MEASUREMENT AND MODELING
OF THE CUTTING FORCE FLUCTUATIONS
DURING MACHINING

by

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ABSTRACT

The dynamic cutting forces in single point metal cutting operations were measured in detail using a specially designed three-component piezoelectric dynamometer. From data reduction analyses performed on the cutting force fluctuations it was concluded that the cutting forces in single point metal cutting can be considered as a stationary, random process with a Gaussian distribution only for finishing operations.

Some of the more important properties of the cutting force fluctuations that were determined are: the probability density, the root mean square value and the power spectral density. Finally a mathematical model is proposed to represent the cutting force fluctuations. This model will serve as a more accurate forcing function in dynamic and stability analyses of machine tools.

ACKNOWLEDGMENTS

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NOMENCLATURE

The following represent symbols and notations utilized in this work. Some of these symbols are also defined as they appear for the first time in the text.

Z   Coordinate direction of the main cutting force

Y   Coordinate direction of the feed force

X   Coordinate direction of the radial force

F_z Main cutting force component

F_y Feed force component

F_x Radial force component

FM   Frequency modulation

Vc   Chip velocity

V Cutting velocity

i   Inclination angle

Q   Charge produced by piezoelectric cells in dynamometer

K   Charge amplifier gain

V   Voltage

κ Angle of approach

κ' End cutting edge angle

α_b Back Rake

γ_c Clearance Angle

γ_s Side rake

r   Tool nose radius

t   Chip thickness

a   Depth of cut

f   Feed and frequency

$\overline{X_i}$ Mean value of the sample points in the ith interval

$\overline{X_i^2}$ Mean square value (or variance) of the sample points in the ith interval

Y(k)   Sample record obtained from digitizing a force signal

R   Autocorrelation function

G   Power spectral density

F_c Critical frequency

p   Amplitude density

σ Variance

τ Time lag

C H A P T E R   I

INTRODUCTION

For over one hundred years researchers in the field of manufacturing science have been attempting to measure accurately the cutting forces during machining. Machinability of workpiece materials, power requirements for machine tools, optimal cutting conditions and, most importantly, the dynamic behaviour and stability of machine tools depend directly on the magnitude and frequency of cutting forces. For a better understanding of the mechanics of metal cutting, it is then necessary to have an exact description of these cutting forces.

Investigations on the cutting forces using the minimum energy principle, Ernst and Merchant [1-3]*, and the slip line solution, Lee and Shaffer [4], consider metal cutting as a steady state process. With such an assumption it is not possible to determine the dynamic response of machine tools for critical problems such as stability, optimal cutting conditions, etc. Investigators using the above theories have developed simple models [5-10] that give only the steady state mean value and neglect the fluctuating component of the cutting forces.

In the past many researchers considered the cutting forces as being static in nature. However, recent experimental investigations [11,12] have shown that they are essentially

* Numbers in brackets [ ] designate references at the end of the thesis.

dynamic, thereby making it possible to obtain a valuable insight into metal cutting phenomena. By measuring the main cutting force component (F_z) and radial force component (F_x) due to the dynamic variations of the uncut chip thickness and cutting speed, Albrecht [13] was able to relate the forces with the uncut chip thickness and cutting speed. Wallace and Andrew [14] show that a relationship between the force oscillation and the undeformed chip thickness oscillation is necessary to predict the onset of instability for a machine tool under given cutting conditions. It was also shown by Andrew [15] that the dynamic cutting force characteristics should be included in the prediction of chatter behaviour in horizontal milling techniques. Kegg [16], on the other hand, gave experimental techniques for measuring the dynamic cutting forces.

De Vries [17] considered the cutting forces both in orthogonal and oblique machining and gave brief descriptions of the different types of dynamometers used. It was shown that semiconductor strain gauge dynamometers have a high natural frequency and a high sensitivity enabling accurate measurements of the small dynamic fluctuations. Hsu and Choi [18] studied the difficult problems of measuring the cutting forces in oblique cutting. They studied the effect of neglecting the interactions of the cutting force components on each other during force measurements and explained a method of correcting these.

It is known that a dynamometer with a very high natural frequency is required for an accurate measurement of metal

cutting forces. Crisp and Seidel [19] designed, tested and used a dynamometer, with a natural frequency of 60 KHz, to measure the exact force fluctuations that occur during cutting, between a single grain on a grinding wheel and the workpiece. Such a high natural frequency was obtained by the use of piezoelectric quartz elements as force transducers.

Piezoeletric quartz elements were first used by Bickel [11] for cutting force measurements and showed that the cutting forces are not only dynamic but also random in character. Figure 1.1 shows Bickel's results. Further experiments carried out by Peklenik and Sata [12] showed clearly the random nature of the cutting force fluctuations both in the main cutting and feed directions. Consequently, Peklenik and Kwiatkowski [20] concluded that no further progress would be possible in predicting machinability, tool life, resistance of machine tools to chatter and optimal cutting conditions unless the cutting forces are considered time dependent.

Based on this reasoning Kwiatkowski and Bennett [21] determined the dynamic characteristics of machine-tool structures with a random force exciting the machine tool. Further, Kwiatkowski and Al Samarai [22] and Opitz and Weck [23] developed relationships between cutting forces and displacements of machine system using probabilistic techniques. Recent work by Sankar and Osman [24,25] characterized the random force fluctuations as stationary and Gaussian. Using such a mathematical model, they determined the response of a machine tool-

spindle system and also developed a short time acceptance test
for machine tools. In all the above mentioned analyses the
cutting forces were modelled as a stationary, Gaussian process
with no experimental justifications.

In the present investigation an experimental verification
of the nature of the metal cutting forces is made and statis-
tical properties, such as probability distributions, spectral
densities, etc., are evaluated. Furthermore, a new mathema-
tical model for the cutting force fluctuations is proposed
based on the experimental evidence.

In the second chapter the principles of static and dy-
namic force measurements in orthogonal and oblique machining
are explained. Then a brief description of different types
of dynamometers developed to measure static and dynamic cut-
ting forces is given, followed by a detailed description of
the dynamometer used in the investigation.

The experimental set-up for measuring the dynamic cut-
ting forces is described in Chapter 3, along with the static
and dynamic calibration procedure used for calibrating the
dynamometer. Also a technique for increasing the force mea-
suring accuracy is explained and experimental results of three
cutting tests are presented.

The experimental results are data processed and statis-
tical analyses are performed, in Chapter 4, for determining
the statistical properties of the cutting force fluctuations.
Based on these statistical properties a mathematical model is

formulated for describing the cutting force fluctuations.

a) f = 0.25mm/rev

b) f = 0.41mm/rev

c) f = 0.75mm/rev
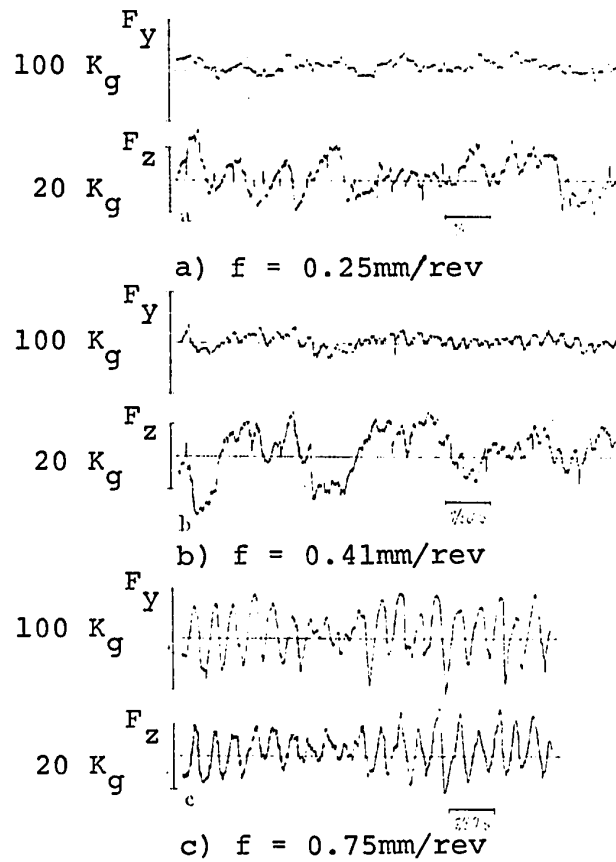
Fig. 1.1   Random fluctuation of the cutting force and

normal force while cutting (after Bickel[11]).

C H A P T E R   2

MEASUREMENT OF THE

DYNAMIC CUTTING FORCES IN TURNING

The dynamometers that were initially used were capable
of measuring only the static component of the cutting forces.
As technology progressed and the need for more accurate mea-
surements developed, better force measuring instruments were
designed. The many types of dynamometers developed may be
classified into two basic categories, (i) strain gauge dyna-
mometers primarily used for static force measurements and (ii)
piezoelectric dynamometers for dynamic force measurements.

The strain gauge dynamometers are widely used in metal
cutting research. Different constructions are used to in-
crease the accuracy and natural frequency so as to measure
not only the static forces but also the dynamic forces. Fig-
ure 2.1 illustrates the features of a three component strain
gauge dynamometer that measures all the cutting force compo-
nents in an oblique turning operation. Natural frequencies
of these dynamometers are of the order of 200-1000 Hz [13,15,
16,18,26] and the response times are large. Therefore they
can only measure the steady components of the cutting forces.
To measure the components of the dynamic forces it is neces-
sary that the natural frequency of the dynamometer be larger
than the highest frequency of that particular component of the
force.

The piezoelectric dynamometers on the other hand have very high natural frequencies and can therefore be employed to measure the dynamic forces. Dynamometers using piezoelectric crystals have been developed by Kistler [27] and Micheletti [28]. Figures 2.2, 2.3, and 2.4 show three dynamometers that use piezoelectric crystals for force measurement.

2.1 Static Force Measurements During Orthogonal Machining

In orthogonal metal cutting the resultant force between the cutting tool and the workpiece is assumed to act along the dashed line, shown in Figure 2.5a. Consider now the free body diagram of the tool-chip-workpiece system. The cutting tool exerts a force on the chip represented by the cutting force vector R_W, as in Figure 2.5b. A force vector R_T equal and opposite to R_W acts on the cutting tool as shown in Figure 2.5c.

In this case, a strain gauge dynamometer will measure simply the vertical and the horizontal static components of R_T. The assumption made here is that the force on the chip is equal in magnitude but opposite in direction to the force on the tool.

This assumption is true if the cutting tool exerts a force only on the chip. However, it is known that the tool may also exert a force directly on the surface of the workpiece. It is this extra force which contributes to plastic deformation on the finished surface of the workpiece [29]. Figure 2.6

shows a typical strain gauge dynamometer used to measure the forces in orthogonal cutting.

2.2 Static Force Measurements During Oblique Machining

The cutting force in orthogonal or two-dimensional machining discussed in the previous section is only a special case of the force in general oblique cutting. Oblique cutting takes place when the inclination angle is not equal to zero. The inclination angle is defined as the angle between the tool cutting edge and the perpendicular to the cutting velocity vector. Figure 2.7 illustrates the difference between orthogonal and oblique cutting and shows how the inclination angle is measured.

In the majority of cutting operations such as single point turning, drilling, milling, etc., the tool is always inclined at some angle to the workpiece and therefore results in oblique machining. In this case an additional force component exists and must be included in force measurements. Figure 2.8 describes all the three components of the force acting on a single point tool. These are: (a) tangential component in a direction tangent to the workpiece, (b) longitudinal component, referred to as the feed component, and is in the direction parallel to the axis of the workpiece and (c) the radial component in a direction normal to both tangential and feed components.

2.3 Dynamic Force Measurements During Machining

The dynamic force measurement essentially follows the same principle as the static force measurement. That is, the resul-

tant cutting forces is resolved into three components for measurement purposes. The main difference is that, for the case of dynamic force measurements, the dynamometer to be used must measure rapid fluctuations of the cutting force, with high sensitivity. To be able to do this adequately, the natural frequency of the dynamometer must be at least twice the value of the highest frequency of the three components. Furthermore, the sensitivity must be high enough to measure even the smallest dynamic variations. These characteristics can be obtained only through a dynamometer using piezoelectric quartz cells as force transducers. Figures 2.2, 2.3 and 2.4 show some of the piezoelectric dynamometers developed for measuring the dynamic force fluctuations [27,28].

2.4 Brief Description of the Dynamometer Used for Measuring Cutting Forces

The dynamometer used is normally of the piezoelectric type. Through light construction and rigid members, a natural frequency of 9-10 KHz in the main cutting force and feed force directions and 13 KHz in the radial force direction is achieved. Figure 2.9 shows a cross-section of the dynamometer, whose main features are:

i)   tool holder,

ii)   three piezoelectric cells,

iii) four bolts for preloading the three piezoelectric cells to avoid the possibility of any separation during operation,

iv)   base plate,

v)   aluminum cover to protect cells against the cutting fluid, and

vi)   main frame, designed to be fastened rigidly by four bolts on the saddle of the machine tool used in the experiment. This is shown in Figure 2.10.

The maximum force that can be measured safely is 500 lbs. in both the X (radial force) and the Y (feed force) directions and 1000 lbs. in the Z (main cutting force) direction.
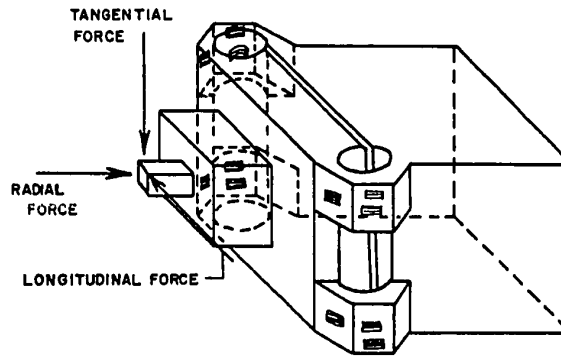
FIG. 2.1 – Three force compo-
nent lathe tool dy-
namometer.
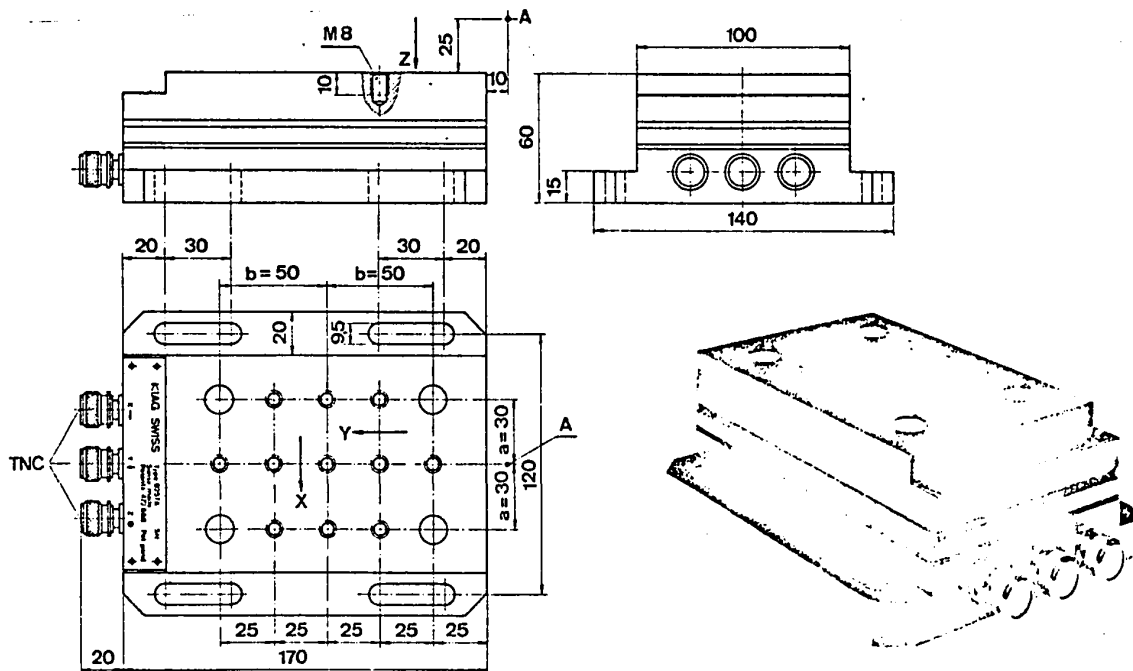
FIG. 2.2 – Piezoelectric three-component measuring platform
for measuring the three orthogonal force components.

Maximum measuring range: $F_x = F_y = F_z = 1,100$ lbs.
Resonant frequency = 4 KHz in all three directions
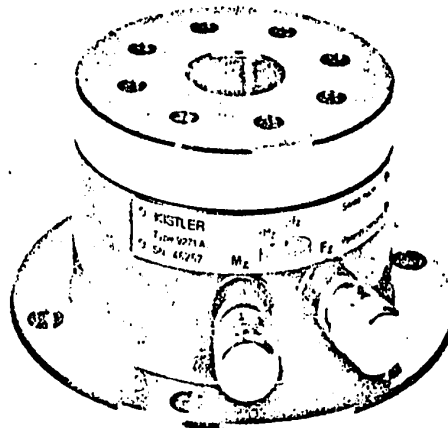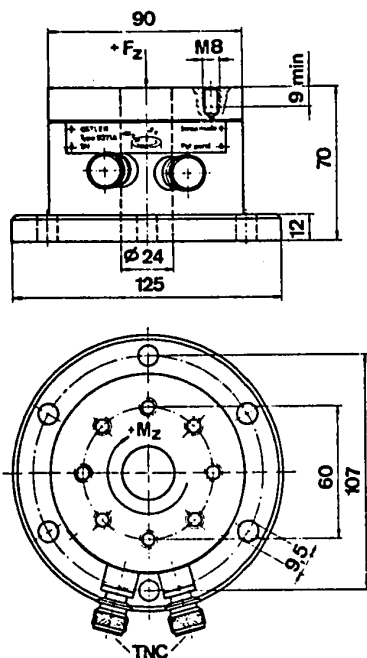
FIG. 2.3 - Piezoelectric two-
component measuring
platform for measuring
a force and a torque
around the force axis.

Maximum measuring range: F_z (pressure) = 4,400 lbs.
                            F_z (tension)   = 1,100 lbs.
                            M_z = ±2,250 lb-cm

Resonant frequency: $F_z > 3$ KH_z; $M_z > 3$ KHz
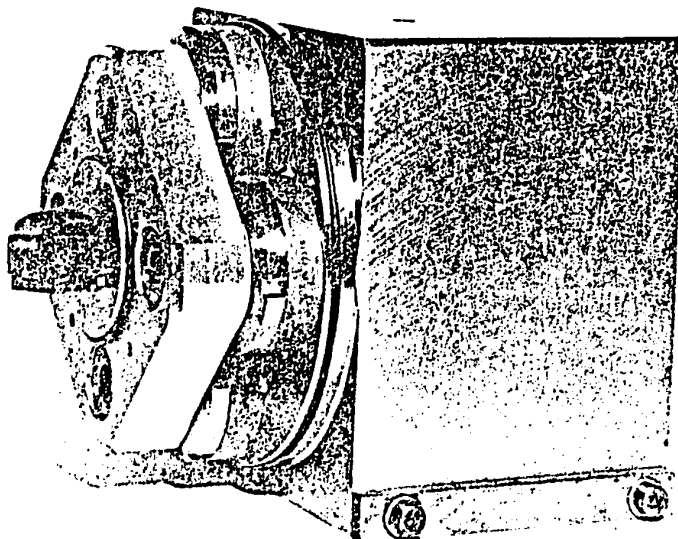
FIG. 2.4 - Piezoelectric
three-component dynamo-
meter for measuring the
three orthogonal force
components.

Maximum measuring range: $F_x = F_y$ = 1,100 lbs.
                              F_z = 2,200 lbs.

