LEARNING CURVE FOR MANUFACTURING COST ESTIMATES

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ABSTRACT

A detailed review of the concept of learning curve and its application to cost estimation for manufacturing industries is presented. The Wright's theory on learning is explained with emphasis on the choice of learning rate and the method of relating individual unit to cumulative averages. The technique of plotting the actual learning curve is shown with illustrative examples. The report also discusses the interpretation of learning curve for multiple phases of product life including all cost elements that could possess some information on learning. Extension of the learning concept to planning of new products is also described. For the sake of completeness, different learning functions proposed as modification to basic Wright's theory are listed and their applications to type of manufacturing cost estimation are discussed. It is concluded that the Wright's learning curve is simple and direct and can be satisfactorily applied to many manufacturing industries, small and large without resorting to any advanced learning theories that have been proposed in the recent years. An appendix on the logical approach and strategy on bidding is included.

RESUME

Un examen approfondi du concept d'accoutumance ainsi que son application à l'estimation des coûts de fabrication sont présentés. La théorie de Wright sur l'accoutumance est expliquée en mettant l'accent sur le choix du taux d'accoutumance et sur la méthode qui fait le rapport entre la valeur unitaire et la moyenne cumulative. La technique pour tracer le graphique de la courbe réelle d'accoutumance est démontrée par des exemples. L'interprétation de la courbe d'accoutumance s'appliquant aux différentes phases de la vie d'un produit, compte tenu de tous les éléments de coût sujets à l'accoutumance, est discutée. L'utilisation du concept d'accoutumance pour la planification de nouveaux produits est aussi décrite. Pour compléter, différentes fonctions d'accoutumance qui ont été proposées comme correctif à la théorie de Wright sont données et leurs applications à différentes entreprises manufacturières sont discutées. En guise de conclusion, il est dit que la théorie d'accoutumance de Wright est simple et directe et peut donner de bons résultats pour différentes entreprises manufacturières sans avoir à faire appel à des théories plus avancées. La majorité de ces théories ont été développées récemmment. A l'appendice, une approche logique et stratégique pour soumissionner est incluse.

ACKNOWLEDGEMENTS

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TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

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One of the basic elements of sound management is accurate estimating of manufacturing costs. Without reliable estimates it is impossible to properly price a product, to plan financial management and cash flow, to budget an operation, to determine product profitability, and to evaluate performance for present and future.

The best estimates in the past came from the experienced mechanic who had been upgraded to the engineering or sales department and who could then provide estimates on the product that were acceptable. The complexity of the manufactured products, the shortage of estimating experience available and the need for more precise management control of operations, have in some cases eliminated the necessity of accurate and reliable estimating technique from the management spectrum. Today, some use carefully compiled cost data, work standards, engineering operations, modern accounting and planning techniques, and computers to develop manufacturing cost estimates [1].

Two significant sources of estimating technology are the fields of building contractors and the job machine shops. In each of these, management is forced to give a firm price for a one time manufacturing project.

The use of carefully collected cost data and standards and the recent application of such techniques as PERT, CPM, Data processing, Probability theories have greatly improved the estimating precision. Another cost estimating tool contributed by the aircraft industry has been the learning curve technique for introducing the effect of long runs into the development of costs.

Learning curve is a technique based on statistics that is utilized to calculate the allowances made for the learning period on operations having a relatively high percentage of labour, or those requiring the learning of a new procedure. The aim of the present study is to outline the importance of the learning curve technique and to describe its many applications in a manufacturing industry.

For example, the process engineer or cost estimator using this technique can predict with a reasonable degree of accuracy the following quantities:

i) the time required to produce any lot quantity;

ii) the time required to produce any one particular unit of the lot;

iii) the time required for the lot if the lot size were either increased or decreased during the production run;

iv) the adjusted time if a design change were introduced during the run.

The curve may be used for production control through material and inventory control, the allocation of machines, direct labor for both new and old product design, and the verification of training progress of the employees.

In chapter 2, Wright's theory on learning is explained together with the basic information for its application. This includes the fundamental consideration for the selection of the learning rate, the method of transfering block experience to unit experience, and methods of fitting a line between plotted points.

Chapter 3 describes the interpretation of the learning curve in the different manufacturing aspects namely in the design and the production phases and to decipher from all the cost elements those data that show the learning aspect. Explained in this chapter is the method of treating learning for an improved design, where many of the components are the same as or similar to the original design.

Chapter 4 explains the difference between progress and learning in order to avoid errors in planning future operations. The step-by-step planning in shop costs for new products is also detailed.

Chapter 5 reviews other learning functions that have been developed in various manufacturing industry. The learning functions that differ from the standard practice are those for mixed model assembly.

In the concluding remarks in Chapter 6 it is stated that though the learning curve technique is relatively simple to understand and is effective, as long as one is careful in its application, it is at present almost exclusively used only by high technology industry.

A brief résumé on bidding strategy, another tool that could help the estimator win a contract, is presented in the appendix.

CHAPTER 2

THE CONCEPT OF THE LEARNING CURVE

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THE CONCEPT OF THE LEARNING CURVE

2.1 Preliminaries

The learning curve is a statistical expression for predicting the time required to produce a specific unit of a production run. It is a line of best fit for actual data, and one must recognize that factual performance will seldom equal the theoretical value, but will be more or less closely distributed around the trend line.

In this chapter Wright's theory of learning curve is explained together with the basic knowledge required for its use. The considerations to be given for the selection of the slope i.e. the learning rate, are accounted for. Because cost experience by individual blocks (lot quantity) is available more often than individual unit the method of transfering block to unit experience is demonstrated. The effect of annual model changes, introduction of new products, extra costs incurred during periods of change in manufacturing operations, new or different technology in material or manufacturing on the learning curves are described. Finally, the slope analyzer and the least square method of fitting the curve are discussed.

2.2 Wright's Theory

Based on practical experiences in aeronautical industry Wright [2] proposed that each time the production of a quantity doubles, the unit man-hours are reduced at a constant rate. This is expressed mathematically as follows:

$$Y_x = K\, x^n \qquad (1)$$

and

$$T_x = \sum_{i=1}^{x} Y_i \qquad (2)$$

or

$$T_x = K \left[\frac{(x + \tfrac{1}{2})^{n+1} - (\tfrac{1}{2})^{n+1}}{n + 1} \right] \qquad (3)$$

where

- Y_x : the man-hours required for production of the single unit x
- T_x : the cumulative total man-hours required
- x : the number designation of the unit for which the man-hours are being determined
- K : the man-hours for the first unit
- n : value of the slope

Defining a quantity 'r', called the improvement ratio,

$$r = \frac{Y_{2x}}{Y_x} \quad \text{, where} \quad 0 < r \leq 1 \qquad (4)$$

and substituting equation (1) in equation (4)

$$r = \frac{Y_{2x}}{Y_x} = \frac{K(2x)^n}{Kx^n} = \frac{(2x)^n}{x^n} = 2^n$$

From which n is found to be

$$n = \frac{\ln r}{\ln 2} \tag{5}$$

Equation (1) can be readily calculated for any specific value of x, but equation (2) involves computing every unit value from 1 to x and has to be done using a computer. Equation (3) is a close approximation of equation (2) and is satisfactory for most practical applications.

To eliminate cumbersome calculations tables have been developed which give the cost for each unit and for various improvement ratios.

The improvement ratios may vary from about 60% (representing high improvement) to 100% (representing no improvement at all). A widely used improvement ratio is about 80%. The simpler the work the less improvement is likely and in such cases a higher improvement ratio is used. On the other hand, the more complex the work the greater is the chance of improvement and therefore, a lower improvement ratio has to be used.

2.3 Basis for Cost Estimating

The method used by many cost estimators [3] consists in determining the expected cost for an individual unit or a quantity at some time early in the production period. The unit or quantity selected may be the actual experienced cost or the forecast cost which will be realized at a point when starting costs have been eliminated and tooling problems solved.

Once the unit and its cost are selected, projections are made on a log-log basis using prior learning rates based upon experience gained from similar products. Figure 2.1 illustrates a hypothetical situation in which a cost of $100.00 at the 250th unit has been projected on a 90% curve, using the theory that the cumulative average is a straight line. A feature of the learning technique is that there exists a constant relationship between the cumulative average line and the individual unit line. The procedure of drawing the curve is as follows:

First, the 250th unit at a value of $100.00 is located. Then the cost of the 500th unit is plotted at a value of $90.00. The cost of the 125th unit (at $111.00) is arrived at by dividing $100.00 by 0.9. Normally, two points should suffice to draw the line. But it is preferable to use three points in order to prevent inaccuracies or errors in plotting. The unit line is then extended to the left as far as unit 10 or as far to the right as required.

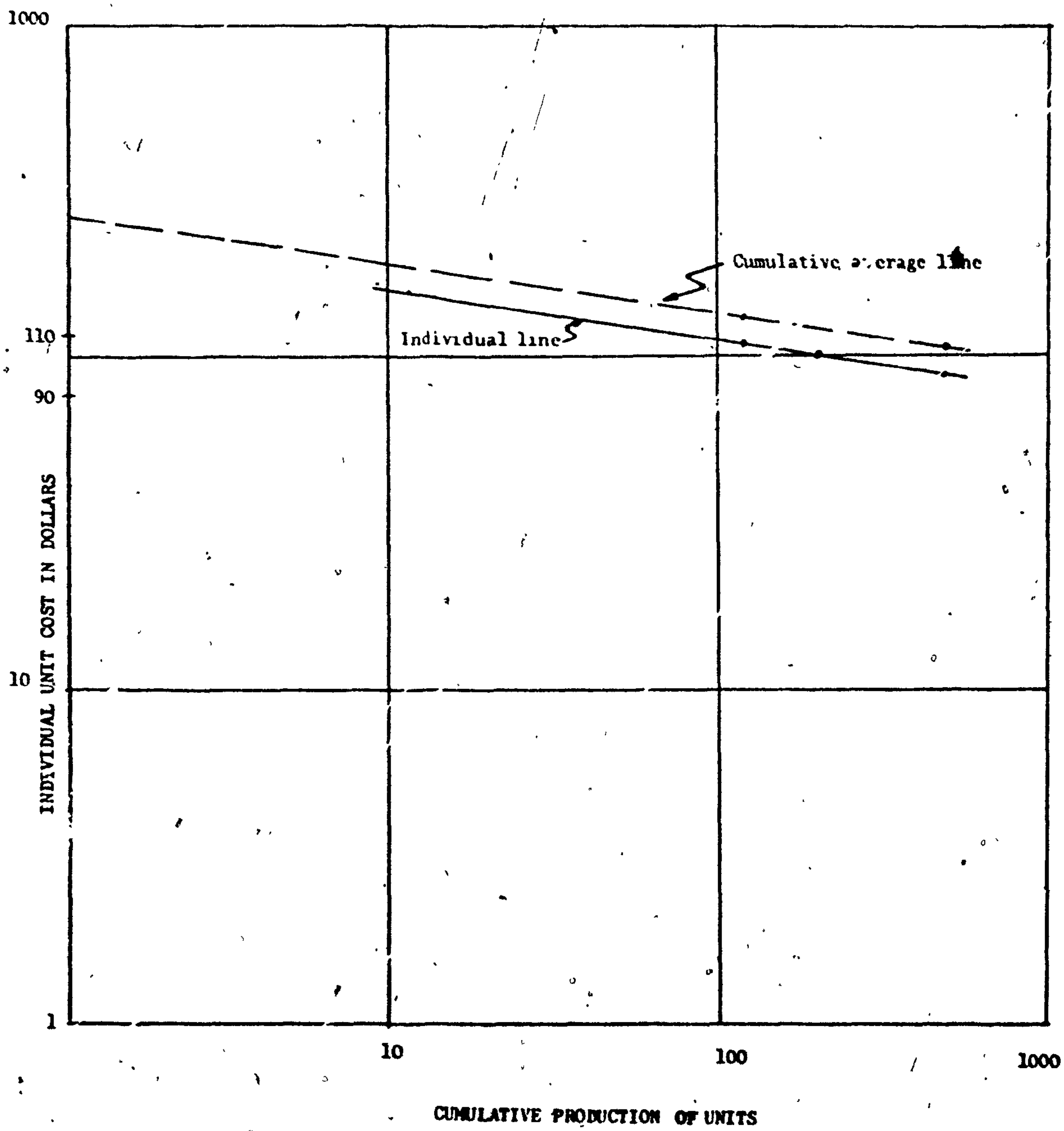


FIGURE 2.1 - Learning Curve for Individual Units and Cumulative Average

Learning curve experts hold that for all practical purposes the two lines (individual unit line and cumulative average line) become parallel at about the 10th unit. It can be shown that this fixed relationship is $1+n$, where n is given by equation (5)

