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Gauge Fields and Feynman Rules in a Fully Left-Right Supersymmetric Extension of the Standard Model

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ABSTRACT

Gauge Fields and Feynman Rules in a Fully Left-Right
Supersymmetric Extension of the Standard Model

René M. Francis

A supersymmetric, left-right symmetric extension of the standard model is proposed. The model is specified by determining its particle content and the hierarchy of VEV's of the Higgs fields and hence the pattern of symmetry breaking. The Lagrangian of the model is then written down for one generation of quarks and leptons. The gauge sector of the theory is investigated and expressions for the low-energy bosons, charginos, and neutralinos are obtained. These expressions are then used to compute the Feynman diagrams of three interactions involving the gauge fields.

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INTRODUCTION

A little over twenty years ago, apart from quantum electrodynamics (QED), particle physics phenomena were described by piecemeal phenomenological models, with no fundamental theory in sight. In stark contrast to this, we have today a very successful model which describes nearly all available data pertaining to the strong, weak, and electromagnetic interactions - the standard model.

Although its name implies a single theory, the standard model is really made up of two separate parts: the Glashow-Weinberg-Salam (GWS) theory, which combines electromagnetism with the weak force in a non-trivial way; and quantum chromodynamics (QCD) which is accepted as the theory of the strong interactions. It is because these are both gauge theories that, with an eye on the possible unification (one day) of all the forces of Nature, they are placed side by side in a "single" gauge theory based on the group $SU(3) \times SU(2) \times U(1)$. The fact that only partial unification is thus achieved, is one of the reasons for speculating that the standard model may be the effective, low energy limit of a more complete theory.

Certain features of the GWS theory also raise questions about the completeness of the model: the "skew" nature, or left-handed symmetry of the weak interactions is one of them; another is the Higgs sector of the theory. Extensive experimental investigation has so far, been unable to confirm the existence of the spin zero elementary Higgs boson responsible for spontaneous symmetry breaking. Although one could argue that it is only a matter of time before it is discovered, it is widely thought that deeper problems exist in connection with the Higgs particle, which make it necessary to look beyond the standard model.

Over the years, considerable effort has gone into finding possible solutions to the problems just mentioned. In this thesis, two such possibilities are considered: left-right symmetry; and supersymmetry. Combining these symmetries, a model is proposed in which the GWS theory is expanded to include a right-handed gauge symmetry group within a globally supersymmetric theory.

The thesis is organized in the following way. The first three chapters provide background material on the GWS model, left-right symmetry, and supersymmetry. In chapter four the proposed $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model is specified, and its Lagrangian is written down. The physical gauge bosons of the theory are identified in chapter five, and in chapters six and seven, the same thing is done first for the charged gauginos then for the neutral gauginos. Finally, in chapter eight, the Feynman rules of three interactions involving gauge fields are computed.

CHAPTER 1

THE GLASHOW-WEINBERG-SALAM MODEL

One of the major elements of the present day theory of electroweak interactions, the *spontaneous breaking of symmetry,* originated in condensed matter physics. In the early 1960's Nambu[1] and Goldstone[2], realising that this concept was of profound importance, speculated on its application to particle physics. Subsequently, in 1964, Higgs[3] pointed out that the consequences of spontaneous symmetry breaking in gauge theories are very different from those in non-gauge theories. Weinberg[4] and Salam[5], building on earlier work of Glashow[6], then applied Higgs' ideas to an $SU(2) \times U(1)$ gauge theory, which they claimed described satisfactorily the weak and electromagnetic interactions together, that is, in a *unified* way. Serious interest was shown in this theory when 't Hooft[7] proved, in 1971, that it was renormalisable. Since then, the model has met with notable experimental success.

The general idea of the theory is that the weak interaction is mediated by vector bosons, which are, "to begin with", massless. The Lagrangian for the theory also contains terms for massless matter fields and is invariant under an internal gauge symmetry group. A scalar field is introduced (the Higgs field) which has a non-vanishing vacuum-expectation-value (VEV.) Its effect is to induce the spontaneous breakdown of symmetry, which gives masses to the gauge bosons and to the matter fields, but not to the photon or the neutrino.

1.1 Isospin symmetry and parity violation

The concept of symmetries which relate "different" particles to each other was introduced by Heisenberg in the 1930's. He argued that if one were to "switch off" the electromagnetic interaction, there would then be very little difference between a proton and a neutron. In analogy with ordinary spin, his argument went on, one could think of these particles as different "isospin" states of the same particle. Just like ordinary spin, this new internal degree of freedom then gives rise to a symmetry group corresponding to the group SU(2).

Heisenberg's symmetry between nucleons is now believed to be an SU(2) symmetry of quarks, relating to the strong interaction. A further symmetry, weak isospin (the word "weak" will henceforth be dropped) also relates to each other particles such as the u- and d-quarks, and additonally the leptons such as the electron and its neutrino. A vital difference, however, between the concept of isospin as it was originally proposed, and the one that is currently used, is that now the symmetry is a local[8] one. Localizing (or "gauging") a symmetry has the effect of introducing into the theory a gauge potential whose quanta, the gauge bosons, mediate the interaction between the particles linked to each other by the symmetry.

Although the electron and neutrino form an isospin pair, there is a peculiarity about the neutrino that distinguishes it from its partner. Until 1956, it had been assumed that parity (left-right) invariance was a universal symmetry of Nature, but after a thorough study of the then available data, Lee and Yang[9] claimed that parity was violated in the weak interaction. The following year, in the first of a series of historic experiments, C.S.Wu[10] demonstrated the validity of their claim. The cumulative evidence of these experiments implies that the neutrino, which interacts only through the weak interaction, must always possess negative helicity. Put another way, the only neutrinos that have ever been observed are the "left-handed" particles, ν_L, and their "right-handed" antiparticles, $\bar{\nu}_R$.

The next few sections contain an outline of the GWS theory of electroweak interactions. This outline is based on the account given by Halzen and Martin[11], and is limited to summarizing the results which are important in the later development of the thesis.

1.2 $SU(2)_L \times U(1)_Y$ gauge symmetry of the matter fields

In order for the theory to reproduce parity violation in the weak interaction, the gauge symmetry $SU(2)_L$ of isospin is introduced, under which, the "isospinor" L =

$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$ transforms as a doublet and $R = e_R$ transforms as a singlet. Similar assignments exist for the other two lepton generations, but these will not be considered here. A further gauge symmetry is introduced into the model, $U(1)_Y$ of hypercharge, which makes possible the identification of electric charge by the quasi-Gell-Mann-Nishijima relation

$$Q = I_L^3 + \frac{Y}{2}, \tag{1.1}$$

where Q is the electric charge, I_L^3 is the third component of isospin, and Y is the hypercharge. The quantum numbers of L and R are

$$L : I_L = 1/2; \quad Y = -1$$

$$R ; I_L = 0; \quad Y = -2 \tag{1.2}$$

The first generation quarks are accommodated into the scheme by the following assignments

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L : I_L = 1/2; \quad Y = 1/3$$

$$u_R : I_L = 0; \quad Y = 4/3$$

$$d_R : I_L = 0; \quad Y = -2/3 \tag{1.3}$$

A right-handed up quark has been included since quarks, unlike neutrinos, have both right and left components.

The gauge invariant Lagrangian for the, massless, electron-neutrino lepton pair takes the form

$$\mathcal{L}_1 = \bar{L}\,\gamma^{\mu}\,(i\,\partial_{\mu} - \tfrac{1}{2}\,g\,\tau.W_{\mu} + \tfrac{1}{2}\,g'B_{\mu})\,L$$
$$+ \bar{R}\,\gamma^{\mu}\,(i\,\partial_{\mu} + g'B_{\mu})\,R - \tfrac{1}{4}\,W_{\mu\nu}.W^{\mu\nu} - \tfrac{1}{4}\,B_{\mu\nu}B^{\mu\nu} \tag{1.4}$$

where: g and g′ are the gauge coupling constants of $SU(2)_L$ and $U(1)_Y$, respectively; W_{μ} and B_{μ} are the respective gauge fields; and the quantum numbers of L and R have been inserted.

1.3 The Higgs field

The Higgs field consists of four real scalar fields ϕ_i arranged in an $SU(2)_L$ doublet, with hypercharge, Y = 1 :

$$\phi = \begin{pmatrix} \phi^{+} \\ \phi^{0} \end{pmatrix} \qquad \text{where} \qquad \begin{aligned} \phi^{+} &= (\phi_1 + i\phi_2)/\sqrt{2} \\ \phi^{0} &= (\phi_3 + i\phi_4)/\sqrt{2} \end{aligned} \tag{1.5}$$

The Higgs potential is given by

$$V(\phi) = \mu^2\,\phi^{\dagger}\phi + \lambda\,(\phi^{\dagger}\phi)^2 \tag{1.6}$$

where $\mu^2 < 0$ and $\lambda > 0$. This is the choice of coefficients which brings about the spontaneous breaking of symmetry, and hence the generation of masses for the particles.

If the VEV, $<\phi>$, of $\phi(x)$ is chosen to be

$$<\phi> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} , \qquad (1.7)$$

the gauge invariant Lagrangian for the Higgs field becomes

$$\mathcal{L}_2 = | \, (i\partial_\mu - \tfrac{1}{2}\, g\, \tau.\mathbf{W}_\mu - \tfrac{1}{2}\, g'\, B_\mu)\ \phi \, |^2 - V(\phi) \qquad (1.8)$$

where $| \quad |^2 \equiv (\quad)^\dagger(\quad)$

1.4 Masses of the gauge bosons

The gauge boson masses are obtained by substituting the VEV $<\phi>$ for $\phi(x)$ in the Lagrangian $\mathcal{L}_2$. The relevant term in (1.8) is

$$| \, (-\frac{ig}{2}\tau.\mathbf{W}_\mu - \frac{ig'}{2} B_\mu)\ \phi \, |^2$$

$$= \frac{1}{8} \left| \begin{pmatrix} gW^3_\mu + g'B_\mu & g(W^1_\mu - iW^2_\mu) \\ g(W^1_\mu + iW^2_\mu) & -gW^3_\mu + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$= \tfrac{1}{8}\, v^2 g^2 \left[(W^1_\mu)^2 + (W^2_\mu)^2 \right] + \tfrac{1}{8}\, v^2 (g'B_\mu - gW^3_\mu)(g'B^\mu - gW^{3\mu})$$

$$= \left(\tfrac{1}{2} vg \right)^2 W^+_\mu\, W^{-\mu} + \tfrac{1}{8}\, v^2 (W^3_\mu\ ,\ B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \qquad (1.9)$$

where $W^\pm = (W^1 \mp iW^2)/\sqrt{2}$ has been used.

The mass term expected for a charged boson is $M_W^2 W_\mu^+ W^{-\mu}$ and for a neutral boson it is $\frac{1}{2} M_A^2 A_\mu^2$, so the first term of (1.9) gives the mass of the charged bosons as

$$M_W = \frac{1}{2} vg \tag{1.10}$$

The neutral boson mass matrix is off-diagonal in the W_μ^3 and B_μ basis, implying that these two fields mix. Diagonalization of the matrix yields the physical fields:

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{(g^2 + g'^2)^{1/2}} , \quad \text{with mass, } M_A = 0 \tag{1.11}$$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{(g^2 + g'^2)^{1/2}} , \quad \text{with mass, } M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2} \tag{1.12}$$

This is the desired result, that is, one neutral gauge boson remains massless and is identified as the photon, whilst the other three bosons acquire a mass.

It is usual to define the weak mixing angle, or Weinberg angle, θ_W , as

$$\frac{g'}{g} = \tan \theta_W \tag{1.13}$$

In terms of θ_W , (1.11) and (1.12) become

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3$$

$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W^3_\mu \tag{1.14}$$

From (1.10) and (1.12), one of the predictions of the model is

$$\frac{M_W}{M_Z} = \cos \theta_W \tag{1.15}$$

1.5 Masses of the fermions

The Lagrangian, $\mathcal{L}_1$, in equation (1.4), describes a massless field. In fact, if a mass term of the form $-m\bar{L}L$ had been included, the Lagrangian would have lost its gauge invariance. An attractive feature of the standard model ("standard model" is the name given to the Weinberg-Salam model, enlarged to include the gauge symmetry group SU(3) of the "colour" interaction) is that the Higgs doublet which generates masses for $W^\pm$ and Z, also gives masses to the quarks and leptons. The gauge invariant term, included in the Lagrangian, which performs this task is

$$\mathcal{L}_3 = -G_e\left[(\bar{\nu} , \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R (\phi^- , \bar{\phi}^0) \begin{pmatrix} \nu \\ e \end{pmatrix}_L \right] \tag{1.16}$$

Spontaneous breaking of symmetry occurs, and the substitution

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.17}$$

is made in (1.15). The neutral Higgs field h(x) is the only remnant of the Higgs doublet ,(1.5), after the symmetry is spontaneously broken, the other three fields having been gauged away. On substitution of ϕ, the Lagrangian becomes

$$\mathcal{L}_3 = - \frac{G_e}{\sqrt{2}} v \left(\bar{e}_L e_R + \bar{e}_R e_L \right) - \frac{G_e}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L \right) h. \tag{1.18}$$

G_e is a free parameter of the theory which can be chosen so that

$$m_e = \frac{G_e v}{\sqrt{2}} \tag{1.19}$$

and hence generate the electron mass,

$$\mathcal{L}_3 = - m_e \bar{e}e - \frac{m_e}{v} \bar{e}eh \tag{1.20}$$

The second term in this equation represents a coupling between the electron and the Higgs. However, the smallness of the ratio m_e/v renders the coupling very weak, and therefore very difficult to detect experimentally.

The quark masses are generated in the same way, except that, in order to generate a mass for the upper member of a quark doublet, it is necessary to construct a new Higgs doublet from ϕ:

$$\phi_c = - i\tau_2\phi^* = \begin{pmatrix} -\bar{\phi}^0 \\ \phi^- \end{pmatrix} \xrightarrow{\text{breaking}} \frac{1}{\sqrt{2}} \begin{pmatrix} v + h \\ 0 \end{pmatrix} \tag{1.21}$$

ϕ_c transforms in an identical way to ϕ under $SU(2)_L$, but has opposite hypercharge (Y = -1.)

Again, the masses of the quarks are not predictions of the theory, but are defined in terms of a free parameter. The same thing is true for the mass of the Higgs particles; whose value is

$$m_h^2 = 2v^2\lambda \tag{1.22}$$

1.6 Beyond the standard model

There is no doubt that the standard model is an extremely successful theory. Not only does it reproduce all the known features of electroweak theory, it makes predictions about some new ones. Its prediction of weak neutral currents, together with their structures and basic parameters, was confirmed experimentally in the 1970's. In 1983, the $W^{\pm}$ and Z^0 bosons were discovered, and their masses were found very close to their calculated values. In fact, to date, the standard model has been found to be consistent with all the experimental tests to which it has been subjected.

Successful as it is, there are many reasons to believe that this theory is not the final word. Firstly, it is not as well determined as it could be. There are of the order of twenty free parameters in the model, covering such things as: the masses of the leptons; quarks; and gauge bosons;

their mixing angles and coupling constants. There is no mechanism in the theory to account for P- or CP-violation, or for the origin of weak interaction symmetry and its breaking.

The Higgs sector of the theory is particularly mysterious, and remains to date the least tested part of the model. If Higgs particles are detected, they will be the first "fundamental" scalar particles shown to exist. Scalar particles have several nice properties, among which is the ability to possess a non-vanishing VEV, without breaking Lorentz invariance. This, as was shown above, is one of the most important ingredients of the standard model. However, scalar particles have another property which is not considered as nice: their masses are subject to quadratic divergences in perturbation theory (details about divergences in perturbative quantum field theory and the subject of renormalisation, can be found in appendix C.) The fact that the standard model is renormalisable, means that it can be valid to very high energies, and this is where the divergences of the scalar field pose a problem. In the literature this is referred to as the "gauge hierarchy problem (GHP)." The subject is discussed in some detail in section 3.2 , but essentially, the problem is that it is very difficult in perturbation theory for two, widely separated, mass scales to co-exist. The reason is that the light scale is "polluted", through vacuum polarization and the exchange of virtual particles , by the heavy scale. In order for this "pollution" to have little effect, it is necessary to "fine tune" the parameters of the model up to very high orders in perturbation theory, a procedure which is thought to be very "unnatural".

For these reasons, and many others, a great deal of research has gone into either, developing new theories to replace the standard model, or, finding extensions to the model which improve its performance. In view of the considerable success that the standard model continues to enjoy, it is the second of these alternatives which is pursued in this thesis.

In the following two chapters, two possible extensions to the standard model are considered: "left-right symmetry;" and "supersymmetry."

CHAPTER 2

LEFT-RIGHT SYMMETRY

The original motivation for the introduction of left-right (L-R) symmetric models[12] based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ was to provide a possible mechanism for parity violation in weak interactions. In this framework, the weak interaction respects all space-time symmetries, as do the strong, electromagnetic, and gravitational interactions. The asymmetry observed in nature at low energies is then attributed to the non-invariance of the vacuum under parity symmetry[13]. A bonus of this approach is that it reproduces all the features of $SU(2)_L \times U(1)_Y$ at low energies.

There are other important reasons for considering this kind of L-R model. Foremost among them is the question of the neutrino mass[14]: if the neutrino has a mass; then this class of models becomes the most natural framework in which to work. In addition, if it turns out that quarks and leptons are themselves the results of a more fundamental substructure, and that the forces operating at the sub-structure level are similar to QCD[15], then there are strong arguments which point to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as the weak interaction symmetry, rather than $SU(2)_L \times U(1)_Y$. The B-L quantum number[16] (baryon number minus lepton number) is the only anomaly free quantum number left ungauged in the standard model, a fact which seems to suggest a deeper symmetry structure. By replacing the gauge generator $U(1)_Y$, which has no physical significance, with $U(1)_{B-L}$, all the generators of the theory acquire a physical meaning.

Another compelling reason to consider L-R models is found in CP-violation. In the Kobayashi-Maskawa parameterization of generation mixing, for three generations, all CP-violations are dependent on only one parameter, δ_{KM} (the K-M phase), and there is no hint as to why the observed CP-violation has milli-weak strength. The L-R model can give rise to CP-violation for only two generations, and can account for its strength by relating it to the suppression of V+A currents[17].

2.1 Particle content of the model

The L-R symmetric model of weak interactions, considered here, is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Isospinors are defined in much the same way as in the standard model, with the exception that there are now "right-handed" doublets too. For the first generation leptons these doublets are

$$L_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L ,$$

with $(SU(2)_L, SU(2)_R, U(1)_{B-L})$ quantum numbers : $(\frac{1}{2}, 0, -1)$

and $L_R \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_R$, with quantum numbers : $(0, \frac{1}{2}, -1)$

(2.1)

The first generation quark doublets are

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \text{with quantum numbers} = (\tfrac{1}{2}, 0, \tfrac{1}{3})$$

$$Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R \quad \text{with quantum numbers} = (0, \tfrac{1}{2}, \tfrac{1}{3}) \tag{2.2}$$

The Gell-Mann-Nishijima formula for this group is

$$Q = I^3_L + I^3_R + \frac{B-L}{2} \tag{2.3}$$

The Higgs sector of the theory, as is discussed on page 19, can be chosen from a number of possibilities. The one that will be considered in this thesis consists of a bi-doublet

$$\Phi \ (\tfrac{1}{2}, \tfrac{1}{2}, 0), \tag{2.4}$$

whose charge decomposition is

$$\begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix} \tag{2.5}$$

and two triplets

$$\Delta_L \ (1, 0, 2) \quad \text{and} \quad \Delta_R \ (0, 1, 2) \tag{2.6}$$

whose charge decompositions (in the matrix form: $\frac{1}{\sqrt{2}} \tau.\Delta$) are

$$\begin{pmatrix} \sqrt{\frac{1}{2}}\,\Delta^{+} & \Delta^{++} \\ \Delta^{0} & -\sqrt{\frac{1}{2}}\,\Delta^{+} \end{pmatrix}_{L,R} \qquad (2.7)$$

The gauge fields consist of an $SU(2)_R$ triplet W_R^{μ}, an $SU(2)_L$ triplet W_L^{μ} and a $U(1)_{B-L}$ singlet V_{μ}. The gauge coupling constants are g_L , g_R , and g_V.

2.2 Symmetry breaking

The model is constructed in such a way that , before symmetry breaking, it contains three gauge symmetries and a discrete parity symmetry, i.e., $g_L = g_R$. The breaking of symmetry is accomplished in three stages[18]

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \xrightarrow[M_P]{} SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \xrightarrow[M_{W_R}]{} SU(2)_L \times U(1)_Y$$

$$SU(2)_L \times U(1)_Y \xrightarrow[M_{W_L}]{} U(1)_{Em} \qquad (2.8)$$

At the first stage only the parity symmetry is broken (M_P is the mass scale at which this breaking occurs, no gauge boson of that mass is produced), this results in $g_L \neq g_R$, and leaves W_L and W_R massless. The second stage breaks $SU(2)_R \times U(1)_{B-L}$ to $U(1)_Y$, and is achieved through $\langle\Delta_R\rangle \neq 0$. Often, and this is the case for the Higgs sector in this thesis, the Higgs multiplets are chosen in such a way that the parity symmetry and $SU(2)_R$ are broken at the same scale, i.e., $M_P = M_{W_R}$. The final stage of breaking is brought about by $\langle\Phi\rangle \neq 0$ and (but this is not essential) $\langle\Delta_L\rangle \neq 0$.

As in the standard model, in order to ensure that $U(1)_{Em}$ remains unbroken, only the neutral Higgs fields are allowed to have non-zero VEV's. These values are

$$\langle\Delta_L\rangle = \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}$$

$$\langle\Delta_R\rangle = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

$$\langle\Phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' e^{i\alpha} \end{pmatrix} \qquad (2.9)$$

$\langle\Phi\rangle$ causes W_L and W_R to mix with a CP-violating phase $e^{i\alpha}$.

It should be noted that all L-R models contain a bi-doublet field Φ, whereas the additional Higgs fields can be members of doublets or triplets. The choice of a triplet representation is preferred because, as Mohapatra and Senjanovic show, it has the ability to generate a large Majorana mass for ν_R and, at the same time, a small one for ν_L, thus providing a natural explanation for the smallness of the ν_L mass. The doublet Higgs representations generate only Dirac masses, and consequently, achieve the objective of a small ν_L mass in a more contrived way. In addition, they have the defect of generating flavour-changing neutral currents [ref. (14)], which are not observed in experiments.

2.3 Hierarchy for the VEV's of the Higgs fields

The phenomenologically required hierarchy[19] for the VEV's of the Higgs fields is

$$v_R \gg \max(\kappa \ , \ \kappa') \gg v_L \tag{2.10}$$

Within these limits, there are strong arguments [ref.(19)] which favour the assignments

$$\kappa' = 0, \text{ and } v_L = 0$$

These values will be used for calculations in the rest of this thesis.

The L-R symmetric model presented here offers several interesting possibilities for a further refinement of electroweak theory. It is particularly appealing for the following two reasons: it restores parity to the status of a conserved quantum number in electroweak theory - just as it is in the other fundamental interactions; and it introduces B-L as a generator of gauge symmetry - the last anomaly-free generator left ungauged.

CHAPTER 3

SUPERSYMMETRY

The goal of theoretical physics is to describe as many phenomena as possible with a simple and natural theory. A classic example of this is Maxwell's unification of magnetism and electrostatics. In elementary particle physics, the hope is that a unified scheme will be found which combines all particles and their interactions into one consistent theory. In the first part of this century it seemed that considerable progress was being made in that direction: the wave-particle duality of quantum mechanics and the subsequent concept of the "exchange particle" in perturbative quantum field theory appeared to have abolished for good the Fermi-Bose, matter-force dichotomy. However, the advent of gauge theories has reintroduced it forces are mediated by gauge potentials, i.e., by vector fields of spin one, whereas matter (including integer spin mesons) is built of quarks and leptons, i.e., spin 1/2 fermions.

A part of the appeal of supersymmetric theories is that they "unite" bosons and fermions into multiplets, and so, once again lift the distinction between matter and interaction. Photinos ("superpartners" of the photons) are thought of as a carriers of the electromagnetic force just as much as photons, except that as fermions they obey the Pauli exclusion principle and so cannot conspire to form a coherent, measurable potential. Matter and force become phenomenological distinctions: bosons manifest themselves as forces because they can build up coherent classical fields; fermions are seen as matter because no two identical ones can occupy the same point in space - an intuitive definition of material existence.

