OBSERVER DESIGN TECHNIQUES FOR TIME INVARIANT LINEAR SYSTEMS

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ABSTRACT

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This dissertation discusses different observer schemes to reconstruct the states of the time invariant linear systems necessary for control. In some cases where the parameters of the system are unknown, adaptive observers are used, and adaptive laws are developed. The effect of these observers on the overall system stability is discussed, and it is shown that the observer does not change the pole location of the system, but it adds its own poles to the system. The rate of convergence, and the effects of observation noise have been discussed.

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

In a control system, if the input or the commanding signal is predetermined, and will not change no matter what the outcome of the control is, the system is said to be an open loop control system. If a proper change in the control signal is required to bring the response of the system to a desired one, such a system whose input signal depends on the outcome of the control is called 'feedback control system'. If a dynamical-equation description of a system is available, it is reasonable to select the input based on the value of the state, the reference input, and possibly on t, because the state and the input determine completely the future behavior of the system. So we can write the control signal as:

$$u(t) = f(r(t), X(t), t)$$

where $r(t)$ = reference input

$X(t)$ = state of the system

t = time.

This relation is called a control law. ~~In~~ linear, time variant systems, we can write the control law as follows:

$$u(t) = r(t) + KX(t)$$

where K is real constant called 'feedback gain matrix'.

Now, in case of state feedback control systems, where the state X is fed back into the input, we assume that the state variables are available as outputs, so that the control input u(t) can be determined. Also, the study of the system behaviour depends on its parameters, such as controllability and observability of the system.

In practical situations where the state variables of the system are not known, and some part of the parameters or all the parameters are not known, the observer and adaptive observer are used to determine the state and parameters of the system. This dissertation discusses the various methods and techniques used in order to observe the states of the time invariant linear systems, and also to identify the parameters of these systems (adaptive observer). In Chapter 2, observers which reconstruct missing state-variable information necessary for control are presented. The special topics of the identity observer, a reduced order observer, and stability properties are discussed. In Chapter 3, it is shown that the design of an observer for a system with m outputs can be reduced to the design of m separate observers for single output subsystems. Also, the application of observers to control design is investigated. It is shown that an observer's estimate of the system state vector can be used in place of the actual state vector in linear feedback designs without loss of stability. In Chapter 4, the identification problem is defined, using a model reference approach. Methods for determining the adaptive laws for adjusting unknown parameters which result in an asymptotically stable overall system are considered. Chapter 5 deals with an adaptive scheme which observes the state and simultaneously identifies all the parameters of single input-single output n^{th}-order linear systems. The adaptive scheme is proved to be globally asymptatically stable. In Chapter 6, the so called parametrized observer is discussed. In this observer, it is shown that the state estimate is a linear function of its parameters. Three such schemes are presented. Chapter 7 is the conclusion which shows the advantages and disadvantages of the different observers presented in this dissertation, and the areas of future work.

CHAPTER 2

AN INTRODUCTION TO OBSERVERS

2.1 INTRODUCTION

It is often convenient when designing feedback control systems to assume initially that the entire state vector of the system to be controlled is available through measurements. Thus for the linear time-invariant system governed by:

$$\dot{X}(t) = A\,X\,(t) + BU(t) \qquad \ldots\ldots\ldots\ (2.1)$$

where	X	denotes	n x 1	state vector
	U	"	r x 1	input vector
	A	"	n x n	system matrix
	B	"	n x r	distribution matrix.

One might design a feedback law of the form $u(t) \triangleq \psi(X(t),t)$ which could be implemented if X(t) were available. If the entire state vector cannot be measured, as is typical in most complex systems, the control law deduced in the form $U(t) = \psi(X(t),t)$ cannot be implemented. So an approximate state vector will be substituted for the unavailable state. This is done by an observer. The observer is a dynamic system whose characteristics are somewhat free to be determined by the designer.

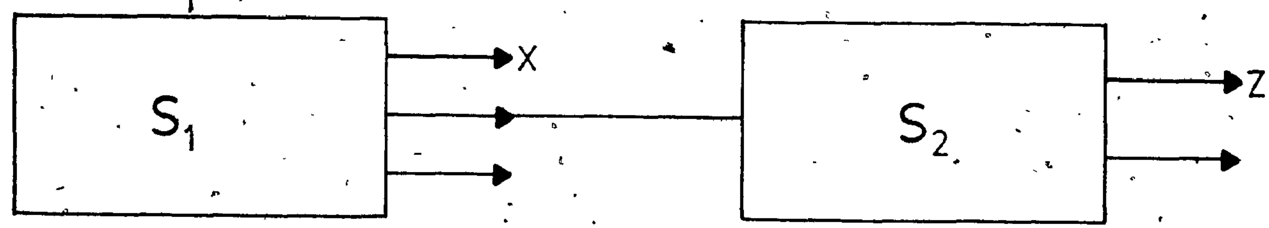


Fig. 2.1 — A Simple Observer.

2.2 BASIC THEORY

Initially, consider the problem of observing a free system S_1, i.e., a system with zero input. If the available outputs of this system are used as inputs to drive another system S_2, the second system will almost always serve as an observer of the first system in that its state will tend to track a linear transformation of the state of the first system, see Fig. 2.1.

2.2.1 THEOREM 1 (Observation of a Free System)

Let S_1 be a free system $\dot{X}(t) = A\,X(t)$, which drives S_2, $\dot{Z}(t) = FZ(t) + H\,X(t)$. Suppose there is a transformation T satisfying $TA - FT = H$. If $Z(0) = TX(0)$, then $Z(t) = T\,X(t)$ for all $t \geq 0$, or more generally:

$$Z(t) = TX(t) + e^{Ft}[Z(0) - T\,X(0)] \qquad \text{. . . . . (2.2)}$$

Proof: Well proved in [23].

It should be noted that the two systems S_1 and S_2 need not have the same dimension. Also, if A and F have no common eigen values, there is a unique solution T to the equation $TA - FT = H$.

Proof: Suppose that the transformation T exists, i.e. suppose that for all t:

$$Z(t) = T\,X(t)$$

$$\therefore \quad \dot{X} = AX \qquad (1)$$

$$\dot{Z} = FZ + HX \qquad (2)$$

by using the relation $Z = TX$

$$\therefore T\dot{X} = TAX$$

$$T\dot{X} = FTX + HX \qquad (3)$$

Since the left side coincides, so must the right side of equation (3).

$$\therefore TA - FT = H.$$

Since A and F have no common eigen values, (4) will have a unique solution T.

Thus any system S_2 having different eigen values from A is an observer for S_1 in the sense of Theorem 1. The result of Theorem 1 for a free system can be easily extended to a forced system by including the input in the observer as well as the original system. Thus, if S_1 is governed by:

$$\dot{X}(t) = A\,X(t) + BU(t) \qquad \ldots\ldots\ldots\ldots (2.3)$$

S_2 governed by:

$$\dot{Z}(t) = FZ(t) + HX(t) + TBU(t) \qquad \ldots\ldots\ldots\ldots (2.4)$$

will satisfy (2.2).

Thus, an observer for a system can be designed by first assuming the system is free, and then incorporating the inputs as in (2.4).

2.3 IDENTITY OBSERVER

An obviously convenient observer would be one in which the transformation T relating the state of the observer to the state of the original system is the identity transformation. This requiers that the observer S_2 be of the same dynamic order as the original system S (full order observer), and that (with $T = I$) $F = A - H$. Specification of such an observer rests, therefore, on specification of the matrix H. The matrix H is determined partly by the fixed output structure of the original system and partly by the input structure of the observer.

If S_1 with m dimensional output vector y is governed by:

$$\dot{x}(t) = A\,x(t) \qquad \text{. . . . . (2.5a)}$$

$$y(t) = C\,x(t) \qquad \text{. . . . . (2.5b)}$$

and S_2, the observer, is governed by:

$$\dot{Z}(t) = FZ(t) + Gy(t) \qquad \text{. . . . . (2.6)}$$

then $H = GC$. In designing the observer the m x n matrix C is fixed and the n x m matrix G is arbitrary. Thus, an identity observer is determined uniquely by selection of G and takes the form:

$$\dot{Z}(t) = (A - GC)Z(t) + Gy(t) \qquad \text{. . . . . (2.7)}$$

Any G leads to an identity observer, but the dynamic response of the observing process is according to Theorem 1, determined by the matrix A - GC.

LEMMAI

Corresponding to the real matrices C and A, the set of eigenvalues of (A-GC) can be made to correspond to the set of eigenvalues of any $n \times n$ real matrix by suitable choice of the real matrix C if and only if (C,A) is completely observable.

Proof: Well proved in [6].

THEOREM 2

An identity observer having arbitrary dynamics can be designed for a linear time invariant system if and only if the system is completely observable.

Proof: Proved in [23].

In practice, the real parts of the eigenvalues of the observer are selected to be negative, so that the state of the observer will converge to the state of the observed system, and they are chosen to be somewhat more negative than the eigenvalues of the observed system so that convergence is faster than other system effects. Theoretically, the eigenvalues can be moved arbitrarily toward minus infinity, yielding extremely rapid convergence. This tends to make the observer act like a differentiator and thereby become highly sensitive to noise.

EXAMPLE: Consider the system shown in Fig. 2.2. This has state-variable representation:

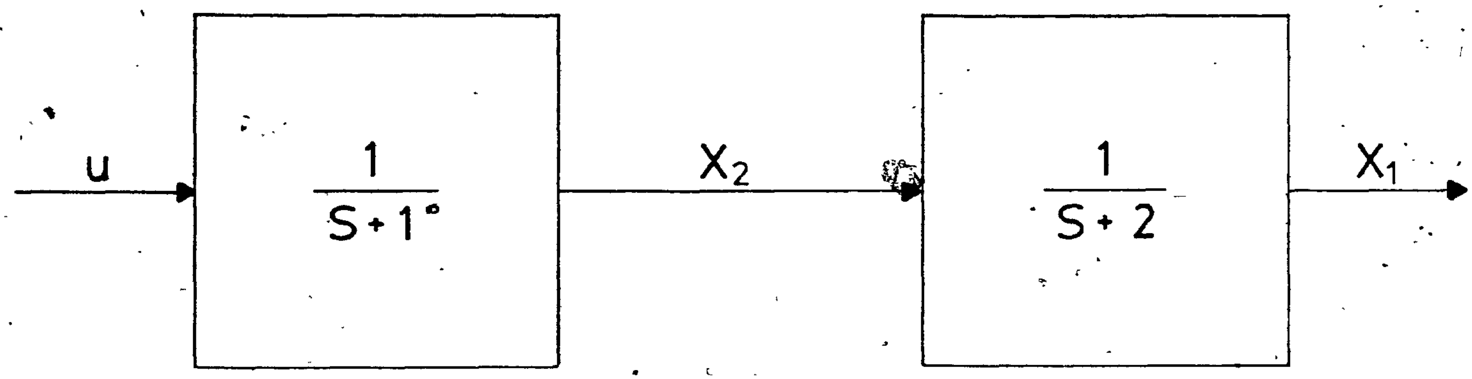


Fig. 2.2 — Example of Second Order Observer.

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad \ldots\ldots (2.8a)$$

$$y = [1 \quad 0] \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (2.8b)$$

An identity observer is determined by specifying the observer input vector:

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix}$$

The resulting observer system matrix is:

$$A - GC = \begin{bmatrix} -2 - g_1 & 1 \\ 0 - g_2 & -1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (2.9)$$

which has the corresponding characteristic equation:

$$\lambda^2 + (3 + g_1)\lambda + 2 + g_1 + g_2 = 0 \quad \ldots\ldots\ldots\ldots (2.10)$$

Suppose we decide to make the observer have two eigenvalues equal to -3. This would give the characteristic equation $(\lambda + 3)^2 = \lambda^2 + 6\lambda + 9$. Matching coefficients from (2.10), yields $g_1 = 3$, $g_2 = 4$. The observer is thus governed by:

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} -5 & 1 \\ -4 & -1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

2.4 Reduced Dimension Observer

The identity observer possesses a certain degree of redundancy. The redundancy stems from the fact that while the observer constructs an estimae of the entire state, part of the state as given by the system outputs is already available by direct measurement. This redundancy can be eliminated and an observer of lower dimension but still of arbitrary dynamics can be constructed. The basic construction of reduced order observer is shown in Fig. 2.3. If y(t) is of dimension m, an observer or order n-m is constructed with state Z(t) that approximates TX(t) for some m x n matrix T, as in Theorem 1. Then an estimate X(t) can be determined through:

$$\hat{X}(t) = \left[\frac{T}{C}\right]^{-1} \cdot \begin{bmatrix} Z(t) \\ y(t) \end{bmatrix} \quad \ldots\ldots\ldots\ldots \quad (2.11)$$

provided that the indicated partitioned matrix is invertible.

Again consider the system:

$$\dot{X}(t) = AX(t) + BU(t) \quad \ldots\ldots\ldots\ldots \quad (2.12a)$$

$$y(t) = C\,X\,(t) \quad \ldots\ldots\ldots\ldots \quad (2.12b)$$

And assume that without loss of generality that m outputs of the system are linearly indepenent or C has rank m. Also C can take the form $C = [I \,\vdots\, 0]$, i.e., C is partitioned into an m x m identity matrix and m x (n-m) zero matrix. An appropriate change of coordinates is obtained by selecting an (n-m) x n matrix D such that:

$$M = \left[\frac{C}{D}\right]$$

is nonsingular and using the variable $\overline{X} = MX$. It is convenient to partition

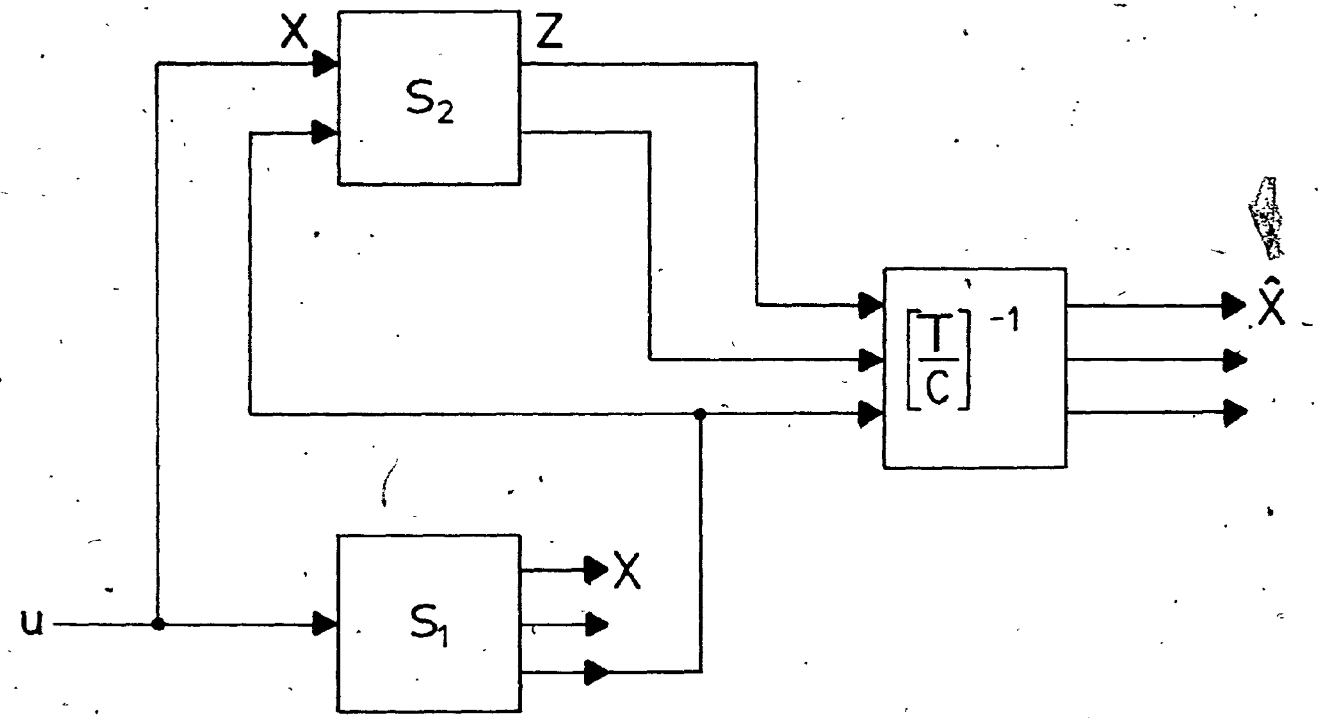


Fig. 2.3 — Structure of Reduced Order Observer.

the state vector as:

$$X = \begin{bmatrix} y \\ w \end{bmatrix}$$

and accordingly write the system in the form:

$$\dot{y}(t) = A_{11}y(t) + A_{12}W(t) + B_1U(t) \qquad \text{. . . . . (2.13a)}$$

$$\dot{w}(t) = A_{21}y(t) + A_{22}W(t) + B_2U(t) \qquad \text{. . . . . (2.13b)}$$

The idea of the construction is then as follows: the vector $y(t)$ is available for measurement and if we differentiate it, so is $\dot{y}(t)$. Since $U(t)$ is also measureable (2.13a) provides the measurement $A_{12}W(t)$ for the system (2.13b) which has the state vector $W(t)$ and input $A_{21}y(t) + B_2U(t)$. An identity observer of order (n-m) is constructed for (2.13b) using this measurement. The justification of the construction is based on the following Lemma:

LEMMA 2:

If (C,A) is completely observable, then so is (A_{12}, A_{22}).

Proof: Well explained in [8].

