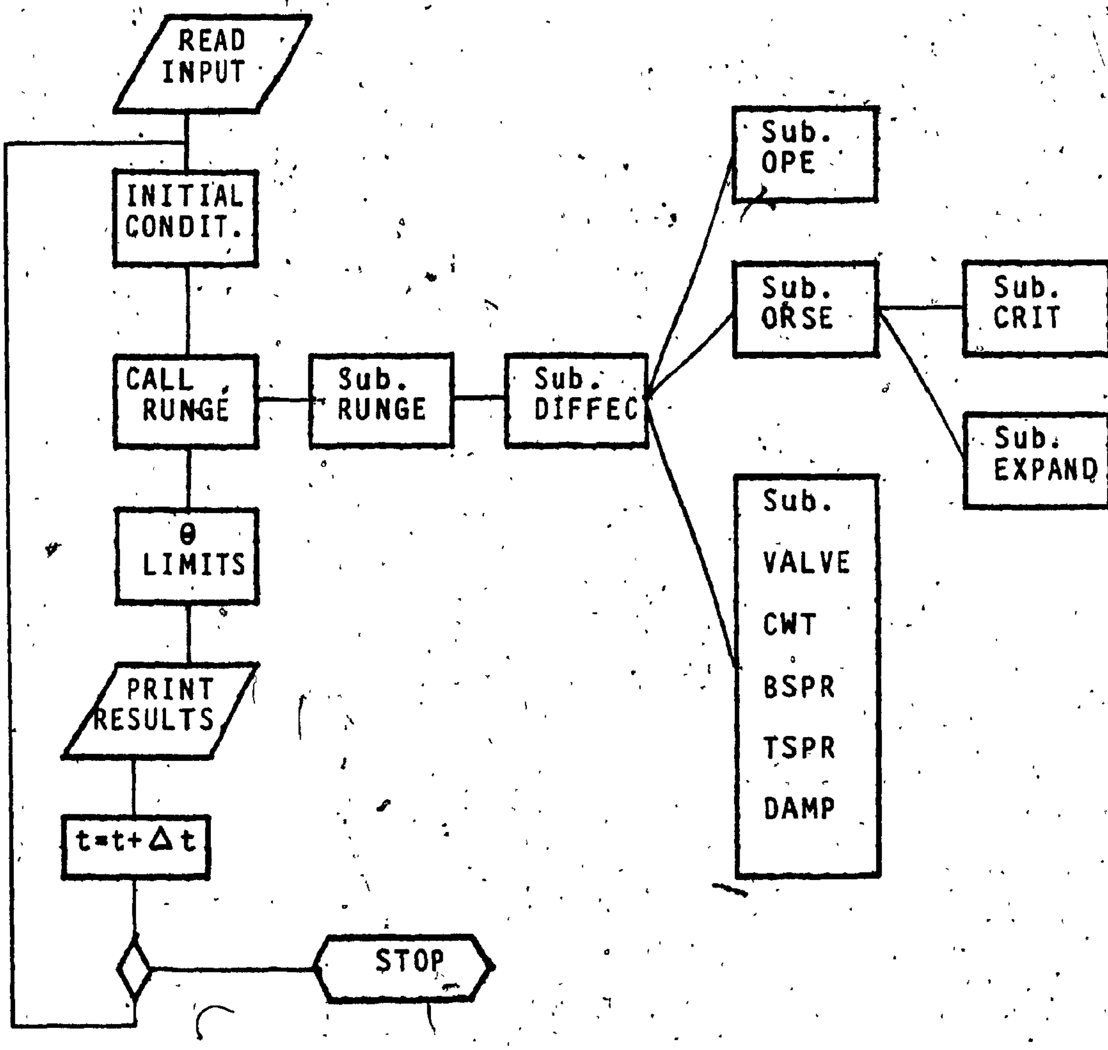



FIG. F 5.3.1.1 Program DYNA

: Bending spring constant and preload,

: Torsional spring geometry and constant,

: Damper geometry and weight,

: Damper oil density,

: Damper hydraulic cyrcuit Coeff. of Resistance,

: Initial value of clapper angle,

: Initial value of clapper angular velocity,

: Time interval,

: Time to reach full operating conditions,

: Maximum time,

: Elasticity of clapper stop,

: Elasticity of seat.

Output:

: Clapper opening angle at each time increment,

: Clapper angular velocity at each time level,

: Elapsed time.

Other parameters like differential pressure, Coefficient of Resistance, flow torque, etc. may be obtained, at each time level, with minor program modifications.

The Main Program, DYNA:

: reads all inputs,

: organizes, for each time level, the calculation of:

: opening angle,

: angular velocity,

using Subroutine RUNGE,

: introduces the upper and lower angular limits,

: sets the "bouncing" value of clapper angular velocity.

Subroutine RUNGE calculates, for each time level, the Runge-Kutta coefficients and computes updated values of clapper opening angle, angular velocity , using Subroutine DIFFEQ.

Subroutine DIFFEQ calculates values of the F function and the system total moment of inertia. The moment of inertia and torque contribution of each system component to the F function are obtained calling:

: Subroutine OPE,

: Subroutine ORSE,

: Subroutine VALVE,

: Subroutine CWT,

: Subroutine BSPR,

: Subroutine TSPR,

: Subroutine DAMP.

These calculations are repeated four times for each time level, each time with values of opening angle, angular velocity and time level modified according to the Runge-Kutta algorithm requirements.

Subroutine OPE calculates upstream pressure, mass flow and density line parameters, according to a predetermined time dependant relation. Subroutine OPE must be eliminated when DYNA is linked to the line unsteady flow simulation. OPE is set to simulate Standard Test Conditions.

Subroutine ORSE calculates:

: flow rate, relative to clapper velocity,

: elementary Coefficints of Resistance,

: elementary and total pressure drop, calling Sub. CRIT,

: seat alone pressure drop,

: clapper flow torque.

When DYNA is linked to the line unsteady flow simulation, the pressure drop becomes for DYNA an input data.

Subroutine VALVE calculates:

: clapper weight torque,

: valve contribution to total inertia.

Subroutine CTW calculates:

: counterweight weight torque,

: counterweight contribution to total inertia.

Subroutine BSPR calculates:

: bending spring torque,

Subroutine TSPR calculates:

: torsional spring torque.

Subroutine DAMP calculates:

: damper viscous torque,

: damper weight torque,

Subroutine CRIT checks whether sonic or subsonic conditions occur. When sonic conditions occur, subroutine CRIT calculates:

: pressure drop,

: upstream pressure and density,

: sonic mass flow (adiabatic flow)

When subsonic conditions occur, CRIT calculates the pressure drop, calling Subroutine EXPAND.

Subroutine EXPAND calculates, by iteration:

: pressure drop,

: net expansion factor,

: drag compressibility coefficient.

for each elementary component of the Orifice Sequence Model.

## 5.4 FIELD CASE

### 5.4.1 OBJECTIVE.

Using Standard Testing Conditions, Program COMPR and Program DYNA can be used to set dynamic specifications of a required check valve, or to compare different engineering alternatives.

The Standard Testing Conditions limit the physical meaning of the procedure to cases where the line-valve interaction is not important. In terms of Cost Function, it means neglecting the valve influence on costs "external to valve (EC) (subparagraph 2.3.3). This assumption is always acceptable for low pressure piping systems and valid for valve rating.

The procedure for using Programs COMPR and DYNA as support for an engineering analysis, will be demonstrated using a field case.

5.4.2 THE FIELD PROBLEM.

Two turbo-blowers feed a pipe system with air. A check valve is installed downstream of each turbo-blower and the line normal operating conditions are:

$p$ = 21.65 psia (149255 Pa)

$\varrho$ = 1.427 Kg/m3

$m$ = 840 lb/min (6.35 Kg/s)

$T$ = 197 F (364.8 K )

A minimum flow rate operating condition is also given:

$p$ = 149255 Pa

$\varrho$ = 1.427 Kg/m3

$m$ = 2.6 Kg/s

The line geometric dimensions are:

$D$ = 18" pipe nominal diameter,

$D$ = 0.431 pipe inside diameter.

It must be decided whether a CHECK RITE wafer check valve (Fig. F 3.3.3.1) is suitable for the application. The valve geometric dimensions are:

$D_1$ = 0.4310 m

$D_2$ = 0.3302 m

DCL = 0.3715 m

$X_e$ = 0.0381 m

$D_e$ = 0.0125 m

LA = 0.2190 m

Other valve characteristics are:

| | | | | |
|---|---|---|---|---|
| WV | = | 129 | N | disc weight |
| JV | = | 0.77 | Kg*m2 | disc-hinge mom. of inertia |
| GY | = | 0.0 | m | Y coord. of center of grav. |
| GX | = | 0.014 | m | X coord. of center of grav. |
| θ | = | 60 | deg. | maximum opening angle. |

## 5.4.3 DYNAMIC ANALISYS OF STD. VALVE.

First, the program DYNA is used to investigate the standard valve behaviour, under the following operating conditions:

Condition A:

p = 149255 Pa

ρ = 1.427 Kg/m3

$\dot{m}$ 6.4 Kg/s

Interval A:

: steady operating conditions,

: initial opening angle: 10 .

Interval B:

: sinusoidal mass flow variation,

Amplitude = 0.2*$\dot{m}$

Frequency =6.28 rad/s

Interval C:

: steady operating conditions

Interval D:

: zero mass flow.

Condition B:

as Condition A but with m = 2.6 Kg/s

Diagram D 5.4.3.1 (lines 1,2) shows the time variation of the clapper opening angle. Excessive oscillations for low flow rate may be observed: an increase of the valve stability at low flow rates is required.

With the same operating conditions , the clapper opening is now limited to 45 deg.. Diagram D 5.4.3.2 (lines 1,2) shows the clapper angle variation. The valve is stable at full operating conditions, while damped oscillations occur at minimum flow rate.

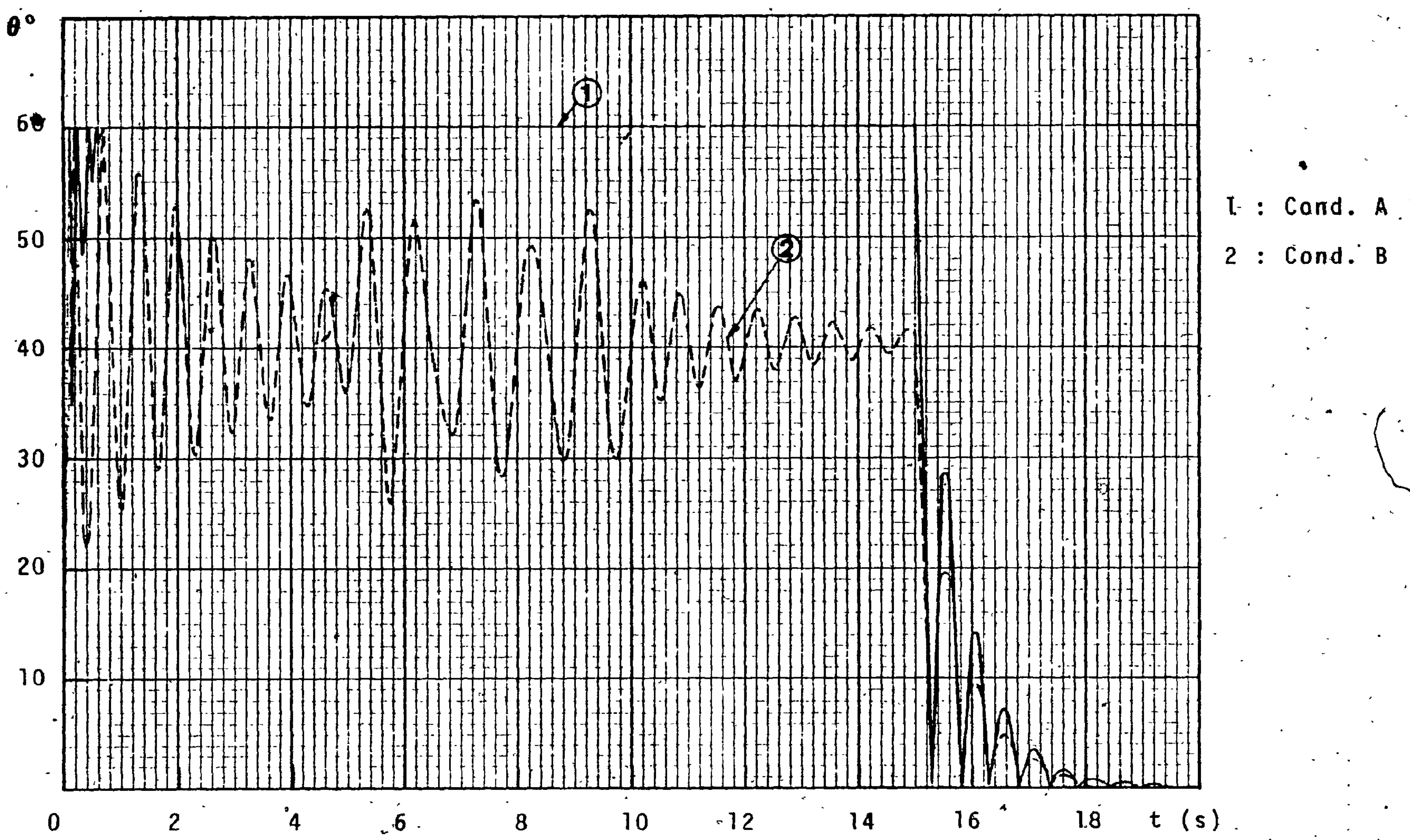



Diagram D 5.4.3.1 18" Valve. Clapper Oscillation. Max. Angle 60°.


Diagram D 5.4/3.2 18" Valve. Clapper Oscillation. Max. Angle 46°.

5.4.4 MODIFICATIONS.

The dynamic diagrams of the standard valve show that, limiting the opening angle to 45 deg., the clapper still oscillates at minimum flow. To increase stability, a further reduction of the maximum opening angle could be made, as the pressure drop would still be negligible, but this is not strictly necessary.

Therefore, a standard CHECK RITE wafer valve with reduced opening angle ($\theta$ = 45 deg) is an acceptable solution.

Nevertheless, a better system stability may be desirable for flow rates lower than the minimum specified.

System stability at low flow rates with the clapper in a semiclosed position may be increased adding the single acting dashpot of subparagraph 4.3.2, with damping effect limited to opening motion. A weak spring and a counterweight are also added, the spring to reduce the balancing effects of the damper weight and Coulomb friction, the counterweight to help the valve opening (Fig. F 5.4.4.1). With the new conditions, the clapper is stable to approximatly 50% of the maximum operating flow, while, at minimum flow (40%), the oscillations have a lower frequency and are damped more effectively (Diagr. D 5.4.4.1). A significant reduction of the disc maximum angular velocity is also obtained.

The auxiliary equipment parameters are:

Dashpot:

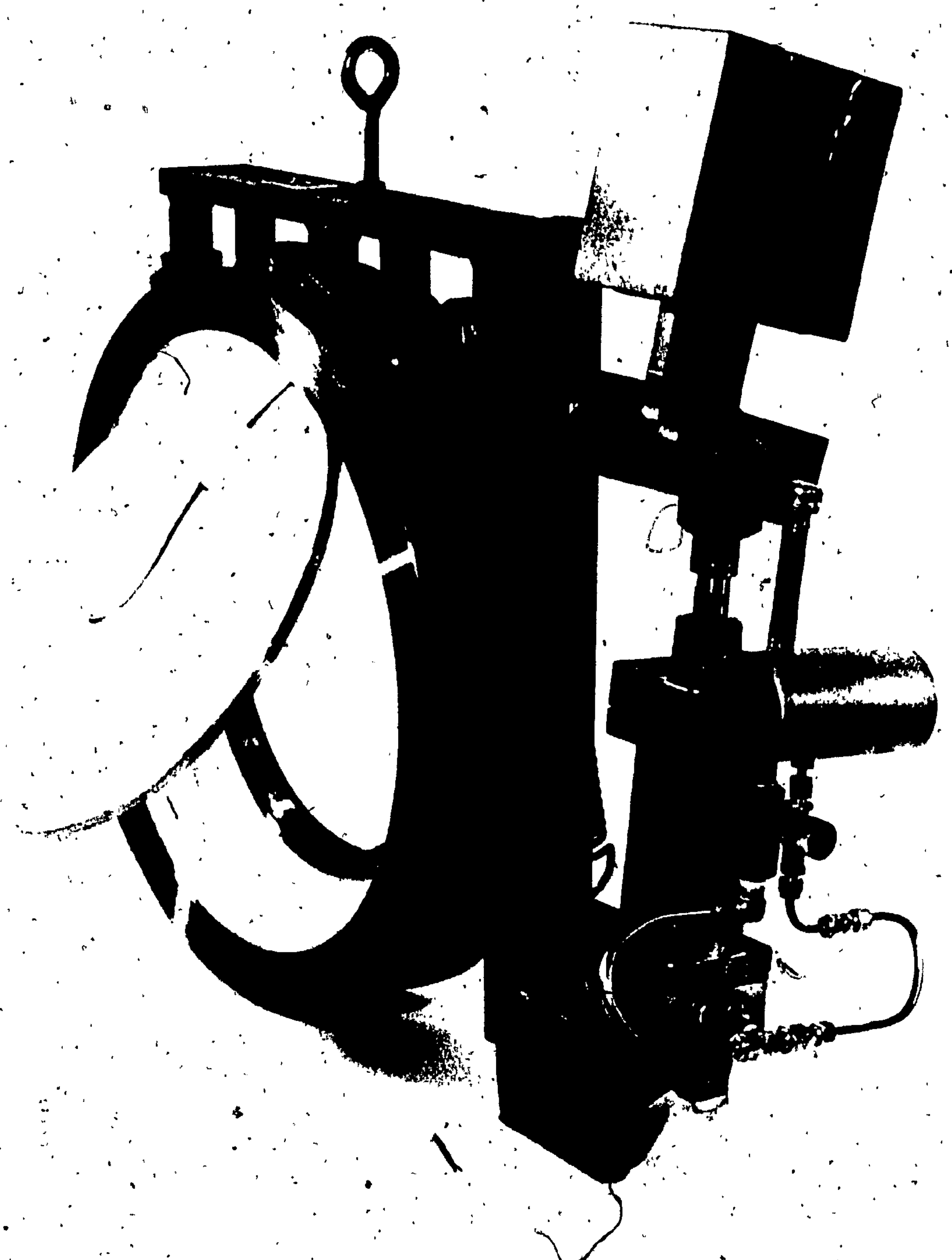
| | | | | |
|---|---|---|---|---|
| DCY | = | 0.080 | m | cylinder inside diameter. |
| DRO | = | 0.025 | m | rod diameter. |
| AMD | = | 3 | Kg | piston-rod mass |
| WD | = | 30 | N | piston-rod weight. |
| AL1D | = | .418 | m | damper fixed arm. |
| AL2D | = | .102 | m | damper mobile arm. |
| ALPD | = | .320 | m | rod length. |
| ABETA | = | 112.5 | | arms angle (closed pos.). |
| Koo | = | 0.4 | | Coeff. of Resistance. |
| DOV | = | 0.005 | m | hydr. piping dia. |
| DENOIL | = | 900 | Kg/m3 | oil density. |

Counterweight:

| | | | | |
|---|---|---|---|---|
| WC | = | 112 | N | weight. |
| ALC | = | .320 | m | arm. |
| AGAMMA | = | 157.5 | | angle in closed position. |

Torsional spring:

| | | | | |
|---|---|---|---|---|
| AKTS | = | 1383 | N/m | spring factor |
| AL1S | = | .418 | m | spring fixed arm |
| AL2S | = | .076 | m | spring mobile arm. |
| ALSO | = | .320 | m | spring freelength. |
| ADELTA | = | 67.5 | ° | arms angle (closed pos.). |



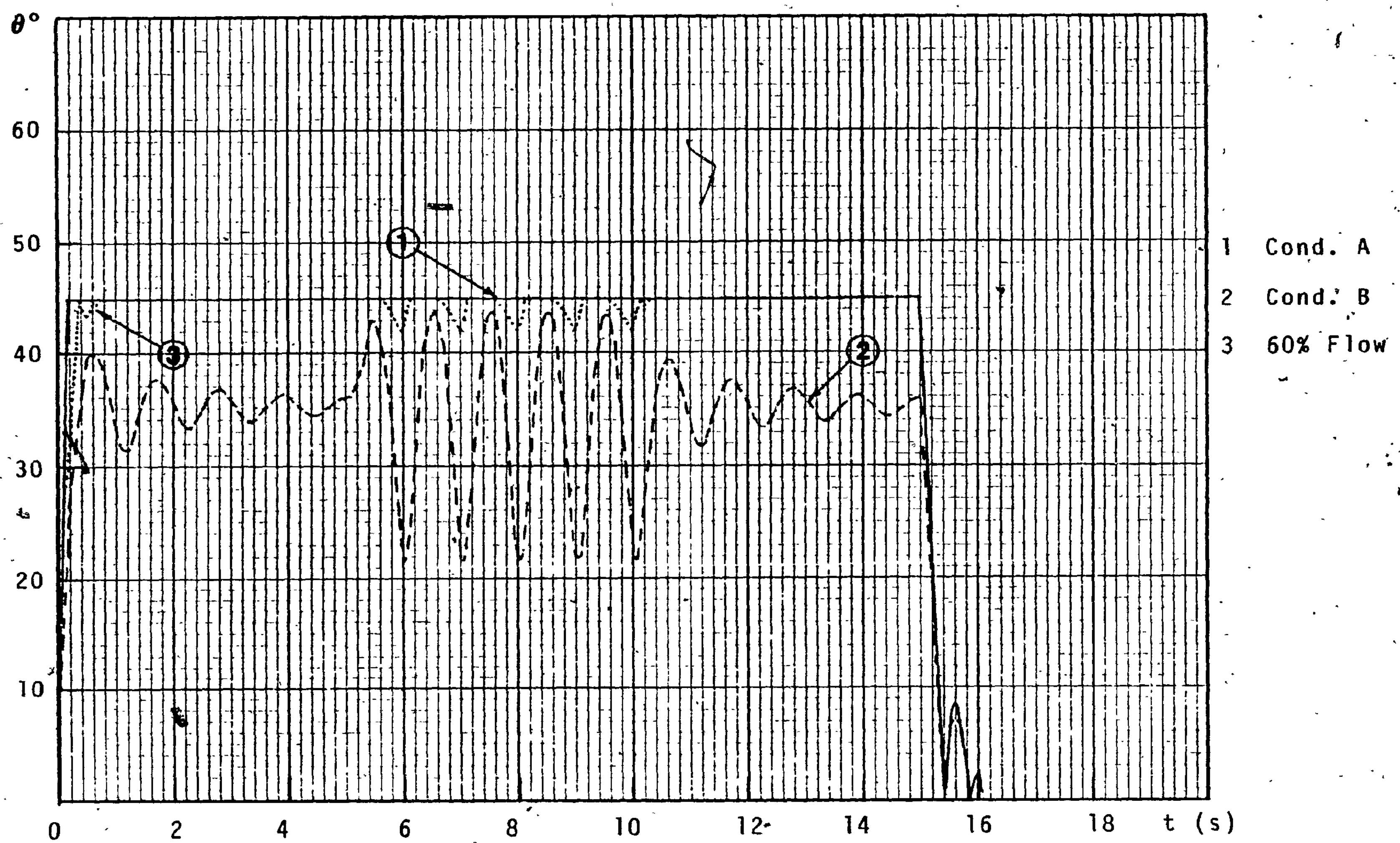



Diagram D 5.4.4.1 18" Valve Clapper Damped Oscillations. Max. Angle 45°.

5.4.5 VALVE SPECIFICATIONS.

Using Programs DYNA and COMPR, the valve dynamic specifications have been defined (Table T 5.45.1 end of chapter). These specifications include:

: TYPE OF VALVE.
: PIPING SYSTEM OPERATING CONDITIONS.
: STANDARD TEST CONDITIONS OF THE NUMERICAL MODEL.
: SIMULATED FLOW RATE VALUES.

: VALVE ENGINEERING ALTERNATIVES, SUCH AS:
: STANDARD VALVE,
: STANDARD VALVE WITH LIMITED CLAPPER ANGLE,
: VALVE WITH ADDITION OF AUXILIARY DEVICES.

: VALVE AND AUXILIARY DEVICES BASIC DIMENSIONS.

: ENERGY EFFICIENCY PARAMETERS AT OPER. COND., AS:
: EQUIVALENT PIPE LENGTH,
: PRESSURE LOSSES,
: AVAILABILITY DESTRUCTION.

: DYNAMIC BEHAVIOUR AT OPERATING CONDITIONS, SUCH AS:
: CLAPPER OPEN ANGLE,
: AMPLITUDE OF CLAPPER OSCILLATION,
: FREQUENCY OF CLAPPER OSCILLATION,

: VALVE CLOSING TIME.

5.4.6 AN ALTERNATIVE SOLUTION.

For the field case, the piping system was already designed and the valve diameter was fixed. Otherwise, it would have been advisable to investigate whether an 18" valve was the most convenient solution.

Programs COMPR and DYNA have been used to investigate an alternative with a 14" valve and reducers (Fig. F 5.4.6.1), giving proper consideration to the reducers additional Coefficient of Resistance.

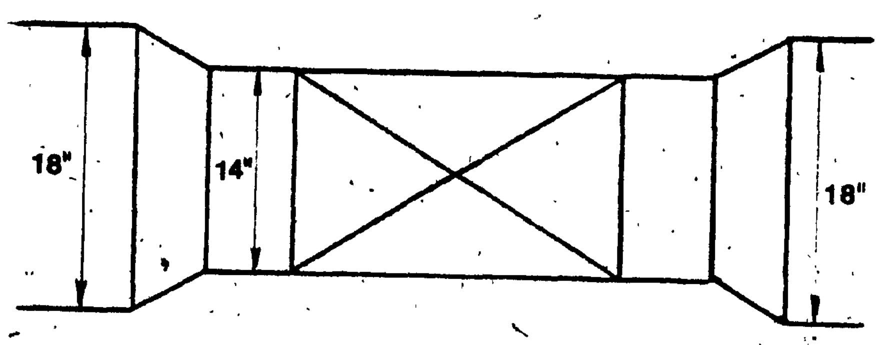



FIG. F 5.4.6.1 14" Valve with Reducers.

While Diagram D 5.4.6.1 shows the 14" valve dynamic behaviour, Table T 5.4.6.1 gives the comparison of the most significant parameters with an 18" valve.

TABLE T 5.4.6.1 Energy Efficiency Comparison:
18" Valve with 45 deg. max. opening,
14" Valve plus reducers.

| | | 18" | 14" |
|---|---|---|---|
| Pressure losses at full flow. | KPa | 2,683 | 3,609 |
| Availab. destr. at full flow. | Mwh/y | 83 | 112 |
| Availab. destr. at full flow. | $/y | 830 | 1119 |

The dynamic behaviour of the 14" valve plus reducer system is excellent: stability is complete for almost all flow rate range (Diagr. D 5.4.6.1) and it is obtained without addition of external devices or disc opening limitations.

Meanwhile, the energy operating costs are less for the 18" valve solution.

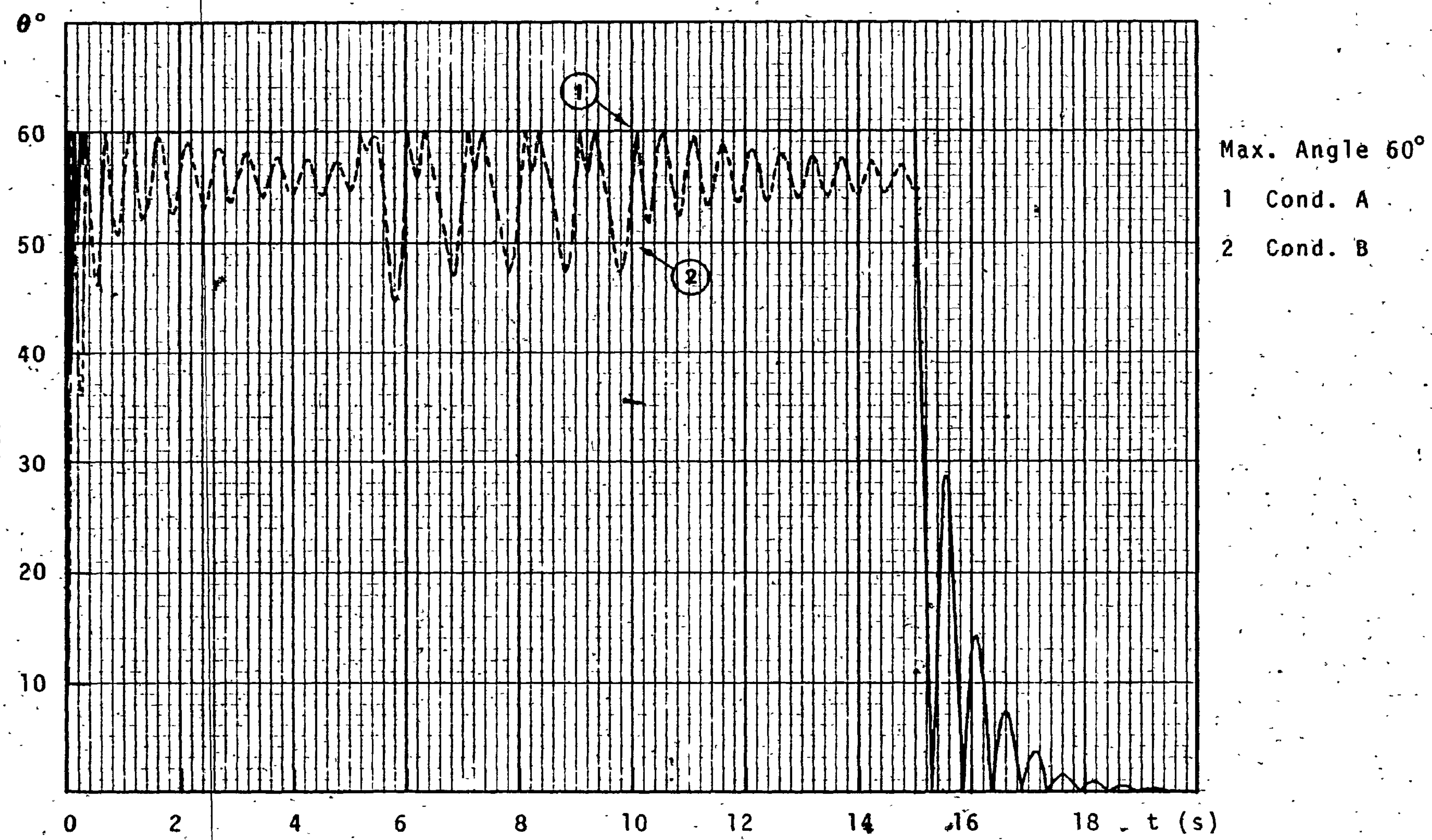



Diagram D 5.4.6.1 14" Valve. Clapper Oscillation

## 5.5 CONCLUSIONS

Three acceptable solutions have been obtained, all in principle suitable for the Field Case blowing system.

i) an 18" check valve with limited disc opening, dashpot, spring,counterweight;

ii) an 18" check valve with limited disc opening;

iii)a 14" valve with reducers.

In Table T 5.5.0.1 the independent-dependent variables relationship is shown for the Partial Cost Function, relative to "valve opening" and "oscillation" blocks of Fig. F 2.3.2.1 and for the three acceptable solutions domain.

The operating costs of solutions i) and ii) (18" valve) are identical, and 290 $/year less expensive than the 14" valve solution iii).

The "Total Present Value" (paragraph 2.3) of the operating cost increase, for a 14" valve, assuming 15 years of plant life and 10% interests, is:

TABLE T 5.5.0.1 Partial Cost Function.

| | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|
| $I : Investment costs | ■ | | | | |
| $R : Recovery costs | ■ | | | | |
| $O : Operating costs | | | | | |
| $M : Maintenance costs | ■ | ■ | ■ | | |
| | | | | | |
| $II : Indirect investment costs | | | | | |
| $IM : Indirect maintenance costs | | | | | |

$$\$P = 280*\left(\frac{1-(1+0.1)^{-15}}{0.1}\right) = 2129\ \$$$

The 18" standard valve with 45 deg. opening angle limit (ii) and the 14" valve with reducer are fully acceptable solutions, therefore, if we were still in the design stage, the best choice of the two would have been the least expensive, considering:

: the purchasing cost of each valve;

: for the 18"valve:

: two 18"flanges with bolting,

: 18" pipe for a length of both reducers;

: for the 14" valve:

: a double reducer,

: 2100 $ of additional operating costs;

For the Field Case the 18" valve IS the best choice.

Solution (i), the valve with external auxiliary devices, allows a "smoother" valve operation and a lower disc angular velocity,and thus a longer fatigue life may be expected. The additional investment cost represents what must be paid to decrease the probability of incurring maintenance costs.

TABLE T 5.4.5.1 Standard Test Conditions Form.

```
*****************************************************************
*                                                               *
*                     VALVE  SPECIFICATION                      *
*                                                               *
*  TYPE   :   Wafer check.                                      *
*  SIZE   :   18".                                              *
*  CLASS  :   ANSI 150.                                         *
*  AUXILIARY DEVICES   : Torsional spring, Counterweight,       *
*                        dashpot. Max. opening angle limited    *
*                        to 45 deg.                             *
*****************************************************************
*                                                               *
*                     OPERATING CONDITIONS                      *
*                                                               *
* PRESSURE          :   149,200  Pa                             *
* DENSITY           :   1.427    Kg/m3                          *
* MASS FLOW         :   6.4      Kg/s                           *
* MASS FLOW (MIN)   :   2.6      Kg/s                           *
* GAS               :   air.                                    *
* OTHER CONDITIONS   :     none.                                *
*****************************************************************
*                                                               *
*                  STANDARD TEST CONDITIONS                     *
*                                                               *
* INTERVAL A   ; Disc is open 10 deg.. Suddenly, full pressure  *
*                and mass flow are introduced.                  *
*                Then all parameters are constant fot 5 s.      *
* INTERVAL B   : Mass flow oscillates for 5 s with:             *
*                     Amplitude: 0,20 of average value,         *
*                     Frequency: 1 cycle/s (value close to      *
*                                valve natural frequency)       *
* INTERVAL C   : All parameters constant for 5 s at same value  *
*                of A.                                          *
* INTERVAL D   : Suddenly flow rate falls to zero.              *
*****************************************************************
*                                                               *
*                      TEST PARAMETERS                          *
*                                                               *
* 1 : Operating pressure and density. Operating mass flow       *
* 2 : Operating pressure and density. Minimum mass flow         *
* 3 : Operating pressure and density. 60% of oper. mass flow    *
*****************************************************************
```

## ALTERNATIVES

- I : Valve with spring, counterweight, dashpot.
- II : Standard valve with full disc opening (60 deg.)
- III : Standard valve with 45 deg. opening.

## GEOMETRY

| | U | I | II | III |
|---|---|---|---|---|
| VALVE | | | | |
| NOMINAL DIAMETER | inch | 18 | 18 | 18 |
| PIPE I.D. | inch | 17 | 17 | 17 |
| SEAT DIAMETER | inch | 13 | 13 | 13 |
| MAXIMUM DISC ANGLE | deg. | 45 | 60 | 45 |
| COUNTERWEIGHT | | | | |
| WEIGHT | lbs | 25 | | |
| ARM LENGTH | inch | 12.6 | | |
| ANGLE TO VERT. (CLSD. CTRCL.) | deg. | 157.5 | | |
| DASHPOT | | | | |
| CYLINDER INSIDE DIA | mm | 80 | | |
| MAXIMUM STROKE | inch | 4 | | |
| VERTICAL ARM | inch | 16.5 | | |
| MOBILE ARM | inch | 4 | | |
| ANGLE TO VERT. (CLSD. CTRCL.) | deg. | 112.5 | | |
| HYDR. CIRC. COEFF. RESIST. | | 0.4 | | |
| HYDR. CIRC. INSIDE DIA. | mm | 5 | | |
| MAXIMUM ALLOW. HYDR. PRESSURE | psig | 150 | | |
| TORSIONAL SPRING | | | | |
| SPRING FACTOR | N/m | 1380 | | |
| VERTICAL ARM | inch | 16.5 | | |
| MOBILE ARM | inch | 3 | | |
| SPRING FREE LENGTH | inch | 11.75 | | |
| FREE LENGTH. SPRING ALONE | inch | 9 | | |
| ANGLE TO VERT. (CLSD. CTRCL.) | deg. | 67.5 | | |

## ENERGY DEGRADATION

| | U | I | II | III |
|---|---|---|---|---|
| EQUIV. LENGT AT MAX. ANGLE | L/D | 348 | 143 | 348 |
| EQUIV. LENGTH AT OPER. ANGLE | L/D | 348 | 143 | 348 |
| PRESSURE DROP AT OPER. COND. | Pa | 2684 | 1108 | 2684 |
| PRESSURE DROP AT MIN. FLOW | Pa | 430 | 177 | 430 |
| AVAILAB. DESTR. AT MAX. FLOW | Mwh/y | 83 | 34 | 83 |
| AVAILAB. DESTR. AT MIN. FLOW | Mwh/y | 5 | 2 | 5 |
| COST OF KWH | $/Kwh | 0.01 | 0.01 | 0.01 |
| DOLLAR LOSSES AT MAX. FLOW. | $/y | 830 | 340 | 830 |

## DYNAMICS

| | U | I | II | III |
|---|---|---|---|---|
| DISC ANGLE AT OPER. COND. | deg. | 45 | 60 | 45 |
| DISC ANGLE AT MIN. FLOW RATE | deg. | 35 | 40 | 40 |
| AMPLIT. OF OSCILL.. OP. COND. | | | | |
| AFTER 5 S | deg. | 0 | 0 | 0 |
| AFTER 10 S | deg. | 0 | 0 | 0 |
| AFTER 15 S | deg. | 0 | 0 | 0 |
| AMPLIT. OF OSCILL.. MIN. FLOW | | | | |
| AFTER 5 S | deg. | 2 | 11 | 3 |
| AFTER 10 S | deg | 22 | 22 | 16 |
| AFTER 15 S | deg. | 2 | 3 | 3 |
| FREQ. OF OSCILL.. OP. COND. | | | | |
| AFTER 15 S | cycl/s | 0 | 0 | 0 |
| FREQ. OF OSCILL.. MIN. FLOW | | | | |
| AFTER 15 S | cycl/s | 0.75 | 1.5 | 1.5 |
| CLOSING TIME FROM FULL OPEN | s | 0.40 | 0.28 | 0.28 |

PART 2

FAST TRANSIENTS IN PIPING

# CHAPTER 6

# FAST TRANSIENTS

## 6.1 SUMMARY

The most significant external (Chap. 2) effect of a check valve closing is its influence on the downstream flow parameters. When a check valve closes, a pressure surge or "gas hammer" may be generated and additional investment and expenditure may be required for devices to counteract its effects.

An investigation of the valve-piping combined system may give useful information to determine, when possible, valve parameters minimizing gas hammer. For a complete investigation, a model of the piping system, to be combined with the valve dynamic model (Chap.5), is required.

Basic equations for unsteady continuous flow in a pipe and for moving shock are provided based on assumption of one dimensional flow.

In addition to the classic continuity, momentum and state equations, a fourth equation is derived combining the energy and entropy equations.

The basic elements of shock theory and the most significant equations are provided but not derived.

All these equations are used (Chap. 8) to build the numerical model of a piping system.

## 6.2 PRESSURE GRADIENTS IN A PIPING SYSTEM

### 6.2.1 ORIGIN OF PRESSURE GRADIENTS.

Most piping systems operate for extended periods of time in industrially steady conditions i.e. with very small variation of the flow parameters about their average value. Furthermore, when changes are requested, pressure and mass flow gradients are controlled and, in general, amplificatin effects avoided. Typical of these conditions are the gas distribution pipelines where slow changes are originated by request variations of a number of users. Small gradients in pipelines have been investigated by STREETER and WYLIE (17) and by STOPER (18) also with respect to the stock in motion or "gas pack" of long pipelines.

In process piping systems, pressure and mass flow gradients are most usually caused by malfunctions or unavoidable conditions.

Breakdown of compressors and piping ruptures are the most obvious, although not the most frequent, of a number of possible malfunctions, all with the common characteristic of producing a sudden pressure drop at a point in the pipeline and a consequent downstream flow inversion. In fluid dynamic terms an expansion wave travels upstream of the troubled point and may be reflected and inverted at any discontinuity in the piping system.

More frequent, although less devastating but still dangerous causes of swift pressure gradients are careless operation of shut off valves or valving and major variations in flow supply, like switching of compressors, culprits of many check valves failures. Fast downstream throttling or upstream increase of the gas stream can activate unstable line conditions.

When fast gradients are investigated theoretically, it is common practice to assume that the duct flow is one dimensional, to avoid mathematical complications (14). This assumtion implies that all fluid properties are considered uniform over a pipe section.

Furthermore, elbows, valves, contractions and expansions may be considered as equivalent to pipe elements of proper length (Chap.4), where a succession of steady but varied motions occur (discrete changes hipothesis).

### 6.2.2 UNSTEADY ONE DIMENSIONAL CONTINOUS FLOW.

One dimensional, shock free, flow through a pipe is governed by:

: the continuity equation,

: the momentum equation,

: the entropy equation,

: the energy equation,

: the state equation.

The principle of conservation of mass may be expressed by:

$$\frac{\partial}{\partial x}(\rho*V*A)*dx=-\frac{\partial}{\partial t}(\rho*A*dx)$$

or, after simplification:

$$\frac{\partial \rho}{\partial t}+\rho*(\frac{\partial V}{\partial x})+V*(\frac{\partial \rho}{\partial x})=0 \qquad (6.2.2.1\ )$$

To write the momentum equation we shall assume that the friction effects are expressed in terms of pipe friction factor "f". The momentum equation may be written as:

$$\frac{\partial V}{\partial t}+V*(\frac{\partial V}{\partial x})+\frac{1}{\rho}*(\frac{\partial p}{\partial x})+F=0 \qquad (6.2.2.2\ )$$

where F is the wall friction term defined by:

$$F=\frac{4*f*V^2*|V|}{D*2*V} \qquad (6.2.2.3)$$

The First Law of Thermodynamic, applied to the control volume (dotted line) of Fig. F 6.2.2.1 gives (14):

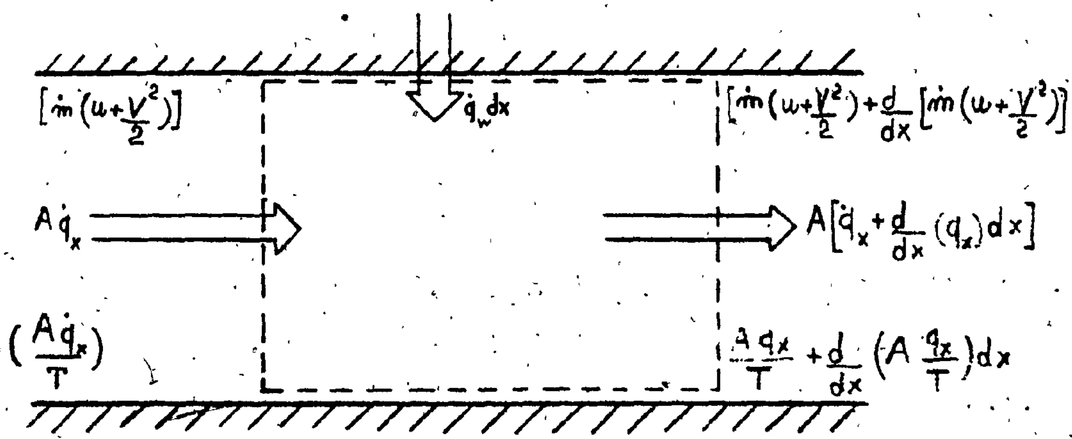



FIG. F 6.2.2.1 : Energy and Entropy Balance.

$$\frac{\partial}{\partial t}\left((q*A*dx)*\left(u+\frac{V^2}{2}\right)\right)=-\frac{\partial}{\partial x}\left((q*A*V)*\left(u+\frac{p}{q}+\frac{V^2}{2}\right)\right)*dx-A*\left(\frac{\partial*\dot{q}_x}{\partial x}\right)*dx+\dot{q}_w*A*dx$$

with:

: $\dot{q}_x$ = fluid conductive heat flux.

: $\dot{q}_w$ = time rate of heat through wall per unit length.

and expanding and using the continuity equation (6.2.2.1 ):

$$\frac{D}{Dt}*\left(u+\frac{V^2}{2}\right)=\frac{\dot{q}_w}{\varrho*A}-\frac{1}{\varrho}*\left(\frac{\partial \dot{q}_x}{\partial x}\right)-\frac{V}{\varrho}*\left(\frac{\partial p}{\partial x}\right)-\frac{p}{\varrho}*\left(\frac{\partial V}{\partial x}\right)$$

The term $\partial p/\partial x$ may be eliminated using the momentum equation (6.2.2.2 ):

$$\frac{D}{Dt}(u)+\frac{p}{\varrho}*\left(\frac{\partial V}{\partial x}\right)=\frac{1}{\varrho}*\frac{\dot{q}_w}{A}-\frac{1}{\varrho}*\frac{\partial \dot{q}_x}{\partial x}+V*F \qquad (6.2.2.4\ )$$

with:

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+V*\frac{\partial}{\partial x}$$

The equation of state for a real gas is given by:

$$\frac{p}{\varrho}=R_g*Z*T \qquad (6.2.2.4\ )$$

with:

Z=compressibility.

For an ideal gas (Z=1) it reduces to:

$$\frac{p}{\varrho}=R_g*T \qquad (6.2.2.5\ )$$

or in differential form:

$$\frac{dp}{p}=\frac{d\rho}{\rho}+\frac{dT}{T} \qquad (6.2.2.6\ )$$

For the control volume (dotted line) of Fig. 6.2.2.1 the entropy balance is given by (14):

$$\frac{\partial}{\partial t}(S)_{cv}=-\frac{\partial}{\partial x}(A*\frac{\dot{q}_x}{T})*dx+(\frac{q_w}{T})*dx-\frac{\partial}{\partial x}((\rho*A*V)*s)*dx+\dot{\sigma}*A*dx$$

where:

$\dot{\sigma}$=time rate of entropy production per unit of volume.

or:

$$s*(\frac{\partial}{\partial t}(\rho*A*dx))+\rho*A*dx*(\frac{\partial s}{\partial t})=-(\rho*A*dx)*V*(\frac{\partial s}{\partial x})-s*dx*(\frac{\partial}{\partial x}(\rho*A*V))-$$

$$-\frac{\partial}{\partial x}(\frac{A*\dot{q}_x}{T})*dx+\frac{\dot{q}_w}{T}*dx+\dot{\sigma}*A*dx$$

Simplifying and using the continuity equation (6.2.2.1 ):

$$\rho*\frac{Ds}{Dt}=-\frac{\partial}{\partial x}(\frac{\dot{q}_x}{T})+\frac{q_w}{A*T}+\dot{\sigma} \qquad (6.2.2.7\ )$$

The first Tds equation gives:

$$T*ds=du+p*d\left(\frac{1}{\rho}\right) \qquad (6.2.2.8\ )$$

$$T*ds=du-\frac{R_g*T}{\rho}*d\rho \qquad (6.2.2.9\ )$$

Using the (6.2.2.9 ) the (6.2.2.7 ) becomes:

$$\rho*\left(\frac{1}{T}*\left(\frac{Du}{Dt}\right)-\frac{R_g}{\rho}*\left(\frac{D\rho}{Dt}\right)\right)+\frac{\partial}{\partial x}\left(\frac{\dot{q}_x}{T}\right)-\frac{\dot{q}_w}{A*T}=\dot{\sigma}$$

and, using the energy equation (6.2.2.4 ):

$$\dot{\sigma}=\rho*\left(\frac{1}{T}*\left(\frac{\dot{q}_w}{\rho*A}-\frac{1}{\rho}*\left(\frac{\partial\ \dot{q}_x}{\partial\ x}+V*F-\frac{p}{\rho}*\left(\frac{\partial\ V}{\partial\ x}\right)\right)\right)-\frac{R_g}{\rho}\left(\frac{D\rho}{Dt}\right)\right)+\frac{\partial}{\partial x}\left(\frac{\dot{q}_x}{T}\right)-\frac{\dot{q}_w}{A*T}$$

$$\dot{\sigma}=\rho*\frac{V*F}{T}-\frac{1}{T}*\left(\frac{\partial\dot{q}_x}{\partial x}\right)+\frac{\partial}{\partial x}\left(\frac{\dot{q}_x}{T}\right)-\frac{p}{T}*\left(\frac{\partial\ V}{\partial\ x}\right)-R_g\frac{D\rho}{Dt}$$

$$\dot{\sigma}=\rho*\frac{V*F}{T}-R_g*\rho*\left(\frac{\partial\ V}{\partial\ x}\right)-R_g*\left(\frac{D\rho}{Dt}\right)+\frac{\dot{q}_x}{T}*\left(\frac{\partial\ T}{\partial\ x}\right) \qquad (6.2.2.10)$$

Using the (6.2.2.10), the entropy balance (6.2.2.7 ) may be written:

$$\rho*\left(\frac{Ds}{Dt}\right)=\frac{\dot{q}_w}{A*T}-\frac{\partial}{\partial x}\left(\frac{\dot{q}_x}{T}\right)+\frac{\partial}{\partial x}\left(\frac{\dot{q}_x}{T}\right)-\frac{1}{T}*\left(\frac{\partial\dot{q}_x}{\partial x}\right)+\frac{\rho*V*F}{T}-\frac{p}{T}*\left(\frac{\partial\ V}{\partial\ x}\right)-R_g*\left(\frac{D\rho}{Dt}\right)$$

$$\rho*\left(\frac{Ds}{Dt}\right)=\frac{\rho*V*F}{T}+\frac{\dot{q}_w}{A*T}-\frac{1}{T}*\left(\frac{\partial\dot{q}_x}{\partial x}\right)-\rho*R*\left(\frac{\partial\ V}{\partial\ x}\right)-R_g*\left(\frac{\partial\ \rho}{\partial\ t}\right)-R_g*V*\left(\frac{\partial\ \rho}{\partial\ x}\right)$$

and, using the continuity equation (6.2.2.1 ):

$$\frac{Ds}{Dt}=\frac{V*F}{T}+\frac{R_g*\dot{q}_w}{A*p}-\frac{1}{T*\varrho}*(\frac{\partial\dot{q}_x}{\partial x}) \qquad (6.2.2.11)$$

For a perfect gas (6.2.2.4 ) and introducing the Tds equation (6.2.2.9 ), the (6.2.2.11) becomes:

$$\frac{1}{T}*(c_v*(\frac{DT}{Dt}))-\frac{R_g}{\varrho}*(\frac{D\varrho}{Dt})=\frac{V*F}{T}+\frac{R_g*\dot{q}_w}{A*p}-\frac{R_g}{p}*(\frac{\partial\dot{q}_x}{\partial x})$$

or, using the differential form of the state equation (6.2.2.6 ):

$$c_v*(\frac{1}{p}*(\frac{Dp}{Dt}))-(R_g+c_v)*(\frac{1}{\varrho}*(\frac{D\varrho}{Dt}))=\frac{V*F}{T}+\frac{R*\dot{q}_w}{A*p}-\frac{R_g}{p}*(\frac{\partial\dot{q}_x}{\partial x})$$

$$\frac{R_g}{\gamma-1}*(\frac{1}{p}*(\frac{Dp}{Dt}))-\frac{R_g*\gamma}{\gamma-1}*(\frac{1}{\varrho}*(\frac{D\varrho}{Dt}))=\frac{V*F}{T}+\frac{R_g*\dot{q}_w}{A*p}-\frac{R_g}{p}*(\frac{\partial\dot{q}_x}{\partial x})$$

and by integration for the same particle:

$$\frac{R_g}{\gamma-1}*\ln(\frac{p}{\varrho^\gamma})\Big|_{t_1}^{t_2}=\int_{t_1}^{t_2}(\frac{V*F}{T}+\frac{R_g*\dot{q}_w}{p*A}-R_g*(\frac{d\dot{q}_x}{dx}))dt$$

or:

$$(\frac{p_2}{\varrho_2^\gamma})=(\frac{p_1}{\varrho_1^\gamma})*e^{\frac{\gamma-1}{R}\int_{t_1}^{t_2}(\frac{V*F}{T}+\frac{R_g}{p}*(\frac{\dot{q}_w}{A}-\frac{d\dot{q}_x}{dx}))dt} \qquad (6.2.2.12)$$

The (6.2.2.12) for an adiabatic process becomes:

$$\left(\frac{p_2}{\rho_2^{\gamma}}\right) = \left(\frac{p_2}{\rho_2^{\gamma}}\right) * e^{(\gamma-1)*\int_{t_1}^{t_2}\left(\frac{V*F}{R*T}\right)*dt} \qquad (6.2.2.13)$$

and, for an isentropic process (inviscid flow):

$$\left(\frac{p_1}{\rho_1^{\gamma}}\right) = \left(\frac{p_2}{\rho_2^{\gamma}}\right) \qquad (6.2.2.14)$$

These equations are all valid only for a given particle, a fact that must be taken into consideration when integration is performed. For limited time increment, dt, and always for an isentropic process, point and particle may be considered coincident.