For some time it was thought that symmetries which would naturally relate forces and fermionic matter would be in conflict with field theory. The progress in understanding elementary particles through the SU(3) classification of the "eight-fold way" (a global symmetry) had led to attempts to find a unifying symmetry which would directly relate to each other several of the SU(3) multiplets (baryon octet, decuplet, etc.), even if these had different spins. The failure of attempts to make those "spin symmetries" relativistically covariant led to the formulation of several no-go theorems, culminating in 1967 in a paper by Coleman and Mandula[32] which was widely understood to show that it is impossible, within the framework of relativistic field theory, to unify space-time symmetry with internal symmetries. More precisely, the theorem says that the charge operators whose eigenvalues represent internal quantum numbers, e.g.,isospin, must be translationally and rotationally invariant. This means that these operators commute with the energy, the momentum, and the angular momentum operators. In fact, the only symmetry generators which transform at all under both translations and rotations are those of the Lorentz transformation themselves. The generators of internal symmetries cannot relate eigenstates with different eigenvalues of the mass and spin operators. This means that irreducible multiplets of symmetry groups cannot contain particles of different mass or of different spin. This no-go theorem seems to rule out exactly the sort of unity sought. There is, however, a loophole in the theorem of Coleman and Mandula: it considers only those symmetry transformations which form Lie groups with real parameters. The charge operators associated with such Lie groups of symmetry transformations (their generators) obey well defined commutation relations with each other. Different spins in the same multiplet are allowed if symmetry operations, whose generators obey anticommutation relations, are included. This was first proposed in 1971 by Gol'fand and Likhtman[33], and followed up by Volkov and Akulov[34] who arrived at what is now called a non-linear realisation of supersymmetry. Their model was not renormalisable, but in 1973 Wess and Zumino[20] presented a renormalisable model of a spin 1/2 particle in interaction with two spin zero particles, where the particles are related by symmetry transformations and therefore "sit" in the same multiplet.

The aesthetic appeal of supersymmetry is obvious, but if that were all, it would not have commanded the staggering attention that it has. Somewhere in the region of two thousand papers have been published on the subject, and this in spite of the fact that not a shred of evidence has yet been found to support it. The two most important factors that fuel this interest are: (a) supersymmetric theories offer the possibility of resolving the gauge hierarchy problem; and (b) local (or gauged) supersymmetry implies gravity (supergravity) and therefore creates a framework in which the four interactions could be unified.

In the sections below, a necessarily, very sketchy outline of some features of supersymmetric theories is given. Much of the material contained in this chapter is based on references (21), (22), (23), and (27)

3.1 Supersymmetry algebra and some of its implications

Supersymmetry is a symmetry that transforms bosons into fermions and vice versa. The generator, Q, of these transformations must therefore have a fermionic character, and thus can be written as, Q_α , a left-handed Weyl spinor, which transforms as a (1/2, 0) representation under Lorentz transformations. Its Hermitean adjoint is denoted by, $\bar{Q}_{\dot{\beta}}$, a right-handed Weyl spinor, which transforms as (0, 1/2).

Since the the supersymmetry generators carry half a unit of spin they obey anticommutation relations. The anticommutator of any operator with its adjoint is non-zero, which implies

$$\{Q_\alpha , \bar{Q}_{\dot{\beta}}\} \neq 0 . \qquad (3.1)$$

$\{Q_\alpha , \bar{Q}_{\dot{\beta}}\}$ transforms as (1/2, 1/2) under Lorentz transformations, and since the only operator which transforms in this way is P_μ (the energy-momentum tensor), these operators must be connected to each other. Hence, the algebra that defines supersymmetry is:

$$[Q_\alpha \, , \, P_\mu] = 0 \; , \tag{3.2}$$

$$\{Q_\alpha \, , \, \bar{Q}_{\dot\beta}\} = 2 \, \sigma^\mu_{\alpha\dot\beta} \, P_\mu \; , \tag{3.3}$$

where, σ, denotes the Pauli matrices, and the factor 2 is chosen for convenience.

Equation (3.2), for $\mu = 0$ shows that Q commutes with the Hamiltonian $H = P^0$, which leads to the observation that states of non-zero energy are paired by the action of Q. In view of the fermionic character of Q, this implies that supersymmetric multiplets contain an equal number of bosonic and fermionic degrees of freedom. Moreover, these supersymmetric partners must have the same mass.

Equation (3.3) relates the supercharges to the Hamiltonian. Using $\sigma^\mu_{\alpha\dot\beta} \, \sigma_\nu^{\alpha\dot\beta} = 2 \, g^\mu_\nu$, the following relation can be derived

$$H = P^0 = \frac{1}{4} \, (\bar{Q}_1 Q_1 + Q_1 \bar{Q}_1 + \bar{Q}_2 Q_2 + Q_2 \bar{Q}_2) \; , \tag{3.4}$$

which implies the important result $H \geq 0$. Defining $|0\rangle$ as the vacuum state of a globally supersymmetric theory (i.e one which is not spontaneously broken), the further implication is

$$Q_\alpha|0\rangle = 0 \tag{3.5}$$

Since H is given by the anticommutator of the charges this leads to

$$E_{vac} \equiv \langle 0|H|0\rangle = 0 \tag{3.6}$$

If supersymmetry is spontaneously broken, then for some Q_α: $Q_\alpha|0\rangle \neq 0$, which implies that $E_{vac} \neq 0$. In summary, global supersymmetry is spontaneously broken if, and only if, the energy of the vacuum is not exactly zero.

3.2 Resolution of the gauge hierarchy problem

As was mentioned in section 1.5, the standard model produces highly accurate phenomenolgy, but provides no explanation for a variety of fundamental questions such as the origin of generations, the values of fundamental constants, the exact equality of the electric charge of the proton and the positron, and the role of gravity. Answers to these questions may come from a more complete theory at some small distance scale, e.g., grand unified theories (GUT). These theories indicate that a complete fundamental synthesis can only take place somewhere between 10^{14} GeV and the Planck scale 10^{19} GeV (the scale at which gravity plays an important role).

What physics may lie between the presently explored scale of 10^2 Gev and the GUT scale is an open question. One view, the "Boring Desert" hypothesis, states that nothing new will be found in that region. However, this view can only be held by ignoring the GHP, or the problem of "naturalness" of the two widely separated scales, represented by the Higgs VEV $\approx$ 250 GeV and the GUT scale.

The GHP is a consequence of the following assumptions:

(1) The standard model correctly describes nature up to energies $\approx$ M very much larger than the weak scale of about 250 GeV.

(2) New physics occurs at the scale M, possibly including GUT theories (e.g., SU(5)), gravity, etc.

(3) The behaviour of the world at ordinary energies is not very sensitive to the values of fundamental parameters. In particular, the very existence of a low energy world, characterized by the Higgs VEV, should not require the fundamental parameters of the microscopic world at scale M to be "fine tuned".

Assumption (3) needs some more explanation. A phenomenological field theory like the standard model must be cut off at small distances to be defined. The formal path integral is meaningless until a prescription is given for regularising its short distance behaviour. The prescription specifies a momentum k as a cut off and a set of parameters (coupling constant g and mass parameter μ)

which generally depend on k. The cutoff theory is only useful for momenta and energies less than k. The central point of renormalisation theory is that the effects of quantum fluctuations involving momenta larger than k, can be lumped into the values of the constants g and μ, rendering them functions of k: g(k); μ(k). This means that the phenomenological parameters, g and μ, are dynamical objects which depend in a complicated way on on all the physics at length scales smaller than k^{-1}.

Applying this idea to some hypothetical unified theory, suppose that the theory contains no light fields other than those of the standard model. In this case if the cutoff k is less than M, the physics of low energies must be the standard model with gauge couplings $g_3(k)$, $g_2(k)$, $g_1(k)$, Yukawa coupling $g_Y(k)$, quartic scalar couplings $\lambda(k)$ and a mass parameter $\mu^2(k)$. The last of these is the usual (negative) mass squared of the Higgs field, which has the potential

$$V(\phi) = \lambda\phi^4 - \mu^2\phi^2 \tag{3.7}$$

These parameters summarize all the known physics at momenta >k. In particular they depend non-trivially on the structure of the unified theory at distances shorter than M^{-1}.

The mass of every particle in the standard theory is proportional to the VEV of the Higgs, $<\phi>$. For example, quark and lepton masses have the form

$$m_q \simeq g_Y \langle\phi\rangle \tag{3.8}$$

The Z and W masses are proportional to

$$g_2 \langle\phi\rangle \tag{3.9}$$

and the Higgs mass is proportional to

$$\sqrt{\lambda}\ \langle\phi\rangle \tag{3.10}$$

The VEV $\langle\phi\rangle$ is not a new parameter, but can be calculated from $\lambda(k)$, $\mu(k)$, $g(k)$, and $g_Y(k)$. Ignoring quantum fluctuations, $\langle\phi\rangle$ can be calculated by minimizing $V(\phi)$. This gives

$$\langle\phi\rangle = \mu(k)/2\sqrt{\lambda(k)} \tag{3.11}$$

This, however, is not a good approximation if the cutoff k is large ($\simeq$ M). It is spoiled by quantum fluctuations of all modes down to wavelengths $\simeq k^{-1}$. A much better approximation is to use a low energy cutoff k', of ,say, the order 1 TeV. For most purposes, k' = 0, is reasonable since k'<<< k, so that

$$\langle\phi\rangle \simeq \mu(0)/2\sqrt{\lambda(0)} \tag{3.12}$$

Thus $\langle\phi\rangle$ may be well approximated by computing the "renormalised" mass $\mu(0)$ and coupling $\lambda(0)$.

For example, consider $\mu(0)$ which is calculated by Feynman graphs.

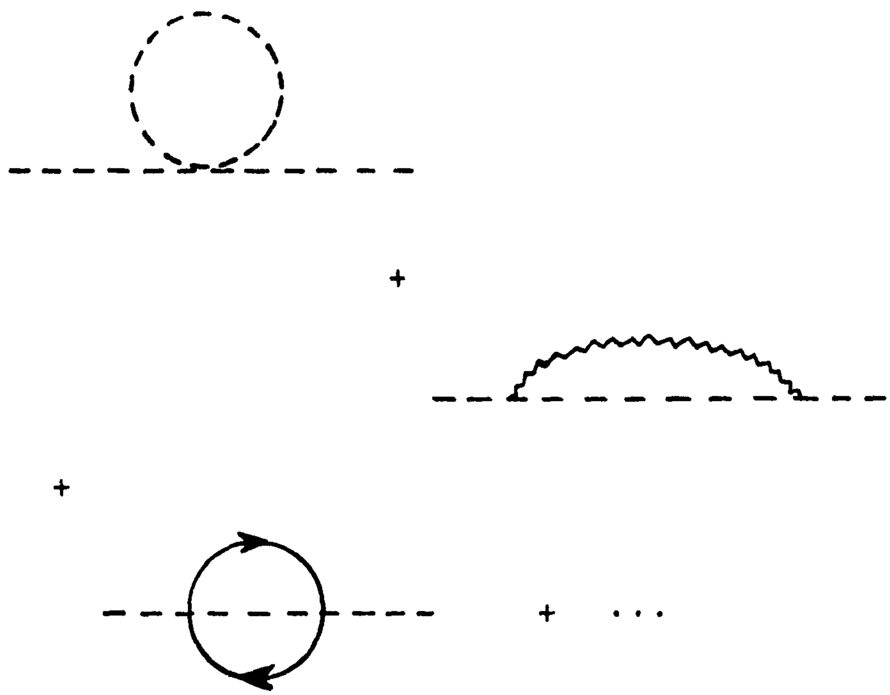


Figure 3.1

Each graph in fig.3.1 is cutoff dependent and proportional to k^2. Thus, $\mu^2(0)$ has the form

$$\mu^2(0) = \mu^2(k) + k^2(C_1\lambda + C_2g^2 + \ldots) \tag{3.13}$$

Now, since it is believed that the values of $(g(k),\ldots,\mu(k))$ are most closely related to the fundamental underlying unified theory when k is as large as possible (say of order M), then eq. (3.13) becomes

$$\mu^2(0) = \mu^2(M) + M^2(C_1\lambda(M) + \ldots) \tag{3.14}$$

From eq. (3.12), $\mu(0)$ should be $\simeq \langle\phi\rangle\sqrt{\lambda(0)}$, and for typical $\sqrt{\lambda} \simeq 1$, $\mu(0)$ is of order 10^2 GeV. Suppose the scale $M \simeq 10^{15}$ GeV. Then eq. (3.14) may be written as

$$\mu^2(0)/M^2 = 10^{-26} = \mu^2(M)/M^2 + (C_1\lambda + \ldots) \qquad (3.15)$$

This is a very unreasonable situation. It requires the dimensionless $\mu^2(M)/M^2$ to cancel the complicated series $(C_1\lambda + \ldots)$ to 26 decimal places in order for the world of ordinary particles to be as light as it is. But $\mu^2(M)/M^2$ only knows about physics at small distances. To obtain such cancellation would require a "miraculous conspiracy" among the parameters of the unified theory. In other words, the existence of a scale $\simeq 10^2$ GeV in the standard model with a second more fundamental scale at 10^{15} GeV is unnatural. This is called the GHP (actually, it is often referred to as the technical GHP, and the GHP is then the name of the more philosophical question of why there should be two, so widely separated, scales).

Supersymmetry resolves the GHP by including a bosonic/fermionic partner for every fermion/boson in the non-supersymmetric theory. These partners have equal masses and couple with the same strength. The divergences in the Higgs masses now obtain contributions from two equal sources: boson loops ; and fermion loops. What is more, these contributions carry opposite signs and therefore cancel each other out! This means that in supersymmetric theories the coefficients C_1, C_2,..., etc. of eq. (3.13) vanish. As will be pointed out in the next section, supersymmetry has to be broken, and superpartners, then, no longer have the same masses. Nonetheless, the divergences in this situation are much more "friendly", and the naturalness problem of GUT's does not re-emerge.

3.3 Breaking supersymmetry

Since superpartners of the same mass have never been observed, it must be concluded that: if supersymmetry exists in nature, it must be broken. There exist only two mechanisms which break supersymmetry spontaneously: (1) the Fayet-Illiopoulos[24] (or D-type) mechanism; and (2) the O'Raifeartaigh[25] (or F-type) mechanism. Both these methods impose very strict requirements on the theory, making them cumbersome to use in building models. In that respect, the preferred way to achieve broken supersymmetry in model building is to do it "by hand", i.e., to include in the Lagrangian explicit mass terms. This, of course, has to be done with care in order to avoid spoiling the renormalisability of the model. Girardello and Grisaru[26] have listed all the supersymmetry breaking terms which do not introduce quadratic divergences to the unrenormalised theory, these "soft-breaking" terms are:

$$\tilde{M}_1 \text{ Re } A^2 + \tilde{M}_2 \text{ Im } A^2 + c(A^3 + \text{h.c.}) + \tilde{M}_3 \ (\lambda^a\lambda^a + \bar{\lambda}^a\bar{\lambda}^a)$$

$$+ \tilde{M}_4 \ (\lambda'\lambda' + \bar{\lambda}'\bar{\lambda}') \qquad (3.16)$$

where A^2 and A^3 are group invariant combinations of the scalar fields A_i (e.g., $A^3 = d_{ijk}A_iA_jA_k$, etc.), and λ is the fermionic partner of the gauge boson, the gaugino. The parameter $\tilde{M}_1$ splits the mass of the complex scalar A_i from its fermionic partner ψ_i. If A_i is expressed in terms of two real spin zero fields, then $\tilde{M}_2$ splits the masses of these two fields. The coupling constant c corresponds to a new (non-supersymmetric) scalar interaction term. Finally, $\tilde{M}_3$ and $\tilde{M}_4$ are Majorana mass terms for the gauginos corresponding to the groups G (non-abelian group) and U(1).

3.4 Supersymmetric lagrangians

Haber and Kane[23] give the following recipe for constructing interaction Lagrangians for supersymmetric gauge theories. These theories consist of gauge bosons V^a_μ and their (two component) gaugino partners λ^a in the adjoint representation of the gauge group G, and matter fields consisting of complex scalar fields A_i and two-component fermions ψ_i which transform under some representation R of G. Kinetic energy terms are omitted, and the summation convention is used throughout.

The interaction Lagrangian consists of three pieces:

(a) Self interaction of the gauge multiplet

This piece contains the usual "three-gluon" and "four-gluon" vertices which are not written down. In addition, the gauginos interact with the gauge field via the term:

$$igf_{abc}\lambda^a \sigma^\mu \bar{\lambda}^b V^c_\mu \qquad (.17)$$

where f_{abc} are the structure constants of G.

(b) Interactions of the gauge and matter multiplets

The following interaction terms arise:

$$-gT^a_{ij}V^a_\mu(\bar{\psi}_i\bar{\sigma}^\mu\psi_j + iA^*_i \overleftrightarrow{\partial}_\mu A_j) + ig\sqrt{2}\,T^a_{ij}(\lambda^a\psi_j A^*_i - \bar{\lambda}^a\bar{\psi}_i A_j)$$

$$+ g^2(T^aT^b)_{ij}V^a_\mu V^{\mu b}A^*_i A_j \qquad (3.18)$$

where T^a is the (Hermitean) group generator in the representation R.

(c) Self interactions of the matter multiplet

It is necessary to introduce some notation here. The superpotential W is some cubic gauge-invariant function of the scalar matter fields A_i (and does not depend on A^*_i). Define the auxiliary functions:

$$F_i = \partial W/\partial A_i \qquad (3.19)$$

$$D^a = g\, A_i^* \, T_{ij}^a \, A_j \tag{3.20}$$

Then the ordinary scalar potential consists of

$$V = \frac{1}{2}\, D^a \, D^a + F_i^* \, F_i \tag{3.21}$$

And the Yukawa interactions and fermion mass terms are contained in

$$-\frac{1}{2}\, [(\partial^2 W/\partial A_i \partial A_j)\psi_i \psi_j + (\partial^2 W/\partial A_i \partial A_j)^* \bar{\psi}_i \bar{\psi}_j] \tag{3.22}$$

3.5 Higgs sector in supersymmetric theories

In the standard model, the Higgs field, ϕ, can be used to give mass to both up and down quarks. This is because ϕ and its complex conjugate form SU(2) doublets. Thus Yukawa couplings of the form

$$g_D Q_L \phi \, D_R^\dagger + g_U Q_L \phi^\dagger U_R^\dagger \tag{3.23}$$

are allowed.

In a supersymmetric theory these couplings arise from cubic terms in the superpotential formed from the chiral supermultiplets of the same chirality. Now if ϕ is a left-handed chiral supermultiplet then ϕ^* is right-handed, since Q_L, $D_R^\dagger$ and $U_R^\dagger$ are left-handed. The coupling $Q_L \phi \, Q_R^\dagger$ is allowed but $Q_L \phi^\dagger U_R^\dagger$ is not. The remedy for this situation is to double the Higgs content by introducing a second Higgs doublet, $\tilde{\phi}$, which transforms like ϕ^* under the electro-weak group but which is left-handed. Eq. (3.23) is then replaced by

$$g_D Q_L \phi \; D_R^\dagger + g_U Q_L \tilde{\phi} \; U_R^\dagger \tag{3.24}$$

in the superpotential.

The focus of this chapter has been on global supersymmetry and on how it can resolve the difficulties associated with the Higgs sector of the standard model. These difficulties appear when the model is extended to very high energy regions, energies at which it may no longer be possible to ignore gravity. This thesis makes no attempt to deal with this very complex problem, but a few words on its possible connection with supersymmetry, are in order.

The unification of gravity with the other fundamental forces has to contend with the purely attractive nature of the gravitational force which, in the context of present day field theory, implies that it must be carried by a field with even integer spin. Since the carriers of the electroweak and strong forces have spin one, it has been shown that such a unification can only be achieved within a locally supersymmetric theory. Thus, an attempt to unify all the forces of Nature automatically leads to the inclusion of supersymmetry.

CHAPTER 4

A LEFT-RIGHT SYMMETRIC SUPERSYMMETRIC MODEL

Taken individually, both left-right symmetry and supersymmetry offer worthwhile avenues of investigation. It therefore makes a lot of sense to consider models in which both these symmetries operate. The model proposed here is a supersymmetric version of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.

In this chapter the model is specified: the fields are listed together with their gauge group quantum numbers; the pattern of symmetry breaking is described; and the Lagrangian of the model is written down.

4.1 Notation

"Component-fields", as opposed to superfields, are used throughout. A field and its superpartner are denoted by the same symbol (except in the case of gauge bosons and gauginos), with a tilde placed over the field introduced into the theory by supersymmetry, e.g.,

$$L_L \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \text{ supersymmetric partner: } \tilde{L}_L \equiv \begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L$$

in this case the field introduced by supersymmetry is bosonic, whereas in the case of, say, the Higgs field Δ_L, the supersymmetric addition $\tilde{\Delta}_L$ is fermionic.

Two component notation is used for spinors, the conventions employed are listed in appendix A.

4.2 The fields

(a) Matter fields

For simplicity, only one generation of quarks and leptons is considered. These are assigned to $SU(2)_{L,R}$ doublets of two-component spinors:

$$Q_{L,R} \equiv \begin{pmatrix} u_{L,R} \\ d_{L,R} \end{pmatrix} \; ; \quad L_{L,R} \equiv \begin{pmatrix} \nu_{L,R} \\ e_{L,R} \end{pmatrix}$$

and their supersymmetric scalar partners:

$$\tilde{Q}_{L,R} \; ; \; \tilde{L}_{L,R}$$

(b) Gauge fields

The $SU(2)_{L,R}$ gauge fields are triplets of vector bosons, $W^{\mu}_{L,R}$, and their two-component fermionic partners, $\lambda_{L,R}$. The $U(1)_{B-L}$ gauge fields are a singlet vector boson V^{μ}, and its partner λ_V .

The gauge coupling constants are: g_L ; g_R ; and g_V .

(c) The Higgs fields

The Higgs sector of the model has the same basic structure as the one described in section 2.1, except that supersymmetry, for two different reasons, requires the addition of more fields.

First, for the reasons discussed in section 3.5, The Higgs fields responsible for the masses of the quarks have to be doubled. This means that there is one Higgs, Φ_u , which generates mass for the upper members of $SU(2)_{L,R}$ doublets, and another, Φ_d , to generate mass for the lower ones. Their fermionic partners are: $\tilde{\Phi}_u$; and $\tilde{\Phi}_d$.

Second, because the electric charges of the members of the $\Delta_{L,R}$ triplets are: 0; +1; and +2 , their fermionic counterparts, $\tilde{\Delta}_{L,R}$ will have the same charge content (superpartners have the same quantum numbers). This implies that the fermion sector will give rise to triangle anomalies which destroy the renormalisability of the theory. The remedy for this situation is to introduce two more Higgs triplets, $\delta_{L,R}$, and with them, $\tilde{\delta}_{L,R}$, with opposite $U(1)_{B-L}$ charge (giving electric charges of: 0; -1; and -2). Now, the "triangle-loop" contributions of $\tilde{\delta}_{L,R}$ carry opposite signs to those of $\tilde{\Delta}_{L,R}$ and therefore cancel them. These new Higgs fields do not acquire VEV's and play no part in spontaneous symmetry breaking, furthermore, because of their quantum numbers, they couple to very few fields and hence play a very "minor" role in the theory.

In table 4.1, a summary is given of the fields, their components, their quantum numbers, and their names.

Table 4.1

Field	Component fields	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum number			Name
Matter					
Q_L	$\begin{pmatrix} u \\ d \end{pmatrix}_L$	1/2	0	1/3	L-h up-quark L-h down-quark
Q_R	$\begin{pmatrix} u \\ d \end{pmatrix}_R$	0	1/2	1/3	R-h up-quark R-h down quark
L_L	$\begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1/2	0	-1	L-h neutrino L-h electron
L_R	$\begin{pmatrix} \nu \\ e \end{pmatrix}_R$	0	1/2	-1	R-h neutrino R-h electron
$\tilde{Q}_L$	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_L$	1/2	0	1/3	L-h up-squark L-h down-squark
$\tilde{Q}_R$	$\begin{pmatrix} \tilde{u} \\ \tilde{d} \end{pmatrix}_R$	0	1/2	1/3	R-h up-squark R-h down squark
$\tilde{L}_L$	$\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_L$	1/2	0	-1	L-h s-neutrino L-h s-electron
$\tilde{L}_R$	$\begin{pmatrix} \tilde{\nu} \\ \tilde{e} \end{pmatrix}_R$	0	1/2	-1	R-h s-neutrino R-h s-electron
Gauge					
W_L	$W_L^+;W_L^-;W_L^0$	Triplet	Singlet	Singlet	Gauge boson
W_R	$W_R^+;W_R^-;W_R^0$	Singlet	Triplet	Singlet	Gauge boson
V	V	Singlet	Singlet	Singlet	Gauge boson
λ_L	$\lambda_L^+;\lambda_L^-;\lambda_L^0$	Triplet	Singlet	Singlet	Gaugino
λ_R	$\lambda_R^+;\lambda_R^-;\lambda_R^0$	Singlet	Triplet	Singlet	Gaugino
λ_V	λ_V	Singlet	Singlet	Singlet	Gaugino

Table 4.1 (continued)

Field	Component fields	$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ quantum number			Name
Higgs					
$\Phi_{u,d}$	$\begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}_{u,d}$	1/2	1/2	0	Higgs boson
Δ_L	$\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_L$	1	0	2	Higgs boson
Δ_R	$\begin{pmatrix} \frac{1}{\sqrt{2}}\Delta^+ & \Delta^{++} \\ \Delta^0 & -\frac{1}{\sqrt{2}}\Delta^+ \end{pmatrix}_R$	0	1	2	Higgs boson
δ_L	$\begin{pmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{pmatrix}_L$	1	0	-2	Higgs boson
δ_R	$\begin{pmatrix} \frac{1}{\sqrt{2}}\delta^- & \delta^0 \\ \delta^{--} & -\frac{1}{\sqrt{2}}\delta^- \end{pmatrix}_R$	0	1	-2	Higgs boson
$\tilde{\Phi}_{u,d}$	$\begin{pmatrix} \tilde{\phi}_1^0 & \tilde{\phi}_1^+ \\ \tilde{\phi}_2^- & \tilde{\phi}_2^0 \end{pmatrix}_{u,d}$	1/2	1/2	0	Higgsino
$\tilde{\Delta}_L$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\tilde{\Delta}^+ \end{pmatrix}_L$	1	0	2	Higgsino
$\tilde{\Delta}_R$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\Delta}^+ & \tilde{\Delta}^{++} \\ \tilde{\Delta}^0 & -\frac{1}{\sqrt{2}}\tilde{\Delta}^+ \end{pmatrix}_R$	0	1	2	Higgsino
$\tilde{\delta}_L$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\delta}^- & \tilde{\delta}^0 \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\tilde{\delta}^- \end{pmatrix}_L$	1	0	-2	Higgsino
$\tilde{\delta}_R$	$\begin{pmatrix} \frac{1}{\sqrt{2}}\tilde{\delta}^- & \tilde{\delta}^0 \\ \tilde{\delta}^{--} & -\frac{1}{\sqrt{2}}\tilde{\delta}^- \end{pmatrix}_R$	0	1	-2	Higgsino

4.3 Symmetry breaking

The breaking of left-right symmetry, in this model, follows much the same pattern as that in sectioı ℨ.2. That is: the VEV ɔf Δ_R is responsible for the first stage of breaking - both parity ($g_L = g_R$) and $SU(2)_R \times U(1)_{B-L}$ are broken.

$$(SUSY)\cdot SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \xrightarrow{\langle\Delta_R\rangle \neq 0} (SUSY)\cdot SU(2)_L \times U(1)_Y$$

The mass scale of this breaking, M_{W_R}, has an experimentally determined lower bound (from $K_L - K_S$ mixing[18]) of 1.6 TeV.

