OPTIMUM DESIGN OF SATellite ANtenna STRUCTURES
SUBJECTED TO RANDOM EXCITATIONS

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ABSTRACT

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Concordia University, 1982

An overall technique is presented for analyses and optimal design of satellite antenna structures subjected to random excitation of arbitrarily varying profiles of power spectral densities. The design which results in the minimum structural weight yet meets all the design reliability requirements has been considered as the optimum design.

A mathematical model for describing the random excitation with arbitrarily varying power spectral densities and for computing the responses of structures to such excitations, is proposed. The capability of performing a complete random response analysis using finite element program SPAR as the base, has been developed.

A technique of handling probabilistic constraints in mass optimization has been developed. The structural analysis finite element program and the optimization procedure have been linked together with several procedure files to form a unified design system.

A typical communication satellite antenna structure was analyzed and designed using this system. The analysis included determination of the eigen values, the mode shapes, the displacements and the stresses. To meet the design requirements, simultaneous constraints were imposed on the minimum frequency, the maximum displacement, and the maximum stress. A weight saving of 43 percent was realized using the proposed approach.
A method for incorporating the fatigue life requirements in the design of satellite antenna structures has been developed. The approach presented here for the analysis and design of satellite antenna structures is versatile in nature and the procedure to extend this approach for other applications is also presented.
TO MY WIFE
RANJANA
AND MY CHILDREN
MUNEESH AND SHALINI
ACKNOWLEDGMENTS

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NOMENCLATURE

b, c 
Fatigue parameters for materials which define S-N curve

[C] 
Damping matrix 

D 
Accumulated fatigue damage 

E[D(t)] 
Expected damage at time t 

f 
Frequency of input excitation 

f_1 
Lower limit frequency of a frequency band 

f_u 
Upper limit frequency of a frequency band 

F(x) 
Objective function at x 

G_j 
jth equality constraint 

H(\omega) 
Frequency response function 

G_j 
Inequality constraint functions 

[k] 
Stiffness matrix 

[K] 
Generalized stiffness matrix 

[m] 
Mass matrix 

[M] 
Generalized mass matrix 

N 
Slope of power spectral density in dB/octave 

P_j 
Probability function 

m 
grth mode shape 

RMS 
Root mean square value 

S_x 
Power spectral density of input excitation 

S_y 
Power spectral density of output excitation 

T 
Time in seconds 

W(\bar{x}) 
Weight of the structure
\( \bar{x} \) Design variable vector

\( \sigma \) mean square value of response

\( \eta_r \) rth generalized coordinates

\( \zeta \) structural damping coefficient

\( n_g \) probability of satisfying the constraints
CHAPTER 1

INTRODUCTION, LITERATURE SURVEY,
AND THE THESIS OBJECTIVES

1.1 General Objective

The concept of designing minimum weight structures for a given strength is gaining wide acceptance in industry. One particular industry where this criterion is of significant benefit and of vital necessity is the aerospace industry. Design and manufacture of communications satellites forms a significant portion of the work performed in the aerospace industry. The antenna of a communication satellite serves the important function of transmitting and receiving the signal to and from earth. The satellite antenna structures must be designed to withstand failure under any of the different environmental conditions encountered by the satellite and hence these structures have to be designed under very strict reliability requirements. At the same time these structures should be as light in weight as possible to minimize the cost of launching the satellite into space. The cost of launching one pound of payload into synchronous orbit is seventy thousand dollars as of 1981. The designer of the satellite structures is thus faced with two seemingly conflicting but essential requirements, namely high structural reliability and the lowest possible weight.

The problem is further complicated by the fact that the satellite structures are subjected to randomly varying excitations and hence the response quantities can be expressed only in a probabilistic sense. As a result, in any structural optimization, the constraints on these
parameters have to be formulated in a probabilistic manner.

The objective of this investigation is to develop a total analysis-optimization approach for designing minimum weight satellite antenna structures capable of withstanding the specified random excitations. For these satellite antennas the imposed random excitations fluctuate in a manner that their power spectral densities have an arbitrarily varying profile. In order to design minimum weight satellite antenna structures, a reliable technique of computing the response statistics of the structures subjected to such random excitations of arbitrarily varying power spectral density must be developed. Since analysis of these complex structures can be carried out using only numerical techniques based on a finite element modelling, a finite element program capable of performing a complete random response analysis must be developed.

There are several computer analysis programs developed for structural analysis and also for corresponding optimization. But there exists an absence of overall computer packages which unify the analysis and the optimization procedure in a well integrated fashion to enable automated optimal structural design. Such a package synthesizing analysis and optimization is necessary for designing the satellite antennas under the specified conditions mentioned above.

In summary, the objective of this investigation is to develop a unified approach for designing complex satellite antenna structures subjected to arbitrary random loads using structural analysis techniques coupled with an optimization procedure for minimum weight and which can handle probabilistic constraints on the response parameters of the
structure. Further, incorporation of certain structural integrity against fatigue failure of the antenna unit will also be attempted.

1.2. Description of a Satellite Antenna

Satellite antennas generally work at Super High Frequencies (SHF) ranging from one giga/Hertz upto 30 giga/Hertz. The electromagnetic waves in this frequency range are called the microwaves. At these frequencies, the same antenna serves the function of both receiving and transmitting communication signals. The complete antenna system consists of a reflector dish and an exciter. The reflector dish is the principal structural component of the antenna and the surface of the reflector has the profile of a paraboloid, that is, a surface described by a parabola rotated about its axis. The surface of the reflector is electrically conductive. The exciter is a small conductor located at the focus of the paraboloid. The unique property of the paraboloid surface is that any signal originating at the focus will be reflected back in a direction parallel to the line joining the vertex of the surface to its focal point. Microwaves follow very closely optical principles of reflection, refraction, etc. The exciter serves the function of transforming the high frequency electrical energy into energy of electromagnetic radiation and vice-versa and the reflector gives the required directivity to the transmitted and the received waves. The microwaves radiated by the exciter arrive at the reflector surface and are reflected back from it as a highly directional beam. Figure 1.1 shows the basic functioning of the antenna.

It is then of utmost importance that the satellite antennas do
Fig. 1.1. An Antenna Reflector.
possess high structural integrity under the action of the dynamic random vibrational environment experienced by the satellite during the launch conditions. The different structural requirements for the design of satellite antenna structures are outlined in the next section.

1.3 Description of the Problem

The solution requirements for the satellite antenna design can be divided into four major areas. Satellite antenna structures are designed with very low margins of safety in order to minimize the weight of the structure and hence a highly reliable estimate of their structural response is of crucial importance. The first requirement, therefore, is to reliably analyze and compute the complete dynamic response of these structures to random excitations of varying power spectral densities. Since the response of the structure has to be numerically computed, a finite element analysis is normally required for this purpose. Thus the second requirement is to develop an overall finite element software capable of carrying out the random response analysis. Thirdly, well formulated optimization algorithm is required to minimize the weight of the structure subject to the required-strength and frequency constraints. Finally, in order to implement the analysis technique, the finite element program and the optimization for the satellite antenna design, these three procedures have to be synthesized into a unified system in a logical sequence. These four essential requirements are now discussed below in detail.

1.3.1 Description of the Random Loading

When satellites with their antenna structures are launched into
space, they are normally placed at the nose cone of the launch vehicle as shown in Fig. 1.2. The acoustical noise loading from the jet exhaust of the rockets as well as the pulsations due to the extremely rapid burning of the fuel in the rocket engines generate severe vibrations in the launch vehicle structure. These vibrations are generally random and broadband in nature. The launch vehicle structure acts as a filter and the vibrations reaching the spacecraft structure in the nose cone region are no longer broadband. Thus the excitations to which the antenna structures are subjected, have a finite bandwidth with arbitrarily varying power spectral densities within this band. The conventional analysis techniques for computing the response of a structure subjected to random excitations, cannot be directly used, because they do not take into account the arbitrarily varying power spectral density of the input excitation. This means an appropriate mathematical model of the excitation process has to be implemented. Further, when the structure has to be designed to meet stringent reliability requirements against structural failure and are at the same time expected to have minimal weight, then a reliable response analysis and computation become very crucial.

Hence, a new analysis technique must be developed for proper determination of the response of these structures for excitations with arbitrarily varying power spectral densities and is attempted in this investigation.

1.3.2 Finite Element Structural Analysis Program

The satellite antenna is a complex structure and the computation
Fig. 1.2. Launch Configuration of a Typical Satellite
of the structural response can only be carried out using a numerical technique. Finite element technique of analyzing structures is a powerful and reliable technique and has been successfully employed in many structural problems for computing the response characteristics. Thus an appropriate finite element analysis for computing the structural response parameters is a necessary requirement for designing such antenna structures.

It is important that the finite element analysis program employed should be efficient in terms of computation time. The efficiency is of concern because the structure is complex and the analysis has to be repeated before an optimum design can be achieved. The flexible nature of the finite element analysis program is also important because it has to be integrated with the optimization program for the optimization procedure. Flexibility may also allow for partitioning of the analysis in such a way that only the necessary analysis is repeated during each iteration of the optimization cycle and thus saving considerable computational time and cost.

### 1.3.3 Optimization Algorithm

Since the weight of the structure has to be a minimum, an algorithm is required to iteratively change the design until an optimum design is achieved. The optimization algorithm selection is again of crucial importance in designing optimum weight structures. The algorithm chosen should be efficient and require minimum number of structural response analyses. Each response analysis is a time consuming operation, depending on the complexity of the finite element arrangement, and hence it is essential to keep the number of iterations down to a
realistic minimum. The algorithm adopted should be very stable, that is, it should have a guaranteed convergence. Any failure during the execution of the optimization process in the last few iterations can lead to frustratingly large time consumption. Since the response of the structure to random excitations are to be expressed in statistical terms, the constraints on the response parameters of the structure such as the stresses and the displacements have to be expressed only in a probabilistic manner. Thus, an appropriate technique for handling such probabilistic constraints must also be developed.

1.3.4 Analysis and Optimization Synthesizer

Even after reliable determination of the dynamic responses of the structure using a finite element approach and after the construction of an optimization algorithm, the actual task of designing the optimum weight antenna structures cannot be completed unless proper software connecting the analysis and the optimization is properly developed. This software development is also an essential component in solving the problem under investigation.

To summarize, it is required that a technique to compute the response of antenna structures to arbitrary random excitations be developed along with a finite element software which can be employed to compute all the structural responses. An optimization algorithm is required to obtain a minimum weight of the structure for a given strength specification. Finally both analysis and optimization must be integrated into a comprehensive design package which can be used to design minimum weight satellite antenna structures which are capable of withstanding the specified random vibration environment without any
structural failure.

For a given application the satellite antenna structure may have to sustain repeated application of random loads, thereby causing a fatigue failure for the structure. Hence a technique to handle fatigue life requirements for the antenna structure should be developed so that appropriate fatigue constraints can be incorporated into the design process.

1.4 Literature Review

The primary subject of discussion here is the minimum weight design of spacecraft antenna structures capable of withstanding specified randomly fluctuating loads without any structural failure. This problem required a review of several topics, such as the computation of response of structures in a random vibration environment, the finite element techniques of structural analysis, the optimization algorithms for minimizing functions, the techniques of handling constraints in optimization, and finally the problem of synthesizing the analysis and optimization procedures. The literature covering different aspects of all these fields is quite numerous and only the significant contributions in various fields directly related to the design problem are discussed in the following subsections 1.4.1 to 1.4.4.

The first subsection 1.4.1 deals with the problem of reliably computing the responses of structures subjected to random vibrations. The papers presenting approaches for computing response of structures to excitations different from wide band and white noise type are reviewed. The literature relating to fatigue considerations in structural
design is also discussed. The second subsection 1.4.2 is devoted to the review of the available software for the structural analysis using finite element techniques. The merits of the various analysis programs are presented in the context of their suitability for application in designing the minimum weight spacecraft antenna structures. In the third subsection 1.4.3, the literature dealing with optimization techniques is reviewed. The problem of handling different types of constraints is also discussed. The final subsection 1.4.4 covers the literature dealing with the problem of synthesizing the optimization and structural analyses for structural systems.

1.4.1 Determination of the Structural Response Under Random Loading Environment

As mentioned earlier, spacecraft antenna structures experience random excitations that have an arbitrarily varying power spectral density profile in the frequency domain. Earliest work on the subject of computing response of structures due to random excitations was done by Samuel and Eringen [1], who proposed a general analytical approach for the determination of structural response to random excitations using a generalized harmonic analysis. Their method assumes the existence of normal modes for structures and obtains the result in the form of a series solution. However, when the excitation is assumed to be of white noise type, the response quantities become infinite, which obviously are not valid in design. Crandall [2] considered different beam models with different damping mechanisms and obtained the response quantities for some cases even though the excitation was of wide band, white noise nature. The approximation of forcing as an equivalent white noise
produces dynamic response results which are not reliable enough for satellite antenna structures considered here. Roberts [3] was one of the earlier investigators who dealt with the problem of determining response due to band limited white noise type excitations. The response of a simple oscillator due to such excitations and the influence of the variation of the bandwidth upon the subsequent response were determined [3]. Bhat and Rao [4] studied the structural response due to acoustically generated random loads such as a typical aircraft jet noise. The power spectral density profile of such loading exhibits a peak at a particular frequency in the specified frequency range. The solution proposed by them was to fit an analytical function to describe the profile and then compute the response also analytically. This investigation [4] is specific to spectral profiles having only a single peak, while the profile of power spectral densities for spacecraft antenna structures may exhibit several peaks and the proposed method [4] cannot be applied in such cases. Another approach of handling a peaked power spectral density profile has been proposed by Jacobs and Lagerquest [5]. Here the forcing function is used in conjunction with a finite element analysis technique for analysing simple structures. Here, the structure is broken into several regions over which the external forcing function remains a constant. Each region is then treated as a separate finite element for analysis. This approach is limited to pressure fields on plate type structures. Pulgrano and Ablowitz [6] obtained the response of a single degree freedom system subjected to random excitations of continuously increasing or decreasing power spectral density profiles. Simple functions representing power spectral density by a straight line-
were used. Makhoul [36] has proposed use of linear prediction techniques for modelling profiles of power spectral densities arbitrarily varying in the frequency domain. In this method the time history of the signal corresponding to the actual power spectral density is expressed as a linear combination of the past outputs and the present and the past inputs. By minimizing the least square error in the time domain an expression for approximated power spectral density is obtained. The mathematical model obtained in this manner cannot be integrated analytically and this makes its application difficult for computing structural response.

1.4.2 Structural Analysis

The design of antenna structures under investigation requires that a finite element program be constructed taking into account all special requirements on the structure. There are several finite element programs that have been employed in industry for different structural dynamic problems. These programs have been reviewed for possible application for the purpose of the design problem under consideration.

The most sophisticated and comprehensive structural analysis program in existence to date is NASTRAN [7]. It is capable of performing both static and dynamic analysis. However, this program cannot be integrated into a particular design system because it lacks flexibility. Particularly, it cannot be partitioned into various segments for modifications and, hence, is not suitable for application here. There are other analysis programs such as STARDYNE [8], ANSYS [9] and EASE [10], which are commercially available. These programs cannot also be altered or modified and hence are not appropriate for this application. The
most appropriate base program for the present application is SPAR [11].

finite element program. This program is written in a modular form, that is, its various modules can be used in any desired combination to perform the required computations. There is a separate module for generating the mass matrix and another one for generating the stiffness matrix, and so on. This modularity in construction provides it with a great deal of flexibility and makes it ideally suited for extension and modification in terms of application to the present problem. But, SPAR does not have the capability of carrying out either the frequency response or a random response analysis. However, because SPAR has a modular nature, it has the capability of writing data into and read data from a user written Fortran program. Using this capability, the mass matrix, the stiffness matrix, the eigen values, and the mode shape information, etc, can be read into a Fortran program. This information can then be used to compute all the structural responses required.

1.4.3 Optimization Procedures and the Corresponding Algorithms

An optimization algorithm is required to obtain a minimum weight of the structure that is capable of withstanding the specified random vibration loads. The optimization techniques broadly fall under two categories, namely the direct search techniques and the gradient search techniques. Direct search techniques are easier to use because they do not require information on the gradients of the function, but are less efficient as compared to the gradient techniques. Most commonly used direct search technique of optimization is the Hooke and Teeves [12] technique. The direction of move during successive iterations is determined using the value of the design variables and the function in the
previous iteration. This method works well on well-behaved functions, but the logic breaks down quickly if a sharp discontinuity in the function is encountered. Nelder and Mead's [13] direct search technique works well on all types of objective functions. This method of search sets up a set of \( n+1 \) points in an \( n \) dimensional space called the simplex. It gropes towards the optimum solution by flipping, expanding or contracting the simplex and the logic used depends upon the evaluation of the function at each corner. This technique has the undesirable feature of requiring comparatively large number of functional evaluations. A gradient search technique due to Fletcher and Powell [14] is commonly employed in many studies. Here, the direction of the search is determined using the partial derivative information of the function at the current and the previous steps. The method was first proposed in 1964, but several modifications [15,16] of this method have been suggested. The method due to Zoutendijk [17] is also based upon gradient information, but this method is for constrained optimization problems. In this technique, the constraints are treated as separate functions. The step determined is such that the move at each step brings the solution or keeps it in the feasible region and hence the method is also known as the method of feasible directions. Vanderplatt and Moses [18] have modified this method to improve its efficiency and stability, by considering only the violated or near violated constraints in the optimization. If no constraints are active, only the function to be minimized is considered. The gradients of functions can be computed numerically or analytically. Arora and Hang [38] have presented techniques of computing analytical derivatives of structural response parameters. The gradients are obtained using the infor-
mation from the stiffness matrix, load matrix and the displacement response of the structure.

Handling of the constraints in optimization is carried out by minimizing a function that combines the objective function and the constraints. Minimization of this combined function then leads to an optimum solution that satisfies all the constraints. One such technique is due to Fiacco and McCormick [19]. The inverse of the constraint function is multiplied with a positive number, and is added to the function to be minimized to form the combined function. The inverse of the constraint function serves the purpose of keeping the design in the feasible region, because if the solution approaches a constraint boundary, the value of the inverse of the constraint function increases and the solution stays away from the constraints. This technique of handling constraints works well but tends to be inefficient because all the constraints are always included in the function to be minimized, hence has not been used for this investigation.

1.4.4 Structural Optimization

The minimum weight design of the spacecraft antenna structure requires simultaneous application of structural analysis and optimization techniques. Most of the work done previously in this area have been aimed at optimizing structures subjected to deterministic loads, that is either static loads or steady state harmonic loads. Dobbs [20] presented a method for automated design of structures subjected to static loads with constraints placed upon nodal displacements of the structure. This method is based upon Kuhn-Tucker [21] necessary conditions for achieving a local optimum. An iterative procedure is used to resize the structure
until the Kuhn-Tucker conditions are satisfied. Each iteration requires solution of a set of linear algebraic equations equal in number to the constraints. Rizzi [22] modified this method by including only the active constraints in the solution of the linear algebraic equations at each iteration, thereby making the method more efficient. Rubin [23] demonstrated the use of gradients in optimization by using a method based upon Lagrange multipliers. The constraint on the design performed was to meet the minimum frequency requirements. The method proceeds by first finding the design with the desired frequency and then by holding the frequency constant while the weight is minimized. The weight and frequency constraints are not simultaneously handled and this is an undesirable feature of this algorithm. These techniques are valid only for problems with constraint on frequency and displacement alone. Rao [37] has summarized approaches for optimizing structures under shock environment. The constraints on dynamic responses are considered. The constraints are deterministic unlike the constraints for the antenna structures which are probabilistic. Moreover the antenna design considered here requires simultaneous application of stress, displacement and frequency constraints.

Several other papers dealing with the structural optimization with frequency constraint exist in the literature. Sipel and Warner [24] proposed a technique of handling constraints which are in the form of partial differential equations. A sandwich plate subject to a constraint on lowest natural frequency was optimized as a continuous structure and this led to the constraints involving partial differential equations. A piecewise linear approximation technique [24] was suggested
to convert the constraints into a set of definable functions. The same technique has been extended to sandwich type plate structures by Cardou and Warner [25]. Treatment of structures as continuous structure is so far restricted to simple beam or plate like structures. Khot [26] compared all of the techniques of structural optimization which use conditions of optimum [21] as the basis. The only difference between various methods was found to be in the manner in which approximations are made to evaluate the Lagrange multipliers.

An approach to synthesize structural analysis and optimization algorithm has been proposed by Sobieski and Bhat [27]. This approach is used to analyze structures dealing with static loads and natural frequency constraints. The satellite antenna design requires that probabilistic constraints be imposed upon the dynamic response of the antenna structure, hence Sobieski and Bhat approach can be extended to include these considerations.

