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**HEDGING EFFECTIVENESS AND PRICING OF  
STOCK INDEX FUTURES IN THE PRESENCE OF INDEX  
PARTICIPATION UNITS**

Sami Akkaoui

A Thesis in the Faculty of Commerce and Administration

Presented in Partial Fulfillment of the Requirements  
for the Degree of Master of Science in Administration  
at  
Concordia University  
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## ABSTRACT

### HEDGING EFFECTIVENESS AND PRICING OF STOCK INDEX FUTURES IN THE PRESENCE OF INDEX PARTICIPATION UNITS

Sami Akkaoui

The problem with stock index arbitrage is that arbitrageurs have to duplicate the index in equal proportions in order to capitalize on any market disequilibrium. If the index is comprised of many stocks, it is very difficult to take advantage of such opportunities in a timely manner in terms of executing orders and transaction costs. Arbitrage activities should increase market efficiency and increase hedging effectiveness.

To increase Hedging effectiveness with stock index futures, the correlation between the cash and futures market has to approach 1 so that basis risk is reduced. Therefore, the presence of index participation units that track the market must increase hedging effectiveness and reduce any mispricing in stock index futures.

Hedging performance of a portfolio comprised of a long position in the Toronto 35 Index and a short position in the Toronto 35 Index Futures for the period before and after TIPs was introduced was evaluated. It was deduced that the hedging performance of

this portfolio resulted in better hedging results in the period before TIPs was introduced than in the period when TIPs was traded. In addition, when comparing the T35 hedging effectiveness to a control group in the U.S., namely, the S&P 500 and the MMI, It was discovered that the U.S. indexes resulted in better hedging performance.

On the other hand, TIPs seems to have reduced the mispricing between actual and theoretical futures prices. By measuring the standard error of the regression of the theoretical price on the actual price, the standard error was significantly reduced in the time period when TIPs was traded. Statistics on the difference between actual and theoretical futures price was computed for the time periods before and after TIPs was introduced.

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## INTRODUCTION

The enhanced participation of institutional investors in equity markets has increased the popularity of "basket trading." A basket is simply a portfolio of stocks that moves with general market trends. Buying and selling baskets of stocks can be expensive in terms of both the time and cost to execute all the trades for each individual holding. In response to the need for a simple product that enables all investors, large and small, to easily and quickly participate in the Canadian market, the Toronto Stock Exchange (TSE) has developed the Toronto 35 Index Participation Units (TIPs) in March 1990. Each TIPs unit represents an interest in a trust that holds baskets of the stocks in the Toronto 35 Index, T35. Thus, TIPs are backed by the underlying stocks and may be redeemable for the underlying basket of stocks at any time.<sup>1</sup>

From their inception, TIPs have attracted significant trading volume, and have been deemed a great marketing success by the TSE. Currently TIPs enjoy the status of being one of the most actively traded shares on the TSE. The substantial demand for TIPs can be contributed to several features that make TIPs an easy tool to track the market movement while avoiding the costs associated with similar products on the market index.

Index mutual funds may not have complete freedom of trading and can impose management fees. Index futures, which do not include dividends of the underlying shares<sup>2</sup>

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<sup>1</sup> Investors holding a minimum of the prescribed number of TIPs, may redeem their TIPs for the underlying basket of stocks at any time. The prescribed number of TIPs is currently about 51,244 units worth approximately \$ 1 million, based on stock market values at February 1992.

<sup>2</sup> TIPs holders, do not pay management fees and receive dividends paid by the 35 underlying stocks. The distribution of dividends is based on a quarterly basis. Thus, TIPs holders miss the interest earned by the interim dividends held by the trust. Also, reinvestment of dividends in

are subject to margin requirements, marking to market, and inaccurate tracking of current market movements. It should be interesting to examine the nature of the structural change experienced by the futures market as a consequence of the introduction of TIPs, which provide a relatively attractive alternative to market indexing.

The objective of this thesis is to investigate whether the introduction of the Toronto Participation Units (TIPs) have made the Toronto futures market more efficient. In essence this research will investigate this problem by looking at the hedging effectiveness and the mispricing of futures contracts. I will follow the methodology presented by Figlewski (1984)<sup>3</sup> to evaluate the hedging effectiveness of the Canadian market as compared to its U.S. counterpart. Figlewski investigated the hedging performance of five stock indexes, namely, S&P 500, NYSE, AMEX, OTC, and DOW by selling the nearest to expiration S&P 500 stock index futures against each index and rolling over to the next contract when the first nearest expires. Figlewski compared an unhedged strategy to a hedged one by using two different hedge ratios; the minimum risk hedge,  $h$ , and the beta of the portfolio,  $\beta$ , as hedge ratios. He concluded that a hedged portfolio was successful in reducing risk. In addition, when the portfolio betas were used as the hedge ratios, hedge performance deteriorated. In all cases, beta hedges were dominated by the minimum risk hedges, which had both lower risk and higher return. In the second part of his paper, Figlewski tested the components of basis risk and concluded

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new units, possible with the conventional funds, would be difficult because of TIPs' closed-end status.

<sup>3</sup> Stephen Figlewski, "Hedging Performance and Basis Risk in Stock Index Futures", *The Journal of Finance*, Vol. XXXIX, NO. 3, July 1984



that basis risk was not negligible even when the cash portfolio being hedged was the underlying index portfolio itself. He found dividend risk to be marginal and attested that basis risk rises in general due to imperfect relationship between the prices of the cash and futures market. In this study, minor modifications will be applied to Figlewski's methodology in order to take into account the presence of TIPs.

Arbitrage activity plays an integral role in market efficiency. For hedging to be effective, the market has to be efficient. Thus, with stock index futures, the existence of any discrepancy between actual and theoretical futures prices will enhance arbitrageurs' behavior. The problem of arbitrage in stock index futures is that arbitrageurs need to duplicate the index in identical proportions and engage in buy or sell orders in a timely manner. Executing buy or sell orders in stock index futures in a timely manner is easier said than done because of time delays in processing orders, transaction costs, and restrictions on short sales. Therefore, in theory, the existence of participation units that track an index (market portfolio) and which can be sold and bought like a share must enhance arbitrage opportunity due to the ease in executing orders and the lower transaction costs involved, i.e., no need to duplicate the individual stocks of an index in their identical proportions when you can buy a share that represent the underlying stocks. Restrictions on short sells will only affect small investors and not institutional investors. Thus, a participation unit must increase arbitrage activity which in turn increases market efficiency and hedging effectiveness

On the other hand, it can be argued that speculators and hedgers could be

expected to see TIPs as an alternative to the Toronto 35 Index Futures Contracts, TXF. The resulting thin trading in the TXF should reduce pricing efficiency of the TXF and hedging effectiveness. But if TIPs indeed make it easy to track the market movement and to arbitrage away any discrepancy between the index and the index products, trading in futures should become more active. With increased activity, the index futures markets may exhibit more efficiency in revealing price information

The objective of this thesis is twofold. First, I will compare the hedging effectiveness in the Canadian market by evaluating the risk-return characteristics of a portfolio comprised of being long in the T35 and short in the Toronto 35 Index futures Contracts, TXF, to another portfolio comprised of a long position in TIPs and a short position in the TXF. In addition, the hedging effectiveness of the T35 will be compared to a control group in the U.S. This control group will be made up of two portfolios, namely, the first portfolio consist of being long in the S&P 500 Index and short in the S&P 500 futures contracts and the second portfolio consists of being long in the Major Market Index (MMI) and short in the Major Market Futures contracts. The reason for this analogy is to compare hedging effectiveness of a country where an index participation unit is traded (Canada) to another country where no index participation unit is traded (U.S.).

By comparing the hedging effectiveness of the T35 to the TIPs, I should be able to draw an analogy between an index and a participation unit. Furthermore, by considering the U.S. market, it will be interesting to test if the existence of a participation unit adds to hedging effectiveness. Like Figlewski, I will be comparing the hedging performance of

unhedged and hedged portfolios using the minimum risk hedge. Risk-return combinations will be reported for the full sample period and for the period before and after TIPs was introduced. One major difference between this research and the one conducted by Figlewski is that the cash index will at all times be the stock index and it will be hedged with its futures index counterpart. This should eliminate one component of basis risk which Figlewski referred to as nonmarket risk. A cross hedge portfolio will be constructed by hedging the MMI with the S&P 500 futures contracts in order to show the significance of basis risk in a cross hedge. Therefore, I will be comparing first the risk-return combination of the T35 to TIPs. Then, hedging effectiveness of the T35 will be evaluated in the period before and after TIPs was introduced. For a hedge to be attractive, basis risk must be less than price risk. Nonetheless, if basis risk is decreased, then the fluctuation between cash and futures prices will narrow and the result is better hedging, thus hedging will be more effective. Therefore, basis risk for the three time periods will be calculated in an effort to draw analogy and measure its significance to hedging effectiveness. Cause of basis risk, namely, nonmarket and dividend risk will be discussed. In addition, the dividend component of basis risk will also be tested for significance.

In the second part of my thesis, the mispricing of futures contracts will be evaluated in order to test if the presence of TIPs mispricing of futures contracts was decreased and the efficiency of futures market was increased. I will utilize the methodology presented by Peters<sup>4</sup> in evaluating the increase in efficiency. The same kind

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<sup>4</sup> Peters, Ed. "The growing efficiency of index futures markets". The Journal of Portfolio

of analysis will be conducted for the U.S. contracts so that an analogy can be drawn between the two markets because at the time this research was conducted no participation units were being traded in the U S.<sup>5</sup>

Therefore, this research examines whether participation units have added to the hedging effectiveness and to the efficiency of stock index futures in the Canadian market. A number of results will be reported about the effectiveness of TIPs in measuring hedging performance and mispricing of stock index futures. Chapter 1 presents the methodology of the cost of carry model which prices futures contracts and discusses the cash-and-carry and reverse cash-and carry strategies. In addition, the no arbitrage zone which is bound by an upper and lower limits is mentioned. Chapter 2 examines hedging, basis risk, and the hedge ratio. Chapter 3 presents stock index futures. Chapter 4 discusses the trading activity of TIPs and Index Futures. Chapter 5 provides a description of the data and the hedging performance methodology. Chapter 6 presents and tests the assumptions that underline the statistical theory in regression analysis. Chapter 7 presents the data and the mispricing methodology to test the efficiency of futures markets. The final section summarizes the results and suggests the implications of participating units on stock index futures.

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Management, summer 1985.

<sup>5</sup> In January 1993, Standard & Poor's Depository Receipts, SPDR, was introduced on the American Stock Exchange.

## CHAPTER 1

### PRICING of FUTURES CONTRACTS

#### I. PRICING OF FUTURES CONTRACTS

The pricing of futures contracts should be done in such a way that arbitrage opportunity does not arise. Arbitrage is a transaction(s) that leads to a riskless profit without cash outlay and it should not exist if markets are efficient. On the other hand, if arbitrage possibilities exist then arbitrageurs will exploit them and in the process prices will adjust to the mispricing mechanism until equilibrium is reached. At equilibrium, no arbitrage opportunities exist. The cost of carry model is used to price futures contracts in order to ensure that arbitrage opportunities do not exist.

#### II. THE COST OF CARRY MODEL

Under perfect market conditions which is characterized by the absence of transaction costs, equal borrowing and lending rates, and no restrictions on short selling, the cost of carry model can be used to price futures contracts and is defined as

Futures price = Spot price + Carry cost - Carry return

or simply,

$$F_{0,T} = S_0 + CC - CR \qquad 1.1$$

where:

$F_{0,T}$  = Futures price at time,  $t=0$  for delivery at time,  $t=T$

$S_0$  = Spot price at time  $t=0$

CC = Carry costs

CR = Future value of dividends

The spot price is the cash price of the asset. The carry costs, CC, are the costs incurred in holding the asset. Some of these costs can be related to the interest charged on borrowed funds to buy the asset, insurance and transportation costs, and storage fees. For stock index futures these costs are limited to the interest rate costs. The carry return applies only to financial futures that earn return. This is the future value of cash inflows. For example, in the case of stock index futures it includes the dividends payments or in the case of Treasury bonds it includes accrued interest. Therefore, in theory the futures price should be higher than the spot price by the carry costs minus the carry return. If the actual futures price is different than the theoretical price then an arbitrage opportunity exists; one strategy is known as cash-and-carry arbitrage and the other is known as reverse cash-and-carry. When the theoretical futures price is less than the actual price, a cash-and-carry arbitrage should be performed by borrowing funds, buying the cash asset and selling futures. Note that since the theoretical price is less than the actual price, the market anticipates that the futures price will decrease and thus traders sell futures. This situation will continue until arbitrageurs have exploited the price difference and no more price discrepancy persists. A reverse cash-and-carry situation arises when the theoretical

price of futures is higher than the actual price. In this situation, the strategy will be to short sell the cash asset, lend the proceeds and buy futures contract. Arbitrageurs will exploit this price difference because they anticipate the actual price will appreciate and thus buy futures contract. This arbitrage opportunity will continue until the price difference is anticipated.

