Regime Switching in Commodity Prices

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Abstract

Regime Switching in Commodity Prices

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During the 1970's, the increase in the price of oil and many other commodities dominated macroeconomic discussions. In the late 1980's and early 1990's, commodity prices generally declined and not much attention was given to the topic. Currently, the surge in the price of oil and many other commodities, both in nominal and real terms, drew back attention to the issue.

This thesis follows the empirical approach in modelling nonlinear behavior of commodity prices. The approach was motivated by the observation that commodity prices tend to move together in groups in response to a common macroeconomic variable or group of variables. The thesis attempts to explain this phenomenon by, first, classifying commodity prices according to their recorded border prices (an issue that has been ignored in previous studies), and then by trying to find the best transition (threshold) variable that can explain this common dynamic in each group.

The observed nonlinearity in commodity prices is modelled using the smooth transition regression (STR) model with external threshold variables. The use of external threshold variables, in addition to the commonly used autoregressive lags of the dependent variable, is a theme that distinguishes this thesis from the majority of the studies in the regime switching literature. The STR model, which technically models

regime switching in the conditional mean equation of the data generating process, is extended to model regime switching in the conditional variance. Both models (the STR in mean and the STR in variance) were fitted to the Grilli & Yang's (1988) commodity price index and to the individual price series forming the index.

Two external transition variables were found successful in capturing the regime switching dynamics of the commodity price index: inflation rate and the price of oil. Using both transition variables in the STR in mean and the STR in variance models, both models displayed the same dynamics in the limiting processes of the commodity price index. This result suggests that both models can be seen as *substitutes* when modelling nonlinearity in the commodity price index. As for the two transition variables, inflation was capable of capturing the early dynamics (between 1900 and 1950) of the commodity index whereas oil price captured the late ones (between 1980 and 2007). This result motivates the use of *external* threshold variables in regime switching models in general and, in particular, the use of inflation and oil price in the STR model when applied to an index of commodity prices.

More insight about the co-movement of commodity prices is gained by studying the individual price processes forming the index. However, it is worth noting that there is no single variable that is capable of explaining the behavior of all commodity prices. The way each price series is recorded and the history of the commodity's major exporter and importer is crucial in styding and modelling its dynamics.

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I would also like to thank my sister, Dr. Gehane Fahmy, for her continuous support. Finally, I would like to dedicate this thesis to the memory of my father and my beloved mother, Nawal Zaki, whose unconditional love gave me the strength to finish this thesis and made me the person I am today. May her soul rest in peace.

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Chapter 1

Introduction

During the 1970's, the increase in the price of oil and many other commodities, such as sugar, cotton, gold and silver, dominated macroeconomic discussions. In the late 1980's and early 1990's, commodity prices generally declined and, as stated by Frankel (2006), '*the topic fell out of favor*'. Currently, the surge in the price of oil and many other commodities, both in nominal and real terms, drew back the attention to the issue.

Studying the dynamic behavior of commodity prices over time and whether or not they are driven by common transition variable(s) are the main objectives of this research. To this end, technical issues are considered in detail. In particular, the research answers questions like: How are the dynamics of commodity prices to be modelled? Is (are) there common driving variable(s) responsible for such changing dynamics? And if these variables exist, what is the economic rationale behind choosing them? Why is treating an index of commodity prices different than treating individual commodities? And why is a border price classification crucial to identify the potential driving variables for *individual* commodities?

The dynamic behavior of commodity prices is characterized by a number of stylized facts including skewness, kurtosis, high asymmetric volatility, lack of trend, and high degree of autocorrelation. For instance, the prices of agricultural crops, such as wheat, rice, cotton, or maize, can be described, as stated by Deaton and Laroque (1996), as 'doldrums' pattern interrupted by upward (not downward) spikes. Prices of metals traded on exchanges, on the other hand, resemble those of financial assets, where volatility clusters (periods of high volatility and low volatility alternate) is a common feature.

The issue of commodity price formation has been studied extensively in the literature. Early theoretical models originate from Gustafson's (1958) work on the theory of competitive storage and the work of Muth (1961), who introduced the rational expectations assumption in a model of commodity price formation. Both contributions formed the basic model of commodity price formation. The competitive storage theory postulates that speculative arbitrage is what generates the observed serial dependence in commodity prices. The idea here is that a risk neutral speculator will carryover the commodity if its future expected price just covers the cost of inventory carryover. This implies that inventory holding will generate the observed high autocorrelations in commodity prices. Extensions (in different directions) to this basic model can be seen from further notable contributions. Samuelson (1971) showed that the solution of the competitive storage problem (in a dynamic-programming framework) is a *nonlinear* first-order Markov price process. Danthine (1977) analyzed a model of commodity markets and showed, among other things, that the martingale property of price changes does not follow from the behavior of economic agents in efficient markets. More recently, focusing on inventory speculation, Williams and Wright (1991) introduced an excellent treatment of the theory of competitive speculation in stocks, where inventory speculation was the key feature of fluctuations in commodity prices. Building on these notable contributions, Deaton and Laroque (1992), in an attempt to confront the theory with evidence, applied a rational expectations competitive storage model to study the actual behavior of thirteen commodities. Under the assumption of independently and identically distributed (i.i.d.) harvest shocks, their model explained most of the above-mentioned stylized facts of commodity prices, but failed to capture the nonlinearity observed in the dynamics of theses price processes.

The previously mentioned attempts, elegant as they were, did not succeed in capturing entirely the dynamic behavior of commodity prices. The general conclusion is that commodity prices are nonlinear and this nonlinearity is attributed to the speculative behavior of agents for holding stocks or to unobserved demand and supply shocks.

This thesis follows a different approach in modelling the dynamics of commodity prices. The approach is empirical in nature and is motivated by the fact that commodity prices tend to move together in groups that can be classified according to the recorded *border price* of the commodity under consideration.

Border prices, also referred to as International Commercial Terms (Incoterms),[1] are the terms of selling that define the obligations of the trading parties engaged in trading contracts. The agreed border price determines the risks and costs incurred by the exporter and the importer of a specific commodity. Among all border prices, two are used the most: Free on board (FOB) prices and cost insurance and freight (CIF) prices. A trading contract effected on a FOB basis implies that the exporter (shipper) bears all the risks and costs of transporting the cargo from the point of origin (e.g., the exporter's factory) to the port of export in the country of origin (exit point of the exporting country). The importer (consignee) bears all the risks and costs of the cargo from that point up to delivery to final destination. FOB price, thus, does not include freight, insurance, and other transportation costs needed to transfer the commodity from one country to another. A CIF price, on the other hand, is a FOB price plus insurance cost plus ocean freight cost; i.e., under a CIF contract, the exporter, in addition to the insurance, bears all risks and costs of transporting the cargo from the point of origin to the port of discharge (entry point of the importing country). Therefore, for simplicity, FOB prices are referred to as export prices at the exit point of a country and CIF prices are referred to as import prices at the entry point of a country.

[1] The Incoterms are published by the International Chamber of Commerce.

The issue of co-movements of commodity prices refers to the tendency of some commodities to move together in response to a common macroeconomic variable. This research seeks to explain this fact by, first, classifying various commodities according to their border price (an issue that has been ignored in previous studies), and then by attempting to find the best common macroeconomic variable that is responsible for this co-movement. Figure 1.1 illustrates this idea. The commodity price index used in this research is the Grilli and Yang (1988) commodity price index, which is a composite index of 24 commodity prices. The individual price processes are classified into different groups based on the recorded border price. The time series in each group may be thought of as remaining in a given regime until pushed to a new regime by a shock, a series of shocks, or by a *common* driving (transition) variable. One can interpret such dynamics as switching behavior between multiple equilibria. Finding those common driving variables and modelling this switching behavior are the main objectives of this research.

The data set published by Grilli and Yang (1988), which is used in this research, was originally developed and used by the authors to study the long-run behavior of the net barter terms of trade series or, in other words, to test the Prebisch (1950) and Singer (1950) hypothesis (PS hypothesis) of a secular decline in the price of primary commodities relative to the price of manufactures. To this end, the authors developed a commodity price index that consists of 24 primary commodity prices and known as the Grilli and Yang Commodity Price Index (GYCPI). The authors deflated the

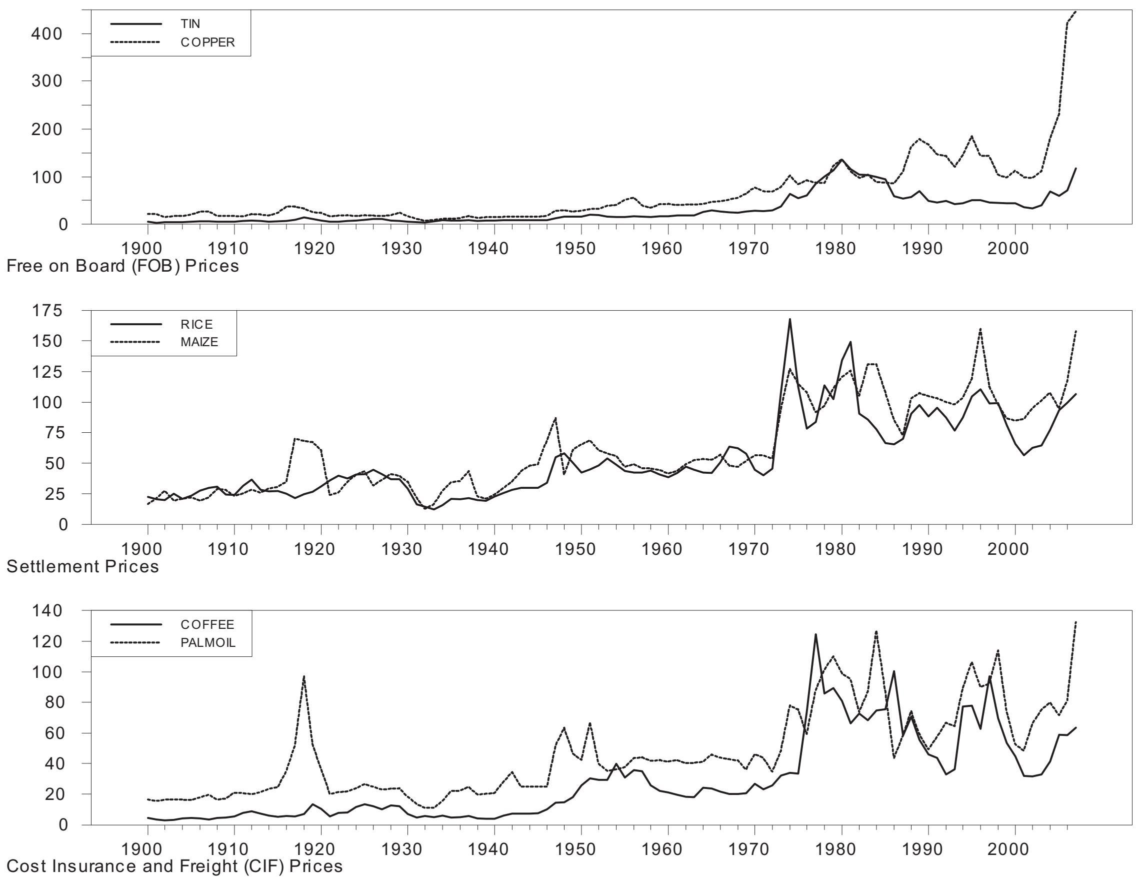


Figure 1.1: Border price classification of some individual commodities in the Grilli & Yang (1988) data set.

GYCPI by an index of manufactured goods' unit values (MUV)[2] and fitted a log-linear time trend model to the ratio GYCPI/MUV as well as to the individual commodities forming the index. They found a significant downward trend in the net barter terms of trade; that is, the estimated coefficient obtained from regressing the logarithm of the ratio GYCPI/MUV (denoted y_t in the text) on a linear time trend has a negative sign. Applying this trend stationary (TS) model to y_t, Grilli and Yang found support for the PS hypothesis.

The PS hypothesis has been re-assessed by a number of authors using the data set published by Grilli and Yang. Generally the literature attempted to model the price of primary commodities relative to manufactures using econometric univariate and bivariate time series models that may or may not allow for structural breaks. The findings varied from one study to another to the extent that a clear cut conclusion could not be reached. In what follows, we present a brief literature review on those attempts.

As mentioned above, Grilli and Yang (1988) applied a TS model to their data set and found support to the PS hypothesis. The authors also found that there is no evidence of structural breaks. Cuddington and Urzua (1989) pointed out that the residuals of the TS model might possibly be nonstationary, which, in turn, renders the OLS estimate of the trend coefficient to be unreliable. Therefore, the authors assumed that the Grilli and Yang series had a unit root and could not reject the unit

[2] The MUV is a trade-weighted index of the five major developed countries' (France, Germany, Japan, United Kingdom, and United States) exports of manufactured commodities to developing countries.

root hypothesis (nonstationarity) in the price series using the Dickey-Fuller (1979) test. Based on this nonstationarity assumption, they fitted a difference stationary (DS) model to y_t, where they regressed the first difference of y_t on a constant, a dummy to account for a structural shift in 1921, and a MA(3) error process. Apart from the one-time drop in the price series after 1920, the PS hypothesis of a secular decline in the price of primary commodities was not supported by their results.

Von Hagen (1989) found that the logarithm of the GYCPI and the MUV index are cointegrated and that the expected long-run net barter terms of trade series, y_t, is stationary. Hence, the PS hypothesis was not supported. Powell (1991) also assumed that the data generating process (DGP) is nonstationarity and fitted the same DS model yet with more than one structural break: in 1921, 1937, and 1975. Powell's findings did not support the PS hypothesis.

Helg (1991) tested the series y_t for stationarity and rejected the nonstationarity hypothesis using the Dickey - Fuller (1979) test. The author also applied Schmidt and Phillips' (1989) test and rejected the unit root hypothesis in y_t. The result was in favour of a trend stationary model with a negative trend coefficient for most of the century (1900 - 1988) and a major structural break at the end of the World War One. Helg's results supported the PS hypothesis.

Ardeni and Wright (1992) pointed out that the TS or DS models, resulting from the Box and Jenkins' (1976) identification framework, require making a preliminary hypothesis regarding the stationarity of the data generating process. To avoid this complication, the authors followed a structural time series approach that does not rest on any prior stationarity assumption (see Harvey (1989, pp. 31-51)). By examining the behavior of y_t over the period from 1900 to 1988, their findings were consistent with Grilli and Yang's (1988) in supporting the PS hypothesis. The authors also reported that the inclusion of a dummy variable to account for the 1921 break claimed by Cuddington and Urzua (1989) had no effect on the results. They concluded that even if the break really occurred in 1921, y_t would still have a negative trend but less steep.

Bleaney and Greenaway (1993) extended the Grilli and Yang data series to 1991. The authors fitted an autoregressive model with a time trend to y_t and rejected the PS hypothesis in favor of a one-off drop in 1980.

Newbold and Vougas (1996) found that the starting point of the econometric analysis is crucial in testing the PS hypothesis; that is, whether the DGP is TS or DS. The authors' results were ambiguous in this matter. In particular, they found that there is strong evidence of the PS hypothesis when the relative price series is TS, but when the series is DS, the PS hypothesis was rejected. The authors also found that allowing for the possibility of structural break in the series does not help in assessing whether the time series is TS or DS. Trivedi (1995) also concluded that the empirical results of whether the relative price process is TS or DS are not clear-cut.

Lutz (1999) gave an excellent summary of the previously mentioned attempts. He also extended the Grilli and Yang data set to cover the period 1900 to 1995. Lutz argued that the reason behind the various findings is the choice of the econometric model. Grilli and Yang (1988), Helg (1991), Ardeni and Wright (1992), and Bleaney and Greenaway (1993) fitted a TS model; Cuddington and Urzua (1989) and Newbold and Vougas (1996) fitted both TS and DS models; Von Hagen (1989) and Powell (1991) used the bivariate approach and tested the hypothesis that the price series are cointegrated. Lutz combined the TS, DS, and the cointegration relation into an encompassing first-order distributed lag model with its error-correction equivalent. Using the Johansen procedure, Lutz supported the PS hypothesis contrary to the findings of other authors who employed the bivariate framework (e.g., Von Hagen (1989) and Powell (1991)).

Persson and Terasvirta (2003) assumed, as a starting point to the analysis, that the series y_t are stationary. Using the extended series of Lutz (1999), the authors, unlike all the previous studies, considered the hypothesis that the series might be nonlinear. In particular, they tested the linearity hypothesis in y_t against a parametric nonlinear model (the smooth transition autoregressive model) and could not reject the nonlinearity in the price series given the stationarity assumption. If the price series is nonlinear and stationary, the resulting mean reversion behavior will contradict the PS hypothesis. Therefore, the authors reached the same conclusion as of Newbold and Vougas (1996) that the findings vary according to the starting point of the analysis.

In this thesis, we follow the same path as of Persson and Terasvirta (2003) that the Grilli and Yang commodity price index as well as the individual price processes forming the index are nonlinear. We test the nonlinearity hypothesis and we model the dynamics of the price processes using regime switching (threshold) models. Threshold models are best suited here because, unlike other models (Markov switching models for instance), they assume that the DGP under consideration is driven by an *observed* transition process. This is consistent with our approach of using common predetermined observed transition variables that are capable of explaining the dynamics of commodity prices. In particular, we employ the smooth transition regression (STR) model pioneered by Granger & Teräsvirta (1993) and Teräsvirta (1994). STR models have been used extensively in the regime switching literature. Most of the studies (see, for instance, Teräsvirta & Anderson (1992), Granger & Teräsvirta (1993), Lutkepohl, Teräsvirta, & Wolters (1999)) have been focusing on modelling nonlinearities in aggregate macroeconomic time series such as GDP, money demand functions, and industrial production. Recently, Sarantis (2001) used these models to explain the cyclical behavior in stock markets. In addition to their popularity, STR models possess some appealing features. They are based on a three-stage modelling procedure starting from a specification stage, estimation, and finally an evaluation stage. Also, as opposed to other regime switching models (e.g. pure threshold models and Markov switching models), STR models have the flexibility of describing processes that can move from one regime to the other such that the transition is smooth.

Another issue concerning nonlinearity is whether it enters the conditional mean or the conditional variance of the commodity price time series under consideration.

Beck (2001), for instance, using a variation of Engle's (1982) Autoregressive Conditional Heteroskedasticity (ARCH) model found nonlinearity in storable and not in non-storable commodity prices. In this thesis, the STR model, which is technically a nonlinear model in the conditional mean, is extended to model nonlinearity in conditional variance. Hence, STR in variance models are also considered.[3] One central theme, however, that distinguishes this study from most of other studies in the regime switching literature is its attempt to look for potential transition variables that can explain the detected nonlinearity (whether in mean or in variance) in commodity prices. The study shows that, in addition to the customarily used transition variables (lags of the dependent variable), external variables like oil price and inflation are capable of capturing the dynamics of commodity prices and can act as driving factors to price processes. The economic rationale behind selecting those potential transition variables lies in their connection with commodity prices. An attempt to establish such a link is also pursued in this study.

When modelling the behavior of commodity prices empirically, either using nonlinear models or any other models, the statistical and econometric techniques are directly applied to the time series under consideration. Two significant particulars are, consequently, overlooked in this context. First, the treatment of an index of commodity prices is different than the treatment of individual time series. Modelling individual price series requires a deeper look into the commodity's history, its

[3]This, however, does not rule out the possibility of modelling nonlinearity using ARCH models. Actually, some of the individual commodities in the data set used in this thesis were best modeled using ARCH or variations of ARCH models. This point is discussed in detail in Chapter 3.

main characteristics, its major exporters and importers, and its main shipping routes. Sometimes the commodity's history can explain some of the high swings perceived in the price process. Valuable information regarding major trading routes and sales terms (usually inferred from the data sources) of a commodity can assist in identifying the potential transition variable responsible for driving the commodity price time series from one regime to the other. For instance, if the recorded commodity price is a cost and freight price, oil price is expected to be the potential variable responsible for the transition of the commodity price time series from one regime to the other. This can be justified through the oil price-commodity price connection. The idea here is that fuel surcharge constitutes a significant portion of ocean freight, which, in turn, is included in the recorded cost and freight price. Therefore, the changing dynamic of the oil price causes the recorded border price to move from one regime to the other. It turned out that oil price is, indeed, the common transition variable for all cost and freight commodities in the Grilli and Yang's (1988) data set used.

The second overlooked issue in modelling the behavior of commodity prices is understanding the way the commodity is traded. Commodities can be classified according to the type of trading into two groups: (1) imported or exported goods, where actual delivery of the merchandise is mandatory, and (2) commodities traded in exchange,[4] where physical delivery is not a must; the trading is usually done over the counter for profit sake and the recorded prices are settlement prices. The

[4]This type of commodities includes metals, mainly, and some grains and agricultural items.

price behavior of the later group resembles that of the financial assets, which can be captured by Engle's (1982) ARCH model. In this research the threshold technology is incorporated into Engle's ARCH model and applied to this group of commodities. Hence, smooth threshold ARCH (ST-ARCH) models are entertained for this type of commodities. The value added of the ST-ARCH models can be seen from their ability to capture the asymmetric responses of the time series to previous positive and negative shocks; a feature that is commonly observed in commodity prices processes.

The plan of the thesis is as follows. Chapter 2 introduces the smooth transition regression model, which is technically a regime switching in conditional mean, and applies it to the Grilli and Yang's (1988) commodity price index using inflation and oil price as predetermined transition variables. The chapter discusses the economic rationale behind selecting inflation and oil price as transition candidates in explaining the nonlinearities in commodity prices through exploring two connections: the commodity price-consumer price connection and the commodity price-oil price connection. The feedback from consumer price to commodity price is also discussed from a regime switching perspective. The smooth transition model is then applied to model the nonlinearities in the 24 individual commodity price processes forming the Grilli and Yang's (1988) commodity price index. To determine the adequate external transition variable that is capable of explaining the dynamics of individual price processes, the 24 time series are classified into different groups based on the recorded border price. Nonlinearity is then tested and modeled within each group. Chapter 3 applies the same smooth transition regression model discussed in Chapter 2 to the conditional variance of the Grilli and Yang commodity price index using inflation and oil price as potential transition variables. The STR in variance model is then applied to the individual processes forming the commodity price index. Following the border price rationale discussed in Chapter 2, the nonlinearities in the conditional variance of the individual processes in each group were characterized and analyzed. Finally, Chapter 4 concludes.

Chapter 2

Regime Switching in Mean

The primary version of the switching regression model was due to Quandt (1958), who used maximum likelihood in estimating one switching point in a two regime regression system. Bacon & Watts (1971) considered two different distinct linear regression lines and developed a smooth transition technique from one linear regime to the other. Beach (1977) considered incorporating structural change in a regression model where the change occurs gradually over a known transition period. Recent accounts include Granger & Teräsvirta (1993), Teräsvirta (1994, 1998), Franses & Van Dijk (2000), and Teräsvirta (2004). Teräsvirta (1994) combined the threshold autoregressive models and the exponential autoregressive models in a single family of models called the smooth transition regression (STR) models. The STR model, described briefly by Teräsvirta (2004) and fully by Teräsvirta (1994), is a nonlinear regression model that describes the changing dynamics of an autoregressive model from one regime to another such that the transition is smooth. The model can be considered as a generalization of the model devised by Bacon & Watts (1971). The model also nests the linear model as a special case.

In this thesis, we will focus on the smooth transition regression model, pioneered by Granger & Teräsvirta (1993), as the regime switching framework that will be used to explain the behavior of commodity prices. Before introducing the model, we will start by introducing a general threshold specification. Consider the following r regimes autoregressive regression model of order p.

$$y_t = \left(\phi_0^1 + \sum_{i=1}^{p} \phi_i^1 y_{t-i}\right) G^1(s_t;\Psi) + ... + \left(\phi_0^r + \sum_{i=1}^{p} \phi_i^r y_{t-i}\right) G^r(s_t;\Psi) + a_t, \qquad (2.1)$$

with

$$\sum_{j=1}^{r} G^j(s_t;\Psi) = 1; \quad G^j(s_t;\Psi) \geq 0, \forall j, \forall t,$$

where $G^j(s_t;\Psi)$ is a state of nature (or regime) j indicator function, for $j = 1, ...r$, s_t is a vector of switching variables, Ψ is a vector of parameters, and $a_t \sim i.i.d.(0, \sigma^2)$.

Threshold models, STR models, and Markov switching models can all be expressed from (2.1) with the exception that s_t is not observed in case of Markov switching models. Hence, the definition of the transition function, $G(\cdot)$, will differ from one model to the other, and, actually, it is what distinguishes these models from one another.

2.1 The Pure Threshold Model

In the pure threshold model, the transition function $G(\cdot)$ is a discrete function that takes the values $\{0, 1\}$. Consider the simple case where we assume two regimes, i.e., $r = 2$. Then, equation (2.1) can be written as

$$y_t = \left(\phi_0^1 + \sum_{i=1}^{p} \phi_i^1 y_{t-i}\right) G^1(s_t; \Psi) + \left(\phi_0^2 + \sum_{i=1}^{p} \phi_i^2 y_{t-i}\right) G^2(s_t; \Psi) + a_t, \tag{2.2}$$

where

$$G^1(\cdot) + G^2(\cdot) = 1,$$

and

$$G^1(s_t; \Psi) = \begin{cases} 1 & \text{if } s_t \in A_1(\Psi), \\ 0 & \text{otherwise,} \end{cases}$$

where $A_1(\Psi)$ is a set that defines regime 1.

When the transition variable is one of the autoregressive lags of the dependent variable, the pure threshold model is referred to as the threshold autoregressive (TAR) model. The TAR model of Tong (1978) is a case in point. A TAR model can also be expressed from our general specification in (2.1). A simple two-regime TAR(p) model with $s_t = y_{t-1}$ so that the delay is 1, and the threshold is c is expressed as

$$y_t = \begin{cases} \phi_0^1 + \sum_{i=1}^{p} \phi_i^1 y_{t-i} + a_t & \text{if } y_{t-1} < c, \\ \phi_0^2 + \sum_{i=1}^{p} \phi_i^2 y_{t-i} + a_t & \text{if } y_{t-1} \geq c, \end{cases} \tag{2.3}$$

where $a_t \sim i.i.d.(0, \sigma^2)$. More thorough treatment of threshold models can be found in Priestly (1988) and Tong (1983, 1990).

2.2 Markov Switching Models

Tong (1983) discussed the idea of using probability switching in nonlinear time series models. Hamilton (1989) proposed a regime switching model in which the unobserved state of nature s_t follows a two-state Markov chain. A time series y_t following a simple two-state Markov switching autoregressive model of order p can be expressed as

$$y_t = \left(\alpha_0^1 + \sum_{i=1}^{p} \alpha_i^1 y_{t-i}\right) s_t + \left(\alpha_0^2 + \sum_{i=1}^{p} \alpha_i^2 y_{t-i}\right)(1 - s_t) + a_t, \tag{2.4}$$

where the transition variable s_t assumes values in $\{0, 1\}$ and is a first order Markov chain with transition probabilities given by the transition probability matrix P as

$$P = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix},$$

where $0 \leq p_{11} \leq 1$ is the probability of being in regime 1 conditional on being in this regime in the previous period, i.e., $p_{11} = (s_t = 1 | s_{t-1} = 1)$, and $0 \leq p_{00} \leq 1$ is the probability of being in regime 2 given we were already in this regime in the previous period, i.e., $p_{00} = (s_t = 0 | s_{t-1} = 0)$. The process a_t is a sequence of *i.i.d.* random variables.

The Markov switching model uses a hidden Markov chain that governs the transition from one regime to another. This idea of unobserved states of nature makes the estimation of Markov switching models harder than other nonlinear model. Hamilton (1990) used an expectation-maximization algorithm to estimate a Markov switching model. Other researchers used the Markov chain Monte Carlo method to estimate a general Markov switching autoregressive model; see, for instance, McCulloh and Tsay (1994).

2.3 Smooth Transition Regression Models

In the pure threshold model in (2.2), the transition function $G(\cdot)$ was a discrete function taking the values $\{0,1\}$. In order to introduce smoothness in the transition between regimes, the STR model assumes that the transition function is continuous, i.e., $G(\cdot) \in [0,1]$.

The standard STR model of order p is expressed as follows:

$$y_t = \left(\phi_0 + \phi_1 y_{t-1} + ... + \phi_p y_{t-p}\right) + \{\theta_0 + \theta_1 y_{t-1} + ... + \theta_p y_{t-p}\} G(s_t;\Psi) + \varepsilon_t,$$

or in a more compact notation

$$y_t = \Phi^{'} z_t + \Theta^{'} z_t G(s_t;\gamma,c) + \varepsilon_t, \tag{2.5}$$

where y_t is a scalar, $z_t = (1, y_{t-1}, ..., y_{t-p})^{'} = (1, \tilde{z}_t^{'}), \Phi^{'} = (\phi_0, \phi_1, ..., \phi_p) = (\phi_0, \tilde{\Phi}^{'}), \Theta^{'} = (\theta_0, \theta_1, ..., \theta_p) = (\theta_0, \tilde{\Theta}^{'})$, and $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. $G(s_t;\Psi)$ is the transition function. It is a bounded function of the continuous transition variable s_t and it is continuous everywhere in the parameter space for any value of s_t. In practice, the transition variable s_t is an element of the autoregressive lags of the dependent variable, i.e., $s_t = y_{t-d}$ or $s_t = \triangle y_{t-d}$, where $d > 0$ is the delay parameter. In this thesis we allow the transition variable s_t to be either an element of the autoregressive lags of y_t or an external variable that is capable of describing the behavior of y_t. The selection of the transition variable, and hence the model type, is performed in the first stage of the modelling process; *the specification stage* (more of that later). The definition of $G(s_t; \Psi)$ is the one governing the empirical applicability of (2.5). Some definitions have been suggested in the literature; see, for instance, Granger and Teräsvirta (1993, Ch. 7). In this thesis, $G(\cdot)$ is a logistic function defined in general as

$$G(s_t; \gamma, \mathbf{c}) = \left(1 + \exp\{-\gamma \prod_{i=1}^{k} (s_t - c_i)\}\right)^{-1}, \qquad \gamma > 0, \tag{2.6}$$

where γ is the slope of the function, and $\mathbf{c} = (c_1, ..., c_k)'$ is a vector of location parameters, such that $c_1 \leq ... \leq c_k$. Given such a definition, the STR model defined in (2.5) is then referred to as the logistic smooth transition regression (LSTR) model.

The transition function in (2.6) is a monotonically increasing function of s_t. The restriction $\gamma > 0$ is an identifying restriction. The choice of k is not only crucial in determining the behavior of the logistic transition function, but also has a significant implication in interpreting the time series under consideration. Two common choices for k are used in the literature; $k = 1$ and $k = 2$. In the LSTR model with $k = 1$ (LSTR(1)), the parameters $\Phi + \Theta G(s_t; \gamma, c)$ in equation (2.5) change monotonically as a function of s_t from Φ to $\Phi + \Theta$. Given such a feature, the LSTR(1) is capable of characterizing asymmetric time series behavior, i.e., processes whose dynamic properties are different in an expansionary regime from what they are in a recessionary regime, and the transition between the two regimes is smooth. The logistic function in the LSTR(1) model takes the form

$$G(s_t; \gamma, c) = \left(1 + \exp\{-\gamma(s_t - c)\}\right)^{-1}, \qquad \gamma > 0. \tag{2.7}$$

Note that when $s_t \longrightarrow -\infty$, $G(\cdot) = 0$; this defines the recessionary regime. On the other hand, when $s_t \longrightarrow +\infty$, $G(\cdot) = 1$ and the time series is said to be in an expansionary regime. The LSTR(1) function in (2.7) is plotted in Figure 2.1, where the threshold $c = 0.5$ and the slope $\gamma = \{2, 1000\}$ for the solid and the dashed lines respectively.

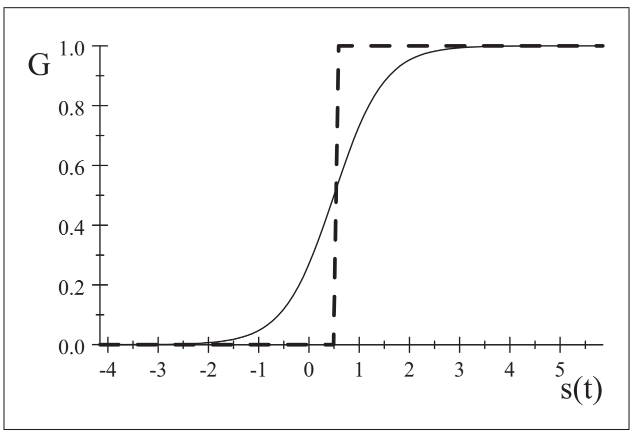


Figure 2.1: The smooth transition logistic function of order 1 with a moderate slope $\gamma = 2$ (the solid line) and with an extremely larger slope $\gamma = 1000$ (the dashed line). The threshold value $c = 0.5$.

In the LSTR model with $k = 2$ (LSTR(2)), the parameters $\Phi + \Theta G(s_t; \gamma, c)$ change symmetrically around the midpoint $\bar{c} = \frac{c_1 + c_2}{2}$, where the logistic function attains its minimum value. The LSTR(2) model is a three-regime switching regression model in which the dynamics of the two outer regimes, associated with large and small values of s_t, are the same while the behavior in the transition period (middle regime) is different. The second-order logistic function in the LSTR(2) model takes the form

$$G(s_t;\gamma,c) = \left(1+\exp\{-\gamma(s_t-c_1)(s_t-c_2)\}\right)^{-1}, \qquad \gamma > 0,\ c_1 \leq c_2, \tag{2.8}$$

where $\gamma > 0$ and $c_1 \leq c_2$ are identifying restrictions as mentioned before. Notice that, unlike the first-order logistic function, the second-order function is not zero at the minimum; it has a value

$$G_{\min} = \frac{1}{1+\exp\{-\gamma\tilde{c}\}},$$

where $\tilde{c} = c_1c_2 - \bar{c}^2$. The behavior displayed by the second-order logistic function depends on the value taken by the transition variable s_t. When the transition variable takes the value of any of the thresholds, i.e., $s_t = c_1$ or $s_t = c_2$, $G(s_t;\gamma,c_1,c_2) = \frac{1}{2}$. This characterizes the middle regime. On the other hand, the two outer regimes are achieved when $s_t \longrightarrow \pm\infty$ and $G = 1$. The LSTR(2) function in (2.8) is plotted in Figure 2.2, where $c_1 = -1, c_2 = 2$ and $\gamma = \{1, 1000\}$ for the solid and dashed lines respectively. Observe how the functions change symmetrically around $\bar{c} = 1/2$ (the midpoint). When $\gamma = 0$, the transition function $G(s_t;\gamma,c_1,c_2) = 1/2$, and the STR model in (2.5) nests the linear model. When $\gamma \rightarrow \infty$ in the LSTR(1) model, the result approaches the pure threshold model in (2.3) (see the dashed line in Figure 2.1); when $\gamma \rightarrow \infty$ in the LSTR(2) model, the result is a pure threshold model with three regimes such that the two outer regimes are similar while the ground regime (middle regime) is different and the transition between regimes is swift (see the dashed line in Figure 2.2).

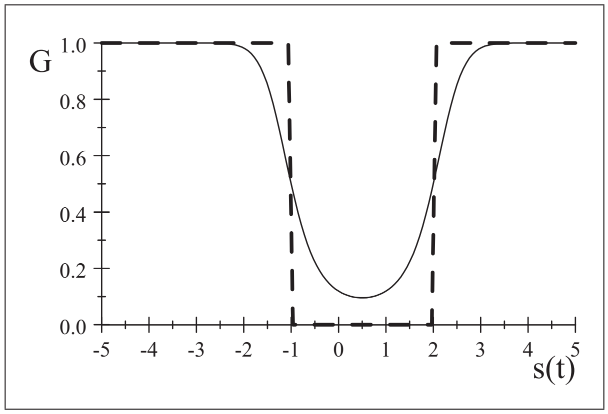


Figure 2.2: The smooth transition logistic function of order 2 with a moderate slope $\gamma = 1$ (the solid line) and with an extremely larger slope $\gamma = 1000$ (the dashed line). The threshold values are $c_1 = -1$ and $c_2 = 2$.

I shall follow the modelling framework proposed by Granger and Teräsvirta (1993), Teräsvirta (1994), and Eitrheim & Teräsvirta (1996), which consists of three stages. The first stage is the specification stage, in which the delay parameter of the nonlinear model is determined by suggesting and testing a linear model against the STR model. If the null hypothesis of linearity is rejected, a short sequence of F-tests is conducted to decide the type of the nonlinear model (LSTR(1) or LSTR(2)). The specified model is estimated in the second stage and the evaluation of the estimated model is performed in the third stage. Nonlinearity testing, estimation, and evaluation of the LSTR model are briefly discussed in the following subsections.

2.3.1 Testing for nonlinearity

The specification stage of the LSTR model involves two phases. First a linear model forming the starting point of the analysis is specified and subjected to linearity tests. Then, the type of the STR model (LSTR(1) or LSTR(2)) is selected.

To begin, a linear autoregressive (AR) model is specified. To this end, y_t is regressed on $y_{t-1}, y_{t-2}, ..., y_{t-p}$, and a constant. A common technique to determine the lag length of the autoregression, p, is to use an order selection criterion like the Akaike information criterion (AIC) (Akaike, 1974) or the Bayesian information criterion (BIC) (Schwarz, 1978). Since the BIC might lead to too parsimonious a model in the sense that the residuals of the preliminary model might not be free from autocorrelation, the researcher can, as a start, fit a linear model using the lag order from the BIC and then test for residual autocorrelation. If it is still present, the researcher can increase the number of lags till the residuals are free from serial correlation. An alternative strategy, which will be used in this study, is to follow the convention in the literature and use the value of p that minimizes the AIC directly. The linear AR model then takes the following form

$$y_t = \phi_0 + \phi_1 y_{t-1} + ... + \phi_p y_{t-p} + a_t, \qquad a_t \sim i.i.d.(0, \sigma_a^2), \tag{2.9}$$

and the LSTR model takes the form

$$y_t = \phi_0 + \phi_1 y_{t-1} + ... + \phi_p y_{t-p} + \{\theta_0 + \theta_1 y_{t-1} + ... + \theta_p y_{t-p}\} G(s_t; \gamma, c) + \varepsilon_t, \tag{2.10}$$

where $G(\cdot)$, as mentioned before, is a logistic transition function with the transition variable s_t and $\varepsilon_t \sim i.i.N.(0, \sigma_\varepsilon^2)$. Before accepting the AR model in (2.9) as the starting point of the LSTR analysis, diagnostic tests should be applied to the model to ensure its adequacy as a starting model. In particular, the Ljung-Box (1978) test of no serial correlation of order $q = 1$ up to $q = 8$ in the residuals ($Q(q)$) and Engle's (1982) Lagrange multiplier test of no autoregressive conditional heteroscedasticity of order $v = 1$ up to $v = 4$ in the residuals ($LM_{ARCH(v)}$) are considered.[1] Once the model is proven to be adequate, we move to the second phase in the specification stage, which is testing for nonlinearity.

