FREIGHT TRAIN OPTIMAL TRAJECTORY CALCULATION BY LINEAR PROGRAMMING

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NOMENCLATURE

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
A	$(3n+1+l+\overline{\tau}_n+\overline{\omega}_n) \times (3n+1+l+\overline{\tau}_n+\overline{\omega}_n)$ constant system matrix of the augmented discrete system: $x(k+1) = Ax(k)+Bu(k)$	-	63
$a_i(t)$	air flow rate in the brake cylinder pipe of the i^{th} train member	in^3/sec.	26
$a_{n+1}(t)$	air flow rate in the pipe between the train brake pipe and auxiliary reservoir of train member J	in^3/sec.	29
B	$(3n+1+l+\overline{\tau}_n+\overline{\omega}_n) \times (l+2)$ constant control matrix of the augmented discrete system: $x(k+1) = Ax(k) + Bu(k)$	-	63
b(k)	state variable constraint parameter	-	77

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$b_i(t)$	air braking effort developed at the i^{th} train member; $b_i(t) = e_i p_i(t)$	lbs.	16
C_i	fluid capacitance of brake cylinder of the i^{th} train member	$in^5/lb.$	26
C_{n+1}	fluid capacitance of auxiliary reservoir of train member J	$in^5/lb.$	29
c	$\{(l+2)K+1\}$ - dimensional cost vector: $c = \{0\ 0\ 0\ \ldots\ 0\ 1\}^T$	-	74
D(k)	$(3n+1+l+\bar{\omega}_n+\bar{\tau}_n)$ X $(l+2)K$ matrix defined by: $D(k) = \{A^{k-1}B\ A^{k-2}B\ \ldots\ AB\ B\ 0\ \ldots 0\}$ where 0 is a $(3n+1+l+\bar{\tau}_n+\bar{\omega}_n)$ - dimensional null column vector	-	76
d	direction vector normal to the separating hyperplane defined as follows in the euclidian space E^n: $\langle d,x \rangle = \langle d,d \rangle$, where $\langle d,x \rangle$ denotes the inner product of the n-dimensional vectors d and x	-	72

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$d\{s_i(t)\}$	the effective grade at the position of the i^{th} member	%	39
d_2	direction vector used to define velocity constraints	-	88
d_{3i+1}	direction vector used to define objective constraints	-	87
d_L	direction vector used to define tractive effort constraints	-	88
d_M	direction vector used to define dynamic braking constraints	-	88
$d^*(t)$	disturbance input to the system (ie. the time dependent function giving the effective grade at the head end of the train = actual grade + equivalent grade due to curvature)	%	40
$d^*(k)$	discrete time equivalent of $d^*(t)$	%	59
E^n	euclidian space of dimension n	-	72
E^+	the half-space region in E^n defined by: $E^+ = \{x: \langle d,x\rangle > \langle d,d\rangle\}$	-	72

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
E^-	the half-space region in E^n defined by: $E^- = \{x: <d,x> < <d,d>\}$	-	72
e_i	constant of proportionality between brake cylinder pressure and braking effort at the i^{th} train member	in^2	27
F	$\bar{m} \times (3n+1+1+\bar{\tau}_n+\bar{\omega}_n)$ terminal constraint matrix of the augmented discrete system: $x(k+1) = Ax(k) + Bu(k)$; $Fx(K) = f$	-	71
f	$\bar{m}$ - dimensional terminal constraint vector of the augmented discrete system: $x(k+1) = Ax(k) + Bu(k)$; $Fx(k) = f$	-	71
$f_i^*(t)$	tractive/dynamic braking effort developed by the i^{th} train member (identically zero for all t if train member i is not a powered locomotive)	lbs.	17

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$\mathcal{F}_i(t)$	force of combined draft gears between train member i and (i+1)	lbs.	16
g	acceleration due to gravity	ft/sec^2	39
H(k)	control variable constraint parameter	-	78
h_i	length of the i^{th} train member	ft.	19
I	the member number of the locomotive nearest train member i which is capable of effecting a reduction in brake pipe pressure	-	24
i	index indicating member number of train; i=1 indicates the leading member in the direction of train motion	-	16
J	the member number of the car furthest from any locomotive capable of effecting an increase in brake pipe pressure	-	29

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
j	index indicating locomotive number (note that it is not necessarily true that $i = j$)	-	50
K	total number of discrete time intervals, each of duration T, in the interval (t_0, t_f)	-	57
KIPS	kilopounds	lbs	91
k	discrete time interval number	-	57
L''	tractive effort linear constraint parameter	lb-secs/ft	66
L''	tractive effort linear constraint parameter	lbs.	66
l	total number of locomotives in the train ($\leq$ no. of cars $= n-1$)	-	50
M'	dynamic braking linear constraint parameter	lbs-secs/ft	66
$\overline{m}$	dimension of terminal constraint vector f		71

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
M''	dynamic braking linear constraint parameter	lbs.	66
m_i	the effective mass of the i^{th} train member (ie. the actual mass of the member plus some allowance for the rotational inertia of the wheelsets)	slugs	16
n	total number of members in the train, including locomotives	-	29
$\underline{0}$	(3n+1) - dimensional null column vector	-	64
0	$(3n+1+1+\overline{\tau}_n+\overline{\omega}_n)$ - dimensional null column vector	-	76
0_{31}	3-dimensional null column vector	-	50
0_{13}	3-dimensional null row vector	-	50
0_{33}	3X3 null matrix	-	50
P_i	maximum horsepower rating of locomotive member i	hp	33

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$p_i(t)$	brake cylinder pressure at the i^{th} train member	lbs/in^2	26
$p_{n+1}(t)$	reduction in auxiliary reservoir pressure of train member 1 (when $p_{n+1}(t) = 0$, all auxiliary reservoirs are completely recharged)	lbs/in^2	29
$q(t)$	(3n+1) - dimensional state vector of the continuous system: $\dot{q} = \Delta q + \Omega r$	-	46
$q(k)$	(3n+1)-dimensional state vector of the discrete system: $q(k+1) = \Delta^* q(k) + \Omega^* r(k)$	-	58
$q'(k)$	$(1+\bar{\tau}_n+\bar{\omega}_n)$- dimensional vector appended to q(k) to represent the time delays in the system	-	62
$r(t)$	(2n+1) - dimensional control vector of the continuous system: $\dot{q} = \Delta q + \Omega r$	-	47
$r(k)$	(2n+1) - dimensional control vector of the discrete system: $q(k+1) = \Delta^* q(k) + \Omega^* r(k)$	-	58

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$r^*(t)$	reduction in equalizing reservoir pressure effected by the engineman, resulting from varying automatic brake valve handle positions	lb/in^2	22
$r_i^*(t)$	reduction in brake pipe pressure at the i^{th} train member	lbs/in^2	24
R_C	curve resistance	lbs.	41
R_{F+A}	friction and air resistance	lbs.	43
R_G	grade resistance	lbs.	39
R_i	fluid resistance of pipe connecting auxiliary reservoir and brake cylinder of the i^{th} train member	$lbs\text{-}secs/in^5$	26
R_{n+1}	fluid resistance of pipe connecting auxiliary reservoir and train brake pipe at train member J	$lbs\text{-}secs/in^5$	29
S_0	reference position of train	ft.	16

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$s_i(t)$	position of the i^{th} train member relative to S_0	ft.	16
T	discrete time interval duration	secs.	57
t	the independent variable time (t_0, t_f denote initial and final times, respectively)	secs.	16
t'	dummy variable	-	57
U	{(l+2)K + 1} - dimensional constant offset vector	-	86
u	{(l+2)K + 1} -dimensional LP vector defined by: $u = \{u(0)\ u(1), \ldots, u(K-1)\ \pi\}^T$	-	74
u'	u' = u + U	-	86
u(k)	(l+2) - dimensional control vector of the augmented discrete system: $x(k+1) = Ax(k) + Bu(k)$	-	60
$\bar{u}$	number of scalar control constraints	-	80

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
V'	minimum permissible train speed	ft/sec.	67
V''	maximum permissible train speed	ft/sec.	67
$v_i(t)$	velocity of the i^{th} train member	ft/sec.	16
W_i	total weight of the i^{th} member	lbs.	33
w_i	zero-force length of the draft gears and couplers between train member i and (i+1)	ft.	19
$x(k)$	$(3n+1+1+\bar{\tau}_n+\bar{\omega}_n)$ - dimensional state vector of the augmented discrete system: $x(k+1) = Ax(k) + Bu(k)$	-	60
x_o	initial condition of x ; $x_o = x(0)$	-	71
$\bar{x}$	number of state trajectory constraints or minimax objectives	-	80

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$y_i(t)$	total resistance to motion of the i^{th} train member due to grades, curves, wind and friction	lbs.	17
Z	upper limit to dynamic braking level which applies to locomotives with extended range over the speed range 6-25 mph	lbs.	35
$z_i(t)$	extension (+) or compression(-) of draft gears between train members i and (i+1)	ft.	19
α_i	spring constant of the combined draft gears between train members i and (i+1)	lbs/ft	19
β_i	damping coefficient of the combined draft gears between train members i and (i+1)	lbs-secs/ft.	19
γ	response time of the automatic brake valve and equalizing reservoir	secs.	23

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
Δ	(3n+1) X (3n+1) constant system matrix of the continuous system: $\dot{q} = \Delta q + \Omega r$	-	48
Δ^*	(3n+1) X (3n+1) constant system matrix of the discrete system: $q(k+1) = \Delta^* q(k) + \Omega^* r(k)$	-	58
δ	transmission speed of the brake pipe reduction signal in the train line	ft/sec.	23
ξ	low speed dynamic braking coefficient (ie., constant of porportionality between velocity of locomotive and dynamic braking limit at low speeds)	lbs-secs/ft	35
η	constant of proportionality between the time delayed equalizing reservoir reduction appearing at the $i\underline{th}$ train member and the supply pressure of that member's auxiliary reservoir.	-	24

SYMBOL	DEFINITION	UNITS	FIRST APREARS ON PAGE:
θ	vertical angle of the track gradient	deg.	39
λ_i	velocity coefficient in linearized version of the Davis formula for resistance, due to air and friction of member i	secs/ft.	43
λ_{0i}	zero-velocity constant in linearized version of the Davis formula for resistance due to air and friction of member i ($\lambda_0 = \lambda_{0i}$ is assumed constant for all members of the train).	-	43
μ	coefficient of friction between wheel and rail (assumed constant for each train member)	-	33
π	dummy scalar variable which is to be minimized in the LP problem corresponding to the original minimax problem	-	74

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$\rho_C\{s_i(t)\}$	track curvature at position $s_i(t)$.	deg.	39
$\rho_G\{s_i(t)\}$	track gradient at position $s_i(t)$	%	38
σ_i	time constant of brake cylinder and connecting pipe system of the i^{th} train member ($\sigma_i = R_i C_i$)	secs.	27
σ_{n+1}	time constant of auxiliary reservoir and connecting pipe system of train member J ($\sigma_{n+1} = R_{n+1}C_{n+1}$)	secs.	30
τ_i	overall time delay which occurs between the reduction made by the engineman and the initiation of braking action at member i ($\tau_i = \gamma + \frac{1}{\delta}\left\| s_1 - s_i \right\|$)	secs.	23
$\bar{\tau}_i$	number of time intervals, each of duration T, represented by the time delay τ_i (ie. $\bar{\tau}_i = \tau_i/T$)	-	59

SYMBOL	DEFINITION	UNITS	FIRST APPEARS ON PAGE:
$\Phi(t)$	state transition matrix of the system (for system matrix A: $\Phi(t) = e^{At}$)	-	57
Ω	(3n+1) X (2n+1) constant control matrix of the continuous system: $\dot{q} = \Delta q + \Omega r$	-	50
Ω^*	(3n+1) X (2n+1) constant control matrix of the discrete system: $q(k+1) = \Delta^* q(k) + \Omega^* r(k)$	-	58
ω_i	the time delay between the appearance of the effective grade $d^*(t)$ at $s_0(t)$ and $s_i(t)$	secs.	40
$\overline{\omega}_i$	the number of time intervals, each of duration T, represented by the time delay ω_i (ie., $\overline{\omega}_i = \omega_i/T$	-	59

ABSTRACT

Freight train separations and derailments attributable to adverse longitudinal dynamics of the train moving over undulating track have become more frequent with the advent of longer and heavier trains in recent years. This thesis addresses the problem of attempting to compute the optimal control trajectory for a specific train over a specific track section, wherein the optimality criterion used is the minimization of the maximum coupler force appearing in the train over the duration of its operation.

A methematical model of the train (including a representation of its braking systems) is first formulated, based on a third-order model for each train member. The discrete equivalent of this continuous model, incorporating the time delays inherent in the system, is then derived. The optimization problem is then reformulated as a linear programming problem and solved on a digital computer.

The complementary relationship of this work to the research in this area currently underway at the Canadian Institute of Guided Ground Transport at Queen's University, Kingston, Ontario, is also delineated.

The procedure developed in this thesis appears to meet the stated objective of designing a method of pre-computing the "best" way to operate a specific train over a particular territory.

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CHAPTER 1

INTRODUCTION

1.1 Background:

Modern freight trains, particularly those in bulk commodity service, are characterized by very high tonnage, great length and increasing speed of operation. This trend has been due to the necessity of increasing both the capacity of single-track main line operations and the utilization of the motive power fleet. Failure to realize these operational objectives would result in the requirement for capital expenditures of such magnitude that rail transport costs for these commodities would soon become prohibitive.

However, the move towards operation of heavier and longer freight trains has unfortunately been marked by a significant increase in the frequency of train separations and derailments. The experience of the Southern Railway (43) is typical evidence of the correlation between train uncoupling rate and train length. One can only conclude that there must be even more frequent occurrences of excessive inter-car forces which, although not resulting in such catastrophic events as train derailments or separations, certainly contribute to the costly incidence of car and lading damage. Estimates of this cost run as high as a quarter of a billion dollars. (60)

Although railbed, track and equipment failures are a major cause of train derailments, a very significant portion of such incidents can be attributed mainly to the adverse dynamic effects of a train in motion, variously described as "train action", "slack action", "run-ins" and "run-outs." The suitability of such descriptive terms becomes immediately apparent when one considers the characteristics of a moving n-car train. Rather than being rigidly connected together, each car in the train is coupled with its neighbour in such a manner as to allow relative displacements between successive cars of approximately 0.5 feet. Hence, as a result of various unbalanced external forces, the individual cars comprising the train are likely to undergo relative velocities and accelerations. Indeed, a 200 car train can vary in length by anywhere up to 100 feet during motion.

Inter-car forces well in excess of 500,000 pounds resulting from such slack action in trains have been reported.[(69)] A serious problem currently confronting railways is to devise a method of operating trains such that these excessive slack action forces are minimized.

1.2 <u>Prior Work</u>:

A specific train moving over a particular section of track may be considered as a system with a specified set of <u>parameter values</u> being subjected to a specified set of <u>input</u> or <u>control functions</u>. In the context of this system viewpoint, the problem defined above may be considered to be an optimization problem, wherein one is concerned with the selection of parameter values and/or control functions which will lead to optimum performance, as defined by an appropriate criterion.

Wilson [50] identifies the most important elements of this system, which may be divided into two groups, as follows:

<u>System Parameters</u>

- i) track condition (ie., effective coefficient of friction between wheel and rail)
- ii) freight car characteristics
- iii) locomotive characteristics
- iv) braking system characteristics
- v) inter-car coupling and draft gear characteristics
- vi) train marshalling (ie., sequential ordering of locomotives and cars)

<u>System Control Functions</u>

- i) track profile (ie., gradient and curvature as a function of time)
- ii) locomotive throttle action, as a function of time
- iii) train braking action, as a function of time

Practically speaking, the parameters (i) to (iv) and control function (i) are fixed; that is, the system designer is usually not free to select these parameter values and control function. Thus, the only system elements available for design in the optimization process are the system parameters (v) and (vi) and the system control functions (ii) and (iii).

Therefore, all prior work on the above-defined problem, whether empirical or analytical in nature, may be classified as effectively either parameter optimization or optimal control approaches.

1.2.1. Parameter Optimization:

It is intuitively evident that the optimum selection of control functions from some class of functions is a more difficult analytical or empirical problem than the optimum selection of parameter values from some admissible set. Perhaps for this reason, most prior work on the slack action problem has in fact been in the area of parameter optimization.

Parker (29) for example, has given an empirical method for estimating the optimum marshalling of locomotives in the train.

Train simulations have been used extensively in attempts to arrive at optimal draft gear characteristics and train marshalling policies. The work of Wilson (50) and Roggeveen (34 - 38) is representative of the efforts that have been made in this direction.

The effects of draft gear characteristics on both freight car lading and longitudinal train motions has also been the subject of much analytical work. For example, Pipes [32] has treated the analysis of longitudinal motions of trains by the electrical analog. Frudenstein [14] has completed a dynamic analysis which relates the principle design parameters of the draft gears to the forces experienced by a resilient lading.

No attempt will be made here to compile an exhaustive history of work in the area of parameter optimization. The American Association of Railroads (AAR) has undertaken a comprehensive ten year program of research on track/train dynamics. The first phase of this program will include the preparation of an extensive bibliography of work in this area and this reference is expected to be available very shortly.

1.2.2 Optimal Control:

Despite much work in the area of parameter optimization, it is generally concluded that train separations caused by excessive coupler forces resulting from slack actions are still very much a problem.

For example, the Southern Railway has decided to abandon using all motive power on the front of long unit trains; instead, they are distributing locomotives through the train by adopting remote-control techniques. Even so, the use of mid-train locomotive units has not in itself eliminated slack action problems. (67) Parker (29) gives an excellent case study of train operation with remote-controlled locomotives and he too concludes that skillful handling of the train control functions is the most important factor in avoiding slack action problems.

In a 1968 panel discussion (67), S.H. Fillion, Chief Engineer for Waugh Equipment, a major producer of draft gears, expressed doubts that upgrading of the knuckle, coupler, yoke and draft gear can be the whole solution. Slack, he said, must be controlled because "if a run-in is going to buckle an essentially unstable train, then stronger inter-car hardware is not going to solve, and may actually complicate, the problem by causing structural failure or derailment."

Again, Wilson (50) recognizes the need to establish "optimal operating programs for a given train on a given route," and that this implies development of optimal brake and locomotive throttle functions such that they be "coordinated to yield a combined energy input and dissipation system which provides the best attainable performance."

Many train handling rules have been empirically evolved as a result of extensive operating experience. Reference (53) contains an excellent example of the practical train braking and throttle control procedures that have resulted from experimental field tests by CP Rail. Goldstone [15] reports that the Southern Railway is even instrumenting one coupler in some of its trains with a strain guage in an attempt to provide the train engineer with some input regarding the internal dynamics of his moving train.

There have been relatively few analytical approaches to the problem of designing optimal braking and throttle control functions for a specific train over a specific route.

Ichikawa [17] has successfully applied optimization methods for determining the optimal control law which minimizes energy consumption for a train operating from one station to the next. Unfortunately, he has had to assume an extremely simple, lumped-mass mathematical model for the train as a whole in order to be able to solve the resulting bounded state variable problem using the calculus of variations. This approach is simply impractical when a more realistic (i.e., more complex) train model is assumed.

To the author's knowledge, by far the most promising research on the problem of designing optimal control functions for freight trains is the work of McLane and Peppard [25], currently in progress at the Canadian Institute of Guided Ground Transport (CIGGT) at Queen's University, Kingston, Ontario. The objective of this work is, through the application of linear regulator theory, to develop a feedback controller which minimizes coupler forces and velocity deviations from some scheduled velocity. The model for the longitudinal dynamics of a train is somewhat deficient in that the air braking system of the train is not represented. Nevertheless, Dr. McLane has indicated to the author that the model will be altered so as to include this important aspect of train operation. When this is accomplished, a practical regulator capable of compensating for model inaccuracies and such system noise as wind and variable track conditions should result.

Although there is a good possibility that the work currently underway at the CIGGT will result in an implementable train regulator, there still remains unresolved the problem of designing the optimal "scheduled velocity" about which this regulator should function. It is precisely this problem which is the subject of this thesis - to develop a procedure for designing for a specific train an ideal velocity schedule (or more accurately, an optimal system trajectory) which may then serve as the reference input for an on-board train regulator such as that being developed by McLane and Peppard.

One very desirable feature of such a design procedure is that the track profile be taken into account directly in the process of determining the optimal trajectory of the system. This track profile, consisting of the gradient and curvature as functions of distance, is one of the most important inputs to the system. Whereas McLane and Peppard are attempting to design a regulator which is capable of operating over non-level terrain by treating gradient and curvature effects as just additional noise to the system, it would seem a better approach to take advantage of the fact that this input is known in the process of deriving the optimal system trajectory. Intuitively, if grades and curves are accounted for in the specification of the "scheduled velocity" as a function of time, then the regulator would have a much better chance of succeeding at compensating for the many other true system noises, such as wind, slippery rail conditions, etc.

1.3 Scope of Thesis and Brief Outline of Approach:

Athans (1) gives an excellent outline of a systematic design process for controlling a non-linear uncertain system about a desired trajectory through the use of the stochastic linear-quadratic-gaussian (LQG) problem. * In his paper, Athans identifies thirteen distinct steps in the design process, comprising the following six separate functions:

- Part A: Deterministic Modelling
- Part B: Stochastic Modelling
- Part C: Linearization Modelling
- Part D: Control Problem Calculations
- Part E: Filtering Calculations
- Part F: Construction of Linearized Dynamic Compensator.

* I am indebted to Dr. McLane for bringing this paper to my attention.

Athans also gives the structure of an overall control system which combines pre-computed signals with on-line measurements generated by the linear dynamic compensator. This structure is repeated here as Figure 1.1 for reference.

Whereas McLane and Peppard are concerned with Parts C, D and F, resulting in the construction of a proportional - integral - derivative dynamic compensator, this thesis addresses the problem of pre-computing the optimal trajectory of the system as defined by the set of vector functions $\{u_0(t), x_0(t), y_0(t)\}$. With reference to Athans' paper, the objective of this thesis is to provide the pre-computed ideal open-loop control input and system output, by accomplishing the following three steps of Part A:

Part A: Deterministic Modelling -

Step 1:

Obtain deterministic model of actuators and plant. In state variable form, this yields the vector differential equation:

$$\dot{x}(t) = f\{x(t), u(t)\} \qquad (1\text{-}1)$$

Step 2:

Obtain deterministic model of plant and sensors:

$$y(t) = g\{x(t)\} \qquad (1\text{-}2)$$

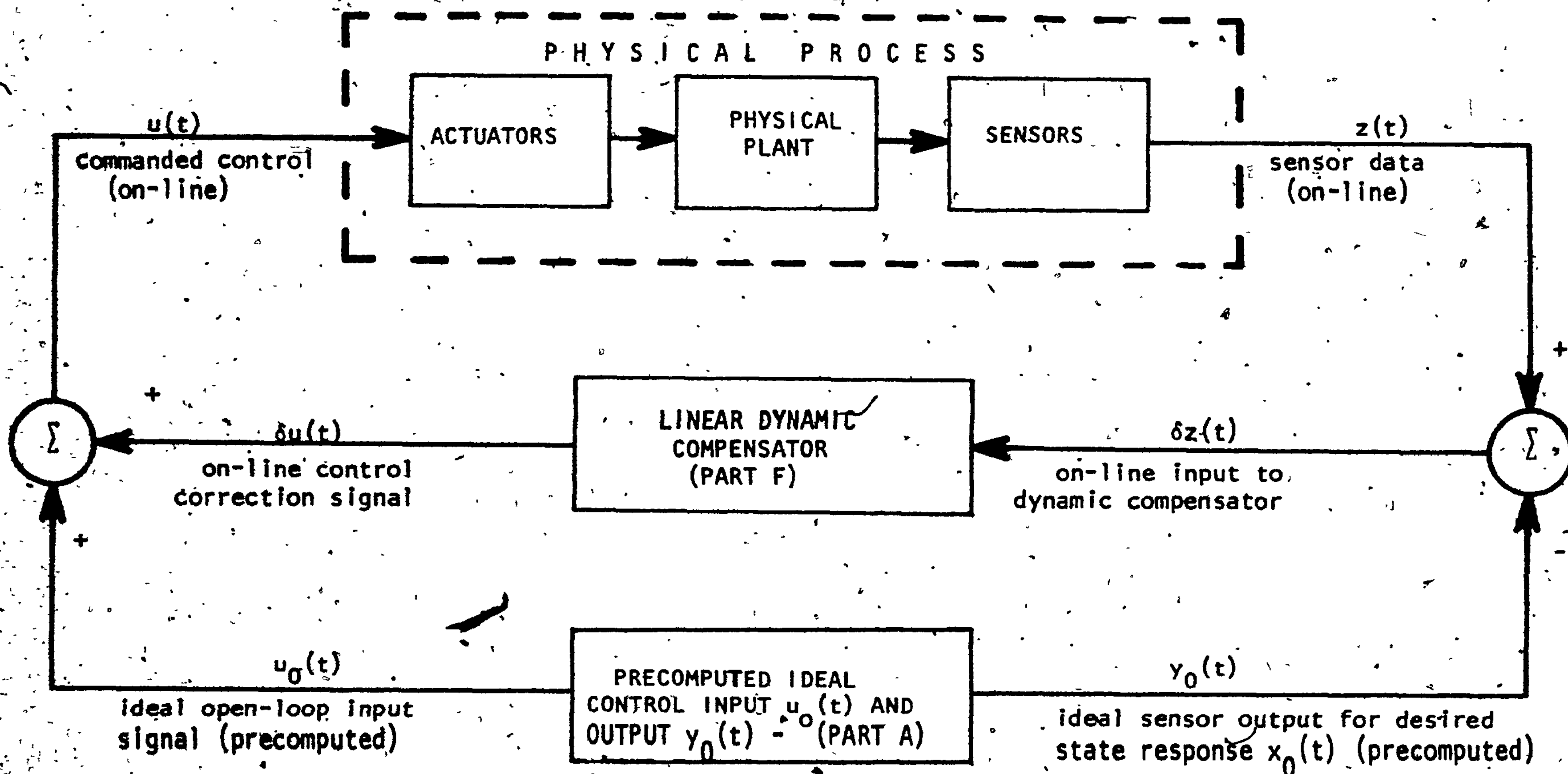


Figure 1.1 Structure of overall control system, as given by Athans [1].

Step 3:

Determine ideal input - state - output response:

u_0 (t) : ideal (open-loop) input

x_0 (t) : ideal state response

y_0 (t) : ideal output response

As pointed out by Athans, the modelling process (steps 1 and 2 combined) is extremely critical to the ultimate success of the control design, and much of this thesis will be concerned with this modelling process.

The usual procedure for accomplishing step 3 is to formulate an optimal control problem of the following form:

> "Given the system described by equation (1-1), find an admissible control vector u_0 (t) which transfers the system from some initial state x_0 (t_0) to some final state x_0 (t_f), such that x_0 (t_f) is a subset of S, where S is a target set of final states, and such that some cost functional defined by the designer is minimized."

It will be shown that the traditional means of obtaining a solution to the problem just formulated will not work in this instance, primarily because of the extremely high dimensionality of the system. A procedure first outlined by Enns [12] will then be employed to obtain a solution. The approach of Enns is to reformulate the given problem as a standard linear programming (LP) problem by an appropriate discretization of the system equation (1-1); the resulting LP problem may then be solved by any of the standard methods commonly available. An important attribute of this solution method is that the dimension of the LP problem is relatively independent of the dimensionality of the system (1-1).

The thesis then concludes with a worked example for a small sample train of three cars (or three sections), and a discussion of the results.

In essence, the problem under consideration in this thesis is to devise a method for controlling excessive inter-car forces caused by relative accelerations between successive cars that may result from the slack-action of a train in motion.

CHAPTER 2.

PHYSICAL SYSTEM AND MODEL FORMULATION

2.1 Assumptions Regarding Overall System Model:

In this chapter, a model of the overall physical system will be formulated. The model so derived will be a one-dimensional representation in which only longitudinal motion and forces will be considered directly.

To be sure, the actual train undergoes vertical displacements as a result of track gradients; but it is only the horizontal components of the forces due to gravity that we are interested in and the effect of these forces is represented in the model as resistance to longitudinal motion. Also, the track itself is assumed to be continuous in that the impact of wheels on rail joints is neglected.

Similarly, lateral forces are certainly encountered between the flanges of the car wheels and the rail guideways, particularly through curved track sections, but the effect of these forces is again represented in the model simply as resistance to longitudinal motion.

Additional assumptions will be noted as each of the elements of this physical system is modelled in the following sections.

2.2. Model for Individual Train Members:

Roggeveen [34] has shown in freight car impact tests that a two-lump model of an individual train member must be assumed in order to depict the coupler force versus time characteristic reasonably accurately for all coupler force levels. However, he has also shown that a first-order lumped model gives a sufficiently accurate representation of the impact of successive train members up to a certain critical coupler force level, corresponding to the point where the spring of the inter-car draft gear is completely compressed. At force levels exceeding this critical value, the two train members are effectively in solid metal-to-metal contact, and it is in this region that the first-order model fails and a second-order representation becomes necessary.

However, the end objective of this whole exercise is to minimize the peak coupler forces in the train. Making the assumption that the optimal control derived in this dissertation will succeed in keeping coupler forces at least below this critical level (approximately 200,000 pounds for average draft-gears) permits us to assume the simpler first-order model. By way of justifying this assumption, Wilson [50] reports excellent agreement between test data from instrumented trains and computer predictions from a model which assumes a first-order representation for each train member.

Let us define the following nomenclature:

t: the independent variable time (in seconds)

i: the subscript indicating the member number in the train; $i = 1$ indicates the leading member in the direction of train motion.

m_i: the effective mass (in slugs) of the i^{th} train member; this is the actual mass of the member plus some allowance for the rotational inertia of the wheelsets (see Taylor [(46)], for instance).

$s_i(t)$: the position (in feet) of the i^{th} train member, relative to some reference position S_0.

$v_i(t)$: the velocity (in ft/sec) of the i^{th} train member.

$b_i(t)$: the air braking effort (in pounds) developed at the i^{th} train member.

$\mathcal{F}_i(t)$: the force (in pounds) on the i^{th} and $(i + 1)^{th}$ train members from the combined draft gears between them.

$f_i^*(t)$: the tractive or dynamic braking effort (in pounds) developed by the i^{th} train member (identically zero for all t if train member i is not a powered locomotive).

$y_i(t)$: the total resistance to motion (in pounds) of the i^{th} train member due to grades, curves, wind and friction.

Considering the model depicted in Figure 2.1, then from Newton's Second Law the equations of motion for the i^{th} train member are:

$$\dot{s}_i(t) = v_i(t) \qquad \text{(2-1a)}$$

$$\dot{v}_i(t) = \frac{1}{m_i}\{\mathcal{F}_{i-1}(t) - \mathcal{F}_i(t) - y_i(t) - b_i(t) + f_i^*(t)\} \qquad \text{(2-1b)}$$

where $(\dot{\cdot}) \equiv \frac{d}{dt}(\cdot)$

2.3 Model for Draft Gears and Couplers between Train Members:

Smith [45] lists over twenty different types of draft gears, but for our purposes we shall assume that the important characteristics of the combined draft gears between any two successive train members

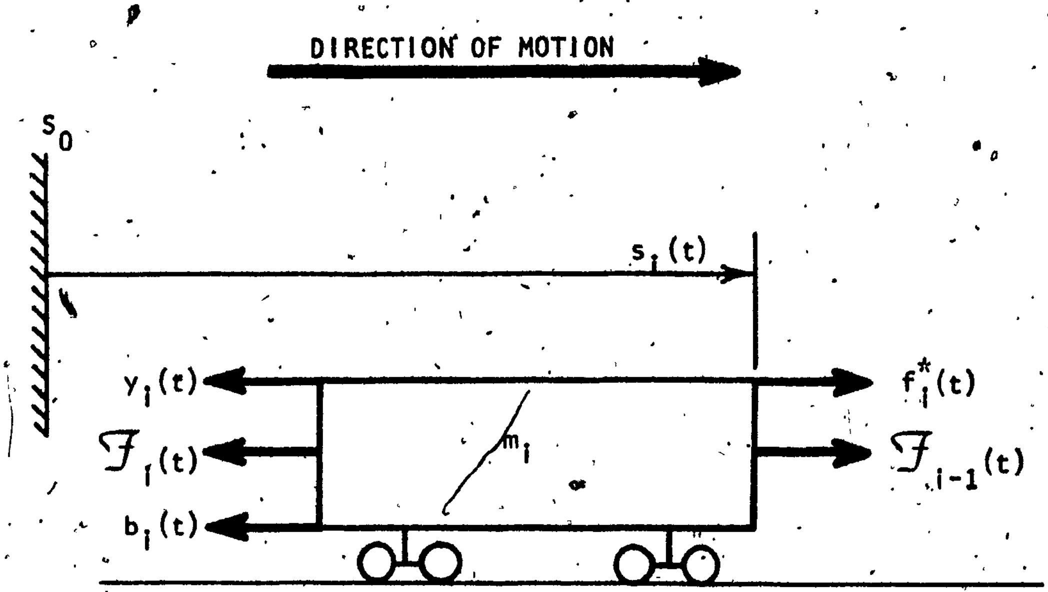


Figure 2.1: Model of the i^{th} member of the train.

can be modelled by a translational pure spring and viscous damper arranged in parallel, as in Figure 2.2 wherein the following additional symbols are defined:

α_i: the spring constant (in pounds/foot) of the combined draft gears between train members i and (i + 1).

β_i: the damping coefficient (in pounds-seconds/foot) of the combined draft gears between train members i and (i + 1).

h_i: the length (in feet) of train member i.

w_i: the zero-force length (in feet) of the draft gears between train member i and (i + 1).

$z_i(t)$: the extension or compression (in feet) of the draft gears between train members i and (i + 1). Note that $\{ z_i(t) + w_i \}$ represents the total separation between the trailing wall of member i and the leading wall of member (i + 1).

