Dynamic Analysis Of Tall Planar And Tube-Type Building Structures

Antonio Mulè

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ABSTRACT

DYNAMIC ANALYSIS OF TALL PLANAR AND TUBE-TYPE BUILDING STRUCTURES

Antonino Mulè

A computer program is developed for linear dynamic analysis of planar and tube-type building structures using the macro-element technique. In this technique the actual discrete frame is replaced by an elastically equivalent orthotropic membrane. The equivalent structure is subsequently analyzed by using the finite element method. Seismic response is obtained through spectrum analysis in conjunction with the lumped mass formulation. The seismic responses obtained from an analysis may include: natural frequencies and mode shapes, displacements, storey shear forces, overturning base moments, and internal forces. The final structural responses are determined by the square-root-sum-of-squares method. Several two and three-dimensional structures consisting of frames and walls have been analyzed, and the results are in excellent agreement with published data.

It has been found that in framed-tube structures, shear lag behaviour is present, and that flexibility of

finite size, joints significantly affect the seismic behaviour. Shear deformations in beams and columns have less influence.

The computer program was used to generate data which are presented in the form of graphs and simplified equations for rapid estimation of the natural frequencies of planar and tube-type structures. Recommendations are also made for the preliminary estimates of natural frequencies of framed-tube buildings.

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NOTATIONS

a, b	dimensions of the elements
A	depth of structure
A_b, A_c	cross-sectional area of beam and column
A_{rb}, A_{rc}	shear area of beam and column
A_m, A_n, C_y, C_{xy}	coefficients used in determining the elastic properties of the equivalent membrane
B	width of structure
c	number of corners in the plan view of the structural model.
$[C]$	damping matrix
$\{d\}, \{\dot{d}\}, \{\ddot{d}\}$	displacement, velocity and acceleration vector
d_b, d_c	depth of beam and column
d_i	distance from neutral axis to column i
D_i	perpendicular distance from the facade to the reference point of level i
$[D]$	strain-diplacement matrix
E, G	elastic moduli of the material of the actual structure
E_x, E_y, G_{xy}	elastic moduli of the equivalent orthotropic membrane
$[E]$	elasticity matrix
f	function
$\{f_s\}$	effective external elastic force vector
h	storey height
$\{h\}$	vector of level elevations measured from the base of the structure

H	structure height
I	moment of inertia
$[I]$	identity matrix
J	in-plane level rotational inertia
$[K]$	stiffness matrix
$[K_{ij}]$	submatrix of $[K]$
l	span of lintel beam
L	number of levels in a structural model
$\{L\}$	vector of earthquake-excitation modal factors
$\overline{m}$	linear mass distribution along the height of a structure
m_i	mass of level i
$[M]$	mass matrix
M_u, M_v	overturning base moment about the x and y axis
$[N]$	shape function
P	column axial force
$\{P\}$	external load vector
$\{P_{eff}\}$	effective external force vector
Q	applied shear force
r	element aspect ratio
$\{r\}$	influence coefficient vector
S_a, S_d, S_v	acceleration, displacement, and pseudo-velocity spectral response
SL	shear lag parameter
t	equivalent orthotropic membrane thickness

t_b, t_c	thickness of beam and column
T	time period
$[T]$	transformation matrix
$[T_{ij}]$	submatrix of $[T]$
u, v	displacements in the x and y directions
$\ddot{u}, \ddot{v}$	acceleration in the x and y directions
$v_g, \ddot{v}_g$	ground displacement and acceleration
V_b, V_c	shear force in beam and column
V_u, V_v	base shear in the x and y direction
V_θ	base torque
w	bay width
α	coefficient used in determining E_y, the angle measured from the positive x axis to the positive direction of a facade
α_i	coefficients used in the displacement functions
β	factor of the inertia of the normal facade
ξ, η	dimensionless element axis system, = x/a, y/b
η	normal coordinate
$\{\sigma\}; \{\epsilon\}$	stress and strain vector
$\sigma_x, \sigma_y; \tau_{xy}$	stress components in the equivalent orthotropic membrane
$\epsilon_x, \epsilon_y, \gamma_{xy}$	strain components in the equivalent orthotropic membrane
$\theta, \ddot{\theta}$	in-plane rotation, and rotational acceleration of a level

$\{\Delta\}$	lateral displacement vector
$\{\phi\}$	mode shape vector
λ	natural angular frequency
λ_s	basic natural frequency
$[\mathcal{M}],[\mathcal{C}],[\mathcal{K}]$	generalized mass, damping, and stiffness matrices
ω	corner vertical displacement
$\mathcal{M}_n, \mathcal{C}_n, \mathcal{K}_n$	generalized mass, damping, and stiffness for mode n
β	fraction of critical damping
Γ	modal participation factor
μ	ductility factor
Π	dimensionless parameter
ν	Poisson's ratio
ρ	material density
γ	angle measured counterclockwise from the x axis to the direction of ground motion

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CHAPTER 1

INTRODUCTION

1.1 PLANAR AND TUBE-TYPE STRUCTURES

Planar and tube-type structures are broad classifications of many existing structures. Planar structures are those which can be analyzed as two-dimensional structures such as shear walls, coupled shear walls, and two-dimensional frames.

Tube-type structures are any assembly of planar structures connected at their edges. Several examples of tube-type structures are framed-tube buildings, frame-shear wall buildings and core supported buildings.

1.2 FRAMED-TUBE BUILDING

The dynamic characteristics of framed-tube buildings are of primary importance in this project and thus a characteristic review of this structure is in order. The framed-tube system was originally developed for the design of the 43-storey De Witt Chestnut Appartment Building in Chicago in 1963 (Fig. 1.1b) (2).

The framed-tube system consists of closely spaced exterior columns connected by deep spandrel beams around the periphery of the building which is usually but not necessarily rectangular.

The structure resembles an assembly of perforated planar exterior walls or a perforated tube (Fig. 1.1a). The exterior walls are designed to withstand the entire lateral load. The frame efficiency of lateral load resistance results from deeper members in conjunction with shorter bay widths and storey heights. Bay widths (or centre to centre column spacings) generally vary from 4 ft. to 10 ft. although structural efficiency is not significantly reduced for column spacings up to 15 ft. (1). Typical column and spandrel beam depths vary from 2 ft. to 5 ft. while column widths vary from 10 inches up to 3 ft. (2). The framed-tube system is an optimum design for buildings up to about 400 ft., however the twin towers of the World Trade Centre in New York are 110 storeys high (3).

Although the structure has a tube-like appearance, the structural behaviour is much more complex than that of a simple cantilever tube. This structural system combines the characteristic behaviour of both multi-storey, multi-bay frames, and walls. The structure thus undergoes both the true cantilever action of shear walls and the shearing action of frames. Consider the framed tube shown in Fig. 1.2. The whole three-dimensional structure resists overturning under lateral load, causing compression and tension in columns. The shear from the lateral load is resisted by the bending of beams and columns primarily in the facades parallel to the direction of loading. Unlike the pure bending behaviour of ideal tubes which exhibit linear normal stress variation in the web the framed tube perforations in the "web" frame cause an inefficiency due to its higher degree of flexibility causing what is known as shear lag.

1.3 SCOPE AND OBJECTIVES

The primary objectives of this project are i) to develop an efficient computer program for earthquake spectral analysis of large planar and three-dimensional tube-type buildings, ii) to study the behaviour of framed-tube building structures subject to seismic forces, and iii) to provide equations and graphs for natural frequency calculation of the above mentioned structures.

The macro-element technique of structural modelling developed by El-Moselhi (1) is adopted and further extended for dynamic analysis. This technique is capable of modelling most existing planar and tubular structural systems. These systems may include multi-storey, multi-bay frames having a wide range of aspect ratios and stiffnesses, shear walls, coupled shear walls, clad frames, planar and tubular structures consisting of frame and shear wall assemblies, and core-supported structures.

The computer program is used to study the behavioural characteristics of framed-tube building structures subjected to earthquake ground motions. The effects of shear deformation in members and flexibility of finite size joints on the seismic responses of framed-tube structures will also be investigated.

Simple methods for estimating the natural frequencies of planar and tube-type structures will be presented.

1.4 GENERAL ASSUMPTIONS

Several general assumptions will be made here which hold throughout this research. Other specific assumptions pertaining to specific topics treated in this work will be presented later.

(1) The structural material is isotropic, homogeneous, and linearly elastic.

(2) Frame members are rigidly connected to form an orthogonal grid system.

(3) Floor diaphrams are infinitely rigid in their own planes. This assumption is widely accepted for the exact analysis of tall building structures (4).

(4) Out-of-plane deformations of walls and frames are ignored. Published results show their negligible contribution to structural behaviour (3, 4, 5, 6, 7).

1.5 ORGANIZATION OF THE THESIS

Chapter II of the thesis introduces the concept of replacing a discrete structural system by elastically equivalent membranes. Equations for the elastic properties of the orthotropic membranes are presented. The effects of column axial deformations, shear and bending deformations in beams and columns, and flexible finite size joints are taken into account.

Chapter III presents two orthotropic rectangular plane stress finite elements incorporating the assumption of infinite horizontal rigidity for computational efficiency purposes. The first is an "ordinary" element with 4 corner nodes and 6 degrees of freedom. The second is a "refined" element with 6 nodes and 9 degrees of freedom.

These elements are used to subdivide each equivalent membrane and consequently discretizing the equivalent structure.

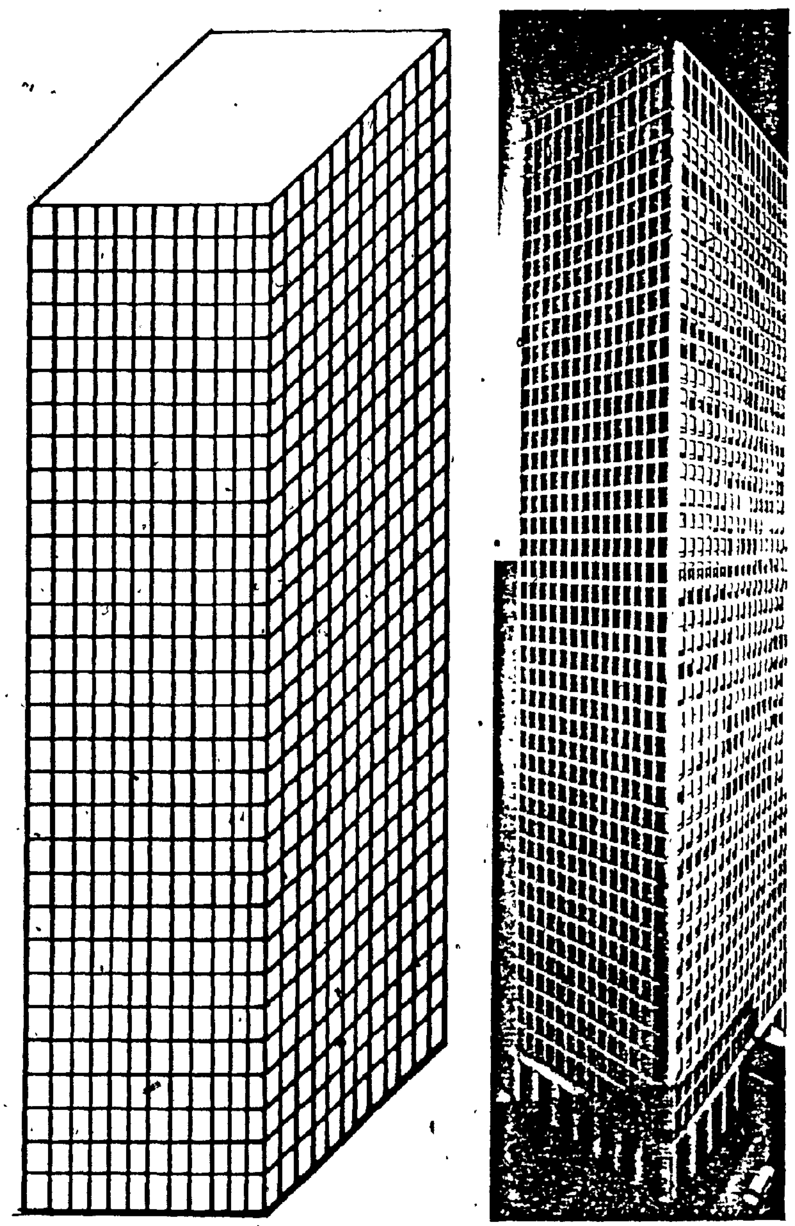
In Chapter IV, a general three-dimensional structure is modelled, and the formulations for its linear static analysis and linear earthquake spectrum analysis are developed. A ductility factor may be used to alter the response spectrum for approximate elastic-plastic earthquake analysis.

Chapter V is designed mainly to establish the validity of the present method. Two and three-dimensional structures are analyzed to show the accuracy, versatility, and efficiency of the present method.

Chapter VI is a study on the behavioural characteristics of framed-tube buildings. A shear lag parameter is introduced to measure the degree of shear lag in the structure, and its effect on internal column axial forces and natural frequencies. The contributions of flexible finite size joints, and shear deformations in beams and columns to the dynamic responses are investigated.

Chapter VII provides designers and researchers with two methods of determining natural frequencies of planar and tube-type structures. The first more accurate method requires the use of graphs and simplified equations. The second method, which applies only to framed-tube structures, requires less computation, but yields more approximate results.

Finally, conclusions for the present study and recommendations for further studies are presented in Chapter VIII.



(a) (b)

Fig. 1.1 (a) Framed tube

(b) De Witt Chestnut apartment building (2)

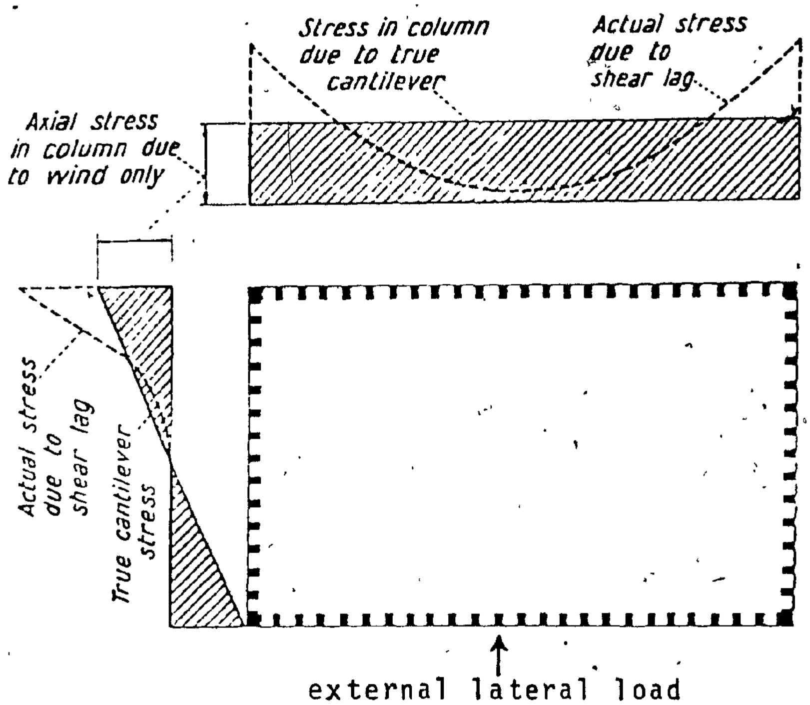


Fig. 1.2 Shear lag in framed tube (2)

CHAPTER II

EQUIVALENT ORTHOTROPIC MEMBRANE

2.1 INTRODUCTION

The concept of modelling a complex, highly redundant discrete system by an elasticaly equivalent continuum is not new. It has been applied to ship and aircraft structures (8), bridges (9), shells (10), space roof trusses (11), coupled shear walls (12, 13), and tall frame-type building structures (14, 15, 16). With particular reference to tube-type building structures, the macro-element technique has been shown (1) to be an efficient and accurate method for static analysis. The method consists of first replacing the discrete structure by an elastically equivalent system. For completeness, the relevant theoretical development related to the elastically equivalent membranes for coupling lintel beams and rectangular framework is presented in this chapter.

2.2 ORTHOTROPIC MEMBRANE FOR COUPLING BEAMS

Lintel beams connecting planar or three-dimensional assemblies of shear walls are replaced by an elastically equivalent orthotropic membrane as shown in Fig. 2.1. Both bending and shear deformations of the connecting beams are.

considered in evaluating the shear modulus of the membrane; this allows modelling of a wide range of coupling elements from slabs to relatively deep lintel beams.

In evaluating the shear modulus of the membrane, it is assumed that the inflection points are located at the midspan of the lintel beams. This assumption is commonly accepted (1,3,12,13). For a general cross-section of lintel beam the membrane shear modulus is

$$G_{xy} = \frac{\dfrac{E}{t\,h}}{\left[\dfrac{l^2}{12 I_b} + \dfrac{1}{A_{rb}}\left(\dfrac{E}{G}\right)\right]} \tag{2.1}$$

where

l	=	span of lintel beam
E,G	=	elastic moduli of lintel beam material
I_b, A_{rb}	=	moment of inertia and reduced (effective) area of the beam cross-section
t	=	membrane thickness
h	=	storey height

For a lintel beam of rectangular cross-section with thickness t_b and depth d_b, the membrane shear modulus is

$$G_{xy} = \frac{E\left(\frac{d_b}{h}\right)}{\left[\left(\frac{\ell}{d_b}\right)^2 + 1.2\left(\frac{E}{G}\right)\right]} \tag{2.2}$$

2.3 ORTHOTROPIC MEMBRANE FOR PLANAR GRIDWORK SYSTEMS

In replacing the perforated wall or planar gridwork system by an elastically equivalent orthotropic membrane, there are several restrictions based on practical considerations. Firstly, w/A and h/H of the structure must be small; A and H are depth and height of the structure respectively. Typical existing structures range from (1/10 to 1/20) and (1/20 to 1/100) respectively. Secondly, the member properties, storey heights, and bay widths may vary only from one zone to another within the building. These restrictions would make the application of the technique more feasible, but are not required for the validity of the theoretical development.

2.3.1 MODULI OF ELASTICITY E_x and E_y

Consider a typical wall-frame unit composed of rectangular concrete sections subjected to a vertical axial load P (Fig. 2.2). The total vertical deformation is obtained considering the uniform stress in parts 1 and 3, and the complex stress variation in part 2, (Fig. 2.2b). Next a rectangular membrane encompassing the wall-frame unit is subjected to a uniform vertical axial stress of P/(wt). Considering the vertical elastic modulus, E_y, of the membrane as unknown, the total deformation of the wall-frame unit, and the membrane are equated to yield

$$E_y = \frac{E\, t_c\, d_c}{t\, w\, C_y} \tag{2.3}$$

in which

$$C_y = 1 - \frac{d_b}{h} + \frac{t_c}{t_b}\left[\frac{d_b d_c}{w h} + \frac{8w}{\pi^2 h}\,\frac{1}{3}\sum_{n=1}^{3}\sum_{m=1}^{\infty} A_n \frac{A_m}{m}\sinh^2(\alpha d_b)\right] \tag{2.4}$$

$$A_n = \cos\left[\alpha d_c (2-n)/2\right] \tag{2.5}$$

$$A_m = \frac{\sinh(\alpha d_c)}{m\left[\sinh(2\alpha d_b) + 2\alpha d_b\right]} \tag{2.6}$$

where

$$\alpha = m\pi/w \qquad (2.7)$$

d_c, t_c = depth and thickness of column respectively

d_b, t_b = depth and thickness of beam respectively

A similar procedure can be carried through to obtain the horizontal equivalent elastic modulus E_x, but in the context of this research, E_x has been taken to be infinity; infinite in-plane rigidity of floors can be assumed in tall building analysis (1, 4).

It can be seen from Eq. (2.4) that for slender members, C_y approaches unity. In this instance, Eq. (2.3) is an expression of the equivalent vertical elastic modulus of the column only. For structures having deep beams and a small column thickness to beam thickness ratio, C_y could be significantly less than one. Typical values of C_y range from 0.8 to 0.96. In the case of steel structures, the expression for C_y can be considerably simplified.

2.3.2 SHEAR MODULUS G_{xy}

Consider the wall-frame unit of Fig. 2.3 subjected to a horizontal shear force Q. Note that the support conditions simulate the midspan inflection point assumption commonly used in literature (3). Again the rectangular membrane is made to deflect with the grid unit. The total lateral deflection of the grid unit is obtained by considering bending and shear deformations of both beam and column, and the flexibility of the finite size joint. Equating this deflection to the lateral deflection expression of the membrane yields the equivalent shear modulus.

$$G_{xy} = \frac{E}{t\, w\, C_{xy}} \tag{2.8}$$

in which

$$C_{xy} = \frac{(h-d_b)^3}{12hI_c} + \frac{h(w-d_c)^3}{12w^2I_b} + \frac{E}{G}\left[\frac{h(w-d_c)}{w^2A_{rb}} + \frac{(h-d_b)}{hA_{rc}} + \frac{h}{A_{rj}d_b}\left(1-\frac{d_c}{w}-\frac{d_b}{h}\right)^2\right] \tag{2.9}$$

where

I_b, I_c	=	moment of inertia of beam and column respectively
A_{rb}, A_{rc}	=	reduced or effective shear area of the beam and column respectively
A_{rj}	=	cross-sectional area of the joint parallel to the acting force Q
G	=	shear modulus of the actual material

Note that C_{xy} is a summation of five terms. Going from left to right they take into account the following deformations: bending in the column, bending in the beam, shear in the beam, shear in the column, flexibility of the finite size joint. In the case of steel structures, the flexibility of finite size joint comes mainly from the so-called "panel zone deformations". This factor can be derived in a similar fashion.

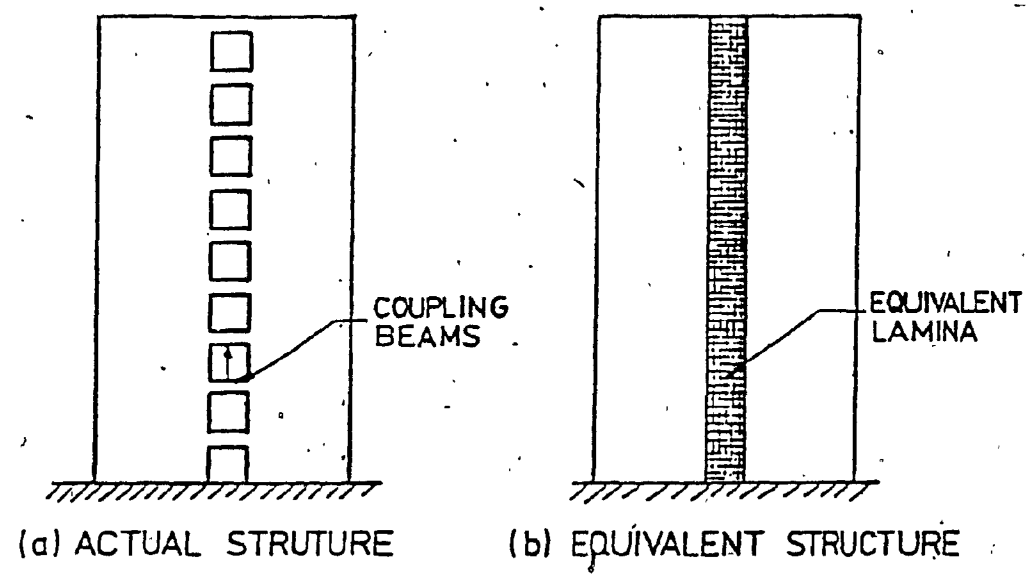


Fig 2.1 Present modelling of coupling beams

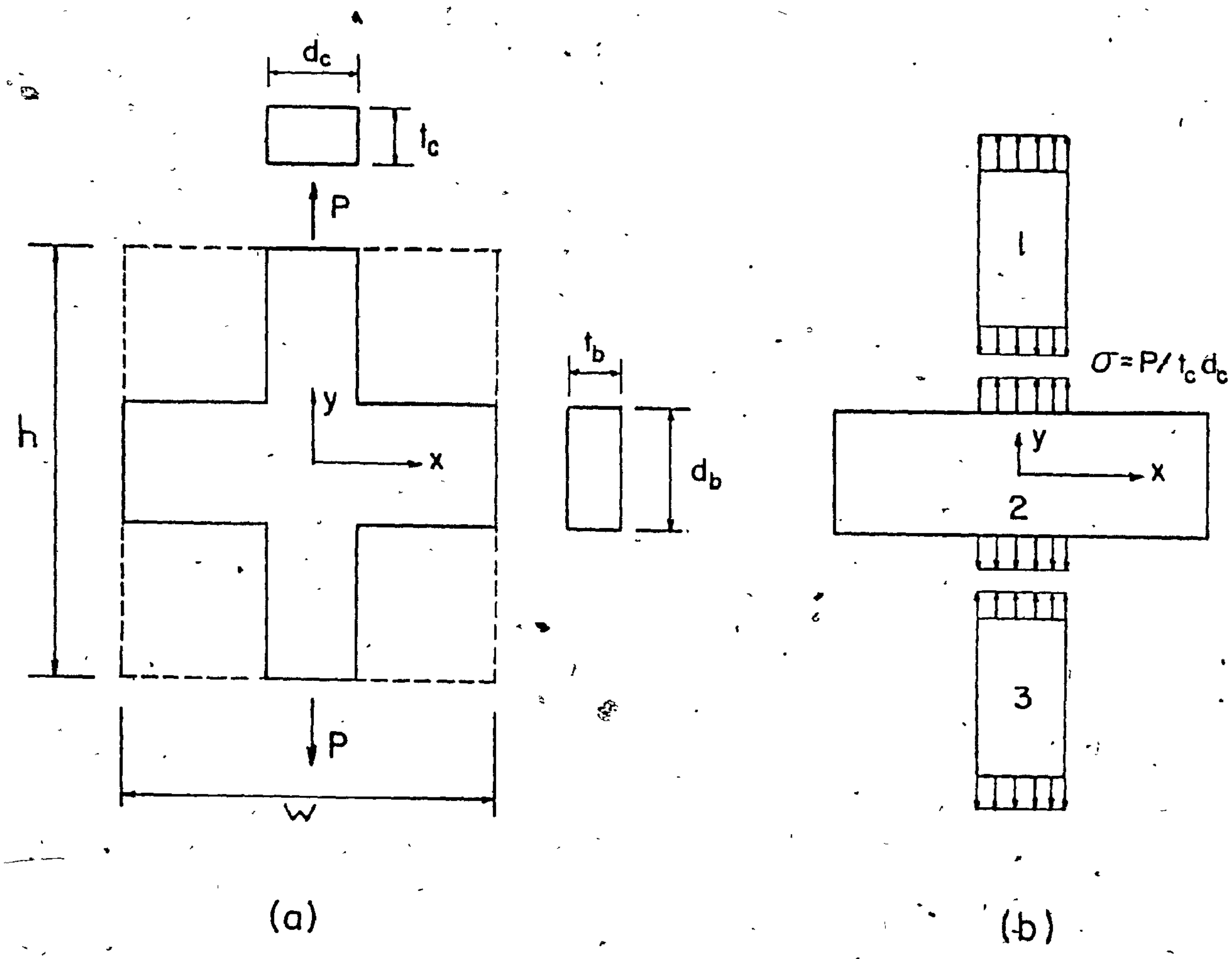


Fig. 2.2 Model for evaluating the equivalent membrane elastic modulus E_y

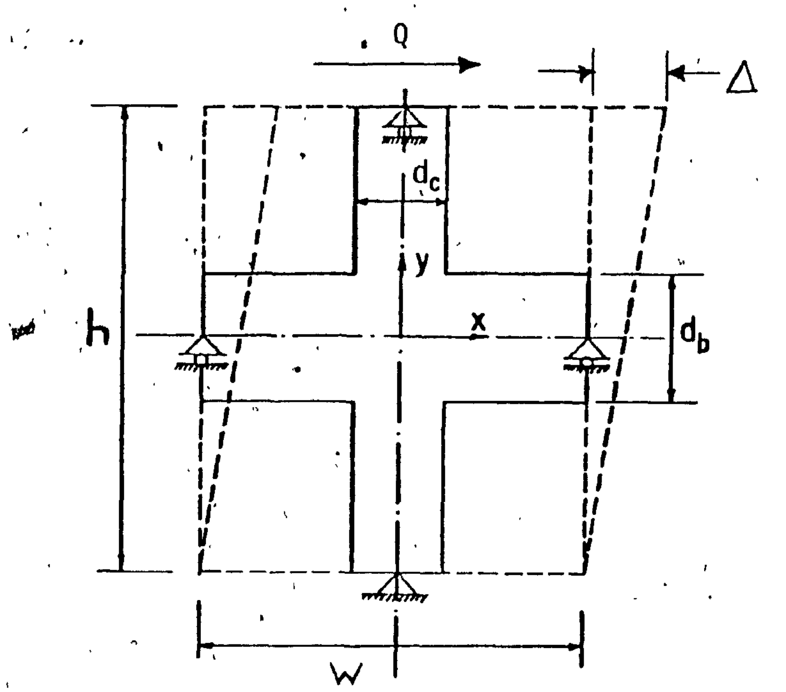


Fig. 2.3 Model for evaluating the equivalent membrane shear modulus G_{xy}

CHAPTER III

SPECIALLY ORTHOTROPIC FINITE ELEMENTS FOR TALL BUILDING ANALYSIS

3.1 INTRODUCTION

For highly regular-planar and tubular structures subjected to static lateral loads, closed form solutions for displacements and internal forces have been presented (1, 2, 3). Most tubular buildings have variations that cannot be realized by closed form solutions; lintel beam sizes may vary along the height of coupled shear walls, member sizes and grid sizes in a frame may change from one zone to another, assemblies of coupled shear walls, frames, and solid shear walls may be difficult to analyze, building configurations may not be rectangular, mass distribution for a dynamic analysis is irregular. Finite element analysis is a good alternative. The discrete idealized structure is replaced by equivalent membranes, (presented in the previous chapter), which are subsequently subdivided into several rectangular finite macro-elements. Note that the actual structure need be only highly regular within each individual membrane. This technique has been shown to be an efficient and accurate method for static analysis. For completeness, the relevant theoretical developments of two macro-elements (1) are presented in this chapter.

3.2 FORMULATION OF THE ORDINARY ELEMENT

Consider the rectangular element of Fig. 3.1. The displacement function which satisfies infinite horizontal rigidity (in accordance with infinitely rigid floors), maintains compatibility along the edges, and satisfies all other requirements for convergence in finite element theory (17) is assumed to be:

$$u = \alpha_1 + \alpha_2 \eta \tag{3.1a}$$

$$v = \alpha_3 + \alpha_4 \xi + \alpha_5 \eta + \alpha_6 \xi \eta \tag{3.1b}$$

in which ξ and η are dimensionless coordinates equal to x/a and y/b respectively. Eq. (3.1) can be expressed in terms of the 6 nodal displacement as

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N]\{d\} \tag{3.2}$$

such that $$[N] = \begin{bmatrix} 1-\eta & \eta & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\xi)(1-\eta) & \xi(1-\eta) & \xi\eta & \eta(1-\xi) \end{bmatrix} \tag{3.3}$$

and $$\{d\}^T = [u_1 \;\; u_2 \;\; v_1 \;\; v_2 \;\; v_3 \;\; v_4] \tag{3.4}$$

Using the strain-displacement relationship

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \partial/\partial x & 0 \\ 0 & \partial/\partial y \\ \partial/\partial y & \partial/\partial x \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \quad (3.5)$$

results in

$$\{\epsilon\} = [D]\{d\} \quad (3.6)$$

where $\{\epsilon\}^T = [\epsilon_x \;\; \epsilon_y \;\; \gamma_{xy}]$ (3.7)

and $$[D] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(1-\xi)/b & -\xi/b & \xi/b & (1-\xi)/b \\ -1/b & 1/b & -(1-\eta)/a & (1-\eta)/a & \eta/a & -\eta/a \end{bmatrix} \quad (3.8)$$

Since infinite in-plane rigidity of floors is assumed, Poisson's ratios ν_{xy} and ν_{yx} of the equivalent orthotropic membrane have zero values and thus the stress-strain matrix relation is given by:

$$\{\sigma\} = [E]\{\epsilon\} \quad (3.9)$$

such that $\{\sigma\}^T = [\sigma_x \;\; \sigma_y \;\; \tau_{xy}]$ (3.10)

and $$[E] = \begin{bmatrix} E_x & 0 & 0 \\ 0 & E_y & 0 \\ 0 & 0 & G_{xy} \end{bmatrix} \quad (3.11)$$

The 6 X 6 ordinary element stiffness matrix (K_e) is obtained from

$$[K_e] = tab\int_0^1\int_0^1 [D]^T[E][D]\,d\xi\,d\eta \tag{3.12}$$

$$[K_e] = \begin{bmatrix} K_1 & & & & & \\ -K_1 & K_1 & & \text{Sym.} & & \\ K_2 & -K_2 & K_3 & & & \\ -K_2 & K_2 & K_4 & K_3 & & \\ -K_2 & K_2 & -\frac{K_3}{2} & K_5 & K_3 & \\ K_2 & -K_2 & K_5 & -\frac{K_3}{2} & K_4 & K_3 \end{bmatrix} \tag{3.13}$$

in which $K_1 = \dfrac{taG_{xy}}{b}$ (3.14a)

$$K_2 = \frac{tG_{xy}}{2} \tag{3.14b}$$

$$K_3 = \frac{t(\frac{a}{b}E_y + \frac{b}{a}G_{xy})}{3} \tag{3.14c}$$

$$K_4 = \frac{t(\frac{a}{2b}E_y - \frac{b}{a}G_{xy})}{3} \tag{3.14d}$$

$$K_5 = \frac{t(\frac{b}{2a}G_{xy} - \frac{a}{b}E_y)}{3} \tag{3.14e}$$

3.3 FORMULATION OF THE REFINED ELEMENT

The need for a more refined element was induced because the ordinary element was unreliable in certain applications. The ordinary element is only capable of describing a linear deformation configuration along its edges. For this reason, the bending deformation of several example shear walls were poorly described. It was found that as the aspect ratio increased, the lateral deflection became more erroneous. Errors for the maximum lateral deflection of over 100% were frequently obtained. It can also be seen that stress variation within an ordinary element is only linear, and thus many elements may have to be used to obtain accurate column axial forces in a framed-tube building characterized by nonlinear column axial force distribution due to shear lag.

Consider the rectangular element of Fig. 3.2. The displacement functions satisfying convergence requirements in finite element theory (17) may be assumed to be

$$U = \alpha_1 + \alpha_2 \eta + \alpha_3 \eta^2 \tag{3.15a}$$

$$V = \alpha_4 + \alpha_5 \xi + \alpha_6 \eta + \alpha_7 \xi^2 + \alpha_8 \xi\eta + \alpha_9 \eta^2 \tag{3.15b}$$

such that $\xi = x/a$ and $\eta = y/b$.

Note that the refined element is capable of describing a quadratic deformation configuration along its edges, which is more representative of bending deformation. In terms of nodal displacements Eq. (3.15) may be written as

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = [N]\{d\} \tag{3.16}$$

where

$$[N]^T = \begin{bmatrix} -\frac{1}{2}\eta(1-\eta) & 0 \\ 1-\eta^2 & 0 \\ \frac{1}{2}\eta(1+\eta) & 0 \\ 0 & -\frac{1}{4}\xi(1-\xi-\eta+\xi\eta) \\ 0 & \frac{1}{2}(1-\eta-\xi^2+\xi^2\eta) \\ 0 & \frac{1}{4}\xi(1+\xi-\eta-\xi\eta) \\ 0 & \frac{1}{4}\xi(1+\xi+\eta+\xi\eta) \\ 0 & \frac{1}{2}(1+\eta-\xi^2-\xi^2\eta) \\ 0 & -\frac{1}{4}\xi(1-\xi+\eta-\eta\xi) \end{bmatrix} \tag{3.17}$$

and

$$\{d\}^T = [u_1 \; u_2 \; u_3 \; V_1 \; V_2 \; V_3 \; V_4 \; V_5 \; V_6] \tag{3.18}$$

The (D) matrix is obtained from the strain-displacement relationship Eq. (3.5)

$$(D)^T = \begin{bmatrix} 0 & 0 & -(1-2\eta)/2b \\ 0 & 0 & -2\eta/b \\ 0 & 0 & (1+2\eta)/2b \\ 0 & \xi(1-\xi)/4b & -(1-2\xi-\eta+2\xi\eta)/4a \\ 0 & -(1-\xi^2)/2b & -\xi(1-\eta)/a \\ 0 & -\xi(1+\xi)/4b & (1+2\xi-\eta-2\xi\eta)/4a \\ 0 & \xi(1+\xi)/4b & (1+2\xi+\eta+2\xi\eta)/4a \\ 0 & (1-\xi^2)/2b & -\xi(1+\eta)/a \\ 0 & -\xi(1-\xi)/4b & -(1-2\xi+\eta-2\xi\eta)/4a \end{bmatrix} \tag{3.19}$$

The 9 X 9 refined element stiffness matrix is

$$[K_e] = t\,ab \int_{-1}^{1}\int_{-1}^{1} [D]^T[E][D]\, d\xi\, d\eta \tag{3.20}$$

$$[K_e] = \begin{bmatrix} 7K_1 & & & & & & \text{Sym.} & & \\ -8K_1 & 16K_1 & & & & & & & \\ K_1 & -8K_1 & 7K_1 & & & & & & \\ 5K_2 & -4K_2 & -K_2 & K_3 & & & & & \\ 0 & 0 & 0 & K_4 & K_9 & & & & \\ -5K_2 & 4K_2 & K_2 & K_5 & K_4 & K_3 & & & \\ -K_2 & -4K_2 & 5K_2 & K_6 & K_7 & K_8 & K_3 & & \\ 0 & 0 & 0 & K_7 & K_{10} & K_7 & K_4 & K_9 & \\ K_2 & 4K_2 & -5K_2 & K_8 & K_7 & K_6 & K_5 & K_4 & K_3 \end{bmatrix} \tag{3.21}$$

in which

$$K_1 = \frac{tG_{xy}}{3r} \tag{3.22a}$$

$$K_2 = \frac{tG_{xy}}{6} \tag{3.22b}$$

$$K_3 = t\left(\frac{2r}{15}E_y + \frac{7}{9r}G_{xy}\right) \tag{3.22c}$$

$$K_4 = \frac{1}{3}t\left(\frac{r}{5}E_y - \frac{8}{3r}G_{xy}\right) \tag{3.22d}$$

$$K_5 = -\frac{t}{6}\left(\frac{r}{5}E_y - \frac{2}{3r}G_{xy}\right) \tag{3.22e}$$

$$K_6 = \frac{t}{6}\left(\frac{r}{5}E_y + \frac{1}{3r}G_{xy}\right) \tag{3.22f}$$

$$K_7 = -t\left(\frac{r}{15}E_y + \frac{4}{9r}G_{xy}\right) \tag{3.22g}$$

$$K_8 = -\frac{t}{6}\left(\frac{4r}{5}E_y - \frac{7}{3r}G_{xy}\right) \tag{3.22h}$$

$$K_9 = \frac{8}{3}t\left(\frac{r}{5}E_y + \frac{2}{3r}G_{xy}\right) \tag{3.22i}$$

$$K_{10} = -\frac{8}{3}t\left(\frac{r}{5}E_y - \frac{1}{3r}G_{xy}\right) \tag{3.22j}$$

$$r = \frac{a}{b} \tag{3.22k}$$

3.4 DETERMINATION OF FORCES IN BEAMS AND COLUMNS

Once all element displacements are obtained, the equivalent membrane strain and stress distributions are determined from Eqs. (3.6) and (3.9) respectively. Referring to Fig. 3.3, axial forces and shear forces in beams and columns are evaluated first at their midlength points by integrating the corresponding equivalent membrane stress component over a bay width w or a storey height h for columns or beams respectively, as follows:

$$P_{ij} = \int_{x_{ci}=-w/2}^{x_{ci}=w/2} t\,(\sigma_y)_{y_{cj}=0}\,dx \tag{3.23a}$$

$$V_{cij} = \int_{x_{ci}=-w/2}^{x_{ci}=w/2} t\,(\tau_{xy})_{y_{cj}=0}\,dx \tag{3.23b}$$

$$V_{bij} = \int_{y_{bj}=-h/2}^{y_{bj}=h/2} t\,(\tau_{xy})_{x_{bi}=0}\,dy \tag{3.23c}$$

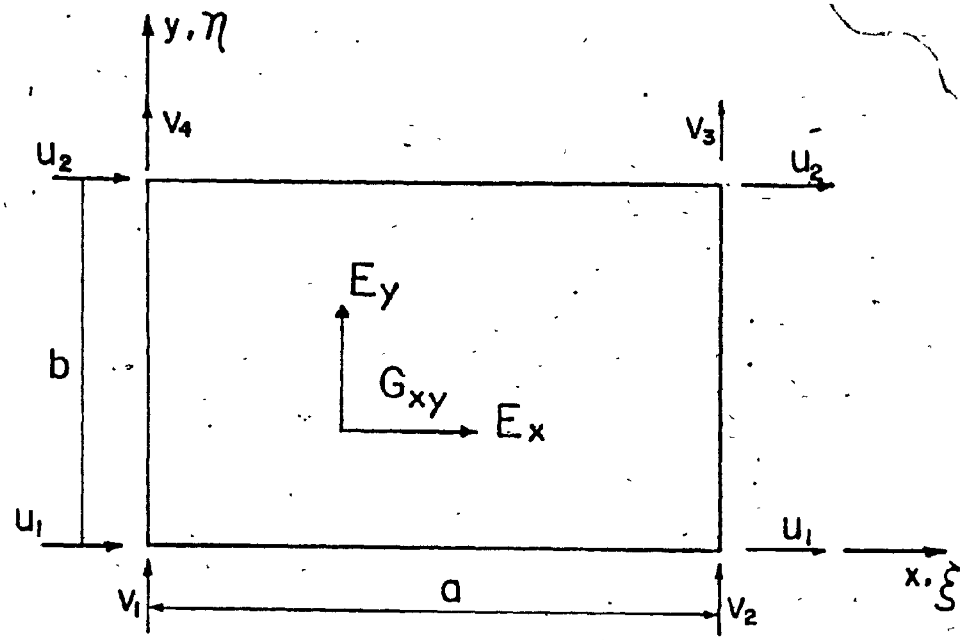
in which

P_{ij}, V_{cij} = the axial and shear forces respectively in the ij column

V_{bij} = the shear force in the ij beam

$x_{bi}, x_{ci}, y_{bj}, y_{cj}$ = the coordinates of the appropriate midlength point, as shown in Fig. 3.3

Because inflection points are assumed at the midspan of beams and columns, the bending moment in beams and columns can be evaluated at any section by multiplying the appropriate midspan shear force V_{bij} or V_{cij} respectively by the distance from the midpoints.



$$\xi = x/a \quad , \quad \eta = y/b$$

Fig. 3.1 The ordinary element

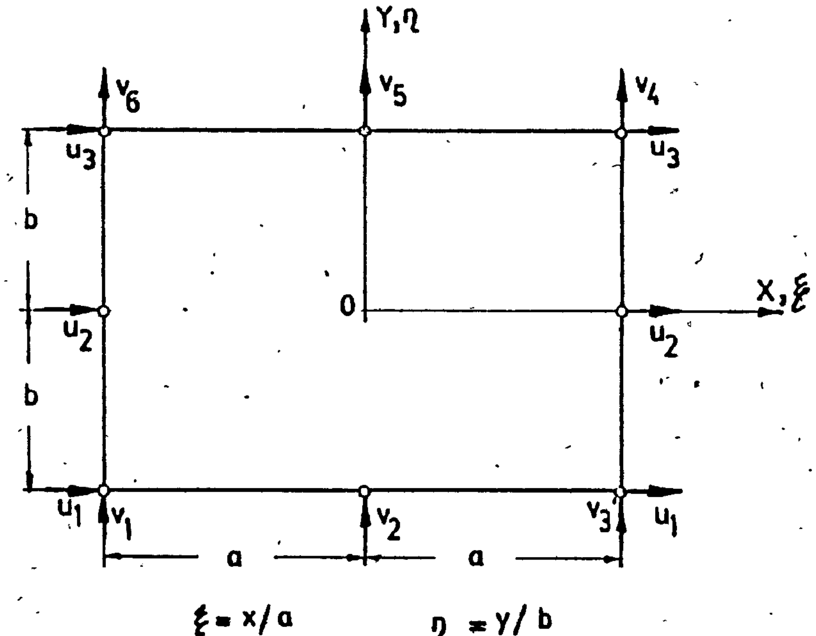


Fig. 3.2 The refined element

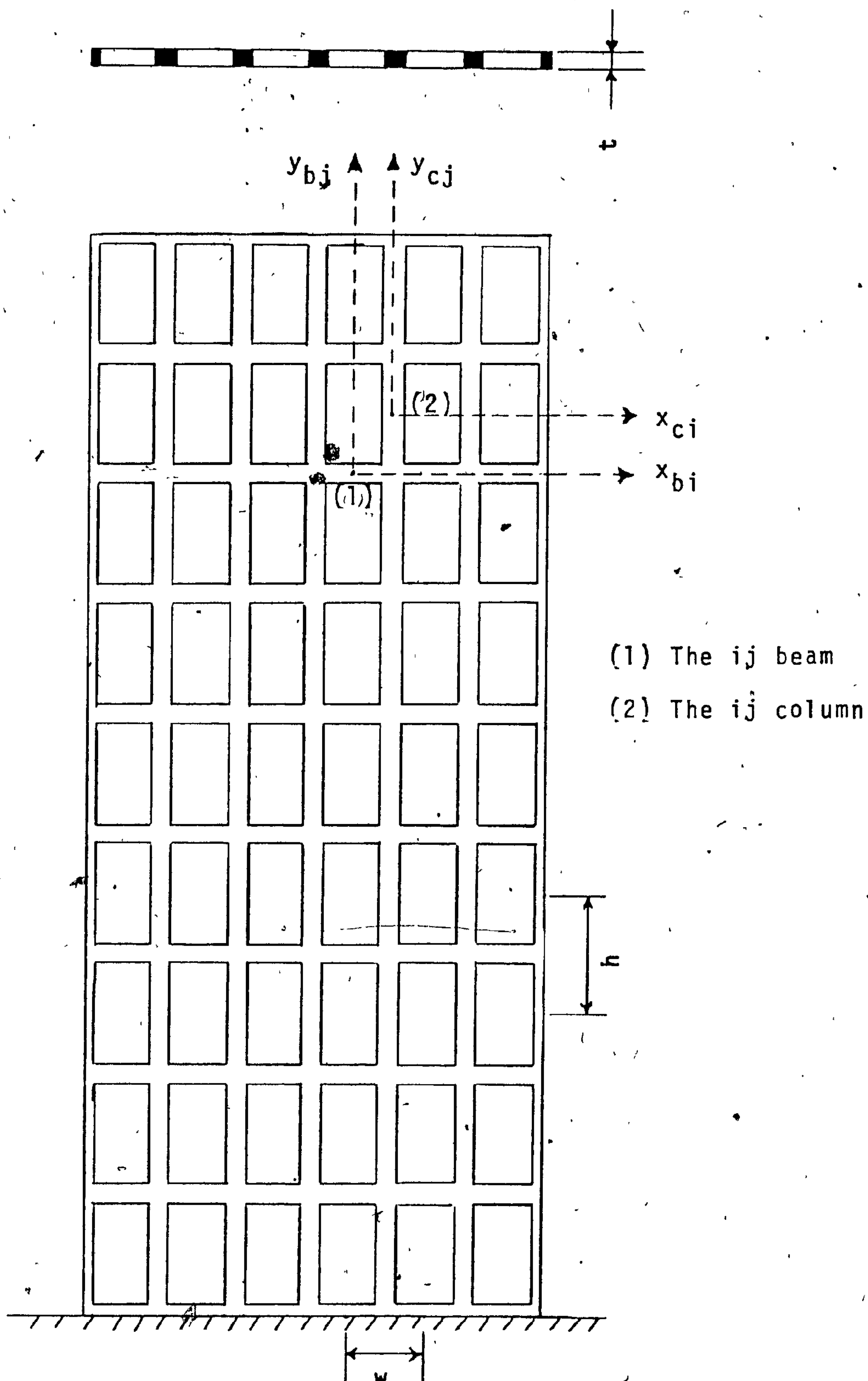


Fig. 3.3 Definitions of symbols in equations 3.23 a, b, c.

CHAPTER IV

STATIC AND EARTHQUAKE ANALYSIS OF PLANAR AND TUBE-TYPE STRUCTURES

4.1 INTRODUCTION

Planar structures, in this thesis, encompass shear walls, coupled shear walls, frames or any combination of these three that form a plane. Tube-type building structures here means any assembly of planar structures connected along their edges. Typical examples of tube-type building structures are framed-tube structures, shear wall structures, shear wall frame structures, rigid tube structures and core-supported structures.

Many approximate and detailed methods of analysis have been developed for these types of structures (3). In the early stages of research on this topic, approximate methods of analysis such as the portal and cantilever methods resulted in relatively large errors, but provided a fairly simple method of analysis. Most approximate methods of analysis are derived from basic assumptions; some are more valid then others. In general, most recent developments in approximate structural analysis methods give fast reliable results, but are too restricted in applicability. On the other hand, detailed methods of analysis alter very good results with a wider range of applicability at the expense of time and effort.

The present equivalent membrane finite element method of analysis yields fairly accurate results with little effort and time, and has wide range of applicability for building structures and structural components.

4.2 SCOPE

The theoretical static and dynamic analysis developments presented in this chapter must satisfy the following conditions:

1) The behaviour is linear elastic.

2) The structure must be planar or tube-type.

    i) Planar structures may be any assembly of rigidly connected shear walls, frames, and lintel beams forming a plane. Frames must consist of rigidly connected horizontal beams and vertical columns.

ii) Tube-type structures may be any assembly of planar structures connected along their edges.

4) Both static and dynamic analyses follow the same general assumptions stated earlier in Chapter I.