In real markets, market imperfections will complicate matters and disturb the equality of the cost of carry model. Traders face transaction costs, restrictions on short selling, unequal borrowing and lending rates, and bid-ask spread. Let  $T$  be a percentage charged by the broker to carry out the transaction, let  $C_B$  and  $C_L$  be the borrowing and lending rates respectively, let  $f$  represent the unusable amount deprived from the short sell (amount held by the broker), and let  $S_B$  and  $S_A$  be the spot bid and ask prices. When a cash-and-carry strategy is involved traders borrow funds at the current borrowing rate,  $C_B$ , buy the spot asset at the asked price,  $S_A$ , and sell futures contract.<sup>6</sup> Note that the amount they borrow is equivalent to the spot asked price plus the transaction cost,  $T$ . Therefore, the no arbitrage cash-and-carry model under imperfect market conditions is:

$$F_{0,1} \leq S_A(1+T)(1+C_B) \quad 1.2$$

On the other hand, a reverse-cash-and-carry strategy requires that traders short sell the cash asset at  $S_B$  and receive only the short sell amount minus the transaction cost, lend the proceeds at current lending rates,  $C_L$ , and buy futures contract.

Mathematically, the no arbitrage reverse cash-and-carry under market imperfections is:

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<sup>6</sup> Kolb, W.R., "Understanding Futures Markets", Kolb Publishing Company, 1991.

$$F_{0,T} \geq S_B(1-T)(1+C_L) \quad 1.3$$

If there are restrictions on short selling, then traders will receive only  $fS_B$ , where  $f$  has values between zero and 1. If the value of  $f$  equals 1 implies no restrictions on short sells. The more the  $f$  value approaches zero the less is the amount received by the trader from the short sell process. Equation 1.3 will then be modified to

$$F_{0,T} \geq fS_B(1-T)(1+C_L) \quad 1.4$$

Combining equations 1.2 and 1.4 gives

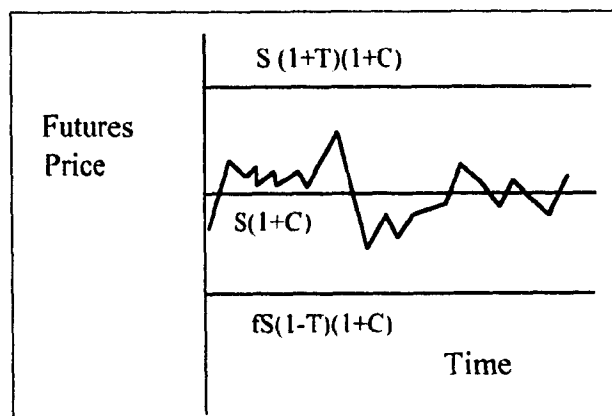
$$fS_B(1-T)(1+C_L) \leq F_{0,T} \leq S_A(1+T)(1+C_B) \quad 1.5$$

Equation 1.5 defines the no arbitrage boundary. This boundary is bounded by upper and lower limits where the futures prices can fluctuate without causing arbitrage opportunities. If futures prices goes above the upper limit, there will be cash-and-carry arbitrage opportunity. This occurs when  $F_{0,T} > S_A(1+T)(1+C_B)$ . Whereas, if the futures price drops below the lower limit, a reverse-cash-and-carry arbitrage will result. This situation is when  $F_{0,T} < fS_B(1-T)(1+C_L)$ . In perfect markets the no arbitrage condition is shown by the middle line in Figure 1.1.<sup>7</sup> In a perfect market the futures price is higher than the spot price by only the carry costs. Notice that as the  $T$  value increases, the  $f$  value approaches zero, and the bid-ask spread widens the band gets wider and arbitrage opportunity vanishes.

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Figure 1.1 shows the no arbitrage bounds by assuming no bid-ask spread.



**Figure 1.1:** The no arbitrage bounds

## **CHAPTER 2**

### **HEDGING, BASIS RISK, and HEDGE RATIO**

#### **I. HEDGING**

Prices today are known with certainty, but future prices are unknown and are engulfed with uncertainty. Hedgers like to remove their risk exposure to price volatility. The futures market can be used by traders as a hedging tool. Typically, hedgers take long or short positions in the futures market in order to hedge a cash position. The hope here is that any loss in the cash market will be offset by gains in the futures market and vice versa. A short hedge (selling hedge) occurs when a firm sells futures to hedge the assets it produces. The objective of this strategy is to reduce losses in the cash market due to price decline. Thus, if the cash position incurs losses it should be offset by gains in the short futures position. A long hedge (buying hedge) is the situation when a consumer of a product buys futures to hedge the assets it uses. The primary purpose is to protect oneself against price increases. When futures are involved, long or short hedgers lock in future prices to avoid price risk.

#### **II. BASIS RISK**

Hedgers replace price risk with basis risk. Basis is defined as cash price minus the futures price. If futures and cash prices always change by the same amount the basis will not change. In this case, any loss in the cash position will be totally offset by a gain in the futures position. This hedge is known as a perfect hedge because it eliminates price risk.

On the other hand, if the basis does change and the hedger can predict the change accurately, the hedge can be used to eliminate price risk. Thus, when constructing a hedge, a common strategy is to minimize basis risk so that the outcome of the hedge will be more reliable. In order to illustrate the difference between price risk and basis risk, it is important to start from the definition of basis. Basis (B) is the difference between cash price (S) and the futures price (F):

$$B_{t,T} = S_t - F_{t,T} \quad 2.6$$

A change in the basis will be defined as

$$\Delta B_{t,T} = \Delta S_t - \Delta F_{t,T}^8 \quad 2.7$$

The change in the basis will be zero if the change in the cash and futures prices are equal, therefore, when

$$\Delta S_t = \Delta F_{t,T} \quad 2.8$$

the change in the basis is

$$\Delta B_{t,T} = \Delta S_t - \Delta F_{t,T} = 0 \quad 2.9$$

When the cash and future price changes are not equal, which is the case in most instances, basis risk will be present. Define the variance of basis risk as  $\sigma^2(B)_{t,T}$

$$\sigma^2(B)_{t,T} = \sigma^2(S_t - F_{t,T}) \quad 2.10$$

This can be expanded to

$$\sigma^2(B)_{t,T} = \sigma^2(S_t) + \sigma^2(F_{t,T}) - 2\rho_{sf} \sigma(S_t)\sigma(F_{t,T}) \quad 2.11$$

---

<sup>8</sup> As described by Franklin R. Edwards and Cindy W. Ma, "Futures & Options", McGraw Hill.

where  $\sigma$  is the standard deviation and  $\rho_{sf}$  is the correlation coefficient between the cash and futures prices. If the variances of the cash and futures prices are equal and the correlation coefficient is 1, the variance of the basis will be zero. We conclude that basis risk will be reduced as the correlation coefficient increases and approaches one. In reality, hedgers face basis risk because there is no perfect correlation between the cash and futures prices. For a hedge to be attractive, basis risk should be less than price risk.

### III. THE HEDGE RATIO

Hedging is a concept whose primary objective is to reduce volatility in one's portfolio due to price change. Volatility is simply risk and risk can be measured by the variance. The hedge ratio, HR,<sup>9</sup> is defined as the number of futures contracts needed to hedge the spot position. Thus,

$$HR = \frac{\text{Futures Position}}{\text{Spot Position}} \quad 2.12$$

After establishing the hedge, the trader has a hedged portfolio that consists of the spot position and the futures position

$$\text{Hedged portfolio} = \Delta \tilde{S} - h \Delta \tilde{F} \quad 2.13$$

The minus sign indicates a short position in the futures market. The risk of the hedged portfolio is represented by the variance

$$\text{var} [ (\Delta \tilde{S}) - (h \Delta \tilde{F}) ] = \sigma^2 [ (\Delta \tilde{S}) - (h \Delta \tilde{F}) ] = \quad 2.14$$

<sup>9</sup> The HR equivalently also represents the proportion of the spot position which is hedged.

$$1^2 \text{var}(\Delta\tilde{S}) + h^2 \text{var}(\Delta\tilde{F}) - 2(1)(h) \text{cov}(\Delta\tilde{S}, \Delta\tilde{F}) \quad 2.15$$

The covariance can be expanded to

$$\text{cov}(\Delta\tilde{S}, \Delta\tilde{F}) = \text{corr}(\Delta\tilde{S}, \Delta\tilde{F}) \sigma(\Delta\tilde{S}) \sigma(\Delta\tilde{F}) \quad 2.16$$

Combining equation 2.15 and equation 2.16 gives

$$1^2 \text{var}(\Delta\tilde{S}) + h^2 \text{var}(\Delta\tilde{F}) - 2(1)(h) \text{corr}(\Delta\tilde{S}, \Delta\tilde{F}) \sigma(\Delta\tilde{S}) \sigma(\Delta\tilde{F}) \quad 2.17$$

In order to find the minimum risk hedge ratio, take the derivative of equation 2.17 and set it equal to zero. Therefore, the minimum hedge ratio,  $h^*$

$$h^* = \frac{\text{cov}(\Delta\tilde{S}, \Delta\tilde{F})}{\text{var}(\Delta\tilde{F})} = \frac{\sigma(\Delta\tilde{S}) \text{corr}(\Delta\tilde{S}, \Delta\tilde{F})}{\sigma(\Delta\tilde{F})} \quad 2.18$$

By regressing the historical change in spot prices on historical change in futures prices, one can estimate the HR which is the coefficient of the change in the futures price in equation 2.19<sup>10</sup>

$$\Delta S_t = \alpha + \beta \Delta F_t + \varepsilon_t \quad 2.19$$

<sup>10</sup> Figlewski et al, 1988, has shown that the traditional hedge ratio technique may be deficient because first, it is assumed that the objective is risk minimization and not maximization of expected utility. Second, the joint distribution of cash and futures prices changes, and therefore the hedge ratio is estimated incorrectly, since there is no adjustment for the fact that it varies substantially over time.

<sup>11</sup> Herbst, A.F. et al, 1989, have shown that the ARIMA methodology is successful in solving the problem of autoregressive disturbances in the data better than the traditional OLS.

The presumption in Equation (2.19) is that the changes in the cash and futures prices are linearly related and are subject to random errors,  $\varepsilon_t$ , that is basis risk. The estimated coefficient,  $\beta$ , of the independent variable, futures, is the minimum hedge ratio.<sup>12</sup> Care must be taken when estimating the HR by regression analysis.<sup>13</sup> There are often complex statistical issues to be resolved and there are always important judgments to be made. The hedger should be confident that a solid relationship exists between the spot price change and the futures price change. The integrity of the HR when using historical price data depends entirely on this price change relationship.  $R^2$ , the coefficient of determination, is a measure of goodness of fit. It is the square of the correlation coefficient of the two variables, spot and futures, in the regression and it has values between 0 and 1. The closer this value to 1 the better is the fit and the hedge. Low values of  $R^2$  have greater basis risk.

<sup>12</sup> Louis Edcington, "The Hedging Performance of the New Futures Market," *Journal of Finance*, March 1979, pp. 157-170.

<sup>13</sup> Following the methodology of bivariate GARCH model by R. Baillie & R. Myers, Dr. L. Switzer of Concordia University ran for me the bivariate GARCH model of the T35 on the TXF. We found that the hedge ratio estimate using the GARCH methodology resulted in a marginal risk reduction when compared to the OLS. The difference in variance of a hedged portfolio (comprised of being long in the T35 and short in the TXF) using the GARCH method to the OLS was decreased by 0.36% for the period before TIPs was introduced and was decreased by 0.23% for the period after TIPs was introduced. Therefore, using OLS technique should not be far off in estimating the hedge ratio.

## **CHAPTER 3**

### **STOCK INDEX FUTURES**

#### **I. TORONTO INDEX PARTICIPATION UNITS, (TIPs)**

As mentioned earlier, the primary objective of this research is to investigate the efficiency of the Toronto 35 Index Futures with the introduction of the Toronto 35 Index Participation Units, TIPs. In general, a participation unit can be defined as a share comprised in the same proportion of the companies that make up the Index. TIPs were introduced in March 1990 and each unit reflects interest in a trust created by the exchange. The trust holds baskets of senior Canadian stocks that represent the Toronto 35 Index. Therefore, investors wishing to participate in a diversified Canadian equity portfolio can engage in buy and sell orders of TIPs which is listed on the Toronto Stock Exchange. In addition, TIPs pay dividend on a quarterly basis and can be bought on margin. Also by implementing specific strategies TIPs can be used with derivative products like futures and options to perform hedging. TIPs will be redeemed for cash at any time, but in order to redeem to receive the basket of shares, a Prescribed Number of TIPs is required. Finally, unlike mutual funds, TIPs has no management fees and since it tracks the market it saves any costs associated with rebalancing the mutual fund. These features have helped in making TIPs one of the mostly traded shares on the TSE. Therefore, it is important to keep in mind at all times that TIPs is a diversified portfolio of major companies in Canada.