The problem with nonlinearity testing is that the nonlinear STR model considered is only identified under the alternative hypothesis. Consider the STR model in (2.5) with $\Psi = \{\gamma, c\}$. Imposing the restriction that $\Theta = 0$ (see page 20) reduces (2.5) into a linear model but Ψ is not identifiable under this null-hypothesis. An alternative way is to set $\gamma = 0$, which also renders the model linear but, again, this leaves $\{\Theta, c\}$ unidentifiable. Luukkonen et al. (1988), based on a paper by Davies (1977), suggested a solution to this problem. Their solution, which was adopted by Teräsvirta (1994), was simply to replace the transition function (2.6) in (2.5) by a Taylor approximation about the null hypothesis $\gamma = 0$. In particular they assumed a first order logistic function, i.e., $k = 1$ in (2.6), and performed a third order Taylor approximation about the null hypothesis $\gamma = 0$. Let $G(s_t, \gamma, c)$ be the first order logistic function

[1] Testing the presence of ARCH up to order $v = 4$ is adequate here since we have annual data.

given as

$$G(s_t, \gamma, c) = \left[(1 + \exp\{-\gamma(s_t - c)\})^{-1} - 1/2\right] \qquad \gamma > 0,$$

where the 1/2 was subtracted to facilitate the Taylor approximation. The third order Taylor approximation about γ is

$$T(s_t, \gamma, c) \simeq G(\cdot)|_{\gamma=0} + \frac{\partial G(\cdot)}{\partial \gamma}|_{\gamma=0}\gamma + \frac{\partial^2 G(\cdot)}{\partial \gamma^2}|_{\gamma=0}\frac{\gamma^2}{2!} + \frac{\partial^3 G(\cdot)}{\partial \gamma^3}|_{\gamma=0}\frac{\gamma^3}{3!}, \tag{2.11}$$

where, at $\gamma = 0$, the first and third terms of the right hand side of equation (2.11) are zeros and will drop. The second and fourth terms are

$$\frac{\partial G(\cdot)}{\partial \gamma}|_{\gamma=0} = \frac{1}{4}(s_t - c) \tag{2.12}$$

and

$$\frac{\partial^3 G(\cdot)}{\partial \gamma^3}|_{\gamma=0} = -\frac{1}{8}(s_t - c)^3. \tag{2.13}$$

Now consider the STR model in (2.5) and replace the transition function $G(\cdot)$ by its Taylor approximation in (2.11) with (2.12) and (2.13). This yields

$$y_t = \Phi' z_t + \frac{1}{4}\gamma\Theta' z_t(s_t - c) - \frac{1}{48}\gamma^3\Theta' z_t(s_t - c)^3 + \varepsilon_t, \tag{2.14}$$

Using $z_t' = (1, \tilde{z}_t')$, $\Phi' = (\phi_0, \tilde{\Phi}')$, and $\Theta' = (\theta_0, \tilde{\Theta}')$, equation (2.14) can be expressed as

$$y_t = \delta_0 + \delta_1' \tilde{z}_t + \pi_1' \tilde{z}_t s_t + \pi_2' \tilde{z}_t s_t^2 + \pi_3' \tilde{z}_t s_t^3 + \varepsilon_t^*, \tag{2.15}$$

where

$$\delta_0 = \phi_0 - \frac{1}{4}\gamma c\theta_0 + \frac{1}{48}\gamma^3 c^3\theta_0,$$

$$\delta_1^{'} = \tilde{\Phi}' - \frac{1}{4}\gamma c\tilde{\Theta}' + \frac{1}{48}\gamma^3 c^3\tilde{\Theta}',$$

$$\pi_1^{'} = \frac{1}{4}\gamma\tilde{\Theta}' - \frac{1}{16}\gamma^3 c^2\tilde{\Theta}',$$

$$\pi_2^{'} = \frac{1}{16}\gamma^3 c\tilde{\Theta}',$$

$$\pi_3^{'} = -\frac{1}{48}\gamma^3\tilde{\Theta}',$$

and $\varepsilon_t^* = \varepsilon_t + R(\gamma, c, s_t)\theta_0$ with $R(\gamma, c, s_t) = \frac{1}{4}\gamma s_t - \frac{1}{48}\gamma s_t^3 + \frac{1}{16}\gamma^3 c s_t^2 - \frac{1}{16}\gamma^3 c^2 s_t$ is the remainder.[2] Because each π_j, $j = 1, 2, 3$, is of the form $\gamma\tilde{\pi}_j$, where $\tilde{\pi}_j \neq 0$ is a function of $\tilde{\Theta}$, the null hypothesis of linearity is then $H_{0L} : \pi_1 = \pi_2 = \pi_3 = 0$. Also note that because $\varepsilon_t^* = \varepsilon_t$ under the null hypothesis, the asymptotic theory is not affected if an LM test is used. Following Luukkonen et al. (1988) and Teräsvirta (1994), a convenient procedure for computing the LM statistic by OLS is to estimate (2.15) under the null hypothesis and compute the sum of squares of the residuals (SSR_0), then estimate (2.15) under the alternative hypothesis and compute SSR_1. The LM statistic is computed as $LM = \frac{T(SSR_0 - SSR_1)}{SSR_1}$, where T is the sample size. The test statistic has an asymptotic χ^2-distribution with $3p$ degrees of freedom when the null hypothesis is valid. However, the F-statistic is recommended because the χ^2-statistic can be size-distorted in small and even moderate samples. In this thesis, we shall use the F-distribution with $3p$ and $T - 4p - 1$ when the null hypothesis H_{0L} is valid. The test is repeated for each transition candidate in the transition set. If the null

[2] Here we assume that s_t is an external variable, i.e., not an element in z_t. If s_t is an element in z_t, then the auxiliary regression in (2.15) will be

$$y_t = \delta_1^{'} z_t + \pi_1^{'} z_t s_t + \pi_2^{'} z_t s_t^2 + \pi_3^{'} z_t s_t^3 + \varepsilon_t.$$

hypothesis of linearity, H_{0L}, using the F-test (F_L) is rejected for at least one of the models, the model against which the rejection, measured in the p-value, is strongest is chosen to be the STR model to be estimated.

Another purpose of conducting the linearity test is to use the test results for model selection. If linearity is rejected and a transition variable is selected, the next step is to choose a model type, i.e., to choose between LSTR(1) or LSTR(2) models.[3] The choice between the two models can be based, again, on the auxiliary regression (2.15). Teräsvirta (1994) showed that when $c = 0$ then $\pi_2 = 0$ when the model is an LSTR(1), where as $\pi_1 = \pi_3 = 0$ when the model is an LSTR(2). The following F-tests sequence was then suggested based on the auxiliary regression in (2.15):

1. Test the null hypothesis: $H_{04} : \pi_3 = 0$ with an ordinary F-test (F_4). A rejection of H_{04} can be interpreted as a rejection of the LSTR(2).

2. Test the null hypothesis that $\pi_2 = 0$ given that $\pi_3 = 0$, $H_{03} : \pi_2 = 0 | \pi_3 = 0$, using another F-test (F_3). Failure to reject H_{03} indicates that the model is an LSTR(1). The idea here is that the s_t^2 terms of a third order Taylor series approximation to a logistic function of order one are zeros when $c = \theta_0 = 0$ (see equation (2.15)). In the LSTR(2), however, these terms will not be zeros. Rejection of H_{03}, on the other hand, is not very informative.

3. The last F-test (F_2) in the sequence is to test the null hypothesis that $\pi_1 = 0$

[3]The exponential smooth transition regression model (ESTR) was not considered in this thesis because the case where the two threshold values are equal, i.e., $c_1 = c_2$, was not encountered when modelling nonlinearity in both the commodity price index and the individual price processes.

given that $\pi_2 = \pi_3 = 0$ as, $H_{02} : \pi_1 = 0|\pi_2 = \pi_3 = 0$. Rejecting H_{02} after accepting H_{03} supports the choice of the LSTR(1) model. Accepting H_{02} after rejecting H_{03} points to the LSTR(2) model.

After carrying out the three F-tests and noting which hypotheses are rejected, if the test H_{03} yields the strongest rejection measured in the p-value, choose the LSTR(2) model; otherwise select the LSTR(1) model.[4]

2.3.2 Estimation and Evaluation

After specifying the type of the STR model and determining the transition variable, the next step is to estimate the model. Consider the STR specification in (2.5), where ε_t is $i.i.d.$ $(0, \sigma^2)$ with a density $p(\varepsilon_t)$. The log-likelihood function at time t can be expressed as

$$l_t(\Phi, \Theta, \sigma, \Psi) = \ln\left(p(y_t - (\Phi' + \Theta' G(z_t; \Psi))z_t)\right). \tag{2.16}$$

Assuming normality of the error term, i.e., if ε_t is $\sim N\ (0, \sigma^2)$, then

$$p(\varepsilon_t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{\varepsilon_t}{\sigma}\right)^2}, \tag{2.17}$$

and the time t log-likelihood function is

$$l_t(\Phi, \Theta, \sigma, \Psi) = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma^2) - \frac{1}{2}\frac{\{y_t - (\Phi' + \Theta' G(z_t; \Psi))z_t\}^2}{\sigma^2}, \tag{2.18}$$

[4]For more details concerning the decision rule for choosing between LSTR(1) and LSTR(2) models, see Teräsvirta (1994).

with $\Psi = \{\gamma, c\}$. Maximum likelihood estimators of the parameters Φ, Θ, σ, and Ψ can be obtained by maximizing the log-likelihood function. However the estimation might be difficult due to the existence of flat segments or numerous local maxima in the surface of the log-likelihood function. Estimation is, therefore, carried out conditional on the Ψ parameters. The conditional log-likelihood[5] is maximized using the iterative Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm. Starting-values for γ and c needed for the algorithm are obtained by constructing a three dimensional grid. The model parameters are estimated conditionally on γ and c, and the sum of squared residuals is computed. The procedure is repeated for N combinations of these parameters. The parameter values that minimize the sum of square residuals is selected.

To facilitate the construction of an effective grid, I follow Teräsvirta's (1998) suggestion of standardizing the exponent of the transition function $G(s_t; \gamma, c)$ by dividing it by the k^{th} power of the sample standard deviation of the transition variable σ_s^k. This is done mainly to render the parameter γ scale-free. The transition function for an LSTR(1), with $k = 1$, then becomes

$$G(s_t; \gamma, c) = \left(1 + \exp\{-\gamma(s_t - c)/\sigma_s\}\right)^{-1}, \qquad \gamma > 0, \tag{2.19}$$

and that for an LSTR(2), with $k = 2$, becomes

$$G(s_t; \gamma, c) = \left(1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)/\sigma_s^2\}\right)^{-1}, \qquad \gamma > 0,\ c_1 \leq c_2. \tag{2.20}$$

[5]Conditions for obtaining a consisent and asymptotically normal estimates can be found in Wooldridge (1994) and Mira and Escribano (2000).

The last stage in the modelling procedure is the diagnostic stage where the adequacy of the fitted model is considered. The misspecification tests for the STR models that have been considered in Eitrheim & Teräsvirta (1996) and Teräsvirta (1988) will be considered in this thesis. Eitrheim & Teräsvirta (1996) have suggested three tests: A Lagrange multiplier test of no error autocorrelation (LM_{AUTO}), an LM-type test of no remaining nonlinearity ($NRNL$) in the fitted STR model, and LM-type test of parameter constancy (PC), where the null hypothesis of parameter constancy is tested against the alternative that allows smooth continuous change in the parameters of the STR model. The following is a brief discussion on these tests.

Test of No Error Autocorrelation

This test, $LM_{AUTO(q)}$,[6] can be viewed as a special case of a general LM test that was first suggested by Godfrey (1988). Teräsvirta (1998) applied this test to the STR model. Briefly, the test consists of regressing the residuals of the STR model on the lagged residuals up to order q and the partial derivatives of the log-likelihood function with respect to the parameters of the model evaluated at the maximizing value. The test has an approximate F-distribution with q and $T-n-q$ degrees of freedom under the null-hypothesis of no error autocorrelation. T is the number of observations and n is model parameters. The test statistic is $F = \frac{SSR_0 - SSR_1/q}{SSR_1/(T-n-q)}$, where SSR_0 is the sum of squared residuals of the STR model and SSR_1 is the sum of squared residuals

[6]The usual Ljung and Box (1978) test of no serial correlation is inapplicable here because the asymptotic distribution of the test statistic is unknown when the residuals from the STR model are used (Eitrheim & Teräsvirta , 1996).

of the auxiliary regression discussed above. For more details on the derivation and the asymptotic distribution of the test statistic, see Eitrheim & Teräsvirta (1996).

Test of No Remaining nonlinearity (NRNL)

A natural question to ask after fitting a STR model is whether the model adequately captures the nonlinearity originally found in the data. To this end, consider the following additive STR model

$$y_t = \Phi^{'} z_t + \Theta^{'} z_t G(\gamma_1, c_1, s_{1t}) + \psi^{'} z_t F(\gamma_2, c_2, s_{2t}) + \varepsilon_t,$$

where $\varepsilon_t \sim i.i.d.N(0, \sigma^2)$ and $F(\gamma_2, c_2, s_{2t})$ is a second transition function, and test whether $\gamma_2 = 0$. In practice, $s_{2t} \in S$ (the set of potential transition variables) and the test can be applied to each transition variable in the set. However, in this research, I will only carry on the $NRNL$ test against the transition variable selected in the specification stage. The null hypothesis of no additive nonlinearity can be defined as $\gamma_2 = 0$. Since the model is only identified under the alternative hypothesis and following Eitrheim & Teräsvirta's (1996) suggestion, the added transition function F is approximated by a third-order Taylor expansion about the null hypothesis $\gamma_2 = 0$. The approximation, after reparameterization and rearranging terms, yields the following auxiliary regression

$$y_t = \pi_0^{'} z_t + \Theta^{'} z_t G(\gamma_1, c_1, s_{1t}) + \pi_1^{'} \tilde{z}_t s_{2t} + \pi_2^{'} \tilde{z}_t s_{2t}^2 + \pi_3^{'} \tilde{z}_t s_{2t}^3 + \varepsilon_t^*,$$

where $\varepsilon_t^* = \varepsilon_t + R_3(\gamma_2, c_2, s_{2t}) \psi^{'} z_t$ with $R_3(\gamma_2, c_2, s_{2t})$ is the remainder from the polynomial approximation. The hypothesis of no additional nonlinear structure becomes

$H_{0L} : \pi_1 = \pi_2 = \pi_3 = 0$ and the associated F-distribution is given in the same way as of the test on linearity in the specification stage. For more details, see Eitrheim & Teräsvirta (1996).

Test of Parameter Constancy (PC)

Nonlinear STR models are estimated under the assumption of constant parameters. Testing the null hypothesis of parameter constancy against the alternative that allows smooth continuous change in parameters is, therefore, a relevant test. A brief description of the test follows.[7]

Consider the nonlinear STR model with changing parameters over time as

$$y_t = \Phi(t)' z_t + \Theta(t)' z_t G(\gamma, c, s_t) + \varepsilon_t, \qquad \varepsilon_t \sim i.i.d.N(0, \sigma^2),$$

where

$$\Phi(t) = \Phi + \lambda_1 H_1(\gamma_\phi, c_\phi, t^*)$$

and

$$\Theta(t) = \Theta + \lambda_2 H_2(\gamma_\theta, c_\theta, t^*),$$

where $t^* = t/T$. The functions $H_1(\gamma_\phi, c_\phi, t^*)$ and $H_2(\gamma_\theta, c_\theta, t^*)$ are logistic functions with $s_t = t^*$. Thus, $\Phi(t)$ and $\Theta(t)$ are time varying parameter vectors whose values vary smoothly from Φ to $\lambda_1 H_1$ and from Θ to $\lambda_2 H_2$ respectively and the null hypothesis of parameter constancy is $\gamma_\phi = \gamma_\theta = 0$. The derivation of the F-statistics, denoted $PC(k)$, of the test depends on the order of the polynomial in the exponent k of the

[7]See Teräsvirta (2004, pp 232) for details.

logistic functions $H_1(\gamma_\phi, c_\phi, t^*)$ and $H_2(\gamma_\theta, c_\theta, t^*)$. Assuming $\gamma_\phi = \gamma_\theta$ and following Eitrheim & Teräsvirta (1996),[8] three alternative transition functions are considered as follows

$$H(\gamma, c, t^*) = \left(1 + \exp\{-\gamma \prod_{k=1}^{3} (t^* - c_k)\}\right)^{-1} - \frac{1}{2}, \qquad \gamma > 0,$$

where the 1/2 was subtracted to facilitate the Taylor approximation. The statistic $PC(1)$, with $k = 1$, tests parameter constancy against a smooth monotonic change; $PC(2)$, with $k = 2$, tests the null hypothesis of parameter constancy against non-monotonic but symmetrical change about $t = c$; and finally $PC(3)$, with $k = 3$, tests parameter constancy against non-monotonic as well as non-symmetrical parameter change. All three alternatives are considered in this thesis.

2.4 The Threshold Variable in the STR Model

The selection of the transition variable s_t in the STR models is crucial as it explains the dynamics of the dependent variable y_t, which is the ultimate goal of fitting such a model. The majority of the researchers in the regime switching literature start their analysis by suspecting a nonlinear pattern in the DGP and conduct a sequence of linearity tests (as mentioned in Section 2.3.1) to confirm it. Once nonlinearity is proven to be present, they are faced with the decision of which transition variable they should employ. Most of the studies employ a lag order autoregressive compo-

[8] According to the authors, the restriction $\gamma_\phi = \gamma_\theta$ does not affect the asymptotic null distribution of the test statistics.

nent of the dependent variable as the transition variable. This is probably because it is difficult to find external transition variables that can explain the dynamics of the DGP, especially when the economic theory does not say much about the relation between the dependent variable under consideration and the potential transition candidates. In this thesis, in addition to the autoregressive lags of the dependent variable, other potential transition candidates that can explain the detected nonlinearities in commodity prices are considered.

Since commodity markets respond to business cycle fluctuations, it was natural to consider, among the external transition candidates, business cycle variables. In particular, current and one period lag unemployment, money supply measures (M1 and M2), current and one period lag interest rates (1, 5, and 10 years U.S. treasury bills), and oil price were considered. Linearity was not rejected for all variables except for oil price and the U.S. inflation rates and, therefore, both variables were selected as the potential transition candidates. The rationale behind using inflation rates and oil price lies in their connection with commodity prices. This connection is briefly discussed in the following two sections.

2.5 Commodity Price-Consumer Price Connection

The commodity price-consumer price connection rests on a number of linkages. The most important one describes commodity prices as assets whose prices react quickly to unanticipated shocks. For instance, a surge in aggregate demand in response to a change in money supply causes commodity prices to initially overshoot their long run equilibrium while final goods prices only respond with a lag (Blomberg and Haris, 1995). This overshooting hypothesis has been used by many authors (see Frankel (1986), Boughton and Branson (1991), and Fuhrer and Moore (1992)) to explain the commodity price-consumer price connection. Blomberg and Haris (1995) gave another reason linking commodity prices and broad inflation. The authors argue that an increase in commodity prices should be eventually passed through to final goods prices because commodities are an important input into production. A third reason, still according to the authors, stems from the first two: commodity prices are seen by investors as a useful inflation hedge because of their quick response to general inflation pressures.

The previous linkages can only explain one way causality from commodity prices to consumer prices. However, a feedback relation also exists. In a recent paper, Kyrtsou and Labys (2006), using monthly U.S. data, found evidence of *nonlinear bidirectional* (feedback) Granger causality between the growth rate (logarithmic differences) of the consumer price index and the commodity price index. The authors modelled this bidirectional relation by constructing a noisy chaotic multivariate model based on the Mackey-Glass (1977) nonlinear time delay differential equation.[9] The authors argue that the advantage of the Mackey-Glass model over simple VAR alternatives is its ability to filter more difficult dependent dynamics (cause and feedback) in a

[9]Mackey-Glass equation was originally developed by the authors to model physiological control systems. The equation was very successful in modelling feedback systems.

time series. We make use of this bidirectional Granger causality to justify the use of inflation rate as a potential transition variable that is capable of explaining the dynamics of commodity prices. We also attempt to verify the feedback effect, yet from a regime switching point of view; that is, we attempt to use the growth rate of commodity prices as the transition variable in modelling the dynamics of inflation rate using the smooth transition regression model.

2.6 Commodity Price-Oil Price Connection

The connection between the price of a commodity and the oil price rests on the term of selling (Incoterm) used in the contract effected between the buyer and the seller of the commodity.[10] As mentioned before (see Chapter 1, page 4), the frequently used terms of selling are FOB prices and CIF prices. Border prices (FOB and CIF) are different from market prices in the sense that the former do not reflect market distortions in the latter. That's why all the international organizations (e.g., the World Bank, the IMF, the OECD) use border prices in their databases and in the construction of their indexes of commodity prices.

In addition to the insurance cost, the CIF price of a commodity includes the transportation cost (ocean freight). A significant portion of the latter is due to the bunker fuel cost. Therefore, one would expect oil prices to play a significant role in

[10] See Appendix A for a brief discussion on the characteristics of the shipping industry and the sales terms used in effecting trading contracts.

explaining the behavior of commodity prices recorded on a CIF basis. This connection is overlooked when modelling the behavior of commodity prices empirically. The statistical and econometric techniques are usually applied directly to the data set under consideration without proper investigation of the sources of the time series.

2.7 The Data

The commodity price index studied in this thesis is an index of 24 primary commodity prices developed by Enzo Grilli and Maw Cheng Yang (1988) and known as the Grilli and Yang Commodity Price Index (GYCPI). Aiming, among other things, at analyzing the long-run movement in the net barter terms of trade series, the authors deflated the GYCPI by an index of manufactured goods' unit values (MUV) between 1900 and 1986. The MUV is a trade-weighted index of the five major developed countries' (France, Germany, Japan, United Kingdom, and United States) exports of manufactured commodities to developing countries. The ratio GYCPI/MUV, or the real GYCPI, measures the purchasing power of the primary commodities in terms of traded manufactures.

In this thesis, we use the extended version of the Grilli and Yang series from 1900 to 2007, developed and provided by Pfaffenzeller, Newbold, and Rayner (2007).[11] We follow the convention in the literature and use the logarithm of the real index defined as $y_t = \log\left(\frac{GYCPI}{MUV}\right)_t$. A plot of y_t is shown in Figure 2.3. The first potential

[11] I am indebted to Stephan Pfaffenzeller for supplying the recent update from 2003 to 2007.

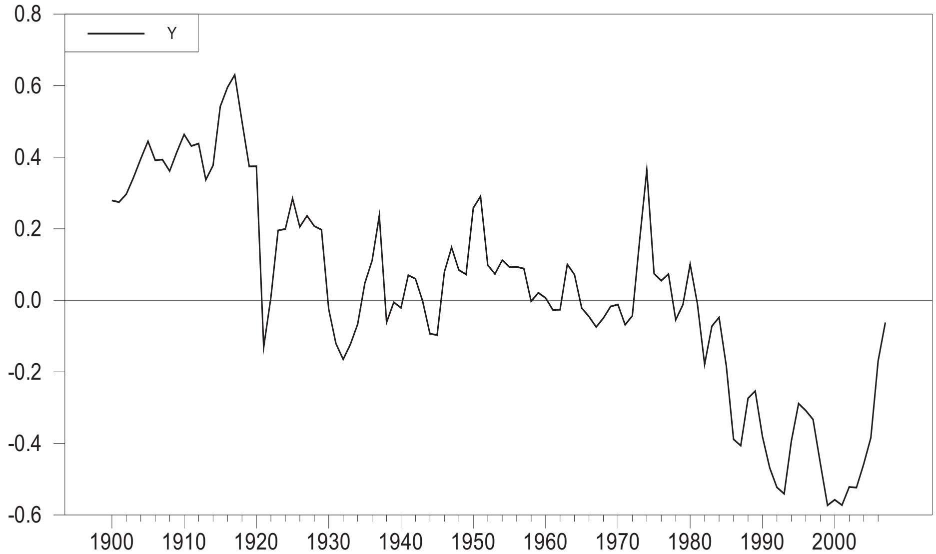


Figure 2.3: The logarithm of the real GYCPI between 1900 and 2007.

transition candidate is inflation rate defined as the first difference of the logarithm of the U.S. consumer price index for all urban consumers (CPI_t), $\Pi_t = \triangle \log(CPI_t)$. The second transition candidate is the logarithm of real crude oil price defined as $R_t = \log\left(\frac{Oil\ price}{CPI}\right)_t$. The stationarity of y_t is confirmed at the 5% level of significance by the ADF, PP, and KPSS tests (see Table 2.1).

A plot of y_t against our two transition candidates between 1900 and 2007 is shown in Figure 2.4. By looking closely at the figure, one can immediately observe the distinction between the behavior of inflation rate and oil price in three periods: (1) In *the early period*, between 1900 and 1950, the fluctuations in the inflation rate are higher in magnitude compared to those in oil prices; (2) in the mid period, 1950

Time Series	**ADF(p)**	**PP**	**KPSS$_\tau$**
$y_t = \log\left(\frac{GYCPI}{MUV}\right)_t$	$-3.99(1)$	-3.75	0.13
$\Pi_t = \triangle\log(CPI_t)$	$-5.65(1)$	-5.25	0.07
$R_t = \log\left(\frac{Oil\ price}{CPI}\right)_t$	$-3.11(0)$	-3.15	0.14
$g_t = \triangle\log\left(\frac{GYCPI}{CPI}\right)_t$	$-7.76(1)$	-8.14	0.04

Table 2.1: ADF, PP, and KPSS are respectively the augmented Dickey-Fuller, the Phillips-Perron, and the Kwiatowski, Phillips, Schmidt and Shin tests with a trend. P is the number of lags of the ADF test. The 5 percent critical values are -3.45 for ADF and PP and 0.146 for KPSS.

to 1970, there is not much to tell as both variables look steady with very small fluctuations; (3) in *the late period*, 1970 to 2007, however, the oil price time series exhibits higher swings, especially around the two well known oil shock periods (1974 and 1984), as compared to the inflation rate, which seems to have stabilized after the second oil shock of 1984. This simple analysis suggests a reverse pattern between both variables. That is, if a connection between the Grilli and Yang commodity price index and both variables is established, one, by using the inflation rate alone as the transition variable, will adequately capture the early period fluctuations in commodity prices, while, using oil price, late period variations will be well modelled. In the following subsections, Teräsvirta's (1994) three stages modelling procedure for the LSTR models will be applied to the Grilli and Yang commodity price index using inflation and oil price as transition variables respectively.

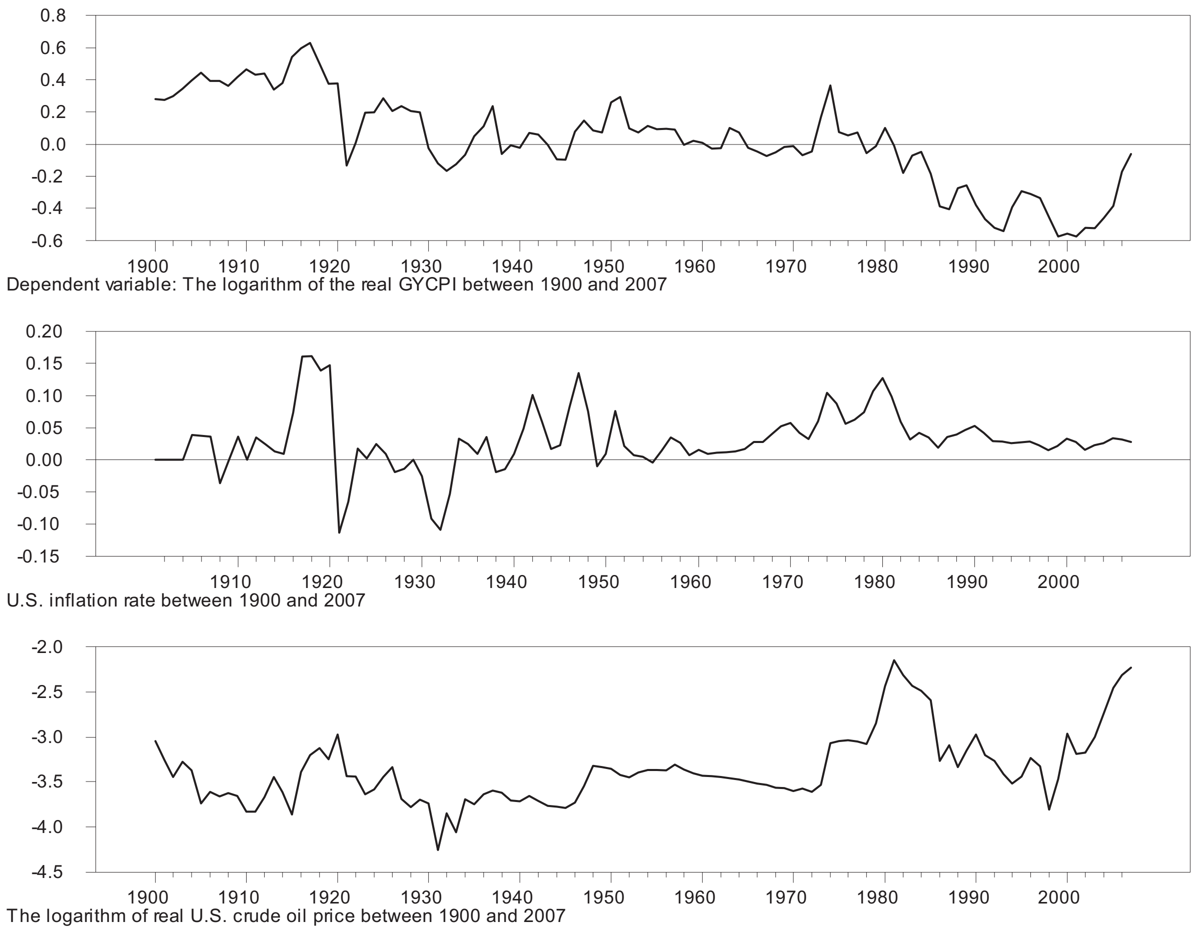


Figure 2.4: The logarithm of real GYCPI, $y_t = \log(\frac{GYCPI}{MUV})_t$, U.S. inflation rate, $\Pi_t = \triangle \log(CPI_t)$, and the logarithm of real U.S. crude oil price, $R_t = \log(\frac{Oil\ price}{CPI})_t$, between 1900 and 2007.

2.8 Regime Switching in the Commodity Price Index

In this section the nonlinearity in the Grilli and Yang commodity price index is modelled using a smooth transition regression model with inflation and oil price as potential transition variables. Following the nonlinearity modelling framework proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994), we start by specifying a linear AR model by regressing $y_t = \log\left(\frac{GYCPI}{MUV}\right)_t$ on a constant and p lags, where p is the value that minimizes the AIC directly. Both AIC and BIC were minimized at a value of $p = 1$. The estimated AR(1) model and the preliminary misspecification tests results are reported as follows.

$$
\begin{aligned}
y_t &= \underset{(0.90)}{-0.001} + \underset{(0.000)}{0.92}\, y_{t-1} + \widehat{a}_t, \qquad (2.21)\\
Q(1) &= 0.91(0.34), \qquad Q(8) = 4.80(0.78),\\
LM_{ARCH(1)} &= 0.13(0.72), \qquad LM_{ARCH(4)} = 1.47(0.83),\\
JB &= 4.56(0.10), \qquad K_3 = -0.25, \quad K_4 = 3.5,
\end{aligned}
$$

where the figures in parentheses beneath the estimated parameters and next to the test statistics are p-values. Judging by the Ljung-Box (1978) statistics $Q(q)$, the null hypothesis of no serial correlation of order $q = 1$ up to $q = 8$ in the residuals series was not rejected at the 5% level of significance. Also, applying Engle's (1982) Lagrange multiplier test of no autoregressive conditional heteroscedasticity of order v

($LM_{ARCH(v)}$), the null-hypotheses of no ARCH(1) up to ARCH(4)[12] were not rejected at the 5% level of significance. Finally, the null-hypothesis of normality of errors was not rejected at the 5% level of significance as seen from the p-value of the JB test statistic. The skewness (K_3) and kurtosis (K_4) results are also reported in (2.21). Notice that the constant drift coefficient is insignificant; other than that, the model passes all preliminary diagnostic tests and can act as a good start for our analysis.[13] The next step is testing for nonlinearity.

2.8.1 Switching Variable: Inflation

The set of the predetermined potential transition variables, Ω_t, in this case consists of the autoregressive variable y_{t-1} and its first difference, $\triangle y_{t-1}$, and the current and the one period lag inflation (Π_t and Π_{t-1} respectively) as the potential transition candidates, i.e., $\Omega_t = \{y_{t-1}, \triangle y_{t-1}, \Pi_t, \Pi_{t-1}\}$. The previously mentioned sequence of nonlinearity tests (see Section 2.3.1) was executed for each of the potential transition variables in Ω_t and the results are reported in Table 2.2. At a first glance, the current inflation rate, Π_t, tagged with the symbol **, seems to be the variable with the strongest test rejection (the smallest p-value; see the second column of Table 2.2) and, therefore, it should be selected as the appropriate transition variable for the STR

[12] Testing the presence of ARCH up to order $v = 4$ is adequate here since we have annual data.

[13] When this preliminary analysis was applied to the 24 commodities forming the Grilli and Yang index, the estimated residuals of the fitted AR models of some price processes exhibited ARCH pattern. These linear models, therefore, could not act as adequate starting points for an LSTR setting; different models (such as ARCH or smooth threshold ARCH models) were, consequently, entertained for such group of commodities. Estimation and misspecification tests results of these models are discussed in detail in Chapter 3.

s_t	F_L	F_4	F_3	F_2	Suggested Model
y_{t-1}	7.31×10^{-1}	4.50×10^{-1}	6.48×10^{-1}	4.74×10^{-1}	$Linear$
$\triangle y_{t-1}$	2.39×10^{-1}	8.29×10^{-1}	6.45×10^{-2}	3.40×10^{-1}	$Linear$
Π_t^{**}	1.06×10^{-4}	2.08×10^{-2}	3.02×10^{-4}	1.57×10^{-1}	$LSTR2$
Π_{t-1}^{*}	5.08×10^{-3}	6.17×10^{-1}	8.56×10^{-1}	1.53×10^{-4}	$LSTR1$

Table 2.2: P-values of the linearity F-tests sequence applied to the logarithm of the real GYCPI when inflation is the transition variable.

model. The associated model is clearly the LSTR(2) model as the p-value of the F_3 test is less than the p-value of the F_2 and F_4. However, using $s_t = \Pi_t$, and proceeding to the estimation stage, the diagnostics of the estimated LSTR(2) model showed, besides some residuals autocorrelations of order 2 and 3, severe parameter change, i.e., the parameter constancy hypothesis was rejected, which is a clear indicator of misspecification. This is, most probably, due to the possible endogenity resulting from the correlation between the period t commodity price index and the period t consumer price index, which might renders the estimates unreliable. Therefore, a better choice is to use the one period lag inflation instead of current inflation as transition variable. Actually, applying our testing procedure confirms that; the second best transition candidate is the one period lag inflation Π_{t-1}, tagged with the symbol * in Table 2.2, and the suggested model is the LSTR(1). The p-values of the F-tests for linearity and model selection sequence previously described are reported in the same table. Based on the previous analysis, the adequate nonlinear LSTR(1) model is then

$$y_t = \phi_0 + \phi_1 y_{t-1} + \{\theta_0 + \theta_1 y_{t-1}\}(1 + \exp\{-\gamma(\Pi_{t-1} - c)\}^{-1} + \varepsilon_t. \tag{2.22}$$

Estimation of the LSTR(1) model in (2.22) is performed using conditional maximum likelihood method, where the log-likelihood function in (2.18) is maximized using the iterative BFGS algorithm. Starting-values for γ and c needed for the algorithm were $\widehat{\gamma} = 0.5$ and $\widehat{c} = 0.01$.

To render the parameter γ scale-free, the exponent of the transition function was divided by the sample standard deviation of the transition variable (the one period lag inflation), σ_Π, as

$$G(s_t; \gamma, c) = \frac{1}{1 + \exp\{-\gamma(s_t - c)/\sigma_\Pi\}}, \qquad \gamma > 0. \tag{2.23}$$

The best fitted LSTR(1) model that we managed to obtain after dropping the insignificant drift parameter in the nonlinear part of y_t and the results of Eitrheim & Teräsvirta (1996) and Teräsvirta's (1988) misspecification tests are reported as follows.

$$y_t = \underset{(0.01)}{0.009} + \underset{(0.04)}{0.98} y_{t-1} - \{\underset{(0.10)}{0.44} y_{t-1}\}(1 + \exp(-\underset{(1278)}{213}(\Pi_{t-1} - \underset{(0.003)}{0.07})/0.046))^{-1} + \widehat{\varepsilon}_t, \tag{2.24}$$

$$\overline{R}^2 = 0.88, \qquad \widehat{\sigma}_\Pi = 0.046, \qquad \widehat{\sigma} = 0.10,$$

$$LM_{ARCH(1)} = 0.26, \qquad LM_{ARCH(4)} = 0.87,$$

$$\begin{aligned} LM_{AUTO(1)} &= 0.29, \quad LM_{AUTO(4)} = 0.40, \quad LM_{AUTO(8)} = 0.82, \quad NRNL = 0.21, \\ PC(1) &= 0.19, \quad PC(2) = 0.43, \quad PC(3) = 0.15, \quad JB = 0.001, \\ K_3 &= -0.5, \quad K_4 = 4.13, \quad RMSE_{Linear} = 0.235, \quad RMSE_{STR} = 0.063, \end{aligned}$$

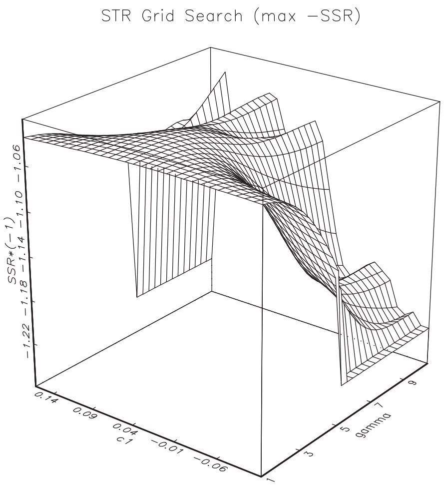


Figure 2.5: Graphical presentation of the constructed grid for the LSTR1 model with one period lag inflation rate as transition variable.

where $\widehat{\sigma}_{\Pi}$ is the sample standard deviation of the transition variable $s_t = \Pi_{t-1}$ and $\widehat{\sigma}$ is the residual standard deviation. The figures in parentheses beneath the parameter values are standard deviations. $LM_{AUTO(\nu)}$ is the p-value for the v^{th} order autocorrelation. $LM_{ARCH(q)}$ is the p-value of the q^{th} order $ARCH$. $PC1, PC2$, and $PC3$ are p-values for parameter constancy tests against monotonic change, non-monotonic symmetrical change, and non-monotonic and non-symmetrical change respectively. $NRNL$ is the p-value for the no remaining nonlinearity test. JB is the p-value of the Jarque-Bera test of normality. K_3 is skewness and K_4 is kurtosis. The original and fitted series of y_t are plotted in Figure 2.6. The root mean square errors (RMSE) of the last seven yearly one-step-ahead forecasts (from year 2001 to 2007) for the linear and the STR model are 0.235 and 0.063 respectively.[14] The RMSE of the STR model is lower than the RMSE of the linear autoregressive model, indicating that the forecasting performance of the STR model is superior to that of the linear model.

Perhaps the most noticeable detail of (2.24) is the large standard deviation of the estimated slope of the logistic function, $\widehat{\gamma} = 213$. It is common, for LSTR models, that the estimated standard deviation of γ tends to be large for large values of γ. This is not crucial, however, as it will not affect either the shape of the logistic function $G(\cdot)$ or the other estimates of the model. Teräsvirta (1994) gave an example of this exact case and provided a discussion on the estimation issues of γ. The message delivered by this enormous slope of the transition function is that y_t will be moving sharply from one regime to the other. This is not surprising since we are modelling the dynamic behavior of a commodity price index, which is flexible by nature. This type of behavior can be observed from the transition function $G(\Pi_{t-1}, \gamma, c)$ plotted in Figure 2.7. One might argue that there is no value added from fitting the smooth transition regression model and that the dynamic behavior of y_t might be adequately captured by the pure threshold model. We verified this argument by fitting a threshold autoregressive model (TAR) to the Grilli and Yang commodity price index using inflation as the switching variable. It turned out, as we shall see in the following subsection, that the commodity price index under the fitted TAR model exhibited the same dynamic behavior as the one obtained from the LSTR model in (2.24). This confirms the ability

[14]The forecasts were made without re-estimating the models during the prediction period (from 2001 to 2007).

of the STR-type models to encompass pure threshold behavior when the slope of the transition function is large enough to produce abrupt transitions between regimes.

Other than the large slope of the transition function, the estimated coefficients are all significant and the model passes the misspecifications tests mentioned before. One exception is the rejection of the null-hypothesis of normality of errors at the 5% level of significance as seen from the p-value of the JB test statistic. But, this is due to the existence of outliers in the time series. A quick look at the standardized residuals time series plotted in Figure 2.8, one can notice the very large outliers in 1921 and 1937 where the absolute value of the standardized residuals is greater than three. These outliers are corresponding to the post WWI and the post great depression periods respectively. This is consistent with the findings of Cuddington and Urzua (1989) and Powell (1991) who both fitted a DS model to y_t accounting for structural breaks in 1921 and 1937 respectively. Also, judging by the negative skewness, $K_3 = -0.5$, the distribution of the standardized residuals is skewed to the right, which means that negative outliers are more likely than positive outliers as seen from the figure.

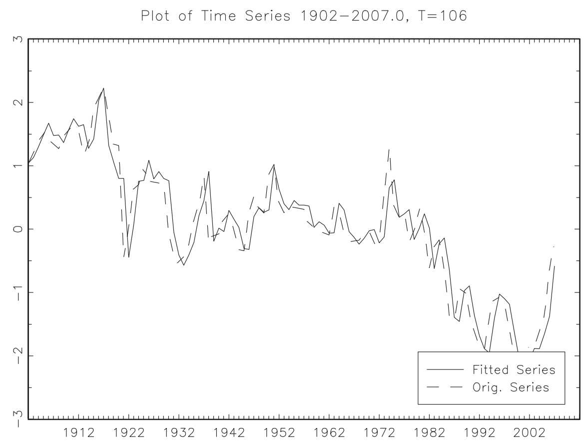


Figure 2.6: Original and fitted values of the real GYCPI between 1900 and 2007 with the one period lag inflation rate as transition variable

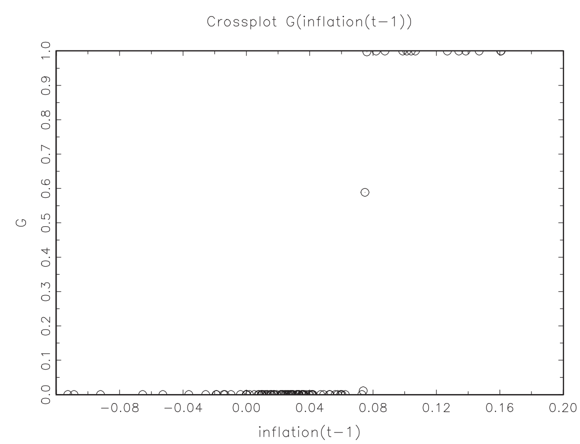


Figure 2.7: Transition function $G(\Pi_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to one observation. The transition variable is the one period lag inflation.