The scale of supersymmetry breaking is a much more complex question. It is often parametrized by $\sqrt{f_{\tilde{G}}}$ [19], where $f_{\tilde{G}}$ is the goldstino decay constant. Its value has been taken to range from as low as < 1 TeV , to as high as $\geq 10^{10}$ GeV. According to Haber and Kane, the more fashionable models use a supersymmetry breaking scale which is very high, but the mass squared splitting of the ordinary and supersymmetric particles is taken to be of the order of $M^2_{W_L}$. Arguments originating from SU(5), also support the view that the effective breaking scale of supersymmetry is close to the weak scale.

For these reasons, it will be assumed in this model that supersymmetry and $SU(2)_L \times U(1)_Y$ are broken together, i.e.,

$$(SUSY)\cdot SU(2)_L \times U(1)_Y \xrightarrow{<\Phi_{u,d}>\neq 0,\ \&\ SSBT} U(1)_{Em}$$

where SSBT stands for "soft supersymmetry breaking terms".

The VEV's of the Higgs fields will be taken as:

$$<\Delta_R> = \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} ; \qquad <\Delta_L> = <\delta_{L,R}> \equiv 0$$

$$<\Phi_u> = \begin{pmatrix} \kappa_u & 0 \\ 0 & 0 \end{pmatrix} ; \qquad <\Phi_d> = \begin{pmatrix} 0 & 0 \\ 0 & \kappa_d \end{pmatrix} \tag{4.1}$$

4.4 The Lagrangian

(a) The gauge Lagrangian

The first part of the lagrangian concerns itself with the gauge fields: it contains the kinetic and self interaction terms for the vector fields, and the Dirac Lagrangian of the gaugino fields. The covariant derivative D_μ is of the general form: $\partial_\mu + igT_aG^a_\mu$; where T_a are the generators of the gauge group; and G_μ is the gauge field.

$$\begin{aligned}\mathcal{L}_{Gauge} = \quad & -\frac{1}{4} W^L_{\mu\upsilon}\cdot W^{\mu\upsilon}_L + \frac{1}{2}\bar{\lambda}_L\,\bar{\sigma}_\mu\, D^L_\mu\,\lambda_L \\ & -\frac{1}{4} W^R_{\mu\upsilon}\cdot W^{\mu\upsilon}_R + \frac{1}{2}\bar{\lambda}_R\,\bar{\sigma}_\mu\, D^R_\mu\,\lambda_R \\ & -\frac{1}{4} V_{\mu\upsilon}\, V^{\mu\upsilon} + \frac{1}{2}\bar{\lambda}_V\,\bar{\sigma}_\mu\,\partial_\mu\,\lambda_V\end{aligned} \tag{4.2}$$

(b) The matter Lagrangian

This piece contains the kinetic terms for the fermionic and bosonic matter fields (The Higgs fields are also included in this category), as well as the interactions of the gauge and matter multiplets.

$$\mathcal{L}_{Matter} = + Q_L^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_V}{6} V_\mu \right) Q_L$$

$$+ Q_R^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R - \frac{ig_V}{6} V_\mu \right) Q_R$$

$$+ L_L^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L + \frac{ig_V}{2} V_\mu \right) L_L$$

$$+ L_R^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R + \frac{ig_V}{2} V_\mu \right) L_R$$

$$+ \mathrm{Tr} \left\{ (\tau . \tilde{\Delta}_L)^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - ig_V V_\mu \right) \tau . \tilde{\Delta}_L \right\}$$

$$+ \mathrm{Tr} \left\{ (\tau . \tilde{\delta}_L)^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L + ig_V V_\mu \right) \tau . \tilde{\delta}_L \right\}$$

$$+ \mathrm{Tr} \left\{ (\tau . \tilde{\Delta}_R)^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R - ig_V V_\mu \right) \tau . \tilde{\Delta}_R \right\}$$

$$+ \mathrm{Tr} \left\{ (\tau . \tilde{\delta}_R)^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R + ig_V V_\mu \right) \tau . \tilde{\delta}_R \right\}$$

$$+ \mathrm{Tr} \left\{ \bar{\tilde{\Phi}}_u \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_R}{2} \tau . W_\mu^R \right) \tilde{\Phi}_u \right\}$$

$$+ \mathrm{Tr} \left\{ \bar{\tilde{\Phi}}_d \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_R}{2} \tau . W_\mu^R \right) \tilde{\Phi}_d \right\}$$

$$+ \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_V}{6} V_\mu \right) \tilde{Q}_L \right|^2$$

$$+ \left| \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R - \frac{ig_V}{6} V_\mu \right) \tilde{Q}_R \right|^2$$

$$+ \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L + \frac{ig_V}{2} V_\mu \right) \tilde{L}_L \right|^2$$

$$+ \left| \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R + \frac{ig_V}{2} V_\mu \right) \tilde{L}_R \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_R}{2} \tau . W_\mu^R \right) \Phi_u \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - \frac{ig_R}{2} \tau . W_\mu^R \right) \Phi_d \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L - ig_V V_\mu \right) \tau . \Delta_L \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_L}{2} \tau . W_\mu^L + ig_V V_\mu \right) \tau . \delta_L \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R - ig_V V_\mu \right) \tau . \Delta_R \right|^2$$

$$+ \mathrm{Tr} \left| \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R + ig_V V_\mu \right) \tau . \delta_R \right|^2$$

$$+ i\, \tilde{Q}_L^\dagger \left(\frac{g_L}{\sqrt{2}}\, \tau.\lambda_L + \frac{g_V}{3\sqrt{2}}\, \lambda_V \right) Q_L + \text{h.c.}$$

$$+ i\, \tilde{Q}_R^\dagger \left(\frac{g_L}{\sqrt{2}}\, \tau.\lambda_R + \frac{g_V}{3\sqrt{2}}\, \lambda_V \right) Q_R + \text{h.c.}$$

$$+ \frac{i}{\sqrt{2}}\, \tilde{L}_L^\dagger \left(g_L\, \tau.\lambda_L - g_V\, \lambda_V \right) L_L + \text{h.c.}$$

$$+ \frac{i}{\sqrt{2}}\, \tilde{L}_R^\dagger \left(g_R\, \tau.\lambda_R - g_V\, \lambda_V \right) L_R + \text{h.c.}$$

$$+ i\sqrt{2}\, \mathrm{Tr} \left\{ (\tau.\Delta_L)^\dagger \left(g_L\, \tau.\lambda_L + 2\, g_V\, \lambda_V \right) \tau.\tilde{\Delta}_L \right\} + \text{h.c.}$$

$$+ i\sqrt{2}\, \mathrm{Tr} \left\{ (\tau.\delta_L)^\dagger \left(g_L\, \tau.\lambda_L - 2\, g_V\, \lambda_V \right) \tau.\tilde{\delta}_L \right\} + \text{h.c.}$$

$$+ i\sqrt{2}\, \mathrm{Tr} \left\{ (\tau.\Delta_R)^\dagger \left(g_R\, \tau.\lambda_R + 2\, g_V\, \lambda_V \right) \tau.\tilde{\Delta}_R \right\} + \text{h.c.}$$

$$+ i\sqrt{2}\, \mathrm{Tr} \left\{ (\tau.\delta_R)^\dagger \left(g_R\, \tau.\lambda_R - 2\, g_V\, \lambda_V \right) \tau.\tilde{\delta}_R \right\} + \text{h.c.}$$

$$+ \frac{i}{\sqrt{2}}\, \mathrm{Tr} \left\{ \Phi_u^\dagger \left(g_L\, \tau.\lambda_L + g_R\, \tau.\lambda_R \right) \tilde{\Phi}_u \right\} + \text{h.c.}$$

$$+ \frac{i}{\sqrt{2}}\, \mathrm{Tr} \left\{ \Phi_d^\dagger \left(g_L\, \tau.\lambda_L + g_R\, \tau.\lambda_R \right) \tilde{\Phi}_d \right\} + \text{h.c.} \qquad (4.3)$$

(c) The Yukawa Lagrangian

This piece involves the self interactions of the matter multiplets, again, this includes the Higgs multiplets.

$$
\begin{aligned}
\mathcal{L}_Y = \{ & h_u^L \, (L_L^\dagger \, \Phi_u \, L_R) + h_d^L \, (L_L^\dagger \, \Phi_d \, L_R) + h_u^Q \, (Q_L^\dagger \, \Phi_u \, Q_R) \\
& + h_d^Q \, (Q_L^\dagger \, \Phi_d \, Q_R) \\
& + h_u^L \, (\tilde{L}_L^\dagger \, \tilde{\Phi}_u \, L_R) + h_d^L \, (\tilde{L}_L^\dagger \, \tilde{\Phi}_d \, L_R) + h_u^Q \, (\tilde{Q}_L^\dagger \, \tilde{\Phi}_u \, Q_R) \\
& + h_d^Q \, (\tilde{Q}_L^\dagger \, \tilde{\Phi}_d \, Q_R) \\
& + h_u^L \, (\tilde{L}_R^\dagger \, \tilde{\Phi}_u \, L_L) + h_d^L \, (\tilde{L}_R^\dagger \, \tilde{\Phi}_d \, L_L) + h_u^Q \, (\tilde{Q}_R^\dagger \, \tilde{\Phi}_u \, Q_L) \\
& + h_d^Q \, (\tilde{Q}_R^\dagger \, \tilde{\Phi}_d \, Q_L) + \mathrm{Tr} \, (\mu_1 \, [\tau_1 \, \tilde{\Phi}_u \, \tau_1]^T \, \tilde{\Phi}_d) \\
& + \mathrm{Tr} \, \{ \mu_2(\tau.\tilde{\Delta}_L) \, (\tau.\tilde{\delta}_L) \} + \mathrm{Tr} \, \{ \mu_3(\tau.\tilde{\Delta}_R) \, (\tau.\tilde{\delta}_R) \\
& + h_{LR} \, (L_L^T \, \tau_1 \tau.\Delta_L \, L_L + L_R^T \, \tau_1 \tau.\Delta_R \, L_R) \\
& + h_{LR} \, (\tilde{L}_L^T \, \tau_1 \tau.\tilde{\Delta}_L \, L_L + \tilde{L}_R^T \, \tau_1 \tau.\tilde{\Delta}_R \, L_R) \} + h.c.
\end{aligned}
\tag{4.4}
$$

(d) The scalar potential

The scalar potential is obtained from eq. (3.21), with the addition of a soft-breaking potential, V_{Soft}, which has its origins in supergravity theory. It becomes

$$
V = |F|^2 + \frac{1}{2} \, |D|^2 + V_{Soft} \tag{4.5}
$$

where:

$$|F|^2 = |\, h_u^Q \tilde{Q}_L \tilde{Q}_R + h_u^L \tilde{L}_L \tilde{L}_R \,|^2 + |\, h_d^Q \tilde{Q}_L \tilde{Q}_R + h_d^L \tilde{L}_L \tilde{L}_R \,|^2$$

$$+ |\, h_u^Q \Phi_u \tilde{Q}_R + h_d^Q \Phi_d \tilde{Q}_R \,|^2 + |\, h_u^Q \Phi_u \tilde{Q}_L + h_d^Q \Phi_d \tilde{Q}_L \,|^2$$

$$+ |\, h_u^L \Phi_u \tilde{L}_R + h_d^L \Phi_d \tilde{L}_R + 2\, h_{LR}\, \tau.\Delta_L\, \tilde{L}_L |^2$$

$$+ |\, h_u^L \Phi_u \tilde{L}_L + h_d^L \Phi_d \tilde{L}_L + 2\, h_{LR}\, \tau.\Delta_R\, \tilde{L}_R |^2 + h.c.;$$

(4.6)

$$\frac{1}{2}\,|D|^2 = \frac{1}{2}\, g_L \sum_L \,|\, \sum_A A^\dagger\, \tau_L\, A \,|^2 + \frac{1}{2}\, g_R \sum_R \,|\, \sum_A A^\dagger\, \tau_R\, A \,|^2$$

$$+ \frac{1}{2}\, g_V \,|\, \sum_A A^\dagger\, V\, A \,|^2 \qquad (4.7)$$

where: $A = \tilde{Q}_L,\ \tilde{Q}_R,\ \tilde{L}_L,\ \tilde{L}_R,\ \Phi_u,\ \Phi_d,\ \Delta_L,\ \Delta_R,\ \delta_L,\ \delta_R$; and τ_L, τ_R, V, are the generators of the gauge groups.

$$V_{Soft} = m_S\, (\ (h_u^Q\, \tilde{Q}_L^\dagger\, \Phi_u\, \tilde{Q}_R + h_d^Q\, \tilde{Q}_L^\dagger\, \Phi_d\, \tilde{Q}_R + h_u^L\, \tilde{L}_L^\dagger\, \Phi_u\, \tilde{L}_R$$

$$+ h_d^L\, \tilde{L}_L^\dagger\, \Phi_d\, \tilde{L}_R + h_{LR}(\, \tilde{L}_L^T\, \tau_1 \tau.\Delta_L\, \tilde{L}_L + \tilde{L}_R^T\, \tau_1 \tau.\Delta_R\, \tilde{L}_R)$$

$$+ \mathrm{Tr}\, [\mu_1 (\tau_1\, \Phi_u\, \tau_1)^T\, \Phi_d]) + \mathrm{Tr}\, [\mu_2 (\tau.\Delta_L)\, (\tau.\delta_L)]$$

$$+ \mathrm{Tr}\,[\mu_3(\tau.\Delta_R)\ (\tau.\delta_R)] + \mathrm{h.c.})$$

$$+ m^2_{QL}\, \tilde{Q}^\dagger_L\, \tilde{Q}_L + m^2_{QR}\, \tilde{Q}^\dagger_R\, \tilde{Q}_R + m^2_{LL}\, \tilde{L}^\dagger_L\, \tilde{L}_L + m^2_{LR}\, \tilde{L}^\dagger_R\, \tilde{L}_R \tag{4.8}$$

(e) The soft-breaking Lagrangian

This is the term which gives Majorana mass to the gauginos:

$$L_{Soft} = m_L\, (\lambda^a_L\, \lambda^a_L + \bar{\lambda}^a_L\, \bar{\lambda}^a_L) + m_R\, (\lambda^a_R\, \lambda^a_R + \bar{\lambda}^a_R\, \bar{\lambda}^a_R)$$

$$+ m_V\, (\lambda_V\, \lambda_V + \bar{\lambda}_V\, \bar{\lambda}_V) \tag{4.9}$$

This concludes the description of the model proposed in this thesis. In the following chapters, the effects of spontaneous symmetry breaking on the gauge sector of the model are investigated.

CHAPTER 5

VECTOR BOSON MASS-EIGENSTATES

After the spontaneous breaking of symmetry, all the gauge bosons (weak interaction eigenstates) acquire mass. Diagonalization of the vector boson mass matrix, then yields the physical states of the theory - the mass eigenstates.

It is the purpose of this chapter, to identify these mass eigenstates. The choice of VEV's of the Higgs fields (section 4.3), i.e., $v_R \gg \kappa_u, \kappa_d$ and $\kappa'_u = \kappa'_d = v_L = 0$, makes it possible to analyse the generation of mass for the gauge bosons in two separate stages. In the first stage, $\langle \Delta_R \rangle$ generates masses for $W_R^{\pm}$, W_R^0, and V. The two neutral states mix (just as in section 1.3), yielding the physical fields Z_R and B. The next stage takes place at a much lower energy scale. Here, $\Phi_{u,d}$, which couple to both left- and right-handed fields, mix W_L and W_R. However, the amount of mixing is so small that, effectively, the right-handed fields can be considered to have decoupled from this part of the theory, and only $W_L^{\pm}$, W_L^0, and B acquire mass from this stage. Once again, the neutral fields mix, and, the familiar, Z_L and A_μ are formed.

5.1 Right-handed vector bosons

For the first stage of symmetry breaking, the relevant terms to consider in the Lagrangian, (4.3), are

$$+ \mathrm{Tr}\left| \left(-\frac{ig_R}{2}\, \tau . W^R_\mu - ig_V\, V_\mu \right) \tau . \Delta_R \right|^2 \qquad (5.1)$$

Substituting the VEV $<\Delta_R>$ for Δ_R in the above gives

$$\frac{1}{4}\,\mathrm{Tr}\left| \begin{pmatrix} -ig_R\, W^0_R - 2ig_V V & -i\sqrt{2}\, g_R W^+_R \\ -i\sqrt{2}\, g_R W^-_R & +ig_R\, W^0_R - 2ig_V V \end{pmatrix} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix} \right|^2$$

$$= \frac{1}{2}\, v_R^2\, g_R^2\, W^+_R\, W^-_R + v_R^2 \left(\frac{1}{2}\, g_R\, W^0_R - g_V\, V \right)^2 \qquad (5.2)$$

This immediately yields the charged boson masses: comparing the first term in eq. (5.2) with the mass term expected for a charged boson, $M_W^2\, W^+\, W^-$,

$$M_{W_R} = \frac{1}{\sqrt{2}}\, g_R\, v_R \qquad (5.3)$$

In order to obtain the masses of the neutral bosons, the second term of (5.2) is written as

$$v_R^2\, (\, W^0_R\, ,\, V\,) \begin{pmatrix} \frac{1}{4}\, g_R^2 & -\frac{1}{2}\, g_R g_V \\ -\frac{1}{2}\, g_R g_V & g_V^2 \end{pmatrix} \begin{pmatrix} W^0_R \\ V \end{pmatrix} \qquad (5.4)$$

The mass matrix in expression (5.4) is off-diagonal in this basis, implying that W^0_R and V mix. The physical fields, Z_R and B, are found by diagonalizing the matrix and computing the corresponding eigenvectors.

The results of this calculation are:

$$Z_R = \frac{g_R W_R^0 - 2 g_V V}{(g_R^2 + 4 g_V^2)^{1/2}} , \tag{5.5}$$

$$B = \frac{g_R V + 2 g_V W_R^0}{(g_R^2 + 4 g_V^2)^{1/2}} \tag{5.6}$$

The masses of these fields are obtained from the eigenvalues of the matrix: $(\frac{1}{4} g_R^2 + g_V^2)$; and 0. Since the expected mass term for a neutral boson is $\frac{1}{2} M_Z^2 Z^2$, the masses are:

$$M_{Z_R} = \frac{1}{\sqrt{2}} v_R (g_R^2 + 4 g_V^2)^{1/2} \tag{5.7}$$

$$M_B = 0 \tag{5.8}$$

The new massless state, B_μ, is the gauge boson of the symmetry group $U(1)_Y$, which survives the breaking of $SU(2)_R \times U(1)_{B-L}$. $W_R^\pm$ and Z_R, being very massive, decouple from the low-energy theory, leaving only B_μ to go through to the next stage of symmetry breaking.

5.2 Left-handed vector bosons

For the second stage of symmetry breaking, the terms to consider in eq. (4.3) are

$$\mathrm{Tr}\left| \left(-\frac{ig_L}{2}\,\tau.W^L_\mu - \frac{ig_R}{2}\,\tau.W^R_\mu \right) \Phi_u \right|^2$$

$$+\ \mathrm{Tr}\left| \left(-\frac{ig_L}{2}\,\tau.W^L_\mu - \frac{ig_R}{2}\,\tau.W^R_\mu \right) \Phi_d \right|^2 \qquad (5.9)$$

However, $W^\pm_R$ and Z_R have effectively decoupled from this part of the theory. As a consequence, the charged bosons emerging from this stage are, to a good approximation, $W^\pm_L$. Their masses are simply calculated as:

$$M_{W_L} \simeq \frac{1}{\sqrt{2}}\, g_L\, (\, \kappa_u^2 + \kappa_d^2\,)^{1/2} \qquad (5.10)$$

As for the neutral bosons, W^0_R now has to be written in terms of the fields B and Z_R. From eqs. (5.5) and (5.6):

$$W^0_R = \frac{g_R\, Z_R + 2\, g_V\, B}{(\, g_R^2 + 4\, g_V^2\,)^{1/2}} \qquad (5.11)$$

Substituting this in eq. (5.9), and retaining neutral terms only (with the exception of Z_R which has decoupled), the following is obtained:

$$\mathrm{Tr} \left| \left\{ \frac{-ig_L}{2} \begin{pmatrix} -W^0_L & 0 \\ 0 & W^0_L \end{pmatrix} - \frac{i\, g_R\, g_V}{(\, g_R^2 + 4\, g_V^2\,)^{1/2}} \begin{pmatrix} B & 0 \\ 0 & -B \end{pmatrix} \right\} \cdots \right.$$

$$\left. \cdots \begin{pmatrix} \kappa_u & 0 \\ 0 & \kappa_d \end{pmatrix} \right|^2 \qquad (5.12)$$

The first term of eq. (5.12) is in fact $-\tau_3.W^L_\mu$, the reason for the change of sign has to do with the way that the quantum numbers in the Higgs bi-doublet matrices are distributed (more is found on this subject in appendix B.) Defining g′, the gauge coupling constant of $U(1)_Y$, as

$$g' \equiv \frac{i\, g_R\, g_V}{(\, g^2 + 4\, g^2\,)^{1/2}} \quad , \tag{5.13}$$

eq. (5.12) becomes

$$(\, \kappa_u^2 + \kappa_d^2\,)\; (\, g_L/2\; W_L^0 - g'\; B\,)^2 \tag{5.14}$$

By the same method that was used on the second term of eq. (5.2), the neutral mass eigenstates are found to be:

$$Z_L = \frac{g_L\; W_L^0 - 2\; g'\; B}{(\, g_L^2 + 4\; g'^2)^{1/2}} \tag{5.15}$$

$$\text{with mass} = \{\; (\, \kappa_u^2 + \kappa_d^2\,)\; (\, g_L^2 + 4\; g'^2)\; \}^{1/2}\; ; \tag{5.16}$$

and the massless photon, A_μ , given by

$$A_\mu = \frac{2\; g'\; W_L^0 + g_L\; B}{(\, g_L^2 + 4\; g'^2)^{1/2}} \tag{5.17}$$

5.3 A consistency check

The model studied by Mohapatra and Senjanovic (ref. (14)) is very similar to this one, and differs only in the fact that, not being supersymmetric, it requires only one Higgs bi-doublet instead of two. It is therefore possible to compare the form of their results, in the gauge boson sector, with the ones obtained here.

To this end, two modifications of the present model are necessary. First, it has to be assumed that $g_L = g_R = g$, even after the breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$ to $SU(2)_L \times U(1)_Y$. Second, g_v and g' have to be rescaled to have twice the values used above. Another point to note is that $\kappa' \neq 0$ in their model, but because there are two bi-doublets in this model, κ_u and κ_d play identical roles, in form, to their κ and κ'.

With these modifications, the vector bosons take the following form:

$$Z_R^\mu = \frac{g\, W_R^3 - g_v\, V}{(\, g^2 + g_v^2\,)^{1/2}} \tag{5.18}$$

$$B_\mu = \frac{g_v W_R^3 - g\, V}{(\, g^2 + g_v^2\,)^{1/2}} \tag{5.19}$$

$$Z_L^\mu = \frac{(\, g^2 + g_v^2\,)^{1/2}\, W_L^3 - g_v\, B}{(2\, g_v^2 + g^2)^{1/2}} \tag{5.20}$$

$$A_\mu = \frac{(g^2 + g_V^2)^{1/2} B + g_V W_L^3}{(2 g_V^2 + g^2)^{1/2}} \tag{5.21}$$

Now define: $$\tan \theta_W \equiv \frac{g_V}{(g^2 + g_V^2)^{1/2}} , \tag{5.22}$$

and the following are obtained for the physical fields of the theory:

$$Z_R^\mu = (\cos \theta_W)^{-1} (\cos 2\theta_W)^{1/2} W_R^3 - \tan \theta_W V$$

$$Z_L^\mu = \cos \theta_W W_L^3 - \sin \theta_W \tan \theta_W W_R^3 - \tan \theta_W (\cos 2\theta_W)^{1/2} V$$

$$A_\mu = \sin \theta_W (W_L^3 + W_R^3) + (\cos 2\theta_W)^{1/2} V \tag{5.23}$$

These are the same as the expressions given by Mohapatra and Senjanovic.

