ON THE APPLICATION OF INTEGRAL METHOD TO THE SOLUTION OF MELTING AND SOLIDIFICATION PROBLEMS IN A FUSION WELDING PROCESS

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ABSTRACT

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Melting and solidification processes during fusion welding are investigated analytically. The procedure of obtaining the solution of heat transfer is divided into two stages: (i) the melting stage, and (ii) the solidification stage. The melting stage is an important stage to obtain an intimate metallurgical combination between the liquid and solid metal. Therefore the investigation of the melting stage is emphasized in the present work.

Integral methods are mainly used for solving the melting and solidification problems. A relation for phase front movement, which is suitable for fusion welding consisting of melting and solidification processes, is found. A criterion on the occurrence of a melting process in fusion welding is established.

The application of results is illustrated by taking an aluminum fusion welding as an example.

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TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF FIGURES

NOMENCLATURE

NOMENCLATURE

a	Thermal diffusivity
c	Specific heat
k	Thermal conductivity
Q_L	Latent heat of the fusion
T_s	Melting point of the metal
L	One half of the width of fusion zone
$T(x,t)$	Temperature distribution
T_0	Initial temperature of the liquid metal
T_∞	Initial temperature of the solid metal
t	Time
x	Dimensional position coordinate
$X(t)$	Coordinate of the phase boundary between solid and liquid phases
α	a_2/a_1, dimensionless
$\delta(t)$	The location of the heat penetration depth front
τ	Dimensionless time
ξ	Dimensionless position coordinate
$\theta(\xi,\tau)$	Dimensionless temperature distribution
θ_0	Dimensionless initial temperature of the liquid metal
θ_∞	Dimensionless initial temperature of the solid metal
$\Delta(\tau)$	Dimensionless coordinate of heat penetration depth front
$\eta(\tau)$	Dimensionless phase boundary between solid and liquid phases
ρ	Density

Subscripts

1	Solid phase
2	Liquid phase
f	Finality of the process
m	Maximum point of phase boundary

Superscripts

'	Solidification process
–	Average value

CHAPTER I

INTRODUCTION

CHAPTER 1

INTRODUCTION

Fusion welding is a process by which two sheets of heavy material are welded together by infusing molten metal between them. Depending on the welding set-up and physical properties of the material, two cases of heat transfer may occur during the process: one is that only solidification of the molten metal occurs, another is that solidification follows the melting of a very small part of the parent plates. Theoretically, a better result is obtained by the latter case due to an intimate metallurgical combination between the liquid and solid metal.

During the process the intensive heat required in the molten metal is transferred by radiation, convection and mainly conduction to the surrounding metal. Compared with conduction heat transfer, radiation and convection are rather small [1]. Therefore, only conduction heat transfer in the fusion welding problem will be considered.

The characteristic features of solidification and melting problems are the coupling of the temperature distributions with the rate of movement of the phase boundary between the solid and liquid phases. As both the temperature and the coordinate of the phase boundary are unknown functions, they make the problem non-linear. Only a few exact analytical solutions have been found for special cases

(see Carslaw and Jaeger [2]). With regard to the non-linear problem, solutions will be generally obtained by analytical approximations and numerical methods. For example, the heat balance integral method was used by Goodman [3], Goodman and Shea [4], Poots [5] and Libby and Chen [6]; the variational method by Biot [7], Biot and Daughaday [8] and Lapadula and Mueller [9]; the method of moving heat sources by Rosenthal [10] and Jackson [11]; the polynomial approximation by Megerlin [12] and Stephan [13]; the perturbation method by Lock [14] and Pedroso and Domoto [15, 16]; the numerical integration method by Beauboeuf and Chapman [17]; the method of similarity by Lin [18] to [23]; also different analytical procedures were treated by Siegel and Savino [24,25,26]. A general review of the field of heat conduction with freezing or melting was given by Muehlbauer and Sunderland [27].

Due to the complication of the combination of melting and solidification processes involved in the fusion welding problem, Eckert and Drake simplified the problem by neglecting the process of phase change at the phase front [28]. In the present work, the fusion welding problem involving both the melting and solidification processes will mainly be solved by the heat balance integral method.

CHAPTER II

THEORETICAL ANALYSIS

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THEORETICAL ANALYSIS

2.1 THE CONCEPT OF HEAT BALANCE INTEGRAL METHOD

The formulation of the integrated heat-conduction equation can easily be shown for the transient case in one space dimension, the semi-infinite body. When a semi-infinite body initially existing at some temperature T_∞ receives heat from the free surface, a temperature field is built up as is shown schematically in Fig.2.1. This temperature field comprises a region within the body wherein the local temperature differs from T_∞. The depth to which the thermal effects are felt is called the penetration depth and following the boundary-layer practice is designated by δ. The penetration depth is a function of time.

The differential equation for the case of the semi-infinite solid with constant properties is

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2} \tag{2.1}$$

Equation (2.1) can be integrated from $x = 0$ to $x = \delta(t)$

$$\int_0^\delta \frac{\partial T}{\partial t}\, dx = \int_0^\delta a\frac{\partial^2 T}{\partial x^2}\, dx \tag{2.2}$$

which may be evaluated to yield

$$\frac{d}{dt}\left[\int_0^{\delta} T(x,t)\,dx\right] - T(\delta,t)\frac{d\delta}{dt} = a\left[\frac{\partial T}{\partial x}(\delta,t) - \frac{\partial T}{\partial x}(0,t)\right] \quad (2.3)$$

2.2 MATHEMATICAL FORMULATION

We consider two extensive sheets of heavy material having the same dimensions, the same material and the same initial temperature T_∞. They are separated at a distance 2L as shown in Fig. 2.2. These sheets will be welded together by infusing molten metal having a uniform initial temperature T_0 between them. It is assumed that the exposed surfaces of the sheets lose heat at a much slower rate than the heat flows by conduction into the metal, and that the material properties are taken as constant in the solid and liquid phase. The temperatures and material properties in the solid and liquid phases are denoted by subscripts 1 and 2, respectively.

The system of equations formulating the problem can be described as follows.

The differential and integrated equations of one-dimensional heat conduction in the solid and liquid phases are respectively written as

$$\frac{\partial T_1}{\partial t} = a_1 \frac{\partial^2 T_1}{\partial x^2} \quad (2.4)$$

for solid part (System 1), and

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial x^2} \tag{2.5}$$

for the liquid part (System 2).

$$\frac{d}{dt}\left[\int_X^{\delta_1} T_1(x,t)\,dx - T_\infty \delta_1(t)\right] = -a_1 \frac{\partial T_1}{\partial x}(X,t) \tag{2.6}$$

and

$$\frac{d}{dt}\left[\int_X^{\delta_2} T_2(x,t)\,dx - T_0 \delta_2(t)\right] = -a_2 \frac{\partial T_2}{\partial x}(X,t) \tag{2.7}$$

The initial conditions are obvious for $t = 0$

$$T_1(x,0) = T_\infty \tag{2.8}$$

$$T_2(x,0) = T_0 \tag{2.9}$$

The boundary conditions at the phase boundary between solid and liquid phases $x = X(t)$ for $t > 0$ are

$$T_1(X,t) = T_s \tag{2.10}$$

$$T_2(X,t) = T_s \tag{2.11}$$

The boundary conditions at the penetration depths, $x = \delta_1$ in the solid phase and $x = \delta_2$ in the liquid phase can be expressed as

$$T_1(\delta_1,t) = T_\infty \tag{2.12}$$

$$T_2(\delta_2,t) = T_0 \tag{2.13}$$

$$\frac{\partial T_1}{\partial x}(\delta_1, t) = 0 \tag{2.14}$$

$$\frac{\partial T_2}{\partial x}(\delta_2, t) = 0 \tag{2.15}$$

At the solid-liquid interface there exists another relation of energy balance for the melting or solidification process

$$\rho_1 c_1 \frac{dX}{dt} = k_1 \frac{\partial T_1(X,t)}{\partial x} - k_2 \frac{\partial T_2(X,t)}{\partial x} \tag{2.16}$$

2.3 INTRODUCTION OF DIMENSIONLESS VARIABLES

In order to reduce the number of parameters in the system of Eqs. (2.4) to (2.16), all the variables are made dimensionless.