4.3 ANALYSIS PROCEDURE

A planar structure may be considered as a tube-type structure with one facade. Since planar structures are special cases of tube-type structures, the latter will be used for the development of the analysis procedure.

4.3.1 MODELLING OF STRUCTURES

The hypothetical framed-tube of Fig. 4.1a is used to illustrate the idealization process. The structure consists of three planar facades intersecting at corners (Fig. 4.1c). Note that the plan form can be any open or closed polygon shape. Facades and corners are numbered in a counterclockwise direction.

The facades of the actual structure are replaced by the equivalent orthotropic membrane which is discretized into a set of rectangular plane stress finite elements (Fig. 4.1b), developed in Chapter III, according to the following rules which apply also to shear walls and lintel beams.

1) Elements may span one or more stories or bays (if applicable), but element boundaries need not coincide with beam and column lines. However, the element boundaries must be aligned throughout the height and width of the structure. The structure is thus divided into a number of "structural levels" which should be numbered consecutively from bottom to top (Fig. 4.1b).

2) Structural properties of members may vary along the building height and width, but must remain constant within each element. (i.e. elastic modulus, shear modulus, storey height, bay width, moment of inertia, shear area, and depths of columns and beams).

3) As shown in Fig. 4.2a, element corner nodes are numbered from bottom to top starting from the

left to the right of each facade. The remaining nodes of the refined element are subsequently numbered in the same fashion (Fig. 4.2b).

4) Element connectively is specified by node numbers in a counterclockwise direction starting from the bottom left corner node.

The discretization process of steps 3 and 4 above are automatically generated from the given number of elements in the horizontal and vertical directions of each facade. Automatic generation reduces effort in data preparation and eliminates possible data errors.

With the assumption of rigid floor diaphragms, each level of the building will acquire three degrees of freedom: two orthogonal translations and one in-plane rotation. These three degrees of freedom can be associated with any convenient point on the level. To ensure vertical compatibility at facade junctions (or corners), a vertical degree of freedom is assigned at each building corner for each level. The above degrees of freedom will be referred to as the "global static degrees of freedom" (Fig. 4.3). When the building is subjected to a horizontal earthquake excitation, the vertical corner degrees of freedom of the global static degrees of freedom are statically condensed and the remaining degrees of freedom are termed as the "global dynamic degrees of freedom". For convenience, these degrees of freedom should be associated with the mass center of each level (Fig. 4.4). Displacements associated with interior nodes within individual facades are called the "internal facade degrees of freedom". These are statically condensed, leaving just global degrees of freedom to form the global structure stiffness matrix.

The primary unknowns in a global analysis are the global degrees of freedom. Once these are known, all the previously condensed degrees of freedom can be recovered and subsequently, the stresses in the membranes as well as member forces in the actual structure can be determined.

The present method of analysis takes advantage of the finite element analysis technique on a macroscale. Finite elements are conventionally associated with only small portions of structural components. The macro-element, conforms to the same rules as a finite element, but encompasses many structural members and large areas of walls.

4.3.2 STATIC ANALYSIS PROCEDURE

The major steps in the static analysis procedure are summarized below.

1) The individual facade stiffness matrices are assembled from their respective element stiffness matrices. Boundary and facade symmetry conditions are directly taken into account. Out-of-plane deformations of facades are not considered.

2) The internal degrees of freedom of each facade stiffness matrix are condensed to leave only those at the edges of the facade.

3) The condensed facade stiffness matrices are transformed into the global coordinate system taking into account the global (structure) restraints and/or symmetry conditions.

4) The global static structure stiffness matrix is assembled from the transformed, condensed facade stiffness matrices obtained in the previous step.

5) The global structure load vector associated with the global static degrees of freedom is assembled.

6) The global static displacements are solved for.

7) The global static displacements are extracted and transformed into local axes for each facade.

8) The internal local displacements of each facade are recovered for subsequent stress calculations within each element.

9) The internal member forces in the actual structure are determined by integrating the appropriate stress component.

4.3.3 EARTHQUAKE SPECTRUM ANALYSIS PROCEDURE

In the dynamic analysis of buildings subjected to horizontal ground excitation, steps 1 to 4 of the static case are equally applicable. The additional steps are as follows:

5) Static condensation of the vertical corner displacements leaves only two translations and

one in-plane rotation per level as the global dynamic degrees of freedom. Lumped masses and rotational inertias are required at each level.

6) Determination of the natural frequencies and vibration mode shapes by solving the eigenvalue problem.

7) For each mode the following responses are determined:

   a) modal participation factor
   b) pseudo-velocity from a suitable response spectrum taking into account elastic or elasto-plastic behaviour
   c) normal coordinate
   d) effective elastic forces associated with the global dynamic degrees of freedom
   e) base shears, torques and moments
   f) global dynamic displacements
   g) corner vertical displacements
   h) facade internal displacements
   i) element stresses

8) Use the square-root-sum-of-squares (SRSS) method to obtain the final response for d, e, f, g, i, of part 7.

9) Determine the internal forces of the actual structure.

All the above mentioned steps for the static and earthquake analysis are described in detail below with particular attention to programming aspects.

4.4 ASSEMBLY OF FACADE STIFFNESS MATRIX IN LOCAL AXES

Since the facade coordinate system coincides with that of its elements, the local element stiffness matrices are directly assembled to form the facade stiffness matrix. Prescribed zero-displacements due to structural supports and symmetry conditions are satisfied by elimination of the corresponding equilibrium equations. This technique saves computation time and computer storage. The bottom nodes are assumed to be fixed, however, foundation flexibility can be taken into account by introducing a bottom row of elements to simulate the supporting soil.

Symmetry exists at two levels: global structure level and local facade level. If local facade symmetry is specified, then global structure symmetry must also be specified (to ensure proper assembly of the global static structure stiffness matrix). There are no other restrictions on the compatibility of these two symmetry levels.

The facade stiffness matrix is assembled element-by-element according to their associated degrees of freedom. These degrees of freedom are automatically generated according to the type of facade symmetry shown in Fig. 4.5. One of the following facade symmetry options may be exercised:

1) No symmetry.

2) Facade symmetry type one means no horizontal nodal deflections of the facade.

3) Facade symmetry type two means no vertical nodal deflections of the facade at edge J.

4) Facade symmetry type three means no vertical nodal deflections of the facade at edge I.

Note that each edge of a facade is designated I or J. The I edge is encountered first when numbering the structure corners counterclockwise.

Since the facade stiffness matrix is symmetrical, only its upper triangle is assembled and stored in a singly subscripted array (i.e. vector) column by column. The mapping function between the stiffness matrix coefficient at row i column j and its vector representation address "l" is

$$l = i + [(j - 1)j]/2 \quad \text{for} \quad j \geq i \qquad (4.1)$$

4.5 CONDENSATION OF FACADE INTERNAL DEGREES OF FREEDOM

All facade internal degrees of freedom (i.e. those not required to maintain compatibility with adjacent facades) are eliminated by the well known process of static condensation (17, 18, 19). Static condensation entails no loss of accuracy, but significantly reduces storage requirements and computation time. In effect, facades are treated as substructures with boundary degrees of freedom at facade edges. An efficient algorithm for static condensation and recovery of unwanted displacements using vector representaion is described in reference (18).

4.6 ASSEMBLY OF THE GLOBAL STATIC STRUCTURE STIFFNESS MATRIX

Since the orientation of a facade in plan is arbitrary, it is necessary to transform the condensed facade stiffness matrix into the global coordinate system (Fig. 4.3). This process involves the transformation between facade degrees of freedom and global static degrees of freedom.

$$\{d_r\} = [T]\{d_r^*\} \tag{4.2}$$

in which

$\{d_r^*\}$ = vector of global degrees of freedom affecting facade deformations.

$\{d_r\}$ = vector of local facade degrees of freedom.

$[T]$ = transformation matrix

A specific ordering system for the global structural degrees of freedom as well as the local facade uncondensed degrees of freedom is chosen to facilitate the presentation and derivation of the transformation matrix [T].

$$\left\{d_r\right\}_{3L\times 1} = \begin{Bmatrix} \{\Delta\}_{L\times 1} \\ \{\omega_I\}_{L\times 1} \\ \{\omega_J\}_{L\times 1} \end{Bmatrix} \tag{4.3}$$

and

$$\left\{d_r^*\right\}_{5L\times 1} = \begin{Bmatrix} \{u\}_{L\times 1} \\ \{v\}_{L\times 1} \\ \{\theta\}_{L\times 1} \\ \{\omega_I\}_{L\times 1} \\ \{\omega_J\}_{L\times 1} \end{Bmatrix} \tag{4.4}$$

in which

L = number of levels.

$\{\Delta\}$ = vector of lateral or horizontal local facade degrees of freedom.

$\{\omega_I\}$ and $\{\omega_J\}$ = vector of vertical degrees of freedom at edges I and J respectively of the facade. These degrees of freedom are also in the vector of global degrees of freedom.

$\{u\}, \{v\}, \{\theta\}$ = vectors of the global degrees of freedom associated with the reference point of each level; u refers to global x-translation, v to global y-translation, and θ to rigid body level rotation. For a dynamic analysis, the reference point must be positioned at the lumped mass locations.

Referring to Fig. 4.3, the lateral displacements of the facade at level i can be expressed in terms of the three global displacements at the same level.

$$\Delta_i = [\cos\alpha \quad \sin\alpha \quad D_i] \begin{Bmatrix} u_i \\ v_i \\ \theta_i \end{Bmatrix} \qquad (4.5)$$

where

α = the angle measured from the global x-axis to the positive direction of the facade. Note that the positive direction of a facade is defined as the direction going from the facade corner I to corner J.

D_i = the perpendicular distance between the facade and the level reference point.

Eq. (4.2) then becomes

$$\left\{\begin{array}{c}\Delta_1\\ \Delta_2\\ \vdots\\ \Delta_L\\ \hline \omega_{I1}\\ \omega_{I2}\\ \vdots\\ \omega_{IL}\\ \hline \omega_{J1}\\ \omega_{J2}\\ \vdots\\ \omega_{JL}\end{array}\right\} = \left[\begin{array}{c|c|c|c|c} \begin{matrix}\cos\alpha & & \\ & \cos\alpha & \\ & & \ddots \\ & & & \cos\alpha\end{matrix} & \begin{matrix}\sin\alpha & & \\ & \sin\alpha & \\ & & \ddots \\ & & & \sin\alpha\end{matrix} & \begin{matrix}D_1 & & \\ & D_2 & \\ & & \ddots \\ & & & D_L\end{matrix} & & \\ \hline & & & \begin{matrix}1 & & \\ & 1 & \\ & & \ddots \\ & & & 1\end{matrix} & \\ \hline & & & & \begin{matrix}1 & & \\ & 1 & \\ & & \ddots \\ & & & 1\end{matrix} \end{array}\right] \left\{\begin{array}{c}u_1\\ u_2\\ \vdots\\ u_L\\ \hline v_1\\ v_2\\ \vdots\\ v_L\\ \hline \theta_1\\ \theta_2\\ \vdots\\ \theta_L\\ \hline \omega_{I1}\\ \omega_{I2}\\ \vdots\\ \omega_{IL}\\ \hline \omega_{J1}\\ \omega_{J2}\\ \vdots\\ \omega_{JL}\end{array}\right\}. \tag{4.6}$$

Let $$[T_{11}]_{L\times L} = \cos\alpha\,[I] \tag{4.7a}$$

$$[T_{12}]_{L\times L} = \sin\alpha\,[I] \tag{4.7b}$$

$$[T_{13}]_{L\times L} = \begin{bmatrix} D_1 & & 0 \\ & D_2 & \\ 0 & & \ddots & D_L \end{bmatrix} \qquad (4.7c)$$

where

[I] = identity matrix

Then

$$[T] = \begin{bmatrix} [T_{11}] & [T_{12}] & [T_{13}] & & \\ & & & [I] & 0 \\ & 0 & & & [I] \end{bmatrix} \qquad (4.8)$$

From Eq. (4.2) the contragradient law of transformation yields

$$\{\bar{P}_r^*\} = [T]^T \{\bar{P}_r\} \qquad (4.9)$$

where $\{\bar{P}_r^*\}$ and $\{\bar{P}_r\}$ are nodal force vectors associated with the global structure degrees of freedom and boundary facade degrees of freedom respectively.

Substituting the following equation into Eq. (4.9)

$$[\bar{K}_F]\{d_r\} = \{\bar{P}_r\} \qquad (4.10)$$

in which $[\bar{K}_F]$ is the facade boundary stiffness matrix, yields

$$\{\bar{P}_r^*\} = [T]^T[\bar{K}_F]\{d_r\} \qquad (4.11)$$

Substituting Eq. (4.2) yields

$$\{\bar{P}_r^*\} = [T]^T[\bar{K}_F][T]\{d_r\} \qquad (4.12)$$

The transformed facade stiffness matrix in global coordinates can be recognized as

$$[\bar{K}_F^*] = [T]^T[\bar{K}_F][T] \qquad (4.13)$$

The direct matrix multiplication above requires large storage and multiplication of large matrices. To alleviate this problem, the sparseness of [T] will be taken advantage of as follows:

Let $[\bar{K}_F]$ be partitioned as

$$[\bar{K}_F] = \begin{bmatrix} [K_{11}] & [K_{12}] & [K_{13}] \\ [K_{21}] & [K_{22}] & [K_{23}] \\ [K_{31}] & [K_{32}] & [K_{33}] \end{bmatrix} \qquad (4.14)$$

corresponding to $\{\Delta\}$, $\{\omega_I\}$, and $\{\omega_J\}$, (i.e. no facade symmetry). Then from Eqs. (4.8), (4.13), and (4.14)

$$[\bar{K}_F^*] = \begin{bmatrix} [T_{11}]^T & & \\ [T_{12}]^T & 0 & \\ [T_{13}]^T & & \\ & [I] & \\ 0 & & [I] \end{bmatrix} \begin{bmatrix} [K_{11}] & [K_{12}] & [K_{13}] \\ [K_{21}] & [K_{22}] & [K_{23}] \\ [K_{31}] & [K_{32}] & [K_{33}] \end{bmatrix} \begin{bmatrix} [T_{11}] & [T_{12}] & [T_{13}] & & 0 \\ & 0 & & [I] & \\ & & & & [I] \end{bmatrix} \tag{4.15}$$

Manually multiplying these matrices, subsequently substituting Eqs. (4.7) and enforcing the symmetric properties of stiffness matrices yields

$$[\bar{K}_F^*] = \begin{bmatrix} \cos^2\alpha[K_{11}] & \cos\alpha\sin\alpha[K_{11}] & \cos\alpha[K_{11}][T_{13}] & \cos\alpha[K_{12}] & \cos\alpha[K_{13}] \\ & \sin^2\alpha[K_{11}] & \sin\alpha[K_{11}][T_{13}] & \sin\alpha[K_{12}] & \sin\alpha[K_{13}] \\ & & [T_{13}][K_{11}][T_{13}] & [T_{13}][K_{12}] & [T_{13}][K_{13}] \\ & & & [K_{22}] & [K_{23}] \\ & \text{Sym.} & & & [K_{33}] \end{bmatrix} \tag{4.16}$$

Having transformed the condensed facade stiffness matrix for each facade, the overall global static structure stiffness matrix $[K_s^*]$ is assembled. Symmetry of $[K_s^*]$ again allows storage of its upper triangular part into a vector column by column.

4.7 TRANSFORMATION MATRICES FOR DIFFERENT STRUCTURE SYMMETRY CONDITIONS

Structure symmetry conditions may be used to discard some global degrees of freedom. Its application results in significant input data reduction and smaller matrices. To take advantage of this option, one of the following conditions must be satisfied.

1) The structural stiffness is symmetric about one or two axes. This condition is usually associated with geometric structural symmetry. For a static analysis, loading must also be symmetric.

2) The structure is externally restrained such that $\{u\}=0$, $\{v\}=0$, or $\{\theta\}=0$.

The second condition is usually not associated with buildings.

For the case of structure symmetry type 1, only $\{u\}$ is considered at the reference point. $\{v\}$ and $\{\theta\}$ are restrained (i.e. set to zero), thus the transformation matrix is modified to

$$[T]_1 = \begin{array}{c} \begin{array}{ccc} u & \omega_I & \omega_J \end{array} \\ \begin{bmatrix} [T_{11}] & & \\ & [I] & \\ & & [I] \end{bmatrix} \end{array} \tag{4.17}$$

and the global facade stiffness matrix is given by

$$[\bar{K}_F^*]_1 = \begin{bmatrix} \cos^2\alpha[K_{11}] & \cos\alpha[K_{12}] & \cos\alpha[K_{13}] \\ & [K_{22}] & [K_{23}] \\ \text{Sym.} & & [K_{31}] \end{bmatrix} \tag{4.18}$$

Structure symmetry type 2 is similar to type 1 except that $\{v\}$ is retained while $\{u\}$ and $\{\theta\}$ are set to zero. The transformation matrix is

$$\begin{array}{cc} & \begin{array}{ccc} v & \omega_I & \omega_J \end{array} \\ [T]_2 = & \begin{bmatrix} [T_{12}] & & \\ & [I] & \\ & & [I] \end{bmatrix} \end{array} \tag{4.19}$$

and the global facade stiffness matrix is given by

$$[\bar{K}_F^*]_2 = \begin{bmatrix} \sin^2\alpha[K_{11}] & \sin\alpha[K_{12}] & \sin\alpha[K_{13}] \\ & [K_{22}] & [K_{23}] \\ \text{Sym.} & & [K_{33}] \end{bmatrix} \tag{4.20}$$

If the loading and geometry are such that only twisting occurs, then $\{\theta\}$ is retained while $\{u\}$ and $\{v\}$ are restrained. The appropriate transformation matrix is

$$\begin{array}{cc} & \begin{array}{ccc} \theta & \omega_I & \omega_J \end{array} \\ [T]_3 = & \begin{bmatrix} [T_{13}] & & \\ & [I] & \\ & & [I] \end{bmatrix} \end{array} \tag{4.21}$$

and the global facade stiffness matrix is

$$[\bar{K}_F^*] = \begin{bmatrix} [T_{13}][K_{11}][T_{13}] & [T_{13}][K_{12}] & [T_{13}][K_{13}] \\ & [K_{22}] & [K_{23}] \\ \text{Sym.} & & [K_{33}] \end{bmatrix} \tag{4.22}$$

It should be noted that the reduced facade stiffness matrix of Eq. (4.14) is for facade symmetry type 0, and thus of order 3L x 3L. If facade symmetry exists, the order is only 2L x 2L. For purposes of compatibility, it is temporarily expanded to 3L x 3L before transformation.

After the global static structure stiffness matrix is assembled, the static and earthquake analysis procedures differ.

4.8 STATIC ANALYSIS

4.8.1 SOLUTION FOR DISPLACEMENTS

Having established the global static stiffness matrix, and the associated load vector, the global static structural displacements can be solved from the linear set of equations

$$\{P^*\} = [K_s^*]\{d^*\} \tag{4.23}$$

Solutions for $\{d^*\}$ are obtain using the Gauss elimination technique described in section 4.5.

Global static structural displacements are then transformed into local facade boundary displacements for each facade using the appropriate transformation matrix described in the previous section. These boundary displacements, are used in conjunction with the full condensed facade stiffness matrix to recover the condensed degrees of freedom for each facade.

4.8.2 DETERMINATION OF STRESSES

After all facade displacements are determined, they are extracted to yield the displacements associated with each element within the facade. The strains and stresses are subsequently determined as in Chapter III. It is noted that these stresses are for the equivalent membrane, and not for the actual structure. Internal member forces in the discrete beam-column system of the actual structure are evaluated based on these stresses as explained in Chapter III.

4.9 EARTHQUAKE ANALYSIS

4.9.1 GLOBAL DYNAMIC STRUCTURE STIFFNESS MATRIX

The global static structure stiffness matrix previously described corresponds to the set of degrees of freedom:

$\{u\}$, $\{v\}$, $\{\theta\}$, $\{\omega_1\}$, $\{\omega_2\}$, ..., $\{\omega_c\}$ where c = the number of corners in plan view. For a dynamic analysis, the reference points must have been located at the mass center of each level. To obtain the global dynamic structure stiffness matrix $[\bar{K}_s^*]$, the vertical corner degrees of freedom $\{\omega_1\}$, $\{\omega_2\}$, ..., $\{\omega_c\}$ are eliminated by static condensation. The global dynamic structure stiffness matrix is now associated only with $\{u\}$, $\{v\}$, $\{\theta\}$ if no structure symmetry exists and only one of these if structure symmetry does exist.

4.9.2 DIAGONAL LUMPED-MASS MATRIX

For simplicity of the analytical formulation, the distributed inertia of the building is lumped at the mass center of each level in the form of lumped masses and rotation inertia. Because the global dynamic degrees of freedom $\{u\}$, $\{v\}$, and $\{\theta\}$ are associated with the center of mass at each level, the mass matrix will be diagonal, thus simplifying the eigenvalue solution for frequencies and mode shapes. A more refined analysis could be achieved via a consistent-mass matrix which contains off-diagonal terms leading to what is called mass coupling. However, in normal circumstances, such refinement does not justify the extra computational effort.

With the above approximation, the diagonal form of the mass matrix can be explained as follows. Let m_{ij} be the mass coefficient in row i, column j of the lumped-mass matrix. The coefficient m_{ij} can be interpreted as the inertia force produced at degree of freedom i due to a unit acceleration at degree of freedom j while restraining all other degrees of freedom. Applying a unit acceleration at j and restraining all other degree of freedom accelerations would cause only an inertia force at j equal to the lumped mass or inertia at j. Thus, the mass influence coefficient $m_{ij} = 0$ for $i \neq j$, and $m_{ij} = m_j$ for $i = j$.

The lumped-mass matrix differs among the possible structural symmetry conditions. Let m_i be the the lumped mass at level i and let J_i be the rotational mass moment of inertia lumped at level i. For no structural symmetry, the mass matrix is:

$$[M]_0 = \begin{bmatrix} m_1 & & & & & & & & & & & \\ & m_2 & & & & & & & & & \\ & & \ddots & & & & & & 0 & & \\ & & & m_L & & & & & & & \\ & & & & m_1 & & & & & & \\ & & & & & m_2 & & & & & \\ & & & & & & \ddots & & & & \\ & & & & & & & m_L & & & \\ & & 0 & & & & & & J_1 & & \\ & & & & & & & & & J_2 & \\ & & & & & & & & & & \ddots & \\ & & & & & & & & & & & J_L \end{bmatrix}_{3L\times 3L} \tag{4.24}$$

For structural symmetry type one and two

$$[M]_{1,2} = \begin{bmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_L \end{bmatrix}_{L\times L} \tag{4.25}$$

For structural symmetry type three

$$[M]_3 = \begin{bmatrix} J_1 & & & 0 \\ & J_2 & & \\ & & \ddots & \\ 0 & & & J_L \end{bmatrix}_{L \times L} \qquad (4.26)$$

Note that the order of the mass matrix in each case must be the same as the number of available global dynamic degrees of freedom in the structure. For programming purposes only the diagonal coefficients of the lumped-mass matrix are stored in a vector.

4.9.3 DYNAMIC EQUILIBRIUM EQUATIONS

The dynamic equilibrium of the structure is described by a set of ordinary second order differential equations in the following form:

$$[M]\{\ddot{d}_s^*\}_a + [C]\{\dot{d}_s^*\} + [\bar{K}_s^*]\{d_s^*\} = \{P(t)\} \qquad (4.27)$$

where $[M]$ = lumped-mass matrix

$[C]$ = damping matrix

$[\bar{K}_s^*]$ = dynamic structure stiffness matrix

$\{P(t)\}$ = structural load vector associated with the global dynamic degrees of freedom

$$\{d_s^*\} = \begin{Bmatrix} \{u\} \\ \{v\} \\ \{\theta\} \end{Bmatrix} \text{ = vector of global displacements relative to support motion} \tag{4.28}$$

$\{d_s^*\}_a$ = vector of absolute displacements along global degrees of freedom

Note tht a dot ($\dot{}$) over a symbol designates its first derivative with respect to time while a double dot ($\ddot{}$) for its second derivative. The two displacment vectors $\{d_s^*\}$ and $\{d_s^*\}_a$ are related in the following fashion:

$$\{d_s^*\}_a = \{r\} v_g + \{d_s^*\} \tag{4.29}$$

and

$$\{\ddot{d}_s\}_a = \{r\} \ddot{v}_g + \{\ddot{d}_s^*\} \tag{4.30}$$

where v_g is the ground displacement and $\{r\}$ is the influence coefficient vector which describes the influence of a unit ground displacement on the global degrees of freedom. These vectors have the following form for a typical level with no structural symmetry (Fig. 4.4):

$$\begin{Bmatrix} u_a \\ v_a \\ \theta_a \end{Bmatrix} = \begin{Bmatrix} \cos\gamma \\ \sin\gamma \\ 0 \end{Bmatrix} v_g + \begin{Bmatrix} u \\ v \\ \theta \end{Bmatrix}_i \qquad (4.31)$$

and

$$\begin{Bmatrix} \ddot{u}_a \\ \ddot{v}_a \\ \ddot{\theta}_a \end{Bmatrix} = \begin{Bmatrix} \cos\gamma \\ \sin\gamma \\ 0 \end{Bmatrix} \ddot{v}_g + \begin{Bmatrix} \ddot{u} \\ \ddot{v} \\ \ddot{\theta} \end{Bmatrix}_i \qquad (4.32)$$

where γ = the angle formed going counterclockwise from the positive x axis to the line of ground motion.

Considering all levels, the influence coefficient vector $\{r\}$ is obtained.

$$\{r\}_o = \left\{ \begin{array}{l} \begin{Bmatrix} \cos\gamma \\ \cos\gamma \\ \vdots \\ \cos\gamma \end{Bmatrix}_{L\times 1} \\ \begin{Bmatrix} \sin\gamma \\ \sin\gamma \\ \vdots \\ \sin\gamma \end{Bmatrix}_{L\times 1} \\ \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{Bmatrix}_{L\times 1} \end{array} \right\} \tag{4.33}$$

For structural symmetry types 1 and 2 and 3 respectively, the influence coefficent vectors are

$$\{r\}_1 = \begin{Bmatrix} \cos\gamma \\ \cos\gamma \\ \cdot \\ \cdot \\ \cdot \\ \cos\gamma \end{Bmatrix}_{L\times 1} \tag{4.34}$$

and

$$\{r\}_2 = \begin{Bmatrix} \sin\gamma \\ \sin\gamma \\ \cdot \\ \cdot \\ \cdot \\ \sin\gamma \end{Bmatrix}_{L\times 1} \tag{4.35}$$

and

$$\{r\}_3 = \begin{Bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{Bmatrix}_{L\times 1} \tag{4.36}$$

Note that for structural symmetry type 3, the influence coefficient vector is null, reflecting the exclusion of rotational ground motion considerations. Usage of this symmetry condition would yield trivial solutions for all parameters except mode shapes and frequencies.

In the case of a seismic analysis, there are no externally applied loads so that $\{P(t)\} = \{0\}$. Eq. (4.27) may now be written as

$$[M]\left(\{r\}\ddot{v}_g + \{\ddot{d}_s^*\}\right) + [C]\{\dot{d}_s^*\} + [\bar{K}_s^*]\{d_s^*\} = \{0\} \tag{4.37}$$

or

$$[M]\{\ddot{d}_s^*\} + [C]\{\dot{d}_s^*\} + [\bar{K}_s^*]\{d_s^*\} = -[M]\{r\}\ddot{v}_g \tag{4.38}$$

Eq. (4.38) can be interpreted as the equation of motion of the structure not subjected to ground motion but resisting the following effective dynamic forces:

$$\{P_{eff}\} = -[M]\{r\}\ddot{v}_g \tag{4.39}$$

The coupled set of ordinary second order differential equations of motion may be solved directly by numerical integration; this technique is commonly used for inelastic analysis. However, for linear structures, a transformation of the degrees of freedom into a set of generalized coordinates usually called normal or modal coordinates via the mode shapes of the system, is much more efficient because the support motions tend to excite strongly only the lowest modes of vibration. This transformation will uncouple the dynamic equilibrium Eqs. (4.38) and the normal coordinates may be solved for independently.

4.9.4 MODE SHAPES AND NATURAL FREQUENCIES

The vibration mode shapes must satisfy the undamped free vibration equation given by

$$[M]\{\ddot{d}_s^*\} + [\bar{K}_s^*]\{d_s^*\} = \{0\} \tag{4.40}$$

Assume a solution of the form

$$\{d_s^*\} = \{\phi\} \cos(\lambda t + \Upsilon) \tag{4.41}$$

where λ = angular frequency of vibration

Then $$\{\ddot{d}_s^*\} = -\lambda^2\{\emptyset\}\cos(\lambda t+\Upsilon) \qquad (4.42)$$

Substituting Eqs. (4.41) and (4.42) into Eq. (4.40) yields

$$-[M]\lambda^2\{\emptyset\}\cos(\lambda t+\Upsilon) + [\bar{K}_s^*]\{\emptyset\}\cos(\lambda t+\Upsilon) = \{0\} \qquad (4.43)$$

Since the cosine term is arbitrary, it can be cancelled yielding

$$\left([\bar{K}_s^*] - \lambda^2[M]\right)\{\emptyset\} = \{0\} \qquad (4.44)$$

This is the classical eigenvalue problem. The solution procedure adopted is applicable to the case where $[\bar{K}_s^*]$ is singular (20) (i.e. for unrestrained structures). Eq. (4.44) may be rewritten in the following form:

$$\lambda^2[M]\{\emptyset\} = [\bar{K}_s^*]\{\emptyset\} \qquad (4.45)$$

If $[\bar{K}_s^*]$ is positive definite, Choleski factorization can be used:

$$[\bar{K}_s^*] = [L][L]^T \tag{4.46}$$

where [L] is a lower triangular matrix. To accommodate the case where $[\bar{K}_s^*]$ is singular, Eq. (4.45) is transformed to

$$\left([L]^{-1}[M][L]^{-T}\right)\left([L]^T\{\phi\}\right) = \frac{1}{\lambda^2}\left([L]^T\{\phi\}\right) \tag{4.47}$$

Letting $[D] = [L]^{-1}[M][L]^{-T}$ (4.48)

and $$\{q\} = [L]^T\{\phi\} \tag{4.49}$$

Eq. (4.47) may be rewritten in the following manner:

$$[D]\{q\} = \frac{1}{\lambda^2}\{q\} \tag{4.50}$$

With the above form the eigenvalues are $1/\lambda_i^2$ and eigenvectors are $\{q\}_i$. The natural frequencies are λ_i and the corresponding mode shapes are $\{\phi\}_i$. The eigenvalues corresponding to the lower natural frequencies may be obtained by simultaneous iteration with matrix [D] which is premultiplied by trail eigenvectors. The computer subroutines implemented into the present computer program for eigenvalue and eigenvector solutions were developed by Corr and Jennings (20). The parseness of the equations is taken advantage of by way of variable bandwidth storage of [D], [M], $[\bar{K}_s{}^*]$, and [L]. For the present purposes, minor modifications of the subroutine were effected for the triangular storage form of $[\bar{K}_s{}^*]$.

4.9.5 UNCOUPLED EQUATIONS OF MOTION

As mentioned in section 4.9.3 the dynamic equilibrium Eq. (4.38) may be uncoupled by a suitable transformation of coordinates. Without any loss of generality, the dynamic global displacement vector may be represented by a superposition of mode shapes:

$$\{d_s^*\} = [\phi]\{\eta(t)\} \tag{4.51}$$

where $[\phi]$ = matrix of mode shapes placed columnwise

$\{\eta(t)\}$ = vector of normal coordinates, representing the modal amptitudes.

Substitution of the above transformation into Eq. (4.38) yields

$$[M][\phi]\{\ddot{\eta}\} + [C][\phi]\{\dot{\eta}\} + [\bar{K}_s^*][\phi]\{\eta\} = -[M]\{r\}\ddot{V}_g \tag{4.52}$$

Multiplying by $[\phi]^T$ yields

$$[\phi]^T[M][\phi]\{\ddot{\eta}\} + [\phi]^T[C][\phi]\{\dot{\eta}\} + [\phi]^T[\bar{K}_s^*][\phi]\{\eta\} = -[\phi]^T[M]\{r\}\ddot{V}_g \tag{4.53}$$

Letting

$$[\mathcal{M}] = [\phi]^T[M][\phi] \tag{4.54}$$

and

$$[\mathcal{C}] = [\phi]^T[C][\phi] \tag{4.55}$$

and

$$[\mathcal{K}] = [\phi]^T[\bar{K}_s^*][\phi] \tag{4.56}$$

and

$$[L] = -[\emptyset]^T[M]\{r\} \tag{4.57}$$

where

$[\mathcal{M}]$ = generalized mass matrix

$[\mathcal{C}]$ = generalized damping matrix

$[\mathcal{K}]$ = generalized global dynamic stiffness matrix

Eq. (4.53) becomes

$$[\mathcal{M}]\{\ddot{\eta}\} + [\mathcal{C}]\{\dot{\eta}\} + [\mathcal{K}]\{\eta\} = \{L\}\ddot{V}_g \tag{4.58}$$

Through the use of Betti's law, $[\mathcal{M}]$ can be proven to be diagonal (21). Applying this condition to the undamped, free vibration Eq. (4.40) reveals that $[\mathcal{K}]$ is also diagonal.

Rayleigh showed that the damping matrix $[C]$ is uncoupled by $[\emptyset]$ through the transformation of Eq. (4.55) (i.e. $[\mathcal{C}]$ is diagonal) if it can be expressed as a linear combination of the mass matrix $[M]$, and the global dynamic stiffness matrix $[\overline{K}_s^*]$. Clough and Penzien (21) further proved that this condition is satisfied by an infinite number of matrices formed from the above mass and stiffness matrices. The diagonal form of [ζ] is adopted in the present formulation. Eq. (4.58) is now an uncouple equation of motion in normal coordinates and can be expressed for each mode n as follows:

$$\mathcal{M}_n \ddot{\eta}_n + \zeta_n \dot{\eta}_n + K_n \eta_n = L_n \ddot{V}_g \qquad (4.59)$$

where $\mathcal{M}_n, \zeta_n, K_n$ = the nth diagonal elements of [$\mathcal{M}$], [ζ], and [K] respectively. They represent, respectively, the generalized mass, generalized damping, and generalized stiffness for mode number n.

L_n = the nth element of $\{L\}$

Making use of Duhamel's Integral (21), and assuming low values of modal damping, the solution to Eq. (4.59) can be expressed as

$$\eta_n(t) = \frac{L_n}{\lambda_n \mathcal{M}_n} V_n(t) \qquad (4.60)$$

where

$$V_n(t) = \int_0^t \ddot{V}_g(t)\, e^{-\zeta_n \lambda_n (t-\tau_n)} \sin \lambda_n (t-\tau_n)\, d\tau_n \qquad (4.61)$$

Once $\eta_n(t)$ is solved for each mode, Eq. (4.51) is used to obtain the global dynamic structural displacements. In practice, only the first few modes need to be considered.

4.9.6 RESPONSE SPECTRUM

Unless the ground acceleration records are available for a given earthquake, a more common solution technique is via the average response spectrum. The response spectrum provides the maximum average response (from many earthquake records) of any given single degree of freedom (in this case normal coordinate) oscillator with specified period of vibration and damping value. From Eq. (4.60), the maximum modal amplitude is given by

$$\eta_{n,max} = \frac{L_n}{\lambda_n \mathcal{M}_n} S_{vn} \qquad (4.62)$$

where $S_{vn} = V_n(t)_{max}$ = the spectral pseudo-velocity response for mode n, which can be read off from the spectrum curves.

Other response measures are the spectral displacement S_d, and the spectral acceleration S_a which are related to the spectral pseudo-velocity S_v as follows:

$$S_d = \frac{S_v}{\lambda} \tag{4.63}$$

$$S_a = \lambda S_v = \lambda^2 S_d \tag{4.64}$$

Examples of response spectrum curves are shown in Fig. (4.6). For a given period T and damping value ζ, the spectral values S_d, S_v, and S_a can be readily obtained from a response spectrum established from the earthquake records of a particular site. Response spectrum curves can also be established for an estimate of the inelastic response by means of a ductility ratio. The response spectrum given in the Supplement to the National Building Code of Canada 1980 (22) will be used in all the examples presented in the latter chapters (Fig. 4.7).

4.9.7 STRUCTURAL MODAL RESPONSES TO EARTHQUAKE GROUND MOTIONS

Once the natural frequencies and mode shapes of the first few predominant modes are determined, the corresponding modal participation factors can be determined as

$$\Gamma_n = \frac{L_n}{m_n} = \frac{\{\phi\}_n^T [M] \{r\}}{\{\phi\}_n^T [M] \{\phi\}_n} \tag{4.65}$$

For a given damping ratio and period, S_v can be extracted from the response spectrum. If an elasto-plastic analysis is necessary, the methodology of (22) may be applied. A ductility factor μ must be selected depending on the type of building and materials (Table 4.1). S_v must then be altered as follows:

$$S_{v_n} \leftarrow \frac{S_{v_n}}{\mu} \qquad \text{valid for } T > 0.5 \text{ sec} \tag{4.66}$$

$$S_{v_n} \leftarrow \frac{S_{v_n}}{\sqrt{2\mu - 1}} \qquad \text{valid for } T < 0.5 \text{ sec} \tag{4.67}$$

The maximum modal amplitude is determined from Eqs, (4.62) and (4.65) yielding

$$\eta_{n,max} = \frac{\Gamma_n}{\lambda_n} S_{v_n} \tag{4.68}$$

which can be converted to the structural physical displacements by Eq. (4.51).

$$\{d_s^*\}_{n,max} = \{\emptyset\}_n \, \eta_{n,max} \tag{4.69}$$

The condensed corner degrees of freedom are then recovered, as explained in section 4.5, to yield the maximum corner displacements $\{\omega_1\}_n$, $\{\omega_2\}_n$, ..., $\{\omega_c\}_n$.

The maximum elastic forces $\{f_s\}_{n,max}$ associated with the global dynamic degrees of freedom are sought next.

$$\{f_s\}_{n,max} = [\bar{K}_s^*]\{d_s^*\}_{n,max} \tag{4.70}$$

Substituting Eq. (4.69) yields

$$\{f_s\}_{n,max} = [\bar{K}_s^*]\{\emptyset\}_n \eta_{n,max} \tag{4.71}$$

Making use of Eq. (4.45) gives

$$\{f_s\}_{n,max} = \lambda_n^2 [M]\{\emptyset\}_n \eta_{n,max} \tag{4.72}$$

Applying Eq. (4.62) produces

$$\{f_s\}_{n,max} = [M]\{\emptyset\}_n \lambda_n \frac{L_n}{m_n} S_{vn} \tag{4.73}$$

Finally substituting Eq. (4.65) yields

$$\{f_s\}_{n,max} = [M]\{\emptyset\}_n \lambda_n \Gamma_n S_{vn} \tag{4.74}$$

This expression for the elastic forces is more convenient because of the diagonal form of the lumped-mass matrix.

It is emphasized that Eq. (4.74) is a completely general expression for the elastic forces developed in a damped structure subjected to arbitrarily varying ground motions; even though it was derived using an expression for undamped free vibrations, its applicability is not limited.

The structure can now be analyzed statically by applying the elastic forces externally for each mode. The maximum base shears in the x and y directions, and maximum base torque respectively are:

$$V_{u,n,max} = \left[\overset{u}{\{1\}^T_{L\times 1}} \quad \overset{v}{\{0\}^T_{L\times 1}} \quad \overset{\theta}{\{0\}^T_{L\times 1}} \right] \{f_s\}_{n,max} \qquad (4.75a)$$

$$V_{v,n,max} = \left[\{0\}^T_{L\times 1} \quad \{1\}^T_{L\times 1} \quad \{0\}^T_{L\times 1} \right] \{f_s\}_{n,max} \qquad (4.75b)$$

$$V_{\theta,n,max} = \left[\{0\}^T_{L\times 1} \quad \{0\}^T_{L\times 1} \quad \{1\}^T_{L\times 1} \right] \{f_s\}_{n,max} \qquad (4.75c)$$

where $\{1\}$ = unit column vector

The maximum base overturning moments about the x and y axes respectively are:

$$M_{u,n,max} = \left[\overset{u}{\{0\}^T_{L\times 1}} \quad \overset{v}{\{h\}^T_{L\times 1}} \quad \overset{\theta}{\{0\}^T_{L\times 1}} \right] \{f_s\}_{n,max} \qquad (4.76a)$$

$$M_{v,n,max} = \left[\{h\}^T_{L\times 1} \quad \{0\}^T_{L\times 1} \quad \{0\}^T_{L\times 1} \right] \{f_s\}_{n,max} \qquad (4.76b)$$

where $\{h\}$ = vector of level elevations measured from the building base.

Once all maximum modal global displacements are obtained, the maximum modal membrane stresses are computed as explaind in section 4.8.2.

4.9.8 FINAL STRUCTURAL RESPONSE TO EARTHQUAKE GROUND MOTIONS

The calculations described in the previous sections are carried out for each mode. The "total" maximum response obtained as the absolute sum of the individual maximum modal responses would be overly conservative because these maximum modal responses do not all occur at the same instant of time.

The square-root-sum-of-squares (SRSS) method is a simple statistical method of estimating the probable maximum response. Suppose a_i's are the maximum modal responses of a certain parameter for modes i = 1, 2, ... n respectively. The absolute maximum response would be

$$a = \sum_{i=1}^{n} |a_i| \tag{4.77}$$

and the probable maximum response would be

$$a = \sqrt{\sum_{i=1}^{n} (a_i)^2} \tag{4.78}$$

The SRSS method will be used in this thesis.

Table 4.1 Ductility factor (22)

Building Type	Structural Ductility Factor
Ductile moment resisting space frame	4
Combined system of 25 percent ductile moment resisting space frame and ductile flexural walls	3
Ductile reinforced concrete flexural walls	3
Regular reinforced concrete structures, cross-braced frame structures and reinforced masonry structures	2
Structures having no ductility and plain masonry structures	1

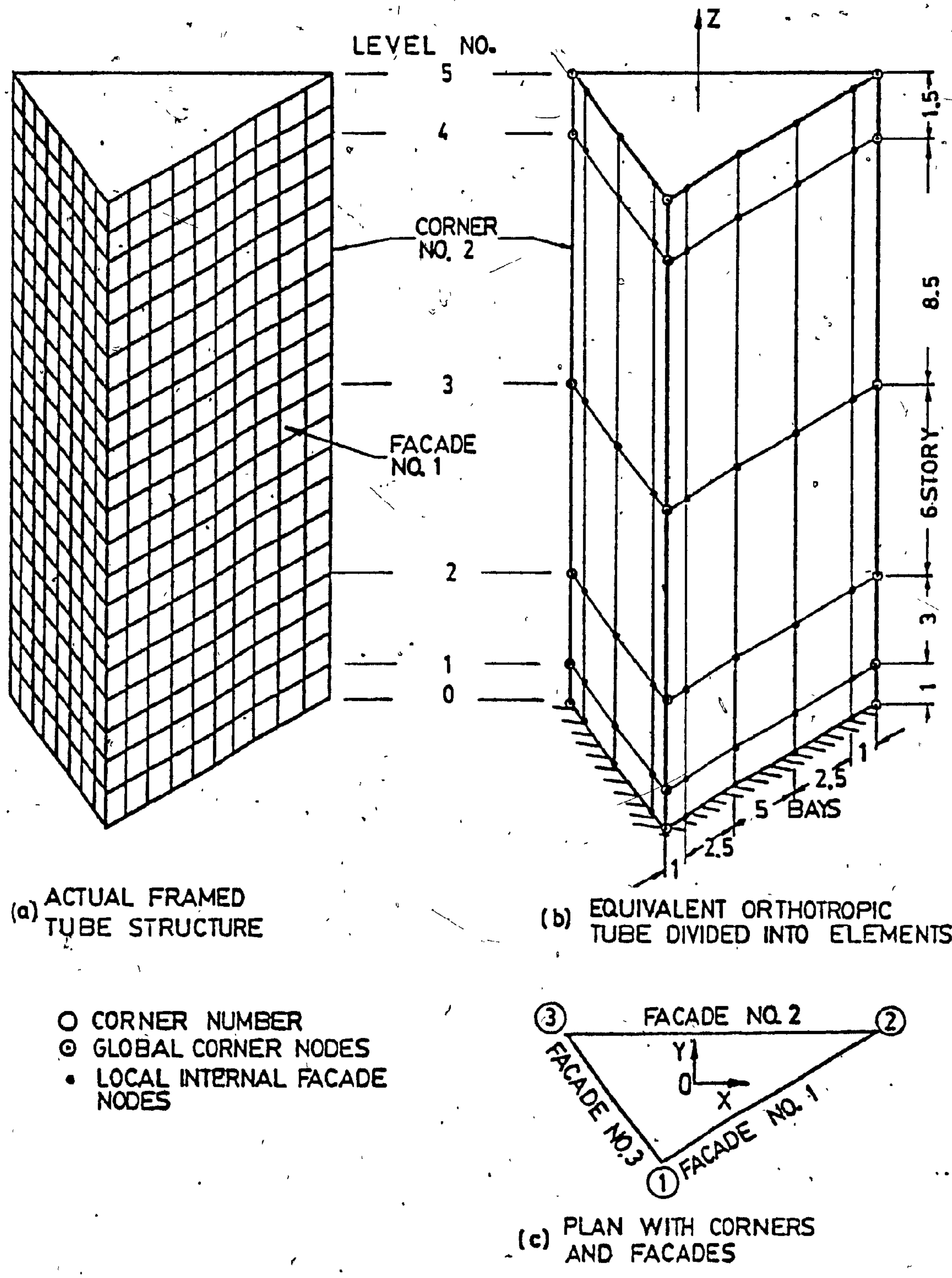
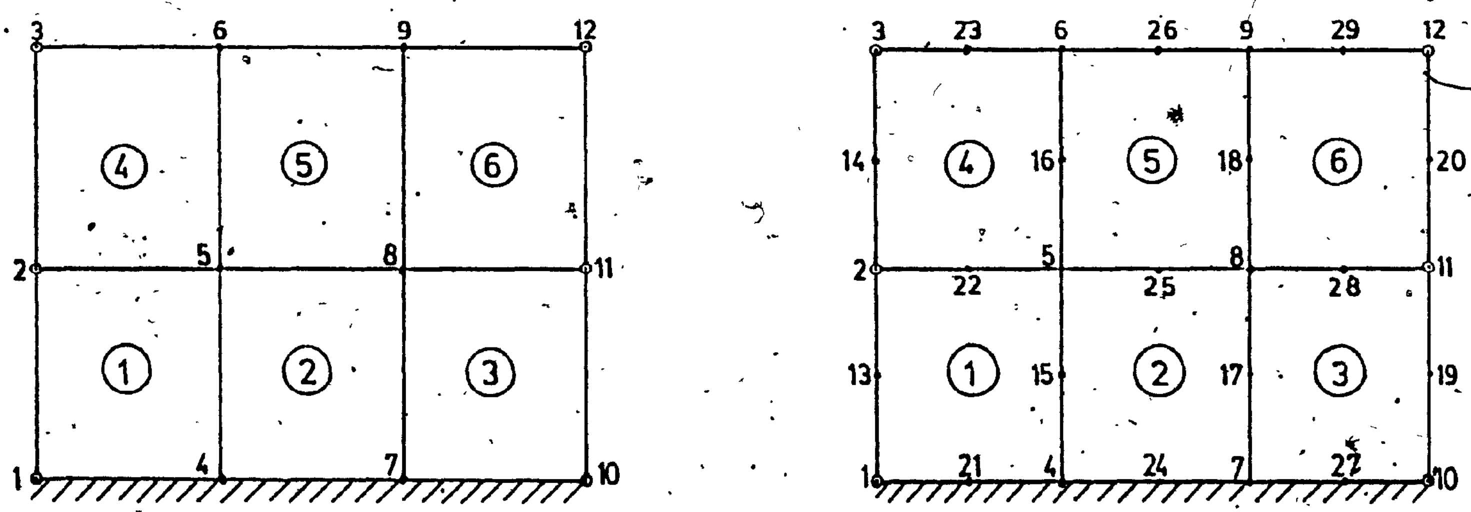