## II. STOCK INDEX FUTURES

Trading stock index futures began early in 1982 on three exchanges in the U.S. Since then, trading has increased substantially. The success of stock index futures can be attributed to its low transaction costs and its flexibility in allowing investors to take different positions according to their personal preferences, namely, arbitrageurs, hedgers, and speculators. The study will concentrate on the three indexes, the Toronto 35 Index (T35), Standard & Poor's 500 (S&P 500), and the Major Market Index (MMI). Two of the three indexes are value weighted indexes, namely, the T35 and S&P 500. The MMI is an equally weighted index. Thus, in an equally weighted index all stocks have the same proportion in the index. In a value weighted index, stocks with higher capitalization value have higher weighting in the index.<sup>14</sup>

## III. THE COST OF CARRY MODEL

The cost of carry model presented earlier and repeated below

$$F_{0,T} = S_0 + CC - CR \quad 1.1$$

will have to be modified in order to be applicable to stock index futures. This kind of modification is essential because all stock indexes exclude dividend payments. Being long in stocks entitles the holder to dividends, but the cash index reflects only the price of the stocks with no dividends. Therefore, the value of stock index is simply a price index. Since the cost of carry model which prices futures contracts is associated with the index price

<sup>14</sup> The weighted index is calculated by summing up the product of each stock price by its number of shares outstanding and dividing by a base value. An equally weighted index is calculated by summing up the prices of each stock and dividing by a divisor.



value, dividends will have to be deducted from the futures prices. The cost of carry model needs to be adjusted by subtracting the future value of dividends from the time of receipt until the futures contract expiration day, T. The carrying costs with stock index futures are the interest rate costs,  $h(0,T)$ , incurred in financing the purchase of the stocks between today until the futures expiration. The cost of carry model to be used in pricing stock index futures is

$$F_{0,T} = S_0(1+h(0,T)) - \sum FV(\text{Div}) \quad 3.1$$

where,

$F_{0,T}$  = Futures price at time,  $t=0$  for delivery at time,  $t=T$

$S_0$  = Spot price at time  $t=0$

$h(0,T)$  = is the interest rate during the period  $t=0$  to  $t=T$ .

$\sum FV(\text{Div})$  = The cumulative future value of all the stocks

This cost-of-carry formula has been simplified because it does not allow for the continuous compounding of interest costs but captures only the simple interest financing cost.<sup>15</sup> A continuous cost-of-carry formulation is shown below

$$F_t^c = S_t e^{r(T-t)} - \sum_{\tau=t+1}^T D(\tau) e^{r(T-\tau)} \quad 3.2$$

where T is the maturity date and the second term reflects the future value of dividends received and reinvested at the risk free rate between the time they were received and T.

The same strategies described in chapter 1 still apply to stock index futures and

<sup>15</sup>

Assuming r is the annual interest rate and there are T days to delivery, then  $h(0,T)=rT/365$

will be repeated for convenience. When the cost of carry model is violated, arbitrage opportunity exists. The cash-and carry strategy allows the trader to buy the spot by borrowing funds and selling index futures. This strategy is attractive when stocks are underpriced compared to the futures. The reverse cash-and carry strategy permits the trader to short sell the stock, lend the proceeds and buy futures. Such a strategy is attractive when stocks are overpriced relative to the futures. Therefore, any discrepancy between the futures prices and the spot market would lead to a profit at expiration date simply by exploiting the appropriate strategy. Since index arbitrage involves the trading of many stocks, program trading is usually used to execute index arbitrage.

#### IV. THE STOCK INDEX HEDGE RATIO

As mentioned in chapter 1, when an investor wishes to hedge with futures, the return on the hedged portfolio,  $R_h$ , will be comprised of the return of the portfolio (to be hedged),  $R_p$ , and the return of the futures position,  $R_f$ . Thus, the expected return on the hedged portfolio will be

$$\bar{R}_h = \bar{R}_p - h \bar{R}_f \quad 3.3$$

and the variance of the hedged portfolio is

$$\sigma_h^2 = \sigma_p^2 + h^2 \sigma_f^2 - 2h\sigma_{pf} \quad 3.4$$

to find the minimum-risk hedge ratio, take the derivative of equation 3.4 with respect to  $h$  and set it equal to zero to obtain

$$h^* = \sigma_{pf} / \sigma_f^2 \quad 3.5$$

as mentioned earlier  $h^*$  can be found by regressing the spot rate of return on the futures rate of return.

## **CHAPTER 4**

### **TRADING ACTIVITY of TIPs and INDEX FUTURES**

#### **I. IMPLICATIONS of TIPs on STOCK INDEX FUTURES**

If participation units provide more diverse payoffs to investors than the existing securities, the capital market will be more complete. One way to observe any benefit arising from the trading of TIPs is to test the market efficiency of products based on the index. If TIPs indeed make it easy to track the market movement and to arbitrage away any discrepancy between the index and the index products, trading in index futures should become more active ( TSE officials have confirmed this by saying that more dealers use index futures and options to arbitrage/hedge TIPs and vice versa). With increased activity, the index futures markets may exhibit more efficiency in revealing price information.

The large daily trading volume of TIPs coupled with their redemption option may generate increased activity in the underlying 35 stocks. However, TIPs trading may take away some trading volume from the underlying 35 stocks if some investors are mainly interested in those stocks for a diversified portfolio that tracks the general market movement. If the TIPs are indeed easily used in index arbitrage, activities of index futures would rise. However, if TIPs take away the market indexing functions of index futures used in portfolio management, the presence of TIPs may depress the activities of futures. Since TIPs holders receive quarterly dividends, they are not subject to dividend uncertainty as is the case with index futures holders. TIPs also do not have to be rolled

over frequently as do futures and they may be attractive to small investors who can not afford large denominations associated with index futures , let alone the high margin requirements of the latter. Finally, studies by Anthony (1988), Harris (1989), and Kawaller et al (1987), on the relationship between the index and index futures have shown that index futures lead the index in price and volume indicating that investors with new information are more likely to use index futures than the stocks underlying the index. With the presence of TIPs, such information trading can be shifted from futures since investors may find the fixed maturity and margin requirement associated with futures unattractive. If TIPs act as a close substitute for index futures, volume in the futures markets would decline. If TIPs create new interest in market indexing and becomes an easy tool for index arbitrage, volume in the futures would be expected to increase.

## **II. TRADING ACTIVITY**

Whether TIPs on balance serve as a substitute product or complement index futures is assessed by examining the trading volume of index futures in a two year window before and after the introduction of TIPs in March 1990. Figure 4.1 shows the monthly trading volume of the T35 stocks and T35 futures (TXF). Although, the observed trading volumes do not exhibit any particular monotonically increasing or decreasing pattern, these volumes are not adjusted by other market factors which would impinge on futures trading irrespective of TIPs' trading. To account for such market factors, trading volumes are adjusted by the trading volumes of Toronto 300 Composite Stocks. For T35 stocks,

the adjusted relative trading volume represents the volume of T35 stocks divided by the volume of the TSE 300 Composite Index without the T35 stocks and TIPs. This relative volume gives us the activity of T35 stocks relative to other stocks in TSE 300. The volumes for the futures is adjusted by dividing by this factor.

Table 4.2 shows the increase in activity for futures is apparent. However, trading activity in the underlying T35 stocks does not show much change. Table 4.1 shows the results of t-tests for changes in the mean trading volumes for the T35 and TXF. The mean difference covers two 23 months periods from April 1988 to February 1990 and from April 1990 to February 1992. Since trading activities may be subject to seasonal patterns, 23 months around March 1990 are deleted. The mean differences in the trading volumes are positive for the two securities. For the futures contracts, the difference is statistically significant at the 1% level. This finding supports the conjecture that TIPs increase trading volume of index futures. Since the volume of T35 did not change after March 1990, it is evident that TIPs created a "new" opportunity for investors to engage in index arbitrage.

The T35 Index Futures market experienced the greatest change in activity after the introduction of TIPs. In the other chapters, the impact of TIPs on hedging effectiveness and pricing of futures contracts will be evaluated.

Figure 4.1

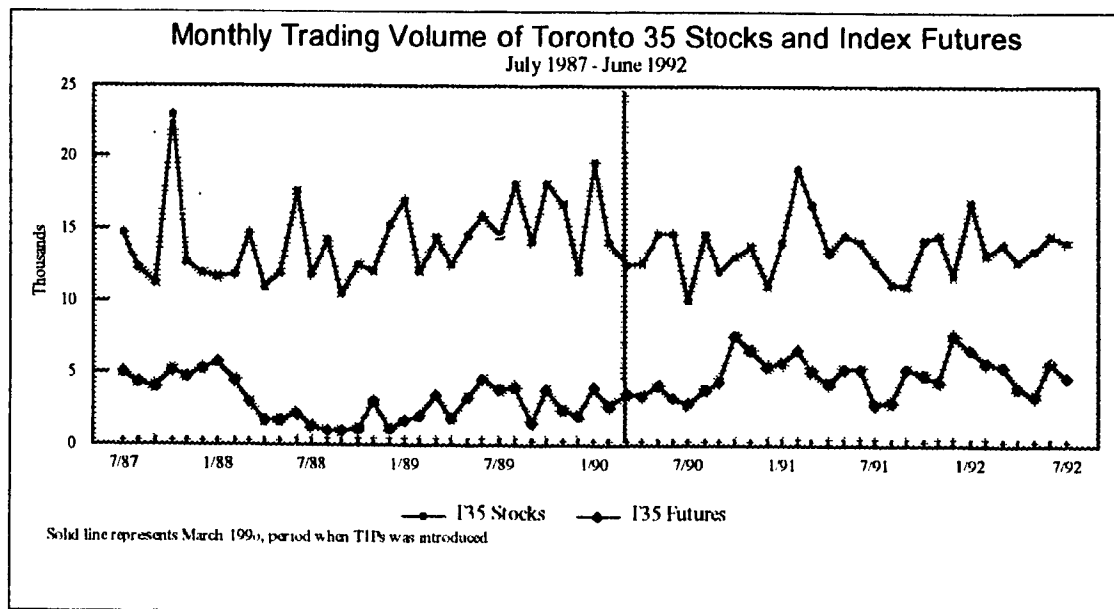
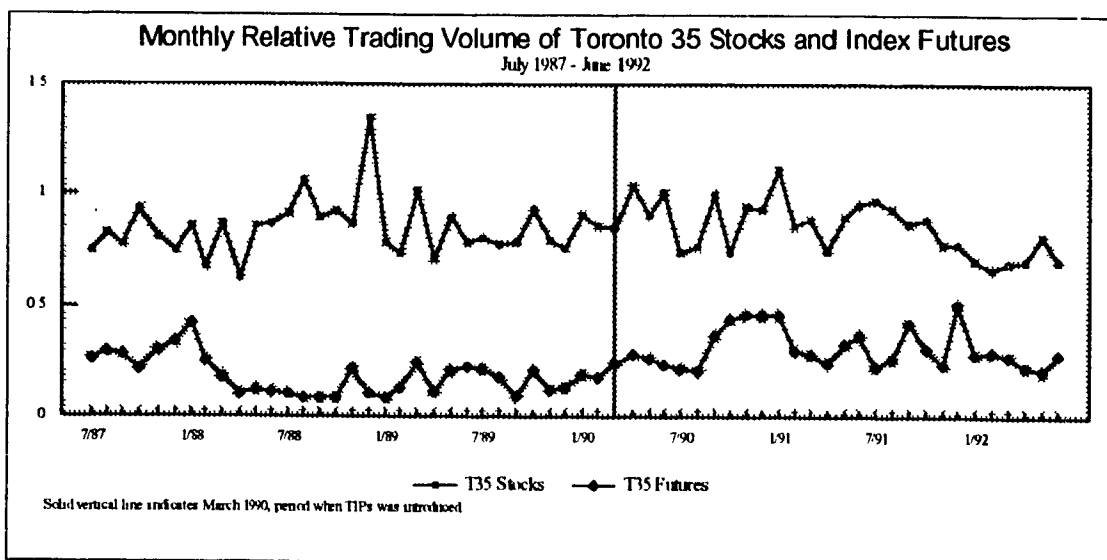


Figure 4.2



**Table 4.1**

Mean Values of Relative Monthly Trading Volumes for T35 Stocks and Futures

April, 1988 - February, 1992

Security	(a) April 88 - Feb. 90	(b) April 90 - Feb. 92	Mean Difference (b) - (a)	t-stat <sup>a</sup> for Difference in Means
T35 Stocks	0.869	0.879	0.009	0.23
T35 Futures (x10 <sup>-4</sup> )	0.142	0.322	0.180	7.99 <sup>b</sup>

<sup>a</sup> This t-stat tests the hypothesis that the true means of the two periods are equal. The underlying assumption here is that the variables are normally and independently distributed within the group.

