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HYPERBOLIC PARABOLOIDAL BASED WATER TANK

CHAN Chee Wah, John

A DISSERTATION
in
The Department
of
Civil Engineering.

Presented in Partial Fulfillment of the Requirement
for the degree of Master of Engineering at
Concordia University
Montreal, Quebec, Canada

August, 1975

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ABSTRACT
ABSTRACT

CHAN Chee Wah, John

HYPERBOLIC PARABOLOIDAL BASED WATER TANK

The main achievements of this dissertation are the analysis of a hyperbolic paraboloidal shell by the membrane theory and the design of the reinforcement with respect to the analysis. Before the analysis, there is a brief account of the derivation of the membrane theory from the general basic shell theory. Then, the membrane theory is applied to the hyperbolic paraboloidal shell.

The hyperbolic paraboloidal shell concerned is the base of the water tank of a water tower. After the internal stresses and forces have been analyzed, the numerical values are solved by computer. Then the reinforcement of the shell is designed by the working stress design.

There is a brief account on the state of the arts of the design of the water tank.

Finally, there is a discussion on the advantages and disadvantages of the method of analysis and design.
To my parents
& betrothed
ACKNOWLEDGEMENTS
ACKNOWLEDGEMENTS

The writer would like to express his gratitude to his supervisor, Dr. Z.A. Zielinski, for his encouragement and advice in the course of this dissertation.
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LIST OF NOTATIONS

x, y, z Directions of axes
N_x, N_y Internal normal forces per unit length
N_x y, N_y x Internal shearing forces per unit length
N_{x y}, N_{y x} Projections of internal normal forces on the x-y plane
N_{x y}, N_{y x} Projections of internal shearing forces on the x-y plane
Q_x, Q_y Internal transverse forces per unit length
M_x, M_y Internal normal moments per unit length
M_{x y}, M_{y x} Internal twisting moments per unit length
r_x, r_y Radii of curvatures
σ_x, σ_y Internal normal stresses
σ_1, σ_2 Principal stresses
τ_x, τ_y Internal shear stresses
U, V, W Displacements in x, y, z-directions, respectively
E, G Moduli of elasticity and rigidity
ν Poisson's ratio
ε_1, ε_2 Strain of middle surface in x, y-directions, respectively
η_x, η_y Change of curvatures before and after the deformation of the middle surface
η_{x y} The change of twist of the middle surface
r Shearing strain at the middle surface
X, Y, Z External loading in x, y, z-directions, respectively
ρ Density of Water
ρ_c Pressure due to concrete weight in z-direction
$f'_c$  Concrete strength

$ft_a$  Allowable tensile stress of concrete

$f_{ca}$  Allowable compressive stress of concrete

$f_s$  Allowable stress of steel

$f_c, f_s$  Internal concrete and steel stresses

$t$  Thickness of shell
CHAPTER 1

INTRODUCTION AND STATE OF THE ARTS
ON THE DESIGN OF WATER TOWERS
CHAPTER 1

INTRODUCTION AND STATE OF THE ARTS ON THE DESIGN OF WATER TOWERS

Elevated water tanks are essential to water supply systems. In the early twentieth century, water towers were built for supplying water to a certain factory or a certain urban area. Water towers are especially important for emergencies, e.g., sudden shortage of regular water supply. Sometimes, they are also built for balancing the water flow rate at peak demand of small inflow. More recently, water towers were built with only economical considerations in view while the architecture was not taken into account. Nowadays, water towers are built according to modern architectural design trends, [1], therefore as well as functioning in urban usage, they also provide a pleasant view in the city.

Although every water tower is unique, they have quite a lot of characteristics in common. Most of them are circular in order to minimize stresses and bending moments. They usually have a main shaft at the centre supporting the tank on top.

There are two ways in which concrete water towers can be constructed. The first method is to build it with ordinary formwork. The other way is to construct the central shaft, and then build the tank at ground level. After this, the tank is lifted up to the top by jacking.
Water towers must be protected from water leakage, therefore, no cracks must be allowed. To achieve this, the tank has to be in working stress design. In cold areas, water should be prevented from freezing in order to prevent shortage of supply, as well as cracking.

Water towers can be built in either steel or concrete but, for economic reasons, most towers are built of concrete. In more recent years water towers have been built of reinforced concrete, but the modern ones are usually built of pre-stressed concrete. One of the advantages of pre-stressed concrete is that it can help to reduce cracks if designed properly.

The tank of a water tower is usually a shell structure. Shell structures can be analyzed either by analytical method or by a numerical technique called finite element method. In this project, the main emphasis will be on the practice of the analysis of shell structure. The method of analysis used will be membrane theory derived from the general shell theory. The material used will be reinforced concrete. The loading considered will be symmetrically acting gravity loads.

The main subject of this project is to analyze and design the hyperbolic-paraboloidal shell, which is the base of the water tank. The design of the beams and the shaft will not be included.
CHAPTER 2

THEORY OF SHELLS
CHAPTER 2
THEORY OF SHELLS

2.1 GENERAL CONCEPT AND NOTATIONS

A shell is a curved surface structure. This definition implies that the thickness "t" is small compared with its other dimensions. The examples of shell structures are: the soap bubble, the body of an aeroplane, the dome roof of a building, the cylindrical water tank, etc.

Consider a small element with dimensions dy x dx, which is cut out from a shell as shown in Fig. 1.

The notations are as follows:
- \( x, y, z \) are mutually orthogonal.
- \( u, v, w \) are components of displacement.
- \( N_x \) and \( N_y \) are normal forces.
- \( N_{yx} \) and \( N_{xy} \) are shear forces.
- \( Q_x \) and \( Q_y \) are transverse forces.
- \( M_x \) and \( M_y \) are normal moments.
- \( M_{yx} \) and \( M_{xy} \) are twisting moments.

All \( N, Q \) and \( M \) values are called stress resultants and are forces and couples per unit length of the middle surface of the shell.

\( r_x \) and \( r_y \) are radii of curvatures.
FIG. 1

THIN SHELL NOTATION
FIG. 2. SECTION A-A OF FIG. 1.
2.2 THE RELATIONSHIP BETWEEN THE STRESS RESULTANTS AND STRESSES

Consider Figure 2; \[ dA = (r_x - z) \frac{dx}{r_x} \, dz \quad \text{(shaded part)} \]

\[ = (1 - \frac{z}{r_x}) \, dx \, dz \]

\[ N_y = \frac{1}{dx} \int_{-y_2}^{y_2} \sigma_y \, dA \]

where \( \sigma_y \) is the normal stress coming out of the paper.

\[ N_y = \frac{1}{dx} \int_{-y_2}^{y_2} \sigma_y (1 - \frac{z}{r_x}) \, dx \, dz \]

\[ N_y = \int_{-y_2}^{y_2} \sigma_y (1 - \frac{z}{r_x}) \, d\theta_y \] \[ \quad \ldots \quad (a) \]

Similarly,

\[ N_x = \int_{-y_2}^{y_2} \sigma_x (1 - \frac{z}{r_y}) \, dz \] \[ \quad \ldots \quad (b) \]

\[ N_{xy} = \int_{-y_2}^{y_2} \tau_{xy} (1 - \frac{z}{r_y}) \, dz \] \[ \quad \ldots \quad (c) \]

\[ N_{yx} = \int_{-y_2}^{y_2} \tau_{yx} (1 - \frac{z}{r_x}) \, dz \] \[ \quad \ldots \quad (d) \]