Fig. 4.1 Structure idealization



(a) ORDINARY ELEMENT (b) REFINED ELEMENT

Fig. 4.2 Automated numbering system of nodes and elements

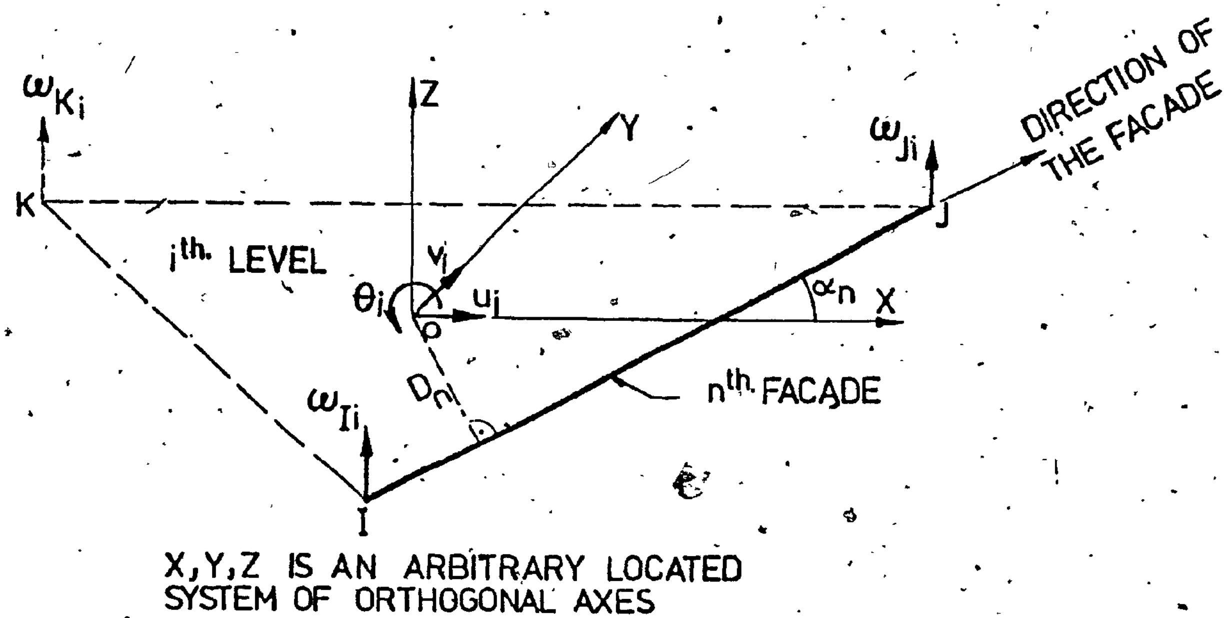


Fig. 4.3 Global static degrees of freedom at the i^{th} level

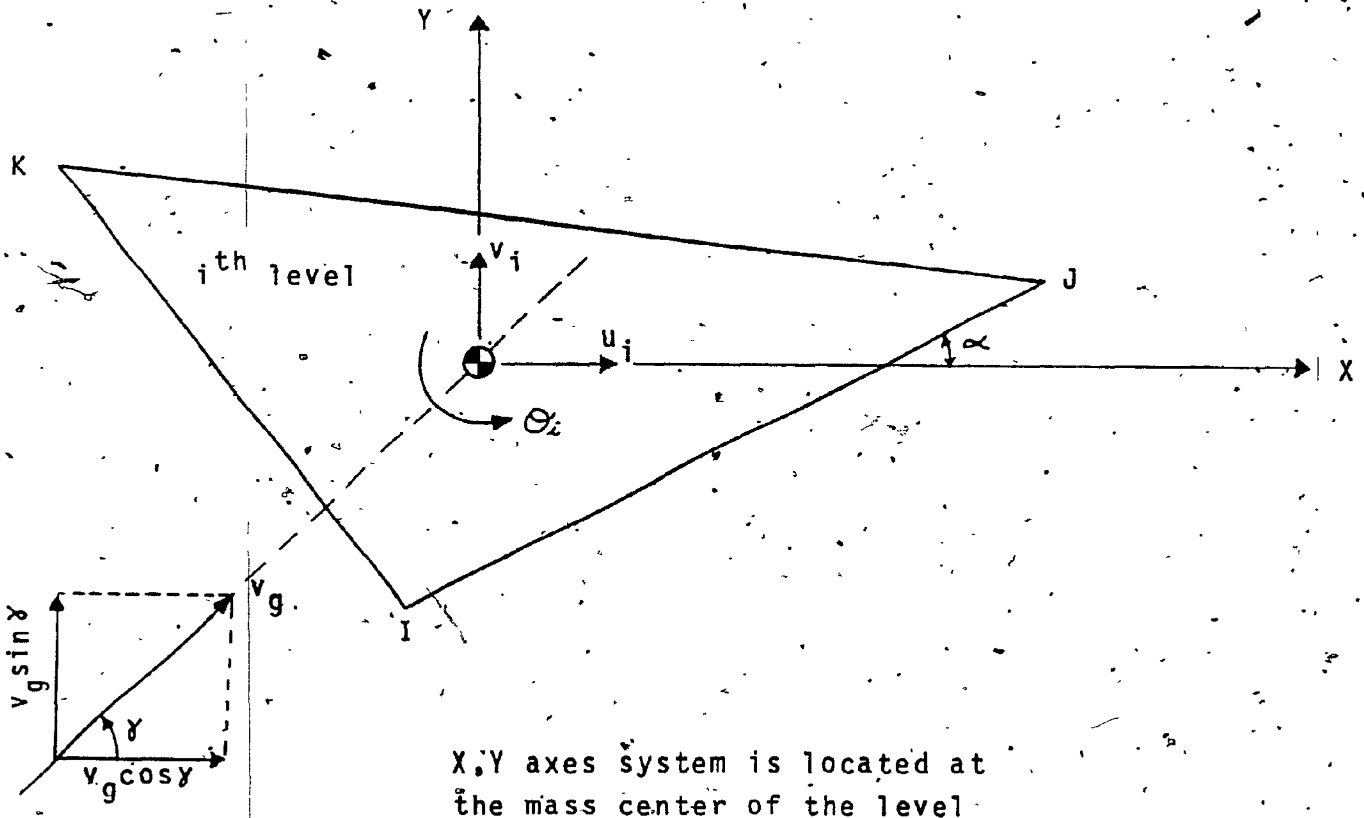
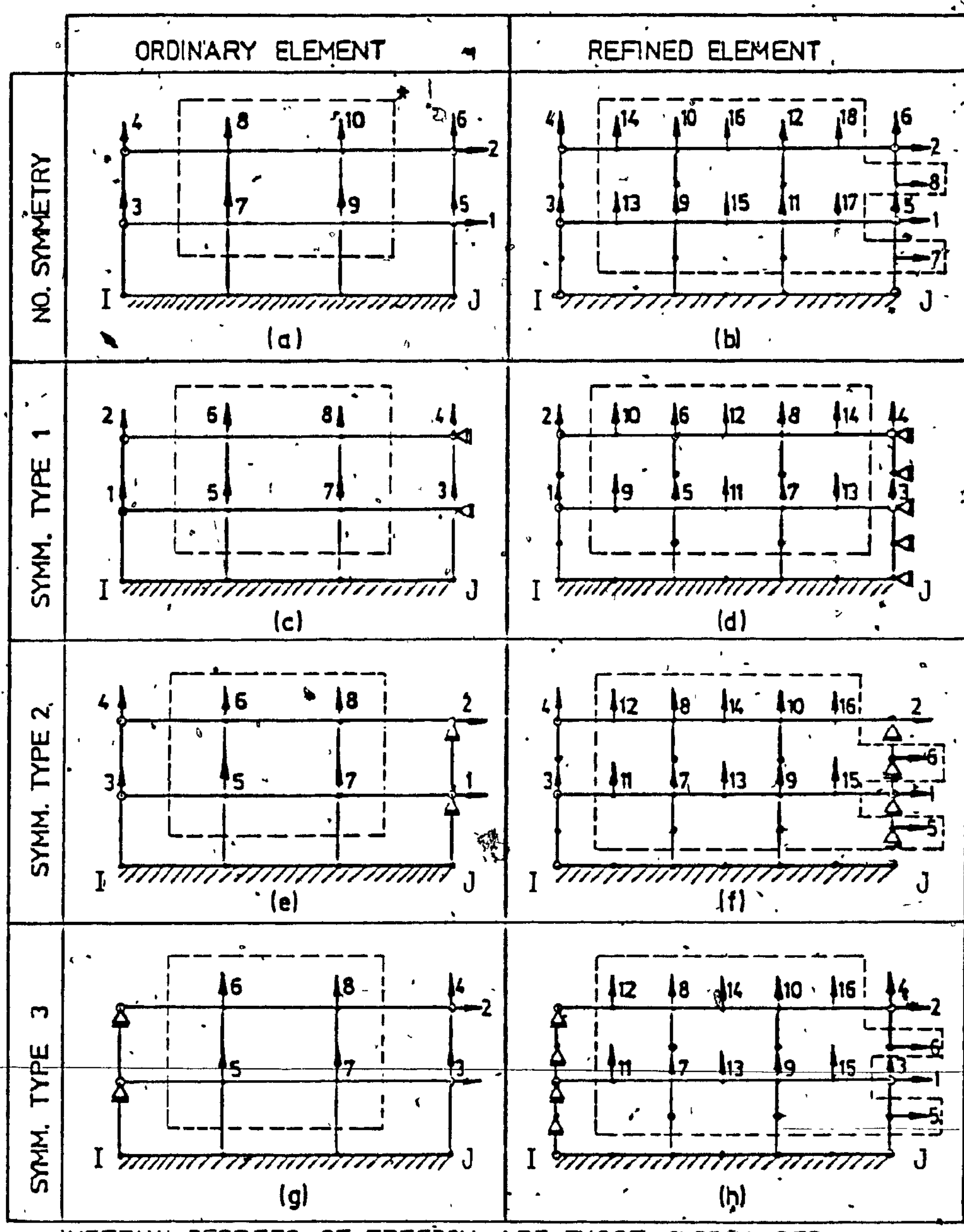


Fig. 4.4 Global dynamic degrees of freedom at level i and the direction of ground motion.



* INTERNAL DEGREES OF FREEDOM ARE THOSE SURROUNDED BY THE DOTTED LINE

Fig. 4.5 Automated facade degrees of freedom system

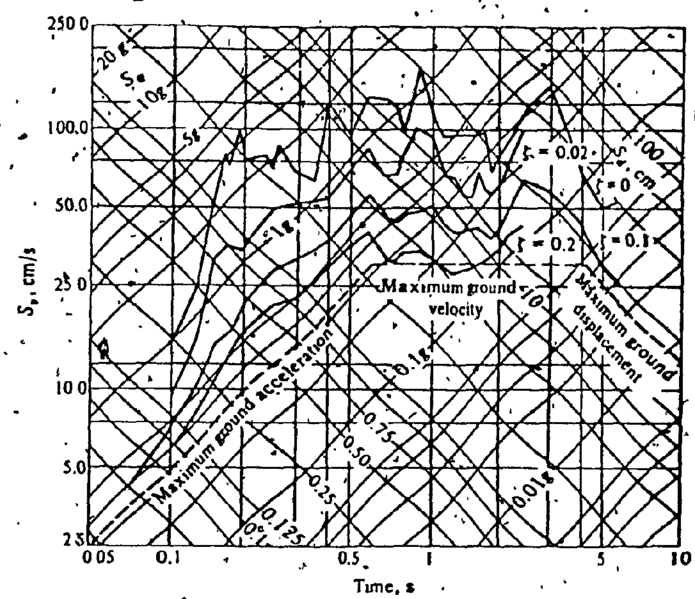


Fig. 4.6 Response spectra for El Centro earthquake, 1940. (21)

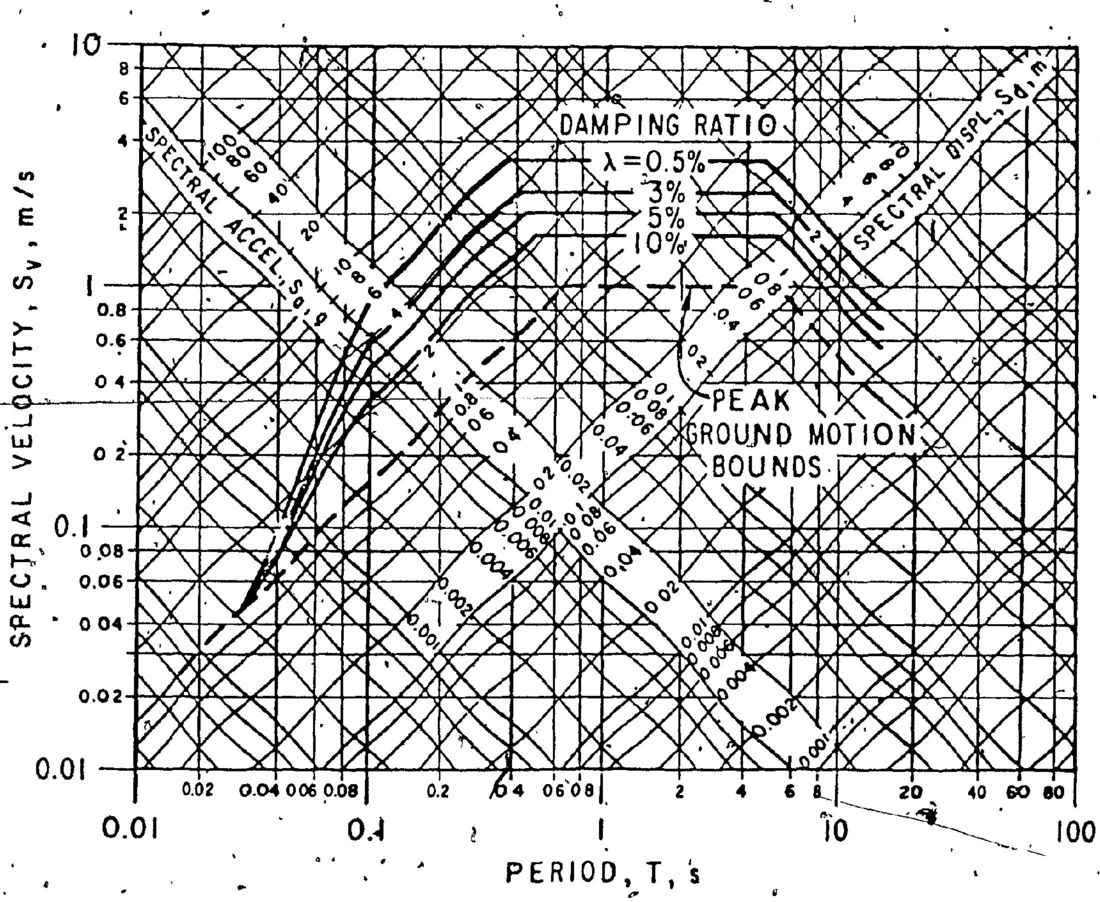


Fig. 4.7 Peak ground motion bounds and elastic average response spectrum for 1.0 g max. ground accel. (22)

CHAPTER V

EXAMPLES OF TUBE PROGRAM APPLICATION

5.1 INTRODUCTION

A program named TUBE has been developed to analyse most planar and tube-type building structures. Several examples of these types of structures have been cited earlier in Section 4.1. The analysis can be carried out for linear elastic static, or linear elastic earthquake spectral responses with an option to take into account the ductility factor. It is noted that response spectrum curves are required for the earthquake analysis. Description of the program input data is given in Appendix A.

In this chapter, four structures are analysed using the present method, and the results will be compared with other techniques. The objective is to verify the validity of the proposed modelling for seismic responses of tall planar and tube-type structures.

In chapter III, the stiffness matrices of two types of rectangular elements were presented: one consisting of 4 nodes and 6 degrees of freedom, and the other consisting of 6 nodes and 9 degrees of freedom. The linear 6 degree of freedom elements are attractive because they have only corner nodes and are simple to use, but they are too stiff in bending. For very tall buildings where the elements may have high aspect ratios, these simple elements may yield unacceptably inaccurate results. Alternative schemes for improvement generally fall into two classes: hybrid elements and incompatible displacement elements. These alternative schemes should be studied, however, they are beyond the scope of the present project. Throughout this chapter, the refined 9 degree of freedom element will be used.

5.2 BOX CANTILEVER BEAM

To verify the validity of the present macro-element technique applied to simple structures, a box cantilever beam is considered. Typical dimensions and properties of the structure are given in Fig. 5.1. Three analysis techniques will be compared: Goodno's and Gere's superelement method (23), the present macro-element method, and beam theory.

In the superelement technique, a basic rectangular element is originally assembled from a plate bending element and a plane stress element each containing 16 degrees of freedom. This basic element has 4 corner nodes each consisting of 8 degrees of freedom: 3 orthogonal translations, 2 out-of-plane rotations, 2 independent in-plane edge rotations, and 1 in-plane nodal twist. To expedite the assembly of the superelement, other specialized elements are generated from the basic element. Four elements are subsequently assembled to form a box-shaped superelement. The superelement stiffness matrix is statically condensed to 4 degrees of freedom representing horizontal translations of the top and bottom edges parallel to one direction (Fig. 5.2). The superelement reduced stiffness matrices are assembled to give the structure stiffness matrix. Three different mass formulations are used: consistent mass (CM), assembled lumped mass (ALM), and input lumped mass (ILM). The CM formulation permits the mass of the actual continuous structure to be lumped at the corner nodes of the rectangular element on an energy equivalent basis. The ALM formulation in contrast, requires physical mass lumping at each of the 4 nodes. The element mass matrices of the 4 basic or specialized elements are then assembled to form the superelement mass matrix which is subsequently dynamically condensed to the 4 parallel horizontal degrees of freedom. The structure mass matrix is then assembled in a similar fashion as the structure stiffness matrix. Unlike the CM and ALM formulations, the ILM formulation permits a diagonal structure mass matrix because it is created from the lumped masses associated with the acceleration of the 4 remaining superelement degrees of freedom.

Goodno and Gere modelled the box cantilever beam using 8 superelements (Fig. 5.2). Sixteen structure degrees of freedom were retained from the 352 nodal degrees of freedom represented in the model.

Using the present method, the box cantilever beam is modelled as follows. Due to symmetry, only one quarter of the structure is analysed. No part of the structure needs to be replaced by an elastically equivalent membrane because no discrete columns or beams exist. For simplicity the remaining structure is analysed using equally sized refined elements. To show convergence, two meshes are used; the first and second meshes are divided into 4 and 8 levels respectively. Each mesh has two elements horizontally spanning the longer facade and one element horizontally spanning the shorter facade (Fig. 5.1c). The input data presented in section A.5 was used to obtain the first four natural frequencies of the structure in the x direction using the first mesh.

Employing a discrete coordinate system for a dynamic response analysis can only yield approximate results because the motions of the system are represented by a limited number of displacement coordinates. In contrast, the beam theory approach renders more accurate results because it considers the behaviour of an infinite number of connected points by means of differential equations; this procedure allows the distributed physical properties of the structure to be described at each one of these points.

Table 5.1 compares the natural frequencies in the x direction obtained from beam theory taking into account shear deformations, the superelement technique, and the present macro-element method.

The first three natural frequencies obtained using mesh 1 are all within 7% of beam theory results, but the fourth natural frequency is 25% less. For the refined mesh 2 the corresponding values are 8% and 11%. Convergence for higher modes can thus be obtained by mesh refinement.

The efficiency of the present method can be exposed when comparing it to the superelement method. For the first mesh, only 4 structure degrees of freedom were retained from the 32 nodal degres of freedom represented in the model. Respectively, these values are 1/4 and 1/11 of those using the superelement technique. Although mesh 1 uses significantly less degrees of freedom than the superelement model, its first two natural frequencies (which are the greatest contributors to the structural responses) are closer to those of beam theory than the ILM formulation. Compared to beam theory, the ALM formulation provides the best results for the superelement method, but does not justify the extra computational effort due to mass coupling. Mesh 2 provides comparable accuracy with lumped masses (i.e diagonal mass matrix).

5.3 CONCRETE WALL-FRAME BUILDING

In this example, a 30-storey concrete wall-frame building is analysed (Fig. 5.3a). Two parallel shear walls are connected to two parallel frames forming a rectangular plan. The structure has the following properties:

storey height	= 12 ft.
width of structure	= 127.5 ft.
depth of structure	= 50.0 ft.
lumped weight per floor	= 1305 kips

Frame properties

16 columns per frame at 7.5 ft. c/c spacing

column size = 1.0 ft. x 1.5 ft.

beam size = 0.5 ft. x 2.0 ft.

Wall properties

thickness = 1.0 ft.

Material Properties

elastic modulus = 3.0×10^6 psi.

Poisson's ratio = 0.25

Although the structure looks like a cantilever tube, classical beam theory cannot be applied for analysis. Due to the flexibility of the spandrel beams in the frames, the column axial forces near the center lag behind (are lower than) the column axial forces near the building edges. This phenomenon is known as shear lag. This example shows that wall-frame combinations forming a tubular structure can be analysed using the present method.

The results of the analysis are compared to those obtained by Chan (3). His formulation is derived from the principle of minimum total potential energy. The strain energy is considered to be composed of:

1) in-plane bending of the shear wall

2) axial deformations of columns

3) in-plane bending of beams and columns

4) out-of-plane bending of columns; found to be negligible

It should be noted that shear deformations in the shear wall, beams and columns are not considered. Finite size joints are considered rigid. In-plane rigidity of floors and midspan inflection points of beams and columns are assumed as in the present method. Mass is distributed throughout the height of the building. Compatibility is imposed at the building edges while the column axial deformations, which take into account the "magnitude of shear lag in the normal frame", are approximated by a hyperbolic cosine variation or a parabolic variation. A sixth order differential equation for the mode shapes in the direction parallel to the shear walls is derived. A summation of 4 hyperbolic and 2 trigonometric terms is presented as the general solution. The mode shape equation is subjected to 6 boundary conditions which produce a homogeneous system of linear equations. The natural frequencies are those which yield a zero determinant for the coefficient matrix (i.e. a trial-and-error procedure).

Using the present method, the structure is modelled as follows. Due to symmetry, only one quarter of the structure need be analysed using global structural symmetry type one. The framed portion of the remaining structure is replaced by an elastically equivalent membrane. This equivalent structure is then divided into refined macro-elements forming a mesh (Fig 5.3b). If a full spectral earthquake analysis for ground motion in the x direction is required, the set of input data presented in section A.5 must be processed with TUBE.
Using a membrane thickness of 0.5 ft., the equivalent elastic properties of the membrane were determined to be:

$$E_y = 159 \times 10^3 \text{ ksf.}$$
$$G_{xy} = 13.8 \times 10^3 \text{ ksf.}$$

Their ratio G_{xy}/E_y indicates the degree of shear lag present in the frame (see section 6.2).

Table 5.2 shows the first three natural frequencies obtained by Chan, the present method, and beam theory. For the latter, the building was first considered as i) a cantilever tube beam (i.e. shear lag is ignored), and then ii) as two parallel shear walls excluding the framed facades. The results for the two parallel shear walls were found to be close to those obtained by Chan and the present method. This suggests that preliminary estimates of the dynamic characteristics of wall-frame structures can be obtained by considering only the shear walls. The cantilever tube beam model yields natural frequencies which are at least 40% greater than any other model presented. The first, second, and third natural frequencies obtained using the present method are approximately 1.11, 0.89, 0.78 times those obtained by Chan's approximate method. This significant difference in results may be due to the following approximations in Chan's formulation:

1) Shear deformation modes in beams and columns are ignored.
2) Finite size joints are considered rigid.
3) An assumed axial column deformation function.

5.4 PLANAR FRAME

In this example, a 16-bay, 40-storey plane frame will be analysed. The structure has the following properties along with those shown in Fig. 5.4:

storey height	=	12 ft.
lumped weight per storey	=	1800 kips
elastic modulus	=	3.0×10^6 psi.
shear modulus	=	1.2×10^6 psi.

As in the previous example, the shear lag effect inhibits the accurate application of beam theory. The purpose of this example is to show that TUBE is capable of analysing planar structures effeciently and accurately. The results will be compared to those of another modified computer program called TABS (24). TABS assumes infinite in-plane rigidity of floors. Joints are considered rigid with finite dimensions. Both bending and shear deformations of beams and columns are recognized. TABS allocates one vertical and one in-plane rotational degree of freedom per joint, and one lateral translational degree of freedom per storey. The joint degrees of freedom are statically condensed to leave the lateral storey translations which are transformed into global degrees of freedom consisting of two orthogonal translations and one in-plane rotation per storey. Mass and in-plane rotational inertia is specified for each storey.

For the present method, only half the structure is considered because of symmetry. The remaining structure is replaced by an equivalent orthotropic membrane which is subsequently divided into equal size macro-elements for ease of data preparation. Here, structure symmetry type one and facade symmetry type three is used. Two analyses will be performed; the first will include the same assumptions as TABS, and the other will consider flexible finite size joints.

The first four natural frequencies are presented in Table 5.3. Note that TABS uses 40 degrees of freedom for the natural frequency calculation, while TUBE uses only 10; this is a very significant difference in computational effort. However, the natural frequencies of the first analysis are comparable with those of TABS. It was expected that the present method would yield higher natural frequencies due to the model's greater stiffness induced by the prescribed element displacement functions; TABS uses no such functions. This would only be true if a consistent-mass formulation was used.

The second analysis, which includes the effects of finite size joint flexibility, alters from 9% to 13% lower natural frequencies compared to TABS. In the next chapter, it will be shown that this percentage difference can be magnified for the internal member forces.

5.5 FRAMED-TUBE BUILDING

In this example, a framed-tube building will be analysed. The structure is square in plan with each facade consisting of a 16-bay, 40-storey frame.

The structure has the following properties along with those shown in Fig. 5.5:

storey height	=	12 ft.
lumped weight per floor	=	3,600 kips
elastic modulus	=	3.0×10^6 psi.
shear modulus	=	1.2×10^6 psi.

Many computer programs such as TABS, do not consider the vertical compatibility at facade junctions (i.e. after loading their edges slide vertically relative to each other). The present method eliminates this action at the corner nodes of the building. The result is a stiffer, but more representative model of the actual structure.

The results obtained by TABS and Chan (3) will be used as comparisons for the present method. Chan's method of analysis is analogous to that presented in section 5.3 except i) out-of- plane bending of columns is ignored, and ii) a hyperbolic sine variation is assumed for the column axial deformations.

Using the present method, only one quarter of the structure is analysed because of symmetry conditions. The remaining structure is replaced by an elastically equivalent orthotropic membrane which is divided into rectangular plane stress refined macro-elements each spanning several bays and storeys (Fig. 5.5). Note that elements of equal size are used here only to simplify data preparation. Some useful rules for better modelling of the structure are presented in section A.4.1. Three analyses were performed for the natural frequencies and mode shapes of the building in the x direction. Each considers a different combination of deformation modes.

i) Analysis 1 : bending of beams and columns, rigid finite size joints (same deformation modes as Chan)

ii) Analysis 2 : bending of beams and columns, shear in beams and columns, rigid finite size joints (same deformation modes as TABS)

iii) Analysis 3 : bending of beams and columns, shear in beams and columns, flexible finite size joints

Table 5.4 displays the first four natural frequencies. Although Chan's method is approximate, the results compare well with analysis 1; the maximum difference being 4.7% for the second mode. The effect of shear deformations in members is found to be negligible when comparing with analysis 2.

Observe that the natural frequencies of TABS are identical to those presented in the preceeding section. This was expected since

i) the frame properties are identical to those of the previous section.

ii) the normal frame does not contribute to the structural stiffness because facades exhibit only in-plane stiffness, and because vertical compatibility at facade junctions are not enforced.

The resulting internal member force in the parallel facade are unreliable, while those of the normal facade are nonexistant. The first natural frequency of analysis 2 is 11.4% higher than TABS' value reflecting a stiffer model induced by the vertical compatibility at the facade junction. If the normal frames are replaced by shear walls, thereby increasing the overall stiffness of the structure, the difference will increase even more.

As for the case of the wall-frame structure of section 5.3, the normal frame has little contribution to the dynamic behaviour. Preliminary estimates of the natural frequencies and mode shapes can be obtained by considering only the frames parallel to the vibration direction.

The results of analysis 3 are the most accurate because flexibility of finite size joints is also considered. The resulting model is more flexible than that of analysis 2. Consequently, the natural frequencies are lower.

5.6 DISCUSSION

The present method was applied to a planar frame, and 4 three-dimensional assemblies of planar frames and planar shear walls. Although not presented here, the present method is versatile enough to consider panels, frames, lintel beams, and shear walls all combined in planar facades.

The efficiency of the present method is significantly greater than others presented (i.e. superelement technique and TABS); only one quarter of the global dynamic degrees of freedom were used, but results were in good agreement.

It was shown that increasing the number of levels in the structural model induces convergence of higher natural frequencies. Enforcing vertical compatibility at facade junctions was found to be important.

In examining the validity of the present method, some important structural behaviour characteristics were found:

1) The effect of shear deformations in members has little contribution to the dynamic behaviour.
2) Finite size joint flexibility significantly affects the dynamic behaviour.
3) Contribution of the normal facades of rectangular buildings to the dynamic behaviour is significant but small so that rough estimates can be obtained by considering only the parallel facades.

Table 5.1 Natural frequencies of box cantilever beam

Mode No.	Natural Frequencies (rad/sec)					
	Beam Theory (23)	TUBE Program		Superelement Method (23)		
		Mesh No. 1	Mesh No. 2	CM	ALM	ILM
1	1186	1271	1284	1279	1267	1368
2	5815	5777	5918	5914	5702	6245
3	13189	11752	12674	14125	12992	12792
4	21488	15976	19047	25093	21162	18000

Table 5.2 Natural frequencies (rad/sec) of wall-frame building.

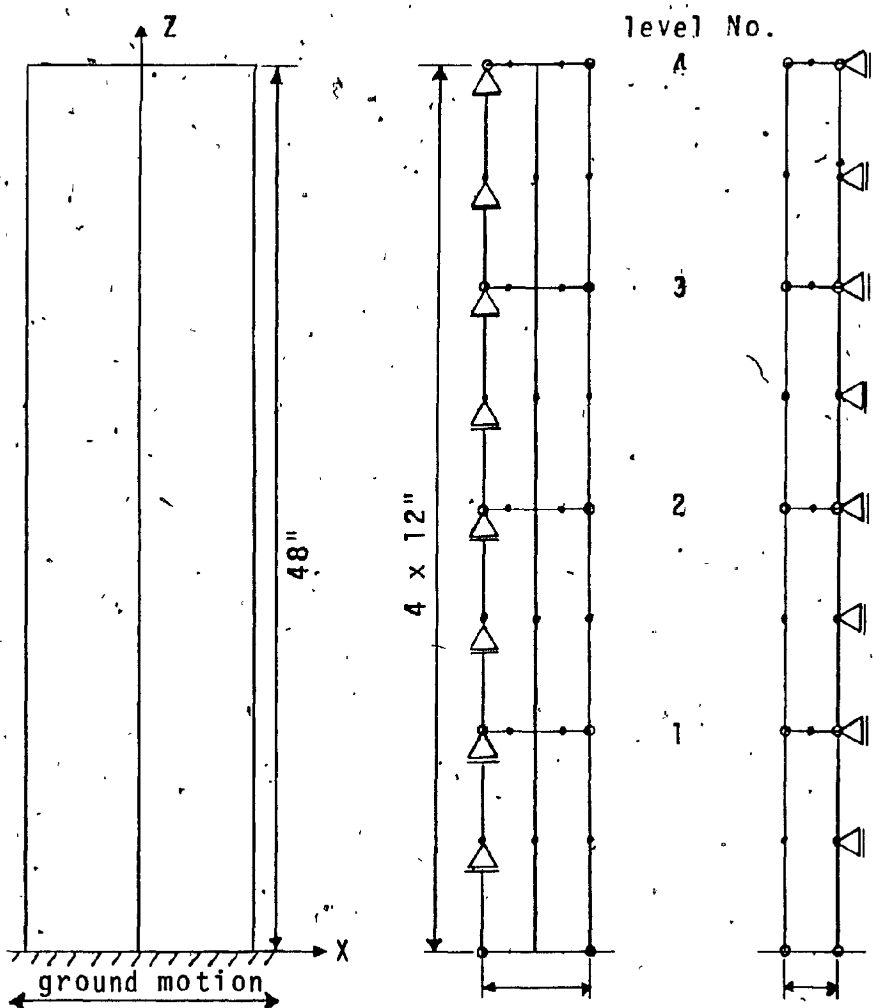
Method		Mode 1	Mode 2	Mode 3
Simple Beam Theory(3)	Shear Wall Alone	1.47	9.18	25.69
	Cantilever tube beam (shear lag ignored)	2.23	13.97	39.11
CHAN (3)	Parabolic Approximation	1.47	10.07	27.88
	Hyberbolic cosine Approximation	1.47	9.82	26.92
TUBE		1.64	8.78	22.27

Table 5.3 Natural frequencies of planar frame

MODE NUMBER	NATURAL FREQUENCY (rad/sec)			% diff. with TABS	
	TABS(24)	TUBE		TUBE	
		FIRST ANALYSIS	SECOND ANALYSIS	FIRST ANALYSIS	SECOND ANALYSIS
1	.710	.697	.644	-1.8	-9.3
2	2.31	2.23	2.08	-3.5	-10
3	4.33	4.10	3.85	-5.3	-11
4	6.21	5.75	5.41	-7.4	-13
NO. OF RETAINED DEGREES OF FREEDOM	40	10	10		

Table 5.4 Natural frequencies of framed-tube building.

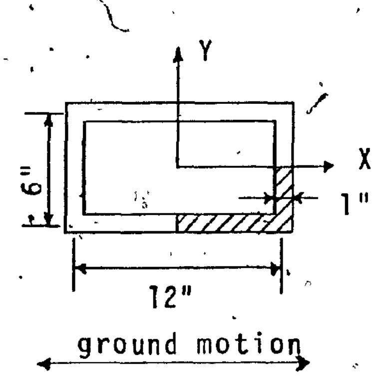
MODE NUMBER	NATURAL FREQUENCY (rad/sec)				
	TABS (24)	CHAN (3)	TUBE		
			ANALYSIS 1	ANALYSIS 2	ANALYSIS 3
1	.710	.792	.803	.791	.740
2	2.31	2.33	2.44	2.40	2.25
3	4.33	4.22	4.23	4.16	3.92
4	6.21	5.96	5.85	5.75	5.43



(b) Elevation

Facade No. 1 Facade No. 2

(c) Structure Idealization (Mesh 1)



(a) Plan
Only the shaded portion
is modelled

(d) Properties (steel)
Young's modulus 30000 ksi.
Poisson's ratio 0.25
specific weight 0.284 lb/in^3

Fig. 5.1 Box cantilever beam and present model

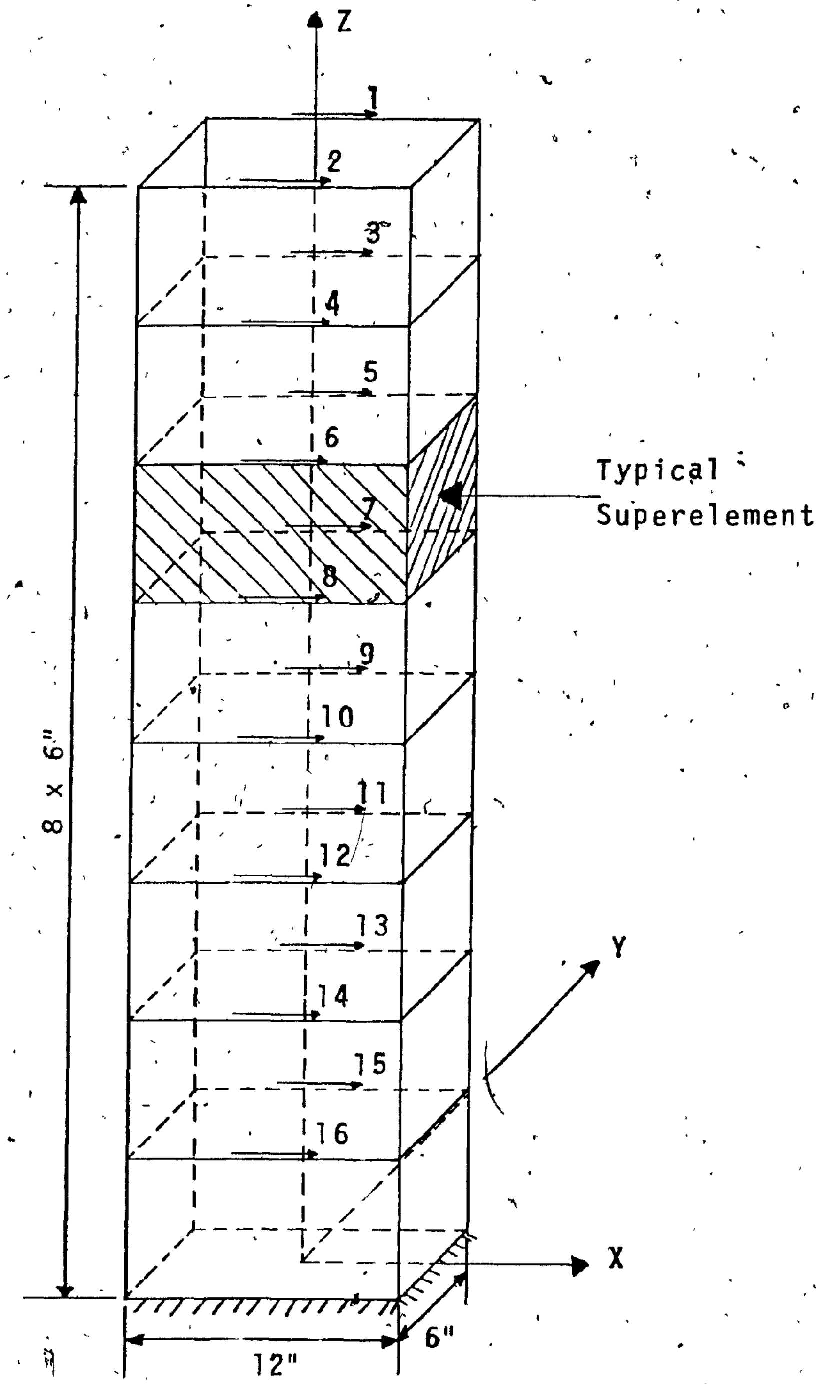


Fig. 5.2 Superelement model of box cantilever beam

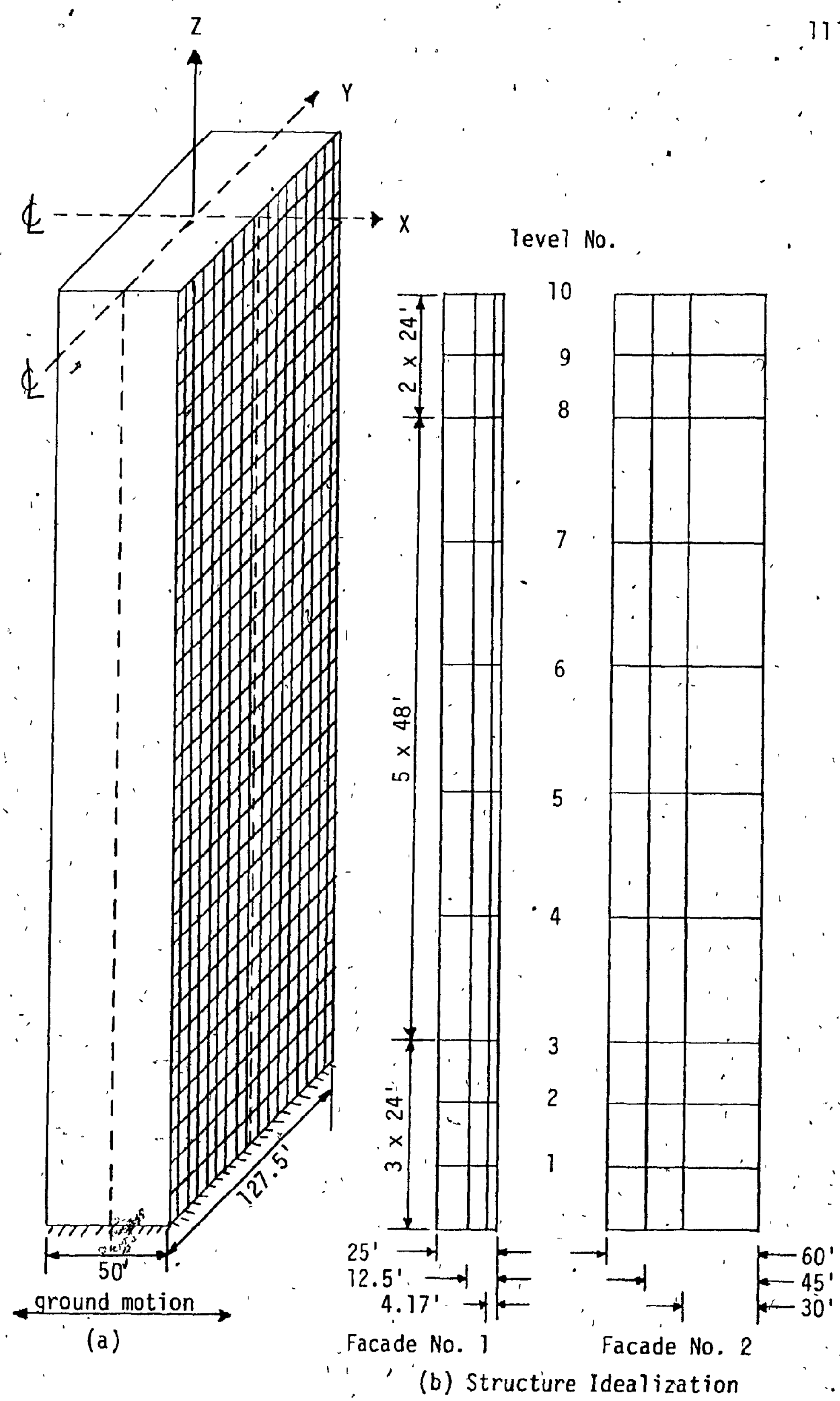


Fig. 5.3 Wall-frame building and the present idealization.

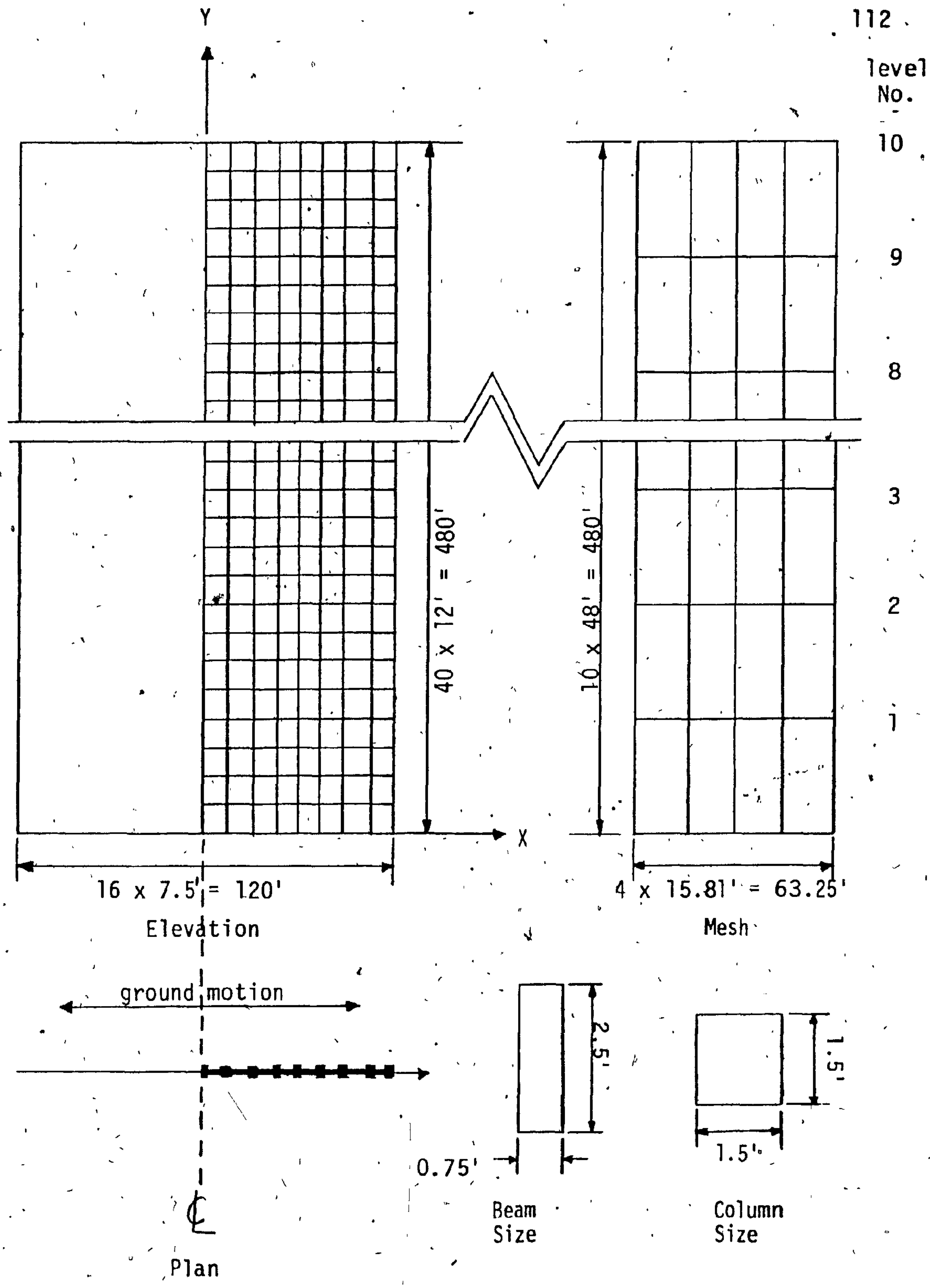


Fig. 5.4 Planar frame, properties, and model.

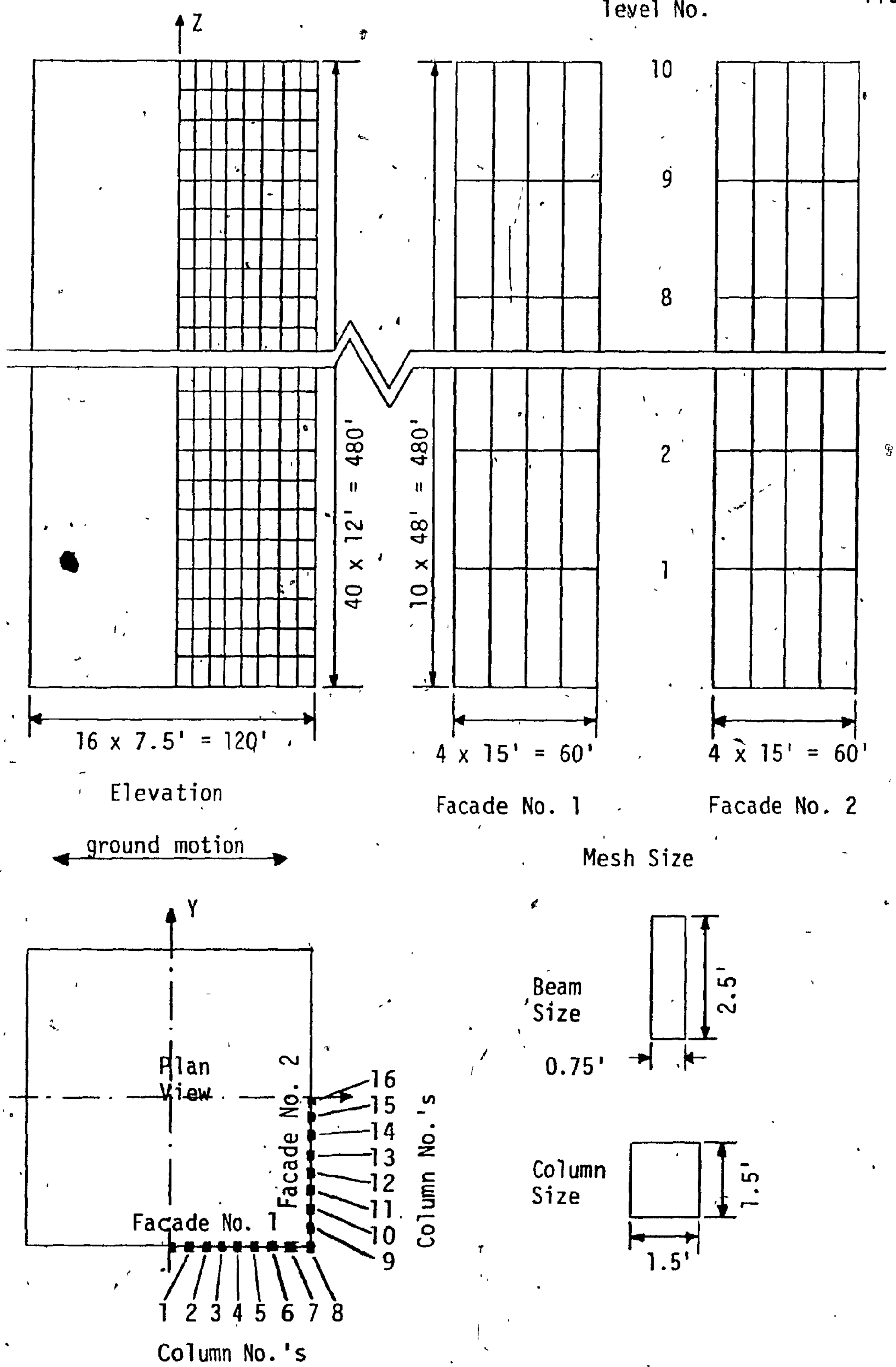


Fig. 5.5 Framed-tube building, properties, and idealization

CHAPTER VI

BEHAVIOURAL CHARACTERISTICS OF TALL FRAMED-TUBE BUILDING STRUCTURES

6.1 INTRODUCTION

In this chapter, the behavioural characteristics of tall framed-tube buildings are investigated. In particular, the shear lag phenomenon, and the effects of shear deformations and of finite size joints on the dynamic responses are of special interest. For this study, the framed-tube structure presented in section 5.5 is used in conjunction with the following additional data:

1) 5% critical damping for all modes
2) risk factor of 0.01 (for Vancouver); an acceleration of 0.08g is recommended (22)
3) the response spectrum as shown in Fig. 4.7 (22).

6.2 SHEAR LAG PHENOMENON

In the case of a slender tube made up of solid faces and subjected to lateral load, the normal stress distribution would be almost linear. However, when the faces are perforated with large openings resembling windows in tall buildings, the shear flexibility of the "window lintel beams" tend to diminish the shear transfer from one "column" to another with the consequence that the interior "columns" take less axial forces than the exterior ones. This nonlinearity of stress distribution is most pronounced at the tube corners, and the term shear lag is used to denote this phenomenon.

To reduce the shear lag effect (i.e. to make the tube more effective in resisting lateral load), the beam stiffnesses should be high, thus requiring relatively deep beams. A measure of the shear lag severity can be effected by means of the ratio

$$SL = \frac{G_{xy}}{E_y} \tag{6.1}$$

The higher the shear lag parameter, the more effective the tube action will be.

The shear lag parameter for the cantilever tube discussed in section 5.2 is 0.40. Fig. 6.1 shows that the vertical stress variation is almost linear.