The polytropic coefficient can now be calculated:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\alpha} = \left(\frac{\rho_2}{\rho_1}\right) * e^{\Phi}$$

$$\alpha * \ln\left(\frac{\rho_2}{\rho_1}\right) = \left(\gamma * \ln\left(\frac{\rho_2}{\rho_1}\right)\right) + \Phi$$

$$\alpha = \gamma + \Phi * \ln\left(\frac{\rho_1}{\rho_2}\right) \qquad (6.2.2.15)$$

with:

$$\Phi = \frac{(\gamma - 1)}{R_g} * \int_1^2 \left( \frac{V*F}{T} + \frac{R_g*}{p} \left( \frac{\dot{q}_w}{A} - \frac{d\dot{q}_x}{dx} \right) \right) dt$$

## 6.3 PRESSURE RISE IN A PIPING SYSTEM

### 6.3.1 VALVE CLOSING AND SUDDEN PRESSURE RISE.

An unsteady one dimensional continuous flow can be described using:

: the continuity and momentum equations,

: the state, energy and/or entropy equations,

: the assumption that all flow properties may be considered continuous.

Compression and expansion continuous waves will be the result of a controlled valve closing.

(14) "A sudden closure of a valve against a fast moving fluid generates a steep compression wave with a steep pressure gradient in space. Travelling continuous compression waves become continuously steeper until ultimately a vertical slope forms at some point of the wave. As the wave steepens toward a vertical tangent, the longitudinal velocity and temperature gradients approach infinity. Hence, no matter how small the coefficients of

viscosity and thermal conductivity are, longitudinal viscous stresses and longitudinal heat conduction, postulated negligible, must ultimately become of the same order of magnitude as the other terms in the dynamic and energy equations. Accordingly, the type of flow originally assumed is severely modified. The vertical tangent to the wave is never formed, because of the "spreading" or "smoothing-out" effects of viscosity and heat conduction. Instead, a zone of rapidly changing fluid properties is formed, within which viscous stresses and heat conduction play controlling roles. As time progresses, more and more of the compressive wave merges into this zone, thus increasing the amplitude of the pressure change within the zone. The part of wave with a nearly vertical tangent is called a shock wave, compression shock, or shock front. The existence of viscous stresses and heat transfer within the zone of shock leads in general to non isentropic changes of state for a fluid particle.

Within the shock wave, all fluid properties change continuously. From a practical point of view, it is almost never necessary to take into account conditions within the shock wave. Theoretical analyses indicate that shock wave thickness is of the order of only a few mean free molecular path lengths. Therefore, it can be assumed, for the purpose of calculation, that the shock wave is so thin that it may be replaced by a simple model in which all fluid properties change discontinuously across a mathematical line. This line is called the shock wave."

6.3.2 ONE-DIMENSIONAL MOVING SHOCK WAVES.

In making calculations according to the shock model, we need consider only conditions before and after the shock wave. Although viscous stresses and heat transfer are significant within the wave, and in fact determine the force of the wave, the change from the initial to final conditions of the discontinuity may be treated as occurring without external friction and heat transfer, because the shock zone is so thin that heat transfer and friction from bodies external to the gas stream (such as the walls of a duct) are negligibly small.

In the special case where a shock of constant pressure rise travels into a zone of constant fluid properties, the shock will travel with constant speed and all fluid properies after the shock line will be constant. Therefore, by imagining that the observer travels with the uniform linear speed of the shock, the phenomenon may be reduced to one of steady flow.

In the more general case where the shock is of variable pressure rise and is travelling into a region of variable fluid properties, the shock will travel with variable speed and the fluid properties after the shock will also be variable from instant to instant. The shock may be treated with good accuracy as a discontinuity. Hence, if the observer moves with the shock, and if continuity, momentum and energy equations are written for a control surface which encloses only the shock, the unsteady flow terms relative to the time rates of change of mass, momentum, and internal plus kinetic energy within the control volume, are equal to zero. From this it follows that the steady flow relations are applicable to the states of the two sides of the variable shock. Therefore, the flow through a shock discontinuity may be considered as quasi-steady (steady at each instant of time) in terms of properties, seen by an observer to whom the shock is stationary.

The governing equations for a moving shock with respect to moving coordinates are (upper sign right travelling waves) (Fig. F 6.3.2.1):

Continuity:

$$q_1 * W = q_2 * (W + (V_1 - V_2)) \qquad (6.3.2.1)$$

Momentum:

$$p_1 + q_1 * W^2 = p_2 + q_2 * (W + (V_1 - V_2))^2 \qquad (6.3.2.2)$$

Energy:

$$h_1+\frac{W^2}{2}=h_2+\frac{(W+(V_1-V_2))^2}{2} \qquad (6.3.2.3)$$

Perfect gas (6.2.2.5 ) and:

$$h_2-h_1=c_p*(T_2-T_1) \qquad (6.3.2.4)$$

$$c_p-c_v=R_g \qquad (6.3.2.5)$$

$$c^2=\gamma*R_g*T \qquad (6.3.2.6)$$

$$\frac{c_p}{c_v}=\gamma \qquad (6.3.2.7)$$

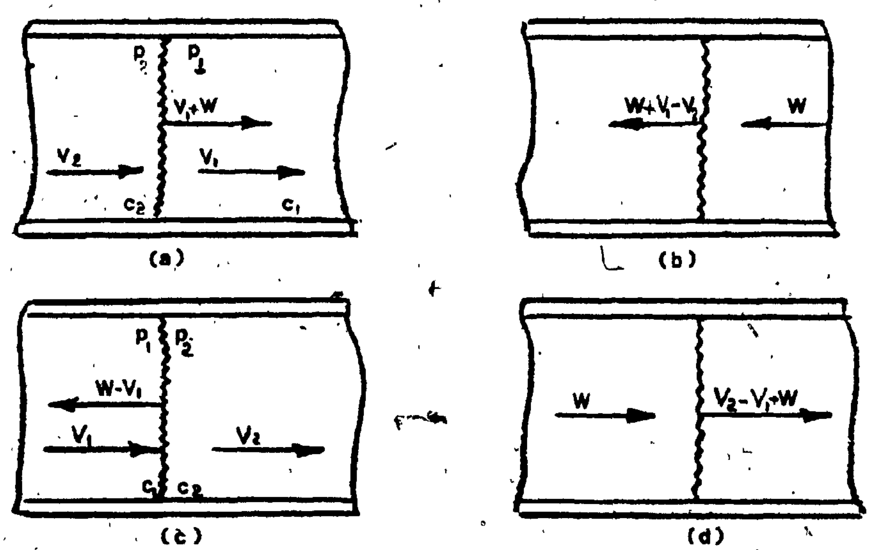



FIG. F 6.3.2.1 : Moving shock.

| | | | |
|---|---|---|---|
| a-b | right travelling | c-d | left travelling |
| a-c | stationary coord. | b-d | moving coord. |

Assuming:

$$M_x=\frac{W}{c} \qquad (6.3.2.9)$$

as independent variable, it can be written:

$$\frac{V_2 - V_1}{c_1} = \pm \frac{2}{\gamma+1} * \left(M_x - \frac{1}{M_x}\right) \qquad (6.3.2.10)$$

$$\left(\frac{c_2}{c_1}\right)^2 = 1+2*\frac{(\gamma-1)}{(\gamma+1)}*\left(\gamma*M_x^2 - \frac{1}{M_x^2} - (\gamma-1)\right) \qquad (6.3.2.11)$$

$$\left(\frac{Q_2}{Q_1}\right) = \frac{1}{1-\left(\frac{2}{(\gamma+1)}\right)*\left(1-\left(\frac{1}{M_x^2}\right)\right)} \qquad (6.3.2.12)$$

$$\frac{(p_2)}{(p_1)} = 1+\frac{2*\gamma}{\gamma+1}*(M_x^2-1) \qquad (6.3.2.13)$$

Only values of $M_x$ greater than unity must be considered, in accordance with the entropy requirement that the gas entering the shock must be travelling at supersonic speed relative to the shock.

For values of $M_x$ close to unity, or, mathematically, for $(M_x-1) \ll 1$ it may be shown (14) that the shock formulae become equivalent to those for continuous flow. Therefore, very weak shocks are approximately equivalent to continuous isentropic compression waves and may be treated as such.

Usually, shock waves propagate in an already unsteady flow. As a general rule, the shock waves always overtake continuous waves of the same family (direction) in the region ahead of the shock, and are always overtaken by continuous waves of the same family in the region behind the shock.

The absolute velocity of propagation of a shock is (Fig. F 6.3.2.1):

$(V_1 + W)$ for right travelling (6.3.2.14)

$(W - V_1)$ for left travelling (6.3.2.15)

When the reverse flow velocity approaches the sound velocity, or:

$$V \approx c$$

a weak shock will be quasi-stationary. Further shocks or compressive waves will overtake the stationary shock, gradually increasing its strength and absolute velocity.

## 6.4 CONCLUSIONS

Pipe ruptures, compressor breakdowns and valving generate continuous waves and flow inversion. The closure of a check valve may originate a travelling shock.

If the valve closes when the reverse flow velocity has already reached values close to the local speed of sound, shock conditions are immediatly originated, no matter how gradual the valve closure is. The shock will gain strength and velocity as it is overtaken by successive compression waves.

Unsteady, continuous flow through a pipe is governed by classic fluid dynamics and thermodynamics equations. When possible, the flow properties should be calculated using:

: continuity equation

: momentum equation

: state equation

: entropy derived equation

One dimensional shock waves are, in general, represented by a specific set of equations and their solution requires a special approach.

CHAPTER 7

# CONTINUOUS FLOW
# TEST CASE AND REFERENCE ALGORITHM

## 7.1 SUMMARY

A solution of the unsteady flow in a piping system is required to determine the mutual influence of a check valve and piping. A numerical model is the logical choice. Sufficient accuracy, adaptability to changes, simplicity of results and reasonable computer use are the realistic requirements of an ideal model.

Experimental testing of the numerical model is, for the time being, discarded as too expensive. An alternative comparison method of the model results for a selected case with the solution being obtained by a commonly accepted method, is proposed.

The test case, a steady flow to a reservoir perturbed by a pipe rupture, and the test algorithm are discussed. Basic concepts of the theory of characteristics are presented and a numerical model based on the Method of Characteristics applied to the test case.

An analysis of the agreement between the Method of Characteristics and the ideal model requirements indicates the areas where new algorithms are justified.

## 7.2 NUMERICAL MODELLING OF UNSTEADY CONTINUOUS FLOWS

### 7.2.1 IDEAL MODEL CHARACTERISTICS.

Unsteady flow through a piping system is defined when three flow properties are known at any time and point of the domain wetted by the fluid. An unidimensional assumption, as in our case, reduces the problem to a time-space 2-D solution domain, as the fluid properties are considered constant in all points of any pipe section.

A wide range of possibilities exists and has been used to select the dynamic and thermodynamic properties describing a compressible flow through a pipe. A.H.

SHAPIRO (14) suggests and uses flow velocity. sound velocity and density, E.B. WYLIE and V.L. STREETER (17) use pressure, density and mass flow for an isothermal analysis of slow transients, and M.A. STONER ((18) chooses pressure,density and flow velocity for its slow transients general solution. "Riemannn" variables, a combination of normalized flow velocity and speed of sound are used by G. RUDINGER (19). J.M. KIRSHNER and S. KATZ (20) use mass flow and mechanical potential for their analysis of fluidic circuits. An ideal choice of variables for fast transients analysis in industrial piping systems is the one which enables a piping engineer to immediately visualize what happens in the pipe. Translation from complex fluid variables is always a solution but decreases the simplicity of the numerical model.

The isentropic assumption is, in many instances, acceptable for an analysis of gaseous flows. Adiabatic non isentropic (viscous) flows of real gases or more complex conditions, may not be, sometimes, simplified to an isentropic process. An ideal model should be easily adaptable to non isentropic conditions.

A solution with a numerical model will consist of "slicing" the 2-D space-time domain (Fig. F 7.2.1.1)in two directions and finding three chosen properties at each corner point.

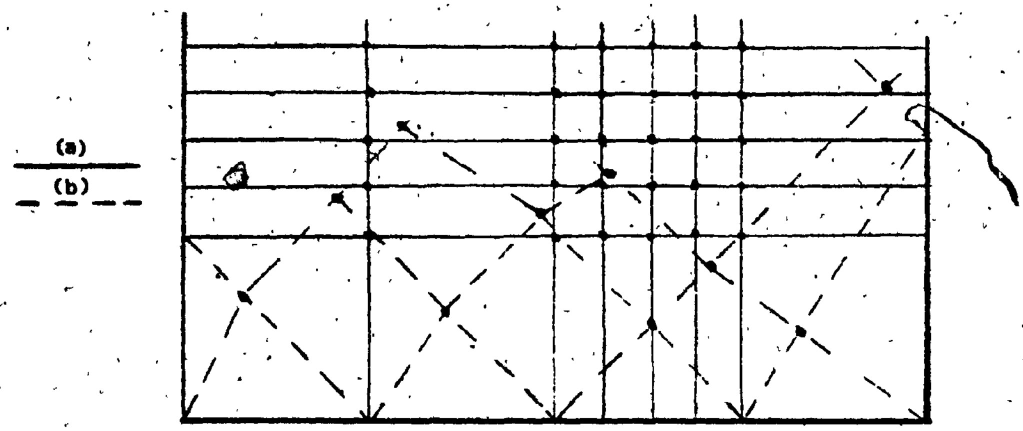



FIG. F 7.2.1.1 Solution Domain.

a) controlled orthogonal subdivision
b) non orthogonal subdivision

An adjustable subdivision is required to obtain in detail particular portions of the time-space domain. An understandable print-out is also a major requirement. Hence, a controlled orthogonal domain subdivision is a most desirable requirement.

Fast transients require high-iteration rates as major pressure and mass flow variations may occur in a few milliseconds. As the ultimate objective is to determine what can be done in terms of valve dynamics to avoid major pressure rises into the piping system, the model accuracy must comply with this practical requirement rather than with a highly precise description of the flow parameters.

The numerical model will be used to simulate many alternative configurations of the same pipeline, in order to find the best valve position and dynamic characteristics. Components may be added to the piping system, removed or changed in position. A good model is expected to handle several piping system configurations without requiring extensive or complex modifications.

Computer time and space are also factors, although fast computers allow implicit and iterative solutions not easily affordable only a few years ago.

Summarizing, an ideal numerical model for fast transients should have:

: understandable physical variables.

: adaptability to non isentropic conditions.

: easily readable results with possibility of magnifying chosen domain zones.

: sufficient accuracy to determine significant pressure and mass flow changes.

: the possibility of accepting modifications.

: a reasonable use of computer time and space.

## 7.2.2 EXPERIMENTAL ANALYSIS.

A numerical model should always be tested on typical cases and and its results compared with experimental data.

An acceptable experimental analysis of fast transients requires:

: an experimental pipeline at least 100 m long and 4" wide.

: flow velocities up to 30 m/s

: pressures of at least 300 KPa

: available compressing power of at least 100 KW.

An often used practice is to install sensors in an industrial piping system and collect data from routine and simulated emergency conditions. In West Germany an abandoned coal coking plant had been used for similar purposes.

Close cooperation with major industries is a necessary condition and preliminary results are a primary requirement to capture their interest.

7.2.3 AN ALTERNATIVE APPROACH.

The unidimensional, isentropic flow of an ideal gas through a constant section pipe has been graphically and numerically solved with the Method of Characteristics. Results obtained with this method have been proven to be very close to reality in many simple as well as complex cases (14).

Applying the Method of Characteristics to a simple piping system, the test case, where only continuous waves occur, the computed results may be assumed very close to reality and used to evaluate the accuracy of other numerical simulations.

As a test case, (Fig. F 7.2.3.1) a straight pipe connected with a reservoir in constant known conditions (infinite reservoir) has been chosen.

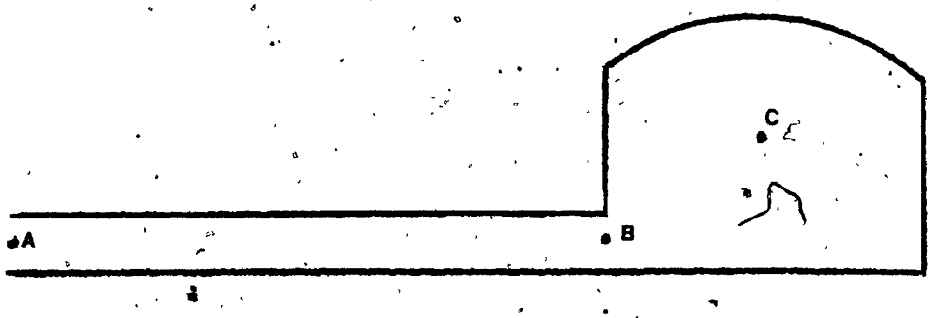


FIG. F 7.2.3.1 Test Case.

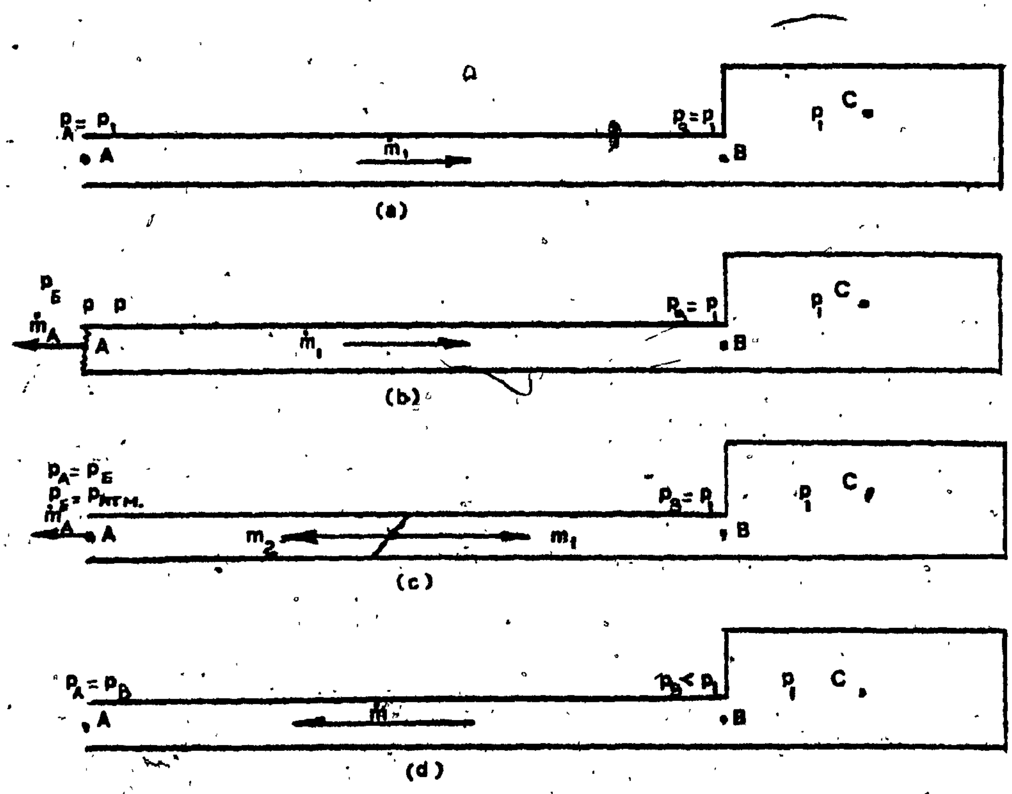


FIG. F7.2.3.2 Test Case: Flow Sequence.

Unsteady flow is obtained simulating a pipe rupture at the left end of the system. The time sequence is:

- : at time $t < t_o$ (Fig. F 7.2.3.2 a) a steady flow of inviscid gas enters the reservoir;
- : at time $t_o$ (Fig. F 7.2.3.2 b) a sudden decrease of pressure begins at point A (rupture);
- : the pressure $p_a$ (Fig. F 7.2.3.2 c) decreases until it reaches the atmospheric value;
- : at time $t_1$ (Fig. F 7.2.3.2 d) the pressures at points B and A are equal.

Three typical operating conditions have been selected and "perfect" air assumed as gas:

| | | | |
|---|---|---|---|
| (A) | p=149000 Pa | ρ=1.43 kg/m | V=30 m/s |
| (B) | p=300000 Pa | ρ=3.60 kg/m | V=30 m/s |
| (C) | p=500000 Pa | ρ=5.81 kg/m | V=30 m/s |

## 7.3 NUMERICAL SOLUTION OF THE TEST CASE
## BY
## METHOD OF CHARACTERISTICS

### 7.3.1 THE METHOD OF CHARACTERISTICS.

The governing equations for a one dimensional, shock free, isentropic, flow through the control surface of Fig. F 7.3.1.1 are:

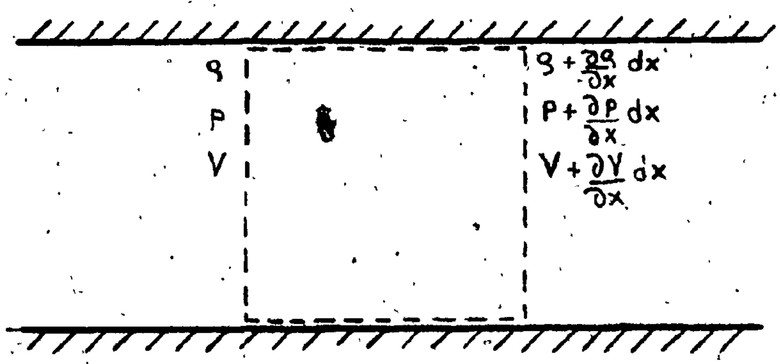



FIG. F 7.3.1.1 Flow Through a Control Surface.

$$\frac{\partial}{\partial x}(\rho*A*V)=-\frac{\partial}{\partial t}(\rho*A) \qquad \text{(continuity)} \qquad (7.3.1.1)$$

$$\frac{\partial V}{\partial t}+V*\left(\frac{\partial V}{\partial x}\right)=-\frac{1}{\rho}*\left(\frac{\partial p}{\partial x}\right) \qquad \text{(momentum)} \qquad (7.3.1.2)$$

$$\frac{p}{\rho^{\gamma}}=\text{const.} \qquad \text{(entropy)} \qquad (7.3.1.3)$$

Let us define a velocity potential $\Phi$ (x, t) through the relation

$$V = \frac{\partial \Phi}{\partial x} = \Phi_x$$

which, used with the governing equations and after rearranging, gives:

$$c^2 = c_o^2 - \left(\frac{\gamma-1}{2}\right)*V^2 - (\gamma-1)*\Phi_t \qquad (7.3.1.4)$$

$$(c^2 - \Phi_x^2)*\Phi_{xx} - 2*\Phi_x*\Phi_{xt} - \Phi_{tt} = 0 \qquad (7.3.1.5)$$

According to the theory of characteristics, if a differential equation has the form:

$$A*\Phi_{xx} + 2*B*\Phi_{xt} + C*\Phi_{tt} = 0$$

where A,B and C are functions of x, t, $\Phi$, and $\Phi_x$, then the characteristic curves are defined by:

$$\left(\frac{dt}{dx}\right)_{I,II} = \frac{B \pm \sqrt{B^2 - A*C}}{A} \qquad (7.3.1.6)$$

$$\left(\frac{d\Phi_t}{d\Phi_x}\right)_{I,II} = \frac{-B \pm \sqrt{B^2 - A*C}}{A} \qquad (7.3.1.7)$$

In our case it is:

$$A = c^2 - V^2 \qquad B = -V \qquad C = -1$$

Then, the characterisic curves are defined by:

$$\left(\frac{dt}{dx}\right)_I = \frac{1}{V+c} \quad ; \quad \left(\frac{dt}{dx}\right)_{II} = \frac{1}{V-c} \qquad (7.3.1.8\ ,\ 9)$$

$$\left(\frac{d\Phi_t}{dV}\right)_I = -(V-c); \quad \left(\frac{d\Phi_t}{dV}\right)_{II} = -(V+c) \qquad (7.3.1.10,\ 11)$$

The characteristics are lines of possible discontinuity in the first derivatives of V and c, and the velocity of propagation of such discontinuities is (V+c). This means that the velocity of propagation relative to the fluid itself is the local speed of sound, c, (14). Interpreting the discontinuity lines as waves, a sign plus indicates a wave travelling to the right and a minus a wave travelling to the left, each with respect to a fluid particle.

Differentiating the (7.3.1.4 ) with respect to V and rearranging, it is:

$$\left(\frac{dc}{dV}\right)_I = -\frac{(\gamma-1)}{2} \quad ; \quad \left(\frac{dc}{dV}\right)_{II} = +\frac{(\gamma-1)}{2} \qquad (7.3.1.12,\ 13)$$

The six remarkably simple equations (7.3.1.8, 9, 10, 11, 12, 13) are the fundamental relations of the popular

Method of Characteristics and are valid under the assumption of:

: perfect gas

: adiabatic isentropic process

: constant duct area

: the absence of friction

Heat exchanges, area changes and friction can be introduced and other complex and interlocked relationships between fluid properties may be devised.

## 7.3.2 TEST CASE : BOUNDARY CONDITIONS.

The point of rupture in the pipe will be referred to as left boundary and the pipe-reservoir common section as right boundary.

On the left boundary, at time $t \leq t_0$ (Fig. F 7.2.3.2 a) the flow parameters are:

$p(1)=p_0$ (initial pressure)

$V(1)=V_0$ (initial flow velocity, positive to reservoir)

and:

$c(1)=c_0$ (initial sound velocity)

At time $t>t_0$, $p(1)$ decreases according to:

$$p(1)=p_o-(p_o-101325)*(t-t_o)/TO$$

with TO adjustable and set as;

$$TO=0.05 \quad s$$

When:

$$p(1)=p_{ATM}=101325 \quad Pa$$

the boundary pressure stays constant until the local flow velocity equalizes the local speed of sound. From this instant point , the boundary condition becomes:

$$|V(1)|=|c(1)|$$

The left boundary flow parameters may be computed solving by iteration (Fig. F 7.3.2.1):

$$\left(\frac{dt}{dx}\right)_{II}=\frac{1}{V-c} \qquad (7.3.1.9)$$

$$\left(\frac{dc}{dV}\right)_{II}=\frac{\gamma-1}{2} \qquad (7.3.1.8)$$

$$\frac{p(1)}{(\rho(1))^{\gamma}}=\text{const.} \qquad (6.2.2.14)$$

$$p(1)=p_o-(p_o-101325)*(t-t_o)/TO \qquad (7.3.2.1)$$

the first two suitably discretized along the II characteristic line and for points $(x(1), t^{n+1})$ and $(x(2)^n, t^n)$.

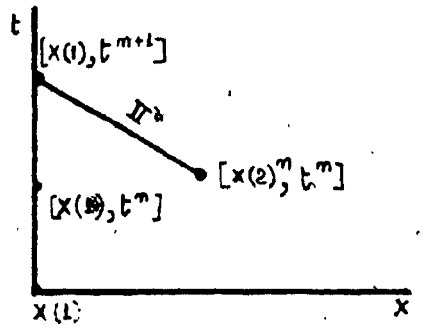



FIG. F 7.3.2.1. Left Bound.

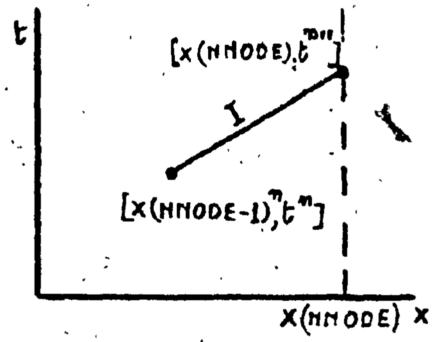



FIG. F7.3.2.2. Right Bound.

On the right boundary at time $t \leq t$ the fluid flows to the reservoir at pressure:

$$p(NNODE)=p_o$$

The flow pressure remains constant until the expansion wave reaches the boundary, i.e. the flow velocity changes direction (from positive to negative). From that instant point on, according to a commonly accepted practice (14), the time dependent relations involving boundary values are approximated to the corresponding steady ones, thus considering the unsteady flow as a succession of steady state conditions. The flow parameters of the right boundary are computed solving by iteration (Fig. F 7.3.2.2):

$$\left(\frac{dt}{dx}\right)_I = \frac{1}{V+c} \qquad (7.3.1.8)$$

$$\left(\frac{dc}{dV}\right)_I = -\frac{\gamma-1}{2} \qquad (7.3.1.12)$$

$$\frac{p(NNODE)}{\rho(NNODE)^{\gamma}} = const. \qquad (6.2.2.14)$$

$$c_o^2 = (c(NNODE))^2 + \left(\frac{\gamma-1}{2}\right) * V^2 \qquad (7.3.1.2)$$

the first two suitably discretized along the I characteristic line and for points $(x(NNODE), t^{n+1})$ and $(x(NNODE)^n, t^n)$.

### 7.3.3 METHOD OF CHARACTERISTICS: NUMERICAL ALGORITHM.

In general, the fundamental operation of the Method of Characteristics is to find the location of a third point using data of two already known points. Repeated application of this procedure yields the complete characteristics net (Fig. F 7.3.3.1). The integration of (7.3.1.12, 13, 8, 9) gives:

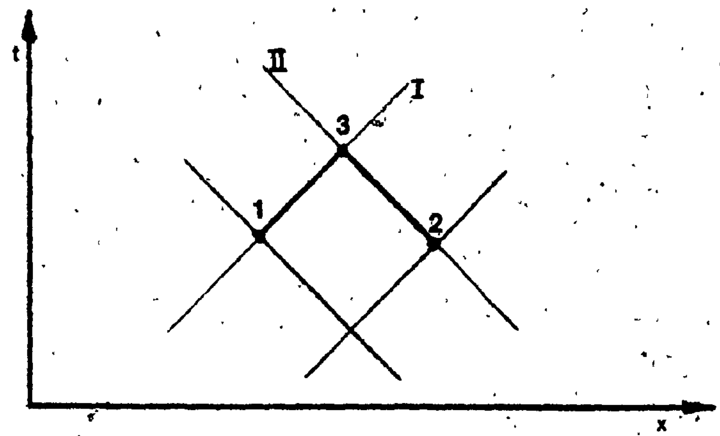



FIG. F 7.3.3.1. Characteristics Points.

$$\int_1^3 \left(\frac{dc}{dV}\right)_I \Rightarrow c_3 - c_1 = -\left(\frac{\gamma - 1}{2}\right)*(V_3 - V_1)$$

$$\int_2^3 \left(\frac{dc}{dV}\right)_{II} \Rightarrow c_3 - c_2 = +\left(\frac{\gamma - 1}{2}\right)*(V_3 - V_2)$$

$$\int_1^3 \left(\frac{dx}{dt}\right)_I \Rightarrow x_3 - x_1 = \left(\frac{(V_3 + c_3) + (V_1 + c_1)}{2}\right)*(t_3 - t_1)$$

$$\int_2^3 \left(\frac{dx}{dt}\right)_{II} \Rightarrow x - x = \left(\frac{(V_3 - c_3) + (V_2 - c_2)}{2}\right)*(t_3 - t_2)$$

and rearranging:

$$t_3 = \frac{2*(x_2 - x_1) - (t_2 * \bar{A}_2 - t * \bar{A}_1)}{\bar{A}_1 - \bar{A}_2} \qquad (7.3.3.1)$$

$$x_3 = x_1 + \bar{A}_1 * (t_3 \underset{2}{-} t_1) \qquad (7.3.3.2)$$

$$c_3 = \frac{AL + AR}{4} \qquad (7.3.3.3)$$

$$V_3 = \frac{AL - AR}{2*(\gamma - 1)} \qquad (7.3.3.4)$$

with:

$$A = (V_3 + c_3) + (V_1 + c_1) \qquad A = (V_3 - c_3) + (V_1 - c_1)$$

$$AL = \frac{2*c_1 + (\gamma - 1)*V_1}{4} \qquad AR = \frac{2*c_2 - (\gamma - 1)*V_2}{4}$$

If the pipe is divided in equal length elements, and the initial conditions are known, it is possible to determine a network of points in the (x-t) domain where the characteristic dependent variables c and V are known.

Program TESTAL (Appendix 4) gives solutions of the test-case with the Method of Characteristics. Typical results are shown in diagrams D 7.3.3.1, 2, 3, and D 7.3.3.4, 5, 6. A typical net of characteristic lines is shown in diagram D 7.3.3.7.

7.3.4 ANALYSIS OF NUMERICAL ALGORITHM.

The accuracy of the Method of Characteristics is recognized excellent (14) for reasonably dense nets of characteristic lines. In its simplest version, an $O(t_{m+1}-t_m)$ approximation is achieved on the time derivatives. An additional error is introduced by averaging V and c along the characteristic lines.

The explicit numerical solution is fast and simple and the handling of the boundary conditions relatively straight forward.

Translation of physical parameters, pressure, mass flow and density to and from the characteristic dependent variables is required.

The net of characteristic lines (Diagr. D 7.3.3.7) gives excellent information for SOME points irregularly distributed on the (x-t) domain, while significant domain portions are neglected. Severe initial conditions, like a high initial pressure, accelerate the network distortion to a point that information along the pipeline is not available after the flow inversion.

When the left boundary reaches the local sound velocity, the I characteristic originating on the boundary point, becomes vertical and the closest internal point "dies" at the next time level, as it is moved to the boundary line (Diagr. D 7.3.3.7). Eventually, all the internal points are concentrated on the left boundary line and unable to "communicate" with the right boundary.

Each network point is placed at a particular time level and space location. Information along all the pipeline at a given time level, or, for internal points, at a given location for a period of time, are not directly available and must be obtained by interpolation combined with proper adjustments of the network.

In complex piping systems, the Method of Characteristics must be specifically adjusted to each branch. The treatment of branch connections is cumbersome. The flow parameters of each branch depend on the past values inside the branch and on the boundary conditions. For an incompressible flow and an equal subdivision of contiguous branches the boundary values at the connecting points can be found at each time level solving in sequence all branches. A similar procedure can be used also for multi-branch connections. When an unequal subdivision of contiguous branches is desired and always with a compressible flow, the boundary values must be found with numerical interpolation and iterative methods.

System modifications and introduction of additional piping components, require substantial modifications of the numerical model

The simple adiabatic isentropic model cannot be extended to different thermodynamic conditions. For non isentropic flows, at each net point an iterative procedure must be introduced.

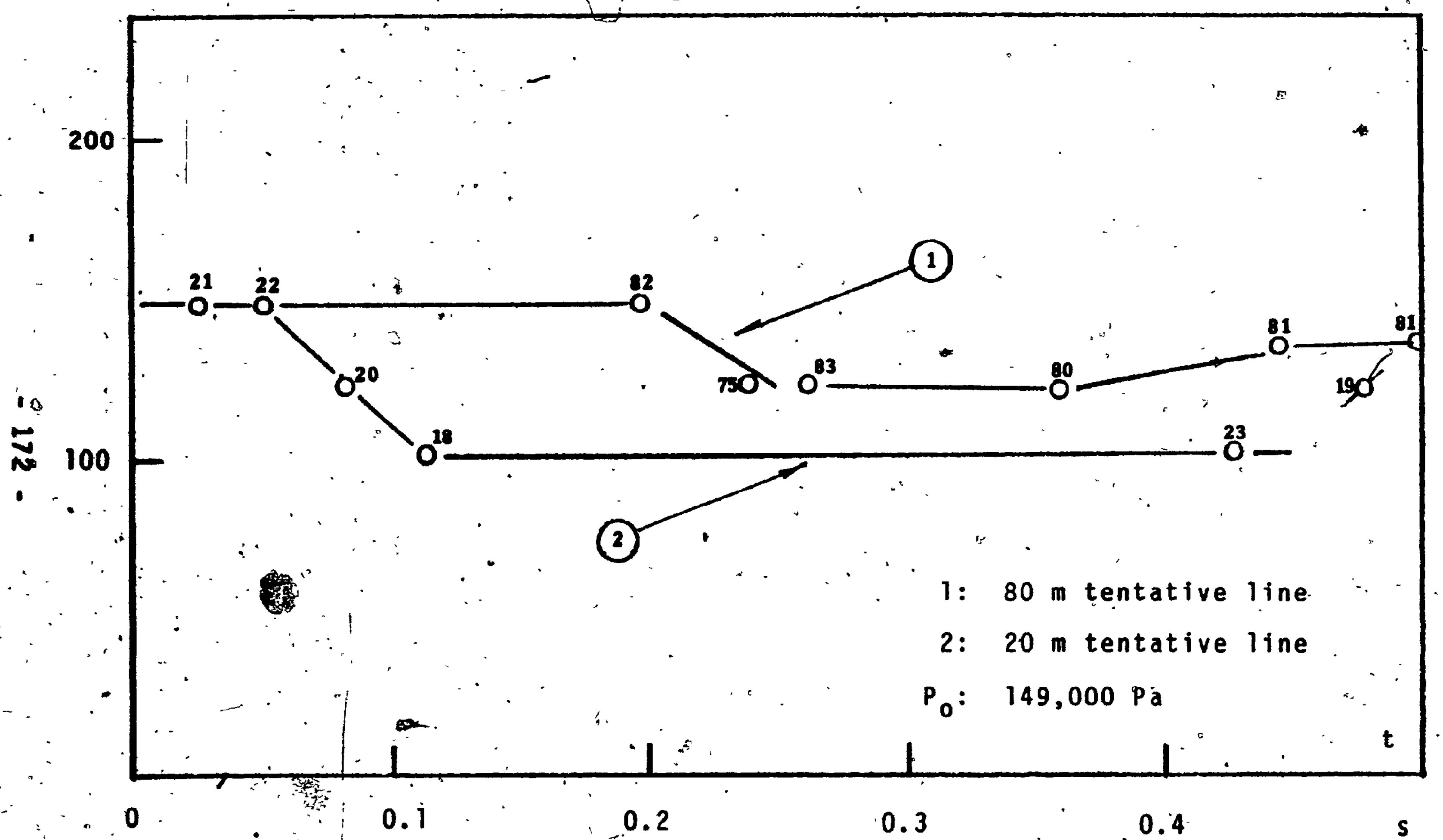



Diagram D 7.3.3.1 Characteristics Method. Pressure vs. Time at Various Pipe Locations. Distances in m from Pipe Left End. Case A.

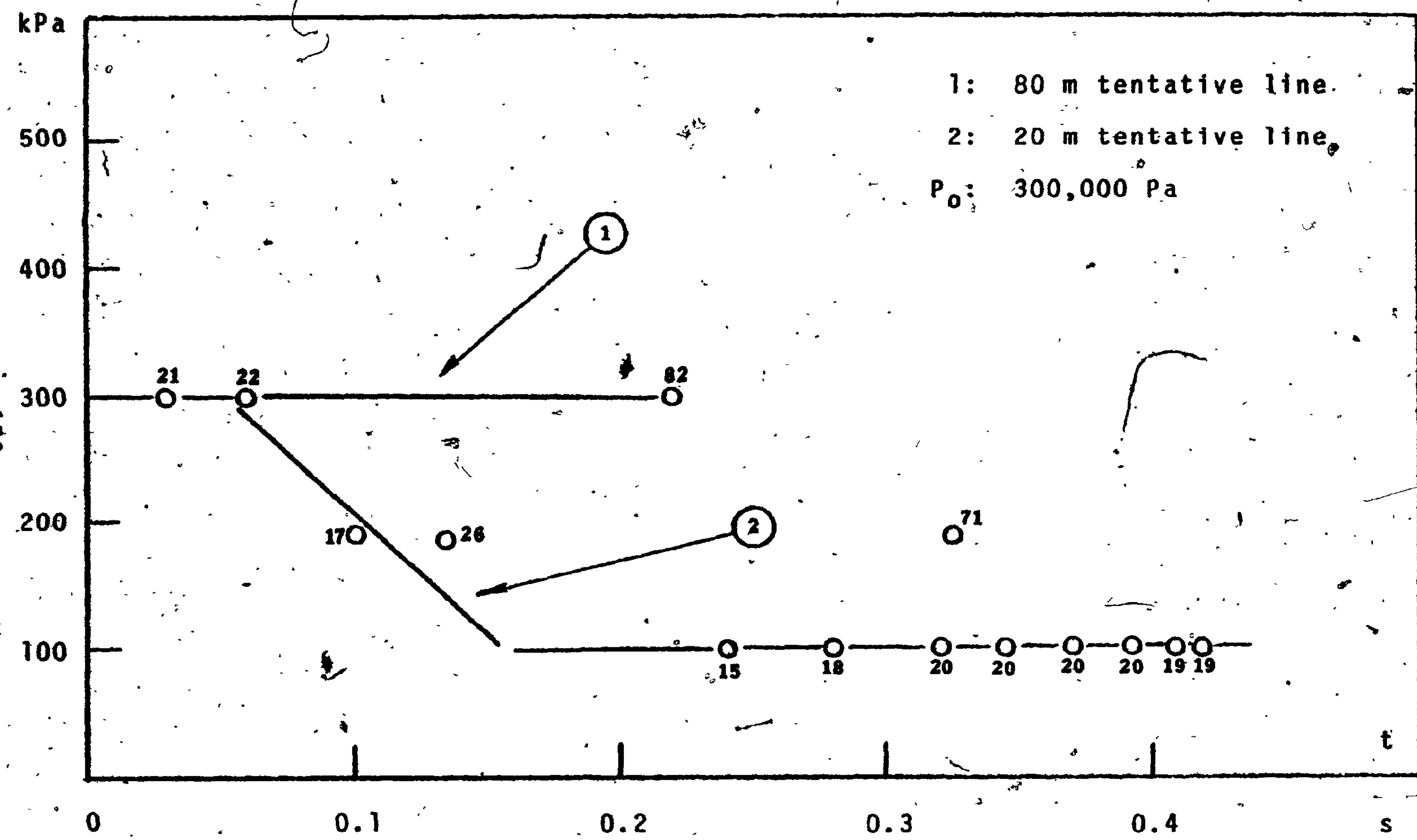



Diagram D 7.3.3.2 Characteristics Method. Pressure vs. Time at Various Pipe Locations. Distances in m from Left End. Case B.

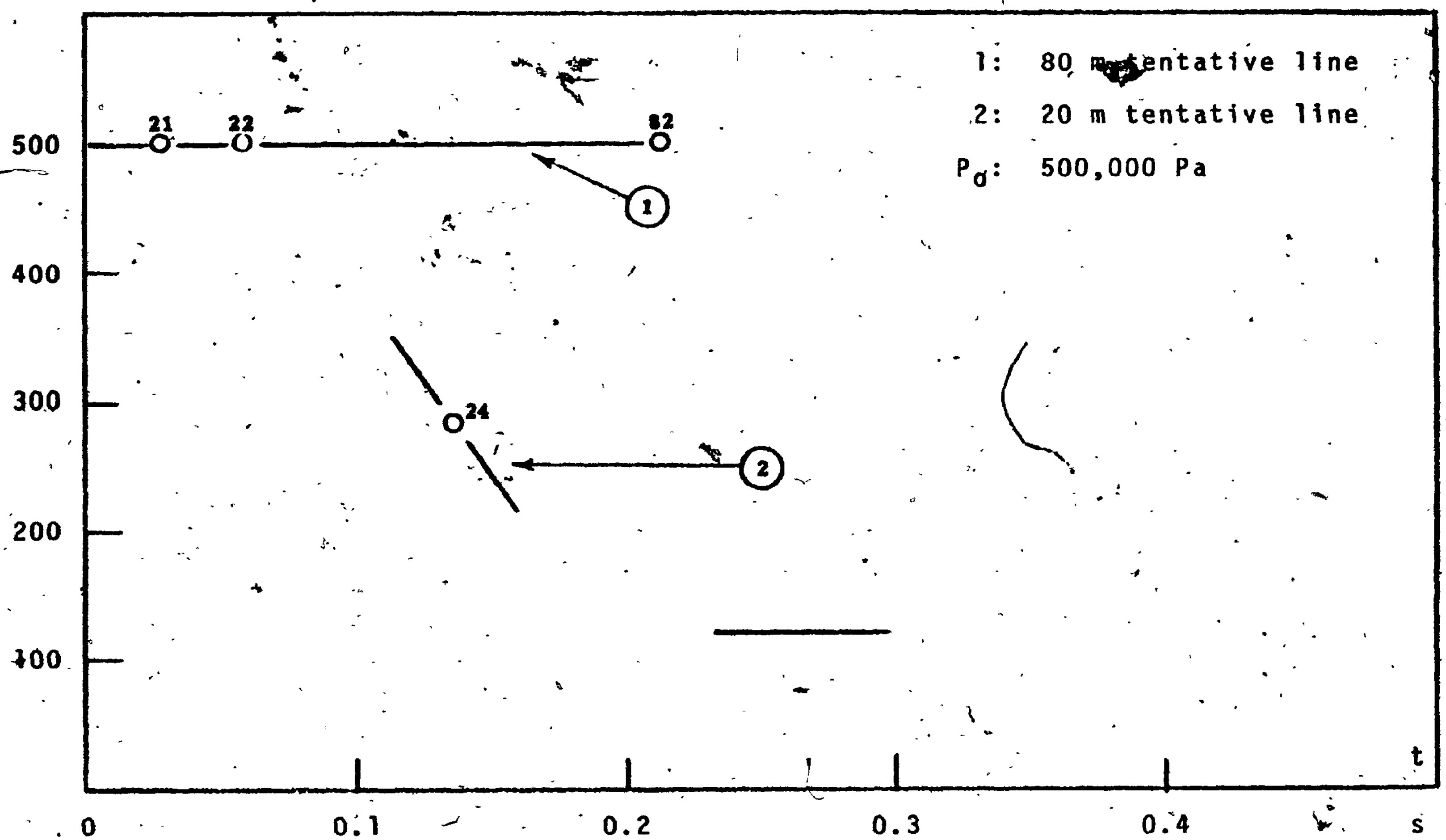



Diagram D 7.3.3.3 Characteristics Method. Pressure vs. Time at Various Pipe Locations. Distances in m from Pipe Left End. Case C.

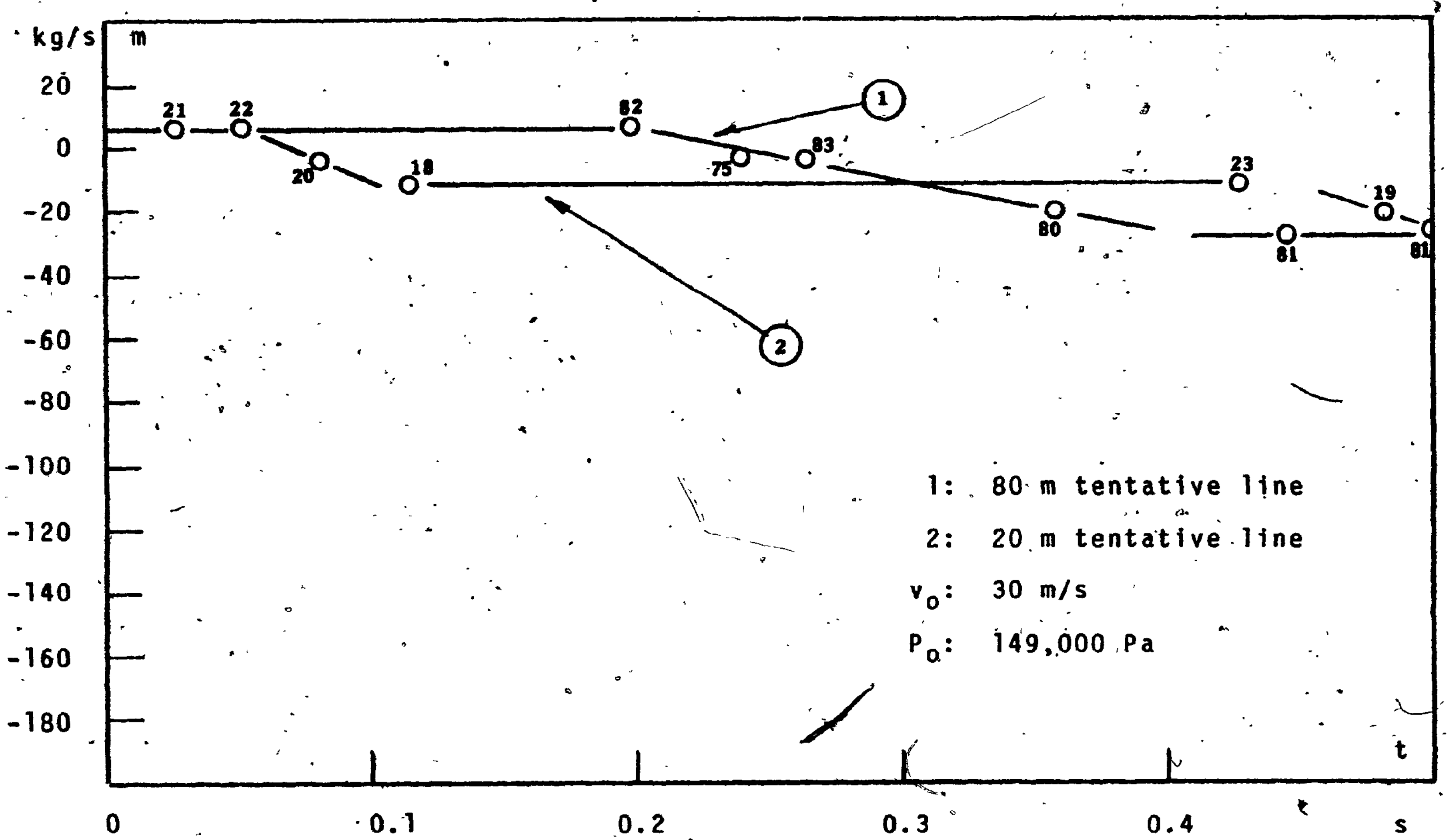



Diagram D 7.3.3.4 Characteristics Method. Flow Rate vs. Time at Various Pipe Locations. Distances in m from Pipe Left End. Case A.