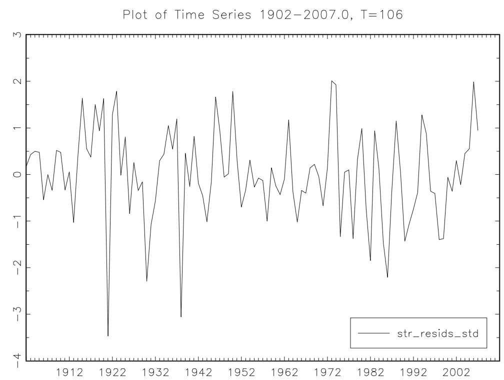


Figure 2.8: Standardized residuals of the fitted LSTR1 model between 1900 and 2007 with one period lag inflation rate as transition variable.

Dynamic Analysis

As mentioned in Section 2.3, the LSTR(1) model suggests two regimes; the upper regime, associated with $G(\cdot) = 1$ and the lower regime, associated with $G(\cdot) = 0$. The parameter c in (2.22) can be interpreted as the threshold or the border between the two regimes, in the sense that the logistic function changes monotonically from 0 to 1 as $s_t = \Pi_{t-1}$ increases. The parameter γ, the slope of the logistic transition function, determines the smoothness of the change in the value of the logistic function and, hence, the smoothness of the transition from one regime to the other. Table 2.3 summarizes the dynamics of the two regimes suggested by the LSTR(1) model with the one period lag inflation as the transition variable. The threshold value $\widehat{c} = 0.07$ indicates that if the one period lag inflation rate increases beyond 7%, the logarithm of the real Grilli and Yang commodity price index will move to the upper stationary regime with unconditional mean of 0.02 and unconditional variance of 0.014. On the other hand, if there is a slow down in economic activities so that inflation decreases below the 7% threshold, the real commodity index will switch to the lower regime with 0.5 unconditional mean and a larger unconditional variance equals to 0.25. The higher variance of the lower regime (17 times larger as compared to the upper regime) indicates that the logarithm of the real commodity price index likes to wander in the recessionary regimes but always revert back to the mean because the time series is stationary. This dynamic behavior can be detected by examining the transition function and the behavior of the transition variable above and below the estimated threshold value, i.e., when $s_t = \Pi_{t-1} = 7\%$, as shown in Figure 2.9. The behavior of the transition function captures the dynamics of the transition variable Π_{t-1} during the periods posts the WWI and WWII. It also exhibits the high swings in inflation during the two well known oil shocks in 1974 and 1984. No fluctuations are present during the great depression of the 1930's, however. This implies that the inflation rate is capable of capturing the fluctuations in commodity prices mainly during the early period (between 1900 and 1950); observe that the Grilli and Yang commodity price index, starting from the early fifties till 2007, is following one contractionary regime (with the exception of the two oil shocks). Therefore, inflation by itself fails to account for the observed late period fluctuations in the Grilli and Yang commodity price index when nonlinearity is modelled in the conditional mean. A simulation of both regimes is plotted in Figure 2.10.

s_t= Π_{t-1}	Upper regime: $G(\cdot) = 1$	Lower regime: $G(\cdot) = 0$
Threshold: $\widehat{c}$	0.07	0.07
Model	y_t= $0.009 + 0.54y_{t-1}$+$\widehat{\varepsilon}_t$	y_t= $0.009 + 0.98y_{t-1}$+$\widehat{\varepsilon}_t$
Behavior of y_t	$Stationary\ AR(1)$	$Near\ Random\ Walk$
Mean	$E(y_t) = 0.02$	$E(y_t) = 0.45$
Variance	$var(y_t) = 0.014$	$var(y_t) = 0.25$

Table 2.3: The upper and lower regimes of the LSTR(1) model of the real GYCPI with inflation as the transition variable.

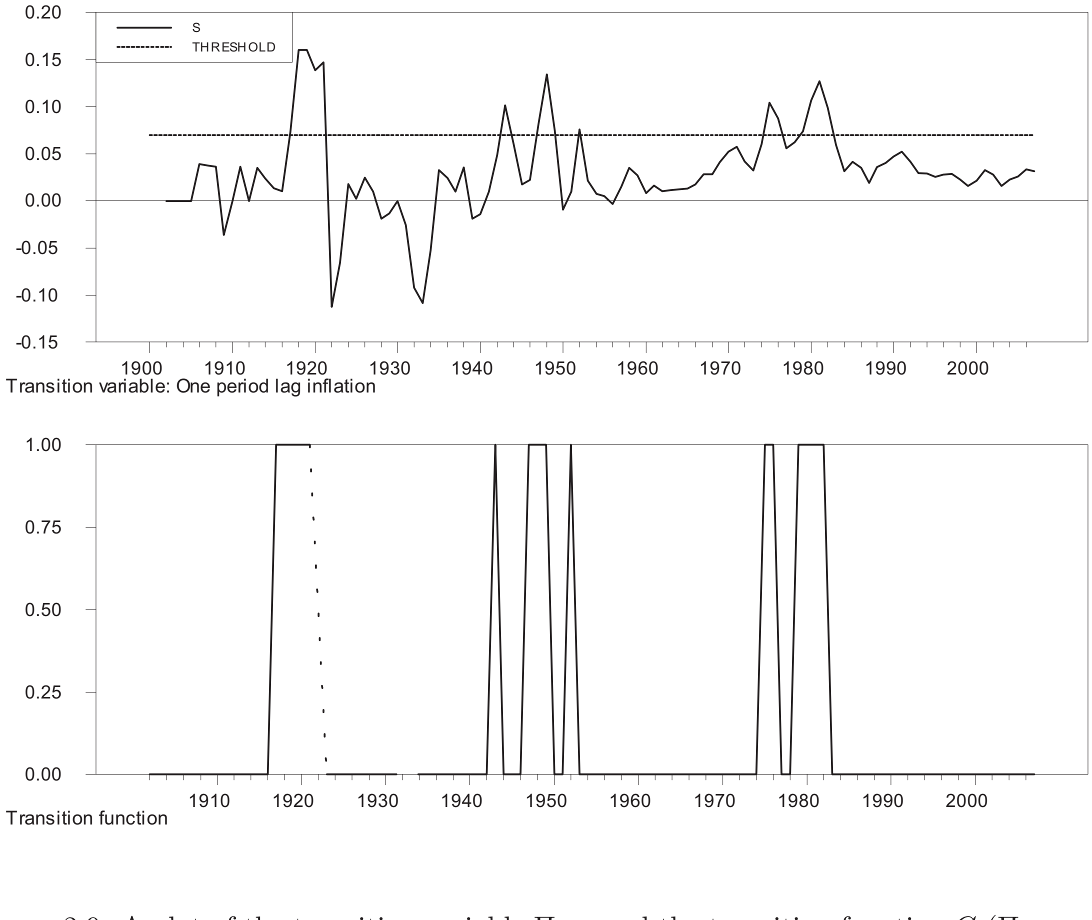
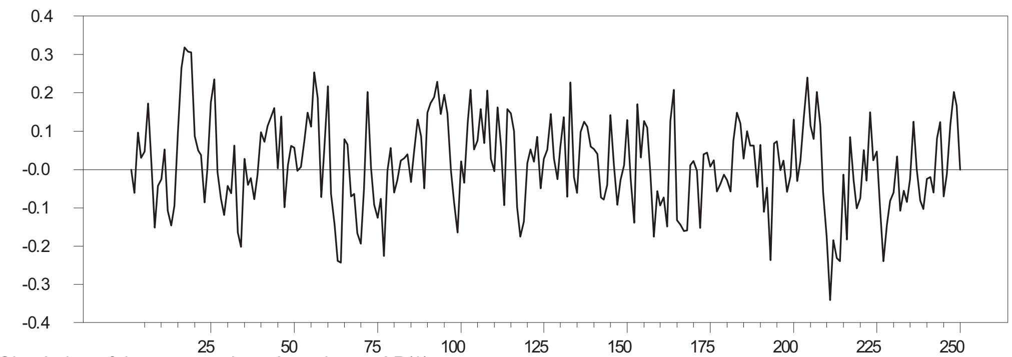


Figure 2.9: A plot of the transition variable Π_{t-1} and the transition function $G_t(\Pi_{t-1}; \gamma, c)$ between 1900 and 2007.



Simulation of the upper regime: A stationary AR(1).

Simulation of the lower regime: A near random walk.

Figure 2.10: Simulation of the upper and lower regimes of the real GYCPI with one period lag inflation rate as transition variable.

2.8.2 TAR Model with Inflation as Switching Variable

Consider the following two-regime TAR(1) model with the one period lag inflation as the threshold variable, that is $s_t = \Pi_{t-1}$, and the threshold is c as

$$y_t = \begin{cases} \phi_0^1 + \phi_1^1 y_{t-1} + a_t & \text{if } \Pi_{t-1} < c, \\ \phi_0^2 + \phi_1^2 y_{t-1} + a_t & \text{if } \Pi_{t-1} \geq c, \end{cases} \tag{2.25}$$

where $a_t \sim i.i.d.(0, \sigma^2)$. To capture nonlinearities, the TAR model allows the parameters $(\phi_0^1, \phi_1^1, \phi_0^2, \phi_1^2)$ to change from one regime to the other according to the value of the threshold variable.[15]

The TAR model was suggested by Tong (1978) and applied by Tong and Lim (1980) on real data. Tsay (1989) proposed a four-step model-building procedure for TAR models in the same spirit as the one proposed by Terasvirta (1994) for modeling STR models. Although Tsay's approach is simple and does not require intensive computations, it involves some human decision when selecting the threshold value. Later, Hansen (1997) introduced a test for threshold nonlinearity and a procedure for estimating TAR models. The appealing feature of Hansen's approach is that the threshold value can be estimated with the other parameters of the model and, therefore, no subjective decision has to be made regarding the value of the threshold parameter. We shall introduce both approaches in the following subsections.

[15] Note that if the threshold variable in one of the autoregressive lags of the dependent variable, the TAR model is referred to as self-exciting TAR or SETAR model.

Tsay's Approach

The first step in Tsay's approach, which involves specifying a linear AR(p) model for y_t using any information criterion, is the standard preliminary step in any nonlinear model-building framework. Following our previous analysis, the lag length of the logarithm of the real Grilli and Yang commodity price index was minimized at $p = 1$ as in (2.21).

The second step is testing for threshold nonlinearity. Tsay (1989) proposed a nonlinearity test that is based on *the standardized predictive residuals* and the idea of *arranged autoregression* that was first introduced by Ertel and Fowlkes (1976). The null hypothesis is that the time series y_t is linear; the alternative is that it follows a two-regime TAR(1) model. The problem, as usual, is that the threshold c is only identified under the alternative. To overcome this problem, Tsay suggested to arrange the equations in (2.25) for $t = \max(p) + 1, ..., n$, where n is the number of observations, according to the threshold variable Π_{t-1}. If there are, say, $k < n$ values in Π_{t-1} that are smaller than the threshold c, then the first k equations in the arranged model will correspond to regime one and the following $n - k$ equations will correspond to the second regime. Next, estimate the arranged autoregressions using recursive least squares and compute the standardized predictive residuals, $\hat{e}_t$. If the predictive residuals are white noise and orthogonal to the regressors, nonlinearity will be rejected. Therefore, Tsay suggested to run an auxiliary regression where the predictive residuals are regressed on the model's regressors as

$$\hat{e}_t = \omega_0 + \omega_1 y_{t-1} + error_t,$$

then nonlinearity can be detected using a conventional F-test.

The F-statistic obtained from applying Tsay's threshold nonlinearity test to the Grilli and Yang commodity price index using the one period lag inflation as threshold variable is 5.51 and the p-value is 0.005. Therefore, the null hypothesis of no threshold nonlinearity is rejected at the 5% level of significance.

Once nonlinearity is detected in the time series, the next step in Tsay's approach is to locate the threshold value using scatter plot of the standardized recursive residuals versus the ordered threshold variable Π_{t-1}. This can be detected from Figure 2.11. The plot exhibits a structural break occurring somewhere between $\Pi_{t-1} = 0.03$ and $\Pi_{t-1} = 0.07$, which is consistent with the estimated threshold value of 0.07 obtained from the LSTR(1) model with inflation as transition variable. Although we can observe the structural break, it is hard to confirm its exact value and a human decision has to be made. To avoid such subjective selection, we apply Hansen's (1997) test for threshold-type nonlinearity in the following subsection.

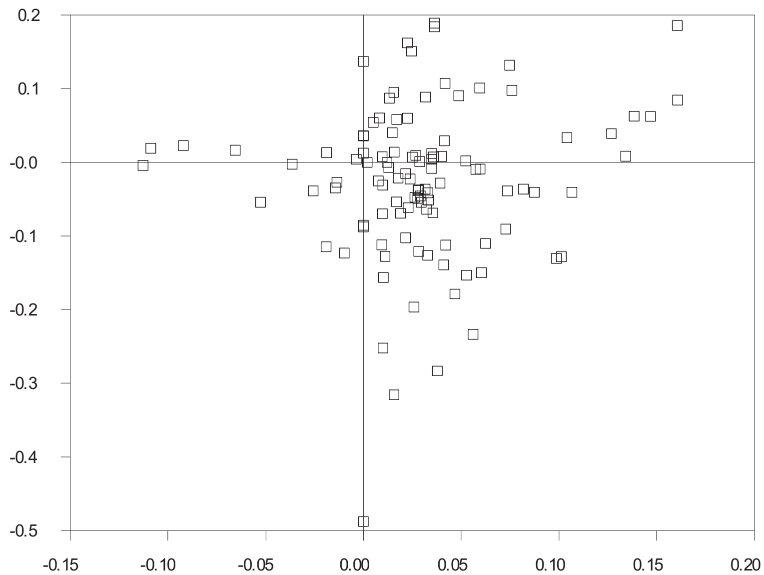


Figure 2.11: Scatter plot of the standardized recursive residuals (y-axis) versus the threshold variable, Π_{t-1}, (x-axis).

Hansen's Approach

Consider again the two-regime TAR(1) model in (2.25). If the threshold c is known, then the model parameters $(\phi_0^1, \phi_1^1, \phi_0^2, \phi_1^2, \sigma^2)$ can be estimated by least squares. Under the assumption that the error terms in (2.25) are normally distributed, we can test the null-hypothesis that the model is a one regime TAR(1) model against the alternative that it is a two-regime TAR(1) model using a likelihood ratio test as

$$F(c) = \frac{SSR_R - SSR_{UN}}{\hat{\sigma}^2_{UN}(c)} = n^* \frac{\hat{\sigma}^2_R - \hat{\sigma}^2_{UN}(c)}{\hat{\sigma}^2_{UN}(c)}, \tag{2.26}$$

where SSR_R is the sum of squares residuals of the restricted model (one-regime TAR(1)), SSR_{UN} is the sum of squares residuals of the unrestricted model (two-regimes TAR(1)) given the threshold c, $\hat{\sigma}_R^2$ and $\hat{\sigma}_{UN}^2(c)$ are the estimated residual variances of the one-regime TAR(1) and the two-regimes TAR(1) models respectively, and n^* is the effective sample size after adjusting for the starting values and the lag length of the transition variable. Since the threshold c is unknown, Hansen (1997) suggests to compute the supremum likelihood ratio by searching over all the possible values of the threshold variable (the one period lag inflation in our case) as

$$F_{\text{sup}} = \text{sup}(c). \tag{2.27}$$

Since the threshold value c is not identified under the alternative hypothesis, the asymptotic distribution of F_{sup} in (2.27) will differ from the chi-square distribution of the F-statistic in (2.26). Hansen (1996) showed that the distribution of F_{sup} is non-standard and can be asymptotically approximated by a bootstrap procedure. Using 1000 bootstrap replications, the LR statistic of Hansen's threshold nonlinearity test applied to the Grilli and Yang commodity price index using the one period lag inflation as threshold variable is 9.47 achieved at a threshold value $\hat{c} = 0.03(3\%)$. The bootstrap p-value is 0.004. Therefore, at the 5% significance level, we reject the null hypothesis of linearity. A plot of the F statistics against the threshold variable, Π_{t-1}, is found in Figure 2.12.

The estimation results from fitting the two-regime TAR model in (2.25) to y_t with Π_{t-1} as threshold variable are reported as

$$y_t = \begin{cases} 0.001 + 0.59y_{t-1} + \hat{a}_t & \text{if } \Pi_{t-1} \geq 3\%; \qquad \hat{\sigma}_1 = 0.04, \\ 0.004 + 0.98y_{t-1} + \hat{a}_t & \text{if } \Pi_{t-1} < 3\%; \qquad \hat{\sigma}_2 = 0.05, \end{cases} \tag{2.28}$$

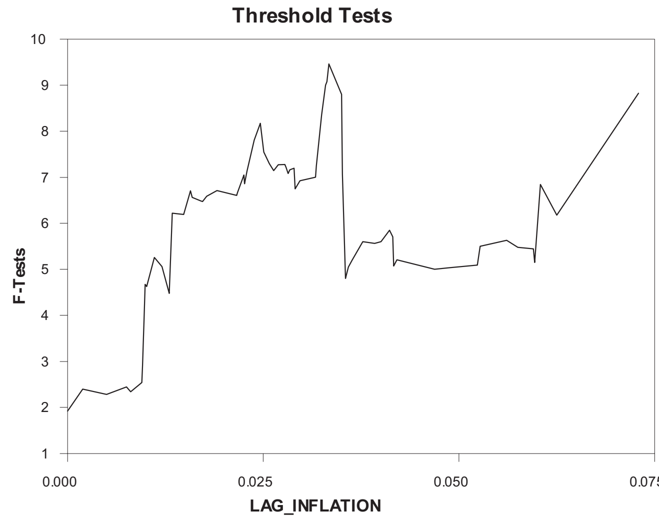


Figure 2.12: Plot of Hansen's (1996) F-statistics against the threshold variable Π_{t-1}.

where $\hat{\sigma}_1$ and $\hat{\sigma}_2$ are the standard errors of residuals from regime 1 and 2 respectively. The fitted values of y_t from the two-regime TAR(1) model exhibit the exact same behavior of the fitted values from the LSTR(1) model obtained in the previous section; that is, the upper regime behaves in a near random walk fashion with an autoregressive coefficient of 0.98, while the lower regime is stationary. The only difference, however, is the estimated threshold value. The threshold value $\hat{c} = 3\%$ obtained from Hansen's TAR test can be seen as the lower bound of Tsay's suggested range (see the scatter plot in Figure 2.11), while the threshold value $\hat{c} = 7\%$ obtained from the fitted LSTR(1) model in equation (2.24) is its upper bound. This result confirms the ability of STR models to capture the TAR models' abrupt switching transition between regimes.

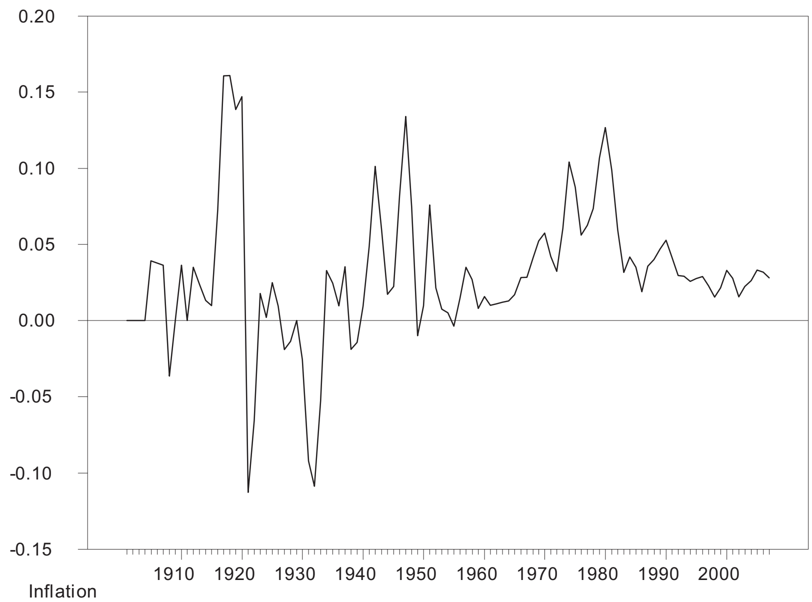


Figure 2.13: U.S. inflation rate between 1900 and 2007.

2.8.3 Feedback: From Commodity Price to Inflation

We have just seen how the growth rate of the CPI (inflation) was capable of modelling the *early* nonlinear dynamics in the logarithm of real Grilli and Yang commodity price index. In this section, we explore the feedback scenario. The question addressed here is: What happens when inflation is the dependent variable and the growth rate of the real Grilli and Yang commodity price index, denoted as $g_t = \triangle \log \left(\frac{GYCPI}{CPI}\right)_t$, is the transition variable? To answer this question, we apply the previously mentioned three-step modelling procedure to the U.S. inflation rate (dependent variable). Let the dependent variable be the inflation rate defined as $\Pi_t = \triangle \log(CPI_t)$. The AIC was minimized at a lag length $p = 5$, however, when fitting the preliminary AR(5)

s_t	F_L	F_4	F_3	F_2	Suggested Model
Π_{t-1}	8.27×10^{-1}	8.40×10^{-1}	7.73×10^{-1}	3.64×10^{-1}	*Linear*
Π_{t-2}	8.07×10^{-2}	3.07×10^{-2}	2.50×10^{-1}	2.43×10^{-1}	$LSTR1$
g^*_{t-1}	3.22×10^{-2}	2.02×10^{-1}	2.48×10^{-1}	2.14×10^{-2}	$LSTR1$
g_{t-2}	4.32×10^{-1}	7.58×10^{-1}	3.39×10^{-1}	2.02×10^{-1}	*Linear*

Table 2.4: P-values of the linearity F-tests sequence applied to the logarithm of the real CPI when the growth rate of the GYCPI is the transition variable.

model, the last three lags were insignificant and, therefore, were dropped. The fitted linear AR(2) model acting as the starting point for the LSTR analysis is given as follows.[16]

$$\Pi_t = \underset{(0.001)}{0.014} + \underset{(0.000)}{0.77}\,\Pi_{t-1} - \underset{(0.002)}{0.23}\,\Pi_{t-2} + \widehat{a}_t. \tag{2.29}$$

Next, we test for nonlinearity. Following the AR(2) model in (2.29), the set of predetermined transition variables, Σ_t, consists of the two autoregressive lags of the dependent variable and the first and second lag of the growth rate of the real Grilli and Yang commodity price index (g_{t-1} and g_{t-2} respectively); that is, $\Sigma_t = \{\Pi_{t-1}, \Pi_{t-2}, g_{t-1}, g_{t-2}\}$.[17] The nonlinearity test in Section 2.3 was executed for each variable in Σ_t. The p-values of the F-tests for linearity and model selection are reported in Table 2.4. The one period lag growth rate of the real Grilli and Yang commodity price index, g_{t-1}, tagged with the symbol *, was the variable with the strongest test rejection (the smallest p-value); see the second column of Table 2.4.

[16] $Q(1) = 0.08(0.77); Q(8) = 6.41(0.60); LM_{ARCH(1)} = 1.29(0.26); LM_{ARCH(4)} = 3.60(0.46)$, where the figures in parentheses beneath the estimated parameters and next to the test statistics are p-values and all the test statistics are defined as before. All the coefficients are significant at the 5% level and the model passes all preliminary diagnostic tests.

[17] The period t growth rate of the real Grilli and Yang commodity price index, g_t, was not employed here to avoid any endogenity that might result from the correlation with the dependent variable.

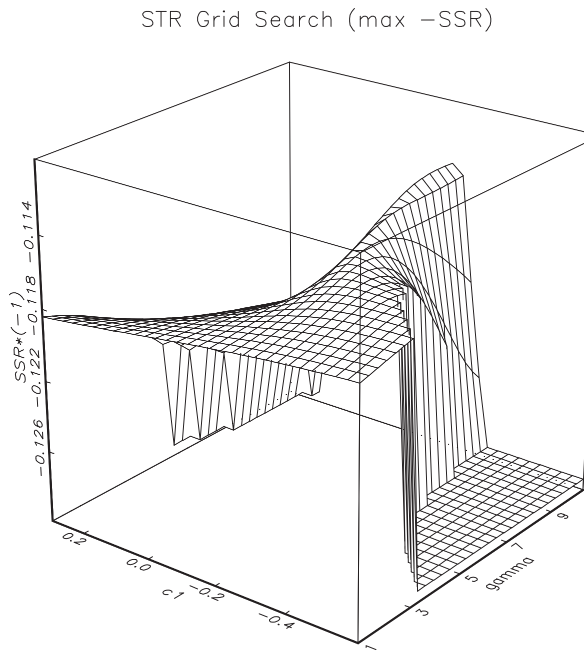


Figure 2.14: Graphical presentation of the constructed grid for the LSTR1 model with inflation as the dependent variable. The transition variable is the one period lag growth rate of the real GYCPI.

Therefore it was selected as the appropriate transition variable for the LSTR model. The associated model is the LSTR(1) model, which can be expressed as

$$\Pi_t = \phi_0 + \phi_1 \Pi_{t-1} + \phi_2 \Pi_{t-2} + \{\theta_0 + \theta_1 \Pi_{t-1} + \theta_2 \Pi_{t-2}\}(1 + \exp\{-\gamma(g_{t-1} - c)\})^{-1} + \varepsilon_t. \tag{2.30}$$

The starting values of γ and c necessary for estimating (2.30) were $\widehat{\gamma} = 8.1$ and $\widehat{c} = -0.16$. The best fitted LSTR(1) model that we managed to obtain after dropping the insignificant parameters in the linear and nonlinear parts of y_t and the misspecification tests results are reported as follows.

$$\Pi_t = -\underset{(0.01)}{0.01} + \underset{(0.09)}{0.81}\Pi_{t-1} + \{\underset{(0.01)}{0.03} - \underset{(0.11)}{0.37}\Pi_{t-2}\}(1+\exp(-\underset{(17.6)}{10.6}(g_{t-1} + \underset{(0.04)}{0.14})/0.13))^{-1} + \widehat{\varepsilon}_t,$$

$$\overline{R}^2 = 0.48, \qquad \widehat{\sigma}_g = 0.13, \qquad \widehat{\sigma} = 0.03,$$

$$AUTO(1) = 0.57, \qquad AUTO(8) = 0.08, \qquad LM_{ARCH(1)} = 0.50, \qquad LM_{ARCH(4)} = 0.55,$$

$$PC(1) = 0.26, \qquad PC(2) = 0.34, \qquad PC(3) = 0.55,$$

$$NRNL = 0.09, \qquad JB = 0.000, \qquad K_3 = -1.7, \qquad K_4 = 13.18, \tag{2.31}$$

where $\widehat{\sigma}_g$ is the sample standard deviation of the transition variable $s_t = g_{t-1}$, $\widehat{\sigma}$ is the residual standard deviation and the figures in parentheses beneath the parameter values are standard deviations of the estimates and the figures after the test statistics are p-values. Observe the moderate slope of the estimated logistic function, $\widehat{\gamma} = 10.6$. Hence, smooth transition matters in this case. This is due to the nature of the dependent variable (inflation rate) and its tendency to move smoothly from one regime to the other driven by the high fluctuations in the growth rate of the real Grilli & Yang commodity price index (transition variable). This smooth transition can be noticed from the plot of the transition function in Figure 2.15. The estimated coefficients are all significant and the model passes the misspecifications tests mentioned above except for the null-hypothesis of normality of errors, which was rejected at the 5% level of significance as seen from the p-value of the JB test statistic. Again, this rejection of the normality assumption is due to the presence of outliers corresponding

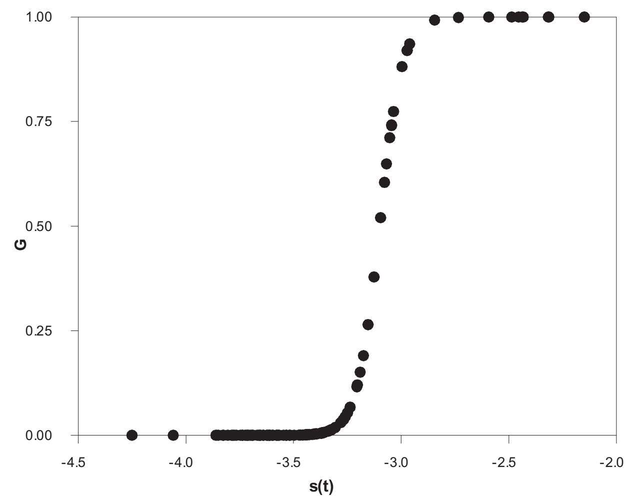


Figure 2.15: Transition function $G(s_t, \gamma, c)$ as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag of the real growth rate of the GYCPI.

to the post WWI as seen from the plot of the standardized residuals in Figure 2.16. The original and fitted values of y_t are plotted in Figure 2.17.

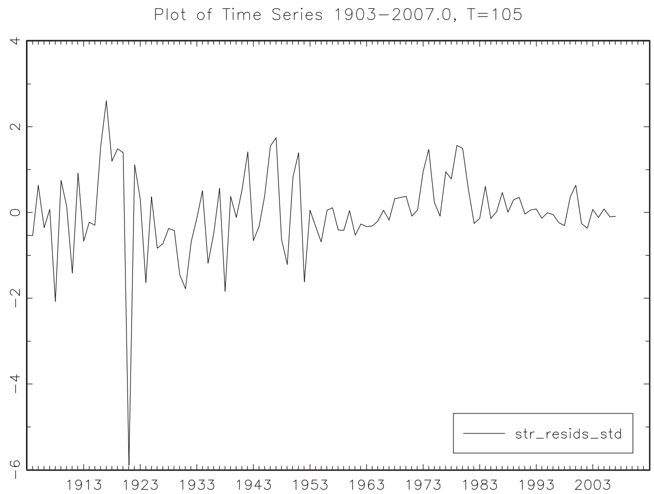


Figure 2.16: Standardized residuals of the fitted LSTR1 model with inflation as the dependent variable between 1900 and 2007. The transition variable is the one period lag growth rate of the real GYCPI.

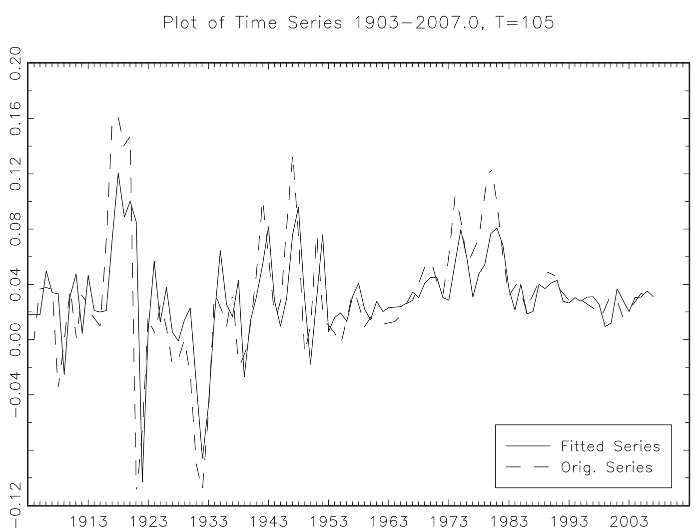


Figure 2.17: Original and fitted values of inflation rate between 1900 and 2007.

Dynamic Analysis

Table 2.5 summarizes the dynamics of the two regimes suggested by the LSTR(1) model with the one period lag growth rate of the real Grilli and Yang commodity price index as transition variable. The inflation rate follows a stationary autoregressive regime when the growth rate of the real Grilli and Yang commodity index moves above and below the threshold value $\widehat{c} = -14\%$. In a recessionary regime, i.e., if the growth rate of the real Grilli and Yang commodity price index is below -14%, inflation rate will follow a stationary AR(1) process with an unconditional mean of -0.05 and variance of 0.003. Inflation does not, however, stay too long in the lower regime; the root of the lower regime has a period of 10 months (0.81 year) (see the third column of Table 2.5). It tends to move back to the upper regime, where it stays much longer (7.5 years), when the growth rate of the price index moves above the threshold value of -14%. In this expansionary regime, the inflation rate follows a stationary AR(2) process characterized by a complex pair of roots with a modulus of 0.61 and a period of 7.5 years. This implies that most of the time the inflation rate is following the stationary AR(2) regime; it moves to the lower regime but quickly returns to the upper regime. This dynamics can be traced from the plotted transition function in Figure 2.18. The above analysis confirms the feedback effect from commodity prices to inflation rate from a regime switching point of view. Next, we examine the second potential transition candidate in our nonlinearity analysis of commodity prices: The logarithm of real crude oil price.

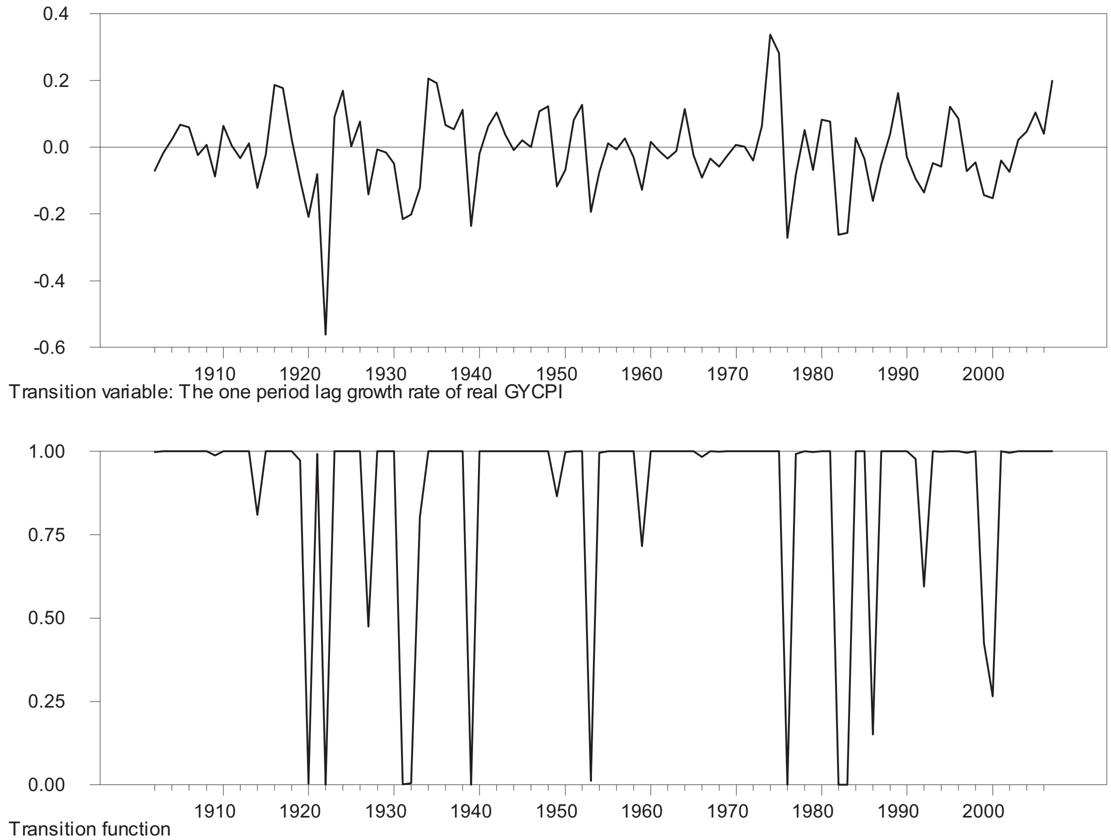


Figure 2.18: A plot of the transition variable g_{t-1} and the transition function $G(g_{t-1};\gamma,c)$ between 1900 and 2007.

$s_t = g_{t-1}$	Upper regime: $G(\cdot) = 1$	Lower regime: $G(\cdot) = 0$
Threshold: $\widehat{c}$	-0.14	-0.14
Model	$\Pi_t = 0.02 + 0.81\Pi_{t-1} - 0.37\Pi_{t-2} + \widehat{\varepsilon}_t$	$\Pi_t = -0.01 + 0.81\Pi_{t-1} + \widehat{\varepsilon}_t$
Roots	$0.41 \pm 0.45i$	0.81
Modulus	0.61	
Period	7.5	0.81
Behavior of Π_t	$Stationary\ AR(2)$	$Stationary\ AR(1)$
Mean	$E(\Pi_t) = 0.04$	$E(\Pi_t) = -0.05$
Variance	$var(\Pi_t) = 0.004$	$var(\Pi_t) = 0.003$

Table 2.5: The upper and lower regimes of the LSTR(1) model of U.S. inflation rate with the one period lag growth rate of the real GYCPI as the transition variable.

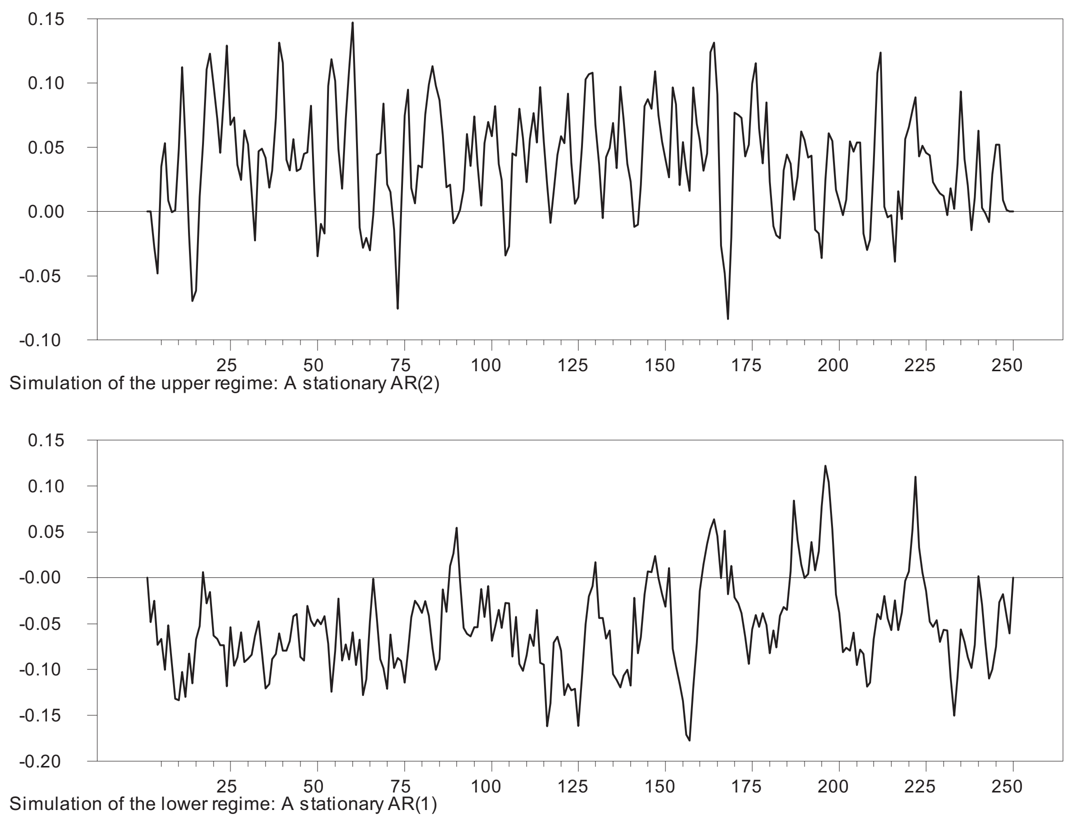


Figure 2.19: Simulation of the upper and lower regimes of the inflation rate with the one period lag growth rate of real GYCPI as transition variable.

s_t	F_L	F_4	F_3	F_2	Suggested Model
y_{t-1}	7.31×10^{-1}	4.50×10^{-1}	6.48×10^{-1}	4.74×10^{-1}	*Linear*
$\triangle y_{t-1}$	2.39×10^{-1}	8.29×10^{-1}	6.45×10^{-2}	3.40×10^{-1}	*Linear*
R_t	1.12×10^{-1}	6.18×10^{-2}	5.00×10^{-1}	1.90×10^{-1}	*Linear*
R^*_{t-1}	4.66×10^{-4}	1.27×10^{-2}	8.29×10^{-2}	4.98×10^{-3}	$LSTR1$

Table 2.6: P-values of the linearity F-tests sequence applied to the logarithm of the real GYCPI when the logarithm of real oil is the transition variable.