$$n = \frac{\ln .9}{\ln 2} = -.152$$

or

$$1+n = .848$$

Therefore, each of the three point values i.e. \$90.00, \$100.00, \$111.00 is divided by 0.848, the factor for converting from the individual unit line to the cumulative average line, and the results are plotted in Figure 2.1 to obtain the cumulative average line. This line is extended to the left until it intercepts the vertical axis. Once the log-log plot has been completed, the estimator is in a position to perform calculations within the accuracy of the graph. The individual unit line allows instant reading once the correct unit number has been selected.

The advantages of the log-log plots are that the results can be obtained quickly and also it allows quick response to requests for cost estimates based on the same conditions i.e. using the same curve.

2.4 The Selection of the Slope

There are a number of considerations before the learning rate or the slope of the learning curve can be determined. First, a prior experience on similar components or processes is essential but most accounting systems are tailored to the needs of the bookkeepers in determining the profit or loss and evaluating the inventory for tax purposes and does not fit into the needs of planners and cost estimators. Accurate cost data are a prerequisite for establishing learning rate. In the absence of such data the estimator is forced to develop and extrapolate his own data from the available records.

The machine operator contributes the least to the learning process. Often, he is limited by the feed and speed of the machine and improvements in his contribution are accomplished only by an increase in the lot sizes which reduces set-up costs. Unless there is an organized program of cost reduction, involving for example value engineering; the likelihood of realizing an improvement under 100% during the performance of the order does not exist.

A job that is not well planned will result in many production difficulties and high cost for the first unit. If there are subsequent units to fabricate the total manufacturing time per unit would normally decrease at a certain rate as long as the workers are learning from the fabrication of the previous unit. Since the first unit was not well planned the time for the second unit should be substantially less than the first unit and hence showing high improvement between the two units. But if a job is well planned the initial starting costs should be less than the badly planned job and the manufacturing time per unit for the subsequent unit fabricated will not diminish as rapidly as in the first case. Therefore, one should be careful in looking only at the learning to evaluate the performance of a shop.

2.5 Inflationary Factors

In recent years, inflation has become an important factor in cost estimating. The ideal situation is an estimate based on current year's labour and expense rates and to include a clause for inflation in the contract. It is a good practice to prepare estimates using learning rates which are free from inflation and leave the guessing to the accountants or to the contract negotiators. There are several reasons for this. It is difficult to construct a constant slope curve with inflationary factors included which vary from year to year. Also, labour rates, with most companies are negotiated with bargaining units at the corporate levels and the estimator will not be in a position to predict the trend of future negotiations. If it becomes necessary to reflect inflation in the estimate, it is recommended that the data be fed to the estimator from the proper corporate source and that these factors be added to the results derived from the use of the learning curve.

2.6 Plotting the Actual Learning Curve

When a point on either the individual unit or cumulative average curves and the degree of learning are known it is simple to chart the learning curve. There will be many occasions where total cost experience by individual blocks (lots) will be available and it will be necessary to incorporate this experience before projecting the cost into the future.

In plotting cumulative average costs, the correct plotting point is always the point represented by the cumulative total produced. In plotting the value for individual blocks i.e. the total manufacturing time required to fabricate a lot of so many units, there are several methods by which one can determine the unit number on the individual unit line that represents the average cost of the block. The most commonly used rule in plotting actual cost is explained below [4].

First block: Plot cost at the one-third point provided production of the first block is greater than 10 units. If production is less than 10 units, plot at the one-half point.

Subsequent blocks: Plot at the cumulative total of all previous blocks plus one-half the current block i.e. plot at the midpoint of the block.

The following example is given to illustrate how shop data can be plotted to obtain the individual unit line that is described here. The data are shown in Table 2.1. They are given in the same terms as it is reported from the shop, namely, the quantity of units per block, the total man-hours per block, and the average man-hours per block.

From those observations, one can find the points on the plot for the learning curve. The first step is to determine the plot points for the first block: since the production of the first block is greater than 10 the plot point for the unit number is at 1/3 of the lot quantity i.e. 1/3 of 60 or unit 20 and the average man-hours per block is 50. The second step is to calculate the plot points for all the subsequent blocks: the unit number is plotted at the cumulative total of all previous blocks plus one half the current block and the average man-hours per block is obtained directly from the Reported Data (Table 2.1). For example, the plot points for individual block no. 2 are computed as follows:

$$\text{cumulative total of previous blocks} \quad + \quad \tfrac{1}{2}(\text{current block})$$

$$60 \quad + \quad \tfrac{1}{2}\,(50) = 85$$

The values for block no. 3 and 4 shown in Table 2.2 are calculated on the same basis as that for block no. 2. Finally, the curve joining all these points is drawn to give the individual unit line (Figure 2.2)

BLOCK NO.	QTY OF UNITS PER BLOCK	TOTAL MAN-HOURS PER BLOCK	AVERAGE MAN-HOURS PER UNIT
1	60	3000	50
2	50	1900	38
3	50	1875	37.5
4	80	2840	35.5

TABLE 2.1 Reported Data from the Shop

		PLOT POINTS	
INDIVIDUAL BLOCK NO.	UNITS PER INDIVIDUAL BLOCK	AVERAGE MAN-HOURS PER UNIT	UNIT NUMBER
1	60	50	20
2	50	38	85
3	50	37.5	135
4	80	35.5	200

TABLE 2.2 Calculated Plot Points

	CUMULATIVE	EMPIRICAL RULE	MATHEMATICAL
First 100 units	100	33 1/3	33.9
Next 100 units	200	150	146.8
Next 100 units	300	250	248.1
Next 100 units	400	350	348.9

TABLE 2.3 Mathematical Empirical Results

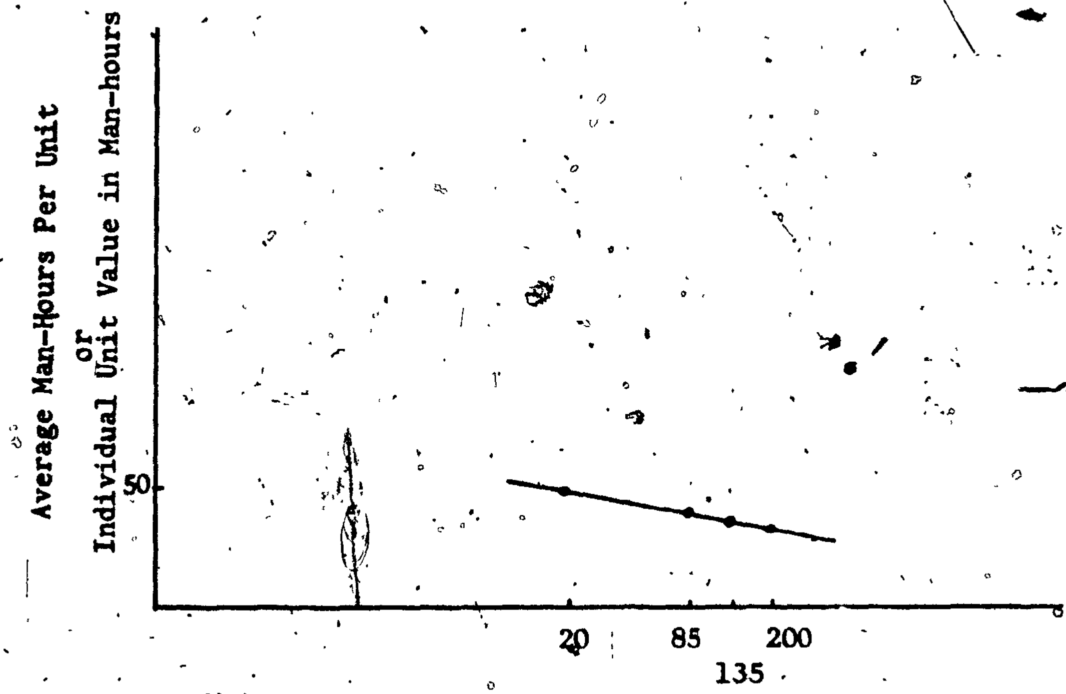


FIGURE 2.2 Individual Unit Line

Although the above empirical rule is a popular method for plotting the actual cost is slightly inaccurate as shown in table 2.3 where the block mid-point and plot points are calculated for a 90% curve, shows that this method becomes more accurate as production increases.

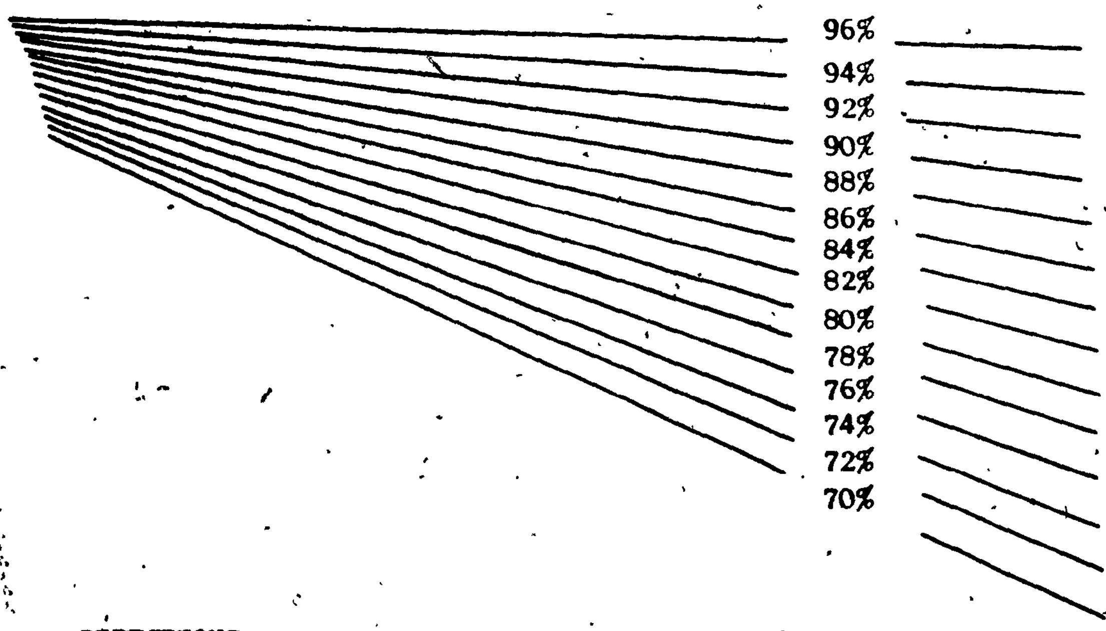
Determination of block plotting points by the mathematical method is done using the following formula

$$\ln x = \frac{\ln (Q_1 + Q_2 + .5)^{1+n} - (Q_1 + .5)^{1+n} - \ln Q_2 - \ln (1+n)}{n} \tag{6}$$

where

- x: plotting point for the present block
- Q_1: previous cumulative quantity
- Q_2: present block
- n : slope of the learning curve
- 1+n : conversion factor to find the individual unit cost when cumulative average is known and vice-versa.