Resonant frequency: $F_x = F_y = F_z$ = 10 KHz
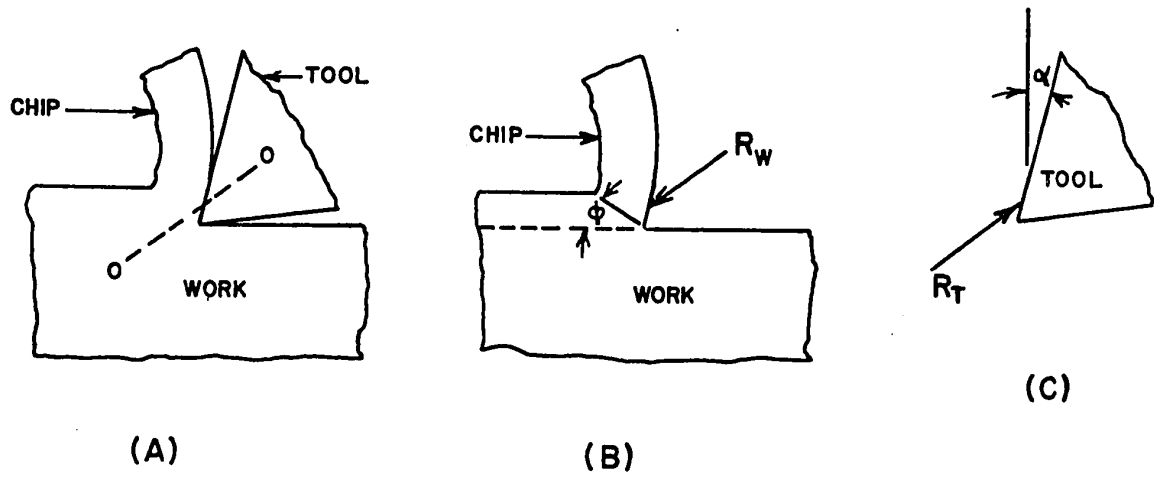
FIG.2.5   Forces between tool and workpiece.
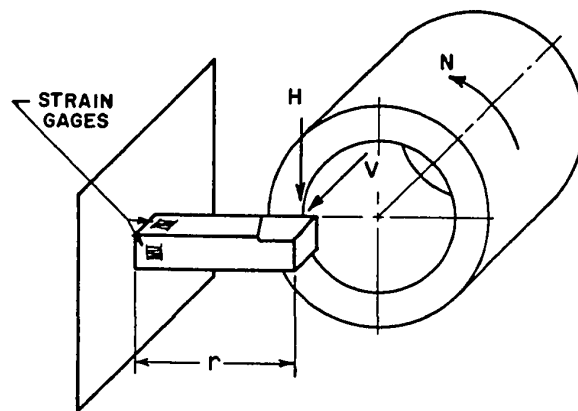
FIG.2.6   Simple two com-
ponent lathe
tool dynamometer.
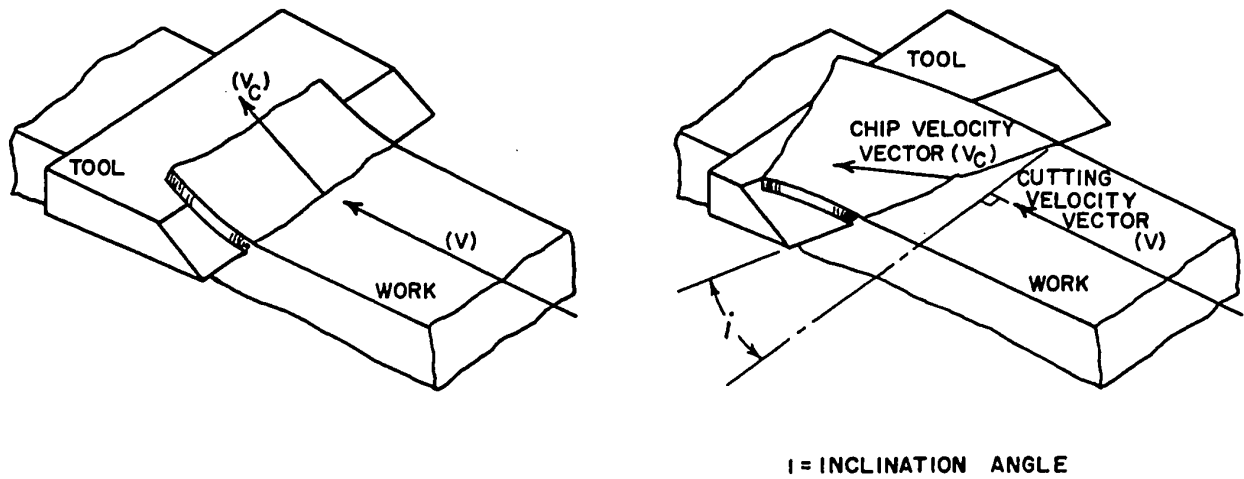
FIG.2.7   Comparison of orthogonal (left)
          and non-orthogonal cutting.
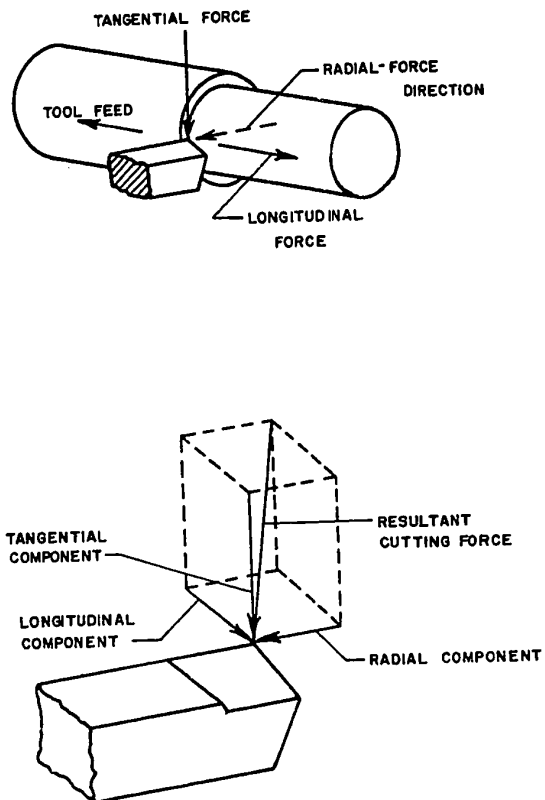
FIG.2.8   Force components act-
          ing on a single point
          turning tool.

FIG. 2.9 - Cross-section of the three-component dynamometer

1. main frame
2. base plate
3. nose
4. tool holder
5. retaining ring
6. cover
7. screw
8. back cover
9. shield
10. screw
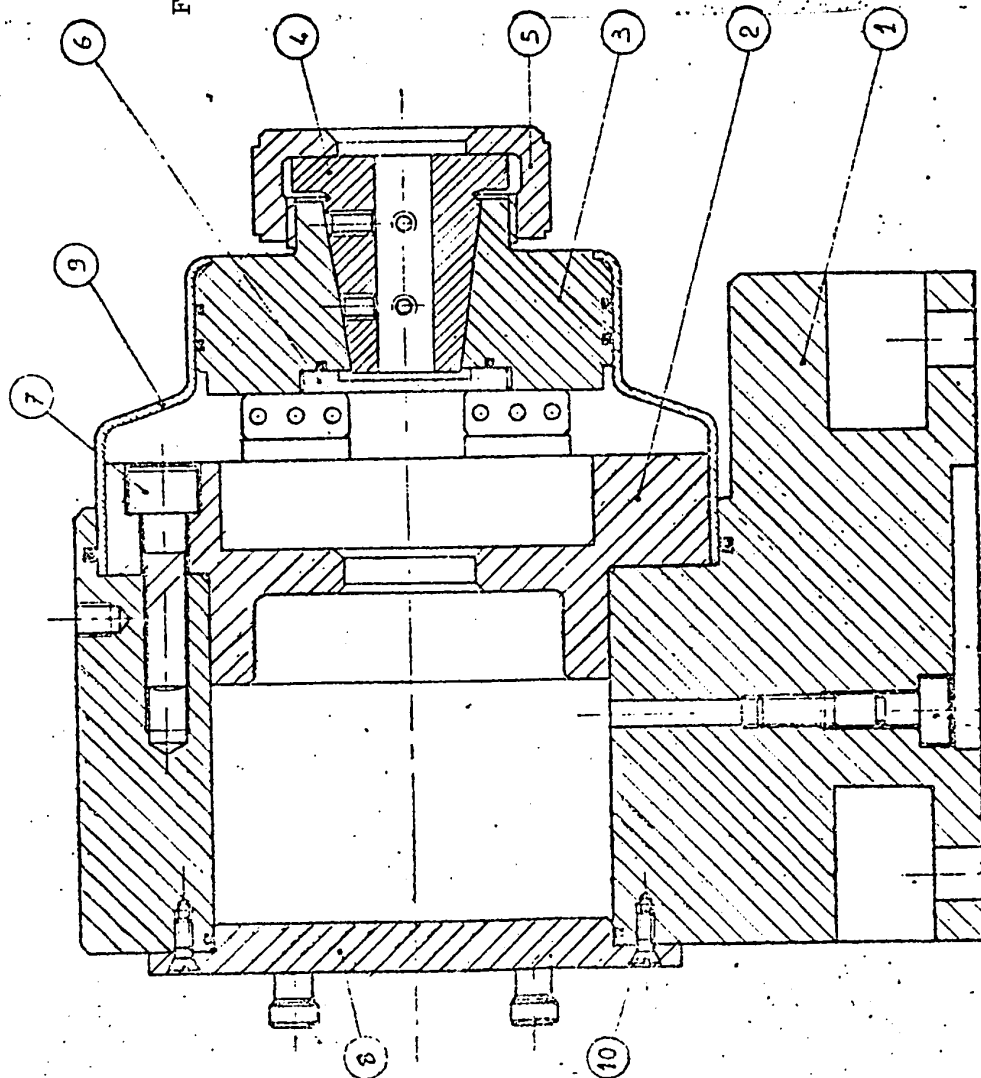
FIG. 2.10 - Machine tool used in the experiments and saddle with four bolts to mount dynamometer.

C H A P T E R   3

EXPERIMENTAL SET-UP AND TEST RESULTS

3.1   The Experimental Set-Up

The set-up used for measuring the dynamic cutting forces
in turning is shown in Figure 3.1. Figure 3.2 shows a block
diagram of this set-up. The main equipment consists of

i)   a lathe

ii)   a special dynamometer

iii)   three charge amplifiers

iv)   one oscilloscope for observing the cutting force
        signals

v)   a magnetic tape for storing the signals.

The lathe used is a 12 hp Demoor type, with a wide range
of speeds and feeds. The charge amplifiers are of the Kistler
type. They are employed to convert the electric charge pro-
duced by the piezoelectric cell transducers in the dynamometer
into proportional voltages. Figure 3.3 shows the circuit dia-
gram for one of the charge amplifiers. The X-Y-Z compensator
includes three operational amplifiers for the three force com-
ponent signal output from the charge amplifiers. These serve
the purpose of tapping any desired portion of any one of the
outputs of the compensator and feeding it as a compensation
signal to the inputs of the others. When this is done for all
three components the outputs from the compensator are the ac-
tual cutting force components. Figure 3.4 shows a functional
schematic of the compensator. The magnetic tape used is a

Hewlett Packard, 7-channel FM recording, with a bandwidth
of 10 KHZ at 60 ips recording speed and a maximum noise level
of 2 mv rms.

In order to relate signal stored on the magnetic tape
to the actual cutting force fluctuations the dynamometer must
be calibrated statically. Also it must be made certain that
the force fluctuations measured by the dynamometer are due to
cutting forces and must exclude vibrations of the tool due to
resonance in the machine tool-dynamometer system. Therefore,
besides static calibration, a dynamic test is also required
to make certain that the frequency response of the system does
not have any resonance points close to the frequencies of the
forces to be measured, so that the test results are valid and
accurate.

3.2 Static Calibration of the Dynamometer

In this investigation, both static calibrations and dy-
namic tests were performed with the three-component piezo-
electric dynamometer mounted on the machine tool. Figure 3.5
shows the general set-up for static calibration. For this
purpose, a special tool holder as shown in Figure 3.6 was de-
signed to allow for uniaxial forces along the three main di-
rections of force measurement. This is necessary for elimin-
ating the cross-sensitivity, as explained below.

Before the static calibration can be performed the cross-
sensitivity has to be compensated for. This was done by apply-
ing uniaxial loads, with the help of the special tool holder,
along each of the three cutting directions, indicated in Figure

3.6. Then the X-Y-Z compensator was used to eliminate the cross-sensitivity between the three directions. When this was done the compensator was locked and was ready for the static calibration.

Figure 3.7 shows the static calibration lines for the three components (F_x, F_y, F_z) of the dynamometer. The ordinates represent the actual electric charge produced by the dynamometer when a uniaxial load is applied. The electrical charge is calculated by the relation

$$Q = KV \tag{3.1}$$

where   Q is the charge in pico-coulombs

V is the voltage read on the oscilloscope

and K is the gain set on the charge amplifiers

The calibration lines were obtained by slowly increasing the applied load on the tool holder and recording the voltage and amplifier setting. The magnitude of the applied uniaxial load was measured by a Kistler force transducer that was introduced between the load and the tool holder. To ensure that any form of hysteresis is totally avoided, the loading was increased in short steps up to a maximum and then decreased similarly down to zero. The loading and unloading curves are linear and coincident for the range tested. This may be seen from Figure 3.7.

3.3 Dynamic Tests

The purpose of these tests is to determine

(a) the natural frequencies of the tool-dynamometer sys-
tem along the three cutting directions, and

(b) whether there could exist cross-interference between
the natural frequencies of the test set-up (Figure
3.8) and the dominant frequencies of the measured
cutting forces.

To determine the natural frequencies of the tool-dynamo-
meter system the impulse load method was employed. This was
necessary since the estimated natural frequencies exceeded the
upper frequency limit of the vibration exciter that is avail-
able. By dropping a steel ball, 19/32 inches in diameter, on
each of the three faces of the tool and counting on the oscil-
loscope the number of oscillations per second, the natural fre-
quencies were found to be 13 KHz along the X direction and 9 -
10 KHz along the Y and Z directions.

Elimination of cross-interference
requires the natural frequencies of the machine tool-dynamo-
meter vibration system (Figure 3.8), obtained through dynamic
tests, be well removed from the frequencies of the forces mea-
sured during the experiments. A method for checking such
cross-interference is to compare the frequency response of the
system with the spectral density of the cutting forces. In
the event that there is interference, the peaks of the spectral
density curve will be close to the peaks of the frequency re-

sponse curve of the system, for the particular cutting conditions employed. Then the experiment has to be repeated using different cutting conditions. Figure 3.9 shows the frequency response of the system up to 4000 Hz. A peak around 30 Hz may generally be expected, due to the machine tool being set in resonance. To determine the frequency response of the machine tool-dynamometer system, Figure 3.8, a dynamic load was applied on the tool tip. The load was kept constant by a feed back loop controlling the Bruel and Kjaer exciter and the output force was observed for an input force frequency range of 20-4000 Hz. Figures 3.10 and 3.11 show the measuring set-up for obtaining the frequency response of the system.

3.4 Accuracy of the Cutting Force Measurements

In order to record accurately the dynamic cutting forces, the static force (steady component) was measured separately from the superimposed fluctuating component. This was done by first setting the time constant of the charge amplifiers to "long". This in effect changed the time constant of the system to a large value enabling the steady component of the force to be measured. The average force was calculated according to the sensitivity setting on the charge amplifiers. Then, for accurate measurement of the small fluctuations in the cutting force over the static component, the charge amplifiers were reset. This caused the output signals corresponding to the average of the cutting force components to go

to zero. Then the position of the time constant switch on the charge amplifiers was changed from "long" to "short", and the sensitivity was increased. This increease in sensitivity allowed the force fluctuations to be measured with greater accuracy. Figure 3.12 shows schematically the effect on the signal of resetting the charge amplifiers to a more sensitive range.

3.5 Experimental Results

A set of experiments was carried out to measure the actual dynamic cutting forces during metal cutting. The experimental set up was calibrated statically and tested dynamically as explained in sections 3.2 and 3.3. The cutting conditions chosen for these tests were as follows:

Machine: 12 hp Demoor lathe

Workpiece: AISI 1020 steel, cold drawn

Tool material and geometry: As shown in Figure 3.13.

Cutting speed, feed and depth of cut: see table 3.1.

Photographs of the cutting force signals were obtained by a camera mounted on the oscilloscope. Figures 3.14, 3.15, and 3.16 show the actual force signals for the three cutting conditions mentioned previously. The photographed signals were also recorded on the magnetic tape for further analysis, as will be described in the next chapter.

These photographs demonstrate clearly how the cutting force varies rapidly with time. Although similar test results were obtained by Bickel [11], the mathematical characteristics

and properties of the fluctuations were not explained. For example, it was not known whether these fluctuations can be considered deterministic or whether they are essentially random. If the cutting forces are predominantly random, then what kind of distribution do they follow? How can they be mathematically modelled?

In the following chapter a rigorous analysis is presented on some of the cutting force signals recorded in order to explain the basic characteristics of the cutting forces. Such information is vital in determining the dynamic responses of machine tools, since the cutting forces constitute the main forcing function.