2.8.4 Switching Variable: Oil Price

We have seen, in Section 2.8.1, how inflation captured the early fluctuations observed in the Grilli and Yang commodity price index. In this section, the analysis is repeated using the logarithm of real crude oil as the predetermined transition variable.[18] The set of predetermined transition variables in this case is $\Xi_t = \{y_{t-1}, \triangle y_{t-1}, R_t, R_{t-1}\}$, where $R_t = \log\left(\frac{Oil\ price}{CPI}\right)_t$. The nonlinearity tests sequence in Section 2.3.1 was executed for each variable in Ξ_t. The p-values of the F-tests for linearity and model selection are reported in Table 2.6. The best transition candidate in Ξ, tagged with the symbol * in Table 2.6, is the one period lag of the logarithm of real oil price, R_{t-1}, and the suggested model is the LSTR(1) expressed as

$$y_t = \phi_0 + \phi_1 y_{t-1} + \{\theta_0 + \theta_1 y_{t-1}\}(1 + \exp\{-\gamma(R_{t-1} - c)\}^{-1} + \varepsilon_t. \tag{2.32}$$

The initial values for γ and c are that are necessary for the estimation of (2.32) are $\widehat{\gamma} = 8.13$ and $\widehat{c} = -3.1$. The fitted LSTR(1) model and the misspecification tests

[18]For consistency purpose, experimenting with the growth rate of real crude oil was also performed here. The estimated model, however, showed siginificant remaining nonlinearity. The logarithm of real crude oil was, therefore, used instead.

results are reported as follows.

$$y_t = \underset{(0.01)}{0.009} + \underset{(0.04)}{0.98} y_{t-1} - \{\underset{(0.04)}{0.10} + \underset{(0.14)}{0.40} y_{t-1}\}(1 + \exp(-\underset{(5.23)}{7.3}(R_{t-1} + \underset{(0.06)}{3.1})/0.38))^{-1} + \widehat{\varepsilon}_t,$$

$$\overline{R}^2 = 0.88, \qquad \widehat{\sigma}_R = 0.38, \qquad \widehat{\sigma} = 0.10,$$

$$LM_{AUTO(1)} = 0.12, \quad LM_{AUTO(8)} = 0.71, \quad LM_{ARCH(1)} = 0.91, \quad LM_{ARCH(4)} = 0.78,$$

$$PC(1) = 0.13, \quad PC(2) = 0.32, \quad PC(3) = 0.51, \quad NRNL = 0.15, \quad JB = 0.26,$$

$$K_3 = -0.29, \quad K_4 = 3.5, \quad RMSE_{Linear} = 0.235, \quad RMSE_{STR} = 0.076, \qquad (2.33)$$

where $\widehat{\sigma}_R$ is the sample standard deviation of the transition variable $s_t = R_{t-1}$, $\widehat{\sigma}$ is the residual standard deviation and the figures in parentheses beneath the parameter values are standard deviations of the estimates. All the coefficients are significant and the model passes all the misspecifications tests at the 5% level of significance as seen from the p-values of the misspecifications tests. The superiority of the forecasting performance of the STR model over that of the linear AR model is confirmed from the lower RMSE of the STR over the in-sample forecasting period from 2001 to 2007. The standardized residuals time series is plotted in Figure 2.21. The original and fitted series, and the transition function $G(R_{t-1}; \gamma, c)$ are plotted in Figures 2.22 and 2.23 respectively.

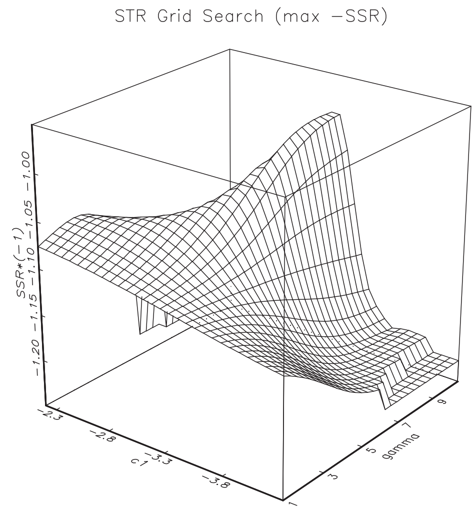


Figure 2.20: Graphical presentation of the constructed grid for the LSTR1 model with the one period lag logarithm of real crude oil as transition variable.

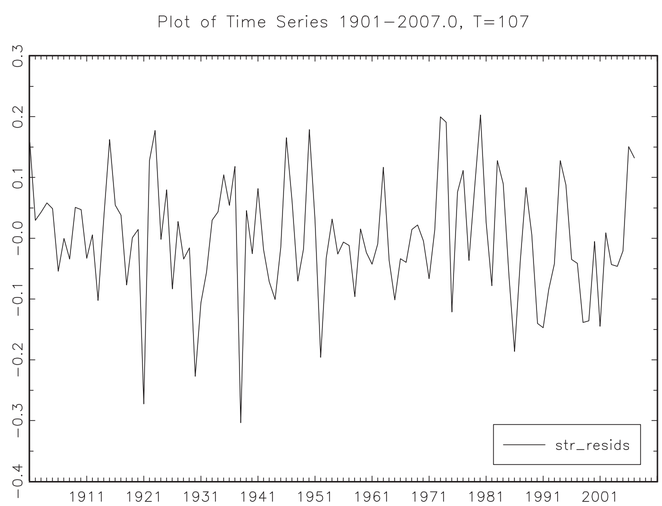


Figure 2.21: Standardized residuals of the fitted LSTR1 model between 1900 and 2007 with the one period lag logarithm of real crude oil as transition variable.

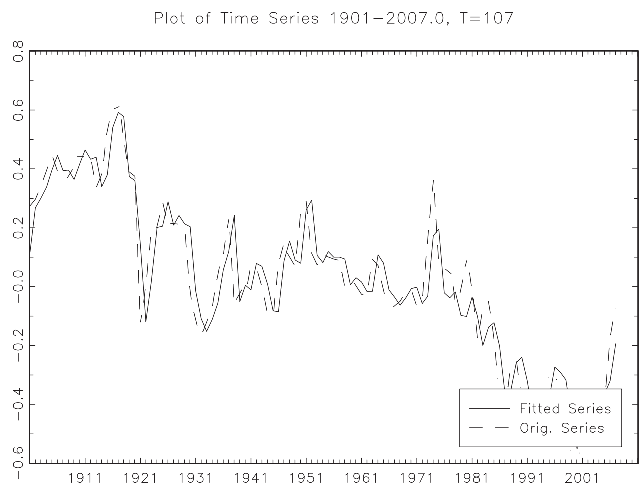


Figure 2.22: Original and fitted values of the logarithm of real GYCPI between 1900 and 2007 with the one period lag logarithm of real crude oil as transition variable.

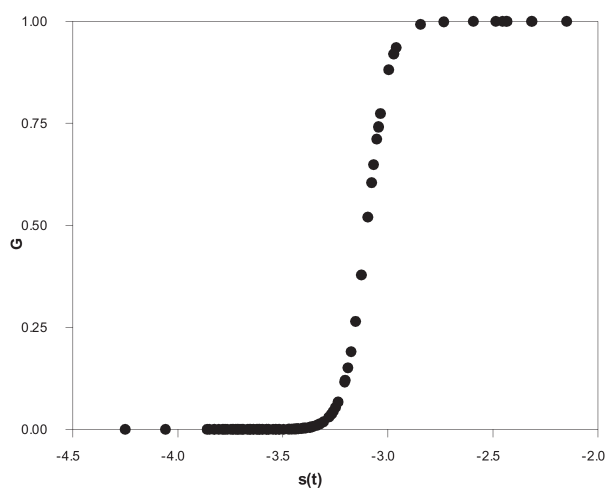


Figure 2.23: Transition function $G(s_t, \gamma, c)$ as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag logarithm of real crude oil.

$s_t = R_{t-1}$	Upper regime: $G(\cdot) = 1$	Lower regime: $G(\cdot) = 0$
Threshold: $\widehat{c}$	-3.1	-3.1
Model	$y_t = -0.09 + 0.58 y_{t-1} + \widehat{\varepsilon}_t$	$y_t = 0.009 + 0.98 y_{t-1} + \widehat{\varepsilon}_t$
Behavior of y_t	$Stationary\ AR(1)$	$Near\ Random\ Walk$
Mean	$E(y_t) = -0.2$	$E(y_t) = 0.45$
Variance	$var(y_t) = 0.015$	$var(y_t) = 0.25$

Table 2.7: The upper and lower regimes of the LSTR(1) model of the real GYCPI with the one period lag real U.S. crude oil price as the transition variable.

Dynamic Analysis

Table 2.7 summarizes the dynamics of the two regimes suggested by the LSTR(1) model with oil price as the transition variable. The estimated threshold $\widehat{c} = -3.1$ means that if the one period lag logarithm of real oil price is higher than -3.1, or, in other words, if the proportion of the one period lag oil price out of the CPI exceeds 5%,[19] the real commodity index will be moving to the *upper* stationary regime with unconditional mean of -0.2 and unconditional variance of 0.015. On the other hand, if the proportion of oil price out of the CPI is less than 5%, the real commodity index will follow a near random walk behavior with 0.45 unconditional mean and a larger unconditional variance equals to 0.25. The higher variance of the lower regime (17 times larger as compared to the upper regime) indicates that the logarithm of the real commodity price index likes to wander in the recessionary regimes but always revert back to the mean because the time series is stationary (see Figure 2.24). This is exactly the same behavior that y_t has exhibited when the one period lag inflation

[19] If $s_t \equiv R_{t-1} = \log(\frac{Oil\ price_{t-1}}{CPI_{t-1}}) = -3.1$, then $\frac{Oil\ price_{t-1}}{CPI_{t-1}} = e^{-3.1} = 0.05$.

was used as a transition variable (see Table 2.3, page 52). This result motivates the use of external transition variables in regime switching models and confirms the claim that both variables (oil and inflation) are capable of explaining the nonlinearity in the Grilli and Yang commodity price index. Inflation captured the early fluctuations in the Grilli and Yang index, but failed to capture the late ones; oil price, on the other hand, captured the late dynamics, but failed to capture the early ones. Therefore, both variables can be seen as *complements* in explaining the dynamic behavior of the Grilli and Yang commodity price index. This complementarity between oil and inflation can be seen from their transition functions in Figure 2.26.

The reason behind having more than one transition candidate is the composition of the commodity price index. Six commodity prices out of the 24 commodities forming the index are recorded on a CIF basis and, therefore, oil price represents an excellent transition candidate for modelling their dynamics. The dynamics of the remaining prices (FOB, settlement, and auction prices) can be captured by macroeconomic news variable(s) (e.g., inflation). Perhaps a better way to understand the behavior of commodity prices is to classify them into groups based on their recorded border price and attempt to find a common transition variable that can model nonlinearity in each group. This is the subject matter of the following section.

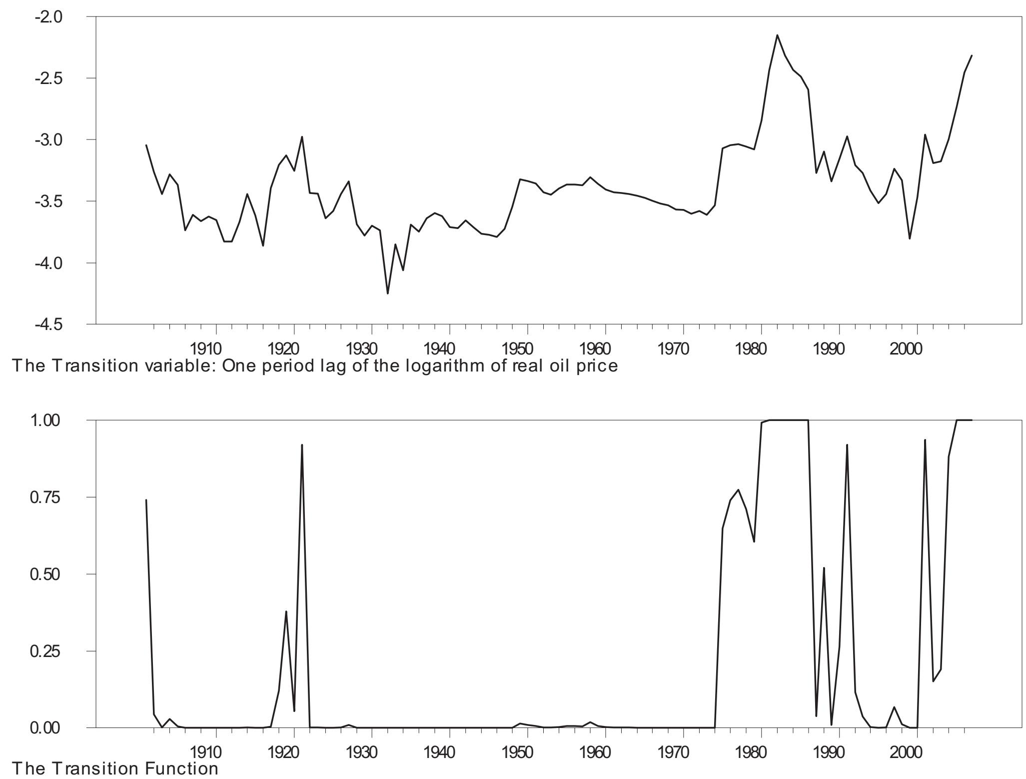


Figure 2.24: A plot of the transition variable R_{t-1} and the transition function $G_t(R_{t-1}; \gamma, c)$ between 1900 and 2007.

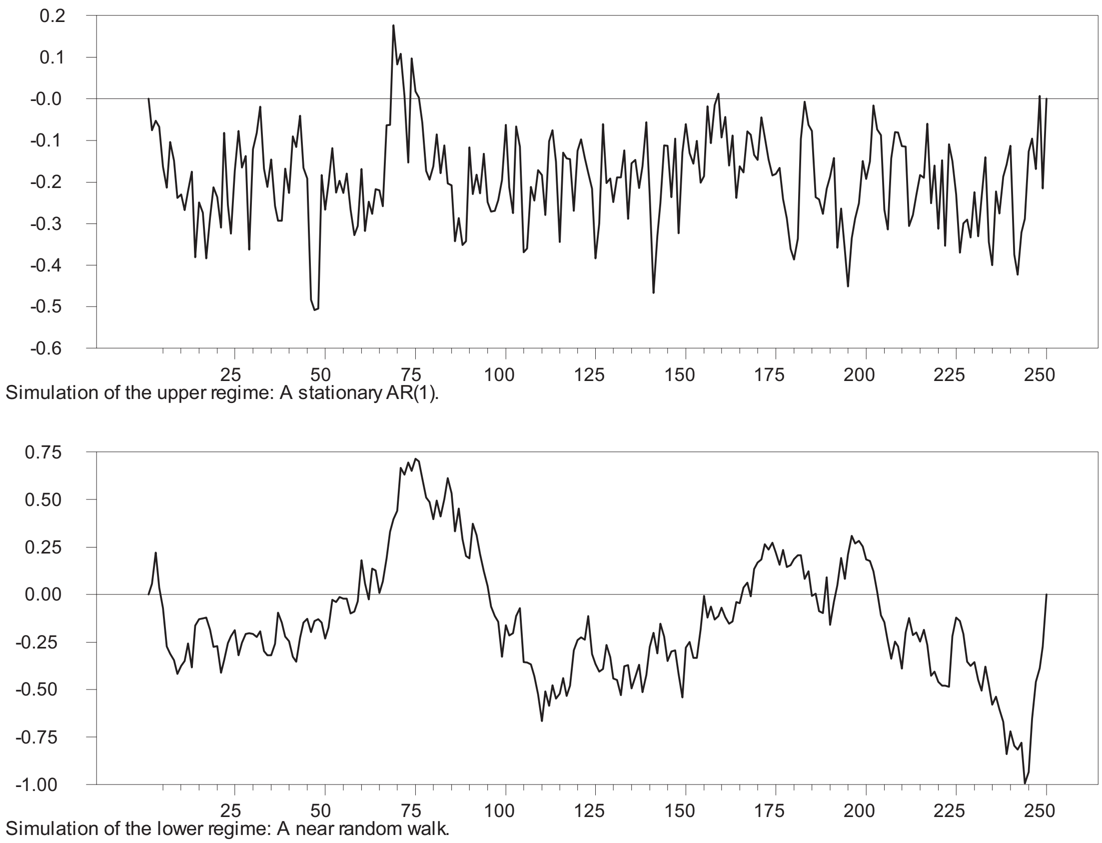


Figure 2.25: Simulation of the upper and lower regimes of the GYCPI with the one period lag logarithm of real crude oil pirce as transition variable.

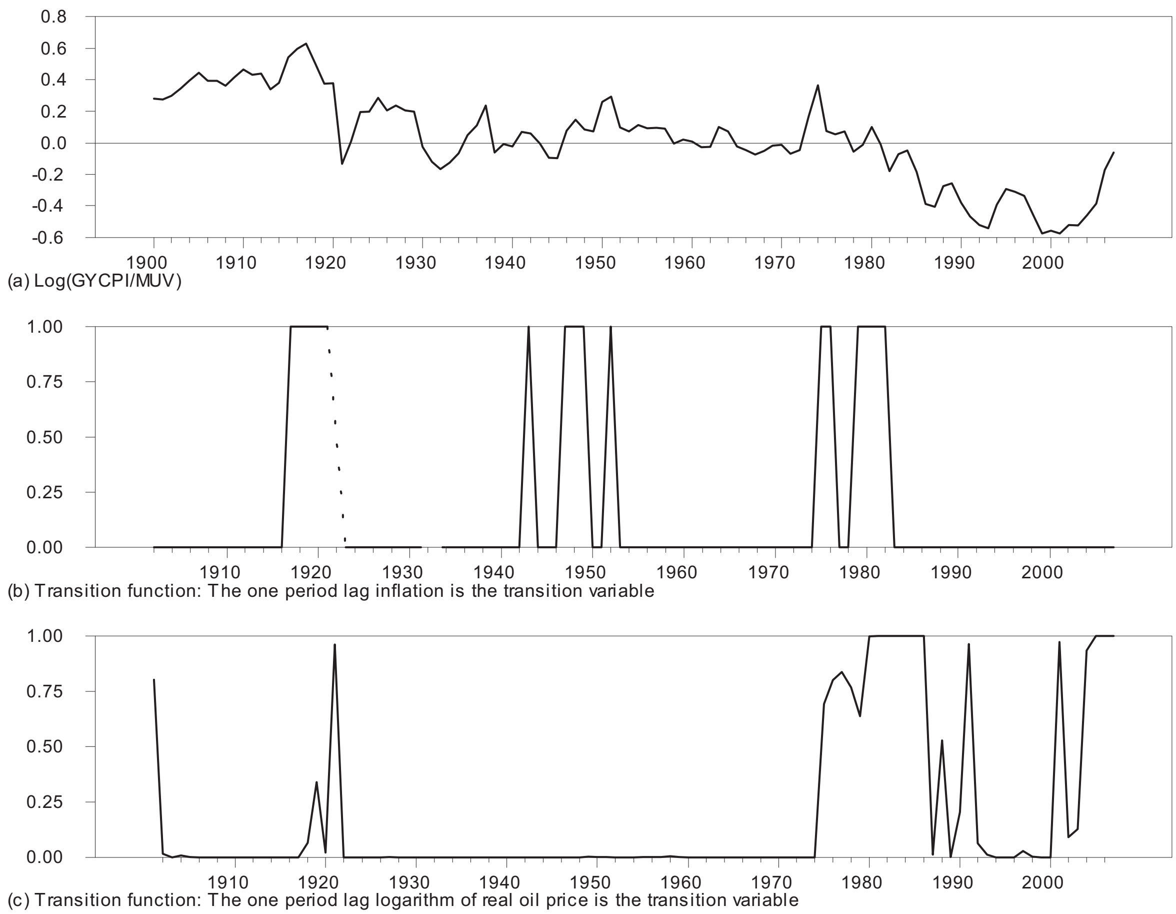


Figure 2.26: Panel (b) and (c) show the first order logisitic transition function with the one period lag inflation rate and the one period lag logarithm of real oil price as transition variables respectively.

2.9 Regime Switching in Individual Commodities

We have just seen how the price of oil and the inflation rate were capable of modelling the nonlinearities in the Grilli & Yang commodity price index. The question now is which transition variable should we employ when modelling nonlinearities in *individual* commodities? To answer this question, we need to classify the individual commodity prices into groups according to their border prices and then device a strategy to model nonlinearity in each group.

2.9.1 Border Price Classification

The rationale behind the border price classification of commodity prices rests on how commodity prices are recorded and it can provide an insight regarding the selection of the transition variable that is capable of modelling their dynamics. For the traded commodities, the standard practice followed by the major institutions when recording data on commodity prices is to select the largest trading route of a commodity and detect whether the trade volume is controlled by a major exporting or importing country. If this route is dominated by a major exporter, the export price at the exit point of the exporting country (FOB price) will be recorded; if the route, on the other hand, is dominated by a major importer of the commodity, the import price at the entry point of the importing country (cost and freight or CIF price) will then be recorded. Of course there are some exceptions, but this is usually the standard practice. For the other commodities that are traded on exchanges (metals and grains mostly), where physical delivery of the commodity is not a must, settlement prices, option prices, or auction prices are used.

Classifying commodities according to their border prices can guide us in determining the potential transition variables that are capable of explaining the dynamics of commodity prices. Commodities that are recorded on a FOB basis suggest that the transition variable is one of the macroeconomic news variables, inflation for instance, in the *exporting* country. Notice here that the price of oil can not be considered among the potential transition candidates in this group of commodities. The reason is that FOB prices do not include the ocean freight which is driven by the price of oil. On the other hand, commodities recorded on a cost and freight or CIF basis suggest that the price of oil and the macroeconomic news variables in the *importing* country are potential transition candidates for nonlinearity analysis. The reason behind choosing oil price for this CIF group of commodities is that a significant portion of it is due to freight cost (shipping cost), which, in turn, is driven by the price of crude oil. Therefore, a reasonable hypothesis here is that ocean freight and, hence, oil price plays a significant role in modelling the dynamics of those commodities falling in this group. This commodity price-oil price connection is demonstrated by means of a simple illustrative example in the following subsection. But, before exploring this connection, let us apply this border price classification to the data set under consideration.

A quick look at the original Grilli and Yang (1988) data sources and their updates, given in detail in Pfaffenzeller, Newbold, and Rayner (2007), one can notice that some commodity prices are free on board prices and cost and freight prices, while others, mostly metals traded over the counter in London Metal Exchange (LME), are settlement prices. In particular, six commodities are traded on a cost and freight basis; these are bananas, palm oil, coffee, timber, cotton, and tobacco; seven commodities are traded on a free on board basis; these are wheat, jute, rice, hides, maize, sugar, and beef; five settlement prices for copper, zinc, lead, tin, and aluminum; two spot prices for rubber and wool; one wholesale price for lamb; one auction price for tea; and one option price for cocoa. Finally the silver price time series is Hary & Harmer, New York, price. A brief description of the 24 commodities forming the Grilli and Yang data set is found in Appendix B. Information on major producer(s) of each commodity as well as the main trading routes are briefly stated to justify the border price classification and to highlight the fact that revising this information is of greater importance in selecting potential transition candidates for nonlinearity analysis. Table 2.8 summarizes each commodity's trading route, its top exporter and importer, the recorded border price, and the threshold variable entailed by the border price classification. Now we have a criterion to select the suitable transition variable(s) for each price process. The next step is to apply the previous three-step modelling procedure to test for and model nonlinearities in individual commodities.

Series	Origin	Destination	Price	Top Exporter	Top Importer	s_t
Bananas	NA†	Gulf ports	CIF	India / Brazil	USA	$r_t; \Pi_t$(USA)
Palm oil	Malaysia	Netherlands	CIF	Malaysia	Netherlands[1]	$r_t; \Pi_t$(Malaysia;Netherland)
Timber	NA†	UK	CIF	NA	NA	$r_t; \Pi_t$(USA)
Coffee	Average‡	New York	CIF[2]	Brazil	US/Germany	$r_t; \Pi_t$(Germany;USA)
Cotton	Memphis	Europe	CIF	USA	China	$r_t; \Pi_t$(USA; Europe)
Lamb	New Zealand	London	Wholesale	New Zealand	UK	$r_t; \Pi_t$(New Zealand)
Tobacco	NA	USA	CIF	Brazil/USA	Russia/USA	$r_t; \Pi_t$(USA)
Wheat	Canada	NA	FOB	US/Canada	China/Japan	Π_t(Canada)
Jute	Bangladesh	NA	FOB	India	Various	Π_t(India)
Rice	Bangkok	NA	FOB	Thailand	Philippines	Π_t(Thailand)
Hides	USA	NA	FOB			Π_t(USA)
Maize	Gulf Port	NA	FOB	USA	Japan	Π_t(USA)
Sugar	Caribbean Ports	Various	FOB	Brazil	Russia	Π_t(USA)
Beef	Argentina	NA	FOB	Australia	USA	Π_t(Argentina,USA)
Copper	London Metal Exchange		Settlement			Macroeconomic news variable
Zinc	London Metal Exchange		Settlement			Macroeconomic news variable
Lead	London Metal Exchange		Settlement			Macroeconomic news variable
Tin	London Metal Exchange		Settlement			Macroeconomic news variable
Aluminum	London Metal Exchange		Settlement			Macroeconomic news variable
Cocoa	London and US Exchange		Option Price			Macroeconomic news variable
Rubber	Rubber Traders Association		Spot Price			Macroeconomic news variable
Tea	NA		Auction			Macroeconomic news variable
Wool	Australia Exchange		Spot quote			Macroeconomic news variable
Silver	Handy & Harry					

s_t is the transition variable suggested by the border price classification.

‡ The price is arithmetic average of El Salvador, Guatemala, and Mexico.

†Not Available. r_t is the growth rate of real oil; $\Pi_t(x)$ is the inflation in country x.

[1] Netherlands is the top importer in Europe; China and India are the World's top importers.

[2] The price here is an average Hamburg and New York Ex-dock price.

Table 2.8: Trading route, border price, top importer and exporter, and the suggested transition variable for each individual commodity in the Grilli and Yang data set.

2.9.2 An Illustrative Example

There is no doubt that transportation cost is the most significant factor affecting the flow of trade from one nation to another. Analyzing the domestic transport industry is a separate topic by itself and is beyond the scope of this analysis. Exporters of commodities care only about the trucking cost (inland transport) of transporting their commodities from one point to another. The rates for inland transportation of 20-feet and 40-feet containers are pretty much standardized and do not fluctuate much over time. Airfreight and ocean freight rates, however, are highly volatile. This high volatility is mainly due to the fluctuations in oil price, which is considered one of the significant determinants of ocean freight rates. The connection between oil prices and commodity prices is established through this fluctuation in oil prices.

This subsection is devoted to illustrate the commodity price-oil price connection by means of a simple example. The purpose is to estimate, within a factor of ten, the relation between the oil price and the price of any commodity recorded on a CIF basis. Although the data on the shipping industry exists, it is not freely available and, therefore, we are not looking for accuracy, but rather for illustration.

Commodities are shipped either in tankers or in dry bulk carriers. In addition to the cost of the vessel itself (capital cost or return on investment), a significant part of the CIF price reflects the cost of bunker fuel. This later relation is what we are trying to estimate in this sub-section. Consider one of the CIF commodities in the Grilli & Yang (1988) data set, price of bananas say. The recorded price is CIF Gulf ports (central and south America) from the primary commodity data base. Since the port of origin was not mentioned in the commodity description, we will consider a frequent port of origin for banana shipments going to central and south America; Rio de Janeiro (Brazil) say. The relation that we are trying to estimate here is how much the bunker fuel cost (oil cost) represents out of the CIF price of bananas? We proceed by dividing the problem into three sub-problems as follows.

How many tons of fuel a medium-sized ship can burn during a round trip from Rio de Janeiro to Gulf ports?

To answer this question, we need to find the distance between the port of origin and the port of discharge, the speed of the vessel, and the duration of the trip. The distance between Rio de Janeiro and the Gulf ports is around 5097 nautical miles. It takes around 16 days to travel this distance with a medium-sized vessel cruising at an actual cruising speed, V, of 14 knots.[20] We were not sure about the nominal maximum cruising speed, V_N, for a medium-sized vessel, but it is definitely between 15 and 18 knots. So, we will choose the geometric mean of 16 knots. A medium-sized vessel burns around 40 tons of bunker fuel, F_N, per day for the main engines at nominal speed (Ronen, 1982). It is well known in ship engineering that the bunker fuel consumption of the main engines of a motor ship, F, is directly related to the

[20] Source: www.searates.com

third power of the speed (Manning, 1956); that is

$$F = \left(\frac{V}{V_N}\right)^3 F_N = \left(\frac{14\ knots}{16\ knots}\right)^3 \times 40\ tons \approx 27\ tons.$$

Therefore, the actual fuel consumption of a medium size vessel carrying bananas from Rio de Janeiro to Gulf ports is around 27 tons a day. It takes around 16 days (one way) to travel this distance, then the total fuel consumption of a round trip (two legs)[21] is

$$Total\ Fuel\ consumption = 2\ legs \times \frac{16\ days}{1\ leg} \times \frac{27\ tons}{1\ day} \approx 864\ tons.$$

The average oil price per barrel over the last three years ranges from a minimum of \$90 per barrel to a maximum of \$170 per barrel. We will take the geometric mean which is \$125 per barrel approximately. Now, let's convert that into cubic meters (cbm) as

$$1\ barrel = 1\ barrel \times \frac{42\ gallons}{1\ barrel} \times \frac{0.004\ cbm}{1\ gallon} = 0.17\ cbm.$$

The cost of one cbm (1 ton) of bunker fuel is about

$$Fuel\ Cost\ per\ cbm\ (two\ legs) = 1\ cbm \times \frac{\frac{1}{0.17} barrel}{1\ cbm} \times \frac{\$125}{1\ barrel} \approx \$750.$$

Therefore, the total bunker fuel cost of a round (two legs) trip is about

$$Total\ Fuel\ Cost\ (two\ legs) = 2\ legs \times \frac{\$750}{ton} \times \frac{864\ tons}{2\ legs} = \$648,000.$$

[21] In the returning trip, the vessel is usually empty yet the fuel cost is the same. Therefore, the carriers consider the two-leg bunker cost in their ocean freight calculations.

Capacity: How many 40 feet containers a medium-size vessel can hold? What is the fuel cost per container? And per one cbm of cargo?

Now, let's try to estimate the cost per container. I will consider a standard 40 feet refrigerated container (also called "refer"). The internal dimensions of a standard 40 feet container are 12.022 m of length, 2.352 m of width, and 2.395 m of height.[22] Therefore,

$$1 \times 40' container = 1 \times 40' container \times \frac{(12.022 \times 2.352 \times 2.395)\ cbm}{1 \times 40' container} \approx 67\ cbm.$$

Only 65% to 70% of the container's space is filled with actual cargo; the rest is devoted to crating, boxes, and other packaging materials. This leaves us with an actual 45 cbm of cargo in a 40 feet container.

The capacity of our medium-sized vessel ranges from a minimum of 350 twenty foot equivalent unit (TEU) to a maximum of 400 TEU. We will take the geometric mean which is about 370 TEU. Dividing this number by 2 yields around 185 container (40 feet each), which can fit around 8300 cbm of cargo

$$\frac{45\ cbm}{1 \times 40' container} \times 185\ container = 8300\ cbm.$$

Therefore, the bunker fuel cost per cbm, F_{cbm}, is

$$F_{cbm} = \frac{Total\ Fuel\ Cost}{Total\ cbms} = \frac{\$648,000}{8300\ cbm} \approx \$80\ per\ cbm.$$

To account for the frequent fluctuations in the oil price, all the shipping lines adjusts the ocean freight per cbm by adding a fuel surcharge fee (also known as the Bunker

[22] Source: www.geocities.com.

Adjustment Factor (BAF)). For a 40 feet container, the BAF is about \$680, or \$15 per one cbm of actual cargo. Thus, the total cost of fuel (including BAF) for one cbm of cargo is around \$95.

Oil price - commodity price

Now, let's check the CIF price of one cbm of bananas shipped from Rio de Janeiro to Gulf ports. From the primary commodity price data base, over the last three years, the CIF price ranged from \$650 to \$800 per metric ton. Taking the geometric mean, we can estimate \$720 per metric ton. Recall that this price includes the insurance cost, customs clearance at the port of origin and port of discharge, handling, other fixed surcharges, and, of course, the cost of the vessel itself (capital cost). The cost of insurance and the other charges ranges from a maximum of 40% of the price and a minium of 30%. We will take the geometric mean which is 35%. Then, the stripped price of bananas is around \$470 ($\$720 \times 0.65$). This implies that the bunker fuel cost represents roughly around 20% ($\frac{\$95}{\$470} \approx 0.20$) of the stripped CIF price of bananas. This is indeed a significant proportion. The previous analysis was performed for all nonlinear CIF commodities in the Grilli and Yang (1988) data set and the results are recorded in Table 2.9. The only exception was the tobacco price series as the data on the origin country was not available. The analysis reveals that the fuel cost represents roughly 20% of the price of commodities recorded on a CIF basis.

Commodity	Bananas	Palm oil	Timber	Coffee	Cotton
Origin	Brazil	Malaysia	Malaysia	N/A†	Memphis, USA
Port of loading	Rio de Janeiro	Kelang	Kelang		Memphis
Destination	US	Netherlands	UK	US/	Northern Europe
Port of discharge	Gulf Ports	Rotterdam	London	NYC	Liverpool, UK
Distance (nautical miles)	5097	8083	7930	2845	5144
Time (days)	16	24	23.5	8.5	15.3
Fuel cost/cbm	\$95	\$250	\$130	\$65	\$90
Price/cbm	\$470	\$890	\$546	\$650	\$660
% of fuel out of the price	20%	28%	24%	10%	14%

†The port of origin was not mentioned explicitly. The price is arithmetic average of shipments from El Salvador, Guatemala, and Mexico. All calculations are done based on the arithmetic means of the available data.

Table 2.9: The percentage of fuel out of the CIF price for all CIF commodities in the Grilli and Yang (1988) data set.

2.9.3 Specification Stage

The first step in our modelling framework is the specification stage. Let P_{it} be commodity i^{th} price and let $y_{it} = \log\left(\frac{P_i}{MUV}\right)_t$ be the logarithm of the real price series i. A linear $AR(p)$ model, where p is the value that minimizes the AIC, is selected for each commodity as the starting point of the analysis. The preliminary AR model is estimated for each commodity price series and the relevant diagnostic tests are applied to each model's residuals to ensure its adequacy as a starting model for the nonlinearity analysis. The value of the lag order of the AIC and the p-values of the diagnostic tests are reported in Table 2.10. Judging by the Ljung-Box (1978) statistics, $Q(q)$, the null hypothesis of no serial correlation of order $q = 1$ up to $q = 8$ in the residuals series for all the 24 commodities is not rejected at the 5%

level of significance. The null hypotheses of no $ARCH(v)$, $v = 1, ..., 4$, were also not rejected at the 5% level of significance for the majority of the 24 commodities; notable exceptions are cotton, wool, silver, tea, and aluminum. Finally, the null-hypothesis of normality of errors was rejected at the 5% level of significance for the majority of commodities as seen from the p-value of the Jarque and Bera (1980), JB, test statistic. This rejection is due to the presence of outliers in the time series.

From the reported results in Table 2.10, we can observe that 8 commodity prices exhibit ARCH pattern in their residuals. In particular, these are tobacco, silver, jute, lead, cotton, wool, aluminum, and tea. This is not surprising as the majority of these prices are settlement or auction prices of commodities traded in exchanges and, therefore, tend to exhibit volatility clusters; a common feature of stock and option prices. Therefore, ARCH or smooth transition ARCH (ST-ARCH) models are suitable models for this type of commodities. We shall classify those 8 commodities into *Group A*, where ARCH and ST-ARCH models are entertained. It is worth mentioning that commodities in this group are all storable commodities. This is consistent with Muth's (1961) hypothesis and with the results obtained by Beck (2001), who applied a variation of (G)ARCH techniques to commodity prices and found an ARCH process in storable but not in non-storable commodity data. Since ARCH models entail regime switching in variance, we shall discuss the modelling and estimation of this group of commodities in the following chapter. We now consider the rest of the price processes and proceed to the second step in our modelling framework, which is nonlinearity testing.

2.9.4 Testing for Nonlinearity

The advantage of the previously mentioned border price classification is to provide guidance on the potential transition variables for each price process. The transition set for the remaining 16 commodities, Λ_t, consists of the autoregressive lags of the dependent variable, the current and one period lag inflation rates, Π_t and Π_{t-1} respectively, the current and one period lag logarithm of real oil, R_t and R_{t-1} respectively, and their first difference; that is, $r_t = \triangle R_t = \triangle \log \left(\frac{Oil\ Price}{CPI}\right)_t$ and r_{t-1}. The models with the growth rate of real oil price (r_t and r_{t-1}) as transition variables outperformed those with R_t and R_{t-1} and, therefore, the later ones were dropped from the analysis. The transition set is then

$$\Lambda_t = \{y_{t-1}, y_{t-2}, ..., y_{t-p}, \Pi_t, \Pi_{t-1}, r_t, r_{t-1}\}. \tag{2.34}$$

Applying the nonlinearity tests sequence discussed in Chapter 2, Section 2.3.1, to each transition candidate in Λ_t, linearity was not rejected for 8 commodities as seen from Table 2.11. In particular, linearity was not rejected for beef, cocoa, lamb, wheat, tin, copper, zinc, and rubber. Therefore, these commodities that passed linenarity tests were classified into *Group B*, where linear AR models are entertained. No further nonlinearity analysis was performed for this group.

For completeness sake, for each individual commodity, the nonlinearity tests sequence was executed for each transition variable in (2.34). The results are reported

P_{it}	$AIC(p)$	Residuals Analysis (AR Model) JB	K_3	K_4	$Q(1)$	$Q(8)$	$ARCH(1)$	$ARCH(4)$
Tobacco	$p=5$	†0.01	0.42	3.84	0.90	0.84	0.29	†0.04
Silver	$p=3$	†0.000	0.66	4.50	0.94	0.82	†6.2×10^{-5}	†0.003
Jute	$p=3$	0.13	-0.07	3.71	0.75	0.70	0.59	†0.03
Lead	$p=1$	†0.01	0.24	4.05	0.27	0.79	0.08	†0.01
Cotton	$p=4$	0.84	-0.14	2.84	0.87	0.89	†0.002	†0.04
Wool	$p=5$	0.42	0.22	3.23	0.92	0.99	†0.04	0.32
Aluminum	$p=3$	†0.000	0.62	6.11	0.98	0.91	†0.02	†0.02
Tea	$p=3$	†0.008	0.16	4.16	0.93	0.56	†6.4×10^{-4}	†0.02
Wheat	$p=9$	†0.003	0.42	4.10	0.99	0.99	0.83	0.89
Lamb	$p=5$	†0.000	0.21	4.89	0.69	0.96	0.37	0.87
Coffee	$p=1$	†0.02	0.51	3.55	0.52	0.80	0.33	0.78
Copper	$p=3$	0.62	0.19	3.05	0.77	0.68	0.58	0.14
Cocoa	$p=3$	†0.004	0.65	3.59	0.92	0.99	0.93	0.05
Timber	$p=1$	0.50	0.10	3.31	0.60	0.29	0.39	0.11
Tin	$p=3$	†0.01	-0.33	3.95	0.86	0.97	0.87	0.52
Zinc	$p=2$	†0.000	1.64	10.2	0.99	0.96	0.63	0.98
Maize	$p=5$	†0.000	-0.38	4.70	0.75	0.95	0.32	0.56
Beef	$p=1$	†0.000	0.57	5.77	0.80	0.72	0.71	0.78
Rice	$p=5$	†0.01	-0.07	4.18	0.84	0.86	0.41	0.39
Bananas	$p=3$	0.08	-0.36	3.52	0.95	0.97	0.76	0.21
Palm oil	$p=3$	†0.000	-0.54	4.25	0.99	0.63	0.12	0.44
Rubber	$p=1$	†0.000	0.75	5.28	0.39	0.47	0.63	0.10
Hides	$p=3$	0.20	-0.33	3.30	0.76	0.64	0.35	0.66
Sugar	$p=3$	†0.000	0.67	5.35	0.93	0.30	0.45	0.89

†The null-hypothesis is rejected at the 5% level of significance.

Table 2.10: The lag order of the AIC and the p-values of the diagnostic tests of the linear AR model's residuals applied to the 24 commodities in the Grilli and Yang (1988) data set.

in Table 2.11. However, they should be interpreted with caution. For instance, in the case of zinc, the transition variable that showed the highest rejection of linearity tests was the current growth of real oil, g_t, and the associated suggested model was the LSTR1. But, the growth rate of oil is not applicable in such a case since the border price is a settlement price; exchange rate or a stock index are suggested instead. However, due to data limitation, they were not employed. Even when we fitted the LSTR1 model with oil price as the transition variable, the estimated parameters and the threshold variable were insignificant.

The only exception in the linear group (*Group B*) is the case of rubber; the one period lag inflation rate, which is a potential transition candidate according to the border price classification, showed the highest rejection of linearity tests and the associated model is the LSTR1. But, when I estimated the model, the estimated coefficients were insignificant and the estimated value of the threshold variable was not lying within the transition variable's data range. Inflation was, therefore, dropped from the transition set and the model was classified as a linear autoregressive model.

The third group in our classification is *Group C*, where the threshold variable is one of the autoregressive lags of the dependent variable. Three commodities fit into this group: Maize, rice, and sugar. All three prices were recorded on a FOB basis and therefore, oil did not play a role in this analysis. The transition variables that showed the highest rejection of linearity tests were the second order autoregressive lag for maize, the fifth order for rice, and the first order for sugar. The estimation and evaluation results of this group are reported in the following subsection.

The last group in our classification is *Group D*, where the threshold variable is an external variable. This group consists of 5 commodities: four are recorded on a CIF basis (bananas, palm oil, timber, and coffee) and one on a FOB basis (hides). Following the border price rationale of the previous section, one expects that all CIF commodities to be driven by oil price. Actually, our results confirm that; the transition variables that showed the highest rejection of linearity tests were oil price for all CIF commodities and inflation for hides (see Table 2.11). The estimation and misspecification tests results of this group of commodities are discussed in the following subsection.

