Discrimination of Planar Shapes using Shape Matrices

Antoine Taza

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ABSTRACT

Discrimination of Planar Shapes using Shape Matrices

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This thesis presents an algorithm to describe and discriminate binary images of planar shapes. The descriptor is a matrix which has dimensions dependent on the maximum radius of the shape. The descriptor is obtained by a polar quantization of the shape, and is independent of the shape's position, orientation, and size. The descriptors are used to discriminate two or more shapes, by measuring their degree of similarity. The shapes experimented include alphabet letters, numerals, geometrical figures, and physical objects. The results obtained from the alphabet letters were compared to those reported in a recent research. This comparison shows that the algorithm proposed in this thesis provides a better shape discrimination.

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CHAPTER 1

INTRODUCTION

Description and discrimination of planar shapes is one of the fundamental problems in computer vision and pattern recognition. To discriminate shapes, a procedure to describe them is usually required. Then, by comparing shape descriptors, shape discrimination is achieved.

Some of the known methods for describing and/or discriminating shapes are:

1. The use of critical points

   The shape is conveyed by the curving of the boundary line, and is considered independent of scale and orientation. The curving is regarded as a concatenation of arcs of varying instantaneous radii of curvature; possibly interspersed occasionally by discontinuities. The description of shape is facilitated by segmenting the boundary line at so-called critical points: curves, points of inflection, curvature maxima, intersections, and points of tangency. These critical points are extracted and used in the development of a shape description system.

2. Template matching in rotated images

   Normalized invariant moments are used for rotationally invariant template matching. If normalized invariant moments in circular windows are used, then template matching in rotated images becomes similar to template matching in translated images. In template matching, the zeroth-order moment is used in the first stage to determine the likely match positions, and the second- and third-order moments are used in the second stage to determine the best match position among the likely ones.

3. Polar quantization

The descriptor is obtained by polar quantization of the shape. The shape contour is sampled with equal angular steps relative to the center of gravity of the shape. Then, distances of the sample points to the center of gravity of the shape are plotted as a function of other angles. The obtained waveform is called the signature of the shape. If, at a given angle, the contour is intersected at more than one point, the largest distance or the sum of the distances is used as the distance value for the angle.

4. Fourier descriptors

Given a closed figure in the complex plane, the contour can be traced, yielding a one-dimensional complex function of time. If the contour is traced repeatedly, the periodic function which results can be expressed in a Fourier series. The obtained Fourier coefficients are used to characterize the shape.

5. Visual perception

Some of the methods related to visual perception use the following features for describing a shape:

1. The ratio of the length of the longest side and the perimeter of the shape.
2. The standard deviation of the length of the sides of the shape.
3. The length of the perimeter and the derivatives of its side lengths.
4. The ratio of the largest to the smallest angle.
5. The ratio of the largest angle to the total degree of angles.
6. The standard deviation of angles.
7. The degree of compactness, defined by: P*P/A, where P and A are the perimeter and area of the shape, respectively. Given different shapes of a constant area, the one with the smallest perimeter is the most compact shape.
8. Regularity, defined as the ratio of the standard deviation of side lengths and standard deviation of all angles.
9. Angular variability, defined as the mean absolute difference of adjacent angles, taken in overlapping

pairs about the boundary, where convex angles are given a positive sign, and concave angles are given a negative sign.

Some of the shape descriptors mentioned above are not information preserving, i.e., it is not possible to reconstruct the original shapes using their descriptors. Other shape descriptors are information preserving.

The shape descriptor presented in this thesis is information preserving; but this property will not be exploited, and it has no practical importance within the scope of the thesis.

Some of the techniques mentioned above for shape description, use the shape's boundaries, and the patterns inside the shape are forgotten. In this work, the shape is described by a polar quantization process. This process takes into consideration the complete geometry of the shape. The resulting descriptor is independent of the shape's position, orientation, and size.

After describing two or more shapes, a proper comparison process is applied on the descriptors to determine their degree of similarity. Fig. 1 shows the step-by-step process to find the degree of similarity between objects.

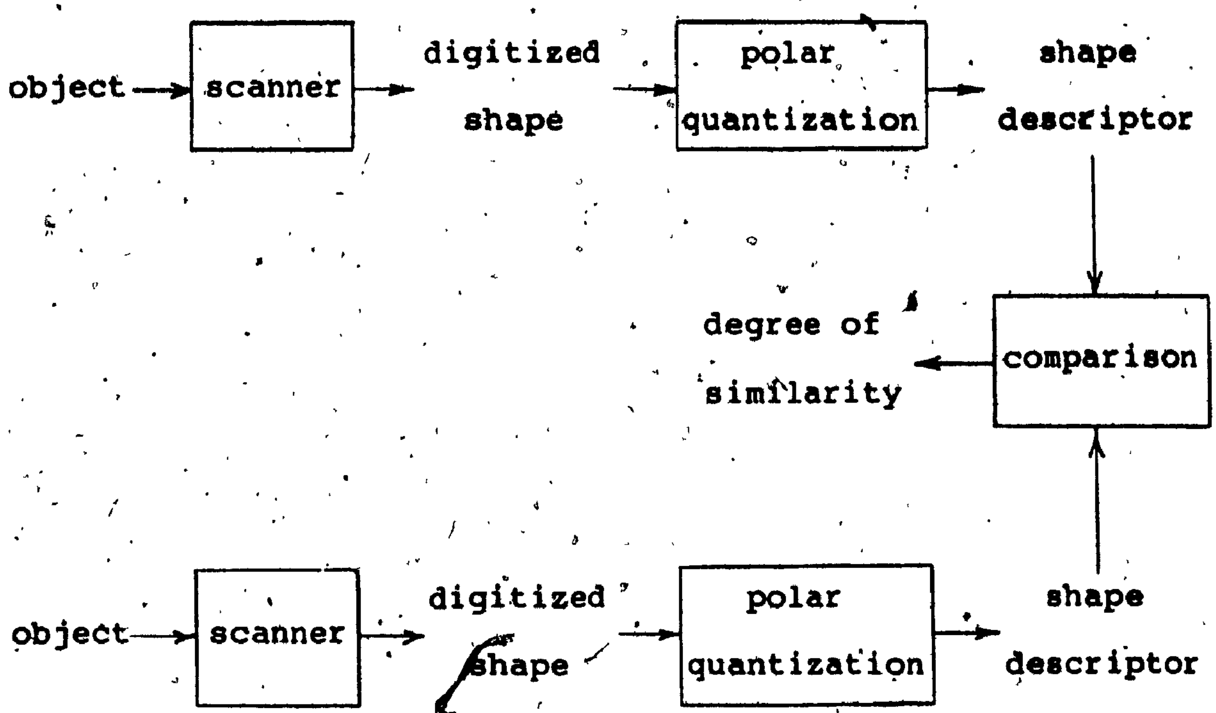


Fig. 1 Block diagram to find the degree of similarity between two objects.

CHAPTER 2

SHAPE DESCRIPTION

The shape descriptor is an m x n matrix that will be referred to as "shape matrix" or "M". It is constructed by the following steps:

1. Let O be the center of gravity of the shape, and OA with length r the maximum radius of the shape.

2. Divide OA into (n-1) equal lengths.

3. Using O as center, draw circles with radii r/(n - 1), 2r/(n-1),...,(n-1)r/(n-1).

4. Starting from OA, and counter-clockwise, draw radii that divide each circle into m equal arcs, each arc being dtheta = 360/m degrees. (the values of n and m are discussed in chapter 3)

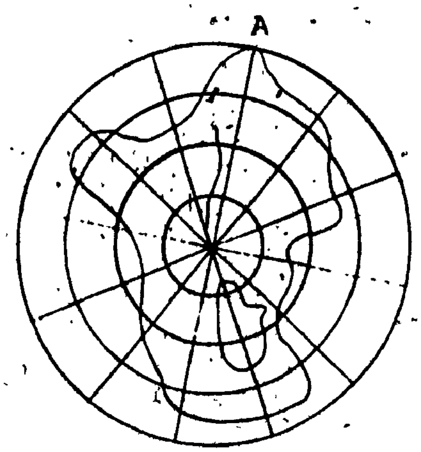
5. Fill in the elements of the shape matrix as follows:

```
FOR i = 0 TO (n-1)
  FOR j = 0 TO (m-1)
    IF the point with polar coordinates
    (ir/(n-1) , j(360/m)) lies inside the shape
    THEN M(i,j) = 1
    ELSE M(i,j) = 0
```

An example of a digitized shape and its shape matrix is given in fig. 2, where n = 5 and m = 12.

It can be seen that if the shape is displaced or rotated,

a digitized shape



its shape matrix

	0	1	2	3	4
0	1	1	1	1	1
1	1	1	1	0	0
2	1	1	1	1	0
3	1	1	0	0	0
4	1	1	0	0	0
5	1	1	1	0	0
6	1	1	1	1	0
7	1	0	0	1	0
8	1	1	0	0	0
9	1	1	0	0	0
10	1	1	1	0	0
11	1	1	1	1	0

Fig. 2 Shape and shape matrix

its shape matrix will not be affected. If it is enlarged or shrinked, the shape matrix will also remain unaffected, provided that the same values of n and m are used.

Therefore, the interesting property of shape matrices is that they are invariant to translation, rotation, and scaling, of their shapes.

The shape matrix will be used in a comparison process to determine the similarity, or to discriminate, with other shapes.

CHAPTER 3

ANALYSIS OF SHAPE MATRICES

The digitized shape is obtained by scanning the object. It is a binary shape that consists of black and white pixels, in a rectangular frame. We will assume a white background for the frame.

Let us take the distance between two adjacent pixels, vertical or horizontal, as unit of length. Every pixel will have coordinates (x,y), where the values of x and y depend on the pixel's relative horizontal and vertical positions, respectively, with respect to a fixed origin.

The center of gravity of the shape, point O, lies on a pixel whose coordinates (ox,oy) are given by:

ox = round(xtotal/k)

oy = round(ytotal/k)

where

k : number of black pixels in digitized shape

xtotal: sum of horizontal coordinates of black pixels

ytotal: sum of vertical coordinates of black pixels

The function "round" is necessary, because (xtotal/k), (ytotal/k) are real; but ox, oy must be integers.

Since round(x) and x are different in general, where x is any real number, O(ox,oy) is an approximation of the real center of gravity.

This approximation is necessary for any point, because we cannot work with a point that does not have coordinates. Therefore, any point we refer to, will be forced on a pixel, by means of a proper approximation.

The resolution of a scanner is defined as the number of pixels generated by this scanner, for a certain length. The higher the resolution, the lesser is the difference between round(x) and x, or trunc(x) and x; and hence, the better the approximation.

Once we have determined the point O, the point A(ax,ay) of the shape's maximum radius OA, is determined by seeking the maximum distance between O and the black pixels.

Notice that A has to be on a black pixel, whereas O can be on either a white or a black pixel.

The length r of OA is given by:

r = SQRT [SQR(ax-ox) + SQR(ay-oy)]

Since we are going to use polar coordinates, we must find theta, the angle between OA and the positive x-axis.

The value of theta is given by arctan [(ay-oy) / (ax-ox)], in addition to other detailed trigonometric calculations to take care of sign changes when the angle falls in different quadrants or on the axes.

Let O be the origin of our polar coordinate system.

Now we have determined the cartesian and the polar coordinates of O and A:

	cartesian	polar
O	(ox,oy)	(0,0)
A	(ax,ay)	(r,theta)

Before proceeding further to construct the shape matrix, there is a caution to be exercised. When we draw circles around the shape, it is possible that these circles extend beyond the frame's boundaries, if the shape lies too close to these boundaries. More precisely, if the distance between the center of gravity, and any of the four sides of the rectangular frame, is shorter than the shape's maximum radius, an overflow will occur if we do nothing to prevent it.