To construct the observer, initially define it in the form:

$$\dot{\hat{W}}(t) = (A_{22} - LA_{12})\hat{W}(t) + A_{21}y(t) + B_2U(t) + L[\dot{y}(t) - A_{11}y(t)]$$

$$- LB_1U(t) \qquad \text{. . . . . (2.14)}$$

In view of Lemmas 1 and 2, L can be selected so that A_{22}- LA_{21} has arbitrary eigenvalues, or:

$$\dot{Z}(t) = (A_{22} - LA_{12})Z(t) + (A_{22} - LA_{12})Ly(t) + (A_{21} - LA_{11})y(t)$$

$$+ (B_2 - LB_1)u(t) \quad \ldots\ldots\ldots\ldots (2.15)$$

$$\text{with } Z(t) = \hat{W}(t) - Ly(t) \quad \ldots\ldots\ldots\ldots (2.16)$$

For this observer $T = [-L \mid I]$ as shown in Fig. 2.4. This construction enables us to state the following theorem.

THEOREM 3

Corresponding to an n^{th} order completely controllable linear time invariant system having m linearly independent outputs a state observer of order $(n-m)$ can be constructed having arbitrary eigenvalues.

Example: Consider the 2nd order system shown in Fig. 2.2. This system has single output so a first order observer with an arbitrary eigenvalue can be constructed. The C matrix already has the required form $C = [1 \;\; 0]$. In this case, $A_{22}-GA_{12} = -1 - G$, which gives the eigenvalue of the observer. Let us select $G = 2$ so that the observer will have its eigenvalues equal to -3. The resulting observer is as shown in Fig. 2.5.

2.5 Closed-Loop Properties

It would be undesirable if stable control design became unstable when it was realized by introduction of an observer. In this section, it will be shown that if a linear time invariant control law is realized with an observer, the resulting eigenvalues of the system are those of the observer itself and those that would be obtained if the control law could be directly implemented. Thus, an observer does not change the closed loop eigenvalues of a design but merely adjoins its own eigenvalues. Suppose we have the system:

$$\dot{X}(t) = A X(t) + BU(t) \qquad \text{(2.17a)}$$

$$y(t) = C X(t) \qquad \text{(2.17b)}$$

and the Central law

$$u(t) = K X(t) \qquad \text{(2.18)}$$

If it were possible to realize this control law by use of available measurements (which would be possible if $K = RC$ for some R), then the closed loop system would be governed by:

$$\dot{X}(t) = (A + BK) X(t) \qquad \text{(2.19)}$$

and hence its eigenvalues would be the eigenvalues of $A + BK$. Now if the central can not be realized directly, an observer of the form:

$$\dot{Z}(t) = FZ(t) + Gy(t) + TBU(t) \qquad \text{(2.20a)}$$

$$U(t) = K\hat{X}(t) = EZ(t) + Dy(t) \qquad \text{(2.20b)}$$

where $TA - FT = GC$ (2.21a)

$$K = ET + DC \qquad \text{(2.21b)}$$

can be constructed.

From the previous theory (C,A) completely observable is sufficient for there to be G, E, D, F, T satisfying (2.21) with F having arbitrary eigenvalues. Setting $U(t) = KX(t)$ leads to the composite system:

$$\begin{bmatrix} \dot{X} \\ \dot{Z} \end{bmatrix} = \begin{bmatrix} A + BDC & BE \\ GC + TBDC & F + TBE \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix} \qquad \text{(2.22)}$$

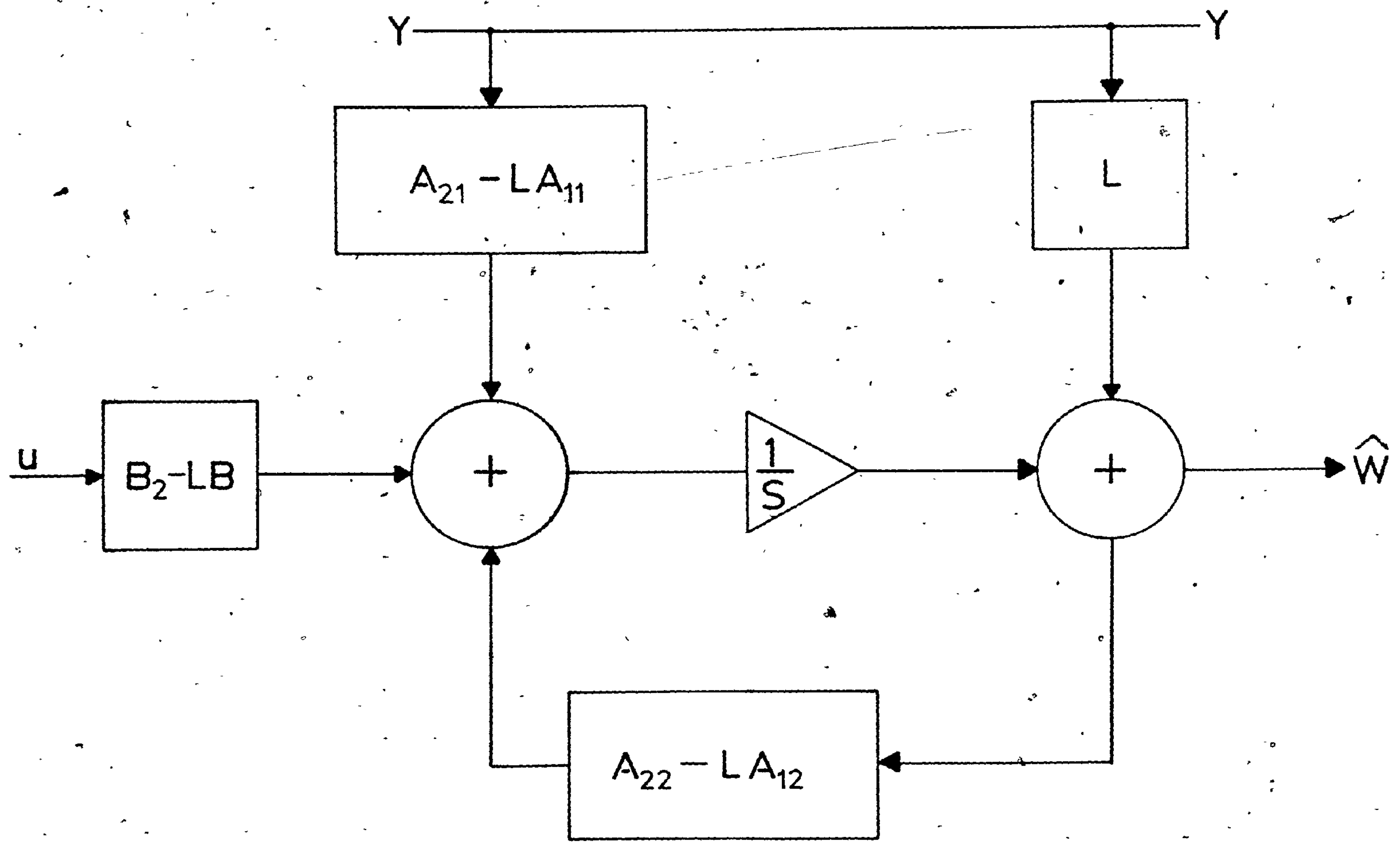


Fig. 2.4 — Reduced Order Observer.

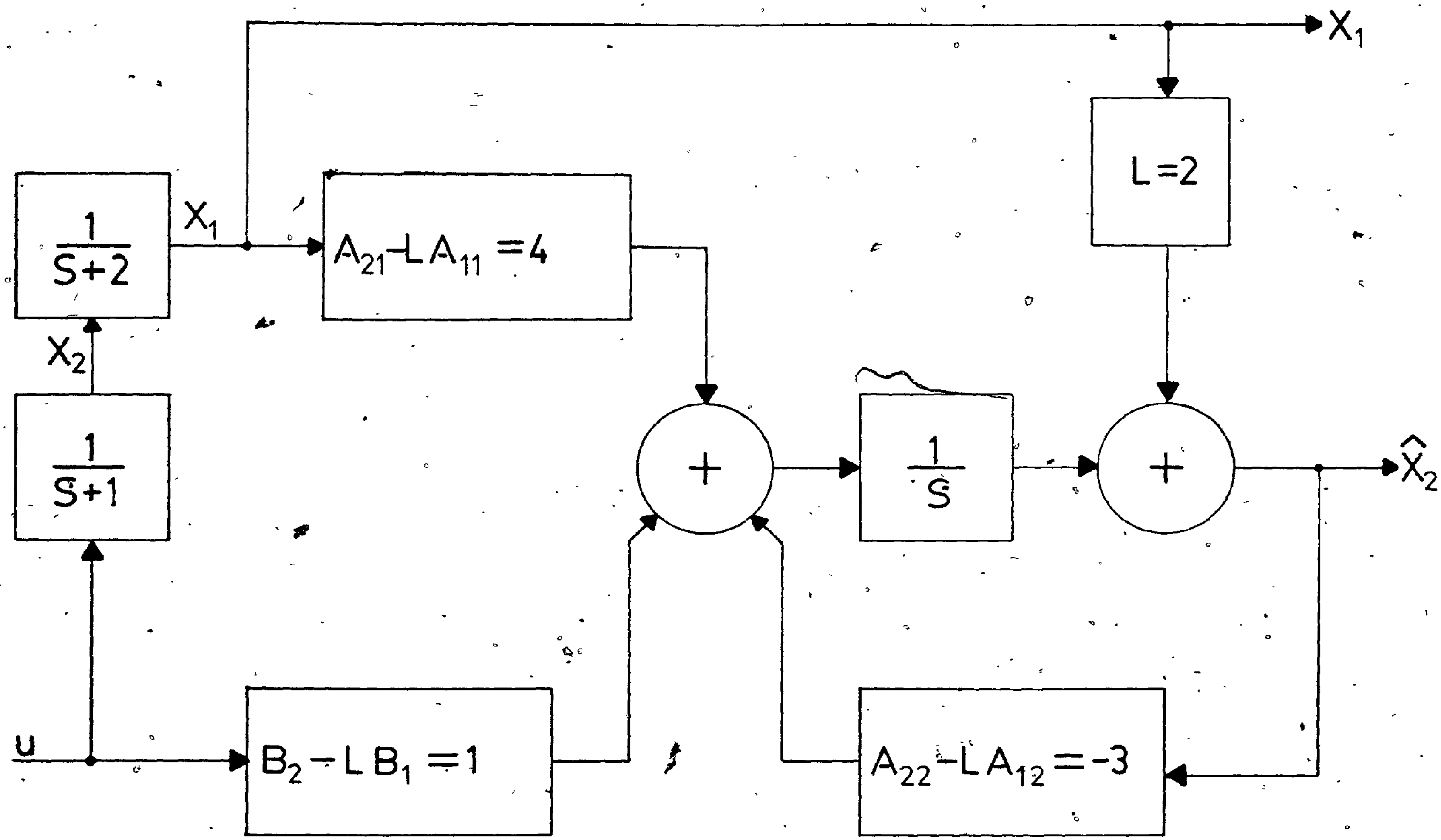


Fig. 2.5 — Reduced Order Observer for Example Fig. 2.2.

This whole structure can be simplified by introducing $\psi = Z - TX$, and using X and ψ as coordinates. Then (2.22) becomes:

$$\begin{bmatrix} \dot{X} \\ \\ \psi \end{bmatrix} = \begin{bmatrix} A + BK & BE \\ \\ 0 & F \end{bmatrix} \begin{bmatrix} X \\ \\ \psi \end{bmatrix} ;$$

Thus, the eigenvalues of the composite system are those of $A + BK$ and F. In view of Lemma 1, if the system (2.17) is completely controllable it is possible to select K to place the closed loop eigenvalues arbitrary. If this control law is not realiazable but the system is completely observable, an observer can be constructed so that the control law can be estimated. Since the eigenvalues of the observer are also arbitrary the eigenvalues of the complete composite system may be selected arbitrarily. The following theory results:

THEOREM 4

Corresponding to an n^{th} order completely controllable and completely observalbe system (2.17) having m linearly independent outputs, a dynamic feedback system of order (n-m) can be constructed such that the (zn-m) eigenvalues of the composite system take any preassigned values.

So as seen from the above, the stability of a linear time invariant system is not affected by an unstable observer.

CHAPTER 3

OBSERVERS FOR MULTIVARIABLE SYSTEMS

3.1 INTRODUCTION

In this chapter, a new procedure for designing observers for systems which have several outputs is explained. The problem is reduced to a series of observer designs for single output systems. The new procedure based on a special canonical form for multiple-output systems, leads to simpler observer designs. Also, the problem of reconstructing a single linear functional of the state rather than the entire state vector is considered. It is shown that considerable reduction in observer complexity is then possible. Finally, it will be shown that observers may be used to realize both linear and nonlinear control laws without loss of stability.

3.2 PROBLEM STATEMENT

In this section the design of an observer for a system with M outputs can be reduced to the design of m separate observers for single output subsystem. The general theory developed in Chapter 2 applies to multiple output system as well as single output systems. As with the single output situation, an essential assumption is that of complete observability with respect to the outputs.

A system $\dot{X} = AX$ . . . . (3.1a)

$y = H^T X$ . . . . (3.1b)

is completely observable if the nx(nm) matrix $[H, A^T H, \ldots (A^T)^{n-1} H]$ has rank n. The observability index ν of the system (3.1) is defined as the least positive integer for which the matrix $[H, A^T H, \ldots (A^T)^{\nu-1} H]$ has rank n. For some systems the extension to the multiple-output case is elementary. For example: consider the fourth order system shown in Fig. 3.1. It is assumed that the two variables X1 and X3 are available for direct measurements.

This system may be regarded as two coupled 2nd order subsystems as indicated by the dashed-line boxes in Fig. 3.1. The output of the first box is the measureable variable X1 and the input is the measureable variable X3. Therefore, since it is possible to measure the input and output of this second-order subsystem, a first order observer may be constructed for this subsystem. Similar considerations apply to the second box, so it is seen that an observer for the total system can be built up from two separate observers, each observing a single output subsystem. In fact, all multiple output observing problems can be reduced to the observation of single output subsystems. This is a result of the following Theorem:

3.2.1 THEOREM 1 (Canonical Representation of Multiple Output Systems).

Suppose that the n^{th} order system $\dot{X} = AX$ with associated output vector $y = H^T X$ is completely observable with observability index ν.

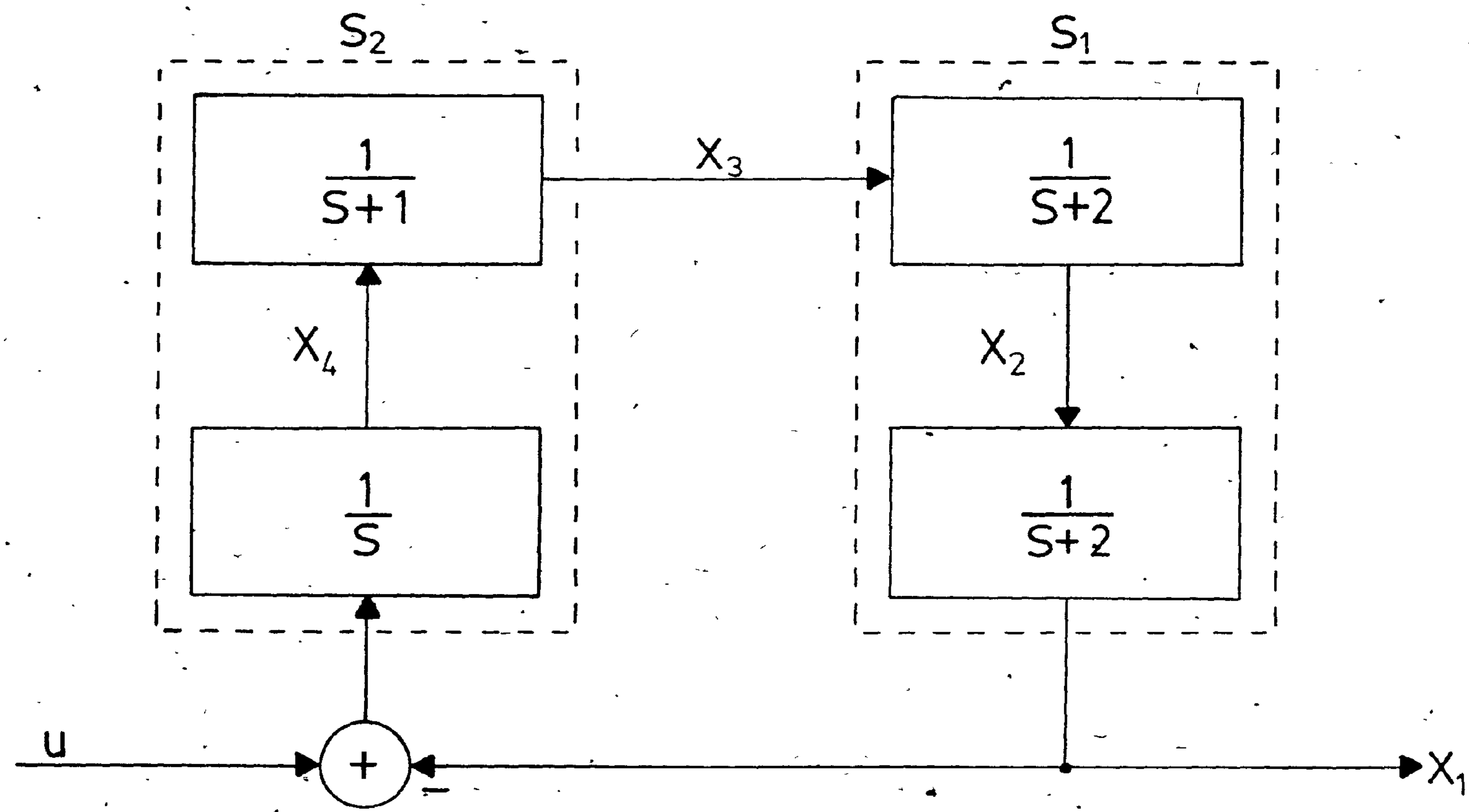


Fig. 3.1 — Fourth Order System Example.