Taking the basic geometry of Chan's builiding, and varying the shear modulus G_{xy}, the column axial stresses are plotted for different values of the shear lag parameter as shown in Fig. 6.2. The shear lag parameter varies from 0.0199 to 0.0819. These values are much less than that of the cantilever tube above resulting in highly nonlinear curves. Note that the linearity increases with increasing SL especially for the parallel facade.

In terms of the actual structure, as opposed to the equivalent membrane model, G_{xy} can be increased while leaving E_y relatively constant simply by increasing the beam depth (Table 6.1). A stiffer beam thus yields a higher shear lag parameter and permits more efficient shear transfer from one column to the other. The shear lag effect diminishes resulting in a more linear column axial force variation.

The magnitude of the maximum internal member forces depend heavily on the natural frequencies which are ultimately derived from the mass and stiffness properties of the structure. A correlation can also be made here between the shear lag parameter and the natural frequencies. Varying G_{xy} as before, the first four natural frequencies are plotted in Fig. 6.3. The natural frequencies increase with increasing shear lag parameter.

Similar curves are presented in Chapter VII to be used in conjunction with simplified equations for natural frequency estimations of planar and tube-type building structures.

6.3 IMPORTANCE OF SHEAR DEFORMATIONS, FLEXIBLE FINITE SIZE JOINTS, AND RESPONSE SPECTRUM CONSIDERATION

The natural frequencies of the framed-tube structure are presented in section 5.5. It was concluded that omitting any kind of deformation component results in a stiffer model, and consequently higher frequencies. However, a designer's concern is with respect to their effects on the internal member forces. The column axial forces of four analyses will be investigated to determine the importance of shear deformations in beams and columns and finite size joint flexibility. The deformation considerations of each analysis are listed below.

Analysis 1: bending in beams and columns,
shear in beams and columns,
flexible finite size joints.

Analysis 2: bending in beams and columns,
shear in columns, (shear in beams is ignored),
flexible finite size joints.

Analysis 3: bending in beams and columns,
shear in beams, (shear in columns is ignored),
flexible finite size joints.

Analysis 4: bending in beams and columns,
shear in beams and columns,
rigid finite size joints.

The results are plotted in Fig. 6.4. Ignoring shear deformations in beams (analysis 2) and columns (analysis 3) increases the corner column (No. 8) axial force by 2.1% and 4.6% respectively. If rigid joints are considered (analysis 3), there is an acute increase of 17%. The joint size of this structure in relation to the storey height and bay width is typical of framed-tube buildings. If the relative size of the joint is increased, the error due to the rigid joint assumption also increases. For a reasonably accurate analysis, joints must be considered flexible. The accuracy may still be slightly enhanced by considering shear deformations in beams and columns.

It was stated in section 4.9.7 that once the elastic forces on the structure are determined, the structure can be analysed statically for each mode. From Eq. (4.74), the elastic force vector for each mode depends on λ, Γ, S_v, $\{\emptyset\}$, [M]. For each of the analyses, [M] remains the same, while $\{\emptyset\}$ was almost identical for corresponding modes; from Eq. (4.65), Γ must also be the same for corresponding modes. We can thus conclude that λ and S_v are the major factors which changed the elastic force vectors from one analysis to another. Table 6.2 displays Γ, λ, S_v, and $(\lambda S_v)_m/(\lambda S_v)_1$ where the subscripts refer to the analysis number. The last ratio is termed the error factor which reveals the factor by which the elastic force vector of analysis m is greater than that of analysis 1 for a specific mode. Because the analyses are linear elastic, the member internal forces of analysis m are also greater by this factor for each corresponding mode. Notice that the frequencies increase with analysis number (m). For mode number one, the pseudo-velocities, which depend on the frequencies, also increase. Thus both effects strengthen the error factor.

For the other modes, the pseudo-velocity stays constant, and thus only the ratio of natural frequencies provide the only error factor. In other words the response spectrum acts in such a way that errors are magnified for modes with periods greater than approximately 5 seconds. As shown above, the first and perhaps the second mode, which are the major contributors to the structural behaviour, lie in this region, and thus consideration of flexible joints is vital. Shear deformation considerations are less essential to the analysis.

6.4 DISCUSSION

Slender planar and tube-type structures with solid (unperforated) faces exhibit an almost linear normal stress distribution. When the faces are perforated, the distribution becomes nonlinear. A shear lag parameter was introduced to measure the degree of linearity. It was found that increasing the beam depth, also increased the shear lag parameter, inducing a more linear distribution especially in the parallel facade.

A higher shear lag parameter was also found to increase the natural frequencies which cause an increase in the pseudo-velocities of the primary modes and consequently increase the maximum effective external elastic forces and internal stresses.

Flexibility of finite size joints was found to significantly affect the internal forces. Rigid joints yielded approximately 20% higher column axial forces at the second storey of a framed-tube building. Shear deformation considerations had substantial but less significant effects and thus may be ignored at least for preliminary purposes.

The nature of the response spectrum was found to magnify errors in the internal forces induced by approximations in modelling the stiffness properties.

Table 6.1 Variation of G_{xy}, E_y, and SL with beam depth.

BEAM DEPTH (ft.)	EQUIVALENT MODULUS+ (psf.)		SL
	G_{xy}	E_y	
2.5*	343	172	.0199
3	433	172	.0251
3.5	536	173	.0312
4	656	174	.0377
5	986	178	.0554
6	1515	185	.0819

+ The membrane thickness is taken as 0.75 ft

* Basic structure

TABLE 6.2 Modal participation factors, natural frequencies, pseudo-velocities, and error factors for framed-tube building.

MODE NO.	ANALYSIS NO.	Γ	λ (rad/sec)	S_v (ft/sec)	$(\lambda S_v)_m/(\lambda S_v)_1$ (error factor)
1	1	1.35	.740	.311	1.000
	2	1.35	.749	.315	1.025
	3	1.35	.759	.319	1.052
	4	1.34	.791	.332	1.141
2	1	.510	2.25	.525	1.000
	2	.511	2.28	.525	1.013
	3	.513	2.31	.525	1.027
	4	.503	2.40	.525	1.067
3	1	.264	3.92	.525	1.000
	2	.265	3.98	.525	1.015
	3	.265	4.03	.525	1.028
	4	.263	4.16	.525	1.061
4	1	.176	5.43	.525	1.000
	2	.176	5.51	.525	1.015
	3	.177	5.59	.525	1.029
	4	.176	5.75	.525	1.059

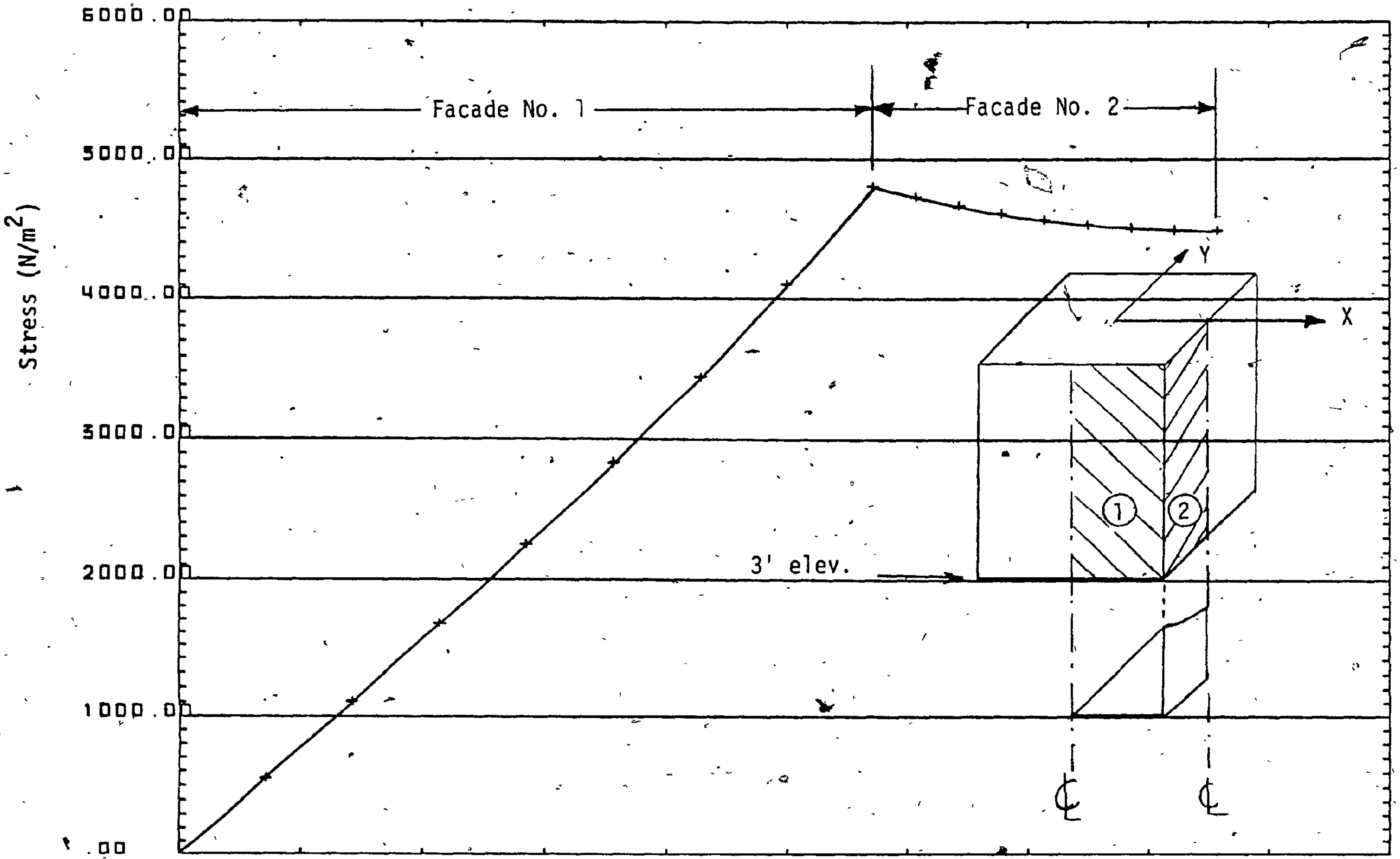


Fig. 6.1 Variation of normal stress at 3' elevation of the box cantilever beam.

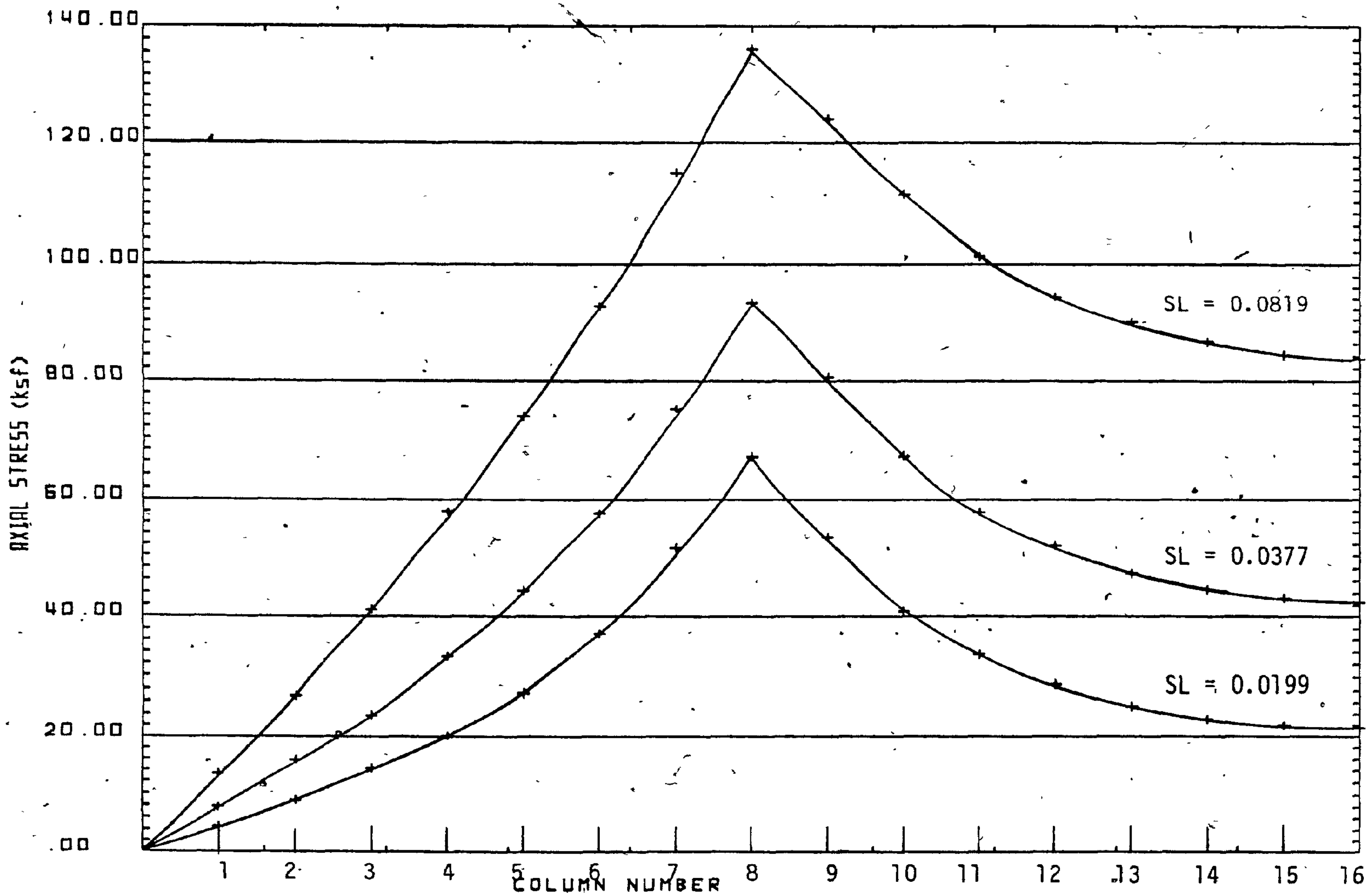


Fig. 6.2 Effect of the shear lag parameter (SL) on the column axial stresses at the fourth storey of the framed-tube building.

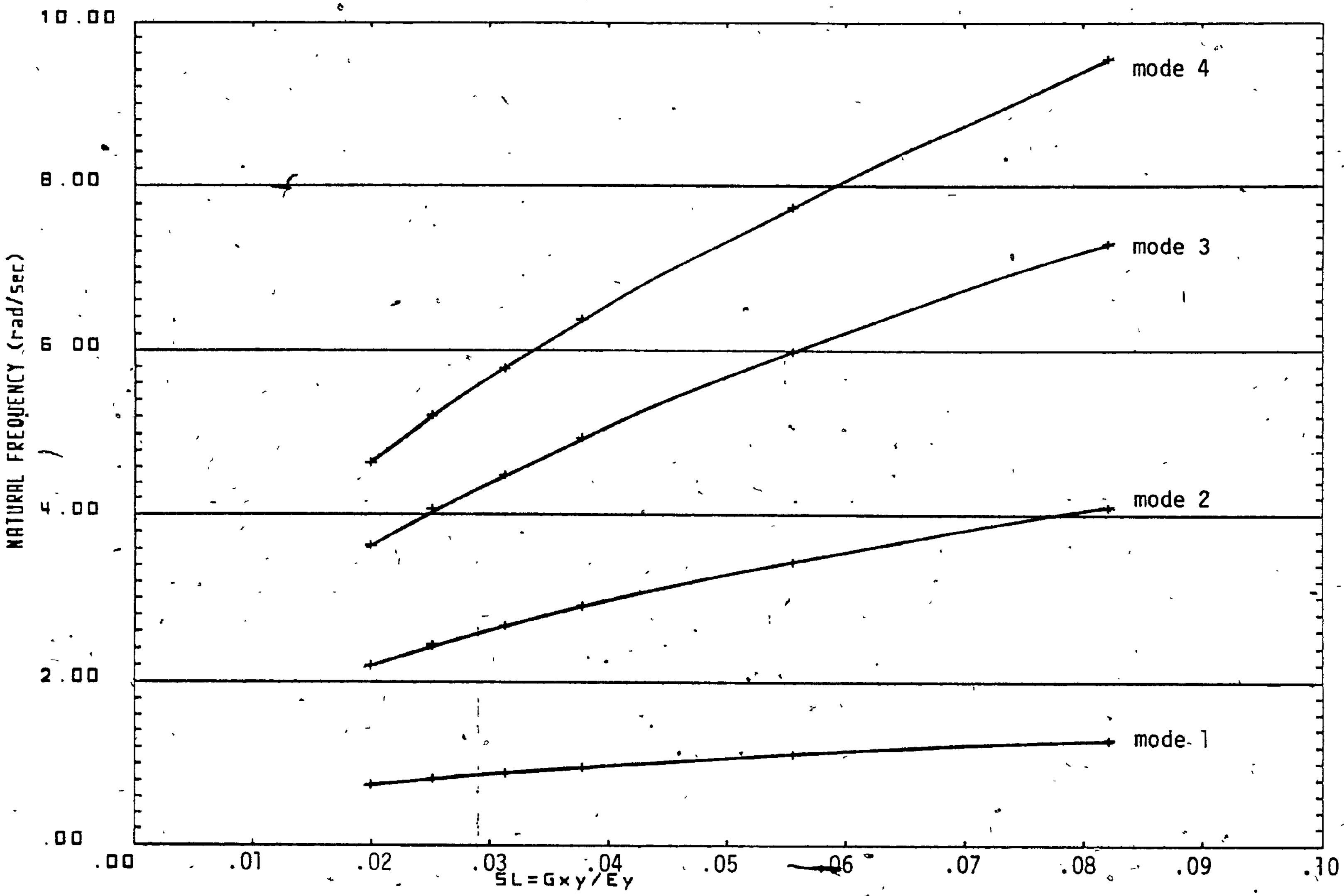


Fig. 6.3 Effect of the shear lag parameter (SL) on the natural frequencies of the framed-tube building.

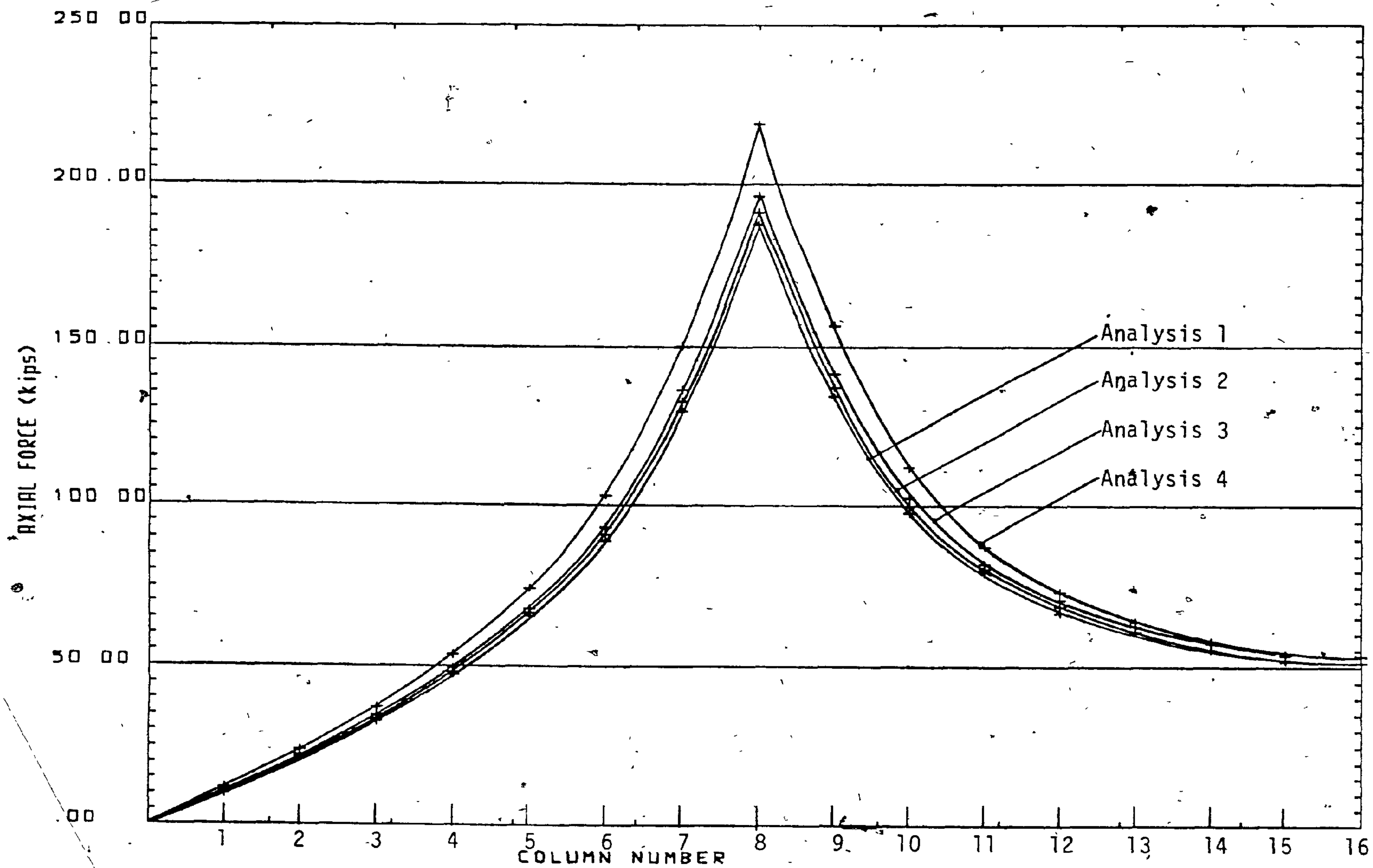


Fig. 6.4 Column axial forces at the second storey of the framed-tube building

CHAPTER VII

NATURAL FREQUENCY DETERMINATION FOR PLANAR AND TUBE-TYPE STRUCTURES

7.1 INTRODUCTION

This chapter provides designers and researchers with two simple methods of determining natural frequencies of planar and tube-type structures. The first, more accurate method requires the use of a simple frequency equation. The Buckingham π theorem (30) is used to partially solve the equation as the product of two functions. The undetermined variation of the second function is found to depend on three dimensionless parameters. Many computer runs varying these parameters provided characteristic points which were plotted to give design curves for the second function. These curves in conjunction with the first function provide the natural frequencies of a wide range of structural characteristics.

The second, more approximate method requires less computations, and applies strictly to framed-tube buildings. The building is modelled in such a way that modified natural frequency equations for cantilever beams can be used. The modelling which yields the most accurate results for different building characteristics is sought.

7.2 SCOPE AND LIMITATIONS

The two methods of natural frequency determination presented, are within the following scope and limitations:

1) Only the first two translational natural frequencies for vibration along the depth (A) of the structure can be determined.
2) Linear elastic behaviour.
3) The stiffness properties must remain constant through the structures (i.e. the structure can be idealized using a single orthotropic membrane).
4) Infinite horizonal rigidity is assumed (i.e. all the points in a horizontal plane displace equally).
5) Structure is fixed to a rigid foundation.
6) Three-dimensional structures must be rectangular in plan
7) Mass distribution along the height of the structure must be uniform or at least concentrated uniformly along many (>4) equally spaced intervals.

8) The first method can be applied to tube-type or planar structures. The second method is strictly for framed-tube buildings with members of rectangular cross-section.

7.3. BUCKINGHAM π THEOREM APPLIED TO NATURAL FREQUENCIES

The variables affecting the natural frequencies of the structure along with their basic dimensions are described below.

variable	dimension	description
A	L	width of structure.
B	L	depth of structure.
t	L	thickness of elastically equivalent orthotropic membrane.
H	L	height of structure
E_y	$ML^{-1}T^{-2}$	equivalent vertical elastic modulus.

G_{xy}	$ML^{-1}T^{-2}$	equivalent shear modulus.
$\overline{m}$	ML^{-1}	mass distribution along the height of the structure
λ	T^{-1}	natural frequency.

where L = length

M = mass

T = time

All the above variables are known to be essential to the solution, and hence some functional relation must exist.

$$f_1(A,B,t,H,E_y,G_{xy},\overline{m},\lambda) = 0 \qquad (7.1)$$

The Buckingham π theorem allows a reduction in variables. There are eight variables and only three dimensions; Eq. (7.1) can be expressed using 8 - 3 = 5 dimensionless parameters (π) as

$$f_2(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \qquad (7.2)$$

$\pi_1, \pi_2, \pi_3, \pi_4, \pi_5$ must be found.

Since there are three dimensions, three repeating variable (which together contain all the dimensions) must be chosen. H, E_y, and $\overline{m}$ yields π's with the best physical meaning.

$$\pi_1 = H^{x_1} E_y^{y_1} \overline{m}^{z_1} A$$
$$\pi_2 = H^{x_2} E_y^{y_2} \overline{m}^{z_2} B$$
$$\pi_3 = H^{x_3} E_y^{y_3} \overline{m}^{z_3} t$$
$$\pi_4 = H^{x_4} E_y^{y_4} \overline{m}^{z_4} G_{xy}$$
$$\pi_5 = H^{x_5} E_y^{y_5} \overline{m}^{z_5} \lambda$$

Since the π's are dimensionless,

for π_1, $(L)^{x_1}(ML^{-1}T^{-2})^{y_1}(ML^{-1})^{z_1}(L) = M^0 L^0 T^0$

from which

$$x_1 - y_1 - z_1 + 1 = 0$$
$$-2y_1 = 0$$
$$y_1 + z_1 = 0$$

thus $x_1 = -1 \quad y_1 = 0 \quad z_1 = 0$

Similarly for π_2, $x_2 = -1 \quad y_2 = 0 \quad z_2 = 0$

Similarly for π_3, $x_3 = -1 \quad y_3 = 0 \quad z_3 = 0$

For Π_4, $$(L)^{x_4}(ML^{-1}T^{-2})^{y_4}(ML^{-1})^{z_4}(ML^{-1}T^{-2}) = L^0\,T^0\,M^0$$

from which

$$\begin{aligned} x_4 - y_4 - z_4 - 1 &= 0 \\ -2y_4 \qquad -2 &= 0 \\ y_4 + z_4 + 1 &= 0 \end{aligned}$$

thus

$$x_4 = 0 \qquad y_4 = -1 \qquad z_4 = 0$$

For Π_5, $$(L)^{x_5}(ML^{-1}T^{-2})^{y_5}(ML^{-1})^{z_5}(T^{-1}) = L^0\,T^0\,M^0$$

from which

$$\begin{aligned} x_5 - y_5 - z_5 \qquad &= 0 \\ -2y_5 \qquad -1 &= 0 \\ y_5 + z_5 \qquad &= 0 \end{aligned}$$

thus

$$x_5 = 0 \qquad y_5 = -1/2 \qquad z_5 = 1/2$$

The five dimensionless Π parameters are

Π_1 = A/H

Π_2 = B/H

Π_3 = t/H

$\Pi_4 = G_{xy}/E_y$

$\Pi_5 = \lambda\sqrt{\dfrac{m}{E_y}}$

Eq. (7.2) can now be expressed as

$$f_2\ (A/H, B/H, t/H, G_{xy}/E_y, \lambda\sqrt{\bar{m}/E_y}) = 0 \tag{7.3}$$

or

$$f_3\ (H/A, H/B, t/H, E_y/G_{xy}, \lambda\sqrt{\bar{m}/E_y}) = 0 \tag{7.4}$$

Eq. (7.4) can further be manipulated to give

$$\lambda\sqrt{\frac{\bar{m}}{E_y}} = f_4(H/A, H/B, t/H, E_y/G_{xy}) \tag{7.5}$$

or

$$\lambda = \sqrt{\frac{E_y}{\bar{m}}}\, f_4(H/A, H/B, t/H, E_y/G_{xy}) \tag{7.6}$$

This is the limit to where the π Buckingham theorem can be used.

Eq. (7-6) can further be simplified by noticing that the element stiffness matrices shown in section 3.2 and 3.3 are directly proportional to t. Since t is constant throughout the structure, the facade stiffness matrices and global structure stiffness matrix are also linearly proportional to t. Recall the eigenvalue problem obtained from free undamped vibrations yielding Eq. (4.45).

$$\lambda^2 [M]\{\emptyset\} = [\bar{K}_s^*]\{\emptyset\} \tag{7.7}$$

Let $[\bar{K}_s^*] = t\,[K]$ (7.8)

Then Eq. (7.7) becomes

$$\lambda^2 [M]\{\phi\} = t[K]\{\phi\} \tag{7.9}$$

or

$$\left(\frac{\lambda}{\sqrt{t}}\right)^2 [M]\{\phi\} = [K]\{\phi\} \tag{7.10}$$

From this equation it can be seen clearly that the natural frequency is directly proportional to the square root of the thickness (i.e. $\lambda \propto \sqrt{t}$). This means that $f_4 \propto \sqrt{t}$. Since f_4 can be factored by $\sqrt{t}$, and one of the dimensionless parameters is t/H, then f_4 can be factored by $\sqrt{t/H}$.

$$\lambda = \sqrt{\frac{E_y t}{\bar{m} H}}\; f(H/A, H/B, E_y/G_{xy}) \tag{7.11}$$

Let E_{ys}, t_s, $\bar{m}_s$, H_s be a set of standard (constant) values, then the natural frequency for this set of values is

$$\lambda_s = \sqrt{\frac{E_{ys}\, t_s}{\bar{m}_s\, H_s}}\; f(H/A, H/B, E_y/G_{xy}) \tag{7.12}$$

which depends on three dimensionless parameters. λ_s is termed "basic frequency". Dividing Eq. (7.11) by Eq. (7.12) yields

$$\lambda = \lambda_s \sqrt{\left(\frac{E_y}{E_{ys}}\right)\left(\frac{t}{t_s}\right)\left(\frac{\overline{m}_s}{\overline{m}}\right)\left(\frac{H_s}{H}\right)} \qquad (7.13)$$

For a given structure, H/A, H/B, and E_y/G_{xy} are known, and λ_s can be determined through Eq. (7.12.) If E_y, t, $\overline{m}$, and H, are also known, then λ can be calculated using Eq. (7.13.)

For convenience the standard constants are taken as

$$E_{ys} = 10^6$$
$$H_s = 10^2$$
$$\overline{m}_s = 10^4$$
$$t_s = 1$$

Eq. (7.13) becomes

$$\lambda = \lambda_s \sqrt{\frac{E_y t}{\overline{m} H}} \qquad (7.14)$$

λ has units of time^{-1} ; λ_s must therefore be unitless, and the standard constants may be taken as unitless.

Many computer runs using TUBE were performed leading to graphs of λ_s for a wide range of H/A, H/B, E_y/G_{xy} values and the standard constants. Because of symmetry only one half of the planar structure and one quarter of the tube-type stucture was analysed. For the remaining model, 10 levels were used in conjuction with 4 refined elements horizontally spanning each facade. Figs. 7.1a/-r show the variation of λ_s for the first and second mode.

7.4 SIMPLE STEPS FOR NATURAL FREQUENCY CALCULATIONS

The following set of steps may be followed chronologically to obtain the natural frequencies of planar or tube-type structures.

STEP 1. Determine tE_y and tG_{xy} . For solid walls, use the elastic properties of the actual material. For orthogonal gridwork of rectangular cross-section, use Eqs. (2.3) and (2.8). These values may always be determined by equating the deflection of the discrete structure obtained experimentally, and the equivalent membrane obtained theoretically (See sections 2.3.1 and 2.3.2).

STEP 2. Determine H/A, H/B, E_y/G_{xy}

STEP 3. Determine the basic natural frequency (λ_s)

(1) For tube-type structures

- For the first mode, use Figs. 7.1a - g
- For the second mode, use Figs. 7.1h - p

(2) For planar structures

- For the first mode, use Fig. 7.1q
- For the second mode, use Fig. 7.1r

STEP 4. Determine $\overline{m}$

$\overline{m}$ = the linear mass density along the height of the full structure

STEP 5. Determine λ for each mode.

$$\lambda_n = \lambda_{sn}\sqrt{\frac{E_y t}{\overline{m} H}} \tag{7.15}$$

where n = mode number

The following examples should familiarize the reader with the method while giving confidence in the results.

The first example to be illustrated is the box cantilever beam of section 5.2.

STEP 1.

$$tE_y = 1 \text{ in.}(30{,}000 \text{ ksi}) = 30{,}000 \text{ kips/in.}$$

$$tG_{xy} = 1 \text{ in.}\frac{(30{,}000 \text{ ksi})}{2(1 + 2.5)} = 12{,}000 \text{ kips/in.}$$

STEP 2.

$$H/A = 48 \text{ in.}/12 \text{ in.} = 4$$

$$H/B = 48 \text{ in.}/6 \text{ in.} = 8$$

$$E_y/G_{xy} = 30{,}000/12{,}000 = 2.5$$

STEP 3. Using Fig. 7.1d, λ_{s1} can be determined.
Using Fig. 7.1k, λ_{s2} can be determined.

$$\lambda_{s1} = 0.262$$

$$\lambda_{s2} = 1.210$$

STEP 4.

$$\bar{m} = \frac{0.284 \text{ lb/in}^3(2)(12\text{in.} + 6\text{in.})(1\text{in.})}{32.2 \text{ ft/s}^2}$$

$$\bar{m} = 0.3175 \text{ slugs/in.}$$

STEP 5. $\sqrt{\dfrac{E_y t}{\overline{m} H}} = \dfrac{(30{,}000{,}000 \text{ slugs } (12 \text{in.})/s^2 \text{in})}{0.3175 \text{ slugs/in } (48 \text{ in})}$

$= 4860$ rad/sec

$\lambda_1 = 4860\ (0.262) = 1{,}270$ rad/sec

$\lambda_2 = 4860\ (1.210) = 5{,}880$ rad/sec

The resulting frequencies are close to those calculated previously.

The second example shown below is for a steel cantilever beam of rectangular cross-section. The following data is required for the analysis:

$E = 30{,}000$ ksi

$\nu = 0.25$

$\rho = 0.00882$ slugs/in

$H = 500$ in.

$A = 50$ in.

$t = 50$ in.

From beam theory, the natural frequencies are

$$\lambda_n = C_n \sqrt{\frac{EI}{\overline{m} H^4}} \tag{7.16}$$

where $C_1 = 1.875^2$ and $C_2 = 4.694^2$

For a solid rectangular cross-section

$$I = \frac{t\,(A)^3}{12}$$

resulting in

$$\lambda_n = C_n \sqrt{\frac{1}{12}\left(\frac{A}{H}\right)^3} \sqrt{\frac{E}{m}\frac{t}{H}} \qquad (7.17)$$

Note the similarity of Eq.(7.15) to Eq. (7.17)

If the present method is to yield equivalent answers as beam theory, then

$$\lambda_{sn} = C_n \sqrt{\frac{1}{12}\left(\frac{A}{H}\right)^3}$$

Note that λ_{sn} for beam theory depends only on H/A (i.e. independent of E_y/G_{xy}). Putting values in the above equation yields

$$\lambda_{s1} = (1.875)^2 \sqrt{\frac{1}{12}\left(\frac{50}{500}\right)^3} = 0.0321$$

$$\lambda_{s2} = (4.694)^2 \sqrt{\frac{1}{12}\left(\frac{50}{500}\right)^3} = 0.201$$

Using the graphs

1) $t = 50$ in.

$$G_{xy} = \frac{30,000 \text{ ksi}}{2(1 + 0.25)} = 12 \text{ ksi}$$

$$tE_y = 50\,\text{in}[3 \times 10^7 \text{slugs}(12\,\text{in})/s^2\text{in}] = 1.8 \times 10^{10} \text{ slugs}/s^2$$

$$tG_{xy} = 50\,\text{in}[1.2 \times 10^7 \text{slugs}(12\,\text{in})/s^2\text{in}] = 0.72 \times 10^{10} \text{ slugs}/s^2$$

2) $H/A = 500/50 = 10$

$B = 0$

$E_y/G_{xy} = 1.8/.72 = 2.5$

3) From Figs. 7.1q and r respectively, $\lambda_{s1} = 0.0318$

$\lambda_{s2} = 0.193$

These values are very close to the ones calculated above using beam theory.

For the planar frame of section 5.4

1) Using Eqs. (2.3) and (2.8), tE_y and tG_{xy} are calculated to be 128.9×10^6 lb/ft and 2.569×10^6 lb/ft respectively.

2) $H/A = 480/120 = 4$

$B = 0$

$E_y/G_{xy} = 429.8/8.563 = 50.2$

3) From Fig. 7.1q, $\lambda_{s1} = 0.082$

From Fig. 7.1r, $\lambda_{s2} = 0.265$

4) $\bar{m} = 1800 \times 10^3 \frac{\text{slugs ft.}}{s^2 \text{ storey}} \times \frac{\text{storey}}{12 \text{ ft.}} \times \frac{1}{32.2 \text{ ft.}/s^2} = 4658 \frac{\text{slugs}}{\text{ft.}}$

5) $\sqrt{\frac{E_y t}{\bar{m} H}} = \sqrt{\frac{128.9 \times 10^6 \text{ slugs}/s^2}{4658 \text{ slugs/ft.} (480 \text{ ft.})}} = 7.593 \text{ rad/sec.}$

$\lambda_1 = (0.082)7.593 = 0.62$ rad/sec.

$\lambda_2 = (0.265)7.593 = 2.01$ rad/sec.

These values are almost identical to those previously calculated for the second analysis of section 5.4.

The last example presented here is for Chan's framed-tube building presented in section 5.5.

1) Using equations (2.3) and (2.8), tE_y and tG_{xy} are calculated to be 128.9×10^6 lb/ft and 2.569×10^6 lb/ft respectively.

2) $H/A = 480/120 = 4$

$H/B = 480/200 = 4$

$E_y/G_{xy} = 429.8/8.563 = 50.2$

3) From Fig. 7.1d and k, $\lambda_{s1} = .138$ and $\lambda_{s2} = .420$ respectively

4) $\overline{m} = 3600 \times 10^3 \dfrac{\text{slugs ft.}}{s^2 \text{ storey}} \times \dfrac{\text{storey}}{12 \text{ ft.}} \times \dfrac{1}{32.2 \text{ ft.}/s^2} = 9317 \dfrac{\text{slugs}}{\text{ft.}}$

5) $\sqrt{\dfrac{E_y \lambda}{\overline{m} H}} = \sqrt{\dfrac{128.9 \times 10^6 \text{ slugs ft.}/s^2 \text{ft.}}{9317 \text{ slugs/ft.} (480 \text{ ft.})}} = 5.369 \text{ rad/sec.}$

$\lambda_1 = (0.138)5.396 = 0.741 \text{ rad/sec.}$

$\lambda_2 = (0.420)5.396 = 2.25 \text{ rad/sec.}$

The first and second natural frequencies obtained in section 5.5 are 0.740 and 2.25 radians per second respectively for analysis 3.

7.5 CANTILEVER BEAM MODELLING OF FRAMED-TUBE BUILDINGS FOR FREQUENCY CALCULATIONS

Natural frequency calculations using the design curves may prove inconvenient for preliminary design. Determining the equivalent elastic orthotropic membrane properties E_y and G_{xy} is often tedious. A simple method of natural frequency calculations using a cantilever beam model is presented here. All limitations presented in section 7.2 apply here also.

For a cantilever beam, the nth natural frequency along the depth (A) is given by Eq. (7.16). Treating Chan's framed-tube building presented in section 6.2 a cantilever beam in which

$$I = \sum_{\substack{\text{all} \\ \text{columns}}} (d^2 A_c)_i \tag{7.18}$$

where d = distance from the neutral axis to column i

A_c = cross-sectional area of column i,

the moment of inertia was determined to be 346,285 ft^4 of which 70% was contributed by the normal facades (normal to the vibration direction), and 30% by the parallel facades (parallel to the vibration direction). From Eq. (7.6), the first and second natural frequencies were determined to be 1.934 rad/sec and 12.12 rad/sec respectively. These values are 2.4 and 5 times greater than those determined from the design curves. These errors are too large for any reasonable preliminary design. From the design curves, frequency is seen to be more sensitive to A than to B; the reverse is true using the beam vibration equation. This is one reason why Eq. (7.16) applied to framed-tube structures is fundamentally incorrect. The form of Eq. (7.16) will be altered to take care of the errors involved.

$$\lambda_n = X C_n \sqrt{\frac{E(I_1 + \beta I_2)}{\bar{m} H^4}} \tag{7.19}$$

where X = a correction factor to be determined

I_1 = moment of inertia of the parallel facades including corner columns.

I_2 = moment of inertia of the normal facades excluding corner columns.

β = a factor to be determined specifying the percentage of I_2 to be considered.

Because it is now known that for framed-tube structures, the parallel facade is of greater importance, β is introduced so that only a fraction of the normal facade moment of inertia is considered. Eq. (7.18) is valid only for a linear distribution of axial column deformations; the shear lag phenomenon inhibits this behavior. The correction factor X takes into account this discrepency along with the effects of β. The β sought is that which yields the smallest variation of X. From Eq. (7.15) and Eq. (7.19).

$$\lambda_{sn} \sqrt{\frac{E_y t}{\bar{m} H}} = X C_n \sqrt{\frac{E(I_1 + \beta I_2)}{\bar{m} H^4}}$$

from which

$$X^2 = \left(\frac{\lambda_{sn}}{C_n}\right)^2 \frac{E_y t H^3}{E(I_1 + \beta I_2)} \qquad (7.20)$$

Substituting Eq. (2.3) yields

$$X^2 = \left(\frac{\lambda_{sn}}{C_n}\right)^2 \frac{E t_c d_c H^3}{W C_y (I_1 + \beta I_2)} \qquad (7.21)$$

but $$I_1 = 4\sum A_c d_i^2 = 4 t_c d_c \sum d_i^2 \tag{7.22}$$

Summation is over one half of one parallel facade.

$$I_1 = 4 t_c d_c w^2 Z \tag{7.23}$$

where w = bay width

$$Z = \sum_{i=1}^{\frac{1}{2}N_{B1}} i^2 \quad \text{if } N_{B1} \text{ is even} \tag{7.24}$$

$$Z = \sum_{i=1}^{\frac{1}{2}(N_{B1}+1)} \left(i - \tfrac{1}{2}\right)^2 \quad \text{if } N_{B1} \text{ is odd} \tag{7.25}$$

N_{B1} = No. of bays in one parallel facade

$$I_2 = 2(N_{B2}-1)A_c\left(\frac{A}{2}\right)^2 = \frac{1}{2} t_c d_c A^2 (N_{B2}-1) \tag{7.26}$$

where N_{B2} = No. of bays in one normal facade

A = depth of one parallel facade

Substituting Eqs. (7.23) and (7.26) into (7.21) yields

$$X^2 = \left(\frac{\lambda_{sn}}{C_n}\right)^2 \frac{H^3}{w C_y\left[4w^2 Z + \frac{1}{2}\beta A^2 (N_{B2}-1)\right]} \tag{7.27}$$

Replacing w with A/N_{B1} gives

$$X^2 = \left(\frac{\lambda_{sn}}{C_n}\right) \frac{2\,(H/A)^3}{C_y\left[8\left(\frac{1}{N_{B1}}\right)^3 Z + \beta\left(\frac{N_{B2}-1}{N_{B1}}\right)\right]} \tag{7.28}$$

At this point, several approximatins are made. An average value of C_y for framed-tube structures is 0.9. From the basic frequency curves (Figs. 7.1a-q), for a given H/A, the λ_{sn} does not vary too much. λ_{sn} of Eq. (7.28) will be replaced in accordance with Table 7.1 derived for H/A = H/B and E_y/G_{xy} = 50. The ratio E_y/G_{xy} of most framed-tube structures varies between 30 and 80. Table 7.1 also displays Eq. (7.27) incorporating the above approximations. Note that for a given H/A, the X value varies only with N_{B1}, N_{B2}, and β, (i.e. $Y=f(N_{B1}, N_{B2}, \beta)$). Values of Y were computed for all combinations of N_{B1} = 8,9,10, ...,24 and N_{B2} = 8,9,10,...,24 and β = 0.,.1,.2,....,1.0. Several of these values are shown in Table 7.2. Note that as β decreases, the percentage variation of Y for different values of N_{B1} and N_{B2} also decreases. For β = 0., this variation is the smallest; an average value for Y is taken as 1.55 for $N_{B1} = N_{B2} = 15$.

Eq. (7.19) now becomes

$$\lambda_n = X \cdot C_n \sqrt{\frac{E\, I_1}{\bar{m} \cdot H^4}} \tag{7.30}$$

The value of X is obtained from Table 7.3. It can now be seen clearly that X takes into account the errors involved in moment of inertia calculations using Eq. (7.18) and the effects of ignoring the normal facades.

Errors in using Eq. (7.30) are usually within 15%, but this value can be significantly decreased. The major source of error is in the approximation of λ_{sn} for a given H/A. For framework with deep members, X should be increased by a factor of 1.1. For framework with shallow members, X should be decreased by a factor of 0.9. The number of bays within the parallel facade should be between 8 and 24. For these two extremes, multiply X by

0.94 or 1.05 respectively. If these two rules are followed, errors will usually remain within 8%.

Eq. (7.29) will be used on Chan's framed-tube building. The following values were computed.

$I_1 = 103{,}300 \text{ ft}^4$

$\overline{m} = 9317$ slug/ft

$E = 432 \times 10^6$ psf.

$H = 480$ ft

$H/A = 4$

From Table 7.3, $X_1 = .725$ and $X_2 = .352$

The framework if fairly normal, and the parallel facades have 16 bays, therefore the values of X_1, and X_2 are unaltered. The first two natural frequencies are

$$\lambda_1 = 0.725\,(3.516)\sqrt{\frac{(432 \times 10^6 \text{ psf.})(103300 \text{ ft.}^4)}{(9317 \text{ slugs/ft.})(480 \text{ ft.})^4}} = 0.725(3.516)(0.30) = 0.765 \text{ rad/sec.}$$

$$\lambda_2 = 0.352(22.034)(0.30) = 2.33 \text{ rad/sec.}$$

These values are respectively 3% and 4% greater than those obtained using the basic natural frequency curves.