If this is the case, we move the shape into the center of the frame. As a result, the cartesian coordinates of all the pixels will change. The polar coordinates are not affected.

Now, if the distance between the center of gravity and any side of the frame, is still shorter than the maximum radius, this means that the frame is too small for the given shape. Again, an overflow will occur if we do not solve this problem.

In this case, we enlarge the frame's dimensions, by adding rows and/or columns of white pixels, adjacent to its boundaries, until we meet the condition for no overflow. This procedure ensures that the outermost circle drawn around the shape lies completely inside the frame's boundaries.

Now, we are going to determine the proper values for n and m, mentioned earlier. These values must be integers, because we divide a length of a radius, or an arc, into an integer number of equal distances.

These values, n and m, are the dimensions of the shape matrix. The larger these values are, the smaller the distances between sample points in the digitized shape, and the bigger the shape matrix.

If these values are too small, our sampling will not be large enough to cover all the pixels, i.e., we will have less than one sample point for some pixels, and hence we will miss some information in the shape descriptor.

If these values are too large, we will have more than one sample point for every pixel. This is unnecessary, and will entail a waste in computer memory space and CPU computing time.

Since we have assigned the unit of length to be the distance between two adjacent (top or bottom, left or right) pixels, we need the distances between any two adjacent (on the same radius or circumference) sample points, to be smaller or equal to the unit of length. This is our guide in finding the suitable values for n and m.

Value of n:

Length of maximum radius = r

Therefore, in principle we have:

$$n = r$$

We know however that r is real and n is integer.

This yields:

$$n = \mathrm{trunc}(r) + 1$$

From the above, we can state the following:

n = length of maximum radius

= number of sample points on maximum radius

= number of pixels on maximum radius

Value of m:

Length of maximum circumference (outermost circle) = $2 * \pi * r$

Therefore, in principle we have:

$$m = 2 * \pi * r$$

Since $(2 * \pi * r)$ is real and m is integer, we have:

$$m = \mathrm{trunc}(2 * \pi * r) + 1$$

from the above, we can state the following:

m = circumference of outermost circle

= number of sample points on this circumference

= number of pixels on this circumference

Now that n and m are defined, we fill in the elements of the shape matrix as described in chapter 2.

CHAPTER 4

SHAPE DISCRIMINATION

Our ultimate goal is to find out how similar, or how different, two or more shapes are. This is achieved by comparing their shape matrices.

When we have two or more shapes for comparison, the first input shape will be called the "master shape". The following shape(s) will be compared to the master shape, and will be called "test shape(s)".

In order to carry out a proper comparison, let us consider the elements of a shape matrix (see fig. 2). Every element corresponds to a sample point in the digitized shape.

Now, consider the digitized shape. We can notice that the density of sample points is higher near the center than near the outermost circle. This is a significant phenomenon. So let us express it more explicitly:

1. The number of sample points on the inner circles is equal to the number of sample points on the outer circles. This can also be confirmed by the fact that there is a one-to-one correspondance between columns in the shape matrix, and circles in the digitized shape; and obviously, all the columns of a given matrix have the same number of elements.

2. The number of pixels on the inner circles is smaller than the number of pixels on the outer circles. This

is clear because the circumference of the inner circles is smaller than that of the outer circles.

3. Recall from chapter 3 that the number of sample points on the outermost circle is equal to the number of pixels on this circle.

From 1, 2, 3 above, we can deduce that there is no redundancy on the outermost circle, but there is on the inner circles. This redundancy increases as we move from the outermost circumference towards the center, to other circles, and it is constant for a given circle.

The redundancy R, for a given circle, is defined by:

$$R = \frac{\text{number of sample points}}{\text{number of pixels}}$$

$$= \frac{\text{number of sample points}}{\text{circle circumference}}$$

Since the number of sample points is constant for any circle, we get

$$R = \frac{\text{constant}}{\text{circle circumference}}$$

$$= \frac{\text{constant}}{2 * \pi * i} \quad \text{where } i \text{ is the circle radius}$$

$$= \frac{\text{constant}}{i}$$

Here we come to the interesting deduction, that the redundancy for a given drawn circle on the digitized shape, is inversely proportional to the radius of this circle:

$$R \propto \frac{1}{i}$$

Every column in the shape matrix corresponds to the circumference of a circle in the digitized shape. Every element of that column corresponds to a sample point on that circle.

Since the number of sample points is constant for all the circles, and the number of pixels is higher on the outer circles, there is more information on the outer circles for the same number of sample points.

Therefore, there is more information, or more pixels, in the higher order columns, than in the lower order columns, even though the number of elements is the same.

Let us define the amount of this information, contained in an element, for a given column, by the "weight".

Therefore, the weight W is defined by:

$$W = \frac{\text{number of pixels}}{\text{number of sample points}}$$

$$W = \frac{1}{R}$$

The weight is the inverse of the redundancy.

$$W \propto i$$

$$W = \text{constant} * i \qquad (i > 0)$$

This is the most interesting point:

Rule 1.a:

" The weight of a point on a nonzero radius circle, in a digitized shape, is directly and linearly proportional to the length of its radius."

We can also state, in parallel:

Rule 1.b:

" The weight of an element in a column, whose order is higher than zero, of a shape matrix, is directly and linearly proportional to the order of this column."

This rule is essential in shape discrimination. When we compare two shape matrices, before comparing corresponding elements, we multiply every element by a factor that represents its weight. This factor is equal to the order of the column, except for the first column which has order 0.

For the highest order column, as an example, which represents the outermost circle, this factor is equal to (n-1), which is the order of this column.

We know however, that the weight of any element in this column is equal to 1, because the number of sample points is equal to the number of pixels, in this column.

This means that the computed weight is greater than the defined weight by a factor of $(n-1)/1 = (n-1)$. This factor applies to all the columns.

There is no disadvantage in obtaining the computed weights greater than the defined weights, by a constant factor. The advantage is purely the simplification of calculations. If we had to compute the real defined weights, we would have to multiply the element (=1) by the order of the column, and divide by $(n-1)$. This is longer and unnecessary, because the weights are used for comparison only, and we are interested in their relative values.

So far, we can calculate the weights for all the columns, except the first, because rule 1 does not apply to it.

This whole column represents a single pixel in the center of gravity. It requires a separate calculation:

$$\text{Defined weight for column } [i=0] = \frac{\text{number of pixels}}{\text{number of sample points}}$$

$$= \frac{1}{m}$$

Computed weight column [i=0] = (n-1) * defined weight column [i=0]

$$= \frac{n - 1}{m}$$

Knowing how to compute the weight of any element, we can compare two shape matrices. The following formula defines the degree of similarity between two shape matrices:

$$sim = 1 - \frac{dif}{tot}$$

where sim: degree of similarity

dif: weight of different elements

tot: weight of "1" elements in master shape matrix

The maximum value for sim is 1. The minimum value is $-\infty$. This happens when tot = 0.

As we see in the formula, the degree of similarity between two shapes depends not only on their differences, but also on which of the two is the master shape.

CHAPTER 5

ALGORITHM

The following is a brief outline of the main points in the algorithm:

1. Read in an input shape. If it is the first, call it "master shape"; otherwise, call it "test shape".

2. Find the center of gravity.

3. Find the maximum radius.

4. If necessary, center the shape.

5. Construct the shape matrix.

6. If this is a test shape, compare its shape matrix with the master shape matrix.

7. If there is another input shape, go to 1; otherwise, terminate.

The actual algorithm is more sophisticated. Some of the important features are:

1. For a master shape, we construct one shape matrix only. For a test shape, we construct a test shape matrix, and compare it to the master shape matrix. If the similarity between the shape matrices lies below a defined threshold, we construct another test shape matrix and compare it to the master shape matrix. We repeat this procedure, for a limited number of times,

until we find a good similarity.

2. In addition to the maximum radius OA, we find a number "X" of reference radii, OA1, OA2,...,OAX, at some defined angles from OA.

We take the set of ratios OA/OA1, OA/OA2,...,OA/OAX, for the master shape. These ratios are used as references to the corresponding ratios obtained from the test shapes. We compare the two sets of ratios, and if there is a match, within a certain tolerance, we proceed to construct a shape matrix for the test shape. If there is no match, we find the next maximum radius, and repeat the procedure.

The reasons are:

1. A digitized shape can have more than one maximum radius. It is necessary to find from the test shape, the maximum radius that corresponds to the one in the master shape.

2. A digitized shape may have many radii whose lengths are very close to the maximum radius. After rotating and reducing the object, and digitizing it, we may find one of these "close" maximum radii in the master shape, to be the maximum radius in the test shape. This imperfection may come from

the scanner's quality and the limitation of its resolution.

Now, our purpose is twofold:

1. To find the "right" maximum radius in the test shape. This is the reason we find another maximum radius and shape matrix, and repeat until we obtain a good similarity between the shape matrices.

2. To save computation time. This is why we do not construct the shape matrix, unless we obtain a good match for the ratios of the reference radii.

Another important feature is the values of n and m.

These values are calculated for the master shape only, and then fixed for all the test shapes. This is the way to make the shape matrix independent of the size of the shape. This is also the way to simplify the task of comparing the shape matrices, since it normalizes them to the same size.

At this point, it is important to clarify what was mentioned in chapter 4 about the value of the redundance on the outermost circle. We mentioned that it is equal to 1. Now, we have to add that this value applies to the master shape only. In general, the redundancy will be different from 1 for test shapes.

CHAPTER 6

DESCRIPTION OF THE LOGICAL STEPS

In this chapter, we start by describing the logical steps in the program (part A); and then, we present a "structured" flow chart: the Nassi-Shneiderman chart for the program (part B).

A) Logical steps

The following are the important constants used:

π : the well-known geometrical constant, value = 3.14159

X : number of reference radii

ITERMAX : maximum number of iterations

TOLERANCE: tolerance between ratios of reference radii

THRESHOLD: value for good similarity

1. Initialize the flag: MASTER to TRUE.

2. Initialize to -1 the elements of SHPMAT: the master shape matrix, and MAT: the test shape matrix.

3. If END-OF-FILE

   then terminate.

   else proceed to the next step.

4. Initialize the following:

   - Number of black pixels in the input shape: K to 0
   - Degree of similarity: SIM to -1
   - Number of iterations: ITER to 0

- Flag: CENTERED to FALSE
- Flag: GOODSIM to FALSE
- Flag: GOODRAD to FALSE

5. Read an input shape.

6. Start a new page on output file.

7. If MASTER = TRUE

   then write: MASTER SHAPE.

   else write: TEST SHAPE.

8. Print the input shape.

9. Write the input frame dimensions.

10. Find the center of gravity: "O", and count the number of black pixels: K.

11. Output the information obtained from the previous step.

12. If (ITER = 0) OR ((NOT MASTER) AND (NOT GOODSIM) AND (ITER < ITERMAX))

    then proceed to the next step.

    else go to step 39.

13. Increment ITER.

14. If ITER > 1

    then proceed to the next step.

    else go to step 17.

15. Start a new page on output file.

16. Write the value of ITER.

17. Find the length of the maximum radius "OA", determine the point "A", and the angle between the segment OA, and the positive x-axis: "THETA".

18. Output the information obtained from the previous step.

19. If NOT CENTERED

    then proceed to the next step.

    else go to step 23.

20. If the shape needs to be centered, then center it, and print it in its centered position.

21. Output the information as in steps 9, 11, 18.

22. CENTERED := TRUE.

23. Find X reference radii: OA1, OA2, ..., OAX, at angles: THETA1 = THETA + PI/4, THETA2 = THETA + 2PI/4, ..., THETAX = THETA + XPI/4, respectively.

