Dynamic Analysis Of Tall Planar And
Tube-Type Building Structures

Antonio Mule

A Thesis
in the
Center for
Building Studies,
Faculty of Engineering

Presented in partial fulfillment of the requirements for
the degree of Master of Engineering
at
Concordia University
Montreal, Quebec, Canada

April 1983

© Antonio Mule, 1983
ABSTRACT

DYNAMIC ANALYSIS OF TALL PLANAR
AND TUBE-TYPE BUILDING STRUCTURES

Antonino Mulè

A computer program is developed for linear dynamic analysis of planar and tube-type building structures using the macro-element technique. In this technique, the actual discrete frame is replaced by an elastically equivalent orthotropic membrane. The equivalent structure is subsequently analyzed by using the finite element method. Seismic response is obtained through spectrum analysis in conjunction with the lumped mass formulation. The seismic responses obtained from an analysis may include: natural frequencies and mode shapes, displacements, story shear forces, overturning base moments, and internal forces.

The final structural responses are determined by the square-root-sum-of-squares method. Several two- and three-dimensional structures consisting of frames and walls have been analyzed, and the results are in excellent agreement with published data.

It has been found that in framed-tube structures, shear lag behaviour is present, and that flexibility of
finite size, joints significantly affect the seismic behaviour. Shear deformations in beams and columns have less influence.

The computer program was used to generate data which are presented in the form of graphs and simplified equations for rapid estimation of the natural frequencies of planar and tube-type structures. Recommendations are also made for the preliminary estimates of natural frequencies of framed-tube buildings.
ACKNOWLEDGEMENTS

I would like to express my deep gratitude to my supervisor, Dr. H.K. Ha, for his continuous encouragement, guidance, suggestions, and critical feedback during the course of this research.

Special thanks goes to Mr. Luong Pham for his excellent technical and programming advice during the development of TUBE. Mr. Ian MacIntosh's assistance with the MEGATECH plotter for the production of graphs is gratefully acknowledged.

The miscellaneous typing services of Mrs. Anna Circelli and Miss Maria Sciascia will not go unrewarded. Additional thanks are due to all my professors, relatives, friends, and colleagues who encouraged me throughout this study.

The financial support of the Natural Science and Engineering Research Council of Canada in the form of a postgraduate scholarship is very much appreciated.

With much respect, love, and admiration, I sincerely thank my dear parents for their encouragement and moral support.
# Table of Contents

## Abstract ................................................................. i

## Acknowledgements ................................................... iii

## List of Figures ....................................................... vii

## List of Tables ....................................................... ix

## Notations ................................................................... x

## I Introduction

1.1 Planar and Tube-Type Structures .................. 1
1.2 Framed-Tube Building ................................. 1
1.3 Scope and Objectives ................................. 3
1.4 General Assumptions ................................. 4
1.5 Organization of the Thesis ......................... 5

## II Equivalent Orthotropic Membrane

2.1 Introduction ....................................................... 10
2.2 Orthotropic Membrane for Coupling Beams ...... 10
2.3 Orthotropic Membrane for Planar Gridwork Systems 12

2.3.1 - Moduli of Elasticity $E_x$ and $E_y$ ....... 13
2.3.2 Shear Modulus $G_{xy}$ ............................ 14

## III Specially Orthotropic Finite Elements for Tall Building Analysis

3.1 Introduction ....................................................... 20
3.2 Formulation of the Ordinary Element .......... 21
3.3 Formulation of the Refined Element .......... 24
3.4 Determination of Forces in Beams and Columns 28

## IV Static and Earthquake Analysis of Planar and Tube-Type Structures

4.1 Introduction ....................................................... 33
4.2 Scope ............................................................... 34
4.3 Analysis Procedure .......................................... 35

4.3.1 Modelling of Structures ......................... 35
4.3.2 Static Analysis Procedure ...................... 39
4.3.3 Earthquake Spectrum Analysis Procedure .... 40
4.4 Assembly of Facade Stiffness Matrix in Local Axes .............. 42
4.5 Condensation of Facade Internal Degrees of Freedom .......... 44
4.6 Assembly of the Global Static Structure Stiffness Matrix ....... 45
4.7 Transformation Matrices for Different Structure Symmetry Conditions .................................. 52
4.8 Static Analysis ........................................................................... 55
4.8.1 Solution for Displacements .................................................... 55
4.8.2 Determination of Stresses ....................................................... 56
4.9 Earthquake Analysis ................................................................. 56
4.9.1 Global Dynamic Structure Stiffness Matrix .................. 56
4.9.2 Diagonal Lumped-Mass Matrix ........................................... 57
4.9.3 Dynamic Equilibrium Equations ......................................... 60
4.9.4 Mode Shapes and Natural Frequencies ......................... 66
4.9.5 Uncoupled Equations of Motion ......................................... 69
4.9.6 Response Spectrum ............................................................. 73
4.9.7 Structural Modal Responses to Earthquake Ground Motions ... 75
4.9.8 Final Structural Response to Earthquake Ground Motions ... 79

V EXAMPLES OF TUBE PROGRAM APPLICATION

5.1 Introduction .............................................................................. 88
5.2 Box Cantilever Beam ................................................................. 89
5.3 Concrete Wall-Frame Building ............................................. 93
5.4 Planar Frame ............................................................................ 97
5.5 Framed-Tube Building ............................................................... 99
5.6 Discussion .................................................................................. 103

VI BEHAVIOURAL CHARACTERISTICS OF TALL FRAMED-TUBE BUILDING STRUCTURES

6.1 Introduction .............................................................................. 114
6.2 Shear Lag Phenomenon ........................................................... 114
6.3 Importance of Shear Deformations, Flexible Finite Size Joints, and Response Spectrum Considerations .................. 117
6.4 Discussion .................................................................................. 120
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>DESCRIPTION</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1a</td>
<td>Framed tube</td>
<td>8</td>
</tr>
<tr>
<td>1.1b</td>
<td>De Witt Chestnut apartment building (2)</td>
<td>8</td>
</tr>
<tr>
<td>1.2</td>
<td>Shear lag in framed tube</td>
<td>9</td>
</tr>
<tr>
<td>2.1</td>
<td>Present modelling of coupling beams</td>
<td>17</td>
</tr>
<tr>
<td>2.2</td>
<td>Model for evaluating the equivalent membrane elastic modulus $E_y$</td>
<td>18</td>
</tr>
<tr>
<td>2.3</td>
<td>Model for evaluating the equivalent membrane shear modulus $G_{xy}$</td>
<td>19</td>
</tr>
<tr>
<td>3.1</td>
<td>The ordinary element</td>
<td>30</td>
</tr>
<tr>
<td>3.2</td>
<td>The refined element</td>
<td>31</td>
</tr>
<tr>
<td>3.3</td>
<td>Definitions of symbols in equations 3.23 a, b, c</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Structure idealization</td>
<td>82</td>
</tr>
<tr>
<td>4.2</td>
<td>Automated numbering system of nodes and elements</td>
<td>83</td>
</tr>
<tr>
<td>4.3</td>
<td>Global static degrees of freedom at the ith level</td>
<td>84</td>
</tr>
<tr>
<td>4.4</td>
<td>Global dynamic degrees of freedom at level $i$ and the direction of ground motion</td>
<td>85</td>
</tr>
<tr>
<td>4.5</td>
<td>Automated facade degrees of freedom system</td>
<td>86</td>
</tr>
<tr>
<td>4.6</td>
<td>Response spectra for El Centro earthquake, 1940. Ref. (21)</td>
<td>87</td>
</tr>
<tr>
<td>4.7</td>
<td>Peak ground motion bounds and elastic average response spectrum for $1.0g_{max.}$ ground accel. (22)</td>
<td>87</td>
</tr>
<tr>
<td>5.1</td>
<td>Box-cantilever beam and present model</td>
<td>109</td>
</tr>
<tr>
<td>5.2</td>
<td>Superelement model of box cantilever beam</td>
<td>110</td>
</tr>
<tr>
<td>5.3</td>
<td>Wall-frame building and the present idealization</td>
<td>111</td>
</tr>
<tr>
<td>5.4</td>
<td>Planar frame, properties, and model</td>
<td>112</td>
</tr>
<tr>
<td>5.5</td>
<td>Framed-tube building, properties, and idealization</td>
<td>113</td>
</tr>
<tr>
<td>FIGURE</td>
<td>DESCRIPTION</td>
<td>PAGE</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>6.1</td>
<td>Variation of normal stress at 3' elevation of the bow cantilever beam</td>
<td>124</td>
</tr>
<tr>
<td>6.2</td>
<td>Effect of the shear lag parameter (SL) on the column axial stresses at the fourth storey of the framed-tube building</td>
<td>125</td>
</tr>
<tr>
<td>6.3</td>
<td>Effect of the shear lag parameter (SL) on the natural frequencies of the framed-tube building</td>
<td>126</td>
</tr>
<tr>
<td>6.4</td>
<td>Column axial forces at the second storey of the framed-tube building</td>
<td>127</td>
</tr>
<tr>
<td>7.1</td>
<td>Basic natural frequency for tube-type structure</td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>mode 1, H/A = 1</td>
<td>155</td>
</tr>
<tr>
<td>b</td>
<td>mode 1, H/A = 2</td>
<td>156</td>
</tr>
<tr>
<td>c</td>
<td>mode 1, H/A = 3</td>
<td>157</td>
</tr>
<tr>
<td>d</td>
<td>mode 1, H/A = 4</td>
<td>158</td>
</tr>
<tr>
<td>e</td>
<td>mode 1, H/A = 6</td>
<td>159</td>
</tr>
<tr>
<td>f</td>
<td>mode 1, H/A = 8</td>
<td>160</td>
</tr>
<tr>
<td>g</td>
<td>mode 1, H/A = 10</td>
<td>161</td>
</tr>
<tr>
<td>h</td>
<td>mode 2, H/A = 1</td>
<td>162</td>
</tr>
<tr>
<td>i</td>
<td>mode 2, H/A = 2</td>
<td>163</td>
</tr>
<tr>
<td>j</td>
<td>mode 2, H/A = 3</td>
<td>164</td>
</tr>
<tr>
<td>k</td>
<td>mode 2, H/A = 4</td>
<td>165</td>
</tr>
<tr>
<td>m</td>
<td>mode 2, H/A = 6</td>
<td>166</td>
</tr>
<tr>
<td>n</td>
<td>mode 2, H/A = 8</td>
<td>167</td>
</tr>
<tr>
<td>p</td>
<td>mode 2, H/A = 10</td>
<td>168</td>
</tr>
<tr>
<td>q</td>
<td>Basic natural frequency for planar structure</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>mode 1</td>
<td>169</td>
</tr>
<tr>
<td>A.1</td>
<td>Definition of NOUT for IEW = 4 and IEH = 3</td>
<td>198</td>
</tr>
<tr>
<td>TABLE</td>
<td>DESCRIPTION</td>
<td>PAGE</td>
</tr>
<tr>
<td>-------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>4.1</td>
<td>Ductility factor (22)</td>
<td>81</td>
</tr>
<tr>
<td>5.1</td>
<td>Natural frequencies of box cantilever beam</td>
<td>105</td>
</tr>
<tr>
<td>5.2</td>
<td>Natural frequencies (rad/sec) of wall-frame building</td>
<td>106</td>
</tr>
<tr>
<td>5.3</td>
<td>Natural frequencies of planar frame</td>
<td>107</td>
</tr>
<tr>
<td>5.4</td>
<td>Natural frequencies of framed-tube building</td>
<td>108</td>
</tr>
<tr>
<td>6.1</td>
<td>Variation of $G_{xy}$, $E_y$, and SL with beam depth</td>
<td>122</td>
</tr>
<tr>
<td>6.2</td>
<td>Modal participation factors, natural frequencies, pseudo-velocities, and error factors for framed-tube building</td>
<td>123</td>
</tr>
<tr>
<td>7.1</td>
<td>Average values of $\lambda_s$ and $X$ for different $H/A$ values taking $E_y/G_{xy} = 50$</td>
<td>152</td>
</tr>
<tr>
<td>7.2</td>
<td>$Y$ values for different $\beta$, $N_{B1}$, and $N_{B2}$ values</td>
<td>153</td>
</tr>
<tr>
<td>7.3</td>
<td>Average $X$ values for different $H/A$ ratios taking $E_y/G_{xy} = 50$, $N_{B1} = N_{B2} = 15^2$</td>
<td>154</td>
</tr>
</tbody>
</table>
NOTATIONS

\(a, b\)  
dimensions of the elements

\(A\)  
depth of structure

\(A_b, A_c\)  
cross-sectional area of beam and column

\(A_r, A_c\)  
shear area of beam and column

\(A_m, A_n, C_y, C_{xy}\)  
coefficients used in determining the elastic properties of the equivalent membrane

\(B\)  
width of structure

\(c\)  
number of corners in the plan view of the structural model

\([C]\)  
damping matrix

\([d], [\dot{d}], [\ddot{d}]\)  
displacement, velocity and acceleration vector

\(d_b, d_c\)  
depth of beam and column

\(d_i\)  
distance from neutral axis to column \(i\)

\(D_i\)  
perpendicular distance from the facade to the reference point of level \(i\)

\([D]\)  
strain-displacement matrix

\(E, G\)  
elastic moduli of the material of the actual structure

\(E_x, E_y, G_{xy}\)  
elastic moduli of the equivalent orthotropic membrane

\([E]\)  
elasticity matrix

\(f\)  
function

\(\{f_s\}\)  
effective external elastic force vector

\(h\)  
storey height

\([h]\)  
vector of level elevations measured from the base of the structure.
\( H \) \hspace{1cm} \text{structure height}

\( I \) \hspace{1cm} \text{moment of inertia}

\([I]\) \hspace{1cm} \text{identity matrix}

\( J \) \hspace{1cm} \text{in-plane level rotational inertia}

\([K]\) \hspace{1cm} \text{stiffness matrix}

\([K_{ij}]\) \hspace{1cm} \text{submatrix of } [K]

\( l \) \hspace{1cm} \text{span of lintel beam}

\( L \) \hspace{1cm} \text{number of levels in a structural model}

\([L]\) \hspace{1cm} \text{vector of earthquake-excitation modal factors}

\( \bar{m} \) \hspace{1cm} \text{linear mass distribution along the height of a structure}

\( m_i \) \hspace{1cm} \text{mass of level } i.

\([M]\) \hspace{1cm} \text{mass matrix}

\( M_{ux}, M_{uy} \) \hspace{1cm} \text{overturning base moment about the } x \text{ and } y \text{ axis}

\([N]\) \hspace{1cm} \text{shape function}

\( P \) \hspace{1cm} \text{column axial force}

\([p]\) \hspace{1cm} \text{external load vector}

\([P_{\text{eff}}]\) \hspace{1cm} \text{effective external force vector}

\( Q_{c} \) \hspace{1cm} \text{applied shear force}

\( r \) \hspace{1cm} \text{element aspect ratio}

\([r]\) \hspace{1cm} \text{influence coefficient vector}

\( S_{a}, S_{d}, S_{v} \) \hspace{1cm} \text{acceleration, displacement, and pseudo-velocity spectral response}

\( S_{l} \) \hspace{1cm} \text{shear lag parameter}

\( t \) \hspace{1cm} \text{equivalent orthotropic membrane thickness}
tb, tc
T
[T]
[Tij]
u, v
\ddot{u}, \ddot{v}
v_g, \dot{v}_g
V_b, V_c
V_u, V_v
V_\theta
w
\alpha
\alpha_l
\beta
{\xi}, {\eta}
\eta
\{\sigma\}; \{\varepsilon\}
\sigma_x, \sigma_y, T_{xy}
\varepsilon_x, \varepsilon_y, \varepsilon_{xy}
\Theta, \dot{\Theta}

thickness of beam and column
time period
transformation matrix
submatrix of [T]
displacements in the x and y directions
acceleration in the x and y directions
ground displacement and acceleration
shear force in beam and column
base shear in the x and y direction
base torque
bay width
coefficient used in determining E_y, the angle measured from the positive x axis to the positive direction of a facade
coefficients used in the displacement functions
factor of the inertia of the normal facade
dimensionless element axis system, = x/a, y/b
normal coordinate
stress and strain vector
stress components in the equivalent orthotropic membrane
strain components in the equivalent orthotropic membrane
in-plane rotation, and rotational acceleration of a level.
\{\Delta\} \quad \text{lateral displacement vector}

\{\phi\} \quad \text{mode shape vector}

\lambda \quad \text{natural angular frequency}

\lambda_s \quad \text{basic natural frequency}

[m],[c],[k] \quad \text{generalized mass, damping, and stiffness matrices}

\omega \quad \text{corner vertical displacement}

\mathcal{M}_n, \mathcal{C}_n, \mathcal{K}_n \quad \text{generalized mass, damping, and stiffness for mode } n

f \quad \text{fraction of critical damping}

\Gamma \quad \text{modal participation factor}

\mu \quad \text{ductility factor}

\Pi \quad \text{dimensionless parameter}

\nu \quad \text{Poisson's ratio}

\rho \quad \text{material density}

\gamma \quad \text{angle measured counterclockwise from the } x \text{ axis to the direction of ground motion}
CHAPTER 1

INTRODUCTION

1.1 PLANAR AND TUBE-TYPE STRUCTURES

Planar and tube-type structures are broad classifications of many existing structures. Planar structures are those which can be analyzed as two-dimensional structures such as shear walls, coupled shear walls, and two-dimensional frames.

Tube-type structures are any assembly of planar structures connected at their edges. Several examples of tube-type structures are framed-tube buildings, frame-shear wall buildings and core supported buildings.

1.2 FRAMED-TUBE BUILDING

"The dynamic characteristics of framed-tube buildings are of primary importance in this project and thus a characteristic review of this structure is in order. The framed-tube system was originally developed for the design of the 43-storey De Witt Chestnut Appartment Building in Chicago in 1963 (Fig. 1.1b) (2)."
The framed-tube system consists of closely spaced exterior columns connected by deep spandrel beams around the periphery of the building which is usually but not necessarily rectangular.

The structure resembles an assembly of perforated planar exterior walls or a perforated tube (Fig. 1.1a). The exterior walls are designed to withstand the entire lateral load. The frame efficiency of lateral load resistance results from deeper members in conjunction with shorter bay widths and storey heights. Bay widths (or centre to centre column spacings) generally vary from 4 ft. to 10 ft. although structural efficiency is not significantly reduced for column spacings up to 15 ft. (1). Typical column and spandrel beam depths vary from 2 ft. to 5 ft. while column widths vary from 10 inches up to 3 ft. (2). The framed-tube system is an optimum design for buildings up to about 400 ft., however the twin towers of the World Trade Centre in New York are 110 storeys high (3).

Although the structure has a tube-like appearance, the structural behaviour is much more complex than that of a simple cantilever tube. This structural system combines the characteristic behaviour of both multi-storey, multi-bay frames, and walls. The structure thus undergoes both the true cantilever action of shear walls and the
shearing action of frames. Consider the framed tube shown in Fig. 1.2. The whole three-dimensional structure resists overturning under lateral load, causing compression and tension in columns. The shear from the lateral load is resisted by the bending of beams and columns primarily in the facades parallel to the direction of loading. Unlike the pure bending behaviour of ideal tubes which exhibit linear normal stress variation in the web the framed tube perforations in the "web" frame cause an inefficiency due to its higher degree of flexibility causing what is known as shear lag.

1.3 SCOPE AND OBJECTIVES

The primary objectives of this project are i) to develop an efficient computer program for earthquake spectral analysis of large planar and three-dimensional tube-type buildings, ii) to study the behaviour of framed-tube building structures subject to seismic forces, and iii) to provide equations and graphs for natural frequency calculation of the above mentioned structures.

The macro-element technique of structural modelling developed by El-Moselhi (1) is adopted and further extended for dynamic analysis. This technique is capable of modelling most existing planar and tubular structural systems. These systems may include multi-storey,
multi-bay frames having a wide range of aspect ratios and stiffnesses, shear walls, coupled shear walls, clad frames, planar and tubular structures consisting of frame and shear wall assemblies, and core-supported structures.

The computer program is used to study the behavioural characteristics of framed-tube building structures subjected to earthquake ground motions. The effects of shear deformation in members and flexibility of finite size joints on the seismic responses of framed-tube structures will also be investigated. Simple methods for estimating the natural frequencies of planar and tube-type structures will be presented.

1.4 GENERAL ASSUMPTIONS

Several general assumptions will be made here which hold throughout this research. Other specific assumptions pertaining to specific topics treated in this work will be presented later.

(1) The structural material is isotropic, homogeneous, and linearly elastic.

(2) Frame members are rigidly connected to form an orthogonal grid system.
(3) Floor diaphragms are infinitely rigid in their own planes. This assumption is widely accepted for the exact analysis of tall building structures (4).

(4) Out-of-plane deformations of walls and frames are ignored. Published results show their negligible contribution to structural behaviour (3, 4, 5, 6, 7).

1.5 ORGANIZATION OF THE THESIS

Chapter II of the thesis introduces the concept of replacing a discrete structural system by elastically equivalent membranes. Equations for the elastic properties of the orthotropic membranes are presented. The effects of column axial deformations, shear and bending deformations in beams and columns, and flexible finite size joints are taken into account.

Chapter III presents two orthotropic rectangular plane stress finite elements incorporating the assumption of infinite horizontal rigidity for computational efficiency purposes. The first is an "ordinary" element with 4 corner nodes and 6 degrees of freedom. The second is a "refined" element with 6 nodes and 9 degrees of freedom.
These elements are used to subdivide each equivalent membrane and consequently discretizing the equivalent structure.

In Chapter IV, a general three-dimensional structure is modelled, and the formulations for its linear static analysis and linear earthquake spectrum analysis are developed. A ductility factor may be used to alter the response spectrum for approximate elastic-plastic earthquake analysis.

Chapter V is designed mainly to establish the validity of the present method. Two and three-dimensional structures are analyzed to show the accuracy, versatility, and efficiency of the present method.

Chapter VI is a study on the behavioural characteristics of framed-tube buildings. A shear lag parameter is introduced to measure the degree of shear lag in the structure, and its effect on internal column axial forces and natural frequencies. The contributions of flexible finite size joints, and shear deformations in beams and columns to the dynamic responses are investigated.

Chapter VII provides designers and researchers with two methods of determining natural frequencies of planar
and tube-type structures. The first more accurate method requires the use of graphs and simplified equations. The second method, which applies only to framed-tube structures, requires less computation, but yields more approximate results.

Finally, conclusions for the present study and recommendations for further studies are presented in Chapter VIII.
Fig. 1.1 (a) Framed tube
(b) De Witt Chestnut apartment building(2)
Fig. 1.2 Shear lag in framed tube (2)
CHAPTER II

EQUIVALENT ORTHOTROPIC MEMBRANE

2.1 INTRODUCTION

The concept of modelling a complex, highly redundant discrete system by an elastically equivalent continuum is not new. It has been applied to ship and aircraft structures (8), bridges (9), shells (10), space roof trusses (11), coupled shear walls (12, 13), and tall frame-type building structures (14, 15, 16). With particular reference to tube-type building structures, the macro-element technique has been shown (1) to be an efficient and accurate method for static analysis. The method consists of first replacing the discrete structure by an elastically equivalent system. For completeness, the relevant theoretical development related to the elastically equivalent membranes for coupling lintel beams and rectangular framework is presented in this chapter.

2.2 ORTHOTROPIC MEMBRANE FOR COUPLING BEAMS

Lintel beams connecting planar or three-dimensional
assemblies of shear walls are replaced by an elastically equivalent orthotropic membrane as shown in Fig. 2.1. Both bending and shear deformations of the connecting beams are considered in evaluating the shear modulus of the membrane; this allows modelling of a wide range of coupling elements from slabs to relatively deep lintel beams.

In evaluating the shear modulus of the membrane, it is assumed that the inflection points are located at the midspan of the lintel beams. This assumption is commonly accepted (1,3,12,13). For a general cross-section of lintel beam the membrane shear modulus is

\[ G_{xy} = \frac{E}{k h} \left[ \frac{1}{2 I_b} + \frac{1}{A_{rb}} \left( \frac{E}{G} \right) \right] \]  (2.1)

where

- \( l \) = span of lintel beam
- \( E, G \) = elastic moduli of lintel beam material
- \( I_b, A_{rb} \) = moment of inertia and reduced (effective) area of the beam cross-section
- \( t \) = membrane thickness
- \( h \) = storey height
For a Tintel beam of rectangular cross-section with thickness $t_b$ and depth $d_b$, the membrane shear modulus is

$$G_{xy} = \frac{E \left( \frac{d_b}{h} \right)}{\left[ \left( \frac{d_b}{k} \right)^2 + 1.2 \left( \frac{E}{G} \right) \right]}$$  \hspace{1cm} (2.2)

2.3 **ORTHOTROPIC MEMBRANE FOR PLANAR GRIDWORK SYSTEMS**

In replacing the perforated wall or planar gridwork system by an elastically equivalent orthotropic membrane, there are several restrictions based on practical considerations. Firstly, $w/A$ and $h/H$ of the structure must be small; $A$ and $H$ are depth and height of the structure respectively. Typical existing structures range from $1/10$ to $1/20$ and $1/20$ to $1/100$ respectively. Secondly, the member properties, storey heights, and bay widths may vary only from one zone to another within the building. These restrictions would make the application of the technique more feasible, but are not required for the validity of the theoretical development.
2.3.1 MODULI OF ELASTICITY $E_x$ AND $E_y$

Consider a typical wall-frame unit composed of rectangular concrete sections subjected to a vertical axial load $P$ (Fig. 2.2). The total vertical deformation is obtained considering the uniform stress in parts 1 and 3, and the complex stress variation in part 2, (Fig. 2.2b). Next a rectangular membrane encompassing the wall-frame unit is subjected to a uniform vertical axial stress of $P/(wt)$. Considering the vertical elastic modulus, $E_y$, of the membrane as unknown, the total deformation of the wall-frame unit, and the membrane are equated to yield

$$E_y = \frac{E \frac{t_x}{t} \frac{d_c}{d}}{t \frac{w}{C_y}}$$

(2.3)

in which

$$C_y = 1 - \frac{d_b}{h} + \frac{t_x}{t_b} \left[\frac{d_c}{d} \frac{d}{d_c} + \frac{8w}{\pi^2 h^2} \frac{1}{3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{A_n A_m}{m} \sinh^2(\alpha d_b)\right]$$

(2.4)

$$A_n = \phi \left[\alpha d_c (2-n)/2\right]$$

(2.5)

$$A_m = \frac{\sinh(\alpha d_c)}{m [\sinh(2\alpha d_b) + 2 \alpha d_b]}$$

(2.6)
where

\[ \alpha = \frac{m \Pi}{N} \]  \hspace{1cm} (2.7)

\[ d_c, t_c \] = depth and thickness of column respectively

\[ d_b, t_b \] = depth and thickness of beam respectively

A similar procedure can be carried through to obtain the horizontal equivalent elastic modulus \( E_x \), but in the context of this research, \( E_x \) has been taken to be infinity; infinite in-plane rigidity of floors can be assumed in tall building analysis (1, 4).

It can be seen from Eq. (2.4) that for slender members, \( C_y \) approaches unity. In this instance, Eq. (2.3) is an expression of the equivalent vertical elastic modulus of the column only. For structures having deep beams and a small column thickness to beam thickness ratio, \( C_y \) could be significantly less than one. Typical values of \( C_y \) range from 0.8 to 0.96. In the case of steel structures, the expression for \( C_y \) can be considerably simplified.

2.3.2 Shear Modulus \( G_{xy} \)

Consider the wall-frame unit of Fig. 2.3 subjected to a horizontal shear force \( Q \). Note that the support conditions simulate the midspan inflection point assumption commonly used in literature (3). Again the
rectangular membrane is made to deflect with the grid unit. The total lateral deflection of the grid unit is obtained by considering bending and shear deformations of both beam and column, and the flexibility of the finite size joint. Equating this deflection to the lateral deflection expression of the membrane yields the equivalent shear modulus.

\[ G_{xy} = \frac{E}{t \cdot w \cdot C_{xy}} \]  \hspace{1cm} (2.8)

in which

\[ C_{xy} = \frac{1}{12hI_c} + \frac{h(w-d_t)^3}{12w^2I_b} + \frac{E}{G} \left[ \frac{h(w-d_t)+(h-d_b)^3}{w^2A_{rb}} + \frac{1}{hA_{rc}} A_{rj} \left( 1 - \frac{d_b}{w} \right)^3 \right] \]  \hspace{1cm} (2.9)

where

- \[ I_b, I_c \] = moment of inertia of beam and column respectively.
- \[ A_{rb}, A_{rc} \] = reduced or effective shear area of the beam and column respectively.
- \[ A_{rj} \] = cross-sectional area of the joint parallel to the acting force Q.
- \[ G \] = shear modulus of the actual material.
Note that $C_{xy}$ is a summation of five terms. Going from left to right they take into account the following deformations: bending in the column, bending in the beam, shear in the beam, shear in the column, flexibility of the finite size joint. In the case of steel structures, the flexibility of finite size joint comes mainly from the so-called "panel zone deformations". This factor can be derived in a similar fashion.
Fig 2.1 Present modelling of coupling beams
Fig. 2.2 Model for evaluating the equivalent membrane
elastic modulus $E_y$
Fig. 2.3 Model for evaluating the equivalent membrane shear modulus $G_{xy}$
CHAPTER III

SPECIALLY ORTHOTROPIC FINITE ELEMENTS FOR TALL BUILDING ANALYSIS

3.1 INTRODUCTION

For highly regular-planar and tubular structures subjected to static lateral loads, closed form solutions for displacements and internal forces have been presented (1, 2, 3). Most tubular buildings have variations that cannot be realized by closed form solutions; lintel beam sizes may vary along the height of coupled shear walls, member sizes and grid sizes in a frame may change from one zone to another, assemblies of coupled shear walls, frames, and solid shear walls may be difficult to analyze, building configurations may not be rectangular, mass distribution for a dynamic analysis is irregular. Finite element analysis is a good alternative. The discrete idealized structure is replaced by equivalent membranes, (presented in the previous chapter), which are subsequently subdivided into several rectangular finite macro-elements. Note that the actual structure need be only highly regular within each individual membrane. This technique has been shown to be an efficient and accurate method for static analysis. For completeness, the relevant
theoretical developments of two macro-elements (1) are presented in this chapter.

3.2 FORMULATION OF THE ORDINARY ELEMENT

Consider the rectangular element of Fig. 3.1. The displacement function which satisfies infinite horizontal rigidity (in accordance with infinitely rigid floors), maintains compatibility along the edges, and satisfies all other requirements for convergence in finite element theory (17) is assumed to be:

\[ u = \alpha_1 + \alpha_2 \eta \]  \hspace{1cm} (3.1a)

\[ v = \alpha_3 + \alpha_4 \xi + \alpha_5 \eta + \alpha_6 \xi \eta \]  \hspace{1cm} (3.1b)

in which $\xi$ and $\eta$ are dimensionless coordinates equal to $x/a$ and $y/b$ respectively. Eq. (3.1) can be expressed in terms of the 6 nodal displacement as

\[
\begin{pmatrix}
  u \\
  v
\end{pmatrix}
= [N]
\begin{pmatrix}
  d
\end{pmatrix}
\]

\hspace{1cm} (3.2)

such that

\[
[N] =
\begin{bmatrix}
  1 & -\eta & 0 & 0 & 0 & 0 \\
  0 & 0 & (1-\xi)(1-\eta) & \xi(1-\eta) & \xi \eta & \xi \eta \eta
\end{bmatrix}
\]

(3.3)

and

\[
\begin{pmatrix}
  d
\end{pmatrix}^T = [u, u, v, v, v, v]
\]

\hspace{1cm} (3.4)
Using the strain-displacement relationship
\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial}{\partial x} & 0 \\
0 & \frac{\partial}{\partial y} \\
\frac{\partial}{\partial y} & \frac{\partial}{\partial x}
\end{bmatrix}
\begin{bmatrix}
u
\end{bmatrix}
\]
results in
\[
\{\varepsilon\} = [D] \{d\}
\]
where \(\{\varepsilon\}^T = [\varepsilon_x \ \varepsilon_y \ \gamma_{xy}]\)

and
\[
[D] =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{(1-\nu)}{\nu} & -\frac{\nu}{\nu} & \frac{\nu}{\nu} & (1-\nu) \frac{\nu}{\nu} \\
-\nu & \nu & -(1-\nu) \frac{\nu}{\nu} & -(1-\nu) \frac{\nu}{\nu} & \frac{\nu}{\nu} & -\frac{\nu}{\nu}
\end{bmatrix}
\]

Since infinite in-plane rigidity of floors is assumed, Poisson's ratios \(\nu_{yx}\) and \(\nu_{xy}\) of the equivalent orthotropic membrane have zero values and thus the stress-strain matrix relation is given by:

\[
\{\sigma\} = [E] \{\varepsilon\}
\]

such that \(\{\sigma\}^T = [\sigma_x \ \sigma_y \ \tau_{xy}]\)

and
\[
[E] =
\begin{bmatrix}
E_x & 0 & 0 \\
0 & E_y & 0 \\
0 & 0 & G_{xy}
\end{bmatrix}
\]
The 6 X 6 ordinary element stiffness matrix \( K_e \) is obtained from

\[
[K_e] = \frac{ta}{b} \int_0^1 \left[ \begin{bmatrix} 0 \end{bmatrix} \left[ \begin{bmatrix} 0 \end{bmatrix} \right] \right] d\xi \, d\eta \tag{3.12}
\]

\[
[K_e] = \begin{bmatrix}
K_1 & -K_1 & K_1 & \text{Sym.} \\
-K_1 & K_2 & -K_2 & K_3 \\
K_2 & K_2 & K_4 & K_3 \\
-K_2 & K_2 & -\frac{K_3}{2} & K_5 \\
K_2 & -K_2 & K_5 & -\frac{K_3}{2} & K_4 & K_3
\end{bmatrix} \tag{3.13}
\]

in which

\[
K_1 = \frac{taG_{xy}}{b} \tag{3.14a}
\]

\[
K_2 = \frac{tG_{xy}}{2} \tag{3.14b}
\]

\[
K_3 = \frac{t(aE_y + bG_{xy})}{3} \tag{3.14c}
\]

\[
K_4 = \frac{t(aE_y - bG_{xy})}{3} \tag{3.14d}
\]

\[
K_5 = \frac{t(bG_{xy} - aE_y)}{3} \tag{3.14e}
\]
3.3 FORMULATION OF THE REFINED ELEMENT

The need for a more refined element was induced because the ordinary element was unreliable in certain applications. The ordinary element is only capable of describing a linear deformation configuration along its edges. For this reason, the bending deformation of several example shear walls were poorly described. It was found that as the aspect ratio increased, the lateral deflection became more erroneous. Errors for the maximum lateral deflection of over 100% were frequently obtained. It can also be seen that stress variation within an ordinary element is only linear, and thus many elements may have to be used to obtain accurate column axial forces in a framed-tube building characterized by nonlinear column axial force distribution due to shear lag.