In summary, from a review of the existing literature, it is found that the problem of designing minimum weight structures capable of withstanding random excitations of arbitrarily varying power spectral densities has not yet been dealt with. An approach towards designing the minimum weight satellite antenna structures has been developed as part of this investigation, the details of this investigation are discussed in the next section.

1.5 Scope of the Present Investigation

The objective of this work is to develop an overall technique for analysis and design of minimum weight spacecraft antenna structures capable of withstanding any specified random vibrational environment, using
an acceptable structural analysis technique coupled with a reliable and efficient optimization procedure. The work leading up to the realization of this objective can be broadly divided into two categories, namely the analysis part and the design sector. First five chapters of this thesis deal with the analysis aspects of the problem while the last three chapters deal specially on the design and application aspects.

Since the random excitations experienced by the spacecraft antenna structure are not of the usual broadband white noise type, but have an arbitrarily varying profile of power spectral density. A mathematical technique to properly represent such excitations and to develop a method to compute the probabilistic responses of the structure to such excitations is then essential. In Chapter 2, a mathematical model is developed to represent such arbitrary random excitations taken from previous records. The proposed mathematical model envelopes the arbitrarily varying profile of random excitations by linear segments of different slopes. The actual excitation is then considered as a net sum of all these linear segments assuming linear behaviour of the structure. Since it is possible to calculate the response for each of these individual linear segments, the net response of the structural system can be determined by summing up the responses due to the excitation caused by each of these linear segments. Analytical expressions have also been developed for a general spectral profile. A simple cantilever was tested with random excitations of varying power spectral densities. The experimentally measured response shows good agreement with the analytically computed response using the proposed model. Experimental results verifying the validity of the mathematical model are also presented.
In Chapter 3, details of the basic development of a structural analysis program are given. The finite element program 'SPAR' has been used as the main frame of the analysis program. SPAR, as it exists, is not found suitable for application in here because it lacked the capability to perform the required frequency and random response analysis. Additional software has been generated, which together with the existing features of SPAR, makes it suitable for the present problem. Details of the program together with all the details of the additionally generated software are given in this chapter.

The fourth chapter describes the optimization algorithm. Since only the RMS value of the response parameters such as the displacement or the stress can be obtained for structures subjected to random excitations, the peak value of these response-parameters can only be expressed in conjunction with the probabilities of their occurrence. This implies that the constraints on the peak values of stress or displacement response parameters can only be expressed with a certain probability of satisfying these constraints. Conventional techniques of handling constraints can not be directly used to handle such probabilistic constraints. A method has been proposed to convert the probabilistic constraints into a set of equivalent deterministic constraints. The conversion is done in such a way that the probabilistic nature of the constraints is indirectly retained. This transformation permits the use of the existing techniques for handling constraints and for a successful design application for industry.

The fifth chapter describes the procedure for synthesizing the
structural analysis program developed and the optimization algorithm into one unified system. This task of synthesis is accomplished with the help of several procedure files which had to be written for this purpose. The logic governing the complete operation of this unified system is described. An example of designing a cantilever beam is included to demonstrate the performance of this analysis-optimization system.

Chapter 6 is devoted to the primary problem of design of a spacecraft antenna structure using the analysis-optimization system developed earlier. A circular dish with the surface of a paraboloid has been taken as a representative shape for the spacecraft antenna structure considered. Actual industrial specifications on the constraints are placed upon the required natural frequency, the peak displacement, and the peak stress. A vibrational environment typical of what an actual spacecraft antenna structure experiences has been used. Using the procedure developed, the complete design of the structure is carried out. The final design information results in the thickness of the face sheet of the antenna as well as the rib sizes of the supporting structure on the back of the antenna dish. Starting from the best guessed design, a saving of 47% in weight is demonstrated.

In Chapter 7, the design of the antenna under a fatigue environment is carried out. Generally, fatigue requirements are not placed upon spacecraft antenna structures due to the short duration of the application of the load. However, with the advent of the space shuttle, it is likely that these structures will have to be designed to withstand many launches and, therefore, fatigue will be an important factor in
design. With this in mind, the fatigue constraints have been developed which can automatically handle the imposed fatigue environment on the structure. The same antenna structure, as specified in Chapter 6, is designed again with the inclusion of the fatigue constraints. It is observed that the weight of the structure increases by 9.8% when fatigue constraints are included.

The technique developed here for designing the satellite antenna structures is general in nature and can be extended for designing other complex structures. The procedure to extend the proposed technique for designing other complex structures is discussed in Chapter 8. Design of a multi-storey building structure has also been discussed to illustrate the use of the proposed design approach. Finally conclusions and recommendations for future work are presented in the ninth chapter.
CHAPTER 2

MATHEMATICAL MODELLING OF RANDOM EXCITATIONS WITH

ARBITRARILY VARYING POWER SPECTRAL DENSITY

Satellite antenna structures are often subjected to randomly varying excitations which have their power spectral density varying in an arbitrary manner in the frequency domain. In order to analyse and design these structures against such loading, it is essential to describe the power spectral densities of excitations analytically in a functional form in order to compute the response process. Although a mathematically exact description of the actual power spectral density variation to which the structures are subjected is not normally feasible, there are means to describe them approximately which are sufficiently reliable in most practical situations. For example, when the power spectral density is almost flat over a very large frequency range of interest, it is meaningful to model it as a pure white noise [28], or if its flatness is limited to a certain discrete range of frequencies it can be approximated as a band limited white noise [3]. Profiles of many typical power spectral density variations which are analytically describable for setting up system response equations are reported in [4]. However, when the power spectral density of the excitation varies arbitrarily and shows discontinuities in the frequency domain, an exact analytical modelling becomes often impossible, for carrying out analytical computations for the response.

A mathematical model for effectively describing excitations having arbitrarily varying power spectral densities in the frequency domain is proposed in this chapter. The mathematical model is developed
analytically by enveloping the profile of the spectral function by linear segments of varying slopes with the power spectral densities being plotted in a logarithmic scale. The net excitation is then the sum of the different excitations contributed by these individually arranged segments. Since it is possible to calculate the response of the system for these individual segments, the net response of the system can be obtained by summing up the individual responses for the linear cases. Analytical expressions are derived for a general profile and experimental verification for the validity of the model is also reported in this chapter.

2.1. Nature of Random Excitations in Spacecraft Applications

In most of the spacecraft structures, random vibrations are generated by the thrust of the rocket engines of the launch vehicle. The techniques of predicting response of the associated structures under such random excitations, are fairly well established [29,30]. These techniques assume that the input excitations are of broad band in nature and are nearly constant in the neighbourhood of the resonant frequencies of the structure. The excitation is assumed essentially as a Gaussian process with a zero mean and a specified variance. However, for satellite antenna structures, the input excitation is seldom broad band of the type mentioned above.

For satellite antennas that is of interest in this investigation, the filtering action of the launch vehicle structure transforms the broad band thrust excitations into certain narrow band random excitations. This transformation from the wide band to the narrow band exci-
tation takes place because the launch vehicle and the space craft structure act as a broad band pass filter, shown in Fig. 2.1. This narrow band excitation then becomes the input forcing to the antenna structure. The power spectral density no longer remains constant or flat over the frequency band of interest. A typical profile of the input excitation experienced by the antenna structure is described in Fig. 2.2. Because the satellite antennas are designed with high reliability and low margins of safety in order to reduce the payload due to the structural weight, it is essential to know the response of the antenna structure to such excitations as reliably and accurately as possible. The standard approaches [29,30] normally valid for other engineering problems cannot be directly taken for application for computing responses under such excitations. The details of the mathematical modelling technique to describe these types of excitations that have an arbitrarily varying power spectral density in the frequency domain and the methods for determination of the response of structures to such inputs are elaborated in the subsequent sections.

2.2 The Mathematical Model for the Excitation Process

A typical profile of the arbitrary power spectral density variation in the frequency domain to which satellite antenna structures are subjected is illustrated in Fig. 2.2. The basic concept behind the proposed analytical model is that any given profile of the power spectral density variation in the frequency domain can be enveloped by a series of appropriately chosen linear segments particularly when the power spectral densities are plotted in a logarithmic scale. These linear
Fig. 2.1. Transformation of Rocket Broadband Excitations into Narrowband Forcing in Satellite Launching.
segments may then have negative, positive or zero slopes. Figure 2.3 shows how a general profile can be enveloped by an arrangement of such linear segments. Peaks in the region of 100-200 Hz can also be enveloped by additional linear segments. However, if the natural frequencies of the structure do not fall in this region, it is appropriate to model the profile as shown in Fig. 2.3.

As mentioned earlier, this envelope model essentially involves three types of profile segments. They are segments with positive slopes, segments with zero slopes, and segments with negative slopes. In Fig. 2.4, these segments are represented by sample line segments, AB, BC, and CD respectively. The relationship between the slope specified in dB/octave, the power spectral density value and the frequency is given by

\[ S_X(f) = S_X(f_e)(f/f_e)^{N/3} \]  \hspace{1cm} (2.1)

where

\[ N = \text{slope of the segment in dB/octave} \]
\[ f = \text{frequency at any point within the segment in Hz or cycles/sound} \]
\[ S_X(f) = \text{power spectral density value at frequency } f \]
\[ f_e = \text{frequency at the beginning of the envelope process} \]
\[ \text{(e.g., } f_1 \text{ in Fig. 2.4 for region AB)} \]
\[ S_X(f_e) = \text{power spectral density value at } f_e \]

2.3 **Determination of the Structural Response**

Assuming linear characterization of the total structure, the net response of the structure is the sum of the response due to various linear segments and can be computed as follows.
Fig. 2.3. Linear Segments Envelope of the Power Spectral Density Profile.
Fig. 2.4. Power Spectral Density of Excitation Enveloped by Three Linear Segments.
The response power spectral density for a single degree linear structural system subjected to random excitations is given by the standard relation

\[ S_Y(f) = |H(f)|^2 \cdot S_X(f) \quad (2.2) \]

where

- \( S_X(f) \) = power spectral density of the input
- \( S_Y(f) \) = power spectral density of the output
- \( H(f) \) = the complex frequency response of the system relating the input and the output.

The mean square value of the response can then be obtained from equation (2.2) as

\[ \sigma^2 = \int_{f_1}^{f_2} S_X(f) |H(f)|^2 \, df \quad (2.3) \]

In general, it is possible to consider any slope, \( N \), of the excitation power spectral density profile to obtain the mean square response using equation (2.3). However, the integral in equation (2.3) can be evaluated analytically only if the numerical value of the term \((N/3)\) in equation (2.1) is an integer. If this is not the case, one can obtain only the mean square value of the response by appropriate numerical evaluation of the integral in equation (2.3).

In the present analysis the value of \((N/3)\) is considered to be an integer to enable closed form evaluation of the integral in equation (2.3) analytically. As an example, a profile is represented completely by three different segments in Fig. 2.4. The expressions after carrying out the necessary integration for these three different types of linear
segments in Fig. 2.4 are given in the following sections.

The general expression for the mean square response in the three regions shown in Fig. 2.4 is:

\[ \sigma^2 = \frac{S_X(f_L)}{f_L^N/3} \int_{f_L}^{f_u} f^{N/3} |H(f)|^2 \, df \]  \hspace{1cm} (2.4)

where

- \( f_L \) = lower limit of the frequency for the segment
- \( f_u \) = upper limit of the frequency for the segment
- \( S_X(f_L) \) = the excitation power spectral density value at the lower frequency limit \( f_L \) of the segment
- \( N \) = slope of the particular linear segment in dB/octaves
- \( H(f) \) = the complex frequency response of the system

The mean square response parameter of the antenna structure calculated through equation (2.4) can be either displacement, velocity or the acceleration, depending upon the type of frequency response function \( H(\omega) \) used in evaluating equation (2.4). In antenna design the design specifications require use of the acceleration and the displacement response for the structure, hence expressions for the acceleration and the displacement response are derived in the following pages.

2.3.1 The Acceleration Response

The frequency response relating the base acceleration excitation and the acceleration response for a single degree of freedom system is given by

\[ H(f) = \frac{f_n^2 + 21\zeta ff_n}{f_n^2 - f^2 + 21\zeta ff_n} \]  \hspace{1cm} (2.5)
where $f_n$ is the natural frequency of the system and $\xi$ the damping ratio. Substituting equation (2.5) into equation (2.4) and expressing frequencies as nondimensional ratios in the form $\Omega = f/f_n$, the mean square acceleration response is computed as

$$\sigma^2 = \frac{S_X(f_2)}{f_2^{N/3}} \cdot \frac{f_n^{(1+N/3)}}{f_n^{N/3}} \Omega \int_0^\Omega \frac{\Omega^{N/3}(1 + 4\xi^2\Omega^2)d\Omega}{(1-\Omega^2)^2 + 4\xi^2\Omega^2}$$  \hspace{1cm} (2.6)$$

For any positive or negative value of the integer $(N/3)$, the expression within the integral in equation (2.6) can be broken down into partial fractions and integrated in closed form. The most common slope used for specifying a given random loading environment in spacecraft structures is 6 dB/octave, and hence the expression in equation (2.6) is integrated for $N = \pm 6$ for the segments AB and CD and $N = 0$ for the segment BC, as shown in Fig. 2.4. The response for each of these segments is calculated as follows:

a) Acceleration response under linear segment AB:

Substituting $N = 6$ in equation (2.6) and noting that $f_2 = f_1$, $f_u = f_2$ and $S_X(f_2) = S_1$ from Fig. 2.4, the mean square response acceleration is

$$\sigma_{AB}^2 = \frac{S_1}{f_1^3} \cdot \frac{f_n^3}{f_1^2} \int_0^\Omega \frac{\Omega^2(1 + 4\xi^2\Omega^2)d\Omega}{(1-\Omega^2)^2 + 4\xi^2\Omega^2}$$  \hspace{1cm} (2.7)$$

Breaking the integrand into partial fractions

$$\frac{\Omega^2(1+4\xi^2\Omega^2)}{(1-\Omega^2)^2+4\xi^2\Omega^2} = 4\xi^2 + \left(\frac{1+8\xi^2-16\xi^4}{41\xi^2}\right) \left\{ \frac{\Omega}{1-\Omega^2-21\xi\Omega} - \frac{\Omega}{1-\Omega^2+21\xi\Omega} \right\}$$

$$- 2\xi^2 \left\{ \frac{1}{1-\Omega^2-21\xi\Omega} + \frac{1}{1-\Omega^2+21\xi\Omega} \right\}$$  \hspace{1cm} (2.8)$$
Using solutions for standard integrations of the type
\[ \frac{\omega d\omega}{1 \pm \omega^2 + 2\zeta \omega} \]
and integrating
\[ \sigma_{AB}^2 = \frac{S_1 \cdot f_n^3}{f_1^2} \left[ I_{AB}(\omega_2, \zeta) - I_{AB}(\omega_1, \zeta) \right] \quad (2.9) \]
where
\[ I_{AB}(\omega, \zeta) = 4\zeta^2 \omega + \frac{(1 + 8\zeta^2 - 16\zeta^4)}{4\zeta^2} \tan^{-1} \left( \frac{2\zeta \omega}{1-\omega^2} \right) \]
\[ - 1 + 16\zeta^2 - 16\zeta^4 \ln \frac{\omega^2 + 2\zeta(1-\zeta)^2 + 1}{\omega^2 - 2\zeta(1-\zeta)^2 + 1} \quad (2.10) \]

b) Acceleration response under linear segment CD

Substituting \( N = -6 \) in equation (2.6) and noting that \( f_y = f_3 \), \( f_u = f_4 \) and \( S_x(f_y) = S_3 \) from Fig. 2.4, the mean square response acceleration is
\[ \sigma_{CD}^2 = \frac{S_x(f_y)}{f_n^2} \int_{\omega_3}^{\omega_4} \frac{(1 + 4\zeta^2 \omega^2) d\omega}{\omega^2 \left[ (1-\omega^2)^2 + 4\zeta^2 \omega^2 \right]} \quad (2.11) \]

Breaking the integral again into partial fractions
\[ \frac{1 + 4\zeta^2 \omega^2}{\omega^2 \left[ (1-\omega^2)^2 + 4\zeta^2 \omega^2 \right]} = \frac{1}{\omega^2} + \frac{1}{4\zeta^2} \left[ \frac{-\omega}{1-\omega^2-2i\zeta \omega} - \frac{\omega}{1-\omega^2+2i\zeta \omega} \right] \]
\[ + \frac{1}{1-\omega^2+2i\zeta \omega} + \frac{1}{1-\omega^2-2i\zeta \omega} \quad (2.12) \]

Using the standard integrals
\[ \sigma_{CD}^2 = \frac{S_3 \cdot f_3^2}{f_n} \left[ I_{CD}(\omega_4, \zeta) - I_{CD}(\omega_3, \zeta) \right] \quad (2.13) \]
where
\[ I_{CD}(\omega, \zeta) = -\frac{1}{\omega} + \frac{1}{4\zeta^2} \tan^{-1} \left( \frac{2\zeta \omega}{1-\omega^2} \right) + \frac{3}{8(1-\zeta)^2} \ln \left[ \frac{\omega^2 + 2\zeta(1-\zeta)^2 + 1}{\omega^2 - 2\zeta(1-\zeta)^2 + 1} \right] \quad (2.14) \]
c) Acceleration response under linear segment BC

Substituting $N = 0$ in equation (2.6) and noting that $f_1 = f_2$, $f_u = f_3$ and $S_x(f_3) = S_2$ from Fig. 2.4, the mean square response acceleration is calculated as:

$$\sigma_{BC}^2 = S_2 f_n [I_{BC}(\alpha_3, \zeta) - I_{BC}(\alpha_2, \zeta)]$$  \hspace{1cm} (2.15)

where

$$I_{BC}(\alpha, \zeta) = \frac{(1 + 4\zeta^2)}{4\zeta} \tan^{-1} \left( \frac{2\zeta \Omega}{1 - \zeta^2} \right)$$

$$+ \frac{(1 - 4\zeta^2)}{8(1 - \zeta^2)\frac{1}{2}} \ln \left[ \frac{\zeta^2 + 2\zeta(1 - \zeta^2)^{1/2} + 1}{\zeta^2 - 2\zeta(1 - \zeta^2)^{1/2} + 1} \right]$$  \hspace{1cm} (2.16)

2.3.2 The Displacement Response

The frequency response relating the base acceleration excitation and the displacement response relative to the base for a single degree of freedom structural system is given by

$$H(f) = \frac{1}{f_n^2 - f^2 + 2i\zeta f f_n}$$  \hspace{1cm} (2.17)

where $f_n$ is the natural frequency of the system and $\zeta$ is the damping of the system. Substituting equation (2.17) in equation (2.4) and expressing the frequencies in the form of nondimensional ratio $\Omega = f / f_n$, the mean square displacement response may be determined through the expression

$$\sigma^2 = \frac{S_x(f_3)}{f_k^{N/3}} \cdot \frac{f_n^{(N/3 - 1)}}{f_k^{N/3}} \int_{\frac{\Omega_k}{(1 - \Omega^2)^{1/2} + 4\zeta^2 \Omega^2}}^{\Omega_u} \Omega^{N/3} d\Omega$$  \hspace{1cm} (2.18)

Equation (2.18) can now be integrated to yield the displacement response for any of the linear segments AB, BC or CD, shown in Fig. 2.4.
a) Displacement response under linear segment AB

Substituting $N = 6$ in equation (2.18) and noting that $f_1 = f_2$, $f_u = f_2$, and $S_X(f_2) = S_1$, the mean square displacement response is

$$
\sigma_{AB}^2 = \frac{S_1}{f_1^2 \cdot f_n} \left[ I_{AB}(\alpha_2, \xi) - I_{AB}(\alpha_1, \xi) \right] \tag{2.19}
$$

where

$$
I_{AB}(\alpha, \xi) = \frac{1}{4\xi} \tan^{-1} \left\{ \frac{8\xi\alpha}{1 - \alpha^2} \right\} - \frac{1}{8\sqrt{1 - \xi^2}} \ln \left[ \frac{\alpha^2 + 2\alpha \sqrt{1 - \xi^2} + 1}{\alpha^2 - 2\alpha \sqrt{1 - \xi^2} + 1} \right] \tag{2.20}
$$

b) Displacement response under linear segment CD

Substituting $N = 6$, in equation (2.18), and noting that $f_1 = f_3$, $f_u = f_3$, and $S_X(f_3) = S_3$, the mean square displacement response is

$$
\sigma_{CD}^2 = \frac{S_3 f_3^2}{f_n^2} \left[ I_{CD}(\alpha_3, \xi) - I_{CD}(\alpha_2, \xi) \right] \tag{2.21}
$$

where

$$
I_{CD}(\alpha, \xi) = \frac{1 - 4\xi^2}{4\xi} \tan^{-1} \left\{ \frac{2\xi\alpha}{1 - \xi^2} \right\} - \frac{1}{\alpha} \\
+ \left( \frac{3 - 4\xi^2}{8\sqrt{1 - \xi^2}} \right) \ln \left[ \frac{\alpha^2 + 2\alpha \sqrt{1 - \xi^2} + 1}{\alpha^2 - 2\alpha \sqrt{1 - \xi^2} + 1} \right] \tag{2.22}
$$

c) Displacement response under linear segment BC

Substituting $N = 6$ in equation (2.18)

$$
\sigma_{BC}^2 = \frac{S_2}{f_n} \left[ I_{BC}(\alpha_2, \xi) - I_{BC}(\alpha_3, \xi) \right] \tag{2.23}
$$

where

$$
I_{BC}(\alpha, \xi) = \frac{1}{4\xi} \tan^{-1} \left\{ \frac{2\xi\alpha}{1 - \alpha^2} \right\} + \frac{1}{8\sqrt{1 - \xi^2}} \ln \left[ \frac{1 + 2\alpha \sqrt{1 - \xi^2} + \alpha^2}{1 - 2\alpha \sqrt{1 - \xi^2} + \alpha^2} \right] \tag{2.24}
$$
Thus, if any general excitation were to be approximated by \( n \) linear segments of positive, negative or zero slopes, then the total response of the structure may be obtained by the superposition principle stated by

\[
\sigma_{\text{total}}^2 = \sum \sigma_i^2
\]

where \( i = 1, n \ldots \) \hspace{1cm} (2.25)

where \( n \) is the number of segments
\( \sigma_i \) is the RMS response of the \( i \)th segment

### 2.4 Design Guidelines

The number of linear segments to be selected in enveloping a particular profile of power spectral density variation is entirely up to the designer's requirement on accuracy of response for assessing reliability and integrity of the system. However, the error in the mathematical modelling of the profile through such segment representation will decrease with an increase in the number of segments chosen. As an example the profile of Fig. 2.2 can be represented by linear segments ranging in total number from 1 to any desired upper limit. For comparison, the RMS error has been computed for varying number of segments. The error sampling has been done at intervals of 100 Hz in a frequency band ranging from 0 - 2000 Hz.