24. Determine the ratios: OA/OA1, OA/OA2, ..., OA/OAX.

25. If NOT MASTER

    then calculate the percent differences between these ratios, and the corresponding ratios in the master shape.

26. Output the information obtained in the three previous steps.

27. If the differences mentioned in step 25 are within the value of TOLERANCE

then GOODRAD := TRUE.

else GOODRAD := FALSE.

28. If (ITER = 1) OR GOODRAD

then proceed to the next step.

else go to step 39.

29. If MASTER

then construct the shape matrix: SHPMAT.

else construct the shape matrix: MAT.

30. Write the values of n and m.

31. If (ITER = 1)

then proceed to the next step.

else go to step 35.

32. Start a new page on output file.

33. If MASTER

then write: MASTER SHAPE MATRIX.

else write: TEST SHAPE MATRIX.

34. If MASTER

then print the shape matrix: SHPMAT.

else print the shape matrix: MAT.

35. If NOT MASTER

then proceed to the next step.

else go to step 39.

36. Compare the two shape matrices

37. If SIM > THRESHOLD

then GOODSIM := TRUE.

38. Print the following results:

- number of "1" elements in master shape matrix
- their weight: TOT
- number of different elements in shape matrices
- their weight: DIF
- degree of similarity: SIM

39. MASTER := FALSE.

40. Go to step 3.

B) Nassi-Shneiderman chart (see fig. 3)

The Nassi-Shneiderman chart is an alternative to conventional flow charts. Its superiority lies in the fact that it is structured, and therefore is compatible with structured design and programming. Its compact format allows an easier overall view of the program.

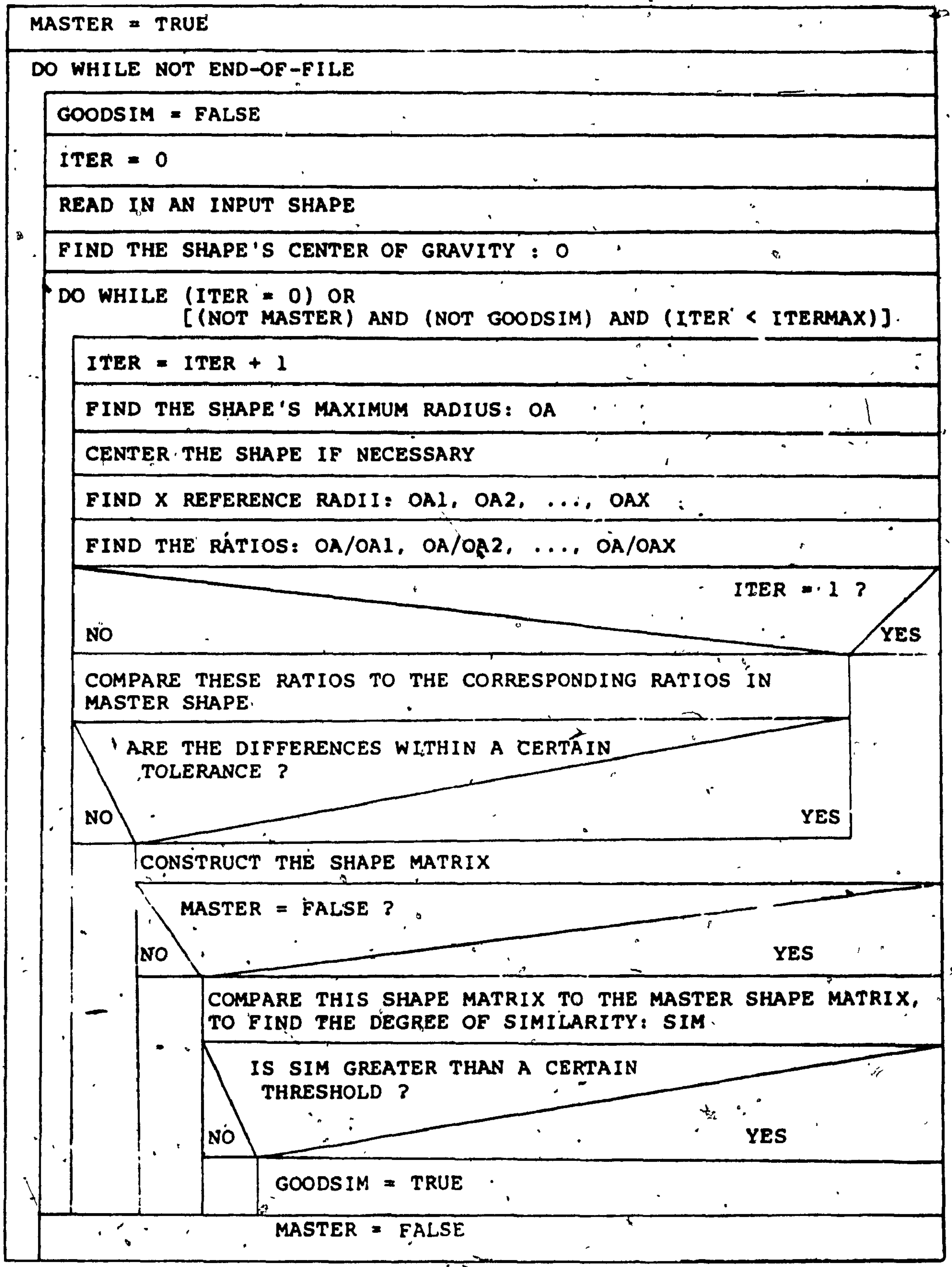


Fig. 3 Nassi-Shneiderman chart

Since this chart has to fit into one page, and our program is relatively sophisticated, the information in the chart was condensed, in order to show the complete "skeleton" of the program. On the other hand, some details like input/output and initializations, were omitted.

CHAPTER 7

RESULTS

The following sets of data have been inputted to the program:

1. The alphabet letters
2. Numeric / alphabetic characters which look alike
3. Geometrical figures set 1
4. Geometrical figures set 2
5. Nuts and screws
6. French curves

We will analyze and interpret the results of each set separately:

1. The alphabet letters: (see fig. 4)

   This table contains two sets of the uppercase letters of the alphabet. The first set was reduced in size and rotated to produce the second set.

   Therefore the two sets are similar, but there are differences in the position, orientation, and size for every two similar letters.

   The main purpose of producing this table is to compare it to table 1 in the main reference [1], which treats the same data. We can notice the following for the ranges of matches in percentages:

