DEVELOPMENT OF A MODEL TO OPTIMIZE IRRIGATION PLANNING FOR MULTI-RESERVOIR SYSTEMS

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ABSTRACT

DEVELOPMENT OF A MODEL TO OPTIMIZE IRRIGATION PLANNING FOR MULTI-RESERVOIR SYSTEMS

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The purpose of this study is to develop an optimization planning model for irrigation multi-reservoir systems. This is a monthly-based model that can be applied to multi-reservoir systems to obtain optimal planning policies to maximize the net benefit resulting from planting different kinds of crops. This model can handle an arbitrary number of reservoirs with no restriction on their layout.

The planning model, developed through this research effort optimizes the total net benefit that can result from planting different kinds of crops, subject to physical constraints such as mass balance, target demands and storage limits. The model is written in ANSI FORTRAN 77 and the Penalty Successive Linear Programming (PSLP) procedure is employed. PSLP is one of the most promising optimization techniques for non-linear non-convex objective functions. The model developed in this study is the first model of its type that applies PSLP to optimize the planning of multi-reservoir systems.

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NOMENCLATURE

$A_{c,n}$: planted area of crop c served by reservoir n, in (L^2).

$ATOT_n$: total area (planted or not) under the responsibility of reservoir n, in (L^2).

B : total dollar value of the net benefit resulting from planting C different kinds of crops.

C : total number of crops.

$D_{c,n,t}$: water demanded by crop c planted in the area served by reservoir n, during month t, in (L^3).

$E_{t,n}$: volume of evaporation from reservoir n during month, t, in (L^3).

F_c : fixed cost of crop c, in ($/$L^2$).

F_t : monthly consumptive use factor.

L_t : normal lake evaporation during month t, in (L).

M : a routing matrix.

$\left[M S_{p_t} \right]_n$: total volume of water spilled into reservoir n. One or more reservoirs can contribute to the spilled volume.

N : total number of reservoirs.

P_c :	unit market price of crop c, in (\$/unit yield).
P_{r_t} :	effective rainfall, i.e., the amount of rain absorbed by the root system, during month t, in (L^3).
P_t :	monthly percentage of daytime hours.
$Q_{t,n}$:	inflow coming into reservoir n during month t, in (L^3).
$R_{c,n,t}$:	release to crop c, from reservoir n, during month t, in (L^3).
$R_{t,n}$:	total agricultural water volume released by reservoir n during month t, in (L^3).
$RMAX_{t,n}$:	lower bound on release for reservoir n.
$RMIN_{t,n}$:	upper bound on release for reservoir n.
$S_{t,n}$:	storage volume of reservoir n at the beginning of the current month t, in (L^3).
$S_{t+1,n}$:	storage volume of reservoir n at the end of the current month t, in (L^3).
$S_{p_{t,n}}$:	volume of water spilled by reservoir n, during month, t, in (L^3).
$SMAX_{t,n}$:	maximum capacity of reservoir n.
$SMIN_{t,n}$:	minimum dead storage of reservoir n.
T :	total number of months of the growing season.
T_t :	mean monthly air temperature, in (°F).

$U_{c,t}$: potential evapo-transpiration of crop c, during month t, in (L).

V_c : variable unit cost of crop c, in (\$/unit yield).

$V_{c,n,t}$: available water for crop c planted in the area served by reservoir n, during month t, in (L^3).

$W_{t,n}$. a positive large value (penalty weight).

$Y_{c,n}$: predicted total yield of crop c, planted at an area served by reservoir n, in (unit yield/L^2).

Y'_c : potential grain yield for grain crop c, in (M/L^2) or (L^3/L^2).

$Y_{p_{c,t}}$: potential dry matter yield for dry matter crop c during month t, in (M/L^2).

$a_{t,n}$: average surface area of reservoir n, during month t, in (L^2).

$b_{c,n,t}$: derivative of the non-linear part of the objective function with respect to $R_{c,n,t}$ at $R_{c,n,t} = R''_{c,n,t}$ and $A_{c,n} = A''_{c,n}$.

c : an index representing the crop number from 1 to C.

d_i's : represent the change in the non-linear variables between any two consecutive iterations.

e : total efficiency from the reservoir to the root system.

$h_{t,n}$: the non-linear constraint (mass balance equation).

$k_{c,t}$: empirical coefficient of consumptive use for crop c, during month t.

$m_{c,n}$: derivative of the non-linear part of the objective function with respect to $A_{c,n}$ at $R_{c,n,t} = R^0_{c,n,t}$ and $A_{c,n} = A^0_{c,n}$.

n : an index representing the reservoir number from 1 to N.

$n_{t,n}$: a new independent positive variable.

p : exact penalty function.

p_l : linear approximation of the exact penalty function.

$p_{t,n}$: a new independent positive variable.

$q_{t,n}$: sum of unregulated or independent flow volumes between reservoir n and the reservoirs immediately upstream during month t, in (L^3).

t : an index representing the month number from 1 to T.

z : a coefficient describing the geometry of reservoir n.

α_ϕ : total area, planted or not, owned by farmer ϕ.

$\alpha_{\phi,c}$: minimum area of crop c that farmer ϕ wants to plant.

β_i's : represent the upper and lower bounds on the change in the non-linear variables.

$\delta_{1_{c,n}}(R_{c,n,t}, A_{c,n})$: non-linear part of the objective function.

$\delta_{2_{c,n}}(R_{c,n,t}, A_{c,n})$: linear part of the objective function.

Φ : total number of farmers.

ϕ : an index representing the farmer number from 1 to Φ.

$\Psi_{t,n}$: derivative of the non-linear constraint (mass balance equation) with respect to $S_{t,n}$ at $S_{t+1,n} = S^0_{t+1,n}$ and $S_{t,n} = S^0_{t,n}$.

$\lambda_{c,t}$: relative sensitivity of crop c to water deficit during month t.

$\mu_{\phi,c}$: minimum fraction of the total area owned by farmer ϕ that must be allocated for crop c.

ρ_n : a coefficient describing the geometry of reservoir n.

ζ_c : minimum percentage of the full crop water requirements to allow the crop to be produced throughout the season.

CHAPTER 1

Introduction

Water is a natural resource serving a wide range of uses in modern-day society. Besides the basic purpose for support of life, water is also used for other purposes such as irrigation, hydropower, navigation, recreation, pollution abatement, etc. Irrigation is the only use of water discussed herein.

The availability of water, and its temporal and spatial distribution are governed by climatic factors outside man's control, at the present state of science and technology. Optional management of water resources is therefore fundamental in ensuring the continuous availability of water for the benefit of humankind both for the present and the future.

The natural availability of water is often not sufficient to satisfy the demand and use imposed by society. Control of water has therefore been practiced since the early days of civilization.

The works of water control include primarily the construction of regulation facilities in the form of storage and diversion channels. For many developing countries, water resources development projects have been and will continue to be very important components of their infrastructure.

1.1 Background:

When demand for water is low compared to its natural supply, water control works are relatively simple. All potential users receive their full requirements and little planning or coordination is required since efficiency is of little concern when there is a surplus of water. Larson (1981) summarized the practice of irrigation management in the private sector. He stated that, when water is abundant and energy is cheap, the approach to irrigation is to apply plenty of water. Little attention was paid to how much water was applied nor how efficiently it was applied. This wasteful situation prompted researchers to develop techniques to avoid over-irrigation, thus reducing water use, energy consumption, and irrigation costs.

The techniques that were designed to avoid waste are called irrigation scheduling. Irrigation scheduling consists of, applying the right amount of water at the right time. The goal is to meet the full water requirement of the crop thus avoiding water stress and maximizing yields.

The yield maximizing approach outlined by Larson has worked relatively well in the past when the intent was to maximize returns from a limited area of agricultural land with an ample water supply. However, this maximizing approach is not appropriate when one wants to maximize returns with a limited amount of water.

1.2 Problem:

This research effort assumes that water is the limiting factor, i.e., crops might not receive their full water requirements during all periods of the growing season. Under unlimited water supply conditions, however, water usage in the current period does not affect the availability of water in the near-term (the period of the planning horizon). Whereas, under conditions of limited water supply, current usage reduces the water available in the near-term, thus affecting subsequent water-use decisions.

Temporal interdependence has major implications for the manner in which the irrigation management problem should be examined. It implies an analysis that considers the time frame of the planning horizon, because the water supply in future periods is directly related to the water amount used in the current period. This implies that the optimal planning for crop irrigation during any period includes the evaluation of both current and subsequent periods. Therefore to obtain the maximum benefit from water under unlimited water conditions, one could plant any acreage available for agriculture with a high return crop and supply the full amount of water required. Whereas, under limited water conditions, the availability of water limits the acreage and type of crops that should be planted. Thus obtaining the optimal acreage of the crops can be done by formalizing the problem and then applying a suitable optimization method to it.

The optimal planning of crop irrigation from multi-reservoir systems can be modeled as a non-linear, constrained optimization problem. The objective is to maximize the crop net benefit resulting from planting different kinds of crops. If hydrological inflows to the system are assumed known or predictable, the resulting problem becomes deterministic. Therefore, deterministic optimization techniques can be applied. They are simpler to apply than their stochastic counterparts.

A typical crop irrigation planning optimization problem is characterized by a non-linear and non-convex objective function with most constraints linear. The non-convex objective function constitute the primary difficulty in finding the successful mathematical solution of this problem. For small systems, discrete dynamic programming is the most reliable method for obtaining the global optimum solution. The real difficulty arises when the system is large with many state variables, more than three, for instance (Hiew, 1987). Choosing a suitable optimization method, for this problem, is the focus of the next chapter.

CHAPTER 2

LITERATURE REVIEW

A literature review is attempted, in this chapter, to choose adequate crop yield functions and a suitable optimization procedure to be applied to the model developed in Chapter 3.

2.1 Crop yield functions:

Since ancient times, farmers have observed that, to get a good harvest, they had to provide enough water to the crops. However it was not until the 20^{th} century that the concept of crop yield function was formalized, i.e., the relationship between crop yield and water supply. Jensen (1968) proposed the following equation to estimate the grain yield of crops.

$$Y = Y_p \prod_{i=1}^{I} \left(\frac{U_c}{U_p} \right)_i^{\lambda_i} \tag{2.1}$$

where, Y is the predicted yield, in (M/L^2) or (L^3/L^2).

Y_p is the potential grain yield, in (M/L^2) or (L^3/L^2).

U_t is the consumptive use or actual evapo-transpiration, in (L).

U_p is the potential consumptive use or potential evapo-transpiration, in (L). It refers to conditions when water is adequate for unrestricted growth and development. U_p represents the rate of maximum evapo-transpiration of a healthy crop, growing in large fields under optimum agronomic and irrigation management.

λ_i is the relative sensitivity of the crop to water deficit during the i^{th} stage of growth.

Hanks (1974) developed an analogous model to predict dry matter yield:

$$Y = Y_p\left(\frac{T}{T_p}\right) \tag{2.2}$$

where, Y_p is the potential dry matter yield, in (M/L^2).

T is the cumulative transpiration by the crop, in (L)

T_p is the potential cumulative transpiration, in (L).

These two models are mostly used to predict the yield from evapo-transpiration or transpiration of crops. They have proven to be reliable, as deviations, between predicted and actual yields, are not significant (Hanks 1974, Rasmussen and Hanks 1978, Stewart et al. 1977).

2.2 Optimization Procedures:

The optimal planning of multi-reservoir systems is a subject of great practical and economic importance in the field of water resources engineering (Yeh, 1985). Mathematical modeling is the most extensively used engineering technique in planning and analysis of large-scale problems. A mathematical model is a set of mathematical or logical expressions used to describe the essential operation and response of a physical system for a specific purpose.

There are two distinct deterministic categories of models, simulation and optimization.

Simulation models are descriptive models that attempt to represent the essential physical and operational characteristics of a real system. They can accurately represent the complex interactions between system components, be they linear, non-linear, convex or non-convex. The flexibility of simulation models makes them widely accepted for many water resources modeling applications. However, the disadvantage of simulation models is that a lot of effort and experience are required of the user in finding the optimal solution. For a complex problem, finding the optimum through several simulation experiments can be difficult, time-consuming and frustrating. Examples of popular simulation models are the Stanford Watershed Model, Storm Water Management Model, Stream Flow Synthesis and Reservoir Regulation Model, Massachusetts Institute Technology Model, and a series of models released by Hydrologic Research Center such as the HEC-1, HEC-3, HEC-5 and HEC-6 Models (Hiew, 1987).

Optimization models, on the other hand, are aimed at identifying the best solution based on a specific index of performance (the objective value) and meeting all the relevant constraints. However, the mathematical structure of optimization models is more restrictive than simulation models and real-life problems often need to be simplified in order to fit into the structure and format of existing algorithms. Such simplification, however has to be carried out with caution to ensure that the essential characteristics of the original problem are retained. Examples of optimization techniques commonly used in water resources engineering are Linear Programming, Dynamic Programming, Optimal Control Theory and Non-linear Programming (Hiew, 1987).

The choice of mathematical model for a particular system is highly dependent on the structure of the problem and the preference and experience of the user. Because many techniques have been developed and applied to optimization studies of multi-reservoir water resources systems, inexperienced users are often faced with the difficult question of deciding which of these algorithms is best suited to a problem. Time and budget limitations often do not allow experimentation with several algorithms to find the best optimal result. Unfortunately there is no easy answer to this question and much depends on the complexity of the particular system and the objective function of the problem. Since most algorithms published in the literature are displayed on different systems, i.e., not all algorithms were applied to each single problem variation to determine which algorithm is the best suited. It is hard to draw conclusions on the relative merits of each optimization procedure applied to a certain problem using strictly its objective criteria.

However, a review is attempted below.

The most widely used optimization technique among engineers is Linear Programming (LP). Its popularity is due, to a large extent, to the existence of standard programming packages capable of solving large problems efficiently. LP can handle large number of variables and constraints but requires that the relationships between these variables be linear, both in the objective function and the constraints. Dantzing (1950) first developed the simplex algorit to solve linear problems.

Most water resources problems are inherently non-linear and therefore LP cannot be directly applied. In order for standard LP to be used for these types of problems, a suitable linearization scheme must be introduced. A widely adopted linearization scheme is based on first-order Taylor's series expansion of a non-linear function about a given initial solution. This method is commonly known as Successive Linear Programming (SLP). The concept of SLP is simple and easy to carry out. The method has been widely used by practicing engineers for many non-linear engineering problems. Palicio-Gomez (1982) studied the theoretical properties of SLP and proved that, for linearly constrained problems, an iterative linear programming algorithm, which yields a feasible solution at each iteration, will converge to a local optimum as the step size is reduced if the objective function is continuously differentiable. In addition, he showed that SLP compares favorably with the generalized reduced gradient code (GRG2) and with MINOS/GRG. Moreover, he found that SLP will be most successful when applied to large problems with relatively low degrees of freedom. Grygier (1983) tested SLP against two other methods: Optimal Control Theory (OCT) and Dynamic Programming-Linear

Programming method (DP-LP) and found that SLP was able to achieve the maximum objective value consistently and with moderate computer time. These conclusions were based on studies involving a single reservoir, two reservoirs in series and three reservoirs in parallel. Although the objective functions were non-linear, they were relatively smooth and did not deviate too far from linearity.

From each category of optimization techniques mentioned above, Hiew (1987) selected the following algorithms: Successive Linear Programming (SLP), Feasible Direction Method (FDM), Optimal Control Theory (OCT), Incremental Dynamic Programming (IDP), and Objective-Space Dynamic Programming (OSDP). In the selection of these algorithms which cover a broad range of techniques, preference was given to methods that are commonly used and possess generalized structures. It was concluded that both (SLP) and (OCT) are more general, provide more consistent performance and better computational efficiency than the other methods tested. Therefore, they are recommended to optimize planning and operation of large-scale multi-reservoir systems characterized by non-linear and non-convex objective functions.

From the above, it can be concluded that SLP is one of the best optimization methods commonly used in water resources engineering problems. It performs most successfully when applied to large scale systems with relatively low degree of freedom, and when a flexible, powerful and reliable LP code is available. However, for non-linearly constrained problems, a suitable SLP algorithm has still to be sought.

A discussion of Successive Linear Programming and the development of the only existing SLP algorithm that has a convergence proof for non-linearly constrained problems follows.

2.2.1 Successive Linear Programming:

The basic idea of SLP is to approximate the objective function and the non-linear constraints, if any, by using first-order Taylor series expansion about an initial or trial solution. This results in an approximated programming problem which is linear in both the decision and state variables. In LP, decision and state variables are treated alike as unknowns to be determined simultaneously from the solution of the resulting constrained optimization problem.

SLP algorithms solve non-linear optimization problems via a sequence of linear problems. Recall that SLP performs better when the non-linearity is confined to the objective function. The proof of convergence with linear constraints is available from Palacios-Gomez et al. (1982). The convergence theorem states that a problem with linear constraints and a continuously differentiable objective function, the sequence of iterates generated by an SLP algorithm will converge to a constrained stationary point if an appropriate step bound-reduction scheme is adopted. Step bound-reduction introduces maximum allowable deviations (measured from one iteration to the next) in the decision variables of the linearized problem with linear programming. Without such a scheme, SLP might experience oscillations or zigzagging about the optimum solution when the optimum point is non-vertex, i.e., lying in the interior of the feasible solution space. The feasible solution space is the locus where the constraints are satisfied.

Palacios-Gomez et al. (1982) proposed some modifications to the conventional SLP algorithm to better cope with non-linear constraints, by using a modified objective function and adding a term to it to account for the sum of feasibility violations caused by the non-linear constraints. The reason of this sum of feasibility violations is the replacement of the non-linear constraints by a term added to the objective function. It is possible to introduce a criteria for rejecting or accepting a successor point and the related changes of the step bound reduction. Palacios-Gomez tested this algorithm known as SLP reject or SLPR on more than forty non-linear problems of different designs and dimensions and reported good results with convergence to optmum (global or local). Nevertheless, he admitted that theoretical proof of convergence is not possible for problems with non-linear constraints. Then Zhang et al. (1985) developed a new SLP algorithm called PSLP (penalty SLP). PSLP represents a significant strengthening and refinement of the SLPR procedure. It can be viewed as a steepest descent procedure applied to the exact penalty function associated with the non-linear problem. The search direction is determined by solving the linearized problem with linear programming, and the distance advanced in each direction is determined by the size of a rectangular trust region (the region that each non-linear variable is additionally constrained by). This region is specified by the same step bounds given in Palacios-Gomez et al. (1982). However, the trust region theory developed by ZHU and Zhang (1982), suggests a new criteria for varying these step bounds and for accepting or rejecting new iterates. This step bound-reduction theorem is more reliable and sound than the one given by Palacios-Gomez et al. (1982). This conclusion was borne out both by extensive computational results and more importantly by a convergence proof. This convergence proof shows that PSLP converges to a stationary point of the exact penalty function. It is the first SLP convergence proof for non-linearly constrained problems of general form. The convergence theory of PSLP was supported by computational performance done at Exxon by Baker and Lasdon (1985) who solved "real world" non-linear problems.

Based on Baker and Lasdon's works, PSLP appears to be a powerful and promising method for solving general non-linear problems, i.e., problems that have non-linearity in both the objective function and the constraints.

Readers interested in greater details are referred to the works of Yeh (1985) and Wurbs, et al. (1985), both of whom have documented excellent state-of-the-art reviews on the general application of engineering techniques to the planning and management of water resources systems.

CHAPTER 3

THEORY AND PROBLEM FORMULATION

Theoretical development and problem formulation in this work will be based on a watershed that has an existing multi-reservoir system. The operation of this system is optimized to maximize the total net benefit resulting from planting different kinds of crops, over a time horizon of T periods, each of one month length, with known unregulated flows $q_{t,n}$ and initial reservoir content $S_{1,n}$. Here, the subscript n refers to reservoir number and t to the time period. This general problem has a total of N reservoirs with no restriction on their layout. This implies that the reservoirs can be in series or parallel. Figure (3.1) shows the input and output components of a typical multi-reservoir system for which the present problem formulation is applicable. The output components of reservoir n are: spilling from reservoir n to downstream, SP_n, total release from reservoir n to the agricultural lands associated with reservoir n, R_n, and the loss/gain of water through evaporation, rain, and infiltration, if any, from reservoir n. Whereas, the input component of reservoir n which is the total regulated inflows into reservoir n, Q_n, is the sum of the unregulated flows from tributaries to reservoir n, q_n, and the spilling from one or more directly connected upstream reservoirs.