Figure 2.5 explains how the errors decrease as the chosen number of segments increases. It can be seen from the figure that with four to five segments, the profile can be represented in an acceptable manner with an error of only 10 percent. Thus as a guideline four to five segments are recommended as reasonable; however, the designer must consider these results in the context of a particular profile he is
Fig. 2.5: Effect of Choice of Number of Segments Upon the RMS Error.
designing the system for.

2.5 Experimental Verification of the Excitation Model

The object of this experimental procedure is to verify the validity of the analytical results stated previously by comparing the experimentally obtained response with the analytically computed response. Since any profile can be represented by a combination of positive, negative and zero slope segments, a profile consisting of these three types of segments was chosen as the input excitation. The profile used for the experiment is reproduced in Fig. 2.4. To simplify the manufacturing aspects of the structure to be tested and for reason of economy, a simple cantilever beam was chosen as the structural system that is subjected to random vibrations of the type described in this investigation.

The test set up, described schematically in Fig. 2.6, consists of a simple cantilever mounted on a special support. The structure was mounted on a vibration table and was subjected to base accelerations. The structure tested was an aluminum cantilever beam of length 30 cm and a cross section of 2.5 x 1.25 cm. The structure was initially subjected to pure harmonic excitation at varying frequencies in order to obtain the natural frequencies of the system. The frequency response of the system is given in Fig. 2.7 which shows that the first two natural frequencies occur at 102 and 645 Hz. The damping ratios of the system associated with the first two natural frequencies were calculated from Fig. 2.7 and are found to be 0.004 and 0.00625. Subsequently, the structure was subjected to a random acceleration input at the support and the response was measured at the tip of the beam. An excitation
Fig. 2.6. The Experimental Set-Up.
Fig. 2.7. The Frequency Response Plot of the Cantilever Structure.
signal having a power spectral density profile same as the one shown in Fig. 2.4 was recreated in the tests. The positive and negative slopes were 6 dB/octave and the limiting frequencies were \( f_1 = 20 \text{ Hz} \), \( f_2 = 200 \text{ Hz} \), \( f_3 = 400 \text{ Hz} \) and \( f_4 = 700 \text{ Hz} \) corresponding to Fig. 2.4. The power spectral density of the excitation from \( f_2 \) to \( f_3 \) was taken as 0.1 \( \text{g}^2/\text{Hz} \). The excitation signal was chosen in such a manner that the two regions with the non-zero slopes contained the first and second natural frequencies of the structure. The root mean square acceleration measured at the tip of the structure was 62 g RMS. The response plot of the structure is shown in Fig. 2.8.

The computed value of root mean square acceleration using the analytical expressions derived previously in this Chapter was 61.3 g RMS, showing a good agreement with the test results. Thus, it can be seen that the analytically obtained response is within 2 percent of the measured response. It was also demonstrated earlier that by choosing sufficiently large number of segments a mathematical model of a general profile can be reliably generated to compute the response of the structure. This experiment verifies the validity of this type of modelling technique for describing any arbitrary power spectral density of excitation for analytically evaluating the responses of a linear dynamic system.

The usefulness of the proposed model is also demonstrated when the measured response results are compared with the computed response using the standard white noise approximation to represent the excitations. The analytically computed response using such wide band approximation is
100.1 grms which is 161 percent of the measured response. As expected, the white noise modelling of the input process gives conservative results which in turn implies that the weight of the structure will be higher if white noise approximation were to be used as compared to the proposed model.

2.6 Conclusions

A mathematical model is proposed for describing any type of power spectral density input which varies rather arbitrarily in the frequency range of interest. The model essentially envelopes the power spectral density with linear segments of rising and falling slopes as well as flat portions in decibel versus octave frequency scale. Mean square acceleration response of a typical dynamic system is evaluated using a sample power spectral density of excitation having one segment of rising slope, one flat segment and one segment of falling slope. Experiments were conducted on the same dynamic system under a similar power spectral density input which is synthesized in the same segmental fashion as described in the analytical study. The agreement between the experimentally measured mean square acceleration and that evaluated analytically using the proposed model is quite good, thereby validating the usefulness of such an approach in evaluating the response of any dynamic system to any arbitrary power spectral density of excitation.

The mathematical model developed in here has been employed in calculating the response of a satellite antenna structure. The finite element program used to mathematically model the satellite antenna structure and the computation of the response process is described in the next Chapter.
CHAPTER 3

STRUCTURAL RESPONSE USING FINITE ELEMENT APPROACH

A primary objective of this investigation is to develop a generalized methodology to design the spacecraft antenna structures which possess minimum possible payload and which are capable of reliably withstanding a specified random vibrational environment without any component structural failure. All optimum structural design problems under dynamic considerations require two essential elements, namely an analyzer and an optimizer. The function of the analyzer is to compute all the necessary dynamic response parameters such as accelerations, velocities or displacements at different locations in the structure particularly subjected to a random vibrational environment. The function of the optimizer is to use this information from the analyzer to calculate the new set of values of the system design parameters which would result in lowest possible weight under certain given criteria. The details on the analyzer are presented in this chapter. The details regarding the optimizer requirements are elaborated in the next chapter.

The analyzer package used in here is based on a finite element program called SPAR, developed by NASA [11]. The SPAR program in its existing form was not found suitable for application in this particular investigation. Additional software was generated to compute the dynamic displacement and the stress response of the system, when it is subjected to random excitations.

Pertinent descriptions of the finite element analysis program SPAR and the additional software developed are discussed in the following sections.
3.1 General Description of the Response Analysis Software

The finite element program SPAR is a versatile general purpose finite element program for structural dynamic analysis. The program consists of a number of modules or processors which perform the basic task of finite element analysis. These processors communicate with each other through the data base. The data base consists of one or more direct access libraries, which contain the data sets output from the different processors. These data sets have a specific identifying name with which any particular processor can access them whenever they are required as inputs for that particular computation. Figure 3.1 shows the schematic of the general structure of the overall finite element program. Each processor of SPAR performs a specific function e.g. processor TAB, creates data sets containing tables of joint locations, section properties, material constants etc. processor K assembles the stiffness matrix of the structure, processor M assembles the mass matrix of the structure, etc. Further, the processor AUS performs various matrix manipulations, such as multiplication, addition, transpose etc. This processor provides a great deal of flexibility to the user in adopting SPAR program to suit any computational needs. Other program and analysis details about the SPAR processors are contained in reference [11].

SPAR executes on a processor by processor basis. Each processor execution is commanded by a separate explicit command. A string of such commands interlaced with the input numerical data is written by the user for a problem at hand, and is called a run stream. Various run streams can be executed in any order. As an example a typical run
Fig. 3.1. SPAR Program Block Diagram.
stream for determining the first natural frequency of a simple cantilever beam will be as described below:

COMMAND
[XQT TAB
START 5$

COMMENTS
Begin execution of processor TAB
There are 5 joints in the structure

JOINT LOCATIONS
1 0. 0. 0.
2 0. 0. 10.
3 0. 0. 20.
4 0. 0. 30.
5 0. 0. 40.

Joint locations

MATC
1 10. .46 .3

Material properties table

BEAM ORIENTATIONS
1 1 1 1 1 1

Cross sectional properties

E21 SECTION PROP.
TUBE 1 2. 2.25

CONSTRAINT DEFINITION 1$
ZERO 1 2 3 4 5 6 1

Boundary conditions, joint #1
is fixed in all six directions

[XQT ELD
E21: 1,2; 2,3; 3,4; 4,5

Begin execution of processor ELD
Element connectivity definition
COMMAND (Continued)

[XQT  TOPO
[XQT  E
[XQT  EKS
[XQT  M
[XQT  K
[XQT  INV
[XQT  AUS

ALPHA:  CASE TITLES
""""""""TRANSVERSE LOAD"

SYSVEC:  APPLIED FORCES
CASE 1:  I = 2, J = 5; 1000. $ 5 in direction 2.

[XQT  SSOL
[XQT  GSF
[XQT  PSF
[XQT  EXIT

Begin execution of the processor in the order they are listed.
Assemble the system mass and stiffness matrices.

Titles in the output

Applied load of 1000 lb at joint 5 in direction 2.

Execute SSOL to get joint displacements.
Execute GSF to get member stresses.
Execute PSF to print the output results.
Terminate execution.

SPAR program contains many different types of finite elements and assembly. A full detail of these finite elements is included in SPAR reference manual [11].

SPAR is an extremely flexible and efficient computer program; it is modular in nature, has data base capability and can be easily modified for any particular application. These features of SPAR make it well suited for its specific application as the analyzer in the structural optimization system that is of interest in this thesis. More
details on the software are contained in reference [11].

SPAR data base contains all the information relevant to the type of structure being analyzed. Data base is located on the drum storage, the location of a particular data in the data base is given by the library number and the data set name. A table of contents (TOC) is used to store and relate the names and addresses of all data sets resident in the data base. A typical TOC listing is given in Appendix A.

There are two basic ways of communicating with the SPAR data complex. The first is to use the utility processors included in SPAR and the second is to use the SPAR data handling routines directly to create the new processors required. The first method is useful for performing general utility operations such as entering data created by other processors into the SPAR data base, printing data sets, moving data sets between libraries, etc. Processors DCU and AUS provide many such capabilities. The second method involves imbedding statements to call the data handling routines directly in the user program from outside of SPAR itself. This method is efficient and is desirable for creating new processors or for enhancing the existing capabilities of SPAR. The second approach has been used in this investigation to expand the analysis capabilities and to enable SPAR to carry out the random response analysis.

There are five data handling routines which provide communication between the SPAR data base and the user written computer programs. The details regarding the usage of these five handling routines are given in reference [31]. These data handling subroutines can be incorporated through user written FORTRAN programs. They have the capacity to
extract data from the SPAR database and this extracted data can be used in the user written programs. The user can also create new sets of data through these subroutines. These subroutines have been employed here to generate the additional coding to enable SPAR to do the random response analysis.

3.2 Determination of RMS Displacement Responses

SPAR has the capability of calculating all the displacements and stresses of complex structures when subjected to static loading conditions. It also has the additional capability of performing eigen value analysis, and various matrix manipulations needed for a basic dynamic response calculations. However, additional software had to be designed and written, which in conjunction with the available capabilities of SPAR, can be used for determining the required responses of a structure under random vibrational environments. A brief account of the different response calculation procedures is provided in the following pages.

The spacecraft antenna structures are normally required to be designed for a specified level of base excitation and hence the response of such structures subjected to base excitations has to be considered. In Fig. 3.2, a typical discrete multi-degree-of-freedom system is identified. Let \( y_i \), \( i = 1, 2, ..., n \), denote the absolute displacements of the different masses. The base displacement is denoted by \( x \). The displacements of masses relative to the base are then given by \( (y_i - x) \), \( i = 1, ..., n \). Considering viscous damping \( c \), the equations of motion are

\[
[-m \cdot \ddot{y}] + c(\dot{y} - \dot{x}) + k(y - x) = 0
\]

(3.1)
Fig. 3.2. Typical Multi Degree of Freedom System.
Defining \( z_i = y_i - x \), the equations of motion are

\[
[-m_-](\ddot{z}) + [c](\dot{z}) + [k](z) = - [-m_-]x
\]  

(3.2)

Since \([-m_-]\) is a diagonal matrix and \(\bar{x}\) can be written as \(\bar{x}[1\ 1\ 1\ \ldots\ n]^T\), the right hand side of equation (3.2) can be written as \(-\bar{x}(m)\), where \(m\) is a column matrix containing all the masses.

Equation (3.2) can now be written as

\[
[-m_-](\ddot{z}) + [c](\dot{z}) + [k](z) = - \bar{x}(m)
\]  

(3.3)

The relative displacement \(z\) can be expressed in the form of normal modes, \(\phi\), and the normal coordinates, \(n\), related by the expression

\[
{z} = \begin{bmatrix} \phi \end{bmatrix}[n]
\]  

(3.4)

Substituting (3.4) in (3.3), the equations of motion are

\[
[m][\phi](\ddot{\bar{n}}) + [c][\phi](\dot{\bar{n}}) + [k][\phi](n) = - \bar{x}(m)
\]  

(3.5)

Premultiplying by \([\phi]^T\)

\[
[\phi]^T[m][\phi](\ddot{\bar{n}}) + [\phi]^T[c][\phi](\dot{\bar{n}}) + [\phi]^T[k][\phi](n) = - \bar{x}(\phi)^T[m]
\]  

(3.6)

Since \([\phi]^T[m][\phi] = [M]\) (generalized mass matrix)

\[
[\phi]^T[c][\phi] = 2[c][K][-\omega^-1]
\]

and \([\phi]^T[k][\phi] = [K]\) (generalized stiffness matrix)

or

\[
[M](\ddot{n}) + 2[c][K][\omega]^{-2}(\dot{n}) + [K](n) = - \bar{x}(\phi)^T[m]
\]  

(3.7)

The \(r\)th equation of motion then is

\[
\ddot{n}_r + 2\omega_r\dot{n}_r + \omega_r^2n_r = - \frac{T_r}{M_r} \bar{x}
\]  

(3.8)
where

\[ T_r = \{ \phi^r \}^T \{ m \} \]

The solution of equation (3.8) is given by

\[ \eta_r(t) = \frac{T_r}{\omega_r^2 M_r} \cdot H_p(\Omega) \cdot \ddot{x}(t) \]  \hspace{1cm} (3.9)

where \( H_p(\Omega) \) is the system transfer function relating the input to the output of the system and \( \Omega \) is the nondimensional frequency ratio, \( \omega/\omega_r \).

The response \( z_i(t) \) of \( i \)th degree of freedom at time \( t \) can be expressed in terms of normal modes

\[ z_i(t) = \sum_{r=1}^{n} \phi_{ir} \eta_r(t) \]  \hspace{1cm} (3.10)

The mean square of the displacement response is then given by the averaging process

\[ \overline{z_i^2(t)} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} z_i^2(t) dt \]  \hspace{1cm} (3.11)

Substituting \( \eta_r(t) \) from (3.9) in (3.10) and (3.11)

\[ \overline{z_i^2(t)} = \lim_{T \to \infty} \frac{1}{2T} \sum_{r=1}^{n} \sum_{s=1}^{n} \phi_{ir} \phi_{is} \int_{-T}^{T} |H_r(\Omega)||H_s(\Omega)|x^2(t) dt \]  \hspace{1cm} (3.12)

For a lightly damped system, the magnification factors \( |H_p(\Omega)| \) have pronounced peaks in the neighborhood of the corresponding natural frequencies \( \omega_r \). The products \( |H_r(\Omega)||H_s(\Omega)| \) for \( r \neq s \) are small in comparison to the products when \( r = s \), thus for lightly damped system, it is reasonable to disregard the cross product terms of the transfer functions. With this assumption, transformation of equation (3.12) from time domain to frequency domain, the solution for frequency response is given by
\[ z_i^2(t) = \sum_{r=1}^{n} \frac{\phi_{ir} T_r^2}{M_r} \cdot \frac{1}{2\pi} \int_0^\infty |H_r(\omega)|^2 \cdot S_x(\omega) d\omega \quad (3.13) \]

\( H_r(\omega) \) will depend upon the type of output function desired. The input to the satellite system is always the base acceleration process given in terms of the power spectral density. The output required could be the response acceleration power spectral density or the displacement power spectral density depending on design criteria employed. The technique proposed and demonstrated in Chapter 2 for modelling arbitrary input processes can be used to handle either the acceleration or the displacement response or both. In equation (3.13), if the desired response is the displacement response, then for a constant power spectral density, indicating a wide band input, equation (3.13) reduces to

\[ z_{i\text{rms}} = \left( \sum_{r=1}^{n} \frac{S_r(\omega) T_r^2 \phi_{ir}^2}{8 \zeta_r \omega_r^3 M_r^2} \right)^{\frac{1}{2}} \quad (3.14) \]

In the proposed analyses system, first the SPAR structural analysis program is executed to generate the structure geometry data, the mass matrix, and the stiffness matrix of the structure. SPAR is then used to compute all the eigen values and the different vibration mode shapes for the given satellite structure. This data is then used for generating the generalized masses for each mode of vibration. This computation is performed within the SPAR program using its matrix manipulation capabilities. The root mean square (RMS) displacement response of the structure is calculated in a separate program developed as part of this research investigation. This probabilistic response determination program reads the user specified loads in the form of discrete power spectral densities of excitation at various frequencies and notes
also the damping coefficients for each mode shape. The different relevant matrices generated during the execution of SPAR are extracted in this program using the data handling subroutines [31]. The flow chart depicting the sequence of these numerical operations is given in Fig. 3.3. The theory developed in here is later used in the computation of the structural stress response. A listing of the displacement analysis program is included in Appendix D.

3.3 Determination of RMS Stress Responses

In order to arrive at an optimal design of a structure satisfying both the displacement and the stress constraints, it is necessary to know completely the displacement and the stress responses of the structural system. The RMS stress response is determined by applying joint displacement loads on the structure equal in magnitude to the computed RMS displacements. This approach of computing stress gives stress values which are slightly conservative as compared to computing stresses directly using the strain matrix of each finite element, because the phase information is retained only up to the displacement calculations. This approach was used because of the ease offered in computing the stress response.

A new data set for SPAR is created within the displacement response calculation program. This data set contains all the RMS displacement responses of the structure. SPAR is then executed to generate the response of the structure corresponding to this displacement data set. This response then is the RMS stress response of the structure under consideration. All the software necessary to carry out these operations
Fig. 3.3. Block Diagram of RMS Displacement Calculation Routine.
Fig. 3.4. Computation of RMS Stress Response.
were generated as a part of this numerical investigation. Listing of
the different programs performing the structural stress computations is
included in Appendix E. A block diagram showing the basic operations
for these stress computations is shown in Fig. 3.4.

3.4 Conclusions

The software to compute the displacement and the stress responses
of any structural system under a specified random vibrational environment
was completely developed and tested for validity on a sample problem to
ensure the accuracy of the results produced by the entire numerical
methodology. A listing of the sample program and the run stream used to
compute the stress responses are included in Appendix H.

Thus a versatile finite element structural analysis program
capable of performing random response analysis has been developed. The
optimization procedure to be used with the foregoing analyzer is ela-
borated in the next chapter. Since the constraints on the structural
design under random loads are to be expressed also probabilistically, a


CHAPTER 4

STRUCTURAL OPTIMIZATION WITH PROBABILISTIC CONSTRAINTS

One of the essential parts of an Optimization-Analysis and synthesis system for structural design is the optimization algorithm. This chapter describes the main features of the optimization algorithm used for the minimum weight satellite antenna design presented in this investigation. The other main requirement in designing such optimal weight structures under a random vibrational environment is that the algorithm should be capable of handling constraints specified in probabilistic terms.