These non-dimensional variables are defined as follows:

$$\left.\begin{aligned}
\theta_1 &= \frac{C_1}{Q_L}(T_1 - T_s) \\
\theta_2 &= \frac{k_2}{k_1}\frac{C_1}{Q_L}(T_2 - T_s) \\
\tau &= \frac{a_1 t}{L^2} \\
\xi &= \frac{x}{L} \\
\Delta_1 &= \frac{\delta_1}{L}
\end{aligned}\right\} \tag{2.17}$$

$$\Delta_2 = \frac{\delta_2}{L}$$

$$\eta = \frac{X}{L}$$

2.4 MATHEMATICAL ANALYSIS

The analysis is divided into two main stages: melting stage and solidification stage. During the melting stage the penetration depths are propagating freely in both the solid and liquid phases. The integral method is applicable for the determination of temperature distributions in this stage. During the solidification stage, the integral method is applicable in the solid phase only. In the liquid phase the problem is simplified in this stage by assuming that there is an apparent latent heat consisting of the heat, which keeps the molten metal in liquid state, and the latent heat of fusion. Thus, the temperature in the liquid phase is averaged at melting point.

2.4.1 Melting Stage

During this period the depths of penetration, δ_1 and δ_2, are freely propagating to the left and right directions of the phase front. The integral method is applicable in both the solid and liquid phases.

By introducing dimensionless parameters, the system of equations (2.4) to (2.16) are transformed as follows.

$$\frac{\partial \theta_1}{\partial \tau} = \frac{\partial^2 \theta_1}{\partial \xi^2} \tag{2.18}$$

$$\frac{\partial \theta_2}{\partial \tau} = \alpha \frac{\partial^2 \theta_2}{\partial \xi^2} \tag{2.19}$$

Where $\alpha = \frac{a_2}{a_1}$,

$$\frac{d}{d\tau}\left[\int_{\eta}^{\Delta_1} \theta_1(\xi,\tau)\,d\xi - \theta_\infty \Delta_1\right] = -\frac{\partial \theta_1}{\partial \xi}(\eta,\tau) \tag{2.20}$$

$$\frac{d}{d\tau}\left[\int_{\eta}^{\Delta_2} \theta_2(\xi,\tau)\,d\xi - \theta_0 \Delta_2\right] = -\alpha \frac{\partial \theta_2}{\partial \xi}(\eta,\tau) \tag{2.21}$$

$$\theta_1(\xi,0) = \theta_\infty \tag{2.22}$$

$$\theta_2(\xi,0) = \theta_0 \tag{2.23}$$

$$\theta_1(\eta,\tau) = 0 \tag{2.24}$$

$$\theta_2(\eta,\tau) = 0 \tag{2.25}$$

$$\theta_1(\Delta_1,\tau) = \theta_\infty \tag{2.26}$$

$$\theta_2(\Delta_2,\tau) = \theta_0 \tag{2.27}$$

$$\frac{\partial \theta_1}{\partial \xi}(\Delta_1,\tau) = 0 \tag{2.28}$$

$$\frac{\partial \theta_2}{\partial \xi}(\Delta_2,\tau) = 0 \tag{2.29}$$

$$\frac{d\eta}{d\tau} = \frac{\partial\theta_1(\eta,\tau)}{\partial\xi} - \frac{\partial\theta_2(\eta,\tau)}{\partial\xi} \tag{2.30}$$

Assume the temperature distribution $\theta_1(\xi,\tau)$ in System 1, can be represented by a polynomial of the second degree

$$\theta_1(\xi,\tau) = \beta_0 + \beta_1\xi + \beta_2\xi^2 \tag{2.31}$$

where the coefficients β_i are functions of τ to be determined by Eqs. (2.24), (2.26) and (2.28).

From Eq. (2.24)

$$\beta_0 + \beta_1\eta + \beta_2\eta^2 = 0 \tag{2.32}$$

From Eq. (2.26)

$$\beta_0 + \beta_1\Delta_1 + \beta_2\Delta_1^2 = \theta_\infty \tag{2.33}$$

From Eq. (2.28)

$$\beta_1 + 2\beta_2\Delta_1 = 0 \tag{2.34}$$

Solving Eqs. (2.32), (2.33) and (2.34), we obtain

$$\beta_0 = \frac{\theta_\infty}{(\Delta_1-\eta)^2}(\eta^2-2\Delta_1\eta) \tag{2.35}$$

$$\beta_1 = \frac{2\Delta_1\theta_\infty}{(\Delta_1-\eta)^2} \tag{2.36}$$

$$\beta_2 = \frac{-\theta_\infty}{(\Delta_1-\eta)^2} \tag{2.37}$$

Thus the temperature distribution in System 1 becomes

$$\theta_1(\xi,\tau) = \frac{\theta_\infty}{(\Delta_1-\eta)^2}\left[\eta(\eta-2\Delta_1) + 2\Delta_1\xi-\xi^2\right] \tag{2.38}$$

Similarly, the temperature distribution in System 2, is

$$\theta_2(\xi,\tau) = \frac{-\theta_0}{(\Delta_2-\eta)^2}\left[\eta(\eta-2\Delta_2) + 2\Delta_2\xi-\xi^2\right] \tag{2.39}$$

where $\eta(\tau)$, $\Delta_1(\tau)$ and $\Delta_2(\tau)$ still remain unknown.

Substitute Eq. (2.38) into Eq. (2.20), gives

$$\frac{\theta_\infty}{3}\frac{d}{d\tau}(\Delta_1 + 2\eta) = \frac{2\theta_\infty}{\Delta_1-\eta}$$

or

$$\frac{d\Delta_1}{d\tau} + 2\frac{d\eta}{d\tau} = \frac{6}{\Delta_1-\eta} \tag{2.40}$$

Similarly, substitute Eq. (2.39) into Eq. (2.21), we obtain

$$\frac{d\Delta_2}{d\tau} + 2\frac{d\eta}{d\tau} = \frac{6\alpha}{\Delta_2-\eta} \tag{2.41}$$

Substituting Eqs. (2.38) and (2.39) into Eq. (2.30), we obtain

$$\frac{d\eta}{d\tau} = \frac{2\theta_\infty}{\Delta_1-\eta} - \frac{2\theta_0}{\Delta_2-\eta} \tag{2.42}$$

These three equations are used to determine Δ_1 Δ_2 and η.

Eliminating terms $(\Delta_1-\eta)$ and $(\Delta_2-\eta)$ from Eqs. (2.40), (2.41) and (2.42), we obtain the relation between η, Δ_1 and Δ_2 as follows:

$$\alpha\theta_\infty\Delta_1 - \theta_0\Delta_2 + (2\alpha\theta_\infty-2\theta_0-3\alpha)\eta = C_1 \qquad (2.43)$$

where C_1 is a constant which can be determined by fitting initial conditions.

For $\tau = 0,\ \Delta_1 = \Delta_2 = \eta = 1$

Therefore

$$C_1 = \alpha\theta_\infty - \theta_0 + (2\alpha\theta_\infty - 2\theta_0 - 3\alpha)$$

thus Eq. (2.43) becomes

$$\alpha\theta_\infty(\Delta_1-1) - \theta_0(\Delta_2-1) + (2\alpha\theta_\infty-2\theta_0-3\alpha)(\eta-1) = 0 \qquad (2.44)$$

It is obviously that the differential equations (2.40), (2.41) and (2.42) are non-linear. In order to be able to solve the problem analytically, the following assumptions are made.

First, assume the solid-liquid interface movement $\eta(\tau)$ will be in the form

$$\eta - 1 = p\sqrt{4\tau} = \gamma\tau \qquad (2.45)$$

Should the coefficients p and γ be determined, $\eta - 1$ would physically fulfill both in the melting and solidifi-

cation processes.

When $\eta - 1$ reaches maximum which is, in other words, at the instantaneousness of the ending of melting process and the commencing of the solidification process, we obtain from Eq. (2.45) by

$$\frac{d(\eta-1)_m}{d\tau} = 0$$

$$p\frac{1}{\sqrt{\tau_m}} - \gamma = 0 \qquad (2.46)$$

or

$$\tau_m = \frac{p^2}{\gamma^2} \qquad (2.47)$$

where τ_m is the time at $(\eta-1)_m$.

Substituting Eq. (2.47) into (2.45) we obtain

$$(\eta-1)_m = \frac{p^2}{\gamma} \qquad (2.48)$$

From Eq. (2.42), we also obtain

$$\left(\frac{d\eta}{d\tau}\right)_{(\eta-1)_m} = \frac{2\theta_\infty}{(\Delta_1-1)_m - (\eta-1)_m} - \frac{2\theta_0}{(\Delta_2-1)_m - (\eta-1)_m} = 0$$

or $$\theta_\infty\left[(\Delta_2-1)_m - \frac{p^2}{\gamma}\right] = \theta_0\left[(\Delta_1-1)_m - \frac{p^2}{\gamma}\right]$$

or $$(\Delta_1-1)_m = \frac{\theta_\infty}{\theta_0}\left[(\Delta_2-1)_m - \frac{p^2}{\gamma}\right] + \frac{p^2}{\gamma} \qquad (2.49)$$

where $(\Delta_1-1)_m$ and $(\Delta_2-1)_m$ are penetration depth at time τ_m.