Table 7.1 Average Values of λ_s and X for different H/A values taking $E_y/G_{xy} = 50$

	MODE 1		MODE 2	
H/A	λ_{s1}	X*	λ_{s2}	X*
10	.065	.872Y	.22	.471Y
8	.081	.777Y	.26	.398Y
6	.103	.642Y	.32	.318Y
4	.138	.468Y	.42	.227Y
3	.166	.366Y	.50	.176Y
2	.210	.252Y	.63	.121Y
1	.310	.131Y	.91	.0616Y

$$* \quad Y = \left[8\left(\frac{1}{N_{\theta 1}}\right)^3 Z + \beta\left(\frac{N_{\theta 2}-1}{N_{\theta 1}}\right)\right]^{-\frac{1}{2}} \qquad (7.29)$$

Table 7.2 Y values for different β, N_{B1}, and N_{B2} values

β	N_{B1}	N_{B2}		
		8	16	24
0.	8	1.46	1.46	1.46
	16	1.58	1.58	1.58
	24	1.63	1.63	1.63
0.5	8	1.05	.843	.724
	16	1.27	1.07	.946
	24	1.38	1.21	1.08
1.0	8	.862	.653	.547
	16	1.09	.865	.738
	24	1.22	.999	.866

Table 7.3 Average X values for different H/A ratios taking $E_y/G_{xy} = 50$, $N_{B1} = N_{B2} = 15$

H/A	X	
	MODE 1	MODE 2
10	1.35	.730
8	1.20	.617
6	.995	.493
4	.725	.352
3	.567	.272
2	.391	.187
1	.203	.0954

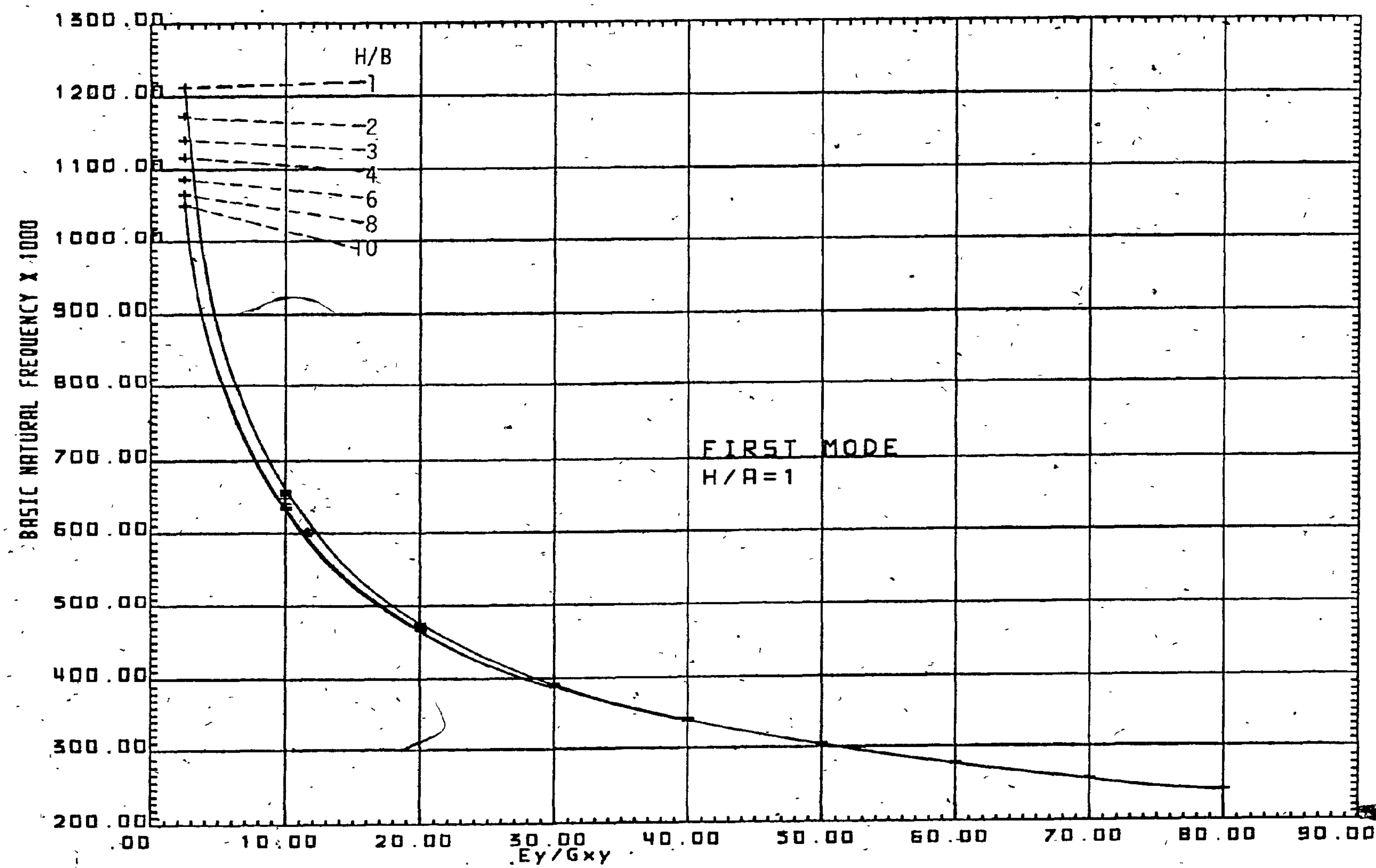


Fig. 7.1a Basic natural frequency for tube-type structure (mode 1, H/A=1)

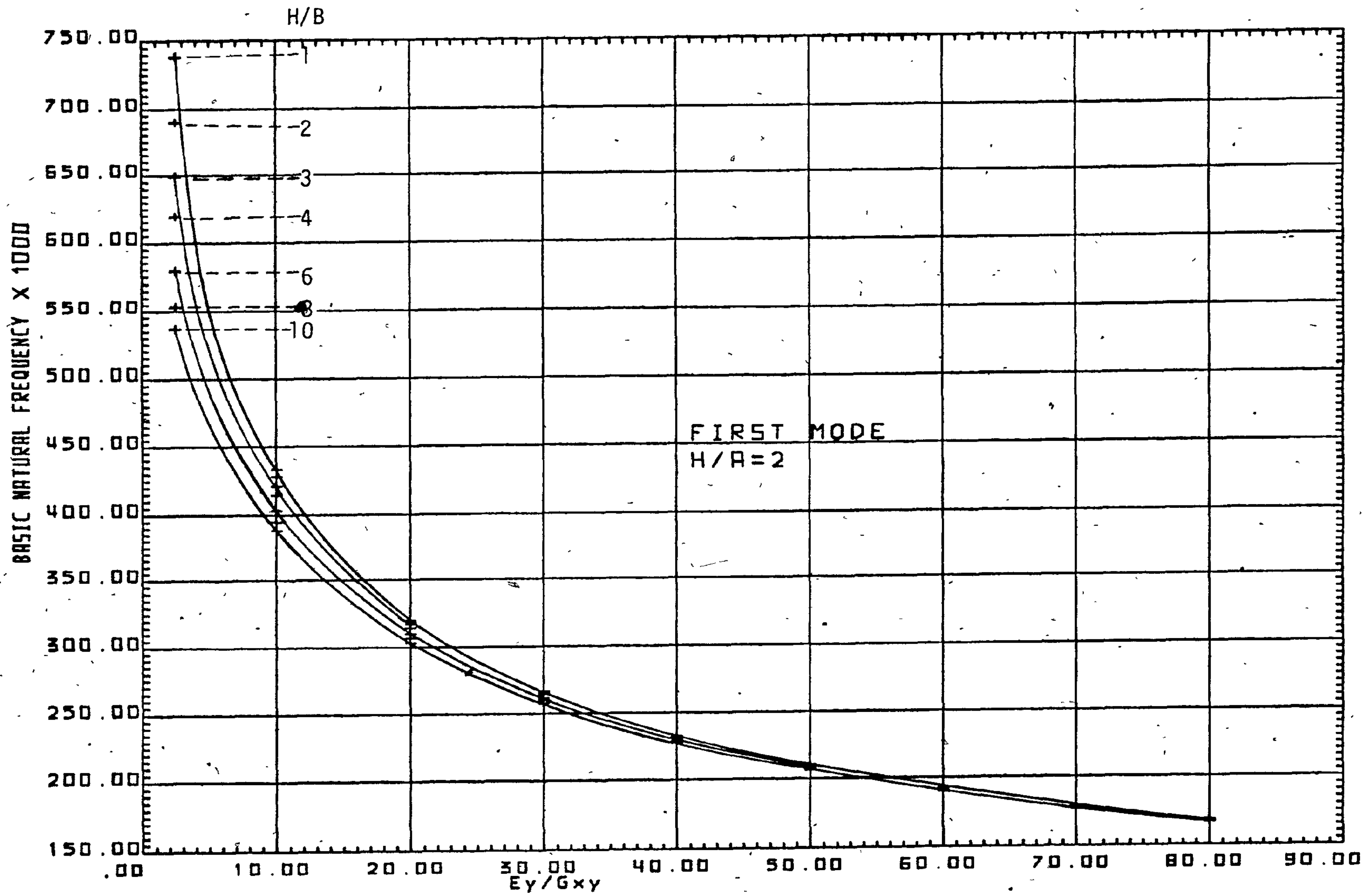


Fig. 7.1b Basic natural frequency for tube-type structure (mode 1, H/A=2)

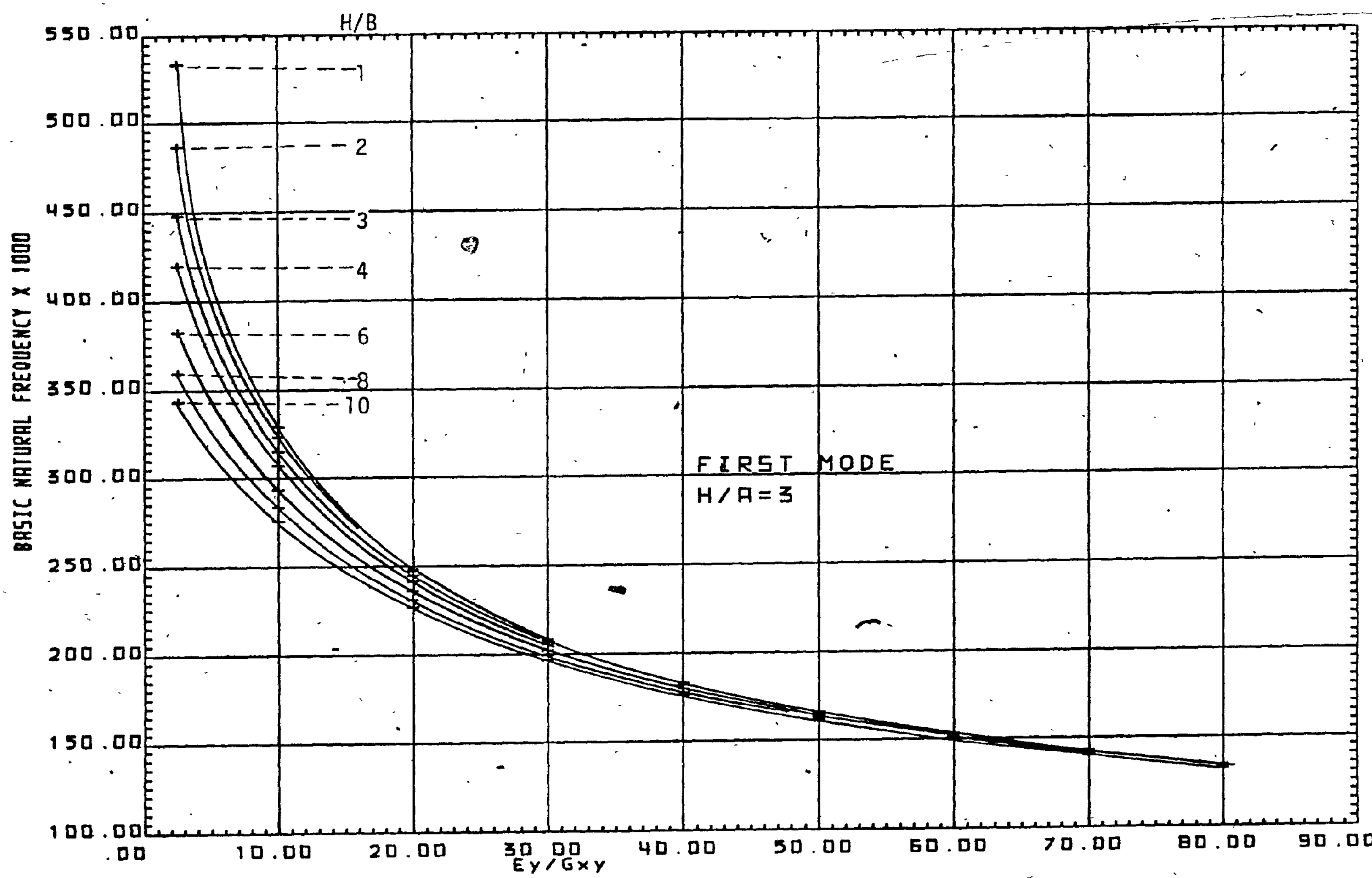


Fig. 7.1c Basic natural frequency for tube-type structure (mode 1, H/A=3)

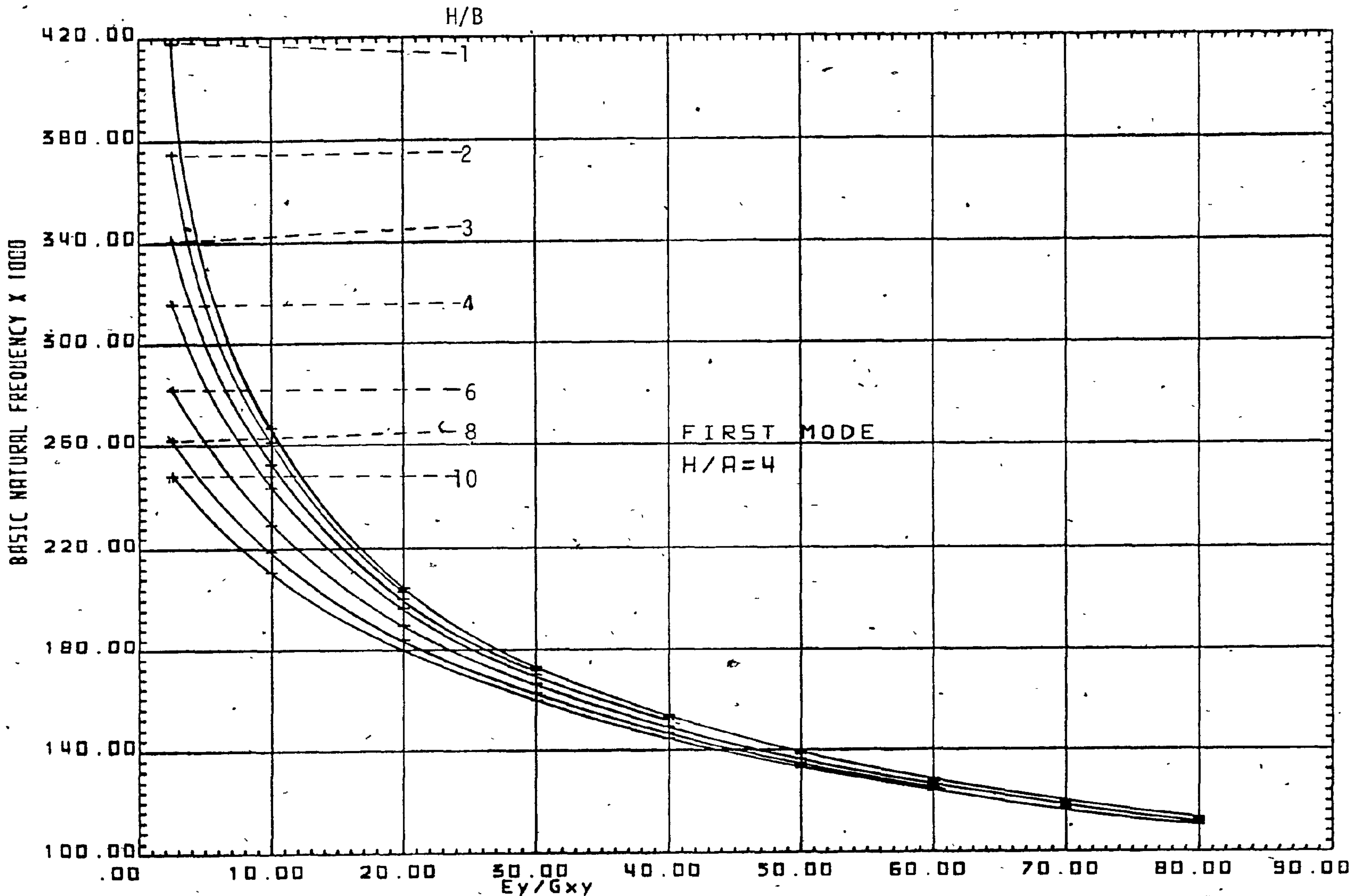


Fig. 7.1d Basic natural frequency for tube-type structure (mode 1, H/A=4)

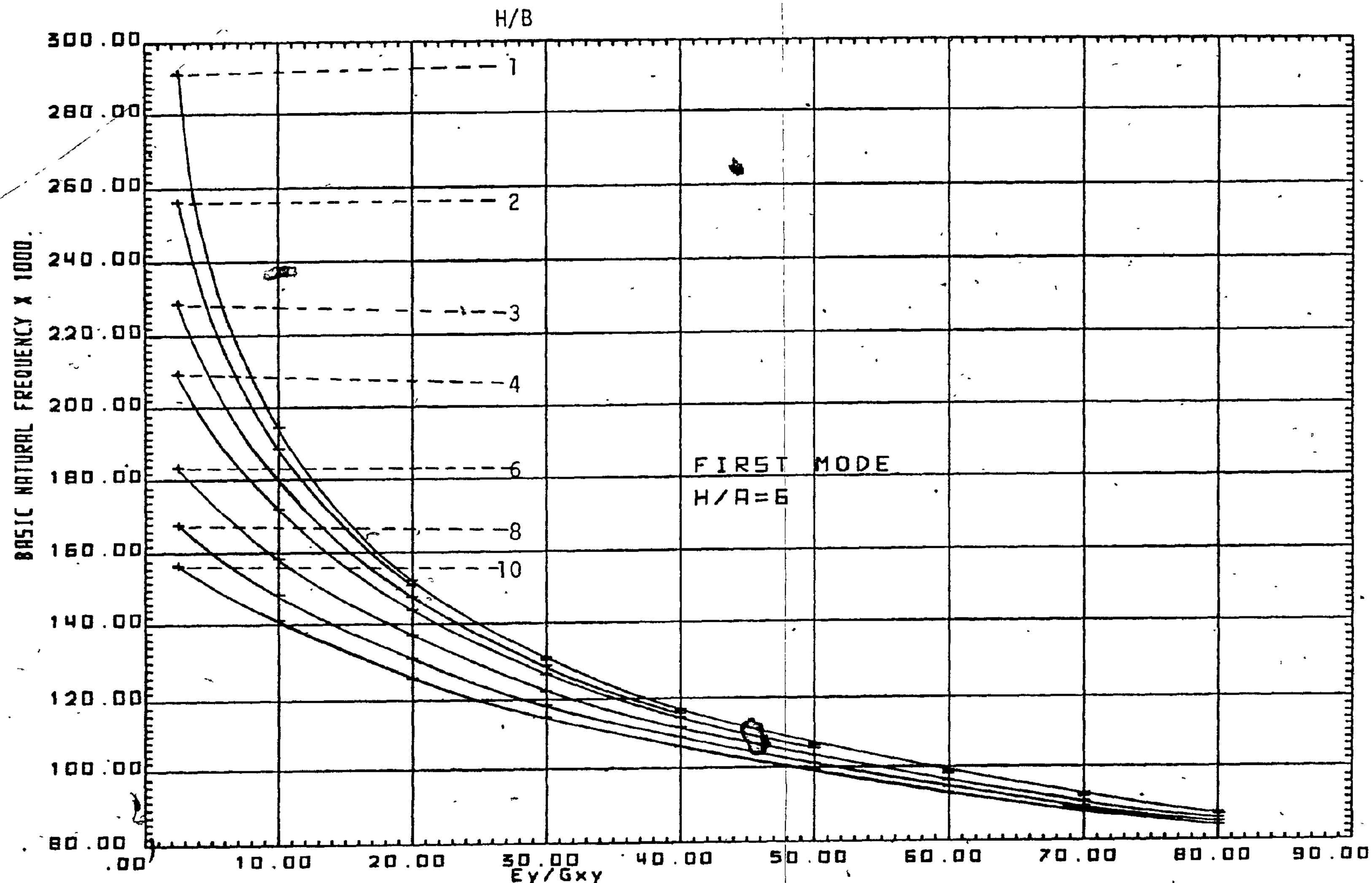


Fig. 7.1e Basic natural frequency for tube-type structure (mode 1, H/A=6)

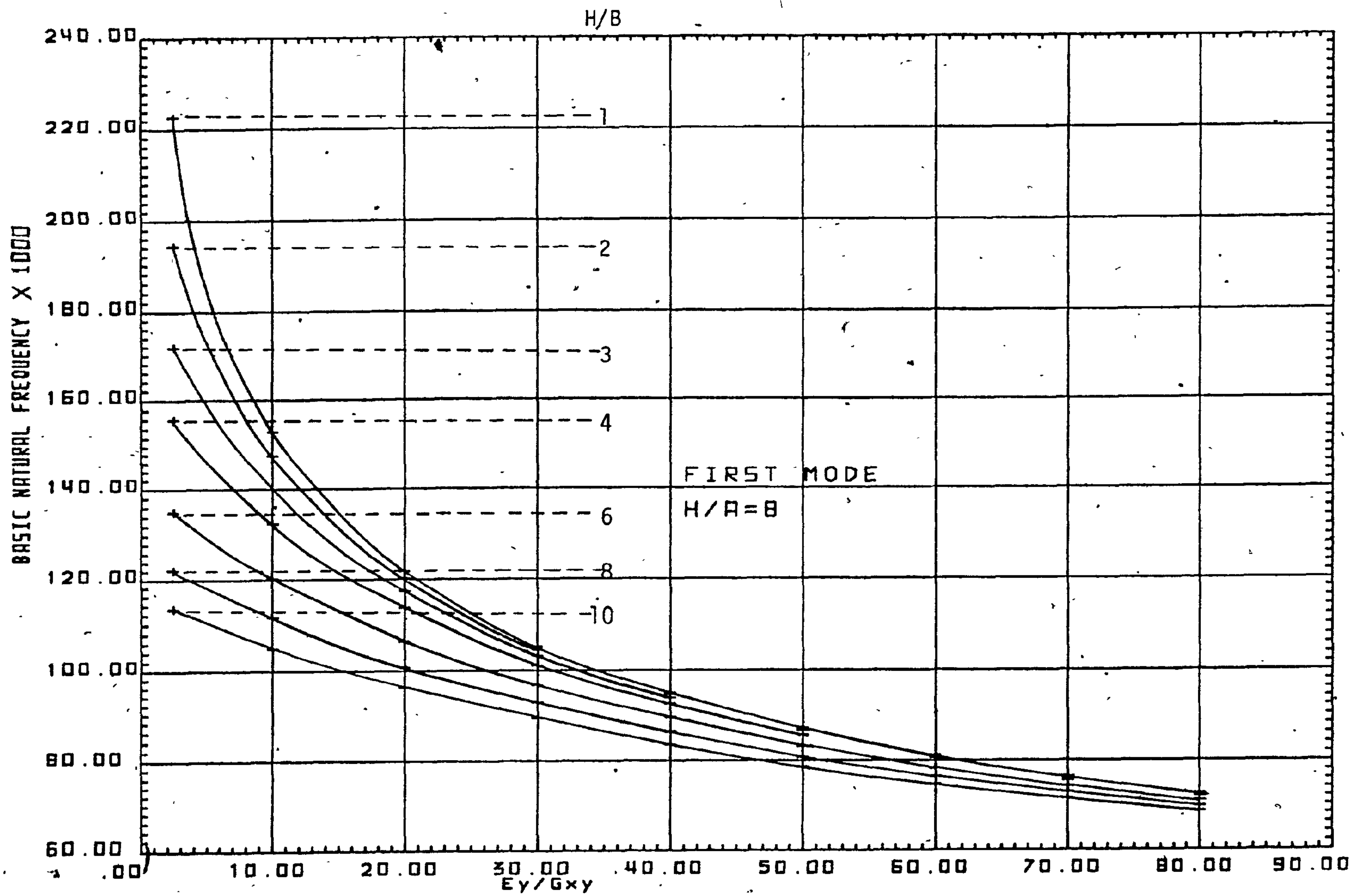


Fig. 7.1f Basic natural frequency for tube-type structure (mode 1, H/A=8)

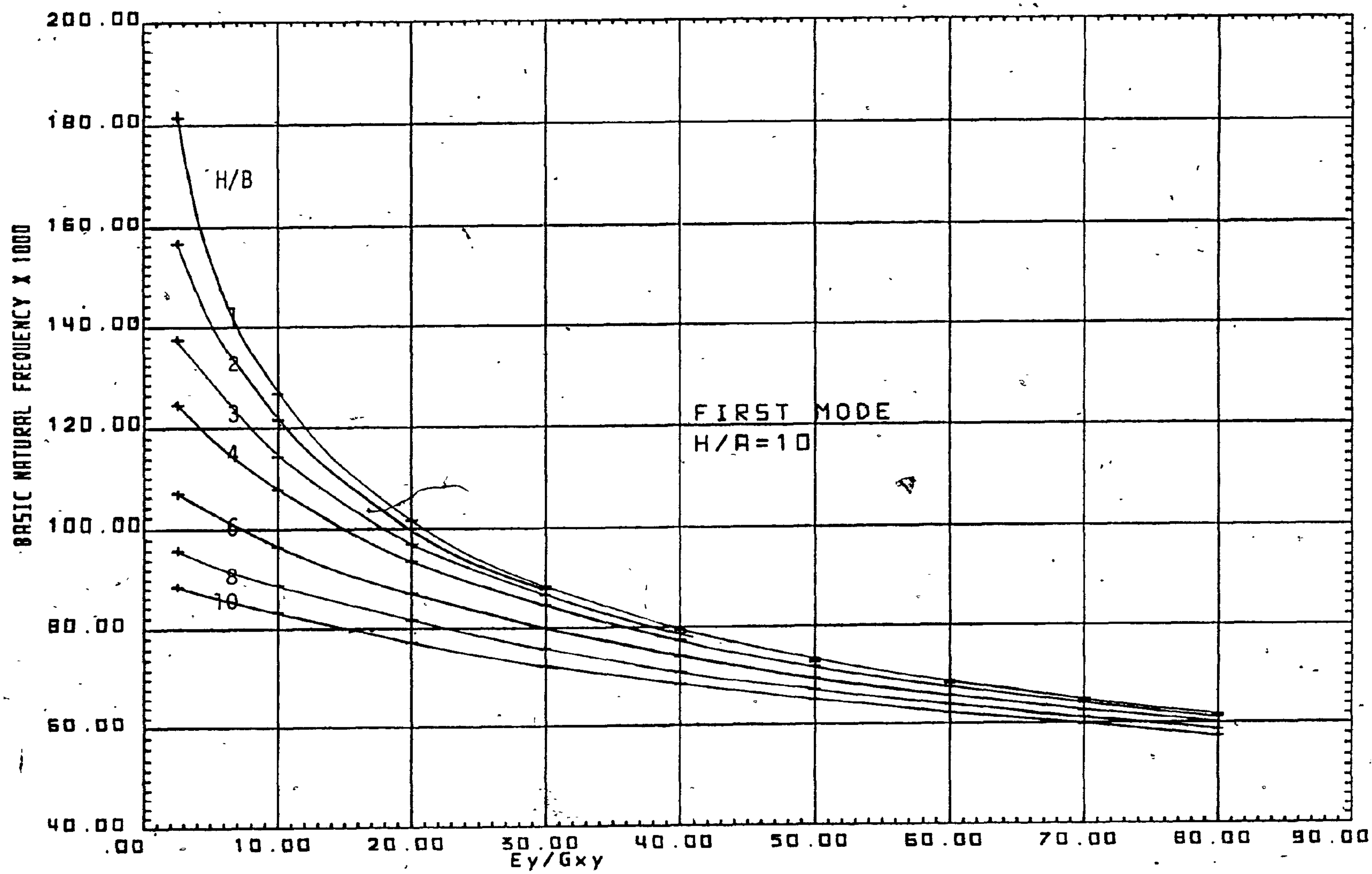


Fig. 7.1g Basic natural frequency for tube-type structure (mode 1, H/A=10)

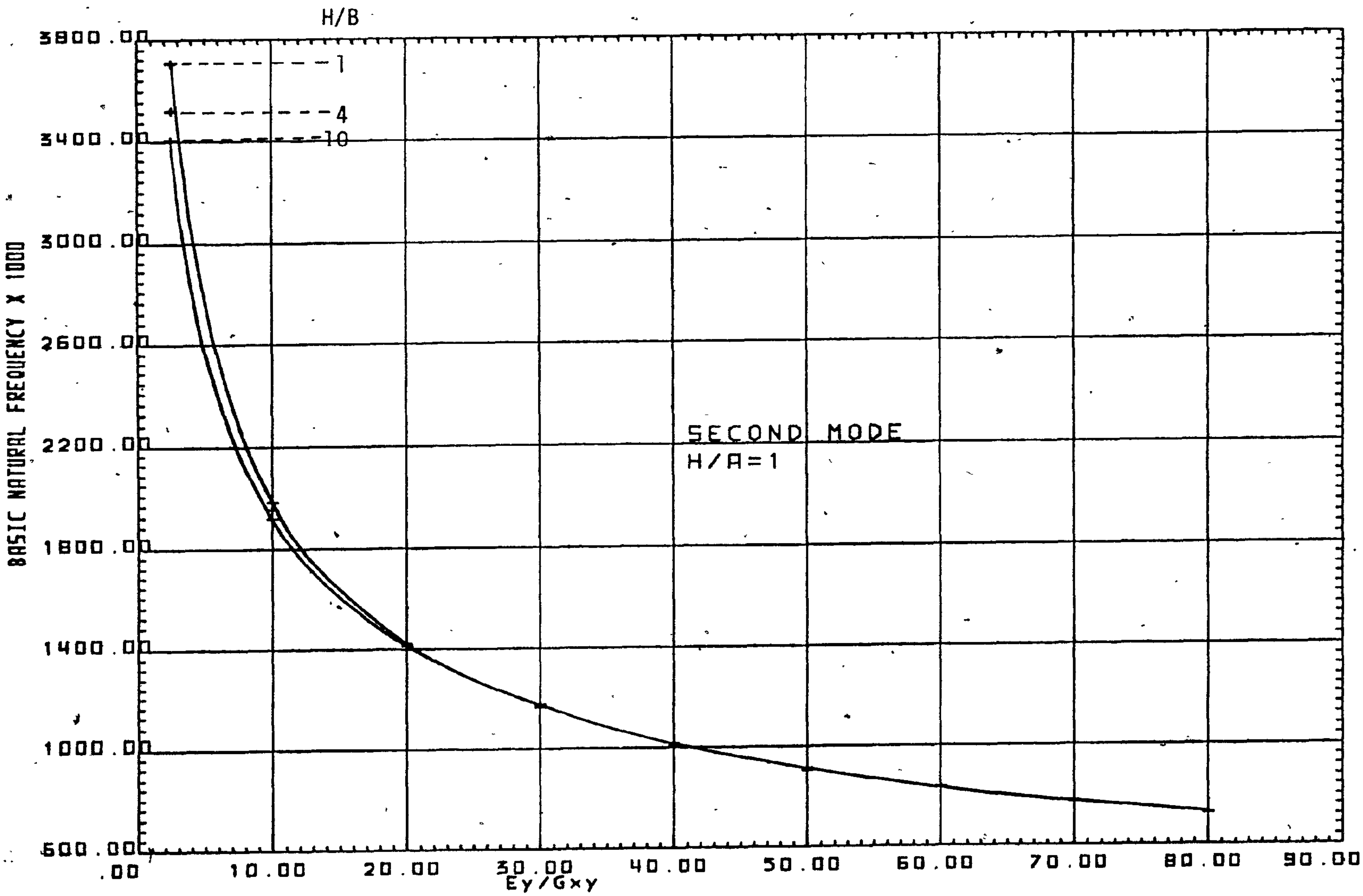


Fig. 7.1h Basic natural frequency for tube-type structure (MODE 2, H/A=1)

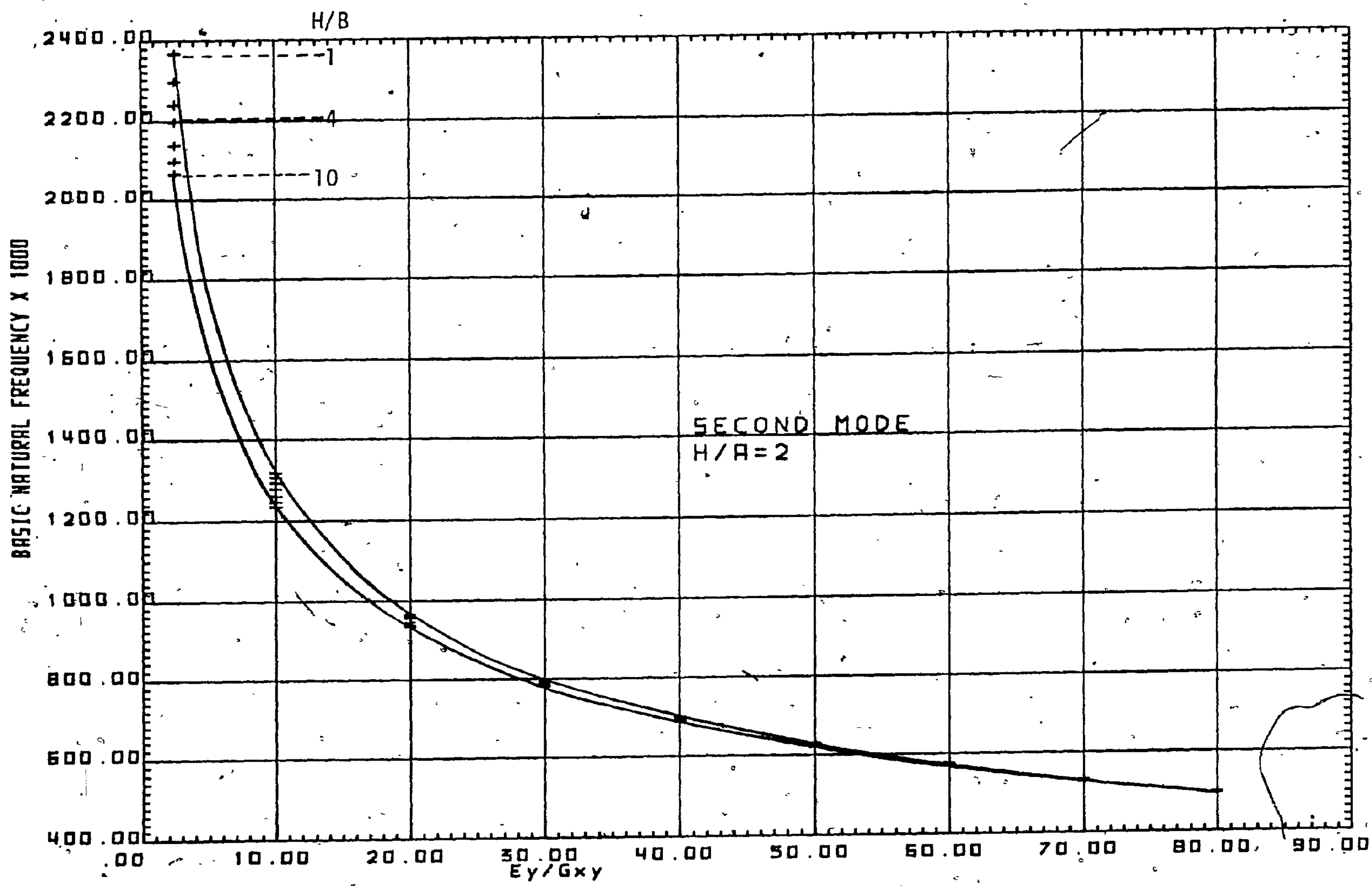


Fig. 7.1i Basic natural frequency for tube-type structure (mode 2, H/A=2)

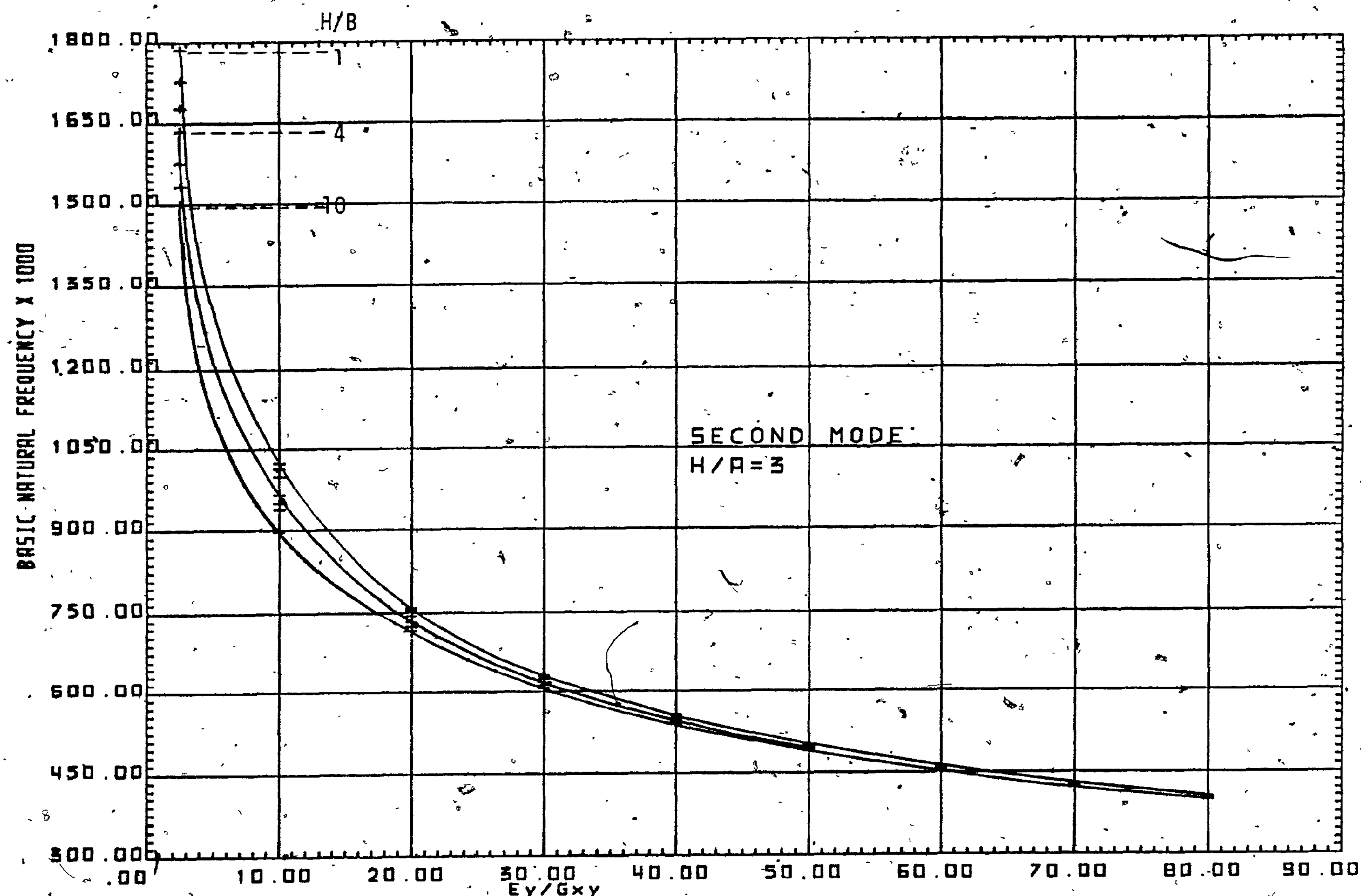


Fig. 7.1j Basic natural frequency for tube-type structure (mode 2, H/A=3)

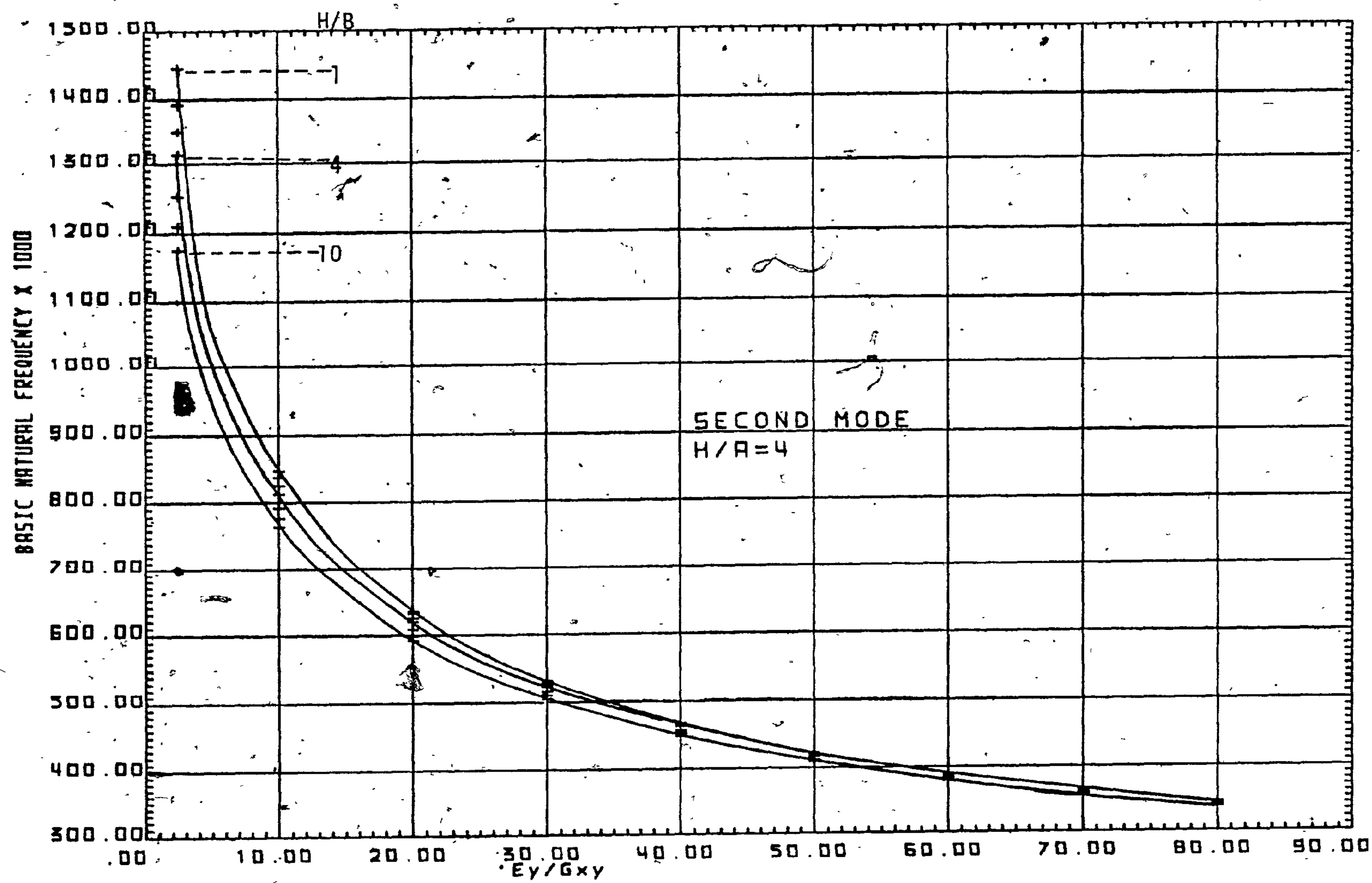


Fig. 7.1k Basic natural frequency for tube-type structure (mode 2, H/A=4)

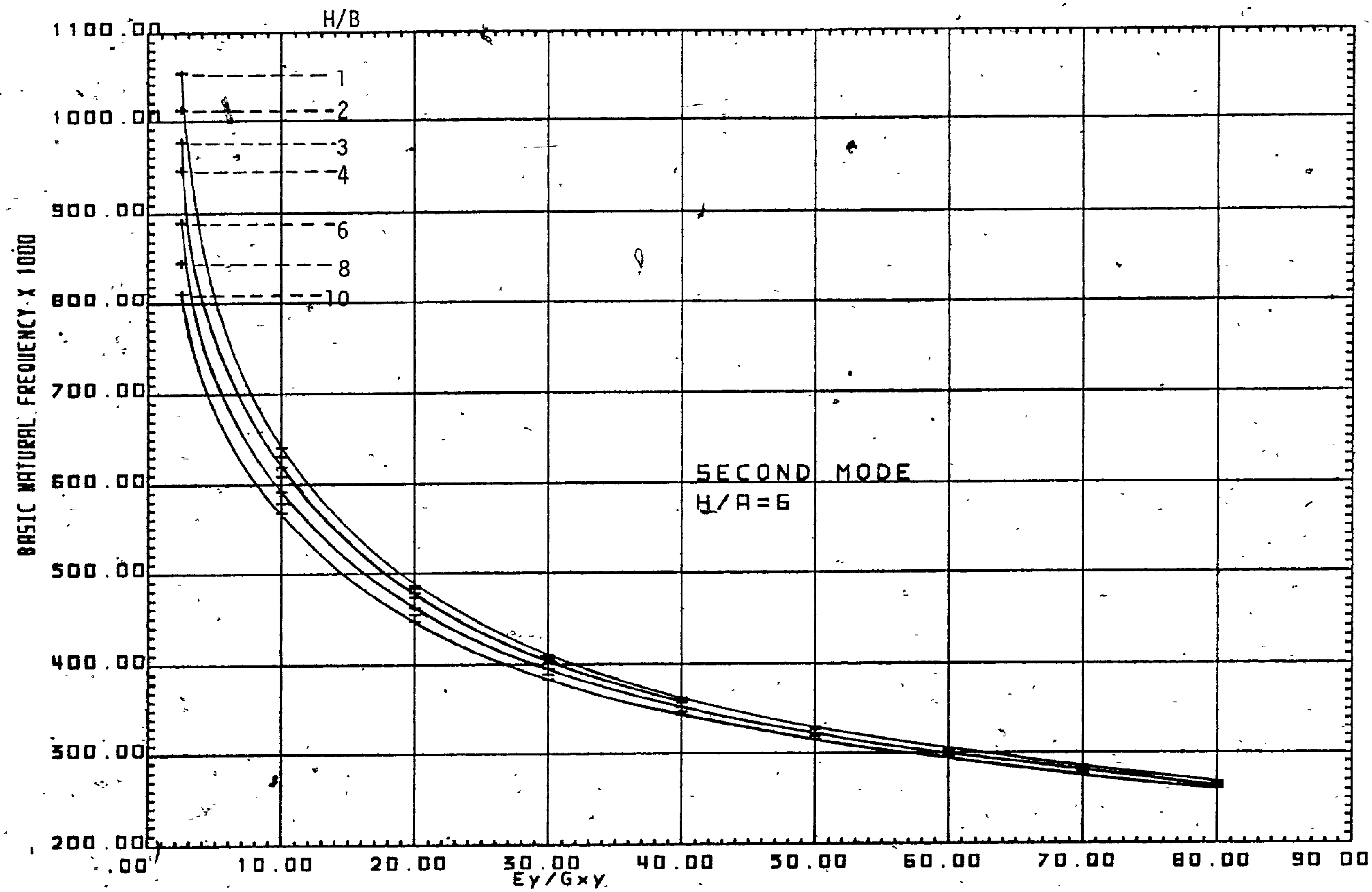


Fig. 7.1m Basic natural frequency for tube-type structure (mode 2, H/A=6)

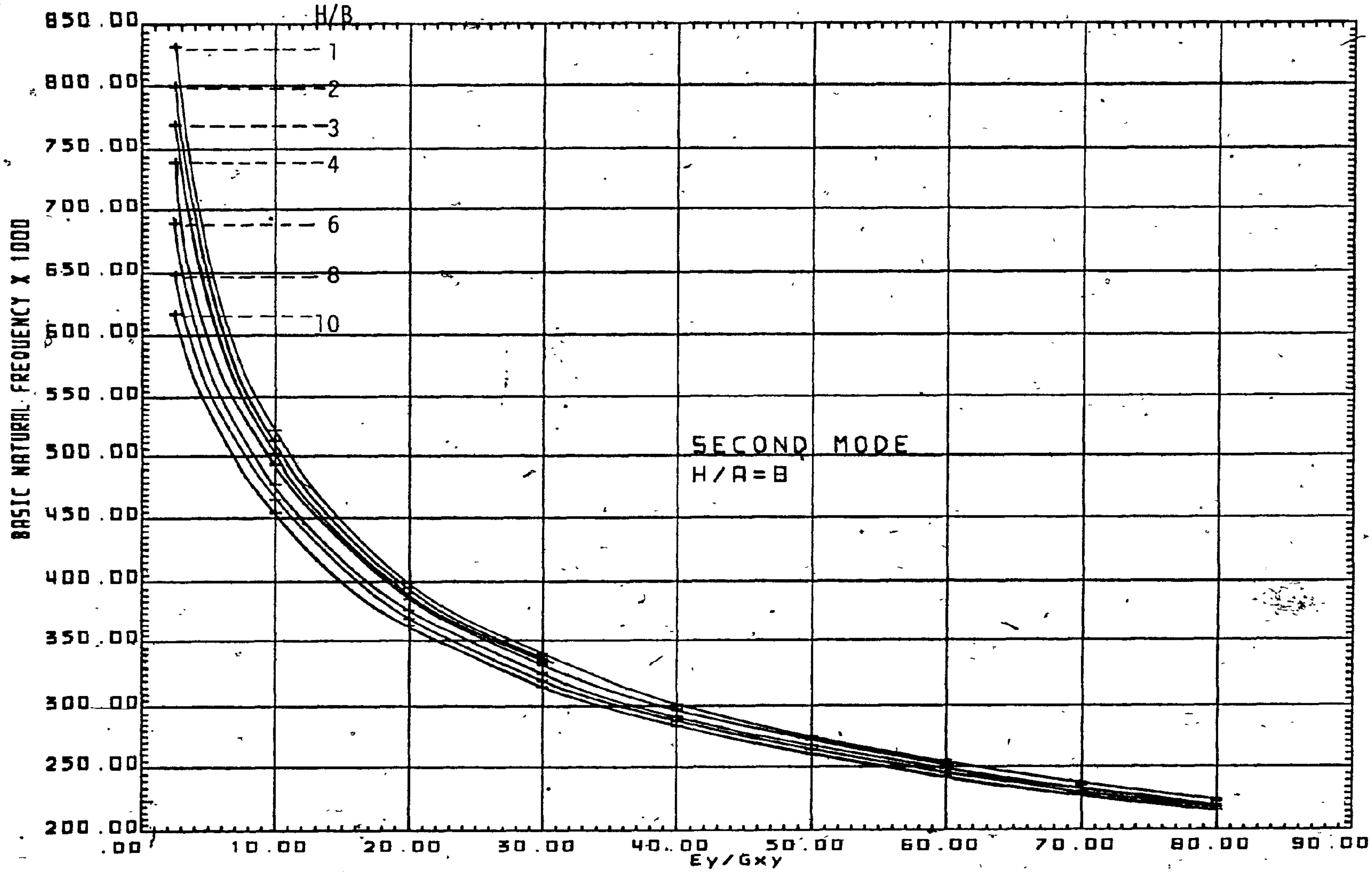


Fig. 7.1n Basic natural frequency for tube-type structure (mode 2, H/A=8)

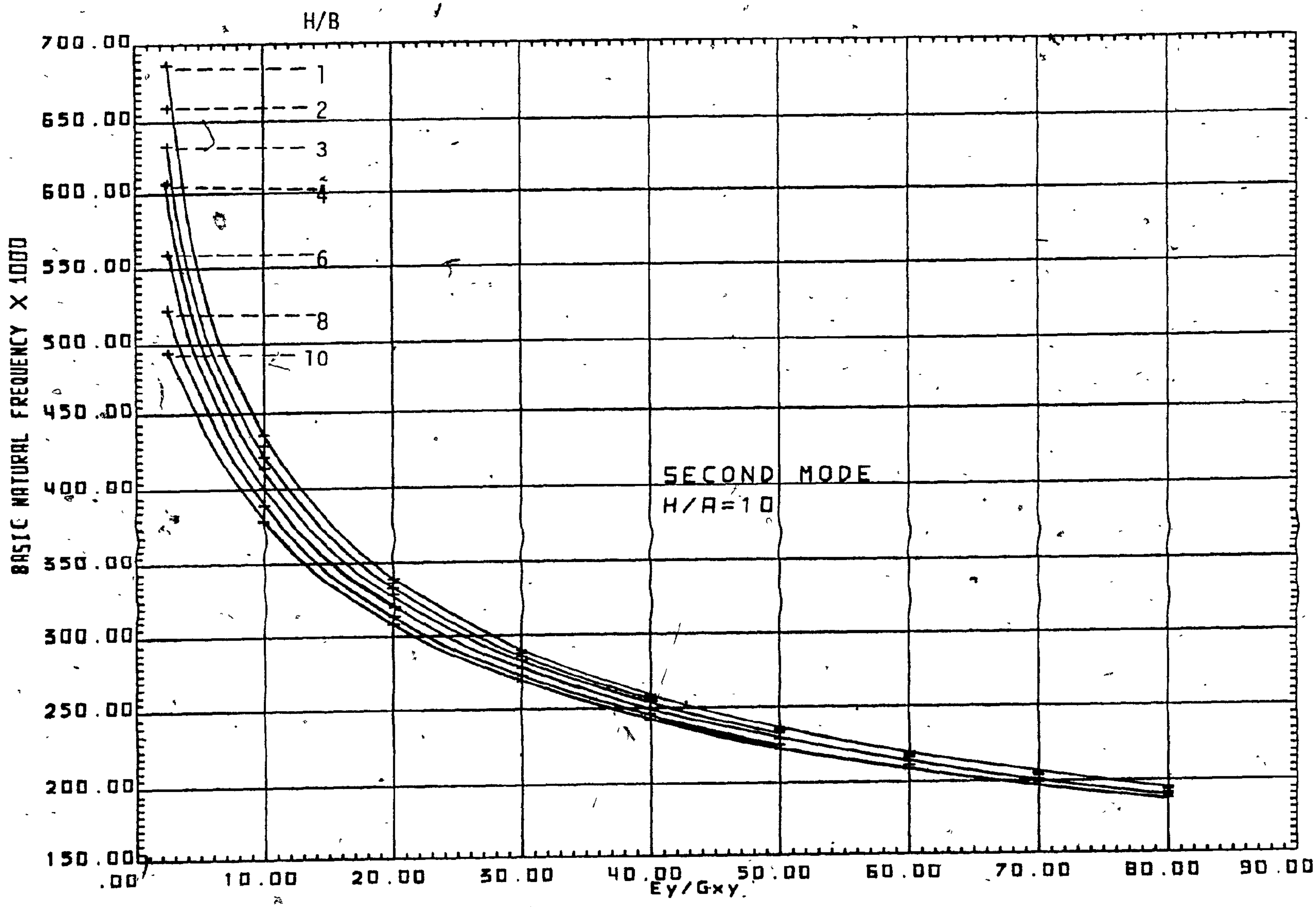


Fig. 7.1p Basic natural frequency for tube-type structure (mode 2, H/A=10)

