Modeling Producer Behavior by Using the Third-Order Translog Cost Function

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ABSTRACT

Modeling Producer Behavior by Using

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The introduction of flexible functional forms based on the second-order Taylor's series approximation, has gone a long way in the study of producer behavior. However, since they are based only on the second-order Taylor's series approximation, they impose some rigidities on the formulas derived and are not equipped to estimate, among other important economic concepts, the curvature of demand and supply curves.

The objectives of this thesis is to explore the contributions of third-order flexible functional forms in the study of producer behavior To this end the third-order translog cost function is examined both theoretically and empirically in the study of producer behavior. The major theoretical findings of the study are that there is (1) a reduction in the bias, (2) more flexibility in the derived economic relations (such as variable share elasticities, Allen Uzawa elasticities of substitution), (3) additional restrictions for more rigorous testing of the maintained hypothesis about the underlying technology and (4) the introduction of sensitivity parameters to estimate the curvature of the demand function.

Next, the performance of the second and third-order translog cost functions are put to an empirical test by using KLEM (capital, labor, energy and intermediate materials) data in the U. S. manufacturing sector 1947-1971. First, the bias in the estimated demand functions from both models is calculated and compared. The results show that there is a reduction in the bias when the third-order function is used. Second, the second-order cost function is tested for specification error, using the likelihood ratio test. This function is rejected in favor of the third-order translog cost function. Third, the share

elasticities are estimated using the new formula and are found to be variable rather than constant, as assumed in the case of the second-order cost function. The flexibility of the share elasticities in turn play an important role in making formulae containing share elasticities, such as the Allen partial elasticities of substitution (AUES), more flexible. The estimated AUES values, by using the new model, ranged from greater than unity, to less than unity. This implies that substitution possibilities between any two inputs have changed over the years. Fourth, the estimated measures of curvatures are used to analyze the rates of change of demands and of the share of inputs as well as measures of the sensitivity of some concepts such as share elasticities.

Finally, the linear and nonlinear restrictions for various separability types are derived for both the second and third-order cost functions. The set of restrictions obtained for each type of separability is then tested for significance. The results obtained from the second-order model led to the rejection of all but one type; the utilized capital specification. On the other hand, the third-order results show the rejection of all separability types. This result is achieved due to more rigorous testing, made possible by additional restrictions provided by the third-order approximation of the cost function and the precision of the variables involved in the formula to determine functional separability. The implication of this result is that the demand analysis of the U.S. manufacturing sector must take all four inputs namely capital, energy, labor and intermediate materials, into account. The forecasting of investment demands for any of the inputs cannot be made by using the information relating only to the sub-set of these inputs.

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TABLE OF CONTENTS

Page

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Empirical analysis of demand and supply has been a rapidly expanding area of research in the past few decades. The analysis of markets in terms of demand and supply suggest that attempts must be made to estimate, not only the slopes of these curves, but also their curvatures and other economic concepts that are relevant to producer and consumer behavior. The use of curvatures in economics will be discussed briefly in Chapter 2.8.

Traditionally economists have been using simple functional forms like the Cobb-Douglas and the constant elasticities of substitution (CES). The simplicity of the above functions was achieved only at the expense of imposing many restrictive assumptions about the underlying technology and producer behavior. It imposes strong separability, hence, unit elasticities of substitution result between any two inputs in the production process. The CES function did relax the assumption of unit elasticities of substitution, but assumed them to be constant and hence equal. This implies that in a multi-input production function the existence of complementarity is ruled out a priori.

In order to eliminate the rigidities of the above forms, flexible functional forms like the translog and generalized Leontief functions (based on the second-order Taylor's series approximation) have been introduced. The advantages of such forms include the following:

(a) the ability to estimate the slopes of demand and supply expressed in terms of their arguments. (b) the ability to estimate and test various maintained hypotheses concerning producer behavior and the underlying technology.

The economic concepts that will be tested are generated from the objective function as first and second derivatives with respect to input prices and output.

The introduction of flexible functional forms based on the second-order Taylor's series approximation, has gone a long way in the study of producer behavior. However, since they are based only on the second-order Taylor's series approximation, they impose some rigidities on the formulas derived and are not equipped to estimate the curvature of demand and supply curves.

Our objectives in this thesis are (1) to identify the short comings of second-order flexible functional forms; (2) to explore the contributions of the third-order functional form in modeling producer behavior; (3) to provide the theoretical justification for considering the third-order translog cost function in applied empirical studies.

The basic shortcomings of second-order flexible functional forms are identified as being the truncation bias, rigidity of some of the economic relationships derived from these functions (such as the constancy of share elasticities, the inability of the formula in measuring Allen elasticities of substitution to give a value both greater than and less than unity as prices change during a study period) and the inability to measure rates of change of functions derived as the first derivative of the objective function (such as input demands and the share of an input). These shortcomings mentioned above are likely to render imprecision in the results calculated from any formula obtained from the second-order cost function and in the restrictions derived from it.

We extended the second-order flexible form to a third-order to review the following :

a) reduction in truncation bias, b) to see if the formulas and the restrictions derived from the objective function are more flexible and c) to see if parameters to measure economic relationships that could not have been measured by using the conventional flexible forms could be introduced.

The questions that must be answered in extending the second-order cost function to a third-order form can be listed as follows:

(1) Does the extended model fulfill the requirements necessary to qualify it as representative of the desired behavior ?

(2) Is the extended model superior to the existing one in terms of reduction in bias, precision of the formulas and restrictions derived from it ?

(3) Does the extension add anything new to existing theory in the study of producer behavior ? In other words, are we able to investigate economic relationships within the extended model that could not have been done by using the existing model ?

(4) Are the theoretical justifications for extending the second-order translog cost function supported by empirical investigations as applied to producer behavior ?

The above questions were all answered affirmatively in the course of our study.

This thesis is organized within the following framework. In Chapter 2, the literature is surveyed. Special emphasis is placed on the methodology of the second-order flexible cost functions that give a locally well behaved region. The review of the literature includes:

(a) a brief survey of duality theory.

(b) the examination of regularity conditions.

(c) the structure and properties of important functional forms within the class of locally well-behaved functions.

(d) the derivation of economic relationships that are of concern to producer behavior.

(e) the examination of criteria used for choosing a particular functional form.

(f) the derivation and examination of the restrictions that are necessary for functional separability.

In Chapter 3, the second-order translog cost function will be extended to a third-order form. The third-order translog cost function is shown to be superior to the second-order form for both theoretical and practical purposes. The new conditions for aggregation and homogeneity (reflecting the budget constraint), symmetry and negativity (reflecting the consistency of choice) were derived from the extended model. All of the economic relationships that could be derived from the second-order cost were derived from the extended model. The models were then compared. The additional economic relationships that could not have been derived from the second-order function, were derived from the alternative model, and their contributions to the analysis of producer behavior were presented. The major objectives of this chapter were to demonstrate theoretically that there will be (1) a reduction in the bias, (2) more flexibility in the derived economic relations (such as variable share elasticities), (3) more flexible formulas (such as Allen

Uzawa elasticities of substitution), (4) additional restrictions for more rigorous testing of the maintained hypothesis about the underlying technology and (5) the introduction of sensitivity parameters to estimate the curvature of the demand function.

In Chapter 4, the performance of the second and third-order translog cost functions were put to an empirical test by using KLEM (capital, labor, energy and intermediate materials) data in the U. S. manufacturing sector 1947-1971.

In this empirical analysis various tests were performed. First, the second-order cost function was tested for specification error, using the likelihood ratio test. This function was rejected in favor of the third-order translog cost function (the alternative model). The implication of this is that the results estimated from the rejected model are no longer reliable and one should turn to the third-order cost function to analyze the U.S. manufacturing sector. Second, positivity and negativity conditions were tested and were all satisfied. Third, the share elasticities were estimated using the new formula and were found to be variable rather than constant, as assumed in the case of the second-order cost function. Fourth, the share elasticities were estimated at every point and at their means for hypothesis testing. The flexibility of the share elasticities in turn played an important role in making formulae containing share elasticities, such as the Allen partial elasticities of substitution, more flexible. Fifth, the Allen partial elasticities of substitution were estimated by using a third-order translog cost function. The Allen partial elasticities of substitution estimated for every data point from the third-order cost function showed more flexibility, since their values ranged from greater than unity, to less than unity This property of Allen partial elasticities, which has far reaching policy implications was obtained due to the variable share elasticity (rather than the constant one derived from the second-order function) embodied in the formula to measure substitution possibilities. This implies that substitution possibilities between any two inputs have changed over the years. The Allen partials were also calculated at their means and hypotheses testing performed. Sixth, the bias in the estimated demand functions from both models was calculated and compared. The results showed that there was a reduction in the bias when the third-order function was used. Seventh, the estimated measures of curvature were used to analyze the rates of change of demands and of the shares of inputs as well as a measure of the sensitivity of some concepts such as share elasticities. Finally, some policy implications were drawn by using the estimates and the formulae derived from the third-order cost function.

In Chapter 5 the linear and nonlinear restrictions for various separability types were derived for both the second and third-order cost functions. The set of restrictions obtained for each type of separability was then tested for significance. The results obtained from the second-order model led to the rejection of all but one type; the utilized capital specification. On the other hand, the third-order results showed the rejection of all separability types. This result was achieved due to more rigorous testing, made possible by additional restrictions provided by the third-order approximation of the cost function and the precision of the variables involved in the formula to determine functional separability. The implication of this result is that the demand analysis of the U.S. manufacturing sector must take all four inputs namely capital, energy, labor and intermediate materials, into account. The forecasting of investment demands for any of the inputs cannot be made by using the information relating only to the subset of these inputs. Chapter 6 includes the summary and conclusions of the thesis.

CHAPTER 2

LITERATURE SURVEY

Traditional functional forms like Cobb - Douglas and CES are simple to use in empirical studies and are globally well behaved. However, they are unable to provide a second-order Taylor series approximation to an arbitrary function and hence, are not flexible in the sense of Diewart (1971). As a result of this inflexibility, they are often inadequate for testing many economic hypotheses relating to the second derivatives.

These shortcomings have led to the development of flexible functional forms such as the translog and the generalized Leontief. The flexible functional forms do not impose any *a priori* restrictions for local first and second-order properties of the function. This allows for the testing of many restrictions which the traditional forms could not meaningfully handle. The flexible functional forms do have their drawbacks. They are well-behaved only locally as opposed to globally. In some cases the local region could be too narrow to allow a meaningful analysis. However, within the region where they are well-behaved, they are said to approximate the underlying producer technology to the second-order. Depending on the point of interest in the study of producer (consumer) behavior, one could use any of these forms to estimate the parameters and test different hypotheses relating to the study in question.

In this chapter I will show that the most popular flexible form, the second-order translog, is not flexible enough when studying factor substitutability over time. The share elasticity derived from a second-order translog cost function is constant and this imposes some restrictions on the values and signs of Allen Uzawa elasticity of substitution (AUES). This observation is one of our motivations for extending the translog to the third-order which is the subject of this thesis[1].

This chapter will examine the following topics. In section (2.1), the theory of duality between cost and production functions will be reviewed briefly and the advantages of using a cost function over a production function will be presented. In section (2.2), the regularity conditions needed for an arbitrary cost function to represent producer technology and some important characteristics of producer technology will be examined in detail. In section (2.3) traditional func'ional forms will be discussed. In sections (2.4) and (2.5) many of the commonly used flexible cost functions will be enumerated and their properties discussed. In section (2.6) the criteria for choosing functional forms will be discussed. This will help us learn more about the structure of most of the applied functional forms. In section (2.7), the derived results from a second-order translog cost function will be discussed. In Section (2.8) the merits and shortcomings of the second-order translog cost function will be discussed. In addition, the third-order translog cost function will be introduced and its merits will be briefly discussed.

2.1 Duality

The development of duality theory between cost and production functions is attributed to Samuelson (1953-4) and Shephard (1953, 1970).

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1 Additional drawbacks will be discussed in the relevant sections and will be summarized in section (2.8).

A given technology can be represented by a production function or by its dual, the cost function. The standard form of a single output primal function can be written as follows:

$$y = f(x), \ x = (x_1, \ldots, x_n)^T, \tag{2.1.1}$$

where f is a production function and y is the maximum amount of the output that can be produced given the vector of inputs, x and if we are also given the vector of input prices,

$w = (w_1, \ldots, w_n)^T > 0$, where T indicates transpose,

the dual minimum cost function can be defined as:

$$C(w,y) = \underset{x}{\mathrm{Min}}(w^T x \ : \ f(x) \geq y, \ x > 0). \tag{2.1.2}$$

The minimum cost function needed to produce a given level of output is represented as a function of input prices and output in (2.1.2).

The cost function has several advantages over its primal representation. First, there is a simple relationship between a cost function and the conditional factor demand functions since by Shephard's lemma,

$$x_i(w,y) = \partial C(w,y)/\partial w_i. \tag{2.1.3}$$

Second, it has been argued that since prices are more exogenous than quantities[2], estimation results that arise from using the cost function are more reliable than those from the production function. Third, the

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[2] Varian, 1984, Chapter 4.

demand elasticity and elasticity of substitution formulae are a lot simpler, due to the explicit nature of the derived input demands. Lastly, given that certain regularity conditions are imposed, a cost function can be given a desirable form, without having to solve the minimization problem [Despotakis, 1986].

We now turn to the investigation of regularity conditions that must be satisfied in order to have theoretical consistency.

2.2 Properties and Characteristics of Cost Functions

Certain regularity conditions are traditionally imposed on neo-classical cost functions. To examine these regularity conditions, a single output, N input prices cost function is considered

1) Domain:- the cost function given must be a positive function for positive prices and output:

$$C(w,y) \geq 0 \,, \tag{2.2.1}$$

2) Monotonicity:- It must be non-decreasing in input prices and output:

$$\text{If } w_1 \geq w_2 \text{ then } C(w_1,y) \geq C(w_2,y),$$

$$\text{and if } y_1 > y_2 \text{ then } C(w,y_1) > C(w,y_2) \tag{2.2.2a}$$

If $C(w,y)$ is differentiable, the monotonicity condition implies that:

$$\partial C(w,y)/\partial w_i \geq 0 \text{ and } \partial C(w,y)/\partial y > 0, \tag{2.2.2b}$$

or if the cost function is in logarithmic form,

$$S_i = \partial \ln C(w,y)/\partial \ln w_i \geq 0, \tag{2.2.2c}$$

(2.2.2a) and (2.2.2b) imply that any increase in factor prices can not lead to a reduction in the minimum cost needed to produce a given level of output, also, producing a higher level of output for given factor prices does not lead to a reduction in the minimum cost. Due to Shephard's lemma the monotonicity condition implies that input demands and input shares in the total cost are non-negative.

3) Homogeneity:- the cost function must be linearly homogeneous in input prices:

$$C(\lambda w,y) = \lambda C(w,y) \text{ , for } \lambda > 0 \tag{2.2.3a}$$

If the cost function is differentiable the above condition can be expressed as follows in levels form by using Shephard's lemma and Euler's theorem:

$$C(w,y) = \sum_{i=1}^{n} w_i \, \partial C(w,y)/\partial w_i = \sum_{i=1}^{n} w_i \, x_i(w,y), \tag{2.2.3b}$$

tnus the following adding up property holds,

$$1 = \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} [\partial \ln C(w,y)/\partial \ln w_i]. \tag{2.2.3c}$$

Conditions (2.2.3a) and (2.2.3b) imply that a proportional change in all factor prices leads to the same proportional change in minimum cost, whereas the logarithmic format indicates that the shares of inputs in the total cost add up to unity

4) Concavity:- for $0 \leq \lambda \leq 1$ and two input price vectors w_1 and w_2:

$C(\lambda w_1 + (1 - \lambda)w_2, y) \geq \lambda C(w_1, y) + (1 - \lambda)\, C(w_2, y)$,

or if $C(w,y)$ is twice differentiable the concavity condition requires the matrix,

$$\partial^2 C(w,y)/\partial w\, \partial w^T \text{ is negative semi definite.} \qquad (2.2.4)$$

Since a negative semi-definite matrix must have non-positive elements on the diagonal, (2.2.4) implies that if the price of a factor is increased with all other factor prices kept constant, the total cost will increase at a decreasing rate. The concavity condition and Shephard's lemma thus imply that factor demands are non-positively sloped

Not only does a cost function satisfy (1) to (4), but any general functional form that satisfies (1) to (4) is a cost function and hence represents a technology. One need not solve a minimization problem to get a desired functional form if one can verify that (1) to (4) are satisfied or if one imposes (1) to (4) on arbitrary function That is, one can take any complicated functional form to represent a specific technology as long as the functional form satisfies (1) to (4). This adds more flexibility in choosing a functional form that is best suited to a particular examination of a producer behavior

The differentiability and symmetry conditions are commonly satisfied for cost functions used in empirical work so that factor demands, factor shares and concavity conditions can be derived as (2.1.3), (2.2.3c) and (2.2.4) respectively. By Young's theorem,

$$\partial^2 C(w,y)/\partial w_i\, \partial w_j = \partial^2 C(w,y)/\partial w_j\, \partial w_i \text{ so that } \partial X_i/\partial w_j = \partial x_j/\partial w_i, \quad \forall\ i \neq j \qquad (2.2.5)$$

A cost function may also exhibit one or more of the following technological characteristics·

5) Homotheticity:- a technology will be homothetic if:

$$C(w,y) = F(y)\ G(w), \qquad (2.2.6a)$$

i.e. factor prices and output independently affect the cost function and therefore, the function representing output can be factored out of the cost function as shown above. If the cost function is differentiable the above conditions imply·

$$\partial^2 \ln C(w,y)/\partial \ln w_i\ \partial \ln y = \partial S_i(w,y)/\partial \ln y = 0 \quad \forall i, \qquad (2.2.6b)$$

i.e. the shares of an input are independent of output:

6) Homogeneity of the technology:- if the function representing output in the total cost above is given the following general form, $F(y) = y^{\tau}$, then the technology is said to exhibit decreasing, constant or increasing retuns to scale as $\tau > 1$, $\tau = 1$, $\tau < 1$, respectively.

7) Substitutability: A cost function can also be used to measure substitutability among inputs. The most popular measure is AUES. Knowing the degree of substitutability among inputs is a matter of great importance for both producers, in making their price policy, and government policy makers in determining the impact of tax and subsidies

on the demands for the inputs in question. The degree of substitutability is measured by the following formula:

$$\sigma_{ij}(W,y) = \frac{C(w,y)\ \{\ \partial^2 C(w,y)/\partial w_i\ \partial w_j\ \}}{\{\partial C(w,y)/\partial w_i\}\ \{\partial C(w,y)/\partial w_j\}} \equiv C(w,y)\ C_{ij}(w,y)/x_i x_j. \quad (2.2.9)$$

This formula measures a normalized response of an input i due to a change in the price of another input j. This and other formulas that are relevant in producer behavior analysis will be discussed for different functional forms in the next section.

8) Functional Separability:

Inputs i, and j are said to be homothetically separable from input k, if

$$\left[\partial/\partial w_k\right]\ \left[C_i/C_j\right] = \left[C_i C_{jk} - C_j C_{ik}\right] \Big/ C^2 = 0 \qquad (2.2.10)$$

where C_i and C_j are first partial derivatives of the cost function with respect to respective factor prices, while C_{jk} and C_{ik} are the first-order derivatives of the factor demand functions with respect to the separable input price, and C represents the cost function[3].

2.3. Traditional functional forms and their properties

In this section some traditional functional forms and their properties will be discussed. Where there is no particular interest in how output affects costs, only the unit cost functions will be discussed.

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[3] A detailed discussion of separability is presented in section (2.4)

Cobb-Douglas (Cobb-Douglas, 1928)

$$\ln C(w,y) = \gamma_0 + \sum_i^n \gamma_i \ln w_i + \ln(F(y)). \tag{2.3.1}$$

This form can be taken as the first-order expansion of lnC(w,y) in powers of lnw and lny (Lau, 1974). (2.3.1) will be linearly homogeneous in input prices, if:

$$\sum_i^n \gamma_i = 1, \qquad i = 1,\dots n. \tag{2.3.2}$$

Since the Cobb-Douglas form is globally consistent, the regularity conditions discussed under (2.2.1 - 2.2.4) will be satisfied. The above function has a constant share , $S_i = \gamma_i$, and a unit elasticity of substitution ($\sigma = 1$).

Constant Elasticity of Substitution, CES, (ACMS, 1961):

$$C(w,1) = \left[\sum_i^n \gamma_i^{\sigma} w_i^{1-\sigma} \right]^{1/(1-\sigma)}, \tag{2.3.3}$$

where $\sum_i^n \gamma_i = 1$ is required for linear homogeneity in input prices and σ is the constant elasticity of substitution.

This form is more flexible than the Cobb-Douglas form as it allows the elasticity of substitution to be different from unity although it restricts them all to be constant and equal. This function is attractive due to its simplicity and its global consistency.

2.4. Flexible functional forms and their properties

Generalized Leontief, Linear, (Diewert, 1971)

$$C(w,y) = F(y)(\sum_{i}^{n}\sum_{j}^{n} \tilde{\gamma}_{ij}\ w_i^{1/2}\ w_j^{1/2}) \qquad (2.4.1),$$

where F is a continuously, monotonically increasing function of y and $\tilde{\gamma}_{ij} = \tilde{\gamma}_{ji}\ \forall\ i \neq j$. For constant returns technology, $F(y) = y$. This function satisfies conditions (2.2.1 - 2.2.4) locally. This function collapses into the fixed proportion Leontief cost function if $\tilde{\gamma}_{ij} = 0\ \forall\ i \neq j$. By Shephard's lemma, a factor demand function can be derived as follows:

$$\partial C(w,y)/\partial(w_i) = x_i(w,y) = \sum_{j}^{n} \tilde{\gamma}_{ij}\ (w_j\ /\ w_i)^{1/2}\ F(y). \qquad (2.4.2)$$

The factor demand function (2.4.2) is non linear in variables but linear in parameters, and thus it is not a difficult function to estimate. However, since it involves output problems may arise if the data on output is of poor quality, or may be impossible to estimate, if the data on output is unavailable. One could alternatively estimate n-1 share equations derived from (2.4.1) which will not have output as an argument assuming homotheticity but will be non-linear in both variables and parameters. The non-linearity in parameters clearly makes computation more difficult and may also present convergence problems. Hence, using the share system of equations in such circumstances may not be a good idea.

The parameter $\tilde{\gamma}_{ij}$ can be related to the partial AUES as follows (Diewert 1974, p 116):

$$\sigma_{ij}(w,y) = C(w,y)\ \tilde{\gamma}_{ij}\ w_i^{-1/2}\ w_j^{-1/2}\ F(y)\ /\ x_i x_j \quad \text{for } \forall\ i \neq j \tag{2.4.3}$$

The magnitude and signs of σ_{ij} will be influenced by the parameter $\tilde{\gamma}_{ij}$ in (2.4.3). The larger $\tilde{\gamma}_{ij}$ is, the greater will be the substitution between two inputs. If we set parameter $\tilde{\gamma}_{ij}$ equal to zero for $\forall\ i \neq j$, the elasticity of substitution will be zero, corresponding to the elasticity of substitution derived from a fixed proportion Leontief cost function.

Generalized Square Rooted, non-linear, (Diewert, 1974)

$$C(w,y) = F(y)\left(\sum_i^n \sum_j^n \tilde{\gamma}_{ij}\ w_i\ w_j \right)^{1/2}. \tag{2.4.4}$$

Equation (2.4.4) is a generalization of a Leontief Cost Function (non-linear version). For the non-linear version, the whole matrix of prices is raised to the power 1/2. Unlike the simple Leontief fixed proportion function, this format does not impose a zero elasticity of substitution. However, the derived conditional input demand from both linear and non-linear formats, will involve output in their arguments. This will present problems similar to those discussed in the previous case.

Generalized Quadratic Mean of Order ρ (Denny, 1974)

$$C(w,y) = F(y) \left[\sum_i^n \sum_j^n \gamma_{ij}\ w_i^{\rho/2}\ \ w_j^{\rho/2} \right]^{1/\rho} \tag{2.4.5},$$

This is an extension of the Generalized Leontief Cost Function, where

(2.4.5) reduces to the linear and non-linear format as $\rho = 1$, and $\rho = 2$, respectively.

Translog (Christensen et al. 1973)

$$\ln C(w,y) = \gamma_0 + \sum_i^n \gamma_i \ln w_i + \gamma_y \ln y + 1/2\, \gamma_{yy} (\ln y)^2$$

$$+ \; 1/2 \sum_i^n \sum_j^n \gamma_{ij} \ln w_i \ln w_j + \sum_i^n \gamma_{iy} \ln w_i \ln y. \qquad (2.4.6)$$

This is a second-order approximation of $\ln C(w,y)$, in powers of $\ln w_i$ and $\ln y$ with symmetry, $\gamma_{ij} = \gamma_{ji}$ $\forall$ $i \neq j$ imposed. The function should be non-negative in input prices and output. Its first logarithmic derivative should be positive (monotonicity requirement) and the matrix formed by a second-order derivative should be negative semi definite (concavity requirement). The necessary and sufficient conditions for the above cost function to be linearly homogeneous in input prices, w, are:

$$\sum_i^n \gamma_i = 1, \; \sum_j^n \gamma_{ij} = 0\,, \text{ and } \sum_i^n \gamma_{iy} = 0, \qquad (2.4.7),$$

where i and j represent input prices.

The translog cost function is linearly homogeneous in output, if we impose the following conditions:

$$\gamma_y - 1 = \gamma_{yy} = \gamma_{iy} = 0, \qquad (2.4.8)$$

Finally, if all the interaction parameters are zero, the Translog Cost Function reduces to a Cobb-Douglas function.

Input Share Equations

If we differentiate (2.4.6) with respect to all components of the logarithm of w, we get the share of the respective input in total cost. The share of input 'i' is given by:

$$\partial \ln C/\partial \ln w_i \equiv S_i(w,y) = \left[\partial C/\partial w_i\right] w_i/C = \gamma_i + \sum_j^n \gamma_{ij} \ln w_j + \gamma_{iy} \ln y,$$

$$i,j = 1,\ldots n \tag{2.4.9}$$

The system of the share functions, (2.4.9), is defined by $S_i(w,y) \equiv w_i x_i/C(w,y)$, where $x_i = \partial C/\partial w_i$ is the input demand functions (using Shephard's lemma). The share function is homogeneous of degree zero in factor prices. The shares must add up to unity and each share must lie between zero and unity, by monotonicity and cost exhaustion respectively.

However, this latter property can not be a global property of the shares derived from a cost function higher than a first-order Taylor's series approximation. This is evident when we examine the share equation given above. If we keep all w_j except one constant, then the share of an input moves with the magnitude of the particular input price: As $w_1 \to 0$, $\ln w_1 \to -\infty \Rightarrow S_1 < 0$, similarly as $w_1 \to \infty$, $\ln w_1 \to \infty \Rightarrow S_1 > 1$. Therefore, the translog cost function is not globally consistent, unless $\gamma_{ij} = \gamma_{iy} = 0$. In this case we get the Cobb-Douglas cost function.[4]

[4] See Appendix to Chapter 2 for a local and global consistency of a translog cost function.

Functional Separability and the Translog Cost Function.

In terms of the translog cost function, the functional separability given by (2.2.10) requires the following:

$$S_i\gamma_{jk} - S_j\gamma_{ik} = 0 \qquad (2.4.10),$$

where, $C_i = CS_i/w_i$, $C_j = CS_j/w_j$, $C_{jk} = C\left[\gamma_{jk} + S_jS_k\right]\Big/w_jw_k$,

$$C_{ik} = C\left[\gamma_{ik} + S_iS_k\right]\Big/w_iw_k. \qquad (2.4.11)$$

If one considers a three input translog cost function, for example, (2.4.10) can be written as (for separability between inputs 1 and 2 with respect to 3):

$$S_1\gamma_{23} - S_2\gamma_{13} = 0. \qquad (2.4.12)$$

From (2.4.10), if one has $\gamma_{jk} = \gamma_{ik} = 0$, which is a linear restriction, we get the sufficient condition for separability. If on the other hand γ_{ik} and γ_{jk} are not equal to zero, then one has to take the share equations into consideration. Therefore, substituting for S_i and S_j and rearranging, one gets the restrictions for global separability:

$$\left[\gamma_i\gamma_{jk} - \gamma_j\gamma_{ik}\right] + \sum_1^n \left[\gamma_{im}\gamma_{jk} - \gamma_{jm}\gamma_{ik}\right] \ln w_m = 0 \qquad (2.4.13)$$

If the equality in (2.4.13) is to hold, the following conditions must hold:

$$\gamma_i\gamma_{jk} - \gamma_j\gamma_{ik} = 0 \text{ and } \gamma_{im}\gamma_{jk} - \gamma_{jm}\gamma_{ik} = 0 \qquad (2.4.14)$$

Therefore, the non-linear global separability restrictions can be obtained by using (2.4.14):

$$\left[\gamma_i/\gamma_j\right] = \left[\gamma_{ik}/\gamma_{jk}\right] = \left[\gamma_{im}/\gamma_{jm}\right] \qquad (2.4.15)$$

If, for example, three inputs are under consideration, the global separability of input 1 and 2 from input 3 can be given as follows:

$$\left[\gamma_1/\gamma_2\right] = \left[\gamma_{13}/\gamma_{23}\right] = \left[\gamma_{11}/\gamma_{21}\right] = \left[\gamma_{12}/\gamma_{22}\right] \qquad (2.4.16)$$

One can then derive three independent restrictions. For local separability, it is only necessary to use the first set of restrictions,

$$\left[\gamma_1/\gamma_2\right] = \left[\gamma_{13}/\gamma_{23}\right] \qquad (2.4.17),$$

since the shares reduce to γ_i at the point of approximation. As for the global restrictions, Denny and Fuss (1977) were able to show that the second set of restrictions yield the following form:

$$\gamma_{ii}\gamma_{jj} - \gamma_{ij}^2 = 0 \qquad (2.4.18)$$

where the first set reduces to the form given by (2.4.18).

$$\begin{aligned} &\gamma_1\gamma_{23} - \gamma_2\gamma_{13} && (12\sim3) \\ &\gamma_1\gamma_{23} - \gamma_3\gamma_{12} && (13\sim2) \qquad (2.4.19) \\ &\gamma_2\gamma_{13} - \gamma_3\gamma_{12} && (23\sim1) \end{aligned}$$

The non-linear restrictions given above, have undesirable consequences on the flexibility of functional forms. Once imposed, they become separably inflexible (e.g. Denny et al.). Instead of an exact test, they suggested the use of an approximate test that does not depend on all values of the parameters. Blackorby et al. (1977) arrived at the same conclusion.

Functional separability was assumed in the traditional functional forms. In order to test for separability in a function, the function must at least be quadratic This is something that is lacking in traditional functional forms like the Cobb-Douglas. The importance of separability is emphasized in the study of demand and supply mainly for the following reasons:

a) it allows optimization in stages, enabling decentralized decision making and avoids having to deal with several variables at the same time.

b) it allows for the use of aggregates when individual input prices (quantities) are unavailable.

c) it justifies the use of net output or value added if primary inputs are separable from intermediate inputs.

d) it plays a major role in functional form specification, and hence, influences generality and simplicity of the form to be used.

The development of flexible functional forms in the seventies led to the testing of functional separability extensively. Among primary studies, we cite Berndt and Christensen (1973a), Berndt and Wood (1975), and Denny and Fuss (1977)[5].

Since the objective of this thesis is to compare and contrast the second-order with the third-order translog cost functions, we defer the detailed discussion of production studies until section (2.7).

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5 For details on various types of separability see Blackorby et al, 1978.