Suppose further that y consists of m independent components. Then there is a non-singular linear coordinate transformation such that in terms of the new coordinates the system has the representation shown in Fig. 3.2. In this form, the system consists of m component subsystems, each with one observable output which is a linear combination of the components of y. The orders of the subsystems satisfy $n_1 + n_2 + \ldots$ $n_m = n$, and the largest subsystem is of order ν. The subsystems are coupled to each other only through their outputs.

<u>Proof</u>: The first step in the proof is the generation of a certain set of n linear independent vectors.

Since the matrix $[H, A^T H, A^{T\ \nu-1}]$ has rank n, n independent vectors can be taken as a certain n columns of this matrix.

To define these vectors precisely:

(a) Start with the columns $h_1, h_2, \ldots h_m$ of the matrix H.

(b) Adjoin to these the columns $A^T h_1$, $A^T h_2$, $A^T h_m$ one by one, checking that each new column is linearly indepdnetnt of the previous ones.

(c) If any of the new columns is found to be independent, omit it from the matrix and go on to the next.

(d) After $A^T h_m$ has been tested, continue with $(A^T)^2 h_1$, $(A^T)^2 h_2$, $\ldots$ $(A^T)^2 h_m$, etc, until n linearly independent columns have been found.

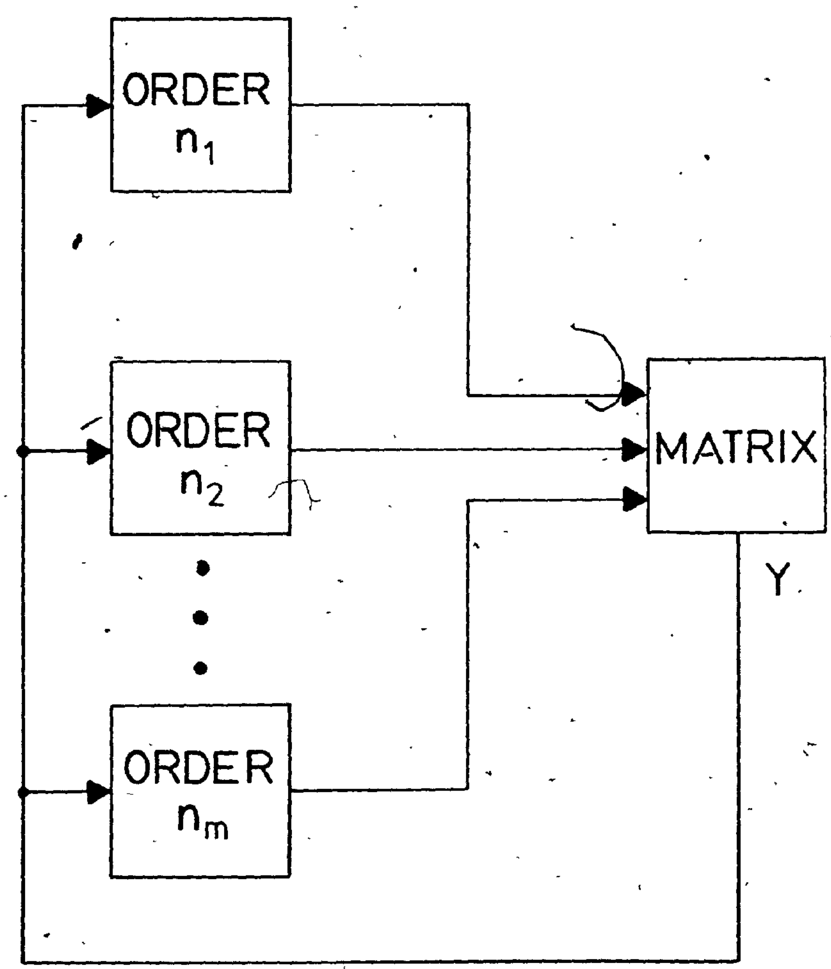


Fig. 3.2 — Canonical Form of Multiple Output System.

(e) If a column $(A^T)^i h_j$ has been skipped because of linear dependence, all columns of the form $(A^T)^k h_j$ where $k > i$ can be skipped, since they also must be dependent on the previous columns.

As a result of this procedure, there is defined an array of n independent vectors:

$$h_1, A^T h_1, \ldots\ldots (A^T)^{\nu_1 - 1} h_1$$

$$h_2, A^T h_2, \ldots\ldots (A^T)^{\nu_2 - 1} h_2$$

$$\vdots$$

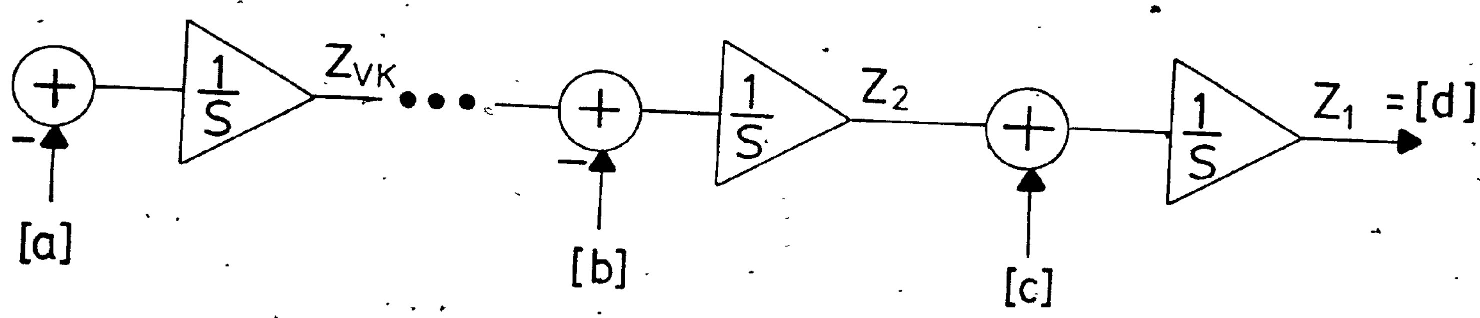
$$h_m, A^T h_m, \ldots\ldots (A^T)^{\nu_m - 1} h_m$$

where for each K, $\nu_K \leqslant \nu$. Furthermore, by construction there are coefficients $\alpha_{iJ}(k)$ such that:

$$(A^T)^{\nu_k} h_k = \sum_{j=1}^{m} \sum_{i=0}^{\nu-1} \alpha_{iJ}(k)(A^T)^i h_J \qquad \ldots\ldots\ldots\ldots (3.1c)$$

where $\alpha_{iJ}(k) = 0$ for $i > \nu_k$, and $\alpha_{iJ}(k) = 0$ for $i = \nu_k$, if $J > k$.

The desired canonical form of the system will have a structure similar to the structure of the above array in that k^{th} subsystem will be of order ν_k. However, the state variables of the k^{th} subsystem will be defined in terms of vectors from the complete array rather than just the k^{th} row. The K^{th} subsystem takes the form shown in Fig. 3.3. The outputs of the m subsystems are each linear combinations of the original outputs



[a] $\sum_{j=1}^{m} \alpha_{o}, j^{(K)} h_j^T X$

[b] $\sum_{j=1}^{m} \alpha_{VK}-2, J^{(K)} h_J^T X$

[c] $\sum_{J=1}^{m} \alpha_{VK}-1, J^{(K)} h_J^T X$

[d] $h_J^T X - \sum_{J=1}^{K-1} \alpha_{VK}, J^{(K)} h_J^T$

Fig. 3.3 — K^{th} Subsystem of Canonical Form.

and hence are themselves measureable quantities. Conversely, the new outputs are linearly independent so the old outputs can be recovered from the new.

The independence of the n outputs follows from the fact that the transformation matrix relating the old and new outputs is triangular with 1's along the diagonal. In order to establish that the proposed canonical form is in fact a linear coordinate change of the original system, it is only necessary to verify that all variables take the form $Z = K^T X$ in the canonical form satisfy $\dot{Z} = A^T K$. That this requirement is satisfied by the K^{th} subsystems shown in Fig. 3.3, follows directly from equation (3.1c).

EXAMPLE: The system shown in Fig. 3.1 is already in canonical form appropriate for design of a second order observer. The poles of the observer are arbitrary and will be both chosen to be -3. The design is carried out separately for each subsystem.

System S is governed by:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} X3 \qquad \dots\dots\dots (3.2)$$

According to results from Chapter 2, an observer with a pole at -3 driven by $X_1 = [1 \quad 0]\ X$ will produce TX where:

$$T \begin{bmatrix} -2 & 1 \\ 0 & -2 \end{bmatrix} + 3T = [1 \quad 0] \qquad \dots\dots\dots (3.3)$$

get $T = [1 \quad -1]$. The observer will be governed according to:

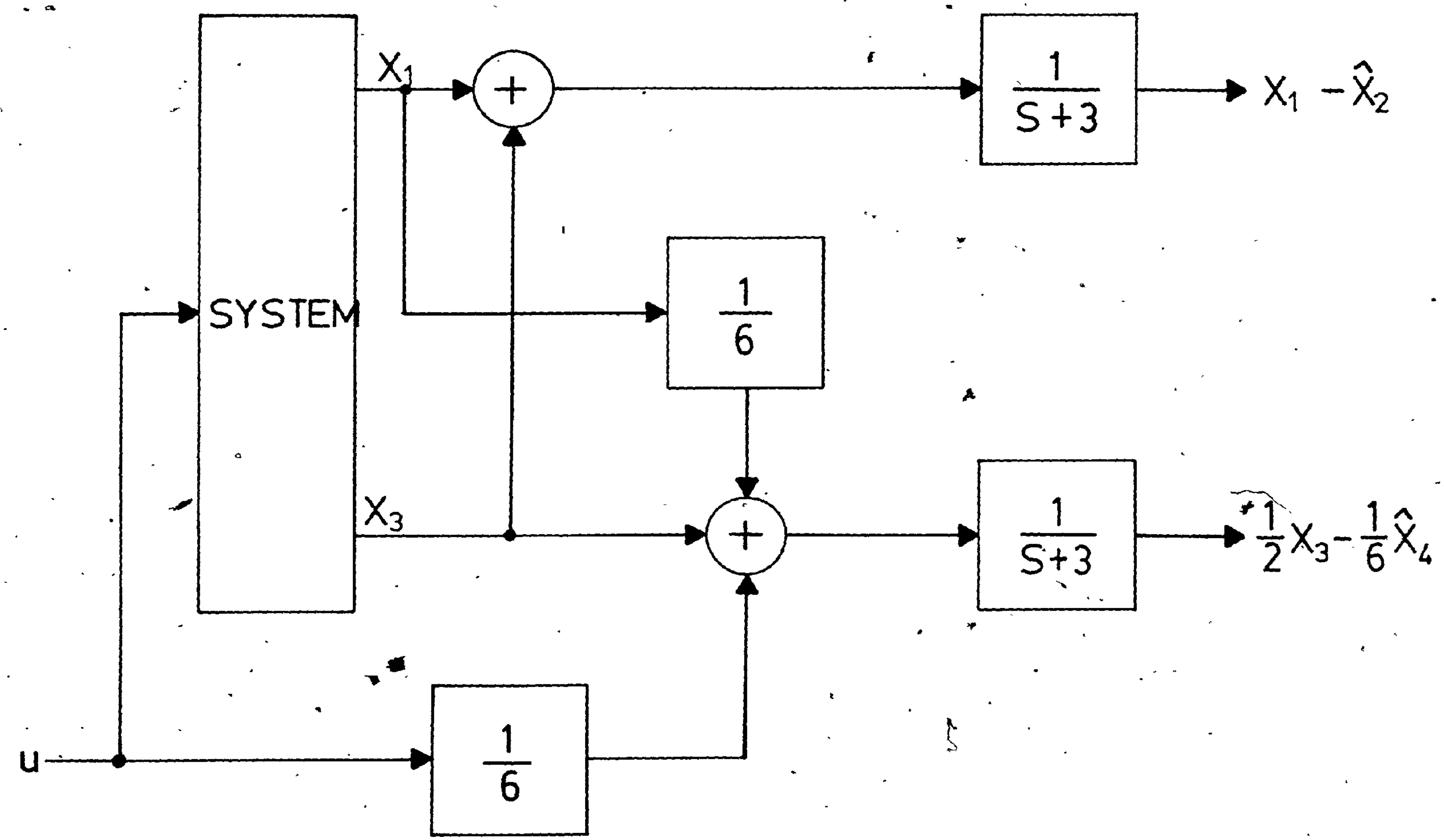


Fig. 3.4 — Observer for Fourth Order System Example.

Suppose these poles are chosen first. Then the poles of each of the other blocks of the observer can be chosen to be a subset of the poles of the largest block. Now corresponding to each output y_k, there is a transfer function of the form $\frac{\Delta_k(s)}{\Delta(s)}$ from y_k through the observer to $a^T X$.

The polynomial $\Delta(s)$ is the characteristic polynomial of the largest block in the observer, and $\Delta_k(s)$ is a polynomial of degree no greater than that of $\Delta(s)$. Thus, the observer shown in Fig. 3.5 may be equivalent to the one shown in Fig. 3.6 when the individual blocks of the original observer have common poles. An observer of this form can be realized by a system of order ν-1, therefore the following theorem is established.

3.3.1 THEOREM 11

A single linear functional of the state of a linear system can be observed by a system with ν-1 arbitrary poles (ν is the observability index of the system). As shown in [1] ν-1 is often considerably less than n-m, the order of complete observer. In fact $(n/m)-1 \leq \nu-1 \leq n-m$. A twenty-fifth order system with five outputs, for example, may require as few as four arbitrary poles to construct an estimate of a single linear functional of the state vector.

Example: Consider again the fourth order system in Fig. 3.4. Suppose it is desired to reconstruct the single linear functional $X_2 + X_1$. According to Theorem 2, an observer with a single arbitrary pole is sufficient. If the pole is chosen to be -3, the observer constructed before

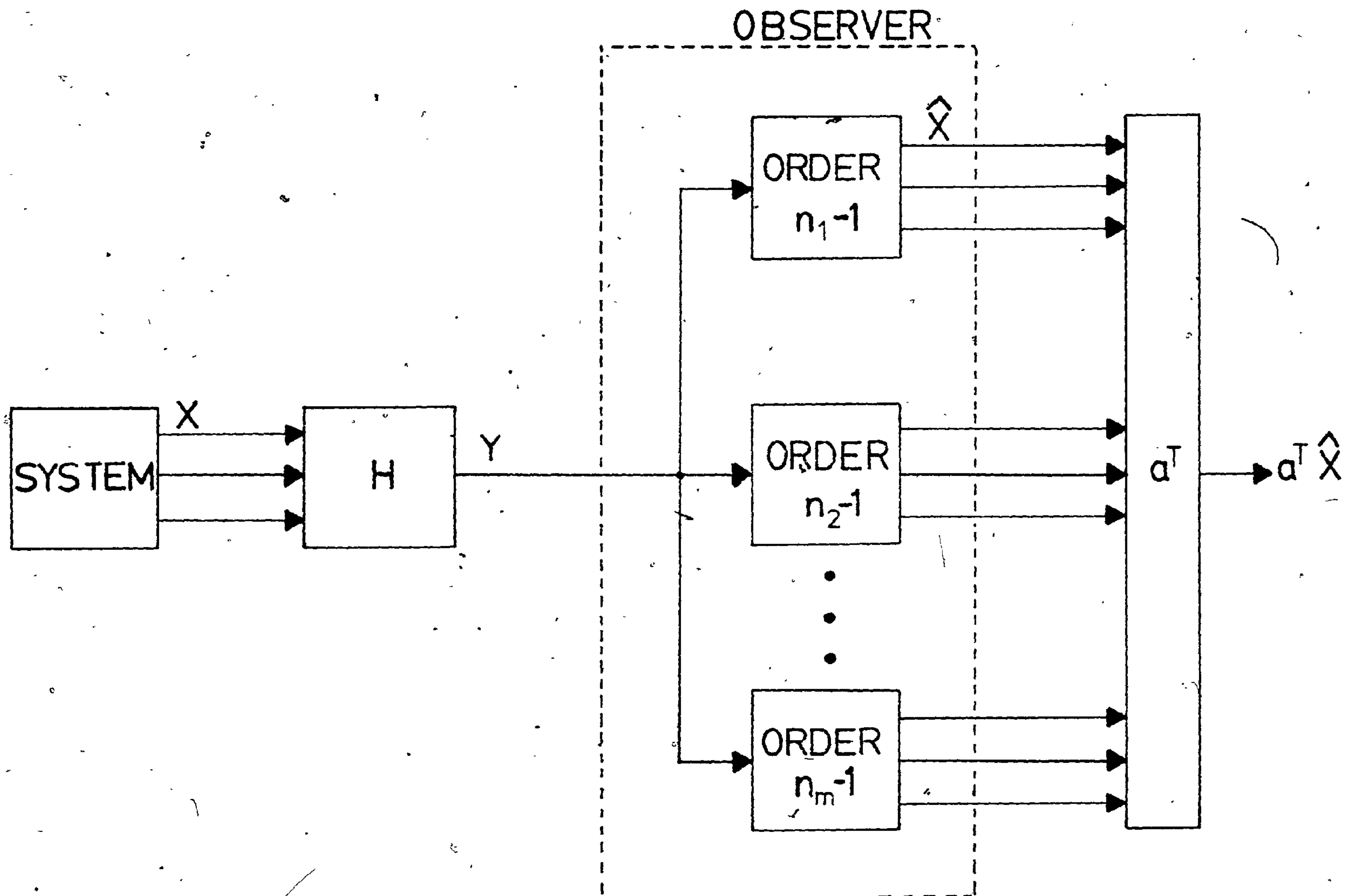


Fig. 3.5 — Observing A Single Linear Functional.