Consider the rectangular element of Fig. 3.2. The displacement functions satisfying convergence requirements in finite element theory (17) may be assumed to be

\[ U = \alpha_1 + \alpha_2 \eta + \alpha_3 \eta^2 \]  \hspace{1cm} (3.15a)

\[ V = \alpha_4 \xi + \alpha_5 \xi^2 + \alpha_6 \xi^2 \eta + \alpha_7 \xi^2 \eta^2 + \alpha_8 \xi^2 \eta^2 \]  \hspace{1cm} (3.15b)

such that \( \xi = x/a \) and \( \eta = y/b \).
Note that the refined element is capable of describing a quadratic deformation configuration along its edges, which is more representative of bending deformation. In terms of nodal displacements, Eq. (3.15) may be written as

\[
\begin{align*}
\begin{bmatrix} u \\ v \end{bmatrix} &= [N] \begin{bmatrix} d \end{bmatrix} \\
\text{(3.16)}
\end{align*}
\]

where

\[
[N]^T = \begin{bmatrix}
-\frac{1}{2} \gamma(1-\gamma) & 0 & 0 & 0 & -\frac{1}{4} \delta(1-\delta-\gamma+\delta \gamma) \\
1-\gamma^2 & 0 & 0 & 0 & \frac{1}{2} (1-\delta+\delta^2+\delta \gamma) \\
\frac{1}{2} \gamma(1+\gamma) & 0 & 0 & 0 & \frac{1}{4} \delta(1+\delta-\gamma-\delta \gamma) \\
0 & -\frac{1}{4} \delta(1-\delta-\gamma+\delta \gamma) & 0 & 0 & \frac{1}{4} \delta(1+\delta+\gamma+\delta \gamma) \\
0 & \frac{1}{2} (1-\delta+\delta^2+\delta \gamma) & 0 & 0 & \frac{1}{2} (1+\delta-\delta^2-\delta \gamma) \\
0 & 0 & 0 & -\frac{1}{4} \delta(1-\delta-\gamma+\delta \gamma) & \frac{1}{4} \delta(1+\delta+\gamma+\delta \gamma)
\end{bmatrix} \tag{3.17}
\]

and

\[
\{d\}^T = [u, u_1, u_2, v_1, v_2, v_3, v_4, v_5, v_6]. \tag{3.18}
\]

The \( (D) \) matrix is obtained from the strain-displacement relationship Eq. (3.5)
\[(D)^T = \begin{bmatrix}
0 & 0 & -(1-2\gamma)/2b \\
0 & 0 & -2\gamma/b \\
0 & 0 & (1+2\gamma)/2b \\
0 & \xi(1-\xi)/4b & -(1-2\xi-\gamma+2\xi\gamma)/4a \\
0 & (1-\xi^2)/2b & -\xi(1-\gamma)/4a \\
0 & \xi(1+\xi)/4b & (1+2\xi-\gamma-2\xi\gamma)/4a \\
0 & \xi(1+\xi)/4b & (1+2\xi+\gamma+2\xi\gamma)/4a \\
0 & (1-\xi^2)/2b & -\xi(1+\gamma)/4a \\
0 & -\xi(1-\xi)/4b & -(1-2\xi+\gamma-2\xi\gamma)/4a \\
\end{bmatrix}\] (3.19)

The 9 X 9 refined element stiffness matrix is

\[ [K_e] = \int_{\Omega} \int_{\Gamma} [D][\varepsilon][D] d\varepsilon d\Omega \] (3.20)

\[ [K_e] = \begin{bmatrix}
7K_1 \\
-8K_1 & 16K_1 \\
K_1 & -8K_1 & 7K_1 \\
-5K_2 & -4K_2 & -K_2 & K_3 \\
0 & 0 & 0 & K_4 & K_9 \\
-5K_2 & 4K_2 & K_5 & K_4 & K_3 \\
-5K_2 & -4K_2 & 5K_2 & K_6 & K_7 & K_8 & K_3 \\
0 & 0 & 0 & K_7 & K_{10} & K_7 & K_4 & K_9 \\
K_2 & 4K_2 & -5K_2 & K_8 & K_7 & K_6 & K_5 & K_4 & K_3 \\
\end{bmatrix} \] (3.21)
in which

\[ K_1 = \frac{tG_{xy}}{3r} \]  

\[ K_2 = \frac{tG_{xy}}{6} \]  

\[ K_3 = t(\frac{2r}{15}E_y + \frac{7}{9r}G_{xy}) \]  

\[ K_4 = \frac{1}{3}t(\frac{r}{5}E_y - \frac{8}{3r}G_{xy}) \]  

\[ K_5 = -\frac{t}{6}(\frac{r}{5}E_y - \frac{2}{3r}G_{xy}) \]  

\[ K_6 = \frac{t}{6}(\frac{r}{5}E_y + \frac{1}{3r}G_{xy}) \]  

\[ K_7 = -t(\frac{r}{15}E_y + \frac{4}{9r}G_{xy}) \]  

\[ K_8 = -\frac{t}{6}(\frac{4r}{5}E_y - \frac{7}{3r}G_{xy}) \]  

\[ K_9 = \frac{8}{3}t(\frac{r}{5}E_y + \frac{2}{3r}G_{xy}) \]  

\[ K_{10} = -\frac{8}{3}t(\frac{r}{5}E_y - \frac{1}{3r}G_{xy}) \]  

\[ r = \frac{a}{b} \]  

(3.22a)  

(3.22b)  

(3.22c)  

(3.22d)  

(3.22e)  

(3.22f)  

(3.22g)  

(3.22h)  

(3.22i)  

(3.22j)  

(3.22k)
3.4 DETERMINATION OF FORCES IN BEAMS AND COLUMNS

Once all element displacements are obtained, the equivalent membrane strain and stress distributions are determined from Eqs. (3.6) and (3.9) respectively. Referring to Fig. 3.3, axial forces and shear forces in beams and columns are evaluated first at their midlength points by integrating the corresponding equivalent membrane stress component over a bay width \( w \) or a storey height \( h \) for columns or beams respectively, as follows:

\[
P_{ij} = \int_{y_i}^{x_i = w/2} t (r_{ij}) y_{ij} = 0 \, dx \quad (3.23a)
\]

\[
V_{cij} = \int_{x_i}^{y_i = w/2} t (I_{xy}) y_{cij} = 0 \, dx \quad (3.23b)
\]

\[
V_{bij} = \int_{y_i}^{y_i = h/2} t (I_{xy}) y_{bij} = 0 \, dy \quad (3.23c)
\]

In which

\( P_{ij}, V_{cij} \) = the axial and shear forces respectively in the \( ij \) column

\( V_{bij} \) = the shear force in the \( ij \) beam

\( x_{bij}, y_{bij}, y_{cij} \) = the coordinates of the appropriate midlength point, as shown in Fig. 3.3
Because inflection points are assumed at the midspan of beams and columns, the bending moment in beams and columns can be evaluated at any section by multiplying the appropriate midspan shear force $V_{bij}$ or $V_{cij}$ respectively by the distance from the midpoints.
\[ \xi = \frac{x}{a}, \quad \eta = \frac{y}{b} \]

Fig. 3.1 The ordinary element
Fig. 3.2. The refined element.
Fig. 3.3 Definitions of symbols in equations 3.23 a, b, c.
CHAPTER IV

STATIC AND EARTHQUAKE ANALYSIS
OF PLANAR AND TUBE-TYPE STRUCTURES

4.1 INTRODUCTION

Planar structures, in this thesis, encompass shear walls, coupled shear walls, frames or any combination of these three that form a plane. Tube-type building structures here means any assembly of planar structures connected along their edges. Typical examples of tube-type building structures are framed-tube structures, shear wall structures, shear wall frame structures, rigid tube structures and core-supported structures.

Many approximate and detailed methods of analysis have been developed for these types of structures. In the early stages of research on this topic, approximate methods of analysis such as the portal and cantilever methods resulted in relatively large errors, but provided a fairly simple method of analysis. Most approximate methods of analysis are derived from basic assumptions; some are more valid than others. In general, most recent developments in approximate structural analysis methods give fast reliable results, but are too restricted in
applicability. On the other hand, detailed methods of analysis alter very good results with a wider range of applicability at the expense of time and effort.

The present equivalent membrane finite element method of analysis yields fairly accurate results with little effort and time, and has wide range of applicability for building structures and structural components.

4.2 SCOPE

The theoretical static and dynamic analysis developments presented in this chapter must satisfy the following conditions:

1) The behaviour is linear elastic.

2) The structure must be planar or tube-type.

i) Planar structures may be any assembly of rigidly connected shear walls, frames, and lintel beams forming a plane. Frames must consist of rigidly connected horizontal beams
and vertical columns.

ii) Tube-type structures may be any assembly of planar structures connected along their edges.

4) Both static and dynamic analyses follow the same general assumptions stated earlier in Chapter I.

4.3 ANALYSIS PROCEDURE

A planar structure may be considered as a tube-type structure with one facade. Since planar structures are special cases of tube-type structures, the latter will be used for the development of the analysis procedure.

4.3.1 MODELLING OF STRUCTURES

The hypothetical framed-tube of Fig. 4.1a is used to illustrate the idealization process. The structure consists of three planar facades intersecting at corners (Fig. 4.1c). Note that the plan form can be any open or closed polygon shape. Facades and corners are numbered in a counterclockwise direction.

The facades of the actual structure are replaced by
the equivalent orthotropic membrane which is discretized into a set of rectangular-plane stress finite elements (Fig. 4.1b), developed in Chapter III, according to the following rules which apply also to shear walls and lintel beams.

1) Elements may span one or more stories or bays (if applicable), but element boundaries need not coincide with beam and column lines. However, the element boundaries must be aligned throughout the height and width of the structure. The structure is thus divided into a number of "structural levels" which should be numbered consecutively from bottom to top (Fig. 4.1b).

2) Structural properties of members may vary along the building height and width, but must remain constant within each element. (i.e. elastic modulus, shear modulus, storey height, bay width, moment of inertia, shear area, and depths of columns and beams).

3) As shown in Fig. 4.2a, element corner nodes are numbered from bottom to top starting from the
left to the right of each facade. The remaining nodes of the refined element are subsequently numbered in the same fashion (Fig. 4.2b).

4) Element connectively is specified by node numbers in a counterclockwise direction starting from the bottom left corner node.

The discretization process of steps 3 and 4 above are automatically generated from the given number of elements in the horizontal and vertical directions of each facade. Automatic generation reduces effort in data preparation and eliminates possible data errors.

With the assumption of rigid floor diaphragms, each level of the building will acquire three degrees of freedom: two orthogonal translations and one in-plane rotation. These three degrees of freedom can be associated with any convenient point on the level. To ensure vertical compatibility at facade junctions (or corners), a vertical degree of freedom is assigned at each building corner for each level. The above degrees of freedom will be referred to as the "global static degrees of freedom" (Fig. 4.3). When the building is subjected to a horizontal earthquake excitation, the vertical corner degrees of freedom of the global static degrees of freedom
are statically condensed and the remaining degrees of freedom are termed as the "global dynamic degrees of freedom". For convenience, these degrees of freedom should be associated with the mass center of each level (Fig. 4.4). Displacements associated with interior nodes within individual facades are called the "internal facade degrees of freedom". These are statically condensed, leaving just global degrees of freedom to form the global structure stiffness matrix.

The primary unknowns in a global analysis are the global degrees of freedom. Once these are known, all the previously condensed degrees of freedom can be recovered and subsequently, the stresses in the membranes as well as member forces in the actual structure can be determined.

The present method of analysis takes advantage of the finite element analysis technique on a macroscale. Finite elements are conventionally associated with only small portions of structural components. The macro-element, conforms to the same rules as a finite element, but encompasses many structural members and large areas of walls.
4.3.2 STATIC ANALYSIS PROCEDURE

The major steps in the static analysis procedure are summarized below.

1) The individual facade stiffness matrices are assembled from their respective element stiffness matrices. Boundary and facade symmetry conditions are directly taken into account. Out-of-plane deformations of facades are not considered.

2) The internal degrees of freedom of each facade stiffness matrix are condensed to leave only those at the edges of the facade.

3) The condensed facade stiffness matrices are transformed into the global coordinate system taking into account the global (structure) restraints and/or symmetry conditions.

4) The global static structure stiffness matrix is assembled from the transformed, condensed facade stiffness matrices obtained in the previous step.
5) The global structure load vector associated with the global static degrees of freedom is assembled.

6) The global static displacements are solved for.

7) The global static displacements are extracted and transformed into local axes for each facade.

8) The internal local displacements of each facade are recovered for subsequent stress calculations within each element.

9) The internal member forces in the actual structure are determined by integrating the appropriate stress component.

4.3.3 EARTHQUAKE SPECTRUM ANALYSIS PROCEDURE

In the dynamic analysis of buildings subjected to horizontal ground excitation, steps 1 to 4 of the static case are equally applicable. The additional steps are as follows:

5) Static condensation of the vertical corner displacements leaves only two translations and
one in-plane rotation per level as the global
dynamic degrees of freedom. Lumped masses and
rotational inertias are required at each level.

6) Determination of the natural frequencies and
vibration mode shapes by solving the eigenvalue
problem.

7) For each mode the following responses are
determined:
   a) modal participation factor
   b) pseudo-velocity from a suitable response
      spectrum taking into account elastic or
      elastico-plastic behaviour
   c) normal coordinate
   d) effective elastic forces associated with the
      global dynamic degrees of freedom
   e) base shears, torques and moments
   f) global dynamic displacements
   g) corner vertical displacements
   h) facade internal displacements
   i) element stresses
8) Use the square-root-sum-of-squares (SRSS) method to obtain the final response for d, e, f, g, i, of part 7.

9) Determine the internal forces of the actual structure.

All the above mentioned steps for the static and earthquake analysis are described in detail below with particular attention to programming aspects.

4.4 ASSEMBLY OF FACADE STIFFNESS MATRIX IN LOCAL AXES

Since the facade coordinate system coincides with that of its elements, the local element stiffness matrices are directly assembled to form the facade stiffness matrix. Prescribed zero-displacements due to structural supports and symmetry conditions are satisfied by elimination of the corresponding equilibrium equations. This technique saves computation time and computer storage. The bottom nodes are assumed to be fixed, however, foundation flexibility can be taken into account by introducing a bottom row of elements to simulate the supporting soil.
Symmetry exists at two levels: global structure level and local facade level. If local facade symmetry is specified, then global structure symmetry must also be specified (to ensure proper assembly of the global static structure stiffness matrix). There are no other restrictions on the compatibility of these two symmetry levels.

The facade stiffness matrix is assembled element-by-element according to their associated degrees of freedom. These degrees of freedom are automatically generated according to the type of facade symmetry shown in Fig. 4.5. One of the following facade symmetry options may be exercised:

1) No symmetry.

2) Facade symmetry type one means no horizontal nodal deflections of the facade.

3) Facade symmetry type two means no vertical nodal deflections of the facade at edge J.

4) Facade symmetry type three means no vertical nodal deflections of the facade at edge I.
Note that each edge of a facade is designated I or J. The I edge is encountered first when numbering the structure corners counterclockwise.

Since the facade stiffness matrix is symmetrical, only its upper triangle is assembled and stored in a singly subscripted array (i.e. vector) column by column. The mapping function between the stiffness matrix coefficient at row i column j and its vector representation address "1" is

\[ l = i + \left[ (j - 1)j \right]/2 \quad \text{for } j \geq i \quad (4.1) \]

4.5 CONDENSATION OF FACADE INTERNAL DEGREES OF FREEDOM

All facade internal degrees of freedom (i.e. those not required to maintain compatibility with adjacent facades) are eliminated by the well known process of static condensation (17, 18, 19). Static condensation entails no loss of accuracy, but significantly reduces storage requirements and computation time. In effect, facades are treated as substructures with boundary degrees of freedom at facade edges. An efficient algorithm for static condensation and recovery of unwanted displacements using vector representation is described in reference (18).
4.6 ASSEMBLY OF THE GLOBAL STATIC STRUCTURE STIFFNESS MATRIX

Since the orientation of a facade in plan is arbitrary, it is necessary to transform the condensed facade stiffness matrix into the global coordinate system (Fig. 4.3). This process involves the transformation between facade degrees of freedom and global static degrees of freedom.

\[
\{d_r\} = [T] \{d_r^*\}
\]  

(4.2)

in which

\[
\{d_r^*\} = \text{vector of global degrees of freedom affecting facade deformations.}
\]

\[
\{d_r\} = \text{vector of local facade degrees of freedom.}
\]

\[
[T] = \text{transformation matrix}
\]

A specific ordering system for the global structural degrees of freedom as well as the local facade uncondensed degrees of freedom is chosen to facilitate the presentation and derivation of the transformation matrix \([T]\).
\[
\{d_r\}_{Lx1} = \begin{bmatrix}
\{\Delta\}_{Lx1} \\
\{\omega_i\}_{Lx1} \\
\{\omega_j\}_{Lx1}
\end{bmatrix}
\]  
(4.3)

and

\[
\{d_r^v\}_{Lx1} = \begin{bmatrix}
\{u\}_{Lx1} \\
\{v\}_{Lx1} \\
\{\theta\}_{Lx1} \\
\{\omega_i\}_{Lx1} \\
\{\omega_j\}_{Lx1}
\end{bmatrix}
\]  
(4.4)

in which

\[L\] = number of levels.

\{\Delta\} = vector of lateral or horizontal local facade degrees of freedom.

\{\omega_i\} and \{\omega_j\} = vector of vertical degrees of freedom at edges I and J respectively of the facade. These degrees of freedom are also in the vector of global degrees of freedom.

\{u\}, \{v\}, \{\theta\} = vectors of the global degrees of freedom associated with the reference
point of each level; $u$ refers to global $x$-translation, $v$ to global $y$-translation, and $\Theta$ to rigid body level rotation. For a dynamic analysis, the reference point must be positioned at the lumped mass locations.

Referring to Fig. 4.3, the lateral displacements of the facade at level $i$ can be expressed in terms of the three global displacements at the same level.

$$\Delta_i = \begin{bmatrix} \cos \alpha & \sin \alpha & D_i \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ \Theta_i \end{bmatrix}$$ (4.5)

where

\[ \alpha = \text{the angle measured from the global } x\text{-axis to the positive direction of the facade. Note that the positive direction of a facade is defined as the direction going from the facade corner I to corner J.} \]

\[ D_i = \text{the perpendicular distance between the facade and the level reference point.} \]
Eq. (4.2) then becomes

\[
\begin{pmatrix}
\Delta_1 \\
\Delta_2 \\
\vdots \\
\Delta_l \\
\omega_{\Delta 1} \\
\omega_{\Delta 2} \\
\vdots \\
\omega_{\Delta l}
\end{pmatrix}
=
\begin{pmatrix}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha \\
\end{pmatrix}
\begin{pmatrix}
D_1 \\
D_2 \\
\vdots \\
D_l \\
1 \\
1 \\
\vdots \\
1
\end{pmatrix}
\begin{pmatrix}
u_1 \\
v_2 \\
\vdots \\
v_l \\
\Theta_1 \\
\Theta_2 \\
\vdots \\
\Theta_l \\
\omega_{\Delta 1} \\
\omega_{\Delta 2} \\
\vdots \\
\omega_{\Delta l}
\end{pmatrix}
\tag{4.6}
\]

Let

\[
[T_1]_{LX} = \cos \alpha [I] \tag{4.7a}
\]

\[
[T_2]_{LX} = \sin \alpha [I] \tag{4.7b}
\]
\[
[T_{13}]_{LxL} = \begin{bmatrix}
D_1 & 0 \\
0 & D_2 \\
0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & D_L
\end{bmatrix}
\] (4.7c)

where

\[
[I] = \text{identity matrix}
\]

Then

\[
[T] = \begin{bmatrix}
[T_1] & [T_{12}] & [T_{13}] & [I] & 0 \\
0 & [I] & 0 & [I] & 0
\end{bmatrix}
\] (4.8)

From Eq. (4.2) the contragradient law of transformation yields

\[
\{\bar{F}^y_r\} = [T]^T \{\bar{F}_r\}
\] (4.9)

where \(\{\bar{F}^y_r\}\) and \(\{\bar{F}_r\}\) are nodal force vectors associated with the global structure degrees of freedom and boundary facade degrees of freedom respectively.

Substituting the following equation into Eq. (4.9)

\[
[K_{r}] \{d_r\} = \{\bar{F}_r\}
\] (4.10)
in which $[\bar{K}_F]$ is the facade boundary stiffness matrix, yields
\[
\{p^*_F\} = [T]^T[\bar{K}_F]\{d_F\} \tag{4.11}
\]

Substituting Eq. (4.2) yields
\[
\{p^*_F\} = [T]^T[\bar{K}_F][T]\{d_F\} \tag{4.12}
\]

The transformed facade stiffness matrix in global coordinates can be recognized as
\[
[\bar{K}_F^\star] = [T]^T[\bar{K}_F][T] \tag{4.13}
\]

The direct matrix multiplication above requires large storage and multiplication of large matrices. To alleviate this problem, the sparseness of $[T]$ will be taken advantage of as follows:
Let $[\bar{K}_F]$ be partitioned as
\[
[\bar{K}_F] = \\
\begin{bmatrix}
[k_{11}] & [k_{12}] & [k_{13}] \\
[k_{21}] & [k_{22}] & [k_{23}] \\
[k_{31}] & [k_{32}] & [k_{33}]
\end{bmatrix}
\tag{4.14}
\]
corresponding to \( \{ \Delta \} \), \( \{ \omega_2 \} \), and \( \{ \omega_3 \} \), (i.e. no facade symmetry). Then from Eqs. (4.8), (4.13), and (4.14)

\[
\begin{bmatrix}
K_F^x
\end{bmatrix} =
\begin{bmatrix}
T_1^T, \\
T_2^T, \\
T_3^T, \\
[1], \\
[0]
\end{bmatrix}
\begin{bmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
1 \\
1
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
K_{11} \\
K_{22} \\
K_{33}
\end{bmatrix}
\begin{bmatrix}
I \\
I
\end{bmatrix}
\]  \hspace{1cm} (4.15)

Manually multiplying these matrices, subsequently substituting Eqs. (4.7) and enforcing the symmetric properties of stiffness matrices yields

\[
\begin{bmatrix}
K_F^x
\end{bmatrix} =
\begin{bmatrix}
\cos^2 \theta \Delta K_{11} & \cos \omega_2 \cos \Delta K_{11} & \cos \omega_2 \cos \Delta K_{12} & \cos \omega_2 \cos \Delta K_{13} \\
\cos \omega_2 \cos \Delta K_{12} & \cos \omega_2 \cos \Delta K_{12} & \cos \omega_2 \cos \Delta K_{12} & \cos \omega_2 \cos \Delta K_{12} \\
\cos \omega_2 \cos \Delta K_{13} & \cos \omega_2 \cos \Delta K_{13} & \cos \omega_2 \cos \Delta K_{13} & \cos \omega_2 \cos \Delta K_{13} \\
T_1 T_2 & T_2 T_3 & T_3 T_1 & T_3 T_1 \\
K_{11} & K_{12} & K_{13} & K_{13}
\end{bmatrix}
\begin{bmatrix}
I \\
1 \\
1
\end{bmatrix}
\]  \hspace{1cm} (4.16)
Having transformed the condensed facade stiffness matrix for each facade, the overall global static structure stiffness matrix \([K_s^M]\) is assembled. Symmetry of \([K_s^M]\) again allows storage of its upper triangular part into a vector column by column.

4.7 TRANSFORMATION MATRICES FOR DIFFERENT STRUCTURE SYMMETRY CONDITIONS

Structure symmetry conditions may be used to discard some global degrees of freedom. Its application results in significant input data reduction and smaller matrices. To take advantage of this option, one of the following conditions must be satisfied.

1) The structural stiffness is symmetric about one or two axes. This condition is usually associated with geometric structural symmetry. For a static analysis, loading must also be symmetric.

2) The structure is externally restrained such that \(\{u\} = 0\), \(\{v\} = 0\), or \(\{\Theta\} = 0\).
The second condition is usually not associated with buildings.

For the case of structure symmetry type 1, only \( \{u\} \) is considered at the reference point. \( \{v\} \) and \( \{\theta\} \) are restrained (i.e. set to zero), thus the transformation matrix is modified to

\[
\begin{bmatrix}
\mathbf{u} & \omega_x & \omega_y \\
\end{bmatrix}
\text{\footnotesize \left[ \begin{array}{c}
\mathbf{T}_{n} \\
\mathbf{I}
\end{array} \right]}
\]

(4.17)

and the global facade stiffness matrix is given by

\[
\begin{bmatrix}
\mathbf{K}_F^2 \\
\end{bmatrix} =
\begin{bmatrix}
\mathbf{K}_{11} & \mathbf{K}_{12} & \mathbf{K}_{13} \\
\mathbf{K}_{12} & \mathbf{K}_{22} & \mathbf{K}_{23} \\
\mathbf{K}_{13} & \mathbf{K}_{23} & \mathbf{K}_{33}
\end{bmatrix}
\text{\footnotesize \left[ \begin{array}{c}
\mathbf{K}_{12} \\
\mathbf{K}_{13} \\
\mathbf{K}_{23}
\end{array} \right]}
\]

(4.18)

Structure symmetry type 2 is similar to type 1 except that \( \{v\} \) is retained while \( \{u\} \) and \( \{\theta\} \) are set to zero.

The transformation matrix is
and the global facade stiffness matrix is given by

\[
[T_{3}] = \begin{bmatrix}
T_{13} \\
0 \\
0
\end{bmatrix}
\]

(4.21)

and the global facade stiffness matrix is

\[
[K_{F}^g]_{3} = \begin{bmatrix}
\sin^2 \alpha [K_{11}] & \sin \alpha \cos \alpha [K_{12}] & \sin \alpha \cos \alpha [K_{13}] \\
\sin \alpha \cos \alpha [K_{12}] & [K_{22}] & [K_{23}] \\
\sin \alpha \cos \alpha [K_{13}] & [K_{23}] & [K_{33}] \\
\end{bmatrix}
\]

(4.20)

If the loading and geometry are such that only twisting occurs, then \{\theta\} is retained while \{u\} and \{v\} are restrained. The appropriate transformation matrix is

\[
[T_{3}] = \begin{bmatrix}
T_{13} \\
0 \\
0
\end{bmatrix}
\]

(4.21)
It should be noted that the reduced facade stiffness matrix of Eq. (4.14) is for facade symmetry type 0, and thus of order $3L \times 3L$. If facade symmetry exists, the order is only $2L \times 2L$. For purposes of compatibility, it is temporarily expanded to $3L \times 3L$ before transformation.

After the global static structure stiffness matrix is assembled, the static and earthquake analysis procedures differ.

4.8 STATIC ANALYSIS

4.8.1 SOLUTION FOR DISPLACEMENTS

Having established the global static stiffness matrix, and the associated load vector, the global static structural displacements can be solved from the linear set of equations

$$\begin{bmatrix} p^* \end{bmatrix} = \begin{bmatrix} K_s^* \end{bmatrix} \begin{bmatrix} d^* \end{bmatrix} \quad (4.23)$$

Solutions for $\{d^*\}$ are obtained using the Gauss elimination technique described in section 4.5.
Global static structural displacements are then transformed into local facade boundary displacements for each facade using the appropriate transformation matrix described in the previous section. These boundary displacements, are used in conjunction with the full condensed facade stiffness matrix to recover the condensed degrees of freedom for each facade.

4.8.2 DETERMINATION OF STRESSES

After all facade displacements are determined, they are extracted to yield the displacements associated with each element within the facade. The strains and stresses are subsequently determined as in Chapter III. It is noted that these stresses are for the equivalent membrane, and not for the actual structure. Internal member forces in the discrete beam-column system of the actual structure are evaluated based on these stresses as explained in Chapter III.

4.9 EARTHQUAKE ANALYSIS

4.9.1 GLOBAL DYNAMIC STRUCTURE STIFFNESS MATRIX

The global static structure stiffness matrix
previously described corresponds to the set of degrees of freedom:
\[ \{u\}, \{v\}, \{\theta\}, \{\omega_1\}, \{\omega_2\}, \ldots, \{\omega_c\} \]
where \( c \) = the number of corners in plan view. For a dynamic analysis, the reference points must have been located at the mass center of each level. To obtain the global dynamic structure stiffness matrix \( [K]^* \), the vertical corner degrees of freedom \( \{\omega_1\}, \{\omega_2\}, \ldots, \{\omega_c\} \) are eliminated by static condensation. The global dynamic structure stiffness matrix is now associated only with \( \{u\}, \{v\}, \{\theta\} \) if no structure symmetry exists and only one of these if structure symmetry does exist.

4.9.2 DIAGONAL LUMPED-MASS MATRIX

For simplicity of the analytical formulation, the distributed inertia of the building is lumped at the mass center of each level in the form of lumped masses and rotation inertia. Because the global dynamic degrees of freedom \( \{u\}, \{v\}, \{\theta\} \) are associated with the center of mass at each level, the mass matrix will be diagonal, thus simplifying the eigenvalue solution for frequencies and mode shapes. A more refined analysis could be achieved via a consistent-mass matrix which
contains off-diagonal terms leading to what is called mass coupling. However, in normal circumstances, such refinement does not justify the extra computational effort.

With the above approximation, the diagonal form of the mass matrix can be explained as follows. Let $m_{ij}$ be the mass coefficient in row $i$, column $j$ of the lumped-mass matrix. The coefficient $m_{ij}$ can be interpreted as the inertia force produced at degree of freedom $i$ due to a unit acceleration at degree of freedom $j$ while restraining all other degrees of freedom. Applying a unit acceleration at $j$ and restraining all other degree of freedom accelerations would cause only an inertia force at $j$ equal to the lumped mass or inertia at $j$. Thus, the mass influence coefficient $m_{ij} = 0$ for $i \neq j$, and $m_{ij} = m_j$ for $i = j$.

The lumped-mass matrix differs among the possible structural symmetry conditions. Let $m_i$ be the the lumped mass at level $i$ and let $J_i$ be the rotational mass moment of inertia lumped at level $i$. For no structural symmetry, the mass matrix is:
For structural symmetry type one and two

\[
[M]_0 = \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_L \\
\end{bmatrix}
\]

(4.24)

\[
[M]_{i,k} = \begin{bmatrix}
m_1 & \text{0} & \text{0} \\
\text{0} & m_2 & \text{0} \\
\vdots & \vdots & \vdots \\
\text{0} & \text{0} & m_L
\end{bmatrix}_{L \times L}
\]

(4.25)
For structural symmetry type three

\[
[M]_3 = \begin{bmatrix}
J_1 & 0 \\
J_2 & & 0 \\
 & & \ddots & \ddots \\
0 & & & J_L \\
\end{bmatrix}_{L \times L}
\]  \hspace{1cm} (4.26)

Note that the order of the mass matrix in each case must be the same as the number of available global dynamic degrees of freedom in the structure. For programming purposes only the diagonal coefficients of the lumped-mass matrix are stored in a vector.

4.9.3 DYNAMIC EQUILIBRIUM EQUATIONS

The dynamic equilibrium of the structure is described by a set of ordinary second order differential equations in the following form:

\[
[M]\{d^y_s\} + [C]\{d^x_s\} + [K^s]\{d^x_s\} = \{P(t)\}
\]  \hspace{1cm} (4.27)
where \( [M] \) = lumped-mass matrix
\( [C] \) = damping matrix
\( [K_s^*] \) = dynamic structure stiffness matrix
\( \{ P(t) \} \) = structural load vector associated with
the global dynamic degrees of freedom

\[
\{ d_s^* \} = \begin{bmatrix} \{ u \} \\ \{ v \} \\ \{ \theta \} \end{bmatrix} = \text{vector of global displacements relative to support motion}
\tag{4.28}
\]
\( \{ d_s \}^* \) = vector of absolute displacements along global degrees of freedom

Note that a dot (\( ' \)) over a symbol designates its first
derivative with respect to time while a double dot (\( '' \))
for its second derivative. The two displacement
vectors \( \{ d_s^* \} \) and \( \{ d_s \}^* \) are related in the following
fashion:

\[
\{ d_s \}^*_a = \{ r \} v_g + \{ d_s^* \} \tag{4.29}
\]
and

\[
\{ \dot{d}_s \}_a = \{ r \} \dot{v}_g + \{ \dot{d}_s^* \} \tag{4.30}
\]

where \( v_g \) is the ground displacement and \( \{ r \} \) is the
influence coefficient vector which describes the influence
of a unit ground displacement on the global degrees of freedom. These vectors have the following form for a typical level with no structural symmetry (Fig. 4.4):

\[
\begin{bmatrix}
U_a \\
V_a \\
\Theta_a
\end{bmatrix} = \begin{bmatrix}
\cos \gamma \\
\sin \gamma \\
0
\end{bmatrix} V_g + \begin{bmatrix}
u \\ v \\ \dot{\theta}
\end{bmatrix}
\]  \(4.31\)

and

\[
\begin{bmatrix}
\ddot{U}_a \\
\ddot{V}_a \\
\ddot{\Theta}_a
\end{bmatrix} = \begin{bmatrix}
\cos \gamma \\
\sin \gamma \\
0
\end{bmatrix} \ddot{V}_g + \begin{bmatrix}
\ddot{u} \\ \ddot{v} \\ \ddot{\theta}
\end{bmatrix}
\]  \(4.32\)

where \(\gamma\) = the angle formed going counterclockwise from the positive x axis to the line of ground motion.
Considering all levels, the influence coefficient vector \( \{ r \} \) is obtained.

\[
\{ r \}_o = \begin{pmatrix}
\cos\gamma \\
\cos\gamma \\
\vdots \\
\cos\gamma_{Lx1} \\
\sin\gamma \\
\sin\gamma \\
\vdots \\
\sin\gamma_{Lx1} \\
0 \\
0 \\
\vdots \\
0
\end{pmatrix}
\] (4.33)
For structural symmetry types 1 and 2 and 3 respectively, the influence coefficient vectors are

\[
\{ r \}_1 = \begin{bmatrix}
\cos \gamma \\
\cos \gamma \\
\vdots \\
\cos \gamma
\end{bmatrix}_{L \times 1}
\]  \hspace{1cm} (4.34)

and

\[
\{ r \}_2 = \begin{bmatrix}
\sin \gamma \\
\sin \gamma \\
\vdots \\
\sin \gamma
\end{bmatrix}_{L \times 1}
\]  \hspace{1cm} (4.35)

and

\[
\{ r \}_3 = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}_{L \times 1}
\]  \hspace{1cm} (4.36)
Note that for structural symmetry type 3, the influence coefficient vector is null, reflecting the exclusion of rotational ground motion considerations. Usage of this symmetry condition would yield trivial solutions for all parameters except mode shapes and frequencies.