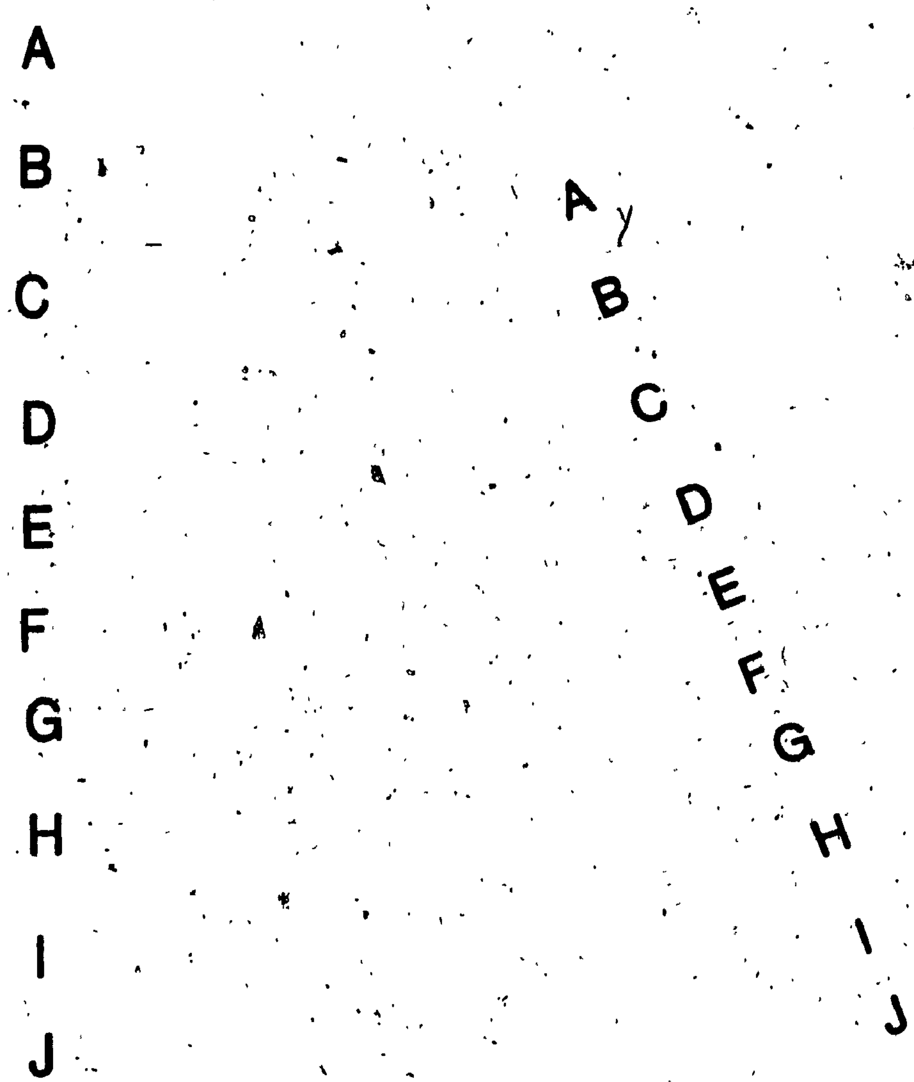


Fig. 4 (a) - 1 The alphabet letters: data

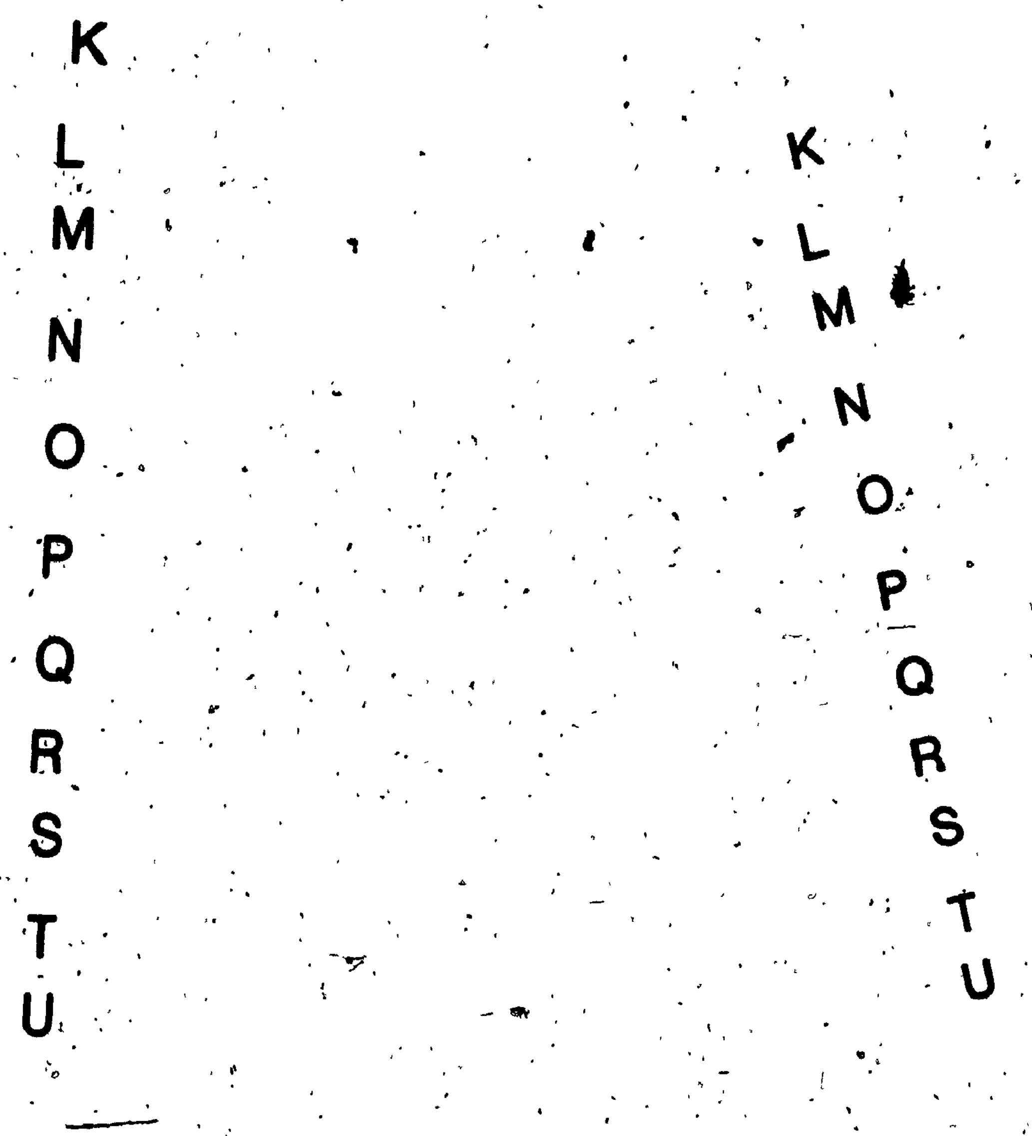


Fig. 4 (a) - 2 The alphabet letters: data

Fig. 4 (a) - 3 The alphabet letters: data

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	72	5	-42	-20	-24	-22	-27	-40	-76	-40	-32	-91	6	-19	-41	10	-37	-11	-28	-63	-20	25	-15	-37	-25	-46	A
B	-27	83	-5	43	-78	-102	31	7	145	137	-98	-172	-17	3	2	-74	-7	-33	-30	-168	-97	-57	-62	-60	-114	-55	B
C	-51	-4	73	6	-65	-87	-13	-52	-137	-102	-101	-150	-13	-10	33	-63	-16	-47	-13	-138	-40	-78	-67	-81	-102	-60	C
D	-33	60	28	85	-31	-75	55	-11	-136	-140	-99	-159	-17	-24	22	-69	42	-32	12	-147	-3	-78	-49	-73	-105	-14	D
E	-20	32	-1	2	80	-30	4	-12	-86	-46	-7	-117	-36	-3	-31	-27	-17	0	4	-84	-25	-69	7	-20	-64	17	E
F	7	7	-26	-20	-9	84	-22	-23	-39	27	-7	-44	-25	-3	-37	42	-23	-7	-8	-31	-22	13	-17	-10	9	-27	F
G	-37	38	22	44	-28	-72	84	3	-114	-150	-84	-135	-3	-20	24	-77	46	-50	18	-133	-40	-78	-38	-67	-87	-30	G
H	-25	48	-4	19	-30	-47	21	73	-74	-107	-43	-116	25	51	-18	-52	1	20	19	-104	13	-51	-13	-40	-67	-46	H
I	-7	0	-23	-19	-18	-13	-8	-4	89	-17	11	-8	1	-21	-21	-13	-16	-4	-5	18	-30	-20	-12	22	-13	-35	I
J	-9	3	-17	-17	-1	43	-25	-19	-1	84	1	-36	-8	-8	-31	31	-11	18	-17	-1	0	-10	-12	-6	14	-25	J
K	4	4	-26	-24	17	-6	-21	-16	-16	-28	80	-33	-8	0	-33	-8	-26	7	-13	-21	-17	-13	-16	22	-13	-7	K
L	18	-4	-22	-18	-21	-3	-9	-20	-2	-28	-1	80	11	-19	-24	-7	-19	-7	-11	30	-23	6	0	0	15	-33	L
M	-29	12	-26	-2	2	-66	4	-36	-97	-80	-52	-121	85	-5	-31	-34	-3	-25	-24	-109	-33	-74	18	-18	-89	-14	M
N	-25	7	-37	-16	-17	-48	-14	37	-132	-84	-51	-148	-9	88	-28	-34	-17	-12	-28	-127	-40	-54	-25	-47	-78	-27	N
O	-89	-10	1	-11	-84	-140	-2	-80	-189	-196	-147	-224	-53	-61	85	-139	-52	-112	17	-213	-115	-126	-116	-107	-145	-55	O
P	29	-1	-27	-18	-7	37	-31	-33	-57	21	-16	-62	-15	5	-45	88	-19	-1	-24	-29	-2	19	-3	-14	-6	-29	P
Q	-33	27	11	42	-39	-65	62	-12	-107	-115	-79	-129	-5	-13	43	-65	80	-47	0	-118	-33	-53	-46	-58	-77	-34	Q
R	-10	53	-16	13	-6	-22	-9	16	-59	-43	-17	-88	0	8	-39	-4	2	78	-8	-63	49	-43	-14	-27	-42	-30	R
S	-65	31	4	-3	-50	-86	13	-56	-150	-168	-129	-185	-47	-53	24	-98	-52	-84	87	-174	-105	-109	-87	-93	-83	-53	S
T	6	-4	-22	-18	-11	-17	-13	-16	26	-20	3	11	5	-22	-29	6	-17	-4	-11	80	-17	-7	-12	2	-1	-35	T
U	3	0	-9	10	-30	-29	1	30	-99	-56	-41	-83	5	35	-26	-6	17	33	-39	-75	82	-11	-3	-40	-59	-36	U
V	54	15	-30	-9	-7	-2	-16	-18	-64	-42	-15	-59	26	-7	-30	-11	-27	7	-21	-42	-4	72	-11	-33	-23	-29	V
W	-8	1	-28	-9	7	-35	-11	-27	-66	-48	-43	-75	-14	0	-39	-11	-16	-7	-23	-56	-21	-47	81	-9	-60	-20	W
X	-14	-5	-33	-23	-11	-24	-20	-28	-14	-43	12	-67	-31	-10	-28	-20	-31	-23	-14	-45	-44	-24	-2	81	-26	20	X
Y	23	-5	-30	-22	-36	0	-17	-16	-33	-27	11	-48	-7	-18	-23	-13	-19	-25	-9	-37	-40	23	-45	-1	81	-27	Y
Z	-30	-16	-32	-23	-59	-55	-1	-35	6	-88	-39	-59	1	-9	-1	-54	-17	-55	-6	-34	-69	-44	-50	-8	-55	80	Z
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	

Fig. 4 (b) The alphabet letters' results

	Reference [1]		Present Work	
	low	high	low	high
1. Similar letters	82 (M)	94 (J)	72 (V)	89 (I)
2. Dissimilar letters	25 (O-Y)	83 (Q-D)	-224 (L-O)	62 (G-Q)

We believe that our results are better because:

1. There is a threshold range of 10 (= 72 - 62) in our results. In the reference, there is no threshold range, but there is an overlap (83 > 82).

2. The difference in the degree of similarity between similar and dissimilar letters, is much bigger in our work than in the reference. In other words, we have a better shape discrimination.

We think there are two reasons for these advantages:

1. Our algorithm uses the concept of weight that neutralizes the effect of the different degrees of redundancy in the shape matrix. We believe that this concept is essential for this method of shape discrimination. This concept does not exist in the reference.

2. The formula that calculates the degree of similarity, in the reference, is

$$sim = 1 - \frac{S}{(m - 2) * n}$$

where S : number of different elements (without weight)

m, n: the same as in our work

Our formula is

$$sim = 1 - \frac{dif}{tot}$$

where dif: weight of different elements

tot: weight of "1" elements in master shape matrix

We believe that our formula is more accurate.

2. Numeric / alphabetic characters which look alike (see fig. 5)

We selected five numerics and five alphabetic letters, that have a high degree of resemblance.

The purpose of this test is to determine the sensitivity of the algorithm to a practical character recognition case, namely, close characters.

The results show a good discrimination for these characters, except for the letter O versus numeral 0.

0

1

2

5

8

0

I

Z

S

B

Fig. 5 (a) Numeric / alphabetic characters which look alike: data

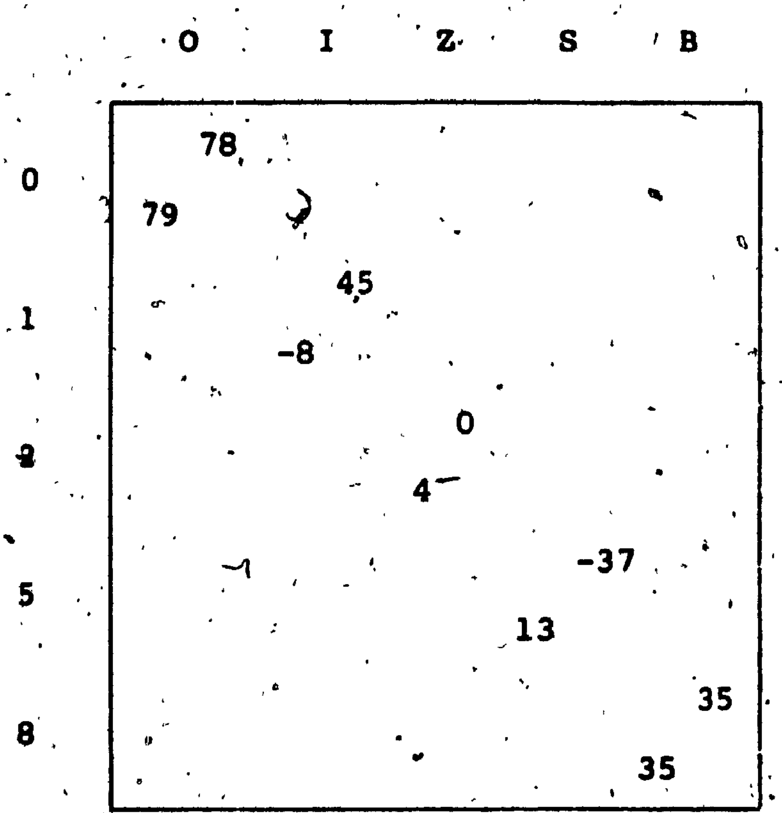


Fig. 5 (b) Numeric / alphabetic characters which look alike:

results

These two characters in the used alphabet set (different from the previous set) look so much alike, that our program fails to discriminate them enough. Their degree of similarity is too high for two different characters.

3. Geometrical figures set 1 (see fig. 6)

This table contains three different shapes of five different geometrical figures. These figures are not ideal, i.e., the output from the scanner has been processed as it is without any enhancement or pre-processing. It can be observed that the straight lines are not perfect in the digitized shape.

The purpose of this data is to verify that our shape descriptor (the shape matrix) is independent of position, orientation, and size of the shape.

This is confirmed by the results. The degrees of similarity range from 89% to 95%. This is very satisfactory for a practical case where the data have not been edited.