FIG. 3.1 - Equipment layout for dynamic
cutting force measurements
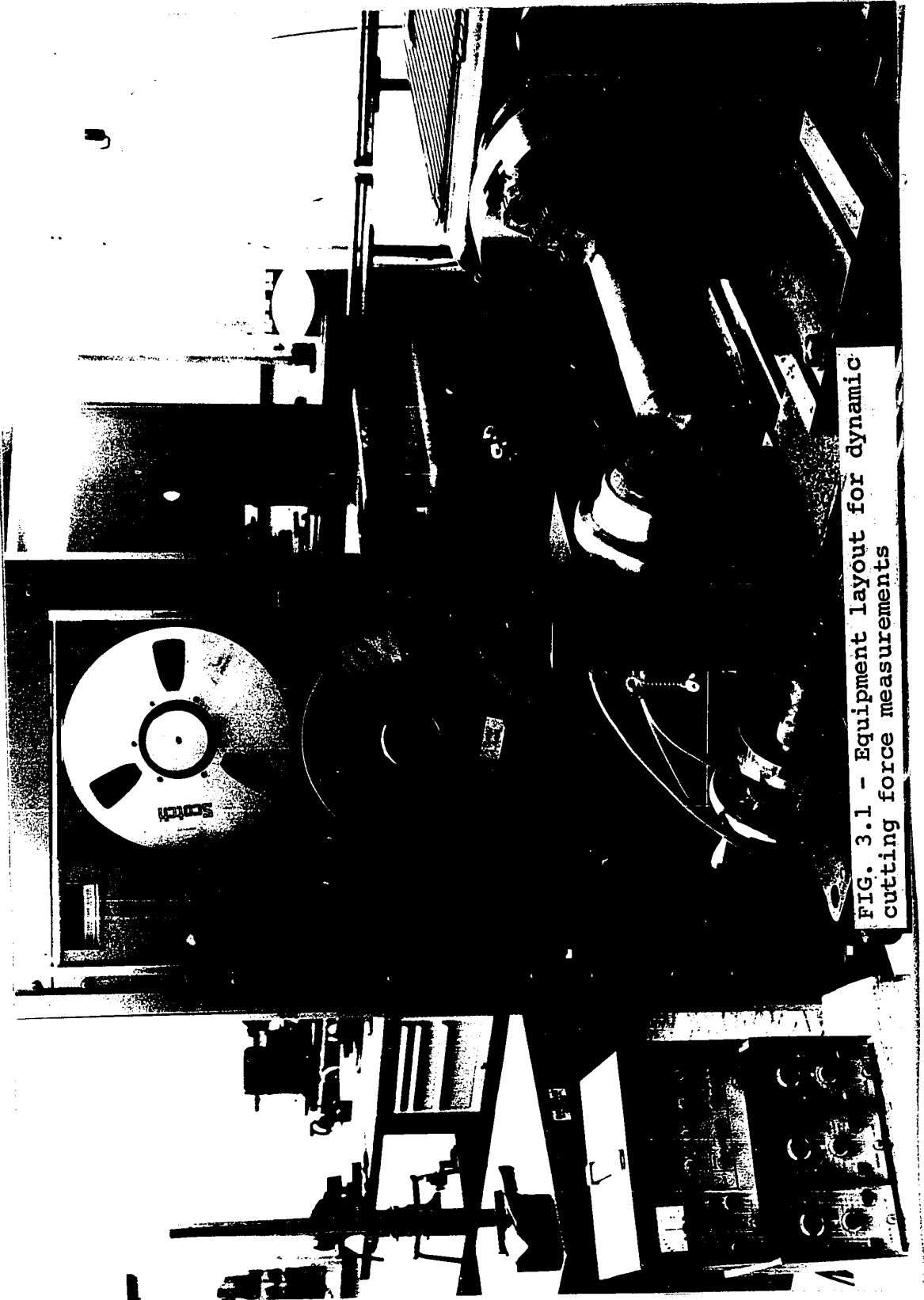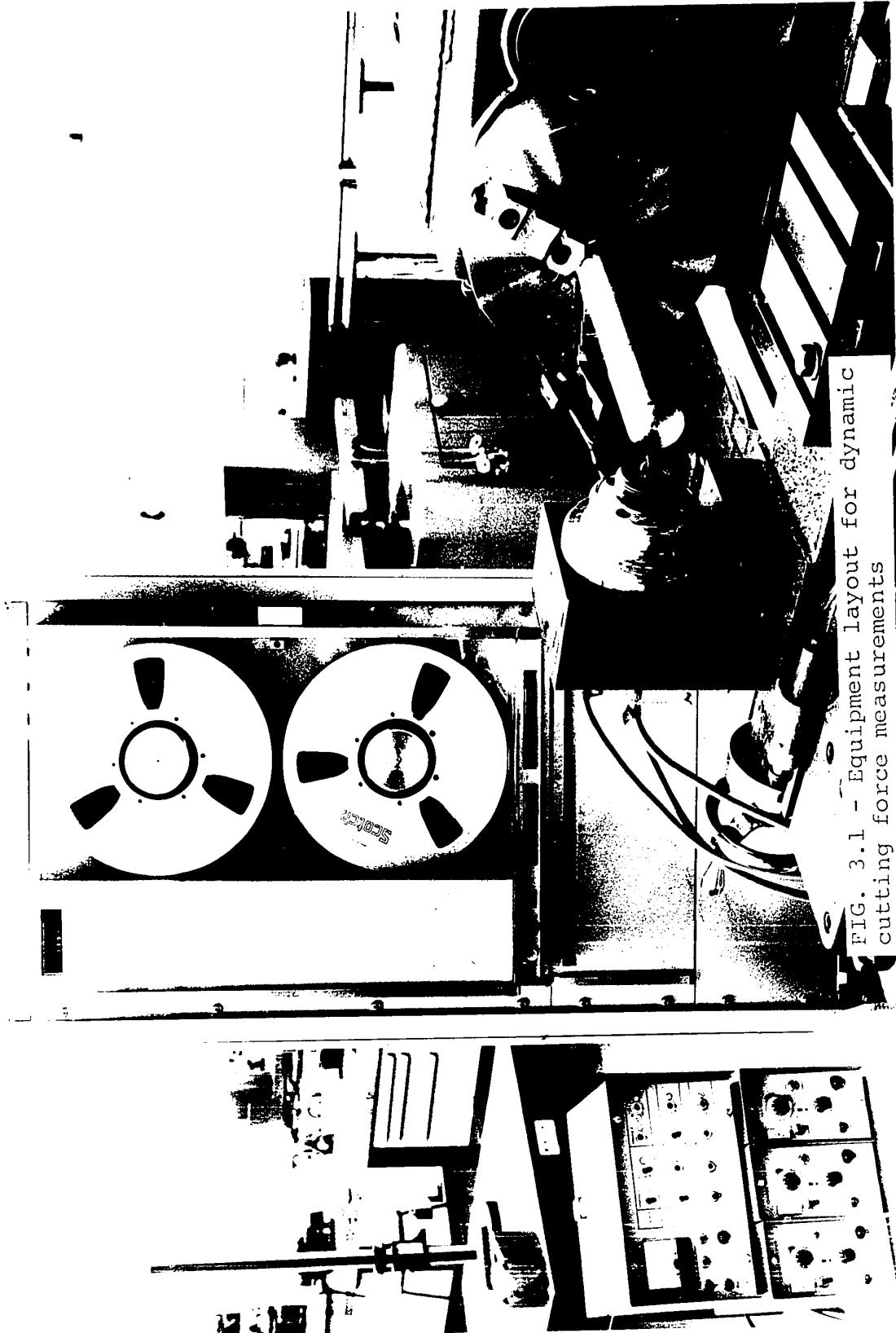
FIG. 3.1 - Equipment layout for dynamic
cutting force measurements
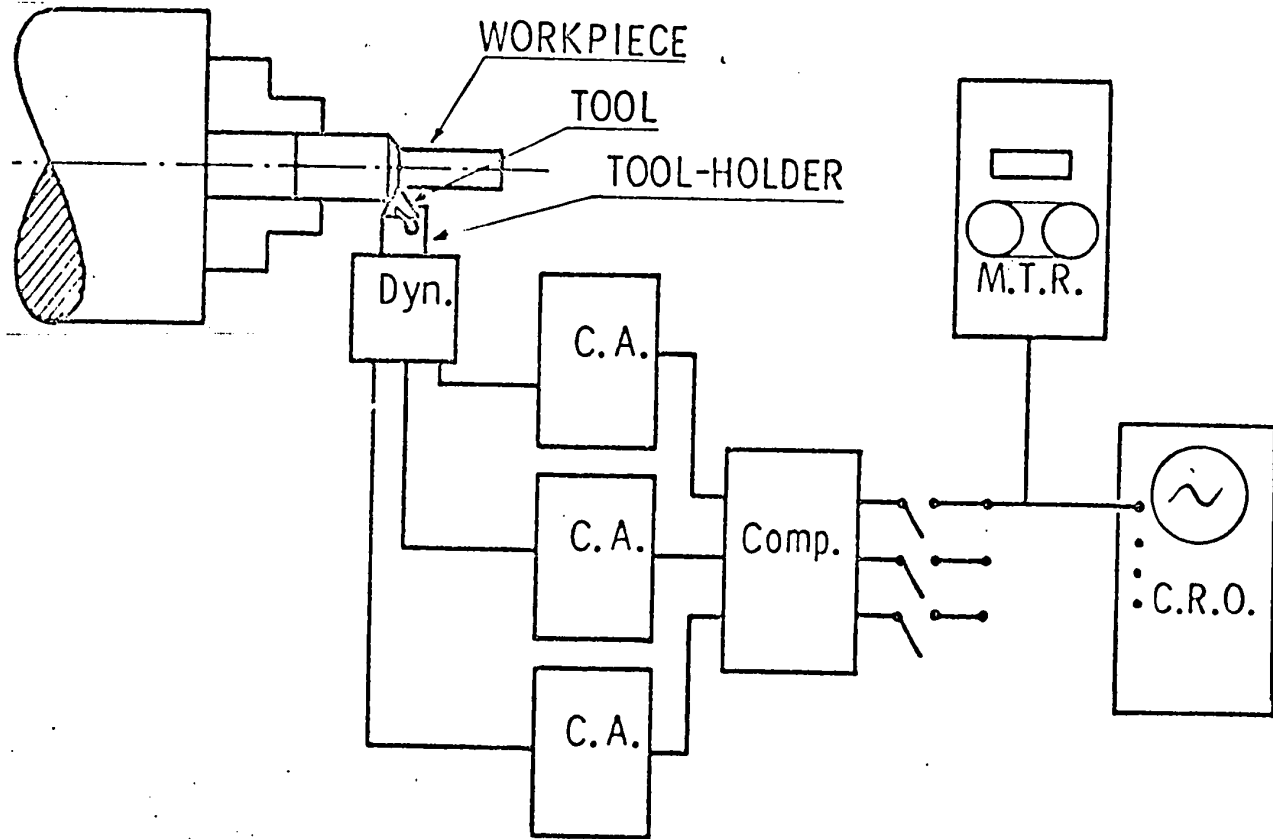
FIG. 3.2 - Block diagram of the test set-up and instrumentation
for the dynamic force measurements

    Dyn: Dynamometer
    C.A.:   Charge amplifier
    Comp: Compensator
    C.R.O.: Cathode ray oscilloscope
    M.T.R.: Magnetic tape recorder
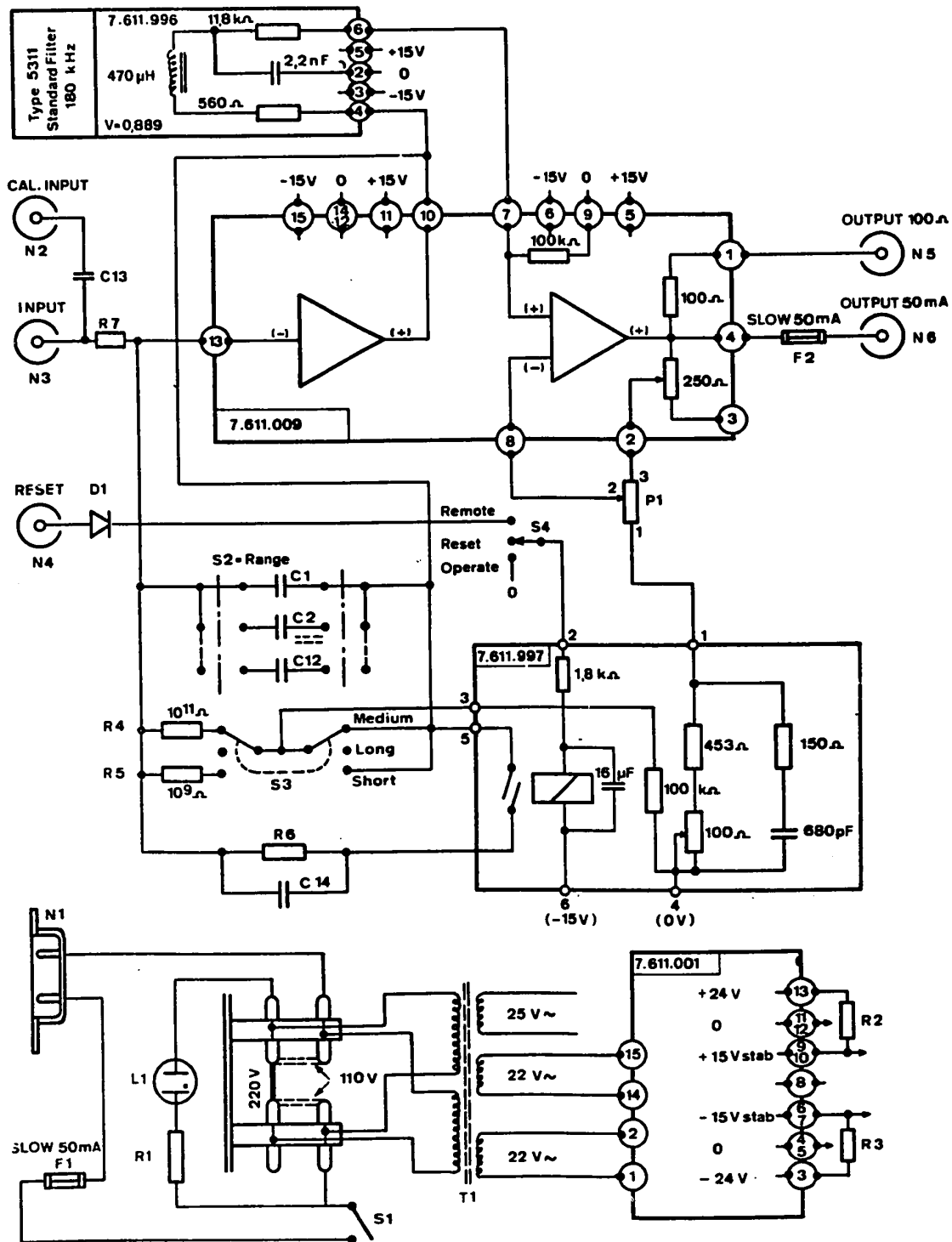
FIG. 3.3 - Circuit diagram of a charge amplifier
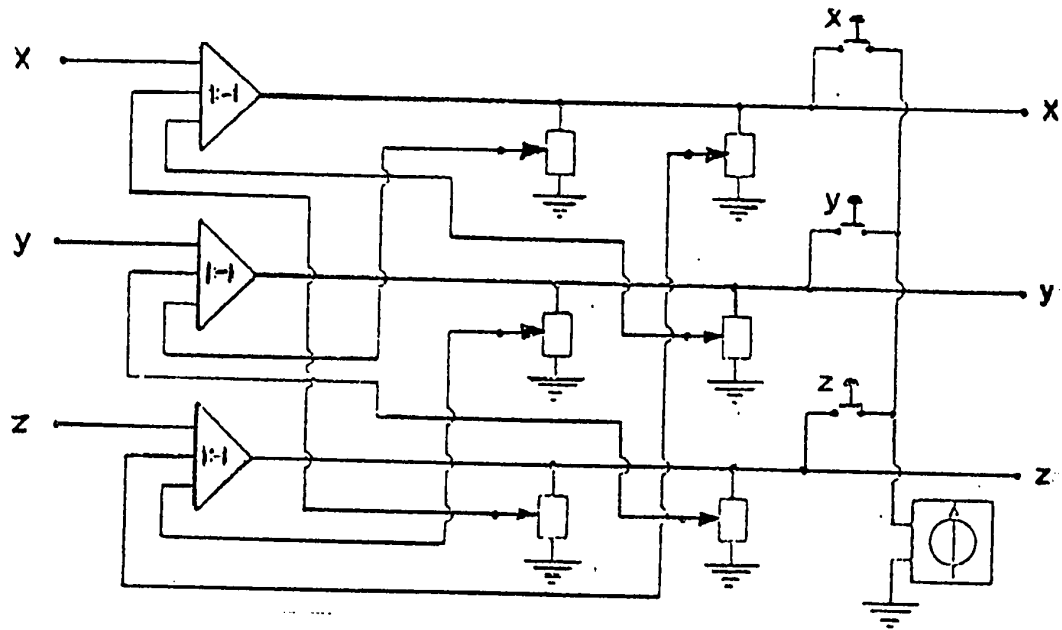
FIG. 3.4 - Functional schematic of the X-Y-Z compensator.

Z (main cutting force F_z)

Y (feed force F_y)

X (radial force F_x)
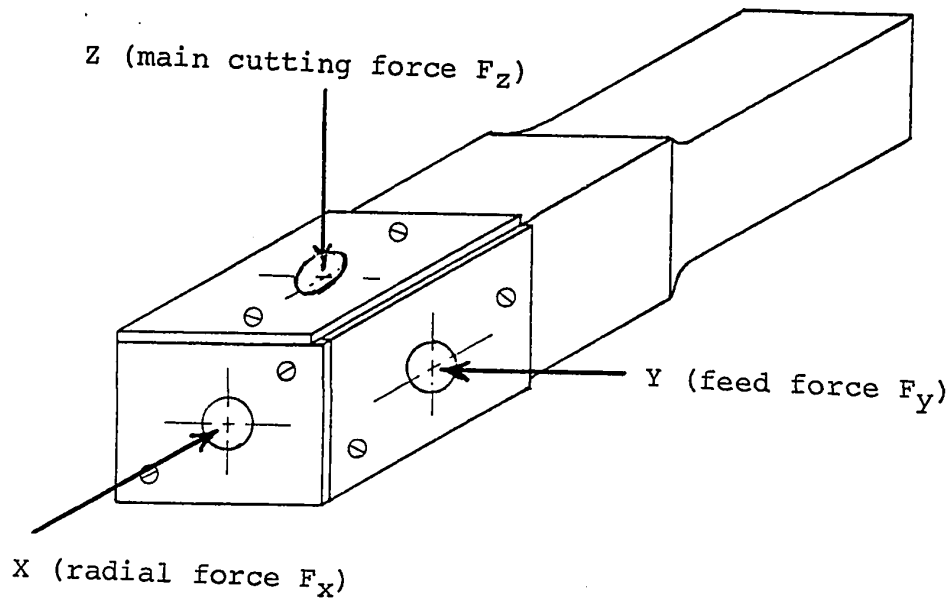
FIG. 3.6 - Specially designed tool holder for applying uniaxial forces along three mutually perpendicular directions.
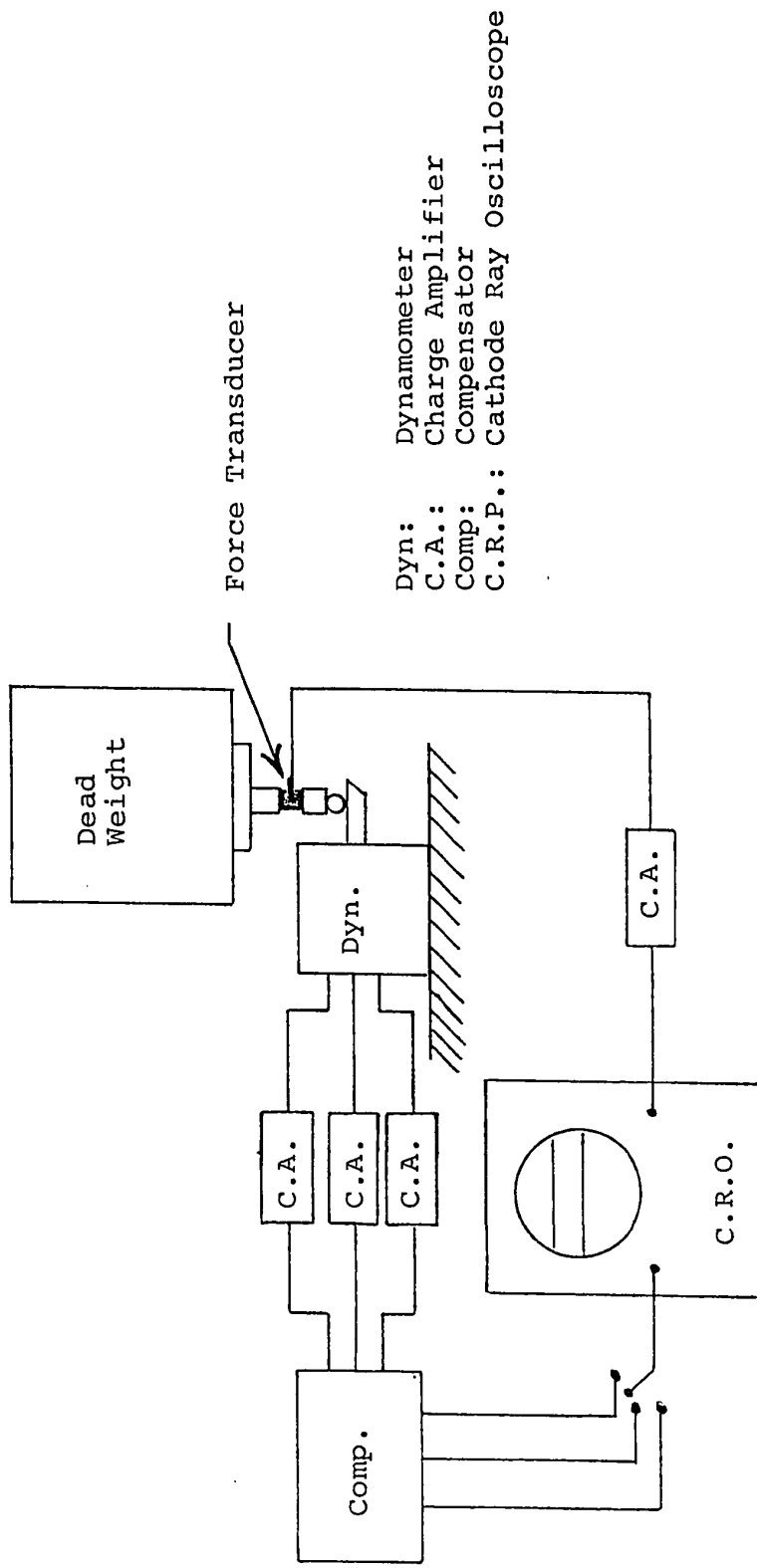
FIG. 3.5 – Set-up for static calibration of the
three-component piezoelectric dynamometer