In the case of a seismic analysis, there are no externally applied loads so that \( \{P(t)\} = \{0\} \). Eq. (4.27) may now be written as:

\[
[M] \{ \ddot{r} v_j + \ddot{\bar{d}}_j \} + [C] \{ \dot{\bar{d}}_j \} + [\bar{R}_s]\{ \ddot{\bar{d}}_j \} = \{0\}
\] (4.37)

or

\[
[M] \{ \ddot{\bar{d}}_j \} + [C] \{ \dot{\bar{d}}_j \} + [\bar{R}_s]\{ \ddot{\bar{d}}_j \} = -[M] \{ r \} \ddot{v}_j
\] (4.38)

Eq. (4.38) can be interpreted as the equation of motion of the structure not subjected to ground motion but resisting the following effective dynamic forces:

\[
\{ P_{eff} \} = -[M] \{ r \} \ddot{v}_j
\] (4.39)

The coupled set of ordinary second order differential equations of motion may be solved directly by numerical
integration; this technique is commonly used for inelastic analysis. However, for linear structures, a transformation of the degrees of freedom into a set of generalized coordinates usually called normal or modal coordinates via the mode shapes of the system, is much more efficient because the support motions tend to excite strongly only the lowest modes of vibration. This transformation will uncouple the dynamic equilibrium Eqs. (4.38) and the normal coordinates may be solved for independently.

4.9.4 MODE SHAPES AND NATURAL FREQUENCIES

The vibration mode shapes must satisfy the undamped free vibration equation given by

\[ [M] \{\ddot{d}_s^*\} + [K^*] \{d_s^*\} = \{0\} \]  \hspace{1cm} (4.40)

Assume a solution of the form

\[ \{d_s^*\} = \{\theta\} \cos(\lambda t + \gamma) \]  \hspace{1cm} (4.41)
where \( \lambda \) = angular frequency of vibration

Then

\[
\{d^*\} = -\lambda^2 \{\theta\} \cos(\lambda t + \gamma) \tag{4.42}
\]

Substituting Eqs. (4.41) and (4.42) into Eq. (4.40) yields

\[
-\lambda^2 \{\theta\} \cos(\lambda t + \gamma) + [\bar{K}^*] \{\theta\} \cos(\lambda t + \gamma) = \{0\} \tag{4.43}
\]

Since the cosine term is arbitrary, it can be cancelled yielding

\[
([\bar{K}^*] - \lambda^2 [M]) \{\theta\} = \{0\} \tag{4.44}
\]

This is the classical eigenvalue problem. The solution procedure adopted is applicable to the case where \([\bar{K}^*]\) is singular (20) (i.e. for unrestrained structures). Eq. (4.44) may be rewritten in the following form:

\[
\lambda^2 [M] \{\theta\} = [\bar{K}^*] \{\theta\} \tag{4.45}
\]
If \([\bar{K}_s^*]\) is positive definite, Choleski factorization can be used:
\[
[\bar{K}_s^*] = [L][L]^T
\]  
(4.46)

where \([L]\) is a lower triangular matrix. To accommodate the case where \([\bar{K}_s^*]\) is singular, Eq. (4.45) is transformed to
\[
([L]^T[M][L]^{-T}) ([L]^T\{\phi\}) = \frac{1}{\lambda^2} ([L]^T\{\phi\})
\]  
(4.47)

Letting \([D] = [L]^{-1}[M][L]^{-T}\)  
(4.48)

and \(\{\varphi\} = [L]^T\{\phi\}\)  
(4.49)

Eq. (4.47) may be rewritten in the following manner:
\[
[D]\{\varphi\} = \frac{1}{\lambda^2} \{\phi\}
\]  
(4.50)
With the above form the eigenvalues are $1/\lambda_i^2$ and eigenvectors are $\{\psi\}_i$. The natural frequencies are $\lambda_i$ and the corresponding mode shapes are $\{\phi\}_i$. The eigenvalues corresponding to the lower natural frequencies may be obtained by simultaneous iteration with matrix $[D]$ which is premultiplied by trail eigenvectors. The computer subroutines implemented into the present computer program for eigenvalue and eigenvector solutions were developed by Corr and Jennings (20). The parseness of the equations is taken advantage of by way of variable bandwidth storage of $[D]$, $[M]$, $[K^*_S]$, and $[L]$. For the present purposes, minor modifications of the subroutine were effected for the triangular storage form of $[K^*_S]$.

4.9.5 **UNCOPLED EQUATIONS OF MOTION**

As mentioned in section 4.9.3 the dynamic equilibrium Eq. (4.38) may be uncoupled by a suitable transformation of coordinates. Without any loss of generality, the dynamic global displacement vector may be represented by a superposition of mode shapes:

$$\{d^*_i\} = [\phi] \{\gamma(i)\} \tag{4.51}$$
where $[\phi]$ = matrix of mode shapes placed columnwise
$\{\eta(t)\} = $ vector of normal coordinates, representing the
modal amplitudes.

Substitution of the above transformation into Eq. (4.38)
yields

$$[M][\phi]\{\ddot{\eta}\} + [C][\phi]\{\dot{\eta}\} + [K_s][\phi]\{\eta\} = -[M]\{\eta\} \dot{\phi}$$

(4.52)

Multiplying by $[\phi]^T$ yields


(4.53)

Letting

$$[\bar{M}] = [\phi]^T[M][\phi]$$

(4.54)

and

$$[\bar{C}] = [\phi]^T[C][\phi]$$

(4.55)

and

$$[\bar{K}_s] = [\phi]^T[K_s][\phi]$$

(4.56)
and

\[
[L] = -[\varnothing]^T[M]\{r\}
\]  (4.57)

where

\[
[M] = \text{generalized mass matrix}
\]

\[
[C] = \text{generalized damping matrix}
\]

\[
[K] = \text{generalized global dynamic stiffness matrix}
\]

Eq. (4.53) becomes

\[
[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = [L]\{\ddot{v}\}
\]  (4.58)

Through the use of Betti's law, \([M]\) can be proven to be diagonal (21). Applying this condition to the undamped, free vibration Eq. (4.40) reveals that \([K]\) is also diagonal.

Rayleigh showed that the damping matrix \([C]\) is uncoupled by \([\varnothing]\) through the transformation of Eq. (4.55) (i.e. \([\varnothing]\) is diagonal) if it can be expressed as a linear combination of the mass matrix \([M]\), and the global dynamic stiffness matrix \([K_s]\). Clough and Penzien (21)
Further proved that this condition is satisfied by an infinite number of matrices formed from the above mass and stiffness matrices. The diagonal form of $[E]$ is adopted in the present formulation. Eq. (4.58) is now an uncoupled equation of motion in normal coordinates and can be expressed for each mode $n$ as follows:

$$M_n \ddot{\xi}_n + C_n \dot{\xi}_n + K_n \xi_n = L_n \ddot{y} \tag{4.59}$$

where $M_n$, $C_n$, $K_n$ are the $n$th diagonal elements of $[M]$, $[C]$, and $[K]$ respectively. They represent, respectively, the generalized mass, generalized damping, and generalized stiffness for mode number $n$.

$L_n$ is the $n$th element of $\{L\}$

Making use of Duhamel's Integral (21), and assuming low values of modal damping, the solution to Eq. (4.59) can be expressed as

$$\eta(t) = \frac{L_n}{\lambda_n M_n} V_n(t) \tag{4.60}$$
where

$$V_n(t) = \int_0^t \dot{\eta}_n(t') e^{-\lambda_n(t-t')} \sin \lambda_n(t-t') \, dt' \, dJ_n$$  \hspace{1cm} (4.61)$$

Once $\eta_n(t)$ is solved for each mode, Eq. (4.51) is used to obtain the global dynamic structural displacements. In practice, only the first few modes need to be considered.

### 4.9.6 RESPONSE SPECTRUM

Unless the ground acceleration records are available for a given earthquake, a more common solution technique is via the average response spectrum. The response spectrum provides the maximum average response (from many earthquake records) of any given single degree of freedom (in this case normal coordinate) oscillator with specified period of vibration and damping value. From Eq. (4.60), the maximum modal amplitude is given by

$$\eta_{n,\text{max}} = \frac{S_{vn}}{\lambda_n M_n}$$  \hspace{1cm} (4.62)$$
where \( S_{vn} = V_n(t)_{\text{max}} \) is the spectral pseudo-velocity response for mode \( n \), which can be read off from the spectrum curves.

Other response measures are the spectral displacement \( S_d \), and the spectral acceleration \( S_a \) which are related to the spectral pseudo-velocity \( S_v \) as follows:

\[
S_d = \frac{S_v}{\lambda} \quad (4.63)
\]

\[
S_a = \lambda S_v = \lambda^2 S_d \quad (4.64)
\]

Examples of response spectrum curves are shown in Fig. (4.6). For a given period \( T \) and damping value \( \xi \), the spectral values \( S_d, S_v, \) and \( S_a \) can be readily obtained from a response spectrum established from the earthquake records of a particular site. Response spectrum curves can also be established for an estimate of the inelastic response by means of a ductility ratio. The response
spectrum given in the Supplement to the National Building Code of Canada 1980 (22) will be used in all the examples presented in the latter chapters (Fig. 4.7).

4.9.7 STRUCTURAL MODAL RESPONSES TO EARTHQUAKE GROUND MOTIONS

Once the natural frequencies and mode shapes of the first few predominant modes are determined, the corresponding modal participation factors can be determined as

\[
\Gamma_h = \frac{\Gamma_n}{m_n} = \frac{[\phi]^T_n [M] [\ddot{r}]}{[\phi]^T_n [M] [\phi]_n}
\]  

(4.65)

For a given damping ratio and period, \( S_V \) can be extracted from the response spectrum. If an elasto-plastic analysis is necessary, the methodology of (22) may be applied. A ductility factor \( \mu \) must be selected depending on the type of building and materials (Table 4.1). \( S_V \) must then be altered as follows:
\[ S_{Vn} \leftarrow \frac{S_{Vn}}{\mu} \quad \text{valid for } T > 0.5 \text{ sec} \quad (4.66) \]

\[ S_{Vn} \leftarrow \frac{S_{Vn}}{\sqrt{2\mu - 1}} \quad \text{valid for } T < 0.5 \text{ sec} \quad (4.67) \]

The maximum modal amplitude is determined from Eqs. (4.62) and (4.65) yielding

\[ \eta_{n, \text{max}} = \frac{\Gamma_n}{\lambda_n} S_{Vn} \quad (4.68) \]

which can be converted to the structural physical displacements by Eq. (4.51).

\[ \{d^*_s\}_{n, \text{max}} = \{\varphi\}_n \eta_{n, \text{max}} \quad (4.69) \]

The condensed corner degrees of freedom are then recovered, as explained in section 4.5, to yield the maximum corner displacements \( \{u\}_n, \{\omega_1\}_n, \ldots, \{\omega_c\}_n \).

The maximum elastic forces \( \{f_s\}_{n, \text{max}} \) associated with the global dynamic degrees of freedom are sought next.
\[
\{f_s\}_{n,\text{max}} = \left[ F_s^* \right] \{d_s^*\}_{n,\text{max}}
\]  
\[(4.70)\]

Substituting Eq. (4.69) yields
\[
\{f_s\}_{n,\text{max}} = \left[ F_s^* \right] \{ \hat{d}_s \}_{n, \text{max}}
\]  
\[(4.71)\]

Making use of Eq. (4.45) gives
\[
\{f_s\}_{n,\text{max}} = \lambda^2_n \{M\} \{\hat{d}\}_n \gamma_{n,\text{max}}
\]  
\[(4.72)\]

Applying Eq. (4.62) produces
\[
\{f_s\}_{n,\text{max}} = \{M\} \{\hat{d}\}_n \lambda_n \frac{\Gamma_n}{M_n} S_v n
\]  
\[(4.73)\]

Finally substituting Eq. (4.65) yields
\[
\{f_s\}_{n,\text{max}} = \{M\} \{\hat{d}\}_n \lambda_n \Gamma_n S_v n
\]  
\[(4.74)\]

This expression for the elastic forces is more convenient because of the diagonal form of the lumped-mass matrix.
It is emphasized that Eq. (4.74) is a completely general expression for the elastic forces developed in a damped structure subjected to arbitrarily varying ground motions; even though it was derived using an expression for undamped free vibrations, its applicability is not limited.

The structure can now be analyzed statically by applying the elastic forces externally for each mode. The maximum base shears in the $x$ and $y$ directions, and maximum base torque respectively are:

$$
V_{u,n,max} = u \{1\}^T_{Lx1} \{O\}^T_{Lx1} \{O\}^T_{Lx1} \{f_s\} n_{max} \tag{4.75a}
$$

$$
V_{v,n,max} = \{1\}^T_{Lx1} \{1\}^T_{Lx1} \{1\}^T_{Lx1} \{f_s\} n_{max} \tag{4.75b}
$$

$$
V_{\theta,n,max} = \{1\}^T_{Lx1} \{O\}^T_{Lx1} \{O\}^T_{Lx1} \{f_s\} n_{max} \tag{4.75c}
$$

where $\{1\}$ = unit column vector

The maximum base overturning moments about the $x$ and $y$ axes respectively are:

$$
M_{u,n,max} = [ \{O\}^T_{Lx1} \{h\}^T_{Lx1} \{O\}^T_{Lx1} \{f_s\} ] n_{max} \tag{4.76a}
$$

$$
M_{v,n,max} = [ \{h\}^T_{Lx1} \{O\}^T_{Lx1} \{O\}^T_{Lx1} \{f_s\} ] n_{max} \tag{4.76b}
$$
where \( \{ h \} \) = vector of level elevations measured from the building base.

Once all maximum modal global displacements are obtained, the maximum modal membrane stresses are computed as explained in section 4.8.2.

4.9.8 FINAL STRUCTURAL RESPONSE TO EARTHQUAKE GROUND MOTIONS

The calculations described in the previous sections are carried out for each mode. The "total" maximum response obtained as the absolute sum of the individual maximum modal responses would be overly conservative because these maximum modal responses do not all occur at the same instant of time.

The square-root-sum-of-squares (SRSS) method is a simple statistical method of estimating the probable maximum response. Suppose \( q_i \)'s are the maximum modal responses of a certain parameter for modes \( i = 1, 2, \ldots, n \) respectively. The absolute maximum response would be

\[
a = \sum_{i=1}^{n} |q_i|
\]  

(4.77)
and the probable maximum response would be

\[ a = \sqrt{\sum_{i=1}^{n} (a_i)^2} \]  \hspace{1cm} (4.78)

The SRSS method will be used in this thesis.
<table>
<thead>
<tr>
<th>Building Type</th>
<th>Structural Ductility Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ductile moment resisting space frame</td>
<td>4</td>
</tr>
<tr>
<td>Combined system of 25 percent ductile moment resisting space frame and ductile flexural walls</td>
<td>3</td>
</tr>
<tr>
<td>Ductile reinforced concrete flexural walls</td>
<td>3</td>
</tr>
<tr>
<td>Regular reinforced concrete structures, cross-braced frame structures and reinforced masonry structures</td>
<td>2</td>
</tr>
<tr>
<td>Structures having no ductility and plain masonry structures</td>
<td>1</td>
</tr>
</tbody>
</table>
Fig. 4.1 Structure idealization

- **Corner Number**
- **Global Corner Nodes**
- **Local Internal Facade Nodes**

(a) Actual Framed Tube Structure

(b) Equivalent Orthotropic Tube Divided into Elements

(c) Plan with Corners and Facades
(a) ORDINARY ELEMENT

(b) REFINED ELEMENT

Fig. 4.2 Automated numbering system of nodes and elements.
X, Y, Z is an arbitrary located system of orthogonal axes.

Fig. 4.3 Global static degrees of freedom at the i^th level.
X, Y axes system is located at the mass center of the level.

Fig. 4.4 Global dynamic degrees of freedom at level i and the direction of ground motion.
<table>
<thead>
<tr>
<th>NO. SYMMETRY</th>
<th>ORDINARY ELEMENT</th>
<th>Refined Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td><img src="image" alt="Ordinary Element" /></td>
<td><img src="image" alt="Refined Element" /></td>
</tr>
<tr>
<td>(c)</td>
<td><img src="image" alt="Symm. Type 1" /></td>
<td><img src="image" alt="Symm. Type 1" /></td>
</tr>
<tr>
<td>(e)</td>
<td><img src="image" alt="Symm. Type 2" /></td>
<td><img src="image" alt="Symm. Type 2" /></td>
</tr>
<tr>
<td>(g)</td>
<td><img src="image" alt="Symm. Type 3" /></td>
<td><img src="image" alt="Symm. Type 3" /></td>
</tr>
</tbody>
</table>

Internal degrees of freedom are those surrounded by the dotted line.

Fig. 4.5 Automated façade degrees of freedom system
Fig. 4.6 Response spectra for El Centro earthquake, 1940. (21)

Fig. 4.7 Peak ground motion bounds and elastic average response spectrum for 1.0 g max. ground accel. (22)
CHAPTER V

EXAMPLES OF TUBE PROGRAM APPLICATION

5.1 INTRODUCTION

A program named TUBE has been developed to analyse most planar and tube-type building structures. Several examples of these types of structures have been cited earlier in Section 4.1. The analysis can be carried out for linear elastic static, or linear elastic earthquake spectral responses with an option to take into account the ductility factor. It is noted that response spectrum curves are required for the earthquake analysis. Description of the program input data is given in Appendix A.

In this chapter, four structures are analysed using the present method, and the results will be compared with other techniques. The objective is to verify the validity of the proposed modelling for seismic responses of tall planar and tube-type structures.

In chapter III, the stiffness matrices of two types of rectangular elements were presented: one consisting of 4
nodes and 6 degrees of freedom, and the other consisting of 6 nodes and 9 degrees of freedom. The linear 6 degree of freedom elements are attractive because they have only corner nodes and are simple to use, but they are too stiff in bending. For very tall buildings where the elements may have high aspect ratios, these simple elements may yield unacceptably inaccurate results. Alternative schemes for improvement generally fall into two classes: hybrid elements and incompatible displacement elements. These alternative schemes should be studied, however, they are beyond the scope of the present project. Throughout this chapter, the refined 9 degree of freedom element will be used.

5.2 BOX CANTILEVER BEAM

To verify the validity of the present macro-element technique applied to simple structures, a box cantilever beam is considered. Typical dimensions and properties of the structure are given in Fig. 5.1. Three analysis techniques will be compared: Goodno's and Gere's superelement method (23), the present macro-element method, and beam theory.

In the superelement technique, a basic rectangular
element is originally assembled from a plate bending element and a plane stress element each containing 16 degrees of freedom. This basic element has 4 corner nodes each consisting of 8 degrees of freedom: 3 orthogonal translations, 2 out-of-plane rotations, 2 independent in-plane edge rotations, and 1 in-plane nodal twist. To expedite the assembly of the superelement, other specialized elements are generated from the basic element. Four elements are subsequently assembled to form a box-shaped superelement. The superelement stiffness matrix is statically condensed to 4 degrees of freedom representing horizontal translations of the top and bottom edges parallel to one direction (Fig. 5.2). The superelement reduced stiffness matrix are assembled to give the structure stiffness matrix. Three different mass formulations are used: consistent mass (CM), assembled lumped mass (ALM), and input lumped mass (ILM). The CM formulation permits the mass of the actual continuous structure to be lumped at the corner nodes of the rectangular element on an energy equivalent basis. The ALM formulation in contrast, requires physical mass lumping at each of the 4 nodes. The element mass matrices of the 4 basic or specialized elements are then assembled to form the superelement mass matrix which is subsequently
dynamically condensed to the 4 parallel horizontal degrees of freedom. The structure mass matrix is then assembled in a similar fashion as the structure stiffness matrix. Unlike the CM and ALM formulations, the ILM formulation permits a diagonal structure mass matrix because it is created from the lumped masses associated with the acceleration of the 4 remaining superelement degrees of freedom.

Goodno and Gere modelled the box cantilever beam using 8 superelements (Fig. 5.2). Sixteen structure degrees of freedom were retained from the 352 nodal degrees of freedom represented in the model.

Using the present method, the box cantilever beam is modelled as follows. Due to symmetry, only one quarter of the structure is analysed. No part of the structure needs to be replaced by an elastically equivalent membrane because no discrete columns or beams exist. For simplicity the remaining structure is analysed using equally sized refined elements. To show convergence, two meshes are used; the first and second meshes are divided into 4 and 8 levels respectively. Each mesh has two elements horizontally spanning the longer facade and one element horizontally spanning the shorter facade (Fig. 5.1c). The input data presented in section A.5 was used
to obtain the first four natural frequencies of the structure in the x direction using the first mesh.

Employing a discrete coordinate system for a dynamic response analysis can only yield approximate results because the motions of the system are represented by a limited number of displacement coordinates. In contrast, the beam theory approach renders more accurate results because it considers the behaviour of an infinite number of connected points by means of differential equations; this procedure allows the distributed physical properties of the structure to be described at each one of these points.

Table 5.1 compares the natural frequencies in the x direction obtained from beam theory taking into account shear deformations, the superelement technique, and the present macro-element method.

The first three natural frequencies obtained using mesh 1 are all within 7% of beam theory results, but the fourth natural frequency is 25% less. For the refined mesh 2 the corresponding values are 8% and 11%. Convergence for higher modes can thus be obtained by mesh refinement.

The efficiency of the present method can be exposed when comparing it to the superelement method. For the first mesh, only 4 structure degrees of freedom were
retained from the 32 nodal degrees of freedom represented in the model. Respectively, these values are 1/4 and 1/11 of those using the superelement technique. Although mesh 1 uses significantly less degrees of freedom than the superelement model, its first two natural frequencies (which are the greatest contributors to the structural responses) are closer to those of beam theory than the ILM formulation. Compared to beam theory, the ALM formulation provides the best results for the superelement method, but does not justify the extra computational effort due to mass coupling. Mesh 2 provides comparable accuracy with lumped masses (i.e. diagonal mass matrix).

5.3 CONCRETE WALL-FRAME BUILDING

In this example, a 30-storey concrete wall-frame building is analysed (Fig. 5.3a). Two parallel shear walls are connected to two parallel frames forming a rectangular plan. The structure has the following properties:

storey height = 12 ft.
width of structure = 127.5 ft.
depth of structure = 50.0 ft.
lumped weight per floor = 1305 kips
Frame properties:
16 columns per frame at 7.5 ft. c/c spacing

Column size = 1.0 ft. x 1.5 ft.
Beam size = 0.5 ft. x 2.0 ft.

Wall properties:
Thickness = 1.0 ft.

Material Properties:
elastic modulus = $3.0 \times 10^6$ psi.
Poisson's ratio = 0.25

Although the structure looks like a cantilever tube, classical beam theory cannot be applied for analysis. Due to the flexibility of the spandrel beams in the frames, the column axial forces near the center lag behind (are lower than) the column axial forces near the building edges. This phenomenon is known as shear lag. This example shows that wall-frame combinations forming a tubular structure can be analysed using the present method.

The results of the analysis are compared to those obtained by Qian (3). His formulation is derived from the principle of minimum total potential energy. The strain energy is considered to be composed of:

1) in-plane bending of the shear wall
2) axial deformations of columns
3) in-plane bending of beams and columns
4) out-of-plane bending of columns; found to be negligible

It should be noted that shear deformations in the shear wall, beams and columns are not considered. Finite size joints are considered rigid. In-plane rigidity of floors and midspan inflection points of beams and columns are assumed as in the present method. Mass is distributed throughout the height of the building. Compatibility is imposed at the building edges while the column axial deformations, which take into account the "magnitude of shear lag in the normal frame", are approximated by a hyperbolic cosine variation or a parabolic variation. A sixth order differential equation for the mode shapes in the direction parallel to the shear walls is derived. A summation of 4 hyperbolic and 2 trigonometric terms is presented as the general solution. The mode shape equation is subjected to 6 boundary conditions which produce a homogeneous system of linear equations. The natural frequencies are those which yield a zero determinant for the coefficient matrix (i.e. a trial-and-error procedure).
Using the present method, the structure is modelled as follows. Due to symmetry, only one quarter of the structure need be analysed using global structural symmetry type one. The framed portion of the remaining structure is replaced by an elastically equivalent membrane. This equivalent structure is then divided into refined macro-elements forming a mesh (Fig 5.3b). If a full spectral earthquake analysis for ground motion in the x direction is required, the set of input data presented in section A.5 must be processed with TUBE.

Using a membrane thickness of 0.5 ft., the equivalent elastic properties of the membrane were determined to be:

$$E_y = 159 \times 10^3 \text{ ksf}.$$  
$$G_{xy} = 13.8 \times 10^3 \text{ ksf}.$$  

Their ratio $G_{xy}/E_y$ indicates the degree of shear lag present in the frame (see section 6.2).

Table 5.2 shows the first three natural frequencies obtained by Chan, the present method, and beam theory. For the latter, the building was first considered as i) a cantilever tube beam (i.e. shear lag is ignored), and then ii) as two parallel shear walls excluding the framed facades. The results for the two parallel shear walls were found to be close to those obtained by Chan and the present method. This suggests that preliminary estimates of the dynamic characteristics of wall-frame structures can be obtained by considering only the shear walls. The
cantilever tube beam model yields natural frequencies which are at least 40% greater than any other model presented. The first, second, and third natural frequencies obtained using the present method are approximately 1.11, 0.89, 0.78 times those obtained by Chan's approximate method. This significant difference in results may be due to the following approximations in Chan's formulation:

1) Shear deformation modes in beams and columns are ignored.
2) Finite size joints are considered rigid.
3) An assumed axial column deformation function.

5.4 PLANAR FRAME

In this example, a 16-bay, 40-storey plane frame will be analysed. The structure has the following properties along with those shown in Fig. 5.4:

storey height \( = 12 \text{ ft.} \)
lumped weight per storey \( = 1800 \text{ kips} \)
elastic modulus \( = 3.0 \times 10^6 \text{ psi.} \)
shear modulus \( = 1.2 \times 10^6 \text{ psi.} \)
As in the previous example, the shear lag effect inhibits the accurate application of beam theory. The purpose of this example is to show that TUBE is capable of analysing planar structures efficiently and accurately. The results will be compared to those of another modified computer program called TABS (24). TABS assumes infinite in-plane rigidity of floors. Joints are considered rigid with finite dimensions. Both bending and shear deformations of beams and columns are recognized. TABS allocates one vertical and one in-plane rotational degree of freedom per joint, and one lateral translational degree of freedom per storey. The joint degrees of freedom are statically condensed to leave the lateral storey translations which are transformed into global degrees of freedom consisting of two orthogonal translations and one in-plane rotation per storey. Mass and in-plane rotational inertia is specified for each storey.

For the present method, only half the structure is considered because of symmetry. The remaining structure is replaced by an equivalent orthotropic membrane which is subsequently divided into equal size macro-elements for ease of data preparation. Here, structure symmetry type one and facade symmetry type three is used. Two analyses will be performed; the first will include the same
assumptions as TABS, and the other will consider flexible finite size joints.

The first four natural frequencies are presented in Table 5.3. Note that TABS uses 40 degrees of freedom for the natural frequency calculation, while TUBE uses only 10; this is a very significant difference in computational effort. However, the natural frequencies of the first analysis are comparable with those of TABS. It was expected that the present method would yield higher natural frequencies due to the model's greater stiffness induced by the prescribed element displacement functions; TABS uses no such functions. This would only be true if a consistent-mass formulation was used.

The second analysis, which includes the effects of finite size joint flexibility, alters from 9% to 13% lower natural frequencies compared to TABS. In the next chapter, it will be shown that this percentage difference can be magnified for the internal member forces.

5.5 FRAMED-TUBE BUILDING

In this example, a framed-tube building will be analysed. The structure is square in plan with each facade consisting of a 16-bay, 40-storey frame.
The structure has the following properties along with those shown in Fig. 5.5:

storey height \(= 12 \text{ ft.}\)
lumped weight per floor \(= 3,600 \text{ kips}\)
elastic modulus \(= 3.0 \times 10^6 \text{ psi.}\)
shear modulus \(= 1.2 \times 10^6 \text{ psi.}\)

Many computer programs such as TABS, do not consider the vertical compatibility at facade junctions (i.e. after loading their edges slide vertically relative to each other). The present method eliminates this action at the corner nodes of the building. The result is a stiffer, but more representative model of the actual structure.

The results obtained by TABS and Chan (3) will be used as comparisons for the present method. Chan's method of analysis is analogous to that presented in section 5.3 except i) out-of-plane bending of columns is ignored, and ii) a hyperbolic sine variation is assumed for the column axial deformations.

Using the present method, only one quarter of the structure is analysed because of symmetry conditions. The remaining structure is replaced by an elastically equivalent orthotropic membrane which is divided into
rectangular plane stress refined macro-elements each spanning several bays and storeys (Fig. 5.5). Note that elements of equal size are used here only to simplify data preparation. Some useful rules for better modelling of the structure are presented in section A.4.1. Three analyses were performed for the natural frequencies and mode shapes of the building in the x direction. Each considers a different combination of deformation modes.

i) Analysis 1: bending of beams and columns, rigid finite size joints (same deformation modes as Chan)

ii) Analysis 2: bending of beams and columns, shear in beams and columns, rigid finite size joints (same deformation modes as TABS)

iii) Analysis 3: bending of beams and columns, shear in beams and columns, flexible finite size joints
Table 5.4 displays the first four natural frequencies. Although Chan's method is approximate, the results compare well with analysis 1; the maximum difference being 4.7% for the second mode. The effect of shear deformations in members is found to be negligible when comparing with analysis 2.

Observe that the natural frequencies of TABS are identical to those presented in the preceding section. This was expected since

i) the frame properties are identical to those of the previous section.

ii) the normal frame does not contribute to the structural stiffness because facades exhibit only in-plane stiffness, and because vertical compatibility at facade junctions are not enforced.

The resulting internal member force in the parallel facade are unreliable, while those of the normal facade are nonexistent. The first natural frequency of analysis 2 is 11.4% higher than TABS' value reflecting a stiffer model induced by the
vertical compatibility at the facade junction. If the normal frames are replaced by shear walls, thereby increasing the overall stiffness of the structure, the difference will increase even more.

As for the case of the wall-frame structure of section 5.3, the normal frame has little contribution to the dynamic behaviour. Preliminary estimates of the natural frequencies and mode shapes can be obtained by considering only the frames parallel to the vibration direction.

The results of analysis 3 are the most accurate because flexibility of finite size joints is also considered. The resulting model is more flexible than that of analysis 2. Consequently, the natural frequencies are lower.

5.6 DISCUSSION

The present method was applied to a planar frame, and 4 three-dimensional assemblies of planar frames and planar shear walls. Although not presented here, the present method is versatile enough to consider panels, frames, lintel beams, and shear walls all combined in planar facades.

The efficiency of the present method is significantly
greater than others presented (i.e. superelement technique and TABS); only one quarter of the global dynamic degrees of freedom were used, but results were in good agreement.

It was shown that increasing the number of levels in the structural model induces convergence of higher natural frequencies. Enforcing vertical compatibility at facade junctions was found to be important.

In examining the validity of the present method, some important structural behaviour characteristics were found:

1) The effect of shear deformations in members has little contribution to the dynamic behaviour.

2) Finite size joint flexibility significantly affects the dynamic behaviour.

3) Contribution of the normal facades of rectangular buildings to the dynamic behaviour is significant but small so that rough estimates can be obtained by considering only the parallel facades.
Table 5.1 Natural frequencies of box cantilever beam

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural Frequencies (rad/sec)</th>
<th>Beam Theory (23)</th>
<th>TUBE Program Mesh No. 1</th>
<th>TUBE Program Mesh No. 2</th>
<th>Superelement Method (23)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2 Natural frequencies (rad/sec) of wall-frame building.

<table>
<thead>
<tr>
<th>Method</th>
<th>Mode 1</th>
<th>Mode 2</th>
<th>Mode 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple Beam</td>
<td>1.47</td>
<td>9.18</td>
<td>25.69</td>
</tr>
<tr>
<td>Theory(3)</td>
<td>2.23</td>
<td>13.97</td>
<td>39.11</td>
</tr>
<tr>
<td>CHAN (3)</td>
<td>1.47</td>
<td>10.07</td>
<td>27.88</td>
</tr>
<tr>
<td>TUBE</td>
<td>1.64</td>
<td>8.78</td>
<td>22.27</td>
</tr>
</tbody>
</table>
Table 5.3 Natural frequencies of planar frame

<table>
<thead>
<tr>
<th>MODE NUMBER</th>
<th>NATURAL FREQUENCY (rad/sec)</th>
<th>% diff. with TABS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TABS(24)</td>
<td>TUBE</td>
</tr>
<tr>
<td>1</td>
<td>.710</td>
<td>.697</td>
</tr>
<tr>
<td>2</td>
<td>2.31</td>
<td>2.23</td>
</tr>
<tr>
<td>3</td>
<td>4.33</td>
<td>4.10</td>
</tr>
<tr>
<td>4</td>
<td>6.21</td>
<td>5.75</td>
</tr>
<tr>
<td>NO. OF RETAINED DEGREES OF FREEDOM</td>
<td>40</td>
<td>10</td>
</tr>
</tbody>
</table>
Table 5.4 Natural frequencies of framed-tube building.

<table>
<thead>
<tr>
<th>MODE NUMBER</th>
<th>NATURAL FREQUENCY (rad/sec)</th>
<th>TABS (24)</th>
<th>CHAN (3)</th>
<th>ANALYSIS 1</th>
<th>ANALYSIS 2</th>
<th>ANALYSIS 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>.710</td>
<td>.792</td>
<td>.803</td>
<td>.791</td>
<td>.740</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.31</td>
<td>2.33</td>
<td>2.44</td>
<td>2.40</td>
<td>2.25</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>4.33</td>
<td>4.22</td>
<td>4.23</td>
<td>4.16</td>
<td>3.92</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>6.21</td>
<td>5.96</td>
<td>5.85</td>
<td>5.75</td>
<td>5.43</td>
</tr>
</tbody>
</table>
(b) Elevation

Facade No. 1  Facade No. 2

(c) Structure Idealization (Mesh 1)

(d) Properties (steel)
- Young's modulus 30000 ksi
- Poisson's ratio 0.25
- Specific weight 0.284 lb/in$^3$

(a) Plan
- Only the shaded portion is modelled

Fig. 5.1 Box cantilever beam and present model
Fig. 5.2 Superelement model of box cantilever beam
(a) Ground motion

(b) Structure Idealization

Fig. 5.3 Wall-frame building and the present idealization.
Fig. 5.4 Planar frame, properties, and model.
Fig. 5.5 Framed-tube building, properties, and idealization.
CHAPTER VI

BEHAVIOURAL CHARACTERISTICS
OF TALL FRAMED-TUBE BUILDING STRUCTURES

6.1 INTRODUCTION

In this chapter, the behavioural characteristics of tall framed-tube buildings are investigated. In particular, the shear lag phenomenon, and the effects of shear deformations and of finite size joints on the dynamic responses are of special interest. For this study, the framed-tube structure presented in section 5.5 is used in conjunction with the following additional data:

1) 5\% critical damping for all modes
2) risk factor of 0.01 (for Vancouver); an acceleration of 0.08g is recommended (22)
3) the response spectrum as shown in Fig. 4.7 (22).