4. Geometrical figures set 2 (see fig. 7)

This table contains a collection of different geometrical figures. These figures, except the circle, are ideal, i.e., the straight lines, and the dimensions

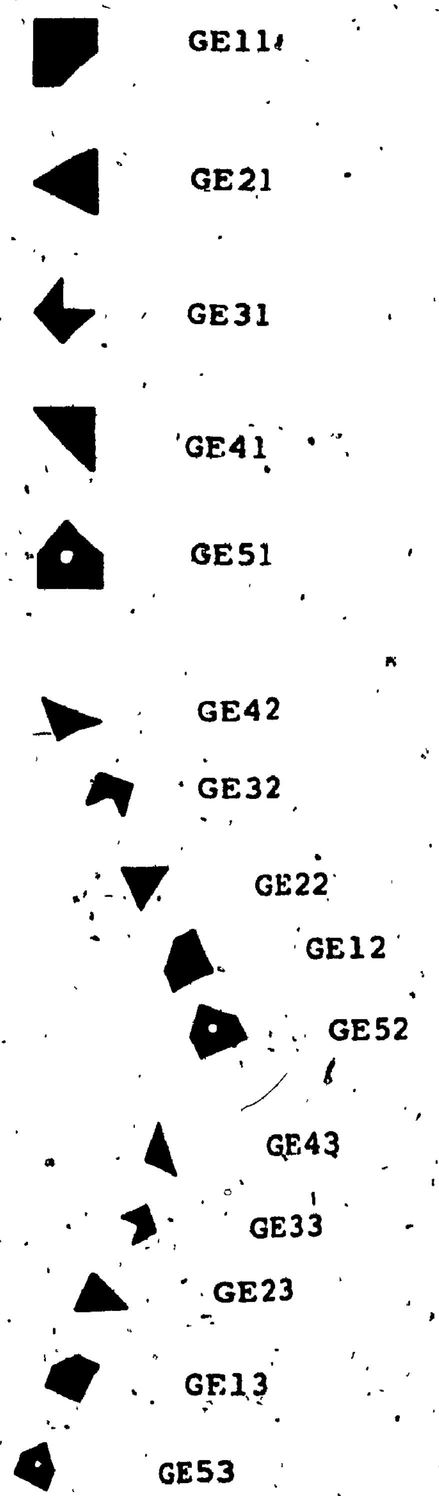


Fig. 6 (a) Geometrical figures set 1: data

	GE13	GE23	GE33	GE43	GE53
GE12	93				
GE22		94			
GE32			91		
GE42				89	
GE52					89
GE11	94				
GE21		93			
GE31			93		
GE41				91	
GE51					95

Fig. 6 (b) Geometrical figures set 1: results

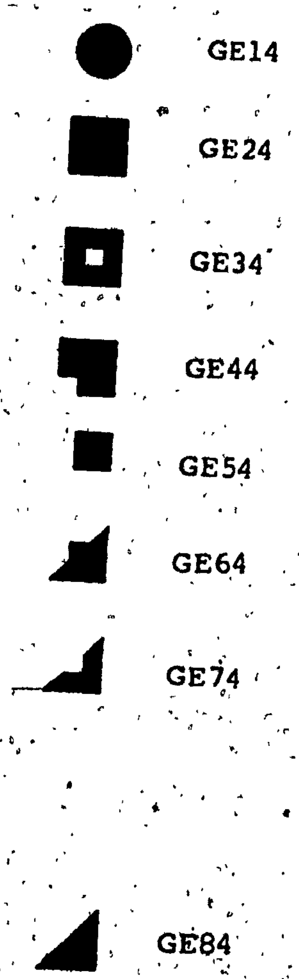


Fig. 7 (a) Geometrical figures set 2: data

	GE14	GE24	GE34	GE44	GE54	GE64	GE74	GE84
GE14					48			
GE24	66				98			
GE34		89			87			
GE44		79	64		79			
GE54								
GE64								80
GE74						66		81
GE84								

Fig. 7 (b) Geometrical figures set 2: results are accurate and well defined. The outputs from the scanner have been edited slightly, except for the circle, to produce this perfection.

The purposes of this data are:

1. To examine the effects of defects or distortions in shapes on the degree of similarity. The degrees of similarity between GE24 versus GE34 and ge44, shows that the similarity is affected not only by the amount of the defect, but also by the location of that defect, i.e., by the displacement of the center of gravity.

   GE34 and GE44 are distorted versions of ge24. The amount of the distortion is the same, but the location is different. In GE34, the center of gravity was not displaced; we obtained a similarity of 89%. In GE44, the center of gravity was displaced; we obtained a similarity of 79%. This is an interesting remark.

2. To confirm that the shape matrix is independent of position and size of the shape. The degree of similarity between GE54 and GE24 is 98%.

3. This is the most interesting purpose. It is to examine the validity and accuracy of the degree of

similarity, as a measurement. This point is treated in detail in chapter 8.

5. Nuts and screws (see fig. 8).

The data consist of two nuts and two screws, in three different shapes.

The purpose of this table is to test a practical case. The results are in general satisfactory, but relatively weak. The reason is that the images are very light in some parts; and after digitization, they are hardly recognizable.

6. French curves (see fig. 9)

The data consist of six French curves, in three different shapes.

The purpose of this table is to test another practical case.

Again, ~~the~~ results are in general satisfactory, but relatively weak. We think that in the case of a thin curve, the artificial distortion introduced by the digitization, is relatively high. In this case, a higher resolution will substantially improve the results.

A sample program output is given in the following

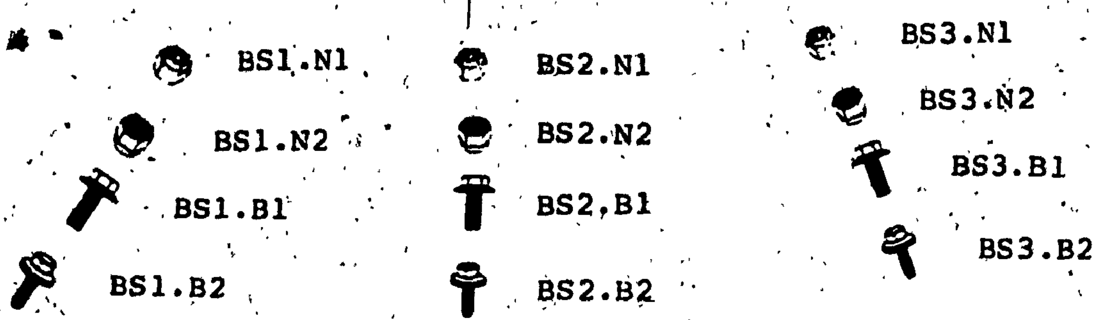


Fig. 8 (a) Nuts and screws: data

	BS1.N1	BS1.N2	BS1.B1	BS1.B2
BS2.N1	62			
BS2.N2		73		
BS2.B1			80	
BS2.B2				68
BS3.N1	60			
BS3.N2		73		
BS3.B1			80	
BS3.B2				72

Fig. 8 (b) Nuts and screws: results

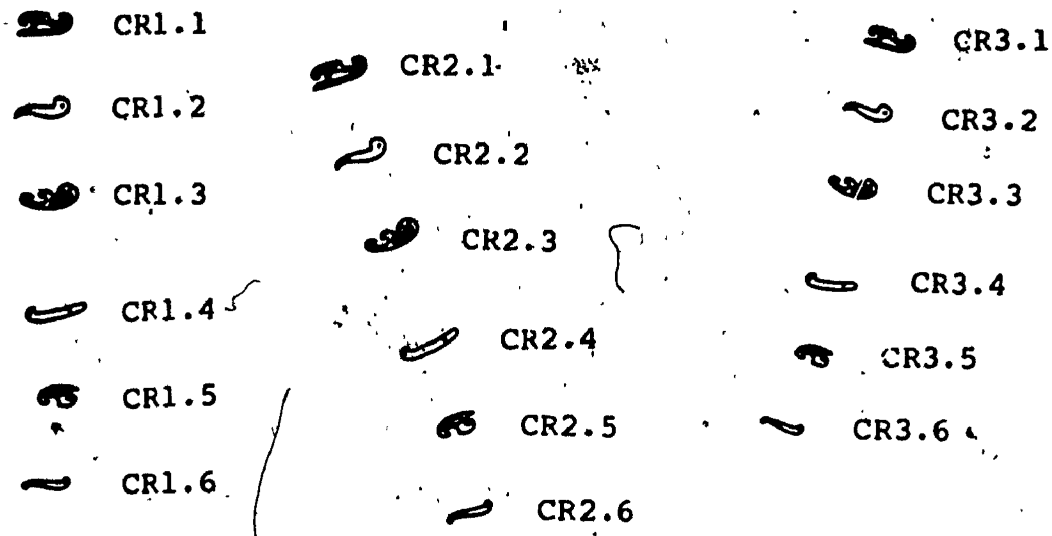


Fig. 9 (a) French curves: data

	CR1.1	CR1.2	CR1.3	CR1.4	CR1.5	CR1.6
CR2.1	69					
CR2.2		65				
CR2.3			57			
CR2.4				48		
CR2.5					76	
CR2.6						77
CR3.1	64					
CR3.2		55				
CR3.3			68			
CR3.4				60		
CR3.5					72	
CR3.6						71

Fig. 9 (b) French curves: results

pages. Other outputs are given in the technical report.

Master shape:

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                111111111122222222223333333333444444444455555555556
       123456789012345678901234567890123456789012345678901234567890

  1    .......................XXXXXXXXX............................
  2    ...................XXXXXXXXXXXXXXX..........................
  3    ...............XXXXXXXXXXXXXXXXXXXX.........................
  4    ...........XXXXXXXXXXXXXXXXXXXXXXXXX........................
  5    ...........XXXXXXXXXXXXXXXXXXXXXXXXXX.......................
  6    ..........XXXXXXXXXXXX.........XXXXXXX......................
  7    ...........XXXXXXX..............XXXXXX......................
  8    ...........XXXXXX................XXXXX......................
  9    ...........XXXXXX................XXXXXX.....................
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59 ....
60 ....

Frame dimensions: 60 x 60

Number of black pixels in input shape: 415

Center of gravity "O": 24 , 15

Length of maximum radius "OA": 22.6

Point "A": 19 , 37

Theta = 103

Master centered shape:

```
               111111111122222222223333333333444444444455555555556
      123456789012345678901234567890123456789012345678901234567890

   1  ............................................................
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  48  .......................XXXXX................................
  49  .......................XXXXXX...............................
  50  ......................,XXXXXX...............................
  51  ........................XXXXX...............................
  52  ........................rXXXX...............................
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  56  ............................................................
  57  ............................................................
  58  ............................................................
```

```
59  ............................................................
60  ............................................................
```

