Essays in Option Pricing and Foreign Exchange Rate Determination

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ABSTRACT

ESSAYS IN OPTION PRICING AND FOREIGN EXCHANGE RATE DETERMINATION

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Garman and Ohlson (1981) derive the relationship for risky asset prices in arbitrage-free economies with and without transaction costs (TC). In Essay one, a model is proposed to capture the effects of TC on option prices. The most important TC-related variables are the rate of change in the hedging rate with a change in the underlying security's price (Υ) and the option's price sensitivity to its underlying security's volatility (λ). For a sample of options on the futures of five major currencies, a linear specification of the model exhibits bias only for short maturity options, and a log-linear specification exhibits no more bias than would be expected by chance.

In Essay two, simplified derivations of models to price physical and future options are presented for various types of European options. These options include call and put options on a single asset, options to exchange one asset for another, and call and put options on the maximum or the minimum of two assets. The covariance structure of the exchange rates of nine major currencies is also studied. No statistical difference in the covariance structure of returns from holding these currencies for one, three, or six months is found. All currencies move in the same direction against the US dollar only in the short run. Currency diversification (or the use of currency baskets) may only reduce FX risk in the long run from a US or British point of

view.

In Essay three, an exchange rate determination model, which is compatible with the empirical regularities found in the literature, is developed and tested. The model is based on purchasing power parity (PPP) in the long run. Short term deviations from this parity are due to financial speculation caused by interest and inflation rates expectations, and are constrained by the possibility of (costly) goods arbitrage. The model accounts for transactions costs, and its risk premia are related to the volume of capital involved in currency speculation. The model satisfactorily explains the levels of exchange rates for the German mark (DM), the Swiss franc (SF), and the Canadian dollar (CD). The model satisfactorily explains exchange rate changes for the DM and the SF, and it usually outperforms a random walk in out-of-sample tests. The results for the CD are less satisfactory, probably due to the political events which occurred in Canada during the studied period.

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List of Symbols

$a_j(\theta)$	=	$P_j(\theta) + D_j(\theta)$
B	=	amount borrowed
BTS	=	Box's [1949] test statistic
C	=	option price; call option price; variable used to compute BTS
C_F	=	call option on a futures contract
$C_{Fmax}(F_1,F_2,X,T)$	=	option to buy the maximum of F_1 and F_2 at a price X at maturity
$C_{Fmin}(F_1,F_2,X,T)$	=	option to buy the minimum of F_1 and F_2 at a price X at maturity
C_m	=	market value of the option
$C_{max}(S_1,X_1,S_2,X_2,T)$	=	option to buy S_1 at a price of X_1 and S_2 at a price of X_2 at maturity
$C_{max}(S_1,S_2,X,T)$	=	option to buy the maximum of S_1 and S_2 at a price X at maturity
$C_{min}(S_1,S_2,X,T)$	=	option to buy the minimum of S_1 and S_2 at a price X at maturity
C_t	=	theoretical price of the option in a transactions-costs-free economy
c_j^s, c_j^b	=	transaction costs incurred if one unit of security j is sold or bought, respectively, at the beginning-of-the-period

$c_j^s(\theta), c_j^b(\theta)$	=	transaction costs incurred if one unit of security j is sold or bought, respectively, at the end-of-the-period in state θ
c_1, c_2	=	standardized currency units
$D_j(\theta)$	=	end-of-period dividend
Dir	=	percentage of times the predicted change of FX is of the correct sign
d_1, d_1'	=	variables used in option price calculations
df	=	degrees of freedom
dR	=	$R_d - R_f$
dr	=	$r_d - r_f$
dz, dz_i	=	standard Gauss-Wiener processes
$d\delta_i$	=	$\delta_i^1 - \delta_i^0$
$d\pi$	=	dR - dr
E( )	=	expectations operator
$E^{rn}(\ )$	=	expected value in a risk neutral world
ER_j	=	expected return on asset j
$Ex(S_1,S_2,T)$	=	option to exchange security S_2 for S_1
$Ex_F(F_1,F_2,T)$	=	option to exchange forward contract F_2 for F_1
F	=	forward price; futures price; forward rate
F^m	=	value of F at the maturity of the option
FX, FX_t	=	foreign exchange rate
$f(\bullet)$	=	log-normal probability distribution
HR	=	return on a hedged portfolio
i	=	nominal interest rate
JTS	=	Jennrich's [1970] test statistic

k	=	round trip transaction cost in Leland's [1985] model; number of correlation matrices
M	=	modified moneyness of an option; variable used to compute BTS; money supply
M_1	=	moneyness of an option
MAE	=	mean absolute error
ME	=	mean error
MSE	=	mean squared error
$N(\bullet)$	=	cumulative standard normal probability distribution
$N(\bullet, \bullet)$	=	cumulative normal probability distribution
$N(\bullet, \bullet, \bullet)$	=	cumulative bivariate standard normal probability distribution
n, n_1	=	sample size
$O(\ )$	=	order of magnitude of the term
P	=	price of a put option; price level
P_F	=	put option on a futures or forward contract
P_0	=	pre-transaction-costs price at the beginning of the period
$P_1(\theta)$	=	pre-transaction-costs end-of-period price in state θ
$P_{max}(S_1, S_2, X, T)$	=	option to sell the maximum of S_1 and S_2 at a price X at maturity
$P_{min}(S_1, S_2, X, T)$	=	option to sell the minimum of S_1 and S_2 at a price X at maturity
PC	=	0 for a call and 1 for a put

PI	=	price index
p	=	ln P; size of the correlation matrix
q	=	real exchange rate
R	=	nominal interest rate
$\bar{R}$	=	weighted average of R_i's
R_i	=	i^{th} correlation matrix
R_j	=	return on asset j
R_m	=	return on market portfolio
RMSE	=	root mean squared error
r	=	interest rate; real interest rate
$\bar{r}_{ij}$	=	element of row i, comumn j of $\bar{R}$
$\bar{r}^{ij}$	=	element of row i, column j of $\bar{R}^{-1}$
S	=	stock price; security price; underlying security; spot rate; variables used to compute JTS and BTS
$\bar{S}$	=	long-term equilibrium exchange rate
S_i	=	covariance matrix for BTS
S^m	=	value of S at the maturity of the option
s	=	ln S
s^{PPP}	=	s if PPP holds immediately
T	=	time
T_{MR}	=	test statistic proposed by Meese and Rogoff [1988]
${}_iTR$	=	$\sum_{j=1}^{i} {}_jdR$
${}_iTr$	=	$\sum_{j=1}^{i} {}_jdR$

t	=	time; a measure of marginal transaction costs
$U(\ )$	=	utility operator
u	=	random variable with zero mean and unit variance
V	=	implicit price system
$Var(\ )$	=	variance
v	=	rate at which the exchange rate converges to equilibrium in Dornbush's [1976] model
w	=	error term
X	=	percentage of times the prediction falls on the same side of the forward as the true exchange rate
x	=	natural logarithm of the real exchange rate
Y	=	domestic income
y_j	=	a portfolio
Z_i	=	variable used to compute JTS
z_j	=	a portfolio
α	=	variance elasticity in a CEV process; significance level of a statistical test
α_1, α_i	=	expected return on a security
α_j	=	$Var(R_j)/Var(R_m)$ = measure of own risk
β_i	=	parameters
β_j	=	$Cov(R_j, R_m)/Var(R_m)$ = measure of market risk
γ	=	$\partial\delta/\partial S$
γ_j	=	measure of the relative concentration of holdings of asset j

Δ_i	=	$i({}_iTr_d - {}_iTr_f)$
Δ_s	=	$s_1 - s_0$
Δt	=	revision interval in Leland's [1985] model
δ	=	$\partial C/\partial S$ = hedge ratio
δ_i	=	$i({}_iTR_d - {}_iTR_f)$
δ_{ij}	=	Kronecker
δ_{inf}	=	$\pi_d - \pi_f$
ε	=	error term
ε_j	=	"fudge factor" function of transaction costs
ε_j^1, ε_j^2	=	variables used to compute the bounds on ε_j
θ	=	$\partial C/\partial T$; state at the end of the period
θ'	=	$\partial \delta/\partial T$
λ	=	$\partial C/\partial \sigma$; risk premium
λ'	=	$\partial \delta/\partial \sigma$; a measure of risk aversion
μ	=	expected return on a security
π	=	inflation rate
ρ	=	discount rate
$\rho_{i,j}$	=	correlation coefficient
σ, σ_i	=	standard deviation; volatility
$\hat{\sigma}$	=	modified volatility in Leland's [1985] model
ϕ	=	$\partial C/\partial r$
ϕ'	=	$\partial \delta/\partial r$
Ω	=	δ S/C = option price elasticity
•	=	expected value
*	=	refers to the variables in the foreign country

$\hat{}$ = transaction-cost-economy counterpart of a variable

D, F = domestic and foreign variable, respectively

d, f = domestic and foreign variable, respectively

Chapter 1

INTRODUCTION

This dissertation consists of three essays dealing with option pricing and with exchange rate determination. The first essay explores the relationship between transaction costs and option prices. It attempts to explain some of the discrepencies between prices generated by various option pricing models and actual market prices. The second essay deals with the pricing of complex options, such as options on the maximum, or minimum, of two assets, and it studies the correlation structure of foreign exchange rates. The third essay presents a new model for the determination of exchange rates, which is based on inflation and interest rate expectations, and where transaction costs play a major role. In all three essays, the empirical work uses data from the foreign exchange markets. Nonetheless, the approaches and the models developed herein may be applied, with appropriate modifications, to other markets.

Essay I attempts to explain the option pricing biases observed in the literature by using a new approach to transaction costs. Garman and Ohlson (1981) derive the relationship between risky asset prices in arbitrage-free economies, with and without transaction costs (TC). To capture the effects of TC on option prices, a class of market participants, financial intermediaries, is assumed to have the smallest TC in the economy, and these TC are assumed to be nonnegligible. Direct TC effects are captured through the costs of hedging and rehedging the financial intermediaries option portfolios. Mayshar (1981) shows that in a TC economy the own risk of an asset may be priced. In this dissertation, Mayshar's definition of TC is used;

namely, taxes on transactions, short sale restrictions, various institutional restraints, subjective costs of managing one's own portfolio, brokers' fees and bid-ask spreads. Indirect TC effects are captured through the risk of options and option portfolios, and through the prices of options which act as a proxy for the differential between borrowing and lending interest rates.

The model developed in Essay I is tested using data on options on foreign currency futures traded on the International Money Market of the Chicago Mercantile Exchange. Options on foreign currency futures are studied because they avoid some of the problems associated with options on other assets. To illustrate, foreign currencies do not pay discrete dividends, the probability of bankruptcy of the governments of major industrial Western countries is negligible, and the concepts of financial and operating leverage (which apply to corporations) do not apply to sovereign governments. Furthermore, no liquidity problem is encountered with currency futures and options on these futures.

Since the empirical tests are conducted on foreign currency futures options, the results may not be directly applicable to options on other securities (such as stock options). However, by appropriately modifying the basic model, the same approach can be used to incorporate the impact of transaction costs into the pricing of stock options.

Essay II deals with the pricing of complex options. Since the pioneering work of Black and Scholes (1973), other option pricing models have been proposed to account for assumptions different from those of Black and Scholes. Models were presented for dividend paying stocks, for American options, for various stochastic processes of the underlying security and of the risk free interest rate, and for the case where transaction costs exist. Another series of models was presented for pricing options on assets, such as foreign currencies, futures, and bonds. Margrabe (1978) developed a model for valuing an option to exchange one asset for another, and Stulz (1982) proposed a model for valuing options on the maximum or minimum of two assets.

Based on the insight of Cox, Ross and Rubinstein (1979), this essay presents simplified, intuitive and rigorous derivations of pricing models for various types of European options, namely, call and put options on a single asset, options to exchange one asset for another, and call and put options on the maximum or the minimum of two assets with the same or different exercise prices. These models are derived for the case where the underlying securities are "physicals" (stocks, bonds or commodities), and for the case where the underlying securities are forward or futures contracts.

In order to assess the parameters required in these models for foreign exchange, an empirical study of the correlation structure of exchange rates is conducted for the major world currencies. These are the US dollar, the German mark, the Japanese yen, the Swiss franc, the British pound, the French franc, the Australian dollar, the Dutch guilder, and the Canadian dollar. The stability of the correlation structure over time, and for different holding periods, is studied.

Essay III presents and empirically tests a foreign exchange rate determination model. This model is compatible with the behaviour of exchange rates during the early 1980's. This is a period when both the inflation and interest rates were higher in the US than in Japan and Germany, yet the dollar was appreciating vis-à-vis the yen and the DM. It is also consistent with the empirical regularities observed in the literature.

The model assumes that exchange rates are determined by relative price levels in the long run, and that deviations from that long run equilibrium are due to financial speculation driven by interest and inflation rate expectations. Excessive deviations from purchasing power parity are prevented by (costly) goods arbitrage. Financial speculation involves risk. Currencies are treated like other primary assets, and the risk premia involved in the model are derived in an economy with transactions costs. The risk premium in the model is dependent on the volume of capital involved in currency speculation, which, in turn, is assumed to be a function of the real interest rate differentials across countries. The model uses the entire term structures of interest rates and inflation rate expectations.

The first form of the model explains exchange rate levels, and the second form explains exchange rate changes. The empirical analysis is conducted on the German mark, the Swiss franc and the Canadian dollar over the period from 1975 to 1987 due to data availability. The model explaining exchange rate changes is tested for changes over one, three, six and twelve months, using both in- and out-of-sample analyses.

The approach used to develop the model in this essay is quite general and could be applied to any asset, though it would probably be most useful for the pricing of assets which have two characteristics. First, their own risk is not usually well diversified, perhaps because of institutional restraints. For example, foreign exchange traders in a bank cannot diversify their risk by buying stocks, and futures traders are usually rewarded for their performance in futures trading. Second, the amount of speculative capital used for trading in these assets must have a substantial variability. Both of these characteristics are generally applicable to futures contracts and to traded commodities.

Chapter 2

ESSAY ONE ON

"TRANSACTION COSTS AND BIASES IN OPTION PRICING"

I. Introduction

Most empirical studies that have compared theoretical prices generated by option pricing models (OPM's) with actual market prices have found systematic biases. These biases have been related to the volatility of the underlying stock, to the degree the option is in- or out-of-the-money, to the maturity of the option, and to whether it was a call or a put Several methodological and theoretical reasons have been suggested to explain these biases. The methodological arguments, which were particularly relevant for the earlier studies, dealt with the non-simultaneity of the prices of options and of their underlying securities, and with the estimation of the volatilities used in the OPM's. The theoretical arguments have mostly centered on the use of European OPM's for pricing American options, on the stochastic process governing the price of the underlying security, and on transaction costs.

Various American OPM's have been proposed by Roll [1977] and Whaley [1986], amongst others. Various stochastic processes have been proposed, such as the constant elasticity of variance (CEV) (Cox and Ross [1975]), diffusion-jump processes (Merton [1976a]), and compound options (Geske [1979]). Solutions have been proposed to the transactions cost problem (e.g., Leland [1985]). Unfortunately, none of these theoretical contributions appears to explain all of the option pricing biases found in the empirical literature.

The primary purpose of this essay is to determine if transaction costs explain most of the biases in the theoretical option pricing models by using the Garman and Ohlson [1981] discussion of the valuation of risky assets in arbitrage-free economies with transaction costs. Garman and Ohlson found that the price of a risky asset in a transaction costs economy ($\hat{P}_j$) is given by its price in a no transaction costs economy (P_j) plus what they call a "fudge factor", ϵ_j, which is a function of transaction costs. The contribution of this paper is to specify the variables which affect transaction costs for option hedging and rehedging in a transactions cost economy (i.e., the determinants of ϵ_j for options), to test whether or not these variables are statistically significant, and to test whether or not these variables explain the observed biases in a theoretical OPM.

The empirical analysis will use data on options on foreign currency (FX) futures. FX futures were chosen because they have no marketability or liquidity problems, foreign currencies do not pay discrete dividends, the probability of bankruptcy of the governments of major industrial western countries is negligible, and the concepts of financial and operational leverage do not apply to sovereign governments. Thus, the results from this study may not be directly transferable to options on stocks. Options on futures were used instead of options on the spot, because the former does not require the knowledge of the foreign risk-free interest rate.

The paper is organized as follows. In section II, the empirical literature is reviewed. In section III, the proposed explanations of these findings are critiqued. In section IV, a model which incorporates the effects of transaction costs is developed. In section V, the data and the methodology are discussed. In section VI, the empirical findings are presented and analyzed. In section VII, some concluding remarks are offered.

II. The Empirical Literature

Black and Scholes (B-S) [1972] find that their model tends to overestimate the value of options on high variance securities and to underestimate the value of options on low variance securities. Black [1975] finds that options that are way out-of-the-money tend to be overpriced, that options that are way into-the-money tend to be underpriced, and that options with less than three months to maturity tend to be overpriced.

Blomeyer and Klemkosky [1983] compare the B-S model with the Roll [1977] model for unprotected American call options on stocks with known dividends. They find no statistically significant ex-post performance difference between the two models. Both models display almost identical biases by undervaluing out-of-the-money call options relative to the market price, and pricing fairly well at- and in-the-money options. According to the authors, their results suggest that the systematic pricing bias observed in the B-S model is not a dividend bias.

Brennan and Schwartz [1977] develop a numerical algorithm which uses a finite difference approach for the pricing of put prices with a finite life, and which may or may not be protected against dividend payments on the underlying stock. They find that if the model prices are accepted as equilibrium prices, then put prices on high variance stocks are systematically underpriced relative to put prices on low variance stocks. This result is consistent with the B-S finding that call prices on high variance stocks are underpriced relative to those on low variance stocks.

Bodurtha and Courtadon [1987] test an American OPM on the foreign currency options traded on the Philadelphia stock exchange. They find that out-of-the-money options tend to be underpriced by the model relative to at- and in-the-money-options, and that the model, on average, overprices put options relative to call options. In addition, they find the degree of mispricing to decrease as time to maturity increases.

Some conflicting results exist in the literature. MacBeth and Merville [1979] find that: (1) The prices predicted by the B-S model are, on average, less (greater) than market prices for in-the-money (out-of-the-money) options; (2) With the exception of out-of-the-money options with less than ninety days to expiration, the extent to which the B-S model underprices (overprices) an in-the-money (out-of-the-money) option increases with the extent to which the option is in-the-money (out-of-the-money), and decreases as the time to expiration decreases; and (3) For out-of-the-money options with less than ninety days to expiration, the B-S model prices are, on average, greater than market prices. However, no consistent relationship appears to exist between the extent to which these options are overpriced by the B-S model and the degree to which these options are out-of-the-money or the time to expiration.

For American options on futures, Whaley [1986] finds that the moneyness bias for calls is the opposite of that reported for stock options. Specifically, out-of-the-money options are underpriced relative to the model, and in-the-money options are overpriced. He finds the reverse for puts; namely, out-of-the-money puts are overpriced relative to the model, and in-the-money puts are underpriced.

Brenner and Galai [1981] examine the properties of the estimated risk of common stocks implied by option prices. They find that the implied standard deviations of longer maturity options tend to be higher than those of shorter maturity options. This implies that options with a long life are overpriced relative to short maturity options.

Other studies find a systematic bias which changes over time. Rubinstein [1985], using non-parametric tests on paired observations, tries to distinguish which pricing formula seems to provide the better explanation for the observed biases from the B-S values. He examines the following models: the displaced diffusion model of Rubinstein [1983], the pure-jump model of Cox and Ross [1976], the diffusion-jump model of Merton [1976a], the compound-option model of Geske [1979], and the constant-elasticity-of-variance model of Cox and Ross [1976]. Using transactions data for the period August 23, 1976 to August 31, 1978, Rubinstein concludes that no model captures all the biases, and that the models that capture the time to expiration biases are disjoint from those that capture the striking price biases. He also finds that, while the striking price biases from the B-S values are significant and tend to go in the same direction for most stocks at any point in time, the direction of the bias changes from period to period.

Finally, Trennepohl [1981] compares listed option premiums and B-S model prices for the period 1973-1979. He finds that studies comparing model and market prices, using different time periods, may produce inconsistent results. His analysis indicates significant underpricing by the model only for out-of-the money options.

III. Possible Sources of Bias Considered in the Literature

III.1 The Use of European OPM's for Evaluating American Options

The argument that the biases found in studies using European OPM's could be due to the fact that most exchange-traded options are actually American does not seem to be supported empirically. Indeed, Blomeyer and Klemkosky [1983] find the same biases in the Roll [1977] model for unprotected American call options as in the B-S model. Rubinstein [1985] finds that no model explains all the biases in the B-S model. Brennan and Schwartz [1987] find the same biases for their American OPM as occurs with the B-S model. Whaley [1986] and Bodurtha and Courtadon [1987] find systematic biases for options on futures and foreign exchange, respectively, when using American OPM's.

The only exception is Whaley [1982], who finds that the moneyness and the maturity biases disappear when the correct American call option valuation model is used. However, Whaley's results on maturity bias may be due to his empirical technique. He computes different implied volatilities for options on the same stock which have different maturities. Since this procedure will absorb any maturity bias in the implied volatility, the residual error will be left with little bias. As to the moneyness bias, he finds that none exists when the B-S model is applied to the stock price net of the present value of the escrowed dividend. This finding is probably due to the fact that too few options were priced with each estimated implied volatility (1.67 on average). The volatility bias found by Whaley may be due to the movement of volatilities from their extreme values towards their long term trend values. Although this does not invalidate Whaley's American OPM, it does question the evidence concerning the capacity of that model to eliminate all maturity and moneyness biases.

III.2 The Underlying Stochastic Process

Several alternative stochastic processes are suggested in the literature for the underlying security. Merton [1976a] proposes an OPM where the underlying stock returns are discontinuous. He specifically assumes that part of the variance could be attributed to a continuous diffusion process and part to discrete price jumps which follow a poisson process. Based on a series of simulations, Merton [1976b] concludes that the general level of the magnitude of the errors is surprising, in that the effect of specification error in the underlying stock returns on option prices will generally be rather small. Beckers [1980] presents an interesting approach for estimating the parameters of the diffusion-jump model. He finds that the resulting differences between the B-S and Merton models are insignificant and confirm the simulation results obtained by Merton. Rubinstein [1985] finds that Merton's model cannot explain the biases observed with the B-S model.

The empirical evidence suggests that the underlying stock's volatility is inversely related to its price. Christie [1982] finds that equity volatility is an increasing function of financial leverage, that financial leverage is sufficient to induce a negative elasticity between equity volatility and the value of equity, and that interest rates have a strong positive impact on volatility. If the variance of the return on the assets of a firm is assumed to be constant, then the variance of the return on equity will increase as the value of the leverage of the firm increases. Leverage can increase if more debt is issued, if the market value of shares drops, or if interest rates increase. Ceteris paribus, an increase in interest rates, can increase the leverage of the firm since the value of equity can be viewed as the present value of the future income stream minus the debt service. The discount factors diminish as interest rates increase, while the cost of debt service increases.[1] Other factors used to explain the negative relationship between volatility and stock prices are the increasing probability of default with decreasing stock prices, and operating leverage which affects the volatility in the same manner as financial leverage.

Geske [1979] addresses this problem by considering the stock of the firm as an option on its assets, and the option on the stock as an option on the option. While Geske's model is conceptually interesting, it has not been very successful empirically (see, for example, Rubinstein [1985]).

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1. This would indicate that the level of interest rates should be incorporated in the modeling of the volatility of an option. However, it has not been included because the variance of interest rates has usually been considered small with respect to the volatility of stocks.

Cox and Ross [1975] present an OPM where the stock price follows a constant elasticity of variance (CEV) stochastic process. This process can be written as:

$$dS = \mu\, Sdt + \sigma\, S^{\alpha/2}\, dZ \qquad 0 \le \alpha < 2 \qquad [1]$$

where S = stock price
μ = expected return on the stock
$\sigma^2/S^{2-\alpha}$ = instantaneous variance of relative price change (volatility squared).

When $\alpha = 0$, the volatility is inversely proportional to the stock price. This case is often referred to as the absolute model. When $\alpha = 1$, the volatility decreases with the square root of the stock price. This case is often referred to as the square root model. When $\alpha = 2$, we simply have the B-S model where volatility is independent of the stock price. If the true stock return generating process is a CEV with $\alpha < 2$ and the B-S model is used to evaluate option prices, out-of-the-money options will be overpriced by the model, while in-the-money options will be underpriced by the model. This is consistent with the bias found by MacBeth and Merville [1979,1980] and by Whaley [1986], and inconsistent with most of the other empirical results.

Empirical testing of the CEV models has not been very successful. MacBeth and Merville [1980] find that the CEV model gives better results than the simple B-S model with values of α ranging between -4.16 and 3.84. Unfortunately, values of α outside the 0 to 2.0 interval are not very meaningful economically. An $\alpha < 0$ means that the variance of the stock price increases when the price of the stock drops. Thus, as the price of a stock goes towards zero (for example, if the firm is liquidating its assets or is about to go bankrupt), the variance of the price of the stock would tend to infinity. The empirical observation that the variance of some stocks have increased after a drop of their prices, (for example, after the stock market crash of October 1987 (see Leland and Rubinstein (1988)) should not be considered as being evidence that $\alpha < 0$. An $\alpha > 2$ means that the volatilities of stock returns increase when stock prices increase. Thus, the variances of stock returns could increase without bound as the stock prices increase. This is not a very satisfactory characteristic of the price diffusion process. Although increases in return volatility have been observed after increases in stock prices, this was probably caused by factors other than α.

MacBeth and Merville find that their estimate of α was not stable over time. For example, it varied from around +5 to around -3 for options on Eastman Kodak within a three month period in 1976. Such results violate the assumption of the CEV option pricing model that σ and α are fixed over time. According to Manaster [1980], the empirical superiority of the CEV model over the B-S may be due to the fact that the former contains the latter as a special case, and has an extra parameter, α, to fit the data, rather than due to its more sound theoretical underpinnings.

Hull and White [1987] present an OPM with stochastic volatilities. As they note, their results can be used to explain the empirical observations of Rubinstein [1985], only if the questionable assumption is invoked that the correlation between volatilities and stock prices reverses from year to year.

III.3 Transaction Costs

Transaction costs have usually been studied in the literature in terms of the Black and Scholes hedged portfolio (BSHP). The derivation of the B-S formula is based on the construction of a portfolio whose instantaneous return is risk free. Such a portfolio would contain one call option and would be short δ (= $\partial C/\partial S$) units of the underlying security, S. To ensure that this portfolio is risk free, an investor needs to continuously adjust the hedge portfolio as δ changes. Transaction costs would be incurred for each readjustment. In the limit, these costs tend to infinity.

One approach includes transaction costs in the differential equation of the option pricing formula. Unfortunately, this yields a complicated partial differential equation with no known solution. An alternative approach (see Leland [1985]) is to readjust the hedge portfolio at discrete intervals. This approach yields a pricing formula similar to that of Black and Scholes, with the exception of a modified instantaneous variance of return given by:

$$\hat{\sigma}^2 = \sigma^2 \, [1 + \sqrt{2/\pi} \; k/\sigma \; \sqrt{\Delta t}] \qquad [2]$$

where k = round trip transaction cost; and

Δt = revision interval.

For either of these approaches to be valid, the price setters in the options markets have to hold the BSHP, or at a minimum, the BSHP should represent the most efficient way to hedge an option.

While each of the proposed theoretical OPM's have not been able to explain all the biases found in the literature, these models should not be rejected. The inclusion of all the relevant omitted factors may remove all the baises of these models.

IV. A New Approach to Incorporating Transaction Costs into Option Pricing

Mayshar's [1981] definition of transaction costs is used herein, namely:

> Transaction costs are often narrowly interpreted as including only brokers' fees and losses due to the bid-ask spread. Taxes on transactions and various other obstacles to trade, however, may also be usefully considered as a form of (sometimes prohibitive) transaction costs. These impediments to trade may include the nondivisibility of assets, short-sale restrictions, various institutional restraints, and even subjective costs of managing one's own portfolio.

IV.1 General Effect of Transaction Costs

OPM's are derived from arbitrage arguments in a transaction-costs-free economy. In such an economy, the creation of derivative securities such as options provides no social benefit. In real markets, the dynamic replication of an option with a portfolio containing the underlying security and the riskless asset may not be feasible and transaction-costs-free. Hence, OPM's derived in a no transaction cost (TC) framework are used to price options which only exist because of TC's. Merton [1988] addresses this problem by assuming that, while many investors cannot trade costlessly, the lowest-cost transactors (by definition, financial intermediaries) can. Merton shows that in this environment a set of feasible contracts exist that permits all investors to achieve optimal consumption-bequest allocations as if they could trade continuously without cost. Although financial intermediaries are likely to be the lowest-cost transactors and the price setters in the economy, they still face some non negligible transaction costs.

Garman and Ohlson [1981] examine the valuation of risky assets in arbitrage-free economies with and without transaction costs. In a TC economy, they show that no portfolio y_j, $j = 1, \ldots, J$, exists such that:

$$\sum_j y_j P_j < 0$$

and $$\sum_j y_j a_j(\theta) \geq 0 \text{ for all } \theta.$$

where P_j = the pre-transactions-costs price of security j at the beginning of the period;

θ = the state at the end of the period which is random and unknown at the beginning of the period. A finite number of states is assumed so that $\theta = 1, \ldots, n$;

$a_j(\theta)$ = $P_j(\theta) + D_j(\theta)$ is the (exogenous) pre-transactions-costs gross end-of-period payoff of security j given state θ, where $P_j(\theta)$ and $D_j(\theta)$ denote the end-of-period price and dividend across different states, respectively.

Using Farkas Lemma, this leads to:

$$P_j = \sum_\theta V(\theta)\ a_j(\theta), \text{ for all } j = 1, \ldots, J;$$

where $V = (V(1), \ldots, V(\theta), \ldots V(n))$ is a non-negative vector and is gene..lly referred to as the implicit price system.

If markets are complete, then $V(\theta)$ is unique and equal to the price of an Arrow-Debreu security which pays off one unit if and only if state θ occurs. For a transactions-costs economy, Garman and Ohlson show that no portfolio y_j, z_j, $j = 1, \ldots, J$, exists such that:

$$\sum_j y_j\ [P_j + c_j^b] - \sum_j z_j\ [P_j - c_j^s] < 0$$

and $$\sum_j y_j\ [a_j(\theta) - c_j^s(\theta)] - \sum_j z_j[a_j(\theta) + c_j^b(\theta)] \geq 0 \quad \text{for all } \theta,$$

such that y_j, $z_j \geq 0$ for all j,

where $c_j^s(\theta)$, $c_j^b(\theta)$ are the transaction costs incurred if one unit of security j is sold or bought, respectively, at the end-of-the-period in state θ;

and c_j^s, c_j^b are the transaction costs incurred if one unit of security j is sold or bought respectively, at the beginning-of-the-period.

This leads to:

$$\hat{P}_j = \sum_\theta \hat{V}(\theta)\ a_j(\theta) + \varepsilon_j,$$

where $\hat{P}_j$ and $\hat{V}(\theta)$ are the transaction cost economy counterparts of P_j and $V(\theta)$ in the no transactions cost economy.

The bounds for ε_j are given by:

$$-\varepsilon_j^2 \leq \varepsilon_j \leq \varepsilon_j^1, \qquad [3]$$

$$|\varepsilon_j| \leq {}^1/_2\ |\ \varepsilon_j^1 - \varepsilon_j^2\ | + {}^1/_2\ |\varepsilon_j^1 + \varepsilon_j^2|,$$

where $$\varepsilon_j^1 \equiv c_j^s + \sum_\theta \hat{V}(\theta)\ c_j^b(\theta), \text{ and}$$

$$\varepsilon_j^2 \equiv c_j^b + \sum_\theta \hat{V}(\theta)\ c_j^s(\theta).$$

Given a fixed implicit price system (i.e., one where $V=\hat{V}$), the relationship between prices in the perfect and imperfect markets becomes:

$$\hat{P}_j = P_j + e_j \qquad [4]$$

Stated more simply: if transaction costs are permitted to "perturb" individual asset (e.g., option) prices, but do not affect the economy at its most fundamental level (i.e., either the values of the underlying security or the risk-free interest rate), then the value of a risky asset in a transaction costs economy is equal to its value in a no TC economy plus a "fudge factor", which depends on the size of transaction costs. In the partial equilibrium framework in which OPM's are usually developed, the assumption that the implicit price system is insensitive to transaction costs is not very restrictive. While these findings indicate that the required modification to the no TC option price depends on the size of transaction costs, they do not prescribe how the "fudge factor" should be computed.

IV.2 Accounting for the Pricing Effects of Transaction Costs

As noted above, financial intermediaries are assumed to be the setters of option prices because they are the lowest cost transactors.[2]

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2. It is further assumed that TC's are the same for all financial intermediaries (since they all are the lowest cost transactors by definition), and that there are no economies of scale to TC's (i.e., the average TC's do not decrease with volume, a reasonable

They are assumed to be maximizers of mean-variance utility, U(E(y), Var(y)). They maximize their wealth by minimizing the volume (cost) of hedging transactions, and by minimizing their risk by hedging and diversifying. Since the costs of hedging and diversifying can not be measured directly, they can be proxied by a number of variables. First, as shown by Mayshar [1981], the own variance of a security (or portfolio) may be priced, and not only its systematic risk, in the presence of transaction costs. Hence, transaction costs will affect option prices directly through the volume of hedging and indirectly through the own risk of the financial intermediary's portfolio.

The option portfolio of a financial intermediary could contain both long and short positions of puts and of calls with different maturities and different exercise prices for each underlying security represented in the intermediary's portfolio. The intermediary can then adjust its bid and ask prices for selected options so as to induce its customers to buy (or sell) the options that reduce the overall need for hedging transactions and/or decrease the risk of its portfolio, or so as to compensate for its TC's and risk.

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assumption for financial intermediaries which are usually large institutions). These assumptions satisfy the restrictions imposed by Garman and Ohlson on TC's.

IV.2.1 Direct Transaction Costs Effects

The first transaction cost incurred is the fixed cost of initiating or transacting an option contract. Transaction costs are also incurred when an option is initially hedged, and when the financial intermediary's portfolio (FIP) is rehedged due to changes in the hedge ratio, δ. Such changes are caused, for example, by movements in the price of the underlying security.

The initial amount of hedging depends on the δ of the option if each option trade is immediately hedged, or on the net δ of the options traded since the last hedging transaction. The FIP may be long for some calls and puts, and short on others. Since the $\delta(=\partial C/\partial S)$ of a call is always positive while that of a put is always negative, the net δ that needs to be hedged may be positive or negative at any point in time. Thus, at some points in time, financial intermediaries may prefer to write calls (buy puts), while at other times they may prefer to write puts (buy calls) so as to decrease the required amount of hedging. Consequently, financial intermediaries may attach positive or negative premia to positive (negative) δ options over time.

Since the transaction costs of the initial hedge are probably small compared to the costs of rehedging which has to take place continuously (or at more or less short time intervals) over the life of the option, δ can be expected to explain only a small part of the difference between market prices and OPM prices in a TC economy.

The amount of rehedging depends on the change in the net δ of the FIP as time passes, as the value of the underlying security changes, and as r and σ change. Although most OPM's (particularly, the B-S model) assume that the risk free rate and the volatily of the underlying security are constant, r and σ do actually change over the life of an option. Thus, the direct transaction costs effects of changes in S, T, r and σ are due to the changes they cause in δ-induced hedging. At the individual option level, these effects are proxied by:

$$\gamma = \frac{\partial\delta}{\partial S} \qquad \theta' = \frac{\partial\delta}{\partial T}$$
$$\lambda' = \frac{\partial\delta}{\partial\sigma} \qquad \phi' = \frac{\partial\delta}{\partial r} \tag{5}$$

The differential between borrowing and lending costs in a TC economy can also cause a TC effect on option prices. Buying an option is equivalent to buying δ units of S and borrowing B dollars so that:

$$C = \delta S - B \tag{6}$$

As C increases in [6], it is easily shown that B decreases. Hence, buying a more expensive option is somewhat similar to giving up part of a loan. Thus, if borrowing is more expensive then lending, more expensive options would be less desirable to hold in a TC economy than in a no TC economy.

IV.2.2 Indirect Transaction Costs Effects

Most OPM's derive option prices in no TC economies on the basis of arbitrage arguments, where a risk-free hedged portfolio is formed containing the option, a fraction ($-\delta$) of the underlying security (US) and the risk free asset. Since this portofio is risk free, the return on the option must be a function of the return on the US. It follows that (e.g., see Cox and Rubinstein [1985]):

Expected rate of return of an option - riskless interest rate
$= \Omega$ (expected rate of return of US - riskless interest rate).

In the context of the Sharpe-Lintner Capital Asset Pricing Model, the relationship is given by:

Beta of an option = Ω (beta of the US).

In the context of Ross's Arbitrage pricing theory:

Beta_i of an option = Ω (beta_i of the US),

where i refers to the i^{th} risk factor, and $\Omega = \delta\ S/C$ is the elasticity of the option.

As noted earlier, the own risk of a security may also be priced in a TC economy. Unexpected changes in an option's price (and thus its own risk) may be caused by unexpected changes in S, σ, and r. While the risk related to S can be hedged with the underlying security, the risk related to σ can only be hedged with other options, and the risk related to r may be hedged with some debt instrument. All these types of hedges require additional TC's. The net (excluding financial intermediaries) market demand for options would usually be such that the FIP is not "naturally" neutral for σ. The financial intermediary may then pay a premium to make its position neutral, or it may accept the risk and demand a reward for bearing this risk. The measures of risk related to σ and r are defined as:

$$\lambda = \partial C/\partial \sigma \quad \text{and} \quad \phi = \partial C/\partial r \qquad [7]$$

Ceteris paribus, the value of an option decreases with the passage of time, so that:

$$\theta = \frac{\partial C}{\partial t} < 0$$

A hedged portfolio is compensated for this deterioration in value by gaining from any upward or downward movement in S. To illustrate, suppose that the value of the underlying security increases so that δ increases from say δ_1 to δ_2. Since the investor is only δ_1 units short of the underlying security, the investor will profit since the gain on the option will more than cover the loss on the short position. Similarly, if the value of the underlying security drops, the gain on the short position will more than offset the loss on the option. The amount of the gain is proportional to the rate of change of δ with respect to S (i.e., to Υ).

In a world where rehedging is continuous and costless, the gains due to Υ and the losses due to θ are such that the return on the hedged portfolio used to derive the theoretical option price is riskless. Such is not the case in a TC economy.

When rehedging is only done at discrete intervals (Δt), Boyle and Emanuel [1980] show that the return on a hedge portfolio, which is long a call option, short $\partial C/\partial S$ of the underlying security and with ($S\ \partial C/\partial S - C$) invested in a risk-free security is:

$$HR = [\partial C/\partial t + (1/2)\ \partial^2 C/\partial S^2\ \sigma^2 S^2 u^2 + r(S\ \partial C/\partial S - C)]\Delta t + O(\Delta t^{3/2}) \qquad [8]$$

where: u is a random variable drawn from a random distribution with zero mean and unit variance.

If higher-order terms in [8] are ignored, [8] becomes:

$$HR = (1/2)\sigma^2 S^2\ \ \partial^2 C/\partial S^2\ \ (u^2-1)\ \Delta t. \qquad [9]$$

In the context of options on forwards or futures, [8] becomes:

$$HR = [\partial C/\partial t + (1/2)\ \partial^2 C/\partial S^2\ \sigma^2 S^2 u^2 - rC]\Delta t + O(\Delta t^{3/2}). \qquad [10]$$

because the proceeds from selling a fraction of a futures contract short are nil.

Replacing the variables in [10] with the variables defined earlier yields:

$$\begin{aligned} HR &= [\ -\theta + \Upsilon(\sigma^2 F^2/2)\ u^2 - rC]\ \Delta t + O(\Delta t^{3/2}) \\ &\approx [\ rC - \Upsilon(\sigma^2 F^2/2) + \Upsilon(\sigma^2 F^2/2)\ u^2\ \ - rC]\ \Delta t \\ &\approx (\sigma^2 F^2/2)\ \Upsilon(u^2-1)\ \Delta t. \qquad [11] \end{aligned}$$

Since u is drawn from a standard normal distribution, HR will have an expected value of zero and will be negative approximately 68% of the time. Hence, the decrease in the value of the option with the passage of time will more than offset the benefits of Υ 68 percent of the time. For the other 32% of the time when there are larger price movements in the underlying security, the gains are, on average, more substantial.

If one hypothesizes that financial intermediaries are, on average, short on options, then they would gain 68% of the time.[3] If financial intermediaries believe that they have better access to the markets, so that they can react before large price swings take place, these financial intermediaries would rather write high Υ options. As a result, options with higher Υ's would be cheaper than would be expected in a no TC market. On the other hand, if financial intermediaries are overly cautious about the market (i.e., if they fear being unable to react before large price swings), higher Υ options may become more expensive. Thus, while it may be hard to assess the exact mechanism through which Υ affects the price of an option, or whether its positive or negative aspects will prevail, Υ is probably an important factor in option pricing.

3. This is especially likely in the foreign exchange markets where there is a demand for options for hedging purposes.

IV.3 Simulations for Υ

Υ appears to be one of the most important factors discussed above.[4] Since the biases found in the literature were related to moneyness, to the volatility of the underlying security, and to time to maturity, the relationship between Υ and each of these variables is examined.

Figure 1 illustrates the relationship between Υ and the moneyness of a call option.[5] For in-the-money calls (out-of-the-money puts), Υ always decreases as the call is more in (out-of) -the-money. Except for very short maturity options, Υ does not drop significantly as the option reaches 10 to 15 % out-of-the-money. Exchange-traded options, especially those with short maturities, are seldom more than 15 % out-of-the-money. If Υ is assumed to be the principal factor used to proxy the price effects of transaction costs for an option, the following observations result.

The moneyness bias implied by Υ in Figure 1 is the same as that found by Black [1975], Blomeyer and Klemkosky [1983], and Bodurtha and Courtadon [1987] if financial intermediaries (i.e., the price setters)

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4. This is supported by the empirical results presented below.

5. The parameters used for the figures are the same as those used by Cox and Rubinstein (1985) in their simulations.

would rather write low Υ options.[6] In order to entice financial intermediaries to write high Υ options, investors would have to give them a premium. Conversely, if financial intermediaires would rather write high Υ options, Figure 1 would imply the moneyness bias found by MacBeth and Merville [1979,1980] and by Whaley [1986]. Thus, the contradictory biases found by the various studies, and the bias reversals observed by Rubinstein [1985], could be explained if the net position (long or short) of financial intermediaries changes over time, or if their confidence in their capacity to react quickly to large price changes, changes over time. Such changes in the net demand for options by end users (excluding financial intermediaries) and in market psychology are not implausible.

In Figure 2, Υ decreases with increasing volatility as long as the annual volatility exceeds about 0.15. This could explain the bias found by Black and Scholes [1972] if financial intermediaries preferred writing low Υ options during the period of their study. Their maturity bias can also be explained with Υ. In Figure 3, Υ decreases as time to maturity increases, except for deeply in- (or out-) of-the-money options with very short maturities. Consistency with their other results would require short maturity options to be overpriced. The opposite results found by Brenner and Galai [1981] can also be explained by Figure 3. Such results would occur, if during the period

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6. Based on the assumption that financial intermediaries are, on average, short on options (i.e., their FIP has a negative Υ). A parallel argument can be made if financial intermediaries are assumed to be long on options.

studied, financial intermediaries preferred, on average, to write high Υ options. The decrease of Υ with time to maturity could also explain why Bodurtha and Courtadon [1987] found that mispricing decreased as time to maturity increased. The relationships between Υ and each of the biases reviewed above are summarized in exhibit 1.

A potential problem with Υ for very short maturity options is depicted in Figure 4. For example, the maximum Υ of the three month options in Figure 1 is 0.056, while Υ can increase by 0.065 (i.e., from 0.294 to 0.359) when the maturity of the option goes from 3 days to 2 days. As the volatility decreases, these changes become even more important. Thus, to avoid the empirical problems associated with intraday changes of Υ, options maturing within short periods (e.g., a week) need to be excluded from empirical analysis.

IV.4 <u>The Model</u>

As discussed earlier, Garman and Ohlson [1981] show that the prices in a TC economy ($\hat{P}_j$) are related to those in a no TC economy (P_j) as follows:

$$\hat{P}_j = P_j + \varepsilon_j \qquad [4]$$

Thus, all the factors which are directly or indirectly related to transaction costs, are determinants of ε_j. If TC's are linear in volume traded, and if the "risk premia" increase linearly with Υ, λ and

ϕ, a linear model would be most appropriate. But there is no compelling reason why these risk premia would be linear. For example, the optimal rehedging interval, Δt, used by financial intermediaries may be different when $\gamma_1 = \gamma_0$ from when $\gamma_2 = 10\ \gamma_0$. Hence, the impact of γ_2 may not be ten times larger than the impact of γ_1. Thus, a log-linear model may be more appropriate. Unfortunately, with a logarithmic specification, the interpretation of the estimated intercept is not possible. Therefore, a square root model is also evaluated.

The full model, incorporating a linear specification of the determinants of ϵ_j in [4], is given by:

$$C_m = C_t + (\beta_0 + \beta_1 C_t + \beta_2 \delta \cdot \beta_3 \gamma + \beta_4 \lambda + \beta_5 \lambda' + \beta_6 \theta + \beta_7 \theta' + \beta_8 \phi + \beta_9 \phi') + \omega$$

or:

$$C_m = \beta_0 + \beta_1' C_t + \beta_2 \delta + \beta_3 \gamma + \beta_4 \lambda + \beta_5 \lambda' + \beta_6 \theta + \beta_7 \theta' + \beta_8 \phi + \beta_9 \phi' + \omega \qquad [12]$$

where: C_m is the market value of the option;
C_t is the theoretical price of the option in a transactions-costs-free economy;
β_i are parameters to be estimated;
ω is an error term, and all the other variables have been defined previously.

Log-linear and square root versions of a reduced form of [12] were also tested.

C_t was proxied by Black's [1976] no TC OPM, which is given by:

$$C_t = e^{-rT}[F.N(d_1) - X.N(d_2)]$$

where: $d_1 = \dfrac{\ln(F/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$;

$d_2 = d_1 - \sigma\sqrt{T}$;

$N(.)$ = is the cumulative standard normal probability distribution;

F = futures price;

X = exercise price;

σ = volatility (daily);

T = time to maturity (in days); and

r = risk-free interest rate (per day).

There are three reasons for using this model. First, according to Ramaswamy and Sundaresan [1985], this model is a useful approximation of the value of American options on futures[7]. Second, more than 90 % of the options in our sample were between 5 % out-of-the-money and 7.5 % in-the-money. Third, transaction costs greatly reduce any early exercise premiums attached to American options.

Black's model was preferred to other European models because it has the fewest parameters to estimate, and no other European model is superior a priori.[8]

An American OPM was not used because it would have made it

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7. In their study, Ramaswamy and Sundaresan consider options on SP 500 futures and assume a 5 % dividend yield. This is similar to the interest rates on the foreign currencies studied herein.

8. The leverage effects or the bankruptcy risks which could justify the use of stochastic processes other than a geometric Brownian motion (e.g., a constant elasticity of variance) do not apply to the currencies of major industrial countries.

prohibitive to calculate implied volatilities using numerical methods for a sample with more than 175,000 observations. The huge increase in computation costs did not seem justified given the very small potential gain in accuracy.

The theoretical value of a put was obtained using the put-call parity equation for European options on futures. Specifically:

$$C = P + (F - X)\ e^{-rt}$$

where C and P are the prices of a call and a put, respectively.

V. Data and Methodology

V.1 Data

The raw data for this study are drawn from the "Quote Capture Report" of the Chicago Mercantile Exchange (CME). The data bank covers all the transactions, bids and offers, on all the FX futures and options traded on the CME between February 1, 1986 and March 31, 1987. Options on the German mark (DM), the Swiss franc (SF) and the British Pound (BP) were traded during the entire period. Options on the Japanese Yen (JY) started to trade in March 1986, while options on the Canadian dollar (CD) only started to trade in June 1986. Only the data relating to actual transactions was kept. For each transaction, the date, the time to the nearest second, the maturity and the price of the contract were available. The exercise price and the nature (put or call) of each option was also specified. The yields on T-Bills maturing closest to the maturity of the option contract, in most cases within a day, as reported in the Wall Street Journal, were used. To avoid the problems associated with intraday changes of Υ, the data for the days when options with fewer than seven days to maturity were traded were removed from the data set.[9]

Each option transaction was matched with the transaction on the underlying futures which immediately preceded it. The maximum time interval allowed between matching transactions was 30 minutes. The resulting data set contained 176,166 observations, with 81,105 for the DM, 46,887 for the SF, 34,216 for the JY, 11,919 for the BP and 2,039 for the CD.[10] The mean time interval between matching transactions (dt) is 66.5 seconds and the median is less than 20 seconds. In more than 75% on the cases, dt does not exceed 1 minute. This matching of transactions should, for all practical purposes, eliminate any non-simultaneity bias.

Since Whaley [1986] found that the moneyness biases of puts and

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9. The time to maturity of the options is measured in days. Thus, an option traded at the opening of the trading day is assumed to have the same time to maturity as an option traded at the close of the trading day, provided both options have the same maturity date. This approximation does not cause major difficulties in most instances. However, it induces large errors in estimating the Υ of options with very short maturities, as discussed in Section IV.3. Moreover, the heaviest transaction volume is usually for options with the shortest maturity which are near the money. Thus, if the options with the shortest maturity are eliminated from the data for a given day, then all the data for that day should be eliminated to alleviate potential biases.

10. Puts represent about 35 percent of all DM and SF option transactions, about 50 percent of all JY and BP option transactions, and about 21 percent of all CD option transactions.

calls are of opposite sign, a variable, M, in addition to the standard definition of moneyness, M_1, is used. They are given by:

$$M_1 = (F-X)(-PC) / X. \qquad [13]$$

where PC = 0 for calls, and 1 for puts.

$$M = (F-X) / X. \qquad [14]$$

Based on the detailed statistics given for this data set in Table 1, M ranges from -0.25 to 0.37 for the DM (mean of 0.006). This reflects the sharp drop of the dollar vs the DM during the studied period. The range for M for the SF is fairly similar to that for the DM. The range is narrower for the JY and the BP, -0.17 to 0.15 and 0.13, respectively. This reflects the greater stability of these currencies versus the dollar. The distribution of M_1 for the CD is almost symmetrical.

The true moneyness of the options, M_1, indicates that options were, on average, slightly in-the-money. More than 90% of the options have a moneyness of between -0.05 and 0.075. Most of the statistics were similar for the DM and the SF, and for the JY and the BP.

V.2 Methodology

The estimation of eq.[12] requires the estimation of 11 parameters, namely, σ and β_0 to β_9. Since all of its variables are computed using σ, equation [12] must be estimated using a non-linear regression procedure.[11]

Non-linear regression procedures require starting values for the estimation of the parameters. To minimize the computations required to estimate equation [12] and to reduce the risk of finding local minima, finding starting values close to the global minimum is desirable.

A three step estimation procedure was used. First, a non-linear procedure was used to estimate an implied starting value for σ from the Black model. Since the error term for this model may not be white noise, this initial starting value estimate of Υ may be biased.

Second, using this estimate, a linear regression for eq.[12] was run assuming σ was known and fixed. Since the market value of options should be constrained to be positive, this estimation should theoretically use a Tobit rather than an Ordinary Least Squares (OLS) procedure. Since no fitted market values estimated in the pilot study

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11. Four non-linear regression methods, which minimize the sum of the squared errors (SSE), were used initially. The "steepest descent" method varies the parameter estimates in the direction of steepest descent of SSE. The "Gauss-Newton" method uses a truncated Taylor series to minimize the SSE. The Marquardt [1963] method is a compromise between the Gauss-Newton and the steepest descent methods. Finally, the "DUD" (Ralston and Jennrich [23]) method is like the Gauss Newton method, except that its derivatives are estimated from the history of iterations, rather than being supplied analytically. All of these four methods are available on the statistical package SAS. In the preliminary analyses, all of the methods yielded fairly similar results. However, since DUD converged most often and was the least sensitive to the initial values provided in the procedure, only the results using DUD are reported below.

were negative, the Tobit procedure was not used.[12]

Third, using the estimate of σ in step 1 and of β_0 to β_9 from step two as initial values, a non-linear regression was run to get the final estimates of the eleven parameters.