6.2 SHEAR LAG PHENOMENON

In the case of a slender tube made up of solid faces and subjected to lateral load, the normal stress
distribution would be almost linear. However, when the faces are perforated with large openings resembling windows in tall buildings, the shear flexibility of the "window lintel beams" tend to diminish the shear transfer from one "column" to another with the consequence that the interior "columns" take less axial forces than the exterior ones. This nonlinearity of stress distribution is most pronounced at the tube corners, and the term shear lag is used to denote this phenomenon.

To reduce the shear lag effect (i.e. to make the tube more effective in resisting lateral load), the beam stiffnesses should be high, thus requiring relatively deep beams. A measure of the shear lag severity can be effected by means of the ratio

\[
SL = \frac{G_{xy}}{F_y}
\]  

(6.1)

The higher the shear lag parameter, the more effective the tube action will be.

The shear lag parameter for the cantilever tube discussed in section 5.2 is 0.40. Fig. 6.1 shows that the vertical stress variation is almost linear.
Taking the basic geometry of Chan's building, and varying the shear modulus $G_{xy}$, the column axial stresses are plotted for different values of the shear lag parameter as shown in Fig. 6.2. The shear lag parameter varies from 0.0199 to 0.0819. These values are much less than that of the cantilever tube above resulting in highly nonlinear curves. Note that the linearity increases with increasing SL especially for the parallel facade.

In terms of the actual structure, as opposed to the equivalent membrane model, $G_{xy}$ can be increased while leaving $E_y$ relatively constant simply by increasing the beam depth (Table 6.1). A stiffer beam thus yields a higher shear lag parameter and permits more efficient shear transfer from one column to the other. The shear lag effect diminishes resulting in a more linear column axial force variation.

The magnitude of the maximum internal member forces depend heavily on the natural frequencies which are ultimately derived from the mass and stiffness properties of the structure. A correlation can also be made here between the shear lag parameter and the natural frequencies. Varying $G_{xy}$ as before, the first four natural frequencies are plotted in Fig. 6.3. The natural frequencies increase with increasing shear lag parameter.
Similar curves are presented in Chapter VII to be used in conjunction with simplified equations for natural frequency estimations of planar and tube-type building structures.

6.3 IMPORTANCE OF SHEAR DEFORMATIONS, FLEXIBLE FINITE SIZE JOINTS, AND RESPONSE SPECTRUM CONSIDERATION

The natural frequencies of the framed-tube structure are presented in section 5.5. It was concluded that omitting any kind of deformation component results in a stiffer model, and consequently higher frequencies. However, a designer's concern is with respect to their effects on the internal member forces. The column axial forces of four analyses will be investigated to determine the importance of shear deformations in beams and columns and finite size joint flexibility. The deformation considerations of each analysis are listed below.

Analysis 1: bending in beams and columns, shear in beams and columns, flexible finite size joints.
Analysis 2: bending in beams and columns, shear in columns, (shear in beams is ignored), flexible finite size joints.

Analysis 3: bending in beams and columns, shear in beams, (shear in columns is ignored), flexible finite size joints.

Analysis 4: bending in beams and columns, shear in beams and columns, rigid finite size joints.

The results are plotted in Fig. 6.4. Ignoring shear deformations in beams (analysis 2) and columns (analysis 3) increases the corner column (No. 8) axial force by 2.1% and 4.6% respectively. If rigid joints are considered (analysis 3), there is an acute increase of 17%. The joint size of this structure in relation to the storey height and bay width is typical of framed-tube buildings. If the relative size of the joint is increased, the error due to the rigid joint assumption also increases. For a
reasonably accurate analysis, joints must be considered flexible. The accuracy may still be slightly enhanced by considering shear deformations in beams and columns.

It was stated in section 4.9.7 that once the elastic forces on the structure are determined, the structure can be analysed statically for each mode. From Eq. (4.74), the elastic force vector for each mode depends on \( \lambda, \Gamma, S_v, \{0\}, [M] \). For each of the analyses, \([M]\) remains the same, while \(\{0\}\) was almost identical for corresponding modes; from Eq. (4.65), \(\Gamma\) must also be the same for corresponding modes. We can thus conclude that \(\lambda\) and \(S_v\) are the major factors which changed the elastic force vectors from one analysis to another. Table 6.2 displays \(\lambda, S_v, (\lambda S_v)_m / (\lambda S_v)_1\) where the subscripts refer to the analysis number. The last ratio is termed the error factor which reveals the factor by which the elastic force vector of analysis \(m\) is greater than that of analysis 1 for a specific mode. Because the analyses are linear elastic, the member internal forces of analysis \(m\) are also greater by this factor for each corresponding mode. Notice that the frequencies increase with analysis number \((m)\). For mode number one, the pseudo-velocities, which depend on the frequencies, also increase. Thus both effects strengthen the error factor.
For the other modes, the pseudo-velocity stays constant, and thus only the ratio of natural frequencies provide the only error factor. In other words the response spectrum acts in such a way that errors are magnified for modes with periods greater than approximately 5 seconds. As shown above, the first and perhaps the second mode, which are the major contributors to the structural behaviour, lie in this region, and thus consideration of flexible joints is vital. Shear deformation considerations are less essential to the analysis.

6.4 DISCUSSION

Slender planar and tube-type structures with solid (unperforated) faces exhibit an almost linear normal stress distribution. When the faces are perforated, the distribution becomes nonlinear. A shear lag parameter was introduced to measure the degree of linearity. It was found that increasing the beam depth, also increased the shear lag parameter, inducing a more linear distribution especially in the parallel facade.

A higher shear lag parameter was also found to increase the natural frequencies which cause an increase in the pseudo-velocities of the primary modes and consequently
increase the maximum effective external elastic forces and internal stresses.

Flexibility of finite size joints was found to significantly affect the internal forces. Rigid joints yielded approximately 20% higher column axial forces at the second storey of a framed-tube building. Shear deformation considerations had substantial but less significant effects and thus may be ignored at least for preliminary purposes.

The nature of the response spectrum was found to magnify errors in the internal forces induced by approximations in modelling the stiffness properties.
Table 6.1 Variation of $G_{xy}$, $E_y$, and SL with beam depth.

<table>
<thead>
<tr>
<th>BEAM DEPTH (ft.)</th>
<th>EQUIVALENT MODULUS* (psf.)</th>
<th>SL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_{xy}$</td>
<td>$E_y$</td>
</tr>
<tr>
<td>2.5*</td>
<td>345</td>
<td>.172</td>
</tr>
<tr>
<td>3</td>
<td>433</td>
<td>.172</td>
</tr>
<tr>
<td>3.5</td>
<td>536</td>
<td>.173</td>
</tr>
<tr>
<td>4</td>
<td>656</td>
<td>.174</td>
</tr>
<tr>
<td>5</td>
<td>986</td>
<td>.178</td>
</tr>
<tr>
<td>6</td>
<td>1515</td>
<td>.185</td>
</tr>
</tbody>
</table>

* The membrane thickness is taken as 0.75 ft  
* Basic structure
### TABLE 6.2
Modal participation factors, natural frequencies, pseudo-velocities, and error factors for framed-tube building.

<table>
<thead>
<tr>
<th>MODE NO.</th>
<th>ANALYSIS NO.</th>
<th>$\Gamma$ (rad/sec)</th>
<th>$\lambda$ (ft/sec)</th>
<th>$S_v$</th>
<th>$(\lambda S_v)_m / (\lambda S_v)_l$ (error factor)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.35</td>
<td>.740</td>
<td>.311</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.35</td>
<td>.749</td>
<td>.315</td>
<td>1.025</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>1.35</td>
<td>.759</td>
<td>.319</td>
<td>1.052</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.34</td>
<td>.791</td>
<td>.332</td>
<td>1.141</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.510</td>
<td>2.25</td>
<td>.525</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.511</td>
<td>2.28</td>
<td>.525</td>
<td>1.013</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.513</td>
<td>2.31</td>
<td>.525</td>
<td>1.027</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.503</td>
<td>2.40</td>
<td>.525</td>
<td>1.067</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0.264</td>
<td>3.92</td>
<td>.525</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.265</td>
<td>3.98</td>
<td>.525</td>
<td>1.015</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.265</td>
<td>4.03</td>
<td>.525</td>
<td>1.028</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.263</td>
<td>4.16</td>
<td>.525</td>
<td>1.061</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.176</td>
<td>5.43</td>
<td>.525</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.176</td>
<td>5.51</td>
<td>.525</td>
<td>1.015</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.177</td>
<td>5.59</td>
<td>.525</td>
<td>1.029</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.176</td>
<td>5.75</td>
<td>.525</td>
<td>1.059</td>
</tr>
</tbody>
</table>
Fig. 6.1 Variation of normal stress at 3' elevation of the box cantilever beam.
Fig. 6.2 Effect of the shear lag parameter (SL) on the column axial stresses at the fourth storey of the framed-tube building.
Fig. 6.3 Effect of the shear lag parameter (SL) on the natural frequencies of the framed-tube building.
Fig. 6.4 Column axial forces at the second storey of the framed-tube building
CHAPTER VII

NATURAL FREQUENCY DETERMINATION FOR PLANAR AND TUBE-TYPE STRUCTURES

7.1 INTRODUCTION

This chapter provides designers and researchers with two simple methods of determining natural frequencies of planar and tube-type structures. The first, more accurate method requires the use of a simple frequency equation. The Buckingham $\pi$ theorem (30) is used to partially solve the equation as the product of two functions. The undetermined variation of the second function is found to depend on three dimensionless parameters. Many computer runs varying these parameters provided characteristic points which were plotted to give design curves for the second function. These curves in conjunction with the first function provide the natural frequencies of a wide range of structural characteristics.

The second, more approximate method requires less computations, and applies strictly to framed-tube buildings. The building is modelled in such a way that modified natural frequency equations for cantilever beams can be used. The modelling which yields the most accurate results for different building characteristics is sought.
7.2 SCOPE AND LIMITATIONS

The two methods of natural frequency determination presented, are within the following scope and limitations:

1) Only the first two translational natural frequencies for vibration along the depth (A) of the structure can be determined.

2) Linear elastic behaviour.

3) The stiffness properties must remain constant through the structures (i.e. the structure can be idealized using a single orthotropic membrane).

4) Infinite horizontal rigidity is assumed (i.e. all the points in a horizontal plane displace equally).

5) Structure is fixed to a rigid foundation.

6) Three-dimensional structures must be rectangular in plan.

7) Mass distribution along the height of the structure must be uniform or at least concentrated uniformly along many (>4) equally spaced intervals.
8) The first method can be applied to tube-type or planar structures. The second method is strictly for framed-tube buildings with members of rectangular cross-section.

7.3. BUCKINGHAM $\pi$ THEOREM APPLIED TO NATURAL FREQUENCIES

The variables affecting the natural frequencies of the structure along with their basic dimensions are described below.

<table>
<thead>
<tr>
<th>variable</th>
<th>dimension</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$L$</td>
<td>width of structure.</td>
</tr>
<tr>
<td>$B$</td>
<td>$L$</td>
<td>depth of structure.</td>
</tr>
<tr>
<td>$t$</td>
<td>$L$</td>
<td>thickness of elastically equivalent orthotropic membrane.</td>
</tr>
<tr>
<td>$H$</td>
<td>$L$</td>
<td>height of structure</td>
</tr>
<tr>
<td>$E_y$</td>
<td>$ML^{-1}T^{-2}$</td>
<td>equivalent vertical elastic modulus.</td>
</tr>
</tbody>
</table>
\[ G_{xy} \quad \text{ML}^{-1} \quad \text{T}^{-2} \quad \text{equivalent shear modulus.} \]

\[ m \quad \text{ML}^{-1} \quad \text{mass distribution along} \]

\[ \lambda \quad \text{T}^{-1} \quad \text{the height of the} \]

\[ \quad \text{structure} \]

\[ \quad \text{natural frequency.} \]

where \( L \) = length

\( M \) = mass

\( T \) = time

All the above variables are known to be essential to the solution, and hence some functional relation must exist.

\[ f_1(A,B,t,H,E_y,G_{xy},m,\lambda) = 0 \quad (7.1) \]

The Buckingham \( \Pi \) theorem allows a reduction in variables. There are eight variables and only three dimensions; Eq. (7.1) can be expressed using \( 8 - 3 = 5 \) dimensionless parameters \((\Pi)\) as

\[ f_2(\Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5) = 0 \quad (7.2) \]

\[ \Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5 \quad \text{must be found.} \]
Since there are three dimensions, three repeating variable
(which together contain all the dimensions) must be
chosen. \( H, E_y, \) and \( m \) yields \( \pi \)'s with the best physical
meaning.

\[
\begin{align*}
\Pi_1 &= H_x^0 E_y^0 m_1^0 \lambda \\
\Pi_2 &= H_x^1 E_y^1 m_2^1 \lambda \\
\Pi_3 &= H_x^2 E_y^2 m_3^2 \lambda \\
\Pi_4 &= H_x^3 E_y^3 m_4^3 \lambda \\
\Pi_5 &= H_x^4 E_y^4 m_5^4 \lambda
\end{align*}
\]

Since the \( \pi \)'s are dimensionless,

for \( \Pi_1 \), \( (L)^x (ML^{-1}T^{-2})^y (ML^{-1})^z (L) = M^0 L^0 T^0 \)

from which

\[
\begin{align*}
X_1 - Y_1 - Z_1 + 1 &= 0 \\
-2Y_1 &= 0 \\
Y_1 + Z_1 &= 0
\end{align*}
\]

thus \( X_1 = -1, \ Y_1 = 0, \ Z_1 = 0 \)

Similarly for \( \Pi_2 \), \( X_2 = -1, \ Y_2 = 0, \ Z_2 = 0 \)

Similarly for \( \Pi_3 \), \( X_3 = -1, \ Y_3 = 0, \ Z_3 = 0 \)
For $\Pi_4$, 
\[
(L)^x_4 (ML^{-1}T^{-2})^y_4 (ML^{-1})^z_4 (ML^{-1}T^{-2}) = L^o T^o M^o
\]
from which
\[
x_4 - y_4 - z_4 - 1 = 0
\]
\[
-2y_4 - 2 = 0
\]
\[
y_4 + z_4 + 1 = 0
\]
thus
\[
x_4 = 0 \quad y_4 = -1 \quad z_4 = 0
\]

For $\Pi_5$, 
\[
(L)^x_5 (ML^{-1}T^{-2})^y_5 (ML^{-1})^z_5 (T^{-1}) = L^o T^o M^o
\]
from which
\[
x_5 - y_5 - z_5 = 0
\]
\[
-2y_5 - 1 = 0
\]
\[
y_5 + z_5 = 0
\]
thus
\[
x_5 = 0 \quad y_5 = -1/2 \quad z_5 = 1/2
\]
The five dimensionless $\Pi_i$ parameters are
\[
\Pi_1 = A/H
\]
\[
\Pi_2 = B/H
\]
\[
\Pi_3 = t/H
\]
\[
\Pi_4 = G_{xy}/E_y
\]
\[
\Pi_5 = \lambda \sqrt{\frac{E}{E_y}}
\]
Eq. (7.2) can now be expressed as
\[ f_2 \left( \frac{A}{H}, \frac{B}{H}, \frac{t}{H}, \frac{G_{xy}}{E_y}, \lambda \sqrt{\frac{m}{E_y}} \right) = 0 \]  

(7.3)

or

\[ f_3 \left( \frac{H}{A}, \frac{H}{B}, \frac{t}{H}, \frac{E_y}{G_{xy}}, \lambda \sqrt{\frac{m}{E_y}} \right) = 0 \]  

(7.4)

Eq. (7.4) can further be manipulated to give

\[ \lambda \sqrt{\frac{m}{E_y}} = f_4 \left( \frac{H}{A}, \frac{H}{B}, \frac{t}{H}, \frac{E_y}{G_{xy}} \right) \]  

(7.5)

or

\[ \lambda = \sqrt{\frac{E_y}{m}} f_4 \left( \frac{H}{A}, \frac{H}{B}, \frac{t}{H}, \frac{E_y}{G_{xy}} \right) \]  

(7.6)

This is the limit to where the Buckingham theorem can be used.

Eq. (7-6) can further be simplified by noticing that the element stiffness matrices shown in section 3.2 and 3.3 are directly proportional to \( t \). Since \( t \) is constant throughout the structure, the facade stiffness matrices and global structure stiffness matrix are also linearly proportional to \( t \). Recall the eigenvalue problem obtained from free undamped vibrations yielding Eq.(4.45).

\[ \lambda^2 [M] \{ \phi \} = [K^R] \{ \phi \} \]  

(7.7)
Let \( [R_s^*] = t [K] \) \hspace{1cm} (7.8)

Then Eq. (7.7) becomes

\[
\lambda^2 \begin{bmatrix} M \end{bmatrix} \{ \emptyset \} = t \begin{bmatrix} K \end{bmatrix} \{ \emptyset \}
\]

or

\[
\left( \frac{\lambda}{t} \right)^2 \begin{bmatrix} M \end{bmatrix} \{ \emptyset \} = \begin{bmatrix} K \end{bmatrix} \{ \emptyset \}
\]

(7.10)

From this equation it can be seen clearly that the natural frequency is directly proportional to the square root of the thickness (i.e. \( \lambda \propto \sqrt{t} \)). This means that \( f_4 \propto \sqrt{t} \).

Since \( f_4 \) can be factored by \( \sqrt{t} \), and one of the dimensionless parameters is \( t/H \), then \( f_4 \) can be factored by \( \sqrt{t/H} \).

\[
\lambda = \sqrt{\frac{E_s t}{m_s H}} f(H/A, H/B, E_y/G_{xy}) \hspace{1cm} (7.11)
\]

Let \( E_s, t_s, m_s, H_s \) be a set of standard (constant) values, then the natural frequency for this set of values is

\[
\lambda_s = \sqrt{\frac{E_s t_s}{m_s H_s}} f(H/A, H/B, E_y/G_{xy}) \hspace{1cm} (7.12)
\]
which depends on three dimensionless parameters. \( \lambda_s \) is termed "basis frequency". Dividing Eq. (7.11) by Eq. (7.12) yields

\[
\lambda = \lambda_s \sqrt{\left( \frac{E_y}{E_{ys}} \right) \left( \frac{H_s}{H} \right) \left( \frac{H_s}{H} \right) \left( \frac{\overline{m}}{m} \right) \left( \frac{\overline{m}}{H} \right)}
\]

(7.13)

For a given structure, \( H/A, H/B \), and \( E_y/G_{xy} \) are known, and \( \lambda_s \) can be determined through Eq. (7.12.) If \( E_y, t, \overline{m}, \) and \( H \), are also known, then \( \lambda \) can be calculated using Eq. (7.13.)

For convenience the standard constants are taken as

\[
\begin{align*}
E_{ys} &= 10^6 \\
H_s &= 10^2 \\
\overline{m}_s &= 10^4 \\
t_s &= 1
\end{align*}
\]

Eq. (7.13) becomes

\[
\lambda = \lambda_s \sqrt{\frac{E_y t}{\overline{m} H}}
\]

(7.14)
\( \lambda \) has units of time\(^{-1} \); \( \lambda \) must therefore be unitless, and the standard constants may be taken as unitless.

Many computer runs using TUBE were performed leading to graphs of \( \lambda \) for a wide range of \( H/A \), \( H/B \), \( E_y/G_{xy} \) values and the standard constants. Because of symmetry only one half of the planar structure and one quarter of the tube-type structure was analysed. For the remaining model, 10 levels were used in conjunction with 4 refined elements horizontally spanning each facade. Figs. 7.1a–r show the variation of \( \lambda \) for the first and second mode.

7.4 SIMPLE STEPS FOR NATURAL FREQUENCY CALCULATIONS

The following set of steps may be followed chronologically to obtain the natural frequencies of planar or tube-type structures.

STEP 1. Determine \( tE_y \) and \( tG_{xy} \). For solid walls, use the elastic properties of the actual material. For orthogonal gridwork of rectangular cross-section, use Eqs. (2.3) and (2.8). These values may always be determined by equating the deflection of the discrete structure obtained
experimentally, and the equivalent membrane obtained theoretically (see sections 2.3.1 and 2.3.2).

STEP 2. Determine $H/A$, $H/B$, $E_y/G_{xy}$

STEP 3. Determine the basic natural frequency ($\lambda_5$)

(1) For tube-type structures
- For the first mode, use Figs. 7.1a - g
- For the second mode, use Figs. 7.1h - p

(2) For planar structures
- For the first mode, use Fig. 7.1q
- For the second mode, use Fig. 7.1r

STEP 4. Determine $\bar{m}$

$\bar{m}$ = the linear mass density along the height of the full structure

STEP 5. Determine $\lambda$ for each mode.

$$\lambda_h = \lambda_{5n} \sqrt{\frac{E_y t}{\bar{m} H}}$$  \hspace{1cm} (7.15)
where \( n = \) mode number

The following examples should familiarize the reader with the method while giving confidence in the results.

The first example to be illustrated is the box cantilever beam of section 5.2.

**STEP 1.**
\[
\begin{align*}
&t_E^y = 1 \text{ in.} \times (30,000 \text{ ksi}) = 30,000 \text{ kips/in.} \\
&t_{G_{xy}} = 1 \text{ in.} \times \frac{30,000 \text{ ksi}}{2(1 + 2.5)} = 12,000 \text{ kips/in.}
\end{align*}
\]

**STEP 2.**
\[
\begin{align*}
&H/A = 48 \text{ in.} / 12 \text{ in.} = 4 \\
&H/B = 48 \text{ in.} / 6 \text{ in.} = 8 \\
&\frac{E_y}{G_{xy}} = \frac{30,000}{12,000} = 2.5
\end{align*}
\]

**STEP 3.** Using Fig. 7.1d, \( \lambda_{31} \) can be determined.

Using Fig. 7.1k, \( \lambda_{32} \) can be determined.

\[
\begin{align*}
\lambda_{31} &= 0.262 \\
\lambda_{32} &= 1.210
\end{align*}
\]

**STEP 4.**
\[
\begin{align*}
\sqrt{m} &= \frac{0.284 \text{ lb/in}^3(2)(12\text{ in.} + 6\text{ in.})(1\text{ in.})}{32.2 \text{ ft/s}^2} \\
\sqrt{m} &= 0.3175 \text{ slugs/in.}
\end{align*}
\]
STEP 5. \[ \sqrt{\frac{E}{m H}} = \frac{(30,000,000 \text{ slugs}\cdot(12\text{ in.})/\text{s}^2\text{in})}{0.3175 \text{ slugs/in (48 in)}} = 4860 \text{ rad/sec} \]

\[ \lambda_1 = 4860 \times (0.262) = 1,270 \text{ rad/sec} \]

\[ \lambda_2 = 4860 \times (1.210) = 5,880 \text{ rad/sec} \]

The resulting frequencies are close to those calculated previously.

The second example shown below is for a steel cantilever beam of rectangular cross-section. The following data is required for the analysis:

- \( E = 30,000 \text{ ksi} \)
- \( \nu = 0.25 \)
- \( \rho = 0.00882 \text{ slugs/in} \)
- \( H = 500 \text{ in.} \)
- \( A = 50 \text{ in.} \)
- \( t = 50 \text{ in.} \)

From beam theory, the natural frequencies are

\[ \lambda_n = C_n \sqrt{\frac{EI}{m H^4}} \quad (7.16) \]

where \( C_1 = 1.875^2 \) and \( C_2 = 4.694^2 \).
For a solid rectangular cross-section

\[ I = \frac{t (A)^3}{12} \]

resulting in

\[ \lambda_n = C_n \sqrt{\frac{1}{12} \frac{(A)^3}{H}} \sqrt{\frac{E + \nu}{m H}} \]  \hspace{1cm} (7.17)

Note the similarity of Eq.(7.15) to Eq. (7.17)

If the present method is to yield equivalent answers as beam theory, then

\[ \lambda_{sn} = C_n \sqrt{\frac{1}{12} \frac{(A)^3}{H}} \]

Note that \( \lambda_{sn} \) for beam theory depends only on \( H/A \) (i.e. independent of \( E/\gamma_{xy} \)). Putting values in the above equation yields

\[ \lambda_{s1} = \left(1.875\right)^2 \sqrt{\frac{1}{12} \frac{\left(50 \right)^3}{500}} = 0.0321 \]

\[ \lambda_{s2} = \left(4.694\right)^2 \sqrt{\frac{1}{12} \frac{\left(50 \right)^3}{500}} = 0.201 \]
Using the graphs

1) \[ t = 50 \text{ in.} \]
\[ G_{xy} = \frac{30,000 \text{ ksi}}{2(1 + 0.25)} \]
\[ \xi E_y = 50 \text{ in} \left( \frac{3 \times 10^7 \text{slug/s}^2 \text{in}}{3 \times 10^6 \text{slug/s}^2 \text{in}^2} \right) = 1.8 \times 10^6 \text{ slug/s}^2 \]
\[ tG_{xy} = 50 \text{ in} \left( \frac{1.2 \times 10^7 \text{slug/s}^2 \text{in}}{3 \times 10^6 \text{slug/s}^2 \text{in}^2} \right) = 0.72 \times 10^6 \text{ slug/s}^2 \]

2) \[ \frac{H}{A} = \frac{500}{50} = 10 \]
\[ B = 0 \]
\[ \frac{E_y}{G_{xy}} = 1.8/0.72 = 2.5 \]

3) From Figs. 7.1q and r respectively, \( \lambda_{s1} = 0.0318 \)
\( \lambda_{s2} = 0.193 \)

These values are very close to the ones calculated above using beam theory.

For the planar frame of section 5.4

1) Using Eqs. (2.3) and (2.8), \( tE_y \) and \( tG_{xy} \) are calculated to be \( 128.9 \times 10^6 \text{ lb/ft} \) and \( 2.569 \times 10^6 \text{ lb/ft} \) respectively.
2) \[ \frac{H/A}{120} = 4 \]
\[ B = 0 \]
\[ E_y/G_{xy} = \frac{429.8}{8.563} = 50.2 \]

3) From Fig. 7.1q, \[ \lambda_{s1} = 0.082 \]
From Fig. 7.1r, \[ \lambda_{s2} = 0.265 \]

4) \[ \bar{m} = 1800 \times 10^6 \text{slugs ft} \times \frac{\text{storey}}{s^2 \text{storey}} \times \frac{1}{12 \text{ft.}} \times \frac{1}{32.2 \text{ft/s}^2} = \frac{4658 \text{slugs}}{\text{ft.}} \]

5) \[ \sqrt{\frac{E_y t}{m H}} = \sqrt{\frac{128.9 \times 10^6 \text{slugs/s}^2}{4658 \text{ slugs/ft.} (180 \text{ft.})}} = 7.593 \text{ rad/sec.} \]
\[ \lambda_1 = (0.082)7.593 = 0.62 \text{ rad/sec.} \]
\[ \lambda_2 = (0.265)7.593 = 2.01 \text{ rad/sec.} \]

These values are almost identical to those previously calculated for the second analysis of section 5.4.

The last example presented here is for Chan's framed-tube building presented in section 5.5.

1) Using equations (2.3) and (2.8), \( tE_y \) and \( tG_{xy} \) are calculated to be \( 128.9 \times 10^6 \text{ lb/ft} \) and \( 2.569 \times 10^6/\text{lb/ft} \) respectively.

2) \[ \frac{H/A}{120} = 4 \]
\[ \frac{H}{B} = \frac{480}{200} = 4 \]
\[ \frac{E_y}{G} = \frac{429.8}{8.563} = 50.2 \]

3) From Fig. 7.1d and k, \( \lambda_{s1} = 0.138 \) and \( \lambda_{s2} = 0.420 \) respectively

4) \[ \bar{m} = \frac{3600 \times 10^3 \text{ slugs} \cdot \text{ft} \cdot \text{s}^2}{\text{storey} \times \frac{\text{storey}}{12 \text{ ft.}} \times \frac{1}{32.2 \text{ ft.}/\text{s}^2}} = 9317 \text{ slugs} \cdot \text{ft} \]

5) \[ \sqrt{\frac{E_y}{\bar{m} H}} = \sqrt{\frac{128.9 \times 10^6 \text{ slugs} \cdot \text{ft} \cdot \text{s}^2}{9317 \text{ slugs} \cdot \text{ft} \cdot (480 \text{ ft.})^2}} = 5.369 \text{ rad/sec} \]

\[ \lambda_1 = (0.138)5.396 = 0.741 \text{ rad/sec} \]

\[ \lambda_2 = (0.420)5.396 = 2.25 \text{ rad/sec} \]

The first and second natural frequencies obtained in section 5.5 are 0.740 and 2.25 radians per second respectively for analysis 3.

7.5 CANTILEVER BEAM MODELING OF FRAMED-TUBE BUILDINGS FOR FREQUENCY CALCULATIONS

Natural frequency calculations using the design curves may prove inconvenient for preliminary design. Determining the equivalent elastic orthotropic membrane
properties $E_y$ and $G_{xy}$ is often tedious. A simple method of natural frequency calculations using a cantilever beam model is presented here. All limitations presented in section 7.2 apply here also.

For a cantilever beam, the $n$th natural frequency along the depth $(A)$ is given by Eq. (7.16). Treating Chan's framed-tube building presented in section 6.2 a cantilever beam in which

$$ I = \sum_{all\ columns} \left( d^2 A_c \right)_i $$  \hspace{1cm} (7.18)

where $d =$ distance from the neutral axis to column $i$

$$ A_c =$ cross-sectional area of column $i$,

the moment of inertia was determined to be 346,285 ft$^4$ of which 70% was contributed by the normal facades (normal to the vibration direction), and 30% by the parallel facades (parallel to the vibration direction). From Eq. (7.6), the first and second natural frequencies were determined to be 1.934 rad/sec and 12.12 rad/sec respectively. These values are 2.4 and 5 times greater than those determined from the design curves. These
errors are too large for any reasonable preliminary design. From the design curves, frequency is seen to be more sensitive to A than to B; the reverse is true using the beam vibration equation. This is one reason why Eq. (7.16) applied to framed-tube structures is fundamentally incorrect. The form of Eq. (7.16) will be altered to take care of the errors involved.

\[ \lambda_n = \sqrt{\frac{E(I_1 + \beta I_2)}{m H^4}} \]  

(7.19)

where \( X \) = a correction factor to be determined

\( I_1 = \) moment of inertia of the parallel facades including corner columns.

\( I_2 = \) moment of inertia of the normal facades excluding corner columns.

\( \beta = \) a factor to be determined specifying the percentage of \( I_2 \) to be considered.
Because it is now known that for framed-tube structures, the parallel facade is of greater importance, \( \beta \) is introduced so that only a fraction of the normal facade moment of inertia is considered. Eq. (7.18) is valid only for a linear distribution of axial column deformations; the shear lag phenomenon inhibits this behavior. The correction factor \( X \) takes into account this discrepancy along with the effects of \( \beta \). The \( \beta \) sought is that which yields the smallest variation of \( X \). From Eq. (7.15) and Eq. (7.19),

\[
\lambda_{sn} \sqrt{\frac{E_{y} \; t}{m \; H}} = X \; C_n \sqrt{\frac{E (1 + \beta I_2)}{m \; H^3}}
\]

from which

\[
X^2 = \left( \frac{\lambda_{sn}}{C_n} \right)^2 \frac{E_{y} \; t \; H^3}{E (1 + \beta I_2)}
\]

(7.20)

Substituting Eq. (2.3) yields

\[
X^2 = \left( \frac{\lambda_{sn}}{C_n} \right)^2 \frac{E \; \frac{I_p}{I_2} \; \frac{d_0}{h} \; H^3}{W \; C_y (1 + \beta I_2)}
\]

(7.21)
but \[ I_1 = 4 \sum A_c d_i^2 = 4 t_c d_c \sum d_i^2 \tag{7.22} \]

Summation is over one half of one parallel façade.

\[ I_1 = 4 t_c d_c w^2 Z \tag{7.23} \]

where \( w = \) bay width

\[ Z = \sum_{i=1}^{\frac{1}{2} N_{B1}} \left( i^2 \right) \quad \text{if } N_{B1} \text{ is even} \tag{7.24} \]

\[ Z = \sum_{i=1}^{\frac{1}{2} (N_{B1}+1)} \left( i - \frac{1}{2} \right)^2 \quad \text{if } N_{B1} \text{ is odd} \tag{7.25} \]

\( N_{B1} = \) No. of bays in one parallel façade

\[ I_2 = 2(N_{B2}-1)A_c \left( \frac{A}{2} \right)^2 = \frac{1}{2} t_c d_c A^2 (N_{B2}-1) \tag{7.26} \]

where \( N_{B2} = \) No. of bays in one normal façade

\( A = \) depth of one parallel façade
Substituting Eqs. (7.23) and (7.26) into (7.21) yields

$$\chi^2 = \left( \frac{\lambda_{sn}}{C_n} \right)^2 \frac{H^3}{\omega C_y \left[ 4 w^2 Z + \frac{1}{2} \beta A^2 (N_{a2} - 1) \right]} \quad (7.27)$$

Replacing \( w \) with \( A/N_{a1} \) gives

$$\chi^2 = \left( \frac{\lambda_{sn}}{C_n} \right)^2 \frac{2 (H/A)^3}{C_y \left[ 8 (N_{a1})^3 Z + \beta (N_{a2} - 1) \right]} \quad (7.28)$$

At this point, several approximations are made. An average value of \( C_y \) for framed-tube structures is 0.9. From the basic frequency curves (Figs. 7.1a-q), for a given \( H/A \), the \( \lambda_{sn} \) does not vary too much. \( \lambda_{sn} \) of Eq. (7.28) will be replaced in accordance with Table 7.1 derived for \( H/A = H/B \) and \( E_y/G_{xy} = 50 \). The ratio \( E_y/G_{xy} \) of most framed-tube structures varies between 30 and 80. Table 7.1 also displays Eq. (7.27) incorporating the above approximations. Note that for a given \( H/A \), the \( \chi \) value varies only with \( N_{a1}, N_{a2}, \) and \( \beta \), (i.e. \( Y = f(N_{a1}, N_{a2}, \beta) \)). Values of \( Y \) were computed for all combinations of \( N_{a1} = 8, 9, 10, \ldots, 24 \) and \( N_{a2} = 8, 9, 10, \ldots, 24 \) and \( \beta = 0, 1, 2, \ldots, 1.0 \). Several of these values are shown.
in Table 7.2. Note that as $\beta$ decreases, the percentage variation of $Y$ for different values of $N_{\beta 1}$ and $N_{\beta 2}$ also decreases. For $\beta = 0$, this variation is the smallest; an average value for $Y$ is taken as 1.55 for $N_{\beta 1} = N_{\beta 2} = 15$.