Translog Multi-output, TLM (Burgess, 1974)

$$\ln C(w,Y) = \gamma_0 + \sum_i^n \gamma_i \ln w_i + \sum_k^n \gamma_k \ln Y_k + 1/2 \sum_k^n \sum_l^n \gamma_{kl} \ln Y_k \ln Y_l$$

$$+ 1/2 \sum_i^n \sum_j^n \gamma_{ij} \ln w_i \ln w_j + \sum_i^n \sum_k^n \gamma_{ik} \ln w_i \ln Y_k. \qquad (2.4.20)$$

This function was suggested by Burgess (1974). It has properties similar to that of a single output version, except that it is not defined if one of the firms does not produce some of the outputs under consideration. This problem was solved by introducing a Box- Cox transformation of the output (Caves et al 1974) given below:

Generalized TLM (Caves et al., 1974)

$$\ln C(w,Y) = \gamma_0 + \sum_i^n \gamma_i \ln w_i + \sum_i^n \gamma_k \left(\left[Y_k^\lambda - 1\right]/\lambda\right)$$

$$+ 1/2 \sum_k^n \sum_l^n \gamma_{kl} \left(\left[Y_k^\lambda - 1\right]/\lambda\right) \left(\left[Y_l^\lambda - 1\right]/\lambda\right)$$

$$+ 1/2 \sum_i^n \sum_j^n \gamma_{ij} \ln w_i \ln w_j + \sum_i^n \sum_k^n \gamma_{ik} \ln w_i \left[Y_k^\lambda - 1\right]/\lambda \qquad (2.4.25)$$

where the zero output level is well defined: $f_k(0) = -(1/\lambda)$. In the limit the Box-Cox function is,

$$[f_k(y) = (Y_k^\lambda - 1)/\lambda] = \ln y_k \text{ as } \lambda \to 0.$$

2.5 Other flexible functional forms and their properties

Generalized Cobb-Douglas (Diewert, 1973b)

$$\ln C(w,y) = \gamma_0 + \gamma_y \ln y + 1/2\, \gamma_{yy} (\ln y)^2$$

$$+ \sum_i^n \sum_j^n \gamma_{ij} \ln(w_i + w_j)/2 + \sum_i^n \gamma_{iy} \ln w_i \ln y. \tag{2.5.1}$$

This function cannot be derived from a Taylor's series approximation, but it allows us to test some of the properties that were imposed, *a priori*, on the Cobb-Douglas function (for example homotheticity and elasticity of substitution). The necessary and sufficient conditions needed for the cost function to be linear homogeneous in input prices, w, are:

$$\sum_i^n \gamma_{ij} = 1, \text{ and } \sum_i^n \gamma_{iy} = 0, \quad i,j = 1,..\ n \tag{2.5.2}$$

where i and j represent input prices.

In order to have a homothetically linear homogeneous cost function in output, the following conditions are needed:

$$\gamma_y - 1 = \gamma_{yy} = \gamma_{iy} = 0, \tag{2.5.3}$$

If $\gamma_{ij} = 0$ in addition to (2.5.3) then the Generalized Cobb-Douglas Cost function reduces to Cobb-Douglas. The estimated parameters and, hence, elasticities from the above function will not be invariant to the arbitrary scaling of factor prices by 1/2 (Wales and Woodland, 1979)

Extended G.C.D (Magnus, 1979)

$$\ln C(w,y) = \gamma_0 + \gamma_y \ln y + 1/2\, \gamma_{yy} (\ln y)^2 + \sum_i^n \sum_j^n \gamma_{ij} \ln(B_i w_i + B_j w_j)$$

$$+ \sum_i^n \gamma_{iy} \ln w_i \ln y. \tag{2.5.4}$$

If $B_k>0\ \forall k$, symmetry requires $\gamma_{ij} = \gamma_{ji}\ \forall\ i\neq j$ and linear homogeneity will be satisfied if $\sum_i \sum_j \gamma_{ij} = 1$ and $\sum_i \gamma_{iy} = 0$. In order for the B_i's to be identified, they must sum to unity. This form was suggested by Magnus, 1979, in order to solve the problem of scaling in the generalized form. He has introduced a scaling factor B_i, to replace 1/2, relating to the second-order parameter. In doing so, he introduced new parameters to be estimated, which is undesirable because of a loss of degrees of freedom. The above cost function will be homothetic if $\gamma_{iy} = 0\ \forall i$, homogeneous if $\gamma_{yy} = \gamma_{iy} = 0\ \forall i$, and exhibit constant returns to scale if,

$$(\gamma_y - 1) = \gamma_{yy} = \gamma_{iy} = 0\ \forall i.$$

The shares from this function can be derived as follows:

$$S_i(w,y) = 2 \sum_j^n \gamma_{ij} B_i w_i (B_i w_i + B_j w_j)^{-1} + \gamma_{iy} \ln y \tag{2.5.5}$$

The derived shares equations are non- linear in parameters and, hence, present another problem in estimation due to lack of convergence and usage of more computer time.

The partial elasticity of substitution can be derived by using the standard formula (2.2.13):

$$\sigma_{ij}(w,y) = 1 - (2\gamma_{ij} B_i B_j w_i w_j) / (B_i w_i + B_j w_j)^2 S_i(w,y) S_j(w,y),$$

$$i \neq j \tag{2.5.6}$$

The sign of the above formula will be determined by the sign of γ_{ij} since the shares are positive by monotonicity and the B_i terms can not be less than 0 by linear homogeneity (Guilkey, 1983, p 595). Therefore, the magnitude of σ_{ij} will be equal to unity if $\gamma_{ij} = 0$, (the Cobb-Douglas case), greater than unity if γ_{ij} is negative and will be less than unity if γ_{ij} is greater than zero. This magnitude can never be greater than unity in one period and less than unity the next regardless of price levels or technological change. This is a weakness suffered by all partial elasticity of substitution formulas derived from less than thrice differentiable functions.

Generalized Box Cox, GBC (Berndt et al. 1979)

$$C(w,Y) = \left[\lambda/2 \sum_{i}^{n} \sum_{i}^{n} \gamma_{ij} w_i^{\lambda/2} w_j^{\lambda/2}\right]^{1/\lambda} Y^{B(W,Y)}. \tag{2.5.7}$$

The above form incorporates different flexible cost functions as λ takes on different values. For example, the linear and non-linear Generalized Leontief, and the translog cost functions are embedded in this function. This form is very useful in comparing flexible functional forms, as it incorporates most of the important functional forms. Its drawback is that the disturbance term is not distributed normally and hence, it may create a problem when carrying out hypotheses testing (Guilkey et al., 1983, p 595).

Modeling producer behavior, by using Taylor series approximation has been criticized by White (1980). He claimed that the Taylor Series approximation provides a poor approximation of the underlying structure. His criticism has led to the use of flexible forms that do not depend on Taylor's expansion such as Jorgenson's differential equation technique, (Jorgenson, 1986), Fourier series approximation, (Gallant, 1981) and Mini flex Laurent expansions, (Barnet et al 1985).

R.P. Byron and A.K. Bera (1983) showed that the calculations made by White were incorrect, and were able to show that the bias tends to disappear as we use a higher order approximation. In their particular example, the third-order Taylor expansion reduced the bias considerably and the second-order forms showed superiority over the first-order forms, (Cobb-Douglas).

2.6 Choice of functional forms

The choice of a functional form depends on the nature of the study in question. The form must adequately describe the problem at hand and must also fulfill certain mathematical requirements (Lau 1986). Among other things, the following criteria are seen to be important in the choice of functional forms.

1) Parsimony in parameters: unnecessary parameters should not be included in the form, since they create multicollinearity and degrees of freedom are lost.

2) The parameters included should be easy to interpret and must have an intuitive economic meaning.

3) Linearity and explicitness in parameters are desirable, since the former renders computation simpler.

4) Interpolative and extrapolative robustness within and without the data set, respectively, are also desirable. The former is important

for hypothesis testing while the latter is important for forecasting.

5) A wider domain of applicability is also desirable. The set of values of independent variables over which theoretical consistency is satisfied, should be large if possible.

6) The function must retain its flexibility. Flexibility can be defined as the ability of the functional form to generate all combinations of economic effects of interest. It should not impose any restrictions, *a priori*, on certain economic effects, for example on the elasticity of substitution or share elasticities.

Depending on the study at hand, some of these criteria are absolutely indispensable while a trade-off can be made with others. Lau (1986) suggests that flexibility and theoretical consistency within the neighborhood of some values of the variables should not be sacrificed. One could however, depending on the case, restrict the domain of applicability, or limit robustness to a smaller range of values

Various criteria were used in past studies in order to determine the appropriate functional form. We discuss some of these studies below.

2.6.1 Data Specific Studies

1) Berndt, Darrough and Diewert (1977) fitted three flexible functional forms (TL, GL, GCD) using Canadian expenditure data. Their study showed the translog function to be superior.

2) Applebaum (1979a) developed a flexible generalized Box-Cox form, that incorporates, CD, CES, TL, GL, GSRQ and Quadratic. Based on fitting the generalized Box-Cox form with the U.S. manufacturing data

(1929- 1971), GL and GSRQ performed relatively better than the other forms considered

3) Berndt and Khaled (1979) used a slightly different generalized Box-Cox form using (1947-1971) U.S. manufacturing data. Their results were inconclusive for TL, but they were able to reject the restrictions of GSRQ.

2.6.2 Ability to Trace a Known Technology

Guilky et al. (1983) assumed a given technology, CES, and investigated the ability of GL and TL to trace it. They were interested in the range of the data set where the two forms approximate the given function. Their Monte Carlo study showed that the performance of GL and TL depended on the value of σ_{ij} (elasticity of substitution). The closer σ_{ij} is to unity, TL out-performed GL; the closer σ_{ij} is to zero, the more GL out-performed TL. This confirms the fact that the TL which is an extension of CD with $\sigma_{ij} = 1$, is better suited when σ_{ij} deviates from zero, or is close to unity. The GL which is an extension of Leontief Fixed Proportion with $\sigma_{ij} = 0$, should perform better when σ_{ij} is close to zero. Therefore, if the elasticity of substitution is known beforehand, it will give us an idea as to which form should be used.

2.6.3 Analytical Approach

Caves and Christensen (1980) used an analytical approach to study the global properties of GL,TL and EGCD. The investigation was conducted in such a way that the tracing ability of each form for a known technology was examined under certain conditions as the function deviates from constant returns to scale, as it deviates from homogeneity, as AUES (σ_{ij}) deviates from 0 and 1, and as AUES deviate from each other. Their controlled study showed that the TL function was superior within the domain of applicability. They have also noticed the deterioration of TL as AUES departs from unity and as they diverge from one another.

2.6.4 Economic Effects at a Point

Despotakis (1986) compared functional forms in terms of their demonstrated economic effects within the domain of applicability. By taking the first derivative of the economic effects at a point, he was able to show the difference between GL and TL. He argued that the first and second derivatives of the flexible forms are not function specific, while the third or higher order derivatives are. (From this, one could conclude that the higher order flexible functional forms are better suited for comparing different forms). When he took the first derivatives of the second-order terms, such as elasticity of substitution, he in effect considered a third-order flexible form, such as the derivatives of C_{ij}, $\partial C_{ij}/\partial w_k$, which are readily found in the third-order forms as C_{ijk}. Based on this line of investigation, he was able to show the following stability conditions for the two forms he considered: AUES= 1 , AUES = 0 for TL and GL respectively

As was expected, there is no particular form that performs well, all the time. This illustrates the point that in absence of *a priori* knowledge about the technology, one has to search for an appropriate functional form corresponding to the study in question. Since the purpose of searching for an appropriate functional form is to be able to express the objectives of producer studies explicitly and be able to test them within the model, the next section will deal with attaining these objectives.

2.7. Further Properties of Second Order Translog Cost Function.

In this section we will examine some of the above mentioned points of interests derived from a second-order translog cost function.

Expansion Elasticity

We get the measure of the biases of scale parameters by differentiating the cost function with respect to all the components of the N+1 arguments·

$$\gamma_{iy} = \partial^2 \ln C(w,y)/\partial \ln w_i \partial \ln y. \tag{2.7.1}$$

(2.7.1) shows the impact of scale on the share of an input in the total cost. If $\gamma_{iy} \gtreqless 0$ then the share of a particular input increases, stays constant, or decreases respectively, as the level of output increases. Expression (2.7.1) can also be interpreted as a measure of the response of cost flexibility with respect to input prices, since the cost flexibility (cost elasticity with respect to output) is defined as:

$$\partial \ln C(w,y)/\partial \ln y = \tau(w,y) = \gamma_y + \gamma_{yy} \ln y + \sum_i^n \gamma_{iy} \ln w_i. \tag{2.7.2}$$

Cost flexibility can be defined as the reciprocal of returns to scale measured as the elasticity of output with respect to all inputs[6]:

$$\tau(w,y) = 1 / (\partial \ln y / \partial \ln x) \qquad (2.7.2)'$$

Thus, for example if $\tau(w,y) < 1$ the dual production function exhibits increasing returns to scale locally, total cost will increase less than in proportion to an increase in output. Since γ_{iy} is the first derivative of $\tau(w,y)$ with respect to log of input prices (also known as scale bias), it reflects the sensitivity of cost flexibility with respect to changes in input prices. If $\gamma_{iy} \gtreqless 0$, one can say that the cost flexibility increases, remains constant, or decreases as input prices increase. If the cost function is homothetic, the cost flexibility will be independent of factor prices. In terms of input demands, a similar exercise yields output elasticity of factor demand:

$$\partial \ln x_i(w,y)/\partial \ln y = \varepsilon_{iy} = (\gamma_{iy} + S_i\tau(w,y))/S_i \qquad (2.7.3)$$

Output Elasticity of Cost Flexibility

The logarithmic second-order derivative of the cost function w.r.t., Y, will give the responsiveness of cost flexibility as output changes

$$\partial^2 \ln C(w,y)/\partial(\ln y)^2 = \gamma_{yy}. \qquad (2.7.4)$$

If the production function exhibits constant returns to scale, the cost flexibility will be independent of output, that is $\gamma_y = 1$, $\gamma_{yy} = 0$. In

6 For more detailed analysis refer to D.W. Jorgenson, 1986, pp 1886-1889

addition, if the cost function is also linearly homogeneous in factor prices the term in (2.7.1) $\gamma_{iy} = 0 \; \forall i$, and the degree of returns to scale and its reciprocal cost flexibility will be equal to unity.

Share Elasticity

The share elasticity is a measure of substitution obtained by differentiating a cost function twice w.r.t. all components of w[7],

$$\partial^2 \ln C(w,y)/\partial \ln w_i \partial \ln w_j = \gamma_{ij} \tag{2.7.5}$$

It shows the response of the share of an input S_i, to a proportional change in the respective input price. If it is greater than zero, the share of a given input increases, it remains constant if it is zero, and decreases if the parameter estimate is less than zero in response to an increase in the corresponding input price. In terms of the derived input demand, a proportional change in the demand of the first input with respect to changes in the price of the second input, results in the following:

$$\partial \ln X_i(w,y)/\partial \ln w_j = (\gamma_{ij} + S_i S_j)/S_i = \varepsilon_{ij}, \tag{2.7.6}$$

where this can be interpreted as the cross elasticity of demand for input i, with respect to the price of input j. The own elasticity of demand for input i can be written as:

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[7] D. W. Jorgenson, 1986, p 1886.

$$\varepsilon_{ii} = (\gamma_{ii} + S_i(1-S_i)) \ / \ S_i \qquad (2.7.6)'$$

Allen-Uzawa Elasticity of Substitution

This formula relates input elasticity and the share of an input to total cost. Unlike the input elasticity, equation (2.7.6), it is symmetric.

$$\sigma_{ij} = \varepsilon_{ij}/S_j, \qquad (2.7.7)$$

where,

$$\varepsilon_{ij} = C_{ij}\, w_j / C_i, \quad S_j = w_j C_j / C \qquad (2.7.8)$$

By substituting (2.7.8) in (2.7.7), one gets a symmetric Allen-Uzawa elasticity of substitution in terms of the cost function, and its first and second-order derivatives:

$$\sigma_{ij} = CC_{ij}/C_i C_j \qquad (2.7.9),$$

in terms of shares, using equations (2.7.6) and (2.7.6)' the formula reduces to:

$$\sigma_{ij} = (\gamma_{ij} + S_i S_j)/S_i S_j = 1 + \gamma_{ij} \ / \ S_i S_j \quad \forall i \neq j,$$

$$i.e. \ \gamma_{ij} < 0 \Rightarrow \sigma_{ij} < 1 \text{ and } \gamma_{ij} > 0 \Rightarrow \sigma_{ij} > 1,$$

$$\sigma_{ii} = (\gamma_{ii} + S_i(S_i - 1))/S_i^2, \text{ for all } i = j. \qquad (2.7.10)$$

where $C_i = (\partial C/\partial w_i) = S_i(C/w_i)$, $C_j = S_jC/W_j$,

$$C_{ij} = \partial^2 C/\partial w_i \partial w_j = (C/w_i w_j)(\gamma_{ij} + S_i S_j). \qquad (2.7.11)$$

Substitutability plays an important role in determining the incidence of taxes. Its magnitude reveals the nature of substitution between inputs and the nature of the underlying functions. As σ_{ij} is positive or negative , factor i and j are said to be substitutes and compliments, respectively. If $\sigma_{ij} = 1$, then the function is Cobb-Douglas. If σ is constant and equal but different from unity, the function is a CES function. Finally if it is zero, the function is a Leontief Fixed Proportions.

In empirical work the signs and magnitude of AUES is used to determine substitution possibilities between inputs. The formula given by (2.7.10) is capable of generating a magnitude equal to unity and less than zero when $\gamma_{ij} < 0$. This implies that two inputs can exhibit a slight substitutability when σ_{ij} between 0 and 1. However, at the same time this magnitude could move to less than zero for different input prices in the same data set. When this happens the two inputs can display complementarity. However, the flexibility of this formula does not go far enough to move inputs from high substitutability, a magnitude greater than one, to complementarity. This is because the elasticity of substitution derived from a second-order translog cost function depends on a constant share elasticity and, hence, can not generate a magnitude greater than one and less than one for a given set of data. This result is obvious from equation (2.7.10). In this formula, the shares are positive and, thus, the sign of σ_{ij} will be determined by the sign of the constant share elasticity γ_{ij} . Once the sign of parameter γ_{ij} is determined, it will stay the same for the whole period, regardless of the levels of input prices and technological change. On the other hand, a third-order translog cost function exhibits a variable share elasticity and, therefore, is capable of correcting this shortcoming.

The formula for σ_{ii} can be used to test for concavity of the cost function in input prices. At the point of approximation, the input shares, $S_i(w,y)$ reduce to γ_i. Thus, given a concave translog cost function σ_{ii} must satisfy the following condition :

$$\sigma_{ii} = \left[\gamma_{ii} + \gamma_i\left(\gamma_i - 1\right)\right] \Big/ \gamma_i^2 \leq 0 \qquad (2.7.12)$$

The points of interests discussed in this section and other important objectives such as technological change, can be tested by using the relevant functional forms. The functional forms were applied in competitive markets such as the U.S manufacturing (Berndt and Wood, 1975) and in regulated industries such as transportation, communication, and electric power both in the U.S and Canada (Christensen and Green, 1976, Stevenson, 1980, Brown, et al, 1979, Fuss, et al, 1981, Christensen, et al, 1983).

The merits of the translog cost function are its ability to measure and test important producer behavior and to avoid the use of output in the share equations when constant returns to scale is imposed. The latter property is important especially when output measures are either unavailable or of poor quality. The shortcoming of the translog cost functions or any cost function derived from a second-order Taylor's series approximation is that the approximation does not go far enough to represent important producer behavior discussed in the next section and in the following Chapter. In addition to these shortcomings, the formulas derived from a second-order cost function have certain rigidities. If relied upon, this could result in serious policy errors. This point will be discussed in section 4.9 Chapter 4.

The merits and the shortcomings of second- and third-order translog cost functions is the subject of the remainder of this chapter and thesis.

2.8 Third Order Approximations

The second-order Taylor series approximation has been used to model producer behavior. The approach does not put many *a priori* restrictions on producer technology. In addition to estimating the slopes of the input demand and output supply functions, it allows one to test the traditionally maintained hypotheses of homogeneity and separability.

Having explored the strength of the functional forms that are based on the second-order approximations one may observe certain shortcomings. The second-order Taylor series expansion may suffer from truncation bias, since it ignores all terms above the second-order. The truncation bias could be reduced through the use of higher order forms . As shown by Kmenta (1971) and Byron et al. (1983) the truncation bias becomes smaller as one goes to the higher order forms.

However, these benefits do not come without costs. The most important cost often mentioned is the loss of degrees of freedom as more explanatory variables are included. If the true underlying cost function is in fact a second-order translog, or if the second-order translog is an adequate approximation, the estimates of the first and second-order parameters as well as functions of them such as the Allen

Uzawa elasticities of substitution will be less efficient so that confidence intervals will be wider and hypothesis tests will have less power. Thus there will be a trade off between reducing bias if the true model is third-order or more, and parameter efficiency if the true model is second-order or less. The problem of loosing degrees of freedom becomes serious when the effective sample size is small compared with the number of factors of production.

The potential seriousness of this problem arises because the number of parameters for a third-order translog function increases with the cube of the number of factors of production. For a unit logarithmic homogeneous cost function without the symmetry assumption the number of free parameters to be estimated will be: $\rho\ (1 + \rho + \rho^2)$, where ρ = the number of share equations to be estimated *i.e.* n - 1. When a function is extended from a Cobb-Douglas to a second-order translog cost function the loss in the degrees of freedom will be ρ^2. This number will increase by ρ^3 when the third-order translog cost function with 'n' factors is considered. The loss could be very large when 'n' is large. However, the number can be considerably reduced using economic theory. For instance, for four factors we only need to estimate three shares reducing the loss in degrees of freedom from 64 to 27. Homogeneity and symmetry restrictions reduce the number of free parameters from 27 to only 10. Without the use of restrictions from economic theory we would have been forced to estimate 84 parameters. With these restrictions, we were only required to estimate 19 free parameters.

Flexible functional forms allow unrestricted estimation of parameters that represent substitutability, technological change, economies of scale, etc.. Estimates from a third-order translog yield derived demands, or input shares which involve second-order terms, as opposed to only the first-order terms in the case of second-order approximations. This should yield better approximations to the true functions. This should give superior estimates of the elasticities of substitution and factor demand price elasticities.

The third-order forms also allow the examination of the curvature of input demands. This requires analysis of the third-order derivatives of the cost function.

One example where knowledge on the curvature of factor demand would be to determine the magnitude of change in a factor price as tax is levied on factors of production. Take the case of monopoly. If factor demand is a linear one as it is the case derived from a second order translog cost function, the change in factor price will be less than the change in the amount of tax as the linear marginal cost curve shifts[1]. This result is due to a constant slope of the factor demand curve. In general a tax may increase the price by more or less the amount of the tax. Factor demand curves derived from a third-order translog cost function are non-linear in variables. Therefore, their slopes change. The change in the slope can be learned by looking at the curvature of the non-linear factor demand curve. Thus depending on the curvature of demand curve, one can find a higher or lower factor price increase compared with the amount of the tax increase on the factors of production. Another interesting feature of higher order forms is found when testing for functional separability. As indicated earlier, the flexible functional forms become inflexible once non-linear separability restrictions are imposed. If we are using the TL function, it collapses to a partial CD (Fuss et al., 1978). Lau (1977) suggested that in order

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[1] J.E. Stiglitz, Economics of public sector, Chapter 17,pp 411-436.

to determine whether the results obtained by using the second-order forms can be maintained, requires the investigation of higher order forms. Lau (1977) also states that regularity conditions can be tested more rigorously, since the higher order forms have more restrictions. Indeed it seems possible that one could find a larger domain of applicability once higher order functions are investigated. These and other related subjects will be investigated in the remainder of this study.

Finally, it should be noted that the additional terms that enter into the third-order forms have an intuitive economic meaning. Extending the flexible functional forms to the third-order approximation not only gives better econometric result but also more flexible economic representations.

Appendix to Chapter 2

Regularity Conditions at the Point of Approximation - Second-order Translog

To clarify the conditions needed for theoretical consistency of the arbitrary cost function, we present Lau's, (1986) analytical approach below for two input, single output second-order translog unit cost function.

First, in order to derive a unit cost function from (2.4.6), it is assumed that (2.4.6) is symmetric in factor prices. Second, it must be linearly homogeneous in factor prices and exhibit constant returns to scale. Hence, the unit cost function can be expressed as follows[9]:

$$\ln c(w_1, w_2) = \gamma_0 + \gamma_1 \ln w_1 + (1-\gamma_1)\ln w_2 + 1/2(\gamma_{11})\ln w_1^2$$

$$-\gamma_{11}\ln w_1 \ln w_2 + 1/2(\gamma_{11})\ln w_2^2. \tag{2.A.1}$$

The local and global theoretical consistency of the translog cost function, can be examined by using (2.A.1), without any loss of generality.

Local Conditions

The local theoretical conditions will be analyzed in some neighborhood of factor prices, for example, for $w_1 = w_2 = 1$.

[9] By using symmetry and adding up property $\gamma_{12} = -\gamma_{11}$ and $\gamma_{22} = \gamma_{11}$.

$$C(1,1) = e^{\gamma_0} > 0 \tag{2.A.2}$$

$$\nabla C(1,1) = \partial C/\partial w \geq 0 \equiv e^{\gamma_0} \begin{bmatrix} \gamma_1 \\ \\ (1-\gamma_1) \end{bmatrix} \tag{2.A.3}$$

$$\nabla^2 C(1,1) = \partial^2 \ln C/\partial(\ln w)^2 . C$$

$$= C \begin{bmatrix} \partial^2 \ln C/\partial(\ln w_1)^2 & \partial^2 \ln C/\partial \ln w_1 \partial \ln w_2 \\ \\ \partial \ln C/\partial \ln w_1 \partial \ln w_2 & \partial^2 \ln C/\partial(\ln w_2)^2 \end{bmatrix}$$

$$= e^{\gamma_0} \begin{bmatrix} \gamma_1(\gamma_1 - 1) + \gamma_{11} & \gamma_1(1-\gamma_1) - \gamma_{11} \\ \\ \gamma_1(1-\gamma_1) - \gamma_{11} & (\gamma_1 - 1)\gamma_1 + \gamma_{11} \end{bmatrix} \leq 0 \tag{2.A.4}$$

Condition (2.A.2) is always satisfied since an exponential function is positive. The monotonicity requirement, (2.A.3), states that, the gradient of C (the first derivative of the cost function with respect to all components of W) must be non-negative. This local non-negativity requirement is satisfied if $0 \leq \gamma_1 \leq 1$. Condition (2.A.4) is a local concavity requirement, which states that the matrix of second-order partial derivatives of the cost function, with respect to all the components of W, must be negative semi-definite. In order for this to hold, the following conditions must be satisfied :

$$\gamma_1(\gamma_1 - 1) + \gamma_{11} \leq 0. \qquad (2.A.5)$$

We know that γ_1 is positive by non-negativity and between zero and one due to monotonicity. Therefore, a sufficient condition for (2.A.5) to hold is γ_{11} be non-positive.

Global Conditions

In order for global consistency to be achieved, the monotonicity and concavity conditions must hold for all price levels. This means that the gradient and Hessian matrices, (2.A.3) and (2.A.4), will be a function of input prices. Hence, as long as γ_{11} is non zero, one can find a value for the variables that violates the concavity and monotonicity condition (this point will be discussed in greater detail in the next chapter in terms of the third-order cost function). Thus, global consistency requires that the interaction parameters, γ_{ij}, must be zero[10]:

$$\gamma_{11} = 0 \text{ and } \gamma_1 \text{ be between } 0 \text{ and } 1. \qquad (2.A.6)$$

If this global consistency requirement is imposed on (2.A.1), then the unit cost function loses its flexibility, since it is reduced to a Cobb-Douglas form. This is the explanation underlying the reason why flexible cost functions are said to be well behaved locally. However, for most studies, the local consistency conditions are sufficient.

To sum up, we make the following observation. The gradient and the second-order partial derivative of the unit cost function with respect

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10 L. J. Lau, 1986, p 1535.

to all input prices at some neighborhood of factor prices were analyzed The analysis showed that the translog unit cost function will be locally consistent if $0 \le \gamma_1 \le 1$, which is the non-negativity requirement. The concavity requirement will be satisfied locally ($w_1 = w_2 = 1$) if $\gamma_1(\gamma_1 - 1) + \gamma_{11} \le 0$. The translog function will not be globally consistent since both the gradient and the second-order derivative of the unit cost function will be functions of input prices and as long as γ_{11} is non zero, one can find a value for the variables that violate the monotonicity and concavity conditions. The simple forms like Cobb-Douglas and CES, on the other hand, are globally consistent since they do not contain the interaction parameters However, the absence of the interaction parameters in these simple forms limits their flexibility

CHAPTER 3

THEORETICAL STRUCTURE OF THE THIRD ORDER TRANSLOG COST FUNCTION

The purpose of this Chapter is to show how some of the shortcomings of the traditional flexible functional forms discussed in Section (2.7) may be remedied by considering third-order translog cost functions (TCF). In particular, we focus on three aspects. First, the third-order forms would significantly reduce the truncation bias introduced by using only the second-order functions. More specifically, the bias would now be of the fourth-order and above. Second, the flexibility of the third-order forms would enable us to study economic relationships rigorously (such as factor demands, factor shares and elasticities) than could be examined by using only a second-order function Finally, a more rigorous test of the hypotheses can be carried out in the present model since there will be more restrictions in every case considered. Thus, with the present model, one can achieve more flexibility and precision in representing producer behavior.

In this Chapter, a third-order translog cost function representing producer technology is developed. The model will be introduced in Section (3.1). In Section (3.2), the theoretical consistency of the model will be examined in its general and particular forms. In Section (3 3) measures used to represent producer behavior will be derived and compared with those derived from the traditional second-order translog cost function In Section (3.4), the restrictions for functional separability will be derived and then compared with the ones derived from the second-order translog cost function. In Section (3.5), concluding remarks will be given.

3.1 The Model

In order to achieve completeness, the third-order Taylor's series approximation of an arbitrary cost function can be expressed as follows:

$$\ln C(w,y) = \ln\gamma_o + \gamma_y \ln y + \sum_{i=1}^{n} \gamma_i \ln w_i + \sum_{i=1}^{n} \gamma_{iy} \ln w_i \ln y$$

$$+ \frac{1}{2}\left[\gamma_{yy}(\ln y)^2 + \sum_{i=1}^{n} \gamma_{iyy} \ln w_i (\ln y)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln w_i \ln w_j\right.$$

$$\left.+ \sum_i\sum_j \gamma_{ijy} \ln w_i \ln w_j \ln y\right] + \frac{1}{6}\left[\sum_i^n\sum_j^n\sum_h^n \gamma_{ijh} \ln w_i \ln w_j \ln w_h + \gamma_{yyy}(\ln y)^3\right] \qquad (3.1\ 1)$$

where, $\gamma_o = \ln C(w^*, y^*)$, $\gamma_i = \frac{\partial \ln C}{\partial \ln w_i}$, $\gamma_{ij} = \frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln w_j}$,

$\gamma_y = \partial \ln C / \partial \ln y$, $\gamma_{iy} = \frac{\partial \ln C}{\partial \ln w_i \partial \ln y}$, $\gamma_{yy} = \frac{\partial^2 \ln C}{\partial (\ln y)^2}$,

$\gamma_{iyy} = \frac{\partial^3 \ln C}{\partial \ln w_i \partial (\ln y)^2}$, $\gamma_{ijy} = \frac{\partial^3 \ln C}{\partial \ln w_i \partial \ln w_j \partial \ln y}$,

$\gamma_{yyy} = \frac{\partial^3 \ln C}{\partial (\ln y)^3}$, $\gamma_{ijh} = \frac{\partial^3 \ln C}{\partial \ln w_i \partial \ln w_j \partial \ln w_h}$,

where all derivatives are evaluated at w^*, y^*, y = aggregate output, w = input prices.