Once the plotting is completed a line may be fitted by the method of least quares and the technique of "slope analyzer" shown on Figure 2.3, can be used for determining the slope visually. The directions for using this analyzer are also explained in the same figure.



DIRECTIONS

1. **Align a ruler exactly along one horizontal line**

2. **Rest Slope Analyzer on ruler**

3. **Slide Analyzer along ruler until it coincides with experience line**

4. **Read answer on extreme right**

FIGURE 2.3 **Slope Analyzer [4]**

2.7 Summary

In this chapter the basic information about the learning curve is given. It consists mainly of the definition of the learning curve concept and the method for obtaining the exact log-log representation. One should now be concerned with the different interpretations of the learning curves, especially its uses under different conditions in a manufacturing industry. This is discussed in the following chapter.

CHAPTER 3

INTERPRETATION OF LEARNING CURVES

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3.1 Preliminaries

In a manufacturing industry when design and/or production time has been expended on a product the knowledge gained should be applicable to the next production phases. This chapter covers primarily the particular utilization of the learning curve through which the industrial cost estimator evaluates the gain of any knowledge for the subsequent fabricating phases. Also discussed are those cost elements that show learning.

3.2 Multiple Phases of Product Life

Industrial cost estimators are faced with the problem of making certain decisions when new products are to be manufactured in more than one phase. These phases normally consist of [3]

i) a development phase involving temporary or general purpose tools

ii) a production phase involving permanent special purpose tools.

Four different techniques are currently employed in industry to reflect the change from the development phase to the production phase using the principles of the learning curve. Figure 3.1 describes a concept in which the production phase is considered having no carry over of experience from the development phase. The production curve starts at the top with the cost of the first production unit at unit number one. These conditions are obtained only if the development phase was carried out in one plant (or sometimes by a subcontractor) and the production phase in another plant different from the first and no development tooling being utilized in the production phase.

Figure 3.2 illustrates the concept where the cost of the last development unit is considered as the cost of the first production unit with the production phase starting over again at unit one. This concept is easily incorporated graphically by moving the cost of the last development unit laterally until it intersects the graph at unit one. From this point the production curve is drawn using the appropriate learning. This is a convenient method for estimating the production phase as it normally requires no production planning.

Figure 3.3 illustrates the concept in which the cumulative development and production phases are continuous from a standpoint of quantity, but there is a downward step i.e. reduction in time, in going from the development to the production phase due to retooling. The magnitude of the step would be determined by the extent to which retooling takes place and facilities and manpower differences between the two phases. This is probably the most realistic of all the three methods presented.

