NEW FOUNDATION MODELS

Walid Hashem Hadid

A Thesis

in

The Faculty

of

Engineering

Presented in Partial Fulfillment of the Requirements
for the degree of Master of Engineering at
Concordia University
Montreal, Quebec, Canada

April 1983

ABSTRACT

NEW FOUNDATION MODELS

Walid Hashem Hadid

Due to the recent development in construction techniques and fabrications of precast concrete units, new foundations modes were proposed. It is believed, that these models will have higher bearing capacity and produce less settlement than the similar conventional ones.

The present study, was limited to the analysis of stress distribution below different models. The result showed a reduction in the vertical, horizontal and shear stresses, within a depth equal to the foundation width up to 20%, 45% and 109%, respectively.

ACKNOWLEDGEMENTS

Sharing science and knowledge is one of the reasons for the continuity of civilization.

I wish to express my sincere gratitude to Professor A.M. Hanna, for his guidance, support and expert advice in this field.

Many thanks go out also to Professors H.B. Poorooshasb and Z. Khalil for their assistance and discussions.

I would like to thank all my friends and especially Mr. Nagie Gergis and Miss Ghislaine Cross for their interest, also Miss Maggie Pierotti for typing this thesis.

My most heartful thanks to my family for their consideration and especially to my brother Nohad, for his untiring dedication and unending patience, without whose help and constant support this would not have been possible.

TABLE OF CONTENTS

Abstract iii

Acknowledgements iv

Table of Contents v

List of Symbols vii

List of Figures ix

List of Tables xiii

CHAPTER I

1. Introduction 1

CHAPTER II

2.1 General 2

2.2 Equilibrium Equation's of Soil Element 3

2.2.1 In Rectangular Coordinates 3

2.2.2 In Cylindrical Coordinates 6

2.2.3 Relations between Stresses in Polar and Rectangular Coordinates 9

2.2.4 Compatibility Equations 11

2.2.5 The Stress Function 14

2.3 Stresses in Soil due to External Loads 17

2.3.1 Geostatic Stress 17

2.3.2 Two-dimensional Problem 17

2.3.3 Three-dimensional Problem 20

2.3.4 In Case of a Vertical Line Loading is Acting Beneath the Semi-infinite Mass 30

2.3.5 In Case of Vertical Point Load Acting Beneath the Surface of a Semi-infinite Mass 32

2.4 Winkler Model or Soil Behaviour 35

CHAPTER III

3.1 General 39

3.2 General Definition of a Surface 39

3.3 Circular Footing 43

3.4 Strip Footing 47

3.5 Cylindrical Strip Footing 52

3.6 Triangular Strip Footing 58

3.7 Spherical Footing 61

3.8 The S-sin Strip Model 68

3.9 Computer Program 70

CHAPTER IV

4.1 General 78

4.2 Program Data 78

4.3 Program Output 81

4.4 Typical Graphs 98

CHAPTER V

Conclusion and Recommendation 117

REFERENCES 118

LIST OF SYMBOLS

A	: area
a	: side length for soil element
α	: angle in the x,z-plane
α_n	: function of α
B	: half width of the foundation
β	: angle in x,y-plane
ψ	: stress function
C	: constant
d	: distance
dA	: unit area
$d\alpha$	: increment
$d\beta$	: increment
ds	: increment
E	: modulus of elasticity
ε_i	: strain in the i direction
F_i	: force
γ_0	: unit weight
γ_{ij}	: strain in the i,j-plane
K	: constant
M	: a point where the stresses are analysed
Mi	: moment
ν	: Poisson's ratio
N	: a point where the load is applied

$O(x,y,z)$	:	system of coordinates
$O(r,\theta,z)$	:	system of coordinates
$\overline{ON}$	:	vector
$\overline{ON},i$	:	the first derivative of vector $\overline{ON}$
P	:	load applied on the foundation
q	:	point load or line load
q_0	:	point load or line load
q_1	:	point load or line load
q_2	:	point load or line load
R	:	radius
R_1	:	distance
R_2	:	distance
S	:	variable in the x direction
σ_i	:	normal stress in the i direction
θ	:	angle in the x,z-plane
u	:	elongation
v	:	elongation
w	:	elongation
w_1	:	deflexion
w_2	:	deflexion
$\overline{X}$	:	component of stresses in the x direction
$\overline{Z}$	:	component of stresses in the z direction.

LIST OF FIGURES

CHAPTER II

Figure 1 : Stress tensor components in rectangular components . 4

Figure 2 : Plane stress components in Cartisien coordinates ... 4

Figure 3 : An element in Polar coordinates 7

Figure 4 (4-a, 4-b, 4-c): Plane stress in cylindrical coordinates 8

Figure 5 : The relationships between plane stresses in rectangular and polar coordinates 10

Figure 6 : An element under loading 12

Figure 7 : Stress - strain graph 12

Figure 8 (8-a, 8-b): Radial distribution of stress as used by "Boussinesq" 18

Figure 9 : Point loading acting on a semi-infinite mass 21

Figure 10: Spherical coordinates 21

Figure 11: Point loading acting at the center of the sphere 24

Figure 12: Uniform pressure on the sphere represented by a load acting in the r, z, direction and in the axis normal to plane (r,z) 26

Figure 13: Vertical line loading beneath the surface of semi-infinite mass 31

Figure 14: Vertical concentrated load beneath the surface of the soil 33

Figure 15: Surface displacement of the Winkler model due to non-uniform load 36

Figure 16: Surface displacement under a cylindrical rigid footing, using Winkler model 37

CHAPTER III

Figure 17: A surface in triaxial system $O_1(X,Y,Z)$ 40
Figure 18: An element of a footing surface 41
Figure 19: An infinitesimal unit area dA 41
Figure 20: Figure showing the vectors and stresses analysis for a circular flat footing 44
Figure 21: Figure showing the vectors and stress analysis for a flat strip footing 48
Figure 22: Shows the cylindrical strip footing 53
Figure 23: A sector divided into equal portions of value dα. ... 53
Figure 24: Figure showing the vectors and stresses for a cylindrical strip footing 56
Figure 25: Shows the triangular strip footing 59
Figure 26: Figure showing the vectors and stresses on a cylindrical strip footing 59
Figure 27: Figure showing a spherical footing 62
Figure 28: Vectors analysis for the unit infinitesimal area dA . 62
Figure 29: Figure showing the vectors and stresses analysis for a spherical footing 65
Figure 30: The shape of the model S-sin 69
Figure 31: Figure showing the vectors and stresses analysis for the shape S-sin 69

CHAPTER IV

Figure 32 (32-a, 32-b), ... 79

(32-c, 32-d): Shows different types of footing 80

Figure 33: This mesh indicates the location of the points, where the stresses are calculated 82

Figure 34: Vertical stresses at depth Z = ·5B for flat and circular models 99

Figure 35: Vertical stresses at depth Z = B for flat and cylindrical models 100

Figure 36: Vertical stresses at Z = 1·5B for cylindrical and flat models 101

Figure 37: Vertical stresses at depth Z = ·5B for flat and triangular models 102

Figure 38: Vertical stresses at depth Z = B for flat and triangular models 103

Figure 39: Vertical stresses at depth Z = 1·5B for flat and triangular models 104

Figure 40: Vertical stresses at depth Z = ·5B for flat and S-sin models 105

Figure 41: Vertical stresses at depth Z = B for flat and S-sin models 106

Figure 42: Vertical stresses at depth Z = 1·5B for flat and S-sin models 107

Figure 43: Vertical stresses at X = 0 for flat and S-sin models 108

Figure 44: Vertical stresses at width X = ·5B for flat and S-sin models 109

Figure 45: Vertical stresses at width X = B for flat and S-in models 110

Figure 46: Horizontal stresses at Z = ·5B for flat and S-sin models 111

Figure 47: Horizontal stresses at Z = B for flat and S-sin models 112

Figure 48: Horizontal stresses at Z = 1·5B for flat and S-sin models 113

Figure 49: Shear stress at depth Z = ·5B for flat and S-sin models 114

Figure 50: Shear stresses at depth Z = B for flat and S-sin models 115

Figure 51: Shear stresses at Z = 1·5B for flat and S-sin models 116

LIST OF TABLES

CHAPTER IV

Table 1 : Vertical stress for a flat strip footing 83

Table 2 : Horizontal stress for a flat strip footing 83

Table 3 : Shear stress for a flat strip footing 84

Table 4 : Vertical stress for a cylindrical strip footing 84

Table 5 : Horizontal stress for a cylindrical strip footing .. 85

Table 6 : Shear stress for a cylindrical strip footing 85

Table 7 : Vertical Stress for a cylindrical strip footing 86

Table 8 : Horizontal stress for a cylindrical strip footing .. 86

Table 9 : Shear stress for a cylindrical strip footing 87

Table 10: Vertical stress for cylindrical strip footing 87

Table 11: Horizontal stress for a cylindrical strip footing .. 88

Table 12: Shear stress for a cylindrical strip footing 88

Table 13: Vertical stress for a triangular strip footing 89

Table 14: Horizontal stress for a triangular strip footing ... 89

Table 15: Shear stress for a triangular strip footing 90

Table 16: Vertical stress for a triangular strip footing 90

Table 17: Horizontal stress for a triangular strip footing ... 91

Table 18: Shear stress for a triangular strip footing 91

Table 19: Vertical stress for a triangular strip footing 92

Table 20: Horizontal stress for a triangular strip footing ... 92

Table 21: Shear stress for a triangular strip footing 93

Table 22: Vertical stress for a flat circular footing 93

Table 23: Vertical stress for a spherical footing 94
Table 24: Vertical stress for a spherical footing 94
Table 25: Vertical stress for a S-sin model strip footing 95
Table 26: Horizontal stress for S-sin model strip footing 95
Table 27: Shear stress for a S-sin model strip footing 96
Table 28: The ratio of vertical stress 96
Table 29: The ratio of horizontal stress 97
Table 30: The ratio of shear stress 97

CHAPTER I

INTRODUCTION

The science of foundations started to exist at the end of the nineteenth century, however, due to the recent development of construction techniques and the need for high-rise buildings, alternatives should be considered rather than the conventional flat ones to increase bearing capacity, and to reduce settlements of the foundations.

In the design of foundations two requirements must be satisfied, the allowable load applied by the structure should be equal to ultimate bearing capacity of the soil divided by a safety factor, and second the settlement of the foundation should be within tolerable range by the structure.

After insuring the safety of the structure (in the form of the above mentioned two requirements), the reduction of the cost is a second goal. This could be achieved by conducting related research which leads to a better understanding of the performance of these foundations, and developing new foundation models to improve such performance.

This investigation is to develop a new type of foundation, which will produce a better uniform stress below the foundation and consequently the settlement.

CHAPTER II

THEORETICAL BACKGROUND

2.1 GENERAL

In this chapter background of the theoretical models is briefly described.

Consider the soil as homogeneous isotropic and linear material, the problem of stresses could be a three-dimensional problem (case of point loading), or two-dimensional problem (case of strip line loading).

The theory of elasticity is used for the purpose of analysis. The stress function will be the solution to the equations of equilibrium with respect to the boundary conditions, and the compatibility equation.

The classical problem of Boussinesq dealing with a normal force applied at the plane boundary of a semi-infinite mass, may be solved by superposing solutions, derived from Kelvin and Boussinesq. The Kelvin solution may be used in studying stresses due to a force applied at a great distance from a boundary, the Boussinesq solution is applicable in the case, where the force acts at the surface. The solution proposed by Melan's (1932) and Mindlin (1936) is applicable for the case, where the load is near the surface.

The inherent complexity in the behaviour of soil has led to the development of many idealized models of soil behaviour. Winkler's model is one of these models, which assumes that the deflexion of the soil is directly proportional to the load applied.

2.2 EQUILIBRIUM EQUATION'S OF SOIL ELEMENT

2.2.1 In Rectangular Coordinates.

Assume an element M Figure 1 at depth Z, three components stresses are acting on each side of the cube, a normal stress σ_1 where (i=1,...,6) two shear stresses σ_{ij} and σ_{ik}, where (j=1,...,6) and (k=1,...,6). On two opposite sides there are increments in the stresses.

For equilibrium, the sum of forces and moments in X, Y and Z directions must be equal to zero (see Figure 1), where dx·dz, dx·dy and dy·dz are the areas of the sides of the cube.

For $\Sigma F_x = 0$

$$\sigma_x - (\sigma_x - \frac{\partial\sigma_x}{\partial x} dx) dydz + (-\tau z_x + \tau z_x + \frac{\partial\tau_{zx}}{\partial z} dz)dxdy$$

$$+ (\tau_{yx} + \frac{\partial\tau_{yx}}{\partial y} dy - \tau_{yx})dxdz = 0$$

After simplification, the sum of $\Sigma F_x = 0$ yields to

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{zx}}{\partial z} + \frac{\partial\tau_{yx}}{\partial y} = 0 \qquad (1)$$

similarly for

$\Sigma F_y = 0$

then

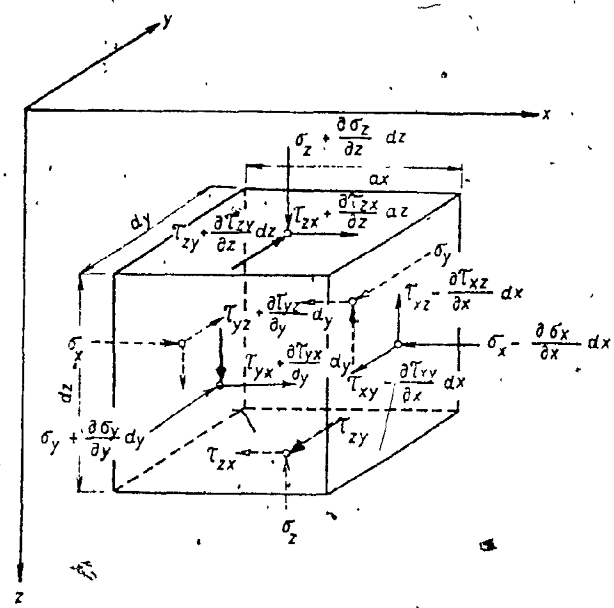


Figure 1. Stress tensor components in rectangular coordinates.

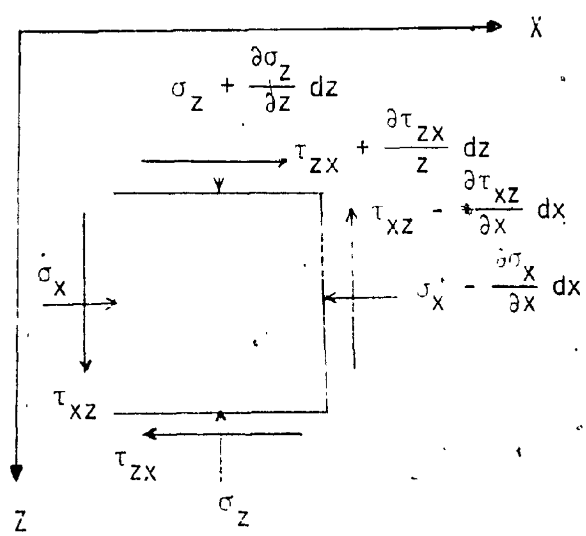


Figure 2. Plane stress components in Cartisien coordinates.

$$\frac{\partial\tau_y}{\partial y} + \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\tau_{zy}}{\partial z} = 0 \qquad (2)$$

and for

$$\Sigma F_z = 0$$

then

$$\frac{\partial\tau_z}{\partial z} + \frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} = \gamma_0 \qquad (3)$$

where γ_0 is the unit weight of the soil.

In case of the two-dimensional problem (Figure 2) (plane (X,Z)), the equations (1, 2 and 3) will be reduced to

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{zx}}{\partial z} = 0 \qquad (1\text{-a})$$

$$\frac{\partial\sigma_z}{\partial z} + \frac{\partial\tau_{xz}}{\partial x} = \gamma_0 \qquad (3\text{-a})$$

Also the sum of the moment in the x direction is given by

$$\Sigma M_x = 0 \qquad (4)$$

Thus

$$(\tau_{zy} + \tau_{zy} + \frac{\partial\tau_{zy}}{\partial z}\,dz)dxdy\,\frac{dz}{2} -$$

$$(\tau_{yz} + \frac{\partial\tau_{yz}}{\partial y}\,dy + \tau_{yz})dxdz\,\frac{dy}{2} = 0$$

After simplification

$$(2\tau_{zy} + \frac{\partial\tau_{zy}}{\partial z}\,dz) - (2\tau_{yz} + \frac{\partial\tau_{yz}}{\partial y}\,dy) = 0$$

$$2(\tau_{zy}-\tau_{yz})+(\frac{\partial\tau_{zy}}{\partial z}\,dz - \frac{\partial\tau_{yz}}{\partial y}\,dy) = 0$$

when dy and dz tend to zero, all terms in the above equation, except τ_{zy} and τ_{yz} approach zero, and thus

$$\tau_{zy} - \tau_{yz} = 0$$

then

$$\tau_{zy} = \tau_{yz} \qquad (4\text{-a})$$

Similarly the sum of the moments in the y and z directions

$$\Sigma M_y = 0 \qquad (5)$$

thus

$$\tau_{zx} = \tau_{xz} \qquad (5\text{-a})$$

and

$$\Sigma M_z = 0 \qquad (6)$$

yields

$$\tau_{yx} = \tau_{xy}. \qquad (6\text{-a})$$

2.2.2 In Cylindrical Coordinates.

Assume an element M located at a depth Z from the ground level at a distance r from the z axis, Figures 3 and 4. These Figures show the stresses acting on the element projected in planes (r,β), (z,r) and (β,z).

For equilibrium, the sum of forces and moments in the radial direction r, in the tangential direction β, and in Z direction, must be equal to zero.

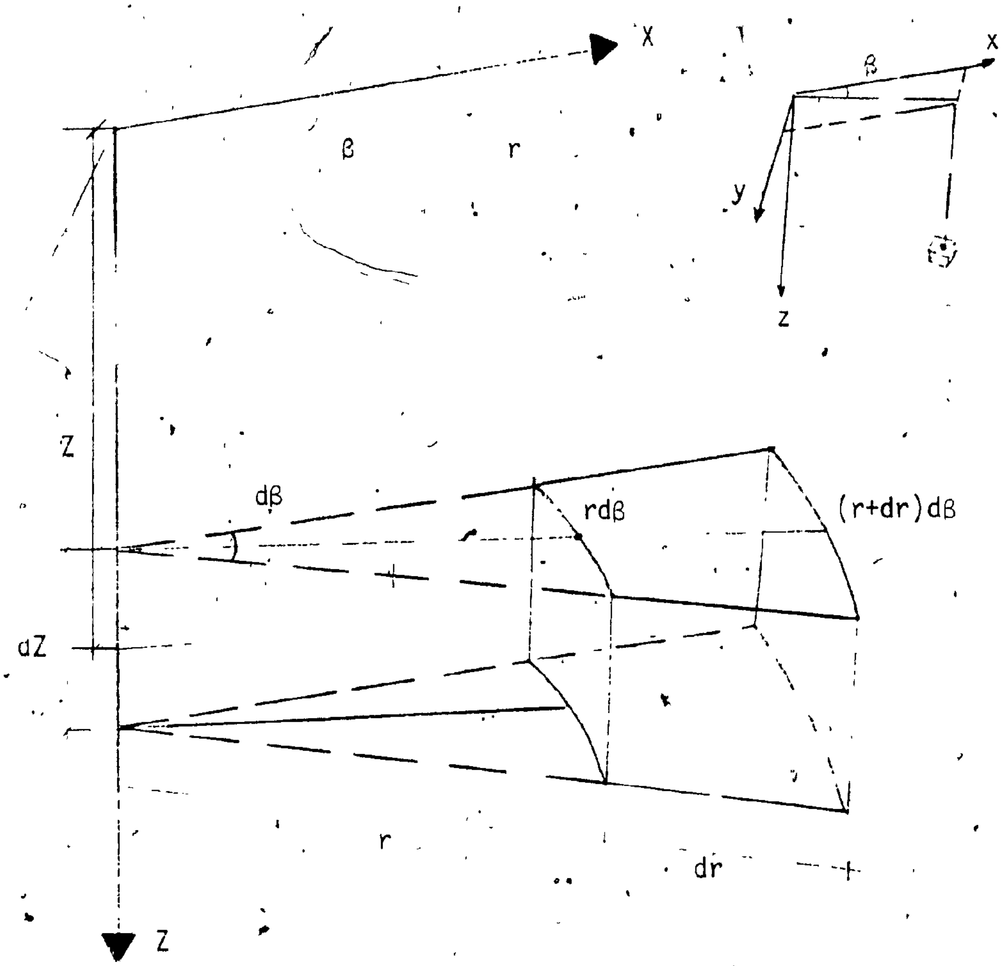
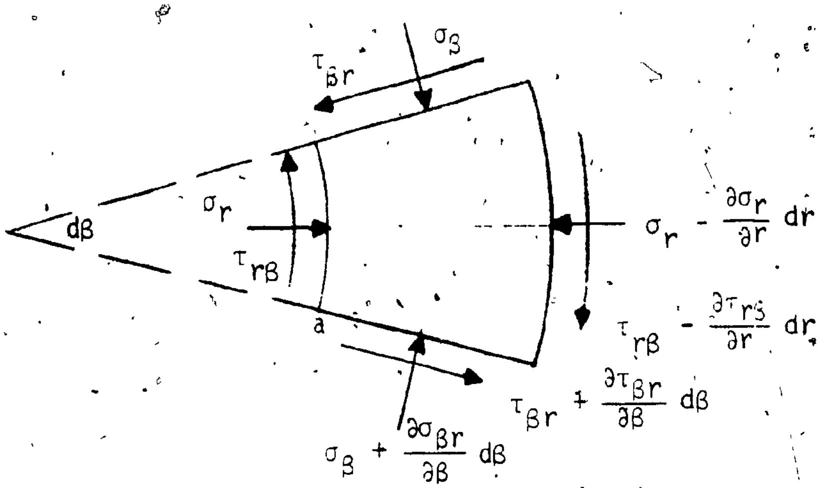
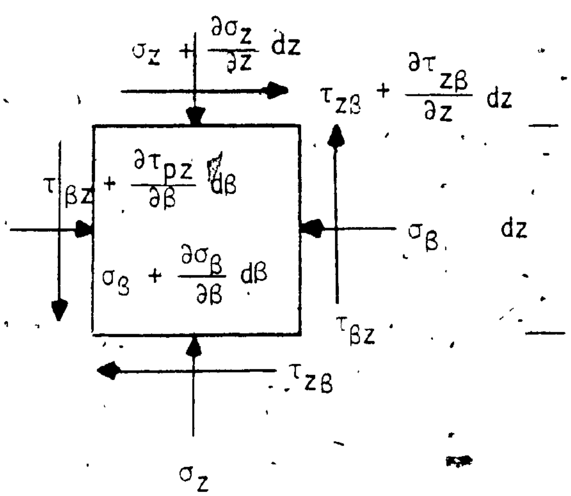


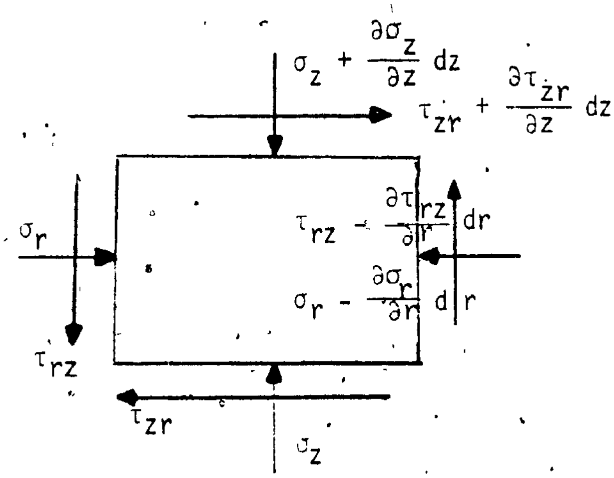
Figure 3: An element in polar coordinates.



4-a. Plane (r,β)



4-b. Plane (z,β)



4-c. Plane (z,r)

Figure 4: Plane stress in cylindrical coordinates.

Hence, we have

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\beta}}{\partial \beta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\beta}{r} = 0 \qquad (7)$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\beta z}}{\partial \beta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \qquad (8)$$

$$\frac{\partial \tau_{r\beta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\beta}{\partial \beta} + \frac{\partial \tau_{\beta z}}{\partial z} + \frac{2\tau_{r\beta}}{r} = 0 \qquad (9)$$

In case of plane stress (x,z), polar coordinates (r,θ) can be used by replacing β with θ in Figure 4-a, the equations of equilibrium become

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (10)$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + 2\frac{\tau_{r\theta}}{r} = 0 \qquad (11)$$

2.2.3 Relations between Stresses in Polar and Rectangular Coordinates.

Assume an element abc where the plane bc is parallel to the y axis (see Figure 5). If A is the area of the side bc and θ is the angle between the normal $\bar{n}$ and x axis, the areas of the two other sides are A cos θ and A sin θ.

If we denote by $\bar{X}$ and $\bar{Z}$, the components of the stresses acting on the side bc. The equilibrium in the x and z directions yields

$$\begin{aligned} \bar{X} &= \sigma_x \cos\theta + \tau_{xz} \sin\theta \\ \bar{Z} &= \sigma_x \sin\theta + \tau_{xz} \cos\theta \end{aligned} \qquad (12)$$

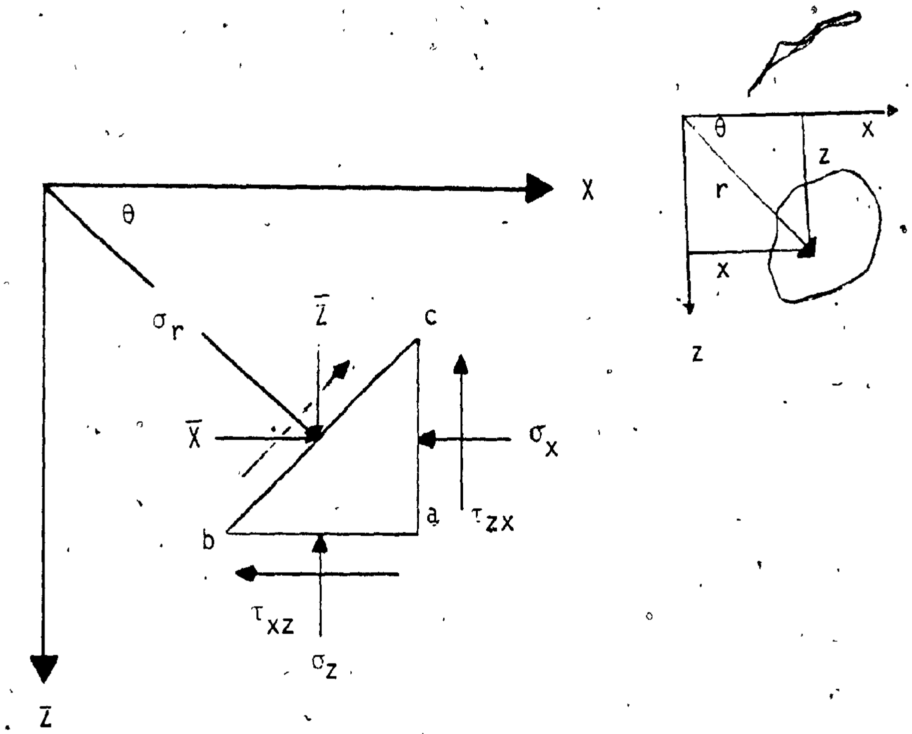


Figure 5. The relationships between plane stresses in rectangular and polar coordinates.

However, the stresses σ_r and $\tau_{r\theta}$, can be written as

$$\sigma_r = \overline{X} \cos \theta + \overline{Z} \sin \theta$$
$$\tau_{r\theta} = \overline{X} \sin \theta + \overline{Z} \cos \theta \qquad (13)$$

Substituting equation 12 into equation 13, yields

$$\sigma_r = \sigma_x \cos^2 \theta + \sigma_z \sin^2 \theta + 2 \tau_{xz} \sin \theta \cos \theta \qquad (14)$$

$$\tau_{r\theta} = \tau_{xy}(\cos^2 \theta - \sin^2 \theta) + (\sigma_z - \sigma_x) \sin \theta \cos \theta \qquad (15)$$

The same analysis could be used in the θ direction yielding

$$\sigma_\theta = \sigma_x \sin^2 \theta + \sigma_z \cos^2 \theta - 2\tau_{xz} \sin \theta \cos \theta \qquad (16)$$

Similarly, σ_x, σ_z and τ_{xz} could be expressed in terms of σ_r, σ_θ and $\tau_{r\theta}$ by

$$\sigma_x = \sigma_r \cos^2 \theta + \sigma_\theta \sin^2 \theta - 2\tau_{r\theta} \sin \theta \cos \theta \qquad (17)$$

$$\sigma_z = \sigma_r \sin^2 \theta + \sigma_\theta \cos^2 \theta + 2\tau_{r\theta} \sin \theta \cos \theta \qquad (18)$$

$$\sigma_{xz} = (\sigma_r - \sigma_\theta) \sin \theta \cos \theta + \tau_{r\theta} (\cos^2 \theta - \sin^2 \theta) \qquad (19)$$

2.2.4 Compatibility Equations

Assume a cube element M in a soil mass is subject to a load F_x, F_y and F_z (as shown in Figure 6), the coordinates of the center of the element are x, y, z, and the cube has a side a.

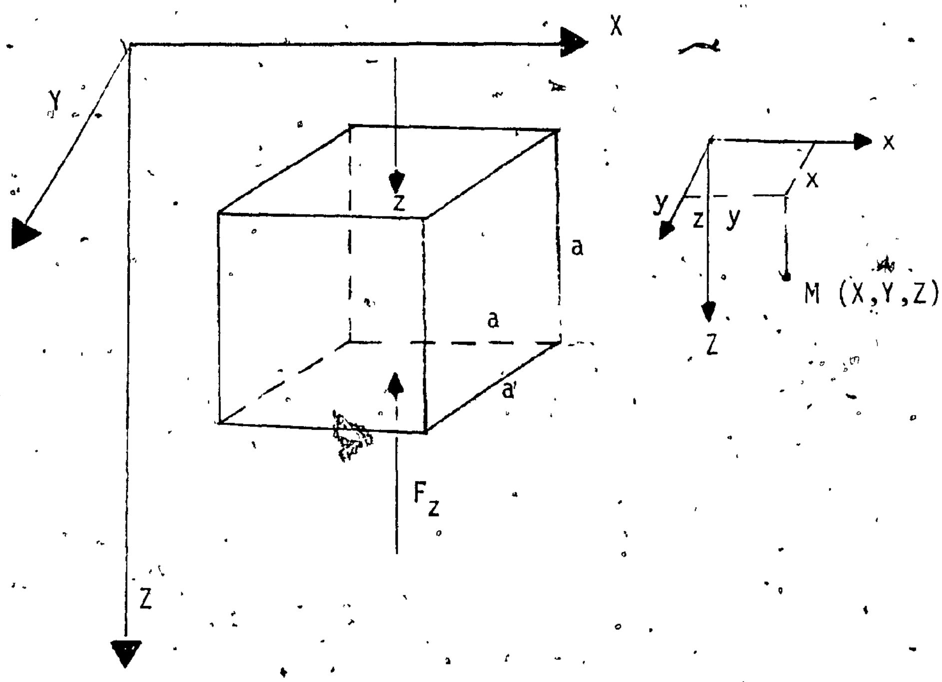


Figure 6. An element under loading.

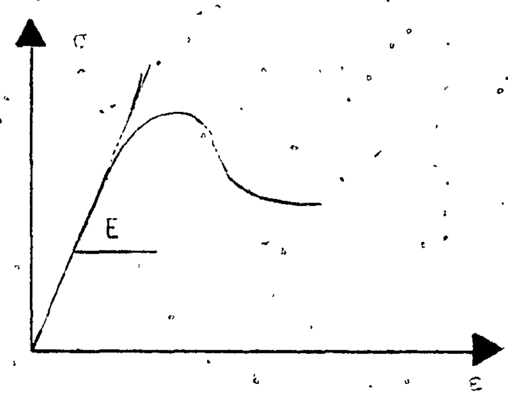


Figure 7. Stress - strain graph.

The deformations demonstrated as an elongation in the x, y and z directions are u, v and w, and an angular deformation. The elongation by unit length (strain) ε are given by the following equations

$$\varepsilon_x = \frac{\partial u}{\partial x} \quad (20)$$

$$\varepsilon_y = \frac{\partial v}{\partial y} \quad (21)$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \quad (22)$$

and the angular deformation (shear strain)

In plane (x,y) $$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \quad (23)$$

In plane (x,z) $$\gamma_{xz} = \frac{\partial w}{\partial x} + \frac{\partial v}{\partial z} \quad (24)$$

In plane (y,z) $$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad (25)$$

when the force F_z is only applied on the cube M, then the stress which is a force per unit area, could be written as a function of strain, as

$$\sigma_z = E\varepsilon_z \quad (26)$$

where E is modulus of elasticity and is the slope of the linear part of Figure 7, the extension of the element in x and y direction is

$$\varepsilon_x = -\nu \frac{\sigma_z}{E} \quad (27)$$

$$\varepsilon_y = -\nu \frac{\sigma_z}{E} \quad (28)$$

where ν is the Poisson's ratio.