Equation (3.1 1) is a third-order approximation of an arbitrary function. If it is to approximate a cost function representing producer technology, the function must be thrice differentiable and the regularity conditions given below must be satisfied in order for theoretical consistency to hold.

3.2 Regularity Conditions

The cost function must be a real valued function for positive input prices; higher output levels should not lead to a lower cost level; higher input prices should lead to a higher cost level and it must be concave in factor prices. These general conditions were given by (2.2.1), (2.2.2a), (2.2.2b) and (2.2.4), respectively for the second-order translog. Those conditions also apply to the third-order translog cost functions (TCF).

(a) The function must be symmetric in input prices, w:

$$\gamma_{ij} = \gamma_{ji}, \ i \neq j, \quad \gamma_{ijh} = \gamma_{ihj} = \gamma_{hij}, \quad \forall\ i = j \neq h,\ \forall\ i = h \neq j,\ \forall\ j = h \neq i,$$

$$\gamma_{ijh} = \gamma_{ihj} = \gamma_{hij} = \gamma_{jhi} = \gamma_{hji} = \gamma_{jih} \quad \forall\ i \neq j \neq h, \quad \gamma_{ijy} = \gamma_{ijy}, \ i \neq j \tag{3.2.1}$$

(b) In order for (3 1.1) to be linearly homogeneous in input prices the following restrictions are needed:

$$\sum_{i=1}^{n} \gamma_{ij} = 0, \quad \sum_{i=1}^{n} \gamma_{ijh} = 0, \quad \sum_{i=1}^{n} \gamma_{iy} = 0, \quad \sum_{i=1}^{n} \gamma_{ijy} = 0, \quad \sum_{i=1}^{n} \gamma_{iyy} = 0$$

$$\text{and } \sum_{i} \gamma_{i} = 1 \tag{3.2.2}$$

If any arbitrary function, such as equation (3.1.1), passes the above-mentioned regularity conditions plus the general conditions stated in the earlier paragraph, it is said to be theoretically consistent in approximating a true cost function. Then, due to the duality between the cost and production functions, equation (3.1.1) can represent producer technology.

In addition to the above properties, the third-order translog approximation is said to be **homogeneous** in output if the following conditions are satisfied:

$$\gamma_{iy} = \gamma_{ijy} = \gamma_{iyy} = 0 \;\forall\; i,j \qquad (3.2.3)$$

The translog approximation will be **linearly homogeneous** in output(constant returns to scale), if the following conditions hold

$$(\gamma_y - 1) = \gamma_{yy} = \gamma_{iy} = \gamma_{ijy} = \gamma_{iyy} = \gamma_{yyy} = 0 \;\forall\; i,j \qquad (3.2.4)$$

Even though conditions (3.2.4) are of some interest, they are not required for the theoretical consistency of the cost function

In order to illustrate all of the above mentioned regularity conditions so that the similarities and differences between properties derived from the second- and third-order functions can be seen, we propose using a simplified version of the general form of the cost function (3.1.1). The assumptions and the derivation are discussed in detail in the appendix to this chapter. In the appendix a linearly homogeneous, two input, third-order translog unit cost function is examined.

Examination of (3.A.10) to (3.A.12) reveals that positivity of the cost function, the share equations and the concavity requirement at the point of approximation (1,1) are similar to the ones derived for the second-order translog cost function. If these conditions are satisfied the translog function is said to be well behaved locally.

However, the conditions (3.A.10) to (3.A.12) are neither necessary nor sufficient for the fulfillment of regularity conditions at any other set of factor prices. Thus, to obtain a meaningful cost function, the unit cost function must be examined at every input price level. The conditions given by (3.A.13) to (3.A.15) are necessary and sufficient for global theoretical consistency of the third-order translog cost function. These conditions include the corresponding conditions derived from the second-order translog function as a special case when the third-order parameter β_{111} is equal to zero. Therefore the conditions derived from the third-order translog function involve more variables and parameters, and hence allow for a more rigorous test of regularity conditions compared with the ones derived from the second-order function. However these conditions cannot be satisfied at every set of factor prices, since one can find a value that will violate these conditions Therefore the third-order form, like the second-order, cannot be said to be well behaved globally. This does not mean that the conditions given by (3.A.13) to (3.A.15) cannot be satisfied for a specified data set. In fact, if they are not satisfied for the data set being used, the results derived are not meaningful. The sufficient condition for global consistency is given by (3.A.16). If this condition is imposed, the third-order function will lose its flexibility, just as the second-order function will lose its flexibility i.e. become Cobb-Douglas. Thus, the advantage of using a third-order function over the second-order one, lies not in gaining global consistency, but rather being able to test the regularity conditions more rigorously for a given data set made possible by the additional restrictions found. This result confirms a suggestion made by Lau (1986).

If a function satisfies the above regularity condition, it also represents production technology according to duality theory. Therefore, several points of interest in the study of producer behavior can be derived and examined. The third-order cost function allows the derivation of additional expressions that are useful for examining the sensitivity of the function with respect to all of the variables. These additional expressions should make the results more reliable as they represent a reduced bias due to a higher order approximation. In addition to the above advantages of the third-order form over the second-order form, some important economic relationships in the study of producer behavior can also be derived from the present model[1]. This added information contained in the third-order TCF makes it even more interesting. In order to appreciate the usefulness of the third-order approximation, equation (3.1.1), its derivatives will be examined in the next section.

Equation (3.1.1) is left in its most general form to enable us to derive and examine economic concepts that concern producer behavior. The

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[1] See equations (3.3.11) to (3.3.17).

only condition that will be imposed is symmetry, (3.2.2), in the variables involved.

3.3 Points of Interest

The main motivation of using flexible functional forms is to be able to examine issues such as input shares and various kinds of elasticities within a given model. These points of interests were reviewed in Chapter 2. To show the advantages of using a third-order translog cost function over the second-order, the traditional points of interests examined in Chapter 2 and some additional ones are re-examined below

Input Share Equations

Given the local theoretical consistency of the model, the share of an input in the total cost is derived by taking the logarithmic differentiation of the cost function (3.1.1) with respect to input prices.

$$\frac{\partial \ln C}{\partial \ln w_i} \equiv \frac{\partial C}{\partial w_i}\frac{w_i}{C} \equiv s_i(w,y)$$

$$= \gamma_i + \sum_{j}^{n} \gamma_{ij} \ln w_j + \gamma_{iy} \ln y + \sum_{j} \gamma_{ijy} \ln w_j \ln y$$

$$+ \frac{1}{2}\left[\sum_{j}^{n}\sum_{h=1}^{n} \gamma_{ijh} \ln w_j \ln w_h + \gamma_{iyy}(\ln y)^2\right] \qquad (3.3.1)$$

Equation (3.3.1) is linear in its parameters, which was also the case in the share equations derived from the second-order translog cost function, but non-linear in variables, unlike the ones derived from the second-order TCF. Furthermore, it contains additional terms that were missing in (2.7.7), due to the higher order. These extra terms make the present share equation a second-order approximation of the true underlying share equation as opposed to usual share equations which are only first-order approximations.

The attractive features of (3.3.1) are the following: first, the shares estimated in the present model tend to reduce the truncation bias. Second, the additional terms in the share equation enable us to examine both the slope and the curvature of this derived function. Third, it responds to changes in the interaction between input prices and output, γ_{ijy}, and between input prices, γ_{ijh}. Fourth, it easily collapses to a share equation derived from the second-order cost function by setting γ_{ijy}, γ_{iyy}, and γ_{ijh} to zero.

The derived input demands can be easily obtained by multiplying both sides of equation (3.3.1) by (C/w). This leaves ($\partial C/\partial w_i$) in the left hand side, which is the i-th input demand by Shephard's Lemma. However, the input demand derived from both the second and third-order cost functions are non-linear in both parameters and variables. Instead, the share equations which are linear in parameters are used for estimation purposes. The shares and input demands are homogeneous of degree zero, and are also non-negative, which is required by monotonicity. Finally, the sum of the shares add up to unity by cost exhaustion.

Cost Elasticity and Returns to Scale

Cost elasticity, also known as cost flexibility, can be derived by differentiating the log of the cost function with respect to the log of output. The reciprocal of this derived expression is known as a measure of returns to scale.

$$\frac{\partial \ln C}{\partial \ln y} = \frac{\partial C}{\partial y}\frac{y}{C} \equiv \tau(w,y) = \gamma_{y} + \gamma_{yy}\ln y + \sum_{i=1}^{n}\gamma_{iy}\ln w_{i}$$

$$+ \frac{1}{2}\left[\gamma_{yyy}(\ln y)^{2} + \sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ijy}\ln w_{i}\ln w_{j}\right] + \sum_{i=1}^{n}\gamma_{iyy}\ln w_{i}\ln y \qquad (3.3.2)$$

The cost flexibility is non-decreasing in output and linear in parameters. If the underlying technology exhibits constant returns to scale, cost flexibility will be equal to returns to scale:

$$\tau(w,y) = \left[\gamma_{y}\right] = 1, \text{ regardless of the order of approximation.}$$

However, if the dual production function exhibits decreasing or increasing returns to scale, then (2.7.2) and (3.3.2) yield flexibility measures that have different magnitudes. This is due to the fact that the latter includes parameters γ_{yyy}, γ_{ijy}, and γ_{iyy} that are not available in the former. Furthermore, the cost flexibility obtained from the third-order cost function is more reliable than the one derived from the second-order for the following reasons: first, it takes into account the interaction between input prices and output, and also between input prices. Second, it reduces truncation bias as it is a second rather than a first-order approximation of the true function

Share Elasticities

The share elasticity, $\gamma^{*}_{ij}(w,y)$, the measure of substitutability between inputs, (i,j), is another important economic relationship whose precise estimate is crucial. This function can be generated by logarithmically differentiating the cost function twice with respect to the log of input prices:

$$\gamma^{*}_{ij}(w,y) \equiv \frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln w_j} = \gamma_{ij} + \sum_{h=1}^{n} \gamma_{ijh} \ln w_h + \gamma_{ijy} \ln y \qquad \forall i \neq j \tag{3.3.3}$$

where the * indicates that the parameter or the variable source is a third-order translog function.

Depending on whether $\gamma^{*}_{ij}(w,y) \gtreqless 0$, the share of a particular input is said to increase, remain constant or decrease as the respective input price changes. Equation (3.3.3) is responsive to changes in input prices and the level of output as opposed to the fact that equation (2.7.7) exhibits a constant share elasticity Since there is no economic theory that suggests it should be constant, the variable share elasticity is more flexible in terms of economic interpretation, whereas the constant share elasticity results when $\sum_{h=1}^{n} \gamma_{ijh} = \gamma_{ijy} = 0$ for $\forall i,j,h$ in (3.3.3). Therefore the variable share elasticity, given in equation (3.3.3) is a first-order approximation that responds to the variables involved.

The first derivative of (3.2.1) with respect to input prices can also represent the derived factor demands. In terms of derived input demands, the cross price elasticity in the case of third-order translog cost function is derived as follows:

$$C(w,y) = \exp\left\{\gamma_0 + \gamma_y \ln y + \sum_{i=1}^{n} \gamma_i \ln w_i + \sum_i^{n} \gamma_{iy} \ln w_i \ln y + \frac{1}{2}\left[\gamma_{yy}(\ln y)^2 \right.\right.$$

$$\left. + \sum_{i=1}^{n} \gamma_{iyy} \ln w_i (\ln y)^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ij} \ln w_i \ln w_j \sum_{i=1}^{n}\sum_{j=1}^{n} \gamma_{ijy} \ln w_i \ln w_j \ln y\right]$$

$$\left. + \frac{1}{6} \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{h=1}^{n} \gamma_{ijh} \ln w_i \ln w_j \ln w_h + \gamma_{yyy}(\ln y)^3\right\} \tag{3.3.4}$$

$$x_i \equiv \frac{C\,\partial \ln C}{w_i\,\partial \ln w_i}(w,y) = \frac{C}{w_i}\left[\gamma_i + \sum_{j=1}^{n} \gamma_{ij} \ln w_j + \gamma_{iy} \ln y + \sum_{j=1}^{n} \gamma_{ijy} \ln w_j \ln y\right.$$

$$\left. + \frac{1}{2}\left[\gamma_{iyy}(\ln y)^2 + \sum_{j=1}^{n}\sum_{h=1}^{n} \gamma_{ijh} \ln w_j \ln w_h\right]\right] \tag{3.3.5}$$

The cross input price elasticity of factor demand is defined as:

$$\frac{\partial x_i}{\partial w_j}\frac{w_j}{x_i} = \varepsilon_{ij} \tag{3.3.6}$$

where $$\frac{\partial x_i}{\partial w_j} = \left[\frac{C}{w_i w_j}\right]\left[\gamma_{ij} + \gamma_{ijy} \ln y + \sum_{h=1}^{n} \gamma_{ijh} \ln w_h + s_i s_j\right] \tag{3.3.7}$$

and is obtained by substituting (3.3.5) and (3.3.7) in (3.3.6):

$$\varepsilon_{ij} = (\gamma_{ij} + \gamma_{ijy}\ln y + \sum_{h=1}^{n}\gamma_{ijh}\ln w_h + s_i s_j)/s_i$$

$$\equiv (\gamma^*_{ij}(w,y) + s_i s_j)/s_i \qquad (3.3.8)$$

The elasticity formula, (3.3.8), depends on the variable share elasticity, $\gamma^*_{ij}(w,y)$, and the share equations which are second-order approximations. However, the corresponding expression obtained from the second-order cost function depends on a constant share elasticity, γ_{ij}, and share equations that are the first-order approximations. Since (3.3.8) depends on the share equations derived from the third-order, our earlier remark with regard to this bias applies here as well.

The cross price elasticity of factor demand corresponding to the second-order cost function is obtained by setting all the sensitivity parameters in (3.3.8) to zero. Furthermore, (3.3.8) is similar to the cross input-price elasticity of factor demand formula provided by Allen (1938) which is expressed in terms of shares and elasticity of substitution, $\varepsilon_{ij} = s_j \sigma_{ij}$. However, the expressions in s_i and σ_{ij} will be different, since in our case they represent the third-order cost function rather than the second-order.

Measure of Scale Bias

This measure is derived by differentiating the log of the cost function twice with respect to input prices and output.

$$\gamma^*_{iy}(w,y) \equiv \frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y} = \gamma_{iy} + \gamma_{iyy}\ln y + \sum_{j=1}^{n}\gamma_{ijy}\ln w_j \qquad (3.3.9)$$

The above measure is known as scale bias or expansion elasticity, and is denoted by $\gamma^{\bullet}_{iy}(w,y)$. It shows the influence of an increasing output level on the input's share in total cost.

Depending on whether $\gamma^{\bullet}_{iy}(w,y) \gtreqless 0$, the share of a particular input in total cost is said to increase, remain constant or decrease, respectively, as the level of output is increased. Comparison of equations (3 3.9) and (2.7.1), shows us the former is flexible since it responds to further changes to w and y. $\gamma^{\bullet}_{iy}(w,y)$ could also be used to measure the responsiveness of cost flexibility, $\tau(w,y)$, with respect to changes in input prices; as

$$\gamma^{\bullet}_{iy}(w,y) = \gamma_{iy} + \gamma_{iyy}\ln y + \sum_{j=1}^{n} \gamma_{ijy}\ln w_j \equiv \text{equation (3.3.9)}.$$

Here too, $\gamma^{\bullet}_{iy}(w,y)$ is responsive to input prices and output, unlike γ_{iy} in equation (2.7.1), which did not respond to further changes in either of the variables.

In terms of input demands, the following measure can be derived:

$$\varepsilon^{\bullet}_{iy}(w,y) = \frac{1}{s_i}\left[\gamma_{iy} + \sum_j \gamma_{ijy}\ln w_j + \gamma_{iyy}\ln y + s_i\tau(w,y)\right]$$

$$\equiv \frac{1}{s_i}\left[\gamma^{\bullet}_{iy}(w,y) + s_i\tau(w,y)\right] \qquad (3.3.10)$$

where $\varepsilon^{\bullet}_{iy}(w,y)$ is elasticity of input demand with respect to y. Expression (3 3 10) depends on a variable expansion elasticity as opposed to a constant one, on the share equation, and on cost flexibility which is of second-order rather than the first-order. Thus, the formula (3.3.10) should result in a better estimate.

Sensitivity of Cost Flexibility

Differentiating the cost function twice with respect to output logarithmically will give the measure of the sensitivity of cost flexibility, $\gamma^{\bullet}_{yy}(w,y)$:

$$\gamma^{\bullet}_{yy}(w,y) \equiv \frac{\partial^2 \ln C(w,y)}{\partial(\ln y)^2} = \gamma_{yy} + \gamma_{yyy} \ln y + \sum_{i=1}^{n} \gamma_{iyy} \ln w_i \qquad (3.3.11)$$

Depending on whether $\gamma^{\bullet}_{yy}(w,y) \gtreqless 0$, the cost flexibility is said to increase, remain constant, or decrease respectively as the level of output changes. In the second-order translog cost function, this measure was again assumed to be constant. In the third-order case, this measure is a testable hypothesis as to whether it is increasing, constant, or decreasing with respect to changes in output level. Therefore, the measure derived in the third-order TCF does not rule out the assumption made in the second-order TCF of being a constant measure, but allows more possibilities.

As shown above, the economic relationships derived from the third-order TCF are more general, containing the properties of the economic relationships from the second-order TCF as a special case. They allow more flexibility because the derived relationships represent more properties. They are also better approximations of important measures, as the derived relationships represent higher order approximations, reducing truncation bias.

In addition to the above differences, other important economic relationships, such as sensitivity of share elasticity, the rate of change of cost flexibility, output sensitivity of share elasticity and biases of scale can be derived only from cost functions of higher than second-order. These issues are examined next.

Sensitivity of Share Elasticity

In the usual second-order cost function, the share elasticity is a constant. The third derivative of the cost function is zero. However, in the present model, the share elasticity, $\gamma^{*}_{ij}(w,y)$, depends on input prices and output. Differentiating (3.3.3) with respect to input prices yields γ_{ijh}. These parameters are useful in examining the sensitivity of the share elasticity as shown below.

$$\frac{\partial^3 \ln C}{\partial \ln w_i \, \partial \ln w_j \, \partial \ln w_h} = \gamma_{ijh} \qquad (3.3.12)$$

The degree of sensitivity of the share elasticity depends on the magnitude of γ_{ijh}. Alternatively, γ_{ijh} could be taken as the measure of the rate of change of a share of an input in the total cost.

One would like to know, not only if the share of an input will decrease as input prices increase, but also if it is going to decrease at an increasing, decreasing or constant rate. This can be learned only if cost functions of higher than second-order approximation are considered. For instance, if $\gamma^{*}_{ij}(w,y) < 0$ and $\gamma_{ijh} > 0$, the share of input will decrease at an increasing rate as input prices change.

The Rate of Change of Cost Flexibility

Differentiating the cost function thrice with respect to output gives the rate of change of cost flexibility

$$\frac{\partial^3 \ln C}{\partial(\ln y)^3} = \frac{\partial}{\partial \ln y}\left[\frac{\partial \tau}{\partial(\ln y)}\right] = \gamma_{yyy} \tag{3.3.13}$$

If, for example, $\dot{\gamma}_{yy}$ (w,y) > 0, and γ_{yyy} < 0, the cost flexibility will increase at a decreasing rate. Thus, we do not have to assume that the rate of change of cost flexibility is constant, even though it may well be the case, if γ_{yyy} = 0. This is a testable hypothesis within the model.

Output Sensitivity of Share Elasticities

The third-order form will generate the response of the share elasticities not only with respect to "w" but also with respect to "y". Differentiating the log of the cost function twice with respect to the log of input prices and once with respect to the log of output gives us the desired measure.

$$\frac{\partial^3 \ln C}{\partial \ln w_i \, \partial \ln w_j \, \partial \ln y} = \gamma_{ijy} \tag{3.3.14}$$

This measure, γ_{ijy}, will give the response of the share elasticity as output changes. Alternatively, γ_{ijy} can be defined as

$$\gamma_{ijy} = \frac{\partial \tau}{\partial \ln w_i \, \partial \ln w_j} \tag{3.3.15}$$

Equation (3.3.15) measures the rate of change of cost flexibility as input prices change. If $\gamma^{\bullet}_{iy}(w,y) > 0$, and $\gamma_{ijy} > 0$, the responsiveness of cost flexibility is said to increase at an increasing rate with respect to input prices. Furthermore, γ_{ijy} can be interpreted as the sensitivity of the scale bias, $\gamma^{\bullet}_{iy}(w,y)$ (expansion elasticity) as input prices change,

$$\gamma_{ijy} = \frac{\partial}{\partial \ln w_j}\left[\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y}\right] \tag{3.3.16}$$

where, $\dfrac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y} \equiv \gamma^{\bullet}_{iy}(w,y)$ (biases of scale)

$\dfrac{\partial}{\partial \ln w_j}(\gamma^{\bullet}_{iy}(w,y))$ measures the slope of biases of scale.

Depending on whether $\gamma_{ijy} \gtreqless 0$, the scale bias may increase, remain constant, or decrease, respectively, as input prices change.

Output Sensitivity of Biases of Scale

Similarly, the response of the biases of scale with respect to a further output change can be found. Differentiating the cost function twice with respect to y and once with respect to w, will generate the appropriate measure,

$$\frac{\partial^3 \ln C}{\partial \ln w_i \partial (\ln y)^2} = \frac{\partial}{\partial \ln y}\left[\frac{\partial^2 \ln C}{\partial \ln w_i \partial \ln y}\right] = \gamma_{iyy} \tag{3.3.17}$$

Equation (3.3.17) evaluates the biases of scale as output changes. The response of output sensitivity will be measured by the magnitude of γ_{iyy} in equation (3.3.17). The above additional economic relationships, equations (3.3.12-3.3.17) can be tested within the model instead of assuming them to be part of the maintained hypothesis.

Technical change

By suitably reinterpreting the constant term ($\ln \gamma_0$) as a function of time, the third-order translog cost function given by (3.1.1) can embody Hicks-neutral technical change. In this case, time would not enter the model interactively, which would prevent the examination of non-neutral technical change. To allow for such a possibility 'T' must be introduced to represent the state of technology in an interactive manner. As well, if constant returns to scale is imposed the number of parameters involved becomes manageable. Thus the underlying technology can be specified with the following third-order, unit-cost, translog approximation.

$$\ln(C/y) = \ln c = \ln \gamma_0 + \gamma_t \ln T + \sum_{i=1}^{n} \gamma_i \ln w_i + \sum_{i=1}^{n} \gamma_{it} \ln w_i \ln T$$

$$+ \frac{1}{2}\left[\sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln w_i \ln w_j + \sum_{i} \gamma_{itt} \ln w_i (\ln T)^2 + \sum_{i} \sum_{j} \gamma_{ijt} \ln w_i \ln w_j \ln T\right.$$

$$\left. + \gamma_{tt}(\ln T)^2 \right] + \frac{1}{6}\left[\sum_{i}^{n}\sum_{j}^{n}\sum_{h}^{n} \gamma_{ijh} \ln w_i \ln w_j \ln w_h + \gamma_{ttt}(\ln T)^3\right] \qquad (3.1.1)'$$

Linear homogeneity in input prices implies that $\Sigma\, \gamma_{it} = \gamma_{ijt} =$

$\gamma_{itt} = 0$, in addition to conditions given by (3.2.4). $\gamma^{\bullet}_{it}(w,T)$ is a measure of bias showing the effect of changing technology on cost shares. This measure is obtained by the logarithmic differentiation of (3.1.1)' first with respect to factor price and then the state of technology 'T'. This measure would be a constant if it were derived from a second-order cost function. It would not respond to further changes to either technology or to different levels of factor prices. In this case it takes the following form.

$$\partial^2 \ln c / \partial \ln w_i \, \partial \ln T = \gamma^{\bullet}_{it}(w,T) = \gamma_{it} + \gamma_{itt} \ln T + \gamma_{ijt} \ln w_j . \quad (3.1.1)''$$

Depending on $\gamma^{\bullet}_{it}(w,T) \lesseqgtr 0$ technical change is said to be input i-saving, i-neutral or i-using. Because of this variable measure of technological change, (compared to a constant one) the effects of technology on the cost shares can be more reliable, as this measure represents a higher order approximation. Further more, if we differentiate (3.1.1)" with respect to T we obtain the measure of the rate of change of technical change, γ_{itt}. We find not only that the technical change was input 'i' saving but also at what rate it has been input saving. This is a feature that could not have been derived from a second-order objective function. Differentiating (3.1.1)" with respect to input prices gives, γ_{ijt}, a measure of sensitivity of technical change as input prices change; again a feature found in an objective function higher than a second-order approximation. Finally, the measure of technological bias derived from a second-order function is given as a special case of (3.1.1)" when $\gamma_{ijt} = \gamma_{itt} = 0$. Having discussed technical change in terms of the third-order translog approximation, Hicks-neutrality will be assumed hereafter

Additional Advantages:

Additional advantages can be gained by considering higher order functions in that they provide better approximating formulae, such as Allen-Uzawa elasticities of substitution (AUES). AUES is a useful way measuring the percentage change in the ratio of the two inputs involved resulting from a one percent change in their relative prices. It is known that AUES depends on the derived relationships from the cost function, such as the share elasticities and shares of inputs. The better these derived relationships approximate the true relationships, the more precise will be the formulae that depend on them. In order to clarify this point, the AUES will be derived in terms of the derived relationships from the cost function in question

$$\sigma_{ij} = c^{\bullet} c^{\bullet}_{ij} / c^{\bullet}_{i} c^{\bullet}_{j}, \tag{3.3.18}$$

where,

$$c^{\bullet}_{i} = \frac{\partial c^{\bullet}}{\partial w_i} = \frac{c^{\bullet}}{w_i} (s^{\bullet}_{i}),$$

$$c^{\bullet}_{j} = \frac{\partial c^{\bullet}}{\partial w_j} = \frac{c^{\bullet}}{w_j} (s^{\bullet}_{j}),$$

$$c^{\bullet}_{ij} = \frac{\partial^2 c^{\bullet}}{\partial w_i \partial w_j} = \frac{c^{\bullet}}{w_i w_j} \left[\gamma_{ij} + \sum_{h=1}^{n} \gamma_{ijh} \ln w_h + \gamma_{ijy} \ln y + s^{\bullet}_{i} s^{\bullet}_{j}\right] \tag{3.3.19}$$

Substituting the set of equations (3.3.19) in equation (3.3.19), the formula for σ_{ij} can be written in terms of shares:

$$\sigma_{ij} = \left(\gamma_{ij} + \gamma_{ijy} \ln y + \sum_{h-1} \gamma_{ijh} \ln w_h + s_i^{\bullet} s_j^{\bullet}\right) / s_i^{\bullet} s_j^{\bullet} . \qquad (3.3.20)$$

Comparing (3.3.20) with (2.7.12), we find that the former has extra expressions in it. These extra expressions incorporate the sensitivity of the share elasticity with respect to both input prices and the level of output. However, the AUES formula derived from the second-order function ignores sensitivity effects. Moreover, the shares in equation (3.3.20) are different from the shares in equation (2.7.7) in that the former represent closer approximations to the true relationship. The formula (3.3.20) may help to reduce the criticism of bias towards the elasticity of substitution estimate in the second-order translog cost function. It may be very useful in policy matters to find a precise formula for this important economic relationship.

3.4 Functional Separability

A production function is said to be weakly separable if the marginal rate of substitution between pairs of factors in the group is independent of the levels of input outside of the group[2]. If such is the case, the function can be divided into subsets, and sequential optimization can be carried out without loss of any information. A

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[2] See M Denny and M. Fuss, 1977 .

sufficient condition for sequential optimization is the existence of weak homothetic separability. If the separable groups are homothetic in their arguments, the condition for weak separability and weak homothetic separability are the same. This permits decentralized decision making based on sequential optimization. The decentralized decision making becomes important if the production process involves a large number of inputs. The presence of separability further allows the use of aggregated data when disaggregated data is unavailable or is of poor quality. However, it would be improper to use aggregated data in the absence of separability, since the various interaction parameters will be ignored. Berndt and Christensen (1979b) defined separability in terms of the Allen-Uzawa partial elasticity of substitution (AUES). The AUES between factors in the group and a factor outside the group must be equal, *i.e.*

$$\sigma_{ik} = \sigma_{jk} \ , \ i,j \in I^s \ , \ k \notin I^s \qquad (3.4.1)$$

where I^s is the weakly separable partition of the input set.

In order to derive the conditions required for separability in terms of the cost function and its derivatives, a generalized unit cost function will be used[3]. The condition for weak separability can be expressed as[4]:

$$\frac{\partial}{\partial w_k}(c^{\bullet}_i/c^{\bullet}_j) = (c^{\bullet}_i c^{\bullet}_{jk} - c^{\bullet}_j c^{\bullet}_{ik})/c^{\bullet}_j = 0, \qquad (3.4.2)$$

---

3 See E. Berndt and D. Wood, 1977.

4 See Berndt and Christensen, 1973b.

where, $c^{\bullet}_i = c\, s^{\bullet}_i / w_i$, $c^{\bullet}_j = c\, s^{\bullet}_j / w_j$, $c^{\bullet}_{jk} = c(\gamma^{\bullet}_{jk} + s^{\bullet}_j\, s^{\bullet}_k)/w_j w_k$,

$c^{\bullet}_{ik} = c(\gamma^{\bullet}_{ik} + s^{\bullet}_i\, s^{\bullet}_k)/w_i w_k$, $\gamma^{\bullet}_{ik} = \dfrac{\partial^2 \ln c}{\partial \ln w_i\, \partial \ln w_k} = \gamma_{ik} + \sum_{m=1}^{n} \gamma_{ikm} \ln w_m$

for $i \neq k$ and $m = 1,2,3$ (3.4.3)

$$\gamma^{\bullet}_{jk} = \frac{\partial^2 \ln c}{\partial \ln w_j\, \partial \ln w_k} = \gamma_{jk} + \sum_{m=1}^{n} \gamma_{jkm} \ln w_m \qquad \text{for } j \neq k \text{ and } m = 1,2,3.$$

s_i, s_j, s_k indicate the shares of the respective input in the total cost.

Equation (3.4.2) can be rewritten in terms of shares, by substituting equation (3 4 3) into equation (3.4.2) and rearranging terms,

$$\left[s^{\bullet}_i\, \gamma_{jk} - s^{\bullet}_j\, \gamma_{ik}\right] + \sum_m \left[s^{\bullet}_i\, \gamma_{jkm} - s^{\bullet}_j\, \gamma_{ikm}\right] \ln w_m = 0. \qquad (3.4.4)$$

If we had a second-order translog function, the equality of both γ_{ik} and γ_{jk} to zero would be a sufficient condition for separability. However, with the third-order cost function, the expressions in the second parenthesis in equation (3.4.4) makes the condition for separability different from those required in the second-order case. Therefore, a sufficient condition for separability is not only that γ_{ik} and γ_{jk} be zero, but that γ_{ikm} and γ_{jkm} be also zero. If any of these parameters is different from zero, global separability will not be satisfied.

