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Junction flow in open channels

Duc minh Tran

A Thesis

in

The Department of
Civil Engineering

Presented in partial fulfillment of the Requirements for the degree of Doctor of Philosophy at Concordia University Montréal, Québec, Canada 1988

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ABSTRACT

Junction flow in open channels

Duc Minh Tran
Concordia University, 1988

Channel junctions are of considerable importance in the study of open channels. The complex patterns of flow that result at the confluence of two streams are difficult to analyse. Usually, one identifies two classes of flow configurations: combining flow and dividing flow. In combining open channel flow, blockage at the junction denotes the rise in the stage due to the effect of the lateral inflow from the branch channel. Blockage phenomenon has been recognized and studied in the past. One serious difficulty encountered in the earlier studies is related to the estimation of the momentum transfer from the branch to the main channel. In the present study, an estimate of the transfer of lateral momentum at rectangular channel junctions is provided on the basis of experimental data. Using this estimation, a relationship between the depth of flow at the junction and the ratio of the lateral discharge to the total discharge is derived on the basis of the momentum principle. Another relationship related to blockage is also derived on the basis of the energy principle. The verification of these relationships is made on the basis of experimental data pertaining to present and past studies.

In the case of dividing open channel flow, the problem is more complicated, hence, comprehensive analytical studies do not seem to exist. In previous studies, the transfer of momentum from the main channel to the branch channel was evaluated experimentally. In the present study, this transfer is evaluated analytically and experimentally. Using this evaluation, a relationship involving the discharge ratio, the Froude numbers in the main channel upstream and downstream of the junction is derived on the basis of the momentum principle. A similar relationship is also derived for open channel flow through floor outlets. These relationships are validated by experimental data. They could be used to predict the division of flow at the branching point of open channel flows. Practical applications of the above flows occur in the study of river hydraulics and also in the design of open channel networks of treatment plants.
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Momentum transfer coefficient \( k = 0.82 \)

1. \( F_{d1} = 0.2 \)
2. \( F_{d1} = 0.3 \)
3. \( F_{d1} = 0.4 \)
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Momentum transfer coefficient \( k = 0.3 \)

1. \( F_{d1} = 0.2 \)
2. \( F_{d1} = 0.3 \)
(3) $F_{d1} = 0.4$
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Momentum transfer coefficient \( k = 0.9 \)

(1) \( F_{d1} = 0.2 \)
(2) \( F_{d1} = 0.3 \)
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The following symbols are used in the thesis:

1) Common notations

\( \alpha \) = kinetic energy flux correction coefficient;
\( \beta \) = momentum flux correction coefficient;
\( \gamma \) = specific weight of water;
\( \theta, \theta' \) = junction angle;
\( \rho \) = density of water;
\( F \) = Froude number;
\( g \) = acceleration due to gravity;
\( n \) = Manning’s coefficient;
\( Q \) = discharge;
\( R \) = hydraulic radius;
\( R_e \) = Reynolds number;
\( V \) = mean velocity;
\( Y \) = flow depth.

2) Combining flow (Fig.3)

\( \delta \) = angle formed by \( V_2' \) and the main channel axis;
\( b \) = width of the channels;
\( b' = b_2/b_1 \);
\( E \) = total head;
\( E_j \) = average total head at the junction;
$h_f$ = friction loss;

$h_l$ = energy loss;

$h_m$ = mixing loss;

$P_u$ = pressure force on the wall CD;

$P_d$ = pressure force on the wall AB;

$R_q = Q_2/Q_3$ (discharge ratio);

$R_y = Y_1/Y_3$ (dimensionless depth rise);

$V_{2'}$ = resultant velocity of the branch channel at section AD;

$X = AH$;

$X' = X/b_1$; and

$Y_c$ = water depth at stagnation zone.

Subscripts:

1 = flow conditions at section EF;
2 = flow conditions at section CB;
3 = flow conditions at section HG; and

$c$ = critical condition.

3) Dividing flow (Fig.5)

$\psi$ = angle between ds and the branch channel axis;

ds = an infinitely small section along dividing streamline EC;

$k$ = momentum transfer coefficient;

$P_1$ = pressure force on the wall DK;

$P_2$ = pressure force on the wall CH;

$R_{dq} = $ discharge ratio ($Q_{d2}/Q_{d1}$);
\[ R_{dy} = \text{depth ratio } (Y_{d2}/Y_{d1}); \]
\[ U = \text{momentum transfer from the main to the branch channel}; \]
\[ w = \text{width of the channels}; \]
\[ w_2 = \text{BE}; \]
\[ w_3 = \text{AE}; \text{ and} \]
\[ z = \text{the distance from ds to Ex}. \]

Subscripts:

\[ d_1 = \text{flow conditions at section AB} \]
\[ d_2 = \text{flow conditions at section FG} \]
\[ d_3 = \text{flow conditions at section HK} \]

4) Flow through floor outlets: (Fig.7)

\[ \phi = \text{angle between } V_f \text{ and the channel axis}; \]
\[ b = \text{width of the channel}; \]
\[ L = \text{length of the outlet}; \]
\[ R_{Oq} = Q_2/Q_1; \]
\[ R_{Oy} = Y_2/Y_1; \]
\[ U_f = \text{the momentum transfer in the axial direction of the main flow through the outlet}; \]

Subscripts:

\[ 01 = \text{flow conditions at section AB} \]
\[ 02 = \text{flow conditions at section CD} \]
\[ f = \text{flow conditions at section GH} \]
CHAPTER I

INTRODUCTION

Flow through an open channel junction is a phenomenon that involves numerous variables, such as the angle of intersection, the width ratio of the adjoining channels, the shape and slope of the channels, the directions and discharges of flow, etc. Chow (1959) stated: "The problem is so complicated that only a few simple and specific cases have been studied. The conclusions of such studies indicate that generalization of the problem is not possible or even desirable".

Since then, using computers, attempts have been made to generalize the problem. Advanced studies have given more useful information to the design of open channel networks of drainage systems. Even though complete generalization is still not possible, models are being created and are applicable to both general and specific situations.

The characteristics of junction flows change drastically from combining flow to dividing flow.

In combining flow, blockage at the junction denotes the rise in the stage due to the effect of lateral inflow from the branch channel. This phenomenon was recognized and studied by previous studies (Lin and Soong - 1979, Carballada - 1979). It is the most important characteristic of combining flow. Comprehensive study of the blockage phenomenon is necessary in the design of combining flow in open channel channel networks.
In dividing flow, the phenomenon is almost inverse. The most important problem is
the division of flow at the junction. The discharge ratio, which is the independent variable
in the study of the blockage effect in combining flow, becomes the dependent variable in
the study of dividing flow.

Flow through transverse floor outlets is a special case of dividing flow. Its flow
pattern is simpler than those of branch channel junctions because of its strictly
two-dimensional characteristics. Study of flow through floor outlets or bottom racks have
direct applications in horizontal trash-racks of hydro power plants, in curb outlets of
divert flow from one stream to another. An understanding of the flow behaviour over
bottom or floor outlets is very important for the proper design of such flow diversion
works. As mentioned in the previous studies (Nasser, et al - 1980, Ramamurthy and
Satish - 1986), the discharge through the outlet is the most important aspect of this
phenomenon.

In chapter II, some previous studies of combining flow, dividing flow and flow
through floor outlets are reviewed and discussed. The more important aspects of
combining flow, dividing flow and flow through floor outlets will be identified and
presented in chapter III, IV and V.
CHAPTER II

Review of literature

In this chapter, some previous studies of combining flow and dividing flow in open channels are presented and discussed subsequently.

A) Taylor's investigation (1944)

A first comprehensive study of junction flow in open channels was provided by Taylor. For subcritical flow passing through the channels at a junction, Taylor investigated the specific cases shown in Fig. 1. The channels are horizontal and of equal widths. In the case of combining flow, the following assumptions were made:

1) The flow is from channels 1 and 2 into channel 3;
2) Channels 1 and 3 lie in a straight line;
3) The flow is parallel to the channel walls, and the velocity is uniformly distributed immediately above and below the junction;
4) Ordinary wall friction is negligible in comparison to other forces involved;
5) The depths in channels 1 and 2 are equal immediately above the junction;

With these assumptions and by the application of the momentum equation to the junction in the direction of 1 to 3, Taylor was able to obtain the following equation:
\[ \frac{F_2^2}{2} = \frac{R_q^2 \left( R_y^2 - 1 \right)}{4R_y \left[ 2R_q - R_q^2 \left( 1 + \cos \theta \right) + R_y - 1 \right]} \] (1)

where \( R_q = \frac{Q_2}{Q_3} \), \( R_y = \frac{Y_1}{Y_3} \), \( F_2 \) is Froude number in the branch channel, \( Y_1 \) is the depth above the junction, \( Y_3 \) is the depth below the junction, and \( \theta \) is the angle between the merging channels.

Taking \( R_q \) as a parameter, \( F_2 \) may be plotted against \( R_y \) for any given \( \theta \).

Equation (1) was verified by experiments on junctions with \( \theta = 45^\circ \) and \( 135^\circ \). It was found that the agreement between theory and experiment was good for \( \theta = 45^\circ \). There was no agreement, however, for \( \theta = 135^\circ \). It was believed that this was because the velocity distribution below the junction was distorted and the flow did not remain parallel to the channel walls. The experimental data showed clearly that assumption 5 above is valid, regardless of the angle of intersection of the channels.

In the case of dividing flow, the problem can not easily be analyzed theoretically because it involves some unknown quantities, and assumptions which might simplify the determination of these quantities, such as assumption 5 for the combining flow, are not available. Basically, the division of flow will depend upon the backwater effects of the two branch channels and the dynamic conditions existing at the junction. For the specific junction shown in Fig. 1, Taylor provided an experimental approach to the problem. His correlation curves could be used to determine the division of flow of a given discharge \( Q_{d1} \) assuming that the inflows to the branches are not controlled by gates but merely by the characteristics of the junction and the branches themselves.
B) Carballada's investigation (1979)

Carballada has investigated the general case of the combining flow phenomenon shown in Fig. 2. His mathematical model is based on the following assumptions:

1) Friction is negligible;
2) Pressure distribution is hydrostatic;
3) The depth in channels 1 and 2 are equal immediately above the junction;
4) The flumes are horizontal and rectangular.

Unlike Taylor (1944), Carballada (1979) did consider the nonuniformity of the velocity distribution in the formulation of the mathematical model related to junction flows.

With the above assumptions and by the application of the momentum equation to the junction in the direction 3 (Fig. 2), Carballada was able to obtain the following equation:

\[
R_y^3 - (1 + 2 \beta_3 F_3^2) R_y + \left[ R_q^2 \beta_2 \frac{\cos \theta}{R_{23}} + (R_q - 1)^2 \beta_1 \frac{\cos \theta}{R_{13}} \right] 2 F_3^2 = 0 \tag{2}
\]

where, \( R_y = \frac{y_1}{y_3}, \) \( R_q = \frac{Q_2}{Q_3}, \) \( R_{13} = \frac{b_1}{b_3}, R_{23} = \frac{b_2}{b_3} \)

\[
F_3 = \frac{V_3}{\sqrt{g Y_3}}
\]

\( Y, b, V, Q, b \) are respectively flow depth, width of flume, mean velocity, discharge, momentum coefficient. Subscripts 1, 2, 3 denote the conditions at channels 1, 2, 3 (Fig. 2).
g is gravity acceleration. \( \theta \) and \( \theta' \) are respectively the angles formed by channel 3 with channels 2 and 1.

For a given geometrical configuration of the junction, the values of \( \theta, \theta', R_{13}, R_{23} \) are known; the dimensionless blockage \( R_y \) is a function of \( F_3, R_q \) and the momentum coefficients \( \beta_1, \beta_2 \) and \( \beta_3 \).

To obtain two dimensional design charts, the dimensionless blockage \( R_y \) is considered as the dependent variable, the Froude number of the downstream channel \( F_3 \) as the independent variable and the discharge ratio \( R_q \) as the group variable. Clearly, the maximum value of \( R_y \) is of interest to the designer. Hence, selecting the maximum value of \( R_y \) as the variable, yet another variable can be included in the graphical presentation of the solution. Thus, the variation of the maximum value of \( R_y \) in terms of \( F_3 \) can be viewed with another group variable such as width ratio, \( \beta_1, \beta_2 \) and \( \beta_3 \).

By this method, Carballada was able to obtain the effect of any variable to his model. The results indicate that \( \beta_1 \) and \( \beta_2 \) are not very sensitive to his model while \( \beta_3 \) is a very important parameter in the blockage phenomenon of combining flow. In fact, upstream of the junction, the flow is nearly uniform and the momentum coefficients \( \beta_1 \) and \( \beta_2 \) can be assumed as unity.
C) Law's investigation (1965)

Law studied analytically and experimentally the dividing flow phenomenon. Using the energy principle, he neglected the energy losses caused by hydraulic jumps and diverging flows. Using the momentum principle, he was able to obtain the following equations:

\[
\left[ \frac{Q_d}{Q_{dl}} \right]^2 \left[ \frac{F_{d1}}{F_{d2}} \right]^2 = \left[ \frac{1 + \frac{Q_d}{Q_{dl}} + 4 \frac{Q_d}{Q_{dl}} F_{d1}^2}{\left( 1 + \frac{Q_d}{Q_{dl}} + 4 F_{d2}^2 \right)^{3/2}} \right]^{3/2}
\]  \hspace{1cm} (3)

and

\[
F_{d1} = \frac{F_{d3}}{\left( \frac{Q_d}{Q_{dl}} \left[ 1 + \frac{2}{c} F_{d3}^2 \right] \right)^4}
\]  \hspace{1cm} (3b)

where \( F \) and \( Q \) are respectively the Froude number and the discharge, \( c \) is the contraction coefficient; subscripts \( d1 \), \( d2 \) denote the conditions of flow in the main channel upstream and downstream of the junction; subscript \( d3 \) denotes the conditions of flow in the branch channel (Fig.5).