If the loads F_x and F_y are applied simultaneously with F_z, the superposition of all strains in the same direction, expressed as follows

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z)) \tag{29}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu(\sigma_x + \sigma_z)) \tag{30}$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu(\sigma_x + \sigma_y)) \tag{31}$$

In case of a two-dimensional problem (plane (x,z)), differentiating equation (20) twice with respect to z, similarly differentiating equation (22) twice with respect to x, and differentiating equation (24), once with respect to x, and once with respect to z, leads to the following equation called the condition of compatibility

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} - \frac{\partial^2 \gamma_{xz}}{\partial x \, \partial z} = 0 \tag{32}$$

Substituting equations (29), (30) and (31) in equation (32) yields to the equation of compatibility

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x \ \sigma_y) \tag{33}$$

2.2.5 The Stress Function

In order to determine the stresses at a given element M in a soil mass, a solution to the differential equations of equilibrium with respect to the compatibility equation and the boundary conditions, can be obtained by using the stress function ψ.

For the case of two-dimensional problem, the equations of equilibrium (1-a), (3-a) and the compatibility equation (33):

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma_0 \qquad \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} = \gamma_0 \tag{34}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)(\sigma_x + \sigma_z) = 0 \tag{35}$$

the stress function ψ can be the solution of equations (34) and (35), and the stresses σ_x, σ_z, τ_{xz} are given by

$$\sigma_x = \frac{\partial^2 \psi}{\partial z^2} + \gamma_0 z \tag{36}$$

$$\sigma_z = \frac{\partial^2 \psi}{\partial x^2} + \gamma_0 z \tag{37}$$

$$\tau_{xz} = -\frac{\partial^2 \psi}{\partial x \partial z} \tag{38}$$

Substituting the value of stresses σ_x, σ_z and τ_{xz} equations (36), (37), (38), into equations (29), (30), (31), (32), and the result into equation 32, the following partial differential equation is obtained.

$$\frac{\partial^4 \psi}{\partial x^4} + \frac{2\,\partial^4 \psi}{\partial x^2 \partial z^2} + \frac{\partial^4 \psi}{\partial z^4} = \nabla^2 \nabla^2 \psi = 0 \tag{39}$$

where ∇^2 is the laplace operator:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{39-a}$$

To obtain the solution in polar coordinates, equation (39) can be used. However, Figure 5 shows

$$x = r\cos\theta \qquad (40\text{-a})$$

$$z = r\sin\theta \qquad (40\text{-b})$$

$$\theta = \tan^{-1}\left(\frac{z}{x}\right) \qquad (40\text{-c})$$

thus

$$\frac{\partial r}{\partial x} = \frac{x}{r} = \sin\theta \qquad (41\text{-a})$$

$$\frac{\partial \theta}{\partial x} = \frac{z}{r^2} = \frac{-\sin\theta}{r} \qquad (41\text{-b})$$

$$\frac{\partial \theta}{\partial z} = \frac{\cos\theta}{r} \qquad (41\text{-c})$$

thus

$$\frac{\partial \psi}{\partial x} = \frac{\partial \psi}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial x} = \cos\theta\,\frac{\partial \psi}{\partial r} - \frac{\sin\theta}{r}\frac{\partial \psi}{\partial \theta} \qquad (42)$$

Repeating the differentiation until the fourth order with respect to x and z, after substituting in equation (39) the following equation is obtained

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)\left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}\right) = \nabla^2\nabla^2\psi = 0 \qquad (43)$$

Solutions to the partial differential equations (39) and (43) are solutions to several problems in foundation, with respect to the compatibility and for various boundary conditions.

2.3 STRESSES IN SOIL DUE TO EXTERNAL LOADS

2.3.1 Geostatic Stress

The vertical geostatic stresses is obtained by integrating equation (3)

$$\sigma_z = \int_0^z \gamma_0 dz - \int_0^z \frac{\partial \tau_{xz}}{\partial x} dz - \int_0^z \frac{\partial \tau_{yz}}{\partial y} dz \quad (44)$$

If the surface of the soil is horizontal and the external force is equal to zero, and γ_0 is constant, then equation (44) becomes

$$\sigma_z = \int_0^z \gamma_0 dz = \gamma_0 z \quad (45)$$

2.3.2 Two-dimensional Problem

In many problems, two-dimensional stress conditions are encountered (case of strip foundation, see Figure 8), the strain in the y direction is zero. From equation (30) ε_y is given by

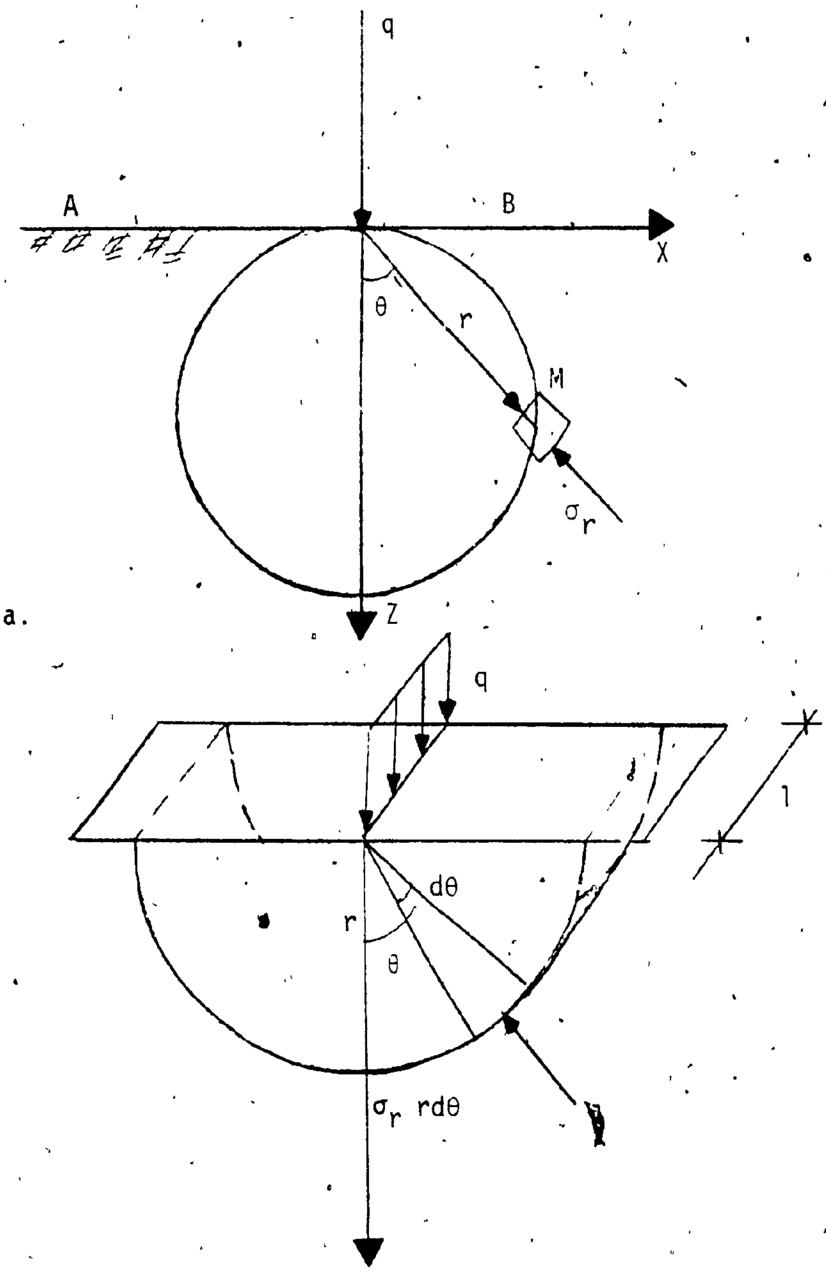
$$\varepsilon_y = \frac{1}{E} (\sigma_y - \nu(\sigma_x + \sigma_z)) = 0$$

thus

$$\sigma_y = \nu(\sigma_x + \sigma_z) \quad (46)$$

Consider a concentrated vertical load q acting on a horizontal plate AB (as shown in Figure 8-a), the thickness of the plate is taken as unity (as shown in Figure 8-b). The distribution of the load along the thickness is uniform.

Any element M at a distance r from the point of application of the load, is subjected to a simple compression in the radial direction.



8-a.

8-b.

Figure 8. Radial distribution of stress as used by "Boussinesq".

The stress components are

$$\sigma_r = \frac{2q}{\pi} \frac{\cos\theta}{r} \tag{47}$$

$$\sigma_\theta = \tau_{r\theta} = 0 \tag{48}$$

the boundary condition is satisfied, the sum of vertical stress acting on the element $rd\theta$ is equal to zero (see Figure 8-b).

$$\int_0^\pi \sigma_r \, r \cos\theta d\theta = \frac{4q}{\pi} \int_0^{\frac{\pi}{2}} \cos^2\theta d\theta = q \tag{49}$$

the equation of compatibility (equation (43)) is also satisfied

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}\right) = 0$$

where

$$\psi = \frac{q}{\pi} r\,\theta \sin\theta \tag{50}$$

In this case the solutions of the equations of equilibrium (7, 8 and 9) are given by

$$\sigma_r = \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial^2\theta} \tag{51}$$

$$\sigma_\theta = \frac{\partial^2\psi}{\partial r^2} \tag{52}$$

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}\right) \tag{53}$$

Substituting equation (50) into equations (51), (52) and (53) yields

$$\sigma_r = \frac{2q}{\pi r}\cos\theta \tag{54}$$

$$\sigma_\theta = \tau_{r\theta} = 0 \tag{55}$$

To obtain the stresses in the rectangular coordinates substitute equation (54) and (55) for equations (17), (18) and (19). Replacing sin (θ) by $\frac{x}{r}$ and cos (θ) by $\frac{z}{r}$, we get

$$\sigma_z = \sigma_r \cos^2\theta = \frac{2q}{\pi}\frac{\cos^3\theta}{r} = \frac{2q}{\pi}\frac{z^3}{r^4} \tag{56}$$

$$\sigma_x = \sigma_r \sin^2\theta = \frac{2q}{\pi}\sin^2\theta \cos^2\theta = \frac{2q}{\pi}\frac{x^2z}{r^4} \tag{57}$$

$$\tau_{xz} = \sigma_r(\sin\theta \cdot \cos\theta) = \frac{2}{\pi} q \frac{\sin\theta \cos^2\theta}{r} = \frac{2q}{\pi}\frac{xz^2}{r^4} \tag{58}$$

2.3.3 Point Loading

The point point loading q is acting on a surface of a semi infinite mass along the Z axis (see Figure 9). Referring to the equations of equilibrium ((7), (8) and (9)) in cylindrical coordinates the stress (σ_r, σ_β and $\sigma_{r\beta}$), and the displacements (u, v and w) respectively in x, y, z directions are independent of β, due to the symmetry. Thus, the strains ε_r, ε_β, ε_z and γ_{rz} can be written as

$$\varepsilon_r = \frac{\partial u}{\partial r} \tag{59}$$

$$\varepsilon_\beta = \frac{u}{r} \tag{60}$$

$$\varepsilon_z = \frac{\partial w}{\partial z} \tag{61}$$

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \tag{62}$$

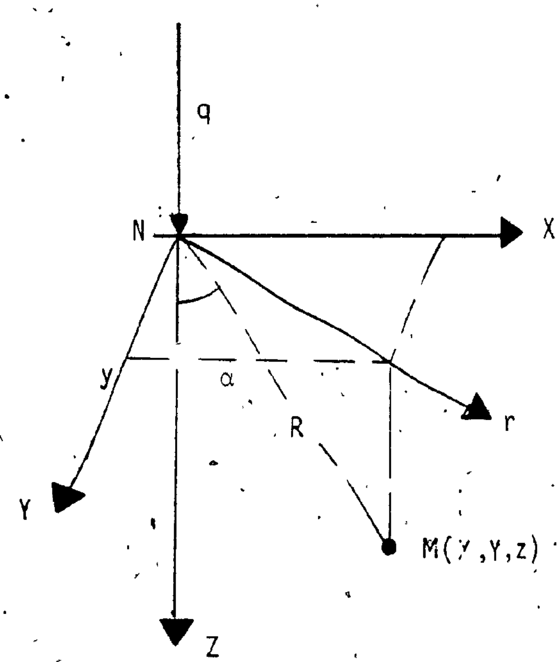


Figure 9. Point load acting on a semi-infinite mass.

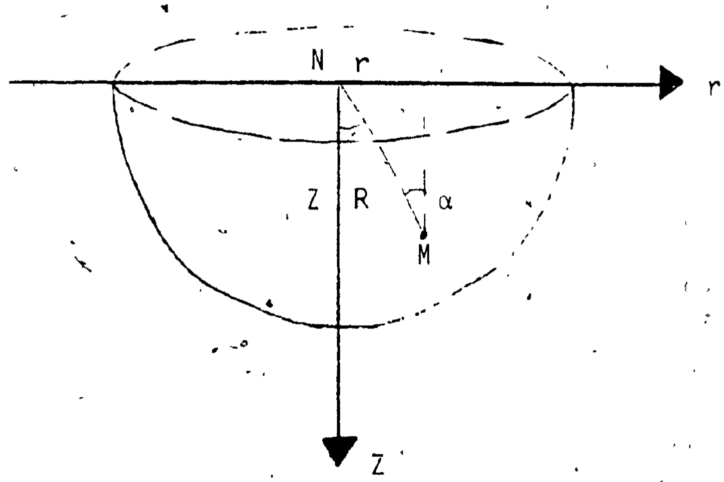


Figure 10. Spherical coordinates.

and the equations of equilibrium can be reduced to

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r2}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad (63)$$

$$\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{rz}}{r} = 0 \qquad (64)$$

The solution to equation (63) and (64) can be found by introducing the stress function ψ as follows

$$\sigma_r = \frac{\partial}{\partial z} \left(\nu \nabla^2 \psi - \frac{\partial^2 \psi}{\partial r^2} \right) \qquad (65)$$

$$\sigma_\beta = \frac{\partial}{\partial z} \left(\nu \nabla^2 \psi - \frac{1}{r} \frac{\partial \psi}{\partial r} \right) \qquad (66)$$

$$\sigma_z = \frac{\partial}{\partial z} \left((2-\nu) \nabla^2 \psi - \frac{\partial \psi}{\partial z^2} \right) \qquad (67)$$

$$\sigma_{rz} = \frac{\partial}{\partial z} \left((1-\nu) \nabla^2 \psi - \frac{\partial^2 \psi}{\partial z^2} \right) \qquad (68)$$

where the function ψ should also satisfy the compatibility equation (43)

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right) \left(\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{\partial^2 \psi}{\partial z^2} \right) = \nabla^2 \nabla^2 \psi = 0 \qquad (43)$$

To solve equation (43) it is convenient to use spherical coordinates (see Figure 10)

$$r = R \sin \alpha$$
$$z = R \cos \alpha \qquad (69)$$

After differentiation, the operator ∇^2 can be determined as

$$\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{1}{R^2} \frac{\partial^2}{\partial \alpha^2} \qquad (70)$$

$$r\frac{\partial}{\partial r} = \frac{1}{R}\frac{\partial}{\partial R} + \frac{\cot g\,\alpha}{R^2}\frac{\partial}{\partial \alpha} \tag{71}$$

Substituting equations (70), (71) in equation (43) results in

$$\frac{\partial^2\psi}{\partial R^2} + \frac{2}{R}\frac{\partial\psi}{\partial r} + \frac{1}{R^2}\cot g\,\alpha\,\frac{\alpha\psi}{\partial\alpha} + \frac{1}{R^2}\frac{\partial^2\psi}{\partial\alpha^2} = 0 \tag{72}$$

A particular solution of equation (72), which is a form of Laplace equation, can be a polynomial of the form

$$\psi_n = R^n\alpha_n \tag{73}$$

where α is the angle ($\widehat{ZOM}$) (see Figure 10).

To find the stresses due to a point loading in a mass, assume the load is acting at the center of a sphere (see Figure 11), due to the fact that some or all of the stresses have singularities, the function ψ_n will be

$$\psi_n = \frac{\alpha_n}{R^{n-2}} \tag{74}$$

Substituting equation (74) in equation (72), and resolving the equation as legender's equation.

$$\psi_n = \frac{K}{R^{n-2}} \tag{75}$$

where K depends on the boundary conditions.

In order to find the stress components in cylindrical coordinates, substituting equation (75) in equations (65), (66), (67) and (68) the following equations are obtained

$$\sigma_r = K\left[\frac{(1-2\nu)z}{R^3} - \frac{3r^2z}{R^5}\right] \tag{76}$$

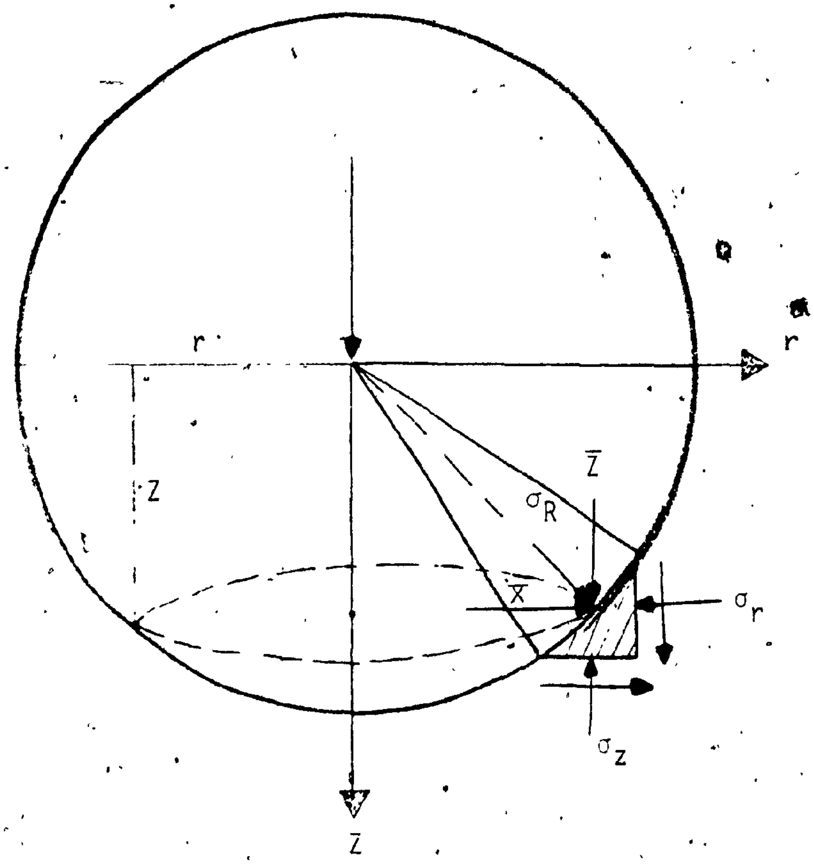


Figure 11. Point load acting at the center of the sphere.

$$\sigma_\beta = K\left[(1-2\nu)\frac{z}{R^3}\right] \tag{77}$$

$$\sigma_z = -K\left[(1-2\nu)\frac{z}{R^3} + \frac{3z^3}{R^5}\right] \tag{78}$$

$$\tau_{rz} = -K\left[(1-2\nu)\frac{r}{R^3} + \frac{3rz^2}{R^5}\right] \tag{79}$$

where

$$R = (r^2+z^2)^{1/2}$$

In case of a point loading at a given depth, consider the point as a small spherical cavity. Figure 11, the vertical stress $\overline{z}$ with respect to the boundary conditions is given by the following equation

$$\overline{Z} = \tau_{rz}\sin\alpha + \sigma_z\cos\alpha \tag{80}$$

Substituting equations (69), (78) and (79) in equation (80) yields

$$\overline{Z} = K\left[\frac{(1-2\nu)}{R^2} + \frac{3z^2}{R^4}\right] \tag{81}$$

The result of these forces over the surface of the cavity is equal to the force applied q integrating from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \frac{\pi}{z}\, r\, 2\pi r\, d\psi = 8K'(1-\nu) = q$$

thus

$$K = \frac{q}{8\pi(1-\nu)} \tag{82}$$

Substituting equation (82) in equations (76), (77), (78) and (79), the solution of Kelvin is obtained.

The radial stress σ_R (see Figure 12) can be written as a function of the stress components represented in equations (76), (78) and (79).

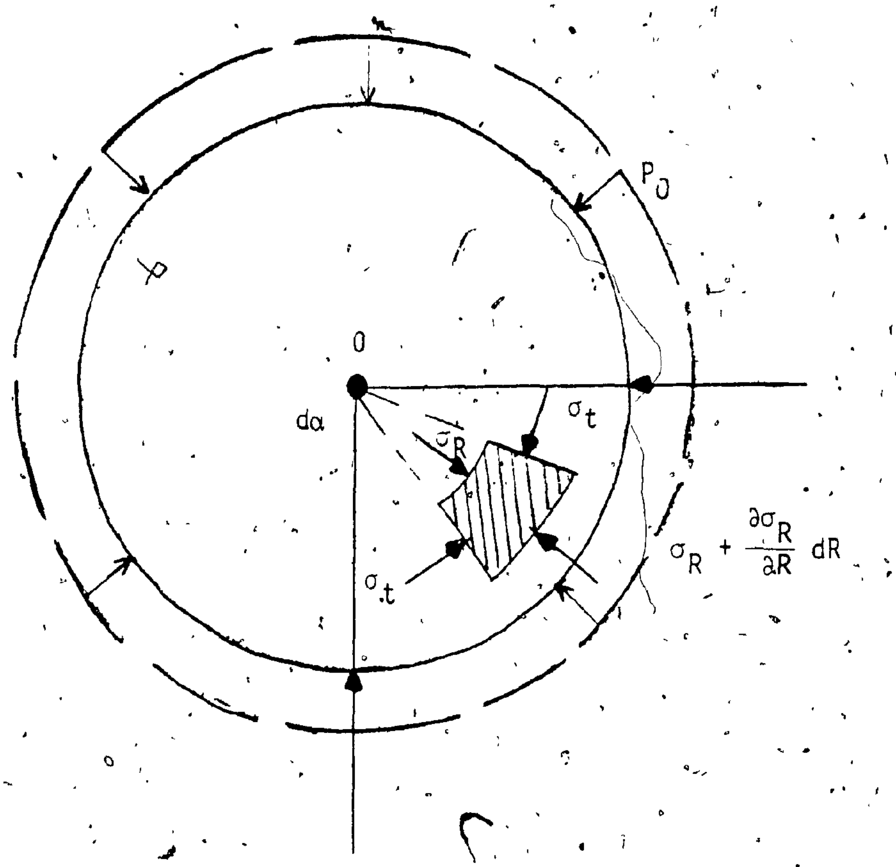


Figure 12. Uniform pressure on the sphere represented by a load acting in the r, z, direction and in the axis normal to plane (r,z).

$$\sigma_R = \sigma_R \sin^2 \alpha + \sigma_z \cos^2 \alpha + 2\tau_{rz} \sin \alpha \cos \alpha$$

$$\sigma_R = \frac{-2(1+\nu)}{R^3} C \left[- \sin^2 \alpha + \frac{2(1-\nu)}{(1+\nu)} \cos^2 \alpha \right] \quad (83)$$

where C is a constant.

If the load q is applied in the r direction, the sin α will be replaced by cos α, and hence equation (83) will be

$$\sigma_R = \frac{-2(1+\nu)}{R^3} C \left[- \cos^2 \alpha + \frac{2(1-\nu)}{1+\nu} \sin^2 \alpha \right] \quad (84)$$

If the load q is applied in the direction perpendicular to the plane (z,r) the angle α will be equal to $\frac{\pi}{2}$, and equation (83) becomes

$$\sigma_R = \frac{2(1+\nu)C}{R^3} \quad (85)$$

Adding equations (83), (84) and (85) to obtain the radial stress σ_r in a sphere under load P

$$\sigma_R = \frac{-4(1-2\nu)C}{R^3} \quad (86)$$

The tangentical stress component σ_t can be obtained from the equation of equilibrium, in the spherical coordinates from Figure 12.

$$\sigma_t = \frac{d\sigma_R}{dR} \frac{R}{2} + \sigma_R \quad (87)$$

Substituting equation (86) in equation (87), yields

$$\sigma_t = - \frac{1}{2} \frac{C}{R^3} \quad (88)$$

The stress components σ_R, σ_z, σ_β and τ_{rz} (in cylindrical coordinates) can be written in functions of σ_R and σ_t by

$$\sigma_r = \sigma_R \sin^2 \alpha + \sigma_z \cos^2 \alpha = C(r^2 - \frac{1}{2z^2})/(r^2+z^2)^{5/2} \quad (89)$$

$$\sigma_z = \sigma_R \cos^2 \alpha + \sigma_t \sin^2 \alpha = C(z^2 - \frac{1}{2r^2})/(r^2+z^2)^{5/2} \quad (90)$$

$$\tau_{rz} = \frac{1}{2} (\sigma_R - \sigma_t) \sin 2\alpha = \frac{3}{2} \frac{Crz}{(r^2+z^2)^5} \quad (91)$$

$$\sigma_\theta = \sigma_t = -\frac{1}{2} \frac{C}{R^3} = -\frac{1}{2} \frac{C}{(r^2+z^2)^3} \quad (92)$$

Assume that centers of pressure are uniformly distributed along the z axis from $z = 0$ to $z = +\infty$. Then by superposition, the stress components produced in an indefinitely extended solid, are from equations (89), (90), (91) and (92) with a new constant C.

$$\sigma_r = \frac{C}{2} \left[\frac{1}{r^2} - \frac{z}{r^2 R} - \frac{z}{R^3} \right] \quad (93)$$

$$\sigma_z = \frac{C}{2} \left[\frac{z}{R^3} \right] \quad (94)$$

$$\tau_{rz} = \frac{C}{2} \frac{r}{R^3} \quad (95)$$

$$\sigma_\theta = -\frac{C}{2} \left[\frac{1}{r^2} - \frac{z}{r^2 R} \right] \quad (96)$$

on the $z = 0$, where the load q is applied. The stresses should satisfy the boundary conditions. The shear stress τ_{rz} given by equations (79) and (95) can be determined as

$$(\tau_{rz})_{z=0} = \frac{K}{r^2} (1-2\nu) \quad (97)$$

$$(\tau_{rz})_{z=0} = \frac{1}{2} \frac{C}{r^2} \quad (98)$$

from equations (97) and (98), the relationship between C and K is given by

$$C = 2K(1-2\nu) \tag{99}$$

the second condition for the boundary conditions is: the sum of the vertical stress is equal to θ, if $\bar{z}$ is the vertical component at the boundary

$$\bar{z} = \sigma_z \cos\alpha + \tau_{rz} \sin\alpha = 3K \frac{z^2}{r^4} \tag{100}$$

integrating $\bar{z}$ from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ yields

$$\phi = \int_{-\frac{\pi}{2}}^{\pi/2} \bar{z}\, r(r^2+z^2)^{1/2}\, d\psi = 2\pi K$$

then

$$K = \frac{q}{2\pi} \tag{101}$$

thus, from equations (99) and (101), we have

$$C = \frac{q}{\pi}(1-2\nu) \tag{102}$$

Finally, substituting the values of K and C in equations (76), (77), (78) and (79), and in equations (93), (94), (95) and (96) respectively, and adding separately for each stress component, the final equations for stresses due to point loading are

$$\sigma_r = \frac{-q}{2\pi R^2}(1-2\nu)\left[\frac{1}{r^2} - \frac{z}{r^2 R} - \frac{3r^2 z}{R^5}\right] \tag{103}$$

$$\sigma_z = \frac{3q\, z^3}{2\pi\, R^5} \tag{104}$$

$$\sigma_\theta = -\frac{q}{2}(1-2\nu)\left(-\frac{1}{r^2} + \frac{z}{r^2}\frac{1}{R} + \frac{z}{R^3}\right) \tag{105}$$

$$\tau_{rz} = \frac{3q}{2\pi} \frac{rz^2}{R^5} \tag{106}$$

Substituting equation equation (103) - (106) in equations (17), (18) and (19), the stress components in rectangular coordinates are obtained.

$$\sigma_x = \frac{3q}{2\pi} \{ \frac{x^2 z}{R^5} + \frac{(1-2\nu)}{3} (\frac{1}{R(R+z)} - \frac{(2R+z)x^2}{R^3(R+z)^2} - \frac{z}{R^3})\} \tag{107}$$

$$\sigma_z = \frac{3q}{2\pi} \frac{z^3}{R^5} \tag{108}$$

$$\tau_{xz} = \frac{3q}{2\pi} \frac{xz^2}{R^5} \tag{109}$$

By replacing y by x the stress components σ_y and τ_{yz} are obtained.

2.3.4 The Case of a Vertical Line Loading Acting Beneath the Semi-infinite Mass

The line loading q is acting at depth d from the surface. To find the stresses acting on an element at depth z, Melan, 1932, developed the following formulas

$$\sigma_x = \frac{-q}{\pi} \left[\frac{1}{2(1-\nu)} \{ \frac{(z-d)x^2}{R_1^4} + \frac{(z+d)(x^2+2d^2) - 2dx^2}{R_2^4} + \frac{8dzx^2(d+z)}{R_2^6} \} + \frac{(1-2\nu)}{4(1-\nu)} \{ (- \frac{z-d}{R_1^2} + \frac{z+3d}{R_2^2} + \frac{4zx^2}{R_2^4}) \} \right] \tag{110}$$

$$\sigma_z = \frac{q}{\pi} \left[\frac{1}{2(1-\nu)} \{ \frac{(z-d)^3}{R_1^4} + \frac{(z+d)((z+d)^2+2dz)}{R_2^4} - \frac{8dzx^2(d+z)}{R_2^6} \} + \frac{1-2\nu}{4(1-\nu)} \{ \frac{(z-d)}{R_1^2} + \frac{3z+d}{R_2^2} - \frac{4zx^2}{R_2^4} \} \right] \tag{111}$$

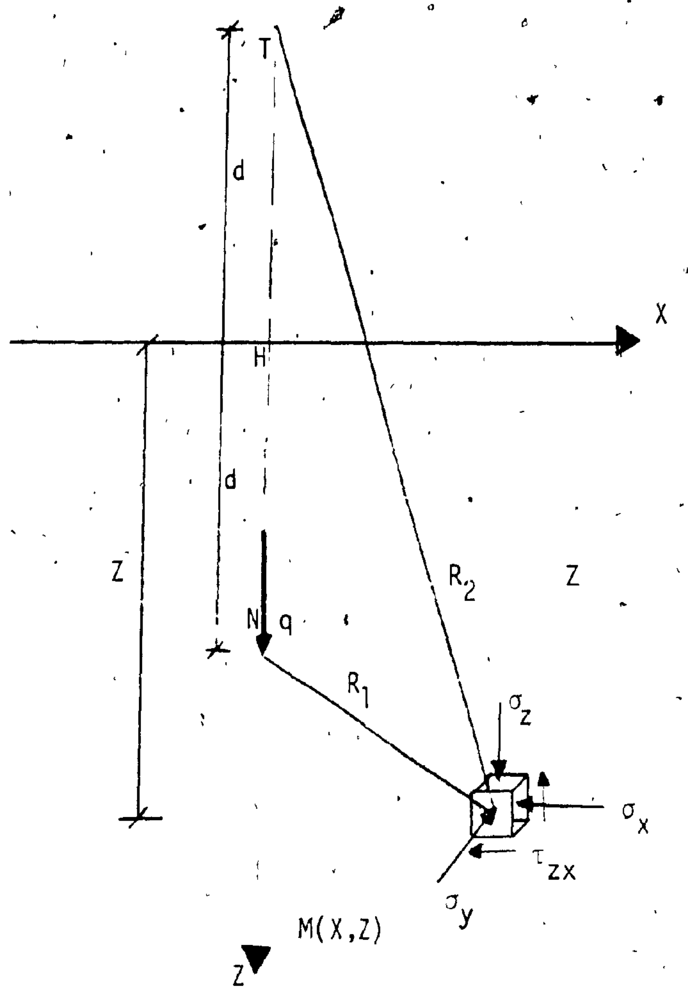


Figure 13. Vertical line loading beneath the surface of semi-infinite mass.

$$\tau_{xz} = \frac{qx}{\pi}\left[\frac{1}{2(1-\nu)}\left\{\frac{(2-d)^2}{R_1^4} + \frac{z^2 - 2dz - d^2}{R_2^4} + \frac{8dz(d+z)^2}{R_2^6}\right\} + \frac{1-2\nu}{4(1-\nu)}\left(\frac{1}{R_1^2} - \frac{1}{R_2^2} + \frac{4z(d+z)}{R_2^4}\right)\right] \quad (112)$$

Assume d = 0 then equations (110), (111) and (112) reduce to the equations developped by Boussinesg for a strip loading represented by

$$\sigma_x = \frac{2q}{\pi}\frac{x^2 z}{R_1^4} \quad (113)$$

$$\sigma_z = \frac{2q}{\pi}\frac{z^3}{R_1^4} \quad (114)$$

$$\tau_{xz} = \frac{2q}{\pi}\frac{xz^2}{R_1^4} \quad (115)$$

as there is no deformation in the y direction the horizontal stress components are given by equation (46) as follows

$$\sigma_y = \nu(\sigma_x+\sigma_z) = \frac{2q\nu}{\pi}\frac{z}{R^2} \quad (116)$$

2.3.5 In Case of Vertial Point Load Acting Beneath the Surface of a Semi-infinite Mass

If the point loading q is acting at depth d (as shown in Figure 14), the stress components at point M located at depth z, are given by (Mindlin, 1936)

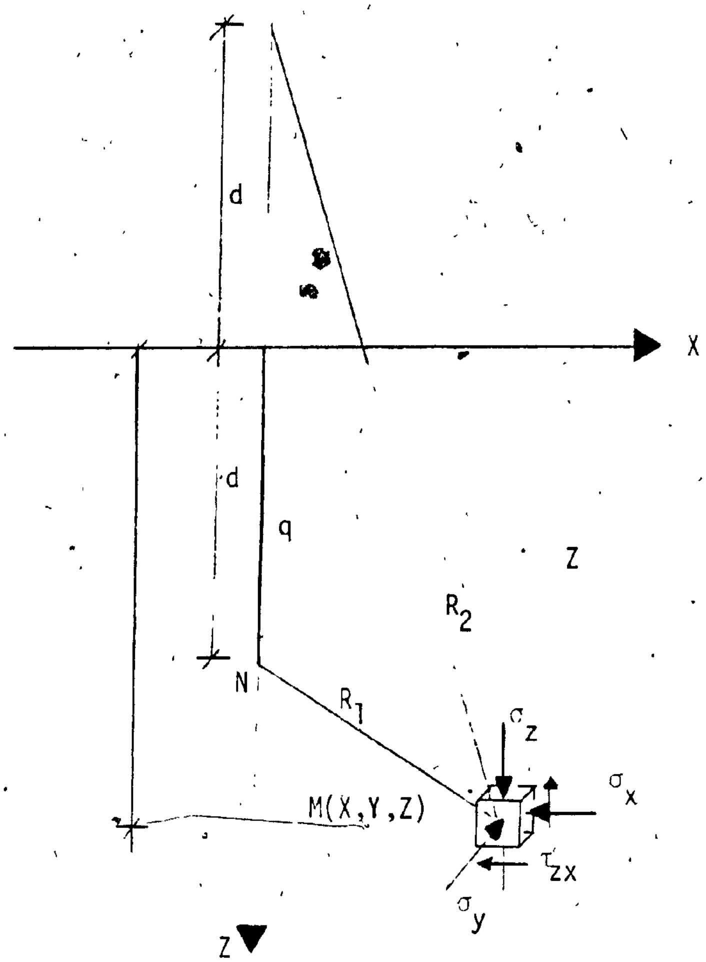


Figure 14. Vertical concentrated load beneath the surface of the soil.

$$\sigma_x = \frac{-q}{8\pi(1-\nu)} \left[\frac{(1-2\nu)(z-d)}{R_1^{\,3}} - \frac{3x^2(z-d)}{R_1^{\,5}} + \frac{(1-2\nu)\{3(z-d) + 4\nu(z+d)\}}{R_2^{\,3}} \right.$$

$$- \frac{3(3-4\nu)x^2(z-d) - 6d(z+d)(1-2\nu)z - 2\nu d}{R_2^{\,5}} \qquad (117)$$

$$\left. - \frac{30d\, x^2 z(z+d)}{R_2^{\,7}} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+d)} \times \left(1 - \frac{x^2}{R_2(R_2+z+d)} - \frac{x^2}{R_2^{\,2}}\right) \right]$$

R_1 and R_2 are shown in Figure 14. Due to the symmetry in the plane x, y, X = Y then $\sigma_x = \sigma_y$

$$\sigma_z = \frac{-q}{8\pi(1-\nu)} \left[\frac{(1-2\nu)(z-d)}{R^3} + \frac{(1-2\nu)(z-d)}{R_2^{\,3}} - \frac{3(z-d)^3}{R_1^{\,5}} \right. \qquad (118)$$

$$\left. - \frac{3(3-4\nu)z(z+d)^2 - 3d(z+d)(5z-d)}{R_2^{\,5}} - \frac{30dz(z+d)^3}{R_2^{\,7}} \right]$$

$$\tau_{xz} = \frac{-qx}{8\pi(1-\nu)} \left[- \frac{(1-2\nu)}{R_1^{\,3}} + \frac{1-2\nu}{R_2^{\,3}} - \frac{3(z-d)^3}{R_1^{\,5}} \right. \qquad (119)$$

$$\left. - \frac{3(3-4\nu)z(z+d) - 3d(3z+d)}{R_2^{\,5}} - \frac{30dz(z+d)^3}{R_2^{\,7}} \right]$$

$\tau_{yz} = \tau_{xz}$ by replacing x by y.