Substituting Eq. (2.49) into Eq. (2.44) it results in

$$(\Delta_2-1)_m = -[2 - \frac{3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)}{\alpha\theta_\infty^2 - \theta_0^2}] \frac{p^2}{\gamma} \qquad (2.50)$$

Physically, the propagation of Δ_2 is much faster than η movement, therefore, we can secondly, assume that when $(\eta-1)$ reaches maximum Δ_2 is approaching 0 or Δ_2 is negligible as compared to the value of 1.

Thus, from Eq. (2.50), we obtain

$$\gamma = [2 - \frac{3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)}{\alpha\theta_\infty^2-\theta_0^2}] p^2 \qquad (2.51)$$

Substituting Eq. (2.51) into Eq. (2.45) we finally obtain

$$\eta - 1 = p\sqrt{4\tau} - [2 - \frac{3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)}{\alpha\theta_\infty^2-\theta_0^2}] p^2\tau \qquad (2.52)$$

Up to this stage every initial and boundary condition has been considered, but constant p in Eq. (2.52) is still unknown. Therefore, in order to determine the constant p, an analytical method for a short time solution will be considered next.

At the beginning of the welding process, t is so small that τ is negligible as compared to $\sqrt{\tau}$.

Thus, Eq. (2.52) can be approximately written as

$$\eta - 1 = p\sqrt{4\tau} \tag{2.53}$$

The heat conduction differential equations (2.18) and (2.19) for System 1 and System 2, are satisfied by the following functions, respectively.

$$\theta_1(\xi,\tau) = A_1 + B_1 \operatorname{erf}\left(\frac{\xi - 1}{\sqrt{4\tau}}\right) \tag{2.54}$$

$$\theta_2(\xi,\tau) = A_2 + B_2 \operatorname{erf}\left(\frac{1 - \xi}{\sqrt{4\alpha\tau}}\right) \tag{2.55}$$

where A_i and B_i are constants to be determined by fitting initial and boundary conditions.

In System 1, letting Eq. (2.54) satisfy the initial and boundary conditions, Eq. (2.22) and (2.24), we obtain

$$A_1 + B_1 = \theta_\infty \tag{2.56}$$

$$A_1 + B_1 \operatorname{erf}\left(\frac{\eta - 1}{\sqrt{4\tau}}\right) = 0$$

where $$\eta - 1 = p\sqrt{4\tau}$$

thus $$A_1 + B_1 \operatorname{erf}(p) = 0 \tag{2.57}$$

Solving Eqs. (2.56) and (2.57) gives

$$A_1 = -\frac{\theta_\infty \operatorname{erf}(p)}{1 - \operatorname{erf}(p)}$$

$$B_1 = \frac{\theta_\infty}{1 - \operatorname{erf}(p)}$$

Thus, Eq. (2.54) becomes

$$\theta_1(\xi,\tau) = \frac{\theta_\infty}{1 - \mathrm{erf}(p)}\left[\mathrm{erf}\left(\frac{\xi-1}{\sqrt{4\tau}}\right) - \mathrm{erf}(p)\right] \quad (2.58)$$

In System 2, by a similar procedure as that for System 1, Eq. (2.55) becomes

$$\theta_2(\xi,\tau) = \frac{\theta_0}{1 + \mathrm{erf}\left(\frac{p}{\sqrt{\alpha}}\right)}\left[\mathrm{erf}\left(\frac{1-\xi}{\sqrt{4\alpha\tau}}\right) + \mathrm{erf}\left(\frac{p}{\sqrt{\alpha}}\right)\right] \quad (2.59)$$

Differentiation of Eq. (2.53) with respect to τ gives

$$\frac{d\eta}{d\tau} = \frac{2p}{\sqrt{4\tau}} \quad (2.60)$$

From Eqs. (2.58) and (2.59) we obtain

$$\frac{\partial\theta_1}{\partial\xi} = \frac{\theta_\infty}{(1-\mathrm{erf}\ p)}\frac{2}{\sqrt{\pi}}e^{-\left(\frac{\xi-1}{\sqrt{4\tau}}\right)^2}\left(\frac{1}{\sqrt{4\tau}}\right)$$

$$\frac{\partial\theta_2}{\partial\xi} = \frac{\theta_0}{1 + \mathrm{erf}\left(\frac{p}{\sqrt{\alpha}}\right)}\frac{2}{\sqrt{\pi}}e^{-\left(\frac{1-\xi}{\sqrt{4\alpha\tau}}\right)^2}\left(\frac{-1}{\sqrt{4\alpha\tau}}\right)$$

For $\xi = \eta$ the above two equations become

$$\left(\frac{\partial\theta_1}{\partial\xi}\right)_{\xi=\eta} = \frac{2\theta_\infty}{1 - \mathrm{erf}\ p}\frac{e^{-p^2}}{\sqrt{\pi}\sqrt{4\tau}} \quad (2.61)$$

$$\left(\frac{\partial\theta_2}{\partial\xi}\right)_{\xi=\eta} = \frac{-2\theta_0}{1 + \mathrm{erf}\frac{p}{\sqrt{\alpha}}}\frac{e^{-p^2/\sqrt{\alpha}}}{\sqrt{\pi}\sqrt{4\alpha\tau}} \quad (2.62)$$

Substituting Eqs. (2.60), (2.61) and (2.62) into Eq. (2.30), it finally results in

$$\frac{2p}{\sqrt{4\tau}} = \frac{2\theta_\infty}{1 - \text{erf } p} \frac{e^{-p^2}}{\sqrt{\pi}\sqrt{4\tau}} + \frac{2\theta_0}{1 + \text{erf } \frac{p}{\sqrt{\alpha}}} \frac{e^{-\frac{p^2}{\sqrt{\alpha}}}}{\sqrt{\pi}\sqrt{\alpha}\sqrt{4\tau}}$$

or

$$\theta_0 = \frac{\sqrt{\alpha}(1+\text{erf } \frac{p}{\sqrt{\alpha}})[\sqrt{\pi}p(1-\text{erf } p) - \theta_\infty e^{-p^2}]}{(1-\text{erf } p)e^{-\frac{p^2}{\alpha}}} \quad (2.63)$$

From Eq. (2.63) the constant p can be determined graphically for every kind of material which possesses a certain value of α. The solution of p from Eq. (2.63) for aluminum is shown as an example, in Fig. 2.3.

2.4.2 Solidification Stage

This stage begins at the instant of melting process finalization, i.e., when η reaches the maximum point, and ends at the instant when η reaches zero. During this stage, we consider that the integral method is applicable in the solid phase only. In the liquid phase the problem is simplified by assuming that there is an apparent latent heat consisting of the heat which keeps the molten metal in liquid state and the latent heat of fusion. Thus, the temperature of the liquid phase is averaged at melting point. [30,31].

Based on this assumption, conduction heat transfer in the solid phase is formulated as follows.

The differential equation is

$$\frac{\partial \theta_1'}{\partial \tau'} = \frac{\partial^2 \theta_1'}{\partial \xi^2} \tag{2.64}$$

The initial temperature distribution in solid phase, from Eq. (12.38).

$$\theta_1'(\xi, \tau_m) = \frac{\theta_\infty}{(\Delta_{1_m} - \eta_m)^2} [\eta_m(\eta_m - 2\Delta_{1_m}) + 2\Delta_{1_m}\xi - \xi^2] \tag{2.65}$$

Boundary conditions are

$$\theta'(\eta_1', \tau') = 0 \tag{2.66}$$

$$\theta_1'(\Delta_1', \tau') = \theta_\infty \tag{2.67}$$

$$\frac{\partial \theta_1(\Delta_1', \tau')}{\partial \xi} = 0 \tag{2.68}$$

The integrated equation of Eq. (2.64) is

$$\frac{d}{d\tau'} \left[\int_{\eta'}^{\Delta_1'} \theta_1'(\xi, \tau') d\xi - \theta_\infty \Delta' \right] = - \left(\frac{\partial \theta_1'}{\partial \xi}\right)(\eta', \tau') \tag{2.69}$$

By a similar procedure as in the melting stage we obtain the temperature distribution in the solid phase as

$$\theta_1'(\xi, \tau') = \frac{\theta_\infty}{(\Delta_1' - \eta')^2} [\eta'(\eta' - 2\Delta_1') + 2\Delta_1'\xi - \xi^2] \tag{2.70}$$

Eq. (2.70) is automatically satisfying the initial

condition, Eq.(2.65).