His analysis was based on two important assumptions:

1) The estimation of the force, acting across the dividing streamline, is based on the average depth of \( Y_{d2} \) and \( Y_{d1} \);

2) The flow depth ratio \( Y_{d2}/Y_{d1} \) in the main channel upstream and downstream of the junction is assumed to be unity.
D) Satish's investigation (1986)

D-1 Branch channel junction

Satish (1986) studied analytically and experimentally the dividing flow phenomenon. He evaluated experimentally the transfer of momentum from the main channel to the branch channel.

Using the momentum principle, he was able to obtain the following equation:

\[
\left[ \frac{Y_d}{Y_d} \right]^3 - \frac{1}{4} \left[ 1 + \frac{Y_{d2}}{Y_{d1}} \right]^2 \frac{Q_d}{Q_d} = (1+2F_{d1}^2) \left[ 1 - \frac{Q_{d1}}{Q_d} \right] \frac{Y_d}{Y_d} - 2F_{d1}^2 \left[ 1 - \frac{Q_{d1}}{Q_d} \right]^2
\]  

(4)

where, \( F, Q, Y \) and subscripts \( d1, d2, d3 \) are as previously defined.

His analysis was based on two important assumptions:

1) The contraction coefficients given by conformal mapping of Mc Nown and Hsu (1950) are applicable for branch channel junction;
2) The existence of a critical section in the branch channel where the maximum contraction occurs.

With these assumptions and Eq. 4, he evaluated \( Q_{d3}/Q_{d1} \) and then \( Y_{d2}/Y_{d1} \) in terms of \( F_{d1} \). From this, he obtained the relationship between the Froude number \( F_{d2} \) in the main channel downstream of the junction and the discharge ratio \( Q_{d3}/Q_{d1} \). However, his model is valid only for \( F_{d3} > 0.35 \).
D-2 Flow through transverse floor outlets:

Based on the characteristics of lateral efflux from a two dimensional conduit (Ramamurthy and Carballada - 1979,1980) and on the approximation by a polynomial fit of the pressure coefficient correction, Satish was able to evaluate the discharge coefficient through the floor outlet as the following:

$$C_{df} = 0.611 + C_1 \eta_f^2 + C_2 \eta_f^4 + C_3 \eta_f^6$$

(5)

where,

$$\eta_f = \frac{F_f}{\sqrt{F_f^2 + 2K}}$$, \hspace{0.5cm} F_f = \frac{U_{fe}}{\sqrt{g Y_{fe}}}$$

K is pressure correction coefficient; $U_{fe}$, $Y_{fe}$, $F_f$ are respectively velocity, depth, Froude number at section EE1 (Fig.7).

K could be evaluated by the polynomial fit equation:

$$K = 1 - 0.35 \frac{L}{Y_c} - 0.95 \left( \frac{L}{Y_c} \right)^2 + 0.90 \left( \frac{L}{Y_c} \right)^3$$

(5b)

where, $L$ is the length of the floor outlet, $Y_c$ is critical depth.
E) Discussion of the previous studies.

1) Combining flow phenomenon:

Consider the case for which $\theta = 90^\circ$. According to previous studies (Taylor-1944, Carballada-1979), the contribution of the lateral momentum to the main channel is:

$$\rho Q_2 V_2 \cos \theta = 0$$

where, $\rho$ is density of water and $Q_2, V_2, \theta$ are as previously defined.

When the branch flow approaches the main channel, its direction changes gradually and its flow pattern is similar to the flow pattern of curved channels. Consequently, there will be some momentum contribution from the lateral channel flow to the main channel flow. Failure to consider this lateral momentum contribution is an important omission in the analysis of combining flow especially when the junction angle is not very small.

In the present study, for right angled junctions of rectangular channels, the pressure forces on the walls of the branch channel were measured to directly obtain the magnitude of the lateral momentum that is transferred to the main channel. This transfer momentum can be large compared to the branch channel momentum $\rho Q_2 V_2$. Because of this, in the formulation of the momentum equation for the analysis of junction flows, one must include the contribution of lateral momentum transfer to get acceptable results.

In addition, in Taylor's study (1944), there was no agreement between theory and experiment for the case where junction angle is $135^\circ$. It was believed that this was because the velocity distribution below the junction was distorted and the flow did not remain parallel to the channel walls. The nonuniformity of the velocity distribution was included in the mathematical model of Carballada (1979). According to the numerical
analysis of this model, the momentum coefficient in section 3 (Fig.2) is very sensitive and one can not assume it to be unity. This analysis could be very useful in the estimation of blockage for combining flow in open channels.

However, in the derivation of this model, Carballada neglected the difference of pressures on the two walls of the branch channel. He also neglected the effect of the energy coefficient of the flow below the junction. In fact, for the reasons stated in chapter III (Combining flow in open channels - section "Discussion of results"), this energy coefficient is very sensitive to the blockage effect.

In the present study, the momentum transfer from the branch to the main channel and the energy coefficient are included in the model to predict the blockage effect for combining flow in open channels.

2) Dividing flow phenomenon.

2-1 Branch channel junction:

Based on experimental results, Taylor (1944) proposed a graphical method to predict flow division. According to it, the inflows to the branches are not controlled by gates but merely by the characteristics of the junction and the branches themselves and such an ideal set of conditions could hardly occur in practice. Furthermore, this method involved the measurement of flow depth $Y_{d3}$ in the branch channel where the flow pattern is very complicated due to flow separation, recirculation, the formation of waves and hydraulic jumps. This means that the measurement of $Y_{d3}$ was not very accurate. It should be recalled that $Y_{d3}$ is an important parameter in Taylor's graphical solution.

In the present model, the problem is studied analytically and experimentally, the flow depth data is confined to sections where the flow is nearly uniform. Furthermore, the present model accommodates common field structures in which the flows in the main channel and the branch channel are regulated by controlled gates.
Law (1965) studied the dividing flow phenomenon analytically and experimentally. From his two basic assumptions mentioned previously about the evaluation of the hydrostatic force acting across the dividing streamline and the ratio of flow depths in the main channel upstream and downstream of the junction, one can apply the momentum principle in the direction of the main channel axis for the control section ADCE (Fig.5) and get the momentum transfer $U$ from the main channel to the branch channel. Accordingly,

$$U = 0$$

In fact, in the present study, it will be shown later in chapter IV that:

$$U = 0.82 \rho Q_{d3} V_{d1}$$

where $\rho$, $Q$, $d1$, $d3$ are defined as previously; $V$ is the mean velocity.

Further, for supercritical flow in the approach channel, a hydraulic jump occurs at the junction. Hence, the flow depth $Y_{d2}$ is much greater than $Y_{d1}$. For subcritical flow in the approach channel, the flow is diverted in the main channel right after the junction. Hence, the flow depth $Y_{d2}$ is in general greater than $Y_{d1}$ even for small values of the discharge ratio $Q_{d3}/Q_{d1}$. The ratio $Y_{d2}/Y_{d1}$ can be much greater than unity for larger discharge ratios $Q_{d3}/Q_{d1}$. In other words, one cannot assume this ratio to be unity.

In the present study, it will be shown that the ratio of flow depths in the main channel upstream and downstream of the junction is a very sensitive and important parameter in the determination of the discharge ratio (chapter IV). This ratio should not be assumed as unity.

Satish (1986) studied analytically and experimentally the dividing flow phenomenon. In the estimation of the hydrostatic force acting across the dividing streamline, he assumed the flow depth as the mean depth $(Y_{d1} + Y_{d2})/2$. In fact, along the dividing streamline,
the flow depth increases from $Y_{d1}$ to the total head $Y_{d1}(1 + \frac{F_{d1}^2}{2})$ at the stagnation point C (Fig.5).

Based on these assumptions and the momentum principle, Satish (1986) was able to obtain a relationship between the discharge ratio and the Froude number in the main channel downstream of the junction. His model is limited by the conditions of Froude number in the branch channel ($F_{d3} > 0.35$).

In the present study, it will be shown that the Froude number in the approach channel is an important parameter for the relationship between the discharge ratio and the Froude number in the main channel downstream of the junction. In other words, the present study is the general case where the flow conditions in the branch channel are not limited.

2-2 Flow through floor outlets:

One important parameter in Satish's study (1986) is the evaluation of the pressure coefficient correction $K$ (Eq.5b). From this equation, for a given discharge of flow through transverse floor outlet, $K$ will be a constant. In fact, if downstream conditions change, obviously, upstream conditions will change except when the flow is supercritical; hence, $K$ will depend on downstream conditions. If the downstream gate is completely closed, the discharge through the floor outlet will be the total discharge and $K$ will change drastically. In other words, Satish's study (1986) is good for the special case where the channel is not controlled by an end gate but merely by the characteristics of the floor outlet and the channel itself. In the present study, it will be shown later that the effect of downstream conditions is an important factor which controls the discharge characteristics of flow through the floor outlet.
CHAPTER III

Combining flow in open channels

III-1 Introduction:

The first comprehensive study of combining flow in open channel was provided by Taylor (1944). He studied the characteristics of flow at the junction of two horizontal channels whose cross section was rectangular. He applied the momentum equation to the analysis of combining flow and verified his predictions with experimental data for junction angles of $45^\circ$ and $135^\circ$. His theoretical predictions were good for the smaller junction angle. However, there was no agreement for the junction angle of $135^\circ$. It was believed that this was because the velocity distribution below the junction was distorted and the flow did not remain parallel to the channel walls. In fact, according to the results of the present study, this disagreement is related to the estimation of the momentum transfer from the branch to the main channel.

The increase of the flow depth in the upstream channel caused by the branch channel flow was studied by Carballada (1979). He tried to generalize the problem by developing a mathematical model for the general case based on the momentum principle. He did consider the velocity distribution effects.

An analytical solution based on complex variable theory has been obtained for the
problem of combining flow by Modi et al (1981) and Webber (1966). Unfortunately, variations of parameters in the vertical direction could not be accommodated in the model as the theoretical analysis was strictly two dimensional.

Using dye injection techniques, Best and Reid (1984) recently conducted experiments to determine the maximum width of flow separation in the main channel at the section downstream of the junction and stated that the theoretical predictions of Modi et al (1981) overestimate the width of separation.

Other previous studies provided useful information in the study of combining flow in open channels. Demuren and Rodi (1983) presented a three-dimensional numerical mathematical model for calculating the near field of side discharges into open channel flow. Lorah (1966) presented a study of hydraulic power losses associated with free surface, subcritical, combining flow in a 90° junction box of circular pipes. Jamison and Villemonte (1971) studied junction losses in laminar and transitional flows. Lin and Soong (1979) studied junction losses in open channel combining flow. They stated that the turbulent mixing loss could be of the same order of magnitude of the boundary friction loss. Joy and Townsend (1981) studied the flow characteristics of a right angled open channel junction and observed that the energy coefficient increases with an increase in the discharge ratio or with a decrease in the width of the lateral channel.

As mentioned above, one serious difficulty encountered in the previous studies (Taylor-1944, Carballada-1979) is related to the estimation of the momentum transfer from the branch to the main channel. This is the main reason for the disagreement between theoretical prediction and experimental data for junction angle of 135° in Taylor's study (1944). Taylor himself, did recognize that failure to measure the pressure on the walls of the branch channel or failure in the estimation of the momentum transfer from the branch to the main channel constitutes an important omission.

In the present study, an estimate of the transfer of lateral momentum at rectangular channel junctions is provided on the basis of experimental data. Using this estimation, a relationship between depth ratio and discharge ratio is derived based on the momentum principle. Another relationship related to the blockage is derived on the basis of the
energy principle. Here, blockage at the junction denotes the rise in the stage due to the effect of lateral inflow from the branch channel. Experimental data from the present and previous studies are provided to validate these relationships.

III-2 Theoretical Investigation

As mentioned in chapter I, flow through an open channel junction is a phenomenon that involves numerous variables, such as the angles of intersection, the width ratios of the adjoining channels, the shape and slope of the channels etc. In analyzing the problem, one has to adopt a simpler version of the phenomenon by designating some of the variables to be more important than others.

In the present study, the specific case shown in Fig. 3 is investigated. The channels are rectangular, horizontal and are of equal width. The junction angle is 90°.

This case is chosen because of the following reasons:

a) wide channels in the field can be assumed as rectangular channels.
b) slopes of field channels are usually small.
c) junction angles in field channels are usually equal to or less than 90°. In the case of right angled junction, the evaluation of the momentum transfer from the branch channel to the main channel is critical because it was neglected in previous studies (Taylor-1944, Carballada-1979).

Fig. 3 shows the right angled junction for which \( Q, V, Y, F, b, \alpha, \beta \) denote the discharge, mean velocity, stage, Froude number, width, energy coefficient and momentum coefficient of the flows. The subscripts 1 and 3 denote the conditions of flows at the sections EF and HG in the main channel upstream and downstream of the junction. The subscript 2 denotes the conditions of flow at section CB in the branch channel. \( V_2 \) is
the resultant velocity of the branch flow turning and entering the main channel. \( \delta \) is the angle formed by the resultant velocity \( V_2' \) and the main channel axis. \( P_u \) and \( P_d \) are the pressure forces on the walls CD and AB.