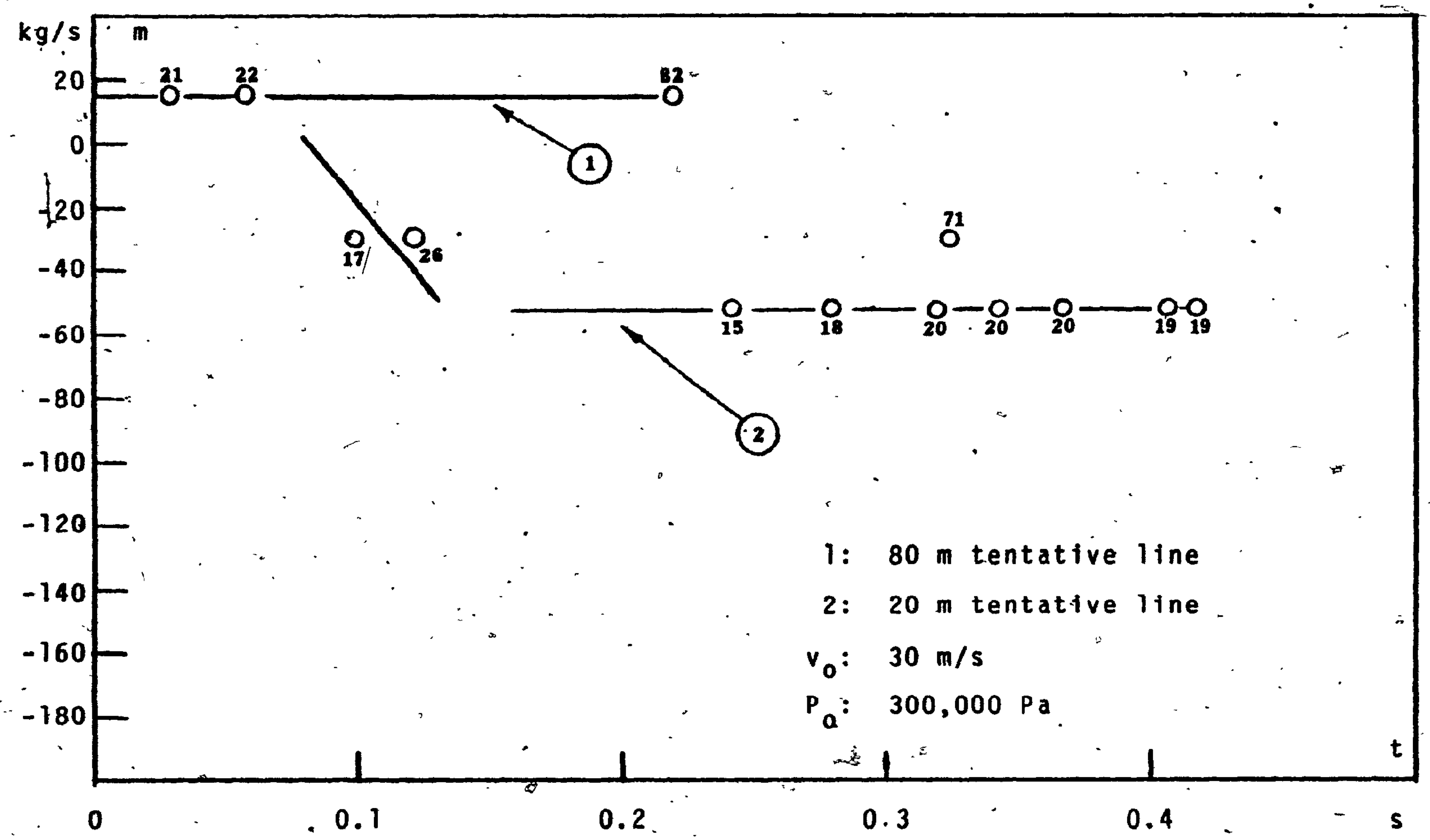



Diagram D 7.3.3.5 Characteristics Method. Flow Rate vs. Time at Various Pipe Locations. Distances in m from Pipe Left End. Case B.

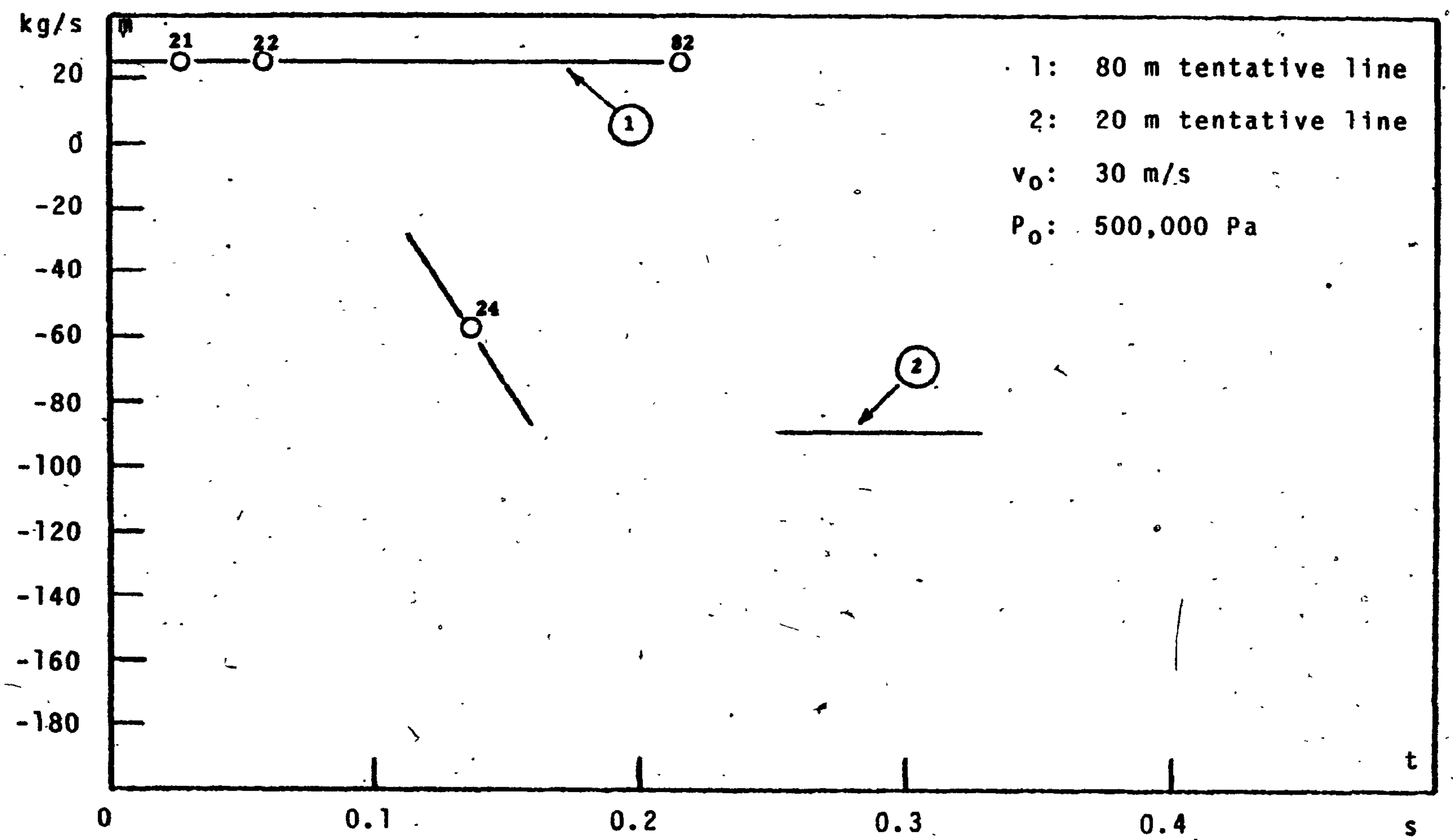



Diagram D 7.3.3.6 Characteristics Method. Flow Rate vs. Time at Various Pipe Locations. Distances in m from Pipe Left End. Case C.

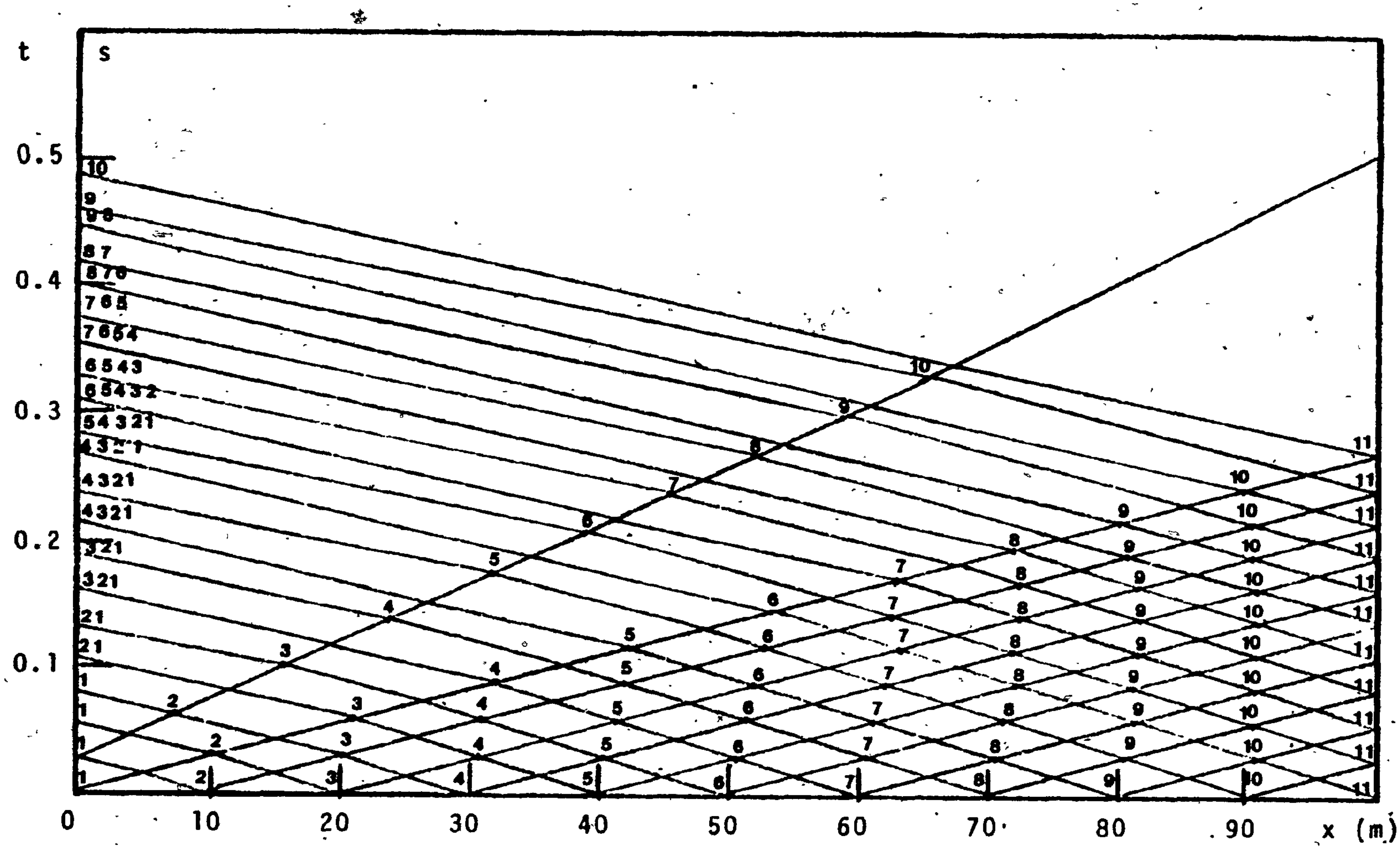



Diagram D 7.3.3.7 Characteristic Lines. Position of Net Modes. 11 Initially Equispaced Nodes are Indicated.

## 7.4 CONCLUSIONS

Numerical solutions of unsteady continuous flow in piping systems may be tested by comparing results for a typical case with the Method of Characteristics.

The Method of Characterisics, although accurate, does not comply with many practical requirements; mainly, it does not allow a regular analysis of the physical system by a proper orthogonal cross sectioning of the time-space domain.

The Test Case and Algorithm Method must be considered an acceptable alternative for an "initial" evaluation of numerical model, a necessary but not sufficient condition. A numerical model should always be experimentally tested. Successful comparison with the Method of Characteristics may be useful to gain industry confidence and cooperation for further developments.

CHAPTER 8

# UNSTEADY CONTINUOUS FLOW IN A PIPE LUMPED ELEMENT METHODS

## 8.1 SUMMARY

The continuity and momentum equations of an unsteady flow of a compressible fluid in a pipe element are represented by an elementary circuit. Pressure and mass flow are assumed as across and through variables respectively. The third unknown, density, requires a third relation: the entropy-energy equation.

An iterative solution, for density, of the discretized circuit integral equations is proposed. A second iteration, for mass flow, due to the non-linear convective term of the momentum equation, is required, but may be performed at the same time as the first one.

Three numerical schemes are proposed and derived:

: Linear conforming approximating functions with weighting function equal to unity (Finite Volumes Method).

: Linear conforming approximating functions with Galerkin weighting function (Finite Element Method).

: Linear conforming and non-conforming approximating functions with weighting function equal to unity (Finite Differences MAC Method). (21)

For a generic piping network and for each numerical approach, a global pseudolinear matrix, valid at a given time level, is obtained. Piping elements, other than pipes, are added as additional pipe sections, and boundary conditions introduced as additional or substitutional nodal relations.

## 8.2 CIRCUIT MODEL

### 8.2.1 UNSTEADY FLOW IN A PIPE.

Let us consider an infinitesimal element of pipe (Fig. F 8.2.1.1).

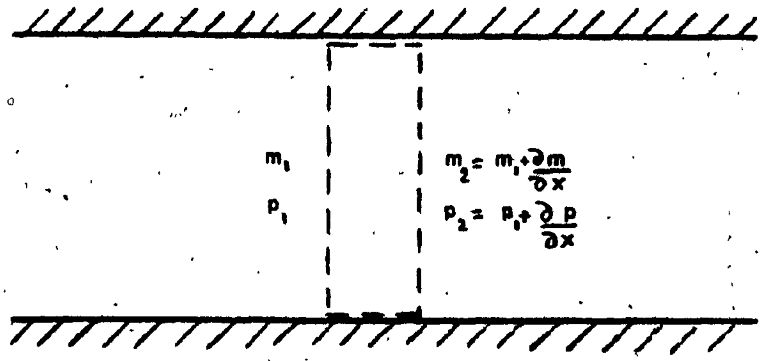



FIG. F8.2.1.1. Pipe Infinitesimal Element.

Using the (6.2.2.1), (6.2.2.2) and (6.2.2.15), The continuity, momentum and entropy equations may be written as:

$$dx*\left(\frac{\partial m}{\partial x}\right)=-\frac{\partial}{\partial t}(A*dx*\rho) \qquad (8.2.1.1\ )$$

$$\frac{\partial}{\partial t}\left(\frac{\dot m}{\rho*A}\right)+\frac{\dot m}{\rho*A}*\frac{\partial}{\partial x}\left(\frac{\dot m}{\rho*A}\right)+\frac{1}{\rho}*\left(\frac{\partial p}{\partial x}\right)+F=0 \qquad (8.2.1.2\ )$$

$$\left(\frac{p}{\rho^{\alpha}}\right)^{t+\Delta t}_{particle}=\left(\frac{p}{\rho^{\alpha}}\right)^{t}_{particle} \qquad (8.2.1.3\ )$$

with:

$$F=\left(\left(\frac{4*f}{D}\right)*\left(\frac{\dot m^2}{2*\rho^2*A^2}\right)*\frac{|\dot m|}{\dot m}\right)$$

$\alpha$= polytropic coefficient

for a viscous flow of an ideal gas. For an isentropic process the entropy equation becomes:

$$\frac{p}{\rho^{\gamma}}= \text{const.} \qquad (6.2.2.14)$$

Therefore,density can always be obtained:

$$(\rho)^{t+\Delta t}=(\rho)^{t}*\left(\frac{p^{t+\Delta t}}{p^{t}}\right)^{\frac{1}{\alpha}} \qquad (8.2.1.4\ )$$

Using a Taylor expansion the time derivative may be rewritten and the (8.2.1.1) and (8.2.1.2) become:

$$\frac{\partial \dot{m}}{\partial x} = -A*\left(\left(\frac{\rho^{n+1}-\rho^{n}}{t^{n+1}-t^{n}}\right)-R_{\rho}\right)$$

$$\frac{1}{\rho*A}*\left(\left(\frac{\dot{m}^{n+1}-\dot{m}^{n}}{t^{n+1}\ t^{n}}\right)-R_{m}\right)-\frac{\dot{m}}{\rho^{2}*A}*\left(\left(\frac{\rho^{n+1}-\rho^{n}}{t^{n+1}\ t^{n}}\right)-R_{\rho}\right)+\frac{\dot{m}}{\rho*A}*\left(\frac{\partial}{\partial x}\left(\frac{\dot{m}}{\rho*A}\right)+\frac{1}{\rho}*\left(\frac{\partial p}{\partial x}\right)+F=0\right.$$

with:

$$R_{\rho}=\frac{\Delta t}{2!}*\left(\frac{\partial^{2}\rho}{\partial t^{2}}\right)+\frac{\Delta t^{2}}{3!}*\left(\frac{\partial^{3}\rho}{\partial t^{3}}\right)+\ldots.$$

$$R_{m}=\frac{\Delta t}{2!}*\left(\frac{\partial^{2}\dot{m}}{\partial t}\right)+\frac{\Delta t^{2}}{3!}*\left(\frac{\partial^{3}\dot{m}}{\partial t^{3}}\right)+\ldots.$$

$$\Delta t=(t^{n+1}-t^{n})$$

or, neglecting all residuals, i.e. introducing $O(\Delta t)$ errors:

$$\frac{\partial m}{\partial x}=-A*\left(\frac{\rho^{n+1}-\rho^{n}}{\Delta t}\right)$$

$$\frac{1}{\rho*A}*\left(\frac{\dot{m}^{n+1}-\dot{m}^{n}}{\Delta t}\right)-\frac{\dot{m}}{\rho^{2}*A}*\left(\frac{\rho^{n+1}-\rho^{n}}{\Delta t}\right)+\frac{\dot{m}}{\rho*A}*\left(\frac{\partial}{\partial x}\left(\frac{\dot{m}}{\rho*A}\right)\right)+\frac{1}{\rho}*\left(\frac{\partial p}{\partial x}\right)+F=0$$

Expanding and combining,we obtain at each point of the 2D domain:

$$\frac{\partial m}{\partial x}+\frac{A}{a^{2}*}\frac{*(p^{n+1}-p^{n})}{\Delta t}=0 \qquad (8.2.1.5)$$

$$\frac{\partial p}{\partial x}+F*\rho-\dot{m}*\left(\left(\frac{\dot{m}}{\rho^{2}*A}*\left(\frac{\partial \rho}{\partial x}\right)\right)+\frac{2}{\rho*A}*\left(\frac{\rho^{n+1}-\rho^{n}}{\Delta t}\right)\right)+\frac{1}{A}*\left(\frac{\dot{m}^{n+1}-\dot{m}^{n}}{\Delta t}\right)=0 \qquad (8.2.1.6)$$

with:

$$a^2 = \frac{(p^{n+1} - p^n)}{(Q^{n+1} - Q^n)} \qquad (8.2.1.7)$$

## 8.2.2 CIRCUIT APPROACH.

Let us consider an infinitesimal circuit with m and p as through and across variables (Fig. F 8.2.2.1).

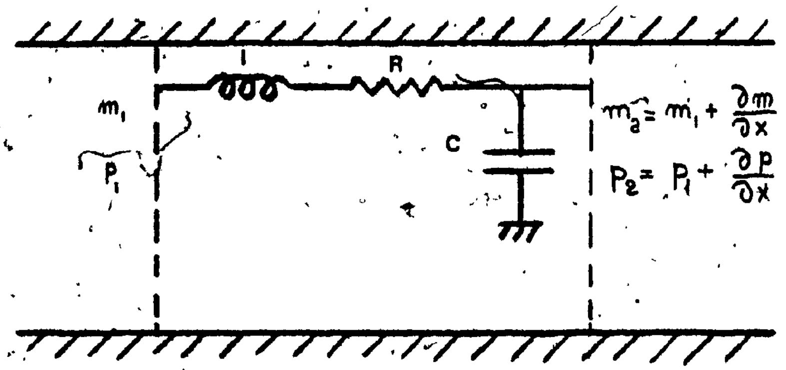



FIG. F 8.2.2.1. Circuit Representation.

The circuit equations are:

$$\dot{m}_1^{n+1} - \dot{m}_2^{n+1} = C*\left(\frac{dp}{dt}\right) \approx C*\left(\frac{p^{n+1} - p^n}{\Delta t}\right) \qquad (8.2.2.1)$$

$$p_2^{n+1} - p_1^{n+1} = I*\left(\frac{d\dot{m}}{dt}\right) + R*\dot{m} \approx I*\left(\frac{\dot{m}^{n+1} - \dot{m}^n}{\Delta t}\right) + R*m \qquad (8.2.2.2)$$

These equations may represent continuity and momentum assuming:

$$C=\frac{A}{a^2}*dx \qquad (8.2.2.3)$$

$$I=\frac{dx}{A} \qquad (8.2.2.4)$$

$$R=R_1+R_2+F*\rho*dx \qquad (8.2.2.5)$$

with:

$$R_1=-\frac{\dot{m}}{\rho^2*A^2}*(\frac{\partial\rho}{\partial x})*dx \qquad (8.2.2.6)$$

$$R_2=-\frac{2}{\rho*A}*(\frac{\rho^{n+1}-\rho^{n}}{\Delta t})*dx \qquad (8.2.2.7)$$

Hence, the pipe element of Fig. F 8.2.2.2 a has the circuit representation of Fig. F 8.2.2.2 b and the element continuity and momentum equations can be written as:

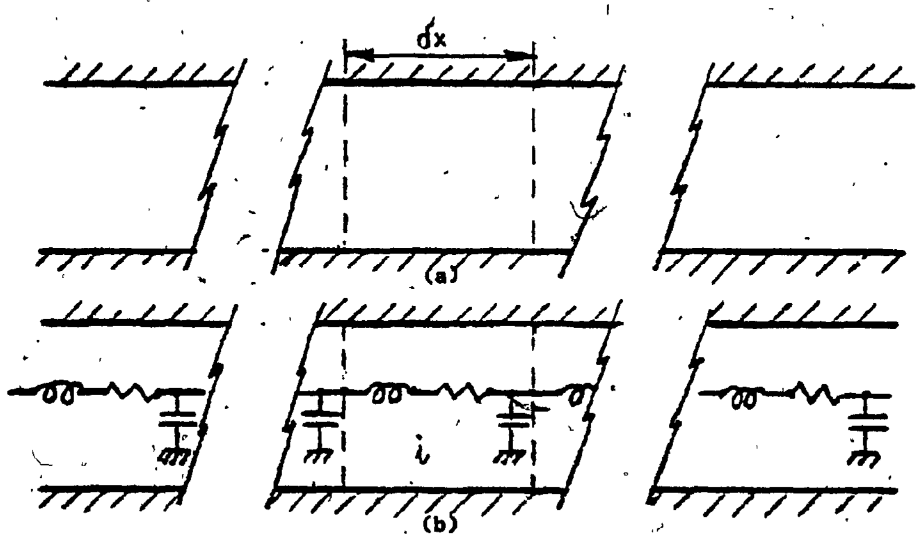



FIG. F8.2.2.2. Pipe element.

$$m_a^{m+1} - m_b^{m+1} = \sum_{i}^{N} C_i^* \frac{(p_i^{m+1} - p_i^{m})}{\Delta t}$$

$$p_a^{m+1} - p_b^{m+1} = \sum_{i}^{N} I_i^* \frac{(m_i^{m+1} - m_i^{m})}{\Delta t} + \sum_{i}^{m} R_i^* m_i^{m+1}$$

or in integral form:

$$m_a^{m+1} - m_b^{m+1} = \int_a^b \bar{C}^* \frac{(p^{m+1} - p^{m})}{\Delta t} dx \qquad (8.2.2.8)$$

$$p_a^{m+1} - p_b^{m+1} = \int_a^b \left( \bar{I}^* \frac{(\dot{m}^{m+1} - \dot{m}^{m})}{\Delta t} + \bar{R}^* \dot{m}^{m+1} \right) dx \qquad (8.2.2.9)$$

with:

$$\bar{C} = \frac{C}{dx} \; ; \; \bar{I} = \frac{I}{dx} \; ; \; \bar{R} = \frac{R}{dx} \qquad (8.2.2.10,11,12)$$

The fluid compressibility is represented by two variable resistors, depending on the density time and space derivatives. The space derivative resistor depends also on the local mass flow and is the non-linear factor of the system equations.

## 8.3 UNSTEADY FLOW IN A PIPE ELEMENT

### 8.3.1 SOLUTION BY ITERATION.

The circuit representation and the numerical approximation of the time derivatives has reduced the continuity and momentum equations to two space integral equations in p, $\dot{m}$, q. The discretized entropy equation gives the third relation.

Using equation (8.2.1.7 ) as density relation, an iterative procedure (Fig. F 8.3.1.1) can be used to solve for p, $\dot{m}$, q the set of equations at a given time level, using the known values of pressure, mass flow and density of the preceding time level.

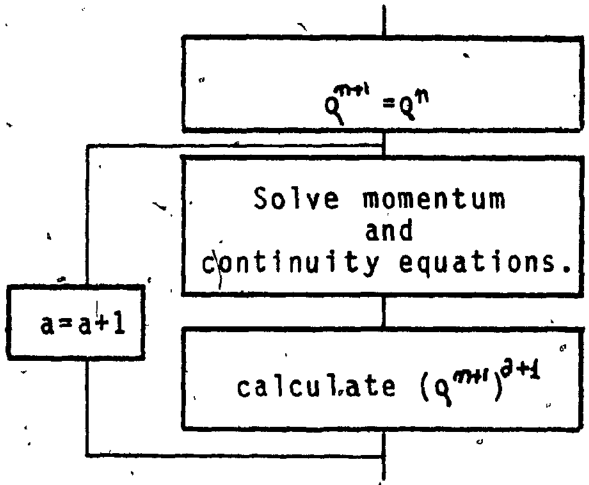



FIG. F 8.3.1.1. Density iteration.

The type of discretization of the continuity and momentum integral equations characterizes the type of algorithm to be solved at each iteration.

8.3.2 LINEAR APPROXIMATING FUNCTIONS.

Let us consider the pipe element of Fig. F 8.3.2.1 and its continuity and momentum equations (8.2.2.6) and (8.2.2.7):

Inside the element, C and I may be considered constant and written as:

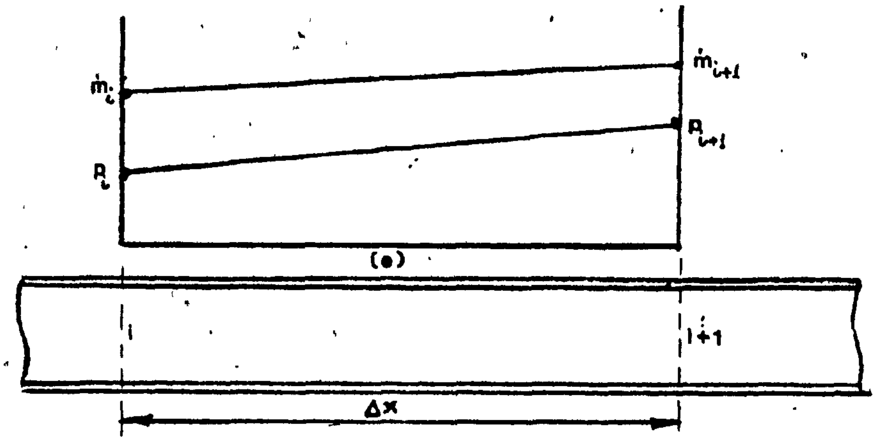



FIG. F 8.3.2.1 Linear approximating Functions.

$$\overline{C}^{(8)} = \frac{4*A}{(a_i + a_{i+1})^2} \quad ; \quad \overline{I}^{(8)} = \frac{1}{A} \qquad (8.3.2.1,\ 2)$$

Remembering the (8.2.2.3) and (8.2.2.10) the resistance $\overline{R}_1$ can be discretized as:

$$\overline{R}_1 = \frac{(-m^{n+1})*(a_{i+1}-a_i)^{n+1}}{(A*(\frac{a_i+a_{i+1}}{2})^{n+1})^2*\Delta x} = R*m \qquad (8.3.2.3)$$

the resistance $\overline{R}_2$ as:

$$\overline{R}_2 = \frac{-4*(a_{i+1}+a_i)^{n+1} - (a_i+a_{i+1})^n}{2*A*(a_i+a_{i+1}) * \Delta t} \qquad (8.3.2.4)$$

and the viscous factor as:

$$\rho*F = \frac{(a_i+a_{i+1})*(f*\dot{m}^{n+1}*|\dot{m}^n|)}{2*D*A*(a_i+a_{i+1})^2} = \overline{R}_3*\dot{m}^{n+1} \qquad (8.3.2.5)$$

The continuity and momentum equations become:

$$\dot{m}_{i+1}^{n+1}-\dot{m}_i^{n+1}=\overline{C}^{(e)}\int_{x_i}^{x_{i+1}}\frac{(p^{n+1}-p^n)}{\Delta t}dx \qquad (8.3.2.6)$$

$$p_{i+1}^{n+1}-p_i^{n+1}=\overline{I}^{(e)}\int_{x_i}^{x_{i+1}}\frac{(\dot{m}^{n+1}-\dot{m}^n)}{\Delta t}dx+\overline{R}_1^{(e)}\int_{x_i}^{x_{i+1}}(\dot{m}^{n+1})^2*dx+\overline{R}_2^{(e)}\int_{x_i}^{x_{i+1}}(\dot{m}^{n+1})*dx \qquad (8.3.2.7)$$

$$+\overline{R}_3^{(e)}\int_{x_i}^{x_{i+1}}(\dot{m}^{n+1})*dx$$

Inside the element, pressure and mass flow may be approximated with linear functions:

$$p^{(e)}=N_i^{(e)}*p_i+N_{i+1}^{(e)}*p_{i+1} \qquad (8.3.2.8)$$

$$\dot{m}^{(e)}=N_i^{(e)}*\dot{m}_i+N_{i+1}^{(e)}*\dot{m}_{i+1} \qquad (8.3.2.9)$$

with the "shape" functions:

$$N_i^{(e)}=\frac{x_{i+1}-x}{x_{i+1}-x_i} \quad ; \quad N_{i+1}^{(e)}=\frac{x-x_i}{x_{i+1}-x_i} \qquad (8.3.2.10,\ 11)$$

Then, continuity and momentum equations become:

$$m_i^{n+1} - m_{i+1}^{n+1} = \overline{C}^{(e)} \int_{x_i}^{x_{i+1}} \frac{((N_i * p_i + N_{i+1} * p_{i+1})^{n+1} - (N_i * p_i + N_{i+1} * p_{i+1})^{n})}{\Delta t} * dx \quad (8.3.2.12)$$

$$p_i^{n+1} - p_{i+1}^{n+1} = \overline{I}^{(e)} \int_{x_i}^{x_{i+1}} \frac{((N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1})^{n+1} - (N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1})^{n})}{\Delta t} * dx \quad (8.3.2.13)$$

$$+ \overline{\overline{R}}_1^{(e)} \int ((N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1})^2) * dx$$

$$+ \overline{R}_2^{(e)} \int (N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1}) * dx$$

$$+ \overline{R}_3^{(e)} \int ((N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1}) * dx$$

and, for an isentropic flow ($\overline{R}_3 = 0$):

$$(m_i - m_{i+1})^{n+1} = \left(\frac{\overline{C}^{(e)} * \Delta x}{2 * \Delta t}\right) * ((p_i + p_{i+1})^{n+1} - (p_i + p_{i+1})^{n}) \quad (8.3.2.14)$$

$$(p_i - p_{i+1})^{n+1} = \left(\frac{I * \Delta x}{2 * \Delta t}\right) * ((m_i + m_{i+1})^{n+1} - (m_i + m_{i+1})^{n}) \quad (8.3.2.15)$$

$$+ \overline{\overline{R}}_1 * \Delta x * \left( \left(\frac{\dot{m}_i}{3} + \frac{\dot{m}_{i+1}}{6}\right) * \dot{m}_i + \left(\frac{\dot{m}_i}{6} + \frac{\dot{m}_{i+1}}{3}\right) * \dot{m}_{i+1} \right)^{n+1}$$

$$+ R_2 * \Delta x * (m_i + m_{i+1})^{n+1}$$

or in matrix form:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \begin{Bmatrix} m \\ p \\ m \\ p \end{Bmatrix}^{n+1} = \begin{Bmatrix} \left(\frac{\overline{C}^{(e)} * \Delta x}{2 * \Delta t}\right) * (p_i + p_{i+1}) \\ \left(\frac{\overline{I}^{(e)} * \Delta x}{2 * \Delta t}\right) * (m_i + m_{i+1}) \end{Bmatrix}^{n} \quad (8.3.2.16)$$

with:

$k_{11} = -1$

$$k_{12} = \left(\frac{\overline{C}^{(e)} * \Delta x}{2 * \Delta t}\right)$$

$k_{13} = -k_{11}$

$k_{14} = k_{12}$

$$k_{21} = \left(\frac{\overline{I}^{(e)} * \Delta x}{2 * \Delta t}\right) + \overline{\overline{R}}_1 * \Delta x * \left(\frac{m_i}{3} + \frac{m_{i+1}}{6}\right)^{m+1} + \overline{R}_2 * \Delta x$$

$k_{22} = k_{11}$

$$k_{23} = \left(\frac{\overline{I}^{(e)} * \Delta x}{2 * \Delta t}\right) + \overline{\overline{R}}_1 * \left(\frac{m_i}{6} + \frac{m_{i+1}}{3}\right)^{m+1} + \overline{R}_2 * \Delta x$$

$k_{24} = 1$

8.3.3 FINITE ELEMENTS.

In numerical terms what has been done in subpar.8.3.1 can be defined as an application of the Finite Volumes Method (22), i.e. a weighted residual method with weighting function equal to unity within the element and zero elsewhere. As the basic differential equation expresses a balance over an infinitesimal control volume, the finite volume equation is simply the finite counterpart of the differential equation.

Along this line, an application of the Galerkin method represents a logical alternative (23), (24). The Galerkin method, a weighted residual non variational finite element approach, is based, as are all the finite element methods, on the concept of approximating a continuous function by a discrete model, composed of a set of piecewise continuous functions which are defined over a finite number of subdomains called elements.

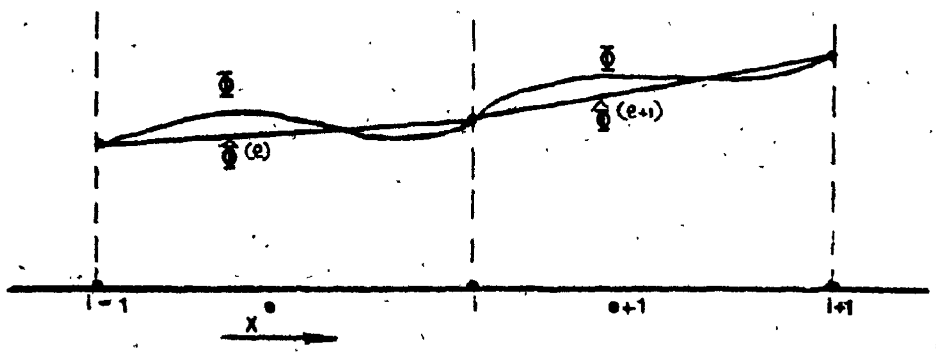



FIG. F 8.3.3.1 Finite Elements.

The most popular form of element function is the polynomial. For a linear element (Fig. F 8.3.3.1):

$$\hat{\Phi}^{(e)} = a^{(e)} + b^{(e)} * x$$

which, as function of the unknown nodal values, becomes:

$$\hat{\Phi} = N^{(e)}_{i-1} * \hat{\Phi}_{i-1} + N_i * \hat{\Phi}_i$$

with:

$$N_i = \frac{x_i - x}{x_i - x_{i-1}} \quad ; \quad N = \frac{x - x_{i-1}}{x_i - x_{i-1}} \qquad (8.3.1.8, 9)$$

called "shape" functions.

The application of the weighted residual method to a differential equation gives:

$$\int W*(\frac{\partial \Phi}{\partial x})*dx=0$$

where O is the assumed distribution of O in terms of nodal values, and W is the weighting function. The integration is performed over the whole calculation domain; however, if W is zero over most of the domain, only a small domain portion remains active into the integral equation. If we choose the shape functions $N_i$ (i=1,2) as the weighting functions, we get the Galerkin method equations, in this case two for each element.

This procedure applied to:

$$-\frac{\partial \dot{m}}{\partial x}=\overline{C}*(\frac{p^{m+1}-p^{m}}{\Delta t}) \qquad \text{(continuity)} \qquad (8.2.2.1)$$

$$-\frac{\partial p}{\partial x}=\overline{I}*(\frac{\dot{m}^{m+1}-\dot{m}^{m}}{\Delta t})+R*\dot{m}^{m+1} \qquad \text{(momentum)} \qquad (8.2.2.2)$$

and to the element (e) of Fig. F 8.32.1, gives:

$$-\int_{x_{i-1}}^{x_i}\left(\frac{\partial \dot{m}}{\partial x}\right)^{n+1} * N_{i-1} * dx = \overline{C}^{(i)}\int_{x_{i-1}}^{x_i}\left(\frac{p^{n+1}-p^{n}}{\Delta t}\right) * N_{i-1} * dx \qquad (8.3.3.1)$$

$$-\int_{x_{i-1}}^{x_i}\left(\frac{\partial \dot{m}}{\partial x}\right)^{n+1} * N_{i} * dx = \overline{C}^{(i)}\int_{x_{i-1}}^{x_i}\left(\frac{p^{n+1}-p^{n}}{\Delta t}\right) * N_{i} * dx \qquad (8.3.3.2)$$

$$-\int_{x_i}^{x_{i+1}}\left(\frac{\partial p}{\partial x}\right)^{n+1} * N_{i-1} * dx = \overline{I}^{(i)}\int_{x_i}^{x_{i+1}}\left(\frac{\dot{m}^{n+1}-\dot{m}^{n}}{\Delta t}\right) * N_{i-1} * dx + \int_{x_i}^{x_{i+1}}\overline{R}^{(i)} * \dot{m}^{n+1} * N_{i-1} * dx \qquad (8.3.3.3)$$

$$-\int_{x_i}^{x_{i+1}}\left(\frac{\partial p}{\partial x}\right)^{n+1} * N_{i} * dx = \overline{I}^{(i)}\int_{x_i}^{x_{i+1}}\left(\frac{\dot{m}^{n+1}-\dot{m}^{n}}{\Delta t}\right) * N_{i} * dx + \int_{x_i}^{x_{i+1}}\overline{R}^{(i)} * \dot{m}^{n+1} * N_{i} * dx \qquad (8.3.3.4)$$

For an isentropic flow ($\overline{R}_s = 0$), with the same notations of subpar.8.32, and introducing:

$$p^{(i)} = N_i * p_i + N_{i+1} * p_{i+1}$$
$$\dot{m}^{(i)} = N_i * \dot{m}_i + N_{i+1} * \dot{m}_{i+1}$$

equations (8.3.3.1, 2, 3, 4) become:

$$\left(\frac{-1}{2}\right) * [-1,1] * \begin{bmatrix} m_i \\ m_{i+1} \end{bmatrix} - \overline{C}^{(i)} * \left(\frac{\Delta x}{\Delta t}\right) * \left[\frac{1}{3},\frac{1}{6}\right] * \begin{bmatrix} p_i \\ p_{i+1} \end{bmatrix} = -\overline{C}^{(i)} * \left(\frac{\Delta x}{\Delta t}\right) * \left[\frac{1}{3},\frac{1}{6}\right] * \begin{bmatrix} p_i \\ p_{i+1} \end{bmatrix}$$

$$\left(\frac{-1}{2}\right) * [-1,1] * \begin{bmatrix} m_i \\ m_{i+1} \end{bmatrix} - \overline{C}^{(i)} * \left(\frac{\Delta x}{\Delta t}\right) * \left[\frac{1}{6},\frac{1}{3}\right] * \begin{bmatrix} p_i \\ p_{i+1} \end{bmatrix} = -\overline{C}^{(i)} * \left(\frac{\Delta x}{\Delta t}\right) * \left[\frac{1}{6},\frac{1}{3}\right] * \begin{bmatrix} p_i \\ p_{i+1} \end{bmatrix}$$

for the continuity, and:

$$(-\frac{1}{2})*[-1,1]*\begin{bmatrix}p_i\\p_{i+1}\end{bmatrix}$$

$$-(\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*[\frac{1}{3},\frac{1}{6}]+\bar{R}_2^{(i)}*\Delta x*[\frac{1}{3},\frac{1}{6}]+\bar{\bar{R}}_1^{(i)}*\Delta x*[(\frac{\dot{m}_i}{4}+\frac{\dot{m}_{i+1}}{12})^{n+1},(\frac{\dot{m}_i}{12}+\frac{\dot{m}_{i+1}}{12})^{n+1}]*\begin{bmatrix}\dot{m}_i\\\dot{m}_{i+1}\end{bmatrix}$$

$$=\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*[\frac{1}{3},\frac{1}{6}]*\begin{bmatrix}\dot{m}_i\\\dot{m}_{i+1}\end{bmatrix}$$

$$(-\frac{1}{2})*[-1,1]*\begin{bmatrix}p_i\\p_{i+1}\end{bmatrix}$$

$$-(\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*[\frac{1}{6},\frac{1}{3}]+\bar{R}_2^{(i)}*\Delta x*[\frac{1}{6},\frac{1}{3}]+\bar{\bar{R}}_1^{(i)}*\Delta x*[(\frac{\dot{m}_i}{12}+\frac{\dot{m}_{i+1}}{12})^{n+1},(\frac{\dot{m}_i}{12}+\frac{\dot{m}_{i+1}}{4})^{n+1}]*\begin{bmatrix}\dot{m}_i\\\dot{m}_{i+1}\end{bmatrix}$$

$$=\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*[\frac{1}{6},\frac{1}{3}]*\begin{bmatrix}\dot{m}_i\\\dot{m}_{i+1}\end{bmatrix}$$

for the momentum.

In matrix form it is:

$$\begin{bmatrix}k_{11}&k_{12}&k_{13}&k_{14}\\k_{21}&k_{22}&k_{23}&k_{24}\\k_{31}&k_{32}&k_{33}&k_{34}\\k_{41}&k_{42}&k_{43}&k_{44}\end{bmatrix}\begin{Bmatrix}\dot{m}_i\\p_i\\\dot{m}_{i+1}\\p_i\end{Bmatrix}^{n+1}=\begin{Bmatrix}-\bar{C}^{(i)}*(\frac{\Delta x}{\Delta t})*(\frac{p_i}{3}+\frac{p_{i+1}}{6})\\-\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*(\frac{\dot{m}_i}{3}+\frac{\dot{m}_{i+1}}{6})\\-\bar{C}^{(i)}*(\frac{\Delta x}{\Delta t})*(\frac{p_i}{6}+\frac{p_{i+1}}{3})\\-\bar{I}^{(i)}*(\frac{\Delta x}{\Delta t})*(\frac{\dot{m}_i}{6}+\frac{\dot{m}_{i+1}}{3})\end{Bmatrix}^{n}$$

with:

$$k_{11} = 0.5$$

$$k_{12} = -\frac{\overline{C}^{(i)}}{3} * \left(\frac{\Delta x}{\Delta t}\right)$$

$$k_{13} = -0.5$$

$$k_{14} = -\frac{\overline{C}^{(i)}}{6} * \left(\frac{\Delta x}{\Delta t}\right)$$

$$k_{21} = -\left(\frac{I}{3} * \left(\frac{\Delta x}{\Delta t}\right) + \frac{R_2^{(i)}}{3} * \Delta x + \frac{\overline{\overline{R}}_1^{(i)}}{4} * \Delta x * \left(\dot{m}_i + \frac{\dot{m}_{i+1}}{3}\right)^{m+1}\right)$$

$$k_{22} = 0.5$$

$$k_{23} = -\left(\frac{\overline{I}^{(i)}}{6} * \left(\frac{\Delta x}{\Delta t}\right) + \frac{R_2^{(i)}}{6} * \Delta x + \frac{\overline{\overline{R}}_1^{(i)}}{12} * \Delta x * \left(\dot{m}_i + \dot{m}_{i+1}\right)\right)$$

$$k_{24} = -0.5$$

$$k_{31} = 0.5$$

$$k_{32} = k_{14}$$

$$k_{33} = -0.5$$

$$k_{34} = k_{12}$$

$$k_{41} = k_{23}$$

$$k_{42} = 0.5$$

$$k_{43} = -\left(\frac{\overline{I}^{(i)}}{3} * \left(\frac{\Delta x}{\Delta t}\right) + \frac{R_2^{(i)}}{3} * \Delta x + \frac{\overline{\overline{R}}_1^{(i)}}{4} * \Delta x * \left(\frac{\dot{m}_i}{3} + \dot{m}_{i+1}\right)\right)$$

$$k_{44} = -k_{11}$$

8.3.4 FINITE DIFFERENCES.

The shape functions, $N_i$, are constructed, for practical reasons, in a piecewise manner (23), i.e. using a different definition within each element of given length. The question of inter-element continuity is important in their choice. Continuity of approximating functions must be such

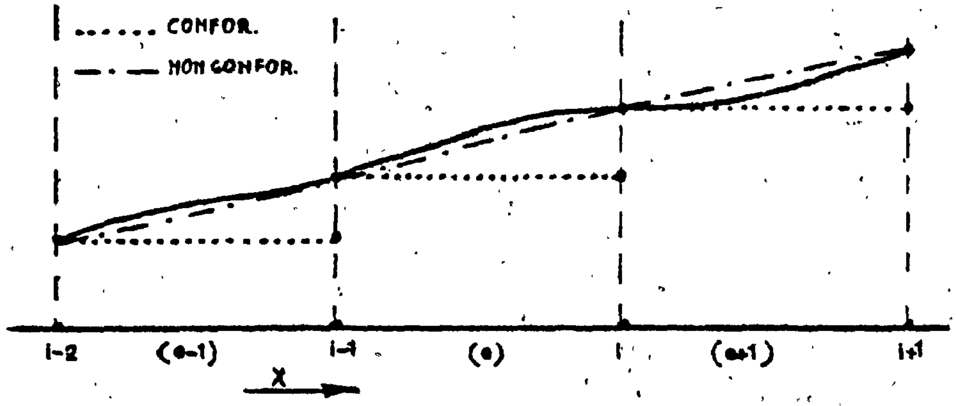



FIG. 8.3.4.1 Conforming and Non Conforming Functions.

that the element integrals can be evaluated directly, without any contribution arising at the element interfaces. Alternatively, such inter-element contribution must be of a kind that decrease continuously with the fineness of element subdivisions. The class of functions satisfying the first condition shall be called conforming and the ones satisfying the second non-conforming (but usually compatible).

Many non-conforming elements have been implemented in practice and convergence proof obtained. Sometime, these non conforming elements have proven to produce results of higher accuracy than corresponding conforming ones. The reason for these apparently disconcerting results has been explained by O.C. ZIENKIEWICZ and used by IRONS and REZZAQUE (25).

In our case, the continuity and momentum equations

(8.3.2.6) and (8.3.2.7) may be discretized and solved assuming the weighting function equal to unity and choosing as approximating functions:

| | |
|---|---|
| $\hat{\dot{m}}=N_i*\dot{m}_i+N_{i+1}*\dot{m}_{i+1}$ | for the mass flow space derivative. (conforming) |
| $\hat{p}=N_i*p_i+N_{i+1}*p_{i+1}$ | for the pressure space derivative. (conforming) |
| $\hat{\dot{m}}=\dot{m}_{i+1}$ | for the mass flow time derivative and R term. (non conforming) |
| $\hat{p}=p_i$ | for the pressure time derivative. (non conforming) |

For an isentropic flow and a generic element equations (8.3.2.6) and (8.3.2.7) become:

$$(m_i-m_{i+1})^{n+1}=\bar{C}^{(n)}*\left(\frac{\Delta x}{\Delta t}\right)*(p_i^{n+1}-p_i^{n})$$

$$(p_i-p_{i+1})^{n+1}=\bar{I}^{(n)}*\left(\frac{\Delta x}{\Delta t}\right)*(m_{i+1}^{n+1}-m_{i+1}^{n})+(\bar{R}_2^{(n)}*\Delta x+\bar{\bar{R}}_1^{(n)}*\Delta x*(m_{i+1})^{n+1})*(m_{i+1})^{n+1}$$

which, reorganized and in matrix form, give a pseudolinear finite difference scheme:

$$\begin{bmatrix} -1 & k_{12} & 1 & 0 \\ 0 & -1 & k_{23} & 1 \end{bmatrix} \begin{Bmatrix} \dot{m}_i \\ p_i \\ \dot{m}_{i+1} \\ p_{i+1} \end{Bmatrix}^{n+1} = \begin{bmatrix} \overline{C}^{(e)} * ((\Delta x)/(\Delta t)) * p_i \\ \overline{I}^{(e)} * ((\Delta x)/(\Delta t)) * \dot{m}_{i+1} \end{bmatrix}^{n} \quad (8.3.4.1)$$

with:

$$k_{12} = \overline{C}^{(e)} * (\Delta x)/(\Delta t)$$

$$k_{23} = \overline{I}^{(e)} * ((\Delta x)/(\Delta t)) + \overline{R}_2 * \Delta x + \overline{\overline{R}}_1 * \Delta x * (\dot{m}_{i+1})^{n}$$

## 8.4 GLOBAL MATRICES

### 8.4.1 LINEAR ELEMENT MATRIX.

Let us consider a piping network divided into N pipe elements (Fig. F 8.4.1.1), with corresponding NNODE nodes.

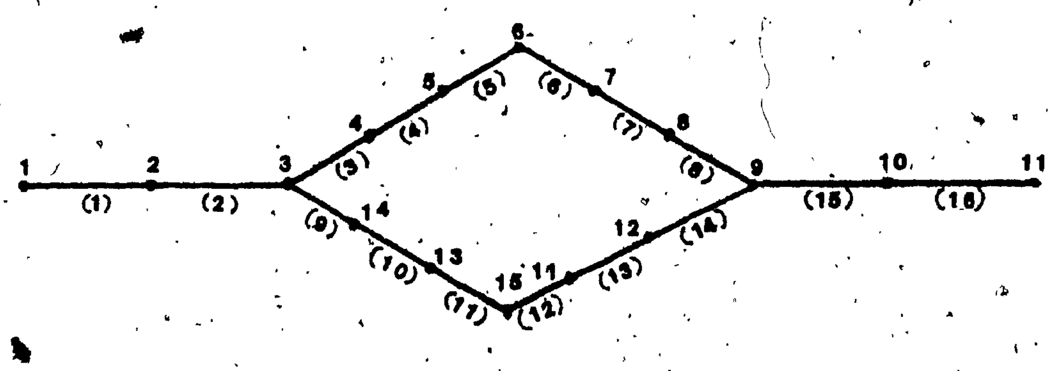



FIG. F 8.4.1.1 Piping Network Divided in Pipe Elements.

The continuity and momentum equations for a generic element are given by:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \end{bmatrix} \begin{Bmatrix} \dot{m}_i \\ p_i \\ \dot{m}_j \\ p_j \end{Bmatrix}^{n+1} = \begin{Bmatrix} \frac{\bar{C}^{(n)}}{2} * \left(\frac{\Delta x}{\Delta t}\right) * (\dot{p}_i + p_j) \\ \frac{\bar{I}^{(n)}}{2} * \left(\frac{\Delta x}{\Delta t}\right) * (\dot{m}_i + \dot{m}_j) \end{Bmatrix}^{n} \quad (8.3.2.16)$$

A pipe branching (nodes 3,9) may be represented by the steady flow relations:

$$\dot{m}_3^{(1)} = \dot{m}_3^{(2)} + \dot{m}_3^{(3)} \quad (8.4.1.1)$$

$$p_3^{(1)} = p_3^{(2)} \quad (8.4.1.2)$$

$$p_3^{(1)} = p_3^{(3)} \quad (8.4.1.3)$$

and these relations introduced into the pseudolinear system as additional relations. Similarly, elbows, obstructions, valves, etc. and other pipe components may also be represented in matrix form by their steady state momentum and continuity equations, i.e. considered as addititonal piping elements of proper length (their equivalent length) in quasi-steady conditions.

At boundaries, i.e. at the entering and exiting ends of the piping system, pressure OR mass flow are known, in general, as time dependent variables.

The element pseudolinear equations, with the addition of branching and piping additional elements steady equations, give the momentum and continuity pseudolinear system of equations for the entire network at a given instant time. In matrix form it is:

$$\left[K_{FV}\right]\begin{Bmatrix}\dot{m}_1\\ p_1\\ \vdots\\ \dot{m}_{NNODE}\\ p_{NNODE}\end{Bmatrix}=\left\{H\right\} \qquad (8.4.1.4)$$

where:

: the matrix $\left[K_{FV}\right]$, of dimension 2*NNODEx2*NNODE, is function of the element parameters $\overline{C}^{(e)}, \overline{I}^{(e)}, \overline{R}^{(e)}, \Delta x^{(e)}$ and of the time increment $\Delta t$.

: the vector $\begin{Bmatrix}\dot{m}_1\\ p_1\\ \vdots\\ \dot{m}_{NNODE}\\ p_{NNODE}\end{Bmatrix}$ contains all the unknowns.