In such a situation, the restriction for local separability is derived by substituting (3.3.1) for s_i^* and the analogous expression for s_j^* in (3.4.4) to obtain.

$$(\gamma_{jk} + \sum_m \gamma_{jkm} \ln w_m)\,(\gamma_i + \sum_m \gamma_{im} \ln w_m + \frac{1}{2} \sum_m \sum_h \gamma_{imh} \ln w_m \ln w_h)$$

$$-(\gamma_{ik} + \sum_m \gamma_{ikm} \ln w_m)\,(\gamma_j + \sum_m \gamma_{jm} \ln w_m + \frac{1}{2} \sum_m \sum_h \gamma_{jmh} \ln w_m \ln w_h) = 0. \qquad (3.4.5)$$

Next, expanding (3.4.5) and rearranging the terms yields the following expression:

$$(\gamma_i \gamma_{jk} - \gamma_j \gamma_{ik}) + \sum (\gamma_{jk} \gamma_{im} - \gamma_{ik} \gamma_{jm}) \ln w_m$$

$$+ \sum_m^n \sum_h^n (\gamma_{jk} \gamma_{imh} - \gamma_{ik} \gamma_{jmh}) \ln w_m \ln w_h + \sum_m^n (\gamma_i \gamma_{jkm} - \gamma_j \gamma_{ikm}) \ln w_m$$

$$+ \sum_m^{n^2} (\gamma_{jkm} \gamma_{im} - \gamma_{ikm} \gamma_{jm}) \left[\ln w_m\right]^2 + \sum_h^n \sum_m^{n^2} (\gamma_{jkm} \gamma_{imh} - \gamma_{ikm} \gamma_{jmh}) \left[\ln w_m\right]^2 \ln w_h = 0 \qquad (3.4.6)$$

In order that equation (3.4.6) become zero, as required by the separability condition, it is sufficient that the expression in the above six parentheses be each equated to zero as shown below

$$\gamma_i \gamma_{jk} - \gamma_j \gamma_{ik} = 0 \qquad (I),$$

$$\gamma_{jk} \gamma_{im} - \gamma_{ik} \gamma_{jm} = 0 \qquad (II),$$

$$\gamma_{jk} \gamma_{imh} - \gamma_{ik} \gamma_{jmh} = 0 \qquad (III),$$

$$\gamma_i \gamma_{jkm} - \gamma_j \gamma_{ikm} = 0 \qquad (IV),$$

$$\gamma_{jkm} \gamma_{im} - \gamma_{ikm} \gamma_{jm} = 0 \qquad (V),$$

$$\gamma_{jkm}\gamma_{imh} - \gamma_{ikm}\gamma_{jmh} = 0 \qquad \text{(VI)},$$

where m, h = 1, 2, 3. In a three-input case, the possibilities are:

i = 1, j = 2, k = 3 $\longrightarrow$ 12-3 (1 and 2 separable from 3),

i = 2, j = 3, k = 1 $\longrightarrow$ 23-1 (2 and 3 separable from 1),

i = 1, j = 3, k = 2 $\longrightarrow$ 13-2 (1 and 3 separable from 2).

In general, if γ_j, γ_{jkh}, γ_{jkm}, γ_{jmh} γ_{jk}, γ_{jm} and γ_{jmh} are not zero, the separability conditions can be expressed in terms of ratios:

$$\frac{\gamma_i}{\gamma_j} = \frac{\gamma_{ik}}{\gamma_{jk}} = \frac{\gamma_{im}}{\gamma_{im}} = \frac{\gamma_{imh}}{\gamma_{jmh}} = \frac{\gamma_{ikm}}{\gamma_{jkm}} = = \frac{\gamma_{ikm}}{\gamma_{jkh}} \qquad (3.4.7)$$

From the above restrictions, only the first three equalities are independent. Given these equalities, the condition for separability may also be expressed in the usual manner:

$$\dot{s}_i\gamma_{jk} - \dot{s}_j\gamma_{ik} = 0 \qquad (3.4.4)'$$

The analogous expression to (3.4.4)' for the case of second-order functions would have produced only the first two sets of restrictions in (3.4.7), while in the third-order case, it will produce the first three sets.

To clarify this point, a three input price-symmetric translog cost function for CRS technology is considered. From this function, the following share equations (with cross-equation symmetry imposed) are shown below:

$$s_1^* = \gamma_1 + \gamma_{11}\ln w_1 + \gamma_{12}\ln w_2 + \gamma_{13}\ln w_3$$

$$+ \frac{1}{2}\left[\gamma_{111}(\ln w_1)^2 + \gamma_{122}(\ln w_2)^2 + \gamma_{133}(\ln w_3)^2\right]$$

$$+ \gamma_{112}\ln w_1 \ln w_2 + \gamma_{113}\ln w_1 \ln w_3 + \gamma_{123}\ln w_2 \ln w_3$$

$$s_2^* = \gamma_2 + \gamma_{12}\ln w_1 + \gamma_{22}\ln w_2 + \gamma_{23}\ln w_3$$

$$+ \frac{1}{2}\left[\gamma_{112}(\ln w_1)^2 + \gamma_{222}(\ln w_2)^2 + \gamma_{233}(\ln w_3)^2\right]$$

$$+ \gamma_{122}\ln w_1 \ln w_2 + \gamma_{123}\ln w_1 \ln w_3 + \gamma_{223}\ln w_2 \ln w_3$$

$$s_3^* = \gamma_3 + \gamma_{13}\ln w_1 + \gamma_{23}\ln w_2 + \gamma_{33}\ln w_3$$

$$+ \frac{1}{2}\left[\gamma_{113}(\ln w_1)^2 + \gamma_{223}(\ln w_2)^2 + \gamma_{333}(\ln w_3)^2\right]$$

$$+ \gamma_{123}\ln w_1 \ln w_2 + \gamma_{133}\ln w_1 \ln w_3 + \gamma_{233}\ln w_2 \ln w_3, \qquad (3\ 4.8)$$

The separability restrictions of the type (12-3) for instance, can be expressed in terms of ratios by using s_1 and s_2 as in (3 4 4)'·

$$\frac{\gamma_1}{\gamma_2} = \frac{\gamma_{13}}{\gamma_{23}} = \frac{\gamma_{11}}{\gamma_{12}} = \frac{\gamma_{12}}{\gamma_{22}} = \frac{\gamma_{111}}{\gamma_{112}} = \frac{\gamma_{112}}{\gamma_{122}} = \frac{\gamma_{122}}{\gamma_{222}} = \frac{\gamma_{113}}{\gamma_{123}} = \frac{\gamma_{123}}{\gamma_{223}} = \frac{\gamma_{133}}{\gamma_{233}} \qquad (3.4\ 9)$$

Similar separability condition can be derived for the varia''les (23-1, 13-2).

If one is interested in finding the exact test for separability of the Berndt and Christensen type, equation (3.4.4)′ must hold for every input price.[5] In order to derive both linear and non-linear exact separability restrictions, the following specific formats of (3.4.4)′ are used; each representing a specific separability type:

$$
\begin{aligned}
&\dot{s}_1\dot{\gamma}_{23} - \dot{s}_2\dot{\gamma}_{13} = 0 \rightarrow (\sigma_{13} = \sigma_{23}), \quad (12\text{-}3)\\
&\dot{s}_1\dot{\gamma}_{23} - \dot{s}_3\dot{\gamma}_{12} = 0 \rightarrow (\sigma_{12} = \sigma_{23}), \quad (13\text{-}2)\\
&\dot{s}_2\dot{\gamma}_{13} - \dot{s}_3\dot{\gamma}_{12} = 0 \rightarrow (\sigma_{12} = \sigma_{13}), \quad (23\text{-}1)
\end{aligned}
\qquad (3.4.10)
$$

The third separability type can be derived from the other two (since $\sigma_{13} = \sigma_{23} = \sigma_{12}$), therefore, only two are independent. In (3.4.10), it is also evident that each separability type imposes equality restrictions between σ_{ik} and σ_{jk}, as was required by the definitions of separability in terms of AUE:

Case (1)

The linear restrictions required in order for separability to hold are derived from (3.4.10)

$$
\begin{aligned}
&\gamma_{13} = 0, \ \gamma_{131} = 0, \ \gamma_{132} = 0, \ \gamma_{133} = 0, \\
&\qquad\qquad\qquad\qquad\qquad\qquad (12\text{-}3)\\
&\gamma_{23} = 0, \ \gamma_{231} = 0, \ \gamma_{232} = 0, \ \gamma_{233} = 0,
\end{aligned}
$$

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5 See Berndt and Christensen 1973a, b.

$$\gamma_{12} = 0, \ \gamma_{121} = 0, \ \gamma_{122} = 0, \ \gamma_{123} = 0,$$

$$(13.2)$$

$$\gamma_{23} = 0, \ \gamma_{231} = 0, \ \gamma_{232} = 0, \ \gamma_{233} = 0,$$

$$\gamma_{12} = 0, \ \gamma_{121} = 0, \ \gamma_{122} = 0, \ \gamma_{123} = 0,$$

$$(23.1)$$

$$\gamma_{13} = 0, \ \gamma_{131} = 0, \ \gamma_{132} = 0, \ \gamma_{133} = 0. \qquad (3.4.11)$$

The linear restrictions corresponding to the second-order case are limited only to the first column of equations (3.4.11). However, in the case of the third-order function the following complete linear restrictions are obtained from (3.4.11)

$$\gamma_{12} = \gamma_{13} = \gamma_{23} = \gamma_{112} = \gamma_{113} = \gamma_{122} = \gamma_{123} = \gamma_{223} = \gamma_{133} = 0 \qquad (3.4.12)$$

If complete linear separability were imposed on the translog unit cost function, the function reduces to Cobb-Douglas. This happens since complete separability in all inputs would imply that all interaction parameters be zero. Thus, all partial AUES must all be equal to unity, ($\sigma_{13} = \sigma_{23} = \sigma_{12} = 1$). In order to test for various types of separability, one should start from the most restrictive case. If this hypothesis is not accepted, one should test to see whether or not any of the three sub-cases in equation (3.4.11) are accepted. If none of these situations are accepted, then the non-linear restrictions must be tested for weak separability

Case (2)

Separability of inputs 1 and 2 from 3, given the violation of condition (3.4.12), can be derived from equations (3.4.9). From the second and third equalities, Berndt and Christensen[6] derived one constraint of the form

$$\gamma_{ii} = (\gamma_{ij})^2/\gamma_{jj} \qquad (3.4.13)$$

Extending this approach to the third-order introduces two more independent non-linear restrictions of the form can be derived:

$$\gamma_{iih} = (\gamma_{ijh})^2/\gamma_{jjh}, \qquad \text{for } i \neq j, \text{ and } h = 2,3, \qquad (3.4.14)$$

if share equations 2 and 3 are to be estimated by deleting share equation 1.

The formula given by (3.4.14) collapses to that of the Berndt and Christensen type (3.4.13) corresponding to the second-order form, that is, for h = 0, since the third-order parameters vanish. The restrictions given in (3.4.13) and in (3.4.14) do not depend on specific separability type. Regardless of specific situations, we will get one restriction from (3.4.13) and two restrictions from (3.4.14). The additional non-linear restrictions found in the case of third-order cost function should enable us to test functional separability more rigorously than would be the case if the function was only of the second-order. Next, from the first set of restrictions in (3.4.9), three more restrictions for the intercept term are found. These restrictions do depend on the specific separability type. They are derived by using the linear homogeneity in prices assumption (see Berndt

6 See Berndt and Christensen, 1973a.

and Christensen, 1971 for derivation)

The complete set of linear and non-linear restrictions for separability are summarized in terms of free parameters (taken from the equation chosen for estimation, that is s_2 and s_3). See Table 3.4.1 below

Table (3.4.1)

Separability Type	Linear Separability Conditions	Non-Linear Separability Conditions
12 3	$\gamma_{13} = 0,\ \gamma_{13h} = 0$	$\gamma_{3} = 1 + (\gamma_{2}\gamma_{23}/\gamma_{22})$
	$\gamma_{23} = 0,\ \gamma_{23h} = 0$	$\gamma_{33} = \gamma_{23}^{2}/\gamma_{22}$
		$\gamma_{332} = \gamma_{232}^{2}/\gamma_{222}$
		$\gamma_{333} = \gamma_{223}^{3}/\gamma_{222}^{2}$
	$\sigma_{13} = \sigma_{23} = 1$	$\sigma_{13} = \sigma_{23} \neq 1$
13 2	$\gamma_{12} = 0,\ \gamma_{12h} = 0$	$\gamma_{3} = (\gamma_{2} - 1)\ \gamma_{23}/\gamma_{22}$
	$\gamma_{23} = 0,\ \gamma_{23h} = 0$	$\gamma_{33} = \gamma_{23}^{2}/\gamma_{22}$
		$\gamma_{332} = \gamma_{232}^{2}/\gamma_{222}$
		$\gamma_{333} = \gamma_{223}^{3}/\gamma_{222}^{2}$
	$\sigma_{12} = \sigma_{13} = 1$	$\sigma_{12} = \sigma_{23} \neq 1$
23 1	$\gamma_{12} = 0,\ \gamma_{12h} = 0$	$\gamma_{3} = \gamma_{2}\ \gamma_{23}/\gamma_{22}$
	$\gamma_{13} = 0,\ \gamma_{13h} = 0$	$\gamma_{33} = \gamma_{23}^{2}/\gamma_{22}$
		$\gamma_{332} = \gamma_{232}^{2}/\gamma_{222}$
		$\gamma_{333} = \gamma_{223}^{3}/\gamma_{222}^{2}$
	$\sigma_{12} = \sigma_{13} = 1$	$\sigma_{12} = \sigma_{13} \neq 1$

where h = 2, 3 represent the shares that are to be estimated.

If one is interested in finding the approximate separability restrictions (Denny and Fuss, 1977), the shares in equation (3.4.4) must be evaluated at the point of approximation, $w_i = 1$. This reduces the shares to their respective intercepts

$$s^*_i = \gamma_i, \quad s^*_j = \gamma_j \tag{3.4.17}$$

Thus, the condition for weak separability will be given as follows.

$$\left(\gamma_i \gamma^*_{jk} - \gamma_j \gamma^*_{ik}\right) = 0 \tag{3.4.18}$$

where $\gamma^*_{ik} = \gamma_{ik} + \sum_m \gamma_{ikm}$, $\gamma^*_{jk} = \gamma_{jk} + \sum_m \gamma_{jkm}$ and $m = 1, \ldots, n$

or

$$\gamma_i \gamma_{jk} - \gamma_j \gamma_{ik} = 0 \text{ and } \gamma_i \gamma_{jkm} - \gamma_j \gamma_{ikm} = 0 \tag{3.4.19}$$

In order for approximate strong separability to hold the following conditions must be met:

$$\gamma_{ik} = \gamma_{jk} = \gamma_{ikm} = \gamma_{jkm} = 0, \qquad m = 1, \ldots, n \tag{3.4.20}$$

In both the weak and strong separability cases, the third order function introduces additional restrictions represented by the third order parameters, γ_{ikm}, and γ_{jkm}. In (3.4.19) we only need the equality between the ratios, $\gamma_i/\gamma_j = \gamma_{ik}/\gamma_{jk} = \gamma_{ikm}/\gamma_{jkm}$ for weak separability to hold. On the other hand, for strong separability, all the interaction parameters must be zero.

3.5 Concluding Remarks

In Section (3.2), we have shown that the third-order cost function has a locally well behaved region just like the second-order cost function. However, since the third-order translog cost function has more restrictions that must be satisfied than the second-order one, it allows for more rigorous testing of the regularity conditions.

In Section (3.3), we have shown that the economic relationships derived from the third-order cost function reduce truncation bias (but fewer degrees of freedom, higher standard error), hence, the estimates are more reliable if the exteded model is the true model. In addition to this, the relationships derived were shown to be more flexible and incorporated the properties of those derived from the second-order cost function as special cases. This enabled us to examine the response of some of those derived functions to further changes in the variables involved. Furthermore, the flexibility and superior estimates of the economic relationships resulting from the third-order cost function should give better approximating formulae, and this may have important consequences for public policy. Allen Uzawa's elasticity of substitution is one example. Since this formula is based on some of the estimated relationships, it is important that the estimates be as accurate as possible. Moreover, the third-order translog function enabled us to derive additional economic relationships that could not have been obtained from the second-order cost functions. Thus, the third-order forms added more flexibility to the already flexible functional forms.

In Section (3.4), we have shown the test for functional separability is richer in the case of the third-order cost function. In this section two additional separability restrictions were found, due to higher order. It is also evident from this section that the function still maintains more flexibility after imposing partial separability restrictions, unlike the second-order function.

Appendix to Chapter 3: Regularity Conditions at the Point of Approximation - Third-Order Translog.

To demonstrate the regularity conditions clearly in terms of the third-order translog cost function, the following assumptions are made.

a) The cost function is linearly homogeneous in input prices (see condition (3.2 3))

b) It exhibits constant returns to scale in production This eliminates all the output variables and the interaction terms between output and input prices in (3 1 1) Thus, the translog approximation can be expressed in terms of its unit cost without any loss of generality

c) It is assumed that there are only two inputs in the production function This reduces the number of parameters in the system The regularity conditions can thus be expressed in a somewhat simple fashion

d) The cost function, and hence, the unit cost function, are symmetric in input prices The symmetry restrictions are

$$\gamma_{12} = \gamma_{21}, \ \gamma_{112} = \gamma_{121} = \gamma_{211}, \ \gamma_{122} = \gamma_{212} = \gamma_{221} \qquad (3\ A.1)$$

where the subscripts 1 and 2 indicate the first and second input prices respectively. Imposing the above four conditions on (3.1.1) yields a third-order translog unit cost function:

$$\ln c\,(w_1, w_2) = \ln\gamma_0 + \gamma_1 \ln w_1 + \gamma_1 \ln w_2 + \frac{1}{2}\left[\gamma_{11}(\ln w_1)^2 + \gamma_{22}(\ln w_2)^2\right]$$

$$\gamma_{12}\ln w_1 \ln w_2 + \frac{1}{2}\left[\gamma_{112}(\ln w_1)^2 \ln w_2 + \gamma_{122}(\ln w_2)^2 \ln w_1\right]$$

$$+ \frac{1}{6}\left[\gamma_{111}(\ln w_1)^3 + \gamma_{222}(\ln w_2)^3\right] \tag{3.A.2}$$

The share equations corresponding to (3.A.2) can be derived by differentiating it with respect to input prices logarithmically :

$$\frac{\partial \ln c}{\partial \ln w_i} \equiv s_i(w_i) = \gamma_i + \sum_{i=1}^{2} \gamma_{ij} \ln w_i + \frac{1}{2}\sum_{j=1}^{2}\sum_{h=1}^{2} \gamma_{ijh} \ln w_j \ln w_h$$

where $i,j,h = 1,2$ (3.A.3)

Written out explicitly, the share equations are

$$s_1 = \gamma_1 + \gamma_{11}\ln w_1 + \gamma_{12}\ln w_2 + \gamma_{112}\ln w_1 \ln w_2$$

$$+ \frac{1}{2}\left[\gamma_{111}(\ln w_1)^2 + \gamma_{122}(\ln w_2)^2\right] \tag{3.A.4}$$

$$s_2 = \gamma_2 + \gamma_{12}\ln w_1 + \gamma_{22}\ln w_2 + \gamma_{122}\ln w_1 \ln w_2$$

$$+ \frac{1}{2}\left[\gamma_{112}(\ln w_1)^2 + \gamma_{222}(\ln w_2)^2\right] \tag{3.A.5}$$

The above equations assume that the following cross equation symmetry holds:

$$\gamma_{12}^{(1)} = \gamma_{12}^{(2)},\ \gamma_{112}^{(1)} = \gamma_{112}^{(2)},\ \gamma_{122}^{(1)} = \gamma_{122}^{(2)}\ . \tag{3.A.6}$$

Superscripts indicate the share equation from which the parameter is taken

In order for the adding up condition to hold, the following must be true:

$$\gamma_1 + \gamma_2 = 1 \Rightarrow \qquad \gamma_2 = (1-\gamma_1);$$

$$\gamma_{11} + \gamma_{12}^{(2)} = 0 \Rightarrow \gamma_{11} = -\gamma_{12}^{(2)} \text{ and } \qquad \gamma_{11} = -\gamma_{12}^{(1)}, \text{ by symmetry;}$$

$$\gamma_{12}^{(1)} + \gamma_{22}^{(2)} = 0 \Rightarrow \gamma_{12} = -\gamma_{22}^{(2)} \text{ and } \qquad \gamma_{11} = -\gamma_{12}^{(1)};$$

therefore, $\gamma_{11} = \gamma_{22}$,

$$\gamma_{111} + \gamma_{112}^{(2)} = 0 \Rightarrow \gamma_{111} = -\gamma_{112}^{(2)} \text{ and } \qquad \gamma_{111} = -\gamma_{112}^{(1)} \text{ by symmetry;}$$

$$\gamma_{112}^{(1)} + \gamma_{122}^{(2)} = 0 \Rightarrow \gamma_{112}^{(1)} = -\gamma_{122}^{(2)} \text{ but } \gamma_{112}^{(1)} = -\gamma_{111};$$

therefore, $\gamma_{111} = \gamma_{122}^{(2)}$,

$$\gamma_{122}^{(1)} + \gamma_{222} \quad 0 \Rightarrow \gamma_{122}^{(1)} = -\gamma_{122} \text{ but } \gamma_{122}^{(1)} = \gamma_{111};$$

therefore, $\gamma_{222} = -\gamma_{111}$ (3 A.7)

Finally, by using the adding up and symmetry conditions, the unit cost function can be reduced to the following:

$$\ln c(w_1, w_2) = \ln\gamma_0 + \gamma_1 \ln w_1 + (1-\gamma_1)\ln w_1 + \frac{1}{2}\left[\gamma_{11}(\ln w_1)^2 + \gamma_{11}(\ln w_2)^2\right]$$

$$- \gamma_{11}\ln w_1 \ln w_2 + \frac{1}{2}\left[\gamma_{111}(\ln w_2)^2 \ln w_1 - \gamma_{111}(\ln w_1)^2 \ln w_2\right]$$

$$+ \frac{1}{6}\left[\gamma_{111}(\ln w_1)^3 - \gamma_{111}(\ln w_2)^3\right] \qquad (3.A.8)$$

Equation (3.A.8) can be used to investigate the local and global consistencies of the third-order translog cost function without loss of generality

Local Consistency

In order to derive the restrictions required for local theoretical consistency, the unit cost function (3.A.8) will be examined in the neighborhood of some input prices as opposed to all prices. For this purpose, we will assume that all prices are equal to unity

$$w_1 = w_2 = 1 \qquad (3.A.9)$$

The conditions that must be met for local theoretical consistency are:

$$c(1,1) = e^{\gamma_o} \geq 0 \qquad (3.A.10)$$

$$\nabla c(1,1) = (\partial c/\partial w) = \left(\frac{\partial \ln c}{\partial \ln w}\frac{c}{w}\right) = \begin{bmatrix} \gamma_1 \\ (1-\gamma_1) \end{bmatrix} \geq 0 \qquad (3.A.11)$$

$$\nabla^2 c(1,1) = \partial^2 c/(\partial w)^2 = \left(\frac{\partial^2 \ln c}{\partial(\ln w)^2}\frac{c}{w^2}\right)$$

$$= e^{\gamma_o}\begin{bmatrix} \gamma_1(\gamma_1-1)+\gamma_{11}\gamma_1(1-\gamma_1) & -\gamma_{11} \\ \gamma_1(1-\gamma_1)-\gamma_{11} & (\gamma_1-1)\gamma_1+\gamma_{11} \end{bmatrix} \leq 0 \qquad (3.A.12),$$

i.e.(3.A.12) must be negative semi-definite; where ∇ indicates the gradient of cost with respect to input prices.

By comparing equations (3 A.10) to (3.A.12) with equations (2.A.2) to (2 A.4) derived from the second-order cost functions, we find that they are similar in structure. Therefore, as in the second-order case, the third-order cost function satisfies the local regularity conditions

Global Consistency

In order to derive the restrictions for global consistency, the third-order cost function is examined in the neighborhood of all sets of input prices.

$$c(w_1,w_2) = \exp\Big[\ln\gamma_o + \gamma_1\ln w_1 + (1-\gamma_1)\ln w_2 + \frac{1}{2}\gamma_{11}(\ln w_1)^2 - \gamma_{11}\ln w_1\ln w_2 + \frac{1}{2}\gamma_{11}(\ln w_2)^2 + \frac{1}{6}\gamma_{111}(\ln w_1)^3 - \frac{1}{2}\gamma_{111}(\ln w_1)^2\ln w_2 + \frac{1}{2}\gamma_{111}(\ln w_2)^2\ln w_1 - \frac{1}{6}\gamma_{111}(\ln w_2)^3\Big] \geq 0 \tag{3.A.13}$$

$$\nabla c(w_1,w_2)' = c.\begin{bmatrix} \Big[\Big\{\gamma_1 + \gamma_{11}\ln w_1 - \gamma_{11}\ln w_2 + \frac{1}{2}\gamma_{111}(\ln w_1)^2 - \gamma_{111}\ln w_1\ln w_2 + \frac{1}{2}\gamma_{111}(\ln w_2)^2\Big\}/w_1\Big] * \\ \\ \Big[\Big\{(1-\gamma_1) - \gamma_{11}\ln w_1 + \gamma_{11}\ln w_2 - \frac{1}{2}\gamma_{111}(\ln w_1)^2 + \gamma_{111}\ln w_1\ln w_2 - \frac{1}{2}\gamma_{111}(\ln w_2)^2\Big\}/w_2\Big] \end{bmatrix} \geq 0 \tag{3.A.14}$$

$$\frac{\partial^2 c}{\partial w_1^2} = c/w_1^2 \left\{ \left[\gamma_1 + \gamma_{11} \ln w_1 - \gamma_{11} \ln w_2 + \frac{1}{2} \gamma_{111} (\ln w_1)^2 - \gamma_{111} \ln w_1 \ln w_2 \right. \right.$$

$$\left. + \frac{1}{2} \gamma_{111} (\ln w_2)^2 \right] \left[(\gamma_1 - 1) + \gamma_{11} \ln w_1 - \gamma_{11} \ln w_2 \right.$$

$$\left. + \frac{1}{2} \gamma_{11} (\ln w_1)^2 - \gamma_{111} \ln w_1 \ln w_2 + \gamma_{111} (\ln w_2)^2 \right]$$

$$\left. + \gamma_{11} + \gamma_{111} \ln w_1 - \gamma_{111} \ln w_2 \right\} \leq 0$$

$$\forall\ w_1,\ w_2 > 0 \qquad (3.A.15)$$

The above three sets of conditions involve all of the input prices. However, equation (3.A.13) remains a real valued function since it is an exponential function In order for monotonicity to hold, both γ_{11} and γ_{111} need to be zero in (3.A.14) As long as these parameters are non-zero, we can find values for w_1 and w_2 that could violate this condition. Hence, global monotonicity requires the following restrictions:

$$1 \geq \gamma_1 \geq 0;\ \gamma_{11} = \gamma_{111} = 0 \qquad (3.A.16)$$

Equation (3.A.16) implies that if a third-order translog cost function is required to fulfill global monotonicity, it will be reduced to a Cobb-Douglas function, since all interaction parameters will vanish by the monotonicity restriction. Although the monotonicity requirement causes inflexibility, as was the case in second-order function, the restrictions are different (compare equation (3.A 16) with equation (2.A.6).

Finally, since the cost function is assumed to be linearly homogeneous in input prices, the condition given in equation (3.A.15) is both necessary and sufficient for global concavity. The restrictions in this case would be the same as equation (3.A.16) imposed on equation (3.A.15) and will give us:

$$c/w_1^2 \left[\gamma_1 \ (\gamma_1 - 1) \right] \leq 0 \qquad (3.A.17)$$

which will always be satisfied given equation (3.A 13) and γ_1 is between zero and one.

CHAPTER 4

ESTIMATION OF THIRD ORDER TRANSLOG COST FUNCTION AND HYPOTHESIS TESTING

In the previous chapter, the theoretical background of the third-order translog cost function was developed. The special features of this model were outlined and discussed. In this chapter, a specific data set, involving capital, labor, energy, and intermediate materials (KLEM) for the period 1947-1971 for the U.S. manufacturing sector will be used to estimate the model discussed in the previous chapter. Various hypotheses will be tested and the results compared with those derived with the traditional second-order translog cost function.

It was found that, due to the higher order cost function, the additional parameter estimates were significantly different from zero; the estimated share elasticities were not constant; and the Allen partial cross elasticity of substitution changed signs during the period. These results suggest that the higher order cost function provides a bet er specification of the underlying technology than the more traditional approach.

In Section 4 1 the model will be discussed. In Section 4.2 the estimation procedure used will be compared with alternative methods. In Section 4.3 the data set that will be used in this study will be discussed and empirical results presented. In Section 4.4 a test for model specification will be carried out. In section 4.5 the estimated share elasticities will be examined and hypothesis testing will be performed. In Section 4.6 the stability of share elasticities with respect to input prices will be examined. In Section 4.7 the estimated Allen partial elasticities of substitution and price elasticities will be analyzed. The issue of capital-energy complementarity will be examined in light of these new results and the new formula for the Allen partial elasticity of substitution derived from the third-order cost function . In section 4.8 the bias in the estimated factor demands from the second and third-order translog cost function will be compared by using the information inaccuracy criterion developed by Thiel (1967). In section 4.9 some policy implications of our findings will be discussed. Finally, section 4.10 will include a summary and conclusions of the chapter.

4.1 The Theoretical Model

Assumptions

The extended theoretical model will be developed under the following assumptions:

(a) The U.S. manufacturing sector could be represented by a thrice differentiable aggregate production function, or a thrice differentiable cost function.

(b) The production function relates the aggregate output, y, with the four inputs previously mentioned, KLEM.

(c) This production function exhibits constant returns to scale and is Hicks neutral with respect to technological change.

(d) The firms are price-takers in the input market.

(e) The cost function can be approximated by a translog function[1].

The Model

The four input KLEM model can be written as follows:

$$\ln C(w,y) = \ln \alpha_0 + \sum_i^n \gamma_i \ln w_i + \frac{1}{2} \sum_i^n \sum_j^n \gamma_{ij} \ln w_i \ln w_j$$

$$+ \frac{1}{6} \sum_i^n \sum_j^n \sum_h^n \gamma_{ijh} \ln w_i \ln w_j \ln w_h + \ln y \qquad (4.1.1)$$

where, C is the total cost, y is aggregate output, w is vector of input prices, α_0, γ_i, γ_{ij}, γ_{ijh} are unknown parameters to be estimated, and i,j,h = K, L, E, M. Equation (4.1.1) is a third-order Taylor series approximation of a cost function. This will also represent the underlying production structure if the assumptions discussed in chapter 3 section (3.2) are satisfied[2].

The cost minimizing share equations for KLEM can be derived as the first-order logarithmic derivative of 4.1.1.

$$\frac{\partial \ln C}{\partial \ln w_i} = \frac{\partial C}{\partial w_i}\left(\frac{C}{w_i}\right)^{-1} = S_i = \gamma_i + \sum_j^n \gamma_{ij} \ln w_j + \frac{1}{2} \sum_j^n \sum_h^n \gamma_{ijh} \ln w_j \ln w_h \quad (4.1.2)$$

where i, j, h = K, L, E, M. As was mentioned earlier, equation (4.1.2) collapses to the formula derived from the second-order translog cost function when $\gamma_{ijh} = 0 \ \forall \ i,j,h$.

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[1] This assumption ensures comparability with recent studies, for example Berndt and Wood (1975).

[2] Recall that the assumptions were symmetry in input prices, positivity of the cost function and the cost shares of all inputs involved with respect to input prices, and the concavity of the cost function with respect to input prices.