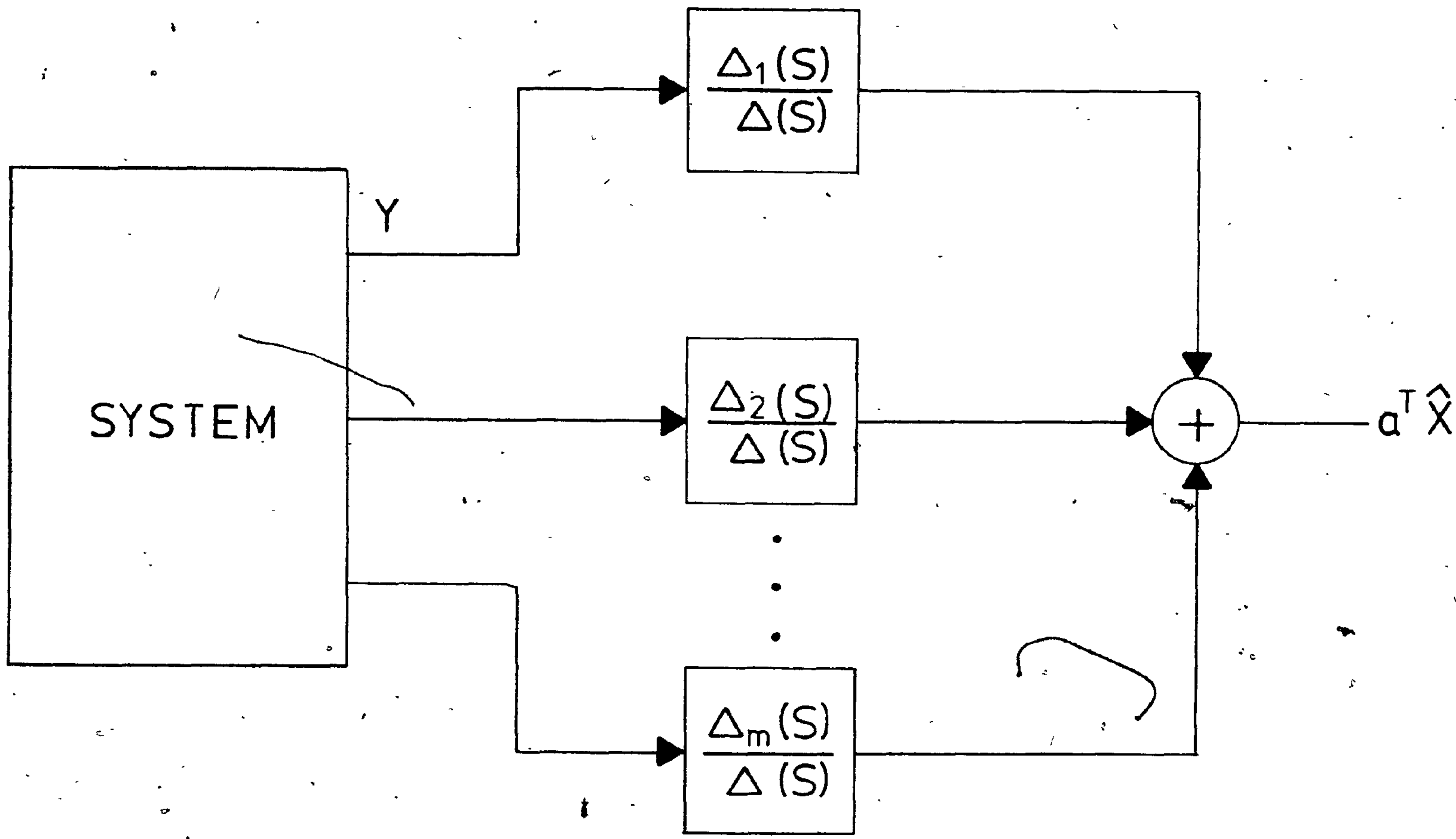


Fig. 3.6 — Reduced Observer for a Single Linear Functional.

$$\dot{Z} = BZ + CX + TDu \qquad \text{(3.4)}$$

which in this case:

$$Z = -3Z + X_1 - X3 \qquad \text{(3.5)}$$

The estimate X_2 is constructed from the measurement X_1 and Z according to:

$$\hat{X}_2 = X_1 - Z \qquad \text{(3.6)}$$

A similar procedure applied to the subsystem S leads to the observer:

$$\dot{w} = -3w + X - \frac{1}{6}(u - X1)$$

$$\hat{X}_4 = 3X3 - 6w$$

The complete observer is shown in Fig. 3.4.

3.3 Observing a Single Linear Functional

Sometimes it is only necessary to estimate a single (but prespecified) linear functional of a system's state vector. This is the situation for example in the design of linear time invariant state feedback for a single input system. In these instances, an observer of considerably reduced complexity can often be constructed which will produce this single quantity.

Imagine an observer constructed for a multiple-output system according to the scheme of Section 3.1. The output of the observer is an estimate of the system state vector X. In order to obtain an estimate of a linear functional of X, say $a^T X$, the same linear functional of the observer output is taken. The result is shown in Fig. 3.5. The largest block in the observer has exactly ν-1 poles which may be chosen arbitrarily.

can be used as a first step for this system in the design procedure. Using the results obtained in the previous example:

$$\hat{X}_2 + \hat{X}_4 = X1 + 3X3 - Z - 6w \quad \text{. . . . . (3.7)}$$

The observer shown in Fig. 3.7 is the result.

3.4 Closed-Loop Stability Properties

Consider the system:

$$\dot{X} = AX + Du \quad \text{. . . . . (3.8a)}$$

$$\text{with} \quad y = H^T X \quad \text{. . . . . (3.8b)}$$

Suppose that a control law of the form $u = F(X)$ has been derived for this system by some design scheme. An appropriate observer for (3.8) is:

$$\dot{Z} = BZ + CX + TDu \quad \text{. . . . . (3.9)}$$

where $TA - BT = C$. CX must be derived from the output vector y, hence $C = GH^T$ for some appropriate matrix G. The estimated state is a linear combination of the system outputs and the state vector of the observer is:

$$\hat{X} = LX + KZ \quad \text{. . . . . (3.10)}$$

where $L + KT = I$ (identity). The control law $u = F(X)$ can be approximated by a control law $\hat{U} = F(\hat{X})$ based on the estimation of the state vector. The complete system is then governed by:

$$\begin{aligned} \dot{X} &= AX + DF(\hat{X}) \\ \dot{Z} &= BZ + CX + TDF(\hat{X}) \\ \hat{X} &= LX + KZ \end{aligned} \quad \text{. . . . . (3.11)}$$

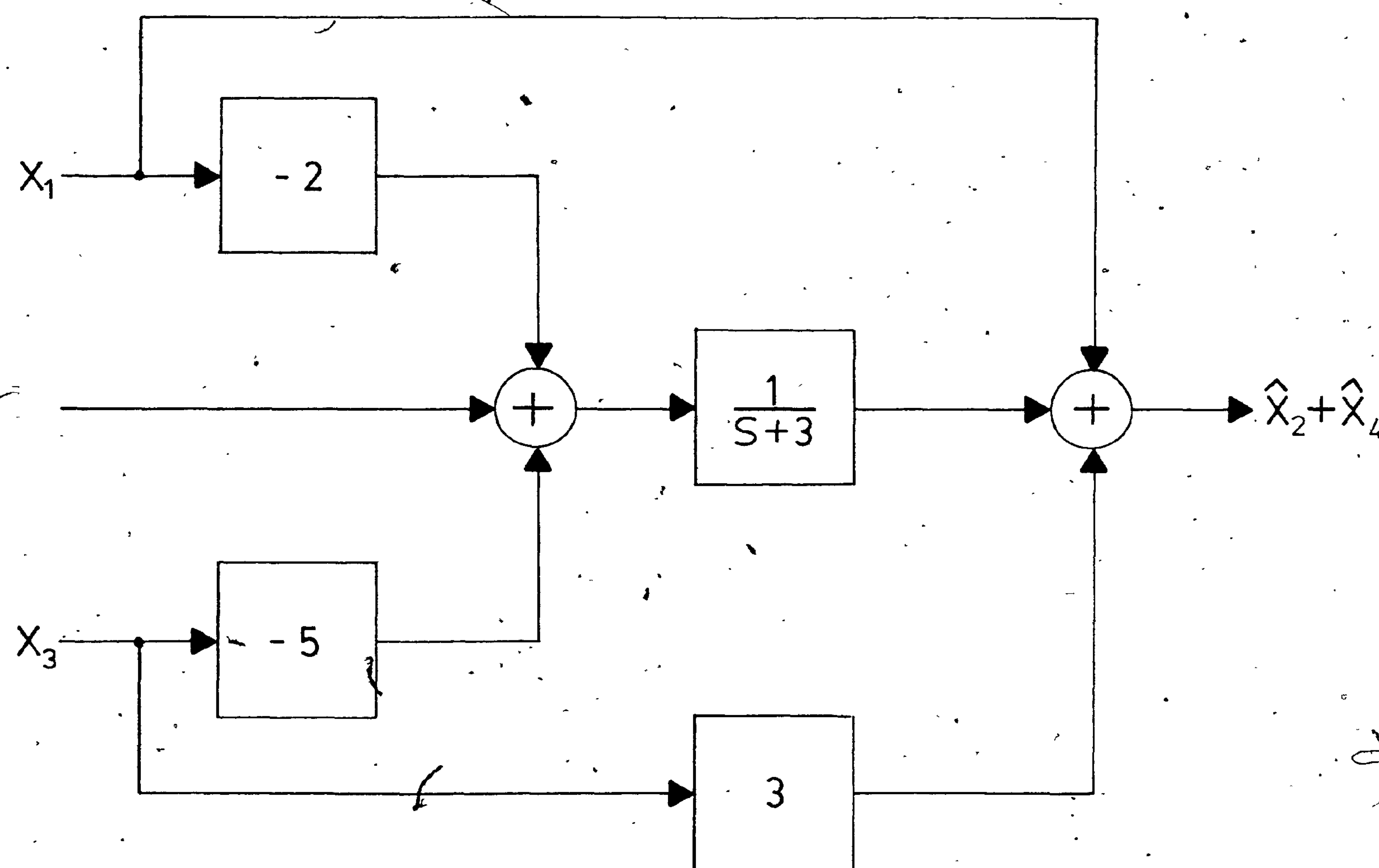


Fig. 3.7 — First Order Observer as an Example.

So let us investigate the stability properties of the control system governed by (3.11). The system equations (3.11) can be rearranged so that many of their stability properties become clearly apparent. Define $\overline{Z} = Z - TX$, $\overline{X} = \hat{X} - X$, and then subtracting T times the first equation in (3.11) from the second leads to:

$$\begin{aligned} \dot{X} &= AX + DF(\hat{X}) \\ \dot{\overline{Z}} &= B\overline{Z} \\ \overline{X} &= K\overline{Z} \end{aligned} \qquad \text{. . . . . (3.12)}$$

If $F(X) = FX$ is linear, the closed loop system using the actual state is $\dot{X} = (A + DF)X$. It has been shown in [21] and in [24] that if an observer with transition matrix B is used to supply an estimate of the state vector, the closed loop poles of the overall system (3.11) are the eigenvalues of $(A + DF)$ and of B. In other words, the observer does not disturb the poles of the original system, but merely adds its own poles.

In similar fashion, it is possible to investigate the effect of an observer in realizing a non-linear control law. Suppose that the closed loop system:

$$\dot{X} = AX + DF(X) \qquad \text{. . . . . (3.13)}$$

is asymptotically stable in the large [16]. It is assumed that the asymptotic stability of (3.13) is established by the construction of a continuously differentiable Liapunov function $V(x)$ for the system which satisfies the following conditions:

1) $V(X) > 0$ for $X \neq \theta$ $V(\theta) = 0$

2) $V(X) \to \infty$ as $||X|| \to \infty$

3) $U(X) \equiv \dot{V}(x) \equiv \nabla_x [AX + DF(x)] < 0$ for $X \neq \theta$

4) $\lim_{||x|| \to \infty} -U(X)/ ||\nabla_X V(X)|| = +\infty$

The first three assumptions are sufficient to guarantee asymptotic stability of (3.13), while the fourth is an additional assumption which is often satisfied in practice. The following theorem shows under relatively mild conditions the observer scheme outlined above leads to an asymptotically stable system.

3.3.1 THEOREM III

Assume that there is available Liapunov function for the system $\dot{X} = AX + DF(X)$ which satisfies the conditions 1) - 4). If $F(X)$ satisfies a uniform Lipschitz condition and the observer is asymptotically stable in the large, i.e., B has its eigenvalues in the left half plane. The complete system (3.12) is asymptotically stable in the large.

Proof: As a first step in the proof, a quadratic Liapunov function is constructed for the observer $\dot{\bar{Z}} = B\bar{Z}$ by the standard procedure for stable, linear, time invariant systems [16], [8]. For this purpose define P as the unique solution to the matrix equation $PB + B^TP = -I$. It is well known that the matrix P so defines is positive definite and that $\bar{Z}^TP\bar{Z} \equiv ||\bar{Z}||^2P$ is a Liapunov function for the system $\dot{\bar{Z}} = B\bar{Z}$ with derivative

$\left(\frac{d}{dt}\right) \|\bar{Z}\|^2 P = -\|\bar{Z}\|^2$. For the overall system (3.12) define $w(X,\bar{z}) = V(X) + \|\bar{Z}\|^2 p$. $W(x,\bar{z})$ is clearly positive definite. Also,

$$\dot{w}(x,\bar{z}) = {}_{X}V(x)\,[AX + DF(\hat{x})] - \|\bar{Z}\|^2$$

$$= U(x) + {}_{X}V(x)\, D\,[F(\hat{x}) - F(x)] - \|\bar{Z}\|^2$$

$$\leq U(x) + C_1 \|\nabla_X V(x)\| \cdot \|\bar{X}\| - \|\bar{Z}\|^2 \quad . . . . (3.14)$$

where the positive constant C_1 is determined by the Lipschitz condition. Using $\bar{X} = K\bar{Z}$ from (3.12), the above inequality can be converted to:

$$\dot{w}(x,\bar{z}) \leq U(x) + C_2 \|{}_{X}V(x)\| \cdot \|\bar{Z}\| - \|\bar{Z}\|^2 \quad . . . . (3.15)$$

Using the function w, it will now be shown that any trajectory of the system (3.12) is bounded. Obviously $\bar{Z}$ is bounded on any trajectory. Condition 4 on the Liapaunov function V implies that for sufficiently larger X and bounded $\bar{Z}$, the function $\dot{w}(X,\bar{z})$ is negative definite. Therefore, since $w(X,\bar{z}) \to \infty$ as $\|X\| \to \infty$, it is impossible that $\|X\|$ increase without bound. Thus, there is an $R > 0$ such taht for all $t > 0$, $\|X(t)\| < R$.

Since $\|\bar{Z}\| \to \infty$ as $t \to \infty$, given $\varepsilon > 0$, there is a finite time T such that for $t > T$, $U(X) + C_2 \|{}_{X}V(X)\| \cdot \|\bar{Z}\| < 0$. Thus for $t > T$ the function $\dot{w}$ is negative definite, and x must tend toward the circle $\|X\| < \varepsilon$. Since ε was arbitrary, X tends to θ. This establishes asymptotic stability in the large.

Chapter 4

STABLE ADAPTIVE OBSERVERS

4.1 Introduction

In this Chapter, minimal and nonminimal realization of the adaptive observer are presented, and their relative advantages in terms of ease of construction, speed of convergence, are discussed. The extension of basic concept of the adaptive observer to multivariable systems, is then treated. In all the above problems, the basic question is one of stability. The error equations of interest in the adaptive observer are linear non-autonomous homogeneous differential equations. The non-autonomous elements are introduced by the input and output of the plant that is to be identified. Here, the stability of these non-autonomous differential equations are presented.

4.2 Identification Problem

Consider an algebraic equation:

$$\alpha^T(t)K = b(t) \qquad \ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

defined for all $t \epsilon T \equiv [0, t_1]$, where $\alpha(t)$ and $b(t)$ are bounded piecewise continuous fractions. $\alpha(.)$ and $b(.)$ take values in R^n and R, and K

is a fixed unknown vector in R^n.

The aim of identification procedure is to determine the constant vector K from observed values of $\alpha(t)$ and b(t) in the interval [0, t₁]. To solve this problem, we can use one of two approaches:

a) Divide the interval t into n distant of time, and measure (t) and b(t), and solve a set of n linearly independent equations for k.

b) An alternative approach is to obtain an estimate K(t) of K at every instant of time t by solving the non-homogeneous differential equation:

$$\dot{\hat{K}} = -[\alpha(t)\alpha^T]\hat{K} + \alpha(t)b(t) \qquad \ldots\ldots\ldots\ldots\ldots (4.2)$$

Throughout this section, we are going to use the second approach.

4.3 IDENTIFICATION OF MULTIVARIABLE SYSTEMS (All States Accessible).

Before discussing the adaptive observer, in the next sections, we shall consider the identification problem of an unknown plant, all its state variables are accessible for measurement. The identification procedure described in this section, gives an idea about the scheme which shall be used later in adaptive observer.

Consider the unknown plant of linear time-variant differential equation:

$$\dot{X}_p(t) = A_p X_p(t) + B_p U(t) \qquad \ldots\ldots\ldots\ldots\ldots (4.3)$$

where A_p and B_p are (nxn), and (nxr) constant matrices with unknown elements:

$$X_p(\cdot) \text{ and } U(\cdot) \text{ are in } R^n \text{ and } R^r.$$

The elements of $U(t)$ are bounded and piecewise continuous. The aim of the identification procedure is to determine the elements of A_p and B_p which are the parameters of the plant.