P_{it}	$AIC(p)$	Border Price	The Transition Variable and the Suggested Model				
			y_{t-j}; $j \in p$	Π_t	Π_{t-1}	r_t	r_{t-1}
Group B: Linear AR Models:							
Beef	$p = 1$	FOB & CIF	Linear	Linear	Linear	Linear	Linear
Cocoa	$p = 3$	Option price	Linear	Linear	Linear	Linear	Linear
Lamb	$p = 5$	CIF	Linear	Linear	Linear	Linear	Linear
Wheat	$p = 9$	FOB	Linear	Linear	Linear	Linear	Linear
Tin	$p = 3$	Settlement	Linear	Linear	Linear	Linear	Linear
Copper	$p = 3$	Settlement	Linear	Linear	Linear	Linear	Linear
Zinc	$p = 2$	Settlement	Linear	Linear	Linear	LSTR1†	Linear
Rubber	$p = 1$	Spot price	Linear	LSTR2	LSTR1‡	Linear	Linear
Group C: STR Models where the threshold variable is one of the autoregressive lags of y_t							
Maize	$p = 5$	FOB	y_{t-2}; LSTR2*	LSTR1	Linear	NA	NA
Rice	$p = 5$	FOB	y_{t-5}; LSTR2*	LSTR2	Linear	NA	NA
Sugar	$p = 3$	FOB	y_{t-1}; LSTR1*	LSTR1	LSTR2	NA	NA
Group D: STR Models where the threshold variable is an external variable							
Hides	$p = 3$	FOB	Linear	LSTR2*	LSTR1**	LSTR1†	Linear†
Bananas	$p = 3$	CIF	Linear	Linear	Linear	Linear	LSTR1*
Palm oil	$p = 3$	CIF	y_{t-3}; LSTR2	LSTR1	Linear	LSTR2*	LSTR1
Timber	$p = 1$	CIF	Linear	LSTR1*	Linear	LSTR1**	Linear
Coffee	$p = 1$	CIF	Linear	LSTR2*	Linear	Linear	LSTR2**

NA≡Not Applicable.

r_t and r_{t-1} are current and one period lag of growth rate in real oil prices respectively.

Π_t and Π_{t-1} are current and one period lag inflation respectively.

‡Transition variable exhibits the highest rejection of linearity tests, but the coefficients of the fitted model were insignificant.

†Although this variable showed, statistically speaking, the highest rejection of linearity tests, it is not applicable in this case.

**Both models were close; but the one tagged with two stars outperforms the one star model.

Table 2.11: Testing for non-linearity in individual commodities.

2.9.5 Estimation and Evaluation

Group C: STR models where the threshold variable is one of the autoregressive lags of the dependent variable

The third group in our classification is Group C, where the transition variable is an autoregressive lag of the dependent variable and the corresponding nonlinear model is the STR model. This group consists of three commodities: Maize, rice, and sugar. All three prices were recorded on a FOB basis. Nonlinearity tests were executed for each transition candidate in the transition set Λ_t (see equation (2.34)) and the results were reported in Table 2.11. The transition variables that showed the highest rejection of linearity tests were the second order autoregressive lag for the maize price, the fifth order for the rice price, and the first order for the sugar price time series. Estimation and misspecification results are reported in Table 2.12.

All estimated coefficients are significant at the 5% percent level of significance and the three models pass all misspecifications tests. At the 5% level of significance, the residuals from the estimated LSTR models exhibit no serial correlation up to order $q = 8$. The null hypothesis of no $ARCH(v)$ up to order $v = 4$ was also not rejected at the 5% level of significance. The hypothesis of no remaining nonlinearity was tested against the alternative of additive nonlinearity in the autoregressive transition variable and, based on the p-value of the $NRNL$ test results, we can conclude that, at the 5% level of significance, the nonlinearity has been adequately captured by the STR models. The null hypothesis of parameter constancy was not rejected for all three commodities, at the 5% level of significance, once tested against the alternative hypothesis of monotonic change, $PC(1)$, non-monotonic symmetrical change, $PC(2)$, and nonmonotonic and nonsymmetrical change, $PC(3)$. The reported p-values confirm the adequacy of the fitted LSTR models. The null-hypothesis of normality of errors was rejected at the 5% level of significance as seen from the p-value of the JB test statistic. This is due to the presence of outliers in the times series. The transition from one regime to the other was smooth in case of rice and sugar as opposed to maize (see Figures 2.28, 2.29, and 2.27 respectively). This is due to the relatively higher value of γ (the slope of the transition function) in the case of maize as opposed to rice and sugar (see Table 2.12). The nonlinearity was captured by the LSTR(1) model in the case of sugar and by the LSTR(2) model in the case of maize and rice. In the later case, the transition function is the second-order logistic function and it is expressed as

$$G(s_t;\gamma,c) = \left(1+\exp\{-\gamma(s_t-c_1)(s_t-c_2)/\sigma^2\}\right)^{-1}, \qquad \gamma>0,\ c_1\le c_2, \tag{2.35}$$

where $\gamma > 0$ and $c_1 \le c_2$ are identifying restrictions as mentioned before. The function achieves its minimum value $G_{\min} = (1+\exp\{-\gamma\tilde{c}\}/\sigma^2)^{-1}$, where $\tilde{c} = c_1c_2 - \bar{c}^2$ and $\bar{c} = \frac{c_1+c_2}{2}$, when the transition variable s_t is equal to $\bar{c}$. To facilitate the interpretation of the regimes, we can apply the following reparameterization of the logistic function in (2.35). Let

$$G(\cdot) = G_{\min} + \tilde{G}(\cdot)\times(1-G_{\min}), \tag{2.36}$$

where

$$\tilde{G}(\cdot) = \frac{G(\cdot) - G_{\min}}{(1 - G_{\min})} \tag{2.37}$$

Substituting (2.36) in the STR model expressed in (2.5) and reparameterizing the model yields

$$y_t = \Lambda^{'} z_t + \Gamma^{'} z_t \tilde{G}(s_t; \gamma, c) + \varepsilon_t, \tag{2.38}$$

where $\Lambda = \Phi + \Theta G_{\min}$ and $\Gamma = (1 - G_{\min})\Theta$. The logistic function in (2.35) displays three regimes depending on the value taken by the transition variable: a ground (middle) regime and two outer regimes. The middle regime is realized when $c_1 \leqslant s_t \leqslant c_2$. When the transition variable takes the value of any of the thresholds, i.e., $s_t = c_1$ or c_2, $G(\cdot) = 1/2$. When the transition variable takes a weighted average of both threshold values; that is, $s_t = \bar{c} = \frac{c_1+c_2}{2}$, then $G(\cdot) = G_{\min}$ and $\tilde{G}(\cdot) = 0$ in (2.37). The STR model in (2.38) is then reduced to the following autoregressive model

$$y_t = \Lambda^{'} z_t + \varepsilon_t. \tag{2.39}$$

The outer regimes are associated with $s_t \to \pm\infty$, which, in turn, implies that $G(\cdot) = 1$ in (2.35) and $\tilde{G}(\cdot) = 1$ in (2.37). Therefore, from (2.38), the behavior of y_t in the outer regimes can be described as

$$y_t = (\Lambda^{'} + \Gamma^{'}) z_t + \varepsilon_t. \tag{2.40}$$

The transition functions for maize, rice, and sugar are plotted in Figures 2.27, 2.28, and 2.29 respectively.

Group C: STR models with autoregressive lags as threshold variables† $y_t = \phi_0+\phi_1 y_{t-1}+\phi_2 y_{t-2}+\phi_3 y_{t-3}+\phi_4 y_{t-4}+\phi_5 y_{t-5}$ $+(\theta_0+\theta_1 y_{t-1}+\theta_2 y_{t-2}+\theta_3 y_{t-3}+\theta_4 y_{t-4}+\theta_5 y_{t-5})G(\gamma, c_1, c_2; s_t)+\varepsilon_t$, where $G(\cdot)=(1+\exp\{-\gamma(s_t-c_1)\}/\sigma_s)^{-1}$ or $G(\cdot)=(1+\exp\{-\gamma(s_t-c_1)(s_t-c_2)\}/\sigma_s^2)^{-1}$			
Commodity Price	Maize	Rice	Sugar
$AIC(p)$	$p=5$	$p=5$	$p=3$
Transition Variable; Model	y_{t-2}; LSTR(2)	y_{t-5}; LSTR(2)	y_{t-1}; LSTR(1)
ϕ_0	0.46 (0.17)	$\phi_0 = 0$	$\phi_0 = 0$
ϕ_1	$\phi_1 = 0$	1.05 (0.09)	1.03 (0.14)
ϕ_2	-0.27 (0.13)	-0.29 (0.09)	-0.40 (0.13)
ϕ_3	$\phi_3 = 0$	$\phi_3 = 0$	0.26 (0.09)
ϕ_4	$\phi_4 = 0$	$\phi_4 = 0$	
ϕ_5	0.20 (0.09)	0.35 (0.09)	
θ_0	-0.46 (0.18)	-0.24 (0.09)	0.94 (0.46)
θ_1	0.99 (0.10)	$\theta_1 = 0$	-1.08 (0.41)
θ_2	$\theta_2 = 0$	$\theta_2 = 0$	$\theta_2 = 0$
θ_3	0.16 (0.02)	$\theta_3 = 0$	$\theta_3 = 0$
θ_4	-0.07 (0.01)	-0.5 (0.15)	
θ_5	$\theta_5 = 0$	$\theta_5 = 0$	
γ	77.22 (56.64)	2.42 (1.15)	2.47 (2.22)
c_1	0.98 (0.02)	-0.62 (0.17)	0.81 (0.42)
c_2	0.77 (0.02)	0.73 (0.05)	
$\widehat{\sigma}_s$‡	0.44	0.45	0.52
$LM_{AUTO(1)}; LM_{AUTO(8)}$	2.84(0.10); 1.77(0.10)	1.22(0.27); 1.11(0.36)	0.11(0.74); 1.51(0.16)
$LM_{ARCH(1)}; LM_{ARCH(4)}$	0.01(0.92); 2.03(0.73)	0.30(0.58); 4.21(0.38)	0.01(0.93); 2.24(0.69)
$NRNL$	0.27	0.97	0.53
$PC(1); PC(2); PC(3)$	0.37; 0.12; 0.05	0.38; 0.41; 0.48	0.12; 0.31; 0.63
$JB; K_3; K_4$	30.3(0.00); 0.67; 5.3	13.8(0.001); -0.25; 4.7	35.1(0.000); 0.8; 5.2
†Figures in parentheses beneath the parameters are standard deviations & those after the test statistics are p-values.			
‡σ_s is the sample standard deviation of the transition variable $s_t = y_{t-j}$, for $j=1,2,...,p$.			

Table 2.12: Estimation and evaluation of Group C: STR models with autoregressive threshold variables.

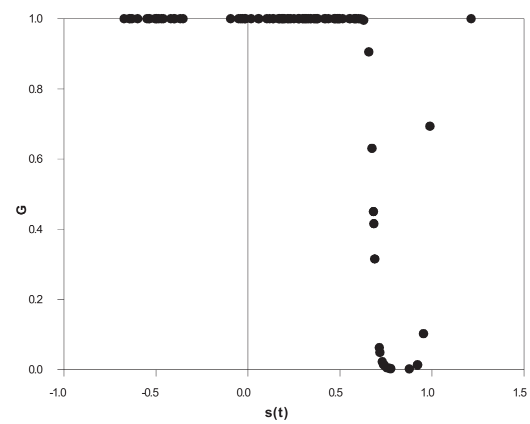


Figure 2.27: Transition function in the LSTR(2) model fitted to the logarithm of real maize price as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the second autoregressive lag y_{t-2}.

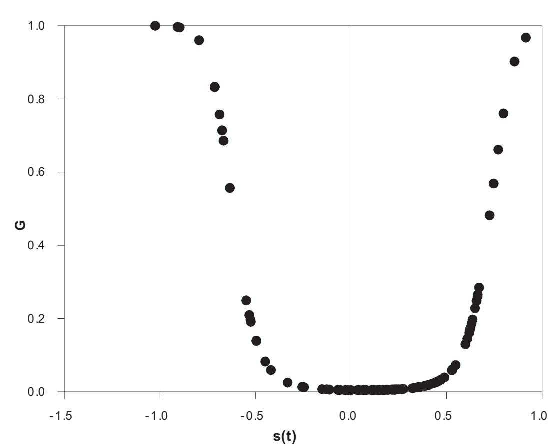


Figure 2.28: Transition function in the LSTR(2) model fitted to the logarithm of real rice price as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the fifth autoregressive lag y_{t-5}.

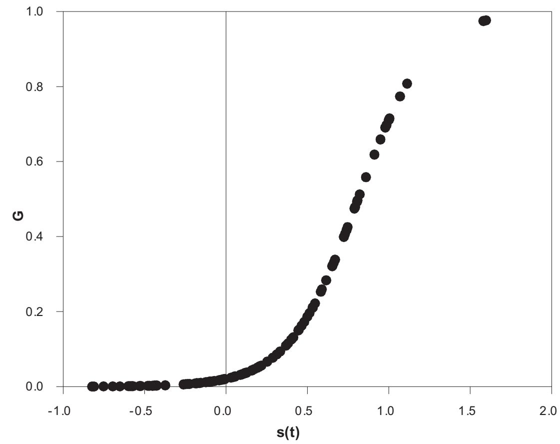


Figure 2.29: Transition function in the LSTR(1) model fitted to the logarithm of real sugar price as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the first autoregressive lag y_{t-1}.

Characteristic roots in each regime for Group C†				
Commodity and Model	Regime	Root(s)	Modulus	Period
Maize: LSTR(2)	Lower (Mid)	$0.31 \pm 0.72i$	0.79	5.4
	Upper (Outer)	1.01	1.01	
		$0.39 \pm 0.61i$	0.73	6.2
Rice: LSTR(2)	Lower (Mid)	$0.47 \pm 0.73i$	0.87	6.3
		1.06	1.06	
	Upper (Outer)	$0.70 \pm 0.61i$	0.93	8.8
		0.68	0.68	
Sugar: LSTR(1)	Lower	$0.06 \pm 0.53i$	0.53	4.3
		0.91	0.91	
	Upper	$-0.24 \pm 0.74i$	0.78	3.3
†Most prominent roots are only reported.				

Table 2.13: The most prominent roots of the characteristic polynomials in the regimes of the estimated LSTR models for the logarithm of the real commodity prices in Group C, 1900-2007.

Dynamic Analysis

Following previous studies (see, for instance, Teräsvirta & Anderson (1992) and Skalin & Teräsvirta (1999)), the dynamic behavior of LSTR models can be explained from the analysis of the characteristic roots of each regime. The roots in each regime are computed in Table 2.13.

Maize Consider the logarithm of real maize price time series. The most prominent pair of complex roots in the ground (mid) regime has a modulus of 0.79 and a period of 5.4 years, see Table 2.13, so that the process is locally stationary in the ground regime. The outer regimes (expansionary or contractionary) are, on the other hand, characterized by a real root with a modulus of 1.01, which is approximately located on the unit circle. This random walk behavior in the outer regimes indicates that the logarithm of the real maize time series moves from an outer regime (upper or lower) to another, passing through the middle (ground) regime, swiftly. This quick transition between regimes is confirmed from the high value of the slope of the transition function, $\gamma = 77.22$, as reported in Table 2.12. The model is similar to the TAR model with three regimes such that the outer ones display similar pattern. The most striking feature of the model is the asymmetry of regimes, which is a common feature of commodity prices. The dynamics of the model can be traced from the plotted transition function, $G(y_{t-2}; \gamma, c_1, c_2)$ in Figure 2.30 or its dot plot presentation in Figure 2.27.

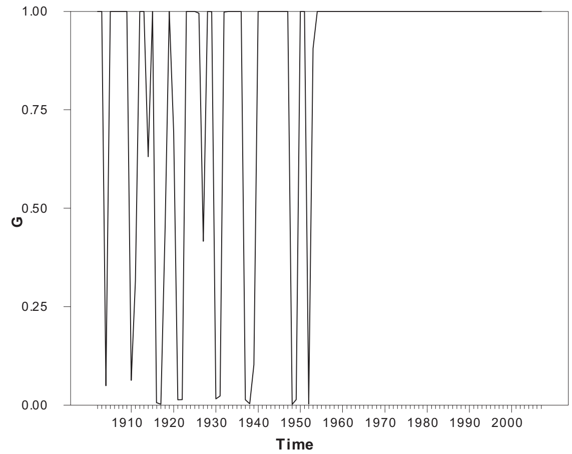


Figure 2.30: Transition function of the LSTR2 model fitted to the logarithm of real maize price time series between 1900 and 2007.

Rice Although the logarithm of the real rice price process is described by the same model of that of the real maize price (LSTR(2) model), the former displays an opposite dynamic. The most prominent root in the ground (mid) regime of the real rice price time series is a real root with a modulus of 1.06. This implies a random walk behavior in the ground regime. The outer regimes however are stationary regimes with a prominent pair of complex roots having a modulus of 0.93 and a period of 8.8 years. The transition function is plotted in Figure 2.31. This behavior implies that the price process is stationary in any expansionary or contractionary regime, yet it can wander randomly in the middle regime. The transition between the regimes is smooth as observed from the moderate value of the slope of the transition function, $\gamma = 2.42$, as reported in Table 2.12. This smooth transition can also be viewed from the dot plot of the transition function in Figure 2.28.

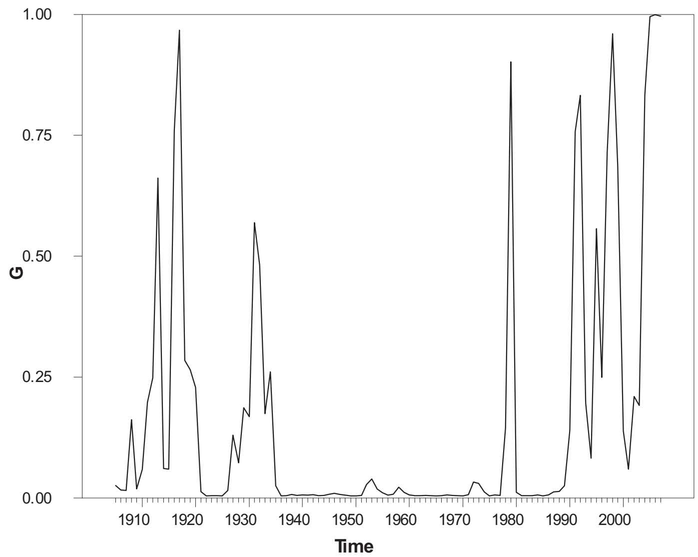


Figure 2.31: Transition function of the LSTR2 model fitted to the logarithm of real rice price time series between 1900 and 2007.

Sugar The best fitted model to the logarithm of real sugar price is still an LSTR model, yet it only displays two stationary regimes (LSTR(1)). The lower regime is dominated by a real root of modulus 0.91; the upper regime is dominated by a pair of complex roots with a modulus of 0.78 and a short period of 3.3 years. The value added of the smooth transition can be viewed from the moderate value of the slope of the transition function, $\gamma = 2.47$, as reported in Table 2.12. The dot plot and the plot of the transition function, in Figures 2.29 and 2.32 respectively, also confirm that.

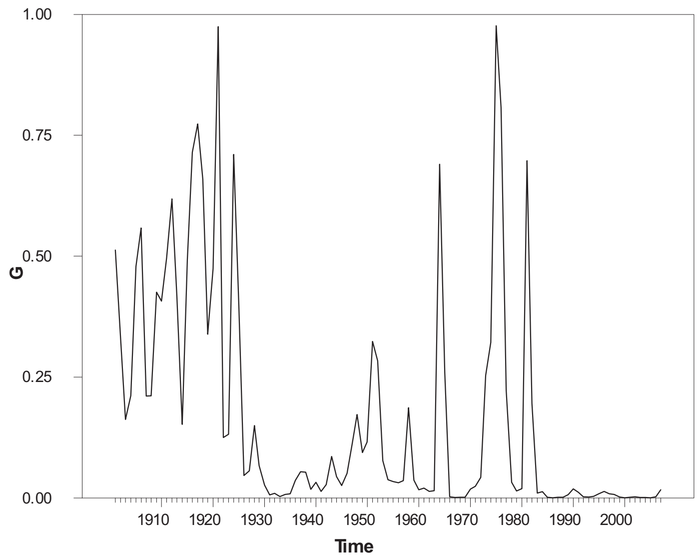


Figure 2.32: Transition function of the LSTR1 model fitted to the logarithm of real sugar price time series between 1900 and 2007.

Group D: STR models where the threshold variable is an external variable

The last and the most important group in our classification is Group D, where the transition variables are external variables that are capable of explaining the dynamics of the price processes. The importance of this group is twofold: First, it highlights the role of the border price classification in selecting a common transition variable for different commodity prices. The group consists of five commodities: Hides, bananas, palm oil, timber, and coffee. All time series were recorded on a CIF basis except hides, which was recorded on a FOB basis. The transition variables that showed the highest rejections of linearity tests were oil price for all CIF commodities and inflation for the FOB price time series (hides). This is consistent with our claim that oil prices play a major role in the behavior of the commodities recorded on a CIF basis. This also confirms the connection between commodity prices, oil, and consumer prices that was discussed before (see page 36-38). Second, the classification of the commodities in Group D motivates the use of external transition variables in the STR models.

The estimation and misspecification tests results for all the five price processes in Group D are reported in Table 2.14. All estimated coefficients are significant at the 5% percent level of significance and the models pass all misspecifications tests. The transition from one regime to the other was smooth in case of banana and coffee. This is confirmed from the moderate slope of the transition functions in both cases (see Table 2.14). Dot plots of the transition functions in this group of commodities are shown in the following figures.

Group D: STR models with external threshold variables

$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + (\theta_0 + \theta_1 y_{t-1} + \theta_2 y_{t-2} + \theta_3 y_{t-3}) G(\gamma, c_1, c_2; s_t) + \varepsilon_t$,

where $G(\cdot) = (1 + \exp\{-\gamma(s_t - c_1)\}/\sigma_s)^{-1}$ or $G(\cdot) = (1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)\}/\sigma_s^2)^{-1}$.

Commodity Price	Hides	Bananas	Palm oil	Timber	Coffee
$AIC(p)$	$p = 3$	$p = 3$	$p = 3$	$p = 1$	$p = 1$
Transition; Model	Π_{t-1}; LSTR(1)	g_{t-1}; LSTR(1)	g_t; LSTR(2)	g_t; LSTR(1)	g_{t-1}: LSTR(2)
ϕ_0	$\phi_0 = 0$	$\phi_0 = 0$	$\phi_0 = 0$	-0.23 (0.06)	-0.11 (0.06)
ϕ_1	0.65 (0.08)	0.96 (0.09)	0.77 (0.05)	0.93 (0.03)	0.83 (0.05)
ϕ_2	$\phi_2 = 0$	-0.31 (0.13)	$\phi_2 = 0$		
ϕ_3	0.25 (0.08)	0.28 (0.09)	$\phi_3 = 0$		
θ_0	-0.20 (0.09)	0.08 (0.03)	0.08 (0.04)	0.22 (0.06)	-0.42 (0.19)
θ_1	$\theta_1 = 0$	$\theta_1 = 0$	0.61 (0.13)	$\theta_1 = 0$	$\theta_1 = 0$
θ_2	$\theta_2 = 0$	$\theta_2 = 0$	-1.18 (0.22)		
θ_3	$\theta_3 = 0$	$\theta_3 = 0$	0.91 (0.18)		
γ	30.6 (132)	9.8 (18.43)	14.7 (21.48)	251 (1420)	0.5 (0.55)
c_1	0.10 (0.01)	0.14 (0.06)	-0.64 (0.20)	-0.37 (0.32)	-0.71 (0.23)
c_2			0.07 (0.01)		0.56 (0.23)
$\widehat{\sigma}_s$†	0.05	0.20	0.20	0.20	0.20
$LM_{AUTO(1)}$	0.72(0.40)	0.74(0.39)	0.39(0.24)	‡NA	0.77(0.38)
$LM_{AUTO(8)}$	0.74(0.65)	1.03(0.42)	0.79(0.62)	‡NA	0.54(0.82)
$LM_{ARCH(1)}$	0.24(0.62)	0.006(0.94)	0.16(0.69)	0.20(0.65)	2.42(0.12)
$LM_{ARCH(4)}$	3.69(0.45)	2.34(0.67)	0.94(0.92)	6.52(0.16)	4.18(0.38)
$NRNL$	0.31	0.25	0.20	‡NA	0.32
$PC(1)$	1.12(0.34)	3.1(0.05)	1.64(0.69)	‡NA	0.15(0.93)
$PC(2)$	1.48(0.19)	1.76(0.11)	1.46(0.17)	‡NA	0.68(0.67)
$PC(3)$	1.21(0.30)	1.31(0.24)	1.36(0.24)	‡NA	0.57(0.82)
JB	3.74(0.15)	2.1(0.36)	1.51(0.36)	1.98(0.37)	9.94(0.007)
$SK; EK$	-0.23; 3.8	-0.23; 3.5	-0.22; 3.4	0.22; 3.51	0.62; 3.9

Figures in parentheses beneath the parameter values are standard deviations and those after the test statistics are p-values.

‡Matrix inversion problem. † σ_s is the sample standard deviation of the transition variable $s_t = y_{t-j}$, for $j = 1, 2, ..., p$.

Table 2.14: Estimation and evaluation of Group D: STR models with external threshold variables.

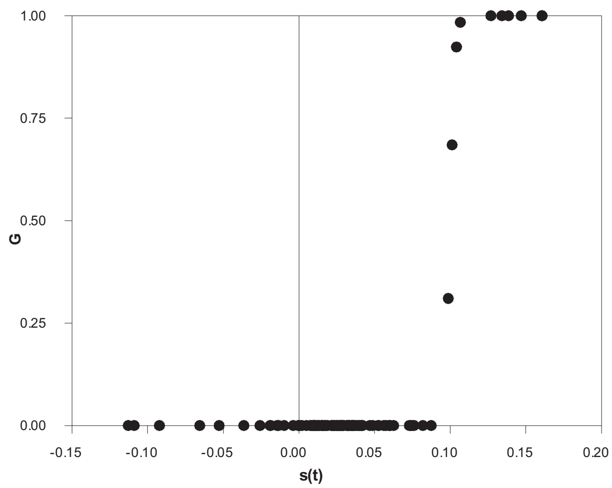


Figure 2.33: Transition function in the LSTR1 model fitted to the logarithm of real hides as a function of observations. Each dot corresponds to one observation. The transition variable s_t is inflation.

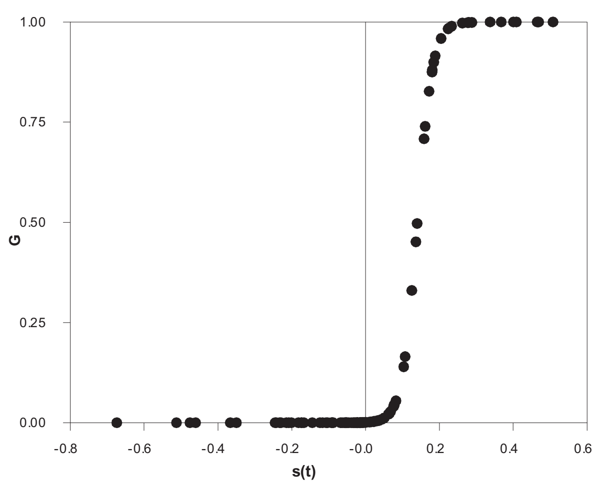


Figure 2.34: Transition function in the LSTR1 model fitted to the logarithm of real bananas as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the growth rate of real oil price.

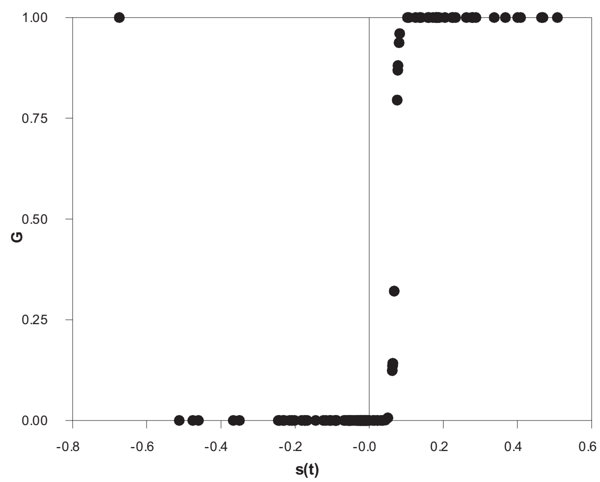


Figure 2.35: Transition function in the LSTR2 model fitted to the logarithm of real palm as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the growth rate of real oil price.

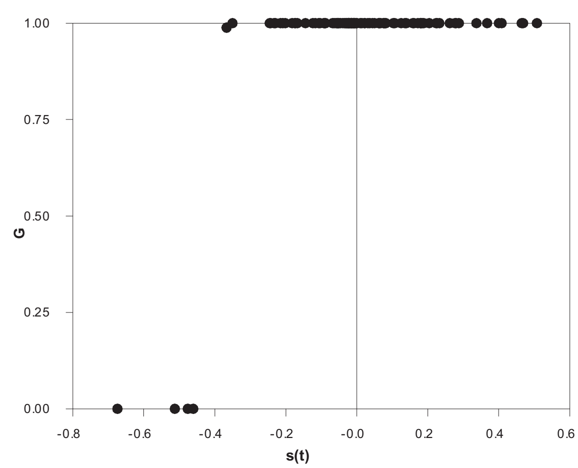


Figure 2.36: Transition function in the LSTR1 model fitted to the logarithm of real timber as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the growth rate of real oil price.

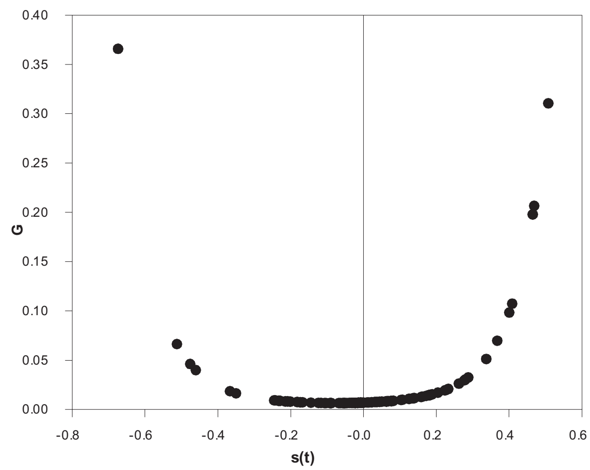


Figure 2.37: Transition function in the LSTR2 model fitted to the logarithm of real coffee as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the growth rate of real oil price.

Characteristic roots in each regime for Group D†				
Commodity and Model	Regime	Root(s)	Modulus	Period
Hides: LSTR(1)	Lower	0.94	0.94	
	Upper	0.94	0.94	
Banana: LSTR(1)	Lower	0.95	0.95	
	Upper	0.95	0.95	
Palm oil: LSTR(2)	Lower (Mid)	$0.15 \pm 0.91i$	0.92	4.5
		1.07	1.07	
	Upper (Outer)	0.77	0.77	
Timber: LSTR(1)	Lower	0.93	0.93	
	Upper	0.93	0.93	
Coffee: LSTR(2)	Lower (Mid)	0.83	0.83	
	Upper (outer)	0.83	0.83	
†Most prominent roots are only reported.				

Table 2.15: The most prominent roots of the characteristic polynomials in the regimes of the estimated LSTR models for the logarithm of the real commodity prices in Group D, 1900-2007.

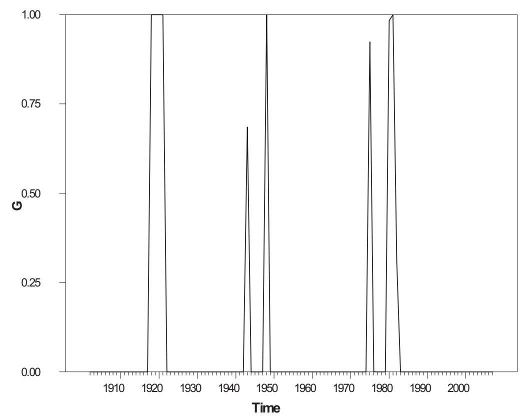


Figure 2.38: Transition function of the LSTR1 model fitted to the logarithm of real hides price time series between 1900 and 2007.

Dynamic Analysis

Hides, Banana, Timber, and Coffee The logarithm of the real price of hides, banana, and timber time series are all characterized by stationary real roots in both upper and lower regimes of the fitted LSTR(1) model, see Table 2.15. The logarithm of the real coffe price process is also locally stationary in the ground regime as well as the outer regimes of the fitted LSTR(2) model. The transition from the lower to the upper regime is smooth in the case of banana. The smooth transition can also be observed in the case of coffee, where the time series moves from an outer regime to another, passing through the ground regime, smoothly as seen from the transition functions in Figures 2.38, 2.39, 2.40, and 2.41 respectively.

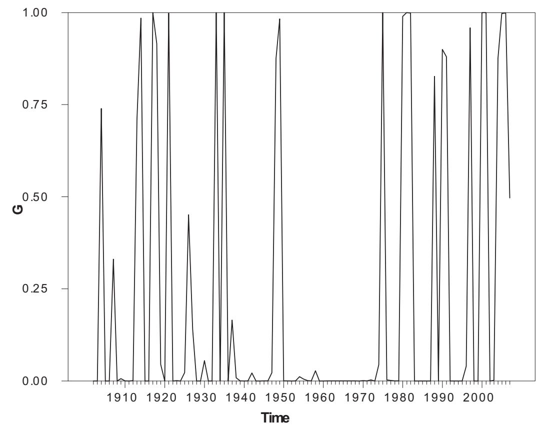


Figure 2.39: Transition function of the LSTR1 model fitted to the logarithm of real banana price time series between 1900 and 2007.

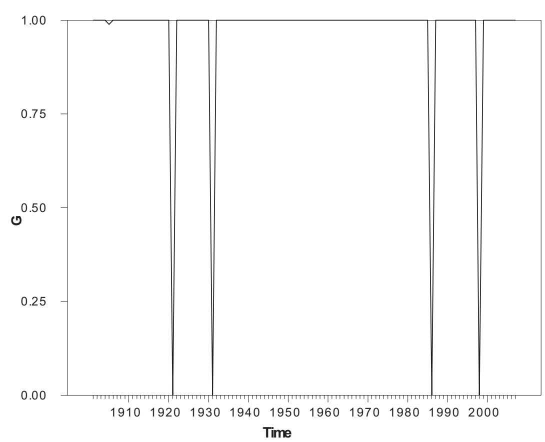


Figure 2.40: Transition function of the LSTR1 model fitted to the logarithm of real timber price time series between 1900 and 2007.

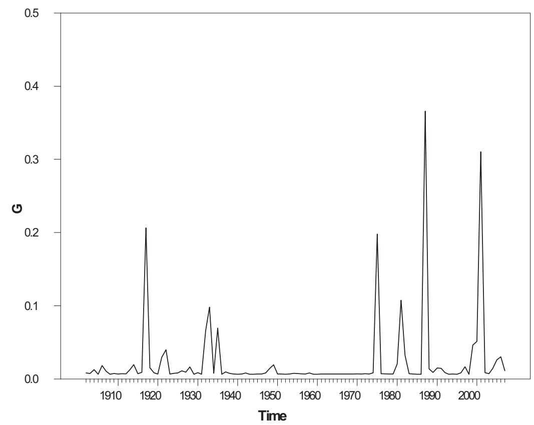


Figure 2.41: Transition function of the LSTR2 model fitted to the logarithm of real coffee price time series between 1900 and 2007.

Palm Oil The logarithm of real palm oil price time series was the only exception in this group. The ground regime is characterized by a real root of 1.07, see Table 2.15, so that the process is behaving in a random walk fashion in the middle regime. The outer regimes (expansionary or contractionary) are, on the other hand, stationary with a prominent real root of 0.77. The near random walk behavior in the middle regime indicates that the passage of the time series process from an outer regime to the other is random; it could stay in the same original regime or swiftly switch to the other outer regime. It could even stay in the ground regime for a while before shifting to another outer regime; see the 1960's period of the plotted transition function in Figure 2.42.

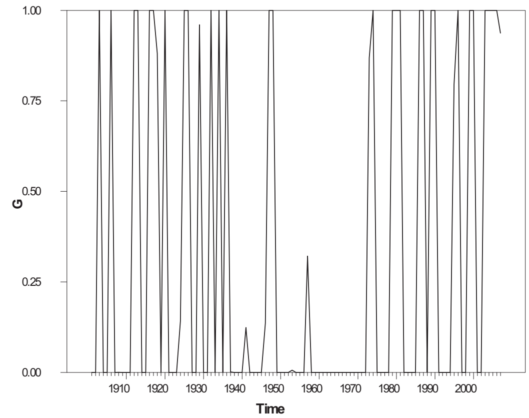


Figure 2.42: Transition function of the LSTR2 model fitted to the logarithm of real palm oil price time series between 1900 and 2007.

2.10 Concluding Remarks

In addition to the autoregressive lags of the dependent variable that are commonly used by the majority of the studies in the smooth transition literature, this study attempts to use external economic variables as potential transition variables that are capable of explaining the nonlinearities in the Grilli and Yang commodity price index. Two particular transition variables were proven successful: Inflation and the price of oil. More specifically, in case of inflation, the one period lag inflation, Π_{t-1}, was the variable with the strongest test rejection among all potential transition candidates and the associated model was the LSTR(1) model. The fitted model passed all misspecification tests and showed how changes in consumer prices (inflation) can explain the dynamics of the logarithm of the real Grilli and Yang commodity price index. The feedback from commodity prices to consumer prices was also demonstrated when inflation was modelled as dependent variable and the growth rate of the real commodity price index was the transition variable. Hence, the bidirectional relation between consumer prices and commodity prices. In case of oil, the one period lag of the logarithm of real oil price, R_{t-1}, was the second successful transition variable that was capable of capturing the nonlinearity in the Grilli and Yang commodity price index. Both variables (inflation and oil) can be seen as complements in the sense that inflation was capable of capturing the early fluctuations in the index while oil price captured the late ones. This result motivates the use of external variables as predetermined transition variables in the smooth transition regression model.

Modelling the dynamic behavior of individual commodity prices processes was slightly different than the index itself. Some price processes were best suited by ARCH or ST-ARCH models (*Group A*). The dynamics of these processes are described in the following chapter. Some price series fitted our nonlinear regime switching framework (*Group C and D*); and others were even linear (*Group B*). The rationale behind such different modelling rests on the recorded border price of the time series under consideration. FOB and CIF price processes were best modelled by the smooth transition regression model with oil price as transition variable in case of CIF prices. Settlement and auction prices were best fitted by ARCH or ST-ARCH models. This result provides further insight into the observed co-movement in commodity prices (see Figure 1.1, page 6) and attempts to explain it through the investigation of the recorded border price of the time series. The analysis also points out to the significance of examining the characteristics of individual price processes prior to fitting the suitable model that is capable of describing their dynamics.

Chapter 3

Regime Switching in Variance

3.1 Introduction

In this chapter, we extend the regime switching framework discussed in Chapter 2 to model the regime switching in the conditional variance of commodity prices. The idea of changing dynamics in the conditional variance equation of a time series emerged in the early financial literature on pricing assets and modelling their returns. A variety of econometric models of changing conditional variance have been suggested in the literature. The first influential attempt was due to Engle (1982), who suggested the Autoregressive Conditional Heteroskedasticity (ARCH) model, in which the conditional variance of a time series is allowed to change over time as a function of past errors.

Let

$$y_t = \Phi' w_t + \xi_t, \tag{3.1}$$

where y_t is a scalar, $w_t = (1, y_{t-1}, ..., y_{t-p})', \Phi' = (\phi_0, \phi_1, ..., \phi_p)$, and

$$\xi_t = \sigma_t Z_t, \qquad Z_t \sim i.i.d.(0,1). \tag{3.2}$$

The conditional variance equation takes the following general form

$$\sigma_t^2 = \sigma^2(\xi_{t-1}, \xi_{t-2}, ..., \sigma_{t-1}^2, \sigma_{t-2}^2, ..., x_t, b), \tag{3.3}$$

where x_t is a vector of predetermined variables and b is a vector of parameters. From (3.3), an ARCH(p) model can be expressed as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \xi_{t-i}^2, \tag{3.4}$$

where ω and all the α's are nonnegative. Engle's model was generally extended by Bollerslev's (1986) generalized ARCH (GARCH) model, which added a more flexible lag structure to the conditional variance equation by expressing σ_t^2 as a linear function of its past values as well as the past squared innovations. A GARCH(p, q) model is expressed as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \xi_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{3.5}$$

where ω is positive, the α's and β's are nonnegative, and $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$. GARCH models captured a common feature of asset returns; volatility clusters (periods of high volatility and low volatility alternate). Although ARCH and GARCH specifications have proven successful in modelling many phenomena (see, for instance, Engle (1982), Bollerslev (1986), Weiss (1984), and Milhoj (1987)), they have many limitations as first noted by Nelson (1991). One important limitation of GARCH models is that the current volatility is only a function of the magnitude (not the sign) of past innovations. Therefore, GARCH models fail to capture the asymmetric response of the conditional variance to positive versus negative news, which is another common feature of stock returns known as the leverage effect (see Black (1976), Christie (1982), and Schwert (1990)). To account for this asymmetry in the conditional variance, many alternative specifications to (3.4) have been suggested in the literature. Nelson (1991) proposed the specification

$$\ln \sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \ln \sigma_{t-j}^2 + \sum_{k=1}^{\infty} \alpha_k g(Z_{t-k}), \tag{3.6}$$

with

$$g(Z_{t-k}) = \theta Z_{t-k} + \gamma(|Z_{t-k}| - E|Z_{t-k}|).$$

Here the logarithm of the conditional variance is expressed as a linear function of a constant, its past values, and some function of the lagged Z_t's. The selected function $g(Z_t) = \theta Z_t + \gamma(|Z_t| - E|Z_t|)$, which is a function of both the size and the magnitude of Z_t, gives rise to different slopes depending on the value of Z_t; the slope of $g(Z_t)$ is $\theta + \gamma$ over the range $0 < Z_t < \infty$ and $\theta - \gamma$ over the range $-\infty < Z_t \leq 0$. Therefore, $g(Z_t)$ allows σ_t^2 to change asymmetrically to positive and negative shocks.