Frame dimensions: 60 x 60

Number of black pixels in input shape: 415

Center of gravity "O": 30 , 30

Length of maximum radius "OA": 22.6

Point "A": 25 , 52

Theta = 103

Length of radius "OA1": 10.3

Point "A1": 21, 35

Theta1 = 148

OA/OA1 = 2.191

Length of radius "OA2": 11.2

Point "A2": 19, 28

Theta2 = 193

OA/OA2 = 2.018

Length of radius "OA3": 14.4

Point "A3": 22, 18

Theta3 = 238

OA/OA3 = 1.564

n̂ = 23   m = 142

Master shape matrix:

```
                            1 1 1 1 1 1 1 1 1 1 2 2 2
        0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2

    0   . . . . * * * * . . . . * * * * * * * * * * *
    1   . . . . * * * * . . * * * * * * * * * * * . .
    2   . . . . * * * * * * * * * * * * * * * * * . .
    3   . . . . * * * * * * * * * * * * * * * . . . .
    4   . . . . * * * * * * * * * * * * * * . . . . .
    5   . . . . * * * * * * * * * * * * * * . . . . .
    6   . . . . * * * * * * * * * * * * . . . . . . .
    7   . . . . * * * * * * * * * * * . . . . . . . .
    8   . . . . . * * * * * * * * * * . . . . . . . .
    9   . . . . . * * * * * * * * * * . . . . . . . .
   10   . . . . . * * * * * * * * * . . . . . . . . .
   11   . . . . . * * * * * * * * . . . . . . . . . .
   12   . . . . . * * * * * * * * . . . . . . . . . .
   13   . . . . . * * * * * * * . . . . . . . . . . .
   14   . . . . . . * * * * * * . . . . . . . . . . .
   15   . . . . . . * * * * * * . . . . . . . . . . .
   16   . . . . . . * * * * * * . . . . . . . . . . .
   17   . . . . . . * * * * * * . . . . . . . . . . .
   18   . . . . . . * * * * * . . . . . . . . . . . .
   19   . . . . . . * * * * * . . . . . . . . . . . .
   20   . . . . . * * * * * * . . . . . . . . . . . .
   21   . . . . . * * * * * * . . . . . . . . . . . .
   22   . . . . . * * * * * . . . . . . . . . . . . .
   23   . . . . . * * * * * * . . . . . . . . . . . .
   24   . . . . . * * * * * * . . . . . . . . . . . .
   25   . . . . . * * * * * * . . . . . . . . . . . .
   26   . . . . . * * * * * * . . . . . . . . . . . .
   27   . . . . . * * * * * * . . . . . . . . . . . .
   28   . . . . . * * * * * * . . . . . . . . . . . .
   29   . . . . . . * * * * * . . . . . . . . . . . .
   30   . . . . . . * * * * * . . . . . . . . . . . .
   31   . . . . . . * * * * * . . . . . . . . . . . .
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   54   . . . . . . . . . . * * * * * . . . . . . . .
   55   . . . . . . . . . . * * * * . . . . . . . . .
   56   . . . . . . . . . . * * * * . . . . . . . . .
   57   . . . . . . . . . * * * * * . . . . . . . . .
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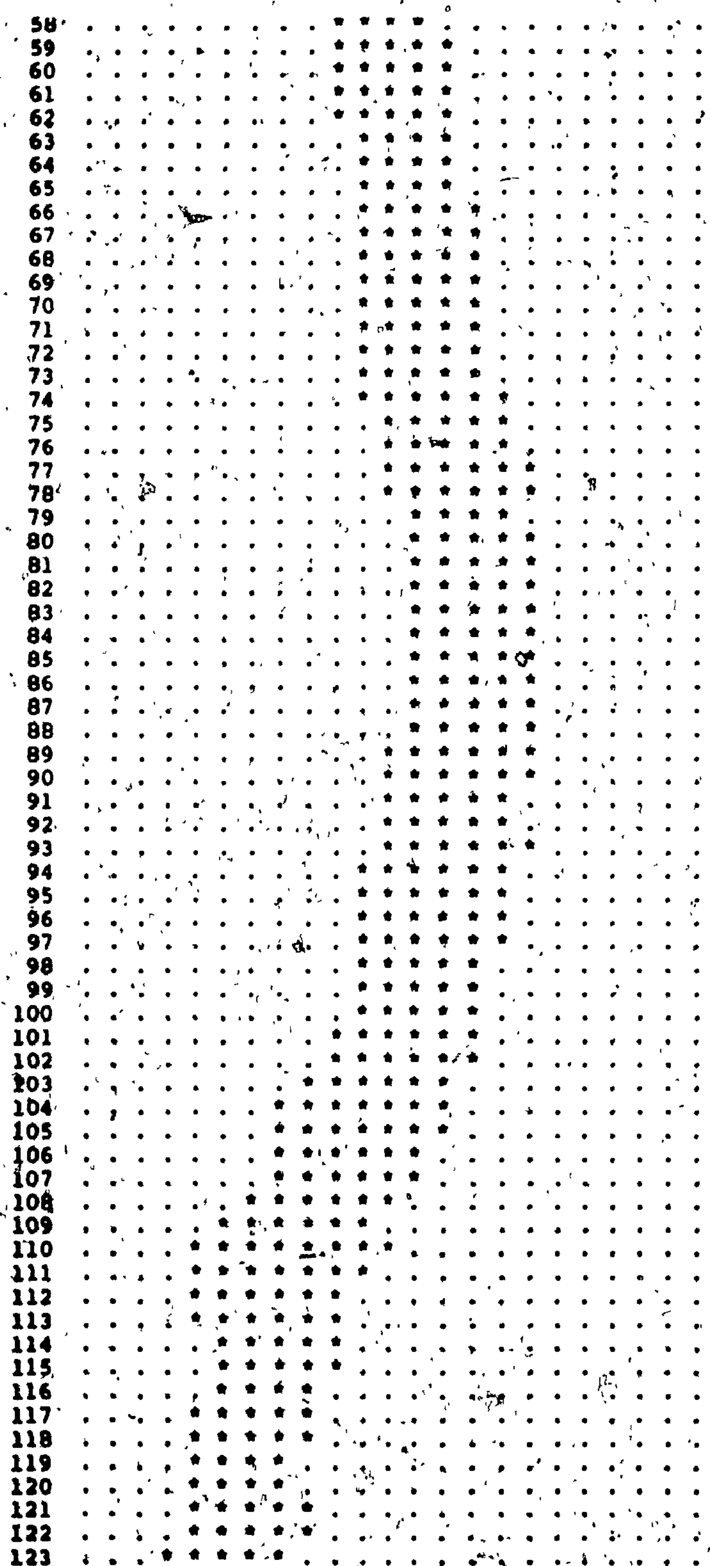

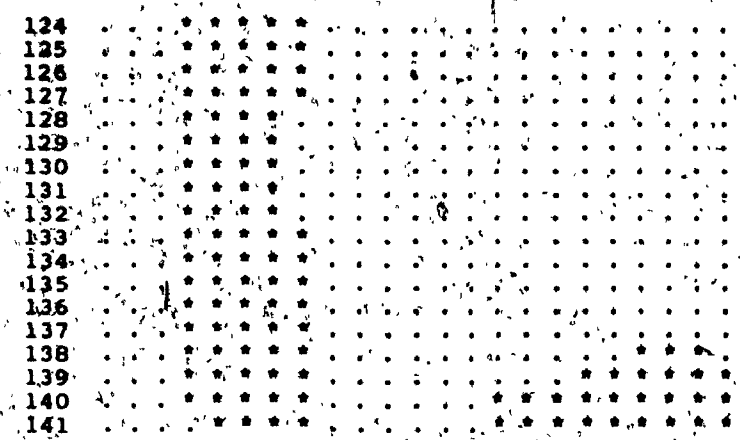
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Test shape:

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             111111111122222222223333333333444444444455555555556
    123456789012345678901234567890123456789012345678901234567890

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58  ............................................................
```

59 ....
60 ....

Frame dimensions: 60 x 60

Number of black pixels in input shape: 539

Center of gravity "O": 32 , 17

Length of maximum radius "OA": 26.4

Point "A": 21 , 41

Theta = 115

Test centered shape:

```
              111111111122222222223333333333444444444455555555556
     123456789012345678901234567890123456789012345678901234567890

  1  ............................................................
  2  ............................................................
  3  ............................................................
  4  ............................................................
  5  ............................................................
  6  ............................................................
  7  ............................................................
  8  ............................................................
  9  ............................................................
 10  ............................................................
 11  ............................................................
 12  ............................................................
 13  ............................................................
 14  ....................XXXXX...................................
 15  ...................XXXXXXXXXXXXXXXXXXXX.....................
 16  ..................XXXXXXXXXXXXXXXXXXXXXXXX..................
 17  ..................XXXXXXXXXXXXXXXXXXXXXXXXX.................
 18  ..................XXXXXXXXXXXXXXXXXXXXXXXXXXX...............
 19  ..................XXXXXXXXXXXXXXXXXXXXXXXXXXXX..............
 20  ..................XXXXXXX............XXXXXXXXX..............
 21  ..................XXXXXX...............XXXXXXXX.............
 22  ..................XXXXXX.................XXXXXX.............
 23  ..................XXXXXX.................XXXXXXX............
 24  ..................XXXXXX..................XXXXXX............
 25  ..................XXXXXX..................XXXXXX............
 26  ..................XXXXXX..................XXXXXX............
 27  ..................XXXXXX..................XXXXXX............
 28  ..................XXXXXX..................XXXXXX............
 29  ..................XXXXXX..................XXXXXX............
 30  ..................XXXXXX.................XXXXXXX............
 31  ..................XXXXXX.................XXXXXX.............
 32  ..................XXXXXX................XXXXXXX.............
 33  ..................XXXXXXXX...........XXXXXXXXX..............
 34  ..................XXXXXXXXXXXXXXXXXXXXXXXXXXXX..............
 35  ..................XXXXXXXXXXXXXXXXXXXXXXXXXXX...............
 36  ..................XXXXXXXXXXXXXXXXXXXXXXXXXX................
 37  .................XXXXXXXXXXXXXXXXXXXXXXXXX..................
 38  .................XXXXXXXXXXXXXXXXXXXXXXX....................
 39  .................XXXXXXX....................................
 40  .................XXXXXXX....................................
 41  .................XXXXXXX....................................
 42  .................XXXXXXX....................................
 43  .................XXXXXXX....................................
 44  .................XXXXXX.....................................
 45  .................XXXXXX.....................................
 46  .................XXXXXX.....................................
 47  .................XXXXXX.....................................
 48  .................XXXXXX.....................................
 49  .................XXXXXX.....................................
 50  .................XXXXXX.....................................
 51  .................XXXXXX.....................................
 52  .................XXXXXX.....................................
 53  .................XXXXXX.....................................
 54  ..................rXXX......................................
 55  ............................................................
 56  ............................................................
 57  ............................................................
 58  ............................................................
```

59 ....
60 ....

Frame dimensions: 60 x 60

Number of black pixels in input shape: 539

Center of gravity "O": 30 , 30

Length of maximum radius "OA": 26.4

Point "A": 19 , 54

Theta = 115

Length of radius "OA1": 11.7

Point "A1": 19, 34

Theta1 = 160

OA/OA1 = 2.256

Percent difference from master shape: 2.9

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 205

OA/OA2 = 2.185

Percent difference from master shape: 8.3

Length of radius "OA3": 17.1

Point "A3": 24, 14

Theta3 = 250

OA/OA3 = 1.545

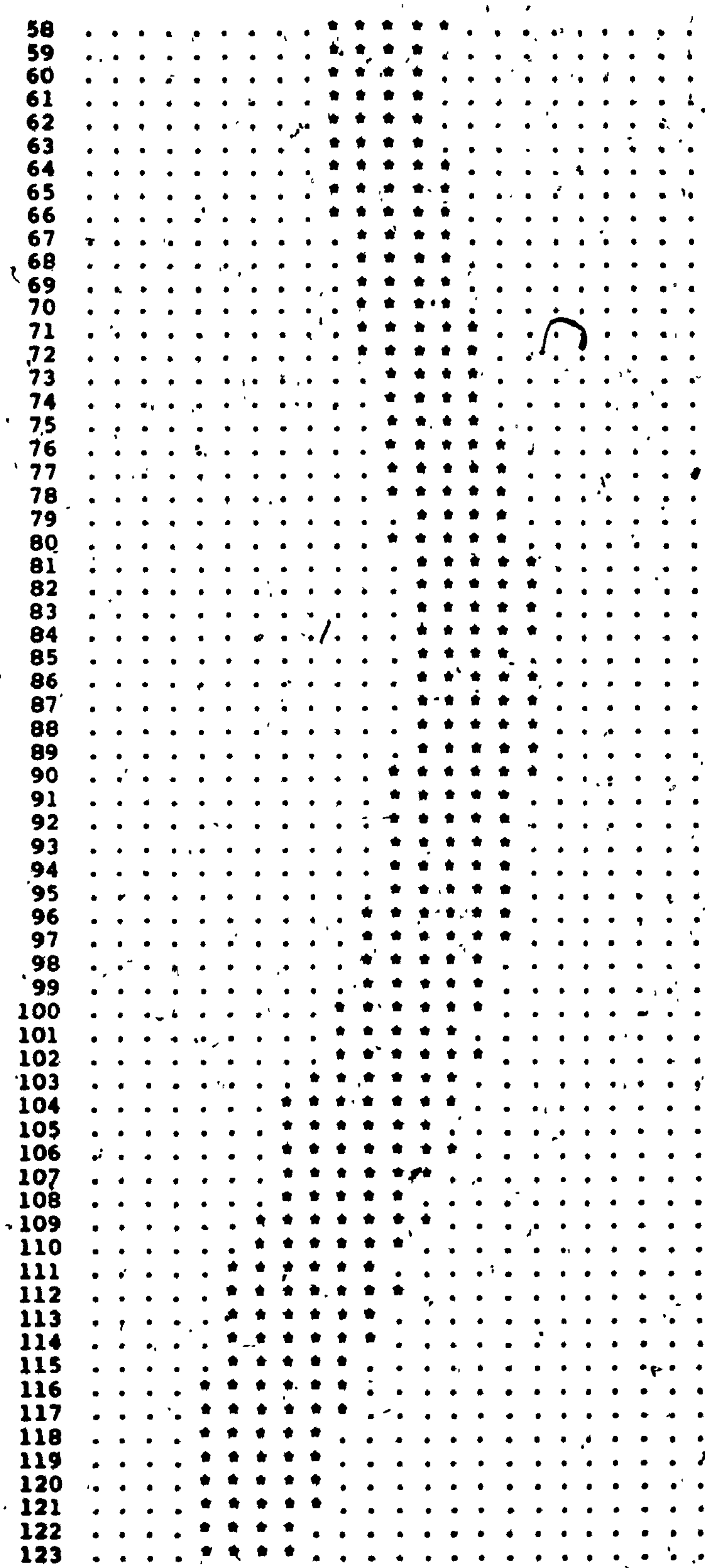
Percent difference from master shape: 1.2