It should be noted that the non-linear effect (namely dead-zone) of slack in the draft gears, as described by Roggeveen [37],

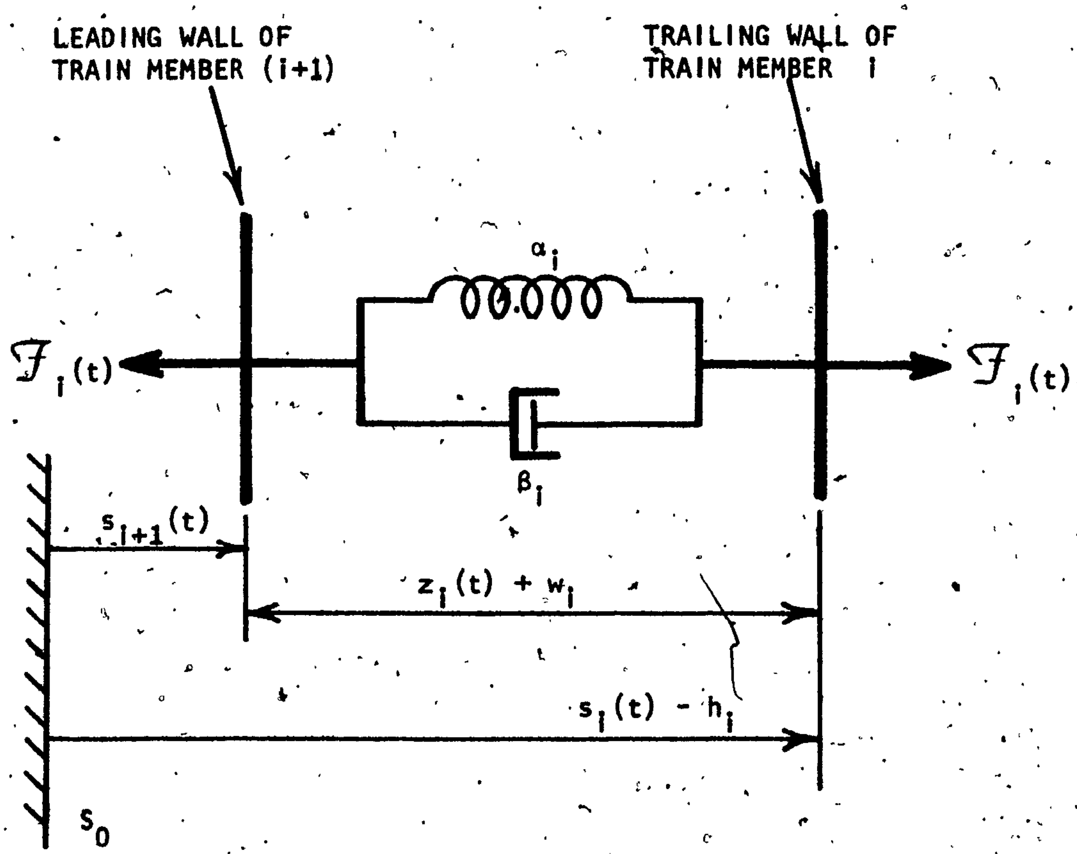


Figure 2.2: Model of the combined draft gears between train members i and (i+1)

is specifically not included in this model. Opinion is currently divided on the question of whether or not this non-linearity is an important factor which should be represented in the model; the current efforts of McLane and Peppard [25] in this regard perhaps will settle this issue.

From Figure 2.2, we have:

$$z_i(t) = s_i(t) - s_{i+1}(t) - h_i - w_i \quad \text{.......................(2-2)}$$

and $\therefore$ $\dot{z}_i(t) = \dot{s}_i(t) - \dot{s}_{i+1}(t) = v_i(t) - v_{i+1}(t)$ ......(2-3)

since h_i and w_i are constants.

Thus, the mathematical model of the inter-car connection is:

$$\mathcal{F}_i(t) = \alpha_i z_i(t) + \beta_i \dot{z}_i(t)$$

$$= \alpha_i z_i(t) + \beta_i \{ v_i(t) - v_{i+1}(t) \} \quad \text{........(2-4)}$$

2.4 Model for Train Air Brake System:

Reference (65) contains an excellent equipment glossary and overall description of the train air brake system. Other relevant material dealing with this important aspect of train operation is given in the bibliography.

To quote from reference (65) :

"The primary source for slowing and stopping trains is the compressed air stored at each car in the train. This air is admitted to the brake cylinder, translated to a mechanical force acting on the brake rigging, and in turn forces the brake shoes against the wheels of the train to retard their rotation. The amount of braking is controlled by the reduction of brake pipe pressure, as determined by the automatic brake valve handle position in the locomotive. This brake pipe reduction throughout the train causes the control valve at each car to respond. The control valve then meters the stored air in the brake cylinders in proportion to the amount of drop in brake pipe pressure."

Let $r^*(t)$ represent the reductions in brake pipe pressure (lbs/in^2) called for by the engineman as a result of varying automatic brake valve handle positions. There are definite restrictions on the manner in which this brake valve handle position can be varied, but these constraints will be dealt with later.

Now consider the following sequence of events which occurs after the engineman has initiated the braking action by moving the brake valve handle from the release position:

i) the locomotive equalizing reservoir pressure reduces;

ii) this in turn causes the train brake pipe pressure at the locomotive to reduce;

iii) this pressure drop is then transmitted through the brake pipe of the train until it reaches the control valve on the i^{th} train member;

iv) the control valve then meters the supply of air from the auxiliary reservoir to the brake cylinder of the i^{th} train member.

Rather than model in detail the above sequence of events, let τ_i represent the overall time delay which occurs between events (i) and (iv). Then, τ_i is known to be of the form:

$$\tau_i = \gamma + \frac{1}{\delta} \left| s_i(t) - s_1(t) \right| \qquad (2\text{-}5)$$

where symbols are defined as follows:

γ : response time (seconds) of the automatic brake valve and equalizing reservoir.

δ : transmission speed (feet/second) of the pressure drop signal in the brake pipe of the train.

I : the member number of the locomotive nearest train member i which is capable of effecting a reduction in brake pipe pressure.

(Typical values for γ and δ are 2 seconds and 500 feet/second, respectively).

The reduction in brake pipe pressure at the i^{th} train member, $r_i^*(t)$, is therefore given by: $r_i^*(t) = r^*(t - \tau_i)$. Thus, the auxiliary reservoir supply pressure to the brake cylinder pipe can be represented as $\eta \cdot r^*(t - \tau_i)$, where τ_i is defined by eg. (2-5) and η is a constant of proportionality between the time delayed equalizing reservoir pressure drop appearing at the control valve of the i^{th} train member and the supply pressure of that member's auxiliary reservoir.

The braking force generated at the wheels of member i is proportional to the pressure in its brake cylinder. In essence, the brake cylinder behaves approximately like a pure fluid capacitance in which energy is stored because of the compressibility of the fluid. The brake cylinder is supplied with air from the auxiliary reservoir through a long, relatively narrow, brake cylinder pipe. Hence, we shall assume the simple model for brake action as illustrated in Figure 2.3.

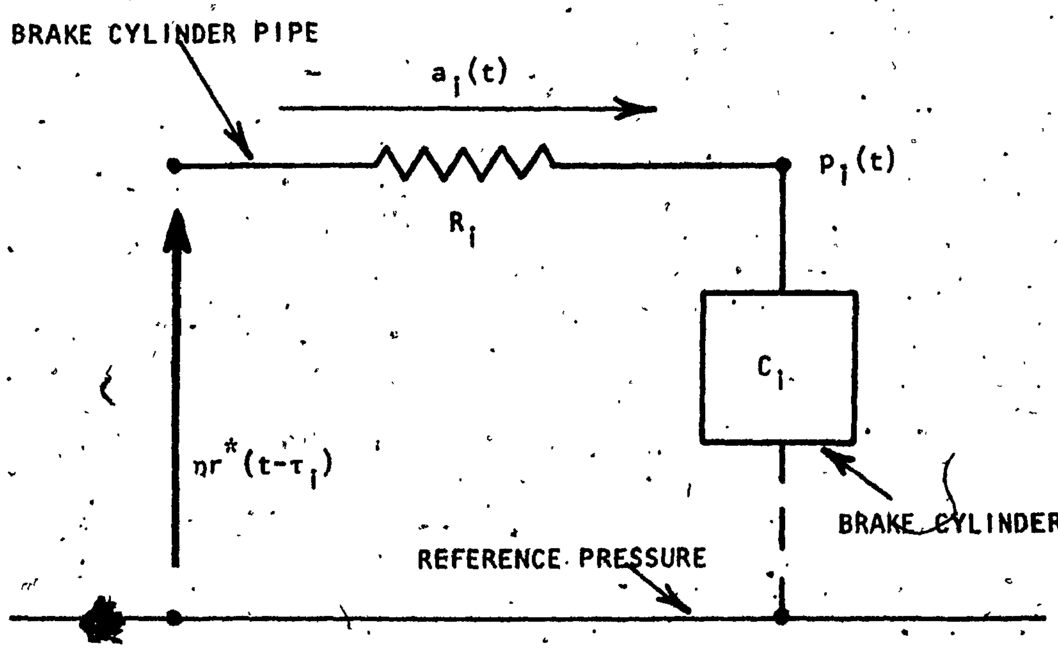


Figure 2.3: Model of brake action at train member i.

The new nomenclature of Figure 2.3 is defined as follows:

C_i : fluid capacitance (in ^{5}/lb.) of the brake cylinder at train member i.

R_i : fluid resistance (lb-secs/in ^{5}) of the brake cylinder pipe at train member i.

a_i (t) : air flow rate (in ^{3}/sec) in the brake cylinder pipe of the $i^{\underline{th}}$ train member.

p_i (t) : brake cylinder pressure (lbs/in^2) at the $i^{\underline{th}}$ train member.

Note that the auxiliary reservoir supply pressure is modelled as an ideal pressure source. In actual fact, the auxiliary reservoir is a fixed volume container, but since its volume is significantly greater than that of the brake cylinder, this representation is a reasonable approximation.

From Figure 2.3, we obtain the mathematical model for brake action at train member i as:

$$a_i\ (t) = C_i\dot{p}_i\ (t) = \frac{1}{R_i}\ \{\eta \cdot r^*\ (t-\tau_i) - p_i\ (t)\ \} \ldots\ldots (2\text{-}6)$$

and ∴ $$\dot{p}_i\ (t) = \frac{1}{\sigma_i}\{\eta \cdot r^*\ (t-\tau_i) - p_i\ (t)\ \} \ldots\ldots\ldots\ldots (2\text{-}7)$$

where $\sigma_i = R_iC_i$ is the time constant (seconds) of the braking system.

Now define e_i to be the constant of proportionality (in^2) between brake cylinder pressure and the braking force developed at the i^{th} train member. Therefore, e_i takes into account such factors as the brake rigging action, the coefficient of friction between the brake shoes and wheels of member i, and the coefficient of friction between wheel and rail. Thus, the braking effort b_i (t) of Figure 2.1 is given by:

$$b_i\ (t) = e_ip_i\ (t) \qquad \text{.................(2-8)}$$

As mentioned previously, in order for the braking action model defined by eqs. (2-7) and (2-8) to be valid, r* (t) must be suitably constrained so as to represent realistic train air brake applications and releases.

For instance, when an application of the train air brakes is called for, the minimum and maximum automatic brake valve handle positions are known [65] to correspond to equalizing reservoir reductions of 7 and 23 psi respectively. Thus we have the first constraint on r* (t) :

$$\left.\begin{array}{ll} \text{either} & r^*(t) = 0 \\ \text{or} & 7 \le r^*(t) \le 23 \end{array}\right\} \qquad \text{.................(2-9)}$$

In addition, although brake application increases are permitted at any time, partial brake releases are not valid. That is, if Δt is some small time increment, then:

$$\left.\begin{array}{ll} \text{either} & r^*(t + \Delta t) = 0 \\ \text{or} & r^*(t) \le r^*(t + \Delta t) \le 23 \end{array}\right\} \quad \text{..........(2-10)}$$

One final constraint on $r^*(t)$ is required, in order to avoid undesired brake releases in the actual train. Peterson [30] gives an excellent account of the causes of such undesired releases, and illustrates that these can be attributed to inappropriate brake applications made under "false gradient" conditions.

Some explanation of terminology is in order before proceeding further. When the automatic brake valve handle is moved to the release position, this causes the brake valve to restore air pressure to the brake pipe. The increasing brake pipe pressure then causes the equipment at each member to move to release, thereby exhausting the brake cylinder pressure and recharging the auxiliary reservoir. Now, "true gradient" may be defined as the difference between the head end and rear car brake pipe pressures when the train is fully charged, and is primarily due to leakage in the piping of the system. Whenever the train is not fully charged (i.e., the recharging function alluded to previously has not been completed), a "false gradient" is said to exist and is defined as the difference between the actual gradient at that point in time and the true gradient for that particular train.

In order to represent this final constraint on $r^*(t)$, it will therefore be necessary to represent in the model this recharging of the auxiliary reservoir farthest from the locomotive capable of supplying air to the brake pipe. To this end, let us define the following nomenclature:

n :	the total number of members in the train, including locomotives.
J :	the member number of the car furthest from any locomotive capable of effecting an increase in brake pipe pressure
C_{n+1}:	fluid capacitance (in^5/lb) of auxiliary reservoir of train member J.
R_{n+1}:	fluid resistance ($lb\text{-}secs/in^5$) of pipe connecting auxiliary reservoir and brake pipe at train member J.
$a_{n+1}(t)$:	air flow rate (in^3/sec) in the pipe connecting the auxiliary reservoir and brake pipe at train member J.
$p_{n+1}(t)$:	reduction in auxiliary reservoir pressure (lbs/in^2) at train member J; as such, $p_{n+1}(t)$ is also the

"false" gradient existing at any time t. When $p_{n+1}(t) = 0$, all auxiliary reservoirs in the train are completely recharged.

The model of this recharging function of the J^{th} member's auxiliary reservoir is therefore assumed as in Figure 2.4.

From Figure 2.4 we have:

$$\dot{p}_{n+1}(t) = \frac{1}{\sigma_{n+1}} \{ r^* (t - \tau_J) - p_{n+1}(t) \} \quad \text{.......... (2-11)}$$

where $\sigma_{n+1} = R_{n+1} C_{n+1}$ is the time constant of the recharging system (secs).

Peterson (30) shows that in order to avoid undesired train brake releases, brake applications required must be increased by an amount equal to any false gradient existing at the time of application. Thus, the final constraints on $r^*(t)$ may be expressed as:

$$\left. \begin{array}{l} \text{If } r^*(t) = 0, \text{ then} \\ \text{either: } r^*(t + \Delta t) = 0 \\ \text{or: } 7 + p_{n+1}(t) \leq r^*(t + \Delta t) \leq 23 \end{array} \right\} \quad \text{.......... (2-12)}$$

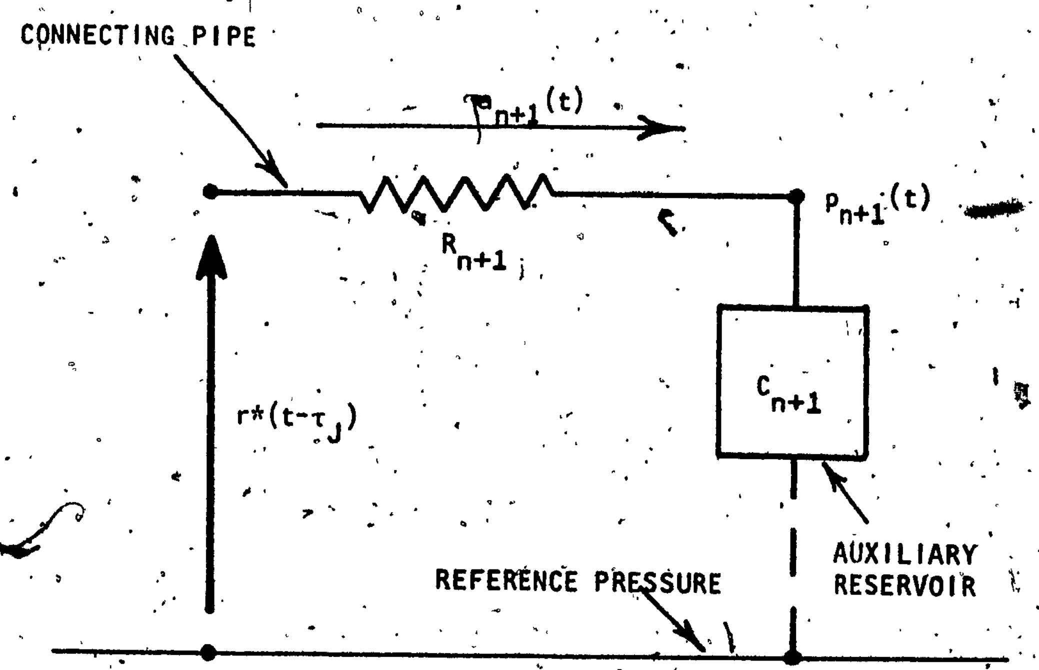


Figure 2.4: Model of train recharging action. When $r^*(t)$ is brought to zero then brake release is initiated and when $p_{n+1}(t) = 0$, then train is fully recharged. Thus, $p_{n+1}(t)$ is a measure of the "false" gradient in the train at any time t.

For convenience, the constraints (2-9), (2-10) and (2-12) are summarized here as follows:

$$\left.\begin{array}{ll} \text{I)} & \text{if } r^*(t) = 0, \text{ then} \\ & \text{either: } r^*(t+\Delta t) = 0 \\ & \text{or: } p_{n+1}(t) + 7 \le r^*(t+\Delta t) \le 23 \\ \text{II)} & \text{if } r^*(t) \ne 0, \text{ then} \\ & \text{either: } r^*(t+\Delta t) = 0 \\ & \text{or: } r^*(t) \le r^*(t+\Delta t) \le 23 \end{array}\right\} \quad \ldots\ldots(2\text{-}13)$$

2.5 Representation of Motive Power Constraints:

Each locomotive in the train represents a potential source of both tractive and braking effort. As in Figure 2.1, let $f_i^*(t)$ represent the force output (lbs) of the locomotive member i at any time t. Then for $f_i^*(t) > 0$ the unit is "motoring", that is, applying tractive force; and for $f_i^*(t) < 0$ the unit is in "dynamic braking", that is, applying braking force. Obviously, if $f_i^*(t) = 0$, then the unit is idling. In order to represent real locomotives, $f_i^*(t)$ must be suitably constrained, as follows.

2.5.1 Motoring Constraints (when $f_i^*(t) > 0$) :

The theoretical maximum force that a locomotive can transmit to the rail is limited by two factors. These are (i) the adhesion limit (coefficient of friction) between the wheel and the rail, beyond which wheel slip will occur, and (ii) the actual power of the locomotive:

i) Adhesion Limit:

If W_i is the effective weight of the locomotive (lbs) and μ is the tractive adhesion coefficient (dimensionless), then:

$$f_i^*(t) \leq \mu \cdot W_i \quad \ldots\ldots\ldots\ldots\ldots (2\text{-}14)$$

ii) Motive Power Limit:

If P_i is the maximum horsepower rating of the locomotive, and $v_i(t)$ is the velocity of the locomotive (ft/sec), then from reference (61):

$$f_i^*(t) \leq \frac{451\, P_i}{v_i(t)} \quad \ldots\ldots\ldots\ldots\ldots (2\text{-}15)$$

where the constant 451 is derived from 550 ft-lbs/sec per horsepower X 0.82 transmission efficiency of the electro-mechanical drive system.

2.5.2. Dynamic Braking Constraints (when $f_i^*(t) < 0$):

There are basically four constraints on the braking force which can be generated by locomotive member i:

i) Adhesion Limit:

As in motoring, the maximum braking force is limited by the adhesion between wheel and rail:

$$f_i^*(t) \geq -\mu W_i \quad \text{....................(2-16)}$$

ii) Motor Heating Limit:

In dynamic braking, the grid current (and hence the braking horsepower developed through I^2R loss (59, 66) in the braking resistor grids) is limited. For instance, from performance curves (65) for the SD-40 locomotive, this limit is known to be as follows:

$$f_i(t) \geq -\frac{638\, P_i}{v_i(t)} \quad \text{....................(2-17)}$$

iii) Motor Field Current Limit:

At low speeds, the motor armature current is less, as the wheels are turning slowly. Hence, the braking horsepower is less, since I^2R is smaller due to the smaller grid current I. This limit on the available braking effort is of the form [65]:

$$f_i^*(t) \geq -\xi v_i(t) \quad \text{......(2-18)}$$

where the value of ξ depends on whether dynamic braking is standard or extended range (see reference (65) for an explanation of these terms). Typical values for a 4 axle, 2250 horsepower unit are $\xi_{std} \approx 1110$ lb-secs/ft., and $\xi_{ext} \approx 3960$ lb-secs/ft.

iv) Extended Range Dynamic Braking Limit:

If extended range dynamic braking is available, then over the speed range of approximately 9 to 37 ft/sec, the following constraint applies:

$$f_i^*(t) \geq -Z \quad \text{......(2-19)}$$

Typically, Z ≃ 40,000 lbs. for a GP-40 unit (4 axles, 2250 horsepower), and Z ≃ 60,000 lbs. for an SD-40 unit (6 axles 3000 horsepower).

2.5.3. Summary of Motive Power Constraints:

For convenience, the constraints on $f_i^*(t)$ are summarized here as follows:-

$$\left.\begin{aligned} &\left| f_i^*(t) \right| \leq \mu \cdot W_i \\ &-\frac{638P_i}{v_i(t)} \leq f_i^*(t) \leq \frac{451P_i}{v_i(t)} \\ &f_i^*(t) \geq -\xi v_i(t) \\ &f_i^*(t) \geq -Z \end{aligned}\right\} \quad \ldots\ldots\ldots(2\text{-}20)$$

It should be noted that some or all of the constraints of (2-30) apply for any particular class of power, depending on the specific values of μ, P_i, ξ and Z. However, in the most general problem, all these constraints may apply, in which case the allowable region for $f_i^*(t)$ has the form represented by the hatched area in Figure 2.5.

Of course, if dynamic braking is not permitted for any particular train, then $f_i^*(t) \geq 0$ must be added as the final constraint.

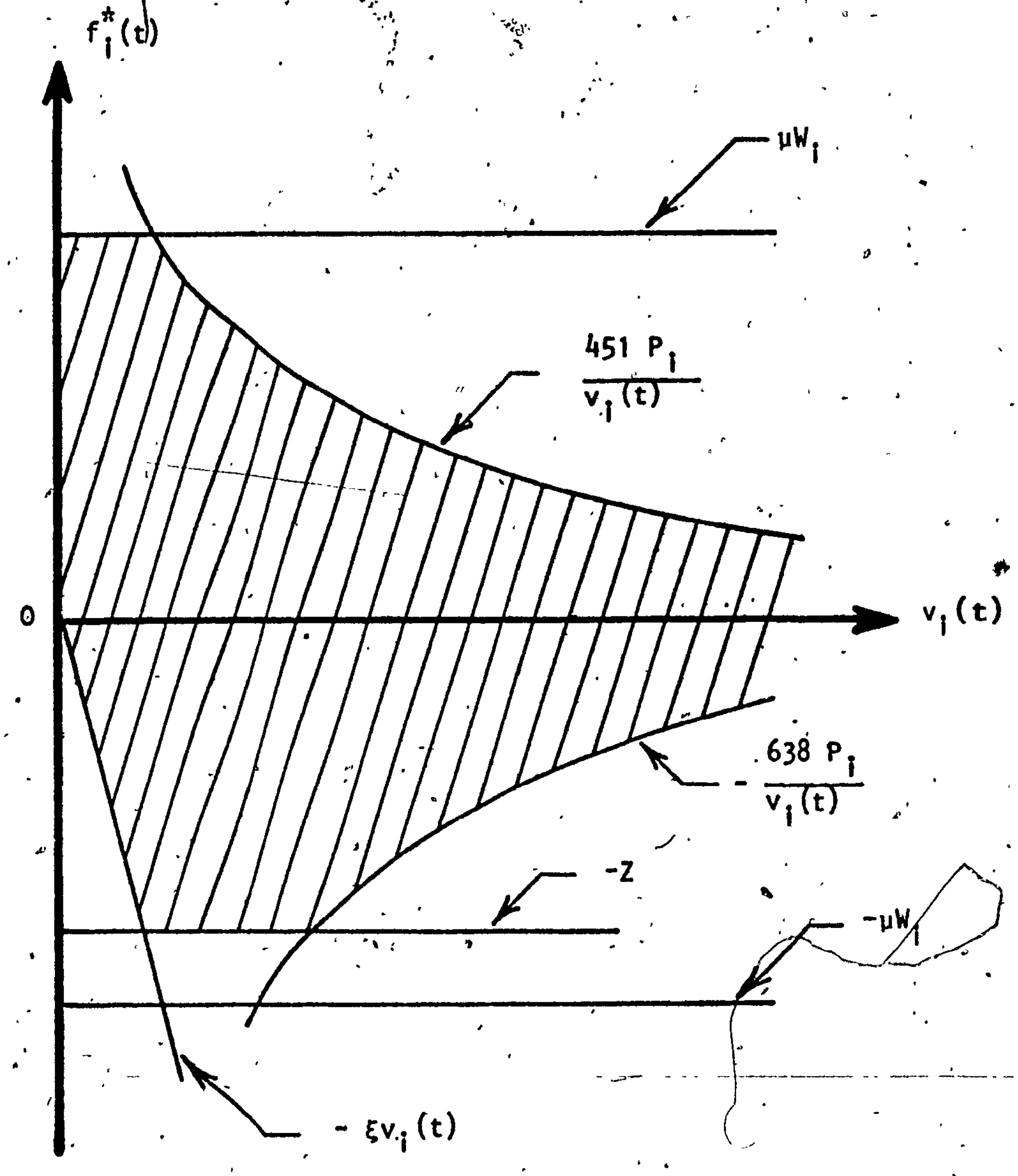


Figure 2.5: Motive power constraints; hatched area is allowable region.

In addition, $f_i^*(t)$ within the allowable region of Figure 2.5 must be quantized, since the engineman's throttle has only eight distinct positions and each corresponds to a specific power output. For instance, McLane and Peppard [25] treat the quantization of locomotive tractive effort, and reference (53) contains an example of this in the case of a locomotive in dynamic braking. However, the approach taken here will be to assume that the allowable region is continuous and then effect the required quantization after the optimal control has been obtained.

2.6 Representation of the Resistance to Motion Function, $y_i(t)$, for the i^{th} Member:

The function $y_i(t)$ is taken to represent the total resistance to motion of the i^{th} train member due to grades, curves, friction and air resistance. Let us consider each component of $y_i(t)$ in the following sections.

2.6.1. Grade Resistance:

If the member i is ascending a grade (slope), its mass is being raised vertically. Let $\rho_G\{s_i(t)\}$ be the grade, expressed in per cent, at position $s_i(t)$. Since the grade $\rho_G\{s_i(t)\}$ gives the vertical rise over 100 ft. of track, then

$$\tan\theta = \frac{\rho_G\{s_i(t)\}}{100} \qquad \text{.....................(2-21)}$$

where θ is the angle of the grade in degrees. Also, for a member of weight W_i ascending a grade of θ degrees, we have for the grade resistance, R_G, in lbs:

$$R_G = W_i \sin\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}22)$$

However, since θ is quite small, we have $\sin\theta = \tan\theta = 0.01\,\rho_G\{s_i(t)\}$ by eq. (2-21). Hence:

$$R_G = 0.01\,W_i \cdot \rho_G\{s_i(t)\} = 0.01\,gm_i\,\rho_G\{s_i(t)\} \ldots.(2\text{-}23)$$

where g is the acceleration due to gravity (ft/sec^2).

2.6.2. Curve Resistance:

The effect of varying degrees of curvature on resistance to motion has been determined empirically [46], and most simply stated, one degree of curvature offers approximately the same resistance to motion as a 0.04 per cent grade, that is 0.8 lbs/ton/degree of curvature. Hence, the effect of curve resistance is represented in the model by constructing an "effective" grade $d\{s_i(t)\}$ which is the sum of the actual grade $\rho_G\{s_i(t)\}$ and the equivalent grade due to curvature. Let $\rho_C\{s_i(t)\}$ represent the actual curvature (degrees) as a function of position on the track. Then we have:

$$d\{s_i(t)\} = \rho_G\{s_i(t)\} + 0.04\ \rho_C\{s_i(t)\} \quad \text{..........(2-24)}$$

As eq. (2-24) now stands, the effective grade is expressed as an explicit function of position. Unfortunately, this is an inconvenient representation, since the functions ρ_G and ρ_C cannot be expressed in closed form and, as we shall see, $s_i(t)$ is one of the state variables of the system. To circumvent this difficulty, we shall arbitrarily assume that the effective grade can be defined as an explicit function of time. Let:

$$d^*(t - \omega_i) = d\{s_i(t)\} \quad \text{......................(2-25)}$$

where ω_i is the time delay (seconds) which occurs before the grade input to the system appears at the $i^{\underline{th}}$ member. It will be found convenient to define $d^*(t)$ with respect to some position $s_0(t)$ one member length ahead of the first train member. If z_0, h_0 and w_0 denote the parameter values of this imaginary "zero$^{\underline{th}}$" member, then we have for $i = 1, 2, \ldots n$:

$$s_i(t) = s_0(t) - \sum_{j=0}^{i-1} (z_j + h_j + w_j) \quad \text{.............(2-26)}$$

and thus for a train travelling at constant speed we have for ω_i :

$$\omega_i = \frac{\sum_{j=0}^{i-1} (z_j + h_j + w_j)}{v_i(t)} \quad \text{.....................(2-27)}$$

Since the objective of this whole exercise is to reduce the maximum absolute value of the z_i's, a reasonable approximation for ω_i is:

$$\omega_i \approx \frac{\sum_{j=0}^{i-1} (h_j + w_j)}{v_i(t)} \quad \text{.....................(2-28)}$$

Thus, we see that the specification of $d^*(t)$ and ω_i for a particular train over a specific piece of track will probably require an iterative approach, since some beforehand assumptions regarding the trajectory of $v_i(t)$ must be made. It is for this reason that the approximation sign is used in (2-25).

To summarize this and the preceding section, if R_C is the resistance due to curvature, then the sum of R_G and R_C for any member i is taken to be given by:

$$R_G + R_C = 0.01\, gm_i d^*(t-\omega_i) \quad \text{.....................(2-29)}$$

2.6.3. Friction and Air Resistance:

To quote from reference (61):

"A railway vehicle moving upon level, tangent track, in still air and at constant speed encounters certain resistances that must be overcome by the tractive effort of the locomotive.

These resistances include:

i) Rolling friction between wheel and rail:
This varies with the surface condition of the rail under load, the horizontal contour of the railhead, and contour and condition of the wheel tread. This can be considered a constant for a given quality of track.

ii) Journal bearing friction:
This varies with the weight of each axle and, at low speed, the type, design and lubrication of the bearing.

iii) Train dynamic losses:
These include flange effects which are associated with lateral motion and the resulting friction and impact of the wheel flanges against the gage side of the rail. They vary with speed, rail alignment, and the tracking effect of the trucks. Also, there are miscellaneous losses due to sway, concussion, buffing and slack action.

iv) Air resistance:

This varies directly with the cross-sectional area of the vehicle, its length and shape, and the square of its speed. It is also influenced by zones of turbulence related to shape."

Davis [10] gives an empirical formula for computing the total of the four classes of resistance noted above. This formula is a second degree polynominal in $v_i(t)$, the speed of the i^{th} member. However, it is reasonable to use a linearized form of the Davis formula as follows [25]:

$$R_{F+A} = gm_i \{\lambda_{0i} + \lambda_i v_i(t)\} \quad \ldots\ldots\ldots (2-30)$$

where we have:

R_{F+A}: friction and air resistance (lbs) of the i^{th} train member.

λ_{0i}: zero-velocity constant (dimensionless) in linearized version of Davis formula for resistance due to friction and air resistance.

λ_i: velocity coefficient (secs/ft) in linearized version of Davis formula.

Hence, the total resistance to motion of the $i^{\underline{th}}$ train member, $y_i(t)$, is given by:

$$y_i(t) = R_G + R_C + R_{F+A}$$

$$= 0.01\ gm_i d^*(t-\omega_i) + gm_i \{ \lambda_{0i} + \lambda_i v_i(t) \} \quad \ldots\ldots (2\text{-}31)$$

2.7 Overall Train Model:

By combining equations (2-1b), (2-3), (2-4), (2-7), (2-8) and (2-31), one derives the following system equations for the $i^{\underline{th}}$ member of the train:

$$\left.\begin{aligned}
\dot{v}_i(t) &= \frac{\alpha_{i-1}}{m_i} \cdot z_{i-1}(t) + \frac{\beta_{i-1}}{m_i} \cdot v_{i-1}(t) \\
&\quad - \frac{\alpha_i}{m_i} \cdot z_i(t) - \left(\frac{\beta_{i-1} + \beta_i}{m_i} + g\lambda_i\right) \cdot v_i(t) \\
&\quad - \frac{e_i}{m_i} \cdot p_i(t) + \frac{\beta_i}{m_i} \cdot v_{i+1}(t) - g\lambda_{0i} \\
&\quad - 0.01\ gd^*(t-\omega_i) + \frac{1}{m_i} \cdot f_i^*(t) \\
\dot{p}_i(t) &= -\frac{1}{\sigma_i} \cdot p_i(t) + \frac{\eta}{\sigma_i} \cdot r^*(t-\tau_i) \\
\dot{z}_i(t) &= v_i(t) - v_{i+1}(t)
\end{aligned}\right\} \quad \ldots\ldots (2\text{-}32)$$

Thus, equations (2-32) constitute the mathematical model for train member i; however, a number of special situations exist which require modification of this system:

I) <u>i=1:</u>

In this situation there is obviously no coupler force on the leading draft gear of the first member, and therefore α_{i-1} and β_{i-1} of eqns. (2-32) should be set to zero for $i = 1$.