: the vector $\left\{H\right\}$ is a function of the element parameters, of the time increment, of the preceding time level pressure and mass flow nodal values and of the boundary conditions.

All the piping system equations must be considered, combined and ordered according to an arbitrary node numeration system.

## 8.4.2 FINITE ELEMENT MATRIX.

With reference to the network of Fig. F 8.4.1.1 the continuity and momentum equations, approximated according to the Finite Element Method, give, for a generic element:

$$\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} \dot{m}_i \\ p_i \\ \dot{m}_j \\ p_j \end{Bmatrix}^{n+1} = \begin{Bmatrix} -\frac{\bar{C}^{(e)}}{3} * \left(\frac{\Delta x}{\Delta t}\right) * \left(p_i + \frac{p_j}{2}\right) \\ -\frac{\bar{I}^{(e)}}{3} * \left(\frac{\Delta x}{\Delta t}\right) * \left(\dot{m}_i + \frac{\dot{m}_j}{2}\right) \\ -\frac{\bar{C}^{(e)}}{3} \left(\frac{\Delta x}{\Delta t}\right) * \left(\frac{p_i}{2} + p_j\right) \\ -\frac{\bar{I}^{(e)}}{3} \left(\frac{\Delta x}{\Delta t}\right) * \left(\frac{\dot{m}_i}{2} + \dot{m}_j\right) \end{Bmatrix}^{n} \quad (8.3.3.2)$$

The combination of all the pipe element matrices or Element Stiffness Matrices, gives the piping system global matrix or global stiffness matrix, and a representation of the piping elements by a pseudolinear system:

$$\left[K^{*}_{FE}\right] \begin{Bmatrix} \dot{m}_1 \\ p_1 \\ \vdots \\ \dot{m}_{NNODE} \\ p_{NNODE} \end{Bmatrix} = \{H\} \quad (8.4.2.1)$$

Branches can be introduced as additional conditions, additional piping elements as additional pipe sections and the boundary conditions considered by substituting their equations or values to corresponding global matrix nodal unknown lines. Automatic techniques as Payne Irons Method or Line and Column Substitution can be used.

After these modifications the piping network will be represented by a pseudolinear system of equations:

$$\left[K_{FE}\right] \begin{Bmatrix} \dot{m}_1 \\ p_1 \\ \vdots \\ \dot{m}_{NNODE} \\ p_{NNODE} \end{Bmatrix} = \left\{H\right\} \qquad (8.4.2.2\ )$$

### 8.4.3 FINITE DIFFERENCES MATRIX.

Still with reference to the network of Fig. F 8.4.1.1, the continuity and momentum equations, approximated for each piping element according to subpar. 8.3.4 procedure, give:

$$\begin{bmatrix} -1 & \overline{C}^{(e)} * \left(\frac{\Delta x}{\Delta t}\right) & 1 & 0 \\ 0 & -1 & \overline{I}^{(e)} * \left(\frac{\Delta x}{\Delta t}\right) + \overline{R}^{(e)} \Delta x & 1 \end{bmatrix} \begin{Bmatrix} \dot{m}_i \\ p_i \\ \dot{m}_j \\ p_j \end{Bmatrix} \quad \begin{bmatrix} \overline{C}^{(e)} * \left(\frac{\Delta x}{\Delta t}\right) * p_i \\ \overline{I}^{(e)} * \left(\frac{\Delta x}{\Delta t}\right) * \dot{m}_j \end{bmatrix}$$

This finite differences matrix has been obtained as a subcase of the weighted residual method of subpar. 8.3.2, thus, applying the same procedure of subpar. 8.4.1, again a pseudolinear matrix representing the network, can be obtained:

$$\left[ K_{FD} \right] \begin{Bmatrix} \dot{m}_{\ell} \\ p_{NNODE} \end{Bmatrix} = \left\{ H \right\} \qquad (8.3.4.1)$$

with $\left[ K_{FD} \right]$ comprehending all the elements and additional pseudolinear equations.

This finite differences procedure yields a numerical scheme identical to the one obtained with the one dimensional application of the MAC method (Marker and Cell) of HARLOW and WELCH (21), a classic solution of fluid dynamic problems in primitive variables.

The tridiagonal matrix has tremendous advantages in computer time. For the particular case of a single pipe (Fig. F 8.4.3.1), the finite difference scheme can be structured to obtain $\left[ K_{FD} \right]$ tridiagonal.

To preserve the tridiagonal characteristic of the network, particular care must be taken in introducing the boundary conditions, as only Dirichlet types can be used.

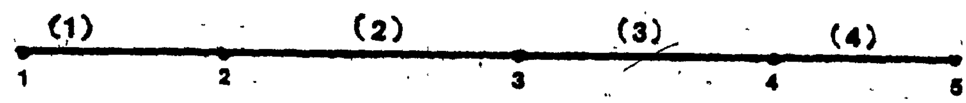



FIG. F 8.4.3.1 Single Pipe Network

## 8.5 CONCLUSIONS

The circuit representation of the unsteady flow of a compressible fluid in an infinitesimal pipe element and its approximate integration along a pipe finite element, performed with proper approximating and weighting functions, gives, at each time level and when extended to a piping system, a pseudolinear system of equations with pressure and mass flow as variables.

The flow density is buried in the system coefficients and some system coefficients are dependant on the unknown mass flow. The entropy equation gives, for each pipe element, the third necessary relationship.

The entropy relation is obtained, in general, by integration along a path line of the entropy equation. For each element, a polytropic coefficient is then derived and used to form the circuit equations.

The pseudolinear system may be solved at each time level by iteration both for density and mass flow.

Three numerical integration methods give different system matrices and right hand side vectors. All matrices are banded and for a single pipe network a finite difference scheme can be reduced to a tridiagonal solver.

Piping elements other than pipe elements, can be added as additional piping elements of proper length and in quasi-steady condition.

Boundary conditions can also be added as additional nodal relationships, as in typical finite volumes, finite elements or finite differences algorithms.

A piping network is now represented, at a given time level, by a numerical model giving, by iteration, the value of the physical variables pressure, density and mass flow at arbitrary nodes of the network.

The density of the node distribution over the time-space domain can be adjusted as necessary, as element length, $\Delta x$, and/or time increment $\Delta t$ need not be held constant.

# CHAPTER 9

# LUMPED ELEMENTS METHOD

# NUMERICAL MODEL

## 9.1 SUMMARY

An iterative model is presented, discussed and used to solve the Test Case in selected operating conditions and its results presented in form of wave diagrams.

The comparison between the three basic Lumped Element Methods algorithms does not give conclusive preference for any one of them, while the comparison with Characteristics Method results shows encouraging perfomance of all three.

The influence of grid fineness is then explored, with results showing heavier weight of the time coordinate discretization and, finally, the Lumped Element Methods performance is rated, with positive results, compared to the Ideal Model (Chap. 7) requirements.

MODELLING OF THE TEST CASE

## 9.2.1 MIXED ITERATIVE SOLUTION.

All lumped element procedures lead to solving a pseudolinear system of equations, i.e. a system where the matrix terms depend on the unknown variables. Specifically, the non linear parts of the momentum equation, convective and friction terms, are represented by circuit resistors, which are quadratically dependant on the mass flow variable (subparagraphs 8.3.2, 8.3.3, 8.3.4).

A matrix solution of the discretized continuity and momentum equations means using an "implicit" numerical scheme (subparagraph 9.3.3), with significant stability advantages and consequent greater freedom of choosing the time increment between two quasi-steady solutions.

The non-linear matrix terms may be linearized, assuming for their evaluation at time level (n)-the mass flow values at time level (n-1) (semi-implicit solution), and acceptable results are obtained for reasonable values of time increments.

Alternatively, the non-linear matrix terms can be solved by a mass flow iteration. For a compressible flow this iteration can be peformed using the density iteration (mixed iteration) with no significant increase in computer time. This procedure will be used to solve the test case.

## 9.2.2 RUPTURE BOUNDARY CONDITION.

The test case left boundary (subparagraph 7.3.2), or "rupture" boundary, consists of:

: A time interval of sudden pressure drop from operating to atmospheric values;

: A time interval of constant pressure, ending when the reverse mass flow reaches the local critical value;

: A time interval of pressure increase with mass flow at local critical value, ending when the flow rate reaches the reservoir critical values.

During the first time interval, the boundary pressure decreases from operating to atmospheric value in 0.05

seconds. Provision is made to limit the pressure drop when local sound velocity is reached during the steep pressure gradient.

When the reversed mass flow exceeds the local critical value, the boundary pressure is adjusted and increased to the value for which the computed mass flow becomes critical (14):

$$p(1)=((\dot{m}(1)/A)**2/(\gamma*\rho_o))**(\gamma/(\gamma+1)) \quad (9.2.2.1)$$

with $\rho_o$ indicating the stagnation density and (1) variables values at pipe left end.

This "rupture" boundary condition is adequate for the purpose of checking the algorithms efficiency by comparison with the Characteristics Method test model solution. More theoretical and experimental work is required for a rigorous and general formulation of the "rupture" model.

9.2.3 RESERVOIR BOUNDARY CONDITION.

The test case right boundary (subparagraph 7.3.2), or "reservoir" boundary, consists of:

: A time period of steady pressure, ending when mass flow inverts.

: A time period of decreasing pressure, where the boundary flow parameters approach the reservoir critical values.

The Characteristics Method (subparagraph 7.3.2) approximates the pressure decreasing period to a sequence of steady conditions satisfying the basic adiabatic and isentropic conditions:

$$\frac{p(R)}{\rho(R)}=\frac{p(NNODE)}{\rho(NNODE)}+\frac{(\gamma-1)}{2*\gamma}*\left(\frac{\dot{m}(NNODE)}{A*\rho(NNODE)}\right)^2$$

$$\frac{p(R)}{(\rho(R))^\gamma}=\frac{p(NNODE)}{(\rho(NNODE))^\gamma}$$

with (R) indicating the reservoir and (NNODE)the end of pipe conditions.

A similar boundary condition is simulated for the Lumped Elements Methods,evaluating the boundary pressure from:

$$(p(NNODE)) = \frac{p(R)}{\rho(R)}*(\rho(NNODE)) - \frac{(\gamma-1)}{2*\gamma}*\left(\left(\frac{\dot{m}(NNODE)}{A}\right)^2\right)*\frac{1}{(\rho(NNODE))}$$

$$\left(\frac{p(NNODE)}{(\rho(NNODE)^{\gamma})}\right) = \frac{p(R)}{(\rho(R))}$$

To compute p(NNODE), a Newton Raphson procedure may be used for most of the field where $\dot{m}(NNODE) \ll \dot{m}(NNODE)$. For values of $\dot{m}$(NNODE) closer to critical mass flow a Bysection procedure is introduced.

Also this boundary procedure must be considered adequate for the limited purpose of model analysis. Undoubtedly, further theoretical and experimental analyses would lead to a more acceptable boundary model.

9.2.4 SOUND VELOCITY INSTABILITY.

For given stagnation conditions, pressure and mass flow of an isentropic flow are related by a curve of the type shown in Fig. F 9.2.4.1 (15).

For mass flow values close to critical conditions, the model iterations may push some pressure nodal values to the left side of the diagram. When it happens, the model deteriorates until convergence fails for negative values of density.

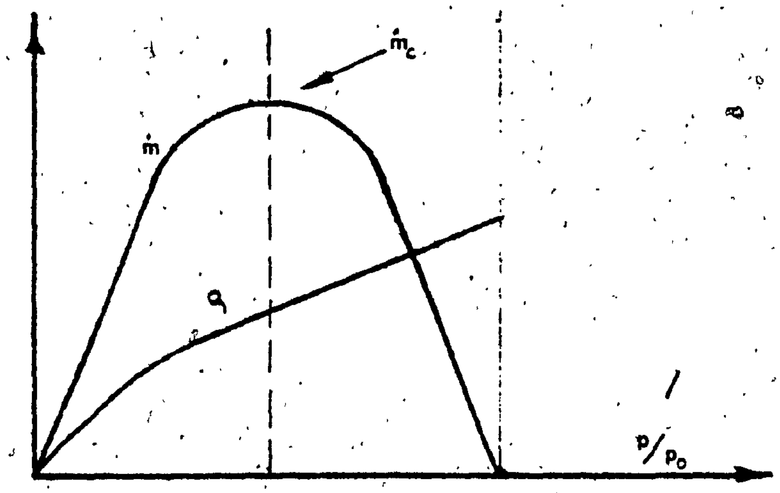



FIG. F 9.2.4.1 Typical Variation of Flow Properties in isentropic flow.

Adequate protection must be introduced in form of velocity and/or density limitations.

## 9.3 NUMERICAL MODEL

### 9.3.1 MODEL STRUCTURE.

A schematic representation of the numerical model simulating the unsteady continuous flow part of the Test Case (Program PIPING) is shown in Fig. F 9.3.1.1.

At each time step the quasi-steady continuity,momentum and entropy equations are numerically solved: first, continuity and momentum, solving a pseudolinear system, then, entropy and non linearity,by iteration. After solving a time level, the model advances by a time step and repeats the solution procedure.

Program PIPING inputs are:

: pipe diameter,

: number of pipe elements,

: maximum time,

: time interval of pressure decrease,

: fineness of discretization,

: element coordinates,

: initial pressure density and velocity.

Program PIPING outputs for each time level are:

:time step,

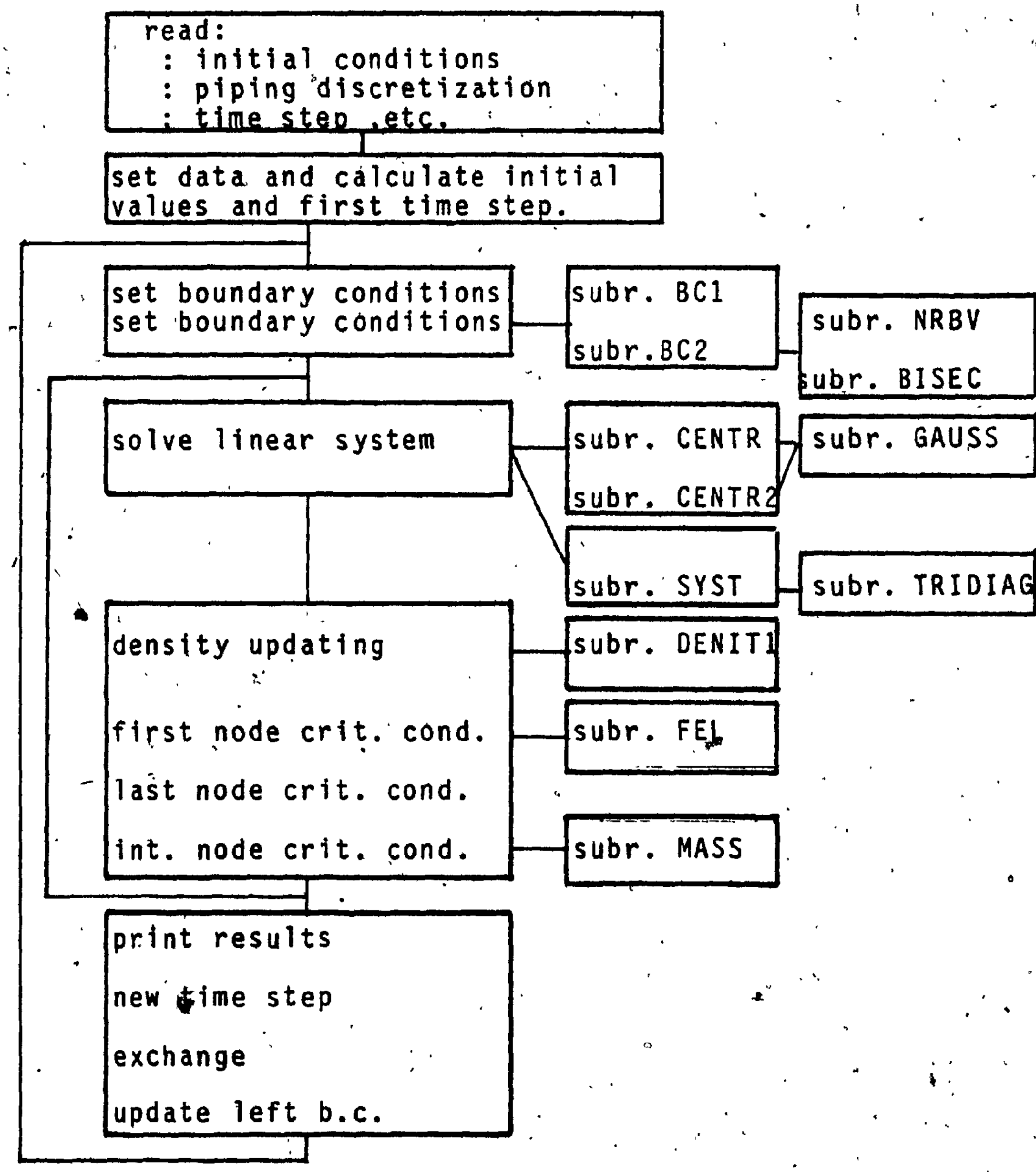



FIG. F 9.3.1.1 Program PIPING

: elapsed time,

: for each node:

: pressure Pa,

: mass flow Kg/s,

: density Kg/m3,

: velocity m/s,

: local sound vel m/s.

Main program PIPING sets flow initial values and the time step procedure.

Inside the time loop:

i ) boundary conditions are set through subroutines BC1 (left boundary) and BC2 (right boundary);

ii ) density and mass flow iteration is performed and the time level flow parameters computed;

iii) a new time step is calculated. The time step is determined as the time required by a pressure signal to travel through the average element when moving at the domain average speed of sound (any reasonable time step can be used instead). Time is then updated and the procedure ended when maximum time is reached;

iv ) last time level parameters values are assumed as known parameters for the new time step (exchange);

v ) a new time step is requested.

Inside the density and mass flow iterative loop:

i ) the linearized system is solved using: a finite volumes method (subroutine CENTR), a finite element method (subroutine CENTR2), a finite differences method (subroutine SYST);

ii ) density is calculated, calling subroutine DENIT1;

iii) local sound velocity conditions at the first node are controlled, calling subroutine FEL. Subroutine FEL is not allowed to interfere, when sound conditions are reached during the pressure decrease;

iv ) last node critical condition is controlled, setting p(NNODE) equal to reservoir discharge critical pressure, when the mass flow at last node becomes greater then the reservoir discharge critical mass flow;

v ) internal nodes critical conditions are controlled calling subroutine MASS.

Subroutine BC1 calculates the left boundary pressure p(1) for each time level. Simulating a pipe rupture, the boundary pressure is linearly decreased from its initial to an-atmospheric value and then kept constant. At each time level, the nodal pressure cannot be lower then the critical pressure, referred to the boundary stagnation conditions of the previous one.

Subroutine BC2 calculates the right boundary pressure p(NNODE) for each time level. As long as the mass flow enters the reservoir (positive mass flow), the boundary pressure is kept equal to the reservoir level. As the gas stream changes direction, the boundary pressure is calculated solving the adiabatic and isentropic relations (subparagraph 9.2.3) through subroutine NRBV (Newton Raphson Method), or, for FLOW(NNODE) ½ 0.9*FLOW(NNODE) , through subroutine BISEC (Bisection Method).

At each time level and for each density-mass flow iteration, subroutine CENTR calculates the matrix coefficients, according to the Linear Approximation Method (subparagrapf 8.3.2), introduces the boundary equations and calls subroutine GAUSS to solve the pseudolinear system.

At each time level and for each density mass flow iteration, subroutine CENTR2 calculates the matrix coefficients, according to the Finite Element Method (subparagraph 8.3.3), introduces the boundary conditions, according to the Payne Irons Method, and calls subroutine

GAUSS to solve the pseudolinear system.

At each time level and for each density-mass flow iteration, subroutine SYST calculates the matrix coefficients, according to the Finite Differences Method (subparagraph 8.3.4), introduces the boundary equations, and calls subroutine TRIDIAG to solve the pseudolinear system.

At each time level and for each density-mass flow iteration, density at each node is updated by subroutine DENIT1, using system results and proper nodal parameters of the previous one. For an adiabatic isentropic flow the entropy equation is:

$$\left(\frac{p(I)}{(\rho(I))}\right)^{n+1}=\left(\frac{p(I)}{(\rho(I))^{\gamma}}\right)^{n}$$

and the nodal density is given by:

$$(\rho(I))^{n+1}=(\rho(I))^{n}*\left\{\frac{(p(I))^{n+1}}{(p(I))}\right\}^{\frac{1}{\gamma}}$$

For a non-adiabatic flow the entropy equation is (subparagraph 6.2.2):

$$\left\{\frac{p(I)}{p(J)}\right\}^{n+1}_{n}=\left\{\frac{(\rho(I))^{n}}{(\rho(J))^{n+1}}\right\}^{\gamma}*e^{\Phi} \qquad (6.2.2.13)$$

Since these relations are valid for an individual particle (subparagraph 6.2.2), the time level (n) nodal properties must be chosen at the proper node, according to the flow velocity indications. For small $\Delta t$ and low velocities (low Mach numbers), the particle position at time level (n) can be confused with the space coordinate of the node in consideration.

At each time increment and for each density-mass flow iteration, subroutine FEL controls the first node mass flow and, when its value is greater then the local critical mass flow, increases the left boundary pressure to the local critical level. Subroutine FEL also does not permit the boundary mass flow absolute value of ever passing the maximum flow obtainable at the reservoir discharge.

As the flow velocity approaches sonic velocity, the numerical approximation may give flow rates slightly greater than critical and the numerical scheme degenerates. To avoid this degeneration, which seems unavoidable, at least for isentropic flows, subroutine MASS imposes critical mass flow values to all nodal mass flows exceeding this limit.

Subroutine GAUSS and TRIDIAG are the linear system solvers. Subroutine TRIDIAG, the fastest known direct solver, can be used only when the system matrix is tridiagonal, a condition reached with the finite differences scheme and for a single pipe system. Subroutine GAUSS is used as solver for all other matrix conditions. Its velocity is limited and its application is justified only for the present model analysis. For industrial applications faster solvers should be developed and used.

### 9.3.2 COMPARISON OF DIFFERENT ALGORITHMS.

Diagrams D 9.3.2.1 and D 9.3.2.2 show typical nodal values of pressure and mass flow, obtained solving the test case with the three numerical algorithms, while typical wave shapes at selected times are shown in diagrams D 9.3.2.3 and D 9.3.2.4.

The Test Case characteristics and initial conditions are:

| | | | |
|---|---|---|---|
| : pipe nominal diameter | : | 18" | |
| : pipe length | : | 100 | m |
| : initial pressure | : | 300000 | Pa |
| : initial density | : | 3.6 | Kg/m3 |
| : initial velocity | : | 30 | m/s |
| : number of equal elements | : | 10 | |
| : "rupture" time | : | 0.05 | s |

The pressure nodal values, obtained with the three different algorithms, are similar to such an extent that is difficult to differentiate most of them. The mass flow nodal values are very close for the Linear Element and Finite Element Methods, while some discrepancies (anticipations) can be observed for the values obtained with the Finite Differences Method. These discrepancies are more pronounced in proximity to the left boundary and are probably due to the mass flow boundary condition adjustment, introduced to obtain a tridiagonal matrix.

The Linear Element Method and, to a lesser degree, the Finite Element Method seems more sensitive to critical flow conditions and degenerates faster at the second node (results from runs not shown). Whether this degeneration, common to all three algorithms, is an effect of an imperfect boundary condition or is unavoidable, at least for inviscid flows, is left to future investigation. Difficulties in signal transmission from downstream at close to critical conditions may also play a role.

Computer times (CYBER 127) for 0.63 seconds of simulation elapsed time are:

| | | |
|---|---|---|
| : Linear Element Method | : | 27.066 s |
| : Finite Element Method | : | 21.646 s |
| : Finite Differences Method | : | 1.627 s |

The great advantage of the Finite Differences Method is all due to the tridiagonal form of the linear system matrix, which allows the use of a particularly fast solver. Therefore, it is limited to the simple cases where this particular matrix may be maintained.

The computer time required by the slower methods can be significantly reduced using iterative solvers (ADI, SOR, etc.) and taking full advantage of the matrices banded form. Matrix partition solvers (CROUT, CHOLESKY, etc.) can probably be used advantageously, concentrating on the right hand side vector the density and mass flow dependant matrix terms and, then, at each iteration, sweeping two triangular linear systems with constant matrix coefficients at each time level.

To reduce computer time, the Finite Differences Method will be assumed, from now on, as representative of the three numerical techniques. density and mass flow depending matrix terms.

For complex cases, the Finite Element Method with a fast solver would be preferred, mainly for its efficient handling of boundary conditions.

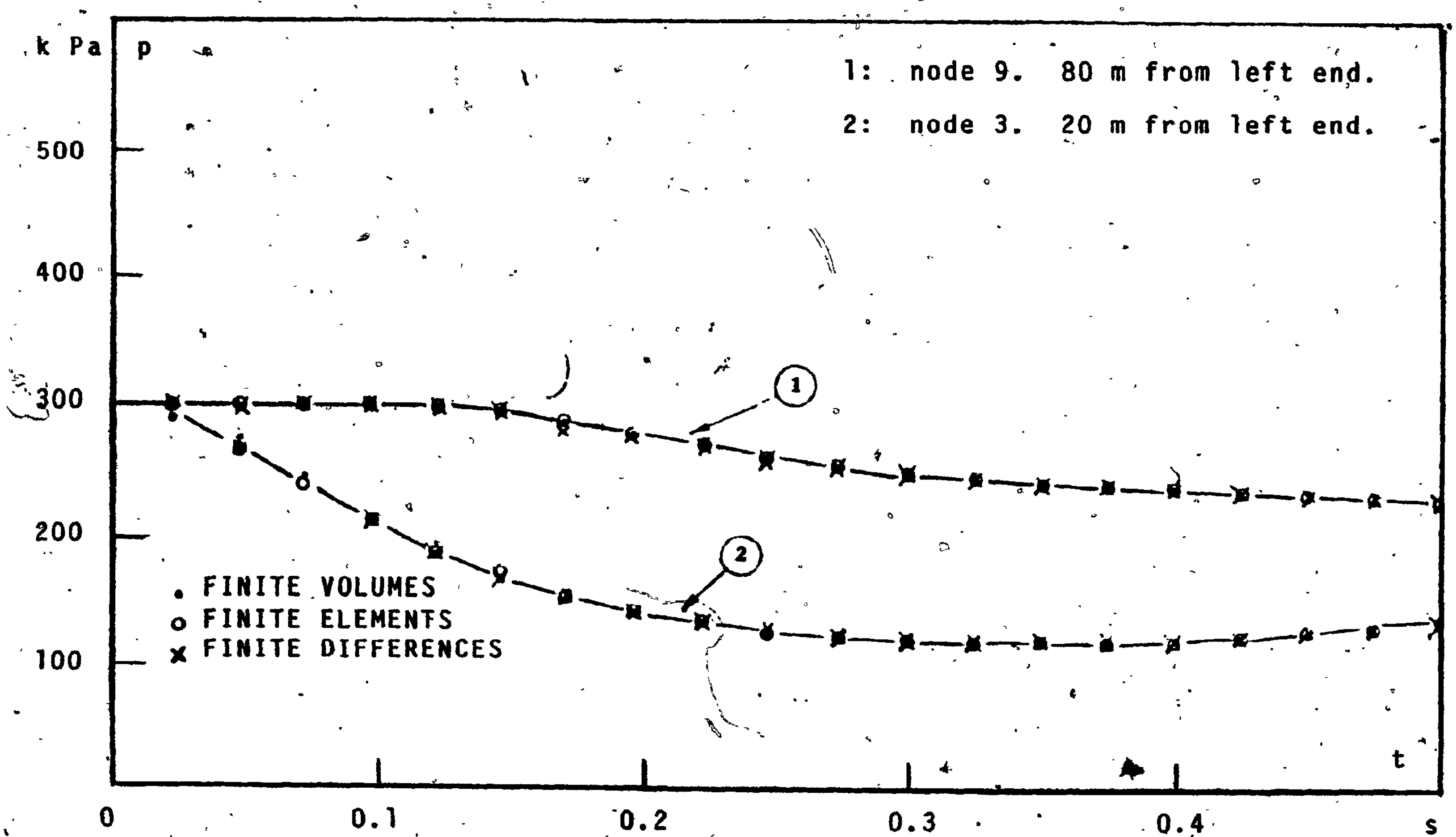



Diagram D 9.3.2.1 Comparison of Different Algorithms. Pressure vs. Time.

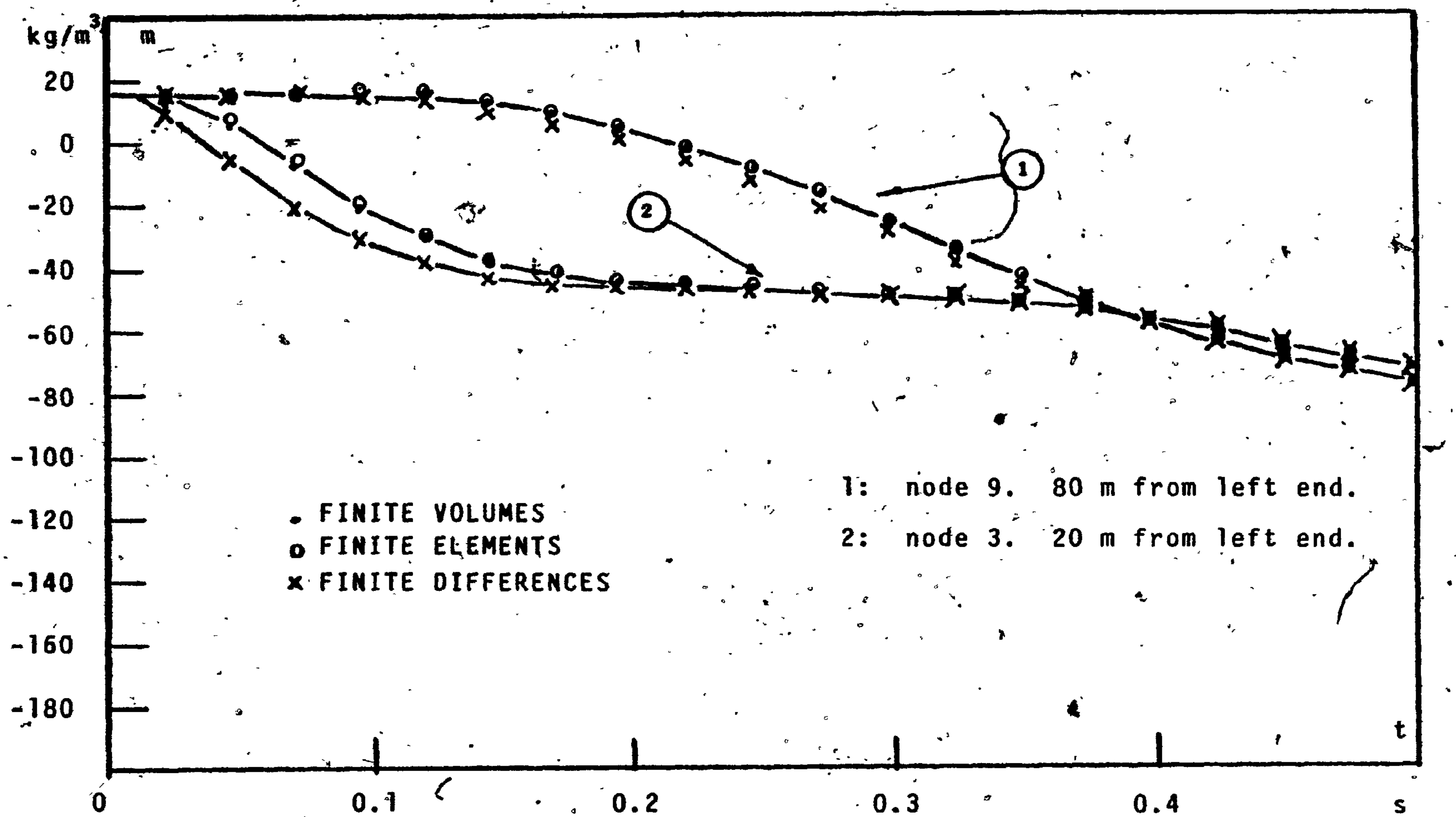



Diagram D 9.3.2.2 Comparison of Different Algorithms. Mass Flow vs. Time.

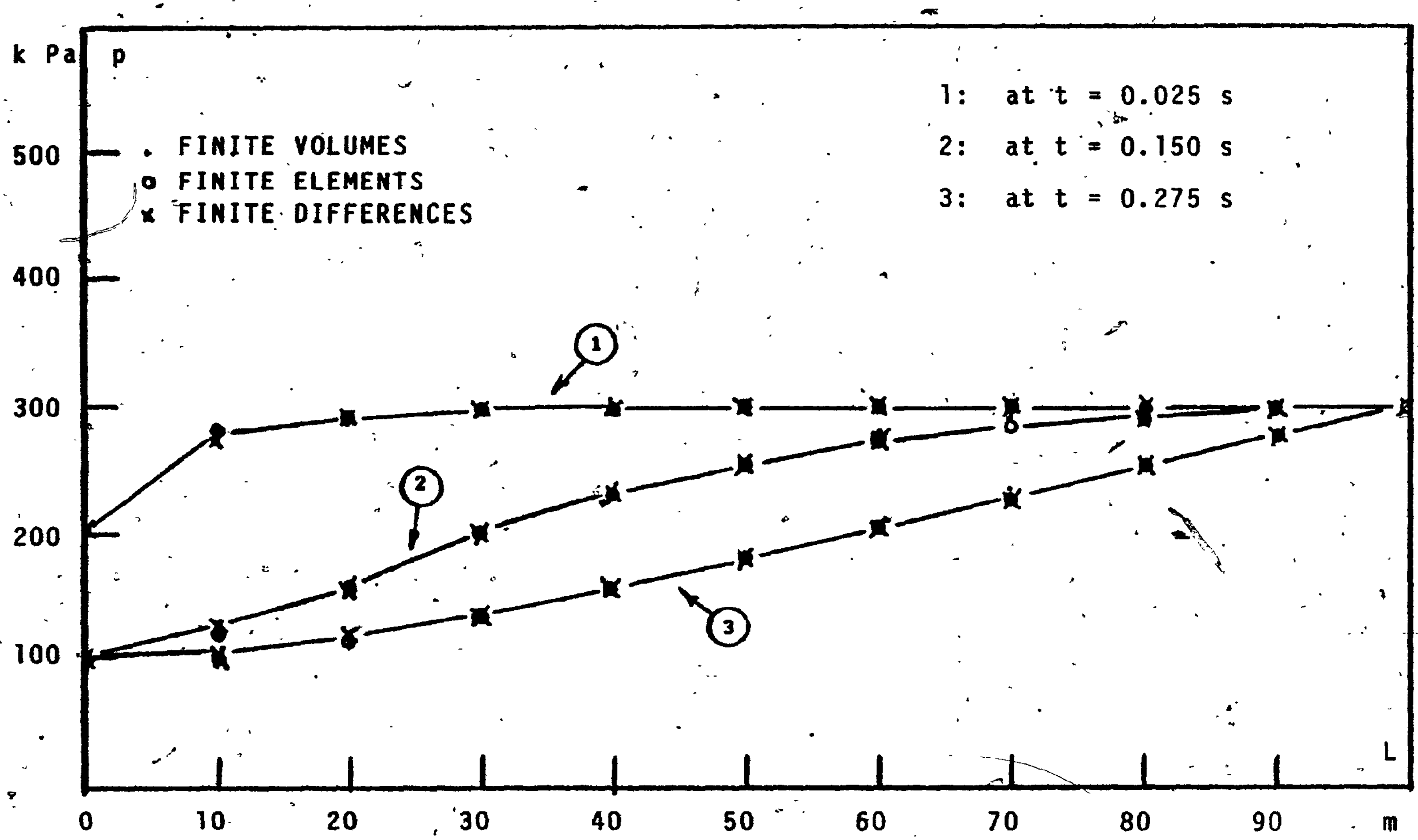



Diagram D 9.3.2.3 Pressure along the Pipe

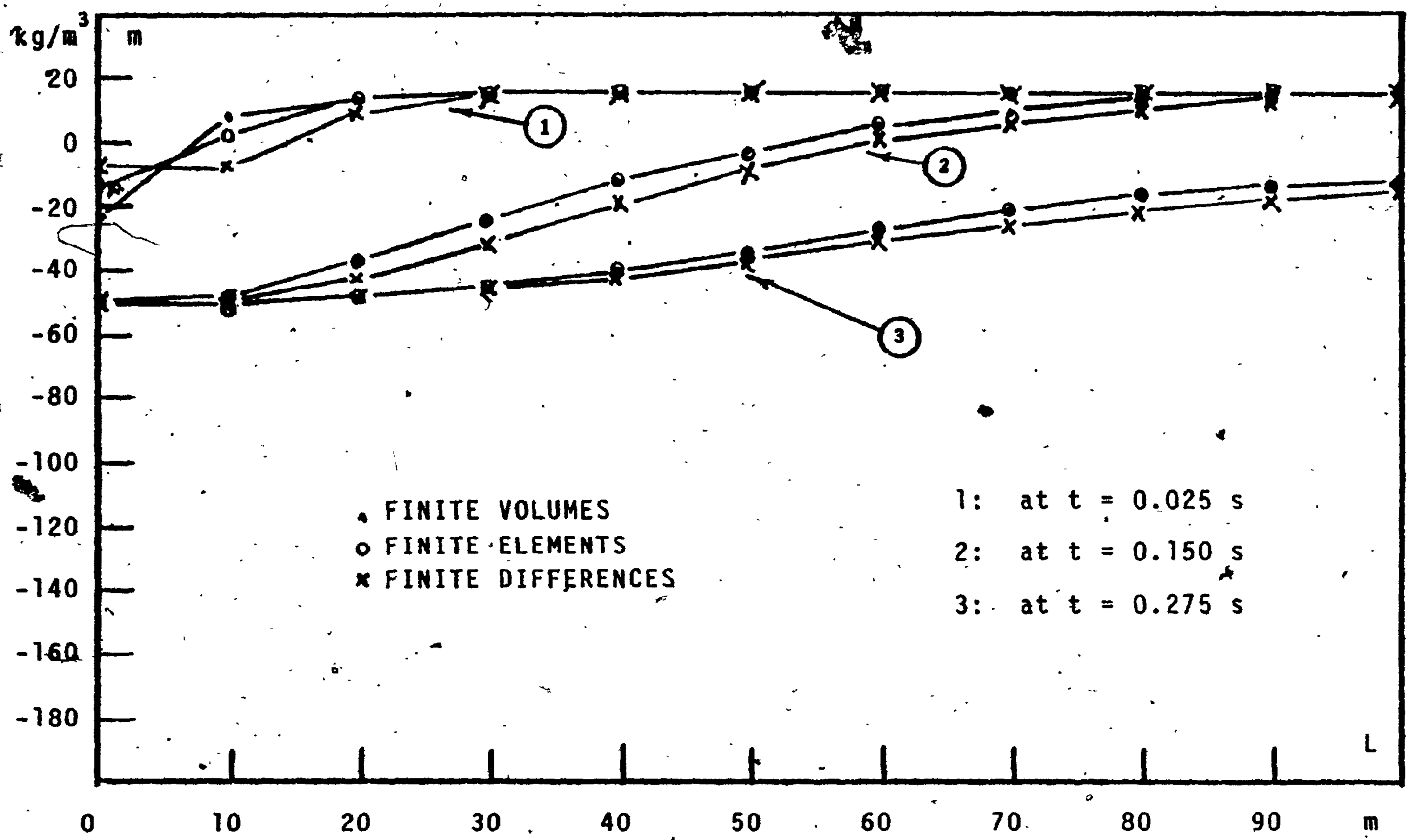



Diagram D 9.3.3.2.4 Mass Flow along the Pipe.

### 9.3.3 COMPARISON WITH TEST ALGORITHMS

Diagrams D 9.3.3.1, D 9.3.3.2, D 9.3.3.3, D 9.3.3.4, D 9.3.3.5, D 9.3.3.6, show a comparison of pressure and mass flow nodal values obtained at X=20 m and X=80 m with the Method of Characteristics and the Lumped Element Methods for the Test Case at the operating conditions:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| : | p = | 149000 | Pa | $\varrho$ = | 1.4 | Kg/m3 | V = | 30 | m/s |
| : | p = | 300000 | Pa | $\varrho$ = | 3.6 | Kg/m3 | V = | 30 | m/s |
| : | p = | 500000 | Pa | $\varrho$ = | 5.8 | Kg/m3 | V = | 30 | m/s |

The values of the Method of Characteristics are the closest available to nodal time-space grid values obtained with the Lumped Element Methods.

Results obtained with the Lumped Element Methods show a smoother trend, and field points far from the front wave are shown perturbed before the perturbation signal can reach them. Nevertheless, the dominant perturbation velocity, calculated where the pressure gradient is greater, shows excellent agreement with the perturbation velocity given by the Method of Characteristics and commonly accepted as very close to reality.

All lumped methods equations have been solved using "implicit" methods (paragraps 8.3 and 8.4). Numerical schemes are called "explicit" when only known values at time level (n), (n-1), ... are needed to advance the calculation to time level (n+1) (21), and "implicit" when advanced values are used in the spatial derivatives, thereby requiring the simultaneous solution of equations at the (n+1) level in order to advance the calculation.

Implicit methods, applied to space derivatives of continuity and momentum equations, result in an infinite propagation speed of the perturbation signal, but the signal intensity decreases with the distance from the maximum gradient. Furthermore, a proper discretization of density depending terms (the density depending resistors of the Lumped Element Scheme of subparagraps 8.3.2, 8.3., 8.3.4) allows a fine self regulation of the travelling velocity of the smooth wave fronts.

The comparison of pressure values shows the lumped systems perturbations always in advance of those of the Characteristics Method, result of the infinite velocity effect already examined. The pressure top and bottom values are generally in good agreement. Diagram D 9.3.3.1 shows an unusual discrepancy of peak values, due to the combined effect of high frequency of perturbation, small pressure variations and "rupture" boundary proximity. A drastic reduction of the time interval (from 0.02 to 0.002) (dotted line) reduces much of the difference and seems to indicate the boundary proximity as the only remaining left differentiating factor. Whether the Method of Characteristics or the Lumped Methods are the most realistic in this case, may be a matter for discussion and only decided by experimental analysis.

The comparison of mass flow values shows the same trend, with better agreement, and without the discrepancies observed in diagram D 9.3.3.1.

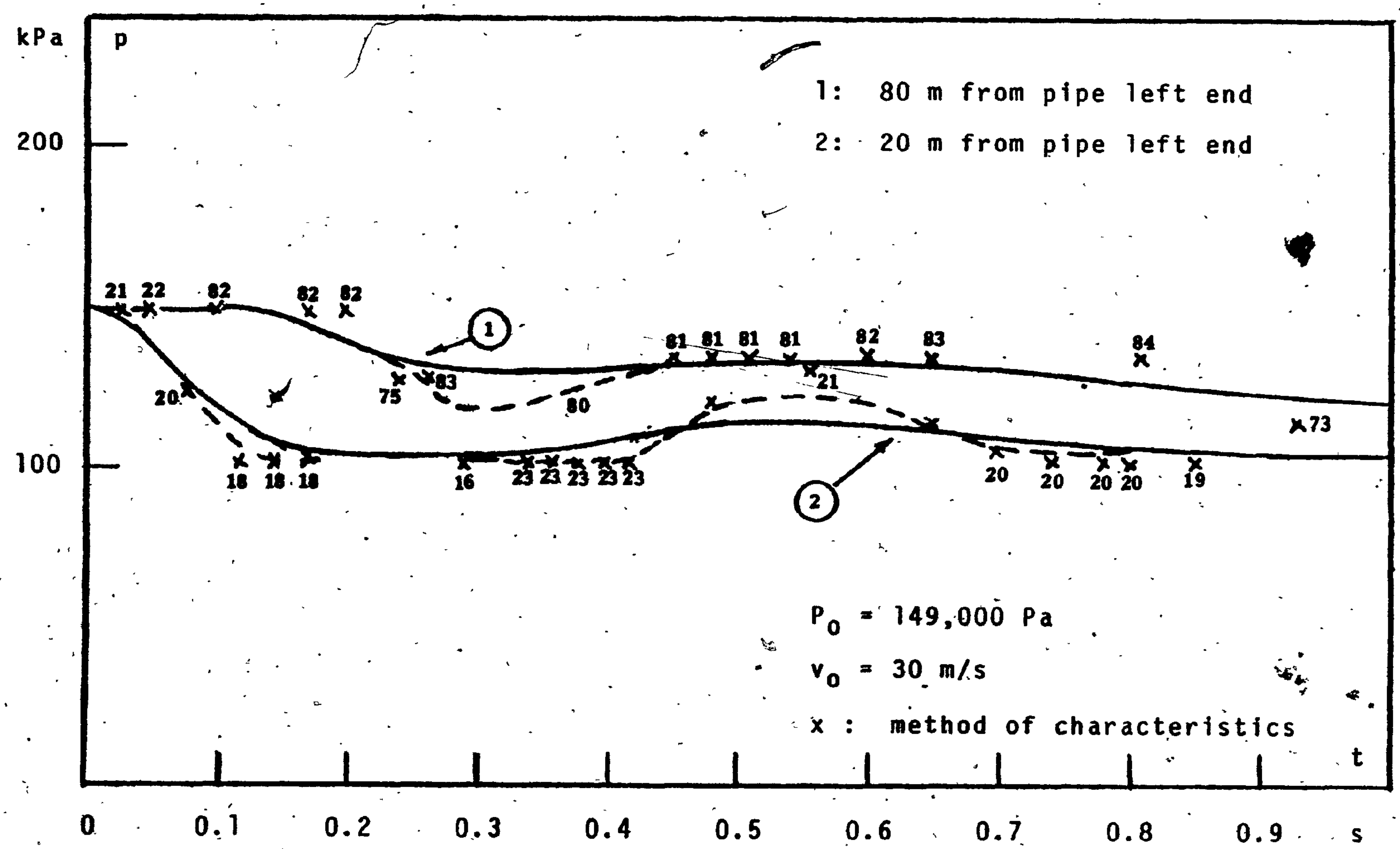



Diagram D 9.3.3.1 18" Valve. Comparison with Method of Characteristics. Pressure vs. Time. Case A

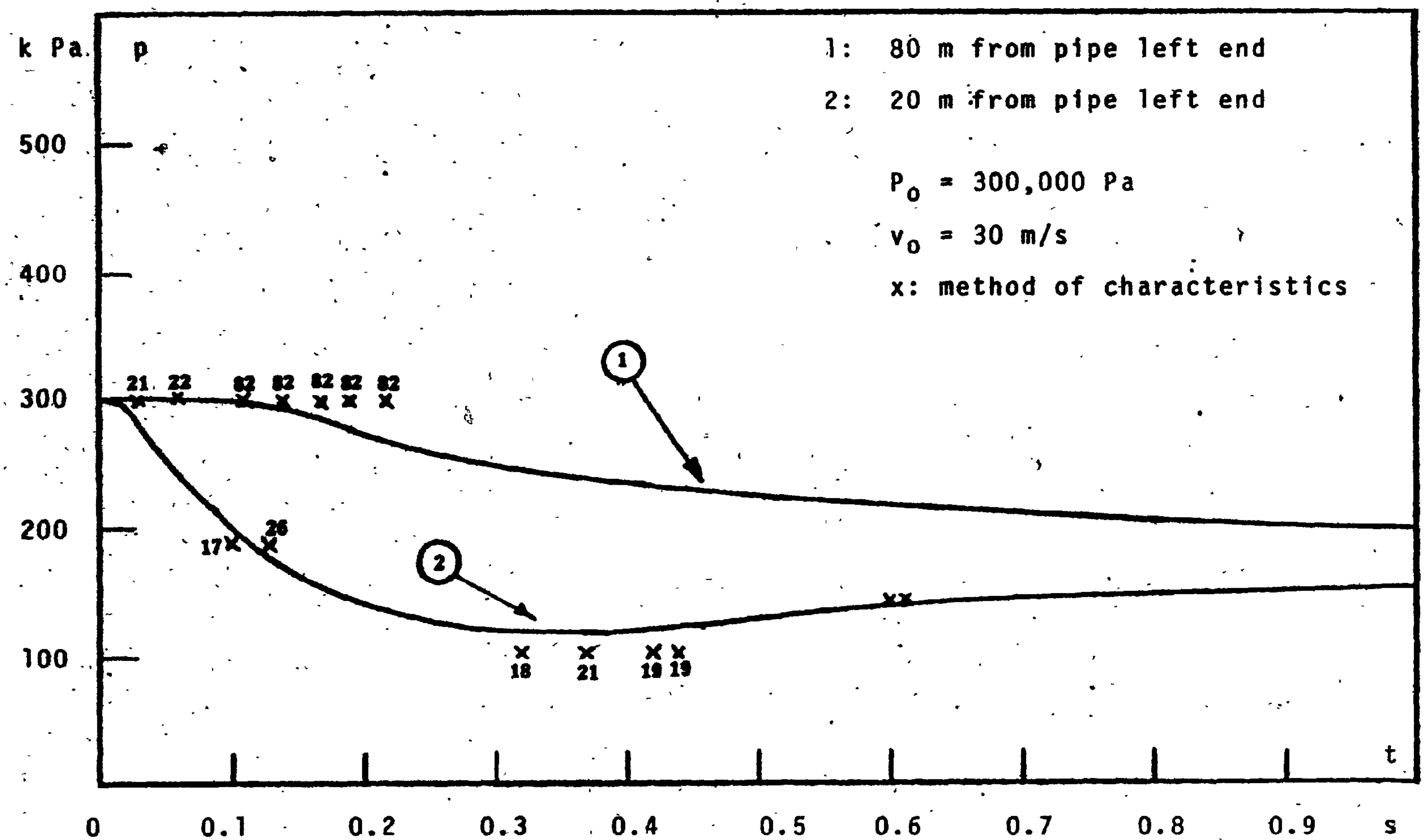



Diagram D 9.3.3.2 18" Valve. Comparison with Method of Characteristics.
Pressure vs. Time. Case B.