Calibrated Range [lb]	F_x	F_y	F_z
	0-500	0-500	0-500
Sensitivity [pc/lb]	13.7	39.0	34.3
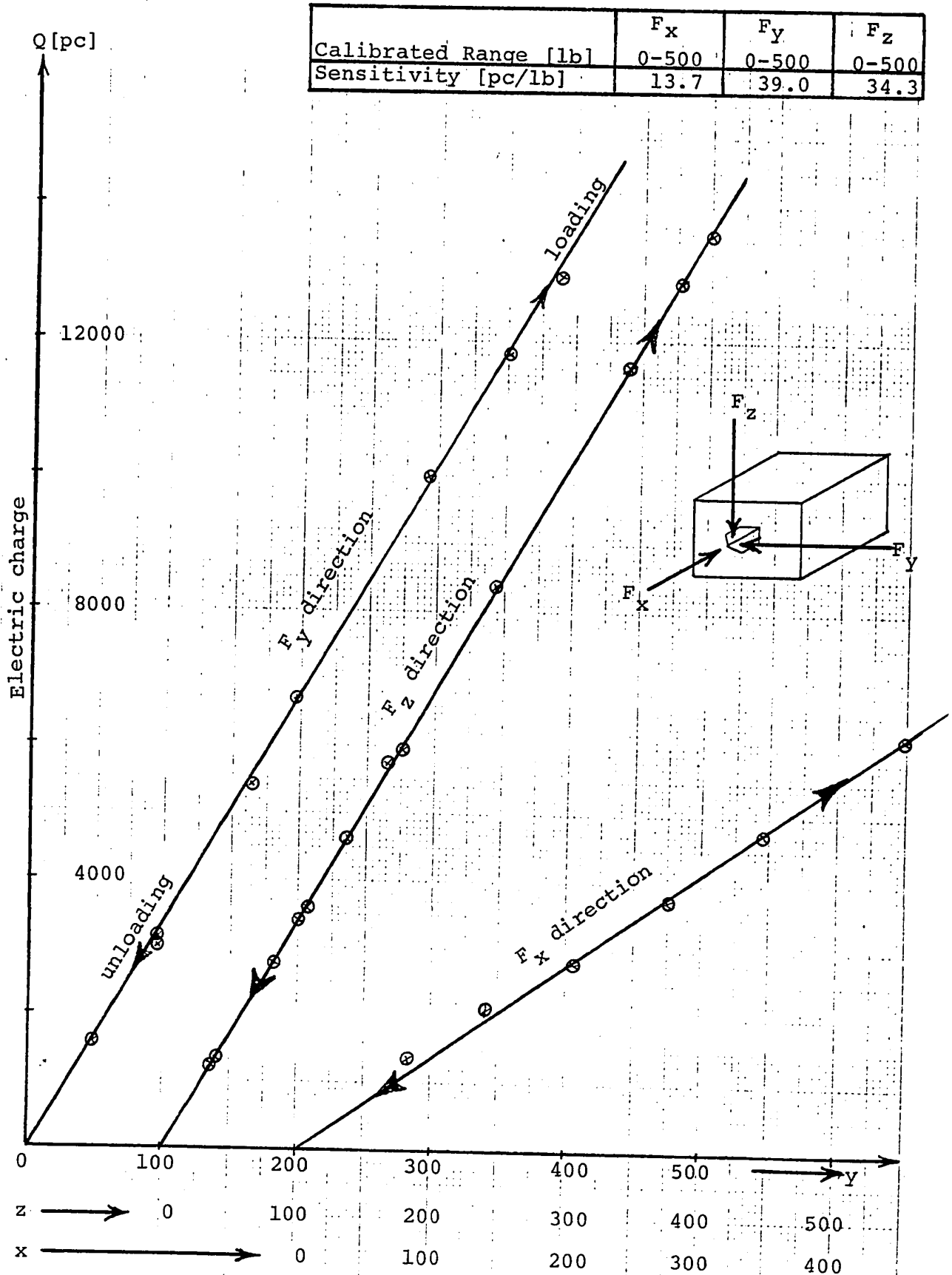
FIG. 3.7 - Static calibration factors for the three-component piezoelectric dynamometer.
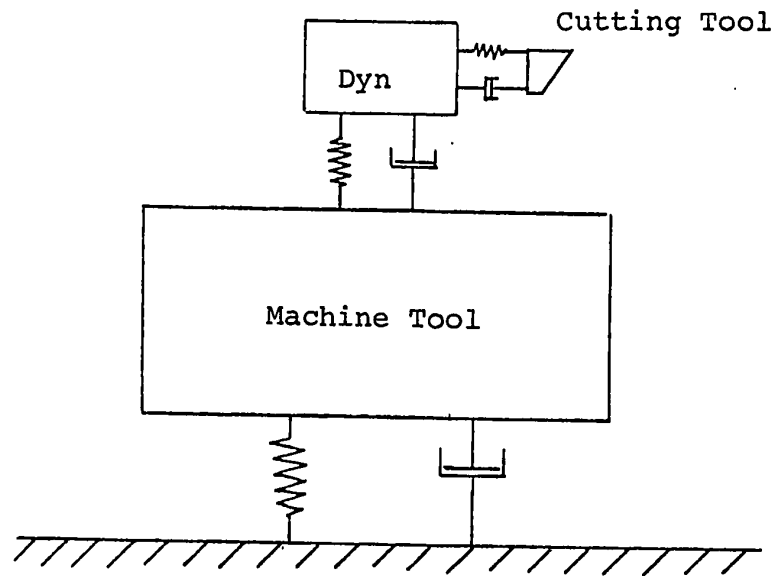
FIG. 3.8 - Machine tool-dynamometer vibration system.
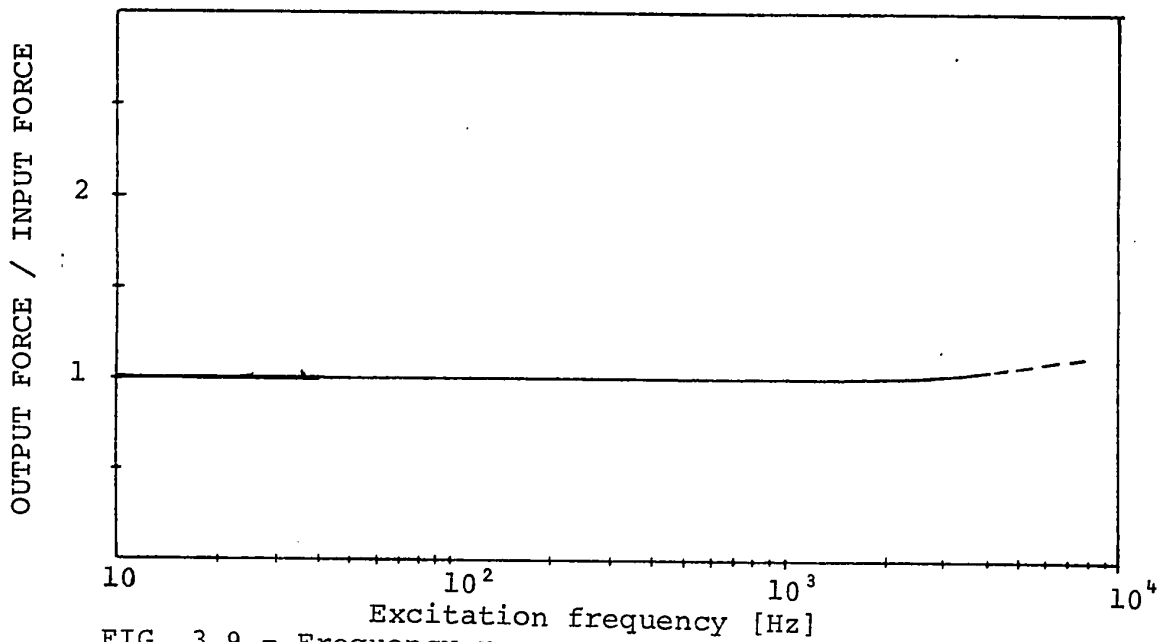
FIG. 3.9 - Frequency response curve of the machine tool-
dynamometer system
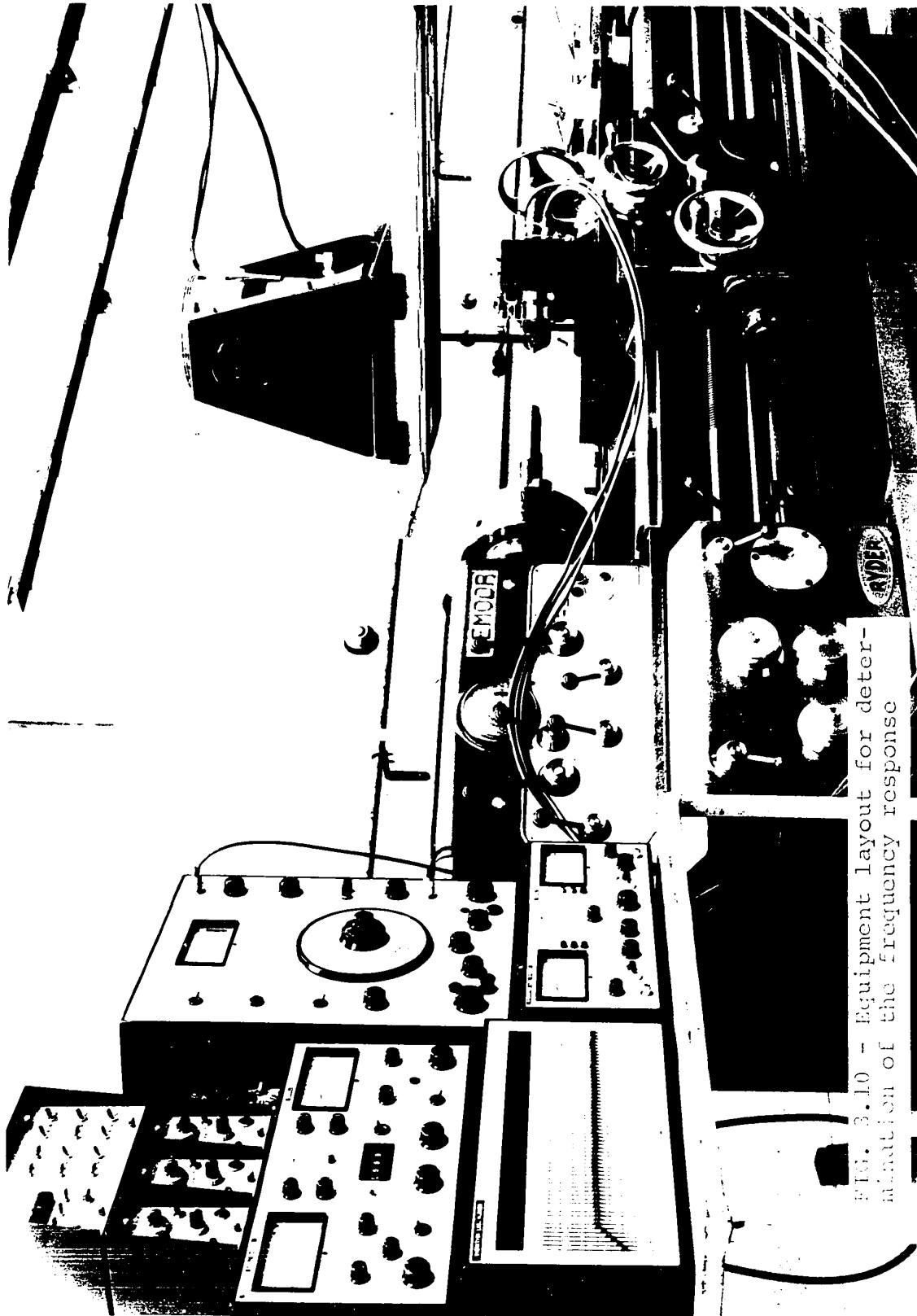
FIG. 3.10 - Equipment layout for deter-
mination of the frequency response
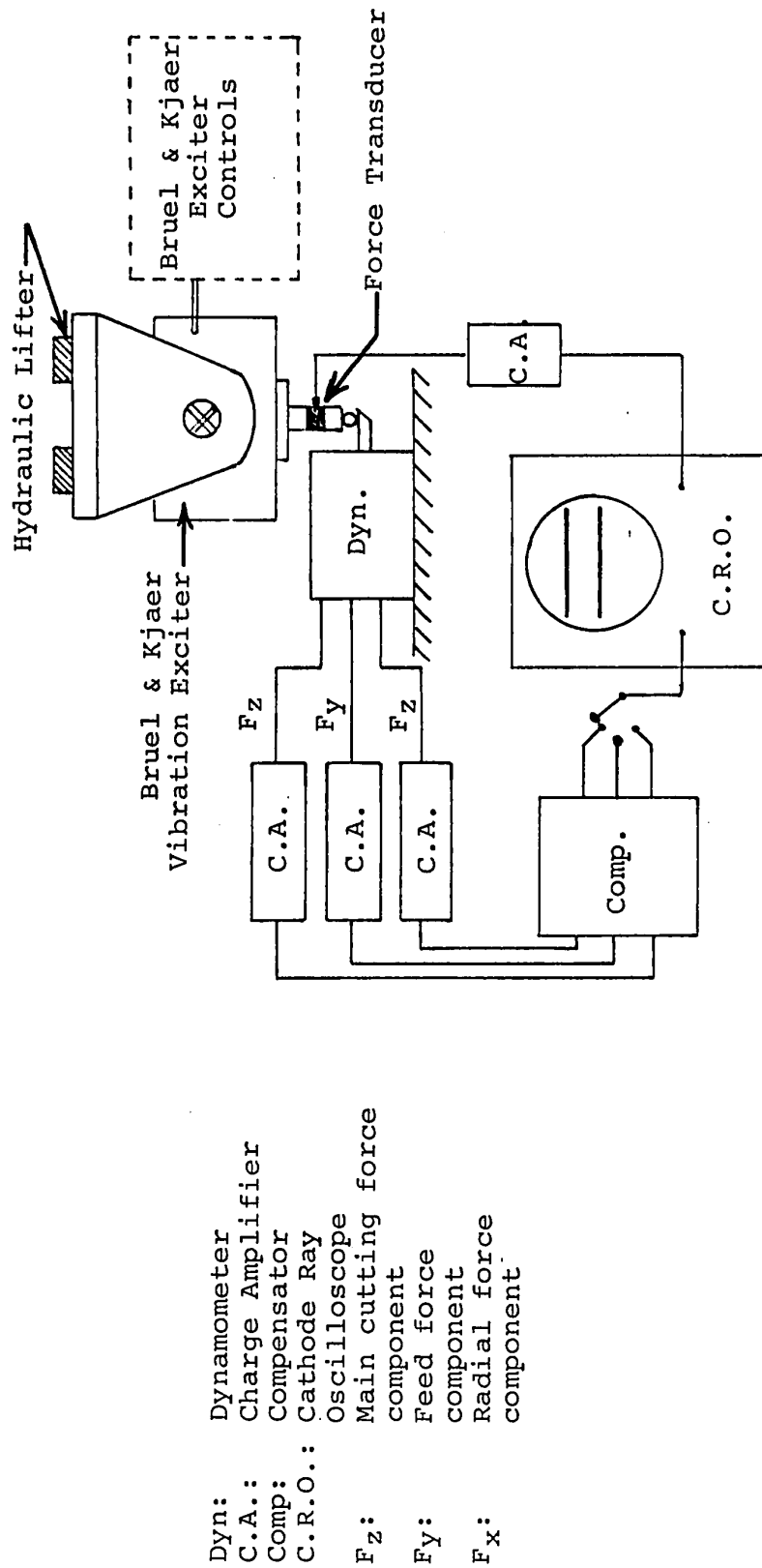
FIG. 3.11 - Block diagram of the set-up for the determination of the frequency response

Dyn: Dynamometer
C.A.:   Charge Amplifier
Comp: Compensator
C.R.O.:   Cathode Ray
           Oscilloscope
F_z: Main cutting force
           component
F_y: Feed force
           component
F_x: Radial force
           component

	FINISHING CUT	MEDIUM CUT	ROUGHING CUT
CUTTING SPEED (ft/min)	49.6	31.4	31.4
FEED PER REV (in/rev)	.002	.010	.025
DEPTH OF CUT (in)	.010	.050	.075

Table 3.1 - Cutting conditions used in the experiments
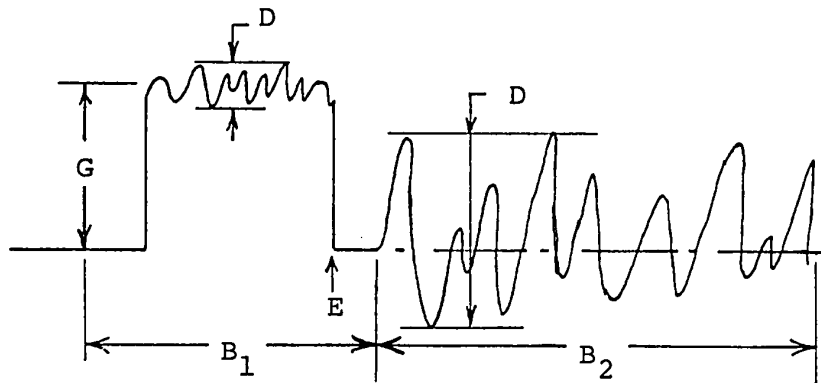
B_1 = Range 1, (eg. 100 lb/v)

B_2 = Range 2, (eg. 5 lb/v)

G = Preload force

D = Superimposed dynamic force

E = Amplifiers reset

FIG. 3.12 - Effect on the signal when resetting the charge amplifier to a higher sensitivity range

SECTION
A-A

γ_c

γ_b

B

A

t

a

κ'

κ

s

B

A

SECTION
B-B

γ_s

α_s
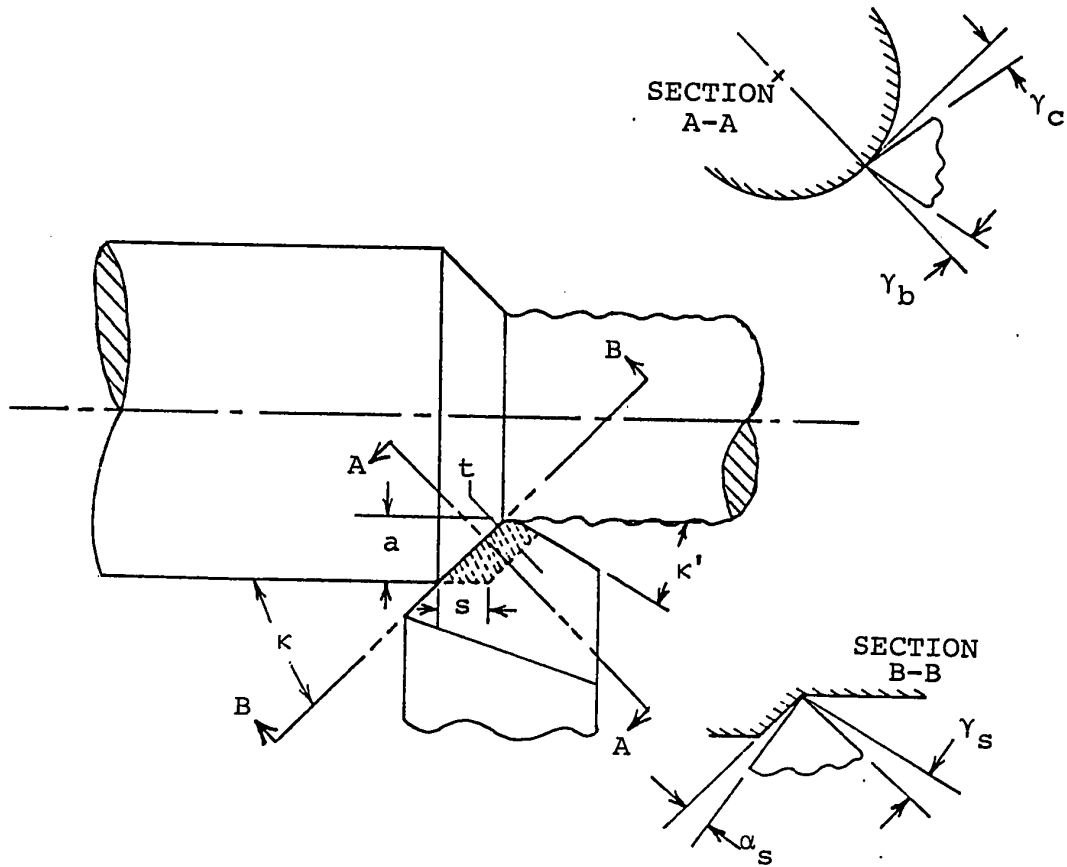
Tool material: carbide insert
κ (angle of approach) = 45^0
κ' (end cutting edge angle) = 35^0
f   (feed)
a   (depth of cut)
t   (chip thickness)
α_b (back rake) = 6^0
γ_c (clearance angle) = 5^0
α_s (side rake) = 0^0
γ_s (side relief) = 11^0
r   (tool nose radius) = 1/32 in.

FIG. 3.13 - Tool material and geometry used for the cutting tests.