II) <u>Member i is not a powered locomotive:</u>

In this case, $f_i^*(t) = 0$ for all t.

III) <u>i=n:</u>

In this instance, there is obviously no coupler force on the trailing draft gear of the last member, and therefore α_i and β_i should be set to zero for $i=n$. Also, the last equation of system (2-32) should be deleted since $z_i(t)$ is not defined for $i=n$.

Now, equation (2-1a) with $i=1$, equation (2-11), and equations (2-32) for each train member, together constitute the overall train model.

2.8. State Variable Representation of Train:

Consideration of the system equations for the train, as derived in the preceding sections, immediately suggests a convenient set of physical variables (see Schultz and Melsa [39], for instance) which may be used to define the state vector of the system. Therefore, let:

$$\begin{bmatrix} s_1(t) \\ v_1(t) \\ p_1(t) \\ z_1(t) \\ \cdot \\ \cdot \\ \cdot \\ v_i(t) \\ p_i(t) \\ z_i(t) \\ \cdot \\ \cdot \\ \cdot \\ v_n(t) \\ p_n(t) \\ p_{n+1}(t) \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ \cdot \\ \cdot \\ \cdot \\ q_{3i-1} \\ q_{3i} \\ q_{3i+1} \\ \cdot \\ \cdot \\ \cdot \\ q_{3n-1} \\ q_{3n} \\ q_{3n+1} \end{bmatrix} = q(t) \quad \text{..........(2-33)}$$

where q is the (3n+1) - dimensional state vector of the n member system.

Let us further define:

$$
\begin{bmatrix}
f_1^*(t) \\
\vdots \\
f_j^*(t) \\
\vdots \\
f_l^*(t) \\
r^*(t-\tau_1) \\
\vdots \\
r^*(t-\tau_i) \\
\vdots \\
r^*(t-\tau_n) \\
0.01\ d^*(t-\omega_1)+\lambda_{01} \\
0.01\ d^*(t-\omega_2)+\lambda_{02} \\
\vdots \\
0.01\ d^*(t-\omega_i)+\lambda_{0i} \\
\vdots \\
0.01\ d^*(t-\omega_n)+\lambda_{0n}
\end{bmatrix}
=
\begin{bmatrix}
r_1 \\
\vdots \\
r_j \\
\vdots \\
r_l \\
r_{l+1} \\
\vdots \\
r_{l+i} \\
\vdots \\
r_{n+l} \\
r_{1+n+l} \\
r_{2+n+l} \\
\vdots \\
r_{i+n+l} \\
\vdots \\
r_{2n+l}
\end{bmatrix}
= r(t) \quad \ldots\ldots\ldots (2\text{-}34)
$$

where r is the (2n+1) - dimensional control vector of the system of n members, l of which are powered locomotives. Note that the last n elements of r are treated as control variables, although in fact they are actually pre-defined inputs.

With the state and control vectors defined as in (2-33) and (2-34), the state variable representation of the train is given by:

$$\dot{q} = \Delta q + \Omega r \quad \ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}35)$$

where Δ is the (3n+1) X (3n+1) constant system matrix:

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & 0_{33} & \cdots & & & & \cdots & 0_{33} & 0_{31} \\ & & & & \vdots & & & & & \\ 0_{33} & \cdots & 0_{33} & \Delta_{i,i-1} & \Delta_{i,i} & \Delta_{i,i+1} & 0_{33} & \cdots & 0_{33} & 0_{31} \\ & & & & \vdots & & & & & \\ 0_{33} & \cdots & & & & & 0_{33} & \Delta_{n,n-1} & \Delta_{n,n} & 0_{31} \\ 0_{13} & \cdots & & & & & 0_{13} & 0_{13} & 0_{13} & \Delta_{n+1,n+1} \end{bmatrix}$$

$$\ldots\ldots\ldots\ldots (2\text{-}36)$$

$$\Delta_{i,i-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{\beta_{i-1}}{m_i} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad i = 2,3,\ldots n \quad \ldots\ldots\ldots(2\text{-}37a)$$

$$\Delta_{i,i} = \left\{ \begin{array}{ll} \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\left(\frac{\beta_1}{m_1} + g\lambda_1\right) & -\frac{e_1}{m_1} \\ 0 & 0 & -\frac{1}{\sigma_i} \end{bmatrix} & i = 1 \\ \begin{bmatrix} 0 & -1 & 0 \\ \frac{\alpha_{i-1}}{m_i} & -\left(\frac{\beta_{i-1}+\beta_i}{m_i} + g\lambda_i\right) & -\frac{e_i}{m_i} \\ 0 & 0 & -\frac{1}{\sigma_i} \end{bmatrix} & i = 2,3,\ldots n-1 \\ \begin{bmatrix} 0 & -1 & 0 \\ \frac{\alpha_{n-1}}{m_n} & -\left(\frac{\beta_{n-1}}{m_n} + g\lambda_n\right) & -\frac{e_n}{m_n} \\ 0 & 0 & -\frac{1}{\alpha_n} \end{bmatrix} & i = n \end{array} \right\} \ldots(2\text{-}37b)$$

$$\Delta_{i,i+1} = \begin{bmatrix} 0 & 0 & 0 \\ -\frac{\alpha_i}{m_i} & \frac{\beta_i}{m_i} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad i = 1,2,\ldots n-1 \quad \ldots\ldots\ldots(2\text{-}37c)$$

$$\Delta_{n+1,n+1} = -\frac{1}{\sigma_{n+1}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}37d)$$

and 0_{33}, 0_{31} and 0_{13} are null matrices of order 3X3, 3X1 and 1X3 respectively.

Also, Ω in (2-35) is the (3n+1) X (2n+1) constant control matrix:

$$\Omega = [\Omega_1 \ldots \Omega_j \ldots \Omega_l \Omega_{l+1} \ldots \Omega_{i+l} \ldots \Omega_{n+l} \Omega_{1+n+l} \ldots \Omega_{i+n+l} \ldots \Omega_{2n+l}] \quad \ldots (2\text{-}38)$$

where Ω_1 through Ω_{2n+l} are defined as follows:

I) Ω_1 through Ω_l:

Each of these (3n+1) - dimensional column vectors is associated with a powered locomotive in the train. Note that the subscript of Ω_l indicates only that there are l such units in the train, and not necessarily that the l^{th} unit is also the l^{th} member of the train; that is, the l powered locomotives could be dispersed throughout the train rather than grouped together at the head end.

Let the j^{th} locomotive be the i^{th} member of the train. Then the only non-zero element of Ω_j is the $(3i-1)^{th}$, which has a value of $\frac{1}{m_i}$.

ii) Ω_{i+1} through Ω_{n+1} :

Each of these n (3n+1) - dimensional column vectors is associated with the braking system for each train member. For the i^{th} member, if $i \neq J$, then the only non-zero element of Ω_{i+1} is the $(3i)^{th}$, which has a value of $\frac{\eta}{\sigma_i}$. If $i = J$, then Ω_{i+1} has one other non-zero element, the $(3n+1)^{th}$, which has a value of $\frac{1}{\sigma_{n+1}}$.

iii) Ω_{i+n+1} through Ω_{2n+1}:

Each of these n (3n+1) - dimensional column vectors is associated with the total resistance to motion at each train member due to grades, curves, friction and air resistance. For the i^{th} member, then, the only non-zero element of Ω_{i+n+1} is the $(3i-1)^{th}$, which has a value of -g.

In summary, with the definitions(2-33), (2-34), (2-36), (2-37) and (2-38), then the state variable representation of the train is given by system (2-35) along with the constraints indicated by (2-13) and (2-20).

CHAPTER 3

FORMULATION AND A METHOD OF SOLUTION OF THE OPTIMAL CONTROL PROBLEM

3.1. The Optimal Control Problem:

Referring to the problem description given in section 1.1, we recall that our objective is to devise a control procedure which minimizes the maximum coupler force experienced anywhere in the train during its operation over a given section of track. This objective will be realized if we can minimize the maximum draft gear extension or compression, since coupler forces are largely proportional to this variable.

If $$\max_{t_0 \leq t \leq t_f} \left[\left|z_1(t)\right|, \left|z_2(t)\right|, \ldots, \left|z_i(t)\right|, \ldots, \left|z_{n-1}(t)\right| \right]$$

represents the maximum draft gear extension or compression experienced anywhere in the train between the start and end of operation at times t_o and t_f respectively, then the optimal control problem may be stated precisely as follows:

"Given the linear system (2-35), it is desired to find an admissible control vector $r(t)$ which transfers the system from its initial state $q(t_0)$ to some desired final state $q(t_f)$ such that the performance measure $\max_{t_0 \le t \le t_f} \left[|z_1(t)|, |z_2(t)|, \ldots, |z_i(t)|, \ldots, |z_{n-1}(t)| \right]$ is minimized." .....(3-1)

3.2. Possible Approaches to the Problem:

The specific problem (3-1) has the following characteristics:

i) The state vector of system (2-35) is of very high dimension; for example, the representation of a 100 member train would require 301 state variables.

ii) There are time delays in the control vector of system (2-35), as is evident in the definition of r given by (2-34). As a matter of fact, the usual description of such a system would be:

$$\dot{q}(t) = \Delta q(t) + \sum_{j=0}^{n} \{\Omega' r'(t-\tau_j) + \Omega'' r''(t-\omega_j)\}$$

where Ω', Ω'', r' and r'' are control matrices and vectors of appropriate dimensions. Olbrot (28) illustrates a procedure for determining the absolute controllability of linear systems with time delays in control, and Sebakhy and Bayoumi [41] give a simplified criterion for the controllability of such systems. However, for the present, it will be convenient to retain the system description given by (2-35) and in particular the control vector definition (2-34).

(iii) The performance measure selected is not of the standard integral form.

(iv) The control vector r is rather severely constrained. In particular, the first l control variables are constrained as in (2-20), the next n as in (2-13), and the final n control variables are actually pre-determined inputs or forcing functions whose value over time is fixed a priori and not subject to manipulation by the optimization process.

(v) We do not seek the optimal control <u>law</u> r{q(t)}; rather, we seek the optimal control <u>function</u> r(t). That is, it would be sufficient for our purposes to determine the optimal open-loop control, rather than the optimal feedback control.

This is consistent with the discussion in section 1.3 regarding the relationship of this work to that of McLane and Peppard [25].

Consideration of these problem characteristics in relation to the available optimization techniques (Pierre[31] provides an excellent catalogue, for example) led the author to the opinion that the only hope for a practical solution method would be to apply some suitable mathematical programming technique to a discrete representation of problem (3-1). For instance, the two point boundary value problem which results from the application of variational techniques is intractable for such large dimensional systems. Similarly, the application of dynamic programming, particularly the state increment form of Larson[22], would have been a very elegant approach, had not Bellman's "curse of dimensionality" made it impractical.

Canon et al[7], Dantzig[9] and Polak[33] provide rigorous treatments of the applicability of mathematical programming (MP) to optimal control problems, and Torng[47], Zadeh and Whalen[52] give specific examples of linear programming (LP) applications in particular.

Wheeler[49] demonstrates a fairly straightforward procedure for transforming any linear optimal control problem into either a linear, quadratic, or convex programming problem. Unfortunately, the method retains the state variables in the MP formulation, so that for other than very low-order systems the matrices involved become extremely large. As an example, application of Wheeler's transformation of the discrete problem analogous to (3-1) for a 100 member train over 100 discrete time intervals would result in an MP matrix containing in the order of two billion elements.

In a procedure outlined by Enns [12,13] and Lack [21], a transformation from state space to control space permits an LP formulation with the crucial property that the dimension of the LP tableau is not directly dependent on the number of state variables. This solution method, to be described in section 3.4, will be applied to a discretized form of problem (3-1).

3.3. Discrete Representation of the Problem:

In order to apply an LP technique, the set of control trajectory values to be optimized must obviously be finite. A discrete form of problem (3-1) must therefore be derived.

3.3.1. System Equations:

Disregarding for the present the time delays present in some elements of the control vector defined in (2-34), let us consider the system (2-35). This representation implies that the control vector is a continuously varying function of time, whereas, in the practical operation of the actual train, this will not be the case. Rather, it is much more realistic to assume that any particular control vector value will be held constant over some minimum time period. Hence, the control variables are actually piecewise - constant signals, and this fact is emphasized by writing:

$$r(t) = r(kT), \quad \left\{ \begin{array}{l} kT \leq t \leq (k+1)T \\ k = 0,1,\ldots,K-1 \end{array} \right\} \quad \ldots\ldots\ldots(3\text{-}2)$$

where $T = t_f/K$ is the minimum time interval over which the control signals are assumed to remain constant, and K is an appropriately specified integer parameter.

It should be stressed that r(kT) is not merely an approximation of r(t); rather, the representation (3-2) is an exact equivalence so long as K is appropriately selected.

With r(t) defined as in (3-2) we no longer require the continuous time response, and in fact need only obtain the values of the state vector at the end of each time interval. To this end, let us first consider the continuous solution of the state equations (2-35), as given in any standard reference (see, for instance, Schwarz and Friedland [40]):

$$q(t) = \Phi(t-t_0)q(t_0) + \int_{t_0}^{t} \Phi(t-t')\Omega r(t')\,dt' \quad \ldots\ldots(3\text{-}3)$$

where t' is a dummy variable and $\Phi(t) = e^{\Delta t}$ is the state transition matrix of the system (2-35).

Now, letting $t_0 = kT$ and $t = (k+1)T$ in (3-3), we have:

$$q\{(k+1)T\} = \Phi(T)q(kT) + \int_{kT}^{(k+1)T} \Phi\{(k+1)T-t'\}\,\Omega r(t')dt' \quad \ldots(3\text{-}4)$$

With r as defined in (3-2):

$$q\{(k+1)T\} = \Phi(T)q(kT) + \left[\int_{kT}^{(k+1)T}\Phi\{(k+1)T-t'\}\Omega dt'\right] r(kT) \quad \ldots(3\text{-}5)$$

The integral term in (3-5) has the same value for all k, and therefore:

$$q\{(k+1)T\} = \Phi(T)q(kT) + \left[\int_0^T \Phi(T-t')\Omega dt'\right] r(kT) \quad \ldots\ldots\ldots\ldots(3\text{-}6)$$

For notational convenience, let us omit the argument T and define:

$$\Delta^* = \Phi(T) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}7)$$

$$\Omega^* = \int_0^T \Phi(T-t')\,\Omega dt' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(3\text{-}8)$$

Then we have the set of (3n+1) difference equations:

$$q(k+1) = \Delta^* q(k) + \Omega^* r(k),\ k = 0,1,\ldots,K-1 \quad \ldots\ldots\ldots\ldots\ldots(3\text{-}9)$$

Enns [12] gives the following equivalent definitions for Δ^* and Ω^*:

$$\Delta^* = \sum_{n=0}^{\infty} \frac{1}{n!} (\Delta T)^n \qquad (3\text{-}10)$$

$$\Omega^* = T\left[\sum_{n=0}^{\infty} \frac{1}{(n+1)!} (\Delta T)^n\right] \cdot \Omega \qquad (3\text{-}11)$$

Equations (3-10) and (3-11) are useful for calculating Δ^* and Ω^* for small values of T, and Johnson and Phillips [18] have derived a practical method of evaluating Ω^* for large T.

The algebraic recursion relation (3-9) is thus an exact discretized representation of system (2-35) for a suitably chosen value of K.

3.3.2. Time Delays in Control:

We may now turn our attention to the time delays in the control vector r. Recalling definitions (2-34) and (3-2), we may write:

$$r_{i+1}(t) = r^*(t-\tau_i) = r^*(k-\overline{\tau}_i) \qquad (3\text{-}12)$$

$$\begin{aligned} r_{i+n+1}(t) &= 0.01d^*(t-\omega_i) + \lambda_{0i} \\ &= 0.01d^*(k-\overline{\omega}_i) + \lambda_0 \qquad (3\text{-}13) \end{aligned}$$

for $i = 1,2,\ldots,n$, and where K has been specified so that $\overline{\tau}_i = \tau_i/T$ and $\overline{\omega}_i = \omega_i/T$ are integers. Also, $\lambda_0 = \lambda_{0i}$ has been assumed constant for all members of the train.

Therefore r_{i+l} is merely the control variable $r^*(k)$ delayed in time by $\overline{\tau}_l$ intervals. Similarly, r_{i+n+l} is the control variable $\{0.01d^*(k)+\lambda_0\}$ delayed in time by $\overline{\omega}_l$ intervals. Since (3-9) is a set of difference equations, we may take advantage of relationships (3-12) and (3-13) to reduce the dimension of the control vector by incorporating the system time delays in the definitions of the system matrix and the state vector (see Noton [26] for example).

To this end, let us define the (l+2) - dimensional control vector u(k) as follows:

$$\begin{bmatrix} f_1^*(k) \\ \vdots \\ f_j^*(k) \\ \vdots \\ f_l^*(k) \\ r^*(k) \\ 0.01d^*(k)+\lambda_0 \end{bmatrix} = \begin{bmatrix} u_1(k) \\ \vdots \\ u_j(k) \\ \vdots \\ u_l(k) \\ u_{l+1}(k) \\ u_{l+2}(k) \end{bmatrix} = u(k) \quad \ldots\ldots (3\text{-}14)$$

Let us further define the $(3n+l+1+\overline{\tau}_n+\overline{\omega}_n)$ - dimensional state vector x(k) by augmenting the vector q of definition (2-33) as follows:

$$\begin{bmatrix} q(k) \\ \hdashline q'_1(k) \\ \vdots \\ q'_j(k) \\ \vdots \\ q'_l(k) \\ q'_{l+1}(k) \\ \vdots \\ q'_{l+\bar{\tau}_n}(k) \\ q'_{l+\bar{\tau}_n+1}(k) \\ \vdots \\ q'_{l+\bar{\tau}_n+\bar{\omega}_1}(k) \\ \vdots \\ q'_{l+\bar{\tau}_n+\bar{\omega}_n}(k) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ \vdots \\ x_{3n+1}(k) \\ \hdashline x_{3n+2}(k) \\ \vdots \\ x_{3n+1+l+\bar{\tau}_n+\bar{\omega}_n}(k) \end{bmatrix} = x(k) \quad \ldots (3\text{-}15)$$

where $q'(k)$ is the vector appended to $q(k)$ and is defined by the following relationships:

$$\left.\begin{array}{lll}
q'_1(k+1) & = f^*_1(k) & = u_1(k) \\
\vdots & & \\
q'_j(k+1) & = f^*_j(k) & = u_j(k) \\
\vdots & & \\
q'_l(k+1) & = f^*_l(k) & = u_l(k) \\
q'_{l+1}(k+1) & = q'_{l+2}(k) & \\
\vdots & & \\
q'_{l+\bar{\tau}_i}(k+1) & = q'_{l+\bar{\tau}_i+1}(k) & \\
\vdots & & \\
q'_{l+\bar{\tau}_n-1}(k+1) & = q'_{l+\bar{\tau}_n}(k) & \\
q'_{l+\bar{\tau}_n}(k+1) & = r^*(k) & = u_{l+1}(k) \\
q'_{l+\bar{\tau}_n+1}(k+1) & = q'_{l+\bar{\tau}_n+2}(k) & \\
\vdots & & \\
q'_{l+\bar{\tau}_n+\bar{\omega}_i}(k+1) & = q'_{l+\bar{\tau}_n+\bar{\omega}_i+1}(k) & \\
\vdots & & \\
q'_{l+\bar{\tau}_n+\bar{\omega}_n-1}(k+1) & = q'_{l+\bar{\tau}_n+\bar{\omega}_n}(k) & \\
q'_{l+\bar{\tau}_n+\bar{\omega}_n}(k+1) & = 0.01d^*(k)+\lambda_0 & = u_{l+2}(k)
\end{array}\right] \quad \ldots\ldots(3\text{-}16)$$

Note that a time delay of T has been introduced in the control variable $u_j(k) = f_j^*(k)$, $j = 1,2,\ldots l$. This will facilitate the expression of constraints (2.20), since $v_i(k)$ and $f_j^*(k-1)$ are now both in state space and, as we shall see, linear state variable constraints can readily be accommodated in the LP formulation. This delay may either be regarded as representative of the reaction time of the engineman and the diesel locomotive, or may be discounted entirely by treating $f_j^*(k-1)$ as the actual system input when the optimal control trajectory has been found.

With x(k) and u(k) defined as in (3-15) and (3-14), the relationships (3-16) may be included in the state variable description of the system (3-9) by writing:

$$x(k+1) = Ax(k) + Bu(k), \; k = 0,1,\ldots,K-1 \qquad \ldots\ldots\ldots(3\text{-}17)$$

where the $(3n+1+l+\bar{\tau}_n+\bar{\omega}_n) \times (3n+1+l+\bar{\tau}_n+\bar{\omega}_n)$ constant system matrix A is given by:

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \qquad \ldots\ldots\ldots(3\text{-}18)$$

and the various sub-matrices of A are defined as follows:

i) A_{11} is the $(3n+1) \times (3n+1)$ matrix:

$$A_{11} = \Delta^* \qquad \text{.........(3-19)}$$

ii) A_{12} is the $(3n+1) \times (1+\bar{\tau}_n+\bar{\omega}_n)$ matrix:

$$A_{12} = \Big[\Omega_1^* \;\ldots\; \Omega_j^* \ldots \Omega_i^* \; \Omega_{n+1}^* \;\ldots \underline{0} \ldots \;\; \Omega_{i+1}^* \ldots \underline{0} \ldots \Omega_{1+1}^* \;\ldots \underline{0} \ldots \Omega_{2n+1}^* \ldots \;\; \underline{0} \ldots \Omega_{i+n+1}^* \ldots \underline{0} \ldots \Omega_{1+n+1}^*\Big] \qquad \text{........(3-20)}$$

in which Ω_1^* through Ω_{2n+1}^* are the column vectors of the matrix Ω^*. Note particularly the inverted order of Ω_{1+1}^* through Ω_{n+1}^*, and of Ω_{1+n+1}^* through Ω_{2n+1}^*. Note also that if $\bar{\tau}_n > n$ there are $(\bar{\tau}_n - n)$ $(3n+1)$ - dimensional null vectors $\underline{0}$ dispersed through the sequence Ω_{1+1}^* through Ω_{n+1}^*. Similarly, if $\bar{\omega}_n > n$ there are $\bar{\omega}_n - n)$ $\underline{0}$ vectors dispersed through the sequence Ω_{1+n+1} through Ω_{2n+1}^*. There are no $\underline{0}$ vectors for $(\bar{\tau}_n+\bar{\omega}_n) \leq 2n$.

iii) A_{21} is an $(1+\bar{\tau}_n+\bar{\omega}_n) \times (3n+1)$ null matrix

iv) $A_{22} = \left[a_{ij}\right]$ is the $(1+\bar{\tau}_n+\bar{\omega}_n) \times (1+\bar{\tau}_n+\bar{\omega}_n)$ matrix:

$$a_{ij} = \begin{cases} 1, & i=(l+1),\ldots,(l+\overline{\tau}_n-1);\ j=(l+2),\ldots,(l+\overline{\tau}_n) \\ 1, & i=(l+\overline{\tau}_n+1),\ldots,(l+\overline{\tau}_n+\overline{\omega}_n-1);\ j=(l+\overline{\tau}_n+2),\ldots,(l+\overline{\tau}_n+\overline{\omega}_n) \\ 0, & \text{otherwise} \end{cases} \qquad \ldots\ldots(3\text{-}21)$$

and where the $(3n+1+l+\overline{\tau}_n+\overline{\omega}_n) \times (l+2)$ constant control matrix $B = \left[b_{ij}\right]$ is defined by:

$$b_{ij} = \begin{cases} 1, & i=(3n+2),\ldots,(3n+1+l);\ j=1,\ldots,l \\ 1, & i=(3n+1+l+\overline{\tau}_n);\ j=(l+1) \\ 1, & i=(3n+1+l+\overline{\tau}_n+\overline{\omega}_n);\ j=(l+2) \\ 0, & \text{otherwise} \end{cases} \qquad \ldots\ldots(3\text{-}22)$$

Thus, (3-17) constitutes a discretization of system (2-35) in which the time delays in control are represented by additional state variables.

3.3.3. System Constraints:

I) The constraints (2-20) on $u_j(k) = f_j^*(k)$, $j=1,\ldots,l$, cannot be expressed in this form since only linear constraints can be accommodated by an LP technique. Instead, the following approximations will be considered as the constraints on the tractive or braking effort:

$$\left.\begin{aligned} \left|x_{3n+1+j}(k)\right| &\leq \mu W_j \\ x_{3n+1+j}(k) + L'x_2(k) &\leq L'' \\ -x_{3n+1+j}(k) + M'x_2(k) &\leq M'' \\ x_{3n+1+j}(k) + \xi x_2(k) &\geq 0 \\ x_{3n+1+j}(k) &\geq -Z \end{aligned}\right\} \quad j=1,\ldots,l \qquad \ldots\ldots\ldots(3\text{-}23)$$

where L', L'', M' and M'' are positive constants chosen so as to best approximate the constraints (2-20) over the speed range of interest, and where $x_{3n+1+j}(k) = u_j(k-1) = f_j^*(k-1)$. Since usually only a specific speed range is desired, not all of the constraints

(3-23) may need to be applied. For example, if the speed of the train (as defined by $x_2(k) = v_1(k)$) is restricted to between V' and V'' in Figure 3.1, then only the indicated subset of (3.23) is required.

If a single train speed is specified rather than a range as in Figure 3.1, then obviously only absolute bounds on $u_j(k)$, $j=1,\ldots,l$, need be applied.

II) The constraints (2-13) on $u_{l+1}(k) = r^*(k)$ are difficult to express simultaneously. Instead, the problem will first be run with the constraint:

$$0 \le u_{l+1}(k) \le 23, \quad k=0,1,\ldots,K-1 \quad \ldots\ldots\ldots(3\text{-}24)$$

Then by examining the optimal trajectory of u(k) and x(k), one or the other of the following constraints will be applied for values of k specified on the the basis of the trajectory of $x_{3n+1}(k)$:

$$\left\{\begin{array}{ll} \text{either} & u_{l+1}(k) = 0 \\ \text{or} & u_{l+1}(k-1) \le u_{l+1}(k) \le 23 \end{array}\right\} \text{(k as specified)} \ldots (3\text{-}25)$$

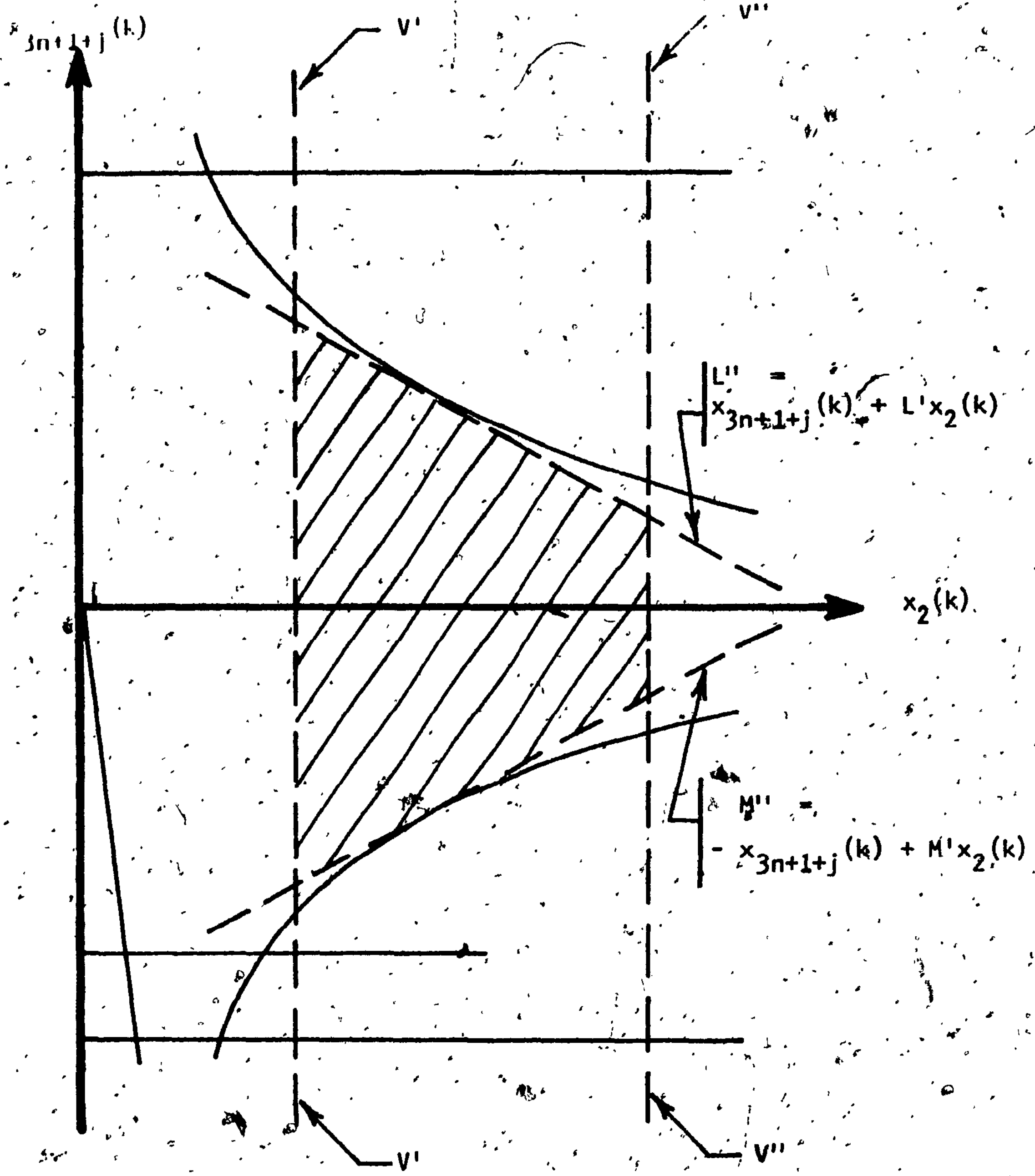


Figure 3.1: Approximation of constraints (2-20) by linear constraints (3-23). Hatched area is allowable region for the indicated V' and V''. Solid lines are constraints (2-20) from Figure 2.5.

With constraints (3-25) specified for appropriate values of k, a new optimal trajectory should then be calculated.

(III) Train speed will usually be restricted by one of the following constraints:

$$\begin{cases} \text{either} & x_2(k) = V' \\ \text{or} & V' \le x_2(k) \le V'' \end{cases} \quad \text{(k as specified)} \;\ldots\ldots (3\text{-}26)$$

(IV) As indicated in section 3.2(iv), the control variable $u_{1+2}(k)$ is actually fixed a priori by the specification of the effective grade $d^*(k)$ as a function of k.

$$u_{1+2}(k) = 0.01d^*(k) + \lambda_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3\text{-}27)$$

If the first of the constraints (3-26) is specified, then $x_1(k) = s_1(k)$ is known and hence $d^*(k)$ as well as $u_{1+2}(k)$ may be defined exactly. However, if the second of the constraints (3-26) is applied to the system then the following iterative approach is required:

i) guess at the probable trajectory of $x_1(k)$.

ii) based on (i), specify $d^*(k)$ and hence $u_{1+2}(k)$.

(iii) calculate the optimal trajectories,

(iv) examine $d^*(k)$ and $x_1(k)$ from step (iii) and re-specify $d^*(k)$ accordingly.

(v) repeat steps (iii) and (iv) until a satisfactory correlation of $x_1(k)$ and $d^*(k)$ is obtained.

3.3.4. Restatement of the Problem

"Given the linear discrete-time system (3-17), it is desired to find a control vector u(k) which transfers the system from its initial state x(0) to some desired final state x(K) such that the specified constraints (3-23) through (3-27) are met, and such that the performance measure $\max_{0 \le k \le K} \left\{ |x_4(k)|, |x_7(k)|, \ldots, |x_{3i+1}(k)|, \ldots, |x_{3n-2}(k)| \right\}$ is minimized. ......(3-28)

3.4. A Method of Solution

Lack [21] and Enns [12,13] give a straightforward procedure for calculating the optimal control trajectory for problems such as (3-28) by linear programming. The reader is referred to reference (3.5) in particular for an excellent detailed description of the method, but a brief summary of the procedure in terms of problem (3-28) is presented here for convenience.

3.4.1. Basic Solution Method:

Consider the system:

$$x(k+1) = Ax(k) + Bu(k), \quad k = 0,1,\dots,K-1 \quad \text{.................} (3\text{-}17)$$

where $x(k)$ is the $(3n+1+1+\overline{\tau}_n+\overline{\omega}_n)$ - dimensional state vector, $u(k)$ is the $(1+2)$ - dimensional control vector, and A and B are the constant matrices of the system as defined by (3-18) and (3-22) respectively.

The initial condition:

$$x(0) = x_0 \quad \text{....................} (3\text{-}29)$$

and the terminal condition:

$$Fx(K) = f \quad \text{.......................} (3\text{-}30)$$

must be satisfied, where f is an $\overline{m}$ - dimensional vector and F is an $\overline{m}x(3n+1+1+\overline{\tau}_n+\overline{\omega}_n)$ matrix.

The optimality criterion is specified by defining a region E^+ in state space and requiring the state trajectory to stay as far from this region as possible. That is, we wish to find the control which maximizes the distance of closest approach of the trajectory $x(k)$ to the region E^+ while meeting conditions (3-29) and (3-30). The region E^+ is the half-space in E^n defined by:

$$E^+ = \{ x : \langle d,k \rangle > \langle d,d \rangle \} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3\text{-}31)$$

where d is a vector normal to the separating hyperplane, as illustrated for the two dimensional case in Figure 3.2. (The half-space E^- is defined analogously). The notation $\langle d,x \rangle$ indicates the inner (scalar) product of the vectors d and x; geometrically, $\langle d,x \rangle$ may be considered to be the "projection" of the vector x on the direction vector d.

For the specific problem (3-28), we ideally require $2(n-1)$ such d vectors to be defined, each specifying the "danger region" for the state variable x_{3i+1}, $i = 1,2,\ldots, n-1$ in turn. This point will be discussed further in chapter 5.