Substituting Eq. (2.70) into Eq. (2.64), results in

$$\frac{d\Delta_1'}{d\tau'} + 2\frac{d\eta'}{d\tau'} = \frac{6}{\Delta_1' - \eta'} \tag{2.71}$$

In the liquid phase an assumption was made in Section 2.4.1 that at

$$\tau' = \tau_m \qquad \Delta_2' = 0$$

Therefore, the initial temperature distribution in the liquid phase, from Eq. (2.39), is

$$\theta_2'(\xi, \tau_m) = \theta_0[1 - \frac{\xi^2}{\eta_m^2}] \tag{2.72}$$

The average temperature in the liquid phase at this moment can be calculated from Eq. (2.72).

$$\overline{\theta_2'} = \frac{1}{-\eta_m} \int_{\eta_m}^{0} \theta_0[1 - \frac{\xi}{\eta_m^2}]d\xi = \frac{2}{3}\theta_0 \tag{2.73}$$

$\overline{T_2 - T_s'} = \frac{2}{3}(T_0' - T_s)$ in dimensional form.

Thus the heat which keeps the molten metal in liquid state is

$$\frac{2}{3} c_2 \rho_2 \chi'(T_0 - T_s) \text{ or } \frac{2}{3}\frac{\theta_0}{\alpha}\eta'$$

in dimensionless form. As indicated previously, this heat is assumed to be a part of the apparent latent heat of the liquid phase.

Hence, a heat balance at the phase boundary between the solid and liquid phases can be formulated as

$$[Q_L\rho_1 + \frac{2}{3}c_2\rho_2(T_0 - T_s)]\frac{dX'}{dt} = k_1(\frac{\partial T_1'}{\partial x})_{x=X'}$$

or in dimensionless form as

$$(1 + \frac{2}{3}\frac{\theta_0}{\alpha})\frac{d\eta'}{d\tau'} = (\frac{\partial\theta_1'}{\partial\xi})_{\xi=\eta'} \qquad (2.74)$$

Substituting Eq. (2.70) into Eq. (2.74) gives

$$(1 + \frac{2}{3}\frac{\theta_0}{\alpha})\frac{d\eta'}{d\tau'} = \frac{2\theta_\infty}{\Delta_1' - \eta'} \qquad (2.75)$$

Eliminating the term $(\Delta_1' - \eta')$ in Eqs.(2.71) and (2.75) it yields

$$\alpha\theta_\infty[(\Delta_1-1) - (\Delta_1-1)_m] + (2\alpha\theta_\infty-2\theta_0-3\alpha)[(\eta'-1)$$

$$-(\eta-1)_m] = 0$$

or

$$\Delta_1' - 1 = (\Delta_1-1)_m - (2-D)[(\eta'-1) -$$

$$- (\eta-1)_m] \qquad (2.76)$$

where

$$D = \frac{2\theta_0 + 3\alpha}{\alpha\theta_\infty} \qquad (2.77)$$

Substitute Eq.(2.76) into Eq. (2.75), thus

$$(1 + \frac{2}{3}\frac{\theta_0}{\alpha})\frac{d(\eta'-1)}{d\tau'} = \frac{2\theta_\infty}{-(3-D)(\eta'-1)+[(\Delta_1-1)_m+(\eta-1)_m(2-D)]}$$

or

$$\{(3-D)(\eta'-1) - [(\Delta_1-1)_m+(2-D)(\eta-1)_m]\}d(\eta'-1) =$$

$$= -\frac{6}{D}d\tau' \qquad (2.78)$$

Integrating Eq. (2.78) from τ_m to τ' it gives

$$(3-D)(\eta'-1)^2-2[(2-D)(\eta-1)_m+(\Delta_1-1)_m](\eta'-1)+[\frac{12}{D}(\tau'-\tau_m) +$$

$$+ (1-D)(\eta-1)_m^2+2(\Delta_1-1)_m(\eta-1)_m] = 0 \qquad (2.79)$$

Solving quadratic Eq. (2.79) finally results in

$$\eta'-1 = \frac{[(2-D)(\eta-1)_m+(\Delta_1-1)_m]-\sqrt{[(\Delta_1-1)_m-(\eta-1)_m]^2-\frac{12}{D}(3-D)(\tau'-\tau_m)}}{3-D}$$

$$(2.80)$$

Substituting Eq. (2.80) into Eq. (2.76) we also obtain

$$\Delta_1'-1 = \frac{[(2-D)(\eta-1)_m+(\Delta_1-1)_m]+(2-D)\sqrt{[(\Delta_1-1)_m-(\eta-1)_m]^2-\frac{12}{D}(3-D)(\tau'-\tau_m)}}{3-D}$$

$$(2.81)$$

where $$D = \frac{2\theta_0 + 3\alpha}{\alpha\theta_\infty} \qquad (2.77)$$

$(\eta-1)m$, $(\Delta_1-1)m$ and τ_m can be obtained from Eqs. (2.48), (2,50), and (2.47) as follows:

$$(\eta-1)_m = \frac{\alpha\theta_\infty^2 - \theta_0^2}{2(\alpha\theta_\infty^2-\theta_0^2)-3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)} \tag{2.82}$$

$$(\Delta_1-1)_m = \frac{-2(\alpha\theta_\infty^2-\theta_0^2)-3(\theta_0^2-\theta_0\theta_\infty-\alpha\theta_\infty)}{2(\alpha\theta_\infty^2-\theta_0^2)-3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)} \tag{2.83}$$

$$\tau_m = \left\{\frac{\alpha\theta_\infty^2-\theta_0^2}{[2(\alpha\theta_\infty^2-\theta_0^2)-3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)]P}\right\}^2 \tag{2.84}$$

Setting $\eta' = 0$ in Eq. (2.80), we can determine the time required for the completion of the whole fusion welding process.

$$\tau_f = \tau_m - \frac{D}{12}[(1-D)(\eta-1)_m^2+2(\eta-1)_m(\Delta_1-1)_m+2(2-D)(\eta-1)_m + 2(\Delta_1-1)_m+(3-D)] \tag{2.85}$$

CHAPTER III

EXAMPLE

ALUMINUM FUSION WELDING

CHAPTER III

EXAMPLE

ALUMINUM FUSION WELDING

We consider two aluminum plates having 0.1 m thickness to be welded together by fusion welding. The fusion zone is set at 0.03 m in width. The plates are wide enough to be considered as infinite. The plates are preheated to 300°C. The initial temperature of the molten aluminum is 1,677°C. We would like to predict all heat activities in both parent plates and the fusion zone. The procedure of calculation is described as follows.

3.1 PROPERTIES, PARAMETERS AND COEFFICIENTS

Many physical properties depend on the purity and physical state (annealed, hard drawn, cast, etc.) of the metal. The data used in this example refers to aluminum in the high state of purity, and is sufficiently accurate for this purpose of exercise.

3.1.1 Physical Properties of Aluminum

Physical properties of aluminum in solid and liquid phase are shown as follows [29]:

Density	ρ_1 = 2,700 Kg/m^3	ρ_2 = 2,310 Kg/m^3
Thermal Conductivity	k_1 = 238.6 w/m °C	k_2 = 83.7 w/m °C
Specific Heat	c_1 = 1.076 KJ/Kg°C	c_2 = 1.084 KJ/Kg°C

Thermal Diffusinity	$a_1 = 8.214 \times 10^{-5}$ m²/sec	$a_2 = 3.343 \times 10^{-5}$ m²/sec
Latent Heat of Fusion	387.7 KJ/Kg	
Melting Point	660°C	

3.1.2 Dimensionless Parameters and Variables

By Equation (2.17) all parameters and variables are transformed to dimensionless form:

$$\alpha = \frac{a_2}{a_1} = 0.407$$

$$\theta_1 = \frac{C_1}{Q_L}(T_1 - T_s) = 0.002775(T_1 - 660)$$

$$\theta_2 = \frac{K_2}{K_1}\frac{C}{Q_L}(T_2 - T_s) = 0.000974(T_2 - 660)$$

$$\theta_\infty = \frac{C_1}{Q_L}(T_\infty - T_s) = -0.999$$

$$\theta_0 = \frac{K_2}{K_1}\frac{C_1}{Q_L}(T_0 - T_s) = 0.991$$

$$\tau = \frac{a_1 t}{L^2} = 0.365t$$

$$\xi = \frac{X}{L} = 66.7X$$

$$\Delta_1 = \frac{\delta_1}{L} = 66.7\delta_1$$

$$\Delta_2 = \frac{\delta_2}{L} = 66.7\delta_2$$

$$\eta = \frac{X}{L} = 66.7X$$

3.1.3 Coefficients p and γ

For $\theta_\infty = -0.999$ and $\theta_0 = 0.991$, from Fig. 2.3, we obtain

$$p = 0.1$$

From Eq. (2.51) we obtain

$$\gamma = [2 - \frac{3\alpha(\theta_\infty^2 - \theta_\infty\theta_0 + \theta_0)}{\alpha\theta_\infty^2 - \theta_0^2}] \cdot p^2$$

$$= 0.083$$

3.2 CHARACTERISTICS OF CONDUCTION HEAT TRANSFER DURING THE WELDING PROCESS

Using the results established in Chapter II, characteristics of conduction heat transfer at various stages during the aluminum fusion welding process can be illustrated as follows.