4.2 Estimation Procedures

First, we assume that the cost shares differ from the logarithmic derivatives of (4.1.1) by a random error term Therefore, an additive disturbance term is added to each equation. Since the shares add to unity, the disturbance terms in the four equations sum to zero for every observation. This will present a problem if one intends to estimate the whole system (4.1.2), since the variance-covariance matrix of the disturbances based on the four cost share equations will be singular.

To overcome this problem, one of the cost share equations is arbitrarily dropped and the remaining n-1 share equations are estimated, subject to the linear homogeneity of the production function, as well as, the general conditions discussed earlier (Chapter 3, section 2) The estimates of the parameters of the deleted equation can be recovered from the other equations[3]. However, invariance problems may arise depending upon the estimation procedure adopted. Invariance arises when the parameter estimates depend on the particular equation deleted This creates a problem since the results depend upon the equation which is dropped. The second problem related to estimating share equations is the existence of common parameters across equations. This renders a single equation estimation technique ineffective, since one cannot impose cross equation symmetry restriction when this method is used Lastly, when n-1 share equations are estimated it is likely that the disturbances are contemporaneously correlated.

To alleviate the above problems, iterative systems estimation

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3 See Berndt and Wood (1975).

procedures, Iterative Efficient Zellner (IEZ), Iterative Three Stage Least Squares (I3SLS), and Full Information Maximum Likelihood (FIML), can be used. These systems methods take the contemporaneous correlation of the disturbances into account. Symmetry restrictions can also be imposed, since equations are simultaneously estimated. As well, given convergence, the parameter estimates will not be sensitive to the particular equation deleted. Although, the systems methods solve the above mentioned problems, they do not perform equally well with regard to the endogeneity and efficiency problems[4].

The I3SLS method would be preferred if the prices that appear on the right hand side of the share equations to be estimated are believed to be endogenous. However, this method has its own drawbacks. Applebaum (1979) indicated that I3SLS estimates could be sensitive to the instruments chosen. In view of the fact that we would like to compare the performance of second and third-order translog cost functions that use a different number of instruments, it would be inappropriate to use a system that is sensitive to the choice of instruments The use of additional instruments in the third-order case is necessitated by the additional right hand variables required. The additional instruments would be needed especially if one follows the procedure used by Berndt and Wood (1975). In that paper, the right hand variables (price ratios) were regressed against the ten instruments chosen, and the fitted values from these regressions were taken as

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[4] Endogeneity problem arise when the right hand variables are determined within the system, while efficiency problem refer to the magnitude of the variance.

instruments. Since our model involves more variables on the right hand side than the second-order cost function, it would require more instruments. Thus, any differences in the estimated results might be attributed to the differences in the number of instruments used, and not necessarily to differences in the two models.

In order to make sure that the two models are invariant to the estimation method, we have chosen the IEZ method of estimation. This method of estimation has been used in several past studies for the U S manufacturing sector, and no significant differences in the values of estimated parameters between this technique and I3SLS were found (Berndt and Wood, 1979). Our results confirm this conclusion

We have also used the FIML (results not reported) method as an alternative estimation technique. This procedure and the IEZ produced identical parameter estimates and log likelihood functions These results confirmed that when convergence is achieved, (i e when the diagonal elements of the co-variance matrix of the residuals equal the sample size) the IEZ estimates converge to the FIML estimates, (Kmenta and Gilbert, 1968). We found the IEZ method to be more economical and efficient in spite of the fact that the results were identical for the purpose of hypothesis testing.

4.3 The Data and Empirical Results

The Data

This study uses the KLEM data on U.S. manufacturing, 1947-1971, where, K, L, E, and M represent capital, labor, energy and intermediate materials respectively. This study uses data on the quantities and prices of the four inputs, and aggregate output which were compiled by

Berndt and Christensen (1975)

Regularity Conditions

The cost function is well behaved if the derived input demands are strictly positive at every data point, $\frac{\partial c}{\partial w_i} > 0$ and if the cost function is concave in input prices. The share equations were found to be positive at every data point and thus the first condition was fulfilled. The Hessian matrix was examined for negative semi-definiteness in order to ensure the concavity condition. This condition was met at every data point except for 1947 and 1948[5]

Comments on parameter estimates

In Table 4 1a we present the IEZ parameter estimates of the KLEM third-order cost function for U.S. manufacturing, 1947-1971, with the restrictions discussed above. In order 'o test for the existence of serial dependence of the disturbance terms in the estimated system, the Durbin-Watson (DW) statistics are reported[6].

The DW statistics are 2.0795 for the L equation, 1.7325 (E), 1.8650 (K), and 1 9675 (M). These values are generally higher than the ones calculated from the second-order translog cost function. The corresponding values are 2.1516 (L), 1.1907 (E), 1.3087 (K), and 1.8517

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[5] The intermediate material and the labor equations failed this test in 1947 and 1948, respectively. Although not all of the share equations fail the concavity test in those two given years, we did not attach any meaning to the results obtained in this particular region.

[6] Durbin, 1957, has suggested that the DW test appropriate to single equation models, may be applied to each equation in simultaneous equation systems

(M). The degrees of freedom for the estimated system is 3N-k, where N is the sample size in each equation and k is the number of parameters in the estimated system Since we have estimated three equations the effective sample size is 75 The corresponding degrees of freedom are 56. The critical value for the two tailed test for α = 025 is 2 01 The estimates and the t statistics for the parameters of the dropped equation were calculated using the information provided in the estimated system

Almost 70/ of the new parameters introduced in the third-order translog cost function were significant By limiting the cost function to the second-order, some explanatory power of these parameters was lost. The fact that several newly introduced sensitivity parameters were different from zero implies that we will have variable share elasticities rather than constant ones The rate of change of the cost shares can no longer be assumed constant, since the sensitivity parameters γ_{ijh} measure these effects

The γ_{ij} were used to represent share elasticities in the second-order cost functions In the present context can no longer be so interpreted, since the second partial derivatives of the third-order translog cost function with respect to input prices are $\gamma^{\bullet}_{ij}$ (defined in 4.5.1) In particular, the γ_{ij}'s, the sensitivity parameters γ_{ijh}'s and the level of all input prices will be factors in determining the magnitudes and signs of variable share elasticities

4.4 Test for Model Specification

The purpose of this section is to determine whether or not the second-order translog cost function was misspecified. Since we are using the IEZ estimator, the likelihood ratio test was used. The likelihood ratio test is computed as follows: $LR = 2(L_1 - L_0)$, where L_1 is the value of the likelihood function for the unconstrained model (third-order translog cost function) and L_0 is the value when the constraints are imposed (the second-order translog cost function). The LR is distributed asymptotically as a chi-squared variable with degrees of freedom equal to the number of restrictions.

After estimating both models, the following values were obtained: $L_1 = 356.460$, $L_0 = 344.567$, with the degree of freedom being 10, (*i.e.* the number of parameters estimated in the unconstrained system less the number of parameters estimated in the constrained system). The null hypothesis that was tested is $\gamma_{ijh} = 0 \ \forall i, j, h$. The calculated LR = 23.79 while the critical value is $\chi^2_{.05} = 18.307$ (df = 10). Since the calculated statistic is well above the critical value, the null hypothesis is rejected. It is also rejected at $\alpha = .01$. This implies that the U.S. manufacturing sector should not be represented by the second-order translog cost function. This result also allows us to claim that the estimates of several economic relationships from our model are superior to the ones obtained from the second-order function.

In the sections to follow, we will compare the results obtained by using the second and third-order translog cost functions. The elasticities calculated from the second-order function are reported for selected years, since the values are stable over the study period. The results obtained from the third-order function are reported for the entire period since the values change from sub-period to sub-period.

4.5 Share Elasticities

The share elasticities derived from the second-order translog cost function are constant, since they are the second partial derivatives. As was argued in the previous chapter, there is no reason to assume constant share elasticities. The share elasticities derived from the third-order translog cost function are variable and contain constant share elasticities as a special case when the third-order parameters are zero. As can be seen in (4.5.1), the share elasticities generally depend on the levels of input prices The general expression for share elasticities after imposing constant returns to scale can be written as:

$$\frac{\partial^2 \ln C}{\partial \ln w_i \, \partial \ln w_j} = \gamma_{ij} + \sum_h \gamma_{ijh} \ln w_h \equiv \gamma^{\bullet}_{ij} \qquad (4.5.1)$$

where, i,j,h = K,L,E,M. Equation (4.5.1) collapses to the traditional formula when $\gamma_{ijh} = 0 \;\forall\; i,\, j,\, h$. However, as was shown in Section (4.3), most of the γ_{ijh} parameter estimates are different from zero. The share elasticities calculated using (4.5.1), are reported for the whole period in Tables 4.3a, 4.3b, and are also calculated at the mean in Table 4.3c.

The share elasticities reported change from year to year in magnitude and at times, even in sign. A change in sign from positive to negative implies a movement from an inelastic region to an elastic region of the input demand function. This flexibility is not permitted when the share elasticities are derived from a second-order function. The variable share elasticities derived from the third-order translog cost function would appear more plausible.

The own-share elasticities are reported in Table 4.3a The own share elasticity of capital, $\gamma^{\bullet}_{kk}$, remained positive for the whole period.[7] The average magnitude of the $\gamma^{\bullet}_{kk}$'s, $\bar{\gamma}^{\bullet}_{kk}$ (= .028), was slightly higher than the constant share elasticity, γ_{kk} (= .025). The estimate of $\bar{\gamma}^{\bullet}_{kk}$ was also statistically significant (t = 4.344). Examining the entries year by year shows that both the magnitudes and significance levels changed. The positive entries indicate that the share of capital increased as its own price increased. Since our model is of the third-order, it is possible to compute the rate of change of the share elasticities, by examining the sensitivity parameter γ_{ijh}. In Table 4.1a, γ_{kkk}, is reported to be insignificant as the t- statistic is only 0.987. Therefore the capital share elasticity does not vary as its own price increases over the years, ceteris paribus. The fact that the share of capital, S_k, increased with respect to its price, also indicates that the underlying input demand is price inelastic (this is confirmed by the fact that the own price elasticity of capital is less than one in absolute value, Table 4.4a).

The values of the own share elasticity of labor, $\gamma^{\bullet}_{LL}$, also remains positive throughout and are highly significant except for the years 1968 and 1971. On average, the results obtained from our model are consistent with the ones found by Berndt and Wood, *i.e.* positive and highly significant. As for the sensitivity of this elasticity, the relevant sensitivity parameter γ_{LLL} was found to be negative and significant (t = - 2.998). This implies that the share of labor increased at a decreasing rate with respect to its own price.

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[7] For 1970 $\gamma^{\bullet}_{kk}$ was negative, however the t-value was insignificant.

The own share elasticity of energy, $\dot{\gamma}_{EE}$, on the other hand changed signs from one sub-period to another, but the values remained low and mostly insignificant with the exceptions of a few cases. Thus, on average, the own share elasticity of energy was found to be insignificant, unlike the result reported by Berndt and Wood, 1975. This finding does not imply that energy demand was insensitive to its own price, but rather that when all other prices are allowed to adjust the own price change does not alter the share of energy in the total cost. As a matter of fact, the own price elasticity and own elasticity of substitution for energy reported in the tables below are significant. Since the estimate of the sensitivity parameter, γ_{EEE}, was positive and significant (t = 2.785), it implies that the factor shares of energy increased at an increasing rate in the period that showed a significant positive value, and it decreased at an increasing rate for the year 1970, the only year that showed a significant negative entry.

The intermediate materials share elasticity remained positive and significant up to 1964 with the exception of the years 1956-58 and 1961-64 which showed positive but insignificant entries. After the year 1964, the signs changed to negative but not all of the entries were significant.

The cross-share elasticities are reported in Tables 4.3(b) and (c). While the cross-share elasticities are symmetric, $\dot{\gamma}_{ij} = \dot{\gamma}_{ji}$, i,j = K,L,E,M; i ≠ j, the rates of change are not. The value of these elasticities change signs and magnitudes over the period. The following conclusions can be drawn:

(1) The share elasticity of labor with respect to the price of

intermediate materials, γ^{*}_{LM}, has a highly significant negative entry up to 1963. This implies that the share of labor decreased at a decreasing rate, since, γ_{LMM} , is also negative and significant (t = -2.444). In the same period the share of intermediate materials, γ^{*}_{ML} , with respect to the price of labor also decreased by symmetry, but at an increasing rate since, γ_{LLM} , is positive and significant (t = 4.398). Between 1964 and 1971, the signs of γ^{*}_{LM} changed to positive, but the values were not significantly different from zero.

(2) The share elasticity of labor and energy with respect to cross prices (*i.e.* γ_{EL}) was positive and significant up to 1956, and then became insignificant for the next two years. Thereafter, the signs changed to an insignificant negative in the year 1959 and then to a highly significant negative. This result is different from the cross-share elasticity estimated using the second-order function which was reported to be negative and insignificant. By contrast, until 1958, the shares of energy and labor increased with respect to cross-prices in the third-order case. The former increased at a constant rate, since γ_{LEE} is insignificant, while the latter increased at a decreasing rate, since γ_{LLE} is negative and significant (t = - 4.26). From 1959 onwards, energy and labor shares decreased at a constant and a decreasing rate, respectively, with respect to cross prices.

(3) The share elasticities of labor and capital showed identical sign reversals, over time, to those discussed in (2). The signs were positive and significant until the year 1955, positive but insignificant for the following three years and then changed to negative but insignificant until the year 1962. The period 1963-1971 showed a

negative, significant relationship. The results presented in table 4.3c show that at the mean, γ^{*}_{LK}, is negative and insignificant which coincides with those in Berndt and Wood (1975). This average tendency definitely hides those significant positive and negative entries reported by our estimated results from the third-order cost function. In terms of our result, the share of labor with respect to the price of capital increased until 1955, and then decreased at a constant rate. On the other hand, the share of capital behaved like the share of labor, but at a decreasing rate, since the estimate of the relevant parameter γ_{LLK} was negative and significant (t = - 3.509).

(4) The sign of γ^{*}_{EK} remained negative for all fifteen significant entries. There were a few positive entries but they were all insignificant. Thus, the overall result in this case was found to be consistent with the ones reported using the second-order cost function. The share of capital decreased at a constant rate (t = 1.737), while the share of energy decreased at an increasing rate with respect to cross prices (t = 2.613).

(5) The share elasticities of materials and capital were found to be mainly insignificant and negative, confirming the results obtained from the second-order cost function with some exceptions. The exceptions were the significant negative entries for the years 1954-55 and the significant positive entries for 1969-70. The relevant sensitivity parameters γ_{KMM} and γ_{KKM} were also insignificant, implying the rate of change will be constant.

(6) Lastly, the share elasticities γ^{*}_{EM} were mainly positive and significant only for the years 1967 and 1969-70, with the signs becoming

negative and significant in the years 1954-1955. The rate of change of the share of energy with respect to the cross-price increased (t = 2.34), while the rate of change of materials decreased since the estimated γ_{EEM} was negative and significant, (t = - 2.731).

In order to demonstrate the similarities of the estimates of share elasticities between the second and third-order cost functions, we have reported the share elasticities calculated at the mean and performed the following tests :

$$H_0: \quad \bar{\gamma}^*_{ij} = 0 \qquad \forall\ i,j = K,L,E,$$

$$H_1: \quad \bar{\gamma}^*_{ij} \neq 0$$

Since the t-statistics and the estimates of share elasticities at the means, $\bar{\gamma}^*_{ij}$, were calculated using the information provided in the estimated system, the degrees of freedom remained the same as above. Hence, the critical value is still $t^* = 2.01$. Based on this critical value, all but $\bar{\gamma}^*_{LK}$, $\bar{\gamma}^*_{EE}$, $\bar{\gamma}^*_{EM}$, and $\bar{\gamma}^*_{MK}$ were found significantly different from zero (Table 4.3c).

By examining the share elasticities at their mean, an interesting comparison can be made with those computed from the second-order cost function. First, the share elasticities, $\bar{\gamma}^*_{LK}$, $\bar{\gamma}^*_{MK}$, and $\bar{\gamma}^*_{EM}$ were found to be insignificant a', with the second-order cost function. Second, γ_{EE} was significant in the second-order cost function, and the corresponding value in the third-order case $\bar{\gamma}^*_{EE}$ was found to be insignificant. Third, the γ_{LE} was found to be insignificant. In our model the corresponding expression, $\bar{\gamma}^*_{LE}$, maintained the same sign but was found to be highly significant (t = - 3.211). Fourth, the signs of all the significant share elasticities ($\bar{\gamma}^*_{LL}$, $\bar{\gamma}^*_{MM}$, $\bar{\gamma}^*_{KK}$, $\bar{\gamma}^*_{LM}$, $\bar{\gamma}^*_{LE}$ and $\bar{\gamma}^*_{EK}$) are the same as the ones reported in the second-order cost estimates reported by Berndt and Wood (1975).

From the above observations, one can conclude that the share elasticities computed from the second-order cost function reflect average tendencies, with the exception of underestimating the cross-share elasticities between labor and energy. Although, average values convey an important message, some detailed information may be lost in the process. The basic importance of calculating the share elasticities from the third-order cost function, aside from precision, is that it brings out all the details lost in the process of averaging. The variable share elasticities computed from our model give the detailed behavior of the shares, depending on the levels of input prices involved and the sensitivity parameter estimates, γ_{ijh}.

4.6 Sensitivity of Share Elasticities

The sensitivity parameters, γ_{ijh}, measures the change in the cross-share elasticity (γ^*_{ij}) with respect to a change in a factor price ($\ln w_h$). For examрle consider the following specific share elasticity .

$$\gamma^*_{KE} = \gamma_{KE} + \gamma_{KLE} \ln w_L + \gamma_{KEE} \ln w_E + \gamma_{KEM} \ln w_M + \gamma_{KKE} \ln w_K \qquad (4.6.1)$$

Thus the variable cross share elasticity, γ^*_{KE}, depends not only on the cross prices directly involved but also the prices of labor and intermediate materials. By examining the estimates of γ_{ijh} (Table 4.1a), we conclude that only the prices of materials (γ_{KEM}, t = -2.039), and capital (γ_{KKE}, t = 2.613) significantly affect the value of γ^{*}_{KE}. By using the relevant formulas for γ^{*}_{KL}, γ^{*}_{KM}, γ^{*}_{LE}, γ^{*}_{LM} , and γ^{*}_{EM}, we conclude that:

(i) γ^{*}_{KL} is sensitive to the price of labor and slightly sensitive to the price of materials;

(ii) γ^{*}_{KM} is sensitive to the price of labor and slightly sensitive to the price of capital;

(iii) γ^{*}_{LE} is sensitive to the price of labor, while γ^{*}_{LM} is sensitive to the prices of labor and materials and also slightly sensitive to the price of capital;

(iv) γ^{*}_{EM} is sensitive to the prices of energy, materials , and capital.

4.7 Energy-Capital Complementarity

The debate as to whether capital and energy are substitutes or complements has produced numerous papers in the seventies and eighties. The differing results have been attributed to differences in data sets, differences in the way the input quantities and prices were constructed, differences in calculating short and long term elasticities and differences in the separability assumptions. Berndt and Wood (1979) attempted to reconcile the different results on energy-capital complementarity obtained by different authors. Their theoretical and empirical study was based on the assumption of utilized capital separability. Based on this assumption they were able to decompose the price elasticity into gross price elasticity and expansion elasticity. They point out that two inputs can be gross substitutes while they can also display net complementarity. The decomposition of the net price elasticity into its components is justified if the separability assumption they made were valid.

In Chapter 5 we show that this specific form of separability is not acceptable. However, the authors have suggested that this debate requires further study, especially in the area of model specification. In this thesis an attempt is made to explain the different results by way of a new model specification Berndt and Wood (1979) indicated that when more than two inputs are involved in a production process, there is no theoretical basis from which one can determine whether any two inputs are net substitutes or complements. Thus, when more than two inputs are involved, attempts should be made to bring the effects of all the inputs involved directly into the measurement of the substitution relationship between any two inputs. The AUES formula generated from the third-order translog cost function involves the variable share elasticities (as opposed to the constant share elasticities) and the shares of the two inputs involved. The variable share elasticities embody the effects of all input prices to determine the substitution possibilities between any two inputs. The AUES derived from the second-order cost function, on the other hand, was shown to be inflexible, in that the value generated can only be greater than unity or less than unity for all observations The implication of this is that two inputs cannot move from a period of high substitutability to a period of complementarity regardless of the level of input prices.

After imposing constant returns to scale on (3.3.20) the AUES can be rewritten as follows:

$$\sigma_{ij} = (\gamma_{ij} + \sum_h \gamma_{ijh} \ln w_h + S_i S_j)/S_i S_j \tag{4.7.1}$$

where, i, j, h = K, L, E, M $i \neq j$

$$\text{and} \quad \sigma_{ii} = (\gamma_{ii} + \sum_h \gamma_{iih} \ln w_h + S_i(S_i - 1))/S_i^2. \tag{4.7.2}$$

The additional relationship that helps us to determine the substitution possibilities is the factor price elasticity. After imposing constant returns to scale, expression (3.3.8) can be rewritten as follows:

$$\varepsilon_{ij} = (\gamma_{ij} + \sum_s \gamma_{ijh} \ln w_h + S_i S_j) / S_i \quad , \; i \neq j \tag{4.7.3}$$

$$\varepsilon_{ii} = (\gamma_{ii} + \sum_h \gamma_{iih} \ln w_h + S_i (S_i - 1)) / S_i \quad , \; i = j \tag{4.7.4}$$

where i,j,h = K, L, E, M

Expressions (4.7.1) to (4.7.4) provide the corresponding expressions for the second-order case when the γ_{ijh}'s are set to zero. The expressions $\sum_h \gamma_{ijh} \ln w_h$ and $\sum_h \gamma_{iih} \ln w_h$ which are absent in the second-order cost function case, allow the value of the AUES to move from less than zero to greater than unity, depending on the signs of the parameters and input price levels. Depending on the sensitivity of the share elasticities with respect to all the prices involved, two inputs may exhibit a period of complementarity and one of substitutability during a given time frame. Using the above formulas we have computed the corresponding values at every data point and also calculated them at their respective means for hypothesis testing. These results are reported in Tables 4.4a-4.4c and 4.5a-4.5e.

Empirical Results

The own AUES and the own factor price elasticities are reported in Tables 4.4a and 4.5a respectively. With the exception of 1947 and 1948, where the concavity requirement failed, the values of own price elasticities were consistently negative. The negative signs attached to the Allen partial elasticities of substitution indicate that the Hessian matrix formed using the parameter estimate was negative semi-definite, a requirement for concavity. Furthermore, the negative signs attached to own price elasticities indicate that the factor demands curves are downward sloping. By examining Table 4.5a, we find that the magnitudes of the own price elasticities are different from the ones derived from the second-order function. We find a substantial responsiveness of energy demand to its own price ($\bar{\varepsilon}_{EE}$ = -.86, t = - 5.524), even higher than what was reported by Berndt and Wood (1975)

The signs and magnitudes of the cross AUES and the cross-price elasticities help us to determine substitution possibilities between factors of production. The signs and magnitudes of various elasticities are changing over time and thus, differ from the ones reported based on second-order cost functions (see Berndt and Wood, 1975, pp 264-265,)

Examining the above results, we observe the following.

(1) Energy and labor show a substantial substitutability up to 1958 which is also highly significant The values for the AUES between

energy and labor (σ_{EL}) range from 1.38 to 4.7 and the cross-price elasticity between energy and labor (ε_{EL}) ranges from .067 to 0.24. The cross-price elasticity between labor and energy (ε_{LE}) range from 0.37 to 1.1. From 1959-1961, the relationship changed to one of slight substitutability ($.17 \le \sigma_{LE} \le .60$, $.21 \le \varepsilon_{LE} \le .80$, $.05 \le \varepsilon_{EL} \le .16$), but only the value in the year 1960 was significantly different from zero. Beginning in 1962, the signs became negative, but remained insignificant up to 1965. The negative signs became significantly different from zero thereafter. This implies that during this period, energy and labor showed substantial complementarity, since the values for the Allen partial elasticity of substitution ranged from - 3.18 to - 1.31. This result is different from the slight substitutability of the two inputs obtained with a second-order cost function.

(2) Labor and capital displayed substitutability up to 1961, and the entries remained positive for the next two years but insignificant. Thereafter they apparently displayed complementarity (Table 4.4b, Table 4.5c). However, a close examination of the significance levels of the relevant Allen partial elasticities reveal that the negative values were not significant. Within this latter period one may conclude that were few substitution possibilities between labor and capital. The results obtained from the second-order cost function predict that these inputs were substitutes for the whole study period due to its rigid formula.

(3) Energy and capital showed substantial complementarity during the periods 1949-53, 1956-1958, and 1969-71. With the exception of the above eleven years, capital and energy did not demonstrate any significant interdependence. The signs of the elasticities changed but

the positive entries were not significant. On average, this finding coincides with those derived from the second-order cost function

(4) The signs of the AUES between energy and intermediate materials remained positive throughout and thus displayed a substantial substitutability in the years 1951-53, 1956-58, 1960-63, 1966-67, and 1969-70. In the remaining years there were no significant substitution possibilities. From the above result we can conclude that the substitution possibilities between the above two inputs were not stable in the study period.

(5) Capital and intermediate materials showed substantial substitutability for all of the eight significant observations in the study period. The other observations were marked by changing signs, which were insignificant

(6) The signs of the relevant elasticities between labor and intermediate materials were consistently positive for all significant entries However, the substitution possibilities changed from slightly substitutable during 1953-63 to quite substitutable during the period 1964-1971.

For completeness, the following hypotheses were tested at the mean of the data :

$$H_0: \bar{\sigma}_{ij} = 0,$$

$$H_a: \bar{\sigma}_{ij} \neq 0.$$

The own-AUES for capital, labor, energy, and intermediate materials are all negative and significant (Table 4.4a). The corresponding own-price elasticities are all negative and also significant (Table 4.5a).

Using Table 4.4b we conclude that:

(1) Energy and capital are highly complementary (σ_{EK} = - 4.73, ε_{EK} = -.25, ε_{KE} = .21, t = - 2.93), which is consistent with the result found from the second-order cost function.

(2) Energy and intermediate materials are quite substitutable. The estimated values are σ_{EM} = 1.64, ε_{EM} = 1.03, ε_{ME} = .07, t = 4.35. These are consistent with the ones derived from the second-order cost function.

(3) Labor and capital display slight substitutability, since the estimated values are σ_{LK} = .68, ε_{LK} = .04 , ε_{KE} = .18, t = 2.8.

(4) Labor and intermediate materials also display slight substitutability, since the estimated values are σ_{LM} = .69, ε_{LM} = .43, and ε_{Ml} = .18, t = 13.94.

(5) Capital and intermediate materials also show slight substitutability. The estimated values are σ_{MK} = .71, ε_{MK} = .04, ε_{KM} =.44 , t = 2.35.

(6) The substitution possibility between labor and energy is insignificant, since the estimated values are σ_{LE} = .31, ε_{LE} = .01, ε_{EL} = .08, t = 1.45. These however, have the same sign reported from the second-order cost function. These results are similar to Berndt and Woods'(1975), except for the slight differences in magnitudes.

For purpose of comparison, a brief summary of earlier studies related to the issue of capital-energy complementarity is reported in

Table 4.6. The different findings reported in these studies have been attributed to distinctions between the long and short-run, and differences in data aggregations. In our view, these explanations are inadequate.

The differences in the results obtained here emanate from the additional flexibility provided by the third-order specification. The flexibility of the formulas used above highlights the loss of information related to input substitution implicit in the rigid formulas derived from the second-order cost specification. We believe that our results reconcile the differing results on substitution possibilities.

4.8 Measure of Bias

In this section, the argument that the third-order translog cost function reduces truncation bias is investigated empirically. We use the "information inaccuracy values" technique advanced by Thiel et al (1967) as goodness of fit indicators. The information inaccuracy of the factor shares for the i^{th} observation is given by

$$I_i = .5 \sum_i (\hat{S}_{ni} - S_{ni})^2 / S_{ni} \qquad (4.8.1)$$

where the $\hat{S}_{ni}$ is the estimated share of an input for the i^{th} observation and S_{ni} represents value for the actual share's of an input.

The information inaccuracy is defined as being the mean inaccuracy over all observations:

$$\bar{I} = (1/T) \sum_{i=1}^{T} I_i \qquad (4.8.2)$$

$$I_{ni} = .5 \sum_{i} (\hat{S}_{ni} - S_{ni})^2 / (S_{ni} (1 - S_{ni})) \quad (4.8.3)$$

where T is the number of observations.

On the other hand, the inaccuracy measure for a specific factor for the i^{th} observation is approximated by:

$$\bar{I}_n = (1/ T) \sum_{i=1}^{T} I_{ni} \quad (4.8.4)$$

The actual and the predicted shares from both models are presented in Tables 4.2a, 4.2b, 4.2c, and 4.2d. Based on the IEZ estimates, the following mean measure of information inaccuracy values are calculated.

	Second Order	Third Order
$\bar{I}$	.000234	.000141
$\bar{I}_K$	.000094	.000064
$\bar{I}_L$	.000073	.000042
$\bar{I}_E$	.000037	.000025
$\bar{I}_M$	.000164	.000073

From the above results the following observations can be made:

(a) The information inaccuracy for the individual factors are smaller than for all the factors combined for both models.

(b) The measure of inaccuracy calculated are smaller in the case of the third-order function compared to the second-order function, since the former are closer to zero. This is true for both individual factors and all factors combined.

The above results support the hypothesis that the bias is smaller when a third-order translog cost function is used rather than the second-order. This implies that the shares of inputs from the third-order function are superior in prediction performance than those derived from the second-order form.

4.9 Policy Implications of our Results

Since the signs and the significance levels of σ_{ij} derived from the third-order cost function changed over the study period, policy implications derived from the second-order cost function estimates may not be appropriate. For example, the estimated AUES from the second-order cost function σ_{EE} and σ_{KE} are negative and σ_{LE} is positive. These signs suggest that energy and capital are complements while energy and labor are substitutes. Hence, the lifting of price ceilings on energy types would reduce the energy and capital-intensiveness and increase the labor intensiveness of producing a given level of output. Examining the signs of the above measures of substitution possibilities, we find similar policy implications only if the predictions were made based on the estimates until the year 1958. After this period, energy-capital complementarity weakened up to the last three years of the study period, *i.e.* there was no significant substitution possibilities among the inputs mentioned. More importantly, the energy-labor substitutability has changed to that of complementarity. Therefore , the lifting of price ceilings on energy types during this period would not alter capital intensiveness, but would reduce labor intensiveness. It would not increase the latter as was suggested by the estimates of the second-order cost function. In addition to the above, it was predicted that the general investment incentive would increase energy and capital demand, since, based on second-order estimates, they were found to be complements. Hence, the investment incentive program was found unattractive to a government with objective of energy conservation. In light of our result, the policy conclusion arrived at using the second-order cost estimates of AUES do not hold, if the predictions were based on the substitution possibilities existing throughout most of the 1960's. Therefore, a fiscal stimulant, such as tax incentives to reduce the price of capital would not affect energy conservation policies.

4.10 Summary and Conclusion

In this chapter a number of significant results were obtained. The third-order parameters that were assumed to be zero were found to be significantly different from zero. The second-order translog cost function that was believed to represent the U.S. manufacturing sector was found to be misspecified. The share elasticities that were assumed to be constant were found to be variable in most cases. The AUES revealed that substitution possibilities did not maintain a uni-directional relationship for the whole period. They moved from a regime of substitutability to one of complementarity. Due to the sensitivity parameters introduced in the third-order function, we were able to analyze the rates of change of important economic relationships such as the cost shares, and also analyze the stability of the share elasticities. Furthermore, unless one is satisfied with average tendencies of the variables, our results suggest that there is a need to examine the movement of important variables such as the AUES period by period. This is made possible due to the flexible formulae derived from the third-order function. Finally, the variable AUES suggests that there is a need to re-evaluate policies from time to time, since the substitution possibilities can change during the policy period.