Enns shows that the maximization of the minimum distance from the trajectory $x(k)$ to the region E^+ is equivalent to the minimax problem:

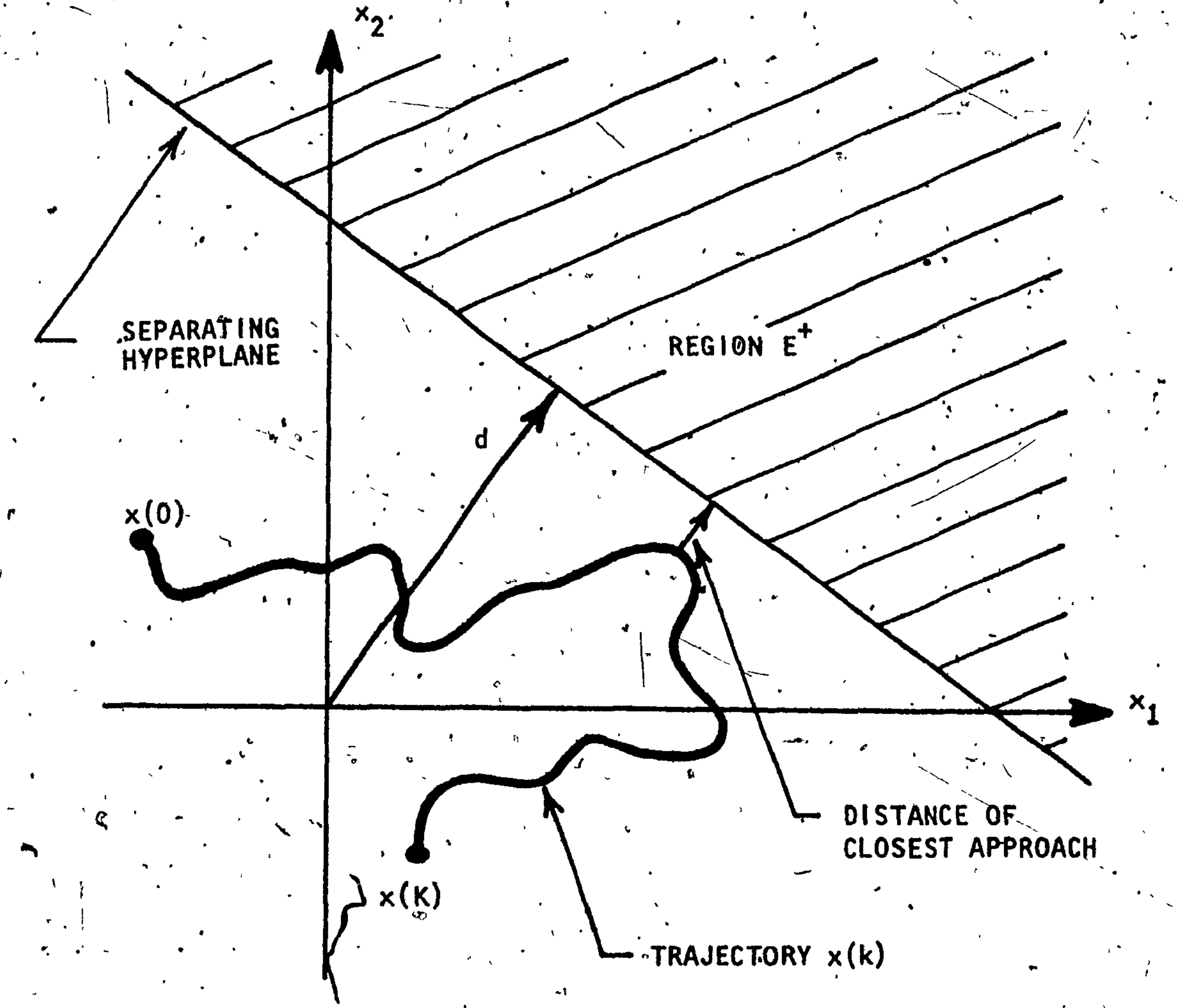


Figure 3.2: Illustration of optimality criterion for state space of dimension 2.

$$\left\{ \begin{array}{l} \text{Find } u(k) \text{ so as to:} \\ \\ \max\limits_{u(k)} \; \min\limits_{k} \left[\dfrac{\langle d, d-x(k) \rangle}{\langle d,d \rangle} \right] \end{array} \right\} \qquad \ldots\ldots\ldots\ldots (3\text{-}32)$$

The key to transforming problem (3-32) to an LP problem is to introduce a dummy variable π (a scalar), and the set of constraints:

$$\langle d, x(k) \rangle \leq \pi \, , \quad k = 0, 1, \ldots, K \qquad \ldots\ldots\ldots\ldots (3\text{-}33)$$

Then (3-32) is equivalent to the constrained minimization problem:

$$\left[\min_{u(k)} \; \pi \, , \text{ subject to } (3\text{-}33) \right] \qquad \ldots\ldots\ldots\ldots (3\text{-}34)$$

Accomplishing (3-34) amounts to "squashing" the state trajectory with a hyperplane normal to the d vector and parallel to the boundary of the region E^+.

To put the problem (3-34) into an LP form, define the $\{(l+2)\cdot K+1\}$ - dimensional vector u and the constant $\{(l+2)\cdot K+1\}$ - dimensional vector c by:

$$u = \begin{bmatrix} u(0) \\ u(1) \\ \cdot \\ \cdot \\ \cdot \\ u(K-1) \\ \pi \end{bmatrix} \qquad \text{.......................(3-35)}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \qquad \text{.......................(3-36)}$$

where it is understood that u(k) is defined by (3-14).

Now, the crucial operation in this procedure is the transformation from state space to control space accomplished by writing the state vector x at any time k as a linear function of u by repeated use of (3-17) with (3-29):

$$x(1) = Ax(0) + Bu(0) = Ax_0 + Bu(0)$$

$$x(2) = Ax(1) + Bu(1) = A^2x_0 + ABu(0) + Bu(1)$$

$$\cdot$$

$$\cdot$$

$$x(k) = Ax(k-1) + Bu(k-1)$$

$$= A^kx_0 + A^{k-1}Bu(0) + A^{k-2}Bu(1) + \ldots + ABu(k-2) + Bu(k-1)$$

or:

$$x(k) = A^kx_0 + \sum_{j=0}^{k-1} A^{k-1-j}\,Bu(j) \qquad \text{.......................(3-37)}$$

By defining the $(3n+1+1+\overline{\tau}_n+\overline{\omega}_n)$ X $(1+2)K$ matrix $D(k)$ as:

$$D(k) = \left[A^{k-1}B\ A^{k-2}B \ldots AB\ B\ 0 \ldots 0\right] \quad \ldots\ldots\ldots\ldots\ldots (3\text{-}38)$$

then (3-37) may be written:

$$x(k) = A^k x_0 + \left[D(k) \middle| 0\right] \cdot u \quad \ldots\ldots\ldots\ldots\ldots (3\text{-}39)$$

and hence the condition (3-30) as:

$$\left[F\ D(K) \middle| 0\right] \cdot u = f - FA^K x_0 \quad \ldots\ldots\ldots\ldots\ldots (3\text{-}40)$$

and the constraints (3-33) as:

$$\left[\langle d, D(k)\rangle \;\middle|\; -1\right] \cdot u \leq -\langle d, A^k x_0\rangle, \; k = 0,1,\ldots,K \quad \ldots\ldots (3\text{-}41)$$

where $\langle d, D(k)\rangle$ indicates the inner product of d with each of the columns of D(k), the result being a row vector.

Thus the control problem (3-32) is shown by Enns to be equivalent to the problem:

$$\left\{\begin{array}{l} \text{Find u to } \min\limits_{u} \langle c,u\rangle \\ \text{subject to (3-40) and (3-41)} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots (3\text{-}42)$$

This is the standard form of linear programming and the optimal control may be found directly by the simplex algorithm (see Künzi [20], for instance).

A distinct advantage of this method is that the problem (3-42) may be readily extended to include the linear constraints associated with problem (3-28) directly as additional LP constraints.

3.4.2 Extension of the Problem to Include State Variable Constraints:

Enns shows that any linear equality or inequality relation for the state variables may be included as additional LP constraints by means of (3-37) and (3-38). In general, the constraint:

$$\langle d, x(k) \rangle \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b, \quad k = 0,1,\ldots,K \qquad \ldots\ldots(3\text{-}43)$$

may be formulated as the set of (K+1) scalar LP constraints:

$$\left.\begin{array}{l} 0 \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b - \langle d, x_0 \rangle \\ \begin{bmatrix} \langle d, D(1) \rangle \,|\, 0 \end{bmatrix} u \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b - \langle d, Ax_0 \rangle \\ \vdots \\ \begin{bmatrix} \langle d, D(k) \rangle \,|\, 0 \end{bmatrix} u \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b - \langle d, A^k x_0 \rangle \\ \vdots \\ \begin{bmatrix} \langle d, D(K) \rangle \,|\, 0 \end{bmatrix} u \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b - \langle d, A^K x_0 \rangle \end{array}\right\} \qquad \ldots\ldots(3\text{-}44)$$

Note that these constraints may be time-varying; that is $b = b(k)$ in general.

3.4.3. Extension of the Problem to Include Control Variable Constraints:

Again, Enns shows that any linear equality or inequality constraint on the controls may be included as additional LP constraints. Although constraints on the control variables of the type (3-43) may be similarly expressed, the more usual situation is that the constraints are simple bounds on the absolute value of a particular control variable. This type of constraint:

$$\left| u_i(k) \right| \leq H(k) \qquad \text{(3-45)}$$

is equivalent to the two linear constraints:

$$\left.\begin{array}{r} u_i(k) \leq H(k) \\ -\,u_i(k) \leq H(k) \end{array}\right\} \qquad \text{(3-46)}$$

and hence may be readily included in problem (3-42) as additional LP constraints. The even simpler constraint :

$$u_i(k) = H(k) \qquad \text{(3-47)}$$

may obviously be incorporated in the LP formulation as well. Constraint (3-47) is the means by which a particular control variable is treated as a fixed input. The notation H(k) has been used in (3-45) to (3-47) to emphasize the fact that the inequality or equality constraint on the control variable may be time varying.

3.5. Linear Programming Problem Equivalent to the Control Problem:

Providing that a suitable set of direction vectors d can be defined, we have thus transformed the original control problem (3-1) to the linear programming problem:

$$\left.\begin{array}{l}\text{"Find } u \text{ to } \min\limits_{u} \langle c,u\rangle \text{ subject to constraints (3-40)} \\ \text{and (3-41), in addition to constraints (3-44),} \\ \text{(3-46) and (3-47) as required."}\end{array}\right\} \quad \ldots\ldots.(3\text{-}48)$$

A remarkable feature of this method is that the dimension of the state vector of problem (3-28), the discrete equivalent of (3-1), does not enter into the dimension of the equivalent LP problem (3-48). In particular, if the:

i) number of control variables = (l+2)

ii) number of scalar control constraints = $\bar{u}$

iii) number of state trajectory constraints or minimax objectives = $\bar{x}$

iv) number of state variable terminal constraints = $\bar{m}$

v) number of discrete time intervals = K

then, as illustrated by Enns [12], the dimensions of the corresponding LP matrix are as follows:

$$\left\{ \begin{array}{ll} \text{Number of rows} = & \{(\bar{x}+\bar{u})K + \bar{x}+\bar{m}\} \\ \text{Number of columns} & \{(1+2)K + 1\} \end{array} \right\} \qquad \text{(3-49)}$$

Thus the limiting factor is now the total number of constraints and discrete time intervals rather than the dimension of state space. This characteristic will be elaborated in Chapter 5, but let us first illustrate the technique with a small example.

CHAPTER 4

EXAMPLE: A THREE-MEMBER TRAIN

To illustrate the solution technique, consider the simple case of a train consisting of just three "members." However, in order that reasonably large coupler forces will be realized, we shall assume that the leading train member actually comprises two 2,000-hp locomotives and ten freight cars, and that the other two members each comprise twelve freight cars. The gross weight of each freight car and locomotive is taken to be 80 tons and 190 tons respectively, and the overall length of each vehicle is assumed to be 44 feet. The velocity at some given initial position is taken to be 45 mph, and we require the train to travel an additional mile in 80 seconds. It should be noted that although we are requiring the average speed to be equal to the initial speed, we are not specifying what the velocity trajectory should be over the run time interval, except that a speed limit of 60 mph is assumed to apply over the given territory.

The parameter values of this 4000 hp, 3100 ton train are given in Table 4.1.

$m_1 = (2\times190) + (10\times80) = 1180 \text{ tons} = 73{,}292 \text{ slugs}$

$m_2 = m_3 = (12\times80) = 960 \text{ tons} = 59{,}627 \text{ slugs}$

$\alpha_1 = \alpha_2 = 500{,}000 \text{ lbs/ft.}$

$\beta_1 = \beta_2 = 15{,}000 \text{ lbs-sec/ft.}$

$e_1 = e_2 = e_3 = 1440 \text{ in.}^2$

$\lambda_0 = 0$

$\lambda_1 = 0.000054 \text{ secs/ft.}$

$\lambda_2 = \lambda_3 = 0.000045 \text{ secs/ft.}$

$\sigma_1 = \sigma_2 = \sigma_3 = 15 \text{ secs.}$

$\sigma_4 = 60 \text{ secs.}$

$\tau_1 = 4 \text{ secs.}$

$\tau_2 = 8 \text{ secs.}$

$\tau_3 = 12 \text{ secs.}$

$\omega_1 = 8 \text{ secs}$

$\omega_2 = 16 \text{ secs.}$

$\omega_3 = 24 \text{ secs.}$

$\eta = 2.5$

Table 4.1: Train Parameter Values

The ω_i's of Table 4.1 are calculated from (2-28), where $h_i + w_i = 12 \times 44 = 528$ ft., for $i = 1,2,3$.

Let the effective grade under consideration be such that its maximum frequency of change in value is once per 350 feet. At the given speed limit of 88 ft/sec., this implies that the effective grade input to each member will be constant for at least four seconds. Since it is reasonable to require the other inputs to the system (ie., air brakes and throttle/dynamic brake applications) to be held constant for at least this maximum time period as well, then it is concluded that a discrete time interval of four seconds, would be a satisfactory assumption in this case. We therefore require the number of time intervals to be $K = t_f/T = 80/4 = 20$.

Let the effective grade as a function of discrete time interval k be specified as in Figure 4.1.

Note from Table 4.1 that λ_0 has been assumed to be zero. Thus the control variable $u_3(k)$, as defined by (3-27), is given simply as 0.01d*(k). This assumption for λ_0 was made merely to simplify the example; any other value would just serve to bias this function.

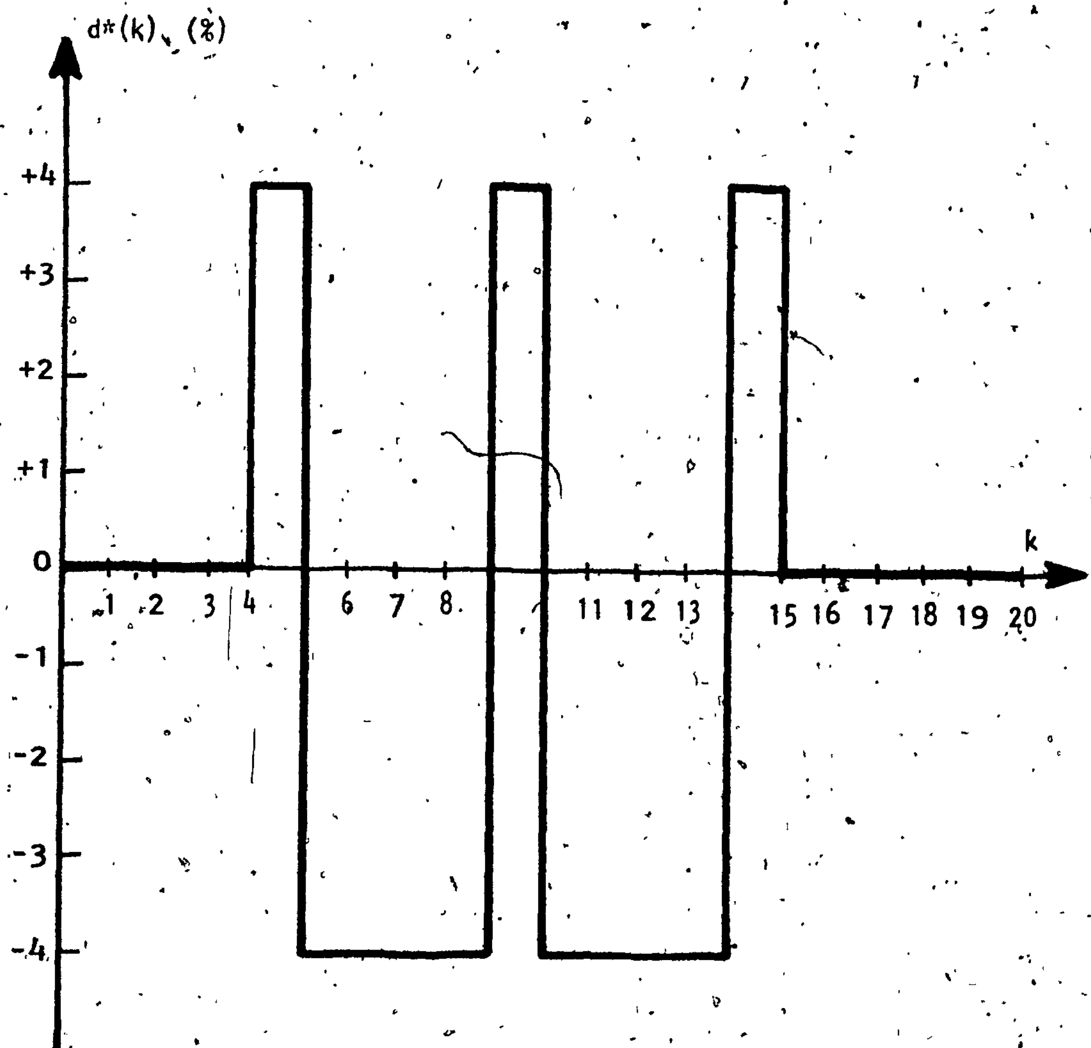


Figure 4.1: Effective grade as a function of discrete time interval k. d*(k) is specified relative to $s_0(k)$ (see discussion in section 2.6.2).

Let us now cast this problem in terms of the previous developments in Chapter 3.

For this case, we have $n = 3$, $l = 1$, $\overline{\tau}_n = \tau_3/T = 3$, and $\overline{\omega}_n = \omega_3/T = 6$. Thus the dimension of the state vector $x(k)$ is 20, and that of the control vector $u(k)$ is 3.

As the initial condition x_0 of the train, we shall assume that the starting position is considered to be zero, the velocities of the three members are equal, the draft gears are neither extended nor compressed and the brake cylinders have all been released for some previous time sufficient to ensure that no false gradient presently exists. Then for x_0 we have:

$$x_0 = [0\ 66\ 0\ 0\ 66\ 0\ 0\ 66\ 0\ 0\ 0\ \ldots\ 0]^T \qquad (4\text{-}1)$$

Now consider for a moment the 61 - dimensional vector u, as defined by (3-35):

$$u = [u_1(0)\ u_2(0)\ u_3(0)\ u_1(1)\ u_2(1)\ u_3(1)\ \ldots\ u_1(19)\ u_2(19)\ u_3(19)\ \pi]^T \qquad (4\text{-}2)$$

In the LP formulation in Chapter 3, it was assumed that all the LP variables (ie., the elements of u) were positive. Except for $u_2(k)$, this is not the case for our train; however, we can establish definite lower bounds for each of the other elements of u. For example, since we have specified that the average speed over the 80 second run should be 45 mph, we may be certain that the train speed will never drop below, say, 20 mph. Hence, by applying inequality (2-17), we know that $u_1(k) \geq -87,000$ for all k. Similarly, from the given effective grade specifications, we know that $u_3(k) \geq -0.04$ for all k. Therefore, u may be offset appropriately so that the resultant vector contains only positive elements. To this end, let u' be defined as:

$$u' = u + U \quad \text{.....................................(4-3)}$$

where U is a constant offset vector, given in this example as:

$$U = \begin{bmatrix} 87,000 & 0 & .04 & 87000 & 0 & .04 & \dots & 87000 & 0 & .04 \end{bmatrix}^T \quad \text{..(4-4)}$$

By substituting u' for u as the LP vector, the requirement for positive LP variables is met. The LP problem may then be solved, and the true solution subsequently may be derived simply by subtracting U from the optimal u' obtained.

If a 61 - dimensional cost vector c is defined as in (3-36), then our problem is to find u to minimize $\langle c,u' \rangle$ subject to the following constraints:

I) Objective Constraints:

Define a set of 20 - dimensional direction vectors d_{3i+1} for $i = 1,2$, where the $(3i+1)^{th}$ element is 1 and the remainder are zero. Then, from (3-41) and (4-3), we have the set of 42 objective constraints:

$$\left[\langle d_{3i+1}, D(k)\rangle \;\middle|\; -1\right] u' \le \left[\langle d_{3i+1}, D(k)\rangle \;\middle|\; -1\right] U - \langle d_{3i+1}, A^k x_0\rangle \quad \text{.................. (4-5)}$$

for $i = 1,2$ and $k = 0,1,\ldots, 20$.

(II) Terminal Constraint:

The only terminal constraint being set for this train $(\bar{m} = 1)$ is that its position at the end of the 80 second time interval should be one mile more than its initial position. From (3-40) and (4-3), this constraint is expressed by:

$$\left[FD(20) \;\middle|\; 0\right] u' = \left[FD(20) \;\middle|\; 0\right] U + f - FA^{20}x_0 \quad \text{............. (4-6)}$$

where the 20 dimensional row vector $F = [1\ 0\ 0\ \ldots\ 0]$ and $f = 5280$.

III) Velocity Constraints:

Define a 20 - dimensional vector d_2 whose second element is equal to 1 and the remainder are 0. Then from (3-44) and (4-3), we have the following 20 velocity constraints:

$$\left[\langle d_{2}, D(k)\rangle \middle| 0\right] u' \leq \left[\langle d_{2}, D(k)\rangle \middle| 0\right] U + V_{MAX} - \langle d_{2}, A^{k} x_{0}\rangle \quad \text{.........(4-7)}$$

for k = 1,2,...,20 and where V_{MAX} = 88 for this example.

IV) Tractive Effort Constraints:

With reference to Figure 3.1, reasonable values for V' and V'' would be 30 and 60 mph respectively. By applying equality (2-15), the parameters of constraints (3-23b) and (3-23c) over this speed range are found to be as follows: L' = 465.91, L'' = 61,500, M' = 659.1 and M'' = 87,000. Define d_L as a 20-dimension vector whose second element is equal to L', whose eleventh element is +1, and whose remaining elements are 0. Similarly, define d_M whose second element is equal to M', whose eleventh element is -1 and whose remaining elements are 0. Then, from (3-44) and (4-3), the following 40 tractive effort constraints apply:

$$\left[\langle d_{L}, D(k)\rangle \middle| 0\right] u' \leq \left[\langle d_{L}, D(k)\rangle \middle| 0\right] U + L'' - \langle d_{L}, A^{k} x_{0}\rangle \quad \text{......(4-8)}$$

$$\left[\langle d_{M}, D(k)\rangle \middle| 0\right] u' \leq \left[\langle d_{M}, D(k)\rangle \middle| 0\right] U + M'' - \langle d_{M}, A^{k} x_{0}\rangle \quad \text{......(4-9)}$$

for k = 1,2,..., 20.

V) Effective Grade Constraints:

From (3-47), (4-3) and (4-4), we have the following 20 grade constraints:

$u'_3(k) = 0.04 + 0.01\, d^*(k)$ ....(4-10)

for k = 0,1,...,19 and for $d^*(k)$ as specified in Figure 4.1 (relative to $s_0(k)$ - see discussion in section 2.6.2).

VI) Air Brake Constraints:

Applying (3-24), we have the following 20 control variable constraints:

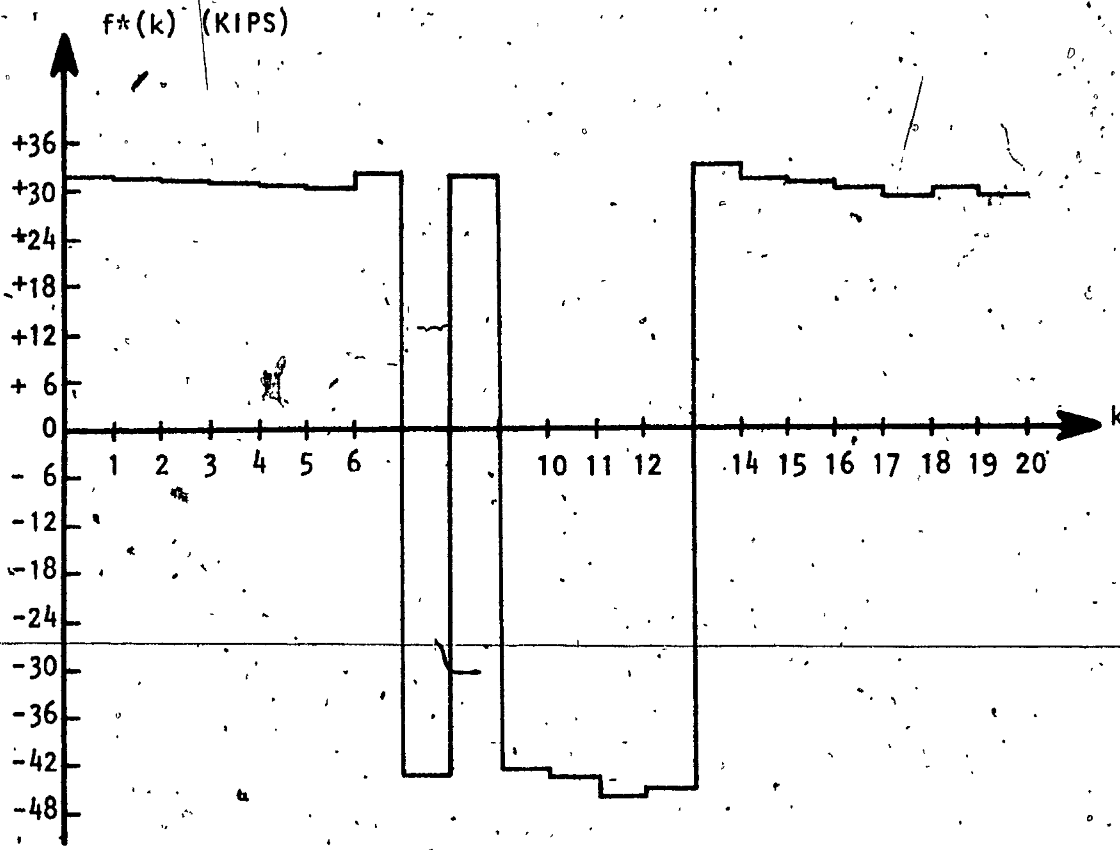
$u'_2(k) \leq 23$ ....(4-11)

for k = 0,1,2,..., 19.

The above defined problem was solved in 2.7 minutes on the Computer Science Canada UNIVAC 1108 computer by employing a system of programs called UMPIRE (Unified Mathematical Programming System Incorporating Refinements and Extensions - see reference (57) and (58)).

The complete solution procedure, including all job control statements, input information, problem definition, and output calculations, is documented in Appendix A for reference.

For convenience, the optimal control trajectory and the resultant force levels in the couplers as a function of k are presented in Fugures 4.2 and 4.3 respectively.



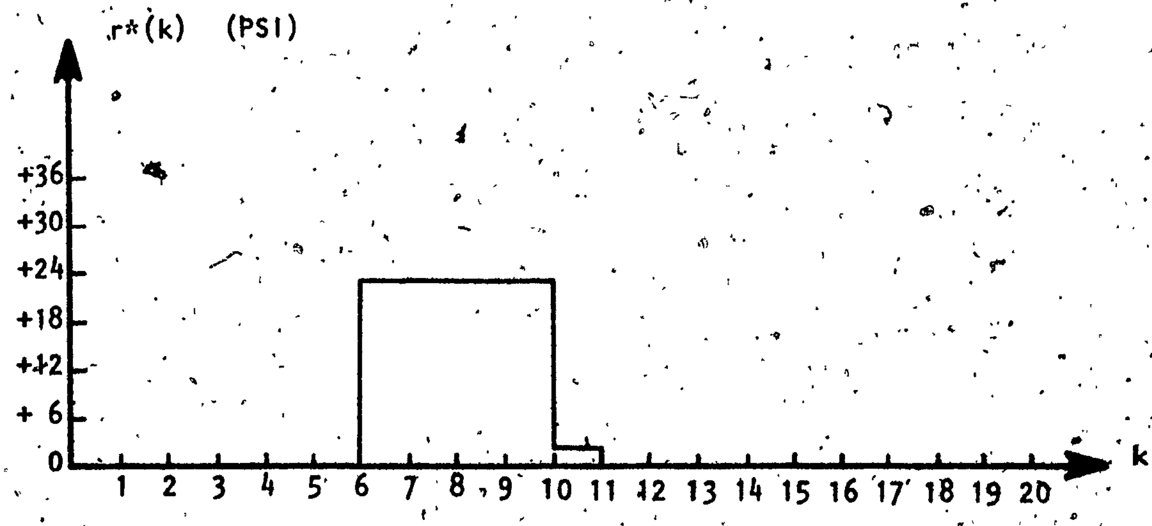


Figure 4.2: Optimal control trajectories u_1 and u_2 (ie., f* and r* respectively). Optimal u_3 is as specified by (3-27) and Figure 4.1.

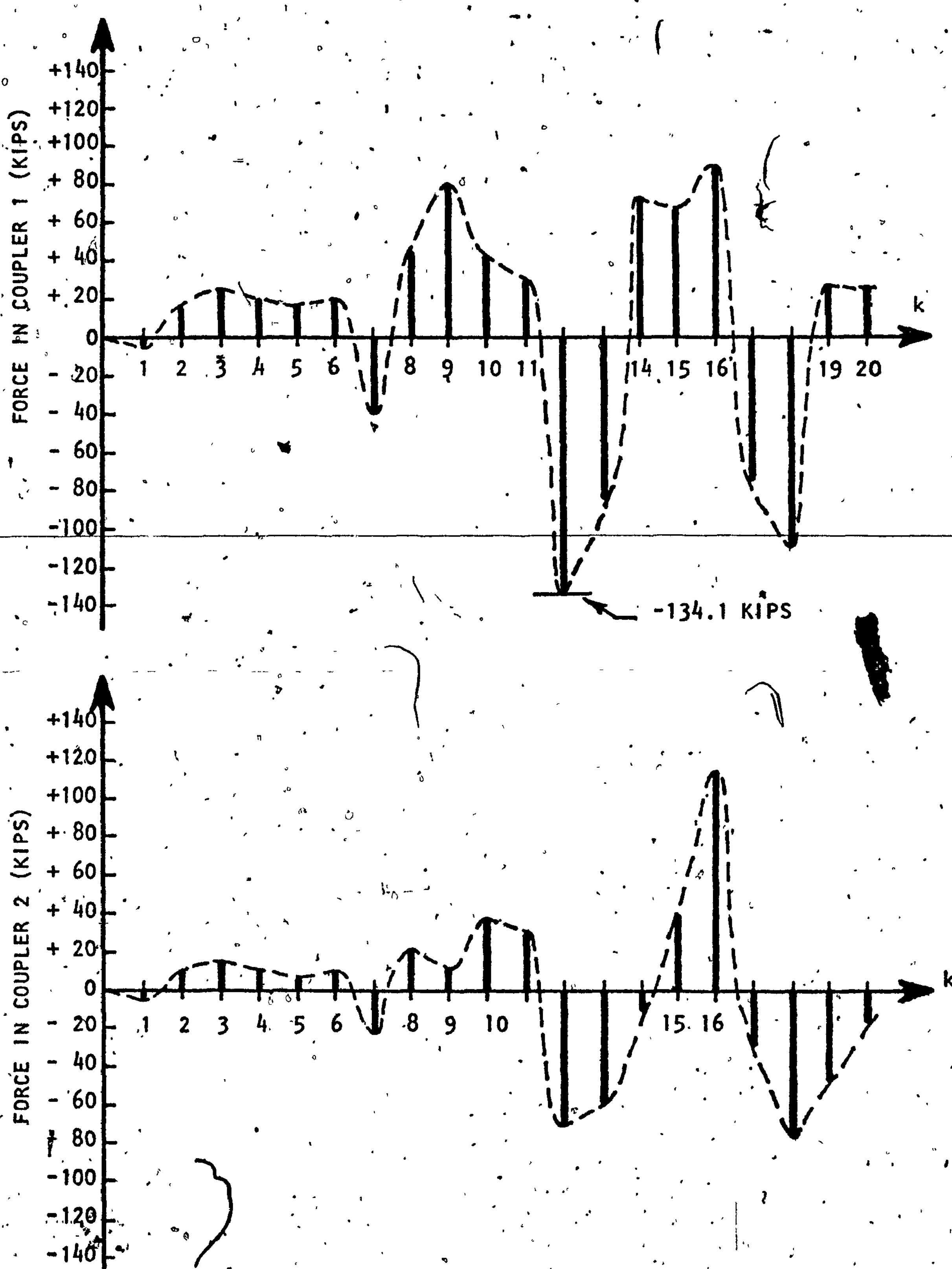


Figure 4.3: Force levels in couplers resulting from the application of the optimal control depicted in Figure 4.2. Exact values are only known at discrete times k; dashed lines are force envelopes.

CHAPTER 5

DISCUSSION OF RESULTS AND SUGGESTIONS FOR FUTURE WORK:

A brief discussion of the salient features of the example problem presented in Chapter 4, as well as the solution obtained, is now in order.