n = 23   m = 142

Test shape matrix:

```
                                  1 1 1 1 1 1 1 1 1 1 2 2 2
          0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2

 0        . . . . * * * * . . . . * . * * * * * * * * *
 1        . . . . * * * * . . . * * * * * * * * * * * *
 2        . . . . * * * * * . * * * * * * * * * * * * .
 3        . . . . * * * * * * * * * * * * * * * * . . .
 4        . . . . * * * * * * * * * * * * * * * . . . .
 5        . . . . * * * * * * * * * * * * * * . . . . .
 6        . . . . * * * * * * * * * * * * * . . . . . .
 7        . . . . * * * * * * * * * * * * . . . . . . .
 8        . . . . . * * * * * * * * * * . . . . . . . .
 9        . . . . * * * * * * * * * * * . . . . . . . .
10        . . . . * * * * * * * * * * . . . . . . . . .
11        . . . . * * * * * * * * * * . . . . . . . . .
12        . . . . * * * * * * * * * . . . . . . . . . .
13        . . . . * * * * * * * * * . . . . . . . . . .
14        . . . . . * * * * * * * * . . . . . . . . . .
15        . . . . . * * * * * * . . . . . . . . . . . .
16        . . . . . * * * * * * . . . . . . . . . . . .
17        . . . . . * * * * * * . . . . . . . . . . . .
18        . . . . . * * * * * * . . . . . . . . . . . .
19        . . . . . * * * * * * . . . . . . . . . . . .
20        . . . . . * * * * * . . . . . . . . . . . . .
21        . . . . . * * * * * . . . . . . . . . . . . .
22        . . . . . * * * * * . . . . . . . . . . . . .
23        . . . . . * * * * * . . . . . . . . . . . . .
24        . . . . . * * * * * . . . . . . . . . . . . .
25        . . . . . * * * * * . . . . . . . . . . . . .
26        . . . . . * * * * * . . . . . . . . . . . . .
27        . . . . . * * * * * . . . . . . . . . . . . .
28        . . . . . * * * * * . . . . . . . . . . . . .
29        . . . . . * * * * * . . . . . . . . . . . . .
30        . . . . . * * * * * . . . . . . . . . . . . .
31        . . . . . * * * * * . . . . . . . . . . . . .
32        . . . . . * * * * * . . . . . . . . . . . . .
33        . . . . . * * * * * * . . . . . . . . . . . .
34        . . . . . * * * * * * . . . . . . . . . . . .
35        . . . . . * * * * * * . . . . . . . . . . . .
36        . . . . . . * * * * * . . . . . . . . . . . .
37        . . . . . . * * * * * . . . . . . . . . . . .
38        . . . . . . * * * * * * . . . . . . . . . . .
39        . . . . . . * * * * * * . . . . . . . . . . .
40        . . . . . . * * * * * * . . . . . . . . . . .
41        . . . . . . * * * * * * * . . . . . . . . . .
42        . . . . . . . * * * * * * . . . . . . . . . .
43        . . . . . . . * * * * * * * . . . . . . . . .
44        . . . . . . . * * * * * * * . . . . . . . . .
45        . . . . . . . * * * * * * * * . . . . . . . .
46        . . . . . . . . * * * * * * * * . . . . . . .
47        . . . . . . . . * * * * * * * . . . . . . . .
48        . . . . . . . . . * * * * * * * . . . . . . .
49        . . . . . . . . . * * * * * * * . . . . . . .
50        . . . . . . . . . . * * * * * * . . . . . . .
51        . . . . . . . . . * * * * * * * . . . . . . .
52        . . . . . . . . . . * * * * * . . . . . . . .
53        . . . . . . . . . . * * * * * . . . . . . . .
54        . . . . . . . . . . * * * * * . . . . . . . .
55        . . . . . . . . . . * * * * * . . . . . . . .
56        . . . . . . . . . * * * * * . . . . . . . . .
57        . . . . . . . . . * * * * * . . . . . . . . .
```



```
124  . . . . * * * * . . . . . . . . . . . . . . .
125  . . . . * * * * . . . . . . . . . . . . . . .
126  . . . . * * * * . . . . . . . . . . . . . . .
127  . . . * * * * * . . . . . . . . . . . . . . .
128  . . . * * * * * . . . . . . . . . . . . . . .
129  . . . * * * * * . . . . . . . . . . . . . . .
130  . . . * * * * * . . . . . . . . . . . . . . .
131  . . . * * * * * . . . . . . . . . . . . . . .
132  . . . * * * * * . . . . . . . . . . . . . . .
133  . . . * * * * * . . . . . . . . . . . . . . .
134  . . . * * * * * . . . . . . . . . . . . . . .
135  . . . * * * * * . . . . . . . . . . . . . . .
136  . . . * * * * * . . . . . . . . . . . . . . .
137  . . . * * * * * . . . . . . . . . . . . . . .
138  . . . . * * * * . . . . . . . . . . . . . . .
139  . . . . * * * * . . . . . . . . . . . * * . .
140  . . . . * * * * . . . . . . . . . * * * * * .
141  . . . . * * * * . . . . . . . * * * * * * * *
```