**Note:** \( \tau_{yx} \) and \( \tau_{xy} \) shown in Fig. 3 are shear stresses.

These shear stresses are equal:

\[ \tau_{yx} = \tau_{xy} \]

\[ Q_x = \int_{-y_2}^{y_2} \tau_{xz} (1 - \frac{z}{r_y}) \, dz \] \[ \quad \ldots \quad (e) \]

\[ Q_y = \int_{-y_2}^{y_2} \tau_{yz} (1 - \frac{z}{r_x}) \, dz \] \[ \quad \ldots \quad (f) \]
FIG. 3. SHEAR STRESS OF AN ELEMENT

\[ M_x = \int_{y_2}^{y_2} \sigma_x z (1 - \frac{z}{r_x}) \, dz \]  \hspace{1cm} \text{(g)}

\[ M_y = \int_{y_2}^{y_2} \sigma_y z (1 - \frac{z}{r_y}) \, dz \]  \hspace{1cm} \text{(h)}

\[ M_{xy} = \int_{y_2}^{y_2} \tau_{xy} z (1 - \frac{z}{r_y}) \, dz \]  \hspace{1cm} \text{(i)}

\[ M_{yx} = \int_{y_2}^{y_2} \tau_{yx} z (1 - \frac{z}{r_x}) \, dz \]  \hspace{1cm} \text{(j)}

There are altogether ten stress resultants that were derived as above. There is one interesting point to be noticed; even though \( \tau_{xy} = \tau_{yx} \), in general, \( N_{xy} \) is not equal to \( N_{yx} \), and \( M_{xy} \) is not equal to \( M_{yx} \) because \( r_x \) is not equal to \( r_y \). However, since \( t \) is much less than \( r_x \) and \( r_y \), \( \frac{z}{r_x} \) and \( \frac{z}{r_y} \) can be neglected if compared to unity, then we have...
\( N_{xy} = N_{yx} \) \text{ and } \ M_{xy} = - M_{yx} \\

We therefore have eight instead of ten stress resultants to be determined in a general case.

From the theory of elasticity [8]

\[
\sigma_x = \frac{E}{1-\nu^2} \left[ \varepsilon_x + \nu \varepsilon_y - \left( \eta_x + \nu \eta_y \right) \right] \quad 2(a)
\]

\[
\sigma_y = \frac{E}{1-\nu^2} \left[ \varepsilon_y + \nu \varepsilon_x - \left( \eta_y + \nu \eta_x \right) \right] \quad 2(b)
\]

\[
\tau_{xy} = G \left( \gamma - 2 \varepsilon \eta_{xy} \right) \quad 2(c)
\]

where \( \eta_x \) and \( \eta_y \) are change of curvatures before and after the deformation of the middle surface

\( \varepsilon_x \) and \( \varepsilon_y \) are strains of the middle surface in the \( x \) and \( y \) directions, respectively.

Other variations in the middle surface include:

\( \gamma \) is the shearing strain of the middle surface

\( \eta_{xy} \) is the change of twist of the middle surface

If we substitute Eq. (2) into Eq. (1)(a), (b), (c), (g), (h), (i), we get
\[ \begin{align*}
N_x &= \frac{Et}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) \\
N_y &= \frac{Et}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) \\
N_{xy} &= N_{yx} = \frac{Et}{2(1+\nu)} \gamma \\
M_x &= -D(\eta_y + \nu \eta_x) \\
M_y &= -D(\eta_x + \nu \eta_y) \\
M_{xy} &= -M_{yx} = D(1-\nu) \eta_{xy}
\end{align*} \]

where
\[ D = \frac{Et^3}{12(1-\nu^2)}. \]

All values of \( \epsilon_x, \epsilon_y, \eta_x, \eta_y, \) and \( \gamma \) are in function of the displacements \( u, v, \) and \( w \), therefore, we can write
\[ \begin{align*}
N_x &= N_x(u,v,w); \quad N_y = N_y(u,v,w); \quad N_{xy} = N_{xy}(u,v,w); \\
M_x &= M_x(u,v,w); \quad M_y = M_y(u,v,w); \quad M_{xy} = M_{xy}(u,v,w).
\end{align*} \]

see [6] and [10].

2.3 STATIC EQUILIBRIUM EQUATION OF A SHELL ELEMENT

Figures 4 and 5 show the middle surface of a differential element. \( d\alpha_x \) and \( d\alpha_y \) are the central angle of the element.

Five equations of equilibrium can be set as follows[6]:

[... continue text]
\[ \sum F_x = 0 \]
\[ \frac{\partial}{\partial x}(N_x r_y) + \frac{\partial}{\partial y}(N_y r_x) + N_{xy} \frac{\partial r_y}{\partial y} - N_y \frac{\partial r_x}{\partial x} - Q_x r_y + X_r r_y = 0 \] \hspace{1cm} 4(a)

\[ \sum F_y = 0 \]
\[ \frac{\partial}{\partial x}(N_y r_x) + \frac{\partial}{\partial y}(N_x r_y) + N_{yx} \frac{\partial r_x}{\partial x} - N_x \frac{\partial r_y}{\partial y} - Q_y r_x + Y_r r_x = 0 \] \hspace{1cm} 4(b)

\[ \sum F_z = 0 \]
\[ \frac{\partial}{\partial x}(Q_x r_y) + \frac{\partial}{\partial y}(Q_y r_x) + N_{rx} r_y + N_{ry} r_x + Z_r r_y = 0 \] \hspace{1cm} 4(c)

\[ \sum M_x = 0 \]
\[ \frac{\partial}{\partial x}(M_{xy} r_y) - \frac{\partial}{\partial y}(M_{yx} r_x) - M_{xy} \frac{\partial r_y}{\partial y} + M_{yx} \frac{\partial r_x}{\partial x} + Q_x r_x r_y = 0 \] \hspace{1cm} 4(d)

\[ \sum M_y = 0 \]
\[ -\frac{\partial}{\partial x}(M_{xy} r_x) + \frac{\partial}{\partial y}(M_{yx} r_y) + M_{xy} \frac{\partial r_x}{\partial x} - M_{yx} \frac{\partial r_y}{\partial y} + Q_y r_y r_x = 0 \] \hspace{1cm} 4(e)

From Equation 3 and Equation 4, altogether we have 11 equations and 11 unknowns, so we can solve the equations. The eleven unknowns include the eight stress resultants and the three displacements in the three directions.
(a) Stress resultants

(b) Stress couples

FIG. 4. DIFFERENTIAL ELEMENT.
FIG. 5 GEOMETRY OF DIFFERENTIAL ELEMENT
CHAPTER 3

THEORY OF SHELL SIMPLIFIED TO MEMBRANE THEORY
CHAPTER 3

THEORY OF SHELL SIMPLIFIED TO MEMBRANE THEORY

In most engineering shells, there are substantially no bending and twisting stress couples and no radial shear stress resultants. Therefore, the load is carried substantially by the membrane forces: \( N_x, N_y \) and \( N_{xy} \). All other stress resultants: \( M_x, M_y, M_{xy}, Q_x \) and \( Q_y \) are neglected and assumed to be zero. Bending and twisting couples and radial shears are only significant in the vicinity of the boundaries or near discontinuities in the loading or in geometry of the shell structure.