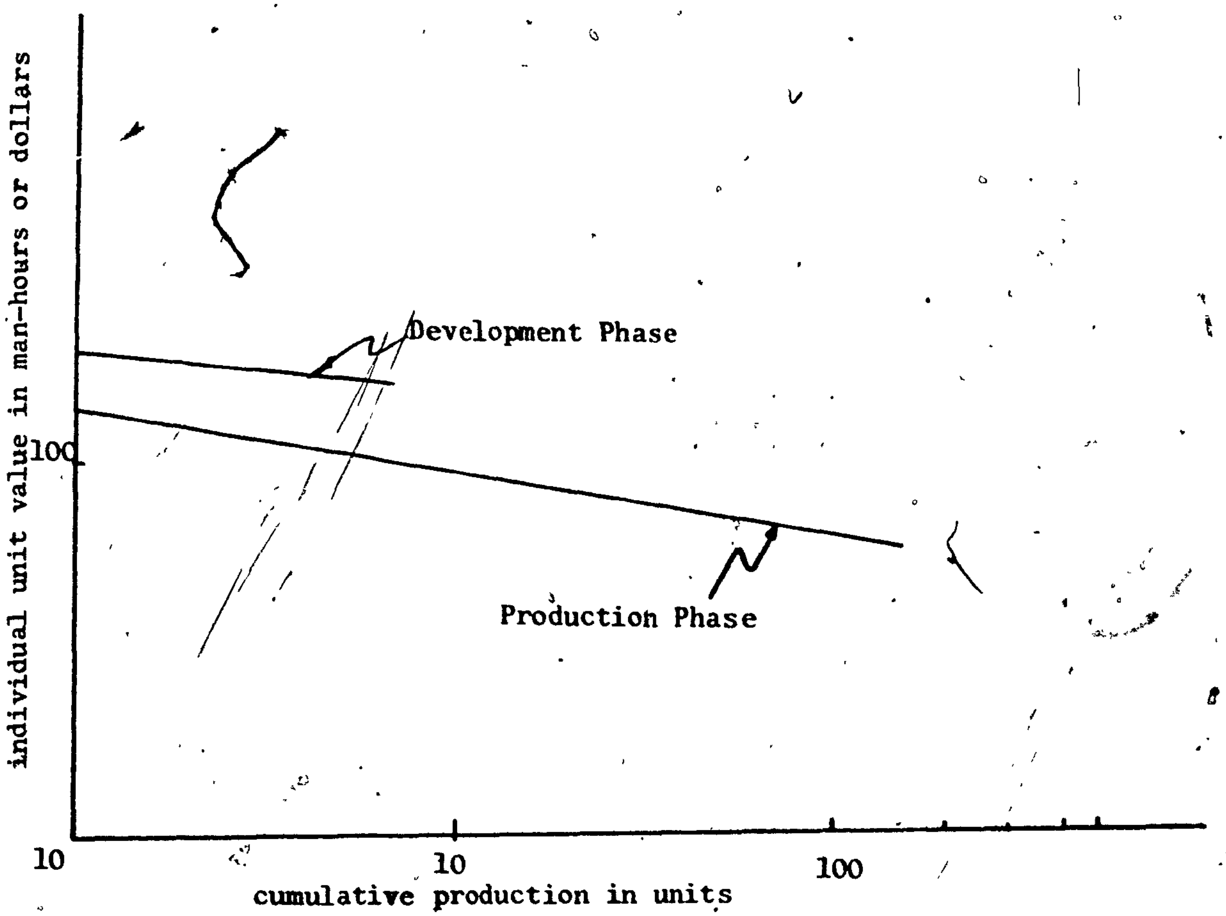

FIGURE 3.1 - Multiple Phases of Product Life without carry over

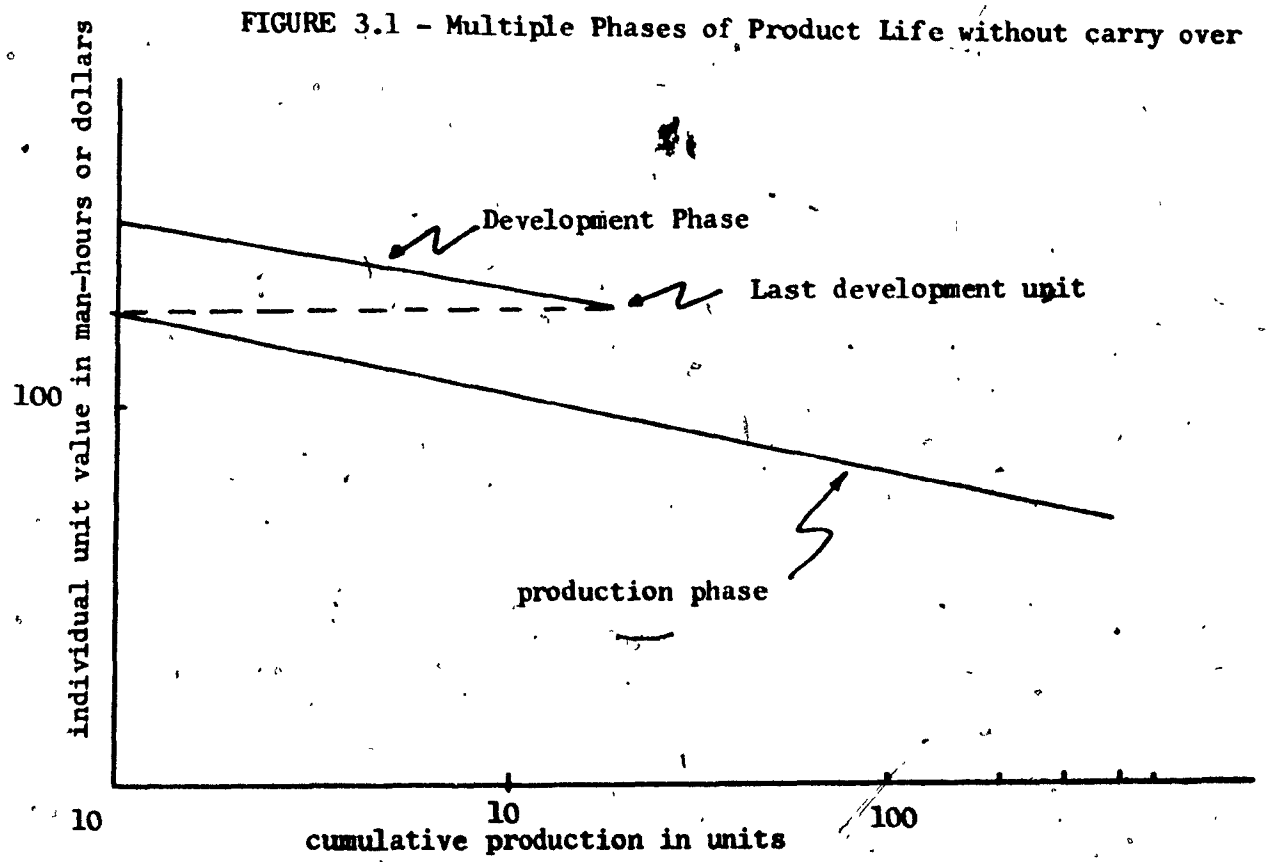

FIGURE 3.2 - Multiple Phases of Product Life with carry over

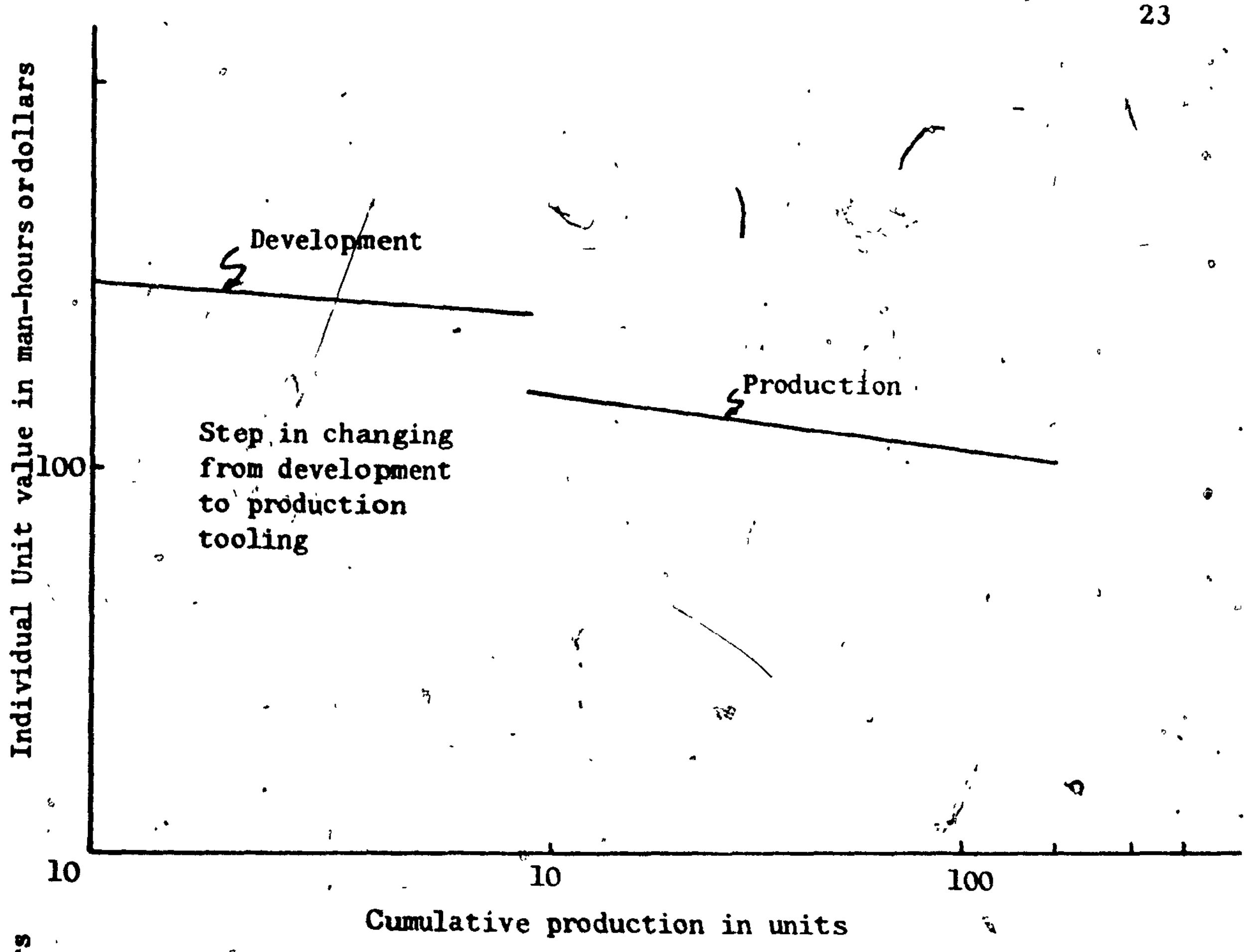

FIGURE 3.3 - Multiple Phases of Product Life with retooling Time Saving

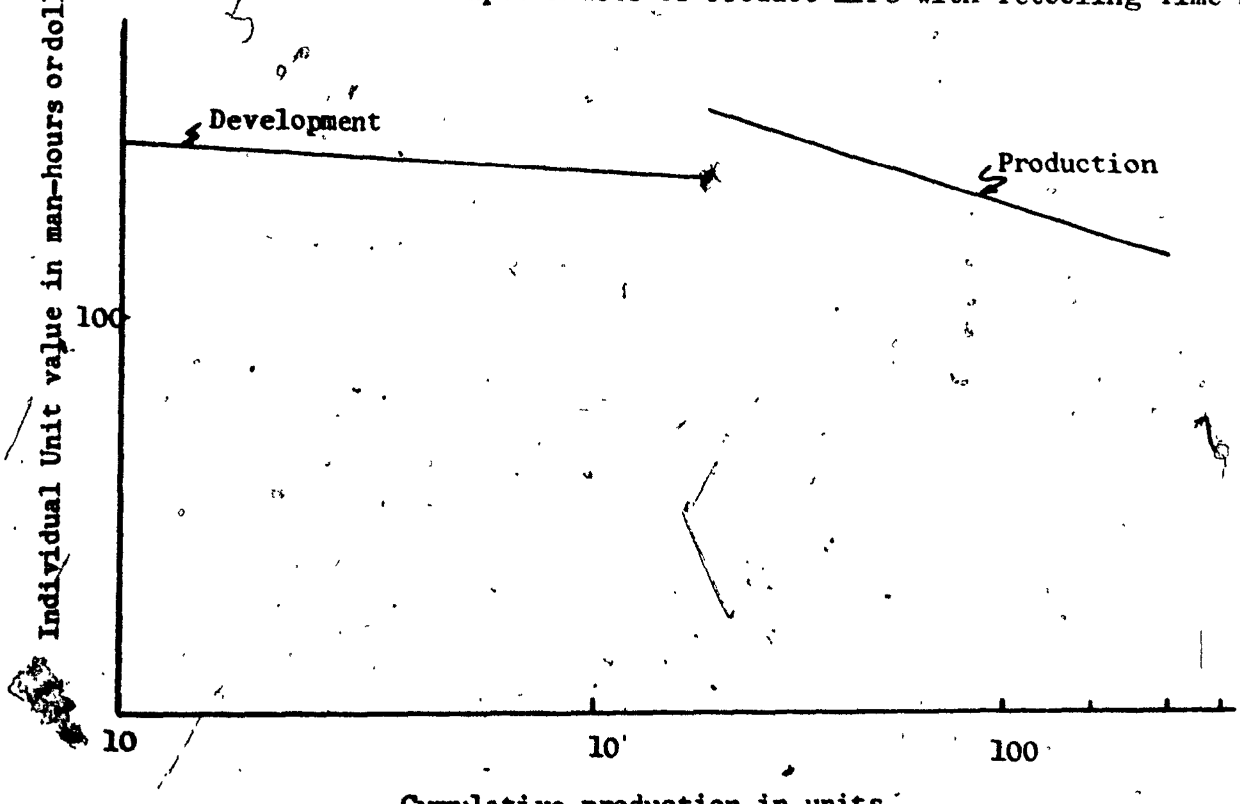

FIGURE 3.4 - Multiple Phases of Product Life with starting Production Costs

Figure 3.4 shows a concept closely related to the one previously explained with the exception that it considers that there will be starting costs on the first production units which will cause them to be more costly than the last development units.

Anyone of these four concepts can fit the situation under which a firm is operating and an analysis of the company is necessary to determine which one of those four methods is the most appropriate. In other words, one must determine what happens after the product is developed i.e. how the knowledge and tooling costs are transferred to the production phase.

3.3 Cost Elements that Show Learning

Many cost estimators consider factory costs (the sum of material, labour and overhead) and sometimes selling prices as the figures for developping learning curve calculations. This is not correct and factory costs alone are to be considered in the learning curve. Material costs must be separated between raw materials and subcontracted items before applying the learning rates. In general, raw materials do not show very good learning since the quantity of the raw material utilized for the first unit does not differ too much from the amount of the raw material utilized for the subsequent units.

Subcontracted items are good examples for learning curve applications. Here, the negotiating ability of both the customer and the supplier, will, howerver,be a major contributing factor. The use of learning curves by both sides will expedite the negotiations by providing common ground for discussion and agreement. Many companies have cost and price analysis units within the purchasing function which measure the performance of sub-contractors based on their performance in the previous orders and calculate the cost of the following order. In such cases, the purchasing agent comes to the negotiation with a preknowledge of what to expect.

Direct labour projections should be on the basis of hours in order to allow flexibility in the application of labour rates.

Overhead or burden is not a good application for the learning curve since this element varies in proportion to the overall business and is usually not related to a specific product. The application of the learning curve separately to material and labour will allow flexibility in applying varying rates of overhead in accordance with future forecasts.

3.4 Handling Design Changes

One of the frustrating experiences for a cost estimator is when an improved design is introduced in which, say 40%, of the components are identical to those of the original design. Treating the new design as a new curve starting over again at unit one is incorrect since the compo-