3.2.1 Movement of Phase Boundary During Melting Stage

Substituting $p = 0.1$ and $\gamma = 0.083$ into Eq. (2.45) results in

$$\eta - 1 = 0.2\sqrt{\tau} - 0.083\tau$$

or

$$X = 0.015 + 0.0018\sqrt{t} - 0.000454t$$

in dimensional form. (curve a, in Fig. 3.1)

3.2.2 Temperature Distributions in Solid and Liquid Phases at the Beginning of the Process

Temperature distributions in solid and liquid phases during the very short period from the beginning of the fusion welding process as shown in Eqs. (2.58) and (2.59) can be found as follows:

$$\theta_1(\xi,\tau) = 0.127 - 1.126 \text{ erf } \frac{\xi-1}{2\sqrt{\tau}}$$

$$\theta_2(\xi,\tau) = 0.148 - 0.842 \text{ erf } \frac{\xi-1}{1.276\sqrt{\tau}}$$

or

$$T_1(x,t) = 705.8 - 405.8 \text{ erf } \frac{66.7x-1}{1.21\sqrt{t}}$$

$$T_2(x,t) = 812 - 864.5 \text{ erf } \frac{66.7x-1}{0.771\sqrt{t}}$$

in dimensional form. (Fig. 2)

3.2.3 Time, Melting Depth and Heat Penetration Depth at the Moment of Finality of Melting Stage

The time, from Eq. (2.47), is

$$\tau_m = \frac{p^2}{\gamma^2} = \left(\frac{0.1}{0.083}\right)^2 = 1.45$$

or

$$t_m = 3.98 \text{ sec.}$$

The melting depth, from Eq. (2.48) is

$$(\eta-1)_m = \frac{p^2}{\gamma} = 0.12$$

or

$$X_m = 0.0168 \text{ m}$$

or the melting depth from the initial phase boundary is

$$X_m - 0.015 = 0.0018 \text{ m}$$

The heat penetration depth, from Eq. (2.83), is

$$(\Delta - 1)_m = 1.25$$

or

$$\delta_{1m} = 0.03375 \text{ m}$$

3.2.4 Temperature Distributions at the Moment of Finality of Melting Stage

At this stage, the temperature distribution in the solid phase, from Eq. (2.65), is

$$\theta_1(\xi, \tau_m) = 2.96 - 3.52\xi + 0.782\xi^2$$

and that in the liquid phase, from Eq. (2.72), is

$$\theta_2(\xi, \tau_m) = 0.991 - 0.789\xi^2$$

or

$$T_1(x, t_m) = (1.73 - 84.56x + 1250x^2)10^3$$

and

$$T_2(x, t_m) = (1.677 - 3{,}600x^2)10^3$$

in dimensional form. (Fig. 3.3)

3.2.5 Movement of Phase Boundary During Solidification Stage

Substituting the following parameters:

$$D = -7.88, \quad (\eta-1)_m = 0.12, \quad (\Delta_1-1)_m = 1.25$$

and $\tau_m = 1.45$ into Eq. (2.80) we obtain

$$\eta'-1 = 0.224 - 0.374\sqrt{\tau'-1.373}$$

or

$$X' = 0.0184 - 0.00339\sqrt{t-3.76}$$

in dimensional form. (curve b in Fig. 3.1)

3.2.6 Penetration Depth in the Solid Phase During Solidification Stage

From Eq. (2.81), we obtain

$$\Delta_1' - 1 = 0.224 + 3.7\sqrt{\tau-1.373}$$

or

$$\delta_1' = 0.0184 - 0.0335\sqrt{t'-3.76}$$

in dimensional form.

3.2.7 Time Required for the Completion of the Whole Fusion Welding Process

From Eq. (2.85), we obtain

$$\tau_f = 12.07$$

or

$$t_f = 33.08 \text{ sec.}$$

3.2.8 Temperature Distribution in the Solid Phase During the Solidification Stage

Substituting $\theta_\infty = -0.999$, τ_m, η' and Δ_1' which we obtained in Sections 3.2.5 and 3.2.6, respectively, Eq. (2.70) results in

$$\theta_1'(\xi,\tau') = \frac{0.06}{\tau' - 1.373}\,[5.5 + 9.06\sqrt{\tau'-1.373} - 2.91\tau'$$

$$- 2(1.224 + 7.4\sqrt{\tau'-1.373})\xi + \xi^2]$$

or

$$T\,(x,t') = 660 + \frac{59.1}{t'-4.76}\,[5.5 + 5.47\sqrt{t'-4.76} - 1.06t'$$

$$- 133.3(1.224 + 4.47\sqrt{t'-4.76})x + 4444x^2]$$

in dimensional form (Fig.3.4)

3.2.9 Temperature Distribution at the Moment of the Finality of Solidification

Substituting $\tau_f = 12.07$ into $\theta_1'(\xi,\tau')$ we obtain

$$\theta_1'(\xi,\tau_f) = 0.00561\xi^2 - 0.285\xi$$

or

$$T_1'(x,t_f) = 660 - 6847x + 8985x^2$$

in dimensional form. (Fig. 3.5)

CHAPTER IV

DISCUSSIONS AND CONCLUSIONS

CHAPTER IV

DISCUSSIONS AND CONCLUSIONS

(1) For the case of fusion welding consisting of melting and solidification processes, the phase front of solid and liquid phases should move first of all, toward the right-hand side (Fig. 2.2) from its initial position during the melting stage and then, move toward the reverse direction during the solidification stage.

The relation generally used in simple melting or solidification process

$$X \sim \sqrt{\tau}$$

cannot be applied to the present case because the phase front moves only toward one direction of either the right-hand or left-hand side. Therefore, the relation of Eq. (2.45) was proposed in order to fulfill the melting and solidification processes. The results shown in Fig. 3.1 indicate that the function of Eq. (2.45) is suitable for such a problem.

(2) The fusion-welding process involves three dimensionless parameters, α, θ_∞ and θ_0. The range of α, the ratio of thermal diffusivities in the liquid and solid phase for various metals is from 0.25 to 0.50 [29]. Therefore, the influences of α in the whole process of the fusion welding are rather small. θ_∞ and θ_0 are the two main decision

factors of the process.

(3) In Fig. 2.3, it is seen that a drastic increase of the value of θ_0 is required in order to increase a small amount of the value of p, when p is beyond 0.3. A high value of θ_0 means that the liquid metal has a high temperature. In a real example, it is neither practical or economical to have an unreasonably high temperature in liquid metal. Therefore, the value of the constant p should be below 0.3 in most fusion welding cases.

(4) Depending on the values of the factors α, θ_∞ and θ_0, a melting process in solid metal during fusion welding may or may not occur. Should melting of the solid metal occur, the following condition

$$(\eta-1)_m > 0 \qquad (4.1)$$

must be satisfied.

We define the limit of this condition as

$$(\eta-1)_m = 0 \qquad (4.2)$$

which is the critical condition for the occurrence of the melting process.

The physical relationship required for the conditions indicated in Eqs. (4.1) and (4.2) can be obtained by the following two ways.

(i) As $(\eta-1)_m \geq 0$, we obtain from Eq. (2.82)

$$\frac{\alpha\theta_\infty^2 - \theta_0^2}{2(\alpha\theta_\infty^2-\theta_0^2) - 3\alpha(\theta_\infty^2-\theta_\infty\theta_0+\theta_0)} \geq 0$$

or

$$\frac{\theta_0^2 - \alpha\theta_\infty^2}{\alpha\theta_0^2 - \theta_0(3\alpha\theta_\infty - 2\theta_0 - 3\alpha)} \geq 0$$

The denominator of this equation is positive in value. Therefore, it requires that

$$\theta_0^2 - \alpha\theta_\infty^2 \geq 0$$

or

$$\theta_0 \geq \sqrt{\alpha}|\theta_\infty|$$

(ii) If the melting process occurs, the value of p in Eq. (2.45) must be positive, that is

$$p > 0$$

The condition $p = 0$ represents the critical condition for the occurrence of the melting process.

Then, from Eq. (2.63) we obtain for $p = 0$

$$\theta_0 = -\sqrt{\alpha}\,\theta_\infty$$

or

$$\theta_0 = \sqrt{\alpha}|\theta_\infty|$$

Therefore, we can conclude that

$$\theta_0 \geq \sqrt{\alpha}|\theta_\infty|$$

is the condition for the occurrence of the melting process.

(5) In the procedure of the determination of η_{max}, it was assumed that Δ_2 approaches zero as η reaches a maximum. The error on this assumption can be estimated as follows.