If we assume d = 0 (i.e., the load is acting on the surface), the equations (117), (118) and (119) are equal to equations (107), (108) and (109) derived by Boussinesq.

2.4 WINKLER MODEL OR SOIL BEHAVIOUR.

The application of an external force system to a soil element induces deformation; and if the soil elements behave as a purely elastic material, the resulting displacement, is given by a linear function.

A soil model proposed by Winkler (1867), assumes that the deflexion w in the z direction is directly proportional to the stress q_0 and independent of stress levels at other locations (see Figure 15), i.e.,

$$q_0(x,y) = k\,w(x,y). \qquad (120)$$

where k represents the modulus of subgrade reaction with units of stress per unit length.

Physically, Winkler's idealization of the soil medium consists of a system of mutually independent spring elements with stiffness, constants k.

Consider a cylindrical shape as a footing (see Figure 16) for loads q_0, applied on the soil located between two parallel surfaces AB and MN.

The cylindrical shape is divided into elements ds, every element is represented as an infinitesmial area of unit length and a width ds, the stress q_0 for element 1 can be also written as

$$q_1 = E \cdot \varepsilon = E \; \frac{w_1(x,y)}{L} \qquad (121)$$

where L is the thickness of the soil under the element to the surface MN.

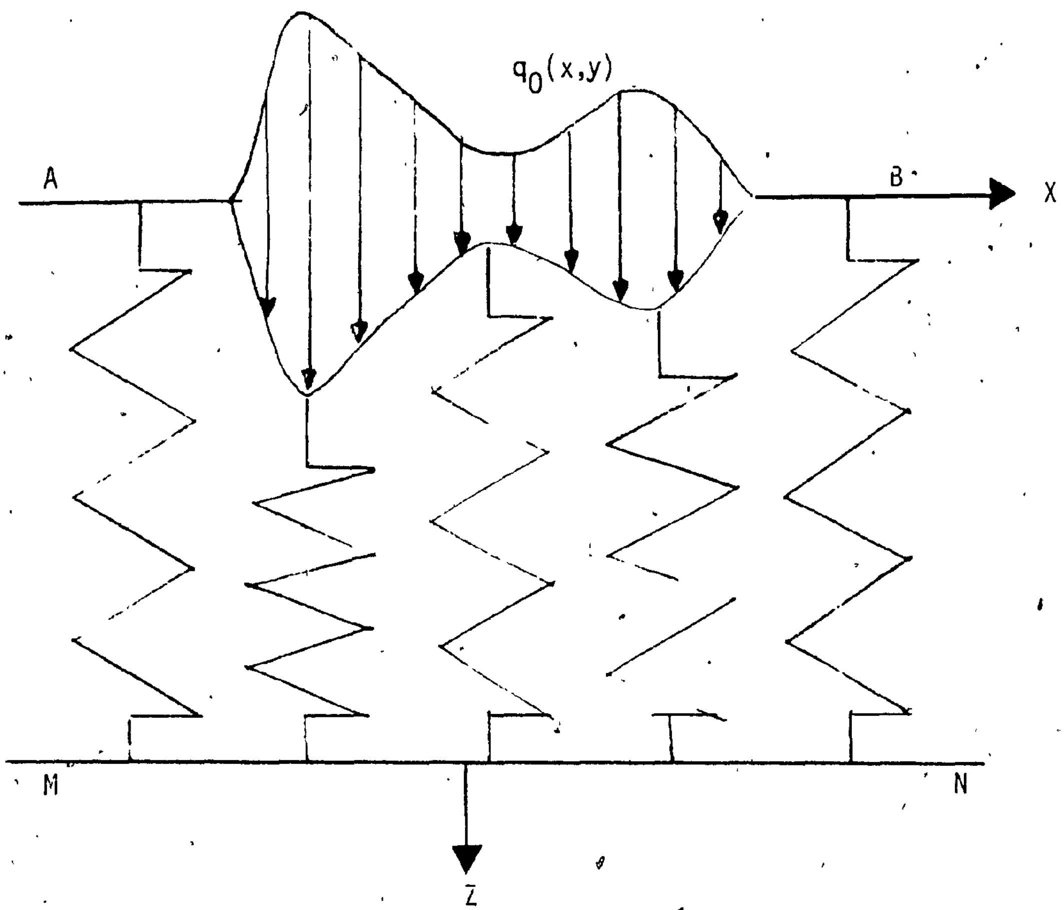


Figure 15. Surface displacement of the Winkler model due to non-uniform load.

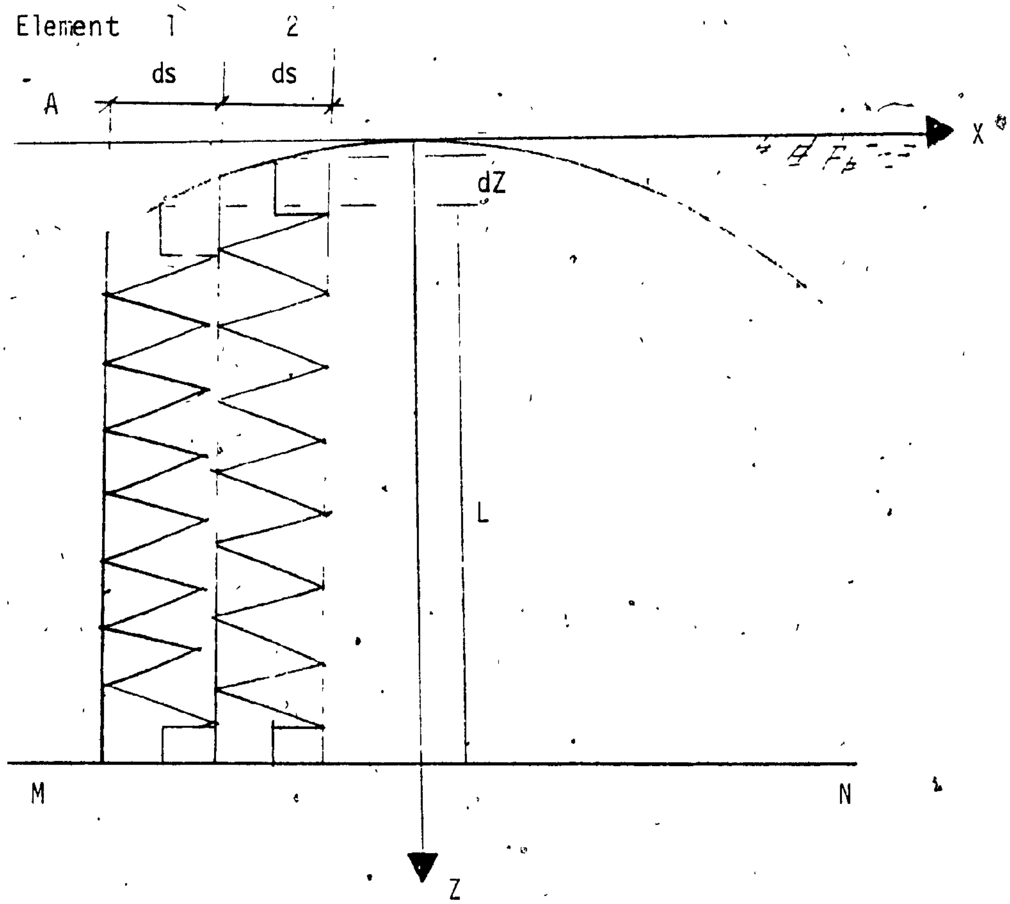


Figure 16. Surface displacement under a cylindrical rigid footing, using Winkler model.

If L + dz is the thickness of the soil under the element 2 next to element 1, then

$$q_2 = E \frac{w_2(x,y)}{L + dz} . \tag{122}$$

Since the cylindrical footing is assumed as a rigid surface, it will settle uniformally by the amount

$$w(x,y) = w_1(x,y) = w_2(x,y). \tag{123}$$

When the surface MN is at infinity, the thickness L + dz will be equal to L, thus,

$$q_0 = q_1 = q_2$$

therefore, the vertical loads q applied by the cylindrical surface (footing) is constant.

CHAPTER III

ANALYSIS OF NEW MODELS

3.1 GENERAL

The concept of a "line loading" can be represented with a load acting on an infinitesmial strip of width ds, while a point loading can be represented with a load acting on an infinitesmial area dA.

In the design of "footing", the structural characteristics of a footing varies with the variation of the shape, also, the intensity and the shape of the load transmitted by the footing to the soil depends on the footing.

In analysing the geometric shape of a footing, the technique of vector analysis is used to compute the infinitesmial area of the footing with a complex shape, i.e., cylindrical, triangular, spherical, etc.

3.2 GENERAL DEFINITION OF A SURFACE

Figure 17, shows a general surface S_0 in O the O(x, y, z) system. A curve on the surface, or the surface itself can be defined by a vector $\overline{ON}$ as follows

$$\overline{ON} = (x, y, z) \tag{124}$$

where x, y, z are the rectangular coordinates, which can be represented by the parameters α, β and r in the polar system of coordinates.

Where α and β are two angles (shown in Figure 19), and r is the modulus of $|\overline{ON}| = r$. A footing is a rigid or elastic soild defined by an average surface (see Figure 18), and a thickness 'h' land on a soil (clay,

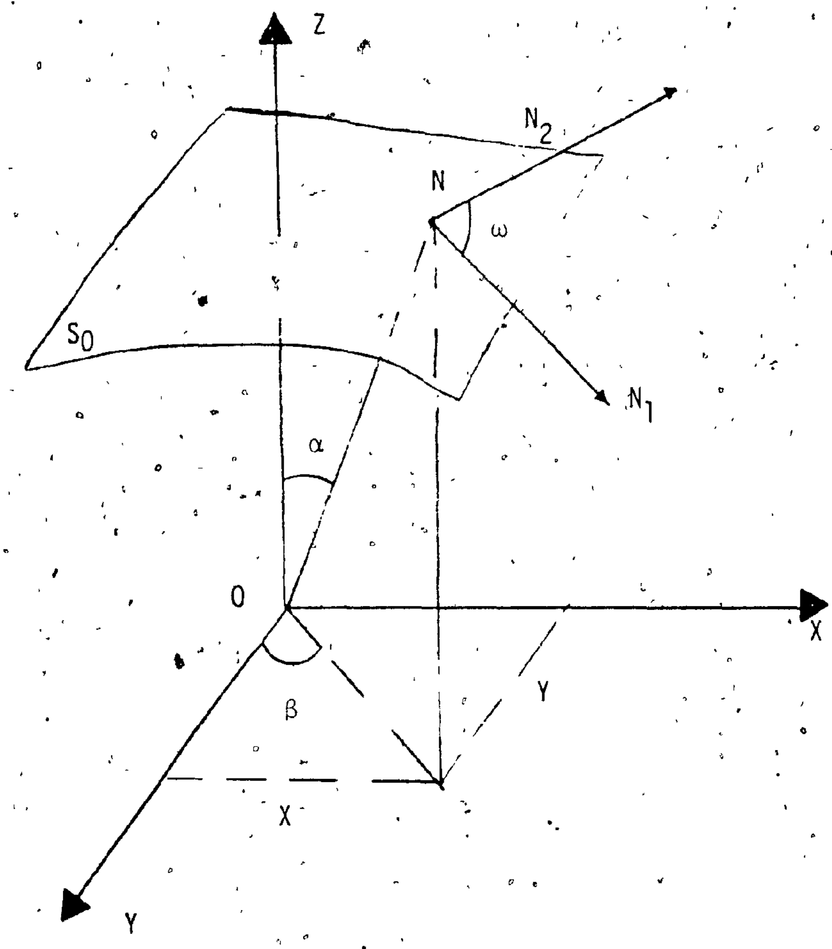


Figure 17. A surface in triaxial system $O_1(X,Y,Z)$.

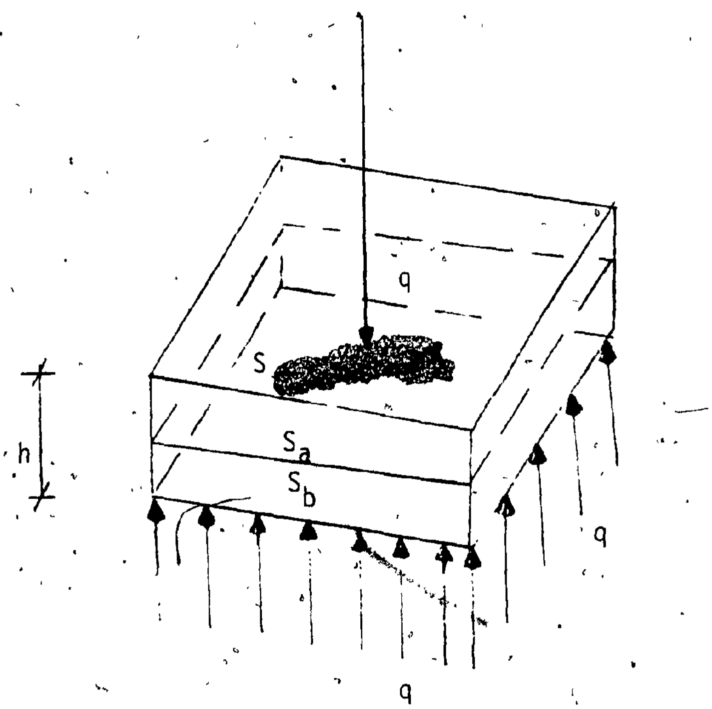


Figure 18. An element of a footing surface.

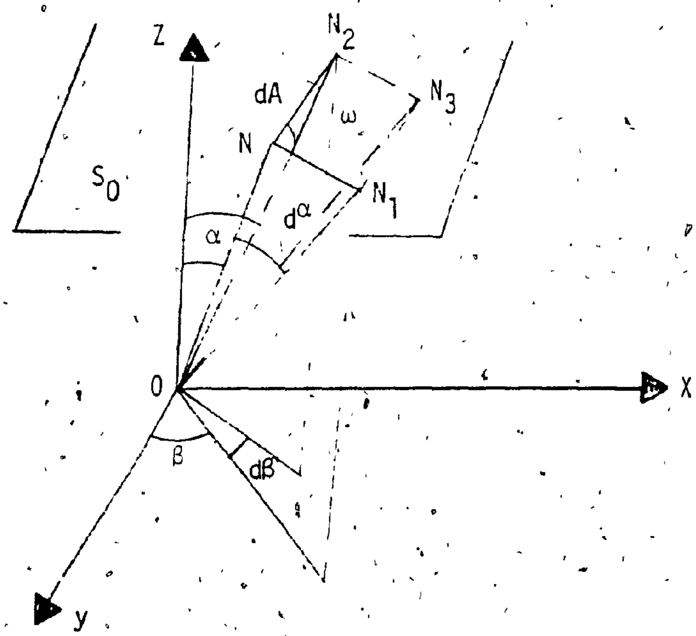


Figure 19. An infinitesimal unit area dA.

sand, etc.) and support a charge P from the structure, the charge P is transmitted to the soil by a unit charge

$$q = \frac{P}{S_b} .$$

Consider the vector where

$$\overline{ON} = (x(\alpha,\beta),\ y(\alpha,\beta),\ z(\alpha,\beta)) \tag{125}$$

taking β as a constant parameter and keeping α as a variable. The first derivative of $\overline{ON}$ is a vector tangent to the surface and directed to the positive direction of α

$$\overline{ON},_\alpha = \frac{\overline{ON}(\alpha+d\alpha,\beta) - \overline{ON}(\alpha,\beta)}{d\alpha} = \frac{d\ \overline{ON}(\alpha)}{d\alpha} \tag{126}$$

for the increment dα the point N moves to a new position N_1 (see Figure 19).

Similarly taking α as a constant and keeping β as a variable, the following is obtained

$$\overline{ON},_\beta = \frac{\overline{ON}(\alpha,\beta+d\beta) - \overline{ON}(\alpha,\beta)}{d\beta} = \frac{d\ \overline{ON}(\beta)}{d\beta} \tag{127}$$

and for the increment dβ the point N moves to N_2.

From equations (126) and (127), the vectors $\overline{NN}_1$ and $\overline{NN}_2$ can be obtained as

$$NN_1 = d(\overline{ON}(\alpha)) = \overline{ON},_\alpha\ d\alpha \tag{128}$$

$$NN_2 = d(\overline{ON}(\beta)) = \overline{ON},_\beta\ d\beta \tag{129}$$

therefore, the area dA represented by the parallelogram $NN_1N_2N_3$ (see Figure 19) is obtained as

$$\begin{aligned} dA &= |\overline{NN}_1 \times \overline{NN}_2| \quad |\overline{ON},_\alpha \times \overline{ON},_\beta|\ dxd\beta \\ &= |\overline{ON},_\alpha|\ |\overline{ON},_\beta|\ d\alpha d\beta\ \sin\omega \end{aligned} \tag{130}$$

where ω is the angle between the two vectors $\overline{ON},_\alpha$ and $\overline{ON},_\beta$.

The vectors $\overline{ON},_\alpha$ and $\overline{ON},_\beta$ can be represented in the following forms, also

$$\overline{ON},_\alpha = (x,_\alpha, y,_\alpha z,_\alpha)$$

and

$$\overline{ON},_\beta = (x,_\beta y,_\beta, z,_\beta)$$

thus

$$|\overline{ON},_\alpha| = (x^2,_\alpha + y^2,_\alpha + z^2,_\alpha)$$

and (131)

$$|\overline{ON},_\beta| = (x^2,_\beta + y^2,_\beta + z^2,_\beta)$$

3.3 <u>CIRCULAR FOOTING</u>

The footing is a circle (0,B) of radius B, the point M is at depth z, and distance r from the z point. The vertical load q applied at point N at a distance r from the center 0 of the circle, and an angle β.

Then

$$\overline{ON}(x,y) = (r \cos \beta, r \sin \beta) \quad (132)$$

$$\overline{ON},\beta = (-r \sin \beta, r \cos \beta)$$

(133)

$$\overline{ON},r = (\cos \beta, \sin \beta)$$

substituting equation (133) in equation (130), yields

$$dA = |\overline{ON},\beta| \; |\overline{ON},r| \; d\beta dr \sin \omega$$

however, $\overline{ON},\beta$ and $\overline{ON},r$ are two orthogonal vectors.

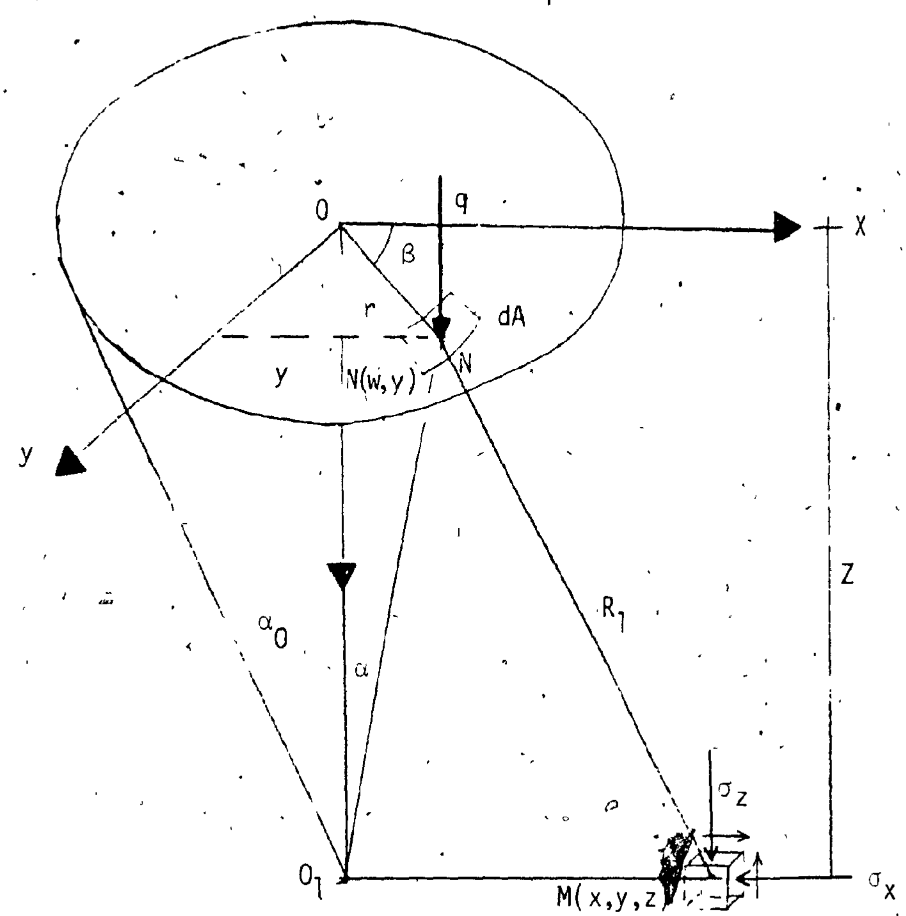


Figure 20. Figure showing the vectors and stresses analysis for a circular flat footing.

$$\overline{ON},_\beta \cdot \overline{ON},_r = -r \cos\beta \sin\beta + r \cos\beta \sin\beta = 0$$

then $\sin \omega = 1$ and

$$dA = |\overline{ON},_\beta| \, |\overline{ON},_r| \, d\beta dr = (r^2 \sin^2\beta + r^2 \cos^2\beta)^{1/2} \cdot (\sin^2\beta + \cos^2\beta)^{1/2} = r dr d\beta$$

therefore

$$dA = r d\beta dr \; . \qquad (135)$$

If P is the load applied on the circular footing of radius B, the load per unit area is equal to

$$q = \frac{P}{\pi B^2} \qquad (136)$$

the load q contributes a vertical stress $d\sigma_z$ at point M, by using equation (108)

$$d\sigma_z = \frac{3q}{2\pi R_1^5} \, dA \; . \qquad (137)$$

thus,

$$\sigma_z = \frac{3q}{2\pi} \int_A \frac{z^3}{R_1} \, dA \qquad (138)$$

where dA is given by equation (135).

$$dA = r d\beta \; dr \; . \qquad (139)$$

From Figure 20,

$$r = z \, tg \, \alpha \qquad (140)$$

therefore

$$dr = \frac{z}{\cos^2 \alpha} \qquad (141)$$

Substituting equations (139), (141) and (143) for equation (138), yields

$$\sigma_z = \frac{3q}{2\pi} \iint \frac{z^5 \sin\alpha}{\cos^3\alpha} \frac{1}{R_1^5}\, d\alpha\, d\beta \tag{142}$$

From Figure 20, the following vectors can be obtained

$$R_1 = |\overline{MN}|$$

$$\overline{ON} = (r\cos\beta,\ r\sin\beta,\ 0)$$

$$\overline{OM} = (x,\ 0,\ z)$$

then

$$\overline{MN} = (r\cos\beta - x,\ r\sin\beta,\ z)$$

thus

$$|\overline{MN}| = R_1 = (r^2 + x^2 - 2\,xr\cos\beta + z^2)^{1/2}$$

but $r = z\,tg\,\alpha$ equation (140), then

$$R_1 = \frac{1}{\cos\alpha}(z^2 + x^2\cos^2\alpha - 2zx\sin\alpha\cos\alpha\cos\beta)^{1/2} \tag{143}$$

Substituting equation (143) in equation (142), yields

$$\sigma_z = \frac{3qz^5}{2\pi}\int_0^{2\pi}\int_0^{\alpha_0} d\beta\, \frac{\cos^2\alpha\sin\alpha\, d\alpha}{(z^2 + x^2\cos^2\alpha - 2zx\sin\alpha\cos\alpha\cos\beta)^{5/2}} \tag{144}$$

In the case where the point M is on the z axis x = 0, equation (144) yields the following

$$\sigma_z = \frac{3qz^5}{2\pi}\cdot 2\pi\int_0^{\alpha_0}\frac{\cos^2\alpha\sin\alpha\, d\alpha}{z^5} = q(1-\cos^3\alpha) \tag{145}$$

the horizontal component σ_x of the stresses is given by equation (107) as

$$\sigma_x = \frac{3q}{\pi} \left[\frac{x^2 z}{R_1^5} + \frac{1-2\nu}{3} \left(\frac{1}{R_1(R_1+z)} - \frac{(2R_1+z)x^2}{R_1^3(R+z)^2} \right) - \frac{z}{R^3} \right] \quad (107)$$

over the unit area dA

$$d\sigma_x = \sigma_x dA$$

integrating as before the vertical stress component is given by

$$\sigma_x = \frac{3q}{\pi} \int_0^{2\pi} d\beta \int_0^{\alpha 0} \frac{x^2 z}{R_1^5} + \frac{(1-2\nu)}{3} \left(\frac{1}{R_1(R_1+z)} - \frac{(2R_1+z)x^2}{R_1^3(R_1+z)^2} - \frac{z}{R^3} \right) d\alpha \quad (146)$$

Also, by using equation (109), the shear stress τ_{xz} is equal to

$$\tau_{xz} = \frac{3q}{\pi} \int_0^{2\pi} d\beta \int_0^{\alpha 0} \frac{xz^2}{R_1^5} d\alpha \quad (147)$$

because of symmetry $\sigma_x = \sigma_y$ and $\tau_{yz} = \tau_{xz}$; and the value of R_1 is given by the equation (143).

3.4 STRIP FOOTING

The strip footing shown in Figure 21, is under a vertical line load p, the load p is transmitted to the soil along the width 2B of the footing. The unit applied load q at the surface is given by

$$q = \frac{P}{2B} \quad (148)$$

Referring to equation (56), the vertical component stress at point M at depth z is equal to

$$\sigma_z = \frac{2q}{\pi} \frac{z^3}{R_1^4} \quad (149)$$

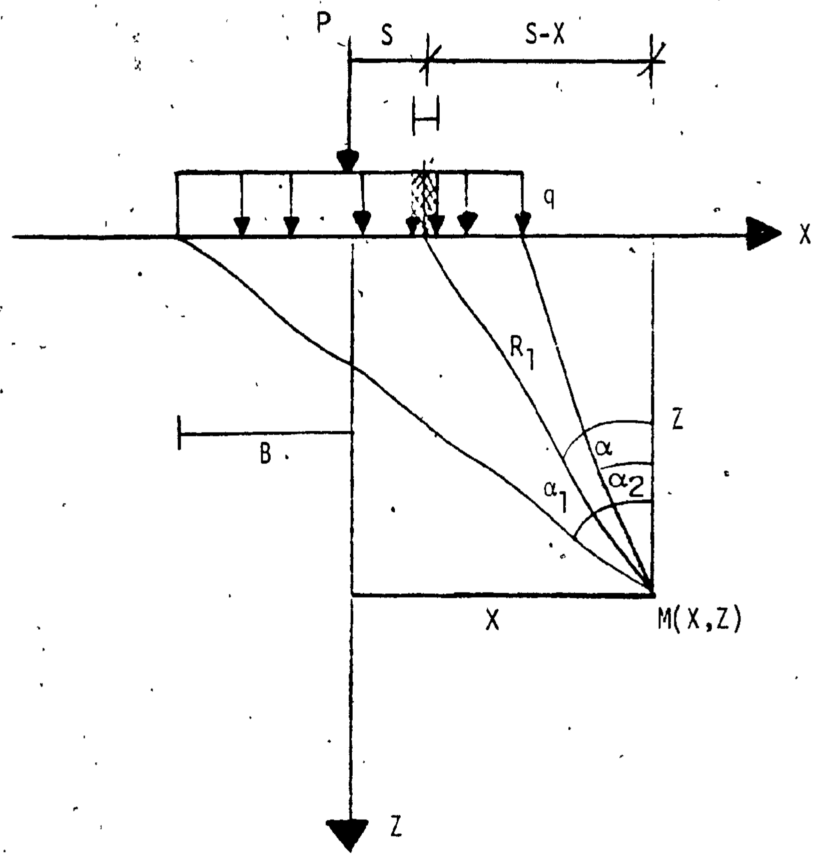


Figure 21. Figure showing the vectors and stresses analysis for a flat strip footing.

An incremental part of the load of width dS and intensity q acts at some distance S, from the center of the load. The vertical stress increment is given by

$$d\sigma_z = \frac{2q}{\pi} \frac{z^3 ds}{R_1^4} \tag{150}$$

Using the polar coordinates and calculating $d\sigma_z$ as a function of $d\alpha$, from Figure 21, the following equation can be deduced

$$tg\alpha = \frac{s-x}{z} \quad \text{OR} \quad (s-x) = z\,tg\alpha \tag{151}$$

$$\cos\alpha = \frac{z}{R_1} \quad \text{OR} \quad R_1 = \frac{z}{\cos\alpha} \tag{152}$$

$$\alpha_1 = \tan^{-1}\left(\frac{-x-B}{z}\right) \tag{153}$$

$$\alpha_2 = \tan^{-1}\left(\frac{-x+B}{z}\right) \tag{154}$$

where α_1 and α_2 are the lower and upper limits of the angle α.