Firstly, as noted in Chapter 3, 2(n-1) direction vectors are ideally required to specify the "danger region" for each of the variables x_{3i+1}, for $i = 1,2,\ldots,(n-1)$ in turn. This is because each x_{3i+1} can take on negative as well as positive values (ie., the draft-gears can be compressed as well as extended). In essence, with reference to Figure 3.2, two hyperplanes are required for each x_{3i+1} in order to minimize its absolute value. However, in order to simplify this example, only the positive values of x_{3i+1} were chosen to be minimized, and therefore, the level of train run-ins was not included directly in the objective constraints. Nevertheless, as shown in Figure 4.3, the maximum coupler force in compression exceeded the maximum coupler force in tension by only 17.6 kips. For practical problems, additional constraints of the form (4-5) should be included in the LP formulation, with direction vectors defined such that the $(3i+1)^{\underline{th}}$ element equals -1 and the remainder are 0. In this manner, the absolute value of coupler forces will be minimized.

Although only two couplers were considered in this problem, the 36 - car 3100 - ton train was subjected to an extremely violent sequence of grade changes, as per Figure 4.1. In fact, this one mile track section is considerably more severe than that likely to be found in the main line of any Canadian railroad. Yet the maximum force levels in these two couplers was only 134 and 116 kips respectively, and both figures are well below the generally accepted critical level of about 250 kips.

However, even though the control trajectory of Figure 4.2 is optimal from the viewpoint of the mathematical problem posed, this solution could not be practically implemented. The reason for this is that there are two characteristics of the solution which conflict with known real constraints of the actual physical system. These are as follows:

i) The air brake release action depicted at time interval 10 is shown as only being a partial release. As discussed in section 2.4, this is not possible in the real system. The problem of course is that we have not completely specified the constraints (2-13) in the formulation of the optimization problem.

2) The instantaneous switching action of the locomotive throttle to dynamic braking at time intervals 7 and 9 is similarly not feasible in the actual physical system. In essence, the traction motors are being asked to instantaneously become generators, and this of course is not possible; the throttle must first be moved to the idle position for some minimal time period (usually 5 seconds) before moving to the dynamic braking region. The problem again, of course, is that this physical restraint was not incorporated as a mathematical constraint in the formulation of the optimization problem.

An obvious and direct way of circumventing these difficulties would be to simply add the appropriate constraints to the LP problem statement. Therefore, it is suggested that the design of these constraint statements is the first extension of this work which should be undertaken.

Alternatively, the most practical way of overcoming these deficiencies in the optimal trajectory might simply be to alter the obtained solution by applying common sense. In the case of our example, for instance, one might make the following modifications:

i) Make the air brake reduction r*(k) go to zero in the tenth interval;

(ii) Rather than switching from braking to tractive effort and back to braking in the 7th, 8th and 9th intervals respectively, make f*(k) go to zero (idle) for the 7th and 8th intervals before going into braking.

These changes would make the resultant control trajectories physically implementatble. To be sure, these controls would be sub-optimal. However, a simulation of the system with the modified inputs (a relatively simple matter, since the difference equation model of the system has been obtained) would be able to quantify the degradation from optimal. Intuitively, from the trivial nature of the changes required, this deterioration should not be too severe. Additional work in this area to test this hypothesis would be well worthwhile.

What is the largest real problem that could be practically solved with the approach outlined in this dissertation? To answer this question, consider again the three separate phases of the solution procedure :

A) obtain discrete model of train

B) formulate LP problem

C) solve LP problem.

Taking these in reverse order, the largest LP problem which UMPIRE can handle is 8,000 rows and 262,000 columns (57). In consideration of this fact and (3-49), one sees that the following inequalities must hold:

$$(\overline{x} + \overline{u}) \cdot K + \overline{x} + \overline{m} \leq 8,000 \qquad \text{..............(5-1)}$$

$$(1+2) \cdot K + 1 \leq 262,000 \qquad \text{..............(5-2)}$$

Generally speaking, $(\overline{x} + \overline{u}) > (1+2)$ so that (5-1) will usually be the limiting constraint. (In order to take full advantage of the capabilities of UMPIRE, some work should be directed towards the formulation of the dual LP problem). Since K will usually be at least two orders of magnitude larger than $\overline{x}$, $\overline{u}$, or $\overline{m}$, then an approximate expression of the limiting constraint for phase C is:

$$K \leq \frac{8000}{(\overline{x}+\overline{u})} \qquad \text{..............................(5-3)}$$

Let us now digress for a moment and consider the difficulties associated with Phase A, ie., obtaining the discrete model of the train. Although the number of state variables does not directly affect the dimension of the LP matrix, it certainly does have considerable import to the problem of calculating the state transition matrix $\Phi(t)$. The evaluation of the Taylor series approximation of (3-10) becomes extremely costly as the dimension of the continuous system matrix Δ grows increasingly large. The author is not aware of what the practical limit for the dimension of Δ might be before the calculation of $\Phi(t)$ becomes infeasible on the basis of cost and/or accuracy. However, it would seem reasonable to assume that a 61 X 61 matrix would not present too much difficulty. Thus, a 20-member train would certainly seem capable of being treated. By considering a 100-car train to be made up of 5-car members, a discrete model for such a train may readily be obtained.

The total number of state and control variable constraints for the example of Chapter 4 was equal to 7, and this number is not likely to exceed 15 for practical trains. Therefore, from (5-3), we have that the maximum value for K should be in the order of 500. It is therefore mandatory to keep K below this approximate limit.

Consider now the effect of the choice of K on T. Obviously, for any particular length of track and overall time interval,

$\{t_0, t_f\}$, these quantities are inversely proportional. Hence, for too small K, T may become so large as to make the accurate representation of the effective grade input impossible. In addition, the control may be too "coarse" for too large T. One is therefore forced to conclude that the limit on K implies a limit on the length of train run which can be optimized.

As an example, for average train speeds of 30 mph and T = 4 secs., we have K = 30 per mile. Hence, optimization over track lengths of about 15 miles would seem to be possible.

In summary, the problem of optimizing the operation of a 100-car train over a 15 mile track section so that the maximum force levels in every fifth coupler are minimized would certainly seem to be feasible. By making the initial condition of the next track section correspond to the terminal values of the preceding section, effectively any track length may be considered.

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APPENDIX A

DOCUMENTATION OF DIGITAL COMPUTER PROCEDURE USED TO SOLVE THE PROBLEM OUTLINED IN CHAPTER 4

```
@ RUN,O THESIS,OAK002,        ,5999,350,10000                    QAK002R2
LOG NUMBER= THESIS

@ ASG R=$00000
FSTRND ASSIGNMENTS*    R= 2/17 $00000

@X ASG,W C,H,D
* TAPE ASSIGNMENTS*    C= 4/00 *        H= 4/02 *        D= 4/03 *

@ ASG A=$MTPAK
FSTRND ASSIGNMENTS*    A= 2/00 $MTPAK

@ ASG B=$UMPLB
FSTRND ASSIGNMENTS*    B= 2/01 $UMPLB
```

```
NOTATION
SUFFICES
C        N = INDEX FOR OBJECTIVE CONSTRAINTS
C        I = INDEX FOR LP VECTOR
C        K = INDEX FOR TIME INTERVAL NUMBER
C        Z = DUMMY INDEX FOR LP VARIABLE PI
    N       NMAX     2
    I       IMAX     60
    K       KMAX     20
    Z       DMAX     1
VARIABLES
C        U  = VECTOR SET OF LLBYNK CONTROL VARIABLES
C        PI = DUMMY LP VARIABLE WHICH IS BEING MINIMIZED
    U(I)        '* II'
    PI(Z)       '** Z'
EXTERNAL VALUES
FIRST CALL PREPIO
    XXXX
DECLARATIONS
    PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20
    PARAMETER MBAR=1
    PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KT=2**NT-1
    PARAMETER MM=M+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS
    PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1
    PARAMETER LLBYNK=LL*NK
    REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11
    REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1
```

CSCX CS0B5Y 19 JUN 73 [illegible]:20:28 USER GAK002 THESIS

CARDS IN 491 OUT 0 LINES 2827 PROCESSING TIME 161.56 SECONDS

FORCE LEVELS IN COUPLERS AS A FUNCTION OF K (T = 4.000 SECONDS)

K	CPLR 1	CPLR 2
1	-.85157+03	-.47988+03
2	.17806+05	.99866+04
3	.24393+05	.14697+05
4	.18137+05	.89690+04
5	.15644+05	.67829+04
6	.18055+05	.90290+04
7	-.38764+05	-.22630+05
8	.44239+05	.21549+05
9	.80390+05	.11510+05
10	.41752+05	.39705+05
11	.28445+05	.77958+05
12	-.13415+06	-.70077+05
13	-.84059+05	-.57636+05
14	.71341+05	-.79235+04
15	.67809+05	.39391+05
16	.88981+05	.11648+06
17	-.77153+05	-.28458+05
18	-.10816+06	-.77397+05
19	.24923+05	-.44758+05
20	.26267+05	-.18515+05

*** MAXIMUM COUPLER FORCE EXPERIENCED IN TRAIN = -134.1 KIPS ***

OPTIMAL SYSTEM TRAJECTORY (T = 4.000 SECONDS)

K	X1(KT)	X2(KT)	X3(KT)	X4(KT)	X5(KT)	X6(KT)	X7(KT)	X8(KT)	X9(KT)	X10(KT)
0	.00000	.66000+02	.00000	.00000	.66000+02	.00000	.00000	.66000+02	.00000	.00000
1	.26318+03	.65592+02	.00000	-.17031-02	.65590+02	.00000	-.95975-03	.65587+02	.00000	.00000
2	.52603+03	.65778+02	.00000	.35613-01	.65824+02	.00000	.19973-01	.65880+02	.00000	.00000
3	.78989+03	.66054+02	.00000	.48785-01	.66053+02	.00000	.29375-01	.66056+02	.00000	.00000
4	.10545+04	.66300+02	.00000	.36274-01	.66279+02	.00000	.17938-01	.66259+02	.00000	.00000
5	.13200+04	.66504+02	.00000	.31288-01	.66504+02	.00000	.13566-01	.66503+02	.00000	.00000
6	.15865+04	.66715+02	.00000	.36109-01	.66724+02	.00000	.18056-01	.66731+02	.00000	.00000
7	.18498+04	.65126+02	.00000	-.77529-01	.64984+02	.00000	-.45261-01	.64814+02	.00000	.00000
8	.21180+04	.66657+02	.13459+02	.88478-01	.66979+02	.00000	.43099-01	.67318+02	.00000	.00000
9	.23787+04	.65306+02	.23768+02	.16078-00	.65222+02	.13459+02	.23019-01	.65226+02	.00000	.00000
10	.26444+04	.67446+02	.31664+02	.83504-01	.67473+02	.23768+02	.77410-01	.67376+02	.13459+02	.14833+01
11	.29103+04	.65606+02	.37711+02	.56889-01	.65700+02	.31664+02	.15592-00	.65668+02	.23768+02	.28710+01
12	.31673+04	.63244+02	.29919+02	-.26829-00	.62640+02	.37711+02	-.14015-00	.62069+02	.31664+02	.41692+01
13	.34200+04	.63412+02	.22916+02	-.16812-00	.63673+02	.29919+02	-.11527+00	.63991+02	.37711+02	.53836+01
14	.36712+04	.61659+02	.17552+02	.14268-00	.61864+02	.22916+02	-.15847-01	.62222+02	.29919+02	.51505+01
15	.39258+04	.65493+02	.13444+02	.13562-00	.65521+02	.17552+02	.78763-01	.65462+02	.22916+02	.48183+01
16	.41893+04	.66036+02	.10297+02	.17796-00	.66334+02	.13444+02	.23297-00	.66432+02	.17552+02	.45076+01
17	.44547+04	.67189+02	.78866+01	-.15431-00	.66589+02	.10297+02	-.56917-01	.66016+02	.13444+02	.42769+01
18	.47263+04	.69128+02	.60406+01	-.21632-00	.69197+02	.78866+01	-.15479-00	.69301+02	.10297+02	.39449+01
19	.50023+04	.68526+02	.46266+01	.49845-01	.68706+02	.60406+01	-.89517-01	.69057+02	.78866+01	.36905+01
20	.52790+04	.70075+02	.35437+01	.52533-01	.70042+02	.46266+01	-.37031-01	.69979+02	.60406+01	.34525+01

OPTIMAL CONTROL TRAJECTORY (T = 4.000 SECONDS)

K	U1(KT)	U2(KT)	U3(KT)
0	.30959+05	.00000	.00000
1	.30884+05	.00000	.00000
2	.30760+05	.00000	.00000
3	.30649+05	.00000	.00000
4	.30537+05	.00000	.40000-01
5	.30436+05	.00000	-.40000-01
6	.31195+05	.23000+02	-.40000-01
7	-.43071+05	.23000+02	-.40000-01
8	.31091+05	.23000+02	-.40000-01
9	-.42519+05	.23000+02	.40000-01
10	-.43693+05	.17699+01	-.40000-01
11	-.45300+05	.00000	-.40000-01
12	-.45199+05	.00000	-.40000-01
13	.32786+05	.00000	-.40000-01
14	.31038+05	.00000	.40000-01
15	.30764+05	.00000	.00000
16	.30245+05	.00000	.00000
17	.29327+05	.00000	.00000
18	.29568+05	.00000	.00000
19	.28849+05	.00000	.00000

OPTIMAL TRAJECTORY CALCULATION FOR A THREE-SECTION FREIGHT TRAIN

RHS. NO. = 1 COST ROW NO. = 1

OBJECTIVE FUNCTION NAME = OBJ AND VALUE = -.152347

```
 1   025234-025331
 2   103103-103110

POSIT /CODE
 0   103111-103145
 1   025332-025607

MGINPT/CODE
 0   103146-105257
 1   025610-027447

MXSCA /CODE
 0   105260-105277
 1   027450-027536

MXADD /CODE
 0   105300-105316
 1   027537-027627

CHECIN/CODE
 0   105317-105355
 1   027630-027732

MOLINE/CODE
 0   105356-105423
 1   027733-030056

MGHEAD/CODE
 0   105424-105520
 1   030057-030352

MRWNDS/CODE
 1   030353-030402
 2   105521-105524

RWTABS/******
 0   105525-106241

MYBLK2/******
 0   106242-106245

MYBLK1/******
 0   106246-112411

MGCOMT/******
 0   112412-112524

MGTABS/******
 0   112525-112554

DRUM LENGTH 020624

END OF ALLOCATION
THIS ALLOCATION WAS DONE ON 19 JUN 73 AT 22:24:00
```

```
NFTVS /CODE
1   016331-016353

NOTIMS/CODE
1   016354-016636
2   101607-101646

NFMTS /CODE
1   016637-017634
2   101647-101765

NLINPS/CODE
1   017635-021511
2   101766-102136

NINIMS/CODE
1   021512-021642
2   102137-102171

NINPTS/CODE
1   021643-023127
2   102172-102224

NEXPS /CODE
0   102225-102311

NOBCS /CODE
1   023130-023172
2   102312-102321

NFLOTS/CODE
1   023173-023256
2   102322-102351

NEPOPT/CODE
0   102352-102615
1   023257-023541

NXMLT /CODE
0   102616-102652
1   023542-023711

RWREAD/CODE
0   102653-102773
1   023712-024615

ITOBCD/CODE
0   102774-103030
1   024616-024747

NEXP1S/CODE
1   024750-025003
2   103031-103031

NWSOL /CODE
0   103032-103102
1   025004-025233

NBKSPS/CODE
```

```
@ XQT RWMAIN

     STARTING ADDRESS  014000

     CORE LIMITS 014000 030402      100000 112554

RWMAIN/CODE
0    100000-100031
1    014000-014132

NSTOPS/CODE

NFINPS/CODE
1    014133-014330
2    100032-100032

NIDIMS/CODE
1    014331-014437
2    100033-100067

SYSSM /CODE

NTABS /CODE

NTABZZ/******
0    100070-100174

NBUFFS/CODE
1    014440-014504
2    100175-101203

NIERS /CODE
1    014505-014657
2    101204-101332

NPAUSS/CODE
1    014660-014701

EDITS /CODE
0    101333-101417

NERRS /CODE
1    014702-015257
2    101420-101552

NOUTS /CODE
1    015260-016330
2    101553-101606
```

```
@ XQT CUR
  1. TEF H
  2. TRW H
  3. ERS
  4. IN R
     END FILE —— UNIT R
     END CUR
```

AGENDUM - STOP 22:23:38 19 JUN 73

OPTIMAL TRAJECTORY CALCULATION FOR A THREE-SECTION FREIGHT TRAIN

BASIC OR N/Z VARIABLE NAME	VARIABLE VALUE	ROW NAME	SHADOW PRICE	ORIGINAL RHS
+ VEL 16	21.990466	+ TEL020	.000000	-104.893899
+ VEL 17	20.975812	+ BRAK01	.000000	23.000000
+ VEL 18	18.906125	+ GRAD01	.766745	.040000
+ VEL 19	19.422947	+ BRAK02	.000000	23.000000
+ VEL 20	17.881572	+ GRAD02	-.422882	.040000
+ TCHI08	157.019430	+ BRAK03	.000000	23.000000
+ TCHI10	155.795790	+ GRAD03	-1.112752	.040000
+ TCHI11	160.097530	+ BRAK04	.000000	23.000000
+ TCHI12	165.992390	+ GRAD04	-.580980	.040000
+ TCHI13	165.613900	+ BRAK05	.000000	23.000000
+ TEL001	113.351101	+ GRAD05	-2.183796	.080000
+ TEL002	113.079330	+ BRAK06	.000000	23.000000
+ TEL003	112.625112	+ GRAD06	.000000	.000000
+ TEL004	112.215756	+ BRAK07	.000112	23.000000
+ TEL005	111.807504	+ GRAD07	.000000	.000000
+ TEL006	111.437596	+ BRAK08	.001091	23.000000
+ TEL007	114.216499	+ GRAD08	.000000	.000000
+ TEL009	113.934569	+ BRAK09	.000000	23.000000
+ TEL014	120.044869	+ GRAD09	.000000	.000000
+ TEL015	113.643324	+ BRAK10	.000000	23.000000
+ TEL016	112.638457	+ GRAD10	.000000	.080000
+ TEL017	110.735859	+ BRAK11	.000000	23.000000
+ TEL018	107.373808	+ GRAD11	.000000	.000000
+ TEL019	108.255967	+ BRAK12	.000000	23.000000
+ TEL020	105.625904	+ GRAD12	.000000	.000000
+ BRAK01	23.000000	+ BRAK13	.000000	23.000000
+ BRAK02	23.000000	+ GRAD13	.000000	.000000
+ BRAK03	23.000000	+ BRAK14	.000000	23.000000
+ BRAK04	23.000000	+ GRAD14	.000000	.000000
+ BRAK05	23.000000	+ BRAK15	.000000	23.000000
+ BRAK06	23.000008	+ GRAD15	.000000	.080000
+ BRAK11	21.231148	+ BRAK16	.000000	23.000000
+ BRAK12	23.000000	+ GRAD16	.000000	.040000
+ BRAK13	23.000000	+ BRAK17	.000000	23.000000
+ BRAK14	23.000000	+ GRAD17	.000000	.040000
+ BRAK15	23.000000	+ BRAK18	.000000	23.000000
+ BRAK16	23.000000	+ GRAD18	.000000	.040000
+ BRAK17	23.000000	+ BRAK19	.000000	23.000000
+ BRAK18	23.000000	+ GRAD19	.000000	.040000
+ BRAK19	23.000000	+ BRAK20	.000000	23.000000
+ BRAK20	23.000000	+ GRAD20	.000000	.040000
EOF				

BASIC OR N/Z VARIABLE NAME	VARIABLE VALUE	ROW NAME	SHADOW PRICE	ORIGINAL RHS
+ OC1 02	.143108	+ VEL 08	.000000	16.590010
+ OC1 03	.178255	+ VEL 09	.000000	13.738256
+ OC1 04	.142899	+ VEL 10	.000000	10.824807
+ OC1 05	.130267	+ VEL 11	.000000	7.961069
+ OC1 06	.144540	+ VEL 12	.000000	5.146317
+ OC1 07	.264983	+ VEL 13	.000000	2.336219
+ OC1 08	.091353	+ VEL 14	.000000	-.468000
+ OC1 10	.066140	+ VEL 15	.000000	-3.251416
+ OC1 11	.137682	+ VEL 16	.000000	-6.011653
+ OC1 12	.412271	+ VEL 17	.000000	-8.756823
+ OC1 13	.317570	+ VEL 18	.000000	-11.486933
+ OC1 14	.025603	+ VEL 19	.000000	-14.199243
+ OC1 15	.040431	+ VEL 20	.000000	-16.893903
+ OC1 16	.030027	+ TEHI01	-.000006	253.139060
+ OC1 17	.330623	+ TEHI02	-.000006	255.221050
+ OC1 18	.384524	+ TEHI03	-.000006	255.735460
+ OC1 19	.130633	+ TEHI04	-.000006	255.885250
+ OC1 20	.130863	+ TEHI05	-.000006	254.422840
+ OC2 01	.153307	+ TEHI06	-.000007	253.123590
+ OC2 02	.213654	+ TEHI07	-.000006	250.108730
+ OC2 03	.238214	+ TEHI08	.000000	247.321100
+ OC2 04	.205652	+ TEHI09	.000000	244.469340
+ OC2 05	.194880	+ TEHI10	.000000	241.555890
+ OC2 06	.207414	+ TEHI11	.000000	238.692160
+ OC2 07	.276939	+ TEHI12	.000000	235.877400
+ OC2 08	.182032	+ TEHI13	.000000	233.067300
+ OC2 09	.148657	+ TEHI14	.000000	230.262190
+ OC2 10	.126456	+ TEHI15	.000000	227.479670
+ OC2 11	.047107	+ TEHI16	.000000	224.719430
+ OC2 12	.302302	+ TEHI17	.000000	221.974260
+ OC2 13	.282139	+ TEHI18	.000000	219.244150
+ OC2 14	.199059	+ TEHI19	.000000	216.531840
+ OC2 15	.157393	+ TEHI20	.000000	213.837180
+ OC2 16	.017518	+ TELO01	.000000	-65.592027
+ OC2 17	.278334	+ TELO02	.000000	-63.510031
+ OC2 18	.368917	+ TELO03	.000000	-62.995625
+ OC2 19	.314529	+ TELO04	.000000	-62.845833
+ OC2 20	.264930	+ TELO05	.000000	-64.308239
+ VEL 02	22.248758	+ TELO06	.000000	-65.607499
+ VEL 03	21.982647	+ TELO07	.000000	-68.622355
+ VEL 04	21.742824	+ TELO08	.000659	-71.409088
+ VEL 05	21.503646	+ TELO09	.000000	-74.261741
+ VEL 06	21.286930	+ TELO10	.000000	-77.175192
+ VEL 07	22.914974	+ TELO11	.000000	-80.038930
+ VEL 08	21.359816	+ TELO12	.000000	-82.853682
+ VEL 09	22.691216	+ TELO13	.000000	-85.663780
+ VEL 10	20.521743	+ TELO14	.000000	-88.468900
+ VEL 11	22.303273	+ TELO15	.000000	-91.251414
+ VEL 12	24.740437	+ TELO16	.000000	-94.011652
+ VEL 13	24.597406	+ TELO17	.000000	-96.756820
+ VEL 14	26.329574	+ TELO18	.000000	-99.486031
+ VEL 15	22.579174	+ TELO19	.000000	-102.199242

OPTIMAL TRAJECTORY CALCULATION FOR A THREE-SECTION FREIGHT TRAIN

RHS. NO. = 1 COST ROW NO. = 1 PROBLEM OPTIMAL

BASIC OR N/Z VARIABLE NAME	VARIABLE VALUE	ROW NAME	SHADOW PRICE	ORIGINAL RHS
Z OBJ	-.152347	Z OBJ	1.000000	.000000
U 01	117958.555000	+ OC1000	.000000	.000000
U 03	.040000	+ OC2000	.000000	.000000
U 04	117884.360000	+ OC1 01	.000000	.001703
U 06	.040000	+ OC1 02	.000000	.108719
U 07	117760.360000	+ OC1 03	.000000	.028379
U 09	.040000	+ OC1 04	.000000	-.045752
U 10	117648.605000	+ OC1 05	.000000	.030219
U 12	.040000	+ OC1 06	.000000	.069[illegible]
U 13	117537.151000	+ OC1 07	.000000	.121608
U 15	.080000	+ OC1 08	.000000	.136084
U 16	117436.159000	+ OC1 09	1.000000	.106517
U 18	.000000	+ OC1 10	.000000	.099146
U 19	118194.840000	+ OC1 11	.000000	.111067
U 20	23.000000	+ OC1 12	.000000	.111181
U 21	.000000	+ OC1 13	.000000	.109528
U 22	43920.928000	+ OC1 14	.000000	.108400
U 23	23.000000	+ OC1 15	.000000	.110366
U 24	.000000	+ OC1 16	.000000	.110917
U 25	118090.540000	+ OC1 17	.000000	.110229
U 26	23.000000	+ OC1 18	.000000	.110110
U 27	.000000	+ OC1 19	.000000	.110486
U 28	44481.382000	+ OC1 20	.000000	.110633
U 29	23.000000	+ OC2 01	.000000	.000960
U 30	.080000	+ OC2 02	.000000	.06[illegible]307
U 31	43307.005000	+ OC2 03	.000000	.0[illegible]872
U 32	1.768851	+ OC2 04	.000000	-.058887
U 33	.000000	+ OC2 05	.000000	-.053356
U 34	81700.436000	+ OC2 06	.000000	-.034270
U 36	.000000	+ OC2 07	.000000	.056154
U 37	41801.256000	+ OC2 08	.000000	.078837
U 39	.000000	+ OC2 09	.000000	.052429
U 40	119785.957000	+ OC2 10	.000000	.045345
U 42	.000000	+ OC2 11	.000000	.056045
U 43	118030.332000	+ OC2 12	.000000	.058830
U 45	.080000	+ OC2 13	.000000	.054561
U 46	117764.002000	+ OC2 14	.000000	.053501
U 48	.040000	+ OC2 15	.000000	.055265
U 49	117244.592000	+ OC2 16	.000000	.055738
U 51	.040000	+ OC2 17	.000000	.055081
U 52	116326.751000	+ OC2 18	.000000	.054043
U 54	.040000	+ OC2 19	.000000	.055257
U 55	116567.582000	+ OC2 20	.000000	.055362
U 57	.040000	A TC	.000000	-1049.659800
U 58	115849.327000	+ VEL 02	.000000	24.489971
U 60	.040000	+ VEL 03	.000000	25.004376
PI 1	.152347	+ VEL 04	.000000	25.154168
+ OC1000	.152347	+ VEL 05	.000000	23.691760
+ OC2000	.152347	+ VEL 06	.000000	22.392501
+ OC1 01	.154050	+ VEL 07	.000000	19.377643

AGENDUM - UNRAVL 22:23:35 19 JUN 73

	TOTAL ITERS	NO. ETAS	ROW IDENT.	CURRENT VALUE	RHS NO.	C/V NO.	PROB. STATE	REMOVED VECTOR	CHOSEN VECTOR	NO. NEG D/J'S	NO OF INFEAS	D/J VALUE (OR THETA)	TIME
	55	37	1	63.276646	1	0	I	155	164	8	1	-.025039	41.6
M	56	38	1	-.226031	1	0	F	77	184	3	0	-.001528	41.7
	57	39	1	-.225888	1	0	F	164	176	3	0	.000054	41.9
M	58	40	1	-.152347	1	0	F	91	71	1	0	-.000466	41.8
M	58	31	1	-.152347	1	0	0	0	0	0	0	.000000	42.0

	TOTAL ITERS	NO. ETAS	ROW IDENT.	CURRENT VALUE	RHS NO.	C/V NO.	PROB. STATE	REMOVED VECTOR	CHOSEN VECTOR	NO. NEG D/J'S	NO OF INFEAS	D/J VALUE (OR THETA)	TIME
	0	1	1	1992.996300	1	0	I	0	0	0	27	.000000	36.3
M	1	2	1	1800.136200	1	0	I	64	145	21	27	-.002596	36.3
	2	3	1	1613.671400	1	0	I	65	148	21	26	-.002541	36.4
	3	4	1	1433.671400	1	0	I	66	151	21	26	-.002485	36.4
	4	5	1	1433.133900	1	0	I	12	205	21	18	-8.000000	36.4
M	5	6	1	1259.581100	1	0	I	67	154	17	17	-.002429	36.4
	6	7	1	1092.590500	1	0	I	68	157	17	16	-.002372	36.4
	7	8	1	932.163360	1	0	I	69	160	17	15	-.002315	36.4
	8	9	1	904.576910	1	0	I	44	163	17	14	-.002258	36.4
M	9	10	1	859.087490	1	0	I	70	155	38	13	-14.150991	36.5
M	10	10	1	856.746480	1	0	I	173	173	29	13	-.101770	36.6
	11	11	1	856.709580	1	0	I	155	170	29	13	-.104617	36.6
M	12	12	1	856.585670	1	0	I	170	191	22	13	-.042860	36.7
M	13	10	1	855.760020	1	0	I	191	173	17	13	.286929	37.5
	14	10	1	855.026620	1	0	I	194	194	17	13	-.031900	37.5
	15	10	1	854.628580	1	0	I	197	197	17	13	-.017393	37.5
	16	10	1	854.497630	1	0	I	200	200	17	13	-.005693	37.5
M	17	10	1	853.489850	1	0	I	188	188	18	13	-.043917	37.7
	18	10	1	852.539840	1	0	I	185	185	18	13	-.041305	37.7
	19	11	1	852.287640	1	0	I	173	182	18	13	-.034730	37.7
M	20	12	1	785.165350	1	0	I	182	175	13	12	-.001532	37.8
	21	13	1	771.085540	1	0	I	94	172	13	12	-.001537	37.8
M	22	14	1	720.179980	1	0	I	73	155	28	11	-18.311741	37.9
M	23	15	1	715.132480	1	0	I	155	176	21	11	-.225575	38.0
M	24	16	1	710.787500	1	0	I	176	169	13	11	-.001569	38.0
	25	16	1	644.902830	1	0	I	179	179	13	10	-5.622135	38.0
M	26	17	1	643.332750	1	0	I	72	173	16	10	-.253690	38.1
M	27	18	1	572.177460	1	0	I	75	178	11	9	-.001519	38.1
	28	19	1	555.515440	1	0	I	173	181	11	9	-.001514	38.2
M	29	20	1	502.786350	1	0	I	17	155	26	8	-25.669329	38.3
M	30	21	1	500.171090	1	0	I	155	170	17	8	-.373660	38.3
	31	22	1	499.462830	1	0	I	96	171	17	8	-20.927098	38.5
M	32	23	1	451.797470	1	0	I	83	202	10	7	-.001517	38.5
	33	24	1	378.636680	1	0	I	170	166	10	6	-.001573	38.5
	34	25	1	378.293230	1	0	I	71	75	10	6	-.020873	38.5
M	35	26	1	377.340050	1	0	I	95	94	13	6	-.011933	38.6
	36	26	1	377.129870	1	0	I	194	194	13	6	.009399	38.7
	37	26	1	377.071130	1	0	I	191	191	13	6	.002554	38.7
M	38	27	1	323.845440	1	0	I	82	199	9	5	-.001524	38.8
	39	27	1	323.528220	1	0	I	197	197	9	5	.013792	38.8
	40	27	1	323.483100	1	0	I	200	200	9	5	.001962	38.9
M	41	21	1	264.869320	1	0	I	94	196	7	4	-.001535	39.8
	42	22	1	242.441370	1	0	I	101	193	7	4	-.001517	39.9
M	43	23	1	202.811320	1	0	I	80	155	23	3	-69.550386	39.9
M	44	23	1	201.300100	1	0	I	185	185	14	3	.065705	40.0
	45	24	1	200.193490	1	0	I	155	167	14	3	-.112006	40.0
M	46	24	1	198.279750	1	0	I	188	188	8	3	.083206	40.1
	47	24	1	196.440420	1	0	I	182	182	8	3	.079971	40.1
	48	25	1	196.247880	1	0	I	167	101	8	3	-.006701	40.1
M	49	26	1	195.775200	1	0	I	81	155	23	3	-.509758	40.2
M	50	27	1	195.663160	1	0	I	155	73	9	3	-.005587	40.4
M	51	28	1	130.730740	1	0	I	79	190	4	2	-.001540	40.4
	52	29	1	75.972208	1	0	I	93	187	4	2	-.001536	40.4
M	53	30	1	64.311239	1	0	I	78	155	27	1	-39.897921	40.6
M	54	36	1	63.629384	1	0	I	179	179	8	1	.029646	41.6

AGENDUM - NORMAL 22:23:27 19 JUN 73

```
THIS PROBLEM HAS -
      61    COLUMNS
     144    ROWS    COST ROW IS        1
       1    RIGHT HAND SIDES -     1 USED
    2573    ELEMENTS
THE DENSITY IS   29.29  PER CENT


PHASE ONE OF INPUT TOOK        34.23    SECONDS
ROW       25 IS REDUNDANT
ROW       26 IS REDUNDANT

PRESOLVE FOUND-

       1    NULL ROWS

      40    SUB ROWS

INPUT TOOK   35.13   SECONDS
```

```
                    KINPUT=13,KEY=3,NOPRNT=1

      AGENDUM - INPUT                                  22:22:51, 19 JUN 73
MAXIMUM ADDRESS ASKED FOR = 163777
      OPTIMAL TRAJECTORY CALCULATION FOR A THREE-SECTION FREIGHT TRAIN
      ROW ID
      MATRIX
      FIRST B
      EOF
```

CSC IPFONET
UMPAGM - UMPIRE/AGENDUM.
PROGRAM NO. 3020 VERSION-07

ALB.B NEWAGH

COMPUTER SCIENCES CANADA LTD.

```
@ XBT CUR
    1. OUT R
    2. TRW R
    3. TRW C
       END CUR
```

AKXO MATRIX FOLLOWS

.496+04	.473+04	.450+04	.426+04	.402+04	.378+04	.354+04	.330+04	.305+04	.281+04
.583+02	.586+02	.590+02	.594+02	.597+02	.601+02	.605+02	.609+02	.612+02	.616+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
-.153-02	-.154-02	-.155-02	-.156-02	-.157-02	-.158-02	-.159-02	-.160-02	-.160-02	-.162-02
.583+02	.586+02	.590+02	.594+02	.597+02	.601+02	.605+02	.609+02	.612+02	.616+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
-.766-03	-.771-03	-.776-03	-.780-03	-.784-03	-.790-03	-.797-03	-.800-03	-.800-03	-.810-03
.583+02	.586+02	.590+02	.594+02	.597+02	.601+02	.605+02	.609+02	.612+02	.616+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
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.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
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.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.256+04	.231+04	.206+04	.181+04	.155+04	.130+04	.104+04	.785+03	.525+03	.263+03
.620+02	.624+02	.628+02	.632+02	.636+02	.640+02	.644+02	.648+02	.652+02	.656+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
-.164-02	-.164-02	-.161-02	-.166-02	-.176-02	-.169-02	-.147-02	-.169-02	-.227-02	-.170-02
.620+02	.624+02	.628+02	.632+02	.636+02	.640+02	.644+02	.648+02	.652+02	.656+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
-.828-03	-.821-03	-.793-03	-.828-03	-.917-03	-.847-03	-.643-03	-.833-03	-.136-02	-.960-03
.620+02	.624+02	.628+02	.632+02	.636+02	.640+02	.644+02	.648+02	.652+02	.656+02
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
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XXXX 1 0.