On the safe side of the error estimation, we assume that

$$(\theta_0 + \sqrt{\alpha}\ \theta_\infty) \quad \text{and} \quad (\eta-1)_m$$

are negligible for a very small melting depth.

From Eq. (2.44) we obtain

$$(\Delta_2-1)_m = -\sqrt{\alpha}(\Delta_1-1)_m$$

Using data in the example

$$\alpha = 0.407 \quad (\Delta_1-1)_m = 1.25$$

we obtain

$$(\Delta_2-1)_m = -0.8$$

Substitute

$$(\Delta_1-1)_m \quad \text{and} \quad (\Delta_2-1)_m$$

values into Eq. (2.44) we obtain

$$(\eta-1)_{m.n} = 0.071$$

or

$$\eta_{m.n} = 1.071$$

Compare to

$$\eta_m = 1.12$$

The error is

$$\varepsilon = \frac{\eta_{m.n}-\eta_m}{\eta_m}$$

$$\doteq 0.044 \text{ or } 4.4\%$$

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FIGURES

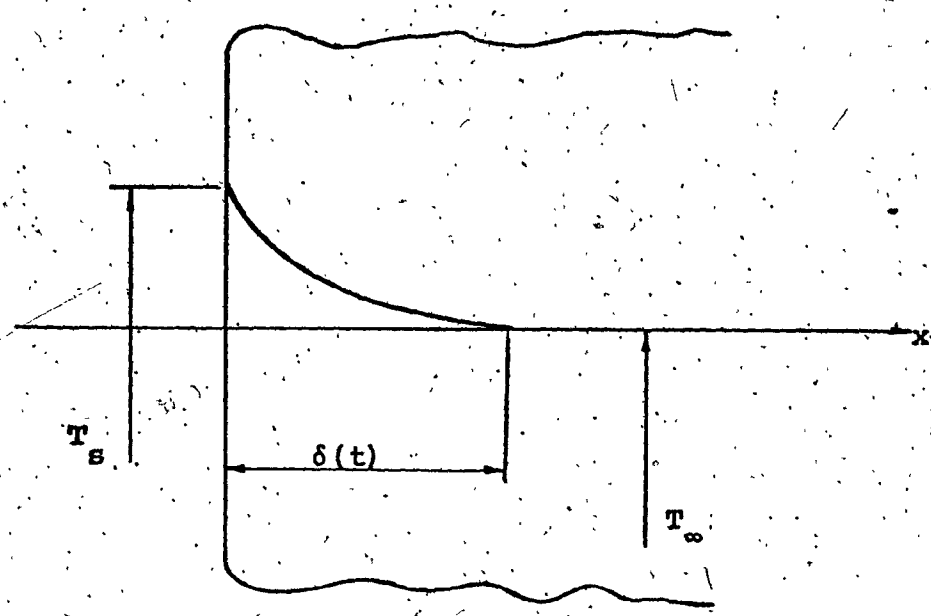


FIG. 2.1 Temperature Distribution in a Semi-Infinite Solid

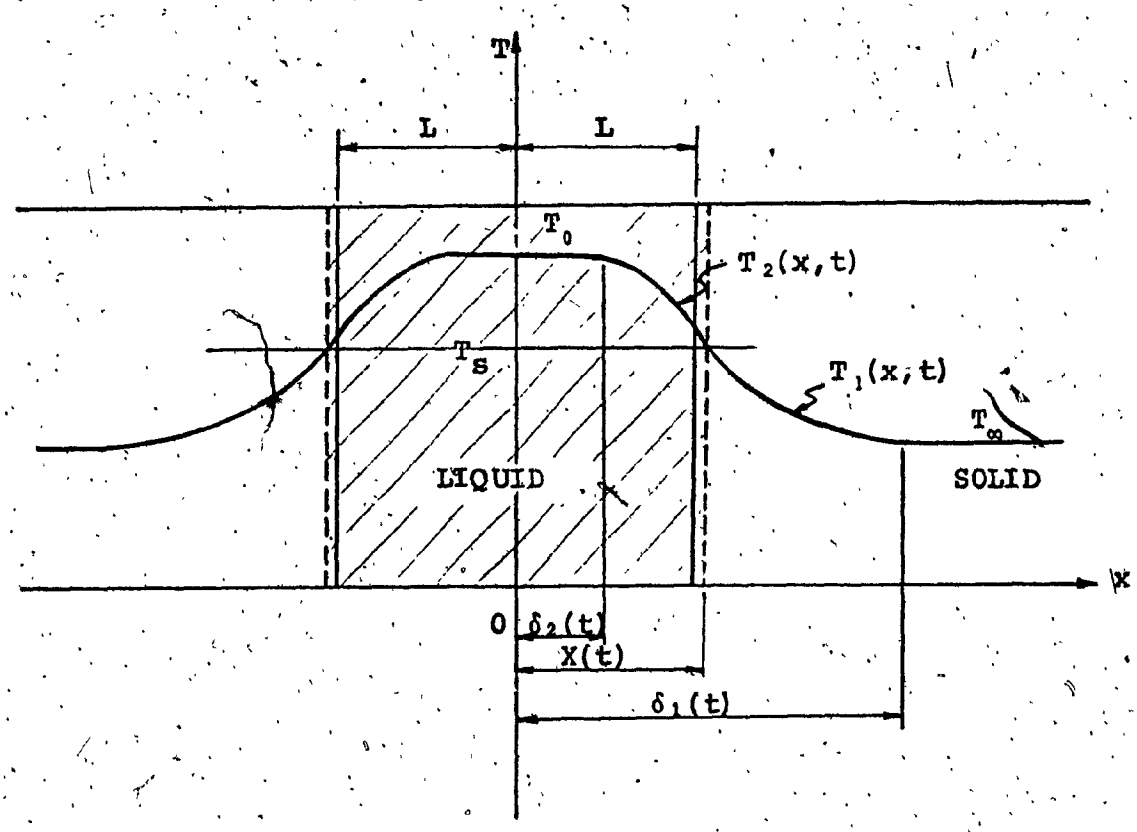


FIG. 2.2 Temperature Distribution in a Fusion Welding

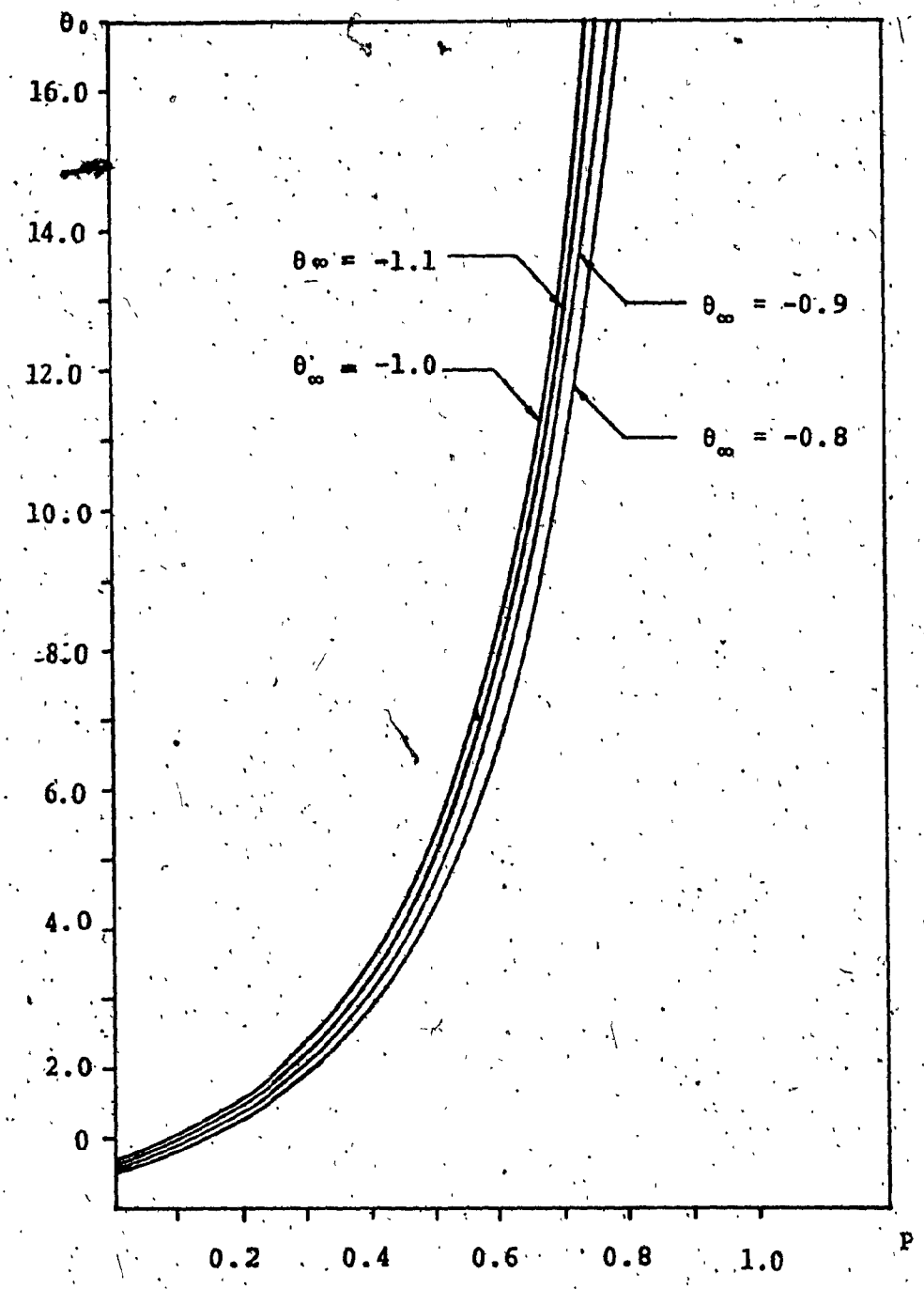
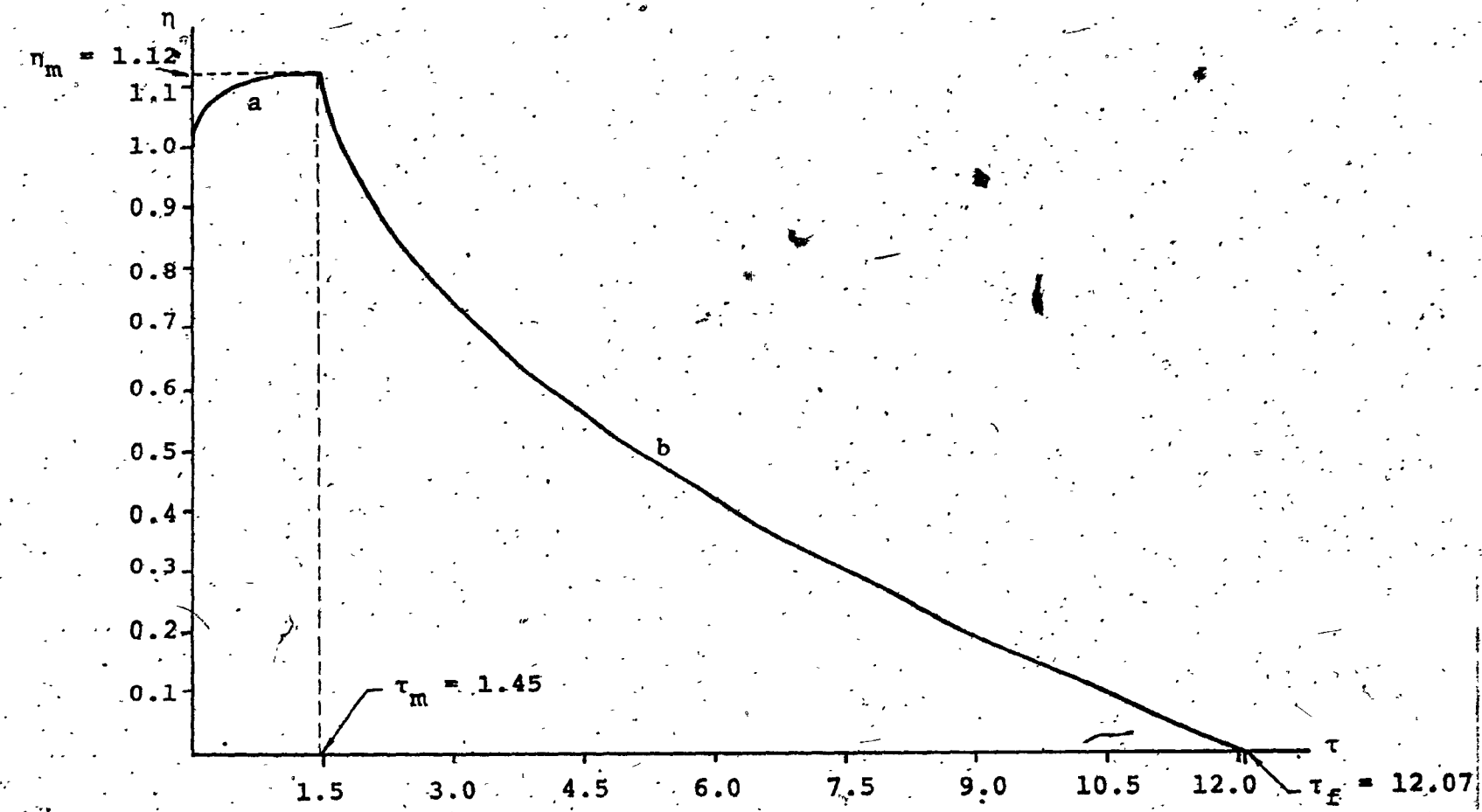
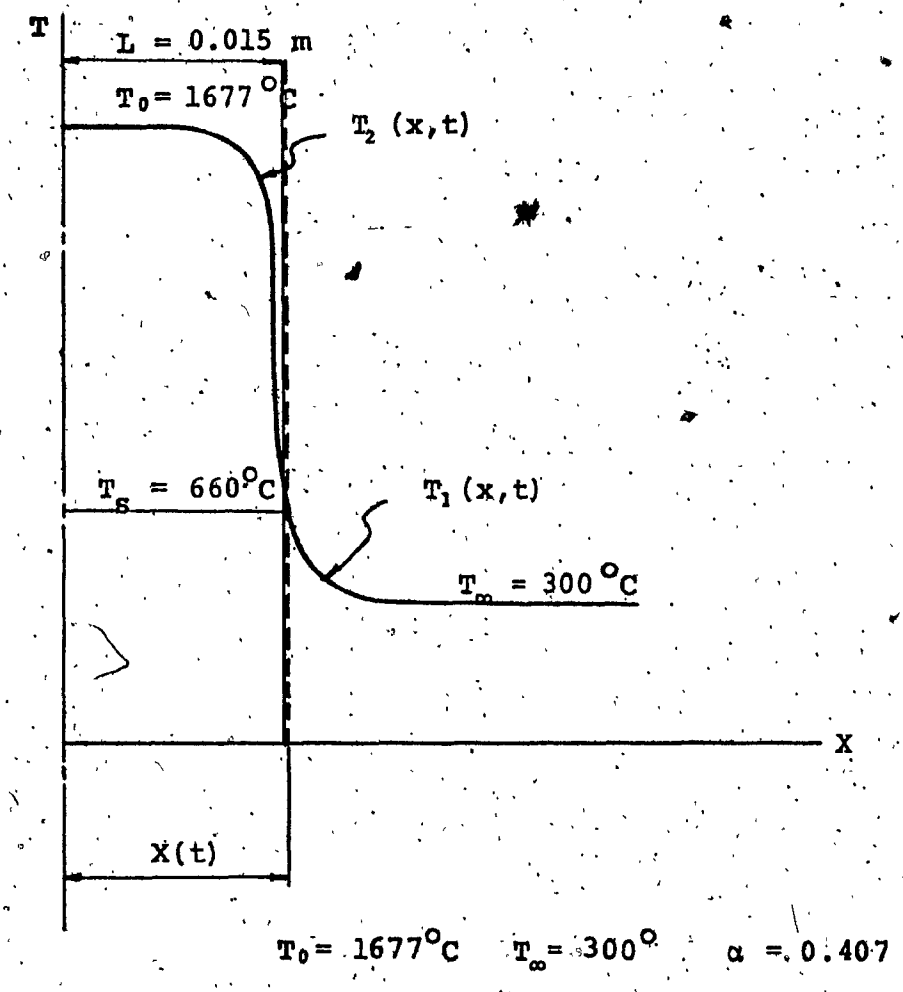


FIG. 2.3 Dimensionless Coordinate of Temperature θ_0 as a Function of Constant P With Dimensionless Temperature, θ_∞ as parameter and $\alpha = 0.407$ (Aluminum)