TYPE OF CUT: ROUGHING
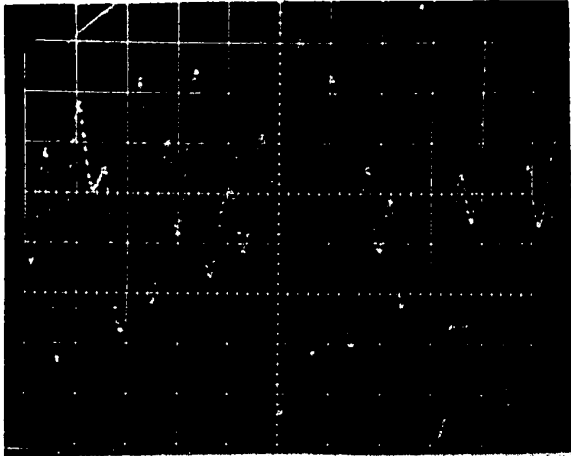
Main Cutting Force Component

F_z

Static Force = 480 lbs

50 lbs

4 m sec
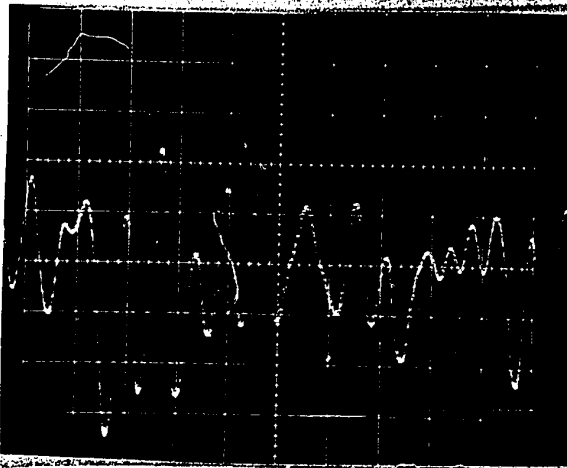
Feed Force Component

F_y

Static Force = 155 lbs

20 lbs

2 m sec
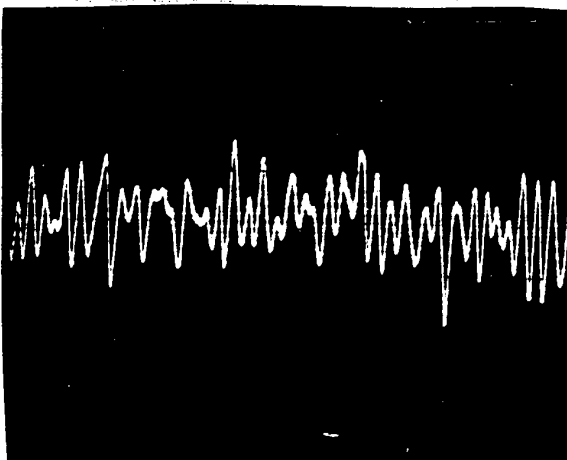
Radial Force Component

F_x

Static Force = 93 lbs

10 lbs

20 m sec

FIG. 3.14   FLUCTUATIONS OF THE CUTTING FORCE
COMPONENTS WHILE CUTTING

TYPE OF CUT: MEDIUM
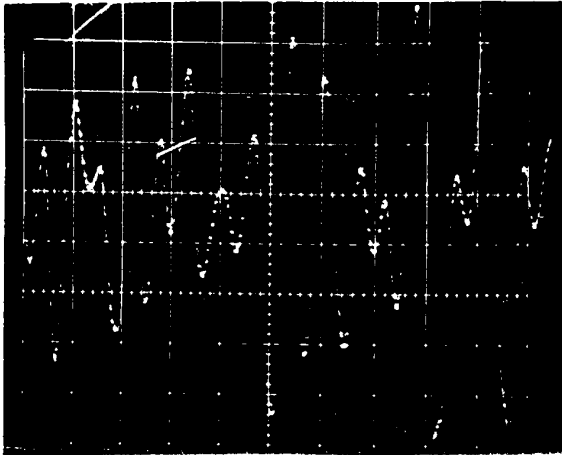
Main Cutting Force Component

$$F_z$$

Static Force = 200 lbs

⊥ 10 lbs

⊢——⊣
1 m sec
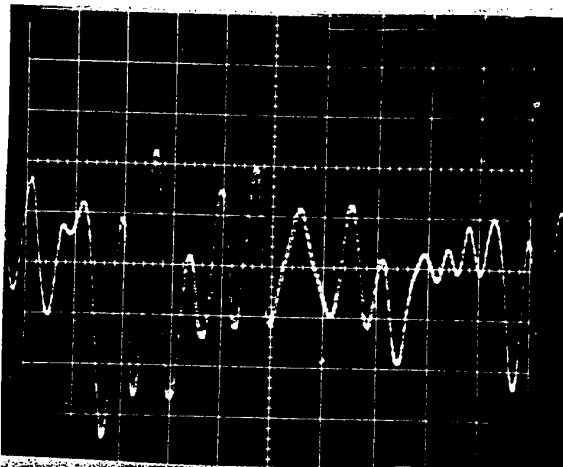
Feed Force Component

$$F_y$$

Static Force = 65 lbs

⊥ 5 lbs

⊢——⊣
1 m sec
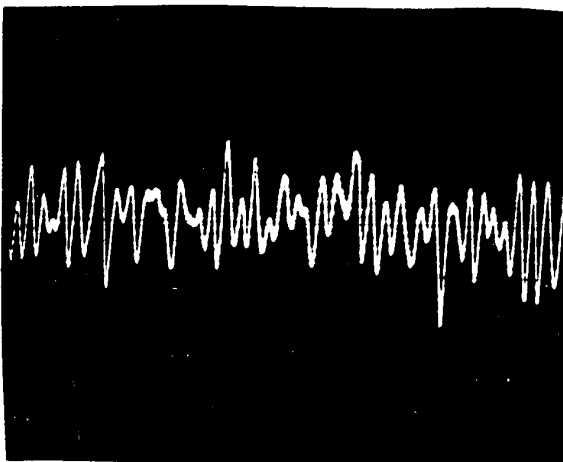
Radial Force Component

$$F_x$$

Static Force = 31.5 lbs

⊥ 5 lbs

⊢——⊣
4 m sec

FIG. 3.15   FLUCTUATIONS OF THE CUTTING FORCE
COMPONENTS WHILE CUTTING

TYPE OF CUT: FINISHING
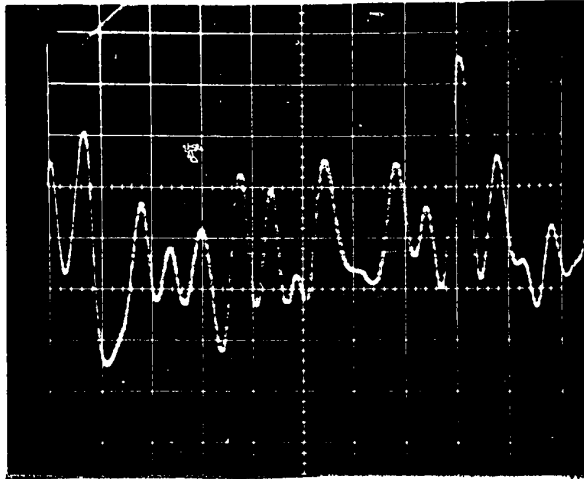
Main Cutting Force Component

F_z

Static Force = 16.7 lbs

$\mathbf{I}$ 2 lbs

$\longmapsto$
.4 m sec
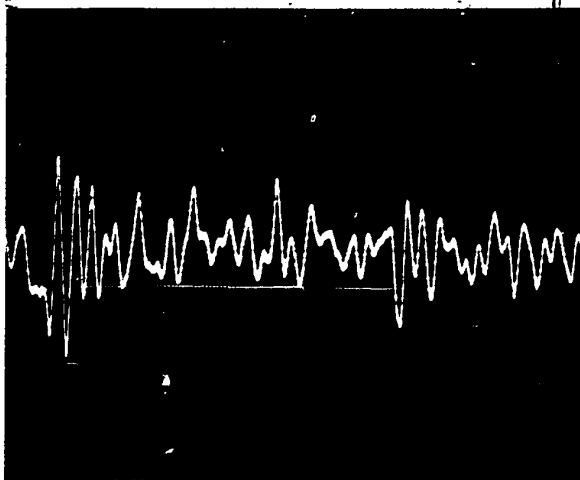
Feed Force Component

F_y

Static Force = 6.0 lbs

$\mathbf{I}$ 1 lb

$\longmapsto$
1 m sec
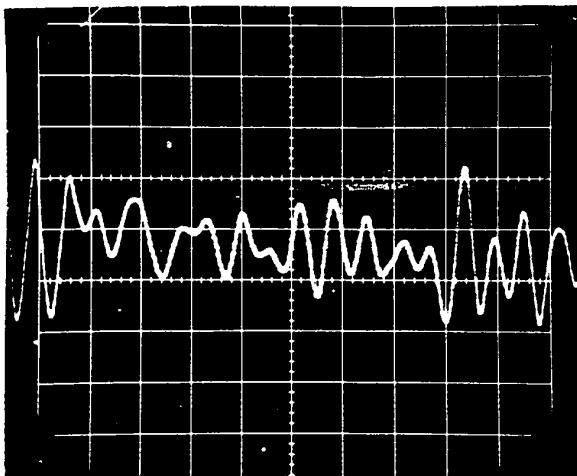
Radial Force Component

F_x

Static Force = 3.9 lbs

$\mathbf{I}$ 1 lb

$\longmapsto$
1 m sec

FIG. 3.16   FLUCTUATIONS OF THE CUTTING FORCE
COMPONENTS WHILE CUTTING

C H A P T E R   4

DATA PROCESSING, ANALYSIS AND MATHEMATICAL

FORMULATION OF THE CUTTING FORCE FLUCTUATIONS

In this chapter the procedure for data processing, data analysis and mathematical formulation of the cutting force fluctuations are explained in detail. This consists basically of three parts:

(i) Analog-to-digital conversion of the cutting force signal stored on the magnetic tape. This requires a special data acquisition system.

(ii) Analysis of the digitized signal obtained from (i), using a specially written computer program to obtain the statistical properties.

(iii) Using the information obtained from (ii), a mathematical model of the dynamic cutting forces in metal cutting is proposed.

4.1 Data Processing of the Cutting Force Signals

The analog signals of the cutting forces recorded on the magnetic tape are converted to sets of discrete data points, for the purpose of analysis. The conversion is made by a sequence of programs running on an Electronics Associate Incorporated 690 digital computer (EAI-690) and a Control Data Corporation 6400 digital computer (CDC-6400). Figure 4.1 shows schematically the arrangement of measuring, sampling

and transmitting the force signals to the CDC-6400 computer for analysis.

Before processing a force signal, the conversion program for the EAI-690 is first loaded. Then the magnetic tape output signal line is connected to an analog-to-digital conversion channel and the magnetic tape is started. Digitizing and storing of the force signal in the EAI-690 memory buffer is initiated by a command from the operator. Both the reading rate on the analog-to-digital channel and the memory buffer are adjustable. These adjustments are explained in detail in a later part of this chapter.

When the memory buffer is full the program on the EAI-690 automatically enters the output phase. In this phase, the contents of the buffer are converted to a form which can be transmitted over telephone lines. Then the transmission is made and the digitized force signal is stored on a disc file on the CDC-6400 computer. The data are transmitted in logical records of eighteen data points at a rate of thirty characters per second.

The EAI-690 computer is limited to approximately eight thousand words of core. The actual acquisition program together with the system reserved areas take a maximum of eight hundred words of core, leaving 7200 words for data points. The number of points required is dependent both on the reading rate and on the length of sample to be analysed. The reading rate, in turn, depends on the maximum force frequency. For

the experiments conducted in this investigation, 4000 Hz is taken to be the maximum reliable force fluctuation since the dynamometer is reliable, according to dynamic tests in section 3.3, only up to this frequency limit.

From the "sampling theorum" of Lathi [30], it is necessary to sample the signal at a minimum rate of one-half the period of the highest order frequency contained in the signal. This implies that 8000 samples per second is the minimum sampling rate and the signal duration is 7200/8000, or 0.9 seconds. If it becomes necessary to increase the number of data points, a second record of 7200 data points may be transmitted and appended to the first record. Such a technique was used in this investigation and thrrefore all the analyses are based on 14400 data points per force signal.

4.2 Statistical Analysis of the Data Points of the Force Signals

Now that the force signals are digitized and stored on a disc file, statistical analysis may be performed. The purpose of the analysis is to obtain the characteristic properties of the force signals. By determining these, a mathematical model of the cutting forces may be found. Therefore the following analysis was carried out on the forces in order to determine their characteristic properties and thus attempt to mathematically model them.

(i) Testing of the stationarity of the cutting forces.

(ii) Testing for the degree of randomness of the cut-
      ting force fluctuations, using autocorrelation
      functions.

(iii) Determination of the probability distribution of
      the amplitudes of the random cutting forces.

(iv) Frequency analysis of the force fluctuations.

4.2.1 Testing for the Stationarity of the Cutting Forces

A proof for stationarity would theoretically involve
verification that all statistical properties of all possible
sample force records (ensemble) are invariant under time
shifts. Such verifications are not feasible in practical
terms since the number of possible sample force records for
a certain cutting condition is infinite. However, by employ-
ing certain assumptions recommended by Bendat [31], which are
generally valid for a majority of random processes, practical
tests for stationarity can be developed. The assumptions are:

(i)   Sample records of the cutting forces to be invest-
      igated are representative of the process and are
      sufficiently long.

(ii) The stationarity test can be carried out over a
      sequence of short time intervals within the same
      signal.

(iii) The stationarity requirement is fulfilled if the
      mean value and the autocorrelation function of the
      signal prove to be invariant under time translations.

(iv)   If the mean square value (or variance) of the
       digitized signal is stationary, then the autocor-
       relation function is also stationary.

The stationarity of each sample record is investigated
under the above assumptions, using the following procedure.

(1)   The sample record is divided into 100 equal time
      intervals and the data in each interval are made
      uncorrelated from the next by allowing a spacing
      between each interval.

(2)   The mean value and the variance for each interval
      is computed and analysed in time sequence as
      follows:

$$\overline{X}_1, \quad \overline{X}_2, \quad \ldots \ldots \overline{X}_i, \quad \ldots \ldots$$

$$\overline{X}_1^2, \quad \overline{X}_2^2, \quad \ldots \ldots \overline{X}_i^2, \quad \ldots \ldots$$

$$i = 1, 2, \ldots 100$$

where $\overline{X}_i$ and $\overline{X}_i^2$ represent the mean value and the variance
of the process in the ith interval

(3)   The above mean and variance sequences are tested
      for stationarity by the "run and trend tests".
      These tests are explained in Bendat [31] and Ap-
      pendix A gives the computer program developed to
      carry out these tests.

According to Bendat [31] the "run test" for the mean value and the variance sequences of one hundred intervals should give a number of runs in the range of 38-63. For the "trend test" this number should be between 2083 and 2866. Table 4.1 shows the results of the computer program for the two tests on the cutting force signals. It can be seen that the force fluctuations for the finishing cut are stationary, while those of the medium cut and roughing cut are not.

4.2.2 Testing for the Degree of Randomness of the Cutting Force Fluctuations Using Autocorrelation Functions

Since the main objective of this investigation is to obtain a mathematical model of the fluctuations of the cutting forces, it is important to determine whether these fluctuations are basically random or otherwise. The determination of the autocorrelation function is a powerful means to draw conclusions concerning the randomness of the cutting force.

The autocorrelation function is defined by

$$R(\tau) \quad = \quad \frac{1}{N-\tau} \sum_{k=0}^{N-\tau} y(k) \; y(k+\tau) \qquad (4.1)$$

$$\tau \quad = \quad 0, \; 1, \; \ldots \; 100$$

where

$$N \quad = \quad \text{number of sample points in a record}$$

$$\tau \quad = \quad \text{time lag}$$

and $y(k)$ is the sample record.

From equation 4.1 it may be noted that the autocorrelation characterizes the dependence of y(k) on y(k+τ). This property is valuable in discriminating between purely random and periodic functions. Another property of the autocorrelation function, not as clearly shown by equation 4.1 is that, at large τ values, the square root of the autocorrelation approaches the mean value of the signal. This is given by the equation

$$m = [\ R(\infty)\ ]^{\frac{1}{2}} \qquad\qquad 4.2$$

where m is the mean of the signal. Therefore, if any signal contains a steady component the autocorrelation function will approach asymptotically this mean value for large τ. Figure 4.2 shows the autocorrelation plots for some well-known signals. From these plots one can conclude that the degree of randomness is a direct function of the rate of decay of the oscillations. The lack of any oscillations in an autocorrelation plot indicates a pure random signal.

To determine the autocorrelations for the three signals recorded from the cutting experiments, a computer program was prepared (Appendix A). This program utilizes 14,400 data points for the analysis of each force signal. Figure 4.3 shows the normalized (with respect to R(0)) autocorrelation plots of the three cutting force signals. From these curves it may be seen that the three functions follow a pattern

similar to that of Figure 4.2c. This indicates that cutting

force fluctuations are basically random and have very little

harmonic content. It may also be seen from the same figure

that the autocorrelation curves asymptotically reach a value

in the order of .05. This suggests that the signals contain

some static component.

4.2.3 The Probability Distribution of the Random Cutting Forces

To study the amplitude density of the forces the ampli-

tude range of each is divided into an appropriate number of

intervals, N y(k). Then the sample points in each interval

are summed, and by plotting the number of data points in each

interval versus the force amplitude the amplitude density

is obtained. Figures 4.4, 4.5, and 4.6 show the amplitude

density curves of the force fluctuations for the three cuts.

It may be seen that the amplitude density for a roughing cut

exhibits high concentration of amplitudes about the mean value

of the cutting force with very small variance. In addition,

it has two symmetrically located peaks with respect to the

mean of the cutting force. As the cutting operation becomes

finer, the force signal contains more and more amplitudes of

the same order of magnitude and the distribution tends to be

Gaussian, as shown in Figures 4.6.

These observations are substantiated from the results

obtained from the normality curves of the three signals shown

in Figures 4.7, 4.8, and 4.9. Figure 4.7 shows the normality

curve for the roughing cut, in which two portions have the same slope and the middle portion has a relatively higher slope. This difference in slopes can be explained from the amplitude density curve Figure 4.4 where the middle portion shows the high concentration of amplitudes and the other two portions give symmetrical distributions with a larger variance. This pattern is maintained for the medium cut and finishing cut, Figures 4.8 and 4.9. However, the slope of the middle sections of the normality curves decreases, and those of the other two sections increase, indicating a trend towards a Gaussian distribution as the cut becomes finer.

4.2.4 Frequency Analysis of the Force Fluctuations

In order to obtain a complete picture regarding the dominating frequencies in the cutting forces the power spectral density is computed analytically. This is done by a computer program, listed in Appendix A, that utilizes the 14,400 data points of each force signal. This program uses the following equation, after Bendat [31], for computing the power spectral density.

$$G\left(\frac{kF_c}{M}\right) = \frac{1}{F_c}\left[\, R(0) + 2\sum_{r=1}^{M-1} R(r)\cos\left(\frac{\pi r h}{M}\right)\right.$$

$$\left. + (-1)^k R(M)\,\right]$$

(4.3)

$$k = 0, 1, \ldots M$$

where F_c = critical frequency

$R(0)$ = autocorrelation at zero time lag

$R(\tau)$ = autocorrelation at τ time lag

Figure 4.10 shows the computed power spectral density for the three cuts. Curve one corresponds to the roughing operation, curve two to the medium cut and curve three to the finishing operation. The medium and the roughing cuts exhibit a dominating frequency at 2400 Hz where as the finishing cut shows a dominating frequency at 2150 Hz. The three curves, also show a second peak at a frequency close to zero. This peak might be arising from the machine tool and may be disregarded in the modelling since the forces are not likely to have frequencies near zero.

Considering the frequencies between 480 - 3800 Hz, it may be concluded that the cutting force fluctuations may not be approximated as a white noise or as a wide band process. If such an assumption has to be made to facilitate the solution of the equations of motion in connection with stability and dynamic analysis of machine tools [32], it is suggested that an equivalent constant power spectrum be computed from the actual spectrum curve. This approximation has to be verified to see if it gives similar system responses as that given by the actual spectrum of the cutting forces.

4.3 A Mathematical Model for the Cutting Force Fluctuations

From the previous analysis of the sample records obtained from the three cutting experiments, it can be concluded that the cutting force fluctuations have the following characteristic properties.

(i) The cutting forces are dynamic in nature and not static.

(ii) The amplitudes of the fluctuations vary randomly with some harmonic content.

(iii) Stationarity can be assumed only for finishing operations. Further experimental work is necessary to establish the range of cutting conditions for which stationarity can be assumed.

(iv) The amplitudes of the force fluctuations in a finishing operation exhibit a Gaussian distribution. However, for a roughing operation, the amplitudes are composed of three superimposed Gaussian distributions — one about the average of the force fluctuations and two symmetrically located with respect to the central distribution.

(v) The cutting forces exhibit similar power spectral densities, with one dominating frequency. Therefore, an assumption of white noise or wide band process can not be made. However, there exists the possibility of computing an equivalent power

spectrum that will approximate and give similar
system responses as the actual spectrum.

From these findings the cutting forces in machining may
be modelled as follows:

(i)   In finishing operations they can be considered as
       stationary, whereas in roughing and medium cuts
       this assumption is not valid and must not be made.

(ii)   The amplitude density of the force fluctuations
       for a finishing operation can be modelled by a
       single Gaussian distribution and for a roughing
       and medium cut by three superimposed Gaussian dis-
       tributions. The superimposition of three Gaussian
       curves is shown in Figure 4.11 and given by the
       following equation.

$$p[y(k)] = \frac{1}{\sigma_1\sqrt{2\pi}} \text{EXP} \left[\frac{-y(k)}{2\sigma_1^2}\right] + \frac{1}{\sigma_2\sqrt{2\pi}} \text{EXP} \left[\frac{-(y(k)-\lambda)}{2\sigma_2^2}\right]$$

4.4

$$+ \frac{1}{\sigma_2\sqrt{2\pi}} \text{EXP} \left[\frac{-(y(k)+\lambda)}{2\sigma_2^2}\right]$$

where $p[y(k)]$ is the amplitude density of the force
          fluctuations

σ_1 is the variance of the central distribution

σ_2 is the variance of the other two distribu-
          tions

λ is the mean of the two side distributions

It may be observed that for the case of a finishing operation the second and third terms of equation (4.4) vanish.