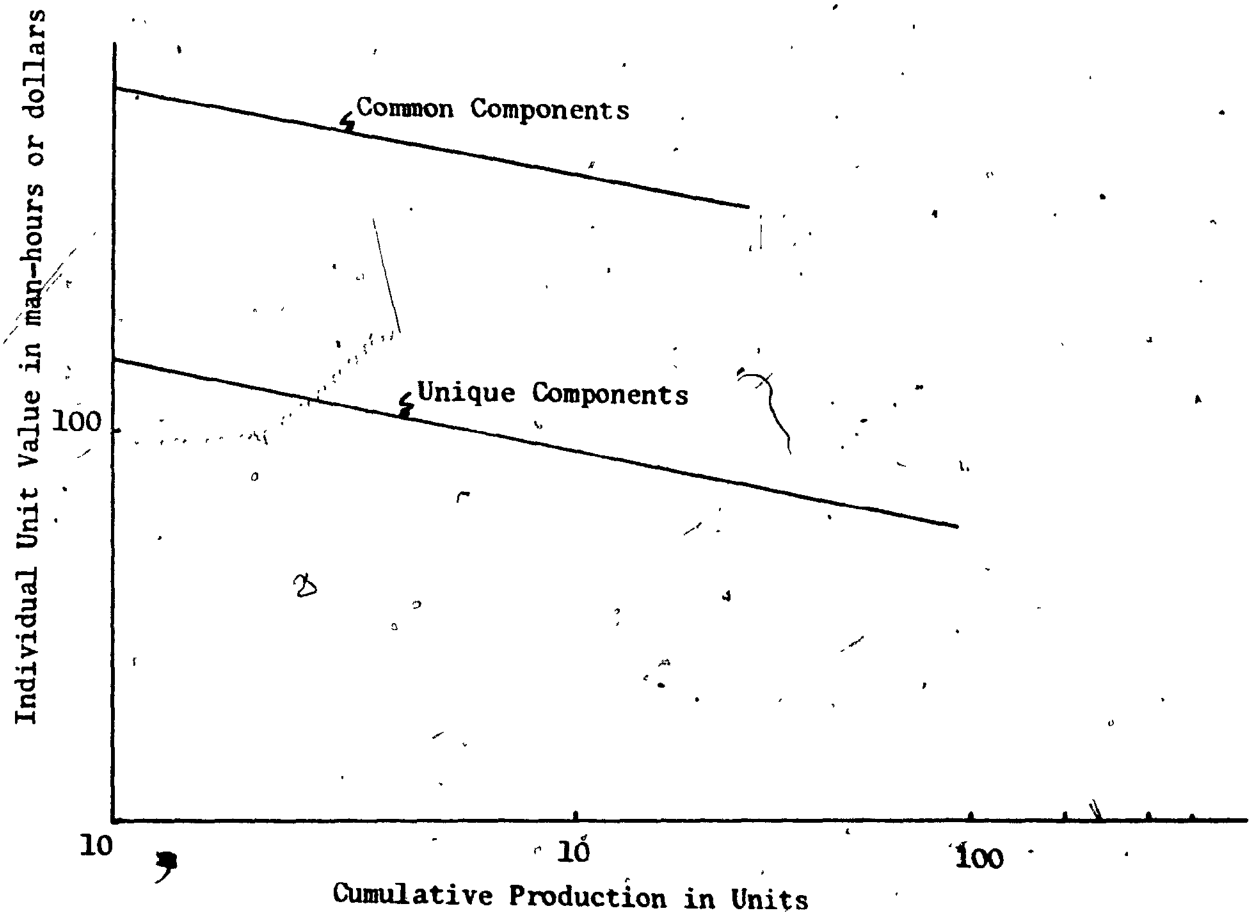

FIGURE 3.5 - Learning Curve with Separated Design Changes

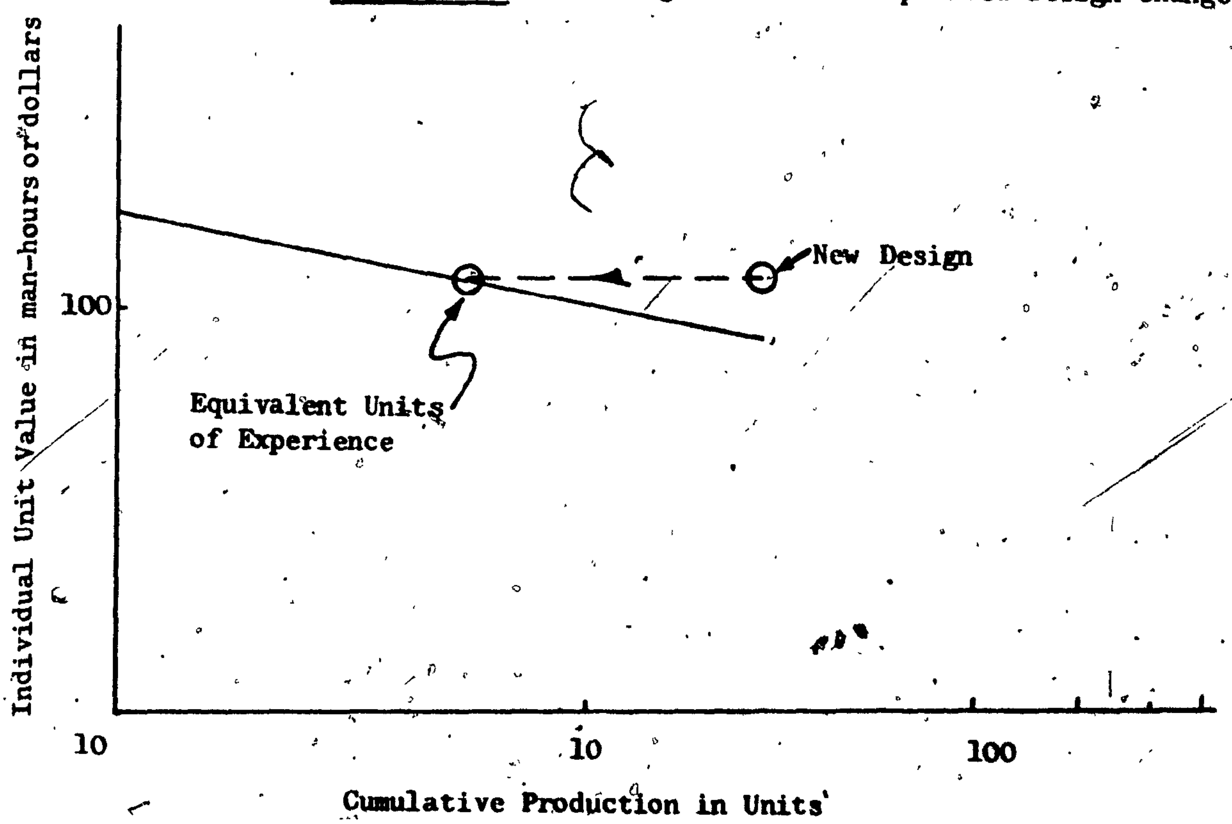

FIGURE 3.6 - Learning Curve with Integrated Design Changes.

nents common to both designs will not learn as fast as the components unique to the new design [3].

If the design changes are minor and when only a few units have been manufactured it is the practice of some companies to start the curve over again, compensating for the slow learning of the common components by raising the slope, for example, from an 85% curve to an 87% curve for a machine shop type of industry.

Figure 3.5 illustrates an example in which the old and the new design costs have been separated and individual curves have been drawn. This method is unwieldy, however, as it requires calculation of the position of the product on both curves and the addition of the results. If material and labour are handled as separate curves, the situation is more complex. One of the impediments to this method is that it cannot be proved since most, if not all, cost systems are designed in such a way that the old and new design costs are not separated.

Figure 3.6 illustrates a more practical method. The cost of an individual unit or the average of the first lot of the new configuration is estimated. The cost is plotted on the experience curve of the previous design at a point equivalent to the total production of both designs. A line is then drawn laterally to the left until it intersects the old curve. This point represents the equivalent number of units of experience which are carried over from the old to the new curves. For example, if the 500th unit is the first unit manufactured to the new configuration, and projection of this cost laterally intersects the old curve at unit 350, there are 350 units of experience contained in the new curve. The first 100 units of the new design would be calculated as units 351–450 of the extrapolation of the old curve.

3.5 Interruptions in Production

A gap in the production schedule exceeding six months can create a problem in the application of the learning curve to cost estimating. Before estimating the cost when production resumes after such a gap, the estimator must consider the following information:

i) whether there has been employee turnover so that experienced personnel are no longer available?

ii) whether the tools are still available or will replacement tools be necessary?

iii) whether the facilities are still available or are they being used to capacity for other products thus requiring the use of less economical facilities and methods?

iv) wheter there has been a change in the lot size compared with the previous production rate?

v) whether the previous vendors and subcontractors are still available?

If many of the answers to the above questions are negative, it will not be possible to start the curve at the point where the prior production ended. The method is similar to the design change method illustrated in figure 3.6. The objective is to determine the number of units of learning which have been lost by interruptions.

3.6 Summary

The different points analyzed in this chapter highlight the fact that the learning curve must definitely reflect the situation in which the firm is operating and one should realize that labour and material alone are the correct cost elements to be considered for learning curve applications. The following chapter is a discussion of specific applications and clarification that should help in the utilization of this technique.

CHAPTER 4

APPLICATION OF LEARNING CURVES

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APPLICATION OF LEARNING CURVES

4.1 Preliminaries

When the manufacturing methods are improved due to new technology or other progress, productivity generally increases. When the employees learn from previous work, productivity is also expected to increase. Therefore, to avoid any misinterpretation between those two totally different cases, a portion of this chapter is devoted to clarify the difference that exists between progress and learning. The second part of this chapter deals with the procedure to be followed in the planning of new products.

4.2 Progress and Learning [5]

The word "progress" has many interpretations. But in manufacturing it can be accurately used to describe cost reduction stemming from the revision of basic shop methods and product design. The specific factors which generate progress are the familiar sources of shop cost reduction. They can be itemized as: i) larger lot sizes, ii) improved techniques, iii) substitute materials, iv) greater mechanization, v) relaxed quality standards, vi) new production processes, and vii) simpler design. Progress is occasionnally measured in mass production plants as a simple annual cut in labour standards or total cost. For example, the model T Ford showed a reduction of 7.4% in total costs per year over 16 years. The bituminous coal industry, as a whole, averaged almost 6% a year in labour reduction per ton, from 1950 to 1968. Some companies currently expect their industrial engineering department to show savings of 5 to 10% a year in labour standards.

But, again, it is long-cycle manufacturing i.e. those in which average operator cycle time exceeds 30 minutes, which exploits the phenomena of progress most effectively. Companies such as aerospace, electronics, shipbuilding etc., often have a high degree of advanced technology in their products,and they accept continued improvement and changes required as an inherent part of their procedure. By plotting the trend of unit or lot costs on log-log scale, these companies can clearly identify the impact of changes leading to progress. Equally important, since their planning employs historical patterns of cost reduction, they automatically budget a continuing trend as progress changes. For example, where the normal slope might run at a fairly smooth 85%, the effect of progress may increase it to 75%, sharply raising the rate of cost reduction.

In studying such cases, it has been found that the ordinary composite cost improvement slope can be separated into two components. The first is the slope of learning; the second is the slope of progress and the product of these two slopes yields the composite slope.

In general terms, the composite slope S_c can be expressed by the equation

$$S_c = S_L \cdot S_p \tag{7}$$

where, S_L represents the slope of the learning curve and S_p, the slope of progress.

This relationship applies to both mass production and low quantity production. In mass production, there is no learning slope ($S_L = 1$) after the break-in period. Accordingly, the composite slope is identical to the progress slope.

Substantial progress slopes often occur in short-cycle industries, where the operator cycle time is less than 30 minutes. For example, a slope of 90% characterized model T production over a 16 year period. The entire auto and steel industries showed even sharper slopes, some up to 60% for 1919-1968 and 1926-1970 respectively. Such slopes indicate definite trends for management policy makers and for manufacturing department objectives. Since cost progress is related to the opportunity for technological improvement and to management policy, it is neither as regular nor as predictable as ordinary learning curve. It may quickly decay or continue for the life of the product with no fixed limitations based on the number of cycles performed.

Effective use of the progress of labour costs requires careful consideration of past history and present policies in relation to the technical opportunity available for progress improvements, and the management support of continuing design and process improvements, and capital expenditures.

Progress may be time-related as well as unit-related. When time related, slope forecasting must also consider future unit output rates. For example, if progress generates a fixed rate of labour reduction per year, a shift from rapid expansion of annual output of stability or decay creates a sharp increase in the progress slope. In any event, failure to measure separately the effect of learning and progress may cause serious errors in planning future operations.

4.3 Planning of new Products [6]

The first step in planning shop costs for new products is to develop a history of cost experience on similar products. This analysis has two phases. In the first, the basic level of unit cost for each product or component is determined for each processing department involved. In the second, the learning slopes are defined. These, in turn, depend on the degree of mechanical control in each processing department. For both phases, departmental cost levels must be established for a specific position of the base unit in the unit sequence represented on the curve. The position chosen is arbitrary, but should represent one by which cost experience normally attains a fairly regular slope of learning.

The widely accepted method of using predetermined standard costs with allocation of variance to represent basic cost levels must be avoided, since it obscures the cost improvement experience that is essential to this analysis. Accurate lot or unit costing should include the cost of normal manpower turnover and other minor disruptions, but the cost penalties of major changes in design or methods must be eliminated statistically, from historical baseline experience. The results of such study should then be expressed as a single "base unit" cost (say for unit 100,500 or 1000) for each product and department, plus a slope for each department.

The second step in planning is to use this experience, plus data on the new design and the new processes, to estimate a reasonable base unit for the work to be done by each process department. This procedure represents cost estimating of the best known type and needs no further discussion here.