Eq. (7.19) now becomes

$$\lambda_n = C_n \frac{E \cdot I_{i}}{\sqrt{m \cdot H^4}} \quad (7.30)$$

The value of $X$ is obtained from Table 7.3. It can now be seen clearly that $X$ takes into account the errors involved in moment of inertia calculations using Eq. (7.18) and the effects of ignoring the normal facades.

Errors in using Eq. (7.30) are usually within 15%, but this value can be significantly decreased. The major source of error is in the approximation of $\lambda_{sn}$ for a given $H/A$. For framework with deep members, $X$ should be increased by a factor of 1.1. For framework with shallow members, $X$ should be decreased by a factor of 0.9. The number of bays within the parallel facade should be between 8 and 24. For these two extremes, multiply $X$ by
0.94 or 1.05 respectively. If these two rules are followed, errors will usually remain within 8%.

Eq. (7.29) will be used on Chan's framed-tube building. The following values were computed.

\[ I_1 = 103,300 \text{ ft}^4 \]
\[ \bar{m} = 9317 \text{ slug/ft} \]
\[ E = 432 \times 10^6 \text{ psf} \]
\[ H = 480 \text{ ft} \]
\[ H/A = 4 \]

From Table 7.3, \( x_1 = .725 \) and \( x_2 = .352 \).

The framework if fairly normal, and the parallel facades have 16 bays, therefore the values of \( x_1 \) and \( x_2 \) are unaltered. The first two natural frequencies are

\[ \lambda_1 = 0.725 \left( 3.5 \pi \right) \sqrt{\frac{932 \times 10^6 \text{ psf}}{(9317 \text{ slugs/ft})(480 \text{ ft})^2}} = 0.725(3.5\pi)(0.30) = 0.745 \text{ rad/sec} \]

\[ \lambda_2 = 0.352(22.034)(0.30) = 2.33 \text{ rad/sec} \]

These values are respectively 3% and 4% greater than those obtained using the basic natural frequency curves.
Table 7.1  Average Values of $\lambda_i$ and X for different H/A values taking $E_y/G_{xy} = 50$

<table>
<thead>
<tr>
<th>H/A</th>
<th>$\lambda_{51}$</th>
<th>$X^*$</th>
<th>$\lambda_{52}$</th>
<th>$X^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>.065</td>
<td>.872Y</td>
<td>.22</td>
<td>.471Y</td>
</tr>
<tr>
<td>8</td>
<td>.081</td>
<td>.777Y</td>
<td>.26</td>
<td>.398Y</td>
</tr>
<tr>
<td>6</td>
<td>.103</td>
<td>.642Y</td>
<td>.32</td>
<td>.318Y</td>
</tr>
<tr>
<td>4</td>
<td>.138</td>
<td>.468Y</td>
<td>.42</td>
<td>.227Y</td>
</tr>
<tr>
<td>3</td>
<td>.166</td>
<td>.366Y</td>
<td>.50</td>
<td>.176Y</td>
</tr>
<tr>
<td>2</td>
<td>.210</td>
<td>.252Y</td>
<td>.63</td>
<td>.121Y</td>
</tr>
<tr>
<td>1</td>
<td>.310</td>
<td>.131Y</td>
<td>.91</td>
<td>.0616Y</td>
</tr>
</tbody>
</table>

\[
Y = \left[ 8 \left( \frac{1}{N_{61}} \right)^3 \beta \left( \frac{N_{62}-1}{N_{61}} \right) \right]^{-\frac{1}{2}}
\]  
(7.29)
Table 7.2  Y values for different $\beta$, $N_{B1}$, and $N_{B2}$ values

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$N_{B1}$</th>
<th></th>
<th>$N_{B2}$</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td></td>
<td>1.46</td>
<td></td>
<td>1.46</td>
<td>1.46</td>
</tr>
<tr>
<td>0.</td>
<td>16</td>
<td></td>
<td>1.58</td>
<td></td>
<td>1.58</td>
<td>1.58</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td>1.63</td>
<td></td>
<td>1.63</td>
<td>1.63</td>
</tr>
<tr>
<td>0.5</td>
<td>8</td>
<td></td>
<td>1.05</td>
<td></td>
<td>.843</td>
<td>.724</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td>1.27</td>
<td></td>
<td>1.07</td>
<td>.946</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td>1.38</td>
<td></td>
<td>1.21</td>
<td>1.08</td>
</tr>
<tr>
<td>1.0</td>
<td>8</td>
<td></td>
<td>.862</td>
<td></td>
<td>.653</td>
<td>.547</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td>1.09</td>
<td></td>
<td>.865</td>
<td>.738</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td></td>
<td>1.22</td>
<td></td>
<td>.999</td>
<td>.866</td>
</tr>
</tbody>
</table>
Table 7.3  Average X values for different H/A ratios

taking $E_y/G_{xy} = 50$, $N_{B1} = N_{B2} = 15$

<table>
<thead>
<tr>
<th>H/A</th>
<th>X</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MODE 1</td>
<td>MODE 2</td>
</tr>
<tr>
<td>10</td>
<td>1.35</td>
<td>.730</td>
</tr>
<tr>
<td>8</td>
<td>1.20</td>
<td>.617</td>
</tr>
<tr>
<td>6</td>
<td>.995</td>
<td>.493</td>
</tr>
<tr>
<td>4</td>
<td>.725</td>
<td>.352</td>
</tr>
<tr>
<td>3</td>
<td>.567</td>
<td>.272</td>
</tr>
<tr>
<td>2</td>
<td>.391</td>
<td>.187</td>
</tr>
<tr>
<td>1</td>
<td>.203</td>
<td>.0954</td>
</tr>
</tbody>
</table>
Fig. 7.1b Basic natural frequency for tube-type structure (mode 1, H/A=2)
Fig. 7.1c Basic natural frequency for tube-type structure (mode 1, H/A=3)
Fig. 7.1d Basic natural frequency for tube-type structure (mode 1, H/A=4)
Fig. 7.1f Basic natural frequency for tube-type structure (mode 1, H/A=8)
Fig. 7.1g Basic natural frequency for tube-type structure (mode 1, \( H/A = 10 \))
Fig. 7.1j Basic natural frequency for tube-type structure (mode 2, $H/A=3$)
Fig. 7.1k Basic natural frequency for tube-type structure (mode 2, H/A=4)
Fig. 7.1m Basic natural frequency for tube-type structure (mode 2, H/A=6)
Fig. 7.1p Basic natural frequency for tube-type structure (mode 2, H/A=10)
Fig. 7.1r Basic natural frequency for planar structure (mode 2)
CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 CONCLUSIONS

The equivalent orthotropic macroelement method has been adopted and implemented for the efficient dynamic analysis of planar and tube-type structures consisting of frames, shear walls, coupled shear walls and their combinations. Frames must consist of rigidly connected horizontal beams and vertical columns. Repetition of dimensions over large portions of the structure is required for efficient implementation of the method.

The technique requires replacing discrete beam-column systems and bands of lintel beams by elastically equivalent orthotropic membranes. Expressions for the elastic properties of the membranes were presented; they take into account axial column deformations, bending and shear deformations in beams and columns, and finite size joint deformations. The equivalent structure is subsequently discretized by either of two specially orthotropic finite elements. It is noted that although the membrane properties have been obtained analytically,
they may also be determined experimentally or by detailed finite element analysis.

A general large capacity computer program (TUBE) was developed for the static and earthquake spectrum analysis of the above structures. The program was developed for minimal memory storage requirements, as well as effort for data preparation.

The efficiency of the present method is significantly greater than others presented; only 1/4 of the global dynamic degrees of freedom were used to obtain comparable results. Convergence of higher natural frequencies was shown when increasing the number of levels in the model. With respect to the translational natural frequencies of wall-frame structures, and framed-tube structures, the facades normal to the vibration direction have little contribution so that preliminary estimates can be obtained by considering only the parallel facades. Enforcing vertical compatibility at facade junctions yielded significantly higher natural frequencies so that facades cannot be assumed to act independently of each other.

Large perforations in planar and tube-type structures cause shear lag so that internal normal forces in columns form a nonlinear variation. A measure of shear lag severity (nonlinearity) was expedited through a shear
lag parameter which is the ratio of the equivalent shear modulus to the equivalent vertical elastic modulus of the membrane. It was discovered that increasing this ratio produced higher natural frequencies and a more linear column axial force distribution especially in the parallel facades. In terms of the actual structure, this can be done simply by increasing the beam depth.

Flexibility of finite size joints in planar frames and framed-tube buildings was found to significantly decrease the natural frequencies and internal column axial forces. Shear deformations in the members were determined to be less significant and can be ignored at least for preliminary design.

Two methods of determining natural frequencies of planar and tube-type structures are presented in the form of graphs and simplified equations. The first method is simply a compilation of data obtained by TUBE using three dimensionless parameters as variables. To ensure good accuracy, the equivalent structures was refined to 10 levels and 4 elements horizontally spanning each facade. The second, more approximate method is strictly for framed-tube structures, and attempts to model them as cantilever beams. The results of the first method are approximated to yield a modified cantilever beam frequency
equation which considers only the parallel facade inertia, and a correction factor. The errors are expected to remain within 15%.

8.2 RECOMMENDATIONS FOR FURTHER STUDIES

The present method is confined to the elastic static and elastic earthquake spectrum analysis with a possible ductility factor consideration for approximate elastic-plastic analysis. Loads and ground motions are considered only in horizontal planes; the analysis can be extended to consider loads and ground motions in vertical planes. Extension of the method for stability analysis and optimization study is also possible. In addition, the following aspects could be investigated.

1) Obtain the equivalent membrane properties of the basic frame unit for members of nonrectangular cross-section. Experimental or detailed finite element analysis are possible routes, and may improve the membrane properties of the unit.
2) Improvement of the finite element stiffness matrices by using hybrid stress elements or incompatible displacement models.

3) Extension of the program procedure for the analysis of tube-in-tube structural systems.

(4) Extend the analysis for a consistent mass formulation.

(5) TUBE can be altered and appended to include earthquake analysis by the direct numerical integration of the equation of motion.

(6) Analysis can be extended to consider plastic behaviour.
REFERENCES


(4) Stafford-Smith, Editor, Planning and Design of Tall Buildings, Vol. CB, Chapter Z1C, Revised Draft 1, March 1976.


A.1 INTRODUCTION

A large capacity computer program (TUBE) has been developed for the static and earthquake spectrum analysis of most planar and tube-type building structures, incorporating the developments presented in chapters II, III, and IV.

The program was written in FORTRAN IV computer language and was run on a CDC 6000 computer using the FTN compiler. Auxiliary storage via nine magnetic tapes, and substructuring techniques along with static condensation procedures allow the efficient use of memory storage.

Reduction of computer time and storage is generally obtained at the expense of some loss in accuracy. Minimization of accuracy loss can be achieved through proper structure idealization. Efficient modelling of a structure via the equivalent orthotropic macroelement method requires an understanding of its characteristic physical behaviour. Structures that can be analysed by TUBE have been previously studied; (1, 2, 3, 14, 25, 26, 27, 28, )
are recommended as references.

Effort in data preparation and possible data errors are minimized through the usage of automatic data generation in the program wherever it is possible. The amount of data required is thus kept to a minimum.

The aim of this appendix is to clearly describe the essential features and logic of TUBE. This will assist the user in implementing TUBE correctly and efficiently while facilitating any further alterations and modifications of its current version.

A.2 SCOPE

The program may perform approximate elastic static analysis of structures subjected to lateral loads or approximate elastic earthquake spectral analysis of structures with an option for ductility considerations. Only planar or tube-type building structures can be analysed. Direct examples of planar structures are frames consisting of slender or relatively deep members, shear walls, coupled shear walls, frames interacting with shear walls, and clad-frames. Some examples of tube-type building structures include framed-tube structures, core-supported structures with open or closed sections,
and structures consisting of interacting shear walls and frames. These tubular systems in general are any three-dimensional assembly of planar systems connected at their edges. Planar frames must consist of horizontal beams rigidly connected to vertical columns. Structures must also satisfy the assumptions stated earlier in Chapter I.

Structures to be analysed by TUBE should preferably possess a high degree of regularity. For example, bay widths, storey heights and member sizes should not change over large portions of framed structures. This is not a heavy limitation; high degree of regularity is inherent in most tall building structures due to aesthetic reasons and to ensure ease of construction and subsequent savings in time and money (26).

A.3 DESCRIPTION OF THE PROGRAM

Most of the program logic follows the "Analysis Procedure" section presented in Chapter IV. The program can be divided into three main stages.

The first stage involves data entry on two levels: global structure level through subroutine DATA1, and local facade and element level through subroutine DATA2.
Basically, in the global level, the geometry of the plan view and the analysis type is specified. Also, DATA1 directly assembles the overall load vector in the case of a static analysis, or the diagonal vector representation of the lumped-mass matrix in the case of a dynamic analysis. At the local level, facade properties (number of elements and their properties) are specified.

The second main stage is the construction of the global structure stiffness matrix which corresponds to the overall load vector or lumped-mass matrix. In this stage, the elastic properties of the equivalent orthotropic membranes, for different element types if not input directly, are evaluated in subroutine EMAT. Element connectivity and type automatically generated in ELCON provide facade discretization. Refined or ordinary element stiffness matrices are generated in RECT and then assembled in ASMBLL to form the local facade stiffness matrix. Assembly is based on the degrees of freedom associated with each element. The numbering of the degrees of freedom are automatically generated in LABEL and adjusted in SYMM if any facade symmetry is specified. The local facade stiffness matrix is then reduced by condensing all internal facade degrees of freedom using GAUSS, and then stored on tape 4. Once all reduced local
Facade stiffness matrices are stored, they are recalled from tape, transformed to global degrees of freedom, and then assembled into the global static structure stiffness matrix one by one within ASMBL2. Note that the transformation procedure for static and dynamic analysis differ due to the latter's possibly varying reference position from level to level. If the analysis is dynamic, the corner vertical degrees of freedom of the global static structure stiffness matrix are further condensed via GAUSS to form the global dynamic structure stiffness matrix which is stored on tape 6.

In the third and final stage, procedures for the static and earthquake analysis differ significantly. For the static analysis, displacements, membrane stresses, and member internal forces are evaluated respectively via GAUSS, STRESS, and FORCE. For the earthquake analysis, frequencies and mode shapes are evaluated through SIVIB2. Subsequent displacements, level shears and torques, base shears and torques, and overturning base moments are determined in MODAL. Finally, membrane stresses and member internal forces are determined. Note that for each parameter above, except member internal forces the square root sum of squares (SRSS) method is used to obtain the final response. The member internal forces are computed from the SRSS of membrane stresses.
A.4 USER'S GUIDE

In an effort to minimize any possible data errors and to efficiently implement TUBE, a detailed description of data preparation along with useful suggestions to improve accuracy of results is provided in this section. These suggestions are based on experience gained through implementing the program.

A.4.1 STRUCTURAL MODELLING

Besides the rules and definitions given previously in Section 4.3.1, the following steps may be followed:

STEP 1:
Sketch a plan view and elevation of the structure. Include the reference axes, and lateral loads or ground motion direction. Number each level, corner, and facade as shown in Figs. 5.1 and 5.5.

STEP 2:
Decide whether the structure is to be discretized using the ordinary element (4-corner nodes, and 6 degrees of freedom) or the refined element (8 nodes and 9 degrees of freedom). The latter is recommend for the cases where the effect of shear lag is significant, and where there is bending of high aspect ratio structures.
STEP 3:

If symmetry exists, restrain the appropriate degrees of freedom. Thus, indicate the symmetry type number for the structure and for each facade. Note that if facade symmetry is specified, then structure symmetry must also be specified.

STEP 4:

Draw a separate elevation for each facade discretized by several elements. Number each element and its type. Note that element boundaries need not coincide with beam or column lines.

STEP 5:

Decide on the type of output (i.e. stresses or internal member forces) and the levels at which they should be evaluated.

STEP 6: (For dynamic analysis only)

Decide on the required output (i.e. level shears and torques, base shears and torques, overturning base moments).
In addition to the above 6 steps, the following suggestions may help in dividing the structure into levels, and subsequently facades into elements.

1) For a static analysis, an element can incorporate as many as 5 to 10 storeys. For a dynamic analysis masses are lumped at levels. To better approximate mass distribution, more levels should be used.

2) Three single-storey levels at the base of the structure is a sound rule (1,14,26) mainly because of the high curvature.

3) A single-storey level or more at the top of the structure is desirable particularly for structures deforming in a predominantly bending mode.

   This is not crucial to the overall behaviour of the structure.

4) Always provide a finer mesh at stress concentration areas such as facade edges. In general, the elements at the corners of buildings should incorporate only one bay especially when using the ordinary element.
A.4.2 INPUT DATA

Units must be consistent and data cards must be in the sequence shown below. Integer and real numbers are input in free-format. Below will be presented the sequence of cards, the variables on each card, and the condition(s) for the existence of that card or group of cards. It is very important to remember that if the condition(s) in the right hand side column is/are not satisfied for a particular card, that card must be excluded. Section A.4.3 provides a description of each variable.
<table>
<thead>
<tr>
<th>CARD GROUP No.</th>
<th>CARD No. of GROUP</th>
<th>INPUT VARIABLES</th>
<th>CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>TITLE1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>NDYN</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>NF NL NDOF NSHAPE NSSYM NSYM23 NOUT</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>M NRQD NOI TOLVEC GES IRENT GAMMA INTER</td>
<td>NDYN=0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1 (X(1) Y(1))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 (X(2) Y(2))</td>
<td></td>
</tr>
<tr>
<td></td>
<td>\vdots</td>
<td>\vdots</td>
<td></td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>N (X(N) Y(N))</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1 (FP(1,1) FP(1,2) FP(1,3) RFP(1,1))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 (FP(2,1) FP(2,2) FP(2,3) RFP(2,1))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>\vdots</td>
<td>\vdots</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NF</td>
<td>NF (FP(NF,1) FP(NF,2) FP(NF,3) RFP(NF,1))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1 (FP(1,1) FP(1,2) FP(1,3))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 (FP(2,1) FP(2,2) FP(2,3))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>\vdots</td>
<td>\vdots</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NF</td>
<td>NF (FP(NF,1) FP(NF,2) FP(NF,3))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>(FX(1) FX(2) \ldots FX(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(FY(1) FY(2) \ldots FY(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(MT(1) MT(2) \ldots MT(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>(FX(1) FX(2) \ldots FX(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(FY(1) FY(2) \ldots FY(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>(MT(1) MT(2) \ldots MT(NL))</td>
<td>NDYN=0</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1 (XMC(1) YMC(1) MAS(1) RMAS(1))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 (XMC(2) YMC(2) MAS(2) RMAS(2))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>\vdots</td>
<td>\vdots</td>
<td></td>
</tr>
<tr>
<td></td>
<td>NL</td>
<td>NL (XMC(NL) YMC(NL) MAS(NL) RMAS(NL))</td>
<td>NDYN=1</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>NPRINT(1) NPRINT(2) \ldots NPRINT(NL)</td>
<td>NDYN=0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>NPRINT(1) NPRINT(2) \ldots NPRINT(NL)</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>NPRINT(1) NPRINT(2) \ldots NPRINT(NL)</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>NPRINT(1) NPRINT(2) \ldots NPRINT(NL)</td>
<td>NDYN=1</td>
</tr>
<tr>
<td></td>
<td>(\text{NPRI(3)=1})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Card group numbers 12, 13, 14, **

**15, 16, 17 must be repeated for **

**each facade.**
<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1</td>
<td>NMAT NTYPE NGROUP NSYM INCODE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>1.1</td>
<td>EMU IBENDB IBENDC ISHEARB ISHEARC ISHEARJ INCODE=0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>BD BD BT</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>HC DC CT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.1</td>
<td>Same as 1.1, 1.2, 1.3,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>but for material</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.3</td>
<td>type-2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMAT.1</td>
<td>Same as 1.1, 1.2, 1.3,</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMAT.2</td>
<td>but for material</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMAT.3</td>
<td>type NMAT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>EX(1) EY(1) GXY(1) TH(1) INCODE=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>EX(2) EY(2) GXY(2) TH(2) INCODE=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NMAT</td>
<td>EX(NMAT) EY(NMAT) GXY(NMAT) TH(NMAT) INCODE=1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>ET(1,1) ET(1,2) ETM(1) IEW(1) IEH(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>ET(2,1) ET(2,2) ETM(2) IEW(2) IEH(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NTYPE</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>NS(1) NE(1) NELT(1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>NS(2) NE(2) NELT(2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NGROUP</td>
<td>NS(NGROUP) NE(NGROUP) NELT(NGROUP)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>U(1,1) U(2,1)......U(NREM,1) IRENT=1 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>U(1,2) U(2,2)......U(NREM,2) IRENT=1 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>U(1,M) U(2,M)......U(NREM,M) IRENT=1 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>1</td>
<td>W(1,1) W(2,1)......W(NREM,1) IRENT=2 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>W(1,2) W(2,2)......W(NREM,2) IRENT=2 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>M</td>
<td>W(1,M) W(2,M)......W(NREM,M) IRENT=2 NDYN=1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>SV(1) SV(2)......SV(NRQD) INTER=0 NDYN=1 any NPRI=1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A.4.3 DESCRIPTION OF TUBE INPUT

TITLE 1 = 80 spaces are provided for an alphanumeric description of the problem.

NDYN = 0 for static analysis
        1 for dynamic analysis

NF = (integer) number of facades in the model after symmetry considerations. (NF ≤ 4)

NL = (integer) number of levels in the model (NL ≤ 10).

NDOF = 0 for the ordinary element (4 nodes and 6 degrees of freedom)
        1 for the refined element (8 nodes and 9 degrees of freedom)

NSHAPE = 0 for closed plan shape of model after symmetry considerations.
          1 for open plan shape of model after symmetry considerations.

NSSYM = 0 for no structural symmetry.
        (If NSSYM ≠ 0, then at least 2 facades must not be parallel). (If NF = 1, then NSSYM = 0)
        1 for structure symmetry type one
        2 for structure symmetry type two
        3 for structure symmetry type three

(See Section 4.7 and Fig. 4.3)
NSYM23 = (integer) number of facade edges that are restrained vertically

NOUT = 0

= 1 element stresses output at element midheight coinciding with vertical divisions specified by IEW of card group 16.

= 2 element stresses output at the centroid of the ordinary element, or at the centroids of each half of the refined element cut vertically.

(See Fig. A.1)

M = (integer) number of trial vectors or trial mode shapes to be used for eigenvector calculations.

NRQD = (integer) number of vectors required to satisfy the vector tolerance criteria (see TOLVEC), and the number of modes to contribute to the structural responses. (NRQD ≤ M ≤ 11)

NOI = (integer) (NOI + 1) is the maximum number of trial vector iterations if the vector tolerance criteria is not achieved (see TOLVEC). A value of 10 is recommended.

TOLVEC = (real) number specifying the maximum error (20) permitted in the trial vectors. A value of 0.0001 is recommended.

GES = (real) random number chosen between -0.5 and +0.5 with at least 4 decimal places used for automatic trial vector generation.

IRENT = 0 for automatic generation of random trial vectors

1 for user input of trial vectors

2 for user input of trial mode shapes

(See section 4.9.4 for the difference between the trial vectors and the mode shapes)
GAMMA = (real) radian angle measured counterclockwise from the X axis to the line of earthquake excitation (Fig. 4.4)

INTER = 0 for pseudo-velocity input from datafile
         = 1 for interactive pseudo-velocity input after time periods are displayed.

X(KK) = (real) X coordinate of corner number KK

Y(KK) = (real) Y coordinate of corner number KK

N = (integer) number of corners in plan view of model after symmetry considerations
   (= NF if NSHAPE = 0)
   (= NF + 1 if NSHAPE = 1)
   (N >= NSYM23 must be satisfied)

FP(K,1) = (integer) number of corner I of facade K

FP(K,2) = (integer) number of corner J of facade K

FP(K,3) = (integer) number of elements horizontally spanning facade K. (NL x FP(K,3) <= 90, i.e. no more than 90 elements per facade)

RFP(K,1) = (real) perpendicular distance from facade K to the reference point. Reference points are assumed to form a vertical line through the structure if a static analysis (NDYN = 0) is specified.

FX(L) = (real) externally applied concentrated load in the x direction at the reference point of level L

FY(L) = (real) externally applied concentrated load in the y direction at the reference point of level L.

MT(L) = (real) externally applied torque applied to level L.

XMC(L) = (real) x-coordinate of the mass center for level L.

YMC(L) = (real) y-coordinate of the mass center for level L.
MAS(L) = (real) lumped mass of level L.

RMAS(L) = (real) rotational in-plane mass moment of inertia of level L with respect to its mass center.

NPRI(MM) =
- 0 response parameter MM is not output for each mode.
- 1 response parameter MM is output for each mode.

MM = 1 reference point displacements
MM = 2 corner displacements of structure
MM = 3 element stresses or internal member forces
MM = 4 inertia or equivalent elastic forces at the reference points of each level
MM = 5 base shears in the x and y direction and base torque
MM = 6 overturning base moments about the x and y axis

NPRIN(MM) =
- 0 the square-root-sum-of-squares of response parameter MM is not output
- 1 the square-root-sum-of-squares of response parameter MM is output

NPRINT(L) =
- 0 don't calculate stresses or member internal forces for the elements between level L and (L - 1)
- 1 do calculate stresses or member internal forces for the elements between level L and (L - 1)

TITLE2 = 80 spaces for an alphanumeric description of the facade
NMAT = (integer) number of different kinds of materials in the facade. More specifically it equals the number of different equivalent membranes in the facade. The membrane properties are \((E_y, G_{xy}, t)\). If member properties are input (see INCODE), the membrane properties are derived from the inputs of card group number 14. Thus any different combination of \(E_y, G_{xy}, t\) or card group number 14 within a facade constitutes a different material. 
\((NMAT \leq 12)\)

NTYPE = (integer) number of different element types in the facade. Element types are characterized by the material kind, height, width, and internal subdivisions which specify stress or force evaluation points. 
\((NTYPE \leq 36)\)
(See input variables for card group 16)

NGROUP = (integer) Designating each element with an element type number starting from 1, rectangular groups of the same element type can be formed. NGROUP is the total number of groups. Often, only one single element or a single line of elements form a group.

NSYM = 0 for no facade symmetry (If NSYM = 0, then NSYM must be 0)
= 1 for facade symmetry type 1
= 2 for facade symmetry type 2
= 3 for facade symmetry type 3
(See section 4.4 and Fig. 4.5)

INCODE = 0 if frame properties will be input for the whole facade (i.e. card group 14)
= 1 if membrane properties will be input for the whole facade (i.e. card group 15)

If lintel beams or framed portions exist in a composite facade, the elastic properties of their equivalent membrane must be evaluated using Eqs. 2.1, 2.2, 2.3, 2.8 prior to executing TUBE.
E = (real) Young's modulus
MU = (real) Poisson's ratio
B = (real) bay width
BD = (real) beam depth
BT = (real) beam thickness. TUBE treats the beam thickness also as the membrane thickness.
H = (real) storey height
CD = (real) column depth
CT = (real) column thickness
IBENDB = consider bending deformations in beams
          (0 = no, 1 = yes)
IBENDC = consider bending deformations in columns
          (0 = no, 1 = yes)
ISHEARB = consider shear deformations in beams
          (0 = no, 1 = yes)
ISHEARC = consider shear deformations in columns
          (0 = no, 1 = yes)
ISHEARJ = consider flexibility of finite size joints
          (0 = no, 1 = yes)
EX(LL) = (real) horizontal elastic modulus for membrane type LL
EY(LL) = (real) vertical elastic modulus for membrane type LL
GXY(LL) = (real) shear modulus for membrane type LL
TH(LL) = (real) thickness of membrane type LL
ET(JJ,1) = (real) width of element type JJ
ET(JJ,2) = (real) height of element type JJ
ETM(JJ) = (integer) material kind for element type JJ
IEW(JJ) = (integer) number of "bays" within an element which partly specifies the internal membrane stresses or member forces evaluation points for element type JJ. (see Fig. A.1)

IEH(JJ) = (integer) same as IEW (above) but for "stories". (see Fig. A.1).

NS(NN) = (integer) number of the bottom left element of the element group NN (see NGROUP)

NE(NN) = (integer) number of the top right element of the element group NN (see NGROUP)

NELT(NN) = (integer) number of element type of the above element group NN

NREM = (integer) number of global dynamic degrees of freedom

= (3 X NL if NSSYM = 0)
= (NL if NSSYM ≠ 0)

U(A,II) = (real) trial vector for mode number II. (See section 4.9.4 for the difference between trial vectors and mode shapes).

W(A,II) = (real) trial mode shape for mode number II.

SV(II) = (real) pseudo-velocity for mode number II.
Fig. A.1 Definition of NOUT for IEW = 4 and IEH = 3.
A.5 INPUT DATA EXAMPLES

This section provides the input data required to analyse the structures presented in sections 5.2 and 5.3.

<table>
<thead>
<tr>
<th>CARD GROUP No.</th>
<th>CARD No. of GROUP</th>
<th>DATA INPUT for box cantilever beam of section 5.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>FREQUENCY ANALYSIS OF TUBULAR CANTILEVER BEAM</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2 4 1 1 1 1 0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4 4 15 .0001 .3145 0 0 0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1 0. -.0762</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>2 .1524 -.0762</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3 .1524 0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1 1 2 2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2 2 3 1</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>1 0. 0.13.86 0</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>2 0. 0.13.86 0</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>3 0. 0.13.86 0</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>4 0. 0.6.93 0</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>0 0 0 0 0</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>facade No. 1</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>1 1 1 3 1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0. 2100000000000. 84000000000. .0254</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>1 .0762 .3048 1 0 0</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1 8 1</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>facade No. 2</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>1 1 1 1</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>0. 2100000000000. 84000000000. .0254</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>1 .0762 .3048 1 0 0</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>1 4 1</td>
</tr>
<tr>
<td>CARD GROUP No.</td>
<td>CARD No. of GROUP</td>
<td>DATA INPUT for wall-frame building of section 5.3</td>
</tr>
<tr>
<td>---------------</td>
<td>-------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>EARTHQUAKE ANALYSIS OF CHAN'S WALL-FRAME BUILDING</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>21011110</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4310.0001.31540.0.1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>10. -63.75</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>225. -63.75</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>325. 0.</td>
</tr>
<tr>
<td>7</td>
<td>1</td>
<td>1123</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2233</td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td>10.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>20.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>30.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>40.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>50.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>60.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>70.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>80.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>90.0.0.20264.0.</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>100.0.0.10132.0.</td>
</tr>
<tr>
<td>11</td>
<td>1</td>
<td>11111111</td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>1111111</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>1111111111</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>facade No. 1</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>16931</td>
</tr>
<tr>
<td>15</td>
<td>1</td>
<td>432000000.432000000.172800000.1.</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>112.524.132</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>28.3324.122</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>34.1724.112</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>412.548.134</td>
</tr>
<tr>
<td>16</td>
<td>5</td>
<td>58.3348.124</td>
</tr>
<tr>
<td>16</td>
<td>6</td>
<td>64.1748.114</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>171</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>282</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>393</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>10224</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>11235</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>12246</td>
</tr>
<tr>
<td>17</td>
<td>7</td>
<td>25281</td>
</tr>
<tr>
<td>17</td>
<td>8</td>
<td>26292</td>
</tr>
<tr>
<td>17</td>
<td>9</td>
<td>27303</td>
</tr>
<tr>
<td>12</td>
<td>1</td>
<td>facade No. 2</td>
</tr>
<tr>
<td>13</td>
<td>1</td>
<td>14610</td>
</tr>
<tr>
<td>14</td>
<td>1.1</td>
<td>432000000.2511111</td>
</tr>
<tr>
<td>14</td>
<td>1.2</td>
<td>7.52.5</td>
</tr>
<tr>
<td>14</td>
<td>1.3</td>
<td>12.1.1.5</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
<td>115.24.122</td>
</tr>
<tr>
<td>16</td>
<td>2</td>
<td>230.24.142</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>315.48.124</td>
</tr>
<tr>
<td>16</td>
<td>4</td>
<td>430.48.144</td>
</tr>
<tr>
<td>17</td>
<td>1</td>
<td>187</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>392</td>
</tr>
<tr>
<td>17</td>
<td>3</td>
<td>10233</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>12244</td>
</tr>
<tr>
<td>17</td>
<td>5</td>
<td>25291</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>27302</td>
</tr>
</tbody>
</table>
A.6 PROGRAM OUTPUT

Most of the input data is printed to allow direct checking. Some automatically generated data is also printed. In case of errors arising from core allocation, operations in algebraic equations in GAUSS, incomptble data, a corresponding message will be printed out.

Some input variables control what is to be printed out.

A.7 PROGRAM LISTING

The FORTAN IV compiled listing of TUBE is documented below.
PROGRAM TUBE

PROGRAM TUBE (TAPE1, TAPE2, INPUT, OUTPUT, TAPE3, TAPE4, TAPE5,
1, TAPE6, TAPE7, TAPE10, TAPE11)

THE PURPOSE OF THE PROGRAM IS TO ANALYZE TUBULAR
/all building structures subjected to earth quake.
ELASTIC AND STATIC.