In the next two chapters, a similar identification will be made of the supersymmetric partners of the gauge bosons, the "charginos" and "neutralinos".

CHAPTER 6

CHARGED GAUGINOS AND HIGGSINOS

In parallel to the rearrangement of the vector boson fields after the breaking of gauge symmetries, the gauginos interact with Higgs bosons and higgsinos to form new combinations of fields. The identification of these new states is a more involved process than the one in the previous chapter, because of the presence of supersymmetry breaking terms in the Lagrangian.

In this chapter, the case of the charged gauginos and higgsinos will be investigated. These particles are often referred to as "charginos", whilst the term "neutralino" applies to their neutral cousins.

6.1 Mixing of gauginos and higgsinos[29]

The mixing of gauginos and higgsinos has its origins in the "$\lambda\psi A$" term of eq. (3.18). As a result of this coupling, fermionic mass terms are generated by the non-zero VEV of the Higgs fields. The, now massive, fermions mix, and in so doing give rise to new combinations of fields. Soft supersymmetry-breaking terms also contribute to the masses of the fermions, and their effect has to be included in the identification of the physical fields of the theory.

The terms in the Lagrangian which are relevant to the mixing will be called $\mathcal{L}_{GH}$, where

$$\mathcal{L}_{CH} =$$

$$i\sqrt{2}\,\mathrm{Tr}\left\{ (\tau.\Delta_L)^\dagger \,(\, g_L\, \tau.\lambda_L + 2\, g_V\, \lambda_V\,)\; \tau.\tilde{\Delta}_L \right\} + \mathrm{h.c.}$$

$$+\, i\sqrt{2}\,\mathrm{Tr}\left\{ (\tau.\Delta_R)^\dagger \,(\, g_R\, \tau.\lambda_R + 2\, g_V\, \lambda_V\,)\; \tau.\tilde{\Delta}_R \right\} + \mathrm{h.c.}$$

$$+\, \frac{i}{\sqrt{2}}\,\mathrm{Tr}\left\{ \Phi_u^\dagger \,(\, g_L\, \tau.\lambda_L + g_R\, \tau.\lambda_R\,)\; \tilde{\Phi}_u \right\} + \mathrm{h.c.}$$

$$+\, \frac{i}{\sqrt{2}}\,\mathrm{Tr}\left\{ \Phi_d^\dagger \,(\, g_L\, \tau.\lambda_L + g_R\, \tau.\lambda_R\,)\; \tilde{\Phi}_d \right\} + \mathrm{h.c.}$$

$$+\, m_L\,(\, \lambda_L^a\, \lambda_L^a + \bar{\lambda}_L^a\, \bar{\lambda}_L^a\,) + m_R\,(\lambda_R^a\, \lambda_R^a + \bar{\lambda}_R^a\, \bar{\lambda}_R^a\,)$$

$$+\, m_V\,(\, \lambda_V\, \lambda_V + \bar{\lambda}_V\, \bar{\lambda}_V\,) + \mathrm{Tr}\,(\, \mu_1 [\tau_1\, \tilde{\phi}_u\, \tau_1]^T\, \tilde{\Phi}_d\,)$$

$$+\, \mathrm{Tr}\,\{\, \mu_2 (\tau.\tilde{\Delta}_L)\;(\tau.\tilde{\delta}_L)\,\} + \mathrm{Tr}\,\{\, \mu_3 (\tau.\tilde{\Delta}_R)\;(\tau.\tilde{\delta}_R)\,\} \qquad (6.1)$$

6.2 Chargino mixing

Chargino mixing is determined by the terms of eq. (6.1) involving charged fields. The first step is to substitute the VEV's of the Higgs fields, eq. (4.1), into eq. (6.1). This gives the chargino-mixing Lagrangian $\mathcal{L}_{CM}$:

$$\mathcal{L}_{CM} = \{\, i\lambda_R^-\; (\sqrt{2}\, g_R\, v_R\, \tilde{\Delta}_R^+ + g_R\, \kappa_d\, \tilde{\phi}_d^+) + i\lambda_L^-\, g_L\, \kappa_d\, \tilde{\phi}_d^+$$

$$+ i\lambda_R^+ g_R \kappa_u \tilde{\phi}_u^- + i \lambda_L^+ g_L \kappa_u \tilde{\phi}_u^- + m_L \lambda_L^+\lambda_L^-$$

$$+ m_R \lambda_R^+\lambda_R^- + \mu_1\tilde{\phi}_u^+ \tilde{\phi}_d^- + \mu_1\tilde{\phi}_u^- \tilde{\phi}_d^+) + \text{h.c.} \qquad (6.2)$$

where, for simplification, it is assumed that $\mu_2 = \mu_3 = 0$.

Once again, the two stages of symmetry breaking are considered separately. At the first stage, the charged term, that comes into play (from eq. (6.2)) is

$$\mathcal{L}_\omega = (i\sqrt{2}\, g_R\, v_R)\, \lambda_R^-\, \tilde{\Delta}_R^+ + \text{h.c.} \qquad (6.3)$$

This is the mass term of the fermionic partner of the W_R^+. It is more easily recognised by translating it into four-component notation. Using eq. (A19), from appendix A, it follows that eq. (6.3) is equal to

$$\mathcal{L}_\omega = -\sqrt{2}\, g_R\, v_R\, \bar{\tilde{W}}_R^+\, \tilde{W}_R^+ \qquad (6.4)$$

$$\text{where } \tilde{W}_R^+ = \begin{pmatrix} \tilde{\Delta}_R^+ \\ i\bar{\lambda}_R^- \end{pmatrix} \qquad (6.5)$$

$\tilde{W}_R^+$ is a four-component Dirac spinor, in fact, all the charged fermions have to combine into four-component Dirac spinors because Majorana fermions cannot carry any conserved additive quantum numbers.

As expected, the mass of the $\tilde{W}_R^+$, $\sqrt{2}\, g_R\, v_R$, is the same as that of the W_R^+. It is interesting to note that an equivalent mass is not generated for a $\tilde{W}_R^-$. This means that supersymmetry has in fact been broken. It may be possible to avoid this breaking by introducing into the theory a $(\Delta_R)_c$ field, in the same way as in the non-supersymmetric standard model. Since $(\Delta_R)_c$ could not couple to fields belonging to a chiral multiplet (section 3.5), it would not generate new masses for the matter fields, but $\tilde{W}_R^-$ belongs to a vector multiplet and, so, could become massive.

The particles produced at the first stage of symmetry breaking are very massive and decouple from the low energy theory. The next step is to look at the remaining terms in eq. (6.2), $\mathcal{L}_c$, defined as

$$\mathcal{L}_c \equiv \text{eq. (6.2) - eq. (6.3)} \tag{6.6}$$

$\mathcal{L}_c$ is rewritten in the form

$$\mathcal{L}_c = -\frac{1}{2}(\psi^+,\psi^-)\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \tag{6.7}$$

where:

$$\psi^+ \equiv (-i\lambda_L^+ , -i\lambda_R^+ , \tilde{\phi}_u^+ , \tilde{\phi}_d^+)$$

$$\psi^- \equiv (-i\lambda_L^- , -i\lambda_R^- , \tilde{\phi}_u^- , \tilde{\phi}_d^-) \tag{6.8}$$

$$X = \begin{pmatrix} m_L & 0 & 0 & g_L\kappa_d \\ 0 & m_R & 0 & g_R\kappa_d \\ g_L\kappa_u & g_R\kappa_u & 0 & -\mu_1 \\ 0 & 0 & -\mu_1 & 0 \end{pmatrix} \qquad (6.9)$$

Let the mass eigenstates be defined as

$$\chi_i^+ = V_{ij}\,\psi_j^+ , \quad \chi_i^- = U_{ij}\,\psi_j^- , \quad i,j = 1,2,3,4 \qquad (6.10)$$

where V and U are unitary matrices chosen such that

$$U^* \, X \, V^{-1} = M_D \qquad (6.11)$$

where M_D is a diagonal matrix with non-negative entries. Equation (6.7) can then be written as [note eq.(A24)]

$$-(\,\chi_i^- \,(M_D)_{ij}\,\chi_j^+ + \text{h.c.}) \qquad (6.12)$$

Using eq. (A19), eq. (6.12) may be written in four-component notation as

$$-\sum_{i=1}^{4} M_i \,\bar{\tilde{\chi}}_i \,\tilde{\chi}_i \qquad (6.13)$$

where the $\tilde{\chi}_i$ are charged four-component Dirac spinors:

$$\tilde{\chi}_1 = \begin{pmatrix} \chi_1^+ \\ \bar{\chi}_1^- \end{pmatrix} \qquad (6.14)$$

It now remains to work out the expressions for the masses M_i, and the matrices U and V.

The eigenvalues of X can be either positive or negative whereas M_D is required to contain only nonnegative entries. It is therefore better to consider the eigenvalue problem for $X^\dagger X$. The positive square roots of the eigenvalues of $X^\dagger X$ will be the diagonal entries of M_D. From eq. (6.12) one obtains

$$M_D^2 = V\, X^\dagger\, X\, V^{-1} = U^*\, X\, X^\dagger\, (U^*)^{-1} \qquad (6.15)$$

Thus, the diagonalizing matrices U^* and V will be obtained by computing the eigenvectors corresponding to the eigenvalues of $X^\dagger X$ and $X X^\dagger$ respectively.

Unfortunately, the eigenvalue problem for the matrix (6.9) results in a quartic equation whose solutions are not particularly illuminating. It is therefore left, as a future problem, either, to insert numerical values for the parameters in (6.9), or, to make further simplifying assumptions about the relationships between the parameters, in order to obtain meaningful answers.

The Feynman rules that will be derived in chapter 8, will be given in terms of unknown elements of the diagonalizing matrices U^* and V.

CHAPTER 7

THE NEUTRALINOS

Finding the neutral states that result from the mixing of massive gauginos and higgsinos, is a very similar problem to the one dealt with in the previous chapter. The noteworthy differences are: first, there is one more gauge field, λ_V , that contributes to this sector; and second, because the neutralinos do not carry electric charge, they can be represented by Majorana spinors.

As was the case for the gauge bosons and charginos, the physical neutralinos fall into two categories: "heavy" neutralinos produced at the first stage of symmetry breaking; and "light" ones produced at the second. The amount of mixing between the heavy and light states is, once again, very small, making it a reasonable approximation to calculate the mass eigenstates of each scale independently of the other.

7.1 The neutralino mass Lagrangian

The neutralino mass terms are contained in $\mathcal{L}_{CH}$ (eq. (6.1)). The analogous expression to eq. (6.2), for the neutralinos, is (again assuming $\mu_2 = \mu_3 = 0$)

$$\mathcal{L}_{NM} = \{ - i\lambda_R^0 \sqrt{2}\, g_R\, v_R\, \tilde{\Delta}_R^0 + i\lambda_V^0\, 2\sqrt{2}\, g_V\, v_R\, \tilde{\Delta}_R^0$$

$$+ i\, \lambda_R^0 \frac{1}{\sqrt{2}} g_R\, \kappa_u\, \tilde{\phi}_{1u}^0 - i\, \lambda_L^0 \frac{1}{\sqrt{2}} g_L\, \kappa_u\, \tilde{\phi}_{1u}^0$$

$$- i\, \lambda_R^0 \frac{1}{\sqrt{2}} g_R\, \kappa_d\, \tilde{\phi}_{2d}^0 + i\, \lambda_L^0 \frac{1}{\sqrt{2}} g_L\, \kappa_d\, \tilde{\phi}_{2d}^0$$

$$+ m_L\, \lambda_L^0\, \lambda_L^0 + m_R\, \lambda_R^0\, \lambda_R^0 + m_V\, \lambda_V^0\, \lambda_V^0$$

$$+ 2\, \mu_1\, \tilde{\phi}_u^0\, \tilde{\phi}_d^0\,) + \text{h.c.} \tag{7.1}$$

7.2 The "heavy" neutralinos

In this section, the mass eigenstates that "condense out" at the first stage of symmetry breaking are calculated. The terms in eq. (7.1) that determine these states (defined as φ_{NMH}) are all those involving v_R, i.e.,

$$\mathcal{L}_{NMH} \quad - i\lambda_R^0\, \sqrt{2}\, g_R\, v_R\, \tilde{\Delta}_R^0 + i\lambda_V^0\, 2\sqrt{2}\, g_V\, v_R\, \tilde{\Delta}_R^0 + \text{h.c.} \tag{7.2}$$

$$\text{Define:} (\xi^0)^T \equiv (-i\lambda_R^0\, , -i\lambda_V^0\, , \tilde{\Delta}_R^0\,) \tag{7.3}$$

Equation (7.2) now takes the form

$$\mathcal{L}_{NMH} = - \frac{1}{\sqrt{2}}\, v_R\, (\xi^0)^T\, Y\, \xi^0 + \text{h.c.} \tag{7.4}$$

$$\text{where } Y = \begin{pmatrix} 0 & 0 & -g_R \\ 0 & 0 & 2g_V \\ -g_R & 2g_V & 0 \end{pmatrix} \tag{7.5}$$

The mass eigenstates are defined by

$$\chi_i^0 = N_{ij}\ \xi_j^0 \qquad (i,j = 1,2,3) \tag{7.6}$$

where N is a unitary matrix satisfying

$$N^{*}\ Y\ N^{-1} = N_D\ , \tag{7.7}$$

where N_D is a diagonal matrix with nonnegative entries. To determine N, it is easiest to square eq. (7.7) obtaining

$$N_D^2 = N\ Y^{\dagger}\ Y\ N^{-1} \tag{7.8}$$

which is analogous to eq. (6.15). Using eqs. (A19) and (7.6), eq. (7.4) may be written in terms of four-component neutral Majorana spinors. Defining

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}\ , \tag{7.9}$$

the mass term becomes

$$-\frac{1}{\sqrt{2}}\ \sum_i M_i\ \bar{\tilde{\chi}}_i^0\ \tilde{\chi}_i^0\ , \tag{7.10}$$

where M_i are the diagonal elements of N_D.

Now, since

$$Y^{\dagger} Y = \begin{pmatrix} g_R^2 & -2g_R g_V & 0 \\ -2g_R g_V & 4g_V^2 & 0 \\ 0 & 0 & g_R^2 + 4g_V^2 \end{pmatrix}, \qquad (7.11)$$

the eigenvalues of eq. (7.11) are: twice $(g_R^2 + 4g_V^2)$; and 0. These are the diagonal entries of N_D^2 , and the diagonalizing matrix N is given by

$$N = \begin{pmatrix} g_R/\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}, & -2g_V/\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}, & \frac{-1}{\sqrt{2}} \\ g_R/\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}, & -2g_V/\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}, & \frac{+1}{\sqrt{2}} \\ 2g_V \ /(g_R^2+ 4g_V^2)^{1/2}, & g_R \ / \ (g_R^2+ 4g_V^2)^{1/2}, & 0 \end{pmatrix}$$

(7.12)

Using eqs. (7.6) and (7.9), the physical neutralinos resulting from the first breaking are:

$$\tilde{\chi}_{Z1} = \begin{pmatrix} \dfrac{-i(g_R\lambda_R^0 - 2g_V\lambda_V)}{\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}} - \dfrac{\tilde{\Delta}_R^0}{\sqrt{2}} \\ \dfrac{+i(g_R\bar{\lambda}_R^0 - 2g_V\bar{\lambda}_V)}{\sqrt{2}(g_R^2+ 4g_V^2)^{1/2}} - \dfrac{\bar{\tilde{\Delta}}_R^0}{\sqrt{2}} \end{pmatrix} \qquad (7.13)$$

with mass $= \frac{1}{\sqrt{2}} v_R \ (g_R^2 + 4g_V^2)^{1/2}$; (7.14)

$$\tilde{\chi}_{Z2} = \begin{pmatrix} \dfrac{-i(g_R\lambda_R^0 - 2g_V\lambda_V^0)}{\sqrt{2}(g_R^2 + 4g_V^2)^{1/2}} + \dfrac{\tilde{\Delta}_R^0}{\sqrt{2}} \\ \dfrac{+i(g_R\bar{\lambda}_R^0 - 2g_V\bar{\lambda}_V^0)}{\sqrt{2}(g_R^2 + 4g_V^2)^{1/2}} + \dfrac{\bar{\tilde{\Delta}}_R^0}{\sqrt{2}} \end{pmatrix} \tag{7.15}$$

with mass = $\frac{1}{\sqrt{2}}\, v_R\, (g_R^2 + 4g_V^2)^{1/2}$; and (7.16)

$$\tilde{\chi}_B = \begin{pmatrix} \dfrac{-i(g_R\lambda_V^0 + 2g_V\lambda_R^0)}{(g_R^2 + 4g_V^2)^{1/2}} \\ \dfrac{+i(g_R\bar{\lambda}_V^0 + 2g_V\bar{\lambda}_R^0)}{(g_R^2 + 4g_V^2)^{1/2}} \end{pmatrix} \tag{7.17}$$

with mass = 0. (7.18)

Thus, the neutralino spectrum, at this mass scale, consists of: two Majorana fermions, degenerate in mass; and a massless Majorana fermion. The two massive states can be written as a single Dirac spinor:

$$\tilde{\zeta}^0 = \begin{pmatrix} \tilde{\Delta}_R^0 \\ \dfrac{i(g_R\bar{\lambda}_R^0 - 2g_V\bar{\lambda}_V^0)}{(g_R^2 + 4g_V^2)^{1/2}} \end{pmatrix} \tag{7.19}$$

In the absence of supersymmetry-breaking terms, this is exactly as expected: $\tilde{\zeta}^0$ is the superpartner of Z_R ; and $\tilde{\chi}_B$, that of B_μ . To complete the symmetry that exists between ordinary particles and their superpartners, $\tilde{\zeta}^0$

decouples from the low energy theory, and the massless* $\tilde{\chi}_B$ goes through to the next stage of symmetry breaking.

7.3 The "light" neutralinos

In view of the rearrangements that have taken place among the fields at the high energy scale, the particles taking part in the low energy interactions are no longer those of eq. (7.1). Specifically, λ_R^0 and λ_V^0, have to be re-written in terms of λ_Z^0 and λ_B^0, which are defined by

$$\lambda_Z^0 \equiv \frac{g_R\ \lambda_R^0 - 2g_V\ \lambda_V^0}{(g_R^2 + 4g_V^2)^{1/2}} \tag{7.20}$$

$$\lambda_B^0 \equiv \frac{g_R\ \lambda_V^0 + 2g_V\ \lambda_R^0}{(g_R^2 + 4g_V^2)^{1/2}} \tag{7.21}$$

giving

$$\lambda_R^0 = \frac{g_R\ \lambda_Z^0 + 2g_V\ \lambda_B^0}{(g_R^2 + 4g_V^2)^{1/2}} \tag{7.22}$$

$$\lambda_V^0 = \frac{g_R\ \lambda_B^0 - 2g_V\ \lambda_Z^0}{(g_R^2 + 4g_V^2)^{1/2}} \tag{7.23}$$

The mass Lagrangian of the light neutralinos, $\mathcal{L}_{NML}$, is obtained from eq. (7.1), by substituting eqs. (7.22) and

---

* one of the reasons for the inverted commas on "heavy".

(7.23) for λ_R^0 and λ_V^0 , and removing the contributions of the fields which have decoupled (i.e., $\tilde{\Delta}_R^0$ and λ_Z^0). The result of this is

$$\mathcal{L}_{NML} = \{- i\ \lambda_L^0 \tfrac{1}{\sqrt{2}}\ g_L\ \kappa_u\ \tilde{\phi}_{1u}^0 + \frac{i\ \lambda_B^0 \sqrt{2}\ g_V\ g_R\ \kappa_u\ \tilde{\phi}_{1u}^0}{(g_R^2 + 4g_V^2)^{1/2}}$$

$$+ i\ \lambda_L^0 \tfrac{1}{\sqrt{2}}\ g_L\ \kappa_d\ \tilde{\phi}_{2d}^0 - \frac{i\ \lambda_B^0 \sqrt{2}\ g_V\ g_R\ \kappa_d\ \tilde{\phi}_{2d}^0}{(g_R^2 + 4g_V^2)^{1/2}}$$

$$+ m_L\ \lambda_L^0\ \lambda_L^0 + \frac{m_R\ 4g_V^2\ \lambda_B^0\ \lambda_B^0}{g_R^2 + 4g_V^2} + \frac{m_V\ g_R^2\ \lambda_B^0\ \lambda_B^0}{g_R^2 + 4g_V^2}$$

$$+ 2\ \mu_1\ \tilde{\phi}_u^0\ \tilde{\phi}_d^0\ \} + \text{h.c.} \tag{7.24}$$

The identification of the mass eigenstates follows the, now, familiar procedure: define

$$(\Omega^0)^T \equiv (-i\lambda_L^0\ , -i\lambda_B^0\ , \tilde{\phi}_{1u}^0\ , \tilde{\phi}_{2d}^0) \tag{7.25}$$

so that eq. (7.24) takes the form

$$\mathcal{L}_{NML} = -\tfrac{1}{2}\ (\Omega^0)^T\ Z\ \Omega^0 + \text{h.c.} \tag{7.26}$$

where

$$Z = \begin{pmatrix} m_L & 0 & \frac{-1}{\sqrt{2}} g_L \kappa_u & \frac{1}{\sqrt{2}} g_L \kappa_d \\ 0 & \frac{m_V g_R^2 + 4m_R g_V^2}{g_R^2 + 4g_V^2} & \frac{\sqrt{2} g_V g_R \kappa_u}{(g_R^2 + 4g_V^2)^{1/2}} & \frac{-\sqrt{2} g_V g_R \kappa_d}{(g_R^2 + 4g_V^2)^{1/2}} \\ \frac{-1}{\sqrt{2}} g_L \kappa_u & \frac{\sqrt{2} g_V g_R \kappa_d}{(g_R^2 + 4g_V^2)^{1/2}} & 0 & -2\mu_1 \\ \frac{1}{\sqrt{2}} g_L \kappa_d & \frac{-\sqrt{2} g_V g_R \kappa_d}{(g_R^2 + 4g_V^2)^{1/2}} & -2\mu_1 & 0 \end{pmatrix} \tag{7.27}$$

All that remains to be done, now, is to find the matrices, M and M^{-1}, that will diagonalize $Z^{\dagger}Z$ ...

With those matrices in hand, the physical states, χ_i^0 , are given by

$$\chi_i^0 = M_{ij}\, \Omega_j^0 \,, \qquad (i,j = 1,\ldots,4) \tag{7.28}$$

These are then written as four-component Majorana spinors [eq. (7.9)], whose masses are the positive square roots of

$$M_D^2 = M\, Z^{\dagger} Z\, M^{-1} \tag{7.29}$$

In the absence of explicit expressions for the physical states, the Feynman rules for interactions involving neutralinos will be given in terms of unknown elements of the matrix M.

CHAPTER 8

FEYNMAN RULES

The next step in the development of the model is to subject it to phenomenological constraints. Frank and Kalman[30], in their recent study of a similar model, pinpoint two areas in which such constraints should be sought: (a) the weak neutral currents; and (b) the anomalous magnetic moment of the muon.

The value of (g-2) for the muon is one of the most precise measurements in physics (the uncertainty being in the eighth significant figure,) and the current theoretical prediction of this value, based on QED calculations involving only known particles, is one of the greatest success stories of theoretical physics. It is therefore essential for a theory which introduces new particles, that it should not spoil this happy state of affairs.

As for the neutral current bounds, they serve to restrict the values of the coupling constants of the model, and the mass of the new boson Z_R(the only constraint implicit in the model being that the coupling constants used, should result in the value "e" as the strength of the electromagnetic interaction.) It is interesting to note that in the comprehensive analysis of neutral current data, published by Amaldi *et al*[31], the left-handed parameters $\varepsilon_L(u)$, $\varepsilon_L(d)$ are in close agreement with the standard model calculations (including radiative corrections), whereas the right-handed $\varepsilon_R(u)$, $\varepsilon_R(d)$ deviate from them. This kind of deviation is naturally corrected in a model containing a Z_R boson.

The rest of this chapter is devoted to the calculation of the Feynman rules of the three interactions: chargino - lepton - slepton; neutralino - lepton - slepton; and W_R- lepton - lepton. These Feynman rules will be used, at some future date, to calculate the anomalous magnetic moment of the muon.