TABLE 4.1a. ITERATIVE ZELLNER PARAMETER ESTIMATES
OF THIRD ORDER TRANSLOG COST FUNCTION
US MANUFACTURING, 1947 - 1971

PARAMETER	ESTIMATE	PARAMETER	ESTIMATE
γ_{K}	.0534	γ_{KLE}	-.0170
	(27.182)		(-0.491)
γ_{KK}	.0396	γ_{KLM}	.1780
	(1.883)		(1.994)
γ_{KL}	.0364	γ_{KEE}	0.1844
	(3.340)		(1.737)
γ_{KE}	.0030	γ_{KEM}	-.3611
	(.202)		(-2.039)
γ_{KM}	-.0791	γ_{KMM}	.4740
	(-2.221)		(1.385)
γ_{L}	.2500	γ_{E}	.0432
	(89.002)		(36.403)
γ_{LL}	.1275	γ_{EE}	-.0586
	(6.023)		(-1.498)
γ_{LF}	.0255	γ_{LLL}	-.2227
	(3.235)		(-2.998)
γ_{LM}	-.1894	γ_{LLE}	-.1354
	(-6.457)		(-4.260)
γ_{M}	.6533	γ_{LLM}	.4961
	(170.857)		(4.398)
γ_{EM}	.0300	γ_{LEE}	.1707
	(0.589)		(1.792)
γ_{MM}	.2385	γ_{LEM}	-.0183
	(2.714)		(-.137)
γ_{KKK}	0.1204	γ_{LMM}	-.6558
	(0.987)		(-2.444)
γ_{KKL}	-.0231	γ_{EEE}	.7598
	(-0.593)		(2.785)
γ_{KKE}	-.1936	γ_{EEM}	-1.1149
	(-2.613)		(-2.731)
γ_{KKM}	-.2909	γ_{EMM}	1.4943
	(0.987)		(2.340)
γ_{MMM}	-1.3124	γ_{KLL}	-.1380
	(1.227)		(-3.509)

t^{*} statistics are given in parentheses

TABLE 4.1b. ITERATIVE ZELLNER PARAMETER ESTIMATES
OF SECOND ORDER TRANSLOG COST FUNCTION
US MANUFACTURING, 1947 - 1971

PARAMETER	ESTIMATE
γ_K	.0570
	(41.900)
γ_{KK}	.0297
	(5.005)
γ_{KL}	-.0004
	(-.096)
γ_{KE}	-.0102
	(-3.014)
γ_{KM}	-.0191
	(-1.935)
γ_L	.2534
	(119.582)
γ_{LL}	.0754
	(11.077)
γ_{LE}	-.0044
	(-1.810)
γ_{LM}	-.0706
	(-6.559)
γ_E	.0443
	(50.119)
γ_{EE}	.0188
	(3.508)
γ_{EM}	-.0041
	(-.479)
γ_M	.6453
	(191.828)
γ_{MM}	.0938
	(4.034)

t^* statistics are given in parentheses

Table 4.2a. COST, ACTUAL AND FITTED SHARES FOR ENERGY AND INTERMEDIATE MATERIALS FROM THIRD ORDER TRANSLOG COST FUNCTION, U.S. MANUFCTURING 1947-1971.

	COST	SEFT	SE	SMFT	SM
1947	182.373	.432397E-01	.425335E-01	.653381	.659127
1948	183.162	.509155E-01	.512662E-01	.626326	.613399
1949	186.532	.489239E-01	.507523E-01	.654308	.644113
1950	221.709	.446461E-01	.460648E-01	.648655	.656084
1951	255.945	.453024E-01	.448221E-01	.645569	.649915
1952	264.670	.458006E-01	.445987E-01	.640547	.639691
1953	291.161	.462156E-01	.436907E-01	.633632	.640707
1954	274.456	.454253E-01	.478693E-01	.631453	.624109
1955	308.908	.457392E-01	.451722E-01	.630454	.637599
1956	328.286	.493863E-01	.457561E-01	.631820	.639399
1957	338 634	.478334E-01	.482025E-01	.628319	.629624
1958	323 319	.462136E-01	.483578E-01	.627562	.618663
1959	358.434	.453467E-01	.456288E-01	.619674	.619483
1960	366.251	.452257E-01	.458453E-01	.621677	.618896
1961	366.161	.450269E-01	.464030E-01	.620523	.616169
1962	390.667	.446258E-01	.452995E-01	.619124	.616127
1963	412 188	.434649E-01	.446979E-01	.617535	.619619
1964	433.767	.432622E-01	.439176E-01	.617758	.618135
1965	474.970	.426733E-01	.411396E-01	.617154	.624233
1966	521.291	.422078E-01	.401368E-01	.616907	.621627
1967	540.341	.415839E-01	.407409E-01	.616892	.618369
1968	585.448	.394924E-01	.397063E-01	.620108	.613884
1969	630.452	.409595E-01	.396303E-01	.615492	.615967
1970	623.445	.426293E-01	.434792E-01	.609927	.606417
1971	658.237	.443653E-01	.447938E-01	.615960	.619401

Table 4.2b. ACTUAL AND FITTED SHARES FOR CAPITAL AND LABOR FROM THIRD ORDER TRANSLOG COST FUNCTION, U.S. MANUFACTURING 1947-1971.

	SKFT	SK	SLFT	SL
1947	.533543E-01	.510655E-01	.250025	.247274
1948	.566414E-01	.581730E-01	.266117	.277162
1949	.434424E-01	.460195E-01	.253325	.259115
1950	.489060E-01	.499073E-01	.257793	.247943
1951	.512496E-01	.503896E-01	.257879	.254873
1952	.505577E-01	.491616E-01	.263095	.266549
1953	.517103E-01	.472836E-01	.268442	.268318
1954	.537277E-01	.563479E-01	.269394	.271674
1955	.533889E-01	.525759E-01	270418	264653
1956	.501885E-01	.460388E-01	.268605	268806
1957	.516183E-01	.503329E-01	272229	.27[illegible]41
1958	.533824E-01	.601535E-01	.272842	.272825
1959	.582297E-01	.618550E-01	.276750	.273033
1960	.558523E-01	.578840E-01	.277245	.277375
1961	.559826E-01	.590339E-01	.278467	.278394
1962	.557475E-01	.557790E-01	.280502	.282795
1963	.570859E-01	.560083E-01	.281914	.279675
1964	.562803E-01	.545189E-01	.282700	.283429
1965	.575560E-01	.546665E-01	.282617	.279961
1966	.574212E-01	.546020E-01	.283464	.283634
1967	.555759E-01	.544279E-01	.285948	.286462
1968	.560139E-01	.575830E-01	.284386	.288827
1969	.545891E-01	.540971E-01	.288959	290306
1970	.531844E-01	.525498E-01	.294259	.297554
1971	.455190E-01	.467507E-01	.294156	.289054

Table 4.2c ACTUAL AND FITTED SHARES FOR ENERGY AND INTERMEDIATE MATERIALS FROM SECOND ORDER TRANSLOG COST FUNCTION, U.S. MANUFCTURING 1947-1971.

	SEFT	SE	SMFT	SM
1947	.442865E-01	.425335E-01	.645290	.659127
1948	.483620E-01	.512662E-01	.639051	.613399
1949	.497967E-01	.507523E-01	.645659	.644113
1950	.473141E-01	.460648E-01	.642055	.656084
1951	.459200E-01	.448221E-01	.641330	.649915
1952	.467630E-01	445987E-01	.638690	.639691
1953	464677E-01	436907E-01	.635043	.640707
1954	.459782E-01	.478693E-01	.633768	.624109
1955	.461085E-01	.451722E-01	.633106	.637599
1956	.471711E-01	.457561E-01	.635825	.639399
1957	.464001E-01	.482025E-01	.632713	.629624
1958	.455863E-01	.483578E-01	.631324	.618663
1959	.438793E-01	.456288E-01	.625170	.619483
1960	.442595E-01	.458453E-01	.626232	.618896
1961	.440293E-01	.464030E-01	.624916	.616169
1962	.438520E-01	.452995E-01	.622683	.616127
1963	.429259E-01	.446979E-01	.619003	.619619
1964	.432336E-01	.439176E-01	.617511	.618135
1965	.424305E-01	.411396E-01	.616457	.624233
1966	.420157E-01	401368E-01	.616068	.621627
1967	421999E-01	.407409E-01	.614996	.618369
1968	.416675E-01	.397063E-01	.609948	.613884
1969	.419069E-01	.396303E-01	.613061	.615967
1970	.424721E-01	.434792E-01	.611720	.606417
1971	.454788E-01	.447938E-01	.609139	.619401

Table 4.2d. ACTUAL AND FITTED SHARES FOR CAPITAL AND LABOR FROM SECOND ORDER TRANSLOG COST FUNCTION, U.S. MANUFCTURING 1947-1971.

	SKFT	SK	SLFT	SL
1947	.570227E-01	.510655E-01	.253401	.247274
1948	.533147E-01	.581730E-01	.259272	277162
1949	.451696E-01	.460195E-01	.259375	.259115
1950	.503945E-01	.499073E-01	.260236	.247943
1951	.522754E-01	.503896E-01	.260475	.254873
1952	.508245E-01	.491616E-01	263723	.266549
1953	.512601E-01	.472836E-01	.267229	.268318
1954	.530812E-01	563479E-01	.267173	.271674
1955	.526820E-01	.525759E-01	.268104	.264653
1956	.485793E-01	.460388E-01	268425	268806
1957	.502399E-01	.503329E-01	.270647	271841
1958	.523390E-01	.601535E-01	.270751	272825
1959	.564567E-01	.618550E-01	.274494	273033
1960	.546521E-01	.578840E-01	.274856	.277375
1961	.548369E-01	.590339E-01	.276218	.278394
1962	.548050E-01	.557790E-01	.278660	.282795
1963	.565723E-01	.560083E-01	.281499	.279675
1964	.560617E-01	.545189E-01	.283194	283429
1965	.574593E-01	.546665E-01	.283653	279961
1966	.576681E-01	.546020E-01	.284248	.283634
1967	.562891E-01	.544279E-01	.286515	.286462
1968	.581502E-01	.575830E-01	.290234	288827
1969	.556413E-01	.540971E-01	.289391	.290306
1970	.528065E-01	.525498E-01	.293002	.297554
1971	.486232E-01	.467507E-01	.296759	.289054

TABLE 4.3a. OWN SHARE ELASTICITY ESTIMATES OF THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 ESTIMATE

	S_{KK}	S_{LL}	S_{EE}	S_{MM}
1947	.0396	.1275	-.0586	.2385
	(1 883)	(6.023)	(-1.498)	(2.714)
1948	.0721	.0860	.1073	.4699
	(3.219)	(5.530)	(3.913)	(4.836)
1949	0178	.1417	-.0194	.1921
	(769)	(7.032)	(-.653)	(2.316)
1950	.0288	.1230	-.0199	.1995
	(3 136)	(7 527)	(-1.046)	(3.701)
1951	.0250	.1231	-.0480	.1481
	(2 735)	(7 740)	(-1.768)	(2.229)
1952	.0266	.1132	-.0200	.1554
	(3.177)	(8.616)	(-1.153)	(3.351)
1953	.0284	.0998	-.0055	.1393
	(4.204)	(10.196)	(-.467)	(4.426)
1954	.0389	.0897	.0132	.1832
	(4.558)	(9.882)	(2.220)	(6.971)
1955	.0368	.0882	.0156	.1741
	(5.078)	(10.457)	(2.710)	(7.177)
1956	.0149	.1108	-.0278	.0704
	(.929)	(10.070)	(-1.302)	(1.207)
1957	.0200	.0972	-.0159	.0706
	(1.900)	(11.981)	(-1.054)	(1.787)
1958	.0271	.0882	-.0073	.0939
	(4.634)	(12.733)	(-.708)	(3.734)
1959	.0417	.0586	.0235	.1202
	(2.579)	(8.227)	(2.052)	(2.805)

TABLE 4.3a. OWN SHARE ELASTICITY ESTIMATES OF THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 ESTIMATE, Continued

	S_{KK}	S_{LI}	S_{EE}	S_{MM}
1960	.0309	.0689	.0009	0646
	(3.593)	(12.204)	(.096)	(2 633)
1961	.0299	.0650	.0004	0467
	(3.449)	(10 993)	(.046)	(1 793)
1962	.0285	.0585	.0035	0207
	(3.423)	(8.405)	(.407)	(727)
1963	.0325	0437	.0130	0085
	(2.450)	(4.361)	(1.019)	(195)
1964	.0336	0379	.0267	.0111
	(2 651)	(3.361)	(2.027)	(226)
1965	0346	.0339	0201	- 0025
	(2.150)	(2.741)	(1.281)	(- 046)
1966	.0304	.0352	0057	-.0375
	(1.999)	(2.802)	(348)	(- 725)
1967	.0218	.0369	-.0064	- 0956
	(2.101)	(2.782)	(-.455)	(-2.017)
1968	.0335	.0127	0324	-.0621
	(1.869)	(.692)	(1 566)	(- 841)
1969	.0130	.0359	- 0229	- 1677
	(1.372)	(2.373)	(-1.482)	(-3.096)
1970	-.0026	.0409	-.0431	-.2648
	(-.155)	(2.372)	(-2.453)	(-3 639)
1971	.0074	0258	.0392	- 1698
	(.324)	(1.239)	(1.315)	(-2.027)

t^{*} statistics are given in parentheses

TABLE 4.3b. CROSS SHARE ELASTICITY ESTIMATES OF THIRD ORDER TRANSLOG CCST FUNCTION. US MANUFACTURING, 1947 - 1971

	S_{LM}	S_{LE}	S_{EM}	S_{LK}	S_{KE}	S_{KM}
1947	-.1664	.0255	.0300	.0364	.0031	-.0791
	(-6.457)	(3.235)	(.589)	(3.340)	(.202)	(-2.221)
1948	-.1578	.0502	-.1880	.0216	.0305	-.1242
	(-5.273)	(2.706)	(-3.733)	(2.342)	(1.632)	(-3.219)
1949	-.2132	.0405	.0243	.0309	-.0454	-.0033
	(-6 000)	(3.678)	(.578)	(2.274)	(-3.169)	(-.094)
1950	-.1789	.0293	.0128	.0660	-.0221	-.0334
	(-7.000)	(3.848)	(.484)	(2.864)	(-2.931)	(-1.809)
1951	-.1694	0200	.0505	.0263	-.0221	-.0291
	(-6.918)	(2.868)	(1.322)	(2.970)	(-2 276)	(-1.418)
1952	-.1541	.0207	.0225	.0203	-.0232	-.0237
	(-7 524)	(3.454)	(.930)	(2.676)	(-3.338)	(-1.477)
1953	-.1280	.0148	.0106	.0134	-.0200	-.0218
	(-8.561)	(3.161)	(.669)	(2.373)	(-3.871)	(-1.829)
1954	-.1167	.0153	-.0227	.0117	-.0059	-.0438
	(-8 845)	(2 826)	(-2.532)	(2.405)	(-1.072)	(-3.313)
1955	-.1127	.0144	-.0222	.0102	-.0078	-.0392
	(-9.000)	(2.683)	(-2.606)	(2.227)	(-1.588)	(-3.352)
1956	-.1371	.0127	.0552	.0136	-.0400	.0116
	(-6.975)	(2.126)	(1.716)	(1.739)	(-3.682)	(.466)
1957	-.1127	.0071	.0397	.0084	-.0308	.0024
	(-8.361)	(1.763)	(1.786)	(1.537)	(-4.155)	(.148)
1958	-.0996	.0048	.0211	.0067	-.0185	-.0127
	(10 051)	(1.631)	(1.501)	(1.677)	(-4.107)	(-1.540)
1959	-.0502	-.0051	-.0250	-.0033	.0065	-.0449
	(-3.749)	(-.957)	(-1.267)	(-.575)	(.705)	(-1.936)

TABLE 4.3b. CROSS SHARE ELASTICITY ESTIMATES OF THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 (CONTINUED)

	S_{LM}	S_{LE}	S_{EM}	S_{LK}	S_{KE}	S_{KM}
1960	-.0598	-.0070	-.0151	-.0021	-.0090	-.0299
	(-7.057)	(-2.636)	(1.215)	(-.541)	(-1.823)	(-1.588)
1961	-.0501	-.0104	.0194	-.0045	- 0094	-.0160
	(-5.399)	(-3.656)	(1.486)	(-1.111)	(-1.923)	(-1.254)
1962	-.0344	-.0152	.0225	-.0089	-.0108	- 0088
	(-3.039)	(-4.410)	(1.708)	(-1.923)	(-2.333)	(-.687)
1963	-.0056	-.0229	.0122	-.0152	-.0023	-.0151
	(-.316)	(-4.169)	(.620)	(-2.152)	(- 317)	(- 756)
1964	.0036	-.0232	-.0015	-.0184	- 0020	- 0132
	(.182)	(-3.696)	(-.068)	(-2 419)	(- 274)	(-.651)
1965	0137	-.0279	.0057	-.0200	.0021	-.0170
	(.614)	(-4.051)	(.232)	(-2.261)	(234)	(- 696)
1966	.0174	-.0323	0285	-.0203	-.0018	-.0084
	(.785)	(-4.679)	(1.199)	(-2.342)	(-.222)	(- 364)
1967	.0233	-.0372	0573	- 0230	- 0138	.0150
	(1.061)	(-5.154)	(2.757)	(-2.828)	(-2.317)	(.834)
1968	.0608	-.0415	.0060	- 0320	.0032	-.0046
	(1.887)	(-4.262)	(.181)	(-2.664)	(.304)	(-.157)
1969	.0367	-.0456	.0928	-.0270	-.0243	.0383
	(1.510)	(-5.056)	(3.800)	(-3.128)	(-3.520)	(2.014)
1970	.0424	-.0524	.1423	-.0309	-.0467	.0802
	(1.5980)	(-4.601)	(4.234)	(-3.383)	(-3.761)	(2.698)
1971	.0506	-.0380	.0435	-.0384	-.0447	.0757
	(1.717)	(-4.640)	(1.045)	(-3.571)	(-3.044)	(1.994)

t^{*} statistics are given in parentheses

TABLE 4.3c. OWN AND CROSS SHARE ELASTICITS AT THE MEAN

ESTIMATES OF THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 ESTIMATE

Share Elasticities	Estimated Coefficient	T-Statistic
$\bar{S}_{LL}$	.067	11.819
$\bar{S}_{EL}$	.004	.625
$\bar{S}_{MM}$	.046	2.039
$\bar{S}_{KK}$	.028	4.344
$\bar{S}_{LM}$	- .054	-6.324
$\bar{S}_{LE}$	-.009	-3.211
$\bar{S}_{EM}$	.018	1.692
$\bar{S}_{LK}$	-.005	-1.303
$\bar{S}_{KE}$	-.014	-3.549
$\bar{S}_{KM}$	-.010	-.961

TABLE 4.4a ESTIMATED OWN ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

	σ_{LL}	σ_{EE}	σ_{MM}	σ_{KK}
1947	-.960	-53.473	.028	-3.830
	(-2.769)	(2.539)	(.158)	(.421)
1948	-1.543	22.759	.601	5.834
	(-7.352)	(2.140)	(2.390)	(.775)
1949	-.739	-27.550	-.079	-12.584
	(-2.491)	(-2.272)	(-.446)	(-1.127)
1950	-1.028	-31.393	-.068	-7.378
	(-3.883)	(3.354)	(-.485)	(-2.015)
1951	-1.027	-44.657	-.194	-9.000
	(-4.200)	(-3.332)	(-1.195)	(-2.503)
1952	-1.166	-30.359	-.182	-8.373
	(-6.270)	(-3.622)	(-1.619)	(2.406)
1953	-1.340	-23.227	-.231	-7.720
	(-9.858)	(-3.996)	(-2.891)	(-2.466)
1954	-1.476	-14.610	-.124	-4.488
	(-11.919)	(-5.438)	(-1.957)	(-1.839)
1955	-1.493	-13.416	-.148	-4.831
	(-12.630)	(-4.793)	(-2.348)	(-1.802)
1956	-1.188	-30.643	-.406	-13.018
	(-7.802)	(-3.348)	(-2.748)	(-1.813)
1957	-1.362	-26.871	-.413	-10.884
	(-12.414)	(-4.090)	(-4.120)	(-2.651)
1958	-1.481	-24.049	-.355	-8.220
	(-15.912)	(-5.179)	(-5.652)	(-5.030)
1959	-1.848	-9.603	-.301	-3.865
	(-19.643)	(-1.743)	(-2.698)	(-1.007)

TABLE 4.4a ESTIMATED OWN ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 (Continued)

	σ_{LL}	σ_{EE}	σ_{MM}	σ_{KK}
1960	-1.711	-20.693	-.441	-7.003
	(-23.237)	(-4.815)	(-6.974)	(-2.751)
1961	-1.752	-20.998	-.490	-7.314
	(-22.962)	(-4.739)	(-7.287)	(-2.951)
1962	-1.822	-19.629	-.561	-6.538
	(-20.756)	(-4.556)	(-7.576)	(-2.912)
1963	-1.998	-15.149	-.597	-6.538
	(-15.762)	(-2.339)	(-5.208)	(-1.533)
1964	-2.063	-7.853	-.590	-6.153
	(-14.635	(-1.162)	(-4.593)	(1.413)
1965	-2.114	-11.396	-.627	-5.942
	(-13.568)	(-1.233)	(-4.352)	(-1.065)
1966	-2.090	-19.512	-.720	-7.184
	(-13.389)	(-2.016)	(-5.275)	(-1.391)
1967	-2.046	-26.718	-.872	-9.937
	(-12.633)	(-3.259)	(-6.994)	(-2.860)
1968	-2.359	-3.566	-.774	-6.185
	(-10.461)	(-.279)	(-4.050)	(-1.161)
1969	-2.031	-37.059	-1.068	-12.958
	(-11.256)	(-3.946)	(-7.463)	(-4.031)
1970	-1.926	-46.189	-1.351	-18.704
	(-9.740)	(-4.819)	(-6.919)	(-3.180)
1971	-2.101	-1.622	-1.071	-17.383
	(-8.625)	(-.120)	(-4.842)	(-1.621)

t^{*} statistics are given in parentheses

TABLE 4.4b ESTIMATED CROSS ALLEN PARTIAL

ELASTICITIES OF SUBSTITUTION THIRD ORDER
TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

	σ_{EM}	σ_{KE}	σ_{KM}	σ_{LE}	σ_{LM}	σ_{KL}
1947	2.061	2.338	-1.268	3.363	-.160	3.727
	(1.140)	(.345)	(-1.276)	(4.567)	(-.901)	(4.499)
1948	-4.895	11.574	-2.502	4.363	.054	2.432
	(-3.108)	(1.792)	(-2.294)	(3.472)	(.409)	(4 091)
1949	1.760	- 20.365	.883	4 266	-.286	3 811
	(1.355)	(-3.006)	(.741)	(4.873)	(-1.303)	(3.150)
1950	1.440	-9.112	-.052	3.542	-.070	3 109
	(1.632)	(-2.620)	(-.034)	(5.351)	(-.633)	(4.197)
1951	2.725	-8.522	.119	2.717	-.018	2.988
	(2.085)	(-2.044)	(.175)	(4.50	(-.155)	(4.421)
1952	1.765	-9.008	.268	2.719	.085	2.524
	(2.113)	(-3 022)	(.483)	(5.437)	(798)	(4 406)
1953	1.362	-7.330	.334	2.196	247	1.962
	(2.436)	(-3.470)	(.708)	(5.659)	(2.936	(4.628)
1954	.208	-1.370	-.291	2.249	.314	1 808
	(.799)	(-.572)	(-.653)	(5 231)	(4.009)	(5.551)
1955	.231	-2.172	-.165	2.160	.339	1 707
	(.778)	(-1.101)	(-.487)	(4.918)	(4.475)	(5.263)
1956	2.768	-15.169	1 365	1.959	.192	2.011
	(2.626)	(-3.488)	(1.651)	(4.182)	(1.769)	(3.317)
1957	2.319	-11.463	1.075	1.542	.341	1.596
	(3.153)	(-3.827)	(2.061)	(5.034)	(4.342)	(4.049)
1958	1.726	-6.511	.544	1.377	418	1.459
	(3.635)	(-3.463)	(2.213)	(6 155)	(6.975)	(5 790)
1959	.110	3.471	-.245	.597	.707	.794
	(.165)	(1.009)	(-.285)	(1.402)	(8 880)	(2.342)

TABLE 4.4b ESTIMATED CROSS ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971 (Continued)

	σ_{EM}	σ_{KE}	σ_{KM}	σ_{LE}	σ_{LM}	σ_{KL}
1960	1.536	-2.552	.428	.446	.653	.867
	(3.502)	(-1.284)	(1.277)	(2.191)	(13.186)	(3.673)
1961	1.694	-2.746	.540	.172	.710	.710
	(3.677)	(-1.365)	(1.602)	(.891)	(13.074)	(2.932)
1962	1.813	-3.344	.746	-.214	.801	.432
	(3.830)	(-1.787)	(2.005)	(-.691)	(12.358)	(1.494)
1963	1.454	.079	.573	-.866	.968	.057
	(2 027)	(.030)	(.986)	(-1.890)	(9.429)	(.070)
1964	.944	.168	.620	-.894	1.021	-.156
	(1.169)	(.050)	(1.010)	(-1.710)	(8.949)	(-.386)
1965	1.217	1.845	.523	-1.313	1.079	-.211
	(1.272)	(.487)	(.707)	(-2.378)	(8.446)	(-.504)
1966	2.092	.253	.764	-1.700	1.100	-.246
	(2 250)	(.047)	(1.115)	(-3.030)	(8.732)	(-.553)
1967	3.235	-4.976	1.438	-2.127	1.132	-.446
	(3.968)	(-1.945)	(2.707)	(-3.536)	(9.137)	(-.910)
1968	1.243	2.431	.868	-2.693	1.345	-1.011
	(.921)	(.524)	(1.048)	(-3.083)	(7.395)	(-1.281)
1969	4.680	-9.874	2.140	-2.849	1.206	-.710
	(4.801)	(-3.209)	(3.766)	(-3.779)	(8.875)	(-1.307)
1970	6.471	-19.605	3.471	-3.179	1.236	-.974
	(5.019)	(-3.577)	(3.771)	(-3.465)	(8.401)	(-1.671)
1971	2.591	-21.151	3.701	-1.908	1.280	-1.870
	(1.712)	(-2.902)	(2.757)	(-3.057)	(7.798)	(-2.315)

t^* statistics are given in parentheses

TABLE 4.4c ESTIMATED OWN AND CROSS ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION AT THE MEAN THIRD ORDER TRANSLOG COST FUNCTION.
US MANUFACTURING, 1947 - 1971

Allen Partial Elasticities	Estimated Coefficint	T-Statistic
$\bar{\sigma}_{LL}$	-1.757	-23.386
$\bar{\sigma}_{EE}$	-19.153	-5.524
$\bar{\sigma}_{MM}$	-.479	-8.416
$\bar{\sigma}_{KK}$	-7.860	-3.473
$\bar{\sigma}_{EM}$	1.635	4.354
$\bar{\sigma}_{KE}$	-4.735	-2.930
$\bar{\sigma}_{KM}$	.710	2.353
$\bar{\sigma}_{LE}$	.311	1.454
$\bar{\sigma}_{LM}$	.689	13.938
$\bar{\sigma}_{KL}$	.682	2.797

TABLE 4.4d ESTIMATED ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION SECOND ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

AES	1955	1960	1965	1970	AT THE MEAN	
σ_{KK}	- 7.2657	- 7.3401	- 7.3954	- 7.2714	-7.3007	(-3.516)
σ_{LL}	- 1.6807	- 1.6400	- 1.5881	- 1.5345	- 1.6427	(-18.166)
σ_{EE}	-11.8620	-12.0152	-12.1455	-12.1428	-11.9723	(-4.493)
σ_{MM}	- .3453	- .3575	- .3751	- .3839	- .3557	(-6.014)
σ_{KL}	.9738	.9754	.9773	.9761	.9748	(3.695)
σ_{KE}	- 3.2131	- 3.2310	- 3.1477	- 3.5632	- 3.2699	(-2.308)
σ_{KM}	.4291	.4658	.4392	.3995	.4297	(1.458)
σ_{LE}	.6426	.6368	.6329	.6450	.6406	(3.226)
σ_{LM}	.5814	.5886	.5959	.6086	.5896	(9.424)
σ_{EM}	.8592	.8517	.8428	.8418	.8537	(2.796)

TABLE 4.5a ESTIMATED OWN PRICE
ELASTICITIES THIRD ORDER
TRANSLOG COST FUNCTION.
US MANUFACTURING, 1947 - 1971

	ε_{LL}	ε_{EE}	ε_{MM}	ε_{KK}
1947	-.237	-2.274	.019	-.196
1948	-.428	1.167	.369	.339
1949	-.191	-1.398	-.051	-.579
1950	-.255	-1.446	-.443	-.368
1951	-.262	-2.002	-.126	-.454
1952	-.311	-1.354	-.117	-.412
1953	-.360	-1.015	-.148	-.365
1954	-.401	-.699	-.078	- 253
1955	-.395	-.606	-.094	-.254
1956	-.319	-1.402	-.260	-.599
1957	-.370	-1.295	-.260	-.548
1958	-.404	-1.163	-.220	- 495
1959	- 505	-.438	-.186	- 239
1960	-.475	- 949	-.273	- 405
1961	-.488	- 974	-.302	-.432
1962	-.515	-.889	-.346	-.434
1963	-.559	-.677	-.370	-.366
1964	-.585	-.345	- 365	-.336
1965	-.592	-.469	- 391	- 325
1966	-.593	-.783	-.447	-.392
1967	-.586	-1.089	-.539	- 541
1968	-.681	-.142	-.475	- 356
1969	-.590	-1.469	-.658	-.701
1970	-.573	-2.008	- 820	- 983
1971	-.607	-.073	-.663	-.813

TABLE 4.5b ESTIMATED CROSS PRICE ELASTICITIES THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

	ε_{LE}	ε_{EL}	ε_{LM}	ε_{ML}
1947	.143	.832	-.105	-.040
1948	.241	1.303	.033	.015
1949	.217	1.105	-.184	-.074
1950	.163	.878	-.046	-.173
1951	.122	.692	-.012	-.005
1952	.121	.725	.055	.023
1953	.096	.560	.159	.064
1954	.107	.611	.196	.085
1955	.976	.572	.216	.090
1956	.090	.527	.123	.052
1957	.074	.419	.215	.093
1058	.067	.376	.259	.114
1959	.027	.163	.438	.193
1960	.021	.124	.404	.181
1961	.008	.048	.437	.198
1962	-.010	-.060	.494	.227
1963	-.039	-.242	.560	.271
1964	-.034	-.253	.631	.289
1965	-.050	-.368	.673	.302
1966	-.068	-.482	.683	.312
1967	-.087	-.609	.670	.324
1968	-.107	-.778	.825	.388
1969	-.113	-.827	.743	.350
1970	-.138	-.946	.750	.368
1971	-.086	-.551	.792	.370

TABLE 4.5c ESTIMATED CROSS PRICE ELASTICITIES THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