So, if we consider Fig. 4(a), neglecting transverse forces \( Q \) and considering the \( x \) and \( y \)-axis intersect at any angle of \( \omega \) instead of \( 90^\circ \), we get the following equations of static equilibrium in the \( x \) and \( y \)-direction, respectively[7]:

\[
\begin{align*}
\left[ N_x \frac{\partial N_x}{\partial x} \frac{dx}{2} \right] dy & - \left[ N_x - \frac{\partial N_x}{\partial x} \frac{dx}{2} \right] dy + \left[ N_{xy} + \frac{\partial N_{xy}}{\partial y} \frac{dy}{2} \right] dx \\
\left[ N_{xy} - \frac{\partial N_{xy}}{\partial y} \frac{dy}{2} \right] dx & + X(x,y) dx dy \sin \omega = 0 \quad \text{... (a)}
\end{align*}
\]

\[
\begin{align*}
\left[ N_y + \frac{\partial N_y}{\partial y} \frac{dy}{2} \right] dx & - \left[ N_y - \frac{\partial N_y}{\partial y} \frac{dy}{2} \right] dx + \left[ N_{xy} + \frac{\partial N_{xy}}{\partial x} \frac{dx}{2} \right] dy \\
\left[ N_{xy} - \frac{\partial N_{xy}}{\partial y} \frac{dy}{2} \right] dy & + Y(x,y) dx dy \sin \omega = 0 \quad \text{... (b)}
\end{align*}
\]

By simplifying:
\[ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -X \sin \omega \quad \text{(5a)} \]

\[ \frac{\partial N_y}{\partial y} + \frac{\partial N_{yx}}{\partial x} = -Y \sin \omega \quad \text{(5b)} \]

Considering the z-direction, and by modifying the equation at the bottom of page 167 [7]

\[ \frac{\partial}{\partial x} (N_x \frac{\partial z}{\partial x}) + \frac{\partial}{\partial y} (N_{xy} \frac{\partial z}{\partial y}) + \frac{\partial}{\partial x} (N_y \frac{\partial z}{\partial y}) + N_z (x, y) \sin \omega = 0 \]

Differentiating the products in the last equation

\[ N_x \frac{\partial^2 z}{\partial x^2} + 2N_{xy} \frac{\partial^2 z}{\partial x \partial y} + N_y \frac{\partial^2 z}{\partial y^2} = -Z \sin \omega \]

\[ -\left( \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \right) \frac{\partial z}{\partial x} - \left( \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) \frac{\partial z}{\partial y} \]

Making use of Eq. 5(a) and (b).

\[ N_x \frac{\partial^2 z}{\partial x^2} + 2N_{xy} \frac{\partial^2 z}{\partial x \partial y} + N_y \frac{\partial^2 z}{\partial y^2} = (X \frac{\partial z}{\partial x} + Y \frac{\partial z}{\partial y} - Z) \sin \omega \quad \text{(5c)} \]

It is important to note that the membrane theory solution is a statically determinate one, defined simply by the above three equilibrium equations and the boundary conditions of the structure.
CHAPTER 4

THE GEOMETRY OF THE HYPERBOLIC PARABOLOID
CHAPTER 4
THE GEOMETRY OF THE HYPERBOLIC PARABOLOID

The hyperbolic paraboloid has a doubly curved surface and the surface can be defined in two ways, either as a surface of translation or as a warped parallelogram. In the first case, the surface can be defined by translating or moving a vertical parabola having upward curvature over another parabola with downward curvature. This is shown in Figure 6, where the saddle-shaped surface is formed by moving parabola ABC over parabola BOP.

The hyperbolic paraboloid surface may also be generated along y-axis, a straight line that remains parallel to the xz plane at all times but pivots while sliding along the straight line ABC, (see Figure 7). The resulting surface is represented in Figure 7 by the grid of straight lines $h_n$ and $i_n$, and every point on it may be considered to be the intersection of two such lines contained in the surface. This surface can be visualized by considering the horizontal plane A'C'E'G' to be warped by vertically depressing corners A' and E' to new positions A and E.

The shape of the base of the water tank in this project will be generated from the second way.

Let us consider a quadrant in Figure 8:
\[ z = \frac{c}{b} y \]

\[ c = \frac{h}{a} x \]

Substitute 7 into 6.

\[ z = \frac{h}{ab} xy \]

If \( a = b \), then \( z = \frac{h}{a^2} xy \)

In Figure 9:

\[ x' = y \cos \frac{\omega}{2} + x \cos \frac{\omega}{2} \] \hspace{2cm} \text{9(a)}

\[ y' = y \sin \frac{\omega}{2} - x \sin \frac{\omega}{2} \] \hspace{2cm} \text{9(b)}

\[ 9(a) \cdot \sin \frac{\omega}{2} \]

\[ x' \sin \frac{\omega}{2} = y \cos \frac{\omega}{2} \sin \frac{\omega}{2} + x \cos \frac{\omega}{2} \sin \frac{\omega}{2} \] \hspace{2cm} \text{10(a)}

\[ 9(b) \cos \frac{\omega}{2} \]

\[ y' \cos \frac{\omega}{2} = y \cos \frac{\omega}{2} \sin \frac{\omega}{2} - x \cos \frac{\omega}{2} \sin \frac{\omega}{2} \] \hspace{2cm} \text{10(b)}

\[ 10(b) - 10(a) \]

\[ x' \sin \frac{\omega}{2} - y' \cos \frac{\omega}{2} = 2x \cos \frac{\omega}{2} \sin \frac{\omega}{2} \]

\[ 10(b) + 10(a) \]

\[ x' \sin \frac{\omega}{2} + y' \cos \frac{\omega}{2} = 2y \cos \frac{\omega}{2} \sin \frac{\omega}{2} \]
FIG. 8. A SKEW HYPERBOLIC PARABOLOID.

FIG. 9. NORMAL VIEW OF PLANE OBC'D OF FIG. 8.

note: $a = b$
(a special case)
\[ x = \frac{(x' \sin \frac{\omega}{2} - y' \cos \frac{\omega}{2})}{2 \cos \frac{\omega}{2} \sin \frac{\omega}{2}} \]

\[ y = \frac{(x' \sin \frac{\omega}{2} + y' \cos \frac{\omega}{2})}{2 \cos \frac{\omega}{2} \sin \frac{\omega}{2}} \]

Substitute into Equation 8:

\[ Z = \frac{h}{a^2} \sin \omega \left( x' \sin \frac{\omega}{2} - y' \cos \frac{\omega}{2} \right) \left( x' \sin \frac{\omega}{2} + y' \cos \frac{\omega}{2} \right) \]

\[ Z = \frac{h}{a^2} \sin \omega \left( x'^2 \sin^2 \frac{\omega}{2} - y'^2 \cos^2 \frac{\omega}{2} \right) \]

If \( x' \) is constant, we have:

\[ Z = \frac{h}{a^2} \sin \omega \left( \text{constant} - y'^2 \cos^2 \frac{\omega}{2} \right) \]

This formula is an expression of a parabola.

If \( y' \) is constant, then the formula also becomes an expression of a parabola.