(iii) For analytical purposes the spectral density of the cutting forces ought to be expressed in a mathematical form. Simple mathematical forms for the three spectral density curves of the cutting operation in this investigation clearly are not possible. Further, these curves indicate neither narrow nor wide band. However it may be possible to reasonably represent approximately the spectrums in terms of a number of flat portions over the frequency range of the force fluctuations. This approximation can then be used for the purpose of dynamic analysis.
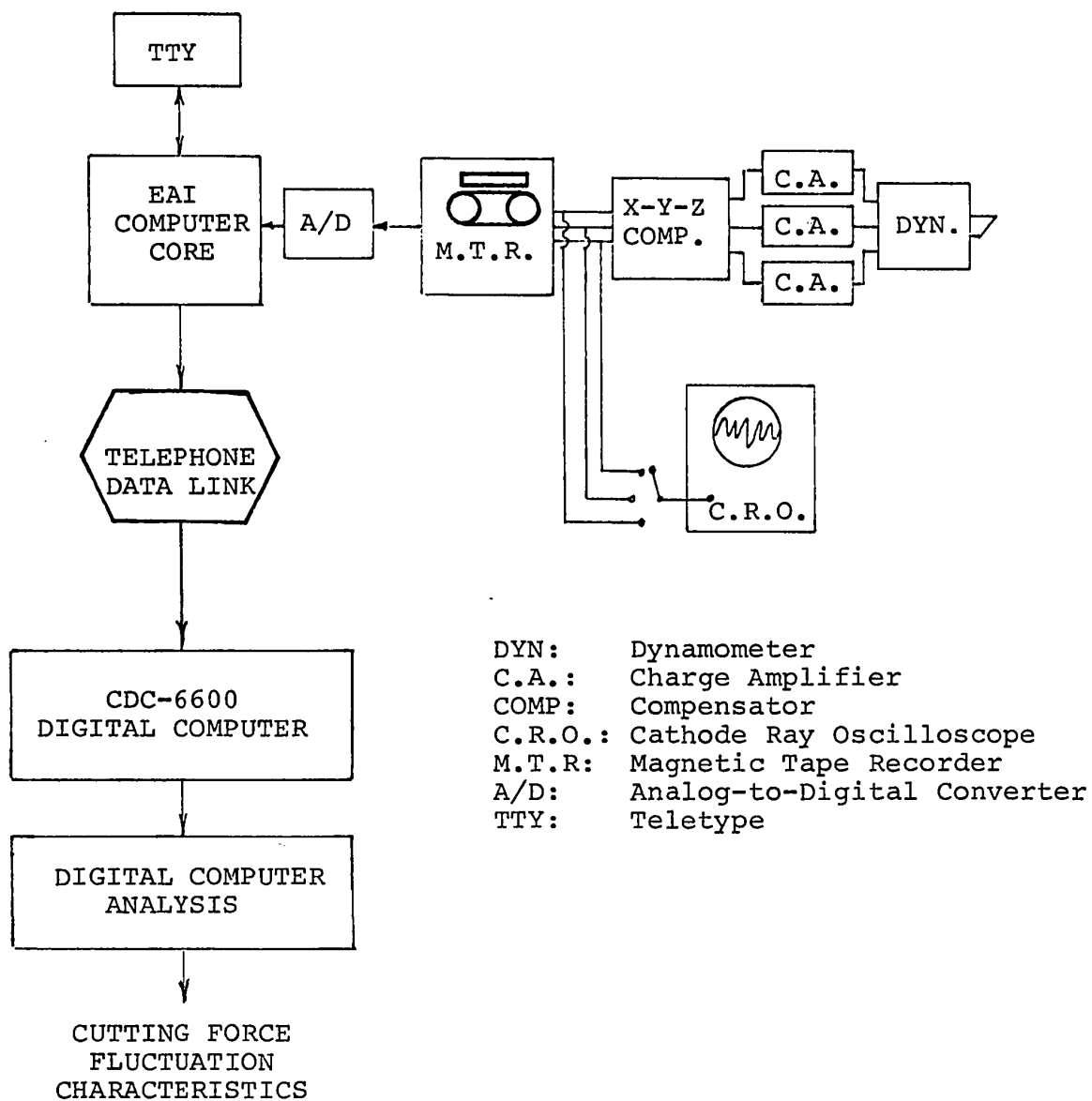
FIG. 4.1 - Schematic arrangement for measuring, digitizing
and transmitting the force signals
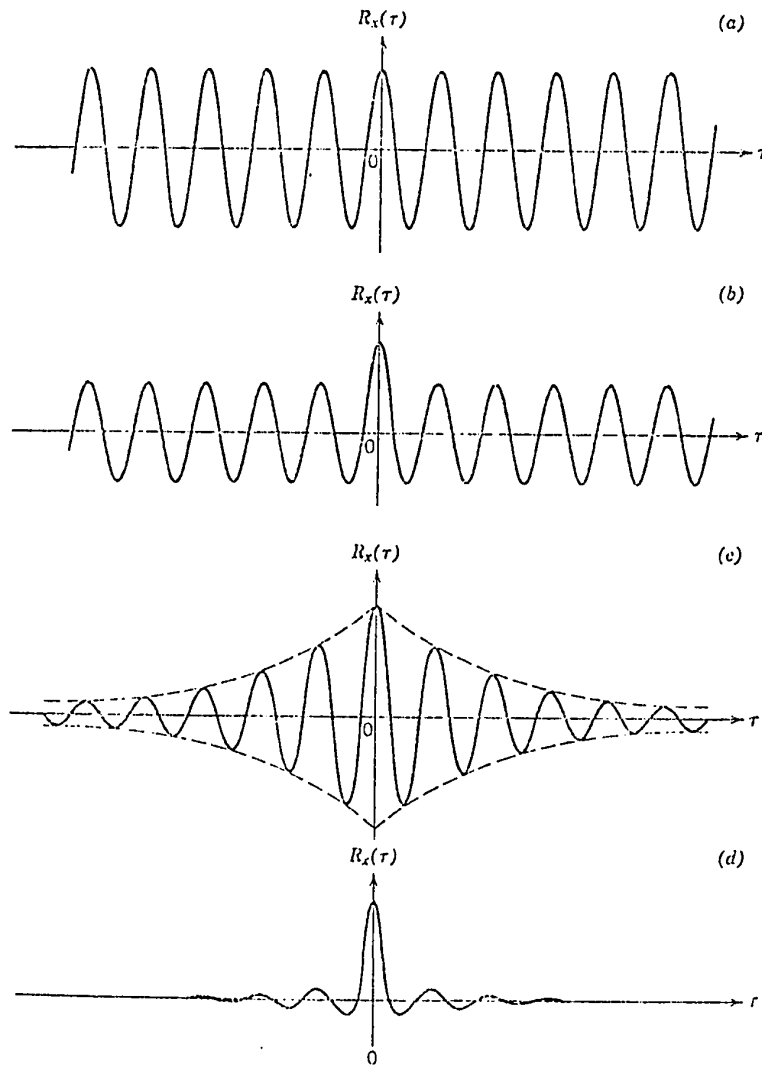
FIG. 4.2 - Plots of autocorrelation function. a) Sine wave.
b) Sine wave plus random noise. c) Narrow-band
random noise. d) Wide-band random noise. (After
Bendat [31]).

	RUN TEST FOR MEAN VALUE SEQUENCE 38-63	RUN TEST FOR VARIANCE SEQUENCE 38-63	TREND TEST FOR MEAN VALUE SEQUENCE 2083-2866	TREND TEST FOR VARIANCE SEQUENCE 2083-2866
FINISHING CUT Main Cutting Force Component F_z	51	50	2229	2553
MEDIUM CUT Main Cutting Force Component F_z	17	2	1919	3094
HEAVY CUT Main Cutting Force Component F_z	19	2	2448	3162

TABLE 4.1 - Results of stationarity tests for the three force signals.
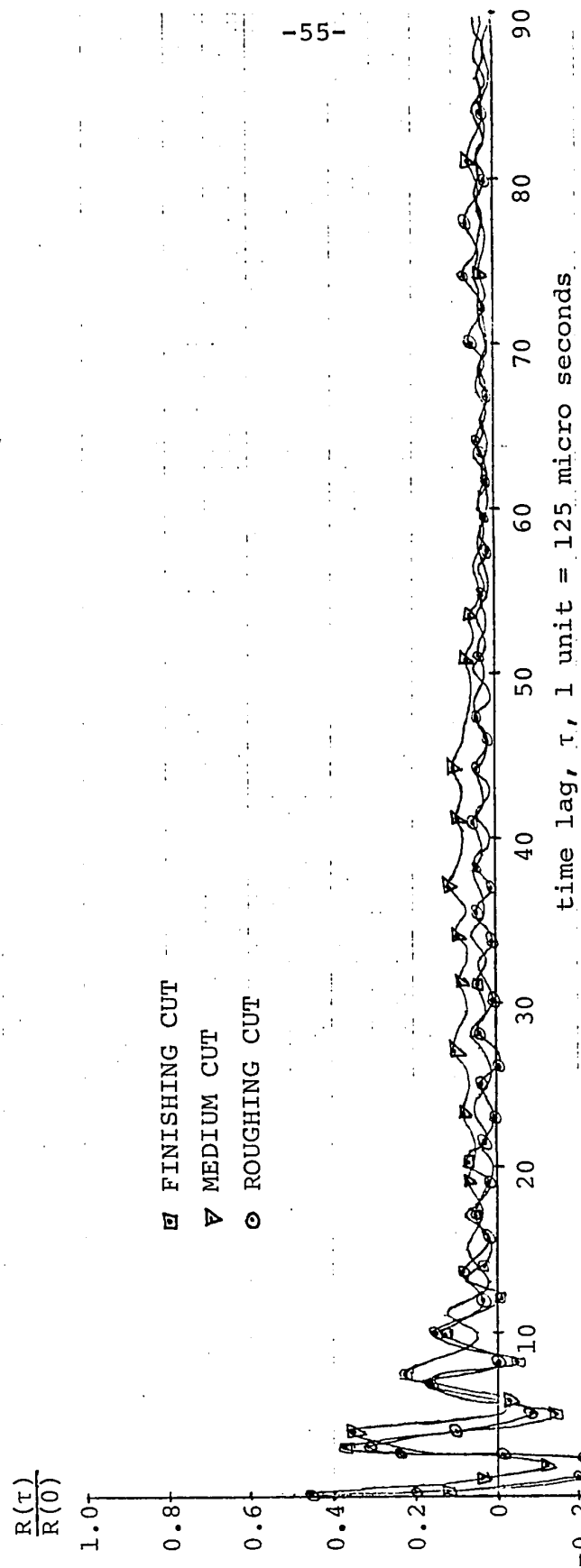
FIG. 4.3 – Autocorrelation plots for the three cutting force signals
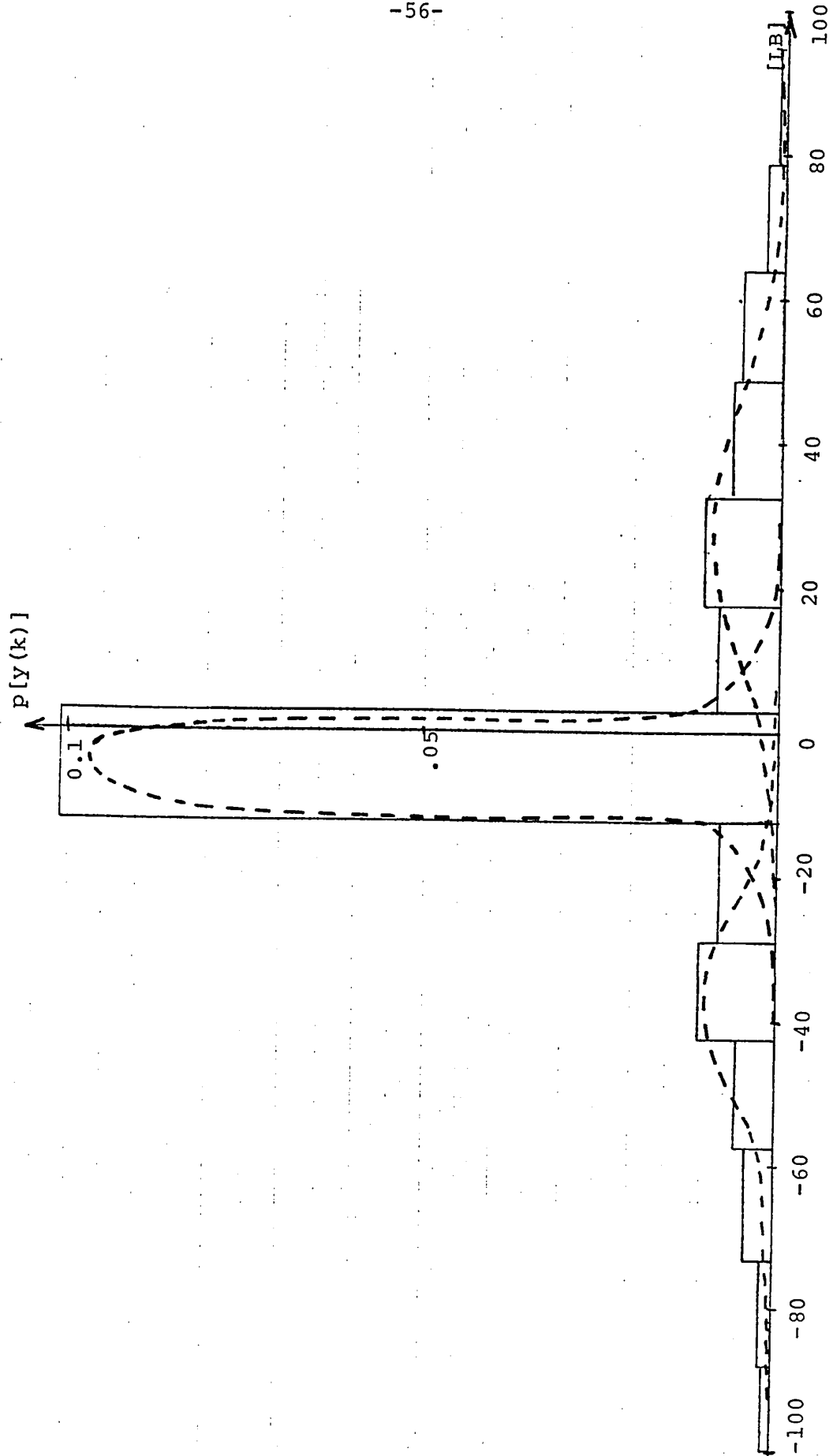
FIG. 4.4 — Amplitude density curve for the force fluctuations for a rough cut
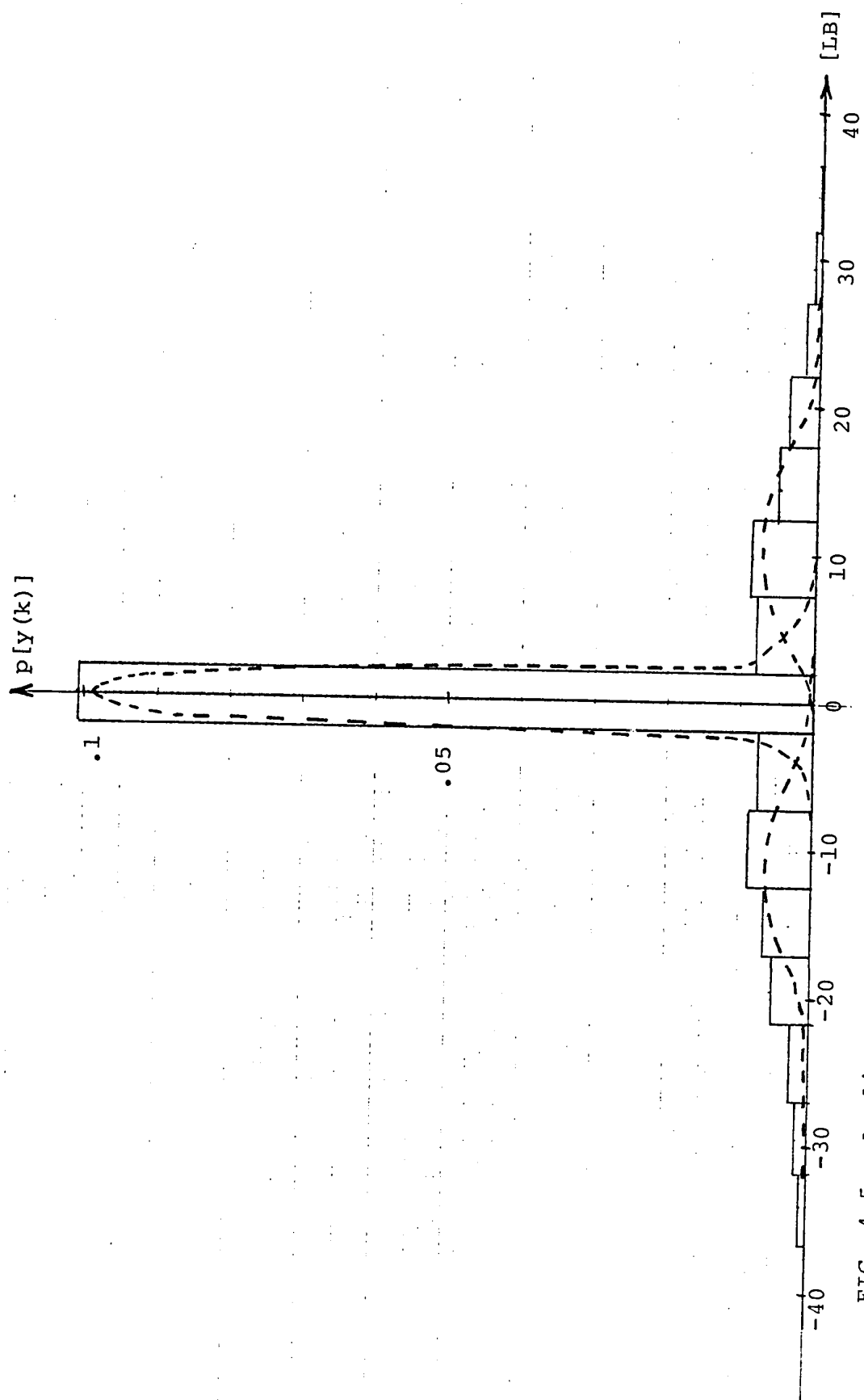
FIG. 4.5 - Amplitude density curve for the force fluctuations for medium cut.
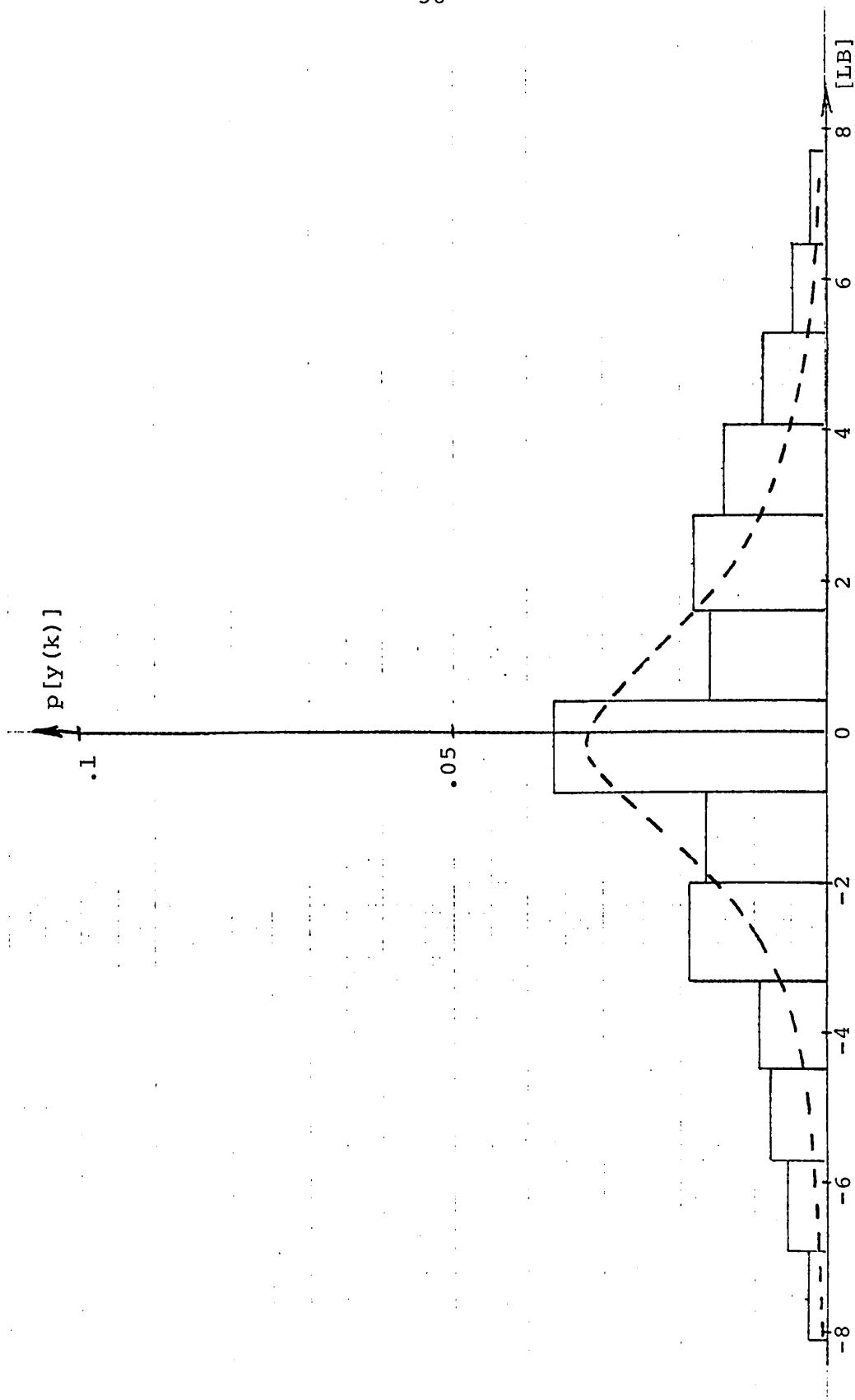
FIG. 4.6 - Amplitude density curve for the force fluctuations for the finishing operation