The following assumptions are made to study the combining flow problem:

1) The branch and main channels have the same rectangular section and are horizontal.
2) The incoming flows \( Q_1 \) and \( Q_2 \) are subcritical in the branch and the main channels.
3) Based on the experimental data of the present and previous studies (Taylor-1944, Lin and Soong-1979), for the entrance sections BC and EF, one can assume that the flow depths are equal and the flows are nearly uniform.
4) Boundary friction is negligible in the control volumes ABCD and EFGH.
5) The cross-sectional area at sections BC and AD are nearly equal such that the mean axial velocity \( V_2 \) of the flow in the branch channel is the same at AD and BC.
6) Flow is not submerged right downstream of the junction (section HG) and the flow at section HG is critical in the range \( 0.3 \leq Q_2/Q_1 \) for the reasons stated in section III-2-2.

### III-2-1 The transfer of lateral momentum at open channel junctions.

The importance of the transfer of lateral momentum at right angled junctions of closed conduits has been recognized by many investigators in the past. For instance, McNown (1954) introduced an unbalanced force term in the formulation of the momentum equation for branching flows in closed conduits. In the study of manifolds, Bajura (1970) states that the momentum balance for the flow in the main channel can not be achieved without the inclusion of the lateral momentum term. He obtained a rough estimate of this term.
indirectly on the basis of experimental data. In open channel flows, Taylor (1944) recognized that failure to measure the pressure on the walls of the branch channel constitutes an important omission. In the present study, to directly obtain the magnitude of the lateral momentum that is transferred to the main channel, the pressure forces on the walls of the branch channel were measured.

Considering Fig.3, the stagnation point occurs near corner D and the flow separates at corner A (Modi et al - 1981). Hence, near corner A, the flow velocity will have reached a high value and this causes the water surface to be lower while near corner D, the velocity of flow will have reached an extremely small value and this causes the water surface to be higher. Consequently, \( P_u \) will be larger than \( P_d \) and one should expect a net transfer of momentum from the branch to the main channel on the basis of the momentum principle.

Consider the control volume ABCD (Fig.3) in the branch channel and apply the momentum equation in the axial direction of the main channel to get the following balance of forces:

\[
\rho Q_2 V_2 \cos \delta = P_u - P_d \tag{6}
\]

where \( \rho \) is the density of water

From assumption 5, it follows that:

\[
V_2' = \frac{V_2}{\sin \delta} \tag{7}
\]

Using Eq.6 in Eq.7, one gets:

\[
P_u - P_d = \frac{\rho Q_2 V_2}{\tan \delta} \tag{8}
\]

Based on the studies made in an experimental facility (Fig.4), a relation between
\( \rho Q_2 V_2 / (P_u - P_d) \) and \( Q_2/Q_1 \) was obtained (Fig.9). For the range \( 0.3 \leq Q_2/Q_1 \leq 1.5 \), this relation can be described approximately by the following equation:

\[
\frac{\rho Q_2 V_2}{P_u - P_d} = \frac{Q_2}{Q_1} \tag{9}
\]

Using Eqs. 8 and 9, and further noting that \( Q_2/Q_1 = V_2/V_1 = F_2/F_1 \) (assumptions 1 and 3), one gets:

\[
\tan \delta = \frac{Q_2}{Q_1} = \frac{V_2}{V_1} = \frac{F_2}{F_1} \tag{10}
\]

From Eqs. 9 and 10,

\[
P_u - P_d = \rho Q_2 V_1 = \rho Q_1 V_2 = \rho Q_1 V_1 \frac{Q_2}{Q_1} \tag{11}
\]

Or

\[
P_u - P_d = \rho Q_2 V_2 \frac{F_1}{F_2} \tag{12}
\]

Eq. 11 indicates that the transfer of lateral momentum can be significant when the ratio \( Q_2/Q_1 \) is large. Clearly, inclusion of the lateral momentum transfer is essential for a proper analysis of junction flows.

**III-2-2 Blockage effect for combining flow in open channels:**

In combining open channel flow, blockage at the junction denotes the rise in the stage
due to effect of lateral inflow from the branch channel. For right angled sharpened edged junctions of rectangular channels, a relation between the ratio of flow depths upstream and downstream of the junction and the ratio of the lateral discharge to the total discharge is derived on the basis of the momentum principle. Using this relationship, one can predict blockage for the main channel if the undisturbed flow conditions in the channels are specified. Experimental data is provided to verify the stage predicted.

Considering the control volume EFGH (Fig.3) and applying the momentum equation in the direction of the main channel axis, one gets:

\[
\frac{1}{2} \gamma b Y_1^2 + \beta_1 \rho Q_1 V_1 + \rho Q_2 V_1 = \frac{1}{2} \gamma b Y_3^2 + \beta_3 \rho Q_3 V_3 \quad (13)
\]

where \( b \) is width of the channels, \( \gamma \) is specific weight of water, \( \rho \) is density of water and \( \rho Q_2 V_1 \) is momentum transfer from the branch to the main channel (Eq.11)

Noting that,

\[
V_1 = \frac{Q_1}{b Y_1}, \quad V_3 = \frac{Q_3}{b Y_3},
\]

\[
\beta_1 = 1 \quad \text{(Assumption 3)} \quad \text{and} \quad \gamma = \rho g
\]

Using these relationships in Eq.13 and simplifying,

\[
b Y_1^2 + \frac{2 Q_1}{g b Y_1} (Q_1 + Q_2) = b Y_3^2 + \frac{2 \beta_3 Q_3^2}{g b Y_3} \quad (14)
\]
Further,

\[ F_3 = \frac{V_3}{\sqrt{g \frac{Y_3}{\alpha_3}}} = \frac{Q_3 \sqrt{\alpha_3}}{b \ Y_3^{1.5} \sqrt{g}} \]

and \( Q_3 = Q_1 + Q_2 \) (continuity), from Eq.14,

\[ Y_1^2 + \frac{2Q_1 Q_3}{g b^2 Y_1} = Y_3^2 + 2 \beta_3 Y_3^2 \frac{F_3^2}{\alpha_3} \] (15)

Simplifying Eq.15, one gets:

\[ \left[ \frac{Y_1}{Y_3} \right]^2 + \frac{2Q_1}{Q_3} \frac{Y_1}{Y_3} \frac{Q_3^2}{g b^2 Y_3^4} = 1 + 2 \beta_3 \frac{F_3^2}{\alpha_3} \] (16)

Or

\[ R_y^2 + \frac{2(1 - R_q)}{R_y} \frac{F_3^2}{\alpha_3} = 1 + \frac{2 \beta_3 F_3^2}{\alpha_3} \] (17)

where \( R_y = Y_1/Y_3 \), \( R_q = Q_2/Q_3 \)

Simplifying,

\[ R_y^3 - (1 + 2 \frac{\beta_3 F_3^2}{\alpha_3}) R_y + \frac{2 F_3^2}{\alpha_3} (1 - R_q) = 0 \] (18)
A contraction occurs in the main channel due to the separation of the lateral flow, and this was studied by Best and Reid (1984), and Modi et al (1981). One should normally expect the critical flow conditions to occur at the contracted section especially when the contraction is severe. However, due to the curvilinear nature of the flow and the mixing process associated with combining flow, the flow field is three dimensional and the flow tends to regain its two dimensional nature only after it reattaches to the side wall of the main channel at HG (Fig.3). For $Q_2/Q_1 \geq 0.3$, a hydraulic jump was always observed in the present tests just after section HG. This implied the existence of supercritical and critical flow conditions just upstream of the jump.

Figs. 11, 12, and 13 provide data related to typical velocity and depth measurements at section HG. The results indeed confirmed the existence of critical conditions in the vicinity of this section. The depth of flow at the critical section is:

$$Y_C = \left[ \frac{1}{\alpha_3 \left( \frac{Q_3}{g \beta_3} \right)^3} \right]$$

(19)

Note that $F_3 = 1$ at the critical section HG, from Eq. 18,

$$R_y^3 - \left( 1 + 2 \frac{\beta_3}{\alpha_3} \right) R_y + \frac{2(1 - R_y)}{\alpha_3} = 0$$

(20)

The values of $\alpha_3$ and $\alpha_3/\beta_3$ depend on many factors such as the rounding of the corners at the junction, junction angle, discharge ratio etc... According to Joy and Townsend (1981), small modifications to the outlet geometry greatly affect the flow conditions at the confluence. Hence, $\alpha_3$ and $\alpha_3/\beta_3$ may change drastically. For a given geometrical channel junction, one can assume that the discharge ratio is the main variable (Joy and Townsend-1981). For right angled junctions of rectangular open channels when
the width ratio is unity, Fig.14 shows the variation of the energy coefficient $\alpha_3$ and of the ratio $\beta_3/\alpha_3$ with the discharge ratio $R_q=Q_2/Q_3$. Clearly, in the range $0.23 \leq Q_2/Q_3 \leq 0.60$, one notices a linear variation of $\alpha_3$ and $\beta_3/\alpha_3$ with $R_q$.

Accordingly, for $0.23 \leq Q_2/Q_3 \leq 0.60$,

$$\alpha_3 = 1.25 + 0.5 \frac{Q_2}{Q_3}$$

(21)

and

$$\frac{\beta_3}{\alpha_3} = 1 - 0.24 \frac{Q_2}{Q_3}$$

(22)

Incidentally, the data of Joy and Townsend (1981) for $b_2/b_1=1/3$ and $2/3$ shown in the insert of Fig.14 indicate that $\alpha_3$ increases approximately linearly with $R_q$ in the range $0.1 < R_q < 0.5$ (Range of Joy and Townsend-1981).

The comparison of the present data with those of Joy and Townsend (1981) should be made only on a qualitative basis as the dimensionless parameter $X'$ denoting the location of the measuring section (Fig.14) is also a secondary factor that determines $\alpha_3$ and $\beta_3$.

Using Eqs.21 and 22 in Eq.20 and noting that $Y_3=Y_c$ at the critical section, one gets the following expression for the dimensionless blockage $R_y = Y_1 / Y_3 = Y_1 / Y_c$. 

\[ R_y^3 + (0.48 R_q - 3) R_y + \frac{1 - R_q}{0.625 + 0.25 R_q} = 0 \quad (23) \]

It may be recalled that Eq.19 yields \( Y_c \) for given discharge \( Q_3 \). In other words, from Eqs.19 and 23, one can predict the flow depth \( Y_1 \) just upstream of the junction for given discharges \( Q_2 \) and \( Q_3 \).

Note that the downstream condition does not affect the characteristics of flow at the junction because of the existence of the critical conditions at section HG (Fig.3).

However, when the flow is submerged, assumption 6 and Eqs.21 and 22 become invalid, \( F_3 \) is not equal to unity, and one should use Eq.18 to evaluate the dimensionless blockage \( R_y \). In this case, assuming \( \alpha_3, \beta_3 \) as unity, Eq.18 becomes:

\[ R_y^3 - (1 + 2F_3^2) R_y + 2F_3^2(1 - R_q) = 0 \quad (24) \]

Considering \( R_q \) as an independent variable, \( R_y \) as a dependent variable and \( F_3 \) as a group variable, one can draw the variation of \( R_y \) in terms of \( R_q \) and \( F_3 \). Fig.16 shows the variation of \( R_y \) as a function of \( R_q \) with \( F_3 \) as a group parameter. From this figure, when \( F_3 \) is equal to 0.2, \( R_y \) will vary from 1 to 1.04. In other words, in the case of submerged flows where \( F_3 \) is relatively small, the blockage effect is negligible. In fact, when the flow is submerged (caused by the downstream gate), the flow depth increases drastically but the effect of combining flow on blockage is negligible.
III-2-3 Alternate approach to study blockage effects for combining flow in open channels:

For combining flow at right angled junctions (Fig.3), according to the ideal fluid flow model of Modi et al (1981), a stagnation point could occur at D or at a location along DC or DE. In particular, according to them, stagnation of flow occurs at D only when discharge ratio \( \frac{Q_2}{Q_3} = 0.618 \) or \( \frac{Q_2}{Q_1} = 1.618 \). In real flows, discontinuity of the boundary can lead to the separation of flow. As the flows from the branch channel and the main channel approach the corner D, they encounter an adverse pressure gradient and this can lead to flow separation from the side walls DC and DE just ahead of the corner D. In other words, a stagnation zone occurs in the vicinity of corner D.

Lin and Soong (1979) did recognize that the side channel flow introduces a significant effect on the water surface profile in the main channel and hence, the blockage phenomenon can not be neglected in the design of combining flow junctions. Blockage can be represented by the depth of water at the stagnation zone or the total head at the junction.

Considering the total heads at sections EF and CB (Fig.3):

\[
E_1 = Y_1 + \frac{V_1^2}{2g} \quad (25)
\]

\[
E = Y + \frac{V^2}{2g} \quad (26)
\]

In subcritical flow, downstream conditions affect the surface profile of the upstream flow. In other words, \( E_1 \) and \( E_2 \) are dependent on the junction conditions. The average total head \( E_J \) at the junction zone will be nearly equal to the stagnation depth near corner D:

\[
E_J = Y_s \quad (27)
\]
where, $Y_s$ is the depth of water at stagnation zone.

From sections CB and EF to the junction, both flow cross sections corresponding to $Q_1$ and $Q_2$ are contracted. Hence, neglecting the power losses in the contracting zones, one gets,

$$\gamma E_j Q_3 = \gamma E_1 Q_1 + \gamma E_2 Q_2$$

or

$$Y_s = E_j = E_1 \frac{Q_1}{Q_3} + E_2 \frac{Q_2}{Q_3}$$

(29)

For a long reach downstream of the junction in the main channel, Lin and Soong (1979) divided the junction losses in two components: the turbulent mixing and the boundary friction. They stated that these two components of energy losses are of the same order of magnitude. In fact, this mixing loss includes the dominant portion of the loss caused by hydraulic jump just downstream of section HG (assumption 6). Further, one also notes that the flow accelerates from the junction zone to section HG which is just upstream of the hydraulic jump. Hence, in the present study, while seeking an approximate solution, one can neglect the mixing loss from the junction to section HG.