To check whether the various biases were removed from the error term, the following linear regressions were run:

$$\omega = \beta_0' + \beta_1' C_t + \beta_2' M + \beta_3' T + \beta_4' PC + \varepsilon' \quad [15]$$

$$\omega = \beta_0'' + \beta_1'' C_t + \beta_2'' M_1 + \beta_3'' T + \beta_4'' PC + \varepsilon'' \quad [16]$$

where ω is the residual error from the model.

These regressions were run to determine whether all the moneyness and maturity biases were eliminated, and whether systematic differences existed for the pricing of calls and puts. If biases remain in either equation [15] or [16], then factors other than the impact of transaction costs (as specified) have a significant effect on option prices.

In the pilot study, the third-step estimates always improved relative to the second-step estimates in that they reduced the SSE and they tended to decrease whatever bias existed. When the step-two

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12. A pilot study was conducted on all options on DM futures which were traded on the CME during November and December, 1984. The sample contained 7,553 observations.

residuals exhibited no bias, neither did the step-three residuals. Thus, when the results from step two were satisfactory, step three was not used. A single non-linear regression of eq.[12] for one month of data for one currency took up to eight hours of CPU time on a VAX 11/785 computer!

VI. Empirical Findings

VI.1 Full Model Estimations Using Data Pooled Monthly

Ideally, empirical tests should be conducted on a daily basis, since the volatility of a currency may change from day-to-day. Unfortunately, problems arise when daily estimation is attempted because very few series of options (i.e., options with different exercise prices and/or different maturities) are traded on any given day. Since options from the same series may differ in price by a few hundredths of a cent, while options from different series may differ in price by several cents, and since several variables are highly correlated (see Table 2), the daily data do not contain enough dispersion for the precise estimation of the parameters. To resolve these problems, the data were pooled on a monthly basis. For the pooled regressions, the volatility was estimated daily, while the other parameters were estimated monthly. The underlying rationale was the belief that volatility is more variable than the parameters of the other variables.

The daily volatilities ranged between 0.00153 for the CD on

November 19, 1986, to 0.00915 for the DM on April 23, 1986 (see Table 3). The standard errors of the volatility estimates are of the order of 5×10^{-5}.

Estimations of equations [15] and [16], using residuals from the first step of the empirical procedure, revealed the presence of significant biases for all but two months for the CD (months where the sample sizes were extremely small). The percentage of the variance in the error term (R^2) explained by C_t, M or M_1, T and PC are given in Table 4. Generally, M resulted in a higher R^2 than M_1. This supports the finding of Whaley [1986] that the moneyness biases of puts and calls on futures are of opposite sign, when models are used that assume the absence of transaction costs.

Eq.[12] was then estimated and its residuals were tested for bias. The root mean squared error (RMSE) of the model, and the R^2 and the significance of the regression are presented in Table 5. Statistically significant biases remain for:

February 1986 for DM and SF,
April 1986 for SF,
May 1986 for DM, SF and BP,
August 1986 for DM, JY and BP,
October 1986 for DM,
November 1986 for DM, SF and JY, and
February 1987 for DM, SF, JY and CD.

A number of tests were then run to determine the source of these biases. First, since the difference in value between American and European options is largest for deep-in-the-money options, options which were more than 15 percent in-the-money, were eliminated from the estimations. This had no effect on the biases. Second, because of the discrete nature of option prices, the biases could be attributable to very cheap options. Option prices are quoted in cents per currency unit, and the smallest tick is 0.01 cents (except for the BP were it is 0.05 cents). Thus, options with a market value of less than 0.05 cents, then 0.10 cents, were removed from the estimations. This had no effect on the observed biases.

Because of the very large sample sizes (up to 9,844 observations), even small R^2 may be statistically significant in Table 5. For example, the statistically significant bias regressions for the months of April 1986 (SF) and October 1986 (DM) have extremely small R^2 values of 0.0023 and 0.0057, respectively.

All of the other months with biases (namely, February, May, August and November 1986 and November 1987), precede option maturity dates (referred to herein as pre-maturity months). In other words, short maturity options are traded during these months. Thus, eliminating options with only seven or less days to maturity may not be sufficient to remove the problem of intra-day changes in the parameters, or other difficulties which may be associated with short maturity options.

VI.2 Reduced Model Estimations Using Data Pooled Monthly

While the use of the full model removed all the biases except for some pre-maturity months, the estimation of the parameters was not satisfactory because of multicollinearity between the independent variables. As is illustrated with the July 1986 correlation matrix for the DM variables (see Table 6), the correlations between the variables are typically high. A good estimation of the parameters is necessary in order to infer the relative importance of each factor and to understand how the market actually values options.

The principal criteria used in the selection of variables for the reduced form model were the frequency with which a variable was statistically significant, and its degree of correlation with the other variables. The first reduced form model that was estimated was:

$$C_m - C_t = \beta_0 + \beta_1 C_t + \beta_2 \delta + \beta_3 \gamma + \beta_4 \lambda + \beta_5 \theta + \beta_6 \phi + \omega \qquad [17]$$

Based on the results presented in Table 5, the model's explanatory power (as measured by the RMSE) is very similar to that of the full model. Any biases that were found in the full model were generally slightly amplified.

When options with up to 21 days to maturity were eliminated, about one-half of the biases in the pre-maturity months were eliminated.[13]

13. Rubinstein [1985] eliminated all options maturing within 21 days.

This supports the conjecture that pricing problems are associated with very short maturity options.

The parameter estimates and the standardized parameters for eq.[17] are presented in *Table 7*.[14] The parameters of Υ and θ are usually the most significant economically (i.e., they usually have the largest standardized estimates), and they appear to move in opposite directions. Specifically, when the parameter of Υ is large (algebraically), θ's parameter is small. The reason may be that Υ and θ are highly correlated. To illustrate, the average correlation between Υ and θ is 0.935 for the DM and 0.928 for the SF (see Table 8). Due to this high correlation, any non-linear estimation of eq.[17] could not converge.

Since Υ appears to be more economically significant than θ, a series of runs was conducted on the following more reduced model:

$$C_m - C_t = \beta_0 + \beta_1 C_t + \beta_2 \delta + \beta_3 \Upsilon + \beta_4 \lambda + \beta_5 \phi + \varepsilon \qquad [18]$$

The results of this estimation are presented in Table 5, and the parameter estimates and their standardized estimates are presented in Table 9. Although the estimated model for eq.[18] has a very similar explanatory power to that for eq.[17], the parameter estimates of Υ are significantly different. These results may indicate that θ is

14. Only options maturing within 7 days were eliminated in the sample used for these estimations.

unimportant, and that its relative importance in eq.[17] may have been due to its correlation with Υ and the well-known problems associated with multicollinearity. On the other hand, θ may actually be important, but its removal does not decrease the explanatory power of eq.[18] because all of its impact is captured by Υ. Since no unambiguous conclusion is possible about θ's importance, the results of both models have been presented.

Some multicollinearity remains among the independent variables for even the most reduced model as can be seen from the condition numbers (the square root of the ratio of the largest eigenvalue of the matrix of the independent variables to its smallest eigenvalue) presented in Table 9. They range from 9.8 for the JY in February 1987 to 40.9 for the CD in June 1986. Thus, the percentage of the variance in the independent variables explained by the smallest eigenvalue is between 1 and 0.06 percent of that explained by the largest eigenvalue. In most cases, only the first eigenvalue was larger than one.

The condition numbers exhibit a quarterly pattern; namely, they are usually largest on the maturity months and smallest on the pre-maturity months. During maturity months beyond the maturity dates, the closest maturity options expire after about three months. Few transactions occur for options with longer maturities. For the pre-maturity months, series of options maturing within one and four months are fairly heavily traded. Unfortunately, the months with the smallest condition numbers (i.e., the pre-maturity months) exhibit the problems associated with short maturity options.

If the criterion for considering a bias significant is taken to be a statistically significant regression and an R^2 of at least 1 percent,[15] the number of non-pre-maturity months with biases is reduced to 3,2,0,2 and 1 for the DM, SF, JY, BP and CD, respectively.[16] These biases are not necessarily due to a misspecification of the model. Since the parameters of the model are estimated monthly, these biases may be due to within month non-stationarity in the parameters.

VI.3 Reduced-Form Linear Model Estimations Using Data Pooled Daily

With the reduced number of parameters in eq.[18], daily estimation is possible, but feasible only for the DM and the SF. For the other currencies, the number of daily transactions and the number of option series traded daily are too small.

Of the 170 days in the non-pre-maturity months, only six and five days exhibited bias at the 5% level of significance for the DM and the SF, respectively (see Table 10). Since 5 percent of the days would be

15. This criterion is used to avoid the somewhat artificial situation where a bias regression is statistically significant (due to large sample size) although its variables explain less than 1 % of the variance in the residuals.

16. All currencies have nine non-pre-maturity months in the sample except for the CD which has seven.

expected to be significant at the 5% level (i.e., 8.5 days) if all the residuals were random, the observed results could be attributed to chance. However, such is not the case since several of these biases are significant at the 0.01% level, and the days with bias occur during the same periods (April 1986 and January 1987) for the DM and the SF. These biases may be related to real economic events. The standard deviations of the daily DM and SF exchange rates were the largest during these time periods (see Table 11).

The problems caused by the short maturity options are clearly depicted in the lower part of Table 10. The 39 and 15 days in the pre-maturity months which had bias in their residuals for the DM and the SF are listed in the table.

The daily estimates of the parameters of model [18] are presented in Tables 12 and 13 for the DM and the SF, respectively, and can be summarized as follows. First, the daily estimates are non stationary, which could explain at least some of the bias remaining in the monthly estimates. Second, the first-order autocorrelations of the parameter estimates given in Table 14 seem to be quite high. This indicates that the changes are not random but result from market pressures which evolve relatively slowly. These autocorrelations are highest (above 0.7) for λ, Υ and the intercept. Third, the parameter estimates are highly correlated (see Table 14), which suggests that the DM and the SF follow the same pattern vis-a-vis the US dollar. Fourth, the estimated parameter for Υ has a positive value, or is close to zero for relatively prolonged periods during two time intervals; namely, from the end of March to the beginning of April 1986, and January 1987. The uncertainty about exchange rates was at its peak during these time periods. This supports the conjecture that "financial intermediaries" demand a positive premium for Υ only when market uncertainty is high.[17]

The importance of a variable can be determined from the product of its standard deviation and its estimated coefficient (β'). This statistic is similar to a standardized β, except that it is not divided by the standard deviation of the dependent variable. The β', which are reported in Tables 15 and 16, provide the actual importance of a variable in terms of dollar and cents. The β' for gamma for the DM on 870324 is -0.0311, which indicates that an increase of one standard deviation in gamma would decrease the option price by approximately 0.03 cents per DM. This β' is 4.3 percent of the median market value of 0.72 cents for a DM option (see Table 1).

Based on Tables 15 and 16, the most important variable is Υ, followed by λ ($=\partial C/\partial \sigma$), ϕ ($=\partial C/\partial r$), C_t (the theoretical no TC option value), and δ ($=\partial C/\partial F$). The means of the absolute values of the β's are:

	Ct	δ	Υ	λ	ϕ
DM	0.0075	0.0026	0.0190	0.0155	0.0080
SF	0.0084	0.0035	0.0200	0.0179	0.0091

17. Alternatively, the direct impact of TC for rehedging as measured by Υ may more than offset the "positive" feature of Υ when markets are volatile.

The mean values of the intercepts are 0.071 cents for the DM, and 0.083 cents for the SF. The most important variables, Υ and λ, have high first-order autocorrelation, and cross-currency correlations. As expected, the risk associated with changes in the volatility of the underlying security is more important than the risk associated with changes in interest rates.

Thus, transaction costs may affect option prices indirectly through: (1) Υ, which is negative or positive depending on whether "financial intermediaries" are confident about their timing abilities; (2) λ and ϕ, which are measures of the own-risk of options; and (3) C_t, the no TC theoretical option price. The parameter of C_t is usually negative. The reason may be that, since borrowing and lending do not occur at the same interest rate, more expensive options are less desirable to hold, or, conversely, more desirable to write. (4) The intercept is positive in more than 99 % of the cases indicating that financial intermediaries incur an initial fixed TC to write any type of option.

VI.4 Non-Linear Estimations of Reduced Form Models Using Data Pooled Daily

As discussed earlier, the linear form of the model may not be appropriate if the actual effects of the variables on the fudge factor are non-linear. The problem can be expected to be especially acute for very short maturity options since Υ, λ and ϕ can have quite different values for short and long maturity options (for example, Υ may vary between 0.01 and 0.40). The log-linear version of model [18] was specified as:[18]

$$C_m = \beta_0 + \beta_1 C_t + \beta_2 \delta + \beta_3 \ln(\Upsilon) + \beta_4 \ln(\lambda) + \beta_5 \ln(-\phi) + \varepsilon \qquad [19]$$

For this model, biases remained in the residuals for only three days (in pre-maturity months) for the DM (86 02 26, 86 05 12 and 86 08 29). The significance of these bias regressions are 0.0423, 0.0121 and 0.0117, respectively. The rejection of the null hypothesis of no bias at the 5 % level three times in more than 200 tests is, however, hardly significant.[19]

Thus, a log-linear model explains all the systematic deviations between market and theoretical option values. Unfortunately, the estimated regression coefficients of the log-linear model are very hard to interpret economically.

The following square root, reduced-form model was also estimated for the days when the linear model did not eliminate the biases:

$$C_m = \beta_0 + \beta_1 Ct + \beta_2 \delta + \beta_3 \sqrt{\Upsilon} + \beta_4 \sqrt{\lambda} + \beta_5 \sqrt{-\phi} + \varepsilon \qquad [20]$$

---

18. A negative sign was attached to ϕ because it is always negative and the logarithm of a negative value is undefined.

19. The non linear regression procedure did not converge for the DM on 86 02 24.

Model [20] exhibited less biases than the linear model but more biases than the log-linear. Specifically bias remained for 7 days for the DM (namely, 86 02 12/13/24/25/27/28 and 86 11 21), and for 8 days for the SF (namely, 86 02 12/13/18/21/24/25/27 and 86 03 18) for pre-maturity months. Since the log-linear model was most successful in bias reduction, the appropriate power of the model is probably closer to 0 than 1.

VII. Concluding Remarks

The biases identified in the option pricing literature were shown to move with the second derivative of the option price with respect to the price of the underlying security. A model was proposed to account for the effect of transaction costs on option pricing based on the valuation framework provided by Garman and Ohlson [1981] for risky assets in arbitrage-free economies with transaction costs. The direct effects of transaction costs were incorporated through the costs of hedging and rehedging, and the indirect effects through measures of the own risk of options and option portfolios, and through the price of the option which acts as a proxy for the differential between borrowing and lending interest rates.

The model was estimated on data pooled monthly for options on the futures of five currencies, and pooled daily for two currencies. The monthly estimations indicated that Υ and the time decay of options (θ) are the most important transaction-costs-related variables affecting option pricing. Because of the very high correlations between Υ and θ, it was impossible to ascertain whether θ is really important by itself. Estimation on a daily basis for the DM and the SF confirmed that Υ is the most significant variable, followed by λ and ϕ which measure the sensitivity of an option's price to the volatility of its underlying security, and to interest rates, respectively. Thus, the effect of transactions costs may be mostly indirect through its impact on own risk.

The only biases which could not be explained by the linear model were those for short maturity options. In contrast, a square-root model specification exhibited less biases, and a log-linear model specification exhibited no more bias than would be expected by chance.

APPENDIX

$$\delta = \frac{\partial Ct}{\partial F} = e^{-rT}\,(N(d_1) - PC)$$

$$\Upsilon = \frac{\partial^2 Ct}{\partial F^2} = \frac{e^{-rT}}{F\sigma\sqrt{T}}\,f(d_1)$$

where f(.) is the standard normal density function.

$$\lambda = \frac{\partial Ct}{\partial \sigma} = F^2\,\sigma T\Upsilon$$

$$\lambda' = \frac{\partial \delta}{\partial \sigma} = \frac{\partial^2 Ct}{\partial F \partial \sigma} = e^{-rT}\,f(d1)\,\left(\frac{-\ln F/X}{\sigma^2\sqrt{T}} + \frac{\sqrt{T}}{2}\right)$$

$$\theta = \frac{\partial Ct}{\partial T} = -rCt + \Upsilon\,\frac{F^2\sigma^2}{2} - r\,(X-F)\,e^{-rT}\,(PC)$$

$$\theta' = \frac{\partial \delta}{\partial T} = \frac{\partial^2 Ct}{\partial F \partial T} = -r\delta - \frac{e^{-rT}}{2T}\,f(d_1)d_2$$

$$\phi = \frac{\partial Ct}{\partial r} = -TC - T\,(X-F)\,e^{-rt}\,(PC)$$

$$\phi' = \frac{\partial \delta}{\partial r} = \frac{\partial^2 Ct}{\partial F \partial r} = -T\delta$$

EXHIBIT 1

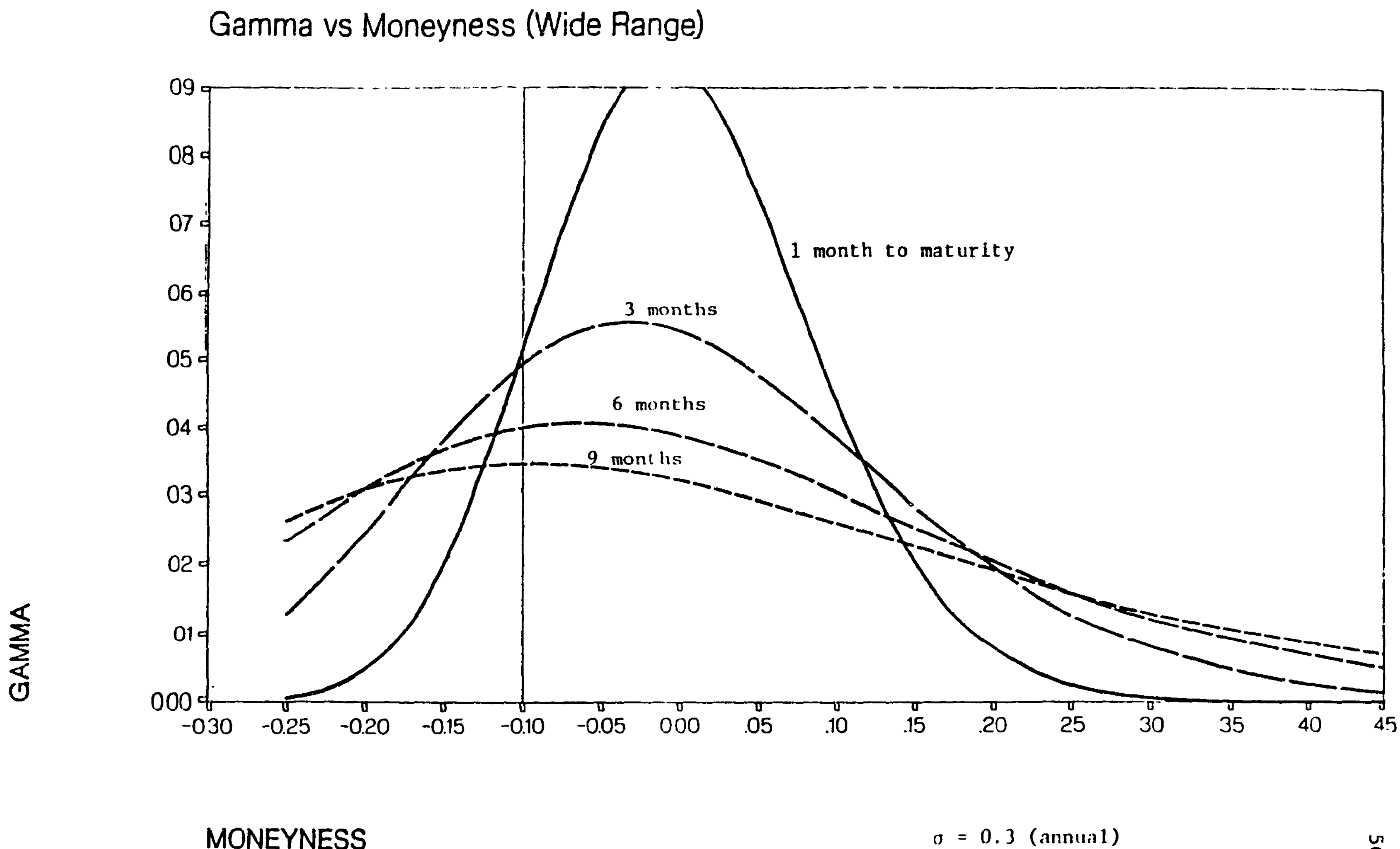
Biases in the Literature and Υ'

Black and Scholes	[1972]	High volatility (low Υ)	underpriced**
Black	[1975]	In-the-money (low Υ) Short maturity (high Υ)	underpriced overpriced
Blomeyer and Klemkosky	[1983]	Out-of-the-money (high Υ)	overpriced
Brennan and Schwartz	[1977]	High volatility puts (low Υ)	underpriced
Bodurtha and Courtadon	[1987]	Out-of-the-money (high Υ)	overpriced
Brenner and Galai	[1981]	Long maturity (low Υ)	overpriced
MacBeth and Merville	[1979, 1980]	In-the-money (low Υ)***	overpriced
Whaley	[1986]	Calls-in-the-money (low Υ) Puts-out-of-the-money (low Υ)	overpriced

* Unless specified otherwise, these biases refer to call prices.

** Market prices are below OPM prices.

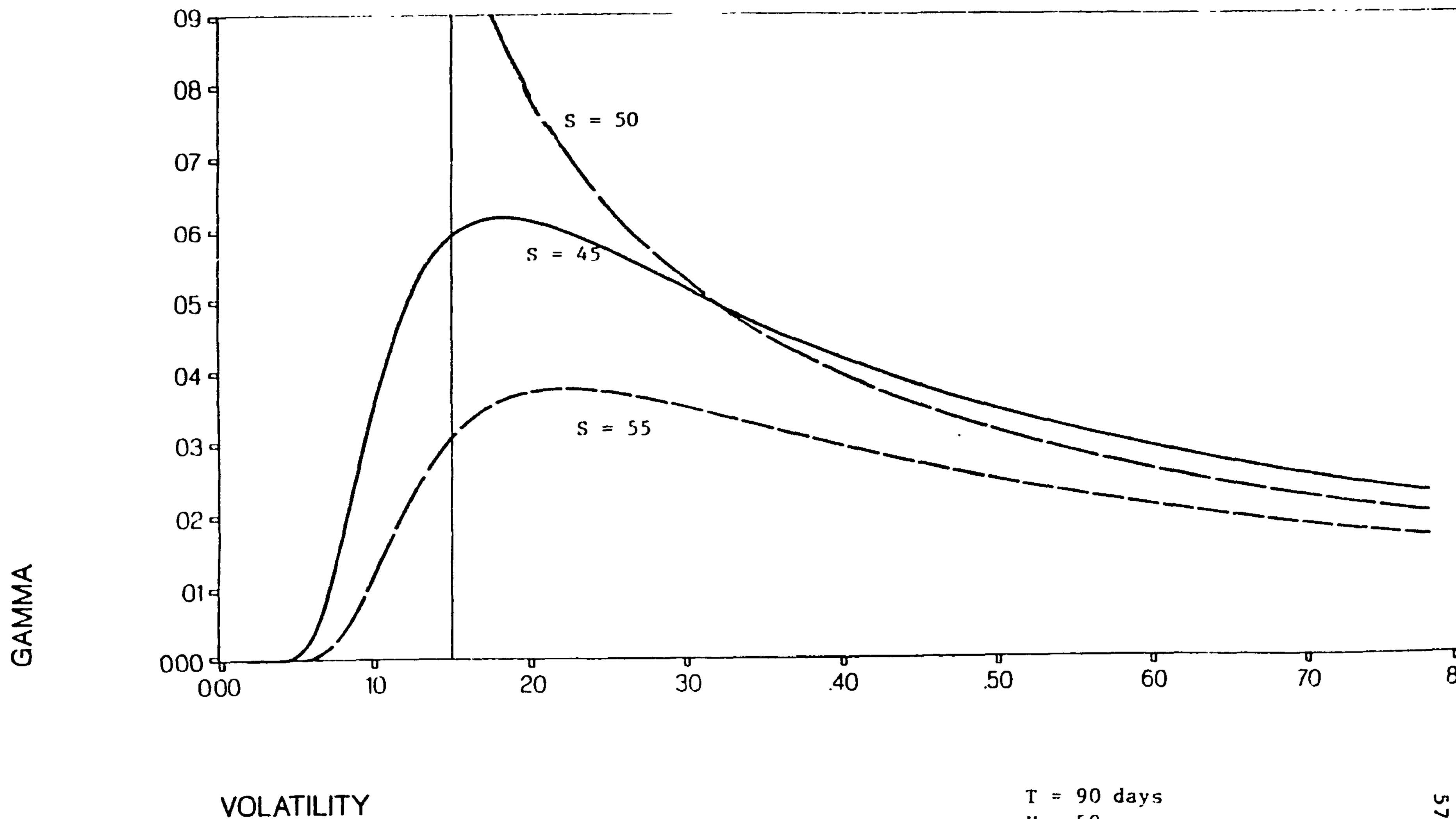
*** Error decreases as time to maturity decreases.



σ = 0.3 (annual)
X = 50
i = 6 %

FIGURE 1

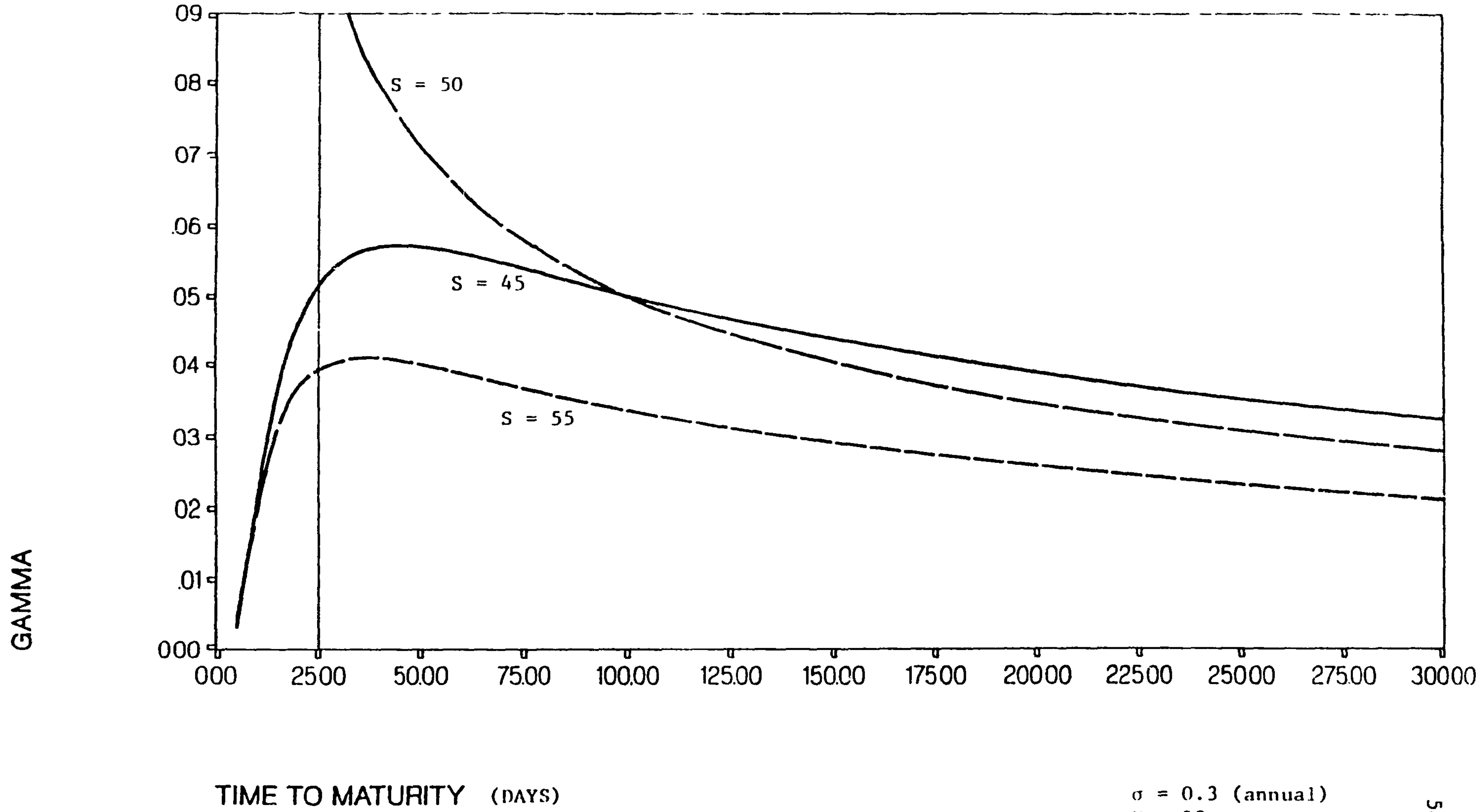
Gamma vs Sigma



T = 90 days
X = 50
i = 6 %

FIGURE 2

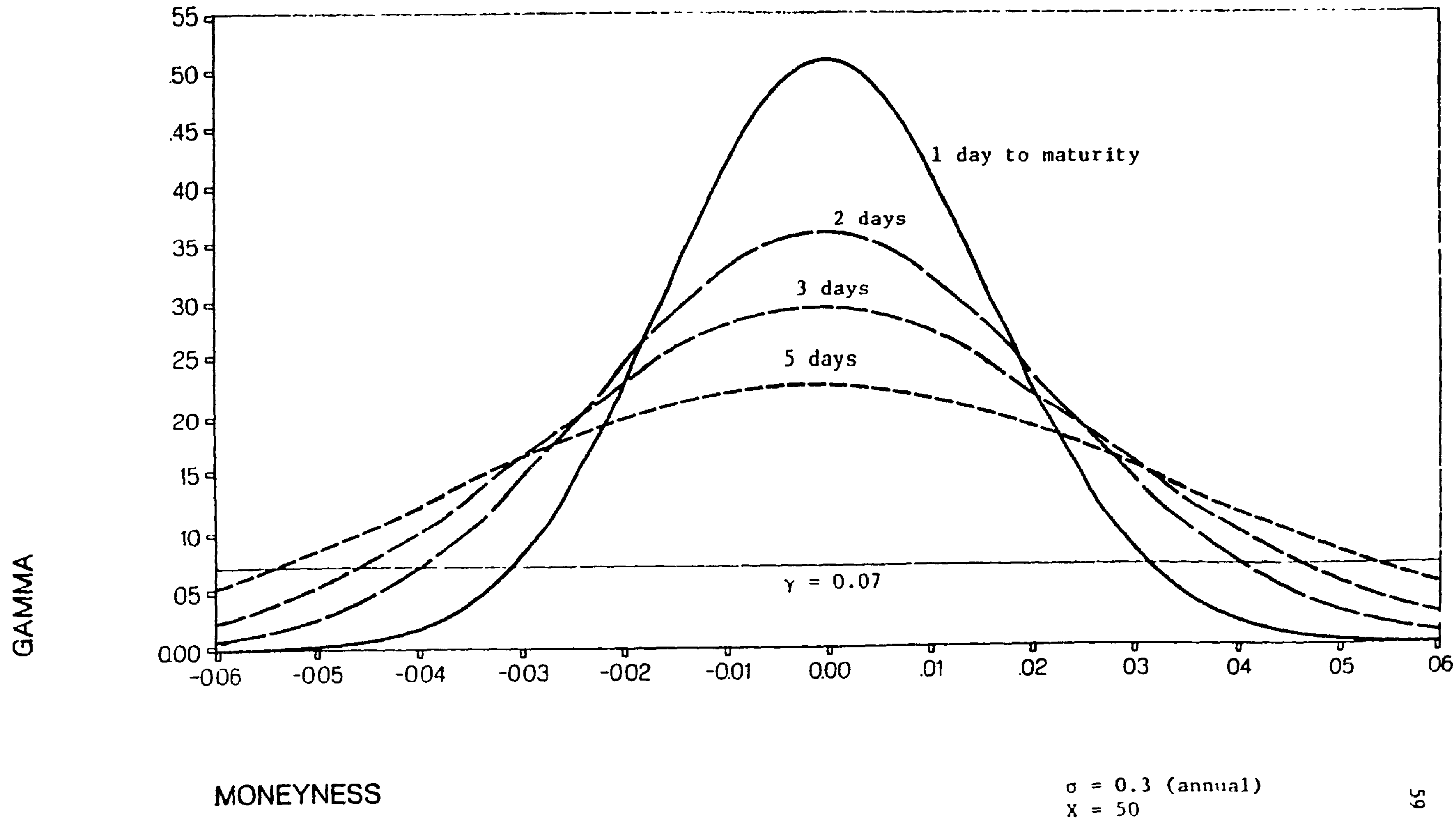
Gamma vs Time to Maturity



σ = 0.3 (annual)
X = 50
i = 6 %

FIGURE 3

Gamma vs Moneyness (Narrow Range)



σ = 0.3 (annual)
X = 50
i = 6 %

FIGURE 4

TABLE 1

Description of the Data

DM: N = 81,105 | 33.87 % puts | Avg int = 5.90 %

	Std dev	mean	min	5 %	25 %	50 %	75 %	95 %	max
				Percentiles					
ΔT	142.5	58.95	0	1	6	17	45	260	1797
CM	0.828	0.929	0.01	0.11	0.38	0.72	1.20	2.52	11.83
M	0.037	-0.014	-0.251	-0.074	-0.035	-0.014	0.005	0.048	0.370
M_1	0.039	0.006	-0.130	-0.052	-0.019	0.002	0.028	0.077	0.370
	SF: N = 46,887			35.28 % puts			Avg int = 5.84 %		
ΔT	146.3	57.05	1	2	6	15	39	263	1783
CM	0.950	1.116	0.01	0.15	0.48	0.88	1.45	2.93	11.74
M	0.034	-0.014	-0.226	-0.066	-0.032	-0.013	0.004	0.046	0.275
M_1	0.036	0.007	-0.123	-0.047	-0.016	0.003	0.028	0.069	0.275
	JY: N = 34,216			47.11 % puts			Avg int = 5.60 %		
ΔT	169.9	73.36	0	1	8	20	56	348	1792
CM	0.877	1.081	0.01	0.13	0.44	0.86	1.46	2.91	8.27
M	0.029	-0.011	-0.170	-0.056	-0.027	-0.012	0.003	0.042	0.150
M_1	0.030	0.005	-0.101	-0.043	-0.015	0.003	0.023	0.058	0.170
	BP: N = 11,919			49.01 % puts			Avg int = 6.02 %		
ΔT	216.9	113.9	0	2	14	39	108	516	1788
CM	1.814	2.589	0.05	0.45	1.30	2.20	3.45	5.90	17.70
M	0.025	-0.011	-0.166	-0.052	-0.026	-0.010	0.003	0.030	0.132
M_1	0.028	0.000	-0.101	-0.045	-0.017	0.000	0.016	0.046	0.166
	CD: N = 2,039			21.20 % puts			Avg int = 5.58 %		
ΔT	289.7	198.9	0	5	30	84	234	876	1793
CM	0.583	0.674	0.01	0.09	0.26	0.51	0.89	1.85	4.25
M	0.015	-0.003	-0.048	-0.025	-0.012	-0.004	0.004	0.023	0.059
M_1	0.015	0.001	-0.048	-0.021	-0.008	-0.001	0.009	0.027	0.059

ΔT is the interval between the matching option and futures transactions.
CM is the market value of the option.
M_1 is the moneyness of the option with the sign inverted for puts: $M_1 = (F-X)/X$.
M is the moneyness of the option: $M = M_1\ (-PC)$
and PC = 1 for puts
= 0 for calls
Avg int is the average interest rate used in computing the option prices.

TABLE 2

Typical Daily Correlation Matrix
SF, April 7th 1986

N = 150

	Ct	δ	γ	λ	λ'	θ	θ'	Φ
Ct								
δ	-0.276							
γ	0.348	-0.067						
λ	0.663	-0.052	0.549					
λ'	-0.918	0.301	-0.301	-0.410				
θ	0.351	-0.347	0.9987	0.544	-0.306			
θ'	-0.884	0.150	-0.279	-0.434	0.971	-0.289		
Φ	-0.396	0.424	-0.129	-0.683	0.183	-0.091	0.141	
Φ'	0.319	-0.937	0.057	0.051	-0.326	0.030	-0.143	-0.359

TABLE 3

Ranges of the Estimated Daily Volatilities

	Minimum		Maximum	
	Volatility	Date	Volatility	Date
DM	0.00504	26/03/87	0.00915	23/04/86
SF	0.00546	26/03/87	0.00902	24/04/86
JY	0.00406	11/03/87	0.00887	12/05/86
BP	0.00460	19/12/86	0.00820	24/03/86
CD	0.00153	19/11/86	0.00356	26/08/86

TABLE 4

Bias in the residual of Black's model.

	DM	SF	JY	BP	CD
8602	0.082	0.071*		0.116	
8603	0.556*	0.656*	0.147	0.108	
8604	0.646	0.590*	0.446	0.130	
8605	0.267	0.118	0.453	0.181	
8606	0.367*	0.219*	0.363	0.141	0.169* [a]
8607	0.348*	0.124	0.186*	0.108	0.258*
8608	0.761*	0.408*	0.641*	0.440	0.235
8609	0.320	0.191	0.250	0.239	0.284
8610	0.310	0.426*	0.160*	0.339*	0.165
8611	0.639*	0.636*	0.444*	0.478*	0.373
8612	0.420*	0.312*	0.360	0.123*	0.138 [b]
8701	0.287	0.277	0.276	0.104*	0.190*
8702	0.214	0.212*	0.171*	0.219*	0.161
8703	0.496	0.286	0.215	0.219	0.235

The table presents the R^2 of: $\varepsilon = \beta_0 + \beta_1 Ct + \beta_2 M + \beta_3 T + \beta_4 PC + \varepsilon_1$
Unless specified otherwise, all regressions are significant at $\alpha = 0.01$.

* Indicates that the R^2 is larger when M_1 is used instead of M. The larger value is indicated in the table.

a. Level of significance = 0.0325, note N = 61.

b. Not significant statistically, in this case N = 47.

TABLE 5
Overall Results for Equations 12, 17 and 18

DM

Month	N	Full Model: Model RMSE	Full Model: Residual R^2	Reduced Model 1: Model RMSE	Reduced Model 1: Residual R^2	Reduced Model 2: Model RMSE	Reduced Model 2: Residual R^2
8602	8200	.0319	.0085[a]	.0335	.0188[a]	.0353	.0231[a]
8603	4805	.0261	.0003	.0267	.0008	.0267	.0008
8604	8830	.0224	.0008	.0233	.0020[a]	.0234	.0026[a]
8605	7515	.0223	.0048[a]	.0248	.0008	.0251	.0038[a]
8606	2998	.0158	.0007	.0159	.0005	.0159	.0009
8607	6329	.0160	.0001	.0161	.0010	.0165	.0044[a]
8608	5134	.0171	.0085[a]	.0180	.0284[a]	.0184	.0405[a]
8609	4089	.0170	.0007	.0173	.0006	.0173	.0030*
8610	5266	.0208	.0057[a]	.0220	.0155[a]	.0220	.0151[a]
8611	4926	.0179	.0371[a]	.0182	.0506[a]	.0182	.0506[a]
8612	3327	.0158	.0007	.0165	.0037	.0171	.0119[a]
8701	9844	.0378	.0001	.0403	.0033[a]	.0408	.0019[a]
8702	7121	.0309	.0279[a]	.0316	.0192*[a]	.0320	.0090[a]
8703	2721	.0217	.0005	.0224	.0021	.0235	.0128[a]

SF

Month	N	Full Model: Model RMSE	Full Model: Residual R^2	Reduced Model 1: Model RMSE	Reduced Model 1: Residual R^2	Reduced Model 2: Model RMSE	Reduced Model 2: Residual R^2
8602	5027	.0424	.0138[a]	.0435	.0194[a]	.0442	.0281[a]
8603	2252	.0244	.0001	.0257	.0015	.0257	.0019*
8604	4621	.0298	.0023[a]	.0318	.0039[a]	.0318	.0047*[a]
8605	3729	.0340	.0075[a]	.0358	.0030*	.0361	.0023*
8606	1574	.0198	.0002	.0199	.0002	.0199	.0007
8607	4057	.0241	.0013	.0245	.0115[a]	.0245	.0117[a]
8608	3709	.0246	.0026	.0248	.0091*[a]	.0251	.0136[a]
8609	2906	.0234	.0000	.0235	.0000	.0235	.0003*
8610	3293	.0263	.0028	.0275	.0104[a]	.0278	.0150[a]
8611	3881	.0222	.0383[a]	.0229	.0586[a]	.0234	.0698[a]
8612	2571	.0192	.0000	.0199	.0007	.0201	.0021
8701	5651	.0414	.0003	.0451	.0014*	.0454	.0013*
8702	4076	.0368	.0059[a]	.0390	.0002	.0397	.0035[a]
8703	1540	.0336	.0002	.0348	.0055*[a]	.0350	.0036

JY

Month	N	Full Model: Model RMSE	Full Model: Residual R^2	Reduced Model 1: Model RMSE	Reduced Model 1: Residual R^2	Reduced Model 2: Model RMSE	Reduced Model 2: Residual R^2
8603	819	.0286	.0011	.0296	.0065	.0297	.0104
8604	2109	.0420	.0028	.0439	.0002	.0463	.0053
8605	3863	.0402	.0002	.0456	.0075*[a]	.0533	.0418[a]
8606	1615	.0245	.0000	.0248	.0001	.0249	.0006
8607	4718	.0269	.0001	.0270	.0013	.0270	.0012
8608	2950	.0314	.0065[a]	.0338	.0131[a]	.0344	.0089[a]
8609	2470	.0259	.0001	.0262	.0007	.0265	.0033
8610	4779	.0400	.0009	.0442	.0044*[a]	.0448	.0032*[a]
8611	3139	.0244	.0408[a]	.0251	.0596[a]	.0264	.0896[a]
8612	1003	.0177	.0003	.0179	.0004	.0182	.0077*
8701	3384	.0435	.0002	.0463	.0033*	.0469	.0018
8702	1549	.0304	.0100[a]	.0311	.0017	.0311	.0030*
8703	1818	.0330	.0003	.0334	.0001	.0334	.0015*

BP

Month	N	Full Model: Model RMSE	Full Model: Residual R^2	Reduced Model 1: Model RMSE	Reduced Model 1: Residual R^2	Reduced Model 2: Model RMSE	Reduced Model 2: Residual R^2
8602	1781	.1139	.0005	.1145	.0007	.1158	.0028
8603	890	.1026	.0001	.1031	.0000	.1031	.0002*
8604	1192	.0897	.0000	.0911	.0002	.0911	.0006*
8605	940	.1108	.0208[a]	.1107	.0199[a]	.1121	.0282[a]
8606	421	.0924	.0004	.0930	.0004	.0931	.0007
8607	975	.0975	.0024	.0978	.0017	.0992	.0090
8608	806	.1053	.0373[a]	.1078	.0422[a]	.1085	.0500[a]
8609	861	.0835	.0010	.0848	.0039	.0858	.0105
8610	780	.0723	.0003	.0723	.0002	.0795	.0300[a]
8611	696	.0779	.0133	.0800	.0218[a]	.0834	.0429[a]
8612	406	.0769	.0001	.0772	.0002	.0772	.0005
8701	901	.0968	.0001	.0988	.0007	.0989	.0008*
8702	524	.0894	.0043	.0927	.0059	.0927	.0057
8703	746	.0885	.0000	.0888	.0003	.0927	.0261[a]

CD

8606	61	.0191	.0016	.0211	.0014	.0210	.0008*
8607	113	.0168	.0001	.0176	.0007	.0178	.0018*
8608	109	.0248	.0125	.0277	.0275	.0280	.0339
8609	64	.0206	.0002	.0203	.0002	.0201	.0003*
8610	151	.0168	.0002	.0173	.0002	.0174	.0085
8611	100	.0221	.0009	.0232	.0198*	.0287	.0634
8612	47	.0330	.0005	.0329	.0005	.0326	.0007
8701	413	.0443	.0077	.0443	.0096	.0442	.0095
8702	468	.0443	.0353[a]	.0455	.0563[a]	.0456	.0577[a]
8703	513	.0409	.0049	.0432	.0296[a]	.0431	.0294[a]

The full model is: $C_m = \beta_0 + \beta_1 Ct + \beta_2\delta + \beta_3\gamma + \beta\mu T\gamma + \beta_5\lambda + \beta_6\lambda' + \beta_7\theta + \beta_8\theta' + \beta_9\Phi + \beta_{10}\Phi' + \varepsilon$

This regression includes one variable ($T\gamma$) not discussed in the paper. $T\gamma$ never contributed significantly to the explanatory power of the model.

The reduced model 1 is: $C_m = \beta_0 + \beta_1 Ct + \beta_2\delta + \beta_3\gamma + \beta_4\lambda + \beta_5\theta + \beta_6\Phi + \varepsilon$

The reduced model 2 is: $C_m = \beta_0 + \beta_1 Ct + \beta_2\delta + \beta_3\gamma + \beta_4\lambda + \beta_6\Phi + \varepsilon$

Model RMSE is the RMSE for the fitted model.

Residual R^2 is the R^2 of the regression: $\varepsilon = \beta_0 + \beta_1 Ct + \beta_2 M + \beta_3 T + \beta_4 PC + \varepsilon_1$
where ε is the residual from the fitted model.

* Indicates that the bias is more severe when measured with M_1.
The reported results are those for the larger bias.

[a] Indicates that residual is significant at the 1 % level. A significance level of 1 % was selected because of the usually large number of degrees of freedom involved.

TABLE 6
Typical Monthly Correlation Matrix
DM. July 1986

	Ct	δ	γ	λ	λ'	θ	ρ'	Φ
Ct								
δ	-0.081							
γ	-0.655	0.061						
λ	-0.348	0.001	0.185					
λ'	-0.770	0.083	0.172	0.243				
θ	-0.695	0.030	0.977	0.231	0.221			
θ'	-0.705	-0.042	0.164	0.228	0.965	0.217		
Φ	-0.259	-0.350	0.315	-0.529	0.161	0.346	0.188	
Φ'	0.101	-0.865	0.045	-0.218	-0.150	0.062	-0.012	0.517

N = 6,329

TABLE 7
Parameter Estimates of Equation 17

DM

	Intercept	Ct	δ	γ	$\lambda^{a}X10^{-4}$	θ	$\Phi^{a}X10^{-5}$
8602	0.053	-0.010 (-0.31)[b]	-0.010 (-0.11)	-0.30 (-0.81)	-1.06 (-0.17)	4.55 (0.58)	3.66 (0.11)
8603	0.117	-0.011 (-0.30)	0.012 (0.10)	0.03 (0.02)	-6.35 (-0.61)	-0.88 (-0.03)	12.86 (0.24)
8604	0.096	-0.010 (-0.27)	-0.007 (-0.07)	-0.01 (-0.01)	-6.17 (-0.60)	1.12 (0.08)	14.41 (0.28)
8605	0.047	0.003 (0.08)	-0.002 (-0.02)	-0.39 (-0.92)	-0.93 (-0.14)	3.40 (0.52)	26.44 (0.53)
8606	0.070	-0.003 (-0.15)	0.001 (0.02)	-0.49 (-0.52)	-1.80 (-0.27)	2.74 (0.18)	8.43 (0.24)
8607	0.070	-0.008 (-0.38)	-0.002 (-0.04)	0.02 (0.04)	1.22 (0.24)	-8.92 (-0.94)	13.92 (0.41)
8608	0.028	-0.001 (-0.03)	-0.004 (-0.04)	-0.77 (-1.55)	3.03 (0.44)	6.52 (0.77)	9.23 (0.15)
8609	0.076	-0.004 (-0.19)	-0.003 (-0.05)	-0.57 (-0.58)	1.06 (0.19)	-2.31 (-0.16)	10.51 (0.34)
8610	0.043	-0.003 (-0.11)	-0.005 (-0.08)	-0.44 (-0.71)	1.18 (0.21)	1.26 (0.10)	4.27 (0.09)
8611	0.022	0.000 (0.01)	-0.005 (-0.07)	-0.23 (-0.71)	1.29 (0.26)	0.12 (0.02)	4.11 (0.08)
8612	0.048	-0.004 (-0.17)	-0.002 (-0.03)	-0.30 (-0.47)	3.80 (0.71)	-10.21 (-0.63)	8.69 (0.24)
8701	0.106	-0.013 (-0.33)	-0.006 (-0.06)	-0.35 (-0.28)	-3.49 (-0.38)	2.66 (0.18)	9.15 (0.23)
8702	0.052	-0.002 (-0.07)	0.000 (0.00)	-0.34 (-0.67)	-1.60 (-0.27)	2.61 (0.38)	12.84 (0.27)
8703	0.092	-0.006 (-0.16)	0.004 (0.04)	-0.49 (-0.44)	3.59 (0.48)	-13.7 (-0.56)	11.50 (0.22)

SF

	Intercept	Ct	δ	γ	$\lambda^a X10^{-4}$	θ	$\phi^a X10^{-5}$
8602	0.085	-0.014 (-0.37)	-0.007 (-0.07)	-0.39 (-0.68)	-1.67 (-0.26)	2.49 (0.33)	2.24 (0.06)
8603	0.129	-0.011 (-0.25)	0.013 (0.11)	0.09 (0.04)	-7.75 (-0.68)	2.21 (0.08)	8.54 (0.15)
8604	0.125	-0.013 (-0.31)	-0.004 (-0.03)	-0.14 (-0.07)	-5.67 (-0.52)	1.10 (0.07)	13.50 (0.25)
8605	0.049	-0.001 (-0.03)	0.001 (-0.01)	-0.47 (-0.78)	-0.64 (-0.09)	2.68 (0.39)	16.19 (0.34)
8606	0.065	-0.002 (-0.09)	-0.004 (-0.07)	-0.77 (-0.59)	-1.67 (-0.26)	5.15 (0.36)	5.04 (0.16)
8607	0.064	-0.005 (-0.25)	-0.005 (-0.09)	-0.62 (-0.70)	-0.13 (-0.03)	1.87 (0.21)	5.52 (0.20)
8608	0.035	-0.002 (-0.[illegible]3)	-0.002 (-0.02)	-0.83 (-1.64)	1.22 (0.28)	4.60 (0.95)	8.12 (0.23)
8609	0.085	-0.004 (-0.19)	-0.002 (-0.04)	-0.93 (-0.58)	0.66 (0.10)	-0.34 (-0.02)	8.06 (0.24)
8610	0.047	-0.002 (-0.06)	0.002 (0.02)	-0.22 (-0.21)	2.36 (0.35)	-6.04 (-0.44)	5.44 (0.10)
8611	0.018	-0.001 (-0.02)	-0.004 (-0.05)	0.23 (0.50)	2.00 (0.38)	-7.73 (-1.12)	7.50 (0.14)
8612	0.059	-0.001 (-0.05)	-0.001 (-0.01)	-0.50 (-0.45)	2.32 (0.39)	-5.50 (-0.32)	10.04 (0.25)
8701	0.118	-0.009 (-0.23)	0.003 (0.02)	-0.29 (-0.15)	-4.85 (-0.49)	2.03 (0.14)	7.28 (0.15)
8702	0.065	-0.001 (-0.03)	0.006 (0.05)	-0.60 (-0.76)	-1.52 (-0.20)	3.074 (0.41)	23.59 (0.36)
8703	0.087	-0.005 (-0.12)	0.012 (0.10)	-0.87 (-0.45)	3.71 (0.42)	-7.78 (-0.30)	16.01 (0.25)

JY

	Intercept	Ct	δ	γ	$\lambda^a X10^{-4}$	θ	$\phi^a X10^{-5}$
8603	0.057	-0.003	0.014	-0.74	1.34	3.58	26.96
		(-0.08)	(0.15)	(-0.39)	(0.16)	(0.12)	(0.61)
8604	0.128	-0.015	-0.001	-0.92	-2.03	7.26	30.42
		(-0.26)	(-0.00)	(-0.44)	(-0.18)	(0.34)	(0.52)
8605	0.093	-0.006	0.007	-1.53	-2.64	13.19	30.81
		(-0.09)	(0.04)	(-1.30)	(-0.26)	(1.18)	(0.46)
8606	0.091	-0.004	-0.008	-0.82	-1.73	4.21	17.25
		(-0.16)	(-0.09)	(-0.41)	(-0.21)	(0.21)	(0.40)
8607	0.103	-0.007	-0.001	-0.85	0.50	0.99	16.85
		(-0.28)	(-0.01)	(-0.68)	(0.01)	(0.09)	(0.53)
8608	0.028	-0.000	-0.007	-1.16	3.02	6.07	10.67
		(-0.00)	(-0.05)	(-1.30)	(0.36)	(0.60)	(0.14)
8609	0.081	-0.000	-0.010	-0.54	1.12	-4.90	14.52
		(-0.01)	(-0.12)	(-0.28)	(0.15)	(-0.25)	(0.29)
8610	0.043	0.006	-0.003	-1.00	2.96	5.61	22.84
		(0.12)	(-0.03)	(-0.74)	(0.33)	(0.28)	(0.28)
8611	0.018	0.009	0.004	0.25	1.29	-10.11	8.02
		(0.20)	(0.06)	(0.73)	(0.24)	(-1.31)	(0.15)
8612	0.057	-0.002	-0.002	-0.34	1.67	-5.54	16.72
		(-0.06)	(-0.03)	(-0.46)	(0.33)	(-0.29)	(0.32)
8701	0.119	-0.009	-0.009	-0.43	-3.35	2.27	20.55
		(-0.18)	(-0.07)	(-0.26)	(-0.34)	(0.15)	(0.25)
8702	0.053	-0.001	0.001	-0.22	-0.14	-0.20	21.74
		(-0.02)	(0.01)	(-0.57)	(-0.03)	(-0.02)	(0.31)
8703	0.126	-0.011	0.007	-0.48	-1.37	-1.62	18.07
		(-0.27)	(0.07)	(-0.34)	(-0.16)	(-0.06)	(0.24)

BP

	Intercept	Ct	δ	γ	$\lambda^{a}X10^{-4}$	θ	$\phi^{a}X10^{-5}$
8602	0.017	0.012	-0.049	-3.27	0.14	5.19	1.73
		(0.22)	(-0.19)	(-0.72)	(0.02)	(0.70)	(0.32)
8603	0.197	-0.002	0.007	-4.33	-0.39	1.40	1.79
		(-0.05)	(0.03)	(-0.19)	(-0.03)	(0.04)	(0.29)
8604	0.172	-0.001	-0.024	-5.19	0.99	1.62	1.84
		(-0.02)	(-0.11)	(-0.47)	(0.12)	(0.11)	(0.42)
8605	0.048	0.002	0.019	1.83	2.72	-6.02	2.51
		(0.03)	(0.07)	(0.39)	(0.38)	(-0.77)	(0.46)
8606	0.007	0.002	0.013	-5.27	7.08	-3.16	3.98
		(0.04)	(0.05)	(-0.30)	(0.66)	(-0.11)	(0.73)
8607	0.138	-0.002	-0.000	4.37	2.77	-14.39	2.62
		(-0.04)	(-0.00)	(0.40)	(0.37)	(-0.82)	(0.60)
8608	-0.002	0.007	0.011	1.50	2.69	-6.83	0.98
		(0.08)	(0.04)	(0.33)	(0.33)	(-0.74)	(0.14)
8609	0.175	0.011	0.021	1.36	1.39	-15.05	1.09
		(0.22)	(0.09)	(0.11)	(0.13)	(-0.56)	(0.19)
8610	0.085	0.009	0.003	0.82	4.12	-14.96	1.58
		(0.14)	(0.02)	(0.14)	(0.51)	(-0.78)	(0.25)
8611	0.055	0.002	0.011	1.49	1.61	-11.27	0.06
		(0.02)	(0.05)	(0.52)	(0.22)	(-1.05)	(0.01)
8612	0.237	-0.009	-0.041	-1.96	-1.40	-2.92	-0.41
		(-0.17)	(-0.21)	(-0.22)	(-0.15)	(-0.08)	(-0.06)
8701	0.274	-0.013	-0.030	-3.15	-1.39	-0.94	1.24
		(-0.29)	(-0.13)	(-0.43)	(-0.05)	(-0.06)	(0.23)
8702	0.104	-0.002	-0.034	-0.71	9.14	-2.81	1.77
		(-0.05)	(-0.14)	(-0.23)	(0.15)	(-0.31)	(0.27)
8703	0.425	-0.016	0.010	-3.17	2.58	-19.29	1.49
		(-0.33)	(0.04)	(-0.29)	(0.22)	(-0.53)	(0.21)

	Intercept	Ct	δ	γ	$\lambda^a X10^{-4}$	θ	$\Phi^a X10^{-5}$
8606	0.057	-0.005 (-0.15)	0.004 (0.06)	0.506 (0.79)	1.66 (0.30)	-37.26 (-1.45)	0.28 (0.04)
8607	0.053	-0.006 (-0.21)	0.002 (0.03)	0.037 (0.11)	-0.16 (-0.04)	-9.58 (-0.67)	1.04 (0.13)
8608	0.004	-0.007 (-0.12)	-0.000 (-0.00)	-0.239 (-0.97)	1.73 (0.43)	8.44 (0.68)	2.91 (0.24)
8609	0.024	0.012 (0.26)	0.008 (0.12)	-0.070 (-0.13)	-0.24 (-0.05)	-0.40 (-0.01)	3.02 (0.35)
8610	0.023	0.001 (0.03)	-0.006 (-0.14)	-0.033 (-0.17)	0.54 (0.19)	-4.90 (-0.26)	2.51 (0.34)
8611	0.089	-0.010 (-0.08)	0.001 (0.02)	-0.002 (-0.02)	-1.18 (-0.30)	-17.07 (-0.95)	0.80 (0.08)
8612	-0.007	0.022 (0.19)	0.038 (0.46)	-0.106 (-0.19)	2.89 (0.56)	-5.57 (-0.10)	8.89 (0.84)
8701	0.111	-0.036 (-0.47)	-0.026 (-0.19)	-0.170 (-0.30)	-1.61 (-0.31)	0.76 (0.02)	0.34 (-0.06)
8702	0.057	-0.017 (-0.25)	-0.012 (-0.10)	-0.021 (-0.06)	-0.11 (-0.02)	-5.15 (-0.31)	1.01 (0.16)
8703	0.112	-0.039 (-0.51)	-0.011 (-0.08)	-0.277 (-0.24)	-0.54 (-0.08)	-3.37 (-0.07)	-0.47 (-0.07)

a) Only the reported parameter estimates must be multiplied by 10^{-4} or 10^{-5} as noted.

b) Standardized parameter estimate are given in parentheses.