In modeling the planning of irrigation multi-reservoir systems, the objective is to calculate the amount of water to be released, stored, and/or spilled to another reservoir according to the economic value. For agricultural purposes, the irrigation multi-reservoir system planning policy should reflect the economic value of stored versus released water in all reservoirs. A planning model has been developed to estimate crop yields, transform yields into net benefits, and maximize the economic value of available water. For a single kind of crop this can be done by developing a model with the objective of maximizing the total crop yield for the areas covered by all the reservoirs, subject to physical constraints such as mass balance, target demands and storage limits. For a multiple crop case, however, the objective is to maximize the total net benefit, B, which reflects the different unit values of crops and differ^nt water requirements per unit of crop land. Therefore, fixed costs, F_c, variable unit costs, V_c, and the crop unit price, P_c must be included to figure out the amount of acreage $A_{c,n}$ and the time and quantity of release to each crop. Variables denoted by subscript c refer to a specific crop, c, numbered between 1 and C.

The objective function for a single crop would be:

$$\text{Maximize } B = \sum_{n=1}^{N} \left[Y_{c,n} \left(P_c - V_c \right) - F_c \right] A_{c,n} \tag{3.1}$$

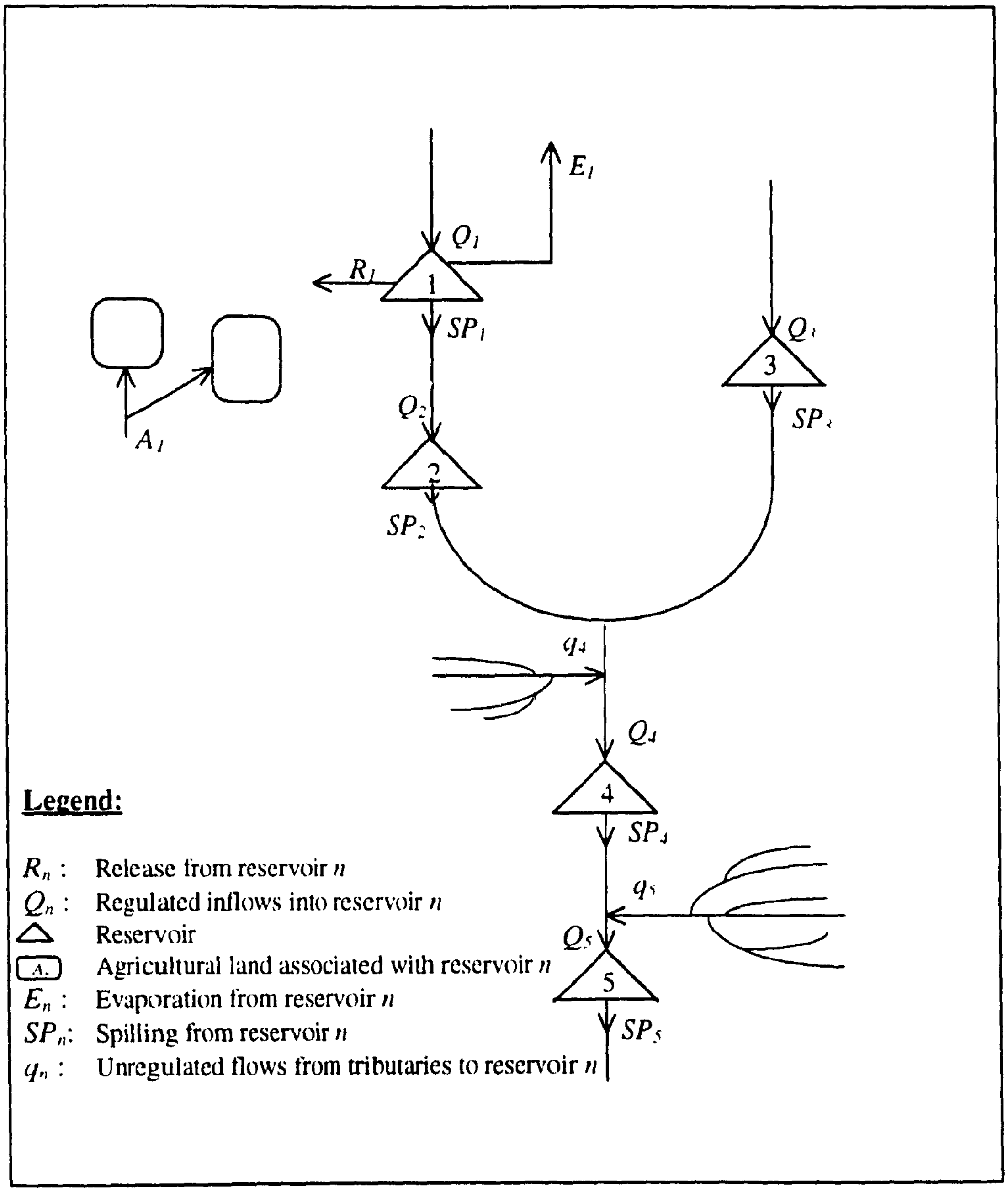

Figure (3.1) Input and Output Components of a Typical Multi-Reservoir System where, B is the total dollar value of the net benefit resulting from planting a certain kind of crop c.

$Y_{c,n}$ is the predicted total yield of crop c, planted at an area served by reservoir n, in (unit yield/L^2).

1. For grain crops, unit yield is measured in terms of either mass or volume, M or L^3, depending on the crop.
2. For dry matter crops, unit yield is measured in terms of mass, M.

P_c, V_c and F_c are constant associated with crop c. So, if area $A_{c,n}$ is constant, then maximizing the net benefit will be nothing but maximizing the total crop yield.

This formulation is extended to several crops in the next paragraph.

3.1 Objective Function:

The objective of the model is to maximize the net benefit from several crops as described by the following equation:

$$\text{Maximize } B = \sum_{n=1}^{N}\sum_{c=1}^{C}\left[Y_{c,n}\left(P_c - V_c\right) - F_c\right] A_{c,n} \tag{3.2}$$

where, B is the total dollar value of the net benefit resulting from planting C different kinds of crops.

P_c is the unit market price of crop c, in (\$/unit yield).

V_c is the variable unit cost of crop c, in (\$/unit yield).

F_c is the fixed cost of crop c, in (\$/L^2).

$A_{c,n}$ is the planted area of crop c served by reservoir n, in (L^2). $A_{c,n}$ is either prescribed or a variable determined by the optimization process.

Crop prices, variable and fixed costs must be known. They can be obtained by compiling information from the farmers in the region.

3.1.1 Crop Yield:

The two equations (2.1) and (2.2) presented in section 2.1 will be used. Inspection of these two equations reveals that the crop yield is a function of the ratio of actual to potential evapo-transpiration. Dariane and Hughes (1991) have adapted these relations for reservoir management by assuming that the ratio of available water to water demanded for a crop is analogous to the ratio of actual to potential evapo-transpiration by that crop. They recommended the following two equations.

1. for grain yield:

$$Y_{c,n} = Y'_c \prod_{t=1}^{T} \left(\frac{V_{c,n,t}}{D_{c,n,t}} \right)^{\lambda_{c,t}} \tag{3.3}$$

2. for dry matter yield:

$$Y_{c,n} = \sum_{t=1}^{T} Y_{P_{c,t}} \left(\frac{V_{c,n,t}}{D_{c,n,t}} \right) \tag{3.4}$$

where, Y'_c is the potential grain yield for grain crop c, in (M/L^2) or (L^3/L^2).

$Y_{p_{c,t}}$ is the potential dry matter yield for dry matter crop c during month t, in (M/L^2).

$V_{c,n,t}$ is the available water for crop c planted in the area served by reservoir n, during month t, in (L^3).

$D_{c,n,t}$ is the water demanded by crop c planted in the area served by reservoir n, during month t, in (L^3).

$\lambda_{c,t}$ is the relative sensitivity of crop c to water deficit during month t. It is an exponent used to give the relative importance on crop yield of water available versus water demanded for each stage of growth for each crop.

Note that actual evapo-transpiration is not exactly balanced with available water for crops. There are some important factors such as soil moisture content at the beginning of the irrigation season and soil water balance throughout the season that are ignored in this formulation. Moreover the value of the relative sensitivity of the crop to water stress $\lambda_{c,t}$ for equation (3.3) may slightly differ from that of equation (2.1), but since grain yield is not very sensitive to the value of relative sensitivity (Hank, 1974) it seems reasonable to use the same $\lambda_{c,t}$.

The available water for a certain crop, $V_{c,n,t}$, is the water received, by the root system of that crop, from both rainfall and reservoir release as described by the following relationship:

$$V_{c,n,t} = R_{c,n,t}\, e + P_{e_t}\, A_{c,n} \tag{3.5}$$

where, $R_{c,n,t}$ is the release to crop c, from reservoir n, during month t, in (L^3).

e is the total efficiency from the reservoir to the root system. It is the fraction of water released from a reservoir that is absorbed by the root system.

P_{e_t} is the effective rainfall, i.e., the amount of rain absorbed by the root system, during month t, in (L^3).

The crop water demand for a certain crop, $D_{c,n,t}$, is the potential evapo-transpiration multiplied by the area planted of that crop as described by equation (3.6).

$$D_{c,n,t} = U_{c,t}\, A_{c,n} \tag{3.6}$$

where, $U_{c,t}$ is the potential consumptive use or potential evapo-transpiration of crop c, during month t, in (L).

The Soil Conservation Service (SCS, 1970) proposed a simple equation to provide good estimates of potential evapo-transpiration, $U_{c,t}$, for long periods. A period of one month is sufficiently long for its application. In that use, the SCS equation takes the form of equation (3.7).

$$U_{c,t} = k_{c,t}\, F_t \tag{3.7}$$

where, $k_{c,t}$ is the empirical coefficient of the consumptive use for crop c, during month t.

F_t is the monthly consumptive use factor. It can be computed with equation (3.8)

$$F_t = \frac{T_t P_t}{100} \tag{3.8}$$

where, T_t is the mean monthly air temperature, in (°F).

P_t is the monthly percentage of daytime hours.

By substituting equation (3.8) into (3.7), one arrives to the following expression for $U_{c,t}$:

$$U_{c,t} = \frac{k_{c,t} T_t P_t}{100} \tag{3.9}$$

3.2 Constraints:

The following constraints will be considered in this optimization problem: mass balance, release, area and storage constraints.

3.2.1 Mass Balance Equation:

In a reservoir problem, the main factors affecting mass balance, are input, output, and storage. The change in storage of a reservoir is calculated by adding all the inflows coming into it, and subtracting the evaporation and the outflows leaving it. Therefore, the mathematical model for the mass balance of a reservoir is given by:

$$\frac{dS}{dt} = Q - R - E - S_p \tag{3.10}$$

where, $\frac{dS}{dt}$ is the rate of change in the storage of a reservoir.

Q is the rate of inflow into the reservoir.

R is the rate of release from the reservoir.

E is the rate of evaporation from the reservoir.

S_p is the rate of spilling from the reservoir.

Integrating both sides of equation (3.10) between the beginning of period t, $t\,\Delta t$, and the end of period t, $(t+1)\,\Delta t$, yields,

$$\int_{t\Delta t}^{(t+1)\Delta t} dS = \int_{t\Delta t}^{(t+1)\Delta t} \left(Q_{(t)} - R_{(t)} - E_{(t)} - S_{p_{(t)}}\right) dt$$

or,

$$S_{t+1} - S_t = \int_{t\Delta t}^{(t+1)\Delta t} Q_{(t)} dt - \int_{t\Delta t}^{(t+1)\Delta t} R_{(t)} dt - \int_{t\Delta t}^{(t+1)\Delta t} E_{(t)} dt - \int_{t\Delta t}^{(t+1)\Delta t} S_{p_{(t)}} dt \tag{3.11}$$

The rate of flows and evaporation (volume/time) can be converted to volumes during a time period of t, as follows,

$$Q_t = \int_{t\Delta t}^{(t+1)\Delta t} Q_{(t)} dt, \quad R_t = \int_{t\Delta t}^{(t+1)\Delta t} R_{(t)} dt, \quad E_t = \int_{t\Delta t}^{(t+1)\Delta t} E_{(t)} dt, \quad S_p = \int_{t\Delta t}^{(t+1)\Delta t} S_{p_{(t)}} dt \tag{3.12}$$

Substituting equation (3.12) into (3.11) yields the discrete mass balance equation of a reservoir.

$$S_{t+1,n} = S_{t,n} + Q_{t,n} - R_{t,n} - E_{t,n} - S_{p_{t,n}} \tag{3.13}$$

where: $S_{t,n}$ and $S_{t+1,n}$ are the storage volumes of reservoir n at the beginning (t) and end ($t+1$) of the current month t, in (L^3).

$Q_{t,n}$ is the inflow coming into reservoir n during month t, in (L^3). It is described in equation (3.14).

$R_{t,n}$ is the total agricultural water volume released by reservoir n during month t, in (L^3).

$E_{t,n}$ is the volume of evaporation from reservoir n during month, t, in (L^3).

$S_{p_{t,n}}$ is the volume of water spilled by reservoir n, during month, t, in (L^3).

The inflow coming into reservoir n during month, t, is the sum of the unregulated flows and spills (from upstream reservoirs) into that reservoir.

$$Q_{t,n} = q_{t,n} + \left[M\,S_{p_t}\right]_n \tag{3.14}$$

where, $q_{t,n}$ is the sum of unregulated or independent flow volumes between reservoir n and the reservoirs immediately upstream during month t, in (L^3).

$\left[M\,S_{p_t}\right]_n$ is the total volume of water spilled into reservoir n. One or more reservoirs can contribute to the spilled volume.

M is a routing matrix described later.

For the multi-reservoir system, S, Q, R, E, S_p and q for a specific time period t, are vectors of size N, where N is the number of reservoirs. The volumes of water spilled into reservoir n is the sum of water spilled by directly-connected upstream reservoirs

A reservoir can receive inflows from one ore more directly-connected upstream reservoirs. Figure (3.1) shows reservoir No. 4 receiving inflows from two directly-connected upstream reservoirs, No. 2 and No. 3. A routing matrix M is used to account for the flow of water from upstream to downstream reservoirs and therefore is constructed to reflect the configuration of the system. For a system with N reservoirs, the routing matrix is a square matrix of size N with elements, 0 or 1. The matrix element S_{p_i} has a numerical value of 1 when reservoir i receives the outflow of reservoir j, or 0 if the two reservoirs are not directly-connected.

Equation (3.14) assumes that there is instantaneous equilibrium between reservoirs. If the times of travel of flows between reservoirs are small compared to the time period used, this assumption is justified. In general, time-discretization periods of one month are used. In such cases, travel times are much shorter than the time period used, and hence the above assumption is valid.

The routing matrix, M, corresponding to the typical multi-reservoir system shown in Figure (3.1) is,

$$M = \begin{bmatrix} S_{P_{1,1}} & S_{P_{1,2}} & S_{P_{1,3}} & S_{P_{1,4}} & S_{P_{1,5}} \\ S_{P_{2,1}} & S_{P_{2,2}} & S_{P_{2,3}} & S_{P_{2,4}} & S_{P_{2,5}} \\ S_{P_{3,1}} & S_{P_{3,2}} & S_{P_{3,3}} & S_{P_{3,4}} & S_{P_{3,5}} \\ S_{P_{4,1}} & S_{P_{4,2}} & S_{P_{4,3}} & S_{P_{4,4}} & S_{P_{4,5}} \\ S_{P_{5,1}} & S_{P_{5,2}} & S_{P_{5,3}} & S_{P_{5,4}} & S_{P_{5,5}} \end{bmatrix}_{5\times5} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{5\times5}$$

In equation (3.14), $Q_{t,n}$, consists of the sum of unregulated flows between reservoir n and the reservoirs immediately upstream, and the total volume of water spilled into reservoir n. $Q_{t,n}$, which is used in the mass balance equation (Eq. 3.13), can be multiplied by a factor less than one to include the channel seepage loss, if any, between any two directly-connected reservoirs. This factor can either be obtained from previous measurements of channel seepage between each two directly-connected reservoirs, or, it can be estimated from an empirical coefficient that gives the seepage per unit length of channel. This coefficient depends on channel bed texture.

The evaporation rate from a reservoir at time t, can be calculated with the following equation,

$$E_{(t)} = a_{(t)}\, L_{(t)} \tag{3.15}$$

where, $a_{(t)}$ and $L_{(t)}$ are the reservoir's surface area and normal lake evaporation rate, at time t.

The reservoir's surface area at time t, $a_{(t)}$ is calculated by,

$$a_t = \rho\, S_{(t)}^{z} \tag{3.16}$$

where, $S_{(t)}$ is the reservoir storage at time t.

ρ and z are coefficients describing the geometry of the reservoir. They can be obtained by fitting the reservoir's surface area versus reservoir storage relationship to available data. See Figure (3.2).

Substitution of equation (3.16) into (3.15), and substitution of the resulting equation into equation (3.12) yield,

$$E_t = \int_{t\Delta t}^{(t+1)\Delta t} E_{(t)}\, dt = \int_{t\Delta t}^{(t+1)\Delta t} a_{(t)}\, L_{(t)}\, dt = \int_{t\Delta t}^{(t+1)\Delta t} L_{(t)}\, \rho\, S_{(t)}^{z}\, dt \tag{3.17}$$

The evaporation rate $L_{(t)}$ is only available as a monthly average L_t. Since the evaporation rate does not vary considerably during one-month period, in equation (3.17) $L_{(t)}$ can be replaced by its average over the period without significantly affecting the results.

$$E_t = L_t\, \rho \int_{t\Delta t}^{(t+1)\Delta t} S_{(t)}^{z}\, dt \tag{3.18}$$

Since the reservoir storage can only be known at the beginning and end of each time period of study, an approximating rule had to be applied, like the trapezoidal rule. That is, the average surface area of reservoir n during month t, can be approximated by assuming that the reservoir's storage during month t is equal to the average of storage at the beginning of a current month t and the end of that month, as shown in Figure (3.2).

$$a_{t,n} = \rho_n \left[\frac{S_{t,n} + S_{t+1,n}}{2} \right]^{z_n} \qquad (3.19)$$

where, $\frac{S_{t,n} + S_{t+1,n}}{2}$ is the average reservoir storage during period t.

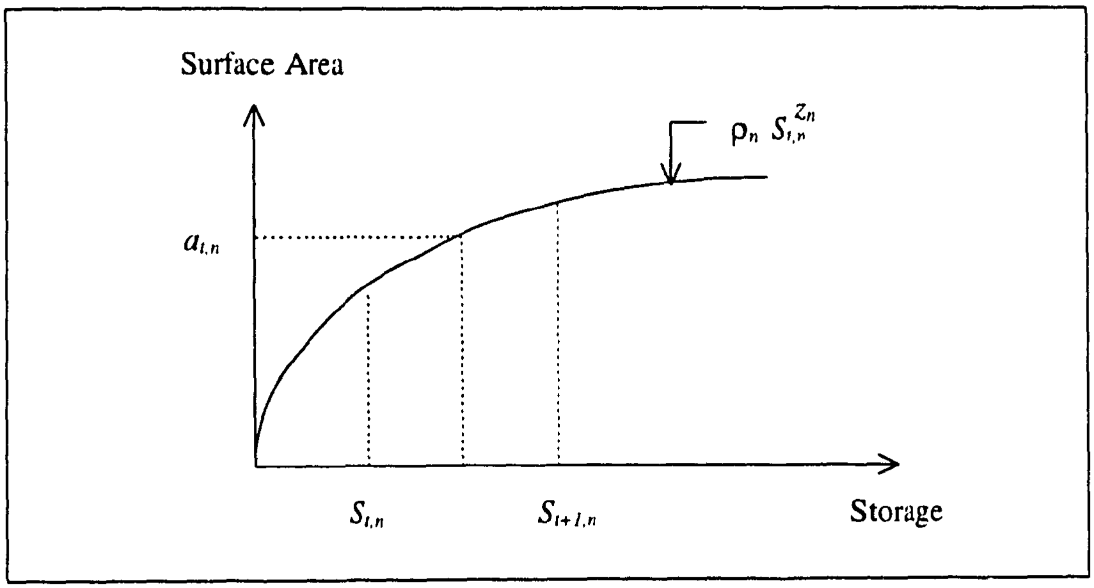

Figure (3.2) Reservoir Surface Area vs. Storage Relationship

Finally, the evaporation volume from reservoir n during period t can be expressed as,

$$E_{t,n} = a_{t,n} \, L_t \qquad (3.20)$$

where, $a_{t,n}$ is the average surface area of reservoir n, during month t, in (L^2).