Differentiation of equation (151), yields

$$ds = \frac{z}{\cos^2\alpha}\, d\alpha \tag{155}$$

The substitution of equations (152) and (155) for equation (150), yields

$$d\sigma_z = \frac{2q}{\pi} \cos^2\alpha\, d\alpha \tag{156}$$

integrating equation (156) between α_1 and α_2, the vertical stress is obtained by the superposition of all the contributions of the loads q. Thus

$$\sigma_z = \int_{\alpha_1}^{\alpha_2} \frac{2q}{\pi} \cos^2\alpha\, d\alpha \tag{157}$$

using any handbook of integration where

$$\int \cos^2 \alpha d\alpha = \frac{1}{2} + \frac{\sin 2\alpha}{4} . \qquad (158)$$

Equation (157), yields

$$\sigma_z = \frac{2q}{\pi} \left[\frac{\alpha_1 - \alpha_2}{z} + \frac{1}{4} (\sin 2\alpha_2 - \sin 2\alpha_1) \right] \qquad (159)$$

or

$$q = \frac{p}{2B} \quad \text{(equation 148)}$$

and

$$\sin A - \sin B = 2 \sin \frac{A-B}{2} \cos \frac{(A+B)}{2} \qquad (160)$$

substituting equations (148) and (160) for equation (159)

$$\sigma_z = \frac{p}{2\pi K} \left[(\alpha_2 - \alpha_1) + \sin \frac{(\alpha_2 - \alpha_1)}{2} \cos \frac{(\alpha_2 + \alpha_1)}{2} \right] \qquad (151)$$

where σ_z is the vertical stress at point M.

In a similar manner referring to equation (57), and replacing da by dS, and x by s - x, the increment of the horizontal stress is given by

$$d\sigma_x = \frac{2q}{\pi} \frac{z(s-x)^2}{R^4} ds \qquad (161)$$

substituting equations (151), (152) and (155) in equation (160), then

$$d\sigma_x = \frac{2qz}{\pi} \left(\frac{z^2 tg^2 \alpha}{\frac{z^4}{\cos^4 \alpha}} \frac{z}{\cos^2 \alpha} d\alpha \right) \qquad (162)$$

$$d\sigma_x = \frac{2q}{\pi} (\sin^2 \alpha \, d\alpha)$$

Integrating from α_1 to α_2

$$\sigma_x = \frac{2q}{\pi} \int_{\alpha_1}^{\alpha_2} \sin^2 \alpha \, d\alpha \tag{163}$$

where

$$\int \sin^2 \alpha = \frac{\alpha}{z} - \frac{\sin 2\alpha}{4} \tag{164}$$

thus,

$$\sigma_x = \frac{2q}{\pi} \quad \frac{\alpha_2 - \alpha_1}{2} - \frac{\sin 2\alpha_2 - \sin 2\alpha_1}{4} \tag{165}$$

Substituting equations (148) and (160) in equation (164), yields

$$\sigma_x = \frac{p}{2\pi B} \left[(\alpha_2 - \alpha_1) - \sin \frac{(\alpha_2 - \alpha_1)}{2} \cos \frac{(\alpha_2 + \alpha_1)}{2} \right] \tag{166}$$

Therefore, the same analysis to calculate the shear stress, equation (58) can be satisfied.

$$\tau_{xz} = \frac{2q}{\pi} \frac{xz^2}{R^4} \tag{58}$$

$$d\tau_{xz} = \frac{2q}{\pi} \frac{z^2(s-x)}{R^4} \, ds \tag{167}$$

$$= \left(\frac{2q}{\pi} z^2\right) \left(\frac{z \operatorname{tg} \alpha}{\frac{z^4}{\cos^4 \alpha}} \frac{z}{\cos^2 \alpha} \, d\alpha \right)$$

$$= \frac{2q}{\pi} \sin \alpha \cos \alpha \, d\alpha$$

thus

$$\tau_{xz} = \frac{2q}{\pi} \int_{\alpha_1}^{\alpha} \sin \alpha \cos \alpha \, d\alpha \tag{168}$$

$$\tau_{xz} = \frac{q}{\pi} (\sin^2 \alpha) \Big|_{\alpha_1}^{\alpha_2}$$

$$\tau_{xz} = \frac{P}{2\pi B} (\sin^2 \alpha_2 - \sin^2 \alpha_1) \tag{169}$$

the horizontal component stress σ_y is given by equation (46), as

$$\sigma_y = \nu(\sigma_x + \sigma_z) = \nu \frac{q}{\pi B} (\alpha_2 - \alpha_1). \tag{170}$$

3.5 CYLINDRICAL STRIP FOOTING.

The strip footing in Figure 22, has a cylindrical shape. The force P per unit length P is applied in the z direction.

Assuming that, the intensity of contact pressure between the soil and the foundation is q_0, the foundation settles uniformly because of symmetric loading and shape, after paragraph 2.4 of Chapter II. The vertical load q by unit length and unit width ds is a constant denoted by q_0.

Dividing the circular foundation into smaller segments, each having an angle $d\alpha$ (as shown in Figure 23). With the lower and upper limits at α are $-\alpha_0$ and α_0, respectively.

Assume: R: is the radius of the circle

S: is the horizontal coordinate of the point N, where the vertical load q_0 is acting

2B: is width of the footing.

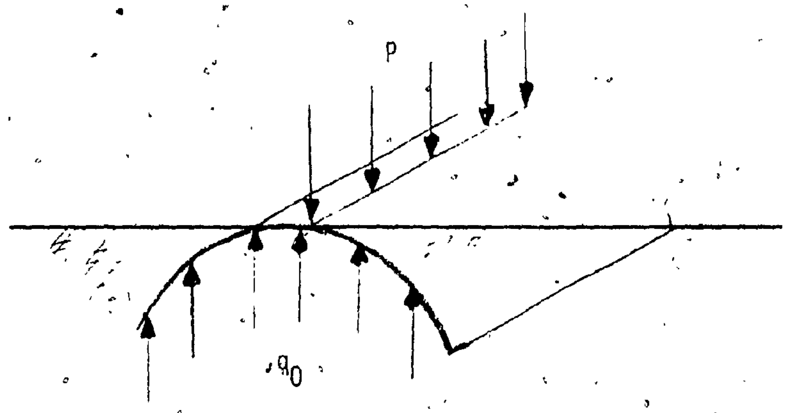


Figure 22. Shows the cylindrical strip footing.

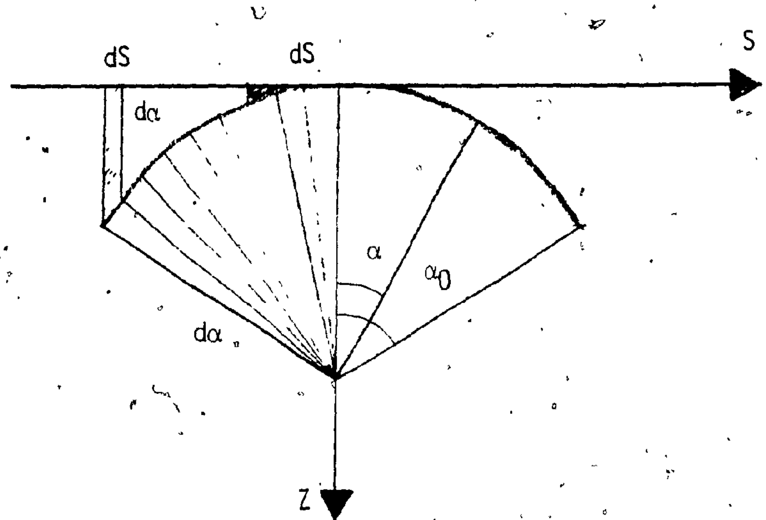


Figure 23. A sector divided into equal portions of value dα.

Figure 23, shows the following

$$s = R \sin \alpha \tag{171}$$

$$\alpha_0 = \sin^{-1} (\frac{B}{R}) \tag{172}$$

Differentiating equation (170), the following is obtained:

$$ds = R \cos \alpha \, d\alpha \tag{173}$$

then

$$q = q_0 \, ds = q_0 R \cos \alpha \, d\alpha \tag{174}$$

For equilibrium on cylindrical foundation, the total vertical line load acting on the soil (q_0 /unit area)$\cdot$ ds is equal to the initial load per unit length.

$$P = \Sigma q \, ds = \int_{-\alpha_0}^{\alpha_1} q_0 \, R \cos \alpha \, d\alpha$$

$$= 2q_0 \, R \sin \alpha_0 .$$

where $R \sin \alpha_0 = A$ (from equation (161)).

and

$$qds = \frac{P}{2B} R \cos \alpha \, d\alpha \tag{175}$$

The stresses due to q at point M in Figure 24, can be evaluated by using equations (110) to (112), developed by Melan's (1932). For the vertical components σ_z using equation (111) $\sigma_z = V \frac{q}{\pi}$ by replacing x by (s-x) = w, where

$$V = \frac{1}{2(1-\nu)} \{\frac{(z-d)^3}{R_1^4} + \frac{(z+d) \ (z+d)^2+2dz}{R_2^4} - \frac{8dz(d+z)w^2}{R_2^6} + \frac{1-2\nu}{4(1-\nu)} (\frac{z-d}{R_1^2} + \frac{3z+d}{R_2^2} - \frac{4zw^2}{R_2^4})\} \tag{176}$$

the Figure 24 shows that

d: is the vertical distance where the load acts.

$$|\overline{MN}| = R_1$$

$$|\overline{MT}| = R_2 \qquad (177)$$

$$|\overline{NT}| = 2d$$

in the (O_1, x, z) system, the vectors O_1N and O_1M are equal to

$$\overline{O_1N} = (R \sin\alpha, R \cos\alpha)$$

$$\overline{O_1M} = (x, r-z)$$

in (O, x, z) system

$$\overline{CN} = (R \sin\alpha, R - R\cos\alpha)$$

$$\overline{OM} = (x, z) = \text{constant}$$

$$\overline{OT} = (x_N, -z_N) = (R\sin\alpha, R\cos\alpha - R)$$

$$\overline{MN} = \overline{ON} - \overline{OM} = ((R\sin\alpha - x), (R - R\cos\alpha - z))$$

$$\overline{MT} = \overline{OT} - \overline{OM} = ((R\sin\alpha - x), (R\cos\alpha - R - z))$$

then the values of R_1, R_2, d and W are equal to

$$R_1 = |\overline{MN}| = ((R\sin\alpha - x)^2 + (R - R\cos\alpha - z)^2)^{1/2} \qquad (178)$$

$$R_2 = |\overline{MT}| = ((R\sin\alpha - x)^2 + (R\cos\alpha - R - z)^2)^{1/2} \qquad (179)$$

$$d = |Z_N| = R - R\cos\alpha. \qquad (180)$$

$$W = R\sin \quad - x \qquad (181)$$

As for a strip footing for an increment $d\alpha$, the vertical stress increment is obtained by using equations (175) and (176) as.

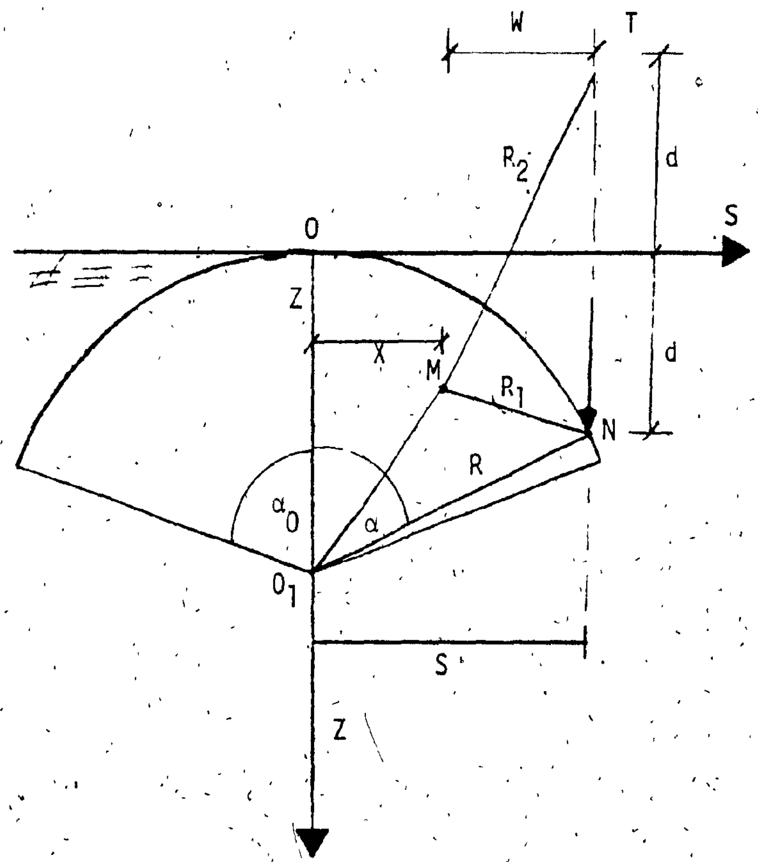


Figure 24. Figure showing the vectors and stresses for a cylindrical strip footing.

$$d\sigma_z = A\,\frac{q}{\pi}\,ds = \frac{AP}{2B\pi}\,R\cos\alpha\,d\alpha \qquad (111)$$

therefore, the total vertical stress is equal to the integration of the above equation from $-\alpha_0$ to α_0, thus

$$\sigma_z = \frac{PR}{2B\pi}\int_{-\alpha_0}^{\alpha_0}\cos\alpha\left[\frac{1}{2(1-\nu)}\left\{\left(\frac{(z+d)^3}{R_1^{\,4}} + \frac{(z+d)((z+d)^2+2dz)}{R_2^{\,4}} - \frac{8dz(d+2)w^2}{R_2^{\,6}}\right\} + \frac{1-2\nu}{4(1-\nu)}\left(\frac{z-d}{R_1^{\,2}} + \frac{3z+d}{R_2^{\,4}} - \frac{4zw^2}{R_2^{\,6}}\right)\right] \qquad (182)$$

where R_1, R_2 and d are given by equations (178), (179) and (180).

Similarly for the horizontal components, using equation (110)

$$\sigma_x = \frac{PR}{2B\pi}\int_{-\alpha_0}^{\alpha_0}\cos\alpha\left\{\frac{1}{2(1-\nu)}\left\{\frac{(z-d)w^2}{R_1^{\,4}} + \frac{(z+d)(w^2+2d^2)-2dw^2}{R_2^{\,4}} + \frac{8dz(d+z)w^2}{R_2^{\,6}}\right\} + \frac{1-2\nu}{4(1-\nu)}\left(\frac{d-z}{R_1^{\,2}} + \frac{z+3d}{R_2^{\,2}} + \frac{4zw^2}{R_2^{\,4}}\right)\right\} \qquad (183)$$

and the shear stress τ_{xz} using equation (112) is equal to

$$\tau_{xz} = \frac{PR}{2B\pi}\int_{-\alpha_0}^{\alpha_0}\cos\alpha\left\{\frac{1}{2(1-\nu)}\left\{\frac{(z-d)^2}{R_1^{\,4}} + \frac{z^2-2dz-d^2}{R_2^{\,4}} + \frac{8dz(d+z)^2}{R_2^{\,6}}\right\} + \frac{1-2}{4(1-\ )}\left(\frac{1}{R_1^{\,2}} - \frac{1}{R_2^{\,2}} + \frac{4z(d+z)}{R_2^{\,4}}\right)\right\} \qquad (184)$$

the value of each integral is obtained by using numerical analysis (the subroutine DCADRE in the library of Computer Science at Concordia University was used).

3.6 TRIANGULAR STRIP FOOTING.

By considering the shape of the footing as triangular (as shown in Figure 25), it is taken that the width of the footing is 2B and the peak angle is 2β. In addition, the triangular footing is divided into equal segments ds.

Assuming q_0 is the vertical load by unit area (taken at point load) the vertical line load q is a constant

$$q = q_0 \, ds \qquad (185)$$

As it is stated in equilibrium conditions, the sum of the vertical loads are equal to zero, therefore

$$\Sigma q - P = 0 \qquad (186)$$

where

$$q_0 = \frac{P}{2B} \text{ / unit area} \qquad (187)$$

$$q = \frac{P}{2B} \text{ ds/unit length} \qquad (188)$$

In order to analyse the stress components at point M below the ground level, the analysis of the footing described above will be similar to the analysis of the strip footing.

From Figure 26, let

$$R_1 = |\overline{MN}| \; ; \quad R_2 = |\overline{TM}| \text{ and } |\overline{TN}| = 2d \qquad (189)$$

where

$$d = \frac{S}{tg\ \beta} \qquad (190)$$

d is always positive, then if s is positive

$$d = \frac{S}{tg\ \beta} \qquad (191)$$

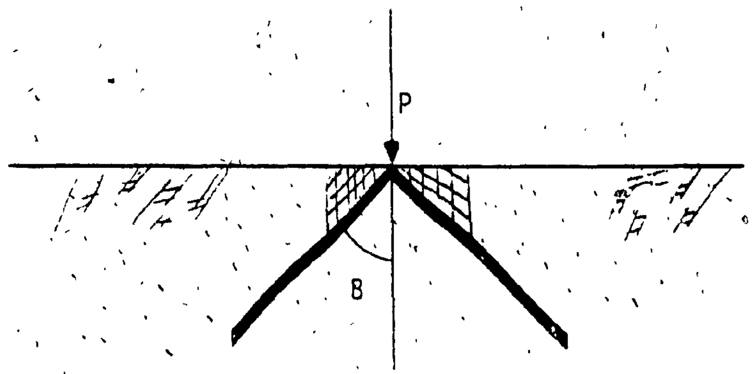


Figure 25. Shows the triangular strip footing.

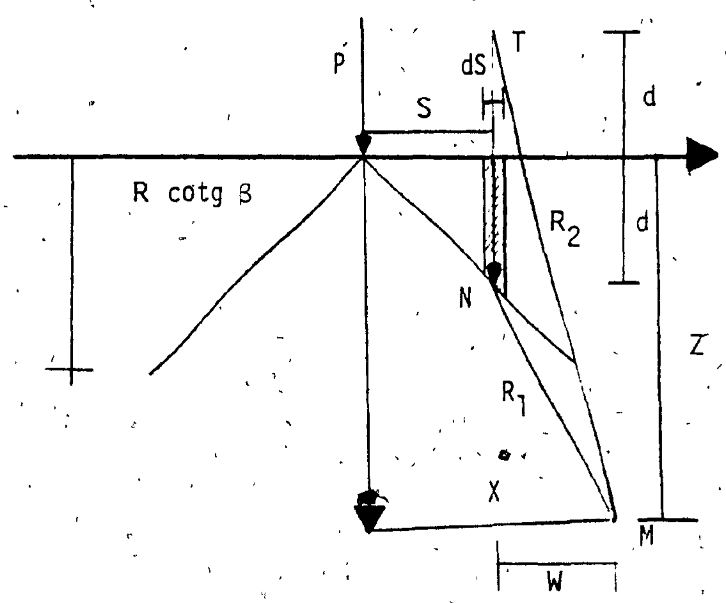


Figure 26. Figure showing the vectors and stresses on a cylindrical strip footing.

and if s is negative

$$d = \frac{-s}{tg\ \beta} \tag{192}$$

The following vectors can be written as

$$\bar{OM} = (x, z) \tag{193}$$

$$\bar{ON} = (s, d) \tag{194}$$

$$\bar{OT} = (s, -d) \tag{195}$$

$$\bar{MN} = \bar{ON} - \bar{OM} = (s-x, z-d) \tag{196}$$

$$\bar{TN} = \bar{ON} - \bar{OT} = (s-x, z+d) \tag{197}$$

Assume that

$$W = s - x \tag{198}$$

then the modulus of the vectors $\bar{NM}$ and $\bar{TM}$ are equal to

$$R_1 = |\bar{NM}| = \left[(s-x)^2 + (z-d)^2\right]^{1/2} \tag{199}$$

$$R_2 = |\bar{TN}| = \left[(x-s)^2 + (z+d)^2\right]^{1/2} \tag{200}$$

assuming

$$C_2 = \frac{P}{2\pi B} \tag{201}$$

an increment part of the load over width ds, the vertical stress increase $d\sigma_z$, integrating $d\sigma_z$ from -B to +B, yields

$$\sigma_z = \int_{-B}^{B} C_2 \left\{\frac{1}{2(1-\nu)} \left\{\frac{(z-d)^3}{R_1^4} + \frac{(z+d)\ (z+d)^2+2dz}{R_2^4} - \frac{8dz(d+z)w^2}{R_2^6} + \frac{1-2\nu}{4(1-\nu)}\ \frac{z-d}{R_1^2} + \frac{3z+d}{R_2^2} - \frac{4zw^2}{R_2^4}\right\} ds \tag{202}$$

Similarly for the horizontal component of the stress, σ_x is equal to

$$\sigma_x = \int C_2 \left[\frac{1}{1-2\nu} \left\{ \frac{(z-d)w^2}{R_1^4} + \frac{(z+d)(w^2+2d^2)-2dz^2}{R_2^4} \right. \right. \tag{203}$$

$$\left. \left. - \frac{8dz(d+z)}{R_2^6} \right\} + \frac{1-2\nu}{4(1-\nu)} \left\{ \frac{d-z}{R_1^2} + \frac{z+3d}{R_2^2} + \frac{4zw^2}{R_2^4} \right\} \right] ds$$

And the shear stress τ_{xz} is given by

$$\tau_{xz} = \int_{-B}^{B} C_2 \, w \left[\frac{1}{2(1-\nu)} \left\{ \frac{(z-d)^2}{R_1^4} + \frac{z^2-2dz-d^2}{R_2^4} + \frac{8dz(d+z)^2}{R_2^6} \right\} \right. \tag{204}$$

$$\left. + \frac{1-2}{4(1-\nu)} \left(\frac{1}{R_2^2} - \frac{1}{R_2^2} + \frac{4z(d+z)}{R_2^4} \right) \right] d\alpha$$

where R_1, R_2, d and C_2 are given by the equations (199), (200), (190) and (201). The upper and lower limits AL and BL are given by the following equation

$$AL = -B \quad \text{and} \quad BL = B. \tag{205}$$

3.7 SPHERICAL FOOTING.

The footing Figure 27, is a portion of a sphere, where the center O_1 is located at depth R from the ground level. The load P is acting on the soil as sum of unit load q acting on a unit surface dA (see Figure 28).

The coordinates system $O_1(x, y, z)$ will be used to determine the geometric characteristics for the surface of the sphere, which is defined

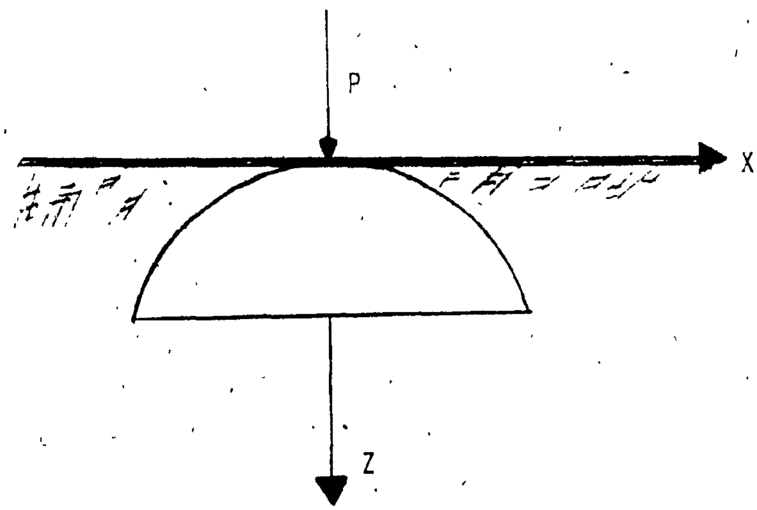


Figure 27. Figure showing a spherical footing.

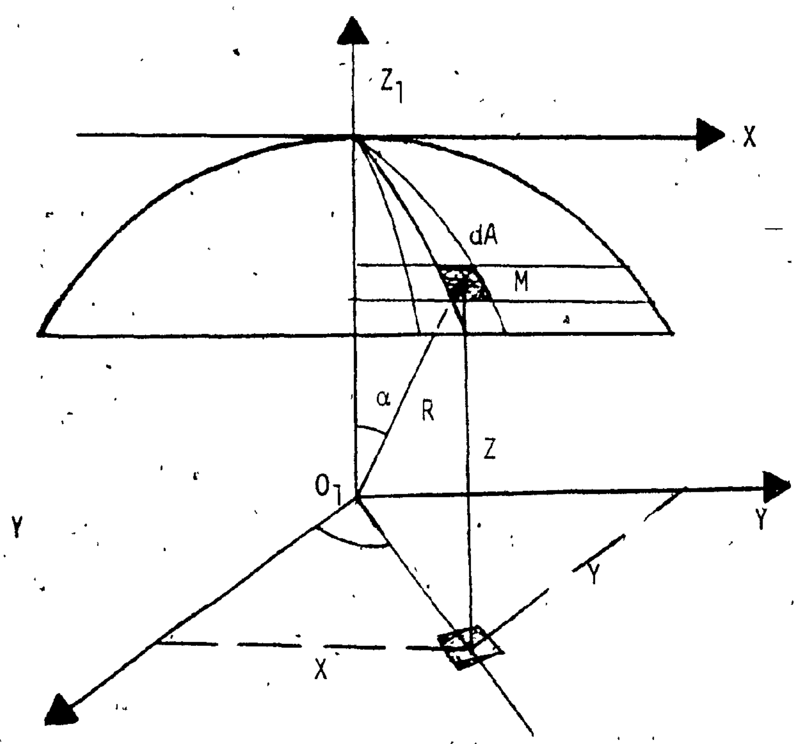


Figure 28. Vectors analysis for the unit infinitesimal area dA.

by a vector $\overline{O_1N}$ as

$$\overline{O_1N} = (x, y, z_1) \tag{204}$$

where

$$x = R \sin\alpha \cos\beta$$
$$y = R \sin\alpha \sin\beta \tag{205}$$
$$z_1 = R \cos\alpha$$

the angles α and β are shown in Figure 28, as the parameters of the coordinates x, y, z_1.

The first derivative of the vector $\overline{O_1N}$ is obtained by

$$\overline{O_1N},_\alpha = (-R \cos\alpha \cos\beta,\ R \cos\alpha \sin\beta,\ -R \sin\alpha) \tag{206}$$
$$\overline{O_1N},_\beta = (-R \sin\alpha \cos\beta,\ R \sin\alpha \sin\beta,\ 0) \tag{207}$$

The unit area dA can be obtained by using the equation (128)

$$dA = |\overline{ON},_\alpha \times \overline{ON},_\beta| = |\overline{ON},_\alpha|\ |\overline{ON},_\beta| \sin\omega\, d\alpha d\beta \tag{128}$$

where ω is the angle between the two vectors $\overline{O_1N},_\alpha$ and $O_1N,_\beta$. However, the scalar product of these two vectors is zero, therefore

$$\omega = \frac{\pi}{2} \quad \text{and} \quad \sin\omega = 1. \tag{208}$$

the modulus $|\overline{O_1N},_\alpha|$ and $|\overline{O_1N},_\beta|$ are given by

$$|\overline{O_1N},_\alpha| = (R^2 \cos^2\alpha \cos^2\beta + R^2 \cos^2\alpha \sin^2\beta + R^2 \sin^2\alpha)^{1/2}$$
$$= (R^2 \cos^2\alpha(\cos^2\beta + \sin^2\beta) + R^2 \sin^2\alpha)^{1/2} = R \tag{209}$$

$$|\overline{O_1N},_\beta| = (R^2 \sin^2\alpha \sin^2\beta + R^2 \sin^2\alpha \cos^2\beta)^{1/2}$$
$$= R \sin\alpha. \tag{210}$$

Substituting equations (208), (209) and (210) for equation (128), the following value is obtained

$$dA = R^2 \sin \alpha \, d\alpha \, d\beta. \tag{211}$$

The projection of the area dA on x y-plane is given by

$$dA = dA \cos \alpha = R^2 \sin \alpha \cos \alpha \, d\alpha \, d\beta. \tag{212}$$

However, α and β are two independent angles, and α varies from zero to α_0 and β varies from zero to 2π, where

$$\alpha_0 = \sin^{-1} \left(\frac{B}{R}\right) \tag{213}$$

The center of the sphere shown in Figure 29 located at the point O_1 it is at a distance of OO_1 from the original center O of the system of coordinates O(x, y, z), with a positive sense of direction of the z-axis pointing downwards.

Let N be the point of contact between the soil and the sphere at which the point load q is acting, q is taken as a unit load. The projection of point N on the x, y-plane is the point H, while T is the point symmetric to N, with respect to the x, y-plane.

Therefore,

$$|\overline{NH}| = |\overline{HT}| = d \tag{214}$$

Let M be any point in the soil with (x, y, z) coordinates, for which the stresses can be calculated. The modulus of the vectors $\overline{NM}$ and $\overline{TM}$ are given by

$$\begin{aligned} |\overline{NM}| &= R_1 \\ |\overline{TM}| &= R_2. \end{aligned} \tag{215}$$

However, the following vectors can be obtained by

$$\overline{OM} = (x_M, y_M, z_M)$$

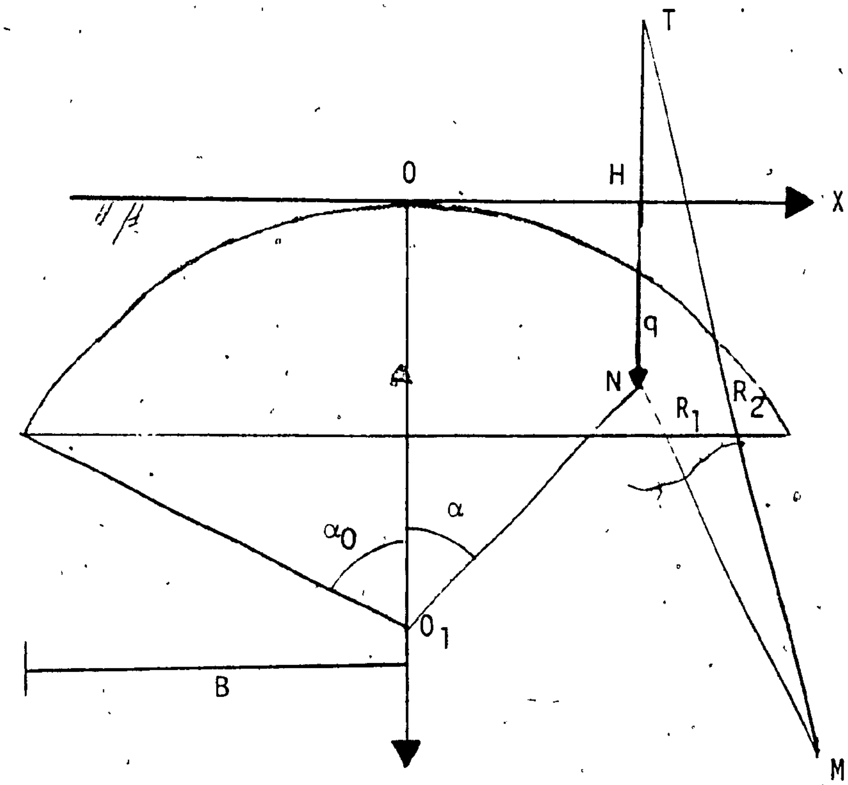


Figure 29. Figure showing the vectors and stresses analysis for a spherical footing.

$$\overline{ON} = (x_N, y_N, z_N) = (R \sin \alpha \cos \alpha, R \sin \alpha \cos \beta, R\text{-}R \sin \alpha)$$

$$\overline{ON} = (x_N, y_N, -z_N)$$

$$\overline{MN} = \overline{ON} - \overline{OM}$$
$$= ((x_N - x_M), (y_N - y_M), (z_N - z_M) \qquad (216)$$

and

$$\overline{TM} = \overline{OT} - \overline{OM}$$
$$= ((x_T - x_M), (y_T - y_M), (z_T - z_M)) \qquad (217)$$

After replacing the coordinates by their values the distance R_1, R_2 and d is found as follows

$$R_1 = |\overline{MN}| = ((R \sin \alpha \cos \beta - x)^2 + (R \sin \alpha \cos \beta - y)^2 + (R\text{-}R \cos \quad - z)^2)^{1/2} \qquad (218)$$

$$R_2 = |\overline{MT}| = ((R \sin \alpha \cos \beta - x)^2 + (R \sin \alpha \cos \beta - y)^2 + (R\text{-}R \cos \alpha + z)^2)^{1/2} \qquad (219)$$

$$d = R\text{-}R \cos \alpha. \qquad (220)$$

The vertical stress at point M can be obtained by superposition of all the partial vertical stresses acting on differential unit area dA on the footing. Let V equal to σ_z as given in equation (118) developea by Mindlin (1936)

$$d\sigma_z = q \; V \; d \; A \qquad (221)$$

However, dA is given by equation (211) as

$$dA = R^2 \sin \alpha \cos \alpha \; d\alpha \; d\beta$$

Substituting equation (211) in equation (221) and integrating along the spherical surface limited by the footing. The vertical stress a point M, is given as follows

$$\sigma_z = \frac{qR^2}{8\pi(1-\mu)} \int_0^{2\pi} \int_0^{\alpha_0} \sin\alpha \cos\alpha \left[\frac{(1-2\nu)(z-d)}{R^3} + \frac{(1-2\nu)(z-d)}{R_2^{\,3}} \right. \tag{222}$$

$$\left. - \frac{3(z-d)^3}{R_1^{\,5}} - \frac{3(3-4\nu)z(z+d)^2 - 3d(z+d)(5z-d)}{R_2^{\,5}} - \frac{30dz(z+d)^3}{R_2^{\,7}} \right] dx$$

Similarly for the horizontal component σ_x can be obtained by using the same procedure as for the vertical stress component, by integrating the value of σ_x given by equation (117)

$$\sigma_x = \frac{qR^2}{8\pi(1-\nu)} \int_0^{2\pi} \int_0^{\alpha_0} \sin\alpha \cos\alpha \left[\frac{(1-2\nu)(z-d)}{R_1^{\,3}} - \frac{3x^2(z-d)}{R_1^{\,5}} \right.$$

$$+ \frac{(1-2\nu)\{3(z-d) + 4\nu(z+d)\}}{R_2^{\,3}} -$$

$$- \frac{(3-4\nu)x^2(2-d) - 6d(z+d)(1-2\nu)z - 2\nu d}{R_2^{\,5}} \tag{223}$$

$$\left. - \frac{30d\,x^2 z(z+d)}{R_2^{\,7}} - \frac{4(1-\nu)(1-2\nu)}{R_2(R_2+z+d)} \times \left(1 - \frac{x^2}{R_2(R_2+z+d)} - \frac{x^2}{R_2^{\,2}}\right) \right] d\alpha$$

also the shear stress can be obtained by using the same procedure as before, by integrating the value of τ_{xz} given by equation (119)

$$\tau_{xz} = \frac{qR^2}{8\pi(1-\mu)} \int_0^{2\pi} \int_0^{\alpha_0} \sin\alpha \cos\alpha \left[-\frac{(1-2\nu)}{R_1^3} + \frac{1-2\nu}{R_2^3} - \frac{3(z-d)^3}{R_1^5} \right. \tag{224}$$

$$\left. - \frac{3(3-4\nu)z(z+d) - 3d(3z+d)}{R_2^5} - \frac{30dz(z+d)^3}{R_2^7} \right] d\alpha$$

the stress components σ_y and τ_{yz} are equal to the stress components σ_x and τ_{xz} respectively, because the load and the footing are symmetric about the z-axis.