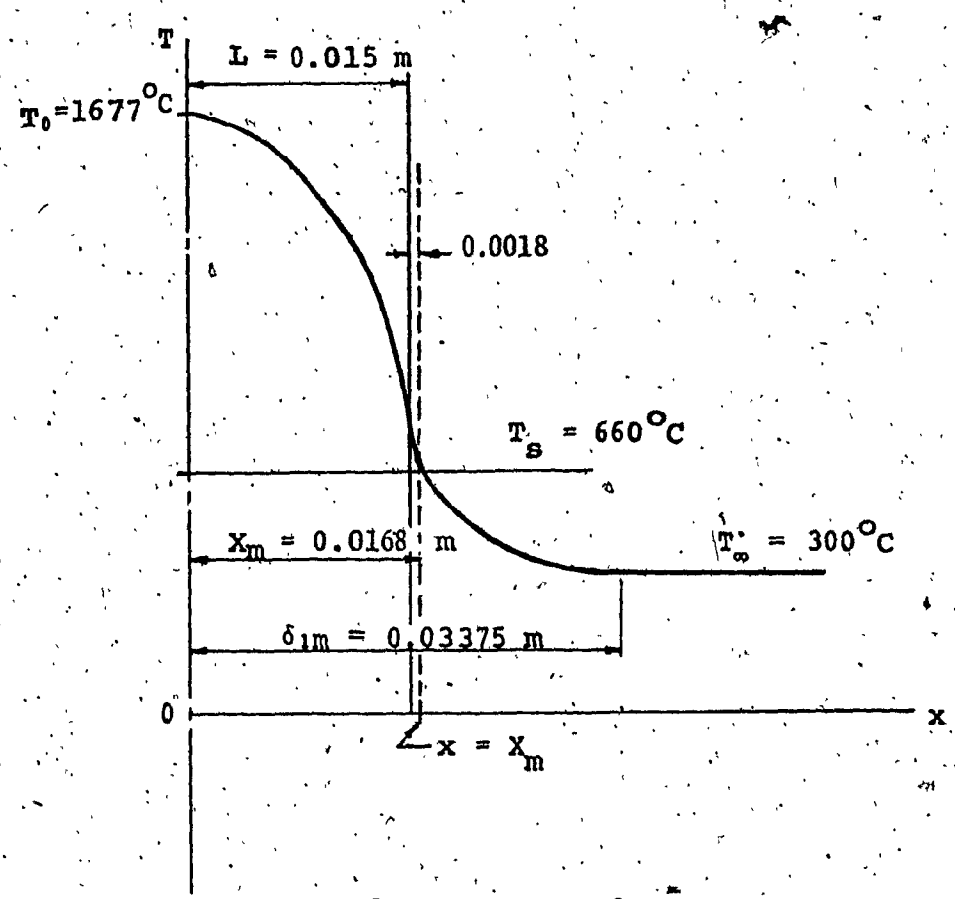
$\theta_\infty = -0.999 \quad \theta_0 = 0.991 \quad \alpha = 0.407$

FIG. 3.1 Dimensionless Coordinate of Phase Boundary η as a Function of Dimensionless Time τ for Aluminum Fusion Welding



$T_0 = 1677^\circ C$ $T_\infty = 300^\circ$ $\alpha = 0.407$

FIG. 3.2 Temperature Distributions in a Very Short Time at the Beginning of an Aluminum Fusion Welding (Schematic Graph)



T_∞ = 300°C T_0 = 1677°C α = 0.407

FIG. 3.3 Temperature Distributions at the Moment of Melting Stage Ending for an Aluminum Fusion Welding

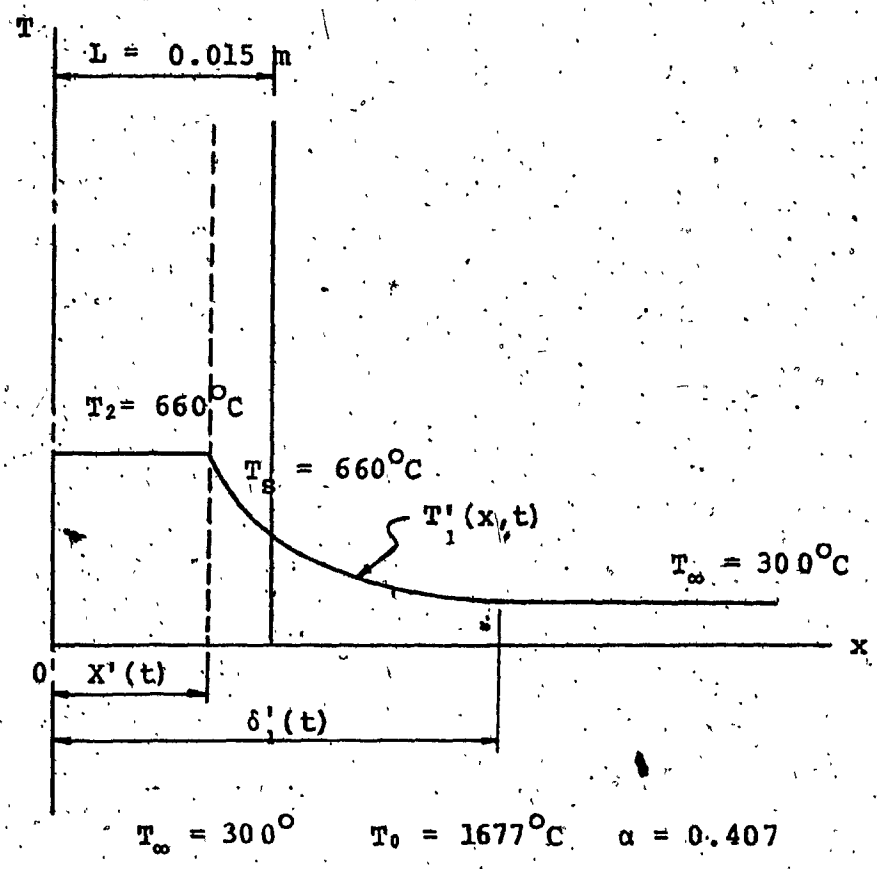


FIG.3.4 Temperature Distributions During Solidification Stage for an Aluminum Fusion Welding (Schematic Graph)

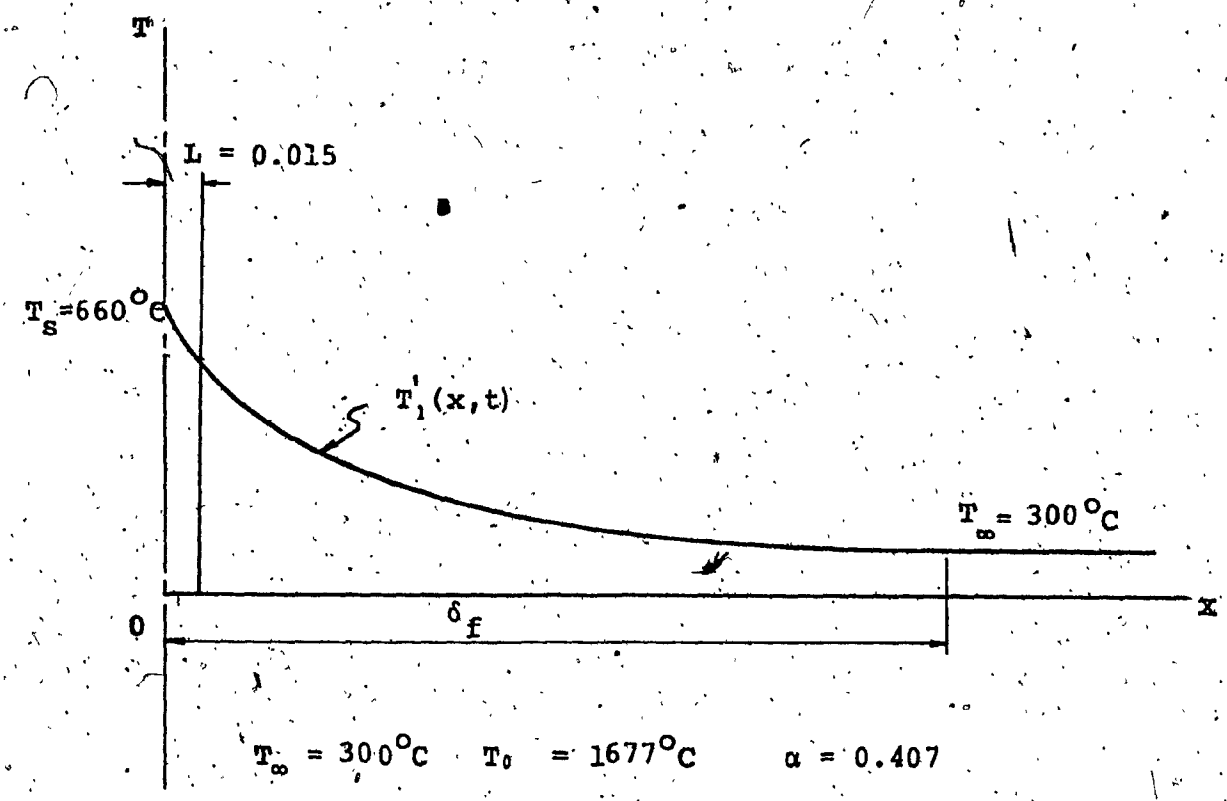


FIG. 3.5 Temperature Distribution at the Moment of Solidification Ending for an Aluminum Fusion Welding