TABLE 8

Correlation Between γ and Θ for the DM and the SF

	DM	SF
8602	0.857	0.856
8603	0.988	0.976
8604	0.913	0.907
8605	0.962	0.956
8606	0.973	0.978
8607	0.977	0.989
8608	0.993	0.993
8609	0.939	0.930
8610	0.978	0.964
8611	0.996	0.993
8612	0.942	0.950
8701	0.718	0.653
8702	0.933	0.914
8703	0.921	0.934
	$\bar{\rho} = 0.935$	$\bar{\rho} = 0.928$

TABLE 9
Parameter Estimates of Equation 18

DM

	Intercept	Ct	δ	γ	λ^{a}X10^{-4}	Φ^{a}X10^{-5}	Condition Number [b]
8602	0.050	-0.009 (-0.28)[c]	-0.010 (-0.12)	-0.11 (-0.29)	-0.97 (-0.16)	0.36 (0.11)	16.5
8603	0.117	-0.011 (-0.30)	0.012 (0.10)	-0.02 (-0.01)	-6.37 (-0.61)	1.27 (0.23)	14.4
8604	0.095	-0.010 (-0.27)	-0.006 (-0.06)	0.08 (0.06)	-6.13 (-0.60)	1.49 (0.29)	13.1
8605	0.051	0.002 (0.08)	0.001 (0.01)	-0.18 (-0.43)	-0.60 (-0.09)	3.03 (0.61)	11.0
8606	0.071	-0.003 (-0.15)	0.002 (0.03)	-0.34 (-0.36)	-1.61 (-0.24)	0.92 (0.27)	19.4
8607	0.065	-0.007 (-0.35)	-0.003 (-0.06)	-0.44 (-0.80)	0.64 (0.12)	1.00 (0.30)	18.9
8608	0.021	-0.001 (-0.01)	-0.005 (-0.05)	-0.38 (-0.77)	3.09 (0.45)	0.90 (0.15)	16.6
8609	0.075	-0.004 (-0.19)	-0.003 (-0.06)	-0.72 (-0.73)	0.99 (0.18)	1.01 (0.33)	23.1
8610	0.043	-0.003 (-0.11)	-0.005 (-0.08)	-0.38 (-0.62)	1.21 (0.22)	0.45 (0.10)	15.8
8611	0.022	0.000 (0.01)	-0.005 (-0.07)	-0.22 (-0.70)	1.30 (0.26)	0.41 (0.08)	11.9
8612	0.045	-0.004 (-0.16)	-0.002 (-0.03)	-0.62 (-0.97)	3.23 0.61)	0.65 (0.18)	18.3
8701	0.122	-0.015 (-0.39)	-0.008 (-0.07)	-0.25 (-0.20)	-3.52 (-0.38)	0.96 (0.24)	19.4
8702	0.059	-0.003 (-0.09)	0.001 (0.01)	-0.17 (-0.33)	-1.38 (-0.24)	1.54 (0.32)	10.3
8703	0.088	-0.006 (-0.16)	0.004 (0.05)	-1.02 (-0.91)	2.88 (0.38)	0.88 (0.17)	17.1

SF

	Intercept	Ct	δ	γ	$\lambda^a X10^{-4}$	$\Phi^a X10^{-6}$	Condition Number [b]
8602	0.088	-0.015 (-0.39)	-0.009 (-0.09)	-0.24 (-0.41)	-1.73 (-0.27)	0.15 (0.04)	19.3
8603	0.129	-0.011 (-0.25)	0.014 (0.11)	0.26 (0.11)	-7.66 (-0.67)	0.91 (0.16)	18.9
8604	0.123	-0.013 (-0.30)	-0.003 (-0.02)	-0.02 (-0.01)	-5.62 (-0.52)	1.40 (0.26)	16.0
8605	0.054	-0.001 (-0.03)	0.004 (0.04)	-0.25 (-0.42)	-0.39 (-0.06)	1.93 (0.41)	11.7
8606	0.063	-0.001 (-0.07)	-0.003 (-0.05)	-0.33 (-0.25)	-1.31 (-0.21)	0.66 (0.20)	27.8
8607	0.064	-0.005 (-0.25)	-0.005 (-0.09)	-0.44 (-0.50)	-0.08 (-0.02)	0.59 (0.21)	29.3
8608	0.027	-0.001 (-0.05)	-0.002 (-0.02)	-0.34 (-0.67)	1.26 (0.29)	0.80 (0.22)	18.5
8609	0.085	-0.004 (-0.19)	-0.003 (-0.04)	-0.97 (-0.60)	0.64 (0.10)	0.80 (0.24)	27.3
8610	0.046	-0.002 (-0.07)	0.002 (0.02)	-0.68 (-0.62)	2.14 (0.31)	0.37 (0.07)	13.7
8611	0.015	-0.001 (-0.01)	-0.005 (-0.05)	-0.28 (-0.60)	1.81 (0.35)	0.42 (0.08)	11.[illegible]
8612	0.059	-0.002 (-0.06)	-0.001 (-0.02)	-0.81 (-0.73)	2.05 (0.34)	0.89 (0.23)	20.9
8701	0.136	-0.011 (-0.28)	0.001 (0.01)	-0.19 (-0.10)	-4.94 (-0.50)	0.76 (0.16)	22.0
8702	0.077	-0.002 (-0.05)	0.006 (0.05)	-0.32 (-0.41)	-1.05 (-0.14)	2.89 (0.45)	11.4
8703	0.085	-0.005 (-0.13)	0.011 (0.10)	-1.36 (-0.71)	3.31 (0.37)	1.47 (0.23)	23.1

JY

	Intercept	Ct	δ	γ	$\lambda^a X10^{-4}$	$\phi^a X10^{-4}$	Condition Number [b]
8603	0.056	0.003 (-0.06)	0.016 (0.17)	-0.57 (-0.30)	1.79 (0.22)	2.93 (0.67)	26.6
8604	0.127	-0.012 (-0.21)	0.013 (0.08)	-0.42 (-0.20)	-1.46 (-0.13)	3.57 (0.61)	19.3
8605	0.121	-0.004 (-0.07)	0.010 (0.05)	-0.25 (-0.21)	-2.26 (-0.22)	3.90 (0.59)	12.1
8606	0.092	-0.004 (-0.15)	0.008 (0.10)	-0.45 (-0.23)	-1.38 (-0.17)	1.83 (0.42)	24.8
8607	0.104	-0.007 (-0.28)	-0.000 (-0.01)	-0.75 (-0.60)	-0.10 (-0.02)	1.72 (0.54)	30.8
8608	0.035	-0.001 (-0.01)	-0.007 (-0.05)	-0.65 (-0.73)	3.01 (0.36)	1.18 (0.16)	16.0
8609	0.074	0.000 (0.01)	-0.009 (-0.11)	-0.83 (-0.44)	0.52 (0.07)	1.24 (0.25)	39.8
8610	0.048	0.006 (0.12)	-0.005 (-0.05)	-0.66 (-0.50)	2.71 (0.30)	2.22 (0.27)	15.4
8611	0.012	0.010 (0.22)	0.004 (0.06)	-0.18 (-0.52)	0.83 (0.16)	0.29 (0.05)	7.6
8612	0.055	-0.002 (-0.06)	-0.003 (-0.05)	-0.50 (-0.68)	1.42 (0.28)	1.60 (0.31)	21.1
8701	0.132	-0.010 (-0.20)	-0.009 (-0.06)	-0.33 (-0.20)	-3.16 (-0.32)	2.28 (0.27)	17.3
8702	0.053	-0.001 (-0.02)	0.001 (0.01)	-0.23 (-0.59)	-0.16 (-0.03)	2.14 (0.30)	9.8
8703	0.124	-0.011 (-0.27)	0.006 (0.06)	-0.51 (-0.36)	-1.57 (-0.19)	1.73 (0.23)	23.6

BP

	Intercept	Ct	δ	Υ	$\lambda^{a}X10^{-4}$	$\Phi^{a}X10^{-4}$	Condition Number [b]
8602	-0.006	0.013 (0.25)	-0.045 (-0.17)	-0.07 (-0.02)	0.04 (0.01)	1.60 (0.30)	14.4
8603	0.198	-0.002 (-0.05)	0.007 (0.03)	-3.41 (-0.15)	-0.37 (-0.03)	1.82 (0.29)	36.8
8604	0.168	-0.000 (-0.01)	-0.024 (-0.10)	-3.99 (-0.36)	0.99 (0.12)	1.86 (0.42)	28.9
8605	0.037	0.000 (0.00)	0.013 (0.05)	-1.69 (-0.36)	2.10 (0.29)	1.74 (0.32)	14.7
8606	0.004	0.002 (0.03)	0.008 (0.03)	-7.00 (-0.39)	6.97 (0.65)	3.87 (0.71)	33.7
8607	0.132	-0.003 (-0.07)	-0.009 (-0.04)	-3.80 (-0.34)	1.58 (0.21)	1.62 (0.37)	18.1
8608	-0.013	0.007 (0.09)	0.015 (0.05)	-1.78 (-0.39)	2.55 (0.32)	0.69 (0.10)	14.5
8609	0.166	0.010 (0.22)	0.023 (0.10)	-5.04 (-0.40)	0.47 (0.05)	0.65 (0.11)	24.3
8610	0.036	0.008 (0.12)	0.009 (0.04)	-2.44 (-0.42)	2.07 (0.26)	0.12 (0.02)	12.6
8611	0.030	-0.000 (-0.00)	0.015 (0.07)	-1.31 (-0.46)	1.27 (0.18)	1.12 (0.12)	11.8
8612	0.231	-0.009 (-0.17)	-0.042 (-0.22)	-2.61 (-0.29)	-1.38 (-0.15)	-0.43 (-0.07)	23.7
8701	0.269	-0.013 (-0.29)	-0.030 (-0.13)	-3.41 (-0.47)	-0.42 (-0.05)	1.20 (0.22)	20.1
8702	0.101	-0.002 (-0.04)	-0.034 (-0.15)	-1.63 (-0.52)	0.87 (0.14)	1.61 (0.20)	15.8
8703	0.385	-0.016 (-0.33)	0.007 (0.03)	-4.75 (-0.43)	-1.83 (-0.16)	0.71 (0.10)	22.9

CD

	Intercept	Ct	δ	γ	$\lambda^{a}X10^{-4}$	$\phi^{a}X10^{-4}$	Condition Number [b]
8606	0.053	-0.007 (-0.22)	0.000 (0.01)	0.04 (0.05)	-2.67 (-0.48)	-0.96 (-0.15)	40.0
8607	0.052	-0.006 (-0.21)	0.001 (0.02)	-0.13 (-0.39)	-1.31 (-0.36)	-0.63 (-0.08)	16.9
8608	0.008	-0.008 (-0.15)	-0.001 (-0.02)	-0.08 (-0.34)	1.73 (0.43)	3.63 (0.30)	12.6
8609	0.024	0.012 (0.26)	0.008 (0.12)	-0.08 (-0.14)	-0.24 (-0.05)	3.02 (0.35)	23.1
8610	0.020	0.001 (0.03)	-0.006 (-0.13)	-0.07 (-0.35)	0.25 (0.09)	1.98 (0.27)	12.0
8611	0.041	-0.006 (-0.05)	-0.003 (-0.04)	-0.07 (-0.62)	-0.43 (-0.11)	0.31 (0.03)	19.9
8612	-0.009	0.023 (0.20)	0.037 (0.46)	-0.15 (-0.27)	2.77 (0.54)	8.55 (0.80)	29.5
8701	0.112	-0.036 (-0.47)	-0.026 (-0.19)	-0.16 (-0.28)	-1.62 (-0.32)	-0.33 (-0.06)	18.1
8702	0.052	-0.016 (-0.23)	-0.012 (-0.09)	-0.13 (-0.33)	-0.09 (-0.02)	0.93 (0.15)	15.2
8703	0.112	-0.039 (-0.50)	-0.011 (-0.08)	-0.35 (-0.31)	-0.54 (-0.08)	-0.48 (-0.08)	17.9

a Only the reported parameter estimates must be multiplied by 10^{-4}.

b The condition number is the square root of the ratio of the largest eigenvalue of the matrix of the independent variables divided by its smallest eigenvalue.

c Standardized parameter estimates are given in parentheses.

TABLE 10

Days for Which the Residuals of Equation 18 Were Biased

Days not in pre-maturity months

DM		SF	
860408	(0.0488)*	860318	(0.0431)
860428	(0.0001)		
		860425	(0.0001)
870112	(0.0001)		
870113	(0.0001)	860715	(0.0284)
870127	(0.0001)	860721	(0.0001)
870129	(0.0002)		
		870114	(0.0084)

* Level of significance of the bias regression:

$$\varepsilon_m = \beta_0 + \beta_1 Ct + \beta_2 M + \beta_3 M_1 + \beta_4 T + \beta_5 PC + \varepsilon'$$

Days in pre-maturity months

DM			SF	
860206	860515	861114	860211	860801
860210	860527	861121	860212	
860211	860529	861124	860213	861125
860212	860530	861125	860214	861126
860213		861128	860218	
860214	860807		860221	
860218	860814	870202	860224	
860219	860820	870205	860225	
860220	860821	870206	860226	
860221	860822	870223	860227	
860224	860825	870224		
860225	860826	870225	860514	
860226	860827		860522	
860227	860829			
860228				

TABLE 11

Monthly Variability of Exchange Rates

	DM			SF		
	Standard[a] Deviation	Minimum Value	Maximum Value	Standard[a] Deviation	Minimum Value	Maximum Value
8602	1.211	41.42¢	45.00¢	1.527	48.94¢	53.30¢
8603	0.783	42.64	45.43	0.805	50.99	53.42
8604	1.600	41.73	46.20	1.918	49.68	55.29
8605	0.901	43.01	46.00	0.988	51.72	55.42
8606	0.587	43.02	45.43	0.859	51.83	55.66
8607	0.648	45.68	47.77	1.143	55.87	59.59
8608	0.383	47.85	49.13	0.492	59.26	60.94
8609	0.575	47.80	50.26	0.840	58.84	62.34
8610	0.657	48.52	50.66	1.107	58.45	61.92
8611	0.697	48.41	50.68	0.889	57.94	60.86
8612	0.749	49.52	51.99	1.036	58.65	61.96
8701	1.409	51.79	55.96	1.664	61.39	66.53
8702	0.424	53.88	55.62	0.512	63.86	65.88
8703	0.500	53.45	55.49	0.746	63.70	66.42

a) Standard deviation of daily exchange rates.

Table 12

Parameter Estimates for the DM

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860203	0.0257	0.0437	-0.0122	-0.0003	-0.286	0.00021	0.000056
860204	0.0175	0.0521	-0.0086	-0.0073	-0.410	0.00022	-0.000039
860205	0.0174	0.0403	-0.0067	-0.0063	-0.384	0.00041	0.000139
860206	0.0194	0.0456	-0.0064	-0.0026	-0.288	0.00008	-0.000036
860207	0.0293	0.0229	-0.0065	-0.0081	-0.218	0.00028	0.000096
860210	0.0175	0.0709	-0.0194	-0.0000	-0.260	-0.00001	-0.000000
860211	0.0166	0.0419	-0.0050	-0.0127	-0.202	0.00002	0.000001
860212	0.0183	0.0327	-0.0002	-0.0146	-0.150	0.00003	0.000060
860213	0.0202	0.0509	-0.0087	-0.0193	-0.171	-0.00007	-0.000064
860214	0.0209	0.0406	-0.0082	-0.0028	-0.193	0.00001	-0.000013
860218	0.0252	0.0262	-0.0048	-0.0075	-0.112	0.00003	0.000020
860219	0.0375	0.0432	-0.0047	-0.0178	-0.019	-0.00008	0.000159
860220	0.0261	0.0551	-0.0072	-0.0142	0.007	-0.00027	0.000034
860221	0.0209	0.0527	-0.0111	-0.0096	-0.149	-0.00008	0.000009
860224	0.0280	0.0621	-0.0127	-0.0090	-0.071	-0.00015	0.000084
860225	0.0287	0.0912	-0.0205	-0.0188	-0.087	-0.00028	-0.000014
860226	0.0353	0.0992	-0.0235	-0.0198	-0.121	-0.00027	-0.000023
860227	0.0467	0.0841	-0.0156	-0.0054	-0.071	-0.00026	0.000031
860228	0.0372	0.0997	-0.0194	-0.0181	-0.096	-0.00031	0.000016
860310	0.0416	0.1931	-0.0165	-0.0043	0.027	-0.00121	-0.000121
860311	0.0185	0.1806	-0.0226	0.0032	-0.464	-0.00058	0.000154
860312	0.0190	0.1613	-0.0197	-0.0047	-0.468	-0.00051	0.000107
860313	0.0162	0.1000	-0.0076	0.0005	-0.497	-0.00014	0.000176
860314	0.0218	0.1747	-0.0189	-0.0040	-0.472	-0.00054	0.000120
860317	0.0190	0.1624	-0.0112	0.0235	-0.445	-0.00056	0.000232
860318	0.0166	0.1615	-0.0193	-0.0001	-0.480	-0.00053	0.000058
860319	0.0190	0.1537	-0.0168	0.0032	-0.468	-0.00049	0.000153
860320	0.0172	0.1343	-0.0193	-0.0038	0.027	-0.00073	0.000059
860321	0.0198	0.1902	-0.0277	0.0140	-0.257	-0.00082	0.000047
860324	0.0156	0.0875	-0.0074	0.0161	0.029	-0.00048	0.000182
860325	0.0306	0.0975	-0.0044	0.0163	0.237	-0.00080	0.000099
860326	0.0207	0.1093	-0.0129	0.0192	0.532	-0.00110	0.000068
860327	0.0218	0.1333	-0.0179	0.0171	0.242	-0.00098	0.000040
860331	0.0206	0.0972	-0.0123	0.0078	0.288	-0.00077	0.000143
860401	0.0200	0.0844	-0.0065	0.0079	0.461	-0.00086	0.000107
860402	0.0137	0.0614	0.0006	0.0001	0.441	-0.00081	0.000152
860403	0.0142	0.0630	-0.0015	0.0003	0.556	-0.00090	0.000173
860404	0.0170	0.0666	-0.0011	0.0056	0.708	-0.00108	0.000059
860407	0.0155	0.0591	-0.0038	-0.0012	0.475	-0.00077	0.000166
860408	0.0170	0.0974	-0.0098	0.0039	0.459	-0.00100	0.000145
860409	0.0132	0.0821	-0.0097	0.0057	0.225	-0.00057	0.00 327

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860410	0.0211	0.1083	-0.0118	0.0008	0.077	-0.00072	0.000187
860411	0.0148	0.0980	-0.0135	-0.0072	0.268	-0.00079	0.000107
860414	0.0136	0.1046	-0.0142	0.0050	0.282	-0.00091	0.000132
860415	0.0153	0.1144	-0.0153	0.0058	-0.005	-0.00065	0.000188
860416	0.0192	0.1427	-0.0206	-0.0133	-0.052	-0.00079	0.000056
860417	0.0265	0.1521	-0.0223	-0.0278	-0.157	-0.00071	0.000026
860418	0.0193	0.1466	-0.0200	-0.0112	-0.255	-0.00055	0.000155
860421	0.0233	0.1829	-0.0252	-0.0119	-0.221	-0.00087	0.000099
860422	0.0219	0.1815	-0.0228	-0.0119	-0.284	-0.00071	0.000230
860423	0.0193	0.1139	-0.0114	-0.0111	0.149	-0.00077	0.000123
860424	0.0156	0.0803	-0.0039	-0.0163	0.263	-0.00072	0.000102
860425	0.0221	0.1367	-0.0132	-0.0122	-0.276	-0.00051	0.000206
860428	0.0216	0.1101	-0.0112	-0.0211	-0.049	-0.00065	-0.000020
860429	0.0168	0.1075	-0.0117	-0.0213	-0.237	-0.00040	0.000012
860430	0.0183	0.1133	-0.0117	-0.0092	-0.268	-0.00042	0.000112
860501	0.0313	0.0750	0.0013	-0.0082	0.025	-0.00055	0.000160
860502	0.0219	0.0732	0.0008	-0.0108	0.058	-0.00062	0.000054
860505	0.0184	0.0926	-0.0058	-0.0057	-0.207	-0.00036	0.000214
860506	0.0211	0.0678	-0.0002	-0.0087	0.039	-0.00046	0.000200
860507	0.0169	0.0685	-0.0025	0.0027	0.008	-0.00030	0.000446
860508	0.0224	0.0894	-0.0050	0.0028	-0.220	-0.00025	0.000428
860509	0.0190	0.0674	-0.0029	-0.0063	-0.107	-0.00029	0.000274
860512	0.0163	0.0698	-0.0010	-0.0111	0.013	-0.00042	0.000214
860513	0.0209	0.0456	0.0036	-0.0160	0.049	-0.00042	0.000042
860514	0.0199	0.0523	0.0017	-0.0033	-0.104	-0.00022	0.000215
860515	0.0184	0.0293	0.0015	-0.0021	-0.058	-0.00009	0.000163
860516	0.0185	0.0314	0.0036	-0.0063	-0.075	-0.00011	0.000168
860519	0.0175	0.0398	0.0061	-0.0032	-0.048	-0.00018	0.000352
860520	0.0218	0.0287	0.0058	-0.0052	-0.079	-0.00001	0.000302
860521	0.0234	0.0171	0.0097	0.0049	-0.106	0.00002	0.000197
860522	0.0163	0.0190	0.0055	-0.0018	-0.163	0.00011	0.000203
860523	0.0200	0.0176	0.0071	-0.0001	-0.187	0.00015	0.000215
860527	0.0178	0.0355	0.0020	0.0032	-0.149	-0.00001	0.000163
860528	0.0153	0.0444	0.0035	-0.0020	-0.239	0.00000	0.000157
860529	0.0188	0.0276	0.0098	0.0068	-0.127	-0.00006	0.000132
860530	0.0180	0.0175	0.0055	0.0033	-0.091	-0.00001	0.000090
860609	0.0144	0.0683	-0.0003	-0.0031	-0.723	0.00010	0.000110
860610	0.0121	0.0932	-0.0032	-0.0007	0.317	-0.00078	0.000051
860611	0.0184	0.0743	-0.0029	-0.0007	0.347	-0.00064	0.000088
860612	0.0115	0.0719	-0.0040	-0.0166	0.410	-0.00066	0.000022
860613	0.0161	0.0917	-0.00[illegible]7	-0.0073	-0.371	-0.00025	0.000073
860616	0.0139	0.0734	-0.00[illegible]5	0.0059	-0.270	-0.00022	0.000095
860617	0.0135	0.0709	-0.0029	-0.0040	-0.262	-0.00024	0.000058
860618	0.0162	0.0529	0.0005	0.0093	-0.359	-0.00004	0.000127
860619	0.0107	0.0528	-0.0028	0.0030	-0.666	0.00020	0.000058
860620	0.0157	0.0678	0.0025	0.0034	21.993	-0.01915	-0.000035
860623	0.0200	0.0672	-0.0026	0.0007	-0.616	0.00011	0.000141
860624	0.0114	0.0737	-0.0065	0.0005	-0.302	-0.00019	0.000076
860625	0.0115	0.0715	-0.0063	-0.0009	-0.266	-0.00021	0.000071
860626	0.0089	0.0477	-0.0033	0.0075	-0.547	0.00025	0.000266
860627	0.0160	0.0669	-0.0044	0.0132	-0.586	0.00012	0.000192
860630	0.0125	0.0776	-0.0079	0.0119	-0.560	0.00007	0.000197
860701	0.0136	0.0955	-0.0090	-0.0037	-0.313	-0.00031	0.000038
860702	0.0122	0.0746	-0.0064	0.0009	-0.520	-0.00001	0.000097
860703	0.0124	0.0981	-0.0107	0.0046	-0.627	0.00001	0.000184

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860707	0.0142	0.0885	-0.0093	-0.0022	-0.582	-0.00004	0.000034
860708	0.0127	0.0738	-0.0084	-0.0083	-0.490	0.00007	0.000198
860709	0.0117	0.0817	-0.0095	-0.0080	-0.385	-0.00014	0.000083
860710	0.0098	0.0845	-0.0095	-0.0073	-0.538	0.00001	0.000128
860711	0.0139	0.0690	-0.0051	0.0035	-0.783	0.00035	0.000233
860714	0.0119	0.0698	-0.0074	0.0046	-0.562	0.00014	0.000199
860715	0.0137	0.1036	-0.0133	-0.0026	-0.423	-0.00023	0.000043
860716	0.0139	0.1010	-0.0137	-0.0037	-0.610	0.00003	0.000118
860717	0.0128	0.0786	-0.0098	0.0010	-0.606	0.00016	0.000112
860718	0.0152	0.1058	-0.0147	0.0034	-0.761	0.00018	0.000160
860721	0.0144	0.1231	-0.0179	-0.0067	-0.615	-0.00009	0.000036
860722	0.0206	0.0804	-0.0113	-0.0049	-0.666	0.00027	0.000103
860723	0.0157	0.0478	-0.0058	-0.0040	-0.511	0.00029	0.000105
860724	0.0148	0.0502	-0.0041	0.0029	-0.593	0.00036	0.000189
860725	0.0126	0.0374	-0.0011	-0.0086	-0.423	0.00027	0.000173
860728	0.0154	0.0539	-0.0051	0.0033	-0.505	0.00038	0.000335
860729	0.0150	0.0578	-0.0051	-0.0059	-0.448	0.00019	0.000127
860730	0.0142	0.0554	-0.0055	-0.0109	-0.366	0.00009	0.000100
860731	0.0147	0.0486	-0.0034	-0.0055	-0.423	0.00021	0.000083
860801	0.0183	0.0409	-0.0015	-0.0142	-0.435	0.00025	0.000043
860804	0.0160	0.0295	-0.0008	-0.0072	-0.365	0.00025	0.000097
860805	0.0137	0.0381	-0.0032	0.0018	-0.487	0.00040	0.000246
860806	0.0179	0.0410	-0.0050	-0.0056	-0.418	0.00032	0.000161
860807	0.0159	0.0335	-0.0020	-0.0078	-0.438	0.00028	0.000052
860808	0.0129	0.0322	-0.0022	-0.0002	-0.565	0.00050	0.000211
860811	0.0155	0.0148	0.0008	-0.0000	-0.392	0.00041	0.000212
860812	0.0152	0.0366	-0.0040	-0.0067	-0.572	0.00049	0.000196
860813	0.0157	0.0173	0.0002	-0.0094	-0.431	0.00040	0.000135
860814	0.0118	0.0235	-0.0000	-0.0026	-0.485	0.00045	0.000127
860815	0.0107	0.0473	-0.0083	-0.0081	-0.566	0.00034	0.000074
860818	0.0147	0.0134	0.0014	-0.0025	-0.387	0.00039	0.000166
860819	0.0152	0.0448	-0.0059	0.0000	-0.546	0.00041	0.000209
860820	0.0162	0.0258	-0.0016	0.0045	-0.449	0.00034	0.000160
860821	0.0148	0.0427	-0.0056	-0.0036	-0.494	0.00024	0.000035
860822	0.0153	0.0370	-0.0029	-0.0039	-0.507	0.00038	0.000112
860825	0.0139	0.0338	-0.0026	-0.0083	-0.440	0.00029	0.000132
860826	0.0158	0.0280	-0.0003	-0.0075	-0.392	0.00025	0.000084
860827	0.0164	0.0177	0.0001	-0.0015	-0.332	0.00026	0.000075
860828	0.0196	0.0419	-0.0048	0.0011	-0.380	0.00021	0.000137
860829	0.0198	0.0409	-0.0057	-0.0113	-0.345	0.00016	0.000046

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860908	0.0119	0.0712	0.0006	-0.0070	-0.961	0.00024	0.000104
860909	0.0119	0.0568	-0.0027	-0.0056	-1.398	0.00065	0.000141
860910	0.0115	0.0735	0.0001	-0.0004	-1.031	0.00026	0.000082
860911	0.0155	0.0453	0.0044	-0.0023	-0.627	0.00013	0.000067
860912	0.0213	0.0690	-0.0025	-0.0036	-0.943	0.00027	0.000140
860915	0.0138	0.1028	-0.0062	-0.0016	-1.101	0.00016	0.000082
860916	0.0150	0.1092	-0.0068	-0.0098	-1.011	0.00009	0.000103
860917	0.0158	0.1307	-0.0123	0.0017	-1.245	0.00019	0.000120
860918	0.0186	0.1445	-0.0160	-0.0116	-1.123	0.00004	0.000051
860919	0.0149	0.1457	-0.0148	-0.0043	-1.476	0.00025	0.000091
860922	0.0152	0.0899	-0.0028	0.0047	-0.964	0.00015	0.000159
860923	0.0144	0.0809	-0.0006	0.0003	-0.685	0.00001	0.000138
860924	0.0125	0.0715	-0.0014	-0.0032	-0.623	0.00004	0.000142
860925	0.0163	0.0575	-0.0011	0.0070	-0.358	-0.00005	0.000133
860926	0.0126	0.0490	-0.0015	0.0052	-0.307	0.00002	0.000211
860929	0.0184	0.0748	-0.0070	-0.0074	-0.488	-0.00000	0.000076
860930	0.0156	0.0654	-0.0040	-0.0130	-0.533	0.00004	0.000045
861001	0.0153	0.0550	-0.0031	-0.0031	-0.716	0.00026	0.000059
861002	0.0146	0.0698	-0.0046	0.0068	-0.929	0.00039	0.000182
861003	0.0151	0.0514	-0.0013	0.0036	-0.738	0.00034	0.000142
861006	0.0102	0.0755	-0.0068	-0.0023	-0.716	0.00020	0.000106
861007	0.0130	0.0713	-0.0069	-0.0021	-0.665	0.00021	0.000158
861008	0.0126	0.0521	-0.0056	-0.0013	-0.656	0.00032	0.000115
861009	0.0128	0.0582	-0.0040	0.0096	-0.784	0.00040	0.000118
861010	0.0187	0.0556	-0.0059	0.0042	-0.807	0.00044	0.000114
861013	0.0162	0.0998	-0.0141	-0.0025	-0.892	0.00041	0.000233
861014	0.0153	0.0938	-0.0127	0.0050	-0.812	0.00023	0.000129
861015	0.0147	0.0704	-0.0099	-0.0061	-0.739	0.00037	0.000136
861016	0.0105	0.0658	-0.0087	-0.0029	-0.680	0.00032	0.000057
861017	0.0109	0.0587	-0.0046	0.0006	-0.916	0.00061	0.000160
861020	0.0111	0.0549	-0.0072	-0.0097	-0.793	0.00050	0.000072
861021	0.0094	0.0437	-0.0015	0.0023	-0.820	0.00066	0.000156
861022	0.0177	0.0535	-0.0081	-0.0049	-0.808	0.00067	0.000138
861023	0.0325	0.0242	0.0027	-0.0024	-0.640	0.00059	0.000128
861024	0.0209	0.0245	0.0048	0.0023	-0.352	0.00022	0.000160
861027	0.0140	0.0315	0.0029	0.0051	-0.243	0.00007	0.000146
861028	0.0123	0.0424	-0.0002	0.0034	-0.277	0.00003	0.000108
861029	0.0129	0.0367	-0.0022	-0.0042	-0.169	-0.00004	0.000030
861030	0.0176	0.0462	0.0018	0.0001	-0.245	-0.00002	0.000136
861031	0.0198	0.0461	0.0009	0.0045	-0.164	-0.00014	0.000044
861103	0.0179	0.0301	0.0017	0.0053	-0.110	-0.00009	0.000042
861104	0.0208	0.0275	0.0044	0.0023	-0.325	0.00018	0.000156
861105	0.0164	0.0275	0.0014	-0.0009	-0.363	0.00028	0.000197
861106	0.0127	0.0287	0.0000	0.0046	-0.368	0.00035	0.000284
861107	0.0166	0.0411	-0.0023	0.0033	-0.395	0.00019	-0.000023
861110	0.0113	0.0381	-0.0031	-0.0048	-0.322	0.00018	0.000091
861111	0.0122	0.0420	-0.0024	-0.0074	-0.364	0.00016	0.000029
861112	0.0143	0.0496	-0.0036	-0.0070	-0.325	0.00005	-0.000029
861113	0.0149	0.0383	-0.0010	-0.0008	-0.364	0.00032	0.000268
861114	0.0137	0.0339	-0.0018	-0.0116	-0.324	0.00026	0.000099
861117	0.0130	0.0310	-0.0011	-0.0058	-0.255	0.00014	0.000090
861118	0.0146	0.0327	-0.0016	-0.0014	-0.265	0.00017	0.000147
861119	0.0145	0.0433	-0.0018	-0.0086	-0.288	0.00009	0.000056
861120	0.0124	0.0259	-0.0014	-0.0050	-0.282	0.00030	0.000260
861121	0.0140	0.0334	0.0013	-0.0044	-0.259	0.00001	-0.000136

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
861124	0.0139	0.0221	0.0012	-0.0077	-0.214	0.00011	0.000024
861125	0.0179	0.0298	-0.0018	-0.0053	-0.220	0.00016	0.000108
861126	0.0190	0.0290	0.0006	-0.0197	-0.176	0.00004	0.000003
861128	0.0127	0.0283	-0.0032	-0.0120	-0.164	0.00011	0.000137
861208	0.0109	0.0658	-0.0027	0.0024	-0.742	0.00023	0.000127
861209	0.0102	0.0651	0.0003	0.0021	-0.869	0.00034	0.000173
861210	0.0117	0.0572	-0.0042	0.0045	-0.813	0.00031	0.000073
861211	0.0173	0.0678	-0.0049	-0.0019	-0.698	0.00014	-0.000040
861212	0.0114	0.0528	-0.0037	0.0004	-0.910	0.00049	0.000172
861215	0.0151	0.0389	-0.0018	-0.0042	-0.753	0.00038	-0.000022
861216	0.0144	0.0279	-0.0020	0.0113	-0.890	0.00072	0.000267
861217	0.0114	0.0400	-0.0027	-0.0015	-0.873	0.00051	0.000002
861218	0.0115	0.0469	-0.0033	0.0002	-0.816	0.00048	0.000058
861219	0.0168	0.0683	-0.0078	-0.0116	-0.827	0.00037	-0.000018
861222	0.0115	0.0669	-0.0074	-0.0031	-0.926	0.00053	0.000112
861223	0.0107	0.0772	-0.0091	-0.0104	-0.925	0.00048	0.000043
861224	0.0141	0.0981	-0.0152	-0.0045	-1.024	0.00048	0.000032
861226	0.0154	0.0353	-0.0027	-0.0011	-0.645	0.00048	0.000128
861229	0.0120	0.1002	-0.0148	-0.0186	-0.913	0.00040	0.000012
861230	0.0152	0.0920	-0.0129	-0.0209	-0.751	0.00023	-0.000032
861231	0.0212	0.0952	-0.0153	-0.0117	-0.798	0.00028	0.000038
870102	0.0132	0.0838	-0.0107	-0.0074	-0.692	0.00021	0.000044
870105	0.0202	0.0860	-0.0093	-0.0054	-0.721	0.00026	0.000115
870106	0.0143	0.1211	-0.0204	-0.0092	-0.727	0.00008	0.000038
870107	0.0130	0.0931	-0.0117	-0.0023	-0.870	0.00036	0.000142
870108	0.0131	0.1040	-0.0171	-0.0046	-0.908	0.00032	0.000074
870109	0.0145	0.1243	-0.0189	0.0002	-0.899	0.00023	0.000125
870112	0.0182	0.0900	-0.0148	-0.0107	-0.458	0.00000	0.000021
870113	0.0207	0.0989	-0.0151	-0.0168	-0.518	0.00004	0.000056
870114	0.0679	0.1166	-0.0122	-0.0364	-0.003	-0.00045	0.000039
870115	0.0399	0.1142	-0.0135	-0.0135	-0.225	-0.00027	0.000096
870116	0.0310	0.1042	-0.0123	-0.0082	-0.228	-0.00024	0.000110
870119	0.0301	0.1638	-0.0198	-0.0026	0.001	-0.00069	0.000070
870120	0.0216	0.0963	-0.0096	-0.0058	0.159	-0.00052	0.000097
870121	0.0227	0.0798	-0.0093	-0.0050	0.192	-0.00049	0.000088
870122	0.0288	0.1219	-0.0161	-0.0095	0.010	-0.00053	0.000079
870123	0.0226	0.1046	-0.0131	0.0005	-0.003	-0.00055	0.000055
870126	0.0162	0.0892	-0.0074	0.0036	0.004	-0.00040	0.000157
870127	0.0290	0.0883	-0.0088	0.0078	-0.059	-0.00041	0.000122
870128	0.0374	0.1358	-0.0162	-0.0042	-0.032	-0.00055	0.000141
870129	0.0241	0.1296	-0.0157	0.0073	0.028	-0.00065	0.000138
870130	0.0586	0.1103	-0.0121	0.0005	0.069	-0.00064	0.000102
870202	0.0202	0.0865	-0.0074	0.0097	-0.091	-0.00037	0.000129
870203	0.0238	0.1169	-0.0129	-0.0003	-0.323	-0.00035	0.000124
870204	0.0179	0.0656	-0.0043	-0.0022	-0.256	-0.00014	0.000064
870205	0.0218	0.0596	-0.0028	0.0157	-0.250	-0.00009	0.000157
870206	0.0241	0.0575	-0.0023	0.0128	-0.020	-0.00036	0.000019
870209	0.0279	0.0685	-0.0045	-0.0025	-0.177	-0.00020	0.000130
870210	0.0314	0.0897	-0.0051	-0.0060	-0.295	-0.00016	0.000201
870211	0.0228	0.0715	-0.0047	-0.0054	-0.367	-0.00000	0.000253
870212	0.0344	0.0420	0.0023	-0.0159	-0.378	0.00014	0.000221
870213	0.0246	0.0487	0.0007	0.0006	-0.512	0.00032	0.000433
870217	0.0206	0.0582	-0.0030	-0.0088	-0.248	-0.00003	0.000152
870218	0.0197	0.0445	0.0023	-0.0000	-0.094	-0.00021	0.000021
870219	0.0318	0.0500	0.0018	0.0017	-0.006	-0.00030	0.000085

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
870220	0.0290	0.0617	-0.0048	-0.0042	0.235	-0.00045	0.000311
870223	0.0218	0.0331	0.0050	-0.0021	-0.123	-0.00015	-0.000048
870224	0.0200	0.0313	0.0071	-0.0019	-0.161	0.00001	0.000214
870225	0.0172	0.0215	0.0004	0.0020	-0.085	0.00004	0.000201
870226	0.0220	0.0351	-0.0031	-0.0095	-0.094	-0.00002	0.000129
870227	0.0406	0.0438	-0.0054	-0.0164	-0.218	-0.00001	0.000002
870309	0.0170	0.0934	0.0001	-0.0065	-1.403	0.00035	0.000033
870310	0.0268	0.0597	0.0008	0.0046	-1.482	0.00065	0.000218
870311	0.0190	0.0894	0.0020	0.0035	-1.792	0.00061	0.000061
870312	0.0135	0.0728	0.0036	0.0003	-1.827	0.00076	0.000050
870313	0.0160	0.0766	-0.0019	0.0080	-1.870	0.00086	0.000099
870316	0.0195	0.1009	-0.0078	-0.0039	-1.505	0.00053	0.000188
870317	0.0168	0.1064	-0.0068	0.0040	-1.295	0.00037	0.000119
870318	0.0160	0.0749	-0.0025	0.0090	-1.036	0.00033	0.000019
870319	0.0146	0.0718	-0.0056	0.0006	-0.898	0.00037	0.000177
870320	0.0127	0.0741	-0.0034	-0.0016	-0.990	0.00041	0.000067
870323	0.0189	0.1037	-0.0073	0.0135	-1.075	0.00026	0.000088
870324	0.0212	0.0996	-0.0045	0.0025	-1.155	0.00033	0.000198
870325	0.0175	0.1007	-0.0058	-0.0063	-1.188	0.00039	0.000094
870326	0.0138	0.0772	-0.0069	-0.0037	-1.321	0.00079	0.000202
870327	0.0240	0.1381	-0.0167	0.0018	-1.410	0.00038	0.000044
870330	0.0188	0.1309	-0.0161	-0.0028	-1.177	0.00029	0.000106
870331	0.0188	0.1219	-0.0144	-0.0026	-1.233	0.00036	0.000153

Notes:
- Sigma is the RMSE of the regression
- The estimates for 860620 are obvious outliers, they should be excluded from any analysis

Table 13

Parameter Estimates for the SF

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860203	0.0170	0.0751	-0.0197	0.0028	-0.536	0.00025	0.000048
860204	0.0141	0.0624	-0.0071	0.0166	-0.538	0.00031	0.000318
860205	0.0164	0.0764	-0.0147	0.0047	-0.532	0.00028	0.000170
860206	0.0183	0.0629	-0.0120	-0.0080	-0.487	0.00022	0.000072
860207	0.0249	0.0258	-0.0098	-0.0127	-0.364	0.00027	-0.000061
860210	0.0206	0.1000	-0.0241	-0.0067	-0.458	0.00007	0.000057
860211	0.0200	0.0651	-0.0120	-0.0047	-0.382	0.00014	0.000094
860212	0.0217	0.0555	-0.0115	0.0009	-0.297	0.00011	0.000083
860213	0.0319	0.0363	-0.0041	-0.0066	-0.148	-0.00000	0.000042
860214	0.0288	0.0726	-0.0136	-0.0001	-0.241	-0.00013	-0.000012
860218	0.0295	0.0590	-0.0106	-0.0077	-0.085	-0.00025	-0.000107
860219	0.0635	0.0866	-0.0109	-0.0062	-0.053	-0.00032	0.000013
860220	0.0320	0.0661	-0.0060	0.0009	-0.051	-0.00015	0.000157
860221	0.0289	0.0691	-0.0112	0.0051	-0.158	-0.00019	0.000010
860224	0.0294	0.0803	-0.0105	-0.0175	-0.082	-0.00025	0.000007
860225	0.0349	0.0667	-0.0105	-0.0153	0.002	-0.00031	-0.000076
860226	0.0405	0.1500	-0.0243	-0.0173	-0.369	-0.00043	-0.000095
860227	0.0687	0.2028	-0.0319	-0.0048	-0.563	-0.00038	0.000089
860228	0.0493	0.1958	-0.0259	-0.0148	-0.570	-0.00033	0.000190
860310	0.0233	0.1527	-0.0132	0.0186	-0.442	-0.00043	0.000160
860311	0.0212	0.1718	-0.0167	0.0054	-0.108	-0.00075	0.000081
860312	0.0235	0.1163	-0.0049	0.0200	0.053	-0.00056	0.000152
860313	0.0221	0.1153	-0.0190	-0.0089	0.371	-0.00073	-0.000050
860314	0.0234	0.1404	-0.0140	0.0003	0.545	-0.00091	0.000068
860317	0.0220	0.1957	-0.0229	-0.0256	0.731	-0.00122	-0.000024
860318	0.0275	0.2231	-0.0258	-0.0243	-0.465	-0.00086	-0.000158
860319	0.0174	0.1598	-0.0155	0.0026	-0.177	-0.00065	0.000145
860320	0.0159	0.1434	-0.0172	0.0048	0.557	-0.00095	0.000053
860321	0.0196	0.2152	-0.0265	0.0097	-0.092	-0.00091	0.000042
860324	0.0198	0.1420	-0.0124	0.0173	0.789	-0.00118	0.000009
860325	0.0241	0.0997	-0.0063	0.0059	0.525	-0.00079	0.000158
860326	0.0189	0.1542	-0.0117	0.0224	0.250	-0.00104	-0.000007
860327	0.0212	0.1411	-0.0143	0.0171	0.671	-0.00118	-0.000014
860331	0.0293	0.0929	-0.0094	-0.0102	0.680	-0.00074	0.000269
860401	0.0183	0.1041	-0.0125	0.0076	0.781	-0.00092	0.000126
860402	0.0203	0.0693	-0.0095	0.0010	0.645	-0.00066	0.000148
860403	0.0279	0.0603	-0.0067	0.0126	0.806	-0.00074	0.000142
860404	0.0196	0.0756	-0.0088	0.0184	0.894	-0.00089	0.000048
860407	0.0228	0.0595	-0.0095	0.0115	0.726	-0.00076	0.000140
860408	0.0197	0.1132	-0.0084	0.0141	0.376	-0.00089	0.000078