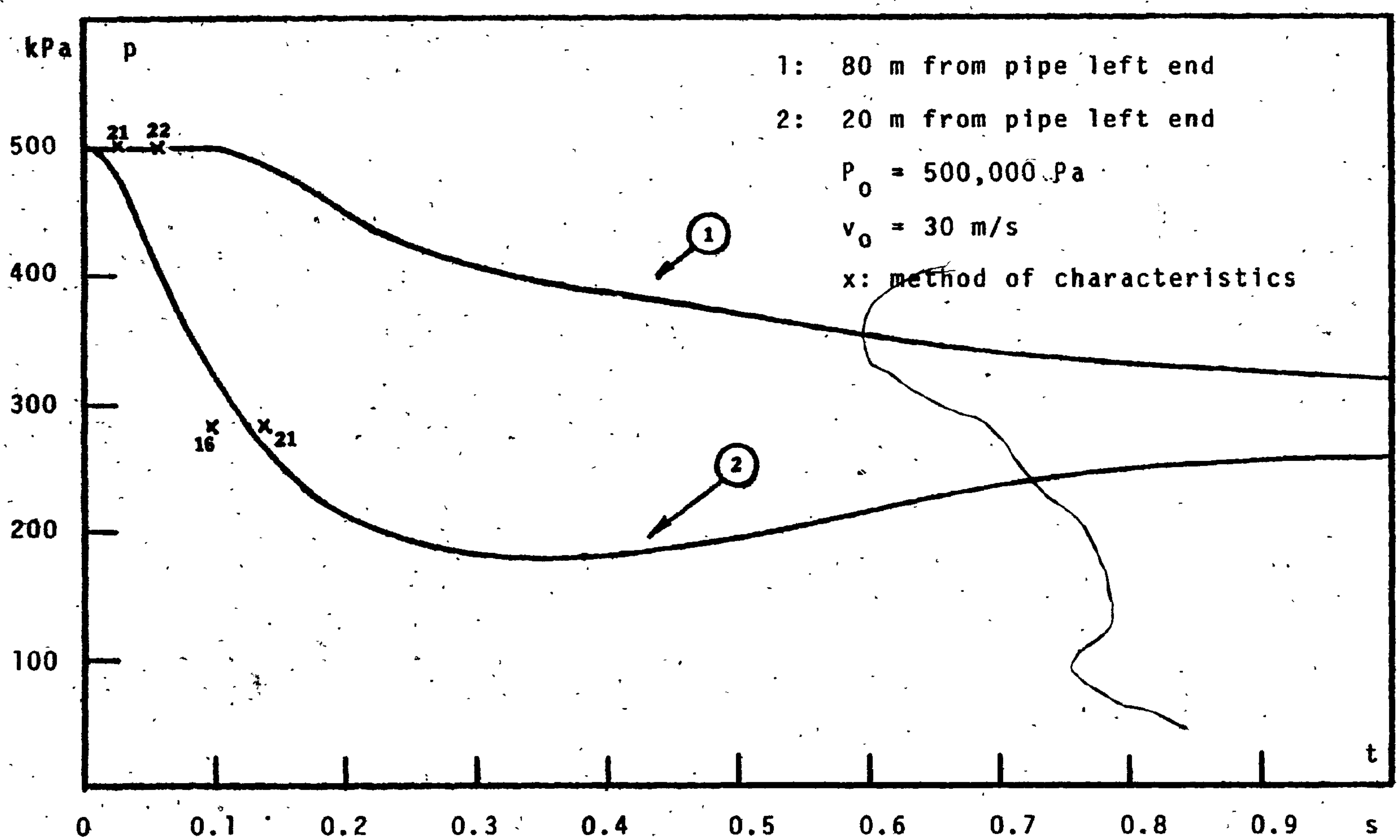
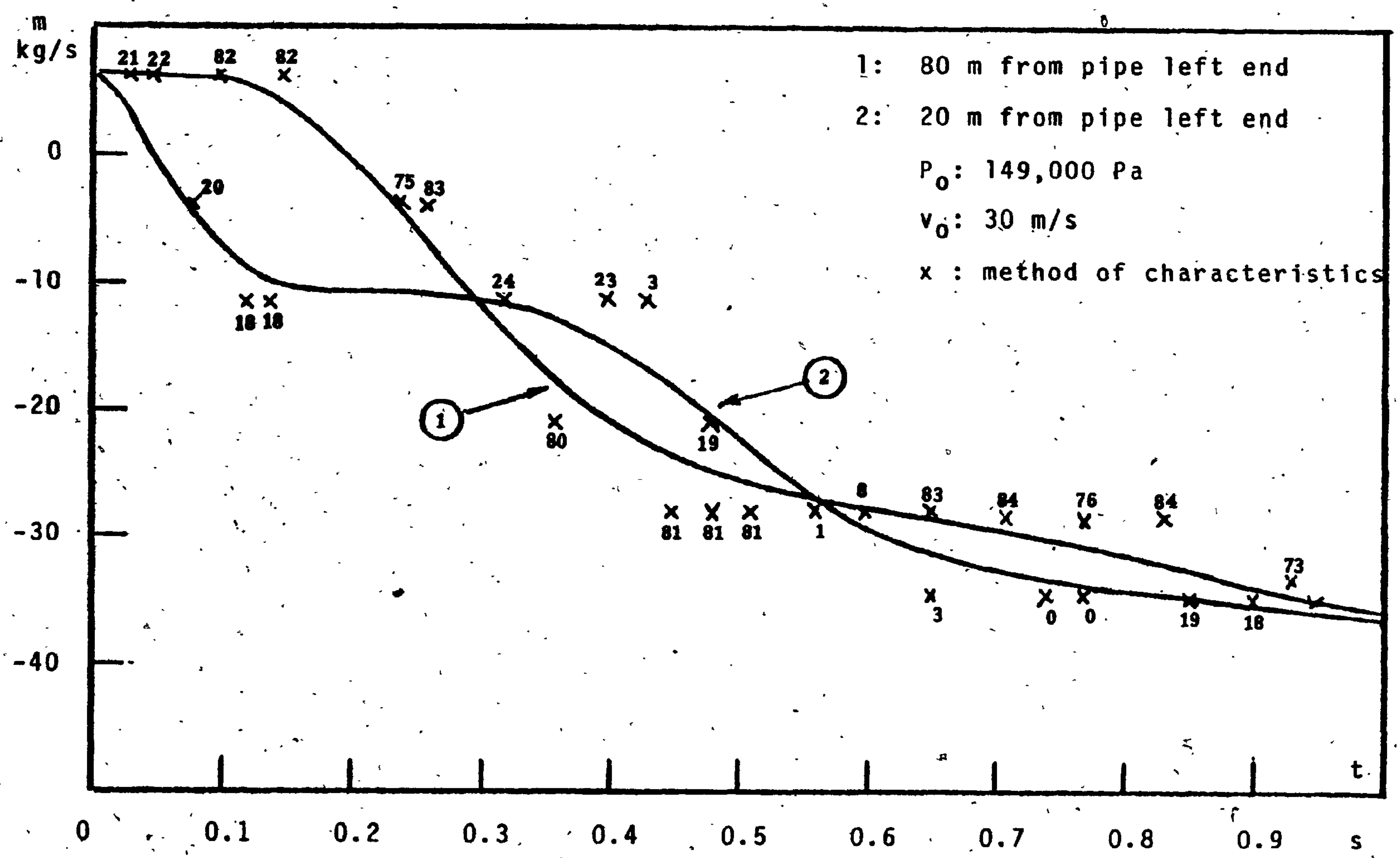



Diagram 9.3.3.3 18" Valve. Comparison with Method of Characteristics. Pressure vs. Time. Case C.




'Diagram 9.3.3.4 18" Valve. Comparison with Method of Characteristics.
Mass Flow vs. Time. Case A.

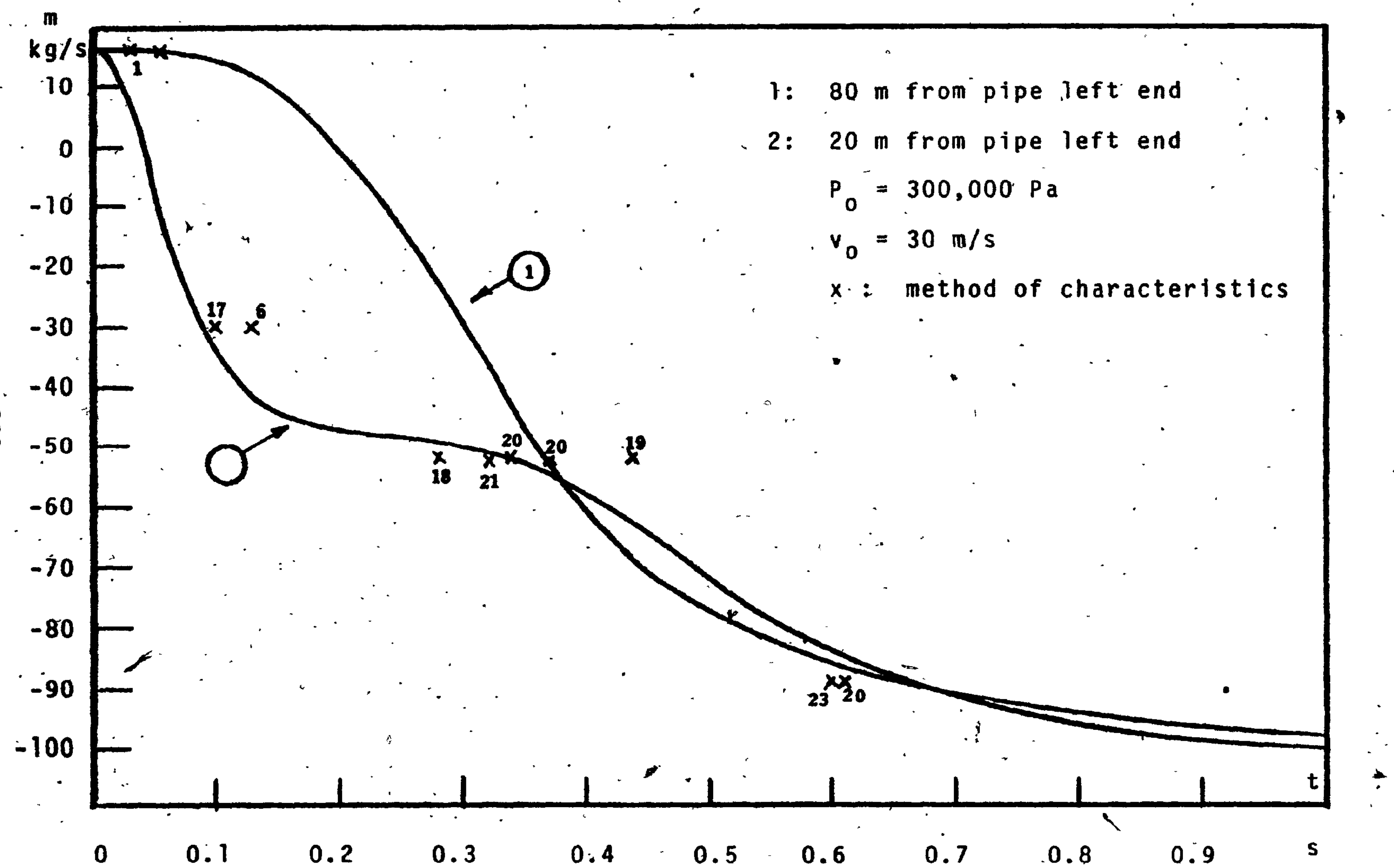



Diagram 9.3.3.5 18" Valve. Comparison with Method of Characteristics. Mass Flow vs. Time. Case B.

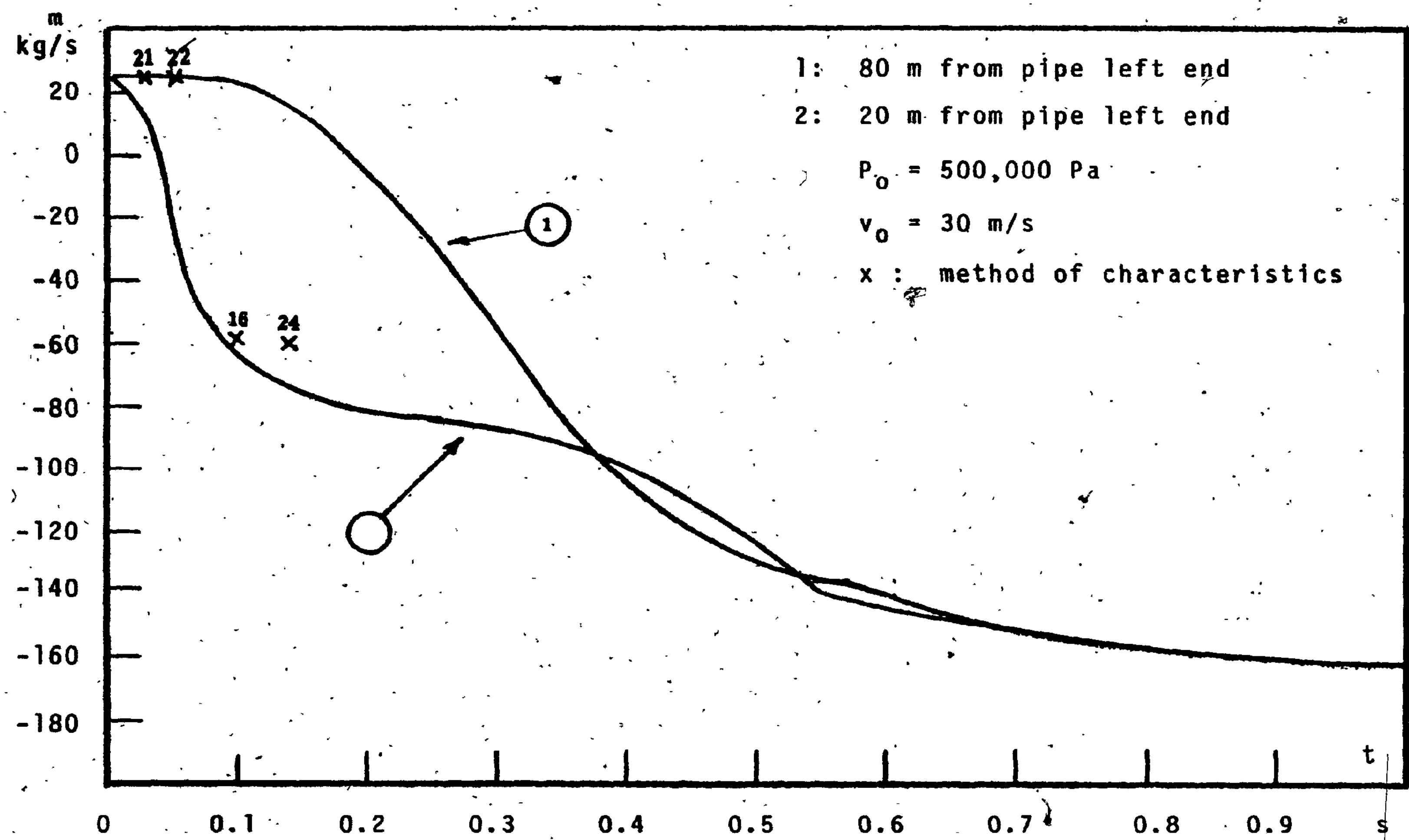



Diagram 9.3.3.6 18" Valve. Comparison with Method of Characteristics.
Mass Flow vs. Time. Case C.

### 9.3.4 MODEL ANALYSIS.

Diagrams D 9.3.4.1 and D 9.3.4.2 show a comparison of pressure and mass flow nodal values obtained solving the Test Case for the operating conditions:

$p = 300{,}000$ Pa   $\rho = 3.6$ Kg/m3   $V = 30$ m/s

with geometric characteristics:

$D = 18"$
$L = 100$ m
$NEL = 10$

but with three different time steps:

$\Delta t = 0.0125$ s
$\Delta t = 0.0250$ s
$\Delta t = 0.0500$ s

Pressure and mass flow nodal values are sensitive, in a significant but not determinant degree, to the time step fineness, with better accuracy for smaller $\Delta$ t. It is worthwhile to notice that reducing the time step by half does not necessarily mean doubling machine time, as significant reductions can be expected in the number of iterations required to converge at each time level.

Diagrams D 9.3.4.3 and D 9.3.4.4 show a comparison of Lumped - Element Methods pressure and mass flow values obtained solving the Test Case at the same operating conditions, with a constant time step of:

$$\Delta t = 0.025 \quad s$$

but with different element lengths:

$$\Delta x = 5 \quad m$$
$$\Delta x = 10 \quad m$$
$$\Delta x = 20 \quad m$$

Under these limits of variation, pressure and mass flow values don't seem to be affected by the element size.

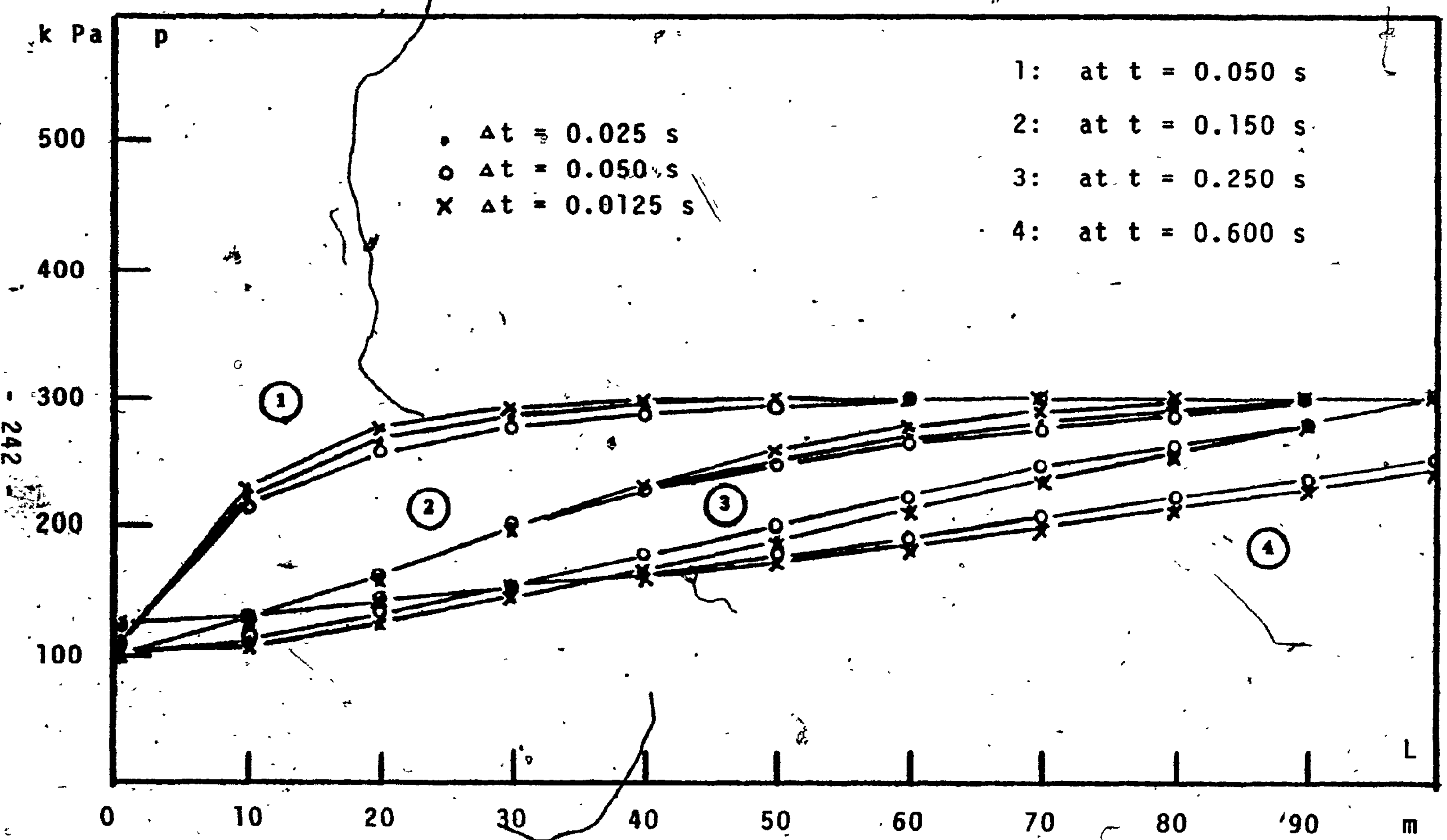



Diagram D 9.3.4.1 Pressure along the Pipe with Different Time Steps.

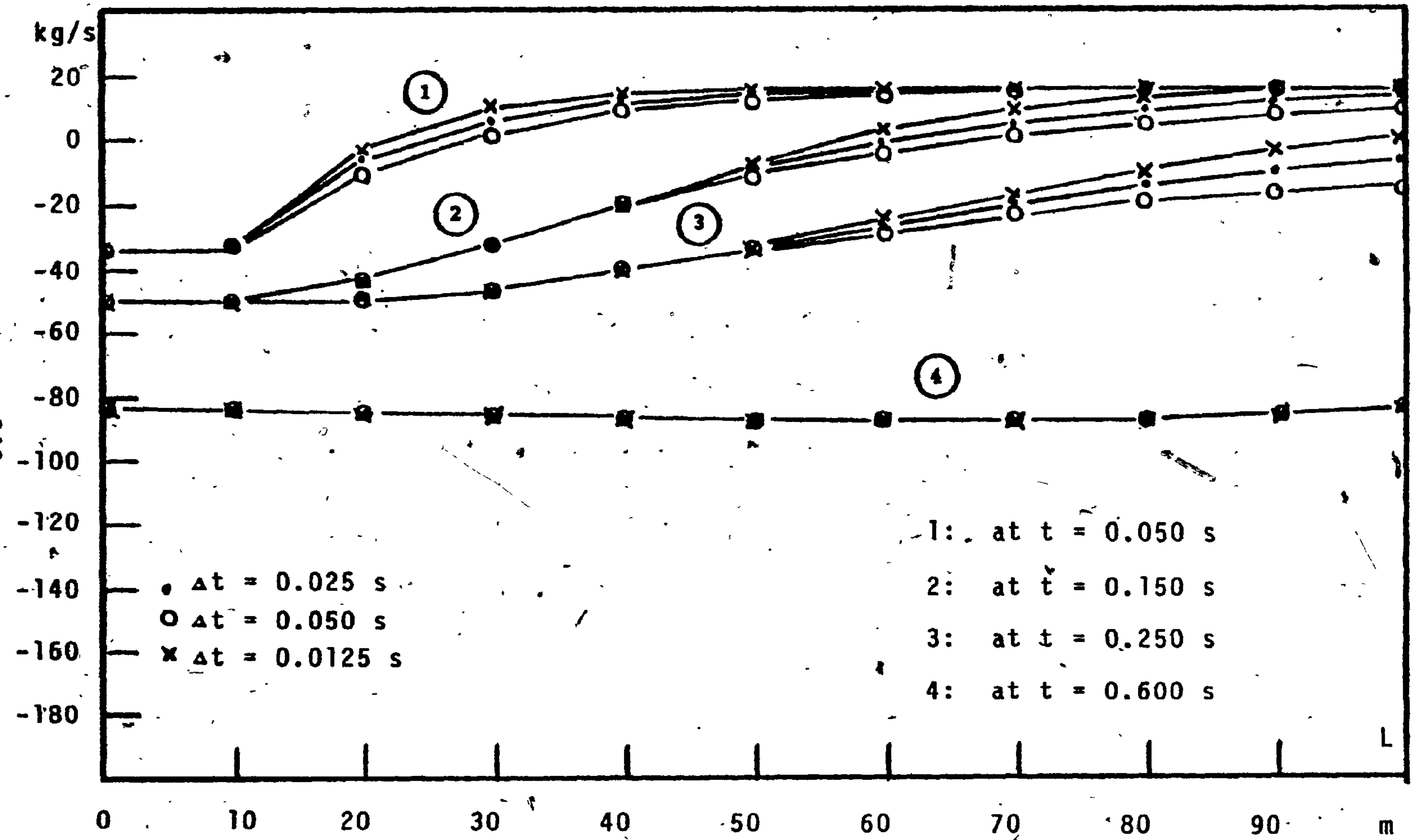



Diagram D 9.3.4.2 Mass Flow along the Pipe with Different Time Steps.

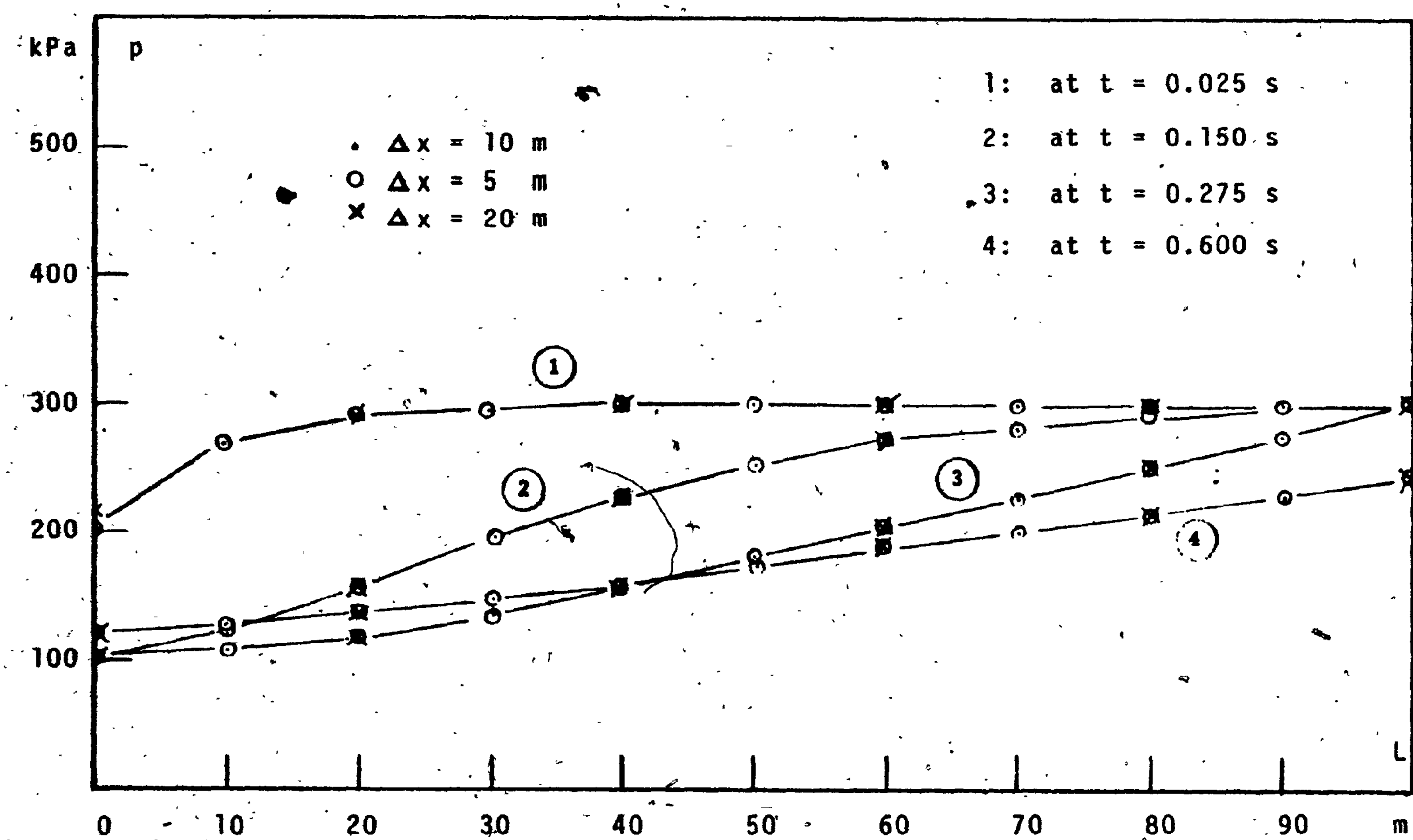



Diagram D 9.3.4.3 Pressure along the Pipe with Different Element Lengths.

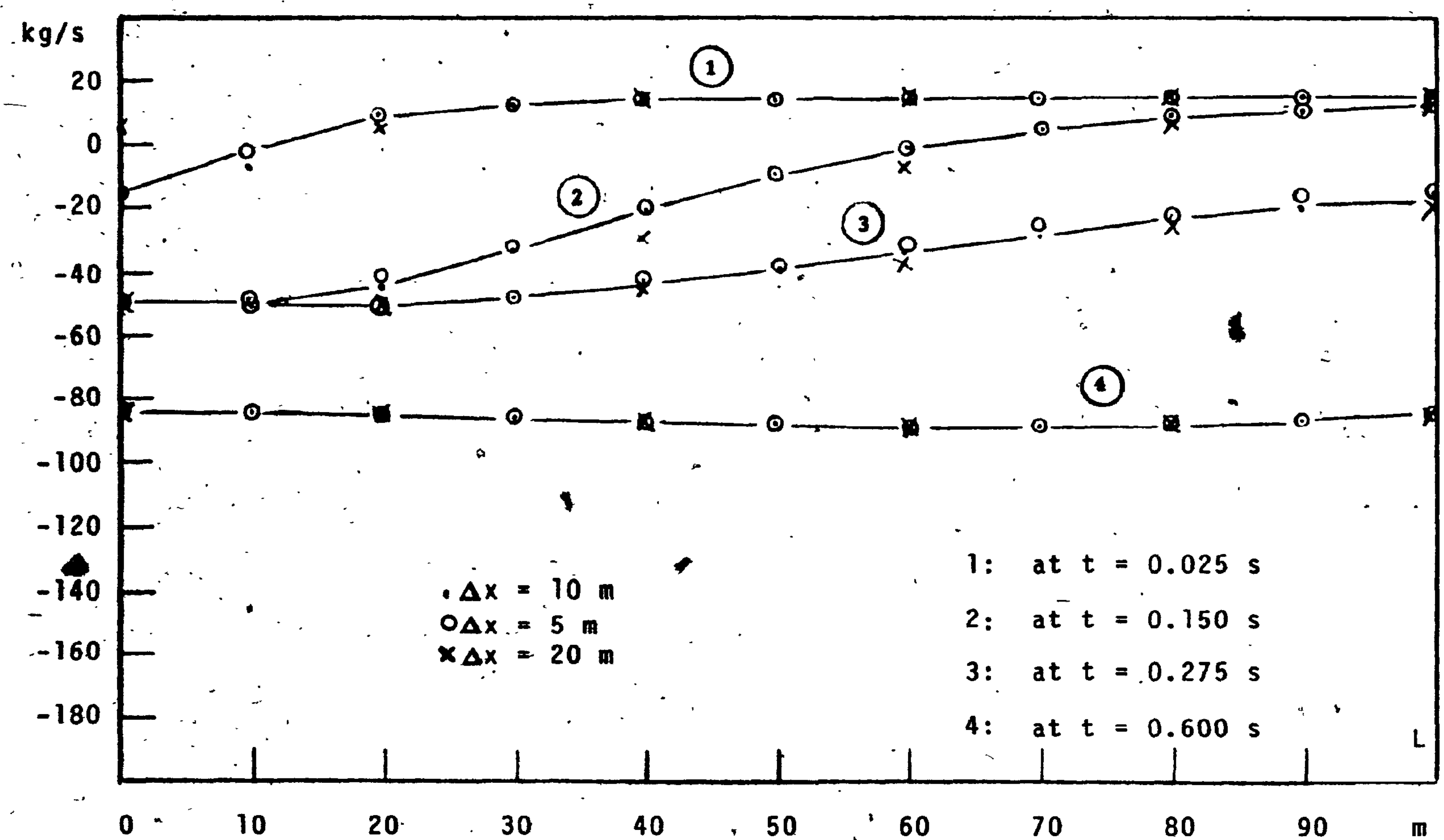



Diagram D 9.3.4.4 Mass Flow along the Pipe with Different Element Lengths.

9.3.5 MODEL EVALUATION.

An ideal numerical model (subparagraph 7.2.1) should have:

: meaningful physical variables,
: adaptability to non-isentropic conditions,
: easily readable results,
: sufficient accuracy,
: the possibility of accepting modifications,
: reasonable computer requirements.

The Lumped Element Methods use pressure and mass flow as dependent variables. Nodal density vector is available and nodal velocities are easily obtained when required.

Viscous flows can be handled introducing an additional non linear resistor to be solved by the already necessary density iteration. Non-adiabatic flows can be solved calculating the (n+1) time level nodal density using proper (n) time level parameters, according the concepts and the formulae of subparagraphs 6.2.2 and 8.2.1.

Results are obtained as a succession of strings of nodal values of constant time level (Appendix 5), and the orthogonal grid dimensions can be modified uniformly or in selected zones in both space and time dimensions.

Accuracy is, in general, acceptable, when values are compared with Method of Characteristics results, and most discrepancies are time shiftings more than differences in peak values. Accuracy can be improved?with finer grids, especially in time.

Modifications of the basic network and addition of piping equipment can be handled, adding to the basic quasilinear system groups of steady state equations, each group representing steady state continuity and momentum equations of additional pipe elements. Branches can also be introduced and treated as an additional group of equations.

Computer time for Linear Element and Finite Element Methods, referred to the Test Case is much higher than that required using the Method of Characteristics, but can be greatly improved using solvers faster then the Gaussian Elimination.

## 9.4 CONCLUSIONS

The Lumped Element Methods, based on the numerical solution of continuity, momentum and energy-entropy equations, gives reasonable values of the flow variables of the unstesdy, continuous flow of real gases in piping systems.

The Lumped Element Methods solutions, when compared with results obtained with commonly accepted methods, show smoother wave fronts, travelling at realistic velocity, with realistic bottom and top values.

The smoother shape of the front wave is mainly due to the intrinsic approximation of the numerical methods used, all of order ($\Delta x$, $\Delta t$), therefore, ignoring errors proportional to $\Delta x$ , $\Delta t$ and to the second order time and space derivatives. These approximations are equivalent (26) (27) (28) to introducing, at least for the mass flow derivatives, an artificial viscosity. In other words, the smooth wave fronts may represent, to a certain extent, the behaviour of a fluid of higher viscosity than the fluid under consideration.

Only additional theoretical analysis and experimental work will allow an accurate efficiency rating of the Lumped

Element Methods which, now, can be considered promising but not fully tested means of investigation of continuous unsteady flows in piping systems.

Whether and how these models can be adapted to shock conditions is the subject of the next chapter.

## CHAPTER 10

## GAS HAMMER

### 10.1 SUMMARY

In this chapter the block "gas hammer" of Fig. F 2.3.2.1 is investigated.

The dynamics of a gravity check valve, closing in a line under the effect of a sudden flow reversal, is examined and its similarity to an instantaneous closure justified.

The travelling shock, generated by a valve sudden closure is analyzed, modelled and integrated in the continuous, unsteady flow numerical model (Program SHOCK).

Program SHOCK is applied to the Test Case, completed with the instantaneous closure of the pipe left end. The influence of delayed instantaneous closures on the gas hammer strength is then studied and the benefits of a fast closing valve demonstrated.

Negative side effects of slow clapper closings are discussed and Partial Cost Functions indicated for both fast closing and slow closing check valves.

## 10.2 VALVE CLOSING

### 10.2.1 SUDDEN VALVE CLOSURE.

The closing time of a check valve may vary through a wide range of values, depending on the valve size, on the auxiliary equipments, springs, counterweights and dampers, and on the flow conditions.

A complete analysis of valve closing requires compounding the valve dynamics (Program DYNA) and the flow instantaneous characteristics of the piping system. In subparagraph 5.2.1 general guidelines have been given for a complete numerical solution.

The closure of a check valve is generated by a mass flow reduction below the limit required to balance the total mechanical effect of valve, counterweight, springs and system friction, and/or pressure differentials. Sudden flow inversions and downstream pressure build-ups increase the valve closing velocity. When a valve closes in reverse flow conditions, a pressure build-up occurs at the entrance side and, therefore, a reduction in valve closing time may be expected.

Pressure build-up at high reverse flow velocities is transmitted against the flow direction at reduced speed and the transmission velocity becomes zero when the reverse flow reaches local sonic conditions. At sonic conditions a shock must occur to transmit a pressure signal against the flow direction.

Compression waves, generated behind the shock front, will tend to overtake the shock to generate a reinforced shock front (14) while, in non shock conditions, compression waves will tend to overtake preceeding waves of the same family to generate steeper and stronger waves until, eventually, shock occurs.

A sudden valve closure (zero closing time), located at the tail end of the real closing time, will be a conservative but realistic approximation of the valve real closure, sufficient to obtain information on valve closing effects before or without using the complete dynamic model.

10.2.2 TRAVELLING SHOCK.

Let us introduce a sudden valve closure at the left end, "rupture" side, of the piping system assumed as Test Case (Fig. F 10.2.2.1).

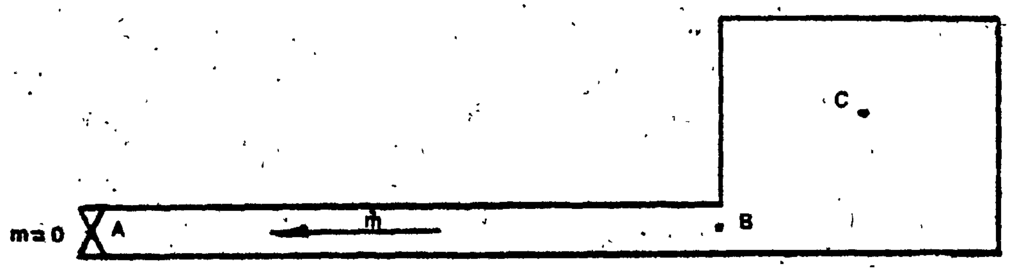
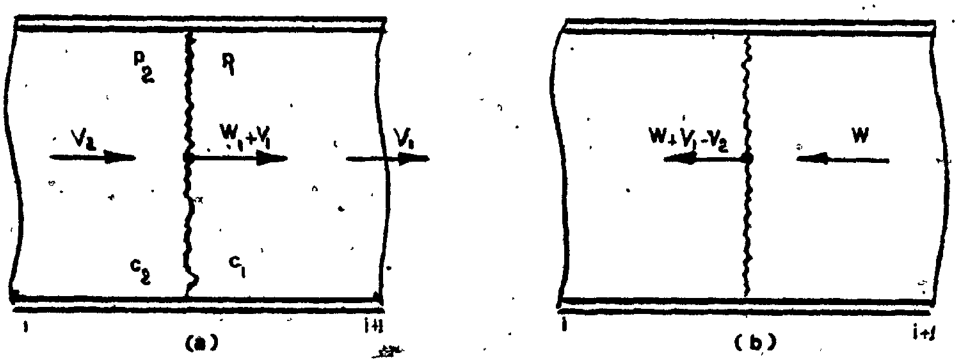



Fig. F 10.22.1 Sudden Valve Closure.

Sudden valve closure for low operating pressure or, in general, at low reverse flow will generate a relatively moderate compression wave and non shock conditions. Therefore, the continuous flow models of program PIPING can still be used to obtain flow and pressure information.

For reverse flow higher velocities, i.e. for Mach numbers closer to unity, a sudden closure generates a shock.




a) Stationary coord. b) Moving coord.

Fig. F 10.2.2.2 Travelling shock.

With reference to Fig. F 10.2.2.1 and Fig. F 10.2.2.2 it is:

: Shock is possible if $(V_i - V_{i+1}) > 0$

: Shock is right travelling if $|V_i| < |V_{i+1}|$.

If shock is right traveling, it may be assumed:

: $V_1 = V_{i+1}$ $\qquad c_1^2 = \gamma * p_{i+1} / q_{i+1}$

: $V_2 = V_i$

If shock is left traveling, it may be assumed:

: $V_1 = V_i$ $\qquad c_i = * p_i / q_i$

: $V_2 = V_{i+1}$

Let us consider the sudden closure, at time $t^n$, of the valve at node i of Fig. F 10.2.2.1. At time $t^n$ it is (subparagraph 6.3.2, and Fig. F 10.2.2.3):

$$M_x = \frac{UDIFF + \sqrt{(UDIFF)^2 + 4}}{2} \qquad (6.3.2.10)$$

with:

$$UDIFF = \left(\frac{\gamma+1}{2}\right) * \left(\frac{V_2 - V_1}{c_1}\right)$$

$$M_x = \frac{W}{c_1}$$

$V_2 = V_i$ (zero at node i)

W= absolute value of shock velocity relative to the gas into which moves (i.e. the gas in state 1)

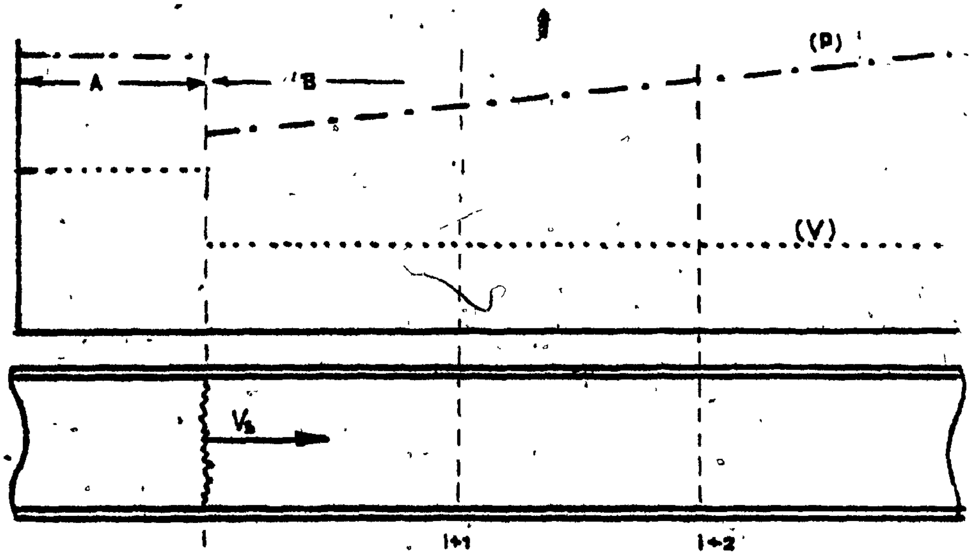



Fig. 10.22.3 Shock at time t

Weak shocks ($(M_x - 1) \ll 1$) may be treated as continuous compression waves. For strong shocks it is (14):

$$VS = V_i + W \qquad \text{for right shock} \qquad (10.2.2.1)$$

$$VS = V_i - W \qquad \text{for left shock} \qquad (10.2.2.2)$$

with:

VS = absolute shock velocity

The node i pressure, behind the shock front, is given by (subparagraph 6.3.2):

$$\frac{p_2}{p_1} = 1 + \frac{2*\gamma*(M_x^2 - 1)}{\gamma+1} \qquad (6.3.2.13)$$

and density:

$$\frac{\rho_2}{\rho_1} = \frac{1}{1-\left(\frac{2}{\gamma+1}\right)*\left(1-\left(\frac{1}{M_x^2}\right)\right)} \qquad (6.3.2.12)$$

with:

$$p_2 = p_i^n \qquad \rho_2 = \rho_i^n$$

$$p_1 = p_i^{n+1} \qquad \rho_1 = \rho^{n+1}$$

Therefore, at time $t^n$, the sudden closure of the check valve at node i has generated, at the same node i, a shock front, with known pressure, density and flow velocity behind it, moving at velocity VS toward node (i+1).

Assuming the traveling shock velocity constant on element (i), the shock front will reach node (i+1) at time:

$$t^{n+1} = t^{n} + \Delta t$$

with:

$$\Delta t = \Delta x / VS$$

At time $t^{n+1}$, the flow parameters, pressure, density and velocity, of node (i+1), before the arrival of the shock front, are not yet affected by the shock conditions and may be known as results of the application of the continuous flow model to the subdomain A (Fig. F 10.2.2.3).

At the same time increment, the flow parameters of node (i+1), non updated but after the arrival of the shock front, may be assumed known, translating to node (i+1) the after shock pressure, density and flow velocity of node i.

It is now possible to determine the after shock parameters of node (i+1) at time $t^{n+1}$, knowing:

$$UDIFF = \left(\frac{\gamma+1}{2}\right) * \left(\frac{V_S - V_1}{C_1}\right)$$

with:

$$V_2 = V_i$$

$$V_1 = V_{i+1}$$

and using the already remembered (6.3.2.10, 12, 13).

The shock front, caused by the valve closing, has moved from node i to node (i+1) at a velocity on its strength. Repeating this procedure, the shock front will travel along the pipe with a speed proportional to its nodal strength.

The shock front divides the space domain in two subdomains, where, assuming that additional shock conditions will not occur, the unsteady flow parameters may be separately determined according to the continuous flow theory and models applied to each of them.

## 10.3 SHOCK MODEL

### 10.3.1 NUMERICAL MODEL.

Subroutine FRONT, added to program PIPING (Appendix 6: PROGRAM SHOCK), introduces the shock front calculations and the subdomain boundary conditions as complements to the regular continuous flow models.

Subroutine FRONT is automatically activated when M, calculated between contiguous nodes, reaches a value greater than a given constant, close to unity, thus excluding weak shocks, which are treated as continuous compression waves (14).

Diagram 10.3.1.1 shows pressure values, obtained with an application of program SHOCK to the test case for the usual operating conditions:

p = 300,000 Pa.
ϱ = 3.6 Kg/m3.
V = 30 m/s.

and for:

D = 18"
L = 100 m

when a valve suddenly closes after 0.4 second of a "rupture" occurred at the pipe left end with pressure dropping from operating to atmospheric values in 0.05 seconds.

The sudden valve closure activates a swift pressure rise, the shock, which is followed by a further, gradual pressure increase. The gradual pressure increase is the result of flow entering the system from the reservoir which is at higher pressure since the rupture expansion wave has reached the pipe right end.

When the shock front reaches the pipe right end the pipe right end assumes a pressure value higher than the reservoir and, therefore, the flow changes direction. The flow inversion spreads all along the pipe and a pressure decrease occurs until a new flow inversion starts.

As a result, at each pipe section,the pressure oscillates with a decreasing amplitude, as the pipe net energy balance is negative, and with a constant frequency, until, eventually, the steady state is reached. The pressure variation at the valve point, the left pipe end, is what is shown in Diagram D-10.3.1.1.

This shock model, although giving realistic and acceptable results, can only be considered a first, simple attempt at representing a moving shock front along a pipe system. The model is not yet experimentally tested and is limited to being able to handle only one shock front.

This limitation and others can be eliminated with further theoretical work, which should be subordinated to an experimental confirmation of the accuracy of the simple one front model.

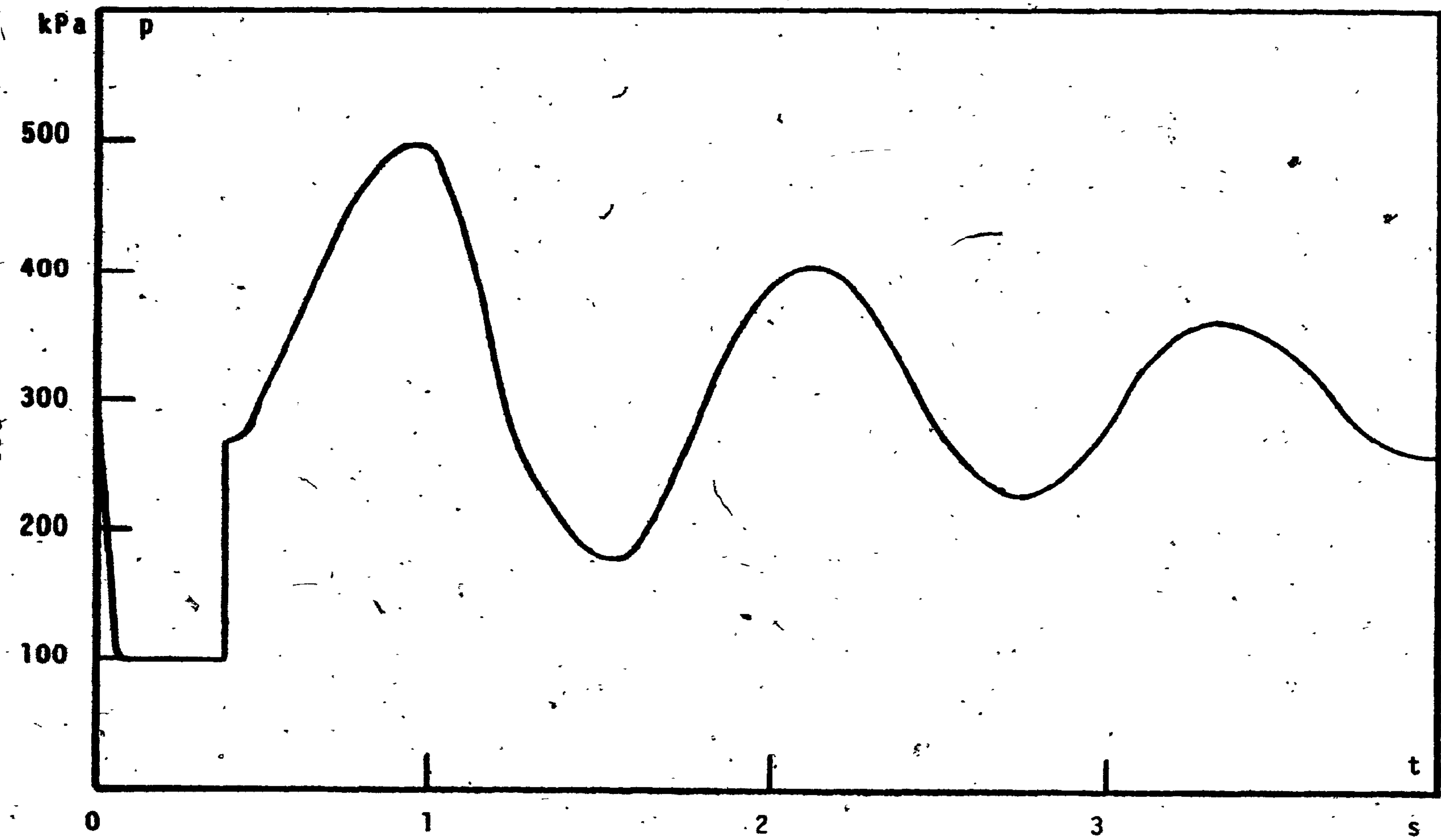



Diagram D 10.3.1.1    Sudden Valve Closure.   Pressure Variation at Valve Point.

10.3.2 EFFECTS OF CLOSING TIME.

Diagrams, D 10.3.2.1, D 10.3.2.2, and D 10.3.2.3 show the pressure variation at the pipe right end of the test case for a sudden valve closure with the usual geometric conditions and pressure drop assumptions, but for three typical operating pressures:

p = 149,000 Pa.

p = 300,000 Pa.

p = 3,000,000 Pa.

and for different sudden closure times.

All readings, and diagram D 10.3.2.4 synthesis, indicate that at all pressures the gas hammer effect increase with the time delay of the instantaneous closure.

Although an accurate quantitative answer requires a combination of the valve dynamic model with the shock model, by inspection it may be asserted that a flow activated check valve increase its closing velocity when a flow inversion occurs in a short time. Therefore, sudden closures from 0.2 to 0.4 seconds after rupture may conservatively represent real line closures of wafer check valves to 36" pipe diameter.

A flow activated check valve, of fast response, and of light construction, is an excellent protection against back-flow and naturally approaches the ideal conditions for limiting gas hammer effects.

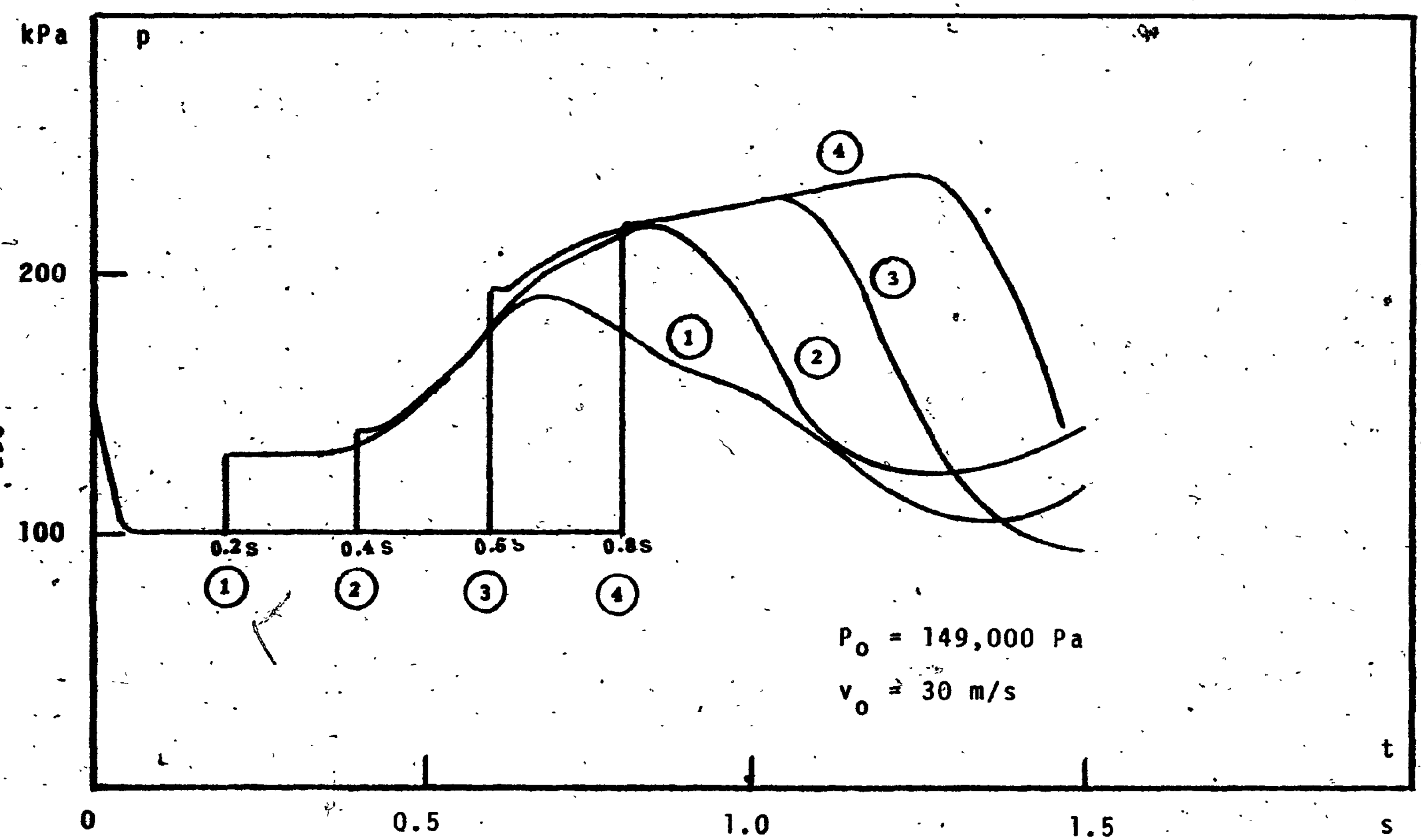



Diagram D 10.3.2.1 Shock Variation with Sudden Closure Delay.
Low Pressure Line.