The response parameters such as the displacements and the stresses for the structure when subjected to random vibrational loading have been computed in the form of root mean square (RMS) values. The constraints on the maximum values of such parameters can only be specified with a given probability that they do not exceed certain allowable or pre-specified peak values. For example, in the case of the cantilever beam subjected to random excitations, it can not be said with certainty that the maximum tip displacement should not exceed a given maximum value because there always exists a finite probability in any random phenomenon that parameters such as the maximum displacement could exceed that given value. Hence the constraints for optimization can only be meaningfully expressed in conjunction with some specified probability associated with the response parameters. But the conventional techniques of optimization can handle only deterministically specified constraints. A method is proposed for transforming the probabilistic constraints into corresponding deterministic constraints in this chapter for optimization of random
structural responses.

4.1 Description of the Optimization Procedure

In choosing an optimization algorithm for solving the general structural design problem, several features are desirable. First, the number of times the structure must be analyzed and reanalyzed with change in parameter values should be kept as few as possible because for complex structures each analysis computational cycle could often be very time consuming. This is particularly so when the constraints are specified on the dynamic characteristics of the structure under consideration. Secondly, the amount of gradient information required in the optimization process should be reduced considerably or, if at all possible, eliminated.

The algorithm used for the optimization procedure here in this investigation has the above mentioned desired features and is based upon the method of feasible directions [32]. Zoutendijk [17] first put forward a generalized method of feasible directions to minimize a constrained function.

In general, there are two possible ways of handling the constraints in a design problem. One is to convert the constrained problem into a sequence of unconstrained problems where the constraints on the objective function are treated indirectly as penalty functions. The second approach is to treat the constraint functions directly in the scheme for the minimization process itself. The latter of the above two approaches was employed by Zoutendijk [17] in the development of the algorithm for the minimization of the constrained function. Vanderplatts and Moses

The general problem of optimization in a structural design similar to one at hand may be stated as follows:

Minimize $F(X)$
subject to
$G_j(X) \leq 0, \quad j = 1, \ldots, \text{NCON}$ \hspace{1cm} (4.1)
$H_j(X) = 0, \quad j = 1, \ldots, \text{NEQU}$ \hspace{1cm} (4.2)

where $X$ is a vector of design variables,
NCON is the number of inequality constraints,
NEQU is the number of equality constraints,
$G_j(X)$ are the inequality constraint functions,
and $H_j(X)$ are the equality constraint functions.

The optimization process iteratively determines the design vector $X$, and minimizes the function $F$ until an optimum solution is found. The design vector at $(q+1)$th iteration is given by the incremental expression

$$X_{q+1} = X_q + \alpha^* S_q$$ \hspace{1cm} (4.3)

where $q$ is the iteration number,
$S_q$ is the gradient defining the direction of the move,
$\alpha^*$ is the optimum value of a scalar multiplier defining the step size.

The direction of the move is determined as explained in the next subsection.
4.1.1 Determination of the Direction of the Move

The gradient \( S_q \) defining the direction of the move is found such that it satisfies the following conditions:

\[
\Delta F(X_q) \cdot S_q \leq 0 \tag{4.4}
\]

\[
\Delta G_j(X_q) \cdot S_q \leq 0, \quad j = 1, \text{ NAC} \tag{4.5}
\]

where NAC is the number of active (violated) constraints. Equation (4.4) ensures that a move in the direction \( S_q \) will decrease the value of the function \( F \) and the equation (4.5) ensures that this move will keep the solution in the feasible region for the problem under consideration.

Zoutendijk [17] has shown that the direction \( S_q \) satisfying equations (4.4) and (4.5) can also be found by solving the following adjoint problem.

Maximize \( \beta \)

subject to

\[
\Delta F(X_q) \cdot S_q + \beta \leq 0 \tag{4.6}
\]

\[
\Delta G_j(X_q) + \theta_j \beta \leq 0, \quad j = 1, \text{ NAC} \tag{4.7}
\]

The scalars \( \theta_j \) in equation (4.7) are referred to as the push-off factors which effectively push the design results away from the active constraints. The values of \( \theta_j \) have a significant effect on the efficiency and stability of the optimization algorithm.

The physical significance of the \( \theta_j \)'s in the process of finding the direction \( S_q \) can be illustrated with the help of Fig. 4.1. If \( \theta_j \) is taken as zero, the search direction \( S_q \) will tend to follow the active constraint and if \( \theta_j \) is very large, the direction \( S_q \) will
Fig. 4.1. Effect of Push-Off Factor $\theta_j$ on Direction $S_q$. 
tend to follow the objective function. Thus, a small value of $\theta_j$ results in a direction which would rapidly reduce the value of the objective function but which may quickly encounter the same constraint once again. However, a large value of $\theta_j$ would reduce the risk of encountering the same constraint again but might not reduce the objective function as fast as in the previous case. Zoutendijk [27] proposed a compromise constant value of $\theta_j$ to be used in the optimization algorithm for an effective solution.

Later Vanderplatt and Moses [32] found out that the algorithm can be made more efficient and stable by varying the $\theta_j$ values according to the constraint to which it applies and suggested using the value of $\theta_j$ as a quadratic function of $G_j(X_q)$ with a constraint width of $2\delta$, $\delta$ being the tolerance within which $G_j(X_q)$ is not considered violated.

The proposed value of $\theta_j$ is then given by

$$\theta_j = \theta_0 \left[ \frac{G_j(X_q)}{\delta} + 1 \right]$$

(4.8)

where $\theta_0$ is a preselected constant.

Thus the direction $S_q$ of the move is determined using equations (4.6), (4.7) and (4.8).

4.1.2 Determination of the Step Size

The step size is the change in the value of the design variables at each iteration step of the optimization routine. Step size in the present algorithm is given by the term $\alpha^*S_q$, where $\alpha^*$ is the optimum
value of a scalar multiplier which establishes the step size at each iteration sequence.

Scalar $\alpha^*$ is determined here using a one dimensional search. The search is carried out in such a way until $\alpha^*$ meets either of the following conditions:

a) the objective function $F(x)$ is minimized;
b) a constraint is encountered;
c) starting from a non feasible region, a feasible solution is found.

The general situation relating to the one dimensional search is shown in Fig. 4.2. The design from the previous iteration is deemed to be located at point 'a'. The direction of move is given by the gradient vector $S_g$. A feasible design is obtained at 'b' and the objective function is a minimum at 'c'. The active constraint $G_3(X_Q)$ is encountered at 'e'. The associated one dimensional search is shown in Fig. 4.3. Point 'c', which gives an optimum value of $\alpha^*$ is determined using a three point quadratic interpolation.

4.1.3 The Optimization Algorithm

The optimization algorithm proposed by Vanderplatts and Moses [18] and used for the optimization-analysis in the present system is summarized in terms of the following procedural steps.

a) choose an initial design vector $X$. It is desirable but not essential to define this initial design in the feasible region. Choose an initial tolerance by which the constraints may be allowed to be violated;
Fig. 4.2. Constrained Design Space.
Fig. 4.3. One Dimensional Search in Direction $S_q$. 

$F(x) = \text{MINIMUM}$
b) determine the value of the objective function, $F(X_q)$ and the constraints $G_j(X_q)$;

c) determine the gradients of the objective function and those of all the active constraint functions;

d) if no constraints are active, set $S_q = -\Delta F(X_q)$, which is the direction of steepest descent, and proceed to step (h);

e) determine the value of the push-off factors using equation (4.8);

f) define the constraints on the problem for finding direction $S_q$ through equations (4.6) and (4.7) if no constraints are violated and by (4.7) alone if one or more constraints are violated;

g) obtain the gradient direction vector $S_q$ using (4.6) and (4.7);

h) solve the one dimensional search problem to determine $\alpha^*$;

i) check the convergence criterion, i.e. when two consecutive iterations fail to reduce the objective function by 1 percent or more. If the convergence criterion is not satisfied, then return to step (b) for the next cycle of computation or otherwise terminate the operation.

4.2 Optimization with Probabilistic Constraints

A typical problem in structural optimization subjected to probabilistic constraints can be stated as following:

Minimize $W(X)$

Subject to

$P(G_j(X) \leq G_{j,\text{spec}}) \geq P_j, \ j = 1, \ldots, N_{\text{CON}}$ (4.9)
where $W(X)$ is the weight of the structure and $P$ denotes the probability distribution.

The constraints specified in the inequality (4.9) imply that the probability that $G_j(X)$, which represents a parameter like stress or displacement, is less than or equal to a specified value $G_j$ is greater than or equal to a probability $P_j$. Here $NCON$ is the number of such constraints in the problem. Constraint (4.9) could also be expressed in an integral form as

$$\int_{-\infty}^{G_{j_{\text{spec}}}} f_j[G_j(X)] \, df_j \geq P_j \quad (4.10)$$

where $f_j[G_j(X)]$ is the probability density function of the parameter $G_j(X)$.

The integral (4.10) defining the probabilistic constraints cannot be used directly as such in the optimization routine and hence must be expressed in a different format.

Since the excitations on the structure have been assumed to be Gaussian and the structure is taken to be a linear system, the response parameters defining the constraints $G_j(X)$ are also by definition Gaussian. Using this important property, it is possible to express the integral representation in equation (4.10) in a different form as explained below.

Let $\overline{G}_j(X)$ be the first moment or the mean value of the parameter $G_j(X)$. Further, let $G_{j_{\text{G}}}^2$ be the second moment or the standard deviation of the parameter $G_j(X)$. For the required limiting probability $P_j$, and the specified constraint value $G_{j_{\text{spec}}}$ on $G_j$, tables for the
unit normal variate give a value of
\[
\frac{G_{j\text{spec}} - \bar{G}_j(X)}{\sigma_{G_j}}
\]
corresponding to the probability level \( P_j \). Let this value be denoted by \( \eta_j \). The condition stated through the probabilistic constraint of equation (4.10) can then also be satisfied by the adjoint deterministic constraint stated by the inequality expression
\[
\frac{G_{j\text{spec}} - \bar{G}_j(X)}{\sigma_{G_j}} \geq \eta_j \quad (4.11)
\]
Since the response quantities considered such as displacement, stress etc. in the present can have a zero mean value \( \bar{G}_j(X) \) is zero and \( \sigma_{G_j} \) then becomes equal to the root mean square value of the parameter \( \{G_j(X)\} \). Hence equation (4.11) may be rewritten in the following manner:
\[
\frac{G_{j\text{spec}}}{G_j \text{rms}} \geq \eta_j \quad (4.12)
\]
which is in a very simple form and can easily be applied to structural optimization problems where constraints are expressed probabilistically. \( G_j \text{rms} \) for any parameter like stress or displacement can be determined from the knowledge of all the characteristics of the excitation process and the transfer functions of the structure in hand.

The value of \( \eta_j \) will vary depending upon the acceptable probability level \( P_j \) specified on the constraint. The value of \( \eta_j \) is equal to 1 if \( P_j \) is equal to 0.65, 2 if \( P_j \) is equal to 0.95 and 3 if \( P_j \) is equal to 0.9927.

The selection of the probability level \( P_j \) will be different
for different situations and applications. Its value depends upon the risks involved if the structure fails, i.e. the risks involving cost, human life, health hazard, etc. With the variation in the specific field probability level, the optimum solution for a structure will change. In general, a high reliability imposition in the probability will result in a heavier structure.

4.3 Conclusions

Thus, using the proposed concept, the probabilistic optimization constraints can be transformed into a set of equivalent deterministic constraints. These deterministic constraints indirectly retain the probabilistic nature of the constraints. This transformation permits the use of existing techniques of optimization which are valid for handling constraints of deterministic nature.

With the employment of the finite element analysis program described in the third chapter, and with the help of the optimization procedure along with the technique proposed in this chapter for handling the probabilistic constraints, an analysis-optimization system can now be put together. Details of such an 'analysis-optimization' system are given in the next chapter.
CHAPTER 5

SYNTHESIS OF THE STRUCTURAL ANALYSIS-OPTIMIZATION SYSTEM

A finite element analysis program capable of evaluating the response of structures subjected to random vibrations and an optimization algorithm capable of handling probabilistic constraints were described in detail in the two previous chapters. However, a successful implementation of either the analysis program or the optimization algorithm for the purpose of achieving a minimum weight design is not possible without the proper synthesis of the analysis and optimization systems.

In many past investigations, structural analysis and optimization applications have been considered as well as developed as two separate fields. A good marriage and integration between the advances in the state of the art of optimization and that of the analysis capabilities did not take place until recently. The first significant work in this area was published by Sobieski and Bhat [27], who demonstrated the feasibility of the integration between an efficient optimization method and a state of the art structural analysis program. Sobieski and Bhat proposed a system which could be employed for the solution of static structures, as well as vibrating structures with frequency constraints. Since most of the problems relating to the spacecraft structures deal with dynamic loading conditions, the structural optimization attempted must consider all the constraints on the dynamic structural response to such loads. Such an optimization system which is capable of dealing with dynamic loads is presented in this chapter.

5.1 Main Framework of the System

The proposed system uses a state of the art finite element program
SPAR combined together with the software generated as part of this investigation for the structural analysis and a technique of optimization employing the method of feasible directions proposed by Vanderplatts [32]. The random response analysis is carried out through a separate program, which uses the natural frequencies and mode shape information generated by SPAR. The principal components of the synthesized system are listed below along with a detailed description of their functions.

5.1.1 The Analyzer

Function of the analyzer is to compute the values of the behavioral response variables, such as frequencies, displacements, stresses, etc., of the structure. The analyzer in a general sense can be any type of a finite element program. Its input comprises of structural cross-sectional dimensions, material properties, element connectivity data, nodal point coordinates, and the loads. The output results generated by the analyzer are the different response parameters. One of the most important output variable generated in the case of spacecraft applications is the weight of the structure which is commonly used as the objective function to be minimized in the later part of the execution. Since the analyzer has to be executed many times in the loop as shown in the flowchart described in Fig. 5.1, it has been conveniently split into non-repeatable and repeatable parts. The non-repeatable part is executed only once, and hence is kept outside the loop, while the repeatable parts are included within the loop as indicated in Fig. 5.1. The dividing line between the repeatable and non-repeatable components of the program can be decided upon by an examination of the finite element
Fig. 5.1. Basic Flow Chart of Analyzer-Optimizer System.
program employed. The SPAR analysis program is very flexible in this respect and can be broken into repeatable and non-repeatable parts almost at any stage. SPAR is actually a system of computer programs and is capable of analyzing most linear types of finite element structural models. It can now, with the additional software developed, perform the required random response analysis, besides the normal static structural analysis. More details about the SPAR analyzer are already included in Chapter 3.

5.1.2 The Optimizer

Function of the optimizer is to calculate the new vector of the design variables on the basis of the initial set of values of the design variables as well as on the basis of the constructed objective function and the specified constraints. The optimizer automatically changes the values of the design variables until an optimum design is achieved that would satisfy all the constraints placed on the problem.

The algorithm used for the optimization is based upon the method of feasible directions. It is a gradient search technique and has been described earlier in detail in Chapter 4. A guide for application of this method in an optimization computer program is described in reference [32]. The organization of the optimization program is shown in Fig. 5.2. The complete optimization program consists of a main program and an optimization subroutine. The main program reads the initial values of the design variables, and the values of various control parameters such as the number of iterations and the convergence criterion. The computation of the values of the objective function, constraints, and their
Fig. 5.2. Flow Chart for the Optimization Program.
gradients can be carried out within the main program if the problem on hand is computationally of smaller magnitude; otherwise these computations might have to be carried out in the form of a subroutine outside the main program.

The program ROPT1, listed in Appendix B, serves the function of the main program for optimization. The operations of ROPT1 are to

a) read optimization control parameters in the first execution;
b) read analyzer output in the second and the subsequent executions;
c) call optimization subroutine;
d) output new design variables for the analyzer;
e) stop itself to permit the external analysis;
f) write and save all data needed for subsequent analysis;
g) generate a message to NOS-JCL to stop execution when the optimization criterion is satisfied.

5.1.3 The Front Processor

The optimizer communicates with the analyzer through a front processor. The front processor is a user-written and problem-dependent program. The function of the front processor is to convert the variables of the optimization process to a set of input parameters, written in a format required by the analyzer. In the case of structural optimization, these parameters are generally the structural member geometry. In summary, the functions of the front processor are to

a) read output of the optimization program, in the form of a vector of the design variables;
b) convert these design variables into structural parameters;
c) output these structural parameters in a format meaningful to
the analyzer.

The listing of the front processor program RFPRS is given in Appendix C.

5.1.4 The Response Program

The response program is a repeatable part of the analysis-optimi-
zation system. The analyzer is obviously branched at this point. The
function of the response program is to extract the information regarding
the mode shapes, eigenvalues, applied random excitations in terms of
their power spectral densities, etc., and using all this data, to compute
the RMS displacement response of the structure at all the nodal points.
These nodal displacements are then fed back into the analyzer where the
stresses corresponding to these displacements are computed. SPAR data
handling routines DAL and FIN have been used in the response program to
retrieve or to create data sets from and into SPAR library. The listing
for the response program is given in Appendix D.

5.1.5 The End Processor

Function of the end processor is to compute the value of the
objective function and the constraints and also to output them in a
format required by the optimization program. The end processor reads
the user supplied constants defining the limits imposed upon the
behavioral variables. It also reads the probabilities associated with
each constraint and that is required to be satisfied. Output data sets
defining the stresses and the displacements are then retrieved from the
SPAR data library using the data handling subroutine DAL. The values of the stresses and the displacements are then compared with the bounds imposed upon them by the design requirements on the application in question. Based upon this comparison, values of the constraint functions are computed. The value of the objective function is computed by retrieving the mass information from the SPAR data sets. The values of the objective function and the constraints are then output in a format meaningful for the optimization subroutine. A listing of the end processor program is included in Appendix E.

5.1.6 The Terminator

Function of the terminator program is to issue a termination command when the termination criterion is satisfied. The termination criterion is actually checked in the optimization subroutine and a flag is raised when the criterion is satisfied. Using this flagged information, instructions are issued to the NOS-JCL to stop further executions.

5.1.7 The Connecting Network

Connecting network is essentially the brain of the whole system. It gets various elements and components of the system to do their individual tasks whenever required. This coordination is accomplished with the help of several procedure files. Following is a summary of the various tasks performed by the connecting network:

a) execution of the various elements/programs in a predetermined sequence of computations;

b) performing logical functions such as branching on an if-test, looking or skipping to a labelled statement; and
c) storing permanently or temporarily, the data generated by the various programs.

The master control is provided by the main procedure file called RPRO1. This file controls the sequence and the logic of the various operations. First, the non-repeatable part of the analysis is executed outside the optimization loop and then the optimization program is executed. The values of the objective function and the constraints required by the optimization program are then computed using the procedure file RXQTEUN. After the optimum solution is reached, the final analysis of the optimum structure is performed using the procedure file RANAL. All the procedure files are controlled by the main procedure file. Figure 5.3 depicts the logic followed by the main procedure file which connects all the different elements of the system.

A full description of the connecting network is included in Appendix F.

### 5.2 Flexibility Features of the Analyzer-Optimizer System

The most important feature of the system proposed here is its flexibility. It is flexible in the sense that this methodology can accommodate any appropriate finite element analysis computer program and any given optimization program. However, some of the well known finite element programs such as SPAR are most suited for such an application because of their restarting capability and also because the analysis can be interrupted at any stage. Similarly any efficient optimization technique can be integrated without difficulty into the proposed system. However, the technique used in the optimization must be efficient and
Fig. 5.3. Master Procedure File Flow Chart.
stable because each iteration of the finite element analysis is a very time consuming process. The flexibility of the present program also enables extension to the area of solution of a wide variety of structural optimization problems. This flexibility is achieved by leaving the problem dependent software to be generated by the user. This problem dependent software is minimal and can be generated by a user with only a limited knowledge of computer programming. The user is also free to choose any terminating criterion depending upon the time available for computation and also on the limits placed on the process of design optimization. This task can be accomplished by choosing a maximum number of iterations and/or by alternately choosing a large or a small value of the optimization convergence criterion.

The system presented here is aimed at structural systems which are to be designed to withstand random vibrational environments of arbitrarily varying power spectral density. By suitably changing the "Response" program, the system can be adapted for use in problems under the action of any other type of dynamic load. Thus the system is quite flexible with respect to the type of loads for which it can be applied.

The system also possesses a complete restart capability, in the sense that all the data that is generated during each iteration is automatically saved in various permanent storage files. Hence the design optimization process can be restarted from the previous iteration. This restart capability safeguards the user against computer system breakdowns or against using optimization criterion which may stop the process sooner than the user wants it to stop, because the user can restart from any
point where the previous run terminated.