A model of the plant is set up which is:

$$\dot{X}_m = KX_m + [A_m(t) - K]\, X_p + B_m(t)U(t) \qquad \ldots\ldots\ldots\ldots (4.4)$$

where K is stable matrix, and $A_m(t)$ and $B_m(t)$ are matrices of adjustable parameters. It is desired to determine adaptive laws for adjusting the time derivatives $\dot{A}_m(t)$ and $\dot{B}_m(t)$ so that:

$$\lim_{t \to \infty} A_m(t) = A_p$$

$$\lim_{t \to \infty} B_m(t) = B_p$$

$$\lim_{t \to \infty} [X_m(t) - X_p(t)] = \lim_{t \to \infty} e(t) = 0$$

The state error equation may be written as:

$$\dot{e}(t) = Ke(t) + \phi(t)Xp(t) + \psi(y)U(t) \qquad \ldots\ldots\ldots\ldots (4.5)$$

where the parameter error matrices are defined by:

$$\phi(t) \triangleq [A_m(t) = A_p]$$

$$\psi(t) \triangleq [B_m(t) - B_p]$$

It has been shown, using [31], [13], using Liapunov function candidate:

$$V(e, \phi, \psi) = \frac{1}{2}\,[e^T Pe + \text{tr}\,\{\phi^T \Gamma_1 \phi + \psi^T \Gamma_2 \psi\}]$$

where P, Γ_1 and Γ_2 are symmetric positive definite matrices, and P satisfies the matrix equation:

$$K^T P + PK = -Q \qquad Q = Q^T > 0$$

and the time derivative $\dot{V}$ of V along a trajectory can be made negative semi-definite by choosing the adaptive laws:

$$\phi(t) = \dot{A}_m(t) = -\Gamma_1^{-1}\, Pe(t)\, X_p^T(t) \qquad (4.6)$$

$$\psi(t) = \dot{B}_m(t) = -\Gamma_2^{-1}\, Pe(t) U^T(t) \qquad (4.7)$$

Using (7.6) and (7.7) it can be shown that:

$\dot{V} = -e^T Qe < 0$, so that stability of the origin in (e, ϕ, ψ), and boundedness of solutions are assured.

4.4 Error Models in Identification

a) Model 1 - Fig. 4.1

Equation (4.5) represents the error models that arise naturally in identification and control problems, using the methods described here. The error between the plant and model states satisfies a differential equation of the form:

$$\dot{e} = Ke + \theta^T(t)V(t) \qquad (4.8)$$

where K is stable matrix

$\theta^T(t)$ is (nxs) matrix of parameter errors, and

$V(t)$ is an s vector.

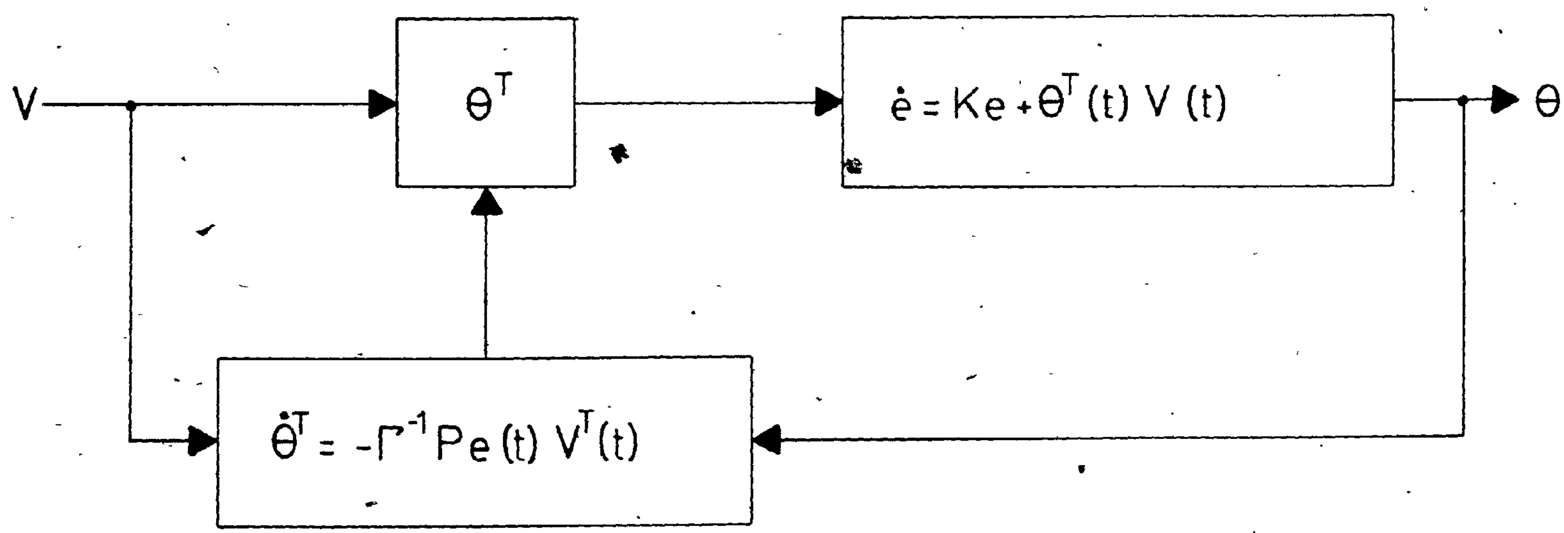


Fig. 4.1 — Gain Matrix Followed by Stable System. "All State Variables Accessible".

Comparing equations (4.5) and (4.8) we get:

$$B^T = [\phi(t) \mid \psi(t)]$$

$$V^T(t) = [X_p(t)^T \mid U^T(t)]$$

From (4.8) $\theta^T(t)$ is unknown, but $\dot{\theta}(t)$ can be adjusted, and it is desired to determine adaptive laws so that $\lim_{t \to \infty} e(t) = 0$, and $\lim_{t \to \infty} \theta(t) = 0$.

An adaptive law which can do that is of the form:

$$\dot{\theta}^T = -\Gamma^{-1} Pe(t)V^T(t) \quad \ldots\ldots\ldots\ldots (4.9)$$

where $\Gamma = \Gamma^T > 0$; $K^T p + PK = -Q$, $Q = Q^T > 0$.

b) Model 2 - Fig. 4.2

In case when only some of the outputs of the plant are accessible, here, the error between plant and model outputs can be described by:

$$\dot{\varepsilon} = K\varepsilon + D\theta^T V, \quad \eta = H\varepsilon \quad \ldots\ldots\ldots\ldots (4.10)$$

The adaptive law for updating the (m x s) matrix $(t)^T$ in this case is found to be:

$$\dot{\theta}^T = -\Gamma^{-1} \eta V^T \qquad \Gamma = \Gamma^T > 0 \quad \ldots\ldots\ldots (4.11)$$

Again, as in the previous case, $e(t)$ and $\theta(t)$ tend to zero as $t \to \infty$, if the input $V(t)$ is sufficiently rich.

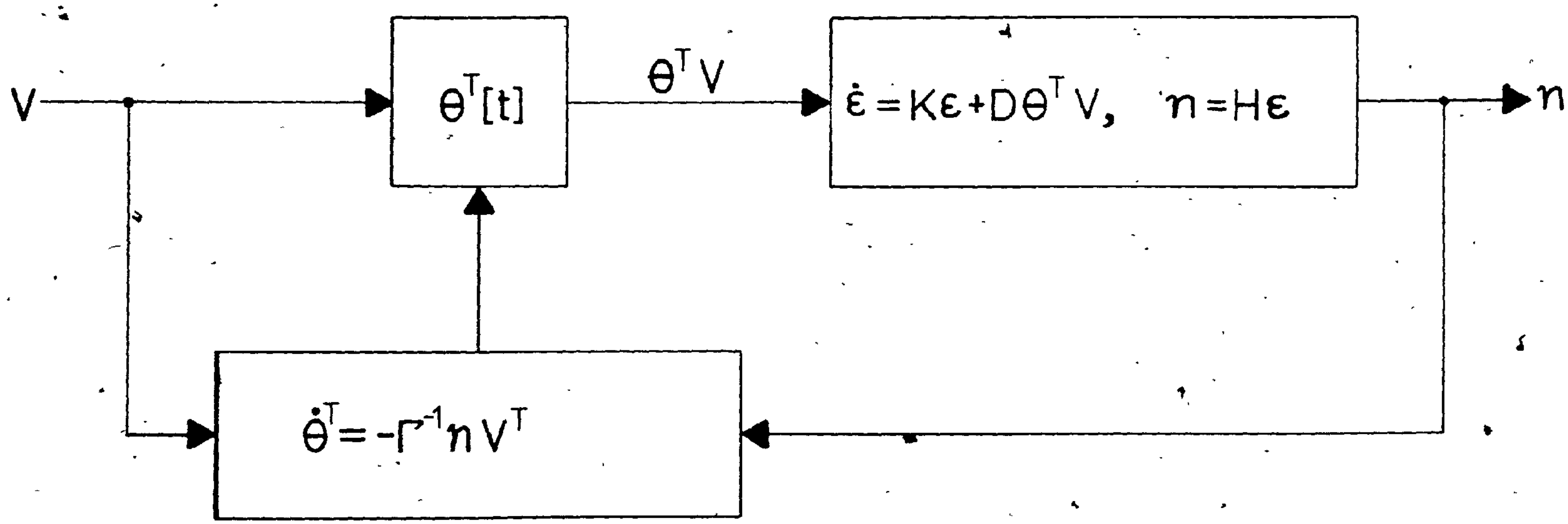


Fig. 4.2 — Identification of Parameters From Input-Output Data "Multivariable Case".

4.5 The Adaptive Observer

The main disadvantages of the above models that it depends on the accessibility of all the state variables of the plant to be identified. But in most practical situations, only some state variables of the plant can be measured. This in turn, leads to the concepts of the adaptive observer in which both the parameters and the state variables of the plant are estimated simultaneously.

During 1973-1974, several seemingly different versions of the adaptive observer appeared in the central literature [13], [5]. Carroll and Lindorff were the first to provide such an observer. Lüders and Narendra suggested an alternative observer [20], and later modified their model to have a simpler structure. The paper by Kudva and Narendra [14] proposed another model and in [30], Narendra and Kudva showed that all these results could be derived in a unified manner. In 4.5.1, an outline of the philosophy of these approaches and indications of the structure of the adaptive observer is given:

4.5.1 Minimal Realization [5], [14]

A single input single output plant is assumed to be described by the differential equation:

$$\dot{x} = [-a \mid \bar{A}]X + bu \qquad y = X_1 = h^T X \quad . \; . \; . \; . \; . \; . \; . \; (4.12)$$

where the constant column vectors a, b represent the unknown parameters of the plant. The $n \times (n-1)$ matrix $\bar{A}$ is known, and $h^T = [1, 0, 0, . . . 0]$. It is desired to construct an observer which could estimate the parameter vectors a and b, and the state X of the plant simultaneously.

The plant equations may be rewritten as:

$$\dot{x} = [-K|\bar{A}]X + [K-a]X_1 + bu$$

$$y = h^T X \qquad \text{.............. (4.13)}$$

In (4.13), the unknown parameters X, a, b and unknown signals, X_1, u are associated together. A convenient structure for the adaptive observer then is:

$$\dot{\hat{X}}(t) = K\hat{X} + (K - \hat{a}(t)X_1(t) + \hat{b}(t)U(t) + w^{(1)}(t) + w^{(2)}(t)$$

$$\text{.............. (4.14)}$$

where $\hat{a}(t)$ and $\hat{b}(t)$ are the estimates of the parameter vectors a and b, $\hat{X}(t)$ is the estimate of $X(t)$, and $w^{(1)}(t)$ and $w^{(2)}(t)$ are auxiliary n-dimensional input signals which are required to stabilize the adaptive observer.

If $e(t) \triangleq \hat{x}(t) - x(t)$

$\phi(t) \triangleq a - \hat{a}(t)$

$\psi(t) \triangleq \hat{b}(t) - b$,

the error equation may be written as:

$$\dot{e} = Ke + \theta Z + w \qquad e_1 = h^T e \qquad \text{.............. (4.15)}$$

where $\theta = [\phi \mid \psi](Z^T = [X_1, u], \& w = w^{(1)} + w^{(2)}$ is n vector. The objective in this case is to determine the signals $w^{(1)}(t)$ and $w^{(2)}(t)$, and the updating laws for $\dot{\theta}(t)$ which will make $\lim_{t \to \infty} e(t)$, $\psi(t)$ and (t) $\to 0$. From Model 2 above, we know that if:

$$\dot{\varepsilon} = K\varepsilon + d\theta^T V \qquad \varepsilon_1 = h^T \varepsilon \qquad \text{.............. (4.16)}$$

The adaptive law is $\dot{\theta} = -\varepsilon_1 v$ . . . . (4.17)

To find the law for updating $\dot{\theta}$ in (4.15), the following proposition is used: Given a 2 vector Z(t) whose elements are bounded functions of time, there exists vectors v(t) and w(t) with

$v(t) = G(pPZ(t) \quad P = \frac{d}{dt}$ and $w = w(\dot{\theta}, V)$ such that:

$$\dot{e} = Ke + \theta Z + w \qquad e_1 = h^T e$$

$$\dot{\varepsilon} = K + d\theta^T V \qquad \varepsilon_1 = h^T \varepsilon_1$$

have the same outputs (i.e., $e_1(t)$, provided the pair (h^T, K) is completely observable. For a proof of this proposition, refer to [30]. Using the above proposition, an adaptive law:

$$\dot{\theta} = -e_1 \dot{v} \quad \text{where} \quad V = \begin{bmatrix} v\ (1) \\ v\ (2) \end{bmatrix} \quad \ldots\ldots\ldots \quad (4.18)$$

which is the same as that used in (4.17). The schematic diagram for this adaptive observer is shown in Fig. 4.3. As shown in Fig. 4.3 , X_1 (t) and u(t) are used as inputs into identical $(n-1)^{th}$ order systems whose outputs are $V^{(1)}(t)$ and $V^{(2)}(t)$. The signals $w^i(t)$, (i = 1, 2), which are functions only of $V^{(i)}(t)$ and $\dot{\theta}(t)$ are added to the input of the observer.

4.5.2 The Adaptive Observer: (Non Minimal Realization) [32]

The presence of $w^{(1)}(t)$ and $w^{(2)}(t)$ makes the practical realization of the observer difficult. Another approach to eliminate this difficulty is by using transfer function rather than a state variable description of the plant. The transfer function of the single input-single output

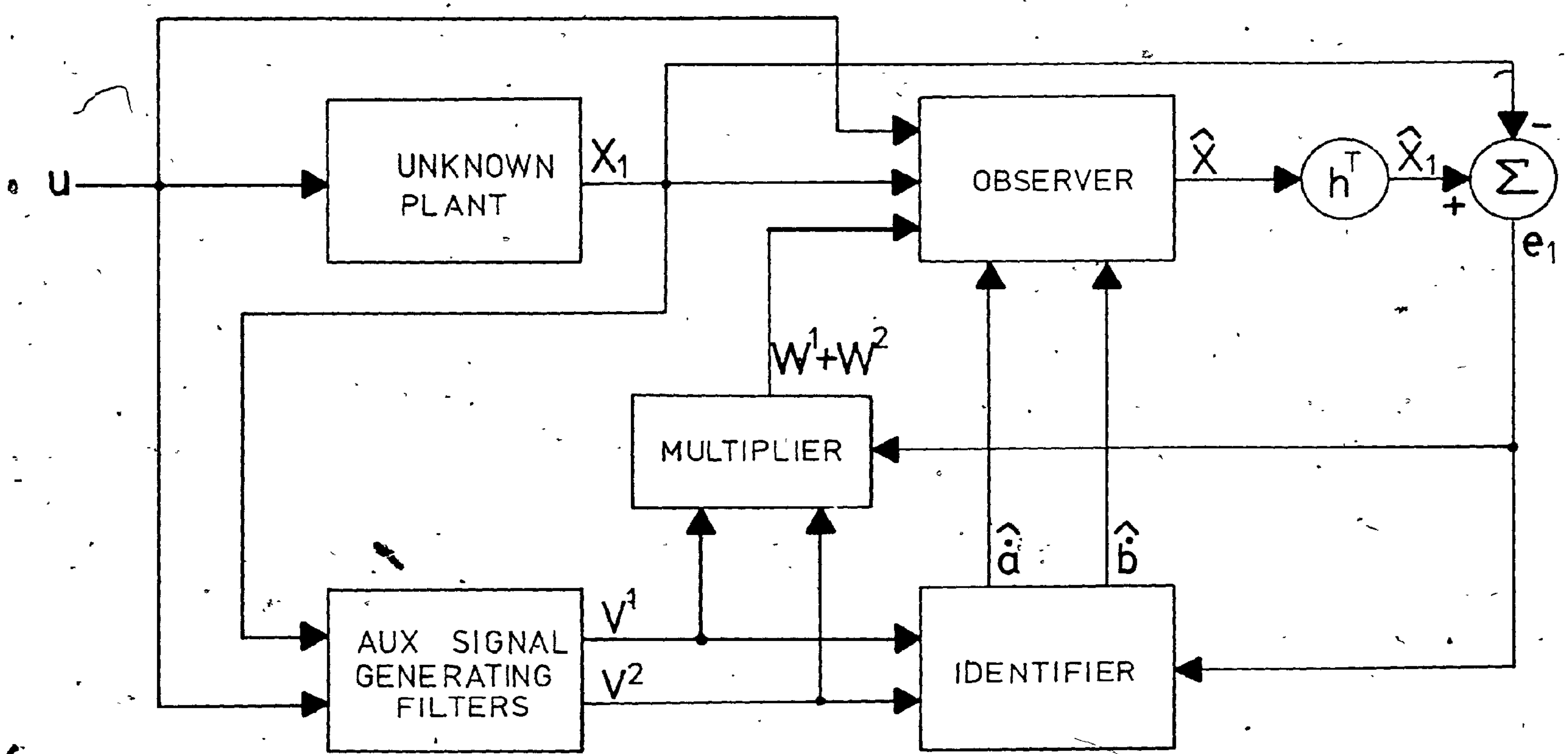


Fig. 4.3 — The Adaptive Observer.

plant is of the form:

$$T_p(s) = \frac{Q_p(s)}{R_p(s)}$$

where $Q_p(s)$, and $R_p(s)$ are respectively $(n-1)^{th}$ and n^{th} order polynomials in s. $T_p(s)$ can take the form:

$$T_p(s) = \frac{Q_p(s)}{R(s)} \left[\frac{\frac{1}{s+\lambda_0}}{1 + \frac{R_p(s)}{R(s)} \cdot \frac{1}{s+\lambda_0}} \right]$$

where $R(s)$ is a Hurwitz polynomial of degree $(n-1)$, and $R(s)(s+\lambda_0) + P_p(s) = R_p(s)$. Thus by adjusting the coefficients of the polynomials $Q(s)$ and $P(s)$ of a model shown in Fig. 4.4, any n^{th} order transfer function $T_p(s)$ can be realized. This is the model used for the identification of the plant in this approach.