	ε_{LK}	ε_{KL}	ε_{EK}	ε_{KF}
1947	3.727	.922	.119	2 338
1948	2.438	.674	.673	15 076
1949	2.835	.988	-.937	-24.369
1950	2.875	.771	-.455	-11 066
1951	3.133	.761	-.430	-10.668
1952	2.518	.673	-.443	-11 523
1953	1 975	.526	- 347	-9 346
1954	1.967	.491	-.077	-1 786
1955	1.883	.452	-.114	-2.917
1956	2.003	.541	-.698	-20.805
1957	1.697	.434	-.577	-15 821
1958	1.687	.398	-.392	-9 072
1959	1.038	217	215	4 748
1960	1 087	.241	-.148	-3 523
1961	.897	.198	- 162	-3 779
1962	547	.122	-.187	-4.605
1963	.076	.016	.004	.107
1964	-.207	-.044	.009	.233
1965	-.297	-.059	.101	2.558
1966	-.356	- 070	.014	.354
1967	-.619	-.128	-.271	-6 926
1968	-1.515	-.292	.140	3.485
1969	-1 030	-.206	-.534	-14 463
1970	-1.291	-.230	-1.030	-28.606
1971	-2.247	-.540	-.989	-34.833

TABLE 4.5d ESTIMATED CROSS PRICE ELASTICITIES THIRD ORDER TRANSLOG COST FUNCTION. US MANUFACTURING, 1947 - 1971

	ε_{EM}	ε_{ME}	ε_{MK}	ε_{KM}
1947	1.359	.088	-.065	-1.268
1948	-3.003	-.251	-.146	-2.640
1949	1.134	.089	.041	.937
1950	.945	.066	-.003	-.058
1951	1.771	.122	.006	.145
1952	1 129	.079	.013	.322
1953	.873	.060	.016	.397
1954	.130	.010	-.016	-.351
1955	.147	010	.009	-.205
1956	.177	.127	.063	1.765
1957	1.460	112	.054	1.406
1958	1.068	.084	.033	.722
1959	.068	.005	-.152	-.321
1960	.951	.070	.248	.574
1961	1.044	.079	.032	.726
1962	1.117	.042	.416	1.005
1963	.901	.065	.032	.763
1964	.583	.415	.034	.839
1965	.760	.050	.286	.720
1966	1.301	.084	.042	1.084
1967	2.001	.132	.782	2.047
1968	.763	.049	.050	1.245
1969	2.883	.186	.116	3.282
1970	3.924	.281	.182	5.372
1971	1 605	.116	.173	5.735

TABLE 4.5e STIMATED OWN AND
CROSS PRICE ELASTICITIES AT THE MEAN THIRD
ORDER TRANSLOG COST FUNCTION.
US MANUFACTURING, 1947 - 1971

Elaticities	Estimated Coefficient	T- Statistic
$\bar{\varepsilon}_{LL}$	-.482	-23.386
$\bar{\varepsilon}_{EE}$	-.858	-5.524
$\bar{\varepsilon}_{MM}$	-.300	-8.416
$\bar{\varepsilon}_{KK}$	-.420	-3.473
$\bar{\varepsilon}_{LK}$	.037	2 797
$\bar{\varepsilon}_{KL}$	.187	2 797
$\bar{\varepsilon}_{EK}$	-.253	2 930
$\bar{\varepsilon}_{KE}$	-.212	2 930
$\bar{\varepsilon}_{EM}$	1.026	4.354
$\bar{\varepsilon}_{ME}$	.073	4 354
$\bar{\varepsilon}_{MK}$	.038	2 353
$\bar{\varepsilon}_{KM}$	445	2.353
$\bar{\varepsilon}_{LE}$	.014	1.454
$\bar{\varepsilon}_{EL}$	086	1.454
$\bar{\varepsilon}_{LM}$	.431	13.938
$\bar{\varepsilon}_{ML}$	.189	13.938

TABLE 4.6 SELECTED STUDIES OF CAPITAL-ENERGY COMPLEMENTARITY DEBATE

Author	Country and Industry	Data and Observation	Assumptions & Cost or Production Function	Main Results
Berndt and Christensen (1973)	U.S. Manufacturing	Time series 1929-1968	Linear Homogeneous and Separable $f(K_i, K_j, L)$	$(K_i:K_j)$, $(K_i, K_j\ L)$ = Substitutes
Hudson and Jorgensen (1974)	U.S. 9 Industrial Sectors	Time Series 1947-1971	Homogeneous and Separable (K,L,E,M)	(K:E) = Complements (L,E) = Substitutes
Berndt and Wood (1975)	U.S. Manufacturing	Time Series 1947-1971	Linear Homogeneous and Separable (K,L,E,M)	(K:L),(K:M),(L:E) (L:M),(E:M) = Substitutes; (K:E) = Complements
Brendt and Wood (1979)	U.S. Manufacturing	Time Series 1947-1971	Linear Homogeneous and Separable (K,L,E,M)	(K:E) = Net Complements, (K:E) = Gross Substitutes
Fuss (1977)	Canadian Manufacturing	Pooled Data 1961-1971	Homothetic (K,L,E,M)	(K:L),(K:M),(L:E) (L:M),$(E_i:E_j)$ = Substitutes, (K:E),(E:M) = Complements
Hazila and Koop (1984)	U.S. 34 Producing Sector	Time Series (1958-1974)	Homothetic and Separable (K,L,E,M)	(K:E),(E:L),(E:I) = Mixed Results
Chung (1987)	U.S. Manufacturing	Time Series 1947-1971	Non Homogeneous and Separable (K,L,E,M)	(K:L),(K:E),(K:M) (L:E),(L:M),(E:M) = Substitutes
Said (1991)	U.S. Manufacturing	Time Series 1947-1971	Linear Homogeneous and Separable (K,L,E,M)	(K:E),(E:M) = Mixed Results, At the Mean = Substitutes; (K:E) = Mixed Results, At the Mean = Mixed Results

Notes: K = capital, K_j = capital i (i = equipment or structures in 1973), L = labor, E = energy, M = materials, I = non-resource intermediate input.

CHAPTER 5

Testing for functional separability.

Specification of aggregate production functions, relating output to capital and labor, proved to be useful in empirical analysis of producer behavior. However, the use of aggregates of labor and aggregates of capital in aggregate production functions like the "CD" and "CES" assumes functional separability (Leontief, 1947 a,b). The multi-input version of "CD" and "CES" also impose functional separability, since there are no parameters to take into account of the interactions among the inputs in question (Berndt, 1973).

In order to resolve the rigidities of traditional functional like "CD" and "CES", various flexible functional forms were developed. The properties of these forms were discussed in Chapter 2 These forms allow us to use disaggregated inputs, and also takes into account the interactions among the inputs involved. Hence, separability becomes a testable parametric restriction.

The theoretical aspects of separability were discussed in both Chapters 2 and 3. In Chapter 2, the linear and non linear separability restrictions were derived by using the second order translog cost function In Chapter 3, the derivation was extended to a third order translog cost function.

In the present chapter, the number of inputs will be extended from three to four and the separability restrictions will be derived for every possible form of aggregation. We will then test for various separability restrictions derived from the third and second-order translog cost functions. The results will then be compared.

The data to be used in examining the separability issue is the same as those used in Chapter 4:KLEM, for U.S. manufacturing sector 1947-1971. The estimation techniques used also remains the iterative Efficient Zellner.

The fitted cost share equations derived from the third-order translog cost function with and without the separability restrictions of various types were estimated. Based on the likelihood ratio test, we were able to reject all types of separability restrictions, including the utilized capital specification[1], which could not be rejected by using the restrictions derived from the second order translog cost function. Our findings suggest that there will be a loss of explanatory power if the U.S. manufacturing sector were specified in terms of capital and labor aggregates, implied by the value added specification. This will also be true if it were specified in terms of the aggregate index formed by energy and capital, on one hand, and labor and intermediate materials on the other, implied by the utilized capital specification.

In section 5.1, a four input production function and the corresponding cost functions will be specified and the assumptions needed will be outlined. In section 5.2, the restrictions needed for various separability tests will be derived from the third-order translog cost function. In section 5.3, the empirical test results will be discussed and compared with the second-order results. In 5.4, the conclusions of the Chapter will be presented.

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[1] The value added specification is based on assumption that the aggregation of capital and labor is permitted, while utilized capital specification allows the aggregation of capital and energy.

5.1 Specifications

It is assumed that a consistent output aggregate (Y) for the U.S. manufacturing sector exists, such that $Y = f(K,L,E,M,T)$, where, K,L,E, and M are capital, labor, energy and intermediate materials respectively and T is the technology index. In order to be able to assume that the U.S. manufacturing sector can be specified in terms of Y, K, L , E, and M, we must assume that the four factors of production are functionally separable from other inputs which might have some influence in the production process. Thus, the four inputs, KLEM, may be taken as a sub-aggregate index. We also assume the existence of a corresponding sub-aggregate cost function which is represented by a third-order translog cost function, (4.1.1). As in the previous chapter, we are also assuming symmetry in input prices, linear homogeneity in input prices, constant returns to scale and Hicks neutral technical change[2].

5.2 Separability Restriction

In this section the linear and non linear separability restrictions will be derived from a four input third-order translog cost function The general formula used by Berndt and Christensen (1973b) to derive the separability restrictions is still applicable:

$$S_j^* \gamma_{ih}^* - S_i^* \gamma_{jh}^* = 0. \tag{5.2.1}$$

where, $\gamma_{ih}^* = \gamma_{ih} + \sum_n \gamma_{ihn} \ln w_n$, $i \neq h$ $\quad \gamma_{jh}^* = \gamma_{jh} + \sum_n \gamma_{jhn} \ln w_n$, $j \neq h$, $i,j,h,n = K, L, E, M$; S_i^* and S_j^* are the shares derived from the

[2] The technology index, T, is thus subsumed in the constant term of the cost function or production function.

third-order translog cost function.

For the value added specification separability takes the form ((K,L), E,M). In this case, K and L will form a weakly separable set under the following conditions :

$$S_K \gamma_{EL} - S_L \gamma_{EK} = 0 \quad \Rightarrow \quad \sigma_{EL} = \sigma_{EK}.$$

and (5.2.1a)

$$S_K \gamma_{LM} - S_L \gamma_{KM} = 0. \quad \Rightarrow \quad \sigma_{LM} = \sigma_{KM}.$$

Berndt and Christensen (1973) show that there is a correspondence between the weak separability restrictions derived from the production function and the cost function if the former function is homothetically separable.

The weak separability restriction given in (5.2.1) can be satisfied in two separate ways:

(a) since cost shares are positive by monotonicity, all of the parameters in γ^*_{ik} equal zero;

(b) the ratio of shares (S^*_j/S^*_i) equals the ratio of γ^*'s $(\gamma^*_{ji}/\gamma^*_{ij})$ Condition (a) results in a linear separability restriction. This restriction is a symmetric one and as was discussed in Chapter 2, imposes a partial Cobb Douglas specification. Condition (b), on the other hand, results in nonlinear separability restrictions, which are not symmetric. These restrictions may force certain Allen partial elasticities of substitution to be equal to each other , but not equal to unity, as in the former restrictions.

5.2a Linear Separability Restrictions

The linear separability restrictions imply symmetric relations in a separable group. For example, if inputs K and L form a weakly separable set in group [(K,L), E, M], the remaining inputs E and M are also weakly separable. This implies that the value added specification is symmetric (Berndt and Christensen, 1973), *i.e.* f[g(K, L), E, M]≡ f[g (K,L), h(E, M)]. Similar results can be shown for the utilized capital specification.

The four inputs under consideration can be put in two groups in seven distinct ways, (Berndt, 1973c). Group one specifies one input against three, while group two considers two against two inputs in terms of the dual cost function·

Group I:

$[(P_K), (P_L, P_E, P_M)]$; $[((P_L), (P_K, P_E, P_M)]$; $[(P_E), (P_K, P_L, P_M)]$, and $[(P_M), (P_K, P_L, P_E)]$

group II:

$[(P_K, P_L), (P_E, P_M)]$; $[(P_K, P_E), (P_L, P_M)]$; $[(P_K, P_M), (P_L, P_E)]$

Since the derivation techniques were discussed in Chapters 2 and 3, we will simply list the restrictions corresponding to each separability type in table 5.1 below.

Table 5.1 Linear Separability restrictions (group I)

Separability Type	Parameters Restricted to Zero		AUES Restricted To Unity
L1 $[(P_K), (P_L, P_E, P_M)]$	γ_{KL}	γ_{KLi}	σ_{KL}
	γ_{KE}	γ_{KEi}	σ_{KE}
	γ_{KM}	γ_{KMi}	σ_{KM}
	$\forall\ i = K, L, E, M.$		
L2. $[(P_L), (P_K, P_E, P_M)]$	γ_{KL}	γ_{KLi}	σ_{KL}
	γ_{LE}	γ_{LEi}	σ_{LE}
	γ_{LM}	γ_{LMi}	σ_{LM}
	$\forall\ i = K, L, E, M.$		
L3 $[(P_E), (P_K, P_L, P_M)]$	γ_{KE}	γ_{KEi}	σ_{KE}
	γ_{LE}	γ_{LEi}	σ_{LE}
	γ_{EM}	γ_{EMi}	σ_{EM}
	$\forall\ i = K, L, E, M.$		
L4 $[(P_M), (P_K, P_L, P_E)]$	γ_{KM}	γ_{KMi}	σ_{KM}
	γ_{LM}	γ_{LMi}	σ_{LM}
	γ_{EM}	γ_{EMi}	σ_{EM}
	$\forall\ i = K, L, E, M.$		

Table 5.1 Linear Separability restrictions (group II)

Separability Type	Parameters Restricted to Zero		AUES Restricted To Unity
L1. $[(P_K, P_L), (P_E, P_M)]$	γ_{KE}	γ_{KEi}	σ_{KE}
	γ_{KM}	γ_{KMi}	σ_{KM}
	γ_{LE}	γ_{LEi}	σ_{LE}
	γ_{LM}	γ_{LMi}	σ_{LM}
	$\forall\ i = K, L, E, M$		
L2. $[(P_K, P_E), (P_L, P_M)]$	γ_{KL}	γ_{KLi}	σ_{KL}
	γ_{KM}	γ_{KMi}	σ_{KM}
	γ_{LE}	γ_{LEi}	σ_{LE}
	γ_{EM}	γ_{EMi}	σ_{EM}
	$\forall\ i = K, L, E, M$		
L3. $[(P_K, P_M), (P_L, P_E)]$	γ_{KL}	γ_{KLi}	σ_{KL}
	γ_{KE}	γ_{KEi}	σ_{KE}
	γ_{LM}	γ_{LMi}	σ_{LM}
	γ_{EM}	γ_{EMi}	σ_{EM}
	$\forall\ i = K, L, E, M$		

The relevant equations to be estimated after imposing the above linear separability restrictions and the homogeneity assumptions are given in the appendix to this chapter.

5.2b Non-Linear Separability Restrictions

Unlike the previous case, the non-linear separability restrictions are not symmetric. Aside from the seven linear separability specifications discussed above, there are six additional distinct ways of aggregating inputs into groups of two:

group III:

$[(P_K, P_L), P_E, P_M]$; $[(P_K, P_E), P_L, P_M]$; $[(P_K, P_M), P_L, P_E]$;

$[(P_L, P_E), P_K, P_M]$; $[(P_L, P_M), P_K, P_E]$ and $[(P_E, P_M), P_K, P_L]$.

In the literature surveyed, the Allen partial elasticities of substitution are noted to be closely related to the concept of separability restrictions. Non-linear separability imposes restrictions on the values of the AUES.

(a) The separability type in which a group is composed of one input against three, for example, [(K), (L, E, M)] requires the equality of all three AUES In the above example the restrictions will be $\sigma_{KE} = \sigma_{KL} = \sigma_{KM}$, which implies that the optimal mix of the three inputs in the separable group be determined. If that is the case then an optimal mix of an input with the sub group aggregate will be determined.

(b) A utilized capital specification, for instance, [(K, E), (L M)], imposes the equality of all relevant AUES·

$$\sigma_{KL} = \sigma_{KM} = \sigma_{LE} = \sigma_{EM}$$

which means that it assumes the separability of both (K, E) and (L, M). The acceptance of this separability type allows for further aggregation of a consistent index of the two of sub-groups, *i.e.*, $c = f(p_K, p_L, p_E, p_M) = f_1(p_K^{*}, p_L^{*})$, where p_K^{*} is the index of p_K and p_E and p_L^{*} is an index of p_L and p_M. Alternatively the sub-groups can be analyzed separately without any loss of valuable information.

(c) In the value added specification, for example, [(K, L), E, M], there are two ways in which AUES can be restricted:

$$\sigma_{KE} = \sigma_{LE} \; ; \; \sigma_{KM} = \sigma_{LM}$$

This type of separability restriction implies two stage optimization. First, the optimal levels of the two inputs in a separable sub-set must be determined. Second, the optimal mix of the sub-group with the rest of the two inputs must be determined. If the separability restrictions could not be rejected, it implies that one can analyze the substitution possibilities between capital and labor by ignoring the information on the remaining two inputs, namely energy and intermediate materials.

The derivation of the non-linear restrictions was shown in Chapter 2 and 3 in terms of three inputs for the second and third-order translog cost functions respectively. In this section, we will extend the number of inputs to four and, hence, more separability specifications will emerge. The corresponding restrictions for the second-order translog cost function were shown by Berndt and Christensen (1973). We will list the restrictions corresponding to each separability type.

In group I (defined in terms of AUES in (a) above), there are twelve independent restrictions in each section, the first five restrictions are those that would have been derived in a four input second-order translog cost function. The next seven restrictions in each section are obtained due to the third-order extension carried out here.

Non-Linear Separability Restrictions

From the Third-Order Translog Cost Function

Separability Type (group NI)

<u>NIa $[(P_K),(P_L,P_E,P_M)]$</u>

$\gamma_L = \gamma_E \, \gamma_{LL} / \gamma_{LE}$, $\quad\gamma_{LM} = \gamma_{EM} \, \gamma_{LL} / \gamma_{LE}$

$\gamma_{EE} = \gamma_{LE}^2 / \gamma_{LL}$, $\quad\gamma_M = \gamma_E \, \gamma_{LL} \gamma_{EM} / \gamma_{LE}^2$

$\gamma_{MM} = \gamma_{EM} \, \gamma_{LL} / \gamma_{LE}^2$, $\quad\gamma_{EEE} = \gamma_{LLE} \, \gamma_{LE}^2 / \gamma_{LL}^2$

$\gamma_{LMM} = \gamma_{EEM} \, \gamma_{LL}^2 / \gamma_{LE}^2$, $\quad\gamma_{LEM} = \gamma_{EEM} \, \gamma_{LL} / \gamma_{LE}$

$\gamma_{EMM} = \gamma_{EM} \, \gamma_{EEM} \, \gamma_{LL}^2 / \gamma_{LE}^2$, $\quad\gamma_{LMM} = \gamma_{EEM} \, \gamma_{LL}^2 / \gamma_{LE}^2$

$\gamma_{LEE} = \gamma_{LE} \, \gamma_{LLE} / \gamma_{LL}$ $\quad\gamma_{MMM} = \gamma_{EM} \, \gamma_{EEM}^2 / \gamma_{LE}^2$

<u>NIb. $[(P_L),(P_K,P_E,P_M)]$</u>

$\gamma_E = (\gamma_L - 1) \, \gamma_{EM} / \gamma_{LM}$, $\quad\gamma_M = (\gamma_L - 1) \, \gamma_{LM} / \gamma_{LL}$

$\gamma_{LE} = \gamma_{LL} \, \gamma_{EM} / \gamma_{LM}$, $\quad\gamma_{EE} = \gamma_{LL} \, \gamma_{EM}^2 / \gamma_{LM}^2$

$\gamma_{MM} = \gamma^2_{LM} / \gamma_{LL}$, $\gamma_{LLE} = \gamma_{LL} \gamma_{EM} \gamma_{LLM} / \gamma^3_{LM}$

$\gamma_{LEE} = \gamma_{LL} \gamma_{LLM} \gamma^2_{EM} / \gamma^3_{LM}$, $\gamma_{EMM} = \gamma_{EEM} \gamma^2_{LM} / \gamma_{LL} \gamma_{EM}$

$\gamma_{LEM} = \gamma_{EM} \gamma_{LLM} / \gamma_{LM}$, $\gamma_{LMM} = \gamma_{LM} \gamma_{LLM} / \gamma_{LL}$

$\gamma_{EEE} = \gamma_{LL} \gamma_{EM} \gamma_{EEM} / \gamma^2_{LM}$, $\gamma_{MMM} = \gamma_{EEM} \gamma^4_{LM} / \gamma^2_{LL} \gamma^2_{EM}$

NIc. $[(P_E), (P_K, P_L, P_M)]$

$\gamma_E = (\gamma_L \gamma_{LM} / \gamma_{LL}) + 1$, $\gamma_M = \gamma_L \gamma_{LE} / \gamma_{LL}$

$\gamma_{EM} = \gamma_{LE} \gamma_{LM} / \gamma_{LL}$, $\gamma_{EE} = \gamma^2_{LE} / \gamma_{LL}$

$\gamma_{MM} = \gamma^2_{LM} / \gamma_{LL}$, $\gamma_{LEM} = \gamma_{LM} \gamma_{LLE} / \gamma_{LL}$

$\gamma_{LMM} = \gamma_{LLM} \gamma_{LM} / \gamma_{LL}$, $\gamma_{EMM} = \gamma_{LLE} \gamma^2_{LM} / \gamma^2_{LL}$

$\gamma_{LEE} = \gamma_{LE} \gamma_{LLE} / \gamma_{LL}$, $\gamma_{EEM} = \gamma_{LE} \gamma_{LM} \gamma_{LLE} / \gamma^2_{LL}$

$\gamma_{EEE} = \gamma_{LLE} \gamma^2_{LE} / \gamma^2_{LL}$, $\gamma_{MMM} = \gamma_{LLM} \gamma^2_{LM} / \gamma^2_{LL}$

NId $[(P_M), (P_K, P_L, P_E)]$

$\gamma_E = \gamma_L \gamma_{LE} / \gamma_{LL}$, $\gamma_M = (\gamma_L \gamma_{LM} / \gamma_{LL}) + 1$

$\gamma_{EM} = \gamma_{LE} \gamma_{LM} / \gamma_{LL}$, $\gamma_{EE} = \gamma^2_{LE} / \gamma_{LL}$

$\gamma_{MM} = \gamma^2_{LM} / \gamma_{LL}$, $\gamma_{LEE} = \gamma_{LE} \gamma_{LLE} / \gamma_{LL}$

$$\gamma_{LEM} = \gamma_{LE}\, \gamma_{LLM} / \gamma_{LL}\ , \qquad \gamma_{EEM} = \gamma_{LLM}\, \gamma_{LE}^{3} / \gamma_{LL}^{3}$$

$$\gamma_{LMM} = \gamma_{LM}\, \gamma_{LLM} / \gamma_{LL}\ , \qquad \gamma_{EMM} = \gamma_{LE}\, \gamma_{LLM}\, \gamma_{LM} / \gamma_{LL}^{2}$$

$$\gamma_{EEE} = \gamma_{LLE}\, \gamma_{LE}^{2} / \gamma_{LL}^{2}\ , \qquad \gamma_{MMM} = \gamma_{LLM}\, \gamma_{LM}^{2} / \gamma_{LL}^{2}$$

Separability Type NII

In this group (defined in terms of AUES in (b) above), there are also twelve independent restrictions. The first five in each section correspond to the second-order translog cost function, while the rest are additions due to the the third-order function. The restrictions in each section are identical, except for the intercept terms. For completeness all restrictions are reported below.

Separability Type (group NII)

NIIa $[(P_K, P_L), (P_E, P_M)]$

$$\gamma_{L} = (\gamma_{M}\, \gamma_{LL} / \gamma_{LM}) - \gamma_{LL} / (\gamma_{LL} + \gamma_{LM})$$

$$\gamma_{E} = \gamma_{M}\, \gamma_{LE} / \gamma_{LM}\ , \quad \gamma_{EM} = \gamma_{LE}\, \gamma_{LM} / \gamma_{LL}, \qquad \gamma_{EE} = \gamma_{LE}^{2} / \gamma_{LL}$$

$$\gamma_{MM} = \gamma_{LM}^{2} / \gamma_{LL}, \quad \gamma_{LEE} = \gamma_{LE}\, \gamma_{LLE} / \gamma_{LL}, \quad \gamma_{EEM} = \gamma_{LE}\, \gamma_{LLE}\, \gamma_{LM} / \gamma_{LL}^{2}$$
$$\gamma_{EMM} = \gamma_{LM}^{2}\, \gamma_{LLE} / \gamma_{LL}^{2}, \quad \gamma_{LMM} = \gamma_{LM}\, \gamma_{LLM} / \gamma_{LL}$$

$$\gamma_{LEM} = \gamma_{LM}\, \gamma_{LLE} / \gamma_{LL}, \quad \gamma_{EEE} = \gamma_{LE}\, \gamma_{LLE}^{2} / \gamma_{LL}^{2}, \quad \gamma_{MMM} = \gamma_{LM}^{2}\, \gamma_{LL''} / \gamma_{LL}^{2}$$

NIIb. $[(P_K, P_F), (P_L, P_M)]$

$$\gamma_L = (\gamma_E \gamma_{LL} / \gamma_{LE}) + \gamma_{LL} / (\gamma_{LL} + \gamma_{LM})$$

$$\gamma_M = (\gamma_{LM} / \gamma_{LL} + \gamma_{LM}) + (\gamma_E \gamma_{LM} / \gamma_{LE}), \quad \gamma_{EM} = \gamma_{LE} \gamma_{LM} / \gamma_{LL}$$

$$\gamma_{EE} = \gamma^2_{LE} / \gamma_{LL}, \quad \gamma_{MM} = \gamma^2_{LM} / \gamma_{LL}, \quad \gamma_{LEE} = \gamma_{LE} \gamma_{LLE} / \gamma_{LL}$$

$$\gamma_{EEM} = \gamma_{LE} \gamma_{LLE} \gamma_{LM} / \gamma^2_{LL}, \quad \gamma_{EMM} = \gamma^2_{LM} \gamma_{LLE} / \gamma^2_{LL}$$

$$\gamma_{LMM} = \gamma_{LM} \gamma_{LLM} / \gamma_{LL}, \quad \gamma_{LEM} = \gamma_{LM} \gamma_{LLE} / \gamma_{LL}$$

$$\gamma_{EEE} = \gamma_{LE} \gamma^2_{LLE} / \gamma^2_{LL}, \quad \gamma_{MMM} = \gamma^2_{LM} \gamma_{LLM} / \gamma^2_{LL}$$

NIIc $[(P_K, P_M), (P_L, P_F)]$

$$\gamma_L = (\gamma_M \gamma_{LL} / \gamma_{LM}) + (\gamma_{LL} / \gamma_{LL} + \gamma_{LE})$$

$$\gamma_E = (\gamma_{LE} / \gamma_{LL} + \gamma_{LE}) + (\gamma_M \gamma_{LE} / \gamma_{LM})$$

$$\gamma_{EM} = \gamma_{LE} \gamma_{LM} / \gamma_{LL}, \qquad \gamma_{EE} = \gamma^2_{LE} / \gamma_{LL}$$

$$\gamma_{MM} = \gamma^2_{LM} / \gamma_{LL}, \qquad \gamma_{LEE} = \gamma_{LE} \gamma_{LLE} / \gamma_{LL}$$

$$\gamma_{EEM} = \gamma_{LE} \gamma_{LLE} \gamma_{LM} / \gamma^2_{LL}, \qquad \gamma_{EMM} = \gamma^2_{LM} \gamma_{LLE} / \gamma^2_{LL}$$

$$\gamma_{LMM} = \gamma_{LM} \gamma_{LLM} / \gamma_{LL}, \qquad \gamma_{LEM} = \gamma_{LM} \gamma_{LLE} / \gamma_{LL}$$

$$\gamma_{EEE} = \gamma_{LE}\ \gamma^{2}_{LLE} \ / \ \gamma^{2}_{LL}\ , \qquad \gamma_{MMM} = \gamma^{2}_{LM}\ \gamma_{LLM} \ / \ \gamma^{2}_{LL}$$

Separability Type NIII

In this group (defined in (c) above) there are eight independent restrictions, of which the first three in each section correspond to the second-order non-linear restrictions. There are five additional independent restrictions arising out of the third-order function. These restrictions are listed below.