8.1 Chargino-lepton-slepton

The piece of the Lagrangian [eqs. (4.3), (4.4)] that determines this interaction (and the one in the next section) will be called $\mathcal{L}_{LSI}$, defined as

$$\mathcal{L}_{LSI} = \frac{i}{\sqrt{2}} \tilde{L}_L^\dagger (g_L \tau.\lambda_L - g_V \lambda_V) L_L + \frac{i}{\sqrt{2}} \tilde{L}_R^\dagger (g_R \tau.\lambda_R - g_V \lambda_V) L_R$$

$$+ h_u^L (\tilde{L}_L^\dagger \tilde{\Phi}_u L_R) + h_d^L (\tilde{L}_L^\dagger \tilde{\Phi}_d L_R) + h_u^L (\tilde{L}_R^\dagger \tilde{\Phi}_u L_L)$$

$$+ h_d^L (\tilde{L}_R^\dagger \tilde{\Phi}_d L_L) + h_{LR}(\tilde{L}_L^T \tau_1 \tau.\tilde{\Delta}_L L_L + \tilde{L}_R^T \tau_1 \tau.\tilde{\Delta}_R L_R)$$

$$+ \text{h.c.} \qquad (8.1)$$

Only the charged terms of eq. (8.1) are relevant to this section, they are:

$$
\begin{aligned}
\mathcal{L}_{CLS} \equiv i \{ & g_L (\lambda^-_L \, \nu_L \, \tilde{e}^*_L - \bar{\lambda}^-_L \, \bar{\nu}_L \, \tilde{e}_L + \lambda^+_L \, e_L \, \tilde{\nu}^*_L \\
& - \bar{\lambda}^+_L \, \bar{e}_L \, \tilde{\nu}_L) + g_R (\lambda^-_R \, \nu_R \, \tilde{e}^*_R - \bar{\lambda}^-_R \, \bar{\nu}_R \, \tilde{e}_R \\
& + \lambda^+_R \, e_R \, \tilde{\nu}^*_R - \bar{\lambda}^+_R \, \bar{e}_R \, \tilde{\nu}_R) \; \} \\
& + h^L_u (\tilde{\phi}^-_u \, \nu_R \, \tilde{e}^*_L + \bar{\tilde{\phi}}^-_u \, \bar{\nu}_R \, \tilde{e}_L + \tilde{\phi}^+_u \, e_R \, \tilde{\nu}^*_L + \bar{\tilde{\phi}}^+_u \, \bar{e}_R \, \tilde{\nu}_L) \\
& + h^L_d (\tilde{\phi}^-_d \, \nu_R \, \tilde{e}^*_L + \bar{\tilde{\phi}}^-_d \, \bar{\nu}_R \, \tilde{e}_L + \tilde{\phi}^+_d \, e_R \, \tilde{\nu}^*_L + \bar{\tilde{\phi}}^+_d \, \bar{e}_R \, \tilde{\nu}_L) \\
& + h^L_u (\tilde{\phi}^-_u \, \nu_L \, \tilde{e}^*_R + \bar{\tilde{\phi}}^-_u \, \bar{\nu}_L \, \tilde{e}_R + \tilde{\phi}^+_u \, e_L \, \tilde{\nu}^*_R + \bar{\tilde{\phi}}^+_u \, \bar{e}_L \, \tilde{\nu}_R) \\
& + h^L_d (\tilde{\phi}^-_d \, \nu_L \, \tilde{e}^*_R + \bar{\tilde{\phi}}^-_d \, \bar{\nu}_L \, \tilde{e}_R + \tilde{\phi}^+_d \, e_L \, \tilde{\nu}^*_R + \bar{\tilde{\phi}}^+_d \, \bar{e}_L \, \tilde{\nu}_R) \\
& + h_{LR} \Big(\tfrac{1}{\sqrt{2}} \tilde{\Delta}^+_L \, \nu_L \, \tilde{e}_L + \tfrac{1}{\sqrt{2}} \bar{\tilde{\Delta}}^+_L \, \bar{\nu}_L \, \tilde{e}^*_L - \tfrac{1}{\sqrt{2}} \tilde{\Delta}^+_L \, e_L \, \tilde{\nu}_L \\
& \qquad - \tfrac{1}{\sqrt{2}} \bar{\tilde{\Delta}}^+_L \, \bar{e}_L \, \tilde{\nu}^*_L + \tilde{\Delta}^{++}_L \, e_L \, \tilde{e}_L + \bar{\tilde{\Delta}}^{++}_L \, \bar{e}_L \, \tilde{e}^*_L \\
& \qquad + \tfrac{1}{\sqrt{2}} \tilde{\Delta}^+_R \, \nu_R \, \tilde{e}_R + \tfrac{1}{\sqrt{2}} \bar{\tilde{\Delta}}^+_R \, \bar{\nu}_R \, \tilde{e}^*_R - \tfrac{1}{\sqrt{2}} \tilde{\Delta}^+_R \, e_R \, \tilde{\nu}_R \\
& \qquad - \tfrac{1}{\sqrt{2}} \bar{\tilde{\Delta}}^+_R \, \bar{e}_R \, \tilde{\nu}^*_R + \tilde{\Delta}^{++}_R \, e_R \, \tilde{e}_R + \bar{\tilde{\Delta}}^{++}_R \, \bar{e}_R \, \tilde{e}^*_R \Big)
\end{aligned}
\tag{8.2}
$$

where $\lambda^{\pm} = (1/\sqrt{2}) \; (\lambda^1 \mp i\lambda^2)$ has been used.

In order to translate eq. (8.2) into four-component notation, it is necessary to define the weak interaction eigenstates:

$$\tilde{W}_L \equiv \begin{pmatrix} -i\lambda_L^+ \\ i\bar{\lambda}_L^- \end{pmatrix} \; ; \quad \tilde{W}_R \equiv \begin{pmatrix} -i\lambda_R^+ \\ i\bar{\lambda}_R^- \end{pmatrix} \; ;$$

$$\tilde{F}_u \equiv \begin{pmatrix} \tilde{\phi}_u^+ \\ \bar{\tilde{\phi}}_u^- \end{pmatrix} \; ; \quad \tilde{F}_d \equiv \begin{pmatrix} \tilde{\phi}_d^+ \\ \bar{\tilde{\phi}}_d^- \end{pmatrix} \; ; \quad \tilde{D}_L^1 \equiv \begin{pmatrix} \tilde{\Delta}_L^+ \\ \bar{\tilde{\Delta}}_L^- \end{pmatrix} \; ;$$

$$\tilde{D}_R^1 \equiv \begin{pmatrix} \tilde{\Delta}_R^+ \\ \bar{\tilde{\Delta}}_R^- \end{pmatrix} \; ; \quad \tilde{D}_L^2 \equiv \begin{pmatrix} \tilde{\Delta}_L^{++} \\ \bar{\tilde{\Delta}}_L^{--} \end{pmatrix} \; ; \quad \tilde{D}_R^2 \equiv \begin{pmatrix} \tilde{\Delta}_R^{++} \\ \bar{\tilde{\Delta}}_R^{--} \end{pmatrix} \; ; \qquad (8.3)$$

and ,in the notation of eq. (A11), the electron and neutrino spinors;

$$e = \begin{pmatrix} e_L \\ \bar{e}_R \end{pmatrix} \; ; \qquad \nu = \begin{pmatrix} \nu_L \\ \bar{\nu}_R \end{pmatrix} \qquad (8.4)$$

Using eq. (A28), eq. (8.2) takes the form:

$$- \{ \; g_L(\bar{\tilde{W}}_L \; P_L \; \nu \; \tilde{e}_L^* + \bar{\nu} \; P_R \; \tilde{W}_L \; \tilde{e}_L + \bar{\tilde{W}}_L^c \; P_L \; e \; \tilde{\nu}_L^* + \bar{e} \; P_R \; \tilde{W}_L^c \; \tilde{\nu}_L)$$

$$+ \; g_R(\bar{\nu} \; P_L \; \tilde{W}_R^c \; \tilde{e}_R^* + \bar{\tilde{W}}_R^c \; P_R \; \nu \; \tilde{e}_R + \bar{e} \; P_L \; \tilde{W}_R \; \tilde{\nu}_R^* + \bar{\tilde{W}}_R \; P_R \; e \; \tilde{\nu}_R)\}$$

$$+ \; h_u^L \; (\bar{\nu} \; P_L \; \tilde{F}_u^c \; \tilde{e}_L^* + \bar{\tilde{F}}_u^c \; P_R \; \nu \; \tilde{e}_L + \bar{e} \; P_L \; \tilde{F}_u \; \tilde{\nu}_L^* + \bar{\tilde{F}}_u \; P_R \; e \; \tilde{\nu}_L)$$

$$+ \; h_d^L \; (\bar{\nu} \; P_L \; \tilde{F}_d^c \; \tilde{e}_L^* + \bar{\tilde{F}}_d^c \; P_R \; \nu \; \tilde{e}_L + \bar{e} \; P_L \; \tilde{F}_d \; \tilde{\nu}_L^* + \bar{\tilde{F}}_d \; P_R \; e \; \tilde{\nu}_L)$$

$$+ \; h_u^L \; (\bar{\tilde{F}}_u \; P_L \; \nu \; \tilde{e}_R^* + \bar{\nu} \; P_R \; \tilde{F}_u \; \tilde{e}_R + \bar{\tilde{F}}_u^c \; P_L \; e \; \tilde{\nu}_R^* + \bar{e} \; P_R \; \tilde{F}_u^c \; \tilde{\nu}_R)$$

$$+ \; h_d^L \; (\bar{\tilde{F}}_d \; P_L \; \nu \; \tilde{e}_R^* + \bar{\nu} \; P_R \; \tilde{F}_d \; \tilde{e}_R + \bar{\tilde{F}}_d^c \; P_L \; e \; \tilde{\nu}_R^* + \bar{e} \; P_R \; \tilde{F}_d^c \; \tilde{\nu}_R)$$

$$+ \frac{1}{\sqrt{2}} h_{LR} (\bar{\tilde{D}}^{1c}_L P_L \nu \tilde{e}_L + \bar{\nu} P_R \tilde{D}^{1c}_L \tilde{e}^*_L - \bar{\tilde{D}}^{1c}_L P_L e \tilde{\nu}_L$$

$$- \bar{e} P_R \tilde{D}^{1c}_L \tilde{\nu}^*_L + \sqrt{2} \bar{\tilde{D}}^{2c}_L P_L e \tilde{e}_L + \sqrt{2} \bar{e} P_R \tilde{D}^{2c}_L \tilde{e}^*_L$$

$$+ \bar{\nu} P_L \tilde{D}^1_R \tilde{e}_R + \bar{\tilde{D}}^1_R P_R \nu \tilde{e}^*_R - \bar{e} P_L \tilde{D}^1_R \tilde{\nu}_R - \bar{\tilde{D}}^1_R P_R \nu \tilde{e}^*_R$$

$$+ \sqrt{2} \bar{e} P_L \tilde{D}^2_R \tilde{e}_R + \sqrt{2} \bar{\tilde{D}}^2_R P_R e \tilde{e}^*_R) \qquad (8.5)$$

where $P_{L,R}$ is defined by eq. (A10), and the charge conjugated states (such as $\tilde{W}^c_L$) are defined by eq. (A12).

$\tilde{W}_L$, $\tilde{W}_R$, $\tilde{F}_u$ and $\tilde{F}_d$ must now be expressed in terms of the mass eigenstates, $\tilde{\chi}_1$, $\tilde{\chi}_2$, $\tilde{\chi}_3$, $\tilde{\chi}_4$ [defined in eq. (6.14).] These are the states that interact with the leptons and sleptons in the low energy theory, they do not mix with the very masive $\tilde{W}^+_R$ whose existence at this scale can be ignored. In the choice of VEV's of this model, the states $\tilde{D}^{1,2}_{L,R}$ also remain unmixed and constitute eigenstates of the theory. Using eqs. (6.8) and (6.10), one obtains:

$$\chi^+_1 = -i V_{11} \lambda^+_L - i V_{12} \lambda^+_R + V_{13} \tilde{\phi}^+_u + V_{14} \tilde{\phi}^+_d \qquad (8.6)$$

$$\chi^-_1 = -i U_{11} \lambda^-_L - i U_{12} \lambda^-_R + U_{13} \tilde{\phi}^-_u + U_{14} \tilde{\phi}^-_d \qquad (8.7)$$

with similar expressions for $\chi^\pm_2$, $\chi^\pm_3$, $\chi^\pm_4$. Now,

$$P_R \tilde{W}_L = \begin{pmatrix} 0 \\ -i\bar{\lambda}^-_L \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ -i\,(U^*_{11}U_{11} + U^*_{21}U_{21} + U^*_{31}U_{31} + U^*_{41}U_{41})\ \bar{\lambda}^-_L \end{pmatrix} \tag{8.8}$$

since, from the properties of unitary matrices, the term in brackets is equal to one. However, from eq. (8.7) and similar ones for $\chi^-_{2,3,4}$, the R.H.S of eq. (8.8) can be written as

$$P_R\ (U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4) \tag{8.9}$$

Therefore,

$$P_R\ \tilde{W}_L = P_R\ (U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4) \tag{8.10}$$

Similarly:

$$\bar{\tilde{W}}_L\ P_L = (U^*_{11}\bar{\tilde{\chi}}_1 + U^*_{21}\bar{\tilde{\chi}}_2 + U^*_{31}\bar{\tilde{\chi}}_3 + U^*_{41}\bar{\tilde{\chi}}_4)\ P_L$$

$$\bar{\tilde{W}}^c_L\ P_L = (V^*_{11}\bar{\tilde{\chi}}^c_1 + V^*_{21}\bar{\tilde{\chi}}^c_2 + V^*_{31}\bar{\tilde{\chi}}^c_3 + V^*_{41}\bar{\tilde{\chi}}^c_4)\ P_L$$

$$P_R\ \tilde{W}^c_L = P_R\ (V_{11}\tilde{\chi}^c_1 + V_{21}\tilde{\chi}^c_2 + V_{31}\tilde{\chi}^c_3 + V_{41}\tilde{\chi}^c_4)$$

$$P_L\ \tilde{W}^c_R = P_L\ (U^*_{12}\tilde{\chi}^c_1 + U^*_{22}\tilde{\chi}^c_2 + U^*_{32}\tilde{\chi}^c_3 + U^*_{42}\tilde{\chi}^c_4)$$

$$\bar{\tilde{W}}^c_R\ P_R = (U_{12}\bar{\tilde{\chi}}^c_1 + U_{22}\bar{\tilde{\chi}}^c_2 + U_{32}\bar{\tilde{\chi}}^c_3 + U_{42}\bar{\tilde{\chi}}^c_4)\ P_R$$

$$P_L\ \tilde{W}_R = P_L\ (V^*_{12}\tilde{\chi}_1 + V^*_{22}\tilde{\chi}_2 + V^*_{32}\tilde{\chi}_3 + V^*_{42}\tilde{\chi}_4)$$

$$\bar{\tilde{W}}_R P_R = (V_{12}\bar{\tilde{\chi}}_1 + V_{22}\bar{\tilde{\chi}}_2 + V_{32}\bar{\tilde{\chi}}_3 + V_{42}\bar{\tilde{\chi}}_4) P_R \qquad (8.11)$$

Using these results, and equivalent ones for the other charginos, eq. (8.5), is now rewritten in terms of the physical states as

$$-g_L \{ (U^*_{11}\bar{\tilde{\chi}}_1 + U^*_{21}\bar{\tilde{\chi}}_2 + U^*_{31}\bar{\tilde{\chi}}_3 + U^*_{41}\bar{\tilde{\chi}}_4) P_L \nu \tilde{e}^*_L$$

$$+ \bar{\nu} P_R (U_{11}\tilde{\chi}_1 + U_{21}\tilde{\chi}_2 + U_{31}\tilde{\chi}_3 + U_{41}\tilde{\chi}_4) \tilde{e}_L$$

$$+ (V^*_{11}\bar{\tilde{\chi}}^c_1 + V^*_{21}\bar{\tilde{\chi}}^c_2 + V^*_{31}\bar{\tilde{\chi}}^c_3 + V^*_{41}\bar{\tilde{\chi}}^c_4) P_L e \tilde{\nu}^*_L$$

$$+ \bar{e} P_R (V_{11}\tilde{\chi}^c_1 + V_{21}\tilde{\chi}^c_2 + V_{31}\tilde{\chi}^c_3 + V_{41}\tilde{\chi}^c_4) \tilde{\nu}_L \}$$

$$-g_R \{ \bar{\nu} P_L (U^*_{12}\tilde{\chi}^c_1 + U^*_{22}\tilde{\chi}^c_2 + U^*_{32}\tilde{\chi}^c_3 + U^*_{42}\tilde{\chi}^c_4) \tilde{e}^*_R$$

$$+ (U_{12}\bar{\tilde{\chi}}^c_1 + U_{22}\bar{\tilde{\chi}}^c_2 + U_{32}\bar{\tilde{\chi}}^c_3 + U_{42}\bar{\tilde{\chi}}^c_4) P_R \nu \tilde{e}_R$$

$$+ \bar{e} P_L (V^*_{12}\tilde{\chi}_1 + V^*_{22}\tilde{\chi}_2 + V^*_{32}\tilde{\chi}_3 + V^*_{42}\tilde{\chi}_4) \tilde{\nu}^*_R$$

$$+ (V_{12}\bar{\tilde{\chi}}_1 + V_{22}\bar{\tilde{\chi}}_2 + V_{32}\bar{\tilde{\chi}}_3 + V_{42}\bar{\tilde{\chi}}_4) P_R e \tilde{\nu}_R \}$$

$$+ h^L_u \{ \bar{\nu} P_L (U^*_{13}\tilde{\chi}^c_1 + U^*_{23}\tilde{\chi}^c_2 + U^*_{33}\tilde{\chi}^c_3 + U^*_{43}\tilde{\chi}^c_4) \tilde{e}^*_L$$

$$+ (U_{13}\bar{\tilde{\chi}}^c_1 + U_{23}\bar{\tilde{\chi}}^c_2 + U_{33}\bar{\tilde{\chi}}^c_3 + U_{43}\bar{\tilde{\chi}}^c_4) P_R \nu \tilde{e}_L$$

$$+ \bar{e}\ P_L\ (V^*_{13}\tilde{\chi}_1 + V^*_{23}\tilde{\chi}_2 + V^*_{33}\tilde{\chi}_3 + V^*_{43}\tilde{\chi}_4)\ \tilde{\nu}^*_L$$

$$+ (V_{13}\bar{\tilde{\chi}}_1 + V_{23}\bar{\tilde{\chi}}_2 + V_{33}\bar{\tilde{\chi}}_3 + V_{43}\bar{\tilde{\chi}}_4)\ P_R\ e\ \tilde{\nu}_L\ \}$$

$$+ h^L_d\{\ \bar{\nu}\ P_L\ (U^*_{14}\tilde{\chi}^c_1 + U^*_{24}\tilde{\chi}^c_2 + U^*_{34}\tilde{\chi}^c_3 + U^*_{44}\tilde{\chi}^c_4)\ \tilde{e}^*_L$$

$$+ (U_{14}\bar{\tilde{\chi}}^c_1 + U_{24}\bar{\tilde{\chi}}^c_2 + U_{34}\bar{\tilde{\chi}}^c_3 + U_{44}\bar{\tilde{\chi}}^c_4)\ P_R\ \nu\ \tilde{e}_L$$

$$+ \bar{e}\ P_L\ (V^*_{14}\tilde{\chi}_1 + V^*_{24}\tilde{\chi}_2 + V^*_{34}\tilde{\chi}_3 + V^*_{44}\tilde{\chi}_4)\ \tilde{\nu}^*_L$$

$$+ (V_{14}\bar{\tilde{\chi}}_1 + V_{24}\bar{\tilde{\chi}}_2 + V_{34}\bar{\tilde{\chi}}_3 + V_{44}\bar{\tilde{\chi}}_4)\ P_R\ e\ \tilde{\nu}_L\ \}$$

$$+ h^L_u\{\ (U^*_{13}\bar{\tilde{\chi}}_1 + U^*_{23}\bar{\tilde{\chi}}_2 + U^*_{33}\bar{\tilde{\chi}}_3 + U^*_{43}\bar{\tilde{\chi}}_4)\ P_L\ \nu\ \tilde{e}^*_R$$

$$+ \bar{\nu}\ P_R\ (U_{13}\tilde{\chi}_1 + U_{23}\tilde{\chi}_2 + U_{33}\tilde{\chi}_3 + U_{43}\tilde{\chi}_4)\ \tilde{e}_R$$

$$+ (V^*_{13}\bar{\tilde{\chi}}^c_1 + V^*_{23}\bar{\tilde{\chi}}^c_2 + V^*_{33}\bar{\tilde{\chi}}^c_3 + V^*_{43}\bar{\tilde{\chi}}^c_4)\ P_L\ e\ \tilde{\nu}^*_R$$

$$+ \bar{e}\ P_R\ (V_{13}\tilde{\chi}^c_1 + V_{23}\tilde{\chi}^c_2 + V_{33}\tilde{\chi}^c_3 + V_{43}\tilde{\chi}^c_4)\ \tilde{\nu}_R\ \}$$

$$+ h^L_d\{\ (U^*_{14}\bar{\tilde{\chi}}_1 + U^*_{24}\bar{\tilde{\chi}}_2 + U^*_{34}\bar{\tilde{\chi}}_3 + U^*_{44}\bar{\tilde{\chi}}_4)\ P_L\ \nu\ \tilde{e}^*_R$$

$$+ \bar{\nu}\ P_R\ (U_{14}\tilde{\chi}_1 + U_{24}\tilde{\chi}_2 + U_{34}\tilde{\chi}_3 + U_{44}\tilde{\chi}_4)\ \tilde{e}_R$$

$$+ (V^*_{14}\bar{\tilde{\chi}}^c_1 + V^*_{24}\bar{\tilde{\chi}}^c_2 + V^*_{34}\bar{\tilde{\chi}}^c_3 + V^*_{44}\bar{\tilde{\chi}}^c_4)\ P_L\ e\ \tilde{\nu}^*_R$$

$$+ \bar{e}\ P_R\ (V_{14}\tilde{\chi}^c_1 + V_{24}\tilde{\chi}^c_2 + V_{34}\tilde{\chi}^c_3 + V_{44}\tilde{\chi}^c_4)\ \tilde{\nu}_R\ \}$$

$$+ \frac{1}{\sqrt{2}} h_{LR} (\bar{\tilde{D}}_L^{1c} P_L \nu \tilde{e}_L + \bar{\nu} P_R \tilde{D}_L^{1c} \tilde{e}_L^* - \bar{\tilde{D}}_L^{1c} P_L e \tilde{\nu}_L$$

$$- \bar{e} P_R \tilde{D}_L^{1c} \tilde{\nu}_L^* + \sqrt{2} \bar{\tilde{D}}_L^{2c} P_L e \tilde{e}_L + \sqrt{2} \bar{e} P_R \tilde{D}_L^{2c} \tilde{e}_L^*$$

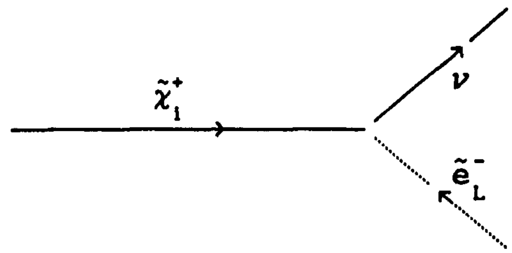
$$+ \bar{\nu} P_L \tilde{D}_R^1 \tilde{e}_R + \bar{\tilde{D}}_R^1 P_R \nu \tilde{e}_R^* - \bar{e} P_L \tilde{D}_R^1 \tilde{\nu}_R - \bar{\tilde{D}}_R^1 P_R \nu \tilde{e}_R^*$$

$$+ \sqrt{2} \bar{e} P_L \tilde{D}_R^2 \tilde{e}_R + \sqrt{2} \bar{\tilde{D}}_R^2 P_R e \tilde{e}_R^*) \qquad (8.12)$$

Figure 8.1 contains the Feynman diagrams for the interactions represented by eq. (8.12). The Feynman rules, which can be read off directly from the equation, are written beside the diagrams.

The conventions used are the following. All fermions are represented by a solid line accompanied by an arrow. In the case of leptons, the direction of the arrow indicates the flow of lepton number, for charginos, it is the direction of positive electric charge. The scalars are represented by broken lines.