5.3 Illustrative Example - Application to Design of a Simple Cantilever Beam

The example presented in this section shows the effectiveness of the proposed system in solving a class of structural optimization problems, where the structure is subjected to random vibration loads and where the constraints, under which the optimization is carried out are probabilistic in nature. It also serves as a test to check the reliability of the proposed analysis technique. In the beam design example presented here, a cantilever beam has been optimally designed using the procedure developed in the form of the proposed system. A cantilever box beam of 40" (100 cm) length has been taken as a test case and is shown in Fig. 5.4. The base of the beam is subjected to the random excitations of magnitudes described in Fig. 5.5. The cross-section of the beam was defined in the program by using four geometry variables \( x_1, x_2, x_3 \) and \( x_4 \) as shown in Fig. 5.4. These variables were to be optimized to yield a minimum weight design or to a minimum area of cross section. The objective function is then the weight of the beam to be designed. The following subsections describe the problem requirements and the solutions obtained.

5.3.1 Design Requirements

The following probabilistic and deterministic constraints were imposed upon the design:
Fig. 5.4. Parameters to be optimized for the Cantilever Beam.
Fig. 5.5. Design Load for Cantilever Beam.
Stochastic Constraints (reliability bounds)

P[Maximum displacement < 0.5"] ≥ 95%
P[Maximum stress < 4000 psi] ≥ 95%

Deterministic Constraints (geometry limits)

0.5" (1.27cm) ≤ x_1 ≤ 100" (254cm)
0.02" (0.05cm) ≤ x_2 ≤ 100" (254cm)
0.1" (0.25cm) ≤ x_3 ≤ 100" (254cm)
0.2" (0.51cm) ≤ x_4 ≤ 100" (254cm)

5.3.2 Analysis of the Initial Design

The starting values of the variables used were

x_1 = 2.0"

x_2 = .06"

x_3 = 0.5"

x_4 = 0.06"

A five node mathematical finite element model was used for the analysis. The model included 4 beam elements. A sketch of the finite element model is shown in Fig. 5.6.

The response parameters corresponding to the starting values of the design variables were determined as follows:

Initial Weight = 1.212 lb (0.55kg)

Natural Frequencies = 17.68 Hz, 45.93 Hz, 109.67 Hz, 281.97 Hz,
304.02 Hz, 582.38 Hz, 770.2 Hz, 1230.2 Hz.
1  2  3  4  5

\[
\begin{array}{cccc}
\Delta & \Delta & \Delta & \Delta \\
1 & 2 & 3 & 4 \\
\end{array}
\]

 NODE NUMBERS

 ELEMENT NUMBERS
 CONSTRANGED: NODE #1
 FREE NODES: 2, 3, 4, 5
 TOTAL NUMBER OF DEGREES OF FREEDOM = 24

Fig. 5.6. Finite Element Method of the Cantilever Beam.
Maximum RMS displacement = 0.21" (0.53cm)

Maximum RMS stress = 1495 psi (212.6kg/cm²)

5.3.3 Optimum Design of the Cantilever

The cantilever beam was optimized for the random loads using the system described earlier in this chapter. The weight of the cantilever beam decreased from the initial design of 1.2 lbs (0.55kg) to the final design of 0.2 lb (0.09kg). The optimum design and the response parameters for the resulting optimum design are given below.

First natural frequency = 13.73 Hz

Minimum weight = 0.172 lb (0.08kg)

Optimum design variables: \( x_1 = 0.5" \) (1.27cm)
\( x_2 = 0.02" \) (0.05cm)
\( x_3 = 0.57" \) (1.45cm)
\( x_4 = 0.02" \) (0.05cm)

Maximum RMS displacement = 0.25" (0.63cm)

Maximum RMS stress = 1700 psi (119kg/cm²)

This solution converged in eight iterations. The rate of convergence is shown in Fig. 5.7. The first eight mode shapes of vibration are plotted in Figs. 5.8 to 5.15.

A simple beam design was used here to test the proposed system. Use of a simple structural example like this permits the checking of the results obtained by using this analysis-optimization system. The validity of results was checked by simple hand calculations showing that the results are valid and correct.
Fig. 5.7. Design Development Cantilever Beam.
COORDINATE SYSTEM

VIEW IN YZ PLANE

VIEW IN XZ PLANE

MODE NUMBER 1
NATURAL FREQUENCY 13.73

Fig. 5.8. Vibration Mode Shape.
Coordinate System

View in YZ plane

View in XZ plane

Mode Number  2
Natural Frequency 16.01 Hz

Fig. 5.9. Vibration Mode Shape.
Coordinate System

View in YZ Plane

View in XZ Plane

Mode Number 3
Natural Frequency 85.82 Hz

Fig. 5.10. Vibration Mode Shape.
Fig. 5.11. Vibration Mode Shape.
COORDINATE SYSTEM

VIEW IN YZ PLANE

VIEW IN XZ PLANE

MODE NUMBER 5
NATURAL FREQUENCY 241.0 Hz

Fig. 5.12. Vibration Mode Shape.
COORDINATE SYSTEM

VIEW IN YZ PLANE

VIEW IN XZ PLANE

MODE NUMBER 6
NATURAL FREQUENCY 280.5 Hz

Fig. 5.13. Vibration Mode Shape.
COORDINATE SYSTEM

VIEW IN YZ PLANE

VIEW IN XZ PLANE

MODE NUMBER 7
NATURAL FREQUENCY 471.7 Hz

Fig. 5.14. Vibration Mode Shape.
COORDINATE SYSTEM

VIEW IN YZ PLANE

VIEW IN XZ PLANE

MODE NUMBER 8
NATURAL FREQUENCY 548.3 Hz

Fig. 5.15. Vibration Mode Shape.
5.4 Discussion of the Results and Conclusions

The computing system combining analysis and optimization procedures for structural dynamic problems has been applied to design an optimal cantilever box-beam. Initial design was chosen by simple hand calculations, and this design was chosen so as to satisfy the specified stress constraint on the problem. The weight of the initial design was 1.2 lb (0.55 kg) as compared to the finally arrived optimum weight of 0.2 lb (0.09 kg). All the probabilistic and deterministic constraints imposed upon the design were fully satisfied. The details on the development of the design optimization shown in Fig. 5.7 reveals the following interesting facts:

i) Most of the reduction in the weight takes place in the first few iterations, reflecting the rapidly converging nature of the gradient search techniques.

ii) The history of the changes in the variables gives some insight into the optimization process. The width \( x_1 \) of the section shows the most rapid decrease in the first iteration and this is obvious on an intuitive basis. In a box section, the width of the section contributes linearly to the area moment of inertia of the section whereas the height of the section contributes in a proportion that is cubic in power.

The cross section of the beam is changed automatically to ensure most efficient use of the material. The design variables \( x_1 \) to \( x_4 \) change in a manner which is obvious on an intuitive basis. The width \( x_1 \) of the section decreases, the height \( x_3 \) of the section increases and the thicknesses \( x_2 \) and \( x_4 \) of the section decrease, resulting in
a combination that would yield a beam of minimum area of cross-section that meets all the design requirements.

The history of the development of the design process, shown in Fig. 5.7, indicates that the width $x_1$ of the section decreases rapidly until it reaches the lower limit value of $0.5"$ (1.27cm). The height $x_3$ of the section shows the expected increase. The height of the section first increases so that all the frequency, stress and displacement constraints are satisfied and then it decreases up to a point where all the constraints are just met. The thickness of the section in the width direction $x_2$, and the thickness of the section in the height direction $x_4$, show a continuous decrease in their values, thereby resulting in a lower weight design. The history of optimization also gives a qualitative feel as to which variables should change most in order to achieve an optimum design of a cantilever beam.

The analysis-optimization system proposed in this chapter is thus validated by employing it in the design of a simple cantilever beam. The example of cantilever beam has been used in this investigation to verify all the analytical and software development presented here. The main task for the analysis-optimization system is for utilization in the design of a satellite antenna structure. An optimum design of such antenna structures subjected to random excitations of arbitrarily varying power spectral densities has been realized employing this analysis-optimization system and the detailed design of the satellite antenna structure is discussed in the next chapter.
CHAPTER 6

ANALYSIS AND DESIGN OF A SATELLITE ANTENNA STRUCTURE

The satellite antenna structures are required to be designed to withstand the applied random excitations of varying power spectral densities without undergoing any structural failure and at the same time these structures are strictly required to have the minimum possible weight. The random vibrations for which satellite antenna structures are designed arise from the transmitted vibrations of the launch vehicle. A sketch of an actual communication satellite is shown in Fig. 6.1. The launch sequence for a typical communication satellite is shown in Fig. 6.2, to facilitate an understanding of the source of such vibrations. An optimum (minimum weight) design of a satellite antenna structure capable of withstanding the specified random vibration environment is described in this chapter. The optimum design has been realized using the analysis-optimization system developed earlier and this investigation and described in the previous chapter.

6.1 General Description of an Antenna System

A satellite antenna system is generally a circular dish, the curved profile of this circular dish following the contours of the generated surface of a paraboloid. The antenna serves the primary function of receiving and transmitting the communication signals reaching and leaving the spacecraft. The antenna is an integral part of the communications subsystem of any satellite. The structural survival of an antenna is of utmost importance for the successful operation of any satellite.
Fig. 6.1. A Typical Communication Satellite.
[Courtesy SPAR Aerospace Ltd].
Fig. 6.2. A Typical Launch Sequence.
The antenna structure is always located on the top deck of the spacecraft structure. The vibrations to the spacecraft structure are transmitted from the launch vehicle at the base of the structure and since the antenna structure is located on the top of the satellite, it experiences the most severe vibrations in comparison to the rest of the spacecraft structure. Antenna structures are positioned just under the nose cone of the launch vehicle and often there exists very limited clearance between the nose cone and the antenna structure. Therefore, the maximum displacement of the antenna structure under the transmitted vibrational load becomes very critical in design. This maximum displacement should be strictly controlled in a manner that it does not exceed the clearance between the antenna structure and the nose cone structure of the launch vehicle. The antenna structure is manufactured out of either solid or honeycomb sheets. The elastic rigidity to the structure is provided by the fan of ribs attached to the back of the antenna dish.

As a single component, the antenna structure is of significant weight in proportion to the total satellite weight and a reasonable weight saving is always sought. Because of this and the nature of the structural dynamic considerations, it has been attempted here to investigate this type of structure for an optimal design.

6.2 Spacecraft Antenna Design Criteria

Design criteria for the spacecraft antenna structures include requirements on frequency, displacement, structural integrity, size, shape, and manufacturability of the antenna.
The requirements on the natural frequencies of the antenna structure are placed so that the natural frequencies of the antenna structure will be above the natural frequencies of the satellite, thus avoiding the possibility of coupling between the satellite and the antenna structure. The loads experienced by the antenna structure would increase significantly if such a coupling did take place. Hence, a requirement on the first natural frequency of the antenna structure is to be imposed.

The requirements on the maximum displacement of the structure are imposed in order to ensure that there would be no interference between the edges of the antenna structure and the inside walls of the nose cone of the launch vehicle during the launch of the satellite into space.

The structural integrity requirements are imposed to ensure that the stresses generated in the antenna structure by the launch vibrations will not cause any structural failure. The random vibrations produce rapidly fluctuating displacements and stresses which can only be estimated in a statistical sense due to the inherent nature of the applied random vibrations. Thus the structural integrity requirements can only be specified with a specified reliability factor or a corresponding probability. Hence, the structure must be designed to meet these structural reliability requirements.

The basic geometry, size and shape requirements are placed on the antenna structure to ensure that the antenna structure will fit within the inside envelope of the launch vehicle. Hence, during the optimum design process, suitable size limiting constraints have to be
included in order to keep the design within the specified envelope.

The manufacturability of the design is an essential implicit requirement in all such applications. The optimum design may yield thicknesses of the cross sections which may be too thin to manufacture. In addition, there may be a few other manufacturability requirements placed on the cross-sectional dimensions of the structure depending on the type. These manufacturability requirements can be met by imposing appropriate side constraints on the design variables governing these thickness parameters.

The above statements summarize the main requirements generally imposed upon an antenna structure. Additional requirements may be placed for any specific design depending on the problem in hand and can be included in the overall program.

6.3 Satellite Antenna Design Stages

The basic shape and size of the antenna dish is first established by the satellite requirements. The factors governing the shape and the size of the antenna are the area of the earth to be covered by the satellite antenna, the pointing angle of the satellite in space and the electrical gain requirements imposed on the electrical performance of the antenna.

The mechanical engineer receives this information about the size and shape of the antenna dish and comes up with the first conceptual design of the antenna structure, including the thickness of the dish, the rib structure pattern to be put on the back of the antenna dish,
and the dimensions of the rib cross sections. The first conceptual design is performed based upon elementary stress calculations done by hand.

Next stage in the design is to generate a mathematical finite element model of the antenna structure. A finite element analysis of this model is performed to determine the natural frequencies of the structure, the displacements at the various points and the stresses at the various locations on the antenna structure. Based upon this analysis, the conceptual design is modified, for example, the sizes of ribs and plate-thicknesses are increased or decreased as deemed necessary. The analysis is then repeated several times until an acceptable design is established. The total duration of such iterative cycles in design may range from two to three months. In the proposed system explained in this investigation, this design analysis and modification cycle gets automated and the two to three months design duration is reduced to two to three days.

The final stage in the design is the preparation of manufacturing drawings for the complete antenna structure. Once all the dimensions are established, the final drawings are produced, the hardware is then built, the antenna dish is electrically checked and then integrated into the satellite.

6.4 Description of the Excitation Process

Various sources which give rise to the structural loads experienced by the antenna structure are as follows:
a) The acoustic noise generated by the rocket engines of the launch vehicle;
b) The shock loads at the time of separation of the satellite from the final stage of the launch vehicle;
c) The random vibrations generated by the pulsations of the rocket engine fuel burning process.

The predominant load on the structure comes from the random vibrations generated by the above sources. To the designer, the loads are specified in the form of a power spectral density profile over the frequency range. The structure must then be designed to withstand these random vibrational loads, assuming that the structure is excited at its mounting points by these excitation processes. A typical profile of random vibrations to which antenna structures are subjected is given in Fig. 6.3.

6.5 Application of the Analysis-Optimization System to Antenna Design

The formulation of the antenna design problem and the use of the design system is included in this section. The details relating to the design requirements, the finite element model and the constraints in optimization are also discussed here.

6.5.1 Design Requirements

The specified requirements on the design are stated as follows:

1) The first natural frequency of the antenna structure should be greater than 15 Hz.
2) The maximum displacement of any point on the antenna structure should not exceed 1.0" (2.54cm), when subjected to the
Fig. 6.3. The Input Power Spectral Density Profile for Antenna Reflector Design.
random vibrations as shown in Fig. 6.3. The confidence level
associated with this requirements should be at least 95%.

iii) The maximum stress in any element of the antenna structure
should not exceed 10,000 psi, when the structure is subjected
to the random vibrational loading specified in Fig. 6.3.

iv) Minimum thickness of any section must be .02" (.05cm). This
requirement is based upon the manufacturability requirements.

The above requirements reflect the typical magnitude and the
genral nature of requirements imposed on the antenna design in the
aerospace industry, at present.

.6.5.2 Description of the Finite Element Model

The finite element mathematical model of the antenna structure
is presented in Figs. 6.4 to 6.6. The model consists of 34 nodes con-
ected with 24 plate elements and 32 beam elements. Each of the 32
nodes have six degrees of freedom and the two remaining nodes are fixed
in all six degrees of freedom. These fixed nodes represent the support
boundary conditions for the structure. The total number of degrees of
freedom of the system used in the present analysis is 192. The size of
the elements near the base has been kept smaller in comparison to the
elements on the outer edges of the structure. The finer divisions near
the base has been done because the most critical stresses are expected
near the base since the smaller element size would ensure greater
accuracy in the estimate of stresses in the plate elements. The coor-
dinate system used for the analysis is shown in Fig. 6.4. The excita-
tion axis of the structure has been taken as the z axis. This is the
Fig. 6.4. Node Locations with Finite Element Model of the Antenna Reflector.

NUMBER OF PLATE ELEMENTS = 24
NUMBER OF BEAM ELEMENTS = 24

Fig. 6.5. Beam Elements in Finite Element Model of the Antenna Reflector.
Fig. 6.6. Plate Elements in the Finite Element Model of the Antenna Reflector.
direction in which the most damaging excitations are experienced by the antenna structure.

The accuracy of the mathematical model with respect to the geometrical locations of the various nodes and with respect to the element connectivity was checked by performing geometry plots of the model. A sample of such geometry plots is given in Fig. 6.7. A listing of the finite element model used is included in Appendix G and Appendix H.

6.5.3 Formulation of the Design Problem for Optimization

The purpose of the optimization is to design an antenna with minimum possible weight and yet capable of meeting all the imposed design requirements. Hence, for the optimization, the particulars are:

The Objective Function is then the weight of the structure and is specified in the form \( f(\bar{x}) \), where \( \bar{x} \) is the vector of design parameters.

The four parameters, describing the antenna design selected for optimization and shown in Fig. 6.8, are stated below:

a) the thickness of the dish;
b) the height of the ribs at the back of the dish;
c) the width of the ribs; and
d) the thickness of the ribs.

The thickness of the dish and the height of the ribs are assumed to be linearly decreasing from the central support of the dish to the outer edge, and the slope parameter defining the thickness and the heights at various locations are to be optimized.
Fig. 6.7. Undeformed Shape Plot of the Antenna Reflector.
The variables for the optimization, thus, are:

$x_1$, the slope defining thickness of the plate;

$x_2$, the slope defining the height of the ribs;

$x_3$, the width of the ribs; and

$x_4$, the thickness of the rib section.

The constraints on the optimization, which reflect all the design requirements are as follows:

The Deterministic Constraints

1. Natural frequency $\geq 15$ Hz
2. $0.002$ radians $\leq x_1 \leq 0.05$ radians
3. $0.02$ radians $\leq x_2 \leq 0.50$ radians
4. $0.30''$ (0.76cm) $\leq x_3 \leq 100.0''$ (254.0cm)
5. $0.02''$ (0.05cm) $\leq x_4 \leq 10.0''$ (25.4cm)

The Probabilistic Constraints

1. $P[\text{Maximum displacement} \leq 1.0] \geq 95$
2. $P[\text{Maximum stress in ribs} \leq 10,000$ psi$] \geq 95$
3. $P[\text{Maximum stress in dish surface} \leq 10,000$ psi$] \geq 95$

Thus the problem is now fully defined for carrying out an optimal design using the procedure established earlier.

6.6 Optimum Design of the Antenna

The analysis-optimization system was updated to reflect the new problem. The necessary subroutines/subprograms/data files were also modified to reflect the problem in hand.
$X_1$ - SLOPE GOVERNING PLATE THICKNESS
$X_2$ - SLOPE GOVERNING RIB HEIGHT
$X_3$ - WIDTH OF THE RIBS
$X_4$ - THICKNESS OF THE RIB SECTION

Fig. 6.8. Design Variables to be Optimized for the Antenna Reflector.
The initial values of the design variables were chosen by using
standard beam formulae to satisfy the stress and displacement constraints.
This was done to choose realistic and meaningful starting values. The
weight saving thus obtained then becomes quite meaningful.

6.6.1 The Initial Design Parameters

The parameters describing the initial design of the antenna are
given by the following:

\[ x_1 = 0.004 \text{ radians} \]
\[ x_2 = 0.05 \text{ radians} \]
\[ x_3 = 0.5" (1.27cm) \]
\[ x_4 = 0.05" (0.13cm) \]

Weight of the structure = 19.6 lb (8.9kg)

First natural frequency = 8.4 Hz

Maximum RMS displacement = 0.047" (0.12cm)

Maximum RMS stress = 5881 psi (427kg/cm²)

Detailed results of the initial analysis are included in Appendix I.
Initial analysis also indicates that the starting design does not
satisfy the frequency and stress constraints.

6.6.2 The Final Design of the Antenna

After the completion of the automated optimal design performed
by employing the proposed system, a feasible design realizing a weight
less than the initial infeasible design is obtained. The design
history of the optimization cycle is shown in Fig. 6.10. The detailed
output of the optimization program is given in Appendix J.
The final optimum design is obtained as follows:

\[ x_1 = 0.002 \text{ radians} \]
\[ x_2 = 0.08 \text{ radians} \]
\[ x_3 = 0.68" (1.72\text{cm}) \]
\[ x_4 = 0.03" (0.076\text{cm}) \]

Objective function = 11.18 lb (5.08kg)
First natural frequency = 15.06 Hz
Maximum RMS displacement = 0.016" (0.04\text{cm})
Maximum RMS stress = 2073 psi (150.5kg/cm²)

This optimum design is depicted in Fig. 6.9. Detailed output of the final analysis contains information on eigen values, mode shapes and dynamic responses and is included in Appendix K.

6.7 Discussion of the Results

It can be seen that the proposed analysis-optimization system can automatically yield an optimum design of an antenna structural system. In the problem solved here, the initial starting design values did not meet all the design requirements, that is, the initial solution was actually infeasible since it did not satisfy the stress and the frequency constraints; yet the analysis-optimization system successfully produced a design which met all the requirements imposed on the design. The results show that starting with an antenna design with a weight of 20.0 lbs (9.09kg) a final optimum design weighing only 12.0 lb (5.5kg) could be realized, thus resulting in a net saving of 43% in weight.