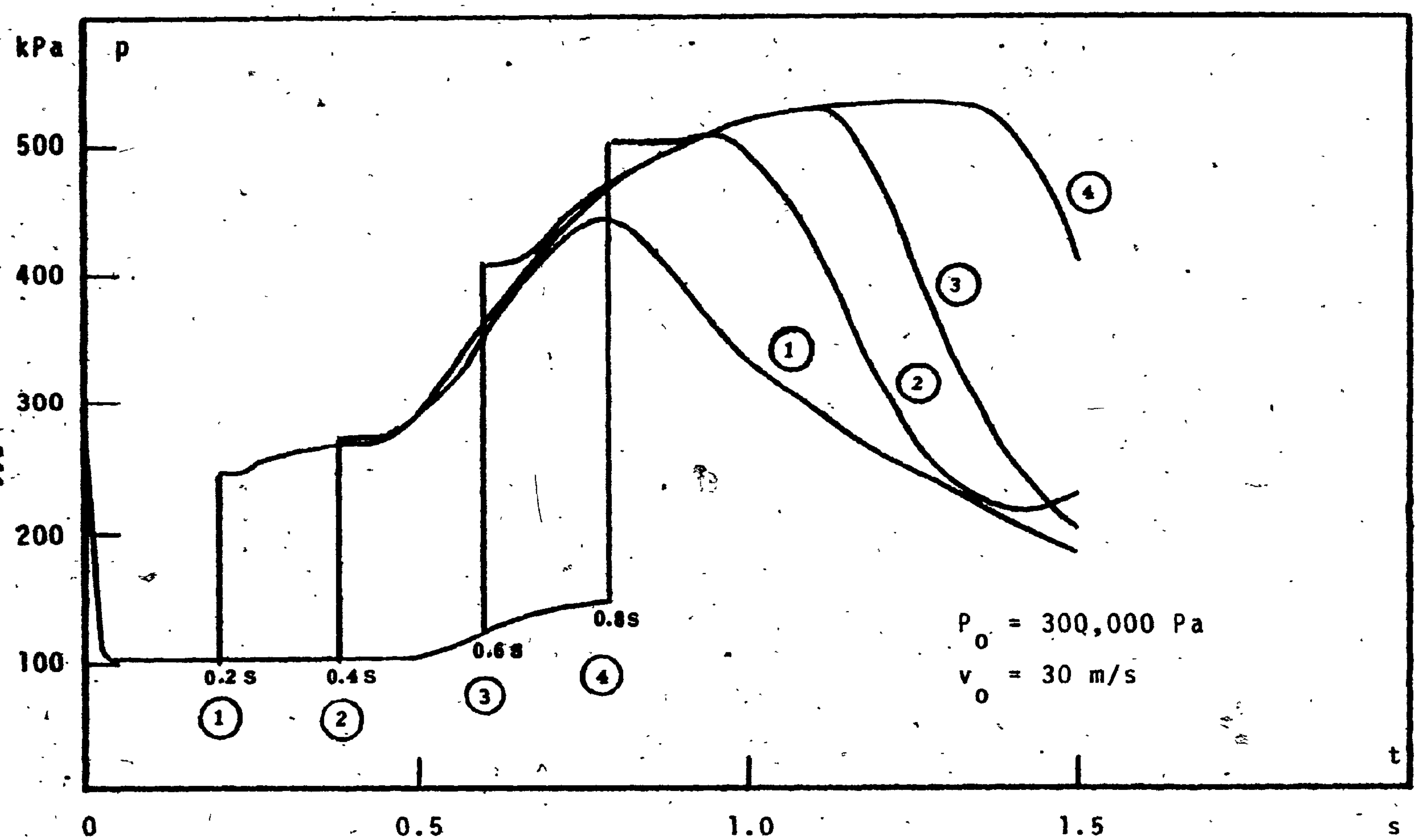



Diagram D 10.3.2.2 Shock Variation with Sudden Closure Delay. Medium Pressure Line.

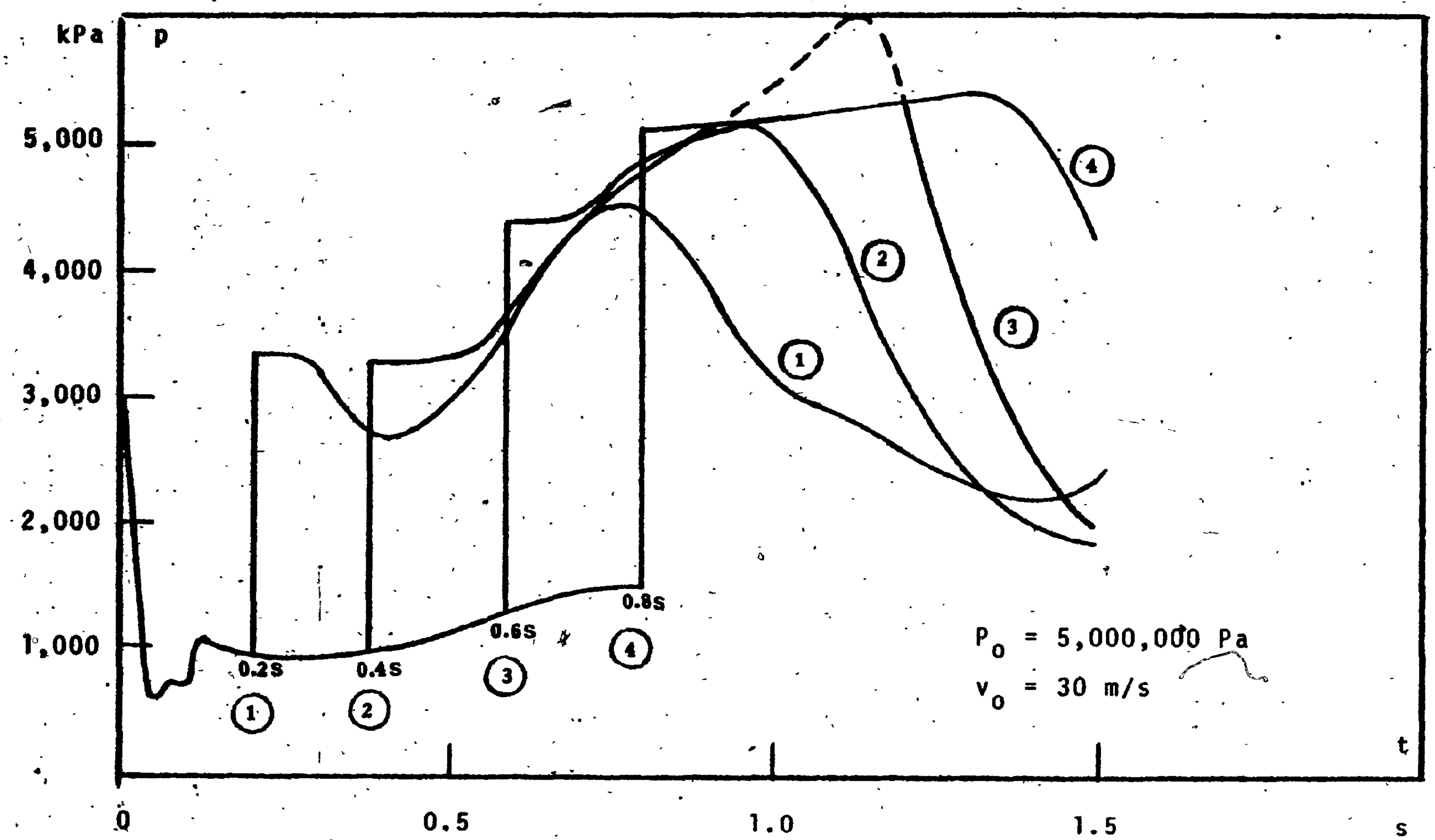



Diagram D 10.3.2.3 Shock Variation with Sudden Closure Delay. High Pressure Line.

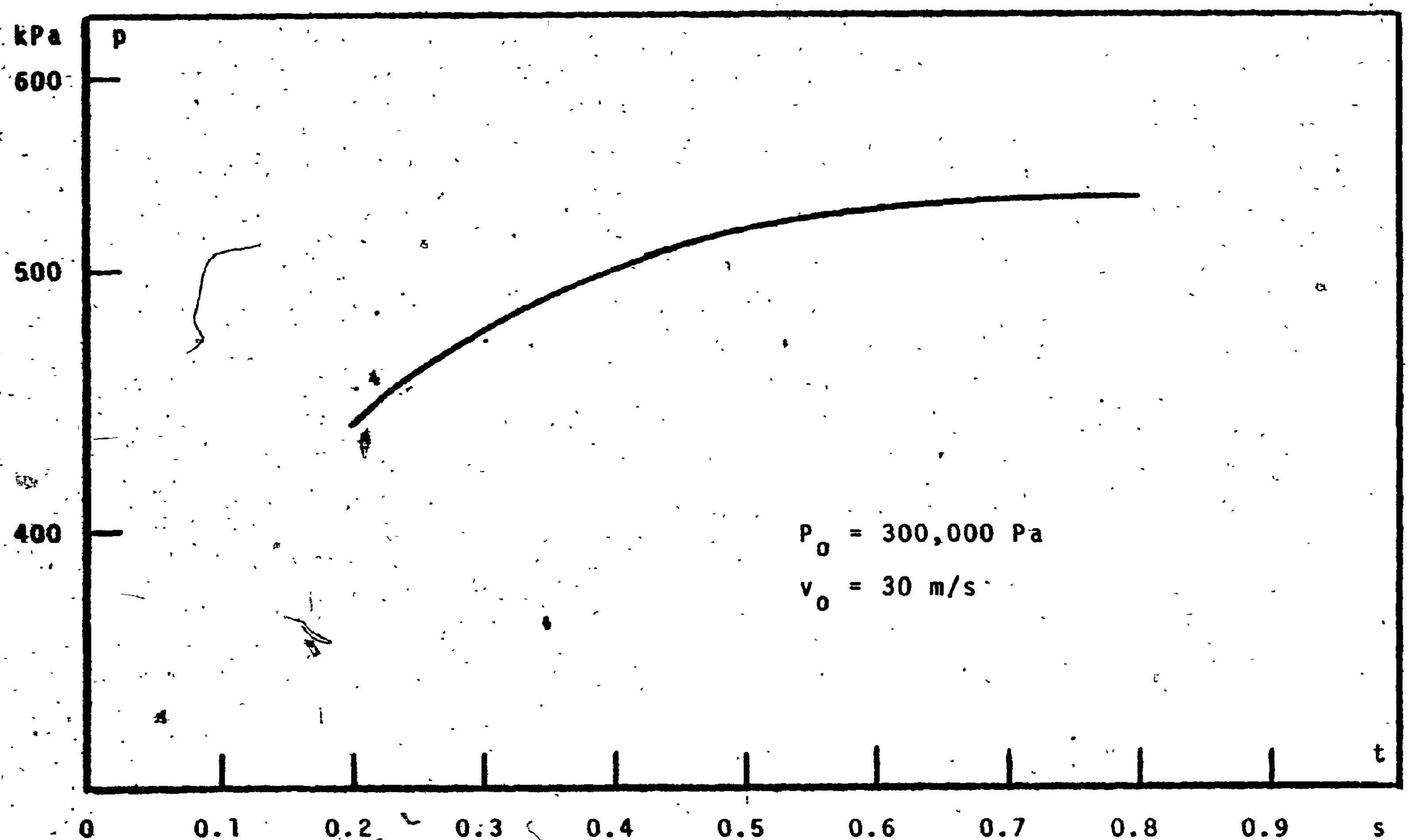



Diagram D 10.3.2.4 Peak Pressure vs. Sudden Closure Delay.

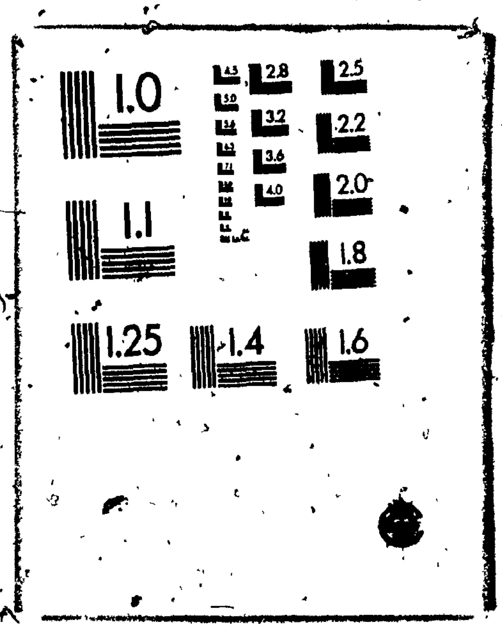
### 10.3.3 MODEL ANALYSIS.

As already mentioned, the program SHOCK can handle only one shock front and, therefore, only a sudden valve closure can be considered.

The amplitude of the continuous waves (Diagram D 10.3.1.1), which are subsequent to the travelling shock, is undoubtedly decreased by the artificial viscosity, introduced with the numerical solver of the flow differential equations. How much of the damping, shown in the pipe pressure oscillations, is due to artificial viscosity, and how much to the right boundary energy outflow, can be determined with further analytical and experimental work.

The reservoir boundary condition is inadequate for demanding applications (subparagraph 9.2.3), which require additional investigation, and can generate model instabilities for finer space discretizations.

4 OF/DE 4



## 10.4 CONCLUSIONS

The shock model and the computer program SHOCK, in spite of their limitations, allow valuable qualitative and, sometimes, quantitative considerations about gas hammer and how it is influenced by the type of valve closing.

Gas hammer is undoubtedly less devastating for piping than water hammer, its equivalent in liquid flow piping systems, and may become negligible for low pressure-density flows. Its strength always increases with the increase of the time delay of an instantaneous pipe interception.

When the intercepting device is a gravity check valve, there is a "natural" adjustment to conditions minimizing the pressure surge.

While a delay on the first part of the closing cycle, equivalent to a delayed sudden closure, must be avoided, the addition of springs and the containment of the valve nominal size are beneficial. On the contrary, high inertia counterweights have a slightly negative effect.

Summarizing, the Partial Cost Function of block "Gas hammer" of figure F 2.3.2.1 for a fast closing valve is shown in Table T 10.4.0.1.

TABLE T 10.4.0.1 Partial Cost Function for Fast Closing Valves.

| | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|
| $I : Investment costs | X | X | | | |
| $R : Recovery costs | X | | | | |
| $O : Operating costs | | | | | |
| $M : Maintenance costs | | | | | |
| | | | | | |
| $II : Indirect investment costs | X | X | X | | |
| $IM : Indirect maintenance costs | | | | | |

It is known, and confirmed by program SHOCK in non reported numerical experiments, that a partial closure, similar to a valve closing cycle slowed in its last part, noticeably reduces the gas hammer or, for liquids, the water hammer.

While the partial closure effect can be used to develop interesting line mufflers, the slowing of the last part of the disc closing cycle, also beneficial for slamming control, has serious limitations in its practical application.

First of all, the disc slowing effect must be limited, to avoid excessive amount of fluid flowing back upstream of the check valve, then, also assuming that the amount of non intercepted fluid does not damage upstream equipment, the backflow can generate a pressure drop, and a flow torque, which, at small disc angles, results in excessive torsion stresses on the clapper pivot shaft.

Table T 10.4.0.2 shows the "Gas hammer' Partial Cost Function for a slowed valve closing.

TABLE T 10.4.0.2 Partial Cost Function for Slowed Valve Closing.

| | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|
| $I : Investment costs | X | X | | | X |
| $R : Recovery costs | X | | | | |
| $O : Operating costs | | | | | |
| $M : Maintenance costs | | | | | X |
| | | | | | |
| $II : Indirect investment costs | X | X | X | | X |
| $IM : Indirect maintenance costs | | | | | |

# CHAPTER 11

# CONCLUSIONS

# AND

# SUGGESTED FURTHER WORK

## 11.1 CONCLUSIONS

### 11.1.1 TOTAL COST FUNCTION.

Table T 11.1.1.1 shows the influence of the wafer valve independent variables:

VS : valve size,
VR : auxiliary devices,
VA : clapper max. opening angle,
VA : valve orifice,
VL : material stress,

on the cost factors of the Total Cost Function.

TABLE T 11.1.1.1 Global Cost Function for Fast closing Valves.

| | VS | VR | VA | VO | VL |
|---|---|---|---|---|---|
| $I : Investment costs | X | X | | | |
| $R : Recovery costs | X | | | | |
| $O : Operating costs | X | | X | X | |
| $M : Maintenance costs | X | X | | | |
| | | | | | |
| $II : Indirect investment costs | X | X | X | | |
| $IM : Indirect maintenance costs | | | | | |

The Total Cost Function is obtained combining the Partial Cost Functions of the blocks of Fig. F 2.3.2.1, examined in the previous chapters:

: Pressure losses,

: Energy degradation,

: Valve opening,

: Oscillations,

: Gas hammer.

The influence of these blocks on the cost factors is shown in Table T 11.1.1.2, with:

AV = "Pressure losses" and "energy degradation",

OO = "Valve opening" and "Oscillations",

GH = "gas hammer".

TABLE T 11.1.1.2 Influence on Dependent Variables of Design Blocks.

| | AV | OO | GH |
|---|---|---|---|
| $I : Investment costs | X | X | X |
| $R : Recovery costs | X | X | X |
| $O : Operating costs | X | | |
| $M : Maintenance costs | | X | |
| | | | |
| $II : Indirect investment costs | | | X |
| $IM : Indirect maintenance costs | | | |

In gas lines, energy degradation, while significant for high pressure-density fluids, is a factor of much less importance, compared to water hammer in liquid lines.

There is a qualitative uniformity between liquid and gases on the influence of energy degradation and other design parameters, while their weight is related to fluid density, directly for the energy degradation, inversely for some other parameters. A liquid may be considered, with the exception of the shock theory, a gas with high density and low compressibility and a liquid line treated with the same numerical algorithms, used for gas, with very little modifications, simply ignoring the shock algorithm.

In this case, negative values of pressure indicate areas of flow discontinuity, where cavitation can occur.

As a consequence, a low inertia, fast closing check valve is suitable for protecting a line and in containing pressure rises for both gases and liquids, while the energy efficiency requirement becomes more important with increase of fluid density.

A fast closing valve is beneficial from most points of view, but, when fast closure is obtained with high disc velocity, this characteristic may be in conflict with the necessity of preventing "slamming" of the disc on the seat surface.

Also without performing any investigation into slamming, it can be said, without doubt, that slowing the disc in the last part of its closing cycle, is a way of protecting the valve seat, as long as this practice does not allow too much flow through the valve and the pivot shaft torsion stresses are kept at an acceptable level.

As an order of magnitude, a pressure drop of one psi through a 36" valve generates more than 1500 ft.lb. of torque, acting on the disc pivot shaft. In a semiclosed position, one psi pressure drop may be reached with a very limited reverse mass flow.

A safe slamming reduction, although only partially effective, is possible using pneumatic shock adsorbers, instead of hydraulic dampers, which may easily become a danger to the valve integrity.

The really sound way of controlling the disc slamming is to use a valve with low inertia and minimum closing arc.

### 11.1.2 EVALUATION OF THE WAFER CHECK VALVE.

Using the conclusions of the previous chapters, the wafer check valve can be compared and rated against other popular types of check valves.

From the energy degradation point of view, it had already been seen (subparagraph 3.2.2) that the wafer check is inferior only to the full bore API 6D gravity swing check valve and, may be, to the larger sizes of the low angle tilting disc check (10).

The dynamic analysis of the wafer check, used for gas lines, shows that the only real drawback is the difficulty to reach the fully open position with low density flows. This inability, shared with all gravity swing check valves, can be overcome either by reducing the disc maximum opening (Chap. 5), or by reducing the valve nominal size, with little harm to the energy dissipation costs.

On the other hand, the wafer check is not heavily dependant on closing springs, as is the case with the double door check, and, without internal springs, can oscillate at low flows with no danger of internal ruptures.

The closing time of the wafer check, and, therefore, its gas hammer efficiency, is good and, when helped with external springs, this valve response is second only to the tilting disc check. Very fast closing times are required mainly when several compressing units are connected with very short branches to a manifold or in some special applications.

The relatively high inertia of the wafer and all other swing check valves, while beneficial to their stability at very low flows, penalizes these valves, when slamming is taken into consideration.

The traditional swing check valve has only one advantage over the wafer check, but that is an important one. It is possible to change clapper, without taking the valve out of the line. Since the wafer check must be taken out of the line, this precludes its utilization for high temperature steam services.

One important advantage of the wafer check, partially shared with the double door check, is the compactness of its longitudinal and transversal dimensions, which makes cost and weight of this valve very attractive.

Summarizing, the wafer check is an excellent investment and performance compromise for all gas services, excluding high temperature (steam) lines and services where very small closing times are required.

## 11.2 SUGGESTIONS FOR FURTHER WORK

### 11.2.1 IMPROVEMENT IN VALVE DESIGN.

An ideal check valve for gas is a gravity type which combines the economical and dimensional advantages of the wafer check with the low closing time and inertia characteristics of the tilting disc check, and with the additional possibility of using its core, seat and disc system, in a conventional body to allow disc replacement in line.

The combination of all these design characteristics will result in a low investment, fast closing and non slamming swing check valve, adaptable to any pressure and temperature gaseous service. The same valve would be excellent also for all clean, non corrosive, liquid services, providing that good energy efficiency would also be obtained.

### 11.2.2 IMPROVEMENT IN SIMULATION MODELS.

An experimental analysis of the Orifice Sequence Model (Chap. 4) is the mandatory requirement, before starting any further modelling work on programs COMPR (Availability destruction) and DYNA (Valve dynamic).

Experimental tuning, with gaseous fluids, is also necessary for programs PIPING and SHOCK, before starting further theoretical and modelling work on unsteady flow in piping.

A major area of improvement is open for the unsteady flow numerical models and programs PIPING and SHOCK.

The shock wave algorithm (Program SHOCK) needs to be modified, to handle more than one shock wave at a time, to simulate finite velocities of clapper closure. In physical terms, it means to reproduce shock waves, travelling at different speeds and overtaking each other. A subdivision of the pipe length domain in a number of continuous subdomains, each limited by two shock fronts, with all fronts travelling at a specific velocity, seems to be a promising area of investigation.

The continuous waves model (Program PIPING) needs to be modified to automatically handle complex networks, in particular multibranches. The conceptual approach is clear, it requires the introduction of additional submatrices (paragraph 8.4). However, the actual development shows some numerical difficulties. Of much less difficulty should be the introduction of a faster matrix solver (subparagraph 9.3.5).

Where the Orifice Sequence Model proved to be not satisfactory for all valve sizes, it would be necessary to find a new algorithm to determine the flow torque, transmitted to the valve clapper. A finite element investigation, in 2-D or in partial 3-D seems to be the most promising approach.

Since all concepts and most algorithms can be easily adapted to handle incompressible flows, the dynamic and unsteady flow models should be combined in a general model, valid for complex piping systems in unsteady condition, for both gases and liquids.

With such a model available, the interaction between valves and piping could be simulated for complex industrial piping networks, allowing the optimization of the entire valving system and the development of unidirectional mufflers to further reduce the pressure surge with less energy dissipation.

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APPENDIX 1

FIELD CASES[1]

(1) Courtesy of RITEPRO Inc.

APPENDIX 1

Programs "COMPR", "PIPING" and "SHOCK" have been adapted and used to select check valves for several industrial applications. The most significant are:

1) Three air blowers in parallel serve an air line in a natural gas processing plant. Three hydraulically cushioned and balanced 12" x 16" CHECK RITE valves have been designed.

2) A water pumping station supplies cooling water to a steel plant. This 36" CHECK RITE valve, with hydropneumatic cushion and controlled closing time, has been designed.

3) An air line supplies solid catalyst to a petro chemical plant. A 36" CHECK RITE valve that is able to intercept the solid parts in all emergency conditions has been designed.

4) A low pressure line supplies an explosive mix of gases and a solid catalyst to a processing unit in a coal gasification plant. The behaviour and the service life of a CHECK RITE valve has been predicted. Actual results confirmed the numerical calculations.

APPENDIX 2

PROGRAM "COMPR"

```
      PROGRAM COMPR(INPUT,OUTPUT)

C  THIS PROGRAM CALCULATES:
C              : COEFFICIENT  OF RESISTANCE
C              : PRESSURE DROP
C              : AVAILABILITY DESTRUCTION
C  OF A WAFER CHECK VALVE.
C  THE PROGRAM IS SET TO HANDLE:
C              : VALVE ALONE
C              : VALVE WITH REDUCER (CASE=5)
C  VARIABLE CASE IS EXTERNAL.

      DIMENSION AKO(5),BKAA(3,4),XVAR(3),AY(4),
     1 ADELP(4),ADENSE(4),AVADES(4),ACOS(4)
     1 ,AKMAT(6,6)
      GAMMA=1.4

  900 FORMAT('1',21X,'PIPE EQIVALENT LENGTH,AVILABILITY'/
     1 22X,'PARTIAL COST FUNCTIONS'////
     1 4X,'VALVE SIZE',20X,':  ',F10.7,8X,'M'/
     1 4X,'PRESSURE  ',20X,':  ',F10.4,6X,'KPA'/
     1 4X,'DENSITY   ',20X,':  ',F10.8,4X,'KG/M3'/
     1 4X,'VELOCITY  ',20X,':  ',F10.4,6X,'M/S'/
     1 4X,'ENVIRON. TEMP.',16X,':  ',F10.4,8X,'C'/
     1 4X,'COST OF KWH',19X,':  ',F10.6,4X,'$/MWH'/
     1 4X,'CASE',26X,':',2X,F10.8/////
     1 32X,'CLAPPER OPENING ANGLE ( DEGREE )'//
     1 39X,'30',5X,'45',5X,'60',5X,F3.0,4X//)
  905 FORMAT(4X,'FRICTION FACTOR',29X,F7.5//
     1 4X,'PIPE EQIV. LENGTH',8X,'DIA',4X,4F7.2//
     1 4X,'KO QUADRATIC FACTORS')
  910 FORMAT(14X,'KOF',I1,25X,F14.7)
  915 FORMAT(4X,'NET EXPANSION FACTOR',12X,4F7.3/
     1 4X,'PRESSURE DROP',12X,'KPA',4X,4F7.3/
     1 4X,'EXIT DENSITY ',12X,'KG/M3',2X,4F7.3//
     1 4X,'AVAILABILITY DESTRUCTION ','MWH/Y',2X,4F7.3/
     1 4X,'AVAILABILITY DESTRUCTION ','$/Y',4X,4F7.0//
     1 4X,'AVAILABILITY QUADRATIC FACTORS')
  916 FORMAT(14X,'MAF',I1,25X,F14.7)
  917 FORMAT(/)
  918 FORMAT(14X,'$AF',I1,25X,F14.7)

C  READ OPERATING CONDITIONS

      PRINT*,'   ENTER VALVE SIZE (M):'
      READ*,VD
      PRINT*,VD
      PRINT*,'   ENTER OPER. PRESS. (KPA):'
      READ*,P
      PRINT*,P
      PRINT*,'   ENTER OPER. DENSITY (KG/M3):'
      READ*,DENS
```

```
      PRINT*,DENS
      PRINT*,'   ENTER VELOCITY (M/S):'
      READ*,VEL
      PRINT*,VEL

C  ENTER GAS CONSTANT

      PRINT*,'ENTER GAS CONSTANT'
      READ*,R
      PRINT*,R

C  READ VALVE DIMENSIONS

      PRINT*,'   ENTER D1,D2,DCL,X,DIS,ALA (ALL M):'
      READ*,D1,D2,DCL,X,DIS,ALA
      PRINT*,D1,D2,DCL,X,DIS,ALA
      PRINT*,'   ENTER WIDE OPEN ANGLE (DEGREE):'
      READ*,TETAF
      PRINT*,TETEF

C  COEFFICIENTS OF RESISTANCE MUST BE ALREADY REFERRED
C  TO DIAMETER.

      PRINT*,'ENTER COEFFICIENT OF RESISTANCE OF CONTRACTION'
      READ*,AKRC
      PRINT*,AKRC
      PRINT*,'ENTER COEFFICIENT OF RESISTANCE OF EXPANSION'
      READ*,AKRE
      PRINT*,AKRE
      PRINT*,'ENTER PIPE DIAMETER'
      READ*,DP
      PRINT*,DP

C  READ AVAILABILITY REFERENCE PARAMETERS

      PRINT*,'   ENTER ENVIRONMENT TEMP (C):'
      READ*,TRIF
      PRINT*,TRIF
      PRINT*,'   ENTER COST OF KWH ($/1000KWH):'
      READ*,CA
      PRINT*,CA

C  REDUCER

      PRINT*,'FOR REDUCER ENTER 5'
      READ*,CASE
      PRINT*,CASE

C  PRINT HEADERS

      PRINT 900,VD,P,DENS,VEL,TRIF,CA,CASE,TETAF

C  CALCULATION OF PIPE EQUIVQLENT LENGTH.
```

```
      TETA =30
      DO 99 I=1,4
      DO 99 J=1,4
       AKMAT(I,J)=0
99    CONTINUE
1     CV1=(D2/D1)**2/(.05+((D2/D1)**2-1)**2)**0.5
      AA=X*((D1/2)**2-X**2)**.5
      AVAL1=2*X/D1
      AVAL2=2*X/DCL
      AA=AA+((D1/2)**2*ASIN(AVAL1))
      AA=AA-COSD(TETA)*(X*((DCL/2)**2-X**2)**.5
     1+(DCL/2)**2*ASIN(AVAL2))
      AA=AA-(1-COSD(TETA))*2*ALA*X
      AA=AA+2*DIS*X*SIND(TETA)
      AA=4*AA/(3.14*D1**2)
      FF=1-(DCL/D1)**2*COSD(TETA)-AA
      IF(D1.LE.0.054)FFA=0.019
      IF(D1.GT.0.054.AND.D1.LE.0.105)FFA=0.0175
      IF(D1.GT.0.105.AND.D1.LE.0.160)FFA=0.015
      IF(D1.GT.0.160.AND.D1.LE.0.210)FFA=0.014
      IF(D1.GT.0.210.AND.D1.LE.0.260)FFA=0.0135
      IF(D1.GT.0.260.AND.D1.LE.0.310)FFA=0.013
      IF(D1.GT.0.310.AND.D1.LE.0.600)FFA=0.012
      IF(D1.GT.0.600)FFA=0.011
      DO=0.7*ALA*TAND(TETA)
      DF=(FF*D1**2)
      DF=SQRT(DF)
      D3=SQRT(D1**2-DF**2)
      S1=3.14*D2*DO
      IF(S1.LT.0.001)S1=0.001
      DS1=SQRT(4*S1/3.14)
      IF(DS1.GT.D1)DS1=D1
      S2=3.14*((D1-D3)**2+DO**2)**0.5*(D1+D3)/2
      DS2=SQRT(4*S2/3.14)
      IF(DS2.GT.D1)DS2=D1
      AK1=0.04/(D2/D1)**4
      BETA=(D2/DS1)**2
      AK2=((1-BETA)**2/BETA**2)/(DS1/D1)**4
      BETA=1/BETA
      IF(BETA.LT.1)AK2=((1-BETA)/BETA**2)
     1 /(D2/D1)**4
      BETA=(DF/DS2)**2
      GAMMAA=90-TETA
      GAMMAA=SIND(GAMMAA)
      AK3=((1-BETA)*SQRT(GAMMAA)/BETA**2)/(DS2/D1)**4
      AK4=(1-FF)**2/FF**2

C  PROVISION FOR REDUCER

      IF(CASE.EQ.5)THEN
       AKR=AKRC+AKRE
       ALPHA=D1/DP
       ALPHA=ALPHA**4
```

```
      ELSE
         ALPHA=1
         AKR=0
      END IF

C  TOTAL COEFFICIENT OF RESISTANCE

      AK=AK1+AK2+AK3+AK4+AKR*ALPHA
      K=K+1
       AKMAT(K,1)=AKRC/FFA
       AKMAT(K,2)=AK1/(FFA*ALPHA)
       AKMAT(K,3)=AK2/(FFA*ALPHA)
       AKMAT(K,4)=AK3/(FFA*ALPHA)
       AKMAT(K,5)=AK4/(FFA*ALPHA)
       AKMAT(K,6)=AKRE/FFA
      AKO(K)=AK/(FFA*ALPHA)
      TETA=TETA+15
      IF(TETA.GE.73)TETA=TETAF
      IF(K.LT.4)GO TO 1
      PRINT905,FFA,(AKO(I),I=1,4)

C  CALCULATION OF QUADRATIC FACTORS

      TETA =30
      DO 2 I=1,3
       BKAA(I,1)=AKO(I)**2
       BKAA(I,2)=AKO(I)*TETA
       BKAA(I,3)=1
       BKAA(I,4)=-TETA**2
      TETA=TETA+15
       IF(TETA.GE.73)TETA=TETAF
 2    CONTINUE
      CALL GAUSS(BKAA,XVAR)
      DO 3 I=1,3
       PRINT 910,I,XVAR(I)
 3    CONTINUE

C  CALCULATION OF NET EXPANSION FACTOR
C  ANDPRESSUR DROP

      P=1000*P
      TETA=30
      DO 4 I=1,4
      DERIF=DENS
      PRIF=P
      SUM=0
     DO 50 JJ=2,5
      IF(TETA.EQ.30)AK=AKMAT(1,JJ)
      IF(TETA.EQ.45)AK=AKMAT(2,JJ)
      IF(TETA.EQ.60)AK=AKMAT(3,JJ)
      IF(TETA.GT.60)AK=AKMAT(4,JJ)
       IF(TETA.EQ.30)AKKK=AKO(1)
       IF(TETA.EQ.45)AKKK=AKO(2)
```

```
      IF(TETA.EQ.60)AKKK=AKO(3)
      IF(TETA.GT.60)AKKK=AKO(4)
      Y=1
      DO 5 J=1,100
       DELTAP=FFA*AK*DENS*VEL**2/(2*Y**2)
       PE=P-DELTAP
       IF(PE.LE.PCR)THEN
        PE=PCR
        DELTAP=P-PCR
       END IF
       DENSE=PE**2+4*(P/DENS+((GAMMA-1)/(2*GAMMA))
     1 *VEL**2)*((GAMMA-1)/(2*GAMMA))
     1 *(VEL*DENS)**2
       DENSE=PE+SQRT(DENSE)
       DENSE=DENSE/(2*(P/DENS+((GAMMA-1)/(2*GAMMA))
     1 *VEL**2))
       REF=(DENS/DENSE-1)*2/AK*FFA+(1+DENS/DENSE)/2
       REF=SQRT(1/REF)
       ERROR=ABS(REF-Y)
       IF(ERROR.LE.0.00001)GO TO 6
       Y=REF
5     CONTINUE
      PRINT*,'EXCESSIVE Y ERROR'
6     DENS=DENSE
      P=PE
      SUM=SUM+DELTAP
50    CONTINUE
      AY(I)=Y
      ADENSE(I)=DENSE
      ADELP(I)=SUM/1000
      TETA=TETA+15
      DENS=DERIF
      P=PRIF
4     CONTINUE

C  CALCULATION OF AVAILABILITY DESTRUCTION
C  AND DOLLAR LOSSES

      TRIF=273.14+TRIF
      TETA=30
      DO 7 I=1,4
       AVAL1=(P-1000*ADELP(I))/P
       AVAL2=DENS/ADENSE(I)
       DELS=(1/(GAMMA-1))*ALOG(AVAL1)
     1 +(GAMMA/(GAMMA-1))*ALOG(AVAL2)
       AVADES(I)=TRIF*0.785*D1**2*DENS*VEL*DELS
       AVADES(I)=R*AVADES(I)*24*365/(1000000)
       ACOS(I)=CA*AVADES(I)
       TETA=TETA+15
       IF(TETA.GE.73)TETA=TETAF
7     CONTINUE
      PRINT
 915,(AY(I),I=1,4),(ADELP(I),I=1,4),(ADENSE(I),I=1,4),
```

```
     1 (AVADES(I),I=1,4),(ACOS(I),I=1,4)
C  CALCULATION OF AVAILABILITY FUNCTION
C  QUADRATIC FACTORS

      TETA=30
      DO 8 I=1,3
       BKAA(I,1)=AVADES(I)**2
       BKAA(I,2)=AVADES(I)*TETA
       BKAA(I,3)=1
       BKAA(I,4)=-TETA**2
       TETA=TETA+15
       IF(TETA.GE.73)TETA=TETAF
 8    CONTINUE
      CALL GAUSS(BKAA,XVAR)
      DO 11 I=1,3
       PRINT 916,I,XVAR(I)
 11   CONTINUE

C  CALCULATION OF COST FUNCTION
C  QUADRATIC FACTORS

      TETA=30
      DO 9 I=1,3
       BKAA(I,1)=ACOS(I)**2
       BKAA(I,2)=ACOS(I)*TETA
       BKAA(I,3)=1
       BKAA(I,4)=-TETA**2
       TETA=TETA+15
       IF(TETA.GE.73)TETA=TETAF
 9    CONTINUE
      CALL GAUSS(BKAA,XVAR)
      PRINT 917
      DO 12 I=1,3
       PRINT 918,I,XVAR(I)
 12   CONTINUE
      STOP
      END

C  SUBROUTINE GAUSS -SOLUTION OF A LINEAR SYSTEM
C  BY GAUSSIAN ELIMINATION-

      SUBROUTINE GAUSS(A,X)
      DIMENSION A(3,4),X(3)
      N=3
      M=N+1
      L=N-1

      DO 12 K=1,L
      KP1=K+1
      DO 11 I=KP1,N
```

```
      QUOT=A(I,K)/A(K,K)
       DO 11 J=KP1,M
11      A(I,J)=A(I,J)-QUOT*A(K,J)
      DO 12 I=KP1,N
12    A(I,K)=0
      X(N)=A(N,M)/A(N,N)
      DO 14 NN=1,L
       SUM=0
       I=N-NN
       IP1=I+1
       DO 13 J=IP1,N
13      SUM=SUM+A(I,J)*X(J)
14     X(I)=(A(I,M)-SUM)/A(I,I)
     RETURN
     END
```