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860409	0.0227	0.1429	-0.0096	0.0013	0.849	-0.00130	0.000052
860410	0.0355	0.1267	-0.0167	-0.0203	0.309	-0.00073	0.000178
860411	0.0268	0.1519	-0.0167	-0.0148	0.363	-0.00097	0.000093
860414	0.0194	0.1363	-0.0111	0.0026	-0.032	-0.00061	0.000393
860415	0.0340	0.1359	-0.0160	0.0191	-0.459	-0.00027	0.000499
860416	0.0197	0.2003	-0.0223	-0.0091	-0.072	-0.00093	0.000061
860417	0.0298	0.2439	-0.0307	-0.0195	-0.468	-0.00081	0.000088
860418	0.0252	0.2128	-0.0230	-0.0073	-0.514	-0.00072	0.000037
860421	0.0206	0.2293	-0.0320	-0.0016	-0.355	-0.00077	0.000133
860422	0.0231	0.2349	-0.0312	-0.0053	-0.358	-0.00084	0.000105
860423	0.0276	0.2113	-0.0261	-0.0019	-0.510	-0.00065	0.000182
860424	0.0248	0.2153	-0.0211	-0.0209	-0.613	-0.00070	0.000015
860425	0.0311	0.1478	-0.0110	-0.0175	-0.723	-0.00017	0.000176
860428	0.0252	0.1671	-0.0195	-0.0179	-0.644	-0.00036	0.000028
860429	0.0269	0.1001	-0.0086	-0.0132	-0.441	-0.00016	0.000081
860430	0.0226	0.1082	-0.0097	0.0011	-0.294	-0.00032	0.000137
860501	0.0297	0.0960	-0.0075	-0.0050	-0.165	-0.00039	0.000109
860502	0.0196	0.0736	0.0034	-0.0049	0.181	-0.00064	0.000036
860505	0.0199	0.1092	-0.0059	-0.0037	-0.277	-0.00040	0.000165
860506	0.0225	0.0681	0.0002	0.0027	-0.143	-0.00015	0.000332
860507	0.0247	0.0687	-0.0028	-0.0043	0.081	-0.00048	0.000051
860508	0.0277	0.1005	-0.0112	-0.0223	0.020	-0.00060	-0.000039
860509	0.0882	-0.0378	0.0019	-0.0396	0.562	-0.00011	-0.000011
860512	0.0202	0.0952	-0.0100	-0.0229	-0.127	-0.00046	-0.000058
860513	0.0547	0.0536	0.0009	-0.0035	-0.067	-0.00040	-0.000094
860514	0.0236	0.0566	-0.0055	-0.0116	-0.003	-0.00036	-0.000114
860515	0.0228	0.0282	0.0049	0.0068	-0.084	-0.00013	0.000082
860516	0.0195	0.0348	0.0055	0.0024	-0.179	0.00002	0.000315
860519	0.0184	0.0322	0.0024	0.0103	-0.057	-0.00008	0.000224
860520	0.0228	0.0440	-0.0019	-0.0049	-0.160	0.00005	0.000378
860521	0.0242	0.0276	0.0011	0.0098	-0.157	0.00003	0.000166
860522	0.0176	0.0315	-0.0010	0.0022	-0.205	0.00017	0.000289
860523	0.0212	0.0235	0.0001	0.0092	-0.320	0.00029	0.000259
860527	0.0204	0.0321	-0.0033	0.0011	-0.243	0.00006	0.000047
860528	0.0189	0.0489	-0.0006	0.0015	-0.290	-0.00007	-0.000017
860529	0.0245	0.0137	-0.0003	0.0126	-0.201	0.00024	0.000231
860530	0.0209	0.0309	0.0035	0.0142	-0.194	0.00004	0.000166
860609	0.0252	0.1244	-0.0085	-0.0194	-0.336	-0.00053	-0.000186
860610	0.0186	0.0910	-0.0048	-0.0035	0.403	-0.00059	-0.000043
860611	0.0190	0.1178	-0.0091	-0.0170	0.231	-0.00064	-0.000052
860612	0.0151	0.0675	-0.0007	-0.0014	0.120	-0.00036	0.000056
860613	0.0159	-0.0045	0.0060	0.0121	-0.123	0.00022	0.000366
860616	0.0139	0.0668	-0.0008	-0.0199	0.055	-0.00033	0.000046
860617	0.0204	0.0315	0.0036	-0.0159	0.504	-0.00042	0.000058
860618	0.0151	0.0292	0.0025	0.0049	-0.049	-0.00006	0.000198
860619	0.0133	-0.0363	0.0113	0.0116	32.283	-0.01708	0.000176
860620	0.0206	0.0669	0.0008	0.0093	-1.345	0.00037	0.000120
860623	0.0147	0.0577	-0.0010	0.0071	0.350	-0.00051	0.000061
860624	0.0227	0.0683	-0.0021	0.0041	-0.473	-0.00006	0.000107
860625	0.0191	0.0719	-0.0035	-0.0046	-0.030	-0.00036	0.000005
860626	0.0151	0.0620	-0.0021	-0.0095	-0.715	0.00012	0.000099
860627	0.0207	0.0956	-0.0075	-0.0140	-0.625	-0.00011	0.000001
860630	0.0189	0.1177	-0.0093	0.0030	-1.076	0.00006	0.000090
860701	0.0171	0.1655	-0.0164	-0.0020	-1.079	-0.00011	0.000089
860702	0.0168	0.0626	-0.0025	0.0003	-0.669	0.00009	0.000095

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860703	0.0196	0.1550	-0.0170	-0.0030	-1.250	0.00002	0.000038
860707	0.0241	0.1293	-0.0125	-0.0165	-0.648	-0.00020	0.000011
860708	0.0145	0.0767	-0.0051	-0.0157	-0.027	-0.00036	0.000030
860709	0.0184	0.0743	-0.0039	-0.0041	-0.715	0.00007	0.000097
860710	0.0166	0.0909	-0.0072	-0.0156	-0.614	-0.00011	-0.000032
860711	0.0213	0.0385	0.0003	0.0192	-0.925	0.00048	0.000220
860714	0.0179	0.0742	-0.0071	0.0018	-0.711	0.00012	0.000109
860715	0.0279	0.1099	-0.0120	-0.0239	-0.764	-0.00006	-0.000083
860716	0.0171	0.1569	-0.0158	0.0034	-1.024	-0.00004	0.000184
860717	0.0248	0.1040	-0.0115	-0.0069	-0.854	0.00009	0.000108
860718	0.0211	0.1643	-0.0220	-0.0044	-1.063	-0.00002	0.000065
860721	0.0290	0.0621	-0.0012	-0.0154	-0.375	-0.00008	0.000010
860722	0.0267	0.1099	-0.0131	-0.0035	-0.713	-0.00004	0.000034
860723	0.0332	0.0550	-0.0037	-0.0099	-0.537	0.00013	0.000112
860724	0.0168	0.0605	-0.0062	-0.0073	-0.364	-0.00001	0.000061
860725	0.0188	0.0412	-0.0017	0.0031	-0.340	0.00008	0.000171
860728	0.0226	0.0677	-0.0073	-0.0078	-0.389	-0.00001	0.000085
860729	0.0270	0.0589	-0.0061	0.0020	-0.472	0.00010	0.000138
860730	0.0265	0.0593	-0.0051	-0.0030	-0.454	0.00014	0.000239
860731	0.0239	0.0462	-0.0024	0.0101	-0.416	0.00008	0.000126
860801	0.0264	0.0139	0.0024	-0.0115	-0.159	0.00004	0.000083
860804	0.0218	0.0268	-0.0003	-0.0121	-0.145	-0.00003	0.000010
860805	0.0222	0.0240	-0.0020	-0.0098	-0.177	-0.00001	-0.000025
860806	0.0220	-0.0180	0.0085	0.0134	-0.147	0.00027	0.000307
860807	0.0178	0.0207	0.0016	-0.0070	-0.332	0.00013	0.000072
860808	0.0288	0.0490	-0.0049	-0.0021	-0.660	0.00033	0.000238
860811	0.0205	-0.0221	0.0073	-0.0012	-0.083	0.00017	0.000127
860812	0.0300	0.0216	-0.0022	-0.0095	-0.401	0.00017	0.000039
860813	0.0175	0.0255	-0.0018	-0.0030	-0.518	0.00020	0.000025
860814	0.0165	0.0193	0.0001	0.0048	-0.463	0.00026	0.000188
860815	0.0173	0.0540	-0.0052	-0.0025	-0.572	0.00016	0.000104
860818	0.0266	0.0329	-0.0070	0.0114	-0.418	0.00030	0.000290
860819	0.0271	0.0546	-0.0056	0.0083	-0.463	0.00013	0.000170
860820	0.0205	0.0580	-0.0099	0.0061	-0.455	0.00014	0.000140
860821	0.0158	0.0692	-0.0118	0.0027	-0.478	-0.00000	-0.000005
860822	0.0243	0.0354	-0.0031	0.0024	-0.372	0.00007	0.000015
860825	0.0218	0.0704	-0.0066	-0.0141	-0.532	0.00005	0.000043
860826	0.0236	0.0323	-0.0017	0.0020	-0.396	0.00011	0.000050
860827	0.0207	0.0385	-0.0025	0.0029	-0.422	0.00017	0.000108
860828	0.0258	0.0390	-0.0054	-0.0075	-0.310	0.00013	0.000138
860829	0.0268	0.0504	-0.0055	0.0010	-0.406	0.00026	0.000333

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
860908	0.0208	0.0523	0.0016	0.0068	-1.116	0.00021	0.000033
860909	0.0216	0.0694	-0.0018	0.0032	-1.259	0.00022	0.000057
860910	0.0142	0.0371	-0.0005	-0.0121	-2.147	0.00094	0.000480
860911	0.0193	0.0707	0.0022	-0.0051	-1.013	0.00009	0.000060
860912	0.0267	0.0950	-0.0021	-0.0104	-0.388	-0.00036	-0.000126
860915	0.0201	0.1063	-0.0042	-0.0022	-0.563	-0.00020	0.000096
860916	0.0173	0.1332	-0.0097	-0.0158	-1.066	-0.00013	-0.000007
860917	0.0218	0.1095	-0.0064	-0.0115	-1.234	0.00011	0.000111
860918	0.0243	0.1931	-0.0175	-0.0227	-1.558	-0.00009	0.000006
860919	0.0207	0.2944	-0.0277	-0.0059	-2.409	-0.00010	0.000085
860922	0.0221	0.1533	-0.0063	0.0112	-1.741	0.00011	0.000193
860923	0.0306	0.1117	-0.0063	0.0081	-1.187	-0.00002	0.000001
860924	0.0220	0.0640	-0.0019	0.0101	-1.390	0.00036	0.000169
860925	0.0202	0.0502	-0.0008	0.0041	-0.822	0.00024	0.000265
860926	0.0203	0.1050	-0.0040	-0.0009	-1.336	0.00012	0.000016
860929	0.0199	0.0570	-0.0042	-0.0055	-0.401	0.00004	0.000259
860930	0.0237	0.0879	-0.0025	-0.0029	-0.832	-0.00000	0.000050
861001	0.0205	0.1014	-0.0074	-0.0045	-0.776	-0.00007	0.000015
861002	0.0228	0.0483	-0.0017	0.0076	-1.403	0.00064	0.000347
861003	0.0279	0.0979	-0.0082	0.0015	-1.415	0.00033	0.000104
861006	0.0216	0.0901	-0.0089	-0.0117	-0.898	0.00012	0.000041
861007	0.0173	0.0970	-0.0067	0.00[illegible]4	-0.868	-0.00001	-0.000030
861008	0.0239	0.0896	-0.0064	0.0116	-1.362	0.00034	0.000071
861009	0.0206	0.1187	-0.0100	-0.0177	-1.464	0.00026	-0.000264
861010	0.0209	0.1292	-0.0146	-0.0124	-1.465	0.00025	-0.000123
861013	0.0200	0.0731	-0.0051	-0.0099	-1.318	0.00045	0.000072
861014	0.0154	0.1045	-0.0151	-0.0008	-1.102	0.00019	-0.000005
861015	0.0168	0.0653	-0.0057	-0.0059	-1.113	0.00040	0.000037
861016	0.0152	0.0972	-0.0120	-0.0099	-1.079	0.00025	-0.000042
861017	0.0178	0.0499	-0.0030	-0.0032	-1.315	0.00070	0.000122
861020	0.0186	0.0741	-0.0052	0.0106	-1.264	0.00050	0.000060
861021	0.0174	0.0267	0.0017	0.0123	-1.110	0.00068	0.000179
861022	0.0280	0.0104	-0.0011	-0.0071	-1.689	0.00168	0.000702
861023	0.0313	0.0259	-0.0007	0.0025	-0.966	0.00060	0.000081
861024	0.0285	0.0352	0.0028	0.0063	-0.695	0.00026	0.000019
861027	0.0220	0.0447	0.0029	0.0182	-0.708	0.00030	0.000187
861028	0.0205	0.0523	0.0004	0.0104	-0.730	0.00030	0.000252
861029	0.0201	0.0583	-0.0000	0.0047	-0.638	0.00013	0.000081
861030	0.0254	0.0285	0.0030	0.0006	-0.614	0.00036	0.000273
861031	0.0256	0.0388	-0.0010	0.0066	-0.480	0.00017	0.000146
861103	0.0249	0.0440	0.0013	0.0057	-0.507	0.00016	0.000177
861104	0.0175	0.0369	-0.0003	-0.0033	-0.520	0.00023	0.000131
861105	0.0192	0.0419	-0.0036	0.0110	-0.472	0.00011	-0.000093
861106	0.0146	0.0304	0.0007	0.0088	-0.483	0.00028	0.000132
861107	0.0189	0.0420	-0.0029	0.0001	-0.546	0.00021	0.000019
861110	0.0164	0.0336	-0.0027	0.0019	-0.467	0.00024	0.000122
861111	0.0136	0.0283	-0.0005	-0.0087	-0.436	0.00024	0.000108
861112	0.0158	0.0391	-0.0046	-0.0098	-0.550	0.0004[illegible]	0.000247
861113	0.0194	0.0322	-0.0061	-0.0088	-0.495	0.000[illegible]1	0.000081
861114	0.0168	0.0379	-0.0028	-0.0189	-0.480	0.00036	0.000240
861117	0.0151	0.0307	-0.0017	-0.0107	-0.452	0.00042	0.000469
861118	0.0146	0.0329	-0.0003	-0.0018	-0.427	0.00030	0.000311
861119	0.0130	0.0270	-0.0004	-0.0093	-0.332	0.00027	0.000175
861120	0.0173	0.0405	-0.0060	0.0031	-0.409	0.00023	0.000088
861121	0.0194	0.0249	-0.0013	0.0095	-0.344	0.00021	0.000091

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
861124	0.0158	0.0167	0.0037	0.0070	-0.257	0.00015	0.000087
861125	0.0177	0.0270	-0.0042	-0.0069	-0.246	0.00023	0.000154
861126	0.0283	0.0022	0.0019	-0.0065	-0.187	0.00024	0.000191
861128	0.0186	0.0395	-0.0030	-0.0204	-0.240	0.00014	0.000162
861208	0.0135	0.0405	0.0003	0.0000	-0.882	0.00032	0.000238
861209	0.0185	0.0580	-0.0030	0.0160	-0.891	0.00022	0.000133
861210	0.0130	0.0490	0.0011	0.0016	-0.786	0.00017	0.000098
861211	0.0168	0.0452	-0.0010	0.0004	-0.497	0.00010	0.000137
861212	0.0204	0.0822	-0.0026	-0.0046	-1.048	0.00012	-0.000052
861215	0.0136	0.0382	-0.0055	0.0048	-0.719	0.00034	0.000319
861216	0.0230	0.0417	-0.0002	0.0054	-1.613	0.00083	0.000219
861217	0.0132	0.0402	0.0005	0.0201	-5.522	0.00333	0.000295
861218	0.0231	0.0571	-0.0072	0.0136	-0.879	0.00025	0.000005
861219	0.0141	0.0745	-0.0005	0.0097	-1.432	0.00051	0.000172
861222	0.0188	0.0858	-0.0051	0.0068	-0.772	0.00009	0.000125
861223	0.0155	0.1057	-0.0091	-0.0072	-1.027	0.00021	0.000132
861224	0.0160	0.1078	-0.0073	-0.0269	-1.077	0.00012	-0.000072
861226	0.0172	0.0857	-0.0069	-0.0110	-1.307	0.00037	0.000043
861229	0.0172	0.0624	-0.0050	-0.0122	-0.828	0.00012	0.000017
861230	0.0140	0.0969	-0.0068	-0.0176	-1.173	0.00027	0.000006
861231	0.0223	0.1019	-0.0088	-0.0106	-1.208	0.00034	0.000137
870102	0.0170	0.1064	-0.0078	-0.0093	-1.314	0.00032	0.000044
870105	0.0264	0.1176	-0.0092	-0.0100	-1.144	0.00016	-0.000024
870106	0.0166	0.0844	-0.0048	-0.0036	-1.200	0.00039	0.000104
870107	0.0166	0.0830	-0.0058	-0.0074	-0.770	0.00009	0.000047
870108	0.0176	0.1058	-0.0105	-0.0184	-0.741	-0.00003	-0.000055
870109	0.0218	0.1012	-0.0093	-0.0066	-0.892	0.00010	0.000029
870112	0.0212	0.1668	-0.0215	-0.0068	-1.006	-0.00006	0.000021
870113	0.0204	0.1522	-0.0174	0.0015	-0.796	-0.00011	0.000101
870114	0.0591	0.1077	-0.0067	-0.0213	-0.246	-0.00036	-0.000024
870115	0.0367	0.1095	-0.0069	0.0018	-0.018	-0.00040	0.000087
870116	0.0378	0.0627	-0.0017	0.0182	0.181	-0.00028	0.000215
870119	0.0337	0.1357	-0.0095	0.0096	0.263	-0.00069	0.000135
870120	0.0328	0.0829	-0.0014	0.0133	1.047	-0.00089	0.000147
870121	0.0311	0.0497	0.0031	0.0059	0.635	-0.00048	0.000189
870122	0.0270	0.1286	-0.0104	-0.0007	0.580	-0.00092	0.000085
870123	0.0217	0.1092	-0.0067	0.0039	0.452	-0.00081	-0.000012
870126	0.0212	0.0792	-0.0029	0.0134	0.675	-0.00072	0.000189
870127	0.0300	0.1246	-0.0125	0.0001	0.353	-0.00079	0.000043
870128	0.0605	0.2139	-0.0197	-0.0096	0.046	-0.00101	-0.000009
870129	0.0259	0.1998	-0.0227	-0.0026	-0.111	-0.00081	0.000093
870130	0.0543	0.1039	-0.002	0.0445	0.586	-0.00084	0.000225
870202	0.0267	0.1029	-0.0064	-0.0017	0.277	-0.00067	0.000083
870203	0.0272	0.1548	-0.0161	-0.0106	0.120	-0.00075	0.000103
870204	0.0254	0.0874	-0.0036	0.0071	-0.257	-0.00027	0.000280
870205	0.0290	0.0931	0.0019	0.0202	0.094	-0.00070	-0.000092
870206	0.0309	0.0723	0.0011	0.0129	-0.269	-0.00010	0.000403
870209	0.0324	0.0942	-0.0056	-0.0005	-0.010	-0.00042	0.000137
870210	0.0446	0.0757	0.0005	0.0272	-0.049	-0.00033	0.000301
870211	0.0206	0.0696	-0.0012	0.0050	-0.178	-0.00024	0.000183
870212	0.0376	0.0574	0.0081	0.0104	-0.307	-0.00023	-0.000039
870213	0.0294	0.0532	0.0083	0.0166	-0.500	0.00003	0.000136
870217	0.0194	0.0558	-0.0016	-0.0061	-0.425	0.00008	0.000176
870218	0.0204	0.0364	0.0037	0.0012	-0.354	0.00019	0.000260
870219	0.0284	0.0444	-0.0050	0.0001	-0.146	0.00001	0.000332

DATE	SIGMA	INTERCEP	CT	DELTA	GAMMA	LAMBDA	FI
870220	0.0395	0.0574	0.0011	-0.0031	0.206	-0.00048	0.000328
870223	0.0450	0.0397	0.002[illegible]	0.0152	-0.364	0.00028	0.000465
870224	0.0179	0.0488	0.001[illegible]	-0.0000	-0.3[illegible]3	0.00013	0.000181
870225	0.0204	0.0244	-0.0010	-0.0018	-0.1[illegible]6	-0.00004	-0.000076
870226	0.0332	0.0580	-0.0077	-0.0100	-0.1[illegible]4	-0.00012	-0.000041
870227	0.0556	0.0532	-0.0074	-0.0297	-0.283	0.00007	0.000137
870309	0.01[illegible]8	0.0929	0.0073	0.0031	-1.656	0.00020	0.000005
870310	0.0227	0.0979	0.0016	-0.0016	-1.412	0.00006	-0.000113
870311	0.0209	0.0785	0.0006	-0.0018	-2.546	0.00077	0.000125
870312	0.0241	0.0389	0.0007	-0.0084	-2.114	0.00090	0.000324
870313	0.0497	0.0471	0.0003	0.0215	-3.951	0.00171	0.000597
870316	0.0141	0.0809	-0.0031	0.0126	-2.172	0.00066	0.000125
870317	0.0182	0.1061	-0.0091	-0.0091	-1.971	0.00052	0.000109
870318	0.0194	0.0915	-0.0055	-0.0055	-2.709	0.00090	0.000071
870319	0.0189	0.0729	-0.0025	0.0392	-35.363	0.01869	0.000712
870320	0.0163	0.0844	-0.0128	0.0157	2[illegible].572	-0.01147	0.000192
870323	0.0351	0.0957	-0.0096	0.0241	-1.853	0.00063	0.000351
870324	0.0266	0.1093	-0.0055	0.0315	-1.952	0.00057	0.000375
870325	0.0256	0.1377	-0.0091	0.0204	-1.870	0.00035	-0.000055
870326	0.0215	0.0813	-0.0045	0.0141	-1.449	0.00050	0.000262
870327	0.0276	0.1292	-0.0096	0.0237	-1.806	0.00043	0.000170
870330	0.0558	0.1497	-0.0168	-0.0026	-1.596	0.00030	0.000199
870331	0.0349	0.1015	-0.0077	0.0103	-1.896	0.00063	0.000279

Notes:
- Sigma is the RMSE of the regression
- The estimates for 860619, 870319 and 870320 are obvious outliers, they should be excluded from any analysis

TABLE 14

First-Order Autocorrelation of Estimated Parameters

	DM	SF
Intercept	0.808	0.732
Ct	0.763	0.623
δ	0.496	0.347
γ	0.886	0.723
λ	0.893	0.737
ϕ	0.211	0.206

Correlation Between the Estimates for the DM and the SF

Intercept	0.799
Ct	0.715
δ	0.354
γ	0.796
λ	0.803
ϕ	0.048

Table 15

Modified Parameter Estimates (β') for the DM

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860203	-0.0075	-0.0001	-0.0196	0.0096	0.0032
860204	-0.0064	-0.0030	-0.0288	0.0112	-0.0027
860205	-0.0052	-0.0025	-0.0298	0.0197	0.0083
860206	-0.0069	-0.0011	-0.0237	0.0045	-0.0024
860207	-0.0045	-0.0037	-0.0173	0.0140	0.0061
860210	-0.0153	-0.0000	-0.0222	-0.0008	-0.0000
860211	-0.0055	-0.0058	-0.0198	0.0015	0.0001
860212	-0.0003	-0.0063	-0.0141	0.0018	0.0045
860213	-0.0086	-0.0087	-0.0164	-0.0038	-0.0068
860214	-0.0069	-0.0012	-0.0173	0.0007	-0.0011
860218	-0.0050	-0.0033	-0.0120	0.0018	0.0016
860219	-0.0054	-0.0081	-0.0020	-0.0047	0.0151
860220	-0.0071	-0.0064	0.0007	-0.0166	0.0035
860221	-0.0115	-0.0039	-0.0150	-0.0052	0.0009
860224	-0.0148	-0.0035	-0.0077	-0.0095	0.0082
860225	-0.0233	-0.0077	-0.0086	-0.0177	-0.0019
860226	-0.0284	-0.0076	-0.0123	-0.0175	-0.0033
860227	-0.0216	-0.0023	-0.0073	-0.0170	0.0047
860228	-0.0246	-0.0070	-0.0079	-0.0190	0.0022
860310	-0.0189	-0.0016	0.0005	-0.0439	-0.0100
860311	-0.0269	0.0011	-0.0098	-0.0204	0.0123
860312	-0.0250	-0.0016	-0.0115	-0.0197	0.0080
860313	-0.0074	0.0001	-0.0113	-0.0052	0.0130
860314	-0.0247	-0.0014	-0.0124	-0.0234	0.0097
860317	-0.0162	0.0084	-0.0120	-0.0232	0.0191
860318	-0.0261	-0.0000	-0.0137	-0.0217	0.0042
860319	-0.0187	0.0011	-0.0105	-0.0159	0.0087
860320	-0.0255	-0.0014	0.0007	-0.0299	0.0048
860321	-0.0373	0.0055	-0.0079	-0.0368	0.0040
860324	-0.0070	0.0061	0.0007	-0.0168	0.0109
860325	-0.0043	0.0063	0.0067	-0.0316	0.0089
860326	-0.0105	0.0071	0.0153	-0.0435	0.0045
860327	-0.0165	0.0055	0.0068	-0.0454	0.0028
860331	-0.0134	0.0028	0.0101	-0.0373	0.0110
860401	-0.0061	0.0027	0.0155	-0.0422	0.0080
860402	0.0006	0.0000	0.0154	-0.0323	0.0105
860403	-0.0013	0.0001	0.0203	-0.0369	0.0097
860404	-0.0010	0.0021	0.0259	-0.0475	0.0051
860407	-0.0035	-0.0005	0.0179	-0.0323	0.0149
860408	-0.0095	0.0013	0.0144	-0.0391	0.0093
860409	-0.0088	0.0019	0.0074	-0.0237	0.0208
860410	-0.0117	0.0002	0.0023	-0.0255	0.0106
860411	-0.0138	-0.0023	0.0083	-0.0322	0.0076
860414	-0.0150	0.0017	0.0092	-0.0349	0.0087
860415	-0.0163	0.0020	-0.0001	-0.0274	0.0132
860416	-0.0221	-0.0045	-0.0015	-0.0282	0.0042
860417	-0.0253	-0.0099	-0.0045	-0.0267	0.0022
860418	-0.0246	-0.0042	-0.0091	-0.0222	0.0117
860421	-0.0335	-0.0042	-0.0074	-0.0353	0.0079
860422	-0.0268	-0.0042	-0.0085	-0.0275	0.0183
860423	-0.0144	-0.0042	0.0049	-0.0319	0.0128
860424	-0.0050	-0.0063	0.0087	-0.0299	0.0087
860425	-0.0158	-0.0045	-0.0095	-0.0200	0.0167
860428	-0.0150	-0.0083	-0.0019	-0.0286	-0.0022
860429	-0.0148	-0.0085	-0.0102	-0.0204	0.0012
860430	-0.0149	-0.0033	-0.0118	-0.0191	0.0094
860501	0.0015	-0.0033	0.0010	-0.0231	0.0127
860502	0.0009	-0.0042	0.0022	-0.0281	0.0039
860505	-0.0078	-0.0021	-0.0090	-0.0161	0.0136
860506	-0.0003	-0.0035	0.0016	-0.0208	0.0150
860507	-0.0026	0.0010	0.0003	-0.0132	0.0282

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860508	-0.0053	0.0010	-0.0089	-0.0098	0.0242
860509	-0.0034	-0.0025	-0.0050	-0.0121	0.0155
860512	-0.0011	-0.0043	0.0006	-0.0203	0.0159
860513	0.0040	-0.0061	0.0025	-0.0206	0.0031
860514	0.0020	-0.0013	-0.0060	-0.0107	0.0142
860515	0.0015	-0.0008	-0.0037	-0.0047	0.0103
860516	0.0032	-0.0028	-0.0047	-0.0054	0.0093
860519	0.0068	-0.0012	-0.0033	-0.0089	0.0185
860520	0.0055	-0.0021	-0.0056	-0.0006	0.0190
860521	0.0075	0.0022	-0.0081	0.0012	0.0128
860522	0.0042	-0.0008	-0.0132	0.0060	0.0115
860523	0.0056	-0.0000	-0.0154	0.0085	0.0131
860527	0.0016	0.0015	-0.0161	-0.0010	0.0102
860528	0.0027	-0.0008	-0.0273	0.0001	0.0106
860529	0.0091	0.0033	-0.0152	-0.0041	0.0097
860530	0.0042	0.0014	-0.0115	-0.0009	0.0057
860609	-0.0003	-0.0011	-0.0128	0.0027	0.0059
860610	-0.0034	-0.0002	0.0056	-0.0207	0.0036
860611	-0.0035	-0.0002	0.0073	-0.0219	0.0061
860612	-0.0048	-0.0066	0.0090	-0.0248	0.0020
860613	-0.0062	-0.0027	-0.0073	-0.0076	0.0039
860616	-0.0025	0.0022	-0.0049	-0.0069	0.0065
860617	-0.0020	-0.0015	-0.0055	-0.0071	0.0032
860618	0.0006	0.0034	-0.0092	-0.0016	0.0080
860619	-0.0029	0.0011	-0.0157	0.0059	0.0031
860620	0.0021	0.0014	0.4767	-0.4904	-0.0016
860623	-0.0025	0.0002	-0.0148	0.0034	0.0079
860624	-0.0069	0.0002	-0.0069	-0.0061	0.0038
860625	-0.0072	-0.0003	-0.0066	-0.0078	0.0044
860626	-0.0028	0.0024	-0.0107	0.0064	0.0109
860627	-0.0039	0.0049	-0.0132	0.0037	0.0091
860630	-0.0065	0.0042	-0.0116	0.0022	0.0118
860701	-0.0096	-0.0013	-0.0075	-0.0102	0.0026
860702	-0.0079	0.0003	-0.0146	-0.0005	0.0058
860703	-0.0127	0.0016	-0.0170	0.0005	0.0129
860707	-0.0103	-0.0007	-0.0165	-0.0018	0.0021
860708	-0.0073	-0.0028	-0.0121	0.0026	0.0099
860709	-0.0116	-0.0028	-0.0118	-0.0050	0.0039
860710	-0.0088	-0.0024	-0.0140	0.0005	0.0065
860711	-0.0045	0.0012	-0.0216	0.0126	0.0122
860714	-0.0069	0.0016	-0.0163	0.0060	0.0120
860715	-0.0126	-0.0009	-0.0121	-0.0083	0.0025
860716	-0.0141	-0.0014	-0.0199	0.0014	0.0079
860717	-0.0103	0.0004	-0.0205	0.0058	0.0070
860718	-0.0137	0.0013	-0.0251	0.0072	0.0108
860721	-0.0205	-0.0027	-0.0258	-0.0042	0.0027
860722	-0.0115	-0.0019	-0.0273	0.0111	0.0054
860723	-0.0058	-0.0017	-0.0205	0.0125	0.0063
860724	-0.0046	0.0011	-0.0262	0.0165	0.0103
860725	-0.0009	-0.0033	-0.0157	0.0106	0.0086
860728	-0.0048	0.0014	-0.0218	0.0150	0.0191
860729	-0.0050	-0.0025	-0.0205	0.0085	0.0084
860730	-0.0046	-0.0044	-0.0163	0.0045	0.0061
860731	-0.0038	-0.0022	-0.0220	0.0096	0.0056
860801	-0.0017	-0.0063	-0.0231	0.0121	0.0029
860804	-0.0008	-0.0032	-0.0200	0.0137	0.0062
860805	-0.0028	0.0008	-0.0253	0.0210	0.0147
860806	-0.0042	-0.0024	-0.0222	0.0168	0.0111
860807	-0.0021	-0.0034	-0.0266	0.0166	0.0047
860808	-0.0022	-0.0000	-0.0340	0.0293	0.0137
860811	0.0009	-0.0000	-0.0263	0.0252	0.0149
860812	-0.0034	-0.0027	-0.0385	0.0312	0.0132
860813	0.0002	-0.0040	-0.0291	0.0257	0.0094
860814	-0.0000	-0.0012	-0.0368	0.0278	0.0085
860815	-0.0071	-0.0035	-0.0403	0.0226	0.0049
860818	0.0012	-0.0012	-0.0307	0.0249	0.0092
860819	-0.0047	0.0000	-0.0437	0.0273	0.0126
860820	-0.0019	0.0020	-0.0368	0.0234	0.0130
860821	-0.0054	-0.0016	-0.0380	0.0161	0.0026
860822	-0.0031	-0.0016	-0.0481	0.0253	0.0082
860825	-0.0023	-0.0039	-0.0434	0.0202	0.0107
860826	-0.0003	-0.0034	-0.0425	0.0175	0.0059
860827	0.0001	-0.0008	-0.0399	0.0188	0.0056
860828	-0.0039	0.0005	-0.0452	0.0145	0.0091
860829	-0.0060	-0.0051	-0.0468	0.0121	0.0036

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860908	0.0005	-0.0028	-0.0172	0.0071	0.0067
860909	-0.0035	-0.0020	-0.0326	0.0244	0.0080
860910	0.0000	-0.0001	-0.0181	0.0080	0.0059
860911	0.0044	-0.0009	-0.0125	0.0044	0.0036
860912	-0.0027	-0.0013	-0.0193	0.0100	0.0087
860915	-0.0072	-0.0006	-0.0252	0.0064	0.0050
860916	-0.0073	-0.0038	-0.0224	0.0041	0.0068
860917	-0.0128	0.0006	-0.0262	0.0073	0.0080
860918	-0.0167	-0.0048	-0.0236	0.0017	0.0042
860919	-0.0134	-0.0017	-0.0268	0.0089	0.0073
860922	-0.0028	0.0018	-0.0183	0.0060	0.0119
860923	-0.0005	0.0001	-0.0134	0.0005	0.0089
860924	-0.0014	-0.0012	-0.0122	0.0016	0.0081
860925	-0.0011	0.0024	-0.0078	-0.0020	0.0084
860926	-0.0021	0.0018	-0.0091	0.0013	0.0154
860929	-0.0087	-0.0028	-0.0146	-0.0004	0.0054
860930	-0.0041	-0.0043	-0.0140	0.0020	0.0027
861001	-0.0042	-0.0010	-0.0245	0.0123	0.0036
861002	-0.0047	0.0024	-0.0272	0.0169	0.0130
861003	-0.0015	0.0014	-0.0240	0.0145	0.0089
861006	-0.0059	-0.0009	-0.0199	0.0084	0.0065
861007	-0.0081	-0.0007	-0.0228	0.0094	0.0081
861008	-0.0068	-0.0005	-0.0256	0.0163	0.0098
861009	-0.0031	0.0033	-0.0257	0.0159	0.0053
861010	-0.0072	0.0015	-0.0327	0.0227	0.0078
861013	-0.0127	-0.0010	-0.0311	0.0175	0.0135
861014	-0.0109	0.0019	-0.0270	0.0115	0.0084
861015	-0.0088	-0.0022	-0.0236	0.0150	0.0068
861016	-0.0089	-0.0011	-0.0278	0.0153	0.0034
861017	-0.0039	0.0002	-0.0337	0.0271	0.0084
861020	-0.0068	-0.0034	-0.0339	0.0252	0.0042
861021	-0.0018	0.0009	-0.0408	0.0321	0.0081
861022	-0.0072	-0.0019	-0.0368	0.0297	0.0064
861023	0.0023	-0.0009	-0.0280	0.0260	0.0062
861024	0.0041	0.0009	-0.0149	0.0099	0.0080
861027	0.0022	0.0021	-0.0106	0.0034	0.0073
861028	-0.0001	0.0012	-0.0114	0.0016	0.0060
861029	-0.0020	-0.0017	-0.0080	-0.0025	0.0017
861030	0.0013	0.0000	-0.0103	-0.0009	0.0071
861031	0.0006	0.0018	-0.0080	-0.0076	0.0026
861103	0.0011	0.0021	-0.0059	-0.0049	0.0025
861104	0.0032	0.0008	-0.0188	0.0115	0.0109
861105	0.0010	-0.0003	-0.0218	0.0161	0.0115
861106	0.0000	0.0016	-0.0226	0.0200	0.0152
861107	-0.0012	0.0014	-0.0248	0.0102	-0.0014
861110	-0.0017	-0.0019	-0.0205	0.0103	0.0046
861111	-0.0022	-0.0031	-0.0269	0.0095	0.0017
861112	-0.0030	-0.0029	-0.0232	0.0033	-0.0022
861113	-0.0007	-0.0003	-0.0258	0.0180	0.0143
861114	-0.0012	-0.0048	-0.0245	0.0150	0.0057
861117	-0.0008	-0.0024	-0.0221	0.0090	0.0051
861118	-0.0013	-0.0006	-0.0233	0.0112	0.0094
861119	-0.0015	-0.0035	-0.0262	0.0057	0.0036
861120	-0.0010	-0.0021	-0.0267	0.0198	0.0133
861121	0.0009	-0.0018	-0.0264	0.0012	-0.0073
861124	0.0009	-0.0034	-0.0246	0.0079	0.0014
861125	-0.0015	-0.0024	-0.0275	0.0109	0.0070
861126	0.0005	-0.0086	-0.0237	0.0033	0.0002
861128	-0.0030	-0.0054	-0.0233	0.0083	0.0088
861208	-0.0017	0.0010	-0.0142	0.0066	0.0061
861209	0.0002	0.0008	-0.0184	0.0095	0.0068
861210	-0.0041	0.0016	-0.0212	0.0118	0.0036
861211	-0.0038	-0.0006	-0.0166	0.0049	-0.0017
861212	-0.0032	0.0001	-0.0242	0.0174	0.0066
861215	-0.0019	-0.0012	-0.0261	0.0166	-0.0009
861216	-0.0015	0.0039	-0.0279	0.0240	0.0077

DATE	CT	DELTA	GAMMA	LAMBDA	FI
861217	-0.0030	-0.0005	-0.0355	0.0254	0.0001
861218	-0.0028	0.0001	-0.0279	0.0199	0.0028
861219	-0.0078	-0.0043	-0.0304	0.0160	-0.0011
861222	-0.0058	-0.0012	-0.0277	0.0186	0.0051
861223	-0.0077	-0.0041	-0.0305	0.0184	0.0025
861224	-0.0124	-0.0019	-0.0336	0.0184	0.0018
861226	-0.0025	-0.0004	-0.0248	0.0199	0.0079
861229	-0.0141	-0.0076	-0.0359	0.0176	0.0008
861230	-0.0123	-0.0089	-0.0282	0.0103	-0.0021
861231	-0.0157	-0.0050	-0.0313	0.0133	0.0029
870102	-0.0114	-0.0033	-0.0275	0.0104	0.0041
870105	-0.0081	-0.0024	-0.0246	0.0105	0.0070
870106	-0.0212	-0.0041	-0.0293	0.0043	0.0038
870107	-0.0091	-0.0010	-0.0287	0.0160	0.0102
870108	-0.0156	-0.0019	-0.0348	0.0163	0.0054
870109	-0.0166	0.0001	-0.0321	0.0102	0.0087
870112	-0.0156	-0.0048	-0.0197	0.0003	0.0022
870113	-0.0169	-0.0072	-0.0222	0.0023	0.0070
870114	-0.0158	-0.0166	-0.0001	-0.0252	0.0066
870115	-0.0184	-0.0061	-0.0080	-0.0157	0.0146
870116	-0.0166	-0.0034	-0.0082	-0.0144	0.0153
870119	-0.0293	-0.0011	0.0000	-0.0411	0.0109
870120	-0.0131	-0.0026	0.0055	-0.0280	0.0148
870121	-0.0131	-0.0022	0.0078	-0.0293	0.0098
870122	-0.0214	-0.0037	0.0003	-0.0311	0.0088
870123	-0.0154	0.0002	-0.0001	-0.0283	0.0054
870126	-0.0092	0.0015	0.0001	-0.0240	0.0148
870127	-0.0142	0.0029	-0.0027	-0.0235	0.0100
870128	-0.0215	-0.0017	-0.0012	-0.0317	0.0187
870129	-0.0200	0.0027	0.0010	-0.0337	0.0128
870130	-0.0127	0.0002	0.0023	-0.0333	0.0089
870202	-0.0088	0.0036	-0.0037	-0.0205	0.0120
870203	-0.0149	-0.0001	-0.0124	-0.0185	0.0087
870204	-0.0056	-0.0009	-0.0114	-0.0083	0.0076
870205	-0.0028	0.0061	-0.0105	-0.0048	0.0112
870206	-0.0022	0.0051	-0.0008	-0.0189	0.0012
870209	-0.0054	-0.0009	-0.0081	-0.0119	0.0105
870210	-0.0057	-0.0023	-0.0129	-0.0103	0.0202
870211	-0.0057	-0.0020	-0.0178	-0.0001	0.0192
870212	0.0025	-0.0061	-0.0202	0.0088	0.0140
870213	0.0007	0.0002	-0.0320	0.0203	0.0233
870217	-0.0030	-0.0035	-0.0174	-0.0022	0.0129
870218	0.0026	-0.0000	-0.0064	-0.0150	0.0017
870219	0.0019	0.0007	-0.0004	-0.0201	0.0058
870220	-0.0048	-0.0015	0.0161	-0.0298	0.0224
870223	0.0042	-0.0008	-0.0118	-0.0105	-0.0030
870224	0.0068	-0.0008	-0.0173	0.0014	0.0137
870225	0.0005	0.0008	-0.0094	0.0034	0.0129
870226	-0.0042	-0.0040	-0.0113	-0.0016	0.0080
870227	-0.0056	-0.0069	-0.0274	-0.0009	0.0001
870309	0.0001	-0.0027	-0.0261	0.0126	0.0021
870310	0.0008	0.0017	-0.0366	0.0299	0.0152
870311	0.0017	0.0013	-0.0426	0.0252	0.0034
870312	0.0034	0.0001	-0.0495	0.0335	0.0048
870313	-0.0021	0.0030	-0.0599	0.0418	0.0066
870316	-0.0086	-0.0015	-0.0435	0.0262	0.0151
870317	-0.0055	0.0015	-0.0307	0.0140	0.0058
870318	-0.0021	0.0033	-0.0317	0.0150	0.0008
870319	-0.0083	0.0002	-0.0397	0.0214	0.0084
870320	-0.0025	-0.0006	-0.0268	0.0129	0.0021
870323	-0.0064	0.0049	-0.0314	0.0111	0.0055
870324	-0.0036	0.0009	-0.0311	0.0150	0.0117
870325	-0.0033	-0.0019	-0.0290	0.0169	0.0060
870326	-0.0073	-0.0013	-0.0508	0.0373	0.0105
870327	-0.0146	0.0006	-0.0431	0.0178	0.0025
870330	-0.0130	-0.0010	-0.0317	0.0120	0.0075
870331	-0.0139	-0.0008	-0.0386	0.0175	0.0093

Table 16

Modified Parameter Estimates (β') for the SF

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860203	-0.0109	-0.0012	-0.0279	0.0128	0.0030
860204	-0.0048	0.0066	-0.0292	0.0166	0.0171
860205	-0.0111	0.0021	-0.0312	0.0150	0.0083
860206	-0.0096	-0.0034	-0.0307	0.0144	0.0043
860207	-0.0091	-0.0056	-0.0253	0.0175	-0.0035
860210	-0.0194	-0.0028	-0.0329	0.0046	0.0038
860211	-0.0120	-0.0022	-0.0303	0.0100	0.0080
860212	-0.0118	0.0004	-0.0228	0.0070	0.0105
860213	-0.0044	-0.0032	-0.0112	-0.0005	0.0045
860214	-0.0150	-0.0000	-0.0171	-0.0091	-0.0012
860218	-0.0144	-0.0033	-0.0071	-0.0191	-0.0118
860219	-0.0159	-0.0028	-0.0040	-0.0251	0.0016
860220	-0.0065	0.0003	-0.0038	-0.0114	0.0180
860221	-0.0127	0.0021	-0.0121	-0.0137	0.0012
860224	-0.0132	-0.0067	-0.0071	-0.0196	0.0009
860225	-0.0151	-0.0066	0.0002	-0.0238	-0.0130
860226	-0.0349	-0.0071	-0.0337	-0.0319	-0.0134
860227	-0.0418	-0.0018	-0.0474	-0.0279	0.0132
860228	-0.0322	-0.0050	-0.0442	-0.0234	0.0240
860310	-0.0122	0.0071	-0.0058	-0.0141	0.0117
860311	-0.0162	0.0017	-0.0015	-0.0262	0.0063
860312	-0.0042	0.0076	0.0007	-0.0174	0.0125
860313	-0.0158	-0.0030	0.0062	-0.0268	-0.0043
860314	-0.0140	0.0001	0.0077	-0.0262	0.0060
860317	-0.0204	-0.0077	0.0092	-0.0333	-0.0015
860318	-0.0374	-0.0096	-0.0082	-0.0303	-0.0176
860319	-0.0136	0.0008	-0.0023	-0.0188	0.0086
860320	-0.0200	0.0017	0.0102	-0.0371	0.0035
860321	-0.0303	0.0037	-0.0017	-0.0350	0.0038
860324	-0.0136	0.0065	0.0150	-0.0483	0.0008
860325	-0.0055	0.0022	0.0117	-0.0320	0.0101
860326	-0.0105	0.0081	0.0047	-0.0385	-0.0005
860327	-0.0138	0.0054	0.0138	-0.0469	-0.0007
860331	-0.0075	-0.0032	0.0158	-0.0370	0.0203
860401	-0.0108	0.0021	0.0198	-0.0493	0.0107
860402	-0.0066	0.0003	0.0162	-0.0318	0.0125
860403	-0.0064	0.0050	0.0212	-0.0355	0.0105
860404	-0.0083	0.0062	0.0237	-0.0452	0.0038
860407	-0.0072	0.0044	0.0206	-0.0347	0.0090
860408	-0.0073	0.0048	0.0083	-0.0360	0.0058
860409	-0.0083	0.0004	0.0179	-0.0560	0.0046
860410	-0.0187	-0.0069	0.0070	-0.0311	0.0131
860411	-0.0153	-0.0054	0.0078	-0.0422	0.0071
860414	-0.0074	0.0009	-0.0007	-0.0246	0.0207
860415	-0.0174	0.0060	-0.0110	-0.0109	0.0317
860416	-0.0251	-0.0032	-0.0016	-0.0399	0.0060
860417	-0.0366	-0.0068	-0.0106	-0.0354	0.0084
860418	-0.0287	-0.0029	-0.0128	-0.0347	0.0037
860421	-0.0349	-0.0005	-0.0077	-0.0350	0.0111
860422	-0.0405	-0.0019	-0.0090	-0.0389	0.0103
860423	-0.0341	-0.0007	-0.0128	-0.0284	0.0170
860424	-0.0236	-0.0073	-0.0150	-0.0321	0.0013
860425	-0.0152	-0.0064	-0.0201	-0.0081	0.0194
860428	-0.0241	-0.0069	-0.0190	-0.0185	0.0033
860429	-0.0115	-0.0057	-0.0148	-0.0085	0.0091
860430	-0.0115	0.0004	-0.0093	-0.0153	0.0148
860501	-0.0092	-0.0021	-0.0051	-0.0189	0.0102
860502	0.0037	-0.0020	0.0052	-0.0324	0.0036
860505	-0.0070	-0.0013	-0.0085	-0.0196	0.0123
860506	0.0001	0.0011	-0.0043	-0.0082	0.0273
860507	-0.0032	-0.0016	0.0026	-0.0225	0.0050
860508	-0.0134	-0.0088	0.0007	-0.0283	-0.0036
860509	0.0022	-0.0163	0.0207	-0.0062	-0.0011

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860512	-0.0109	-0.0088	-0.0046	-0.0228	-0.0050
860513	0.0013	0.0013	-0.0029	-0.0233	-0.0106
860514	-0.0055	-0.0046	-0.0001	-0.0239	-0.0128
860515	0.0057	0.0025	-0.0044	-0.0071	0.0081
860516	0.0048	0.0009	-0.0086	0.0012	0.0184
860519	0.0020	0.0039	-0.0031	-0.0049	0.0153
860520	-0.0019	-0.0022	-0.0089	0.0033	0.0233
860521	0.0008	0.0041	-0.0095	0.0022	0.0111
860522	-0.0008	0.0010	-0.0135	0.0096	0.0191
860523	0.0001	0.0042	-0.0213	0.0190	0.0172
860527	-0.0026	0.0005	-0.0214	0.0047	0.0038
860528	-0.0005	0.0006	-0.0264	-0.0054	-0.0013
860529	-0.0003	0.0057	-0.0204	0.0163	0.0153
860530	0.0032	0.0067	-0.0222	0.0034	0.0130
860609	-0.0104	-0.0070	-0.0047	-0.0161	-0.0105
860610	-0.0057	-0.0013	0.0057	-0.0188	0.0034
860611	-0.0121	-0.0062	0.0039	-0.0263	-0.0048
860612	-0.0009	-0.0005	0.0018	-0.0166	0.0053
860613	0.0069	0.0043	-0.0016	0.0070	0.0235
860616	-0.0010	-0.0078	0.0007	-0.0103	0.0031
860617	0.0032	-0.0061	0.0052	-0.0094	0.0031
860618	0.0030	0.0017	-0.0008	-0.0022	0.0108
860619	0.0146	0.0041	0.6185	-0.6131	0.0072
860620	0.0009	0.0033	-0.0245	0.0146	0.0067
860623	-0.0013	0.0026	0.0068	-0.0176	0.0030
860624	-0.0026	0.0016	-0.0082	-0.0021	0.0086
860625	-0.0045	-0.0016	-0.0005	-0.0139	0.0004
860626	-0.0019	-0.0033	-0.0087	0.0036	0.0064
860627	-0.0088	-0.0055	-0.0118	-0.0046	0.0001
860630	-0.0111	0.0012	-0.0219	0.0024	0.0059
860701	-0.0180	-0.0008	-0.0210	-0.0046	0.0076
860702	-0.0027	0.0001	-0.0127	0.0037	0.0079
860703	-0.0196	-0.0013	-0.0264	0.0009	0.0031
860707	-0.0164	-0.0066	-0.0158	-0.0106	0.0012
860708	-0.0060	-0.0065	-0.0005	-0.0130	0.0019
860709	-0.0057	-0.0017	-0.0179	0.0024	0.0051
860710	-0.0118	-0.0061	-0.0165	-0.0037	-0.0061
860711	0.0003	0.0081	-0.0184	0.0148	0.0100
860714	-0.0080	0.0007	-0.0182	0.0063	0.0086
860715	-0.0140	-0.0096	-0.0195	-0.0031	-0.0066
860716	-0.0174	0.0013	-0.0245	-0.0020	0.0136
860717	-0.0121	-0.0029	-0.0215	0.0049	0.0097
860718	-0.0244	-0.0019	-0.0290	-0.0015	0.0055
860721	-0.0019	-0.0067	-0.0110	-0.0047	0.0010
860722	-0.0176	-0.0016	-0.0226	-0.0020	0.0037
860723	-0.0050	-0.0044	-0.0181	0.0068	0.0095
860724	-0.0076	-0.0030	-0.0120	-0.0006	0.0053
860725	-0.0017	0.0012	-0.0102	0.0043	0.0123
860728	-0.0095	-0.0033	-0.0143	-0.0006	0.0071
860729	-0.0076	0.0009	-0.0165	0.0069	0.0152
860730	-0.0059	-0.0014	-0.0160	0.0096	0.0233
860731	-0.0033	0.0043	-0.0153	0.0054	0.0152
860801	0.0033	-0.0055	-0.0058	0.0036	0.0083
860804	-0.0004	-0.0053	-0.0058	-0.0025	0.0014
860805	-0.0022	-0.0044	-0.0069	-0.0008	-0.0028
860806	0.0117	0.0062	-0.0065	0.0205	0.0259
860807	0.0016	-0.0031	-0.0130	0.0092	0.0063
860808	-0.0057	-0.0009	-0.0296	0.0255	0.0245
860811	0.0106	-0.0005	-0.0045	0.0129	0.0114
860812	-0.0024	-0.0037	-0.0204	0.0132	0.0034
860813	-0.0024	-0.0012	-0.0286	0.0161	0.0020
860814	0.0001	0.0019	-0.0250	0.0203	0.0136
860815	-0.0040	-0.0010	-0.0282	0.0121	0.0080
860818	-0.0070	0.0052	-0.0249	0.0241	0.0258
860819	-0.0060	0.0033	-0.0255	0.0101	0.0144
860820	-0.0131	0.0026	-0.0290	0.0114	0.0120
860821	-0.0150	0.0011	-0.0291	-0.0002	-0.0005
860822	-0.0044	0.0009	-0.0249	0.0055	0.0014
860825	-0.0079	-0.0063	-0.0411	0.0043	0.0045
860826	-0.0022	0.0008	-0.0321	0.0092	0.0049
860827	-0.0026	0.0013	-0.0351	0.0145	0.0101
860828	-0.0052	-0.0034	-0.0288	0.0117	0.0121
860829	-0.0064	0.0004	-0.0405	0.0220	0.0239