<sup>b</sup> statistically significant at 1% confidence level.

$$\text{Relative Trading Volume} = \frac{\text{Trading Volume of a Security}}{\text{Volume of (Toronto Composite - T35 Stocks - TIPs)}}$$



## CHAPTER 5

### DATA and METHODOLOGY

#### I. DATA and METHODOLOGY

Daily prices were provided by the TSE for the T35, TIPs<sup>16</sup>, and the TXF in Canada and daily prices were also obtained for the S&P 500 Index, S&P 500 futures contracts, the MMI, and the MMI futures contracts in the U.S. for the period starting June 1, 1988 until December 31, 1991. The second step was to find weekly rates of return for each data set. The structure of the methodology is based on hedging the spot index by selling futures contracts. Only the nearest futures contract to maturity is used for hedging and subsequent roll over to the next contract when the original contract expires. The TXF and the MMI futures contracts used expire monthly while the S&P 500 futures contract expire quarterly. The next step was to find weekly rates of return on both the stock indexes and the stock index futures.<sup>17</sup> Furthermore, total weekly rates of return (i.e., including dividends) for the spot indexes were calculated. To find the minimum-risk hedge ratio, I ran a regression of the rate of return of the cash market on the futures market rate of return. The dividend payment on the T35 was deduced from total return of the T35.<sup>18</sup> The NYSE dividend payments were used as proxy for the S&P 500 and the MMI stock index. In order to find the hedge ratio, I ran a regression of the T35 index on the TXF for

<sup>16</sup> TIPs started trading on the Toronto Stock Exchange on March 9, 1990.

<sup>17</sup> The futures rate of return is defined as the change in the price of the futures contract divided by the stock index. Since the basis is small during the last three days of trading, the rate of return during this period is calculated using the second nearest contract.

<sup>18</sup> The equation to find the dividends on the T35 in Canada is shown in chapter 6.

the whole period. The T35 data was also divided into 21 month intervals for the period before TIPS was introduced and after TIPS was introduced. TIPS rates of return were also regressed on the TXF to find the hedge ratio. The U.S. data was also divided in the same manner to find the hedge ratio. Hence, I will compare the risk-return combinations for unhedged and hedged portfolios and a hedging performance comparison will be deduced.

## II. RISK and RETURN of HEDGED and UNHEDGED PORTFOLIOS<sup>19</sup>

As stated earlier, the coefficient of the independent variable is the hedge ratio (HR). The calculated risk and return combinations which can be achieved by selling TXF

**Table 5.1<sup>20</sup> : Risk Return for Unhedged and Hedged Portfolios**

Notes: Mean returns and standard deviations are in percent, at annual rates.

Portfolio	Unhedged		Minimum-Risk Hedge		
	E(R)	$\sigma$	h	E(R)	$\sigma$
T35	4.73	11.70	0.89	7.16	3.08
T35 B	6.92	10.85	0.91	7.80	2.18
T35 A	1.88	12.56	0.87	6.46	3.54
TIPS	2.52	13.18	0.94	7.48	4.34
S&P 500	13.73	14.37	0.96	4.50	2.21
S&P 500 B	13.47	12.58	0.92	5.03	2.33
S&P 500 A	12.19	16.12	0.97	3.95	2.00
MMI	17.01	14.61	0.98	3.57	2.30
MMI B	17.51	13.54	0.97	4.32	1.93
MMI A	15.09	15.80	0.99	2.78	2.62
MMISP	12.48	14.65	0.90	8.11	4.41

<sup>19</sup>

<sup>20</sup>

The risk-return combination, and the hedge ratio are found using the methodology on page 20. The letters B and A after the portfolio's name indicate period June 1, 88-February 28, 90 and April 1, 90-December 31, 91 respectively. MMISP represents hedging the MMI Index with S&P 500 Futures, cross hedge.

against the T35 and TIPs, by selling S&P500 futures against S&P 500 index and by selling MMI futures against the MMI are shown in Table 5.1. By examining Table 5.1, it is apparent that standard deviation of the minimum risk hedge was reduced substantially when compared to the unhedged portfolios. Consider first the short hedge involving a long position in the T35, T35 B and the T35 A and a short position in the TXF. Risk reduction is significant, from a standard deviation of 11.7% to 3.08%, 10.85% to 2.18% and 12.56% to 3.54% respectively. Reasonably, good risk reduction was also achieved for the TIPs. Table 5.2 shows the risk reduction for the portfolios in question. In the Canadian market, risk reduction varied from period to period with a dispersion from 71.8% to 79.9% for the T35. TIPs risk reduction is 67.07%. Nonetheless, Table 5.1 and Table 5.2 show that TIPs has the highest standard deviation for the unhedged and for the minimum risk hedge and the least risk reduction when compared to the T35, T35 B and

**Table 5.2: Risk Reduction, Basis Risk,<sup>21</sup> and Correlation Coefficient**

Portfolio	Risk Reduction	Basis Risk ( $\sigma$ )	$\rho$
T35	73.6%	3.18%	0.965
T35 B	79.9%	2.62%	0.972
T35 A	71.8%	3.67%	0.960
TIPs	67.07%	4.4%	0.944
S&P 500	84.62%	2.3%	0.988
S&P 500 B	81.5%	2.51%	0.982
S&P 500 A	87.6%	2.02%	0.992
MMI	84.25%	2.3%	0.987
MMI B	85.74%	1.96%	0.989
MMI A	83.41%	2.61%	0.986
MMISP	69.80%	4.47%	0.954

<sup>21</sup>

Basis risk is calculated using the basis risk methodology on page 12.

to the T35 A. The difference between the variances of the futures and cash prices will result in some basis risk. In practice, however, the magnitude of the basis risk depends mainly on the degree of correlation between the cash and futures prices: the higher the correlation, the less the basis. It is interesting to notice that by comparing each country's portfolios separately, risk reduction increases, basis risk decreases as  $\rho$  approaches 1. This reinforces the fact that risk reduction is most effective when basis risk is zero. In the U.S. market, the standard deviation of the minimum risk hedge was significantly lower than the unhedged. Risk reduction varied not only from period to period but also from index to index. The hedged S&P 500 index risk reduction varied from 81.5% to 87.6%. The hedged MMI risk reduction varied from 83.41% to 85.74%. As mentioned earlier, as the correlation coefficient approaches 1, risk reduction increases and basis risk decreases. The S&P 500 B has the highest  $\rho$ , the highest risk reduction, and the lowest basis risk. Similarly, the MMI B shows the same results. The MMISP is a portfolio designed by being long in the MMI and short in the S&P 500 futures. This is a cross hedge. In a cross hedge, the cash position is not tied directly to the index. Therefore, nonmarket risk is present and thus basis risk. The  $\rho$  of the MMISP is lower than the other correlations in the American market, risk reduction seems to have deteriorated, and basis risk has increased significantly

So far, I have concluded that the hedging effectiveness of the T35 has performed better than the TIPs in Canada. In the U.S. I have found that a cross hedge does not provide better hedging performance than a direct hedge. The important point to remember

is that the  $\rho$  is the crucial factor because it tells us how much risk is reduced by using the futures market as a hedging mechanism. Hence, the higher the  $\rho$  the lower the basis risk.

Next, I would like to compare the hedging performance of the T35 for the period before and after TIPs was introduced and also to compare the performance of the indexes in both countries in the period after TIPs was present. By examining Table 5.2, it is apparent that the T35 B has enjoyed a higher risk reduction and a lower basis risk than the T35 A. In the U.S. where there was no participation unit being traded at the time of the study, it is interesting to see that the T35 A has provided a lower  $\rho$ , lower risk reduction, and higher basis risk when compared to the U.S. indexes. Thus, the T35 B provided better hedging performance than the T35 A and the T35 A resulted in lower hedging effectiveness compared to the S&P 500 index and the MMI.

To quantify the hedging effectiveness, I need to start from the definition of the HR as described by Ederington. Ederington points out that the HR chosen by a hedger will determine the reduction in risk. Therefore, the futures markets' potential for risk reduction can be measured by comparing the risk on an unhedged portfolio with the minimum risk portfolio. Thus, the measure of hedging effectiveness is<sup>22</sup>

$$e = 1 - \text{Var}(R_h)/\text{Var}(R_u) \quad 5.1$$

where  $R_h$  and  $R_u$  are the hedged and unhedged portfolios. As  $e$  approaches 1 the better is the hedge because more risk reduction can be explained by hedging with futures.

Table 5.3 lists the hedging effectiveness.

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<sup>22</sup> It is hypothesized that  $e$  will be greater for longer durations because absolute changes in cash prices should be greater and futures prices would have more time to respond over longer periods.

TIPs has the lowest hedging effectiveness in the Canadian market, namely, 89.1% of risk was reduced when hedging with futures. The T35 in the period before TIPs was introduced had a higher measure of hedging effectiveness, 95.96%, than in the second time period, 92.05%. Therefore, hedging effectiveness of the T35 provided better results than TIPs. Nonetheless, we can state that hedging TIPs with TXF is not as effective as hedging T35 with TXF (for the same time period). This result is anticipated because the T35 is tied directly to the TXF. In the U.S. market, hedging effectiveness was increased in the second time period for the S&P 500 and it was decreased for the MMI. MMISP has the lowest hedging effectiveness than the other portfolios surely due to its status as a cross hedge. Again the hedging effectiveness of the T35 A was no match to its U.S. counterparts. Different economic factors in both countries can not be ruled out for the different hedging effectiveness in both countries.

**Table 5.3: Measure of Hedging Effectiveness**

Portfolio	Hedging Effectiveness
T35	93.07%
T35 B	95.96%
T35 A	92.05%
TIPs	89.17%
S&P 500	97.63%
S&P 500 B	96.56%
S&P 500 A	98.46%
MMI	97.52%
MMI B	97.96%
MMI A	97.25%
MMISP	90.93%

When it was assumed that the existence of index participation units would increase arbitrage possibilities and result in more efficient markets and better hedging effectiveness, the Canadian experience showed otherwise. That is, in the period before TIPs was traded, hedging effectiveness of the T35 was higher than in the period when TIPs was present. This is supported by the higher basis risk and the lower  $e$  value in the period when TIPs was present. Therefore, TIPs not only did not add to hedging effectiveness but also it might have reduced the hedging performance.

An interesting finding in Table 5.1 is that the hedged return in the Canadian market had a higher return than the unhedged return. At first this might sound very surprising but by examining the futures return during the three different time periods, one finds the return to be negative.<sup>23</sup> In contrast, the spot return was positive during the same time periods. Since the correlation between the spot and futures prices was higher than 0.94, it implies that when the spot price decreased so did the futures prices but because of the higher volatility in the futures market the decrease was larger and therefore negative.

Table 5.4 shows the futures return for the designated time periods. Therefore, an investor holding a cash position in the T35 or TIPs and hedges by selling futures and since futures return was negative, the loss incurred in the spot position was more than offset by gains in the futures market. Since the second time period coincides with the recession period in Canada, hedging with stock index futures during a bear market resulted in far

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<sup>23</sup>

I argue that since the hedge ratio  $< 1$  implies that the change in futures prices is larger than the spot. Since the economy was in a recession at that time period, the change in the futures prices fluctuated much more than the spot in Canada than in the U.S. which resulted in higher variation and greater gain for a short hedger.

**Table 5.4:** Futures rate of return

Portfolio	Futures rate of return
TXF	-2.25 %
TXF B	-0.89 %
TXF A	-4.89 %
S&P 500	9.23 %
S&P 500 B	8.71 %
S&P 500 A	8.15 %
MMI	13.13 %
MMI B	12.98 %
MMI A	12.05 %

superior results not only in reducing risk but also increasing return. This result can not be explained by portfolio financial theory and it could have been attributed to some unique economic factors. On the other hand, ex post results need not be equal to ex ante. That is, when it was anticipated that the futures return will be positive, ex post results showed that it was negative and as such, the expected return of a hedged portfolio resulted in higher value than the unhedged.

When hedging, basis risk will always be present. As seen above basis risk can be present even if the hedge involves a position in the index portfolio itself and there is no nonmarket risk. Of course the MMISP provided a higher basis risk because of the nonmarket risk factor in the U.S. In Canada TIPs can be considered a cross hedge because it does not have a one to one relationship with the TXF. Hence where does basis risk come from? The answer lies in the next section.



## II. SOURCES of BASIS RISK

Figlewski<sup>24</sup> attributed the sources of basis risk to nonmarket risk and dividends risk. He also stressed the fact that futures prices are not directly tied to the underlying index, except at expiration. Daily price fluctuations between the spot and futures markets will increase basis risk and unless arbitrage possibilities can be executed in a timely manner, market equilibrium will be reached and basis risk will be reduced. But for stock index futures, a perfect arbitrage appears to be infeasible because you need to duplicate the index in the same proportion and be engaged in sell and buy orders of all the stocks simultaneously in order to capitalize on short term deviations between actual and theoretical futures prices. Hedging TIPs with TXF and the MMI with S&P 500 futures is considered a cross hedge and thus must have higher basis risk. This is supported by the results in Table 5.2 which shows that TIPs has highest basis risk in the Canadian market and MMISP has highest basis risk in the U.S. market.