APPENDIX 3

PROGRAM "DYNA"

```
      PROGRAM DYNA(INPUT,OUTPUT)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA

C  THIS PROGRAM ALLOWS TO COMPUTE THE VALVE
C  DIFFERENTIAL EQUATION. THE FOLLOWING DE-
C  VICES CAN BE ADDED:
C          : BENDING SPRING
C          : TORSIONAL SPRING
C          : COUNTERWEIGHT
C          : DAMPER
C  EACH AUXILIARY MAY BE IGNORED SETTING AT
C  ZERO ITS WEIGHT OR, FOR SPTINGS, THE SPRING
C  CONSTANT.


C  READ STEADY DOWNSTREAM OPERATING CONDITIONS

      READ*,PO,FLOWO,DENSO,PA,DENSA,OMEGA,AMPL
      PRINT 501,PO,FLOWO,DENSO,PA,DENSA,OMEGA,AMPL
  501 FORMAT('1',2X,'INPUT'//4X,'STEADY'
     1 ,' OPERATING CONDITIONS.'/4X,'DOWNSTREAM CONDITONS'
     1 ,' ARE SET IN OPE'/5X,'PO     ',F10.3/
     1 5X,'FLOWO ',F10.3/5X,'DENSO ',F10.3/
     1 5X,'PA    ',F10.3/5X,'DENSA ',F10.3/
     1 5X,'OMEGA ',F10.3/5X,'AMPL  ',F10.3)

C  READ VALVE GEOMETRY,WEIGHT AND INERTIA

      READ*,VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV
     1 ,ALG,ATETAV
      PRINT 502,VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV
     1 ,ALG,ATETAV
  502 FORMAT(//4X,'VALVE GEOMETRY WEIGHT AND INERTIA'
     1 /5X,'VD    ',F10.3/5X,'D1    ',F10.3/
     1  5X,'D2    ',F10.3/5X,'DCL   ',F10.3
     1 /5X,'XD    ',F10.3/5X,'DIS   ',F10.3
     1 /5X,'ALA   ',F10.3/5X,'TETAF ',F10.3
     1 /5X,'WV    ',F10.3/5X,'AJV   ',F10.3
     1 /5X,'ALG   ',F10.3/5X,'ATETAV',F10.3)

C  READ COUNTERWEIGHT GEOMETRY AND WEIGHT

      READ*,ALC,WC,AGAMMA
      PRINT 503,ALC,WC,AGAMMA
  503 FORMAT(//4X,'COUNTERWEIGHT GEOMETRY AND WEIGHT'/
```

```
     1 5X,'ALC   ',F10.3/5X,'WC    ',F10.3/
     1 5X,'AGAMMA',F10.3)

C  READ BENDING SPRING DATA

       READ*,AKS,ATETAS
       PRINT 504,AKS,ATETAS
 504   FORMAT(//4X,'BENDING SPRING DATA'/
     1 5X,'AKS   ',F10.3/5X,'ATETAS',F10.3)

C  READ TORSIONAL SPRING DATA

       READ*,AKTS,ALSO,AL1S,AL2S,ADELTA
       PRINT 505,AKTS,ALSO,AL1S,AL2S,ADELTA
 505   FORMAT(//4X,'TORSIONAL SPRING DATA'/
     1 5X,'AKTS  ',F10.3/5X,'ALSO  ',F10.3/
     1 5X,'AL1S  ',F10.3/5X,'AL2S  ',F10.3/
     1 5X,'ADELTA',F10.3)

C  READ DAMPER DATA

       READ*,AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,AL2D,
     1 ALPD,ABETA
       PRINT 506,AMD,WD,DCY,DRO,DENOIL,DOV,AL1D,AL2D,
     1 ALPD,ABETA
 506   FORMAT(//4X,'DAMPER DATA'/
     1 5X,'AMD   ',F10.3/5X,'WD    ',F10.3/
     1 5X,'DCY   ',F10.3/5X,'DRO   ',F10.3/
     1 5X,'DENOIL',F10.3/5X,'DOV   ',F10.3/
     1 5X,'AL1D  ',F10.3/5X,'AL2D  ',F10.3/
     1 5X,'ALPD  ',F10.3/5X,'ABETA ',F10.3)

C  READ INITIAL CONDITIONS,TIME INTERVAL,END OF INITIAL
C  CONDITIONS,TIME INTERVAL AND ELASTICITY FACTORS.

       READ*,TETAO,OMEO,DT,TIN,TMAX,ELS,ELB
       PRINT 507,TETAO,OMEO,DT,TIN,TMAX,ELS,ELB
 507   FORMAT(//4X,'INITIAL CONDITIONS'/4X,
     1 'TIME INTERVAL'/4X,
     1 'END OF INITIAL CONDITIONS'/4X,'MAX. TIME'/
     1 4X,'ELASTICITY FACTORS.'/
     1 5X,'TETAO ',F10.3/5X,'OMEO  ',F10.3/
     1 5X,'DT    ',F10.3/5X,'TIN   ',F10.3/
     1 5X,'TMAX  ',F10.3/5X,'ELS   ',F10.3/
     1 5X,'ELB   ',F10.3)
       TETA=TETAO
       OME=OMEO
       TIME=0
       PRINT 200
 200   FORMAT('1',30X,'RESULTS'////14X,'DISC ANGLE',
     1 3X,'FLOW TORQUE',3X,'ANG. VELOC. ',
     1 3X,'TIME'/17X,'DEG.',11X,'NM',11X,'RAD/S',
     1 8X,'S')
```

```
C  SYSTEM SOLUTION - RUNGE KUTTA -

  1     CALL RUNGE(TIME,TETA,OME,DT,TIN)

C  GEOMETRIC LIMITS

        IF(TETA.GE.TETAF)THEN
         TETA=TETAF
         OME=-ELB*OME
        END IF
        IF(TETA.LE.0)THEN
         TETA=0
         OME=-ELS*OME
        END IF

C  PRINT RESULTS
C  EXCESS TORQUE IS WITH FLOW TORQUE SIGN

        PRINT 100,TETA,DELTAF,OME,TIME
  100   FORMAT(14X,F10.5,4X,F10.3,4X,F10.5,4X,F6.3)
        IF(TIME.LT.TMAX)GO TO 1
        RETURN
        END


C  SUBROUTINE RUNGE

        SUBROUTINE RUNGE(TIME,TETA,OME,DT,TIN)

        COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1  PO,FLOWO,DENSO,OMEGA,AMPL
        COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1  ,ATETAV
        COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1  ,AL2S,ADELTA
        COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1  AL2D,LPD,ABETA
        RTETA=TETA*(3.14/180)
        F1=0
        F2=0
        CALL DIFFEQ(TIME,TIN,TETA,OME,F1,F2)
        AK11=DT*F1
        AK12=DT*F2
        TIMER=TIME+0.5*DT
        RTETAR=RTETA+0.5*AK11
        TETAR=(180/3.14)*RTETAR
        OMER=OME+0.5*AK12
        CALL DIFFEQ(TIMER,TIN,TETAR,OMER,F1,F2)
        AK21=DT*F1
        AK22=DT*F2
        TIMER=TIME+0.5*DT
        RTETAR=RTETA+0.5*AK21
```

```
      TETAR=(180/3.14)*RTETAR
      OMER=OME+0.5*AK22
      CALL DIFFEQ(TIMER,TIN,TETAR,OMER,F1,F2)
      AK31=DT*F1
      AK32=DT*F2
      TIMER=TIME+DT
      RTETAR=RTETA+AK31
      TETAR=(180/3.14)*RTETAR
      OMER=OME+AK32
      CALL DIFFEQ(TIMER,TIN,TETAR,OMER,F1,F2)
      AK41=DT*F1
      AK42=DT*F2
      RTETA=RTETA+(AK11+2*AK21+2*AK31+AK41)/6
      TETA=RTETA*(180/3.14)
      OME=OME+(AK12+2*AK22+2*AK32+AK42)/6
      TIME =TIME+DT
      RETURN
      END


C  DIFFERENTIAL EQUATION.SUBROUTINE DIFFEQ.

      SUBROUTINE DIFFEQ(TIME,TIN,TETA,OME,F1,F2)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      DC=0
      AC=0
      IF(WC.NE.0)CALL CTW(AC,DC,TETA)
      DSB=0
      IF(AKS.NE.0)CALLBSPR(DSB,TETA)
      DST=0
      IF(AKTS.NE.0)CALL TSPR(DST,TETA)
      AD=0
      DD=0
      IF(WD.NE.0)CALL DAMP(TETA,OME,AD,DD)
      CALL OPE(TIME,TIN)
      CALL ORSE(TOF,TETA,
     1 DELTAP,OME,TIME)
      CALL VALVE(TETA,AV,DV)
      A=AC+AD+AV
      IF(TIME.GT.15)TOF=0
      D=-DV-DC-DSB-DST-DD+TOF
      F2=D/A
      F1=OME
      RETURN
```

```
      END

C  OPERATING CONDITIONS

      SUBROUTINE OPE(TIME,TIN)
C  PO = STEADY OPERATING PRESSURE
C  DENSO = STEADY OPERATING DENSITY
C  FLOWO = STEADY OPERATING MASS FLOW
C  TIN = LINE START UP TIME (TO REACH STEADY OPERATING COND.)
C  FLOW = FLOW RATE
C  P = UPSTREAM PRESSURE
C  PE = DOWNSTREAM PRESSURE

C  UPSTREAM PRESSURE IS THE PRESSURE AT THE UPSTREAM
C  SIDE (STEADY FLOW) OF THE VALVE.
C  UPSTREAM AND DOWN STREAM PRESSURES BECOME INPUTS
C  WHEN THE MODEL IS INTEGRATED INTO THE PIPING SI-
C  MULATION.
C  OUTPUTS ARE:
C      : COEFFICIENT OF RESISTANCE IN SUBSONIC CONDITIONS,
C      : MASS-FLOW IN SONIC CONDITIONS.

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      GAMMA=1.4
      ST=PO/DENSO+((GAMMA-1)/(2*GAMMA))*(FLOWO/(0.7854*D1**2
     1 *DENSO))**2

C  START UP CONDITIONS

      IF(TIN.GT.TIME.AND.TIN.NE.0.)THEN
       P=(TIME/TIN)*(PO-PA)+PA
       FLOW=(TIME/TIN)*(FLOWO-0.00001)+0.00001
       DENS=(TIME/TIN)*(DENSO-DENSA)+DENSA
      ELSE

C  STEADY CONDITIONS

       TIS=TIN+5
       TIR=TIS+5
       IF(TIME.LT.TIS.OR.TIME.GT.TIR)THEN
        P=PO
        DENS=DENSO
        FLOW=FLOWO
       ELSE
```

```
         ALPHA=OMEGA*TIME
         FLOW=FLOWO+AMPL*FLOWO*SIN(ALPHA)
         P=PO
         AC=((GAMMA-1)/(2*GAMMA))*(FLOWO/(0.7854*D1**2))**2
         BC=PO
         CC=-ST
         X=(-BC+SQRT(BC**2-4*AC*CC))/(2*AC)
         DENS=1/X
        END IF
       END IF
C   DOWN STREAM PRESSURE IS SET EQUAL TO UPSTREAM PRESSURE.
C   THEY BECOME INPUTS  WHEN DYNA IS INTEGRATED WITH THE
C   PIPING MODEL.
       IF(TIME.EQ.0)THEN
        PE=P
        DENSE=DENS
       END IF
       RETURN
       END


C   SUBROUTINE ORSE. - ORIFICE SEQUENCE MODEL -

       SUBROUTINE ORSE(TOF,TETA,
     1 DELTAP,OME,TIME)
       DIMENSION AKPAR(4)
       COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
       COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
       COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
       COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
       GAMMA=1.4
       FLOWR=FLOW
       D1=D1
       FLOW=FLOW-DENS*0.7854*D1**2*OME*ALA*COSD(TETA)
C   CALCULATION OF COEFFICIENTS OF RESISTANCE
C   FOR TETA=15 AND TETA=10

  10   IF(INDEX.EQ.0.)THEN
        TETAN=TETA
        TETA=15
        INDEX=1
       END IF

C   A) CALCULATION OF GEOMETRIC PARAMETERS

  20   AVAL1=2*XD/D1
```

```
      AVAL2=2*XD/DCL
      AA=XD*((D1/2)**2-XD**2)**0.5
      AA=AA+((D1/2)**2*ASIN(AVAL1))
      AA=AA-COSD(TETA)*(XD*((DCL/2)**2-XD**2)**0.5
     1 +(DCL/2)**2*ASIN(AVAL2))
      AA=AA-(1-COSD(TETA))*2*ALA*XD
      AA=AA+2*DIS*XD*SIND(TETA)
      AA=4*AA/(3.14*D1**2)
      FF=1-(DCL/D1)**2*COSD(TETA)-AA
      IF(D1.LE.0.054)FFA=0.019
      IF(D1.GT.0.054.AND.D1.LE.0.105)FFA=0.0175
      IF(D1.GT.0.105.AND.D1.LE.0.160)FFA=0.015
      IF(D1.GT.0.160.AND.D1.LE.0.210)FFA=0.014
      IF(D1.GT.0.210.AND.D1.LE.0.260)FFA=0.0135
      IF(D1.GT.0.260.AND.D1.LE.0.310)FFA=0.013
      IF(D1.GT.0.013.AND.D1.LE.0.600)FFA=0.012
      IF(D1.GT.0.600)FFA=0.011
      D0=0.7*ALA*TAND(TETA)
      DF=(FF*D1**2)
      DF=SQRT(DF)
      D3=SQRT(D1**2-DF**2)
      S1=3.14*D2*D0
      IF(S1.LT.0.001)S1=0.001
      DS1=SQRT(4*S1/3.14)
      IF(DS1.GT.D1)DS1=D1
      S2=3.14*((D1-D3)**2+D0**2)**0.5*(D1+D3)/2
      DS2=SQRT(4*S2/3.14)
      IF(DS2.GT.D1)DS2=D1
      IF(INDEX.EQ.2)GO TO 30

C   B) DIRECT FLOW

      IF(FLOW.GE.0.OR.INDEX.EQ.1)THEN
       IF(TETA.LT.15)THEN
        TETAD=TETA
        IF(TETAD.LT.3)TETAD=3
       END IF
       AK1=0.04/(D2/D1)**4
       BETA=(D2/DS1)**2
       AK2=((1-BETA)**2/BETA**2)/(DS1/D1)**4
       BETA=1/BETA
       IF(BETA.LT.1)AK2=((1-BETA)/BETA**2)
     1 /(D2/D1)**4
       BETA=(DF/DS2)**2
       GAMMAA=90-TETA
       GAMMAA=SIND(GAMMAA)
       AK3=((1-BETA)*SQRT(GAMMAA)/BETA**2)/(DS2/D1)**4
       AK4=(1-FF)**2/FF**2
       AK=+AK1+AK2+AK3+AK4
       IF(INDEX.EQ.1)THEN
        AKD15=AK
        TETA=10
        INDEX=2
```

```
          GO TO 20
         END IF
         AKO=AK/FFA
        END IF
   30   IF(FLOW.LT.O.OR.INDEX.EQ.2)THEN
C   C) REVERSE FLOW
         IF(TETA.LT.10)THEN
          TETAR=TETA
          IF(TETAR.LT.3)TETAR=3
         END IF
         BETA=(D2/D1)**2
         AK1=(1-BETA)**2/BETA**2
         BETA=(DS1/D2)**2
         AK2=((1-BETA)**2/BETA**2)/(D2/D1)**4
         BETA=1/BETA
         IF(BETA.LT.1)AK2=((1-BETA)/BETA**2)/
     1   (DS1/D1)**4
         BETA=(DF/DS2)**2
         AK3=((1-BETA)**2/BETA**2)/(DS2/D1)**4
         BETA=1/BETA
         IF(BETA.LT.1)AK3=((1-BETA)/BETA**2)/(DF/D1)**4
         AK4=(1-FF)/FF**2
         AK=AK1+AK2+AKS1+AK3+AK4
         IF(INDEX.EQ.2)THEN
          TETA=TETAN
          AKR10=AK
          INDEX=3
          GO TO 20
         END IF
         AKO=AK/FFA
        END IF
C   D) SMALL ANGLES
        IF(TETAD.NE.O)THEN
         AK=AKD15*15/TETAD
         AKO=AK/FFA
         TETAD=O
        END IF
        IF(TETAR.NE.O)THEN
         AK=AKR10*10/TETAR
         AKO=AK/FFA
         TETAR=O
        END IF
C   CRITICAL MASS FLOW
        IF(FLOW.GE.O)THEN
         AKMC=AK1
         DOO=D2
         IF(DS1.LE.D2.AND.DS1.LE.DF)THEN
```

```
      AKMC=AK2
      DOO=DS1
     END IF
     IF(DF.LE.D2.AND.DF.LE.DS1)THEN
      AKMC=AK3
      DOO=DF
     END IF
     IF(TETA.LT.15)THEN
      AKMC=AK2
      DOO=DS1
     END IF
    ELSE
     AKMC=AK4
     DOO=DF
     IF(DS1.LE.D2.AND.DS1.LE.DF)THEN
      AKMC=AKS1
      DOO=DS1
     END IF
     IF(D2.LE.DS1.AND.D2.LE.DF)THEN
      AKMC=AK2
      DOO=D2
     END IF
     IF(TETA.LT.10)THEN
      AKMC=AK3
      DOO=DS1
     END IF
    END IF
    IF(AKMC.GT.AK)AKMC=AK

C   DRAG FORCES

    IF(FLOW.LT.0)THEN
     AKPAR(1)=AK4
     AKPAR(2)=AK3
     AKPAR(3)=AK2
     AKPAR(4)=AK1
    ELSE
     AKPAR(1)=AK1
     AKPAR(2)=AK2
     AKPAR(3)=AK3
     AKPAR(4)=AK4
    END IF
    IND=0
    CALL CRIT(IND,AKPAR,FDT,AK,AKMC,GAMMA,DOO
   1 ,PU,FLOWU,DENSU,DELTAP)
    IF(FLOW.EQ.0)THEN
     FDT=0
     GO TO 14
    END IF
    FDTT=FDT

C   EQUIVALENT TORQUE ARM
```

```
 14     IF(FLOW.GE.0)THEN
         TAC=0.74
         IF(TETA.GE.20)TAC=0.0209*TETA+0.3220
        ELSE
         TAC=0.65
          IF(TETA.GE.20)TAC=0.0117*TETA+0.4160
         IF(TETA.GE.45)TAC=0.0583*TETA-1.6820
        END IF

C  ORIFIC NET EFFECT

        IF(FLOW.GE.0.AND.TETA.LE.15)THEN
         FDT=0
         GO TO 90
        END IF
        IF(FLOW.LT.0.AND.TETA.LE.10)THEN
         FDT=0
         GO TO 90
        END IF
        IND=1
        CALL  CRIT(IND,AKPAR,FDT,AK,AKMC,GAMMA,DOO
      1 ,PU,FLOWU,DENSU,DELTAP)
 90     FDTT=FDTT-FDT

C  DYNAMIC TORQUE.TORQUE IS POSITIVE WHEN FLOW IS
C  POSITIVE.

        TOF=TAC*ALA*FDTT
        FLOW=FLOWR
        RETURN
        END


C  COUNTERWEIGHT

        SUBROUTINE CTW(AC,DC,TETA)

        COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
      1 PO,FLOWO,DENSO,OMEGA,AMPL
        COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
      1 ,ATETAV
        COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
      1 ,AL2S,ADELTA
        COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
      1 AL2D,LPD,ABETA
        AC=WC*ALC**2/9.81
        ASUM=(AGAMMA+TETA)
        DC=ALC*WC*SIND(ASUM)
        RETURN
        END


C  BENDING SPRING
```

```
      SUBROUTINE BSPR(DSB,TETA)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      ASUM=ATETAS+TETA
      DSB=AKS*ASUM
      RETURN
      END

C   TORSIONAL SPRING

      SUBROUTINE TSPR(DST,TETA)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      ASUM=TETA+ADELTA
      ALS=(AL1S**2+AL2S**2-2*AL1S*AL2S*COSD(ASUM)
     1 )**0.5
      FACT=(AL2S**2+ALS**2-AL1S**2)/(2*ALS*AL2S)
      ADEL1=ACOS(FACT)
      DST=AKTS*(ALS-ALSO)*AL2S*SIN(ADEL1)
      RETURN
      END

C   DASHPOT

      SUBROUTINE DAMP(TETA,OME,AD,DD)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      ASUM=(ABETA-TETA)
      ALD=(AL1D**2+AL2D**2-2*AL1D*AL2D*
     1 COSD(ASUM))**0.5
```

```
      FACT=(ALD**2+AL2D**2-AL1D**2)/(2*ALD*AL2D)
      ADEL1=ACOS(FACT)
      VEL=OME*AL2D*SIN(ADEL1)
      ADEL2=ABETA+TETA+ADEL1*180/3.14
      TWD=WD*(AL2D*SIND(ASUM)-LPD*SIND(ADEL2))
      AD=AMD*SIN(ADEL1)*AL2D
      B=(CRR*DENOIL/2)*(VEL*DCY**2/DOV**2)**2
     1 *0.7854*DCY**2
      BREF=B/(0.7854*DCY**2)
      B=B*AL2D*SIN(ADEL1)

C  DAMPER DOWNSTREAM

      B=-B
      TWD=-TWD

C  LIMITED DAMPING IN DISC CLOSING DIRECTION

      IF(OME.LT.0.0.AND.BREF.GT.100000)
     1 B=-100000*(0.7854*DCY**2)*AL2D*SIN(ADEL1)

C  GENERAL RELIEF VALVE

      IF(BREF.GT.900000)B=-900000*(0.7854*DCY**2)
     1 *AL2D*SIN(ADEL1)
      IF(OME.EQ.0)THEN
       DD=TWD
       GO TO 10
      END IF
      DD=TWD-B*OME/ABS(OME)
 10   RETURN
      END

C  VALVE CLOSING SYSTEM

      SUBROUTINE VALVE(TETA,AV,DV)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      ADIFF=TETA-ATETAV
      DV=WV*ALG*SIND(ADIFF)
      AV=AJV
      RETURN
      END
```

```
C  THIS SUBROUTINE CALCULATES THE FLOW TORQUE AND IN
C  SONIC CONDITIONS CORRECTS THE MASS FLOW.

      SUBROUTINE CRIT(IND,AKPAR,FDT,AK,AKMC,GAMMA,DOO
     1 ,PU,FLOWU,DENSU,DELTAP)

      DIMENSION AKPAR(4)
      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      C1=GAMMA/(GAMMA-1)
      C2=-1/GAMMA
      C3=1/C1
      IF(FLOW.LE.0)THEN
       PU=PE
       DENSU=DENSE
      ELSE
       PU=P
       DENSU=DENS
      END IF
      DOO=PU
      DENSOO=DENSU

C  STAGNATON CONDITIONS

      KOUNT=0
   22 PS=(PU**C3+PU**C2*(C3/2)*FLOW**2/
     1 (DENSU*(0.7854*D1**2)**2))**C1
      DENSS=DENSU*(PS/PU)**C2
      KOUNT=KOUNT+1
      IF(KOUNT.GT.100)THEN
       PRINT*,'PROGRAM DIVERGES. STOP AND SEE CRIT'
      END IF

C  SONIC VELOCITY

      VC=SQRT((2*GAMMA/(GAMMA+1))*PS/DENSS)
      FLOWU=FLOW

C  SUBSONIC CONDITIONS

      DPSUM=0
      DTSUM=0
      IF(IND.EQ.0)THEN
       I1=1
       I2=4
      END IF
      IF(IND.EQ.1.AND.FLOW.LT.0)THEN
```

```
        I1=3
        I2=4
       END IF
       IF(IND.EQ.1.AND.FLOW.GE.0)THEN
        I1=1
        I2=2
       END IF
      DO 50 I=I1,I2
       AK=AKPAR(I)
       CALL EXPANU (PU,FLOWU,DENSU,Y,DELTAP,AH,GAMMA,AK
     1 ,PD,DENSD,TETA)
       DPSUM=DPSUM+AK*FLOWU**2/(2*(0.7854
     1 *D1**2*Y)**2*DENSU)
       DTSUM=DTSUM+AK*AH**2*FLOWU**3/(2*
     1 0.7854*D1**2*DENSU*ABS(FLOWU))
       IF(AK.NE.AKMC)GO TO 23
C  POSSIBILITY OF SONIC CONDITIONS

       VEL1=FLOW/(0.7854*D1**2*DENSOO)
       VEL1=ABS(VEL1)
       VEL=FLOW/(0.7854*DOO**2*DENSD)
       VEL=ABS(VEL)
       IF(VEL.GT.VC)THEN
        PRINT*,'SONIC CONDITIONS'
        DO 20 K=1,50
         PC=POO-DPSUM-DENSD*VC**2*(1-VEL1/VC)*(DOO/D1)**2
         DENSC=DENSOO*(VEL1/VC)*(D1/DOO)**2
         AMC=VC*0.7854*DOO**2*DENSC
         AM=ABS(FLOW)
         ERROR=(AM-AMC)
         ERROR=ABS(ERROR)
         IF(ERROR.LT.0.01)GO TO 21
         AC=(PS/DENSS)
         BC=-POO
         CC=-(MC/(DENSOO*0.7854*D1**2))**2*C3/2
         DENSOO=(-BC+SQRT(BC**2-4*AC*CC))/(2*AC)
         POO=PC*(DENSOO/DENSC)+(C3/2)*(VC**2-(AMC/
     1   (DENSOO*0.7854*D1**2))**2)
         VEL1=MC/(DENSOO*0.7854*D1**2)
 20     CONTINUE
        PRINT*,'ECCESSIVE ERROR IN SONIC COND.'
 21     ERROR=POO-PU
        ERROR=ABS(ERROR)
        PU=POO
        DENSU=DENSOO
        FLOW=AMC*FLOW/ABS(FLOW)
        IF(ERROR.GT.10)GO TO 22
       END IF
 23    DENSU=DENSD
       PU=PD
 50   CONTINUE
       DELTAP=DTSUM
```

```
      FDT=DTSUM
 14   FLOW=FLOWU
      IF(FLOW.LE.0.)THEN
       PE=PU
       DENSE=DENSU
       P=PD
       DENS=DENSD
      ELSE
       P=PU
       DENS=DENSU
       PE=PD
       DENSE=DENSD
      END IF
 45   RETURN
      END

C  CALCULATION OF NET EXPANSION FACTOR AND
C  DRAG COMPRESSIBILITY COEFFICIENT.
C  UPSTREAM PARAMETERS

      SUBROUTINE EXPANU (PU,FLOWU,DENSU,AY,DELTAP,AH,GAMMA,AK,
     1 PD,DENSD,TETA)

      COMMON/A1/DENSE,DELTAF,PS,PE,PA,DENSA,P,FLOW,DENS,
     1 PO,FLOWO,DENSO,OMEGA,AMPL
      COMMON/A2/ VD,D1,D2,DCL,XD,DIS,ALA,TETAF,WV,AJV,ALG
     1 ,ATETAV
      COMMON/A3/ ALC,WC,AGAMMA,AKS,ATETAS,AKTS,ALSO,AL1S
     1 ,AL2S,ADELTA
      COMMON/A4/ AMD,WD,DCY,DRO,DENOIL,CRR,DOV,AL1D,
     1 AL2D,LPD,ABETA
      AY=1
      VEL=FLOWU/(0.7854*D1**2*DENSU)
      C1=GAMMA/(GAMMA-1)
      C2=-1/GAMMA
      C3=1/C1
      PS=(PU**C3+PU**C2*(C3/2)*FLOW**2/
     1 (DENSU*(0.7854*D1**2)**2))**C1
      DENSS=DENSU*(PS/PU)**C2
      ST=PS/DENSS
      DO 2 I=1,20
        DELTAP=AK*DENSU*VEL**2/(2*AY**2)
        PD=PU-DELTAP
        AC=((GAMMA-1)/(2*GAMMA))*(FLOWU/(0.7854*D1**2))**2
        BC=PD
        CC=-ST
        X=(-BC+SQRT(BC**2-4*AC*CC))/(2*AC)
        DENSD=1/X
       REF=(DENSU/DENSD-1)*2/AK+(1+DENSU/DENSD)/2
       REF=SQRT(1/REF)
       ERROR=ABS(REF-AY)
       IF(ERROR.LE.0.001)GO TO6
```

```
      AY=REF
2     CONTINUE
      PRINT*,' EXCESSIVE Y ERROR '
6     AH=(1/AY**2-2*(1-DENSD/DENSU)/AK)
      AH=SQRT(AH)
15    RETURN
      END
```

# APPENDIX 4

## PROGRAM "TESTAL"

```
      PROGRAM TESTAL (INPUT,OUTPUT)
C  PROGRAM TESTAL COMPUTES UNSTEADY FLOW PARAMETERS
C  AT NODAL POINTS OF A STRAIGHT PIPE ENTERING A
C  RESERVOIR. THE PIPE IS DEVIDED IN EQUAL LENGTH
C  ELEMENTS AS INITIAL CONDITION. BOUNDARY CONDITIONS
C  ARE@D
C           @D "RUPTURE"
C           @D INFINITE RESERVOIR
C  THE PROCESS IS CONSIDERED ISENTROPIC AND THE FLUID
C  AN IDEAL GAS.
C  THE SOLVING ALGORITHM IS BASED ON THE METHOD OF
C  CHARACTERISTICS.
      DIMENSION XLONG(40),PREF(40),INDEX(80),
    1 GVEL(40,DZ(40),X(80)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
    1 FLOW(40),DENS(40),DENS1(40)
    1
,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),FLOWC(40),KD,GAMMA,PEO,PCRR
    1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO

C  READ EXTERNAL DATA
C         @D PIPE INSIDE DIAMETER
C         @D NUMBER OF INTERVALS
C         @D MAXIMUM TIME
C         @D PRESSURE DROP TIME INTERVAL
C         ; MULTIPLIER OF NUMBER OF INTERVALS.

      READ*,DIA,N,TMAX,TO,KD,CLO
      DO 1 I=2,N+1
       READ*,XLONG(I)
       DX(I-1)=XLONG(I)-XLONG(I-1)
  1   CONTINUE
      XTOT=XLONG(N+1)
      DO 19 I=1,N
       DO 20 J=1,KD
        JJ=KD-J
        DZ(KD*I-JJ)=DX(I)/KD
  20   CONTINUE
  19  CONTINUE
      NOO=N
      N=N*KD
      DO 21 I=1,N
       DX(I)=DZ(I)
  21  CONTINUE
      DO 22 I=1,N
  22  CONTINUE

C  READ FLOW INITIAL CONDITIONS.

      READ*,PO,DENSO
```

```
      READ*,VELO
 778  PRINT 987,DIA,XTOT,N,PO,DENSO,VELO,CLO
 987  FORMAT('1',2X,'STRAIGHT PIPE RESERVOIR PROBLEM '////3X,
     1 'INPUT'//
     1 4X,'DIAMETER OF PIPE          =',F10.5,'  M'/
     1 4X,'PIPE LENGTH               =',F10.2,'  M'/
     1 4X,'NUMBER OF INTERVALS       =',I10/
     1 4X,'PRESSURE                  =',F10.0,'  PA'/
     1 4X,'DENSITY                   =',F10.5,'  KG/M3'/
     1 4X,'VELOCITY                  =',F10.3,'  M/S'/
     1 ////////////////
     1 3X,'OUTPUT'////
     1 4X,'PRESSURE      PA'/
     1 4X,'MASS FLOW   KG/S'/
     1 4X,'DENSITY    KG/M3'/
     1 4X,'VELOCITY     M/S'/
     1 4X,'SOUND VEL.   M/S'/)
      PRINT 989
 989  FORMAT('1'////50X,'R E S U L T S'////)
C  INITIAL CONDITIONS

      N=N+1
      A=3.14*(DIA**2)/4
      GAMMA=1.4
      TETA=1/(GAMMA-1)
      C1=1/GAMMA
      C2=(GAMMA-1)/(2*GAMMA)
      C3=GAMMA/(GAMMA-1)
      C4=(GAMMA+1)/(2*(GAMMA-1))
      C5=(GAMMA-1)
      C6=1/(GAMMA-1)
      C11=GAMMA/(GAMMA+1)
      PR=PO
      DENSR=DENSO
      FLOWO=A*DENSO*VELO
      DO 2 I=1,N
       P(I)=PO
       POLD(I)=PO
       FLOW(I)=FLOWO
       PREF(I)=PO
       FLOLD(I)=FLOWO
 2    CONTINUE
      ASO=SQRT(GAMMA*PO/DENSO)
      DO 3 I=1,N
       AS(I)=ASO
       DENS(I)=DENSO
       FLOWC(I)=-SQRT(GAMMA*P(I)/DENS(I))*DENS(I)*A
       DENS1(I)=DENSO
 3    CONTINUE
      DENSE=(DENSO**C5+(DENSO**GAMMA/PO)*C2*VELO**2)**C6
      PE=PO*(DENSE/DENSO)**GAMMA
      PPE=PO
```

```
      PEOLD=PE
      DTO=0.
      PEO=PO

C  CRITICAL CONDITIONS WITH RESPECT TO RESERVOIR

      PCRR=PR*(2/(GAMMA+1))**C3
      FLOCRR=A*SQRT(GAMMA*PR*DENSR)*(2/(GAMMA+1))**C4

C  SET INITIAL VALUES FOR CHARACTERISICS METHOD

      WI(1,1)=0
      DO 8  ,I=2,N
       WI(1,I)=XLONG(I)
  8   CONTINUE
       DO 16 I=1,N
        WI(2,I)=0
        WI(3,I)=AS(I)
        WI(4,I)=FLOW(I)/(A*DENS(I))
        WI(5,I)=DENS(I)
        WI(6,I)=P(I)
        WI(7,I)=FLOW(I)
        WI(8,I)=0
 16   CONTINUE
      KOUNT=0

C  MAIN LOOP - ONE LOOP FOR EACH TIME INCREMENT -

      T=0
      DT=.1
  5   CONTINUE
      KOUNT=KOUNT+1

C  SOLUTION

      IF(KOUNT.EQ.3)THEN
       PRINT 989
       KOUNT=1
      END IF
       CALL CHAR(INDEX,X)
       DTO=WI(2,1)
       T=DTO
      IF(T.GT.TMAX)GO TO 10
      DTO=DTO+DT
      PEOLD=PE
      RHOLD=DENSE
      PE=PPE-((PPE-101325.)/TO)*DTO
      IF(PE.LE.101325.)PE=101325.
      DENSE=RHOLD*((PE/PEOLD)**C1)
      PEO=P[illegible]
      DO [illegible]=1,2*N+1
       INDEX(I)=0
 11   CONTINUE
```

```
      GO TO 5
10    STOP
      END

C  SUBROUTINE CHARACTERISTICS

      SUBROUTINE CHAR(INDEX,X)
      DIMENSION INDEX(80),X(80),W(10,40)
      COMMON/A1/N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1
,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO
      GAMMA=1.4
      C1=2/(GAMMA-1)
      C2=1/GAMMA
      C4=(GAMMA/(GAMMA-1))
      C5=(GAMMA+1)/(2*(GAMMA-1))
      NNODE=N
      NELEM=NNODE-1

C  CRITICAL CONDITIONS

      PCRR=PR*(2/(GAMMA+1))**C4
      FLOCRR=A*SQRT(GAMMA*PR*DENSR)*(2/(GAMMA+1))**C5
      DCR=DENSR*(PCRR/PR)**C2
      UCR=FLOCRR/(A*DCR)

C  SET PARAMETERS MATRIX

C           W SECOND ROW   @D  TIME
C           W THIRD ROW    @D  SOUND VELOCITY
C           W FORTH ROW    @D  VELOCITY
C           W FIFTH ROW    @D  DENSITY
C           W SIXTH ROW    @D  PRESSURE
C           W SEVENTH ROW  @D  MASS FLOW
C           W EIGTH ROW    @D  ITERATIONS

C  INTERNAL POINTS

      DO 4 I=2,NNODE-1
       AL=2*WI(3,(I-1))+(GAMMA-1)*WI(4,(I-1))
       AR=2*WI(3,(I+1))-(GAMMA-1)*WI(4,(I+1))
       W(3,I)=(AL+AR)/4
       W(4,I)=(AL-AR)/(2*(GAMMA-1))
       AW4I=ABS(W(4,I))
      IF(AW4I.GT.W(3,I))W(4,I)=W(3,I)*W(4,I)/AW4I
       AA1=(W(4,I)+W(3,I))+(WI(4,(I-1))+WI(3,(I-1)))
       AA2=(W(4,I)-W(3,I))+(WI(4,(I+1))-WI(3,(I+1)))
```

```
      W(2,I)=(2*(WI(1,(I+1))-WI(1,(I-1)))-(WI(2,(I+1))*AA2-
     1 WI(2,(I-1))*AA1))/(AA1-AA2)
      W(1,I)=WI(1,(I-1))+AA1*(W(2,I)-WI(2,(I-1)))/2
      W(5,I)=WI(5,(I-1))*(W(3,I)/WI(3,(I-1)))**C1
      W(6,I)=WI(6,(I-1))*(W(5,I)/WI(5,(I-1)))**GAMMA
      W(7,I)=W(5,I)*A*W(4,I)
      W(8,I)=WI(8,I)+1
      P(I)=W(6,I)
      FLOW(I)=W(7,I)
      AS(I)=W(3,I)
      DENS(I)=W(5,I)
4     CONTINUE
      IF(AW4I.GT.UCR)THEN
       W(6,I)=PCRR
       W(4,I)=UCR
       W(5,I)=DCR
       W(3,I)=(PCRR/DCR)*GAMMA
       W(3,I)=SQRT(W(3,I))
       W(7,I)=FLOCRR
      END IF
C  LEFT BOUNDARY CONITION

      C3=(GAMMA-1)/(2*GAMMA)
      DO 5 I=1,20
      P(1)=PE
       W(3,1)=WI(3,1)*(P(1)/WI(6,1))**C3
       W(6,1)=P(1)
       W(5,1)=WI(5,1)*(W(3,1)/WI(3,1))**C1
       W(1,1)=WI(1,1)
       W(4,1)=(W(3,1)-WI(3,2))*C1+WI(4,2)
       AW41=ABS(W(4,1))
       IF(AW41.GT.W(3,1))W(4,1)=W(3,1)*W(4,1)/AW41
       W(2,1)=(W(1,1)-WI(1,2))*2/((W(4,1)-W(3,1))+(WI(4,2)
     1 -WI(3,2)))+WI(2,2)
       PE=PO-((PO-101325.)/TO)*W(2,1)
       IF(PE.LT.101325..AND.AW41.LT.W(3,1))PE=101325
       PL1=AW41**2*W(5,1)/GAMMA
       IF(PL1.GE.PE)PE=PL1
       ERROR=ABS(PE-P(1))
       IF(ERROR.LT.10.)GO TO 6
5     CONTINUE
      PRINT*,' ERROR OF LEFT BOUNARRY ',ERROR
6     W(7,1)=W(4,1)*W(5,1)*A
C  RIGHT BOUNDARY CONDITION
      IF(WI(7,NNODE).GE.0)THEN.
       P(NNODE)=PR
       W(3,NNODE)=WI(3,NNODE)*(P(NNODE)/WI(6,NNODE))**C3
       W(5,NNODE)=WI(5,NNODE)*(PR/WI(6,NNODE))**C2
       W(6,NNODE)=P(NNODE)
       W(1,NNODE)=WI(1,NNODE)
```

```
       W(4,NNODE)=-(W(3,NNODE)-WI(3,(NNODE-1)))*C1+
     1 WI(4,(NNODE-1))
       W(2,NNODE)=(W(1,NNODE)-WI(1,(NNODE-1)))*2/((W(4,NNODE)
     1 +W(3,NNODE))+(WI(4,(NNODE-1))+WI(3,(NNODE-1))))+
     1 +WI(2,(NNODE-1))
       IF(W(4,NNODE).LT.0)GO TO 9
      END IF
 9    IF(WI(7,NNODE).LT.0.OR.W(4,NNODE).LT.0)THEN
       DO 7 I=1,20
        PRIF=P(NNODE)
        W(3,NNODE)=WI(3,NNODE)*(PRIF/WI(6,NNODE))**C3
        W(6,NNODE)=PRIF
       W(5,NNODE)=WI(5,NNODE)*(PRIF/WI(6,NNODE))**C2
        W(1,NNODE)=WI(1,NNODE)

W(4,NNODE)=-(W(3,NNODE)-WI(3,(NNODE-1)))*C1+WI(4,(NNODE-1))
        AW4N=ABS(W(4,NNODE))
        W(2,NNODE)=(W(1,NNODE)-WI(1,(NNODE-1)))*2/((W(4,NNODE)
     1  +W(3,NNODE))
     1  +(WI(4,(NNODE-1))+WI(3,(NNODE-1))))+WI(2,(NNODE-1))
        W(3,NNODE)=SQRT((PR/DENSR)*GAMMA-W(4,NNODE)**2/C1)
        P(NNODE)=W(3,NNODE)**2*W(5,NNODE)/GAMMA
        ERROR=ABS(PRIF-P(NNODE))
        IF(ERROR.LT.10.)GO TO 8
 7     CONTINUE
 8     IF(AW4N.GT.UCR)THEN
        W(6,NNODE)=PCRR
        W(4,NNODE)=UCR
        W(5,NNODE)=DCR
        W(3,NNODE)=SQRT((PCRR/DCR)*GAMMA)
        W(7,NNODE)=FLOCRR
       END IF
      END IF
      W(7,NNODE)=W(4,NNODE)*W(5,NNODE)*A

C  EXCHANGE

      DO 11 I=1,8
       DO 12 J=1,NNODE
        WI(I,J)=W(I,J)
 12    CONTINUE
 11   CONTINUE

C  PRINT RESULTS

      PRINT 100,W(2,1),PE
 100  FORMAT(3X//F8.5,2X,F8.0//)
      PRINT 200,((W(I,J),J=1,NNODE,KD),I=1,8)
 200  FORMAT(1X,11F10.3)
      RETURN
      END
```

APPENDIX 5

PROGRAM "PIPING"

```
      PROGRAM PIPING (INPUT,OUTPUT)
C  PROGRAM PIPING COMPUTES UMSTEADY FLOW PARAMETERS
C  AT NODAL POINTS OF A STREIGHT PIPE ENTERING A
C  RESERVOIR. THE PIPE IS DEVIDED IN EQUAL LENGTH
C  ELEMENTS. BOUNDARY CONDITIONS ARE
C         "RUPTURE"
C         INFINITE RESERVOIR
C  THE PROCESS IS CONSIDERED ISENTROPIC AND THE
C  FLUID AN IDEAL GAS.
C  SOLVING ALGORITHMS ARE
C          FINITE VOLUMES
C          FINITE ELEMENTS
C          FINITE DIFFERENCES.

      DIMENSION XLONG(40),PREF(40),INDEX(80),
     1 GVEL(40),DZ(40),X(80)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO

C  READ EXTERNAL DATA
C         PIPE INSIDE DIAMETER
C        NUMBER OF INTERVALS
C        MAXIMUM TIME
C        PRESSURE DROP TIME INTERVAL
C        MULTIPLIER OF NUMBER OF INTERVALS


      READ*,DIA,N,TMAX,TO,KD,CLO
      DO 1 I=2,N+1
       READ*,XLONG(I)
       DX(I-1)=XLONG(I)-XLONG(I-1)
 1    CONTINUE
      XTOT=XLONG(N+1)
      DO 19 I=1,N
       DO 20 J=1,KD
        JJ=KD-J
        DZ(KD*I-JJ)=DX(I)/KD
 20    CONTINUE
 19   CONTINUE
      NOO=N
      N=N*KD
      DO 21 I=1,N
       DX(I)=DZ(I)
 21   CONTINUE
      DO 22 I=1,N
 22   CONTINUE

C  READ
```

```
C         FLOW INITIAL CONDITIONS
C         TIME INCREMENT REGULATOR
C   SET SOLVING ALGORITHM
C         1 FOR FINITE DIFFERENCES
C         3 FOR FINITE VOLUMES
C         4 FOR FINITE ELEMENTS

      READ*,PO,DENSO
      READ*,VELO
      READ*,TREG
      READ*,ICASE
      PRINT 987,DIA,XTOT,N,PO,DENSO,VELO
  987 FORMAT('1',2X,'STRAIGHT PIPE RESERVOIR PROBLEM '////3X,
     1 'INPUT'//
     1 4X,'DIAMETER OF PIPE            =',F10.5,'  M'/
     1 4X,'PIPE LENGTH                 =',F10.2,'  M'/
     1 4X,'NUMBER OF INTERVALS         =',I10/
     1 4X,'PRESSURE                    =',F10.0,'  PA'/
     1 4X,'DENSITY                     =',F10.5,'  KG/M3'/
     1 4X,'VELOCITY                    =',F10.3,'  M/S'/
     1 ///////////////
     1 3X,'OUTPUT'////
     1 4X,'PRESSURE       PA'/
     1 4X,'MASS FLOW   KG/S'/
     1 4X,'DENSITY   KG/M3'/
     1 4X,'VELOCITY     M/S'/
     1 4X,'SOUND VEL.   M/S'/////)
      PRINT 989
  989 FORMAT('1'////50X,'R E S U L T S'////)

C  INITIAL CONDITIONS

      N=N+1
      A=3.14*(DIA**2)/4
      GAMMA=1.4
      TETA=1/(GAMMA-1)
      C1=1/GAMMA
      C2=(GAMMA-1)/(2*GAMMA)
      C3=GAMMA/(GAMMA-1)
      C4=(GAMMA+1)/(2*(GAMMA-1))
      C5=(GAMMA-1)
      C6=1/(GAMMA-1)
      C11=GAMMA/(GAMMA+1)
      PR=PO
      DENSR=DENSO
      FLOWO=A*DENSO*VELO
      DO 2 I=1,N
       P(I)=PO
       POLD(I)=PO
       FLOW(I)=FLOWO
       PREF(I)=PO
       FLOLD(I)=FLOWO
    2 CONTINUE
```

```
      ASO=SQRT(GAMMA*PO/DENSO)
      DO 3 I=1,N
       AS(I)=ASO
       DENS(I)=DENSO
       FLOWC(I)=-SQRT(GAMMA*P(I)/DENS(I))*DENS(I)*A
       DENS1(I)=DENSO
    3 CONTINUE
      DENSE=(DENSO**C5+(DENSO**GAMMA/PO)*C2*VELO**2)**C6
      PE=PO*(DENSE/DENSO)**GAMMA
      PPE=PO
      PEOLD=PE
      DTO=0.
      PEO=PO

C  CRITICAL CONDITIONS WITH RESPECT TO RESERVOIR

      PCRR=PR*(2/(GAMMA+1))**C3
      FLOCRR=A*SQRT(GAMMA*PR*DENSR)*(2/(GAMMA+1))**C4
      KOUNT=0

C  MAIN LOOP - ONE LOOP FOR EACH TIME INCREMENT -

      T=0
      DT=.1
    5 CONTINUE
      KOUNT=KOUNT+1

C BOUNDARY CONDITIONS

      CALL BC1(INDEX,X,IFLAG,ICASE)
      CALL BC2(INDEX,X,IFLAG,ICASE)

C  SYSTEM SOLUTION

    9 IF(ICASE.EQ.1)CALL SYST(INDEX,X)
      IF(ICASE.EQ.3)CALL CENTR(INDEX,X)
      IF(ICASE.EQ.4)CALL CENTR2(INDEX,X)
      ERROR=0.
      DO 15 I=1,N
      ERROR1=ABS(P(I)-PREF(I))
   15 ERROR=AMAX1(ERROR,ERROR1)
      IF(ERROR.GT.10.)THEN

C  DENSITY UPDATING

     CALL DENIT1
       DO 6 I=1,N
        PREF(I)=P(I)
        FLOWC(I)=-SQRT(GAMMA*P(I)/DENS(I))*DENS(I)*A
    6  CONTINUE

C  PIPE RUPTURE  PRESSURE DROP LIMITATION.
```

```
      AFLOW=ABS(FLOW(1))
      AFLOCR=ABS(FLOWC(1))
      TREF=T0+DT
      IF(AFLOW.GT.AFLOCR.AND.T.LT.TREF)THEN
       PRIF=(P(1)/DENS(1)+((GAMMA-1)/(2*GAMMA))
     1  *(FLOW(1)/(DENS(1)*A))**2)
       AEX=1/GAMMA
       BEX=GAMMA/(GAMMA-1)
       PRIF=(DENS(1)/P(1)**AEX)*PRIF
       PRIF=PRIF**BEX
       CEX=GAMMA/(GAMMA+1)
       DENRIF=DENS(1)*(PRIF/P(1))**AEX
       PER=((FLOW(1)/A)**2*PRIF**AEX/(GAMMA*DENRIF))**CEX
       X(2)=X(2)+0.3*(PER-X(2))
       GO TO 9
      END IF

C  CRITICAL CONDITIONS - FIRST ELEMENT -

      CALL FEL(INDEX,X,ICASE)

C  CRITICAL CONDITIONS -INTERNAL NODES -

     CALL MASS(INDEX,X)

C  CRITICAL CONDITIONS - LAST ELEMENT-

      ACOMP=ABS(FLOW(N))
      BCOMP=A*SQRT(GAMMA*POLD(N)*DENS1(N))*(2/(GAMMA+1))**C4
      IF(ACOMP.GT.BCOMP)P(N)=POLD(N)*(2/(GAMMA+1))**C3
      GO TO 9
     END IF

C  PRINT RESULTS

     DO 62 IND=1,N
      GVEL(IND)=FLOW(IND)/(A*DENS(IND))
62   CONTINUE
     IF(KOUNT.EQ.3)THEN
      PRINT 989
      KOUNT=1
     END IF
     PRINT 100,DT,T,(XLONG(I),I=2,NOO+1),
   1  (P(I),I=1,N,KD),(FLOW(I),I=1,N,KD)
   1 ,(DENS(I),I=1,N,KD),(GVEL(I),I=1,N,KD),(AS(I),I=1,N,KD)
100  FORMAT(///4X,'TIME INCREASED OF  =',F8.4,'  S'/
   1 4X,'ELAPSED TIME          =',F8.4,'  S'///
   1 45X,'DISTANCE FROM RUPTURE ( M )'//
   1 6X,'        0  ',10F8.0///
   1 1X,'PRESS.  ',11F8.0//1X,'MASS FL.',11F8.2//
   1 1X,'DENS.    ',11F8.5//1X,'GAS VEL.',11F8.1//
   1 1X,'SOUND V.',11F8.1///)
```

```
C  UPDATE TIME

      SUM=0.
      DO 7 I=1,N
       SUM=SUM+SQRT(GAMMA*P(I)/DENS(I))
  7   CONTINUE
      ASM=SUM/N
      DT=XLONG((N-1)/KD)/(((N-1)/KD)*ASM)
      DT=DT/TREG

C  UPDATE PARAMETERS

      DO 25 I=1,N
       DENS1(I)=DENS(I)
       FLOLD(I)=FLOW(I)
       POLD(I)=P(I)
 25   CONTINUE
      T=T+DT
      IF(T.GT.TMAX)GO TO 10
      DTO=DTO+DT
      PEOLD=PE
      RHOLD=DENSE
      PE=PPE-((PPE-101325.)/TO)*DTO
      IF(PE.LE.101325.)PE=101325.
      DENSE=RHOLD*((PE/PEOLD)**C1)
      PEO=PE
      DO 11 I=1,2*N+1
       INDEX(I)=0
 11   CONTINUE
      GO TO 5
 10   STOP
      END


C  SUBROUTINE SYSTEM SOLUTION

      SUBROUTINE SYST(INDEX,X)
      DIMENSION INDEX(80),X(80),AB(80),BE(80),D(80),RHS(80)
     1 ,R1(80),R2(80)

COMMON/A1/N,A,DX(40),AS(40),DT,P(40),FLOW(40),DENS(40),DENS1(40)
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NN=2*N+1

C  BASIC MATRIX

      DO 1 I=3,NN-2,2
      R1(I)=(4*DX((I+1)/2)/(A*DT))*(DENS((I+1)/2)
     1 -DENS1((I+1)/2))/
```

```
     1 (DENS((I+1)/2)+DENS1((I+1)/2))
       R2(I)=-FLOW((I+1)/2)*(DENS((I+1)/2)-DENS((I-1)/2))/(A*DENS
     1 ((I+1)/2))**2
       D(I)=DX((I+1)/2)/(A*DT)
       RHS(I)=D(I)*FLOLD((I+1)/2)
 1     D(I)=D(I)-R1(I)+R2(I)
       D(1)=1
       D(NN+1)=1
       RHS(1)=1
       RHS(NN+1)=1
       DO 2 I=2,NN,2
       D(I)=A*DX((I+2)/2)/(DT*AS((I)/2)*ABS(AS((I)/2)))
 2     RHS(I)=D(I)*POLD(I/2)
       DO 3 I=1,NN+1
       AB(I)=1
 3     BE(I)=-1
       AB(1)=0
       AB(NN)=0
       D(NN)=1
       BE(NN+1)=0
C  BOUNDARY CONDITIONS
       DO 5 I=1,NN+2
       IF(INDEX(I).EQ.0)GO TO 5
       D(I)=1
       RHS(I)=X(I)
       AB(I)=0
       BE(I)=0
 5     CONTINUE
       NN=NN+1
       CALL TRIDIAG(AB,BE,RHS,D,NN)
        NN=NN-1
       DO 6 I=2,NN,2
 6     P((I)/2)=RHS(I)
       DO 7 I=1,NN,2
 7     FLOW((I+1)/2)=RHS(I)
       IF(FLOW(1).EQ.0)THEN
       FLOW(1)=FLOW(2)
       END IF
       IF(FLOW(N).EQ.0)FLOW(N)=FLOW(N-1)
       RETURN
       END

C  SUBROUTINE CENTRAL -CENTRAL POINTS

       SUBROUTINE CENTR(INDEX,X)
       DIMENSION BKAA(80,80),INDEX(80),X(80),NODE(40,2),R1(80)
     1 ,R2(80),RHS(80),STIFF(4,4)
       COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),FLOW(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
```

```
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 FLOCRR,AGAM(40)
       NNODE=N
       NELEM=N-1
       NDPEREL=2
       NN=2*NNODE
C  NODE ELEMENT RELATINSHIP
       DO 1 IE=1,NELEM
        DO 2 I=1,NDPEREL
         NODE(IE,I)=IE-1+I
 2      CONTINUE
 1     CONTINUE
C  ASSEMBLING OF GLOBAL INFLUENCE MATRIX
       DO 3 IE=1,NELEM
        RHO=DENS(NODE(IE,1))+DENS(NODE(IE,2))
        RHOLD=DENS1(NODE(IE,1))+DENS1(NODE(IE,2))
        R1(IE)=-2*(RHO-RHOLD)/(RHO*A*DT)
        R2(IE)=-4*(DENS(NODE(IE,2))-DENS(NODE(IE,1)))/(DX(IE)*
     1  (A*RHO)**2)
        SV=(AS(NODE(IE,1))+AS(NODE(IE,2)))/2
        STIFF(1,1)=1
        STIFF(1,2)=-(A*DX(IE)/(2*SV*ABS(SV)*DT))
        STIFF(1,3)=-STIFF(1,1)
        STIFF(1,4)=STIFF(1,2)
        STIFF(2,1)=-(DX(IE)/(2*A*DT)+R1(IE)*DX(IE)/2+
     1  R2(IE)*DX(IE)*
     1  (FLOW(NODE(IE,1))+FLOW(NODE(IE,2))/2)/3)
        STIFF(2,2)=1
        STIFF(2,3)=-(DX(IE)/(2*A*DT)+R1(IE)*DX(IE)/2+
     1  R2(IE)*DX(IE)*
     1  (FLOW(NODE(IE,1))/2+FLOW(NODE(IE,2)))/3)
        STIFF(2,4)=-STIFF(2,2)
        RHS(2*NODE(IE,1))=STIFF(1,2)*(POLD(NODE(IE,1))
     1 +POLD(NODE(IE,2)))
        RHS(2*NODE(IE,1)+1)=-(DX(IE)/(2*A*DT))*(FLOLD(NODE(IE,1))
     1 +FLOLD(NODE(IE,2)))
        DO 4 K=1,2
         NROW=2*NODE(IE,1)-1+K
         DO 5 J=1,NDPEREL
          DO 6 KK=1,2
           NCOL=2*NODE(IE,J)-2+KK
           BKAA(NROW,NCOL)=STIFF(K,2*J-2+KK)
 6        CONTINUE
 5       CONTINUE
 4      CONTINUE
 3     CONTINUE
       BKAA(1,1)=1
       BKAA(NN,NN)=1
       DO 10 I=2,NN-1
```

```
       BKAA(I,NN+1)=RHS(I)
10    CONTINUE
      BKAA(1,NN+1)=1
      BKAA(NN,NN+1)=1

C  ENTER BOUNDARY CONDITIONS

      IF(INDEX(2).EQ.1)THEN
       DO 20 I=1,2*NNODE+1
        BKAA(1,I)=BKAA(2,I)
20     CONTINUE
      END IF
      DO 13 I=1,2*NNODE
       IF(INDEX(I).EQ.0)GO TO 13
       DO 14 J=1,7
        JJ=I+4-J
        IF(JJ.LT.1)JJ=1
        IF(JJ.GT.NN)JJ=NN
        BKAA(I,JJ)=0
14     CONTINUE
       BKAA(I,I)=1
       BKAA(I,NN+1)=X(I)
13    CONTINUE

C  SOLVE MATRIX

      CALL GAUSS(BKAA,NN)

C  UNLOAD RESULTS

      DO 15 I=2,NN,2
       P(I/2)=BKAA(I,NN+1)
15    CONTINUE
      DO 16 I=1,NN,2
       FLOW((I+1)/2)=BKAA(I,NN+1)
16    CONTINUE

C  CLEAR MATRIX

      DO 17 I=1,NN
       RHS(I)=0
       DO 18 J=1,NN+1
        BKAA(I,J)=0
18     CONTINUE
17    CONTINUE
      RETURN
      END


C  SUBROUTINE CENTR2 - FINITE ELEMENTS -

      SUBROUTINE CENTR2(INDEX,X)
```

```
      DIMENSION BKAA(80,80),INDEX(80),X(80),NODE(40,2),R1(80),
     1 R2(80),RHS(80),STIFF(4,4)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NNODE=N
      NELEM=N-1
      NDPEREL=2
      NN=2*NNODE

C  NODE ELEMNT RELATIONSHIP

      DO 1 IE=1,NELEM
       DO 2 I=1,NDPEREL
        NODE(IE,I)=IE-1+I
2      CONTINUE
1     CONTINUE

C  STIFFNESS MATRIX PER ELEMENT - EACH ELEMENT IN SEQUENCE -
C  ASSEMBLING OF GLOBAL INFLUENCE MATRIX

      DO 3 IE=1,NELEM
       RHO=DENS(NODE(IE,1))+DENS(NODE(IE,2))
       RHOLD=DENS1(NODE(IE,1))+DENS1(NODE(IE,2))
       R1(IE)=-2*(RHO-RHOLD)/(RHO*A*DT)
       R2(IE)=-4*(DENS(NODE(IE,2))-DENS(NODE(IE,1)))
     1 /(DX(IE)*(ARHO)
     1 **2)
       SV=(AS(NODE(IE,1))+AS(NODE(IE,2)))/2
       STIFF(2,1)=.5
       STIFF(2,2)=-(A*DX(IE)/(3*SV*ABS(SV)*DT))
       STIFF(2,3)=-STIFF(2,1)
       STIFF(2,4)=STIFF(2,2)/2
       STIFF(1,1)=-(DX(IE)/(3*A*DT)+R1(IE)*DX(IE)/3+
     1 R2(IE)*DX(IE)
     1 *(FLOW(NODE(IE,1))+FLOW(NODE(IE,2))/3)/4)
       STIFF(1,2)=.5
       STIFF(1,3)=-(DX(IE)/(6*A*DT)+R1(IE)*DX(IE)/6+
     1 R2(IE)*DX(IE)
     1 *(FLOW(NODE(IE,1))+FLOW(NODE(IE,2)))/12)
       STIFF(1,4)=-STIFF(1,2)
       STIFF(4,1)=STIFF(2,1)
       STIFF(4,2)=STIFF(2,2)/2
       STIFF(4,3)=STIFF(2,3)
       STIFF(4,4)=2*STIFF(2,4)
       STIFF(3,1)=STIFF(1,3)
       STIFF(3,2)=STIFF(1,2)
       STIFF(3,3)=-(DX(IE)/(3*A*DT)+R1(IE)*DX(IE)/3+
     1 R2(IE)*DX(IE)*
     1 (FLOW(NODE(IE,1))/3+FLOW(NODE(IE,2)))/4)
       STIFF(3,4)=STIFF(1,4)
```

```
      RHS(2*NODE(IE,1))=RHS(2*NODE(IE,1))+STIFF(2,2)*(POLD(NODE
     1 (IE,1))+POLD(NODE(IE,2))/2)
      RHS(2*NODE(IE,1)-1)=RHS(2*NODE(IE,1)-1)-
     1 (DX(IE)/(3*A*DT))*(FLOLD
     1 (NODE(IE,1))+FLOLD(NODE(IE,2))/2)
      RHS(2*NODE(IE,2))=RHS(2*NODE(IE,2))+STIFF(2,2)*(POLD(NODE
     1 (IE,1))/2+POLD(NODE(IE,2)))
      RHS(2*NODE(IE,2)-1)=RHS(2*NODE(IE,2)-1)-
     1 (DX(IE)/(3*A*DT))*(FLOLD
     1 (NODE(IE,1))/2+FLOLD(NODE(IE,2)))
      DO 4 I=1,NDPEREL
       DO 5 K=1,2
        NRW=2*NODE(IE,I)-2+K
        DO 6 J=1,NDPEREL
         DO 7 KK=1,2
          NCOL=2*NODE(IE,J)-2+KK
          BKAA(NRW,NCOL)=BKAA(NRW,NCOL)+STIFF(2*I-2+K,2*J-2+KK)
 7       CONTINUE
 6      CONTINUE
 5     CONTINUE
 4    CONTINUE
 3   CONTINUE
     DO 10 I=1,NN
      BKAA(I,NN+1)=RHS(I)
 10  CONTINUE

C  ENTER BOUNDARY CONDITIONS

     DO 13 I=1,2*NNODE
      IF(INDEX(I).EQ.0)GO TO 13
      DO 14 J=1,7
       JJ=I+4-J
       IF(JJ.LT.1)JJ=1
       IF(JJ.GT.NN)JJ=NN
       BKAA(I,JJ)=0
 14   CONTINUE
      BKAA(I,I)=1
      BKAA(I,NN+1)=X(I)
 13  CONTINUE

C  SOLVE MATRIX

     KKK=0
     IF(KKK.EQ.1)THEN
      PRINT 100,((BKAA(I,J),J=1,NN),I=1,NN)
      PRINT 100,((BKAA(I,J),J=NN+1,NN+1),I=1,NN)
 100  FORMAT(2X,11F10.3)
     END IF
     CALL GAUSS(BKAA,NN)

C  UNLOAD RESULTS

     DO 15 I=2,NN,2
```

```
        P(I/2)=BKAA(I,NN+1)
   15  CONTINUE
       DO 16 I=1,NN,2
        FLOW((I+1)/2)=BKAA(I,NN+1)
   16  CONTINUE

C   CLEAR MATRIX

       DO 17 I=1,NN
        RHS(I)=0
        DO 18 J=1,NN+1
         BKAA(I,J)=0
   18   CONTINUE
   17  CONTINUE
       RETURN
       END

C   BOUNDARY CONDITION - RUPTURE -

       SUBROUTINE BC1(INDEX,X,IFLAG,ICASE)
       DIMENSION INDEX(80),X(80)
       COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
       COMMON/B1/WI(10,40),PO,TO,DTO

       P(1)=PEO
       INDEX(2)=1
       X(2)=P(1)
       INDEX(1)=1
       X(1)=0
       IF(ICASE.EQ.3)INDEX(1)=0
       IF(ICASE.EQ.4)INDEX(1)=0
       RETURN
       END

C   BOUNDARY CONDITION - RESERVOIR -

       SUBROUTINE BC2(INDEX,X,IFLAG,ICASE)
       DIMENSION INDEX(80),X(80)
       COMMON/A1/
N,A,DX(40),AS(40),DT,P(40),FLOW(40),DENS(40),DENS1(40)
     1
,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
       COMMON/B1/WI(10,40),PO,TO,DTO
       IF(FLOW(N).LT.O)THEN
        AMASS=FLOW(N)
        PRIF=P(N)/PR
        REF=0.9*FLOCRR
        AMASS=ABS(AMASS)
```

```
       IF(AMASS.LT.REF)CALL NRBV(PRIF,AMASS,PR,DENSR,A)
       IF(AMASS.GE.REF)CALL BISEC(PRIF,AMASS,PR,DENSR,A)
       P(N)=PRIF*PR
       INDEX(2*N)=1
       X(2*N)=P(N)
      ELSE
       P(N)=PR
       INDEX(2*N)=1
       X(2*N)=P(N)
      END IF
      RETURN
      END

C SUBROUTINE DENIT1   DENSITY UPDATING - ADIABATIC
C                     ISENTROPIC CASE -

      SUBROUTINE DENIT1
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),ENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON7B1/WI(10,40),PO,TO,DTO
      C1=1/GAMMA
      DO 1 I=1,N
       DENS(I)=DENS1(I)*(P(I)/POLD(I))**C1
        C2=1/(GAMMA-1)
       AS(I)=(GAMMA*P(I)/DENS(I))
       AS(I)=SQRT(AS(I))
 1    CONTINUE
      RETURN
      END

C  CRITICAL CONDITIONS - FIRST ELEMENT -

      SUBROUTINE FEL(INDEX,X,ICASE)
      DIMENSION INDEX(80),X(80)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO
      C3=GAMMA/(GAMMA-1)
      C11=GAMMA/(GAMMA+1)
      C1=1/GAMMA
      AA=ABS(FLOW(1))
      BB=ABS(FLOWC(1))
      PRIF=(P(1)/DENS(1)+((GAMMA-1)/(2*GAMMA))
     1 *(FLOW(1)/(DENS(1)*A))**2)
      AEX=1/GAMMA
      BEX=GAMMA/(GAMMA-1)
      PRIF=(DENS(1)/P(1)**AEX)*PRIF
```

```
      PRIF=PRIF**BEX
      CEX=GAMMA/(GAMMA+1)
      DENRIF=DENS(1)*(PRIF/P(1))**AEX
      IF(AA.GT.BB.AND.PE.EQ.101325)THEN
       FLOW(1)=(BB+1.3*(AA-BB))*FLOW(1)/AA
       PER=((FLOW(1)/A)**2*PRIF**C1/(GAMMA*DENRIF))**C11
       X(2)=X(2)+0.3*(PER-X(2))
       INDEX(2)=1
       INDEX(1)=1
       IF(ICASE.EQ.2)INDEX(1)=1
       IF(ICASE.EQ.3)INDEX(1)=0
       IF(ICASE.EQ.4)INDEX(1)=0
       X(1)=0
       IF(X(2).GT.PCRR)THEN
        X(2)=PCRR
        INDEX(1)=1
        X(1)=-FLOCRR
       END IF
      END IF
      RETURN
      END

C  SUBROUTINE CRITICAL MASS FLOW

      SUBROUTINE MASS(INDEX,X)
      DIMENSION INDEX(80),XX(40),IINDEX(80),X(80)
      COMMON/A1/N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NN=2*N
      GAMMA=1.4
      C3=GAMMA/(GAMMA-1)
      C4=(GAMMA+1)/(2*(GAMMA-1))
      PCR=PR*(2/(GAMMA-1))**C3
      IFLAG=0
      DO 2 I=2,N
       IINDEX(I)=0
       XX(I)=0
2     CONTINUE
      DO 1 I=2,N
       XX(I)=ABS(FLOWC(I))
       ABX=ABS(FLOW(I))
       ABFLOC=ABS(FLOWC(I))
       XX(I)=ABFLOC+0.5*(ABX-ABFLOC)
       IF(ABX.GT.ABFLOC)THEN
        IINDEX(I)=1
        PRIF=(P(I)/DENS(I)+((GAMMA-1)/(2*GAMMA))
     1   *(FLOW(I)/(DENS(I)*A))**2)
        AEX=1/GAMMA
        BEX=GAMMA/(GAMMA-1)
```

```
      PRIF=(DENS(I)/P(I)**AEX)*PRIF
      PRIF=PRIF**BEX
      CEX=GAMMA/(GAMMA+1)
      DENRIF=DENS(I)*(PRIF/P(I))**AEX
      PER=((XX(I)/A)**2*PRIF**AEX/(GAMMA*DENRIF))**CEX
      XX(I)=XX(I)*FLOW(I)/ABS(FLOW(I))
     END IF
 1   CONTINUE
     DO 3 J=3,NN-2,2
      IF(IINDEX((J+1)/2).EQ.1)THEN
       X(J)=XX((J+1)/2)
       X(J+1)=X(J+1)+(PER-X(J+1))
       INDEX(J)=IINDEX((J+1)/2)
       INDEX(J+1)=1
      END IF
 3   CONTINUE
     RETURN
     END


     SUBROUTINE GAUSS(A,N)

C  SOLUTION OF CAPACITANCE AND INERTANCE SYSTEM BY
C  GAUSSIAN ELIMINATION

     DIMENSION A(80,80),RHS(80)
     M=N+1
     L=N-1
     DO 12 K=1,L
     JJ=K
     BIG=ABS(A(K,K))
     KP1=K+1

C  SEARCH FOR LARGEST POSSIBLE PIVOT ELEMENT

     LLL=1
     IF(LLL.EQ.1)GO TO 10
     DO 7 I=KP1,N
     ABB=ABS(A(I,K))
     IF(BIG-ABB)6,7,7
 6   BIG=ABB
     JJ=I
 7   CONTINUE

C  DECISION OF NECESSITY OF ROW INTERCHANGE

     IF(JJ-K)8,10,8

C  ROW INTERCHANGE

 8   DO 9 J=K,M
     TEMP=A(JJ,J)
```

```
      A(JJ,J)=A(K,J)
9     A(K,J)=TEMP
C  CALCULATION OF ELEMENTS OF NEW MATRIX
10    DO 11 I=KP1,N
      QUOT=A(I,K)/A(K,K)
      DO 11 J=KP1,M
11    A(I,J)=A(I,J)-QUOT*A(K,J)
      DO 12 I=KP1,N
12    A(I,K)=0
C  FIRST STEP IN BACK SUBSTITUTION
      RHS(N)=A(N,M)/A(N,N)
C  REMAINDER OF BACK SUBSTITUTION
      DO 20 I=1,N
20    CONTINUE
      DO 14 NN=1,L
      SUM=0
      I=N-NN
      IP1=I+1
      DO 13 J=IP1,N
13    SUM=SUM+A(I,J)*RHS(J)
14    RHS(I)=(A(I,M)-SUM)/A(I,I)
      DO 15 I=1,N
15    A(I,N+1)=RHS(I)
      RETURN
      END


C  SUBROUTINE TRIDIAG
      SUBROUTINE TRIDIAG(A,B,C,D,N)
      DIMENSION A(80),B(80),C(80),D(80)
      REF=3
      IF(REF.GT.10)THEN
      DO 1 I=1,N+2
      PRINT*,I,'  ',B(I),'  ',D(I),'  ',A(I),'  ',C(I)
1     CONTINUE
      END IF
      DO 10 I=2,N
      RATIO=B(I)/D(I-1)
      D(I)=D(I)-RATIO*A(I-1)
10    C(I)=C(I)-RATIO*C(I-1)
      C(N)=C(N)/D(N)
      DO 20 I=2,N
      J=N-I+1
20    C(J)=(C(J)-A(J)*C(J+1))/D(J)
      RETURN
```

```
      END
C  NEWTON RAPHSON METHOD - BOUNDARY VALUES
      SUBROUTINE NRBV(PRIF,AMASS,PR,DENSR,A)
      GAMMA=1.4
      C2=GAMMA/(GAMMA-1)
      C4=GAMMA-1
      C5=GAMMA-2
      C7=(GAMMA-1)/GAMMA
      C8=-2/GAMMA
      C9=-1/GAMMA
      C10=-(GAMMA+2)/GAMMA
      DO 1 I=1,30
      FX=PRIF**C7+(C7/2)*(AMASS/A)**2*PRIF**C8/(PR*DENSR)-1
DFX=C7*PRIF**C9+(C7/2)*C8*(AMASS/A)**2*PRIF**C10/(PR*DENSR)
      REF=PRIF-FX/DFX
      ERROR=ABS(REF-PRIF)
      IF(ERROR.LT.0.0001)GO TO 2
  1   PRIF=REF
      PRINT*,' BOUNDARY SPECIFIC PRESSURE ERROR ',ERROR
      PRINT*,' REF ',REF,' PRIF ',PRIF,'AMASS',AMASS,' PR ',PR
  2   RETURN
      END
C  SUBROUTINE BISECTION
      SUBROUTINE BISEC(PRIF,AMASS,PR,DENSR,A)
      GAMMA=1.4
      C3=GAMMA/(GAMMA-1)
      C7=(GAMMA-1)/GAMMA
      C8=-2/GAMMA
      C9=-1/GAMMA
      C10=-(GAMMA+2)/GAMMA
C  EVALUATE FX FOR PSEUDO CRITICAL CONDITIONS
      A0=(2/(GAMMA+1))**C3
      FX1=A0**C7+(C7/2)*(AMASS/A)**2*A0**C8/(PR*DENSR)-1
C  BYSECTION
      IF(FX1.LT.0)THEN
      KOUNT=0
      A1=A0
      A2=PRIF
  1   KOUNT=KOUNT+1
      IF(KOUNT.GT.20)GO TO 3
      FX1=A0**C7+(C7/2)*(AMASS/A)**2*A0**C8/(PR*DENSR)-1
      P=A1+(A2-A1)/2
      FX2=P**C7+(C7/2)*(AMASS/A)**2*P**C8/(PR*DENSR)-1
      AFX2=ABS(FX2)
```

```
      IF(AFX2.LT.0.0001)GO TO 2
      REF=FX1*FX2
      IF(REF.GT.0)THEN
      A1=P
      GO TO 1
      ELSE
      A2=P
      GO TO 1
      END IF
      ELSE
      P=(2/(GAMMA+1))**C3
      END IF
3     PRINT*,' BYSECTION ERROR ',AFX2
2     PRIF=P
      RETURN
      END
```