L_t is the normal lake evaporation during month t, in (L).

3.2.2 Release:

Equation (3.21) describes the total release to all crops from reservoir n during month t.

$$R_{t,n} = \sum_{c=1}^{C} R_{c,n,t} \tag{3.21}$$

This release is physically limited by upper and lower bounds. The upper bound depends on the maximum channel flow rate, which in turn depends on the channel design. The lower bound is either zero or a small flow rate, sufficient to guarantee that the channel remains clear of debris.

$$RMIN_{t,n} \leq R_{t,n} \leq RMAX_{t,n} \tag{3.22}$$

where, $RMIN_{t,n}$ and $RMAX_{t,n}$ are the upper and lower bounds on release for reservoir n.

Furthermore, the available water for crop c should never exceed the water demand by that crop:

$$V_{c,n,t} \leq D_{c,n,t} \tag{3.23}$$

Substituting equations (3.5) and (3.6) into (3.23) and making some rearrangements yield the following expression for releases:

$$R_{c,n,t} \leq \left[U_{c,t} - P_{e_t}\right]\frac{A_{c,n}}{e} \tag{3.24}$$

Under limited water supply conditions, an additional constraint (equation 3.25) must be added to guarantee that all the crops will at least receive the minimum water requirements during all the periods of the growing season.

$$R_{c,n,t} \geq \zeta_c \left[U_{c,t} - P_{e_t}\right]\frac{A_{c,n}}{e} \tag{3.25}$$

where, ζ_c is the minimum percentage of the full crop water requirements to allow the crop to be produced throughout the season.

3.2.3 Area:

The total area to be irrigated by each reservoir can be either specified or calculated for each crop. At the beginning of each planting season, it is best to specify the total area that each reservoir is responsible for, as an upper bound and let the model allocate it among the crops.

$$\sum_{c=1}^{C} A_{c,n} \leq ATOT_n \tag{3.26}$$

where, $ATOT_n$ is the total area (planted or not) under the responsibility of reservoir n, in (L^2).

The model allocates all of the unconstrained areas to the crop with the highest net revenue. However, this ignores that some farmers may want to plant some crops in which they have special interest. Equation (3.27) provides a means to accommodate this situation.

$$\alpha_{\phi,c} \geq \mu_{\phi,c}\,\alpha_{\phi} \quad \text{for } \phi \in [\Phi_{n-1}+1, \Phi_n] \text{ and } n \in [1, N] \tag{3.27}$$

where, the farmers are enumerated starting from the first reservoir to the last. Φ_n represents the number of farmers enumerated in reservoir 1 to n. Since no farmer is enumerated before reservoir 1, $\Phi_0 = 0$.

$\mu_{\phi,c}$ is the minimum fraction of the total area owned by farmer ϕ that must be allocated for crop c.

$\alpha_{\phi,c}$ is the minimum area of crop c that farmer ϕ wants to plant.

α_{ϕ} is the total area, planted or not, owned by farmer ϕ.

The planted area of crop c, served by reservoir n, is equal to the sum of areas planted with crop c, by each farmer whose land is irrigated by reservoir n.

$$A_{c,n} = \sum_{\phi=\Phi_{n-1}+1}^{\Phi_n} \alpha_{\phi,c} \quad \text{for } \phi \in [\Phi_{n-1}+1, \Phi_n] \text{ and } n \in [1, N] \tag{3.28}$$

Using new variables for farmers will dramatically increase the number of variables and constraints. This could make the computer program much slower. Therefore, an alternative solution, with less farmer variables, was sought. This alternative solution states that the minimum area requirement for crop c, served by reservoir n, should at least be equal to the sum of the minimum area requirements, for crop c, of each of the farmers whose land is irrigated by reservoir n. Therefore, equations (3.27) and (3.28) can be replaced by the following expression,

$$A_{c,n} \geq \sum_{\phi=\Phi_{n-1}+1}^{\Phi_n} \mu_{\phi,c}\,\alpha_\phi \qquad \text{for } \phi \in [\Phi_{n-1}+1, \Phi_n] \text{ and } n \in [1,N] \tag{3.29}$$

The total number of variables and constraints for both equations (3.27) and (3.28) is (CxN + ϕxC) whereas the number of variables and constraints for equation (3.29) is (CxN). This means that the above alternative solution reduces the number of variables by ϕxC variables and also reduces the number of constraints by ϕxC constraints.

3.2.4 Storage:

Maximum capacity and minimum dead storage of each reservoir must be specified to determine storage limits.

$$SMIN_{t,n} \leq S_{t,n} \leq SMAX_{t,n} \tag{3.30}$$

where, $SMIN_{t,n}$ and $SMAX_{t,n}$ are the upper and lower bounds on the storage of reservoir n.

3.3 Summary of the Planning Model:

The model presented above can be summarized with the following equations.

Objective:

$$\text{Maximize B} = \sum_{n=1}^{N}\sum_{c=1}^{C}\left[Y_{c,n}(P_c - V_c) - F_c\right]A_{c,n} \tag{3.2}$$

where,

1. for grain yield:

$$Y_{c,n} = Y'_c \prod_{t=1}^{T}\left(\frac{V_{c,n,t}}{D_{c,n,t}}\right)^{\lambda_{c,t}} \tag{3.3}$$

2. for dry matter yield:

$$Y_{c,n} = \sum_{t=1}^{T} Y_{P_{c,t}}\left(\frac{V_{c,n,t}}{D_{c,n,t}}\right) \tag{3.4}$$

Subject to:

Mass Balance:

$$S_{t+1\,n} = S_{t,n} + Q_{t,n} - R_{t,n} - E_{t,n} - S_{P_{t,n}} \tag{3.13}$$

Release:

$$R_{t,n} = \sum_{c=1}^{C} R_{c,n,t} \tag{3.21}$$

$$RMIN_{t,n} \leq R_{t,n} \leq RMAX_{t,n} \tag{3.22}$$

$$R_{c,n,t} \leq \left(U_{c,t} - P_{e_t}\right)\frac{A_{c,n}}{e} \tag{3.24}$$

$$R_{c,n,t} \geq \zeta_c \left(U_{c,t} - P_{e_t}\right)\frac{A_{c,n}}{e} \tag{3.25}$$

Area:

$$\sum_{c=1}^{C} A_{c,n} \leq ATOT_n \tag{3.26}$$

$$A_{c,n} \geq \sum_{\phi=\Phi_{n-1}+1}^{\Phi_n} \mu_{\phi,c}\,\alpha_\phi \tag{3.29}$$

Storage:

$$SMIN_{t,n} \leq S_{t,n} \leq SMAX_{t,n} \tag{3.30}$$

Inspection of the model summary reveals that the objective function is non-linear in the yield function for grain crops. In addition, there is a non-linear constraint, the mass balance equation (Eq. 3.13) because of the non-linear evaporation term. All other terms in both the objective function and the constraints are linear. To circumvent the non-linearity of the problem, the penalty successive linear programming (PSLP) method was selected because of its simplicity and for the reasons presented in Chapter 2.

3.4 Updating the Model:

The model consists of two planning modes. The first mode is to run the model at the start of the growing season and the second mode is to update the model at the beginning of each month with recently collected data, for the reasons explained in subsequent paragraphs.

Designers can improve the predictive performance of the model by updating it at the beginning of each month with data that has become available. Before the start of each growing season the model should be run to apportion the area to each crop from the agricultural areas associated with each reservoir and to predict the approximate quantities and times, of releases to the cultivated lands, and spilling to other reservoirs. To figure out the final estimated yields of the crops for the different areas planted, during the season the model should be updated at the beginning of each month in order to adjust the storage in each reservoir. Updating does not necessarily mean an increase in the net benefit but rather helps to improve predictions and makes the adjustments that are necessary.

In the updating mode, the optimal crop areas calculated in the first mode must be considered to be known and fixed throughout the growing season. However, the level of reservoir storage must be adjusted, since there may be some difference between calculated and actual storage levels. The expected difference may be partially due to deviations from expected climatic factors, such as levels of precipitation and evaporation; or could be due to the difference between actual and estimated inflows and releases.

3.5 Application of Successive Linear Programming:

The application of Successive Linear Programming to the model is presented in the following section.

3.5.1 Application of PSLP to the Model:

Based on the literature review presented in Chapter 2, PSLP appears to be a promising method for solving this multi-reservoir optimization problem. However, first some transformations should be made to the objective function to bring to a form suitable to the PSLP method.

Substituting equations (3.5) and (3.6) into equations (3.3) and (3.4) yields the following expressions for grain and dry matter yield,

1. For grain yield:

$$Y_{c,n} = Y_c' \prod_{t=1}^{T} \left(\frac{R_{c,n,t}\, e + P_{e_t} A_{c,n}}{U_{c,t} A_{c,n}} \right)^{\lambda_{c,t}} \tag{3.31}$$

2. For dry matter yield:

$$Y_{c,n} = \sum_{t=1}^{T} Y_{p_{c,t}} \left(\frac{R_{c,n,t}\, e + P_{e_t} A_{c,n}}{U_{c,t} A_{c,n}} \right) \tag{3.32}$$

Substituting equations (3.31) and (3.32) into equation (3.2) and numbering grain yield crops from 1 to C_1 and dry matter crops from C_1+1 to C yield the following objective function formulation,

$$B = \sum_{n=1}^{N} \sum_{c=1}^{C_1} \left[Y_c' (P_c - V_c) \prod_{t=1}^{T} \left(\frac{R_{c,n,t}\, e + P_{e_t} A_{c,n}}{U_{c,t} A_{c,n}} \right)^{\lambda_{c,t}} - F_c \right] A_{c,n}$$

$$+\sum_{n=1}^{N}\sum_{c=C_1+1}^{C}\left[(P_c - V_c)\sum_{t=1}^{T} Y_{p_{c,t}}\left(\frac{R_{c,n,t}\,e + P_{e_t}\,A_{c,n}}{U_{c,t}}\right) - F_c\,A_{c,n}\right] \tag{3.33}$$

Equation (3.33) consists of a first part, which is non-linear, and a second part, which is linear. This equation's independent variables are $R_{c,n,t}$ and $A_{c,n}$; all other parameters should be known and must be specified. Therefore equation (3.33) can be characterized as a sum of functions involving the unknowns, $R_{c,n,t}$ and $A_{c,n}$.

$$B = \sum_{n=1}^{N}\sum_{c=1}^{C_1}\delta_{1_{c,n}}(R_{c,n,t}, A_{c,n}) + \sum_{n=1}^{N}\sum_{c=C_1+1}^{C}\delta_{2_{c,n}}(R_{c,n,t}, A_{c,n}) \tag{3.34}$$

where, $\delta_{1_{c,n}}(R_{c,n,t}, A_{c,n}) = \left[Y_t'\,(P_c - V_c)\prod_{t=1}^{T}\left(\frac{R_{c,n,t}\,e + P_{e_t}\,A_{c,n}}{U_{c,t}\,A_{c,n}}\right)^{A_{c,t}} - F_c\right]A_{c,n}$

and $\delta_{2_{c,n}}(R_{c,n,t}, A_{c,n}) = (P_c - V_c)\sum_{t=1}^{T} Y_{p_{c,t}}\left(\frac{R_{c,n,t}\,e + P_{e_t}\,A_{c,n}}{U_{c,t}}\right) - F_c\,A_{c,n}$

All the non-linear constraints in this model (the mass balance equation 3.13) are multiplied by positive large values, $W_{t,n}$, and subtracted from the objective function (Eq. 3.34) to yield the exact penalty function, p, below:

$$p = \sum_{n=1}^{N}\sum_{c=1}^{C_1}\delta_{1_{c,n}}(R_{c,n,t}, A_{c,n}) + \sum_{n=1}^{N}\sum_{c=C_1+1}^{C}\delta_{2_{c,n}}(R_{c,n,t}, A_{c,n}) - \sum_{n=1}^{N}\sum_{t=1}^{T} W_{t,n}\left|h_{t,n}\right| \tag{3.35}$$

where, $h_{t,n} = E_{t,n} + S_{t+1,n} - S_{t,n} - Q_{t,n} + R_{t,n} + S_{p_{t,n}}$

Hence, the linearly constrained penalty problem (LCP) maximizes p subject to linear constraints in the feasible solution space, F.

$F = \{ S_{t+1,n}, R_{t,n}, S_{p_{t,n}} \mid h_{t,n} = 0$, for $t \in [1,T]$, $n \in [1,N]$ and subject to all the linear constraints$\}$.

PSLP attempts to solve the above LCP problem by first linearizing it using a first order Taylor series expansion for the non-linear variables, about a base vector ($R^0_{c,n,t}$, $A^0_{c,n}$, $S^0_{t+1,n}$). In this case, the vector is made of C_1xNxT values of $R^0_{c,n,t}$, C_1xN values of $A^0_{c,n}$ and TxN values of $S^0_{t+1,n}$. Note that $\delta_{1_{c,n}}$ depends only on $R_{c,n,t}$ and $A_{c,n}$ that R and A have the same c and n indices. Application of the PSLP method to equation (3.35) yields the following linear approximation of the penalty function, p_l .

$$p_l = \sum_{n=1}^{N}\sum_{c=1}^{C_1}\left[\delta_{1_{c,n}}\left(R^0_{c,n,t}, A^0_{c,n}\right) + \sum_{t=1}^{T} b_{c,n,t}\, d_{1_{c,n,t}} + m_{c,n}\, d_{2_{c,n}}\right] + \sum_{n=1}^{N}\sum_{c=C_1+1}^{C} \delta_{2_{c,n}}\left(R_{c,n,t}, A_{c,n}\right) - p_{l_1}$$

$$\begin{aligned} p_{l_1} &= \sum_{n=1}^{N} W_{1,n}\left|E_{1,n} + S^0_{2,n} - S_{1,n} - Q_{1,n} + R_{1,n} + S_{P_{1,n}} + \Psi_{2,n} d_{3_{2,n}}\right| \\ &+ \sum_{n=1}^{N}\sum_{t=2}^{T} W_{t,n}\left|E_{t,n} + S^0_{t+1,n} - S^0_{t,n} - Q_{t,n} + R_{t,n} + S_{P_{t,n}} + \Psi_{t+1,n} d_{3_{t+1,n}} + \Psi_{t,n} d_{3_{t,n}}\right| \end{aligned} \tag{3.36}$$

where,

$$b_{c,n,t} = \left.\frac{\partial \delta_{1_{c,n}}\left(R_{c,n,t}, A_{c,n}\right)}{\partial R_{c,n,t}}\right|_{R_{c,n,t}=R^0_{c,n,t}, A_{c,n}=A^0_{c,n}} \qquad m_{c,n} = \left.\frac{\partial \delta_{1_{c,n}}\left(R_{c,n,t}, A_{c,n}\right)}{\partial A_{c,n}}\right|_{R_{c,n,t}=R^0_{c,n,t}, A_{c,n}=A^0_{c,n}}$$

$$\Psi_{2,n} = \left.\frac{\partial h_{1,n}}{\partial S_{2,n}}\right|_{S_{2,n}=S^0_{2,n}} \qquad \Psi_{t+1,n} = \left.\frac{\partial h_{t,n}}{\partial S_{t+1,n}}\right|_{S_{t+1,n}=S^0_{t+1,n}, S_{t,n}=S^0_{t,n}}$$

$$\Psi_{t,n} = \left.\frac{\partial h_{t,n}}{\partial S_{t,n}}\right|_{S_{t+1,n}=S^0_{t+1,n},\, S_{t,n}=S^0_{t,n}} \qquad d_{1_{c,n,t}} = R_{c,n,t} - R^0_{c,n,t}$$

$$d_{2_{c,n}} = A_{c,n} - A^0_{c,n} \qquad d_{3_{t,n}} = S_{t,n} - S^0_{t,n}$$

$$E_{1,n} = E_{1,n}(S^0_{2,n}) \qquad E_{t,n} = E_{t,n}(S^0_{t+1,n}, S^0_{t,n})$$

d_i's represent the change in the non-linear variables between any two consecutive iterations.

and $S_{1,n}$ is the known initial reservoir storage.

Since the non-linear parts are continuously differentiable, p_l is a good approximation to p if d_i's are not large. Thus p can be maximized by a sequence of maximization of p_l, with upper bounds on d_i's. This leads to the approximating problem:

Problem LP:

Maximize p_l

subject to:

1. all the linear constraints.

2. $-\beta_i \le d_i \le \beta_i$ *for i = number of types of non-linear variables.*

The mass balance equation (equation 3.13) can be written as:

$$E_{t,n} + S_{t+1,n} - S_{t,n} - Q_{t,n} + R_{t,n} + S_{p_{t,n}} = 0 \tag{3.37}$$

The first order Taylor series approximations of equation (3.37) is applied to the non-linear variables.

For convenience, new independent positive variables n and p are introduced. ($n_{t,n}$-$p_{t,n}$) is added to the left side of equation (3.37).

$$p_{1,n} - n_{1,n} = E_{1,n} + S^0_{2,n} - S_{1,n} - Q_{1,n} + R_{1,n} + S_{P_{1,n}} + \Psi_{2,n} d_{3_{2,n}} \qquad \text{if } t = 1 \quad (3.38)$$

$$p_{t,n} - n_{t,n} = E_{t,n} + S^0_{t+1,n} - S^0_{t,n} - Q_{t,n} + R_{t,n} + S_{P_{t,n}} + \Psi_{t+1,n} d_{3_{t+1,n}} + \Psi_{t,n} d_{3_{t,n}} \text{ if } t > 1 \quad (3.39)$$

Later in this section, it will be shown that the objective function of the linearized planning model (equation 3.42) minimizes the values of $n_{t,n}$'s and $p_{t,n}$'s variables.

The right hand side of constraints (3.38) and (3.39), which are the only constraints that involve the $n_{t,n}$'s and $p_{t,n}$'s variables, can either be: 1) zero 2) a positive value or 3) a negative value.

Case 1: If the right hand side of equations (3.38) and (3.39) has a zero value, then, $p_{t,n} - n_{t,n} = 0$. Even though this equation has an infinite number of solutions, the optimization planning model which minimizes the values of ($n_{t,n}$'s + $p_{t,n}$'s) variables will lead to the following unique solution for $n_{t,n}$ and $p_{t,n}$.

$$p_{t,n} = n_{t,n} = 0$$

Case 2: If the right hand side of equations (3.38) and (3.39) has a positive value, r, then, $p_{t,n} - n_{t,n} = r$ or $p_{t,n} = n_{t,n} + r$. Since the optimization model minimizes $p_{t,n} + n_{t,n}$ which is equal to $2n_{t,n} + r$, it can be seen that the minimum will occur when $n_{t,n}$ is minimum and $p_{t,n}$ is equal to r:

$$p_{t,n} = r \text{ and } n_{t,n} = 0$$

Case 3: If the right hand side of equations (3.38) and (3.39) has a negative value, $-r$, then, $p_{t,n} - n_{t,n} = -r$. By applying the same procedure described in case 2, it can be shown that, the optimization planning model will have the following solution.

$$p_{t,n} = 0 \text{ and } n_{t,n} = r$$

In all the above cases, either $n_{t,n}$, $p_{t,n}$ or both will be zero, and thus $(p_{t,n})(n_{t,n}) = 0$.