To examine whether dividends risk is a major determinant of basis risk, I will evaluate the risk-return combinations for the T35 portfolio, the S&P 500 portfolio, and the MMI portfolio for the whole period. Table 5.5 presents the effect of dividends risk on basis risk. The inclusion of dividends in the portfolios does not seem to have changed the risk component of the unhedged portfolios. As for the risk minimizing hedges, dividend risk is of little importance for the U.S. data and is slightly higher in the Canadian data. We also notice that the HR is almost the same for the U.S. data with slight difference for the

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<sup>24</sup> S. Figlewski, "Hedging Performance and Basis Risk in Stock Index Futures". *The Journal of Finance*, July 1984.

T35. Hedged returns differ only by the amount of the dividend yield. This result is very similar to that of Figlewski and thus dividend risk is small. Therefore, basis risk can be attributed to a "noise" factor due to non synchronous trading and also due to price

**Table 5.5:** The effect of dividends risk on basis risk

Portfolio	Unhedged		Minimum Risk Hedge		
	E(R)	$\sigma$	h	E(R)	$\sigma$
T35	4.73	11.70	0.89	7.16	3.08
T35 W/D	8.45	11.67	0.80	10.72	4.70
S&P 500	13.73	14.37	0.96	4.50	2.21
S&P 500 W/D	18.16	14.40	0.96	8.57	2.22
MMI	17.01	14.61	0.98	3.57	2.30
MMI W/D	21.63	14.60	0.99	7.66	2.29
MMISP	12.48	14.65	0.90	8.11	4.41
MMISP W/D	21.62	14.61	0.90	12.33	4.37

mismatches between the spot and the futures market prior to delivery date.

I can conclude that the minimum risk hedge has provided a lower variance than the unhedged portfolio for any portfolio in either country. It can also be deduced that the T35 A provided better hedging effectiveness than TIPs. TIPs has a lower risk reduction and a higher basis risk than the T35 A. In addition, TIPs not only had a higher basis risk than the index but also basis risk has increased in the second time period. This view discredits the presence of TIPs in increasing hedging effectiveness. When comparing the two time periods using a measure of hedging effectiveness, T35 B had a higher measure than the T35 A. Not surprisingly, TIPs had a lower measure than the T35 A. Therefore,

TIPs did not provide a better alternative than the index as a hedging mechanism. Hedging TIPs with TXF can be thought of as a cross hedge because TIPs track the T35 and TXF is drawn on the T35. Thus, there is no one to one relationship between TIPs and TXF. By comparing the T35 with the U.S. indexes, I deduced that in the Canadian market, where TIPs is present, the T35 did not submit better results in terms of basis risk, risk reduction and the correlation coefficient. In the U.S. market, the MMISP portfolio showed exactly the same results as TIPs did in the Canadian market. The MMISP is a cross hedge due to the fact of hedging the MMI with S&P 500 futures. On the other hand, we have shown that the major component of basis risk can be attributed to fluctuations between the spot and futures prices. The dividend component was shown to be marginal.

## CHAPTER 6

### DIAGNOSTICS TESTS

#### I. ASSUMPTIONS BEHIND STATISTICS

Statistical analysis tells us that in order to run regressions and test hypothesis, certain assumptions about the error terms need to be satisfied. Consider the following regression equation:

$$S_t = \alpha + \beta F_t + \varepsilon_t \quad 6.1$$

where  $\varepsilon_t$  are independent  $N(0, \sigma^2)$ . The symbol  $N(0, \sigma^2)$  stands for normally distributed with mean zero and variance  $\sigma^2$ . Thus,  $E(\varepsilon_t, \varepsilon_s) = 0$ , for  $t \neq s$  and  $E(\varepsilon_t, \varepsilon_s) = \sigma^2$ , for  $t = s$ . These two assumptions translate into no autocorrelation between the error terms and that the variance is constant and equals  $\sigma^2$  respectively. The assumption of normality is justifiable because the testing procedures which are based on the t-statistic (distribution) is not sensitive to moderate departures from normality. Thus, unless the departures from normality are serious particularly with respect to skewness, the actual risk of errors will be close to the levels for exact normality. The hedge ratio was estimated by regressions analysis for 11 portfolios. Theoretically, the error terms have to meet the assumptions mentioned above. Research on the distributions of futures prices by Stevenson and Bear (1970) and Dusak (1973) have found the distributions to be leptokurtic. They concluded the distributions belong to the family of stable Paretian distribution. In fact, normal distribution is a special case of the stable paretian distribution when the parameter  $\alpha = 2$ .

In another study by Helms and Martell,<sup>25</sup> they examined the distribution of both price changes and log of price changes in various futures contracts by analyzing the characteristics of the distribution of several financial and nonfinancial contracts. They rejected the assumption of normality in all cases. Under the stable distribution if the parameter  $\alpha=2$ , then the distribution is normal. Helms and Martell estimated the parameters of the stable distribution and found  $\alpha$  to equal 2, implying the distribution was normal. In my case, however, the normality assumption was rejected by the data in both countries. On the other hand, the coefficient of skewness was not far from zero. The skewness coefficient ranged in absolute value from 0.005 to 0.9288. In addition, since the sample size is large, by the Central Limit Theorem, the distribution can be approximated by a normal distribution. The non-autocorrelation assumption was violated for all the data at hand. After attempting to correct for AR(1), the correlogram still showed significant rho's for higher lags. After correcting for higher lags, the correlograms did not show any significant rho's. The homoskedasticity assumption was satisfied by using the Goldfeld-Quandt test. This test is based on the idea that if the sample observations have been generated under the conditions of homoskedasticity, then the variance of the disturbances of one part of the sample observations is the same as the variance of the disturbances of another part of the observations. Thus, a test for homoskedasticity becomes a test for equality of two variances. Complete analysis and testing of each of the significance of the model(s), normality, autocorrelation and homoskedasticity assumptions follows.

<sup>25</sup> Billy Helms and Terrence Martell. "An Examination of the Distribution of Futures Price Changes," *Journal of Futures Markets*, 5:2, summer 1985, pp. 259-272

## II. TESTING of HYPOTHESIS

### Ila. Test for the Overall Significance of the Model(s)

Table 6.1 lists the regression results and the F-statistics for the portfolios.

**Table 6.1: Regression results and the F-statistic**

Testing for the overall significance of the models (portfolios):

Portfolio	Beta (HR )	F <sub>p</sub> <sup>*</sup> -statistic
T35	0.88869	11619.005
T35 B	0.91647	7148.325
T35 A	0.87612	4850.033
TIPs	0.94299	3351.698
S&P 500	0.96040	36123.636
S&P 500 B	0.92730	11865.757
S&P 500 A	0.97493	27926.896
MMI	0.98984	34476.235
MMI B	0.97657	20384.97
MMI A	0.99260	15240.737
MMISP	0.90107	8889.255

test for the overall significance of the model(s):<sup>26</sup>

$$H_0 : \beta_p = 0$$

$$H_a : \text{not } H_0$$

$$F_p^* = \text{MSR/MSE}$$

$$F_{1-\alpha, p, n-p-1} = F_{0.9, 1, 111} = 2.71$$

If  $F_p^* > F_{0.9, 1, 111}$ , reject  $H_0$  and conclude a single linear regression model

<sup>26</sup>

Where  $\beta_p$  stands for the beta of the portfolios listed in Table 5.1.

exists. Since according to Table 6.1,  $F_p^* > 2.71$  for all the portfolios, I conclude that the model(s) exist. Therefore, I now proceed to test the assumptions about the error terms, namely, normality, autocorrelation and homoskedasticity.

### Iib. Normality Test for the Error terms

Table 6.2 shows the skewness, kurtosis and the normality test for each portfolio.

**Table 6.2:** Skewness, Kurtosis & Normality test<sup>27</sup>

Portfolio	Skewness	Kurtosis	Normality Test
T35	-0.1935	41.5832	0.89
T35 B	0.7880	2.4930	0.88
T35 A	-0.5115	46.2364	0.87
TIPs	-0.9288	19.3963	0.85
S&P 500	-0.2328	7.2637	0.90
S&P 500 B	-.0680	7.4194	0.89
S&P 500 A	0.2542	4.3927	0.87
MMI	-0.2182	62.6392	0.88
MMI B	-0.4185	15.0993	0.87
MMI A	-0.0050	71.9206	0.86
MMISP	0.4611	2.2095	0.85

Test for normality:

$$H_0 : Z_{resid} \sim N(0,1)$$

$$H_a : \text{not } H_0$$

<sup>27</sup>

The normality test used is the Shapiro-Wilk statistic.

$$W_{n,\alpha} = W_{100,0.1} = 0.989$$

If  $W(\text{normal}) > W_{100,0.1}$ , accept  $H_0$

Since  $W(\text{normal}) < W_{100,0.1}$ , reject  $H_0$  and conclude the residuals are not normally distributed. As mentioned before, Futures prices have been shown not to follow the normal distribution. But since the distribution is symmetric (the coefficient of skewness does not deviate much from zero, see Table 6.2) and since the study by Helms and Martell have concluded that the distribution of futures prices can be approximated by a normal distribution, I will also conclude that the distribution is not far from normality.

### IIC. Test for the Autocorrelation of the Error terms

Table 6.3 lists the number of lags that were required for each model to correct for

**Table 6.3: LM test for AR(P)**

Portfolio	Chi-square with 15 D.F., $\chi^2^*$	Nbr. of lags
T35	7.303	10
T35 B	3.448	8
T35 A	6.096	10
TIPs	3.316	10
S&P 500	6.440	10
S&P 500 B	7.423	10
S&P 500 A	4.046	10
MMI	2.369	15
MMI B	1.703	15
MMI A	3.081	15
MMISP	6.622	15



autocorrelation. It also provides the Chi-square value for the LM test. Without any exception, all 11 portfolios were autocorrelated with AR(1). After correcting for AR(1), higher levels of autocorrelation were revealed. Table 6.3 shows the number of lags performed to correct for autocorrelation.

Test for autocorrelation using the LM test<sup>28</sup>:

$$H_0 : \rho_1 = \rho_2 = \dots = \rho_{15} = 0$$

$$H_a : \text{not } H_0$$

$$\chi^2_{\text{adf}} = \chi^2_{0.1,15} = 8.55$$

If  $\chi^2^* > \chi^2_{0.1,15}$ , reject  $H_0$  and the models are correlated. Since we accept  $H_0$ , all the  $\rho$ 's are equal to zero and I conclude the models (portfolios) error terms are non-autocorrelated.

### IId. Test for the Homoskedasticity of the Error Terms

Table 6.4: lists the Goldfeld-Quandt test statistic,  $F^*$

Test for heteroskedasticity using the Goldfeld-Quandt test:

$$H_0 : \sigma_1^2 = \sigma_2^2 = \dots = \sigma_n^2 = \sigma^2$$

$$H_a : \text{not } H_0$$

<sup>28</sup>

The LM (Lagrange multiplier) test is the Breusch (1978) - Godfrey (1978b) test.

$$F_{1, \alpha; n1-p-1, n2-p-1}^{29} = F_{0.9, \text{INF}, \text{INF}} = 1$$

If  $F^* > 1$ , reject  $H_0$  and the variance of each portfolio model is heteroskedastic. Since  $H_0$  is accepted, I conclude that the variance is homoskedastic.

**Table 6.4:** The Goldfeld-Quandt test for homoskedasticity

Portfolio	Goldfeld-Quandt test, $F^*$
T35	0.52877
T35 B	0.95953
T35 A	0.47883
TIPs	0.73057
S&P 500	0.97954
S&P 500 B	0.48020
S&P 500 A	0.99527
MMI	0.56347
MMI B	0.54580
MMI A	0.62442
MMISP	0.90659

Therefore, I can proceed testing hypothesis about the models since the three assumptions can be adopted in the regression analysis. Next, I will test if the Beta (HR) of the portfolios is different from 1 (naive strategy).

Ile. Comparing the significance of the hedge ratio to determine whether a naive hedge is different from the minimum-risk hedge.

<sup>29</sup> The values of  $n1$  and  $n2$  vary from portfolio to portfolio depending on the value of the sample  $N$ .