The principal contribution of [32] is the realization that the plant output rather than the model output should be used in the feedback path of the model to simplify the determination of the model parameters. The identification of a single input-single output plant takes the form shown in Fig. 4.5 where: b_i and a_i are adjusted according to the law:

$$\begin{aligned} \hat{a}_i &= -e_1 V_i^{(1)} \\ & \qquad\qquad i = 1, 2, \ldots\ldots n \\ \hat{b}_i &= -e_1 V_i^{(2)} \end{aligned} \qquad \ldots\ldots\ldots (4.19)$$

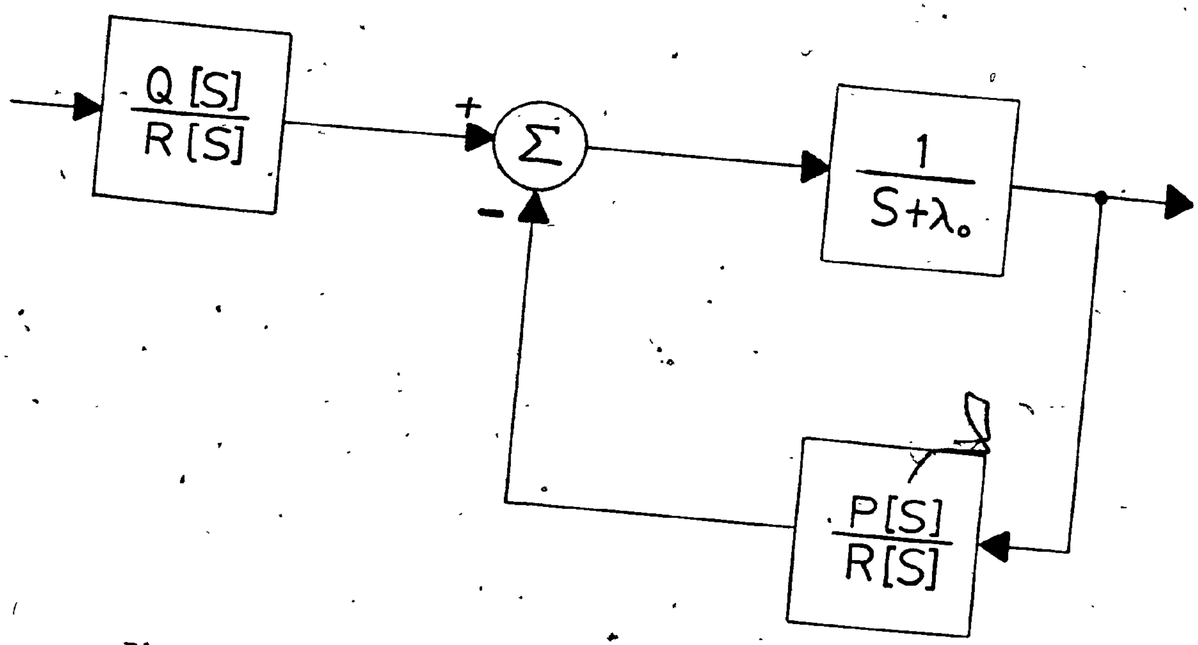


Fig. 4.4 — Non-Minimal Realization of A Single Input-Output Plant.

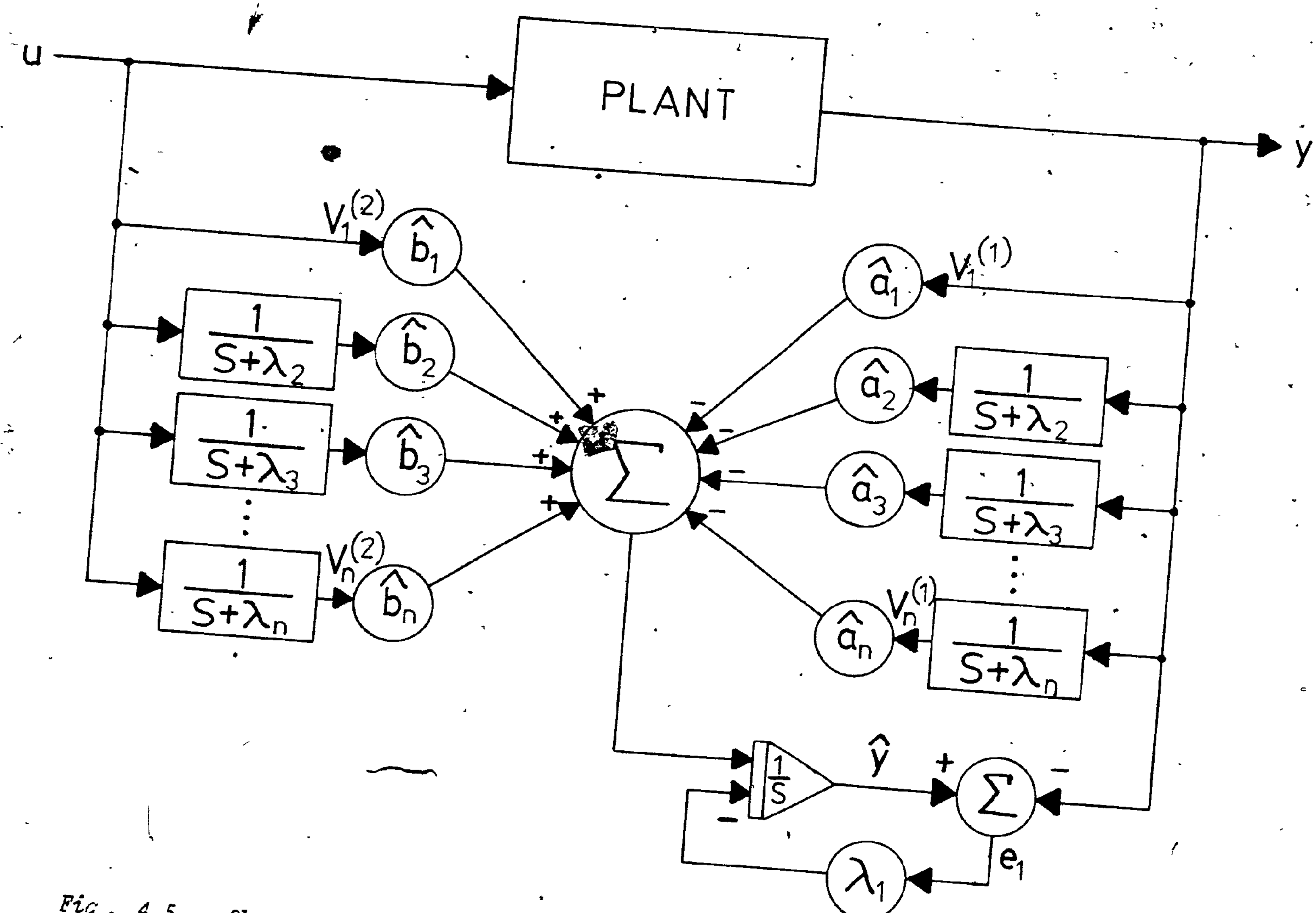


Fig. 4.5 — Observer Structure For Case 4.5.2.

4.6 Stability Properties of the Adaptive Observer

In the cases discussed above, the state and parameter errors (between plant and model) are seen to be described by a homogeneous non-autonomus linear differential equation. For example, consider the equations (4.5), (4.6), (4.7) when K, Γ_1, Γ_2 and P are scalars, we have:

$$\begin{bmatrix} \dot{e} \\ \dot{\phi} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} -K & X_p(t) & u(t) \\ -\gamma_1 X_p(t) & 0 & 0 \\ -\gamma_2 u(t) & 0 & 0 \end{bmatrix} \begin{bmatrix} e \\ \phi \\ \psi \end{bmatrix} \qquad \text{. . . . . (4.20)}$$

or $\dot{E\varepsilon} = A(t)E$

where $E^T = (e, \phi, \psi)$.

The matrix A(t) is time varying since its elements contain the input u(t) and the plant output $X_p(t)$. To study the behaviour of the equation (4.20) it is necessary to assume that the matrix A(t) is bounded and this accounts for the fact that all observer schemes developed so far can be applied only to stable plants. In almost all the schemes discussed in this Chapter, the Liapunov function candidate V is positive definite, and radially unbounded, but $\dot{V}$ (its time derivative) is negative semidefinite. This assures the boundedness of the solutions of the equation. LaSalle's theorem for a symptotic stability [17] applied to autonomous systems, and systems which can be described by equations of the form:

$$\dot{X} = f(X,t), \quad \text{where} \quad f(X,t+T) = f(X,t) \ \text{for same constant T.}$$

A great deal of effort [5], [30], [10], was spent in considering periodic inputs into the system so that LaSalle's theorem could be applied to prove asymptotic stability, or convergence of the parameters of the system to their true values. In all cases, the result is expressed as a richness condition on the input, which must contain a sufficient number of distinct frequencies (i.e., $U(t) = C \sin(w_i t + b_i)$. Morgan and Narendra discussed in detail two classes of systems [25], [26], which are described by equations of the form:

$$\text{(i)} \quad \dot{X} = -B(t)\, B^T(t)\, X,$$

$$\text{(ii)} \quad \begin{bmatrix} \dot{X} \\ \\ \dot{y} \end{bmatrix} = \begin{bmatrix} A & -B^T(t) \\ \\ B(t) & 0 \end{bmatrix} \begin{bmatrix} X \\ \\ y \end{bmatrix} \quad \ldots\ldots\ldots \quad (4.21)$$

These equations include most of the adaptive observers discussed so far. The necessary and sufficient conditions for uniform and non-uniform asymptotic stability are given in terms of richness condition on the matrix $B(t)$. For the first equation in (4.21), the condition is expressed as a linear growth of an integral:

$$\int_0^t \| B(\tau)B^T(\tau)\, w \|\, d\tau \geqslant a(t-t_0) + b$$

where w is any unit vector, a and b are constants $a > 0$. For the uniform asymptotic stability of the second equation in (4.21) positive nos. t_0, ε_0 and δ_0 exist such that given $t_1 \geqslant 0$ and any unit vector

$w \varepsilon R^n$, there is a $t_2 \varepsilon [t_1, t_1 + t_0]$ such that:

$$\left| \int_{t_2}^{t_2 + \delta_0} B(\tau)^T w d\tau \right| \geqslant \varepsilon_0$$

or for any fixed unit vector w, $B^T(t)$ w is periodically large and maintains the same sign for a fixed interval of time δ_0.

CHAPTER 5

AN ADAPTIVE OBSERVER AND IDENTIFIER FOR A LINEAR SYSTEM

5.1 INTRODUCTION

The design of model reference adaptive system using Liapunov's direct method has the important advantage over other adaptive schems in that the global stability of these systems is automatically guaranteed [19]. However, the main disadvantage of adaptive designs using Liapunov's method as compared to other methods has been the fact that they require the complete state of the controlled system for their implementation. In most cases, only the output of the systems can be measured. The solution to the above problem is presented in terms of a new canonical state representation, and permits easier implementation.

5.2 PROBLEM STATEMENT

Consider the following system (single input-single output),

$$\dot{X} = AX + bU(t) \qquad \ldots\ldots\ldots\ldots\ (5.1a)$$

$$y = h^T X \qquad \ldots\ldots\ldots\ldots\ (5.1b)$$

X = n^{th} order state vector

$U(t)$ = single input

y = single output

All that is known about the system is:

a) $U(t)$ c) n

b) y d) time invariant

It is required to design a system which would do the following:

1) estimate state vector X

2) identify the parameters A, h, b.

In the problem stated above, since one is only interested in the input output characteristics of (5.1), there is considerable freedom in choosing the internal state representation X or equivalently, in the choice of h, A, b. If the system (5.1) is completely observable, then it can always be represented in the following form:

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ \dot{X}_n\end{bmatrix} = \left[\begin{array}{c|ccc} a_1 & 1 & 1 \; . \; . \; . & 1\\ \hline a_2 & & & \\ \cdot & & \Lambda & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ a_n & & & \end{array}\right] \begin{bmatrix}X_1\\ X_2\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ x_n\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ b_n\end{bmatrix} U \quad \text{. . . . . (5.2a)}$$

$$y = (1 \quad 0 \; . \; . \; . \; . \; 0)X = X1 \quad \text{. . . . . (5.2b)}$$

where is an $(n-1)x(n-1)$ diagonal matrix with arbitrary but known constant and negative diagonal elements $-\lambda_i (i = 2, \ldots n)$ and $a = (a_1, a_2, \ldots a_n)^T$, and $b = (b_1, b_2, \ldots b_n)^T$ are the 2n unknown parameters to be identified. The reason for the form shown in (5.2) is to make the right hand side of (5.2) have the special form that all terms are products of two quantities, one known, and the other unknown.

5.3 THE ADAPTIVE OBSERVER AND IDENTIFIER

Now consider a model whose form is similar to (5.2), and whose parameters $\alpha = (\alpha_1, \alpha_2, \ldots \alpha_n)^T$ and $\beta = (\beta_1, \beta_2, \ldots \beta_n)^T$ will be adjusted adaptively in order to match those of (5.2) as $t \to \infty$.

$$\begin{bmatrix} \dot{\hat{X}}_1 \\ \dot{\hat{X}}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{\hat{X}}_n \end{bmatrix} = \left[\begin{array}{c|c} \alpha_1 & 1 \quad 1 \ldots 1 \\ \hline \alpha_2 & \\ \cdot & \Lambda \\ & \\ \alpha_n & \end{array}\right] \begin{bmatrix} X_1 \\ \hat{X}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \hat{X}_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \beta_n \end{bmatrix} U \begin{matrix} -\lambda_1(\hat{X}_1 - X) \\ w_2 \\ \\ \\ \\ \\ + \quad w_n \end{matrix} \qquad (5.3)$$

where $w = (w_2, w_3, \ldots w_n)^T$ are signals added to assure the stability of the overall adaptive scheme and will be defined later. Subtracting (5.2) from (5.3) will give the state error $e = \hat{X} - X$:

$$
\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{e}_n \end{bmatrix}
=
\left[\begin{array}{c|c} -\lambda_1 & 1 \quad 1 \; \cdots \; 1 \\ \hline 0 & \\ \cdot & \\ \cdot & \Lambda \\ \cdot & \\ \cdot & \\ 0 & \end{array}\right]
\begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e_n \end{bmatrix}
+
\begin{bmatrix} \phi_1 \\ \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \phi \end{bmatrix} X
+
\begin{bmatrix} \psi_1 \\ \psi_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \psi_n \end{bmatrix} U
+
\begin{bmatrix} \\ \\ \\ W \\ \\ \\ \\ \end{bmatrix}
\qquad \ldots\ldots\ldots\ldots \quad (5.4)
$$

where $\phi_i = \alpha_i - a_i$

$$i = 1, \ldots n$$

$\psi_i = \beta_i - b_i$

are the parameters errors between the system (5.2) and the model (5.3).

It is now required to find adaptive updating equations for the parameter vectors α and β such that $t \rightarrow \infty$, $\alpha \rightarrow a$, $\beta \rightarrow b$, and $e = \hat{X} - X \longrightarrow 0$. Since these adaptive equations as well as the model equation will have to be implemented, it is important that they are functions of signals which are available or can be generated from known signals. However $(e_2, \ldots e_n)$ are unavailable because $(X_2, \ldots X_n)$ are not measureable, and therefore it is necessary to eliminate them from (5.4). It can be shown that if w has a special form, the equations of $\dot{e} = (e_2, \ldots e_n)^T$ can be integrated analytically, i.e., let

$$w = \{[PI - \Lambda]^{-1} \, \dot{\bar{\phi}} + \{[PI - \Lambda]^{-1} u\} \, \dot{\bar{\psi}} \qquad \ldots\ldots \quad (5.5)$$

where $\bar{\phi} = (\phi_2, \ldots \phi_n)^T$

$\bar{\psi} = (\psi_2, \ldots \psi_n)^T$

$p = \frac{d}{dt}$

If we define the auxiliary signals v_i and s_i ($i = 2, \ldots n$) as generated by first-order linear filters:

$$\dot{v}_i + \lambda_i v_i = X_1 \qquad i = 2, \ldots n \qquad \ldots \ldots (5.6a)$$

$$\dot{s}_i + \lambda_i s_i = u \qquad i = 2, \ldots n \qquad \ldots \ldots (5.6b)$$

then $\dot{w} = (w_2, \ldots w_n)^T$ can be rewritten in a more simple form:

$$w_i = -(C_i v_i^2 + d_i s_i^2)e \qquad i = 2, \ldots n \qquad \ldots \ldots (5.7)$$

To obtain $\bar{e}$, we substitute (6.5) into (6.4)

$$\dot{\bar{e}} = \bar{e} + \bar{\phi}X_1 + \{[PI - \mathbf{\Lambda}]^{-1} X_1\} \dot{\bar{\phi}} + \bar{\psi}U + \{[PI - \mathbf{\Lambda}]^{-1}U\} \dot{\bar{\psi}} \qquad \ldots \ldots (5.8)$$

and integrated:

$$\bar{e} = \{[PI - \mathbf{\Lambda}]^{-1} x_1\} \bar{\phi} + \{[PI - \mathbf{\Lambda}]^{-1}u\} \bar{\psi} + \exp[\ t]\ \bar{e}\ (t_0) \qquad \ldots \ldots (5.9)$$

Finally, substituting $\bar{e}$ into the first equation of (5.4) yields

$$\dot{e}_1 = \lambda_1 e + X_1\phi_1 + \{h_1^T[PI - \mathbf{\Lambda}]^{-1}X_1\} \bar{\phi} + U\psi_1$$
$$+ \{h_1^T[PI - \mathbf{\Lambda}]^{-1} u\} \bar{\psi} + h_1^T \exp[\mathbf{\Lambda}t]\bar{e}(t_0) \qquad \ldots \ldots (5.10)$$

where $h_1^T = (1, 1, \ldots 1)$ is an $(n-1)$ vector, and $\bar{e}(t_0)$ is the initial error $\bar{e}$ at time t_0 when the adaptation was started.