The history of the design cycle is shown in Fig. 6.10. Most of the reduction in the weight takes place in the first four iterations.
Fig. 6.9. Optimum Design of the Antenna.
Fig. 6.10. Design History for Antenna Reflector Structure.
The optimization calculations could have been stopped at this stage by choosing a larger value as the convergence criterion. The parameter governing the thickness of the plate sections of the antenna dish is denoted by $x_1$. $x_1$ shows a rapid decrease during the first iteration and then stays at that value for subsequent iterations. This fixed value of $x_1$ is the minimum constraint value of $x_1$. The manufacturability of the dish dictates that certain minimum thickness be kept. The parameter governing the height of the rib, $x_2$, shows an oscillating pattern. Its final value is higher than its starting value, which suggests that an increase in the height of the rib provides a more effective structure. The width of the ribs, $x_3$, also shows an oscillating pattern in the beginning. The thickness of the rib section is given by $x_4$, and this parameter shows a continuous decrease in its value as the iterations continue indicating the direct relationship of the thickness to the weight of the structure. Thus Fig. 6.10 gives good insight into the total optimization process.

The size of the finite element chosen for this antenna design is not large enough to include sufficient nodes on the skin of the dish. This precludes the possibility of getting local plate vibration modes. The size of the model was kept small to limit the computation time. In actual practice it is possible to have a larger size finite element model for more accurate results.

6.8 Conclusions

This satellite antenna design has been accomplished using the design, analysis and optimization techniques developed as part of this overall
investigation. The optimum design for the antenna yields a weight of only 12.0 lb (5.5kg), which is 43% lower in weight in comparison to the best guessed initial design which had a weight of 20.0 lb (9.09kg). This optimum antenna design meets all the functional requirements in terms of the frequency, stress and displacement constraints imposed upon the design. Normally the duration of the design process in industry for typical satellite antenna structures is approximately two to three months and this duration can be considerably reduced to practically a few days by employing the proposed analysis-optimization computational system. Many other similar complex structures subjected to certain forms of random excitations of varying power spectral densities can also be designed for optimum considerations using the proposed system. The system can handle stochastic constraints as design parameters as has been illustrated for the case of satellite antenna design presented here.

Presently, the fatigue considerations are not included in the design requirements for the satellite antenna structures such as the one considered here because the loads are applied only for a very short duration. However, with the advent of the space shuttle program developed by NASA, it is essential that fatigue considerations will have to be incorporated into the design requirements in the future. Inclusion of the fatigue constraints in the structural design and the corresponding re-analysis including optimization of the satellite antenna structure with specified fatigue constraints is considered in the following chapter.
CHAPTER 7

DESIGN OPTIMIZATION WITH FATIGUE CONSTRAINTS

The major objective of this research investigation is to develop an overall technique to design a satellite antenna structure that weighs the least but yet can withstand any specified random vibrational environment with a maximum structural integrity or a minimum failure probability.

Two possible events may cause a structural system to fail. Firstly, the maximum value of the structural response parameters, such as the displacements or the stresses may exceed an upper bound level such failures have been discussed in the previous chapters and can be incorporated in the design procedure through previous analyses. Secondly, the structure may also fail when the accumulated damage due to repeated dynamic loads on the structure exceeds a fixed total. Such damage is accumulated when the structure goes through stress excursions which may not be large enough to cause a direct failure of the first type. The failure of the second type is termed as the fatigue failure and is an important factor in design. The design of an antenna structure for achieving minimum weight for a given strength under specified fatigue constraints is discussed in this chapter.

7.1. Formulation of the Fatigue Constraints

The failure criterion used in modelling the fatigue constraint is based upon the hypothesis proposed by Palmgren [33] and Miner [34]. This is a simple deterministic criterion and has been considered appropriate in formulating the fatigue constraint in many structural dynamic problems. Here it is assumed that each cycle of the random stress response inflicts
an incremental damage which depends upon the peak amplitude of the excursion. Each succeeding cycle inflicts additional damage and the failure occurs when the total damage reaches one hundred percent.

To quantitatively establish the fatigue strength for a specific material, a large number of identical samples are to be tested with varying stress amplitudes. The results of such tests when plotted define the S-N (stress vs number of cycles to failure) curve or the fatigue curve for the material. A typical S-N curve is shown in Fig. 7.1. The fixed stress amplitude is \( S \), the number of cycles until failure occurs at stress \( S \) is \( N \). For many materials, the curve is well approximated by a straight line when \( \log S \) is plotted against \( \log N \), that is, S-N curve may be approximated by the equation

\[
N S^b = C
\]  \hspace{1cm} (7.1)

where \( b \) and \( C \) are material dependent constants.

According to Palmgren-Miner hypothesis, when \( n \) cycles of stress amplitude \( S \) have been experienced, the material has used up a fraction of its fatigue life equal to \( n/N \), where \( N \) is the number of cycles at which failure occurs under uniform stress amplitude \( S \), as indicated by the S-N curve. Thus, if the material experiences \( n_i \) cycles of stress amplitude \( S_i \) for \( i = 1, 2, \ldots, M \), the total cumulative damage is given by

\[
D = \sum_{i=1}^{M} \frac{n_i}{N_i}
\]

(7.2)

According to Palmgren-Miner hypothesis the material will undergo a fatigue failure when the value of the total cumulative damage \( D \) reaches the value equal to unity. Palmgren-Miner hypothesis imposes no
Fig. 7.1 Typical S-N Curve
restrictions regarding the order of application of various stress levels, and is thus applicable to random loading processes in which the stress may vary from one cycle to other.

In order to use the Miner's criterion in formulating the fatigue constraint, it is to be assumed that the response of the structure may be considered as a narrow band process. The validity of this assumption will be later checked before applying the fatigue constraint. The fatigue constraint is then developed in the following manner.

Let $f_0$ be the expected frequency of the narrow band response process in cycles/sec and let $T$ be the time in seconds for which the structure has to withstand the fatigue environment. Then the expected number of stress cycles in time $T$ will be given as $f_0T$.

Let $p(a)$ be the probability density of stress peaks. Then the fraction of the total cycles having a peak stress between $a$ and $a+da$ is $p(a)da$.  

\[ E = \int_{a}^{a+da} p(a) \cdot da = f_0T \cdot \int_{a}^{a+da} p(a) \cdot da \
\]

Let $N(a)$ be the number of cycles at which failure will occur for a constant amplitude stress of $a$. Then according to the Miner's criterion, the accumulated damage for cycles in the range of $a$ and $a+da$ is

\[ \frac{n(a)}{N(a)} = f_0T \cdot \frac{p(a) \cdot da}{N(a)} \]

The total expected damage $E[D(t)]$ is given by

\[ E[D(t)] = f_0 \int p(a) \frac{da}{N(a)} \]

(7.6)
If the response is assumed to be a Gaussian stationary random process, then the peaks have a Rayleigh distribution, given by
\[ p(a) = \frac{a}{\sigma_y^2} \exp(-a^2/2\sigma_y^2) \]  
(7.7)
where \( \sigma_y \) is the RMS response of the stress. Substituting (7.1) and (7.7) in (7.6)
\[ E[D(t)] = \frac{f_0}{C} \int_0^\infty a^{b+1} \exp(-a^2/2\sigma_y^2) \, da \]
(7.8)
\[ = \frac{f_0}{C} (\sqrt{2} \sigma_y)^b \Gamma(1 + b/2) \]
(7.9)
The Miner's hypothesis states that the failure will occur when the total damage will be equal to unity. Hence the condition of failure may be stated as
\[ \frac{f_0}{C} (\sqrt{2} \sigma_y)^b \Gamma(1 + b/2) \geq 1 \]
(7.10)
Thus the fatigue constraint specifying that the structure should not fail for \( T \) seconds can be stated as follows:
\[ \frac{f_0}{C} (\sqrt{2} \sigma_y)^b \Gamma(1 + b/2) \leq 1 \]
(7.11)

### 7.2 Optimization of the Antenna Structure

The fatigue constraint developed as per equation (7.11) can now be used together with other design constraints to arrive at an optimum weight structure which will satisfy all the design constraints including fatigue. In the process of deriving the fatigue constraint, it was assumed that the response of the antenna structure is a narrow band random process. Before using the fatigue constraint given in (7.11),
this assumption regarding the narrow band response is to be verified.

7.2.1 Frequency Response Analysis

The frequency response of a multi-degree of freedom system subjected to harmonic excitations is given by [30]

\[
y_f = \sum_{r=1}^{n} \phi_{2r} \frac{T_R}{M_R \omega_r^2} \frac{\ddot{x}(\omega)}{\sqrt{[1 - (\omega/\omega_r)^2 + 4\zeta^2]}}
\]  

(7.12)

where

- \(y_f\) is the peak displacement,
- \(T_R\) is the participation factor for the \(r\)th mode,
- \(M_R\) is the generalized mass for the \(r\)th mode,
- \(\zeta\) is the structural damping,
- \(\ddot{x}\) is the peak acceleration of the excitation,
- \(\omega_r\) is the \(r\)th natural frequency for the system, and
- \(\omega\) is the frequency of excitation in rad/sec.

The frequency response computation was carried out in the present context using the SPAR finite element program. Additional software had to be generated to enable SPAR to compute the frequency response. A plot of the frequency response is shown in Fig. 7.2. As can be seen from this figure, the response of the antenna is a narrow band process and the predominant natural frequency of the structure is 22.5 Hz. Thus the fatigue constraint developed as per (7.11) can be justifiably used for the antenna structure.

7.2.2 Definition of the Optimization Problem

The satellite antenna structure described in the previous chapter
RESONANCE FREQUENCY 22.5 Hz

Fig. 7.2. Frequency Response of the Antenna Structure.
is now considered to include the application of the fatigue constraint. The initial values chosen for the design parameters are same as the optimum design parameters calculated in the previous chapter. The initial values of the design parameters have been chosen in this manner to clearly bring out the effect upon the optimized weight of the structure due to inclusion of the fatigue constraint also. All the other constraints imposed on the design in the previous chapter have been retained. To recapitulate, the optimization problem for designing the antenna with the fatigue constraint can now be stated as follows:

Minimize - Weight of the antenna structure subject to the following constraints:

a) The first natural frequency ≥ 15 Hz;
b) P[Maximum displacement ≤ 1.0"] ≥ 95%;
c) P[Maximum stress ≤ 10,000 psi] ≥ 95%;
d) No fatigue failure for T = 36000 seconds;
e) Limits on design parameters

- 0.002 radians ≤ x₁ ≤ 0.05 radians
- 0.02 radians ≤ x₂ ≤ 0.50 radians
- 0.3" (0.76cm) ≤ x₃ ≤ 100" (254.0cm)
- 0.02" (0.05cm) ≤ x₄ ≤ 10.0" (25.4cm).

The above parameters x₁ to x₄ are described in the previous chapter.

Following starting values were chosen for x₁ to x₄:

x₁ = 0.002 radians
x₂ = 0.08 radians
x₃ = 0.68" (1.72cm)
\[ x_0 = 0.03" \ (0.076 \text{cm}) \]

Initial values of other antenna parameters were:

- Weight of the structure = 11.18 lb (5.08kg)
- First natural frequency = 15.06 Hz
- Maximum RMS displacement = 0.016" (0.04 cm)
- Maximum RMS stress = 2073 psi (145.7 kg/cm²)

The only change that is to be made in the software of the system, discussed earlier in Chapter 6, was in the end processor program REPRS2, where the constraints are defined. The fatigue constraint as derived in equation (7.11) was added to the existing constraints. The master procedure file RPR01 was then executed which resulted in the following optimum solution.

7.2.3 The Optimum Design

On the basis of the computations carried out including the constraint developed in here, the following optimum solution is arrived at:

- Minimum weight = 12.3 lb (5.6kg)
- First natural frequency = 17.6 Hz
- Maximum RMS displacement = 0.004" (0.010 cm)
- Maximum RMS stress = 512 psi (37.2 kg/cm²)
- Design variables \[ x_1 = 0.002 \text{ radians} \]
  \[ x_2 = 0.089 \text{ radians} \]
  \[ x_3 = 0.629" \ (1.597 \text{cm}) \]
  \[ x_4 = 0.032" \ (0.081 \text{cm}) \]

The final analysis for the minimum weight design of the antenna including
the fatigue constraint is automatically produced by the software package developed. The change in the weight of the antenna structure and also the change in the values of the design parameters during various iterations are plotted in Fig. 7.3. The weight of the antenna continuously increases through various iterations. This is due to the fact that the starting design here was the optimum design without the fatigue constraints and to satisfy the fatigue constraint the structure must be made stronger in comparison to the previous optimum design that was achieved without the fatigue constraints. The design variables $x_1$ to $x_n$ show very little change in their values because the starting values correspond to the optimum design without fatigue constraints and hence there is very little room for change in their values.

7.3 Discussions and Conclusions

The minimum weight of the antenna structure considering the fatigue constraint is 12.3 lb (5.59kg) as compared to the minimum weight of 11.2 lb (5.09kg) for the structure without the fatigue constraint. Thus an increase of 9.8 percent in weight is the penalty for including the fatigue constraint. The weight of the structure will also depend upon the time limits for the fatigue environment. The weight of the structure will increase with the increase in the duration of the fatigue environment. At present, fatigue normally does not enter into the design requirement for spacecraft antennas because the loads are applied only for a very short duration. However, with the advent of the space shuttle it is very likely that some structure will have to undergo loads for more than one launch, and then fatigue will become an important
Fig. 7.3 Design History - Optimization of the Reflector With Fatigue Constraints

NOTE: STARTING DESIGN IS THE OPTIMUM DESIGN WITHOUT FATIGUE CONSTRAINTS
Thus the proposed system of designing spacecraft structure can be used for designing structure to include fatigue requirements. The general nature of the analysis-optimization system and its user-oriented aspects are discussed in the following chapter. The potential design applications of this analysis-optimization system are also noted in the next chapter.
CHAPTER 8

EXTENSION OF THE METHODOLOGY TO

GENERAL STRUCTURAL DESIGN UNDER ARBITRARY RANDOM LOADS

The design of complex structures under dynamic loads requires proper analytical tools to first perform a dynamic analysis and then using this analysis to optimally design the structures to withstand the specified loading environment. Many of these structures in most cases have to withstand random type of loading environment which can only be described by power spectral densities varying in different manners in the frequency domain. The approach developed to analyze and optimally design the satellite antenna structure as part of this investigation can be extended to perform dynamic analysis including an optimal design for such complex structures.

The random loads experienced by many structures originate from a variety of sources. For example, in buildings, the random loading is produced by the wind load and the seismic loads; in automobiles by the irregularities of the road surface; and for offshore drilling platforms such loads are produced by the tidal waves of the sea. In order to analyze and design these structures against such loadings, it is essential to describe the power spectral densities of the imposed excitations in a mathematical form, establish a finite element formulation of the system and develop an appropriate finite element software to compute the response process. Further, an optimization scheme is necessary to ensure the most efficient usage of the material in the construction of such structures, without sacrificing reliability under the specified loading environment. The approach developed for analyzing and designing the
satellite antenna structure can be extended to handle similar problem for analysis and design of other complex structures because of the flexibility in approach and its user oriented nature. Such an extension of the method is discussed in this chapter.

8.1 User Oriented Nature of the System

The system of analysis-optimization presented earlier has been developed keeping the user or the designer in mind. Several features of the system reflect clearly this user oriented nature.

The program software for analysis-optimization system is basically problem independent, that is, for a new problem, the system can be used as is. The areas requiring modifications are where the details of the finite element model are specified and where constraints on the design are defined. The step by step instructions for required modifications to the system for adapting it to a new problem are included in the next section of this chapter.

Several procedure files have been written as part of this system. These files serve the function of calling and executing various programs using a predetermined logical sequence. These procedure files greatly reduce the user's tasks, for example the complete analysis-optimization system can be executed by a single call of the master procedure file RPRO1. This master procedure file calls several other procedure files which remain transparent to the user.

The system is modular in nature. That is, one module of the system can be easily replaced by another user written module. For example, if a user wants to change the technique developed here and employed
for computing the random response analysis, then a new Fortran program similar to REPRSI (Appendix D) can be written and used instead of the existing REPRSI. Similarly, other parts of the program can be changed, if required. This modular nature provides flexibility to the system and makes it easily adaptable for any new problem.

8.2 Guidelines for Adaptation to General Structural Design

This analysis-optimization system can be adapted to any new structural design applications. However, the assumptions made in the process of the development should be borne in mind. The system is applicable for the analysis and design of linear mechanical systems with small structural damping only. The assumption of the small damping has been made to neglect the cross coupling between the various modes of vibrations while deriving expressions for the probabilistic response of the structure. The assumption of the linear behaviour of the structure has been used all through the development of this investigation. The random excitations, for which structures are to be designed, have been considered to be stationary and normally distributed. If the excitations cannot be treated as stationary, then this approach may not give meaningful results. The system developed is capable of handling structures upto 10,000 degrees of freedom, however this is not a real limitation. The core in the finite element program can be increased using the core reset commands to increase the capacity of the program to handle a greater number of degrees of freedom. Such core reset commands are described in the SPAR user's manual [11].

Various steps which should be taken to adapt this system for any other application are summarized below. A sequence of these steps
also shown in Fig. 8.1 in the form of a flow chart.

(i) First a finite element model of the structure to be analyzed and designed should be constructed. The finite element model data in the format required by SPAR [11] should then be generated.

(ii) The finite element model data is then split into two parts. One of these parts contains the commands which have to be executed only once such as data defining node locations, element connectivity, material properties, etc. All the data pertaining to the nonrepeatable part is put on a file called RNREP. The other part of the data that is to be executed within the optimization loop is called the repeatable data and is put on a file called RREP1.

(iii) A front processor FORTRAN program should then be written. The front processor transforms the optimization design variables into structural parameters in a format suitable for executing SPAR. The input to the front processor is the design vector \( x \) and the output contains the structural parameters which are put on a file called RFPOUT. An example of the front processor is given in Appendix C.

(iv) The edit commands to update the data corresponding to the repeatable part during each optimization cycle should be written. These commands merge the data generated by the front processor to the repeatable analysis data and replace the old data with this new data.
NEW PROBLEM

GENERATE THE FINITE ELEMENT MODEL

PREPARE DATA IN A FORMAT SUITABLE FOR SPAR.
DIVIDE DATA INTO REPEATABLE AND NONREPEATABLE PARTS

SELECT DESIGN VARIABLES

WRITE FRONT PROCESSOR

PREPARE DATA FILES RINT & RINP

UPDATE MAIN PROGRAM
FOR OPTIMIZATION

EXECUTE PROCEDURE
FILE RPRO1

OPTIMUM SOLUTION
ON FILE RFINOT

Fig. 8.1. Sequence of Operations to Adapt the System to General Structural Design.
(v) The data relating to the applied loading on the structure, the damping of the structure, and the direction of the random excitations is contained on a file called RINT. This data can also be given directly in response calculation program, as shown in the sample program included in Appendix D.

(vi) A data file RINP contains the information regarding the limiting values of the stress and the displacements as well as the probability bounds with which the constraints have to be satisfied. This information is used in generating the constraint functions.

(vii) The initial data required in the main optimization program such as the number of variables, their starting values, the convergence criterion etc., should be updated in the main program to reflect the new problem. The main program used for the satellite structure optimization scheme is included in Appendix B. The dimension statements in the main optimization should be in accordance with those described in the optimization program reference manual [32].

(viii) The fatigue constraints imposed on the design can be handled in a manner proposed in Chapter 7. Additional constraints, if any can also be incorporated into this approach. This task of adding constraints is accomplished through the Fortran program REPRS2. An example of REPRS2 is included in Appendix E.

All the other software for the system remain unchanged for the new application. The system is then executed by simply submitting the
file RPRO1 for execution. It is advisable to set ITMAX = 1 for the first execution to check out various modifications. Once the system is working, ITMAX may be increased to any appropriate number and the system can be executed in total. At the end of the execution, the optimization results are contained in a file called TAPE6 and the final analysis is contained in a file called FINOUT. These files are automatically saved. These files contain the optimum values of the design variables, the final weight of the structure, the natural frequencies, the displacement response and the stress response for the final design of the structure.