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 155

Their weight: 1653

Degree of Similarity: 0.82

Trial number 2

Length of maximum radius "OA": 26.0

Point "A": 20 , 54

Theta = 113

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 158

OA/OA1 = 2.152

Percent difference from master shape: 1.8

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 203

OA/OA2 = 2.152

Percent difference from master shape: 6.6

Length of radius "OA3": 17.1

Point "A3": 24, 14

Theta3 = 248

OA/OA3 = 1.522

Percent difference from master shape: 2.7

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 185

Their weight: 2069

Degree of Similarity: 0.77

Trial number 3

Length of maximum radius "OA": 25.9

Point "A": 18 , 53

Theta = 118

Length of radius "OA1": 11.4

Point "A1": 19, 33

Theta1 = 163

OA/OA1 = 2.275

Percent difference from master shape: 3.8

Length of radius "OA2": 12.5

Point "A2": 19, 24

Theta2 = 208

OA/OA2 = 2.070

Percent difference from master shape: 2.6

Length of radius "OA3": 16.8

Point "A3": 25, 14

Theta3 = 253

OA/OA3 = 1.548

Percent difference from master shape: 1.1

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 113

Their weight: 1052

Degree of Similarity: 0.88

Trial number 4

Length of maximum radius "OA": 25.6

Point "A": 21, 54

Theta = 111

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 156

OA/OA1 = 2.121

Percent difference from master shape: 3.2

Length of radius "OA2": 11.7

Point "A2": 19, 26

Theta2 = 201

OA/OA2 = 2.190

Percent difference from master shape: 8.5

Length of radius "OA3": 17.5

Point "A3": 23, 14

Theta3 = 246

OA/OA3 = 1.468

Percent difference from master shape: 6.2

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 250

Their weight: 2866

Degree of Similarity: 0.68

Trial number 5

Length of maximum radius "OA": 25.5

Point "A": 19 , 53

Theta = 116

Length of radius "OA1": 11.7

Point "A1": 19, 34

Theta1 = 161

OA/OA1 = 2.178

Percent difference from master shape: 0.6

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 206

OA/OA2 = 2.110

Percent difference from master shape: 4.6

Length of radius "OA3": 17.1

Point "A3": 24, 14

Theta3 = 251

OA/OA3 = 1.492

Percent difference from master shape: 4.6

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 152

Their weight: 1559

Degree of Similarity: 0.83

Trial number 6

Length of maximum radius "OA": 25.3

Point "A": 22 , 54

Theta = 108

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 153

OA/OA1 = 2.094

Percent difference from master shape: 4.5

Length of radius "OA2": 11.7

Point "A2": 19, 26

Theta2 = 198

OA/OA2 = 2.161

Percent difference from master shape: 7.1

Length of radius "OA3": 17.9

Point "A3": 22, 14

Theta3 = 243

OA/OA3 = 1.414

Percent difference from master shape: 9.6

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 305

Their weight: 3571

Degree of Similarity: 0.60

Trial number 7

Length of maximum radius "OA": 25.1

Point "A": 20 , 53

Theta = 113

Length of radius "OA1": 11.7

Point "A1": 19, 34

Theta1 = 158

OA/OA1 = 2.143

Percent difference from master shape: 2.2

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 203

OA/OA2 = 2.076

Percent difference from master shape: 2.9

Length of radius "OA3": 17.1

Point "A3": 24, 14

Theta3 = 248

OA/OA3 = 1.468

Percent difference from master shape: 6.2

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 209

Their weight: 2332

Degree of Similarity: 0.74

Trial number 8

Length of maximum radius "OA": 25.1

Point "A": 18 , 52

Theta = 119

Length of radius "OA1": 11.4

Point "A1": 19, 33

Theta1 = 164

OA/OA1 = 2.198

Percent difference from master shape: 0.3

Length of radius "OA2": 12.5

Point "A2": 19, 24

Theta2 = 209

OA/OA2 = 2.000

Percent difference from master shape: 0.9

Length of radius "OA3": 16.8

Point "A3": 25, 14

Theta3 = 254

OA/OA3 = 1.495

Percent difference from master shape: 4.4

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 144

Their weight: 1438

Degree of Similarity: 0.84

Trial number 9

Length of maximum radius "OA": 24.7

Point "A": 21 , 53

Theta = 111

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 156

OA/OA1 = 2.044

Percent difference from master shape: 6.7

Length of radius "OA2": 11.7

Point "A2": 19, 26

Theta2 = 201

OA/OA2 = 2.110

Percent difference from master shape: 4.6

Length of radius "OA3": 17.5

Point "A3": 23, 14

Theta3 = 246

OA/OA3 = 1.414

Percent difference from master shape: 9.6

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 270

Their weight: 3141

Degree of Similarity: 0.65

Trial number 10

Length of maximum radius "OA": 24.6

Point "A": 19 , 52

Theta = 117

Length of radius "OA1": 11.7

Point "A1": 19, 34

Theta1 = 162

OA/OA1 = 2.101

Percent difference from master shape: 4.1

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 207

OA/OA2 = 2.036

Percent difference from master shape: 0.9

Length of radius "OA3": 16.8

Point "A3": 25, 14

Theta3 = 252

OA/OA3 = 1.467

Percent difference from master shape: 6.2

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 183

Their weight: 1986

Degree of Similarity: 0.78

Trial number 11

Length of maximum radius "OA": 24.4

Point "A": 22 , 53

Theta = 109

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 154

OA/OA1 = 2.015

Percent difference from master shape: 8.0

Length of radius "OA2": 11.7

Point "A2": 19, 26

Theta2 = 199

OA/OA2 = 2.080

Percent difference from master shape: 3.1

Length of radius "OA3": 17.9

Point "A3": 22, 14

Theta3 = 244

OA/OA3 = 1.361

Percent difference from master shape: 13.0

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 347

Their weight: 4154

Degree of Similarity: 0.54

Trial number 12

Length of maximum radius "OA": 24.2

Point "A": 18 , 51

Theta = 120

Length of radius "OA1": 11.4

Point "A1": 19, 33

Theta1 = 165

OA/OA1 = 2.121

Percent difference from master shape: 3.2

Length of radius "OA2": 12.5

Point "A2": 19, 24

Theta2 = 210

OA/OA2 = 1.930

Percent difference from master shape: 4.3

Length of radius "OA3": 15.5

Point "A3": 26, 15

Theta3 = 255

OA/OA3 = 1.558

Percent difference from master shape: 0.4

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 214

Their weight: 2344

Degree of Similarity: 0.74

Trial number 13

Length of maximum radius "OA": 24.2

Point "A": 20 , 52

Theta = 114

Length of radius "OA1": 11.7

Point "A1": 19, 34

Theta1 = 159

OA/OA1 = 2.065

Percent difference from master shape: 5.8

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 204

OA/OA2 = 2.000

Percent difference from master shape: 0.9

Length of radius "OA3": 17.1

Point "A3": 24, 14

Theta3 = 249

OA/OA3 = 1.414

Percent difference from master shape: 9.6

n = 23   m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 253

Their weight: 2912

Degree of Similarity: 0.68

Trial number 14

Length of maximum radius "OA": 24.0

Point "A": 23 , 53

Theta = 107

Length of radius "OA1": 13.9

Point "A1": 18, 37

Theta1 = 152

OA/OA1 = 1.731

Percent difference from master shape: 21.0

Length of radius "OA2": 11.4

Point "A2": 19, 27

Theta2 = 197

OA/OA2 = 2.109

Percent difference from master shape: 4.5

Length of radius "OA3": 17.9

Point "A3": 22, 14

Theta3 = 242

OA/OA3 = 1.344

Percent difference from master shape: 14.1

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 410

Their weight: 5034

Degree of Similarity: 0.44

Trial number 15

Length of maximum radius "OA": 23.8

Point "A": 21 , 52

Theta = 112

Length of radius "OA1": 12.1

Point "A1": 19, 35

Theta1 = 157

OA/OA1 = 1.967

Percent difference from master shape: 10.2

Length of radius "OA2": 12.1

Point "A2": 19, 25

Theta2 = 202

OA/OA2 = 1.967

Percent difference from master shape: 2.5

Length of radius "OA3": 17.5

Point "A3": 23, 14

Theta3 = 247

OA/OA3 = 1.361

Percent difference from master shape: 13.0

n = 23 m = 142

Number of "1" elements in master shape matrix: 891

Their weight: 8987

Number of different elements in shape matrices: 334

Their weight: 4060

Degree of Similarity: 0.55

CHAPTER 8

GEOMETRICAL ARGUMENTS

As mentioned in chapter 7, we want to examine the validity and the accuracy of the degree of similarity between two shapes, as produced by our program. In other words, we intend to test and evaluate the output: SIM as a measurement of the similarity between these two shapes.

To do this, we will compare the results produced by our program, to those obtained from geometrical calculations.

We will take two examples:

1. Circle versus square [ge14 versus ge24] (see fig. 10)

We will take the circle [ge14] as the master shape, and the square [ge24] as the test shape.

Ater digitization, the degree of similarity between these two shapes, geometrically, is defined by:

$$\text{sim} = 1 - \frac{\text{different pixels}}{\text{black pixels in circle}}$$

$$= 1 - \frac{\text{difference in area}}{\text{circle area}}$$

$$= 1 - \frac{\text{circle area} - \text{square area}}{\text{circle area}}$$

$$= 1 - \frac{\pi * r * r - \mathrm{SQR}(\sqrt{2} * r)}{\pi * r * r}$$

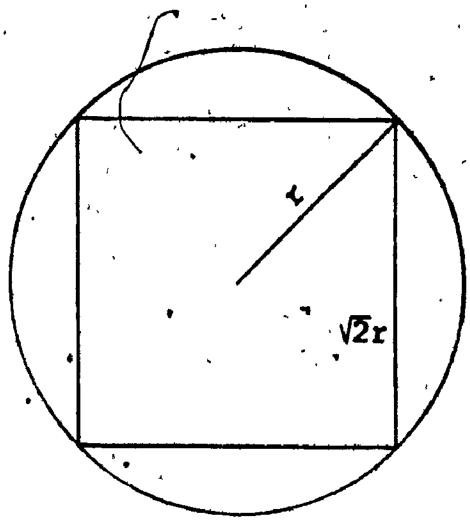


Fig. 10 Circle versus square

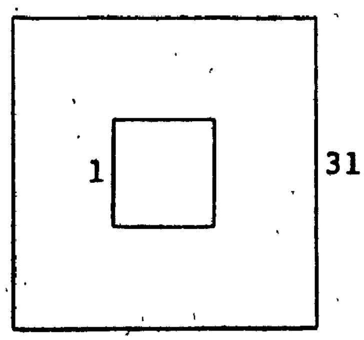


Fig. 11 Square versus hollow square

$$= 1 - \frac{\pi - 2}{\pi}$$

$$= 0.64$$

The result of our program is 0.66.

The reason for this small difference is that, after digitization, the circle is no longer a geometrically perfect shape.

2. Square versus hollow square [ge24 versus ge34] (see fig. 11)

We will take the square [ge24] as the master shape, and the hollow square [ge34] as the test shape.

After digitization, the degree of similarity between these two shapes, geometrically, is defined by:

$$\text{sim} = 1 - \frac{\text{different pixels}}{\text{black pixels in square}}$$

$$= 1 - \frac{\text{difference in area}}{\text{square area}}$$

$$= 1 - \frac{\text{square area - hollow square area}}{\text{square area}}$$

$$= 1 - \frac{SQR(31) - [SQR(31) - SQR(1)]}{SQR(31)}$$

$$= 1 - \frac{9 - (9 - 1)}{9}$$

$$= 1 - \frac{1}{9}$$

$$= 0.89$$

The result of our program is 0.89.

The two results are identical.

In each of the two above examples, the geometrical formula was fairly simple. The difference between the two compared shapes, in each case, did not affect the center of gravity. The maximum radius and the drawn circles, in our algorithm, were the same for the two compared shapes. Otherwise, the geometrical calculations would have been a lot more complicated.

Therefore, the above two examples deal with specific and theoretical cases. These shapes were selected on purpose, in order to easily establish the geometrical arguments.

The results of the two examples demonstrate the validity of our algorithm, and the accuracy of the degree of similarity computed by our program.

CHAPTER 9

APPLICATIONS

Many useful applications can be found in shape description and discrimination, using shape matrices, in accordance with the algorithm presented in chapter 5.

Some suggested applications are:

1. Detecting defective objects

This is the case of a production line where one kind of object is being produced and laid on a conveyor belt.

The object's darkness should contrast with the belt, i.e., if the object is dark, the belt should be light, and vice-versa. This will enable an easy extraction of the object from its background.

A scanner is mounted above the belt, and a sensor (e.g. a photocell) immediately before to trigger it when an object arrives. A master shape matrix is constructed from a model object, only once, in the beginning of run-time. Every object will be processed as a test shape; and its shape matrix will be compared to the master shape matrix.

The position and orientation of objects on the belt can be arbitrary.

The expected degree of similarity for a good object is 100%. In practice, this can vary depending on the quality and resolution of the scanner. A practical test for good and defective objects must be carried out in order to determine a proper threshold.

The coordinates and the degree of defection can be deduced from the shape matrix, and automatically outputted. This, however, would require some enhancement of the algorithm.

2. Template matching in rotated images

With a little modification, the algorithm can be used to locate templates in a rotated binary image. The template, however, should be circular. Hence when the template is shifted on the image at the match position, the template and the window underneath it cover the same pattern, independent of the orientation of the image.

The template is located in an image by comparing its shape matrix to the shape matrix of a window in that image.

3. Character recognition

Alphabet and numeric characters in any language constitute a set of patterns that could have disconnected parts. They could have curves or straight lines, or closed or open lines. In all these instances, the shape matrices can be used as a tool to recognize the characters.

Depending on the size of the set of characters, the required shape matrices would be larger or smaller. The bigger the set, the larger the shape matrix.

Character recognition and discrimination can be carried out independent of the position, orientation, and size of characters. Nevertheless, all the shapes belonging to a certain character, must be derived from the same image of that character.

4. Measurement of scanner accuracy

In ideal conditions, i.e., with a perfect scanner and an infinite resolution, the degree of similarity between different shapes of a given object, will be 100%. In practice, this value will never be reached, because we are dealing with real scanners and finite resolutions. However, a better scanner will produce more accurate shapes, and hence, better similarity.

Based on the above reasoning, we can use a fixed set of patterns of a given object, to rank the quality of a given scanner, by using it to digitize these patterns. The quality of the scanner used, is proportional to the degree(s) of similarity obtained.

5. Robotics

One of the desired features of robots, is the ability to "see" or to recognize objects.

A robot, provided with cameras or scanners, can extract information about objects. This information consists of digitized images. If the robot moves, these images, of the same objects, will be altered. The images will change in position, orientation, and size.

Using this algorithm, the different digitized images of a given object, will all yield the same shape matrix. Thus, the robot will be able to recognize a certain object while it moves.

It is important to notice that our algorithm deals with planar shapes, i.e., shapes moving in two dimensions only; and in general, the robot will encounter images moving in three dimensions. In this case, the present algorithm is not sufficient. However, it can serve as a good basis to be developed into a general algorithm that can deal with three-dimensional shapes.

CHAPTER 10

CONCLUDING REMARKS

This work deals with two-dimensional shapes. Description and discrimination of shapes have been examined.

A two-dimensional object can have different shapes depending on its position, orientation, and size. The amount of translation and rotation can be obtained by moving the object in its plane, and scanning it. The difference in size can be obtained by moving the scanner perpendicular to the plane, and scanning the object at different distances. These differences can be combined to produce different shapes of the same object.

The role of the shape matrix is to filter out these differences. Ideally, the shape matrices of different shapes of the same object should all be identical, provided n and m are fixed. In practice, there will be some differences among these shape matrices for the following reasons:

1. The resolution is limited.
2. The scanner is not perfect.

In order to make shape matrices 100% identical, for a given object, we must have ideal conditions, where the resolution is infinite and the scanner is perfect. Obviously, these conditions cannot be found in the real world, although it is possible to get close to them.

The data used and the results obtained show that the algorithm is sensitive to distinctions between shapes. It becomes more sensitive when these distinctions affect the center of gravity.

In some instances, e.g., detection of defective objects, this sensitivity is highly desirable, because it is important to report the smallest defection in a produced object.

In other instances, e.g., character recognition, it is still desirable to have a sensitive algorithm, in order to discriminate different characters; but it would also be desirable to decrease this sensitivity in case of similar characters. The degree of similarity between similar characters would be boosted up. The gap between similar and different characters would be wider, and the threshold range in the alphabet characters (fig. 4) would be larger.

This suggestion can be realized in an additional work.

Another suggestion was mentioned in chapter 9: to make the algorithm more general to process three-dimensional shapes.

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