PROGRAM VARIABLES

NF = NO. OF FACADES
NL = NO. OF LEVELS
HJL = NO. OF LOADED LEVELS
NHEL = NO. OF horizontal elements in one facade
NVEL = NO. OF vertical elements in one facade
NEAT = NO. OF different materials in one facade
NTYPE = NO. OF element types in one facade
NGROUP = NO. OF groups of element types in one facade
NEO = NO. EQUATIONS FOR THE TUBE STRUCTURE FOR STATIC ANAL
NREM = NO. EQUATIONS FOR THE TUBE STRUCTURE FOR DYNAMIC ANAL

PROGRAM ARRAYS

FP(N,1) = NO. OF FIRST CORNER LINE
FP(N,2) = NO. OF SECOND CORNER LINE
FP(N,3) = NL
FP(N,4) = HJL
RFPP(N) = DISTANCE OF FACADE FROM CENTRE LINE OF TUBE STRUCTURE
X(N) = X-COORDINATE OF A CORNER LINE, N=NO. OF FACADE
Y(N) = Y-COORDINATE OF A CORNER LINE, N=NO. OF FACADE
EX(M),
EY(M),
GXY(M),
EC(NP) = ELEMENT CONNECTIVITY, NP=NODE NO.
E(KND) = ELEMENT COORDINATE NO. OF ELEMENT DOF
OK = ELEMENT STIFFNESS MATRIX
S = GLOBAL STRUCTURAL STIFFNESS MATRIX
ETN(N) = ELEMENT TYPE NO. IN SEQUENCE FROM 1 TO N
BB = STRAIN DISPLACEMENT MATRIX
C = STRESS STRAIN HATRIX
R = GENERALIZED LOAD OR DISPLACEMENT VECTOR
AK = FACADE STIFFNESS MATRIX BEFORE AND AFTER CONDENSATION
D, SIG = ELEMENT NODAL DISPLACEMENTS AND C.G. STRESSES ON
B, BBLK, BNL, FD, FLO, ... ARRAYS FOR MATRIX MANIPULATION

C

C

C
DIMENSION TITLE(8)
COMMON/STRUCTURAL LEVEL...........INPUT
   COMMON NF,NL,NDOF,NSHAPE,NSYM,NOUT
C COMMON ON FACADE LEVEL...........INPUT
   COMMON/P2/NHEL,NVEL,MAN,NMAT,HTYPE,HEGF,NPR,NDAY,NSTOR
   1EX(12),ET(12),QY(12),TH(12),EC(8),EL(4),LEL(9),
   2ET(36,2),ETM(12),ETN(90),QK(9,9),IEW(12),IEH(12)
C COMMON ON STRUCTURAL LEVEL........SOLUTION
   DIMENSION S(240),CA(4),SA(4),LD(30),BD(10),ERR(10),U(30,10)
1,W(30,10),V(30,10),RFP(4,10),R(80,10),GC(10)
2,SU(10) INTEGER EC,EL,ETM,ETN,ICOUNT(4),NPRI(6),NPRINT(10)
3,INTER GD(30),FP(4,4)
REAL HAS(30)
DATA MAXNF, MAXNL, MAXMAT, MAXTYP, MAXFEL
1 / 4, 10, 12, 36, 90/

C PROBLEM IDENTIFICATION AND DESCRIPTION
READ(1,100) (TITLE(I),I=1,8)
READ(1,19) NDYN
WRITE(2,9) *NDYN=*,NDYN
WRITE(2,200) (TITLE(I),I=1,8)

C READ OVERALL STRUCTURAL DATA
CALL DATA1(MAXNF,MAXNL,ISTOP1,NDYN,CA,SA,H,NUL,NRD)
1,TLVEC,DES,IREN,ALPHA,HAS,INTER,NEG,NPRI
2,NPRINT,NPRINT,RFP,FP,R)
1,ISTOP1,GT,0 GO TO 999

C COMPUTE REQ. STORAGE FOR GLOBAL TUBE STIFF MATRIX
IMAX = (HEN+NEQ+NEQ)/2
WRITE(2,9) "STORAGE REQUIRED FOR STRUCTURAL STIFF MATRIX=",IMAX
C
REWRITE 3
REWRITE 4
REWRITE 5
REWRITE 7
DO 500 N=1,NF
NHEL=FP(N,3)
C READ DATA FOR EACH FACADE
CALL DATA2(MAXMAT,MAXTYP,MAXFEL,ISTOP2,NDOF,NHEL,LMAX,NSYM)
1,ISTOP2,GT,0 STOP

C ASSEMBLE EACH FACADE STIFFNESS MATRIX
C CONDENSE INTERNAL DOF
SAVE COMPRESSED FACADE STIFFNESS MATRIX ON TAPE 4

CALL ASMLI(NL,NDOF,NHEL,LMAX,NSYM,R,NEQ)
500 CONTINUE
C
C. ASSEMBLE OVERALL STATIC STRUCTURAL STIFFNESS MATRIX AK
   CALL ASHBL2(NL, NEQ, NREM, IMAX, NF, NSHAPE, NSSYM, S, CA, SA,
          1, ICOUNT, NDYN, RFP, R)
C
120   IF(NDYN, NEQ, 0) GOTO 10
   CALL STAT(6, R, NEQ, IMAX, NSSYM, NL)
   CALL STRESS(NL, NF, NDOF, NSHAPE, NSSYM, NOUT, CA, SA, ICOUNT, RFP
          1, NPRINT, NEQ, R, 1, NDYN)
   GOTO 999
125   C. GENERATE STRUCTURAL STIFFNESS MATRIX ADDRESSES FOR SKYLINe STORAGE
10   NN=0
   DO 5 I=1, NREM
      GD(I)=1
      NN=NN+1
      LD(I)=NN
   CONTINUE
5   C. COMPUTE STORAGE REQUIRED FOR DYNAMIC STRUCTURAL STIFFNESS MATRIX
   IMAX2=NSHAPE-(NREM+1)/2
   WRITE(2, *) NREM, GD(1), I=1, NREM, (MAS(I), I=1, NREM)
   WRITE(2, *) IMAX2, LD(I), I=1, NREM, (S(I), I=1, IMAX2)
   WRITE(2, *) N, NORD, NOI, TOLVEC, IRENT, GES
   IF(IRENT, NEQ, 0) GOTO 20
   GOTO 50
20   IF(IRENT, NEQ, 1) GOTO 30
   DO 60 J=1, M
      READ(1, *) (W(J, I), J=1, NREM)
      CONTINUE
   GOTO 50
60   C. CALCULATE FREQUENCIES AND MODE SHAPES
30   IF(IRENT, NEQ, 2) GOTO 40
   DO 70 J=1, M
      READ(1, *) (W(J, I), J=1, NREM)
      CONTINUE
   GOTO 50
70   C. PERFORM A SPECTRAL ANALYSIS
40   WRITE(2, *) '**********INPUT DATA ERRORS**********'
   WRITE(2, *) '**********INPUT FOR INTER IS NOT EQUAL TO 0,1,OR 2 ***'
50   CALL SIVIR(MAS, GD, S, ID, BD, ERR, U, W, IMAX2, NREM, M, NORD, NOI,
          1, TOLVEC, IRENT, GES)
   IF(1(NPRI(1), EQ, 0), AND, (NPRI(2), EQ, 0), AND, (NPRI(3), EQ, 0), AND, 
           1(NPRI(4), EQ, 0), AND, (NPRI(5), EQ, 0), AND, (NPRI(6), EQ, 0)) GOTO 999
60   C. PERFORM A SPECTRAL ANALYSIS
70   CALL MODAL(ALPHA, NORD, M, IMAX2, MAS, NSSYM, NL, NPRI, NPRIN, NPRINT,
          1, NREM, NEO, ETN, NHEL, BD, ICOUNT, INTER, ERR, U, V, W, BC, SV, R, S, NF, NDOF
          2, NSHAPE, NOUT, CA, SA, RFP)
100  FORMAT(8A10)
200  FORMAT(/8A10/)
999   STOP
C END
SUBROUTINE DATA       FTN 4,8+552  83/03/21, 15:51:27  PAGE 1

* THIS SUBROUTINE READS PROPERTIES OF THE TUBE STRUCTURE *

COMMON NF,NL,NDOF,NSHAPE,NSYM,NOUT
COMMON/P2/NEC,NEU,NMAT,NYPE,NEOF,NPR,NBAY,NSYM,NSTOR,
1   IEX(12),EY(12),GXY(12),TH(12),EC(6),EL(6),LEL(9),
2   ETH(36),ETHM(90),OK(9),IEF(12),IEH(12)
INTEGER FP(4),EC,EL,ETH,NPR,NPR(6),NPRINT(10)
READ(*,14)SA(4),AG,EN(5),AT(5),HAS(30),R(80)
1   RFP(4,10)

READ(*,14)NF,NL,NDOF,NSHAPE,NSYM,NSYM23,NOUT
WRITE(*,24)"NF,NL,NDOF,NSHAPE,NSYM,NSYM23,NOUT"
WRITE(*,24)NF,NL,NDOF,NSHAPE,NSYM,NSYM23,NOUT
NEV=NL
WRITE(2,101)NF,NL
IF(NSYM.EQ.1)READ(*,1)M,NRQD,NOI,TOLVEC,GES,IRENT
1   ALPHA,INTER
   IF(NSYM.EQ.1)WRITE(2,*)M,NRQD,NOI,TOLVEC,GES,IRENT
      ALPHA,INTER
   IF(NSYM.EQ.1)WRITE(2,*M,NRQD,NOI,TOLVEC,GES,IRENT
      ALPHA,INTER
   IF(NSYM.EQ.0)GO TO 90
WRITE(2,95)
GO TO 91
90 CONTINUE
WRITE(2,100)
91 CONTINUE

CHECK TO BE SURE INPUT DATA DOES NOT EXCEED STORAGE CAPACITY

ISTOP1=0
10   IF(NF.LE.MAXNF)GO TO 10
ISTOP1=ISTOP1+1
 WRITE(2,20)MAXNF
10   IF(NL.LE.MAXNL)GO TO 11
ISTOP1=ISTOP1+1
 WRITE(2,21)MAXNL
11   IF(ISTOP1.EQ.0)GO TO 12
WRITE(2,22)ISTOP1
STOP
12 CONTINUE

COMPUT TUBE DOF (NO. OF EQUATIONS)

IF(NSYM.NE.0)GO TO 60
IF(NSHAPE.NE.0)GO TO 50
NED=(4*NF)*NL
GO TO 61
50 CONTINUE
NED=(4+NF)*NL

C
SUBROUTINE DATA

GO TO 61
60 CONTINUE
IF (NSHAPE.EQ.1) GO TO 110
NEQ=NL*(NF+1-HSYM23)
GO TO 61
110 NEQ=NL*(NF+2-HSYM23)
61 CONTINUE
WRITE(2,*)(NEQ="",NEQ

C
READ AND PRINT PROPERTIES OF FACADES

70 IF (NSHAPE.EQ.0) GO TO 14
I=NF
GO TO 15
14 CONTINUE
I=NF+1
75 CONTINUE
DO 29 N=1,I
READ(1,*), NN,X(N),Y(N)
WRITE(2,*), "NN,X(N),Y(N)"
WRITE(2,*) NN,X(N),Y(N)
WRITE(2,52) N,X(N),Y(N)
29 CONTINUE
IF (NDYN.EQ.0) GOTO 140
DO 150 N=1,NF
READ(1,*), NN,FP(N,1),FP(N,2),FP(N,3)
WRITE(2,*), "NN,FP(N,1),FP(N,2),FP(N,3)"
WRITE(2,*) NN,FP(N,1),FP(N,2),FP(N,3)
FP(N,4)=NVEL
150 CONTINUE
GOTO 160
90 CONTINUE
140 WRITE(2,103)
DO 30 N=1,NF
READ(1,*), NN,FP(N,1),FP(N,2),FP(N,3),RFP(N,1)
WRITE(2,*), "NN,FP(N,1),FP(N,2),FP(N,3),RFP(N,1)"
WRITE(2,*) NN,FP(N,1),FP(N,2),FP(N,3),RFP(N,1)
RFP(N,4)=NVEL
WRITE(2,53) N,F,RFP(N,1),I=1,4,RFP(N,1)
30 CONTINUE
C
COMPUTE ORIENTATION OF FACADES: CA AND SA
160 DO 170 N=1,NF
III=FP(N,1)
JJJ=FP(N,2)
XX=X(JJJ)-X(III)
YY=Y(JJJ)-Y(III)
FW=SORT(XX*XX*YY*YY)
CA(N)=XX/FW
SA(N)=YY/FW
170 CONTINUE
IF (NDYN.EQ.0) GOTO 180
C
COMPUTE
C......1...PERPENDICULAR DISTANCE FROM FACADE TO SHEAR CENTER : RFP
C......2...LUMPED MASS MATRIX IN DIAGONAL VECTOR FORM : MAS
DO 190 L=1,NL
SUBROUTINE DATA

READ(*,*) NN,XMC,YMCH,MAS(L),RMASS
WRITE(*,*) 'NN,XMC,YMCH,MAS(L),RMASS'
WRITE(*,*) NN,XMC,YMCH,MAS(L),RMASS
IF((NSSYM.EQ.1).OR.(NSSYM.EQ.2)) GOTO 220
IF(NSSYM.NE.0) GOTO 230
MAS(L+NL)=MAS(L)-MAS(L+2*NL)+RMASS
GOTO 220
230 IF(NSSYM.NE.3) GOTO 220
MAS(L)=RMASS
220 DO 200 I=1,NF
200 FP(N,L)=(YMC-Y(FP(N,1)))*CA(N)-(XMC-X(FP(N,1)))*SA(N)
CONTINUE
190 READ(*,*) (NPRI(I),I=1,6)
WRITE(*,*) 'NPRI(I),NPRI(1),NPRI(3),NPRI(4),NPRI(5),NPRI(6)'
WRITE(*,*) (NPRI(I),I=1,6)
READ(*,*) (NPRI(I),I=1,6)
WRITE(*,*) 'NPRI(I),NPRI(1),NPRI(3),NPRI(4),NPRI(5),NPRI(6)'
WRITE(*,*) (NPRI(I),I=1,6)
IF(NPRI(3).EQ.1) WRITE(*,*) 'NPRINT(I),I=1,NL'
C
GO TO 240
180 WRITE(*,104)
C ASSEMBLE STRUCTURAL LOAD VECTOR
C
N=1
IF(NSSYM.EQ.0) N=3
J=1
DO 40 J=1,N
R(I)=0.
40 CONTINUE
DO 41 I=1,N
JJ=(I-1)*NL+1
II=J+NL-1
READ(*,*) (R(J),J=JJ,II)
41 CONTINUE
WRITE(*,*) '***********************************************************************LOAD VECTOR'
WRITE(*,*) (R(I),I=1,N)
READ(*,*) (NPRI(I),I=1,6)
WRITE(*,*) 'NPRI(I),NPRI(I),NPRI(I),NPRI(I),NPRI(I),NPRI(I)'
WRITE(*,*) (NPRI(I),I=1,6)
C
101 FORMAT(34H INPUT TABLE 1, BASIC PARAMETERS //
1 40SX, 40H NUMBER OF FACADES , 15/
2 5X, 40H NUMBER OF LEVELS , 15)
100 FORMAT(//# ANALYSIS IS PERFORMED USING THE ORDINARY ELEMENT WITH
6 1NOF//)
95 FORMAT(//# ANALYSIS IS PERFORMED USING THE REFINED ELEMENT WITH
1 9NOFS//)
29 FORMAT(// 28H TOO MANY FACADES, MAXIMUM =
20 FORMAT(// 28H TOO MANY LEVELS, MAXIMUM =
22 FORMAT(// 28H EXCUTION HALTED BECAUSE OF,15,13H Fatal Errors //)
102 FORMAT(//34H INPUT TABLE 2, FACADE PROPERTIES //
1 12H CORNER N:,8X,12HX-COORDINATE,8X,12HY-COORDINATE)
SUBROUTINE DATA

52 FORMAT(I12,215,4)

103 FORMAT(/ 12H FACADE NO., 4X, 1HI, 4X, 1HJ, 4X, 4HNHEL, 4X, 4HNVEL, 4X,
175 12HDIST, 4X, FROM CENTRE/)

53 FORMAT(I12,215,218,4)

104 FORMAT(/ 26H INPUT TABLE 3., LOAD DATA //
240 RETURN
1 10H LEVEL NO., 5X, 7HFORCE-X, 5X, 7HFORCE-Y, 5X, 7HTORSION)

END
SUBROUTINE DATA2(MAXMAT,MAXTYP,MAXFEL,ISTOP2,NDOF,NEL,LMAX,NSYM)

******************************************************************************

* THIS SUBROUTINE READS FACADE PROPERTIES

******************************************************************************

COMMON/P2,NHEL,NVEL,NMAT,NTYPE,NEQF,NPR,NBAY,NSTOR,
1EX(12),EY(12),GXY(12),TH(12),EC(8),EL(6),EL(9),
2ET(36,2),ETN(12),ETH(90),QK(9,9),IEW(12),IETH(12)
DIMENSION TITLE(8)
INTEGER EC,EL,ETN,ETH

READ(1,99) TITLE
WRITE(2,100) TITLE
READ(1,*4) NHAT,NMAT,NTYPE,NGROUP,NSYM,INCDE
WRITE(2,*) 'NHAT,NMAT,NTYPE,NGROUP,NSYM,INCDE'
WRITE(2,*) NHAT,NMAT,NTYPE,NGROUP,NSYM,INCDE
NHEL=NHEL,NVEL
WRITE(2,101) NHEL,NVEL,NMAT,NTYPE

CHECK TO BE SURE THAT INPUT DATA DOES NOT EXCEED STORAGE CAPACITY

ISTOP2=0
IF(NHEL.LE.MAXFEL) GO TO 10
ISTOP2=ISTOP2+1
WRITE(2,20) MAXFEL
10 IF(NMAT.LE.MAXMAT) GO TO 11
ISTOP2=ISTOP2+1
WRITE(2,21) MAXMAT
11 IF(NTYPE.LE.MAXTYP) GO TO 12
ISTOP2=ISTOP2+1
WRITE(2,22) MAXTYP
12 IF(ISTOP2.EQ.0) GO TO 13
ISTOP2=ISTOP2+1
WRITE(2,23) ISTOP2
STOP

13 CONTINUE

C READ OR COMPUTE ELASTIC PROPERTIES OF ELEMENT

45 IF(INCODE.NE.0) GO TO 14
CALL EMAT
GO TO 15

14 CONTINUE
READ(1,1) (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
WRITE(2,*) (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)
WRITE(2,*) (EX(I),EY(I),GXY(I),TH(I),I=1,NMAT)

15 CONTINUE
WRITE(2,102)
WRITE(2,103)

C READ AND PRINT ELEMENT TYPE PROPERTIES

}
SUBROUTINE DATA2

WRITE(2,103)
DO 30 J=1,HTYPE
READ(4,*) NN,(ET(J,I),I=1,2),ETH(J),IEW(J),IAM(J)
WRITE(2,53) J,(ET(J,I),I=1,2),ETH(J),IEW(J),IAM(J)
30 CONTINUE
WRITE(2,104)
C....GENERATE ELEMENT TYPE NO. FOR EACH ELEMENT OF FACADE
DO 35 I=1,NGROUP
READ(1,*) NS,NE,MELT
WRITE(2,1) 'NS,NE,MELT'
WRITE(2,1) NS,NE,MELT
LL=NE-NS
L=MOD(LL,NHEL)
INC=NS+L
DO 35 II=INC,NE,NHEL
II=II-L
DO 35 J=I,II
ETN(J)=MELT
35 CONTINUE
C....GENERATE AND PRINT ELEMENT, NODE CONNECTIVITY, AND TYPE
DO 40 I=1,NEL
CALL ELCOM(I,NDOF)
IF(NDOF.NE.0) GO TO 41
NNP=4
GO TO 42
41 CONTINUE
NNP=8
42 CONTINUE
WRITE(2,54) I,(EC(J),J=1,NNP),ETN(I)
40 CONTINUE
C....EVALUATE NO. OF FACADE DOF'S: NEOF
C....THE REQ. STORAGE FOR ITS STIFFNESS MATRIX
IF(NDOF.NE.0) GO TO 210
IF(NSYM.NE.0) GO TO 215
NEOF=NVEL+(NHEL+1)*NHEL
GO TO 205
215 CONTINUE
NEOF=NVEL+(1*NHEL)
GO TO 205
210 CONTINUE
IF(NSYM.NE.0) GO TO 220
NEOF=NVEL*(2*NHEL+3)
GO TO 205
220 CONTINUE
IF(NSYM.EQ.1) GO TO 225
NEOF=NVEL*(2*NHEL+2)
GO TO 205
225 CONTINUE
NEOF=NVEL*(2*NHEL+1)
205 CONTINUE
LMAX=(NEOF*NEOF+NEOF)/2
WRITE(2,*) 'NO. OF FACADE DOF = ',NEOF
WRITE(2,*) 'SPACE REQUIRED FOR FACADE STIFF MATRIX = ',LMAX
SUBROUTINE DATA2  FTM 4.8552  83/03/21.15,51.27  PAGE 3

115  
100 FORMAT('8A10')
101 FORMAT(/'2X,8A10')
102 FORMAT(/'3H INPUT TABLE A., FACADE PROPERTIES //
1   5X, 40H NUMBER OF HORIZONTAL ELEMENTS   15/
2   5X, 40H NUMBER OF VERTICAL ELEMENTS     15/
3   5X, 40H NUMBER OF DIFFERENT MATERIALS    15/
4   5X, 40H NUMBER OF DIFFERENT ELEMENT TYPES 15//)
103 FORMAT(/'3H INPUT TABLE B., MATERIAL PROPERTIES //
1   10H MATERIAL, 5X, 10H MODULUS, 5X, 10H MODULUS,
2   8X, 7H SHEAR, 7X, 8H MATERIAL/
3   4X, 6H NUMBER, 10X, 2H HEX, 13X, 2HEY, 11X, 7H MODULUS, 6X, 8H THICKNESS)
104 FORMAT(/'3H INPUT TABLE C., ELEMENT TYPES //
1   10H ELEMENT, 8X, 7H ELEMENT, 8X, 7H ELEMENT, 2X, 8H MATERIAL,
2   4X, 6H NO., 4X, 6H NO., 4F/
3   6X, 4H TYPE, 8X, 7H WIDTH-A, 7X, 7H HEIGHT-B, 6X, 4H TYPE,
3   6X, 4H BAYS, 4X, 7H STORIES//)
105 FORMAT(/'110 A10, 4E15.4')
106 FORMAT(/'10H INPUT CONNECTIVITY AND TYPE//)
107 FORMAT(/'10I15')
108 FORMAT(/'10H TOOMANY ELEMENTS IN FACADE, MAXIMUM=*15)
109 FORMAT(/'10H TOOMANY DIFFERENT MATERIALS, MAXIMUM=*15)
110 FORMAT(/'10H TOOMANY ELEMENT TYPES, MAXIMUM=*15)
111 FORMAT(/'10H EXECUTION HALTED BECAUSE OF *15, *FATAL ERRORS//)

RETURN END
SUBROUTINE EMAT

*****************************************************************************

* THIS SUBROUTINE EVALUATES THE ELASTIC PROPERTIES
  OF THE EQUIVALENT ORTHOTROPIC MEMBRANE
*****************************************************************************

COMMON/P2,NHEL,NHEL,NHA,HTYPE,NEQF,NPR,NRAY,NSTOR,
1E2(12),EY(12),GXY(12),TH(12),ECB,EL(6),LEL(9),
2ET(36,2)/ETM(12),ETN(50,9,9),IEM(12),IEH(12)
INTEGER FP,EC,EL,ETM,ETN
REAL HU

DO 2999 I=1,NMAT
   READ(1,I*) E,HU,IBEND,IBENDAR,ISHEAR,ISHEARJ
   READ(1,I*) B,BD,BT
   READ(1,I*) H,CD,CT
   WRITE(2,I*) E,HU,IBEND,IBENDC,ISHEAR,ISHEARJ,B,BD,BT
   -H,CD,CT
   WRITE(2,I*) E,HU,IBEND,IBENDC,ISHEAR,ISHEARJ,B,BD,BT
   -H,CD,CT
C....... ELEMENT THICKNESS (TT) IS TAKEN TO BE THE BEAM THICKNESS (BT)
   TT=BT
   Q=1./(CD*BT)
   X=CD/2.
   Y=BD/2.
C
C DELTA OF PARTS 1 AND 3
DCOL=(H-BD)/(CDCTE)
WRITE(2,91) DCOL
   IF(ISHEARJ,M.E.0.) GOTO 97
   VUNIF=0.
   VFUNC=0.
   GOTO 94
97 CONTINUE
C DELTA OF PART 2 DUE TO UNIFORM STRESS ONLY
   VUNIF=4.0*(CD*BD)/(B*B)
   WRITE(2,92) VUNIF
   92 FORMAT(1X,1,5E15.4)
C DELTA OF PART 2 DUE TO INFINITE FOURIER SERIES ONLY
TTER=0.
   DO 10 M=1,50
      AL=M*22./((7.*B/2.)
      ALC=AL*CD/2.
      ALX=AL*Y
      T1=SIN(ALX)/((AL)*
      T2=1.+H1*(ALC*COSH(ALC)*SINH(ALY)
      T3=1.+H1*SINH(ALC)*SINH(ALY)
SUBROUTINE EMAT

T4=(1.+HU)*(ALY*COSH(ALY)-SINH(ALY))*SINH(ALC)
T5=COS(ALY)/(SINH2(ALC)+2.*ALC)
TERM=TERM+T1*(T2+T3-T4)*T5
10 CONTINUE
VFUNC=TERM4.*Q/(E**22./7.)
WRITE(2,93) VFUNC
93 FORMAT( 6E15.4)
WRITE(2,999) EY(I)
999 FORMAT(///** EQUIVALENT MODULUS EY = **F15.4)

C EXTR SHEAR MODULUS GXY

T1=(H-BD)/H
CI=CT*CD**3.
BI=BT*BD**3.
BEND=0.
BENDC=0.
SHEAR=0.
SHEARJ=0.
10 IF((BENDR.EQ.0).AND.(BENDC.EQ.0)) GO TO 20
20  BENDR=(H/8.)*(B-CD)/(B-CD)*(B-CD)/BI
BENDC=(T1**(H-BD))/(H-BD)/CI
30 IF((ISHEAR.EQ.0).AND.(ISHEARC.EQ.0)) GO TO 40
40 IF((ISHEAR.EQ.0).AND.(ISHEARC.EQ.0)) GO TO 50
SHEAR=(H*(B-CD)/(B**2*BT*BD/1.2))**(2./((1.+HU))
SHEARC=(1.2*T1/(CT*CD))**(2./(1.+HU))
50 IF((ISHEAR.EQ.0).AND.(ISHEARC.EQ.0)) GO TO 60
SHEARJ=(((B-CD)/B+T1-1)**2.)/(BT*CD*1-1)**(2./(1.+HU))
60 GXY=BENDR+BENDC+BEND2+SHEAR+SHEARC+SHEARJ
111 FORMAT(///** EQUIVALENT SHEAR MODULUS = **F15.4)
WRITE(2,111) GXY(I)
111 CONTINUE
9999 CONTINUE
RETURN
END
SUBROUTINE ELCON(I, NDOF)
    COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NDOF,NPR,NBAY,NSTOR,
    LEX(12),EY(12),EXY(12),TH(12),EC(8),EL(6),LEL(9),
    2ET(36),ET(12),ETN(90),QK(9,9),IEW(12),IEH(12)
C      INTEGER FP,EC,EL,ETM,ETN
C
C*****************************************************************************
C
* THIS SUBROUTINE GENERATE ELEMENT CONNECTIVITY *
C
C*****************************************************************************
C
C THIS CAN BE ACHIEVED BY RELATING THE ELEMENT NO. TO ITS FIRST NODE (LOWER, LEFT)
C
JL=(I-1)/NHEL
II=I-JL*NHEL
IF(NDOF,N.E.0) GO TO 10
EC(1)=(II-1)*(NVEL+1)+JJ+1
EC(2)=EC(1)+NVEL+1
EC(3)=EC(2)+1
EC(4)=EC(3)+1
GO TO 999
10 CONTINUE
EC(1)=(II-1)*(NVEL+1)+JJ+1
EC(2)=EC(1)+(NHEL+1)*(2*NVEL+1)
EC(3)=EC(1)+NVEL+1
EC(4)=(NVEL+1)*(NHEL+1)+II*NVEL+JJ+1
EC(5)=EC(3)+1
EC(6)=EC(2)+1
EC(7)=EC(1)+1
EC(8)=EC(4)-NVEL
999 CONTINUE
RETURN
END
SUBROUTINE ASHBL1(NL,NDOF,NEL,LMAX,NSYM,NEO)

*******************************************************************************
* THIS SUBROUTINE ASSEMBLES EACH FACADE STIFF. MATRIX,                     *
* CONDENSE IT, AND FORMS LOCAL(K) LEVEL SUBMATRICES AND                   *
* AND GLOBAL SUBMATRICES (S).                                            *
*******************************************************************************
COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEOF,NPR,NBAY,NSTOR,                        
  IEX(12),EY(12),OXY(12),TH(12),EC(8),EL(6),IEL(9),                      
  2ET(36,2),ETM(12),ETM(90),OK(9,9),IEW(12),IEW(12)
REAL K,KAK(6105),R(NEO)
INTEGER FP,E6,E7,ETM,ETN

WRITE(2,*) 'NSYM=',NSYM

C INITIALIZE FACADE STIFF. MATRIX AK
DO 1 I=1,LMAX
  AK(I)=0.0
1 CONTINUE
WRITE(2,100)
IF(NDOF,NE,0) GO TO 5
LIM=6
DO 99 M=1,NEL

C PROCEDURE IN THIS SUBROUTINE WILL BE AS FOLLOWS.,
  1.. FORM LOCAL ELEMENT STIFFNESS MATRIX OK
  2.. ASSEMBLE EACH OK TO FORM FACADE STIFF. MATRIX AK
  3.. CONDENSE AK

C FORM ELEMENT LABEL VECTOR- EL OR LEL -
CALL LABEL(H,NDOF,NSYM)
WRITE(2,200) H,(EL(I),I=1,6)

C FORM ELEMENT STIFF. MATRIX OK
CALL REC(T(H,NDOF))

C ASSEMBLE EACH OK ACCORDING TO THE CORRESPONDING LABEL VECTOR 'EL'
  NOTE- LIM= NO. OF ELEMENT DOF
DO 10 LL=1,LIM
  I=EL(LL)
10 DO 10 HH=1,LIM
      J=EL(HH)
C ASSEMBLE ONLY THE LOWER TRIANGLE, ROW BY ROW
C OR
C THE UPPER TRIANGLE COL. BY COL. ... NOTICE THAT BOTH ARE IDENTICAL
IF(I.GT.J) GO TO 10
SUBROUTINE ASMBLI

IF(I.EQ.0) GO TO 10
LOC=(J-1)*J)/2+1
AK(LOC)=AK(LOC)+QK(LL,MM)
CONTINUE
GO TO 6

5 CONTINUE
LLH=9
DO 88 N=1,NEL
CALL LABEL(N,NDOF,NSYM)
WRITE(2,200) M*(LEL(I),I=1,9)
CALL RECT(N,NDOF)
DO 8 8 LL=1,LLH
I=LEL(II)
J=LEL(MM)
IF(I.GT.J) GO TO 8
IF(I.EQ.0) GO TO 8
LOC=(J-1)*J)/2+1
AK(LOC)=AK(LOC)+QK(LL,MM)
CONTINUE
88 CONTINUE
CONTINUE
6 CONTINUE
C......FACADE STIFFNESS MATRIX (AK) IS NOW FORMED

C CONDENSE INTERNAL DOF IN ABOVE AK

IF(NDOF.EQ.1) GOTO 30
 IF(NHEL.EQ.1) GOTO 20
30 IF(NSYM.NE.0) GO TO 17
NR=3*NL
GO TO 16
17 CONTINUE
NR=2*NL
16 CONTINUE
CALL GAUSS(AK,R,LMAX,NEGF,0,NR,DET,1)
C......FACADE STIFFNESS MATRIX IS NOW REDUCED
20 WRITE(3*) NSYM,NEGF,LMAX,NEL,NMAT,N1,NTYPE,NR
 WRITE(5*) (ETM(J),J=1,NEL)
 WRITE(5*) (EX(I),I=1,NMAT,XY(I),TH(I),I=1,NMAT)
 WRITE(5*) (EM(I),I=1,NMAT,EM(I),I=1,NMAT)
 WRITE(4*) (AK(I),I=1,LMAX)
100 FORMAT(///* ELEMENT NO. DEGREES OF FREEDOM */)
200 FORMAT(11A,5X,9I5)
RETURN
END
SUBROUTINE LABEL(M,HDOF,NSYM)
COMMON/P2/NHEL,NHAT,NTYPE,NEOF,NPR,NBAY,HSTOR,
/EXE(12),EY(12),GXY(12),TH(12),EC(8),EL(6),EL(9),
/2ET3(32,2),ETM(12),ETH(90),NKL(9,9),IEW(12),IEH(12)
INTEGER FP,EC,EL,ETH,ETM

******************************************************************************

THIS SUBROUTINE FORMS THE LABEL(NUMBERING OF DOF) FOR EACH ELEMENT

******************************************************************************

FIRST HORIZONTAL DOF

L=(M-1)/NHEL
NPR=L+1
C LIN=NO. OF DOF PER ELEMENT
C NPL= NO. OF NODES PER ELEMENT
LIL=6
NPL=LIL-2
DO 1 I=1,NPL
IF(I.GT.2) GO TO 2
EL(1)=L
GO TO 1
2 EL(2)=L+1
1 CONTINUE

SECOND VERTICAL DOF

LL=M-L*NHEL
LLL=NVEL*(LL+1)

... ELEMENTS FIXED TO FOUNDATION

IF(L.LE.0) GO TO 10
EL(3)=0
EL(4)=0
IF(NET.EQ.1) GO TO 11
IF(NET.EQ.NHEL) GO TO 12
EL(4)=LLL+1
EL(5)=EL(6)+NVEL
GO TO 111
11 EL(5)=3*NVEL+1
IF(NHEL.EQ.1) EL(5)=2*NVEL+1
EL(6)=NVEL+1
GO TO 111
12 EL(5)=NVEL+2+1
EL(6)=NVEL*(NHEL+1)+1
GO TO 111
10 CONTINUE

... REST OF ELEMENTS
IF(NHEL.EQ.1) GOTO 21

A... ELEMENTS ON THE LEFT MOST SIDE
IF(LL.NE.1) GO TO 20
EL(3)=NVEL+1
EL(4)=NVEL+1
EL(5)=EL(4)+1
EL(6)=EL(3)+1
GO TO 111

B. ELEMENTS ON THE RIGHT MOST SIDE

20 IF(LL.NE.NHEL) GO TO 30
EL(3)=NVEL*(NHEL+1)+1
EL(4)=NVEL*2+1
EL(5)=EL(4)+1
EL(6)=EL(3)+1
GO TO 111

C. INTERIOR ELEMENTS

30 EL(3)=EL(1)+1
EL(4)=EL(3)+NVEL
EL(5)=EL(4)+1
EL(6)=EL(3)+1
GO TO 111

111 CONTINUE
IF(NDOF.NE.0) GO TO 70
IF(NSYM.NE.0) CALL SYM(NDOF,LL,NSYM)
WRITE(7,*) EL,NPR
GO TO 999

70 CONTINUE

C GENERATE LABELLING VECTOR FOR THE Refined ELEMENT *** BASED **
ON THOSE OF THE ORDINARY ELEMENT

LEV(1)=EL(1)
LEV(2)=NVEL*2+1
LEV(3)=EL(2)

1. ELEMENTS FIXED TO FOUNDATION

100 IF(LL.NE.0) GO TO 80
LEV(4)=0
LEV(5)=0
LEV(6)=0
105 IF(LL.EQ.NHEL) GO TO 81
LEV(7)=EL(5)+NVEL
GO TO 82
81 CONTINUE
LEV(7)=EL(5)

82 CONTINUE
LEV(8)=NVEL*(3+NHEL)+(LL-1)*NVEL+1
IF(LL.EQ.1) GO TO 83
LEV(9)=EL(6)+NVEL
GO TO 84
SUBROUTINE LABEL

CONTINUE
LEL(9) = EL(6)

CONTINUE
GO TO 91

C 2,..., REST OF ELEMENTS
C
CONTINUE
LEL(4) = EL(3)
LEL(5) = NVEL*(3+NHEL)+(LL-1)*NVEL+L
LEL(6) = EL(4)
IF(NHEL.EQ.1) GOTO 22
LEL(4) = LEL(4)+NVEL
LEL(6) = LEL(6)+NVEL

CONTINUE
LEL(7) = LEL(4)+1
LEL(8) = LEL(5)+1
LEL(9) = LEL(4)+1
IF(LL.NE.1) GO TO 90
LEL(4) = EL(3)
LEL(9) = LEL(4)+1

CONTINUE
IF(LL.NE.NHEL) GO TO 91
LEL(6)+#LEL(6)+1

CONTINUE
IF(NSYM.NE.0) CALL SYM(NDOF,LL,NSYM)
WRITE(*,10) LEL,NPR

CONTINUE
RETURN

END
SUBROUTINE SYM(X)

* THIS SUBROUTINE MODIFIES ELEMENT LABELS TO ACCOUNT FOR
  * FACADE SYMMETRY CONDITIONS

C

COMMON/P2/NHEL,NVEL,NMAT,NTYPE,NEDF,NPR,NBAY,NSTOR,
  LEX(2),ELY(12),OBX(12),TH(12),EC(8),EL(6),EL(9),
  LEM(2),ETN(12),ETN(90),OK(9,9),IEW(12),IEH(12)
INTEGER FP,EC,EL,ETM,ETN

*** NOTE:
*** NSYM = 0 IMPLIES NO FACADE SYMMETRY OF ACTUAL FACADE
*** NSYM = 1 IMPLIES FACADE SYMMETRY OF ACTUAL FACADE
*** NSYM = 2 OR 3 IMPLIES ANTI-SYMMETRY OF ACTUAL FACADE

C

IF(KDOF.NE.0) GO TO 80
IF(NSYM.NE.1) GO TO 40

C FOR CASE OF SYMM. NSYM=1

EL(1)=0
EL(2)=0
EL(5)=EL(3)-NVEL
EL(4)=EL(4)-NVEL
EL(5)=EL(5)-NVEL
EL(6)=EL(6)-NVEL
GO TO 444

40 CONTINUE

C FOR CASE OF ANTI-SYMM. NSYM=2 OR 3

IF(LI.EQ.1) GO TO 41
IF(LI.EQ.NWEJ) GO TO 42

C A....... INTERNAL ELEMENTS

EL(3)=EL(3)-NVEL
EL(4)=EL(4)-NVEL
EL(5)=EL(5)-NVEL
EL(6)=EL(6)-NVEL
GO TO 444

41 CONTINUE

C B....... ELEMENTS ON THE LEFT MOST SIDE

EL(4)=EL(4)-NVEL
EL(5)=EL(5)-NVEL
IF(NSYM.NE.3) GO TO 444
EL(3)=0
EL(6)=0
GO TO 444

42 CONTINUE

C C....... ELEMENTS ON THE RIGHT MOST SIDE

C
SUBROUTINE SYMM

EL(3)=EL(3)-NVEL
EL(4)=EL(4)-NVEL
EL(5)=EL(5)-NVEL
EL(6)=EL(6)-NVEL
IF(NSYM.NE.2) GO TO 444
EL(4)=0
EL(5)=0

GO TO 444
CONTINUE
DD 4444 I=1+6
IF(EL(I).GE.0) GO TO 4444
EL(I)=0

GO TO 9999
CONTINUE

**** MODIFICATION FOR REFINED ELEMENT.
IF(NSYM.NE.1) GO TO 81

FOR CASE OF SYMM.