DATE	CT	DELTA	GAMMA	LAMBDA	FI
860908	0.0018	0.0026	-0.0149	0.0070	0.0022
860909	-0.0032	0.0012	-0.0272	0.0119	0.0047
860910	-0.0007	-0.0042	-0.0327	0.0373	0.0292
860911	0.0019	-0.0020	-0.0119	0.0029	0.0042
860912	-0.0027	-0.0037	-0.0052	-0.0124	-0.0069
860915	-0.0060	-0.0008	-0.0093	-0.0086	0.0063
860916	-0.0138	-0.0059	-0.0170	-0.0056	-0.0005
860917	-0.0085	-0.0046	-0.0189	0.0044	0.0081
860918	-0.0252	-0.0092	-0.0269	-0.0045	0.0006
860919	-0.0362	-0.0024	-0.0395	-0.0053	0.0092
860922	-0.0059	0.0045	-0.0178	0.0036	0.0135
860923	-0.0065	0.0029	-0.0160	-0.0009	0.0001
860924	-0.0024	0.0033	-0.0204	0.0138	0.0093
860925	-0.0010	0.0015	-0.0142	0.0106	0.0175
860926	-0.0068	-0.0003	-0.0298	0.0057	0.0014
860929	-0.0058	-0.0020	-0.0074	0.0017	0.0111
860930	-0.0029	-0.0010	-0.0148	-0.0001	0.0033
861001	-0.0079	-0.0016	-0.0132	-0.0028	0.0009
861002	-0.0021	0.0028	-0.0293	0.0270	0.0211
861003	-0.0087	0.0006	-0.0278	0.0154	0.0071
861006	-0.0098	-0.0047	-0.0176	0.0047	0.0027
861007	-0.0085	0.0012	-0.0194	-0.0006	-0.0024
861008	-0.0069	0.0042	-0.0293	0.0155	0.0040
861009	-0.0084	-0.0063	-0.0330	0.0095	-0.0099
861010	-0.0225	-0.0050	-0.0477	0.0144	-0.0111
861013	-0.0067	-0.0040	-0.0411	0.0226	0.004[illegible]
861014	-0.0180	-0.0003	-0.0313	0.0112	-0.0004
861015	-0.0071	-0.0023	-0.0349	0.0221	0.0031
861016	-0.0172	-0.0037	-0.0383	0.0141	-0.0050
861017	-0.0031	-0.0012	-0.0392	0.0346	0.0064
861020	-0.0036	0.0045	-0.0328	0.0233	0.0038
861021	0.0015	0.0050	-0.0318	0.0318	0.0097
861022	-0.0013	-0.0026	-0.0599	0.0646	0.0160
861023	-0.0006	0.0010	-0.0307	0.0294	0.0045
861024	0.0026	0.0027	-0.0232	0.0143	0.0011
861027	0.0020	0.0073	-0.0247	0.0161	0.0092
861028	0.0004	0.0042	-0.0260	0.0162	0.0142
861029	-0.0000	0.0018	-0.0193	0.0073	0.0051
861030	0.0021	0.0002	-0.0230	0.0213	0.0180
861031	-0.0008	0.0028	-0.0131	0.0102	0.0092
861103	0.0009	0.0022	-0.0214	0.0107	0.0117
861104	-0.0002	-0.0011	-0.0228	0.0140	0.0073
861105	-0.0027	0.0043	-0.0243	0.0067	-0.0054
861106	0.0004	0.0037	-0.0234	0.0179	0.0079
861107	-0.0021	0.0000	-0.0296	0.0142	0.0012
861110	-0.0018	0.0007	-0.0256	0.0168	0.0083
861111	-0.0004	-0.0030	-0.0257	0.0165	0.0067
861112	-0.0037	-0.0035	-0.0301	0.0255	0.0129
861113	-0.0050	-0.0035	-0.0293	0.0203	0.0054
861114	-0.0020	-0.0075	-0.0281	0.0236	0.0160
861117	-0.0014	-0.0044	-0.0305	0.0268	0.0232
861118	-0.0002	-0.0007	-0.0291	0.0239	0.0241
861119	-0.0004	-0.0041	-0.0286	0.0190	0.0121
861120	-0.0056	0.0013	-0.0306	0.0157	0.0047
861121	-0.0009	0.0039	-0.0291	0.0149	0.0046
861124	0.0029	0.0028	-0.0253	0.0115	0.0045
861125	-0.0037	-0.0032	-0.0247	0.0169	0.0104
861126	0.0013	-0.0028	-0.0204	0.0201	0.0135
861128	-0.0037	-0.0096	-0.0304	0.0117	0.0148
861208	0.0003	0.0000	-0.0187	0.0155	0.0129
861209	-0.0023	0.0057	-0.0138	0.0070	0.0057
861210	0.0008	0.0005	-0.0142	0.0073	0.0054
861211	-0.0005	0.0001	-0.0058	0.0026	0.0061
861212	-0.0020	-0.0016	-0.0180	0.0042	-0.0024
861215	-0.0038	0.0015	-0.0148	0.0124	0.0102
861216	-0.0001	0.0019	-0.0370	0.0324	0.0091

DATE	CT	DELTA	GAMMA	LAMBDA	FI
861217	0.0003	0.0068	-0.1560	0.1499	0.0107
861218	-0.0054	0.0044	-0.0191	0.0098	0.0002
861219	-0.0004	0.0031	-0.0314	0.0184	0.0068
861222	-0.0046	0.0025	-0.0157	0.0033	0.0058
861223	-0.0078	-0.0027	-0.0196	0.0070	0.0074
861224	-0.0098	-0.0106	-0.0288	0.0054	-0.0062
861226	-0.0063	-0.0046	-0.0259	0.0162	0.0028
861229	-0.0052	-0.0054	-0.0211	0.0057	0.0013
861230	-0.0055	-0.0080	-0.0239	0.0114	0.0004
861231	-0.0094	-0.0045	-0.0304	0.0164	0.0114
870102	-0.0080	-0.0039	-0.0313	0.0144	0.0029
870105	-0.0096	-0.0043	-0.0285	0.0067	-0.0017
870106	-0.0048	-0.0016	-0.0309	0.0188	0.0079
870107	-0.0054	-0.0031	-0.0179	0.0042	0.0030
870108	-0.0106	-0.0072	-0.0186	-0.0017	-0.0041
870109	-0.0083	-0.0029	-0.0214	0.0049	0.0019
870112	-0.0223	-0.0027	-0.0274	-0.0036	0.0023
870113	-0.0197	0.0006	-0.0225	-0.0061	0.0124
870114	-0.0095	-0.0101	-0.0071	-0.0221	-0.0035
870115	-0.0099	0.0007	-0.0004	-0.0252	0.0132
870116	-0.0026	0.0082	0.0047	-0.0170	0.0253
870119	-0.0154	0.0041	0.0068	-0.0393	0.0144
870120	-0.0022	0.0057	0.0253	-0.0461	0.0224
870121	0.0047	0.0025	0.0185	-0.0328	0.0195
870122	-0.0139	-0.0002	0.0137	-0.0451	0.0076
870123	-0.0096	0.0014	0.0124	-0.0479	-0.0012
870126	-0.0043	0.0050	0.0216	-0.0481	0.0145
870127	-0.0193	0.0000	0.0113	-0.0490	0.0044
870128	-0.0353	-0.0041	0.0014	-0.0616	-0.0014
870129	-0.0359	-0.0010	-0.0031	-0.0475	0.0073
870130	-0.0034	0.0179	0.0155	-0.0455	0.0198
870202	-0.0081	-0.0006	0.0084	-0.0393	0.0069
870203	-0.0204	-0.0042	0.0034	-0.0461	0.0084
870204	-0.0046	0.0027	-0.0083	-0.0140	0.0148
870205	0.0022	0.0075	0.0026	-0.0400	-0.0066
870206	0.0011	0.0052	-0.0087	-0.0061	0.0266
870209	-0.0063	-0.0002	-0.0003	-0.0268	0.0139
870210	0.0006	0.0099	-0.0016	-0.0200	0.0222
870211	-0.0013	0.0018	-0.0055	-0.0130	0.0166
870212	0.0087	0.0039	-0.0117	-0.0104	-0.0028
870213	0.0097	0.0061	-0.0231	0.0020	0.0145
870217	-0.0021	-0.0023	-0.0227	0.0050	0.0093
870218	0.0031	0.0005	-0.0163	0.0110	0.0191
870219	-0.0063	0.0000	-0.0080	0.0010	0.0184
870220	0.0014	-0.0011	0.0098	-0.0284	0.0189
870223	0.0015	0.0061	-0.0260	0.0193	0.0234
870224	0.0009	-0.0000	-0.0266	0.0091	0.0098
870225	-0.0011	-0.0007	-0.0147	-0.0034	-0.0048
870226	-0.0080	-0.0041	-0.0171	-0.0096	-0.0024
870227	-0.0065	-0.0115	-0.0296	0.0060	0.0077
870309	0.0069	0.0011	-0.0239	0.0075	0.0002
870310	0.0015	-0.0005	-0.0247	0.0028	-0.0081
870311	0.0006	-0.0006	-0.0529	0.0386	0.0080
870312	0.0007	-0.0032	-0.0405	0.0405	0.0179
870313	0.0002	0.0074	-0.0680	0.0777	0.0337
870316	-0.0046	0.0042	-0.0515	0.0345	0.0067
870317	-0.0089	-0.0033	-0.0312	0.0178	0.0056
870318	-0.0068	-0.0017	-0.0621	0.0474	0.0033
870319	-0.0025	0.0117	-0.7124	0.7043	0.0182
870320	-0.0086	0.0043	0.4263	-0.4378	0.0060
870323	-0.0098	0.0083	-0.0390	0.0317	0.0276
870324	-0.0049	0.0098	-0.0356	0.0248	0.0247
870325	-0.0114	0.0072	-0.0439	0.0147	-0.0034
870326	-0.0035	0.0046	-0.0250	0.0201	0.0139
870327	-0.0101	0.0081	-0.0421	0.0202	0.0079
870330	-0.0196	-0.0010	-0.0367	0.0154	0.0140
870331	-0.0076	0.0037	-0.0385	0.0300	0.0224

Chapter 3

ESSAY TWO ON

"THE PRICING OF COMPLEX OPTIONS AND THE CORRELATION STRUCTURE OF EXCHANGE RATES"

I. Introduction

The last few years have witnessed the creation of many new financial instruments such as options, caps, floors, collars, dual-currency bonds and currency warrants. Many of these instruments have incorporated option-like features which have been used for hedging purposes or as "sweeteners". Whatever the reason for the creation of these instruments, the knowledge of option pricing is critical for their valuation.

Since the pioneering work of Black and Scholes (1973), many other option pricing models have been proposed. The basic assumptions for the Black and Scholes model are that markets are frictionless, that the risk free interest rate is constant, that the price of the underlying security follows a Geometric Brownian Motion, that the underlying security makes no payouts (e.g., dividends), and that options are European (i.e., they can only be exercised at their maturity). Subsequent models relaxed one or more of these assumptions. Merton (1973) presents a model for the case where the underlying security pays a continuous dividend (e.g., a continuous interest rate), and a model for the case where the risk free interest rate is not constant. Schwartz (1977) and Cox, Ross and Rubinstein (1979) present numerical solutions to the valuation of American options. Whaley (1981) evaluates American calls on stocks with known dividends. Geske (1977) presents a model where stocks are considered as options on the value of the firm and stock options are considered as compound options. Cox and

Ross (1976) present a model where the variance of the underlying security has a constant elasticity. Merton (1976) assumes that the underlying security follows a diffusion-jump process. Hull and White (1987) consider the volatility of the underlying security to be stochastic. Leland (1985) introduces transaction costs into his model. Geske and Johnson (1984) and Barone-Adesi and Whaley (1987) present approximations to the valuation of American puts and American options in general, respectively.

Brennan and Schwartz (1977), Courtadon (1982), Ball and Torous (1983), and Shaefer and Schwartz (1987) present models for pricing interest rate options. Black (1976) values options on commodity contracts. Garman and Kohlhagen (1983) value foreign exchange options, and Biger and Hull (1983) value options on foreign exchange forwards. Complex options are evaluated by Margrabe (1978) and Stulz (1982). Margrabe presents a model for valuing an option to exchange one asset for another, and Stulz values options on the maximum or minimum of two assets.

This essay presents simplified, intuitive and rigorous derivations of pricing models for various types of European options, namely, call and put options on a single asset, options to exchange one asset for another, and call and put options on the maximum or the minimum of two assets, with the same or different exercise prices. In order to assess the parameters required in these models when the underlying securities are foreign currencies, the correlation structure of exchange rates for the major world currencies (namely, the US dollar, the German mark, the Japanese yen, the Swiss franc, the British pound, the French franc, the Australian dollar, the Dutch guilder and the Canadian dollar), and the stability of that structure are studied over time, and for different holding periods. The data covers the period from January 1974 to December 1987. The practical usefulness of the various option pricing models is also assessed. Since the usefulness of these instruments depends on their relative costs, these costs are evaluated under different assumptions.

The essay is organized as follows. In Section II, the various option pricing models are derived for the case when the underlying securities are traded on the spot market (i.e., require cash outlays to be held), and for the case when the underlying security is a forward or futures contract (i.e., a security not requiring a cash outlay to be held). In Section III, the correlation structure of exchange rates for nine major world currencies is studied. In Section IV, price estimates of the various types of options and their practical implications are discussed. In Section IV, some concluding remarks are offered.

II. Option Pricing Models

II.1 Option on a Single Asset

To derive the price of an European option on a foreign currency, consider a forward contract as the underlying security as opposed to the spot currency. Based on a covered interest arbitrage argument, the interest rate parity theorem can be derived:

$$F = S\, e^{(r_D - r_F)T} \qquad [1]$$

where F = Forward rate

r_D, r_F = Domestic and foreign interest rates, respectively

T = Time to maturity

S = Spot rate

Using F instead of S in the evaluation of the option makes the knowledge of r_F unnecessary. The assumptions needed for the derivation of all the pricing models in this essay are that markets are frictionless (i.e., there are no transaction costs), that the r_D (henceforth, r) is constant, and that the price(s) of the underlying security(ies) follows a Geometric Brownian Motion.

Markets are not open twenty four hours a day, and transactions are not costless. As demonstrated in the first essay, transaction costs cause systematic differences between theoretical and observed market prices. However, to the extent transaction costs are not too large, the pricing models developed below provide a good approximation. Although the risk free interest rate is not constant, its variability is usually very small when compared to the variability of exchange rates or to the variability of the return on most securities. Since changes in interest rates do not significantly affect option prices, assuming a constant r does not seem to induce any significant error. Geometric Brownian motion requires that the returns on the underlying securities conform to a normal distribution. Evidence indicates that security returns follow normal distributions with non-constant variance. However, as shown by Merton (1976), even using a diffusion jump process with a few large jumps instead of a Geometric Brownian Motion significantly changes theoretical option prices only for very short maturity options. This finding is supported in the first essay of this dissertation.[1]

Given these assumptions, the price dynamics of the forward contract can be expressed as:

$$dF = \alpha F\, dt + \sigma F\, dz \qquad [2]$$

where α = expected return on F

σ^2 = instantaneous variance of α

dz = standard Gauss-Wiener process

II.1.1 Calls

The value of a call option at maturity is:

$$C_F(F,\ X,\ 0) = \max\ (0,\ F^m - X) \qquad [3]$$

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1. Most of the arguments for the use of compound option models or constant elasticity of variance models do not hold for foreign currency options.

where $C_F(F, X, 0)$ = call option on F

X = exercise price

F^m = value of F at the maturity of the option

The value of C_F, T periods before its maturity, is then:

$$C_F\ (F, X, T) = \text{Present value of } E[\max(0, F^m-X)]$$

$$= e^{-\rho T}\ E[\max(0, F^m-X) \qquad [4]$$

The discount rate, ρ, and the expected value of the security (here the forward contract) at the maturity of the option need to be determined in [4].

Cox, Ross, and Rubinstein (1979) (CRR) show that the value of the call can be <u>interpreted</u> as the expectation of its discounted future value in a risk-neutral world. This result does not assume that the return on the underlying security or on the option is the risk free rate of return. CRR obtain their result by forming a hedge portfolio for a security having returns which follow a binomial probability distribution. The value of the option is independent of the expected return on the security. In general, the expected returns on both the underlying security and on the option, are not equal to the risk free return.

Based on CRR, equation [4] can be rewritten as:

$$C_F(F, X, T) = e^{-rT}\ E^{rn}\ [\max\ (0, F^m-X)] \qquad [5]$$

where E^{rn} = Expected value at maturity in a risk neutral world

The expected value of the forward rate at maturity in a risk neutral world is F, because no investment is required for holding a forward contract.[2] Since the return distribution for F is assumed to follow a normal distribution, the expected value is given by:

$$E^{rn}[\max(0, F^m - X)] = \int_{x}^{\infty} (F^m - X)\, f(F^m)\, dF^m$$

$$= \int_{-\infty}^{x} F^m f(-F^m)\, dF^m - X\int_{-\infty}^{x} f(-F^m) dF^m$$

where $f(\bullet)$ = the log-normal distribution

Solving the integral (see Appendix I) yields:

$$E^{rn}[\max(0, F^m - X)] = F\, N(d_1) - X\, N(d_2)$$

where $N(\bullet)$ = the cumulative standard normal distribution

$d_1 = [\ln (F/X) + (\sigma^2/2)\, T] \,/\, \sigma\sqrt{T}$

$d_2 = [\ln (F/X) - (\sigma^2/2)\, T] \,/\, \sigma\sqrt{T}$

σ = instantaneous variance of return on F

Hence: $C_F(F, X, T) = e^{-rT}\, [FN(d_1) - XN(d_2)]$ [6]

For the case where holding security S requires an investment of S (for example, a stock), the expected value of the security in a risk neutral world is $S\, e^{rT}$. Hence equation [5] becomes:

$$C(S, X, T) = e^{-rt}\, E^{rn}[\max(0, S^m - X)] \qquad [7]$$

and equation [6] becomes:

---

2. In practice, a bank may require a deposit to guarantee the contract, and future contracts require a margin deposit in, for example, the form of T-Bills. As is generally done in the literature on futures options, the no "performance bond" assumption is invoked herein.

$$C(S, X, T) = S\ N(d_1^{\prime}) - e^{-rT}\ X\ N(d_2^{\prime}) \qquad [8]$$

where $d_1^{\prime} = [\ln (S/X) + (r + \sigma^2/2)T]/\sigma\sqrt{T}$

$d_2^{\prime} = [\ln (S/X) + (r - \sigma^2/2)T]/\sigma\sqrt{T}$

II.1.2 Puts

The valuation of a European put option on a single asset can be derived from put-call parity. For options on a stock, we have:

$$C = P + S - X\ e^{-rt} \qquad [9]$$

This can be seen by verifying that the payoff of a portfolio containing a call and X e^{-rt} dollars invested in the risk free security has the same payoff in all states of nature as a portfolio containing the underlying security and a put. Hence the value of a put is:

$$\begin{aligned} P &= C - S + X\ e^{-rt} \\ &= S(N(d_1^{\prime})-1) - e^{-rt}\ X(N(d_2^{\prime})-1) \\ &= e^{-rt}\ X\ N(-d_2^{\prime}) - S\ N(-d_1^{\prime}) \qquad [10] \end{aligned}$$

In the case of options on forwards, put-call parity becomes:

$$P_F = C_F + (X-F)e^{-rt} \qquad [11]$$

This can be seen by verifying that the payoff of a portfolio containing a call and (X-F) e^{-rt} invested in the risk free security is the same in all states of nature as that of a portfolio containing a put and a forward contact which requires no investment. If $F^m > X$, the portfolio long the put will be worth $F^m - F$ (i.e., the gain on the forward contract). The portfolio long the call will be worth $X - F$ (i.e., the proceeds from the risk free security) plus $F^m - X$ (i.e., the gain on the exercise of the option). If $F^m < X$, the portfolio long the put will be worth $X - F^m$ from exercising the put plus $F^m - F$ (i.e., the loss on the forward contract). This is equal to $X - F$ (i.e., the value of the portfolio long the call) since the call will be worthless. The value of a put on a forward contract can be expressed as:

$$P_F = e^{-rt} [F(N(d_1) - 1) - X(N(d_2) - 1)]$$

$$= e^{-rt} [XN(-d_2) - FN(-d_1)] \qquad [12]$$

When $X=F$, $P_F=C_F$. Intuitively, since the expected value of the forward price at maturity in a risk neutral world is F, the option to sell at this price must have the same value as the option to buy at this price.

II.2 Option to Exchange One Asset for Another

A model for pricing an option to exchange one asset for another was first presented by Margrabe (1978). Assume two assets, S_1 and S_2, with the following price dynamics:

$$dS_1 = \alpha_1 S_1 \, dt + \sigma_1 S_1 \, dz_1 \qquad [13]$$

$$dS_2 = \alpha_2 S_2 \, dt + \sigma_2 S_2 \, dz_2$$

$\rho_{1,2}$ = correlation coefficient of the two Wiener processes

An option Ex (S_1, S_2, T) to exchange security S_2 for S_1 has the following pay-off at maturity:

$$Ex(S_1, S_2, 0) = \max(0, S_1 - S_2) \quad [14]$$

Define $S = S_1/S_2$. Hence: $Ex(S_1, S_2, 0) = S_2 \max(0, S-1)$. Based on CRR:

$$Ex(S_1, S_2, T) = \text{Present value of } E^{rn} [S_2^m \max(0, S^m - 1)] \quad [15]$$

But: $E^{rn}(S_2^m) = S_2 e^{rt}$

$E^{rn}(S^m) = S_1 e^{rt}/S_2 e^{rt} = S_1/S_2 = S$

Hence: $Ex(S_1, S_2, T) = e^{-rt} E^m[S_2 e^{rt} \max(0, S^m - 1)]$

$$= S_2 E^m[\max(0, S^m - 1)], \quad [16]$$

Equation [16] is identical to equation [5] when e^{-rt}, F and X are replaced by S_2, S and 1, respectively.[3]

Hence: $Ex(S_1, S_2, T) = S_2 [S N(d_1) - 1N(d_2)]$

$$= S_1 N(d_1) - S_2 N(d_2) \quad [17]$$

where $d_1 = [\ln S/1 + (\sigma^2/2) T]/\sigma\sqrt{T}$

$= [\ln(S_1/S_2) + (\sigma^2/2) T]/\sigma\sqrt{T}$

$d_2 = [\ln(S_1/S_2) + (\sigma^2/2) T]/\sigma\sqrt{T}$

3. Since S^m is the ratio of two log-normally distributed variables (S_1^m and S_2^m), its probability distribution is simply the difference between two log-normal distributions.

and $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\ \sigma_1\ \sigma_2$

For options on forward contracts, replace S_1 and S_2 by F_1 and F_2. Equation [15] then becomes:

$$Ex_F\ (F_1,\ F_2,\ T) = \text{Present Value of } E^{rn}[F_2^m \max\ (0,\ F^m-1)]$$

where: $F = F_1/F_2$

$E^{rn}\ (F_2^m) = F_2$

$E^{rn}\ (F^m) = F_1/F_2 = F$

Hence: $Ex\ (F_1,\ F_2,\ T) = e^{-rT}\ [F_1\ N(d_1) - F_2\ N(d_2)]$ [18]

where: $d_1 = [\ln\ (F_1/F_2) + (\sigma_F^2/2)\ T]\ /\sigma_F\ \sqrt{T}$

$d_2 = [\ln\ (F_1/F_2) - (\sigma_F^2/2)\ T]\ /\sigma_F\ \sqrt{T}$

and $\sigma_F^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\ \sigma_1\ \sigma_2$

Equation [18] can be illustrated as follows. Suppose the numeraire currency is the US dollar, F_1 is 5 FF/\$ and F_2 is 0.5 £/\$. Hence, $Ex_F(F_1, F_2, T)=Ex_F(5\ FF, 0.5\ £, T)$ is the option to exchange 0.5 £ for 5 FF at maturity. For this option, the numeraire currency (the US \$) is irrelevant. This option is just a call option on 5 FF with an exercise price of 0.5 £. Thus,

$$Ex_F(5\ FF,\ 0.5\ £,\ T) = C_F(5\ FF,\ 0.5\ £,\ T) \quad [19]$$

and σ_F^2 (as given in equation [18]) is simply the instantaneous variance of the return of the French franc expressed in British pounds.

II.3 Call Option on the Maximum of Two Assets

A model for pricing options on the maximum of two assets with the same exercise price was first presented by Stulz (1982). Models for pricing options on the maximum of two assets with the same or different exercise prices are derived herein using a slightly different approach. Define a call option C_{max} (S_1, X_1, S_2, X_2, T) as an option to buy S_1 at a price of X_1 or S_2 at a price of X_2 at maturity (after T periods). The pay off of such an option at maturity is:

$$C_{max}\ (S_1,\ X_1,\ S_2,\ X_2,\ 0) = \max\ [(S_1 - X_1),\ (S_2 - X_2),\ 0] \qquad [20]$$

Based on CRR, equation [20] becomes:

$$C_{max}(S_1,\ X_1,\ S_2,\ X_2,\ T) = e^{-rt}\ E^{rn}\ \{\max\ [(S_1^T - X_1),\ (S_2^T - X_2),\ 0]\} \qquad [21]$$

The investor would purchase S_1 if and only if S_1^T (S_1 at maturity) > X_1 and $S_1^T > S_2^T - X_2 + X_1$ (i.e., $S_2^T < S_1^T - X_1 + X_2$). The investor would purchase S_2^T only if $S_2^T > X_2$ and $S_1^T < S_2^T - X_2 + X_1$. If S_1 and S_2 follow Geometric Brownian diffusion processes and the correlation between the two processes is constant, their probability distribution at maturity is joint bivariate log-normal. Thus, equation [21] can be rewritten as:

$$C_{max}(S_1,X_1,S_2,X_2,T)=e^{-rt}\{\int_X^{\infty}(S_1^T-X_1)\int_{-\infty}^{S_1^T-X_1+X_2} g(S_1^T,S_2^T,\rho)dS_2^T\ dS_1^T$$

$$+ \int_X^{\infty} (S_2^T - X_2) \int_{-\infty}^{S_2^T-X_2+X_1} g(S_1^T,S_2^T,\rho)dS_1^T\ dS_2^T\} \qquad [22]$$

where $g(S_1^T,S_2^T,\rho)$ = the bivariate log-normal density function

ρ = correlation between the instantaneous returns on S_1 and S_2

Since equation [22] has no known closed form solution, it must be evaluated using numerical integration. Evaluations of [22] for different parameter values are presented in section IV of this essay.

If $X_1 = Y_2 = X$ (as in Stulz (1982)), equation [22] can be solved. Equation [22] with the modified integration limits is:

$$C_{max}(S_1,X_1,S_2,X_2,T)=C_{max}(S_1,\ S_2,\ X,\ T)\ e^{-rt}\{\int_X^\infty (S_1^m-X) \int_{-\infty}^{S_1^m} g(S_1^m,S_2^m,\rho)dS_2^m dS_1^m$$

$$+ \int_X^\infty (S_2^m - X)\int_{-\infty}^{S_2^m} g(S_1^m,S_2^m,\rho)dS_1^m\ dS_2^m\} \qquad [23]$$

Equation [23] can be simplified somewhat since the option will expire without being exercised only when both S_1^m and S_2^m are less than X. Thus, the expected value of the cash outlay for exercising the option is:

$$- X + X\int_{-\infty}^{X} \int_{-\infty}^{X} g(S_1^m,\ S_2^m,\ \rho)\ dS_1^m\ dS_2^m \qquad [24]$$

The second term of equation [24] is a simple cumulative bivariate lognormal distribution. Equation [23] can then be rewritten as:

$$C_{max}(S_1,S_2,X,T)=e^{-rt}\{\int_X^\infty S_1^m\int_{-\infty}^{S_1^m} g(S_1^m,S_2^m,\rho)dS_2^m\ dS_1^m$$

$$+ \int_X^\infty S_2^m\int_{-\infty}^{S_2^m} g(S_1^m,S_2^m,\rho)dS_1^m\ dS_2^m\}$$

$$- X + X \int_{-\infty}^{X} \int_{-\infty}^{X} g(S_1^m,\ S_2^m,\ \rho)\ dS_1^m\ dS_2^m\} \qquad [25]$$

Making the appropriate changes of variables (see Appendix II) and solving, the value of the option is:

$$C_{max}(S_1,S_2,X,T) = S_1\, N_2(d_1,\, d_2,\, \rho_1) + S_2\, N_2(d_3,\, d_4,\, \rho_2)$$
$$+ e^{-rT}X\, (N_2\, (d_5,\, d_6,\, \rho) - 1) \qquad [26]$$

where $N_2(.,.,.)$ = the cumulative bivariate normal distribution

$d_1 = [\ln (S_1/X) + (r + \sigma_1^2/2)\, T] \,/\, \sigma_1\sqrt{T}$

$d_2 = [\ln (S_1/S_2) + (\sigma^2/2)\, T] \,/\, \sigma\sqrt{T}$

$d_3 = [\ln (S_2/X) + (r + \sigma_2^2/2)\, T] \,/\, \sigma_2\sqrt{T}$

$d_4 = [\ln (S_2/S_1) + (\sigma^2/2)\, T] \,/\, \sigma\sqrt{T}$

$d_5 = -d_1 + \sigma_1\, \sqrt{T}$

$d_6 = -d_3 + \sigma_2\, \sqrt{T}$

$\rho_1 = (\sigma_1 - \rho\sigma_2)/\sigma$

$\rho_2 = (\sigma_2 - \rho\sigma_1)/\sigma$

and $\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$

A direct check of the first two terms of equation [26] is to compare this option pricing model with the option pricing model for the exchange of one asset for another. The following three portfolios have identical payoffs:

Portfolio 1: $C_{max}(S_1,\, S_2,\, 0,\, T)$

Portfolio 2: $S_2 + Ex\, (S_1,\, S_2,\, T)$

Portfolio 3: $S_1 + Ex\, (S_2,\, S_1,\, T)$

Portfolio 1 contains an option on the maximum of S_1 or S_2 with zero exercise price. This is equivalent to holding S_2 with the option to exchange it for S_1 (portfolio 2) or holding S_1 with the option to exchange it for S_2 (portfolio 3).

The value of portfolio 1 can be computed from equation [26]

$$C_{max}(S_1,\ S_2,\ 0,\ T) = S_1N_2(\infty,\ d_2,\ \rho_1) + S_2N_2(\infty,\ d_4,\ \rho_2) + 0$$
$$= S_1\ N(d_2) + S_2\ N(d_4)$$

where $d_2 = [\ln(S_1/S_2) + (\sigma^2/2)\ T]/\sigma\sqrt{T}$

$d_4 = [\ln(S_2/S_1) + (\sigma^2/2)\ T]/\sigma\sqrt{T}$

and $\sigma = \sigma_1^2 + \sigma_2^2 - 2\ \rho\sigma_1\ \sigma_2$

The value of portfolio 2 can be computed from equation [17]:

$$S_2 + Ex(S_1,\ S_2,\ T) = S_2 + S_1\ N(d_1) - S_2\ N(d_2)$$
$$= S_1\ N(d_1) + S_2\ (1-N(d_2))$$
$$= S_1\ N(d_1) + S_2\ N(-d_2)$$

where $d_1 = [\ln\ (S_1/S_2) + (\sigma^2/2)T]/\sigma\sqrt{T}$

$-d_2 = -[\ln\ (S_1/S_2) - (\sigma^2/2)T]/\sigma\sqrt{T}$

$= [\ln\ (S_2/S_1) + (\sigma^2/2)T]/\sigma\sqrt{T}$

and $\sigma = \sigma_1^2 + \sigma_2^2 - 2\ \rho\sigma_1\sigma_2$

The same can be shown for portfolio 3. Q.E.D.

II.4 Call Option on the Minimum of Two Assets

An investor would exercise an option on the minimum of two assets S_1 and S_2, and purchase S_1 if and only if $S_1^T > X$ and $S_2^T > S_1^T$, and conversely for S_2. Thus, the option will only be exercised when both S_1^T and S_2^T are larger than X. To determine the value of an option on the minimum of two assets, the integration limits must be changed in [25]. Changing its limits, equation [25] becomes:

$$
\begin{aligned}
C_{min}(S_1, S_2, X, T) = e^{-rt} \{ & \int_X^\infty S_1^T \int_{S_1^T}^\infty g(S_1^T, S_2^T, \rho)\, dS_2^T\, dS_1^T \\
& + \int_X^\infty S_2^T \int_{S_2^T}^\infty g(S_1^T, S_2^T, \rho)\, dS_1^T\, dS_2^T \\
& - X \int_X^\infty \int_X^\infty g(S_1^T, S_2^T, \rho)\, dS_1^T\, dS_2^T \}
\end{aligned} \qquad [27]
$$

Equation [27] can be solved to obtain:

$$
\begin{aligned}
C_{min}(S_1, S_2, X, T) = {} & S_1 N_2(d_1, -d_2, -\rho_1) + S_2 N_2(d_3, -d_4, -\rho_2) \\
& - e^{-rT} X N_2(-d_5, -d_6, \rho)
\end{aligned} \qquad [28]
$$

where all the variables are as in equation [26].

This equation is identical to equation [12] in Stulz (1982) except for probable typographical errors in the second term of his first two bivariate normal distributions where the expressions $(-{}^1/_2\, \sigma^2\sqrt{T})$ should read $(-{}^1{,}_2\, \sigma^2 T)$. Stulz does not provide an equation for the direct evaluation of an option on the maximum of two assets.

Verification of the compatibility of equations [26] and [28] proceeds as follows. Since holding a call option on the maximum of two assets is similar to holding a call option on each asset and selling a call option on the minimum of the two assets, one obtains:

$$C_{max}(S_1, S_2, X, T) = C(S_1, X, T) + C(S_2, X, T) - C_{min}(S_1, S_2, X, T)$$

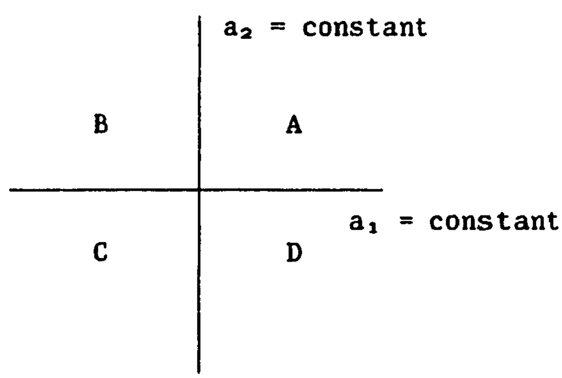
stated differently:

$$S_1\ N_2\ (d_1, d_2, \rho_1) + S_2\ N_2\ (d_3, d_4, \rho_2) + e^{-rT}\ (-X + X\ N_2(d_5, d_6, \rho))$$

$$= S_1\ N(d_1) - e^{-rT}\ X\ N\ (-d_5) + S_2\ N(d_3) - e^{-rT}\ X\ N(-d_6)$$

$$-[S_1\ N_2\ (d_1, -d_2, -\rho_1) + S_2\ N_2\ (d_3, -d_4, -\rho_2) - e^{-rT}\ X\ N_2\ (-d_5, -d_6, \rho)$$

The areas over which the probability density function is accumulated can be determined from a graph.



where a_1 and a_2 are the first and second terms of the bivariate distribution

Examination of the terms in S_1 reveals that $N_2\ (d_1, d_2, \rho_1)$ is the volume above C, $N(d_1)$ is the volume above C and D, $N_2(d_1, -d_2, -\rho_1)$ is the volume above D, thus $N_2\ (d_1, d_2, \rho_1) = N(d_1) - N(d_1, -d_2, -\rho_1)$.

Similar results are obtained for the terms in S_2. Examination of the terms in X, reveals that $N_2(d_5, d_6, \rho)$ is the volume above C, $N(-d_5)$ is the volume above B and A, $N(-d_6)$ is the volume above A and D, $N_2(-d_5, -d_6, \rho)$ is the volume above A. Thus:

$N(-d_5) + N(-d_6) - N(-d_5, -d_6, \rho) = 1 - N(d_5, d_6, \rho)$

Q.E.D.

For options on forward contracts, equations [26] and [28] need to be modified. Equation [26] becomes:

$$C_{Fmax}(F_1, F_2, X, T) = e^{-rT} \{F_1 N_2(d_1, d_2, \rho_1) + F_2 N_2 (d_3, d_4, \rho_2) + X [N_2 (d_5, d_6, \rho) - 1]\} \qquad [29]$$

where $d_1 = [\ln(F_1/X) + (\sigma_1^2/2)T]/\sigma_1\sqrt{T}$

$d_2 = [\ln(F_1/F_2) + (\sigma^2/2)T]/\sigma\sqrt{T}$

$d_3 = [\ln(F_2/X) + (\sigma_2^2/2)T]/\sigma_2\sqrt{T}$

$d_4 = [\ln(F_2/F_1) + (\sigma^2/2)T]/\sigma\sqrt{T}$

$d_5 = -d_1 + \sigma_1\sqrt{T}$

$d_6 = -d_3 + \sigma_2\sqrt{T}$

ρ_1, ρ_2 and σ are as in equation [26]

Such an option is included in a bond that can be redeemed at the option of the holder in any of three currencies. Any of the three currencies can then be chosen as the numeraire, and the two others as F_1 and F_2.

Equation [28] becomes:

$$C_{Fmin}(F_1, F_2, X, T) = e^{-rT}[F_1 N_2(d_1, -d_2, \rho_1) + F_2 N_2(d_3, -d_4, \rho_2) - X N_2(-d_5, -d_6, \rho)] \qquad [30]$$

where all the variables are as in equation [29].

II.5 Put Options on the Maximum or Minimum of Two Assets

The values of put options on the maximum or minimum of two assets are obtained from replicating portfolios. The value of a put option on the maximum of two assets is:

$$P_{max}(S_1, S_2, X, T) = X\ e^{-rt} - C_{max}(S_1, S_2, 0, T) + C_{max}(S_1, S_2, X, T)$$

Portfolio 1, which contains $P_{max}(S_1, S_2, X, T)$, has the same value as portfolio two which is long $C_{max}(S_1, S_2, X, T)$, short $C_{max}(S_1, S_2, 0, T)$ and has $X\ e^{-rt}$ invested in the riskless security. If at maturity $\max(S_1^m, S_2^m) > X$, then both portfolios are worthless. If $\max(S_1^m, S_2^m) < X$, then both portfolios will be worth $X - \max(S_1^m, S_2^m)$.

The value of the option with zero exercise price is:

$$C_{max}(S_1, S_2, 0, T) = S_1\ N(d_2) + S_2\ N(d_4)$$

Hence:

$$\begin{aligned} P_{max}(S_1, S_2, X, T) &= S_1\ (N_2(d_1, d_2, \rho_1) - N(d_2)) \\ &+ S_2\ (N_2(d_3, d_4, \rho_2) - N(d_4)) \\ &-e^{-rt}\ X\ N_2(d_5, d_6, \rho) + X\ e^{-rt} \end{aligned} \qquad [31]$$

Using the Figure on page 18, examination of the terms in S_1 indicates that $N_2(d_1, d_2, \rho_1)$ is the volume above C, and $N(d_2)$ is the volume above B and C. Thus, the coefficient of S_1 equals $-N_2(-d_1, d_2, -\rho_1)$. Similarly, the coefficient for S_2 equals $-N_2(-d_3, d_4, -\rho_2)$, and:

$$P_{max}(S_1, S_2, X, T) = Xe^{-rt} \ (1-N_2(d_5, d_6, \rho) - S_1 \ N_2(-d_1, d_2, -\rho_1)$$
$$- S_2 \ N_2(-d_3, d_4, -\rho_2)$$

where all the variables are as defined in equation [26]. The value of a put option on the minimum of two assets is:

$$P_{min}(S_1, S_2, X, T) = X e^{-rt} - C_{min}(S_1, S_2, 0, T) + C_{min}(S_1, S_2, X, T)$$

Portfolio three, which contains $P_{min}(S_1, S_2, X, T)$, has the same value as portfolio four, which is long $C_{min}(S_1, S_2, X, T)$, short $C_{min}(S_1, S_2, X, T)$ and has $X e^{-rt}$ invested in the riskless security. If at maturity min $(S_1^T, S_2^T) < X$, then both portfolios are worth X - min (S_1^T, S_2^T). The value of the option with the zero exercise price is:

$$C_{min} (S_1, S_2, 0, T) = S_1 \ N(-d_2) + S_2 \ N(-d_4)$$

Hence:

$$P_{min}(S_1, S_2, 0, T) = S_1 \ (N_2(d_1, - d_2, -\rho_1) - N(-d_2))$$
$$+ S_2 \ (N_2(d_3, -d_4, - \rho_2) - N(-d_4))$$
$$- e^{-rt} N_2(-d_5, -d_6, \rho)$$

$$= X e^{-rt} \ (1 - N_2(d_5, d_6, \rho)) - S_1(N_2(-d_1, -d_2, \rho_1))$$
$$- S_2(N_2(-d_3, -d_4, \rho_2)) \qquad [32]$$

III. Correlation Structure of FX Rates

Before evaluating various types of options discussed in this paper, the correlations and covariances between the exchange rates of the major currencies are studied. Since the volatilities of FX rates seem to increase or decrease for all currencies simultaneously (see Essay I), the correlation matrices will be reported in this section instead of the covariance matrices because they are easier to interpret. The correlation (and covariance) of the returns on the various currencies is important for option valuation purposes. For long-term investments in foreign countries, the correlations of the levels of the FX rates is more important than the correlations of changes during intermediate periods (i.e., periodic returns). This analysis will discuss the correlations of both the returns[4] on foreign currencies and the levels of FX rates.

III.1 Statistical Methodology

Two statistical tests are used to test the equality of the correlation matrices. The Jennrich (1970) test for the equality of several correlation matrices converges to a X^2 when the sample sizes tend to infinity. His test statistic is:

$$JTS = \sum_{i=1}^{k} \left(\frac{1}{2} tr(Z_i^2) - dg'(Z_i) S^{-1} dg(Z_i) \right) \qquad [33]$$

4. Returns are computed as: $(FX_{t+1}/FX_t)-1$

where tr (A) = the trace of matrix A

dg (A) = the diagonal of a square matrix A, written as a column vector

dg'(A) = the transpose of dg(A)

$Z_i = \sqrt{n_i}\ \bar{R}^{-1} (R_i - \bar{R})$

R_i = the i^{th} correlation matrix based on a sample of size n_i

$$\bar{R} = \sum_{i=1}^{k} \frac{n_i R_i}{n}$$

k = the number of correlation matrices

$$n = \sum_{i=1}^{k} n_i$$

$S = (\delta_{ij} + \bar{r}_{ij}\ \bar{r}^{ij})$

δ_{ij} = the Knonecker delta, which is equal to 1 when i=j and 0 otherwise

$\bar{r}_{ij}$ = the element of row i, column j of $\bar{R}$

$\bar{r}^{ij}$ = the element of row i, column j of $\bar{R}^{-1}$

JTS is distributed as a X^2 with (k-1)p(p-1)/2 degrees of freedom, where p is the size of the correlation matrix.

Although the sum in equation [33] converges in distribution to a X^2 variable, the terms in the sum need not converge to X^2 variables. When sample sizes are vastly different, this may cause some problems. When daily, monthly, quarterly and semi-annual correlation matrices were compared, the test statistic sometimes came out negative!

The second statistical test is a modified Box (1949) test for the equality of covariance matrices. Box's test statistic is given as:

$$BTS = M\ C^{-1} \qquad [34]$$

$$\text{where } M = \sum_{i=1}^{k} (n_i - 1) \ln |S| - \sum_{i=1}^{k} (n_i - 1) \ln |S_i|$$

S_i is the p by p covariance matrix.

$$S = \sum_{i=1}^{k} \frac{n_i S_i}{n}$$

$$C^{-1} = 1 - \frac{2p^2 + 3p - 1}{6(p+1)\ (k-1)} \left(\sum_{i=1}^{k} \frac{1}{n_i} - \frac{1}{n} \right)$$

and all the other variables are as defined in equation [33]. The BTS is approximately distributed as a X^2 with $(k-1)p(p+1)/2$ degrees of freedom as n_i becomes large. This X^2 approximation appears good if k and p do not exceed four or five, and each n_i is twenty or more.[5] Cho and Taylor (1987) test the equality of correlation matrices by simply replacing the covariance matrices in [28] by the correlation matrices. This is not acceptable since the number of degrees of freedom of the X^2 distribution must be adjusted to recognize that all the diagonal terms of the correlation matrices are ones. Jennrich (1970) shows that even when the n_i's tend to infinity (and hence BTS $\doteq$ M), the BTS does not, in general, converge to a X^2. The critical values taken from the X^2 distribution may be too small by a factor of two. This indicates that a bias may exist towards rejecting the hypothesis that the correlation matrices are equal when in fact they are.

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5. These figures are drawn from Morrison [1976], p. 252.

Since neither test can be used with complete confidence, both tests were used whenever possible to confirm the results. The number of degrees of freedom for the BTS of the correlation matrices was adjusted to (k-1)p(p-1)/2. When the BTS exceeded $\chi^{2(df)}_{\alpha}$, the BTS was compared to two times $\chi^{2(df)}_{\alpha}$ to account for the findings of Jennrich (1970).

III.2 The Data

The FX data, which were provided by I.P. Sharp Associates, included the daily exchange rates in New York for the German Mark (DM), the Swiss Franc (SF), the Dutch Guilder (FL), the French Franc (FF), the Japanese Yen (JY), the British Pound (BP), the Canadian Dollar (CD) and the Australian Dollar (AD). The data covered the period from January 2nd 1974 to December 31st 1987.

III.3 Results

III.3.1 Comparison of Holding Periods

The daily, monthly and quarterly correlations of the returns on the various currencies for the entire period, using the US dollar as the numéraire, are presented in Table 1. Except for the Canadian and the Australian dollar, the correlations for the various holding periods are quite similar. Using both statistical tests, no significant difference exists between the correlation (and covariance) matrices of the monthly, quarterly and semi-annual holding periods for the return on the eight currencies.[6] The covariance matrices were divided by t, where t is the average holding period in days, before being compared across holding periods. This correction assumes that the variance of the FX returns is proportional to the holding period (i.e., consistent with a Geometric Brownian motion). No statistically significant differences were found for the currency covariance matrices across holding periods.

When the daily matrices were also used, the Jennrich statistic became negative (JTS = -100.81, df = 84). This is probably due to the large number of daily observations (3,505) compared to the number of monthly, quarterly and semi-annual observations (168, 56 and 28, respectively). The modified Box statistic was 248.36 (84df, $\alpha <$ 0.0005). The BTS was highly significant ($\alpha \approx 0.005$) even when it was adjusted for the maximum possible bias.

While this suggests that the correlation matrices of daily returns may be different from those for longer holding periods, the difference may be due to the vastly different sample sizes. In addition, the correlation (and covariance) matrices for the levels of FX rates are not statistically different across holding periods (see Table 2 for the daily and monthly correlation matrices of FX levels). Table 3 presents

6. The specific results for the correlation matrices are: JTS = 29.44, df = 56, α = 0.9987; and BTS = 30.37, df = 56, $\alpha > 0.95$.

the results of the tests of equality of the correlation and covariance matrices of FX levels for all the holding periods combined, and for the one, three and six months holding periods combined. Based on these results, it seems that (with the exception of daily returns) the covariance (and correlation) matrices for levels and for returns are not different for different holding periods, and that the hypothesis that returns follow a Geometric Brownian motion cannot be rejected with these tests. To reduce computational costs and the potential problems associated with daily data,[7] the remainder of the analysis will be based on a monthly holding period.

Tables 1 and 2 can be summarized as follows: while all currencies move in the same direction in the short run vis-à-vis the US dollar (as indicated by all the positive correlations between the returns), many move in opposite directions over the long run (as indicated by the negative correlations between the levels). The DM and the SF, and the DM and the FL are highly correlated in all cases, unlike other pairs of currencies. For example, while the correlations of the returns for the FF with the DM and the JY are 0.812 and 0.603, respectively, the corresponding figures for their levels are 0.140 and −0.205, respectively. A possible explanation for the FF-DM results is that the FF is closely connected to the DM through the European Monetary Systems (EMS). While each relatively large (official) parity change appears only once in the return series, it has a permanent impact on the level

7. The correlation structure of returns for daily data may be different from that for all other holding periods.

series. The parity changes are clearly a departure from a Geometric Brownian diffusion process. However, to the extent that the timing of official parity changes can be forecast with reasonable accuracy, option pricing can be based on the returns correlations during the periods between official parity changes. The BP and JY also move together in the short run and apart in the long run. The BP-JY correlations for returns (ρ_r) and levels (ρ_ℓ) are 0.487 and -0.261, respectively.

Except for the Canadian and Australian dollars, currency diversification only works well in the long run. This may indicate a need for short term hedging instruments such as options. The high correlation between the returns on the various currencies suggests that options on the maximum of two currencies would be relatively inexpensive.[8] This result also indicates that the use of baskets of currencies may only reduce currency risk in the long term.

These results only apply for an individual using the US dollar as the numéraire. Verification is required to determine if the situation is the same for other numéraire currencies.

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8. Stulz (1982) shows that the value of an option on the minimum of two assets increases as the correlation increases. The reverse occurs for an option on the maximum of two assets.

III.3.2 Evolution Over Time

During the period of floating exchange rates, the world economy has experienced several major changes such as the oil shocks, the third world debt crisis, the reinforcement of the EMS, and the sharp rise then fall of the US dollar. These events suggest that the correlation structure of exchange rates may have changed over time. Statistical tests show that the covariance and correlation matrices for levels of exchange rates for the two pairs of sub-periods tested (namely, 1974-1980 vs 1981-1987 and 1981-1985 vs 1986-1987) have changed. The tests of equality of the matrices of returns produce weaker results. The Jennrich test does not reject the equality of the correlation matrices of returns for 1981-1985 and 1986-1987 at the 10 % level (see Table 4). The ends of 1980 and of 1985 correspond approximately with the beginnings of a sharp rise and a sharp drop of the US dollar. The political-economic events which marked these periods are the election of Ronald Reagan in November 1980, and the "Plaza Accord" between the G5 countries[9] to lower the value of the US dollar.

Table 5 shows the correlation matrices for the levels of the FX rates for the sub-periods 1974-1980 and 1981-1987. Since the correlations of exchange rate levels are mainly useful in the long run, the results for the shorter sub-periods are not presented. The correlation structure is fairly stable for the JY, the DM, the SF, the

9. These countries are: the United States, Japan, Germany, the United Kingdom and France.

FF and the FL, but it is unstable for the BP, and very unstable for the CD and the AD.

The correlation matrices of the returns on the currencies for the sub-periods 1974-1980, 1981-1987, 1981-1985 and 1986-1987 are given in Table 6. These matrices are significantly different. Three observations can be made from Table 6. First, the strong relationships between the DM, the SF, the FL and the FF are getting stronger over time. For example, the FF ρ_r with the DM increased from 0.798 (1974-1980), to 0.938 (1981-1985), to 0.975 (1986-1987). Second, the JY and the BP are getting more correlated with the Continental European currencies. Third, the correlation of the returns on the AD with all the other currencies (with the exception of the CD) turns negative in 1986-1987. With the exception of the AD, the correlations of returns do not change qualitatively with the passage of time.

III.3.3 Results Using Numéraires Other Than the US Dollar

From a DM numéraire point of view (see Table 7), return correlations are generally positive. Except for the correlations between the CD and the USD, and to a lesser extent between the AD and the USD and the AD and the CD, the correlations amongst currencies are much smaller than when the USD is considered as the numéraire. Thus, a greater scope for diversification exists with a DM numéraire.

The correlation structure of FX rates expressed in DM is provided in Table 8 for the longer sub-periods. From a German numéraire point of view, the following pairs of currencies move together; the USD and the CD, the BP and the AD, and the CD and the AD. The SF moves against all the currencies except the JY. The correlation between the FF and the BP is extremely stable at approximately 0.85, and the correlation between the FF and the FL is large over a long period (e.g. ρ_1 = 0.945 for the period 1974-1987).

From the British numéraire point of view, the European currencies are identifiable as a block. For example, over the entire period, the correlations of the returns amongst the European currencies range between 0.73 and 0.975 (see Table 9), and over the period 1986 to 1987 ρ_r ranges between 0.88 and 0.999. The US and the Canadian dollars move closely together, and the AD is somewhat correlated with the USD and the CD. Based on the correlations of the levels of the FX rates reported in Table 10, the European block (DM, SF, FL and FF) is replaced by a strong currency block including the DM, the SF, the FL and the JY.

From a Canadian numéraire point of view, in the short run (see Table 11), the USD moves independently of the other currencies, the EMS currencies move very closely together, and, with the exception of the USD and the AD, the correlations amongst all the pairs of currencies are increasing over time. These results are similar to those using the USD as a numéraire. In the long run, there is a clear change in the correlation between the USD and the other currencies between 1974 to 1980 and 1981 to 1987. This may be explained by the political events in Canada which caused the Canadian dollar to drop against the USD and all the other currencies between 1974 and 1980. From 1981 onwards the correlations between the USD and the other currencies are weaker.

IV. Comparision of Option Types

In this section, the prices of options on the maximum of two assets are compared with those of options on the sum of two assets and with those of simple options. Each type of option has specific advantages and is most appropriate in certain circumstances. In some cases, they can substitute for each other, though imperfectly, for hedging purposes. Thus, it is useful to compare the prices of these options. First, the pricing of options on portfolios will be discussed, then option prices will be compared and discussed.