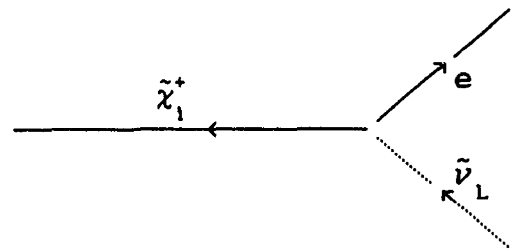
(a)



$$-ig_L\, U_{i1}\, \tfrac{1}{2}\, (1 + \gamma_5)$$

where, i, runs from 1 to 4

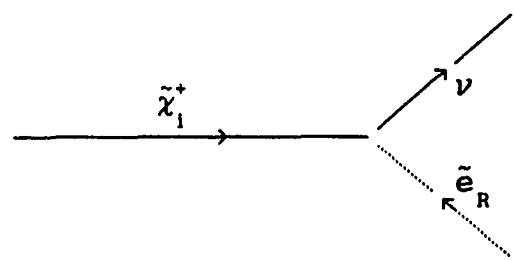
(b)



$$-ig_L\, V_{i1}\, \tfrac{1}{2}\, (1 + \gamma_5)\ C$$

where C is the charge conjugation matrix [eq. (A15)]

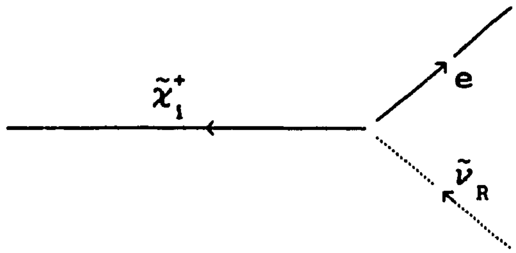
(c)



$$-ig_R\, U_{i2}\, \tfrac{1}{2}\, (1 + \gamma_5)\ C$$

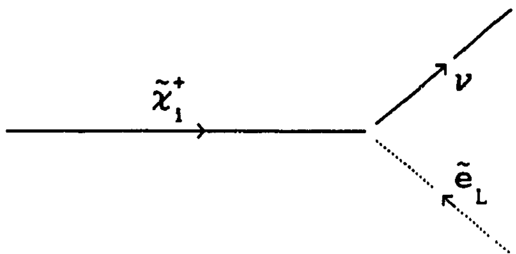
Figure 8.1

(d)



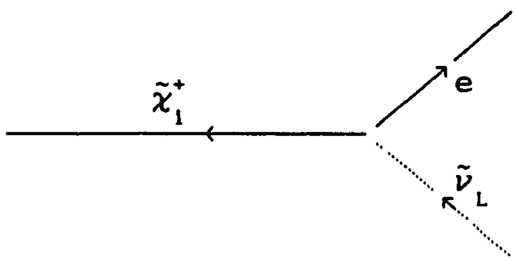
$$-ig_R \, V_{12} \, \frac{1}{2} \, (1 + \gamma_5)$$

(e)



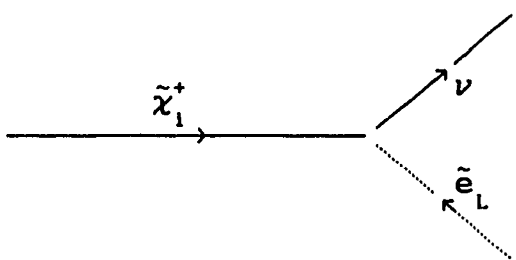
$$ih_u^L \, U_{13} \, \frac{1}{2} \, (1 + \gamma_5) \, C$$

(f)



$$ih_u^L \, V_{13} \, \frac{1}{2} \, (1 + \gamma_5)$$

(g)



$$ih_d^L \, U_{14} \, \frac{1}{2} \, (1 + \gamma_5) \, C$$

Figure 8.1 (continued)

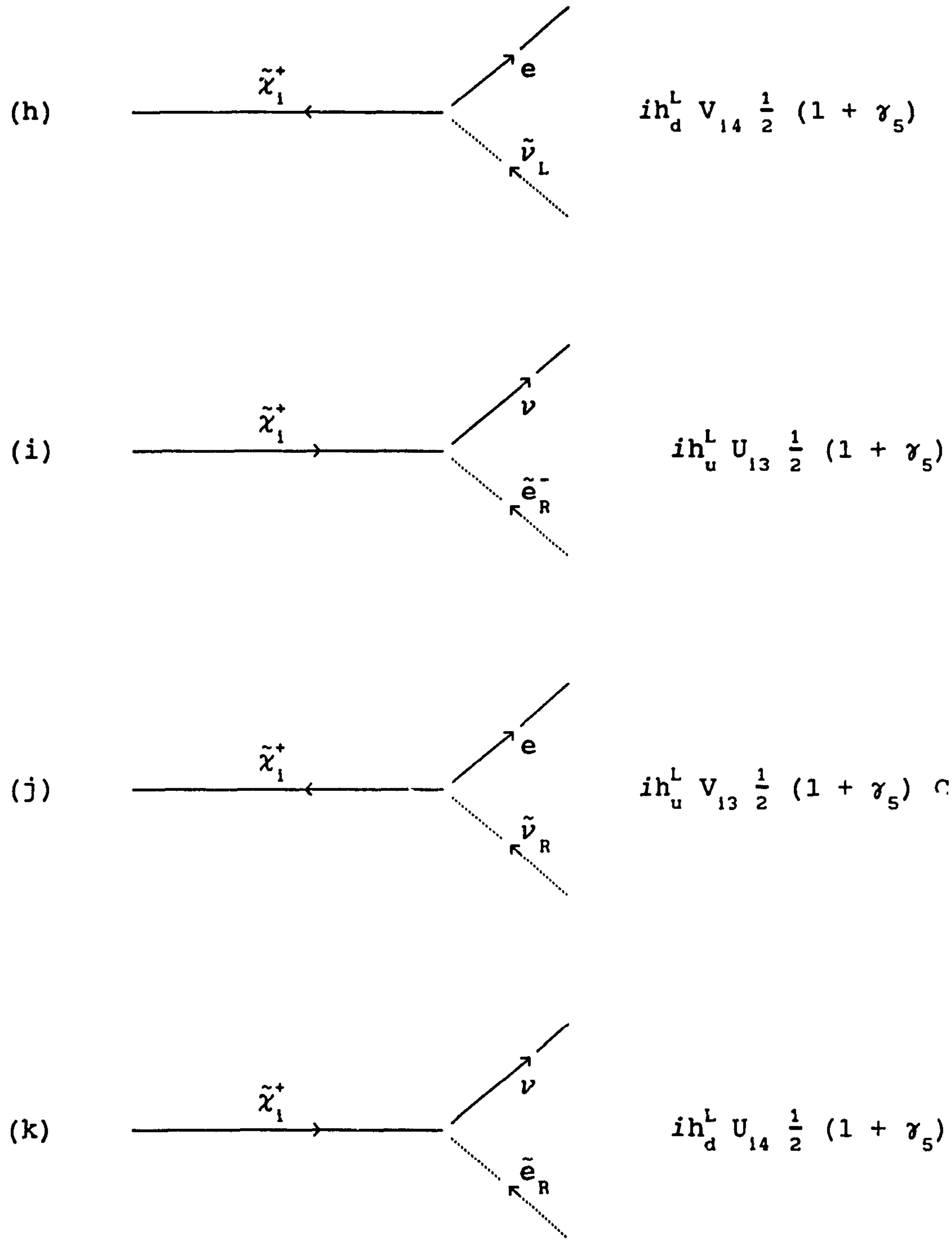
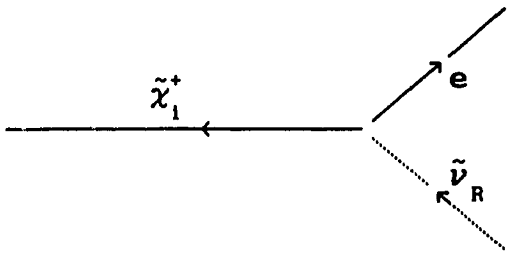


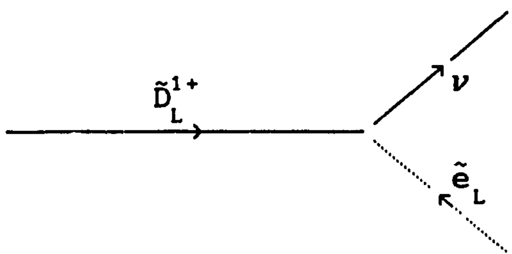
Figure 8.1 (continued)

(l)



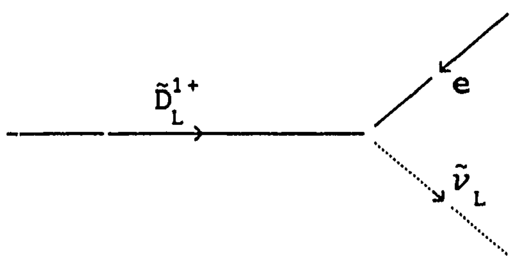
$$i h_d^L V_{14} \frac{1}{2} (1 + \gamma_5) C$$

(m)



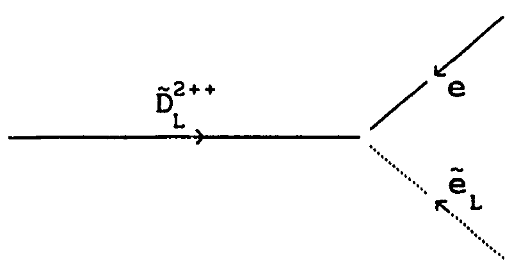
$$\frac{i}{\sqrt{2}} h_{LR} \frac{1}{2} (1 - \gamma_5) C$$

(n)



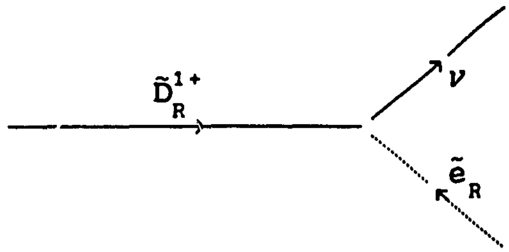
$$\frac{-i}{\sqrt{2}} h_{LR} \frac{1}{2} (1 - \gamma_5) C$$

(o)

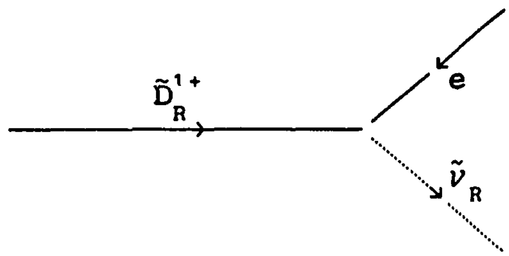


$$i\, h_{LR} \frac{1}{2} (1 - \gamma_5) C$$

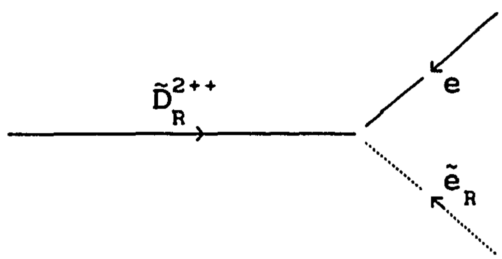
Figure 8.1 (continued)

(p) 

$$\frac{i}{\sqrt{2}} h_{LR} \frac{1}{2} (1 - \gamma_5)$$

(q) 

$$\frac{-i}{\sqrt{2}} h_{LR} \frac{1}{2} (1 - \gamma_5)$$

(r) 

$$i\, h_{LR} \frac{1}{2} (1 - \gamma_5)$$

Figure 8.1 (continued)

In theories with fermion number conservation, all Feynman graphs will contain a continuous flow of fermion number (indicated by the direction of the arrows.) In supersymmetric theories, fermion number, in the conventional sense, is violated. This is usually most easily seen when neutral Majorana fermions are present in the spectrum. However, this feature can already be seen here, in the "clashing" fermion lines of diagrams like (b) in fig. 8.1. The Feynman rules for these diagrams have to include the charge conjugation matrix, C, because it is $\tilde{\chi}^c$ that takes part in the interaction.

8.2 Neutralino-lepton-slepton

The Feynman rules for these interactions are obtained in much the same way as the previous ones. $\mathcal{L}_{NLS}$, which consists of the neutral terms in eq. (8.1), defines the Lagrangian of the interaction:

$$
\begin{aligned}
\mathcal{L}_{NLS} = & \frac{i}{\sqrt{2}} \tilde{\nu}_L^* (g_L \lambda_L^0 - g_V \lambda_V^0) \nu_L - \frac{i}{\sqrt{2}} \tilde{e}_L^* (g_L \lambda_L^0 + g_V \lambda_V^0) e_L \\
& + \frac{i}{\sqrt{2}} \tilde{\nu}_R^* (g_R \lambda_R^0 - g_V \lambda_V^0) \nu_R - \frac{i}{\sqrt{2}} \tilde{e}_R^* (g_R \lambda_R^0 + g_V \lambda_V^0) e_R \\
& + h_u^L (\tilde{\nu}_L^* \tilde{\phi}_{1u} \nu_R + \tilde{e}_L^* \tilde{\phi}_{2u} e_R + \tilde{\nu}_R^* \tilde{\phi}_{1u} \nu_L + \tilde{e}_R^* \tilde{\phi}_{2u} e_L) \\
& + h_d^L (\tilde{\nu}_L^* \tilde{\phi}_{1d} \nu_R + \tilde{e}_L^* \tilde{\phi}_{2d} e_R + \tilde{\nu}_R^* \tilde{\phi}_{1d} \nu_L + \tilde{e}_R^* \tilde{\phi}_{2d} e_L) \\
& + h_{LR} (\tilde{\nu}_L \tilde{\Delta}_L^0 \nu_L + \tilde{\nu}_R \tilde{\Delta}_R^0 \nu_R) + \text{h.c.} \qquad (8.13)
\end{aligned}
$$

In the low energy theory, the spinors, $\tilde{\lambda}_R^0$ and $\tilde{\lambda}_V^0$, have to be replaced by the mixed states: λ_Z^0 and λ_B^0 [eqs. (7.22) and (7.23)]. However, since the very massive, λ_Z^0 , decouples from this scale, only λ_B^0 enters into the picture. Equation (8.13) therefore becomes

$$\frac{i}{\sqrt{2}}\, \tilde{\nu}^*_L\, (g_L\, \lambda^0_L - g'\, \lambda^0_B)\, \nu_L - \frac{i}{\sqrt{2}}\, \tilde{e}^*_L\, (g_L\, \lambda^0_L + g'\, \lambda^0_B)\, e_L$$

$$+\frac{i}{\sqrt{2}}\, \tilde{\nu}^*_R\, (g'\lambda^0_B)\, \nu_R - \frac{i}{\sqrt{2}}\, \tilde{e}^*_R\, (3\, g'\lambda^0_B)\, e_R$$

$$+ h^L_u\, (\tilde{\nu}^*_L\, \tilde{\phi}^0_{1u}\, \nu_R + \tilde{e}^*_L\, \tilde{\phi}^0_{2u} e_R + \tilde{\nu}^*_R\, \tilde{\phi}^0_{1u}\, \nu_L + \tilde{e}^*_R\, \tilde{\phi}^0_{2u} e_L)$$

$$+ h^L_d\, (\tilde{\nu}^*_L\, \tilde{\phi}^0_{1d}\, \nu_R + \tilde{e}^*_L\, \tilde{\phi}^0_{2d} e_R + \tilde{\nu}^*_R\, \tilde{\phi}^0_{1d}\, \nu_L + \tilde{e}^*_R\, \tilde{\phi}^0_{2d} e_L)$$

$$+ h_{LR}\, (\tilde{\nu}_L\, \tilde{\Delta}^0_L\, \nu_L + \tilde{\nu}_R\, \tilde{\Delta}^0_R\, \nu_R) + \text{h.c.} \tag{8.14}$$

$$\text{where } g' = \frac{g_R\, g_V}{(g_R^2 + 4g_V^2)^{1/2}} \tag{8.15}$$

The conversion to four-component notation requires, first, the definition of the four-component Majorana spinors (weak interaction eigenstates):

$$\tilde{W}^0_L \equiv \begin{pmatrix} -i\lambda^0_L \\ i\bar{\lambda}^0_L \end{pmatrix} ; \qquad \tilde{W}^0_B \equiv \begin{pmatrix} -i\lambda^0_B \\ i\bar{\lambda}^0_B \end{pmatrix}$$

$$\tilde{F}^0_{1u} \equiv \begin{pmatrix} \tilde{\phi}^0_{1u} \\ \bar{\tilde{\phi}}^0_{1u} \end{pmatrix} ; \ \tilde{F}^0_{1d} \equiv \begin{pmatrix} \tilde{\phi}^0_{1d} \\ \bar{\tilde{\phi}}^0_{1d} \end{pmatrix} ; \ \tilde{F}^0_{2u} \equiv \begin{pmatrix} \tilde{\phi}^0_{2u} \\ \bar{\tilde{\phi}}^0_{2u} \end{pmatrix} ; \ \tilde{F}^0_{2d} \equiv \begin{pmatrix} \tilde{\phi}^0_{2d} \\ \bar{\tilde{\phi}}^0_{2d} \end{pmatrix} ;$$

$$\tilde{D}^0_L \equiv \begin{pmatrix} \tilde{\Delta}^0_L \\ \bar{\tilde{\Delta}}^0_L \end{pmatrix} ; \qquad \tilde{D}^0_R \equiv \begin{pmatrix} \tilde{\Delta}^0_R \\ \bar{\tilde{\Delta}}^0_R \end{pmatrix} \tag{8.16}$$

With the help of eq. (A28), eq. (8.14) becomes

$$\left\{ \frac{-1}{\sqrt{2}} (g_L\bar{\tilde{W}}^0_L - g'\bar{\tilde{W}}^0_B) P_L \nu \tilde{\nu}^*_L + \frac{1}{\sqrt{2}} (g_L\bar{\tilde{W}}^0_L + g'\bar{\tilde{W}}^0_B) P_L e \tilde{e}^*_L\right.$$

$$- \frac{1}{\sqrt{2}} \bar{\nu} P_L (g'\tilde{W}^0_B) \tilde{\nu}^*_R + \frac{3}{\sqrt{2}} \bar{e} P_L (g'\tilde{W}^0_B) \tilde{e}^*_R$$

$$+h^L_u(\bar{\nu} P_L\tilde{F}^0_{1u} \tilde{\nu}^*_L + \bar{e} P_L\tilde{F}^0_{2u} \tilde{e}^*_L + \bar{\tilde{F}}^0_{1u} P_L\nu \tilde{\nu}^*_R + \bar{\tilde{F}}^0_{2u} P_L e \tilde{e}^*_R)$$

$$+h^L_d(\bar{\nu} P_L\tilde{F}^0_{1d} \tilde{\nu}^*_L + \bar{e} P_L\tilde{F}^0_{2d} \tilde{e}^*_L + \bar{\tilde{F}}^0_{1d} P_L\nu \tilde{\nu}^*_R + \bar{\tilde{F}}^0_{2d} P_L e \tilde{e}^*_R)$$

$$\left. + h_{LR}(\bar{\tilde{D}}^0_L P_L \nu \tilde{\nu}_L + \bar{\nu} P_R \tilde{D}^0_R \tilde{\nu}_R) \right\} + \text{h.c.} \tag{8.17}$$

By the same argument that led from eq. (8.6) to eq. (8.11):

$$\bar{\tilde{W}}^0_L P_L = (M_{11} \bar{\tilde{\chi}}^0_1 + M_{21} \bar{\tilde{\chi}}^0_2 + M_{31} \bar{\tilde{\chi}}^0_3 + M_{41} \bar{\tilde{\chi}}^0_4) P_L$$

$$\bar{\tilde{W}}^0_B P_L = (M_{12} \bar{\tilde{\chi}}^0_1 + M_{22} \bar{\tilde{\chi}}^0_2 + M_{32} \bar{\tilde{\chi}}^0_3 + M_{42} \bar{\tilde{\chi}}^0_4) P_L$$

$$P_L \tilde{W}^0_B = P_L (M^*_{12} \tilde{\chi}^0_1 + M^*_{22} \tilde{\chi}^0_2 + M^*_{32} \tilde{\chi}^0_3 + M^*_{42} \tilde{\chi}^0_4)$$

$$P_L \tilde{F}^0_{1u} = P_L (M^*_{13} \tilde{\chi}^0_1 + M^*_{23} \tilde{\chi}^0_2 + M^*_{33} \tilde{\chi}^0_3 + M^*_{43} \tilde{\chi}^0_4)$$

$$\bar{\tilde{F}}^0_{1u} P_L = (M_{13} \bar{\tilde{\chi}}^0_1 + M_{23} \bar{\tilde{\chi}}^0_2 + M_{33} \bar{\tilde{\chi}}^0_3 + M_{43} \bar{\tilde{\chi}}^0_4) P_L$$

$$P_L \tilde{F}^0_{2d} = P_L (M^*_{14} \tilde{\chi}^0_1 + M^*_{24} \tilde{\chi}^0_2 + M^*_{34} \tilde{\chi}^0_3 + M^*_{44} \tilde{\chi}^0_4)$$

$$\bar{\tilde{F}}^0_{2d} P_L = (M_{14} \bar{\tilde{\chi}}^0_1 + M_{24} \bar{\tilde{\chi}}^0_2 + M_{34} \bar{\tilde{\chi}}^0_3 + M_{44} \bar{\tilde{\chi}}^0_4) P_L \tag{8.18}$$

With these equations, eq. (8.17) can be written in terms of the physical fields as

$$\frac{-1}{\sqrt{2}} \Big\{ g_L (M_1 \, \bar{\tilde{\chi}}^0_1 + M_{21} \, \bar{\tilde{\chi}}^0_2 + M_{31} \, \bar{\tilde{\chi}}^0_3 + M_{41} \, \bar{\tilde{\chi}}^0_4)$$

$$- g' (M_{12} \, \bar{\tilde{\chi}}^0_1 + M_{22} \, \bar{\tilde{\chi}}^0_2 + M_{32} \, \bar{\tilde{\chi}}^0_3 + M_{42} \, \bar{\tilde{\chi}}^0_4) \Big\} \, P_L \, \nu \, \tilde{\nu}^*_L$$

$$\frac{-1}{\sqrt{2}} \, \bar{\nu} \, P_R \Big\{ g_L (M^*_{11} \, \tilde{\chi}^0_1 + M^*_{21} \, \tilde{\chi}^0_2 + M^*_{31} \, \tilde{\chi}^0_3 + M^*_{41} \, \tilde{\chi}^0_4)$$

$$- g' (M^*_{12} \, \tilde{\chi}^0_1 + M^*_{22} \, \tilde{\chi}^0_2 + M^*_{32} \, \tilde{\chi}^0_3 + M^*_{42} \, \tilde{\chi}^0_4) \Big\} \, \tilde{\nu}_L$$

$$+ \frac{1}{\sqrt{2}} \Big\{ g_L (M_{11} \, \bar{\tilde{\chi}}^0_1 + M_{21} \, \bar{\tilde{\chi}}^0_2 + M_{31} \, \bar{\tilde{\chi}}^0_3 + M_{41} \, \bar{\tilde{\chi}}^0_4)$$

$$+ g' (M_{12} \, \bar{\tilde{\chi}}^0_1 + M_{22} \, \bar{\tilde{\chi}}^0_2 + M_{32} \, \bar{\tilde{\chi}}^0_3 + M_{42} \, \bar{\tilde{\chi}}^0_4) \Big\} \, P_L \, e \, \tilde{e}^*_L$$

$$+ \frac{1}{\sqrt{2}} \, \bar{e} \, P_R \Big\{ g_L (M^*_{11} \, \tilde{\chi}^0_1 + M^*_{21} \, \tilde{\chi}^0_2 + M^*_{31} \, \tilde{\chi}^0_3 + M^*_{41} \, \tilde{\chi}^0_4)$$

$$+ g' (M^*_{12} \, \tilde{\chi}^0_1 + M^*_{22} \, \tilde{\chi}^0_2 + M^*_{32} \, \tilde{\chi}^0_3 + M^*_{42} \, \tilde{\chi}^0_4) \Big\} \, \tilde{e}_L$$

$$- \frac{1}{\sqrt{2}} \, g' \, \bar{\nu} \, P_L \, (M^*_{12} \, \tilde{\chi}^0_1 + M^*_{22} \, \tilde{\chi}^0_2 + M^*_{32} \, \tilde{\chi}^0_3 + M^*_{42} \, \tilde{\chi}^0_4) \, \tilde{\nu}^*_R$$

$$- \frac{1}{\sqrt{2}} \, g' (M_{12} \, \bar{\tilde{\chi}}^0_1 + M_{22} \, \bar{\tilde{\chi}}^0_2 + M_{32} \, \bar{\tilde{\chi}}^0_3 + M_{42} \, \bar{\tilde{\chi}}^0_4) \, P_R \, \nu \, \tilde{\nu}_R$$

$$+ \frac{3}{\sqrt{2}} \, g' \, \bar{e} \, P_L \, (M^*_{12} \, \tilde{\chi}^0_1 + M^*_{22} \, \tilde{\chi}^0_2 + M^*_{32} \, \tilde{\chi}^0_3 + M^*_{42} \, \tilde{\chi}^0_4) \, \tilde{e}^*_R$$

$$+ \sqrt{\tfrac{3}{2}}\, g' (M_{12} \bar{\tilde{\chi}}_1^0 + M_{22} \bar{\tilde{\chi}}_2^0 + M_{32} \bar{\tilde{\chi}}_3^0 + M_{42} \bar{\tilde{\chi}}_4^0)\ P_R\, e\, \tilde{e}_R$$

$$+ h_u^L \{ \bar{\nu}\, P_L\, (M_{13}^* \tilde{\chi}_1^0 + M_{23}^* \tilde{\chi}_2^0 + M_{33}^* \tilde{\chi}_3^0 + M_{43}^* \tilde{\chi}_4^0)\ \tilde{\nu}_L^*$$

$$+ (M_{13} \bar{\tilde{\chi}}_1^0 + M_{23} \bar{\tilde{\chi}}_2^0 + M_{33} \bar{\tilde{\chi}}_3^0 + M_{43} \bar{\tilde{\chi}}_4^0)\ P_R\, \nu\, \tilde{\nu}_L$$

$$+ \bar{e}\, P_L\, \tilde{F}_{2u}^0\, \tilde{e}_L^* + \bar{\tilde{F}}_{2u}^0 P_R\, e\, \tilde{e}_L + \bar{\tilde{F}}_{2u}^0\, P_L\, e\, \tilde{e}_R^* + \bar{e}\, P_R\, \tilde{F}_{2u}^0\, \tilde{e}_R$$

$$+ (M_{13} \bar{\tilde{\chi}}_1^0 + M_{23} \bar{\tilde{\chi}}_2^0 + M_{33} \bar{\tilde{\chi}}_3^0 + M_{43} \bar{\tilde{\chi}}_4^0)\ P_L\, \nu\, \tilde{\nu}_R^*$$

$$+ \bar{\nu}\, P_R\, (M_{13}^* \tilde{\chi}_1^0 + M_{23}^* \tilde{\chi}_2^0 + M_{33}^* \tilde{\chi}_3^0 + M_{43}^* \tilde{\chi}_4^0)\ \tilde{\nu}_R \}$$

$$+ h_d^L \{ \bar{e}\, P_L\, (M_{14}^* \tilde{\chi}_1^0 + M_{24}^* \tilde{\chi}_2^0 + M_{34}^* \tilde{\chi}_3^0 + M_{44}^* \tilde{\chi}_4^0)\ \tilde{e}_L^*$$

$$+ (M_{14} \bar{\tilde{\chi}}_1^0 + M_{24} \bar{\tilde{\chi}}_2^0 + M_{34} \bar{\tilde{\chi}}_3^0 + M_{44} \bar{\tilde{\chi}}_4^0)\ P_R\, e\, \tilde{e}_L$$