Providing that, $(p_{t,n})(n_{t,n}) = 0$, and taking the absolute value of both sides of equations (3.38) and (3.39) the following can be written:

$$\left|\Psi_{2,n} d_{3_{2,n}} + E_{1,n} + S''_{2,n} - S_{1,n} - Q_{1,n} + R_{1,n} + S_{P_{1,n}}\right| = p_{1,n} + n_{1,n} \quad \text{if } t=1 \quad (3.40)$$

$$\left|\Psi_{t+1,n} d_{3_{t+1,n}} + \Psi_{t,n} d_{3_{t,n}} + E_{t,n} + S''_{t+1,n} - S''_{t,n} - Q_{t,n} + R_{t,n} + S_{P_{t,n}}\right| = p_{t,n} + n_{t,n} \quad \text{if } t>1 \quad (3.41)$$

Substituting equations (3.40) and (3.41) into (3.36) and omitting the constant terms (which have no effect on the optimum search) from equation (3.3) yield the linearized planning model (LP):

$$\text{Maximize } LP = \sum_{n=1}^{N} \sum_{c=1}^{C_1} \sum_{t=1}^{T} b_{c,n,t} R_{c,n,t} + \sum_{n=1}^{N} \sum_{c=1}^{C_1} n_{c,n} A_{c,n} + \sum_{n=1}^{N} \sum_{c=C_1+1}^{C} \delta_{2_{c,n}} (R_{c,n,t}, A_{c,n})$$

$$- \sum_{n=1} \sum_{t=1}^{T} W_{t,n} (p_{t,n} + n_{t,n}) \quad (3.42)$$

subject to all the linear constraints in the planning model, to the linearized constraints (3.38) and (3.39), and to the following constraints, which improve convergence:

$$-\beta 1 + R^0_{c,n,t} \le R_{c,n,t} \le \beta 1 + R^0_{c,n,t} \qquad \text{for } c \in [1, C_1] \tag{3.43}$$

$$-\beta 2 + A^0_{c,n} \le A_{c,n} \le \beta 2 + A^0_{c,n} \qquad \text{for } c \in [1, C_1] \tag{3.44}$$

$$-\beta 3 + S^0_{t+1,n} \le S_{t+1,n} \le \beta 3 + S^0_{t+1,n} \qquad \text{for } c \in [1, C_1] \tag{3.45}$$

3.5.2 The PSLP Algorithm:

The penalty successive linear programming algorithm works as follows:

1. Select initial values for the variables, x^0, satisfying all the linear constraints.

   Select positive $\beta 1$, $\beta 2$, $\beta 3$, β' and a vector of positive weights $W_{t,n}$.

   Choose scalars $0<\rho_0<\rho_1<\rho_2<1$ and m>1.

   Set $k = 0$.

2. Solve $LP(x^k, \beta^k)$ to obtain an optimal solution x^{k+1}.

3. Compute the actual change in the exact penalty function:

$$\Delta p_k = p(x^{k+1}) - p(x^k)$$

   and the predicted change by its piecewise linear approximation, p_l:

$$\Delta p_{l_k} = p_l(x^{k+1}) - p_l(x^k)$$

note that $p_l(x^k) = p(x^k)$.

If any one of the following criteria occurs the program terminates:

a. $|\mathrm{obj}(x^{k+1})\text{-}\mathrm{obj}(x^k)| < \varepsilon_1(1+|\mathrm{obj}(x^k)|)$,

b. $|\mathrm{p}(x^{k+1})\text{-}\mathrm{p}(x^k)| < \varepsilon_1(1+|\mathrm{p}(x^k)|)$,

c. $|x^{k+1}\text{-}x^k| < \varepsilon_2(1+x^k)$.

Where, obj and p are the objective function value and the exact penalty function value respectively.

$k+1$ and k indicate the current and previous iterate.

Conditions (a) and (b) must be satisfied for three consecutive iterations in order to terminate the algorithm.

If none of the above criteria takes place then, compute the ratio of actual to predicted change:

$$r_k = \frac{\Delta p_k}{\Delta p_{l_k}}$$

4. If $r_k < \rho_0$, then replace β^k with $\dfrac{\beta^k}{m_2}$, go back to step 2; otherwise update β^{k+1} by:

$$\beta^{k+1} = \begin{cases} \beta^k m_1 & \text{if} \begin{cases} i) \ |1 - r_k| < \rho_1 \\ ii) \ |x^{k+1} - x^k| = \beta^k \end{cases} \\ \\ \dfrac{\beta^k}{m_2} & \text{if } |1 - r_k| > \rho_2 \\ \\ \beta^k & \text{otherwise.} \end{cases}$$

Condition (*ii*) must be satisfied, for any variable, for three consecutive iterations.

Replace β^{k+1} with the maximum of β^{k+1} and $\beta^{'}$.

Replace k by k+1 and return to step 2.

This algorithm is of the trust region or restricted step type, which is defined by the constraints (3.43), (3.44) and (3.45). The ratio r_k is used to judge if β^k is of a proper size.

Theoretically, ρ_0 and $\beta^{'}$ can be arbitrary small positive numbers, and the algorithm is not sensitive to their precise value.

Figure (3.3) represents a simplified flow chart of the PSLP algorithm.

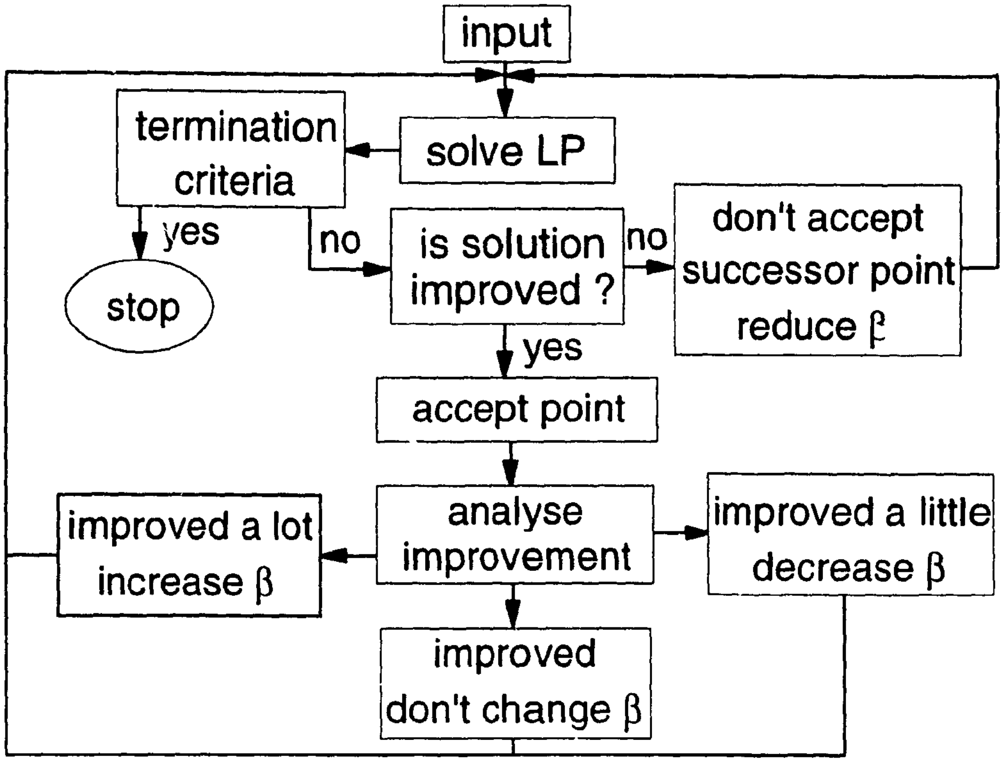

Figure (3.3) Flow Chart of the PSLP Algorithm

A computer code for the PSLP algorithm described above was written using ANSI FORTRAN 77 and it included a Simplex subroutine adapted from Press et. al. (1992). The mathematical formulation of the planing model explained in Chapter 3 was implemented in a computer program. The resulting MAXCROP model was validated and then applied to a realistic problem.

CHAPTER 4

CODE AND MODEL VALIDATION

Program validation is an integral part of software development. Without it, it is impossible to make any judgment about the validity of the results produced by the program. It is therefore crucial that a significant verification of the model be undertaken before making any conclusion about the quality of its results, and its usefulness.

The following presents the test that were performed with MAXCROP model, described in Chapter 3.

4.1 Tests:

The computer code was tested against non-linear problems published in the optimization literature. Problems resembling the mathematical formulation of the MAXCROP model were chosen. All the tested problems gave the best obtained objective values. Two non-linear problems (problem No. 1 and problem No. 2) will be discussed in subsequent paragraphs.

4.1.1 Non-linear Problem No. 1:

Non-linear problem No. 1 corresponds to problem No. 40 of Hock and Schittkowski (1981). The objective function of this problem is a simple product of the variables, x_1, x_2, x_3 and x_4 :

$$\text{MIN } f(x) = -x_1 x_2 x_3 x_4 \tag{4.1}$$

subject to:

$$x_1^3 + x_2^2 - 1 = 0 \tag{4.2}$$

$$x_1^2 x_4 - x_3 = 0 \tag{4.3}$$

$$x_4^2 - x_2 = 0 \tag{4.4}$$

Since the computer code maximizes objective functions, the minimization problem is transformed into a maximization problem, simply by changing the sign of the objective function. Therefore, equation (4.1) should be replaced by:

$$\text{MAX } f(x) = x_1 x_2 x_3 x_4 \tag{4.5}$$

subject to the above constraints.

This problem was chosen because, the objective function of the MAXCROP model contains a non-linear multiplication term $\prod_{t=1}^{T}\left(\frac{R_{c,n,t}\, e + P_{e_t} A_{c,n}}{U_{c,t} A_{c,n}}\right)^{\lambda_{c,t}}$ which is more complex but still resembles that of problem No. 1 (shown in equation 4.5), and because the mass balance equation in the MAXCROP model (equation 3.13) is a non-linear equality constraint where its non-linearity is in the storage term raised to the power z (s^z), which resembles constraints (4.2), (4.3) and (4.4) of problem 1.

Note that under the following special conditions, the objective function of the MAXCROP model becomes identical to that of problem No. 1 (equation 4.5).

1. the watershed consists of one reservoir, only one grain crop will be planted and the relative sensitivity of that crop to water stress, $\lambda_{c,t}$, is equal to one;
2. the growing season consists of four periods;
3. the fixed costs of the crop is very small compared to the variable unit costs and unit price of that crop;
4. the effect of rainfall on crop growth during the growing season is neglected.

Under the above conditions, the objective function of the MAXCROP model becomes, MAX $B = \sum_{t=1}^{4} c_t R_{c,n,t}$, where $c_t = \dfrac{Y_c'(P_c - V_c)e}{U_{c,t}}$ = constant for each t, which is identical to equation (4.5) where, $x_t = c_t\ R_{c,n,t}$ for t = 1,4.

The same initial values given in Hock and Schittkowski (1981) to each variable were used: $x^0_1 = 0.8$, $x^0_2 = 0.8$, $x^0_3 = 0.8$, $x^0_4 = 0.8$.

Hock and Schittkowski reported the best known solution, of the studied non-linear problems, obtained by applying different optimization procedures.

A comparison of the results is shown in Table (4.1), where the relative difference is equal to the difference between MAXCROP and the best known solution divided by the best known solution. It can be observed in Table (4.1) that the maximum absolute relative difference is $5x10^{-5}$.

Table (4.1) A Comparison of MAXCROP and Best Known Solution for Test No. 1

Objective Function Variables	MAXCROP solution	Best Known Solution	Relative Difference
x_1	0.79367507	0.79370056	-0.00003
x_2	0.70714074	0.70710676	0.00005
x_3	0.52971023	0.52973155	-0.00004
x_4	0.84091663	0.84089643	0.00002
$f(x)$	0.25000000	0.25000000	0.00000

4.1.2 Non-linear Problem No. 2:

Non-linear problem No. 2 corresponds to problem No. 86 of Hock and Schittkowski (1981). This problem was selected because it resembles the objective function of the MAXCROP model (equation 3.31) which consists of two parts, the first part contains summations of multiplications of non-linear variables and the second part contains summations of linear variables.

Problem No. 2 can be stated as:

$$\text{MIN} \quad f(x) = \sum_{j=1}^{5} e_j x_j + \sum_{i=1}^{5} \sum_{j=1}^{5} c_{ij} x_i x_j + \sum_{j=1}^{5} d_j x_j^3 \tag{4.6}$$

subject to:

$$\sum_{j=1}^{5} a_{ij} x_j - b_i \geq 0 \qquad i = 1,\dots,10 \tag{4.7}$$

where, a_{ij}, b_i, c_{ij}, d_j and e_j are constant values given in Table (4.2).

Table (4.2) Data for Non-linear Problem No. 2.

j	1	2	3	4	5
e_j	-15	-27	-36	-18	-12
c_{1j}	30	-20	-10	32	-10
c_{2j}	-20	39	-6	-31	32
c_{3j}	-10	-6	10	-6	-10
c_{4j}	32	-31	-6	39	-20
c_{5j}	-10	32	-10	-20	30
d_j	4	8	10	6	2
a_{1j}	-16	2	0	1	0
a_{2j}	0	-2	0	4	2
a_{3j}	-3.5	0	2	0	0
a_{4j}	0	-2	0	-4	-1
a_{5j}	0	-9	-2	1	-2.8
a_{6j}	2	0	-4	0	0
a_{7j}	-1	-1	-1	-1	-1
a_{8j}	-1	-2	-3	-2	-1
a_{9j}	1	2	3	4	5
a_{10j}	1	1	1	1	1
b_j	-40	-2	-0.25	-4	-4
b_{5+j}	-1	-40	-60	5	1

Since MAXCROP maximizes objective functions, equation (4.6) should be replaced by the following equation.

$$\text{MAX } f(x) = -\sum_{j=1}^{5} e_j x_j - \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij} x_i x_j - \sum_{j=1}^{5} d_j x_j^3 \tag{4.8}$$

Equation (4.8) is subject to the constraints of test No. 2.

The initial values given in Hock and Schittkowski (1981) to the variables were used: $x^0_1 = 0, x^0_2 = 0, x^0_3 = 0, x^0_4 = 0, x^0_5 = 1$.

A comparison of the results is shown in Table (4.3). The relative difference is equal to the difference between MAXCROP and the best known solution divided by the best known solution. The maximum absolute relative difference is $4.5\text{x}10^{-4}$.

Table (4.3) A Comparison of MAXCROP and Best Known Solution for Test No. 2

Objective Function Variables	**MAXCROP Solution**	**Best Known Solution**	**Relative Difference**
x_1	0.30000001	0.30000000	0.00000
x_2	0.33351102	0.33346761	0.00013
x_3	0.40000001	0.40000000	0.00000
x_4	0.42841557	0.42831010	0.00024
x_5	0.22386311	0.22396487	-0.00045
$f(x)$	32.34867859	32.34867897	0.00000

4.2 Comparison to an Existing Model:

The MAXCROP model was tested against an existing one-reservoir planning model (Dariane, 1989). Dariane's model is the only existing model that resembles MAXCROP in both the objective function and the constraints. Dariane used GAMS which is an NLP optimization package obtained from Brook, et al. (1988) to solve his problem. The objective of Dariane's model is to calculate the acreage for each crop, the amount of water to be stored in the reservoir and the monthly release to each crop to maximize the total net benefit to be obtained from planting different kinds of crops subject to mass balance, target demand and storage limits. Dariane uses both surface water and groundwater to maximize the value of multiple crop yields as described by the following objective function:

$$\text{MAX } B = \sum_{c=1}^{C} [(P_c - V_c)\, Y_c - F_c]\, A_{c,n} - \text{Pumping Costs} \tag{4.9}$$

Dariane's model was applied in a semi-arid zone to the Sevier Bridge reservoir located in south-central Utah. The size of that reservoir is 236,150 acre-ft ($291.2\text{x}10^6$ m^3). and it is responsible for irrigating an agricultural land of about 60,000 acres (24,282 ha). Three different kinds of crops were to be planted. One of them is alfalfa which has an already planted area of 28,000 acres. Therefore, the model has to use this as a fixed area without considering any fixed costs besides the other three crops. This makes the model as if it had four different kinds of crops: two dry matter (fxalfalfa and alfalfa), where fxalfalfa is the already planted alfalfa (no fixed costs) and two grain crops (corn and wheat).

The only difference between the objective functions of Dariane's model and the MAXCROP model applied to a one-reservoir watershed (equations 4.9 and 3.2 for n=1) is the usage of groundwater in Dariane's model. To make the MAXCROP model comparable to Dariane's, the effect of groundwater attribution to crop irrigation was replaced by its equivalent reservoir storage. Also, the equation for reservoir evaporation was modified in the MAXCROP model to match that of Dariane's problem.

Dariane's estimate of the reservoir surface area at time t (a_t) for the Sevier Bridge reservoir by the best-fit line of the surface area versus storage curve was as follows:

$$a_t = 450 + 0.044\left(\frac{S_t + S_{t+1}}{2}\right) \tag{4.3}$$

which can still be presented in MAXCROP.

$$a_t = \rho\left(\frac{S_t + S_{t+1}}{2}\right)^z \tag{3.16}$$

with the following modification:

$$a_t = constant + \rho\left(\frac{S_t + S_{t+1}}{2}\right)^z \tag{4.4}$$

where, $\rho = 0.044$, $z = 1$ and *constant* = 450.

Notice that the derivatives of equations (4.4) and (3.16) are equal. Moreover, if we substitute $\rho = 0.044$ and $z = 1$ and add the constant 450 into equation (3.16) then.

equation (3.16) will be identical to equation (4.3).

After the above modifications were made, the MAXCROP model was run to maximize the total benefit that can be obtained from planting those four kinds of crop according to the data given in Dariane (inflow data, climatic data, crop data,..., etc.).

4.2.1 Comparison of Results:

In this section, the results obtained from MAXCROP will be compared with those obtained from GAMS.

4.2.1.1 Releases:

A comparison of the releases during each month to corn, wheat, fxalfalfa and alfalfa by both MAXCROP and GAMS is shown in Tables (4.4), (4.5), (4.6) and (4.7) respectively, refer that the relative difference is equal to the difference between releases obtained from MAXCROP and GAMS divided by the releases obtained from GAMS.

It can be noticed from those Tables that the maximum absolute relative difference is 1.4×10^{-4} for corn, 3.0×10^{-4} for wheat, 0.9×10^{-4} for fxalfalfa and 2.5×10^{-4} for alfalfa. These are very small and their effect to the crop net benefit is insignificant as it is shown in section 4.2.1.4.

Table (4.4) A Comparison of Releases to Corn During Each Month by MAXCROP and GAMS, in acre-ft.

Month	MAXCROP Model	GAMS Model	Relative Difference
April	0.0	0.0	0.00000
May	694.1	694.0	0.00014
June	3921.6	3922.0	-0.00010
July	9196.8	9197.0	-0.00002
August	7359.5	7360.0	-0.00007
September	0.0	0.0	0.00000
October	0.0	0.0	0.00000

Table (4.5) A Comparison of Releases to Wheat During Each Month by MAXCROP and GAMS, in acre-ft.

Month	MAXCROP Model	GAMS Model	Relative Difference
April	3177.0	3178.0	-0.00030
May	9064.3	9064.0	0.00003
June	13037.8	13039.0	-0.00009
July	3327.9	3328.0	-0.00003
August	0.0	0.0	0.00000
September	0.0	0.0	0.00000
October	0.0	0.0	0.00000

Table (4.6) A Comparison of Releases to Fxalfalfa During Each Month by MAXCROP and GAMS, in acre-ft.

Month	MAXCROP Model	GAMS Model	Relative Difference
April	0.0	0.0	0.00000
May	11132.0	11133.0	-0.00009
June	17815.8	17816.0	-0.00001
July	24310.0	24311.0	-0.00004
August	19901.3	19901.0	0.00002
September	11202.0	11201.0	0.00009
October	0.0	0.0	0.00000

Table (4.7) A Comparison of Releases to alfalfa During Each month by MAXCROP and GAMS, in acre-ft.

Month	MAXCROP Model	GAMS Model	Relative Difference
April	0.0	0.0	0.00000
May	1192.7	1193.0	-0.00025
June	1908.8	1909.0	-0.00010
July	2604.6	2605.0	-0.00015
August	2132.3	2132.0	0.00014
September	1200.2	1200.0	0.00017
October	0.0	0.0	0.00000

4.2.1.2 Planted Areas:

The planted areas obtained for each crop for both models are identical, as shown in Table (4.8).