If \( z \) is constant, then the formula becomes an expression of a hyperbola. That is how this structural shell gets its name.
CHAPTER 5

MEMBRANE THEORY APPLIED TO HYPERBOLIC PARABOLOID [11,12]
CHAPTER 5
MEMBRANE THEORY APPLIED TO HYPERBOLIC PARABOLOID [11,12]

In Chapter 3, it is shown that shell theory is simplified to membrane theory. In the last Chapter 4, the geometry of the hyperbolic-paraboloid is expressed as:

\[ z = kxy \sin \omega \quad \text{where} \quad k = \frac{h}{a^2 \sin \omega} \]

Note: In this dissertation, \( h = 14 \) ft. \( a = 70 \) ft.

\[ \frac{\partial z}{\partial x} = p = kxy \sin \omega \]

\[ \frac{\partial^2 z}{\partial x^2} = 0 \]

\[ \frac{\partial z}{\partial y} = q = kx \sin \omega \]

\[ \frac{\partial^2 z}{\partial y^2} = 0 \]

\[ \frac{\partial^2 z}{\partial x \partial y} = k \sin \omega \]

Substitute into Equation 5

\[ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -X \sin \omega \quad \text{..}12(a) \]

\[ \left( \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \right) = -Y \sin \omega \quad \text{..}12(b) \]

\[ 2k \sin \omega N_{xy} = (ky \sin \omega X + kx \sin \omega Y - Z) \sin \omega \quad \text{..}12(c) \]
FIG. 10  A SHELL ELEMENT WITH PROJECTION ON THE x-y PLANE
\[
\begin{align*}
\bar{N}_x &= N_x \sqrt{\frac{1 + q^2}{1 + p^2}} \\
\bar{N}_y &= N_y \sqrt{\frac{1 + p^2}{1 + q^2}}
\end{align*}
\]

and

\[
\bar{N}_{xy} = N_{xy}
\]

where \( N_x \) and \( N_y \) are stress resultants of the shell and \( \bar{N}_x \) and \( \bar{N}_y \) are the projections of the stress resultants on the \( x-y \) plane.
CHAPTER 6

THE SHAPE OF THE WATER TOWER
CHAPTER 6

THE SHAPE OF THE WATER TOWER.

In this project, the water tower being designed will be composed of a central shaft of one hundred feet in height, and ten feet in radius. The thickness of the wall of the shaft will be two feet. A tank will be supported on the top of the shaft. The base of the tank will be slanting outward at about thirty-five degrees from a height of about fifty feet to the top, (for exact dimensions, please refer to Figures 11 to 15). The base of the tank will be divided into six equal sectors radially. Between two sectors, there will be a beam slanting radially outward at a slope of $\frac{4}{3}$ from the shaft to the edge. Each sector will be a hyperbolic paraboloid. The shape of this water tower will be approximately the same as the Lahti Reservoir in Finland, (see [1], page 1441).
FIG. 10.a. THE LAHTI RESERVOIR IN FINLAND.
FIG. 11. SKETCH OF ONE SECTOR OF THE WATER TANK

FIG. 12. PLANE OF CE
(REFER TO FIG. 11)

Dimensions:
OE = 56'-0"
OB = 70'-0"
EB = 42'-0"
CB = 21'-0"
OC = 66'7757''
CF = 3'-6"

note: both diagrams are not to scale
FIG. 14. PLANE OECF (REFER TO FIG. 11)
SECTION A-A (REFER TO FIG. 13)

Scale: $\frac{1}{8''} = 1''-0''$

note: thickness not yet determined
FIG. 15. SECTION OF A BEAM  
SECTION B-B (REFER TO FIG. 13)  
Scale: \[ \frac{1}{12} = 1\text{-}0'\]  
Note: thicknesses not yet determined.
CHAPTER 7

ANALYSIS OF THE BASE OF THE TANK
BY, MEMBRANE THEORY.
CHAPTER 7
ANALYSIS OF THE BASE OF THE TANK
BY MEMBRANE THEORY

Before going into the analysis, a few assumptions have
to be made in order to simplify the problem.

(1) A triangular water pressure is formed normal to
plane ABD.

(2) The beams are flexible to prevent secondary moments
in the shells.

(3) No moment exists in the shell, or say, the moment can
be neglected, so that the membrane theory can be
applied. All moments will be sustained by the beam at
the connection with the shaft.

Recalling the formulii for hyperbolic paraboloid:

\[
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = -X \sin \omega \quad \ldots 12(a)
\]

\[
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = -Y \sin \omega \quad \ldots 12(b)
\]

\[
2k \sin \omega N_{xy} = (k_y \sin \omega X + k_x \sin \omega Y - Z) \sin \omega \quad \ldots 12(c)
\]

First, consider the water load:

\[
X = Y = 0
\]
Consider Figures 13, and 14:

\[ Z = \rho \left( 56 - \frac{64}{76 - 3151} x' \right) \quad \text{where} \quad \rho = \text{density of the water,} \]
as proven before:

\[ x' = x \cos \frac{\omega}{2} + y \cos \frac{\omega}{2} \]

\[ Z = \rho \left( 56 - \frac{64}{76 - 3151} x \cos \frac{\omega}{2} - \frac{64}{76 - 3151} y \cos \frac{\omega}{2} \right) \]

Therefore

\[ \frac{\partial \bar{N}_x}{\partial x} + \frac{\partial \bar{N}_{xy}}{\partial y} = 0 \quad \ldots 13(a) \]

\[ \frac{\partial \bar{N}_{xy}}{\partial x} + \frac{\partial \bar{N}_y}{\partial y} = 0 \quad \ldots 13(b) \]

\[ \bar{N}_{xy} = -\frac{Z \sin \omega}{2k \sin \omega} \]

\[ \bar{N}_{xy} = -\frac{\rho \left[ 56 - \frac{64 x}{76 - 3151} \cos \frac{\omega}{2} - \frac{64 y}{76 - 3151} \cos \frac{\omega}{2} \right]}{2k} \quad \ldots 13(c) \]

\[ \bar{N}_{xy} = -\frac{\rho \left[ 56 - (x+y) \frac{64}{76 - 3151} \cos \frac{\omega}{2} \right]}{2k} \quad \ldots 13(c) \]

Substitute \( \bar{N}_{xy} \) into Eq. 13(a) and (b).
\[
\frac{\partial N_x}{\partial x} - \frac{\partial}{\partial y} \left[ \phi \left( \frac{64}{76.3151} \frac{(x+y) \cos \omega}{2k} \right) \right] = 0
\]
\[
\frac{\partial N_y}{\partial y} - \frac{\partial}{\partial x} \left[ \phi \left( \frac{64}{76.3151} \frac{(x+y) \cos \omega}{2k} \right) \right] = 0
\]

by transferring and integrating, we get:

\[
N_x = -\frac{32}{76.3151} \frac{\cos \frac{\omega}{2}}{k} x + f_1(y) \quad \text{(a)}
\]
\[
N_y = -\frac{32}{76.3151} \frac{\cos \frac{\omega}{2}}{k} y + f_2(x) \quad \text{(b)}
\]

Referring to Fig. 16(a) on page 33, the boundary condition is determined as follows.

At the free edge, the shear force \( N_x \) and the force \( (N_x') \) normal to the edge are equal to zero. When these forces are resolved in the \( x' \)-direction, we get an equation of static equilibrium, as follows:

\[
N_x' \cos \frac{\omega}{2} + N_y' \cos \frac{\omega}{2} + N_{xy}' \cos \frac{\omega}{2} + N_{yx}' \cos \frac{\omega}{2} = N_x \cdot 1
\]

\[
(N_x + \bar{N}_y + \bar{N}_{xy} + \bar{N}_{yx}) \cos \frac{\omega}{2} = \bar{N}_x \cdot 2 \cdot \sin \frac{\omega}{2}
\]

But \( \bar{N}_x = 0 \) and \( \bar{N}_{xy} = \bar{N}_{yx} \)
\[ \bar{N}_x + \bar{N}_y + 2 \bar{N}_{xy} = 0 \] (i)

In the \( y' \)-direction, the equation of static equilibrium is as follows:

\[ \frac{1}{2} \sin \frac{\omega}{2} \left( \bar{N}_x + \bar{N}_{xy} \right) - \frac{1}{2} \sin \frac{\omega}{2} \left( \bar{N}_y + \bar{N}_{xy} \right) = \bar{N}_{x'y'} \cdot 1 \]