This idea of changing slopes generated a number of threshold ARCH and GARCH specifications. Zakoïan (1994), for instance, proposed a threshold GARCH (T-GARCH(p, q)) model where the conditional standard deviation is driven by linear combinations of its past values and the sign of past innovations as

$$\sigma_t = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j} + \sum_{i=1}^{p} \left(\alpha_i^+ \xi_{t-i}^+ - \alpha_i^- \xi_{t-i}^- \right), \tag{3.7}$$

where $\xi_t^+ = \max(\xi_t, 0)$ and $\xi_t^- = \min(\xi_t, 0)$ are the positive and negative parts of ξ_t respectively. If the distribution of Z_t is symmetric, the effect of a shock ξ_{t-k}, for $k \leq p$, on the present volatility is proportional to $(\alpha_k^+ - \alpha_k^-)$. Consider, for instance, the case where $p = 1$ and $\alpha_1^- > \alpha_1^+$. Then negative news have high impact on volatility compared to positive news. A similar threshold model but in the conditional variance (instead of the standard deviation) was proposed by Glosten, Jagannathan, and Runkle (1993), where the conditional variance is written as a linear function of the squared positive and negative parts of the noise as

$$\sigma_t^2 = \omega + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \left(\alpha_i \xi_{t-i}^2 + \gamma_i S_{t-i}^- \xi_{t-i}^2 \right), \tag{3.8}$$

where $S_t^- = 1$ if $\xi_t < 0$ and $S_t^- = 0$ otherwise.

The common characteristic shared by the previous threshold models is, as stated by González-Rivera (1998), the existence of only two regimes: low volatility regime which is triggered by positive shocks and high volatility regime which is triggered by negative shocks. So, they are pure threshold models where the threshold is known and is equal to zero. Another common feature, still according to González-Rivera (1998), is that all these models are customary applied to stock returns. In her paper, she accounts for intermediate regimes or states (in addition to the two volatility regimes) by introducing a smooth-transition specification to the conditional variance equation. She also applies the model to exchange-rate data. Smooth transition models in conditional variance can be found in the work of González-Rivera (1998), Lee and Degennaro (2000), Hagerud (1996), Lundbergh and Terasvirta (1998), Anderson et al. (1999), Lubrano (2001), and Lanne and Saikkonen (2005) among others. Those models share the common characteristic that they all allow for intermediate states or regimes in the conditional variance equation. They, however, differ in defining the smooth transition function that describes the transition between the regimes. A smooth transition GARCH(p, d, q) is defined as

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_{1i}\xi_{t-i}^2 + \left(\sum_{i=1}^{p} \alpha_{2i}\xi_{t-i}^2\right) F(s_{t-1}, c, \gamma) + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{3.9}$$

where $F(s_{t-1}, c, \gamma)$ is the smooth transition function, s_{t-1} is the smooth transition variable, c is the threshold parameter, and γ is the smoothness parameter. A popular choice of the smooth transition function that is employed by the advocates of the smooth transition GARCH models is the logistic function expressed in general as

$$F(s_{t-1}, c, \gamma) = [1 + \exp\{-\gamma(s_{t-1} - c)\}]^{-1} - \frac{1}{2}, \qquad \gamma > 0. \tag{3.10}$$

González-Rivera (1998) set $s_{t-1} = \xi_{t-d}$, $d \leq p$, $c = 0$, and entered the smooth parameter γ with a positive sign in (3.10). In particular, she specified

$$F(\xi_{t-d}, \gamma) = \left[1 + \exp\{\gamma\xi_{t-d}\}\right]^{-1} - \frac{1}{2}, \qquad \gamma > 0. \tag{3.11}$$

Observe that when $\xi_{t-d} \to -\infty$, $F(\xi_{t-d}, \gamma) = 1/2$ and when $\xi_{t-d} \to +\infty$, $F(\xi_{t-d}, \gamma) = -1/2$. Therefore, the transition function is a bounded function between $-1/2$ and $1/2$.

Note also that the α_{1i}'s in (3.9) have to be nonnegative but the α_{2i}'s can be positive or negative that is because the sufficient conditions[1] to ensure that the conditional variance in (3.9) is strictly positive are: $\omega > 0$, $\alpha_{1i} \geq 0$, $\alpha_{1i} \geq \frac{1}{2}|\alpha_{2i}|$, for $i = 1, ..., p$, and $\beta_j \geq 0$, for $j = 1, ..., q$. Since her objective was to make the effect of bad news (negative ξ_{t-d}) larger on conditional variance σ_t^2 than positive news (positive ξ_{t-d}) and she already set a positive γ in (3.11), the α_{2i}'s should have the same sign as γ and, therefore, were also assumed positive.

Hagerud (1996) set $s_{t-1} = \xi_{t-i}$ and $c = 0$ in (3.10). He suggested two specifications: the logistic specification

$$F(\xi_{t-i}, \gamma) = \left[1 + \exp\{-\gamma \xi_{t-i}\}\right]^{-1} - \frac{1}{2}, \qquad \gamma > 0, \tag{3.12}$$

and the exponential specification

$$F(\xi_{t-i}, \gamma) = 1 - \exp\{-\gamma \xi_{t-i}^2\}, \qquad \gamma > 0. \tag{3.13}$$

If the logistic specification in (3.12) is used, the dynamics of the conditional variance will change according to the *sign* of the news. If, on the other hand, the symmetric exponential specification in (3.13) is used, the conditional variance will change according to the *magnitude* of the innovations. To introduce the possibility of asymmetry in the conditional variance to positive versus negative news, Lubrano (2001) used a threshold parameter c in (3.13) as

$$F(\xi_{t-i}, \gamma) = 1 - \exp\{-\gamma (\xi_{t-i} - c)^2\}, \qquad \gamma > 0.$$

[1]See Milhoj (1985) and Tjostheim (1986).

Lee and Degennaro (2000), Lundbergh and Terasvirta (1998), Anderson et al. (1999), Lubrano (2001), and Lanne and Saikkonen (2005) also used the logistic specifications to model the conditional variance.

In this chapter, we follow a somewhat similar approach in modeling the regime switching in variance. we use the smooth transition logistic function in modelling the dynamics of the regimes but we do not restrict the transition variable to be one of the lagged innovations or squared innovations as in the previously mentioned studies; we also consider the possibility that the transition variable could be an external variable such as inflation and oil price. Unlike most of the studies on the regime switching on conditional variance where the models are empirically applied to stock returns data, this thesis applies the smooth transition regression models to commodity data. In particular, the regime switching in variance model is applied to the Grilli & Yang commodity price index and to the individual commodities forming the index. Another distinction between this thesis and previous work is that, in addition to the first order logistic function that is widely used, it also considers the second order logistic function when modelling the regime switching in the conditional variance of the price processes under consideration. The second order logistic function allows for three regimes; a mid regime and two outer regimes that can characterize the variance equation.

The plan of this chapter is as follows. Section 3.2 introduces the smooth transition regression in variance model and applies it to the logarithm of the real Grilli & Yang commodity price index with inflation and oil price as potential external transition variables. Section 3.3 applies the model to the individual commodities forming the index and Section 3.4 concludes.

3.2 Regime Switching in the Commodity Price Index

When Engle's (1982) LM test of no ARCH was applied to the error terms of the autoregressive specification of the commodity price index in the preliminary stage of modelling nonlinearity in mean (see Chapter 2, page 43), the null hypothesis of no ARCH was accepted at the 5% level of significance. This implies that including past innovations or past square innovations in the conditional variance equation of the commodity index is not required. Therefore, to model nonlinearity in the conditional variance, we suggest a simple smooth threshold model where the conditional variance takes different values in different regimes following the behavior of a transition variable around a threshold value such that the transition between the regimes is smooth. The model captures the asymmetric response of the conditional variance of the commodity price index to changes in the external transition variables. Following the rationale discussed in chapter 2, inflation rate and oil price are the external potential transition variables that will be used in our nonlinearity analysis. The model is introduced in the following section and then applied to the Grilli & Yang commodity index using inflation and oil price as potential external transition candidates.

3.2.1 The Smooth Transition Regression in Variance Model

Let

$$y_t = \Phi' w_t + \xi_t, \tag{3.14}$$

where y_t is a scalar, $w_t = (1, y_{t-1}, ..., y_{t-p})'$, $\Phi' = (\phi_0, \phi_1, ..., \phi_p)$, and let $\xi_t = Z_t\sqrt{h_t}$, $Z_t \sim i.i.d.(0,1)$. Define the conditional variance equation as

$$h_t = \alpha_1 F(s_t; \gamma, c) + \alpha_2(1 - F(s_t; \gamma, c)), \tag{3.15}$$

where α_1 and α_2 are parameters and $F(s_t; \gamma, c)$ is a first order logistic transition function.[2] A first order logistic transition function is defined as

$$F(s_t; \gamma, c) = [1 + \exp\{-\gamma(s_t - c)]^{-1}, \qquad \gamma > 0, \tag{3.16}$$

where c is a threshold parameter, γ is the smoothness parameter and $\gamma > 0$ is an identifying restriction. s_t is the transition variable; it could be one of the past innovations ξ_{t-i}, or the squared innovations ξ^2_{t-i}, or even an external variable. Serving our purpose of modelling the nonlinearities in commodity prices and following the framework of Chapter 2, we use the inflation rate and the oil price as the potential external transition candidates that are capable of explaining the dynamics of the commodity price index.[3] The first order logistic function $F(s_t; \gamma, c)$ defined in (3.16) is a bounded function between zero and one, $0 < F(s_t; \gamma, c) < 1$. The value of the smoothness parameter γ is the one that governs the definition of $F(s_t; \gamma, c)$. For large values of γ; that

[2] Experimenting with the second order logisitic function did not yield satisfactory results. The fitted models failed to pass the misspecification tests and the estimated parameters were insignificant.

[3] We have also tried to use the past innovations and the past squared innovations as potential transition variables, but the models did not converge.

is, when $\gamma \to +\infty$ and the transition variable is above the threshold value, i.e., $s_t > c$, then $F(s_t; \gamma, c) = 1$ and the conditional variance in (3.15) is $h_t = \alpha_1$; this defines the upper regime. If the transition variable is below the threshold value, i.e., $s_t < c$, and γ is still large, then $F(s_t; \gamma, c) = 0$ and $h_t = \alpha_2$; this defines the lower regime. Also note that when $\gamma = 0$ (its minimum value), $F(s_t; \gamma, c) = \frac{1}{2}$ and the conditional variance $h_t = \frac{1}{2}(\alpha_1 + \alpha_2)$. So, in general, γ takes any value between 0 and $+\infty$, and $F(s_t; \gamma, c)$ takes any value between 0 and 1. This implies that the smooth transition regression (STR) in variance model allows for time-varying shifts in the parameters of the conditional variance equation. The model also allows for asymmetric response of the conditional variance equation to positive versus negative news. The news in this context represent the change in the behavior of the transition variable. Suppose, for instance, that y_t is a commodity price index and the external transition variable in (3.16) is the one period lag inflation rate ($s_t = \Pi_{t-1}$). In case of a negative news, i.e., in case that $s_t > c$, the transition function approaches 1 and the conditional variance $h_t = \alpha_1$, whereas in case of a positive news, $h_t = \alpha_2$. The two parameters α_2 and α_1 in equation (3.15) change monotonically as a function of s_t from α_2 to α_1 and allows for asymmetric effect of external news on the conditional variance of the commodity price index. To ensure the non-negativity of the conditional variance equation, we impose the restrictions that $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$.

Following the nonlinearity analysis framework developed in Chapter 2, we start by testing the null hypothesis that the conditional variance of the commodity price index is linear against the alternative that it follows the STR model defined in (3.15). An LM test for nonlinearity is introduced and applied to the commodity index in the following subsection. The estimation and misspecification tests results from fitting the STR in variance model are reported and analyzed in the subsequent sections.

3.2.2 Testing for Nonlinearity

In this subsection we derive an LM test for the nonlinearity in the conditional variance equation. We follow the same procedure of Granger and Teräsvirta (1993). We test the null hypothesis $H_0 : \gamma = 0$ against the alternative $H_1 : \gamma > 0$. The problem with testing for nonlinearity in variance is similar to the one encountered when testing for nonlinearity in mean (see Chapter 2, page 26); that is, the STR in variance model is only identified under the alternative hypothesis because the nuisance parameters c and α_2 are only identified under the alternative. To solve this problem, we follow Luukkonen et al. (1988) who, based on a paper by Davies (1977), suggested to replace the transition function in (3.16) by a Taylor approximation about the null hypothesis $\gamma = 0$.

Let

$$F(s_t; \gamma, c) = \left[(1 + \exp\{-\gamma(s_t - c)\})^{-1} - 1/2 \right], \tag{3.17}$$

where the 1/2 was subtracted to facilitate the Taylor approximation. The third order

Taylor approximation about γ is

$$T_3(s_t,\gamma,c) \simeq F(\cdot)|_{\gamma=0} + \frac{\partial F(\cdot)}{\partial \gamma}|_{\gamma=0}\gamma + \frac{\partial^2 F(\cdot)}{\partial \gamma^2}|_{\gamma=0}\frac{\gamma^2}{2!} + \frac{\partial^3 F(\cdot)}{\partial \gamma^3}|_{\gamma=0}\frac{\gamma^3}{3!}, \tag{3.18}$$

where, at $\gamma = 0$, the first and third terms of the right hand side of equation (3.18) are zeros and will drop. The second and fourth terms are

$$\frac{\partial F(\cdot)}{\partial \gamma}|_{\gamma=0} = \frac{1}{4}(s_t - c) \tag{3.19}$$

and

$$\frac{\partial^3 F(\cdot)}{\partial \gamma^3}|_{\gamma=0} = -\frac{1}{8}(s_t - c)^3. \tag{3.20}$$

The test procedure can be summarized in the following steps. First, apply OLS to (3.14) and obtain the residuals $\hat{\xi}_t$ and the squared residuals $\hat{\xi}_t^2$. Next, since the conditional variance h_t is not observed, we use the squared residuals $\hat{\xi}_t^2$ as a proxy for h_t in (3.15) as

$$\hat{\xi}_t^2 = \alpha_1 F(s_t;\gamma,c) + \alpha_2(1 - F(s_t;\gamma,c))$$

or simply

$$\hat{\xi}_t^2 = \alpha_2 + \delta F(s_t;\gamma,c), \tag{3.21}$$

where $\delta = (\alpha_1 - \alpha_2)$. Replace $F(s_t;\gamma,c)$ in (3.21) by its third order Taylor approximation expressed in (3.18) and run the following auxiliary regression

$$\hat{\xi}_t^2 = \alpha_2 + \delta T_3(s_t;\gamma,c) + \epsilon_t. \tag{3.22}$$

After reparameterization and collection of terms, equation (3.22) can be expressed as

$$\hat{\xi}_t^2 = \psi_0 + \psi_1 s_t + \psi_2 s_t^2 + \psi_3 s_t^3 + \epsilon_t, \tag{3.23}$$

where

$$\psi_0 = \alpha_2 - \frac{1}{4}\gamma c\delta + \frac{1}{48}\gamma^3 c^3 \delta,$$

$$\psi_1 = \frac{1}{4}\gamma\delta - \frac{1}{16}\gamma^3 c^2 \delta,$$

$$\psi_2 = \frac{1}{16}\gamma^3 c\delta,$$

and

$$\psi_3 = -\frac{1}{48}\gamma^3\delta.$$

Because each ψ_j, $j = 1, 2, 3$, is of the form $\gamma\tilde{\psi}_j$, where $\tilde{\psi}_j \neq 0$ is a function of δ, the null hypothesis of linearity is then $H_{0L} : \psi_1 = \psi_2 = \psi_3 = 0$. To compute the LM statistic, estimate (3.23) under the null hypothesis using OLS and compute the sum of squares of the residuals (SSR_0). Repeat the same procedure under the alternative hypothesis and compute SSR_1. The LM statistic is computed as $LM = \frac{T(SSR_0 - SSR_1)}{SSR_1}$, where T is the number of observations. The test statistic has an asymptotic χ^2-distribution with 3 degrees of freedom when the null hypothesis is valid. The LM can be approximated by an F-distribution with 3 and $T - 4$ degrees of freedom when the null hypothesis H_{0L} is valid.

The STR in variance model is estimated using maximum likelihood method. The log-likelihood function at time t can be expressed as

$$l_t(\Phi, \alpha_1, \alpha_2, \gamma, c) = -\frac{1}{2}\ln(h_t) - \frac{1}{2}\frac{\xi_t^2}{h_t}. \tag{3.24}$$

Maximum likelihood estimators of the parameters $\Phi, \alpha_1, \alpha_2, \gamma$, and c can be obtained by maximizing the log-likelihood function in the same way as in Chapter 2. Also, following the economic rationale of Chapter 2, we use the one period lag inflation, $\Pi_{t-1} = \triangle \log(CPI_{t-1})$, and the one period lag of the logarithm of real oil price, $R_{t-1} = \log\left(\frac{Oil\ price}{CPI}\right)_{t-1}$, as our transition candidates. The model is applied and estimated in each case in the following subsections.

3.2.3 Switching Variable: Inflation

Consider the specification in (3.14), (3.15), and (3.16), with $y_t = \log\left(\frac{GYCPI}{MUV}\right)_t$ and $s_t = \Pi_{t-1} = \triangle \log(CPI_{t-1})$. Applying the nonlinearity test discussed in the previous subsection, the computed $SSR_0 = 0.06849$ and the $SSR_1 = 0.061224$. The computed F-statistics equals 4.11 whereas, at the 5% level of significance with 3 and 104 degrees of freedom, the tabulated $F = 2.70$. Therefore, we reject the null hypothesis that the conditional variance equation is linear against the alternative of an STR in variance model with lag inflation as transition variable. The estimated model and the misspecification tests results are reported as follows.

$$
\begin{aligned}
y_t &= \underset{(0.01)}{0.001} + \underset{(0.03)}{0.97} y_{t-1} + \widehat{\xi}_t, \\
h_t &= \underset{(0.004)}{0.02}\, F_t + \underset{(3\times10^{-4})}{0.006}\, (1 - F_t), \qquad (3.25) \\
F_t(\Pi_{t-1}, \gamma, c) &= \left(1 + \exp(-\underset{(19.6)}{82}\, (\Pi_{t-1} - \underset{(6\times10^{-4})}{0.03})/0.047)\right)^{-1}, \\
Q(1) = 0.18, \quad & Q(8) = 0.55, \quad K_3 = -0.4, \quad K_4 = 3.9, \quad JB = 0.06,
\end{aligned}
$$

where $\widehat{\sigma}_{\Pi}= 0.047$ is the standard deviation of the transition variable. The figures in parentheses beneath the model's parameters are standard deviations. $Q(q)$ are the p-values of Ljung-Box (1978) test of no serial correlation of order q in the standardized residuals $\xi_t/\sqrt{h_t}$. JB is the p-value of the Jarque-Bera test of normality of $\xi_t/\sqrt{h_t}$ and K_3 and K_4 are skewness and kurtosis respectively. All model parameters are significant as seen from the standard deviations reported. The estimated slope of the transition function is large, $\widehat{\gamma} = 82$ indicating that the conditional variance moves swiftly from one regime to another. This can be viewed from the transition function plotted in Figure 3.2. The estimated threshold parameter of the STR in variance model ($\widehat{c} = 3\%$) takes the same value as the estimated threshold parameter from the TAR(1) model (see equation (2.28) page 59). However, it is lower than the value of the estimated threshold parameter in the STR in mean model (7%) (see equation (2.24), page 47). Both the transition variable and the transition functions of the STR in mean model and the STR in variance model are plotted in Figure 3.1. The two estimated threshold parameters from both models are also shown in Panel (a) in Figure 3.1.

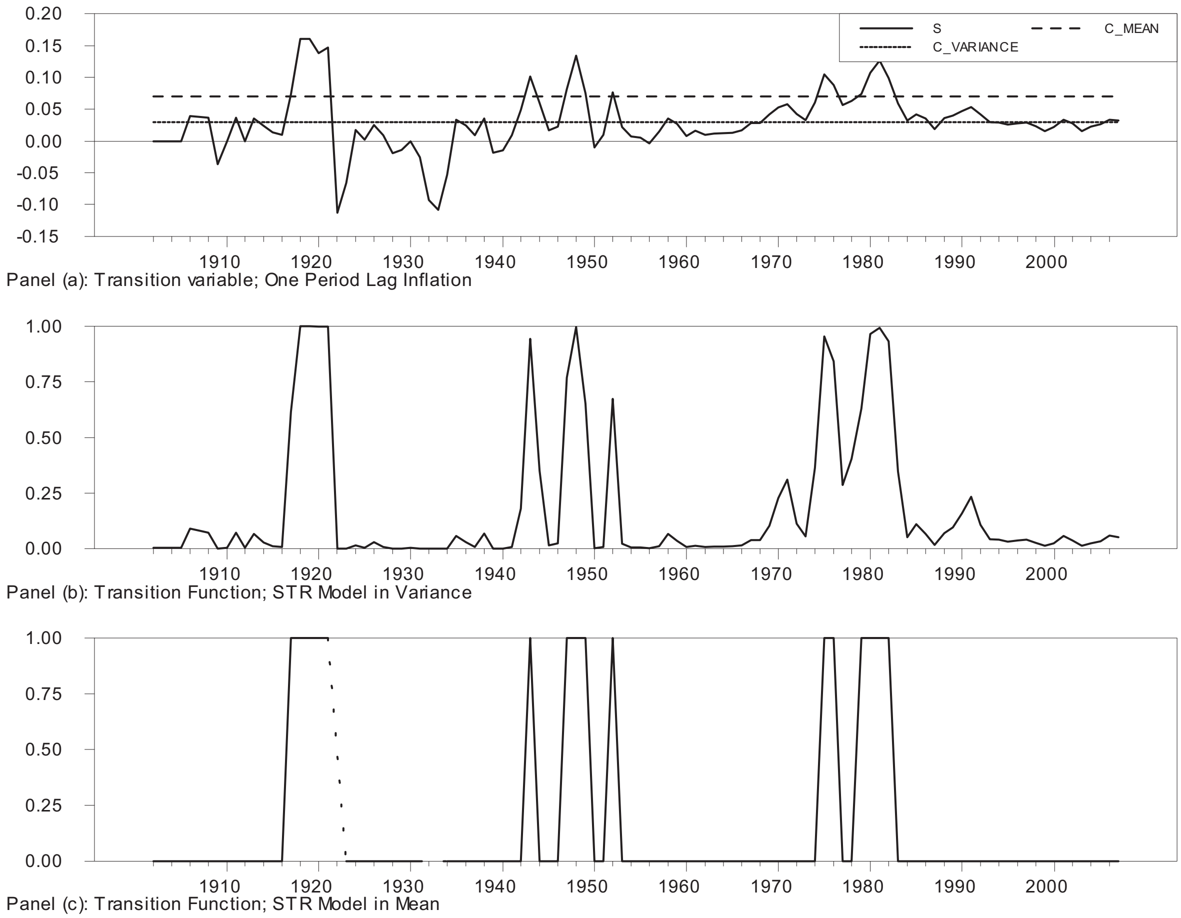


Figure 3.1: A plot of the transition variable Π_{t-1} (panel (a)), the transition function from the STR in variance model (panel (b)), and the transition function from the STR in mean model (panel (c)).

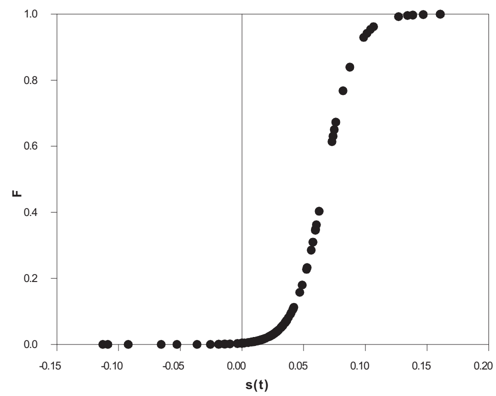


Figure 3.2: The transition function $F(s_t, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable s_t is the one period lag inflation.

Dynamic Analysis

Suppose that we are in the expansionary regime such that the one period lag inflation was higher than the threshold value, that is, $\Pi_{t-1} > 3\%$. The transition function $F_t(\Pi_{t-1}, \gamma, c)$ tends to 1 and the conditional variance of the error term $h_t = 0.02$, which is approximately three times its value in the lower regime ($h_t = 0.006$), where the one period lag inflation falls below 3% and the transition function tends to zero. This result implies that the conditional variance equation of the commodity index is larger in case of negative news (when inflation is above its threshold value) as opposed to positive news or, in other words, high inflation rates have stronger impact on the conditional variance of the commodity price index as opposed to low rates.

A closer look at the transition function, $F(\cdot)$ in Figure 3.2, Panel (b), one can observe roughly the same dynamics observed in the transition function, $G(\cdot)$, of the regime switching in mean model of Chapter 2 (Panel (c) in the same figure). Observe how both transition functions display the same dynamics during the periods 1918 till 1921, during the 1940s and early 1950s, and during the two oil shocks in 1974 and pre 1984. Also in the late period, both functions exhibit one lower regime. We can conclude that inflation as a transition variable in both models (the STR in mean and the STR in variance) failed to capture the *late dynamics* in the commodity price index time series. This is, perhaps, because of the intervention of the U.S. central bank to keep its main monetary policy target (inflation) at its target level. Observe, in Figure 3.1 (panel (a)), the steady inflation rate fluctuating in a small band around 3%. This, in turn, justifies the low regimes observed in the mean and variance equations of the commodity index.

The previous result also implies that the two models can be seen as substitutes when modelling nonlinearity in the commodity price index using inflation as threshold variable. Although the transition functions of both models displayed similar dynamics in the limiting regimes, it should be noted that the two models (the STR in mean and the STR in variance) explain the behavior of commodity prices from two different angles: in the regime switching in variance model, the transition function, $F(\cdot)$, is modelling switching regimes in the conditional variance equation while the mean equation is unchanged, whereas, in the regime switching in mean model, the transition function, $G(\cdot)$, is modelling switching regimes in mean leaving the variance equation unchanged. Therefore, the former explains how the variance of the Grilli &

$s_t = \Pi_{t-1}$	STR in mean model	STR in variance model
Threshold value	0.07	0.03
Upper Regime:		
Model	$y_t = 0.009 + 0.54y_{t-1} + \widehat{\varepsilon}_t$	$y_t = 0.001 + 0.97y_{t-1} + \widehat{\xi}_t$
Error variance	$\sigma^2 = 0.01$	$h_t = 0.02$
Behavior of y_t	$Stationary\ AR(1)$	$Near\ Random\ Walk$
Lower Regime:		
Model	$y_t = 0.009 + 0.98y_{t-1} + \widehat{\varepsilon}_t$	$y_t = 0.001 + 0.97y_{t-1} + \widehat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.006$
Behavior of y_t	$Near\ Random\ Walk$	$Near\ Random\ Walk$

Table 3.1: Comparison between the STR in mean and the STR in variance model with one period lag inflation rate as the transition variable.

Yang commodity price index responds to fluctuations in lag inflation rate around a threshold, while the later explains how the index itself does so. Table 3.1 summarizes the results obtained from fitting both models.

3.2.4 Switching Variable: Oil

In this subsection, we re-estimate the STR in variance model using the one period lag logarithm of real oil price as the transition variable; that is, $s_t = R_{t-1} = \log\left(\frac{Oil\ price}{CPI}\right)_{t-1}$. The estimated model and the misspecification tests results are reported as follows.

$$
\begin{aligned}
y_t &= \underset{(0.01)}{0.005} + \underset{(0.03)}{0.95} y_{t-1} + \widehat{\xi}_t, \\
h_t &= \underset{(0.01)}{0.03} F_t + \underset{(0.01)}{0.008} (1 - F_t), \qquad (3.26) \\
F_t(R_{t-1}, \gamma, c) &= \left(1 + \exp(-\underset{(23.8)}{17}\,(R_{t-1} + \underset{(0.07)}{3.1})/0.38)\right)^{-1},
\end{aligned}
$$

$$
Q(1) = 0.27, \qquad Q(8) = 0.35, \qquad K_3 = -0.3, \qquad K_4 = 3.9, \qquad JB = 0.05,
$$

where $\hat{\sigma}_R = 0.38$ is the standard deviation of the transition variable. The figures in parentheses beneath the model's parameters are standard deviations. The moderate slope of the transition function $\hat{\gamma} = 17$ suggests a smooth transition between the regimes. The transition function is plotted in Figure 3.3.

In case of a negative news (upper regime), i.e., when the oil price is above its threshold value, the transition function $F_t(R_{t-1}; \gamma, c)$ tends to 1 and the conditional variance of the error term $h_t = 0.03$, which is approximately the same result that was obtained in the STR in variance model with inflation as transition variable (see Table 3.1). On the other hand, in case of a positive news (lower regime), the oil price is below its threshold value and the conditional variance $h_t = 0.008$, which is, again, approximately equal to the same value obtained from the STR in variance

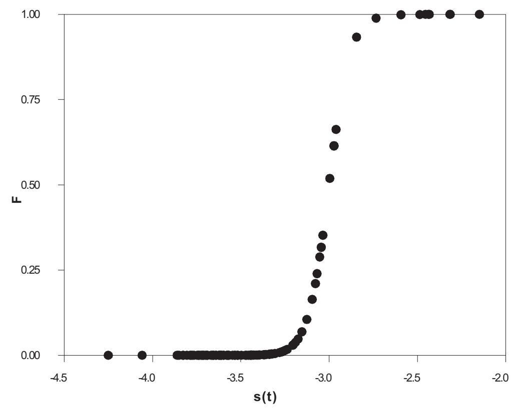


Figure 3.3: The transition function $F(s_t, \gamma, c)$ as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag logarithm of real oil price.

model with inflation as transition variable. This result implies that the price of oil as a transition variable in both the STR in mean and the STR in variance models failed to capture the *early dynamics* in the commodity price index. Figure 3.4, Panel (b) shows the transition function in the STR in variance model with oil as transition variable. Observe how the function captures only the late dynamics after the 1974 oil shock. This same conclusion was reached when the price of oil was used in the STR in mean model (see Figure 3.4, Panel (c)). Table 3.2 summarizes the results obtained from fitting both models with $s_t = R_{t-1}$. These results also imply that the two models can be seen as substitutes when modelling nonlinearity in the commodity price index using the oil price as threshold variable.

To sum up, we can conclude that both models (the STR in mean and the STR in variance) can be seen as *substitutes* when modelling nonlinearity in the Grilli & Yang commodity price index in the sense that both the mean and variance equations display the same dynamics. On the other hand, the transition variables (the inflation rate and the price of oil) can be seen as *complements* in characterizing this nonlinear dynamics in the sense that inflation is capable of capturing the early dynamics whereas the price oil can capture the late ones. The reason we don't have only one common variable that can explain the behavior of the Grilli & Yang commodity price index is that the index itself consists of 24 individual price series each of which was recorded based on a different border price (FOB, CIF, settlement, and auction price). Further insight can be gained by classifying the 24 individual price series into groups according to their border prices and then modelling nonlinearity in each group using a common transition variable. This is the subject matter of the following section.

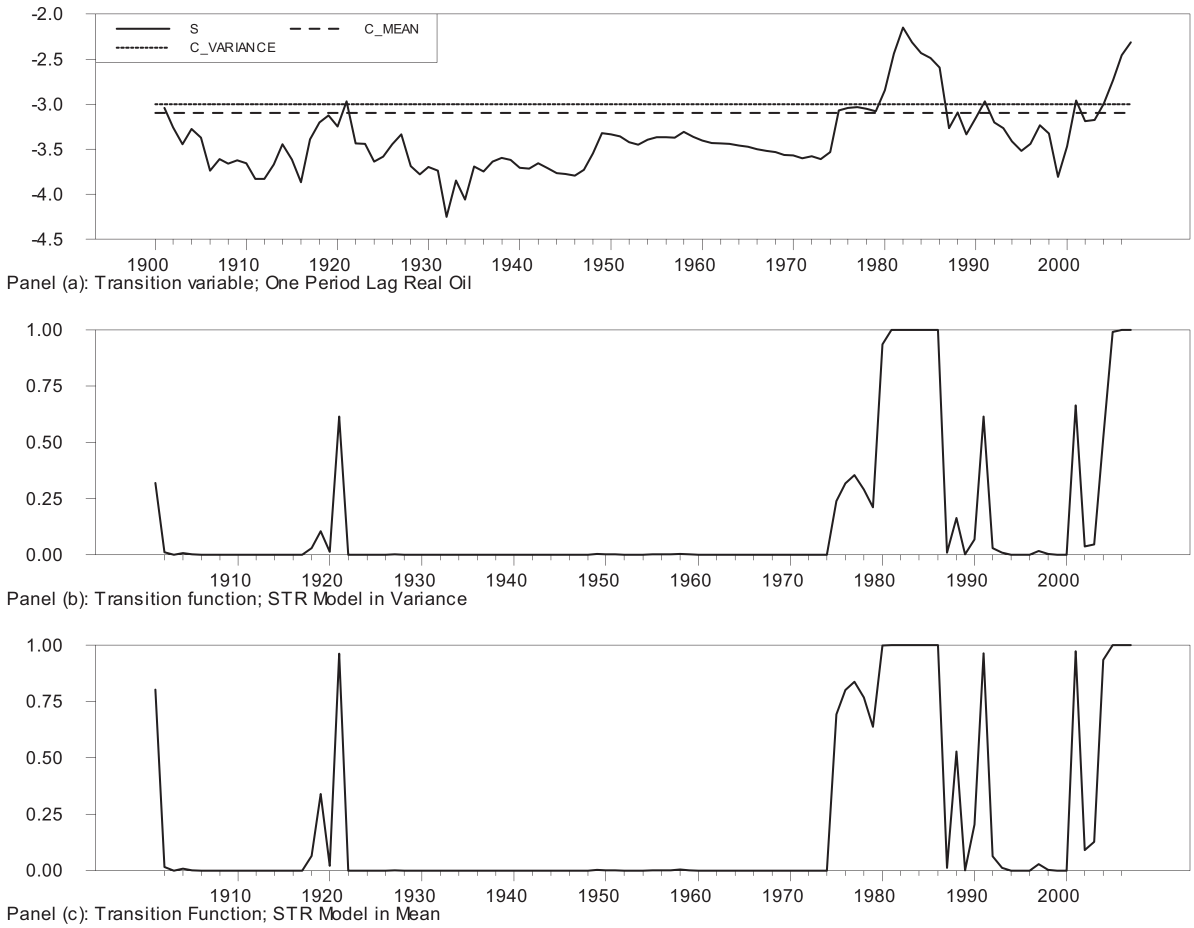


Figure 3.4: A plot of the transition variable R_{t-1} (panel (a)), the transition function from the STR in variance model (panel (b)), and the transition function from the STR in mean model (panel (c)).

$s_t = R_{t-1}$	STR in mean model	STR in variance model
Threshold	-3.1	-3.1
Upper Regime		
Model	$y_t = -0.09 + 0.58 y_{t-1} + \widehat{\varepsilon}_t$	$y_t = 0.005 + 0.95 y_{t-1} + \widehat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.03$
Behavior of y_t	$Stationary\ AR(1)$	$Near\ Random\ Walk$
Lower Regime		
Model	$y_t = 0.009 + 0.98 y_{t-1} + \widehat{\varepsilon}_t$	$y_t = 0.005 + 0.95 y_{t-1} + \widehat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.008$
Behavior of y_t	$Near\ Random\ Walk$	$Near\ Random\ Walk$

Table 3.2: Comparison between the STR in mean and the STR in variance model with the one period lag logarithm of real oil price as the transition variable.

3.3 Regime Switching in Individual Commodities

As mentioned in Chapter 2, Section 2.3, page 20, the first stage in modelling nonlinearity using regime switching models is the specification stage, in which a linear autoregressive model is selected as the starting point of the nonlinearity analysis. If the linear model is adequate, i.e., if it passes all preliminary diagnostic tests, the analysis is then taken to the next step, where nonlinearity is tested. On the other hand, if the preliminary linear model fails the misspecification tests, the model is no longer adequate for nonlinearity analysis and the researcher should look for a different model. Out of the 24 individual commodities forming the Grilli & Yang index, 8 commodities displayed ARCH patterns in the residuals of their fitted linear preliminary autoregressive models and, accordingly, those preliminary linear AR models were not suitable starting models for nonlinearity analysis. Those processes were classified in *Group A*, where ARCH or smooth transition ARCH (ST-ARCH) models are entertained. The value added of ST-ARCH models is their ability to capture the time varying volatility of a time series and its asymmetric responses to previous positive or negative shocks. Analyzing the dynamics of this group of commodities involves modelling the conditional variance of the time series processes and, therefore, is best suited in this chapter. The characteristics of the ST-ARCH model and the estimation and misspecification tests results of fitting this model to the individual commodities forming *Group A* are presented and discussed in Subsection 3.3.2. Some other processes did not pass our nonlinearity tests and were classified in *Group*

B (see Table 2.11, page 94). Consequently, they were dropped from the analysis. The remaining processes were classified into two groups: *Group C* and *Group D*. In the former group, all commodities were recorded on a FOB basis and the threshold variable was one of the autoregressive lags of the dependent variable. In the later group *(Group D)*, all commodities were recorded on a CIF basis and oil was the threshold variable that was capable of explaining their dynamics. In what follows we will model the nonlinearities in the conditional variances of the commodity price processes in those two groups. We will test for nonlinearity first and once proven to be present, we will fit the STR in variance and perform diagnostic tests and analyze the dynamics of the limiting processes.

3.3.1 Smooth Transition Regression in Variance Model

In this subsection, we will apply the smooth transition regression model to the conditional variance of the individual commodities times series in *Group C* and *Group D*. We will start first by testing the null hypothesis of linearity in the conditional variance against the alternative of STR in variance model. The test procedure was developed in Section 3.2.2, page 126. Another purpose of conducting the linearity test is to use the test results in model selection. We will use Teräsvirta's (1994) selection criteria discussed in Chapter 2, page 29, to select the type of the model; that is, to choose between LSTR(1) or LSTR(2) models. Once the model type is selected, the next step is to estimate the model, apply diagnostic tests, and analyze the dynamics of the results.

Group C: FOB Group

Group C consists of three commodity prices: rice, maize and sugar. In the case of rice, the null hypothesis of linearity in conditional variance was not rejected against the alternative of STR in variance with oil and with inflation as external transition variables. No further nonlinearity analysis was performed for the rice price time series. In case of maize and sugar, the null hypothesis of linearity was not rejected when the alternative hypothesis was the STR in variance model with oil as threshold variable. This is consistent with our border price rationale that oil plays no role in explaining the dynamics of commodity prices recorded on FOB basis. When the same nonlinearity test was applied to both commodity series with inflation as threshold variable, the null hypothesis of linearity was rejected and the selected nonlinear model was the LSTR(1) for both series. However, the estimated STR in variance model resulted in a poor fit for both series as the estimated parameters were insignificant and the log likelihood function did not converge. We could not also fit an ARCH or threshold ARCH model to those price processes as there were no ARCH pattern in there error terms; the null hypothesis of no ARCH up to order 4 could not be rejected for both time series (see Table 2.10, page 90). Therefore, the nonlinearity in this group of commodities is only captured in the mean and not in the conditional variance equation.

Group D: CIF Group

Group D consists of four commodity prices: banana, palm oil, timber, and coffee. The first step in our nonlinearity analysis is to test for nonlinearity. The LM test for nonlinearity discussed in Section 3.2.2, page 126, was applied to each of the four commodities in this group. The null hypothesis of linearity in conditional variance was tested against the alternative hypothesis of smooth transition regression model with external threshold variable. The set of potential transition variables included the current and the one period lag inflation rate, the current and the one period lag logarithm of real oil price, the current and the one period lag growth rate of real oil price. The best transition variable that was capable of modeling the nonlinearity in the conditional variance of all individual price processes in this group was the one period lag growth rate of real oil price defined as $r_{t-1} = \triangle \log \left(\frac{Oil\ price}{CPI}\right)_{t-1}$. In particular, in case of banana, coffee, and timber price processes, the null hypothesis of linearity in conditional variance was not rejected when inflation was the transition variable. The null hypothesis was rejected, however, when the test was repeated with the growth rate of real oil price as transition variable. As for the palm oil price process, the null hypothesis of linearity in conditional variance was rejected with both inflation and oil, but oil yielded the highest rejection. These results confirm our suggestion that commodity prices recorded on CIF basis are to a great extent driven by the price of oil.

Following the model selection criteria discussed in Chapter 2, page 26, the previous linearity test was used in model selection, i.e., in choosing between LSTR(1) or LSTR(2) models. The best model that fitted the four price processes in this group was the LSTR(2) model. In what follows, we introduce the LSTR(2) in variance model and discuss its characteristics. The estimation results are then reported and analyzed.