Considering Fig. 3 and applying energy equation between the junction and section HG, one gets:

$$E_j = Y_C + \alpha_3 \frac{V_C^2}{2g} + h_l$$

(30)

where, $Y_C$, $V_C$, $\alpha_3$ are respectively water depth, velocity, energy coefficient at critical
section HG; \( h_1 \) is energy loss from D to HG. In Eq.30, the value of \( h_1 \) depends on many factors such as roughness of the channels, junction angle, width ratio of the channels, discharge ratio and Reynolds number of the flow.

The total losses \( h_1 \) includes the mixing loss \( (h_m) \) and friction loss \( (h_f) \) from the junction to section HG.

As mentioned earlier, the mixing loss \( h_m \) is negligible. One notes that \( h_f \) is Reynolds number \( (R_e) \) dependent and hence, \( h_f \) decreases drastically with increasing values of \( R_e \). Figs.17 and 18, and Tables 4 and 5 show that \( h_f \) decreases drastically when Reynolds number \( R_e \) increases. In field channels, where \( R_e \) is very large, one can expect \( h_f \) between the junction and section HG to be insignificant. Hence, from Eq.30, one can formulate the following approximate relationship:

\[
E_j = 1.5 \ Y_C
\]  

(31)

Or

\[
\frac{Y_s}{Y_C} = \frac{E_j}{Y_C} = 1.5
\]  

(32)

It may be recalled that Eq.19 yields \( Y_C \) for given discharges \( Q_2 \) and \( Q_3 \). Therefore Eq.32 can be used to predict the blockage phenomenon in combining flow.

III-2-4 Generalization of the problem:

Flow through a channel junction is a complicated phenomenon that involves numerous variables and generalization of the problem is not possible or even desirable (Chow, 1959).
However, with the use of computer to solve engineering problems, Carballada (1979) could generalize the problem of combining flow. Based on his model (Eq.2), he provided the design charts to predict the blockage effect for combining open channel flow. From the results of the present study, one should include in his model the transfer of lateral momentum and the proper value of the energy coefficient at the section downstream of the junction for more precise results. According to the experimental data of the present and previous studies (Joy and Townsend-1981), this energy coefficient could be very high ($\alpha_3 = 2$) and could not be assumed as unity. Further, the transfer of lateral momentum and the energy coefficient depend on numerous variables such as the junction angle, the shape and slope of the channels, the rounding of the corners at the junction, the ratio of the channel widths and could be estimated only on the basis of the experimental data.

Considering Eqs.31 and 32, the dimensionless blockage is constant i.e. it does not depend on any variable. However, in the evaluation of $Y_c$, the energy coefficient $\alpha_3$ is a significant variable. The effects of the numerous variables such as the junction angle, the shape and slope of the channels, the rounding of the corners at the junction, the ratio of channel widths etc... could be replaced by their effects on the energy coefficient at section HG (Fig.3).

According to Joy and Townsend (1981) the energy coefficient $\alpha_3$ increases with an increase in the discharge ratio or with a decrease in the width ratio. The accurate evaluation of the energy coefficient $\alpha_3$ at the critical section HG (Fig.3) in any specific case will yield the accurate prediction of blockage for combining open channel flow.

In the case where the junction angle is much greater than $90^\circ$, the energy loss from the junction to section HG can not be neglected. Fortunately, in field channels, the junction angle is usually smaller or equal $90^\circ$.

Eq.32 is based on the existence of the critical section HG (Fig.3) which depends on
many factors: junction angle, width ratio, discharge ratio etc... However, the blockage phenomenon is only important when the combining flow is severely contracted i.e. only when the critical section exists in the region downstream of the junction.

In any specific case one could approximate the energy coefficient \( \alpha_3 \) based on the study of Joy and Townsend (1981) and then use Eqs.19 and 32 to predict blockage in combining open channel flow.
III.3 Experimental facility

Fig. 4 shows the experimental set up for combining flow. The main channel was 0.248 m wide and 6.10 m long. The branch channel was 0.245 m wide and 1.30 m long and the junction angle was 90°. The channels were made of smooth stainless steel plates. The discrepancy of the widths between the main and the branch channel is of the order 1%. The edges of the junction were sharp. The flumes were 0.48 m deep and a large number of 1.6 mm diameter wall static taps were provided at 40 mm horizontal intervals and 50 mm vertical intervals along the walls AB and CD. Near the corners A and D, the tap intervals were even closer. These taps were connected to an inclined manometer bank which had an inclination of 1:6. The inclined manometer bank could register wall pressures to the nearest 0.16 mm of the water depth. The discharges from the channels were measured with standard V-notches. The point gages, used to measure the water levels in the channels and the V-notch tanks could be read to the nearest 0.3 mm. The V-notches were constructed in accordance with ASME standards. For the low range of discharges (0.0028 m³/s to 0.0084 m³/s) an independent gravimetric method of determining the flow rate was used for comparison. The ASME calibration curves (adjusted to account for tank dimensions) indicated that the deviation of the flow rate was less than 3%. The total head and the static head were measured with standard pitot tubes 3 mm in diameter. Raffles and screens were used in the upstream sections of the channels to ensure the flow to be free from large scale turbulence in the channel reaches upstream of the junction.
III-4 DISCUSSION OF RESULTS

III-4-1 The transfer of lateral momentum at open channel junctions:

Considering Fig. 3, the pressure forces $P_u$ and $P_d$ on the side walls of the branch channel were calculated using the integration of the wall pressure recorded on the inclined manometer bank. The pressure distribution was hydrostatic along the wall CD and nearly hydrostatic along the wall BA. Table 1 shows results related to the momentum transfer corresponding to the range of discharge ratio covered by the tests. The data of Table 1 is used to plot Fig. 9 which indicates the variation of the dimensionless form of lateral momentum $\rho Q_2 V_2 / (P_u - P_d)$ with the discharge ratio $Q_2 / Q_1$. Clearly, in the range $0.3 \leq Q_2 / Q_1 \leq 1.5$, the variation of $\rho Q_2 V_2 / (P_u - P_d)$ with $Q_2 / Q_1$ can be closely approximated by a straight line whose slope is unity. The data in Table 1 and Fig. 9 indicate that the contribution of lateral momentum $\rho Q_2 V_2$ to the main flow at right angled junction can be large compared to the branch channel momentum $\rho Q_2 V_2$. Hence, in the formulation of the momentum equation for the analysis of junction flows, one must include the contribution of lateral momentum transfer to get acceptable predictions of flow behaviour.

III-4-2 Blockage effects for combining flow at open channels (momentum relationship):

Tables 2 and 3 indicate that the maximum deviation of the measured stages from the predicted values (Eqs. 19 and 23) is generally limited to 3% and 5%. This validates the relationship between the dimensionless blockage and the discharge ratio (Eq. 23).
Fig. 15 shows the experimental data related to $Y_1 / Y_c$ of the present tests and those of Lin and Soong (1979) plotted as a function of $Q_2 / Q_3$.

For Lin's data, $Y_1$ was measured at a distance of 1.6 meter from the junction. Hence, a minor correction was applied to reduce the value of the total head to conform with the present data recorded at section EF (Fig. 3). This minor correction was of the order of 1% in the value of $Y_1$. It was computed by assuming the roughness coefficient $n$ to be 0.01.

The solid line in Fig. 15 denotes Eq. 23. The dashed line in Fig. 15 denotes the ± 5% bands. The two dark circles in Fig. 15 are associated with the largest deviations from the predicted trend. They correspond to the smallest discharges reported by Lin and Soong (1979). Generally, test data associated with very low discharge tend to be less accurate. One should also note that the ratio $Q_2 / Q_3$ for these tests are close to the limits of applicability of Eq. 23. Lastly, the Reynolds number $Re = VR / \nu$ associated with these two tests are relatively small: $25 \times 10^3$ (Table 3). Here, $\nu$ denotes the kinematic viscosity of water, $R$ is the hydraulic radius of the channel. Tables 2 and 3 shows that for larger $Re$, the errors in the prediction of blockage are relatively smaller as one would expect. As such, it may be added that the prediction of blockage will be more accurate in the field channel where the Reynolds number will be very large.

The conditions for existence of the critical section at HG depend on many variables such as discharge ratio, Froude number of flow in the main channel at section EF (Fig. 3), junction angle, junction energy loss, channel slopes etc... However, the discharge ratio is the most important variable. It controls the contraction of the flow in the main channel to a large extent. When the ratio $Q_2 / Q_1$ is very small (say $Q_2/Q_1 < 0.3$), the existence of critical condition is doubtful and Eqs. 19 and 23 could be invalid. Fortunately, the phenomenon of blockage has practical importance only for larger discharge ratios.
III-4-3 Alternate approach to study blockage effects for combining open channel flow:

In the case of right angled junctions, for given discharges $Q_2$ and $Q_3$, Eq.19 yields the critical depth. From Eq.32, one can predict the water depth of the stagnation zone in the vicinity of the junction. Test variables were chosen to avoid very low Reynolds numbers, for which boundary friction effects are significant even for short reaches; because in practical applications, one seldom encounters very low Reynolds numbers.

Table 6 and Fig. 19 show the variation of experimental values of blockage (present study) in terms of the discharge ratio. There is a fair agreement between these values and Eq.32. Table 7 and Fig. 20 show the variation of experimental values of blockage (Lin and Soong -1979) in terms of the discharge ratio. The agreement of these values with Eq.32 seems to be acceptable.

In the general case, "flow through a channel junction is a complicated phenomenon that involves numerous variables and generalization of the problems is not possible or even desirable"(Chow-1959). However, in the present model (Eqs.19 and 32), numerous variables are reduced to only one: the energy coefficient at section HG.

The accuracy of the prediction of the blockage $Y_s$ depends on the evaluation of the energy coefficient $\alpha_3$ which is not easy to measure with high accuracy. Fortunately, according to Eqs. 19 and 32 the critical depth $Y_c$ and the blockage $Y_s$ vary with the one third power of the energy coefficient. In other words, inherent errors associated in the evaluation of $\alpha_3$ for more complex junction flow will result in acceptable prediction of blockage.
III-4-4 Generalization of the problem:

Carballada (1979) tried to generalize the combining flow problem. In this study, the momentum transfer from the branch channel to the main channel was neglected and the energy coefficient $\alpha_3$ was assumed to be unity. In the present study, this momentum transfer is evaluated by the direct measurement of pressure forces on the walls of the branch channel. Unfortunately, this transfer will change with the geometrical configurations of the junction; hence, one can not include this transfer in Carballada's mathematical model in the generalization of the problem.

Applying the mathematical model of Carballada (Eq.2) to the present physical model where $\theta=90^\circ$, $\theta'=0^\circ$, $R_{23}=1$, $R_{13}=1$; one gets:

$$R_y^3 - (1 + 2 \beta_3 F_3^2) R_y + 2 F_3^2 (1 - R_q)^2 = 0 \quad (33)$$

Comparing the two Eqs. 18 and 33, and noting that $\beta_3$ is very significant in Eq.33 (Carballada, 1979); one can conclude that the ratio $\beta_3/\alpha_3$ of Eq.18 is a very significant variable for the present model. In other words, the energy coefficient $\alpha_3$ is also very sensitive to the blockage phenomenon. According to experimental data of the present study and of previous studies (Joy and Townsend-1981), this coefficient could be much greater than unity and hence, it should be included in the proper analysis of junction flows.

Comparing the two different cases namely Eq.33 and Eq.18, one can conclude that the lateral momentum transfer can not be neglected. These equations are derived for the specific case where the junction angle is $90^\circ$. Obviously, if the junction angle is much smaller than $90^\circ$, the effects of the momentum transfer will be negligible. This fact explains the agreement between theoretical predictions and experimental data of Taylor’s study in
the case where the junction angle is equal to $45^\circ$. In the case where the junction angle is greater than $90^\circ$, clearly, the effect of the momentum transfer will increase drastically. That is the main reason for the disagreement between theoretical predictions and experimental data of Taylor's study in the case where the junction angle is equal to $135^\circ$. It was believed earlier that this disagreement was solely caused by the nonuniformity of the velocity distribution downstream of the junction.
III-5 Conclusions and applications:

For right angled junctions of rectangular open channels of equal width related to combination of flows, the magnitude of the momentum transferred from the branch flow to the main flow can be estimated on the basis of the measurement of pressure forces on the walls of the branch channel. For $0.23 \leq R_{Q} \leq 0.6$, and critical flow at the downstream branch, the lateral momentum transfer is linearly proportional to the discharge in the branch channel and to the mean velocity in the main channel. Inclusion of the lateral momentum transfer is essential for a proper analysis of junction flows.

The prediction of water depth just upstream of the junction is important in river engineering and in the design of open channel networks of treatment plants and drainage systems. Eq.23 provides information related to the dimensionless depth rise $Y_1/Y_c$ for the range of discharge ratio $0.3 \leq Q_2/Q_1 \leq 1.5$ in right angled junction of rectangular open channels. The proposed relationship between $Y_1/Y_c$ and $Q_2/Q_3$ is validated by experimental data of present and previous studies.

Eq.32 provides an estimate of the water depth in the stagnation zone at the corner upstream of the junction. For right angled junction of rectangular open channels, the proposed relationship between $Y_s$ and $Y_c$ is validated by experimental data of present and previous studies.