But \( \bar{N}_{x'y'} = 0 \), \( \bar{N}_{xy} = \bar{N}_{yx} \)

which gives

\[ \bar{N}_x = \bar{N}_y \] (ii)

Substitute (ii) into (i), and we get

\[ \bar{N}_x = -\bar{N}_{xy} \]

or \( \bar{N}_y = -\bar{N}_{xy} \)

Therefore, the boundary condition of the problem is:

when \( x + y = 70 \) ft. \( \bar{N}_x \) or \( \bar{N}_y = -\bar{N}_{xy} \) (iii)

substitute (iii) into Eq. (14).

\[
\frac{9}{2 k} \left[ \frac{56 - 70 \cdot \frac{64}{16.3151} \cos \frac{\omega}{2}}{2 k} \right] = - \left( \frac{P \cdot 32}{16.3151} \frac{\cos \frac{\omega}{2}}{k} \right)(70 - y) + f(y)
\]

\[
\frac{9}{2 k} \left[ \frac{56 - 70 \cdot \frac{64}{16.3151} \cos \frac{\omega}{2}}{2 k} \right] = - \left( \frac{P \cdot 32}{16.3151} \frac{\cos \frac{\omega}{2}}{k} \right)(70 - x) + f(x)
\]
FIG. 16a. AN ELEMENT AT THE BOUNDARY.

FIG. 16b. DEAD LOAD PER UNIT AREA.
\[ f_1(y) = \frac{\rho(56 - 70 \cdot \frac{64}{76.3151} \cos \frac{\omega}{2})}{2k} + \left( \frac{\rho \frac{32}{76.3151} \cos \frac{\omega}{2}}{k} \right)(70 - y) \]

and \[ f_2(x) = \frac{\rho(56 - 70 \cdot \frac{64}{76.3151} \cos \frac{\omega}{2})}{2k} + \left( \frac{\rho \frac{32}{76.3151} \cos \frac{\omega}{2}}{k} \right)(70 - x) \]

\[ N_x = \frac{\rho(56 - 70 \cdot \frac{64}{76.3151} \cos \frac{\omega}{2})}{2k} + \left( \frac{\rho \frac{32}{76.3151} \cos \frac{\omega}{2}}{k} \right)(70 - y - x) \] \[ \ldots 15(a) \]

and \[ N_y = \frac{\rho(56 - 70 \cdot \frac{64}{76.3151} \cos \frac{\omega}{2})}{2k} + \left( \frac{\rho \frac{32}{76.3151} \cos \frac{\omega}{2}}{k} \right)(70 - x - y) \] \[ \ldots 15(b) \]

7.1 DETERMINATION OF X, Y AND Z (CONCRETE'S OWN WEIGHT)

Consider Fig. 16(b)

\[ W_g = \text{weight of 1 sq.ft. of concrete projected on the } x-y \text{ plane.} \]

\[ W_g = \rho_c \sin \omega \sqrt{\phi} \] \[ \text{(see [11])} \]

where \( \rho_c \) is the pressure due to concrete weight in the z-direction.

\[ \phi = 1 + k^2x^2 + k^2y^2 - 2kxy \cos \omega \]

\[ z = \frac{4 + 56.72}{76.3151} \rho_c \sin \omega \sqrt{\phi} \]
\[ x' = -\frac{64}{76.3151} \rho_c \sin \omega \sqrt{\phi} \]

\[ X = Y = -\frac{64}{76.3151} \rho_c \sin \omega \sqrt{\phi} \frac{\sin \omega}{\sin(180^\circ - \omega)} \]

(see Fig. 16(c))

\[ X = Y = x' \frac{\sin \omega}{\sin(180^\circ - \omega)} \]

\[ X' = x' = -\frac{64}{76.3151} \rho_c \sin \omega \sqrt{\phi} \]

Fig. 16(c) COMPONENTS OF X'

Substitute X, Y, and Z into Eq. 12(c):

\[ \mathbf{N}_{xy} = \left( \frac{X}{2} \right) X + \left( \frac{Y}{2} \right) Y - \frac{Z}{2k \sin \omega} \sin \omega \]

\[ = -\frac{X}{2} \frac{64}{76.3151} \rho_c \sin \omega \sqrt{\phi} \frac{\sin \omega}{\sin(180^\circ - \omega)} \sin \omega \]

\[ -\frac{Y}{2} \frac{64}{76.3151} \rho_c \sin \omega \sqrt{\phi} \frac{\sin \omega}{\sin(180^\circ - \omega)} \sin \omega \]

\[ -\frac{Z}{2k \sin \omega} \frac{41.5692}{76.3151} \rho_c \sin \omega \sqrt{\phi} \sin \omega \]

\[ \ldots 16 \]

Let \( C = \frac{\sin \omega}{\sin(180^\circ - \omega)} \)
\[
\frac{\partial N_{xy}}{\partial x} = -\frac{32}{76.3151} p_c \sin^2 \omega C y \left[ \frac{1}{\phi} k(x - ky \cos \omega) \right]
- \frac{32}{76.3151} p_c \sin^2 \omega C \left[ \frac{x}{\phi} k(x - ky \cos \omega) \right]
- \frac{41.5692}{76.3151} \frac{p_c}{2} \left[ \frac{(kx - ky \cos \omega)}{\phi} \right] \sin \omega
\]

\[
\frac{\partial N_y}{\partial y} = -\gamma \sin \omega - \frac{\partial N_{xy}}{\partial x}
= +\frac{64}{76.3151} p_c \sin^2 \omega \sqrt{\phi} C
- \frac{32}{76.3151} p_c \sin^2 \omega C y \left[ \frac{1}{\phi} k(x - ky \cos \omega) \right]
+ \frac{32}{76.3151} p_c \sin^2 \omega C \left[ \frac{x}{\phi} k(x - ky \cos \omega) \right]
- \frac{41.5692}{76.3151} \frac{p_c}{2} \left[ \frac{(kx - ky \cos \omega)}{\phi} \right] \sin \omega
\]

If we let \( m = kx, n = ky; \frac{dx}{k} = \frac{dm}{k}, \frac{dy}{k} = \frac{dn}{k} \);
then \( \phi = \frac{1 + m^2 + n^2 - 2mn \cos \omega}{k} \)

Substitute \( \phi \) into Eq. (17).
\[
\frac{\partial N_y}{\partial n} = \frac{1}{k} \left( \frac{32}{76.3151} P_e \sin^2 \omega C \sqrt{\phi} - \frac{32}{76.3151} P_e \sin^2 \omega \frac{C m}{\sqrt{\phi}} \right) \\
+ \frac{32}{76.3151} P_e \sin^2 \omega C \cos \omega \frac{m}{\sqrt{\phi}} + \frac{32}{76.3151} P_e \sin^2 \omega C \frac{m n}{\sqrt{\phi}} \\
- \frac{32}{76.3151} P_e \sin^2 \omega C \frac{m^2}{\sqrt{\phi}} + \frac{32}{76.3151} P_e \sin^2 \omega C \cos \omega \frac{m n}{\sqrt{\phi}} \\
- \frac{41.5692}{76.3151} P_e \sin \omega \frac{m}{\sqrt{\phi}} + \frac{41.5692}{76.3151} P_e \cos \omega \sin \omega \frac{n}{\sqrt{\phi}} \right)
\]