8.3 Potential Applications

As stated, the proposed system can be used conveniently for designing several types of structures besides the satellite antennas. An application procedure of this system for designing multistorey building structures is discussed here. Multistorey building structures are designed to withstand also the earthquake and wind loads, in addition to the normal static and live loads. These earthquake and wind loads randomly fluctuate in nature and are described by different profiles of power spectral densities of excitations which could vary arbitrarily in the frequency domain. Since the satellite antenna structures have been designed for such types of random excitations, the approach developed in this investigation can be easily extended to the design of multistorey building structures. The detailed procedure explaining application of this design approach to multistorey building structures is described in the next section.
8.4 Design of a Multistorey Building Structure

The multistorey building structure to be designed has been assumed to have eight floors for the purpose of illustration. However, the procedure described here is applicable to buildings with any number of floors and columns. Normally, various design parameters have to be established for a complete design of the building structure. Here, it has been assumed that the major objective is to determine the cross sectional areas for various column and beam members at various floor levels of the multistorey building structure. The detailed steps are as follows:

8.4.1 Generation of the Finite Element Model for the Structure

The first step is to construct an appropriate finite element mathematical model of the structure. For the eight storey structure supported on sixteen columns, a model of 144 nodes connecting 272 beam elements is constructed as shown in Fig. 8.2. The next step is to write the runstream of SPAR codes for this finite element model. The runstream will be similar to the one shown in Appendix G and can be accomplished with the help of the SPAR manual [11]. This runstream should be divided in two parts. The first part defines the geometrical location of each of the 144 nodes, the element connectivity and the material properties for various beam elements. This part of the finite element model will be executed only once and hence is the nonrepeatable part. This data should be stored in the file called RNREP. The second portion of the runstream will consist of statement which require executions of the mass matrix, the stiffness matrix, eigen values, mode shapes etc. This
Fig. 8.2. Finite Element Model of a Building Structure.
portion is executed within the optimization loop and the data is stored in file named RREP1. The data is similar to the run shown in Appendix H.

8.4.2 Selection of the Design Parameters

It is assumed that the shape of the column cross-section is an I-section. The required design variables are the sizes governing the I-section and are as follows:

\[ x_1 \] - the height of the I-section.
\[ x_2 \] - the width of the I-section.
\[ x_3 \] - the thickness of the web and the flange of the I-section.

The size of the column is assumed to vary for each floor. Thus the number of design variables is equal to 3 per floor and is equal to 24 for the complete structure.

8.4.3 Generation of the Front Processor Program

The front processor is a Fortran program and is unique to each application. The function of the front processor is to convert the design variables into section property cards for the beam elements. For the multistory building the front processor will convert the section variables, into section property cards. An example of a front processor is given in Appendix C. The name of this program file is FPRS.

8.4.4 Incorporation of the Design Requirements

The data relating to the applied loading on the structure is given in the response program shown in Appendix D. The limiting values of the stress and the displacements as well as the probability bounds with which the constraints have to be satisfied are incorporated through a
data file RINP. The constraints are formulated in Fortran program REPRS2. The fatigue constraints and any other constraint on the building structure are to be included in this program as shown in Appendix E.

8.4.5 Choosing the Initial Design

The initial design values for the variables governing the I-section of the columns of the building structure are determined using simple hand calculations or by an approximate guess based on experience. The information for this initial design goes on a file called RINT.

8.4.6 Execution of the System for Design

The final step in solving this problem is the execution of the master procedure file RPRO1. A single execution of this file results in the final design information for the structure. The final analysis and the information about design parameters is stored in the file RFINOT. The final results contain the information about the optimum sizes of the design variables, including the minimum weight for the structure. The final structural analysis giving the natural frequencies, mode shapes, displacement and the stress response for the optimum structure is also on the file RFINOT. Using this information the detailed design for the building is generated.

8.5 Conclusions

It is shown that the analysis-optimization technique developed can easily be extended to general structural design as demonstrated in this chapter. Multistorey building structures are not the only structures where this methodology can be employed. It can be applied to
many other structural design problems. For example, this methodology can be applied to design of nuclear power plant structures, offshore drilling platforms, automobile and aircraft structures, etc.

To summarize, there exists an array of potential applications for the analysis-optimization system developed. The methodology presented here is a comprehensive analysis-design system which can both reliably determine the response of general structures as well as produce an optimum design capable of meeting all the different design requirements for those structures.
CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

A computer based analytical approach towards designing satellite antenna structures has been proposed where the satellite antenna structures have to meet several demanding requirements. These satellite antenna structures have very strict reliability requirement, but at the same time these structures must be as light as possible to minimize the cost of launching the satellite into space. These antenna structures have also to be designed to sustain random excitations that have an arbitrarily varying profile of power spectral densities in the frequency domain. In order to achieve the best design for a minimum weight antenna structure, a reliable technique for computing the dynamic response of the structure is essential. Mathematical techniques to model the forces and to compute the response of these complex structures have been developed and presented here to meet this requirement. The finite element program SPAR has been used as a base for the purpose of computerizing the analysis for the response of antenna structure subject to arbitrary random excitations.

Since the satellite antenna structures are subjected to randomly varying excitations from launch conditions, the response quantities can only be expressed in a probabilistic manner, and as a result, the constraints on these parameters have to be formulated also in a probabilistic manner. A technique of handling the probabilistic constraints in the mass optimization procedure has been developed and presented.

A specific satellite antenna structure with a minimum weight, yet capable of sustaining the specified random vibrations of arbitrarily
varying power spectral densities was designed. A method for incorporating the required fatigue life in the design of the satellite antenna structure has also been developed.

The method used in this investigation is an extension of the approach proposed in [27], which is adapted to include probabilistic constraints upon the dynamic response of the structure. Major highlights of the work are presented in the following section.

9.1 Major Highlights

9.1.1 The Mathematical Model for Random Excitations of Arbitrarily Varying Power Spectral Density

A mathematical modelling technique for describing randomly varying excitations with varying power spectral densities in the frequency domain has been proposed. The mathematical model essentially envelopes in a linear manner the arbitrarily varying spectral density profile. The envelope consists of a number of linear segments of varying slopes when power spectral densities are plotted in a logarithmic scale. Since it is possible to calculate the response for these individual linear segments, the net response for the structure is obtained by summing up the responses due to excitations by these linear segments, assuming that the structure has a linear behaviour. The model was verified experimentally by subjecting a cantilever beam to excitations of varying power spectral densities. The experimentally measured response was within 2% of the analytically evaluated response using the proposed model, thereby validating the usefulness of such an approach in evaluating the response of structures subjected to such excitations of arbitrarily varying power spectral
density.

9.1.2 The Random Response Analysis

Analysis and software to compute the displacement and the stress responses of any structural system under a specified random vibrational environment has been completely developed. The finite element program SPAR developed by NASA has been used as the main frame of the analysis program. The RMS displacement response of the structure is generated using the analytical expressions developed for this investigation. These RMS displacements are then treated as enforced joint displacements to compute the RMS stress response for the structure.

9.1.3 Optimization with Probabilistic Constraints

Normal optimization techniques can only handle deterministic constraints, whereas the constraints on the parameters such as stress or the displacement of the structures subjected to random excitations can only be expressed in the form of associated probabilities. A technique of handling such constraints has been proposed. The proposed technique transforms probabilistic constraints into a set of deterministic constraints in such a way that the deterministic constraints retain their probabilistic nature. This transformation permitted the use of existing techniques for optimization and the handling of imposed constraints.

9.1.4 Synthesis of Analysis and Optimization Procedures

In order to achieve an optimum (minimum weight) structural design capable of withstanding a specified random loading environment, it is necessary to synthesize the operations of the random response structural analysis and the optimization scheme into one system. Various procedure
files have been developed which serve the function of a connecting network for the finite element program, the optimization subroutine and the response analysis programs. This connecting network is essentially the brain of the whole system. It coordinates the various elements and components of the design system to do their individual tasks whenever required. The final synthesized system makes it possible to start from any arbitrary design and obtain the optimum solution in one single operation.

9.1.5 The Design of a Satellite Antenna Structure

Prime objective of this investigation was to develop the capability of designing a satellite antenna structure, capable of withstanding a specified random vibration environment. Optimum values of the design parameter for the antenna such as the dimensions of the rib cross-section and the thickness of the face sheet for the antenna structure were computed using the technique developed for this investigation. The structure was required to have the first natural frequency greater than 15 Hz and at the same time not to exceed the displacement and stress limiting constraints. The optimum structure realized using the proposed technique meets all the design requirements and weighs 12.0 lb (5.5kg) as compared to a weight of 20.0 lb (9.09kg) corresponding to the initial design for the structure, thus resulting in a net weight saving of 43%.

9.1.6 Design of the Antenna Structure Including Fatigue Constraints

Under certain circumstances, fatigue constraints may also be imposed upon the design of antenna structures along with other constraints on peak displacement and peak stresses. Such constraints specify the minimum design life required. Fatigue constraints suitable for applica-
tion to the design of an antenna structure and based upon the Miner's criteria of accumulated damage have been developed. A frequency response analysis of the structure was performed to ensure the validity of the assumptions regarding the narrow band nature of the response. As expected, a slight increase in the weight of the structure took place when the fatigue constraints were applied in addition.

9.1.7 Other Applications

The versatile nature of the design approach is illustrated by employing the procedure for an optimal design of a multistorey building under dynamic loads. Extension to other applications are also indicated.

9.2 Limitations of the Investigation

The design approach of synthesizing the structural analysis and the optimization scheme, that is developed for satellite antenna structures and applicable to other complex structures has the following limitations.

a) The random excitations for which the structures are designed have been treated as a stationary process described through their spectral density variations in the frequency domain.

b) All through the development of this investigation, the structure has been assumed to exhibit linear behaviour.

c) The damping in the system has been assumed to be small, thereby making it appropriate to neglect the coupling of the different modes when computing the response of the structure.

d) The size of mathematical finite element model has a limitation
of up to a maximum of 10,000 degrees of freedom, which is sufficient for most applications.

9.3 Recommendations for Future Work

The techniques developed here for designing complex structures assume that the random excitations are stationary. This is a reasonable assumption and holds true in most practical situations. However, if it were necessary to treat the excitations as nonstationary, the analysis gets mathematically complex. However, the forces could be treated as a series of segments of stationary excitations in an approximate sense and the technique developed here can then be used in a step by step or piecewise manner to analyze and design structures subjected to nonstationary random excitations. The summation of responses due to different stationary segments would require still assumption of linearity.

The structural response for the antenna structure has been derived assuming a linear behavior of the structure. The proposed technique can perhaps be extended to nonlinear structures in future. However, the nonlinear behavior of complex structures under random loading is a difficult topic. An approximate method would be to consider a separate analysis over several small displacement ranges where linearity may be assumed.

The assumption of small damping is quite valid for satellite antenna structures, thereby making it possible to neglect the coupling of different modes for computing the response of the structure. This limitation can be eliminated by including the terms providing the contributions due to coupling of different modes in the total response calculation. In this case, the dynamic modal analysis aspect of the present study has
to be completely reworked.

It may be possible to obtain an improved mathematical model for the arbitrarily varying profiles of excitations by using linear prediction techniques described in [36]. In this method the time history of the signal corresponding to the actual power spectral density is expressed as a linear combination of the past outputs and the present and the past inputs. The analytical expressions for approximated power spectral densities are obtained by minimizing the least square error. The analytical integration of the best fit function obtained in this manner may not be possible and techniques of numerical integration may have to be employed for computing the response of the structure subjected to such excitations.

The gradients of the objective function and the constraints used in the optimization algorithm have been computed using the finite difference approach. These gradients can be computed analytically [38], by manipulating the structural mass and stiffness matrices. The use of analytical gradients in optimization will reduce the computer time used in calculating the gradients and will also improve the efficiency of the algorithm. The possibility of incorporating this approach of calculating gradients should be considered in future.
REFERENCES


APPENDIX A

SAMPLE LISTING OF TABLE OF CONTENTS OF THE
FINITE ELEMENT LIBRARY

This appendix gives a listing of table of contents stored in the SPAR data base. Each line of the table contains information denoting when the data was created, the name of the data set and the number of words in the data sets. This listing tells the user what data already exists in the SPAR data base. A reference to this appendix was made in section 3.1 of Chapter 3.
APPENDIX A

SAMPLE LISTING OF TABLE OF CONTENTS OF THE
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APPENDIX B

MAIN PROGRAM FOR OPTIMIZATION

This is the main program which calls the optimization subroutine CONMIN. The starting values of various parameters are defined in this program. This program requires updating for every new application of this design system. This appendix has been referred to in section 5.1.2 of Chapter 5.
MAIN PROGRAM FOR OPTIMIZATION

PROGRAM OPTMAIN(INPUT, OUTPUT, TAPE6, TAPE7, TAPE9, TAPE10, TAPE11)
COMMON /CHM1/, DELFUN, DABFUN, FBCH, FDCH, CT, CTHIN, CTL, CTLMIN, ALPHAX
& AOBJ1, THETA, OBJ, NDV, NCON, NSIDE, IPRINT, NFDG, NSCAL, LINOBJ, ITMAX, IT
& & RND, ICNDIR, IGOTO, MAC, INFO, INF00, ITER
DIMENSION X(6), VLB(6), VUB(6), B(12), SCAL(6), DF(6), A(6, 51),
3B(6), B1(12), G2(12), D(12), S(12), C(51), ISC(12), IC(51), NS1(202)
COMMON/CONSAV/RSAV(50), RSAV(25)
NAMELIST/PASSAGE/NPASS
NAMELIST/LINKE/OBJ
NAMELIST/LINFF/NDV
NAMELIST/SAVE/IPRINT, NDV, ITMAX, NCON, NSIDE, ICNDIR, NSCAL, NFDG,
1FDCH, FBCH, CT, CTHIN, CTL, CTLMIN, THETA, PHI, MAC, DELFUN, DABFUN,
2LINOBJ, IT0R, ITER, IINFO, IGOTO, INFO, OBJ, X, DF, G, ISC, IC, A, S,
33B, G2, C, NS1, B, VLB, VUB, SCAL, RSAV, IBAV, NCOUNT, M1, M2, M3, M4,
4N5, ALPHAX, AOBJ1, KF
READ(10, PASSAGE)
C NPASS=1 FOR INITIAL PASS
GO TO(500, 400), NPASS

500 CONTINUE
IPRINT=3
NDV=4
KF=0
ITMAX=40
NCON=4
NFDG=0
M1=6
M2=11
M3=51
M4=51
M5=102
ALPHAX=0.1
ABOBJ1=0.1
IGOTO=0
INF00=1
MACNX=51
NSIDE=1
VLB(1)=.002
VLB(2)=.02
VLB(3)=.03
VLB(4)=.02
VUB(1)=.05
VUB(2)=.05
VUB(3)=.10
VUB(4)=.10
X(1)=.002
X(2)=.002
X(3)=.002
X(4)=.0308
ICNDR=0
NBL=0
LINOBJ=0
ITM=FDCHM=CT=CTMIN=0
FDCHM=0.01
CTMIN=CT=CTMIN=0
NBL=5
THETA=1.0
DO 10 J=1,NCON
10 ISC(J)=0
DO 21 I=1,N1
21 SCAL(I)=1.0
GO TO 700
600 CONTINUE
READ(7,SAVE)
REWIND 7
READ(9,LINKE)
700 NPASS=2
REWIND 10
WRITE(10,PASSAGE)
30 CONTINUE
100 FORMAT (5E16.8)
CALL CONMIN(X,VLB,VUB,B,SCAL,DF,A,S,G1,G2,B,C,
1ISC,IC,MB1,N1,N2,N3,N4,N5)
KFKF=1
WRITE(7,SAVE)
REWIND;9
WRITE(9,LINKF)
REWIND 9
101 FORMAT (*GO TO 2,*
IF(IGOTO.EQ.0)WRITE(11,101)
REWIND 11
STOP
END
APPENDIX C

FRONT PROCESSOR PROGRAM

This appendix contains the listing of the Front Processor program. This program serves the function of transforming the optimization variable vector $X$ into the property data cards required by the Finite Element Program SPAR. This program is written individually for every new application. A reference to this appendix has been made in section 5.1.3 of Chapter 5.
APPENDIX C

FRONT PROCESSOR PROGRAM

PROGRAM FPRS(INPUT, OUTPUT, TAPE9, TAPE13)
DIMENSION X(6)
NAMELIST/LINKF/NDV,X
READ(9, LINKF)
WRITE(13, 1)
1 FORMAT(*SHELL SECTION PROPERTIES* COMES FROM FRPS*)
   T1=(35.,-7.5)*TAN(X(1))
   T2=(35.,-15.)*TAN(X(1))
   T3=(35.,-27.5)*TAN(X(1))
   H1=(35.,-7.5)*TAN(X(2))
   H2=(35.,-15.)*TAN(X(2))
   H3=(35.,-27.5)*TAN(X(2))
WRITE(13, 2) T1, T2, T3
2 FORMAT(*1*,F10.5/*2*,F10.5/*3*,F10.5)
WRITE(13, 10)
10 FORMAT(*BEAM ORIENTATIONS*/FORMAT=2*/1 0 0 5*/
   1*E21 SECTION PROPERTIES*)
   H=H1
   DB 5 I=1,3
WRITE(13, 3) I, X(3), X(4), H, X(4)
3 FORMAT(*BOX*/,I1, I2, I3, 4F10.5)
IF(I.EQ.1)H=H2
IF(I.EQ.2)H=H3
5 CONTINUE
   WRITE(13, 6)
6 FORMAT(*BOX 4 5, 2.1, 2.1*)
STOP
END
APPENDIX D

RANDOM RESPONSE CALCULATION PROGRAM

This Fortran program computes the stochastic displacement response for the structure under random loading. It extracts the natural frequencies (eigen values), the mode shapes (eigen vectors), and the mass matrix information from the SPAR data base using the subroutine DAL. All these different information are suitably combined within the program to compute the response. The random response calculation program has been referenced in section 5.1.4 of Chapter 5.
PROGRAM RESPON(INPUT,TAPE5,TAPE6)
COMMON A(2500)
DIMENSION ZETA(20),OTR(20),OSNM(204,8),ONM(204,8),GNM(8)
DIMENSION OMG(8),X(204),PSD(8),C(204),OM2(204)
PI=4.0*ATAN(1.0)
NF=8
KFL=3
NJ=34
NJ6=204
K=0
DO 2000 J=1,NJ
DO 2000 I=1,NF
K=K+1
C(K)=0.
IF(I.EQ.KFL)C(K)=1.0
2000 CONTINUE
DO 999 I=1,NF
ZETA(I)=.01.
PSD(I)=4000.
999 CONTINUE
CALL DAL(1,11,A(1),O,IEA,KADR,IER,NNDS,NE,LB,ITYPE,
13HM2T,3HAUS,1,1)
SUM=0.0
DO 1 I=1,NJ6
SUM=SUM+A(I)/3.0
1 
OM2(I)=A(I)/3B6.
WRITE(6,10)(OM2(I),I=1,NJ6)
10 FORMAT(1X,M10.10)
CALL DAL(1,11,SUM,0,1,KADR,IER,1,1,1,1,1,1,3HOBJ,3HAUS,1,1)
CALL DAL(1,11,A(1),0,IEA,KADR,IER,NNDS,NE,LB,ITYPE,
13HSNM,3HAUS,1,1)
K=0
DO 3 J=1,NF
DO 3 I=1,NJ6
K=K+1
OSNM(I,J)=A(K)
3 CONTINUE
WRITE(6,20)((OSNM(I,J),I=1,NJ6),J=1,NF)
20 FORMAT(5X,16I2)
CALL DAL(1,11,A(1),0,IEA,KADR,IER,NNDB,NE,LB,ITYPE,
14HNORM,4HMODE,1,1)
K=0
DO 4 J=1,NF
DO 4 I=1,NJ6
K=K+1
ONM(I,J)=A(K)
4 CONTINUE
WRITE(6,30)((ONM(I,J),I=1,NJ6),J=1,NF)
30 FORMAT(5X,16I2)
DO 3000 J=1,NF
OTR(J)=0.
3000 CONTINUE
DO 3000 I=1,NJ6
  OTR(J)=OTR(J)+GNM(I,J)*C(I)*OM2(I)
3000 CONTINUE
  WRITE(6,5000)(OTR(I),I=1,NF)
5000 FORMAT(5X,*PARTICIPATION FACTORS*/8(E16.8/,/))
  CALL DAL(1,11,A(1),0,IEA,KADR,IERR,NWDS,NE,LIB,ITYPE,
   12HGM,3HAUS,1,1)
  WRITE(6,40)
40 FORMAT(2X,*GENERALIZED MASS*)
  WRITE(6,50)(A(I),I=1,8)
50 FORMAT(6E10.4)
  DO 8 I=1,NF
  GNM(I)=A(I)
8  CALL DAL(1,11,A(1),0,IEA,KADR,IERR,NWDS,NE,LIB,ITYPE,
   14HVIBR,4HEVL,1,1)
  DO 9 I=1,NF
  OMG(I)=SORT(A(I))
9  DO 100 I=1,NJ6
  X(I)=0.
  DO 101 J=1,NF
  S1=GNM(I,J)*(OTR(J)**2)*PSD(J)
  S2=OMG(J)**3*(GNM(J)**4)*&ZETA(J)
  X(I)=X(I)+(S1/S2)
101 CONTINUE
  X(I)=SORT(X(I))
100 CONTINUE
  WRITE(6,200)(X(I),I=1,NJ6)
200 FORMAT(*RMS DISPLACEMENT*/5(6E10.4,/))
  CALL DAL(1,1,X,0,1,KADR,IERR,204,34,204,-1,4HAPPL,4HMTI,1,1)
  CALL FIN(0,0)
END
APPENDIX E