.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.529-03	-.234+01	-.185+04	.449-03	-.170+01	-.134+04	.369-03	-.112+01	-.916+03	.288-03	-.648-00	-.564+03
.200-04	-.168-00	-.123+03	.203-04	-.152-00	-.129+03	.200-04	-.152-00	-.858+02	.197-04	-.105+00	-.894+02
.000	.118+00	.000	.000	.154-00	.000	.000	.201-00	.000	.000	.263-00	.000
-.669-07	.195-03	.507-00	.573-07	.273-03	.117+01	.162-06	.281-03	.639-00	-.149-06	.314-03	.222+01
.199-04	-.168-00	-.126+03	.200-04	-.152-00	-.126+03	.202-04	-.132-00	-.871+02	.204-04	-.105+00	-.886+02
.000	.154-00	.000	.000	.201-00	.000	.000	.263-00	.000	.000	.343-00	.000
-.599-07	.170-03	.698-00	.531-07	.240-03	.215+01	.149-06	.246-03	.151-00	-.132-06	.272-03	-.709-01
.198-04	-.168-00	-.129+03	.198-04	-.152-00	-.124+03	.205-04	-.132-00	-.897+02	.210-04	-.105+00	-.855+02
.000	.201-00	.000	.000	.263-00	.000	.000	.343-00	.000	.000	.448-00	.000
.000	.494-01	.000	.000	.528-01	.000	.000	.564-01	.000	.000	.603-01	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.100+01	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.100+01
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.206-03	-.295-00	-.293+03	.124-03	-.936-01	-.100+03	.424-04	-.133-01	.000	.000	.000	.000
.211-04	-.692-01	-.521+02	.221-04	-.335-01	-.453+02	.192-04	-.930-02	.000	.000	.000	.000
.000	.343-00	.000	.000	.448-00	.000	.000	.585-00	.000	.000	.000	.000
-.405-06	.599-03	-.966-00	.410-06	.106-03	-.289+01	.122-05	-.109-02	.000	.000	.000	.000
.204-04	-.695-01	-.484+02	.205-04	-.336-01	-.489+02	.207-04	-.908-02	.000	.000	.000	.000
.000	.448-00	.000	.000	.585-00	.000	.000	.000	.000	.000	.000	.000
-.372-06	.514-03	-.680-00	.288-06	-.855-03	-.162+01	.689-06	-.571-03	.000	.000	.000	.000
.197-04	-.608-01	-.443+02	.188-04	-.334-01	-.531+02	.225-04	-.895-02	.000	.000	.000	.000
.000	.585-00	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.645-01	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.100+01	.000	.000
.000	.000	.000	.000	.100+01	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.100+01	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.100+01	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.100+01	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.100+01	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.100+01	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.100+01

D MATRIX FOLLOWS

.145-02	-.118+02	-.767+04	.138-02	-.110+02	-.720+04	.130-02	-.102+02	-.673+04	.123-02	-.937+01	-.625+04
.185-04	-.204-00	-.117+03	.186-04	-.204-00	-.117+03	.187-04	-.205-00	-.118+03	.189-04	-.205-00	-.119+03
.000	.482-02	.000	.000	.629-02	.000	.000	.821-02	.000	.000	.107-01	.000
-.760-09	.127-04	.556-02	-.255-09	.151-04	.102-01	.190-09	.177-04	-.315-02	-.108-08	.213-04	-.146-01
.185-04	-.204-00	-.117+03	.186-04	-.204-00	-.117+03	.187-04	-.205-00	-.118+03	.189-04	-.205-00	-.119+03
.000	.629-02	.000	.000	.821-02	.000	.000	.107-01	.000	.000	.140-01	.000
-.493-09	.915-05	.391-02	-.315-10	.112-04	.804-02	.375-09	.135-04	-.415-02	-.783-09	.167-04	-.146-01
.185-04	-.204-00	-.117+03	.186-04	-.204-00	-.117+03	.187-04	-.205-00	-.118+03	.189-04	-.205-00	-.119+03
.000	.821-02	.000	.000	.107-01	.000	.000	.140-01	.000	.000	.183-01	.000
.000	.222-01	.000	.000	.237-01	.000	.000	.254-01	.000	.000	.271-01	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.115-02	-.855+01	-.578+04	.108-02	-.773+01	-.530+04	.999-03	-.691+01	-.481+04	.922-03	-.611+01	-.433+04
.190-04	-.205-00	-.120+03	.191-04	-.204-00	-.121+03	.192-04	-.203-00	-.121+03	.193-04	-.201-00	-.122+03
.000	.140-01	.000	.000	.183-01	.000	.000	.239-01	.000	.000	.311-01	.000
-.220-08	.269-04	.183-01	.973-09	.340-04	.474-01	.373-08	.409-04	-.362-01	-.421-08	.503-04	-.107+00
.190-04	-.205-00	-.120+03	.191-04	-.204-00	-.120+03	.192-04	-.203-00	-.121+03	.193-04	-.201-00	-.122+03
.000	.193-01	.000	.000	.239-01	.000	.000	.311-01	.000	.000	.407-01	.000
-.180-08	.216-04	.159-01	.109-08	.279-04	.419-01	.361-08	.339-04	-.343-01	-.363-08	.421-04	-.989-01
.190-04	-.205-00	-.120+03	.191-04	-.204-00	-.120+03	.192-04	-.203-00	-.121+03	.194-04	-.201-00	-.122+03
.000	.239-01	.000	.000	.311-01	.000	.000	.407-01	.000	.000	.531-01	.000
.000	.290-01	.000	.000	.310-01	.000	.000	.331-01	.000	.000	.354-01	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.845-03	-.531+01	-.384+04	.766-03	-.453+01	-.334+04	.688-03	-.376+01	-.285+04	.609-03	-.303+01	-.235+04
.195-04	-.198-00	-.122+03	.196-04	-.193-00	-.124+03	.197-04	-.187-00	-.125+03	.197-04	-.179-00	-.124+03
.000	.407-01	.000	.000	.531-01	.000	.000	.693-01	.000	.000	.905-01	.000
-.111-07	.687-04	.102+00	.877-08	.914-04	.280-00	.258-07	.106-03	-.240-00	-.238-07	.128-03	-.688-00
.195-04	-.198-00	-.123+03	.196-04	-.193-00	-.123+03	.197-04	-.187-00	-.124+03	.198-04	-.179-00	-.125+03
.000	.531-01	.000	.000	.693-01	.000	.000	.905-01	.000	.000	.118+00	.000
-.987-08	.584-04	.913-01	.820-08	.786-04	.250-00	.237-07	.910-04	-.229-00	-.215-07	.110-03	-.614-00
.194-04	-.198-00	-.123+03	.195-04	-.193-00	-.123+03	.197-04	-.187-00	-.123+03	.199-04	-.179-00	-.126+03
.000	.693-01	.000	.000	.905-01	.000	.000	.118+00	.000	.000	.154-00	.000
.000	.378-01	.000	.000	.404-01	.000	.000	.432-01	.000	.000	.462-01	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

AUGMENTED CONTROL MATRIX OF DISCRETE SYSTEM FOR T = 4.000 SECONDS

.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.10000+01	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.10000+01	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.00000
.00000	.00000	.10000+01

.42433-04	-.11625-01	-.12196-01	-.13287-01	-.77736+02	.00000	-.79058+02	.00000	-.10006+03	.00000
.19223-04	-.89499-02	-.90772-02	-.92981-02	-.43216+02	.00000	-.39752+02	.00000	-.45329+02	.00000
.00000	.00000	.00000	.58518-00	.00000	.00000	.00000	.00000	.00000	.00000
.12336-05	.70259-03	.63911-03	-.10909-02	.16267+01	.00000	.12621+01	.00000	-.28853+01	.00000
.20721-04	-.91066-02	-.91925-02	-.90772-02	-.38958+02	.00000	-.40476+02	.00000	-.49862+02	.00000
.00000	.00000	.58518-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.68906-06	.12104-02	-.50781-03	-.57128-03	.25840+01	.00000	-.95736-00	.00000	-.16249+01	.00000
.22526-04	-.93492-02	-.91066-02	-.89499-02	-.36220+02	.00000	-.3[illegible]58+02	.00000	-.53118+02	.00000
.00000	.58519-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.64493-01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.10000+01	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.10000+01	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.10000+01	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.10000+01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.10000+01
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

AUGMENTED SYSTEM MATRIX OF DISCRETE SYSTEM FOR T = 4.000 SECONDS

.10000+01	.14039+01	-.55789-01	-.61179-00	.12355+01	-.54463-01	-.34453-00	.13432+01	-.53699-01	.00000
.00000	.40823-00	-.23961-01	.74992-00	.27608-00	-.26207-01	.90256-00	.31551-00	-.28857-01	.00000
.00000	.00000	.76593-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	-.10979+00	-.13256-02	.59264-01	-.22518-01	.69172-03	-.95015-01	.13229-00	.93998-03	.00000
.00000	.33197-00	-.26207-01	.18882-00	.40219-00	-.26705-01	-.39554-00	.25963-00	-.25600-01	.00000
.00000	.00000	.00000	.00000	.00000	.76593-00	.00000	.00000	.00000	.00000
.00000	-.13230-00	-.76373-03	-.95015-01	.47170-01	-.51548-03	.77010-01	.85116-01	.14555-02	.00000
.00000	.38781-00	-.28857-01	-.11093+01	.25963-00	-.25600-01	-.71374-00	.34631-00	-.23497-01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.76593-00	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.93551-00
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000

CONTROL MATRIX OF DISCRETE SYSTEM FOR T = 4.000 SECONDS

.42433-04	-.13287-01	-.12196-01	-.11625-01	-.10006+03	-.79058+02	-.77736+02
.19223-04	-.92981-02	-.90772-02	-.89499-02	-.45329+02	-.39752+02	-.43215+02
.00000	.58518-00	.00000	.00000	.00000	.00000	.00000
.12236-05	-.10909-02	.63911-03	.70258-03	-.28853+01	.12621+01	.16267+01
.20721-04	-.90772-02	-.91925-02	-.91066-02	-.48862+02	-.40476+02	-.38958+02
.00000	.00000	.58518-00	.00000	.00000	.00000	.00000
.68906-06	-.57128-03	-.50781-03	.12104-02	-.16249+01	-.95736-00	.25840+01
.22526-04	-.89499-02	-.91066-02	-.93492-02	-.53118+02	-.38958+02	-.36220+02
.00000	.00000	.00000	.58518-00	.00000	.00000	.00000
.00000	.00000	.00000	.64493-01	.00000	.00000	.00000

INTEGRAL OF SYSTEM TRANSITION MATRIX FOR T = 4.000 SECONDS

.40000+01	.31100+01	-.79723-01	-.25674+01	.24572+01	-.73177-01	-.13363+01	.24161+01	-.69750-01	.00000
.00000	.14089+01	-.55789-01	-.61179-00	.12355+01	-.54463-01	-.34453-00	.13432+01	-.53699-01	.00000
.00000	.00000	.35111+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.89679-01	-.65455-02	-.81416-01	-.39228-01	.38346-02	-.12936-00	-.50559-01	.42155-02	.00000
.00000	.15187+01	-.54463-01	.32895-00	.12580+01	-.55155-01	-.24951-00	.12109+01	-.54639-01	.00000
.00000	.00000	.00000	.00000	.00000	.35111+01	.00000	.00000	.00000	.00000
.00000	.50503-01	-.34277-02	-.12936-00	.29756-01	-.30469-02	-.57310-01	-.80315-01	.72624-02	.00000
.00000	.16510+01	-.53699-01	.42396-00	.12109+01	-.54639-01	.67348-00	.11257+01	-.56095-01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.35111+01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.38696+01

SYSTEM TRANSITION MATRIX FOR T = 4.000 SECONDS

.10000+01	.14089+01	-.55789-01	-.61179-00	.12355+01	-.54463-01	-.34453-00	.13432+01	-.53699-01	.00000
.00000	.40823-00	-.23961-01	.74902-00	.27008-00	-.26207-01	.90256-00	.31551-00	-.28857-01	.00000
.00000	.00000	.76593-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	-.10979+00	-.13256-02	.59264-01	-.22518-01	.69172-03	-.95015-01	.13229-00	.93998-03	.00000
.00000	.33197-00	-.26207-01	.18882-00	.40219-00	-.26705-01	-.39554-00	.25963-00	-.25600-01	.00000
.00000	.00000	.00000	.00000	.00000	.76593-00	.00000	.00000	.00000	.00000
.00000	-.13230-00	-.76373-03	-.95015-01	.47170-01	-.51548-03	.77010-01	.85116-01	.14555-02	.00000
.00000	.38781-00	-.28857-01	-.11093+01	.25963-00	-.25600-01	-.71374-00	.34631-00	-.23447-01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.76593-00	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.93551-00

INTEGRAL OF SYSTEM TRANSITION MATRIX FOR T = .250 SECONDS

.25000-00	.29693-01	-.49285-04	-.16491-01	.14960-02	-.19921-05	-.67598-03	.51176-04	-.51165-07	.00000
.00000	.22776-00	-.58021-03	-.19006-00	.21151-01	-.35995-04	-.12115-01	.10394-02	-.12325-05	.00000
.00000	.00000	.24793-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.27860-01	-.47293-04	.21407-00	-.26084-01	.55746-04	.18609-01	-.17759-02	.23858-05	.00000
.00000	.25999-01	-.35995-04	.21873-00	.19819-00	-.66272-03	-.21595-00	.25761-01	-.43963-04	.00000
.00000	.00000	.00000	.00000	.00000	.24793-00	.00000	.00000	.00000	.00000
.00000	.17759-02	-.19409-05	.18609-01	.25753-01	-.59302-04	.21060-00	-.27529-01	.57687-04	.00000
.00000	.12776-02	-.12325-05	.14892-01	.25761-01	-.43963-04	.23084-00	.22292-00	-.70495-03	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.24793-00	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.24948-00

SYSTEM TRANSITION MATRIX FOR T = .250 SECONDS

.10000+01	.22776-00	-.58021-03	-.19006-00	.21151-01	-.35996-04	-.12116-01	.10394-02	-.12323-05	.00000
.00000	.76825-00	-.44361-02	-.13764+01	.21414-00	-.50840-03	-.16865-00	.17173-01	-.25020-04	.00000
.00000	.00000	.98347-00	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.20176-00	-.54421-03	-.59121-00	-.17704-00	.62622-03	.70384-00	-.24721-01	.42731-04	.00000
.00000	.26322-00	-.50840-03	.14846+01	.47712-00	-.47423-02	-.14459+01	.25929-00	-.61919-03	.00000
.00000	.00000	.00000	.00000	.00000	.98347-00	.00000	.00000	.00000	.00000
.00000	.24721-01	-.34763-04	.20384-00	.17243-00	-.61825-03	.55320-00	-.19716-00	.66098-03	.00000
.00000	.21109-01	-.25020-04	.20730-00	.25929-00	-.61919-03	.16532+01	.71924-00	-.53365-02	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.98347-00	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.99584-00

TRUNCATION ERROR IN SERIES APPROXIMATION OF STM AND ITS INTEGRAL FOR T = .250 SECONDS

.00000	-.40140-07	-.18872-09	-.23014-06	.83241-07	.46757-09	.24658-06	-.43104-07	-.23560-09	.00000
.00000	-.20091-06	.80123-09	.97185-06	.41566-06	-.20415-08	-.10595-05	-.21474-06	.10567-08	.00000
.00000	.00000	.45575-24	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	-.14246-06	-.65629-09	-.81614-06	.29774-06	.16711-08	.88911-06	-.15529-06	-.86436-09	.00000
.00000	.51092-06	-.20415-08	-.24967-05	-.10711-05	.52603-08	.27394-05	.56018-06	-.27512-08	.00000
.00000	.00000	.00000	.00000	.00000	.45575-24	.00000	.00000	.00000	.00000
.00000	.15530-06	.70317-09	.88911-06	-.32668-06	-.18323-08	-.98194-06	.17139-06	.96791-09	.00000
.00000	-.26395-06	.10567-08	.13022-05	.56018-06	-.27512-08	-.14372-05	-.29622-06	.14525-08	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.45575-24	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.43463-30

CONTROL MATRIX OF CONTINUOUS SYSTEM

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.13644-04	.00000	.00000	.00000	-.32174+02	.00000	.00000
.00000	.16667-00	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	-.32174+02	.00000
.00000	.00000	.16667-00	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.000[illegible]	.00000	.00000	.00000	.00000	-.32174+02
.00000	.00000	.00000	.16667-00	.00000	.00000	.00000
.00000	.00000	.00000	.16667-01	.00000	.00000	.00000

SYSTEM MATRIX OF CONTINUOUS SYSTEM

.00000	.10000+01	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	-.20640-00	-.19647-01	-.68220+01	.20466-00	.00000	.00000	.00000	.00000	.00000
.00000	.00000	-.66667-01	.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.10000+01	.00000	.00000	-.10000+01	.00000	.00000	.00000	.00000	.00000
.00000	.25156-00	.00000	.83855+01	-.50458-00	-.24150-01	-.83855+01	.25156-00	.00000	.00000
.00000	.00000	.00000	.00000	.00000	-.66667-01	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.10000+01	.00000	.00000	-.10000+01	.00000	.00000
.00000	.00000	.00000	.00000	.25156-00	.00000	.83855+01	-.25301-00	-.24150-01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	-.66667-01	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	.00000	-.16667-01

```
TITLE OPTIMAL TRAJECTORY CALCULATION FOR A THREE-SECTION FREIGHT TRAIN
UNITS       6   13    5   15        99999.0        0.000001
      NMAX=2
      IMAX=60
      KMAX=20
      DMAX=1
DATA
```

```
 0   102667-102723
 1   024743-025074

 HEXP1S/CODE
 1   025075-025130
 2   102724-102724

 MGINPT/CODE
 0   102725-105036
 1   025131-026770

 MLINPS/CODE
 1   026771-030645
 2   105037-105207

 MXMLT /CODE
 0   105210-105244
 1   030646-031015

 MXSEA /CODE
 0   105245-105264
 1   031016-031104

 MXADD /CODE
 0   105265-105303
 1   031105-031175

 CHECIN/CODE
 0   105304-105342
 1   031176-031300

 MGLINE/CODE
 0   105343-105410
 1   031301-031424

 MGHFAD/CODE
 0   105411-105505
 1   031425-031720

 MYBLK2/******
 0   105506-105511

 MYBLK1/******
 0   105512-111655

 MGCONT/******
 0   111656-111770

 MGTABS/******
 0   111771-112020

DRUM LENGTH 022120

END OF ALLOCATION
THIS ALLOCATION WAS DONE ON 19 JUN 73 AT 22:21:47
```

```
NOUTS /CODE
1   015275-016345
2   101545-101600
NFTVS /CODE
1   016346-016370
NOTIMS/CODE
1   016371-016653
2   101601-101640
NEMTS /CODE
1   016654-017651
2   101641-101757
NGOUT /CODE
0   101760-102060
1   017652-021202
COE1  /CODE
0   102061-102332
1   021203-022454
NGELMT/CODE
0   102333-102402
1   022455-022611
CHECKC/CODE
0   102403-102451
1   022612-023120
SCALFC/CODE
0   102452-102452
1   023121-023145
NCODES/CODE
1   023146-023175
2   102453-102453
NINIMS/CODE
1   023176-023326
2   102454-102506
NINPTS/CODE
1   023327-024613
2   102507-102541
NFLOTS/CODE
1   024614-024677
2   102542-102571
NEXPS /CODE
0   102572-102656
NDBCS /CODE
1   024700-024742
2   102657-102666
ITOBCD/CODE
```

```
@ XQT NGMAIN

STARTING ADDRESS  014000

CORE LIMITS 014000 031720     100000 112020

NGMAIN/CODE
0   100000-100016
1   014000-014132

NFOOTS/CODE
1   014133-014315
2   100017-100020

NIOIMS/CODE
1   014316-014424
2   100021-100055

SYSSH /CODE

NTABS /CODE

NTABZZ/******
0   100056-100162

NBUFFS/CODE
1   014425-014471
2   100163-101171

NRWHDS/CODE
1   014472-014521
2   101172-101175

NSTOPS/CODE

NIERS /CODE
1   014522-014674
0   101176-101324

NPAUSS/CODE
1   014675-014716

NDITS /CODE
0   101325-101411

NERRS /CODE
1   014717-015274
2   101412-101544
```

```
@I FRS  MGOT,MGOT
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00101     1*        BLOCK DATA
00102     2*        INCLUDE MGCOMM,LIST
00103     2*        PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20MGCOMM  *INCLUDED*
00104     2*        PARAMETER NBAR=1                                                  MGCOMM  *INCLUDED*
00105     2*        PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KT=2**NT-1     MGCOMM  *INCLUDED*
00106     2*        PARAMETER MM=M+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS        MGCOMM  *INCLUDED*
00107     2*        PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1         MGCOMM  *INCLUDED*
00110     2*        PARAMETER LLBYNK=LL*NK                                            MGCOMM  *INCLUDED*
00111     2*        COMMON/MGTABS/                                                    MGCOMM  *INCLUDED*
00112     2*        COMMON/MGTABS/                                                    MGCOMM  *INCLUDED*
00112     2*        *   XXXX   ,                                                      MGCOMM  *INCLUDED*
00112     2*        X   SPAREC                                                        MGCOMM  *INCLUDED*
00113     2*        COMMON/MGCOMT/                                                    MGCOMM  *INCLUDED*
00113     2*        1   KREAD ,KPRINT,MATRIX,                                         MGCOMM  *INCLUDED*
00113     2*        2   BIG   ,SMALL ,NERROR,LISTOP,                                  MGCOMM  *INCLUDED*
00113     2*        3   KTITLE(12),MGSWCH(50),                                        MGCOMM  *INCLUDED*
00113     2*        4   NAMID ,NAMCOL,NAMROW,KELEM (2),KGOMM                          MGCOMM  *INCLUDED*
00114     2*        COMMON/MGTABS/                                                    MGCOMM  *INCLUDED*
00114     2*        1   N     ,I     ,K     ,Z     ,                                  MGCOMM  *INCLUDED*
00114     2*        2   NMAX  ,IMAX  ,KMAX  ,DMAX  ,                                  MGCOMM  *INCLUDED*
00114     2*        X   SPAREA                                                        MGCOMM  *INCLUDED*
00115     2*        INTEGER                                                           MGCOMM  *INCLUDED*
00115     2*        1   N     ,I     ,K     ,Z     ,                                  MGCOMM  *INCLUDED*
00115     2*        2   NMAX  ,IMAX  ,KMAX  ,DMAX  ,                                  MGCOMM  *INCLUDED*
00115     2*        X   SPAREA                                                        MGCOMM  *INCLUDED*
00116     2*        COMMON/MGTABS/ MGIND,MGLUS( 12)                                   MGCOMM  *INCLUDED*
00117     2*        DIMENSION MGCEQV(74),MGTEQV(     24)                              MGCOMM  *INCLUDED*
00120     2*        EQUIVALENCE (MGCEQV,KREAD),(MGTEQV,XXXX  )                        MGCOMM  *INCLUDED*
00121     2*        DATA KREAD,KPRINT,MATRIX,SMALL,BIG/5,6,13,1.0E-6,9999.0/          MGCOMM  *INCLUDED*
00127     2*        REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11  MGCOMM  *INCLUDED*
00130     2*        REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1   MGCOMM  *INCLUDED*
00131     2*        COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11  MGCOMM  *INCLUDED*
00132     2*        COMMON/MYBLK2/ M1,M11,FV,T1                                       MGCOMM  *INCLUDED*
00133     2*        END                                                               MGCOMM  *INCLUDED*
00134     3*        DATA XO/0,0,66,0,2*0,0,66,0,2*0,0,66,0,12*0,0/
00136     4*        DATA UBIAS/87000.0,0.0,0.04/
00140     5*        DATA GRADE/4*0.0,0.0,4*-4.0,4.0,4*-4.0,4.0,5*0.0/
00142     6*        DATA ALPHA/2*500000.0/,VMAX/88.0/,FV/5280.0/
00146     7*        DATA L1,L11,M1,M11/465.91,61500.0,659.1,87000.0/
00153     8*        END
END COMPILATION **      0 MESSAGES
```

```
00144          DIMENSION FORCE(20,2)
00145     2010 FORMAT (1H1,'OPTIMAL CONTROL TRAJECTORY (T =',F7.3,' SECONDS)'///
00145         *4X,'K',3X,'U1(KT)',6X,'U2(KT)',6X,'U3(KT)'//)
00146     2011 FORMAT (1H0,I4,3E12.5)
00147     2020 FORMAT (1H1,T40,'OPTIMAL SYSTEM TRAJECTORY (T =',F7.3,' SECONDS)'/
00147         *//4X,'K',3X,'X1(KT)',6X,'X2(KT)',6X,'X3(KT)',6X,'X4(KT)',6X,
00147         *'X5(KT)',6X,'X6(KT)',6X,'X(KT)',6X,'X8(KT)',6X,'X9(KT)',6X,
00147         *'X10(KT)'//)
00150     2021 FORMAT (1H0,I4,10E12.5)
00151     2030 FORMAT (1H0////' *** MAXINUM COUPLER FORCE EXPERIENCED IN TRAIN =',
00151         *F7.1,' KIPS ***')
00152     2050 FORMAT (1H1,' FORCE LEVELS IN COUPLERS AS A FUNCTION OF K (T =',
00152         *F7.3,' SECONDS)'///4X,'K',3X,'CPLR 1',6X,'CPLR 2'//)
00153     2051 FORMAT (1H0,I4,2E12.5)
00154          DO 510 I = 1,NK
00157          DO 510 J = 1,LL
00162          K = 3*(I-1) + J
00163      510 BU(K) = BU(K) + UBIAS(J)
00166          WRITE (6,2010) T1
00171          KK = NK - 1
00172          WRITE (6,2011) (K,BU(3*K+1),BU(3*K+2),BU(3*K+3),K=0,KK)
00203          WRITE (6,2020) T1
00206          DO 520 I = 0,NK
00211          IF (I.EQ.0) GO TO 515
00213          DO 511 J = 1,LL
00216          K = 3*(I-1) + J
00217      511 UBIAS(J) = BU(K)
00221          CALL MXMLT(A,X0,X1,MMM,MMM,1,MMM,MMM)
00222          CALL MXMLT(B,UBIAS,X2,MMM,LL,1,MMM,LL)
00223          DO 513 J = 1,MMM
00226      513 X0(J) = X1(J) + X2(J)
00230          FORCE(I,1) = X0(4) * ALPHA(1)
00231          FORCE(I,2) = X0(7) * ALPHA(2)
00232      515 WRITE (6,2021) I,(X0(J),J=1,M)
00241      520 CONTINUE
00243          WRITE (6,2050) T1
00246          FORCEM = 0.0
00247          DO 530 I = 1,NK
00252          DO 525 J = 1,2
00255          IF ( ABS(FORCE(I,J)) .GT. ABS(FORCEM) ) FORCEM = FORCE(I,J)
00257      525 CONTINUE
00261      530 WRITE (6,2051) I,FORCE(I,1),FORCE(I,2)
00267          FORCEM = 0.001 * FORCEM
00270          WRITE (6,2030) FORCEM
00273          RETURN
00274          END
END COMPILATION **     0 MESSAGES **
```