\[
N_y = \frac{1}{k} \left( \frac{32}{76.3151} P_e \sin^2 \omega C \left[ \frac{1}{2} - \frac{1 + m^2 \sin^2 \omega}{1 + m^2 \sin^2 \omega} \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \right] \\
- \frac{1}{k} \frac{32}{76.3151} P_e \sin^2 \omega C \frac{m}{\sqrt{\phi}} \left[ \frac{\phi + m \cos \omega \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right)}{2} \right] \\
+ \frac{1}{k} \frac{32}{76.3151} P_e \sin^2 \omega C \cos \omega \left[ \frac{n}{2} + \frac{3 m \cos \omega}{2} \right] \sqrt{\phi} \\
+ \frac{3 m^2 \cos^2 \omega - 1 - m^2}{2} \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \right) \\
+ \frac{1}{k} \frac{32}{76.3151} P_e \sin^2 \omega C \left[ \frac{1}{2} - \frac{1 + m^2 \sin^2 \omega}{1 + m^2 \sin^2 \omega} \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \right] \\
- \frac{1}{k} \frac{32}{76.3151} P_e \sin^2 \omega C \frac{m}{\sqrt{\phi}} \left[ \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \right] \\
+ \frac{1}{k} \frac{32}{76.3151} P_e \sin^2 \omega C \cos \omega \left[ \frac{\phi + m \cos \omega \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right)}{2} \right] \\
- \frac{1}{k} \frac{41.5692}{76.3151} P_e \sin \omega \frac{m}{\sqrt{\phi}} \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \\
+ \frac{1}{k} \frac{41.5692}{76.3151} P_e \sin \omega \frac{m}{\sqrt{\phi}} \sinh^{-1} \left( \frac{n - m \cos \omega}{1 + m^2 \sin^2 \omega} \right) \\
+ f(m)
\]
From the boundary condition,
when \( x + y = 70 \), \( \vec{N}_y = -\vec{N}_{xy} \)
we get \( f(m) = -\vec{N}_y(m, 70k-m) - \vec{N}_{xy}(x, 70k-x) + f(m) \)

where the terms \( \vec{N}_y(m, 70k-m) \) and \( \vec{N}_{xy}(x, 70k-x) \) are obtained by
substituting \( 70k-m \) into \( n \) in Eqs. 18, and \( 70k-x \) into \( y \) in
Eq. 16, respectively.

Similarly, \( \vec{N}_x \) can be analyzed in the same way. In the
computer program, stress resultants \( \vec{N}_x, \vec{N}_y \) and \( \vec{N}_{xy} \) are computed
by Eq. 12(d).

7.2 DETERMINATION OF PRINCIPAL STRESSES [11]

Angle \( \alpha \) in Figure 10 formed by two intersecting generatrix is obtained by the following formula:

\[
\cos \alpha = \frac{\rho q + \cos \omega}{\sqrt{(1+p^2)(1+q^2)}}
\]

Consider Figure 17, normal and tangential stresses \( \sigma_\beta \),
\( \tau_\beta \) on any section whose normal forms an angle \( \beta \) with the normal
to the x-axis, are obtained by consideration of the equilibrium
in \( \sigma_\beta \) and \( \tau_\beta \) directions, respectively (see Fig. 17).
\[ \sigma_\beta = N_x \frac{\sin \beta}{\sin \alpha} + 2 N_y \frac{\sin \beta \sin (\beta - \alpha)}{\sin \alpha} + N_y \frac{\sin^2 (\beta - \alpha)}{\sin \alpha} \quad \ldots \text{19(a)} \]

\[ \tau_\beta = -N_x \frac{\sin \beta \cos \beta}{\sin \alpha} - N_y \frac{\sin (2 \beta - \alpha)}{\sin \alpha} - N_y \frac{\sin (\beta - \alpha) \cos (\beta - \alpha)}{\sin \alpha} \quad \ldots \text{19(b)} \]

Directions of principal stresses are determined by setting \( \tau_\beta \) equal to zero as in Eq. 19(b).

\[ \tan 2\theta = \frac{2 N_y \sin \alpha + N_x \sin 2\alpha}{N_x + 2 N_y \cos \alpha + N_y \cos 2\alpha} \quad \ldots \text{20} \]

where \( \theta \) and \( 90^\circ - \theta \) are the angles of principal stresses with the normal to the \( x \)-axis.

Values of principal stresses are:

\[ \sigma_1 = N_x \frac{\sin^2 \theta}{\sin \alpha} + 2 N_y \frac{\sin \theta \sin (\theta - \alpha)}{\sin \alpha} + N_y \frac{\sin^2 (\theta - \alpha)}{\sin \alpha} \quad \ldots \text{21(a)} \]

\[ \sigma_2 = N_x \frac{\cos^2 \theta}{\sin \alpha} + 2 N_y \frac{\cos \theta \cos (\theta - \alpha)}{\sin \alpha} + N_y \frac{\cos^2 (\theta - \alpha)}{\sin \alpha} \quad \ldots \text{21(b)} \]

Values are calculated in the program, as shown in the Appendix.

The projection of angle \( \theta \) onto the \( x-y \) plane can be
found by the following formula [12]:

\[
\tan \bar{\theta} = \frac{\xi \sin \theta \sin \phi}{\cos \phi \sin \phi + \sin \theta (\xi \cos \phi - \cos \phi)}
\]

where \( \xi = \frac{1 + p^2}{1 + q^2} \)

Values of \( \bar{\theta} \) are computed in the computer program and the output is shown in Appendix 3.
CHAPTER 8

DESIGN OF THE HYPAR- SHELL
CHAPTER 8
DESIGN OF THE HYPAR-SHELL

Although the thickness of the whole shell is assumed to be two feet, yet in order to save material, the thickness and the amount of steel can be reduced from the shaft to the edge of the tank.

Say, the thickness of the shell starts from 18 inches at the shaft and reduces towards the edge, as shown in Figure 19. From the output of the computer program, values of the principal stresses PS1 and PS2 are calculated. Since the loading and the shell is symmetrical along the $x'$-axis, consequently, the principal stresses and their directions are also symmetrical on both sides. The directions of the principal stresses are approximately shown in Figure 18.

The layout of reinforcement is arranged in the direction of the $x$ and $y$-axis. Since the directions of the principal stresses deviate minutely from the $x'$-axis, they are all assumed to be parallel or perpendicular to the $x'$-axis. The reinforcements will be designed with respect to the components of the principal stresses resolved into the $x$ and $y$-direction. The principal compressive stresses (PS2) are all so small, the design of reinforcement will be governed by the principal tensile stresses (PS1).
MAGNITUDE OF PRINCIPAL STRESS

- tension
- compression

DIRECTION OF PRINCIPAL STRESS
angles in degrees.

FIG. 18.
Say $f'c = 4,000$ psi

$f_{ta} =$ allowable tensile stress of concrete

$= 20\%$ of $f'c = 0.2 \times 4,000 = 800$ psi $[15]$. 

$f_{cd} =$ allowable compressive stress of concrete

$= 0.45 f'c = 1.8$ psi

$fs =$ allowable stress of steel $= 20$ ksi

Consider $x = 0$, $y = 10$ ft, $t = 18"$

$P_{sl} =$ 0.677 ksi

Internal tensile force per foot $= 0.677 \times 12 \times 24 = 194.98$ kips

Force resisted by concrete $= 12 \times 18 \times 0.8 = 172.8$ kips/ft

Force resisted by steel $= 194.98 - 172.8 = 21.68$ kips/ft

Area of steel required $= \frac{21.68}{2} \times \frac{1}{\sin^2(\frac{\pi}{2})} \times \frac{1}{20} = 6.02$ sq. in/ft.