Consider the smooth transition regression in variance model specified in equation (3.14) and (3.15) with the conditional variance equation expressed as

$$h_t = \alpha_1 F(s_t; \gamma, c_1, c_2) + \alpha_2(1 - F(s_t; \gamma, c_1, c_2)), \tag{3.27}$$

where α_1 and α_2 are parameters and $F(s_t; \gamma, c_1, c_2)$ is a transition function. An LSTR(2) model defines $F(s_t; \gamma, c_1, c_2)$ as a second order logistic transition function expressed as

$$F(s_t; \gamma, c_1, c_2) = \left(1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)\}\right)^{-1}, \qquad \gamma > 0,\ c_1 \leq c_2, \tag{3.28}$$

where $\gamma > 0$ and $c_1 \leq c_2$ are identifying restrictions. Notice that, unlike the first-order logistic function, the second-order function is not zero at the minimum; it has a value

$$F_{\min} = \frac{1}{1 + \exp\{-\gamma \tilde{c}\}}, \tag{3.29}$$

where $\tilde{c} = c_1 c_2 - \bar{c}^2$ and $\bar{c} = \frac{c_1 + c_2}{2}$. The behavior displayed by the second-order logistic function depends on the value taken by the transition variable s_t. When the transition variable takes the value of any of the thresholds, i.e., $s_t = c_1$ or $s_t = c_2$, $F(s_t; \gamma, c_1, c_2) = \frac{1}{2}$. This characterizes the middle regime. The two outer regimes are achieved when $s_t \longrightarrow \pm\infty$ and $F(s_t; \gamma, c_1, c_2) = 1$. Thus, $F(s_t; \gamma, c_1, c_2)$ is a bounded function between $F_{\min}$ and 1.

To facilitate the interpretation of the regimes, we can apply the following reparameterization to the logistic function in (3.28). Let

$$F(\cdot) = F_{\min} + \tilde{F}(\cdot) \times (1 - F_{\min}), \tag{3.30}$$

where

$$\tilde{F}(\cdot) = \frac{F(\cdot) - F_{\min}}{(1 - F_{\min})}. \tag{3.31}$$

Substituting (3.30) in the conditional variance equation in (3.27) and reparameterizing yields

$$h_t = \lambda_1 + \lambda_2 \tilde{F}(s_t; \gamma, c_1, c_2), \tag{3.32}$$

where $\lambda_1 = \alpha_2 + (\alpha_1 - \alpha_2)F_{\min}$ and $\lambda_2 = (\alpha_1 - \alpha_2)(1 - F_{\min})$. Notice that $\tilde{F}(s_t; \gamma, c_1, c_2)$ is bounded between zero and one. In the ground (middle) regime, the transition variable can take the value of any of the thresholds parameters or a weighted average of both; that is, $s_t = \bar{c} = \frac{c_1 + c_2}{2}$, then $F(\cdot) = F_{\min}$ and $\tilde{F}(\cdot) = 0$. The conditional variance in this case is $h_t = \lambda_1$. The outer regimes are associated with $s_t \rightarrow \pm\infty$, which, in turn, implies that $F(\cdot) = 1$ and $\tilde{F}(\cdot) = 1$. The conditional variance in the outer regimes takes the value $h_t = \lambda_1 + \lambda_2 = \alpha_1$.

The results of applying the LSTR(2) in variance model to the individual commodity prices in this group are discussed and analyzed in the following subsections.

Banana

Let $y_t = \log\left(\frac{P_{banana}}{MUV}\right)_t$ and the transition variable $r_{t-1} = \triangle \log\left(\frac{Oil\ price}{CPI}\right)_{t-1}$. The estimation and misspecification tests results of the LSTR(2) in variance model are reported as follows.

$$
\begin{aligned}
y_t &= \underset{(0.013)}{0.03} + \underset{(0.09)}{0.89} y_{t-1} - \underset{(0.14)}{0.12} y_{t-2} + \underset{(0.10)}{0.12} y_{t-3} + \widehat{\xi}_t, \\
h_t &= \underset{(0.015)}{0.03} F(r_{t-1}, \gamma, c_1, c_2) + \underset{(0.001)}{0.007}(1 - F(r_{t-1}, \gamma, c_1, c_2)), \qquad (3.33)\\
F(r_{t-1}, \gamma, c_1, c_2) &= \left(1 + \exp\{-\underset{(15.1)}{9.8}(r_{t-1} + \underset{(0.04)}{0.22})(r_{t-1} - \underset{(0.03)}{0.48})/0.04\}\right)^{-1}, \\
Q(1) = 0.57, &\qquad Q(8) = 0.98, \qquad K_3 = -0.008, \qquad K_4 = 3.21, \qquad JB = 0.91,
\end{aligned}
$$

where $\hat{\sigma}_r^2 = 0.04$ is the sample variance of the transition variable. The figures in parentheses beneath the model's parameters are standard deviations. $Q(q)$ is the p-values of Ljung-Box (1978) test of no serial correlation of order q in the standardized residuals $\xi_t/\sqrt{h_t}$. JB is the p-value of the Jarque-Bera test of normality of $\xi_t/\sqrt{h_t}$ and K_3 and K_4 are skewness and kurtosis respectively. All model parameters are significant as seen from the standard deviations reported. To render the parameter γ scale-free, the exponent of the transition function was divided by the sample variance of the transition variable $\hat{\sigma}_r^2 = 0.04$. Following our reparameterization introduced above, the conditional variance in (3.33) can be expressed as

$$
h_t = 0.012 + 0.018 \tilde{F}(s_t; \gamma, c_1, c_2), \qquad (3.34)
$$

where $\lambda_1 = \alpha_2 + (\alpha_1 - \alpha_2)F_{\min} = 0.012$ and $\lambda_2 = (\alpha_1 - \alpha_2)(1 - F_{\min}) = 0.018$. The estimated two threshold values, $\widehat{c}_1 = -0.22$ and $\widehat{c}_2 = 0.48$, define the ground regime;

that is when the one period lag growth rate of real oil price is within the range of -0.22% and 0.48%, the variance of the time series is in the ground regime. Note that if the one period lag growth rate of oil is equal to the weighted average of the two thresholds, i.e., $r_{t-1} = \bar{c} = 0.13\%$, then $F(\cdot) = F_{\min}$ and $\tilde{F}(\cdot) = 0$, which implies that $h_t = \lambda_1 = 0.012$. The conditional variance will approximately double; that is, $h_t = \lambda_1 + \lambda_2 = 0.012 + 0.018 = 0.03$ in case of severe negative or positive shock where the one period lag growth rate of oil tends to $\pm\infty$. This defines the two outer regimes. The transition between the regimes is governed by the slope of the transition function $\gamma = 9.8$, which is a moderate slope in this case. The logarithm of real banana time series, the transition variable, and the transition function are plotted in Figure 3.5. Observe how the ground regime dominated the period between 1940s and the early 1980s. The late period witnessed high swings from one outer regime to another passing swiftly by the ground regime. The switching from one regime to the other is swift as seen from the dot plot of the transition function in Figure 3.6.

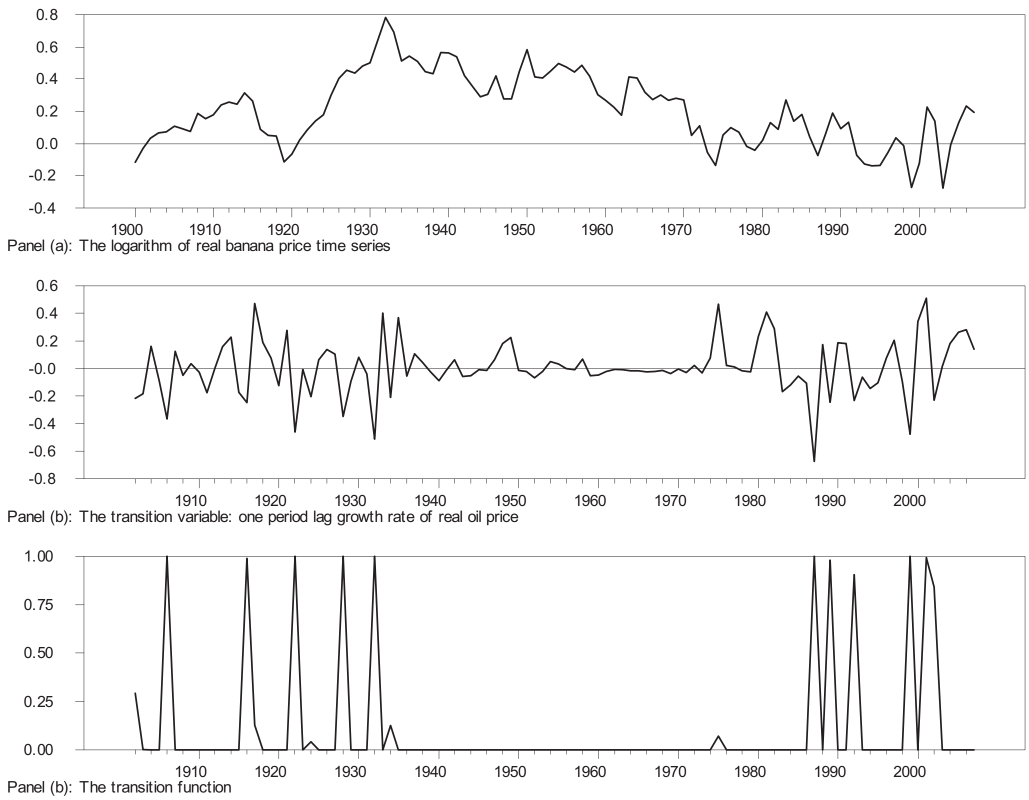


Figure 3.5: The logarithm of real banana price time series (Panel (a)), the one period lag growth rate of real oil price as transition variable (Panel (b)), and the transition function (Panel (c)) between 1900 and 2007.

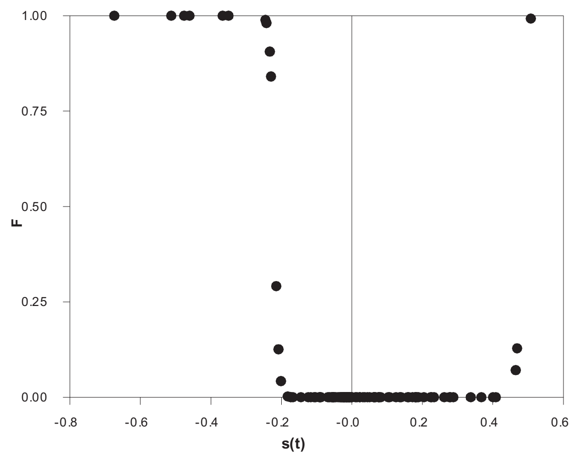


Figure 3.6: The transition function $F(s_t, \gamma, c_1, c_2)$ in case of banana price as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag growth rate of real oil price.

Palm oil

In case of palm oil, the transition variable that yielded the highest rejection was also the one period lag growth rate of real oil price $r_{t-1} = \triangle \log \left(\frac{Oil\ price}{CPI}\right)_{t-1}$. The estimation and misspecification tests results are reported as follows

$$
\begin{aligned}
y_t &= \underset{(0.016)}{0.003} + \underset{(0.09)}{1.03} y_{t-1} - \underset{(0.12)}{0.29} y_{t-2} + \underset{(0.08)}{0.25} y_{t-3} + \widehat{\xi}_t, \\
h_t &= \underset{(0.008)}{0.05} F(r_{t-1}, \gamma, c_1, c_2) + \underset{(0.001)}{0.001} (1 - F(r_{t-1}, \gamma, c_1, c_2)), \qquad (3.35) \\
F(r_{t-1}, \gamma, c_1, c_2) &= \left(1 + \exp\{-\underset{(88.5)}{173} (r_{t-1} - \underset{(0.007)}{0.10})(r_{t-1} - \underset{(0.005)}{0.18})/0.04\}\right)^{-1}, \\
Q(1) &= 0.74, \quad Q(8) = 0.45, \quad K_3 = 0.2, \quad K_4 = 3.48, \quad JB = 0.12,
\end{aligned}
$$

and

$$
h_t = 0.02 + 0.03 \tilde{F}(s_t; \gamma, c_1, c_2). \qquad (3.36)
$$

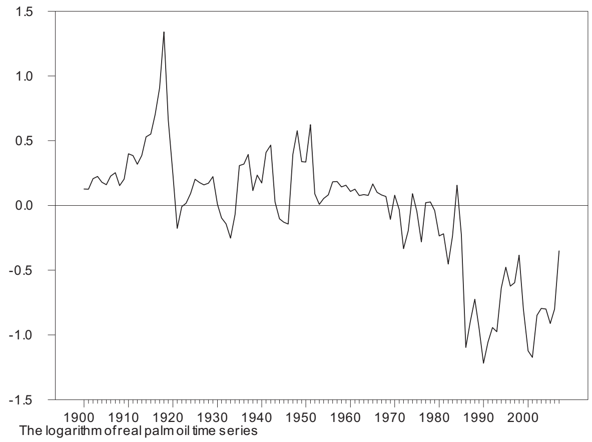


Figure 3.7: The logarithm of real palm oil time series between 1900 and 2007.

Here the ground regime is defined when the one period lag growth rate of real oil price takes a value between $\widehat{c}_1 = 0.10\%$ and $\widehat{c}_2 = 0.18\%$. The transition function $\tilde{F}(\cdot)$ attains its minimum when $r_{t-1} = \bar{c} = 0.14\%$ and the conditional variance $h_t = \lambda_1 = 0.02$. In the two outer regimes, the conditional variance takes the value $h_t = \lambda_1 + \lambda_2 = 0.02 + 0.03 = 0.05$. The transition between the regimes is swift due to the large slope of the transition function $\gamma = 173$; this can be detected from the dot plot of the transition function in Figure 3.9. Unlike the case of banana price time series, the palm oil price series switches from one outer regime to another swiftly and never stays for a long period of time in one regime. This is because the difference between the estimated two threshold values is negligible (approximately 0.1%). The sharp swings between regimes can be detected from Figure 3.8.

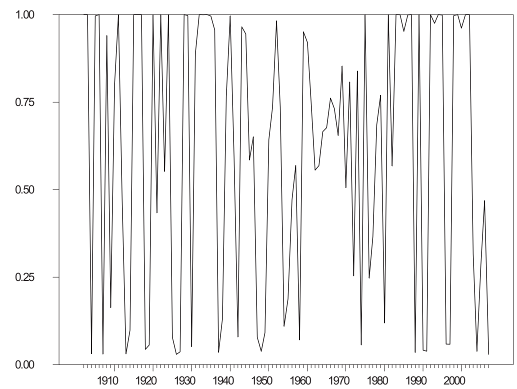


Figure 3.8: The transition function $F(r_{t-1}, \gamma, c_1, c_2)$ of the LSTR(2) in variance model in case of palm oil price time series between 1900 and 2007.

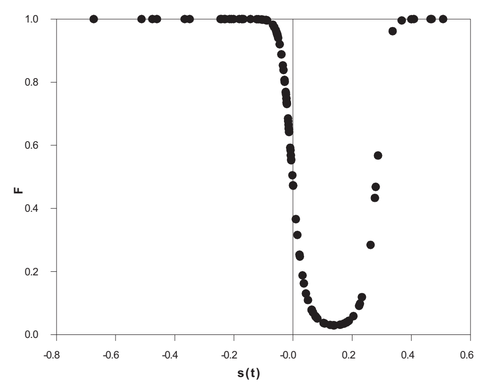


Figure 3.9: The transition function $F(s_t, \gamma, c_1, c_2)$ in case of palm oil price as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag growth rate of real oil price.

Coffee

The estimation and misspecification tests results for the coffee price time series are reported as follows.

$$
\begin{aligned}
y_t &= \underset{(0.04)}{-0.12} + \underset{(0.04)}{0.87} y_{t-1} + \widehat{\xi}_t, \\
h_t &= \underset{(0.012)}{0.08} F(r_{t-1}, \gamma, c_1, c_2) + \underset{(0.004)}{0.01} (1 - F(r_{t-1}, \gamma, c_1, c_2)), \qquad (3.37) \\
F(r_{t-1}, \gamma, c_1, c_2) &= \left(1 + \exp\{-\underset{(961)}{638} (r_{t-1} + \underset{(0.002)}{0.02})(r_{t-1} + \underset{(0.003)}{0.12})/0.04\}\right)^{-1}, \\
Q(1) &= 0.80, \quad Q(8) = 0.77, \quad K_3 = 0.25, \quad K_4 = 3.1, \quad JB = 0.57,
\end{aligned}
$$

and

$$
h_t = 0.02 + 0.06 \tilde{F}(s_t; \gamma, c_1, c_2). \qquad (3.38)
$$

The two threshold values in the case of coffee price series are $\widehat{c}_1 = -0.02\%$ and $\widehat{c}_2 = -0.12\%$; they both define the ground regime. The transition function in (3.38) attains its zero value (minimum value) when the transition variable attains a negative rate of growth of 0.07%; that is, $r_{t-1} = \bar{c} = -0.07\%$, and the conditional variance $h_t = \lambda_1 = 0.02$. In the outer regimes, however, the conditional variance is four times its value in the ground regime; that is, $h_t = \lambda_1 + \lambda_2 = 0.02 + 0.06 = 0.08$. Perhaps the most noticeable detail of (3.37) is the large standard deviation of the estimated slope of the logistic function, $\widehat{\gamma} = 638$. It is common, for LSTR models, that the estimated standard deviation of γ tends to be large for large values of γ. This is not crucial, however, as it will not affect either the shape of the logistic function or

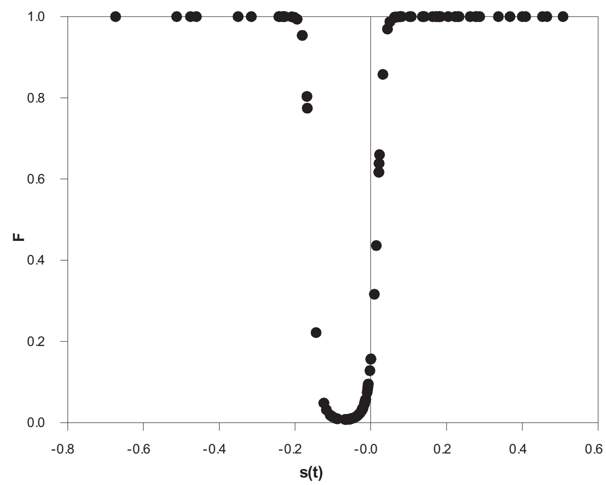


Figure 3.10: The transition function $F(s_t, \gamma, c_1, c_2)$ in case of coffee as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag growth rate of real oil price.

the other estimates of the model. Teräsvirta (1994) gave an example of this exact case and provided a discussion on the estimation issues of γ. The message delivered by this enormous slope of the transition function is that the conditional variance h_t will be moving sharply from one regime to the other. This can be detected from the transition function and its dot plot Figure 3.11, panel (c) and Figure 3.10 respectively.

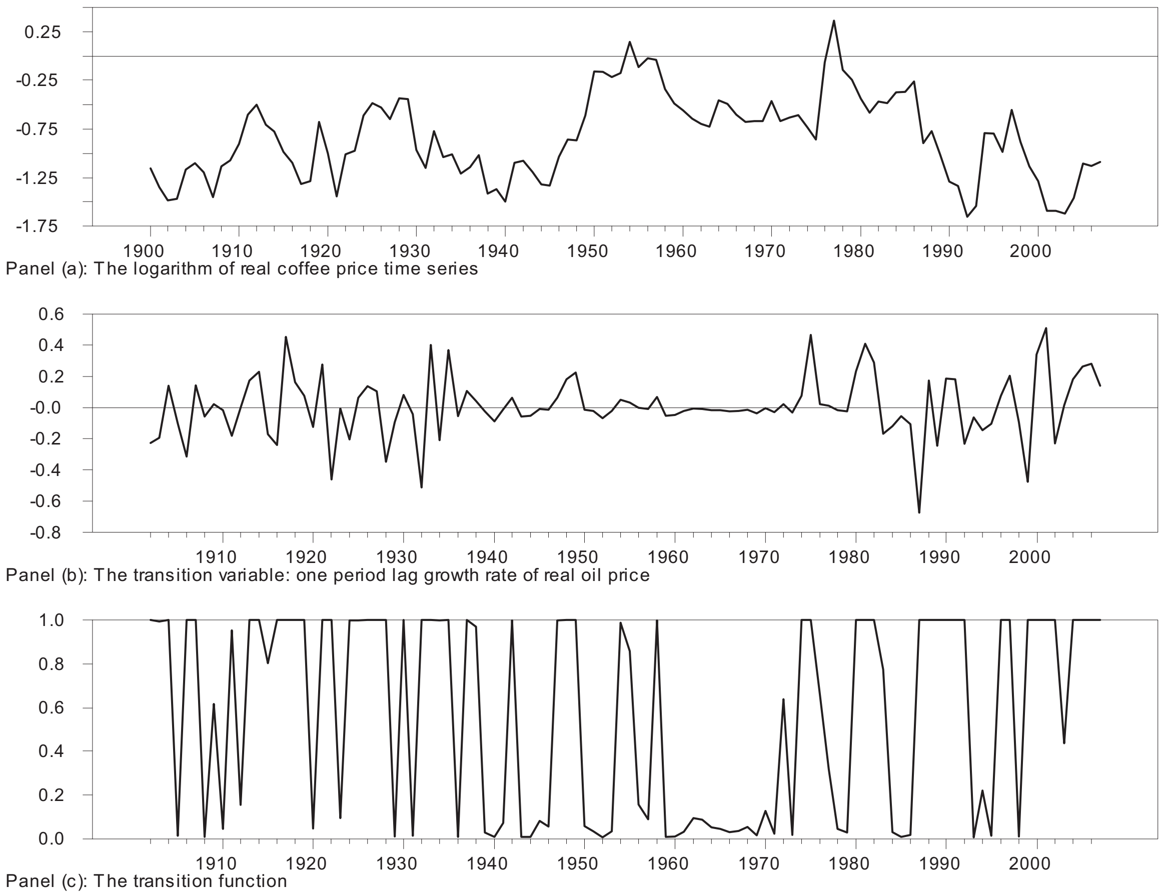


Figure 3.11: The logarithm of real coffee price time series (Panel (a)), the one period lag growth rate of real oil price as transition variable (Panel (b)), and the transition function (Panel (c)) between 1900 and 2007.

Timber

The estimation and misspecification tests results for the timber price time series are reported as follows.

$$y_t = \underset{(0.02)}{-0.03} + \underset{(0.03)}{0.92} y_{t-1} + \widehat{\xi}_t,$$

$$h_t = \underset{(0.004)}{0.02} F(r_{t-1}, \gamma, c_1, c_2) + \underset{(0.002)}{0.01} (1 - F(r_{t-1}, \gamma, c_1, c_2)), \tag{3.39}$$

$$F(r_{t-1}, \gamma, c_1, c_2) = \left(1 + \exp\{-\underset{(250)}{117}(r_{t-1} + \underset{(0.008)}{0.02})(r_{t-1} - \underset{(0.032)}{0.21})/0.04\}\right)^{-1},$$

$$Q(1) = 0.94, \qquad Q(8) = 0.51, \qquad K_3 = 0.08, \qquad K_4 = 3.04, \qquad JB = 0.95,$$

and

$$h_t = 0.011 + 0.008\tilde{F}(s_t; \gamma, c_1, c_2). \tag{3.40}$$

The conditional variance of the logarithm of real timber price series displayed similar dynamics to the case of banana and coffee especially in the ground regime that dominated the period between the 1940s and the early 1970s as seen from Figure 3.12. During this period, the growth rate of real oil price time series was pretty much stable as it fluctuated between -0.02% and 0.21% (the two threshold values). The late period witnessed high swings from one outer regime to another passing through the ground regime. The transition was swift as seen from the dot plot of the transition function in Figure 3.13.

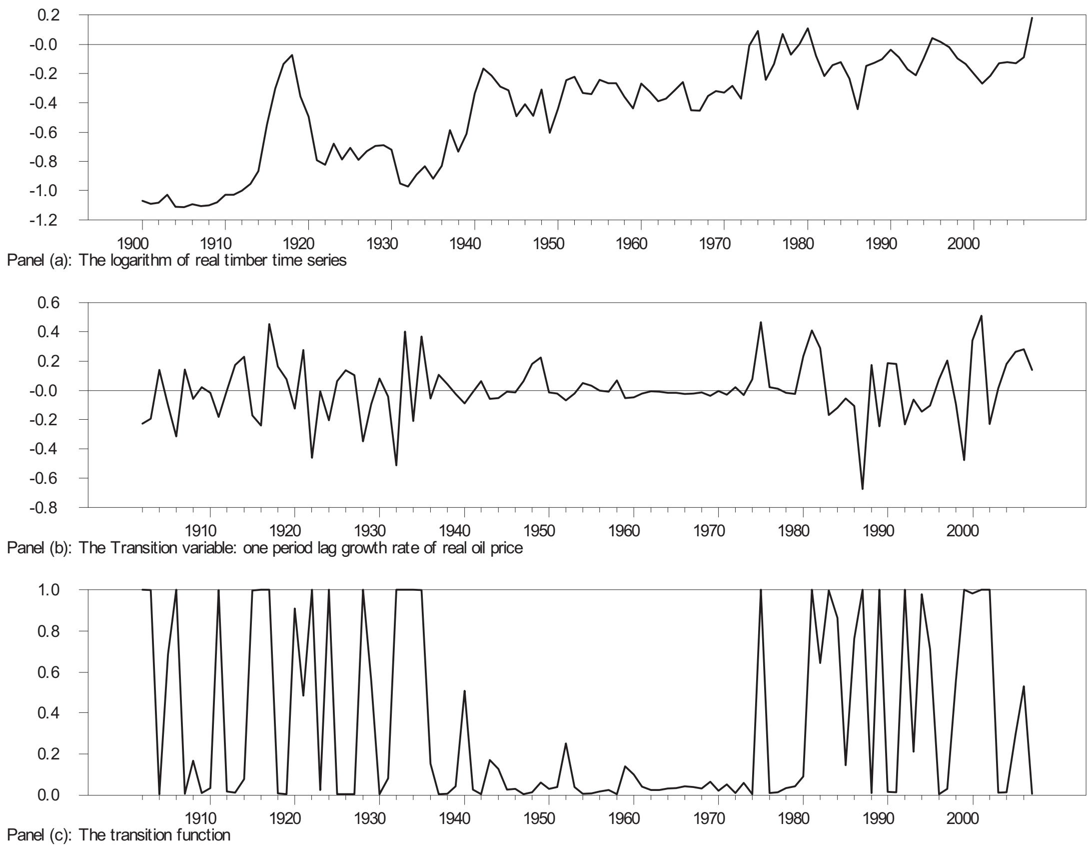


Figure 3.12: The logarithm of real timber price time series (Panel (a)), the one period lag growth rate of real oil price as transittion variable (Panel (b)), and the transition function (Panel (c)) between 1900 and 2007.

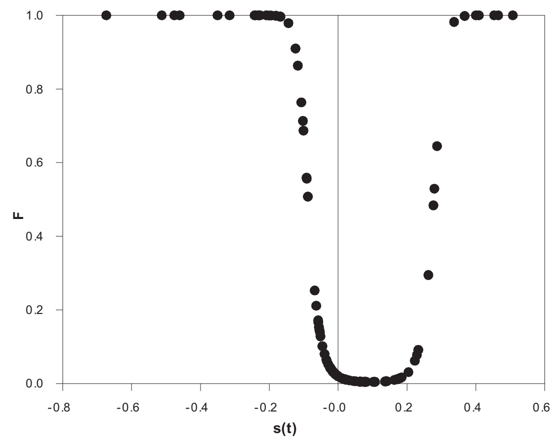


Figure 3.13: The transition function $F(s_t, \gamma, c_1, c_2)$ in case of timber as a function of observations. Each dot corresponds to one observation. The transition variable s_t is the one period lag growth rate of real oil price.

3.3.2 Smooth Transition ARCH Models

In the specification stage of modeling nonlinearity discussed in Chapter 2, page 25, the first step was to specify a linear adequate model forming the starting point of the analysis. Adequacy here refers to spherical disturbances. Out of the 24 individual commodity prices, 8 commodities (tobacco, silver, jute, lead, cotton, wool, aluminum, and tea) displayed ARCH processes in the residuals of their fitted linear preliminary autoregressive models and, accordingly, those linear autoregressive specifications were not suitable starting models for nonlinearity analysis. Those processes were classified in *Group A*, where ARCH or smooth transition ARCH models are entertained.

Since the null hypothesis of no ARCH up to order $p = 4$ was not rejected for those commodities (see Table 2.10, page 90), ARCH, GARCH, or even smooth threshold

(G)ARCH models are best suited for this group. The idea here is to introduce a smooth transition regime switching specification in the conditional variance of a time series admitting an ARCH pattern in its error terms. The value added of using smooth transition ARCH (ST-ARCH) models is that, in addition to capturing the asymmetric effects of the innovations, those models allow for intermediate states or regimes in the conditional variance equation. The smooth transition pattern is introduced through the use of a logistic transition function. When the smooth parameter (the slope of the logistic transition function) is large, the transition from one regime to the other is abrupt and the ST-ARCH model approaches the pure threshold model. An ST-ARCH model can be defined in general as follows.

Let

$$y_t = \Phi' w_t + u_t, \tag{3.41}$$

where $u_t = Z_t\sqrt{h_t}$, $Z_t \sim i.i.d.(0,1)$. Assume that u_t is distributed conditionally normal; that is, $u_t|\Omega_{t-1} \sim N(0, h_t)$, where Ω_{t-1} is the information set up to time $t-1$. A smooth transition threshold ARCH model of order p (ST-ARCH(p)) is expressed as

$$h_t = \omega + \sum_{i=1}^{p}\alpha_{1i}u_{t-i}^2 + \left(\sum_{i=1}^{p}\alpha_{2i}u_{t-i}^2\right) F(u_{t-d}, c, \gamma), \tag{3.42}$$

where $F(u_{t-d}, c, \gamma)$ is a smooth logistic transition function of order 1 or 2 defined as

$$F(u_{t-d}; \gamma, c) = \left(1 + \exp\{-\gamma(u_{t-d} - c)\}\right)^{-1}, \qquad \gamma > 0 \tag{3.43}$$

or

$$F(u_{t-d};\gamma,c_1,c_2) = (1+\exp\{-\gamma(u_{t-d}-c_1)(u_{t-d}-c_2)\})^{-1}, \qquad \gamma>0,\ c_1\leq c_2, \tag{3.44}$$

respectively. All the variables are defined as in (3.17) and (3.28); the only difference is that the transition variable $s_t = u_{t-d}$, where $d \leq p$ is the delay parameter. The specification in (3.42) is general in the sense that there are choices that have to be made based on the data. In particular, choices have to be made regarding the type of the logistic function used (first or second order logistic function); the maximum lag length of the ARCH process (p); and the value of the delay parameter (d) that determines the transition variable.

ST-ARCH specifications can be found, for instance, in the work of Hagerud (1996), González-Rivera (1998), Lee and Degennaro (2000), Lundbergh and Terasvirta (1998), and Lubrano (2001). The difference between these models lies in the specfication of the transition function and the choice of the transition variable. González-Rivera (1998) chose $p = 1$, $d = 1$ (hence, $s_t = u_{t-1}$) and the first order logistic function defined in (3.43) with $c = 0$. She also restricted the slope parameter of the logistic function to enter the equation with a positive sign. Hagerud (1996) used the same specification but without the positive sign of the parameter γ. He also suggested the exponential specification with $s_t = u^2_{t-1}$. The logistic specification captures the effect of the sign of the news on the conditional variance whereas the exponential specification captures the effect of the magnitude of the news. Lubrano (2001) suggested an exponential specification with a threshold parameter. Lee and Degennaro (2000) suggested a first order logistic function with a transition variable equals to the weighted average of past innovations. Lundbergh and Terasvirta (1998) suggested a STAR-STGARCH model that can characterize nonlinear behavior both in the conditional mean and the conditional variance of a time series. They used the first and second order logistic functions defined in (3.43) and (3.44) respectively in both the conditional mean and the conditional variance equations.

In this group of commodities, after experimenting with different lag lengths, the best model that I managed to fit for aluminum and cotton was the ARCH(1) model. Other ARCH and GARCH specifications did not converge and gave insignificant estimates. As for the remaining six commodity series (tobacco, wool, silver, tea, lead, and jute), the best model that I managed to fit among the ARCH, GARCH, and threshold (G)ARCH models is the smooth transition ARCH model of order $p = 1$ (ST-ARCH(1)) defined as

$$h_t = \omega + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-1}^2 F(u_{t-1}, \gamma), \tag{3.45}$$

with

$$F(u_{t-1}, \gamma) = \left[1 + \exp\left(-\gamma u_{t-1}\right)\right]^{-1}, \qquad \gamma > 0. \tag{3.46}$$

The first order logistic function $F(u_{t-1}, \gamma)$ defined in (3.46) is a bounded function between zero and one, $0 < F(u_{t-1}, \gamma) < 1$. In case of a positive shock, i.e., when the transition variable $u_{t-1} \to +\infty$, the transition function takes its extreme value

$F(u_{t-1}, \gamma) = 1$ and the conditional variance in (3.45) becomes

$$h_t = \omega + (\alpha_1 + \alpha_2)u_{t-1}^2; \tag{3.47}$$

this defines the upper regime. In case of a negative shock, $u_{t-1} \to -\infty$, $F(u_{t-1}, \gamma) = 0$, and the conditional variance takes the form

$$h_t = \omega + \alpha_1 u_{t-1}^2; \tag{3.48}$$

this defines the lower regime. Intermediate values of u_{t-1} give rise to a conditional variance process that is a mixture of both regimes. To ensure that the conditional variance h_t is nonnegative, we impose the restrictions $\omega > 0$, $\alpha_1 \geq 0$, and $(\alpha_1+\alpha_2) \geq 0$. Note that α_2 could be positive or negative. A negative α_2 implies that the effect of negative news on the conditional variance is larger than the effect of positive news. If $\alpha_1 = -\alpha_2$, the upper regime in equation (3.47) will not have a time-varying conditional variance and, therefore, an off-ARCH effect will be displayed in case of positive news. The smoothness of the transition from one regime to another is governed by the value of the parameter γ; at low and moderate values of γ, the transition is smooth whereas at high values of γ, the transition is abrupt. Figure 3.14 shows the transition function $F(u_{t-1}, \gamma)$ with low, moderate, and high values of the smoothness parameter γ.

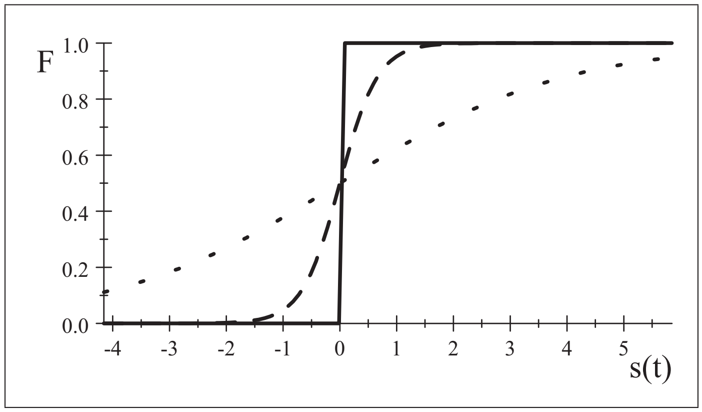


Figure 3.14: The smooth transition logistic function $F(\gamma, u_{t-1})$ with a smal slope $\gamma = 0.5$ (the dotted line), a moderate slope $\gamma = 3$ (the dashed line), and with an extremely larger slope $\gamma = 1000$ (the solid line).

The dynamics of the smooth transition ARCH model is traced through the analysis of the limiting processes of the model. In the upper regime, $F(u_{t-1}, \gamma) = 1$ and the conditional variance in (3.47) is stationary if $\alpha_1 + \alpha_2 < 1$. In the lower regime, $F(u_{t-1}, \gamma) = 0$ and the conditional variance in (3.48) is stationary if $\alpha_1 < 1$.

The log-likelihood function at time t can be expressed as

$$l_t(\Phi, \omega, \alpha_1, \alpha_2, \gamma) = -\frac{1}{2}\ln(h_t) - \frac{1}{2}\frac{u_t^2}{h_t}, \tag{3.49}$$

where maximum likelihood estimators of the parameters $\Phi, \omega, \alpha_1, \alpha_2$, and γ can be obtained by maximizing the log-likelihood function in (3.49). The estimation and evaluation results of fitting the ST-ARCH model to the individual commodities in *Group A* are reported in Table 3.3.

Group A: ARCH and ST-ARCH Models:†

$$y_t = \Phi' w_t + u_t, \qquad u_t = Z_t\sqrt{h_t}, \qquad Z_t \sim i.i.d.(0,1), \text{ where}$$

$$h_t = \omega + \alpha_1 u_{t-1}^2 + (\alpha_2 u_{t-1}^2) F(u_{t-1}, \gamma),$$

with

$$F(u_{t-1}, \gamma) = [1 + \exp\{-\gamma u_{t-1}\}]^{-1}$$

	Tobacco	Wool	Lead	Tea	Silver	Jute	Aluminum	Cotton
$AIC(p)$	$p=5$	$p=5$	$p=1$	$p=3$	$p=3$	$p=3$	$p=3$	$p=4$
Model	ST-ARCH	ST-ARCH	ST-ARCH	ST-ARCH	ST-ARCH	ST-ARCH	ARCH(1)	ARCH(1)
ϕ_0	−0.006 (0.01)	·	·	−0.01 (0.009)		·	·	·
ϕ_1	1.32 (0.05)	0.87 (0.06)	0.94 (0.03)	1.17 (0.03)	1.06 (0.003)	0.76 (0.002)	1.21 (0.87)	1.25 (0.10)
ϕ_2	−0.54 (0.03)	·		−0.38 (0.002)	−0.18 (0.005)	−0.18 (0.004)	−0.26 (0.14)	−0.66 (0.16)
ϕ_3	·	·		0.17 (0.008)	0.12 (0.005)	0.35 (0.07)	0.03 (0.07)	0.41 (0.09)
ϕ_4	·	−0.13 (0.1)						·
ϕ_5	0.15 (0.04)	0.23 (0.08)						
γ	7.1 (6.5)	190 (91.8)	332 (178)	1201 (214)	1213 (658)	12 (0.009)		
ω	0.006 (0.001)	0.022 (0.004)	0.02 (0.004)	0.01 (0.002)	0.02 (0.0003)	0.04 (0.001)	0.009 (0.002)	0.014 (0.003)
α_1	0.89 (0.39)	0.46 (0.26)	0.13 (0.12)	0.26 (0.04)	0.19 (0.008)	0.41 (0.002)	0.79 (0.27)	0.35 (0.18)
α_2	−0.79 (0.32)	−0.01 (0.36)	0.84 (0.47)	0.36 (0.04)	0.34 (0.02)	−0.40 (0.002)		
$Q(1)$‡	0.09	0.46	0.6	0.40	0.39	0.13	0.57	0.39
$Q(8)$‡	0.13	0.65	0.45	0.71	0.40	0.23	0.84	0.76
*Skewness**	0.27	0.28	0.08	0.23	0.09	0.13	0.5	0.02
*Kurtosis**	3.4	3.06	3.52	3.8	3.6	3.3	3.4	2.93
JB**	0.13	0.22	0.52	0.16	0.46	0.71	0.13	0.98

†A missing value in the table means that the corresponding parameter has been set to zero.

‡p-values of the Ljung-Box (1978) statistics, $Q(q)$, of the standardized residuals $u_t/\sqrt{h_t}$ at the 5% level of significance.

*Skewness and Kurtosis coefficients of the standardized residuals $u_t/\sqrt{h_t}$.

** p-values of the Jarque-Bera test for normality of $u_t/\sqrt{h_t}$.

All the estimated parameters are significant at the 5% level of significance.

The figures in parentheses beneath the model's parameters are standard deviations.

Table 3.3: Estimation and evaluation of Group A: ARCH and ST-ARCH models.

	Lower Regime: $F = 0$	Upper Regime: $F = 1$
	α_1	$\alpha_1+\alpha_2$
Tobacco	0.89	0.10
Wool	0.46	0.45
Lead	0.13	0.97
Tea	0.26	0.62
Silver	0.19	0.53
Jute	0.41	0.01

Table 3.4: Dynamic analysis of the ST-ARCH(1) model applied to commodity prices in Group A.

The transition from one regime to another was smooth in the case of tobacco and jute as seen from the dot plot of their transition functions in Figures 3.15 and 3.16 respectively. As for wool, lead, tea, and silver, the smoothness parameter γ was extremely large (see Table 3.3) which implies abrupt transition between regimes as seen from the Figures 3.17 to 3.20.

The estimated coefficient α_2 was negative in the case of tobacco and wool price series (see Table 3.3). This implies that the effect of negative news on the conditional variance is larger than positive news. An off-ARCH effect in the upper regime can be observed in the case of jute price series as $\alpha_1 = -\alpha_2$ and the conditional variance in the (3.48) is constant; that is, $h_t = \omega = 0.04$. The dynamics of the limiting processes are summarized in Table 3.4.