COMPUTER SCIENCES CANADA LTD

```
QI FRS  REPORT,REPORT
COMPILED BY LEVEL 23.5  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00101   1*        SUBROUTINE REPORT
00103   2*        INCLUDE MGCOMM,LIST
00104   2*        PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20MGCOMM *INCLUDED*
00105   2*        PARAMETER MBAR=1                                                    MGCOMM *INCLUDED*
00106   2*        PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KT=2**NT-1       MGCOMM *INCLUDED*
00107   2*        PARAMETER MM=M+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS          MGCOMM *INCLUDED*
00110   2*        PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1           MGCOMM *INCLUDED*
00111   2*        PARAMETER LLBYNK=LL*NK                                              MGCOMM *INCLUDED*
00112   2*        COMMON/MGTABS/                                                      MGCOMM *INCLUDED*
00113   2*        COMMON/MGTABS/                                                      MGCOMM *INCLUDED*
00113   2*       *   XXXX  ,                                                          MGCOMM *INCLUDED*
00113   2*       X   SPAREC                                                           MGCOMM *INCLUDED*
00114   2*        COMMON/MGCONT/                                                      MGCOMM *INCLUDED*
00114   2*       1     KREAD  ,KPRINT,MATRIX,                                         MGCOMM *INCLUDED*
00114   2*       2     BIG   ,SMALL  ,NERROR,LISTOP,                                  MGCOMM *INCLUDED*
00114   2*       3     KTITLE(12),MGSWCH(50),                                         MGCOMM *INCLUDED*
00114   2*       4     NAMID ,NAMCOL,NAMROW,KELEM (2),KCOMM                           MGCOMM *INCLUDED*
00115   2*        COMMON/MGTABS/                                                      MGCOMM *INCLUDED*
00115   2*       1     N      ,I      ,K      ,Z                                      MGCOMM *INCLUDED*
00115   2*       2     NMAX   ,IMAX   ,KMAX   ,DMAX   ,                               MGCOMM *INCLUDED*
00115   2*       X     SPAREA                                                         MGCOMM *INCLUDED*
00116   2*        INTEGER                                                             MGCOMM *INCLUDED*
00116   2*       1     N      ,I      ,K      ,Z                                      MGCOMM *INCLUDED*
00116   2*       2     NMAX   ,IMAX   ,KMAX   ,DMAX   ,                               MGCOMM *INCLUDED*
00116   2*       X     SPAREA                                                         MGCOMM *INCLUDED*
00117   2*        COMMON/MGTABS/ MGIND,MGLUS( 12)                                     MGCOMM *INCLUDED*
00120   2*        DIMENSION MGCEQV(74),MGTEQV(     24)                                MGCOMM *INCLUDED*
00121   2*        EQUIVALENCE (MGCEQV,KREAD),(MGTEQV,XXXX  )                          MGCOMM *INCLUDED*
00122   2*        DATA KREAD,KPRINT,MATRIX,SMALL,BIG/5,6,13,1.0E-6,9999.0/            MGCOMM *INCLUDED*
00130   2*        REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11    MGCOMM *INCLUDED*
00131   2*        REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1     MGCOMM *INCLUDED*
00132   2*        COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11    MGCOMM *INCLUDED*
00133   2*        COMMON/MYBLK2/ M1,M11,FV,T1                                         MGCOMM *INCLUDED*
00134   2*        END                                                                 MGCOMM *INCLUDED*
00135   3*        INCLUDE RWCOMM,LIST
00136   3*        COMMON/RWTABS/                                                      RWCOMM *INCLUDED*
00136   3*       1   IOBJ   ,IRHS   ,OBJVAL,NOBJF                                     RWCOMM *INCLUDED*
00137   3*        COMMON/RWTABS/                                                      RWCOMM *INCLUDED*
00137   3*       1     RU     ( 60      ),                                            RWCOMM *INCLUDED*
00137   3*       2     BRI    ( 1       ),                                            RWCOMM *INCLUDED*
00137   3*       *     RWDUMA                                                         RWCOMM *INCLUDED*
00140   3*        COMMON/RWTABS/                                                      RWCOMM *INCLUDED*
00140   3*       1     SOC    ( 2     ,20      ),                                     RWCOMM *INCLUDED*
00140   3*       2     POC    ( 2     ,20      ),                                     RWCOMM *INCLUDED*
00140   3*       3     SVEL   ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       4     PVEL   ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       5     STCHI  ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       6     PTCHI  ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       7     STCLO  ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       8     PTCLO  ( 20     ),                                             RWCOMM *INCLUDED*
00140   3*       9     PGRAD  ( 20     )                                              RWCOMM *INCLUDED*
00141   3*        COMMON/PWTABS/                                                      RWCOMM *INCLUDED*
00141   3*       1     SBRAK  ( 20     ),                                             RWCOMM *INCLUDED*
00141   3*       2     PBRAK  ( 20     ),                                             RWCOMM *INCLUDED*
00141   3*       *     PWDUMB                                                         RWCOMM *INCLUDED*
00142   3*        COMMON/RWTABS/                                                      RWCOMM *INCLUDED*
00142   3*       *   SOBJ   ,SOC100,POC100,SOC200,POC200,PTC                          RWCOMM *INCLUDED*
00143   3*        END                                                                 RWCOMM *INCLUDED*
```

```
C
      FUNCTION F07(K)
      F07 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK = KK + LL*(II-1) +III
   10 F07 = F07 + (D(2,KKK) + D(11,KKK)/L1)*UBIAS(III)
      KK = NK-K+1
      F07 = F07 + L11/L1 - (AKXO(2,KK) + AKXO(11,KK)/L1)
      RETURN
C
      FUNCTION F08(I,K)
      F08 = 0.0
      KK = LL*(NK-K)+I
      IF (KK.GT.LLBYNK) GO TO 10
      F08 = D(2,KK) - D(11,KK)/M1
   10 RETURN
C
      FUNCTION F09(K)
      F09 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK = KK + LL*(II-1) + III
   10 F09 = F09 + (D(2,KKK) - D(11,KKK)/M1)*UBIAS(III)
      KK = NK-K+1
      F09 = F09 + M11/M1 - (AKXO(2,KK) - AKXO(11,KK)/M1)
      RETURN
C
      FUNCTION F10(I,K)
      F10 = 0.0
      IF (I.NE.K*LL) RETURN
      F10 = 1.0
      RETURN
C
      FUNCTION F11(I,K)
      F11 = 0.0
      IF (I.NE.K*LL-1) RETURN
      F11 = 1.0
      RETURN
      END
END COMPILATION **    0 MESSAGES **
```

```
00224     C
00225           ENTRY C10(DUMMY)
00227             COF1 = F10(I,K)
00230           RETURN
00230     C
00231           ENTRY C19(DUMMY)
00233             COF1 = F11(I,K)
00234           RETURN
00234     C
00235           FUNCTION F01(N,I,K)
00240           F01 = 0.0
00241           KK = LL*(NK-K)+I
00242           IF (KK.GT.LLBYNK) GO TO 10
00244           F01 = D(3*N+1,KK)
00245        10 RETURN
00245
00246           FUNCTION F02(N,K)
00251           F02 = 0.0
00252           KK = LL*(NK-K)
00253           DO 10 II = 1,K
00256           DO 10 III = 1,LL
00261           KKK= KK+LL*(II-1)+III
00262        10 F02 = F02 + D(3*N+1,KKK)*UBIAS(III)
00265           KK = NK-K+1
00266           F02 = F02 - AKXO(3*N+1,KK)
00267           RETURN
00267     C
00270           FUNCTION F03(Z)
00273           F03 = 0.0
00274           DO 10 II = 1,NK
00277           DO 10 III = 1,LL
00302           KK =  LL*(II-1)+III
00303        10 F03 = F03 + D(1,KK)*UBIAS(III)
00306           F03 = F03 + FV - AKXO(1,1)
00307           RETURN
00307     C
00310           FUNCTION F04(I,K)
00313           F04 = 0.0
00314           KK = LL*(NK-K)+I
00315           IF (KK.GT.LLBYNK) GO TO 10
00317           F04 = D(2,KK)
00320        10 RETURN
00320     C
00321           FUNCTION F05(K)
00324           F05 = 0.0
00325           KK = LL*(NK-K)
00326           DO 10 II = 1,K
00331           DO 10 III = 1,LL
00334           KKK = KK + LL*(II-1) + III
00335        10 F05 = F05 + D(2,KKK)*UBIAS(III)
00340           KK = NK-K+1
00341           F05 = F05 + VMAX - AKXO(2,KK)
00342           RETURN
00342     C
00343           FUNCTION F06(I,K)
00346           F06 = 0.0
00347           KK = LL*(NK-K)+I
00350           IF (KK.GT.LLBYNK) GO TO 10
00352           F06 = D(2,KK) + D(11,KK)/L1
00353        10 RETURN
```

```
QI PRS.* COF1
COMPILED BY LEVEL 23.5  CSGX FORTRAN V ON 19 JUN 73 AT 22:21
00101            FUNCTION   COF1(DUMMY)
00103            INCLUDE MGCOMM
00103       C
00135            ENTRY C01(DUMMY)
00137              COF1 = -XO(4)
00140            RETURN
00140       C
00141            ENTRY C02(DUMMY)
00143              COF1 = -XO(7)
00144            RETURN
00144       C
00145            ENTRY C03(DUMMY)
00147              COF1 = F01(N,I,K)
00150            RETURN
00150       C
00151            ENTRY C04(DUMMY)
00153              COF1 = F02(N,K)
00154            RETURN
00154       C
00155            ENTRY C05(DUMMY)
00157              COF1 = D(1,I)
00160            RETURN
00160       C
00161            ENTRY C06(DUMMY)
00163              COF1 = F03(Z)
00164            RETURN
00164       C
00165            ENTRY C08(DUMMY)
00167              COF1 = F04(I,K)
00170            RETURN
00170       C
00171            ENTRY C09(DUMMY)
00173              COF1 = F05(K)
00174            RETURN
00174       C
00175            ENTRY C11(DUMMY)
00177              COF1 = F06(I,K)
00200            RETURN
00200       C
00201            ENTRY C12(DUMMY)
00203              COF1 = F07(K)
00204            RETURN
00204       C
00205            ENTRY C14(DUMMY)
00207              COF1 = F08(I,K)
00210            RETURN
00210       C
00211            ENTRY C15(DUMMY)
00213              COF1 = F09(K)
00214            RETURN
00214       C
00215            ENTRY C16(DUMMY)
00217              COF1 = UBIAS(3) + 0.01*GRADE(K)
00220            RETURN
00220       C
00221            ENTRY C17(DUMMY)
00223              COF1 = UBIAS(2) + 23.0
00224            RETURN
```

```
00406          NAMPOW =  6HTELO
00407          D0  1200 K      = 1, KMAX
00412          CALL ITOBCD(NAMROW,K      ,24,12)
00413          CALL RwPI(NAMROW,PI)
00414          PTFLO ( K)          =PI
00415     1200 CONTINUE
00415     C
00415     C-ROW(S) FOR GPAD
00415     C
00417          NAMROW =  6HGRAD
00420          DO  1270 K      = 1, KMAX
00423          CALL ITOBCD(NAMROW,K      ,24,12)
00424          CALL RwPI(NAMROW,PI)
00425          PGFAD ( K)          =PI
00426     1270 CONTINUE
00426     C
00426     C-ROW(S) FOR BPAK
00426     C
00430          NAMPOW =  6HBRAK
00431          DO  1340 K      = 1, KMAX
00434          CALL ITOBCD(NAMROW,K      ,24,12)
00435          CALL RwRI(NAMROW,PI)
00436          PBRAK ( K)          =PI
00437     1340 CONTINUE
00437     C
00437     C-FIND EOF MARK
00437     C
00441          CALL RWSOL(999)
00441     C
00442          RETURN
00443          END
END COMPILATION **    0 MESSAGES **
```

```
00330   C-READ PI VALUES
00330   C
00331         CALL RWSOL(LUNIT)
00331   C
00331   C
00331   C-ROW(S) FOR OBJ
00331   C
00332         NAMROW =  6HOBJ
00333         CALL RWPI(NAMROW,PI)
00333   C
00333   C-ROW(S) FOR OC1000
00333   C
00334         NAMROW =  6HOC1000
00335         CALL RWPI(NAMROW,PI)
00336         POC100=PI
00336   C
00336   C-ROW(S) FOR OC2000
00336   C
00337         NAMROW =  6HOG2000
00340         CALL RWPI(NAMROW,PI)
00341         POC200=PI
00341   C
00341   C-ROW(S) FOR OC
00341   C
00342         NAMROW =  6HOC
00343         DO   920 N      = 1, NMAX
00346         DO   910 K      = 1, KMAX
00351         CALL ITOBCD(NAMROW,N      ,12, 6)
00352         CABL ITOBCD(NAMROW,K      ,24,12)
00353         CALL RWPI(NAMROW,PI)
00354         POC    ( N,K)         =PI
00355     910 CONTINUF
00357     920 CONTINUF
00357   C
00357   C-ROW(S) FOR TC
00357   C
00361         NAMROW =  6HTC
00362         CALL RWPI(NAMROW,PI)
00363         PTC    =PI
00363   C
00363   C-ROW(S) FOR VEL
00363   C
00364         NAMROW =  6HVEL
00365         DO  1060 K      = 1, KMAX
00370         CALL ITOBCD(NAMROW,K      ,24,12)
00371         CALL RWPI(NAMROW,PI)
00372         PVEL   ( K)           =PI
00373    1060 CONTINUE
00373   C
00373   C-ROW(S) FOR TEHI
00373   C
00375         NAMROW =  6HTEHI
00376         DO  1130 K      = 1, KMAX
00401         CALL ITOBCD(NAMROW,K      ,24,12)
00402         CALL RWPI(NAMROW,PI)
00403         PTEHI  ( K)           =PI
00404    1130 CONTINUE
00404   C
00404   C-ROW(S) FOR TELO
00404   C
```

```
      CALL RWBETA(NAMROW,BETA)
      SOC100=BETA
C
C-ROW(S) FOR OC2000
C
      NAMROW =  6HOC2000
      CALL RWBETA(NAMROW,BETA)
      SOC200=BETA
C
C-ROW(S) FOR OC
C
      NAMROW =  6HOC
      DO   360 N        = 1, NMAX
      DO   350 K        = 1, KMAX
      CALL ITOBCD(NAMROW,N       ,12, 6)
      CALL ITOBCD(NAMROW,K       ,24,12)
      CALL RWBETA(NAMROW,BETA)
      SOC   ( N,K)         =BETA
  350 CONTINUE
  360 CONTINUE
C
C-ROW(S) FOR VEL
C
      NAMROW =  6HVEL
      DO   430 K        = 1, KMAX
      CALL ITOBCD(NAMROW,K       ,24,12)
      CALL RWBETA(NAMROW,BETA)
      SVEL  ( K)           =BETA
  430 CONTINUE
C
C-ROW(S) FOR TEHI
C
      NAMROW =  6HTEHI
      DO   500 K        = 1, KMAX
      CALL ITOBCD(NAMROW,K       ,24,12)
      CALL RWBETA(NAMROW,BETA)
      STEHI ( K)           =BETA
  500 CONTINUE
C
C-ROW(S) FOR TELO
C
      NAMROW =  6HTELO
      DO   570 K        = 1, KMAX
      CALL ITOBCD(NAMROW,K       ,24,12)
      CALL RWBETA(NAMROW,BETA)
      STELO ( K)           =BETA
  570 CONTINUE
C
C-ROW(S) FOR BRAK
C
      NAMROW =  6HBRAK
      DO   640 K        = 1, KMAX
      CALL ITOBCD(NAMROW,K       ,24,12)
      CALL RWBETA(NAMROW,BETA)
      SBRAK ( K)           =BETA
  640 CONTINUE
C
C-BACKTRACK 6HPAVEL
      CALL RWSOL(0)
C
```

```
@I FRS,* RWREAD
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00101      1*        SUBROUTINE RWREAD(KUNIT)
00101      2*  C
00101      3*  C-THIS SR READS ONE SOLUTION INTO CORE
00101      4*  C
00103      5*        INCLUDE MGCOMM, LIST
00104      5*        PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20MGCOMM *INCLUDED*
00105      5*        PARAMETER MBAR=1                                                  MGCOMM *INCLUDED*
00106      5*        PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KT=2**NT-1    MGCOMM *INCLUDED*
00107      5*        PARAMETER MM=4+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS       MGCOMM *INCLUDED*
00110      5*        PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1        MGCOMM *INCLUDED*
00111      5*        PARAMETER LLBYNK=LL*NK                                            MGCOMM *INCLUDED*
00112      5*        COMMON/MGTABS/                                                    MGCOMM *INCLUDED*
00113      5*        COMMON/MGTABS/                                                    MGCOMM *INCLUDED*
00113      5*       *  XXXX   ,                                                        MGCOMM *INCLUDED*
00113      5*       X  SPAREC                                                          MGCOMM *INCLUDED*
00114      5*        COMMON/MGCONT/                                                    MGCOMM *INCLUDED*
00114      5*       1  KREAD  ,KPRINT,MATRIX,                                          MGCOMM *INCLUDED*
00114      5*       2  BIG    ,SMALL ,NERROR,LISTOP,                                   MGCOMM *INCLUDED*
00114      5*       3  KTITLE(12),MGSWCH(50),                                          MGCOMM *INCLUDED*
00114      5*       4  NAMID ,NAMCOL,NAMROW,KELEM (2),KCOMM                            MGCOMM *INCLUDED*
00115      5*        COMMON/MGTABS/                                                    MGCOMM *INCLUDED*
00115      5*       1  N      ,I     ,K     ,Z     ,                                   MGCOMM *INCLUDED*
00115      5*       2  NMAX   ,IMAX  ,KMAX  ,DMAX  ,                                   MGCOMM *INCLUDED*
00115      5*       X  SPAREA                                                          MGCOMM *INCLUDED*
00116      5*        INTEGER                                                           MGCOMM *INCLUDED*
00116      5*       1  N      ,I     ,K     ,Z     ,                                   MGCOMM *INCLUDED*
00116      5*       2  NMAX   ,IMAX  ,KMAX  ,DMAX  ,                                   MGCOMM *INCLUDED*
00116      5*       X  SPAREA                                                          MGCOMM *INCLUDED*
00117      5*        COMMON/MGTABS/ MGIND,MGLUS( 12)                                   MGCOMM *INCLUDED*
00120      5*        DIMENSION MGCEQV(74),MGTEQV(     24)                              MGCOMM *INCLUDED*
00121      5*        EQUIVALENCE (MGCEQV,KREAD),(MGTEQV,XXXX  )                        MGCOMM *INCLUDED*
00122      5*        DATA KREAD,KPRINT,MATRIX,SMALL,BIG/5,6,13,1.0E-6,9999.0/          MGCOMM *INCLUDED*
00130      5*        REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11  MGCOMM *INCLUDED*
00131      5*        REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1   MGCOMM *INCLUDED*
00132      5*        COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11  MGCOMM *INCLUDED*
00133      5*        COMMON/MYBLK2/ M1,M11,FV,T1                                       MGCOMM *INCLUDED*
00134      5*        END                                                               MGCOMM *INCLUDED*
00135      6*        INCLUDE RWCOMM, LIST
00136      6*        COMMON/RWTABS/                                                    RWCOMM *INCLUDED*
00136      6*       1  IOBJ   ,IRHS  ,OBJVAL,NOBJE                                     RWCOMM *INCLUDED*
00137      6*        COMMON/RWTABS/                                                    RWCOMM *INCLUDED*
00137      6*       1  BU     ( 60     ),                                              RWCOMM *INCLUDED*
00137      6*       2  BPI    ( 1      ),                                              RWCOMM *INCLUDED*
00137      6*       *  RWDUMA                                                          RWCOMM *INCLUDED*
00140      6*        COMMON/RWTABS/                                                    RWCOMM *INCLUDED*
00140      6*       1  SOC    ( 2     ,20     ),                                       RWCOMM *INCLUDED*
00140      6*       2  POC    ( 2     ,20     ),                                       RWCOMM *INCLUDED*
00140      6*       3  SVEL   ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       4  PVEL   ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       5  STEHI  ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       6  PTEHI  ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       7  STELO  ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       8  PTELO  ( 20    ),                                               RWCOMM *INCLUDED*
00140      6*       9  PCPAD  ( 20    )                                                RWCOMM *INCLUDED*
00141      6*        COMMON/RWTABS/                                                    RWCOMM *INCLUDED*
00141      6*       1  SBRAK  ( 20    ),                                               RWCOMM *INCLUDED*
00141      6*       2  PBRAK  ( 20    ),                                               RWCOMM *INCLUDED*
00141      6*       *  RWDUMB                                                          RWCOMM *INCLUDED*
```

```
00143          KUNRVL = 8
00144          KCOMM = 0
00145          NSELEC=-1
00146          NAMELIST/PARAM/ KCOMM,KUNRVL,NSELEC
00147          READ(5,PARAM,END=10)
00152       10 IF(KCOMM) 50,30,40
00155       30 CONTINUE
00156          CALL MGHEAD(IFEND)
00157          IF(IFEND .EQ.0) CALL MGINPT
00161          IF(NERROR .LE. 0) GO TO 50
00163          WRITE (KPRINT,32)
00165       32 FORMAT(41H NERROR .GT. 0 IN RWMAIN. JOB TERMINATED.)
00166       40 CONTINUE
00167          READ(KCOMM) MGCEQV
00175          READ(KCOMM)MGTEQV
00203          READ (KCOMM) A,B,XO,UBIAS,ALPHA,T1                     *NEW
00232       50 CALL POSIT(NSELEC,KUNRVL)
00233          CALL RWREAD(KUNRVL)
00234          CALL REPORT
00235          GOTO 50
00236          END
END COMPILATION **    0 MESSAGES **
```

```
BI FRS,* RWMAIN
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00100     1*   C-MAIN PROGRAM RWMAIN
00101     2*          INCLUDE MGCOMM, LIST
00103     2*          PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20MGCOMM *INCLUDED*
00104     2*          PARAMETER MBAR=1                                                 MGCOMM *INCLUDED*
00105     2*          PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KT=2**NT-1    MGCOMM *INCLUDED*
00106     2*          PARAMETER MM=M+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS       MGCOMM *INCLUDED*
00107     2*          PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1        MGCOMM *INCLUDED*
00110     2*          PARAMETER LLBYNK=LL*NK                                           MGCOMM *INCLUDED*
00111     2*          COMMON/MGTABS/                                                   MGCOMM *INCLUDED*
00112     2*          COMMON/MGTABS/                                                   MGCOMM *INCLUDED*
00112     2*        *    XXXX   ,                                                      MGCOMM *INCLUDED*
00112     2*        X    SPAREC                                                        MGCOMM *INCLUDED*
00113     2*          COMMON/MGCOUT/                                                   MGCOMM *INCLUDED*
00113     2*        1    KREAD  ,KPRINT,MATRIX,                                        MGCOMM *INCLUDED*
00113     2*        2    BIG    ,SMALL  ,NERROR,LISTOP,                                MGCOMM *INCLUDED*
00113     2*        3    KTITLE(12),MGSWCH(50),                                        MGCOMM *INCLUDED*
00113     2*        4    NAMID  ,NAMCOL,NAMROW,KELEM (2),KCOMM                         MGCOMM *INCLUDED*
00114     2*          COMMON/MGTABS/                                                   MGCOMM *INCLUDED*
00114     2*        1    M      ,I      ,K      ,Z      ,                              MGCOMM *INCLUDED*
00114     2*        2    NMAX   ,IMAX   ,KMAX   ,DMAX   ,                              MGCOMM *INCLUDED*
00114     2*        X    SPAREA                                                        MGCOMM *INCLUDED*
00115     2*          INTEGER                                                          MGCOMM *INCLUDED*
00115     2*        1    N      ,I      ,K      ,Z      ,                              MGCOMM *INCLUDED*
00115     2*        2    NMAX   ,IMAX   ,KMAX   ,DMAX   ,                              MGCOMM *INCLUDED*
00115     2*        X    SPAREA                                                        MGCOMM *INCLUDED*
00116     2*          COMMON/MGTABS/ MGIND,MGLUS( 12)                                  MGCOMM *INCLUDED*
00117     2*          DIMENSION MGCEQV(74),MGTEQV(     24)                             MGCOMM *INCLUDED*
00120     2*          EQUIVALENCE (MGCEQV,KREAD),(MGTEQV,XXXX  )                       MGCOMM *INCLUDED*
00121     2*          DATA KREAD,KPRINT,MATRIX,SMALL,BIG/5,6,13,1.0E-6,9999.0/         MGCOMM *INCLUDED*
00127     2*          REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11 MGCOMM *INCLUDED*
00130     2*          REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1  MGCOMM *INCLUDED*
00131     2*          COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11 MGCOMM *INCLUDED*
00132     2*          COMMON/MYBLK2/ M1,M11,FV,T1                                      MGCOMM *INCLUDED*
00133     2*          END                                                              MGCOMM *INCLUDED*
00134     3*          INCLUDE PWCOMM,LIST
00135     3*          COMMON/RWTABS/                                                   RWCOMM *INCLUDED*
00135     3*        1    IOBJ   ,IRHS   ,OBJVAL,NOBJF                                  RWCOMM *INCLUDED*
00136     3*          COMMON/RWTABS/                                                   RWCOMM *INCLUDED*
00136     3*        1    RU     ( 60     ),                                            RWCOMM *INCLUDED*
00136     3*        2    BPI    ( 1      ),                                            RWCOMM *INCLUDED*
00136     3*        *    RWDUMA                                                        RWCOMM *INCLUDED*
00137     3*          COMMON/RWTABS/                                                   RWCOMM *INCLUDED*
00137     3*        1    SOC    ( 2      ,20      ),                                   RWCOMM *INCLUDED*
00137     3*        2    POC    ( 2      ,20      ),                                   RWCOMM *INCLUDED*
00137     3*        3    SVCL   ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        4    PVCL   ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        5    STEHI  ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        6    PTEHI  ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        7    STELO  ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        8    PTELO  ( 20     ),                                            RWCOMM *INCLUDED*
00137     3*        9    PGPAD  ( 20     )                                             RWCOMM *INCLUDED*
00140     3*          COMMON/RWTABS/                                                   RWCOMM *INCLUDED*
00140     3*        1    SBRAK  ( 20     ),                                            RWCOMM *INCLUDED*
00140     3*        2    PBRAK  ( 20     ),                                            RWCOMM *INCLUDED*
00140     3*        *    RWDUMB                                                        RWCOMM *INCLUDED*
00141     3*          COMMON/RWTABS/                                                   RWCOMM *INCLUDED*
00141     3*        *    SOBJ   ,SOC100,POC100,SOC200,POC200,PTC                       RWCOMM *INCLUDED*
00142     3*          END                                                              RWCOMM *INCLUDED*
```

```
00712          DO 190 I = 1,NLOCOS
00715          K = M + I
00716      190 B(K,I) = 1.0
00720          B(MM,NLOCOS + 1) = 1.0
00721          B(MMM,NLOCOS + 2) = 1.0
00722          WRITE (6,1007) T1
00725          WRITE (6,300) ((B(I,K),K=1,LL),I=1,MMM)
00736          DO 200 I = 1,MMM
00741          X1(I) = X0(I)
00742          DO 200 II = 1,LL
00745      200 B1(I,II) = B(I,II)
00750          KK = LL*(NK-1)
00751          DO 205 I = 1,MMM
00754          DO 205 II = 1,LL
00757          K = KK+II
00760      205 D(I,K) = B(I,II)
00763          DO 220 I = 1,NK
00766          CALL MXMLT (A,X1,X2,MMM,MMM,1,MMM,MMM)
00767          CALL MXMLT (A,B1,B2,MMM,MMM,LL,MMM,MMM)
00770          DO 210 II = 1,MMM
00773          X1(II) = X2(II)
00774          DO 210 III = 1,LL
00777      210 B1(II,III) = B2(II,III)
01002          K = NK-I+1
01003          KK = LL*(NK-I-1)
01004          DO 220 II = 1,MMM
01007          AKXO(II,K) = X2(II)
01010          IF (KK.LT.0) GO TO 220
01012          DO 219 III = 1,LL
01015          KKK = KK+III
01016          D(II,KKK) = B2(II,III)
01017      219 CONTINUE
01021      220 CONTINUE
01024          WRITE (6,1014)
01026          DO 221 K = 1,5
01031          K1 = 12*(K-1)+1
01032          K2 = 12*K
01033      221 WRITE (6,1012) ((D(I,KK),KK=K1,K2),I=1,MMM)
01045          WRITE (6,1015)
01047          WRITE (6,1013) ((AKXO(I,K),K=1,10),I=1,MMM)
01060          WRITE (6,1013) ((AKXO(I,K),K=11,20),I=1,MMM)
01071          RETURN
01072          END
END COMPILATION **     0 MESSAGES **
   MGIWPT  CODE   RELOCATABLE                 09 FEB 71  11:08:44   1  01112604   24   1 (DELETED)
                                                                   0  01112634   14   3
```

```
00430         WRITE (6,1000) ((DEL2 (I,K),K=1,M),I=1,M)
00441         DO 70 I = 1,M
00444         DO 70 K = 1,M
00447      70 DELTAD(I,K) = DEL1(I,K)
00452         DO 80 I = 1,NT
00455         CALL MXMLT(DELTAD,DELTAD,DEL,M,M,M,M,M)
00456         DO 80 K = 1,M
00461         DO 80 KK = 1,M
00464      80 DELTAD(K,KK) = DEL(K,KK)
00470         WRITE (6,1003) T1
00473         WRITE(6,1000) ((DELTAD(I,K),K=1,M),I=1,M)
00504         DO 90 I = 1,M
00507         DO 90 II = 1,M
00512         DEL(I,II) = 0.0
00513         DELTA(I,II) = 0.0
00514         IF (I.NE.II) GO TO 90
00516         DEL(I,I) = 1.0
00517         DELTA(I,I) = 1.0
00520      90 CONTINUE
00523         DO 100 I = 1,KT
00526         CALL MXMLT(DEL1,DELTA,DEL3,M,M,M,M,M)
00527         DO 95 II = 1,M
00532         DO 95 III=1,M
00535      95 DELTA(II,III) = DEL3(II,III)
00540     100 CALL MXADD(DEL,DELTA,DEL,M,M,M)
00542         CALL MXMLT(DEL2,DEL,DELTA,M,M,M,M,M)
00543         WRITE (6,1004) T1
00546         WRITE(6,1000) ((DELTA(I,K),K=1,M),I=1,M)
00557         CALL MXMLT(DELTA,OMEGA,OMEGAD,M,M,L,M,M)
00560         WRITE (6,1005) T1
00565         WRITE(6,700) ((OMEGAD(I,K),K=1,L),I=1,M)
00574         DO 120 I = 1,M
00577         DO 120 K = 1,M
00602     120 A(I,K) = DELTAD(I,K)
00605         DO 130 I = 1,NLOCOS
00610         K = I + M
00611         DO 130 II = 1,M
00614     130 A(II,K) = OMEGAD(II,I)
00617         KKK = M + NLOCOS - INCTAU + 1
00620         DO 140 I = 1,NCARS
00623         K = KKK + INCTAU + I
00624         KK = NCARS + NLOCOS + 1 - I
00625         DO 140 II = 1,M
00630     140 A(II,K) = OMEGAD(II,KK)
00633         KKK = M + NLOCOS + TAUN - INCOGA + 1
00634         DO 150 I = 1,NCARS
00637         K = KKK + INCOGA + I
00640         KK = L + 1 - I
00641         DO 150 II = 1,M
00644     150 A(II,K) = OMEGAD(II,KK)
00647         DO 170 I = 2,TAUN
00652         K = M + NLOCOS + I
00653     170 A(K-1,K) = 1.0
00655         DO 180 I = 2,OMEGAN
00660         K = MM + I
00661     180 A(K-1,K)= 1.0
00663         WRITE (6,1006) T1
00666         WRITE (6,1000) ((A(I,K),K=1,10),I=1,MMM)
00677         WRITE (6,1011)
00701         WRITE (6,1000) ((A(I,K),K=11,20),I=1,MMM)
```

```
 1008 FORMAT (1H1////T16,' TRUNCATION ERROR IN SERIES APPROXIMATION OF S
     *TM AND ITS INTEGRAL FOR T =',F7.3,' SECONDS'//////)
 1011 FORMAT (1H1//////////)
 1012 FORMAT (12E10.3)
 1013 FORMAT(10E10.3)
 1014 FORMAT(1H1,' D MATRIX FOLLOWS')
 1015 FORMAT(1H1,' AKXO MATRIX FOLLOWS')
      T1 = T*(KT + 1)
      DELTA(1,2) = 1.0
      DELTA(M,M) = -1.0 / SIGMA(NCARS + 1)
      DO 10 I = 1,NCARS
      DELTA(3*I-1,3*I) = -E(I) / MASS(I)
      DELTA(3*I,3*I)   = -1.0 / SIGMA(I)
      IF (I.EQ.1) GO TO 2
      DELTA(3*I-2,3*I-4) = 1.0
      DELTA(3*I-2,3*I-1) = -1.0
      DELTA(3*I-1,3*I-4) = BETA(I-1) / MASS(I)
      DELTA(3*I-1,3*I-2) = ALPHA(I-1)/ MASS(I)
      IF (I.EQ.NCARS) GO TO 6
    4 DELTA(3*I-1,3*I+1) = -ALPHA(I) / MASS(I)
      DELTA(3*I-1,3*I+2) =   BETA(I)  / MASS(I)
      IF (I.EQ.1) GO TO 10
      DELTA(3*I-1,3*I-1) = -(BETA(I-1)+BETA(I))/MASS(I)-G*LAMBDA(I)
      GO TO 10
    2 DELTA(3*I-1,3*I-1) = -BETA(I) / MASS(I)    - G*LAMBDA(I)
      GO TO 4
    6 DELTA(3*I-1,3*I-1) = -BETA(I-1) / MASS(I) - G*LAMBDA(I)
   10 CONTINUE
      WRITE(6,1001)
      WRITE(6,1000) ((DELTA(I,K),K=1,M),I=1,M)
      DO 20 K = 1,NLOCOS
      I = LOCON(K)
   20 OMEGA(3*I-1,K) = 1.0 / MASS(I)
      DO 30 I = 1,NCARS
      K = I + NLOCOS
   30 OMEGA(3*I,K) = ETA / SIGMA(I)
      I = NLOCOS + JL
      OMEGA(M,I) = 1.0 / SIGMA(NCARS+1)
      DO 40 I = 1,NCARS
      K = I + NCARS + NLOCOS
   40 OMEGA(3*I-1,K) = -G
      WRITE(6,1002)
      WRITE(6,700) ((OMEGA(I,K),K=1,L),I=1,M)
      DO 50 I = 1,M
   50 DELTAD(I,I) = 1.0
      DO 60 I = 1,NSUM
      TI = I
      TT = T / TI
      CALL MXADD(DELTAD,DEL1,DEL1,M,M,M)
      CALL MXSCA(DELTAD,M,M,M,TT)
      CALL MXADD(DELTAD,DEL2,DEL2,M,M,M)
      CALL MXMLT(DELTA,DELTAD,DEL,M,M,M,M,M)
      DO 60 K = 1,M
      DO 60 KK = 1,M
   60 DELTAD(K,KK) = DEL(K,KK)
      WRITE (6,1009) T
      WRITE (6,1000) ((DELTAD(I,K),K=1,M),I=1,M)
      WRITE (6,1003) T
      WRITE (6,1000) ((DEL1(I,K),K = 1,M),I=1,M)
      WRITE (6,1004) T
```

```
BI FRS,* MGINPT
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00101          SUBROUTINE MGSUFR
00101     C
00101     C-THIS S-R READS CURRENT MAXIMUMS FROM FILE NUMBER 13
00101     C
00103          INCLUDE MGCOMM
00135          NAMELIST/MGSUFS/
00135         1     MMAX  ,IMAX  ,KMAX  ,DMAX  ,
00135         X     SPAPEA
00136          READ(13,MGSUFS)
00141          RETURN
00141     C
00142          ENTRY MGINPT
00142     C
00142     C-THIS S-R READ EXTERNAL VALUES FROM CARDS AND PRINT THE DATA
00142     C
00143          CALL PREPIO
00143     C
00143     C-XXXX    -CARDS
00143     C
00144          IF (MGSWCH(1).EQ.2) GOTO  110
00146          CALL MGLINE(1)
00147          WRITE(KPPINT,  115)
00151      115 FORMAT(2X,
00151         * 6HXXXX  /2X,
00151         *                                                   1X)
00152      110 CONTINUE
00153          READ(KREAD,  125) KCH,IN,      XXXX
00160      125 FORMAT(2A6, F5.0)
00161          CALL CHFCIN(KCH,IN,6HXXXX  ,     1)
00162          IF(MGSWCH(1).EQ.2)GOTO   120
00164          WRITE(KPPINT,  135) KCH,IN,    XXXX
00171      135 FORMAT(2X,2A6,    F7.0)
00172      120 CONTINUE
00173          RETURN
00174          SUBROUTINE PREPIO
00177          REAL DELTA(M,M),DELTAD(M,M),DEL(M,M),DEL1(M,M),DEL2(M,M),DELX(M,M)
00200          REAL OMEGA(M,L),OMEGAD(M,L),MASS(NCARS),BETA(LESS1),LAMBDA(NCARS)
00201          REAL SIGMA(PLUS1),E(NCARS),B1(MMM,LL),B2(MMM,LL)
00202          INTEGER LOCON(LOCOS)
00203          DATA MASS/73292.0,2*59627.0/
00205          DATA BETA/2*15000.0/,LOCON/1/
00210          DATA LAMBDA,SIGMA,E/0.000054,2*0.000045,3*15.0,60.0,3*1440.0/
00214          DATA ETA,G,T/2.5,32.174,0.25/
00220      300 FORMAT (1H0,3E12.5)
00221      700 FORMAT (1H0,7E12.5)
00222     1000 FORMAT (1H0,10E12.5)
00223     1001 FORMAT (1H1////T43,'SYSTEM MATRIX OF CONTINUOUS SYSTEM'//////)
00224     1002 FORMAT (1H1////T25,'CONTROL MATRIX OF CONTINUOUS SYSTEM'//////)
00225     1003 FORMAT (1H1////T37,'SYSTEM TRANSITION MATRIX FOR T =',F7.3,
00225         *' SECONDS'//////)
00226     1004 FORMAT (1H1////T31,'INTEGRAL OF SYSTEM TRANSITION MATRIX FOR T =',
00226         *F7.3,' SECONDS'//////)
00227     1005 FORMAT (1H1////T19,'CONTROL MATRIX OF DISCRETE SYSTEM FOR T =',
00227         *F7.3,' SECONDS'//////)
00230     1006 FORMAT (1H1////T28,'AUGMENTED SYSTEM MATRIX OF DISCRETE SYSTEM FOR
00230         * T =',F7.3,' SECONDS'//////)
00231     1007 FORMAT (1H1////1H ,'AUGMENTED CONTROL MATRIX OF DISCRETE SYSTEM FO
00231         *R T =',F7.3,' SECONDS'//////)
```

```
00512   239*         DO 31290 K       = 1, KMAX
00515   240*         CALL ITOBCD(IIAMROW,N      ,12, 6)
00516   241*         CALL ITOBCD(IIAMROW,K      ,24,12)
00517   242*         CALL MGFLMT(  C04(DUMMY), 0 , 0 )
00520   243*   31290 CONTINUE
00522   244*   31300 CONTINUE
00522   245*   C
00522   246*   C        ROW(S) FOR TC
00522   247*   C
00524   248*         NAMPOW =  6HTC
00525   249*         CALL MGFLMT(  C06(DUMMY), 0 , 0 )
00525   250*   C
00525   251*   C        ROW(S) FOR VEL
00525   252*   C
00526   253*         NAMPOW =  6HVEL
00527   254*         DO 31440 K       = 1, KMAX
00532   255*         CALL ITOBCD(IIAMROW,K      ,24,12)
00533   256*         CALL MGFLMT(  C09(DUMMY), 0 , 0 )
00534   257*   31440 CONTINUE
00534   258*   C
00534   259*   C        ROW(S) FOR TEHI
00534   260*   C
00536   261*         NAMPOW =  6HTEHI
00537   262*         DO 31510 K       = 1, KMAX
00542   263*         CALL ITOBCD(IIAMROW,K      ,24,12)
00543   264*         CALL MGFLMT(  C12(DUMMY), 0 , 0 )
00544   265*   31510 CONTINUE
00544   266*   C
00544   267*   C        ROW(S) FOR TELO
00544   268*   C
00546   269*         NAMPOW =  6HTELO
00547   270*         DO 31580 K       = 1, KMAX
00552   271*         CALL ITOBCD(IIAMROW,K      ,24,12)
00553   272*         CALL MGFLMT(  C15(DUMMY), 0 , 0 )
00554   273*   31580 CONTINUE
00554   274*   C
00554   275*   C        ROW(S) FOR GRAD
00554   276*   C
00556   277*         NAMPOW =  6HGRAD
00557   278*         DO 31650 K       = 1, KMAX
00562   279*         CALL ITOBCD(IIAMROW,K      ,24,12)
00563   280*         CALL MGFLMT(  C16(DUMMY), 0 , 0 )
00564   281*   31650 CONTINUE
00564   282*   C
00564   283*   C        ROW(S) FOR BRAK
00564   284*   C
00566   285*         NAMPOW =  6HBRAK
00567   286*         DO 31720 K       = 1, KMAX
00572   287*         CALL ITOBCD(IIAMROW,K      ,24,12)
00573   288*         CALL MGFLMT(  C17(DUMMY), 0 , 0 )
00574   289*   31720 CONTINUE
00576   290*         WRITE(MATRIX,24)
00600   291*         RETURN
00601   292*         END
END COMPILATION **     0 MESSAGES **
```

```
00432        C
00432        C      OBJ    ROW
00432        C
00433               NAMROW =  6HOBJ
00434               IF(Z      .NE.     1)GO TO 16020
00436               CALL MGELMT( 1.0          , 0 , 0 )
00437        16020 CONTINUE
00437        C
00437        C      OC1000 ROW
00437        C
00440               NAMROW =  6HOC1000
00441               IF(Z      .NE.     1)GO TO 16040
00443               CALL MGELMT( -1.0         , 0 , 0 )
00444        16040 CONTINUE
00444        C
00444        C      OC2000 ROW
00444        C
00445               NAMROW =  6HOC2000
00446               IF(Z      .NE.     1)GO TO 16060
00450               CALL MGELMT( -1.0         , 0 , 0 )
00451        16060 CONTINUE
00451        C
00451        C      OC     ROW
00451        C
00452               NAMROW =  6HOC
00453               IF(Z      .NE.     1)GO TO 16080
00455               DO 16078 N     = 1, NMAX
00460               DO 16076 K     = 1, KMAX
00463               CALL ITOBCD(NAMROW,N      ,12, 6)
00464               CALL ITOBCD(NAMROW,K      ,24,12)
00465               CALL MGELMT( -1.0         , 0 , 0 )
00466        16076 CONTINUE
00470        16078 CONTINUE
00472        16080 CONTINUE
00473        31078 CONTINUE
00473        C
00473        C         FIRST B
00473        C         -------
00475               WRITE(MATRIX,23)
00475        C
00475        C-SET MGIND TO 1001 FOR FIRST B SECTION
00475        C
00477               MGIND = 1001
00500               NAMCOL = 6H
00501               NAMID = 6H
00501        C
00501        C         ROW(S) FOR OC1000
00501        C
00502               NAMROW =  6HOC1000
00503               CALL MGELMT(  C01(DUMMY), 0 , 0 )
00503        C
00503        C         ROW(S) FOR OC2000
00503        C
00504               NAMROW =  6HOC2000
00505               CALL MGELMT(  C02(DUMMY), 0 , 0 )
00505        C
00505        C         ROW(S) FOR OC
00505        C
00506               NAMROW =  6HOC
00507               DO 31300 N     = 1, NMAX
```

```
00334          NAMROW = 6HOC
00335          DO   938 N       = 1, NMAX
00340          DO   936 K       = 1, KMAX
00343          CALL ITOBCD(NAMROW,N      ,12, 6)
00344          CALL ITOBCD(NAMROW,K)     ,24,12)
00345          CALL MGFLMT(  C03(DUMMY), 0 , 0 )
00346      936 CONTINUE
00350      938 CONTINUE
00350    C
00350    C     TC     ROW
00350    C
00352          NAMROW = 6HTC
00353          CALL MGFLMT(  C05(DUMMY), 0 , 0 )
00353    C
00353    C     VEL     ROW
00353    C
00354          NAMROW =  6HVEL
00355          DO   978 K       = 1, KMAX
00360          CALL ITOBCD(NAMROW,K      ,24,12)
00361          CALL MGFLMT(  C08(DUMMY), 0 , 0 )
00362      978 CONTINUE
00362    C
00362    C     TEHI    ROW
00362    C
00364          NAMROW =  6HTEHI
00365          DO   998 K       = 1, KMAX
00370          CALL ITOBCD(NAMROW,K      ,24,12)
00371          CALL MGFLMT(  C11(DUMMY), 0 , 0 )
00372      998 CONTINUE
00372    C
00372    C     TELO   ROW
00372    C
00374          NAMROW =  6HTELO
00375          DO  1018 K       = 1, KMAX
00400          CALL ITOBCD(NAMROW,K      ,24,12)
00401          CALL MGFLMT(  C14(DUMMY), 0 , 0 )
00402     1018 CONTINUE
00402    C
00402    C     GRAD   ROW
00402    C
00404          NAMROW =  6HGRAD
00405          DO  1038 K       = 1, KMAX
00410          CALL ITOBCD(NAMROW,K      ,24,12)
00411          CALL MGFLMT(  C18(DUMMY), 0 , 0 )
00412     1038 CONTINUE
00412    C
00412    C     BRAK   ROW
00412    C
00414          NAMROW =  6HBRAK
00415          DO  1058 K       = 1, KMAX
00420          CALL ITOBCD(NAMROW,K      ,24,12)
00421          CALL MGFLMT(  C19(DUMMY), 0 , 0 )
00422     1058 CONTINUE
00424    15998 CONTINUE
00424    C
00424    C     PI     VARIABLES
00424    C     ---------------
00426          NAMCOL = 6HPI
00427          DO 31078 Z       = 1, DMAX
00432          CALL ITOBCD(NAMCOL,Z      ,18, 6)
```

```
00227            NAMROW =  6HVEL
00230            DO   610 K        = 1, KMAX
00233            CALL ITOBCD(NAMROW,K      ,24,12)
00234            IFLAG  =  6H       +
00235            WRITE(MATRIX,25)  IFLAG,NAMROW
00241        610 CONTINUE
00241      C
00241      C        ROW(S) FOR TEHI
00241      C
00243            NAMROW =  6HTEHI
00244            DO   680 K        = 1, KMAX
00247            CALL ITOBCD(NAMROW,K      ,24,12)
00250            IFLAG  =  6H       +
00251            WRITE(MATRIX,25)  IFLAG,NAMROW
00255        680 CONTINUE
00255      C
00255      C        ROW(S) FOR TELO
00255      C
00257            NAMROW =  6HTELO
00260            DO   750 K        = 1, KMAX
00263            CALL ITOBCD(NAMROW,K      ,24,12)
00264            IFLAG  =  6H       +
00265            WRITE(MATRIX,25)  IFLAG,NAMROW
00271        750 CONTINUE
00271      C
00271      C        ROW(S) FOR GRAD
00271      C
00273            NAMROW =  6HGRAD
00274            DO   820 K        = 1, KMAX
00277            CALL ITOBCD(NAMROW,K      ,24,12)
00300            IFLAG  =  6H
00301            WRITE(MATRIX,25)  IFLAG,NAMROW
00305        820 CONTINUE
00305      C
00305      C        ROW(S) FOR BRAK
00305      C
00307            NAMROW =  6HBRAK
00310            DO   890 K        = 1, KMAX
00313            CALL ITOBCD(NAMROW,K      ,24,12)
00314            IFLAG  =  6H       +
00315            WRITE(MATRIX,25)  IFLAG,NAMROW
00321        890 CONTINUE
00321      C
00321      C          MATRIX
00321      C          ------
00323            WRITE(MATRIX,22)
00323      C
00323      C-- SET MGIND TO 0 FOR MATRIX SECTION
00323      C
00325            MGIND = 0
00326            NAMID = 6H
00326      C
00326      C     U       VARIABLES
00326      C     -----------------
00327            NAMCOL = 6HU
00330            DO 15920 I       = 1, IMAX
00333            CALL ITOBCD(NAMCOL,I      ,12,12)
00333      C
00333      C     OC      ROW
00333      C
```

```
WI FR5,* MGOUT
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
 00101       1*          SUBROUTINE MGOUT
 00103       2*          INCLUDE MGCOMM
 00135       3*       20 FORMAT(1H*,12A6)
 00136       4*       21 FORMAT(6HROW ID ,72X)
 00137       5*       22 FORMAT(6HMATRIX ,72X)
 00140       6*       23 FORMAT(7HFIRST B,72X)
 00141       7*       24 FORMAT(6HEOF    ,72X)
 00142       8*       25 FORMAT(6X,2A6)
 00143       9*       32 FORMAT(12X,6HSETEND)
 00143      10*    C
 00144      11*          WRITE(MATRIX,20) KTITLE
 00144      12*    C
 00144      13*    C        ROW ID
 00144      14*    C        ------
 00152      15*          WRITE(MATRIX,21)
 00152      16*    C
 00152      17*    C--SET MGIND TO -1000 FOR ROW ID SECTION
 00152      18*    C
 00154      19*          MGIND = -1000
 00154      20*    C
 00154      21*    C       ROW(S) FOR OBJ
 00154      22*    C
 00155      23*          NAMROW = 6HOBJ
 00156      24*          IFLAG  =  6H     Z
 00157      25*          WRITE(MATRIX,25)  IFLAG,NAMROW
 00157      26*    C
 00157      27*    C       ROW(S) FOR OC1000
 00157      28*    C
 00163      29*          NAMROW =  6HOC1000
 00164      30*          IFLAG  =  6H     +
 00165      31*          WRITE(MATRIX,25)  IFLAG,NAMROW
 00165      32*    C
 00165      33*    C       ROW(S) FOR OC2000
 00165      34*    C
 00171      35*          NAMROW =  6HOC2000
 00172      36*          IFLAG  =  6H     +
 00173      37*          WRITE(MATRIX,25)  IFLAG,NAMROW
 00173      38*    C
 00173      39*    C       ROW(S) FOR OC
 00173      40*    C
 00177      41*          NAMROW =  6HOC
 00200      42*          DO   470 N        = 1, NMAX
 00203      43*          DO   460 K        = 1, KMAX
 00206      44*          CALL ITOBCD(NAMROW,N      ,12, 6)
 00207      45*          CALL ITOBCD(NAMROW,K      ,24,12)
 00210      46*          IFLAG  =  6H     +
 00211      47*          WRITE(MATRIX,25)  IFLAG,NAMROW
 00215      48*      460 CONTINUE
 00217      49*      470 CONTINUE
 00217      50*    C
 00217      51*    C       ROW(S) FOR TC
 00217      52*    C
 00221      53*          NAMROW =  6HTC
 00222      54*          IFLAG  =  6H
 00223      55*          WRITE(MATRIX,25)  IFLAG,NAMROW
 00223      56*    C
 00223      57*    C       ROW(S) FOR VEL
 00223      58*    C
```

```
WI FRS,* MGMAIN
COMPILED BY LEVEL 23.8  CSCX FORTRAN V ON 19 JUN 73 AT 22:21
00100     1*    C-MAIN PROGRAM MGMAIN
00101     2*          INCLUDE MGCOMM
00134     3*          CALL MGHEAD(IFEND)
00135     4*          IF(IFEND.EQ.0)  CALL MGINPT
00137     5*          IF(NERROR .EQ. 0) GO TO 20
00141     6*          WRITE (KPPIIIT,10)
00143     7*       10 FORMAT(41H NERROR .GT. 0 IN MGMAIN. JOB TERMINATED.)
00144     8*          STOP
00145     9*       20 CALL MGOUT
00146    10*          IF(MATRIX.EQ.6) GOTO 50
00150    11*          END FILE MATRIX
00151    12*          REWIND MATRIX
00152    13*       50 IF(KCOMM.EQ.0) STOP
00154    14*          WRITE(KCOMM)MGCEQV
00162    15*          WRITE(KCOMM)MGTEQV
00170    16*          WRITE (KCOMM) A,B,XO,UBIAS,ALPHA,T1                    *NEW
00217    17*          END FILE KCOMM
00220    18*          REWIND KCOMM
00221    19*          STOP
00222    20*          END
END COMPILATION **    0 MESSAGES **
```