IV.1 Pricing of Options on a Portfolio

Knowing the standard deviations of the returns on assets in a portfolio and the correlation of these returns, the standard deviation of the return on the portfolio can be determined. If the market value of each asset is given by:

$$S_i = \mu_i + dS_i \qquad \text{where } dS_i \quad N(0, sig_i) \qquad [35]$$

and sig_i is not large

then $\ln (S_i/\mu_i) \approx dS_i/\mu_i$

and $\rho(dS_i/\mu_i,\ dS_j/\mu_j)$, the correlation of the returns equals $\rho(dS_i,\ dS_j)$ and $\rho(S_i,\ S_j)$, and equals approximately $\rho(\ln S_i,\ \ln S_j)$.

The standard deviation of the returns σ_i is a function of sig_i, (the standard deviation of the price of the asset):

$$\sigma_i^2 = \ln \left(1 + \frac{sig_i^2}{\mu_i^2}\right) \qquad [36]$$

If sig_i/μ_i is not large, say ≤ 0.30,

$$\ln \left(1 + \frac{sig_i^2}{\mu_i^2}\right) \approx \frac{sig_i^2}{\mu_i^2}$$

and $$\sigma_i \approx \frac{sig_i}{\mu_i} \qquad [37]$$

Empirically, the daily volatility of currencies is very rarely larger than 0.01 (see Essay I). Hence, for a 90 day option, σ_i would rarely exceed $(0.01\sqrt{90}=)$ 0.09. For a one year option, σ_i could reach $(0.01\sqrt{365}=)$ 0.19. The case in which this derivation may be inappropriate is for very long maturity options on high volatility assets, such as high volatility stocks.

The variance of the price of the portfolio containing n assets can then be expressed as:

$$Sig_p^2 = \sum_{i=1}^{n} \mu_i^2 \sigma_i^2 + \sum_{i \neq j}^{n} \sum^{n} \mu_i \mu_j \rho_{ij} \sigma_i \sigma_j$$

and the volatility of the returns can be expressed as:

$$\sigma_p = \frac{\left[\sum_{i=1}^{n} \mu_i^2 \sigma_i^2 + \sum_{i \neq j}^{n} \sum^{n} \mu_i \mu_j \rho_{ij} \sigma_i \sigma_j\right]^{1/2}}{\sum_{i=1}^{n} \mu_i} \qquad [38]$$

The price of an option on a portfolio of forwards is then given by equation [6] and that of an option on a portfolio of stocks is given by equation [8], when the variables are redefined as:

$$F = \sum_{i=1}^{n} F_i \qquad [39]$$

$$X = \sum_{i=1}^{n} X_i$$

and σ is as given by equation [38] with μ_i replaced by F_i.

Alternatively: $S = \sum_{i=1}^{n} S_i$ [40]

$$X = \sum_{i=1}^{n} X_i$$

and σ is as given by equation [38] with μ_i replaced by S_i.

For the case of options on two currencies forwards, equation [6] can be used with:

$$F = F_1 + F_2$$

$$X = X_1 + X_2$$

and $\sigma = (F_1\ \sigma_1 + F_2\ \sigma_2 + 2\ F_1 F_2\ \rho_{12}\ \sigma_1\ \sigma_2)^{1/2}/(F_1 + F_2)$

IV.2 Comparison of Option Prices

To facilitate the discussion, hypothetical standard currencies will be used, where the standardization is such that the exercise price is fixed at 50 ¢/unit. Suppose that the 90 day forward rates are 20 ¢/FF and 200 ¢/£, and that a firm will have to make a payment of either FF 5 Million or £ 0.5 Million in 90 days. To hedge this payment, the firm could buy a call option for FF 5 Million or £ 0.5 Million with an exercise price of $ 1 Million. This option could be standardized as an option to buy 2 Million units of currency c_1 or c_2 with an exercise price of 50 ¢/unit. The volatilities of c_1 and c_2 would be the same as those of the FF and the £, but the current forward rates would be:

$$F_1 = 2.5 \times \text{FF forward rate}$$

$$F_2 = 0.25 \times \text{£ forward rate.}$$

In this example, an option on the maximum of £ 0.5 Million or FF 5 Million will provide a complete hedge for the payment. It may actually leave the firm with a "speculative" profit if, for example, the actual payment has to be eventually made in FF while the £ appreciated relatively more. If the firm were risk neutral and the markets were perfect, all types of hedging (or no hedging at all) would have the same value in the long run. Practically, firms are not risk neutral, since they often attach much more importance to FX losses than to FX gains.[10] Also, markets are not perfect (for example, bankruptcy costs are not nil). When firms are designing a hedge, they often consider the cost of an option as an expense and typically do not attach great value to "speculative" FX gains. Hence, minimizing the cost of an option hedge becomes important.

In the above example, an alternative hedge would be to buy an option for the francs and one for the pounds. This hedge could yield more "speculative" profit, but it is also more costly. Another hedge could be to buy an option on the sum of FF 5 M and £ 0.5 M. This hedge could be inadequate if, for example, the payment ends up being in £ while the £ went up sharply and the FF declined. However, if the £ and the FF are positively correlated (which they actually are like most pairs of currencies vis-à-vis the US dollar), then this hedge may he worth considering. Let us now look at the costs of these hedges. Assuming the daily volatilities of both currencies are 0.008 and that interest rates are about 11.6 % per annum (0.03 % per day), the costs of 90 day hedges are reported in Table 13. Table 13 reports prices when both currencies are 2 % in-the-money, at-the-money, 2 % out-of-the-money, and combinations of 2 % in-the-money/at the money, 2 % in the money/2 % out of the money, and at-the-money/2 % out-of-the-money. The results are fairly similar in all cases, so the case of at-the-money options will mainly be discussed.

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10. Cezairli and Erdilek (1988) find that in a survey of Fortune 500 companies, 60.8 % of the respondents considered "minimizing cash losses identified with specific foreign currency transactions" as very important, while only 3.9 % considered "profiting from exchange rate volatility" as being very important.

Negative correlations are fairly rare (especially large ones). However, for the sake of completeness, it can be said that at-the-money options on the sum of 2 assets with large negative correlations are very cheap. They are actually free if $\rho = -1$, since, in that case, $c_1 + c_2$ always equals $x_1 + x_2$. At-the-money options on the maximum of 2 currencies which have a large negative correlation are very expensive, since this option will always be valuable unless both currencies do not move at all. A more interesting (and common) scenario is when the correlation is positive. Examining at-the-money options, if we take the price of the option on the maximum of the 2 currencies as 100 when $\rho = 0.6$, an option on the sum of the 2 currencies which would not provide a guaranteed hedge would cost 122.6, buying an option on each of the currencies would cost 137.0, while the price of a single option, which has a large probability of being inadequate, is 68.5. When $\rho = 0.9$, such as amongst the currencies of the European Monetary System (EMS), an option on the sum of the currencies will most probably be adequate but it will cost 158.5. Actually, in such a case, an option on a fraction of the sum of the two currencies may be considered. The cost of buying 2 simple options is 162.6, while the cost of an option on only one currency is 81.3.

When $\rho = 0.95$, as for the FF and the DM during 1981-1987, the price of the option on the maximum of the two currencies is only slightly larger than the price of a single option. The prices of the option on the sum of the currencies, of 2 simple options and of 1 simple option are 169.8, 172.0 and 86.0, respectively. For the case where c_1 is at the money, c_2 is 2 % out of the money and $\rho = 0.95$, the price of the complex option is only 0.06 ¢ per unit. This is 4 % more expensive than a simple option on c_1. This means that, if a firm is willing to issue, for example, bonds in USD with redemption in USD or DM at the option of the holder, then adding an out-of-the-money option for redemption in FF would be very inexpensive for the issuer and could be a valuable hedge for the bond holder.

The advantage of options on the maximum of two currencies is most evident when the amounts in the two currencies are not identical. Table 14 shows what happens when the smaller (X_2) amount is 90 % or 75 % of the larger amount (X_1). For $X_2 = 0.9\ X_1$, when $\rho = 0.6$, the price of the option on the maximum is approximately 30 %, 33 % and 35 % larger than the price of the option on X_1, when both currencies are in-the-money, at-the-money and out-of-the-money, respectively. When $\rho = 0.9$, the increased cost of the option on the maximum as opposed to a simple option is approximately 12 %, 13 % and 14 % when both currencies are in-, at- or out-of-the money, respectively. Obviously, as the correlation gets higher, or as X_2 decreases as a fraction of X_1, the premium gets even smaller.

When c_2 is out-of-the money and c_1 is in-the money, the premium for the option on the maximum becomes very small. If $X_2 = 0.90\ X_1$, and $\rho = 0.6$, the premium is 10 %. If $\rho = 0.9$, the premium is 0.9 %.

A useful example is with c_1 at-the-money and c_2 out-of-the-money. In that case, for $X_2 = 0.90X_1$ and $\rho = 0.9$, the premium is 4 % or 0.06 cents per unit. It should be remembered that a correlation of 0.9 between the European currencies vis-à-vis the US dollar is not uncommon. This indicates that by using complex options, firms could hedge at a very low cost some of the FX risk which previously was unhedged. This also means that financial institutions can "sweeten" simple options by adding complex features at a very low extra cost.

V. Concluding Remarks

In this essay, simplified derivations of pricing models for simple and complex options (such as options to exchange one asset for another or options on the minimum or maximum of two assets) were presented. These derivations are based on the insight provided by Cox, Ross and Rubinstein (1979) that the value of an option can be interpreted as the expectation of its discounted future value in a risk-neutral world. They do not require the solution of differential equations, nor the use of stochastic calculus.

The correlation structure of the exchange rates of nine major currencies was then studied. Several results were found. First, the covariance structure of returns from holding these currencies is not different for either one, three, or six months holding periods. The results for a one day holding period are more ambiguous. These results confirm the finding of Essay I that a Geometric Brownian motion may be appropriate to describe exchange rate movements, especially when longer periods are considered. All currencies were found to move together vis-à-vis the US dollar in the short run, but not in the long run. This may be due to the behaviour of FX traders who quote all currencies versus the US dollar and follow (or cause) its short run fluctuations versus the rest of the world. Alternatively, it may be due to the behaviour of central banks, especially in the European Monetary System, which tend to keep their currencies within a certain range with respect to each other and then make large discrete parity changes. These discrete changes do not affect the correlation of returns significantly, but they affect the correlation of FX levels. The practical consequence of this for business is that from a US or British point of view, currency diversification, or the use of currency baskets, does not reduce FX risk a lot in the short run, although it does in the long run.

From a German or Canadian point of view, currency diversification may reduce FX risk even in the short run. From a Canadian point of view, the correlation structure of FX rates estimated between 1974 and 1980 was dramatically different from that estimated between 1981 and 1987. It was also found that the correlation structure of FX rates changed over time while the structure of FX returns was relatively more stable and reflected the increasing integration of the European economies.

Finally, the costs of using complex options was assessed through some examples. Options on the maximum of two assets may be relatively expensive when the correlation of the returns on the two assets is small or negative. However, the returns on most currencies are highly correlated, especially vis-à-vis the US dollar This indicates that for US corporations, it may be relatively inexpensive to hedge some of the FX risk which used to remain unhedged when the currency of an eventual payment or receipt is not known with certainty, even when the levels of the foreign currencies may be diverging over time. This is particularly true when the amounts to be hedged in the foreign currencies are not equal. The fact that options on the maximum of two currencies are quite inexpensive may also be significant for financial institutions which could use them as marketing tools for the sale of their other more basic financial instruments, or as sweeteners on new security issuances (especially on the euromarkets).

Appendix I

This appendix shows how the solution to the B-S model can be derived. Let:

$$C = \int_X^\infty S^m - X\ f(S^m)\ dS^m$$

where S^m follows a lognormal distribution.

Define: $s^m = \ln S^m \qquad \sigma^t = \sigma\sqrt{T}$

$$\mu_m = \ln S\ e^{rT} - \frac{\sigma^2}{2} T = s + (r - \frac{\sigma^2}{2})\ T$$

Then $\quad n(k) = f(-S^m)$

where $\quad n(\bullet)$ is the standard normal distribution

$$k = -[\ln S^m - (\ln S\ e^{rT} - \sigma^2/2\ T)]/\sigma\sqrt{T}$$

$$= [\ln S/S^m + (r - \sigma^2/2\ T)]\sigma\sqrt{T}$$

$$\int_{-\infty}^{X} -X\ f(-S^m)\ dS^m = -X \int_{-\infty}^{X} n\ (\frac{\ln S/S^m + (r - \sigma^2/2)\ T}{\sigma\sqrt{T}})dS^m = -X\ N(d_2^1)$$

where $d_2^1 = \dfrac{\ln S/X + (r - \sigma_1^2/2)T}{\sigma\sqrt{T}}$

$$\int_S^\infty S^m\ f(S^m)\ dS^m = \int_X^\infty e^{S^m} \exp -1/2\ (\frac{S^m - \mu_m}{\sigma^t})^2\ dS^m$$

$$= \int_X^\infty \exp -1/2 \left[\frac{s^{m^2} - 2\ \mu_m\ s^m + \mu_m^2 - 2\sigma^{t2}\ s^m}{\sigma^{t2}}\right] dS^m$$

$$= \int_X^\infty \exp -1/2 \left[\frac{(s^m - \mu_m - \sigma^{t2})^2 - \sigma^{t4} - 2\mu_m\ \sigma^{t2}}{\sigma^{t2}}\right] dS^m$$

$$= \int_X^\infty \underbrace{\exp\ (\mu_m + 1/2\ \sigma^{t2})}_{= S\ e^{rT}} \exp -1/2\ (\frac{S^m - \mu_m - \sigma^{t2}}{\sigma^t})^2\ dS^m$$

$$= S\ erT \int_X^\infty \exp -1/2\ (\frac{S^m - \mu_m - \sigma^{t2}}{\sigma^t})^2\ dS^m$$

$$= S\ e^{rt} \int_{-\infty}^{X} \exp -1/2\ (\frac{\mu_m + \sigma^{t2} - S^m}{\sigma^t})^2\ dS^m$$

$$= S\ e^{rt}\ N\ (d_1')$$

$$\text{where } d_1' = \left[\ln S\ e^{rt} - \frac{\sigma^2 T}{2} + \sigma^2 T - \ln X\right] / \sigma\sqrt{T}$$

from integration bound

$$= \frac{\ln S/X + (r + \sigma_1^2/2)\ T}{\sigma\sqrt{T}}$$

The difference in the derivation of the formula for options on futures is that S^m is replaced with F^m, and $\mu_r = \ln F - \frac{\sigma^2}{2}\ T$.

Appendix II

This appendix shows how the solution to the first term of equation [21] is derived. Let:

$$A = \int_X^\infty S_1^T \int_{-\infty}^{S_1^m} g(S_1^T, S_2^m, \rho)\, dS_2^m\, dS_1^m \qquad [A1]$$

Since we can act as if we were in a risk neutral world:

$E(S_1^T) = S_1 e^{rt}$ and $E(S_2^m) = S_2 e^{rt}$

define:

$$\begin{array}{llll} S & = S_2/S_1 & S^m & = S_2^m/S_1^m \\ s & = \ln S & s^m & = \ln S^m \\ s_1 & = \ln S_1 & s_1^m & = \ln S_1^m \\ s_2 & = \ln S_2 & s_2^m & = \ln S_2^m \end{array}$$

Replacing S_2^m by S^m in [A1]:

$$A = \int_X^\infty S_1^T \int_{-\infty}^{1} g(S_1^T, S^m, \rho_1)\, dS^m\, dS_1^m \qquad [A2]$$

where: $$g(S_1^T, S^m, \rho_1) = \frac{1}{\sqrt{2\pi}\,\sigma_1^t} \exp\left[-\frac{1}{2}\left(\frac{s_1^m - \mu_1}{\sigma_1^t}\right)^2\right]$$

$$\bullet\ \frac{1}{\sqrt{2\pi}\,\sigma^t\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2}\left(\frac{s^m - \mu - \rho_1\,(\sigma/\sigma_1)\,(s_1^m - \mu_1)}{\sigma^t\sqrt{1-\rho^2}}\right)^2\right]$$

or: $$g(S^m{}_1, S^m, \rho_1) = \exp\left[-\frac{1}{2}\left(\frac{s_1^m - \mu_1}{\sigma_1^t}\right)^2\right] \bullet B \qquad [A3]$$

and: $\sigma_\cdot^t = \sigma_\cdot\sqrt{T}$

$$\mu_1 = \ln S_1 e^{rt} - \frac{\sigma_1^2}{2} T = s_1 + [r - (\sigma_1^2/2)]T$$

$$\mu_2 = s_2 + [r - (\sigma_2^2/2)]T$$

$$\mu = s - (\sigma^2/2)T$$

Define $\rho_1 \equiv \rho_{s_1^m,\, s^m}$

$$\frac{\mathrm{Var}(s^m)}{T} = \frac{\mathrm{Var}(s_2^m - s_1^m)}{T} = \sigma_1^2 + \sigma_2^2 - 2\,\rho\sigma_1\sigma_2 = \sigma^2$$

$$\frac{\mathrm{Cov}(s_1^m, s^m)}{T} = \frac{\mathrm{Cov}(s_1^m, s^m{}_2 - S_1^m)}{T} = \sigma_{1,2} - \sigma_1^2$$

$$==> \rho_1 = \frac{\rho\sigma_1\sigma_2 - \sigma_1^2}{\sigma_1\sigma} = \frac{\rho\sigma_2 - \sigma_1}{\sigma} \qquad [A4]$$

Note that g(.,.,.,.) is the bivariate lognormal density function (see for example Ang and Tang (1975), p. 139, with the normal distribution replaced by the lognormal).

If g(.,.,.,.) were not premultiplied by S_1^m in [A2] the solution would be simply a cumulative bivariate normal distribution. To see the effect of S_1^m on g(.,.,.,.), multiply g(.,.,.,.) in A[3] by S_1^m:

$$S_1^m\, g(S_1^m, S^m, \rho_1) = \exp(\ln S^m{}_1) \bullet \exp\left[-\frac{1}{2}\left(\frac{S_1^m - \mu_1}{\sigma_1^t}\right)^2\right] \bullet B$$

$$= \exp\left[S_1^m - \frac{1}{2}\frac{(S_1^{m2} - 2\,S_1^m\,\mu_1 + \mu_1^2)}{\sigma_1^{t2}}\right] \bullet B$$

$$= \exp\left[-\frac{1}{2}\frac{(S_1^{m2} - 2\,S_1^m\,\mu_1 + \mu_1^2 - 2\sigma_1^{t2}\,S_1^m)}{\sigma_1^{t2}}\right] \bullet B$$

$$= \exp\left\{-\frac{1}{2}\left[\frac{(S_1^m - \mu_1 - \sigma_1^{t2})^2 - \sigma_1^{t4} - 2\mu_1\sigma_1^{t2}}{\sigma_1^{t2}}\right]\right\} \bullet B$$

$$= \exp\left(\mu_1 + \frac{1}{2}\sigma_1^{t2}\right) \bullet \exp\left[-\frac{1}{2}\left(\frac{S_1^m - \mu_1 - \sigma_1^{t2}}{\sigma_1^t}\right)^2\right] \bullet B$$

$$= S_1\, e^{rt} \exp\left[-\frac{1}{2}\left(\frac{S_1^m - \mu_1 - \sigma_1^{t2}}{\sigma_1^t}\right)^2\right] \bullet B \qquad [A5]$$

Note that the first integration limits are from X to ∞. In order to give the answer in terms of a cumulative bivariate normal distribution, the limits need to be inverted and the sign of the term between the parentheses in [A5] needs to be inverted. Hence:

$$A = S_1\, e^{rt}\, N_2\,(d_1, d_2, \rho_1^{\prime}) \qquad [A6]$$

where d_1 and d_2 are standardized variables.

$$d_1 = -\left(\frac{S_1^m - \mu_1 - \sigma_1^{t2}}{\sigma_1^t}\right)$$

$$= -\left(\frac{\ln S_1^m - (\ln S_1 + [r - (\sigma_1^2/2)]T) - \sigma_1^2 T}{\sigma_1\sqrt{T}}\right)$$

$$= -\left(\frac{\ln S_1^m/S_1 - r + (\sigma_1^2/2)\ T}{\sigma_1\sqrt{T}}\right)$$

$$= \frac{\ln S_1/S_1^m + [r + \sigma_1^2/2]\ T}{\sigma_1\sqrt{T}}$$

replacing S_1^m by its value from the integration limit (i.e., X):

$$d_1 = \frac{\ln S_1/X + [r + (\sigma_1^2/2)]T}{\sigma_1\sqrt{T}} \qquad [A7]$$

d_2 is not affected by the premultiplication by S_1^m, hence it is simply:

$$d_2 = \frac{S^m - \mu}{\sigma^t}$$

$$= \frac{\ln S^m - (\ln S_2/S_1 - (\sigma^2/2)T)}{\sigma\sqrt{T}}$$

replacing S^m by its value from the integration limit (i.e., 1):

$$d_2 = \frac{0 + \ln S_1/S_2 + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$= \frac{\ln S_1/S_2 + (\sigma^2/2)T}{\sigma\sqrt{T}} \qquad [A8]$$

and $\rho_1' = -\rho_1$ as given in [A4]

Similar derivations can be made for the other terms of equation [24].

Table 1

Correlation matrices for returns on different currencies using the USD as the numéraire for various holding periods. The data covers 740101 to 871231.

a) One day holding period

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.87	1.00						
RFL	0.937	0.83	1.00					
RFF	0.83	0.75	0.82	1.00				
RJY	0.61	0.58	0.58	0.54	1.00			
RBP	0.66	0.60	0.65	0.61	0.46	1.00		
RCD	0.25	0.23	0.25	0.23	0.14	0.24	1.00	
RAD	0.22	0.19	0.21	0.20	0.22	0.23	0.18	1.00

b) One month holding period

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.89	1.00						
RFL	0.982	0.88	1.00					
RFF	0.89	0.81	0.900	1.00				
RJY	0.57	0.58	0.56	0.60	1.00			
RBP	0.62	0.60	0.64	0.62	0.49	1.00		
RCD	0.27	0.23	0.28	0.24	0.15*	0.26	1.00	
RAD	0.29	0.27	0.29	0.31	0.28	0.28	0.37	1.00

c) Three month holding period

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.88	1.00						
RFL	0.985	0.85	1.00					
RFF	0.89	0.81	0.911	1.00				
RJY	0.65	0.69	0.64	0.66	1.00			
RBP	0.65	0.61	0.67	0.66	0.55	1.00		
RCD	0.16*	0.16*	0.14*	0.11*	0.14*	0.13*	1.00	
RAD	0.35	0.36	0.33*	0.37	0.47	0.30*	0.24	1.00

* Indicates that the correlation coefficient is not significant at the 1 % level.

Table 2

Correlation matrices for the levels of different currencies using the USD as the numéraire for various holding periods. The data covers 740101 to 871231.

a) One day holding period

	DM	SF	FL	FF	JY	BP	CD	AD
DM	1.00							
SF	0.88	1.00						
FL	0.982	0.79	1.00					
FF	0.55	0.14	0.69	1.00				
JY	0.61	0.82	0.49	-0.21	1.00			
BP	0.43	0.05	0.55	0.88	-0.26	1.00		
CD	-0.12	-0.52	0.05	0.69	-0.67	0.65	1.00	
AD	-0.02*	-0.40	0.13	0.75	-0.66	0.81	0.900	1.00

b) One month holding period

	DM	SF	FL	FF	JY	BP	CD	AD
DM	1.00							
SF	0.88	1.00						
FL	0.982	0.79	1.00					
FF	0.54	0.14	0.68	1.00				
JY	0.62	0.82	0.50	-0.21	1.00			
BP	0.41	0.03*	0.53	0.87	-0.26	1.00		
CD	-0.13	-0.52	0.05*	0.69	-0.67	0.66	1.00	
AD	-0.04*	-0.42	0.12*	0.75	-0.66	0.82	0.900	1.00

Table 3

Statistical tests of the equality of the covariance matrices of the FX levels over the entire sample period (1974-1987).

The test results combining all the holding periods are:

JTS	= 12.06	df = 84	$\alpha \approx 1.0$
BTS (correlation)	= 14.10	df = 84	$\alpha > 0.995$
BTS (covariance)	= 11.58	df = 108	$\alpha > 0.995$

The results excluding the daily holding period are:

JTS	= 9.68	df = 56	$\alpha \approx 1.0$
BTS (correlation)	= 7.81	df = 56	$\alpha > 0.995$
BTS (covariance)	= 5.51	df = 72	$\alpha > 0.995$

Table 4

Statistical tests of the equality of the correlation and covariance matrices over time

1974-1980 vs 1981-1987

Levels:	JTS	= 244.66	df = 28	α ≈ 0
	BTS (correlation)	= 526.96*	df = 28	α ≈ 0
	BTS (covariance)	= 723.15	df = 36	α ≈ 0
Returns:	JTS	= 52.98	df = 28	α = 0.0030
	BTS (correlation)	= 118.52*	df = 28	α ≈ 0
	BTS (covariance)	= 101.41	df = 36	α ≈ 0

1981-1985 vs 1986-1987

Levels:	JTS	= 122.18	df = 28	α ≈ 0
	BTS (correlation)	= 322.63*	df = 28	α ≈ 0
	BTS (covariance)	= 328.24	df = 36	α ≈ 0
Returns:	JTS	= 37.42	df = 28	α = 0.1099
	BTS (correlation)	= 110.40*	df = 28	α ≈ 0
	BTS (covariance)	= 111.92	df = 36	α ≈ 0

* Even if the test statistic is divided by 2 to respond to Jennrich's concern, it remains significant.

Table 5

Correlation matrices for the levels of different currencies using the USD as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1980

	DM	SF	FL	FF	JY	BP	CD	AD
DM	1.00							
SF	0.973	1.00						
FL	0.995	0.964	1.00					
FF	0.68	0.66	0.71	1.00				
JY	0.83	0.89	0.83	0.48	1.00			
BP	0.18	0.07*	0.21	0.54	0.04*	1.00		
CD	-0.927	-0.934	-0.921	-0.57	-0.87	-0.04*	1.00	
AD	-0.62	-0.68	-0.61	-0.17*	-0.51	0.54	0.72	1.00

b) For the period 1981 to 1987

	DM	SF	FL	FF	JY	BP	CD	AD
DM	1.00							
SF	0.983	1.00						
FL	0.998	0.977	1.00					
FF	0.76	0.68	0.79	1.00				
JY	0.89	0.910	0.86	0.44	1.00			
BP	0.50	0.41	0.55	0.914	0.12*	1.00		
CD	-0.05*	-0.10*	-0.00*	0.44	-0.44	0.67	1.00	
AD	-0.17*	-0.24*	-0.11	0.45	-0.54	0.68	0.910	1.00

Table 6

Correlation matrices for the returns on different currencies using the USD as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1980

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.86	1.00						
RFL	0.969	0.82	1.00					
RFF	0.80	0.74	0.83	1.00				
RJY	0.41	0.44	0.41	0.47	1.00			
RBP	0.54	0.52	0.55	0.56	0.43	1.00		
RCD	0.23	0.12*	0.22*	0.14*	-0.01*	0.11*	1.00	
RAD	0.32	0.27*	0.31	0.27*	0.30	0.20	0.45	1.00

b) For the period 1981 to 1987

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.920	1.00						
RFL	0.993	0.927	1.00					
RFF	0.950	0.87	0.946	1.00				
RJY	0.68	0.70	0.68	0.69	1.00			
RBP	0.67	0.66	0.70	0.66	0.53	1.00		
RCD	0.32	0.34	0.33	0.34	0.30	0.41	1.00	
RAD	0.27*	0.27*	0.28	0.34	0.27*	0.33	0.31	1.00

c) For the period 1981 to 1985

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.89	1.00						
RFL	0.988	0.901	1.00					
RFF	0.938	0.83	0.933	1.00				
RJY	0.62	0.60	0.60	0.64	1.00			
RBP	0.68	0.64	0.71	0.66	0.45	1.00		
RCD	0.36	0.39	0.38	0.38	0.33	0.41	1.00	
RAD	0.41	0.47	0.43	0.47	0.43	0.36	0.37	1.00

d) For the period 1986 to 1987

	RDM	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RDM	1.00							
RSF	0.963	1.00						
RFL	0.999	0.964	1.00					
RFF	0.975	0.942	0.972	1.00				
RJY	0.75	0.86	0.75	0.75	1.00			
RBP	0.60	0.67	0.61	0.59	0.67	1.00		
RCD	0.10*	0.10*	0.11*	0.06*	0.12*	0.29*	1.00	
RAD	-0.08*	-0.19*	-0.08*	-0.03*	-0.10	-0.20*	0.14*	1.00

Table 7

Correlation matrices for the returns on different currencies using the DM as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	RUS	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RUS	1.00							
RSF	-0.03*	1.00						
RFL	0.10*	0.04*	1.00					
RFF	0.32	0.09*	0.33	1.00				
RJY	0.47	0.18*	0.07*	0.37	1.00			
RBP	0.51	0.11*	0.22	0.33	0.41	1.00		
RCD	0.92	-0.04*	0.11*	0.30	0.44	0.51	1.00	
RAD	0.66	0.01*	0.11*	0.31	0.41	0.43	0.70	1.00

b) For the period 1974 to 1980

	RUS	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RUS	1.00							
RSF	-0.08*	1.00						
RFL	-0.11*	-0.07*	1.00					
RFF	0.47	0.14*	0.40	1.00				
RJY	0.57	0.14*	0.10*	0.46	1.00			
RBP	0.58	0.07*	0.16*	0.45	0.52	1.00		
RCD	0.91	-0.13*	0.10*	0.40	0.48	0.53	1.00	
RAD	0.68	-0.05*	0.07*	0.34	0.51	0.42	0.74	1.00

c) For the period 1981 to 1987

	RUS	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RUS	1.00							
RSF	0.03*	1.00						
RFL	0.10*	-0.28*	1.00					
RFF	0.15*	-0.01*	0.11*	1.00				
RJY	0.38	0.26*	0.00*	0.22*	1.00			
RBP	0.45	0.16*	0.35	0.15*	0.30	1.00		
RCD	0.936	0.07*	0.14*	0.18	0.39	0.51	1.00	
RAD	0.65	0.08*	0.18*	0.31	0.34	0.44	0.68	1.00

d) For the period 1981 to 1985

	RUS	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RUS	1.00							
RSF	0.06*	1.00						
RFL	0.11*	0.31*	1.00					
RFF	0.03*	-0.01*	0.12*	1.00				
RJY	0.40	0.16*	-0.00*	0.22	1.00			
RBP	0.37	0.12*	0.39	0.09*	0.21*	1.00		
RCD	0.61	0.26*	0.24	0.24	0.43	0.33*	1.00	
RAD	0.924	0.12*	0.14*	0.09*	0.43	0.43	0.65	1.00

e) For the period 1986 to 1987

	RUS	RSF	RFL	RFF	RJY	RBP	RCD	RAD
RUS	1.00							
RSF	-0.10*	1.00						
RFL	0.42*	0.05*	1.00					
RFF	0.58	-0.04*	0.03*	1.00				
RJY	0.31*	0.69	0.14*	0.24*	1.00			
RBP	0.59	0.31*	0.44*	0.37*	0.51	1.00		
RCD	0.72	-0.36*	0.17*	0.54	0.19*	0.60	1.00	
RAD	0.953	-0.08*	0.50*	0.52	0.32*	0.64	0.72	1.00

Table 8

Correlation matrices for the levels of different currencies using the DM as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	US	SF	FL	FF	JY	BP	CD	AD
US	1.00							
SF	-0.06*	1.00						
FL	0.06*	-0.89	1.00					
FF	0.04*	-0.88	0.945	1.00				
JY	0.22	0.72	-0.75	-0.80	1.00			
BP	0.33	-0.77	0.77	0.82	-0.59	1.00		
CD	0.79	-0.59	0.61	0.60	-0.30	0.73	1.00	
AD	0.52	-0.73	0.76	0.80	-0.56	0.91	0.89	1.00

b) For the period 1974 to 1980

	US	SF	FL	FF	JY	BP	AD	CD
US	1.00							
SF	-0.80	1.00						
FL	0.86	-0.76	1.00					
FF	0.89	-0.70	0.89	1.00				
JY	0.15*	0.16*	0.05*	0.04*	1.00			
BP	0.73	-0.79	0.72	0.82	0.01*	1.00		
AD	0.926	-0.84	0.82	0.89	0.17*	0.900	1.00	
CD	0.991	-0.82	0.85	0.89	0.10*	0.76	0.94	1.00

c) For the period 1981 to 1987

	US	SF	FL	FF	JY	BP	AD	CD
US	1.00							
SF	0.07*	1.00						
FL	-0.12*	-0.31	1.00					
FF	0.06*	-0.53	0.76	1.00				
JY	-0.04*	0.38	-0.83	-0.77	1.00			
BP	0.39	-0.42	0.70	0.89	-0.76	1.00		
AD	0.64	-0.22*	0.51	0.71	-0.67	0.87	1.00	
CD	0.949	0.00*	0.11*	0.28	-0.30	0.60	0.83	1.00

d) For the period 1981 to 1985

	US	SF	FL	FF	JY	BP	AD	CD
US	1.00							
SF	0.35	1.00						
FL	-0.75	-0.28*	1.00					
FF	-0.78	-0.55	0.71	1.00				
JY	0.84	0.33*	-0.83	-0.67	1.00			
BP	-0.72	-0.55	0.71	0.900	-0.65	1.00		
AD	-0.21*	-0.13*	0.32*	0.50	-0.33	0.42	1.00	
CD	0.942	0.39	-0.67	-0.72	0.71	-0.66	0.03*	1.00

e) For the period 1986 to 1987

	US	SF	FL	FF	JY	BP	AD	CD
US	1.00							
SF	-0.30*	1.00						
FL	-0.16*	0.27*	1.00					
FF	0.953	-0.40*	-0.19*	1.00				
JY	0.29*	0.71	0.28*	0.19*	1.00			
BP	0.82	-0.14*	0.29*	0.83	0.45*	1.00		
AD	0.906	-0.49*	-0.00*	0.923	0.19*	0.88	1.00	
CD	0.991	-0.32*	-0.08*	0.937	0.30*	0.85	0.925	1.00

Table 9

Correlation matrices for the returns on different currencies using the BP as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RCD	RAD
RUS	1.00							
RDM	0.38	1.00						
RSF	0.28	0.84	1.00					
RFL	0.36	0.975	0.82	1.00				
RFF	0.41	0.84	0.73	0.86	1.00			
RJY	0.45	0.50	0.49	0.48	0.55	1.00		
RCD	0.91	0.40	0.29	0.39	0.42	0.43	1.00	
RAD	0.62	0.35	0.27	0.34	0.39	0.40	0.67	1.00

b) For the period 1974 to 1980

	RUS	RDM	RSF	RFL	RFF	RJY	RCD	RAD
RUS	1.00							
RDM	0.39	1.00						
RSF	0.25*	0.82	1.00					
RFL	0.38	0.963	0.78	1.00				
RFF	0.47	0.76	0.67	0.80	1.00			
RJY	0.49	0.39	0.38	0.38	0.48	1.00		
RCD	0.89	0.43	0.26*	0.42	0.46	0.41	1.00	
RAD	0.66	0.43	0.31*	0.42	0.44	0.48	0.74	1.00

c) For the period 1981 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RCD	RAD
RUS	1.00							
RDM	0.37	1.00						
RSF	0.31	0.87	1.00					
RFL	0.36	0.988	0.88	1.00				
RFF	0.37	0.923	0.80	0.916	1.00			
RJY	0.43	0.60	0.62	0.58	0.61	1.00		
RCD	0.931	0.37	0.32	0.36	0.39	0.43	1.00	
RAD	0.60	0.27*	0.24*	0.27*	0.35	0.34	0.62	1.00

d) For the period 1981 to 1985

	RUS	RDM	RSF	RFL	RFF	RJY	RAD	RCD
RUS	1.00							
RDM	0.45	1.00						
RSF	0.39	0.84	1.00					
RFL	0.43	0.983	0.86	1.00				
RFF	0.41	0.908	0.76	0.901	1.00			
RJY	0.52	0.61	0.57	0.58	0.62	1.00		
RAD	0.64	0.45	0.50	0.46	0.50	0.54	1.00	
RCD	0.930	0.46	0.43	0.45	0.44	0.54	0.67	1.00

e) For the period 1986 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RAD	RCD
RUS	1.00							
RDM	0.30*	1.00						
RSF	0.16*	0.938	1.00					
RFL	0.31*	0.999	0.936	1.00				
RFF	0.42*	0.960	0.88	0.957	1.00			
RJY	0.20*	0.62	0.76	0.62	0.61	1.00		
RAD	0.53	-0.04*	-0.29*	-0.05*	0.08*	-0.16*	1.00	
RCD	0.930	0.24*	0.10*	0.25*	0.34	0.14*	0.51	1.00

Table 10

Correlation matrices for the levels of different currencies using the BP as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	US	DM	SF	FL	FF	JY	CD	AD
US	1.00							
DM	0.59	1.00						
SF	0.64	0.967	1.00					
FL	0.55	0.992	0.935	1.00				
FF	-0.32	0.11*	-0.07*	0.22	1.00			
JY	0.72	0.911	0.938	0.87	-0.18*	1.00		
CD	0.82	0.24	0.22	0.24	-0.05*	0.32	1.00	
AD	0.10*	-0.51	-0.51	-0.49	0.10*	-0.49	0.54	1.00

b) For the period 1974 to 1980

	US	DM	SF	FL	FF	JY	AD	CD
US	1.00							
DM	0.53	1.00						
SF	0.38	0.966	1.00					
FL	0.61	0.993	0.941	1.00				
FF	0.83	0.77	0.70	0.82	1.00			
JY	0.55	0.89	0.900	0.89	0.72	1.00		
AD	0.67	-0.18	-0.30	-0.10*	0.31	-0.01*	1.00	
CD	0.89	0.09*	-0.07*	0.18	0.56	0.14*	0.89	1.00

c) For the period 1981 to 1987

	US	DM	SF	FL	FF	JY	AD	CD
US	1.00							
DM	0.31	1.00						
SF	0.34	0.988	1.00					
FL	0.28	0.999	0.987	1.00				
FF	-0.13*	0.78	0.73	0.79	1.00			
JY	0.42	0.961	0.958	0.95	0.68	1.00		
AD	0.30	-0.62	-0.58	-0.63	-0.60	-0.60	1.00	
CD	0.950	0.12*	0.16*	0.09*	-0.30	0.20	0.53	1.00

Table 11

Correlation matrices for the returns on different currencies using the CD as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RBP	RAD
RUS	1.00							
RDM	0.12*	1.00						
RSF	0.12*	0.89	1.00					
RFL	0.11*	0.981	0.87	1.00				
RFF	0.16*	0.88	0.80	0.89	1.00			
RJY	0.23	0.56	0.58	0.56	0.60	1.00		
RBP	0.16*	0.60	0.58	0.62	0.61	0.49	1.00	
RAD	0.05*	0.21	0.21	0.21	0.25	0.24	0.20	1.00

b) For the period 1974 to 1980

	RUS	RDM	RSF	RFL	RFF	RJY	RBP	RAD
RUS	1.00							
RDM	0.21*	1.00						
RSF	0.23*	0.87	1.00					
RFL	0.22*	0.969	0.83	1.00				
RFF	0.33	0.80	0.76	0.84	1.00			
RJY	0.43	0.46	0.50	0.46	0.55	1.00		
RBP	0.37	0.57	0.56	0.58	0.61	0.52	1.00	
RAD	0.04*	0.24*	0.24*	0.23*	0.23*	0.33	0.16*	1.00

c) For the period 1981 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RBP	RAD
RUS	1.00							
RDM	0.02*	1.00						
RSF	-0.02*	0.909	1.00					
RFL	0.01*	0.992	0.917	1.00				
RFF	0.00*	0.943	0.85	0.939	1.00			
RJY	0.04*	0.65	0.67	0.64	0.66	1.00		
RBP	-0.06*	0.62	0.61	0.65	0.61	0.47	1.00	
RAD	0.06*	0.18*	0.18*	0.20*	0.26*	0.19*	0.22*	1.00

d) For the period 1981 to 1985

	RUS	RDM	RSF	RFL	RFF	RJY	RBP	RAD
RUS	1.00							
RDM	0.00*	1.00						
RSF	-0.06*	0.87	1.00					
RFL	-0.01*	0.987	0.88	1.00				
RFF	-0.04*	0.925	0.80	0.919	1.00			
RJY	0.03*	0.56	0.54	0.55	0.58	1.00		
RBP	-0.07*	0.62	0.57	0.66	0.59	0.36	1.00	
RAD	0.04*	0.31*	0.38	0.34	0.38	0.34	0.24	1.00

e) For the period 1986 to 1987

	RUS	RDM	RSF	RFL	RFF	RJY	RBP	RAD
RUS	1.00							
RDM	0.22*	1.00						
RSF	0.20*	0.964	1.00					
RFL	0.21*	0.9994	0.964	1.00				
RFF	0.29*	0.975	0.941	0.972	1.00			
RJY	0.19*	0.76	0.86	0.76	0.76	1.00		
RBP	0.09*	0.61	0.68	0.61	0.60	0.67	1.00	
RAD	0.17*	-0.05*	-0.16*	-0.06*	0.01*	-0.08*	0.17*	1.00

Table 12

Correlation matrices for the levels of different currencies using the CD as the numéraire and a monthly holding period for various time periods.

a) For the period 1974 to 1987

	US	DM	SF	FL	FF	JY	BP	AD
US	1.00							
DM	0.57	1.00						
SF	0.74	0.959	1.00					
FL	0.48	0.992	0.923	1.00				
FF	-0.36	0.48	0.26	0.57	1.00			
JY	0.81	0.83	0.907	0.77	-0.01*	1.00		
BP	-0.16	0.47	0.30	0.53	0.79	0.07*	1.00	
AD	-0.69	-0.33	-0.46	-0.25	0.49	-0.66	0.56	1.00

b) For the period 1974 to 1980

	US	DM	SF	FL	FF	JY	BP	AD
US	1.00							
DM	0.967	1.00						
SF	0.967	0.986	1.00					
FL	0.968	0.998	0.983	1.00				
FF	0.901	0.930	0.921	0.939	1.00			
JY	0.935	0.913	0.940	0.914	0.84	1.00		
BP	0.59	0.66	0.60	0.66	0.75	0.54	1.00	
AD	0.08*	0.11*	0.08*	0.12*	0.26*	0.18*	0.68	1.00

c) For the period 1981 to 1987

	US	DM	SF	FL	FF	JY	BP	AD
US	1.00							
DM	0.34	1.00						
SF	0.37	0.987	1.00					
FL	0.30	0.998	0.983	1.00				
FF	-0.14*	0.78	0.71	0.80	1.00			
JY	0.59	0.933	0.946	0.914	0.56	1.00		
BP	-0.40	0.49	0.41	0.53	0.895	0.23*	1.00	
AD	-0.84	-0.45	-0.50	-0.41	0.16	-0.67	0.44	1.00

Table 13

Comparison of option prices

$X_1 = X_2 = 50$ ¢, $\sigma_1 = \sigma_2 = 0.008$, Maturity = 90 days,
Interest = 0.03 % per day (≈ 11.6 % per annum)

ρ	Option on the maximum of c_1 and c_2	Option on $c_1 + c_2$	
	$c_1 = c_2 = 51$ ¢		
-0.9	3.91	2.05	Option on c_1 = 2.03
-0.5	3.67	2.66	Option on c_2 = 2.03
-0.2	3.48	3.01	Option on c_1
0	3.35	3.22	+ option on c_2 = 4.05
0.4	3.05	3.58	
0.6	2.86	3.75	
0.8	2.61	3.90	
0.9	2.44	3.98	
0.95	2.32	4.01	
0.99	2.16	4.04	
	$c_1 = c_2 = 50$ ¢		
-0.9	2.92	0.66	Option on c_1 = 1.47
-0.5	2.77	1.47	Option on c_2 = 1.47
-0.2	2.64	1.86	Option on c_1
0	2.54	2.08	+ option on c_2 = 2.95
0.4	2.30	2.47	
0.6	2.15	2.64	
0.8	1.95	2.80	
0.9	1.81	2.87	
0.95	1.71	2.91	
0.99	1.58	2.94	
	$c_1 = c_2 = 49$ ¢		
-0.9	2.04	0.09	Option on c_1 = 1.02
-0.5	1.98	0.56	Option on c_2 = 1.02
-0.2	1.90	1.03	Option on c_1
	1.83	1.23	+ option on c_2 = 2.05
0.4	1.66	1.59	
0.6	1.55	1.75	
0.8	1.40	1.90	
0.9	1.29	1.98	
0.95	1.21	2.01	
0.99	1.11	2.04	

ρ	Option on the maximum of c_1 and c_2	Option on $c_1 + c_2$	
	$c_1 = 0.51$, $c_2 = 0.50$		
-0.9	3.43	1.26	Option on c_1 = 2.03
-0.5	3.23	2.02	Option on c_2 = 1.47
-0.2	3.08	2.40	Option on c_1
0	2.97	2.62	+ option on c_2 = 3.50
0.4	2.77	3.00	
0.6	2.54	3.16	
0.8	2.33	3.32	
0.9	2.19	3.40	
0.95	2.03	3.44	
0.99	2.01	3.47	
	$c_1 = 0.51$, $c_2 = 0.49$		
-0.9	3.02	0.66	Option on c_1 = 2.03
-0.5	2.88	1.47	Option on c_2 = 1.02
-0.2	2.75	1.86	Option on c_1
0	2.66	2.08	+ option on c_2 = 3.05
0.4	2.45	2.47	
0.6	2.32	2.64	
0.8	2.16	2.80	
0.9	2.08	2.87	
0.95	2.04	2.91	
0.99	2.03	2.94	
	$c_1 = 0.50$, $c_2 = 0.49$		
-0.9	2.49	0.28	Option on c_1 = 1.47
-0.5	2.38	1.03	Option on c_2 = 1.02
-0.2	2.28	1.41	Option on c_1
0	2.20	1.62	+ option on c_2 = 2.50
0.4	2.00	2.00	
0.6	1.87	2.16	
0.8	1.71	2.32	
0.9	1.60	2.40	
0.95	1.53	2.43	
0.99	1.48	2.46	

Table 14

Comparison of option prices
$X_1 = X_2 = 50$ ¢, $\sigma_1 = \sigma_2 = 0.008$, Maturity = 90 days,
Interest = 0.03 % per day (≈ 11.6 % per annum).

Option on the maximum of
c_1 and $A \times c_2$

$c_1 = c_2 = 51$ ¢

ρ	A = 0.90	A = 0.75
-0.9	3.58	3.15
-0.5	3.36	3.97
-0.2	3.19	2.83
0	3.07	2.73
0.4	2.80	2.52
0.6	2.63	2.39
0.8	2.42	2.24
0.9	2.27	2.15
0.95	2.18	2.10
0.99	2.07	2.05

- Option on c_1 = 2.03
- Option on c_2 = 2.03
- Option on c_1 + 0.9 option on c_2 = 3.85
- Option on c_1 + 0.75 option on c_2 = 3.54

$c_1 = c_2 = 50$ ¢

ρ	A = 0.90	A = 0.75
-0.9	2.64	2.28
-0.5	2.50	2.17
-0.2	2.39	2.08
0	2.30	2.01
0.4	2.09	1.85
0.6	1.96	1.75
0.8	1.79	1.63
0.9	1.67	1.56
0.95	1.59	1.52
0.99	1.50	1.48

- Option on c_1 = 1.47
- Option on c_2 = 1.47
- Option on c_1 + 0.9 option on c_2 = 2.80
- Option on c_1 + 0.75 option on c_1 = 2.58

$c_1 = c_2 = 49$ ¢

ρ	A = 0.90	A = 0.75
-0.9	1.81	1.53
-0.5	1.76	1.49
-0.2	1.69	1.43
0	1.63	1.39
0.4	1.48	1.28
0.6	1.38	1.21
0.8	1.26	1.13
0.9	1.17	1.08
0.95	1.11	1.05
0.99	1.04	1.03

- Option on c_1 = 1.02
- Option on c_2 = 1.02
- Option on c_1 + 0.9 option on c_2 = 1.94
- Option on c_1 + 0.75 option on c_2 = 1.79

c_1 = 51 ¢ c_2 = 50 ¢

ρ	A = 0.90	A = 0.75
-0.9	3.15	2.80
-0.5	2.98	2.66
-0.2	2.84	2.56
0	2.74	2.48
0.4	2.51	2.31
0.6	2.38	2.22
0.8	2.21	2.11
0.9	2.17	2.06
0.95	2.05	2.04
0.99	2.03	2.02

Option on c_1 = 2.03
Option on c_2 = 1.47
Option on c_1 + 0.9 option on c_2 = 3.35
Option on c_1 + 0.75 option on c_2 = 3.13

c_1 = 51 ¢ c_2 = 49 ¢

ρ	A = 0.90	A = 0.75
-0.9	2.80	2.43
-0.5	2.67	2.52
-0.2	2.57	2.35
0	2.49	2.30
0.4	2.31	2.17
0.6	2.21	2.11
0.8	2.10	2.05
0.9	2.04	2.03
0.95	2.03	2.03
0.99	2.03	2.03

Option on c_1 = 2.03
Option on c_2 = 1.02
Option on c_1 + 0.9 option on c_2 = 2.95
Option on c_1 + 0.75 option on c_2 = 2.79

c_1 = 50 ¢ c_2 = 49 ¢

ρ	A = 0.90	A = 0.75
-0.9	2.26	1.97
-0.5	2.17	1.91
-0.2	2.08	1.84
0	2.01	1.79
0.4	1.84	1.68
0.6	1.74	1.61
0.8	1.61	1.53
0.9	1.53	1.49
0.95	1.49	1.48
0.99	1.47	1.47

Option on c_1 = 1.47
Option on c_2 = 1.02
Option on c_1 + 0.9 option on c_2 = 2.39
Option on c_1 + 0.75 option on c_2 = 2.24

Chapter 4

ESSAY THREE ON THE

"PRICING OF FOREIGN CURRENCIES AS PRIMARY ASSETS IN TRANSACTION COST ECONOMIES: THEORY AND TEST"

I. Introduction

This essay presents and empirically tests a foreign exchange (FX) rate determination model. This model is compatible with the behaviour of exchange rates during the early 1980's. While both the inflation and interest rates were higher in the US than in Japan and Germany during this period, the dollar appreciated vis-à-vis the Yen and the DM. The model is also consistent with the empirical regularities that have been reported in the literature.

The model developed herein draws on three strands of literature; namely, international economics, asset pricing and the pricing of futures contracts. Currencies are treated like other primary assets, and the risk premia involved in the model are derived in an economy with transactions costs. Simply stated, the model assumes that exchange rates are determined by relative price levels in the long run, and that deviations from that long run equilibrium are due to financial speculation driven by interest and inflation rate expectations. Excessive deviations from purchasing power parity are prevented by (costly) goods arbitrage.

The first form of the model explains exchange rate levels and the second explains exchange rate changes. The empirical tests of the model use the German mark (DM), the Swiss franc (SF) and the Canadian dollar (CD) for the period from January 1975 (1975-01) to December 1987 (1987-12). The choice of exchange rates was constrained by the availability of data for the entire term structure of interest rates. The data covers the recent floating exchange rates period, which followed the first oil shock. The model is tested for changes over one, three, six and twelve months using in- and out-of-sample tests.

The paper is organized as follows. In Section II, a selective review of the literature on FX rates models is presented. In Section III, the model is developed. In Section IV, the empirical analysis is presented. In Section V, some concluding remarks are offered.

II. Literature Review

The literature on FX rates uses either of two theoretical approaches. The first approach uses a macroeconomic perspective. The balance of payments models relate FX rates to the balance of payments situation, while the monetary models relate the determination of FX rates to the supply and the demand of money in the countries under consideration. The second (or micro economic) approach utilizes the rational behaviour of profit maximizing individual agents,[1] and arbitrage arguments in efficient markets. There are basically two "arbitrage" theories; namely, the Purchasing Power Parity (PPP) theory, and the Interest Rate Parity Theory (IRPT).

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1. More specifically, this approach relies on the existence of utility maximizing individual agents.

II.1 Macroeconomic Approaches

II.1.1 Balance of Payments Models

The balance of payments models of FX rate determination (flow models) have had very little success empirically. One of the empirical regularities of FX rate behaviour reported by Mussa (1984) is that the relationship between movements in nominal (or real) exchange rates and current account balances is not strong and systematic. This relationship does not explain a substantial proportion of actual FX rate movements.