Separability Type (group NIII)

NIIIa. $[(P_K, P_L),\ P_E, P_M\ ]$

$$\gamma_{L} = (\gamma_{LL} / \gamma_{LE} + \gamma_{LM})\ (\gamma_{E} + \gamma_{M} - 1)$$

$$\gamma_{EE} = \gamma_{LE}[(\gamma_{LE} + \gamma_{LM}) \ / \ \gamma_{LL}\ ] - \gamma_{EM}$$

$$\gamma_{MM} = \gamma_{LM}[(\gamma_{LE} + \gamma_{LM}) \ / \ \gamma_{LL}] - \gamma_{EM}$$

$$\gamma_{LEE} = [(\ \gamma_{LLE} \ / \ \gamma_{LL})\ (\ \gamma_{LE} + \gamma_{LM})] - \gamma_{LEM}$$

$$\gamma_{LMM} = [(\ \gamma_{LLM} \ / \ \gamma_{LL})\ (\gamma_{LE} + \gamma_{LM})] - \gamma_{LEM}$$
$$\gamma_{EMM} = [(\ \gamma_{LEM} \ / \ \gamma_{LL})\ (\gamma_{LE} + \gamma_{LM})] - \gamma_{EEM}$$

$$\gamma_{EEE} = [(\ \gamma_{LE} + \gamma_{LM})\ (\gamma_{LLE}\ (\ \gamma_{LE} + \gamma_{LM}) - (\gamma_{LEM} \ / \ \gamma_{LL}))] - \gamma_{EEM}$$

$$\gamma_{MMM} = [(\ \gamma_{LE} + \gamma_{LM})\ (\gamma_{LLM}\ (\ \gamma_{LE} + \gamma_{LM}) - (2\gamma_{LEM} \ / \ \gamma_{LL}))] - \gamma_{EEM}$$

NIIIb. $[(P_K, P_E), P_L, P_M]$

$$\gamma_L = (1 - \gamma_M) + \gamma_E (\gamma_{LL} + \gamma_{LM}) / \gamma_{LE}$$

$$\gamma_{EE} = \gamma_{LE} / (\gamma_{LL} + \gamma_{LM}) (\gamma_{LE} + \gamma_{LM})$$

$$\gamma_{MM} = \gamma_{EM} (\gamma_{LL} + \gamma_{LM}) / \gamma_{LE}) - \gamma_{LM}$$

$$\gamma_{LEE} = \gamma_{LE} (\gamma_{LLE} + \gamma_{LEM}) / (\gamma_{LL} + \gamma_{LM})$$

$$\gamma_{LMM} = \gamma_{LEM} [(\gamma_{LL} + \gamma_{LM}) / \gamma_{LE}] - \gamma_{LEM}$$

$$\gamma_{EMM} = [\gamma_{EEM} (\gamma_{LL} + \gamma_{LM}) / \gamma_{LE}] - \gamma_{LEM}$$

$$\gamma_{EEE} = (\gamma_{LE} / \gamma_{LL} + \gamma_{LM})^2 (\gamma_{LLE} + \gamma_{LEM}) + \gamma_{EEM}$$

$$\gamma_{MMM} = (\gamma_{LL} + \gamma_{LM})^2 (\gamma_{EEM} / \gamma_{LE}) - (2\gamma_{EM} / \gamma_{LE}) + \gamma_{LLM}$$

NIIIc. $[(P_K, P_M), P_L, P_E]$

$$\gamma_L = 1 - \gamma_E + (\gamma_M / \gamma_{LM}) (\gamma_{LL} + \gamma_{LE})$$

$$\gamma_{EE} = \gamma_{EM} (\gamma_{LL} + \gamma_{LE} / \gamma_{LM}) - \gamma_{LE}$$

$$\gamma_{MM} = \gamma_{LM} (\gamma_{LM} + \gamma_{EM} / \gamma_{LL} + \gamma_{LE})$$

$$\gamma_{LEE} = \gamma_{LEM} (\gamma_{LL} + \gamma_{LE} / \gamma_{LM}) - \gamma_{LLE}$$

$$\gamma_{LMM} = \gamma_{LM} (\gamma_{LLM} + \gamma_{LEM} / \gamma_{LL} + \gamma_{LE})$$

$$\gamma_{EMM} = \gamma_{LM} (\gamma_{LEM} + \gamma_{EEM} / \gamma_{LL} + \gamma_{LE})$$

$$\gamma_{EEE} = (\gamma_{LL} + \gamma_{LE}) (\gamma_{EEM} - \gamma_{LEM}) / \gamma_{LM}) - \gamma_{LLE}$$

$$\gamma_{MMM} = (\gamma_{LM} / \gamma_{LL} + \gamma_{LE})^2 (\gamma_{LLM} + \gamma_{EEM} + 2 \gamma_{LEM}).$$

<u>NIIId. $[(P_L, P_E), P_K, P_M]$</u>

$$\gamma_L = \gamma_E \gamma_{LL} / \gamma_{LE}$$

$$\gamma_{EE} = \gamma_{LE}^2 / \gamma_{LL} , \qquad \gamma_{LM} = \gamma_{EM} \gamma_{LL} / \gamma_{LE}$$

$$\gamma_{LEE} = \gamma_{LE} \gamma_{LLE} / \gamma_{LL} , \qquad \gamma_{LEM} = \gamma_{LL} \gamma_{LE} / \gamma_{EEM}$$

$$\gamma_{LMM} = \gamma_{LL} \gamma_{EMM} / \gamma_{LE} , \qquad \gamma_{LLM} = (\gamma_{LL} / \gamma_{LE})^2 \gamma_{EEM}$$

$$\gamma_{EEE} = (\gamma_{LE} / \gamma_{LL})^2 \gamma_{LLE}$$

<u>NIIIe. $[(P_L, P_M), P_K, P_E]$</u>

$$\gamma_L = \gamma_M \gamma_{LL} / \gamma_{LM} , \qquad \gamma_{LE} = \gamma_{LL} \gamma_{EM} / \gamma_{LM}$$

$$\gamma_{MM} = \gamma_{LM}^2 / \gamma_{LL} , \qquad \gamma_{LEE} = \gamma_{LL} \gamma_{EEM} / \gamma_{LM}$$

$$\gamma_{LLE} = \gamma_{LL}^2 \gamma_{EMM} / \gamma_{LM}^2$$

$$\gamma_{LEM} = \gamma_{LL} \gamma_{EMM} / \gamma_{LM}, \qquad \gamma_{LMM} = \gamma_{LM} \gamma_{LMM} / \gamma_{LL}$$

$$\gamma_{MMM} = \gamma_{LM}^2 \gamma_{LLM} / \gamma_{LL}^2$$

NIIIF. $[(P_E, P_M), P_K, P_L]$

$$\gamma_E = \gamma_M \gamma_{LE} / \gamma_{LM}, \qquad \gamma_{EE} = \gamma_{LE} \gamma_{EM} / \gamma_{LM}$$

$$\gamma_{MM} = \gamma_{LM} \gamma_{EM} / \gamma_{LE}, \qquad \gamma_{LEE} = \gamma_{LE} \gamma_{LEM} / \gamma_{LM}$$

$$\gamma_{LMM} = \gamma_{LM} \gamma_{LEM} / \gamma_{LE}, \qquad \gamma_{EMM} = \gamma_{LM} \gamma_{EEM} / \gamma_{LE}$$

$$\gamma_{EEE} = \gamma_{LE} \gamma_{EEM} / \gamma_{LM}, \qquad \gamma_{MMM} = \gamma_{LM}^2 \gamma_{EEM} / \gamma_{LE}^2$$

5.3 Empirical Results

We have estimated the third-order translog cost function using data for the U.S. manufacturing sector with the linear and non-linear separability restrictions imposed respectively. The function was found to be well behaved in that the positivity, as well as the concavity requirement were satisfied at all points except for the years 1947 and 1948. The convergence criteria were met in every case. The likelihood ratios (LR) are reported in tables 5.2 and 5.3 below for all the separability restrictions given above.

Based on the likelihood-ratio test, we were able to reject all types of linear and non linear separability restrictions, including the one case that Berndt and Wood (1975) could not reject: the separability restrictions derived for the utilized capital specification. The utilized capital specification has played a major role in modeling industrial demand for energy, since this specification has, in general not been rejected. This utilized capital specification replaced the value added specification since the latter specification was rejected by previous empirical studies (for example, Berndt and Wood, 1975). The utilized capital specification was also used to reconcile the important debate on energy-capital complementarity. Berndt and Wood (1979) argued that it is possible for two inputs to display both gross substitutability and net complementarity. Their argument was mainly based on their ability to decompose the net price elasticity into gross price elasticity and the expansion elasticity. This enabled them to test their proposition empirically. The gross price elasticity was calculated by taking the logarithmic partial derivative of an input with respect to a relevant input price assuming the constancy of all other inputs outside the separable set. This implies that the effects of labor and the intermediate materials were ignored in the exercise in determining energy-capital complementarity. This exercise would be valid only if the utilized capital specification could not be rejected. Hence, the rejection of this specification in the present study implies that future work on energy demand or any of the remaining factors of production in U S manufacturing should take all inputs into account.

However, our model in general does not over reject compared to the second-order translog cost function. This fact is revealed by examining the entries of the LR obtained for the two models. Comparing the absolute values of the LR obtained by imposing the linear restriction it may seem that the third-order function over rejects. However, since we are comparing these LR values with the critical values based on the degrees of freedom, we find that this is not the case. The LR value obtained from the non-linear restrictions are clearly smaller, even in absolute value, when compared to the ones from the second-order function.

Table 5.2 Test Statistics For Linear separability Restrictions

Separability Type	Number of Parameters restricted to Zero		Test Statistics (LR).	
	3^{rd} Order	2^{nd} Order	3^{rd} Order	2^{nd} Order.
$(P_K),(P_I,P_E,P_M)$	12	3	67	46.79
$(P_I),(P_K,P_I,P_M)$	12	3	285.39	291 62
$(P_I),(P_K,P_L,P_M)$	12	3	318.08	296.02
$(P_M),(P_K,P_I,P_E)$	12	3	285.80	268.54
$(P_K,P_L),(P_E,P_M)$	16	4	81.45	58.51
$(P_K,P_F),(P_I,P_M)$	16	4	34.00	10.32^{*}
$(P_E,P_M),(P_K,P_L)$	16	4	70.77	49.08

$\chi^2_{.01}(3) = 11.34$, $\chi^2_{.01}(4) = 13.28$, $\chi^2_{.01}(12) = 26.22$, $\chi^2_{.01}(16) = 32.$

Table 5.3 Test Statistics For NON Linear separability Restrictions

Separability Type	Number of Parameters restricted to Zero		Test Statistics (LR)	
	3^{rd} Order	2^{nd} Order	3^{rd} Order	2^{nd} Order
$(P_K),(P_L,P_F,P_M)$	12	5	274.87	61.00
$(P_L),(P_K,P_L,P_M)$	12	5	42.47	68.26
$(P_E),(P_K,P_L,P_M)$	12	5	83.20	106.10
$(P_M),(P_K,P_L,P_E)$	12	5	320.89	309.63
$(P_K,P_L),(P_E,P_M)$	12	5	84.47	69.39
$(P_K,P_E),(P_L,P_M)$	12	5	104.29	251.07
$(P_K,P_M),(P_K,P_L)$	12	5	54.66	78.62
$(P_K,P_L),P_E,P_M$	8	3	22.91	45.17
$(P_K,P_E),P_L,P_M$	8	3	21.81	234.13
$(P_K,P_M),P_L,P_E$	8	3	38.43	43.76
$(P_L,P_M),P_K,P_E$	8	3	29.71	50.97
$(P_L,P_E),P_K,P_M$	8	3	45.34	311.64
$(P_E,P_M),P_K,P_L$	8	3	33.17	18.12

$\chi^2_{.01}(3) = 11.34$, $\chi^2_{.01}(5) = 15.09$, $\chi^2_{.01}(8) = 20.09$, $\chi^2_{.01}(12) = 26.22$

5.4 Summary and Conclusion

In this chapter we have derived and tested various separability restrictions both from the second and third-order translog cost functions The results from the second-order function were consistent with the ones reported by Berndt and Wood (1975). However, we have included four cases that were not dealt with in their paper. It has been our contention that the second-order translog cost function is biased as shown in Chapter 4; hence, results obtained from it are not reliable

Our separability results emerge from the precision of the formulas used to derive the required restrictions. Expression (5.2.1), which defines the necessary conditions for separability, embodies variable share elasticities and the share of inputs. The advantages of the third-order function lies both in the variability of share elasticities and in that the estimated share equations are far more accurate compared to the second-order This point was established in Chapter 4. In addition to this, the restrictions derived from the third-order translog cost function are numerically greater than those from the second-order. Therefore, the third-order specification allows us to perform a more rigorous testing of functional separability.

In this chapter we found that the results differ between the second and third-order functions, which have important policy implications. The analysis of substitution possibilities and forecasting of investment demands of various inputs relies on separability results. The issue of the value added specification was settled, since like Berndt and Wood (1975) we find that the data did not substantiate it. However, at least for the U.S. manufacturing sector, they could not reject the grouping of capital and energy (what they called the utilized capital specification). However, we find little support for this hypothesis. We note that although, our model has more separability restrictions, it does not over reject these restrictions when compared to the second-order cost function. This fact can be discovered by examining the likelihood ratios given for various separability tests. The LR entries are actually smaller in absolute magnitudes. In the case of linear tests the absolute numbers seem to be larger than the second-order case, but for hypothesis testing we have to take the degrees of freedom into account. When this was done, we found that our model has smaller log-likelihood ratio values compared to its counterpart. Thus, the present rejection of the only kind of separability (utilized capital specification) that has survived earlier analysis, is not due to over rejections, but because the estimates are superior and the formulas used to test separability are more flexible and precise. Therefore, our result on utilized capital specification suggests that any future empirical work on the demand analysis of the U.S. manufacturing must take all four inputs into account. Evidently, the above remark does not necessarily hold if one were to adopt a different set of inputs (and/or use different definitions of inputs than used here). Our analysis suggests, however, that one has to undertake a very careful testing of the possible separability restrictions before one is entitled to use indexes to support separable groups of inputs.

Appendix to Chapter 5.

The Imposition of Linear Separability in the Four Input Translog Cost function:

1.1. The zero restrictions listed in table 5.1 with the linear homogeneity restriction with respect to prices, force the parameters γ_{KK} and γ_{KKK} to be zero. Thus, the share of capital equation will be left only with the intercept term The rest of the system takes the following form:

$$S_L = \gamma_L + \gamma_{LE} (\ln w_E - \ln w_L) + \gamma_{LM} (\ln w_M - \ln w_L) +$$

$$\gamma_{LLE} [\ln w_L \ln w_E - \ln w_E \ln w_M - .5(\ln w_L)^2] +$$

$$\gamma_{LLM} [\ln w_L \ln w_M - .5(\ln w_L)^2 + \gamma_{LEE} (.5(\ln w_E)^2$$
$$- \ln w_E \ln w_M] + \gamma_{LLM} .5(\ln w_M)^2$$

$$S_E = \gamma_E + \gamma_{LE} (\ln w_L - \ln w_E) + \gamma_{EM} (\ln w_M - \ln w_E) +$$

$$\gamma_{LLE} [.5(\ln w_L)^2 - \ln w_L \ln w_M] + \gamma_{LEE} [\ln w_L \ln w_E -$$

$$\ln w_L \ln w_M - .5(\ln w_E)^2] + \gamma_{EEM} (\ln w_E \ln w_M - .5(\ln w_E)^2]$$

$$+ \gamma_{EMM} .5(\ln w_M)^2 .$$

$$S_M = \gamma_M + \gamma_{LM} (\ln w_L - \ln w_M) + \gamma_{EM} (\ln w_E - \ln w_M) +$$

$$\gamma_{EEM} [.5(\ln w_E)^2 - \ln w_L \ln w_E] + \gamma_{EMM} [\ln w_E \ln w_M$$

$$- \ln w_L \ln w_E - .5(\ln w_M)^2] + \gamma_{LMM} [\ln w_L \ln w_M$$

$$- .5(\ln w_M)^2] \qquad (5.2.2)$$

L2. In this section the separability and homogeneity restriction render both γ_{LL} and γ_{LLL} to be zero. Thus, only the intercept term survive these zero restrictions in the labor share equation. Since we have used the parameters of the deleted equation (capital) to get the equality of γ_{LLL} to be equal to zero, the restricted version takes the following form (The estimates of parameters of capital equation can still be recovered using linear homogeneity restrictions.)

$$S_E = \gamma_E + \gamma_{EE} (\ln w_E - \ln w_K) + \gamma_{EM} (\ln w_M - \ln w_K)$$

$$+ \gamma_{EEE} [.5(\ln w_E)^2 - \ln w_K \ln w_E + .5(\ln w_K)^2] + \gamma_{EEM} [\ln w_E$$

$$\ln w_M + (\ln w_K)^2 - \ln w_K \ln w_E - \ln w_K \ln w_M]$$

$$S_M = \gamma_M + \gamma_{EM} (\ln w_E - \ln w_K) + \gamma_{MM} (\ln w_M - \ln w_K)$$

$$+ \gamma_{EEM} [.5(\ln w_E)^2 + .5(\ln w_K)^2 - \ln w_K \ln w_E]$$

$$+ \gamma_{EMM} [\ln w_E \ln w_M + (\ln w_K)^2 - \ln w_K \ln w_E$$

$$+ \gamma_{MMM} [.5(\ln w_E)^2 + .5(\ln w_K)^2 - \ln w_K \ln w_M] \qquad (5.2.3)$$

L3 After imposing I3 and the linear homogeneity requirement on the share equations, the share equation for energy is reduced to its intercept term, since γ_{EE} and γ_{EEE} will be zero. The corresponding estimated system of equations will be:

$$S_L \quad \gamma_L + \gamma_{LL} (\ln w_L - \ln w_K) + \gamma_{LM} (\ln w_M - \ln w_K) +$$

$$\gamma_{LLL} [5(\ln w_L)^2 + 5(\ln w_K)^2 - \ln w_K \ln w_L] +$$

$$\gamma_{LLM} [(\ln w_K)^2 + \ln w_L \ln w_M - \ln w_K \ln w_L - \ln w_K \ln w_M]$$

$$+ \gamma_{LMM} [(5(\ln w_M)^2 + (\ln w_K)^2 - \ln w_k \ln w_M]$$

$$S_M = \gamma_M + \gamma_{LM} (\ln w_L - \ln w_K) + \gamma_{MM} (\ln w_M - \ln w_K)$$

$$+ \gamma_{LLM} [5(\ln w_L)^2 + .5(\ln w_K)^2 - \ln w_K \ln w_L - \ln w_K \ln w_M]$$

$$+ \gamma_{LMM} [\ln w_L \ln w_M + (\ln w_K)^2 - \ln w_K \ln w_L]$$

$$+ \gamma_{MMM} [.5(\ln w_M)^2 + .5(\ln w_K)^2 - \ln w_K \ln w_M] \qquad (5.2.4)$$

L4 In this case, the intermediate materials equation is reduced to its intercept term, $\gamma_{MM} = \gamma_{MMM} = 0$. The corresponding restricted version takes the following form.

$$S_L \quad \gamma_L + \gamma_{LL} (\ln w_L - \ln w_K) + \gamma_{LE} (\ln w_E - \ln w_K) +$$

$$\gamma_{LLL} [(\ln w_L)^2 + .5(\ln w_K)^2 - \ln w_K \ln w_L] +$$

$$\gamma_{LLE} [(\ln w_K)^2 + \ln w_L \ln w_E - \ln w_K \ln w_L - \ln w_K \ln w_E]$$

$$+ \gamma_{LEE} [.5 (\ln w_E)^2 + .5(\ln w_K)^2 \ln w_K \ln w_E]$$

$$S_E = \gamma_E + \gamma_{LE} (\ln w_L - \ln w_K) + \gamma_{EE} (\ln w_E - \ln w_K)$$

$$+ \gamma_{LLE} [.5 (\ln w_L)^2 - \ln w_K \ln w_L + .5(\ln w_K)^2]$$

$$+ \gamma_{LEE} [(\ln w_K)^2 + \ln w_L \ln w_E - \ln w_K \ln w_L - \ln w_K \ln w_E]$$

$$+ \gamma_{EEE} [.5 (\ln w_E)^2 + .5 [(\ln w_E)^2 - .5(\ln w_K)^2$$

$$- \ln w_K \ln w_E] \qquad (5.2.5)$$

Group II Linear Restrictions.

L1. Restricting parameters to zero and linear homogeneity restrictions allow us to express all the parameters of the restricted system only in terms of γ_{LL}, γ_{LLL}, γ_{EE}, γ_{EEE} and their respective intercept terms. Thus, we get the following restricted system to be estimated.

$$S_L = \gamma_L + \gamma_{LL} (\ln w_L - \ln w_K) + \gamma_{LLL} [.5(\ln w_K)^2 + .5(\ln w_L)^2$$

$$- \ln w_K \ln w_L]$$

$$S_E = \gamma_E + \gamma_{EE} (\ln w_E - \ln w_M) + \gamma_{EEE} [.5(\ln w_E)^2 - \ln w_E \ln w_M + .5(\ln w_M)^2].$$

$$S_M = \gamma_M + \gamma_{EE} (\ln w_M - \ln w_E) + \gamma_{EEE} [\ln w_E \ln w_M - .5(\ln w_M)^2 -.5(\ln w_E)^2]. \qquad (5.2.6)$$

The non zero parameters can be recovered using the linear separability and linear homogeneity restrictions.

L2 The parameters in this group can be expressed in terms of the intercepts and γ_{LL}, γ_{LLL}, γ_{EE}, γ_{EEE}. The restricted three share equations are.

$$S_L = \gamma_L + \gamma_{LL} (\ln w_L - \ln w_M) + \gamma_{LLL} [.5(\ln w_L)^2 + .5(\ln w_M)^2 - \ln w_L \ln w_M]$$

$$S_E = \gamma_E + \gamma_{EE} (\ln w_E - \ln w_K) + \gamma_{EEE} [.5(\ln w_E)^2 - \ln w_K \ln w_E + 5(\ln w_K)^2].$$

$$S_M = \gamma_M + \gamma_{LL} (\ln w_M - \ln w_L) + \gamma_{LLL} [\ln w_L \ln w_M - .5(\ln w_M)^2 -.5(\ln w_L)^2]. \qquad (5.2.7)$$

L3 The restricted above equations in this group takes the following form:

$$S_L = \gamma_L + \gamma_{LL}\,(\ln w_L - \ln w_E) + \gamma_{LLL}\,[.5(\ln w_L)^2 + .5(\ln w_E)^2$$

$$- \ln w_L \ln w_E].$$

$$S_E = \gamma_E + \gamma_{LL}\,(\ln w_E - \ln w_L) + \gamma_{LLL}\,[-.5(\ln w_E)^2 + \ln w_L \ln w_E$$

$$-.5(\ln w_L)^2].$$

$$S_M = \gamma_M + \gamma_{MM}\,(\ln w_M - \ln w_K) + \gamma_{MMM}\,[.5(\ln w_M)^2 - \ln w_K \ln w_M$$

$$+ 5(\ln w_K)^2]. \qquad (5.2.8)$$

CHAPTER 6

SUMMARY AND CONCLUSION

The use of flexible functional forms based on second-order approximation has received wide applicability in the empirical investigation of producer behavior since the early seventies. The attractive features of these functions include: the ability to include the effects of the interaction of the right hand variables on the dependent variables, the ability to test the traditionally maintained hypothesis within the model (such as functional separability, homogeneity in both prices and inputs, returns to scale, and technological change), the ability to derive input demands clearly expressed in terms of their arguments, and the ability to measure and test the significance of different types of elasticities, including the Allen Uzawa elasticities of substitution, which are not assumed to be either equal to unity or constants. This flexibility in the derived Allen partial elasticities of substitution has made a major contribution in determining the substitution possibilities among inputs involved in the production process.

The advantages of using a third-order cost function in terms of (I) econometric modeling, (II) theoretical and (III) empirical analysis of producer behavior are summarized below.

(I) Econometric modeling

Modeling producer behavior by using a third-order translog cost function includes parameters that are not considered in the second-order translog cost function, and all of these parameters have economic meaning. The disadvantage of the loss of degrees of freedom was minimized by using various restrictions such as symmetry and linear homogeneity in input prices and output.

The extended model and the restrictions needed to qualify any arbitrary function as representative of production technology were given in sections (3.1) and (3.2). The additional restrictions were found to be helpful in testing more rigorously certain requirements that must be fulfilled by any arbitrary function.

(II) Theoretical justification

(a) Reduction in bias:

Kmenta (1971) and Byron et al (1983) have rightfully argued that limiting functions to a second-order Taylor series approximation introduces truncation bias. Using an arbitrary Taylor series approximations they were able to show a significant reduction as one goes to the third-order. We put their claim to an empirical test based on the U. S. manufacturing sector and the results obtained confirm their claim. The reduction in bias implies superior estimates Many time-honoured studies in demand analysis like Berndt and Wood (1979) and Lau (1986) have suggested the need to investigate an alternative model specification in order to settle many unresolved issues and allow for a more rigorous examination of producer behavior.

(a) Flexibility and Precision

(1) Functions derived as the first-order partial derivatives of the

extended function are better approximations of the underlying function they represent, than ones derived from the second-order cost function. The reason is that these functions are second-order rather than the first-order approximation . These functions include input demands, the share of inputs, cost flexibility, measure of technical change, and measures of returns to scale.

(2) The functions derived as second-order partial derivatives from the objective function (cost function) are not constant but flexible. Hence, we get variable share elasticities, variable measure of economies of scale, variable technical change, variable expansion elasticities (scale bias), and variable output sensitivity of cost flexibility as a result of the third-order nature of our model. All these measures are assumed to be constant in the second-order cost functions.

Finally, due to the above variable measures, we also get more flexibility in the following quantities

(i) The Allen Uzawa elasticities of substitution formula (AUES) now includes a variable share elasticity that allows adjustment of output and input prices. The variable share elasticity included in the AUES formula allows the measure to exhibit a magnitude greater or less than unity, depending on the level of input prices and output involved. This result is a major contribution in determining the substitution possibilities between inputs over a given period. The Allen partial elasticities of substitution formula derived from a second-order cost function, on the other hand, limits the magnitude to be either less than unity or greater than unity. This implies that any two inputs in a set that include many inputs in the production process will remain substitutes or complements regardless of what happens to the level of input prices, output, and technological change over the whole period under consideration

(ii) The own and cross price elasticities of factor demands will also be more flexible, as they too depend on variable share elasticities The relevant formula derived from our model will be flexible enough to indicate a movement from an inelastic to an elastic region of the factor demand curve, since the value of the own price elasticities can now range from greater than unity to less than unity

(iii) Output elasticities of input demand are also more flexible than the ones that are derived from the second-order function The flexibility of the former depends on the variable expansion elasticity while the latter depends on the constant expansion elasticity

(3) Taking the third derivative of the cost function is a new addition to the analysis of producer behavior The resulting new measures (that are of great importance in the study of producer behavior) include (i) The rate of change of input demand with respect to prices and output, (ii) the rates of change of the share of an input with respect to input prices and output, and (iii) the rate of cost flexibility with respect to output The list also includes measures of sensitivity such as those of share elasticities with respect to input prices, output and input price sensitivity of bias of scale These measures now allow us to examine the following

(a) the rates of change of functions derived as the first-order partial derivative of the cost function and also

(b) the sensitivity of the functions derived as the second-order partial derivative of the objective function with respect to its argument

In the case of the second-order cost functions, these measures cannot be derived since they are assumed, *a priori*, to be zero.

(4) Functional separability

The precision of the estimated parameters, shares and share elasticities, imply the unbiasedness of the expressions from which various separability restrictions are derived. The restrictions necessary for various kinds of functional separability were derived for both the second and third-order translog functions in terms of three and four input prices. The restrictions for the second-order function are obviously the same as the ones given by the existing literature in the analysis of producer behavior. When the objective function is extended to a third-order form, we are able to derive additional restrictions for every separability type. The additional separability restrictions derived can easily be recognized as extensions of the second-order ones. The numbers and the ways in which these restrictions are derived are given in Chapters 2, 3, and 5. The larger number of separability restrictions derived, allow for more rigorous testing of the existence of separability among inputs and if the test passes these strict restrictions, the results can be more reliable than ones obtained from using the second-order cost function (since the restrictions depend on variable share elasticities and shares which are better approximation of the true functions).

(III) Empirical Validation

In order to give more credibility to our theoretical contentions, we have in Chapters 4 and 5, analyzed the U.S. manufacturing sector

1947-1971 empirically, using the third-order translog cost function. We compared the results of the third-order translog cost function with those of the second-order translog cost function. The main findings were as follows.

(1) In order for any arbitrary function to be called a cost function, it must be a real valued function. It must increase with respect to input prices and be concave in factor prices. The positivity requirement was met at every data point since the arbitrary function and the shares derived from it were positive. The concavity requirement was also met at every data point except for 1947 and 1948. This requirement was checked by using the negative semi-definiteness of the Hessian matrix based on the IZ estimates.

(2) Once the regularity conditions for the third-order function were satisfied, the specification test was carried out. The second-order translog cost function was clearly rejected in favor of the third-order cost function. This result was an important one, in that the results derived from the rejected model were not as reliable as the ones derived from the alternative model.

(3) In Chapter 3, we have argued that the truncation bias of the economic relationships estimated from the third-order translog cost function will be smaller than the one derived from the second-order cost function. We have taken the predicted share of inputs from the two models and computed the measure of inaccuracy given by Theil et al. (1966). Based on this measure we found a smaller level of inaccuracy in the predicted shares derived from the third-order cost function. Since the shares of inputs are closely related to factor demands and also enter into most of the important formulas that are used to analyze

producer behavior, the reduced bias should make a significant difference in the results.

(4) The variable share elasticities derived from the third-order function contain the constant share elasticities as special cases. The empirical results showed that the share elasticities were in fact changing over the years, although, the constant share elasticities were not excluded from the third-order model. Because of the variable share elasticity formula, we were able to investigate if in fact the shares responded to factor price changes at a constant rate. The results obtained by examining the significance of the sensitivity parameters (included in the respective formulas) showed otherwise. It is true that some shares exhibited a constant rate of change with respect to some input prices. However, they also responded at a decreasing or increasing rate with respect to the remaining prices. Thus, the constant rate of change of shares assumed in the case of the second-order function is incorrect.

We have also performed hypothesis testing to find out whether or not the estimated share elasticities were different from zero. The elasticities were estimated at every point and at their respective means. The ones estimated at every data point showed a significant variability for a number of cases, while the ones calculated at their means showed results similar to the ones obtained from the second-order cost function on a year to year basis, with the exception of the own share elasticity of energy and the cross share elasticities of labor and energy. In the second-order case, the own share elasticity of energy was found to be significant, while the cross share elasticities of labor and energy was found to be insignificant. The share elasticities estimated at every data point and at their respective means by using a third-order function showed opposite results for both the former and latter. However, the second-order cost function results reflect an average tendency and fail to reflect what is going on year by year or from one sub-period to the next sub-period.

(5) The energy-capital complementarity debate has been the focal point in the analysis of producer behavior. The use of flexible forms in empirical analysis certainly allows us to examine complementarity and substitutability in the production process. However, the rigidities on the AUES formula do not allow for the possibility that any two inputs in the production process which exhibit substitutability in one period, may actually exhibit complementarity in another sub-period. The AUES values calculated from the second-order translog cost function remained stable in magnitude and sign while the ones calculated from the more flexible third-order translog cost function did not remain stable in both magnitude and sign. Examination of the significance levels of the estimated Allen partial elasticities of substitution between energy and capital at every point reveals that these two inputs did not remain complements for the whole period. During most of 1960s, there was no significant relationship between these two inputs. Note that energy and labor were found to be substitutes by the estimates of the relevant formula derived from second-order cost function. Based on the third-order formula, the substitution possibilities have changed during the study period. The two inputs remained substitutes until the mid 1960's and then showed high complementarity for most of the remaining period. The estimated values of factor price elasticities have also changed in magnitude and sign over the period. The above results imply

that there were changing substitution possibilities between any two inputs during the twenty five year period under consideration This and similar findings in this thesis, suggest that there is a need to examine policies, periodically, to see if the substitution possibilities have remained the same as expected

However, energy and capital complementarity analyzed at the mean of the third-order function was in agreement with the results derived from the second-order function, displaying substantial complementarity, while energy and labor substitutability estimated at the mean was not significantly different from zero, since the positive and negative values for Allen elasticities of substitution were almost evenly matched Hence, on the average, there was no apparent dependence between these two variables Thus, average tendencies again cover up what has happened during each sub-period

From the above results, one can conclude that the contradictory results on the question of substitutability reported by different authors are all reconcilable The different results obtained by different authors appear to be due to the different data sets they used and to the difference between the short run and the long run values of the variables under investigation The debate on substitution possibilities took a wrong turn, simply because the models used in the past studies were rigid in that they excluded the possibility of observing the relationship between the variables period by period. Thus, it is entirely possible to report energy-labor substitutability based on a data set dominated in the early years of our study, while it also possible to get energy-labor complementarity if the study is based on the latter years of our study period. It is equally plausible to get different results depending on whether one examines short term and long term substitution possibilities An advantage of our model is that it nests all these different tendencies into one and reveals the internal structures of the variables in question, thereby, reconciling the different results obtained in the past

(b) We have also found different estimated separability results that could not have been found by using a second-order translog cost function We tested seven linear separability types and thirteen non-linear separability types from both the second and third order translog cost functions In the case of the second order function we have rejected all but one linear separability type $[(P_K, P_E), (P_L, P_M)]$ In the case of the third-order translog cost function, all of the separability types were rejected, decisively By examining the LR of the linear separability restrictions the lowest entries obtained were for the separability type $[(P_K, P_E), (P_L, P_M)]$ for both models However, in the case of the third-order cost function it was not low enough for acceptance In the case of nonlinear separability, group NI type, the lowest entry was for the grouping of capital with the aggregates of labor, energy and intermediate materials $[(P_K), (P_L, P_E, P_M)]$ for both models For the group NII types, the grouping of capital and labor with energy and intermediate materials $[(P_K, P_L), (P_E, P_M)]$ does less damage to the data in both models. In the last nonlinear separability type NIII, there is a consistency between the linear and non-linear separability types in the case of the third-order cost function in that the lowest entries are for $[(P_K, P_E), P_L, P_M]$ The lowest entry in this group for the second-order cost function, linear case was like the above while the lowest LR entry for non linear case in the second-order cost function was $[(P_E, P_M), P_K, P_L]$ Comparisons of the LR entries of the models showed that our model did not over reject the separability restrictions when the degrees of freedom were taken into account. Hence, the rejection of the utilized capital specification by our model was not due to the over rejection but was due to the precision of the estimates and the restrictions derived from our third-order cost function.

Areas for future Research

Estimates based on third-order translog function revealed results on some of the important questions concerning the study of producer behavior. It could be worthwhile to revisit some of the contentious issues in the study of producer behavior. Such issues include the testing of the neo-classical theory, the choice of functional forms (preferably by extending the Box-Cox function used by Applebaum or Berndt and Khaled to a third-order form), the question of technological change, and the measurement of the inputs and input prices involved in the U.S. manufacturing sector.

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