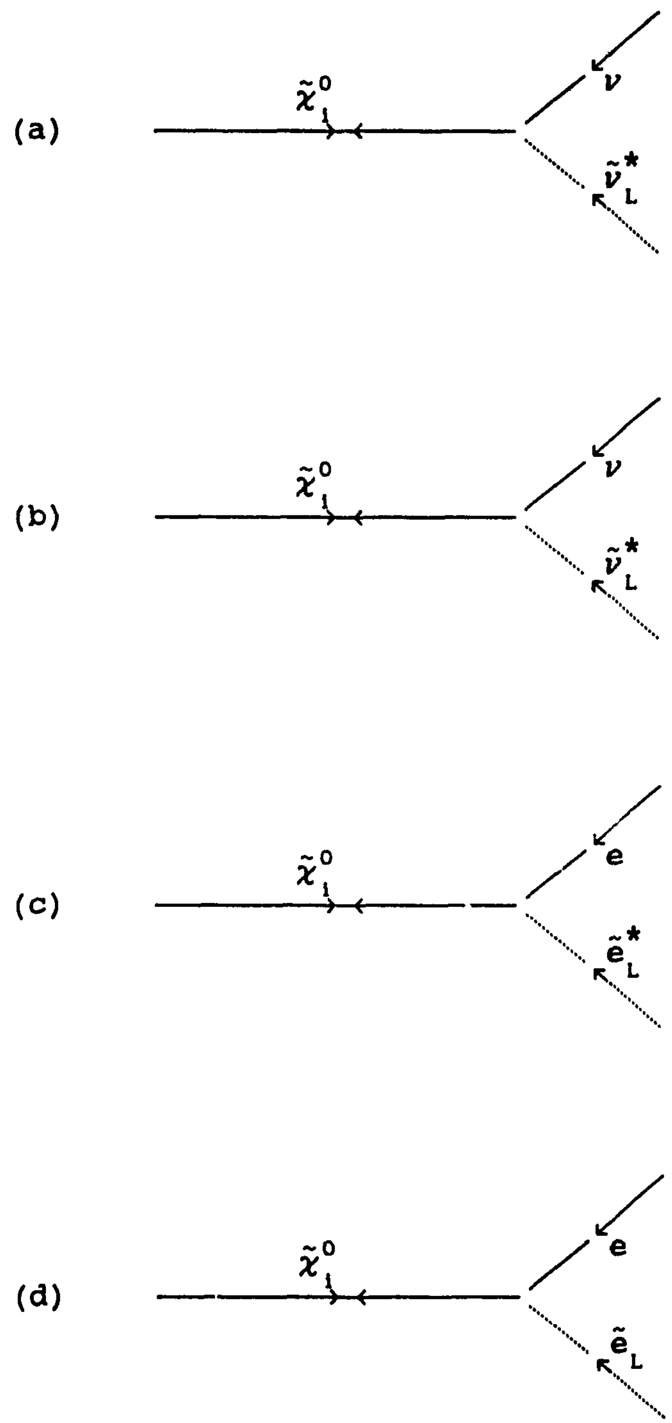
$$+ \bar{\nu}\, P_L\, \tilde{F}_{1d}^0\, \tilde{\nu}_L^* + \bar{\tilde{F}}_{1d}^0 P_R\, \nu\, \tilde{\nu}_L + \bar{\tilde{F}}_{1d}^0\, P_L\, \nu\, \tilde{\nu}_R^* + \bar{\nu}\, P_R\, \tilde{F}_{1d}^0\, \tilde{\nu}_R$$

$$+ (M_{14} \bar{\tilde{\chi}}_1^0 + M_{24} \bar{\tilde{\chi}}_2^0 + M_{34} \bar{\tilde{\chi}}_3^0 + M_{44} \bar{\tilde{\chi}}_4^0)\ P_L\, e\, \tilde{e}_R^*$$

$$+ \bar{e}\, P_R\, (M_{14}^* \tilde{\chi}_1^0 + M_{24}^* \tilde{\chi}_2^0 + M_{34}^* \tilde{\chi}_3^0 + M_{44}^* \tilde{\chi}_4^0)\ \tilde{e}_R \}$$

$$+ h_{LR} (\bar{\tilde{D}}_L^0\, P_L\, \nu\, \tilde{\nu}_L + \bar{\nu}\, P_R\, \tilde{D}_L^0\, \tilde{\nu}_L^* + \bar{\nu}\, P_R\, \tilde{D}_R^0\, \tilde{\nu}_R + \bar{\tilde{D}}_R^0\, P_L\, \nu\, \tilde{\nu}_R^*) \qquad (8.19)$$

The Feynman rules of these interactions are shown in figure 8.2 .



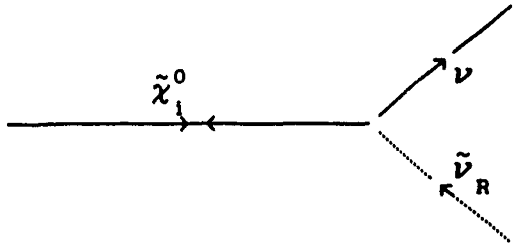
(a) $\frac{-i}{2\sqrt{2}} g_L \, M_{11} \, (1 - \gamma_5)$

(b) $\frac{i}{\sqrt{2}} g' \, M_{12} \, (1 - \gamma_5)$

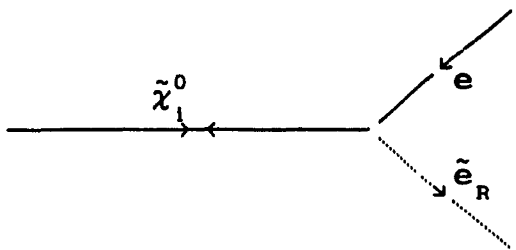
(c) $\frac{-i}{2\sqrt{2}} g_L \, M_{11} \, (1 - \gamma_5)$

(d) $\frac{-i}{2\sqrt{2}} g' \, M_{12} \, (1 - \gamma_5)$

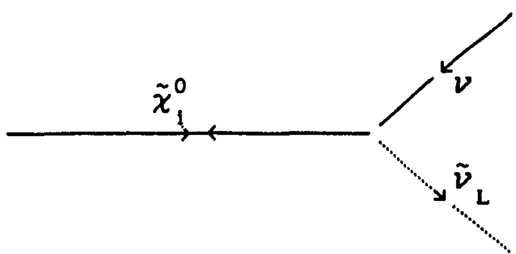
Figure 8.2

(e) 

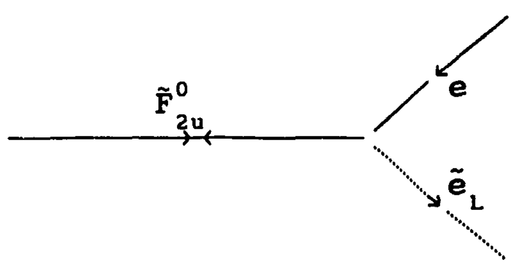
$$\frac{+i}{2\sqrt{2}}\, g'\, M_{12}\, (1 + \gamma_5)$$

(f) 

$$\frac{-3i}{2\sqrt{2}}\, g'\, M_{12}\, (1 + \gamma_5)$$

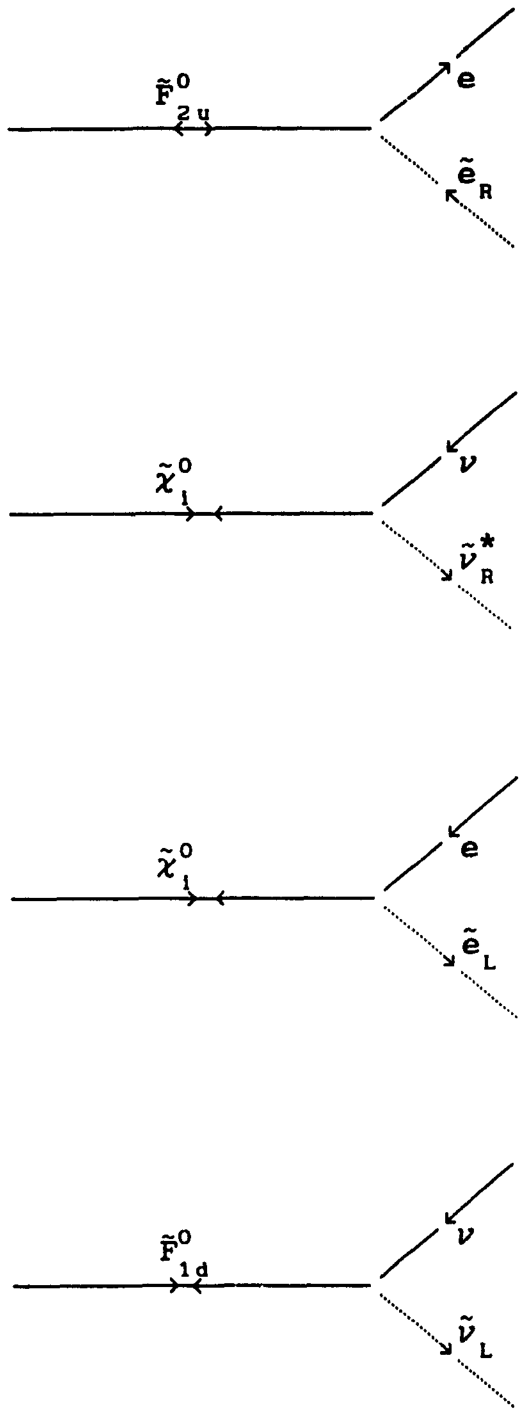
(g) 

$$-i\, h_u^L\, M_{13}\, \frac{1}{2}\, (1 + \gamma_5)$$

(h) 

$$-i\, h_u^L\, \frac{1}{2}\, (1 + \gamma_5)$$

Figure 8.2 (continued)



(i) $-i \; h_u^L \; \frac{1}{2} \; (1 + \gamma_5)$

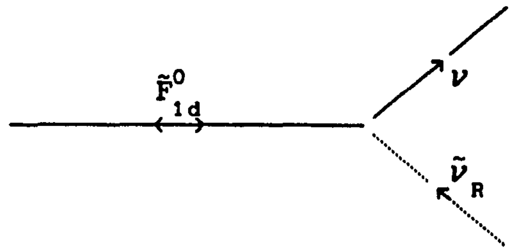
(j) $-i \; h_u^L \; M_{13} \; \frac{1}{2} \; (1 - \gamma_5)$

(k) $-i \; h_d^L \; M_{14} \; \frac{1}{2} \; (1 + \gamma_5)$

(l) $-i \; h_d^L \; \frac{1}{2} \; (1 + \gamma_5)$

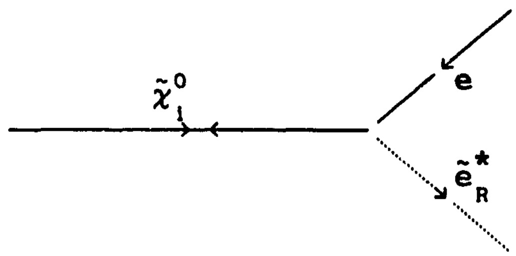
Figure 8.2 (continued)

(m)



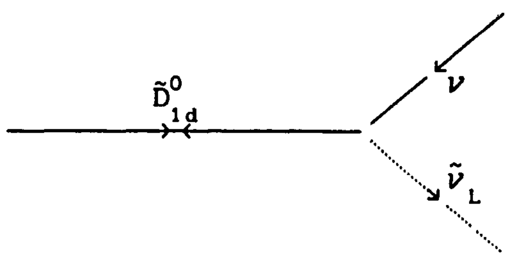
$-i\ h^L_d\ \frac{1}{2}\ (1 + \gamma_5)$

(n)



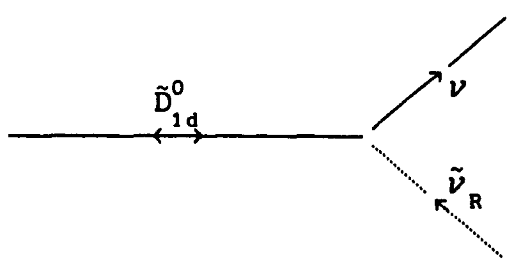
$-i\ h^L_d\ M_{14}\ \frac{1}{2}\ (1 - \gamma_5)$

(o)



$-i\ h_{LR}\ \frac{1}{2}\ (1 - \gamma_5)$

(p)



$-i\ h_{LR}\ \frac{1}{2}\ (1 + \gamma_5)$

Figure 8.2 (continued)

The arrows on the neutralino lines in fig. 8.2 point in both directions to indicate that since these particles are their own antiparticles, the direction of motion is not clearly defined.

8.3 W_R-lepton-lepton

The last of the interactions to be considered, has its roots in the following term of eq. (4.2):

$$+ L_R^\dagger \bar{\sigma}_\mu \left(\partial_\mu - \frac{ig_R}{2} \tau . W_\mu^R + \frac{ig_V}{2} V_\mu \right) L_R \tag{8.20}$$

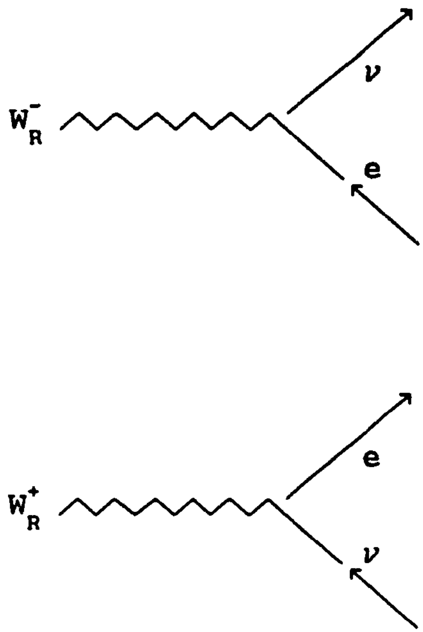
The part of this expression involving interactions between charged particles is

$$\frac{ig_R}{\sqrt{2}} \left\{ \bar{e}_R \bar{\sigma}^\mu W_\mu^- \nu_R + \bar{\nu}_R \bar{\sigma}^\mu W_\mu^+ e_R \right\} \tag{8.21}$$

Using eq. (A28), the equivalent expression in four-component notation is

$$- \frac{ig_R}{\sqrt{2}} \left\{ \bar{\nu} \gamma^\mu P_R e W_\mu^- + \bar{e} \bar{\sigma}^\mu P_R \nu W_\mu^+ \right\} \tag{8.22}$$

The Feynman rules for these interactions are given in fig.8.3



$$-\frac{ig_R}{2\sqrt{2}}\gamma^\mu(1+\gamma_5)$$

$$-\frac{ig_R}{2\sqrt{2}}\gamma^\mu(1+\gamma_5)$$

Figure 8.3

With these Feynman rules, the objectives set out for this thesis have been achieved. Their use in subjecting the model to phenomenological constraints, will be the task of a future project.

CONCLUSION

In proposing a supersymmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model, the intention is to produce a theory which retains the successful features of the GWS theory whilst at the same time providing mechanisms which may give answers to some important questions. By making the theory supersymmetric, one of the problems associated with the Higgs particle - the gauge hierarchy problem of GUT's - is circumvented. In addition, the inclusion of a right-handed gauge symmetry reinstates parity conservation in the basic theory, while its breaking leads to the required: small neutrino mass; and weak CP-violation. The fact that the new symmetries of the model make it more aesthetically appealing is, at this stage, a welcome bonus.

There remains, of course, a great deal of work to be done before the model's value can be assessed. In this thesis the initial steps of the investigation have been taken. The basic structure of the model is specified: its particle content is given; the hierarchy of VEV's of the Higgs fields and the pattern of symmetry breaking are established; and the Lagrangian for one generation of quarks, leptons, and their superpartners is written down. The gauge bosons of the theory are identified, and these are found to be in agreement with the results obtained by Mohapatra and Senjanovic. The gaugino sector is investigated, and expressions are obtained for the physical charginos and neutralinos. Finally, in order to subject the model to phenomenological constraints, the Feynman rules of some interactions involving the gauge fields are computed.

In the immediate future, the value of g-2 for the muon of the model will be calculated. This will decide whether or not further investigation is worthwhile. In the event that a positive result is obtained, a number of exciting areas of research lie ahead. Among these, the left-right symmetry of the model promises to have some interesting consequences in terms of CP-violation. Also, in common with other supersymmetric models, work on pinpointing signatures and production of superpartners will contribute to the debate on the relevance of supersymmetry in elementary particle physics.

APPENDIX A

SPINOR NOTATION AND CONVENTIONS

These are the conventions used by Haber and Kane[23]. In essence, they are the same as those used by Wess and Bagger[35], but with a different metric convention.

The metric is taken to be

$$g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1) \tag{A1}$$

The momentum four-vector is $p^\mu = (E\ ;\ \mathbf{p})$. Introducing the Pauli matrices:

$$\sigma^\mu = (1,\ \vec{\sigma}\ )\ , \qquad \bar{\sigma}^\mu = (1,\ -\vec{\sigma}\ ) \tag{A2}$$

The two-component spinor ξ_α transforms under a matrix M of SL(2,C). Similarly, the spinors $\bar{\xi}_{\dot{\alpha}}$, ξ^α , and $\bar{\xi}^{\dot{\alpha}}$ transform under M^* , M^{-1}, and $(M^{-1})^*$ respectively. In fact, it is possible to define:

$$\bar{\xi}_{\dot{\alpha}} \equiv \xi^*_\alpha\ ,\ \text{etc.}$$

The Dirac equation in two-component notation is:

$$(\bar{\sigma}_\mu\ p^\mu)^{\dot{\alpha}\beta}\ \xi_\beta = m\bar{\eta}^{\dot{\alpha}},\quad (\sigma_\mu p^\mu)_{\alpha\dot{\beta}}\ \bar{\eta}^{\dot{\beta}} = m\xi_\alpha \tag{A3}$$

This allows the introduction of four-component notation.

Usually, a four-component spinor is introduced which satisfies

$$(\gamma_\mu p^\mu - m)\psi = 0 \tag{A4}$$

It follows that

$$\psi = \begin{pmatrix} \xi_\alpha \\ \bar{\eta}^{\dot{\alpha}} \end{pmatrix}, \quad \gamma_\mu = \begin{pmatrix} 0 & \sigma_{\mu\alpha\dot{\beta}} \\ \bar{\sigma}_\mu^{\dot{\alpha}\beta} & 0 \end{pmatrix} \tag{A5}$$

$$\gamma_5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A6}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\,[\gamma^\mu, \gamma^\nu] = 2i \begin{pmatrix} \sigma_\alpha^{\mu\nu\beta} & 0 \\ 0 & \bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}} \end{pmatrix} \tag{A7}$$

where

$$\sigma_\alpha^{\mu\nu\beta} = \frac{1}{4}\,(\sigma^\mu_{\alpha\dot{\alpha}}\,\bar{\sigma}^{\nu\dot{\alpha}\beta} - \sigma^\nu_{\alpha\dot{\alpha}}\,\bar{\sigma}^{\mu\dot{\alpha}\beta})\,, \tag{A8}$$

$$\bar{\sigma}^{\mu\nu\dot{\alpha}}_{\dot{\beta}} = \frac{1}{4}\,(\bar{\sigma}^{\mu\dot{\alpha}\alpha}\,\sigma^\nu_{\alpha\dot{\beta}} - \bar{\sigma}^{\nu\dot{\alpha}\alpha}\,\sigma^\mu_{\alpha\dot{\beta}}) \tag{A9}$$

This is called the chiral representation of the γ-matrices. Define the left- and right-handed projection operators by

$$P_L \equiv \frac{1}{2}\,(1 - \gamma_5)\,, \qquad P_R \equiv \frac{1}{2}\,(1 + \gamma_5) \tag{A10}$$

Then using the notation $\psi_{L,R} \equiv P_{L,R}$, it is seen that

$$\psi \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{A11}$$

(A word of caution, here. Most books define $\sigma_\mu = (1, \vec{\sigma})$ and $\bar{\sigma}_\mu = (1, -\vec{\sigma})$ as opposed to eq. (A2). This would lead to an interchange of ψ_L and ψ_R in eq. (A11))

As usual one defines $\bar{\psi} = \psi^\dagger \gamma^0$. The charge conjugation operator C allows the definition of the charge conjugated spinor:

$$\psi^c = C\,\bar{\psi}^T \tag{A12}$$

In the chiral representation, $C = -i\gamma^2\gamma^0$. In two component notation, one defines an antisymmetric tensor ($\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}$):

$$\varepsilon^{\alpha\beta} = -\varepsilon_{\alpha\beta} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{A13}$$

The $\varepsilon_{\alpha\beta}$ can raise and lower spinor indices

$$\xi^\alpha = \varepsilon^{\alpha\beta}\,\xi_\beta\,, \qquad \xi_\beta = \varepsilon^{\beta\alpha}\,\xi^\alpha \tag{A14}$$

Identical relations hold when replacing undotted with dotted spinor indices in eqs. (A13) and (A14).

$$C = -i\gamma^2\gamma^0 = \begin{pmatrix} \varepsilon_{\beta\alpha} & 0 \\ 0 & \varepsilon^{\dot{\beta}\dot{\alpha}} \end{pmatrix} \tag{A15}$$

and

$$\bar{\Psi}^T = \begin{pmatrix} \eta^\alpha \\ \bar{\xi}_{\dot{\alpha}} \end{pmatrix} \tag{A16}$$

It follows that

$$\Psi^c = \begin{pmatrix} \eta_\beta \\ \bar{\xi}^{\dot{\beta}} \end{pmatrix} \tag{A17}$$

A four-component Majorana spinor has the property that $\eta = \xi$ which implies that $\Psi^c = \Psi$. That is,

$$\Psi_M = \begin{pmatrix} \xi_\alpha \\ \bar{\xi}^{\dot{\alpha}} \end{pmatrix} = \begin{pmatrix} \psi_L \\ i\sigma^2 \psi_L^* \end{pmatrix} \tag{A18}$$

The translation of two-component formalism into four-component formalism is completed with the following set of equations:

$$\bar{\Psi}_1 \Psi_2 = \eta_1 \xi_2 + \bar{\eta}_2 \bar{\xi}_1 \tag{A19}$$

$$\bar{\Psi}_1 \gamma_5 \Psi_2 = - \eta_1 \xi_2 + \bar{\eta}_2 \bar{\xi}_1 \tag{A20}$$

$$\bar{\Psi}_1 \gamma^\mu \Psi_2 = \bar{\xi}_1 \bar{\sigma}^\mu \xi_2 - \bar{\eta}_2 \bar{\sigma}^\mu \eta_1 \tag{A21}$$

$$\bar{\Psi}_1 \gamma^\mu \gamma_5 \Psi_2 = - \bar{\xi}_1 \bar{\sigma}^\mu \xi_2 - \bar{\eta}_2 \bar{\sigma}^\mu \eta_1 \tag{A22}$$

$$- \frac{1}{2} i \bar{\psi}_1 \sigma^{\mu\nu} \psi_2 = \eta_1 \sigma^{\mu\nu} \xi_2 + \bar{\eta}_2 \sigma^{\mu\nu} \bar{\xi}_1 \tag{A23}$$

where the subscripts 1 and 2 label two different four-component spinors and their associated two-component spinors. In eqs. (A19)-(A23), the following have been used:

$$\eta\ \xi = \eta^{a}\ \xi_{a} = \xi\ \eta \tag{A24}$$

$$\bar{\eta}\ \bar{\xi} = \bar{\eta}_{\dot{a}}\ \bar{\xi}^{\dot{a}} = \bar{\xi}\ \bar{\eta} \tag{A25}$$

$$\bar{\eta}_{2}\ \bar{\sigma}^{\mu}\ \eta_{1} = \bar{\eta}_{2\dot{a}}\ \bar{\sigma}^{\mu\dot{a}b}\ \eta_{1b} = \eta_{1}\ \bar{\sigma}^{\mu}\ \bar{\eta}_{2} \tag{A26}$$

$$\bar{\eta}\ \ \bar{\sigma}^{\mu\nu}\ \bar{\xi}\ = \bar{\eta}_{\dot{a}}\ \ \bar{\sigma}^{\mu\nu\dot{a}}{}_{\dot{b}}\ \bar{\xi}^{\dot{b}} = -\ \bar{\xi}\ \ \bar{\sigma}^{\mu\nu}\ \bar{\eta} \tag{A27a}$$

$$\eta\ \ \sigma^{\mu\nu}\ \xi\ = \eta^{a}\ \ \bar{\sigma}^{\mu\nu b}{}_{a}\ \xi_{b} = -\ \xi\ \ \sigma^{\mu\nu}\ \eta \tag{A27b}$$

In eqs. (A24)-(A27), the first equality is one of definition. The second equality follows from eqs. (A13), (A14), and the fact that spinors anticommute. Manipulations of the two-component spinors is illustrated in detail in appendix A of ref. (..). The definition of $\bar{\eta}\bar{\xi}$ has been chosen so that $(\eta\xi)^{\dagger} = \bar{\eta}\bar{\xi}$. Using eqs. (A19)-(A23), all Lagrangians written in two-component form can be converted to four component form, to satisfy the present conventions. In this respect it is particularly useful to rewrite eqs. (A19)-(A22) as follows:

$$\bar{\psi}_{1}\ P_{L}\ \psi_{2} = \eta_{1}\xi_{2}\ ,$$

$$\bar{\psi}_{1}\ P_{R}\ \psi_{2} = \bar{\eta}_{2}\bar{\xi}_{1}\ ,$$

$$\bar{\psi}_1 \gamma^\mu P_L \psi_2 = \bar{\xi}_1 \bar{\sigma}^\mu \xi_2 ,$$

$$\bar{\psi}_1 \gamma^\mu P_R \psi_2 = -\bar{\eta}_1 \bar{\sigma}^\mu \eta_2 . \qquad \text{(A28)}$$

where $P_{L,R}$ are the projection operators defined in eq. (A10). From this, one can build up four-component Dirac and Majorana spinors and interactions from a Lagrangian expressed in two component notation.