3.8 THE S-SIN STRIP MODEL

The model strip foundation has a width of 2B (see Figure 30), and the curvature is a function of sin S as

$$Z = \sin^2 (\pi S). \tag{225}$$

The vector analysis for this model (see Figure 31), is similar to the analysis mode for the triangular models. Again, a rigid foundation is considered and the load intensity is linearly proportional to the unit length, dS. Referring to the equations (202), (203) and (204), the modulus of the vectors $\overline{MN}$ and $\overline{MT}$ is given by

$$|\overline{MN}| = R1 = [W^2 + (z-d)^2] \tag{226}$$

$$|\overline{MT}| = R2 = [W^2 + (z+d)^2] \tag{227}$$

where

$$d = \sin^2 (\pi S) \tag{228}$$

$$W = S - X \tag{229}$$

and

S varies between -B to B.

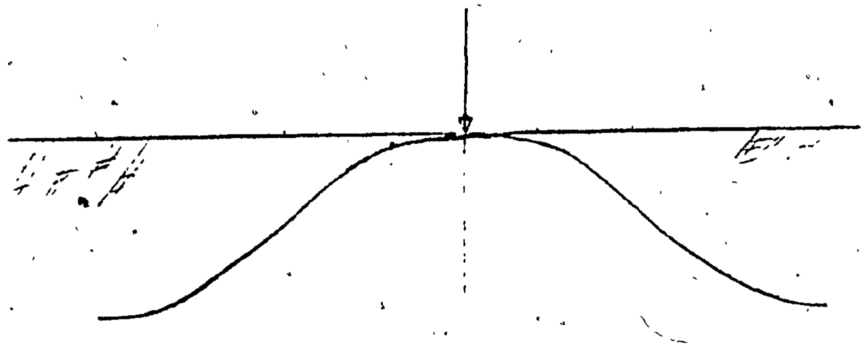


Figure 30. The shape of the model S sin.

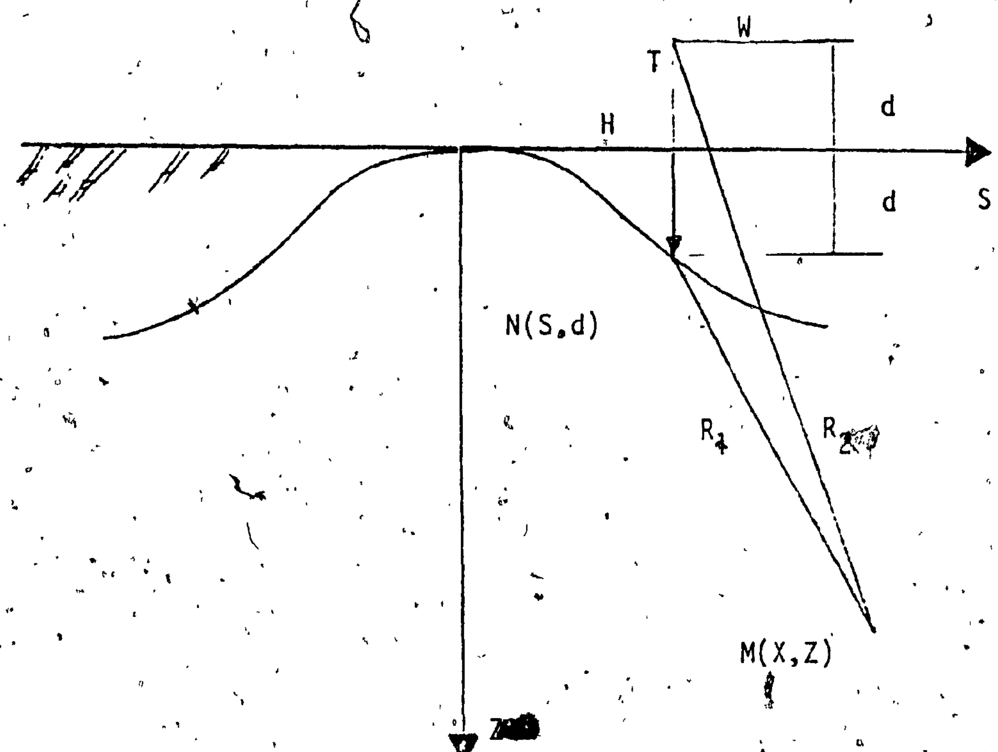


Figure 31. Figure showing the vectors and stresses analysis for the shape S-sin.

3.9 COMPUTER PROGRAM

The following program is coded to analyze stresses for the models proposed in this paper.

Five main programs are used, each consists of a main program and 9 subroutines. All programs are similar, except data input and the subroutine which computes the geometric characteristics of each model.

As an example, a typical program is shown below as a flat strip, and cylindrical modes.

The rectangular coordinates are used in case of flat, triangular and S-sin models.

The polar coordinates system is used in case of cylindrical models.

The polar coordinates system is used in case of cylindrical strip and circular plate modes.

The spherical coordinates system is used in case of spherical models.

All the details of the main program and the subroutines are explained in the program itself as comments.

```
      PROGRAM STRESS(INPUT,OUTPUT)
      COMMON R,Q,B,Z,X,MU
      REAL MU
      EXTERNAL VSC,VSF
      DIMENSION  VC(8,9),VF(8,9),RAV(8,9),RAH(8,9),RAT(8,9)
      DIMENSION HF(8,9),TAF(8,9),HC(8.9) TAC(8,9)
C         *****************************************************
C         *                                                   *
C         *                                                   *
C         *   THIS PROGRAM IS TO CALCULATE THE STRESSES       *
C         *   DISTRIBUTION AT LEVEL Z                         *
C         *                                                   *
C         *****************************************************
C
C
C---------- -------------------------
C         DATA OF THE PROGRAM
C------ ---------------------------
C---- ----------------------------
C
C       Q : THE LOAD PER UNIT LENGTH
C       Z : THE DEPTH OF THE ELEMENT M
C       R : THE RADIUS OF THE FOOTING
C       S : THE DISTANCE FROM THE SYMMETRICAL AXIS AT DEPTH Z
C       2B: THE WIDTH OF THE FOOTING
C       MU: THE POISSON.S RARIO
C-------- --------------------------
C
C       READING OF THE DATA
C_____________________________________
      READ*,MU
      Q=1
      B=.5
C---------------------------------------------------------
C       TO CALCULATE THE DISTRIBUTION OF THE STRESSES
C---------------------------------------------------------
      BW=2*B
C--- --- ---------------------------
C           STRESSES
C       FOR THE FLAT RECTANGULAR STRIP
C----------------------------------
      CALL SIGMAF(VF,HF,TAF)
C--- ---- -- ----------------------
C           STRESSES
C         FOR A CYLINDRICAL STRIP
C-------- ---------------------------------
      DO 10 KK=1,3
      READ*,R
      CALL SIGMAC(VC,HC,TAC)
C---------- ------------------------------
C       COMPUTE THE RATIO OF THE STRESSES
C------- ----------------------------------
> ?
```

```
C------ -- ---- ------------------------------
      DO 1 I=1,8
      DO 1 J=1,9
      RAV(I,J)=VF(I,J)/VC(I,J)
      RAH(I,J)=HF(I,J)/HC(I,J)
1     RAT(I,J)=TAF(I,J)/TAC(I,J)
C------------- ----------------------------
C        PRINT  ALL THE STRESSES
C        AND THEIR RATIOS
C------- ---------------------------------
       PRINT 3,MU,Q,BW
       CALL PRINT(VF)
       PRINT 4
      PRINT 5,R,BW,Q
      CALL PRINT(VC)
C     CALL PRINTR (RAV)
C
       PRINT 31,MU,Q,BW
      CALL PRINT(HF)
       PRINT 41
       PRINT 5,R,BW,Q
       CALL PRINT(HC)
C      PRINT 61
C      CALL PRINTR(RAH)
C
      PRINT 32,MU,Q,BW
      CALL PRINT(TAF)
      PRINT 42
        PRINT 5,R,BW,Q
        CALL PRINT (TAC)
C     PRINT 62
C     CALL PRINTR(RAT)
3      FORMAT('1  /,10X,42('='),/,10X,'VERTICAL STRESS FOR A '
    $,'FLAT STRIP FOOTING',/,10X,42('='),/.10X,'THE POISSON.S RATIO'
    $ ,'    IS EQUAL  TO    : ',F3.1,/,10X, THE LOAD APPLIED IS '
    $ , EQUAL   TO        :',F4.1,/,10X,'THE WIDTH OF THE FOOTING IS ',
    $ 'EQUAL TO :',F4.1,/,10X,42('='))
C
4     FORMAT('1' /,10X,48('='),/,10X, VERTICAL STRESS FOR A'
    $    ,' CYLINDRICAL STRIP  FOOTING ',/,10X,48('='))
C
31       FORMAT('1',/,10X,42('='),/,10X,'HORIZONTAL STRESS FOR A '
    $,'FLAT STRIP FOOTING',/,10X,42('=')./,10X,'THE POISSON.S RATIO'
    $  ,'   IS EQUAL TO   : ',F3.1,/,10X,'THE LOAD APPLIED IS '
    $ ,'EQUAL   TO        :'.F4.1,/,10X,'THE WIDTH OF THE FOOTING IS ',
    $ 'EQUAL TO :'.F4.1,/,10X,42('='))
C
41      FORMAT('1'./,10X,50('='),/,10X, HORIZONTAL STRESS FOR A'
    $    ,' CYLINDRICAL STRIP  FOOTING './,10X,50('='))
C
32       FORMAT('1',/,10X,42('='),/,10X,'SHEAR STRESS FOR A '
    $,'FLAT STRIP FOOTING',/,10X,42('='),/,10X,'THE POISSON.S RATIO'
    $  ,'   IS EQUAL  TO    : ',F3.1,/.10X, THE LOAD APPLIED IS '
```

```
     $ ,'EQUAL  TO        :',F4.1,/,10X, THE WIDTH OF THE FOOTING IS '
     $ 'EQUAL TO :'.F4.1,/.10X,42('='))
C
42      FORMAT('1'./,10X,45('='),/,10X,'SHEAR STRESS FOR A'
     $    ,' CYLINDRICAL STRIP  FOOTING ',/,10X,45('='))
C
5       FORMAT(9X,' THE RADIUS OF THE CIRCLE IS EQUAL TO :',
     $  F6.1,/,9X, THE WIDTH OF THE FOOTING IS EQUAL TO :'.F6.1,
     $ /,9X,' THE LINE LOAD APPLIED IS EQUAL TO    :',F6.1
     $ ,/,10X,44('='))
C
6        FORMAT('1'./,44('*'),/,' THE RARIO OF VERTICAL STRESS OF '
     $ , /,' (FLAT RECTANGULAR STRIP / CYLINDRICAL STRIP)',/,44('*'),/)
61        FORMAT('1',/,44('*'),/,' THE RARIO OF HORIZONTAL STRESS OF '
     $ ,/,' (FLAT RECTANGULAR STRIP / CYLINDRICAL STRIP)',/,44('*'),/)
62        FORMAT('1',/.44('*'),/,' THE RARIO OF SHEAR STRESS OF '
     $ ,/,' (FLAT RECTANGULAR STRIP / CYLINDRICAL STRIP)' /.44('*'),/)
10      CONTINUE
        STOP
        END
C
C
C
      SUBROUTINE SIGMAF(VF,HF,TAF)
        DIMENSION VF(8,9) HF(8,9),TAF(8,9)
C
C        THIS SUBROUTINE IS TO EVALUATE THE STRESSES FOR.
C        A FLAT STRIP FOOTING
C
        REAL MU
        COMMON R,Q,B,Z,X,MU
        EXTERNAL VSF
        EXTERNAL HSF
        EXTERNAL TSF
C
C        THE UPPER AND LOWER LIMITS
C
        AL=-B
        BL=-B/10000
        CL=B/10000
        DL=B
        Z=0
        DO 1 I=1,8  .
        X=-2*B
        Z=Z+B/2
         DO 1 J=1,9
C--------------------------------------------
C        TO EVALUTE THE INTEGRALS OF:
C--------------------------------------------
C  A.     VERTICVAL STRESS
C         ================
         F2=DCADRE(VSF,AL,BL,.0001,.0001,ERROR,IER)
         F3=DCADRE(VSF,CL,DL,.0001,.0001,ERROR,IER)
> ?
```

```
         VF(I,J)=F2+F3
C-------------------------------------------
C  B.    HORIZONTAL STRESS
C        =================
         H1=DCADRE(HSF,AL,BL,.0001,.0001,ERROR,IER)
         H2=DCADRE(HSF,CL,DL, 0001,.0001,ERROR,IER)
         HF(I,J)=H1+H2
C--------------------------------------------
C  C.    SHEAR STRESS
C        ============
         T1=DCADRE(TSF,AL,BL,.0001,.0001,ERROR,IER)
         T2=DCADRE(TSF,CL,DL,.0001,.0001,ERROR,IER)
         TAF(I,J)=T1+T2
1        X=X+B/2
         RETURN
        END
C
C
C
          FUNCTION VSF(S)
C--------------------------------------------------------------
C         THIS SUBROUTINE IS TO EVALUATE THE VERTICAL
C         FUNCTION FOR A FLAT STRIP FOOTING
C------------------------------------------------------------------
          COMMON R,Q,B,Z,X,MU
          C1=Q/(3.14159*B)
          VSF=C1*((Z**3)/((S-X)**2+Z**2)**2)
          RETURN
         END
C
C
C
          FUNCTION HSF(S)
           COMMON R,Q,B,Z,X,MU
         REAL MU
C---------------------------------------------------------------
C         THIS SUBROUTINE IS TO EVALUATE THE HORIZONTAL
C         FUNCTION FOR A FLAT STRIP SHAPE
C---------------------------------------------------------------
         C1=Q/(3.14159*B)
         HSF=C1*Z*(S-X)**2/((S-X)**2+Z*Z)**2
         RETURN
         END
C
C
C
         FUNCTION TSF(S)
         REAL MU
C----------------------------------------------------------------
C        THIS SUBROUTINE IS TO EVALUETE THE SHEAR
C         FUNCTION FOR A FLAT STRIP FOOTING
C---------------------------------------------------------------
          COMMON R,Q,B Z,X,MU
> ?
```

```
      C1=Q/(3.14159*B)
      TSF=C1*(S-X)*Z*Z/((S-X)**2+Z*Z)**2
      RETURN
      END
C
C
C
      SUBROUTINE SIGMAC(VC,HC,TAC)
       DIMENSION VC(8,9),HC(8,9),TAC(8,9)
C
C     THIS SUBROUTINE IS TO EVALUATE THE STRESSES FOR
C     A CYLINDRICAL STRIP FOOTING
C
      REAL MU
       COMMON R,Q,B,Z,X,MU
      EXTERNAL VSC
      EXTERNAL HSC
      EXTERNAL TSC
C
C      THE UPPPER AND LOWER LIMITS
C
       AL=ASIN(-B/R)
      BL=AL/1000
      CL=-BL
       DL=-AL
       Z=0
        DO 1 I= 1,8
         X=-2*B
           Z=Z+B/2
           DO 1 J=1,9
C-------------------------------------------
C      TO EVALUATE THE INTEGRALS OF:
C-------------------------------------------
C  A.   VERTICAL STRESS
C       ===============
                  FI=DCADRE(VSC,AL,BL,.0001,.0001,ERROR,IER)
                  FJ=DCADRE(VSC,CL,DL,.0001,.0001,ERROR,IER)
                  VC(I,J)=FI+FJ
C-------------------------------------------
C  B.  HORIZONTAL STRESS
C      =================
       H1=DCADRE(HSC,AL,BL,.0001,.0001,ERROR,IER)
       H2=DCADRE(HSC,CL,DL,.0001,.0001,ERROR,IER)
       HC(I,J)=H1+H2
C-------------------------------------------
C  C.  SHEAR STRESS
C      ============
       T1=DCADRE(TSC,AL,BL,.0001,.0001,ERROR,IER)
       T2=DCADRE(TSC,CL,DL,.0001,.0001,ERROR,IER)
       TAC(I,J)=T1+T2
 1        X=X+B/2
        RETURN
       END
```

```
      FUNCTION VSC(AP)
      REAL MU
      COMMON R,Q,B,Z,X,MU
C--------------------------------------------------------------
C       THIS SUBROUTINE IS TO EVALUATE THE VERTICAL
C       FUNCTION FOR A STRIP FOOTING WITH A CYLINDRICAL SHAPE
C--------------------------------------------------------------
      CALL CARCYL(AP,SM,TM,C2,W,D,R1,R2)
C       VALUE OF THE FUNCTION VSC
C------------------------------------------------
       IF (R1.EQ.Ø.OR.R2.EQ.Ø) THEN
       VSC=Q*(R-Z)/2/B
       ELSE
      VSC=C2*COS(AP)*(SM*((Z-D)**3/(R1**4)+(Z+D)*
     $((Z+D)**2+2*D*Z)/(R2**4)-8*D*Z*(D+Z)*W*W
     $ /(R2**6))+TM*((Z-D)/R1/R1+(3*Z+D)
     $      /R2/R2-4*Z*W*W/(R2**4)))
      END IF
      RETURN
      END
C
C
C
      FUNCTION HSC(AP)
      REAL MU
      COMMON R,Q,B,Z,X,MU
C--------------------------------------------------------------
C       THIS SUBROUTINE IS TO EVALUATE THE HORIZONTAL
C       FUNCTION FOR A STRIP FOOTING WITH A CYLINDRICAL SHAPE
C--------------------------------------------------------------
      CALL CARCYL(AP,SM,TM,C2,W,D,R1,R2)
C-------------------------------------
C       VALUE OF THE FUNCTION HSC
C-------------------------------------
      IF(R1.EQ.Ø.OR.R2.EQ.Ø) THEN
      HSC=Ø
      ELSE
      HSC=C2*COS(AP)*(SM*((Z-D)*W*W/(R1**4)+(Z+D)*
     $((W*W+2*D*D)-2*D*W*W)/(R2**4)+8*D*Z*(D+Z)*W*W
     $ /(R2**6))+TM*((D-Z)/R1/R1+(3*D+Z)
     $      /R2/R2-4*Z*W*W/(R2**4)))
      END IF
      RETURN
      END
C
C
C
      FUNCTION TSC(AP)
      REAL MU
      COMMON R,Q,B,Z,X,MU
C--------------------------------------------------------------
C       THIS SUBROUTINE IS TO EVALUATE THE SHEAR
C       FUNCTION FOR A STRIP FOOTING WITH A CYLINDRICAL SHAPE
> ?
```

```
C-----------------------------------------------------------------
      CALL CARCYL(AP,SM,TM,C2,W,D,R1,R2)
C      VALUE OF THE FUNCTION TSC
C-----------------------------
      IF(R1.EQ.Ø.OR.R2.EQ.Ø) THEN
      TSC=Ø
      ELSE
      A1=(Z-D)**2/(R1**4)
      A2=(Z*Z-2*D*Z-D*D)/(R2**4)
      A3=8*D*Z*(D+Z)**2/(R2**6)
      TSC=C2*((W)*(SM*(A1+A2+A3)+
     $ TM*(1/R1/R1-1/R2/R2+4*Z*(D+Z)/R2**4)))
      END IF
      RETURN
      END
C
C
C
      SUBROUTINE CARCYL(AP,SM,TM,C2,W,D,R1,R2)
       REAL MU
         COMMON R,Q,B,Z,X,MU
C ---------------------------------------------------------------
C        THIS SUBROUTINE IS TO COMPUTE THE CHARACTERISTICS
C        OF THE CYLINDRICAL FOOTING
C-----------------------------------------------------------------
      SM=1/(2*(1-MU))
      TM=(1-2*MU)/4/(1-MU)
       C2=Q*R/(3.14159*2*B)
      W=R*SIN(AP)-X
      D=R-R*COS(AP)
      R1=SQRT(W*W+(Z-D)**2)
      R2=SQRT(W*W+(Z+D)**2)
      RETURN
      END
C
C
C
      SUBROUTINE PRINT(F)
      DIMENSION F(8,9),TK(9)
      COMMON BT,Q,B,Z,X,MU
       Z=Ø
      BW=2*B
      TK(1)=-2*BW
      DO 3 I=2,9
 3    TK(I)=TK(I-1)+BW/2
         PRINT 12,(TK(J),J=1,9)
 12      FORMAT(/,' Z/B ','X/B=',1X,F4.1,8(2X,F4.1),/,63('-'))
       DO 2  I=1,8
       Z=Z+B/2
       T=Z/B
         PRINT 13,T,(F(I,J),J=1,9)
 13    FORMAT(1X,F3.1,4X,9(F6.4),/,63('-'))
 2        CONTINUE
      RETURN
      END
```

CHAPTER IV

RESULTS AND ANALYSIS

4.1 GENERAL

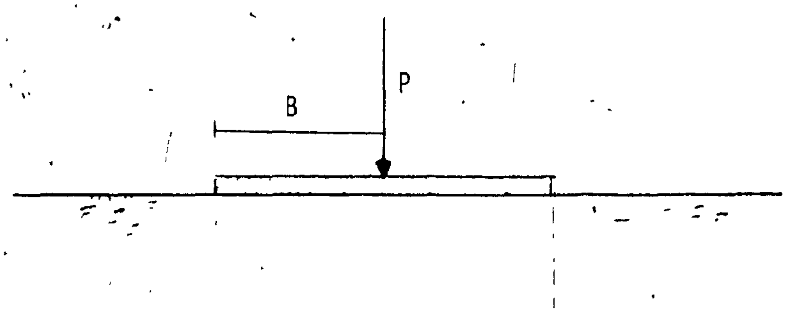
The computer program developed in the present study (Chapter III) to determine stress distribution. Below different foundation levels have been used extensively to generate data, for the purpose of comparison between different foundation models and to draw a conclusion.

4.2 PROGRAM DATA

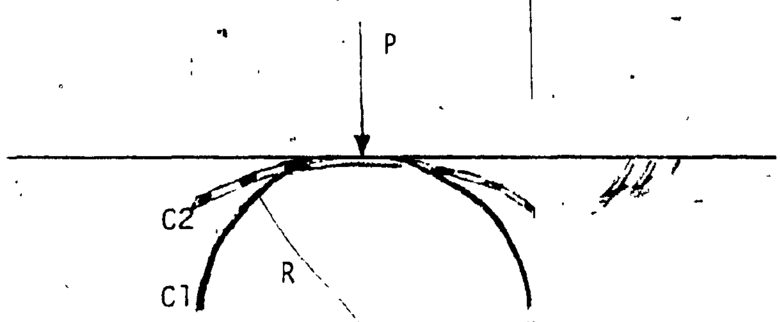
The program is divided into subroutines to evaluate the stress distribution, the load applied is: a unit line load per unit length, or a unit load per unit area. The width of the foundation is always equal to 2B.

For the flat footing strip or circular, only Poisson's ratio ν is needed to execute this part of the program.

For the new proposed models only one variable is required, which depends on the shape of the foundation. For example, for the cylindrical foundation (see Figure 32-b). The radius of the circle R is the variable and for the triangular foundation, the angle peak has a value of 2β, the same data is used for the spherical and the circular foundation, where the variable is the radius R.

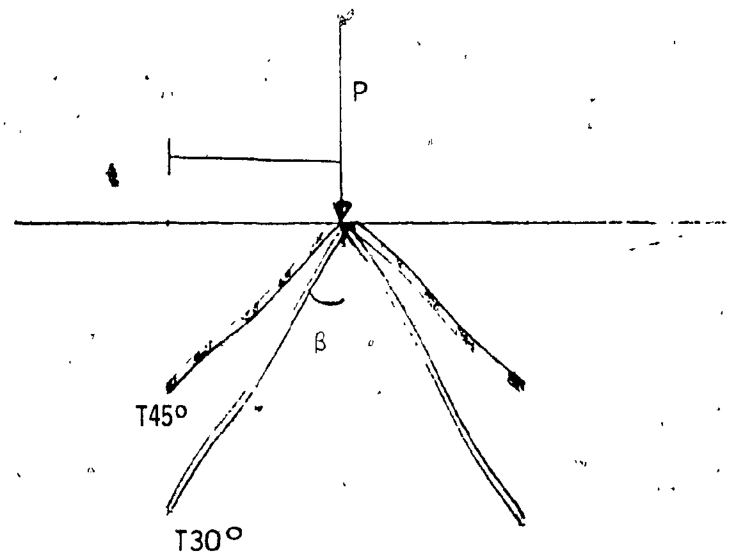


a. Flat strip foundation.

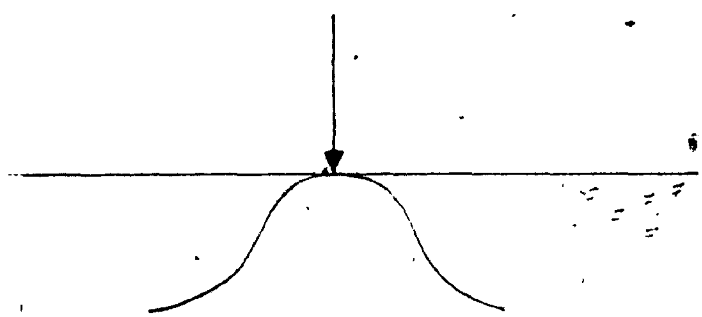


b. C1: cylindrical model with radius R = B.

C2: cylindrical model with radius R = 2B.



c. T30: Triangular model with angle $\beta = 30^0$.
T45: Triangular model with angle $\beta = 45^0$.



d. S cos shape foundation.

Figure 32. Shows different types of footing.

Example of data:

For the cylindrical model

·5	(μ)
1	R
3	R
2000	R

For the triangular model

·5	(μ)
30	(β)
45	(β)
90	(β)

NOTE: Where R = 2000 or $\beta = 90^{0}$, the two above models will work as a flat strip footing.

For the strip footing, where the shape is a function of sin πS called S sin

$$Z = \sin^2(\pi S)$$

the Poisson's ratio μ is the only data required.

4.3 PROGRAM OUTPUT

The mesh of location of the points, where the stresses are analysed (shown in Figure 33). In the horizontal direction, the first point is at S = -2B and the last point is at S = 2B, and the distance between two successive is ·5B. However, in the Z direction the analysis is done at eight levels, the first level is Z = ·5B, and the last one is Z = 4B.

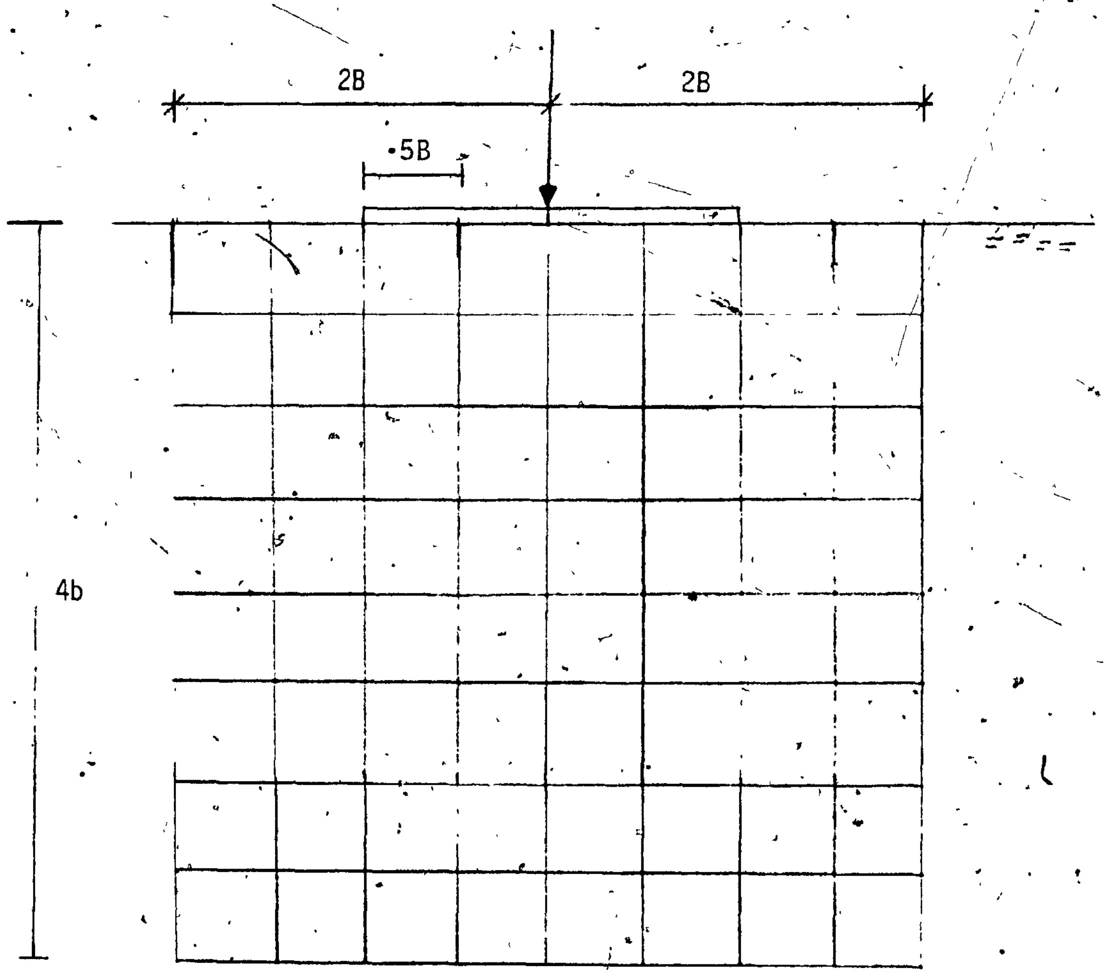


Figure 33. This mesh indicates the location of the points, where the stresses are calculated.