In the general case, the application of the momentum principle relies on the approximation of the momentum transfer from the branch channel to the main channel. The accuracy of the prediction of the blockage phenomenon may be limited by this approximation. However, in field flows, the junction angle is usually equal or smaller than $90^\circ$; hence, Eq.11 will give an upper bound of the momentum transfer.
One can also use the energy principle to predict this blockage, provided the experimental values of $\alpha_3$ can be obtained. Further, the prediction of blockage is expected to be reasonable even though the estimation of $\alpha_3$ has limited accuracy.
CHAPTER IV

Dividing flow in open channels
(Branch channel junction)

IV-1 Introduction:

The study of the division of flows in open channels has a direct application in the
design of water and waste water treatment plants and open channel networks of irrigation
and drainage systems. In dividing flow, the flow at the junction is characterized by a large
number of variables and features like flow separation, recirculation in the branch and wave
formation in both the main and the branch channel. Furthermore, depending on the
downstream conditions in the main and the branch channel, the flow in the junction may be
subcritical or supercritical. Hence, comprehensive analytical studies related to dividing
flow in open channels do not seem to exist.

Taylor (1944) studied the problem of dividing flow by analyzing experimental data
and proposed a graphical solution for the specific case of rightangled junctions. The
application of his study is limited, since he assumed that the inflows to the branches are not
controlled by gates but are merely governed by the characteristics of the junction and the
branches themselves. In other words, it is admittedly a very special case, as such an ideal
set of conditions could hardly be expected to occur in practice. Grace and Priest (1958)
presented experimental results for different width ratios of the branch with the main
channel and classified dividing flow into two regimes based on the existence of surface
waves. Conformal transformation techniques have been used to solve analytically the
problem of the branch channel (Milne Thomson - 1949; Murota - 1958; Tanaka - 1957).
However, the assumption of flow depth being constant in all channels makes it unrealistic for use in practice.

Some earlier investigators (Krishnappa and Seetharamiah - 1963; Pattabhiramiah - 1960; Pattabhiramiah and Rajaratnam - 1966; Rajaratnam -1967) also studied dividing flow where the branch flow was treated as flow through a side weir of zero sill height to obtain an experimental coefficient. Hager (1983) proposed a simplified model to evaluate loss coefficients for flow through the branch channel.

Law and Reynolds (1966) conducted both analytical and experimental investigations to study dividing flow. In this study, unfortunately, the ratio of the depths in the main channel upstream ($Y_{d1}$) and downstream ($Y_{d2}$) of the junction was assumed to be unity. In fact, in general $Y_{d2}$ is greater than $Y_{d1}$ even for small values of the discharge ratio $Q_{d3}/Q_{d1}$. Here, $Q_{d3}$ is the branch channel discharge and $Q_{d1}$ is the discharge in the approach channel. The ratio $Y_{d2}/Y_{d1}$ can be much greater than unity for larger ratios $Q_{d3}/Q_{d1}$. Further, in their study, the reported values of $Y_{d2}$ were less than $Y_{d1}$ in a very large number of tests. This indicates the possible influence of the drawdown on the measurement of $Y_{d2}$.

Recently, Ramamurthy and Satish (1987), and Satish (1986) have studied the dividing flow problem in open channels with short branches. Their studies were limited to the case where the Froude number in the branch channel is greater than 0.35, which ensures unsubmerged flow conditions at the entrance to the branch.

In the present study, the solution to the general case of dividing flow through right angled junctions is obtained for all branch flow conditions. To this end, the discharge ratio $Q_{d2}/Q_{d1}$ is expressed in terms of the Froude number $F_{d1}$ and the depth ratio $Y_{d1}/Y_{d2}$. Alternatively, the discharge ratio can be expressed in terms of the Froude numbers in the main channel upstream and downstream of the junction. The study is based on the transfer of momentum from the main channel to the branch channel. Data from the present and previous studies are used to verify the model.
IV-2 Theoretical investigation:

As mentioned in chapter I, flow through an open channel junction is a phenomenon that involves numerous variables, such as the angles of intersection, the width ratios of the adjoining channels, the shape and slope of the channels etc. In analyzing the problem, one has to adopt a simpler version of the phenomenon by designating some of the variables to be more important than others. In the present study, the specific case shown in Fig.5 is investigated. The channels have the same rectangular section and are horizontal. The junction angle is equal to $90^\circ$. This case is chosen because of the following reasons:

a) wide channels in the field are usually assumed as rectangular channels;
b) the slopes of real channels are usually small;
c) this case was chosen in previous studies (Law-1965, Sridharan-1966) whose experimental data can be used as independent checks for the present model;

Fig.5 shows a right angled junction in which $Q$, $V$, $Y$, $F$ are respectively discharge, mean velocity, flow depth and Froude number; subscripts $d1$, $d2$, $d3$ denote the conditions of flow at sections $AB$, $GF$, $HK$; PEC is the dividing streamline, $w$ is the width of both channels, $EB = w_2$ and $EA = w_3$. The direction $Ex$ is parallel to the main channel axis.

The following assumptions are made to study the dividing flow phenomenon:

1) The main and branch channels have the same rectangular section and are horizontal;
2) The incoming flow $Q_{d1}$ is subcritical;
3) The flows at sections $AB$, $GF$ and $HK$ are nearly uniform; hence, $Q_{d1}/w=Q_{d2}/w_2=Q_{d3}/w_3$. The momentum coefficients at these sections are close to unity;
4) Friction loss in section ADCE and the energy loss along the streamline EC are negligible.

5) For the dividing streamline PEC (Fig.5), the stagnation point is at C; the depth \( Y \), at an indefinitely small section \( ds \), varies from \( Y_{d1} \) at E to \( Y_{d1} (1 + F_{d1}^{2}/2) \) at C. This variation is assumed to follow a very simple curvilinear relation such as:

\[
Y = Y_{d1} + az^2 \tag{34}
\]

where \( a \) is a constant and \( z \) is the distance from \( ds \) to Ex.

IV-2-1 Momentum transfer from the main to the branch channel:

For the control section ADCE in Fig.5, the application of the momentum equation in the direction Ex yields the following relation:

\[
U = \frac{1}{2} \gamma w Y_{d1}^2 + \rho Q_{d} V_{d1} - \int_{E}^{C} \frac{1}{2} \gamma Y^2 \, ds \cos \psi \tag{35}
\]

where \( U \) is the unknown magnitude of the momentum transfer from the main to the branch channel, \( \rho \) is density of water, \( \gamma \) is specific weight of water, \( \psi \) is the angle between \( ds \) and the branch channel axis.

From assumption 5, it follows that,

\[
Y = Y_{d1} + \frac{F_{d1}^2 Y_{d1}}{2 \, w_3^2} \, z^2 \tag{36}
\]

Using Eq.36 in Eq.35, and noting that \( dz = ds \cos \psi \) (Fig.5),
\[ U = \frac{1}{2} \gamma w_3 Y_{d1}^2 + \rho Q_{d3} V_{d1} - \frac{1}{2} \int_0^{w_3} \gamma \left[ Y_{d1} + \frac{F_{d1}^2 Y_{d1}}{2 w_3^2} z^2 \right]^2 \, dz \]  \quad (37)

Simplifying the above expression after integration and noting that:

\[ \gamma w_3 Y_{d1}^2 F_{d1}^2 = \rho Q_{d3} V_{d1} \]

one gets,

\[ U = \rho Q_{d3} V_{d1} \left( \frac{5}{6} - \frac{F_{d1}^2}{40} \right) \]  \quad (38)

Alternatively, Eq.38 can be written in the following form:

\[ U = k \rho Q_{d3} V_{d1} \]  \quad (39)

where, \( k \) denotes the momentum transfer coefficient,

\[ k = \frac{5}{6} - \frac{F_{d1}^2}{40} \]  \quad (39-a)

Eqs.38 and 39 denote the momentum transfer from the main to the branch channel in the direction of the main channel axis.
IV-2-2 Momentum relationship:

Applying the momentum equation to section ABGF in the direction \( x \) (Fig.5), one gets:

\[
\frac{1}{2} \gamma w Y_{d1}^2 + \rho Q_{d1} V_{d1} = U + \rho Q_{d2} V_{d2} + \frac{1}{2} \gamma w Y_{d2}^2
\]  
(40)

Noting that:

\[
\rho Q_{d1} V_{d1} = \gamma w Y_{d1}^2 F_{d1}, \quad \rho Q_{d2} V_{d2} = \gamma w Y_{d2}^2 F_{d2}, \quad U = k \rho Q_{d3} V_{d1}
\]

and simplifying,

\[
\left( \frac{Y_{d1}}{Y_{d2}} \right)^2 + 2 \left( \frac{Y_{d1}}{Y_{d2}} \right)^2 F_{d1}^2 \left[ 1 - k \frac{Q_{d3}}{Q_{d1}} \right] = 1 + 2 F_{d2}^2
\]  
(41)

Further, one notes that,

\[
F_{d2}^2 = \frac{F_{d1}^2 R_{d1}^2}{R_{dy}^3}
\]  
(42)

where: \( R_{dq} = Q_{d2} / Q_{d1}, R_{dy} = Y_{d2} / Y_{d1} \)

Using Eqs.41 and 42, and noting that \( Q_{d3} / Q_{d1} = 1 - R_{dq} \),

\[
\frac{1}{R_{dy}^2} + \frac{2 F_{d1}^2 \left[ 1 - k (1 - R_{dq}) \right]}{R_{dy}^2} = 1 + \frac{2 F_{d1}^2 R_{dq}^2}{R_{dy}^3}
\]  
(43)
Simplifying,

\[ 2 F_{dl}^2 R_{dq}^2 \left[ 1 + 2 F_{dl}^2 (1 - k (1 - R_{dq})) \right] R_{dy} + R_{dq}^3 = 0 \quad (44) \]

where,

\[ k = \frac{5}{6} - \frac{F_{dl}^2}{40} \]

Hence, \( R_{dq} \) can be expressed directly in terms of \( F_{dl} \) and \( R_{dy} \).

From Eq.44,

\[ \frac{1 - R_{dl}}{40} F_{dl}^4 + \left[ \frac{1}{6} + \frac{5}{6} R_{dq} - \frac{R_{dq}^2}{R_{dy}} \right] F_{dl}^2 + \frac{1 - R_{dy}^2}{2} = 0 \quad (45) \]

Hence, \( F_{dl} \) can be expressed in terms of \( R_{dq} \) and \( R_{dy} \) (Fig.21).

Alternatively, from Eqs.41 and 42, one gets:

\[ \frac{4}{F_{dq}} = \frac{4}{F_{dl}^3} \frac{4}{R_{dq}^3} \frac{1 + 2 F_{dl}^2}{1 + 2 F_{dl}^2 \left[ 1 - k (1 - R_{dq}) \right]} \quad (46) \]

Hence, \( R_{dq} \) can be expressed directly in terms of \( F_{dl} \) and \( F_{d2} \) (Fig.31).
IV-2-3 Generalization of the problem.

Flow through a channel junction is a complicated phenomenon that involves numerous variables; hence, the generalization of the problem does not seem exist.

When the junction angle \( \theta \), the width ratio \( R_w \) and the other geometrical configurations change, the transfer of momentum from the main to the branch channel \( (U=k\rho Q_{d3}V_{d1}) \) will change; i.e. the momentum transfer coefficient \( (k) \) is a function of these variables. In other words, the applications of hydraulic theory to the problem encounters limitations and a model study will give the best solution for the flow characteristics involved.

However, with a given geometry of channel junction, one can determine \( k \) for the prescribed conditions. For example, in the present study, since \( 0 < F_{d1} < 1 \), using Eq.39-a, one can obtain the approximation: \( k = 0.82 \pm 0.01 \) or \( k = 0.82 \)

Figs. 21 to 26 show the variation of Froude number \( F_{d1} \) in terms of discharge ratio \( R_{dq} \) with depth ratio \( R_{dy} \) as a group variable, for different values of \( k \). Figs. 27 to 32 show the variation of discharge ratio \( R_{dq} \) in terms of Froude number \( F_{d2} \) with Froude number \( F_{d1} \) as a group variable, for different values of \( k \).
IV-3 Experimental facility

The main and the branch channels were 25.4 cm wide and 43.2 cm deep. The channels were horizontal and made of polished stainless steel plates (Fig. 6). The junction angle was 90 degree and its edges were sharp. A large number of 1.6 mm diameter wall static taps were provided at 40 mm horizontal intervals and 50 mm vertical intervals along the walls KD and HC (Fig. 6). Near the corner C and D, the tap intervals were even closer. These taps were connected to an inclined manometer bank which had an inclination of 1:6. Baffles and screens were used in the upstream sections of the channels to ensure the flow to be free from large scale turbulence in the channel reaches upstream of the junction.

The measurement techniques used in the study of dividing flow are the same techniques described in Chapter III, page 28.
**IV-4 Discussion of results:**

**IV-4-1 Momentum transfer from the main to the branch channel**.

Using experimental data, two independent methods are used to check the validity of Eqs. 38 and 39:

1) An estimate of the lateral momentum transfer \( U \) in Eq.40 can be obtained by direct measurement of the other variables related to the main channel only.

2) Alternatively, applying the momentum equation in direction \( x \) to the branch channel section CDKH in Fig.5, the momentum transfer from the main to the branch channel will be:

\[
U = P_2 - P_1 \quad (47)
\]

where, \( P_2 \) is the pressure force on the wall CH and \( P_1 \) is the pressure force on the wall DK.

These pressure forces can be calculated using the integration of the recorded wall pressures.