Use #8 bars at 6 in c/c (4 layers).

Consider $x = 0$, $y = 20$ ft, $t = 15$

$P_{sl} =$ 0.570 ksi

Internal tensile force $= 0.570 \times 12 \times 24 = 166.18$ kips/ft

Force resisted by concrete $= 12 \times 15 \times 0.8 = 144$ kips/ft

Force resisted by steel $= 166.18 - 144 = 22.18$ kips/ft

Area of steel required $= \frac{22.18}{2} \times \frac{1}{\sin^2(\frac{\pi}{2})} \times \frac{1}{20} = 6.16$ sq. in/ft.

Use #8 bars at 6 in c/c (4 layers).
Similarly, at \( x = 0, \quad y = 30 \text{ ft}, \quad t = 12'' \)

\[ PS1 = 0.463 \text{ ksi} \]

Internal tensile force = 133.34 kips/ft
Force resisted by concrete = 115.2 kips/ft
Force resisted by steel = 18.14 kips/ft
Area of steel required = 5.04 sq.in/ft

Use \( \#8 \) bars at 5\( \frac{1}{2} \) in c/c (3 layers).

At \( x = 0, \quad y = 40 \text{ ft}, \quad t = 12'' \)

\[ PS1 = 0.356 \text{ ksi} \]

Internal tensile force = 102.5 kips/ft
Force resisted by concrete = 115.2 kips/ft
Therefore, no steel is required.

So, to the edge of the tank, 2 layers of \( \#6 \) bars at 12'' c/c are used. Also, in order to be safe, four \( \#6 \) bars are added along the edge, (see Figure 20).
FIG. 19. THICKNESS OF THE SHELL
SECTION A-A (REFER TO FIG. 13)
FIG. 20. REINFORCEMENT ARRANGEMENT OF THE SHELL.

Note: A similar arrangement of steel will run parallel to the x-axis.
CHAPTER 9

DISCUSSION
CHAPTER 9
DISCUSSION

Having seen the great advantages of hyperbolic paraboloidal shells, the writer ventured to apply this shell to the water tank being designed and to estimate the significance in structural and economical points-of-view.

Many hyperbolic paraboloidal shell roofs, having a square shape on the x-y plane, and having a uniformly distributed load in the z-direction, if analyzed by the membrane theory, consist only of shear forces ($N_{xy}$). In contrast, the base of the water tank being designed in this dissertation, is a skew hyperbolic paraboloidal shell, having an isosceles triangle (half rhombic) on the x-y plane, and having a triangular distributed live load in the z-direction and dead load in the x and y-direction. After being analyzed by the membrane theory, it consists of direct forces ($N_x, N_y$) as well as shear forces ($N_{xy}$).

The other reason which the writer has applied a hyperbolic paraboloidal shell to the base of the water tank is: having noticed most of the tanks of water towers are in inverted cone shape. Thus, writer would like to investigate whether a hyperbolic paraboloidal shell in the tank could help in the saving of materials which imply to a hyperbolic paraboloidal shell roof, such as that described in the previous paragraph. The result of analysis and design shows, in this particular case, a saving of a lot of material. The thickness of this shell is one-and-a
half feet at the shaft which is much less than the thickness of many other water tanks which are of inverted cone shape. The amount of steel used in this water tank is also less.

The membrane theory is not an ideal method used for analyzing this particular shell structure. Cracks which must be free from in-water tanks may occur in the 'x'–direction on the outer face and 'y'–direction in the inner face of the tank due to the moments which are neglected in the membrane theory. Therefore, the writer suggests that at the present stage, finite element method, which moments can be included, is the best tool to analyze this shell structure. Due to its simplicity, the membrane theory is best used as a preliminary analysis and design, so as to estimate the amount of materials being used.

The generation of hyperbolic paraboloids by straight lines will provide an advantage on the construction of the shell. If formwork and reinforcements are placed in these directions of straight lines, no bending of wooden plates and steel bars are required.

If the meshes of reinforcement are designed to run along the straight lines which are in the direction of the x and y-axes, a lot of steel will be required. Reinforcement can be reduced by placing bars in the direction of the principal stresses, which are in the x' and y'-directions. Furthermore, prestressed steel can be applied to the direction of the principal tensile stresses to reduce requirements of steel.
In conclusion, a hyperbolic paraboloidal shell does furnish some advantage to the construction, design and economy of this water tank, as well as providing a graceful shape to the structure.
BIBLIOGRAPHY
BIBLIOGRAPHY

References for the General Information of Water Towers


References for the Theory of Shell and Membrane Theory


References for The Analysis of Hypobolic Paraboloid
By Membrane Theory


References for the Design of the Shell


APPENDIX 1

COMPUTER PROGRAM - NOTATIONS
APPENDIX B

LIST OF NOTATIONS IN THE COMPUTER PROGRAM

T       Thickness of shell in feet
DC      Pressure due to concrete weight in z-direction
DW      Density of water
SOMD2   $\sin \frac{\omega}{2}$
COMD2   $\cos \frac{\omega}{2}$
COSOM   $\cos \omega$
SINOM   $\sin \omega$
TH      Angle $\theta$
PS1, PS2  Principal stresses
XK      Constant $k$
SINAF, COAF  $\sin \alpha \cdot \cos \alpha$
SNXBD, SNYBD  Direct stresses due to dead load
SNXBL, SXYBL  Direct stresses due to live load (water)
SNXBT, SNYBT  Total direct stresses
SNXYD  Shear stress due to dead load
SNXYL  Shear stress due to live load
SNXYT  Total shear stress
APPENDIX 2

COMPUTER PROGRAM
PROGRAM MEMTHE (INPUT, OUTPUT).
DIMENSION COSA(15,15),
DIMENSION SNYBD(16,16), SNYBL(16,16), SNYBT(16,16), SNXYD(16,16)
DIMENSION SNXYT(16,16), SNXYL(16,16), TH(16,16), PSI(15,15)
DIMENSION PS2(16,16), SNXBT(16,16), TB(16,16)
DIMENSION SO(15,15), FF(15,15)

C T IS THE THICKNESS OF THE SHELL.
T=2.
DC=150.*T

C = DIMENSION 62.5

C CALCULATION OF COEFFICIENTS
SOMD2= 3.
COMD2= SQR1(-3.2)
COSOM= 1.-2.*SOMD2**2
SINOM= 2.*SOMD2*COMD2
XK= 14.*77.1**2/SINOM
C1 = SQR1(80.**2-24.*2)
CK1 = 32.*DC*SINOM*SOMD2/XK/C1
CK2 = CK1/2.
CK3 = CK2*SINOM
CK4 = CK1*COSOM

CK5 = 41.5692*SINOM/XK/C1/2.
CK6 = CK5*SINOM
CK7 = CK1*CK2.
CK8 = (CK1*CK8)*70.*CK5
CK9 = 6.*(CK1*COSOM)
CK10 = 2.*CK

CK11 = = (-1)**2.*COMD2/C1/XK

C CLEAR ARRAY
DO 9 I=1,15
DO 9 J=1,15
SNYBD(I,J)=0.0
SNYBL(I,J)=0.0
SNYBT(I,J)=0.0
SNXYD(I,J)=0.0
SNXYL(I,J)=0.0
SNXYT(I,J)=0.0
TH(I,J)=0.0