**Table 6.5:** Testing if beta value is significantly different from zero

Portfolio	BETA	Test statistic, $t^*$
T35	0.88869	-10.07 <sup>a</sup>
T35 B	0.91647	-5.85
T35 A	0.87612	-7.72
TIPs	0.94299	-3.2
S&P 500	0.96040	-8.38
S&P 500 B	0.92730	-7.96
S&P 500 A	0.97493	-4.95
MMI	0.98984	-1.94
MMI B	0.97657	-3.22
MMI A	0.99260	-1.03
MMISP	0.90107	-8.56

$$^a \text{ Test statistic} = (\beta - 1)/S.E._\beta$$

test if  $\beta$  is significantly different from 1:

$$H_0 : \beta_p = 1$$

$$H_a : \text{not } H_0$$

$$t_{1-\alpha/2, n-2} = t_{0.95, 197} = 1.645$$

$$\text{If } |t^*| > 1.645, \text{ reject } H_0$$

From Table 6.5, I conclude that only the beta of the MMI A portfolio is insignificant and is not different from 1. All other portfolios beta is different from 1. Therefore, the minimum risk hedge of the MMI A portfolio matches the naive hedge, that is, for every unit of the spot, a unit of the futures is sold.

## CHAPTER 7

### The Mispricing of Futures Contracts

#### I. DATA and METHODOLOGY

Daily prices were collected for the T35, TIPs, and the TXF in Canada and daily prices were also obtained for the S&P 500 Index, S&P futures, the MMI, and the MMI futures in the U.S. The second step was to find the number of days to maturity for each contract. Only the nearest futures contract to maturity is used. The TXF and the MMI futures contracts used expire monthly while the S&P 500 futures contracts expire quarterly. Dividends in the Canadian market were calculated according to a formula given by the Toronto exchange:

$$\text{Dividend} = (\text{TRIV}_t / \text{TRIV}_{t-1}) (\text{SPIV}_{t-1}) - (\text{SPIV}_t)$$

where  $\text{TRIV}_t$  is total return index value and  $\text{SPIV}_t$  is stock price index value at time  $t$ .

On the other hand, the U.S. data used the NYSE dividend as a proxy for the S&P 500 Index and the MMI. In order to price futures contract at equilibrium where no arbitrage profit can be realized, equation 3.2 will be restated below

$$F_t^e = S_t e^{r(T-t)} - \sum_{\tau=t+1}^T D(\tau) e^{r(T-\tau)} \quad 7.1$$

The first part of equation 7.1 represents the spot value and the costs associated with holding the spot and the second part represents the future value of dividends received assuming reinvestment at the risk free rate between the time the dividend is acquired and maturity date,  $T$ .

For the risk free rate, I used the short-term Banker's Acceptance (BA) rates in Canada and the Certificate of Deposit (CD) rates in the U.S. Daily one, three, and six month BA and CD rates were obtained from the Financial Post and interpolated when the number of days to futures' maturity,  $T-\tau$ , were different from the one, three, or six months. In order for equation 7.1 to be in index units and not dollar amounts, it was divided by 500, the multiplier that underlies the futures contract.

Modest and Sunderesan (1983) addressed most of the issues in determining the efficiency of prices in the market. They explored real market data and applied the carrying charges model to form permissible bounds for futures prices and took into account the actual transaction costs that would be incurred in trading the futures and the stocks in the index. Modest and Sunderesan assumed that the trader does not have full use of the proceeds from short sales due to margin requirements, then the interest on the proceeds that can not be used has a market impact on the analysis. Essentially, an arbitrage opportunity might require the short sale of the stock index, which means that the individual stocks comprising the index are sold short in the stock market. In this situation, the short seller might not receive full use of the proceeds from the short sale, because the broker will hold a significant fraction of those proceeds as protection against default by the short seller. Therefore, the success of any such arbitrage depends critically upon the assumptions regarding the use of short sale proceeds. Modest and Sunderesan examined alternative assumptions about the use of short sale proceeds. Thus, no arbitrage band arises because of transaction costs and restrictions on short sales. Moreover, tests of

market efficiency depend critically on careful estimations of these transaction costs. Within the context of their assumptions, they also revealed that arbitrage opportunities exist for traders who have full use of the proceeds, for example, large brokerage houses and institutional investors.

Figlewski (1984) found a puzzling result when testing the equilibrium price of futures. He discovered that in the early days of trading the market value of the stock index futures to be well below its theoretical market prices. In other words, when most people, expected the futures contracts pricing to conform to the cost of carry model (that is, futures price will be higher than the spot price by the amount incurred in carrying the asset which is equivalent to the risk free rate minus the dividend amount on the index portfolio), reality proved otherwise. A number of explanations can be cited for this violation of pricing. First, in theory where perfect capital market assumptions are made, arbitrage profits can be obtained. But in reality every transaction is influenced by transaction costs, time delays in executing orders and restrictions on short sales which will affect the arbitrage argument supporting the cost of carry model. Thus, there should be a region bounded by the theoretical prices around which actual futures prices can fluctuate without inducing any arbitrage possibilities. Second, Cornell and French (1983) point out the difference in tax treatment of stocks and futures. All profits in the futures market are marked to market and taxed accordingly at the end of the fiscal year. On the other hand, a stock portfolio offers a tax related "timing option" that a futures contract does not enjoy. An investor who invests in a stock portfolio has the option to defer any capital

gains by extending the holding period to take advantage of the long term capital gains rate. Or he can sell off the stocks at a loss and deduct the loss at ordinary short term rates. "According to the equilibrium relationship for the taxable investor, stock index futures may be priced below their theoretical level by an amount equal to the value of the timing option".<sup>30</sup> Valuing the timing option can prove to be a complicated task because of its dependence on unknown parameters like the average holding period and average investor's marginal tax rate. Several factors suggest that this effect might not be large enough. " Finally, there seems to be another set of resources based on investor's expectations and preferences to explain why they might be willing to sell stock index futures at prices that are "too low" and to buy stocks at prices that are "too high".<sup>31</sup> The majority of investors carefully select their portfolios such that they will outperform the market and thus hold long positions in stocks. Such investors will not substitute futures contracts for their portfolios just because they can sell their stocks at values lower than they truly are in order to buy underpriced futures. The only time they will perform such an action is when the discount on futures became significantly greater than the expected excess returns on their stocks. But if their beliefs are pessimistic about the market, then they might sell futures at a discount to hedge their portfolios because they expect to do better than the market even when it drops. It can be inferred that in the early days the stock index futures market experienced a situation of disequilibrium. This disequilibrium

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<sup>30</sup> Cornell B., French K. "The Pricing of Stock Index futures." *The Journal of Futures markets*, Vol. 3, No. 1, 1-14 (1983).

<sup>31</sup> Figlewski, S. "Explaining the Early Discounts on Stock Index Futures: The Case for Disequilibrium," *Financial Analyst Journal*, July-August, 1984

can be regarded more of a long term phenomenon than a short term one. Which implies that prices will not adjust in the short run to equate supply and demand. but a long term disequilibrium seems to have existed in such a way that the actions of investors already in the market created profitable investment opportunities for outside investors who for one reason or another were slow to take them up. Under this view, the discount on index futures is largely a transitory phenomenon caused by unfamiliarity with the new markets during the early stages of futures trading. In fact, with regard to the to the pricing of stock index futures, Figlewski (1984) found that significant underpricing that was documented in the early days of trading to have disappeared and that deviations from the equilibrium pricing to have narrowed. This implies that the underpricing did not reflect an equilibrium differential that would have resulted from the tax timing option mentioned by Cornell and French. Rather, it was a transitory phenomenon associated with the early stages of trading in the new market.

In order to test that over the years trading in futures contracts has grown to be efficient, I will utilize the methodology mentioned by Peters<sup>32</sup>. Peters measured the standard error between the closing actual and theoretical prices for different contracts. He calculated the standard error by regressing the theoretical price on the actual price. If there has been an increase in efficiency of the markets, then the standard errors should be falling over time and the  $R^2$  should be rising. It is worth explaining why the standard error should be falling if efficiency of the markets has indeed been increased. To answer this

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<sup>32</sup> Peters, Ed. "The growing efficiency of index futures markets", *The Journal of Portfolio Management*, summer 1985.



question, we have to go to the definition of Ordinary Least Squares (OLS) regression.

OLS regression is a statistical tool which estimates a linear relationship between two variables; the independent and dependent variables. The regression model looks like

$$Y_i = a + b X_i + e$$

$Y_i$  is the dependent variable and  $X_i$  is the independent variable. In the context of the above discussion, the dependent variable is the theoretical price daily rate of return and the independent variable is the actual price daily rate of return. The standard error of the estimate measures how well  $Y$  estimates  $X$ . The standard error,  $SS^2$ , can be expressed as

$$SS^2 = \left(\frac{1}{n-2}\right) \sum_{i=1}^n [Y_i - (a+bX_i)]^2$$

where,

$SS^2$  = the standard error

$n$  = the number of observations

$Y_i$  = the theoretical price daily rate of return of the future on day  $i$ , and

$X_i$  = the actual price daily rate of return of the future on day  $i$

A closer look at the standard error tells us that it represents the difference between observed and estimated values. Thus, as this difference is made negligible,  $S^2$  approaches zero. In this study, as the markets become more efficient, the spread between the theoretical and actual values of the future should narrow. Then the standard error should approach zero over time. To test if TIPs made the markets more efficient, I will evaluate the standard error for the periods before TIPs was introduced, that is, June 1, 1988 till

February 28, 1990 and for the periods after TIPs was introduced, that is, April 1, 1990 till December 31, 1991. If TIPs has indeed caused the futures market to be efficient, then the standard error over the second time frame should be lower than the first as well as the  $R^2$  value should increase in value (approach 1).

Table 7.1 lists the results of the regression, namely, the standard error and the  $R^2$ .

**Table 7.1: Standard error and  $R^2$  value**

Portfolio	Standard Error	$R^2$
T35 B	0.002751	0.843
T35 A	0.002618	0.839
S&P 500 B	0.002523	0.913
S&P 500 A	0.002438	0.938
MMI B	0.002777	0.916
MMI A	0.002459	0.939

As expected, the standard error of the daily rates of return declined from the first half to the second half time period. The  $R^2$  value of the rates of returns, while rising for the U.S. data, showed a slight decrease for the Canadian data. Otherwise they also behaved as expected. I will test the hypothesis for the significance of the decline in variance between the first and second half for the concerned portfolios below.

In order to test if the standard error in the second half,  $\sigma_2^2$ , is less than the standard error in the first half,  $\sigma_1^2$ , I will use the F Distribution. The F Distribution for this test is defined as

$$F = \sigma_1^2 / \sigma_2^2$$

Table 7.2 list the F Distribution for the concerned portfolios.

**Table 7.2:** F Distribution values

Portfolio	F* Distribution
T35	1.05
S&P 500	1.034
MMI	1.129

test for the significance of the decline in standard error

$$H_0 : \sigma_1^2 \leq \sigma_2^2$$

$$H_a : \sigma_1^2 > \sigma_2^2$$

$$F_{1-\alpha, n1-1, n2-1} = F_{0.9, INF, INF} = 1$$

If  $F^* > F_{0.9, INF, INF} = 1$ , reject  $H_0$  and conclude  $H_a$ .

Since  $F^*$  value in Table 7.2  $> F_{0.9, INF, INF} = 1$ , conclude that  $\sigma_1^2 > \sigma_2^2$  and that the standard error declined in the second half for all the portfolios in question.

I can conclude from this information that the futures contracts on both sides of the border have indeed become more efficient with time. This brings into question whether TIPs has contributed to market efficiency in Canada. The results above clearly shows that in the U.S. market where no participation index is being traded, the markets have also experienced greater efficiency in the second time period.

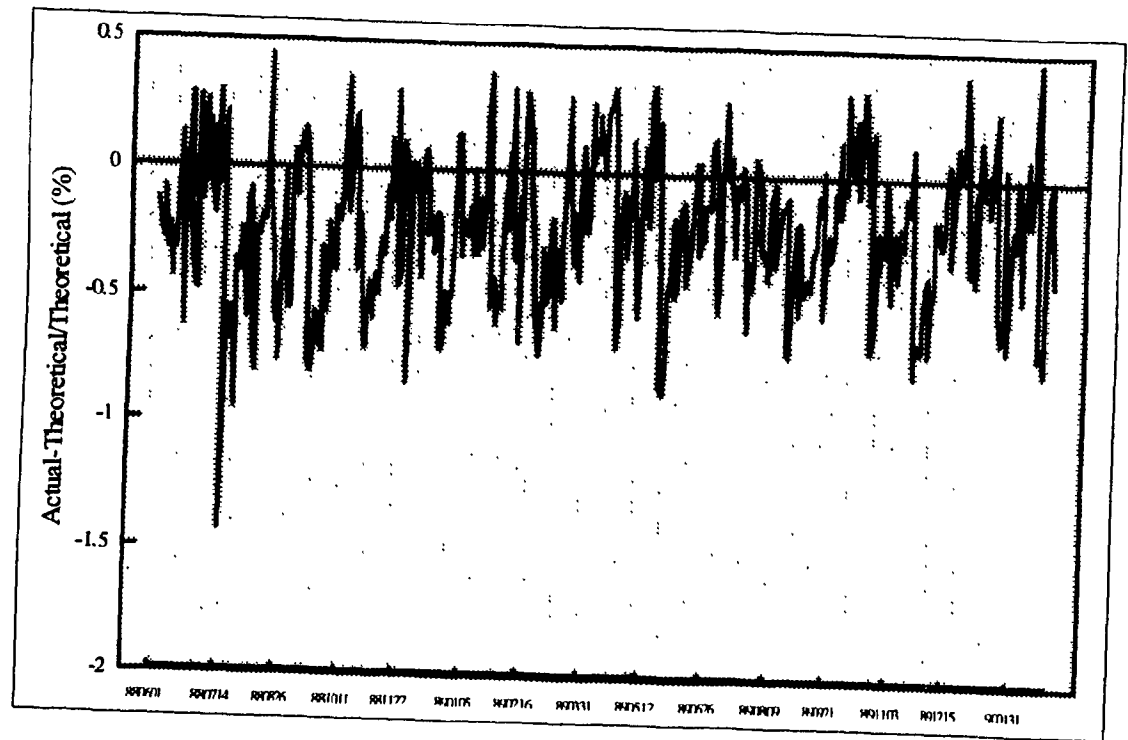
Graph 7.1a and 7.1b depicts the percentage price change between actual and

theoretical prices for the two time periods. The TXF was overvalued by 0.5% and undervalued by 1.4% in the first half.