TABLE-1 VERTICAL STRESS FOR A FLAT STRIP FOOTING

THE POISSON.S RATIO IS EQUAL TO : .5
THE LOAD APPLIED IS EQUAL TO : 1.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0193	.0892	.4969	.9022	.9592	.9022	.4969	.0892	.0193
1.0	.0839	.2137	.4797	.7346	.8182	.7346	.4797	.2137	.0839
1.5	.1457	.2705	.4479	.6070	.6681	.6070	.4479	.2705	.1457
2.0	.1848	.2876	.4091	.5104	.5498	.5104	.4091	.2876	.1848
2.5	.2045	.2851	.3700	.4365	.4617	.4365	.3700	.2851	.2045
3.0	.2112	.2735	.3340	.3791	.3958	.3791	.3340	.2735	.2112
3.5	.2102	.2582	.3023	.3338	.3453	.3338	.3023	.2582	.2102
4.0	.2047	.2421	.2749	.2976	.3057	.2976	.2749	.2421	.2047

1

TABLE-2 HORIZONTAL STRESS FOR A FLAT STRIP FOOTING

THE POISSON.S RATIO IS EQUAL TO : .5
THE LOAD APPLIED IS EQUAL TO : 1.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.1707	.2851	.3471	.3929	.4502	.3929	.3471	.2851	.1707
1.0	.2112	.2488	.2251	.1862	.1817	.1862	.2251	.2488	.2112
1.5	.1848	.1806	.1424	.0978	.0805	.0978	.1424	.1806	.1848
2.0	.1456	.1269	.0908	.0551	.0405	.0551	.0908	.1269	.1456
2.5	.1109	.0892	.0595	.0332	.0227	.0332	.0595	.0892	.1109
3.0	.0839	.0636	.0403	.0212	.0138	.0212	.0403	.0636	.0839
3.5	.0638	.0463	.0281	.0142	.0090	.0142	.0281	.0463	.0638
4.0	.0490	.0343	.0203	.0100	.0062	.0100	.0203	.0343	.0490

1

TABLE-3 SHEAR STRESS FOR A FLAT STRIP FOOTING

THE POISSON.S RATIO IS EQUAL TO : .5
THE LOAD APPLIED IS EQUAL TO : 1.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0551	-.1469	.2996	.1273	0.0000	-.1273	-.2996	-.1469	-.0551
1.0	.1273	.2107	.2546	.1567	.0000	-.1567	-.2546	-.2107	-.1273
1.5	.1567	.2022	.2037	.1273	.0000	-.1273	-.2037	-.2022	-.1567
2.0	.1567	.1753	.1591	.0959	0.0000	-.0959	-.1591	-.1753	-.1567
2.5	.1439	.1469	.1242	.0720	.0000	-.0720	-.1242	-.1469	-.1439
3.0	.1273	.1218	.0979	.0551	0.0000	-.0551	-.0979	-.1218	-.1273
3.5	.1108	.1012	.0783	.0430	.0000	-.0430	-.0783	-.1012	-.1108
4.0	.0959	.0845	.0637	.0343	.0000	-.0343	-.0637	-.0845	-.0959

1

TABLE-4 VERTICAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 1000.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0193	.0892	.4969	.9016	.9569	.9016	.4969	.0892	.0193
1.0	.0839	.2136	.4795	.7339	.8170	.7339	.4795	.2136	.0839
1.5	.1455	.2703	.4476	.6064	.6673	.6064	.4476	.2703	.1455
2.0	.1847	.2874	.4088	.5099	.5492	.5099	.4088	.2874	.1847
2.5	.2043	.2848	.3697	.4360	.4613	.4360	.3697	.2848	.2043
3.0	.2110	.2732	.3337	.3787	.3954	.3787	.3337	.2732	.2110
3.5	.2100	.2580	.3021	.3335	.3450	.3335	.3021	.2580	.2100
4.0	.2045	.2419	.2746	.2973	.3054	.2973	.2746	.2419	.2045

1

TABLE-5 HORIZONTAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO :1000.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.1706	.2849	.3468	.3924	.4503	.3924	.3468	.2849	.1706
1.0	.2111	.2486	.2248	.1860	.1817	.1860	.2248	.2486	.2111
1.5	.1847	.1804	.1422	.0977	.0805	.0977	.1422	.1804	.1847
2.0	.1455	.1267	.0907	.0551	.0405	.0551	.0907	.1267	.1455
2.5	.1108	.0891	.0594	.0332	.0227	.0332	.0594	.0891	.1108
3.0	.0838	.0636	.0402	.0212	.0138	.0212	.0402	.0636	.0838
3.5	.0637	.0462	.0281	.0142	.0090	.0142	.0281	.0462	.0637
4.0	.0489	.0343	.0202	.0100	.0062	.0100	.0202	.0343	.0489

1

TABLE-6 SHEAR STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO :1000.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0550	.1468	.2994	.1266	0.0000	-.1266	-.2994	-.1468	-.0550
1.0	.1272	.2106	.2543	.1563	.0000	-.1563	-.2543	-.2106	-.1272
1.5	.1566	.2020	.2035	.1271	.0000	-.1271	-.2035	-.2020	-.1566
2.0	.1565	.1752	.1590	.0957	.0000	-.0957	-.1590	-.1752	-.1565
2.5	.1438	.1468	.1241	.0719	.0000	-.0719	-.1241	-.1468	-.1438
3.0	.1272	.1217	.0978	.0550	0.0000	-.0550	-.0978	-.1217	-.1272
3.5	.1107	.1011	.0783	.0430	.0000	-.0430	-.0783	-.1011	-.1107
4.0	.0958	.0844	.0636	.0343	.0000	-.0343	-.0636	-.0844	-.0958

> ?

> ?

TABLE-7 VERTICAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 1.Ø
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.Ø
THE LINE LOAD APPLIED IS EQUAL TO : 1.Ø

Z/B X/B=	-2.Ø	-1.5	-1.Ø	-.5	Ø.Ø	.5	1.Ø	1.5	2.Ø
.5	.Ø1Ø6	.Ø643	.552Ø	.92ØØ	.9146	.92ØØ	.552Ø	.Ø643	.Ø1Ø6
1.Ø	.Ø728	.2Ø51	.5Ø22	.75Ø9	.81Ø5	.75Ø9	.5Ø22	.2Ø51	.Ø728
1.5	.1391	.27Ø2	.4594	.6191	.6751	.6191	.4594	.27Ø2	.1391
2.Ø	.1819	.2894	.4167	.52Ø3	.5592	.52Ø3	.4167	.2894	.1819
2.5	.2Ø36	.2875	.376Ø	.4446	.47Ø3	.4446	.376Ø	.2875	.2Ø36
3.Ø	.2115	.2761	.3391	.3857	.4Ø3Ø	.3857	.3391	.2761	.2115
3.5	.211Ø	.26Ø8	.3Ø66	.3393	.3512	.3393	.3Ø66	.26Ø8	.211Ø
4.Ø	.2Ø59	.2446	.2786	.3Ø21	.31Ø6	.3Ø21	.2786	.2446	.2Ø59

1

TABLE-8 HORIZONTAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 1.Ø
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.Ø
THE LINE LOAD APPLIED IS EQUAL TO : 1.Ø

Z/B X/B=	-2.Ø	-1.5	-1.Ø	-.5	Ø.Ø	.5	1.Ø	1.5	2.Ø
.5	.173Ø	.31Ø2	.417Ø	.46Ø8	.5159	.46Ø8	.417Ø	.31Ø2	.173Ø
1.Ø	.2211	.2695	.2415	.21Ø3	.2188	.21Ø3	.2415	.2695	.2211
1.5	.193Ø	.19Ø8	.15Ø8	.1Ø93	.Ø965	.1Ø93	.15Ø8	.19Ø8	.193Ø
2.Ø	.151Ø	.1326	.Ø96Ø	.Ø611	.Ø476	.Ø611	.Ø96Ø	.1326	.151Ø
2.5	.1144	.Ø927	.Ø627	.Ø364	.Ø261	.Ø364	.Ø627	.Ø927	.1144
3.Ø	.Ø862	.Ø658	.Ø422	.Ø23Ø	.Ø156	.Ø23Ø	.Ø422	.Ø658	.Ø862
3.5	.Ø652	.Ø477	.Ø293	.Ø153	.Ø1ØØ	.Ø153	.Ø293	.Ø477	.Ø652
4.Ø	.Ø499	.Ø352	.Ø21Ø	.Ø1Ø6	.ØØ68	.Ø1Ø6	.Ø21Ø	.Ø352	.Ø499

1
> ?

TABLE-9 SHEAR STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 1.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0418	.1300	.3472	.0926	.0000	-.0926	-.3472	-.1300	-.0418
1.0	.1267	.2241	.2701	.1402	.0000	-.1402	-.2701	-.2241	-.1267
1.5	.1640	.2161	.2143	.1266	.0000	-.1266	-.2143	-.2161	-.1640
2.0	.1659	.1871	.1684	.0995	.0000	-.0995	-.1684	-.1871	-.1659
2.5	.1530	.1568	.1322	.0761	.0000	-.0761	-.1322	-.1568	-.1530
3.0	.1356	.1301	.1046	.0587	.0000	-.0587	-.1046	-.1301	-.1356
3.5	.1181	.1081	.0838	.0460	.0000	-.0460	-.0838	-.1081	-.1181
4.0	.1022	.0903	.0681	.0368	.0000	-.0368	-.0681	-.0903	-.1022

1

TABLE-10 VERTICAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 3.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0157	.0783	.5152	.9099	.9385	.9099	.5152	.0783	.0157
1.0	.0797	.2107	.4877	.7401	.8141	.7401	.4877	.2107	.0797
1.5	.1433	.2703	.4517	.6108	.6699	.6108	.4517	.2703	.1433
2.0	.1837	.2881	.4115	.5135	.5526	.5135	.4115	.2881	.1837
2.5	.2041	.2857	.3718	.4389	.4643	.4389	.3718	.2857	.2041
3.0	.2112	.2742	.3355	.3811	.3980	.3811	.3355	.2742	.2112
3.5	.2103	.2589	.3036	.3355	.3471	.3355	.3036	.2589	.2103
4.0	.2050	.2428	.2760	.2989	.3072	.2989	.2760	.2428	.2050

1
> ?

1

TABLE-11 HORIZONTAL STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 3.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.1728	.2989	.3684	.4192	.4791	.4192	.3684	.2989	.1728
1.0	.2152	.2562	.2300	.1942	.1956	.1942	.2300	.2562	.2152
1.5	.1877	.1840	.1449	.1015	.0860	.1015	.1449	.1840	.1877
2.0	.1474	.1287	.0924	.0570	.0428	.0570	.0924	.1287	.1474
2.5	.1121	.0903	.0604	.0342	.0238	.0342	.0604	.0903	.1121
3.0	.0846	.0643	.0408	.0217	.0144	.0217	.0408	.0643	.0846
3.5	.0642	.0467	.0285	.0145	.0093	.0145	.0285	.0467	.0642
4.0	.0492	.0346	.0205	.0101	.0063	.0101	.0205	.0346	.0492

1

TABLE-12 SHEAR STRESS FOR A CYLINDRICAL STRIP FOOTING

THE RADIUS OF THE CIRCLE IS EQUAL TO : 3.0
THE WIDTH OF THE FOOTING IS EQUAL TO : 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0498	.1408	.3111	.1123	0.0000	-.1123	-.3111	-.1408	-.0498
1.0	.1257	.2131	.2579	.1508	.0000	-.1508	-.2579	-.2131	-.1257
1.5	.1574	.2047	.2056	.1264	.0000	-.1264	-.2056	-.2047	-.1574
2.0	.1580	.1774	.1608	.0963	.0000	-.0963	-.1608	-.1774	-.1580
2.5	.1454	.1486	.1256	.0727	.0000	-.0727	-.1256	-.1486	-.1454
3.0	.1286	.1233	.0991	.0557	.0000	-.0557	-.0991	-.1233	-.1286
3.5	.1120	.1024	.0793	.0436	.0000	-.0436	-.0793	-.1024	-.1120
4.0	.0969	.0855	.0644	.0348	.0000	-.0348	-.0644	-.0855	-.0969

1
> ?

TABLE-13 VERTICAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 90.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0193	.0892	.4969	.9022	.9592	.9022	.4969	.0892	.0193
1.0	.0839	.2137	.4797	.7346	.8182	.7346	.4797	.2137	.0839
1.5	.1456	.2705	.4479	.6070	.6681	.6070	.4479	.2705	.1456
2.0	.1848	.2876	.4091	.5104	.5498	.5104	.4091	.2876	.1848
2.5	.2045	.2851	.3700	.4365	.4617	.4365	.3700	.2851	.2045
3.0	.2112	.2735	.3340	.3791	.3958	.3791	.3340	.2735	.2112
3.5	.2102	.2582	.3023	.3338	.3453	.3338	.3023	.2582	.2102
4.0	.2047	.2421	.2749	.2976	.3057	.2976	.2749	.2421	.2047

1

TABLE-14 HORIZONTAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 90.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.1707	.2851	.3471	.3929	.4502	.3929	.3471	.2851	.1707
1.0	.2112	.2488	.2251	.1862	.1817	.1862	.2251	.2488	.2112
1.5	.1848	.1806	.1424	.0978	.0805	.0978	.1424	.1806	.1848
2.0	.1456	.1269	.0908	.0551	.0405	.0551	.0908	.1269	.1456
2.5	.1109	.0892	.0595	.0332	.0227	.0332	.0595	.0892	.1109
3.0	.0839	.0636	.0403	.0212	.0138	.0212	.0403	.0636	.0839
3.5	.0638	.0463	.0281	.0142	.0090	.0142	.0281	.0463	.0638
4.0	.0490	.0343	.0203	.0100	.0062	.0100	.0203	.0343	.0490

> ?

1

TABLE-15 SHEAR STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 90.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0551	.1469	.2996	.1273	0.0000	-.1273	-.2996	-.1469	-.0551
1.0	.1273	.2107	.2546	.1567	0.0000	-.1567	-.2546	-.2107	-.1273
1.5	.1567	.2022	.2037	.1273	0.0000	-.1273	-.2037	-.2022	-.1567
2.0	.1567	.1753	.1591	.0959	.0000	-.0959	-.1591	-.1753	-.1567
2.5	.1439	.1469	.1242	.0720	0.0000	-.0720	-.1242	-.1469	-.1439
3.0	.1273	.1218	.0979	.0551	.0000	-.0551	-.0979	-.1218	-.1273
3.5	.1108	.1012	.0783	.0430	.0000	-.0430	-.0783	-.1012	-.1108
4.0	.0959	.0845	.0637	.0343	-.0000	-.0343	-.0637	-.0845	-.0959

1

TABLE-16 VERTICAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 30.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0176	.0313	.0398	.0609	.7441	.0609	.0398	.0313	.0176
1.0	.0571	.1021	.1162	.7038	.6981	.7038	.1162	.1021	.0571
1.5	.0991	.1827	.2452	.6557	.6297	.6557	.2452	.1827	.0991
2.0	.1422	.2547	.5453	.5814	.5691	.5814	.5453	.2547	.1422
2.5	.1829	.2936	.4357	.5042	.5133	.5042	.4357	.2936	.1829
3.0	.2060	.2919	.3803	.4360	.4520	.4360	.3803	.2919	.2060
3.5	.2140	.2788	.3400	.3816	.3956	.3816	.3400	.2788	.2140
4.0	.2132	.2627	.3072	.3378	.3485	.3378	.3072	.2627	.2132

> ?

> ? U

1

TABLE-17 HORIZONTAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 30.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0748	.1318	.2217	.3206	.2193	.3206	.2217	.1318	.0748
1.0	.1245	.1575	.2071	.1450	.1608	.1450	.2071	.1575	.1245
1.5	.1646	.1987	.2284	.1798	.1761	.1798	.2284	.1987	.1646
2.0	.1741	.2014	.1472	.1452	.1460	.1452	.1472	.2014	.1741
2.5	.1453	.1351	.0970	.0842	.0874	.0842	.0970	.1351	.1453
3.0	.1107	.0932	.0676	.0526	.0496	.0526	.0676	.0932	.1107
3.5	.0833	.0668	.0477	.0343	.0301	.0343	.0477	.0668	.0833
4.0	.0632	.0489	.0341	.0234	.0196	.0234	.0341	.0489	.0632

1

TABLE-18 SHEAR STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 30.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0338	.0779	.1412	.2037	-.0000	-.2037	-.1412	-.0779	-.0338
1.0	.0683	.1289	.2239	.0440	-.0000	-.0440	-.2239	-.1289	-.0683
1.5	.0996	.1520	.3123	.0727	.0000	-.0727	-.3123	-.1520	-.0996
2.0	.1337	.1902	.2054	.0580	.0000	-.0580	-.2054	-.1902	-.1337
2.5	.1476	.1737	.1466	.0655	.0000	-.0655	-.1466	-.1737	-.1476
3.0	.1403	.1443	.1162	.0607	.0000	-.0607	-.1162	-.1443	-.1403
3.5	.1256	.1199	.0940	.0508	.0000	-.0508	-.0940	-.1199	-.1256
4.0	.1101	.1002	.0767	.0415	0.0000	-.0415	-.0767	-.1002	-.1101

> ?

1

TABLE-19 VERTICAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 45.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0166	.0554	.0407	-.0701	.9173	-.0701	.0407	.0554	.0166
1.0	.0544	.1382	1.0545	.7846	.7883	.7846	1.0545	.1382	.0544
1.5	.1115	.2422	.5212	.6710	.6901	.6710	.5212	.2422	.1115
2.0	.1657	.2893	.4514	.5635	.5950	.5635	.4514	.2893	.1657
2.5	.1977	.2954	.4019	.4800	.5069	.4800	.4019	.2954	.1977
3.0	.2116	.2865	.3608	.4150	.4346	.4150	.3608	.2865	.2116
3.5	.2145	.2719	.3255	.3637	.3775	.3637	.3255	.2719	.2145
4.0	.2112	.2555	.2950	.3225	.3324	.3225	.2950	.2555	.2112

1

TABLE-20 HORIZONTAL STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 45.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.1119	.1677	.2571	.3636	.3815	.3636	.2571	.1677	.1119
1.0	.1949	.2768	1.2400	.3144	.2869	.3144	1.2400	.2768	.1949
1.5	.2076	.2404	.1946	.1702	.1725	.1702	.1946	.2404	.2076
2.0	.1713	.1623	.1227	.0943	.0893	.0943	.1227	.1623	.1713
2.5	.1305	.1116	.0807	.0561	.0480	.0561	.0807	.1116	.1305
3.0	.0978	.0785	.0543	.0351	.0280	.0351	.0543	.0785	.0978
3.5	.0736	.0564	.0376	.0231	.0177	.0231	.0376	.0564	.0736
4.0	.0558	.0413	.0267	.0158	.0118	.0158	.0267	.0413	.0558

```
> ?
```

1

TABLE-21 SHEAR STRESS FOR A TRIANGULAR STRIP FOOTING

THE ANGLE OF THE TRIANGLE IS EQUAL TO: 45.0
THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-1.0	-.5	0.0	.5	1.0	1.5	2.0	2.5	3.0
.5	.0206	.0740	.1847	.1311	.0000	-.1311	-.1847	-.0740	-.0206
1.0	.0756	.1406	1.2437	.1151	.0000	-.1151	******	-.1406	-.0756
1.5	.1342	.2089	.2327	.1005	.0000	-.1005	-.2327	-.2089	-.1342
2.0	.1566	.1925	.1768	.0950	.0000	-.0950	-.1768	-.1925	-.1566
2.5	.1522	.1630	.1392	.0783	.0000	-.0783	-.1392	-.1630	-.1522
3.0	.1378	.1359	.1106	.0619	0.0000	-.0619	-.1106	-.1359	-.1378
3.5	.1212	.1132	.0887	.0489	.0000	-.0489	-.0887	-.1132	-.1212
4.0	.1054	.0946	.0720	.0391	0.0000	-.0391	-.0720	-.0946	-.1054

1

TABLE-22 VERTICAL STRESS FOR A FLAT CIRCULAR FOOTING

THE POISSON.S RATIO IS EQUAL TO : .5
THE LOAD APPLIED IS EQUAL TO : 1.0
THE RADIUS OF THE CIRCLE IS EQUAL TO : .5

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0104	.0604	.4175	.8396	.9106	.8396	.4175	.0604	.0104
1.0	.0418	.1267	.3322	.5622	.6464	.5622	.3322	.1266	.0418
1.5	.0650	.1379	.2562	.3750	.4240	.3750	.2562	.1379	.0650
2.0	.0733	.1265	.1960	.2589	.2845	.2589	.1960	.1265	.0733
2.5	.0722	.1090	.1510	.1859	.1996	.1859	.1510	.1090	.0722
3.0	.0666	.0919	.1181	.1384	.1462	.1384	.1181	.0919	.0666
3.5	.0596	.0771	.0940	.1064	.1110	.1064	.0940	.0771	.0596
4.0	.0526	.0649	.0761	.0840	.0869	.0840	.0761	.0649	.0526

> ?

1

TABLE-23 VERTICAL STRESS FOR A SPHERICAL FOOTING

THE RADIUS OF THE SPHERE IS EQUAL TO :2000.0
THE WIDTHE OF THE FOOTING IS EQUAL TO:1.0
THE POINT LOAD APPLIED IS EQUAL TO :1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0104	.0604	.4175	.8396	.9105	.8396	.4175	.0604	.0104
1.0	.0418	.1267	.3323	.5622	.6464	.5622	.3323	.1267	.0418
1.5	.0650	.1379	.2562	.3751	.4240	.3751	.2562	.1379	.0650
2.0	.0733	.1265	.1960	.2589	.2845	.2589	.1960	.1265	.0733
2.5	.0722	.1090	.1510	.1859	.1996	.1859	.1510	.1090	.0722
3.0	.0666	.0919	.1181	.1384	.1462	.1384	.1181	.0919	.0666
3.5	.0596	.0771	.0940	.1064	.1110	.1064	.0940	.0771	.0596
4.0	.0526	.0649	.0761	.0840	.0869	.0840	.0761	.0649	.0526

1

TABLE-24 VERTICAL STRESS FOR A SPHERICAL FOOTING

THE RADIUS OF THE SPHERE IS EQUAL TO : 1.0
THE WIDTHE OF THE FOOTING IS EQUAL TO:1.0
THE POINT LOAD APPLIED IS EQUAL TO :1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0061	.0453	.4664	.8533	.8750	.8533	.4664	.0453	.0061
1.0	.0366	.1160	.3537	.5895	.6609	.5895	.3537	.1160	.0366
1.5	.0628	.1403	.2698	.3961	.4454	.3961	.2698	.1403	.0628
2.0	.0734	.1302	.2056	.2734	.3006	.2734	.2056	.1302	.0734
2.5	.0732	.1127	.1580	.1958	.2106	.1958	.1580	.1127	.0732
3.0	.0681	.0951	.1233	.1453	.1537	.1453	.1233	.0951	.0681
3.5	.0611	.0798	.0978	.1113	.1163	.1113	.0978	.0798	.0611
4.0	.0540	.0670	.0790	.0876	.0907	.0876	.0790	.0670	.0540

> ?

1

TABLE-25 VERTICAL STRESS FOR A SSIN MODEL STRIP FOOTING

THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0147	.0232	.0443	.1526	.7240	.1526	.0443	.0232	.0147
1.0	.0535	.0837	.1244	.4661	.6112	.5058	.1244	.0837	.0535
1.5	.1000	.1607	.1735	.5466	.5669	.5466	.1735	.1607	.1000
2.0	.1364	.2218	.6943	.5458	.5158	.5458	.6943	.2218	.1364
2.5	.1702	.2757	.4800	.5061	.4826	.5061	.4800	.2757	.1702
3.0	.1980	.2903	.3947	.4418	.4456	.4418	.3947	.2903	.1980
3.5	.2104	.2799	.3467	.3869	.3980	.3869	.3467	.2799	.2104
4.0	.2119	.2642	.3114	.3425	.3529	.3425	.3114	.2642	.2119

1

TABLE-26 HORIZONTAL STRESS FOR A SSIN MODEL STRIP FOOTING

THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0759	.1406	.2412	.3557	.2424	.3557	.2412	.1406	.0759
1.0	.1073	.1458	.1932	.1225	.0956	.1225	.1932	.1458	.1073
1.5	.1263	.1334	.1470	.0669	.0828	.0669	.1470	.1334	.1263
2.0	.1486	.1734	.6323	.1515	.1344	.1515	.6323	.1734	.1486
2.5	.1460	.1539	.1029	.1076	.1174	.1076	.1029	.1539	.1460
3.0	.1172	.1028	.0728	.0650	.0695	.0650	.0728	.1028	.1172
3.5	.0890	.0727	.0528	.0422	.0404	.0422	.0528	.0727	.0890
4.0	.0676	.0533	.0383	.0285	.0254	.0285	.0383	.0533	.0676

> ?

1

TABLE-27 SHEAR STRESS FOR A SSIN MODEL STRIP FOOTING

THE WIDTH OF THE FOOTING IS EQUAL TO: 1.0
THE LINE LOAD APPLIED IS EQUAL TO : 1.0.

X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	.0410	.0777	.1373	.2602	-.0000	.2602	-.1373	-.0777	-.0410
1.0	.0800	.1390	.2127	.0269	.0000	-.0269	-.2127	-.1390	-.0800
1.5	.1009	.1594	.2414	.0136	.0000	-.0136	-.2414	-.1594	-.1009
2.0	.1157	.1523	.4380	.0446	.0000	-.0446	-.4380	-.1523	-.1157
2.5	.1353	.1742	.1572	.0368	.0000	-.0368	-.1572	-.1742	-.1353
3.0	.1371	.1484	.1171	.0508	.0000	-.0508	-.1171	-.1484	-.1371
3.5	.1255	.1224	.0947	.0484	-.0000	-.0484	-.0947	-.1224	-.1255
4.0	.1109	.1020	.0778	.0414	-.0000	-.0414	-.0778	-.1020	-.1109

1

TABLE-28 RATIO OF VERTICAL STRESS OF
FLAT RECTANGULAR STRIP / SSIN MODEL STRIP

Z/B X/B=	-2.0	-1.5	-1.0	-.5	0.0	.5	1.0	1.5	2.0
.5	1.31	3.84	11.23	5.91	1.32	5.91	11.23	3.84	1.31
1.0	1.57	2.55	3.86	1.58	1.34	1.45	3.86	2.55	1.57
1.5	1.46	1.68	2.58	1.11	1.18	1.11	2.58	1.68	1.46
2.0	1.36	1.30	.59	.94	1.07	.94	.59	1.30	1.36
2.5	1.20	1.03	.77	.86	.96	.86	.77	1.03	1.20
3.0	1.07	.94	.85	.86	.89	.86	.85	.94	1.07
3.5	1.00	.92	.87	.86	.87	.86	.87	.92	1.00
4.0	.97	.92	.88	.87	.87	.87	.88	.92	.97

1
> ?

1

****

TABLE-29 RATIO OF HORIZONTAL STRESS OF
-------- FLAT RECTANGULAR STRIP / SSIN MODEL STRIP
-------- ***

Z/B X/B=	-2.Ø	-1.5	-1.Ø	-.5	Ø.Ø	.5	1.Ø	1.5	2.Ø
.5	2.25	2.Ø3	1.44	1.1Ø	1.86	1.1Ø	1.44	2.Ø3	2.25
1.Ø	1.97	1.71	1.16	1.52	1.9Ø	1.52	1.16	1.71	1.97
1.5	1.46	1.35	.97	1.46	.97	1.46	.97	1.35	1.46
2.Ø	.98	.73	.14	.36	.3Ø	.36	.14	.73	.98
2.5	.76	.58	.58	.31	.19	.31	.58	.58	.76
3.Ø	.72	.62	.55	.33	.2Ø	.33	.55	.62	.72
3.5	.72	.64	.53	.34	.22	.34	.53	.64	.72
4.Ø	.72	.64	.53	.35	.24	.35	.53	.64	.72

1

****

TABLE-3Ø RATIO OF SHEAR STRESS OF
-------- FLAT RECTANGULAR STRIP / SSIN MODEL STRIP
-------- ***

Z/B X/B=	-2.Ø	-1.5	-1.Ø	-.5	Ø.Ø	.5	1.Ø	1.5	2.Ø
.5	1.34	1.89	2.18	.49	Ø.ØØ	.49	2.18	1.89	1.34
1.Ø	1.59	1.52	1.2Ø	5.82	.ØØ	5.82	1.2Ø	1.52	1.59
1.5	1.55	1.27	.84	9.35	.ØØ	9.35	.84	1.27	1.55
2.Ø	1.35	1.15	.36	2.15	Ø.ØØ	2.15	.36	1.15	1.35
2.5	1.Ø6	.84	.79	1.95	.ØØ	1.95	.79	.84	1.Ø6
3.Ø	.93	.82	.84	1.Ø8	Ø.ØØ	1.Ø8	.84	.82	.93
3.5	.88	.83	.83	.89	-.ØØ	.89	.83	.83	.88
4.Ø	.86	.83	.82	.83	-.ØØ	.83	.82	.83	.86

*5Ø LINES.
> ?

4.4 TYPICAL GRAPHS

Tests summarized in section 4.3 are presented, some typical in graphical forms.

Figure 32, shows the model configurations used.

Figure 33, shows the location of the points, where the stresses are calculated.

Figures 34, 35 and 36, show the vertical stresses respectively at levels equal to ·5B, B and 1·5B respectively, for the flat and cylindrical strip models.

Figures 37, 38 and 39, show the vertical stresses at level z equal to ·5B, B and 1·5B respectively, for the flat and triangular models.

Figures 40, 41 and 42, show the vertical stresses at level z equal to ·5B, B and 1·5B respectively, for a S-sin model.

Figures 43, 44 and 45, show the vertical stresses at the middle, at x equal to ·5B and at x equal to B respectively, for the flat and S-sin models.

Figures 46, 47 and 48, show the horizontal stresses at level z equal to ·5B, B and 1·5B respectively, for the flat and S-sin models.

Figures 50, 51 and 52, show the shear stresses at level z equal to ·5B, B and 1·5B respectively, for the flat and S-sin models.