TB(I,J)=0.0
PSI(I,J)=0.0
9 P52(I,J)=0.0

C CALCULATION OF STRESSES
DO 10 I=1,15
X = I**5-5
XM=XK*X
N=16-I
DO 10 J=1,N;
Y = J**5-5
XM=XK*X
GNC=M = XM-COSOM
SOPH = SQR1(1.0+XM**2+XM**2-2.*XM*XN*COSOM)
SOM = SQR1(1.0+XM**2*SINOM**2)
GDSQ = GNC/SOM
LOGD = ALOG(GDSQ+SQR1(GDSQ**2+1.0))

C SNXYD IS THE SHEAR STRESS DUE TO DEAD LOAD
SNXYD(I,J) = -(CK1*XX*X(Y)*CK5*SOPH/1000.)
V1 = CK7*(GNMC*SQPH+SQ1**2*LOGD)
V2 = (CK4*XK-CK1*XM+CK6)*SQPH+XMCOSOM*LOGD
V3 = CK3*((XN+3*XMCOSOM)*SQPH+(3*(XMCOSOM)**2-1*-XMCOSOM)**2)*LOGD
1)
V4 = -(CK1*XM**2+CK5*XK)*LOGD
XN = 70.0 * XK - XM
GNMC = XN.*XM*COSOM
SQPH = SORT(1.0*XMCOSOM)**2-2.*XM*XN*COSOM)
GSQ = GNMC/SQ1M
LOGD = ALOGD(GDSQ+SORT(GDSQ**2+1.0))
V5 = -CK7*(GNMC*SQPH+SQ1**2*LOGD)
V6 = -(CK4*XK+CK1*XM+CK6)*SQPH+XMCOSOM*LOGD
V7 = -CK3*((XN+3*XMCOSOM)*SQPH+3*(XMCOSOM)**2-1*-XMCOSOM)**2)*LOGD
1)
V8 = (CK1*XM**2+CK5*XK)*LOGD
V9 = CK6*SQPH
VYD = (V1+V2+V3+V4+V5+V6+V7+V8+V9)
N=CK*KY*SINOM
Q=CK*KX*SINOM
QS=Q**2
PS=P**2

C SNYBD IS THE DIRECT STRESS DUE TO DEAD LOAD Y DIR
SNDYBD(I,J) = VYD*SORT(1.0*QS)/(1.0*PS)/1000.
VII = CK11*(70.0-X-Y)
C SNYBL IS THE DIRECT STRESS DUE TO LIVE LOAD Y DIR
SNDYBL(I,J) = VYD*SORT(1.0*QS)/(1.0*PS)/1000.
C SNYBT IS THE TOTAL DIRECT STRESS DUE TO DEAD PLUS LIVE LOAD Y DIR
SNDYBT(I,J) = SNYBD(I,J)+SNYBL(I,J)
C SNXYL IS THE SHEAR STRESS DUE TO LIVE LOAD
SNXYL(I,J) = -0.6*(CK9*(X+Y))/CK10/1000.
C SNYXT IS THE SHEAR STRESS DUE TO LIVE PLUS DEAD LOAD
10 SNXYT(I,J) = SNXYD(I,J)+SNXYL(I,J)
DO 20 I=1,15
20 SNXBT(J,J)=SNYRT(I,J)
DO 21 J=1,15
X=I**5+5
N=1+1
DO 21 J=1,N
Y=J*5+5
P=KX*KY*SINOM
Q=KX*KX*SINOM
QS=Q**2
PS=P**2
FF(I,J) = P*Q*COSOM
SQ(1,J) = SORT(1.0*PS)/(1.0*QS))
COSAF=FF(I,J)/SQ(I,J)
COSA(I,J) = COSAF
SINA = SORT(1.0-COSAF**2)
SIN2A = 2*COSAF*SINA
COS2A = 1.0**2*SINA
XX = 2.0*SNXYT(I,J)+SINA+SNYBT(I,J)*SIN2A
YY = SNXBT(I,J)+2.0*SNXYT(I,J)*COSAF+SNYBT(I,J)*COS2A
TAN2 = XX/YY
THI = ATAN(TAN2)
TH(I,J) = THI/2.
}
SINTH = SIN(TE) 
PHI = SQRT((1.0*PS)/(1.0*QS)) 
TANB = PHI*SIN(TE)*SINOM/(COS(TE)*SINAF+sin(TE)*(PHI*COSOM-COSAF)) 

TB(I,J) = ATAN(TANB) 
STHMA = -SINAF*COS(TE)*SIN(TE)*COSAF 
C = COS(TE)*COSAF + SINAF*SIN(TE) 

PS1 AND PS2 ARE THE PRINCIPAL STRESSES DUE TO DEAD PLUS LIVE LOAD 

PP = SNXBT(I,J)*SIN(TE)**2 
QQ = 2.0*SNXYT(I,J)*SIN(TE)*STHMA 
RR = SNYBT(I,J)*STHMA**2 

PS1(I,J) = PP*QQ/RR 

C PRINT OUT OF RESULTS

PRINT 888 
PRINT 998, 
DO 11 I = 1,15 
PRINT 999, (SNYBD(I,J), J=1,15)
999 FORMAT (6X, 15F7.1) 
II = I*5-5 

11 PRINT 997+II 
PRINT 887 
PRINT 998 
DO 12 I = 1,15 
PRINT 999, (SNYBL(I,J), J=1,15)
II = I*5-5 

12 PRINT 997+II 
PRINT 886 
PRINT 998 
DO 13 I = 1,15 
PRINT 999, (SNYBT(I,J), J=1,15)
II = I*5-5 

13 PRINT 997+II 
PRINT 885 
PRINT 998 
DO 14 I = 1,15 
PRINT 999, (SNXYD(I,J), J=1,15)
II = I*5-5 

14 PRINT 997+II 
PRINT 884 
PRINT 998 
DO 15 I = 1,15 
PRINT 999, (SNXYL(I,J), J=1,15)
II = I*5-5 

15 PRINT 997+II 
PRINT 883 
PRINT 998 
DO 16 I = 1,15 
PRINT 999, (SNXYT(I,J), J=1,15)
II = I*5-5 

16 PRINT 997+II 

888 FORMAT ("II", " DIRECT STRESS DUE TO DEAD LOAD KIPS PER FT") 
887 FORMAT ("II", " DIRECT STRESS DUE TO LIVE LOAD KIPS PER FT") 
886 FORMAT ("II", " TOTAL DIRECT STRESS KIPS PER FT") 
885 FORMAT ("II", " SHEAR STRESS DUE TO DEAD LOAD KIPS PER FT") 
884 FORMAT ("II", " SHEAR STRESS DUE TO LIVE LOAD KIPS PER FT")
883 FORMAT ("1", " TOTAL SHEAR STRESS KIPS PER FT")
996 FORMAT (6X,16F7.3)
998 FORMAT("0","
  Y = 0  Y = 5  Y = 10  Y = 15  Y = 20  Y = 25  Y = 30
  1  Y = 35  Y = 40  Y = 45  Y = 50  Y = 55  Y = 60  Y = 65  Y = 70")
997 FORMAT ("*", "X = ", 12)
STOP
END
APPENDIX 3

COMPUTER OUTPUT
**COMPUTER OUTPUT**

**DIRECT STRESS DUE TO DEAD LOAD, KIPS PER FT.**  (1)

**DIRECT STRESS DUE TO LIVE LOAD, KIPS PER FT.**  (2)
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PS1 TOTAL PRINCIPAL STRESS, KIPS PER SQ. IN. (3)

PS2 TOTAL PRINCIPAL STRESS, KIPS PER SQ. IN. (4)
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<th>(5)</th>
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(7)