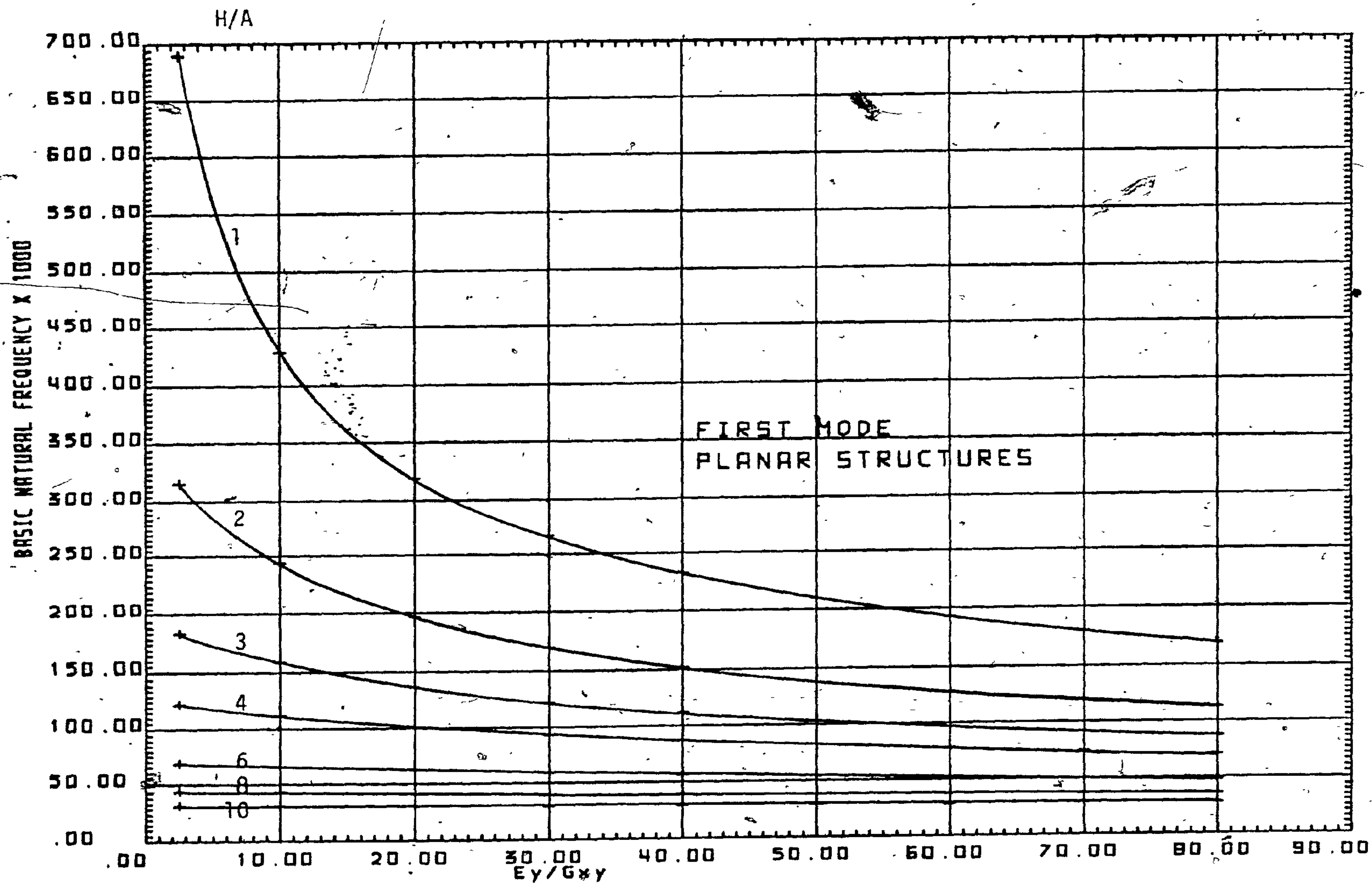


Fig. 7.1q Basic natural frequency for planar structure (mode 1)

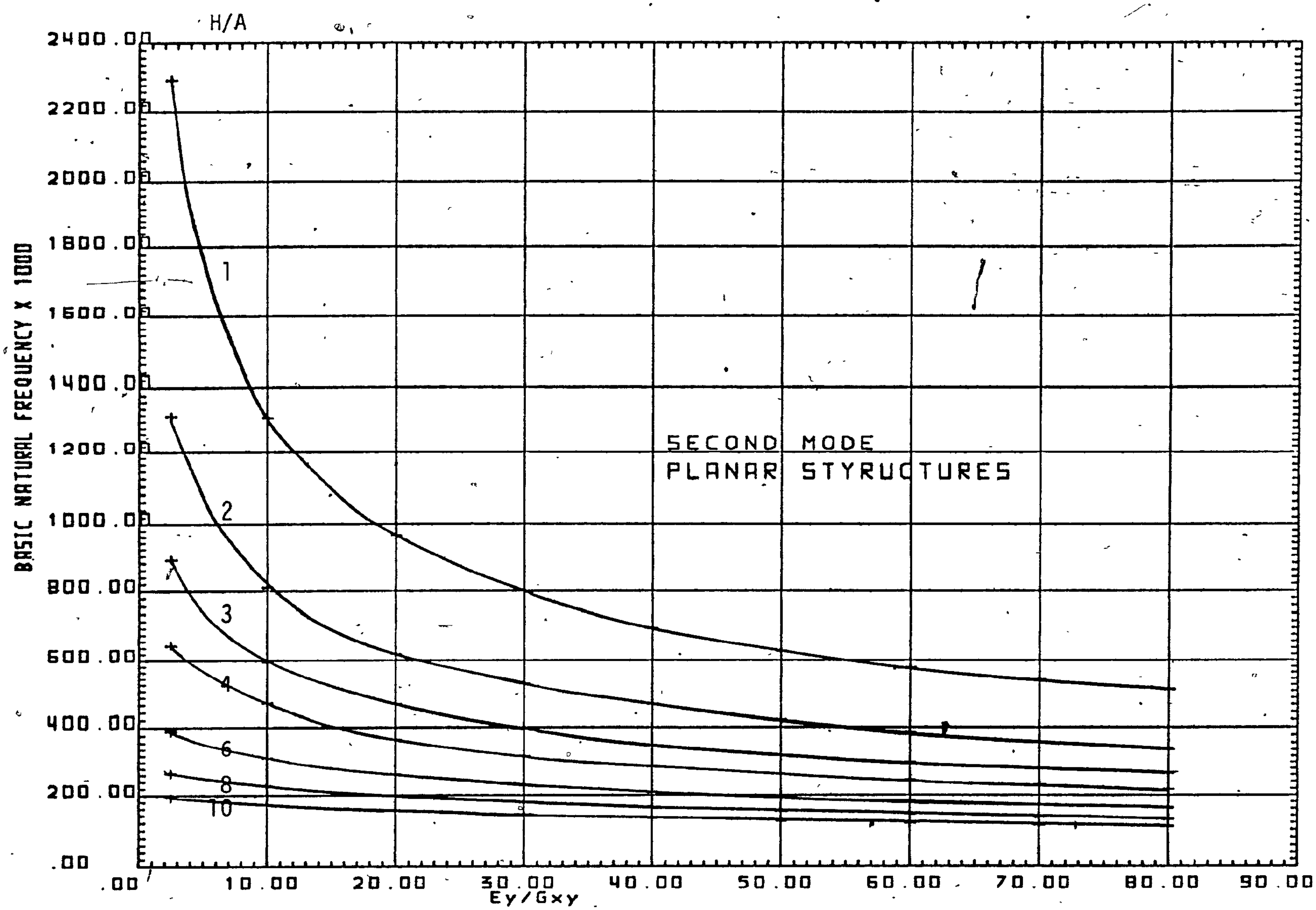


Fig. 7.1r Basic natural frequency for planar structure (mode 2)

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

The equivalent orthotropic macroelement method has been adopted and implemented for the efficient dynamic analysis of planar and tube-type structures consisting of frames, shear walls, coupled shear walls and their combinations. Frames must consist of rigidly connected horizontal beams and vertical columns. Repetition of dimensions over large portions of the structure is required for efficient implementation of the method.

The technique requires replacing discrete beam-column systems and bands of lintel beams by elastically equivalent orthotropic membranes. Expressions for the elastic properties of the membranes were presented; they take into account axial column deformations, bending and shear deformations in beams and columns, and finite size joint deformations. The equivalent structure is subsequently discretized by either of two specially orthotropic finite elements. It is noted that although the membrane properties have been obtained analytically, they may also be determined experimentally or by detailed finite element analysis.

A general large capacity computer program (TUBE) was developed for the static and earthquake spectrum analysis of the above structures. The program was developed for minimal memory storage requirements, as well as effort for data preparation.

The efficiency of the present method is significantly greater than others presented; only 1/4 of the global dynamic degrees of freedom were used to obtain comparable results. Convergence of higher natural frequencies was shown when increasing the number of levels in the model. With respect to the translational natural frequencies of wall-frame structures, and framed-tube structures, the facades normal to the vibration direction have little contribution so that preliminary estimates can be obtained by considering only the parallel facades. Enforcing vertical compatibility at facade junctions yielded significantly higher natural frequencies so that facades cannot be assumed to act independently of each other.

Large perforations in planar and tube-type structures cause shear lag so that internal normal forces in columns form a nonlinear variation. A measure of shear lag severity (nonlinearity) was expedited through a shear lag parameter which is the ratio of the equivalent shear modulus to the equivalent vertical elastic modulus of the membrane. It was discovered that increasing this ratio produced higher natural frequencies and a more linear column axial force distribution especially in the parallel facades. In terms of the actual structure, this can be done simply by increasing the beam depth.

Flexibility of finite size joints in planar frames and framed-tube buildings was found to significantly decrease the natural frequencies and internal column axial forces. Shear deformations in the members were determined to be less significant and can be ignored at least for preliminary design.

Two methods of determining natural frequencies of planar and tube-type structures are presented in the form of graphs and simplified equations. The first method is simply a compilation of data obtained by TUBE using three dimensionless parameters as variables. To ensure good accuracy, the equivalent structures was refined to 10 levels and 4 elements horizontally spanning each facade. The second, more approximate method is strictly for framed-tube structures, and attemps to model them as cantilever beams. The results of the first method are approximated to yield a modified cantilever beam frequency equation which considers only the parallel facade inertia, and a correction factor. The errors are expected to remain within 15%.

8.2 RECOMMENDATIONS FOR FURTHER STUDIES

The present method is confined to the elastic static and elastic earthquake spectrum analysis with a possible ductility factor consideration for approximate elastic-plastic analysis. Loads and ground motions are considered only in horizontal planes; the analysis can be extended to consider loads and ground motions in vertical planes. Extension of the method for stability analysis and optimization study is also possible. In addition, the following aspects could be investigated.

1) Obtain the equivalent membrane properties of the basic frame unit for members of nonrectangular cross-section. Experimental or detailed finite element analysis are possible routes, and may improve the membrane properties of the unit.

2) Improvement of the finite element stiffness matrices by using hybrid stress elements or incompatible displacement models.

3) Extension of the program procedure for the analysis of tube-in-tube structural systems.

(4) Extend the analysis for a consistent mass formulation.

(5) TUBE can be altered and appended to include earthquake analysis by the direct numerical integratin of the equation of motion.

(6) Analysis can be extended to consider plastic behaviour.

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APPENDIX A

COMPUTER PROGRAM MANUAL AND LISTING

A.1 INTRODUCTION

A large capacity computer program (TUBE) has been developed for the static and earthquake spectrum analysis of most planar and tube-type building structures incorporating the developments presented in chapters II, III, and IV.

The program was written in FORTRAN IV computer language and was run on a CDC 6600 computer using the FTN compiler. Auxiliary storage via nine magnetice tapes, and substructuring techniques along with static condensation procedures allow the efficient use of memory storage.

Reduction of computer time and storage is generally obtained at the expense of some loss in accuracy. Minimization of accuracy loss can be achieved through proper structure idealization. Efficient modelling of a structure via the equivalent orthotropic macroelement method requires an understanding of its characteristic physical behaviour. Structures that can be analysed by TUBE have been previously studied; (1,2,3,14,25,26,27,28,) are recommended as references.

Effort in data preparation and possible data errors are minimized through the usage of automatic data generation in the program wherever it is possible. The amount of data required is thus kept to a minimum.

The aim of this appendix is to clearly describe the essential features and logic of TUBE. This will assist the user in implementing TUBE correctly and efficiently while facilitating any further alterations and modifications of its current version.

A.2 SCOPE

The program may perform approximate elastic static analysis of structures subjected to lateral loads or approximate elastic earthquake spectral analysis of structures with an option for ductility considerations. Only planar or tube-type building structures can be analysed. Direct examples of planar structures are frames consisting of slender or relatively deep members, shear walls, coupled shear walls, frames interacting with shear walls, and clad-frames. Some examples of tube-type building structures include framed-tube structures, core-supported structures with open or closed sections, and structures consisting of interacting shear walls and frames. These tubular systems in general are any three-dimensional assembly of planar systems connected at their edges. Planar frames must consist of horizontal beams rigidly connected to vertical columns. Structures must also satisfy the assumptions stated earlier in Chapter I.

Structures to be analysed by TUBE should preferably possess a high degree of regularity. For example, bay widths, storey heights and member sizes should not change over large portions of framed structures. This is not a heavy limitation; high degree of regularity is inherent in most tall building structures due to aesthetic reasons and to ensure ease of construction and subsequent savings in time an money (26).

A.3 DESCRIPTION OF THE PROGRAM

Most of the program logic follows the "Analysis Procedure" section presented in Chapter IV. The program can be divided into three main stages.

The first stage involves data entry on two levels: global structure level through subroutine DATA1, and local facade and element level through subroutine DATA2.

Basically, in the global level, the geometry of the plan view and the analysis type is specified. Also, DATA1 directly assembles the overall load vector in the case of a static analysis, or the diagonal vector representation of the lumped-mass matrix in the case of a dynamic analysis. At the local level, facade properties (number of elements and their properties) are specified.

The second main stage is the construction of the global structure stiffness matrix which corresponds to the overall load vector or lumped-mass matrix. In this stage, the elastic properties of the equivalent orthotropic membranes, for different element types if not input directly, are evaluated in subroutine EMAT. Element connectivity and type automatically generated in ELCON provide facade discretization. Refined or ordinary element stiffness matrices are generated in RECT and then assembled in ASMBL1 to form the local facade stiffness matrix. Assembly is based on the degrees of freedom associated with each element. The numbering of the degrees of freedom are automatically generated in LABEL and adjusted in SYMM if any facade symmetry is specified. The local facade stiffness matrix is then reduced by condensing all internal facade degrees of freedom using GAUSS, and then stored on tape 4. Once all reduced local façade stiffness matrices are stored, they are recalled from tape, transformed to global degrees of freedom, and then assembled into the global static structure stiffness matrix one by one within ASMBL2. Note that the transformation procedure for static and dynamic analysis differ due to the latter's possibly varying reference position from level to level. If the analysis is dynamic, the corner vertical degrees of freedom of the global static structure stiffness matrix are further condensed via GAUSS to form the global dynamic structure stiffness matrix which is stored on tape 6.

In the third and final stage, procedures for the static and earthquake analysis differ significantly. For the static analysis, displacements, membrane stresses, and member internal forces are evaluated respectively via GAUSS, STRESS, and FORCE. For the earthquake analysis, freqencies and mode shapes are evaluated through SIVIB2. Subsequent displacements, level shears and torques, base shears and torques, and overturning base moments are determined in MODAL. Finally membrane stresses and member internal forces are determined. Note that for each parameter above, except member internal forces the square root sum of squares (SRSS) method is used to obtain the final response. The member internal forces are computed from the SRSS of membrane stresses.

A.4 USER'S GUIDE

In an effort to minimize any possible data errors and to efficiently implement TUBE, a detailed description of data preparation along with useful suggestions to improve accuracy of results is provided in this section. These suggestions are based on experience gained through implementing the program.

A.4.1 STRUCTURAL MODELLING

Besides the rules and definitions given previously in Section 4.3.1, the following steps may be followed:

STEP 1:

Sketch a plan view and elevation of the structure. Include the reference axes, and lateral loads or ground motion direction. Number each level, corner, and facade as shown in Figs. 5.1 and 5.5.

STEP 2:

Decide whether the structure is to be discretized using the ordinary element (4-corner nodes, and 6 degrees of freedom) or the refined element (8 nodes and 9 degrees of freedom). The latter is recommend for the cases where the effect of shear lag is significant, and where there is bending of high aspect ratio structures.

STEP 3:

If symmetry exists, restrain the appropriate degrees of freedom. Thus, indicate the symmetry type number for the structure and for each facade. Note that if facade symmetry is specified, then structure symmetry must also be specified.

STEP 4:

Draw a separate elevation for each facade discretized by several elements. Number each element and its type. Note that element boundaries need not coincide with beam or column lines.

STEP 5:

Decide on the type of output (i.e. stresses or internal member forces) and the levels at which they should be evaluated.

STEP 6: (For dynamic analysis only)

Decide on the required output (i.e. level shears and torques, base shears and torques, overturning base moments).

In addition to the above 6 steps, the following suggestions may help in dividing the structure into levels, and subsequently facades into elements.

1) For a static analysis, an element can incorporate as many as 5 to 10 storeys. For a dynamic analysis masses are lumped at levels. To better approximate mass distribution, more levels should be used.

2) Three single storey levels at the base of the structure is a sound rule (1,14,26) mainly because of the high curvature.

3) A single-storey level or more at the top of the structure is desirable particularly for structures deforming in a predominantly bending mode.
This is not crucial to the overall behaviour of the structure.

4) Always provide a finer mesh at stress concentration areas such as facade edges. In general, the elements at the corners of buildings should incorporate only one bay especially when using the ordinary element.

A.4.2 INPUT DATA

Units must be consistent and data cards must be in the sequence shown below. Integer and real numbers are input in free-format. Below will be presented the sequence of cards, the variables on each card, and the condition(s) for the existance of that card or group of cards. It is very important to remember that if the condition(s) in the right hand side column is/are not satisfied for a particular card, that card must be excluded. Section A.4.3 provides a description of each variable.

CARD GROUP No.	CARD No. of GROUP	INPUT VARIABLES	CONDITIONS
1	1	TITLE1	
2	1	NDYN	
3	1	NF NL NDOF NSHAPE NSSYM NSYM23 NOUT	
4	1	M NRQD NOI TOLVEC GES IRENT GAMMA INTER	NDYN=0
5	1 2 : N	1 X(1) Y(1) 2 X(2) Y(2) : : : N X(N) Y(N)	
6	1 2 : NF	1 FP(1,1) FP(1,2) FP(1,3) RFP(1,1) 2 FP(2,1) FP(2,2) FP(2,3) RFP(2,1) : : NF FP(NF,1) FP(NF,2) FP(NF,3) RFP(NF,1)	NDYN=0 NDYN=0 : NDYN=0
7	1 2 : NF	1 FP(1,1) FP(1,2) FP(1,3) 2 FP(2,1) FP(2,2) FP(2,3) : : : : NF FP(NF,1) FP(NF,2) FP(NF,3)	NDYN=1 NDYN=1 : NDYN=1
8	1 1 1 1 2 3	FX(1) FX(2)......FX(NL) FY(1) FY(2)......FY(NL) MT(1) MT(2)......MT(NL) FX(1) FX(2)......FX(NL) FY(1) FY(2)......FY(NL) MT(1) MT(2)......MT(NL)	NDYN=0 NSSYM=1 NDYN=0 NSSYM=2 NDYN=0 NSSYM=3 NDYN=0 NSSYM=0 NDYN=0 NSSYM=0 NDYN=0 NSSYM=0
9	1 2 : NL	1 XMC(1) YMC(1) MAS(1) RMAS(1) 2 XMC(2) YMC(2) MAS(2) RMAS(2) : : : : : NL XMC(NL) YMC(NL) MAS(NL) RMAS(NL)	NDYN=1 NDYN=1 : NDYN=1
10	1	NPRINT(1) NPRINT(2)......NPRINT(NL)	NDYN=0
11	1 2 3	NPRI(1) NPRI(2)........NPRI(NL) NPRIN(1) NPRIN(2).......NPRIN(NL) NPRINT(1) NPRINT(2)......NPRINT(NL)	NDYN=1 NDYN=1 NDYN=1 NPRI(3)=1
		*** Card group numbers 12, 13, 14, *** *** 15, 16, 17 must be repeated for *** *** each facade. ***	
12	1	TITLE2	

13	1	NMAT NTYPE NGROUP NSYM INCODE	
14	1.1 1.2 1.3	E MU IBENDB IBENDC ISHEARB ISHEARC ISHEARJ B BD BT H CD CT	INCODE=0 INCODE=0 INCODE=0
	2.1 2.2 2.3 ⋮	Same as 1.1, 1.2, 1.3, but for material type 2.	INCODE=0 INCODE=0 INCODE=0
	NMAT.1 NMAT.2 NMAT.3	Same as 1.1, 1.2, 1.3, but for material type NMAT	INCODE=0 INCODE=0 INCODE=0
15	1 2 ⋮ NMAT	EX(1) EY(1) GXY(1) TH(1) EX(2) EY(2) GXY(2) TH(2) ⋮ ⋮ ⋮ ⋮ EX(NMAT) EY(NMAT) GXY(NMAT) TH(NMAT)	INCODE=1 INCODE=1 ⋮ INCODE=1
16	1 2 ⋮ NTYPE	1 ET(1,1) ET(1,2) ETM(1) IEW(1) IEH(1) 2 ET(2,1) ET(2,2) ETM(2) IEW(2) IEH(2) ⋮ ⋮ ⋮ ⋮ ⋮	
17	1 2 ⋮ NGROUP	NS(1) NE(1) NELT(1) NS(2) NE(2) NELT(2) ⋮ ⋮ ⋮ NS(NGROUP) NE(NGROUP) NELT(NGROUP)	
18	1 2 ⋮ M	U(1,1) U(2,1)......U(NREM,1) U(1,2) U(2,2)......U(NREM,2) ⋮ ⋮ ⋮ U(1,M) U(2,M)......U(NREM,M)	IRENT=1 NDYN=1 IRENT=1 NDYN=1 ⋮ ⋮ IRENT=1 NDYN=1
19	1 2 ⋮ M	W(1,1) W(2,1)......W(NREM,1) W(1,2) W(2,2)......W(NREM,2) ⋮ ⋮ ⋮ W(1,M) W(2,M)......W(NREM,M)	IRENT=2 NDYN=1 IRENT=2 NDYN=1 ⋮ ⋮ IRENT=2 NDYN=1
20	1	SV(1) SV(2)......SV(NRQD)	INTER=0 NDYN=1 any NPRI=1

A.4.3 DESCRIPTION OF TUBE INPUT

TITLE 1	=	80 spaces are provided for an alphanumeric description of the problem.
NDYN	=	0 for static analysis
	=	1 for dynamic analysis
NF	=	(integer) number of facades in the model after symmetry considerations. (NF $\leq$ 4)
NL	=	(integer) number of levels in the model (NL $\leq$ 10).
NDOF	=	0 for the ordinary element (4 nodes and 6 degrees of freedom)
	=	1 for the refined element (8 nodes and 9 degrees of freedom)
NSHAPE	=	0 for closed plan shape of model after symmetry considerations.
	=	1 for open plan shape of model after symmetry considerations.
NSSYM	=	0 for no structural symmetry. (If NSSYM = 0, then at least 2 facades must not be parallel). (If NF = 1, then NSSYM = 0)
	=	1 for structure symmetry type one
	=	2 for structure symmetry type two
	=	3 for structure symmetry type three
		(See Section 4.7 and Fig. 4.3)

NSYM23 = (integer) number of facade edges that are restrained vertically

NOUT = 0 member forces output.

= 1 element stresses output at element midheight coinciding with vertical divisions specified by IEW of card group 16.

= 2 element stresses output at the centroid of the ordinary element, or at the centroids of each half of the refined element cut vertically.

(See Fig. A.1)

M = (integer) number of trial vectors or trial mode shapes to be used for eigenvector calculations.

NRQD= (integer) number of vectors required to satisfy the vector tolerance criteria (see TOLVEC), and the number of modes to contribute to the structural responses.
$(NRQD \leq M \leq 11)$

NOI = (integer) (NOI + 1) is the maximum number of trial vector iterations if the vector tolerance criteria is not acheived (see TOLVEC). A value of 10 is recommended.

TOLVEC= (real) number specifying the maximum error (20) permitted in the trial vectors. A value of 0.0001 is recommended.

GES = (real) random number chosen between -0.5 and +0.5 with at least 4 decimal places used for automatic trial vector generation.

IRENT = 0 for automatic generatin of random trial vectors

1 for user input of trial vectors

2 for user input of trial mode shapes

(See section 4.9.4 for the difference between the trial vectors and the mode shapes)

GAMMA = (real) radian angle measured counterclockwise from the X axis to the line of earthquake excitation (Fig. 4.4)

INTER = 0 for pseudo-velocity input from datafile

= 1 for interactive pseudo-velocity input after time periods are displayed.

X(KK) = (real) X coordinate of corner number KK

Y(KK) = (real) Y coordinate of corner number KK

N = (integer) number of corners in plan view of model after symmetry consideratins
(= NF if NSHAPE = 0)
(= NF + 1 if NSHAPE = 1)
($N \geq$ NSYM23 must be satisfied)

FP(K,1) = (integer) number of corner I of facade K

FP(K,2) = (integer) number of corner J of facade K

FP(K,3) = (integer) number of elements horizontally spanning facade K. (NL x FP(K,3) $\leq$ 90, i.e. no more than 90 elements per facade)

RFP(K,1) = (real) perpendicular distance from facade K to the reference point. Reference points are assumed to form a vertical line through the structure if a static analysis (NDYN = 0) is specified.

FX(L) = (real) externally applied concentrated load in the x direction at the reference point of level L

FY(L) = (real) externally applied concentrated load in the y direction at the refernce point of level L.

MT(L) = (real) externally applied torque applied to level L.

XMC(L) = (real) x-coordinate of the mass center for level L.

YMC(L) = (real) y-coordinate of the mass center for level L.

MAS(L) = (real) lumped mass of level L.

RMAS(L) = (real) rotational in-plane mass moment of inertia of level L with respect to its mass center.

NPRI(MM) = 0 response parameter MM is not output for each mode.

= 1 response parameter MM is output for each mode.

MM = 1 reference point displacements

MM = 2 corner displacements of structure

MM = 3 element stresses or internal member forces

MM = 4 inertia or equivalent elastic forces at the reference points of each level

MM = 5 base shears in the x and y direction and base torque

MM = 6 overturning base moments about the x and y axis

NPRIN(MM) = 0 the square-root-sum-of-squares of response parameter MM is not output

= 1 the square-root-sum-of-squares of response parameter MM is output

NPRINT(L) = 0 don't calculate stresses or member internal forces for the elements between level L and (L - 1)

= 1 do calculate stresses or member internal forces for the elements between level L and (L - 1)

TITLE2 = 80 spaces for an alphanumeric description of the facade

NMAT = (integer) number of different kinds of materials in the facade. More specifically it equals the number of different equivalent membranes in the facade. The membrane properties are (E_y, G_{xy}, t). If member properties are input (see INCODE), the membrane properties are derived from the inputs of card group number 14. Thus any different combination of E_y, G_{xy}, t or card group number 14 within a facade constitutes a different material. (NMAT $\leq$ 12)

NTYPE = (integer) number of different element types in the facade. Element types are characterized by the material kind, height, width, and internal subdivisions which specify stress or force evaluation points. (NTYPE $\leq$ 36)
(See input variables for card group 16)

NGROUP = (integer) Designating each element with an element type number starting from 1, rectangular groups of the same element type can be formed. NGROUP is the total number of groups. Often, only one single element or a single line of elements form a group.

NSYM = 0 for no facade symmetry (If NSSYM = 0, then NSYM must be 0)

= 1 for facade symmetry type 1

= 2 for facade symmetry type 2

= 3 for facade symmetry type 3

(See section 4.4 and Fig. 4.5)

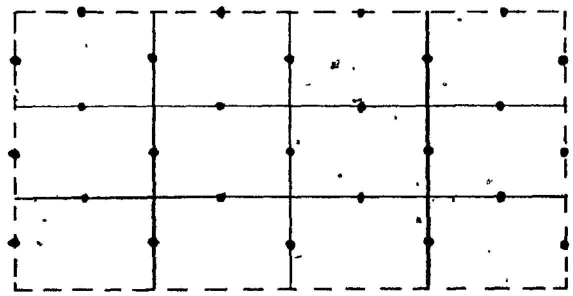
INCODE = 0 if frame properties will be input for the whole facade (i.e. card group 14)

= 1 if membrane properties will be input for the whole facade (i.e. card group 15)

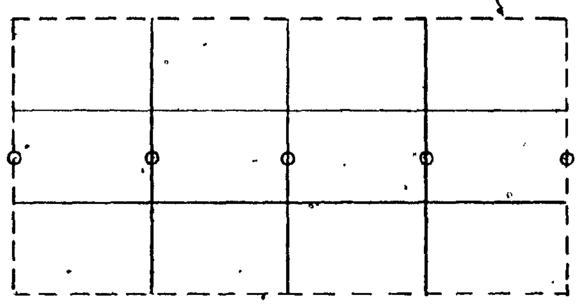
If lintel beams or framed portions exist in a composite facade, the elastic properties of their equivalent membrane must be evaluated using Eqs. 2.1, 2.2, 2.3, 2.8 prior to executing TUBE.

E	=	(real) Young's modulus
MU	=	(real) Poisson's ratio
B	=	(real) bay width
BD	=	(real) beam depth
BT	=	(real) beam thickness. TUBE treates the beam thickness also as the membrane thickness.
H	=	(real) storey height
CD	=	(real) column depth
CT	=	(real) column thickness
IBENDB	=	consider bending deformations in beams (0 = no, 1 = yes)
IBENDC	=	consider bending deformations in columns (0 = no, 1 = yes)
ISHEARB	=	consider shear deformations in beams (0 = no, 1 = yes)
ISHEARC	=	consider shear deformations in columns (0 = no, 1 = yes)
ISHEARJ	=	consider flexibility of finite size joints (0 = no, 1 = yes)
EX(LL)	=	(real) horizontal elastic modulus for membrane type LL
EY(LL)	=	(real) vertical elastic modulus for membrane type LL
GXY(LL)	=	(real) shear modulus for membrane type LL
TH(LL)	=	(real) thickness of membrane type LL
ET(JJ,1)	=	(real) width of element type JJ
ET(JJ,2)	=	(real) height of element type JJ
ETM(JJ)	=	(integer) material kind for element type JJ

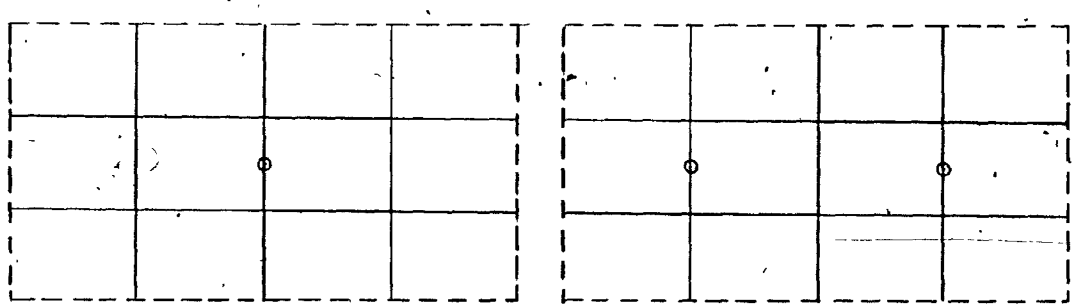
IEW(JJ)	=	(integer) number of "bays" within an element which partly specifies the internal membrane stresses or member forces evaluation points for element type JJ. (see Fig. A.1)
IEH(JJ)	=	(integer) same as IEW (above) but for "stories". (see Fig. A.1).
NS(NN)	=	(integer) number of the bottom left element of the element group NN (see NGROUP)
NE(NN)	=	(integer) number of the top right element of the element group NN (see NGROUP)
NELT(NN)	=	(integer) number of element type of the above element group NN
NREM	=	(integer) number of global dynamic degrees of freedom
	=	(3 X NL if NSSYM = 0)
	=	(NL if NSSYM ≠ 0)
U(A,II)	=	(real) trial vector for mode number II. (See section 4.9.4 for the difference between trial vectors and mode shapes).
W(A,II)	=	(real) trial mode shape for mode number II.
SV(II)	=	(real) pseudo-velocity for mode number II.



NOUT = 0



NOUT = 1
(IEH is redundant)



ordinary element

refined element

NOUT = 2
(IEH and IEW are redundant)

— — — element boundary

■ points of member force evaluation

o points of element stress evaluation

Fig. A.1 Definition of NOUT for IEW = 4 and IEH = 3

A.5 INPUT DATA EXAMPLES

This section provides the input data required to analyse the structures presented in sections 5.2 and 5.3

CARD GROUP No.	CARD No. of GROUP	DATA INPUT for box cantilever beam of section 5.2
1	1	FREQUENCY ANALYSIS OF TUBULAR CANTILEVER BEAM
2	1	1
3	1	2 4 1 1 1 1 0
4	1	4 4 15 .0001 .3145 0 0. 0
5	1	1 0. -.0762
5	2	2 .1524 -.0762
5	3	3 .1524 0.
7	1	1 1 2 2
7	2	2 2 3 1
9	1	1 0. 0. 13.86 0.
9	2	2 0. 0. 13.86 0.
9	3	3 0. 0. 13.86 0.
9	4	4 0. 0. 6.93 0.
11	1	0 0 0 0 0 0
11	2	0 0 0 0 0 0
12	1	facade No. 1
13	1	1 1 1 3 1
15	1	0. 210000000000. 84000000000. .0254
16	1	1 .0762 .3048 1 0 0
17	1	1 8 1
12	1	facade No. 2
13	1	1 1 1 1 1
15	1	0. 210000000000. 84000000000. .0254
16	1	1 .0762 .3048 1 0 0
17	1	1 4 1

CARD GROUP No.	CARD No. of GROUP	DATA INPUT for wall-frame building of section 5.3
1	1	EARTHQUAKE ANALYSIS OF CHAN'S WALL-FRAME BUILDING
2	1	1
3	1	2 10 1 1 1 1 0
4	1	4 3 10 .0001 .3154 0.0. 1
5	1	1 0. -63.75
5	2	2 25. -63.75
5	3	3 25. 0.
7	1	1 1 2 3
7	2	2 2 3 3
9	1	1 0. 0. 20264. 0.
9	2	2 0. 0. 20264. 0.
9	3	3 0. 0. 20264. 0.
9	4	4 0. 0. 20264. 0.
9	5	5 0. 0. 20264. 0.
9	6	6 0. 0. 20264. 0.
9	7	7 0. 0. 20264. 0.
9	8	8 0. 0. 20264. 0.
9	9	9 0. 0. 20264. 0.
9	10	10 0. 0. 10132. 0.
11	1	1 1 1 1 1 1
11	2	1 1 1 1 1 1
11	3	1 1 1 1 1 1 1 1 1 1
12	1	facade No. 1
13	1	1 6 9 3 1
15	1	432000000. 432000000. 172800000. 1.
16	1	1 12.5 24. 1 3 2
16	2	2 8.33 24. 1 2 2
16	3	3 4.17 24. 1 1 2
16	4	4 12.5 48. 1 3 4
16	5	5 8.33 48. 1 2 4
16	6	6 4.17 48. 1 1 4

17	1	1 7 1
17	2	2 8 2
17	3	3 9 3
17	4	10 22 4
17	5	11 23 5
17	6	12 24 6
17	7	25 28 1
17	8	26 29 2
17	9	27 30 3
12	1	facade No. 2
13	1	1 4 6 1 0
14	1.1	432000000. .25 1 1 1 1 1
14	1.2	7.5 2. .5
14	1.3	12. 1. 1.5
16	1	1 15. 24. 1 2 2
16	2	2 30. 24. 1 4 2
16	3	3 15. 48. 1 2 4
16	4	4 30. 48. 1 4 4
17	1	1 8 1
17	2	3 9 2
17	3	10 23 3
17	4	12 24 4
17	5	25 29 1
17	6	27 30 2

A.6 PROGRAM OUTPUT

Most of the input data is printed to allow direct checking. Some automatically generated data is also printed. In case of errors arising from core allocation, operations in algebraic equations in GAUSS, incomptable data, a corresponding message will be printed out.

Some input variables control what is to be printed out.