Table (4.8) A Comparison of Planted areas of Each Crop by MAXCROP and GAMS, in acres.

Crop	MAXCROP Model	GAMS Model	Relative Difference
Corn	11000.0	11000.0	0.00000
Wheat	18000.0	18000.0	0.00000
Fxalfalfa	28000.0	28000.0	0.00000
Alfalfa	3000.0	3000.0	0.00000

4.2.1.3 Reservoir Storage:

A comparison of reservoir storage at the beginning of each month between MAXCROP and GAMS is shown in Table (4.9). It can be noticed from that Table that the maximum absolute relative difference is 2.0×10^{-4}.

4.2.1.4 Crop Net Benefits:

A comparison of crop net benefit between MAXCROP and GAMS is shown in Figure (4.1) and in Table (4.10). The relative difference of crop net benefit can be seen in Figure (4.2).

Table (4.9) A Comparison of Reservoir Storage at the Beginning of Each Month by MAXCROP and GAMS, in acre-ft.

Month	MAXCROP Model	GAMS Model	Relative Difference
April	120000.0	120000.0	0.00000
May	125439.8	125439.0	0.00001
June	104676.8	104677.0	0.00000
July	68147.9	68147.0	0.00001
August	29635.5	29634.0	0.00005
September	5961.2	5960.0	0.00020
October	0.0	0.0	0.00000

Dariane only reported the crop yield values, not the crop net benefit. The latter was obtained by multiplying the difference between the average price of crop and variable cost by the crop yield. The obtained result was subtracted from the multiplication of the fixed cost and the planted area of that crop. It can be noticed from Table (4.10) that the maximum relative difference is 1.2×10^{-7}.

Table (4.10) A Comparison of Crop Net Benefit by MAXCROP and GAMS, in US \$.

Crop	MAXCROP Model	GAMS Model	Relative Difference
Corn	3,402,520.0	3,402,520.0	0.0000000
Wheat	1,843,560.0	1,843,560.0	0.0000000
Fxalfalfa	9,775,225.7	9,775,225.6	0.00000001
Alfalfa	822,345.7	822,345.6	0.00000012

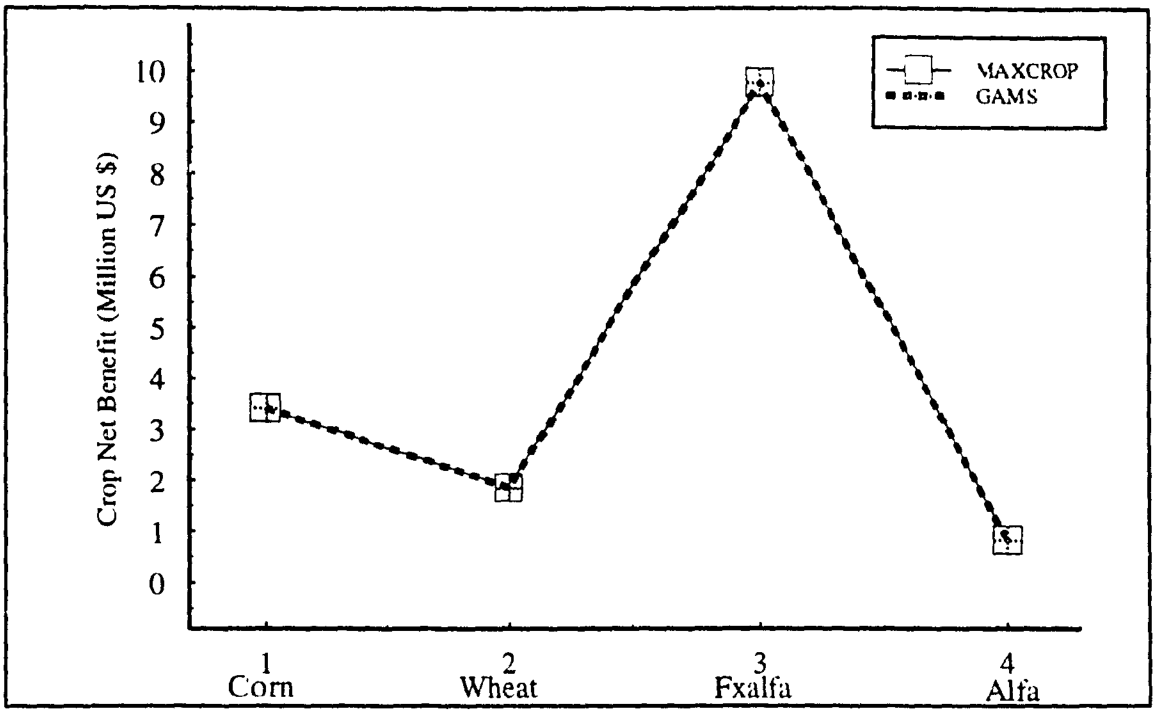

Figure (4.1) Comparison of Crop Net Benefit between MAXCROP and GAMS

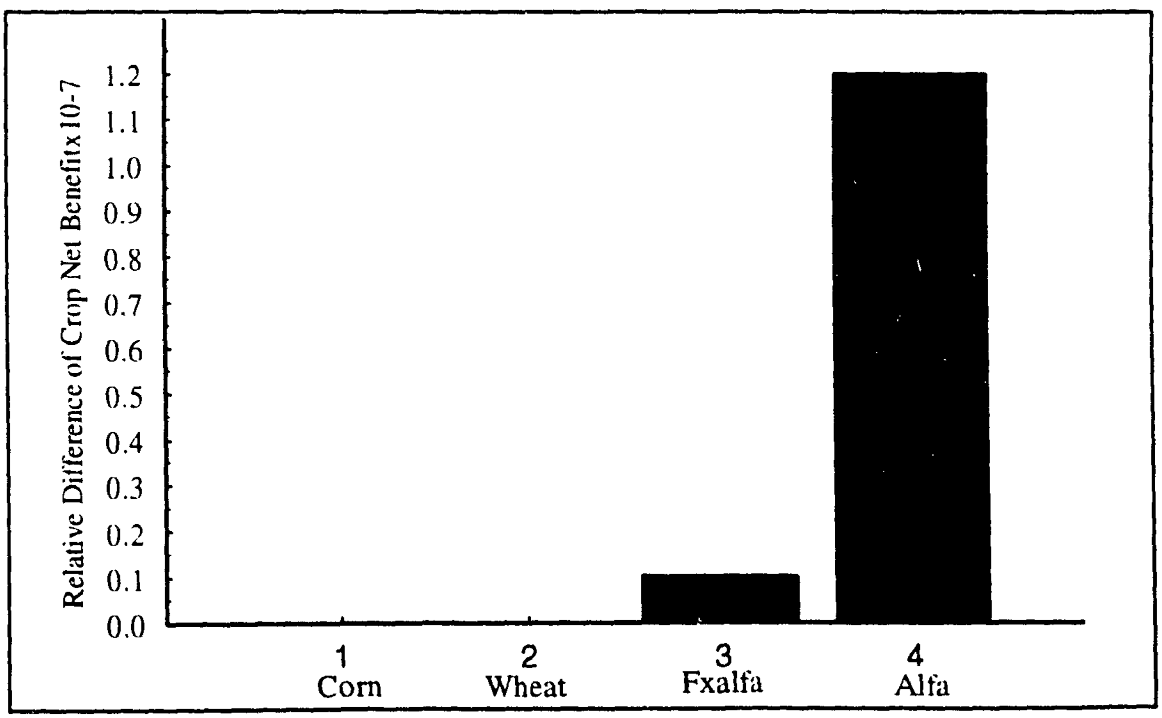

Figure (4.2) Relative Difference of Crop Net Benefit between MAXCROP and GAMS

4.3 Conclusions Regarding Validation:

These tests show no significant difference between the results obtained from MAXCROP and other codes when solving the same problem.

Although these tests do not fully validate the MAXCROP model, they represent the best possible validation of MAXCROP. The objective functions and the constraints of the non-linear problems selected for the comparison were similar to the mathematical formulation of MAXCROP model. Moreover, the only existing model that resembles the MAXCROP model is a one-reservoir planning model called GAMS. The results obtained from the MAXCROP model were consistent in every way with those obtained from GAMS with a maximum relative difference of $3x10^{-4}$ that could be simply due to the optimization procedures approximations or the computer rounding error.

CHAPTER 5

A CASE STUDY: THE MUWAQQAR WATERSHED

5.1 Description of Muwaqqar Watershed:

The Muwaqqar watershed is located, 30 km south-east of Amman, the capital of Jordan. It is a part of the Azraq basin which has an area of 7200 hectares, and is located at the upstream end of the basin. The Muwaqqar area is located in the 100-200 mm rainfall zone (arid zone). In addition, the climate of the area is characterised by irregular, sporadic and unpredictable rainfall. Rain falls during the winter season in the form of intensive storms of short duration causing high rates of runoff. The intensity of rainfall is high and infiltration index is very low due to soil surface crust. The net result is large floods although rainfall is low.

There are three small earth-fill reservoirs in the downstream section of the Muwaqqar watershed about 10 km east of Muwaqqar village. They were constructed to capture flood waters and to store them for irrigation purposes. These reservoirs are connected in series where reservoir 1 lies upstream of reservoir 2 which in turn lies upstream of reservoir 3. In addition, they are permeable in order to allow water infiltration into the ground to impound floods. The capacities of reservoirs 1, 2 and 3 are 2.8940, 2.3200 and 3.2500 ha-m respectively and the cultivated areas irrigated by reservoirs 1, 2 and 3 are about 15, 13 and 11 hectares respectively. Figure (5.1) represents the approximate locations of the six gauge sites and the three reservoirs in the Muwaqqar watershed.

5.2 Data Estimation:

Some of the data obtained from the Muwaqqar watershed was limited. Therefore, some manipulation and personal judgement were employed. Moreover, because of the scarcity of available data, the precipitation and inflow data were used for 1994-95 season.

5.2.1 Reservoir Evaporation:

Reservoir evaporation is an important parameter of the mass balance equation (Eq. 3.13). A general method to estimate evaporation from reservoirs was discussed in section 3.2.1. The application of the method to the three reservoirs is briefly discussed here. The evaporation $E_{t,n}$ was calculated by equation (3.17):

$$E_{t,n} = a_{t,n} L_t \qquad (3.17)$$

where, the reservoir normal lake evaporation rate (L_t) for Muwaqqar watershed is given in Table (5.1).

Figure (5.1) The Muwaqqar Watershed

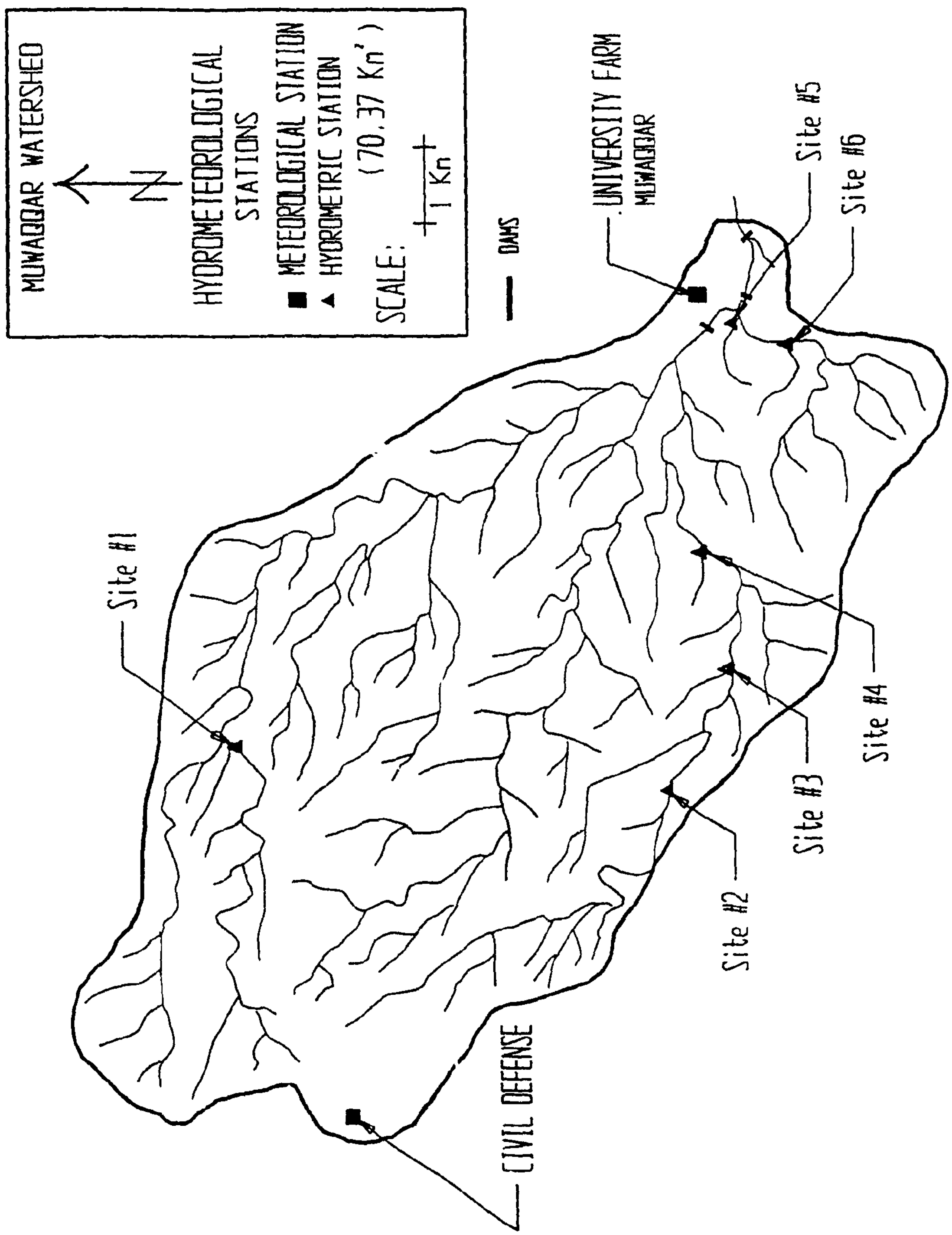

Table (5.1) Monthly Normal Lake Evaporation Rate (mm)

Normal Lake Evaporation	Month							
	Nov.	**Dec.**	**Jan.**	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**
L_t	66	31	31	36	74	156	217	240

The reservoir surface area, $a_{t,n}$, is estimated with equation (3.16):

$$a_{t,n} = \rho_n \left[\frac{S_{t,n} + S_{t+1,n}}{2} \right]^{z_n} \tag{3.16}$$

Values of ρ_n and z_n for the three reservoirs were obtained by applying the surface area versus storage data for each reservoir as shown in Table (5.2) to the Table Curve (TC) software. The best fit curve for equation (3.16) has goodness of fit criteria (squares of correlation coefficients, r^2) of 99.57, 99.85 and 99.26 % as shown in Figures (5.2), (5.3) and (5.4) for reservoirs 1, 2 and 3 respectively. This indicates the high correlation between surface area and reservoir storage in equation (3.16). The values obtained for ρ_n and z_n are shown in Table (5.3).

5.2.2 Estimating the Inflows:

Precipitation data in the developing countries is usually more available and realistic than inflow data. Therefore, a simulation model is required to estimate the inflows at the

Table (5.2) Surface Area versus Storage Data for Each Reservoir

Data No.	Reservoir 1		Reservoir 2		Reservoir 3	
	Area (ha)	**Storage (ha-m)**	**Area (ha)**	**Storage (ha-m)**	**Area (ha)**	**Storage (ha-m)**
1.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2.	0.0100	0.1130	0.0190	0.1890	0.0200	0.1390
3.	0.1355	0.4280	0.1800	0.4730	0.1010	0.2180
4.	0.4600	0.8400	0.6960	0.7350	0.2330	0.3760
5.	0.9305	1.0280	0.8095	0.7740	0.5300	0.7900
6.	1.4520	1.2060	1.2470	0.9400	1.0000	1.1000
7.	2.1390	1.3970	1.7350	1.0700	1.6350	1.3700
8.	2.8940	1.6100	2.320	1.2150	2.3740	1.6300
9.	-	-	-	-	3.2500	1.8800

Table (5.3) Reservoirs Coefficients

Reservoirs Coefficients	Reservoir 1	Reservoir 2	Reservoir 3
ρ_n	1.046078	0.864693	1.026926
z_n	0.404116	0.387935	0.534822

desired locations. In an ideal case, the output of the simulation model should be used as an input to the MAXCROP model. However, because of the insufficient data available and the uncertainty of the exact locations of the gauge sites at the time when this case study was applied, applying simulation models would not give any realistic results. Therefore,