FIG. 4.7 - Normality curve for the
fluctuations in roughing
operation.
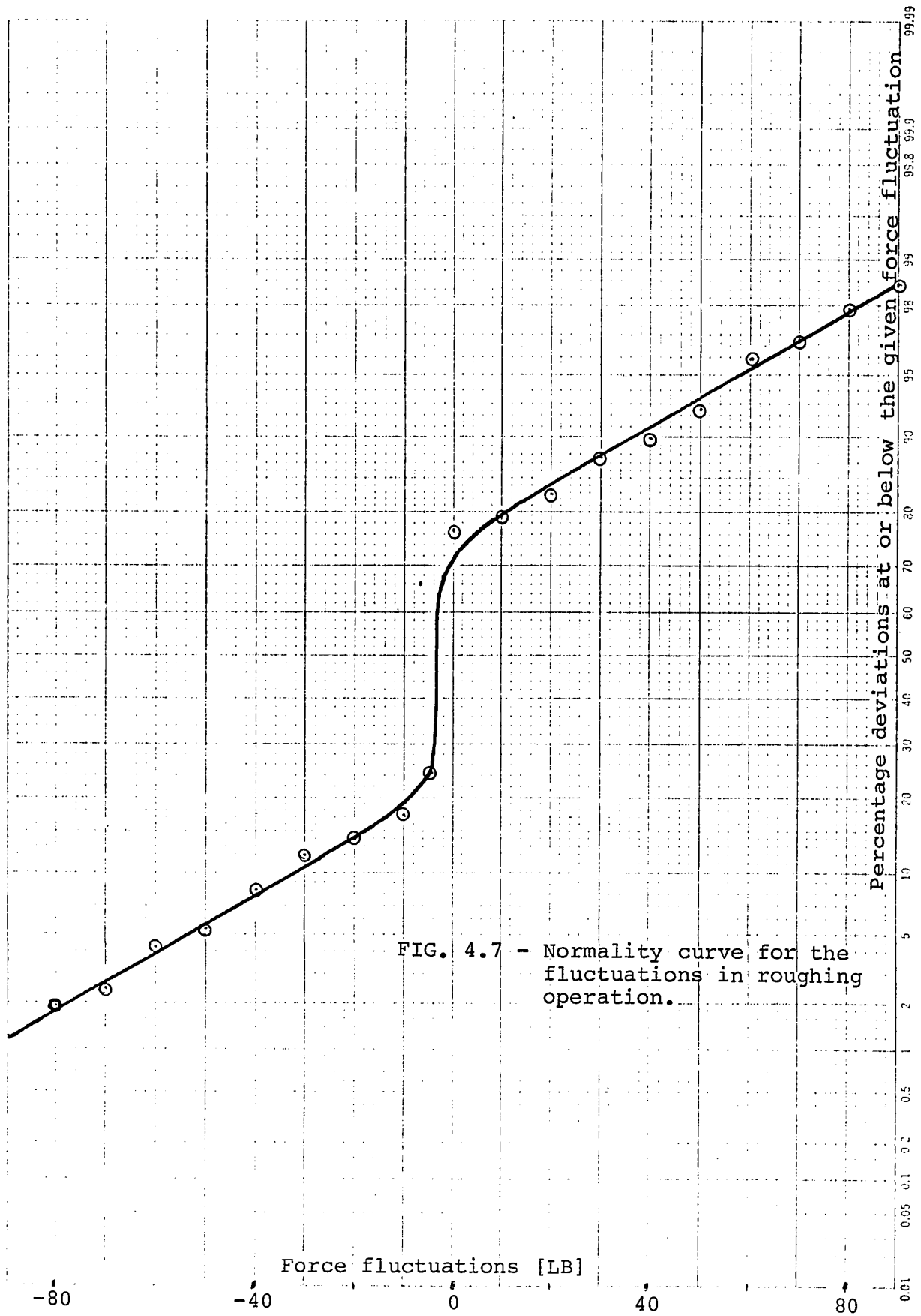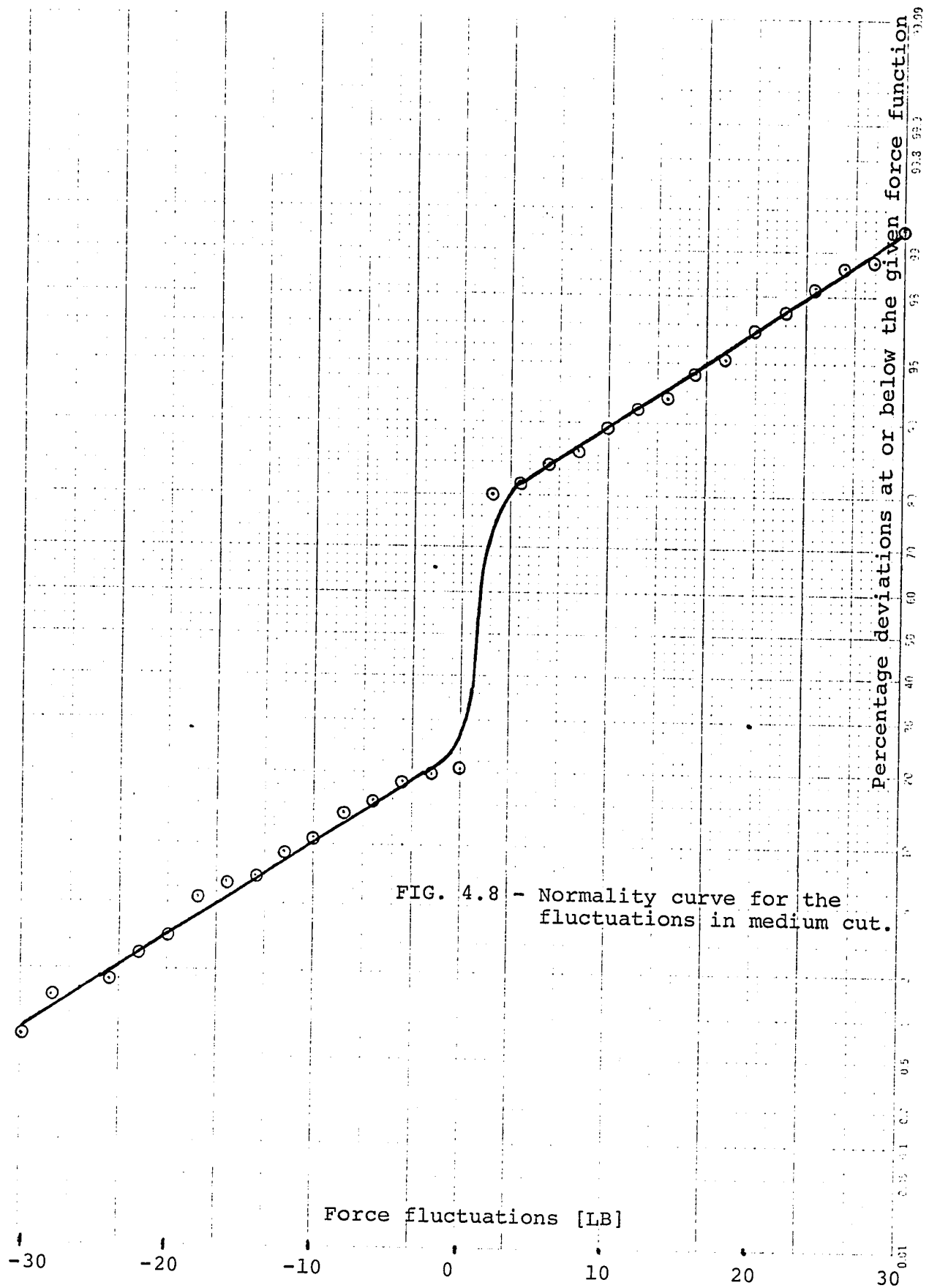
FIG. 4.8 - Normality curve for the
fluctuations in medium cut.

FIG. 4.9 - Normality curve for the fluctu-
ations in finishing cut.
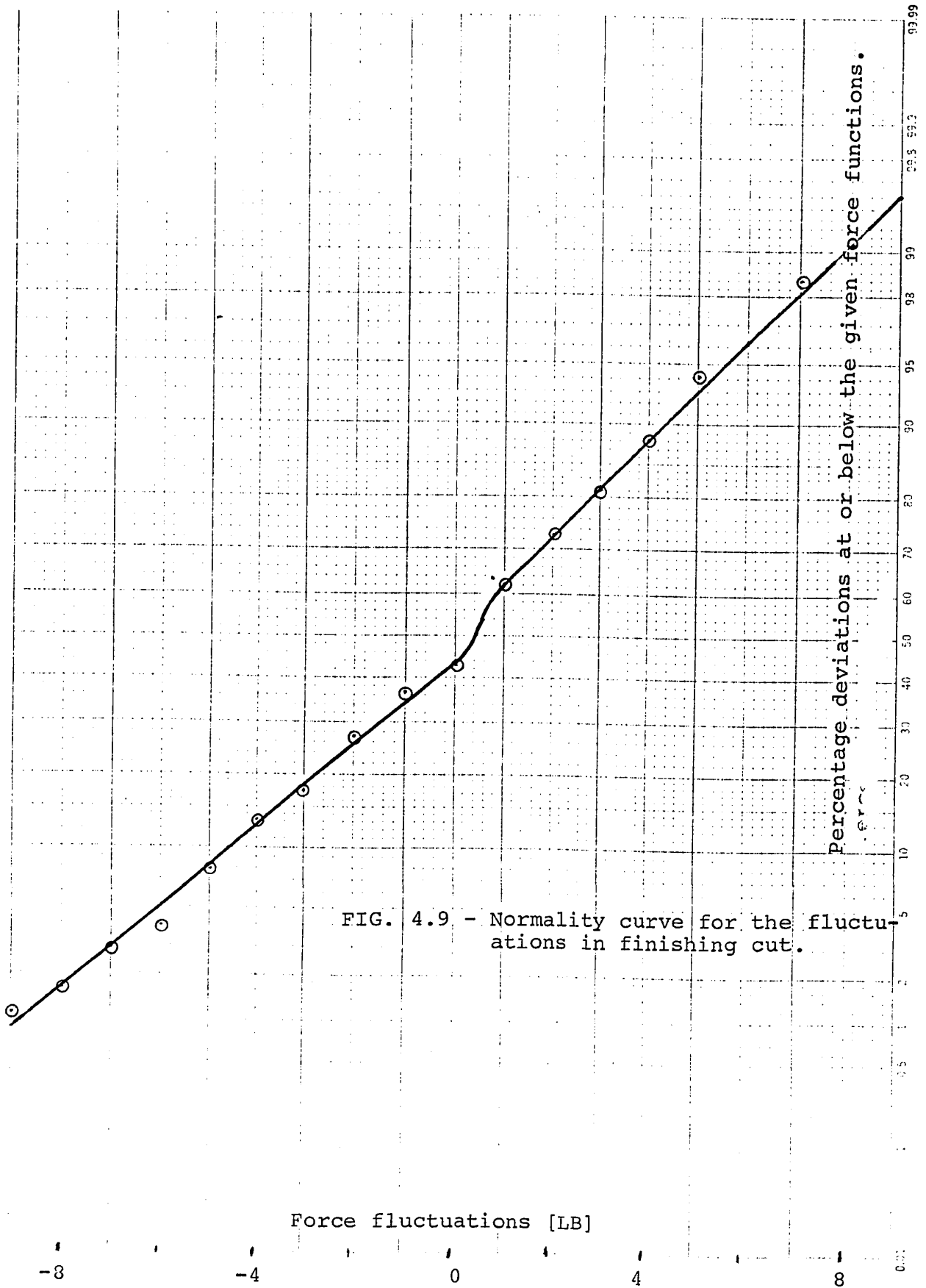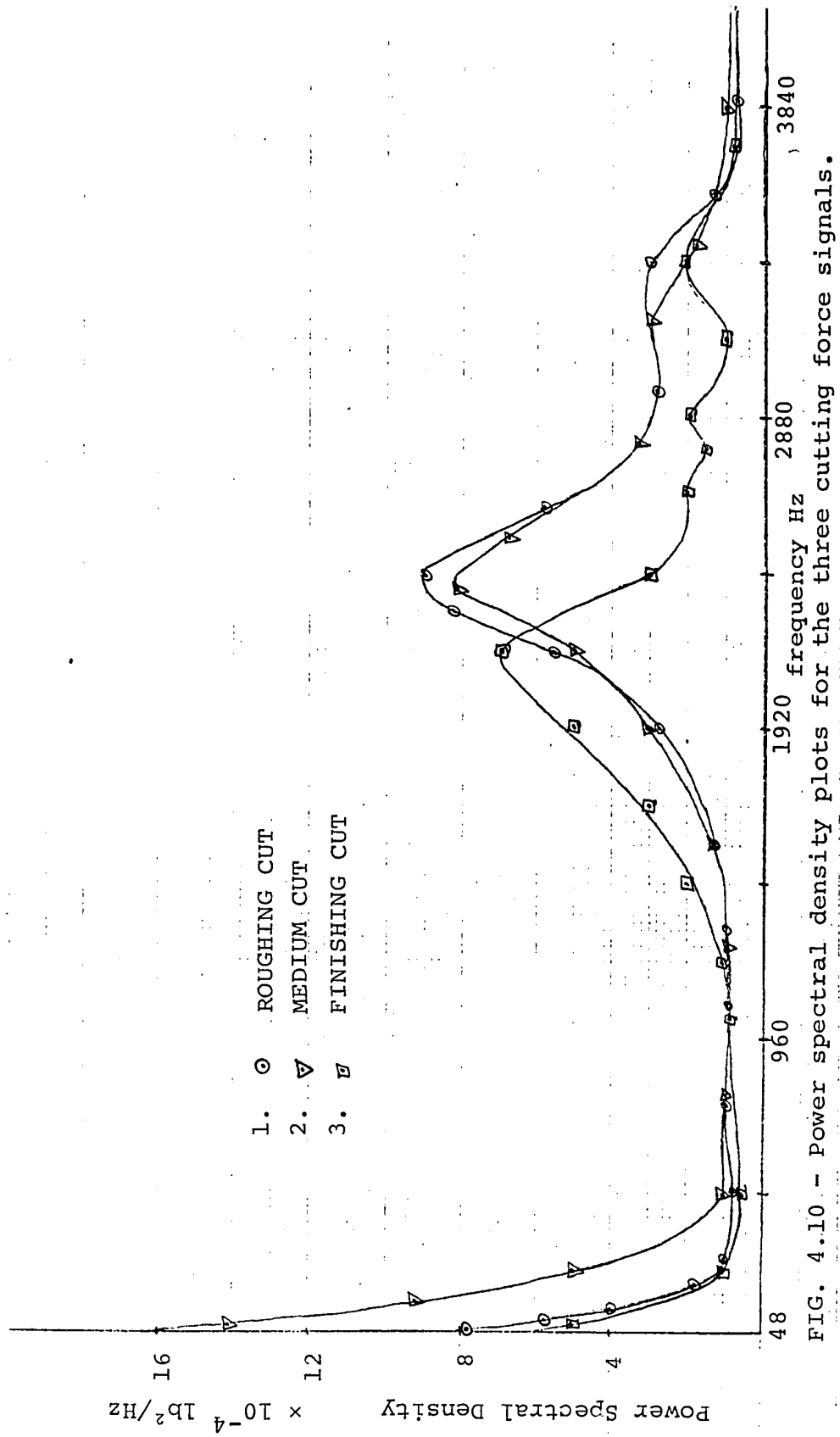
FIG. 4.10 - Power spectral density plots for the three cutting force signals.
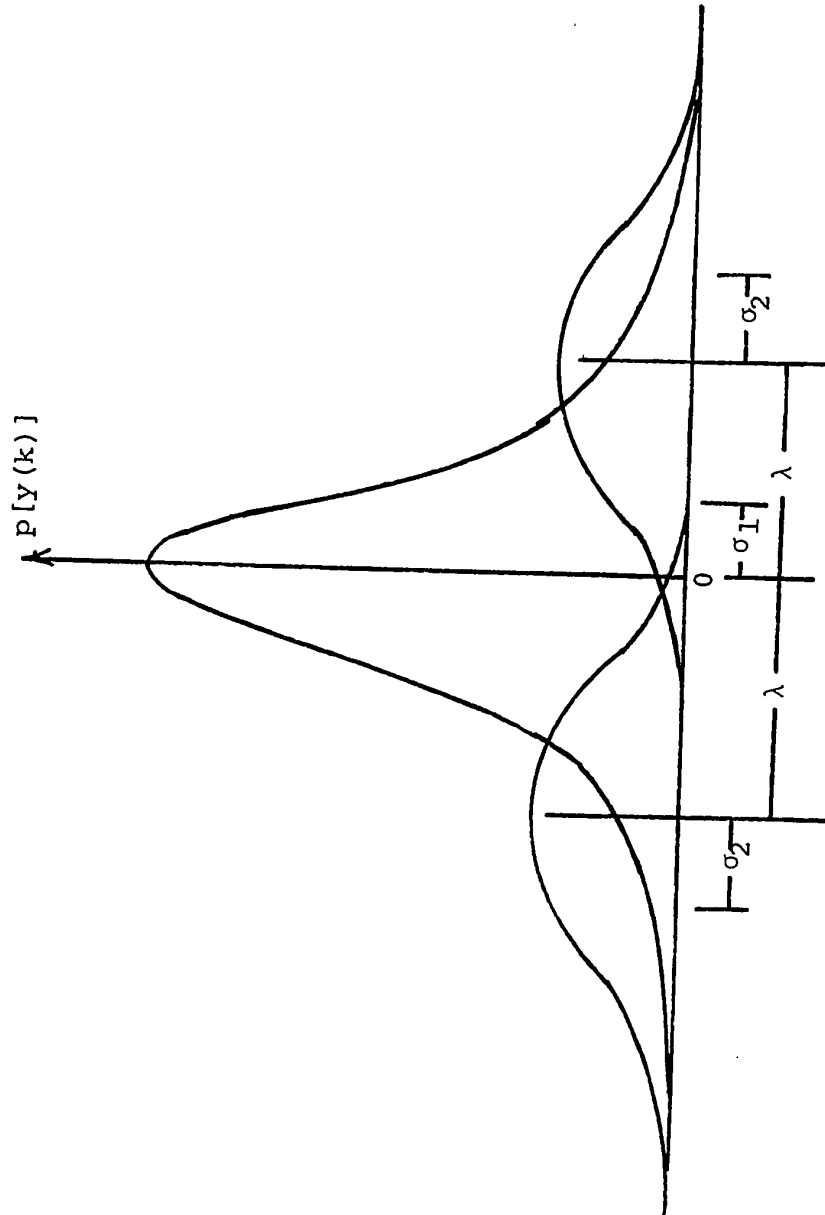
FIG. 4.11 - Three superimposed Gaussian distributions

C H A P T E R   5

CONCLUSIONS AND RECOMMENDATIONS

FOR FUTURE WORK

The cutting forces in single point machining were mea-
sured using a specially designed three-component piezoelectric
dynamometer. Data reduction was then performed on the force
signals to determine their characteristic properties. This
was done, using a computer program written specially for this
purpose.