The third step is to apply the appropriate learning slopes to each departmental base unit cost for the new product. The result is a complete cost estimate of labour on each unit or lot and the cumulative total for the anticipated quantity. For shorter cycle products learning may cease early in the unit sequence.

The final step involves adjusting the subsequent learning curve cost for the impact of cost reduction changes in design and methods. The slope (S_p) of progress in each department must be projected from:

i) past accomplishments;

ii) the technological opportunity felt to exist on the new product and processes;

iii) management policies or capital outlays and other items;

iv) in some cases the anticipated rate of production.

The projected learning slope (S_L) in each department must then be adjusted to obtain the final composite slope. Naturally, the penalties inherent in progress changes must be provided for, but the net saving in labour cost obtained by compounding progress with learning should be substantial.

4.4 Summary

The discussion on progress and learning shows how misleading it can be if the reduction in total manufacturing time is not clearly attributed to either progress or learning. In the section on planning of new products, it is indicated how one could use the learning curve technique for this particular case. To complete this study, the next chapter will deal with other learning functions that have been proposed by different authors.

CHAPTER 5

OTHER LEARNING FUNCTIONS

CHAPTER 5

OTHER LEARNING FUNCTIONS

5.1 Preliminaries

The Wright's formula, explained in Chapter 2,works satisfactorily for complete assemblies. These assemblies consist of a number of different operations and the actual production time is high. This law is also valid for small batches.

According to Wright, the manufacturing time per unit decreases as the size of the batch increases; it is obvious that the production time per unit cannot diminish for ever. Consequently, to avoid a such inconsistencies other learning formulae have been proposed. Since Wright's theory was limited to the aerospace industry, other authors have proposed similar learning curve techniquesfor different manufacturing industries. A review of some of these formulae is presented here.

5.2 De Jong's Law

Recognizing that the reduction in production time cannot continue forever, De Jong [7] introduced the concept by which the learning curve theory becomes more realistic. This law is defined by the following relation:

$$t_n = t_1 \left(M + \frac{1-M}{n^{\alpha}}\right) \tag{8}$$

where

t_n: time required to produce the nth unit of the considered series

t_1: time to produce the first unit

n: order of the unit in the series

α: exponent for decreasing value

M: irreducible constant

In comparaison to Wright's law, the model proposed by De Jong is more realistic since the incorporation of the irreducibility constant gives an asymptotic value to the manufacturing time, therefore it assigns a limit to the learning of the workman. Furthermore, the improvement does not follow the same rate for all the duration of the work; in fact, each time that n doubles, only the reducible portion diminishes proportionally to the decision percentage that corresponds to the value of α, while the irreducible portion Mt_1 remains constant.

The irreducible constant M, has a value between 0 and 1. It assumes different values for each learning curve and is a function of the type of work considered. For example for assembly operations M = 0.75, welding operations M = 0.50, turning operations M = 0.40, and for packaging in food industry M = 0.075.

An operation that is controlled by a machine has little possibilities for improvement and will therefore, have a high degree of irreducibility. M will approach 1 and the n unit will necessitate a manufacturing time almost identical to the time of the first unit. On the other hand, if M = 0, De Jong's law is identical to Wright's. In other words De Jong proposed a version of the power function which generates two components, a fixed component which is set equal to the irreducible portion of the task, and a variable component, which is subject to learning. For practical usage of this formula, De Jong has established tables to obtain t_n for a value of α and for different values of M. Finally, De Jong finds that for series greater than 30 units the average unit manufacturing time for a series of n units is equal to the manufacturing time of the unit of rank 0.3n.

It should be noted that contrary to the studies made in the aeronautical industry, the experimental data on which De Jong's law is based are almost exclusively short cycle operations (between one minute and 10 minutes) and for very large number of units (between 10^3 and 10^6).

5.3 Lazard's Law

Lazard [8] developed a general law of learning for work done in series that could satisfy all possible cases whatever the degree of repetition and the length of the operation cycle. This law has been verified experimentally on 40 work stations in aeronautical industry and seems to apply particularly on long cycle operations which are executed in short series.

The basis of this law is that the decreasing value of the manufacturing time is an equilateral hyperbola or a network of equilateral hyperbolas that obey the following formula

$$Y = M + \frac{Q}{n + P} \qquad (9)$$

Where M, P and Q are three positive constants with M, being the aymptotic value of Y for the unit of rank n approaching infinity. These three constants are obtained from the tables [8].

5.4 Pegels' Law

Pegels [9] proposed an alternative algebraic function, of an exponential type, to complement or replace the power function approach of De Jong [7]. The exponential function was derived from the theory of difference equations and is given by

$$MC(x) = \alpha a^{x-1} + \beta \qquad (10)$$

where MC(x): cost per unit for the xth unit,

α, β and a : are empirically based parameters.

The exponential function has twice as many parameters as the power function. For new processes or new operations, especially those with large amounts of uncertainty, the parameter estimation problem presents the same difficulty, regardless of which function is applied. The exponential functions have the rational feature of leveling out to a constant value. The power function continues to decrease as output increases for both unit time and average manufacturing time. Pegels compares his formula with Levy's and De Jong's power function, using Levy's production and cost data for two pressmen on a new press. This comparison clearly shows that only De Jong's method approaches Levy's results.

5.5 Levy's Law

Levy's main purpose is to point out how managers may use the concept of learning on a particular process and its implications as a tool to aid in such decisions as formal training and equipment replacement. To accomplish this, Levy proposes a function showing the relationship between a firm's rate of learning and the variables that may influence it. Levy's learning function [10] termed the "adaptation function" has the form

$$Q(q) = P\left[1 - \exp\ (a + \mu q)\right] \qquad (11)$$

where

$Q(q)$: rate of output after q units have been produced

P : maximum rate of output

μ : rate of adaptation

q : rank of the unit being manufactured

a : constant of integration

The rate of adaptation will depend upon the variables that can influence how fast the firm adopts to the chosen P. Some of the variables that influence firm learning are:

i) pre-production planning used by the firm to improve the initial efficiency of a process;

ii) industrial engineering techniques such as time and motion studies, quantity purchasing of raw materials, etc.;

iii) employee selection and training instead of assigning employees by seniority;

iv) the improvement due to on – the – job learning or training of employees.

Levy shows that the adaptation function is useful to firms in their budgeting of training and in aiding their equipment replacement decisions.

5.6 Glover's Law

Glover's approach [11] commenced with a study of the gain in skill of individual workers. This was subsequently extended to groups of individuals and then to complete departments of companies. This work, in effect, started with experiments based on the methods of experimental psychology and the model which was found best to incorporate such data was

$$\sum_{i=1}^{n} Y_i + C = a\left(\sum_{i=1}^{n} x_i\right)^m \qquad (12)$$

where Y_i and x_i are interchangeable in the sense that either may represent time or quantity, depending on the format devised. If $\sum Y_i$ is used to represent the total elapsed time, then the x_i will normally be unity.

That is,

$$\sum_{i=1}^{n} x_i = n \qquad (13)$$

so that

$$\sum_{i=1}^{n} Y_i + C = an^m \qquad (14)$$

where C: "work commencement" factor

a: time of the first cycle

m: index of the curve = 1 + b

b: value of the slope .

5.7 Cubic Learning Curve

In most companies the changes in the cost in time and money of producing a new product is estimated by the use of the classic learning curve $Y = Kx^n$. When the "learning trend" is plotted on log-log paper the resulting curve is often S-shaped (Figure5.1) rather than linear, largely because of the training of operators, the cost of tooling, etc. As problems are corrected and production begins to flow on schedule, a cost improvement program can be introduced which will accelerate learning. Then the slope begins to steepen. As production moves a great distance out on the curve, a point of diminishing return is reached. Then the slope begins to level off, the curve becomes asymptotic.

The S-shaped curve can best be represented by a third order polynomial. Consequently, it is called the cubic learning curve. The resulting curve is:

$$Y = Ax^3 + Bx^2 + Cx + D \qquad (15)$$

Where:

Y: time for the xth unit

A,B,C,D, :coefficients of the equation

Miller [12] suggests the estimation of the coefficients of the equation by constructing a system of equations comprising of two points on the curve and the estimated slopes at these points. Thomas [13] sees the disadvantage in using this technique, as with all graphical techniques, as the potential source of significant human error in fitting curves to obtain the points and slopes required in the analysis and proposes the application of regression analysis.

5.8 The Mixed Model Learning Curve

Thomopoulos and Lehman [14] introduced an extension of the learning curve concept by considering the rate of reduction in direct labor assembly time for mixed model assembly situations. Here, more than one model is assembled on the same assembly line; hence, the repetitions of the assembly work are not always the same. Some elements of work are unique to some models, while other elements are common to two or more models. As a consequence, the rate at which operators are learning (and hence complete their work on the units) varies from element to element and from

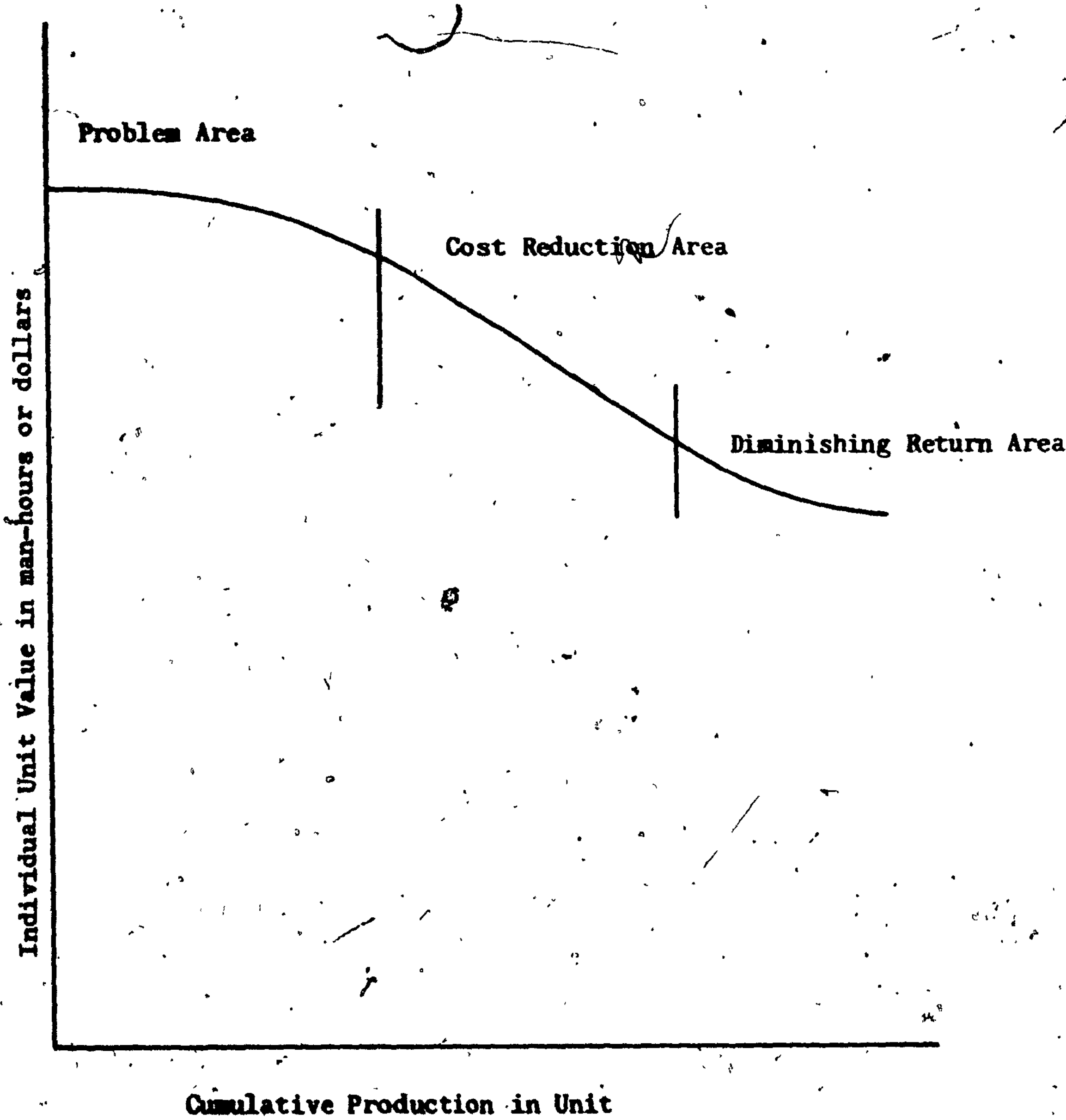

FIGURE 5.1 - Example of Cubic Learning Curve

model to model.