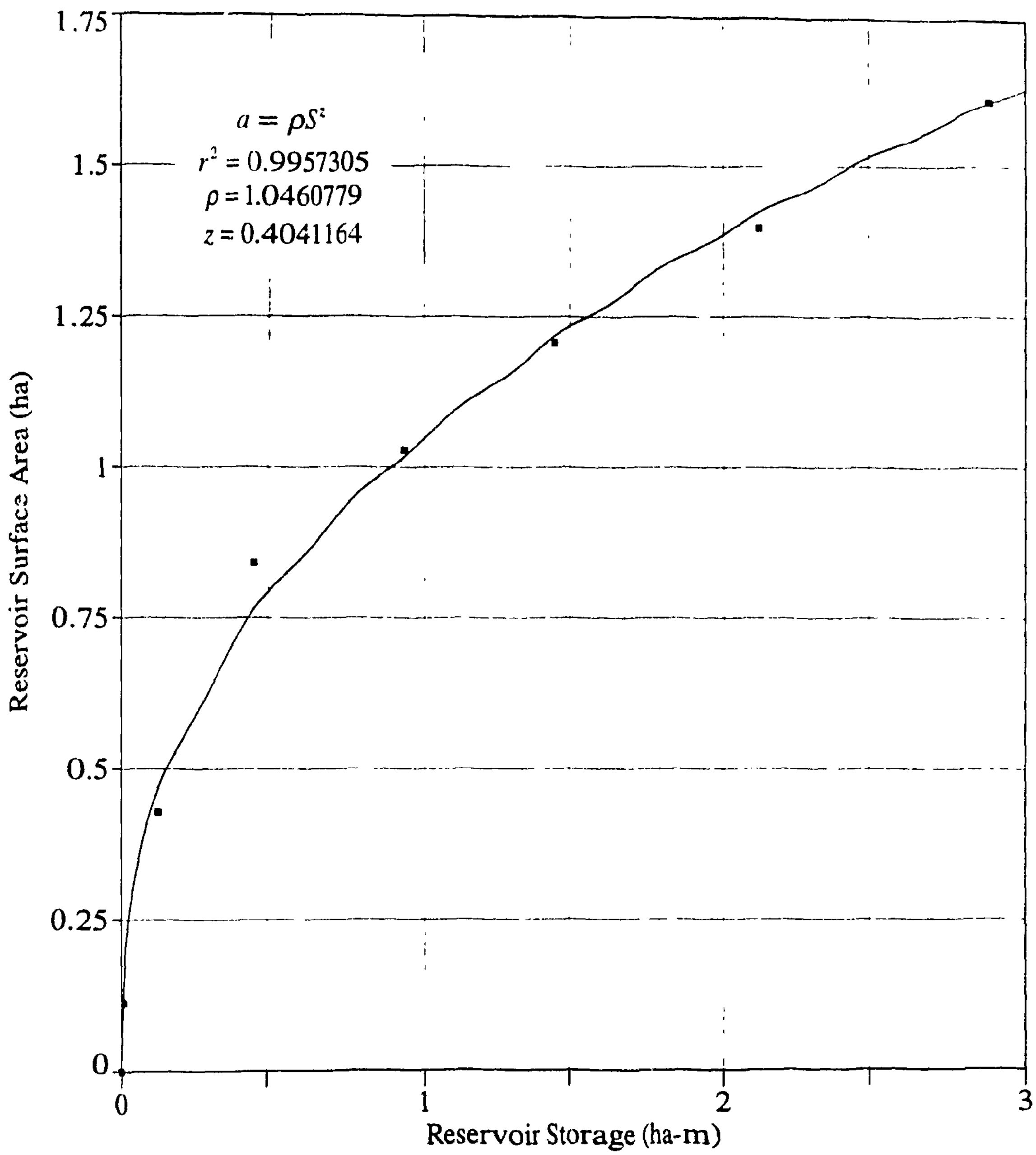

Figure (5.2) Storage versus Surface Area Relationship for Reservoir 1

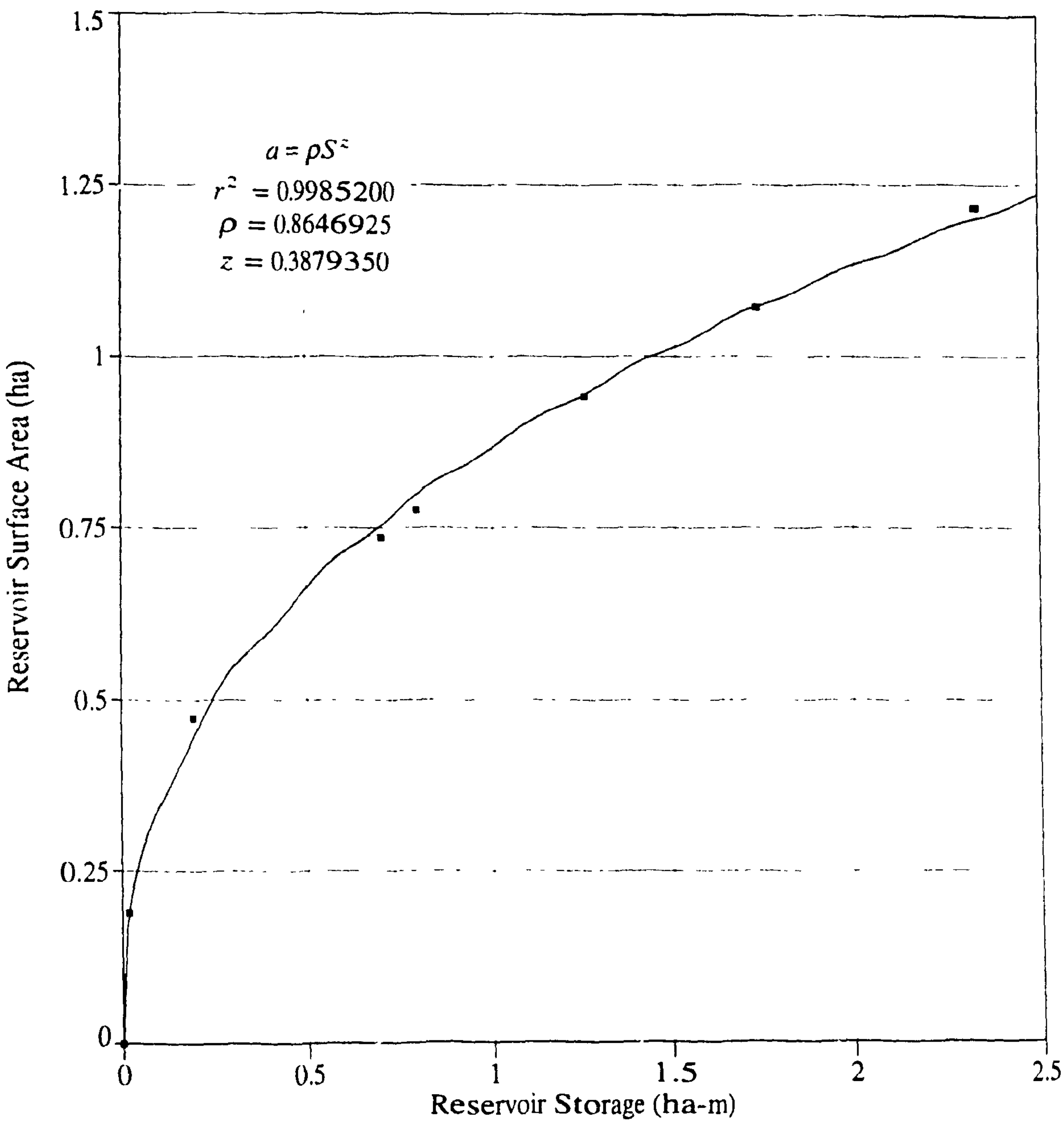

Figure (5.3) Storage versus Surface Area Relationship for Reservoir 2

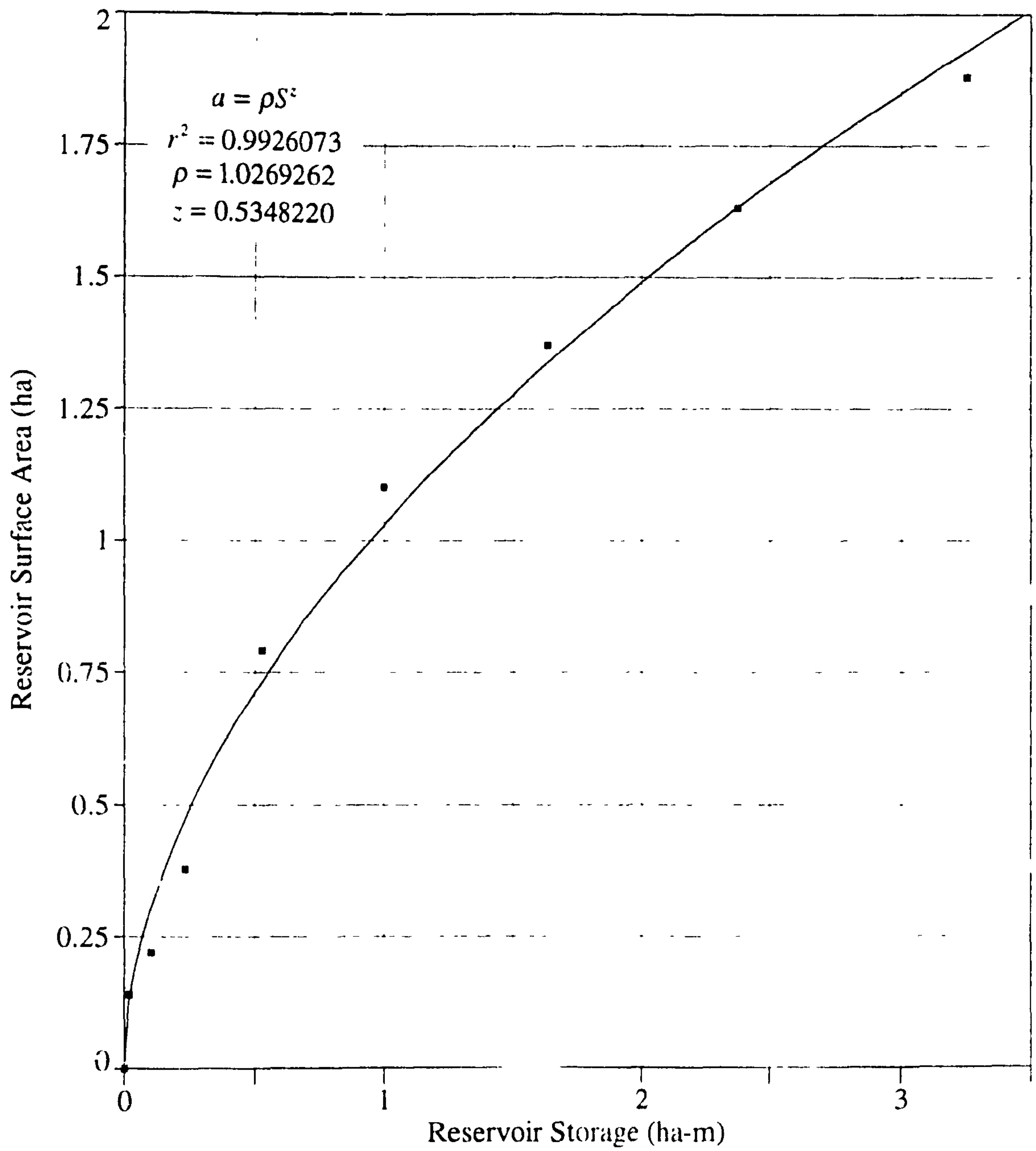

Figure (5.4) Storage versus Surface Area Relationship for Reservoir 3

the following assumptions were made to illustrate the application of MAXCROP model to a case study (Muwaqqar watershed).

The watershed was divided into two sub-watersheds by using the Thiessen method for the locations of rainfall measuring stations. The rainfall data obtained from the Civil Defense station was assumed to be uniform throughout sub-watershed 1 and the rainfall data obtained from University Farm station was assumed to be uniform throughout sub-watershed 2. Then, the runoff coefficients method (Varshney 1986) was used:

$$\Omega = K P$$

where, Ω, K and P are runoff, runoff coefficient and rainfall respectively.

Furthermore, it was assumed that K for each month and for each sub-watershed had a constant average value, i.e., the infiltration rate during each month in each sub-watershed was constant. Noticing that gauge stations 1 and 2 are located in sub-watershed 1 while stations 3, 4, 5 and 6 are located in sub-watershed 2 and from the above, one could arrive to the following expression for the average monthly infiltration coefficients for sub-watersheds 1 and 2 respectively.

$$K_1 = \frac{\left[\sum_{g=1}^{2} \Omega_g \Big/ \sum_{g=1}^{2} a_g \right]}{P_1} \tag{4.10}$$

$$K_2 = \frac{\left[\sum_{g=3}^{6} \Omega_g / \sum_{g=3}^{6} a_g\right]}{P_2} \qquad (4.11)$$

where, K_1 and K_2 are the average monthly infiltration coefficients for sub-watersheds 1 and 2 respectively.

Ω_g is the monthly runoff for gauge g.

a_g is the area covered by gauge g.

P_1 and P_2 are the monthly rainfall at Civil Defense station and University Farm station respectively.

Considering the distance to be traveled and knowing that the soil in the Muwaqqar watershed has a low infiltration capacity because of the crust at the surface which makes the soil almost impervious (Martin and El-jabi 1995), it was assumed that 20% of the runoff from the gauge sites in sub-watershed 1, which is about twice further from reservoir 1 than sub-watershed 2, and 10% from the gauge sites in sub-watershed 2 evaporated and infiltrated. The accuracy of this assumption is not crucial because most of the time there was excess runoff into the reservoirs whenever there was a storm. Moreover, the average yearly runoff-precipitation ratio that arrived to reservoir 1 was found to be equal to 13.2% when the total area of the watershed was considered which does not differ much from the calculated inflows for reservoir 1. This justifies the previous assumptions.

The streams upstream of reservoir 1 flow into reservoir 1. The streams between reservoirs 1 and 2 flow into reservoir 2. The streams between reservoirs 2 and 3 flow into reservoir 3.

Considering the above, the following relationships for the monthly inflows into reservoirs 1, 2 and 3 were estimated as follows.

The monthly inflow into reservoir 1 was estimated to be equal to the sum of 80% of the stream flows in sub-watershed 1 and 90% of the stream flows in sub-watershed 2.

$$q_1 = 0.8k_1 a_{s_1} p_1 + 0.9k_2 a_{s_{12}} p_2$$

where, q_1 is the monthly inflow into reservoir 1.

a_{s_1} is the area of sub-watershed 1 ≅ 3425 ha.

$a_{s_{12}}$ is the area of sub-watershed 2 that contains the streams going into reservoir 1 ≅ 2840 ha.

The monthly inflow into reservoir 2 was estimated to be equal to the stream flows at gauge sites 5 and 6.

$$q_2 = \Omega_5 + \Omega_6$$

where, q_2 is the monthly inflow into reservoir 2.

The monthly inflow into reservoir 3 was estimated to be equal to the stream flows going into reservoir 3.

$$q_3 = k_2 a_{s_{12}} p_2$$

where, q_3 is the monthly inflow into reservoir 3.

$a_{s_{12}}$ is the area that contains the streams going into reservoir 3 ≡ 45 ha.

The calculated monthly inflows into reservoirs 1, 2 and 3 are shown in Table (5.4). Only site 2 was used for sub-watershed 1, because the inflow data for site 1 was missing.

Table (5.4) Monthly Inflows into Reservoirs 1, 2 and 3 (ha-m)

Month	inflow into reservoir 1 (ha-m)	inflow into reservoir 2 (ha-m)	inflow into reservoir 3 (ha-m)
Oct. 94	0.8870	0.2855	0.0156
Nov. 94	59.2994	7.8013	0.4604
Dec. 94	27.5729	5.5282	0.2984
Jan. 95	0	0	0
Feb. 95	0.9286	0.3088	0.0164
Mar. 95	0	0	0
Apr. 95	0	0	0
May 95	0	0	0
Jun. 95	0	0	0

5.2.3 Precipitation Data:

The rainfall data at the University Farm Station was applied, because it is the nearest station to the agricultural lands that are served by the three reservoirs. The rainfall data for the 1994-95 season at the University Farm Station is shown in Table (5.5).

Table (5.5) Monthly Rainfall at University Farm Station for 1994-95 Season (mm)

Rainfall	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
$Rain_t$	42.7	36.0	0.0	6.35	0.0	0.0	0.0	0.0

5.2.4 Sensitivity of the Model to Crop Data:

Four crops have been considered in this case study: tomatoes as vegetable, alfalfa as legume and corn and barley as cereals crops. Crop data, which is required in the model consists of potential evapo-transpiration, potential crop yield, weighting factors, price, fixed and variable costs. The model shows high sensitivity mostly to prices, costs and potential crop yield. As has been noted in section 3.1.1, the model is not very sensitive to weighting factors. The reliability of the results depends greatly on the accuracy of the input data, especially those that show great sensitivity. Tables (5.6), (5.7) and (5.8) show some crop data, monthly potential yield for alfalfa and monthly potential crop evapo-transpiration.

5.2.5 Efficiency:

It is usually difficult to estimate both the total efficiency from the reservoir to crop root system and the rain efficiency. Such is the case in the Muwaqqar watershed. Therefore, for this case study reasonable values of 70% for total efficiency and 30% for rain efficiency were assumed (Doorenbos et al. 1979 and Jensen 1973 .

Table (5.6) Some Crop Data

Crop	Unit Price ($/ton)	Fixed Cost ($/ha)	Variable Unit Cost ($/ton)	Crop Weighting Factor	Potential Yield (ton/ha)
Alfalfa	70	42	1.90	-	15
Tomatoes	38	59	3.20	0.21	30
Corn	97	48	2.15	0.18	8
Barely	92	44	2.55	0.12	6

Table (5.7) Monthly Potential Yield for Alfalfa (ton/ha)

Kind of	Month							
Crop	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.
Alfalfa	-	2.36	3.19	3.78	3.43	2.24	-	-

Table (5.8) Monthly Potential Crop Evapo-Transpiration (mm)

Month	Alfalfa	Tomatoes	Corn	Barley
November	-	-	-	18.6
December	100.0	-	36.0	41.1
January	135.0	-	47.0	64.2
February	160.0	-	87.0	86.9
March	145.0	28.0	113.0	81.0
April	95.0	63.0	53.0	32.5
May	-	115.0	-	-
June	-	106.0	-	-

5.2.6 Routing Matrix:

The routing matrix M for use with the mass balance equation (Eq. 3.13) was discussed in Chapter 3. It is defined explicitly for the three-reservoir system as follows:

$$M = \begin{bmatrix} S_{P_{1,1}} & S_{P_{1,2}} & S_{P_{1,3}} \\ S_{P_{2,1}} & S_{P_{2,2}} & S_{P_{2,3}} \\ S_{P_{3,1}} & S_{P_{3,2}} & S_{P_{3,3}} \end{bmatrix}_{3x3} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}_{3x3}$$

The three reservoirs are connected in series where reservoir 1 lies upstream of reservoir 2 which in turn lies upstream of reservoir 3. Hence, reservoirs 2 and 3 receive the water spilled from reservoirs 1 and 2 respectively. That illustrates why the value of $S_{P_{2,1}}$ and $S_{P_{3,2}}$ is 1, whereas the value of all other elements in the matrix is zero.

5.2.7 Initial Storage:

Usually, initial storage should be given as input data but in this case study it was not. Nevertheless, it can be calculated very easily. The only inflow that took place before the planting season was on Oct. 9. Therefore, the initial storage of each reservoir (storage at the beginning of Nov.94) can be calculated by the mass balance equation (Eq. 3.13):

$$S_{t+1,n} = S_{t,n} + Q_{t,n} - R_{t,n} - E_{t,n} - S_{P_{t,n}} \qquad (3.13)$$

where, t stands for Oct. 9, 1994 and $t+1$ stands for Nov. 1, 1995.

On the right hand side of the above equation (Eq. 3.13) the only non-zero terms are storage $S_{t,n}$ and evaporation $E_{t,n}$. Therefore, equation (3.13) can be written as:

$$S_{t+1,n} = S_{t,n} - E_{t,n} \tag{5.1}$$

where, $S_{t,n}$ is nothing but $q_{t,n}$.

From equations (3.16) and (3.17) the evaporation term $E_{t,n}$ can be calculated by:

$$E_{t,n} = L_t \rho_n \left[\frac{S_{t,n} + S_{t+1,n}}{2} \right]^{\xi_n} \tag{5.2}$$

From equations (5.1) and (5.2) the following can be written:

$$S_{t+1,n} = S_{t,n} - L_t \rho_n \left[\frac{S_{t,n} + S_{t+1,n}}{2} \right]^{\xi_n} \tag{5.3}$$

where, L_t is the normal lake evaporation from Oct. 9 to Nov. 1.

In equation (5.3) the only unknown term is $S_{t+1,n}$. It can be calculated for the three reservoirs by applying the fixed-point iteration method (Gerald 1994). The calculated initial storage is given in Table (5.9).

Table (5.9) Reservoir Initial Storage (ha-m)

Initial Storage	Reservoir 1	Reservoir 2	Reservoir 3
$S_{1,n}$	0.8338	0.2574	0.0102

5.2.8 Minimum Area Requirements:

Total area to be irrigated by each reservoir can be either known data or variable for each crop to be determined by the optimization process. In this case study, the percentage, for each crop, of the minimum area requirement to the total area that reservoir *n* is responsible for irrigation is given in Table (5.10).

Table (5.10) Minimum Area Requirements for Each Crop (%)

Crop	Reservoir 1	Reservoir 2	Reservoir 3
Alfalfa	0.15	0.10	0.00
Tomatoes	0.10	0.05	0.07
Corn	0.07	0.08	0.05
Barley	0.30	0.25	0.25

5.3 Two Case Studies:

The MAXCROP model was applied to a representative multi-reservoir system (Muwaqqar) to demonstrate a case study. The data presented in previous sections were used. First, as a multi-crop case study, the four crops (alfalfa, tomatoes, corn and barley) were studied. Then, as a specific-crop case study, tomatoes, the crop that has the highest net revenue, were studied alone.

5.3.1 Multi-Crop Case Study:

In the PSLP algorithm, the initial selected values must satisfy all the linear constraints. Therefore the initial area values of the crops should be at least equal to the minimum area requirements of the crops. Different initial value combinations were tried to obtain the best local optimal solution. Different optimization attempts with different initial values were tried in a way to satisfy the minimum area requirements for the four crops and to divide the rest of the agricultural area among the crops. The initial area values, in the first run, were the minimum area requirements for the four crops, then the unconstrained agricultural area was divided by giving 25% of it to each crop and increasing it by 10% to one crop and decreasing it by the same amount to another crop at a time in each successive run.

The best obtained local optimal solution gave a total net benefit of US $18,871.3 for the season. The best obtained local optimal areas of crops, monthly release to crops, total monthly release from each reservoir, reservoirs storage, spilling from each reservoir and the total net benefit of the crops are shown in Tables (5.11) to (5.19). The results for release, storage and spills are published to six decimals to help in any future model comparisons with MAXCROP. Figures (5.5), (5.6) and (5.7) represent the best obtained local optimal monthly water available to the four crops from reservoirs 1, 2 and 3 respectively. Whereas, Figures (5.8), (5.9) and (5.10) represent the best obtained monthly percentage of water available versus water demanded.