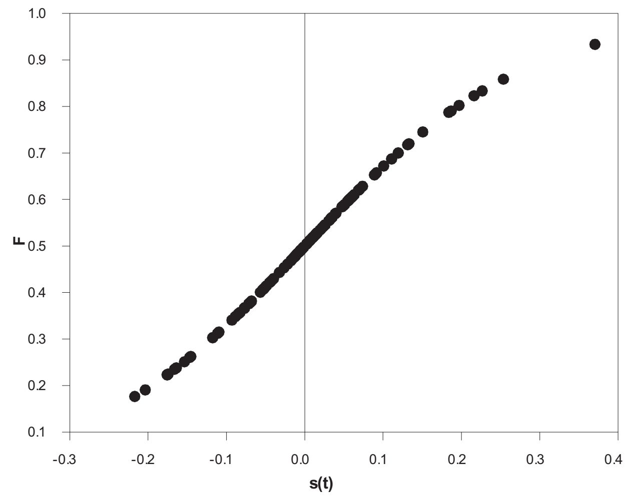


Figure 3.15: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real tobacco price series.

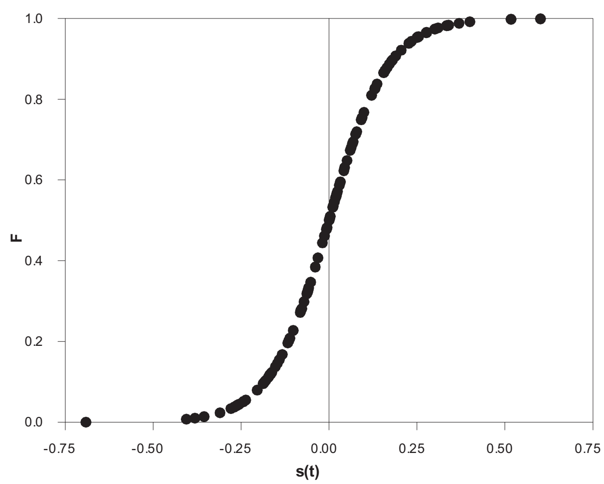


Figure 3.16: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real jute price series.

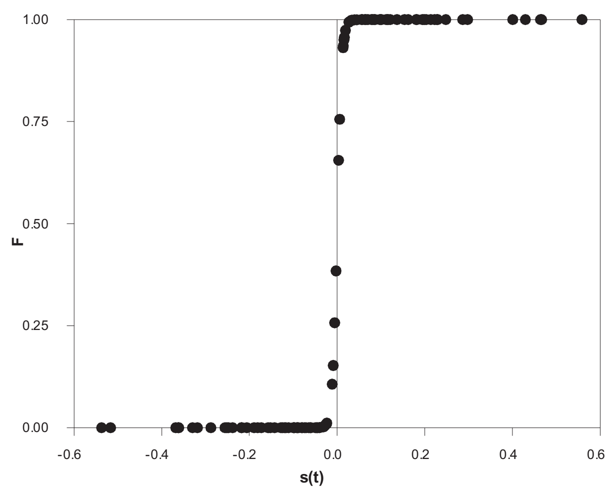


Figure 3.17: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real wool price series.

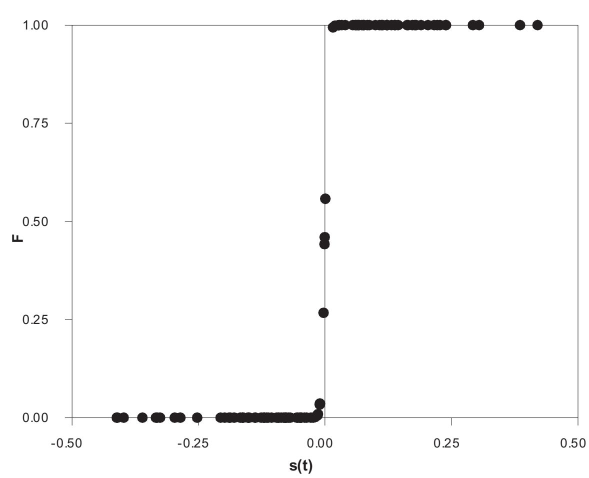


Figure 3.18: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real lead price series.

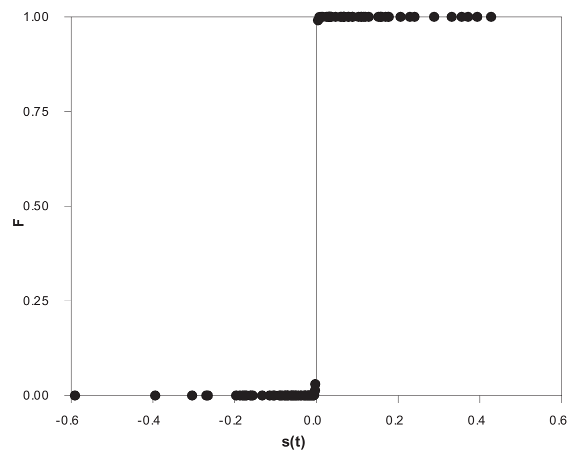


Figure 3.19: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real tea price series.

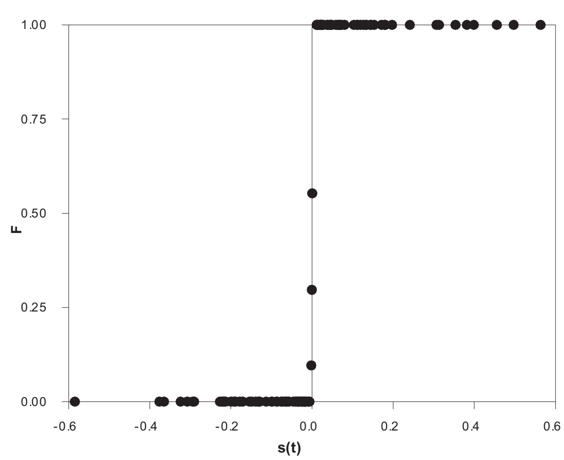


Figure 3.20: The transition function $F(u_{t-1}, \gamma)$ as a function of observations. Each dot corresponds to one observation. The function displays the regime switching dynamics in the conditional variance of the logarithm of real silver price series.

The fact that the Grilli & Yang commodity price index is a composite index of 24 commodities with different border prices and the need to follow a standard framework to model nonlinearities in individual commodity prices processes urge us to propose a general framework, in the same spirit as the one proposed by Lundbergh & Teräsvirta (1998), as follows. The first step is the specification step, where a preliminary linear AR model is specified using an information criterion (AIC say) and diagnostic tests are performed to the residuals of the fitted model. If the null hypothesis of no ARCH in the residuals is rejected and an ARCH pattern is detected, then ARCH or smooth transition ARCH models should be entertained. A preliminary model that passes the misspefication tests is considered an adequate model that can be used as a starting model for nonlinearity analysis. The next step is testing for nonlinearity in mean and/or in variance using potential external transition variables in addition to the autoregressive lags of the dependent variable. In particular, we suggest to perform a border price classification to the price processes under consideration in order to determine which transition variables are potential candidates for nonlinearity tests; that is, if the recorded border price is a free on board price, then, in addition to the autoregressive lags, the transition set will consist of macroeconomic news variables in the country of *origin*,[4] whereas if the border price is a cost and freight (or cost, insurance, and freight) price, the transition set will include autoregressive lags, news variables in country of *destination*, and the price of oil. Finally, if the border price

[4]Note that the use of oil price as transition candidate is inapplicable in such a case.

is a settlement or auction price, the set will include the autoregressive lags of the dependent variable and the exchange rate or other information news in the country of origin and destination. Once the transition sets are determined, the nonlinearity tests sequence proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994) are applied to each predetermined transition variable in the transition set in each group. If linearity is not rejected for all the variables in the set, the model is linear and no further nonlinearity analysis should be conducted. If linearity is rejected, the transition variable that yields the strongest rejection should be selected as the transition variable in the regime switching model. We should try modelling nonlinearity in the mean and in the variance equations of price process under consideration and perform diagnostic tests to the results. The model that gives better results should be selected. It should be noted that some high frequency processes could be best modeled by a model that can characterize nonlinear behavior in both the conditional mean and the conditional variance of the time series. The STAR-STGARCH model of Lundbergh & Teräsvirta (1998) is a case in point.

Chapter 4

Conclusion

In this thesis we followed the empirical approach in modelling the dynamics of commodity prices. The approach was motivated by the observation that commodity prices tend to move together in groups in response to a common macroeconomic variable or group of variables. We attempted to explain this phenomenon by, first, classifying commodity prices according to their border price (an issue that has been ignored in previous studies), and then by trying to find the best common macroeconomic variable that can explain this common dynamic in each group.

The rationale behind the border price classification rests on how the border price can provide an insight regarding the macroeconomic variables that drive the individual price series in each group from one regime to the other. Inflation rate and the autoregressive lags of the dependent variable were the best variables (transition variables) that were capable of modelling the nonlinearity in the individual price series that were recorded on a FOB basis. The price of oil succeeded in modelling the nonlinearity in the price processes that were recorded on a CIF basis.

The rationale behind using inflation and oil price in modelling the dynamics of commodity prices is based on two connections: the commodity price-consumer price connection and the commodity price-oil price connection respectively. The former rests on the bidirectional causality between consumer prices and commodity prices. This causality was explored and confirmed from a regime switching perspective. The later rests on the fact that a significant portion of the ocean freight cost (which is included in the CIF price of commodities) is due to the bunker fuel cost. Therefore, a reasonable hypothesis here is that ocean freight, and, hence, oil price plays a significant role in modelling the dynamics of those commodity prices recorded on a CIF basis. This commodity price-oil price connection was demonstrated by means of a simple illustrative example and the results showed that fuel costs roughly 20% of the price of commodities recorded on CIF basis.

Based on the previously-mentioned border price classification, inflation rate and the price of oil were selected as potential transition variables that can explain the behavior of commodity prices and were employed as external threshold variables in the STR model. This motivates the use of external threshold variables in regime switching models in addition to the autoregressive lags of the dependent variable that are commonly used by the majority of the studies in the regime switching literature.

The study attempted to model the nonlinearity in both the conditional mean and the conditional variance of the price series. Hence, the STR in mean and the STR in variance models. Both models were fitted to the Grilli & Yang commodity price index using the two potential transition candidates (inflation and oil price). The analysis was also extended to model the nonlinearities in the individual price series forming the index.

In case of the commodity index, using either inflation rate or oil price as potential transition candidate, both models (the STR in mean and the STR in variance) displayed the same dynamics in the limiting processes of the commodity index. When nonlinearity was modelled in the conditional mean (STR in mean model), leaving the conditional variance equation constant, the conditional mean equation of the commodity index displayed a stationary AR(1) behavior in the upper regime and a near random walk with a drift parameter in the lower regime. The behavior did not change with either inflation or oil price. When nonlinearity was modelled in the conditional variance equation of the commodity index (STR in variance model), leaving the conditional mean equation unchanged, the conditional variance was approximately three times higher in the upper regime as opposed to the lower regime. These results suggest that both models can be seen as *substitutes* when modelling nonlinearity in the commodity price index using inflation rate or the price of oil as external transition variables.[1]

As for the two transition variables, we can notice a pattern of complementarity

[1] See Tables 3.1 and 3.2 for more details.

in their dynamics in the sense that when both variables were employed in the STR model to characterize the nonlinearity in both the mean and the variance of the Grilli & Yang commodity price index, inflation was capable of capturing the early dynamics (between 1900 and 1950) of the index whereas oil price captured the late ones (between 1980 and 2007). The reason why inflation failed to account for the observed recent fluctuations in the Grilli & Yang commodity price index is, perhaps, due to the continuous monitoring of the FED to the U.S. inflation rate as part of its monetary policy. This result motivates the use of *external* threshold variables in the smooth transition regression model in general and, in particular, the use of inflation and oil price in the smooth transition model when applied to commodity prices. It should be noted, however, that the use of external threshold variable is advisable as long as it can be justified, i.e., as long as there exists a relationship between the threshold variable and the transition variable under consideration.

Since the Grilli & Yang commodity price index consists of 24 primary commodity prices, more insight can be gained by studying the individual commodities forming the index. To this ends, we classified the individual price series into four groups according to the entertained model in each group. The results were as follows:

The first group was *Group A*, where ARCH or ST-ARCH models were entertained. This group was comprised of eight price series: tobacco, silver, jute, lead, cotton, wool, aluminum, and tea. These processes exhibited ARCH pattern in the residuals of their preliminary AR models. This is not surprising as the majority of these prices are settlement or auction prices of commodities traded in exchanges and, therefore, tend to exhibit volatility clusters (a common feature of stock and option prices). Therefore, ARCH or ST-ARCH models were suitable for this type of commodities. The ARCH(1) model was fitted for aluminum and cotton while the ST-ARCH(1) model was entertained for the rest of the series. The second group in our classification is *Group B*, where linear AR models were entertained. This group was comprised of eight commodity prices: beef, cocoa, lamb, wheat, tin, copper, zinc, and rubber. All commodities in this group failed the nonlinearity tests and, therefore, were classified as linear series. The third group was *Group C,* where STR models with autoregressive lags of the dependent variable as transition variable were entertained. This group included three commodity prices recorded on a free on board basis. These were maize, rice, and sugar. Since the border price does not include the ocean freight cost, the oil price did not play any role in this analysis. The transition variables that showed the highest rejection of linearity tests were the second order autoregressive lag for maize, the fifth order for rice, and the first order for sugar. The last group was *Group D*, where STR models with external threshold variables were entertained. This group comprised of five primary commodities; four were recorded on a cost and freight basis (bananas, palm oil, timber, and coffee) and one on a free on board basis (hides). The price of oil was the transition variable that showed the highest rejection of linearity tests for all cost and freight commodities. This result confirms the commodity price-oil price connection and the rationale of the border price classification that was suggested earlier. The four groups and the associated models are summarized in Table 4.1.

The previous analysis reveals some observations: First, all settlement prices were best fitted by an ARCH or ST-ARCH model (see Group A in Table 4.1) except for tin, copper, and zinc. Those three time series in particular failed our nonlinearity tests and were classified in Group B, where linear autoregressive models are entertained. Second, excluding the heteroskedastic and the linear time series, i.e., Group A and B, we can observe that all commodities recorded on a CIF basis were driven by one common transition variable; the price of oil (see Group D in Table 4.1). Third, all commodities recorded on a FOB basis were driven by inflation or by one of the autoregressive lags of the dependent variable (see Group C and D in Table 4.1). The previous two observations are consistent with the border price rationale discussed before and can provide an explanation on the observed co-movement of commodity prices.

The conclusion that we can draw from the previous observations is that the observed co-movement in commodity prices can be explained by carefully analyzing the way the data are recorded. The idea here is that the standard practice followed by the major institutions when recording data on commodity prices is to select the largest trading route of a commodity and detect whether the trade volume is controlled by a major exporting or importing country. If this route is dominated by a major exporter, the export price at the exit point of the exporting country (FOB price) will be recorded; if the route, on the other hand, is dominated by a major importer of the commodity, the import price at the entry point of the importing country (cost and freight or CIF price) will then be recorded. Of course there are some exceptions, but this is usually the standard practice. This way of recording the border prices suggests that the behavior of FOB prices can be best explained by the macroeconomic news variables (inflation for instance) in the major exporting country; whereas, the behavior of CIF prices can be best explained by the price of oil or news variables in the major importing country. This is consistent with our findings in Groups C and D in Table 4.1.

Finally, it is worth mentioning that there is no single common variable that can explain the behavior of all commodity prices. Inflation rate and the price of oil succeeded in capturing the dynamics of the commodity index, but the picture was more clear when the index was disaggregated into groups of commodities and the behavior was modelled within each group. In my opinion, the best strategy to model the dynamics of commodity prices is to consider each commodity case by case and to study the way the time series is recorded and the history of the major exporting or importing country of the commodity under consideration. This analysis is significant in determining the best transition variable that is capable of explaining the behavior of the price series under consideration. In what follows, we illustrate this last point by analyzing the case of sugar price time series.[2]

[2] For more details regarding the major trading routes and the main importer and exporter of each of the individual commodities in the Grilli & Yang data set, please revert to Appendix B.

Series	Border Price	The Model	Transition Variable: s_t
Group A: ARCH and ST-ARCH Models			
Tobacco	CIF	ST-ARCH(1)	
Cotton	CIF	ARCH(1)	
Jute	FOB	ST-ARCH(1)	
Lead	Settlement	ST-ARCH(1)	
Wool	Spot quote	ST-ARCH(1)	
Aluminum	Settlement	ARCH(1)	
Tea	Auction	ST-ARCH(1)	
Silver	Handy & Harry	ST-ARCH(1)	
Group B: Linear Models			
Beef	FOB	Linear	
Wheat	FOB	Linear	
Tin	Settlement	Linear	
Copper	Settlement	Linear	
Zinc	Settlement	Linear	
Rubber	Spot Price	Linear	
Lamb	Wholesale	Linear	
Cocoa	Option Price	Linear	
Group C: STR Models with Autoregressive Transition Variables			
Maize	FOB	STR	y_{t-2} or inflation†
Rice	FOB	STR	y_{t-5} or inflation†
Sugar	FOB	STR	y_{t-1} or inflation†
Group D: STR Models with External Transition Variables			
Bananas	CIF	STR	Oil Price
Timber	CIF	STR	Oil Price
Palm oil	CIF	STR	Oil Price
Coffee	CIF	STR	Oil Price
Hides	FOB	STR	Inflation
†Both variables passed the linearity tests but the variables that showed the highest rejection were the AR lags.			

Table 4.1: The border price, the model type, and the transition variable for each individual commodity in the Grilli and Yang data set.

Case 1 *The Case of Sugar*

The description of the sugar price time series recorded in the Grilli & Yang data set is: International Sugar Agreement (ISA) daily price, raw, FOB and stowed at greater Caribbean ports, from the primary commodity price database. According to our conclusion, one would expect that inflation or any macroeconomic news in the *exporting* country is the potential transition candidate. But, before rushing into this conclusion, let's take a deeper look at the source of the recorded data on sugar.

As noted, the source of the sugar data is the International Sugar Agreement. According to Desmarchelier (1970), the International Sugar Agreement does not attempt to regulate all world production or exports of sugar, rather, it only regulates that sugar that enters the world 'free' market. Not all world traded sugar enter the world 'free' market though; part of the exported sugar takes the form of transfers between states (domestic trade) and another part is within special pricing and/or quota arrangements. The residual amount of sugar is sold on the basis of the world 'free' market price and it is this quantity that the agreement attempts to regulate. Since the quantity of sugar that enters directly into determining the world price of sugar is insignificant (less than 10% of the world production), supply conditions will have small effect on the world price and, in turn, on the recorded sugar price in the Grilli & Yang data set.

The above short analysis on the data source gives the researcher an idea on which variables should be included in the nonlinearity analysis. First, since the effect of supply conditions on that particular sugar price time series is insignificant, they should not be considered. Second, macroeconomic news variables acting as potential transition candidates in the origin or destination countries should be also ruled out as the United States (largest importer of sugar in the world) is not a member of the International Sugar Agreement and it does not buy sugar from the world 'free' market but imports sugar under special quota arrangements at prices well in excess of the world price (Desmarchelier (1970)). Therefore, U.S. inflation will not play any role in our nonlinearity analysis; the implications of the International Sugar Agreement actions are what matter in such case. Finally, the use of oil price as transition variable should not be considered, since the recorded sugar price is a free on board price. That being said, if nonlinearity is rejected for sugar, one would exclude the possibility of an external transition variable and expect an autoregressive lag. Indeed, that was exactly the case; the nonlinearity analysis revealed that the transition variable that showed the strongest nonlinearity test rejection was the first autoregressive lag of the logarithm of real sugar time series, y_{t-1}, and the associated model was the LSTR(1) model (see Group C in Table 4.1)

The message addressed from this example is that detailed analysis of the commodity price series can provide a better understanding of the behavior of the series and the factor influencing their dynamics. The border price classification and the study of the commodity history and the major players (importer or exporter) influencing it serve as guidelines for the researcher to select the best variable or group of variables that can explain the dynamics of a commodity price series. The challenge here, in addition to fitting the best econometric model, is to identify those variables and incorporate them in a model that can mimic the observed behavior of the price process.

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Chapter 5

Appendices

5.1 Appendix A: Characteristics of the Shipping Industry

Subject to a contractual agreement, usually a letter of credit, the shipper (owner of the goods) agrees to export a consignment (a shipment) to the consignee (importer) in exchange of the agreed selling price paid by the consignee. In addition to identifying the obligations of each party, the agreement states the type of the cargo shipped, the transport mode used, the agreed term of selling, and any other details considered by both parties.

Three transportation modes are used to ship cargo from one point to another: Inland transport via trucks or rail, air transport via airlines, and sea transport via shipping lines. The sea freight and the airfreight industries differ from the domestic transportation industry in the sense that there are no competing modes in the formers. Airlines and shipping lines, however, compete in terms of the services they offer.

The international commercial terms (Incoterms for short), published by the International Chamber of Commerce, are a series of international sales terms that define the obligations of both; the shipper and the consignee. There are many sales terms used in the shipping industry; the most frequently used are ex-works (EXW), free on board (FOB), cost and freight (CFR), and cost insurance and freight (CIF).

According to the ex-works term of selling, the shipper is obliged to deliver the goods outside his/her factory and it is the consignee's responsibility to pick up the cargo from that place and move it to its final destination. The consignee bears all the risks and shipping costs from the pick up point up to delivery at the final destination. Ex-works terms feature many varieties; ex-inventory and ex-dock are examples. Ex-inventory is an ex-work variant term used when the cargo is picked-up from the shipper's inventory, i.e., the inventory area inside the factory. This is usually the case when the inventory area is large and the cost of handling and moving the cargo inside the warehouse is significant. Ex-dock is another ex-work variant; the shipper bears all costs and risks of delivering the cargo ex-dock the port of origin in this case.

Another common term of selling is free on board, in which the shipper is obliged to load the goods on board the ship nominated by the consignee. This means that the shipper, in addition to bearing all the risks and costs of transporting the cargo from the pick up point to the port of origin, has to clear the goods for export and load them on board the nominated ship. The consignee bears all risks and costs starting from the time the goods are loaded on board the nominated ship up to delivery at final destination.

Another common term of selling is cost and freight, in which the shipper bears all risks and costs of transporting the cargo from the point of origin up to the final port of destination. The consignee bears the cost of the cargo and the transportation costs up to delivery at the port of destination. Then, it remains the consignee's responsibility to clear the cargo from the port of destination (port of discharge) and transport it to its final destination point. The cost insurance and freight term is a cost and freight plus insurance. Ensuring the safety of the cargo is the responsibility of the shipper, however, the insurance premium reflected in the cost insurance and freight price is paid by the consignee. See Figure 5.1 for illustration.

The handling, packing, inland transport costs, and freight are, of course, reflected in the Incoterm used (EXW, FOB, CFR or CIF), which is also referred to as *'the border price'*. The border price is different from the market price in the sense that the former does not reflect the market distortions of the later. All the international organizations (e.g., the World Bank, the IMF, the OECD) use the border price in their databases and in the construction of price indexes. Although border prices outperform the market distorted prices, yet transportation cost, driven mainly by oil prices, is a significant factor influencing them; especially cost and freight prices.

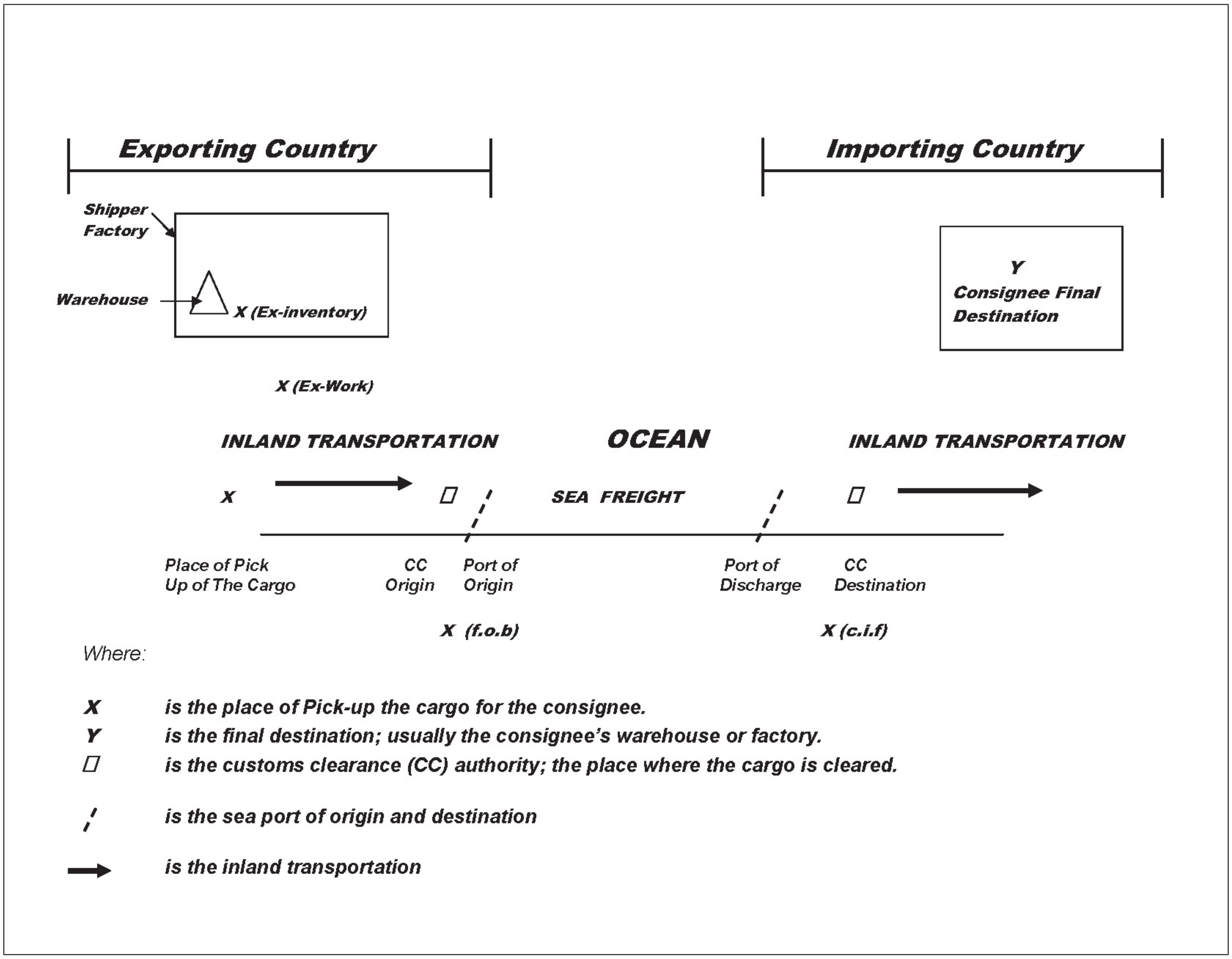

Figure 5.1: A typical outbound ocean freight operation.

5.2 Appendix B: Individual Commodities in the Grilli and Yang Data Set

In this appendix the descriptions and data sources of the 24 primary commodities forming the Grilli & Yang data set are exactly reproduced from their original source (Grilli and Yang, 1988) and their update (Pfaffenzeller, Newbold, and Rayner, 2007). Information on major producer(s) of each commodity as well as the main trading routes are briefly stated to justify the border price classification of Chapter 2, Subsection 2.9.1. Unless otherwise stated, all information on major producing countries for each commodity were obtained from the online FAOSTAT database, 2005.[1] The 24 commodities are grouped for convenience into four categories: Cost and freight prices, free on board prices, settlement prices, and finally spot and auction prices. A brief description of each group is introduced below.

5.2.1 Cost and Freight Prices

Banana

The top banana producing nations are India, Brazil, and China; U.S. is the major importer. The banana prices recorded in the Grilli & Yang data set are U.S. import prices, major brands, free on truck (FOT)[2] U.S. Gulf ports, from the primary

[1]The website address is www.fao.org.

[2]Free on truck is a term of selling that goes one step beyond the cost and freight (CFR) term. The term indicates that the place of delivery of the goods is free on truck port of discharge. This means that the shipper bears the responsibility of transporting the cargo from the point of pickup

commodity price data base. The origins were not mentioned in the data description, but the time series is technically a cost and freight price. This, in turn, implies that the suggested transition variables are the U.S. macroeconomic news variables and the oil price since the ocean freight is clearly part of the recorded time series. After experimenting with various potential macroeconomic variables (including U.S. inflation rate), the one period lag growth rate of real oil price has proven to be the best predetermined transition variable capable of capturing the nonlinearity in the logarithm of real banana price time series.

Palm oil

In 2004, Malaysia was the largest exporter of palm oil in the world followed by Indonesia in the second place.[3] The palm oil recorded price in the Grilli & Yang data set is 5 percent bulk, Malaysian, CIF *Northwest* Europe, from the primary commodity data base. As of 2007, the world's largest importers of palm oil were China and India. The Netherlands, located in *northwest* Europe, is the largest importer of palm oil in the European Union.[4] The one period lag growth rate of real oil price was the variable with the strongest test rejection among all the potential transition variables used in the study. A plot of the real palm oil time series and the transition variable is found in Figure 5.2. Although oil price gave us satisfactory results, there

till the port of discharge, clears the cargo at destination, and places it on truck port of discharge. It is then the consignee's responsibility to deliver the cargo to its final destination (see Figure 5.1).

[3]Source: www.plantation.simedarby.com.

[4]Source: www.bioenergywiki.net.

are other factors playing a major role in the Malaysian palm oil price. In 1988, for instance, the primary industries Minister Keng Yaik announced the government's intention to maintain the international prices for Malaysian palm oil firm for the year 1989; he said: "The currency exchange rate controls imposed by the government have enabled the commodity to fetch a high price of US$700 per tonne for the current year's production of palm oil".[5] In that particular year, as seen from Figure 5.2, the oil price was moving in the opposite direction. In 2004, the two largest oil palm producers in the world, Malaysia and Indonesia, agreed to cooperate to control palm oil prices in the international market.[6] These are examples of the effects of governments policies on commodity prices, especially when we are dealing with major exporters that can control their terms of trade as in our case. The message addressed from these examples is the importance of studying the history and the main characteristics of the individual commodity price time series under consideration.

[5] Source: New Straits Times; December, 22, 1998.

[6] Source: Xinhuanet, Jakarta, December 8, 2004.

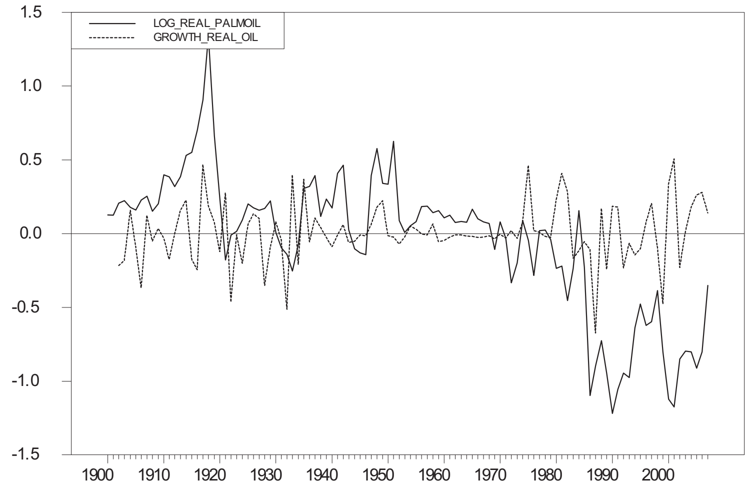

Figure 5.2: The logarithm of real palm oil and the one period lag growth rate of real oil price series between 1900 to 2007.

Coffee (mild Arabica)

According to the recent statistics, the top exporters of coffee in the world are Brazil, Vietnam, Indonesia, and Colombia. The top importer was and still is the United States.[7] The coffee price used in the Grilli & Yang data set is an indicator price of mild Arabica coffee from the International Coffee Organization (ICO). It is an average of New York and Hamburg markets, ex-dock. Nonlinearity tests results showed that the one period lag growth rate of real oil price was the best transition variable and the associated model was the LSTR(2).

[7]Source: World Trade Analyzer data base, 1985-2003, Statistics Canada.

Timber

In 2001, the world's top producer of timber was the United States followed by India in the second place.[8] The recorded border price in the Grilli & Yang data set is a UK import unit value (CIF price) of sawn wood (coniferous species) from the OECD international trade by commodities statistics. The source did not mention the origin of the traded timber, however, guided by the amount of production, U.S. is a major player in the international timber market. Consequently, oil price and U.S. inflation are potential transition candidates in this case. This was confirmed by the nonlinearity tests results (see Table 2.11); the U.S. inflation rate and the growth rate of real oil price were the variables that showed the highest test rejection. The model with real oil price outperformed the one with inflation in explaining the dynamics of the real timber price.

Cotton

In 2004, the U.S. and China were the top cotton exporter and importer respectively in the world.[9] There are many styles for cotton. The cotton in the Grilli & Yang data set is described as: Outlook A cotton index, which is a composite index based on 15 styles of cotton, cost and freight, traded in Far East, from the primary commodity price data base. At the 5% significance level, the hypothesis of no ARCH was rejected in the preliminary analysis for cotton and, therefore, cotton was classified in *Group*

[8] Source: www.mapsofworld.com.

[9] Source: www.internationaltrade.suite101.com.

A, where Both ARCH and ST-ARCH models were entertained. The best fitted model was the ARCH(1) model.

Tobacco

The U.S. is considered the largest importer of unmanufactured tobacco leaves. The Grilli & Yang tobacco prices recorded are U.S. import unit values, unmanufactured leaves, from the primary commodity price database of the World Bank's Development Prospects Group. The hypothesis of no ARCH was also rejected in the preliminary analysis for tobacco and the best model that was capable of describing the dynamics of the time series was the ST-ARCH(1) model.

Lamb

The recorded lamb price time series in the Grilli & Yang data set is: New Zealand, frozen whole carcasses, wholesale price; London; from the primary commodity price database. This is not a surprise since New Zealand is considered among the major exporters of lamb (FAOSTAT, 2004) and the majority of its exports are directed to the United Kingdom. The trade history between both countries goes back to 1882.[10] Technically the price is a CIF price with some distortions being a wholesale price. The nonlinearity tests sequence were executed for all potential transition candidates and were rejected. The commodity price time series was, accordingly, classified in

[10]For details, see Clements, R. and Babcock, B.A., (2004), "Country of Origin as a Brand: The Case of New Zealand Lamb", MATRIC Briefing Paper 04-MBP 9, USDA.

Group B, where linear models are entertained.

5.2.2 Free on Board Prices

Wheat

The wheat price recorded in the Grilli & Yang data was described as: No. 1 Canadian western red spring, in store, St. Lawrence, export price, from the primary commodity price data base. After experimenting with Canadian inflation, U.S. inflation, and other Canadian macroeconomic transition variables, nonlinearity was rejected and, therefore, wheat price was classified in the linear group.

Jute

The major producer of Jute is India followed by Bangladesh in the second place. The recorded jute price time series in the Grilli & Yang data set is FOB Chittagong; Bangladesh's main seaport. The autoregressive model acting as the starting model of the nonlinearity analysis suffered from ARCH in the residuals. The best fitted model was an ST-ARCH(1) model and the jute price time series was, accordingly, classified in *Group A*.

Rice

The Grilli & Yang rice price time series is an indicative price based on weekly surveys of export transactions, FOB Bangkok (one of the major producer in the world), from the primary commodity database. The best model I managed to obtain is an LSTR(2) model with the fifth order lag autoregressive variable as the threshold variable.

Hides

The exact data description for hides, as shown in Pfaffenzeller, Newbold, and Rayner, 2007, is: U.S., Chicago packer's heavy native steers, over 53 lbs., wholesale dealer's price, (formerly over 58 lbs.), FOB shipping point (Wall Street Journal, New York). Prior to November 1985, U.S. Bureau of Labor Statistics, Washington, D. C. The predetermined transition variable with the strongest nonlinearity test rejection was the period t U.S. inflation rate and the associated model was the LSTR(2) model.

Maize

The maize price time series recorded in the Grilli & Yang data set is: U.S. no. 2 yellow, FOB gulf port, from the primary commodity price database. The U.S. is the top maize exporter and Japan is the top importer. The transition variable with the strongest nonlinearity test rejection was the second lag order autoregressive variable and the associated model was the LSTR(2) model.

Sugar

The description of the sugar price time series recorded in the Grilli & Yang data set is: International Sugar Agreement (ISA) daily price, raw, FOB and stowed at greater

Caribbean ports, from the primary commodity price database. The nonlinearity analysis revealed that the transition variable that showed the strongest nonlinearity test rejection was the first autoregressive lag of the logarithm of real sugar time series and the associated model was the LSTR(1) model.

Beef

The data on beef prices in the original Grilli and Yang (1988) study were Argentinean export prices from Argentinean national statistics. Since the Argentinean data were not accessible, the time series were updated by Pfaffenzeller, Newbold, and Rayner (2007) from the IMF commodity price tables series PBEEF, which are Australian and New Zealand U.S. import prices FOB port of entry. This means that part of the time series is FOB Argentinean prices and the other part is cost and freight (in U.S. dollars) Australia and New Zealand prices. This mix of border prices makes the categorization of the commodity into one of the above mentioned groups a difficult task. However, since most of the data were recorded on FOB basis, I decided to classify beef price in the free on board group. Nonlinearity tests were rejected when executed for all potential predetermined transition variables. Therefore, the beef price time series was classified in *Group B*, where linear autoregressive models are entertained.

5.2.3 Settlement Prices

Aluminum

The world's top producing countries of aluminum in 2006 were: China, Russia, U.S., and Canada.[11] The recorded price is a settlement price, London Metal Exchange, and is obtained from the primary commodity price database. Inflation rates and oil prices are unlikely to play a role in determining the aluminum price; a metal index or stock index might be a suitable candidate. The hypothesis of no ARCH was not rejected in the specification stage and no further nonlinearity analysis was performed. The best model that was capable of describing the dynamics of the aluminum price time series was the ARCH(1) model.

Zinc, copper and tin

The prices of the three processes (zinc, copper, and tin) are settlement prices from London Metal Exchange. The behavior of the three series is quite analogous; see Figure 5.1, where the three time series are plotted in levels. A common metal index or a stock index would be an ideal transition candidate in our nonlinearity analysis. However, due to data limitation, the nonlinearity tests sequence was executed for the available transition candidates (the autoregressive lags of the dependent variable, the U.S. inflation, and the price of crude oil (though not relevant here)) and the null-hypothesis of linearity was not rejected for all transition candidates (see Table 2.11).

[11]Source: Altech Company, www.altech.is.com.

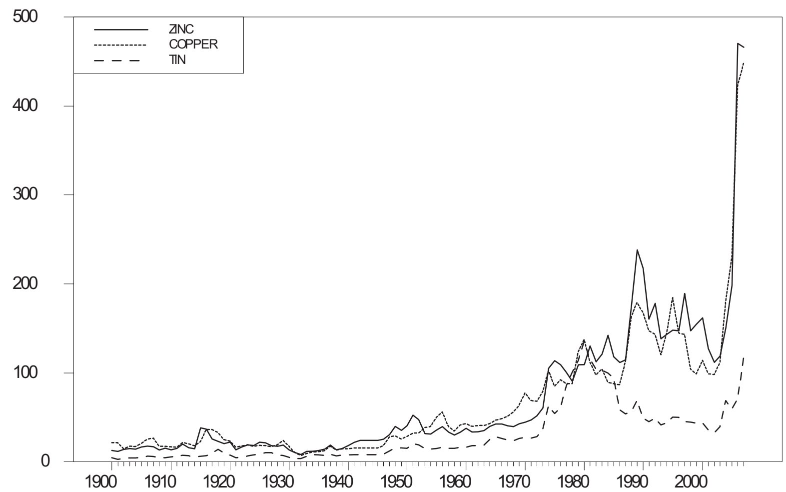

Figure 5.3: Levels of tin, copper, and zinc time series.

Therefore, the three processes were classified in *Group B*, where linear autoregressive models are entertained.

Lead

The top lead producing countries, as of 2008, were Australia, China, USA, Peru, and Canada.[12] The description of lead price time series in the Grilli &Yang data set was: London Metal Exchange, refined, 99.97 percent purity, settlement price, from the primary commodity price database. The specification stage revealed the presence of ARCH process in the residuals of the preliminary fitted AR model. The best model that was capable of describing the dynamics of the lead price time series was the ST-ARCH(1) model.

[12]Source: LDA International; www.ldaint.org.

5.2.4 Auction, Spot, and Option Prices

Cocoa

The world's largest cocoa bean producing countries are Côte d'Ivoire, Ghana, and Indonesia. The cocoa time series is the International Cocoa Organization (ICCO) daily price for cocoa beans, which is computed as the average of the quotations of the nearest three active futures trading months on London International Financial Futures and Options Exchange (LIFFE) and Intercontinental Exchange (ICE) Futures U.S. at the time of London close. The suggested transition variable here could be a stock index in London Exchange or U.S. Exchange. However, due to data limitation, the nonlinearity tests sequence were executed for the autoregressive lags, the current and one period lag inflation rate, and (though not relevant here) the current and one the period lag growth rate of real oil price. The nonlinearity was rejected for all potential transition candidates and, therefore, the time series process was considered linear.

Rubber

The rubber price time series in the Grilli & Yang data set is a spot price from the primary commodity database. The one period lag inflation rate was the transition variable with the strongest test rejection and the associated model was the LSTR(1) model. The fitted model, however, did not yield satisfactory results; the threshold parameter was insignificant and the dynamics showed one prevailing regime from 1920 till 2007. The time series was classified as a linear series.

Wool

The recorded wool price data in the Grilli & Yang data set are Australian Wool Exchange spot quotes. The hypothesis of no ARCH was rejected in the specification stage and the best model that was capable of describing the dynamics of the time series was the ST-ARCH(1) model.