APPENDIX 6

PROGRAM "SHOCK"

```
      PROGRAM SHOCK (INPUT,OUTPUT)
C  THIS PROGRAM COMPUTES FLOW PARAMETERS IN A
C  STRAIGHT PIPE WHEN A VALVE SUDDENDLY CLOSES.
C  THE CLOSING INSTANT IS GIVEN BY VARIABLE "CLO"
C  IN SECONDS FROM INITIAL TIME. SEVERAL SOLU
C  TIONS ARE PROCESSED INCREASING CLO OF 0.2 S
C  EACH TIME UNTILL 1 SEC IS REACHED.
C  SUBROUTINE "FRONT" IS BURIED IN THE MAIN
C  PROGRAM.
      DIMENSION
XLONG(40),PREF(40),INDEX(80),GVEL(40),DZ(40),X(80)
      COMMON/A1/
N,A,DX(40),AS(40),DT,P(40),FLOW(40),DENS(40),DENS1(40)
     1
,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO

C  READ EXTERNAL DATA

      READ*,DIA,N,TMAX,TO,KD,CLO
      DO 1 I=2,N+1
       READ*,XLONG(I)
       DX(I-1)=XLONG(I)-XLONG(I-1)
  1   CONTINUE
      XTOT=XLONG(N+1)
      DO 19 I=1,N
       DO 20 J=1,KD
        JJ=KD-J
        DZ(KD*I-JJ)=DX(I)/KD
  20   CONTINUE
  19  CONTINUE
      NOO=N
      N=N*KD
      DO 21 I=1,N
       DX(I)=DZ(I)
  21  CONTINUE
      DX(N+1)=DX(N)
      DX(N+2)=DX(N)
      DO 22 I=1,N
  22  CONTINUE
      READ*,PO,DENSO
      READ*,VELO
      READ*,ICASE
  778 PRINT 987,DIA,XTOT,N,PO,DENSO,VELO,CLO
  987 FORMAT('1',2X,'STRAIGHT PIPE RESERVOIR PROBLEM '////3X,
     1 'INPUT'//
     1 4X,'DIAMETER OF PIPE          =',F10.5,'  M'/
     1 4X,'PIPE LENGTH               =',F10.2,'  M'/
     1 4X,'NUMBER OF INTERVALS       =',I10/
     1 4X,'PRESSURE                  =',F10.0,'  PA'/
     1 4X,'DENSITY                   =',F10.5,'  KG/M3'/
     1 4X,'VELOCITY                  =',F10.3,'  M/S'/
```

```
     1 4X,'VALVE CLOSES AT TIME      =',F10.3,'  S'//
     1 10X,'NOTE@D IT IS ASSUMED A VALVE SUDDEN CLOSURE'////
     1 3X,'OUTPUT'//
     1 4X,'PRESSURE       PA'/
     1 4X,'MASS FLOW   KG/S'/
     1 4X,'DENSITY    KG/M3'/
     1 4X,'VELOCITY      M/S'/
     1 4X,'SOUND VEL.    M/S'/)
       PRINT 989
  989  FORMAT('1',50X,'R E S U L T S'///)
C  INITIAL CONDITIONS
       N=N+1
       A=3.14*(DIA**2)/4
       GAMMA=1.4
       TETA=1/(GAMMA-1)
       C1=1/GAMMA
       C2=(GAMMA-1)/(2*GAMMA)
       C3=GAMMA/(GAMMA-1)
       C4=(GAMMA+1)/(2*(GAMMA-1))
       C5=(GAMMA-1)
       C6=1/(GAMMA-1)
       C11=GAMMA/(GAMMA+1)
       RT=PO/DENSO+C2*VELO**2
       PR=PO
       DENSR=DENSO
       FLOWO=A*DENSO*VELO
       DO 2 I=1,N+1
        P(I)=PO
        POLD(I)=PO
        FLOW(I)=FLOWO
        PREF(I)=PO
        FLOLD(I)=FLOWO
    2  CONTINUE
       ASO=SQRT(GAMMA*PO/DENSO)
       DO 3 I=1,N+1
        AS(I)=ASO
        DENS(I)=DENSO
        FLOWC(I)=-SQRT(GAMMA*P(I)/DENS(I))*DENS(I)*A
        DENS1(I)=DENSO
    3  CONTINUE
       DENSE=(DENSO**C5+(DENSO**GAMMA/PO)*C2*VELO**2)**C6
       PE=PO*(DENSE/DENSO)**GAMMA
       PPE=PO
       PEOLD=PE
       DTO=0.
       PEO=PO
C  CRITICAL CONDITIONS WITH RESPECT TO RESERVOIR
       PCRR=PR*(2/(GAMMA+1))**C3
       FLOCRR=A*SQRT(GAMMA*PR*DENSR)*(2/(GAMMA+1))**C4
```

```
C  MAIN LOOP - ONE LOOP FOR EACH TIME INCREMENT -

       T=0
       DT=.1
  5    CONTINUE

C SHOCK GENERATION @D
C   VALVE CLOSURE @D SUDDEN CLOSURE

  82   IF(T.GT.CLO)THEN
  81    DO 83 I=1,N
         POLD(I)=P(I)
         FLOLD(I)=FLOW(I)
         DENS1(I)=DENS(I)
  83    CONTINUE
        DO 84 I=1,2*N+1
         INDEX(I)=0
  84    CONTINUE
        INDEX(1)=1
        X(1)=-1
        FLOW(1)=X(1)
        IISE=2
        IF(IISE.EQ.1)GO TO 79

C  ITERATION TO DETERMINE PRESSURE AT CLOSING POINT

        IF(ISIG.EQ.0)THEN
         U1=FLOLD(1)/(DENS1(1)*A)
         U2=U1*FLOW(1)/FLOLD(1)
  80     UREF=U2
          SC=SQRT(GAMMA*P(2)/DENS(2))
         UDIFF=((GAMMA+1)/2)*(U2-U1)/SC
         UDIFF=ABS(UDIFF)
         AMACH=(UDIFF+SQRT(UDIFF**2+4))/2
         DENS(1)=DENS1(1)*(1/(1-(2/(GAMMA+1))*(1-(1/AMACH**2))))
         U2=FLOW(1)/(DENS(1)*A)
         ERROR=ABS(U2-UREF)
         IF(ERROR.GT..1)GO TO 80
         SC=SQRT(GAMMA*P(2)/DENS(2))
         SW=AMACH*SC
         VS=U1+SW
         ISIG=1
         P(1)=POLD(1)*(1+(2*GAMMA/(GAMMA+1))*(AMACH**2-1))
         AS(1)=SQRT(GAMMA*(P(1)/DENS(1)))
         T=T-DT
         DO 60 IND=1,N
          GVEL(IND)=FLOW(IND)/(A*DENS(IND))
  60     CONTINUE
         PRINT 100,T,DT,
     1   (P(I),I=1,N,KD),(FLOW(I),I=1,N,KD)
         GO TO 81
        END IF
        INDEX(2)=0
```

```
      GO TO 17
     END IF
C  SHOCK SEARCH - EXISTENCE AND DIRECTION
C            SW= FLUID VELOCITY RELATIVE TO SHOCK
C            VS= SHOCK VELOCITY RELATIVE TO FIXED COORDINATES
 17  DO 70 I=1,N-1
      TR=0
      U1=FLOW(I)/(A*DENS(I))
      U2=FLOW(I+1)/(A*DENS(I+1))
      U1LU2=U1-U2
      AU1=ABS(U1)
      AU2=ABS(U2)
      IF(U1LU2.LT.0)GO TO 70
      IF(AU1.LT.AU2)THEN
       U1=U2
       U2=FLOW(I)/(A*DENS(I))
       SC=SQRT(GAMMA*P(I+1)/DENS(I+1))
       UDIFF=((GAMMA+1)/2)*(U2-U1)/SC
       UDIFF=ABS(UDIFF)
       AMACH=(UDIFF+SQRT(UDIFF**2+4))/2
       IF(AMACH.GT.1.1)THEN
        SW=AMACH*SC
        VS=U1+SW
       ELSE
        GO TO 70
       END IF
       TR=1
      ELSE
       SC=SQRT(GAMMA*P(I)/DENS(I))
       UDIFF=((GAMMA+1)/2)*(U2-U1)/SC
       UDIFF=ABS(UDIFF)
       AMACH=(UDIFF+SQRT(UDIFF**2+4))/2
       IF(AMACH.GT.1.1)THEN
        SW=AMACH*SC
        VS=U1-SW
       ELSE
        GO TO 70
       END IF
       TR=2
      END IF
      IF(TR.EQ.1)II=I+1
      IF(TR.EQ.2)II=I-1
C  SYSTEM MODIFICATION FOR ENTERING THE SHOCK.FIRST ATTEMPT
      DT=DX(I)/VS
      DT=ABS(DT)
      IF(ISIG.EQ.0)CALL BC1(INDEX,X,IFLAG,ICASE)
      CALL BC2(INDEX,X,IFLAG,ICASE)
      IF(TR.EQ.1)THEN
       INDEX(2*I)=1
       INDEX(2*I+1)=1
```

```
       INDEX(2*(I+1))=1
       X(2*I+1)=FLOLD(I+1)
       X(2*I)=POLD(I)
       X(2*(I+1))=POLD(I+1)
      END IF
      IF(TR.EQ.2)THEN
       INDEX(2*I)=1
       INDEX(2*I-1)=1
       INDEX(2*(I-1))=1
       X(2*I)=POLD(I)
       X(2*I-1)=FLOLD(I)
       X(2*(I-1))=POLD(I-1)
      END IF
C  MODIFIED SYSTEM SOLUTION.FIRST ATTEMPT
 73   DO 71 J=1,N
       PREF(J)=P(J)
 71   CONTINUE
      CALL SYST(INDEX,X)
      CALL DENIT1
      ERROR=0
      DO 72 J=1,N
       ERROR1=ABS(P(J)-PREF(J))
       ERROR=AMAX1(ERROR,ERROR1)
       IF(ERROR.LT.10)GO TO 74
 72   CONTINUE
      GO TO 73
C  SYSTEM MODIFICATION FOR ENTERING SHOCK.FINAL
 74   IF(TR.EQ.1)THEN
       P(I)=P(I)+(P(I-1)-POLD(I-1))/2
       P(I+1)=P(I+1)+(P(I+2)-POLD(I+2))/2
       X(2*I)=P(I)
       X(2*(I+1))=P(I+1)
      END IF
      IF(TR.EQ.2)THEN
       P(I)=P(I)+(P(I+1)-POLD(I+1))/2
       P(I-1)=P(I-1)+(P(I-2)-POLD(I-2))/2
       X(2*I)=P(I)
       X(2*(I-1))=P(I-1)
      END IF
C  SOLUTION OF MODIFIED SYSTEM.FINAL ATTEMPT
 78   DO 75 J=1,N
       PREF(J)=P(J)
 75   CONTINUE
      CALL SYST(INDEX,X)
       CALL DENIT1
      ERROR=0
      DO 76 J=1,N
```

```
      ERROR1=ABS(P(J)-PREF(J))
      ERROR=AMAX1(ERROR,ERROR1)
      IF(ERROR.LT.10)GO TO 77
76    CONTINUE
      IF(ISIG.EQ.0)CALL FEL(INDEX,X,IFLAG,ICASE)
      GO TO 78
77    IF(TR.EQ.2)THEN
       U1=FLOW(I)/(A*DENS(I))
       U2=FLOW(I+1)/(A*DENS(I+1))
       U1LU2=U1-U2
       AU1=ABS(U1)
       AU2=ABS(U2)
       SC=SQRT(GAMMA*P(I)/DENS(I))
       UDIFF=((GAMMA+1)/2)*(U2-U1)/SC
       UDIFF=ABS(UDIFF)
       AMACH=(UDIFF+SQRT(UDIFF**2+4))/2
       IF(AMACH.LE.1.1)THEN
        DO 26 JJ=1,N
         P(JJ)=POLD(JJ)
         FLOW(JJ)=FLOLD(JJ)
         DENS(JJ)=DENS1(JJ)
26      CONTINUE
        GO TO 70
       END IF
       SW=AMACH*SC
       VS=U1-SW
       U1=U2
       DENS(I)=DENS(I)*(1/(1-(2/(GAMMA+1))*(1-(1/AMACH**2))))
       P(I)=P(I)*(1+(2*GAMMA/(GAMMA+1))*(AMACH**2-1))
       FLOW(I)=U1*DENS(I)*A
       CC=P(I)/POLD(I)
       DD=DENS(I)/DENS1(I)
       CCD=ABS(1-CC)
       DDD=ABS(1-DD)
       IF(CCD.LT.0.001.OR.DDD.LT.0.0004)THEN
        AGAM(I)=AGAM(I)
       ELSE
        ACOMP=ALOG(CC)/ALOG(DD)
        IF(ACOMP.LE.0)THEN
         AGAM(I)=AGAM(I)
        ELSE
         AGAM(I)=ACOMP
        END IF
       END IF
      ELSE
       U1=FLOW(I+1)/(A*DENS(I+1))
       U2=FLOW(I)/(A*DENS(I))
       U1LU2=U1-U2
       AU1=ABS(U1)
       AU2=ABS(U2)
       SC=SQRT(GAMMA*P(I+1)/DENS(I+1))
       UDIFF=((GAMMA+1)/2)*(U2-U1)/SC
       UDIFF=ABS(UDIFF)
```

```
            AMACH=(UDIFF+SQRT(UDIFF**2+4))/2
            IF(AMACH.LE.1.1)THEN
             DO 27 JJ=1,N
              P(JJ)=POLD(JJ)
              FLOW(JJ)=FLOLD(JJ)
              DENS(JJ)=DENS1(JJ)
   27        CONTINUE
             GO TO 70
            END IF
            SW=AMACH*SC
            VS=U1+SW
            U1=U2
            DENS(I+1)=DENS(I+1)*(1/(1-(2/(GAMMA+1))
        1   *(1-(1/AMACH**2))))
            P(I+1)=P(I+1)*(1+(2*GAMMA/(GAMMA+1))*(AMACH**2-1))
            FLOW(I+1)=U1*DENS(I+1)*A
            CC=P(I+1)/POLD(I+1)
            DD=DENS(I+1)/DENS1(I+1)
            CCD=ABS(1-CC)
            DDD=ABS(1-DD)
            IF(CCD.LT.0.00.OR.DDD.LT.00.00004)THEN
             AGAM(I+1)=AGAM(I+1)
            ELSE
             ACOMP=ALOG(CC)/ALOG(DD)
             IF(ACOMP.LT.0)THEN
              AGAM(I+1)=AGAM(I+1)
             ELSE
              AGAM(I+1)=ACOMP
             END IF
            END IF
           END IF
           DT=DX(I)/VS
           DT=ABS(DT)
           T=T+DT
           DO 86 KK=1,N
            FLOWC(KK)=SQRT(GAMMA*P(KK)/DENS(KK))*A*DENS(KK)
        1   *FLOW(KK)/ABS(FLOW(KK))
   86      CONTINUE
           DO 61 IND=1,N
            GVEL(IND)=FLOW(IND)/(A*DENS(IND))
   61      CONTINUE
           PRINT 100,T,DT,
        1  (P(J),J=1,N,KD),(FLOW(J),J=1,N,KD)
           GOTO 82
   70     CONTINUE

C  UPDATE PARAMETERS AND UPDATE TIME

          IF(ISIG.EQ.1)THEN
           DO 87 I=1,N
            DENS1(I)=DENS(I)
            FLOLD(I)=FLOW(I)
            POLD(I)=P(I)
```

```
 87     CONTINUE
        SUM=0
        DO 88 I=1,N
         SUM=SUM+SQRT(GAMMA*P(I)/DENS(I))
 88     CONTINUE
         ASM=SUM/(N-1)
         DT=XLONG((N-1)/KD)/(((N-1)/KD)*ASM)
         IF(T.GE.0.AND.T.LT.20)DT=DT
         T=T+DT
         IF(T.GE.TMAX)GO TO 10
         GO TO 79
        END IF

C BOUNDARY CONDITIONS

        CALL BC1(INDEX,X,IFLAG,ICASE)
 79     CALL BC2(INDEX,X,IFLAG,ICASE)

C  SYSTEM SOLUTION

  9     IF(ICASE.EQ.1)CALL SYST(INDEX,X)
        IF(ICASE.EQ.3)CALL CENTR(INDEX,X)
        IF(ICASE.EQ.4)CALL CENTR2(INDEX,X)
        ERROR=0.
        DO 15 I=1,N
        ERROR1=ABS(P(I)-PREF(I))
 15     ERROR=AMAX1(ERROR,ERROR1)
        IF(ERROR.GT.10.)THEN

C  DENSITY UPDATING

       CALL DENIT1
         DO 6 I=1,N
          PREF(I)=P(I)
          FLOWC(I)=-SQRT(GAMMA*P(I)/DENS(I))*DENS(I)*A
  6      CONTINUE

C  PIPE RUPTURE  PRESSURE DROP LIMITATON

         AFLOW=ABS(FLOW(1))
         AFLOCR=ABS(FLOWC(1))
         TREF=T0+DT
         IF(AFLOW.GT.AFLOCR.AND.T.LT.TREF)THEN
          PRIF=(P(1)/DENS(1)+((GAMMA-1)/(2*GAMMA))
     1    *(FLOW(1)/(DENS(1)*A))**2)
          AEX=1/GAMMA
          BEX=GAMMA/(GAMMA-1)
          PRIF=(DENS(1)/P(1)**AEX)*PRIF
          PRIF=PRIF**BEX
          CEX=GAMMA/(GAMMA+1)
          DENRIF=DENS(1)*(PRIF/P(1))**AEX
          PER=((FLOW(1)/A)**2*PRIF**AEX/(GAMMA*DENRIF))**CEX
          X(2)=X(2)+0.3*(PER-X(2))
```

```
        GO TO 9
       END IF
C  CRITICAL CONDITIONS - FIRST ELEMENT -
       CALL FEL(INDEX,X,ICASE)
C  CRITICAL CONDITIONS -INTERNAL NODES -
      CALL MASS(INDEX,X)
C  CRITICAL CONDITIONS - LAST ELEMENT-
       ACOMP=ABS(FLOW(N))
       BCOMP=A*SQRT(GAMMA*POLD(N)*DENS1(N))*(2/(GAMMA+1))**C4
       IF(ACOMP.GT.BCOMP)P(N)=POLD(N)*(2/(GAMMA+1))**C3
       GO TO 9
      END IF
C  PRINT RESULTS
      DO 62 IND=1,N
       GVEL(IND)=FLOW(IND)/(A*DENS(IND))
  62  CONTINUE
      PRINT 100,T,DT,
     1  (P(I),I=1,N,KD),(FLOW(I),I=I,N,KD)
 100  FORMAT(
     1 1X,2F5.3,1X,'PR.  ',11F10.0/12X,'FLOW.',11F10.2/)
C  UPDATE TIME
      IF(ISIG.EQ.1)GO TO 82
      SUM=0.
      DO 7 I=1,N
       SUM=SUM+SQRT(GAMMA*P(I)/DENS(I))
  7   CONTINUE
      ASM=SUM/(N-1)
      DT=XLONG((N-1)/KD)/(((N-1)/KD)*ASM)
      IF(T.GE.0.AND.T.LT.20.)DT=DT
C  UPDATE PARAMETERS
      DO 25 I=1,N
       DENS1(I)=DENS(I)
       FLOLD(I)=FLOW(I)
       POLD(I)=P(I)
  25  CONTINUE
      T=T+DT
      IF(T.GT.TMAX)GO TO 10
      DTO=DTO+DT
      PEOLD=PE
      RHOLD=DENSE
      PE=PPE-((PPE-101325.)/TO)*DTO
```

```
      IF(PE.LE.101325.)PE=101325.
      DENSE=RHOLD*((PE/PEOLD)**C1)
      PEO=PE
      DO 11 I=1,2*N+1
       INDEX(I)=0
11    CONTINUE
      GO TO 5
10    CLO=CLO+0.2
      ISIG=0
      N=N-1
      TTMAX=TMAX -.3
      IF(CLO.GT.TTMAX)GO TO 777
      GO TO 778
777   STOP
      END
```

```
C  SUBROUTINE SYSTEM SOLUTION

      SUBROUTINE SYST(INDEX,X)
      DIMENSION INDEX(80),X(80),AB(80),BE(80),D(80),RHS(80)
     1 ,R1(80),R2(80)
COMMON/A1/N,A,DX(40),AS(40),DT,P(40),FLOW(40),DENS(40),DENS1(40)
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NN=2*N+1
C  BASIC MATRIX
      DO 1 I=3,NN-2,2
      R1(I)=(4*DX((I+1)/2)/(A*DT))*(DENS((I+1)/2)
     1 -DENS1((I+1)/2))/
     1 (DENS((I+1)/2)+DENS1((I+1)/2))
      R2(I)=-FLOW((I+1)/2)*(DENS((I+1)/2)-DENS((I-1)/2))/(A*DENS
     1 ((I+1)/2))**2
      D(I)=DX((I+1)/2)/(A*DT)
      RHS(I)=D(I)*FLOLD((I+1)/2)
 1    D(I)=D(I)-R1(I)+R2(I)
      D(1)=1
      D(NN+1)=1
      RHS(1)=1
      RHS(NN+1)=1
      DO 2 I=2,NN,2
      D(I)=A*DX((I+2)/2)/(DT*AS((I)/2)*ABS(AS((I)/2)))
 2    RHS(I)=D(I)*POLD(I/2)
      DO 3 I=1,NN+1
      AB(I)=1
 3    BE(I)=-1
      AB(1)=0
      AB(NN)=0
      D(NN)=1
      BE(NN+1)=0
C  BOUNDARY CONDITIONS
      DO 5 I=1,NN+2
      IF(INDEX(I).EQ.0)GO TO 5
      D(I)=1
      RHS(I)=X(I)
      AB(I)=0
      BE(I)=0
 5    CONTINUE
      NN=NN+1
      CALL TRIDIAG(AB,BE,RHS,D,NN)
      NN=NN-1
      DO 6 I=2,NN,2
 6    P((I)/2)=RHS(I)
```

```
      DO 7 I=1,NN,2
7     FLOW((I+1)/2)=RHS(I)
      IF(FLOW(1).EQ.0)THEN
      FLOW(1)=FLOW(2)
      END IF
      IF(FLOW(N).EQ.0)FLOW(N)=FLOW(N-1)
      RETURN
      END

C  SUBROUTINE CENTRAL -CENTRAL POINTS -

      SUBROUTINE CENTR(INDEX,X)
      DIMENSION BKAA(80,80),INDEX(80),X(80),NODE(40,2),R1(80)
     1 ,R2(80),RHS(80),STIFF(4,4)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),FLOW(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NNODE=N
      NELEM=N-1
      NDPEREL=2
      NN=2*NNODE

C  NODE ELEMENT RELATINSHIP

      DO 1 IE=1,NELEM
       DO 2 I=1,NDPEREL
        NODE(IE,I)=IE-1+I
2      CONTINUE
1     CONTINUE

C  ASSEMBLING OF GLOBAL INFLUENCE MATRIX

      DO 3 IE=1,NELEM
       RHO=DENS(NODE(IE,1))+DENS(NODE(IE,2))
       RHOLD=DENS1(NODE(IE,1))+DENS1(NODE(IE,2))
       R1(IE)=-2*(RHO-RHOLD)/(RHO*A*DT)
       R2(IE)=-4*(DENS(NODE(IE,2))-DENS(NODE(IE,1)))/(DX(IE)*
     1 (A*RHO)**2)
       SV=(AS(NODE(IE,1))+AS(NODE(IE,2)))/2
       STIFF(1,1)=1
       STIFF(1,2)=-(A*DX(IE)/(2*SV*ABS(SV)*DT))
       STIFF(1,3)=-STIFF(1,1)
       STIFF(1,4)=STIFF(1,2)
       STIFF(2,1)=-(DX(IE)/(2*A*DT)+R1(IE)*DX(IE)/2+
     1 R2(IE)*DX(IE)*
     1 (FLOW(NODE(IE,1))+FLOW(NODE(IE,2))/2)/3)
       STIFF(2,2)=1
       STIFF(2,3)=-(DX(IE)/(2*A*DT)+R1(IE)*DX(IE)/2+
     1 R2(IE)*DX(IE)*
     1 (FLOW(NODE(IE,1))/2+FLOW(NODE(IE,2)))/3)
```

```
      STIFF(2,4)=-STIFF(2,2)
      RHS(2*NODE(IE,1))=STIFF(1,2)*(POLD(NODE(IE,1))
     1 +POLD(NODE(IE,2)))
      RHS(2*NODE(IE,1)+1)=-(DX(IE)/(2*A*DT))*(FLOLD(NODE(IE,1))
     1 +FLOLD(NODE(IE,2)))
      DO 4 K=1,2
       NROW=2*NODE(IE,1)-1+K
       DO 5 J=1,NDPEREL
        DO 6 KK=1,2
         NCOL=2*NODE(IE,J)-2+KK
         BKAA(NROW,NCOL)=STIFF(K,2*J-2+KK)
6       CONTINUE
5      CONTINUE
4     CONTINUE
3    CONTINUE
     BKAA(1,1)=1
     BKAA(NN,NN)=1
     DO 10 I=2,NN-1
      BKAA(I,NN+1)=RHS(I)
10   CONTINUE
     BKAA(1,NN+1)=1
     BKAA(NN,NN+1)=1

C  ENTER BOUNDARY CONDITIONS

     IF(INDEX(2).EQ.1)THEN
      DO 20 I=1,2*NNODE+1
       BKAA(1,I)=BKAA(2,I)
20    CONTINUE
     END IF
     DO 13 I=1,2*NNODE
      IF(INDEX(I).EQ.0)GO TO 13
      DO 14 J=1,7
       JJ=I+4-J
       IF(JJ.LT.1)JJ=1
       IF(JJ.GT.NN)JJ=NN
       BKAA(I,JJ)=0
14    CONTINUE
      BKAA(I,I)=1
      BKAA(I,NN+1)=X(I)
13   CONTINUE

C  SOLVE MATRIX

     CALL GAUSS(BKAA,NN)

C  UNLOAD RESULTS

     DO 15 I=2,NN,2
      P(I/2)=BKAA(I,NN+1)
15   CONTINUE
     DO 16 I=1,NN,2
      FLOW((I+1)/2)=BKAA(I,NN+1)
```

```
16    CONTINUE
C  CLEAR MATRIX
      DO 17 I=1,NN
       RHS(I)=0
       DO 18 J=1,NN+1
        BKAA(I,J)=0
18     CONTINUE
17    CONTINUE
      RETURN
      END

C  SUBROUTINE CENTR2 - FINITE ELEMENTS -
      SUBROUTINE CENTR2(INDEX,X)
      DIMENSION BKAA(80,80),INDEX(80),X(80),NODE(40,2),R1(80),
    1 R2(80),RHS(80),STIFF(4,4)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
    1 FLOW(40),DENS(40),DENS1(40)
    1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
    1 FLOWC(40),KD,GAMMA,PEO,PCRR
    1 ,FLOCRR,AGAM(40)
      NNODE=N
      NELEM=N-1
      NDPEREL=2
      NN=2*NNODE
C  NODE ELEMNT RELATIONSHIP
      DO 1 IE=1,NELEM
       DO 2 I=1,NDPEREL
        NODE(IE,I)=IE-1+I
2      CONTINUE
1     CONTINUE
C  STIFFNESS MATRIX PER ELEMENT - EACH ELEMENT IN SEQUENCE -
C  ASSEMBLING OF GLOBAL INFLUENCE MATRIX
      DO 3 IE=1,NELEM
       RHO=DENS(NODE(IE,1))+DENS(NODE(IE,2))
       RHOLD=DENS1(NODE(IE,1))+DENS1(NODE(IE,2))
       R1(IE)=-2*(RHO-RHOLD)/(RHO*A*DT)
       R2(IE)=-4*(DENS(NODE(IE,2))-DENS(NODE(IE,1)))
    1  /(DX(IE)*(ARHO)
    1  **2)
       SV=(AS(NODE(IE,1))+AS(NODE(IE,2)))/2
       STIFF(2,1)=.5
       STIFF(2,2)=-(A*DX(IE)/(3*SV*ABS(SV)*DT))
       STIFF(2,3)=-STIFF(2,1)
       STIFF(2,4)=STIFF(2,2)/2
```

```
      STIFF(1,1)=-(DX(IE)/(3*A*DT)+R1(IE)*DX(IE)/3+
     1 R2(IE)*DX(IE)
     1 *(FLOW(NODE(IE,1))+FLOW(NODE(IE,2))/3)/4)
      STIFF(1,2)=.5
      STIFF(1,3)=-(DX(IE)/(6*A*DT)+R1(IE)*DX(IE)/6+
     1 R2(IE)*DX(IE)
     1 *(FLOW(NODE(IE,1))+FLOW(NODE(IE,2)))/12)
      STIFF(1,4)=-STIFF(1,2)
      STIFF(4,1)=STIFF(2,1)
      STIFF(4,2)=STIFF(2,2)/2
      STIFF(4,3)=STIFF(2,3)
      STIFF(4,4)=2*STIFF(2,4)
      STIFF(3,1)=STIFF(1,3)
      STIFF(3,2)=STIFF(1,2)
      STIFF(3,3)=-(DX(IE)/(3*A*DT)+R1(IE)*DX(IE)/3+
     1 R2(IE)*DX(IE)*
     1 (FLOW(NODE(IE,1))/3+FLOW(NODE(IE,2)))/4)
      STIFF(3,4)=STIFF(1,4)
      RHS(2*NODE(IE,1))=RHS(2*NODE(IE,1))+STIFF(2,2)*(POLD(NODE
     1 (IE,1))+POLD(NODE(IE,2))/2)
      RHS(2*NODE(IE,1)-1)=RHS(2*NODE(IE,1)-1)-
     1 (DX(IE)/(3*A*DT))*(FLOLD
     1 (NODE(IE,1))+FLOLD(NODE(IE,2))/2)
      RHS(2*NODE(IE,2))=RHS(2*NODE(IE,2))+STIFF(2,2)*(POLD(NODE
     1 (IE,1))/2+POLD(NODE(IE,2)))
      RHS(2*NODE(IE,2)-1)=RHS(2*NODE(IE,2)-1)-
     1 (DX(IE)/(3*A*DT))*(FLOLD
     1 (NODE(IE,1))/2+FLOLD(NODE(IE,2)))
      DO 4 I=1,NDPEREL
       DO 5 K=1,2
        NRW=2*NODE(IE,I)-2+K
        DO 6 J=1,NDPEREL
         DO 7 KK=1,2
          NCOL=2*NODE(IE,J)-2+KK
          BKAA(NRW,NCOL)=BKAA(NRW,NCOL)+STIFF(2*I-2+K,2*J-2+KK)
 7       CONTINUE
 6      CONTINUE
 5     CONTINUE
 4    CONTINUE
 3   CONTINUE
     DO 10 I=1,NN
      BKAA(I,NN+1)=RHS(I)
 10  CONTINUE

C  ENTER BOUNDARY CONDITIONS

     DO 13 I=1,2*NNODE
      IF(INDEX(I).EQ.0)GO TO 13
      DO 14 J=1,7
       JJ=I+4-J
       IF(JJ.LT.1)JJ=1
       IF(JJ.GT.NN)JJ=NN
       BKAA(I,JJ)=0
```

```
 14    CONTINUE
       BKAA(I,I)=1
       BKAA(I,NN+1)=X(I)
 13   CONTINUE

C  SOLVE MATRIX

      KKK=0
      IF(KKK.EQ.1)THEN
       PRINT 100,((BKAA(I,J),J=1,NN),I=1,NN)
       PRINT 100,((BKAA(I,J),J=NN+1,NN+1),I=1,NN)
 100   FORMAT(2X,11F10.3)
      END IF
      CALL GAUSS(BKAA,NN)

C  UNLOAD RESULTS

      DO 15 I=2,NN,2
       P(I/2)=BKAA(I,NN+1)
 15   CONTINUE
      DO 16 I=1,NN,2
       FLOW((I+1)/2)=BKAA(I,NN+1)
 16   CONTINUE

C  CLEAR MATRIX

      DO 17 I=1,NN
       RHS(I)=0
       DO 18 J=1,NN+1
        BKAA(I,J)=0
 18    CONTINUE
 17   CONTINUE
      RETURN
      END

C  BOUNDARY CONDITION - RUPTURE -

      SUBROUTINE BC1(INDEX,X,IFLAG,ICASE)
      DIMENSION INDEX(80),X(80)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
    1 FLOW(40),DENS(40),DENS1(40)
    1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
    1 FLOWC(40),KD,GAMMA,PEO,PCRR
    1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO

      P(1)=PEO
      INDEX(2)=1
      X(2)=P(1)
      INDEX(1)=1
      X(1)=0
      IF(ICASE.EQ.3)INDEX(1)=0
      IF(ICASE.EQ.4)INDEX(1)=0
```

```
      RETURN
      END

C  BOUNDARY CONDITION - RESERVOIR -

      SUBROUTINE BC2(INDEX,X,IFLAG,ICASE)
      DIMENSION INDEX(80),X(80)
      COMMON/A1/
N,A,DX(40),AS(40),DT,P(40),FLOW(40),DENS(40),DENS1(40)
     1
,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO
      IF(FLOW(N).LT.0)THEN
       AMASS=FLOW(N)
       PRIF=P(N)/PR
       REF=0.9*FLOCRR
       AMASS=ABS(AMASS)
       IF(AMASS.LT.REF)CALL NRBV(PRIF,AMASS,PR,DENSR,A)
       IF(AMASS.GE.REF)CALL BISEC(PRIF,AMASS,PR,DENSR,A)
       P(N)=PRIF*PR
       INDEX(2*N)=1
       X(2*N)=P(N)
      ELSE
       P(N)=PR
       INDEX(2*N)=1
       X(2*N)=P(N)
      END IF
      RETURN
      END

C SUBROUTINE DENIT1    DENSITY UPDATING - ADIABATIC
C                      ISENTROPIC CASE -

      SUBROUTINE DENIT1
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),ENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO
      C1=1/GAMMA
      DO 1 I=1,N
       DENS(I)=DENS1(I)*(P(I)/POLD(I))**C1
        C2=1/(GAMMA-1)
       AS(I)=(GAMMA*P(I)/DENS(I))
       AS(I)=SQRT(AS(I))
 1    CONTINUE
      RETURN
      END

C  CRITICAL CONDITIONS - FIRST ELEMENT -
```

```
      SUBROUTINE FEL(INDEX,X,ICASE)
      DIMENSION INDEX(80),X(80)
      COMMON/A1/ N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      COMMON/B1/WI(10,40),PO,TO,DTO
      C3=GAMMA/(GAMMA-1)
      C11=GAMMA/(GAMMA+1)
      C1=1/GAMMA
      AA=ABS(FLOW(1))
      BB=ABS(FLOWC(1))
      PRIF=(P(1)/DENS(1)+((GAMMA-1)/(2*GAMMA))
     1 *(FLOW(1)/(DENS(1)*A))**2)
      AEX=1/GAMMA
      BEX=GAMMA/(GAMMA-1)
      PRIF=(DENS(1)/P(1)**AEX)*PRIF
      PRIF=PRIF**BEX
      CEX=GAMMA/(GAMMA+1)
      DENRIF=DENS(1)*(PRIF/P(1))**AEX
      IF(AA.GT.BB.AND.PE.EQ.101325)THEN
       FLOW(1)=(BB+1.3*(AA-BB))*FLOW(1)/AA
       PER=((FLOW(1)/A)**2*PRIF**C1/(GAMMA*DENRIF))**C11
       X(2)=X(2)+0.3*(PER-X(2))
       INDEX(2)=1
       INDEX(1)=1
       IF(ICASE.EQ.2)INDEX(1)=1
       IF(ICASE.EQ.3)INDEX(1)=0
       IF(ICASE.EQ.4)INDEX(1)=0
       X(1)=0
       IF(X(2).GT.PCRR)THEN
        X(2)=PCRR
        INDEX(1)=1
        X(1)=-FLOCRR
       END IF
      END IF
      RETURN
      END

C  SUBROUTINE CRITICAL MASS FLOW

      SUBROUTINE MASS(INDEX,X)
      DIMENSION INDEX(80),XX(40),IINDEX(80),X(80)
      COMMON/A1/N,A,DX(40),AS(40),DT,P(40),
     1 FLOW(40),DENS(40),DENS1(40)
     1 ,PR,DENSR,PE,DENSE,POLD(40),FLOLD(40),
     1 FLOWC(40),KD,GAMMA,PEO,PCRR
     1 ,FLOCRR,AGAM(40)
      NN=2*N
      GAMMA=1.4
      C3=GAMMA/(GAMMA-1)
```

```
      C4=(GAMMA+1)/(2*(GAMMA-1))
      PCR=PR*(2/(GAMMA-1))**C3
      IFLAG=0
      DO 2 I=2,N
       IINDEX(I)=0
       XX(I)=0
2     CONTINUE
      DO 1 I=2,N
       XX(I)=ABS(FLOWC(I))
       ABX=ABS(FLOW(I))
       ABFLOC=ABS(FLOWC(I))
       XX(I)=ABFLOC+0.5*(ABX-ABFLOC)
       IF(ABX.GT.ABFLOC)THEN
        IINDEX(I)=1
        PRIF=(P(I)/DENS(I)+((GAMMA-1)/(2*GAMMA))
     1  *(FLOW(I)/(DENS(I)*A))**2)
        AEX=1/GAMMA
        BEX=GAMMA/(GAMMA-1)
        PRIF=(DENS(I)/P(I)**AEX)*PRIF
        PRIF=PRIF**BEX
        CEX=GAMMA/(GAMMA+1)
        DENRIF=DENS(I)*(PRIF/P(I))**AEX
        PER=((XX(I)/A)**2*PRIF**AEX/(GAMMA*DENRIF))**CEX
        XX(I)=XX(I)*FLOW(I)/ABS(FLOW(I))
       END IF
1     CONTINUE
      DO 3 J=3,NN-2,2
       IF(IINDEX((J+1)/2).EQ.1)THEN
        X(J)=XX((J+1)/2)
        X(J+1)=X(J+1)+(PER-X(J+1))
        INDEX(J)=IINDEX((J+1)/2)
        INDEX(J+1)=1
       END IF
3     CONTINUE
      RETURN
      END


      SUBROUTINE GAUSS(A,N)

C  SOLUTION OF CAPACITANCE AND INERTANCE SYSTEM BY
C  GAUSSIAN ELIMINATION

      DIMENSION A(80,80),RHS(80)
      M=N+1
      L=N-1
      DO 12 K=1,L
      JJ=K
      BIG=ABS(A(K,K))
      KP1=K+1

C  SEARCH FOR LARGEST POSSIBLE PIVOT ELEMENT
```

```
      LLL=1
      IF(LLL.EQ.1)GO TO 10
      DO 7 I=KP1,N
      ABB=ABS(A(I,K))
      IF(BIG-ABB)6,7,7
 6    BIG=ABB
      JJ=I
 7    CONTINUE
C  DECISION OF NECESSITY OF ROW INTERCHANGE
      IF(JJ-K)8,10,8
C  ROW INTERCHANGE
 8    DO 9 J=K,M
      TEMP=A(JJ,J)
      A(JJ,J)=A(K,J)
 9    A(K,J)=TEMP
C  CALCULATION OF ELEMENTS OF NEW MATRIX
 10   DO 11 I=KP1,N
      QUOT=A(I,K)/A(K,K)
      DO 11 J=KP1,M
 11   A(I,J)=A(I,J)-QUOT*A(K,J)
      DO 12 I=KP1,N
 12   A(I,K)=0
C  FIRST STEP IN BACK SUBSTITUTION
      RHS(N)=A(N,M)/A(N,N)
C  REMAINDER OF BACK SUBSTITUTION
      DO 20 I=1,N
 20   CONTINUE
      DO 14 NN=1,L
      SUM=0
      I=N-NN
      IP1=I+1
      DO 13 J=IP1,N
 13   SUM=SUM+A(I,J)*RHS(J)
 14   RHS(I)=(A(I,M)-SUM)/A(I,I)
      DO 15 I=1,N
 15   A(I,N+1)=RHS(I)
      RETURN
      END

C  SUBROUTINE TRIDIAG
```

```
      SUBROUTINE TRIDIAG(A,B,C,D,N)
      DIMENSION A(80),B(80),C(80),D(80)
      REF=3
      IF(REF.GT.10)THEN
      DO 1 I=1,N+2
      PRINT*,I,'  ',B(I),'  ',D(I),'  ',A(I),'  ',C(I)
1     CONTINUE
      END IF
      DO 10 I=2,N
      RATIO=B(I)/D(I-1)
      D(I)=D(I)-RATIO*A(I-1)
10    C(I)=C(I)-RATIO*C(I-1)
      C(N)=C(N)/D(N)
      DO 20 I=2,N
      J=N-I+1
20    C(J)=(C(J)-A(J)*C(J+1))/D(J)
      RETURN
      END
C  NEWTON RAPHSON METHOD - BOUNDARY VALUES

      SUBROUTINE NRBV(PRIF,AMASS,PR,DENSR,A)
      GAMMA=1.4
      C2=GAMMA/(GAMMA-1)
      C4=GAMMA-1
      C5=GAMMA-2
      C7=(GAMMA-1)/GAMMA
      C8=-2/GAMMA
      C9=-1/GAMMA
      C10=-(GAMMA+2)/GAMMA
      DO 1 I=1,30
      FX=PRIF**C7+(C7/2)*(AMASS/A)**2*PRIF**C8/(PR*DENSR)-1

DFX=C7*PRIF**C9+(C7/2)*C8*(AMASS/A)**2*PRIF**C10/(PR*DENSR)
      REF=PRIF-FX/DFX
      ERROR=ABS(REF-PRIF)
      IF(ERROR.LT.0.0001)GO TO 2
1     PRIF=REF
      PRINT*,' BOUNDARY SPECIFIC PRESSURE ERROR ',ERROR
      PRINT*,' REF ',REF,' PRIF ',PRIF,'AMASS',AMASS,' PR ',PR
2     RETURN
      END

C  SUBROUTINE BISECTION

      SUBROUTINE BISEC(PRIF,AMASS,PR,DENSR,A)
      GAMMA=1.4
      C3=GAMMA/(GAMMA-1)
      C7=(GAMMA-1)/GAMMA
      C8=-2/GAMMA
      C9=-1/GAMMA
      C10=-(GAMMA+2)/GAMMA
```

```
C  EVALUATE FX FOR PSEUDO CRITICAL CONDITIONS

      A0=(2/(GAMMA+1))**C3
      FX1=A0**C7+(C7/2)*(AMASS/A)**2*A0**C8/(PR*DENSR)-1

C  BYSECTION

      IF(FX1.LT.0)THEN
      KOUNT=0
      A1=A0
      A2=PRIF
 1    KOUNT=KOUNT+1
      IF(KOUNT.GT.20)GO TO 3
      FX1=A0**C7+(C7/2)*(AMASS/A)**2*A0**C8/(PR*DENSR)-1
      P=A1+(A2-A1)/2
      FX2=P**C7+(C7/2)*(AMASS/A)**2*P**C8/(PR*DENSR)-1
      AFX2=ABS(FX2)
      IF(AFX2.LT.0.0001)GO TO 2
      REF=FX1*FX2
      IF(REF.GT.0)THEN
      A1=P
      GO TO 1
      ELSE
      A2=P
      GO TO 1
      END IF
      ELSE
      P=(2/(GAMMA+1))**C3
      END IF
 3    PRINT*,' BYSECTION ERROR ',AFX2
 2    PRIF=P
      RETURN
      END
```