A.7 PROGRAM LISTING

The FORTAN IV compiled listing of TUBE is documented below.

```
      PROGRAM TUBE (TAPE1,TAPE2,INPUT,OUTPUT,TAPE3,TAPE4,TAPE5
     1,TAPE6,TAPE7,TAPE10,TAPE11)
C
C     ***********************************************************************
C     *                                                                     *
C     **********THE PURPOSE OF THE PROGRAM IS TO ANALYZE TUBULAR      ******
C     **********TALL BUILDING STRUCTURES SUBJECTED TO EARTHQUAKE      ******
C     **********OR STATIC LOADS. THE ANALYSIS IS LINEAR ELASTIC.      ******
C     **********ELASTIC AND STATIC.     ***********************************
C     *                                                                     *
C     ***********************************************************************
C
C
C     ***********************************************************************
C     *                                                                     *
C     *    PROGRAM VARIABLES                                                *
C     *         NF  =NO. OF FACADES                                         *
C     *         NL  =NO. OF LEVELS                                          *
C     *         NJL = NO. OF LOADED LEVELS                                  *
C     *         NHEL=NO. OF  HORIZONTAL ELEMENTS IN ONE FACADE              *
C     *         NVEL=NO.  OF VERTICAL ELEMENTS IN ONE FACADE                *
C     *         NMAT=NO. OF DIFFERENT MATERIALS IN ONE FACADE               *
C     *         NTYPE=NO. OF ELEMENT TYPES IN ONE FACADE                    *
C     *         NGROUP=NO. OF GROUPS OF ELEMENT TYPES IN ONE FACADE         *
C     *         NEQ  =NO.  EQUATIONS FOR THE TUBE STRUCTURE FOR STATIC ANAL *
C     *         NREM =NO. EQUATIONS FOR THE TUBE STRUCTURE FOR DYNAMIC ANAL *
C     *                                                                     *
C     ***********************************************************************
C
C
C     ***********************************************************************
C      ****PRGRAM ARRAYS                                                    *
C     *         FP(N,1) =NO. OF FIRST CORNER LINE                           *
C     *         FP(N,2) =NO. OF SECOND CORNER LINE                          *
C     *         FP(N,3) =NHEL                                               *
C     *         FP(N,4) =NVEL                                               *
C     *         RFP(N) =DISTANCE OF FACADE FROM CENTRE LINE OF TUBE STRUCTURE*
C     *         X(N)   = X-COORDINATE OF A CORNER LINE, N=NO. OF FACADE     *
C     *         Y(N)   = Y-COORDINATE OF A CORNER LINE, N=NO. OF FACADE     *
C     *         EX(M)..                                                     *
C     *         EY(M)..                                                     *
C     *         GXY(M).=ELASTIC PROPERTIES OF EQUIVALENT ORTHOTROPIC        *
C    *                   MEMBRANE, M= MATERIAL TYPE NO.                     *
C    *          EC(NP) = ELEMENT CONNECTIVITY, NP=NODE NO.                  *
C    *          EL(ND) = ELEMENT LABEL, ND=NO. OF ELEMENT DOF               *
C    *          QK     = ELEMENT STIFFNESS MATRIX                           *
C    *          S------- GLOBAL STRUCTURAL STIFFNESS MATRIX                 *
C    *          ETN(N)  = ELEMENT TYPR NO IN SEQUENCE FROM 1 TO N           *
C    *          BB     = STRAIN DISPLACEMENT MATRIX                          *
C    *          C      = STRESS STRAIN MATRIX                                *
C    *          R      = GENERALIZED LOAD OR DISPLACEMENT VECTOR             *
C     *                  OR LOCAL FACADE DISPLACEMENT VECTOR
C    *          AK       FACADE STIFFNESS MATRIX BEFORE AND AFTER CONDENSATION
C    *          Q, SIG = ELEMENT NODAL DISPLACEMENTS AND C.G. STRESSES ON  *
C    *          B,BBCLK,LN, FD, FLO,  ...  ARRAYS FOR MATRIX MANIPULATION  *
C     *                                                                     *
C   ***************************************************************************
```

```
      C
      C
            DIMENSION TITLE(8)
      C COMMON ON STRUCTURAL LEVEL.........INPUT
            COMMON NF,NL,NDOF,NSHAPE,NSSYM,NOUT
      C COMMON ON FAVADE LEVEL ......INPUT
            COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
           1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
           2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      C COMMON ON STRUCTURAL LEVEL ........SOLUTION
            DIMENSION S(3240),CA(4),SA(4),LD(30),BD(10),ERR(10),U(30,10)
           1,W(30,10),V(30,10),RFP(4,10),R(80,10),GC(10)
           2,SV(10)
            INTEGER EC,EL,ETM,ETN,ICOUNT(4),NPRI(6),NPRIN(6),NPRINT(10)
            INTEGER GD(30),FP(4,4)
            REAL MAS(30)
            DATA MAXNF,MAXNL,MAXMAT,MAXTYP,MAXFEL
           1 /     4,    10,      12,     36,    90/
      C
      C
      C PROBLEM IDENTIFICATION AND DESCRIPTION
      C
            READ(1,100) (TITLE(I),I=1,8)
            READ(1,*) NDYN
            WRITE(2,*) 'NDYN=',NDYN
            WRITE(2,200) (TITLE(I),I=1,8)
      C
      C READ OVERALL STRUCTURAL DATA
      C
            CALL DATA1(MAXNF,MAXNL,ISTOP1,NDYN,CA,SA,M,NOI,NROD
           1,TOLVEC,GES,IRENT,ALPHA,MAS,INTER,NEQ,NPRI
           1,NPRIN,NPRINT,RFP,FP,R)
            IF(ISTOP1.GT.0) GO TO 999
      C
      C COMPUTE REQ. STORAGE FOR GLOBAL TUBE STIFF MATRIX
      C
            IMAX = (NEQ*NEQ+NEQ)/2
            WRITE(2,*) 'STORAGE REQUIRED FOR STRUCTURAL SFIFF MATRIX=',IMAX
      C
            REWIND 3
            REWIND 4
            REWIND 5
            REWIND 7
            DO 500 N=1,NF
            NHEL=FP(N,3)
      C READ DATA FOR EACH FACADE
            CALL DATA2(MAXMAT,MAXTYP,MAXFEL,ISTOP2,NDOF,NEL,LMAX,NSYM)
            IF(ISTOP2.GT.0)  STOP
      C
      C
      C        1....ASSEMBLE EACH FACADE STIFFNESS MATRIX
      C        2....CONDENSE INTERNAL DOF,
      C        3....SAVE CONDENSED FACADE STIFF. MATRIX ON TAPE 4
      C
            CALL ASMBL1(NL,NDOF,NEL,LMAX,NSYM,R,NEQ)
        500 CONTINUE
      C
```

```
      C ASSEBLE OVERALL STATIC STRUCTURAL STIFFNESS MATRIX AK
      C
            CALL ASMBL2(NL,NEQ,NREM,IMAX,NF,NSHAPE,NSSYM,S,CA,SA
           1,ICOUNT,NDYN,RFP,R)
      C
            IF(NDYN.NE.0) GOTO 10
               CALL STAT(S,R,NEQ,IMAX,NSSYM,NL)
               CALL STRESS(NL,NF,NDOF,NSHAPE,NSSYM,NOUT,CA,SA,ICOUNT,RFP
           1,0,NPRINT,NEQ,R,1,NDYN)
            GOTO 999
      C.....GENERATE STRUCTURAL STIFFNESS MATRIX ADDRESSES FOR SKYLINE STORAGE
         10    NN=0
               DO 5 I=1,NREM
                  GD(I)=I
                  NN=NN+I
                  LD(I)=NN
          5    CONTINUE
      C.....COMPUTE STORAGE REQUIRED FOR DYNAMIC STRUCTURAL STIFFNESS MATRIX
               IMAX2=NREM*(NREM+1)/2
            WRITE(2,*) NREM,(GD(I),I=1,NREM),(MAS(I),I=1,NREM)
            WRITE(2,*) IMAX2,(LD(I),I=1,NREM),(S(I),I=1,IMAX2)
            WRITE(2,*) M,NRQD,NOI,TOLVEC,IRENT,GES
               IF(IRENT.NE.0) GOTO 20
                  GOTO 50
        20     IF(IRENT.NE.1) GOTO 30
                  DO 60 I=1,M
                     READ(1,*) (U(J,I),J=1,NREM)
        60        CONTINUE
                  GOTO 50
        30     IF(IRENT.NE.2) GOTO 40
                  DO 70 I=1,M
                     READ(1,*) (W(J,I),J=1,NREM)
        70        CONTINUE
                  GOTO 50
        40     WRITE(2,*) '***********INPUT DATA ERROR************'
               WRITE(2,*) '***INPUT FOR INTER IS NOT EQUAL TO 0,1,OR 2 ***'
      C.....CALCULATE FREQUENCIES AND MODE SHAPES
        50     CALL SIVIB2(MAS,GD,S,LD,BD,ERR,U,V,W,IMAX2,NREM,M,NRQD,NOI
           1,TOLVEC,IRENT,GES)
               IF((NPRI(1).EQ.0).AND.(NPRI(2).EQ.0).AND.(NPRI(3).EQ.0).AND.
           1(NPRI(4).EQ.0).AND.(NPRI(5).EQ.0).AND.(NPRI(6).EQ.0)) GOTO 999
      C.....PERFORM A SPECTRAL ANALYSIS
               MM=NRQD+1
               CALL MODAL(ALPHA,NRQD,MM,IMAX2,MAS,NSSYM,NL,NPRI,NPRIN,NPRINT
           1,NREM,NEQ,ET,ETN,NHEL,BD,ICOUNT,INTER,ERR,U,V,W,GC,SV,R,S,NF,NDOF
           2,NSHAPE,NOUT,CA,SA,RFP)
        100 FORMAT(8A10)
        200 FORMAT(/8A10/)
      C
        999 STOP
            END
```

```
      SUBROUTINE DATA1(MAXNF,MAXNL,ISTOP1,NDYN,CA,SA,M,NOI,NRQD
     1,TOLVEC,GES,IRENT,ALPHA,MAS,INTER,NEQ,NPRI
     1,NPRIN,NPRINT,RFP,FP,R)
C        ******************************************************
C        *                                                    *
C        *  THIS SUBROUTINE READS PROPERTIES OF THE TUBE STRUCTURE  *
C        ******************************************************
C
      COMMON NF,NL,NDOF,NSHAPE,NSSYM,NOUT
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9),IEW(12),IEH(12)
      INTEGER FP(4,4),EC,EL,ETM,ETN,NPRI(6),NPRIN(6),NPRINT(10)
      REAL MT,CA(4),SA(4),X(5),Y(5),MAS(30),R(80)
     1,RFP(4,10)
C
C
      READ(1,*)    NF,NL,NDOF,NSHAPE,NSSYM,NSYM23,NOUT
      WRITE(2,*) 'NF,NL,NDOF,NSHAPE,NSSYM,NSYM23,NOUT'
      WRITE(2,*) NF,NL,NDOF,NSHAPE,NSSYM,NSYM23,NOUT
      NVEL=NL
      WRITE(2,101) NF,NL
      IF(NDYN.EQ.1) READ(1,*) M,NRQD,NOI,TOLVEC,GES,IRENT
     1,ALPHA,INTER
      IF(NDYN.EQ.1) WRITE(2,*) 'M,NRQD,NOI,TOLVEC,GES,IRENT
     -,ALPHA,INTER'
      IF(NDYN.EQ.1) WRITE(2,*) M,NRQD,NOI,TOLVEC,GES,IRENT
     -,ALPHA,INTER
      IF(NDOF.EQ.0) GO TO 90
      WRITE(2,95)
      GO TO 91
   90 CONTINUE
      WRITE(2,100)
   91 CONTINUE
C
C CHECH TO BE SURE INPUT DATA DOES NOT EXCEED STORAGE CAPACITY
C
      ISTOP1=0
      IF(NF.LE.MAXNF) GO TO 10
      ISTOP1=ISTOP1+1
      WRITE(2,20) MAXNF
   10 IF(NL.LE.MAXNL) GO TO 11
      ISTOP1=ISTOP1+1
      WRITE(2,21) MAXNL
   11 IF(ISTOP1.EQ.0) GO TO 12
      WRITE(2,22) ISTOP1
      STOP
   12 CONTINUE
C
C COMPUT TUBE DOF (NO. OF EQUATIONS)
C
      IF(NSSYM.NE.0) GO TO 60
      IF(NSHAPE.NE.0) GO TO 50
      NEQ= (3+NF)*NL
      GO TO 61
   50 CONTINUE
      NEQ=(4+NF)*NL
```

```
      GO TO 61
   60 CONTINUE
      IF (NSHAPE.EQ.1)GO TO 110
      NEQ=NL*(NF+1-NSYM23)
      GO TO 61
  110 NEQ=NL*(NF+2-NSYM23)
   61 CONTINUE
      WRITE(2,*) 'NEQ=',NEQ
C
C READ AND PRINT PROPERTIES OF FACADES
C
      WRITE(2,102)
      IF(NSHAPE.NE.0) GO TO 14
      I=NF
      GO TO 15
   14 CONTINUE
      I=NF+1
   15 CONTINUE
      DO 29 N=1,I
      READ(1,*)     NN,X(N),Y(N)
      WRITE(2,*) 'NN,X(N),Y(N)'
      WRITE(2,*) NN,X(N),Y(N)
      WRITE(2,52) N,X(N),Y(N)
   29 CONTINUE
      IF(NDYN.EQ.0) GOTO 140
          DO 150 N=1,NF
              READ(1,*) NN,(FP(N,I),I=1,3)
      WRITE(2,*) 'NN,FP(N,1),FP(N,2),FP(N,3)'
      WRITE(2,*) NN,(FP(N,I),I=1,3)
              FP(N,4)=NVEL
  150     CONTINUE
          GOTO 160
C
  140 WRITE(2,103)
      DO 30 N=1,NF
      READ(1,*)      NN,(FP(N,I),I=1,3),RFP(N,I)
      WRITE(2,*) 'NN,FP(N,1),FP(N,2),FP(N,3),RFP(N,1)'
      WRITE(2,*) NN,(FP(N,I),I=1,3),RFP(N,1)
      FP(N,4)=NVEL
      WRITE(2,53) N,(FP(N,I),I=1,4),RFP(N,1)
   30 CONTINUE
C
C.....COMPUTE ORIENTATION OF FACADES: CA AND SA
  160 DO 170 N=1,NF
          III=FP(N,1)
          JJJ=FP(N,2)
          XX=X(JJJ)-X(III)
          YY=Y(JJJ)-Y(III)
          FW=SQRT(XX*XX+YY*YY)
          CA(N)=XX/FW
          SA(N)=YY/FW
  170 CONTINUE
      IF(NDYN.EQ.0) GOTO 180
C.....COMPUTE
C.....1...PERPENDICULAR DISTANCE FROM FACADE TO SHEAR CENTER : RFP
C.....2...LUMPED MASS MATRIX IN DIAGONAL VECTOR FORM : MAS
      DO 190 L=1,NL
```

```
                READ(1,*) NN,XMC,YMC,MAS(L),RMAS
             WRITE(2,*) 'NN,XMC,YMC,MAS(L),RMAS'
             WRITE(2,*) NN,XMC,YMC,MAS(L),RMAS
                IF((NSSYM.EQ.1).OR.(NSSYM.EQ.2)) GOTO 220
                IF(NSSYM.NE.0) GOTO 230
                   MAS(L+NL)=MAS(L)
                   MAS(L+2*NL)=RMAS
                   GOTO 220
       230      IF(NSSYM.NE.3) GOTO 220
                   MAS(L)=RMAS
       220      DO 200 N=1,NF
                   RFP(N,L)=(YMC-Y(FP(N,1)))*CA(N)-(XMC-X(FP(N,1)))*SA(N)
       200      CONTINUE
       190 CONTINUE
           READ(1,*)  (NPRI(I),I=1,6)
           WRITE(2,*) 'NPRI(1),NPRI(2),NPRI(3),NPRI(4),NPRI(5),NPRI(6)'
           WRITE(2,*) (NPRI(I),I=1,6)
           READ(1,*) (NPRIN(I),I=1,6)
           WRITE(2,*) 'NPRIN(1,2,3,4,5,6)'
           WRITE(2,*) (NPRIN(I),I=1,6)
           IF(NPRI(3).EQ.1) READ(1,*) (NPRINT(I),I=1,NL)
           IF(NPRI(3).EQ.1) WRITE(2,*) 'NPRINT(1,2,3,.....,NL)'
           IF(NPRI(3).EQ.1) WRITE(2,*) (NPRINT(I),I=1,NL)
     C
           GOTO 240
       180 WRITE(2,104)
     C ASSEMBLE STRUCTURAL LOAD VECTOR
     C
           N=1
           IF(NSSYM.EQ.0) N=3
           J=N*NL+1
           DO 40 I=J,NEQ
              R(I)=0.
        40 CONTINUE
           DO 41 I=1,N
              JJ=(I-1)*NL+1
              II=JJ+NL-1
              READ(1,*)   (R(J),J=JJ,II)
        41 CONTINUE
           WRITE(2,*) '************************LOAD VECTOR'
           WRITE(2,*) (R(I),I=1,NEQ)
           READ(1,*)   (NPRINT(I),I=1,NL)
           WRITE(2,*) 'NPRINT(1,2,3,....,NL)'
           WRITE(2,*) (NPRINT(I),I=1,NL)
     C
       101 FORMAT(34H INPUT TABLE 1.. BASIC PARAMETERS //
          1         5X, 40H NUMBER OF FACADES . . . . . . . . . . . ,I5/
          2         5X, 40H NUMBER OF LEVELS. . . . . . . . . . . . ,I5)
       100 FORMAT(//* ANALYSIS IS PERFORMED USING THE ORDINARY ELEMENT WITH
          1 6DOF*//)
        95 FORMAT(//* ANALYSIS IS PERFORMED USING THE REFINED ELEMENT WITH
          1 9DOF*//)
        20 FORMAT(//// 28H TOO MANY FACADES, MAXIMUM =              ,I5)
        21 FORMAT(//// 28H TOO MANY LEVELS., MAXIMUM =              ,I5)
        22 FORMAT(//// 28H EXCUTION HALTED BECAUSE OF,I5,13H FATAL ERRORS /)
       102 FORMAT(//34H INPUT TABLE 2.. FACADE PROPERTIES //
          1         12H  CORNER NO.,8X,12HX-COORDINATE,8X,12HY-COORDINATE)
```

```
    52 FORMAT(I12,2F20.4)
   103 FORMAT(// 12H  FACADE NO.,4X,1HI,4X,1HJ,4X,4HNHEL,4X,4HNVEL,4X,
     1         17HDIST. FROM CENTRE/)
    53 FORMAT(I12,2I5,2I8,F20.4)
   104 FORMAT(//26H INPUT TABLE 3.. LOAD DATA //
     1         10H LEVEL NO. ,5X,7HFORCE-X,5X,7HFORCE-Y,5X,7HTORSION)
   240 RETURN
       END
```

```
      SUBROUTINE DATA2(MAXMAT,MAXTYP,MAXFEL,ISTOP2,NDOF,NEL,LMAX,NSYM)
C
C         ***********************************************************
C         *                                                         *
C         *   THIS SUBROUTINE READS FACADE PROPERTIES               *
C         *                                                         *
C         ***********************************************************
C
C
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      DIMENSION TITLE(8)
      INTEGER EC,EL,ETM,ETN
C
C
      READ(1,99) TITLE
      WRITE(2,100) TITLE
      READ(1,*)   NMAT,NTYPE,NGROUP,NSYM,INCODE
      WRITE(2,*) 'NHEL,NMAT,NTYPE,NGROUP,NSYM,INCODE'
      WRITE(2,*) NHEL,NMAT,NTYPE,NGROUP,NSYM,INCODE
      NEL=NHEL*NVEL
      WRITE(2,101) NHEL,NVEL,NMAT,NTYPE
C
C CHECK TO BE SURE THAT INPUT DATA DOES NOT EXCEED STORAGE CAPACITY
C
      ISTOP2=0
      IF(NEL.LE.MAXFEL) GO TO 10
      ISTOP2=ISTOP2+1
      WRITE(2,20) MAXFEL
   10 IF(NMAT.LE.MAXMAT) GO TO 11
      ISTOP2=ISTOP2+1
      WRITE(2,21) MAXMAT
   11 IF(NTYPE.LE.MAXTYP) GO TO 12
      ISTOP2=ISTOP2+1
      WRITE(2,22) MAXTYP
   12 IF(ISTOP2.EQ.0) GO TO 13
      ISTOP2=ISTOP2+1
      WRITE(2,23) ISTOP2
      STOP
C
   13 CONTINUE
C READ OR COMPUTE ELASTIC PROPERTIES OF ELEMENT
C
      IF(INCODE.NE.0) GO TO 14
      CALL EMAT
      GO TO 15
   14 CONTINUE
      READ(1,*)  (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
      WRITE(2,*) 'EX(I),EY(I),GXY(I),TH(I)  I=1,NMAT'
      WRITE(2,*) (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
   15 CONTINUE
      WRITE(2,102)
      WRITE(2,52) (I,EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
C
C READ AND PRINT ELEMENT TYPE PROPERTIES
C
```

```
      WRITE(2,103)
      DO 30 J =1,NTYPE
      READ(1,*)  NN,(ET(J,I),I=1,2),ETM(J),IEW(J),IEH(J)
      WRITE(2,53) J,(ET(J,1),I=1,2),ETM(J) ,IEW(J),IEH(J)
   30 CONTINUE
      WRITE(2,104)
C.....GENERATE ELEMENT TYPE NO. FOR EACH ELEMENT OF FACADE
      DO 35 I=1,NGROUP
         READ(1,*)    NS,NE,NELT
      WRITE(2,*) 'NS,NE,NELT'
      WRITE(2,*) NS,NE,NELT
         LL=NE-NS
         L=MOD(LL,NHEL)
         INC=NS+L
         DO 35 II=INC,NE,NHEL
            M=II-L
            DO 35 J=M,II
               ETN(J)=NELT
   35 CONTINUE
C
C GENERATE AND PRINT ELEMENT, NODE CONNECTIVITY, AND TYPE
C
      DO 40 I=1,NEL
      CALL ELCON(I,NDOF)
      IF(NDOF.NE.0) GO TO 41
      NNP=4
      GO TO 42
   41 CONTINUE
      NNP=8
   42 CONTINUE
      WRITE(2,54) I,(EC(J),J=1,NNP),ETN(I)
   40 CONTINUE
C
C EVALUATE NO. OF FACADE DOF'S: NEQF
C THE REQ. STORAGE FOR ITS STIFF. MATRIX
C
      IF(NDOF.NE.0) GO TO 210
      IF(NSYM.NE.0) GO TO 215
      NEQF=NVEL+(NHEL+1)*NVEL
      GO TO 205
  215 CONTINUE
      NEQF=NVEL*(1+NHEL)
      GO TO 205
  210 CONTINUE
      IF(NSYM.NE.0) GO TO 220
      NEQF=NVEL*(2*NHEL+3)
      GO TO 205
  220 CONTINUE
      IF(NSYM.EQ.1) GO TO 225
      NEQF=NVEL*(2*NHEL+2)
      GO TO 205
  225 CONTINUE
      NEQF=NVEL*(2*NHEL+1)
  205 CONTINUE
      LMAX=(NEQF*NEQF+NEQF)/2
      WRITE(2,*) 'NO. OF FACADE DOF =',NEQF
      WRITE(2,*) 'SPACE REQUIRED FOR FACADE STIFF MATRIX=',LMAX
```

```
SUBROUTINE DATA2     FTN 4.8+552    83/03/21. 15.51.27    PAGE     3

   99 FORMAT(8A10)
  100 FORMAT(///2X,8A10)
  101 FORMAT(35H INPUT  TABLE A.. FACADE PROPERTIES //
     1           5X, 40H NUMBER OF  HORIZONTAL ELEMENTS . . . . . ,I5/
     2           5X, 40H NUMBER OF  VERTICAL ELEMENTS . . . . . . ,I5/
     3           5X, 40H NUMBER OF  DIFFERENT MATERIALS . . . . . ,I5/
     4           5X, 40H NUMBER OF  DIFFERENT ELEMENT TYPES . . . ,I5/)
  102 FORMAT(//36H INPUT TABLE B.. MATERIAL PROPERTIES //
     1         10H  MATERIAL,5X,10H   MODULUS,5X,10H   MODULUS,
     2         8X,7H  SHEAR,7X,8HMATERIAL/
     3         4X,6HNUMBER,10X,2HEX,13X,2HEY,11X,7HMODULUS,6X,8HTHICKNES)
   52 FORMAT(I10,4E15.4)
  103 FORMAT(//30H INPUT TABLE C.. ELEMENT TYPES //
     1          10H   ELEMENT,8X,7HELEMENT,8X,7HELEMENT,2X,8HMATERIAL,
     1          4X,6HNO. OF,4X,6HNO. OF/
     2          6X,4HTYPE,8X,7HWIDTH-A,7X,8HHEIGHT-B,6X,4HTYPE,
     3          6X,4HBAYS,4X,7HSTORIES/)
   53 FORMAT(I10,2F15.4,3I10)
  104 FORMAT(//*   ELEMENT CONNECTIVITY AND TYPE*//)
   54 FORMAT(10I5)
   20 FORMAT(////*TOOMANY ELEMENTS IN FACADE, MAXIMUM=*,I5)
   21 FORMAT(////*TOO MANY DIFFERENT MATERIALS, MAXIMUM= *,I5)
   22 FORMAT(////*TOO MANY ELEMENT TYPES  ,MAXIMUM= *,I5)
   23 FORMAT(////*EXECUTION HALTED BECAUSE OF *,I5,*FATAL ERRORS*/)
C
      RETURN
      END
```

```
      SUBROUTINE EMAT
C
C        ***********************************************************
C        *                                                         *
C        *  THIS SUBROUTINE EVALUATES THE ELESTIC PROPERTIES       *
C        *  OF THE EQUIVALENT ORTHOTROPIC MEMBRANE                 *
C        *                                                         *
C        ***********************************************************
C
C
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NRAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      INTEGER FP,EC,EL,ETM,ETN
      REAL MU
C
C
      DO 9999 I=1,NMAT
      READ(1,*)    E,MU,IBENDB,IBENDC,ISHEARB,ISHEARC,ISHEARJ
      READ(1,*)    B,BD,BT
      READ(1,*)    H,CD,CT
      WRITE(2,*) 'E,MU,IBENDB,IBENDC,ISHEARB,ISHEARC,ISHEARJ,B,BD,BT
     -,H,CD,CT'
      WRITE(2,*) E,MU,IBENDB,IBENDC,ISHEARB,ISHEARC,ISHEARJ,B,BD,BT
     -,H,CD,CT
C.....ELEMENT THICKNESS (TT) IS TAKEN TO BE THE BEAM THICKNESS (BT)
      TT=BT
      Q=1./(CD*BT)
      X=CD/2.
      Y=BD/2.
C
C
C   DELTA OF PARTS 1 AND 3
      DCOL=(H-BD)/(CD*CT*E)
      WRITE(2,91) DCOL
   91 FORMAT(//* DEFORMATION OF PARTS 1 AND 3 = *,E15.4)
      IF(ISHEARJ.NE.0) GOTO 97
      VUNIF=0.
      VFUNC=0.
      GOTO 94
   97 CONTINUE
C DELTA OF PART 2 DUE TO UNIFORM STRESS ONLY
      VUNIF=Q*CD*BD/(B*E)
      WRITE(2,92) VUNIF
   92 FORMAT( *    DEFORMATION DUE TO UNIFORM STRESS ..PART 2.. *,
     1E15.4)
C DELTA OF PART 2 DUE TO INFINITE FOURIER SERIES ONLY
      TERM=0.
      DO 10 M=1,50
      AL=M*22./(7.*B/2.)
      ALA=AL*CD/2.
      ALC=AL*BD/2.
      ALX=AL*X
      ALY=AL*Y
      T1=SIN(ALA)/(AL*M)
      T2=(1.+MU)*ALC*COSH(ALC)*SINH(ALY)
      T3=(1.-MU)*SINH(ALC)*SINH(ALY)
```

```
          T4=(1.+MU)*(ALY*COSH(ALY)-SINH(ALY))*SINH(ALC)
          T5=COS(ALX)/(SINH(2.*ALC)+2.*ALC)
          TERM=TERM+T1*(T2+T3-T4)*T5
   10 CONTINUE
          VFUNC=TERM*4.*Q/(E*22./7.)
          WRITE(2,93) VFUNC
   93 FORMAT( *   EDFORMATION DUE TO FOURIER TERMS .. PART 2 ..*.,
     1E15.4)
   94 CONTINUE
          EY(I)=H/(B*TT*(DCOL+VUNIF+2.*VFUNC))
          WRITE(2,999) EY(I)
  999 FORMAT(////*     EQUIVALENT ELASTIC MODULUS EY =   *,F15.4)
C
C E*NT SHEAR MODULUS  GXY
C
          T1=(H-BD)/H
          CI=CT*CD**3.
          BI=BT*BD**3.
          BENDB=0.
          BENDC=0.
          SHEARB=0.
          SHEARC=0.
          SHEARJ=0.
          IF(IBENDB.EQ.0) GOTO 20
             BENDB=(H/B)*((B-CD)/B)*((B-CD)*(B-CD)/BI)
   20 IF(IBENDC.EQ.0) GOTO 30
             BENDC=(T1*(H-BD)*(H-BD)/CI)
   30 IF(ISHEARB.EQ.0) GOTO 40
             SHEARB=(H*(B-CD)/(B*B*BT*BD/1.2))*(2./(1.+MU))
   40 IF(ISHEARC.EQ.0) GOTO 50
             SHEARC=(1.2*T1/(CT*CD))*(2./(1.+MU))
   50 IF(ISHEARJ.EQ.0) GOTO 60
             SHEARJ=((((B-CD)/B+T1-1)**2.)/(BT*CD*(1-T1)))*(2./(1.+MU))
   60 CXY=BENDB+BENDC+SHEARB+SHEARC+SHEARJ
          GXY(I)=E/(TT*B*CXY)
          TH(I)=TT
          WRITE(2,111) GXY(I)
  111 FORMAT(//*    EQUIVALENT SHEAR MODULUS =   *,F15.4)
 9999 CONTINUE
          RETURN
          END
```

```
      SUBROUTINE ELCON(I,NDOF)
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
C
      INTEGER FP,EC,EL,ETM,ETN
C
C     ****************************************************************
C     *                                                              *
C     *   THIS SUBROUTINE GENERATE ELEMENT CONNECTIVITY              *
C     *                                                              *
C     ****************************************************************
C
C
C THIS CAN BE ACHIEVED BY RELATING THE ELEMENT NO. TO ITS
C      FIRST NODE(L/WER, LEFT)
C
      JJ=(I-1)/NHEL
      II=I-JJ*NHEL
      IF(NDOF.NE.0) GO TO 10
      EC(1)=(II-1)*(NVEL+1)+JJ+1
      EC(2)=EC(1)+NVEL+1
      EC(3)=EC(2)+1
      EC(4)=EC(1)+1
      GO TO 999
   10 CONTINUE
      EC(1)=(II-1)*(NVEL+1)+JJ+1
      EC(2)=EC(1)+(NHEL+1)*(2*NVEL+1)
      EC(3)=EC(1)+NVEL+1
      EC(4)=(NVEL+1)*(NHEL+1)+II*NVEL+JJ+1
      EC(5)=EC(3)+1
      EC(6)=EC(2)+1
      EC(7)=EC(1)+1
      EC(8)=EC(4)-NVEL
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ASMBL1(NL,NDOF,NEL,LMAX,NSYM,R,NEQ)
C
C    ****************************************************************
C
C    *     THIS SUBROUTINE ASSEMBELS EACH FACADE STIFF. MATRIX,
C    *     CONDENSE IT, AND FORMS LOCAL(K) LEVEL SUBMATICES AND
C    *     AND GLOBAL SUBMATRICES (S).
C
C    ****************************************************************
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
C
      REAL K,AK(6105),R(NEQ)
      INTEGER FP,EC,EL,ETM,ETN
C
C
      WRITE(2,*) 'NSYM=',NSYM
C
C INITIALIZE FACADE STIFF. MATRIX AK
      DO 1 I=1,LMAX
      AK(I)=0.0
    1 CONTINUE
      WRITE(2,100)
      IF(NDOF.NE.0) GO TO 5
      LIM=6
      DO 99   M=1,NEL
C
C PROCEDURE IN THIS SUBROUTINE WILL BE AS FOLLOWS..
C                1.. FORM LOCAL ELEMENT STIFFNESS MATRIX QK
C                2.. ASSEMBEL EACH QK TO FORM FACADE STIFF. MATRIX AK
C                3.. CONDENSE AK
C
C
C FORM ELEMENT LABEL VECTOR- EL OR LEL -
C
      CALL LABEL(M,NDOF,NSYM)
      WRITE(2,200) M,(EL(I),I=1,6)
C
C FORM ELEMENT STIFF. MATRIX QK
C
      CALL RECT(M,NDOF)
C
C
C ASSEMBEL EACH QK ACCORDING TO THE CORRESPONDING LABEL VECTOR 'EL'
C NOTE , LIM= NO. OF ELEMENT DOF
C
      DO 10 LL=1,LIM
      I=EL(LL)
      DO 10 MM=1,LIM
      J=EL(MM)
C
C ASSEMBEL ONLY THE LOWER TRIANGLE , ROW BY ROW
C OR
C THE UPPER TRIANGLE COL. BY COL. ... NOTICE THAT BOTH ARE IDENTICAL
C
      IF(I.GT.J) GO TO 10
```

```
      IF(I.EQ.0) GO TO 10
      LOC=((J-1)*J)/2+I
      AK(LOC)=AK(LOC)+QK(LL,MM)
   10 CONTINUE
   99 CONTINUE
      GO TO 6
C
C
    5 CONTINUE
      LIM=9
      DO 88 M=1,NEL
      CALL LABEL(M,NDOF,NSYM)
      WRITE(2,200) M,(LEL(I),I=1,9)
      CALL RECT(M,NDOF)
      DO 8 LL=1,LIM
      I=LEL(LL)
      DO 8 MM=1,LIM
      J=LEL(MM)
      IF(I.GT.J) GO TO 8
      IF(I.EQ.0) GO TO 8
      LOC=((J-1)*J)/2+I
      AK(LOC)=AK(LOC)+QK(LL,MM)
    8 CONTINUE
   88 CONTINUE
    6 CONTINUE
C.....FACADE STIFFNESS MATRIX (AK) IS NOW FORMED
C
C CONDENSE INTERNAL DOF IN ABOVE AK
C
      IF(NDOF.EQ.1) GOTO 30
      IF(NHEL.EQ.1) GOTO 20
   30 IF(NSYM.NE.0) GO TO 17
      NR=3*NL
      GO TO 16
   17 CONTINUE
      NR=2*NL
   16 CONTINUE
      CALL GAUSS(AK,R,LMAX,NEQF,0,NR,DET,1)
C.....FACADE STIFFNESS MATRIX IS NOW REDUCED
   20 WRITE(3,*) NSYM,NEQF,LMAX,NEL,NMAT,NTYPE,NR
      WRITE(5,*) (ETN(J),J=1,NEL)
      WRITE(5,*) (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
      WRITE(5,*) (ETM(I),IEW(I),IEH(I),(ET(I,J),J=1,2),I=1,NTYPE)
      WRITE(4,*) (AK(I),I=1,LMAX)
  100 FORMAT(//*   ELEMENT NO.          DEGREES OF FREEDOM     *//)
  200 FORMAT(I14,5X,9I5)
      RETURN
      END
```

```
      SUBROUTINE LABEL(M,NDOF,NSYM)
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      INTEGER FP,EC,EL,ETM,ETN
C
C     ******************************************************************
C
C               THIS SUBROUTINE FORMS THE LABEL(NUMBERING OF DOF) FOR EACH ELEMENT
C
C     ******************************************************************
C
C FIRST  HORIZONTAL DOF
C
      L=(M-1)/NHEL
      NPR=L+1
C LIM=NO. OF DOF PER ELEMENT
C NP= NO. OF NODES PER ELEMENT
      LIM=6
      NP=LIM-2
      DO 1 I=1,NP
      IF(I.GT.2) GO TO 2
      EL(1)=L
      GO TO 1
    2 EL(2)=L+1
    1 CONTINUE
C
C SECOND VERTICAL DOF
C
      LL=M-L*NHEL
      LLL=NVEL*(LL+1)
C
C
C  1... ELEMENTS FIXED TO FOUNDATION
C
      IF(L.NE.0) GO TO 10
      EL(3)=0
      EL(4)=0
      IF(M.EQ.1) GO TO 11
      IF(M.EQ.NHEL) GO TO 12
      EL(6)=LLL+1
      EL(5)=EL(6)+NVEL
      GO TO 111
   11 EL(5)=3*NVEL+1
      IF(NHEL.EQ.1) EL(5)=2*NVEL+1
      EL(6)=NVEL+1
      GO TO 111
   12 EL(5)=NVEL*2+1
      EL(6)=NVEL*(NHEL+1)+1
      GO TO 111
   10 CONTINUE
C
C  2... REST OF ELEMENTS
      IF(NHEL.EQ.1) GOTO 21
C
C         A.. ELEMENTS ON THE LEFT MOST SIDE
C
```

```
      IF(LL.NE.1) GO TO 20
      EL(3)=NVEL+L
      EL(4)=NVEL*3+L
      EL(5)=EL(4)+1
      EL(6)=EL(3)+1
      GO TO 111
C
C        B.. ELEMENTS ON THE RIGHT MOST SIDE
C
   20 IF(LL.NE.NHEL) GO TO 30
      EL(3)=NVEL*(NHEL+1)+L
      EL(4)=NVEL*2+L
      EL(5)=EL(4)+1
      EL(6)=EL(3)+1
      GO TO 111
C
C        C.. INTERIOR ELEMENTS
C
   30 EL(3)=LLL+L
      EL(4)=EL(3)+NVEL
      EL(5)=EL(4)+1
      EL(6)=EL(3)+1
      GOTO 111
   21 EL(3)=NVEL+L
      EL(4)=2*NVEL+L
      EL(5)=EL(4)+1
      EL(6)=EL(3)+1
  111 CONTINUE
      IF(NDOF.NE.0) GO TO 70
      IF(NSYM.NE.0) CALL SYMM(NDOF,LL,NSYM)
      WRITE(7,*) EL,NPR
      GO TO 999
   70 CONTINUE
C
C
C GENERATE LABELLING VECTOR FOR THE REFINED ELEMENT *** BASED **
C ON THOSE OF THE ORDINARY ELEMENT
C
      LEL(1)=EL(1)
      LEL(2)=NVEL*3+1+L
      LEL(3)=EL(2)
C 1.... ELEMENTS FIXED TO FOUNDATION
C
      IF(L.NE.0) GO TO 80
      LEL(4)=0
      LEL(5)=0
      LEL(6)=0
      IF(LL.EQ.NHEL) GO TO 81
      LEL(7)=EL(5)+NVEL
      GO TO 82
   81 CONTINUE
      LEL(7)=EL(5)
   82 CONTINUE
      LEL(8)=NVEL*(3+NHEL)+(LL-1)*NVEL+1
      IF(LL.EQ.1) GO TO 83
      LEL(9)=EL(6)+NVEL
      GO TO 84
```

```
   83 CONTINUE
      LEL(9)=EL(6)
   84 CONTINUE
      GO TO 91
C
C 2.... REST OF ELEMENTS
C
   80 CONTINUE
      LEL(4)=EL(3)
      LEL(5)=NVEL*(3+NHEL)+(LL-1)*NVEL+L
      LEL(6)=EL(4)
      IF(NHEL.EQ.1) GOTO 22
      LEL(4)=LEL(4)+NVEL
      LEL(6)=LEL(6)+NVEL
   22 CONTINUE
      LEL(7)=LEL(6)+1
      LEL(8)=LEL(5)+1
      LEL(9)=LEL(4)+1
      IF(LL.NE.1) GO TO 90
      LEL(4)=EL(3)
      LEL(9)=LEL(4)+1
   90 CONTINUE
      IF(LL.NE.NHEL) GO TO 91
      LEL(6)=LEL(6)+1
   91 CONTINUE
      IF(NSYM.NE.0) CALL SYMM(NDOF,LL,NSYM)
      WRITE(7,*) LEL,NPR
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SYMM(NDOF,LL,NSYM)
C      ************************************************************
C      *                                                          *
C      *    THIS SUBROUTINE MODIFIES ELEMENT LABEL TO ACCOUNT FOR *
C      *    FACADE SYMETRY CONDITIONS                             *
C      *                                                          *
C      ************************************************************
C
C
C
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2EI(36,2),ETM(12),ETN(90),RK(9,9) ,IEW(12),IEH(12)
      INTEGER FP,EC,EL,ETM,ETN
C
C
C *** NOTE:
C *** NSYM = 0 IMPLIES NO FACADE SYMETRY OF ACTUAL FACADE
C *** NSYM = 1 IMPLIES FACADE SYMETRY OF ACTUAL FACADE
C *** NSYM = 2 OR 3 IMPLIES ANTI-SYMETRY OF ACTUAL FACADE
C
      IF(NDOF.NE.0) GO TO 80
      IF(NSYM.NE.1) GO TO 40
C
C FOR CASE OF SYMM. NSYM=1
C
      EL(1)=0
      EL(2)=0
      EL(3)=EL(3)-NVEL
      EL(4)=EL(4)-NVEL
      EL(5)=EL(5)-NVEL
      EL(6)=EL(6)-NVEL
      GO TO 444
   40 CONTINUE
C
C FOR CASE OF ANTI-SYMM. NSYM=2 OR 3
C
      IF(LL.EQ.1) GO TO 41
      IF(LL.EQ.NHEL) GO TO 42
C A...... INTERNAL ELEMENTS
      EL(3)=EL(3)-NVEL
      EL(4)=EL(4)-NVEL
      EL(5)=EL(5)-NVEL
      EL(6)=EL(6)-NVEL
      GO TO 444
   41 CONTINUE
C
C B...... ELEMENTS ON THE LEFT MOST SIDE
      EL(4)=EL(4)-NVEL
      EL(5)=EL(5)-NVEL
      IF(NSYM.NE.3) GO TO 444
      EL(3)=0
      EL(6)=0
      GO TO 444
   42 CONTINUE
C
C C..... ELEMENTS ON THE RIGHT MOST SIDE
C
```

```
      EL(3)=EL(3)-NVEL
      EL(4)=EL(4)-NVEL
      EL(5)=EL(5)-NVEL
      EL(6)=EL(6)-NVEL
      IF(NSYM.NE.2) GO TO 444
      EL(4)=0
      EL(5)=0
  444 CONTINUE
      DO 4444 I=1,6
      IF(EL(I).GE.0) GO TO 4444
      EL(I)=0
 4444 CONTINUE
      GO TO 9999
C
C
   80 CONTINUE
C
C *****   MODIFICATION FOR REFINED ELEMENT.
C
      IF(NSYM.NE.1) GO TO 81
C
C FOR CASE OF SYMM.
C
      LEL(1)=0
      LEL(2)=0
      LEL(3)=0
      NN=2*NVEL
      IF(LL.EQ.1) GO TO 82
      IF(LL.EQ.NHEL) GO TO 83
      LEL(4)=LEL(4)-NN
      LEL(5)=LEL(5)-NN
      LEL(6)=LEL(6)-NN
      LEL(7)=LEL(7)-NN
      LEL(8)=LEL(8)-NN
      LEL(9)=LEL(9)-NN
      GO TO 888
   82 CONTINUE
C
C LEFT MOST SIDE ELEMENTS
C
      LEL(4)=LEL(4)-NVEL
      LEL(5)=LEL(5)-NN
      IF(NHEL.NE.1) GOTO24
      LEL(6)=LEL(6)-NVEL
      LEL(7)=LEL(7)-NVEL
      GOTO 25
   24 CONTINUE
      LEL(6)=LEL(6)-NN
      LEL(7)=LEL(7)-NN
   25 CONTINUE
      LEL(8)=LEL(8)-NN
      LEL(9)=LEL(9)-NVEL
      GO TO 888
83    CONTINUE
C RIGHT MOST SIDE ELEMENTS
      LEL(4)=LEL(4)-NN
      LEL(5)=LEL(5)-NN
```

```
            LEL(6)=LEL(6)-NVEL
            LEL(7)=LEL(7)-NVEL
         LEL(8)=LEL(8)-NN
         LEL(9)=LEL(9)-NN
         GO TO 888
         81 CONTINUE
      C
      C FOR CASE OF ANTI-SYMM.
            LEL(2)=LEL(2)-NVEL
            IF(LL.EQ.1) GO TO 84
            IF(LL.EQ.NHEL) GO TO 85
            LEL(4)=LEL(4)-NVEL
            LEL(5)=LEL(5)-NVEL
            LEL(6)=LEL(6)-NVEL
            LEL(7)=LEL(7)-NVEL
            LEL(8)=LEL(8)-NVEL
            LEL(9)=LEL(9)-NVEL
            GO TO 888
         84 CONTINUE
            LEL(5)=LEL(5)-NVEL
            LEL(6)=LEL(6)-NVEL
            LEL(7)=LEL(7)-NVEL
            LEL(8)=LEL(8)-NVEL
            IF(NSYM.NE.3) GO TO 888
            LEL(4)=0
            LEL(9)=0
      C
            GO TO 888
         85 CONTINUE
            LEL(4)=LEL(4)-NVEL
            LEL(5)=LEL(5)-NVEL
            LEL(6)=LEL(6)-NVEL
            LEL(7)=LEL(7)-NVEL
            LEL(8)=LEL(8)-NVEL
            LEL(9)=LEL(9)-NVEL
            IF(NSYM.NE.2) GO TO 888
            LEL(6)=0
            LEL(7)=0
        888 CONTINUE
            DO 8888 I=1,9
            IF(LEL(I).GE.0) GO TO 8888
            LEL(I)=0
       8888 CONTINUE
       9999 CONTINUE
            RETURN
            END
```

```
      SUBROUTINE RECT(M,NDOF)
      COMMON/P2/NMEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      INTEGER FP,EC,EL,ETM,ETN
C
C        **************************************************************
C
C
C               THIS SUBROUTINE FORMS EACH ELEMENT STIFFNESS MATRIX
C
C
C
C        **************************************************************
C
C
C ELEMENT PROPERTIES
C
      IT=ETN(M)
      A=ET(IT,1)
      B=ET(IT,2)
      IM=ETM(IT)
      E2=EY(IM)
      G12=GXY(IM)
      TT=TH(IM)
      R=A/B
C
      IF(NDOF.NE.0) GO TO 50
C
C
C INITIALIZE ELEMENT STIFFNESS MATRIX QK
C
      DO 10 I=1,6
      DO 10 J=1,6
      QK(I,J)=0.0
   10 CONTINUE
C
C FORM STIFFNESS MATRIX OF ORDINARY ELEMENT
C
      QK(1,1)=TT*R*G12
      QK(2,1)=-QK(1,1)
      QK(3,1)=.5*TT*G12
      QK(4,1)=-QK(3,1)
      QK(5,1)=-QK(3,1)
      QK(6,1)= QK(3,1)
C
      QK(2,2)=QK(1,1)
      QK(3,2)=-QK(3,1)
      QK(4,2)= QK(3,1)
      QK(5,2)=QK(3,1)
      QK(6,2)=-QK(3,1)
C
      QK(3,3)=(E2*R+G12/R)*TT/3.
      QK(4,3)=(.5*E2*R-G12/R)*TT/3.
      QK(5,3)=-.5*QK(3,3)
      QK(6,3)=(.5*G12/R-E2*R)*TT/3.
C
      QK(4,4)=QK(3,3)
```

```
      QK(5,4)=QK(6,3)
      QK(6,4)=-.5*QK(3,3)
C
      QK(5,5)=QK(3,3)
      QK(6,5)=QK(4,3)
C
      QK(6,6)=QK(3,3)
C
      DO 20 I=1,6
      DO 20 J=1,6
      IF(J.LE.I) GO TO 20
      QK(I,J)=QK(J,I)
20    CONTINUE
      GO TO 999
C
C
   50 CONTINUE
C
C FORM STIFFNESS MATRIX OF THE REFINED ELEMENT
C
C INTIALIZE QK 9X9
      DO 55 I=1,9
      DO 55 J=1,9
      QK(I,J) =0.0
   55 CONTINUE
C
C
      S1=TT*G12*R/12.
      S2=TT*G12/12.
      S3=TT*E2*R/60.
      S4=TT*G12/(R*36.)
C
      QK(1,1)=28.*S1
      QK(2,1)=-32.*S1
      QK(3,1)=4.*S1
      QK(4,1)=10.*S2
      QK(6,1)=-10.*S2
      QK(7,1)=-2.*S2
      QK(9,1)=2.*S2
C
      QK(2,2)=64.*S1
      QK(3,2)=QK(2,1)
      QK(4,2)=-8.*S2
      QK(6,2)=-QK(4,2)
      QK(7,2)=QK(4,2)
      QK(9,2)=-QK(4,2)
C
      QK(3,3)=QK(1,1)
      QK(4,3)=QK(7,1)
      QK(6,3)=-QK(7,1)
      QK(7,3)=QK(4,1)
      QK(9,3)=-QK(4,1)
C
      QK(4,4)=8.*S3+28.*S4
      QK(5,4)=4.*S3-32.*S4
      QK(6,4)=-2.*S3+4.*S4
```

```
      QK(7,4)=2.*S3+2.*S4
      QK(8,4)=-4.*S3-16.*S4
      QK(9,4)=-8.*S3+14.*S4
C
      QK(5,5)=32.*S3+64.*S4
      QK(6,5)=QK(5,4)
      QK(7,5)=QK(8,4)
      QK(8,5)=-32.*S3+32.*S4
      QK(9,5)=QK(8,4)
C
      QK(6,6)=QK(4,4)
      QK(7,6)=QK(9,4)
      QK(8,6)=QK(8,4)
      QK(9,6)=QK(7,4)
C
      QK(7,7)=QK(4,4)
      QK(8,7)=QK(5,4)
      QK(9,7)=QK(6,4)
C
      QK(8,8)=QK(5,5)
      QK(9,8)=QK(5,4)
C
      QK(9,9)=QK(4,4)
C
C
      DO 60 I=1,9
      DO 60 J=1,9
      IF(J.LE.I) GO TO 60
      QK(I,J)=QK(J,I)
   60 CONTINUE
  999 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GAUSS(AK,R,IMAX,NEQ,NLO,NR,DET,NCODE)
      DIMENSION R(NEQ,NLO),AK(IMAX)
C
C       *****************************************************************
C
C             AK  CONTAINS COEFFICIENTS IN ONE TRIANGLE OF SYMETRIC
C             MATRIX COL. BY COL. IF UPPER TRIANGLE , ROW BY ROW
C             IF LOWER TRIANGLE.
C             R= RIGHT HAND SIDE MATRIX IN (AK) (X)=(R)
C             NEQ= NO. OF EQUATIONS
C             IMAX=STORAGE REQUIRED FOR AK
C             NLO=NO. OF LOAD CASES, I.E. COLS. OF  R
C             NR= NO. OF EQUATIONS REMAINED IN CONDENSATION
C             **** NOTICE***** ELEMINATED DOF MUST BE STORED LAST
C             DET= DETERMINANT
C             NCODE= 1 FORWARD ELIMINATION AND BACK SUBSTITUTION
C                    2 ELIMINATION OF RIGHT HAND SIDE AND BACK SUBSTITUTE
C             NR.LT.0=MODIFY MATRIX AK AND BACK SUBSTITUTE TO
C                         RECOVER ELIMINATED UNKNOWNS. KNOWN VALUES OF
C                        R MUST BE IN POSITION BEFORE.
C
C
C
C             ******CALLING ARGUMENTS *********
C
C
C             FOR SOLUTIONS OF (AK)(X)=(R) USE GAUSS(AK,R,IMAX,NEQ,NLO,1,DET,1)
C             FOR NEW RIGHT HAND SIDE  GAUSS(AK,R,IMAX,NEQ,NLO,1,DET,2)
C             FOR DETERMINANT GAUSS(AK,0,IMAX,NLO,0,1,DET,1)
C             FOR CONDENSATION GAUSS(AK,R,IMAX,NEQ,NLO,NR,DET,1)
C              NLO MAY BE 0
C             FOR RECOVERY GAUSS(AK,R,IMAX,NEQ,NLO,-NR,DET,2)
C
C        ****************************************************************
C
C
      IF(NR.LT.0) GO TO 15
      NE=NEQ-NR
      DET=1.
      DO 500 M=1,NE
      MAX=NEQ-M
      N=MAX+1
      L=(N*(N+1))/2
      LN=L-N
      IF(AK(L).GT.1.E-20) GO TO 600
      WRITE(2,1) N
      GO TO 500
  600 PIVOT=1./AK(L)
      IF(NCODE.GT.1) GO TO 400
      DO 300 J=1,MAX
      T=AK(LN+J)*PIVOT
      IF(T.EQ.0.) GO TO 300
      DO 200 I=J,MAX
      K=(I*(I-1))/2+J
      KN=LN+I
      AK(K)=AK(K)-AK(KN)*T
  200 CONTINUE
```

```
      SUBROUTINE GAUSS      FTN 4.8+552     83/03/21. 15.51.27     PAGE     2

  300 CONTINUE
      IF(NLO.NE.0) GO TO 400
      IF(NR.NE.1) GO TO 500
      DET=DET*AK(L)
  400 DO 360 J=1,NLO
      T=R(N,J)*PIVOT
      IF(T.EQ.0) GO TO 360
      DO 350 I=1,MAX
      K=LN+I
  350 R(I,J)=R(I,J)-AK(K)*T
  360 CONTINUE
  500 CONTINUE
      IF(NR.GT.1) RETURN
      DO 10 L=1,NLO
   10 R(1,L)=R(1,L)/AK(1)
      M=2
   15 IF(NR.LT.0)  M=1-NR
      DO 20 I=M,NEQ
      K=I-1
      KI=(I*K)/2
      KN=KI+I
      DO 30 L=1,NLO
      DO 25 J=1,K
      KJ=KI+J
   25 R(I,L)=R(I,L)-R(J,L)*AK(KJ)
   30 R(I,L)=R(I,L)/AK(KN)
   20 CONTINUE
    1 FORMAT(*  ZERO DIAGONAL AT ROW *,I5)
      RETURN
      END
```

```
      SUBROUTINE ASMBL2(NL,NEQ,NREM,IMAX,NF,NSHAPE,NSSYM,S,CA,SA
     1,ICOUNT,NDYN,RFP,R)
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETH(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      DIMENSION AK(6105),COF(3,5),S(IMAX),CA(4),SA(4)
     1,RFP(4,10),R(NEQ)
      INTEGER FP,EC,EL,ETH,ETN,ICOUNT(4)
C
C        ***************************************************************
C
C
C
C              THIS SUBROUTINE RETREIVES THE CONDENSED LOCAL
C              FACADE STIFFNESS MATRICES, TRANSFORMS THEM TO
C              GLOBAL FACADE STIFFNESS MATRICES, AND THEN
C              ASSEMBLES THEM INTO THE STRUCTURAL STIFFNESS
C              MATRIX S
C
C
C
C        ***************************************************************
C
C
C
      REWIND 3
      REWIND 4
C
C INITIALIZE  S
C
      DO 100 I=1,IMAX
         S(I)=0.
  100 CONTINUE
      ICO=0
      NSYMB=2
      DO 9999 N=1,NF
         READ(3,*) NSYM,NEQF,LMAX,NEL,NMAT,NTYPE,NR
         READ(4,*) (AK(I),I=1,LMAX)
         IF(NSYM.EQ.1) GOTO 5
         IF(NSSYM.NE.0) GOTO 10
C.....IF NSSYM=0 , EXPAND THE 3NL*3NL REDUCED FACADE STIFFNESS MATRIX AK
C     CORRESPONDING TO LOCAL LATERAL DEFLECTION, AND THE TWO CORNER
C     VERTICAL DEFLECTIONS INTO A 5NL*5NL STIFFNESS MATRIX AK
C     CORESPONDING TO THE THREE SHEAR CENTER DEFLECTIONS AND THE
C     TWO CORNER VERTICAL DEFLECTIONS BY USING THE FOLLOWING
C     MAPPING PROCEDURE IN SEQUENCE :
C     NOTE : INDECIE*RIX ADDRESSES
C     1......AK(4,4)=AK(2,2)
C            AK(4,5)=AK(2,3)
C            AK(5,5)=AK(3,3)
C     2......AK(1,4)=AK(1,2)
C            AK(2,4)=AK(1,2)
C            AK(3,4)=AK(1,2)
C            AK(1,5)=AK(1,3)
C            AK(2,5)=AK(1,3)
C            AK(3,5)=AK(1,3)
C     3......AK(1,2)=AK(1,1)
C            AK(1,3)=AK(1,1)
C            AK(2,2)=AK(1,1)
C            AK(2,3)=AK(1,1)
```

```
      C           AK(3,3)=AK(1,1)
      C.....THE OBJECTIVE OF THIS MAPPING PROCEDURE IS TO LATTER
      C     TRANSFORM THE REDUCED FACADE STIFFNESS MATRIX FROM
      C     LOCAL TO GLOBAL ORIENTATION.
      C
      C
      C........THIS IS MAPPING # 1
                DO 200 L=1,2
                  DO 330 NN=1,NL
                    LOC=NL*L+NN-1
                    LOC=LOC*(LOC+1)/2+NL
                    LN1=NL*(L+2)+NN-1
                    LN1=LN1*(LN1+1)/2+3*NL
                    DO 340 I=1,L
                      IF(I.EQ.L) GOTO 230
                      DO 240 J=1,NL
                        LOC=LOC+1
                        LN1=LN1+1
                        AK(LN1)=AK(LOC)
        240           CONTINUE
                      GOTO 340
        230           DO 250 J=1,NN
                        LOC=LOC+1
                        LN1=LN1+1
                        AK(LN1)=AK(LOC)
        250   *     CONTINUE
        340             CONTINUE
        330         CONTINUE
        200       CONTINUE
      C
      C
      C........THIS IS MAPPING # 2
                DO 260 L=1,2
                  DO 270 NN=1,NL
                    LOC=L*NL+NN-1
                    LOC=LOC*(LOC+1)/2
                    LN1=(2+L)*NL+NN-1
                    LN1=LN1*(LN1+1)/2
                    LN2=LN1+NL
                    LN3=LN2+NL
                    DO 280 I=1,NL
                      LOC=LOC+1
                      LN1=LN1+1
                      LN2=LN2+1
                      LN3=LN3+1
                      AK(LN1)=AK(LOC)
                      AK(LN2)=AK(LOC)
                      AK(LN3)=AK(LOC)
        280         CONTINUE
        270       CONTINUE
        260     CONTINUE
      C
      C
      C........THIS IS MAPPING # 3 (UPPER TRIANGLE INCLUSIVE)
                LOC=0
                DO 290 NN=1,NL
                  LN1=NL+NN-1
```

```
                    LN1=LN1*(LN1+1)/2
                    LN2=LN1+NL
                    LN3=2*NL+NN-1
                    LN3=LN3*(LN3+1)/2
                    LN4=LN3+NL
                    LN5=LN4+NL
                    DO 300 I=1,NN
                       LOC=LOC+1
                       LN1=LN1+1
                       LN2=LN2+1
                       LN3=LN3+1
                       LN4=LN4+1
                       LN5=LN5+1
                       AK(LN1)=AK(LOC)
                       AK(LN2)=AK(LOC)
                       AK(LN3)=AK(LOC)
                       AK(LN4)=AK(LOC)
                       AK(LN5)=AK(LOC)
  300               CONTINUE
  290            CONTINUE
C
C
C.......THIS IS MAPPING # 3 (LOWER TRIANGLE)
                 LG=0
                 LN4=NL-1
                 DO 310 NN=1,LN4
                    LG=LG+NN
                    LOC=LG
                    LN1=NL+NN-1
                    LN1=LN1*(LN1+1)/2+NN
                    LN2=2*NL+NN-1
                    LN2=LN2*(LN2+1)/2+NN
                    LN3=LN2+NL
                    DO 320 I=NN,LN4
                       LOC=LOC+I
                       LN1=LN1+1
                       LN2=LN2+1
                       LN3=LN3+1
                       AK(LN1)=AK(LOC)
                       AK(LN2)=AK(LOC)
                       AK(LN3)=AK(LOC)
  320               CONTINUE
  310            CONTINUE
      WRITE(2,*) 'INITIALIZE FACADE STIF MAT TO BE ROTATED TO GLOB AX'
      LB=5*NL
      DO 122 L=1,LB
         J=L*(L+1)/2
         KKK=J-L+1
         WRITE(2,*) L,J,(AK(K1),K1=KKK,J)
  122 CONTINUE
                 IA=5
                 IH=4
                 COF(1,1)=CA(N)*CA(N)
                 COF(1,2)=CA(N)*SA(N)
                 COF(1,4)=CA(N)
                 COF(1,5)=CA(N)
                 COF(2,2)=SA(N)*SA(N)
```

```
            COF(2,4)=SA(N)
            COF(2,5)=SA(N)
            IF(NDYN.NE.0) GOTO 810
               COF(1,3)=CA(N)*RFP(N,1)
               COF(2,3)=SA(N)*RFP(N,1)
               COF(3,3)=RFP(N,1)*RFP(N,1)
               COF(3,4)=RFP(N,1)
               COF(3,5)=RFP(N,1)
               GOTO 20
  810          IC=3
               COF(1,3)=CA(N)
               COF(2,3)=SA(N)
               GOTO 510
   10    CONTINUE
         IF((NSSYM.EQ.1).OR.(NSSYM.EQ.2)) GOTO 850
            IF(NDYN.EQ.1) GOTO 820
  850       IF(NSSYM.EQ.1) V=CA(N)
            IF(NSSYM.EQ.2) V=SA(N)
            IF(NSSYM.EQ.3) V=RFP(N,1)
            COF(1,1)=V*V
            COF(1,2)=V
            IF(NSYM.NE.0) GOTO 15
             IA=3
             IH=2
             COF(1,3)=V
             GOTO 20
   15       CONTINUE
             IA=2
             IH=2
             GOTO 20
  820       IC=1
            IF(NSYM.NE.0) GOTO 830
               IA=3
               IH=2
         GOTO 510
  830          IA=2
               IH=2
            GOTO 510
   20    CONTINUE
C.....ROTATE REDUCED FACADE STIFFNESS MATRIX INTO GLOBAL FACADE
C     STIFFNESS MATRIX EITHER FOR STATIC ANALYSIS OR IF
C     NSSYM = 1 OR NSSYM = 2
C.....NOTE THAT THE REFERENCE POINTS ON EACH LEVEL FORMS A VERTICAL
C     LINE THROUGH THE BUILDING
         LOC=0
         DO 165 L=1,IA
            DO 160 NN=1,NL
               DO 155 I=1,L
                  IF(I.NE.IH) GOTO 130
                     LN=(L-1)*NL+NN
                     LOC=LN*(LN+1)/2
                     GOTO 160
  130             IF(I.EQ.L) GOTO 150
                  DO 145 J=1,NL
                     LOC=LOC+1
                     AK(LOC)=AK(LOC)*COF(I,L)
  145             CONTINUE
```

```
                        GOTO 155
         150            DO 135 MM=1,NN
                           LOC=LOC+1
                           AK(LOC)=AK(LOC)*COF(I,L)
         135            CONTINUE
         155         CONTINUE
         160      CONTINUE
         165   CONTINUE
               GOTO 5
         510   CONTINUE
      C.....ROTATE REDUCED FACADE STIFFNESS MATRIX INTO GLOBAL FACADE
      C     STIFFNESS MATRIX FOR A DYNAMIC ANALYSIS WITH
      C     NSSYM = 0 OR NSSYM = 3
      C.....NOTE THAT REFERENCE POINTS OF EACH LEVEL DO NOT NECCESARILY
      C     FORM A VERTICAL LINE
               LOC=0
               DO 520 L=1,IA
                  DO 530 NN=1,NL
                     DO 540 I=1,L
                        IF(I.NE.IH) GOTO 550
                           LN=(L-1)*NL+NN
                           LOC=LN*(LN+1)/2
                           GOTO 530
         550            IF(L.NE.IC) GOTO 570
                           IF(I.EQ.L) GOTO 580
                              DO 590 J=1,NL
                                 LOC=LOC+1
                                 AK(LOC)=AK(LOC)*COF(I,L)*RFP(N,NN)
         590                  CONTINUE
                              GOTO 540
         580               DO 600 MM=1,NN
                              LOC=LOC+1
                              AK(LOC)=AK(LOC)*RFP(N,MM)*REP(N,NN)
         600               CONTINUE
                           GOTO 540
         570            IF((L.LE.IC).OR.(I.NE.IC)) GOTO 840
                           DO 610 J=1,NL
                              LOC=LOC+1
                              AK(LOC)=AK(LOC)*RFP(N,J)
         610               CONTINUE
                     GOTO 540
         840            IF(I.EQ.L) GOTO 620
                           DO 630 J=1,NL
                              LOC=LOC+1
                              AK(LOC)=AK(LOC)*COF(I,L)
         630               CONTINUE
                           GOTO 540
         620          LOD0L0C01MM*
                      AK(LOC)=AK(LOC)*COF(I,L)
         640               CONTINUE
         540         CONTINUE
         530      CONTINUE
         520   CONTINUE
           5   CONTINUE
      C.....GLOBAL FACADE STIFFNESS MATRIX HAS NOW BEEN FORMED
               IF(NSYMB.NE.2) GOTO 410
```

```
              ICO=ICO+1
              GOTO 430
      410   IF((NSYMB.NE.0).AND.(NSYMB.NE.1)) GOTO 420
              IF(NSYM.EQ.3) ICO=ICO+2
              ICO=ICO+1
              GOTO 430
      420   IF(NSYMB.NE.3) GOTO 430
              IF(NSYM.EQ.3) ICO=ICO+1
      430   ICOUNT(N)=ICO
            IF(NSSYM.NE.0) GOTO 90
              IB=N-1
              ID=N+2
              IG=3*NL+1
              IH=4
              IJ=5
              IK=0
              IL=4
              IM=5
              GOTO 125
       90   IF(NSYM.NE.0) GOTO 95
              IB=ICOUNT(N)-1
              ID=ICOUNT(N)
              IG=NL+1
              IH=2
              IJ=3
              IK=0
              IL=2
              IM=3
              GOTO 125
       95   IF(NSYM.NE.1) GOTO 105
              IB=ICOUNT(N)
              ID=ICOUNT(N)
              IG=NL+1
              IH=1
              IJ=2
              IK=0
              IL=1
              IM=2
              GOTO 125
      105   IF(NSYM.NE.2) GOTO 110
              IB=ICOUNT(N)-1
              ID=ICOUNT(N)
              IG=0
              IH=2
              GOTO 125
      110   IF(NSYM.NE.3) GOTO 120
              IB=ICOUNT(N)-1
              ID=ICOUNT(N)
              IG=NL+1
              IH=2
              IJ=1
              IK=0
              IL=2
              IM=3
              GOTO 125
      120   WRITE(2,*) '****** ERROR IN INPUT DATA ******'
            WRITE(2,*) 'FACADE SYMETRY WAS NOT SPECIFIED AS 0,1,2,OR 3 FOR
```

```
     2 FACADE NUMBER   ',N
       GOTO 819
C
C.....ASSEMBLE GLOBAL FACADE STIFFNESS* STRUCTURAL
C      STIFFNESS MATRIX (S) CORRESPONDING TO REFERENCE POINT DISPLACEMENTS
C      AND CORNER VERTICAL DISPLACEMENTS OF EACH LEVEL
C
  125    LG=0
         IN=0
         LOC=0
         DO 30 L=1,IA
            IC=L
            IF(L.NE.IH) GOTO 210
             LL=NL*ID+1
             GOTO 85
  210       IF(L.NE.IH) GOTO 65
             IF(NSYM.EQ.2) GOTO 65
               IF((N.NE.NF).OR.(NSHAPE.EQ.1)) GOTO 65
                  IP=0
                  IR=0
                  DO 215 NN=1,NL
                     IP=IG+NL+NN-1
                     IP=IP*(IP-1)/2+IG-1
                     IR=ID*NL
                     IR=IR*(IR-1)/2+IG+NN-1
                     DO 220 I=1,NL
                        IP=IP+1
                        IR=IR+ID*NL+I-1
                        S(IR)=S(IR)+AK(IP)
  220                CONTINUE
  215             CONTINUE
                  IB=0
                  IC=IL
                  LL=IG
                  IF(NSYM.EQ.3) GOTO 85
                     IN=NL
   85          LG=LL*(LL-1)/2
   65          DO 35 NN=1,NL
                  DO 40 I=1,IC
                     IF(I.NE.IH) GOTO 205
                        LG=LG+IB*NL
                        LOC=LOC+IN
  205                IF(I.EQ.IC) GOTO 45
                     DO 50 J=1,NL
                        LG=LG+1
                        LOC=LOC+1
                        S(LG)=S(LG)+AK(LOC)
   50                CONTINUE
                     GOTO 40
   45                DO 60 MM=1,NN
                        LG=LG+1
                        LOC=LOC+1
                        S(LG)=S(LG)+AK(LOC)
   60                CONTINUE
   40             CONTINUE
   35          CONTINUE
   30    CONTINUE
```

```
                NSYMB=NSYM
 9999 CONTINUE
      IF(NDYN.EQ.0) GOTO 730
         NREM=NL
         IF(NSSYM.EQ.0) NREM=3*NL
      WRITE(2,*) (S(I),I=1,IMAX)
C.....IF ANALYSIS IS DYNAMIC, CONDENSE THE STATIC STRUCTURAL STIFFNESS
C     MATRIX TO DYNAMIC STRUCTURAL STIFFNESS MATRIX (REFERENCE POINT DOF'S ONLY)
         CALL GAUSS(S,R,IMAX,NEQ,0,NREM,DET,1)
  730 REWIND 6
      WRITE(6,*) (S(I),I=1,IMAX)
      WRITE(2,*) 'GLOBAL STRUCT STIF MAT IN VECT FORM COL BY COL'
      DO 23 I=1,NEQ
         J=(I*(I+1))/2
         KKK=J-I+1
         WRITE(2,*) I,J,(S(K1),K1=KKK,J)
   23 CONTINUE
  819 RETURN
      END
```

```
      SUBROUTINE MODAL(ALPHA,M,MM,NK,MAS,NSSYM,NL,NPRI,NPRIN,NPRINT,NREM
     1,NEQ,ET,ETN,NHEL,BD,ICOUNT,INTER,ERR,U,V,W,GC,SV,R,S,NF,NDOF
     2,NSHAPE,NOUT,CA,SA,RFP)
      DIMENSION VV(3),ERR(M),W(30,M),GC(M),SV(M),R(80,MM),S(NK)
     1,U(30,M),V(30,M),ET(36,2),X(10),BD(M),CA(4),SA(4),RFP(4,10)
      INTEGER ETN(90),ICOUNT(4),NPRI(6),NPRIN(6),NPRINT(10)
      REAL MAS(NREM),MOM(3,30),MPF,MPFSVF
      IF(INTER.EQ.0) GOTO 510
         DO 520 J=1,M
            PRINT*,'MODE NUMBER ',J
            PRINT*,'---- ------'
            PRINT*,'      FREQUENCY = ',BD(J)
            PRINT*,'      TIME PERIOD = ',6.28318/BD(J)
            PRINT*,'      INPUT PSEUDO VELOCITY '
            READ*,SV(J)
      WRITE(2,*) 'SV(',J,')=',SV(J)
  520GOTOCB*0
 510     READ(1,*) (SV(J),J=1,M)
      WRITE(2,*) 'PSEUDO-VELOCITIES FOR INCREASING MODE NUMBER'
      WRITE(2,*) (SV(J),J=1,M)
  530 ND=1
      VV(1)=COS(ALPHA)
      IF(NSSYM.EQ.2) VV(1)=SIN(ALPHA)
      IF(NSSYM.NE.0) GOTO 540
         ND=3
         VV(2)=SIN(ALPHA)
         VV(3)=0.
      GOTO 550
  540    IF(NSSYM.EQ.3) VV(1)=0.
  550 DO 10 J=1,M
      WRITE(2,13)
   13 FORMAT(T1)
         WRITE(2,*) 'MODE NUMBER',J
C........CALCULATE MODAL PARTICIPATION FACTOR 'MPF'
         Z1=0
         Z3=0
         DO 20 I=1,ND
            Z2=0
            DO 30 K=1,NL
               KK=(I-1)*NL+K
               ERR(KK)=W(KK,J)*MAS(KK)
               Z1=ERR(KK)*W(KK,J)+Z1
               Z2=Z2+ERR(K)
   30       CONTINUE
            Z2=Z2*VV(I)
            Z3=Z3+Z2
   20    CONTINUE
         MPF=Z3/Z1
      WRITE(2,*) 'MODAL PARTICIPATION FACTOR FOR MODE #',J
      WRITE(2,*) MPF
C........GC(J)=GENERALIZED COORDINATE FOR MODE J
         GC(J)=MPF*SV(J)/BD(J)
      WRITE(2,*) 'GENERALIZED COORDINATE FOR MODE',J
      WRITE(2,*) GC(J)
         L1=J
         IF(NPRIN(1).EQ.0) L1=1
```

```
 C........R(I,L1)= DISPLACEMENT I FOR MODE J
          DO 40 I=1,NREM
             R(I,L1)=W(I,J)*GC(J)
   40     CONTINUE
       WRITE(2,*) 'REFERENCE POINT DISPLACEMENTS FOR MODE #',J
          WRITE(2,*) L1,(R(I,L1),I=1,NREM)
          IF((NPRI(2).EQ.0).AND.(NPRI(3).EQ.0).AND.(NPRI(4).EQ.0).AND.
      -(NPRI(5).EQ.0).AND.(NPRI(6).EQ.0)) GOTO 10
             REWIND 6
             READ(6,*) (S(I),I=1,IMAX)
 C...........RECOVER CORNER VERTICAL DISPLACEMENTS
             CALL GAUSS(S,R(1,L1),NK,NEQ,1,-NREM,DET,2)
       WRITE(2,*) 'REFERENCE POINT AND CORNER DISPLACEMENTS FOR MODE #',J
             WRITE(2,*) L1,(R(I,L1),I=1,NEQ)
          L4=J
          IF(NPRIN(4).EQ.0) L4=1
          IF((NPRI(4).EQ.0).AND.(NPRI(5).EQ.0).AND.
      -(NPRI(6).EQ.0)) GOTO 10
             MPFSVF=MPF*SV(J)*BD(J)
 C...........U(I,L1)= EQUIVALENT ELASTIC FORCE I FOR MODE J
             DO 50 I=1,NREM
                U(I,L4)=ERR(I)*MPFSVF
   50        CONTINUE
       WRITE(2,*) 'EQUIVALENT REFERENCE POINT ELASTIC FORCES '
             WRITE(2,*) (U(I,L4),I=1,NREM)
          IF((NPRI(5).EQ.0).AND.(NPRI(6).EQ.0)) GOTO 10
 C...........BASE SHEARS AND TORQUE
 C..FOR NSSYM=0
 C...........V(1,J)=MAX BASE SHEAR IN X DIRECTION FOR MODE J
 C...........V(2,J)=MAX BASE SHEAR IN Y DIRECTION FOR MODE J
 C...........V(3,J)=MAX BASE TORQUE FOR MODE J
 C...........V(4,J)=MAX COMBINED BASE SHEAR X,Y FOR MODE J
 C..FOR NSSYM=1  V(1,J)=BASE SHEAR IN X DIRECTION FOR MODE J
 C..FOR NSSYM=2  V(1,J)=BASE SHEAR IN Y DIRECTION FOR MODE J
 C..FOR NSSYM=3  V(1,J)=BASE TORQUE FOR MODE J
             DO 60 I=1,ND
                Z=0
                DO 70 K=1,NL
                   KK=(I-1)*NL+K
                   Z=Z+U(KK,J)
   70           CONTINUE
                V(I,J)=Z
   60        CONTINUE
       WRITE(2,*) 'SHEARS AND TORQUES FOR MODE #',J
             IF(NSSYM.NE.0) GOTO 80
                V(ND+1,J)=SQRT(V(1,J)*V(1,J)+V(2,J)*V(2,J))
                WRITE(2,*) V(ND+1,J)
   80        WRITE(2,*) (V(I,J),I=1,ND)
          IF(NPRI(6).EQ.0) GOTO 10
             IF(NSSYM.NE.3) GOTO 560
                WRITE(2,*) 0.0,0.0
                GOTO 10
 C...........X(I)=HEIGHT OF LEVEL I
  560        NB=1
             IF(NSSYM.EQ.0) NB=2
          X(1)=ET(ETN(1),2)
          K=1
```

```
                DO 100 I=2,NL
                  K=K+NHEL
                  X(I)=X(I-1)+ET(ETN(K),2)
  100           CONTINUE
             WRITE(2,*) 'HEIGHT OF EACH LEVEL FROM 1 TO NL'
             WRITE(2,*) (X(I),I=1,NL)
C..FOR NSSYM=0
C...........MOM(1,J*SE MOMENT ABOUT Y AXES FOR MODE J
C...........MOM(2,J)=MAX BASE MOMENT ABOUT X AXES FOR MODE J
C...........MOM(3,J)=MAXIMUM COMBINED BASE MOMENT X,Y FOR MODE J
C..FOR NSSYM=1  MOM(1,J)=MAX BASE MOMENT ABOUT Y AXES FOR MODE J
C..FOR NSSYM=2  MOM(1,J)=MAX BASE MOMENT ABOUT X AXES FOR MODE J
             WRITE(2,*) ' BASE MOMENTS FOR MODE #',J
                   DO 110 I=1,NB
                      Z=0
                      DO 120 K=1,NL
                         KK=(I-1)*NL+K
                         Z=Z+U(KK,J)*X(K)
  120                 CONTINUE
                      MOM(I,J)=Z
  110              CONTINUE
                   IF(NSSYM.NE.0) GOTO 130
                      MOM(NB+1,J)=SQRT(MOM(1,J)*MOM(1,J)+MOM(2,J)*MOM(2,J))
                      WRITE(2,*) MOM(NB+1,J)
  130              WRITE(2,*) (MOM(I,J),I=1,NB)
   10        CONTINUE
C
C
C.....THE REMAINDER OF THIS SUBROUTINE IS FOR SRSS CALCULATIONS
C
C
             WRITE(2,13)
             IF(NPRIN(1).EQ.0) GOTO 570
C........DISPLACEMENTS
                II=1
                JJ=NEQ
                IF(NPRIN(2).EQ.0) JJ=NREM
             GOTO 590
  570           IF(NPRIN(2).EQ.0) GOTO 580
                II=NREM+1
                JJ=NEQ
  590 DO 140 I=II,JJ
             WRITE(2,*) 'DISPLACEMENT',I,'FOR ALL MODES'
*  WRITE(2,*) I,(R(I,J),J=1,M)
           Z=0.
                DO 150 J=1,M
                   Z=Z+R(I,J)*R(I,J)
  150           CONTINUE
                R(I,M+1)=SQRT(Z)
  140 CONTINUE
             WRITE(2,*) 'SRSS OF DISPLACEMENTS'
             WRITE(2,*) (R(I,M+1),I=II,JJ)
  580 CONTINUE
             IF(NPRIN(4).EQ.0) GOTO 160
C........ELASTIC FORCES
                DO 170 I=1,NREM
                   Z=0.
```

```
            DO 180 J=1,M
               Z=Z+U(I,J)*U(I,J)
 180        CONTINUE
            U(I,1)=SQRT(Z)
 170     CONTINUE
      WRITE(2,*) 'SRSS OF ELASTIC FORCES'
         WRITE(2,*) (U(I,1),I=1,NREM)
 160  IF(NPRIN(5).EQ.0) GOTO 190
C........BASE SHEARS AND TORQUE
         II=ND
         IF(NSSYM.EQ.0) II=ND+1
         DO 200 I=1,II
            Z=0.
            DO 210 J=1,M
               Z=Z+V(I,J)*V(I,J)
 210        CONTINUE
            V(1,1)=SQRT(Z)
 200     CONTINUE
      WRITE(2,*) 'SRSS OF BASE SHEARS AND TORQUE'
         WRITE(2,*) (V(I,1),I=1,II)
 190  IF(NPRIN(6).EQ.0) GOTO 999
C........BASE MOMENTS
         II=NB
         IF(NSSYM.EQ.0) II=NB+1
         DO 220 I=1,II
            Z=0.
            DO 230 J=1,M
               Z=Z+MOM(I,J)*MOM(I,J)
 230        CONTINUE
            MOM(I,1)=SQRT(Z)
 220     CONTINUE
      WRITE(2,*) 'SRSS OF BASE MOMENTS'
      WRITE(2,*) (MOM(I,1),I=1,II)
C.....COMPUTE STRESSES FOR EACH MODE AND ALSO SRSS STRESSES
      IF(NPRI(3).EQ.1) CALL STRESS(NL,NF,NDOF,NSHAPE,NSSYM,NOUT,CA,SA
     1,ICOUNT,RFP,NPRIN(3),NPRINT,NEQ,R,M,1)
 999  RETURN
      END
```

```
      SUBROUTINE SIVIB2(G,GD,L,LD,BD,ERR,U,V,W,NK,N,M,NROD,NOI
     1,TOLVEC,IRENT,GES)
      REAL G(N),L(NK),BD(M),ERR(M),U(30,M),W(30,M),V(30,M)
      INTEGER GD(N),LD(N)
      WRITE(2,*) N,(GD(I),I=1,N),(G(I),I=1,N)
      WRITE(2,*) NK,(LD(I),I=1,N),(L(I),I=1,NK)
      WRITE(2,*) M,NROD,NOI,TOLVEC,IRENT,GES
      CALL REDUCE(L,LD,NK,N)
      IF(N.LT.0) GOTO 124
      LET=0
      INT=1
      LOCK=1
      IF(IRENT.EQ.2)CALL PREMULT(W,U,L,LD,NK,M,N,1,IRENT)
      CALL RANDON(U,N,M,IRENT,GES)
      CALL ORTHOG(U,N,M,1)
   50 CALL BACKSUB(U,V,L,LD,NK,N,M,LOCK)
      CALL PREMULT(V,W,G,GD,N,N,M,LOCK,0)
      CALL FORSUB(W,V,L,LD,NK,N,M,LOCK)
      CALL DECOUPL(U,V,W,BD,N,M,LOCK,TOLVEC)
      CALL RANDON(W,N,N,IRENT,GES)
      CALL ORTHOG(W,N,M,LOCK)
      CALL ERROR(U,W,BD,ERR,N,M,NROD,LOCK,TOLVEC,LET)
C     MONITOR PRINT INSTRUCTIONS
      WRITE(2,21) INT
   21 FORMAT(///,5X,'EIGENFACTORS AFTER ',I3,'  ITERATIONS')
      WRITE(2,22) (BD(I),I=1,M)
      WRITE(2,23)
   23 FORMAT(,5X,'VECTOR ERRORS ')
      WRITE(2,22) (ERR(I),I=1,M)
   22 FORMAT(5X,7F16.8)
      DO 20 I=1,N
      DO 20 J=LOCK,M
   20 U(I,J)=W(I,J)
      IF((LET-1).EQ.0) GOTO 10
      IF((NOI-INT).LT.0) GOTO 11
      INT=INT+1
      GOTO 50
C     MONITOR PRINT INSTRUCTIONS
   10 WRITE(2,100) INT
  100 FORMAT(/,5X,'CONVERGENCE OBTAINED',/,5X,'VECTOR TOLERANCE CRITERIA
     1 SATISFIED AFTER',I4,'   ITERATIONS')
      GOTO 120
   11 WRITE(2,101) INT
  101 FORMAT(/,5X,'CONVERGENCE NOT OBTAINED ',/,5X,'VECTOR TOLERANCE CRI
     1TERIA NOT SATISFIED AFTER',I4,'   ITERATIONS')
  120 CALL BACKSUB(U,W,L,LD,NK,N,M,1)
      WRITE(2,126)
  126 FORMAT(////,'MODE #',2X,'FREQUENCY (RAD/SEC)',35X,'MODE SHAPE',/,6
     -(1H*),2X,19(1H*),2X,100(1H*),//)
      DO 121 J=1,M
      EL=0.0
      DO 122 I=1,N
  122 IF(ABS(W(I,J)).GT.EL) EL=ABS(W(I,J))
      EL=1.0/EL
      DO 123 I=1,N
  123 W(I,J)=W(I,J)*EL
      BD(J)=1./SQRT(BD(J))
```

```
      WRITE(2,127) J,BD(J),(W(I,J),I=1,N)
  127 FORMAT(/,1X,I3,7X,F13.6,6(5X,F13.6),2(/,24X,6(5X,F13.6)))
  121 CONTINUE
  124 RETURN
      END
```

```
      SUBROUTINE RANDOM(U,N,M,IRENT,GES)
      REAL U(30,M)
      K=1
      IF(IRENT.NE.0) K=M
      IRENT=1
      DO 1 I=1,N
      DO 1 J=K,M
      IF(GES.EQ.0.0) GES=0.31415926
      Y=GES*GES
    2 Y=Y*10.0
      IF((Y-1.0).LT.0.0)  GOTO 2
      Y=Y-AINT(Y)
      U(I,J)=Y
    1 GES=Y
      RETURN
      END
```