Meese and Rogoff (1988) incorporate cumulative trade balances into a model which relates real exchange rates to real interest rates. They find that the coefficient of the US cumulated trade balance is of the wrong sign (but not statistically significant) for the three currencies they studied.

II.1.2 Monetary Models

The monetary approach can be viewed as a descendant of the asset view of PPP.[2] Since prices are a direct effect of money supply and demand, prices and FX rates can be viewed as endogenous variables determined by money supply and demand. The monetary models consider FX rates as the relative price of two moneys rather than the relative

2. This terminology was suggested by Hodrick (1978).

prices of national outputs. Since the equilibrium FX rate is reached when the existing stocks of two monies are willingly held, the theory of exchange rate determination can be derived from the supply and demand for these monies. In general, these models can be expressed as:

$$S = f(M, M^*, Y, Y^*, i, i^*, k, k^*) \tag{1}$$

where S = spot exchange rate

M = money supply

$*$ = refers to the variables in the foreign country

Y = domestic income

i = interest rate

k = other factors

Meese and Rogoff (1983) test three monetary models; namely, a flexible-price model (Frenkel-Bilson), a sticky-price model (Dornbusch-Frankel), and a sticky price model incorporating the current account (Hooper-Morton). They find that the random walk outperforms all of these models for all tested forecasting horizons (namely, 1 month, 6 months, and 12 months).

Somanath (1986) tests an expanded number of structural models and lagged adjustments. Depending on the chosen horizon, currency, and criterion used, different models outperform the predictions from a random walk. However, no model is systematically superior to the random walk. Wolff (1988) examines the "news" formulation of the monetary FX rate model. His results compare favorably with those obtained from the naive random walk forecasting rule. Finn (1986) finds that instrumental-variable estimates of the "simple" monetary model are not supported by the data. She also finds that the full information maximum-likelihood estimates of the simple monetary model's rational-expectations counterpart forecast as well as the random walk model.

Magee (1976) argues that the users of monetary models should not believe that they will beat forward market estimates of future spot rates, since their models, even if correct, would constitute only a subset of the information incorporated in the market price. Kouri (1984) concludes that the "monetarist model has failed ... clearly as an empirically relevant theory...". According to Mussa (1984), the failure of the monetary models can be attributed to two problems; namely, the instability of the demand function for money, and the absence of expectations.

Dornbusch (1976) deals partially with the problem of expectations. In an economy with perfect capital mobility, sticky goods prices, and consistent expectations, he shows that a monetary expansion causes the exchange rate to depreciate. An initial overshooting occurs to account for the expected subsequent appreciation of the exchange rate due to the temporary drop of interest rates after the monetary expansion. Dornbusch specifies the time path of the exchange rate as:

$$S_{t+\Delta t} = \bar{S} + (S_t - \bar{S}) \exp(-v\Delta t) \qquad (2)$$

where $\bar{S}$ is the long-term equilibrium exchange rate

v is the rate at which the exchange rate converges to equilibrium (which is a function of the structural parameters of the economy).

II.2 Arbitrage Approaches

II.2.1 Purchasing Power Parity

The concept of PPP has been traced back by Einzig (1970) to Spanish economists in the sixteenth century. Its rediscovery in the twentieth century is generally credited to Cassel (1914) who places PPP within a systematic framework. PPP is based on a goods arbitrage argument. In the absolute version of PPP, the FX rate between two currencies must be such that the price of goods in both countries is equal. Thus, no abnormal profit can be obtained by simultaneously buying and selling the same goods in different countries. The absolute version of the PPP is given by:

$$S = P_d/P_f \tag{3}$$

where S = spot FX rate given as the price of foreign currency in the domestic currency

P_d = domestic (d) price level

P_f = foreign (f) price level

In practice, absolute PPP is not expected to hold because transportation costs, tariff and non-tariff trade barriers, and other market imperfections allow prices in two countries to be consistently different. The relative price version of the theory maintains that changes in the FX rate will reflect relative rates of change in price levels between countries. This may be expressed as:

$$S_{t-1}/S_t = (1+\pi_d)/(1+\pi_f) \tag{4}$$

where π_d = domestic inflation

π_f = foreign inflation

Empirical tests of PPP have encountered several difficulties, such as which price index should be used. Possibilities include the GDP deflator, the consumer price index, the wholesale price index, other indices of the price of traded goods, the unit labour cost, and the unit factor cost. A case has been made in the literature for each of these indices.

The argument for the use of a traded goods price index is that goods arbitrage can only take place for traded goods. Taken to the limit, this version of PPP is a tautology. For example, if one considers gold as an example of a tradable good,[3] arguing that the Canada/US exchange rate should be such that the price of gold in Canada is the same as in the US (after transaction and transportation costs) does not say anything about the equilibrium exchange rate. Indeed, the price of gold in both countries will be the same, even if actual and equilibrium exchange rates diverge.

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3. Gold is a very homogeneous good, it is heavily traded on financial markets, and is fairly easy to transport.

The arguments for using production cost indices are that they are less subject to adjustments to exchange rate changes than are prices of traded goods, and that costs exclude the volatile profit component. As a result, costs are more likely than product prices to represent long-run prices (see Stern (1973) and Officer (1976)). The problems associated with using production cost indices are that production costs are harder to arbitrage internationally, and they may cause a productivity bias.[4] If the relative change of productivity between the traded and non-traded sectors is not the same across countries, the use of production cost indices may cause PPP to appear biased.

Officer (1976) suggests that the GDP deflator should be used, since it is the broadest price index. Also Cassel's theory is based on the notion that the value of a currency is determined fundamentally by the amount of goods and services that a unit of the currency can buy in the country of issue (that is, by its internal purchasing power).

Due to its greater availability, some authors (e.g., Hakkio (1984)) use the consumer price index (CPI) instead of the GDP deflator. Unfortunately, a productivity bias may arise with these price indices as with all the production cost indices.

A compromise index, the wholesale price index (WPI), is used by many authors (e.g., Adler and Lehmann (1983) and Gailliot (1970)). Use

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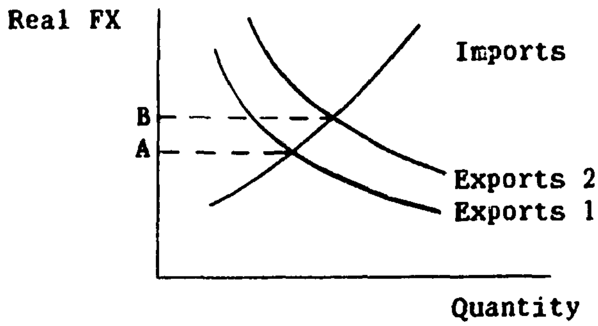
4. For discussions of productivity bias, see Balassa (1964), Officer (1976) and Genberg (1978).

of this index reduces the problem of productivity bias because the index only considers goods prices. Since it considers the (wholesale) prices of all the goods produced in the economy, and not only those belonging to the export or import sectors, use of this index alleviates the problems associated with the use of a purely (internationally) traded goods index.

Several factors, in addition to productivity bias, may explain deviations from PPP. Some of these factors could be directly related to the FX market (such as investor expectations leading to speculation, and central bank intervention to strengthen or weaken given currencies). Other factors may indirectly influence FX rates (such as interest rates, trade policies and practices, changes in technology or consumer tastes, discovery of natural resources, and political events).

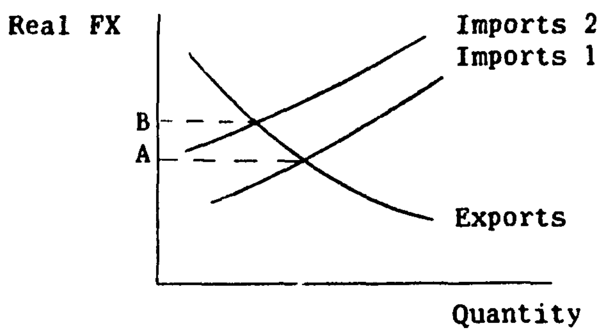
Some factors may cause long term shifts in FX rates. For example, the sudden discovery of oil in a given country would, ceteris paribus, increase the net exports (exports-imports) of that country. Hence, the country could afford to export less and/or import more non-oil products than before the oil discovery. Since relatively less exports are needed and/or relatively more imports can be afforded, the price of local goods can increase above what would be expected from PPP, provided goods markets are not perfectly elastic (see Fig. 1). In the long run, the markets are much more elastic because of the possibilities of altering production capacities, technologies and location, and due to the possibilities of adapting marketing and

Figure 1
Hypothetical Impact of an Oil Discovery on the Real FX Rate of a Country

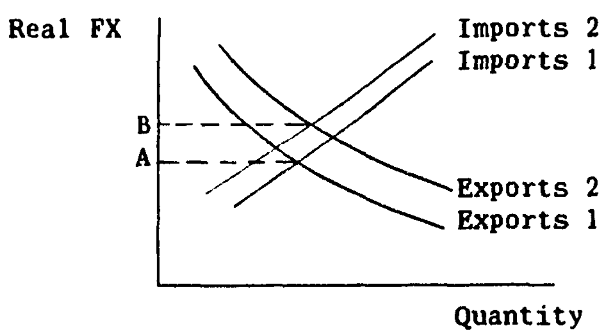


Short term

If the oil discovery shifts the exports supply curve, the real exchange rate will go from its PPP level at A to B.[a]



If the oil discovery shifts the imports demand curve, the real exchange rate will go from its PPP level at A to B.



If the oil discovery shifts both the exports demand and the imports supply curves, the real exchange rate will go from its PPP level at A to B.

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a. These graphs make the simplifying assumption that, prior to the oil discovery, the exchange rate was at its PPP level and exports equalled imports. The arguments would still hold if these assumptions did not hold.

general business strategies. As a result, the impact of the oil discovery would tend to dissipate over time.[5]

The empirical evidence rejects PPP as a short term theory of exchange rate determination. The evidence on long term PPP is more supportive (see, e.g., Hakkio (1984), Officer (1978, 1980) and Kohlhagen (1978), among others). Usually, the nonsupportive evidence rejects PPP as the <u>only</u> factor determining exchange rates as opposed to rejecting PPP as a major factor determining FX rates in the long run.

Adler and Lehmann (1983) test PPP against the competing theory that real FX rates follow a martingale. They assume that the differences between ex-ante real interest rates across countries are constant through time. Based on this assumption, they conclude that short term deviations from PPP should be autocorrelated if PPP holds. If no autocorrelation is found, the martingale theory would be supported. More specifically, they define:

$$y_t = s_t + \pi_t^* - \pi_t \qquad (5)$$

where s_t = actual exchange rate change from t to t+1

π_t = actual inflation rate from t to t+1

$*$ denotes a foreign currency quantity

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5. The effects of other factors are even harder to quantify. For example, what and how strong should the impact of turmoil in Iran be on the Swiss franc?

They test whether the b_i, $i = 1$ to n, are jointly insignificantly different from zero in:

$$y_t = \sum_{i=1}^{n} b_i \, y_{t-i} + v_t \tag{6}$$

The data set used by Adler and Lehmann consists of 43 exchange rates against the US dollar over the period from 1870 to 1981 for the Canadian dollar and from 1966 to 1981 for most of the other currencies. In most cases, they cannot reject the hypothesis based on the martingale theory. They conclude that they "have substantial evidence against the hypothesis of stable but highly autocorrelated autoregression in monthly data". However, the need for a stable autoregressive structure derives directly from their hypothesis that the differences between ex-ante real interest rates across countries are constant through time. In contrast, the primary motivation behind the model developed herein is that the differences between these ex-ante real interest rates are not constant. Also, as noted by Hakkio (1984), the empirical tests for a random walk are not very strong. This is particularly true when a lag structure containing 6 to 18 terms is tested simultaneously (as is the case for the tests in Adler and Lehmann (1983)).

Edison (1987) studies the US dollar/pound exchange rate over the period between 1890 and 1978. He finds that the hypothesis that exchange rate changes are proportional (constant of proportionality = 1) to relative price changes and that the deviation from PPP is amended each year, can not be rejected. He concludes that "PPP provides a fair, though rough, approximation of the long-run exchange rate". He finds that the deviations from PPP are significantly influenced by cash balances and by relative income levels. Cash balances are related to real interest rates (which are major determinants of exchange rates in our model), and the significance of relative income levels. The latter variable may "pick up" productivity bias since Edison uses the GDP price deflator.

Hakkio (1984) tests PPP for four countries by using:

$$\ln S_{it} = \alpha_i + \beta_i \ln(P_t/P_{it}) + u_{it} \tag{7}$$

$$u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it}$$

where S is the exchange rate

P_t is the price level in the USA

P_{it} is the price level in country i

Based on a test of PPP for each individual currency between 1973 and 1982, the null hypotheses that $\beta_i = 0$ and $\beta_i = 1$ could never be rejected statistically. When PPP was tested for the four countries simultaneously with β constrained to be equal across countries, β equalled 1.04 ($\sigma_\beta = 0.30$). Also, the hypotheses that all the β's are equal, and all the β's are equal to one could not be rejected. For data between 1921 and 1925, Hakkio rejects PPP for two out of the four countries. His main conclusion is that many authors have erroneously concluded that PPP did not hold during the 1970's due to a lack of precision in their parameter estimates.

In the presence of transportation costs, Aizenman (1986) shows that traditional regression analysis will tend to reject the PPP hypothesis even if goods markets are well arbitraged. This occurs because the values of the regression coefficients are affected systematically by considerations that are independent of the degree to which markets are arbitraged.

In conclusion, most researchers seem to agree with Kohlhagen (1978) that PPP seems to hold in the long run. In particular, Klein, Fardoust and Filatov (1981) assume that PPP holds in the long run (and on average), in their use of the LINK system for a simulation of the world economy over the period from 1980 to 1990. However, more research is required to explain the reasons for temporary deviations from PPP in either fixed or floating rate periods.[6]

II.2.2 Interest Rate Parity Theory

The interest rate parity theory (IRPT), which is based on a currency arbitrage argument, can be derived using either covered or uncovered arbitrage. Keynes (1923) derived the first expression of a covered interest arbitrage or riskless IRPT. In his model, forward (F)

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6. Reviews of the PPP literature are found in Officer (1976b, 1982), Kohlhagen (1978), amongst others.

and spot (S) FX rates are related to foreign (R_f) and domestic (R_d) interest rates as follows:

$$F/S = (1+R_d)/(1+R_f) \tag{8}$$

Violation of the condition expressed in equation (8) would represent riskless profit opportunities for arbitrageurs who could borrow in one currency and lend the proceeds in the other. The IRPT based on uncovered interest arbitrage relates the expected future spot rate (S^e_{t+1}) to the present spot rate (S_t) as follows:

$$S^e_{t+1}/S_t = (1+R_d)/(1+R_f) \tag{9}$$

Covered interest arbitrage has received substantial empirical support. Most of the small deviations from (8) disappear when transaction costs are taken into account (Frenkel and Levich (1977)), when the correct bid and ask prices are used (Agmon and Bronfeld (1975)), and when interest rates on the euromarkets are used (Martson (1976), Aliber (1973)). This is not unexpected, because bankers derive their forward premiums or discounts from interest rate parity.

Uncovered arbitrage is expected to hold as in (9) only if markets are efficient and no risk premium exists. The evidence on the existence of a risk premium in the FX market is mixed. Cornell (1977) and Engel (1984) fail to find any premium, while Hansen and Hodrick (1980) reject the simple market efficiency hypothesis (i.e., that traders have rational expectations and charge no risk premium in the forward exchange market). Using a statistical procedure which is consistent under a large class of heteroscedasticity, Hsieh (1984) also rejects the joint hypothesis of simple efficiency. Even if a risk premium exists, Frankel (1986) argues that it would be too small to be detectable empirically.[7]

Hakkio and Leiderman (1986) present a theory of the term stucture of exchange rates and interest rates which is based on a consumption-based intertemporal asset pricing model. Their econometric analysis of data from the eurocurrency market generally indicates that the implications of their model are not supported.

Lyons (1988) presents a model with a variable risk premium based on the variance-covariance matrix of exchange rates. He measures the variances from the implied volatilities of options on foreign currencies and covariances from these variances and a correlation matrix assumed to be constant. For each of the mark, the pound and the yen FX rate vis-à-vis the US dollar, Lyons finds that a single risk term is statistically significant, specifically: the covariance between the mark rate and the pound rate for the mark, the variance of the pound rate for the pound, and the covariance between the yen rate

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7. If economic agents are mean variance optimizers, if the covariance of returns does not change over time, and if the coefficient of risk aversion is in the range estimated by earlier studies, Frankel (1986) argues that empirically derived risk premiums would be too small to account for the rejection of the joint hypothesis of efficiency and no risk premium. His argument assumes a no transactions cost economy.

and the pound rate for the yen. This may indicate that the variance of the pound rate may be proxying for some other variable.

Bomhoff and Koedijk (1988) present a model where the risk premium in a bilateral exchange rate is a function of the variance of that rate and of its covariances with the returns on the asset markets of the two countries. By specifying a real exchange framework, they link changes in the risk premium to macroeconomic variables like interest rates and inflation rates. They find that the variance of the short-term US interest rate and the variance of changes in the expected US inflation rate are explanatory variables in regressions for the difference between the spot rate and the lagged forward rate. Hardouvelis (1988) finds that FX markets respond to monetary news, to news about the trade deficit, domestic inflation and variables that reflect the state of the business cycle.

Huang (1989) presents an intertemporal latent-variable asset-pricing model (ILVM) to examine the risk characteristics of the term structure of forward foreign exchange contracts. The ILVM is similar to the Sharpe-Lintner single-period capital asset pricing model (CAPM) in that it measures the risk of an asset relative to a portfolio that lies on the mean-variance frontier as a single constant β. However, Huang explicitly allows for shifts in the opportunity set over time, and treats the benchmark portfolio as a latent variable because it is unobservable. In a test of eight currencies, Huang finds that the constraints imposed by the ILVM are generally not rejected, except for the one month forward premium. He suggests that this rejection may be due to the instability of beta and that this could be caused by such factors as short-term foreign real interest rates or market irrationality.

II.2.3 Combinations of PPP and IRP

Isard (1983) develops a framework of approximate accounting identities which are used to discuss the limitations of existing empirical models for exchange rate determination. His framework can be divided into two parts, "an anchor" and "a rope". The anchor is the expectation that PPP will hold in the long run, and the rope explains the deviations from PPP in the short run. These deviations are due to interest rates and an eventual risk premium. His model is given by:

$$
\begin{aligned}
s_t - s_{t-1} = {} & (p_d - p_f)_t - (p_d - p_f)_{t-1} \\
& + (R_d - R_f)_{t,t+n} - (R_d - R_f)_{t-1,t+n-1} \\
& - (\pi_d - \pi_f)^e_{t,t+n} - (\pi_d - \pi_f)^e_{t-1,t+n-1} - \mathrm{resid}_t
\end{aligned}
\tag{10}
$$

where resid = unexplained error term

$s = \ln S$

$_{d,f}$ = domestic and foreign variables, respectively

$p = \ln$ (price level)

R = nominal interest rate

π = rate of inflation

n = time horizon considered

$(R_d)_{t1,t2}$ = interest rate at time t_1 for a debt instrument with no default risk maturing at t_2

$^{e}_{t1,t2}$ = value expected at time t_1 to be realized at t_2

In this model, changes in FX rates are due to the actual change in price levels, to the shift in long term interest rate differentials, and to the shift in long-term inflation rate differentials. Unfortunately, Isard only uses his model to show that FX rates are affected by long term interest rates (of between 2 and 5 years according to his study) as opposed to the traditionnal view that short-term interest rates determine FX rates.

Shaefer and Loopesko (1983) assume that PPP will hold in the long run and that current deviations from the PPP exchange rate are due to the real interest rate differentials. They then fit the following model:

$$q = a + b\,[(R - \pi^{e}) - (R^{*} - \pi^{e*})] \qquad (11)$$

where q = the real exchange rate

This is done for monthly observations of the mark, yen and pound exchange rates for the period August 1973 to March 1982. They use two proxies for the expected inflation rate; namely, a naive proxy (i.e., the centered twelve-month moving average of actual inflation), and a "rational expectations" proxy based on a vector autoregressive model. Although the naive proxy performs best, it is only successful for the mark and the yen. Sachs (1985) estimates a comparably specified equation for quarterly mark exchange rate data over the period 1977 Q1 to 1984 Q4 using a two-year centered moving average to proxy inflation expectations. Hooper (1984) uses the Federal Reserve Board's ten-country trade-weighted, effective dollar exchange rate with quarterly data from 1974 Q2 to 1983 Q4. He proxies expected inflation by a three-year centered moving average of consumer price inflation. He reports very similar results for a backward-looking moving average. The principal results of these studies are reported in Table 1.

Coe and Golub (1986) extend the analysis in these studies. They recognize that a risk premium may exist, which will be captured in the constant term of equation (11). They use semi-annual data from 1973 S2 to 1983 S2 for 18 currencies (including the effective exchange rate of the US dollar). They measure expected inflation as the three or six semester moving average of the annual growth rate of the GDP deflator.[8] They conclude that the specification "worked", in th sense that the estimated coefficients of the real interest rate differential have the correct sign and are larger than unity (their definition) for Japan, Germany, France, Austria, Belgium, Denmark, Ireland, the Netherlands, Norway, Sweden and Switzerland. Their model failed for the remaining six countries (the UK, Italy, Canada, Australia, Finland and Spain). Coe and Golub suggest several possible reasons for the failure of their model for these six currencies. Adaptive measures of inflation

8. Coe and Golub report that the three semester moving average usually performs better.

expectations may be inadequate in some instance For some countries, interest rates may respond endogenously to exchange rates if exchange rates enter monetary policy reaction functions. For example, the Bank of England often tightens monetary policy when sterling is weak. Thus, if sterling weakens for some reason other than interest rate differentials, increases in interest rate differentials may be associated with sterling depreciation. Another problem is the absence of proxies for the risk premium.

Driskill and McCafferty (1987) present an equilibrium model of exchange rates characterized by imperfect goods and asset substitutability across countries where decreases in risk aversion increase relative price (and exchange rate) variability when the fundamental shocks to the system are monetary.[9] They illustrate their results by considering two polar cases; namely, infinite risk aversion and no risk aversion. If speculators have infinite risk aversion, the capital flows between countries will be nil. As a result, the account must balance at every moment, and exchange rates must remain at the PPP level. If speculators are risk neutral, relative prices show maximum variance as interest-rate shocks must be met by a current movement in the exchange rate sufficiently large to yield an offsetting expectation of capital gain or loss. Based on this insight, they conclude that, even in more general models, they would expect that changes in risk aversion <u>could</u> lead to higher relative price variability.

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9. This result is unambiguous only when shocks are monetary.

III. Model Development

FX rates are determined by the supply of and the demand for foreign currencies caused by the import and export of goods and services, short and long term capital flows[10] and unilateral transfers such as foreign aid or the remittances of expatriate workers to their families. If all markets were perfectly efficient and there were no transaction (including transportation) costs, then the resulting exchange rate would satisfy PPP due to goods arbitrage and capital market integration. If capital markets were perfectly integrated, the real risk-free interest rate, the relevant (or priced) riskiness of assets, and the price of risk would be the same across countries.

However, actual markets (particularly, goods markets) are probably not perfectly efficient, and are definitely not free of transaction costs. As discussed earlier, empirical evidence suggests that PPP holds in the long run.[11] Short-run deviations from PPP can be attributable in large part to the large transaction costs associated with goods arbitrage. These costs include the cost of transportation, tariff and non-tariff trade barriers, the costs of developing new markets, and the limited flexibility in expanding (or contracting) production capacity over the short run.

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10. Capital flows include portfolio investments, direct investments and central bank intervention.

11. Depending on the price index used, a productivity bias may exist.

Evidence that capital markets are not perfectly integrated has been presented by several authors (e.g., Adler and Dumas (1983), Errunza and Losq (1985), and Cho, Eun and Senbet (1986)). Partial segmentation may be due to transaction costs (particularly, information costs), investment barriers, the failure of PPP in the short run, different consumption patterns, and internal relative price differences across countries.

Financial arbitrage across countries is most efficient on the euromarkets (particularly, the interbank market) where transaction costs are very small and investment barriers are virtually non-existent. Consequently, arbitrage on the euromarkets occurs until the exchange rates and interest rates are such that the expected reward on any risky position is equal to the risk-free interest rate plus a risk premium.[12] Due to financial arbitrage, the relationship between changes in the exchange rates of any two currencies will be:

$$ds^{*} = (R_a - R_f + \lambda)\, dt = \mu\, dt \qquad (12)$$

where ds^{*} = expected change in the logarithm of the exchange rate (defined as the price of one unit of foreign currency in the domestic currency)

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12. Arbitrage is more difficult in the euro-equity market because of the possibility of controls and larger transaction costs (e.g., information costs). Moreover, taking a position in foreign equities is akin to taking a position in the foreign currency plus a position in some business risk. The only way to speculate purely on exchange rates is through risk-free bonds. For the purposes of our discussion, a foreign investment in equities (portfolio or direct) can be viewed as an international purchase of bonds, and a local sale of these bonds for the purchase of the equities.

R = continuously compounded (default-free) interest rate over dt

λ = risk premium

III.1 Specification of the Risk Premium

If $\lambda = 0$, the simple (non covered) IRPT would be expected to hold. The risk premium may be specified in different ways. For example, it may be based on the covariance of s with some market return in the context of the Sharpe-Lintner capital asset pricing model, or on the relation of s with the underlying risk factors in the context of Ross's arbitrage pricing theory. The particular specifications of the risk premium suggested by Hakkio and Leiderman (1986), Lyons (1988) and Bomhoff and Koedijk (1988) were discussed earlier.

In this paper, Mayshar's (1981) formulation of the expected return on a risky asset when transaction costs are volume-related will be used. Mayshar considers taxes on transactions and various other obstacles to trade, including short-sale restrictions, institutional restraints, and even the subjective costs of managing one's own portfolio as a form of transactions costs.[13] Mayshar's formulation of the expected return on a risky asset in the context of the CAPM is:[14]

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13. Once short-sale restrictions are included in transaction costs, Mayshar does not impose any further short-sale restrictions in his derivations.

14. This is Mayshar's (1981) equation (32). The approximations are given by Mayshar, and are based on the assumption that V_j/W is sufficiently small.

$$ER_j = r + t + \lambda' \ (\delta_j \beta_j + \Upsilon_j \alpha_j) \tag{13}$$

where ER_j = expected return on asset j

r = the risk-free interest rate

t = a measure of the marginal transactions cost

λ' = a measure of market risk aversion

$\beta_j = \text{Cov}\ (R_j,\ R_m)/\text{Var}(R_m)$

R_m = the return on the market portfolio

$\alpha_j = \text{Var}(R_j)/\text{Var}(R_m)$

$$\Upsilon_j \approx \frac{V_j/n_j}{W/n}$$

$\delta_j \approx 1 - \Upsilon_j$

V_j = total market value of asset j

n_j = number of investors in asset j

W = market value of all the risky assets in the market

n = number of investors in the market

This model implies that the expected return on an asset depends on its own variance through the term $(\lambda' \Upsilon_j \alpha_j)$, and on the relative concentration of holdings of the asset (Υ_j). Carter, Rausser and Schmitz (1983) provide indirect support for Mayshar's model for some commodity futures markets. They find that the expected returns on commodity futures are a function of the net market position of speculators (as opposed to hedgers).[15]

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15. Carter, Rausser and Schmidtz (1983) find that the non-market component of the excess return of a futures contract (i.e., the component not related to market risk in a CAPM framework) is given by: $\alpha_j = \alpha_j + \delta_j Z_t$, where α_j and δ_j are constants and Z_t is the percentage of reporting speculators in the wheat, corn and

Applying equation (13) to the return on default-risk-free foreign bonds[16] maturing at the end of one period, and recognizing that the return on such bonds equals the foreign interest rate plus the expected change in exchange rate, yields:

$$ER_f = R_f + ds^{e} = R_d + t + \lambda'(\delta_j\beta_j + \Upsilon_j\alpha_j).$$

Hence: $$ds^{e} = dR + t + \lambda'(\delta_j\beta_j + \Upsilon_j\alpha_j), \tag{14}$$

where $dR = R_d - R_f$

If the number of speculators in the FX market (n_j) is fixed, an increase in the amount of invested risky FX arbitrage capital (V_j) would linearly increase the own risk premium ($\lambda'\Upsilon_j\alpha_j$), and would linearly decrease the impact of the market risk premium.[17] If a larger real interest rate differential requires a larger flow of capital to make the risk-adjusted IRPT hold, then equation (14) becomes:[18]

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soybeans futures markets who are net long. They also find that the expected value of α_j is near zero when $Z_t = 0.5$, positive when speculators are net long ($Z_t > 0.5$), and negative when speculators are net short ($Z_t < 0.5$).

16. These foreign currency bonds can be viewed as any other risky assets. Their only peculiarity is that their entire riskiness comes from changes in FX rates.

17. An increase in V_j would also have an effect on R_m. In turn, this would affect (R_j, R_m) and β_j. However, if V_j is small compared to W, then these latter effects would not be large.

18. Some authors (e.g., Bomhoff and Koedijk (1988)) measure the amount of capital involved in currency arbitrage as the cumulative current account balance (CCAB). This measure is suspect because a significant part of the CCAB may be invested in equities, and equities may not be completely exposed to FX risk. For example, shares in a Canadian gold mine do not necessarily lose value in US dollars if the Canadian dollar depreciates. The US dollar value may even increase if production costs are in Canadian dollars and gold is priced in US dollars. Another example would be a Canadian firm which produces in Canada and sells in the US. The value of

$$ds^{*} = dR + t + \lambda'[(1-f(dr))\beta_j + f(dr)\alpha_j] \tag{15}$$

where $$f(dr) = \Upsilon_j = \frac{V(dr)/n_j}{W/n}$$

and V(dr) = volume of capital used in FX arbitrage, as a function of the real interest rate differential.

Alternatively:

$$ds^{*} = dR + t + \lambda'\beta_j + f(dr)\ \lambda'(\alpha_j - \beta_j). \tag{16}$$

Equation (16) gives the expected change in the exchange rates between two countries in a risk averse, transaction cost economy. While a single exchange rate is referred to explicitly in equation (16), the model does account for the existence of other exchange rates. Since a currency can be viewed as being an asset, equation (16) gives the return on a given asset (currency), and the effect of the other assets (currencies) is incorporated into the "market" portfolio. In the special case where FX investors only invest in foreign currencies, their "market portfolio" becomes the portfolio of currencies, and the model becomes similar to that presented by Lyons (1988). However, differences remain because the Lyons model is derived in a no transactions costs economy.

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such a firm in US dollars may increase after a devaluation of the Canadian dollar. The stock of foreign holdings of debt instruments may also be a poor proxy for the volume of speculative capital involved in risky FX arbitrage, since some of this stock may be held by firms as a hedge against operating exposure to FX risk. The flow of short-term capital into Canada seems to be quite closely related to the differential in interest rates between the US and Canada. Based on this observation, a larger interest rate differential is assumed to cause a larger capital flow for FX arbitrage. Furthermore, this relationship is assumed to be linear.

Assume that f(dr) is a linear function of dr, say:

$$f(dr) = a_o + a_1 \, dr \tag{17}$$

Then equation (16) can be rewritten as:

$$ds^* = b_o + b_1 \, dr + d\pi \tag{18}$$

where $b_o = t + \lambda'\beta_j + a_o \, \lambda'(\alpha_j - \beta_j)$, and

$b_1 = 1 + a_1 \, \lambda'(\alpha_j - \beta_j)$

$d\pi = dR - dr$ = inflation rate differential

Using the format of equation (12), equation (16) can be rewritten as:

$$ds^* = [dR + \lambda]dt = \mu \, dt \tag{19}$$

where $\lambda = b_o + (b_1 - 1) \, dr$

Equation (19) provides the expected change in exchange rates. However, the actual change which is stochastic is given by:

$$ds = \mu \, dt + \varepsilon \tag{20}$$

where μ = as specified in equation (19), and

ε = a random error term.

The error term may be specified as a diffusion process:

$$\varepsilon = \sigma dz \quad \text{where } dz \text{ is a Wiener process.} \tag{21}$$

Alternatively, the error term may be specified as a diffusion jump process, such as:

$$\varepsilon = \sigma dz + dq, \quad \text{where } dq \text{ follows a Poisson distribution} \tag{22}$$

A jump component may be important if central bank intervention occurs or if some news (for example, political news) can cause jumps in FX rates.

In the short run (say monthly), changes in exchange rates are often quite large and are almost random (see, for example, Mussa (1984)). Hence, ϵ in equation (20) may be the predominant factor. As stated earlier, the main objective of this essay is to provide and test a model for explaining ds. For that purpose, the current spot FX rate, needs to be related to its long term equilibrium rate. To this point, a single default-free interest rate was considered in each country. In the following discussion, multiple (i.e., the term structure of) interest rates will be considered. In the interest of clarity, the case of an economy with risk neutral investors will first be discussed in order to introduce some important concepts. Subsequently, this will be extended to the case of an economy with risk averse investors.

III.2 Relationship Between the Present Spot Exchange Rate and the Expected Long Term Equilibrium Rate

III.2.1 Risk Neutral Market Participants

Based on the empirical evidence, PPP is assumed to hold in the long run.[19] In a risk neutral economy, current deviations from PPP will only be due to real interest rate differentials. If relative PPP is expected to hold at period M,[20] the present PPP exchange rate (which may not be the actual exchange rate) is given as:[21]

$$s_0^{PPP} = s_M^{e\ PPP} - \sum_{i=1}^{M} ({}_i\pi_d^e - {}_i\pi_f^e)dt \tag{23}$$

where s_0^{PPP} = natural logarithm of the exchange rate at time 0 if PPP had held immediately[22]

$s_M^{e\ PPP}$ = PPP exchange rate expected to hold at period M

${}_i\pi^e$ = continously compounded inflation rate, which is expected to occur during period i

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19. Such an assumption is generally supported in the literature. Inter alia, Bomhoff and Koedijk (1988), Isard (1983) and Shaefer and Loopesko (1983) invoke such an assumption.

20. The market need not attach a great amount of confidence to this expectation. The only requirement is that, given the present set of information, the market has no reason to believe that the FX rate at M will be higher or lower than its PPP level.

21. Without loss of generality, dt will be taken as being equal to 1.

22. Throughout this essay, a lower case s refers to the natural logarithm of the FX rate. For the sake of readability, s will be sometimes simply referred to as the exchange rate.

In such a risk neutral economy, IRP is expected to hold at all times. Hence:

$$s_o = s_M^{e\ PPP} - \sum_{i=1}^{M} ({}_i dR) \tag{24}$$

where s_o = exchange rate at time 0

${}_i dR = {}_i R_d - {}_i R_f$

The terms, ${}_i R_d$ and ${}_i R_f$, can be computed from the term structure of interest rates. If $({}_i TR_d/i)$ is defined as the yield to maturity of a domestic bond maturing after i periods, ${}_i R_d$ can be computed as:

$${}_i R_d = {}_i TR_d - {}_{i-1} TR_d \tag{25}$$

and $\sum_{i=1}^{M} {}_i R_d = {}_M TR_d$

and $\sum_{i=1}^{M} {}_i dR = {}_M TdR$.

Hence, the difference between the actual exchange rate and the PPP exchange rate at time 0 is given by:

$$s_o - s_o^{PPP} = - \sum_{i=1}^{M} [({}_i R_d - {}_i \pi_d^e) - ({}_i R_f - {}_i \pi_f^e)]$$

$$= - \sum_{i=1}^{M} {}_i dr \tag{26}$$

where ${}_i dr$ is the expected real interest rate differential between the two countries during period i.

Fisher (1930) argues that real interest rates will be equal across countries if capital markets are perfect. Market imperfections allow real interest rates to differ across countries in the short run, but capital flows would tend to equalize them in the long run.[23] If at period 0 (i), real interest rates are expected to become equal at period ${}_0N$ (${}_1N$), equation (26) will hold for all $M \geq {}_0N$ (${}_1N$).

In this model, short-run deviations from PPP are not caused by imperfections in the goods markets. Instead, they are caused by real interest rate differentials. Imperfections in the goods markets only allow these deviations to exist. Consequently, increased efficiency in the goods markets will tend to decrease the maximum possible deviations from PPP (i.e., the maximum possible cumulative expected future real interest rate differentials).

III.2.2 Risk Averse Market Participants

In an economy where participants are risk averse and transaction costs exist in both the financial and goods markets, PPP is still expected to hold in the long run, and equation (23) still applies. However, since financial arbitrage will not cause IRP to hold, risk premia will exist. Equation (18), which yields the expected changes in exchange rates, is expected to hold for any time interval. Assuming it

23. The authors cited in footnote 19 invoke such an assumption.

holds over M periods, equation (18) can be decomposed over the M periods.[24] As a result, equation (24) becomes:

$$s_O = s_M^{*\,PPP} - \sum_{i=1}^{M} ({}_ib_O + {}_ib_1\ {}_idr + {}_id\pi) \tag{27}$$

$$= s_O^{PPP} - b_O - \sum_{i=1}^{M} {}_ib_1\ {}_idr$$

where $b_O = \sum_{i=1}^{M} {}_ib_O$,

$$\sum_{i=1}^{M} {}_ib_1\ {}_idr = b_1\ {}_MTdr$$

and ${}_ib_1$ is the b_1 which is attached to the dR of the i^{th} future period.

The term ds^* in equation (18) (over M periods) now equals (s_O - $s_M^{*\,PPP}$) in equation (27). The difference between the actual exchange rate and the PPP exchange rate at time 0 is given from equation (27) as:

$$s_O - s_O^{PPP} = -b_O - \sum_{i=1}^{M} {}_ib_1\ {}_idr \tag{28}$$

Due to transaction costs and risk aversion, the deviations of actual from PPP exchange rates are a function of both the real interest rates and the risk premia.

Since real exchange rates are expected to be equal in the long run, M in equations (27) and (28) must be at least as long as the

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24. In this case, the risk premia are not due solely to FX risk as for one period bonds, but rather to both FX and interest rate risks.

horizon over which real interest rates are expected to be different. This is the same condition as in the case of risk neutral market participants. Arbitrage in the goods market does not appear explicitly in equation (28). Its implicit role is to limit the maximum size of $(s_0 - s_0^{PPP})$. As goods arbitrage takes place over time, real and nominal interest rate differentials will evolve so that $(s - s^{PPP})$ becomes small.

III.3 Ex-Post Changes in Exchange Rates

The expected changes in exchange rates are given by equation (18). As is demonstrated below, the actual changes are derivable from equation (24) for a risk neutral economy and from equation (27) for a risk averse economy.

III.3.1 Risk Neutral Market Participants

Equation (24) gives the relationship between the current spot exchange rate and the PPP exchange rate expected to hold after M periods. To observe the change of the exchange rate over a single period, equation (24) can be rewritten for one period ahead as follows:

$$s_1 = {}_1s_M^{*\,PPP} - \sum_{i=2}^{M} ({}_i dR^1) \tag{29}$$

where $M \geq \max({}_0N, {}_1N)$

$^{1}s_M^{*\,PPP}$ = the expectation at time 1 of the PPP exchange rate at time M

$_{i}dR^{1}$ = is the interest rate differential during period i, which is observed from the term structure at time 1

The change in exchange rates is then given by subtracting equation (24) from (29) to yield:

$$s_1 - s_0 = {}^{1}s_M^{*\,PPP} - s_M^{*\,PPP} - \sum_{i=2}^{M} ({}_{i}dR^{1} - {}_{i}dR) + {}_{1}dR \tag{30}$$

The shift in the expected period M PPP exchange rate from time 0 to time 1 is due to changes in the expected relative price levels at period M:

$$\begin{aligned} {}^{1}s_M^{*\,PPP} - s_M^{*\,PPP} &= ({}^{1}_{M}p_d^{*} - {}^{1}_{M}p_f^{*}) - ({}_{M}p_d^{*} - {}_{M}p_f^{*}) \\ &= {}^{1}\Delta dp^{*} \end{aligned} \tag{31}$$

where ${}^{1}_{M}p_d^{*}$ = expectation at time 1 of the domestic price level at time M

Substituting (31) into (30) yields:

$$s_1 - s_0 = {}_{1}dR + {}^{1}\Delta dp^{*} - \sum_{i=2}^{M} ({}_{i}dR^{1} - {}_{i}dR) \tag{32}$$

At time 0, the expected value of ${}^{1}\Delta dp^{*}$ is zero, and the term structure of interest rates is not expected to shift. (In this risk neutral economy, the expectations hypothesis of the term structure holds.)

Hence, the expected value of $(s_1 - s_0)$ is ${}_1dR$, and the unexpected change in s is caused by the other terms.

Alternatively, $(s_1 - s_0)$ can be determined by rewriting equation (26) one period ahead by subtracting the time 0 equation from the time 1 equation and rearranging. This yields:

$$s_1 - s_0 = s_1^{PPP} - s_0^{PPP} - \sum_{i=2}^{M} ({}_idr^1 - {}_idr) + {}_1dr \tag{33}$$

Since the shift in the PPP exchange rate is simply the inflation rate differential, equation (33) can be rewritten as:

$$s_1 - s_0 = ({}_1\pi_d - {}_1\pi_f) + {}_1dr - \sum_{i=2}^{M} ({}_idr^1 - {}_idr)$$

$$= {}_1dR - \sum_{i=2}^{M} ({}_idr^1 - {}_idr) \tag{34}$$

Equation (34) shows that, in a risk neutral economy, the expected change in the exchange rate is simply given by IRP (${}_1dR$), and the unexpected change is given by the change in the expected real interest rate differentials.

VII.3.2 Risk Averse Market Participants

Equation (27) gives the relationship between the current spot exchange rate and the PPP exchange rate expected to hold after M periods. Rewriting equation (27) for one period ahead yields:

$$s_1 = s_1^{ppp} - b_0^1 - \sum_{i=2}^{M} {}_{i-1}b_i^1 \; {}_i dr^1 \tag{35}$$

Equation (35) can be rewritten with M replaced by (M+1). However, as discussed earlier, when M is very large, real interest rates are expected to be equal during M+1. Subtracting equation (27) from equation (35), and substituting from equation (31), yields:

$$s_1 - s_0 = s_1^{ppp} - s_0^{ppp} - (b_0^1 - b_0)$$

$$- \left[\sum_{i=2}^{M} {}_{i-1}b_i^1 \; {}_i dr^1 - \sum_{i=1}^{M} {}_i b_i \; {}_i dr \right] \tag{36}$$

Assume that b_0 and ${}_ib_i$ are constant over time (i.e., that $b_0^n = b_0^m = b_0$ and ${}_ib_i^n = {}_ib_i^m = {}_ib_i \; \forall \; n, m$). Stated differently assume that the risk premium changes over time only as a function of dr. Then equation (36) can be rewritten as

$$s_1 - s_0 = {}_1 d\pi - \sum_{i=2}^{M} {}_{i-1}b_i ({}_i dr^1 - {}_{i-1}dr) + {}_M b_i \; {}_M dr \tag{37}$$

or $$s_1 - s_0 = {}_1 d\pi + {}_1 b_1 \; {}_1 dr - \sum_{i=2}^{M} [{}_{i-1}b_i \; {}_i dr^1 - {}_i b_i \; {}_i dr]$$

$$= dR + ({}_1 b_1 - 1) \; {}_1 dr - \sum_{i=2}^{M} [{}_{i-1}b_i \; {}_i dr^1 - {}_i b_i \; {}_i dr] \tag{38}$$

Interestingly, the only difference in the total change in the exchange rate between a risk averse (equation (38)) and a risk neutral (equation (32)) economy is the presence of the terms ${}_ib_i$ in equation (38). From equation (18), ${}_ib_i$ depends on the investors' risk aversion, on the own and market risk of the exchange rate, on the number of risk-taking speculators in the FX market, on the volume of speculative capital used in the FX market, and on the market's confidence in its expectations. The risk neutral solution is simply a special case of the risk averse solution, when ${}_ib_1 = 1 \;\forall\; i$.

At time 0, the expected value of (s_1-s_0) is given by:

$$(s_1 - s_0)^{e} = {}_1d\pi^{e} - \sum_{i=2}^{M} {}_{i-1}b_1 \; ({}_idr^{e1} - {}_{i-1}dr^{e}) + {}_Mb_1 \; {}_Mdr^{e} \qquad (39)$$

where ${}_idr^{e1}$ is the expectation at time 0 of the dr of period i which will be observed at time 1

Because of the risk premia included in the term structure of interest rates, ${}_idR$ are generally different from ${}_idR^{e1}$, and so ${}_idr^{e}$ are generally different from ${}_idr^{e1}$.

For consistency, the expected FX rate change given by equation (39) must equal that given by equation (18) when applied for one period. Hence:

$$ {}_1b_0 = -{}_1b_1 \; {}_2dr^{e1} + {}_Mb_1 \; {}_Mdr^{e} - \sum_{i=3}^{M} {}_{i-1}b_1({}_idr^{e1} - {}_{i-1}dr^{e}) \qquad (40)$$

M equations like equation (40) can be determined for ${}_ib_0$, where i = 1 to M. These equations would impose the sufficient constraints on the differentials in the term structure of interest rates to satisfy our specification of the risk premium.

The unexpected change in the exchange rate in a risk averse economy is given by the difference between the expected and realized inflation rates, and by the cumulative effects of the unexpected changes in the differentials of the term structure of real interest rates.

Equation (39) can be compared with the expected FX rate change in Dornbusch's (1976) model (equation (2) in this paper). Unlike Dornbusch's model, our model does not predict exponential convergence towards PPP and is not compatible with a distributed lag specification of PPP. Depending on the term structures of interest rates and their expected shifts, the exchange rate may sometimes be expected to increase its divergence from its PPP level before converging back to it. Moreover, the rate of convergence (and perhaps initial divergence) is not constant but depends on present real interest rate differentials and their expected change.

It should be noted that this model does not refer explicitly to political factors such as election results, policy changes or wars, nor to real economic changes such as the discovery of new resources, changes in technology or taste, or central bank intervention. These factors are however accounted for implicitly since their only relevant effects relate to interest rates, price levels, and possibly to risk and risk aversion, provided financial speculation is not prohibited. The empirical problems due to these factors include the difficulty in determining how inflation rate expectations, and hence real interest rates expectations, and risk perception and risk aversion are affected by these factors.

III.4 Illustration

An illustration shows how this model can be consistent with the observed behaviour of FX rates during the 1980's when both PPP and IRP seemed to be violated. The same example will be used first in the risk neutral model, then in the risk averse model. The basic data are provided in Table 2.

III.4.1 Risk Neutral Market Participants

Since at time 0 real interest rates are expected to be equal after year 3, and at time 1 they are expected to be equal after year 6, any value of $M \geq 6$ can be used in the model. Assume M=10. From equation (23):

$$S_{10}^{PPP} = S_{0}^{PPP} = 50 \text{ ¢/DM}$$

since inflation rates are expected to be equal in both countries, and from equation (24)

$$\ln S_o = \ln 50 - 3\ (0.15-0.10)$$

Hence: $S_o = 43.04$ ¢/DM

Thus, the present spot rate of the US dollar is overvalued relative to PPP vis-à-vis the German mark (DM).

At the end of the first year, the PPP exchange rate expected at year 10 has changed so that:

$$ {}^{1}\Delta dp^{*} = {}^{1}s_{10}^{*PPP} - s_{10}^{*PPP} = 0.05 $$

and the actual exchange rate will have changed according to the equation (32) by:

$$ \ln S_1 - \ln S_0 = 0.05 + 0.05 - \sum_{i=2}^{3} (0.05 - 0.05) - \sum_{i=4}^{6} (0.05 - 0.0) + \sum_{i=7}^{10} (0.0 - 0.0) $$

Hence $S_1 = 43.04\ e^{-0.05} = 40.94$ ¢/DM

This result may seem surprising in that it seems to violate both PPP and IRP. Indeed the US had both an inflation rate and an interest rate higher than Germany, yet the US dollar appreciated vis-à-vis the mark. Moreover, the FX rate did not tend to move towards the PPP exchange rate, indeed the gap between the actual rate and the PPP rate increased! Yet this exchange rate behaviour is not irrational, its explanation is actually fairly simple. Interest rate differentials, without corresponding inflation rate differentials, are expected to last 3 years more at time 1 than they were at time 0. This has pushed the DM down to allow an expected future appreciation to compensate for the interest rate differential. The higher inflation rate in the US between time 0 and time 1 reduced the effect of the interest rates, but it did not completely offset it.

III.4.2 Risk Averse Market Participants

In order to use the model for risk averse investors, the values of b_o and ${}_1b_1$ defined in equation (6) need to be known.

For the sake of simplicity, assume:

$b_o = 0.01$

${}_1b_1 = {}_2b_1 = {}_3b_1 = {}_4b_1 = {}_5b_1 = 0.90$

The PPP exchange rate is the same whether market participants are risk neutral or risk averse, but the relationship between the present spot and the PPP exchange rate is not. According to equation (27):

$\ln S_o = \ln 50 - 0.01 - 3\ (0.9)(0.10-0.05)$

Hence $S_o = 50\ e^{-0.145} = 43.25$ ¢/DM

At the end of the first year, the PPP exchange rate will have changed as in the risk neutral case, but the actual exchange rate will have changed according to equation (37) by:

$$\ln S_1 - \ln S_o = 0.05 - \sum_{i=2}^{4} 0.9(0.05-0.05) - \sum_{i=5}^{6} 0.9(0.05-0.0)$$

$$= 0.05 - 0 - 0.09$$

Hence $S_1 = 43.25\ e^{-0.04} = 41.55$ ¢/DM

Two interesting remarks can be made about these results. First, as in the case of Driskill and McCafferty (1987) an increase in risk aversion (a smaller $_ib_1$) decreases exchange rate changes. Second, the hypothetical behaviour of the US dollar/DM exchange in this example, in both the risk neutral and the risk averse market participant cases, is similar to the actual behaviour of the US dollar vis-à-vis the Yen, the DM, the SF and several other currencies during the early 1980's. Moreover, this behaviour does not need to assume any "destabilizing" speculation as suggested by Dornbusch (1984).

III.5 <u>Empirical Regularities</u>

Before testing the model empirically, it would be interesting to verify its consistency with the five "empirical regularities" reported by Mussa (1986). These regularities are:

1. Monthly changes in exchange rates are often quite large and are almost entirely random.

2. Spot and forward rates tend to move in the same direction and by approximately the same amount, especially when changes are relatively large.

3. Monthly changes in nominal and real exchange rates are closely correlated, and cumulative changes in real exchange rates over a period of a year have been quite large.

4. The relationship between movements in nominal or real exchange rates and current account balances is not strong and systematic. It does not provide an explanation for a substantial fraction of actual exchange rate movements.

5. Except possibly for very highly inflationary economies, movements in nominal and real exchange rates are not closely related to differential rates of monetary expansion.

Regularity 1. The model allows for large monthly changes in exchange rates if anticipations about long term interest rates and inflation move by even a small amount. For example in the risk neutral case, ceteris paribus, a 1 % increase in the 5 year domestic interest rate, combined with a 1 % decrease in expected domestic inflation over the next 5 years could cause the present spot rate to increase by about 10 %.

Regularity 2. In the above example, if the 1 % interest rate increase also applied to short term rates, the premium or discount on the 3 months forward rate would only change by approximately 0.25 % while both spot and forward rates move by about 10 %.

Regularity 3. The relationship between nominal and real exchange rates is quite close since nominal FX rates could change radically. Since inflation rates are fairly stable, real FX rates follow nominal rates quite closely.

Regularity 4. This model does not consider the current account balances as such, although their effect could, eventually, be felt through interest rates and inflation rates.

Regularity 5. In the model the effects of monetary policy are only considered through their impact on prices and interest rates. Temporary short-term changes in the rates of monetary expansion are not the principal factor affecting long-term interest and inflation rates. Thus, they should not be expected to explain much of the changes in the nominal or real exchange rates.

IV. Empirical Analysis

IV.1 The Data

In order to conduct this analysis, monthly observations of the term structure of interest rates, of exchange and forward rates, and of the consumer price index were needed. The term structure of interest rates was only available for the following four currencies: the US dollar (US), the German mark (DM), the Swiss franc (SF) and the Canadian dollar (CD) for the period January 1974 (1974-01) to December 1987 (1987-12).