5.4 Adaptive Equations and Proof of Stability

It is now straightforward to devise an adaptive scheme which would guarantee global asymptotical stability in the 3n-dimensional error space (e^T, ϕ^T, ψ^T). Define a Liapunov function candidate:

$$V = \frac{1}{2} e_1^2 + \frac{1}{2} \sum_{i=1}^{n} \left[\frac{1}{c_i} \phi_i^2 + \frac{1}{d_i} \psi_i^2 \right] \qquad \ldots\ldots\ldots\ldots (5.11)$$

$$C_i, d_i > 0, \quad i = 1, \ldots n$$

Then, if we choose:

$$\dot{\phi}_1 = \dot{\alpha}_1 = -C_1 e_1 X_1 \qquad \dot{\bar{\phi}} = -e_1[PI - \Lambda]^{-1} \bar{C} X1$$

$$\dot{\psi}_1 = \dot{\beta}_1 = -d_1 e_1 U \qquad \dot{\bar{\psi}} = -e_1[PI - \Lambda]^{-1} \bar{d}\, u$$

$$\bar{C} = (C, \ldots C_n)^T \qquad \bar{d} = (d_2, \ldots d_n)^T$$

$$\ldots\ldots\ldots\ldots (5.12)$$

then:

$$\dot{V} = -\lambda_1 e_1^2 + e_1 h_1^T \exp[\Lambda t]\bar{e}(t_0) \qquad \ldots\ldots\ldots\ldots (5.13)$$

let $f(t) = h_1^T \exp[\Lambda t]\bar{e}(t_0)$.

Then from (5.11) and (5.12) we have:

$$\dot{V} = -\lambda_1 e_1^2 + e_1 f(t) \leqslant e_1 f(t) \leqslant \sqrt{V}\, |f(t)| \qquad \ldots\ldots\ldots (5.14)$$

and since $f(t) \longrightarrow 0$ exponentially fast as $t \longrightarrow \infty$, it is easily shown that also $\dot{U} \longrightarrow 0$ which together with (5.13) implies $e_1 \longrightarrow o$ as $t \longrightarrow \infty$.

Finally using (5.10) it can be shown that:

$$e_1 \longrightarrow 0, \quad \phi = 0 \quad \psi = 0 \quad \ldots\ldots\ldots\ldots (5.15)$$

if U contains at least n distinct frequencies (real or complex).

Hence, we have the final result that if the system (5.1) is completely observable, and if U is sufficiently general input, then the adaptive observer described by (5.3), (5.5) and (5.12) will asymptotically yield the state X and identify all the parameters a_i and b_i ($i = 1, \ldots n$) of the system (5.1), i.e.

$$\lim_{t \to \infty} a_i = \alpha_i$$

$$\lim_{t \to \infty} b_i = \beta_i$$

$$\lim_{t \to \infty} \hat{X}_i = X_i \quad \ldots\ldots\ldots\ldots (5.16)$$

5.5 PRACTICAL IMPLEMENTATION

The practical implementation of the adaptive observer and identifier defined by (5.3), (5.5), and (5.12), involves the generation of signals to update the parameters α_i and β_i ($i = 1, \ldots n$), as well as those required to construct the signals w_i ($i = 2, \ldots n$) fed back into the observer. To illustrate these aspects and particulars of the adaptive scheme, consider Fig. 5.1, which is the block diagram representation for an n^{th} order system, and Fig. 5.2, which shows the details corresponding to a second-order system. From (5.12) it is seen that

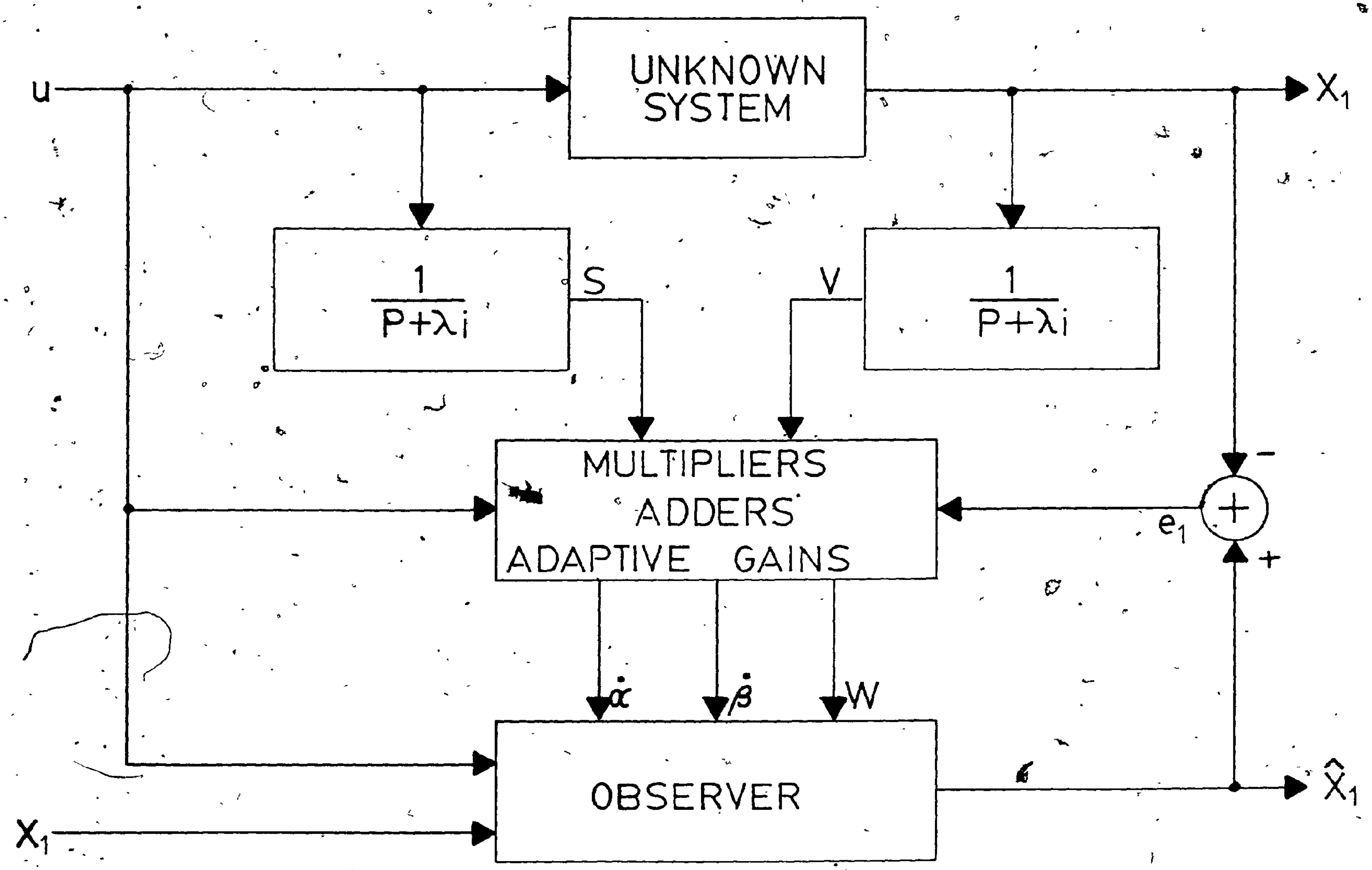


Fig. 5.1 — Block Diagram for an Adaptive Observer and Identification Scheme.

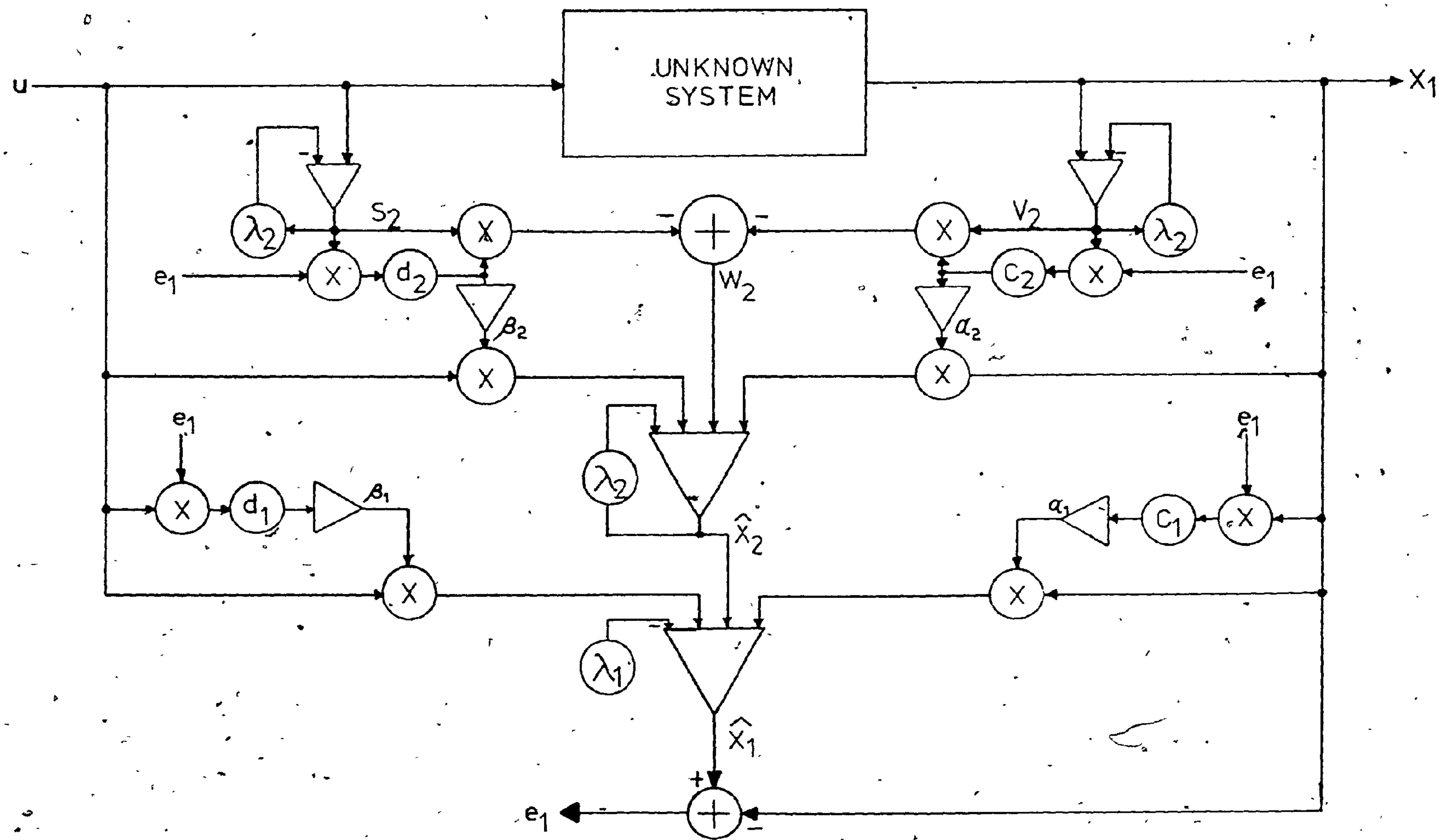


Fig. 5.2 — Implementation of a Second-Order Observer and Identification Scheme.

$2(n-1)$ signals, v_i and $\mathring{s}_i$ $(i = 2 \ldots n)$ are required to generate $\dot{\bar{\phi}}$ and $\dot{\bar{\psi}}$, these are the elements of the diagonal matrices $[PI - \Lambda]^{-1}$ X1, and $[Pi - \Lambda]^{-1}$ U, respectively.

Further, from (5.5) it is observed that the same signals are needed to obtain the feedback vector w. Hence, for the implementation of the entire adaptive process these are the only auxiliary signals to be generated which together with $4(n-1)$ multiplications, represent the heart of the adaptive procedure.

Chapter 6

ADAPTIVE OBSERVERS WITH EXPONENTIAL RATE OF CONVERGENCE

6.1 Introduction

A Luenberger observer as shown in the previous chapters, allows asymptotic reconstruction of the state of a linear system from measurements of its input and output, provided that the system parameters are known.

For the case where no a priori knowledge of the system parameters is available, the so called adaptive observer is used. [The access to the adaptive observer problem taken here is different to [29], [4], not to continuously adopt the parameters in a Luenberger observer]. Instead, a parametrized observer is used, which is an equivalent but structurally different representation of a Luenberger observer, and the parameters are continuously adapted. Thereby, the observation process is well separated from the adaptation process, and suitable adaptation schemes can be developed in a general fashion.

Three such schemes are presented, which are proven to be exponentially rather than asymptotically convergent.

6.2 OBSERVABLE CANONICAL FORM REALIZATION OF $g(s) = \frac{N(s)}{D(s)}$

Consider n^{th} order differential equation $D(P)y(t) = N(P)U(t)$

$$P^i = \frac{d^i}{dt^i} \quad \ldots\ldots\ldots\ldots \text{(I)}$$

where $D(P) = P^n + \alpha_1 P^{n-1} + \ldots\ldots \alpha_n$, $N(P) = \beta_1 P^{n-1} + \beta_2 P^{n-2} + \ldots P_n$

If we have n initial conditions, $y(t_0)$, $y^{(1)}(t_0)$, ... $y^{(n-1)}(t_0)$ then for any input $U(t_0,t]$, the output $y[t_0,t]$ is completely determinable. If we choose $y(t)$, $y^{(1)}(t)$, . . . $y^{(n-1)}(t)$ as state variables, then we cannot obtain a dynamical equation of the form $\dot{X} = AX + bu$, $y = CX$. Instead we will obtain an equation of the form:

$$\dot{X} = AX + bu$$

$$y = CX + d\,u + d\,U^{(1)} + d\,U^{(2)} + \ldots\ldots$$

Taking the Laplace transformation of equation A, and grouping the terms associated with the same power of S, we will obtain:

$$y(s) = \frac{N(s)}{D(s)}u(s) + \frac{1}{D(s)}\Big\{y(0)\,s^{n-1} + [y^{(1)}(0) + \alpha_1 y(0) - \beta_1 u(0)]s^{n-2}$$

$$+ \ldots [y^{(n-1)}(0) + \alpha_1 y^{(n-2)}(0) - \beta_1 u^{(n-2)}(0) + \alpha_2 y^{(n-3)}(0)$$

$$- \beta_2 u^{(n-3)}(0) + \ldots + \alpha_{n-1}\,y(0) - \beta_{n-1}\,u(0)]\Big\} \ldots\ldots \text{(II)}$$

The right hand side of equation 6.2 gives the response due to the input $u(s)$; the remainder gives the response due to the initial conditions. Thus, if all coefficients associated with s^{n-1}, s^{n-2}, . . . s^0 are known, then for any u, a unique y can be obtained.

Now if we consider the state variables as:

$$X_n(t) = y(t)$$

$$X_{n-1}(t) = y^{(1)}(t) + \alpha_1 y(t) - \beta_1 u(t)$$

$$X_{n-2}(t) = y^{(2)}(t) + \alpha_1 y(t) - \beta_1 u^{(1)}(t) + \alpha_2 y(t) - \beta_2 u(t)$$

$$\vdots$$

$$X\ (t) = y^{(n-1)}(t) + \alpha_1 y^{(n-2)}(t) - \beta_1 u^{(n-2)}(t) + \ldots$$

$$+ \alpha_{n-1} y(t) - \beta_{n-1} u(t) \qquad \ldots\ldots\ldots\ldots \text{(III)}$$

The equation III can be rewritten as:

$$y = X_n$$

$$X_{n-1} = \dot{X}_n + \alpha_1 X_n - \beta_1 u$$

$$X_{n-2} = \dot{X}_{n-1} + \alpha_2 X_n - \beta_2 u$$

$$\vdots$$

$$X_1 = \dot{X}_2 + \alpha_{n-1} X_n - \beta_{n-1} u$$

Differentiating X in equation 6.3, once, and using equations 6.1, we get:

$$\dot{X} = -\ {}_n X_n + {}_n u$$

The above equations can be arragned in matrix form as follows:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \vdots \\ \dot{X}_n \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & -\alpha_n \\ 1 & 0 & 0 & \dots & 0 & -\alpha_{n-1} \\ 0 & 1 & 0 & \dots & 0 & -\alpha_{n-2} \\ & & & & & \\ 0 & 0 & 0 & \dots & 1 & -1 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} \beta_n \\ \beta_{n-1} \\ \\ \\ \beta_1 \end{bmatrix}$$

$$y = [\,0 \quad 0 \quad 0 \quad 0 \quad -1\,]X \qquad \dots\dots\dots\dots\dots \quad (IV)$$

The equation IV is known as observable canonical form.