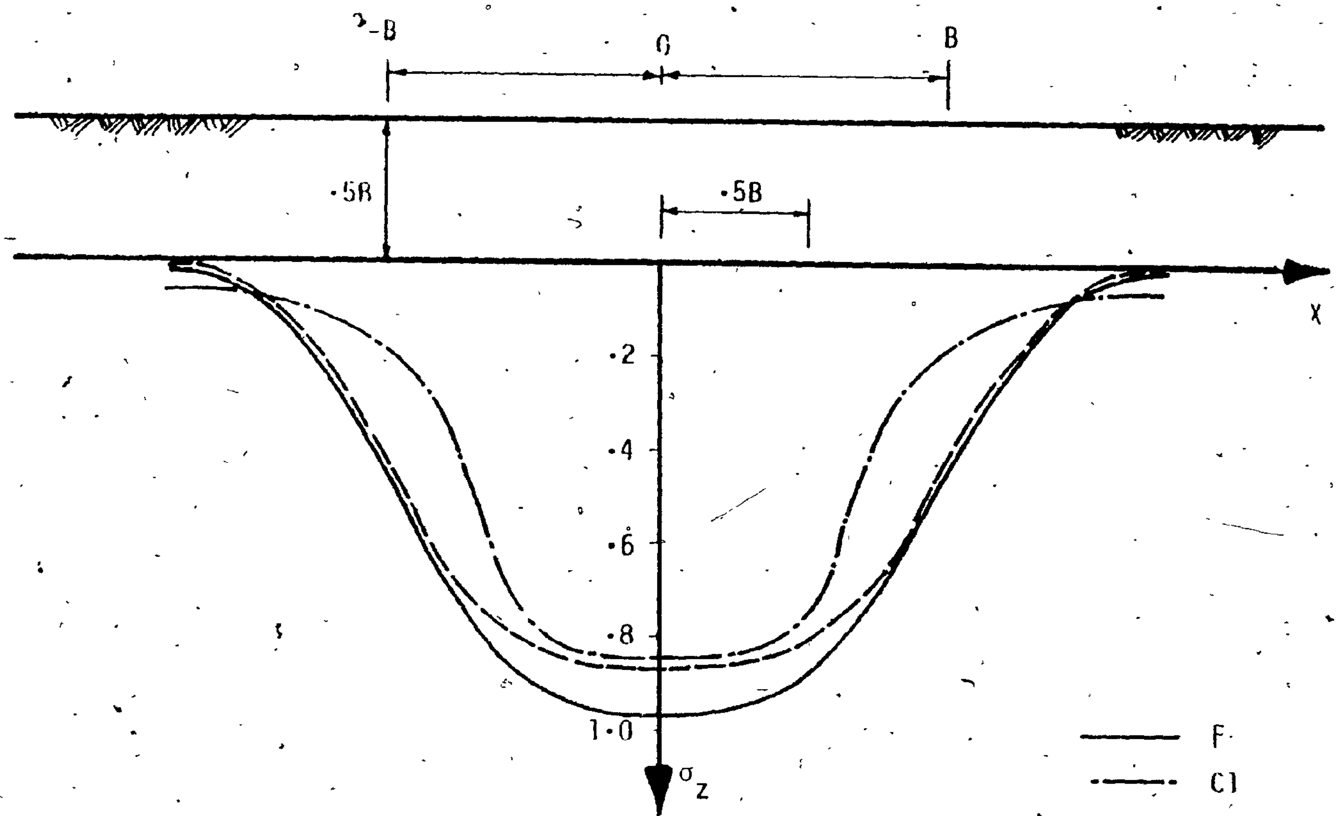


Figure 34. Vertical stresses at depth Z = ·5B for flat and cylindrical models

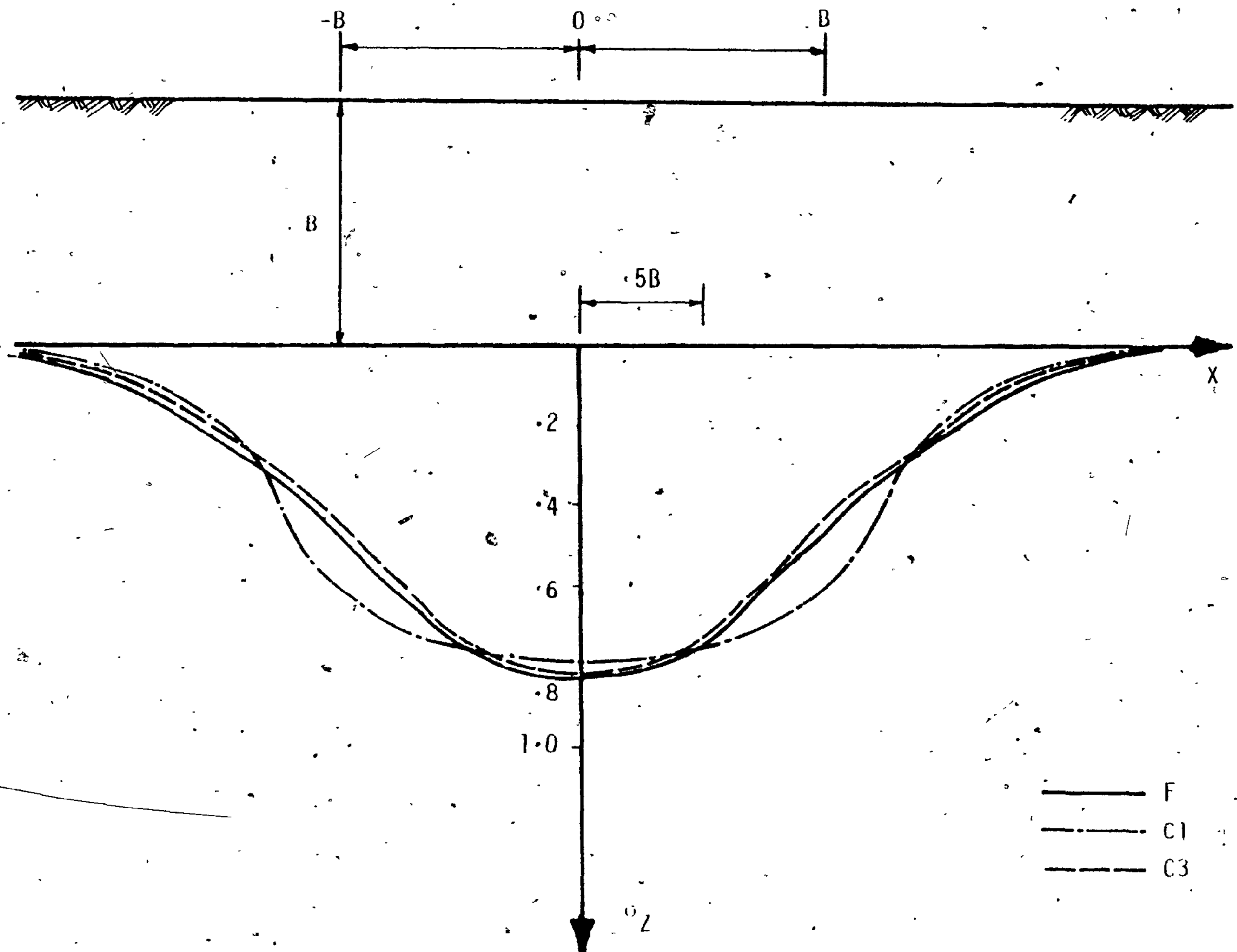


Figure 35. Vertical stresses at depth Z = B for flat and cylindrical models

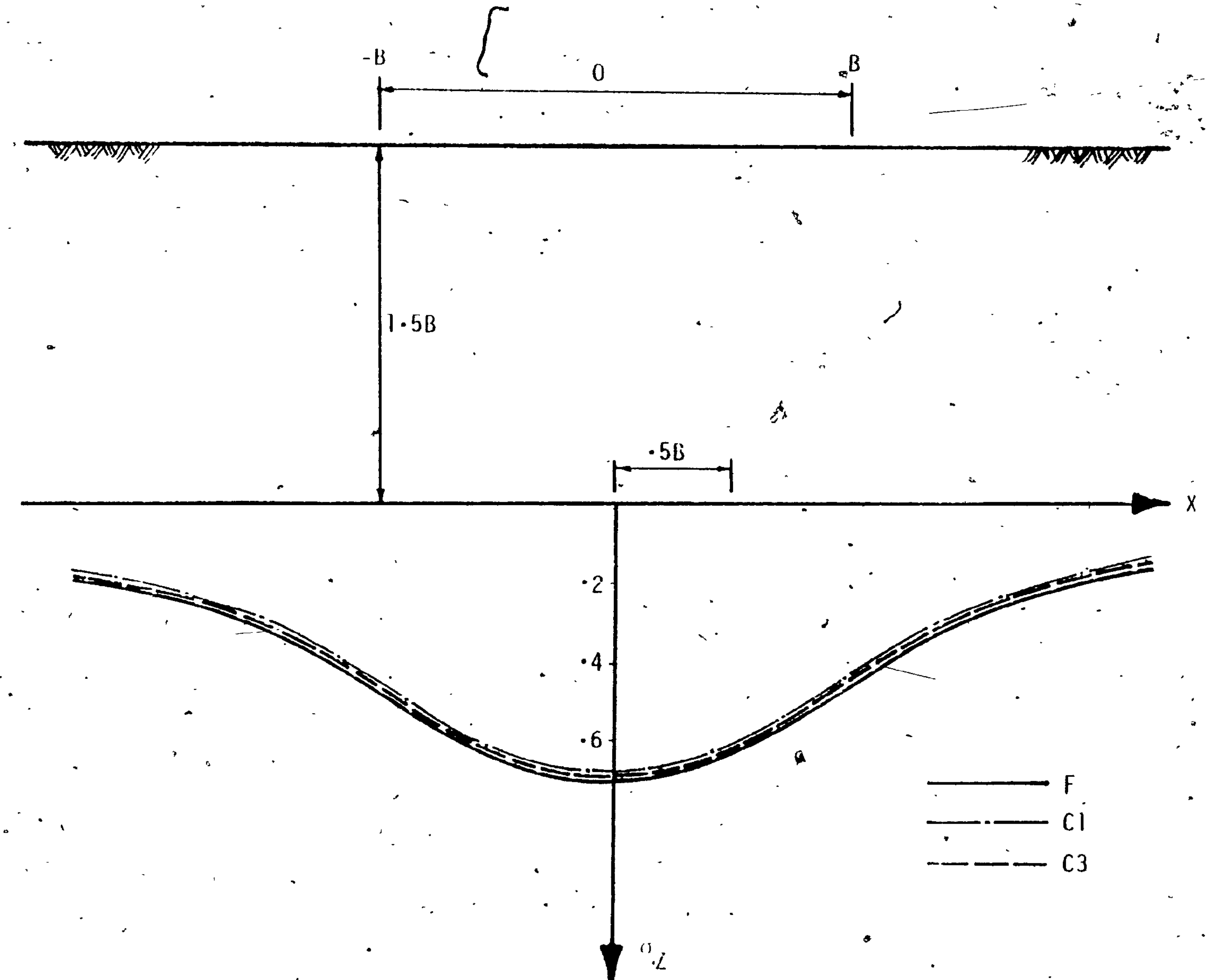


Figure 36. Vertical stresses at Z = 1·5B for cylindrical and flat models.

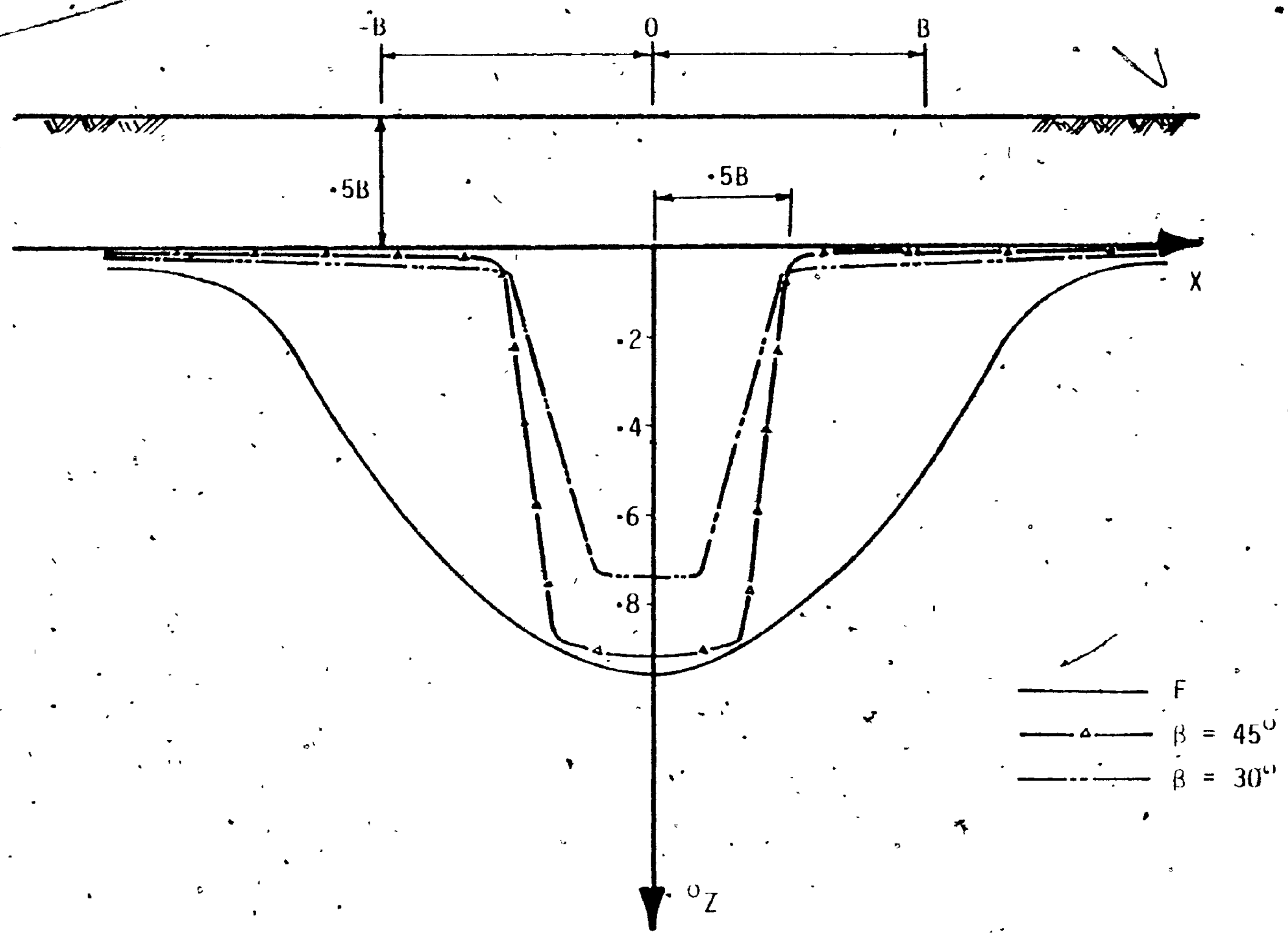


Figure 37. Vertical stresses at depth Z = ·5B for flat and triangular models.

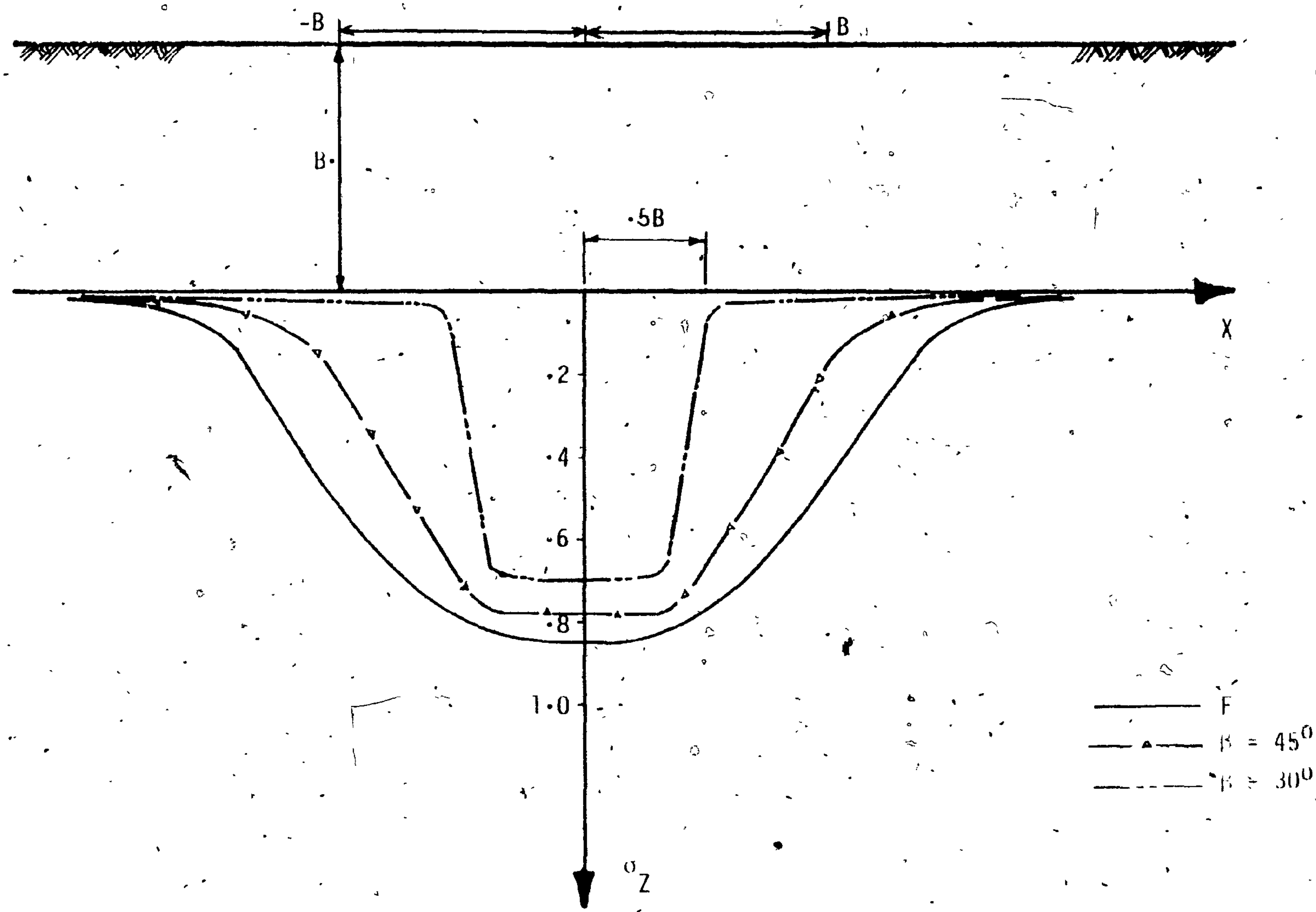


Figure 38. Vertical stresses at depth Z = B for flat and triangular models.

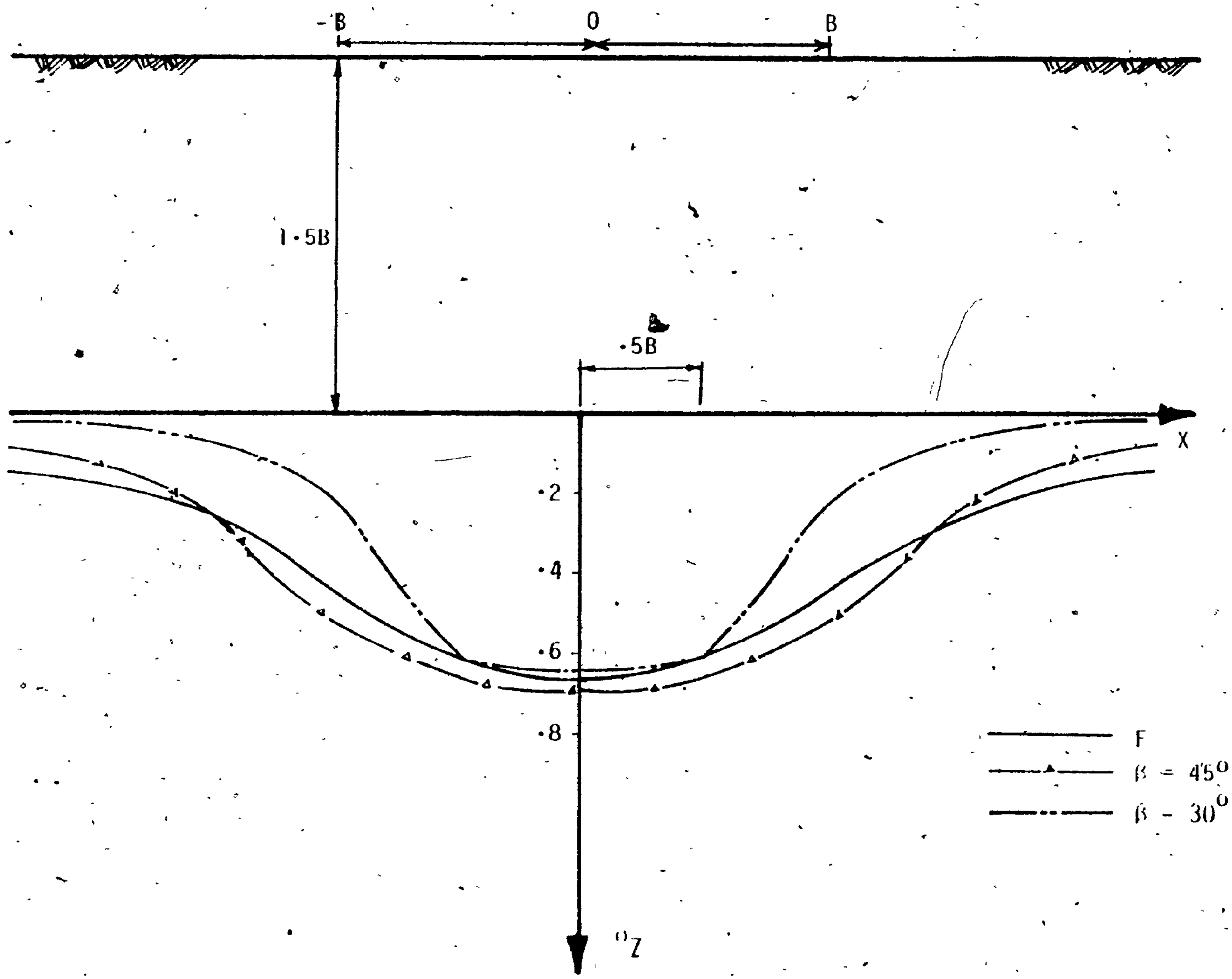


Figure 39. Vertical stresses at depth Z = 1·5B for flat and triangular models.

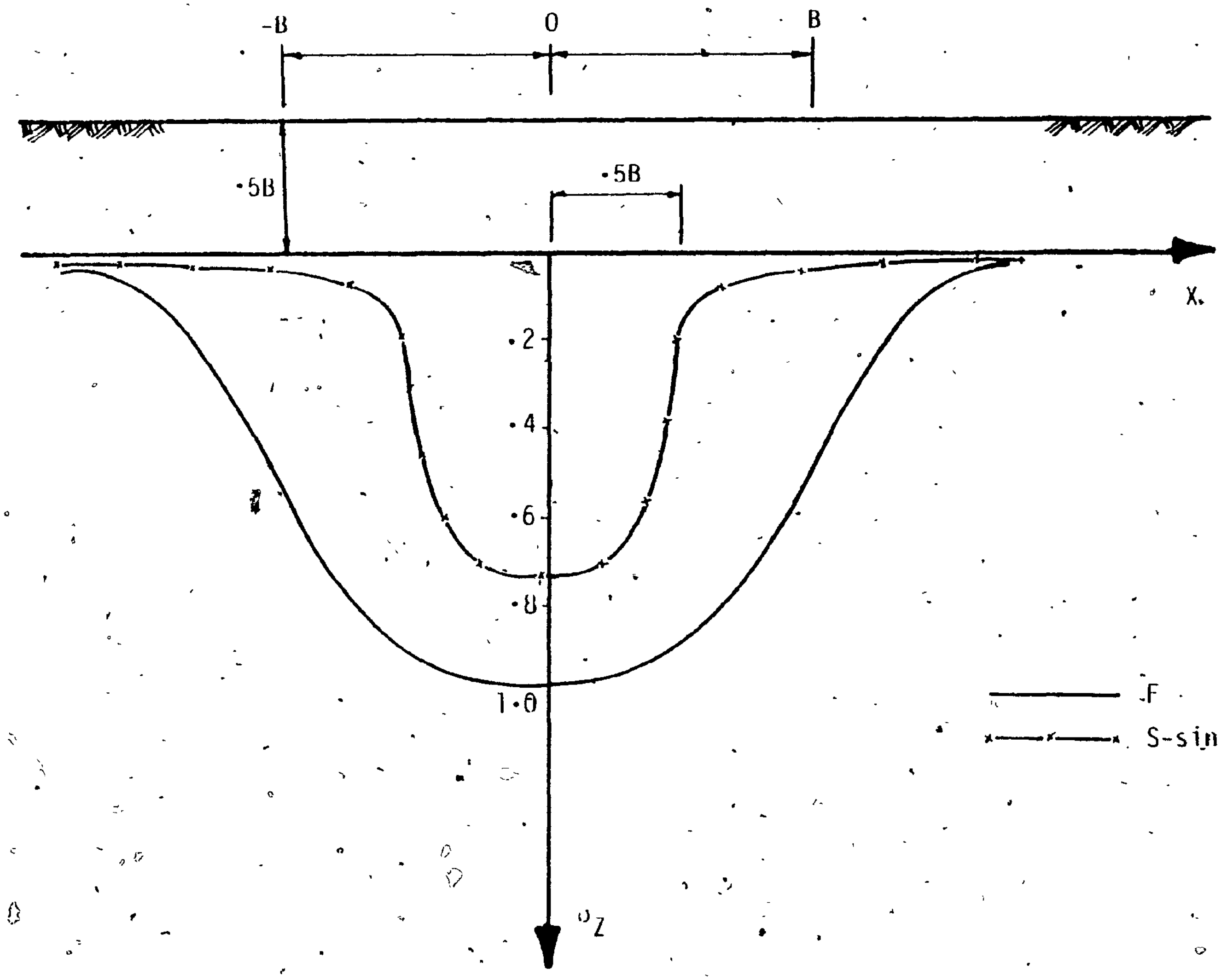


Figure 40. Vertical stresses at depth Z = ·5B for flat and S-sin models.

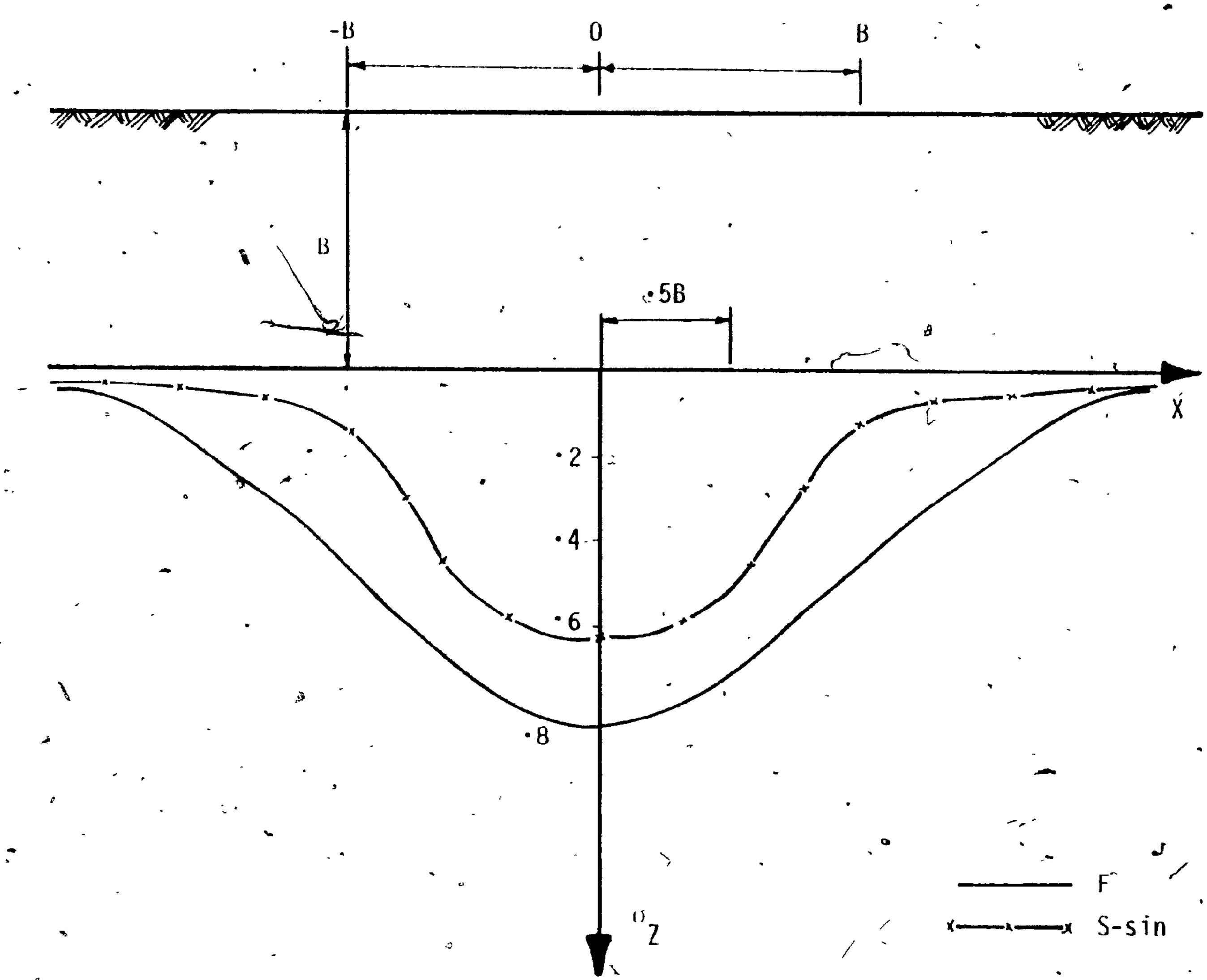


Figure 41. Vertical stresses at depth Z = B for flat and S-sin models.

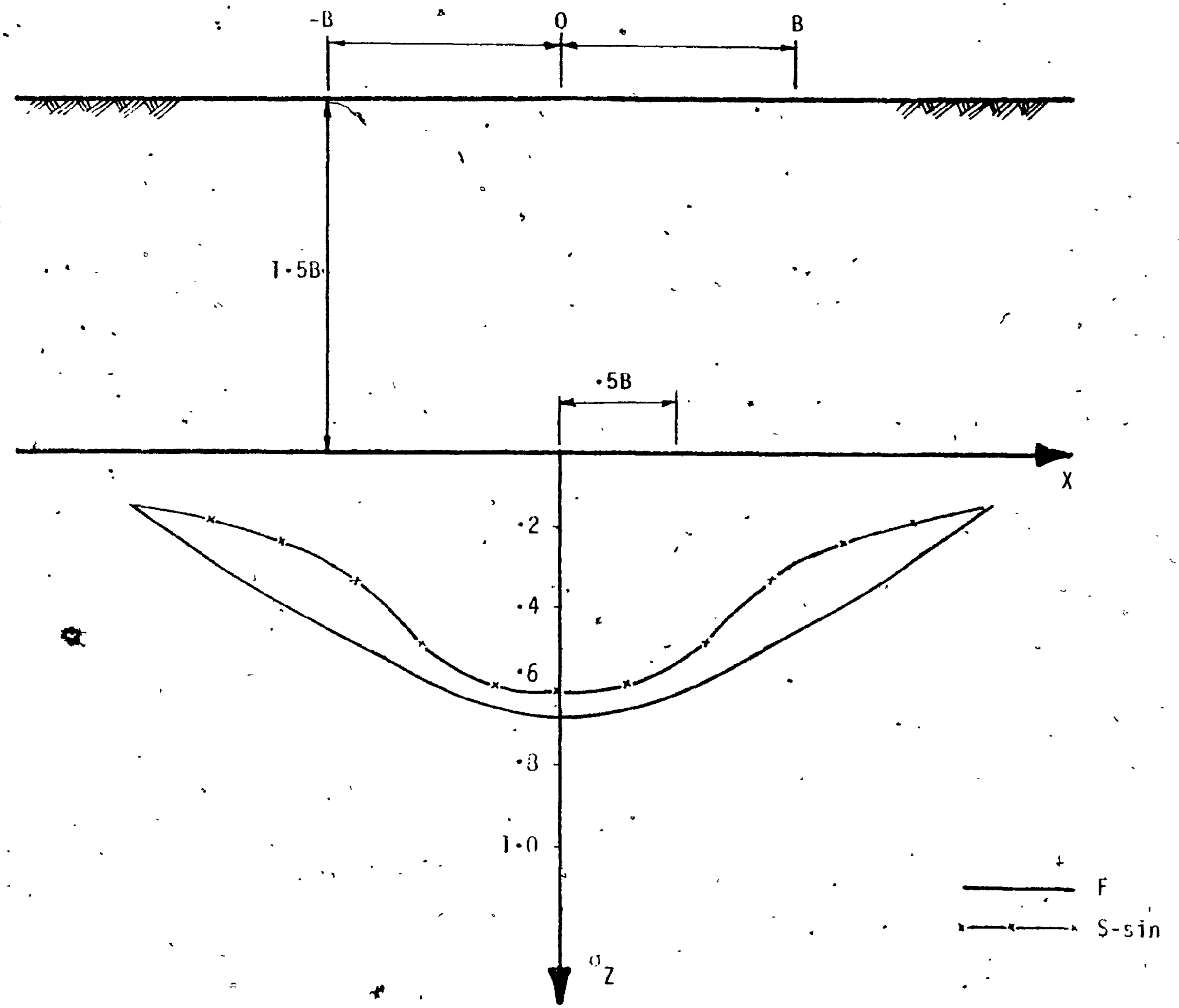


Figure 42. Vertical stresses at depth Z = 1·5B for flat and S-sin models.

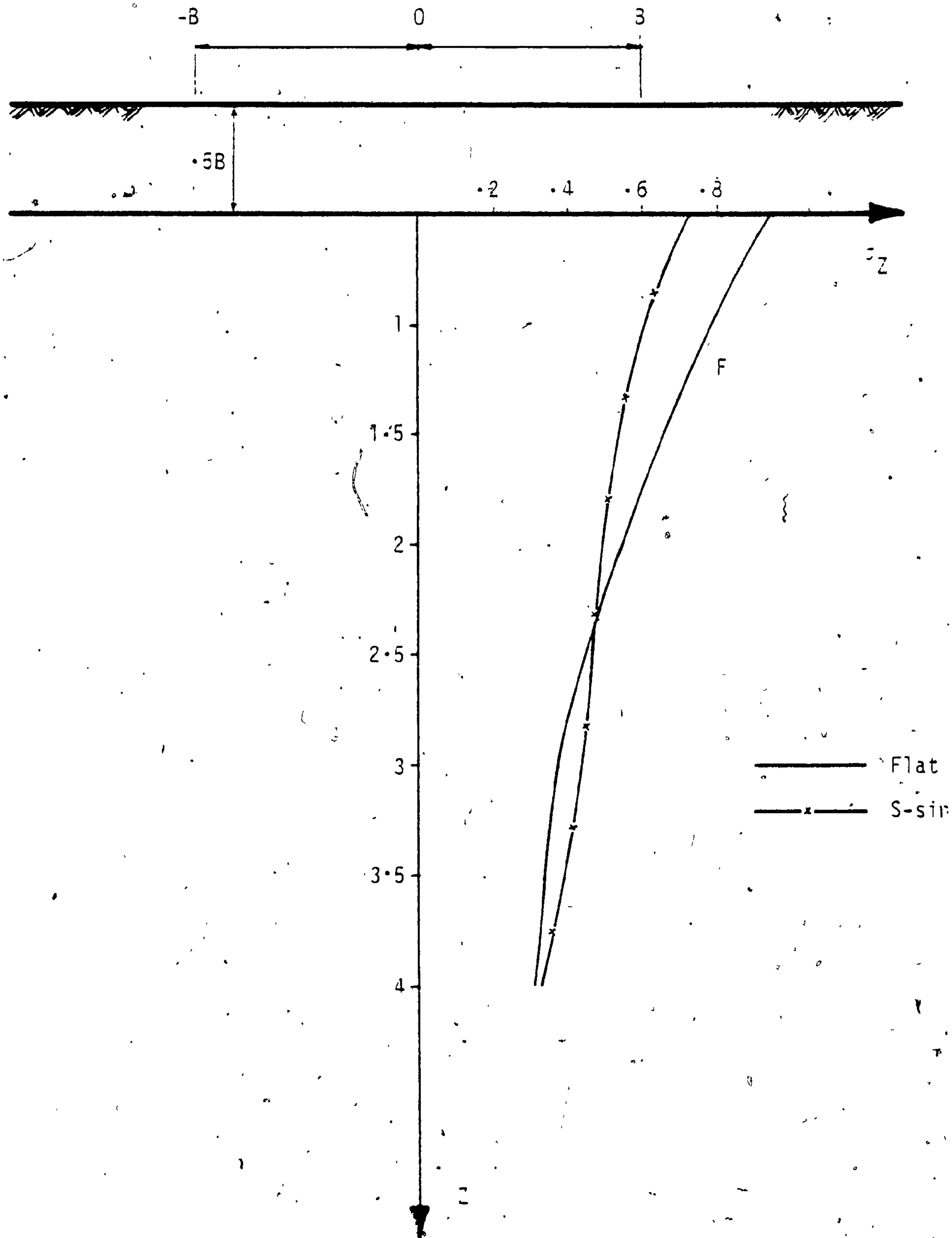


Figure 43. Vertical stresses at X = 0 for flat and S-sin models.