Finally, here are some useful identities which follow from eqs, (A18)-(A23). *If ψ_1 and ψ_2 are anticommuting four-component Majorana spinors, then:*

$$\bar{\psi}_1 \psi_2 = \bar{\psi}_2 \psi_1 \qquad \text{(A29)}$$

$$\bar{\psi}_1 \gamma_5 \psi_2 = \bar{\psi}_2 \gamma_5 \psi_1 \qquad \text{(A30)}$$

$$\bar{\psi}_1 \gamma_\mu \psi_2 = - \bar{\psi}_2 \gamma_\mu \psi_1 \qquad \text{(A31)}$$

$$\bar{\psi}_1 \gamma_\mu \gamma_5 \psi_2 = \bar{\psi}_2 \gamma_\mu \gamma_5 \psi_1 \qquad \text{(A32)}$$

$$\bar{\psi}_1 \sigma_{\mu\nu} \psi_2 = - \bar{\psi}_2 \sigma_{\mu\nu} \psi_1 \qquad \text{(A33)}$$

One useful consequence of the above is:

$$\bar{\psi}_1 \gamma_\mu P_L \psi_2 = - \bar{\psi}_2 \gamma_\mu P_R \psi_1 \qquad \text{(A34)}$$

APPENDIX B

MANIPULATION OF BIDOUBLETS

The bidoublets $\Phi_{u,d}$ are represented by 2 × 2 matrices, with the $(I_L^3\ ,\ I_R^3)$ quantum number distribution:

$$\begin{pmatrix} \phi_1^0[\,-\frac{1}{2}\ ,\ \frac{1}{2}\,] & \phi_1^+[\,\frac{1}{2}\ ,\ \frac{1}{2}\,] \\ \phi_2^-[\,-\frac{1}{2}\ ,\ -\frac{1}{2}\,] & \phi_2^0[\,\frac{1}{2}\ ,\ -\frac{1}{2}\,] \end{pmatrix}_{u,d} \tag{B1}$$

The operator τ_3 , when acting on the bidoublets, picks up the correct eigenvalues of I_R^3 , but those of I_L^3 have the wrong sign. In order to obtain the appropriate values of I_L^3 in operations such as the ones leading to eq.(5.12), the operator τ_3 is replaced with $-\tau_3$.

APPENDIX C

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Renormalizability as a Criterion for Choosing a Realistic Quantum Field Theory: ϕ^N Theories as an Example of the Selection Process

R.M. Francis, M.A. Hussain, and C.S. Kalman

Abstract

The theory of massive, spinless, self-interacting scalar fields is considered. The path integral derivation of Green's functions of the theory is outlined. This is done first for the "free" field and then for self-interactions of the form $\lambda\phi^N/N!$. Diagrammatic notation is introduced, and the Feynman rules for $\lambda\phi^N/N!$ are given. Divergent integrals in the calculation of scattering amplitudes are considered, and a formula for the "superficial degree of divergence" is given. The concept of renormalization is introduced, and field theories are categorized as: renormalizable, nonrenormalizable, or superrenormalizable.

Key words: renormalization of field theories, ϕ^N theories, gauge field theories, path integration, scalar field theories

1. INTRODUCTION

Quantum field theories have been developed that seem to adequately describe strong and electromagnetic interactions. Weak interactions can be described by the Weinberg-Salam theory, and naturally, there is currently a great deal of speculation about extensions to this. Many grand unified theories are presently being investigated, and presumably many more will be developed in the future. If the methods of quantum field theory are appropriate in an attempt to describe the real world, the fundamental question becomes: "Which field theory is the correct one?" For this reason, it is very useful to develop techniques and concepts applicable to any field theory. This can be done not only by considering actual theories, but also by working with simpler theories or models. These models make it easier to investigate certain aspects of the theory by freeing them from other complications.

The models that are the most frequently used are the so-called ϕ^3 and ϕ^4 theories. These theories are particularly convenient for the discussion of renormalization within perturbation theory. The presence of interactions in perturbative field theories gives rise to divergent terms in the perturbation expansion. There are two kinds of divergences: ultraviolet (UV) and infrared (IR). The IR divergences are confined to massless theories and will not be considered in this article. The UV divergences have their origin in short-distance singularities occurring in the product of field operators such as $\phi(x_1)\phi(x_2)$, as x_1 approaches x_2.

It is essential that the problems raised by divergences be satisfactorily resolved. A theory for which this is possible is called renormalizable. Physically, renormalization implies that observable quantities (e.g., masses, coupling constants, Green's functions) obtained from the perturbation solution of the theory are insensitive to its short-distance behavior. A viable theory has to be renormalizable for the simple reason that no one has yet been able to make any sense of a theory that is not. This fact may turn out to be due to an inherent deficiency in the perturbation method. On the other hand, if renormalizability is a constraint that any meaningful quantum field theory must satisfy, this criterion can be used to select a useful quantum field theory out of the whole class of potential theories. The aim of this article is to show how renormalizability can be exploited to select the ϕ^3 and ϕ^4 theories out of the whole class of the ϕ^N theories.

2. THE FREE FIELD

The theory considered deals with a massive, spinless, self-interacting scalar particle. Its Lagrangian can be split into two parts:

$$L = L_0 + L_{INT}, \tag{1}$$

where L_0 is the free Klein-Gordon Lagrangian, and L_{INT} contains the self-interaction. (In this paper we use $\hbar = c = 1$.)

In order to obtain Green's functions of the free field, it is usual to add to

L_0 a "source" term $J(x)$; i.e.,

$$L_0 = \frac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x) + J(x)\phi(x). \tag{2}$$

This technique introduced by Schwinger[1] greatly facilitates calculations. The physical reasoning behind it is that in elementary particle physics, situations are presented in which particles are created (e.g., by collision), they interact, and are then destroyed (on detection). The source term therefore accounts for the acts of creation and destruction.

Define now the transition amplitude[2]-[4] $Z_0[J]$ for the vacuum at $t = -\infty$ to develop into the vacuum at $t = +\infty$ in the presence of a source $J(x)$ as:

$$Z_0[J] = \langle 0, \infty | 0, -\infty \rangle_J$$
$$= \frac{\int D[\phi]\exp\left[i\int d^4x\left(\frac{1}{2}\right)\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x) + J(x)\phi(x)\right]}{\int D[\phi]\exp\left[i\int d^4x\left(\frac{1}{2}\right)\partial_\mu\phi(x)\partial^\mu\phi(x) - \frac{1}{2}m^2\phi^2(x)\right]} \tag{3}$$

The precise meaning of $D[\phi]$ is not clear, but Eq. (3) implies that $Z_0[J]$ is a functional integral performed on the space of functions $\phi(x)$.

Equation (3) can be restructured[4] into the more useful form:

$$Z_0[J] = \exp\left[\frac{i}{2}\int J(x_1)\Delta_F(x_1 - x_2)J(x_2)dx_1dx_2\right] \tag{4}$$

(in the above it is implied that $dx \equiv d^4x$), where $\Delta_F(x_1 - x_2)$, the Feynman propagator, satisfies

$$(\Box + m^2)\Delta_F(x_1 - x_2) = -\delta^{(4)}(x_1 - x_2) \tag{5}$$

The solution of Eq. (5) is obtained by the Fourier transformation to momentum space, giving

$$\Delta_F(x_1 - x_2) = \int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip(x_1 - x_2)}}{p^2 - m^2 + i\epsilon}. \tag{6}$$

When Eq. (4) is expanded as a functional power series, the result is

$$Z_0[J] = \sum_{n=0}^{\infty}\frac{(-i/2)^n}{n!}\int d^4x_1\ldots d^4x_n J(x_1)\ldots J(x_n)G_n(x_1\ldots x_n), \tag{7}$$

where the $G_n(x_1, \ldots, x_n)$ are Green's functions or n-point functions, which are obtained by functionally differentiating $Z_0[J]$ with respect to $J(x)$ and evaluating this at $J = 0$, i.e.,

$$G_n(x_1, \ldots, x_n) = \frac{1}{i^n}\frac{\delta^n Z_0[J]}{\delta J(x_1)\ldots\delta J(x_n)}\bigg|_{J=0}. \tag{8}$$

For this reason $Z_0[J]$ is known as the "generating functional for the free particle Green's functions." Greater insight into the meaning of Green's functions can be gained by using the definition of $Z_0[J]$ given in Eq. (3) to perform the functional differentiation. This gives

$$\frac{\delta^n Z_0[J]}{\delta J(x_1)\ldots\delta J(x_n)}\bigg|_{J=0} = i^n\langle 0|T[\phi(x_1)\ldots\phi(x_n)]|0\rangle. \tag{9}$$

Therefore,

$$G_n(x_1, \ldots, x_n) = \langle 0|T[\phi(x_1)\ldots\phi(x_n)]|0\rangle. \tag{10}$$

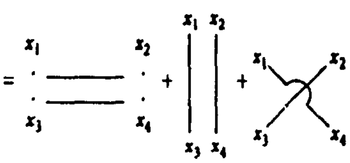
The time-ordering operator T ensures that the operators $\phi(x_1)$ to $\phi(x_n)$ are in chronological order, with the earlier times to the right of the product. The right-hand side of Eq. (10) is known as the "vacuum expectation value of the time-ordered product." Taking the two-point function as an example,

$$G_2(x_1, x_2) = \langle 0|T[\phi(x_1)\phi(x_2)]|0\rangle$$
$$= \frac{1}{i^2}\frac{\delta^2 Z_0[J]}{\delta J(x_1)\delta J(x_2)}\bigg|_{J=0} = i\Delta_F(x_1 - x_2). \tag{11}$$

Since $\Delta_F(x_1 - x_2)$ is a solution of Eq. (5), the following physical interpretation of the two-point function can be made: $G_2(x_1, x_2)$ represents the probability amplitude of a particle being created at x_1 and propagating freely to x_2 where it is destroyed. In diagrammatic notation this is equivalent to:

$$G_2(x_1, x_2) = x_1 \text{\_\_\_\_\_\_} x_2. \tag{12}$$

Green's functions corresponding to odd-valued n are identically zero, and Green's functions higher than G_2 turn out to be the sums of products of G_2; for example, the four-point function is:

$G_4(x_1, x_2, x_3, x_4) =$  (13)

3. INTERACTING FIELDS

Returning now to Eq. (1), consider a self-interaction term L_{INT} of the general form

$$L_{INT} = \sum_{i=3}^{\infty}\lambda_i[\phi(x)]^i/i!. \tag{14}$$

In Eq. (14), the coupling constant λ is a measure of the strength of the interaction, and $1/i!$ is included for combinatorial reasons. The $i = 1$ term is absent in order to ensure that the current $j(x)$ derived from the interaction has zero vacuum expectation value,[5] while the $i = 2$ term is omitted because it is already contained in L_0 (its inclusion in L_{INT} would merely change the value of m in L_0).

It is sufficient to consider a single term of Eq. (14). Set

$$L_{INT} = \frac{\lambda\phi(x)^N}{N!}. \tag{15}$$

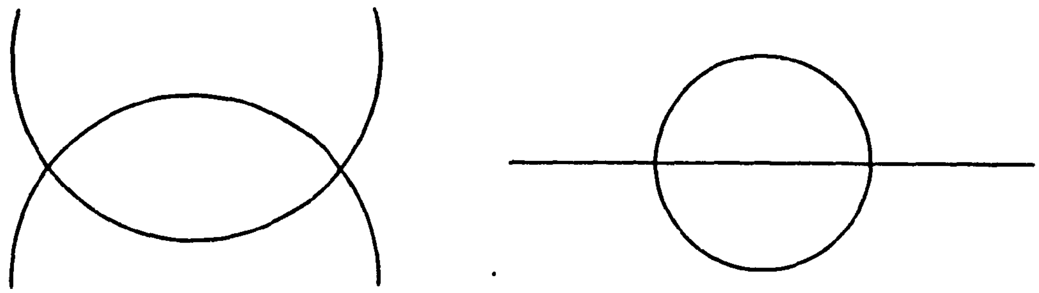
In order to derive an expression for $Z[J]$, the generating functional of Green's functions of the theory, it is useful to make the replacement[2].

$$\phi \rightarrow (1/i)(\delta/\delta J) \tag{16}$$

so that

$$L_{INT}(\phi) \rightarrow L_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J}\right).$$

The normalized generating functional now takes the form



Figures 1 (left) and 2. Second-order connected diagrams in $\lambda\phi^4/4!$ theory.

$$Z[J] = \frac{\exp\left[i\int L_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)dz\right] Z_0[J]}{\left\{\exp\left[i\int L_{INT}\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)dz\right] Z_0[J]\right\}_{J=0}}. \quad (17)$$

At this stage it is necessary to turn to perturbation theory, because the only way of treating $\exp(i\int L_{INT})$ is as a power series in the coupling constant λ.

Substituting (15) into (17) and expanding in powers of λ, the numerator of $Z[J]$ is

$$\left[1 - \frac{i\lambda}{N!}\int\left(\frac{1}{i}\frac{\delta}{\delta J(z)}\right)^N dz + O(\lambda^2)\right]Z_0[J] \quad (18)$$

To order λ^0 the free particle generating functional is recovered. To order λ and taking the specific example of $N = 4$, functionally differentiate $Z_0[J]$ with respect to $J(z)$ four times, so that (18) becomes

$$\left[1 - \frac{i\lambda}{4!}\int\left\{-3(\Delta_F(0))^2 + 6i\Delta_F(0)\left[\int\Delta_F(z-x)J(x)dx\right]^2 + \left[\int\Delta_F(z-x)J(x)dx\right]^4\right\}dz\right]Z_0[J] \quad (19)$$

Since $\Delta_F(0) = \Delta_F(x - x)$, it can be represented diagrammatically as a line that begins and ends at the same point; i.e. a loop

$$\Delta_F(0) \rightarrow \text{O}.$$

Similarly,

$$[\Delta_F(0)]^2 \rightarrow \text{OO}.$$

Using diagrammatic notation to express (19), write

$$(19) \rightarrow \left[1 - \frac{i\lambda}{4!}\int\left(-3\,\text{OO} + 6i\,\underline{\text{O}} + \text{X}\right)dz\right]Z_0[J]. \quad (20)$$

The denominator of (17) becomes

$$1 - \frac{i\lambda}{4!}\int\left(-3\,\text{OO}\right)dz. \quad (21)$$

When (20) is divided by (21) to obtain the full expression for $Z[J]$ to order λ, the vacuum graphs disappear, leaving

$$Z[J] = \left[1 - \frac{i\lambda}{4!}\int\left(6i\,\underline{\text{O}} + \text{X}\right)dz\right]Z_0[J]. \quad (22)$$

In $\lambda\phi^4/4!$ theory four lines meet at every vertex. All the terms generated by this self-interaction have as their basic feature the graph X. The actual graph corresponding to a particular term is then obtained by considering the number of sources $J(x)$ it contains; this, in turn, gives the number of external legs attached to the graph, i.e.,

OO : has no sources; no external lines;

$\underline{\text{O}}$: has two sources, two external lines;

X : has four sources; four external lines

The numerical coefficients (3, 6, 1) in Eqs (19) and (20) are known as "symmetry factors." They can be calculated by counting the number of different ways in which the basic vertex X can be transformed into the particular graph of the term.

Having obtained $Z[J]$, Green's functions of the theory can be calculated Following are some examples:

$$G_2(x_1, x_2) = -\left.\frac{\delta^2 Z[J]}{\delta J(x_1)\delta J(x_2)}\right|_{J=0}$$

$$= i\,.\underline{\qquad}. - \frac{\lambda}{2}\cdot\underline{\text{O}}. + O(\lambda^2)$$

$$G_4(x_1, x_2, x_3, x_4) = \left.\frac{\delta^4 Z[J]}{\delta J(x_1)\delta J(x_2)\delta J(x_3)\delta J(x_4)}\right|_{J=0}.$$

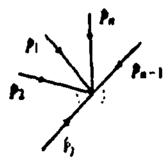
The order λ contribution to this function is

$$\frac{-i\lambda}{4!}\left[12 \times 6\left(\underline{\text{O}}\ \overline{\underline{\qquad}}\right) + 24(\text{X})\right]. \quad (23)$$

Of the two graphs in (23), the first does not contain an interaction (or scattering) between the two particles created, whereas the second does. The first graph is referred to as a "disconnected" diagram and the second as a "connected" diagram. Since connected diagrams are of concern, it is convenient to define a new generating functional, which generates only connected diagrams This functional is denoted by $W[J]$ and is defined by:

$$W[J] = -i\ln Z[J] \quad (24)$$

Table I: Feynman rules for $\lambda\phi^N/N!$ theory.

	Diagram element	*Multiplicative factor*
jth external line		$\frac{i}{p_j^2 - m^2 + i\epsilon}$
lth internal line		$\frac{i}{k_l^2 - m^2 + i\epsilon}$
lth loop integration		$\int \frac{d^4 k_l}{(2\pi)^4}$
Vertex		$-i\lambda\delta\left(\sum_{j=1}^{n} p_j\right)$
Symmetry factor		$S/N!$

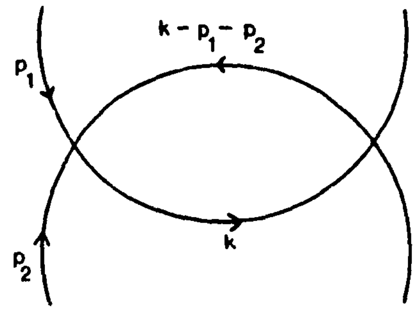


Figure 3. Loop in $\lambda\phi^4/4!$ theory.

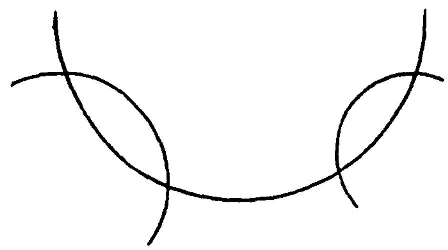


Figure 4 "Superficially" convergent diagram

Going to higher orders in λ increases the number of vertices in each diagram; to order λ^j there are j vertices in each diagram. Examples of second-order connected diagrams in $\lambda\phi^4/4!$ are shown in Figs 1 and 2

4. FEYNMAN RULES FOR $\lambda\phi^N/N!$

As seen in the previous section, diagrammatic representation (Feynman diagrams) is a much more direct and convenient way of writing Green's functions. Listed in Table 1 are the momentum space Feynman rules for $\lambda\phi^N/N!$ theory.

The momentum space Green's functions $G_n(p_1, \ldots, p_n)$ can be found by drawing all possible, topologically distinct diagrams and then assigning multiplicative factors to the various elements of each diagram

4.1 Divergences and Renormalization

Attempts to calculate scattering amplitudes from Feynman diagrams soon lead to divergent integrals Consider the diagram in Fig. 3; the loop contribution of this diagram is proportional to

$$\frac{\lambda^2}{(2\pi)^8} \int \frac{d^4k}{[k^2 - m^2 + i\epsilon][(k - p_1 - p_2)^2 - m^2 + i\epsilon]}. \quad (25)$$

This expression diverges for large-loop momenta (i.e., as $k \to \infty$) In coordinate space, large-loop momenta are equivalent to short distances, so it is $\Delta_F(0)$ that is responsible for ultraviolet divergences. The integral (25) contains four powers of k in the numerator (in the measure) and four powers of k in the denominator, thus making it "logarithmically" divergent. The only other source of divergent integrals in ϕ^4 theory is the two-point function; i.e.,

$$\rightarrow \frac{\lambda}{(2\pi)^4} \int \frac{d^4k}{k^2 - m^2 + i\epsilon}. \quad (26)$$

This integral with four powers of k in the numerator and only two in the denominator is even more divergent (quadratically divergent)

The power counting that has been done in these two examples can be generalized to determine the "superficial" degree of divergence D of any Feynman diagram for any $\lambda\phi^N/N!$ theory. A formula for D is fairly easily obtained[4] in terms of V, the number of vertices, N, as in $\lambda\phi^N$; E, the number of external lines; d, the number of space-time dimensions (which for this purpose becomes a variable, making the measure of the integral d^dk). The formula for D is

$$D = d - (E/2)(d - 2) + V[(d/2)(N - 2) - N] \quad (27)$$

For the case of $N = 4$ and $d = 4$, (27) reduces to $D = 4 - E$.

The four-point function has $E = 4$, giving $D = 0$ (logarithmic divergence), and the two-point function has $E = 2$, giving $D = 2$ (quadratic divergence) This confirms the results that were obtained previously for (25) and (26) and shows that $\tilde{G}_2$ and $\tilde{G}_4$ are the only intrinsically divergent functions in ϕ^4 theory. Having said this, it does not mean that higher Green's functions ($\tilde{G}_6, \tilde{G}_8 \ldots$) in which D is negative, converge. Consider the $\tilde{G}_6$ diagram shown in Fig. 4. It has $D = -2$ but clearly diverges because of the two divergent loop integrations. This is why D is called the "superficial" degree of divergence. However, the divergence of the expression represented in Fig 4 is due to the presence of two, hidden, four-point functions. In fact, the divergence of any Green's function, from G_6 up, can be traced to the presence of hidden two- and four-point functions, which for this reason are referred to as "primitive" divergences.

In contrast to the limited sources of divergence in ϕ^4 theory, in four space-time dimensions, D for $\phi^6 = 4 - E + 2V$. The divergence of Feynman diagrams in ϕ^6 depends on V and, therefore, increases with increasing order in the perturbation expansion. This means that there is an infinity of different types of divergent terms

The apparent disaster of divergences in calculations of Green's functions is not, however, the end of the story. This is where renormalization enters the scene. There is more than one way to renormalize a theory, but a widely used technique is the method of counterterms. This involves the redefinition of the Lagrangian by the inclusion of terms of the same form as those that give rise to the divergences. The effect of these divergent counterterms is to cancel out, order by order in the perturbation expansion, the original divergences in the theory. As a result, a renormalized theory will contain redefinitions of physical observables, such as mass and coupling constant The original value of, say, mass is the one that would be observed in the absence of interaction. But in the absence of interaction there is no observation. So the finite, renormalized mass is regarded as the physical value, whereas the original infinite mass is unphysical

Although the process of renormalization can be applied to any field theory, those in which the number of different types of divergent terms is limited (i.e., in which the number of primitive divergences is finite, as in ϕ^4) are called renormalizable theories

By the application of formula (27) field theories can be categorized as: 1) superrenormalizable, 2) renormalizable, and 3) nonrenormalizable

Table II: Renormalizability properties of $\lambda\phi^N/N!$ theories corresponding to different numbers of space-time dimensions.

N	*Dimensions in which theory is:*		
	Superrenormalizable	*Renormalizable*	*Nonrenormalizable*
3	2, 3, 4, 5	6	$d > 6$
4	2, 3	4	$d > 4$
>4	2	...	$d > 2$

Superrenormalizable applies to theories for which D decreases with increasing V. In Table II the renormalizability properties of $\lambda\phi^N/N!$ theories are listed corresponding to different numbers of space-time dimensions.

6. DISCUSSION

In the introduction, the importance of renormalizability for the viability of a field theory was stressed. Among ϕ^N theories it is seen that only two are renormalizable. ϕ^4 in four space-time dimensions and ϕ^3 in six space-time dimensions. In addition, ϕ^3 is superrenormalizable in four space-time dimensions There is, however, a serious problem with this theory: Because of the ϕ^3 interaction, the energy is not bounded from below, implying that any state must catastrophically decay. In view of this, ϕ^4 stands out as a very useful model in quantum field theory

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Résumé

Les auteurs examinent la théorie de champs scalaires massifs, dépourvus de spin et auto-interactifs On décrit brièvement la dérivation sur la trajectoire intégrale des fonctions de Green. Pour ce faire, on considère d'abord un champ "libre," puis les auto-interactions de la forme $\lambda\phi^N/N!$ Une notation de graphe est introduite, et les règles de Feynman pour $\lambda\phi^N/N!$ sont données. Des intégrales divergentes sont retenues dans le calcul des amplitudes de diffusion, et une formule applicable au "degré de divergence superficiel" est donnée. Le concept de renormalisation est introduit, et les théories de champ sont caractérisées comme renormalisables, non renormalisables, ou super renormalisables.

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