6.3 Problem Statement

Consider a dynamic system which has one input $u(t)$ and one output $y(t)$. All that is known about this system is that it is time invariant, completely controllable and observable linear system of the form:

$$\dot{X}(t) = AX(t) + bU(t) \qquad X(o) = X_o \qquad \dots\dots \quad (6.1)$$

$$y(t) = C^T X(t)$$

where $X(t)$ is a static vector of known dimension n. No information, however is available about the parameters of the system, i.e. the elements of the matrices A, b, and C.

Since no further information about the system is available, there is no loss of generality in choosing a particular state representation for it, where the system matrices have the form:

$$A = \begin{bmatrix} -a_1 & 1 & 0 & 0 & 0 & 0 \\ -a_2 & 0 & 1 & & & \\ \cdot & \cdot & & \ddots & & \\ \cdot & \cdot & & & \ddots & \\ \cdot & \cdot & & & & 1 \\ -a_n & 0 & & & & 0 \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_n \end{bmatrix} \quad c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \end{bmatrix} \qquad (6.2)$$

In this form only the parameters a_i, b_i ($i = 1, 2, \ldots n$) are unknown and therefore only the minimum number of parameters necessary to describe the system behaviour is involved. The above choice of A, b, c simultaneously fixes the coordinate system and thus defines the state X(t) of (6.1). The problem is to reconstruct this state of the unknown system using only measurements of the input u(t), and the output y(t).

6.4 PARAMETERIZED OBSERVER

Since the system (4.1) is time invariant, its state can be reconstructed asymptotically by means of a Luenberger observer of the form:

$$\dot{\hat{X}} = F\hat{X} + gy(t) + hu(t) \qquad \hat{X}(o) = \hat{X}_o \qquad (6.3)$$

where the (constant) observer matrices F, g, and h, are defined as:

$$F = \begin{bmatrix} -f_1 & 1 & 0 & \cdots & 0 \\ -f_2 & 0 & 1 & & \\ \cdot & \cdot & & \ddots & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & 1 \\ -f_n & 0 & 0 & & 0 \end{bmatrix} \qquad g = \begin{bmatrix} g_1 \\ g_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ g_n \end{bmatrix} \qquad h = \begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ h_n \end{bmatrix} \qquad (6.4)$$

F has a desired set of eigenvalues with real parts less than $-\sigma$ $(\sigma > o)$.

The state observation error $\varepsilon(t) = \hat{X}(t) - X(t)$, vanishes exponentially according to $\varepsilon(t) = \exp(Ft)\,_0$, if the observer paramenters g and h satisfy $gC^T = A - F$, and $h = b$. The matching point g^*, h^* of the observer is thus:

$$\begin{aligned} g_i^* &= f_i - a_i \\ h_i^* &= b_i \end{aligned} \qquad (6.5)$$

Of course, g^* and h^* are unknown, since a_i and b_i are unknown.

Now consider the set of all (possibly mismatched), Luenberger observer (6.3) with arbitrary but constant observer parameter values g and h. For this set the following representation can be made.
Let e_i denote the i^{th} unit vector, i.e., $g^T e_i = g_i$, and define:

$$\xi(t) = F\xi_i(t) + e_i y(t) \qquad \xi_i(0) = 0 \qquad \ldots\ldots\ldots\ (6.6a)$$

$$\xi_{i+n}' = F\xi_{i+n}(t) + e_i\, u(t) \qquad \xi_{i+n}(0) = 0 \qquad \ldots\ldots\ldots\ (6.6b)$$

Then by the linearity of (6.3), and application of the superposition theorem, the state of the observer can be written as:

$$\hat{X}(t) = [\xi_1(t), \xi_2(t) \ldots \xi_{2n}(t)]\, P + \mathrm{Exp}(Ft)\hat{X}_0 \qquad \ldots\ldots\ldots\ (6.7)$$

where $p = [g^T, h^T]^T$.

And hence,

$$(SI - F^T)^{-1} e_1 = \frac{1}{\det (SI-F)} [S^{n-1}, S^{n-2} \dots S, 1]^T \quad \dots\dots (6.8)$$

$$(SI - F)^{-1} e_i = T_i(SI - F^T)^{-1} e_1 \qquad i = (1, 2, \dots n) \quad \dots\dots (6.9)$$

where T_i are constant matrices, the elements of which are the numerator polynomial coefficients of $(SI-F)^{-1} e_i$. Therefore (6) reduces to two n^{th}-order differential equations:

$$\dot{\eta}_1(t) = F^T\eta_1(t) + e_1 y(t) \qquad \eta_1(0) = 0 \quad \dots\dots\dots\dots (6.10a)$$

$$\dot{\eta}_2(t) = F^T\eta_2(t) + e_1 u(t) \qquad \eta_2(0) = 0 \quad \dots\dots\dots\dots (6.10b)$$

and the linear relations:

$$\xi_i(t) = T_i\eta_1(t) \; \xi_{i+n}(t) = T_i\eta_2(t) \quad \dots\dots\dots\dots (6.10c)$$

$$i = (1, 2, \dots n)$$

Equations (6.7) and (6.10) defined the parametrized observer, the state estimate appears as a linear function of the observer parameters.

Because the parameterized observer is equivalent to the Luenberger observer, $p = p^*$ (p^* being the matching parameter point) implies (t) = exp(Ft) $_0$. It follows from (6.7), substituted in the relation

$$X(t) = \chi(t) - \varepsilon(t), \text{ that:}$$

$$x(t) = [\xi_1(t), \dots \xi_{2n}(t)]p^* + \exp(Ft)x_0 \quad \dots\dots\dots\dots (6.11)$$

Subtracting (6.11) from (6.7), the state observation error becomes:

$$\varepsilon(t) = [\xi_1(t), \ldots \xi_{2n}(t)]\ (p-p^*) + \exp(Ft)\varepsilon_0 \quad \ldots\ldots\ldots\ldots (6.12)$$

This relation reveals that the state observation error splits into two components. The first one, which is a parameter induced error is proportional to the parameter misalignment. The second component is the observation error, which occurs in an observer with matching observer parameters. It vanishes with the observer dynamics regardless of what the actual observer parameter values are.

It is thus seen that the dynamic part (6.10) performs an essential portion of the observation process independent of P. The observer output is $y(t) = C^T X(t)$. Define $Z(t) \triangleq [\eta_1^T(t), \eta_2^T(t)]^T$, and using the identity $C^T(SI - F)^{-1}e_i = e_i T(SI - F^T)^{-1} e_1$, it follows from (6.6) and (6.10) that:

$$C^T[\xi_1(t), \ldots \xi_{2n}(t)] = Z^T(t)$$

Thus we have from (4.7):

$$\hat{y}(t) = Z^T(t)\ P + C^T \exp(Ft)\,\hat{X}_0 \quad \ldots\ldots\ldots\ldots (6.13)$$

which becomes particularly simple, if $X_0 = 0$. From (6.12), the output observation error $\theta(t) = y(t) - y(t)$ is obtained as:

$$\theta(t) = Z^T(t)[P-P^*] + C^T \exp(Ft) t_0 \quad \ldots\ldots\ldots\ldots (6.14)$$

The parameterized observer developed so far is only an alternative, equivalent representation of the Luenberger observer. Its different structure is shown in Fig. 6.1.

In what follows, we use this parametrized representation of the observer and adjust the parameters in it to observe the state of the

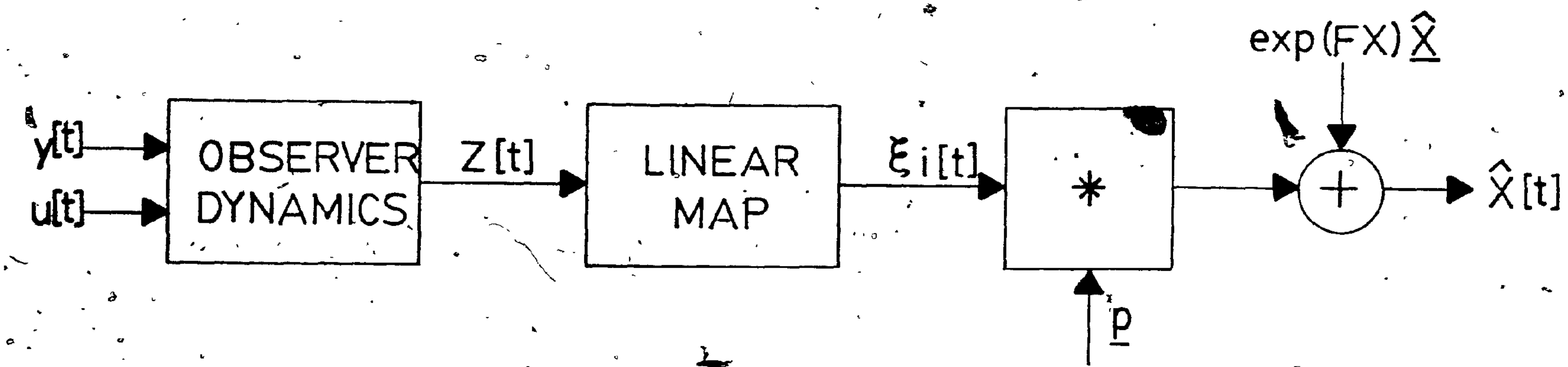


Fig. 6.1 — Structure of the Parameterized Observer.

unknown system (6.1). The adaptive observer setup in this way is defined to have the state:

$$\hat{X}(t) = [\xi_1(t), \xi_2(t), \ldots \xi_{2n}(t)] \hat{P}(t) + \exp Ft \hat{X}_0 \qquad \ldots\ldots (6.15a)$$

with ξ_i from (6.10) and P(t), being the current parameter estimate which is continuously adapted by one of the adaptive laws developed below. The output of the adaptive observer is:

$$\hat{y}(t) = Z^T(t) \hat{P}(t) + C^T \exp(Ft) \hat{X}_0 \qquad \ldots\ldots (6.15b)$$

and the state and output observation errors are defined to be:

$$\tilde{\varepsilon}(t) = \hat{X}(t) - X(t) \text{ and } \tilde{\theta}(t) = \hat{y}(t) - y(t).$$

For any adaptive law, it remains to be shown that:

$$\tilde{\varepsilon}(t) \longrightarrow 0 \quad \text{and } \tilde{P}(t) \longrightarrow P^* \text{ as } t \longrightarrow \infty .$$

6.5 First Adaptation Scheme

The most natural criterion to orientate the adaptation of the observer parameters on is the square of the instantaneous output observation error, i.e., $\theta^2(t)$. Its gradient with respect to p(t) is obtained by use of (4.15b) as $2Z(t)\theta(t)$. Chossing the parameter descent direction proportional to this gradient, we obtain the adaptive law:

$$\dot{P}(t) = -GZ(t) [\hat{y}(t) - y(t)] \qquad \ldots\ldots (6.16)$$

where G is a symmetric, positive definite gain matrix.

THEOREM: If there exist constants K_1, K_2 and T such that

$$0 < K_1 I \leq \int_t^{t+T} Z(\tau) Z^T(\tau) d\tau \leq K_2 I \qquad \text{for all } t \geq 0 \tag{4.17}$$

then the adaptive observer, which is defined from (4.10), (4.15), (4.16), is globally exponentially stable, i.e. $p(t) \longrightarrow p^*$, and $\varepsilon(t) \longrightarrow 0$, exponentially fast.

Proof: Well explained in [11].

6.6 SECOND ADAPTIVE SCHEME

In this section, a generalized output error vector is defined, which can vanish only if certain generalized output signals of the adaptive observer coincide with the unknown system output and its derivatives. Thereby the error criterion can be made positive definite, which results in an adaptive scheme with advanced convergence properties. This idea is due to Lion [18] in connection with input-output identification of linear and non-linear systems. In what follows, a corresponding adaptive scheme in terms of the parametrized observer is developed, where the derivatives of u(t) and y(t) are involved directly. The resulting scheme is shown to be realizable without signal differentiation at the end of this section.

Let $Y(t) = [y(t), \dot{y}(t), \ldots, y^{(2n-1)}(t)]^T$, and define a generalized output vector $\hat{Y}(t)$ of the adaptive observer by the relations:

$$\hat{Y}(t) = \mathcal{Z}^T(t)\, P(t) + \mathcal{C}^T \exp(Ft)\, \hat{X}_0 \tag{6.18a}$$

$$\mathcal{Z}(t) = [Z(t), \dot{Z}(t), \ldots, Z^{(2n-1)}(t)] \tag{6.18b}$$

$$\mathcal{C} = [C, F^T C, \ldots, (F^T)^{2n-1} C] \tag{6.18c}$$

Then the generalized output error $\tilde{E}(t) = \hat{Y}(t) - T(t)$ gives rise to a quadratic error criterion of the form $\tilde{E}(t)Q\tilde{E}(t)$ where Q is p.d. weighting matrix. Differentiating this criterion with respect to $\tilde{P}(t)$, and using the steepest descent technique, we obtain the adaptive law:

$$\dot{P}(t) = -GZ(t)\, Q[\tilde{Y}(t) - Y(t)] \quad \ldots\ldots\ldots\ldots (6.19)$$

where $G = G^T > 0$. This law is directly analogous to (6.16).

6.7 Third Adaptive Scheme

The adaptive schemes of the previous sections made use of the current values of $Z(t)$ and its time derivatives. In this section an adaptation scheme is presented which is based on the time history of $Z(t)$ in the sense of a limited time memory. Consider an error criterion of the form:

$$J(t) = \int_0^t \{Z^T(\tau)\,\tilde{P}(t) + C^T \exp(F\tau)\,\tilde{X}_0 - Y(\tau)\}^2 \exp\{-q(t-\tau)d\tau \quad \ldots\ldots\ldots\ldots (6.20)$$

where q is a positive constant. Note that the squared term in (6.20) is an error signal, which reduces to the actual output observation error $\tilde{y}(\tau) - y(\tau)$ if $\dot{\tilde{P}}(t) \equiv 0$, i.e., if the adaptation has concluded.

Furthermore, using (6.11) premultiplied by C^T and substituted for $y(\tau)$ in (6.20) gives:

$$J(t) = \int_0^t \{Z^T(\tau)\Delta P(t) + C^T \exp(F\tau)\,\tilde{\varepsilon}_0\}^2 \exp\,[illegible]\tau)d\tau \quad \ldots\ldots\ldots\ldots (6.21)$$

Hence, reducing the value of $J(t)$ by a proper adaptation of $p(t)$ tends to satisfy both design objectives of the adaptive observer, namely $\Delta p(t) \longrightarrow 0$, and $\theta(\tau) \longrightarrow 0$. This motivates us to choose an adaptive law, which is based on the graident of J(t) with respect to p(t). From (6.20) the gradient is obtained as:

$$\frac{\partial J(t)}{\partial p(t)} = 2 \{\tilde{p}(t)\ p(t) + r(t)\}$$

where

$$R(t) = \int_0^t Z(\tau)\ Z^T(\tau)\ \exp\{-q(t-\tau)d\tau \qquad \ldots\ldots\ (6.22a)$$

$$r(t) = \int_0^t Z(\tau)\ [C^T \exp(F\tau)\tilde{X}_0 - y(\tau)]\ \exp\{-q(t-\tau)d\tau \quad \ldots\ (6.22b)$$

The adaptive law is chosen as:

$$\dot{p}(t) = -G\ \{R(t)\ \tilde{P}(t) + r(t)\} \qquad \ldots\ldots\ (6.23)$$

where G is a symmetric, positive definite gain matrix.

The integrals occurring in (6.22) can be interpreted convalution integrals. Therefore R(t) and r(t) can be generated by solving the time invariant differential equations:

$$\dot{R}(t) = -qR(t) + Z(t)Z^T(t) \qquad R(0) = 0 \qquad \ldots\ldots\ (6.24a)$$

$$\dot{r}(t) = -qr(t) + Z(t)[C^T \exp(Ft)\tilde{X}_0 - y(t)] \qquad r(0) = 0$$

$$\ldots\ldots\ (6.24b)$$

R(t) and r(t) represent the limited time memory of the adaptation scheme.

CHAPTER 7

CONCLUSIONS

It has been shown in Chapter 2 and 3 that the state vector of a system can be reconstructed from observations of its inputs and outputs. Also, it has been shown that the dynamic order of an observer which observes an n^{th} order system with m outputs is n-m. Hence, when more outputs are available a simpler observer may be reconstructed. The incorporation of an observer to reconstruct the state does not change the pole location of the system, but it adds its own poles to the system. Also, it has been shown that stable observer poles do not affect overall system stability.

In the case of adaptive observer schemes, two major questions have to be resolved before the observer schemes that have been considered so far can be used in practical situations. The first one concerns the speed of convergence of the schemes, and the second deals with observation noise. As has been shown in the model reference adatptive scheme shown in Chapter 4, the unknown plant is specified in terms of its inputs U(t) and outputs y(t). Using this data, adaptive laws are generated for adjusting the parameters of the system or a model. In all cases, the differential equations governing the output and parameter errors are considered and has been shown that they are uniformly asymptotically stable.

In the case of the parametrized observer shown in Chapter 6, it has been shown that if the system input is sufficiently exciting, all schemes shown converge in an exponential rather than in an asymptotic fashion. So by choice of the observer eigenvalues and the adpative gains, fast convergence rates can be obtained in the second and third scheme.

Still some work has to be done in the area of stochastic stability in case of the existence of noise, speed of response, and time varying systems which offer areas of future work.

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