The mixed model learning curve, developed by Thomopoulos and Lehman, predicts the time required to assemble the uth unit under the assumption that the production up to the uth unit contains the same relative proportions of models as that designated for the total production run and is expressed by the following equation

$$F(u) = \frac{a}{n+1} Q\, u^{n+1}, \text{ with } u = 1, 2, \ldots \qquad (16)$$

where

F(u): time required for the first u units

a : assembly time on unit one given by

$$a = \frac{k}{N} \sum_{j=1}^{m} N_j T_j \qquad (17)$$

where

k: constant for all models

m: total number of models in the assembly line in production at any one time

N_j: number of units for the jth model

T_j: work content times for the jth model

$$n = \frac{\ln r}{\ln 2} \qquad (18)$$

where

r: learning rate

u: the unit of assembly, $u = 1, 2, 3, \ldots$

and

$$Q = \frac{\sum_{i=1}^{s} t_i K_i^{b+1}}{N^n \sum_{j=1}^{m} N_j T_j} \tag{19}$$

where

t_i: standard times for the ith element $i = 1, 2, \ldots, s$

K_i: number of repetitions of the ith element over N_j units

Potential applications for this type of learning include the comparison of learning costs on single and mixed model lines, and the selection of models to mixed model lines which tend to minimize the percentage increase in assembly time.

5.9 Summary

In comparison to Wright's law, the model proposed by De Jong is more realistic since the incorporation of the irreducibility constant gives the asymptotic value of the manufacturing time, therefore assigning a limit to the learning of the workman. However, if the irreducible constant M has a zero value De Jong's law is identical to Wright's formula.

Lazard's law has been verified on long cycle operations executed in short series while De Jong's law is valid almost exclusively for short cycle operations of high numerical value. Pegels', Levy's and Glover's hypotheses are all variation of the aforementionned laws.

The Cubic Curve brings forward the fact that a constant slope curve is purely hypothetical. Learning curves generally are "S" shaped over a long period of time.

Thomopoulos and Lehman introduced the mixed model learning curve to predict the time required to assemble the rth unit under the assumption that the production up to the rth unit contains the same relative proportions of models as that designated for the total production run.

CHAPTER 6

SUMMARY AND DISCUSSION

CHAPTER 6

SUMMARY AND DISCUSSION

6.1 Concluding remarks

Learning curve theory is simple and easy to use, but unfortunately not enough industries are utilizing it. It is an interesting fact that the basic learning process represents a straight line on log-log graph, consequently making it easy for plotting such a curve. Furthermore, it allows rapid extrapolation just by extending the straight line, for both small and large unit costs.

The selection of the slope for a learning curve highlights the following facts:

i) the estimator is forced to manufacture his own historical records in order to be able to evaluate the learning rate with different production schedules;

ii) unless there is an organized program of cost reduction, the likelihood of realizing a slope under 100% does not exist when the machine operator is limited by the feed and speed of the machine;

When studying the learning pattern from the development phase to the production phase, one should be very careful in selecting the proper technique to be used. It should be realized that labour and material alone are the only cost elements to be considered for learning applications.

The learning curve is a very useful tool for determining manpower requirements, for controlling shop labour, for checking the training progress of employees and for deciding whether or not the component can be manufactured internally at a lower cost than through an external contract.

It is highly important to measure separately the effect of learning and the effect of progress. Here progress is considered to be related to the opportunity for technological improvement and to management policy or to cost reduction stemming from the revision of basic shop methods and product design.

In planning new products predetermined standard costs with allocation of variance to represent basic cost levels must be avoided since it obscures the cost improvement experience so crucial to data analysis for learning. It is recommended that cost analysts develop their own history of cost experience in order to express the cost for a single "base unit" for each product and department, plus a learning rate for each department.

6.2 Discussion

All the learning functions mentioned in this report seem to be fairly useful as long as they are used in the context in which each law or formula has been developed. According to Pegels, a number of alternative functions have been proposed and most of these were intended only for specific applications and therefore have not affected the popularity of the power function approach to any degree. Some authors have proposed new terms like adaptation function, improvement curve, progress functions, manufacturing progress etc. for emphasising particular aspects of learning curves.

For a firm that is starting to incorporate the learning aspect in their estimating procedure it is advisable that they use Wright's formula at the beginning, even though it might not be the most exact one, nevertheless it is a tool that gives useful results especially for early stages. Then additional data should be collected and eventually equations should be fitted to represent those data, in order to:

i) obtain interpolation formulae or calibration curves;

ii) confirm or refute a theoretical relation to compare several sets of data in terms of the constants in their representing equations and to aid in the choice of a theoretical model [16].

Recently Bevis et al [15] recognized the fundamental need for a predictive technique which will reliably estimate learning parameters.

from data recorded during the early part of the production run. Bevis proposes the Taylor series approach to predict best estimates in the least square error sense. For an input of a given number of data points, the predictive technique iterates until the increments sought in rise time and final value are both less than some chosen value. Where the data forms an oscillatory time series, the predictions are suitably smoothed, and it results in an increase in accuracy.

Though the theory of learning curve has wide applications, it has been restricted, like many management tools, almost exclusively to high - technology industry (e.g. United Aircraft of Canada Ltd., National Cash Register Co., IBM). In heavy industries it is a recognized fact that if only a single item, whatever its size and complexity, is ordered and manufactured the company will probably not make any profit. But, if there are more than one item to be manufactured then the company can make a profit. This is a direct application of the learning theory.

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APPENDIX

A BIDDING STRATEGY FOR INDUSTRIES

After preparing an estimate of the work to be done there remains a final and crucial operation before securing successfully a contract, namely the presentation of the tender or bid. Some aspects of bidding strategy are discussed below [17, 18].

There are two kinds of competitive bidding situations that occur. One is closed bidding in which two or more bidders submit independent bids for the rights to property or to render service. In most cases only one bid per competitor is allowed and the highest or lowest bid is accepted as dictated by the rules. The other kind of bidding is auction or open bidding in which two or more bidders continue to bid openly on an item of value until nobody is willing to increase the bid. The last bid is then considered the winning bid.

The problem of bidding on one contract is discussed and it shall be assumed that the company's sole objective is to maximize total expected profit. This is certainly one of the most common objectives and one of the simplest to handle in a bidding situation of this type.

THE GENERAL MODEL

Let C represent the estimated cost of fulfilling the contract. The actual cost determined after completion of the job will, of course, differ from the estimated cost. It is important, therefore, to determine the bias and variability of the cost estimate. This can be done by studying past data on estimates and actual costs. The probability distribution of the true cost as a fraction of the estimated cost can then be obtained and is described in Figure A.1

Let h(S) be the probability that the ratio of the true cost to the estimated cost is between S and S + dS. Let x be the amount bid for the contract. Then if a bid of x wins, the profit will ultimately be x - SC.

Now let P(x) be the probability that a bid of x will be lowest and win the contract. Then the expected profit, $E\left[x\right]$, if a bid of x is made, will be

$$E\left[x\right] = \int_0^{\infty} (x-SC)\ P(x)\ h(S)\ dS \qquad (20)$$

The value of x for which this expected profit is maximum is the value of x which should be bid. Since P(x) is independent of S, and by normality law

$$\int_0^{\infty} h(S)dS = 1$$

equation (20) becomes

$$E\left[x\right] = P(x)\ (x-C'), \qquad (21)$$

where C' is the estimated cost corrected for bias and is given by

$$C' = C \int_{0}^{\infty} S\,h(S)\,dS \qquad (22)$$

In general, $E[x]$ will appear similar to the curve shown in figure A.2. Once the expected profit curve is determined, it is relatively simple to find the bid that maximizes the profit. The difficulty in determining the expected profit lies in determining $P(x)$, the probability of winning as a function of the amound bid.

PROBABILITY OF WINNING [17]

One way of determining the probability of winning with a given bid lies in studying previous bidding data. Presumably the results of previous bidding on contracts are always announced, and from these announced bids the "bidding patterns" of potential competitors may be studied. Suppose Competitor A is being studied. On every previous contract on which the bidding company made a cost estimate, the ratio of Compagny A's bid to the bidder's cost estimate is determined. If there are enough previous contracts on which A has bid, a pattern of A's bidding behavior relative to the present Company's cost estimates will emerge as a distinct distribution. These patterns can be made for all potential competitors. An example is shown in Figure A.3

Now if it is known exactly which competitors are going to submit bids, the probability of winning for a given bid is relatively easy to compute. Assuming that each competitor is likely to bid as he has done in the past, which is the best assumption in the absence of additional information, the probability of being lower than competitor A by bidding x is the area to the right of the ratio x/C on A's bidding distribution curve. Similarly the probability of being lower than B is the area to the right of the ratio x/C on B's distribution curve. The probability of being the lowest bidder with a bid of x, when the competitors are known, is simply the product of the probabilities of defeating each of the known competitors.

If it is not known exactly how many competitors will submit bids, the problem becomes more difficult. In this case, it is necessary to use the concept of an "average" bidder. The bidding distribution of the "average" bidder is found by combining all previous ratios of an opposition bid to the bidder's cost estimate and obtaining one distribution function as shown in Figure A.4.

Let p(r) be the probability density function of the ratio of the average bidder's bid to the present bidder's cost estimate. Then the probability of a bid x being lower than one average bidder is given by the expression

$$\int_{x/C}^{\infty} p(r)dr \qquad (23)$$

The probability of having an estimate lower than k average bidders is then

$$\left[\int_{x/C}^{\infty} p(r)dr \right]^k \qquad (24)$$

Now assume that one can determine the probability of k bidders submitting bids. Then, if this probability is g(k), the probability P(x) of a bid x being the lowest bid becomes

$$P(x) = \sum_{k=0}^{\infty} g(k) \left[\int_{x}^{\infty} p(r)\,dr \right]^{k} \tag{25}$$

Now p(r) can be found by fitting a curve to the data available. A Gamma distribution will frequently furnish a good fit to data of this sort. Where this is so,

$$p(r) = \frac{a^{b+1}}{b!} r^{b} \exp(-ar) \tag{26}$$

where a and b are constants obtained from fitting the best curve to the frequency data distribution.