International rates (LIBOR) were available for the US, the DM, and the SF for maturities of 1, 3 and 6 months and 1, 2, 3, 4 and 5 years. In addition, eurobond rates with maturities of 3 to 7 years and 7 to 15 years were available for the US and the DM. Domestic rates were available for the US and the CD. These were:

- For the US, 30 and 90 day certificates of deposit, 3, 6 and 12 months T-Bills, and 1, 2, 3, 5, 7, 10, 20, 30 and over 10 years T-Bonds.
- For the CD, 30 and 90 day CD's 3, 6 and 12 months T-Bills, 1-3, 3-5, 5-10 and over 10 years Government Bonds.

The principal sources of data were: Data Resources International, I.P. Sharp and Associates, Bank of Canada Review, Deutsche Bundesbank Security Statistics, and OECD Financial Statistics.

For the CD analysis, the US and Canadian interest rate series had to be made compatible, so the 1, 2 and 3 years US T-Bond rates were averaged to be used in conjunction with the Canadian 1-3 years government bond yields, the average of the 3 and 5 years US T-Bond rates were used with the Canadian 3-5 years yield and the 5, 7 and 10 years US rates were averaged and used with the 5-10 year Canadian rates.

A few interest rate observations were missing. They were generated from the 2 interest rate series with the closest maturities. For example, the few missing 2 year US T-Bonds rate observations (R2YTB) were generated as follows: first, estimate on the data available:

$$R2YTB = \beta_0 + \beta_1\ R1YTB + \beta_2\ R3YTB$$

where RiYTB is the i year US T-Bond yield to maturity; and second, generate the missing values of R2YTB by using β_0, β_1 and β_2 estimated in the previous step in conjunction with observed values of R1YTB and R3YTB.

IV.2 Operationalization of the Models

The models to be studied empirically are given by equations (28) and (37), respectively. They are reproduced here for convenience:

$$s_0 - s_0^{PPP} = -b_0 - \sum_{i=1}^{M} {}_ib_i\ {}_idr \tag{28}$$

$$s_1 - s_0 = {}_1d\pi - \sum_{i=2}^{M} {}_{i-1}b_i({}_idr^1 - {}_{i-1}dr) + {}_Mb_i\ {}_Mdr \tag{37}$$

IV.2.1 Model for Exchange Rate Levels

Equation (28) relates the level of the FX rate to its PPP equilibrium rate. The first problem is in determining s_0^{PPP} which can not be observed. Equation (23) gives the relationship between s_0^{PPP} and s_M^{PPP} for any M. Defining x as the natural logarithm of the real exchange rate, i.e.,

$$x = s - PI_d + PI_f, \text{ where PI is a price index,} \tag{41}$$

yields: $x_0^{PPP} = x_M^{PPP} = \text{constant}\ \forall\ M$

and $s_0 - s_0^{PPP} = x_0 - x_0^{PPP}$.

Hence, equation (28) can be rewritten as:

$$x_o = x_o^{PPP} - b_o - \sum_{i=1}^{M} {}_i b_i \, {}_i dr \qquad (42)$$

The price index selected in this study is the consumer price index because of its availability, and because there is no compelling reason to use any other index.[25]

The second problem is in estimating the expected term structure of inflation rates needed to compute the term structure of real interest rates. Based on Fama (1975) and Fama and Gibbons (1984), short term interest rates can be used as predictors of inflation. Extending this argument, it can be stated that:

Term structure of expected inflation = f(term structure of interest rates) (43)

Moreover, several authors (e.g., Coe and Golub (1986), Hooper (1984), Sachs (1985), and Shaefer and Loopesko (1983)) have used various naïve proxies successfully, and generally with more success than more sophisticated econometric forecasts (Shaefer and Loopesko (1983)). The expected inflation rate used for computing real interest rates in this study is the actual inflation realized during the previous twelve months. Since the same value of inflation was subtracted from all nominal interest rate terms on a given date for a given currency to give the real interest rates, the real and the nominal term structures of interest rates could not be kept simultaneously in the model because of perfect multicollinearity. The model can then be rewritten as:

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25. See Section II.2.1 which discusses the literature dealing with PPP.

$$x_o = \beta_o + \sum_{i=1}^{M} \beta_i \ {}_i dr + \beta_{M+1} \ \delta_{inf} \tag{44}$$

where $\beta_o = x_o^{PPP} - b_o$

δ_{inf} = inflation rate differential between the two countries. With both ${}_i dr$ and δ_{inf} included in the model, the impact of any ${}_i dR$ ($= {}_i dr + \delta_{inf}$) is captured by the paramaters of ${}_i dr$ and δ_{inf}.

Finally, the ${}_i dr$ terms should be the interest rate differentials (${}_i r_d - {}_i r_f$) during period i (see equation (25) for the exact definition of ${}_i R_d$). However, the interest rate of a given period can be expressed as a linear combination of the interest rates observed in the term structure. For example, if the 5 year rate (${}_5 Tr$) and the 4 year rate (${}_4 Tr$) are known, then the rate ${}_5 r$ is given by:

$${}_5 r = 5 \ {}_5 Tr - 4 \ {}_4 Tr \tag{45}$$

Hence, a new variable (Δ_i) is defined for practical purposes as:

$$\Delta_i = i({}_i Tr_d - {}_i Tr_f) \tag{46}$$

so that:

$${}_i dr = \Delta_i - \Delta_{i-1}$$

Thus, the empirical specification of equation (28) is:

$$x = \beta_o + \sum_{i=1}^{M} \beta_i \ \Delta_i + \beta_{M+1} \ \delta_{inf} + \varepsilon \tag{47}$$

where i = 0 for the period when x is observed.

IV.2.2 Model for Exchange Rate Changes

Equation (37) gives the period to period change of the exchange rate. The main problem in estimating equation (37) is the assessment of the term structure of expected inflation. Equation (37) can be rewritten as:

$$s_1 - s_0 = {}_1d\pi - \sum_{i=2}^{M} {}_{i-1}b_1 \left[\left({}_i dR^1 - {}_{i-1}dR \right) - \left({}_i d\pi^{*1} - {}_{i-1}d\pi^{*} \right) \right]$$

$$+ {}_M b_1 \left[{}_M dR - {}_M d\pi^{*} \right] \qquad (48)$$

Based on the argument made earlier for equation (43) and the fact that naïve proxies of expected inflation often outperform more complex ones, assume that the equation given in (43) is linear. With this assumption, the $d\pi$ terms are linear combinations of the dR terms. Hence, for statistical purposes, the $d\pi$ terms of all the periods, except the period 0–1, are redundant.

Replace the ${}_i dR$ terms with a new variable, δ_i, similar to Δ_i in the case of real interest rates, where:

$$\delta_i = i({}_i TR_d - TR_f) \qquad (49)$$

Thus, the empirical counterpart of equation (37) is:

$$\Delta s = \beta_0 + \beta_1 \delta_1^1 + \sum_{i=1}^{M} \beta_{i+1} d\delta_i + \beta_{M+2} \delta_M^0 + \beta_{M+3} \delta_{inf} + \varepsilon \qquad (50)$$

where $d\delta_i = \delta_i^1 - \delta_i^0$

δ_i^0, δ_i^1 = the δ_i observed during the previous and the present period, respectively.

In equation (50), δ_1^1, the shortest maturity interest rate differential, acts as one proxy for expected inflation.

IV.3 Empirical Results

IV.3.1 Model for Exchange Rate Levels

Although the data are available starting in January 1974, the results presented here are for the period 1975-01 to 1987-12 because the oil embargo was still in effect in 1974 causing severe distortions in the world economy.[26] The first step was to fit the full model to the monthly data. However, because of the very high correlation between the interest rate terms, most parameters appeared non-significant. The model could explain 76 %, 58 % and 46 % of the variance in x for the DM, the SF and the CD, respectively. Three interesting observations are evident from Table 3 which presents the correlations for all the variables in the models. First, the multicollinearity between the interest rate terms is evident. The correlations between the RHS variables of equation (47) are often in the 0.90 range and sometimes exceed 0.99. Second, the real exchange rate is most highly correlated with the longest term real interest rate differential available, namely Δ_{7-15x} for the DM, Δ_{5x} for the SF and Δ_{010x} for the CD. Third, the correlation between the real Canadian exchange rate and most interest rate terms is positive but not

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26. Most of the analyses were also conducted on the entire sample. The qualitative results are generally the same and the quantitative results are fairly similar.

significant.[27,28]

To obtain more meaningful results, non-significant variables were eliminated and reduced models were estimated.[29] The estimated reduced form model for the DM is:[30]

$$x = 3.77 - \underset{(18)}{65}\ \Delta_{1m} + \underset{(4.4)}{19.4}\ \Delta_{3m} - \underset{(0.69)}{2.27}\ \Delta_{3y} - \underset{(0.11)}{0.28}\ \Delta_{7-15y}$$

$$n = 156, \quad R^2 = 0.75.$$

The order of magnitude of the Δ's is of the order of magnitude of the maturity to which they are related (e.g., Δ_{3y} is of an order of magnitude 36 times larger than Δ_{1m}). Interestingly, the entire term structure is covered by the reduced model. Unfortunately, collinearity

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27. Khoury and Melard (1985) find that the Treasury Bill Rates in Canada have provided good assessments of expected inflation in the U.S. rather than in Canada during the period 1953 to 1975.

28. In an earlier version of the model where the flow of arbitrage capital was function of the nominal interest rate differential, the real FX rate was negatively related to the real interest rate and positively related to nominal interest rates. Given the manner in which the model was operationalized, it is impossible to distinguish between the parameters of the real interest rates and those of the nominal interest rates. In the theoretical development, it was argued that the risk premium would be a function of the volume of capital used in FX arbitrage. This result could have indicated that there is more FX (and interest rate) arbitrage between Canada and the US than between the US and either Germany or Switzerland. Moreover, the inflation rate differential between the US and Canada is the smallest and the least volatile (see Table 4). This makes the emphasis on real interest rate expectations less relevant.

29. Variables were eliminated using a backward elimination procedure. A variant of forward selection of variables gave exactly the same results. All variables remaining in the model are significant at α=5%.

30. Standard errors are given in parentheses.

remains as witnessed by the positive parameter of Δ_{3m}, and a correlation of 0.99 between Δ_{3m} and Δ_{1m}.

The reduced form model estimated for the SF is:

$$x = \underset{}{3.92} - \underset{(22)}{45}\ \Delta_{1m} + \underset{(8.9)}{31.4}\ \Delta_{3m} - \underset{(0.32)}{1.87}\ \Delta_{3y} + \underset{(0.57)}{1.52}\ \delta_{inf}$$

$$n = 156,\ R^2 = 0.57$$

This model is qualitatively similar to that of the DM, except that the term with the longest maturity (Δ_{5y}) is not in the model. However, the correlation between Δ_{5y} and Δ_{3y} is 0.995.

The reduced form model estimated for the CD is:

$$x = 4.52 - \underset{(6.6)}{49.6}\ \Delta_{1m} + \underset{(5.4)}{26.2}\ \Delta_{3m} - \underset{(3.8)}{18.1}\ \Delta_{6m} + \underset{(1.4)}{7.4}\ \Delta_{1y}$$

$$n = 156,\ R^2 = 0.43.$$

The absence of all long term rates may be due to the multicollinearity problem, or it may be due to the abundance of arbitrage capital flows across the Canada-US border, especially if these capital flows respond primarily to short term interest rate differentials.

The analysis for the CD was repeated using nominal interest rates. Except for the presence of the inflation rate differential in the reduced form model, the results of the models with real and nominal rates are very similar. The estimated model with nominal interest rates is:

$$x = 4.53 - \underset{(6.8)}{50.9}\ \delta_{1m} + \underset{(5.4)}{26.8}\ \delta_{3m} - \underset{(3.8)}{17.9}\ \delta_{6m} + \underset{(1.4)}{7.6}\ \delta_{1y} - \underset{(0.25)}{0.73}\ \delta_{inf}$$

$$n = 156,\ R^2 = 0.43.$$

A regression of real exchange rates on the longest available interest rate differential (which is also the term which is most highly correlated with x) yielded:[31]

$$\text{DM} \quad x = \underset{(0.009)}{3.774} - \underset{(0.26)}{5.35}\ \Delta_{7-15Y} \qquad R^2 = 0.72$$

$$\text{SF} \quad x = \underset{(0.012)}{4.013} - \underset{(0.31)}{3.41}\ \Delta_{5Y} \qquad R^2 = 0.45$$

$$\text{CD} \quad x = \underset{(0.006)}{4.489} + \underset{(0.32)}{0.85}\ \Delta_{010Y} \qquad R^2 = 0.04,\ \alpha = 0.0087$$

The results for the CD are unsatisfactory, because they indicate that the long maturity real interest rate is not very useful for explaining the level of the Canadian dollar. However, as shown earlier, a combination of interest rates may be more useful.

The one variable models for the DM and the SF explain a large part of the DM and SF exchange rates. Since they do not suffer from any multicollinearity problems, the problem of simultaneous determination of exchange rates and interest rates can now be addressed. The one variable models for the DM and SF were estimated using a maximum likelihood method with lagged values of x and Δ as instruments. The resulting estimates are:

$$\text{DM} \quad x = \underset{(0.009)}{3.769} - \underset{(0.29)}{6.02}\ \Delta_{7-15Y} \qquad R^2 = 0.74$$

$$\text{SF} \quad x = \underset{(0.013)}{4.035} - \underset{(0.39)}{4.73}\ \Delta_{5Y} \qquad R^2 = 0.49$$

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31. Δ is given here in annual terms to facilitate interpretation of the results.

These estimates may be biased because the error terms of these models are not independent. The residuals of models using exchange rate levels are highly autocorrelated when observations are collected at short intervals (such as one month). Indeed, the first order autocorrelations for the DM and the SF are 0.873 and 0.905, respectively. In order to determine whether these autocorrelations cause any estimation bias, the maximum likelihood estimations were repeated using non-overlapping data for 3, 6 and 12 month intervals.[32] The results, which are reported in table 5, indicate parameter stability. This suggests that the autocorrelation of the error terms caused no bias. Some authors (e.g., Coe and Golub (1986)) have interpreted the coefficient of Δ as the horizon of real interest rates which affect exchange rates. For example, they would conclude that the real interest rate differential over 6 years affects the present exchange rate of the DM. While this interpretation is correct in a risk-neutral world, this coefficient may also pick up the risk premium which appears in front of the real interest rate differential in equation (28) in a risk averse world.

These results suggest that long term real interest rate differentials are the major determinants of the real exchange rate of the DM and the SF. Because of the large volume of capital involved in currency (and interest) arbitrage between the US and Canada, these results suggest that a combination of interest rates is needed to

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32. The three possible samples of data using observations three months apart were used.

explain the level of the CD.

IV.3.2 Model for Exchange Rate Changes

IV.3.2.1 In-Sample Analysis

Equation (50) gives exchange rate changes over a given period. One month, three months, six months and one year holding periods are considered herein. The explanatory powers (R^2 values) of the full model for the various currencies and holding periods are presented in Table 6. The model has the highest explanatory power for the DM, followed by the SF and the CD. Except for the CD, the model works better for longer holding periods because random errors tend to cancel out over longer holding periods. In the case of the CD, the results are best for the one month holding period. If events not captured by the model have relatively long lasting effects,[33] several observations are affected when changes are measured over periods longer than one month. The methodology used in this section uses overlapping data, so error terms are not independent. Although OLS still provides unbiased estimates of the parameters, standard statistical tests of significance may be misleading.

Since multicollinearity exists for the full model[34] (see Table 7

33. Between 1975 and 1987, Canada has witnessed the rise and then decline of a very powerful separatist movement in Quebec.

34. In most cases, the correlation coefficients are larger for longer holding periods.

for the correlation matrices for one month holding periods), reduced models need to be used. Although standard t statistics tend to overstate the significance of the estimated parameters, they were used to eliminate all variables not "seemingly"[35] significant at α=5 %. Based on the results for the reduced models presented in Table 8, the significant terms measuring the shift in interest rates ($d\delta_i$) tend to cover the entire term structure of interest rates for the DM and the SF. The inflation rate differential (δ_{inf}) is usually significant for longer holding periods (3 months, 6 months and 1 year for the DM, 3 months and 1 year for the SF and 6 months and 1 year for the CD). Moreover, when δ_{inf} is included in the model, it has the correct sign, and is less than two standard deviations away from unity for the CD and the SF.[36] This indicates that inflation rates only play a minor role in short term exchange rate changes, and a more important role in the long run. Whenever the short term interest rate (δ_1^1) entered the model, it had a negative sign. This indicates that δ_1^1 probably did not pick up inflation rate expectations,[37] but rather picked up the impact of short term capital flows caused by short maturity interest rate differentials. The parameter estimates of the other variables in the

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35. Because of the use of overlapping data, some variables remaining in the model may not be actually significant at α=5 %. The elimination of variables was done using a backward elimination procedure and then confirmed with a variant of a forward selection procedure.

36. δ_{inf} is measured here over a period equal to the holding period. For example, δ_{inf} is measured over 3 months for the 3 month holding period, and over 12 months for the 12 month holding period.

37. Except for the one month holding period for the SF, δ_1^1 and δ_{inf} simultaneously remained in the model.

model are difficult to interpret because of the multicollinearity problem.

As was the case with the full models, the explanatory power of the reduced models for the CD for 3, 6 and 12 month holding periods is not satisfactory. Both the 3 and the 12 month holding period reduced models do not include a single term measuring the shift in interest rates ($d\delta_1$). In the model for the 6 month holding period, the only $d\delta$ term in the model is the least significant of the variables remaining in the model. In order to verify the quality of the models, an out-of-sample analysis was conducted.

IV.3.2.2 Out-of-Sample Analysis

The out-of-sample analysis was conducted by first estimating the model over a calibrating period. Then these parameter estimates along with the actual interest and inflation rate observations over the following year were used to explain the FX rate changes over that period. The results cannot be considered as forcasts, since they are just measures to indicate the fit of the model out-of-sample. All the calibrating periods start in 1975-01 and end on the month prior to the out-of-sample year for which the results are measured. The model is recalibrated for each year from 1980 to 1987.[38]

The out-of-sample results are compared to a random walk and to the

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38. Recalibrating the model semi-annually produced similar results.

forward rate because these are the usual benchmarks against which most models in the FX literature are compared. The following statistics are used to assess the performance of the model: the mean error (ME), the mean absolute error (MAE), the mean squared error (MSE), the percentage of time the predicted change is of the correct sign (Dir), and the percentage of time the prediction falls on the same side of the forward rate as the true exchange rate (X).

The ME only allows for a determination of whether the predictions are biased because it uses the simple arithmetic average of the prediction errors. The MSE does not allow positive and negative errors to cancel since the errors are squared. The MSE gives more weight to the larger errors and is the standard regression criterion. The MAE is used to determine whether a single (or a few) outlier is unduely affecting the MSE. The MAE is an important criterion when the distribution of the errors is non-normal stable paretian. Usually the MAE and the MSE point in the same direction. Dir indicates the frequency with which the model predicts a change of the correct sign. The model outperforms a random walk if Dir is significantly larger than 50 %.

Levich (1985) uses the variable X to assess the performance of forecasts. He argues that a possible criterion for the performance of a forecast is how often a decision to speculate or hedge based on that forecast is correct. Such a decision will be correct as long as the forecast and the exchange rate which actually occurs are on the same side of the forward rate.

When overlapping data are used (for 3, 6 and 12 month holding periods), standard statistical tests are inappropriate. A test proposed by Meese and Rogoff (1988) was used to determine whether the model significantly outperformed the random walk and the forward rate. This statistic, which exploits the MA (k-1) behaviour of optimal k-step-ahead forecasts, is given by:

$$T_{MR} = \hat{cov}(x(t), y(t)) \;/\; (\hat{B}/T)^{0.5}$$

where $x(t) = e(1, t) - e(2, t)$

$y(t) = e(1, t) + e(2, t)$

$e(1,t)$ = period-t forecast error from model 1

$e(2,t)$ = period-t forecast error from model 2

T = number of known forecast errors

$$\hat{B} = \sum_{s=-k+1}^{k-1} (1 - |s|/T) \{\hat{cov}(x(t), x(t-s))\hat{cov}(y(t), y(t-s)) + \hat{cov}(x(t), y(t-s))\hat{cov}(y(t), x(t-s))\}.$$

For large T, T_{MR} is approximately N(0,1).

The out-of-sample results for the period 1980-01 to 1987-12 are presented in Table 9. For the DM, the model exhibits the least bias except for the 6 month holding period. Based on the MSE and Dir, the model outperforms the forward rate for all holding periods, and the random walk for all except the one month holding period. For the one month holding period, the model's MSE is 1 % larger than that of the random walk. The model is statistically superior to both the random walk and the forward rate at the 5 % level for the 3 months holding period. The model predicts the sign of the monthly changes of the FX rate correctly 66.7 % of the time, which is significantly (t = 3.37) better than 50 %. X is also significantly better than 50 %.

For the SF, the mean error is smallest in absolute value for the forward rate for the 1 and 12 month holding periods, for the model for the 3 month holding period, and for the random walk for the 6 month holding period! The MSE of the model is worse than that of the RW for a 1 month holding period (MSE(model)/ MSE(RW) = 1.15), about the same for the 3 month holding period, and smaller by 10 % and 11 % for the 6 and 12 month holding periods. The forward rate has a larger MSE than the RW in all cases. Unfortunately, none of these results are statistically significant. The model predicts the direction of change for a monthly period correctly 54.2 % of the time, compared to 42.7 % for the forward rate. Dir is larger for the model for longer holding periods.

The out-of-sample results for the Canadian dollar confirm the in-sample results which showed that the models for 3, 6 and 12 month holding periods were not satisfactory. Indeed, for the twelve month holding period, both the random walk and the forward rate outperform the model significantly. However, the one month holding period model significantly outperforms the random walk and the forward rate out-of-sample. Dir is also significantly larger than 50 % for the model for a one month holding period.

To determine whether the quality of the model "predictions" changed over time, the differences between the model MSE and the random walk MSE were calculated on a yearly basis (see Table 10). For all three currencies and for almost all holding periods, the results for 1980 were the worst. Events during 1980, which was a year of great turmoil, included the second oil shock and the holding of American hostages in Iran. Statistics comparing our model to the RW and the forward rate for the period 1981-01 to 1987-12 are given in Table 11. For the DM and the SF, the model's errors were always the smallest. For the DM, the model outperformed the RW and the forward rate at the 5 % level of significance for the 3 month holding period, and at the 10 % level for the 6 month holding period. For the SF, the model outperformed the forward rate for the 1, 3 and 6 month holding periods at the 10 % level of significance. For the CD, the model outperformed the RW and the forward rate at $\alpha = 5$ % for the 1 month holding period, and was outperformed by both at $\alpha = 10$ % for the 12 month holding period.

Based on Table 10, the performance of the model tends to improve over time for the DM and the SF. It is unclear whether this is due to real changes in the economy, to the empirical technique used,[39] or to chance.

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39. The variables of the reduced models were selected using data from 1975-01 to 1987-12 and then these reduced models were calibrated over shorter periods (e.g., 1975-01 to 1979-12) for the out-of-sample analysis.

The results of the out-of-sample predictions for Canada are fairly disappointing for holding periods longer than one month. This is probably due to the political events which took place in that country during the studied period (1975-01 to 1987-12). To illustrate, the mean exchange rate change during that period is -0.15 cents, and the standard deviation of the exchange rate changes is 1.16. The two largest drops of the Canadian dollar of 6.28 and 3.41 cents were in November 1976 (the month of the first election of the Parti Québécois) and March 1980 (the month of the referendum on the independence of Quebec), respectively. The first polls which showed that the separatists would win the independence referendum were published in October 1977. During that month, the Canadian dollar dropped by 2.91 cents! Although these (and similar) events only affected one observation each when one looked at a one month holding period, they affected 3, 6 and 12 observations when one considers 3, 6 and 12 month holding periods. Thus, it is not surprising that the models for longer holding periods do not perform adequately.

V. Concluding Remarks

In this paper, an exchange rate determination model has been developed which is compatible with the behaviour of exchange rates during the 1980's and is consistent with the empirical regularities observed in the literature. The model is developed in the context of an economy where investors are risk averse and face transactions costs. In the model, purchasing power parity is expected to hold in the long run, and short term deviations from this parity caused by real interest rate differentials are constrained by the possibility of (costly) goods arbitrage. This model differs from previous models in the specification of its risk premium and by the use of the entire term structure of interest rates.

The model was tested in the form specifying the levels of exchange rates, and in the form specifying changes of exchange rates for the exchange rates of the German mark, the Swiss franc, and the Canadian dollar vis-à-vis the US dollar for the period from 1975-01 to 1987-17. In its levels form, the model was quite successful. It indicated that long term real interest rates can explain the levels of the DM and the SF, while several interest rate terms are needed to explain the level of the CD. A possible reason for the CD result is that relatively more capital flows between the US and Canada for exchange (and interest) rate (risky) arbitrage (or speculation), than between the US and Germany or Switzerland.

In its FX rate changes form, the model explains fairly well the changes of the DM and SF. These results usually improve when the holding period lengthens. Based on an out-of-sample fit of the model over the period 1981-01 to 1987-12, the model outperformed a random walk and the forward rate for all holding periods. This superior performance was statistically significant at the 10 % level of significance in 2 out of 8 cases for the random walk and in 5 out of 8 cases for the forward rate. For the CD, the FX rate changes form of the model was only successful for the one month holding period. The failure of the model for longer holding periods may be due to the multi-observation effect of political shocks due to the methodology used.

Table 1

REAL EXCHANGE RATES DETERMINED BY LONG-TERM REAL INTEREST DIFFERENTIALS: RESULTS FROM THREE OTHER STUDIES

Study	Dependent variable (real)[a]	Constant	L-T real interest differential	R^2	DW
Shafer & Loopesko (monthly, 8/73 to 3/82)	dollar-mark	-1.49 (0.01)[b]	2.74 (0.32)	0.24	na
	dollar-yen	-6.04 (0.01)	2.19 (0.21)	0.51	na
Sachs (quarterly, 77Q1 to 84Q4)	dollar-mark	4.62 (0.01)	6.5 (0.56)	0.81	0.71
Hooper (quarterly, 74Q2 to 83Q4)	effective dollar (x100)	457.0 (0.9)	5.9 (0.5)	0.80	0.70

Source: Coe and Golub (1986)

(a) All the real exchange-rate variables are in logarithms. In Shafer and Loopesko, the exchange rate is defined as the price of a dollar in units of domestic currency. Coe and Golub have multiplied Shafer and Loopesko's coefficients on the long-term real interest differential by -1 to make them comparable to the other studies.

(b) Standard errors in parentheses.

Table 2

Data for Illustration

Time 0:	- R_{US}	= 15 % for the next 3 years 10 % thereafter[a]
	- R_{Ger}	= 10 % indefinitely
	- π^{e0}_{US}	= π^{e0}_{Ger} = 5% indefinitely[b]
	- S^{PPP}_{0}	= 50 ¢/DM
Time 1:	- R_{US}	= 15 % for the next 5 years, 10 % thereafter
	- R_{Ger}	= 10 % indefinitely
	- π^{e1}_{US}	= π^{e0}_{Ger} = 5 % indefinitely

- The realized inflation during the first year was 10 % in the US and 5 % in Germany.

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a. R_{US} is as defined in equation 25, it is an annual rate compounded continuously.

b. Annual rate compounded continuously.

Table 3

Correlation Matrix of the Levels of Real Interest Rates and Real Exchange Rates (Period: 1975-01 to 1987-12)

A) DM

	x	δ_{inf}	Δ_{1m}	Δ_{3m}	Δ_{6m}	Δ_{1y}	Δ_{2y}	Δ_{3y}	Δ_{4y}	Δ_{5y}	Δ_{3-7y}
δ_{inf}	-0.69										
δ_{1m}	-0.65	-0.68									
Δ_{3m}	-0.65	-0.71	0.99								
Δ_{6m}	-0.66	-0.74	0.97	0.993							
Δ_{1y}	-0.70	-0.79	0.93	0.97	0.99						
Δ_{2y}	-0.78	-0.83	0.89	0.92	0.94	0.98					
Δ_{3y}	-0.81	-0.83	0.86	0.89	0.92	0.95	0.993				
Δ_{4y}	-0.83	-0.83	0.84	0.87	0.89	0.93	0.98	0.996			
Δ_{5y}	-0.83	-0.84	0.83	0.86	0.89	0.93	0.98	0.993	0.999		
Δ_{3-7y}	-0.84	-0.79	0.81	0.84	0.86	0.90	0.95	0.97	0.98	0.98	
Δ_{7-15y}	-0.85	-0.79	0.78	0.80	0.82	0.87	0.93	0.96	0.97	0.97	0.99

B) SF

	x	δ_{inf}	Δ_{1m}	Δ_{3m}	Δ_{6m}	Δ_{1y}	Δ_{2y}	Δ_{3y}	Δ_{4y}
δ_{inf}	0.68								
Δ_{1m}	-0.43	-0.53							
Δ_{3m}	-0.44	-0.55	0.98						
Δ_{6m}	-0.48	-0.60	0.97	0.992					
Δ_{1y}	-0.55	-0.67	0.94	0.97	0.99				
Δ_{2y}	-0.62	-0.73	0.89	0.92	0.95	0.98			
Δ_{3y}	-0.65	-0.76	0.86	0.90	0.93	0.96	0.994		
Δ_{4y}	-0.66	-0.78	0.85	0.88	0.91	0.95	0.99	0.998	
Δ_{5y}	-0.67	-0.79	0.84	0.87	0.90	0.95	0.98	0.995	0.999

C) CD

	x	δ_{inf}	Δ_{1m}	Δ_{3m}	Δ_{6m}	Δ_{1y}	Δ_{1-3y}	Δ_{3-5y}	Δ_{5-10y}
δ_{inf}	-0.15*								
Δ_{1m}	-0.12*	-0.71							
Δ_{3m}	0.10*	-0.83	0.92						
Δ_{6m}	0.09*	-0.84	0.90	0.98					
Δ_{1y}	0.17**	-0.85	0.87	0.96	0.98				
Δ_{1-3y}	0.18**	-0.91	0.83	0.94	0.96	0.97			
Δ_{3-5y}	0.16**	-0.93	0.80	0.91	0.93	0.94	0.99		
Δ_{5-10y}	0.16**	-0.95	0.79	0.90	0.92	0.93	0.98	0.996	
Δ_{o10y}	0.21	-0.96	0.74	0.87	0.87	0.88	0.94	0.96	0.97

* not significant at α=5%
** not significant at α=1%

Table 4

Inflation Rate Differentials Between the US and Germany, Switzerland and Canada

	U.S.	Germany	Switzerland	Canada
Mean		2.99 %	3.26 %	-0.99 %
Standard Deviation		2.23 %	2.82 %	2.01 %
Minimum		-0.79 %	-1.24 %	-5.18 %
Maximum		8.24 %	10.15 %	4.89 %
CPI[a] 1984-12	247.1	161.1	155.4	281.6

a. CPI 1975-01 = 100.

Table 5

Estimates of the Most Reduced Model of FX Rate Levels for Various Samples

	Interval between the observations	Parameter Estimate for intercept	Δ	n	R^2	1^{st} order autocorrelation
DM:	3 months 1[a]	3.768 (0.015)	-5.73 (0.48)	52	0.74	0.71
	3 months 2	3.767 (0.016)	-6.06 (0.52)	52	0.73	0.70
	3 months 3	3.771 (0.016)	-6.30 (0.53)	52	0.74	0.64
	6 months 1	3.756 (0.024)	-5.56 (0.70)	26	0.72	0.59
	12 months 1	3.763 (0.040)	-5.37 (1.18)	13	0.65	0.32[b]
SF:	3 months 1	4.024 (0.021)	-4.35 (0.63)	52	0.48	0.77
	3 months 2	4.038 (0.023)	-4.84 (0.70)	52	0.49	0.78
	3 months 3	4.044 (0.024)	-5.04 (0.72)	52	0.50	0.76
	6 months 1	4.009 (0.029)	-4.08 (0.83)	26	0.50	0.67
	12 months 1	3.983 (0.044)	-2.53 (1.57)	13	0.19	0.37[b]

a. Indicates that the first month of the overall sample is in the reduced sample.

b. Not significant at $\alpha = 10$ %.

Table 6

Explanatory Power of the Full Model of Exchange Rate Changes for the DM, the SF, and the CD for Various Holding Periods

Holding Period	DM	SF	CD
1 month	0.24	0.17	0.25
3 months	0.39	0.25	0.16
6 months	0.50	0.37	0.19
12 months	0.60	0.43	0.18

Table 7
Correlation Matrix of Changes in Interest Rate Differentials and Exchange Rates

A) DM

	Δs	δ_{inf}	δ^{o}_{7-15y}	δ^{1}_{1m}	$d\delta_{1m}$	$d\delta_{3m}$	$d\delta_{6m}$	$d\delta_{1y}$	$d\delta_{2y}$	$d\delta_{3y}$	$d\delta_{4y}$	$d\delta_{5y}$	$d\delta_{3-7y}$
δ_{inf}	−0.07												
δ^{o}_{7-15y}	−0.11	−0.09											
δ^{1}_{1m}	−0.25	0.30	0.44										
$d\delta_{1m}$	−0.16	−0.03	−0.02	0.30									
$d\delta_{3m}$	−0.17	−0.02	−0.02	0.28	0.91								
$d\delta_{6m}$	−0.14	−0.01	−0.03	0.26	0.86	0.97							
$d\delta_{1y}$	−0.07	0.00	−0.05	0.22	0.77	0.90	0.94						
$d\delta_{2y}$	−0.02	−0.08	−0.08	0.10	0.54	0.65	0.71	0.74					
$d\delta_{3y}$	−0.05	−0.05	−0.08	0.07	0.48	0.59	0.65	0.69	0.93				
$d\delta_{4y}$	−0.01	−0.07	−0.10	0.04	0.43	0.54	0.59	0.64	0.91	0.94			
$d\delta_{5y}$	0.36	−0.29	−0.04	−0.34	−0.16	−0.14	−0.15	−0.10	0.18	0.20	0.24		
$d\delta_{3-7y}$	0.03	−0.00	−0.14	0.08	0.17	0.29	0.30	0.33	0.34	0.31	0.32	0.06	
$d\delta_{7-15y}$	0.14	0.00	−0.21	0.03	0.10	0.17	0.19	0.24	0.37	0.34	0.41	0.09	0.72

B) SF

	Δs	δ_{inf}	δ^{o}_{5y}	δ^{1}_{1m}	$d\delta_{1m}$	$d\delta_{3m}$	$d\delta_{6m}$	$d\delta_{1y}$	$d\delta_{2y}$	$d\delta_{3y}$	$d\delta_{4y}$
δ_{inf}	−0.08										
δ^{o}_{5y}	−0.18	−0.05									
δ_{1m}	−0.25	0.23	0.69								
$d\delta_{1m}$	−0.10	0.05	−0.13	0.20							
$d\delta_{3m}$	−0.09	0.03	−0.16	0.17	0.85						
$d\delta_{6m}$	−0.06	0.06	−0.17	0.15	0.80	0.96					
$d\delta_{1y}$	−0.01	0.06	−0.17	0.14	0.69	0.87	0.94				
$d\delta_{2y}$	−0.17	0.04	−0.17	0.14	0.46	0.65	0.72	0.76			
$d\delta_{3y}$	−0.22	0.07	−0.17	0.14	0.45	0.63	0.70	0.72	0.94		
$d\delta_{4y}$	−0.21	0.07	−0.17	0.14	0.45	0.60	0.69	0.71	0.93	0.96	
$d\delta_{5y}$	−0.19	0.04	−0.18	0.14	0.44	0.58	0.66	0.68	0.90	0.92	0.96

C) CD

	Δs	δ_{inf}	δ^{o}_{010y}	δ^{1}_{1m}	$d\delta_{1m}$	$d\delta_{3m}$	$d\delta_{6m}$	$d\delta_{1y}$	$d\delta_{1-3y}$	$d\delta_{3-5y}$	$d\delta_{5-10y}$
δ_{inf}	−0.04										
δ^{o}_{010y}	−0.22	0.19									
δ_{1m}	−0.23	0.16	0.37								
$d\delta_{1m}$	−0.14	−0.11	−0.01	0.31							
$d\delta_{3m}$	0.15	−0.13	−0.06	0.08	0.68						
$d\delta_{6m}$	0.27	−0.03	−0.14	0.05	0.56	0.87					
$d\delta_{1y}$	0.25	−0.01	−0.15	0.08	0.44	0.69	0.83				
$d\delta_{1-3y}$	0.18	−0.05	−0.16	0.15	0.43	0.62	0.73	0.71			
$d\delta_{3-5y}$	0.21	−0.03	−0.19	0.12	0.23	0.42	0.57	0.57	0.90		
$d\delta_{5-10y}$	0.22	−0.03	−0.20	0.08	0.16	0.37	0.49	0.48	0.82	0.93	
$d\delta_{010y}$	0.27	−0.05	−0.24	0.00	−0.01	0.12	0.30	0.29	0.59	0.73	0.84

Table 8
Estimations of the Reduced Models
for Exchange Rate Changes

<u>DM</u>

1 month:

$$\Delta s = 0.002 - \underset{(2.22)}{5.94}\, d\delta_{3m} + \underset{(0.93)}{2.63}\, d\delta_{1y} - \underset{(0.22)}{0.46}\, d\delta_{3y} + \underset{(0.16)}{0.78}\, d\delta_{5y}$$

$R^2 = 0.19$[a]

3 months:

$$\Delta s = -3.32 + \underset{(0.64)}{3.34}\, \delta_{inf} - \underset{(1.07)}{3.70}\, \delta^1_{3m} + \underset{(0.83)}{2.99}\, d\delta_{2y} - \underset{(0.62)}{2.17}\, d\delta_{3y} + \underset{(0.16)}{0.97}\, d\delta_{5y}$$

$R^2 = 0.36$

6 months:

$$\Delta s = -5.01 + \underset{(0.55)}{5.01}\, \delta_{inf} - \underset{(0.83)}{3.86}\, \delta^1_{6m} + \underset{(1.34)}{3.43}\, d\delta_{6m} + \underset{(1.26)}{3.00}\, d\delta_{2y} - \underset{(0.83)}{2.49}\, d\delta_{3y} + \underset{(0.17)}{1.35}\, d\delta_{5y} - \underset{(0.09)}{0.28}\, d\delta_{7-15y}$$

$R^2 = 0.50$

12 months:

$$\Delta s = -5.83 + \underset{(0.48)}{5.79}\, \delta_{inf} - \underset{(0.67)}{2.79}\, \delta^1_{1y} + \underset{(1.02)}{4.68}\, d\delta_{1y} - \underset{(1.29)}{3.49}\, d\delta_{3y} + \underset{(0.93)}{2.27}\, d\delta_{4y} + \underset{(0.19)}{1.89}\, d\delta_{5y} - \underset{(0.11)}{0.76}\, d\delta_{7-15y}$$

$R^2 = 0.60$

---

a. All regressions are estimated over 156 observations.

<u>SF</u>

1 month:
$$\Delta s = 0.023 - \underset{(1.21)}{3.82}\,\delta^1_{1m} + \underset{(0.57)}{1.65}\,d\delta_{1y} - \underset{(0.22)}{0.84}\,d\delta_{3y} \qquad R^2 = 0.14$$

3 months:
$$\begin{aligned}\Delta s = -1.50 &+ \underset{(0.60)}{1.56}\,\delta_{inf} - \underset{(0.82)}{4.23}\,\delta^1_{3m} + \underset{(0.66)}{2.05}\,d\delta_{1y}\\ &- \underset{(0.23)}{0.72}\,d\delta_{4y}\end{aligned} \qquad R^2 = 0.23$$

6 months:
$$\Delta s = 0.12 - \underset{(0.07)}{0.38}\,\delta^0_{5y} + \underset{(0.90)}{4.97}\,d\delta_{1y} - \underset{(0.37)}{2.27}\,d\delta_{3y} \qquad R^2 = 0.35$$

12 months:
$$\begin{aligned}\Delta s = -1.57 &+ \underset{(0.38)}{1.74}\,\delta_{inf} - \underset{(0.51)}{3.26}\,\delta^1_{1y} + \underset{(1.08)}{8.79}\,d\delta_{1y}\\ &+ \underset{(0.46)}{2.86}\,d\delta_{3y}\end{aligned} \qquad R^2 = 0.43$$

<u>CD</u>

1 month:
$$\Delta s = -0.005 - \underset{(0.012)}{0.026}\,\delta^0_{10y} - \underset{(1.24)}{5.77}\,d\delta_{1m} + \underset{(0.39)}{2.10}\,d\delta_{6m} \qquad R^2 = 0.22$$

3 months:
$$\Delta s = -0.017 - \underset{(0.019)}{0.076}\,\delta^0_{10y} \qquad R^2 = 0.09$$

6 months:
$$\Delta s = -0.69 + \underset{(0.20)}{0.66}\,\delta_{inf} - \underset{(0.03)}{0.12}\,\delta^0_{10y} - \underset{(0.06)}{0.13}\,d\delta_{m-10y} \qquad R^2 = 0.16$$

12 months:
$$\Delta s = -0.67 + \underset{(0.19)}{0.62}\,\delta_{inf} - \underset{(0.04)}{0.15}\,\delta^0_{10y} - \underset{(0.31)}{1.27}\,\delta^1_{1y} \qquad R^2 = 0.14$$

Table 9

Out-of-Sample Results (1980-01 to 1987-12)

DM

	ME	MAE	MSE	T_{MR}[a]	Dir %	t_{Dir}[b]	X %	t_X[b]
1 month[c]								
Model	-0.024	1.30	2.77		66.7	3.37	62.5	2.55
RW	0.060	1.26	2.74	-0.10				
Forward	-0.089	1.29	2.79	0.05	46.9	-0.50		
3 months								
Model	-0.002	1.97	6.52		69.8		72.9	
RW	0.118	2.44	9.13	2.01				
Forward	-0.331	2.55	9.63	2.21	47.9			
6 months								
Model	-0.163	3.13	14.89		68.8		72.9	
RW	0.082	3.69	20.25	1.20				
Forward	-0.823	3.95	22.66	1.42	47.9			
12 months								
Model	-0.175	5.15	38.80		71.9		71.9	
RW	0.181	6.11	54.94	0.93				
Forward	-1.635	6.89	62.38	1.07	39.6			

a. This is the statistic suggested by Meese and Rogoff (1988) to compare the predictions of different models. The number opposite RW compares our model to the RW, and the model opposite Forward, compares our model to the predictions of the forward rate. For a large n (n=96 here), B is normally distributed. The critical values are 1.645 (α=10 %) and 1.96 (α=5 %).

b. t_{Dir} is computed as: $[((Dir/100)(96) + 0.5)-48]/(0.5\ (0.5)96)^{0.5}$.
t_{Dir} tests the hypothesis that Dir = 50 %.
t_X is computed the same way as t_{Dir}, with X replacing Dir.
t_{Dir} and t_X can only be computed for one month holding periods, because the observations of the overlapping periods are not independent.

c. Refers to a 1 month holding period.

SF

	ME	MAE	MSE	T_{MR}	Dir %	t_{Dir}	X %	t_x
1 month								
Model	-0.213	1.71	4.97		54.2	0.93	59.4	1.94
RW	0.166	1.60	4.34	-1.50				
Forward	-0.088	1.68	4.49	-1.04	42.7	-1.33		
3 months								
Model	-0.326	3.10	15.33		60.4		71.9	
RW	0.390	3.05	15.26	-0.17				
Forward	-0.389	3.34	16.67	0.47	45.8			
6 months								
Model	-0.607	4.23	28.23		70.8		74.0	
RW	0.517	4.69	31.46	0.47				
Forward	-1.011	5.20	36.78	1.02	47.9			
12 months								
Model	1.467	6.61	63.21		67.7		74.0	
RW	0.952	6.72	71.35	0.10				
Forward	-0.283	7.13	74.35	0.02	45.8			

CD

	ME	MAE	MSE	T_{MR}	Dir %	t_{Dir}	X %	t_x
1 month								
Model	-0.064	0.69	0.84		60.4	2.14	56.3	1.34
RW	-0.090	0.74	1.07	-2.00				
Forward	0.016	0.76	1.09	-2.18	51.0	0.30		
3 months								
Model	-0.087	1.24	2.60		53.1		56.3	
RW	-0.272	1.23	2.56	0.26				
Forward	-0.166	1.24	2.58	0.08	57.3			
6 months								
Model	0.010	1.78	4.80		54.2		54.2	
RW	-0.584	1.63	4.46	0.71				
Forward	-0.481	1.61	4.37	0.67	61.5			
12 months								
Model	2.123	3.50	17.96		70.8		69.8	
RW	-1.226	2.57	9.60	2.44				
Forward	-1.125	2.56	9.51	2.45	68.8			

Table 10

MSE (model) - MSE (RW) for Various Holding Periods

	Holding period			
	1 month	3 months	6 months	12 months
DM				
1980	1.61	-0.77	17.22	45.19
1981	0.71	-8.84	-38.95	-85.46
1982	-0.27	1.93	12.32	34.04
1983	-0.36	-1.49	-4.07	-3.99
1984	-0.05	-1.28	-1.64	-4.68
1985	-0.06	-1.18	-6.70	-8.56
1986	-0.79	-4.02	-12.12	-66.86
1987	-0.57	-5.19	-8.98	-39.83
SF				
1980	8.49	23.69	46.47	38.43
1981	-1.40	-5.84	7.86	43.17
1982	0.83	-1.37	-13.83	13.94
1983	0.38	3.77	5.79	4.71
1984	0.46	0.24	-1.36	4.47
1985	-0.54	-4.37	-17.63	-13.91
1986	-1.92	-8.59	-31.74	-64.05
1987	-1.28	-6.97	-21.39	-91.88
CD				
1980	-0.20	0.98	0.56	32.71
1981	-0.20	0.14	4.03	-2.53
1982	-0.28	0.50	-1.13	15.01
1983	0.07	0.21	2.13	33.54
1984	-0.19	-1.31	-4.53	-11.72
1985	-0.68	-0.63	-2.33	-11.70
1986	0.08	-0.22	0.32	1.37
1987	-0.47	0.61	3.58	10.22

Table 11

t Statistic For Tests of Whether the Model Outperforms a RW or the Forward Rate for Predictions Excluding 1980

	Holding period			
	1 month	3 months	6 months	12 months
DM				
RW	0.95	2.21	1.64	1.22
Forward	1.27	2.35	1.77	1.26
SF				
RW	1.33	1.38	1.57	0.33
Forward	1.68	1.77	1.75	0.39
CD				
RW	1.95	0.07	-0.67	-1.67
Forward	2.15	0.19	-0.62	-1.66

Chapter 5

CONCLUDING REMARKS

In this chapter, the findings of each of the three essays are briefly reviewed, and some avenues for future research are suggested. In essay I, a model was proposed to account for the effects of transaction costs on option pricing based on the valuation framework provided by Garman and Ohlson (1981) for risky assets in arbitrage free economies with transaction costs. The direct effects of transaction costs were incorporated through the costs of hedging and rehedging, and the indirect effects were incorporated through measures of the own risk of options and option portfolios, and through the price of the option which acts as a proxy for the differential between borrowing and lending interest rates.

The model was estimated on data pooled monthly for options on the futures of five currencies, and pooled daily for two currencies. The monthly estimations indicated that Υ and the time decay of options (θ) are the most important transaction-costs-related variables affecting option pricing. Because of the very high correlations between Υ and θ, it was impossible to ascertain whether θ is important by itself. Estimation on a daily basis for the DM and the SF confirmed that Υ is the most significant variable, followed by λ and ϕ which measure the sensitivity of an option's price to the volatility of its underlying security and to interest rates, respectively. Thus, the effect of transactions costs may be mostly indirect through it impact on own risk. The only biases which could not be explained by the linear model were those for short maturity options. In contrast, a square-root model specification exhibited less biases, and a log-linear model specification exhibited no more bias than would be expected by chance. Moreover, the reversal of the option pricing biases (relative to theoretical no-transactions-costs prices) documented by Rubinstein (1985) for stock options also appeared for foreign exchange futures options. These bias reversals, which could be explained by the model, were shown to be related to macro-economic factors, and particularly to the volatility of the foreign exchange markets.

An interesting extension of this study would be the application of the same approach to stock options. Appropriate modifications could be made to account for the different diffusion processes followed by stocks as opposed to currencies.

In essay II, simplified derivations of pricing models for simple and complex options were presented. These derivations are based on the insight provided by Cox, Ross and Rubinstein (1979) that the value of an option can be interpreted as the expectation of its discounted future value in a risk-neutral world. They do not require the solution of differential equations, nor the use of stochastic calculus.

The correlation structure of the exchange rates of nine major currencies was studied. Several interesting results were found. First, there is no statistical difference in the covariance structure of returns from holding these currencies for one, three or six months. The results for a one day holding period are more ambiguous. These results support the use of a Geometric Brownian motion to describe exchange rate movements. All currencies move together vis-à-vis the US dollar only in the short run. This may be due to the behaviour of FX traders who quote all currencies versus the US dollar and follow (or cause) its short run fluctuations versus the rest of the world. It may also be due to the behaviour of central banks, especially in the European Monetary System, which tend to keep their currencies within a certain range with respect to each other and then make large discrete parity changes. While these discrete changes do not affect the correlation of returns, they may affect the correlation of FX levels. The practical consequence of this for business is that from a US or British point of view, currency diversification (or the use of currency baskets) may reduce FX risk significantly only in the long run. A similar observation can be made from the British point of view. From a German or Canadian point of view, currency diversification may reduce FX risk even in the short run. The correlation structure of FX rates changed over time, while the structure of FX returns was more stable and reflected the increasing integration of the European economies.

Finally, the costs of using complex options was assessed. For US corporations, it may be relatively inexpensive to hedge some of the FX risk which used to remain unhedged when the currency of an eventual payment or receipt is not known with certainty. This is the case even when the levels of the foreign currencies may be diverging over time because of the high correlations found among foreign currency returns. This is especially true when the amounts to be hedged in the foreign currencies are not equal. The fact that options on the maximum of two currencies are quite inexpensive may also be very significant for financial institutions which could use them as marketing tools for the sale of their other more basic financial instruments.

An interesting study related to this essay would be a survey of some of the complex options available on the market, whether imbeded in other financial instruments or not. The prices of these options could then be compared to theoretical prices. Since the market in complex options does not seem to be very well developed, large pricing "errors" may be uncovered.

In essay III, an exchange rate determination model was developed and tested. The model is based on the assumption that purchasing power parity is expected to hold in the long run, while short term deviations from this parity are due to risky financial speculation caused by interest and inflation rate expectations. The short term deviations are constrained by the possibility of (costly) goods arbitrage. This model uses the entire term structure of interest rates and of inflation rate expectations. The risk premia are developed in the context of an economy where transaction costs exist in the financial markets. These risk premia are related to the volume of capital involved in currency and interest rate speculation.

The model was tested in the form specifying the levels of exchange rates, and in the form specifying changes of exchange rates for the exchange rates of the German mark, the Swiss franc, and the Canadian dollar vis-à-vis the US dollar for the period 1975-01 to 1987-12. In its levels form, the model was quite successful. It indicated that long term real interest rates satisfactorily explain the levels of the DM and the SF, while several interest rate terms are needed to explain the level of the CD. A possible reason for the CD result is that relatively more capital flows between the US and Canada for exchange (and interest) rate (risky) arbitrage (or speculation) than between the US and Germany (or Switzerland).

In its FX rate changes form, the model explains reasonably well the changes of the DM and the SF. The results usually improve as the holding period lengthens. With regard to the out-of-sample fit of the model over the period 1981-01 to 1987-12, the model outperformed a random walk and the forward rate for all holding periods. These results were statistically significant at the 10 % level in 2 out of 8 cases for the random walk and in 5 out of 8 cases for the forward rate. For the CD, the FX rate changes form of the model was only successful for the one month holding period. The failure of the model for longer holding periods (especially the 12 months periods) may be due to the political shocks which, because of the methodology used, each affected multiple observations.

The analysis in this essay could be extended in two different directions. First, this essay was concerned with the determination of FX rates and their changes. A future study could attempt to measure the volume of capital involved in FX speculation (perhaps from the volume of daily transactions), and to relate it to the risk premia. More research is required to assess the various parameters underlying the model presented herein. Second, the research on FX rates could be extended to the pricing of other assets, such as futures contracts and commodities. The arguments implicit in this model are quite general and would apply to any asset. For example, if transaction costs are nil, this model would simply converge to the Sharpe-Lintner Capital Asset Pricing Model. However, the specific derivation of this model would make it particularly useful for assets which, because of transactions costs (in the Mayshar sense), are not held in well diversified portfolios and attract amounts of capital which vary significantly over time.

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