Table (5.11) Area of Crops at Each reservoir (ha)

Crop	**Reservoir 1**	**Reservoir 2**	**Reservoir 3**
Alfalfa	2.25	1.30	0.00
Tomatoes	7.20	7.41	7.70
Corn	1.05	1.04	0.55
Barley	4.50	3.25	2.75

Table (5.12) Monthly Release to Alfalfa from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	-	-	-
December	0.286714	0.165657	-
January	0.043393	0.025071	-
February	0.050816	0.029361	-
March	0.046607	0.026929	-
April	0.030536	0.017643	-
May	-	-	-
June	-	-	-

Table (5.13) Monthly Release to Tomatoes from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	-	-	-
December	-	-	-
January	-		-
February	-	-	-
March	0.288000	0.296400	0.308000
April	0.502422	0.535300	0.693000
May	0.460941	0.506360	0.701737
June	0.390978	0.443617	0.596494

Table (5.14) Monthly Release to Corn from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	-	-	-
December	0.037800	0.037440	0.019800
January	0.070437	0.069432	0.036929
February	0.066798	0.066788	0.039560
March	0.048016	0.047964	0.033707
April	0.046485	0.046485	0.032580
May	-		-
June	-	-	-

Table (5.15) Monthly Release to Barley from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	0.037221	0.026882	0.022746
December	0.194786	0.140679	0.119036
January	0.105882	0.075655	0.069923
February	0.092579	0.066765	0.062110
March	0.072787	0.051954	0.056642
April	0.069762	0.050627	0.054598
May	-	-	-
June	-	-	-

Table (5.16) Total Monthly Release from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	0.037221	0.026882	0.022746
December	0.519300	0.343776	0.138836
January	0.219762	0.170158	0.106851
February	0.210193	0.162913	0.101671
March	0.455410	0.423247	0.398349
April	0.649205	0.650055	0.780178
May	0.460941	0.506366	0.701737
June	0.390978	0.443617	0.596494
Summation	2.94301	2.72701	2.84686

Table (5.17) Monthly Storage of Each Reservoir at the Beginning of Each Month (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	0.833800	0.257400	0.010200
December	0.513672	0.513672	0.677730
January	2.894000	2.320000	3.250000
February	2.418797	2.320000	3.084165
March	2.894000	2.320000	3.250000
April	2.063406	2.073392	2.715300
May	1.214941	1.258901	1.690677
June	0.538743	0.571248	0.741482
July	0.000000	0.000000	0.000000

Table (5.18) Monthly Spilling from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	59.561535	67.065758	66.822350
December	24.679756	28.062801	25.657467
January	0.207312	0.000000	0.000000
February	0.197175	0.307525	0.000000
March	0.263470	0.000000	0.000000
April	0.000000	0.000000	0.000000
May	0.000000	0.000000	0.000000
June	0.000000	0.000000	0.000000

Table (5.19) Total Net Benefit of Each Crop at Each Reservoir (US $)

Crop	Reservoir 1	Reservoir 2	Reservoir 3
Alfalfa	467.0	269.8	-
Tomatoes	4288.7	4619.2	5716.2
Corn	464.4	462.0	279.6
Barley	963.8	694.2	646.6
Summation	6183.9	6045.2	6642.4

5.3.2 Specific-Crop Case Study:

Under unlimited water supply conditions, tomatoes have the highest return among the other three studied crops. This time the MAXCROP model was applied to the Muwaqqar watershed by considering tomatoes as the only crop to be planted in that

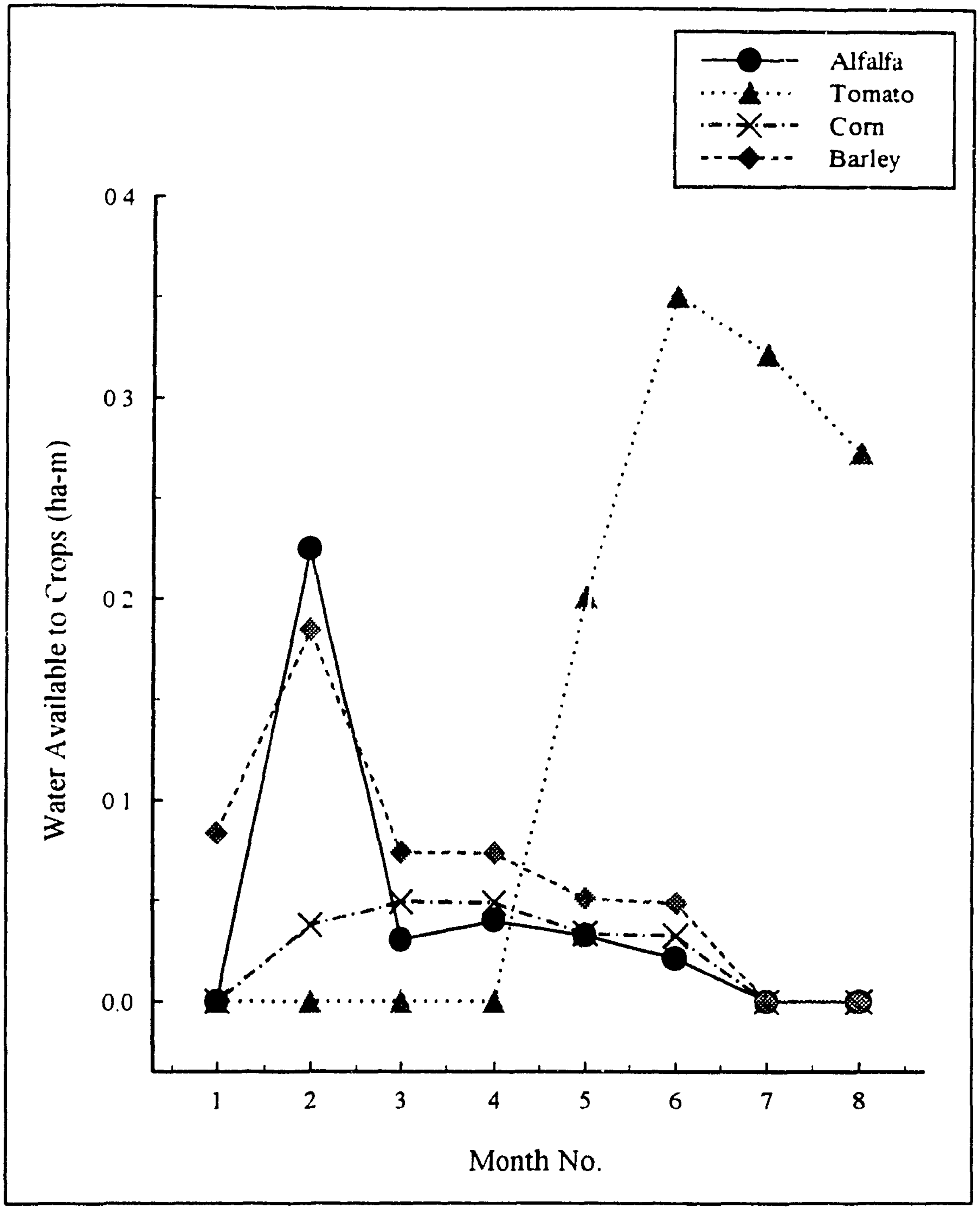

Figure (5.5) Monthly Water Available to Crops from Reservoir 1

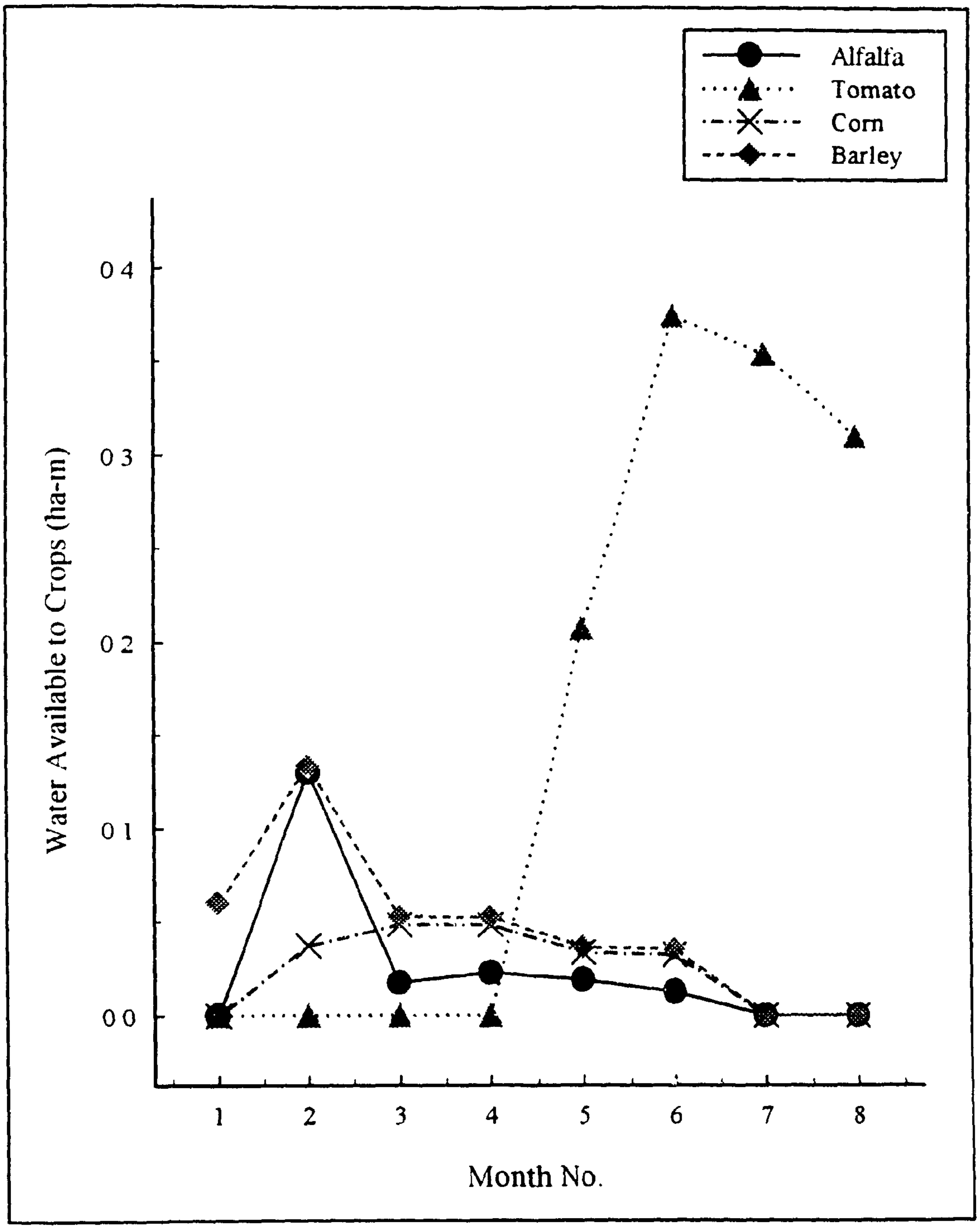

Figure (5.6) Monthly Water Available to Crops from Reservoir 2

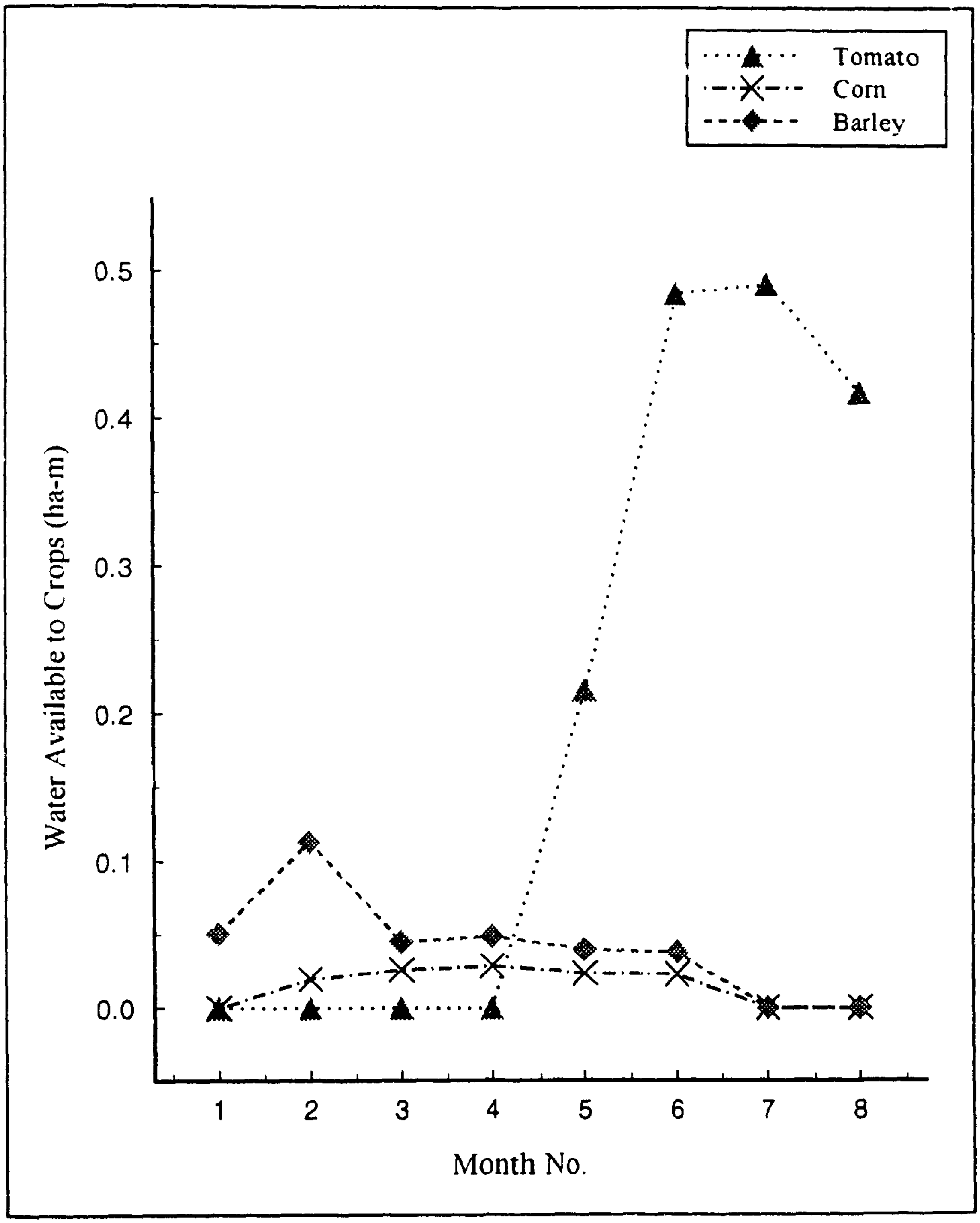

Figure (5.7) Monthly Water Available to Crops from Reservoir 3

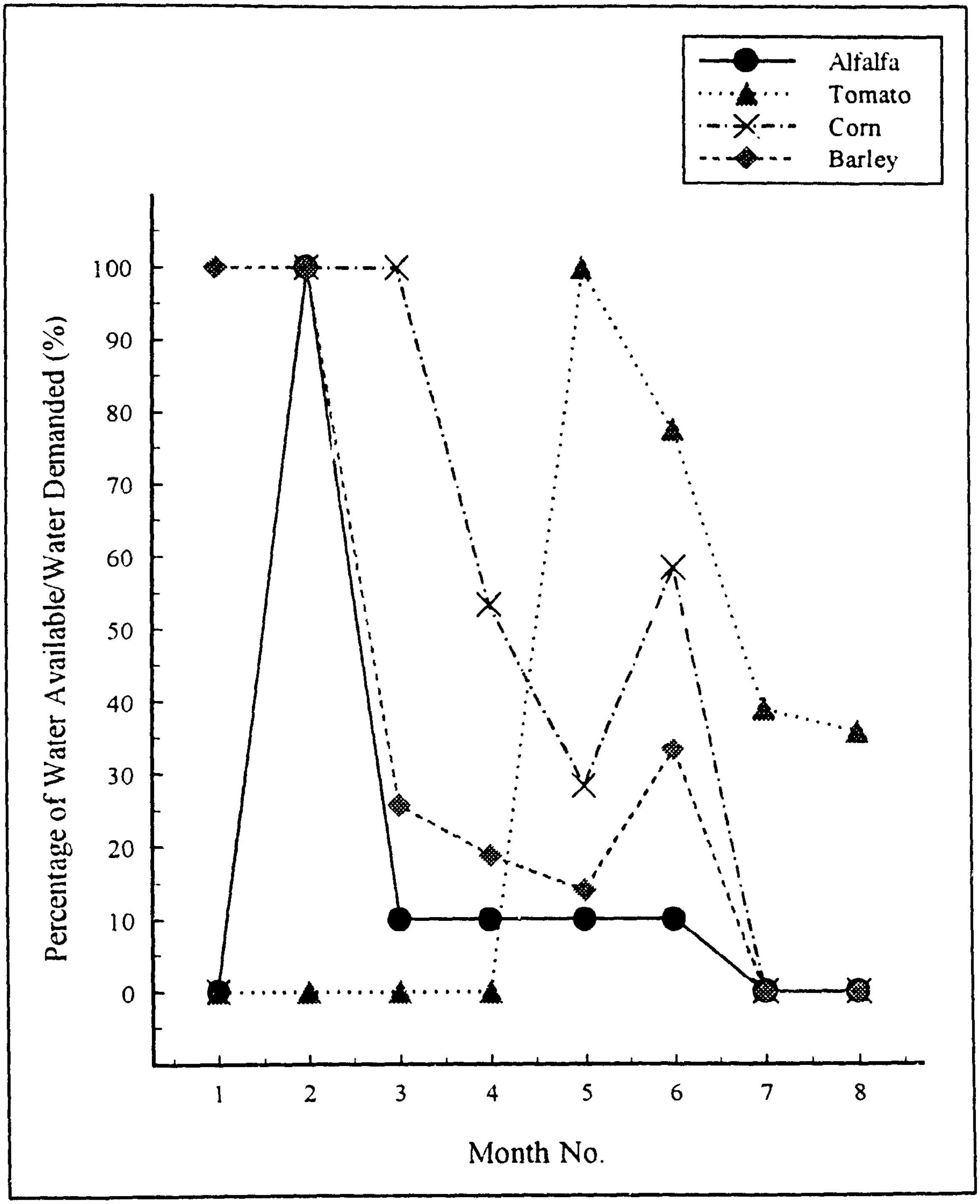

Figure (5.8) Monthly Percentage of Water Available/Water Demanded to Crops from Reservoir 1