Experimental data from the present study and from previous studies (Satish-1986) indicate that there is a fair agreement between the measured values of \( U \) (Eqs.40 and 47) and the predicted ones (Eq.38).

**IV-4-2 Momentum relationship**:

Based on experimental results, Taylor (1944) proposed a graphical solution for dividing flow in a very special case where the inflows to the branches are not controlled by
gates but merely by the characteristics of the junction and the branches themselves. Furthermore, the flow pattern of the branch channel is very complicated due to flow separation, recirculation, the formation of waves and hydraulic jump. Hence, the measurement of the flow depth in the branch channel \( Y_{d3} \) is not very accurate. It should be recalled that, for the problem of dividing flow, \( Y_{d3} \) is an important parameter in Taylor's graphical solution.

On the other hand, in the present study, the measurement of depth is confined to sections AB and GF where the flow is nearly uniform when \( F_{d1} \) is not very large. It is conceded that the accuracy of \( Y_{d2} \) is considerably affected when the flow at section FG tends to be non-uniform. Such situations occur for \( F_{d1} > 0.75 \). Consequently, test data for which \( F_{d1} > 0.75 \) was not included for verifying the proposed model.

The limit of \( F_{d1} \) prescribed in the above analysis is quite high to include most practical situations. Furthermore, the present model accommodates common field structures in which the flow in the main and the branch channel are regulated by control gates. Lastly, the present analysis requires only two depth measurements compared to three depth measurements in Taylor's experimental study. Figs.21 to 26 show the dependence of discharge ratio on Froude number and depth ratio.

One also notes that the flow depth right after the junction increases in the main channel and decreases in the branch channel. In other words, the flow depth in the main channel, right after the junction, is the highest depth of flow at the dividing flow phenomenon. For a given discharge ratio and Froude number \( F_{d1} \); using Eq.44, one can evaluate the depth ratio and hence, the highest flow depth \( Y_{d2} \) near the junction. Table 8 shows the results of the present tests. One notes a fair agreement between the measured depth \( Y_{d2} \) and the predicted one. Tables 9 and 10 compare the data of previous studies (Sridharan-1966; Law-1965) with the proposed model. The agreement seems to be reasonable.
IV-4-3 Generalization of the problem.

In the case of right angled junction, the momentum transfer coefficient:

\[ k = \frac{5}{6} - \frac{F_{d1}^2}{40} \]

can be approximated as 0.82 (section IV-2-3). This is verified by the experimental data of present and previous studies (Satish-1986).

In the general case, the momentum transfer coefficient \( k \) will be a function of numerous variables, such as the junction angle, the channel width ratio, and other geometrical configurations.

For any specific case, one should evaluate the value of the momentum transfer coefficient \( k \).

Figs. 21 to 26 show the variation of Froude number \( F_{d1} \) in terms of discharge ratio \( R_{dq} \) with depth ratio \( R_{dy} \) as a group variable. Each figure correspond with a specific value of \( k \) (assuming \( k \) varies from 0.1 to 0.9).

Figs. 27 to 32 show the variation of discharge ratio \( R_{dq} \) in terms of Froude number \( F_{d2} \) with Froude number \( F_{d1} \) as a group variable. Each figure correspond with a specific value of \( k \) (assuming \( k \) varies from 0.1 to 0.9). These figures represent the effects of downstream conditions on discharge ratio or on the division of flow in open channels. They can be used to generalize the model of previous studies (Ramamurthy and Satish, 1987; Satish, 1986).
IV-5 Conclusions and applications:

For dividing flow in rectangular open channels, a theoretical model was developed to relate the discharge ratio $Q_{d2}/Q_{d1}$ with the Froude number $F_{d1}$ and the depth ratio $Y_{d2}/Y_{d1}$. For $0<F_{d1}<0.75$ the model is validated by experimental data. The proposed method is more useful and practical than Taylor's method because it accommodates the existence of control gates and it does not require the measurement of flow depths in the branch channels where the flow is extremely complicated.

Alternatively, the model can relate the discharge ratio with the Froude numbers in the main channel, upstream and downstream of the junction. In other words, the downstream effects are incorporated in the present model. Hence, the present case has much wider applications than the previous studies (Ramamurthy and Satish - 1987; and Satish - 1986), since there are no restriction to the nature of flow in the branch channel.

Figs. 21 to 32 can be used as design charts in the study of dividing flow in open channels.
Chapter V

Open channel flow through transverse floor outlets

V.1 Introduction:

Study of subcritical flow through floor outlets or bottom racks in rectangular open channels have direct applications in horizontal trash-racks of hydro power plants, in curb outlets of streets (Townsend-1984) and in stream diversion works. An understanding of the flow behaviour over bottom racks or floor outlets is very important for the proper design of such flow diversion structures.

Design charts for multiple outlets (Townsend, R.D., 1984; Metcalf and Eddy Inc., 1972) have been developed based on experimental results. Nasser et al. (1980) proposed an expression for the discharge through a single tranverse floor outlet located in an open channel, disregarding the effect of the ratio of the outlet width to the flow depth on the outflow. In a recent study (Ramamurthy and Satish, 1986), this effect was incorporated in the expression for the diverted discharge on the basis of an existing hydrodynamic model (Mc Nown and Hsu, 1950) for the lateral efflux from a two dimensional conduit. In these studies (Nasser et al., 1980, Ramamurthy and Satish, 1986), the downstream conditions consisted of a free overfall at the end of the channel. In a more general situation, the downstream depth will be the normal depth for the downstream channel which could vary over a wide range and hence affect the outflow. Alternatively, one may come across an artificial control such as an end gate which may also drastically
increase the diverted discharge. Hence, to solve the general problem of flow through floor outlets, the downstream conditions are incorporated in the present model while deriving an expression for the outflow. Experimental data are provided to verify the model.
V-2 Theoretical consideration:

Fig. 7 shows a rectangular floor outlet in an open channel through which part of the approach flow is diverted. Let $Q, V, Y$ and $F$ denote respectively the discharge, mean velocity, water depth and Froude number of the flows. The subscripts 01 and 02 denote the conditions of flows at section AB and CD. $Q_f$ and $V_f$ denote respectively the discharge and mean velocity at section GH. $L$ and $b$ are respectively the length of the outlet and the width of the channel. $\phi$ is the angle between $V_f$ and the channel axis.

The following assumptions are made to study open channel flow through a transverse floor outlet:
1) The channel is rectangular and horizontal; the outlet edges are sharp.
2) Boundary friction is negligible in the control volume ABDC.
3) The flows are nearly uniform at sections AB and CD which are far away from the zone of disturbance.
4) The incoming flow $Q_{01}$ is subcritical.

V-2-1) The momentum transfer through the outlet:

Considering the control volume EKHG and applying the momentum equation in direction $x$, one gets:

$$U_f = \rho Q_f V_f \cos \phi$$  \hspace{1cm} (48)

where, $U_f$ is the momentum transfer in the axial direction of the main flow through the outlet, $\rho$ is the density of water.
According to previous studies (Nasser et al -1980; Venkataraman-1977), the following approximation is valid:

\[ V_r \cos \phi = V_{01} \quad \text{(49)} \]

Hence, the momentum \( U_f \) transferred through the outlet is:

\[ U_f = \rho Q_f V_{01} \quad \text{(50)} \]

**V-2-2) Effect of downstream conditions on the discharge through the outlet:**

Considering the control volume ABCD in Fig. 7 and applying the momentum equation in the direction \( x \), one gets another independent relation which involves \( U_f \):

\[ \frac{1}{2} \gamma b Y_{01}^2 + \rho Q_{01} V_{01} = U_f + \frac{1}{2} \gamma b Y_{02}^2 + \rho Q_{02} V_{02} \quad \text{(51)} \]

where \( \gamma \) is specific weight of water; \( \rho, b, Y_{01}, Q_{01}, V_{01}, U_f, Y_{02}, Q_{02}, V_{02} \) are defined as above.

Noting that:

\[ \rho Q_{01} V_{01} = \gamma b Y_{01}^2 F_{01}^2 \quad \text{(52)} \]

and

\[ \rho Q_{02} V_{02} = \gamma b Y_{02}^2 F_{02}^2 \quad \text{(53)} \]
Using Eqs. 50, 52, 53 in Eq. 51 and simplifying,

\[
\left( \frac{Y_\alpha}{Y_{\alpha}} \right)^2 + 2 \left( \frac{Y_{01}}{Y_{\alpha}} \right)^2 F_{01}^2 \left[ 1 - \frac{Q_f}{Q_{01}} \right] = 1 + 2 F_{02}^2 \quad (54)
\]

Further,

\[
F_{02}^2 = \frac{F_{01}^2 R_{eq}^2}{R_{\alpha y}^3} \quad (55)
\]

where \( R_{eq} = Q_{02} / Q_{01} \), \( R_{\alpha y} = Y_{02} / Y_{01} \).

Noting that \( Q_f / Q_{01} = (1 - R_{eq}) \) and using Eqs. 54 and 55,

\[
\frac{\frac{4}{3}}{F_{02}} \frac{\frac{4}{3}}{F_{02}} = \frac{\frac{4}{3}}{F_{01}} \frac{\frac{4}{3}}{F_{01}} \frac{R_{eq}}{R_{eq}} \quad (56)
\]
V-3- Experimental facility and procedure:

The channel was 25.4 cm wide and 12.19 m long. It was rectangular, horizontal and made of polished stainless steel plates (Fig.8). Transverse rectangular slots fixed in the channel floor were made from machined plexiglass sheets. The edges of the floor outlet were sharp. Two different outlet lengths (1.27 cm and 2.54 cm) were used. Baffles and screens were used in the upstream sections of the channels to ensure the flow to be free from large scale turbulence in the channel reaches upstream of the floor outlet.

The measurement techniques used in the study of open channel flow through floor outlets are the same techniques described in Chapter III, page 28.

In each test, values of $Y_{01}, Y_{02}, Q_{01}$ and $Q_{02}$ were determined. These in turn yielded $F_{01}, F_{02}, U_f, R_{oq}$ (Tables 11, 12, 13, 14).
V-4-Discussion and application of results

V-4-1. The momentum transfer through the floor outlet:

An estimate of the experimental value of $U_f$, the momentum transfer in the axial
direction of the main flow was obtained by direct measurement of the variables in Eq.51
which are related to the main channel only. Both the experimental values (Eq.51) and the
theoretical values of $U_f$ (Eq.50) are shown in Tables 11 and 12. The agreement between
the two values seem to validate the proposed model.

V-4-2. Relationship between Froude numbers and discharge ratio:

Tables 13 and 14 show the experimentally determined variation of the discharge ratio
$R_{Oq}$ as a function of the Froude number $F_{02}$ and the approach channel Froude number
$F_{01}$. The tables also indicate the theoretical values of $R_{Oq}$ for given values of $F_{02}$ and
$F_{01}$ (Eq.56). The results shown provide a verification of the proposed model for the
variation of $R_{Oq}$ with $F_{01}$ and $F_{02}$.

Fig.42 show the variation of Froude number $F_{02}$ in terms of discharge ratio $R_{Oq}$ with
Froude number $F_{01}$ as a group variable. In other words, the outlet conditions do affect the
downstream conditions of the channel.

Fig.33 was developed using Eq.56. It shows the dependence of $R_{Oq}$ on $F_{02}$ with
$F_{01}$ as the group variable. Fig.33 can be used in the design of floor outlets in the general
case where the channel is controlled by an end gate.

The results find direct applications in the design of engineering structures such as horizontal trash-racks of hydro power plants and curb outlets of streets. New application may also be found in the design of open channel networks of water and waste water treatment plant. According to Benefield et al. (1984), "in water and waste water treatment plants, open channels rather than manifolds are often used to distribute incoming flow to parallel treatment units, such as aeration tanks, sedimentation basins, filters and flocculation basins". Such systems employ short open channels in conjunction with additional flow controlling devices (Benefield et al. 1984).

In a practical situation, the characteristics of the approach flow like $Q_{01}$ and $F_{01}$ are known. The designer chooses a specified value of $Q_2$ and $R_{oq}=Q_2/Q_{01}$, for desired flow diversion, and determines the value of $F_{02}$ using Fig.33. Further, $F_{02}$ and $Q_2$ yield $Y_{02}$. Finally, for a control device like an end gate, the rating curve can be used to obtain the required gate opening.
V-Conclusions:

Based on the momentum principle, the momentum transfer in the axial direction of the main flow through floor outlets in a rectangular channel can be evaluated. Further, a relationship between discharge ratio and Froude numbers upstream and downstream of the outlet can be derived.

This relationship represents the effects of downstream conditions on the discharge characteristics of the outlet. The results are useful in the design of floor outlets in the general case where the downstream channel depth is altered by a controlling device or when the downstream channel is very long. Experimental data appear to validate the proposed model.
CHAPTER VI

Conclusions and Scope for further study

This chapter summarises the highlights of the present study related to junction flows in open channels.

VI-1 Junction flows in open channels:

VI-1-1 Combining flow:

In the application of the momentum principle, inclusion of the lateral momentum transfer from the branch channel to the main channel is essential for a proper analysis of combining flow in open channels.

The prediction of blockage phenomenon in combining flow is verified by experimental data of the present and previous studies.

VI-1-2 Dividing flow:

In the case of dividing flow, the transfer of momentum from the main channel to the branch channel can be evaluated theoretically and experimentally. Hence, the division of flow can be evaluated in terms of the Froude numbers in the main channel, upstream and downstream of the junction. This theoretical relationship is derived on the basis of the momentum principle and is verified by experimental data of the present and previous studies.
VI-1-3 Flow through transverse floor outlets:

Open channel flow through transverse floor outlets is a special case of dividing flow. In this case, the momentum transfer coefficient is equal to unity.