It is also reasonable to assume that the number of bidders might have a Poisson distribution. That is, if λ is the estimated number of bidders then

$$g(k) = \frac{\lambda^{k} \exp(-\lambda)}{k!} \tag{27}$$

Both of these distributions can be tested to determine whether they agree with the past data. Assuming that there is a good way of estimating λ, it can be found that [17]

$$P(x) = \exp(-\lambda) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\lambda\left(\int_{x/C}^{\infty} \frac{a^{b+1}}{b!} r^{b} \exp(-ar)\,dr\right)\right]^{k} \tag{28}$$

$$= \exp(-\lambda)\, \exp\left[\lambda\left(\int_{x/C}^{\infty} \frac{a^{b+1}}{b!} r^{b} \exp(-ar)\,dr\right)\right] \tag{29}$$

$$= \exp\left[-\lambda\left(1 - \sum_{m=0}^{b} \left\{\frac{ax}{C}\right\}^{m} \exp\left(\frac{-ax}{C}\right)\right)\right] \tag{30}$$

The summation is simply the cumulative of the Poisson distribution which may be found from standard tables. Figure A.5 shows curves of $P(x)$ for several values of λ.

DETERMINATION OF OPTIMUM BID [17]

The expected profit, E.P., now becomes

$$E.P. = (x-C')\,P(x) = (x-C')\,\exp\left[-\lambda\left(1 - \sum_{m=0}^{b} \left\{\frac{ax}{C}\right\} \exp\left(\frac{-ax}{C}\right)\right)\right], \tag{31}$$

where C' is defined in equation (22).

A graph of expected return plotted against amount bid can now be prepared and the optimum value for x is easily determined using the concept in Figure A.2. An analytic solution for the maximum may be attempted but is not available in closed form. The important parameters in this method of determining the optimum bid are the estimated cost and the expected number of bidders.

The estimated cost has been discussed previously. There are several possible methods of getting an estimated number of bidders. The size of the contract may have some correlation with the number of bidders. Thus a regression of number of bidders can be plotted against the bidder's cost estimates on previous bids. If this is significant, the present cost estimate can be used to estimate the number of bidders from the regression equation. For example, a study of number of bidders versus cost estimates might be that shown in figure A.6.

A linear regression is made on the data, and the above line gives a satisfactory fit. This line is then used to estimate the number of bidders from knowledge of the estimated cost.

The bidding strategy does not and will not supplement sales manager's judgment; it merely complements his experience and intuition. Shrewd bidding and sales techniques are valuable, but the human factor can still reduce precise mathematical calculations to a hypothetical theory.

Individual prejudice sometimes influences bidding so much that it fits no normal pattern. Bias can be traced to a close personal relationship between a salesman and a buyer, to the fact that one bidder has supplied the buyer most of the time, or to real or fancied differences in service. Regardless of the reason, any sales manager can soon detect bias by observing whether he gets a reasonable share of the purchaser's business. When bias against his company is known to exist, the sales manager must adjust the success probability upward to perhaps 80% or 90% for a particular bid.

Pricing can be used as a competitive weapon to create favorable bias in a preferred customer. Realizing that certain customers have more potential than others, an astute sales manager will depart from a broad pricing policy to obtain a major share of that customer's business. Becoming the major supplier for a key customer can result in future favorable bias, permitting the supplier to obtain not only most of the customer's business but the full market price as well.

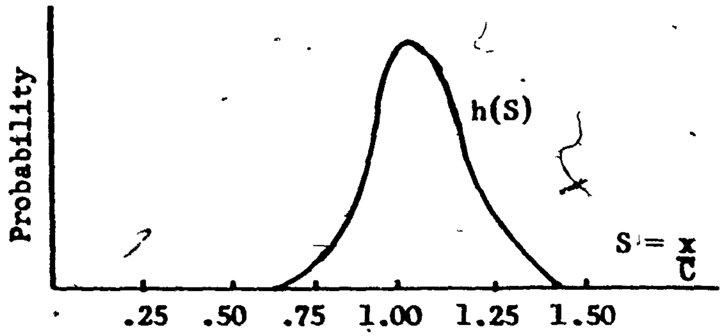

FIGURE A.1 - Reliability of Cost Estimate

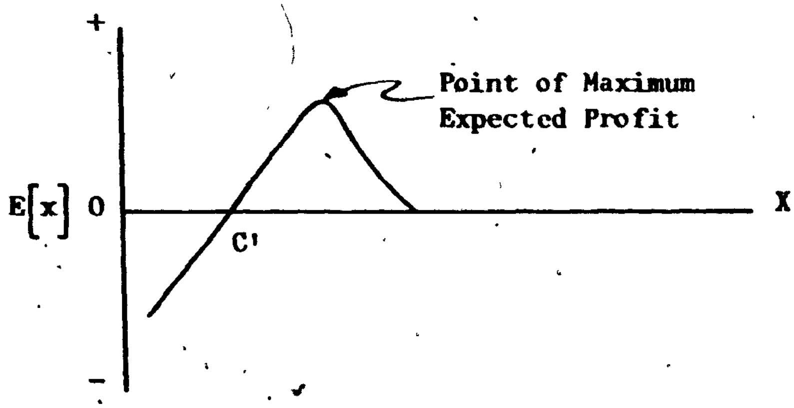

FIGURE A.2 - Expected Profit vs Amount Bid

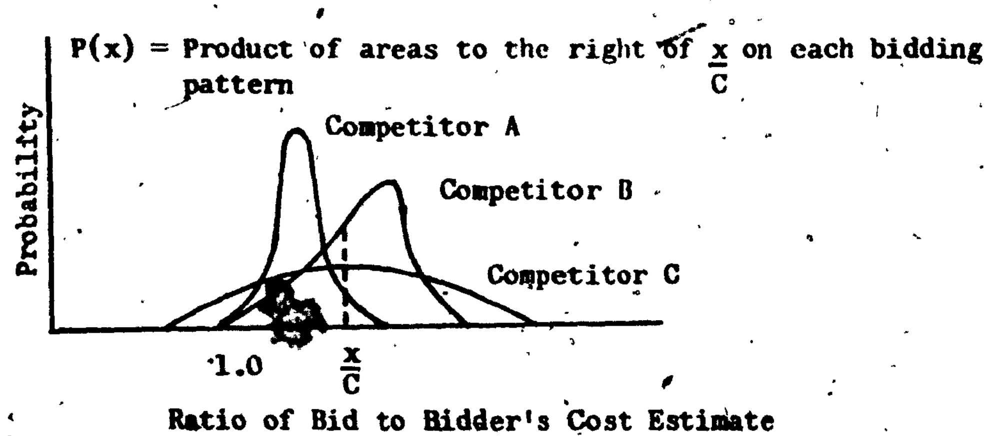

FIGURE A.3 - Bidding Patterns of Competitors

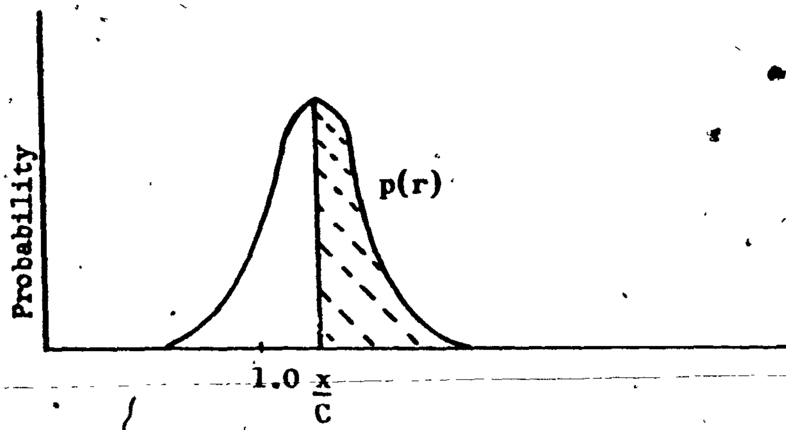

FIGURE A.4 - Bidding Pattern of Average Bidder

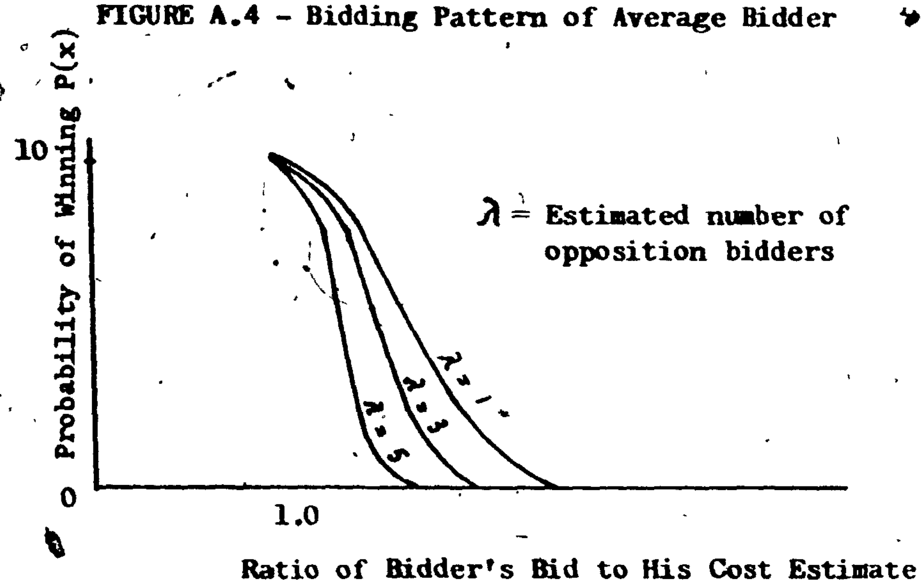

FIGURE A.5 - Probability of Winning vs Bid, for Various Estimated Number of Opposition Bidders

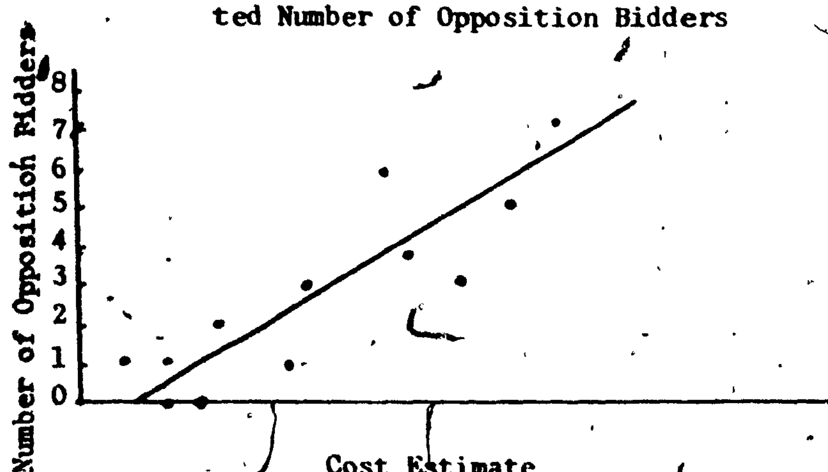

FIGURE A.6 - A Method of Obtaining an Estimate of the Number of Bidders based on Previous Bidding History