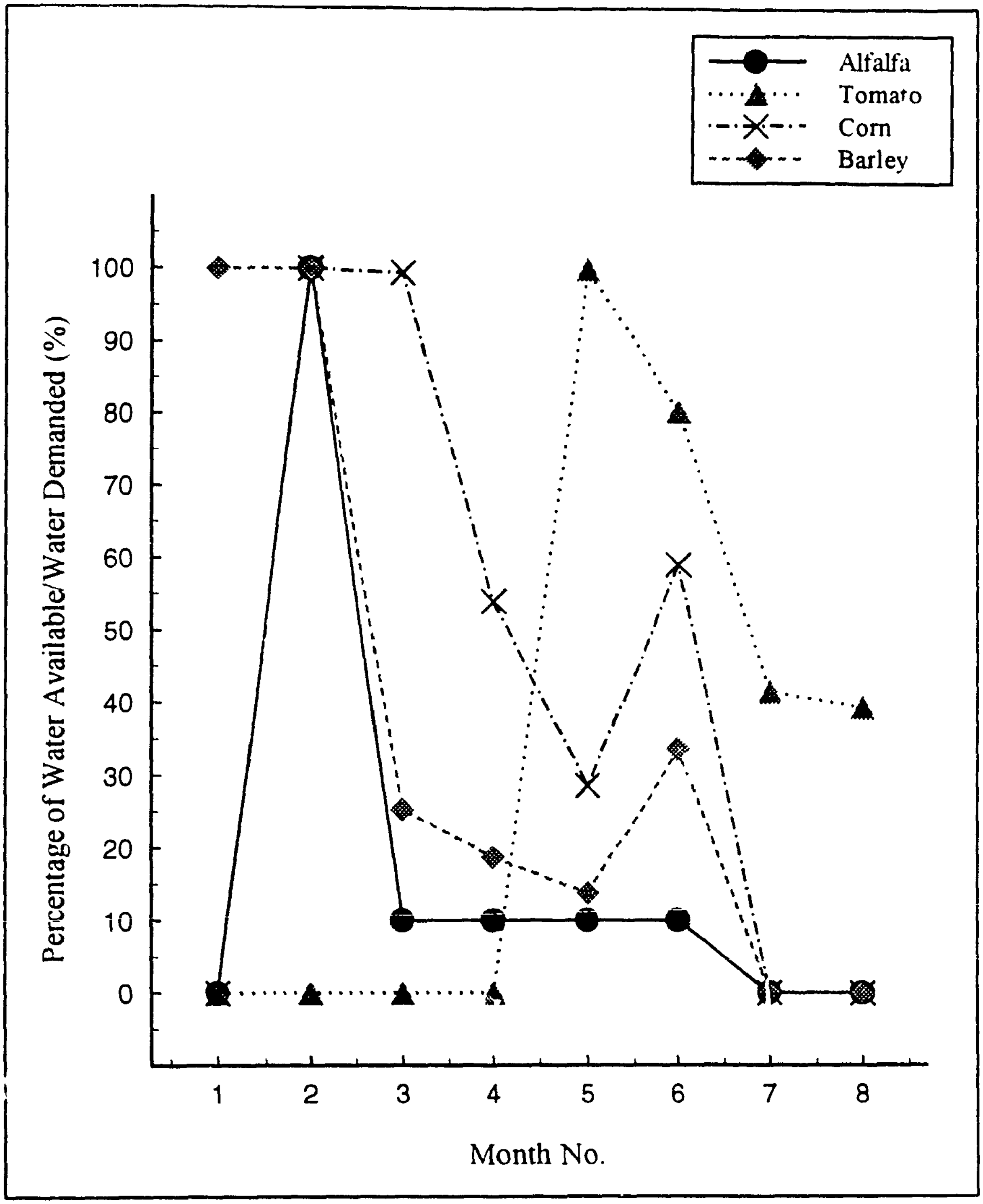

Figure (5.9) Monthly Percentage of Water Available/Water Demanded to Crops from Reservoir 2

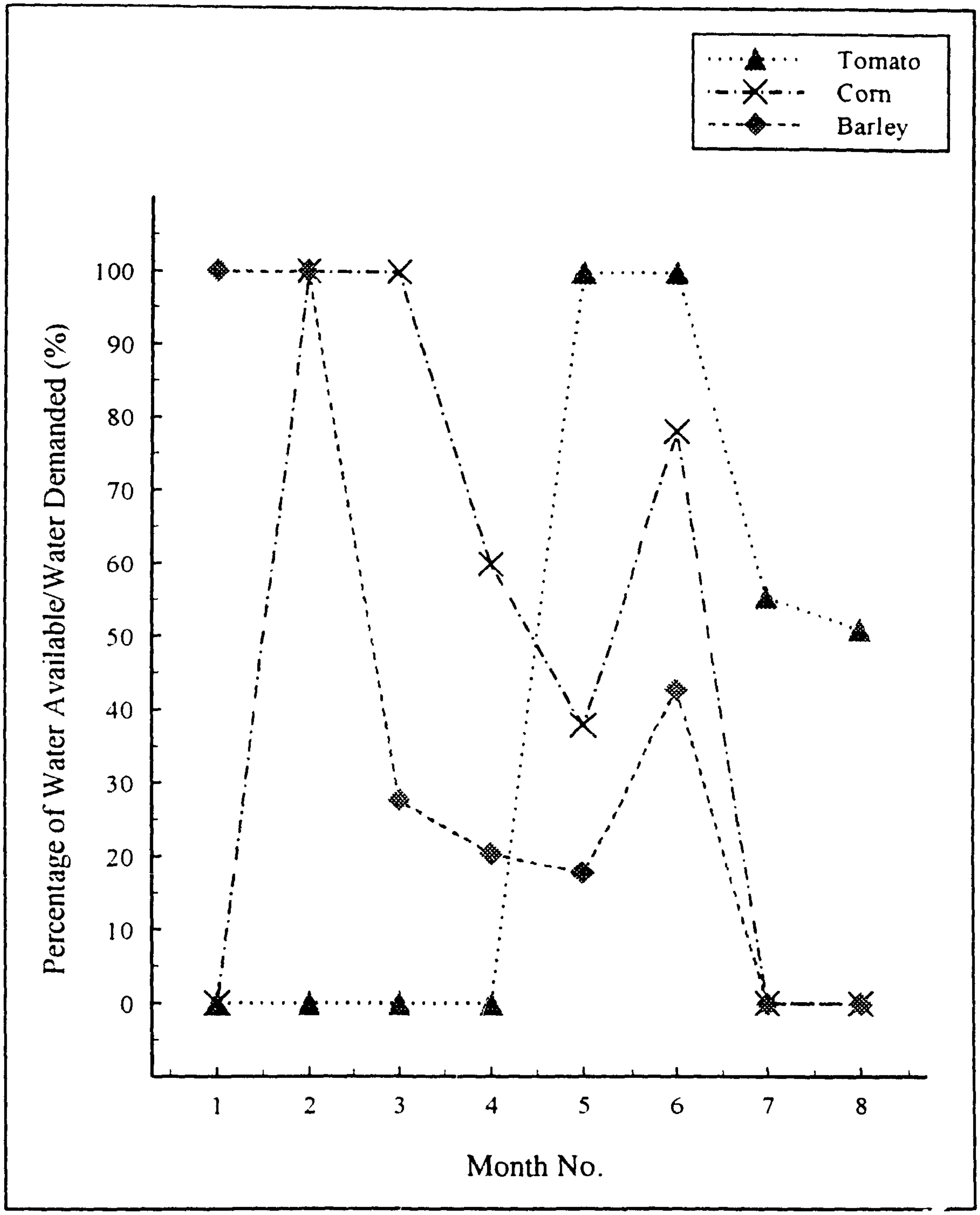

Figure (5.10) Monthly Percentage of Water Available/Water Demanded to Crops from Reservoir 3

region.

The best obtained local optimal results gave a total net benefit of US \$17,330.6 for the season. The best obtained local optimal areas for tomatoes, monthly release to tomatoes, reservoirs storage, spilling from each reservoir and total net benefit of tomatoes are shown in Tables (5.20) to (5.24). Figures (5.11) and (5.12) represent the best obtained local optimal monthly water available to tomatoes and the best obtained monthly percentage of water available versus water demanded respectively.

Table (5.20) Area of Tomatoes at Each Reservoir (ha)

Crop	Reservoir 1	Reservoir 2	Reservoir 3
Tomatoes	15.0	13.0	11.0

Table (5.21) Monthly Release to Tomatoes from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	-	-	-
December	-	-	-
January	-	-	-
February	-	-	-
March	0.560628	0.520000	0.440000
April	0.541755	0.514730	0.756835
May	0.502369	0.483126	0.694514
June	0.430466	0.424104	0.589702

Table (5.22) Monthly Storage of Each Reservoir at the Beginning of Each Month (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	0.833800	0.257400	0.010200
December	0.000000	0.000000	3.132813
January	2.857801	2.046564	2.792015
February	2.529201	2.241289	2.770916
March	2.894000	2.320000	3.250000
April	2.050221	1.886179	2.674149
May	1.307276	1.211603	1.674573
June	0.583024	0.549859	0.733894
July	0.000000	0.000000	0.000000

Table (5.23) Monthly Spilling from Each Reservoir (ha-m)

Month	Reservoir 1	Reservoir 2	Reservoir 3
November	60.116085	68.165688	65.473000
December	24.721140	28.207138	28.855520
January	0.280203	0.049444	0.015518
February	0.517385	0.712173	0.194582
March	0.171557	0.000000	0.000000
April	0.000000	0.000000	0.000000
May	0.000000	0.000000	0.000000
June	0.000000	0.000000	0.000000

Table (5.24) Net Benefit of Tomatoes at Each Reservoir (US $)

Crop	Reservoir 1	Reservoir 2	Reservoir 3
Tomatoes	5550.7	5289.2	6490.7

5.4 Discussion of the Results:

It can be observed in Table (5.11) that the best obtained local optimal solution corresponded to the minimum area requirements for alfalfa, corn and barley whereas, tomatoes obtained the remained area of the agricultural land. Some other optimization attempts gave local optimal values slightly less than the best obtained local optimal value (the relative difference was less than 0.5%). The relative difference is the difference between the best obtained local optimal value and any run's local optimal value divided by the best obtained local optimal value. In those optimization attempts the obtained area for alfalfa and barley were again equal to the minimum area requirements for those two crops. However, this time corn shared with tomatoes the remained area of the agricultural land.

The relative difference between the best obtained total net benefit for the specific-crop case was more than 8% less than that for the multi-crop case. This is because even though under unlimited water supply conditions, tomatoes are the crop that gives the best net benefit among the other studied crops, it requires water in the last four months of the season where the availability of water is limited. This limitation arises from the significant amount of evaporation in those periods and the lack of rainfall and therefore inflows. This also justifies why the relative difference between the local optimal values for some runs

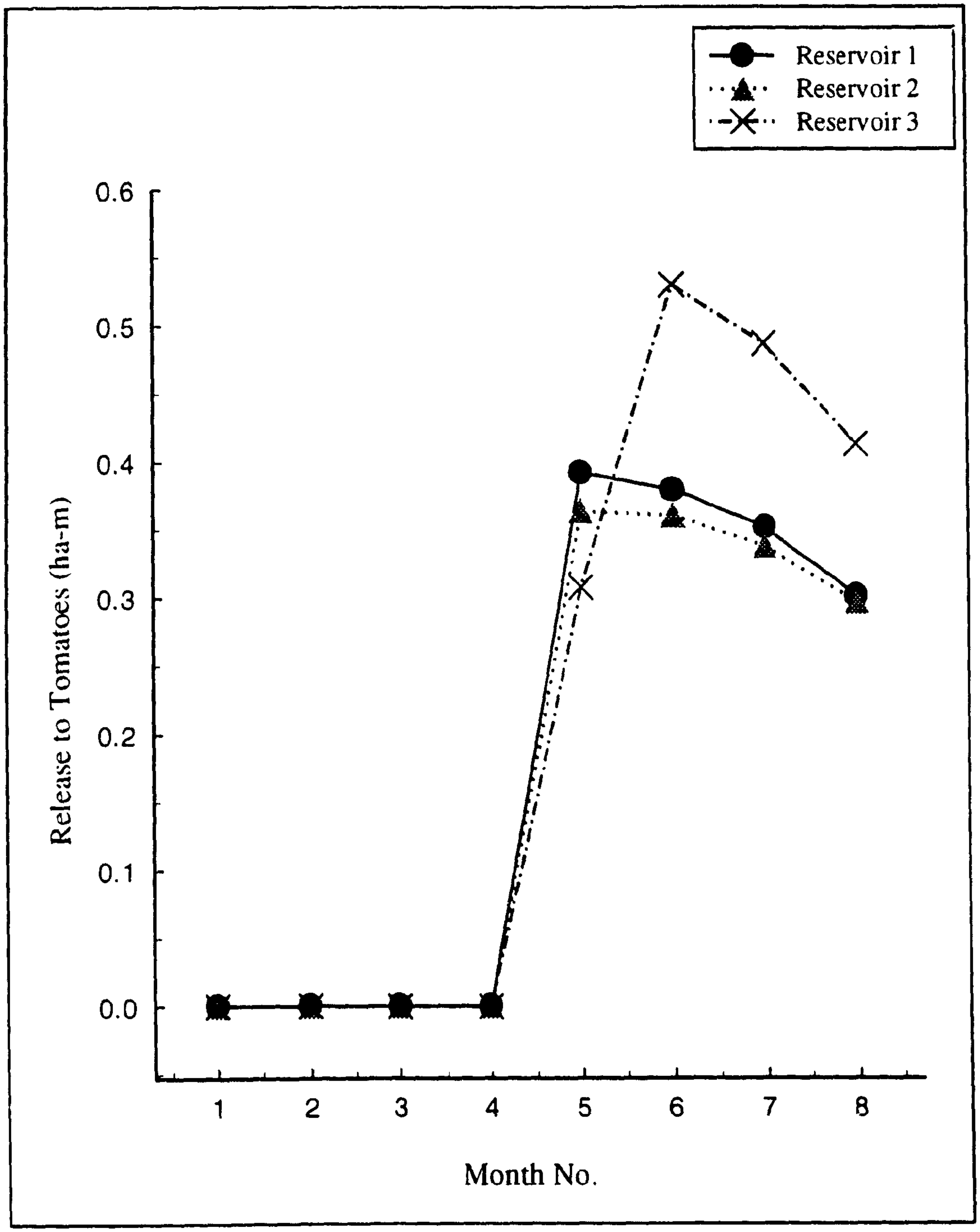

Figure (5.11) Monthly Water Available to Tomatoes from Each Reservoir

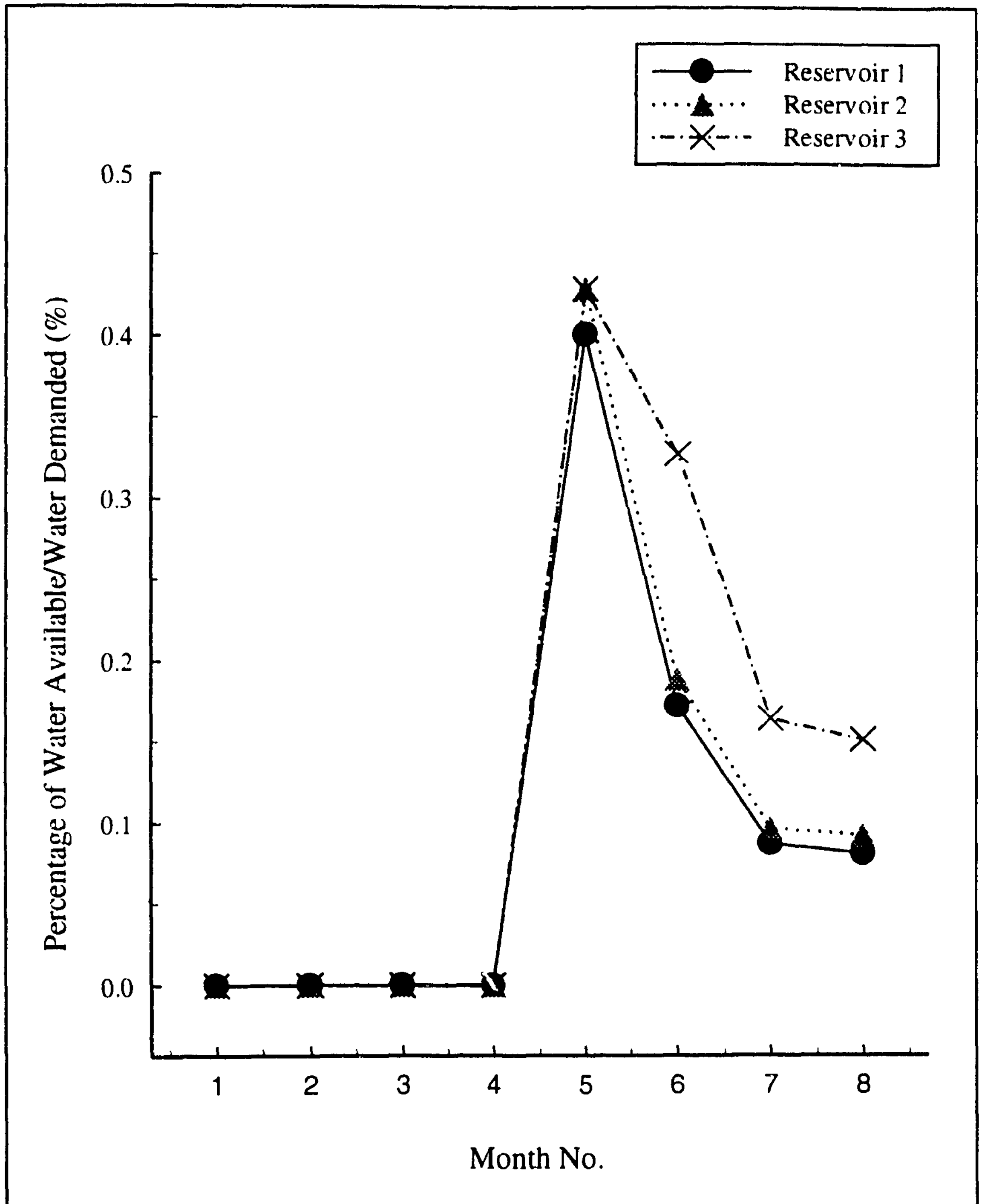

Figure (5.12) Monthly Percentage of Water Available/Water Demanded to Tomatoes from Each Reservoir that gave less planted area for tomatoes and the best obtained local optimal value was sometimes less than 0.5%. Because under unlimited water conditions the same amount of less planted area for tomatoes would have affected the total net benefit more than 0.5% by far.

For both case studies (specific and multi-crop cases), from Figures (5.5) to (5.12) it can be observed that the best obtained optimal release for all crops did not meet the maximum water requirement of crops during all the months in the season. This means increased water availability during each month of the growing season would give higher net benefit even for the same cultivated acreage In addition, when the water available for each crop was not constrained, the optimal water available for each crop decreased over time to decrease the release, apparently as a way for the system to reduce water loss through evaporation, thus making more water available to crops and consequently increasing the total net benefit.

For the multi-crop case study, the total release to crops from the three reservoirs was calculated from Table (5.16), to be 8.5169 ha-m Whereas, from Table (5.18), the water leaving the watershed, which is equal to the summation of the monthly spilling from reservoir 3 was equal to 92.4798 ha-m. Thus, the amount of water left the watershed as reservoirs' outflow was more than 10 times the amount used for irrigation and it was even more than that for the specific-crop case. This is because most of the inflow occurred in the first two months of the growing season and the existing three reservoirs were too small to store that huge amount of water (see Table 5.4). Therefore, some other reservoirs should be built to store the overflow water. The location and size of the reservoirs should be studied in a way to meet the water demand, minimize the cost of constructions, and optimize the water use including minimizing reservoirs evaporation.

CHAPTER 6

Conclusions

The model developed in this research effort (MAXCROP) is a monthly based planning model. It can be applied to existing irrigation multi-reservoir systems to obtain optimal planning policy for maximizing the net benefit to be obtained from planting different kinds of crops, subject to the constraints given in Chapter 3.

The model is used at the beginning of the planting season to determine the monthly spilling from each reservoir, the monthly release to each crop from each reservoir and the area to be allocated for each crop at each reservoir. A lower bound on the acreage for each crop at each reservoir is applied to consider crop diversification's requirements, i.e., crop nutrient cycling and on-farm animal feed needs and to satisfy farmers' needs.

The model is written in ANSI FORTRAN 77 and the Penalty Successive Linear Programming (PSLP) procedure is employed. PSLP is among the most promising optimization techniques in achieving the optimal solution of non-linear non-convex objective functions. Unlike other developed SLP algorithms PSLP has a convergence proof for non-linearly constrained problems of general form. This is the case with this model where non-linearity appears in both the objective function and the constraints.

The code and model validation were checked. First, the computer code was tested for general non-linear problems. The objective values obtained were exactly equal to the best known solution in all the tested problems. Then, the performance of the model was compared to an existing one reservoir planning model. The results were consistent between the two models. The objective values obtained were equivalent, with a maximum absolute relative difference of 3×10^{-4} between all the variables. Finally, as a case study, the model was applied to the Muwaqqar watershed in Jordan. This watershed consists of three small farming reservoirs. It is located in an arid zone where rain falls with high intensity and short duration. The results of the application of the Muwaqqar watershed were illustrated in Chapter 5.

From the above it can be concluded that MAXCROP is potentially a reliable and valid model that can help designers choose the optimal acreage, crop types and monthly release to irrigate each crop, taking into considerations crop diversification's requirements and farmers' needs. More importantly, it is the first model of its type that applies PSLP method to optimize the planning of irrigation multi-reservoir systems. Although, the current version of the model only considers seasonal crops it could be enhanced to consider permanent crops such as fruit trees. In this case, the ideal solution can be obtained by considering the time horizon of the problem, *T*, as the number of months of the life of fruit trees instead of the number of months of the growing season. This would increase dramatically the number of variables and constraints. Therefore, an alternative solution that uses a smaller time horizon should be studied.

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