VI-2 Scope for further study:

The direct determination of momentum transfer in open channels has a great application for combining flow as well as dividing flow and flow through transverse floor outlets. It can be considered as a new method in the analysis of junction flows.

Significant studies have been done in the past to understand the behavior of junction flows in closed conduits (Bajura, 1970, 1976; Mc Nown, 1950, 1954) which involved the concept of the momentum transfer. The present method of direct determination of momentum transfer can also be used in the study of combining flow and dividing flow in closed conduits. One can expect similar momentum relationships to predict flow behaviors at junctions.
Appendix I - References


21. Law, S. W., "Dividing flow in open channel", master thesis presented to the Faculty of graduate studies, McGill University, Montreal, Canada, Aug., 1965.


Fig. 1 Simple channel junctions
   a) Combining flow
   b) Dividing flow
Fig. 2 A channel junction
Fig. 3 Combination of flows at right angled, rectangular open channel junction

\[ EF = ED = DA = AH = AB = 25.4 \text{ cm} \]
Fig. 4 Experimental facility for Combining flow
All dimensions - cm
(not to scale)
Fig. 5  Dividing flow in rectangular open channels
All dimensions in cms
Fig. 6 Experimental facility for dividing flow
All dimensions in cms
(not to scale)
Fig. 7  Open channel flow through transverse floor outlets

$L$ : variable

$AE = KC = 25.4 \text{ cm}$

(Not to scale)
Fig. 8 Experimental facility
Open channel flow through transverse floor outlets
All dimensions in cms
(not to scale)
Discharge ratio $Q_2/Q_1$

Fig. 9 Variation of dimensionless lateral momentum $\rho Q_2 V_2/(P_{u1} - P_d)$ with discharge ratio $Q_2/Q_1$
Fig. 10  Verification of the relationship between the depth ratio 
and the discharge ratio  
All dimensions in cms  
Data of Lin and Soong (1979)  
Present data
Fig. 11 Velocity distribution at section II for selected case: medium total discharge $Q_3=0.0190$ m$^3$/s, $Q_2/Q_3=0.55$.
Fig. 12 Velocity distribution at section IIIG for selected case: large total discharge $Q_3 = 0.0310$ m$^3$/s, $Q_2/Q_3 = 0.45$
Fig. 13 Velocity distribution at section HG for selected case:
small total discharge $Q_3 = 0.0129 \text{ m}^3/\text{s}$, $Q_2/Q_3 = 0.40$
Fig. 14 Variation of $\alpha_3$ and of $\beta_3$ vs. discharge ratio $R_q$

Present data, width ratio is unity
Data of Joy and Townsend (1981)
Fig. 15 Variation of depth ratio \( \frac{Y_1}{Y_c} \) with discharge ratio \( \frac{Q_2}{Q_3} \)

- ○ - Data of Lin and Soong (1979)
- △ - Present data
Fig. 16 Variation of dimensionless blockage in terms of discharge ratio and Froude number downstream of the junction:
(1) $F_3=0.2$
(2) $F_3=0.4$
(3) $F_3=0.6$
Fig. 17  Variation of Energy loss in terms of Reynolds number (present data)
Fig. 18 Variation of Energy loss in terms of Reynolds number
Data of Lin and Soong (1979)
Fig. 19 Variation of depth ratio ($Y_s/Y_c$) in terms of discharge ratio ($Q_2/Q_3$)
Present data
Fig. 20 Variation of depth ratio ($Y_s/Y_c$) in terms of discharge ratio ($Q_2/Q_3$)
Data of Lin and Soong (1979)
Fig. 21  Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable

Momentum transfer coefficient $k = 0.82$

(1) $R_{dy} = 1.16$
(2) $R_{dy} = 1.12$
(3) $R_{dy} = 1.09$
(4) $R_{dy} = 1.05$
(5) $R_{dy} = 1.02$
Fig. 22 Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable.

Momentum transfer coefficient $k = 0.1$

1. $R_{dy} = 1.16$
2. $R_{dy} = 1.12$
3. $R_{dy} = 1.09$
4. $R_{dy} = 1.05$
5. $R_{dy} = 1.02$
Fig. 23 Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable.
Momentum transfer coefficient $k = 0.3$
(1) $R_{dy} = 1.16$
(2) $R_{dy} = 1.12$
(3) $R_{dy} = 1.09$
(4) $R_{dy} = 1.05$
(5) $R_{dy} = 1.02$
Fig. 24 Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable. 
Momentum transfer coefficient $k = 0.5$

1. $R_{dy} = 1.16$
2. $R_{dy} = 1.12$
3. $R_{dy} = 1.09$
4. $R_{dy} = 1.05$
5. $R_{dy} = 1.02$
Fig. 25 Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable

Momentum transfer coefficient $k = 0.7$

1. $R_{dy} = 1.16$
2. $R_{dy} = 1.12$
3. $R_{dy} = 1.09$
4. $R_{dy} = 1.05$
5. $R_{dy} = 1.02$
Fig. 26 Variation of Froude number ($F_{d1}$) in terms of discharge ratio ($R_{dq}$) with depth ratio ($R_{dy}$) as a group variable.

Momentum transfer coefficient $k = 0.9$

1. $R_{dy} = 1.16$
2. $R_{dy} = 1.12$
3. $R_{dy} = 1.09$
4. $R_{dy} = 1.05$
5. $R_{dy} = 1.02$
Fig. 27 Variation of discharge ratio ($Q_{d3}/Q_{d1}$) in terms of Froude number $F_{d2}$

with Froude number $F_{d1}$ as a group variable

Momentum transfer coefficient $k = 0.82$

1. $F_{d1} = 0.2$
2. $F_{d1} = 0.3$
3. $F_{d1} = 0.4$
4. $F_{d1} = 0.5$
5. $F_{d1} = 0.6$
6. $F_{d1} = 0.7$
Fig. 28 Variation of discharge ratio $(Q_{d3}/Q_{d1})$ in terms of Froude number $F_{d2}$

with Froude number $F_{d1}$ as a group variable

Momentum transfer coefficient $k = 0.1$

(1) $F_{d1} = 0.2$
(2) $F_{d1} = 0.3$
(3) $F_{d1} = 0.4$
(4) $F_{d1} = 0.5$
(5) $F_{d1} = 0.6$
(6) $F_{d1} = 0.7$
Fig. 29 Variation of discharge ratio \( \left( Q_{d3}/Q_{d1} \right) \) in terms of Froude number \( F_{d2} \)

with Froude number \( F_{d1} \) as a group variable

Momentum transfer coefficient \( k = 0.3 \)

1. \( F_{d1} = 0.2 \)
2. \( F_{d1} = 0.3 \)
3. \( F_{d1} = 0.4 \)
4. \( F_{d1} = 0.5 \)
5. \( F_{d1} = 0.6 \)
6. \( F_{d1} = 0.7 \)
Fig. 30 Variation of discharge ratio $(Q_{d3}/Q_{d1})$ in terms of Froude number $F_{d2}$ with Froude number $F_{d1}$ as a group variable

Momentum transfer coefficient $k = 0.5$

(1) $F_{d1} = 0.2$
(2) $F_{d1} = 0.3$
(3) $F_{d1} = 0.4$
(4) $F_{d1} = 0.5$
(5) $F_{d1} = 0.6$
(6) $F_{d1} = 0.7$
Fig. 31 Variation of discharge ratio ($Q_d^3/Q_d^1$) in terms of Froude number $F_{d2}$

with Froude number $F_{d1}$ as a group variable

Momentum transfer coefficient $k = 0.7$

1. $F_{d1} = 0.2$
2. $F_{d1} = 0.3$
3. $F_{d1} = 0.4$
4. $F_{d1} = 0.5$
5. $F_{d1} = 0.6$
6. $F_{d1} = 0.7$
Fig. 32 Variation of discharge ratio \( \frac{Q_{d3}}{Q_{d1}} \) in terms of Froude number \( F_{d2} \)

with Froude number \( F_{d1} \) as a group variable

Momentum transfer coefficient \( k = 0.9 \)

(1) \( F_{d1} = 0.2 \)
(2) \( F_{d1} = 0.3 \)
(3) \( F_{d1} = 0.4 \)
(4) \( F_{d1} = 0.5 \)
(5) \( F_{d1} = 0.6 \)
(6) \( F_{d1} = 0.7 \)
Fig. 33 Variation of discharge ratio \((Q/Q_0)\) in terms of Froude number \(F_{02}\) with Froude number \(F_{01}\) as a group variable

Momentum transfer coefficient \(k = 1.0\)

(1) \(F_{01} = 0.2\)
(2) \(F_{01} = 0.3\)
(3) \(F_{01} = 0.4\)
(4) \(F_{01} = 0.5\)
(5) \(F_{01} = 0.6\)
(6) \(F_{01} = 0.7\)
Table 1: Variation of dimensionless lateral momentum $\rho Q_2 V_2/(P_u - P_d)$ in terms of discharge ratio $Q_2/Q_1$ in combining flow

<table>
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Table 2: Comparison of predicted and measured stages in combining flow
Present data

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<th>$Q_3$ (cm)</th>
<th>$Y_c$ (cm)</th>
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<th>$R_y$</th>
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Table 3: Comparison of predicted and measured stages in combining flow
Data of Lin and Soong (1979)

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Data of Lin and Soong (1979)

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Table 5 (continued)

| $R_q$ | $Q_3$ $\text{m} / \text{s}$ | $Y_c$ $\text{cm}$ | $Y_1$ $\text{cm}$ | $Y_2$ $\text{cm}$ | $E_j$ $\text{cm}$ | $Re$ $(10^{-3})$ | Energy loss $\%$
|-------|-----------------|----------------|----------------|----------------|----------------|-----------------|----------------
| 0.15  | 0.0170          | 5.7            | 8.6            | 8.6            | 9.2            | 28              | 6.9            |
| 0.16  | 0.0173          | 5.8            | 8.8            | 8.7            | 9.3            | 29              | 7.0            |
| 0.20  | 0.0180          | 6.0            | 9.1            | 9.0            | 9.6            | 29              | 6.3            |
| 0.23  | 0.0185          | 6.1            | 9.2            | 9.1            | 9.6            | 30              | 4.8            |
| 0.26  | 0.0195          | 6.3            | 9.6            | 9.5            | 10.0           | 32              | 4.5            |
| 0.29  | 0.0204          | 6.6            | 9.9            | 9.8            | 10.3           | 33              | 4.3            |
| 0.31  | 0.0210          | 6.7            | 10.1           | 10.1           | 10.5           | 34              | 3.8            |
| 0.33  | 0.0215          | 6.8            | 10.3           | 10.2           | 10.6           | 35              | 3.6            |
| 0.35  | 0.0225          | 7.1            | 10.6           | 10.5           | 10.9           | 36              | 3.6            |
| 0.37  | 0.0230          | 7.2            | 10.9           | 10.8           | 11.2           | 37              | 3.7            |
| 0.39  | 0.0237          | 7.3            | 11.1           | 11.0           | 11.4           | 38              | 3.3            |
| 0.41  | 0.0245          | 7.5            | 11.3           | 11.3           | 11.6           | 39              | 2.9            |
| 0.43  | 0.0255          | 7.7            | 11.7           | 11.6           | 12.0           | 40              | 2.8            |
| 0.46  | 0.0267          | 8.0            | 12.0           | 11.9           | 12.2           | 40              | 1.8            |
| 0.11  | 0.0205          | 6.4            | 9.4            | 9.4            | 10.2           | 33              | 5.2            |
| 0.12  | 0.0208          | 6.5            | 9.6            | 9.6            | 10.4           | 33              | 5.9            |
| 0.14  | 0.0211          | 6.6            | 9.8            | 9.7            | 10.5           | 34              | 5.6            |
| 0.16  | 0.0216          | 6.7            | 10.0           | 10.0           | 10.7           | 34              | 5.8            |
| 0.19  | 0.0225          | 6.9            | 10.3           | 10.3           | 10.9           | 36              | 5.0            |
| 0.22  | 0.0233          | 7.1            | 10.5           | 10.5           | 11.1           | 37              | 3.9            |
| 0.25  | 0.0242          | 7.3            | 10.8           | 10.8           | 11.3           | 38              | 3.1            |
| 0.25  | 0.0241          | 7.3            | 10.7           | 10.6           | 11.2           | 38              | 2.3            |
| 0.26  | 0.0247          | 7.4            | 11.1           | 11.1           | 11.6           | 38              | 4.0            |
| 0.28  | 0.0253          | 7.6            | 11.2           | 11.2           | 11.7           | 39              | 2.9            |
| 0.30  | 0.0258          | 7.7            | 11.5           | 11.4           | 11.9           | 40              | 3.1            |
| 0.32  | 0.0266          | 7.9            | 11.7           | 11.6           | 12.1           | 41              | 2.7            |
| 0.33  | 0.0272          | 8.0            | 11.9           | 11.9           | 12.3           | 41              | 2.8            |
| 0.35  | 0.0281          | 8.2            | 12.3           | 12.2           | 12.7           | 42              | 3.3            |
Table 6: Variation of blockage in terms of discharge ratio (combining flow)
Present data

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<th>( Y_2 ) cm</th>
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Table 7: Variation of blockage in terms of discharge ratio (combining flow)

Data of Lin and Soong (1979)

For data where \( R_q < 0.23 \) or \( Re < 33000 \), the model is not applicable

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n.a. : not applicable
Table 8: Comparison of experimental data of $Y_{d2}$ and the predicted ones (dividing flow)
Present data