LEL(1)=0
LEL(2)=0
LEL(3)=0
NN=2*NVEL
IF(LEL.EQ.1) GO TO 82
IF(LEL.EQ.NVEL) GO TO 83
LEL(4)=LEL(4)-NN
LEL(5)=LEL(5)-NN
LEL(6)=LEL(6)-NN
LEL(7)=LEL(7)-NN
LEL(8)=LEL(8)-NN
LEL(9)=LEL(9)-NN
GO TO 888

CONTINUE

LEFT HOST SIDE ELEMENTS
LEL(4)=LEL(4)-NVEL
LEL(5)=LEL(5)-NN
IF(NVEL.NE.1) GOTO24
LEL(6)=LEL(6)-NVEL
LEL(7)=LEL(7)-NVEL
GOTO 25

CONTINUE

LEL(6)=LEL(6)-NN
LEL(7)=LEL(7)-NN

CONTINUE

LEL(8)=LEL(8)-NN
LEL(9)=LEL(9)-NVEL
GO TO 888

RIGHT HOST SIDE ELEMENTS
LEL(4)=LEL(4)-NN
LEL(5)=LEL(5)-NN
SURROUTINE SYMM

115    LEL(6)=LEL(6)-NVEL
        LEL(7)=LEL(7)-NVEL
        LEL(8)=LEL(8)-NN
        LEL(9)=LEL(9)-NN
        GO TO 888
120    B1 CONTINUE

C FOR CASE OF ANTI-SYMM.

125    LEL(2)=LEL(2)-NVEL
        IF(LEL.0.1) GO TO 84
        IF(LEL.EQ.NHEL) GO TO 85
        LEL(4)=LEL(4)-NVEL
        LEL(5)=LEL(5)-NVEL
        LEL(6)=LEL(6)-NVEL
        LEL(7)=LEL(7)-NVEL
        LEL(8)=LEL(8)-NVEL
        LEL(9)=LEL(9)-NVEL
        GO TO 888
B4 CONTINUE

135    LEL(5)=LEL(5)-NVEL
        LEL(6)=LEL(6)-NVEL
        LEL(7)=LEL(7)-NVEL
        LEL(8)=LEL(8)-NVEL
        IF(NSYM.NE.3) GO TO 888
        LEL(4)=0
        LEL(9)=0
140    C GO TO 888
B5 CONTINUE

145    LEL(4)=LEL(4)-NVEL
        LEL(5)=LEL(5)-NVEL
        LEL(6)=LEL(6)-NVEL
        LEL(7)=LEL(7)-NVEL
        LEL(8)=LEL(8)-NVEL
        LEL(9)=LEL(9)-NVEL
        IF(NSYM.NE.2) GO TO 888
        LEL(6)=0
        LEL(7)=0
888 CONTINUE

150    DO 8888 I=1,9
155    IF(LEL(I).GE.0) GO TO 8888
        LEL(I)=0
8888 CONTINUE
9999 CONTINUE
RETURN
END

160
SUBROUTINE RECT

**THIS SUBROUTINE FORMS EACH ELEMENT STIFFNESS MATRIX**

ELEMENT PROPERTIES

IT = ET(N)
A = ET(IT) + 1
B = ET(IT) + 2
IM = ETM(IT)
E2 = ETM(IM)
G12 = GXY(IM)
R = A/B

IF(HDOF,HE,0) GO TO 50

INITIALIZE ELEMENT STIFFNESS MATRIX QK

DO 10 I = 1,6
  DO 10 J = 1,6
    QK(I,J) = 0.0
  10 CONTINUE

FORM STIFFNESS MATRIX OF ORDINARY ELEMENT

QK(1,1) = TT*R*G12
QK(2,1) = QK(1,1)
QK(3,1) = 3*TT*G12
QK(4,1) = -QK(3,1)
QK(5,1) = -QK(3,1)
QK(6,1) = QK(3,1)

QK(2,2) = QK(1,1)
QK(3,2) = -QK(3,1)
QK(4,2) = QK(3,1)
QK(5,2) = QK(3,1)
QK(6,2) = -QK(3,1)

QK(3,3) = (E2*R+G12/R)*TT/3.
QK(4,3) = (.5*E2*R-G12/R)*TT/3.
QK(5,3) = -.5*QK(3,3)
QK(6,3) = (.5*G12/R-E2*R)*TT/3.

QK(4,4) = QK(3,3)
SUBROUTINE RECT

OK(5,4)=OK(6,3)
OK(6,4)=-.5*OK(3,3)

C

OK(5,5)=OK(3,3)
OK(6,5)=OK(4,3)

C

OK(6,6)=OK(3,3)

C

DO 20 I=1,6
DO 20 J=1,6
IF(J.LE.I) GO TO 20
OK(I,J)=OK(J,I)
CONTINUE
GO TO 999

C

50 CONTINUE

C

C FORM STIFFNESS MATRIX OF THE REFINED ELEMENT

C

INTIALIZE OK 9X9

DO 55 I=1,9

C

DO 55 J=1,9
OK(I,J) = 0.0

55 CONTINUE

C

S1=1T*612*R/12.
S2=1T*612/12.
S3=1T*E2*R/60.
S4=1T*612/(R*36.)

C

OK(1,1)=28.*S1

C

OK(2,1)=32.*S1
OK(3,1)=4.*S1

C

OK(4,1)=10.*S2
OK(6,1)=-10.*S2

C

OK(7,1)=-2.*S2
OK(9,1)=2.*S2

C

OK(2,2)=64.*S1
OK(3,2)=OK(2,1)

C

OK(4,2)=-8.*S2
OK(6,2)=-OK(4,2)
OK(7,2)=OK(4,2)
OK(9,2)=-OK(4,2)

C

OK(3,3)=OK(1,1)

C

OK(4,3)=OK(7,1)
OK(6,3)=-OK(7,1)
OK(7,3)=OK(4,1)

C

OK(9,3)=-OK(4,1)

C

OK(4,4)=8.*S3+28.*S4
OK(5,4)=4.*S3-32.*S4
OK(6,4)=-2.*S3+4.*S4
SUBROUTINE RECT

115  OK(7,4)=2.*S3+2.*S4  
     OK(8,4)=-4.*S3+16.*S4  
     OK(9,4)=-8.*S3+14.*S4

120  C
     OK(5,5)=32.*S3+64.*S4
     OK(6,5)=OK(5,4)  
     OK(7,5)=OK(8,4)  
     OK(8,5)=-32.*S3+32.*S4
     OK(9,5)=OK(8,4)

125  C
     OK(6,6)=OK(4,4)
     OK(7,6)=OK(9,4)
     OK(8,6)=OK(8,4)
     OK(9,6)=OK(7,4)

130  C
     OK(7,7)=OK(4,4)
     OK(8,7)=OK(6,4)
     OK(9,7)=OK(6,4)

135  C
     OK(8,8)=OK(5,5)
     OK(9,8)=OK(5,4)

140  C
     DO 60 I=1,9
     DO 60 J=1,9
     IF(J.LE.I) GO TO 60
     OK(I,J)=OK(J,I)

145  60 CONTINUE
     CONTINUE
     RETURN
     END
SUBROUTINE GAUSS

DIMENSION R(NEQ,NLO),AK(IMAX)

*****************************************************************************
AK CONTAINS COEFFICIENTS IN ONE TRIANGLE OF SYMMETRIC
MATRIX COL. BY COL. IF UPPER TRIANGLE, ROW BY ROW
IF LOWER TRIANGLE.
R= RIGHT HAND SIDE MATRIX IN (AK)(X)=(R)
NEO= NO. OF EQUATIONS
ILO=STORAGE REQUIRED FOR AK
NLO=NO. OF LOAD CASES, I.E. COLS. OF R
N= NO. OF EQUATIONS REMAINED IN CONDENSATION
**** NOTICEXX ELEIMINATED DOF MUST BE STORED LAST
DET= DETERMINANT
NCODE= 1 FORWARD ELIMINATION AND BACK SUBSTITUTION
2 ELIMINATION OF RIGHT HAND SIDE AND BACK SUBSTITUTE
NR.LT.0 MODIFY MATRIX AK AND BACK SUBSTITUTE TO
RECOVER ELIMINATED UNKNOWNS, KNOWN VALUES OF
R MUST BE IN POSITION BEFORE.

******CALLING ARGUMENTS ******

FOR SOLUTIONS OF (AK)(X)=(R) USE GAUSS(AK,R,IMAX,NEQ,NLO,1,DET;1)
FOR NEW RIGHT HAND SIDE GAUSS(AK,R,IMAX,NEQ,NLO,1,DET;2)
FOR DETERMINANT GAUSS(AK,R,IMAX,NEQ,NLO,1, DET;1)
FOR CONDENSATION GAUSS(AK,R,IMAX,NEQ,NLO,NR, DET;1)
NLO MAY BE 0
FOR RECOVERY GAUSS(AK,R,IMAX,NEQ,NLO,-NR, DET;2)

*****************************************************************************

IF(NR.LT.0) GO TO 15
NE=NEQ-NR
DET=1.
DO 50 500 M=1,NE
MAX=NEQ-M
N=MAX+1
L=(N*(N+1))/2
LN=L-N
IF(AK(L,M),GT.1.E-20) GO TO 600
WRITE(2,1) N
GO TO 500
10
600 PIVOT=1./AK(L)
IF(NCODE,GT.1) GO TO 400
DO 300 J=1,MAX
T=AK(L+J)/PIVOT
IF(T,EQ.0.) GO TO 300
DO 200 I=J,MAX
K=(I*(I-1))/2J
KN=LN+I
AK(K)=AK(K)-AK(KN)*T
CONTINUE
SUBROUTINE GAUSS

300 CONTINUE
   IF(NLO.NE.0) GO TO 400
   IF(NR.NE.1) GO TO 500
   DET=DET*AK(L)
400  DO 360 J=1,NLO
      T=R(N,J)*PIVDT
      IF(T.EQ.0) GO TO 360
      DO 350 I=1,MAX
      K=LN+I
      350 R(I,J)=R(I,J)-AK(K)*T
   360 CONTINUE
   500 CONTINUE
   IF(NR.GT.1) RETURN
   DO 10 L=1,NLO
      10 R(I,L)=R(I,L)/AK(I)
   M=2
   15 IF(NR.LT.0) M=1-NR
   DO 20 I=M,NEQ
      K=I-1
      KI=(I*K)/2
      KN=KI+I
      DO 30 L=1,NLO
      DO 25 J=1,K
      KJ=KI+J
      25 R(I,L)=R(I,L)-R(J,L)*AK(KJ)
      30 R(I,L)=R(I,L)/AK(KN)
   20 CONTINUE
   85 FORMAT(* ZERO DIAGONAL AT ROW **,IS)
   RETURN
END
SUBROUTINE ASHBL2 (NL, NEQ, NREN, IMAX, NF, NSHAPE, NSSYM, S, CA, SA
1, ICOUNT, NDYN, RF2, R)
COMMON/F2/HNEL, HVEL, NMAT, NTYPE, NEQF, NF, NBAR, NSTOR,
1Ex(12), Ey(12), BXY(12), TH(12), EC(8), EL(6), LE(9),
2ET(36, 2), ETN(12), ETN(90), QK(9, 9), IE(12), IEN(12)
DIMENSION AK(105), COF(3, 5), S(1MAX), CA(4), SA(4)
1, RF(4), 10, R(NEQ)
INTEGER FP, EC, EL, ETN, IEN, ICOUNT(4)

*********************************************************************

THIS SUBROUTINE RETRIEVES THE CONDENSED LOCAL
FAUCADE STIFFNESS MATRICES, TRANSFORMS THEM TO
GLOBAL FACADE STIFFNESS MATRICES, AND THEN
ASSEMBLES THEM INTO THE STRUCTURAL STIFFNESS
MATRIX S

*********************************************************************

REWIND 3
REWIND 4

C INITIALIZE S

DO 100 I=1, IMAX
S(I) = 0.
100 CONTINUE
ICO = 0
NSYM = 2
DO 9999 N=1, NF
READ(3, *) NSYM, NEQF, LMAX, HNEL, NMAT, NTYPE, NR
READ(4, *) (AK(I), I=1, LMAX)
IF(NSYM, EQ, 1) GOTO 5
IF(NSYM, NE, 0) GOTO 10
C IF NSYM=0, EXPAND THE 3NL*3NL REDUCED FACADE STIFFNESS MATRIX AK
C CORRESPONDING TO LOCAL LATERAL DEFLECTION, AND THE TWO CORNER
C VERTICAL DEFLECTIONS INTO A 5NL*5NL STIFFNESS MATRIX AK
C CORRESPONDING TO THE THREE SHEAR CENTER DEFLECTIONS AND THE
C TWO CORNER VERTICAL MEL:EMENTS BY USING THE FOLLOWING
C MAPPING PROCEDURE IN SEQUENCE:
C NOTE: INDEX IN RREFERENCES

1 AK(4, 4) = AK(2, 2)
AK(4, 5) = AK(2, 3)
AK(5, 5) = AK(3, 3)

2 AK(1, 4) = AK(1, 2)
AK(2, 4) = AK(1, 2)
AK(3, 4) = AK(1, 2)
AK(1, 5) = AK(1, 3)
AK(2, 5) = AK(1, 3)
AK(3, 5) = AK(1, 3)

3 AK(1, 2) = AK(1, 1)
AK(1, 3) = AK(1, 1)
AK(2, 2) = AK(1, 1)
AK(2, 3) = AK(1, 1)

9999 CONTINUE
C SUBROUTINE ASMBL2
C
C AK(3,3)=AK(1,1)
C THE OBJECTIVE OF THIS MAPPING PROCEDURE IS TO LATER
C TRANSFORM THE REDUCED FACADE STIFFNESS MATRIX FROM
C LOCAL TO GLOBAL ORIENTATION.
C
C........THIS IS MAPPING # 1.
C
DO 200 L=1,12
  DO 330 NN=1,NL
    LOC=NL*L+NN-1
    LOC=LOC*(LOC+1)/2+NL
    LN1=NL*(L+2)+NN-1
    LN1=LN1*(LN1+1)/2+3*NL
    DO 340 I=1,LC
      IF(I.EQ.L) GOTO 230
      DO 240 J=I+NL
        LOC=LOC+1
        LN1=LN1+1
        AK(LN1)=AK(LOC)
        CONTINUE
      GOTO 340
    240
    230
      DO 250 J=I+NN
        LOC=LOC+1
        LN1=LN1+1
        AK(LN1)=AK(LOC)
        CONTINUE
      250 CONTINUE
      340 CONTINUE
    330 CONTINUE
  200 CONTINUE
C
C........THIS IS MAPPING # 2
C
DO 240 L=1,12
  DO 270 NN=1,NL
    LOC=L*NL+NN-1
    LOC=LOC*(LOC+1)/2
    LN1=(2*L)*NL+NN-1
    LN1=LN1*(LN1+1)/2
    LN1=LN1+NL
    LN3=LN2+NL
    DO 280 I=1,NL
      LOC=LOC+1
      LN1=LN1+1
      LN2=LN2+1
      LN3=LN3+1
      AK(LN1)=AK(LOC)
      AK(LN2)=AK(LOC)
      AK(LN3)=AK(LOC)
      CONTINUE
    280 CONTINUE
    270 CONTINUE
  260 CONTINUE
C
C........THIS IS MAPPING # 3 (UPPER TRIANGLE INCLUSIVE)
C
LOC=0
DO 290 NN=1,NL
   LN1=NL+NN-1
   CONTINUE
290
SUBROUTINE ASMBL2  FTN 4.8+552  83/03/21, 15:51:27  PAGE  3

115    LN1=LN1*(LN1+1)/2
120    LN2=LN1+1
125    LN3=2*NL+NN-1
130    LN4=LN3*(LN3+1)/2
135    LN5=LN4+NL
       DO 300 I=1,NN
       LOC=LOC+1
       LN1=LN1+1
       LN2=LN2+1
       LN3=LN3+1
       LN4=LN4+1
       LN5=LN5+1
       AK(LN1)=AK(LOC)
       AK(LN2)=AK(LOC)
       AK(LN3)=AK(LOC)
       AK(LN4)=AK(LOC)
       AK(LN5)=AK(LOC)
290   CONTINUE
300   CONTINUE
320   CONTINUE

C........THI S IS MAPPING # 3 (LOWER TRIANGLE)
140    LG=0
145    LN4=NL-1
       DO 310 NN=1,NN
       LB=LG+NN
       LOC=LB
       LN1=NL+NN-1
       LN1=LN1*(LN1+1)/2+NN
       LN2=2*NL+NN-1
       LN2=LN2*(LN2+1)/2+NN
       LN3=LN2+NL
       LN3=LN3+1
       LN4=LN4+1
       LN4=LN4+1
       LN5=LN5+1
       AK(LN1)=AK(LOC)
       AK(LN2)=AK(LOC)
       AK(LN3)=AK(LOC)
       AK(LN4)=AK(LOC)
       AK(LN5)=AK(LOC)
150   CONTINUE
310   CONTINUE
       WRITE(2,*)'INITIALizes FACADE STIF MAT TO BE ROTATED TO GLOB AX'
       LB=5*NL
160   DO 122 L=1,LB
       J=L*(L+1)/2
       KKK=J-L+1
       WRITE(2,*)'L,J,(AK(K1),K1=KKK,J)
122   CONTINUE
165    IA=5
       IH=4
       COF(1,1)=CA(N)*CA(N)
       COF(1,2)=CA(N)*SA(N)
       COF(1,4)=CA(N)
       COF(1,5)=CA(N)
       COF(2,2)=SA(N)*SA(N)
SUBROUTINE ASHBL2

COF(2:4)=SA(N)
COF(2:5)=SA(N)

175 IF(NSYM.NE.0) GOTO 810
   COF(1:3)=CA(N)*RFP(N,1)
   COF(2:3)=SA(N)*RFP(N,1)
   COF(3:3)=RFP(N,1)*RFP(N,1)
   COF(3:5)=RFP(N,1)
   GOTO 20

810 IC=3
   COF(1:3)=CA(N)
   COF(2:3)=SA(N)
   GOTO 510

185 10 CONTINUE
   IF((NSYM.EQ.1).OR.(NSYM.EQ.2)) GOTO 850
   IF(NSYM.NE.1) GOTO 820

850 IF(NSYM.EQ.1) U=CA(N)
   IF(NSYM.EQ.2) V=SA(N)

190 IF(NSYM.EQ.3) U=RFP(N,1)
   COF(1:1)=U*V
   COF(1:2)=V
   IF(NSYM.NE.0) GOTO 15

195 IA=3
   IH=2
   COF(1:3)=V
   GOTO 20

15 CONTINUE
   IA=2
   IH=2
   GOTO 20

200 IC=1
   IF(NSYM.NE.0) GOTO 830

205 IA=3
   IH=2

830 IA=2
   IH=2
   GOTO 510

210 CONTINUE

C....ROTATE REDUCED FACADE STIFFNESS MATRIX INTO GLOBAL FACADE
C....STIFFNESS MATRIX EITHER FOR STATIC ANALYSIS OR IF
C....NSYM = 1 OR NNSYM = 2
C....NOTE THAT THE REFERENCE POINTS ON EACH LEVEL FORMS A VERTICAL
C....LINE THROUGH THE BUILDING
   LOC=0

220 DO 165 L=1,IA
      DO 155 I=1,L

165 IF(I.NE.IH) GOTO 130
      LN=(L-1)*NL+NN
      LOC=LN*(LN+1)/2
      GOTO 160

130 IF(I.EQ.L) GOTO 150
      DO 145 J=1,NL
           LOC=LOC+1
           AK(LOC)=AK(LOC)*COF(I,L)

145 CONTINUE
SUBROUTINE ASHBL2  FTN 4.8+552  83/03/21. 15.51.27  PAGE 5

230    150  GOTO 155
       DQ 135 MH=1,NN
       LOC=LOC+1
       AK(LOC)=AK(LOC)*COF(I,L)
235    160  CONTINUE
       GOTO 5
       510  CONTINUE

C......ROTATE REDUCED FACADE STIFFNESS MATRIX INTO GLOBAL FACADE
C     STIFFNESS MATRIX FOR A DYNAMIC ANALYSIS WITH
C     NSYM = 0 OR NSYM = 3
C     NOTE THAT REFERENCE POINTS OF EACH LEVEL DO NOT NECESSARILY
C     FORM A VERTICAL LINE

240    245  DO 520 L=1,IA
       DO 530 NN=1,NL
       DO 540 I=1,NL
          IE(I,NE,IH) GOTO 550
       LN=LN+(N+1)/2
       GOTO 530
250    550  IF(L,NE,IC) GOTO 570
       IF(I,EQ,L) GOTO 580
       DO 590 J=1,NL
          LOC=LOC+1
          AK(LOC)=AK(LOC)*COF(I,L)*RFP(N,NN)
255    590  CONTINUE
       GOTO 540
260    580  DO 600 NN=1,NN
       LOC=LOC+1
       AK(LOC)=AK(LOC)*RFP(N,NN)
       CONTINUE
       GOTO 540
265    570  IF((I,LE,IC).OR,(I,NE,IC)) GOTO 840
       DO 610 J=1,NL
          LOC=LOC+1
          AK(LOC)=AK(LOC)*RFP(N,J)
260    610  CONTINUE
       GOTO 540
270    840  IF(I,EQ,LI) GOTO 620
       DO 630 J=1,NL
          LOC=LOC+1
          AK(LOC)=AK(LOC)*COF(I,L)
275    630  CONTINUE
       GOTO 540
280    620  LDDR@L@1MM#AK(LOC)=AK(LOC)*COF(I,L)
280    640  CONTINUE
285    540  CONTINUE
       550  CONTINUE
       560  CONTINUE

C......GLOBAL FACADE STIFFNESS MATRIX HAS NOW BEEN FORMED
       IF(NSYMB,NE,2) GOTO 410
IC0=IC0+1
GOTO 430
410 IF((NSYMB.NE.0).AND.(NSYMB.NE.1)) GOTO 420
IF(NSYM.EQ.3) IC0=IC0+2
GOTO 430
420 IF(NSYMB.NE.3) GOTO 430
IF(NSYM.EQ.3) IC0=IC0+1
430 ICOUNT(N)=IC0
IF(NSYM.NE.0) GOTO 90
IB=H-1
ID=I+2
IG=3*NL+1
IH=4
IJ=5
IK=0
IL=4
IM=5
GOTO 125
305 90 IF(NSYM.NE.0) GOTO 95
IB=ICOUNT(N)-1
ID=ICOUNT(N)
IG=NL+1
IH=2
IJ=3
IK=0
IL=2
IM=3
GOTO 125
310 95 IF(NSYM.NE.1) GOTO 105
IB=ICOUNT(N)
ID=ICOUNT(N)
IG=NL+1
IH=3
IJ=2
IK=0
IL=1
IM=2
GOTO 125
315 105 IF(NSYM.NE.2) GOTO 110
IB=ICOUNT(N)-1
ID=ICOUNT(N)
IG=0
IH=3
GOTO 125
320 110 IF(NSYM.NE.3) GOTO 120
IB=ICOUNT(N)-1
ID=ICOUNT(N)
IG=NL+1
IH=2
IJ=1
IK=0
IL=2
IM=3
GOTO 125
325 120 WRITE(2,2) '***** ERROR IN INPUT DATA *****'
WRITE(2,2) 'FAÇADE SYMÉTRY WAS NOT SPECIFIED AS 0,1,2, OR 3 FOR'
2 FACADE NUMBER

GOTO 819

C \begin{verbatim}
C \ldots ASSEMBLE GLOBAL FACADE STIFFNESS \& STRUCTURAL
C STIFFNESS MATRIX (S) CORRESPONDING TO REFERENCE POINT DISPLACEMENTS
C AND CORNER VERTICAL DISPLACEMENTS OF EACH LEVEL
\end{verbatim}

350 125 LG=0
IN=0
LOC=0
DO 30 L=1,IA
IC=L
IF(L.NE.IH) GOTO 210
LL=NL*ID+1
GOTO 85
210 IF(L.NE.IH) GOTO 65
IF(NSYM.EQ.2) GOTO 65
IF((N,NE,NF).OR.((NSHAPE.EQ.1))) GOTO 65
IP=0
IR=0
DO 215 NN=1,NL
IP=IP*(IP-1)/2+IG-1
IR=IR*(IR-1)/2+IG+NN-1
DO 220 I=1,NL
IF(IP=1)
IR=IR+ID*NN+L-1
S(IR)=S(IR)+AK(IP)
220 CONTINUE
215 CONTINUE
375 CONTINUE
IC=IL
LL=IG
IF(NSYM.EQ.3) GOTO 85
IN=NL
85 LG=LL*(LL-1)/2
DO 35 NN=1,NL
DO 40 I=1,IC
IF(I.NE.IH) GOTO 205
LG=LG*IN
LOC=LOC+IN
385 205 IF(I.EQ.IC) GOTO 45
DO 50 J=1,NL
LG=LG+1
LOC=LOC+1
S(LG)=S(LG)+AK(LOC)
390 50 CONTINUE
GOTO 40
45 DO 60 NN=1,NN
LG=LG+1
LOC=LOC+1
S(LG)=S(LG)+AK(LOC)
395 60 CONTINUE
40 CONTINUE
35 CONTINUE
30 CONTINUE
SUBROUTINE ASMUL2  FTH 4.8+552  83/03/21. 15.51.27  PAGE 8

400 NSYMBS=NSYM
9999 CONTINUE
   IF(NDYN.EQ.0) GOTO 730
   NREM=NL
   IF(NSSYM.EQ.0) NREM=3*NL
   WRITE(2,*) (S(I),I=1,IMAX)
C.....IF ANALYSIS IS DYNAMIC, CONDENSE THE STATIC STRUCTURAL STIFFNESS
C MATRIX TO DYNAMIC STRUCTURAL STIFFNESS MATRIX (REFERENCE POINT DOF'S ONLY)
         CALL GAUSS(S,RO,IMAX,NEQ,O,NREM,DET,1)
    730 REWIND 6
   WRITE(6,*) (S(I),I=1,IMAX)
   WRITE(2,*) 'GLOBAL STRUCT STIF MAT IN VECT FORM COL BY COL'
   DO 23 I=1,NEQ
         J=(I*(I+1))/2
         KKK=J-J+1
   23 CONTINUE
   WRITE(2,*) I,J,(S(K),K=KKK,J)
23 CONTINUE
819 RETURN
END
SUBROUTINE MODAL  FTN 4,8552  03/03/21, 15.51.27  PAGE 1

SUBROUTINE MODAL(ALPHA, H, HM, NK, NASH, NNSYM, NL, NPRI, NPRINT, NREM, 
1, NRET, ET, MHEL, BD, ICOUNT, INTER, ERR, U, V, W, GC, SV, R, S, NF, NDOF, 
2, NSHAPE, NOUT, CA, SA, RFPI) 
DIMENSION VV(3), ERR(H, N(K)), GC(H, SV(H), R(BD, H), S(NK)), 
E(36, 2), C(10), BD(H), CA(4), SA(4), RFPI(4, 10) 
INTEGER I(TN(90), I, COUNT(4), NPRINT(6), NPRINT(10)) 
REAL NASH(NREM), NOUT(3, 30), HPF, HPSVF 
IF(INTER.EQ.0) GOTO 510 
DO 520 J = 1, H 
520 PRINT* * MODE NUMBER G J 
PRINT* ' FREQUENCY = ', BD(J) 
PRINT* * TIME PERIOD = ', 6.28318/B(D(J) 
PRINT* * INPUT PSEUDO VELOCITY 
READ* * SV('J', 'J') = SV(J) 
WRITE(2, *) 
520 GOTO 530 
530 READ(1, *) (SV(J), J = 1, H) 
WRITE(2, *) ' PSEUDO VELOCITIES FOR INCREASING MODE NUMBER' 
WRITE(2, *) (SV(J), J = 1, H) 
530 VV(1) = COS(ALPHA) 
IF(NNSYM.EQ.2) VV(1) = SIN(ALPHA) 
540 IF(NNSYM.EQ.0) GOTO 540 
NO = 3 
VV(2) = SIN(ALPHA) 
VV(3) = 0. 
GOTO 550 
550 IF(NNSYM.EQ.3) VV(1) = 0. 
550 DO 10 J = 1, H 
WRITE(2, 13) 
13 FORMAT(* MODE NUMBER', J 
WRITE(2, 35) 
CALCULATE MODAL PARTICIPATION FACTOR *HPF* 
ZI = 0 
Z3 = 0 
DO 20 I = 1, ND 
Z2 = 0 
20 CONTINUE 
DO 30 K = 1, NL 
KK = (I - 1) * HL + K 
ERR(KK) = W(KK, J) * NASH(KK) 
ZI = ERR(KK) * W(KK, J) + ZI 
Z2 = Z2 * ERR(KK) 
CONTINUE 
Z3 = Z3 + Z2 
20 CONTINUE 
HPF = Z3/Z1 
WRITE(2, 50) * MODAL PARTICIPATION FACTOR FOR MODE #', J 
WRITE(2, 50) HPF 
WRITE(2, 50) * GENERALIZED COORDINATE FOR MODE J 
WRITE(2, 50) GC(J) 
WRITE(2, 50) * GENERALIZED COORDINATE FOR MODE #', J 
WRITE(2, 50) GC(J) 
LI = J 
IF(NPRINT(1), NEQ.0) LI = 1
SUBROUTINE MODAL  FTN 4.8±552  83/03/19. 15.51.27  PAGE 2