**Graph 7.1a**

Daily mispricing (%) of Toronto 35 Index Futures

June 1, 1988-February 28, 1990



On the other hand, the TXF for the second half was overpriced by 1% and under priced by 1.3%. More generally, it can be stated that during the first half future prices were mostly underpriced while in the second half they were less underpriced. This is evident in the mostly negative fluctuations in the first half as compared to the second half. This result reinforces the standard error conclusion , that is, future prices became more

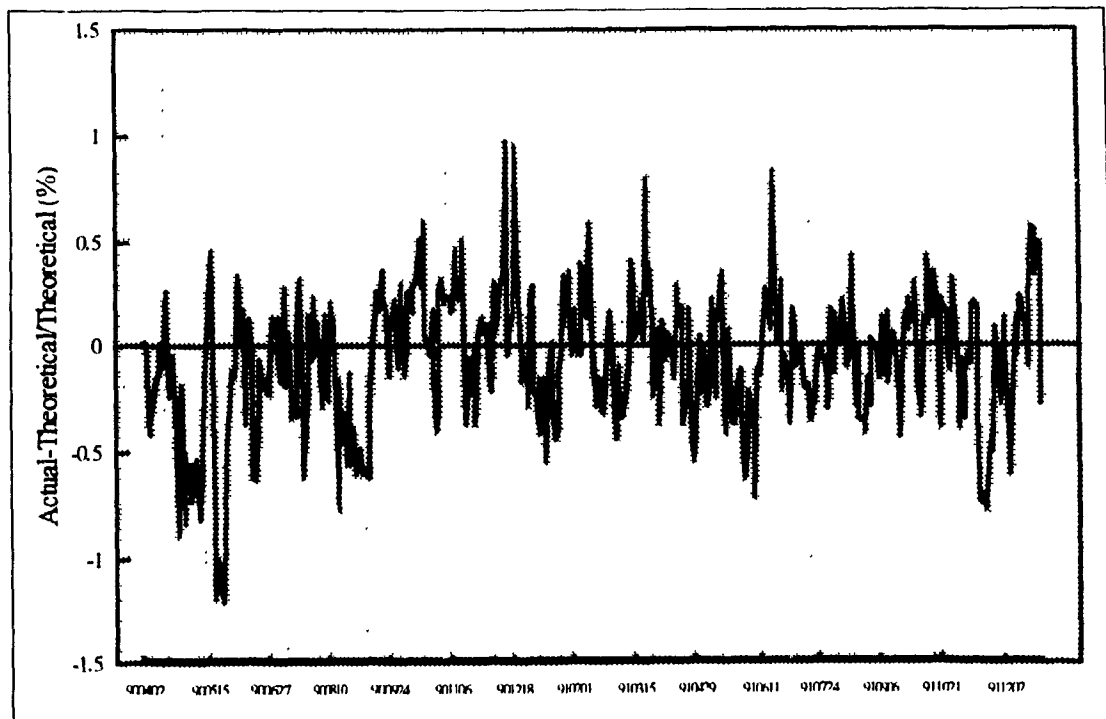
efficient after TIPs was introduced and future prices became much less undervalued..

Graph 7.2a, 7.2b, 7.3a and 7.3b shows the difference between actual and theoretical price expressed as a percent of the theoretical price for the U.S. data. The S&P 500 data for the first half illustrates that the future price was overpriced by 0.6% and underpriced by 2.3%. The second time period illustrates that the future price was overvalued by 0.7% and undervalued by 1.3%. The MMI was overpriced by 0.4% and

### Graph 7.1b

Daily mispricing (%) of Toronto 35 Index Futures

April 1, 1990-December 31, 1991



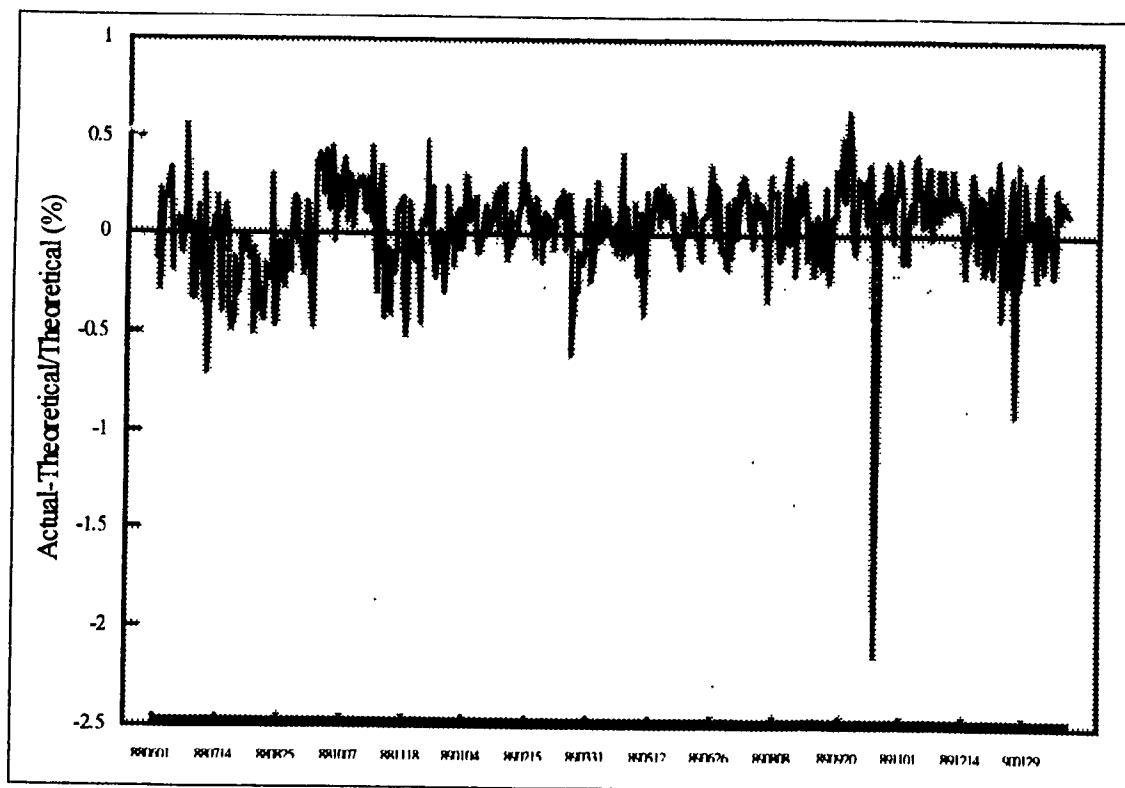
underpriced by 2.5% in the first half and was overpriced by 0.4% and underpriced by

1.6% in the second half. The U.S. data shows that in the second half futures prices became less undervalued. The standard error analysis discussed above supports this finding that future prices became more efficient. It is apparent that all three indexes have shown a significant decrease in underpricing between the two time periods.