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Note: 1 ft = 0.305 meter
Table 9: Comparison of experimental data of $Y_{d2}$ and the predicted ones (dividing flow-Eq.44)
Data of Sridharan (1966)

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Note: 1 ft = 0.305 meter
Table 10: Comparison of experimental data of $Y_{d2}$ and the predicted ones (dividing flow-Eq.44)

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Note: 1 ft = 0.305 meter
Table 10 (continued)

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Note: 1 ft = 0.305 meter
Table 11: Verification of the momentum transfer through the transverse floor outlet
(outlet length = 1.27 cm = 0.5 inch)

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<td>+6</td>
</tr>
<tr>
<td>.282</td>
<td>.292</td>
<td>.294</td>
<td>.226</td>
<td>.157</td>
<td>.165</td>
<td>-5</td>
</tr>
<tr>
<td>.206</td>
<td>.224</td>
<td>.296</td>
<td>.239</td>
<td>.195</td>
<td>.191</td>
<td>+2</td>
</tr>
<tr>
<td>.302</td>
<td>.32</td>
<td>.487</td>
<td>.417</td>
<td>.247</td>
<td>.263</td>
<td>-6</td>
</tr>
<tr>
<td>.401</td>
<td>.411</td>
<td>.484</td>
<td>.397</td>
<td>.256</td>
<td>.245</td>
<td>+5</td>
</tr>
<tr>
<td>.681</td>
<td>.685</td>
<td>.483</td>
<td>.366</td>
<td>.200</td>
<td>.193</td>
<td>+4</td>
</tr>
</tbody>
</table>

Note: 1 ft = 0.305 meter
1 lb = 4.45 Newtons
Table 12: Verification of the momentum transfer through the transverse floor outlet (outlet length = 2.54 cm = 1.0 inch)

<table>
<thead>
<tr>
<th>$Y_{01}$ (ft)</th>
<th>$Y_{02}$ (ft)</th>
<th>$Q_{01}$ (cfs)</th>
<th>$Q_{02}$ (cfs)</th>
<th>$U_f$ (measur.) (lb)</th>
<th>$U_f$ (predicted) (lb)</th>
<th>Discrepancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.134</td>
<td>.147</td>
<td>.138</td>
<td>.072</td>
<td>.154</td>
<td>.158</td>
<td>+3</td>
</tr>
<tr>
<td>.157</td>
<td>.175</td>
<td>.193</td>
<td>.114</td>
<td>.226</td>
<td>.228</td>
<td>+1</td>
</tr>
<tr>
<td>.172</td>
<td>.195</td>
<td>.248</td>
<td>.157</td>
<td>.319</td>
<td>.305</td>
<td>-4</td>
</tr>
<tr>
<td>.188</td>
<td>.219</td>
<td>.297</td>
<td>.194</td>
<td>.361</td>
<td>.376</td>
<td>+4</td>
</tr>
<tr>
<td>.223</td>
<td>.264</td>
<td>.407</td>
<td>.286</td>
<td>.494</td>
<td>.517</td>
<td>+5</td>
</tr>
<tr>
<td>.244</td>
<td>.287</td>
<td>.460</td>
<td>.331</td>
<td>.537</td>
<td>.566</td>
<td>+5</td>
</tr>
<tr>
<td>.260</td>
<td>.305</td>
<td>.509</td>
<td>.376</td>
<td>.581</td>
<td>.607</td>
<td>+5</td>
</tr>
<tr>
<td>.272</td>
<td>.319</td>
<td>.545</td>
<td>.405</td>
<td>.621</td>
<td>.651</td>
<td>+5</td>
</tr>
<tr>
<td>.288</td>
<td>.335</td>
<td>.594</td>
<td>.450</td>
<td>.686</td>
<td>.693</td>
<td>+1</td>
</tr>
</tbody>
</table>

Note: 1 ft = 0.305 meter
1 lb = 4.45 Newtons
Table 13: Verification of the momentum relationship of flow through transverse floor outlet (outlet length = 1.27 cm = 0.5 inch)

<table>
<thead>
<tr>
<th>$R_{oq}$</th>
<th>$F_{01}$</th>
<th>$F_{02}$</th>
<th>$R_{oq}$</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp.</td>
<td>Exp.</td>
<td>Exp.</td>
<td>predicted</td>
<td>%</td>
</tr>
<tr>
<td>.759</td>
<td>.587</td>
<td>.398</td>
<td>.755</td>
<td>-1</td>
</tr>
<tr>
<td>.687</td>
<td>.549</td>
<td>.338</td>
<td>.683</td>
<td>-1</td>
</tr>
<tr>
<td>.604</td>
<td>.313</td>
<td>.182</td>
<td>.603</td>
<td>0</td>
</tr>
<tr>
<td>.591</td>
<td>.293</td>
<td>.168</td>
<td>.592</td>
<td>0</td>
</tr>
<tr>
<td>.944</td>
<td>.120</td>
<td>.113</td>
<td>.943</td>
<td>0</td>
</tr>
<tr>
<td>.769</td>
<td>.415</td>
<td>.303</td>
<td>.767</td>
<td>0</td>
</tr>
<tr>
<td>.807</td>
<td>.670</td>
<td>.477</td>
<td>.809</td>
<td>0</td>
</tr>
<tr>
<td>.856</td>
<td>.621</td>
<td>.483</td>
<td>.853</td>
<td>-1</td>
</tr>
<tr>
<td>.820</td>
<td>.403</td>
<td>.319</td>
<td>.823</td>
<td>0</td>
</tr>
<tr>
<td>.758</td>
<td>.182</td>
<td>.137</td>
<td>.760</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 14: Verification of the momentum relationship of flow through transverse floor outlet (outlet length = 2.54 cm = 1.0 inch)

<table>
<thead>
<tr>
<th>$R_{eq}$ (exp.)</th>
<th>$F_{01}$ (exp.)</th>
<th>$F_{02}$ (exp.)</th>
<th>$R_{eq}$ predicted</th>
<th>Discrepancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.522</td>
<td>.595</td>
<td>.270</td>
<td>.518</td>
<td>-1</td>
</tr>
<tr>
<td>.591</td>
<td>.656</td>
<td>.329</td>
<td>.589</td>
<td>0</td>
</tr>
<tr>
<td>.633</td>
<td>.736</td>
<td>.386</td>
<td>.640</td>
<td>+1</td>
</tr>
<tr>
<td>.653</td>
<td>.771</td>
<td>.400</td>
<td>.646</td>
<td>-1</td>
</tr>
<tr>
<td>.683</td>
<td>.763</td>
<td>.419</td>
<td>.678</td>
<td>-1</td>
</tr>
<tr>
<td>.703</td>
<td>.818</td>
<td>.446</td>
<td>.694</td>
<td>-1</td>
</tr>
<tr>
<td>.720</td>
<td>.807</td>
<td>.455</td>
<td>.711</td>
<td>-1</td>
</tr>
<tr>
<td>.739</td>
<td>.812</td>
<td>.472</td>
<td>.732</td>
<td>-1</td>
</tr>
<tr>
<td>.743</td>
<td>.813</td>
<td>.476</td>
<td>.737</td>
<td>-1</td>
</tr>
<tr>
<td>.758</td>
<td>.813</td>
<td>.491</td>
<td>.756</td>
<td>0</td>
</tr>
</tbody>
</table>
Example 1:

Determine the rise of flow depths for combining open channel flows at right angled junction, given discharge in the main channel \( Q_1 = 15 \text{m}^3/\text{s} \), discharge in the branch channel \( Q_2 = 9.5 \text{m}^3/\text{s} \), width of both channels \( b = 20 \text{m} \), slope of both channels \( S = 0.3\% \), Manning's coefficient for both channels \( n = 0.05 \). Assuming flows are subcritical and both channels are rectangular.

Solution:

Applying Manning's equation in the upstream section of the main channel, far from the junction:

\[
Q_1 = \frac{1}{n} A_{n1} R_{n1}^2 S^2
\]

where, \( A_{n1}, R_{n1} \) are area and hydraulic radius of the transversal section.

For wide and rectangular channels, \( R_{n1} = Y_{n1} \) (normal depth) and \( A_{n1} = b Y_{n1} \). Hence,

\[
Q_1 = \frac{1}{n} \left(b Y_{n1}\right) \left(Y_{n1}\right)^2 S^2
\]
Or,

\[ Y_{n1} = \left[ \frac{Q_1 n}{b S^2} \right]^{\frac{3}{5}} \]

\[ Y_{n1} = \left[ \frac{15 \times 0.05}{20 \sqrt{0.003}} \right]^{\frac{3}{5}} = 0.8 \text{ m} \]

Similarly, in the upstream section of the branch channel, far from the junction:

\[ Y_{n2} = \left[ \frac{Q_2 n}{b S^2} \right]^{\frac{3}{5}} = \left[ \frac{9.5 \times 0.05}{20 \sqrt{0.003}} \right]^{\frac{3}{5}} = 0.61 \text{ m} \]

Discharge ratio:

\[ R_q = \frac{Q_2}{Q_3} = \frac{Q_2}{Q_1 + Q_2} = \frac{9.5}{15 + 9.5} = 0.39 \]

From Eq.23 or Fig.15:

\[ \frac{Y_1}{Y_c} = 1.5 \]

From Eq.21,

\[ \alpha_3 = 1.25 + 0.5 \times \frac{Q_2}{Q_3} = 1.445 \]
From Eq. 19,

\[ y_c = \left( \frac{\alpha_3 Q_3^2}{g b^2} \right)^{\frac{1}{3}} = \left( \frac{1.445 \times 24.5^2}{9.81 \times 20^2} \right)^{\frac{1}{3}} = 0.61 \text{ m} \]

Hence, the flow depth near the junction is:

\[ y_1 = 1.5 \cdot y_c = 0.92 \text{ m} \]

The rise of the flow depth in the main channel:

\[ y_1 - y_{n1} = 0.92 - 0.80 = 0.12 \text{ m or } 15\%. \]

The rise of the flow depth in the branch channel:

\[ y_1 - y_{n2} = 0.92 - 0.61 = 0.31 \text{ m or } 51\%. \]

Example 2:

For dividing open channel flow at right angled junction, determine the flow depth downstream of the junction in the main channel, given discharge \( Q_{d1} = 20 \text{ m}^3/\text{s} \), flow depth \( y_{d1} = 0.8 \text{ m} \), in the main channel upstream of the junction, and discharge \( Q_{d3} = 13.74 \text{ m}^3/\text{s} \) in the branch channel. Width of both channels: \( b = 15 \text{ m} \).
Solution:

In the main channel upstream of the junction, flow velocity:

\[ V_{d1} = \frac{Q_{d1}}{b Y_{d1}} = \frac{20}{15 \times 0.8} = 1.667 \text{ m}^3/\text{s} \]

Froude number:

\[ F_{d1} = \frac{V_{d1}}{\sqrt{g Y_{d1}}} = \frac{1.667}{\sqrt{9.81 \times 0.8}} = 0.6 \]

From Eq.44, given discharge ratio \( R_{d4} = (Q_{d1} - Q_{d3})/Q_{d1} = 0.313 \) and Froude number \( F_{d1} = 0.6 \), one gets the flow depth ratio:

\[ R_{dy} = \frac{Y_{d2}}{Y_{d1}} = 1.115 \]

Hence, flow depth in the main channel downstream of the junction is:

\[ Y_{d2} = 0.89 \text{ m} \]

Example 3:

Determine the discharge through transverse floor outlet, given discharge \( Q_{01} = 0.2 \text{ m}^3/\text{s} \), Froude number \( F_{01} = 0.77 \), in the section upstream of the outlet, and \( F_{02} = 0.4 \) in the section downstream of the outlet.
Solution:

From Eq.56, with $F_{01}=0.77$, $F_{02}=0.4$, one gets:

$$R_q = \frac{Q_{02}}{Q_{01}} = 0.646.$$ 

Hence, discharge through transverse floor outlet:

$$Q_{of} = Q_{01} \left( 1 - 0.646 \right) = 0.0708 \text{ m}^3/\text{s}.$$

Example 4:

For flow through transverse floor outlet, determine the flow depth downstream of the outlet, given discharge $Q_{01} = 0.4 \text{ m}^3/\text{s}$, Froude number $F_{01} = 0.74$ in the section upstream of the outlet, width of the channel $b=0.4 \text{ m}$ and discharge through floor outlet $Q_{of} = 0.1440 \text{ m}^3/\text{s}$.

Solution:

Flow depth upstream of the outlet:

$$Y_{01} = \frac{Q_{01}}{V_{01}b} = \frac{Q_{01}}{F_{01}\sqrt{g} Y_{01}b}$$

Or:

$$Y_{01}^{1.5} = \frac{Q_{01}}{F_{01}b\sqrt{g}} = \frac{0.4}{0.74 \times 0.4 \times \sqrt{9.81}}$$

Hence,

$$Y_{01} = 0.571 \text{ m}$$
Discharge ratio:

\[ R_{oq} = \frac{Q_{o2}}{Q_{o1}} = \frac{0.4 - 0.1440}{0.4} = 0.64 \]

From Eq.56 or from Fig.33, with \( F_{01} = 0.74, R_{oq} = 0.64 \), one gets \( F_{02} = 0.39 \).

Flow depth downstream of the outlet:

\[ y_{o2} = \left( \frac{Q_{o2}}{F_{o2} b \sqrt{g}} \right)^{\frac{2}{3}} = \left( \frac{0.4 - 0.144}{0.39 \times 0.4 \times \sqrt{9.81}} \right)^{\frac{2}{3}} = 0.65 \text{ m} \]