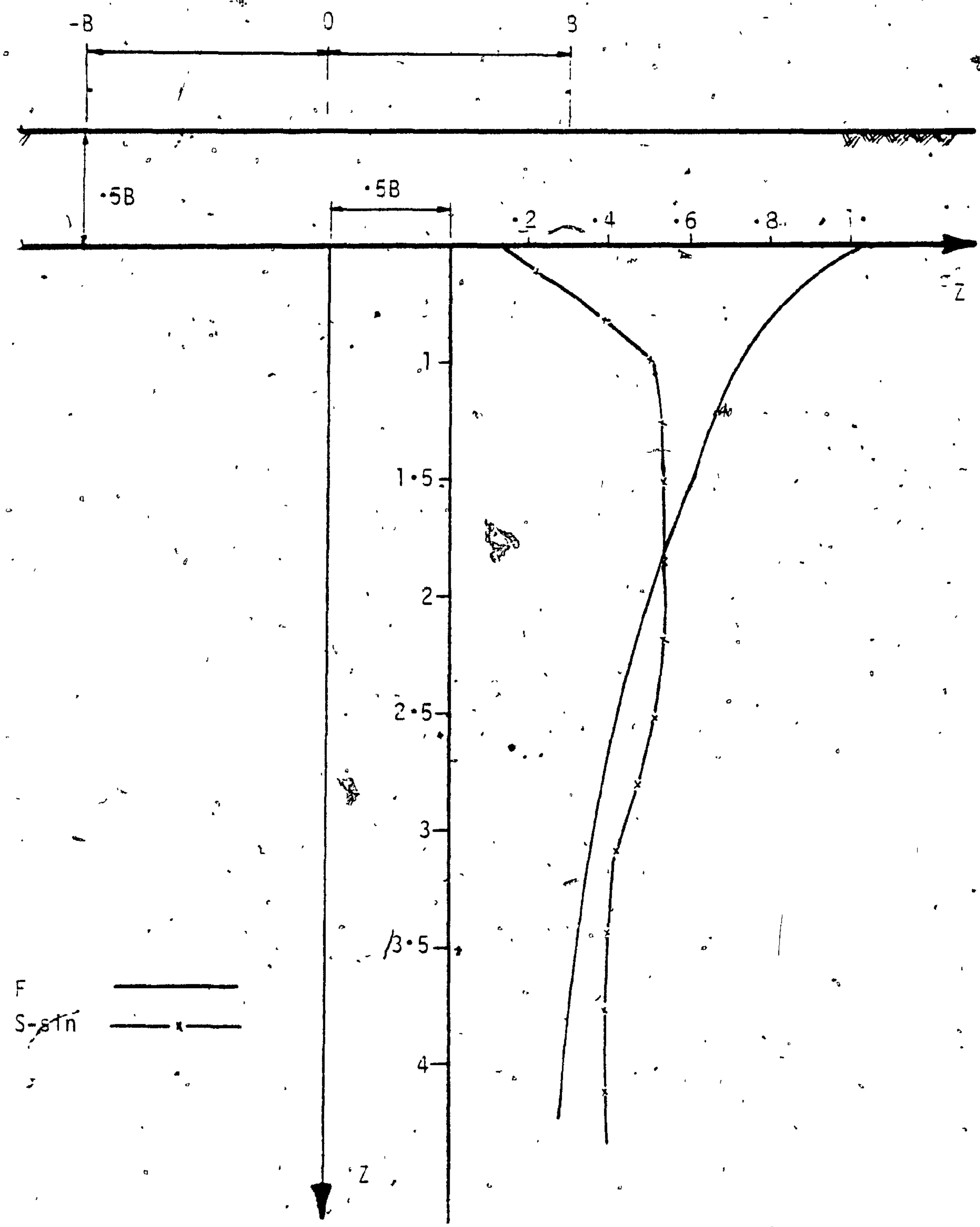


Figure 14. Vertical stresses at width λ = ·5B
for flat and S-sin models.

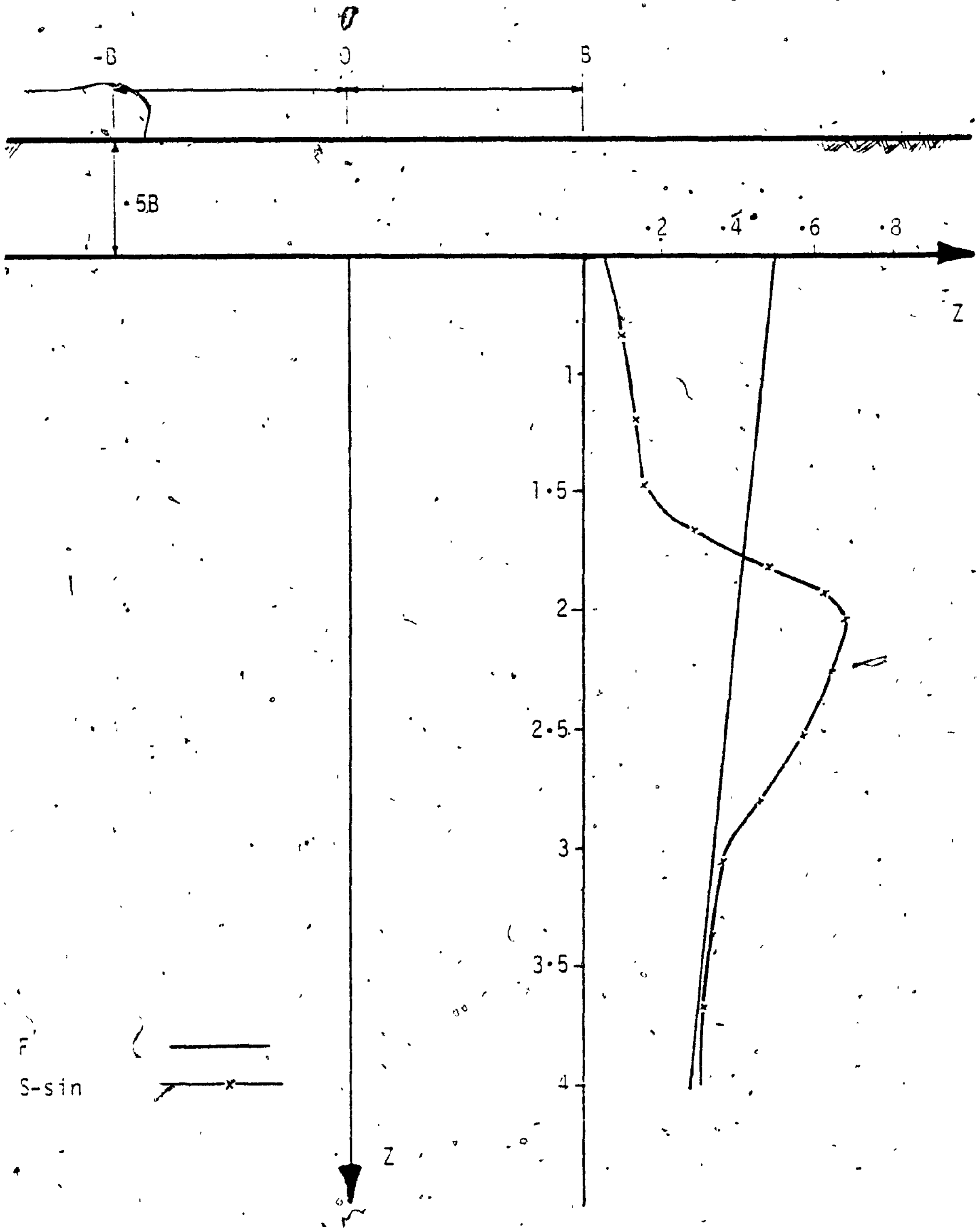


Figure 15. Vertical stresses at width X = B for flat and S-sin models.

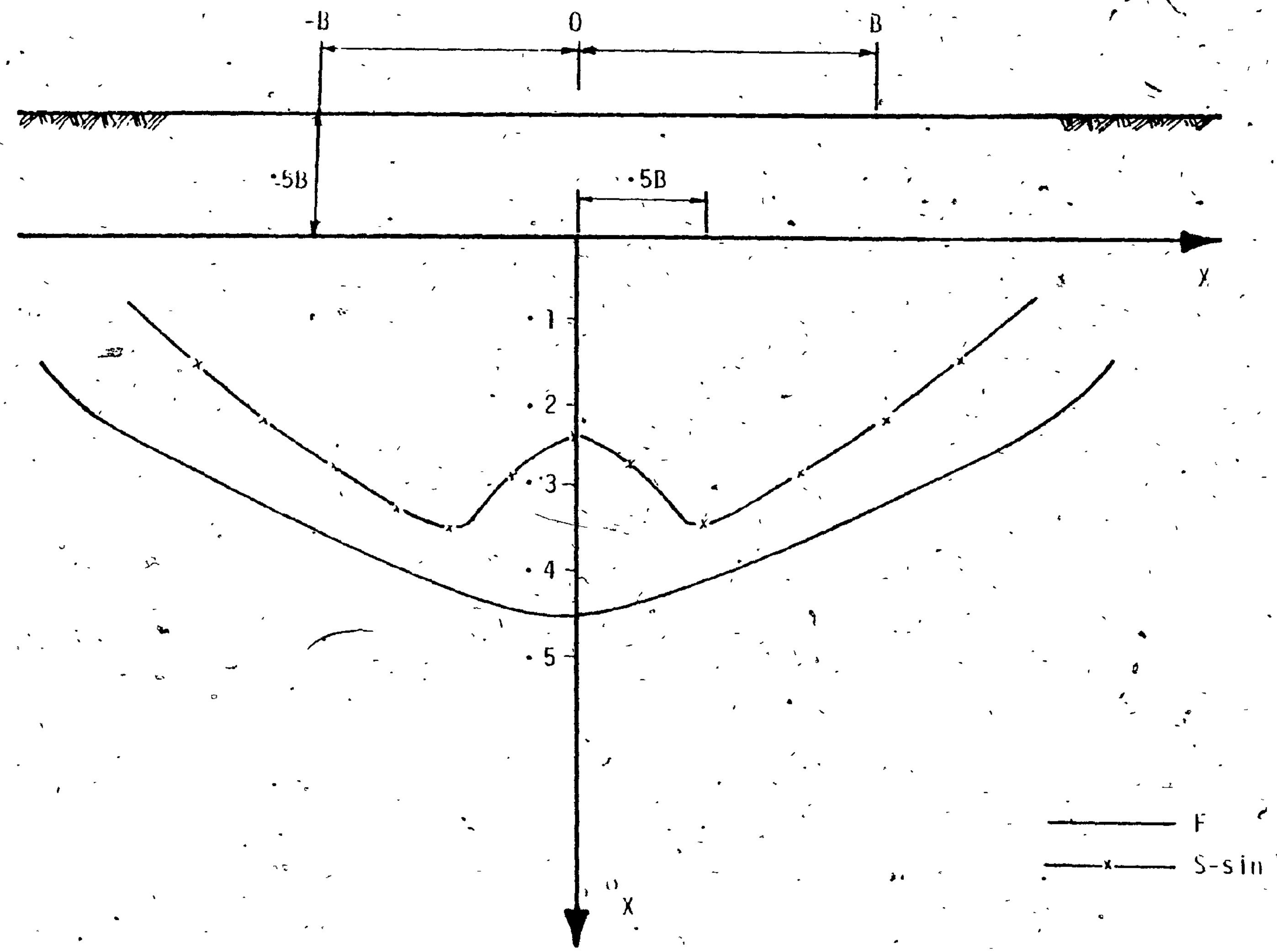


Figure 46. Horizontal stresses at Z = ·5B for flat and S-sin models.

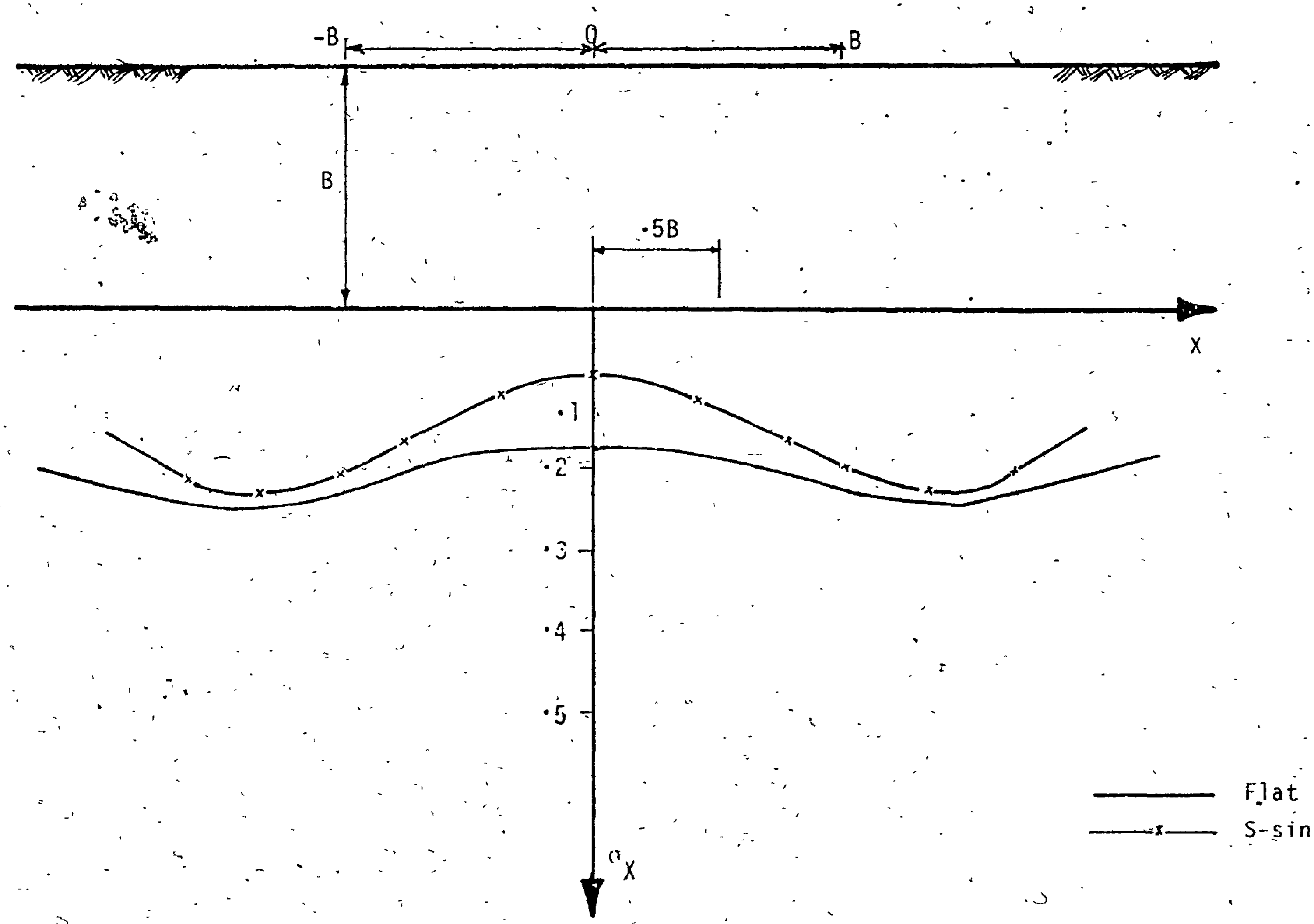


Figure 47. Horizontal stresses at $Z = B$ for flat and S-sin models.

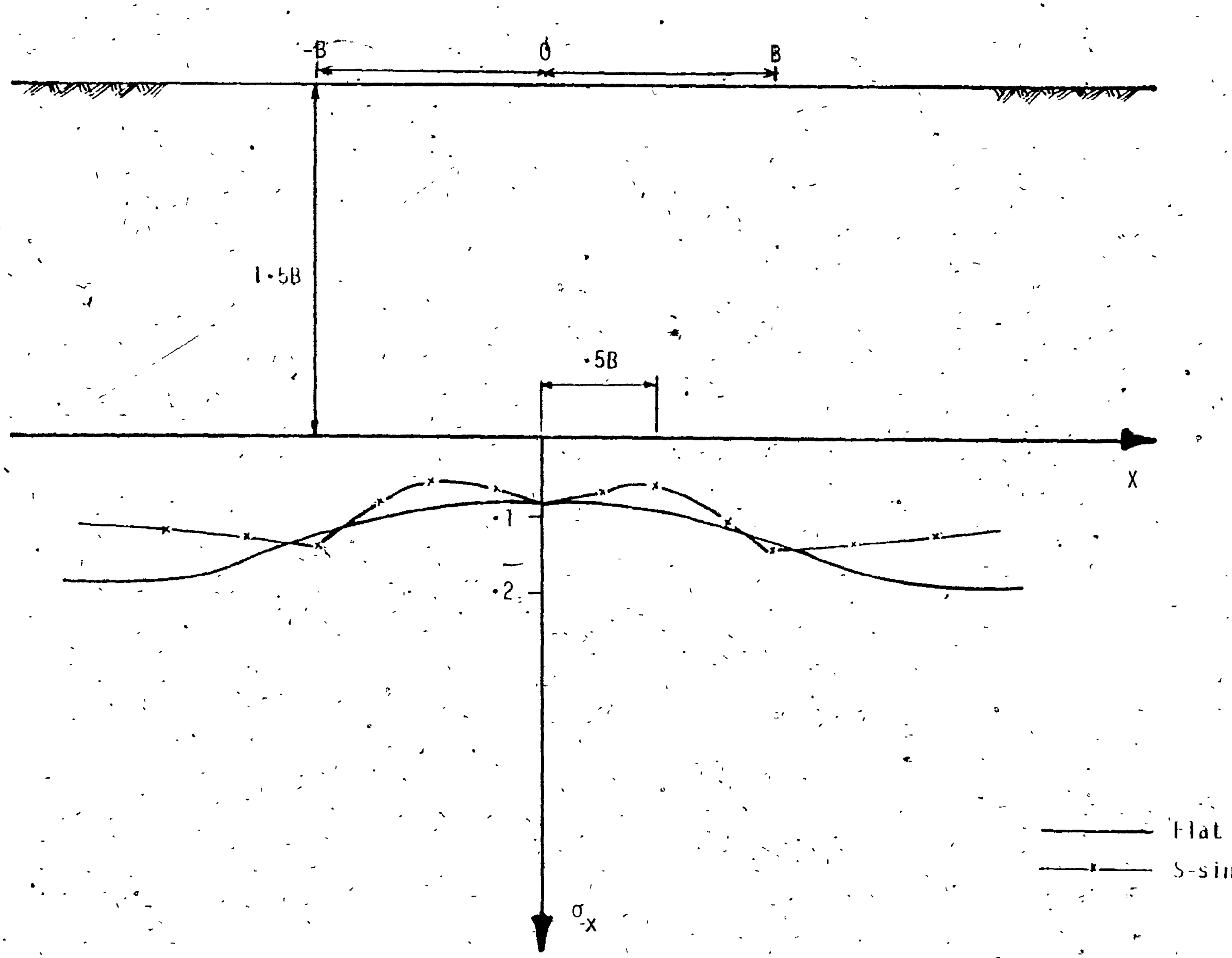


Figure 48. Horizontal stresses at Z = 1·5B for flat and S-sin models.

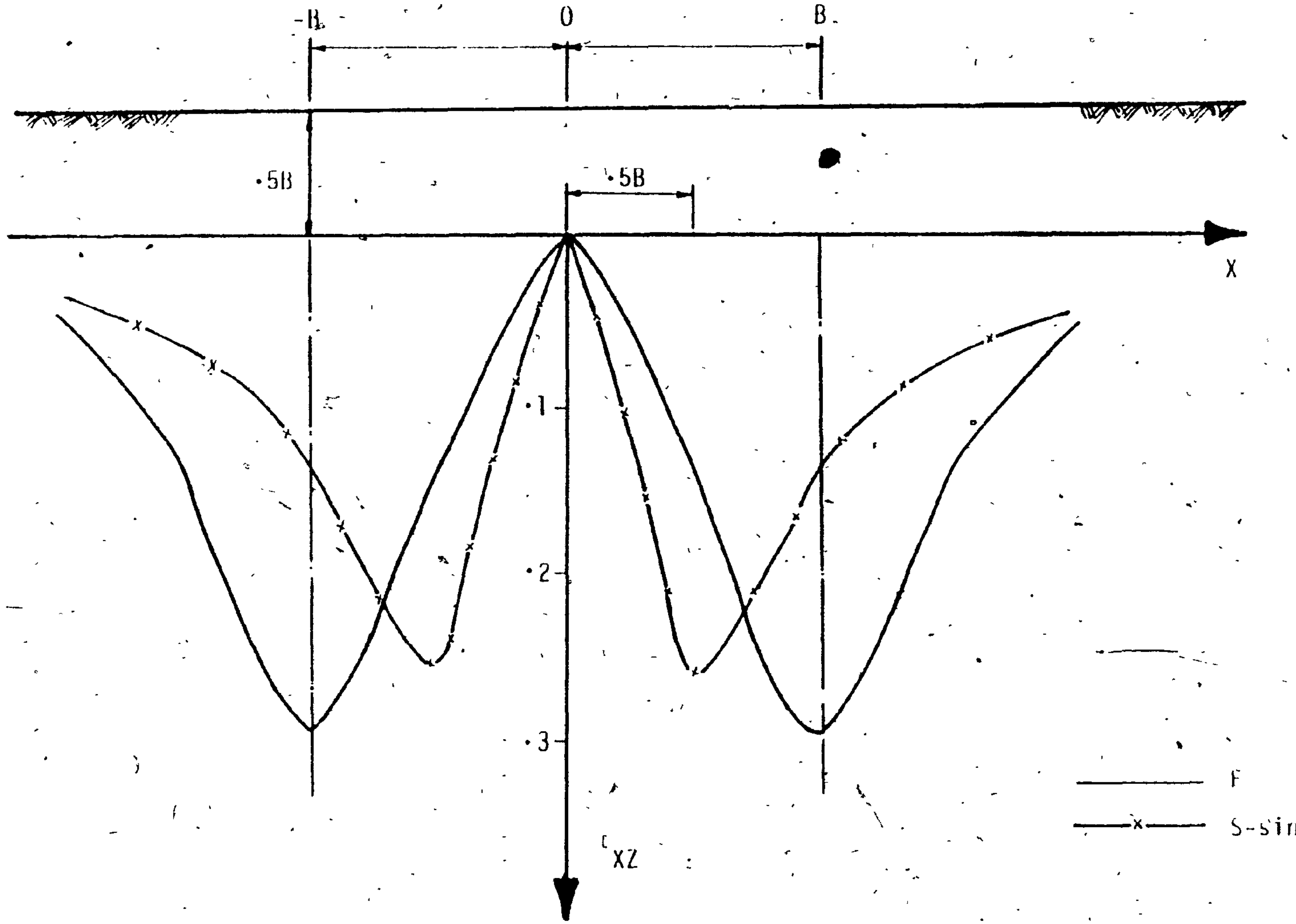


Figure 49. Shear stress at depth Z = ·5B for flat and S-sin models.

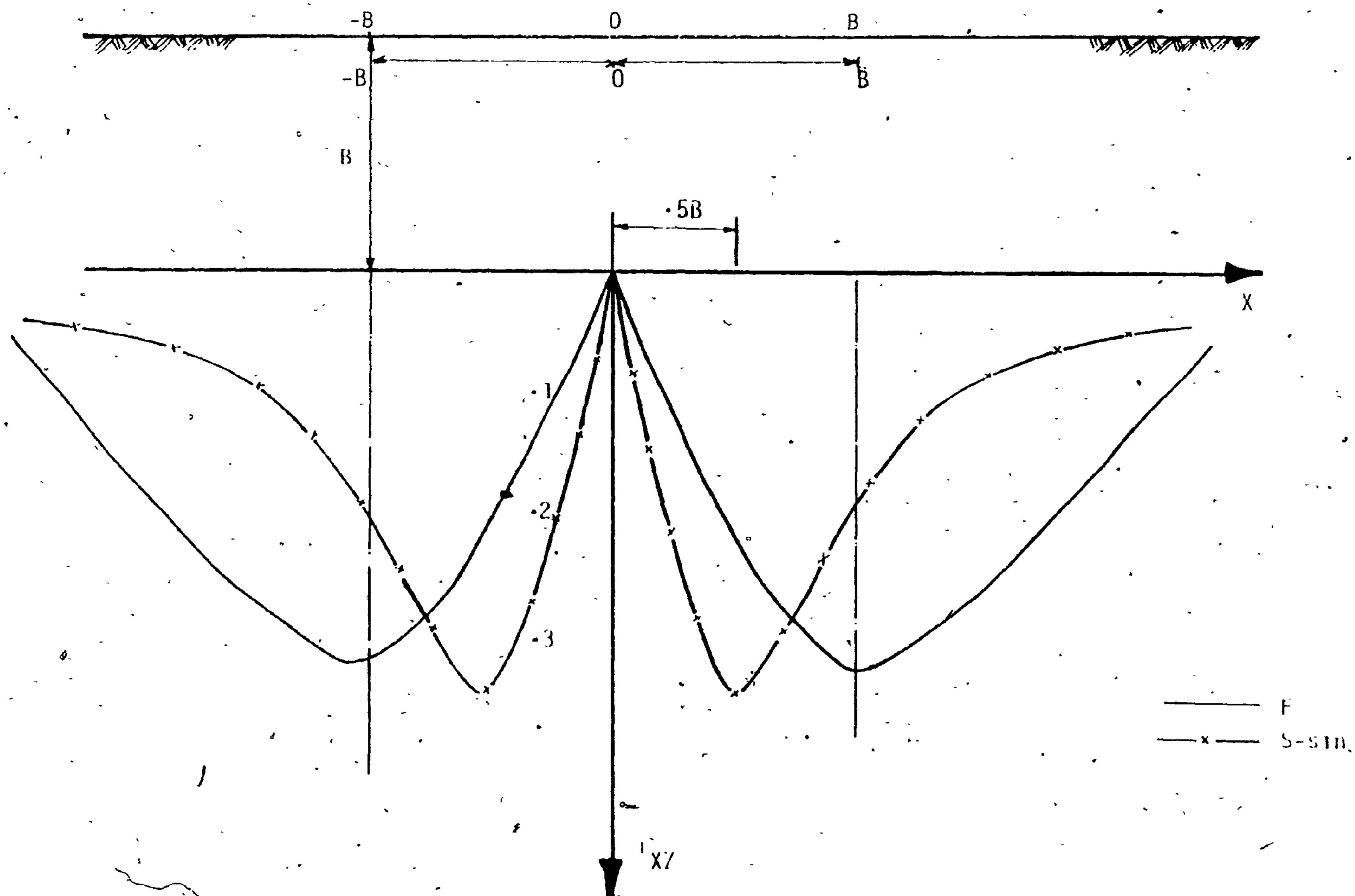


Figure 50. Shear stresses at depth Z = B for flat and S-sin models.

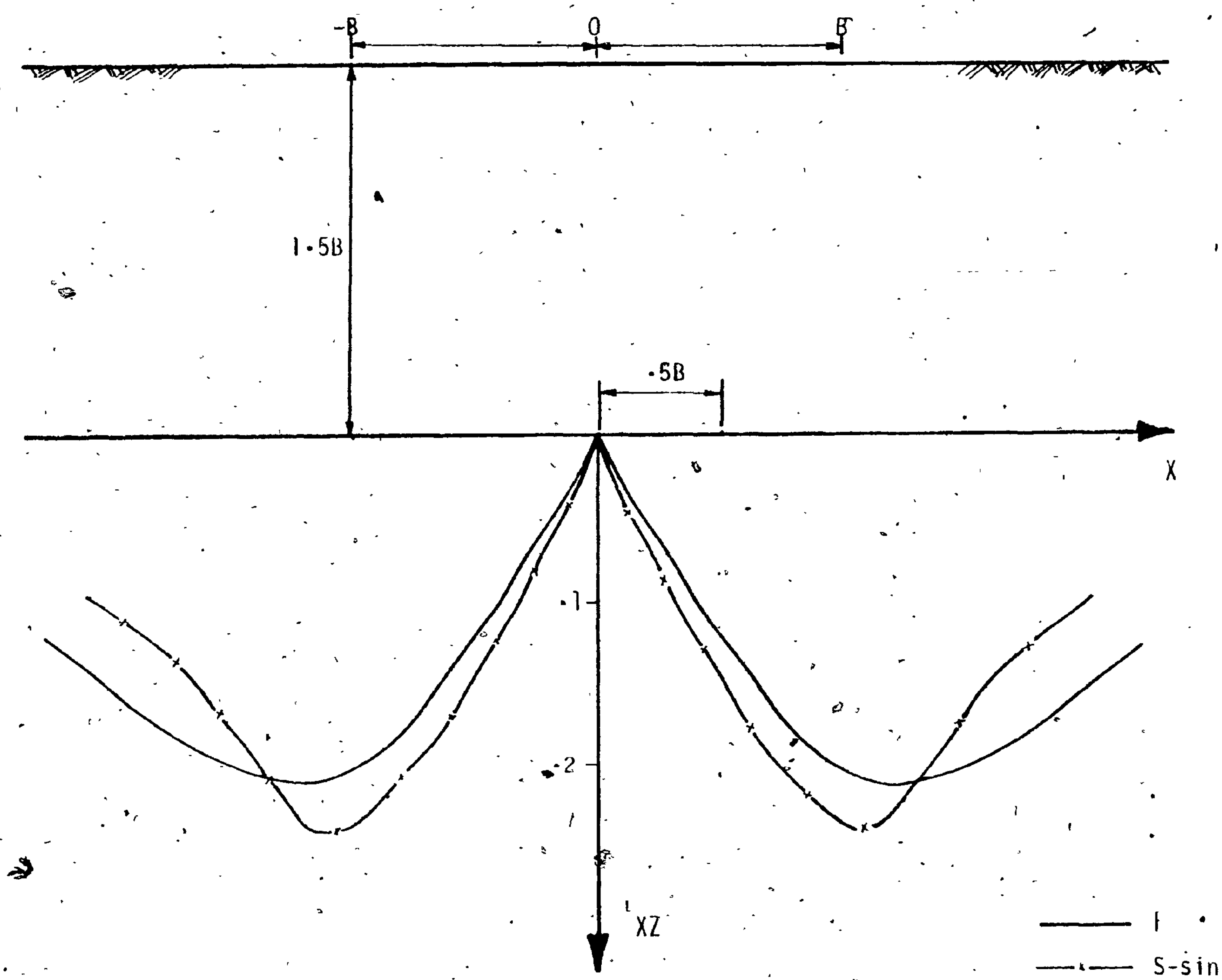


Figure 51. Shear stresses at Z = 1·5B for flat and S-sin models.

CHAPTER V

CONCLUSION AND RECOMMENDATION

The present study is an attempt to introduce new foundation models, to increase bearing capacity and to reduce settlement, resulting in economical foundation design. These models had shapes of arched and sloped ones.

Stress analysis was conducted on the soil beneath these proposed models and the conventional flat footings. The results were presented in the form of a ratio of the deduced stress at a given point, due to the proposed model and the conventional flat foundations. The results were presented in Tables and Figures, where the following conclusion can be drawn:

1 - The proposed models have provided better distribution and lesser in amount of the vertical compression stress up to a depth of about 2.5 of the foundation width. It is believed that the compression of this depth produces up 90% of the foundation settlements. Thus, it can be stated that the proposed foundation models, will produce less settlement, than the conventional footing which has the same width and under the same loads.

2 - The proposed models produce less shear and horizontal stresses than the conventional flat one. Thus. the proposed foundation models will have higher bearing

capacity than the conventional footings.

It is submitted, however, that the work presented here is the first attempt in searching for foundation shapes, to increase its bearing capacity and to reduce its settlements. Thus, efforts should be made in the future:

1 - Utilize the computer program coded for the propose of this study, to optimize models' shapes in order to achieve better improvements. Other shapes may be considered, such as steped ones.

2 - Study of feasability on the construction of these new foundation models, should be conducted and construction techniques should be developed to facilitate and economize the field construction.

REFERENCES

Bowles, J.E. (1982). Foundation Analysis and Design. McGraw-Hill Publishing Company.

Das, B. (1983). Advanced Soil Mechanics. McGraw-Hill Publishing Company.

Feda, Jaroslav. (1978). Stress in Subsoil and Methods of Final Settlement Calculation. Elsevier Scientific Publishing Company.

Lambe, T.W. and Whitman, R.V. (1969). Soil Mechanics. John Wiley & Sons Inc.

Mindlin, R.D. (1936). Force at Point in the Interior of a Semi-infinite Solid. J. Appl. Phys. Vol. 7, No. 5.

Poulos, H.G. and Davis, E.H. (1974). Elastic Solutions for Soil and Rock Mechanics. John Wiley & Sons Inc.

Selvadurai, A.P.S. (1979). Elastic Analysis of Soil Foundation interaction. Elsevier Scientific Publishing Company.

Spiegel, M.R. (1968). Theory and Problems of Mathematical Handbook of Formulas and Tables. Schaum's outline series. McGraw-Hill Publishing Company.

Spiegel, M.R. (1959). Vector Analysis. Schaum's outline series. McGraw-Hill Publishing Company.

Timoshenko, S.P. and Goodier, J.N. (1970). Theory of Elasticity. McGraw-Hill Publishing Company.