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```
@I POP,* P.COMM
UNIVAC 1108 PROCEDURE DEFINITION PROCESSOR DATED MAY 5, 1969
THIS PROC ELEMENT PROCESSED ON 19 JUN 73 AT 22:21:20
         PROC ORIGIN 1        ENTRY POINT 1     RWCOMM* FCOPY
                                                  COMMON/RWTABS/
                                                  1   IOBJ   ,IRHS   ,OBJVAL,NOBJF
                                                  COMMON/RWTABS/
                                                  1    RU     ( 60      ),
                                                  2    BPI    ( 1       ),
                                                  *  RWDUMA
                                                  COMMON/RWTABS/
                                                  1    SOC    ( 2      ,20     ),
                                                  2    POC    ( 2      ,20     ),
                                                  3    SVEL   ( 20     ),
                                                  4    PVEL   ( 20     ),
                                                  5    STEHI  ( 20     ),
                                                  6    PTEHI  ( 20     ),
                                                  7    STELO  ( 20     ),
                                                  8    PTELO  ( 20     ),
                                                  9    PGRAD  ( 20     )
                                                  COMMON/RWTABS/
                                                  1    SBRAK  ( 20     ),
                                                  2    PBRAK  ( 20     ),
                                                  *  RWDUMB
                                                  COMMON/RWTABS/
                                                  *  SOBJ  ,SOC100,POC100,SOC200,POC200,PTC
                                                  END
PROCESSING TIME =  0 SECONDS
```

```
@I POP,* MGCOMM
UNIVAC 1108 PROCEDURE DEFINITION PROCESSOR DATED MAY 5, 1969
THIS PROC.ELEMENT PROCESSED ON 19 JUN 73 AT 22:21:19
      PROC ORIGIN 1      ENTRY POINT 1    MGCOMM* FCOPY
                                          PARAMETER NCARS=3,NLOCOS=1,JL=3,TAUN=3,OMEGAN=6,NSUM=10,NT=4,NK=20
                                          PARAMETER MBAR=1
                                          PARAMETER L=2*NCARS+NLOCOS,M=3*NCARS+1,LL=NLOCOS+2,KJ=2**NT-1
                                          PARAMETER MM=M+NLOCOS+TAUN,MMM=MM+OMEGAN,INCTAU=TAUN/NCARS
                                          PARAMETER INCOGA=OMEGAN/NCARS,LESS1=NCARS-1,PLUS1=NCARS+1
                                          PARAMETER LLBYNK=LL*NK
                                          COMMON/MGTABS/
                                          COMMON/MGTABS/
                                         *   XXXX   ,
                                         X   SPAREC
                                          COMMON/MGCONT/
                                         1   ,KREAD, ,KPRINT,MATRIX,
                                         2    BIG    ,SMALL ,NERROR,LISTOP,
                                         3    KTITLE(12),MGSWCH(50),
                                         4    NAMID ,NAMCOL,NAMROW,KELEM (2),KCOMM
                                          COMMON/MGTABS/
                                         1    N    ,I      ,K     ,Z     ,
                                         2    NMAX ,IMAX   ,KMAX  ,DMAX  ,
                                         X    SPAREA
                                          INTEGER
                                         1    N    ,I      ,K     ,Z     ,
                                         2    NMAX ,IMAX   ,KMAX  ,DMAX  ,
                                         X    SPAREA
                                          COMMON/MGTABS/ MGIND,MGLUS( 12)
                                          DIMENSION MGCEQV(74),MGTEQV(     24)
                                          EQUIVALENCE (MGCEQV,KREAD),(MGTEQV,XXXX )
                                          DATA KREAD,KPRINT,MATRIX,SMALL,BIG/5,6,15,1.0E-6,9999.0/
                                          REAL A(MMM,MMM),B(MMM,LL),D(MMM,LLBYNK),AKXO(MMM,NK),XO(MMM),M11
                                          REAL X1(MMM),X2(MMM),UBIAS(LL),GRADE(NK),ALPHA(LESS1),L1,L11,M1
                                          COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11
                                          COMMON/MYBLK2/ M1,M11,FV,T1
                                          END

PROCESSING TIME =  1 SECONDS
```

```
@ XQT CUR
  1. IN B
     END FILE --- UNIT B
  2. IN,A
     END FILE --- UNIT A
     END CUR
```

```
 99 COMMON STORAGE LOCATIONS IN MG
426 COMMON STORAGE LOCATIONS IN RW
```

```
      FUNCTION F08(I,K)
      F08 = 0.0
      KK = LL*(NK-K)+I
      IF (KK.GT.LLBYNK) GO TO 10
      F08 = D(2,KK) - D(11,KK)/M1
   10 RETURN
C
      FUNCTION F09(K)
      F09 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK = KK + LL*(II-1) + III
   10 F09 = F09 + (D(2,KKK) - D(11,KKK)/M1)*UBIAS(III)
      KK = NK-K+1
      F09 = F09 + M11/M1 - (AKXO(2,KK) - AKXO(11,KK)/M1)
      RETURN
C
      FUNCTION F10(I,K)
      F10 = 0.0
      IF (I.NE.K*LL) RETURN
      F10 = 1.0
      RETURN
C
      FUNCTION F11(I,K)
      F11 = 0.0
      IF (I.NE.K*LL-1) RETURN
      F11 = 1.0
      RETURN
      END
```

```
C
      FUNCTION F05(K)
      F05 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK = KK + LL*(II-1) + III
   10 F05 = F05 + D(2,KKK)*UBIAS(III)
      KK = NK-K+1
      F05 = F05 + VMAX - AKXO(2,KK)
      RETURN
C
      FUNCTION F06(I,K)
      F06 = 0.0
      KK = LL*(NK-K)+I
      IF (KK.GT.LLBYNK) GO TO 10
      F06 = D(2,KK) + D(11,KK)/L1
   10 RETURN
C
      FUNCTION F07(K)
      F07 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK = KK + LL*(II-1) +III
   10 F07 = F07 + (D(2,KKK) + D(11,KKK)/L1)*UBIAS(III)
      KK = NK-K+1
      F07 = F07 + L1I/L1 - (AKXO(2,KK) + AKXO(11,KK)/L1)
      RETURN
C
```

```
      IF (KK.GT.LLBYNK) GO TO 10
      F01 = D(3*N+1,KK)
   10 RETURN
C
      FUNCTION F02(N,K)
      F02 = 0.0
      KK = LL*(NK-K)
      DO 10 II = 1,K
      DO 10 III = 1,LL
      KKK= KK+LL*(II-1)+III
   10 F02 = F02 + D(3*N+1,KKK)*UBIAS(III)
      KK = NK-K+1
      F02 = F02 - AKXO(3*N+1,KK)
      RETURN
C
      FUNCTION F03(Z)
      F03 = 0.0
      DO 10 II = 1,NK
      DO 10 III = 1,LL
      KK =  LL*(II-1)+III
   10 F03 = F03 + D(1,KK)*UBIAS(III)
      F03 = F03 + FV - AKXO(1,1)
      RETURN
C
      FUNCTION F04(I,K)
      F04 = 0.0
      KK = LL*(NK-K)+I
      IF (KK.GT.LLBYNK) GO TO 10
      F04 = D(2,KK)
   10 RETURN
```

```
      SUM(I) C15 *U(I+I) .EQ.C16
FOR ALL K
*     BRAK          '****KK'
      SUM(I) C19 * U(I) .LE.C17
FOR ALL K
C
ELEMENTS
      C01 = -X0(4)
      C02 = -X0(7)
      C03 = F01(M,I,K)
      C04 = F02(N,K)
      C05 = D(1,I)
      C06 = F03(2)
      C08 = F04(I,K)
      C09 = F05(K)
      C11 = F06(I,K)
      C12 = F07(K)
      C14 = F08(I,K)
      C15 = F09(K)
      C16 = UBIAS(3) + 0.01*GRADE(K)
      C17 = UBIAS(2) + 23.0
      C18 = F10(I,K)
      C19 = F11(I,K)
C
FUNCTIONS
C
      FUNCTION F01(M,I,K)
      F01 = 0.0
      KK = LL*(NK-K)+I
```

```
PROBLEM
MINIMISE
C        DUMMY LP VARIABLE PI
*    OBJ          '***'
     1.0 * PI(1)
C
SUBJECT TO
*    OC1000       '******'
     -1.0 * PI(1) .LE.C01
*    OC2000       '******'
     -1.0 * PI(1) .LE.C02
*    OC           '**II KK'
     SUM(I) C03 * U(I) - 1.0 * PI(1) .LE.C04
FOR ALL N,K
*    TC           '**'
     SUM(I) C05 * U(I) .EQ.C06
*    VEL          '*** KK'
     SUM(I) C08 * U(I) .LE.C09
FOR ALL K
*    TEHI         '****KK'
     SUM(I) C11 * U(I) .LE.C12
FOR ALL K
*    TELO         '****KK'
     SUM(I) C14 * U(I) .LE.C15
FOR ALL K
*    GRAD         '****KK'
```

```
      KKK = KK+III
      D(II,KKK) = D2(II,III)
  219 CONTINUE
  220 CONTINUE
      WRITE (6,1014)
      DO 221 K = 1,5
      K1 = 12*(K-1)+1
      K2 = 12*K
  221 WRITE (6,1012) ((D(I,KK),KK=K1,K2),I=1,MMM)
      WRITE (6,1015)
      WRITE (6,1013) ((AKXO(I,K),K=1,10),I=1,MMM)
      WRITE (6,1013) ((AKXO(I,K),K=11,20),I=1,MMM)
      RETURN
```

```
      WRITE (6,1000) ((A(I,K),K=11,20),I=1,MMM)
      DO 190 I = 1,NLOCOS
      K = M + I
  190 B(K,I) = 1.0
      B(MM,NLOCOS + 1) = 1.0
      B(MMM,NLOCOS + 2) = 1.0
      WRITE (6,1007) T1
      WRITE (6,300) ((B(I,K),K=1,LL),I=1,MMM)
      DO 200 I = 1,MMM
      X1(I) = X0(I)
      DO 200 II = 1,LL
  200 B1(I,II) = B(I,II)
      KK = LL*(NK-1)
      DO 205 I = 1,MMM
      DO 205 II = 1,LL
      K = KK+II
  205 D(I,K) = B(I,II)
      DO 220 I = 1,NK
      CALL MXM[illegible](A,X1,X2,MMM,MMM,1,MMM,MMM)
      CALL MXMLT (A,B1,B2,MMM,MMM,LL,MMM,MMM)
      DO 210 II = 1,MMM
      X1(II) = X2(II)
      DO 210 III = 1,LL
  210 B1(II,III) = B2(II,III)
      K = NK-I+1
      KK = LL*(NK-I-1)
      DO 220 II = 1,MMM
      AKXO(II,K) = X2(II)
      IF (KK.LT.0) GO TO 220
      DO 219 III = 1,LL
```

```
C     SET UP A AND B MATRICES
C
      DO 120 I = 1,M
      DO 120 K = 1,M
  120 A(I,K) = DELTAD(I,K)
      DO 130 I = 1,NLOCOS
      K = I + M
      DO 130 II = 1,M
  130 A(II,K) = OMEGAD(II,I)
      KKK = M + NLOCOS - INCTAU + 1
      DO 140 I = 1,NCAPS
      K = KKK + INCTAU * I
      KK = NCARS + NLOCOS + 1 - I
      DO 140 II = 1,M
  140 A(II,K) = OMEGAD(II,KK)
      KKK = M + NLOCOS + TAUN - INCOGA + 1
      DO 150 I = 1,NCARS
      K = KKK + INCOGA * I
      KK = L + 1 - I
      DO 150 II = 1,M
  150 A(II,K) = OMEGAD(II,KK)
      DO 170 I = 2,TAUN
      K = M + NLOCOS + I
  170 A(K-1,K) = 1.0
      DO 180 I = 2,OMEGAN
      K = MM + I
  180 A(K-1,K)= 1.0
      WRITE (6,1006) T1
      WRITE (6,1000) ((A(I,K),K=1,10),I=1,MMM)
      WRITE (6,1011)
```

```
   80 DELTAD(K,KK) = DEL(K,KK)
      WRITE (6,1003) T1
C
      WRITE(6,1000) ((DELTAD(I,K),K=1,M),I=1,M)
C     CALCULATE INTEGRAL OF STM FOR T1 IN DELTA
C
      DO 90 I = 1,M
      DO 90 II = 1,M
      DEL(I,II) = 0.0
      DELTA(I,II) = 0.0
      IF (I.NE.II) GO TO 90
      DEL(I,I) = 1.0
      DELTA(I,I) = 1.0
   90 CONTINUE
      DO 100 I = 1,KT
      CALL MXMLT(DEL1,DELTA,DEL3,M,M,M,M,M)
      DO 95 II = 1,M
      DO 95 III= 1,M
   95 DELTA(II,III) = DEL3(II,III)
  100 CALL MXADD(DEL,DFLTA,DEL,M,M,M)
      CALL MXMLT(DEL2,DEL,DELTA,M,M,M,M,M)
      WRITE (6,1004) T1
      WRITE(6,1000) ((DELTA(I,K),K=1,M),I=1,M)
C
C     CALCULATE DISCRETE CONTROL MATRIX
C
      CALL MXMLT(DELTA,OMEGA,OMEGAD,M,M,L,M,M)
      WRITE (6,1005) T1
      WRITE(6,700) ((OMEGAD(I,K),K=1,L),I=1,M)
C
```

```
C     CALCULATE DFL2 = INTEGRAL OF STM FOR T
C
      DO 50 I = 1,M
   50 DELTAD(I,I) = 1.0
      DO 60 J = 1,NSUM
      TI = I
      TT = T / TI
      CALL MXADD(DELTAD,DEL1,DEL1,M,M,M)
      CALL MXSCA(DELTAD,M,M,M,TT)
      CALL MXADD(DELTAD,DEL2,DEL2,M,M,M)
      CALL MXMLT(DELTA,DELTAD,DEL,M,M,M,M,M)
      DO 60 K = 1,M
      DO 60 KK = 1,M
   60 DELTAD(K,KK) = DFL(K,KK)
      WRITE (6,1009) T
      WRITE (6,1000) ((DELTAD(I,K),K=1,M),I=1,M)
      WRITE (6,1003) T
      WRITE (6,1000) ((DEL1(I,K),K = 1,M),I=1,M)
      WRITE (6,1004) T
      WRITE (6,1000) ((DEL2 (I,K),K=1,M),I=1,M)
C
C     CALCULATE STM FOR T1
C
      DO 70 I = 1,M
      DO 70 K = 1,M
   70 DELTAD(I,K) = DEL1(I,K)
      DO 80 I = 1,NT
      CALL MXMLT(DELTAD,DELTAD,DEL,M,M,M,M,M)
      DO 80 K = 1,M
      DO 80 KK = 1,M
```

```
      IF (I.EQ.NCARS) GO TO 6
    4 DELTA(3*I-1,3*I+1) = -ALPHA(I) / MASS(I)
      DELTA(3*I-1,3*I+2) =  BETA(I)  / MASS(I)
      IF (I.EQ.1) GO TO 10
      DELTA(3*I-1,3*I-1) = -(BETA(I-1)+BETA(I))/MASS(I)-G*LAMBDA(I)
      GO TO 10
    2 DELTA(3*I-1,3*I-1) = -BETA(I) / MASS(I)    - G*LAMBDA(I)
      GO TO 4
    6 DELTA(3*I-1,3*I-1) = -BETA(I-1) / MASS(I) - G*LAMBDA(I)
   10 CONTINUE
      WRITE(6,1001)
      WRITE(6,1000) ((DELTA(I,K),K=1,M),I=1,M)
C
C     SET UP OMEGA MATRIX
C
      DO 20 K = 1,NLOCOS
      I = LOCOM(K)
   20 OMEGA(3*I-1,K) = 1.0 / MASS(I)
      DO 30 I = 1,NCARS
      K = I + NLOCOS
   30 OMEGA(3*I,K) = ETA / SIGMA(I)
      I = NLOCOS + JL
      OMEGA(M,I) = 1.0 / SIGMA(NCARS+1)
      DO 40 I = 1,NCARS
      K = I + NCARS + NLOCOS
   40 OMEGA(3*I-1,K) = -G
      WRITE(6,1002)
      WRITE(6,700) ((OMEGA(I,K),K=1,L),I=1,M)
C
C     CALCULATE DEL1 = STM FOR T
```

```
     *' SECONDS'//////)
 1004 FORMAT (1H1////T31,'INTEGRAL OF SYSTEM TRANSITION MATRIX FOR T =',
     *F7.3,' SECONDS'//////)
 1005 FORMAT (1H1////T14,'CONTROL MATRIX OF DISCRETE SYSTEM FOR T =',
     *F7.3,' SECONDS'//////)
 1006 FORMAT (1H1////T28,'AUGMENTED SYSTEM MATRIX OF DISCRETE SYSTEM FOR
     * T =',F7.3,' SECONDS'//////)
 1007 FORMAT (1H1//4/1H ,'AUGMENTED CONTROL MATRIX OF DISCRETE SYSTEM FO
     *R T =',F7.3,' SECONDS'//////)
 1008 FORMAT (1H1///T16,' TRUNCATION ERROR IN SERIES APPROXIMATION OF S
     *TM AND ITS INTEGRAL FOR T =',F7.3,' SECONDS'//////)
 1011 FORMAT (1H1//////////)
 1012 FORMAT (12E10.3)
 1013 FORMAT(10E10.3)
 1014 FORMAT(1H1,' D MATRIX FOLLOWS')
 1015 FORMAT(1H1,' AKXO MATRIX FOLLOWS')
      TI = T*(KT + 1)
C
C     SET UP DELTA MATRIX
C
      DELTA(1,2) = 1.0
      DELTA(M,M) = -1.0 / SIGMA(NCARS + 1)
      DO 10 I = 1,NCARS
      DELTA(3*I-1,3*I) = -E(I) / MASS(I)
      DELTA(3*I,3*I)   = -1.0  / SIGMA(I)
      IF (I.EQ.1) GO TO 2
      DELTA(3*I-2,3*I-4) = 1.0
      DELTA(3*I-2,3*I-1) = -1.0
      DELTA(3*I-1,3*I-4) = BETA(I-1) / MASS(I)
      DELTA(3*I-1,3*I-2) = ALPHA(I-1)/ MASS(I)
```

```
DATA-CARD  LAYOUT
   OPTIONAL CARDS IN ANY ORDER
TITLE *(PROBLEM TITLE COL.7-78)
OPTIONS  *(OPTION SWITCHES IN COL.11-60)
UNITS    *(KPRINT COL.11-15,MATRIX COL.16-20, KREAD COL. 21-25,KCOMM COL.26-30,BIG COL.36-45,SMALL COL.51-60)
*(CURRENT MAXIMUMS IN COL.2-78)

   COMPULSORY CARDS IN INPUT ORDER
DATA

NAME    DESCRIPTION                              IMPLIED IND.                     EXTERN. IND.
XXXX

CARD: KCH,IM,XXXX
FORMAT:(2A6,F5.0)

         1111111111222222222233333333334444444444555555555566666666667777777777778
12345678901234567890123456789012345678901234567890123456789012345678901234567890
XXXX     1,    *****************************************************************
        NG PROGRAM EXPECTS UP TO     1  CARDS.

      SUBROUTINE PREPIO
      REAL DELTA(M,M),DELTAD(M,M),DEL(M,M),DEL1(M,M),DEL2(M,M),DEL3(M,M)
      REAL OMEGA(M,L),OMEGAD(M,L),MASS(NCARS),BETA(LESS1),LAMBDA(NCARS)
      REAL SIGMA(PLUS1),E(NCARS),B1(MMM,LL),B2(MMM,LL)
      INTEGER LOCON(NLOCOS)
      DATA MASS/73292.0,2*59627.0/
      DATA BETA/2*15000.0/,LOCON/1/
      DATA LAMBDA,SIGMA,E/0.000054,2*0.000045,3*15.0,60.0,3*1440.0/
      DATA ETA,G,T/2.5,32.174,0.25/
  300 FORMAT (1H0,3E12.5)
  700 FORMAT (1H0,7E12.5)
 1000 FORMAT (1H0,10E12.5)
 1001 FORMAT (1H1////T43,'SYSTEM MATRIX OF CONTINUOUS SYSTEM'//////)
 1002 FORMAT (1H1////T25,'CONTROL MATRIX OF CONTINUOUS SYSTEM'//////)
 1003 FORMAT (1H1///T37,'SYSTEM TRANSITION MATRIX FOR T =',F7.3,
```

```
COMMON/MYBLK1/ A,B,D,AKXO,XO,X1,X2,UBIAS,GRADE,ALPHA,VMAX,L1,L11
COMMON/MYBLK2/ M1,M11,FV,T1
DATAMODIFICATIONS
```