**Graph 7.2a**

Daily Mispricing (%) of the S&P 500 Index Futures

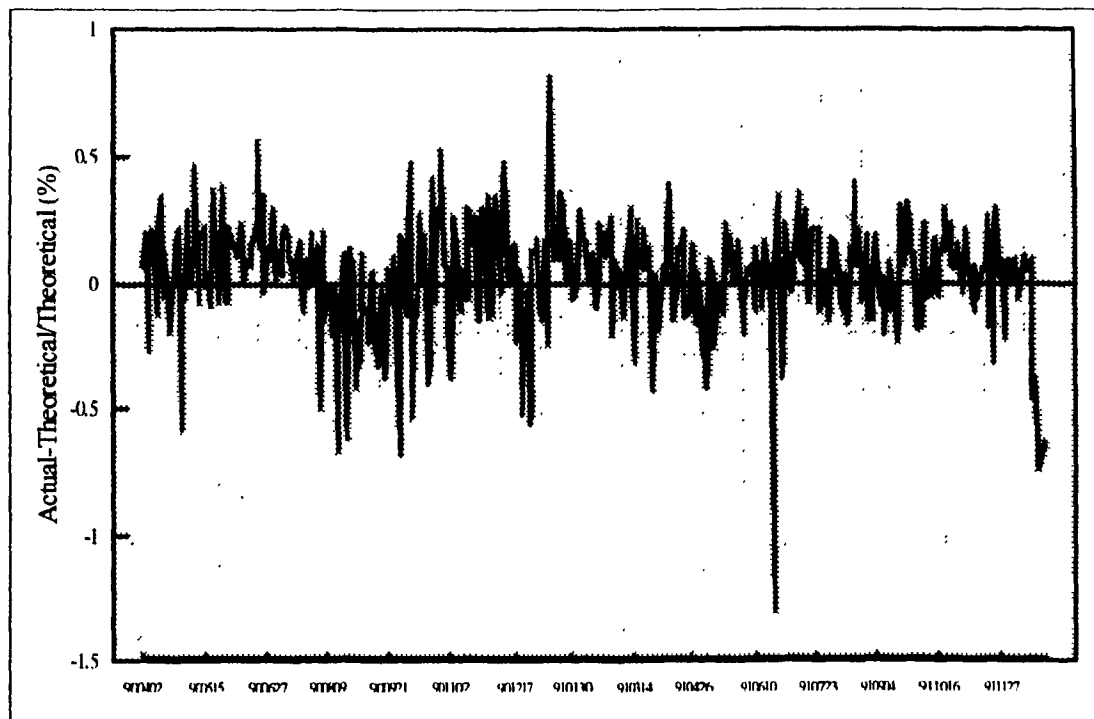
June 1, 1988 - February 28, 1990



**Graph 7.2 b**

Daily Mispricing (%) of the S&amp;P 500 Index Futures

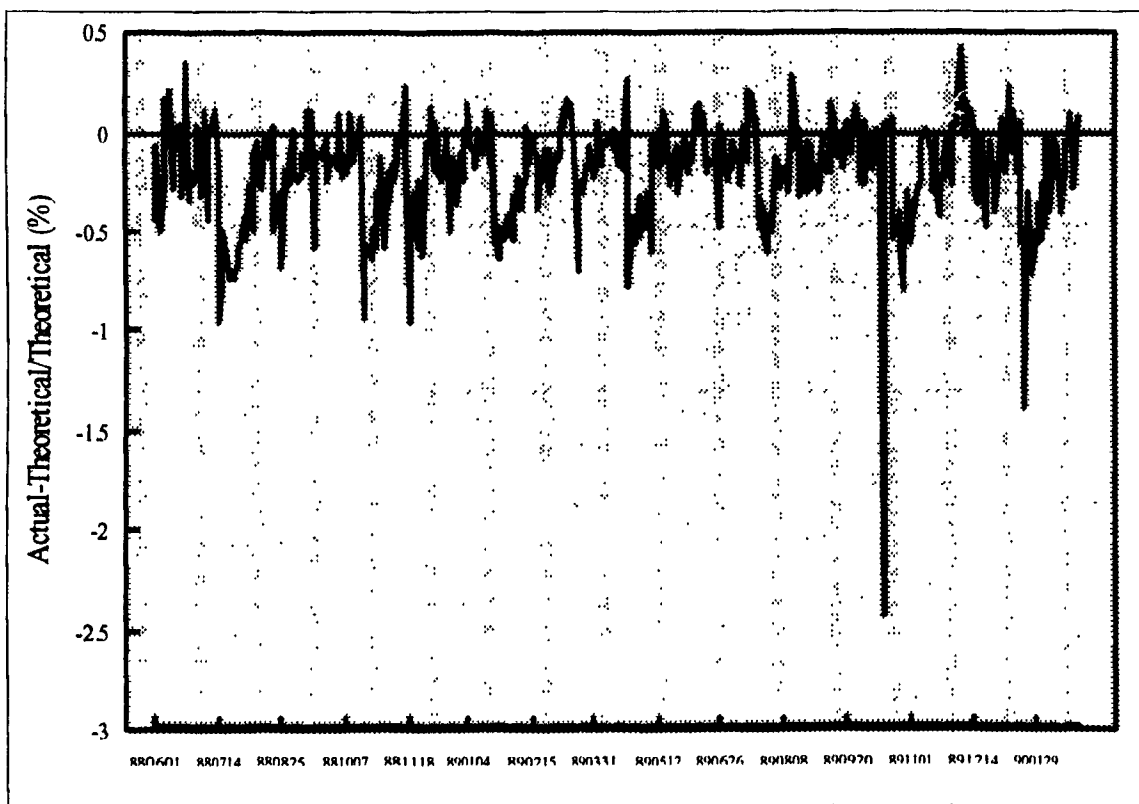
April 1, 1990 - December 31, 1991



**Graph 7.3 a**

Daily Mispricing (%) of the MMI Index Futures

June 1, 1988 - February 28, 1990

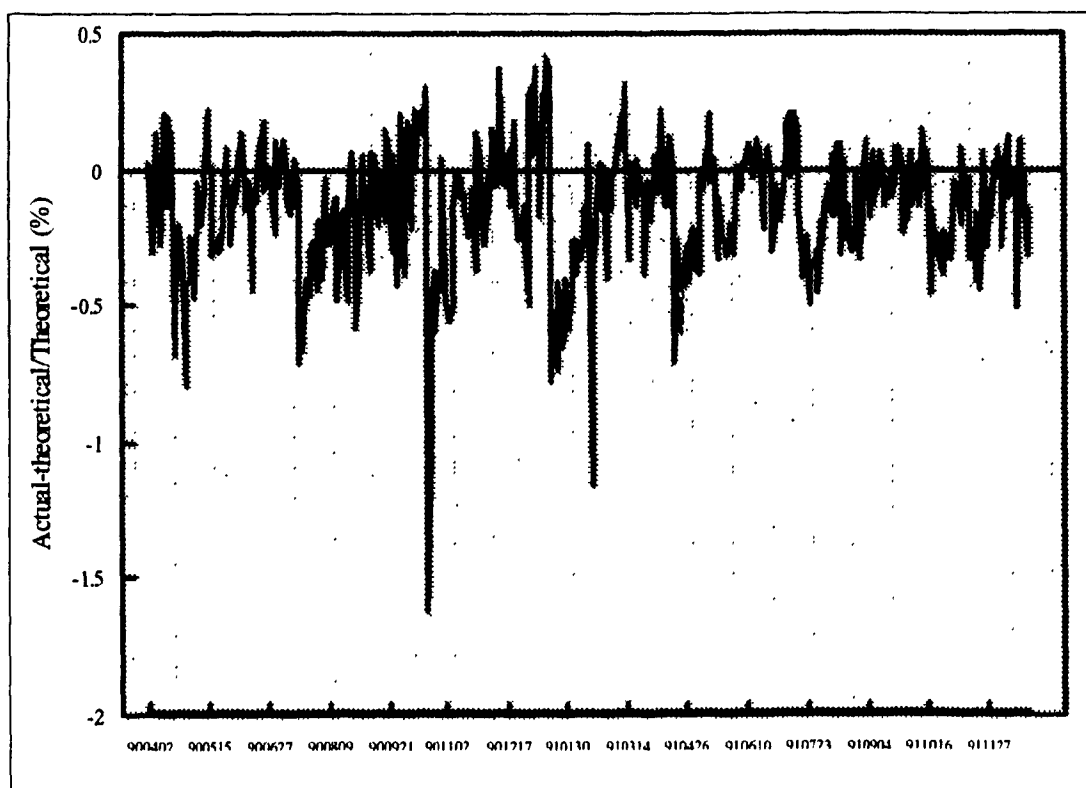




**Graph 7.3 b**

Daily Mispricing (%) of the MMI Index Futures

April 1, 1990 - December 31, 1991



The summary statistics on the average mispricing are reported in Table 7.2a, b, and c. The mispricing is calculated in exactly the same manner as above. The difference is divided by the theoretical price so that the inflationary factor can be eliminated.

**Table 7.2 a**

Summary statistics on the differences between Actual and Theoretical Future Price (Toronto 35 Index Futures) for three different time periods

	First Half <sup>a</sup>	Second Half <sup>b</sup>	Full Period <sup>c</sup>
Number of observations	441	440	903 <sup>d</sup>
Average mispricing	-0.21 (0.28) <sup>e</sup>	-0.06 (0.32)	-0.14 (0.31)
Average absolute values of mispricing	0.28 (0.21)	0.25 (0.21)	0.27 (0.21)
Average days to maturity	15.16 (9.15)	15.16 (9.12)	15.21 (9.16)

<sup>a</sup> includes the period June 1, 1988-February 28, 1990

<sup>b</sup> includes the period April 1, 1990-December 31, 1991

<sup>c</sup> includes the period June 1, 1988-December 31, 1991

<sup>d</sup> includes March 1990

<sup>e</sup> standard deviations are in parentheses

**Table 7.2 b**

Summary statistics on the differences between Actual and Theoretical Futures

Price (S&P 500 Index Futures) for three different time periods

	First Half	Second Half	Full Period
Number of observations	442	443	907
Average mispricing	0.05 (0.23)	0.03 (0.21)	0.04 (0.00)
Average absolute values of mispricing	0.17 (0.16)	0.15 (0.15)	0.16 (0.00)
Average days to maturity	45.95 (26.27)	46.37 (26.71)	46.20 (26.82)

**Table 7.2 c**

Summary statistics on the differences between Actual and Theoretical Futures

Price (MMI Index Futures) for three different time periods

	First Half	Second Half	Full Period
Number of observations	442	443	907
Average mispricing	-0.19 (0.25)	-0.14 (0.23)	-0.16 (0.25)
Average absolute values of mispricing	0.23 (0.23)	0.19 (0.18)	0.21 (0.20)
Average days to maturity	17.76 (9.05)	15.67 (9.06)	15.77 (9.10)

MacKinlay and Ramaswamy<sup>33</sup> found that the mispricing increases on average with maturities. The average maturities before and after March 1990 are computed in order to make the maturities of the futures within the two time periods comparable. As Table 6.2a, b, and c shows, the average days to maturities of the three indexes are not different between the first half and the second half of the testing period. However, the average daily mispricing of the T35 index futures has decreased significantly in the second half. The U.S. daily mispricing has also dropped. Consequently, we can say that the Canadian data shows less underpricing in the second half than in the first half. But it also shows that mispricing was apparent for the whole time period. The U.S. data shows that the S&P 500 overpricing has decreased and the MMI underpricing has decreased between the two time periods. A more reliable figure for mispricing is the average absolute values of mispricing because it eliminates the effect of positive and negative values. The absolute values of mispricing has also decreased in the second half for the all three indexes but the variability of mispricing has not changed in the Canadian case but decreased in the U.S. The t-stat for the significance of the difference in means between the first and second half is shown in Table 7.3. The t-stat to be used for testing the significance of the difference between two means of two normally distributed variables is given below

$$t\text{-stat} = \frac{\bar{X}_1 - \bar{X}_2}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1 + n_2 - 2}$$

where

$$S = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

<sup>33</sup> Mackinlay A. C., Ramaswamy, K. "Index-Futures Arbitrage and the Behavior of Stock Index Futures Prices." *The review of Financial Studies*, Vol. 1, No. 2, 1988

test hypothesis of difference of two means

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_a: \mu_1 - \mu_2 \neq 0$$

$$t_{1-\alpha/2, n_1 + n_2 - 2} = t_{0.975, 441 + 440 - 2} = t_{0.975, 879} = 1.960$$

If  $|t\text{-stat}| > 1.960$  reject  $H_0$  and conclude  $H_a$

Under the average mispricing, the values of the t-stat  $> 1.960$  (see Table 6.3) for the T35, and the t-stat  $< 1.960$  for the U.S. Thus, reject  $H_0$  and conclude the means were not the same between the two time periods for the Canadian data. On the other hand, the null hypothesis,  $H_0$ , is accepted for the U.S. data.

Nonetheless, the t-stat under the average absolute mispricing accepts the null hypothesis and we conclude that there is no difference in means between the two time periods between the two countries.

**Table 7.3**

t-stat for the difference in means between the two time periods  
under the average mispricing and average absolute mispricing

Portfolio	t-stat Average Mispricing	t-stat Average Absolute Values of Mispricing
T35	-4.065	0.971
S&P 500	0.634	0.755
MMI	1.528	1.134

These two results are in conflict and thus there is no one clear cut solution under this scenario to confirm that the presence of TIPs has reduced mispricing.

The difference in means hypothesis states on the one hand that TIPs has reduced the mispricing when looking at the average mispricing. On the other it concludes that there was no difference in mispricing when looking at the average absolute values of mispricing. The standard error tests conducted above in addition to the graphs concluded that mispricing has decreased but not eliminated in the second half. Therefore, there is no conclusive evidence that TIPs has contributed to the decrease in mispricing. It is difficult to find the source of the mispricing empirically. Investors can easily obtain the current index value and the risk free rate during the trading day, but they must estimate the dividend stream. The difficulty investors face in estimating the dividend stream is a major source of inefficiency in the market.

## CONCLUSION

This paper examines the effects of TIPs trading on index futures. After the introduction of TIPs, market participants began to wonder how index participation units are going to fare at the presence of index futures, options and mutual funds. The main question is whether TIPs can attract enough trading volume and if it does, whether it will take any significant volume away from similar products based on market index. Easy availability of a security that tracks the movement of stock index can contribute to the increased activity and market efficiency of other index products. But if such security substitutes more or less the role of index products in tracking the market movement, other index products will show reduced activity and market efficiency. Two attributes of efficiency on the Toronto 35 Index futures markets, namely, hedging efficiency and mispricing are tested to observe any change after the introduction of TIPs.

When measured with respect to hedging efficiency, it was determined that the minimum risk hedge portfolio was successful in reducing risk than the unhedged portfolio. Nonetheless, TIPs have shown to be less favorable in risk reduction than the T35. In other words, hedging TIPs with TXF resulted in less risk reduction than hedging the T35 with TXF. TIPs have also shown to have a lower correlation coefficient with the TXF and a higher basis risk than the T35. In addition, when comparing the two time periods in Canada, the second period has shown higher basis risk than the first. Therefore, when TIPs was assumed to increase hedging effectiveness and thus lower basis risk, reality

proved otherwise.

I reasoned that hedging TIPs with TXF is a cross hedge when compared to hedging T35 with TXF. Because TIPs track the T35 and the TXF is drawn on the T35 . Even though TIPs track the T35 it does not have a one to one connection with the TXF. In addition, I not only found TIPs to have a lower measure of hedging effectiveness than the T35 but also the period were TIPs was present had a lower measure of hedging effectiveness than the first period. The above analysis discredits TIPs in increasing hedging effectiveness. On the other hand, although TIPs as a portfolio was less superior to the index in terms of hedging effectiveness, it will be interesting to compare TIPs performance against, say, a mutual fund that is comprised of the same proportion as the T35.

The same kind of analysis was done in the U.S. and it was also shown that the minimum risk hedge resulted in lower variance than the unhedged portfolio. A cross hedge was formulated, namely, hedging the MMI with S&P 500 futures. The purpose of this strategy was to show that basis risk was higher than a direct hedge. This conclusion support the hypothesis that hedging TIPs with TXF can be viewed as a cross hedge.

When measured with respect to mispricing, no clear cut solution was deduced. First, the standard error was shown to have dropped from the first time period to the second one. Thus, concluding that mispricing has indeed decreased. The graphs also showed that underpricing was reduced in the period after TIPs was introduced. Furthermore, under the average mispricing hypothesis, it was shown that underpricing has decreased in the period when TIPs was being traded and the result was significant at the



1% level. Although, under the average absolute value of mispricing hypothesis underpricing was decreased this result was found to be insignificant. The U.S. data also showed that mispricing was reduced on the MMI and overpricing was reduced on the S&P 500 under both hypothesis. The U.S. data where no participation unit is being traded have shown similar conclusions to the Canadian market, that is, mispricing was reduced. Therefore, there is no conclusive evidence that TIPs has increased the efficiency of pricing futures contracts. Of course, this result may have been caused by some other economic conditions in the U.S.

In order to arrive at a solid conclusion on the effects of TIPs, one may need to examine other factors that are attributes of market efficiency such as bid/ask spreads. Also it might be appropriate to study the effects of index participation units in other markets. In the U.S., the American Stock Exchange recently (January 1993) introduced the S&P 500 Depository receipts, SPDRs, which represents a share in a unit trust that tracks the movement of the S&P 500 Index. Studies on the effects of the SPDRs on the S&P 500 Index futures may shed more light on the economic role of index participation units.

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