From the characteristic properties of the forces a math-
ematical model is proposed that describes the cutting forces
more accurately than in the past. The results of the data
analysis are as follows:

   i)   Cutting forces in machining are dynamic and pre-
       dominantly random.

  ii)   The amplitudes of the force fluctuations are random
       with some harmonic content.

 iii)   In a finishing operation the forces are stationary
       but not in medium and roughing operations.

  iv)   While the amplitudes of the cutting forces in fin-
       ishing operations exhibit a single Gaussian distrib-
       ution, those for medium and roughing operations
       show three superimposed Gaussian distributions: one
       about the mean value and two symmetrically located
       about the mean.

v)   The cutting forces exhibit similar power spectra

density curves, with most of the power concentrated

between 1800 Hz and 3400 Hz.

From these results it may be concluded that forces in machin-
ing have to be considered as a stationary random process in
the case of finishing operations, and can be considered to
have a Gaussian distribution. Medium and roughing operations
exhibit three superimposed Gaussian distributions.

Further work is needed for determining the range of cut-
ting conditions for which the dynamic forces can be modelled
by single Gaussian distribution. Also, the degree of error
in the system responses when approximating the actual spectral
density by a number of flat ones should be investigated.
Further investigations are also required to develop mathema-
tical models of cutting forces in other metal cutting operations,
such as milling, drilling and grinding processes.

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A P P E N D I X   A

COMPUTER PROGRAM AND RESULTS

FOR FINISHING OPERATION

```
       PROGRAM RAND(INPUT,OUTPUT,TAPE10)
       DIMENSION Y(14400),X(100),XX(100),R(100),G(101),GG(101)
       DIMENSION HIST(100),PO(100),F(101)   ,D(100),SHIST(100)
C
C      SUBMITED AS PARTIAL FULFILMENT FOR THE DEGREE OF MASTER OF
C      ENGINEERING AT SIR GEORGE WILLIAMS UNIVERSITY, APRIL 1973,
C      BY   SPIROS KONSTANTINOS MARAGOS
C
C      Y=SAMPLE DATA RECORD,OF DIMENSION 14400
C      READ RECORD IN
       READ (10,30) (Y(I),I=1,14400)
30     FORMAT(E15.9,5E16.9)
C      CONVERT MACHINE UNITS TO VOLTS TO FORCE
       SUM=0
       FCF=10
       DO 40 I=1,14400
       Y(I)=    Y(I)*FCF
40     SUM=Y(I)+SUM
       AVGY=SUM/14400.
       PRINT 50,AVGY
50     FORMAT(1H1,///,15X,*THE AVERAGE OF THE SAMPLE RECORD =*,E11.4)
C      CONVERT DATA TO ZERO MEAN VALUE
       DO 51 I=1,14400
  51   Y(I)=Y(I)-AVGY
       SUM=0
       DO 3 I=1,14400
3      SUM=Y(I)*Y(I)+SUM
       R0=SUM/14400.
       PRINT 170, R0
170    FORMAT(/,15X,*THE VARIANCE OF THE SAMPLE RECORD =*,E10.4,/)
C
C      FIRST TEST FOR STATIONARITY
C      THE RUN TEST
       DO 69 J=1,100
       L=J*144
       K=L-139
       SUM=0
       DO 52 I=K,L
52     SUM=Y(I)+SUM
69     X(J)=SUM/144.
       SUM=0
       DO 70 I=1,100
70     SUM=SUM+X(I)
       AVG=SUM/100.
       K=1
       I=1
       IF(X(1).GE.AVG) GO TO 12
13     I=I+1
       IF(X(I).GE.AVG) GO TO 11
       IF(I.EQ.100) GO TO 14
       GO TO 13
11     K=K+1
12     IF(I.EQ.100) GO TO 14
       I=I+1
       IF(X(I).GE.AVG) GO TO 12
```

```
      K=K+1
      IF(I.EQ.100) GO TO 14
      GO TO 13
14    PRINT 16,K
16    FORMAT(15X,#NUMBER OF RUNS FOR THE SEQUENCE X=#,I3,/)
      DO 80 J=1,100
      L=J*144
      K=L-139
      SUM=0
      DO 90 I=K,L
90    SUM=Y(I)**2+SUM
80    XX(J)=SUM/140.
      SUM=0
      DO 91 I=1,100
91    SUM=SUM+XX(I)
      AVG=SUM/100.
      K=1
      I=1
      IF(XX(I).GE.AVG) GO TO 120
130   I=I+1
      IF(XX(I).GE.AVG) GO TO 110
      IF(I.EQ.100) GO TO 140
      GO TO 130
110   K=K+1
120   IF(I.EQ.100) GO TO 140
      I=I+1
      IF(XX(I).GE.AVG) GO TO 120
      K=K+1
      IF(I.EQ.100) GO TO 140
      GO TO 130
140   PRINT 160,K
160   FORMAT(15X,#NUMBER OF RUNS FOR THE SEQUENCE XX=#,I2,/)
C
C     SECOND TEST FOR STATIONARITY
C     THE TREND TEST
      K=0
      DO 21 J=1,99
      JJ=J+1
      DO 21 I=JJ,100
      IF(X(J).GT.X(I)) K=K+1
21    CONTINUE
      PRINT 22,K
22    FORMAT(15X,#NUMBER OF REVERSE ARRANGEMENTS FOR THE SEQUENCE X=#,I4
     1,/)
      K=0
      DO 23 J=1,99
      JJ=J+1
      DO 23 I=JJ,100
      IF(XX(J).GT.XX(I)) K=K+1
23    CONTINUE
      PRINT 24,K
24    FORMAT(15X,#NUMBER OF REVERSE ARRANGEMENTS FOR THE SEQUENCEXX=#,
     1I4)
C
C     TEST FOR RANDOMNESS
```

```
C          THE AUTOCORRELATION TEST
           N=14400
           DO 1 IR=1,100
           SUM=0
           JJ=N-IR
           DO 2 J=1,JJ
2          SUM=Y(J)*Y(J+IR)+SUM
           Z=N-IR
1          R(IR)=SUM/(Z*RO)
           RO=RO/RO
           PRINT 4,RO
4          FORMAT(1H1,20X,*ESTIMATES OF THE AUTOCORRELATION AT DIFFERENT LAG
          1NUMBERS*,//,40X,E10.4)
           DO 29 I=1,50
           M=I+50
29         PRINT 180,I,R(I),M,R(M)
180        FORMAT(25X,I3,E12.4,10X,I3,E12.4)
C
C          TEST FOR NORMALITY
           PRINT 15
15         FORMAT(1H1,10X,*DEVIATION FROM THE    PERCENTAGE OF DEVIATIONS AT O
          1R*,13X,*AVERAGE VALUE        BELOW THE GIVEN DEVIATION*/)
           Z=125
           DO 20 J=1,49
           K=0
           Z=Z-5
           DO 10 I=1,14400
           IF(Y(I).LE.Z) K=K+1
10         CONTINUE
           A=K
           DEV=(A/14400.)*100.
           PRINT 17,Z,DEV
17         FORMAT(16X,F6.1,19X,F6.1)
20         CONTINUE
C
C          POWER SPECTRAL DENSITY ANALYSIS
           FC=4000
           DO 5 IK=1,101
           K=IK-1
           F(IK)=(FC*K)/100.
           PI=(3.1415926536*K)/100.
           SUM=0
           DO 6 IR=1,99
6          SUM=SUM+R(IR)*COS(PI*IR)
5          G(IK)=(1./FC)*(RO+2.*SUM+(-1**K)*R(100))
C
C          SMOOTHING OF THE POWER SPECTRAL DENSITY
           GG(1)=.5*G(1)+.5*G(2)
           DO 7 K=2,100
7          GG(K)=.25*G(K-1)+.5*G(K)+.25*G(K+1)
           GG(101)=.5*G(100)+.5*G(101)
           PRINT 171
171        FORMAT(1H1,15X,*SMOOTH    SPECTRAL    DENSITY*/,
          113X,*FREQ MAGNITUDE*,4X,*FREQ    MAGNITUDE*/,
          214X,*HZ*,4X,*LB/HZ*,7X,*HZ*,4X,*LB/HZ*)
```

```
        DO 100 I=1,50
        NI=I+50
 100    PRINT89,F(I),GG(I),F(NI),GG(NI)
 89     FORMAT(12X,F5.0,F9.4,4X,F5.0,F9.4)
C
C       COMPUTATIONS FOR THE PROBABILITY DENSITY HISTOGRAM
        K=98
        K2=K+2
        DO 88 I=1,K2
 88     HIST(I)=0
        C=K
        A=-125
        B=125
        C=(B-A)/C
        DO 9 I=1,14400
        YI=Y(I)
        IF(A.LT.YI.AND.YI.LE.B) GO TO 18
        IF(YI.LE.A) GO TO 19
        IF(YI.GT.B) HIST(K2)=HIST(K2)+1.
        GO TO 9
 19     HIST(1)=HIST(1)+1.
        GO TO 9
 18     E=(YI-A)/C
        II=E
        EE=E-II
        IF(EE.GT.0.001) E=E+1
        IE=E
        HIST(IE)=HIST(IE)+1.
 9      CONTINUE
        SHIST(1)=.5*(HIST(1)+HIST(2))
        DO 215 K=2,99
 215    SHIST(K)=.25*HIST(K-1)+.5*HIST(K)+.25*HIST(K+1)
        SHIST(100)=.5*(HIST(99)+HIST(100))
C
C       COMPUTES PROBABILITY DENSITY IN TERMS OF PERCENTAGE OF DATA IN
C       EACH CLASS INTERVAL
        N=14400
        DO 27 I=1,K2
        HIST(I)=HIST(I)/N
 27     SHIST(I)=SHIST(I)/N
        PRINT 25
 25     FORMAT(1H1,15X,*PROBABILITY  DENSITY  HISTOGRAM*/,17X,*FORCE   DENS
       1ITY*,3X,*FORCE   DENSITY*/,19X,*LB      NO.     LB      NO.*)
        DO 194 J=2,99
 194    D(J)=A+(J-2)*C+0.5*C
        DO 195 J=2,50
        JJ=J+49
 195    PRINT 196,D(J),SHIST(J),D(JJ),SHIST(JJ)
 196     FORMAT(17X,F6.2,F8.4,3X,F6.2,F8.4)
C
C       COMPUTES THE PROBABILITY DISTRIBUTION
        PD(1)=HIST(1)
        DO 26 I=2,K2
 26     PD(I)=PD(I-1)+HIST(I)
        PRINT 198
```

```
198   FORMAT(1H1,15X,*PROBABILITY     DISTRIBUTION*/,17X,*FORCE   DIST
     1    FORCE   DIST*/,19X,*LB*12X,*LB*)
      DO 197 J=2,50
      JJ=J+49
197   PRINT 212,D(J),PD(J),D(JJ),PD(JJ)
212   FORMAT(17X,2(F5.1,F6.3,3X))
      STOP
      END
```

PROBABILITY DENSITY HISTOGRAM

FORCE	DENSITY	FORCE	DENSITY
LR	NO.	LR	NO.
-9.90	.0016	.10	.0841
-9.69	.0004	.31	.0477
-9.49	.0013	.51	.0097
-9.29	.0023	.71	.0136
-9.08	.0015	.92	.0202
-8.88	.0005	1.12	.0186
-8.67	.0010	1.33	.0136
-8.47	.0016	1.53	.0194
-8.27	.0016	1.73	.0292
-8.06	.0013	1.94	.0226
-7.86	.0026	2.14	.0152
-7.65	.0042	2.35	.0174
-7.45	.0032	2.55	.0153
-7.24	.0026	2.76	.0116
-7.04	.0031	2.96	.0180
-6.84	.0025	3.16	.0256
-6.63	.0020	3.37	.0162
-6.43	.0061	3.57	.0064
-6.22	.0110	3.78	.0084
-6.02	.0077	3.98	.0113
-5.82	.0035	4.18	.0102
-5.61	.0044	4.39	.0080
-5.41	.0062	4.59	.0102
-5.20	.0066	4.80	.0129
-5.00	.0058	5.00	.0083
-4.80	.0101	5.20	.0037
-4.59	.0153	5.41	.0043
-4.39	.0114	5.61	.0049
-4.18	.0068	5.82	.0039
-3.98	.0090	6.02	.0026
-3.78	.0120	6.22	.0049
-3.57	.0103	6.43	.0078
-3.37	.0076	6.63	.0048
-3.16	.0176	6.84	.0017
-2.96	.0301	7.04	.0019
-2.76	.0208	7.24	.0018
-2.55	.0130	7.45	.0014
-2.35	.0177	7.65	.0014
-2.14	.0176	7.86	.0016
-1.94	.0146	8.06	.0011
-1.73	.0195	8.27	.0005
-1.53	.0246	8.47	.0005
-1.33	.0181	8.67	.0008
-1.12	.0106	8.88	.0005
-.92	.0124	9.08	.0002
-.71	.0153	9.29	.0006
-.51	.0135	9.49	.0010
-.31	.0106	9.69	.0005
-.10	.0465	9.90	.0003

ESTIMATES OF THE AUTOCORRELATION AT DIFFERENT LAG NUMBERS

.1000E+01

1	.1346E+00	51	.7218E-01
2	-.1183E+00	52	.5777E-01
3	.2121E+00	53	.6144E-01
4	.3671E+00	54	.5235E-01
5	.1355E+00	55	.4548E-01
6	-.3044E-01	56	.5136E-01
7	.2082E+00	57	.3010E-01
8	.2061E+00	58	.2710E-01
9	.8689E-01	59	.3085E-01
10	.4779E-01	60	.3680E-01
11	.1351E+00	61	.3782E-01
12	.1192E+00	62	.3173E-01
13	.1654E-01	63	.4150E-01
14	.5737E-01	64	.4864E-01
15	.7794E-01	65	.3920E-01
16	.6622E-01	66	.5407E-01
17	.5414E-01	67	.4661E-01
18	.6537E-01	68	.4338E-01
19	.7304E-01	69	.6069E-01
20	.4576E-01	70	.7581E-01
21	.4668E-01	71	.7402E-01
22	.5850E-01	72	.6487E-01
23	.7831E-01	73	.7787E-01
24	.6415E-01	74	.8250E-01
25	.7478E-01	75	.6287E-01
26	.8908E-01	76	.6755E-01
27	.1119E+00	77	.6990E-01
28	.9276E-01	78	.6850E-01
29	.6472E-01	79	.6668E-01
30	.9179E-01	80	.6266E-01
31	.9215E-01	81	.6614E-01
32	.7866E-01	82	.5676E-01
33	.8941E-01	83	.4505E-01
34	.1029E+00	84	.4089E-01
35	.9536E-01	85	.3419E-01
36	.9371E-01	86	.4281E-01
37	.1193E+00	87	.3193E-01
38	.1006E+00	88	.3290E-01
39	.8042E-01	89	.4661E-01
40	.8731E-01	90	.3720E-01
41	.9582E-01	91	.3353E-01
42	.8865E-01	92	.2247E-01
43	.8450E-01	93	.3118E-01
44	.1000E+00	94	.2490E-01
45	.9036E-01	95	.2175E-01
46	.8281E-01	96	.1770E-01
47	.8015E-01	97	.2136E-01
48	.7780E-01	98	.3361E-01
49	.6317E-01	99	.2765E-01
50	.6011E-01	100	.1056E-01

SMOOTH SPECTRAL DENSITY			
FREQ	MAGNITUDE	FREQ	MAGNITUDE
HZ	LB/HZ	HZ	LB/HZ
0.	.0023	2000.	.0006
40.	.0015	2040.	.0007
80.	.0005	2080.	.0007
120.	.0003	2120.	.0007
160.	.0004	2160.	.0007
200.	.0006	2200.	.0006
240.	.0005	2240.	.0005
280.	.0003	2280.	.0004
320.	.0003	2320.	.0004
360.	.0002	2360.	.0004
400.	.0001	2400.	.0003
440.	.0001	2440.	.0002
480.	.0001	2480.	.0002
520.	.0001	2520.	.0002
560.	.0001	2560.	.0002
600.	.0001	2600.	.0002
640.	.0001	2640.	.0002
680.	.0001	2680.	.0002
720.	.0001	2720.	.0002
760.	.0001	2760.	.0001
800.	.0001	2800.	.0002
840.	.0001	2840.	.0002
880.	.0002	2880.	.0002
920.	.0002	2920.	.0001
960.	.0001	2960.	.0001
1000.	.0001	3000.	.0001
1040.	.0001	3040.	.0001
1080.	.0001	3080.	.0001
1120.	.0001	3120.	.0001
1160.	.0001	3160.	.0002
1200.	.0001	3200.	.0002
1240.	.0001	3240.	.0002
1280.	.0001	3280.	.0002
1320.	.0001	3320.	.0002
1360.	.0001	3360.	.0002
1400.	.0002	3400.	.0002
1440.	.0002	3440.	.0002
1480.	.0002	3480.	.0002
1520.	.0002	3520.	.0002
1560.	.0002	3560.	.0001
1600.	.0002	3600.	.0001
1640.	.0002	3640.	.0001
1680.	.0003	3680.	.0001
1720.	.0003	3720.	.0001
1760.	.0004	3760.	.0001
1800.	.0004	3800.	.0001
1840.	.0004	3840.	.0001
1880.	.0005	3880.	.0001
1920.	.0005	3920.	.0001
1960.	.0006	3960.	.0001

PROBABILITY		DISTRIBUTION	
FORCE	DIST	FORCE	DIST
LB		LB	
-9.9	.006	.1	.595
-9.7	.006	.3	.604
-9.5	.006	.5	.616
-9.3	.011	.7	.621
-9.1	.011	.9	.652
-8.9	.012	1.1	.666
-8.7	.012	1.3	.682
-8.5	.015	1.5	.690
-8.3	.016	1.7	.736
-8.1	.018	1.9	.754
-7.9	.018	2.1	.762
-7.7	.025	2.3	.788
-7.4	.027	2.6	.798
-7.2	.029	2.8	.814
-7.0	.034	3.0	.819
-6.8	.035	3.2	.865
-6.6	.038	3.4	.870
-6.4	.039	3.6	.879
-6.2	.059	3.8	.882
-6.0	.062	4.0	.900
-5.8	.066	4.2	.906
-5.6	.068	4.4	.917
-5.4	.077	4.6	.920
-5.2	.082	4.8	.943
-5.0	.090	5.0	.946
-4.8	.092	5.2	.951
-4.6	.120	5.4	.953
-4.4	.124	5.6	.961
-4.2	.134	5.8	.963
-4.0	.137	6.0	.966
-3.8	.157	6.2	.968
-3.6	.162	6.4	.982
-3.4	.173	6.6	.984
-3.2	.177	6.8	.985
-3.0	.229	7.0	.987
-2.8	.241	7.2	.988
-2.6	.248	7.4	.991
-2.3	.274	7.7	.991
-2.1	.285	7.9	.994
-1.9	.306	8.1	.994
-1.7	.312	8.3	.995
-1.5	.354	8.5	.995
-1.3	.362	8.7	.996
-1.1	.377	8.9	.997
-.9	.382	9.1	.997
-.7	.406	9.3	.997
-.5	.413	9.5	.999
-.3	.429	9.7	.999
-.1	.433	9.9	.999

THE AVERAGE OF THE SAMPLE RECORD = -.2934E+00

THE VARIANCE OF THE SAMPLE RECORD = .1199E+02

NUMBER OF RUNS FOR THE SEQUENCE X= 51

NUMBER OF RUNS FOR THE SEQUENCE XX=50

NUMBER OF REVERSE ARRANGEMENTS FOR THE SEQUENCE X=2229

NUMBER OF REVERSE ARRANGEMENTS FOR THE SEQUENCEXX=2553

DEVIATION FROM THE AVERAGE VALUE	PERCENTAGE OF DEVIATIONS AT OR BELOW THE GIVEN DEVIATION
10.0	99.9
9.0	99.6
8.0	99.1
7.0	98.4
6.0	96.2
5.0	94.2
4.0	88.1
3.0	80.1
2.0	73.2
1.0	62.1
0.0	42.9
-1.0	36.2
-2.0	27.5
-3.0	17.5
-4.0	13.2
-5.0	7.9
-6.0	3.9
-7.0	2.9
-8.0	1.6
-9.0	1.1
-10.0	.4