C...........R(I,L1)= DISPLACEMENT I FOR MODE J
DO 40 I=1,NREM
  R(I,L1)=W(I,J)*SC(J)
  CONTINUE
  WRITE(2,*) 'REFERENCE POINT DISPLACEMENTS FOR MODE #', J
  WRITE(2,*) I, (R(I,L1), I=1,NREM)
  IF(NPRI(2).EQ.0).AND.(NPRI(3).EQ.0).AND.(NPRI(4).EQ.0).AND.
  -(NPRI(5).EQ.0).AND.(NPRI(6).EQ.0)) GOTO 10
  REWIND 6
  READ(6,*) (S(I), I=1,IAX)
C.............RECOVER CORNER VERTICAL DISPLACEMENTS
  CALL GAUSS(6,R(I,L1),NK,NEQ,I,-NREM,DET,2)
  WRITE(2,*) 'REFERENCE POINT AND CORNER DISPLACEMENTS FOR MODE #', J
  WRITE(2,*) I, (R(I,L1), I=1,NEQ)
  L4=1
  IF(NPRI(4).EQ.0) L4=1
  IF(NPRI(4).EQ.0).AND.(NPRI(5).EQ.0).AND.
  -(NPRI(6).EQ.0)) GOTO 10
  HPSVF=HPSV(J)*RD(J)
C.............U(I,L1)= EQUIVALENT ELASTIC FORCE I FOR MODE J
  DO 50 I=1,NREM
    U(I,L1)=ERR(I)*HPSVF
  CONTINUE
  WRITE(2,*) 'EQUIVALENT REFERENCE POINT ELASTIC FORCES'
  WRITE(2,*) (U(I,L4), I=1,NREM)
  IF(NPRI(5).EQ.0).AND.(NPRI(6).EQ.0)) GOTO 10
C.............BASE SHEARS AND TORQUE
  IF(NSSYM=0
    V(1,J)=MAX BASE SHEAR IN X DIRECTION FOR MODE J
  C.............V(2,J)=MAX BASE SHEAR IN Y DIRECTION FOR MODE J
  C.............V(3,J)=MAX BASE TORQUE FOR MODE J
  C.............V(4,J)=MAX COMBINED BASE SHEAR X,Y FOR MODE J
  C.............FOR NSSYM=1 V(1,J)=BASE SHEAR IN X DIRECTION FOR MODE J
  C.............V(1,J)=BASE SHEAR IN Y DIRECTION FOR MODE J
  C.............V(1,J)=BASE TORQUE FOR MODE J
  DO 90 I=1,ND
    Z=0
    DO 70 K=1,NL
      KK=(I-1)*NL+K
      Z=HU(KK,J)
    CONTINUE
    V(I,J)=Z
  CONTINUE
  WRITE(2,*) 'SHEARS AND TORQUES FOR MODE #', J
  IF(NSSYM.NE.0) GOTO 80
  V(ND+1,J)=SORT((V(1,J)+V(2,J)+V(3,J))
  WRITE(2,*) V(ND+1,J)
  GOTO 10
  WRITE(2,*) 'V', (V(I,J), I=1,ND)
  IF(NPRI(6).EQ.0) GOTO 10
  IF(NSSYM.NE.3) GOTO 560
  WRITE(2,*) 0.0,0.0
  GOTO 10
C.............X(I)=HEIGHT OF LEVEL I
  NB=1
  IF(NSSYM.EQ.0) NB=2
  X(I)=ET(ETN(I),2)
  K=1
DO 100 I=2,NL
   K=K*HEL
   X(I)=X(I-1)+ETN(K),2
100  CONTINUE
WRITE(2,*) 'HEIGHT OF EACH LEVEL FROM I TO NL'
WRITE(2,*) (X(I),I=1,NL)

C..FOR NSSYM=0
C..........HOM(1) = MAX MOMENT ABOUT Y AXES FOR MODE J
C..........HOM(2,J) = MAX BASE MOMENT ABOUT X AXES FOR MODE J
C..........HOM(3,J) = MAXIMUM COMBINED BASE MOMENT X,Y FOR MODE J
C..FOR NSSYM=1
C..........HOM(1,J) = MAX MOMENT ABOUT Y AXES FOR MODE J
C..........HOM(2,J) = MAX BASE MOMENT ABOUT X AXES FOR MODE J
WRITE(2,*) ' BASE MOMENTS FOR MODE #: J'
DO 110 I=1,NR
   Z=0.
   DO 120 K=1,NL
      Z=0.
      K=K*HEL
      Z=Z+U(KK,1)*X(K)
120  CONTINUE
   HOM(I,1,J)=Z
110 CONTINUE

IF(NSSYM,EE,0) GOTO 130
   HOM(NB+1,J)=SQRT(HOM(1,J)*HOM(1,J)+HOM(2,J)*HOM(2,J))
WRITE(2,*) HOM(NB+1,J)
130 WRITE(2,*) (HOM(I,J),I=1,NB)
140 CONTINUE

C........THE REMAINDER OF THIS SUBROUTINE IS FOR SRSS CALCULATIONS
C
WRITE(2,13)
IF(NPRIN(1),EE,0) GOTO 570
C..........DISPLACEMENTS
150 JJ=NEQ
   IF(NPRIN(2),EE,0) JJ=NREM
   GOTO 590
570 IF(NPRIN(2),EE,0) GOTO 580
   JJ=NREM+1
590 DO 140 I=II,JJ
   WRITE(2,*) 'DISPLACEMENT',I,'FOR ALL MODES'
140 WRITE(2,*) I,(RI(I,J),J=1,H)
   Z=0.
   DO 150 J=1,M
      Z=Z+RI(I,J)*RI(I,J)
   150   CONTINUE
   Ri(I,H+1)=SQRT(Z)
140 CONTINUE
   WRITE(2,*) 'SRSS OF DISPLACEMENTS'
   WRITE(2,*) (RI(I,H+1),I=II,JJ)
580 CONTINUE
IF(NPRIN(4),EE,0) GOTO 160
C........ELASTIC FORCES
170 CONTINUE
   Z=0.
DO 180 J=1,M
   Z=2*U(I,J)*U(I,J)
180  CONTINUE
   U(I,1)=SOR(T(Z)
175 CONTINUE
WRITE(2,*) 'SSRS OF ELASTIC FORCES'
WRITE(2,*) (U(I,1),I=1,NREK)
160 IF(NPRINT(5),EQ.0) GOTO 190
180 C........BASE SHEARS AND TORQUE
   II=M0
   IF(NSYM.EQ.0) II=MD+1
   DO 200 I=1,II
      Z=0.
      DO 210 J=1,M
         Z=Z+V(I,J)*V(I,J)
210    CONTINUE
   V(I,1)=SOR(T(Z)
200 CONTINUE
190 WRITE(2,*) 'SSRS OF BASE SHEARS AND TORQUE'
   WRITE(2,*) (V(I,1),I=1,II)
195 IF(NPRINT(6),EQ.0) GOTO 999
C........BASE MOMENTS
   II=NB
   IF(NSYM.EQ.0) II=NB+1
   DO 220 I=1,II
      Z=0.
      DO 230 J=1,M
         Z=Z+MOM(I,J)*MOM(I,J)
230    CONTINUE
   MOM(I,1)=SOR(T(Z)
220 CONTINUE
190 WRITE(2,*) 'SSRS OF BASE MOMENTS'
   WRITE(2,*) (MOM(I,1),I=1,II)
205 C........COMPUTE STRESSES FOR EACH MODE AND ALSO SSRS STRESSES
   IF(NPRINT(3),EQ.1) CALL STRESS(NL,NF,NDOF,NSHAPE,NSYM,NOUT,CA,SA
      I,ICOUNT,RFP,NPRINT(3),NPRINT,NEG+R+M+1)
999 RETURN
END
SUBROUTINE GIVI82

1  SUBROUTINE GIVI82(G, GD, LD, BD, ERR, U, V, W, N, M, NROD, NOI, 
2  TOLVEC, IRENT, GES)
3  REAL (G(N), LD(N), BD(M), ERR(M), U(30, M), V(30, M))
4  INTEGER GD(N), LD(N)
5  WRITE(2, 1) (G(I), I = 1, N), (G(I), I = 1, M)
6  WRITE(2, 1) N, LD(I), (1 = 1, N), (L(I), I = 1, M)
7  WRITE(2, 1) M, NROD, NOI, TOLVEC, IRENT, GES
8  CALL REDUCE(L, LD, N, M)
9  IF(N, LT, 0) GOTO 124
10 LET = 0
11 INT = 1
12 LOCK = 1
13 IF(INERM, EQ, 2) CALL PREMULT(W, U, L, LD, N, M, N, I, IRENT)
14 CALL Random(U, N, M, IRENT, GES)
15 CALL ORTHOG(U, N, M)
16 CALL BACKSUB(U, V, L, LD, N, M, LOCK)
17 CALL PREMULT(V, W, GD, N, M, LOCK, 0)
18 CALL ESRSUB(W, V, L, LD, N, M, LOCK)
19 CALL DECOUPLE(U, V, W, BD, N, M, LOCK, TOLVEC)
20 CALL Random(W, N, M, IRENT, GES)
21 CALL ORTHOG(W, N, M, LOCK)
22 CALL Error(U, W, BD, ERR, N, M, NROD, LOCK, TOLVEC, LET)
C MONITOR PRINT INSTRUCTIONS
25 WRITE(2, 1) INT
26 FORMAT(///, 5X, "EIGENFACTORS AFTER ", I3, " ITERATIONS")
27 WRITE(2, 22) (BD(I), I = 1, M)
28 WRITE(2, 23)
29 FORMAT(5X, "VECTOR ERRORS ")
30 WRITE(2, 22) (ERR(I), I = 1, M)
31 FORMAT(5X, /F16.8)
32 DO 20 I = 1, N
33 DO 20 J = LOCK, M
34 U(I, J) = W(I, J)
35 IF(LET, EQ, 0) GOTO 10
36 IF(LET, INT) = 1, LT, 0) GOTO 11
37 INT = INT + 1
38 GOTO 50
C MONITOR PRINT INSTRUCTIONS
40 WRITE(2, 100) INT
41 FORMAT(///, 5X, "CONVERGENCE OBTAINED", /, 5X, "VECTOR TOLERANCE CRITERIA 
42 SATISFIED AFTER", I4, " ITERATIONS")
43 GOTO 120
44 WRITE(2, 101) INT
45 FORMAT(///, 5X, "CONVERGENCE NOT OBTAINED ", /, 5X, "VECTOR TOLERANCE CRITERIA 
46 NOT SATISFIED AFTER", I4, " ITERATIONS")
47 WRITE(2, 126)
48 FORMAT(///, 5X, "MODE ", 2X, "FREQUENCY (RAD/SEC)", 35X, "MODE SHAPE", /, 6 
49 (1H#), 2X, 19(1H#), 2X, 100(1H#), //)
50 DO 121 J = 1, M
51 EL = 0.0
52 DO 122 I = 1, N
53 IF(ABS(W(I, J)), GT, EL) EL = ABS(W(I, J))
54 EL = 1.0/EL
55 DO 123 I = 1, N
56 W(I, J) = W(I, J) * EL
57 BD(J) = 1./SORT(BD(J))
58 22 CONTINUE 
59 23 CONTINUE
SUBROUTINE SIVIB2

WRITE(2,127) J,BD(J),(W(I,J),I=1,N)
127 FORMAT(/,1X,13,7X,F13.6,6(5X,F13.6),2(/,24X,6(5X,F13.6)))
121 CONTINUE
124 RETURN
END
SUBROUTINE RANDOM(U,N,K,IRENT,GES)
REAL U(30,M)
K=1
IF(IRENT.NE.0) K=M
IRENT=1
DO 1 I=1,N
DO 1 J=K,M
IF(GES.EQ.0.0) GES=0.31415926
Y=GES*GES
1 Y=Y*10.0
IF(((Y-1.0).LT.0.0) GOTO 2
Y=Y-AINT(Y)
U(I,J)=Y
GES=Y
15 RETURN
END
SUBROUTINE ORTHOG

REAL W(30,N,N,M,LOCK)
INTEGER I,LOCK,N
DO 1 I=LOCK*N
1    DO 1 J=I,I
     EL=0.0
     DO 4 K=1,N
     4 EL=EL+W(K,J)*W(K,I)
     IF((I-J).EQ.0) GOTO 5
     DO 6 K=1,N
8     W(K,I)=W(K,I)-EL*W(K,J)
     GOTO 1
6    EL=EL/SORT(EL)
     DO 5 K=1,N
9     W(K,I)=D*W(K,I)
5    CONTINUE
1    CONTINUE
RETURN
END
SUBROUTINE DECOUPL

REAL U(30,M), V(30,M), W(30,M), BD(M)

C THIS SECTION CALCULATES THE EIGENVALUES
DO 1 J=1, M
   EL=0.0
   DO 2 I=1, N
      EL=EL+U(I,J)*V(I,J)
   2 CONTINUE
   BD(J)=EL
   DO 3 J=1, M
      JJ=JJ+1
  3 CONTINUE
1  IF(JJ.LELOCK) J=J

C THIS SECTION TESTS THAT THE EIGENVALUES ARE
C IN DESCENDING ORDER AND IF NOT SORTS THEM
DO 4 I=1, M
   EL=EL+BD(I)
   4 CONTINUE
5 IF(ABS(BD(K))-ABS(BD(I))).GT.0.0) GOTO 3
   EL=BD(K)
   BD(I)=BD(K)
   DO 5 J=1, M
      EL=EL+U(J,J)*V(J,J)
   5 CONTINUE
   U(J,J)=U(J,J)*EL
   V(J,J)=V(J,J)*EL
   DO 6 K=1, M
      EL=EL-EL*EL
   6 CONTINUE
   BD(K)=BD(K)-EL
   7 IF((ABS(Q+BD(K))-TOLVEC).LT.0.0) GOTO 10
   EL=EL-EL
   10 IF((ABS(EL/BD(J))-TOLVEC).LT.0.0) GOTO 10
   EL=0.0
25 CONTINUE

C THIS SECTION CALCULATES THE INTERACTION
C BETWEEN THE ITERATION VECTORS AND DECOUPLES THEM
DO 9 I=1, M
   DO 8 J=1, N
      W(I,J)=V(I,J)
   8 K=1, M
   9 CONTINUE

C THIS SECTION TESTS THAT THE EIGENVALUES ARE
C IN DESCENDING ORDER AND IF NOT SORTS THEM
DO 10 J=1, M
   EL=EL+BD(J)
   10 IF(ABS(BD(K))-ABS(BD(I))).GT.0.0) GOTO 10
   EL=EL+EL
   BD(J)=BD(J)-EL
   DO 13 K=1, M
      W(K,J)=W(K,J)-EL*V(K,J)
   13 CONTINUE
   W(K,J)=W(K,J)+EL*V(K,J)
   15 CONTINUE

RETURN
END
SUBROUTINE ERROR

REAL ERR(N), BD(N), U(30*N), W(30*N)

DO 2 J=LOCK+1, N
   ER=0.0
   IF(BD(J).LE.0.0) GOTO 6
   DO 4 I=1,N
      EL=U(I,J)-W(I,J)
      1' ER=ER+EL
      GOTO 2
   6 EL=U(I,J)-W(I,J)
   8 ER=ER+EL
   2 ERR(J)=SORT(ERR/N)
   5 IF((TOLVEC-ERR(LOCK)).LT.0.0) GOTO 4
      ERR(LOCK)=ERR(LOCK)
      LOCK=LOCK+1
      IF((NROD-LOCK).GE.0) GOTO 5
      LET=1
   4 RETURN

END
SUBROUTINE REDUCE

INTEGER LD(N),Q
REAL L(NK)

L(1)=SORT(L(1))
DO 1 I=2,N
Q=LD(I-1)-Q
M1=1+LD(I-1)-Q
DO 2 J=M1,1
10 NN=0
EL=L(J+Q)-Q
KK=LD(J)-J
NN=MAX0(NN,M1)
IF((NN-M1).EQ.0) GOTO 2
J=M1-1
2 DO 3 K=NN,J
3 EL=EL-L(J+K)(KK+K)
2 L(J+Q)=EL/L(LD(J))
IF(EL.LE.0) GOTO 5
XCM=XCM.
IF((XCM.GT.0).AND.(XCM.LT.0)) GOTO 1
25 XCM=XCM
JCM=J
1 L(LD(I))=SORT(EL)
MONITOR PRINTING
ICH=INT(ALG10(XCM))+1
WRITE(2,11) ICH,JCM
10 FORMAT(/'10X,'ACCURACY LOSS =',I3,' DECIMAL PLACES AT PIVOT NUM'
1BER',I3) GOTO 6
5 WRITE(2,11) I
11 FORMAT(/'20X,' REDUCTION FAILURE : PIVOT ','I4,' NOT POSITIVE')
N=N-N
6 RETURN
END
SUBROUTINE FORSUB

REAL U(M), V(M), L(M), N, K
INTEGER LD(M), Q
DO 12 K = 1, N
12 V(1+K) = U(1+K)/L(1)
DO 13 I = 2, N
Q = LD(I-1) + Q
M = I + LD(I-1) - Q
DO 13 K = LOCK, M
EL = 0.0
IF((M-I).EQ.0) GOTO 13
II = I - 1
DO 14 J = M, II
14 EL = EL + (J+Q)*V(J+K)
13 V(1+K) = (U(1+K)-EL)/L(LD(I))
RETURN
END
SUBROUTINE PREMULT

REAL U(LD),V(L),L,D,NA,N,M,LOCK,IRENT
INTEGER Q,LD(N)
DO I=1,N
    LK=L(LD(I))
    DO K=1,LOCK*M
        V(I,K)=U(I,K)*LK
    END DO
    Q=LD(I)-I
    M=I+LD(I-1)-Q
    DO K=1,LOCK*M
        IF((M-I)*EQ.0) GOTO 7
        IF((IRENT-2)*EQ.0) GOTO 6
    END DO
    II=I-1
    DO J=II,II+5
        V(J,K)=V(J,K)+L(J+Q)*U(J,K)
    END DO
    CONTINUE
END
SUBROUTINE BACKSUB

REAL L(NK), U(LD*NK), N(H, LOCK)
INTEGER LD(30), Q
DO 20 K = LOCK, H
DO 20 I = 1, N
  20 V(I, K) = U(I, K) / L(LD(I))
DO 21 IT = 2, N
  I = N + 2 - IT
  Q = LD(I) - I
  M1 = I + LD(I - 1) - Q
  IF ((M1, M), EQ, 0) GOTO 21
  II = I - 1
  DO 22 J = M1, II
  22 K = LOCK + H
  DO 22 K = LOCK, H
  V(J, K) = V(J, K) - L(J+Q) * V(I, K) / L(LD(J))

21 CONTINUE
20 CONTINUE
RETURN
END
SUBROUTINE STRESS

SUBROUTINE STRESS(NL, HF, NDHF, NSHAPE, NSYM, NOUT, CA, SA, ICOUNT, RFP
1, KPRINT, KPRINT, NEQ, R, NDHF, NDYN)

COMMON/P2/NHEL, NUEL, NMAT, NTYPE, HUEOF, NPR, NBRAY, NSCTOR,
1EX(12), Ey(12), GXY(12), TH(12), ET(8), EI(6), ELE(9),
2ET(36, 2), ET(12), ETN(90), O(9, 9), IEW(12), IEH(12)

DIMENSION BR(3, 9), C(3, 3), BAC(3, 9), R(9), SIGS(3), BR(210), CA(4)
1, SA(4), R(80, NDHF), AK(8105), RFP(4, 10), NDISP(90, 9), HP(90), SIGS(3)

INTEGER FF, EC, EL, ET, ETN, ICOUNT(4), HPRINT(10)

******************************************************************************

THIS SUBROUTINE EVALUATES STRESSES
IN EACH ELEMENT IN EACH FACE.
SUBROUTINE STRESS  FTN 4.8+552  83/03/21, 15:51.27

420  JJJ=1
440  IF(NSSYM.NE.0) GOTO 5
   IP=3*NL+1
   IF(NSSYM.EQ.1) GOTO 240
   DO 10 L=1,NL
     J=L+NL
     K=J+NL
     DR(L)=CA(N)*R(L,1)+SA(N)*R(J,1)+RFP(N,1)*R(K,1)
   10    CONTINUE
   GOTO 250
240  DO 270 L=1,NL
     J=L+NL
     K=J+NL
     DR(L)=CA(N)*R(L,NN)+SA(N)*R(J,NN)+RFP(N,L)*R(K,NN)
270  CONTINUE
   DD 15 L=1,2
   LL=L*NL
   LL=NL+1
   IF((N.EQ.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.2)) LL=3
   LLN=LL*NL
   DO 20 J=1,NL
     DR(L+J)=R(LLN+J,NN)
   20    CONTINUE
   GOTO 25
C
5  IF(NSSYM.NE.1) GOTO 30
   IP=2*NL+1
   DD 75 L=1,2
   LL=(L-1)*NL
   LL=ICOUNT(N)+L-1
   EQ.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.2)) LL=1
   LLN=LL*NL
   DD 80 J=1,NL
     DR(LH+J)=R(LLH+J,LLL)
   80    CONTINUE
   GOTO 25
30  IF(NSSYM.EQ.1) V=CA(N)
   IF(NSSYM.EQ.2) V=SA(N)
   IF(NSSYM.EQ.3) V=RFP(N,1)
   IF(NSSYM.NE.0) GOTO 35
   IP=3*NL+1
   IA=2
   IB=2
   GOTO 50
35  IF(NSSYM.NE.2) GOTO 40
   IP=2*NL+1
   IA=1
   IB=2
   GOTO 50
40  IP=2*NL+1
   IA=1
   IB=1
50  DO 55 L=1,NL
     DR(L)=V*R(L,JJJ)
55  CONTINUE
SUBROUTINE STRESS  

DO 65 L=1,IA.
    LN=1&NL
    LL=ICOUNT(N)+L-1
    IF(N.EQ.NF).AND.(NSHAPE.EQ.0).AND.(L.EQ.IP)) LL=1
    LLN=LL&NL
70  DD·70 J=1,NL
    DR(LLN+J)=R(LLN+J,JJJ)
 CONTINUE
65  CONTINUE
25  CONTINUE
125 DO 155 L=IP,NEQF
    DR(L)=0.
 CONTINUE
155 C...RETRIEVE REDUCED FACADE DISPLACEMENTS
 CALL GAUSS2(AK,DR,LMAX,NEQF,1,-NR,DET,2)
130 WRITE(2,*)(*'ALL FACADE DISPLACEMENTS FOR MODE **',NN)
 WRITE(2,*)(* (DR(I),I=1,NEQF)
C
223 WRITE(2,*)(*STRESSES')
 DO 777 N=1,NEL
135 C
C ELEMENT PROPERTIES
C
IT=ETM(M)
A=ET(IT,1)
D=ET(IT,2)
140 C NOTE... D=B-HEIGHT OF ELEMENT
IM=ETM(IT)
E1=EXM(IM)
E2=EY(IM)
145 G12=GXY(IM)
TT=TH(IM)
NBAY=IEW(IT)
NSTOR=IEH(IT)
BU=8/NBAY
SH=D/NSTOR
150 C
IF(NDOF.NE.0) GO TO 140
IF(NDTW.EQ.1) GO TO 290
READ(7,E) EL,NPR
IPRINT=NPRINT(NPR)
GOTO 310
290 IPRINT=NPRINT(NP(M))
310 IF(IPRINT.EQ.0) GO TO 777
155 ISTRS=1
JSTRS=1
RB=1/NBAY
RS=1./NSTOR
C INITIALIZE STRAIN -DISP, MATRIX BB 3X6
C
165 DO 90 I=1,3
70  CONTINUE
160 DO 90 J=1,6
BB(I,J)=1.
90  CONTINUE
170 C C EXTRACT EACH ELEMENT NODAL DISP.
SUBROUTINE STRESS

DO 95 I=1,LIM
Q(I)=0,
J=EL(I)
175 IF(NDYN.EQ.1) J=NDISP(M,I)
IF(J.EQ.0) GOTO 95
Q(I)=Q(I)+DR(J)
95 CONTINUE

C
GO TO 71
140 CONTINUE
IF(NDYN.EQ.1) GOTO 320
READ(7,*) EL,NPR
IPRT=NPRT(NPR)
GOTO 330
185 IPRT=NPRT(NP(M))
330 IF(IPRT.EQ.0) GO TO 777
ISTR=1
JSTR=2
D=1/2.
A=A/2.
RB=2./NBAY
RS=2./NSTOR

C
195 INITIALIZE STRAIN - DISPL. MATRIX RB 3X9
C
DO 72 I=1,3
DO 72 J=1,9
BB(I,J)=0.0

200 72 CONTINUE
C
C EXTRACT EACH ELEMENT Nodal Displacements
DO 105 I=1,LIM
Q(I)=0.0
J=EL(I)
105 CONTINUE

205 IF(NDYN.EQ.1) J=NDISP(M,I)
IF(J.EQ.0) GOTO 105
Q(I)=Q(I)+DR(J)

210 71 CONTINUE
C
C EXTRACT STRAIN MATRIX C 3X3

215 DO 110 I=1,3
DO 110 J=1,3
C(I,J)=0.0

110 CONTINUE
C(1,1)=E1.
C(2,2)=E2.
C(3,3)=G12

220 C
IF(NOUT.NE.0) GO TO 115
ISTR=2*NSTOR
115 CONTINUE
DO 888 IT=1,ISTR
IF(NOUT.EQ.2) GO TO 88
ISIGN=(-1)*IT
IF(ISIGN.LT.1) GO TO 81

888 Continue
SUBROUTINE STRESS  FTN 4.8+552  83/03/21, 15.54.27  PAGE 5

230  JSTRS=NBAB
    IZ=1
    GO TO 88
81 CONTINUE
    JSTRS=NBAB+1
    IZ=0

88 CONTINUE
    DO 88 JJ=1,JSTRS
    IF((IN.EQ.NMOD1).AND.(NDYN.EQ.1)) GOTO 430
    E=.5
    IF(NDOF.NE.0) GO TO 73
    IF(NOUT.EQ.0) GO TO 82
    GO TO 83
82 CONTINUE
    E=II*RS*0.5
83 CONTINUE

245  C  FORM THE BB MATRIX

C  IF(NOUT.EQ.2) GO TO 120
250  Z=(JJ-1)*RB+IZ*RB*-0.5
    GO TO 66
120 CONTINUE
    Z=.5
76 CONTINUE
    BB(2,3)=-(1,-Z)/D
    BB(2,4)=-Z/D
    BB(2,5)=Z/D
    BB(2,6)=(1,-Z)/D

C  BB(3,1)=-1/D
    BB(3,2)=1/D
    BB(3,3)=-(1-E)/A
    BB(3,4)=(1,-E)/A
    BB(3,5)=E/A
    BB(3,6)=-E/A

265  C  GO TO 74
    73 CONTINUE
    IF(NOUT.EQ.0) GO TO 84
    GO TO 85
270  84 CONTINUE
    E=II*RS*0.5-1.0
85 CONTINUE
    E=0.

C  FORM THE BB MATRIX

C  NOTE THAT BOTH D AND A REPRESENT ONLY ONE HALF THE
C  HEIGHT AND THE LENGTH OF THE ELEMENT RESPECTIVELY

280  C  IF(NOUT.EQ.2) GO TO 76
    Z=(JJ-1)*RB+IZ*RB*-0.5-1.0
    GO TO 77
76 CONTINUE
    Z=JJ*1,-1.5

285
SUBROUTINE STRESS  FTN 4.8+552  83/03/21, 15:51:27  PAGE 6

77 CONTINUE
BB(2,4)=Z*(1-Z)/(4.*D)
BB(2,5)=-(1-Z*Z)/(2.*D)
BB(2,6)=-(1+Z)*Z/(4.*D)
BB(2,7)=-BB(2,6)
BB(2,8)=-BB(2,5)
BB(2,9)=-BB(2,4)

290 C
BB(3,1)=-(1-2.*E)/(2.*D)
BB(3,2)=-2.*E/D
BB(3,3)=(1+2.*E)/(2.*D)
BB(3,4)=-((1-2.*E-Z+2.*Z*E)/(4.*A))
BB(3,5)=-Z*(1-E)/A
BB(3,6)=-(1+2.*Z-E-2.*Z*E)/(4.*A)
BB(3,7)=(1+2.*Z+E+2.*Z*E)/(4.*A)
BB(3,8)=-2*(1+E)/A
BB(3,9)=-(1-2.*Z+E-2.*Z*E)/(4.*A)

74 CONTINUE

305 C
C MULTIPLY C TIMES RB AND STORE IN RCC
DO 125 I=1,3
DO 125 J=1,LIM
BCC(I,J)=BB(I,J).
DO 125 I=1,3
BCC(I,J)=BCC(I,J)+C(I,L)*BB(L,J)
125 CONTINUE
C
C CALCULATE STRESSES, SIG=(BCC)*((R)
DO 130 I=1,3
SIG(I)=0.
DO 130 J=1,LIM
SIG(I)=SIG(I)+BCC(I,J)*Q(J)
320 130 CONTINUE
C
IF(NDYN.EQ.0) GOTO 470
WRITE(2,1010) H(SIG(I),I=1,3)
IF(NPRTN3.EQ.0) GOTO 888
IF(NN.EQ.1) GOTO 490
DO 450 I=1,3
READ(NTAPE1,*), SIGS(I)
SIGS(I)=SIGS(I)+SIG(I)*SIG(I)
WRITE(NTAPE2,*), SIGS(I)
450 CONTINUE
GOTO 888
490 DO 480 I=1,3
SIGS(I)=SIGS(I)*SIG(I)
WRITE(NTAPE2,*), SIGS(I)
480 CONTINUE
GOTO 888
430 DO 460 I=1,3
READ(NTAPE1,*), SIG(I)
SIG(I)=SQRT(SIG(I))
WRITE(NTAPE2,*), SIG(I)
460 CONTINUE
SUBROUTINE STRESS  FTN 4.8+552  03/03/21. 15:51.27  PAGE 7

470 IF(NOUT.NE.0) GOTO 135
   WRITE(2,E10) 'FORCES OR SRSS FORCES GOTTEN FROM SRSS OF STRESSES'
   CALL FORCE(SIG,II,JJ,N,JSTRO,TT,BW,SH,IZ)
   GOTO 888
345 CONTINUE
   WRITE(2,1010) M,(SIG(I),I=1,3)
   135 CONTINUE
   WRITE(2,1010) M,(SIG(I),I=1,3)
   888 CONTINUE
350 CONTINUE
   777 CONTINUE
   230 CONTINUE
   9999 CONTINUE
C
360 FORMAT(47H OUTPUT TABLE 2., STRESSES AT ELEMENT CENTROIDS )
   200 FORMAT(7H 20X,15H FACADE NUMBER )
      1  3X,7H ELEMENT,7X,8HSIGMA(X),7X,8HSIGMA(Y),
      2  7X,8H TAU(X,Y)
355 FORMAT(53H OUTPUT TABLE 2. INTERNAL FORCES IN COLUMNS AND BEAMS )
   600 FORMAT(7H 34X,15H FACADE NUMBER )
      1  3X,7H ELEMENT,7X,8HSIGMA(X),7X,8HSIGMA(Y),
      2  7X,8H TAU(X,Y)
360 FORMAT(10H 10X,15H SHEAR,15H MOMENT )
   1000 FORMAT(10H 7HELEMENT,7X,8HSIGMA(X),7X,8HSIGMA(Y),
      1  7X,8H TAU(X,Y))
   1010 FORMAT(10H 10X,15H SHEAR,15H MOMENT )
365 C
   RETURN
   END
SUBROUTINE FORCE

**SUBROUTINE FORCE(SIG,II,JJ,K,JSTRS,TT,BW,SH,IZ)**

*THIS SUBROUTINE EVALUATES INTERNAL FORCES IN BEAMS AND COLUMNS*

DIMENSION SIG(3)

IF(IZ.EQ.0) GO TO 10

EVALUATE BEAM INTERNAL FORCES

SF = SIG(3) * TT * SH
BM = SF * BW / 2.
WRITE(2,100) K,II,JJ,SF,BM
GO TO 999
CONTINUE

EVALUATE COLUMN INTERNAL FORCES

IF((JJ, EQ, 1), OR, (JJ, EQ, JSTRS)) GO TO 20
AF = SIG(2) * TT * BW
SF = SIG(3) * TT * BW
GO TO 30
CONTINUE

AF = SIG(2) * TT * BW / 2.
SF = SIG(3) * TT * BW / 2.
CONTINUE

BM = SF * SH / 2.
WRITE(2,200) M,II,JJ,AF,SF,BM
999 CONTINUE

100 FORMAT(315,4X,E2(4X,E10.3))
200 FORMAT(315,4X,E13.3,2(4X,E10.3))
RETURN
END
SUBROUTINE STAT  
FTN 4.8+552  83/03/21. 15,51.27  PAGE 1

1  SUBROUTINE STAT(S,R,NEQ,IMAX,NSSYN,NL)
   DIMENSION S(IMAX),R(NEQ)
   C....OBTAIN GLOBAL STATIC DISPLACEMENTS
   CALL GAUSS(3*R,IMAX,NEQ,1,1,DET,1)
   WRITE(2,*) 'STRUCTURAL DISPLACEMENT VECTOR IS'
   WRITE(2,*) (R(II),II=1,NEQ)
   ND=NEQ/NL
   IF(NSSYN.NE.0) GOTO 55
   WRITE(2,300)
   DO 50 I=1,NL
      L=(ND-1)*NL+I
      WRITE(2,400) J,(R(J),J=I,L,NL)
   50     CONTINUE
   GOTO 444
   55 IF(NSSYN.NE.1) GOTO 60
      WRITE(2,600)
      GOTO 75
   60 IF(NSSYN.NE.2) GOTO 65
      WRITE(2,700)
      GOTO 75
   65 IF(NSSYN.NE.3) GOTO 70
      WRITE(2,800)
      GOTO 75
   70 WRITE(2,*) '************ ERROR IN INPUT DATA ************'
      WRITE(2,*) 'STRICT SYMMETRY WAS NOT SPECIFIED AS 0,1,2,OR 3'
   GOTO 999
   75     DO 80 I=1,NL
      L=(ND-1)*NL+I
      WRITE(2,900) J,(R(J),J=I,L,NL)
   80     CONTINUE
   444 CONTINUE
   C
   300 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
   1  2X,5HLEVEL,BX,7HX-DISP.,BX,7HY-DISP.,BX,7H ROT-O ,16X,
   2 25HVERTICAL DISP. OF CORNERS) 
   400 FORMAT(17,3E15.3)
   600 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
   1  1H LEVEL NO.  X-DISPL.  DISPLACEMENT OF
   2  1H CORNERS $$/
   700 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
   1  3H LEVEL NO.  Y-DISPL.  DISPLACEMENT OF
   2  1H CORNERS $$/
   800 FORMAT(38HOUTPUT TABLE 1.. GLOBAL DISPLACEMENTS //
   1  2H LEVEL NO.  ROTATION DISPLACEMENT OF
   2  1H CORNERS $$/
   900 FORMAT(110,4X,E10.3,5(4X,E10.3))
   999 RETURN
   END