SUBROUTINE ORTHOG FTN 4.8+552 83/03/21. 15.51.27 PAGE 1

```
      SUBROUTINE ORTHOG(W,N,M,LOCK)
      REAL W(30,M)
      INTEGER LOCK
      DO 1 I=LOCK,M
      DO 1 J=1,I
      EL=0.0
      DO 4 K=1,N
    4 EL=EL+W(K,J)*W(K,I)
      IF((I-J).EQ.0) GOTO 5
      DO 6 K=1,N
    6 W(K,I)=W(K,I)-EL*W(K,J)
      GOTO 1
    5 D=1.0/SQRT(EL)
      DO 3 K=1,N
      W(K,I)=D*W(K,I)
    3 CONTINUE
    1 CONTINUE
      RETURN
      END
```

```
SUBROUTINE DECOUPL   FTN 4.8+552   83/03/21. 15.51.27   PAGE   1

      SUBROUTINE DECOUPL(U,V,W,BD,N,M,LOCK,TOLVEC)
      REAL U(30,M),V(30,M),W(30,M),BD(M)
C     THIS SECTION CALCULATES THE EIGENVALUES
      DO 1 J=LOCK,M
      EL=0.0
      DO 2 I=1,N
    2 EL=EL+U(I,J)*V(I,J)
    1 BD(J)=EL
C     THIS SECTION TESTS THAT THE EIGENVALUES ARE
C     IN DESCENDING ORDER AND IF NOT SORTS THEM
      JJ=LOCK+1
      DO 3 J=JJ,M
      I=J
    4 K=I-1
      IF((ABS(BD(K))-ABS(BD(I))).GT.0.0) GOTO 3
      EL=BD(I)
      BD(I)=BD(K)
      BD(K)=EL
      DO 5 II=1,N
      EL=U(II,I)
      EL1=V(II,I)
      U(II,I)=U(II,K)
      U(II,K)=EL
      V(II,I)=V(II,K)
      V(II,K)=EL1
    5 CONTINUE
      I=I-1
      IF((I-1-LOCK).GT.0) GOTO 4
    3 CONTINUE
C     THIS SECTION CALCULATES THE INTERACTION
C     BETWEEN THE ITERATION VECTORS AND DECOUPLES THEM
      DO 6 I=1,N
      DO 6 J=LOCK,M
    6 W(I,J)=V(I,J)
      DO 7 I=LOCK,M
      IF((I-1).EQ.0) GOTO 7
      II=I-1
      DO 8 J=LOCK,II
      EL=0.0
      DO 9 K=1,N
    9 EL=EL+U(K,I)*V(K,J)
      EL=-2.0*EL
      Q=BD(I)-BD(J)
      IF((ABS(Q/BD(J))-TOLVEC).GT.0.0) GOTO 10
      IF((ABS(EL/BD(J))-TOLVEC).GT.0.0) GOTO 10
      EL=0.0
      GOTO 8
   10 ELL=SQRT(Q*Q+4.0*EL*EL)
      IF(Q.LT.0.0) ELL=-ELL
      EL=EL/(Q+ELL)
      DO 13 K=1,N
      W(K,I)=W(K,I)-EL*V(K,J)
   13 W(K,J)=W(K,J)+EL*V(K,I)
    8 CONTINUE
    7 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ERROR(U,W,BD,ERR,N,M,NRQD,LOCK,TOLVEC,LET)
      REAL ERR(M),BD(M),U(30,M),W(30,M)
      DO 2 J=LOCK,M
      ER=0.0
      IF(BD(J).LE.0.0) GOTO 6
      DO 1 I=1,N
      EL=U(I,J)-W(I,J)
    1 ER=ER+EL*EL
      GOTO 2
    6 DO 8 I=1,N
      EL=U(I,J)+W(I,J)
    8 ER=ER+EL*EL
    2 ERR(J)=SQRT(ER/N)
    5 IF((TOLVEC-ERR(LOCK)).LT.0.0) GOTO 4
      ERR(LOCK)=-ERR(LOCK)
      LOCK=LOCK+1
      IF((NRQD-LOCK).GE.0) GOTO 5
      LET=1
    4 RETURN
      END
```

```
      SUBROUTINE REDUCE(L,LD,NK,N)
      REAL L(NK)
      INTEGER LD(N),Q
      XCM=1.0
      L(1)=SQRT(L(1))
      DO 1 I=2,N
      Q=LD(I)-I
      M1=1+LD(I-1)-Q
      DO 2 J=M1,I
      NN=0
      EL=L(J+Q)
         X).EQ.0) GOTO 2
   KK=LD(J)-J
   NN=1+LD(J-1)-KK
      NN=MAX0(NN,M1)
      IF((NN-J).EQ.0) GOTO 2
      JJ=J-1
      DO 3 K=NN,JJ
    3 EL=EL-L(Q+K)*L(KK+K)
    2 L(J+Q)=EL/L(LD(J))
      IF(EL.LE.0.0) GOTO 5
      XC=1.0/L(I+Q)
      IF((XC-XCM).LT.0.0) GOTO 1
      XCM=XC
      JCM=I
    1 L(LD(I))=SQRT(EL)
C     MONITOR PRINTING
      ICM=INT(ALOG10(XCM))+1
      WRITE(2,10) ICM,JCM
   10 FORMAT(///,10X,'ACCURACY LOSS =',I3,'  DECIMAL PLACES AT PIVOT NUM
     1BER',I3)
      GOTO 6
    5 WRITE(2,11) I
   11 FORMAT(/,20X,' REDUCTION FAILURE :    PIVOT ',I4,'  NOT POSITIVE')
      N=-N
    6 RETURN
      END
```

```
SUBROUTINE FORSUB    FTN 4.8+552   83/03/21. 15.51.27    PAGE    1

      SUBROUTINE FORSUB(U,V,L,LD,NK,N,M,LOCK)
      REAL U(30,M),V(30,M),L(NK)
      INTEGER LD(N),Q
      DO 12 K=LOCK,M
   12 V(1,K)=U(1,K)/L(1)
      DO 13 I=2,N
      Q=LD(I)-I
      M1=1+LD(I-1)-Q
      DO 13 K=LOCK,M
      EL=0.0
      IF((M1-I).EQ.0) GOTO 13
      II=I-1
      DO 14 J=M1,II
   14 EL=EL+L(J+Q)*V(J,K)
   13 V(I,K)=(U(I,K)-EL)/L(LD(I))
      RETURN
      END
```

```
      SUBROUTINE PREMULT(U,V,L,LD,NA,N,M,LOCK,IRENT)
      REAL L(NA),U(30,M),V(30,M),LK
      INTEGER Q,LD(N)
      DO 1 I=1,N
      LK=L(LD(I))
      DO 1 K=LOCK,M
    1 V(I,K)=U(I,K)*LK
      DO 4 I=2,N
      Q=LD(I)-I
      M1=1+LD(I-1)-Q
      DO 7 K=LOCK,M
      IF((M1-I).EQ.0) GOTO 7
      IF((IRENT-2).EQ.0) GOTO 6
      II=I-1
      DO 5 J=M1,II
    5 V(I,K)=V(I,K)+L(J+Q)*U(J,K)
    6 CONTINUE
      DO 8 J=M1,II
      V(J,K)=V(J,K)+L(J+Q)*U(I,K)
    8 CONTINUE
    7 CONTINUE
    4 CONTINUE
      RETURN
      END
```

SUBROUTINE BACKSUB FTN 4.8+552 83/03/21. 15.51.27 PAGE 1

```
      SUBROUTINE BACKSUB(U,V,L,LD,NK,N,M,LOCK)
      REAL L(NK),U(30,M),V(30,M)
      INTEGER LD(30),Q
      DO 20 K=LOCK,M
      DO 20 I=1,N
   20 V(I,K)=U(I,K)/L(LD(I))
      DO 21 IT=2,N
      I=N+2-IT
      Q=LD(I)-I
      M1=1+LD(I-1)-Q
      IF((M1-I).EQ.0) GOTO 21
      II=I-1
      DO 22 J=M1,II
      DO 22 K=LOCK,M
      V(J,K)=V(J,K)-L(J+Q)*V(I,K)/L(LD(J))
   22 CONTINUE
   21 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STRESS(NL,NF,NDOF,NSHAPE,NSSYM,NOUT,CA,SA,ICOUNT,RFP
     1,NPRIN3,NPRINT,NEQ,R,NMOD,NDYN)
      COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
     1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),LEL(9),
     2ET(36,2),ETM(12),ETN(90),QK(9,9) ,IEW(12),IEH(12)
      DIMENSION BB(3,9),C(3,3),BBC(3,9),Q(9),SIG(3),DR(210),CA(4)
     1,SA(4),R(80,NMOD),AK(6105),RFP(4,10),NDISP(90,9),NP(90),SIGS(3)
      INTEGER FP,EC,EL,ETM,ETN,ICOUNT(4),NPRINT(10)
C        ***************************************************************
C
C                THIS SUBROUTINE EVALUATES STRESSES
C                IN EACH ELEMENT IN EACH FACADE
C
C
C        ***************************************************************
C
C
      LIM=6
      IF(NDOF.EQ.1) LIM=9
      IF(NDYN.EQ.0) NMOD1=0
      REWIND 3
      REWIND 4
      REWIND 5
      REWIND 7
      DO 9999 N=1,NF
         READ(3,*) NSYM,NEQF,LMAX,NEL,NMAT,NTYPE,NR
         IF(NDYN.EQ.0) GOTO 210
            DO 220 NN=1,NEL
               READ(7,*) (NDISP(NN,I),I=1,LIM),NP(NN)
 220        CONTINUE
            NMOD1=NMOD+1
            NT=1
 210     READ(4,*) (AK(L),L=1,LMAX)
         READ(5,*) (ETN(L),L=1,NEL)
         READ(5,*) (EX(L),EY(L),GXY(L),TH(L),L=1,NMAT)
         READ(5,*) (ETM(L),IEW(L),IEH(L),(ET(L,J),J=1,2),L=1,NTYPE)
         DO 230 NN=1,NMOD1
            IF(NN.EQ.NMOD1) GOTO 16
            WRITE(2,13) N,NN
            GOTO 17
  16        WRITE(2,14) N
  13        FORMAT(T1,20X,' FACADE #',I2,10X,'MODE #',I2)
  14        FORMAT(T1,20X,'SRSS  FOR FACADE #',I2)
  17        IF(NDYN.EQ.0) GOTO 420
            NT=-NT
            NTAPE1=11
            NTAPE2=10
            IF(NT.EQ.1) GOTO 415
            NTAPE1=10
            NTAPE2=11
 415        REWIND NTAPE1
            IF(NN.EQ.NMOD1) GOTO 223
            REWIND NTAPE2
            JJJ=NN
            GOTO 440
C.....TRANSFORM THE GLOBAL REFERENCE POINT DISPLACEMENTS TO LOCAL ORIENTATION.
C      NOTE THAT CORNER VERTICAL DISPLACEMENTS ARE IN GLOBAL AS WELL
C      AS LOCAL ORIENTATION.
```

```
  420         JJJ=1
  440      IF(NSSYM.NE.0) GOTO 5
              IP=3*NL+1
              IF(NDYN.EQ.1) GOTO 240
              DO 10 L=1,NL
                 J=L+NL
                 K=J+NL
                 DR(L)=CA(N)*R(L,1)+SA(N)*R(J,1)+RFP(N,1)*R(K,1)
   10         CONTINUE
              GOTO 250
  240         DO 270 L=1,NL
                 J=L+NL
                 K=J+NL
                 DR(L)=CA(N)*R(L,NN)+SA(N)*R(J,NN)+RFP(N,L)*R(K,NN)
  270         CONTINUE
  250         DO 15 L=1,2
                 LN=L*NL
                 LL=N+L+1
                 IF((N.EQ.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.2)) LL=3
                 LLN=LL*NL
                 DO 20 J=1,NL
                    DR(LN+J)=R(LLN+J,NN)
   20            CONTINUE
   15         CONTINUE
              GOTO 25
C
    5      IF(NSYM.NE.1) GOTO 30
              IP=2*NL+1
              DO 75 L=1,2
                 LN=(L-1)*NL
                 LL=ICOUNT(N)+L-1
                    *Q.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.2)) LL=1
                 LLN=LL*NL
                 DO 80 J=1,NL
                    DR(LN+J)=R(LLN+J,JJJ)
   80            CONTINUE
   75         CONTINUE
              GOTO 25
   30      IF(NSSYM.EQ.1) V=CA(N)
           IF(NSSYM.EQ.2) V=SA(N)
           IF(NSSYM.EQ.3) V=RFP(N,1)
           IF(NSYM.NE.0) GOTO 35
              IP=3*NL+1
              IA=2
              IB=2
              GOTO 50
   35      IF(NSYM.NE.2) GOTO 40
              IP=2*NL+1
              IA=1
              IB=2
              GOTO 50
 40           IP=2*NL+1
              IA=1
              IB=1
   50      DO 55  L=1,NL
              DR(L)=V*R(L,JJJ)
   55      CONTINUE
```

```
            DO 65 L=1,IA
               LN=L*NL
               LL=ICOUNT(N)+L-1
               IF((N.EQ.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.IB))  LL=1
               LLN=LL*NL
               DO 70 J=1,NL
                  DR(LN+J)=R(LLN+J,JJJ)
   70          CONTINUE
   65       CONTINUE
   25       CONTINUE
            DO 155 L=IP,NEQF
               DR(L)=0.
  155       CONTINUE
C.....RETRIEVE REDUCED FACADE DISPLACEMENTS
            CALL GAUSS(AK,DR,LMAX,NEQF,1,-NR,DET,2)
         WRITE(2,*) 'ALL FACADE DISPLACEMENTS FOR MODE #',NN
         WRITE(2,*) (DR(I),I=1,NEQF)
C
  223 WRITE(2,*) '                              STRESSES'
         DO 777 M=1,NEL
C
C ELEMENT PROPERTIES
C
         IT=ETN(M)
         A=ET(IT,1)
         D=ET(IT,2)
C NOTE.... D=B=HEIGHT OF ELEMENT
         IM=ETM(IT)
         E1=EX(IM)
         E2=EY(IM)
         G12=GXY(IM)
         TT=TH(IM)
         NBAY=IEW(IT)
         NSTOR=IEH(IT)
         BW=A/NBAY
         SH=D/NSTOR
C
         IF(NDOF.NE.0) GO TO 140
         IF(NDYN.EQ.1) GOTO 290
         READ(7,*) EL,NPR
         IPRNT=NPRINT(NPR)
         GOTO 310
  290 IPRNT=NPRINT(NP(M))
  310 IF(IPRNT.EQ.0) GO TO 777
         ISTRS=1
         JSTRS=1
         RB=1./NBAY
         RS=1./NSTOR
C INITIALIZE STRAIN -DISP. MATRIX  BB 3X6
C
         DO 90 I=1,3
         DO 90 J=1,6
         BB(I,J)=0.
   90 CONTINUE
C
C C EXTRACT EACH ELEMENT NODAL DISP.
C
```

```
      DO 95 I=1,LIM
      Q(I)=0.
      J=EL(I)
      IF(NDYN.EQ.1) J=NDISP(M,I)
      IF(J.EQ.0) GOTO 95
      Q(I)=Q(I)+DR(J)
   95 CONTINUE
C
      GO TO 71
  140 CONTINUE
      IF(NDYN.EQ.1) GOTO 320
      READ(7,*) LEL,NPR
      IPRNT=NPRINT(NPR)
      GOTO 330
  320 IPRNT=NPRINT(NP(M))
  330 IF(IPRNT.EQ.0) GO TO 777
      ISTRS=1
      JSTRS=2
      D=D/2.
      A=A/2.
      RB=2./NBAY
      RS=2./NSTOR
C
C INITIALIZE STRAIN - DISPL. MATRIX BB 3X9
C
      DO 72 I=1,3
      DO 72 J=1,9
      BB(I,J)=0.0
   72 CONTINUE
C
C EXTRACT EACH ELEMENT NODAL DISPLACEMENTS
      DO 105 I=1,LIM
      Q(I)=0.0
      J=LEL(I)
      IF(NDYN.EQ.1) J=NDISP(M,I)
      IF(J.EQ.0) GOTO 105
      Q(I)=Q(I)+DR(J)
  105 CONTINUE
   71 CONTINUE
C
C  F*-STRAIN MATRIX C 3X3
C
      DO 110 I=1,3
      DO 110 J=1,3
      C(I,J)=0.
  110 CONTINUE
      C(1,1)=E1
      C(2,2)=E2
      C(3,3)=G12
C
      IF(NOUT.NE.0) GO TO 115
      ISTRS=2*NSTOR
  115 CONTINUE
      DO 888 II=1,ISTRS
      IF(NOUT.EQ.2) GO TO 88
      ISIGN=(-1)**II
      IF(ISIGN.LT.1) GO TO 81
```

```
      JSTRS=NBAY
      IZ=1
      GO TO 88
   81 CONTINUE
      JSTRS=NBAY+1
      IZ=0
   88 CONTINUE
      DO 888 JJ=1,JSTRS
      IF((NN.EQ.NMOD1).AND.(NDYN.EQ.1)) GOTO 430
      E=.5
      IF(NDOF.NE.0) GO TO 73
      IF(NOUT.EQ.0) GO TO 82
      GO TO 83
   82 CONTINUE
      E=II*RS*0.5
   83 CONTINUE
C
C FORM THE BB MATRIX
C
      IF(NOUT.EQ.2) GO TO 120
      Z=(JJ-1)*RB+IZ*RB*0.5
      GO TO 66
  120 CONTINUE
      Z=.5
   66 CONTINUE
      BB(2,3)=-(1.-Z)/D
      BB(2,4)=-Z/D
      BB(2,5)=Z/D
      BB(2,6)=(1.-Z)/D
C
      BB(3,1)=-1/D
      BB(3,2)=1/D
      BB(3,3)=-(1-E)/A
      BB(3,4)=(1.-E)/A
      BB(3,5)=E/A
      BB(3,6)=-E/A
C
      GO TO 74
   73 CONTINUE
      IF(NOUT.EQ.0) GO TO 84
      GO TO 85
   84 CONTINUE
      E=II*RS*0.5-1.0
   85 CONTINUE
      E=0.
C
C FORM THE BB MATRIX
C
C
C NOTE THAT BOTH D AND A REPRESENT ONLY ONE HALF THE
C HEIGHT AND THE LENGTH OF THE ELEMENT RESPECTIVELY
C
      IF(NOUT.EQ.2) GO TO 76
      Z=(JJ-1)*RB+IZ*RB*0.5-1.0
      GO TO 77
   76 CONTINUE
      Z=JJ*1.-1.5
```

```
   77 CONTINUE
      BB(2,4)=Z*(1.-Z)/(4.*D)
      BB(2,5)=-(1.-Z*Z)/(2.*D)
      BB(2,6)=-(1.+Z)*Z/(4.*D)
      BB(2,7)=-BB(2,6)
      BB(2,8)=-BB(2,5)
      BB(2,9)=-BB(2,4)
C
      BB(3,1)=-(1.-2.*E)/(2.*D)
      BB(3,2)=-2.*E/D
      BB(3,3)=(1.+2.*E)/(2.*D)
      BB(3,4)=-(1.-2.*Z-E+2.*Z*E)/(4.*A)
      BB(3,5)=-Z*(1.-E)/A
      BB(3,6)=(1.+2.*Z-E-2.*Z*E)/(4.*A)
      BB(3,7)=(1.+2.*Z+E+2.*Z*E)/(4.*A)
      BB(3,8)=-Z*(1.+E)/A
      BB(3,9)=-(1.-2.*Z+E-2.*Z*E)/(4.*A)
   74 CONTINUE
C
C  MULTIPLY C TIMES BB AND STORE IN BBC
C
      DO 125 I=1,3
      DO 125 J=1,LIM
      BBC(I,J)=0.
      DO 125 L=1,3
      BBC(I,J)=BBC(I,J)+C(I,L)*BB(L,J)
  125 CONTINUE
C
C  CALCULATE STRESSES, SIG=(BBC)X((Q)
C
      DO 130 I=1,3
      SIG(I)=0.
      DO 130 J=1,LIM
      SIG(I)=SIG(I)+BBC(I,J)*Q(J)
  130 CONTINUE
C
      IF(NDYN.EQ.0) GOTO 470
      WRITE(2,1010) M,(SIG(I),I=1,3)
      IF(NPRIN3.EQ.0) GOTO 888
      IF(NN.EQ.1) GOTO 490
      DO 450 I=1,3
         READ(NTAPE1,*) SIGS(I)
         SIGS(I)=SIGS(I)+SIG(I)*SIG(I)
         WRITE(NTAPE2,*) SIGS(I)
  450 CONTINUE
      GOTO 888
  490 DO 480 I=1,3
         SIGS(I)=SIG(I)*SIG(I)
         WRITE(NTAPE2,*) SIGS(I)
  480 CONTINUE
      GOTO 888
  430 DO 460 I=1,3
         READ(NTAPE1,*) SIG(I)
         SIG(I)=SQRT(SIG(I))
         WRITE(NTAPE2,*) SIG(I)
  460 CONTINUE
C
```

```
  470 IF(NOUT.NE.0) GOTO 135
      WRITE(2,*) 'FORCES OR SRSS FORCES GOTTEN FROM SRSS OF STRESSES'
      CALL FORCE(SIG,II,JJ,N,JSTRS,TT,BW,SH,IZ)
      GOTO 888
  135 CONTINUE
      WRITE(2,1010) N,(SIG(I),I=1,3)
  888 CONTINUE
  777 CONTINUE
  230 CONTINUE
 9999 CONTINUE
C
  100 FORMAT(47H OUTPUT TABLE 2.. STRESSES AT ELEMENT CENTROIDS )
  200 FORMAT(/// 20X,15HFACADE NUMBER  ,I5///
     1          3X,7HELEMENT,7X,8HSIGMA(X),7X,8HSIGMA(Y),
     2            7X,8HTAU(X,Y))
  500 FORMAT(53H OUTPUT TABLE..2 INTERNAL FORCES IN COLUMNS AND BEAMS )
  600 FORMAT(//34X,*FACADE NUMBER   *,I5///
     1 *    ELEMENT NO.     COL. AXIAL F.     COL. SHEAR     COL. MOMENT
     2  BEAM SHEAR    BEAM MOMENT */)
 1000 FORMAT(1H1,    7HELEMENT,7X,8HSIGMA(X),7X,8HSIGMA(Y),
     1          7X,8HTAU(X,Y))
 1010 FORMAT(I10,1P3E15.4)
C
      RETURN
      END
```

```
      SUBROUTINE FORCE(SIG,II,JJ,M,JSTRS,TT,BW,SH,IZ)
C
C        ******************************************************************
C        *                                                                *
C        *    THIS SUBROUTINE EVALUATES INTERNAL FORCES IN                *
C        *    BEAMS AND COLUMNS                                           *
C        *                                                                *
C        ******************************************************************
C
C
      DIMENSION SIG(3)
C
C
      IF(IZ.EQ.0) GO TO 10
C
C EVALUATE BEAM INTERNAL FORCES
C
      SF=SIG(3)*TT*SH
      BM=SF*BW/2.
      WRITE(2,100)  M,II,JJ,SF,BM
      GO TO 999
   10 CONTINUE
C
C EVALUATE COLUMN INTERNAL FORCES
C
      IF((JJ.EQ.1).OR.(JJ.EQ.JSTRS))  GO TO 20
      AF=SIG(2) *TT*BW
      SF=SIG(3)*TT*BW
      GO TO 30
   20 CONTINUE
      AF=SIG(2)*TT*BW/2.
      SF=SIG(3)*TT*BW/2.
   30 CONTINUE
      BM=SF*SH/2.
      WRITE(2,200) M,II,JJ,AF,SF,BM
  999 CONTINUE
  100 FORMAT( 3I5  ,47X,2(4X,E10.3))
  200 FORMAT( 3I5  ,4X,E13.3,2(4X,E10.3))
      RETURN
      END
```

```
      SUBROUTINE STAT(S,R,NEQ,IMAX,NSSYM,NL)
      DIMENSION S(IMAX),R(NEQ)
C.....OBTAIN GLOBAL STATIC DISPLACEMENTS
      CALL GAUSS(S,R,IMAX,NEQ,1,1,DET,1)
      WRITE(2,*) 'STRUCTURAL DISPLACEMENT VECTOR IS'
      WRITE(2,*) (R(II),II=1,NEQ)
      ND=NEQ/NL
      IF(NSSYM.NE.0) GOTO 55
        WRITE(2,300)
        DO 50 I=1,NL
          L=(ND-1)*NL+I
          WRITE(2,400) I,(R(J),J=I,L,NL)
   50   CONTINUE
        GOTO 444
   55 IF(NSSYM.NE.1) GOTO 60
        WRITE(2,600)
        GOTO 75
   60 IF(NSSYM.NE.2) GOTO65
        WRITE(2,700)
        GOTO 75
   65 IF(NSSYM.NE.3) GOTO 70
        WRITE(2,800)
        GOTO 75
   70 WRITE(2,*) '********** ERROR IN INPUT DATA **********'
      WRITE(2,*) '      STRUC SYMETRY WAS NOT SPECIFIED AS 0,1,2,OR 3'
      GOTO 999
C
   75   DO 80 I=1,NL
          L=(ND-1)*NL+I
          WRITE(2,900) I,(R(J),J=I,L,NL)
   80   CONTINUE
  444 CONTINUE
C
  300 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
     1       2X,5HLEVEL,8X,7HX-DISP.,8X,7HY-DISP.,8X,7H ROT-O ,16X,
     2      25HVERTICAL DISP. OF CORNERS/ )
  400 FORMAT(I7,3E15*5(E10.3,3X))
  600 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
     1           *  LEVEL NO.   X-DISPL.        DISPLACEMENT OF
     1 CORNERS  *//)
  700 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
     1           *  LEVEL NO.   Y-DISPL.        DISPLACEMENT OF
     1 CORNERS  *//)
  800 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
     1           *   LEVEL NO.   ROTATION        DISPLACEMENT OF
     1 CORNERS  *//)
  900 FORMAT(I10,4X,E10.3,5(4X,E10.3))
  999 RETURN
      END
```