END PROCESSOR PROGRAM

This program extracts the mass, displacement and stress information from the SPAR data base. This information is used for formulating the objective function and the constraints for optimization. The output of this program is the weight of the structure and the design constraints. This appendix has been referenced in section 5.1.5 of Chapter 5.
PROGRAM STRESS(INPUT, TAPES1, TAPE2, TAPE3)
COMMON A(2500)
DIMENSION STRS(52,32), STRS1(33,24), G(4), DISP(6,34), X(6)
NAMELIST/LINKE/OBJ,G
NAMELIST/LINKE/NDV,X
NAMELIST/PREPIN/E21AL,ALDISP,E43AL,ALFREQ
RETRND 9
READ(9,LINKF)
READ(5,PREPIN)
CALL DAL(1,11,A(1),O,IEA,KADR,IERR,NWDS,NE,LB,ITYPE,
14HSTRS,4HE42 ,1,1)
K=0
DO 10 J=1,24
 DO 10 I=1,33
 K=K+1
10 STRS1(I,J)=A(K)
 CALL DAL(1,11,A(1),O,IEA,KADR,IERR,NWDS,NE,LB,ITYPE,
14HSTRS,4HE21 ,1,1)
 K=0
 DO 1 J=1,52
 DO 2 I=1,32
 K=K+1
2 STRS(I,J)=A(K)
1 CONTINUE
 WRITE(6,30)
30 FORMAT(2X, *RMS STRESS RESPONSE, PLATE ELEMENTS *)
 D1=7.5
 D2=15.0
 D3=27.5
 T1=(35.-D1)*TAN(X(1))
 T2=(35.-D2)*TAN(X(1))
 T3=(35.-D3)*TAN(X(1))
 T1=T1*T1/6.
 T2=T2*T2/6.
 T3=T3*T3/6.
 T=T1
 SPMAX=0.
 DO 300 J=1,24
 DO 200 I=29,31
 STRS1(I,J)=STRS1(I,J)/T
 IF(J .GT. B) T=T2
 IF(J .GT. 16) T=T3
200 CONTINUE
 B1=STRS1(29,J)
 B2=STRS1(30,J)
 B3=STRS1(31,J)
 SP=0.5*(B1+B2)+SQRT((B1-B2)**2+4.*X3**2))
 IF(ABS(SP).GT.BMAX) SPMAX=ABS(SP)
300 CONTINUE
 DO 100 J=1,24
 WRITE(6,35) J, (STRS1(I,J), I=29,31)
35 FORMAT(5X, 15,5X,3E16.6)
100 CONTINUE
WRITE(6,40)
FORMAT(2X,*RMS STRESS RESPONSE, BEAM ELEMENTS*)
SBMAX=0.
DO 400 J=1,32
DO 400 I=5,6
IF(ABS(STRS(I,J)) .GE. SBMAX) SBMAX=ABS(STRS(I,J))
400 CONTINUE
DO 3 J=1,32
WRITE(6,45)(STRS(I,J),I=1,10)
45 FORMAT(5X,10E12.6)
3 CONTINUE
CALL DAL(1,11,A(1),0,IEA,KADR,IERR,NWDS,NE,LB,ITYPE,
14HAPPL,4HBQI,1,1)
K=0
DMAX=0.
DO 600 J=1,34
DO 500 I=1,6
K=K+1
DISP(I,J)=A(K)
500 CONTINUE
DI=(DISP(1,J)**2+DISP(2,J)**2+DISP(3,J)**2)**0.5
IF(DJ .GE. DMAX) DMAX=DJ
600 CONTINUE
DO 800 J=1,34
WRITE(6,46)(DISP(I,J),I=1,6)
800 CONTINUE
FORMAT(5X,6E12.6)
CALL DAL(1,11,A(1),0,IEA,KADR,IERR,NWDS,NE,LB,ITYPE,
13HOBJ,3HAUB,1,1)
DBJ=A(1)
CALL DAL(1,11,A(1),0,IEA,KADR,IERR,NWDS,NE,LB,ITYPE,
14HVBR,4HEVAL,1,1)
FREQ=80RT(A(1))/(2.0*4.0*ATAN(1.0))
B(1)=2.0*SBMAX/E43AL-1.0
B(2)=2.0*SBMAX/E21AL-1.0
B(3)=2.0*DMAX/ALDISP-1.0
B(4)=(ALFREQ/FREQ)-1.0
RENEW 9
WRITE(9,LINKE)
CALL FIN(0,0)
END
APPENDIX F
DETAILS OF THE CONNECTING NETWORK

All the procedure files with their function and the listing are included in this appendix. Procedure files serve the function of interconnecting various different programs. The master control to the system is provided by the main procedure file RPRO1. The following is a sequence of procedure files presented in the following pages.

F.1 RPRO1
F.2 RANAL
F.3 RXQTFUN
F.4 RXQTFPX
F.5 RMERGE
F.6 RXQTEPX
F.7 RXQTEP2

The flow charts detailing the logic of these procedure files are also included in this appendix. The following flow charts are included.

F.8 Master Procedure File 'RPRO1'
F.9 Function Calculation Procedure File 'RXQTFUN'
F.10 Front Processor Execution Procedure File 'RXQTFPX'
F.11 Procedure File to Update Analysis Data 'RMERGE'
F.12 Displacement Calculations Procedure File 'RXQTEPX'
F.13 Objective Function and Constraints Computation Procedure File 'RXQTEP2'

This appendix has been referred to in section 5.1.7 of Chapter 5.
F.1 RPRO1

This is the master procedure file and controls the sequence and logic of various operations.

01/03/24. 14.10.46,
PROGRAM TEMPI

JOB=T500.
ACCOUNT=CAYFI27*KAR.
ATTACH:SPAR=SPAR14;DCU=DCU14/UN=C33F86.
GET=RNREP.
SPAR,RNREP,OUT1.
REWIND,SPARLA.
REPLACE,SPARLA=RSPLA.
GET,ROPT1=BCONMIN.
FTN,1=ROPT1;L=LIST;D=ROPT1.
REWIND,ROPT1.
COPYL,ROPT1,BCONMIN,A0,RA.
RETURN,ROPT1=BCONMIN.
GET,RFPRB=REPRB1,REPRB2.
FTN,1=RFPRB;L=LIST;D=BI.
FTN,1=REPRB1,L=LIST;D=GET1.
FTN,1=REPRB2,L=LIST;D=GET1.
RETURN,RNREP,RFPRB,REPRB1,REPRB2.
GET,TAPE7,TAPE9,TAPE10.
RETURN,RNREP,RSPLA,ROPT1,LIST.
CALL,RXQTFUN.
CALL,RANAL(A=AINOUT).
1:COMMENT.
GET,SPARLA=RSPLA.
GET,TAPE9.
REWIND,TAPE7.
AO.
IF(FILE(TAPE11;LO))GO TO,2.
REWIND,TAPE9,TAPE10.
REPLACE,TAPE9.
GET,TAPE9.
CALL,RXQTFUN.
RETURN,SPARLA.
REPLACE,TAPE9.
GOTO,1.
2:COMMENT.
RETURN,B1,BGET,BGET1,SPARLA.
PACK,TAPE6.
REPLACE,TAPE6.
CALL,RANAL(A=AFINOT).
TWXMESS JOB IS FINISHED.
PURGE,DAFIL.
DEFINE,DAFIL.
DAYFILE,DAFIL.
REPLACE,TAPE7.
EXIT.
PACK,TAPE6.
REPLACE,TAPE7.
REPLACE,TAPE6.
PURGE,DAFIL.
RETURN,DAFIL.
DEFINE,DAFIL.
DAYFILE,DAFIL.
TWXMESS JOB BOMBED.
READY.
F.2 Procedure File - RANAL

Function - This procedure file computes the complete structural response analysis of the structure and is called at the beginning and end of the execution of the analysis-optimization system to compute the initial and final response of the structure.

81/03/24. 14.09.56.
PROGRAM RANAL

REWIND,OUT1,OUT2,OUT3,BOUT1,BOUT2.
COPY,OUT1,A.
COPY,OUT2,A.
COPY,OUT3,A.
COPY,BOUT1,A.
COPY,BOUT2,A.
PACK,A.
REPLACE,A.
RETURN,OUT2,OUT3,BOUT1,BOUT2,A.
READY.
F.3 Procedure File - RXQTFUN

Function - This procedure file computes the value of the objective function and the constraints for a specified value of optimization parameter. These values of the objective function and the optimization parameters are used in the optimization program.
F.4  Procedure File - RXQTFPX

Function - This procedure file executes the front processor which converts the optimization variable into variables acceptable for the SPAR analyzer.

old, rxatfp

READY.
1nh

REWIND, B1.
B1.
REWIND, TAPE13.
REPLACE, TAPE13=RFPOUT.
RETURN, TAPE13.
READY.
F.5 Procedure File - RMERGE

Function - This procedure file updates the repeatable analysis data for SPAR to reflect the new value of the design variables computed by the optimization program.

01/03/24. 14.11.47.
PROGRAM RMERGE.
EDIT, FN=F1, I=RCOMD.
REPLACE, F1,
READY.
F.6 Procedure File - RXQTEPX

Function - This file executes the end processor which computes the RMS displacement response of the structures.

81/03/24, 14,12,12.
PROGRAM RXQTEPX

COPYBR,A, TEMP.
COPYBR,SCOMBLK, TEMP.
COPYBF,A, TEMP.
REWIND, TEMP.
LIBRARY, SPARLIB.
TEMP., B.
READY.
F.7  Procedure File - RXQTEP2

Function - This file computes the RMS stress response of the structure.

81/03/24, 14.12.32.
PROGRAM RXQTEP2
COPYBR,A,TEMP,
COPYBR,SCомBLK,TEMP,
COPYBF,A,TEMP,
REWIND,TEMP,
LIBRARY,SPARLIB,
GET,TAPES=C,
TEMP,=B,
REWIND,TAPE9,
RENAME,D=TAPE9,
READY.
Fig. F.8. Master Procedure File.
Fig. F.9. Function Calculation Procedure Flow Chart
Fig. F.10. Front Processor Execution Flow Chart.

- RXQTFPX
- 'EXECUTE FRONT PROCESSOR "B1"'
- GET DATA "TAPE 9"
- OUTPUT ON "RFPOUT"
RMERGE

EDIT RREP1 AND UPDATE WITH RFPOUT

REPLACE REPEATABLE DATA RREP1

Fig. F.5. Updating Repeatable Analysis Data.
Fig. F.11. Displacement Calculations.
APPEND 'SPARLIB' AND 'SCOMBLK' TO END PROCESSOR REPRS2 AND EXECUTE

PUT OBJECTIVE FUNCTION AND CONSTRAINTS ON 'TAPE 9'

Fig. F.6. Objective Function and Constraints Computation.
APPENDIX G

ANTENNA FINITE ELEMENT MODEL - REPEATABLE PART

This appendix contains the finite element model data on the antenna under investigation. This data is updated after each iteration of the optimization. SPAR is executed with this data after each optimization cycle. A reference to this appendix has been made in section 6.5.2 of Chapter 6.
APPENDIX G

ANTENNA FINITE ELEMENT MODEL - REPEATABLE PART

[XQMT TAB
UPDATE=1
*MERGE NEW SECTION PROP. HERE*
SHELL SECTION PROPERTIES* COMES FROM FRPS
1 .05500
2 .04000
3 .01500
BEAM ORIENTIONS
FORMAT=2
1 1 0. 0. 5.
E21 SECTION PROPERTIES
BOX /1 .68500 .03080 2.29329 .03080
BOX /2 .68500 .03080 1.66785 .03080
BOX 3 .68500 .03080 .62544 .03080
BOX 4 .5 2. 1. 2.
UPDATE=0
[XQMT TOPO
[XQMT E
T=1. -20., 05., 02., 02., 20., 1.-4., 1.-4., 1.-4%
[XQMT EKS
[XQMT K
[XQMT M
RESET G=386.
[XQMT 'KD' [XQMT INV
[XQMT EIG
RESET PROB=VIBR,M=CEM,INIT=8
[XQMT AUS
DEFINE VM=1 VIBR MODE 1 1 1 0
TABLE(NI=204,NJ=0)NORM MODE 1 1
TRANSFER(SOURCE=VM)
DEFINE NM=NORM MODE 1 1
SNM=SQUARE(NM)
DEFINE M1=DEM DIAG
TABLE(NI=1,NJ=204)DMAA MASS 1 1
TRANSFER(SOURCE=M1)
DEFINE M2=DMAA MASS 1 1
NMT=RTRAN(NM)
M2T=RTRAN(M2)
[XQMT DCU
PRINT 1 M2T
PRINT 1 NMT
[XQMT AUS
TR=RPROD(NMT,M2T)
[XQMT DCU]
APPENDIX H

ANTENNA FINITE ELEMENT MODEL - NONREPEATABLE PART

This appendix also contains the finite element model data for the antenna. However, this data is used only once to execute SPAR. The information regarding the geometrical locations of various nodes, and the element connectivity is contained in this data. This appendix has been referenced in section 6.5.2 of Chapter 6.
APPENDIX H:

ANTENNA FINITE ELEMENT MODEL – NONREPEATABLE PART

[listing of numerical data]

MATC
1 10. +6 3 .1 .1-4
2 10. +6 3 .0 .1-4

FORMAT=ISOTROPIC

SHELL SECTION PROPERTIES
1 .2
2 .15
3 .1

BEAM ORIENTATIONS
FORMAT=2
1 1 0. 0. 0. 5.

E21 SECTION PROPERTIES
BOX 1 .5 .05 2.0 .05
BOX 2 .5 .05 1.0 .05
BOX 3 .5 .05 .5 .05
BOX 4 5.2 1.2 2.0
CONSTRAINT DEFINITION 1
ZERO 1 2 3 4 5 6 133 34
\*DXDT ELD
E42
NSECT=1
NMAT=1
1 9 10 2
2 10 11 3
3 11 12 4
4 12 13 5
5 13 14 6
6 14 15 7
7 15 16 8
8 16 9 1
NSECT=2
9 17 18 10
10 18 19 11
11 19 20 12
12 20 21 13
13 21 22 14
14 22 23 15
15 23 24 16
16 24 17 9
NSECT=3
17 25 26 18
18 26 27 19
19 27 28 20
20 28 29 21
21 29 30 22
22 30 31 23
23 31 32 24
24 32 25 17
E21
NSECT=1
1 9
2 10
3 11
4 12
5 13
6 14
7 15
8 16
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10 18
11 19
12 20
13 21
14 22
15 23
16 24$
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17 25$
18 26$
19 27$
20 28$
21 29$
22 30$
23 31$
24 32$
NSECT=4
NMAT=2$
33 1$
33 2$
33 3$
33 4$
33 5$
33 6$
33 7$
33 8$
EXIT EXIT
APPENDIX I

INITIAL ANALYSIS

This appendix contains the structural analysis results for the initial design of the antenna. The analysis includes eigen values, mode shapes, RMS displacement response and the RMS stress response for the antenna structure.

The sequence in which the output data is presented is as follows:

a) Geometry details of the Finite Element Model
b) Initial weight of the structure
c) Initial eigen values for the antenna structure
d) Mode shapes
e) RMS displacement response
f) RMS stress response.

A reference to this appendix has been made in section 6.6.1 of Chapter 6.
APPENDIX I

GEOMETRY DETAILS

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Library of member (beam) element reference frame orientation specifications

ENTRY
### THIRD POINT COORDINATES RELATIVE TO GLOBAL REFERENCE FRAME

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**I Beam Cross-Section Properties (Type BA)**

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**SUMMARY OF CONSTRAINT CONDITIONS AND JOINT REFERENCE FRAME ALIGNMENTS. (CONSTRAINT SET 1)**

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**EXIT: 435 115 7**

**NEEXECUTE ELD**

**DATA SPACE: 10112**

**IE42 GROUP 1**

**CONNECTED JOINTS**

**SECTION NON-BTR WT**

---

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**1E21 GROUP 1**

**CONNECTED JOINTS**

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| 3     | 3  | 11 | 12 | 4  | 1  | 1  | 0  | 0  | 1 |
| 4     | 4  | 12 | 13 | 5  | 1  | 1  | 0  | 0  | 1 |
| 5     | 5  | 13 | 14 | 6  | 1  | 1  | 0  | 0  | 1 |
| 6     | 6  | 14 | 15 | 1  | 1  | 0  | 0  | 1 |
| 7     | 7  | 15 | 1  | 1  | 0  | 0  | 1 |
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### OEXIT 3.062 5 8

### OEXECUTE TOP0

- DATA SPACE= 10240
- NO. OF 2-NODE ELEMENTS= 32
- NO. OF 4-NODE ELEMENTS= 24
- TOTAL NO. OF ELEMENTS= 56
- TIME 0.021 3.084
- OMAXCON: MAXSUB: ILMAX= 65 1400 32
- TIME 0.036 3.120
- OSIZE HRS= 29 2 976
- OMAXCON: MAXSUB: ILMAX= 56 1400 32
- TIME 0.092 3.212
- OSIZE INDEX= 78 10 1 0 30 2 1 T= .1000E+01 .5000E-01 .2000E-01 .2000E+00 .2000E+02 .1000E+03 .1000E+03 .1000E-03
- ERROR LEVELS: 2222222222
- 0
TYPE GROUP AREA SUM WEIGHT WEIGHT
E42 1 .336633E+04 .165172E+02 0
E21 1 .285418E+03 .365777E+01 0

TOTAL .1957E+02 0 TOTAL INITIAL WEIGHT

OEXIT 3.367 19 33
OEXECUTE EKS
DATA SPACE= 2112
E42 COMPLETED
E21 COMPLETED
OEXIT 6.072 26 42
OEXECUTE K
DATA SPACE= 8512
OEXIT 6.826 31 103
OEXECUTE M
G .38400000E+03
DATA SPACE= 7936
OEXIT 8.556 39 186
OEXECUTE KG
DATA SPACE= 9216
OEXIT 8.906 45 207
OEXECUTE INV
DATA SPACE= 11712
ONING, NNEG= 0 0
OEXIT 10.680 52 218
OEXECUTE EIG
PRED=VIBR
M =CEN
INIT= 8
DATA SPACE= 8704
OACTIVITY COUNT= 8
OACTIVITY COUNT= 16
OACTIVITY COUNT= 24
OACTIVITY COUNT= 32
OROOTS CONVERGED= ITERATION 4

2 .2858462E+04
3 .2858072E+04
4 .2923953E+04
OACTIVITY COUNT= 37
OROOTS CONVERGED= ITERATION 5

1 .2797762E+04
2 .2858046E+04
3 .2858068E+04
4 .2923952E+04
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SEQ IT 5 IT 4 ERR
1 .2797776E+04 .2797805E+04 .10323217E-04 8.418343
2 .2858046E+04 .2858046E+04 .97308192E-07 8.508534
3 .2858068E+04 .2858072E+04 .94084654E-06 8.305870
4 .2923952E+04 .2923953E+04 .47722621E-06 8.606079
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6 .5219468E+04 .52259270E+04 .12374394E-02 11.498291
7 .5406210E+04 .5577868E+04 .32160039E-01 11.706011
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APPENDIX J

OPTIMIZATION ITERATION SUMMARY

This appendix contains the output produced by the optimization subroutine. The optimization process as it goes through various iterations is shown. The output includes the values of optimization function, the design variables and the constraint functions for each iteration. This appendix has been referred to in section 6.6.2 of Chapter 6.
APPENDIX J

OPTIMIZATION ITERATIONS SUMMARY

CONMIN

FORTRAN PROGRAM FOR

CONSTRANDED FUNCTION MINIMIZATION

CONTRAINEE FUNCTION MINIMIZATION

CONTROL PARAMETERS

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LOWER BOUNDS ON DECISION VARIABLES (VLB)

| 1) | 2.00000E-02 | 2.60000E-01 | 3.00000E+00 | 2.00000E-01 |

UPPER BOUNDS ON DECISION VARIABLES (VUB)

| 1) | 5.00000E-01 | 5.00000E+00 | 1.00000E+02 | 1.00000E+01 |

ALL CONSTRAINTS ARE NON-LINEAR

INITIAL FUNCTION INFORMATION

OBJ = 1.95750E+02
DECISION VARIABLES (X-VECTOR)
1)  .40000E-02  .50000E-01  .50000E+00  .50000E-01

CONSTRAINT VALUES (G-VECTOR)
1)  -.99835E+00  .13527E+01  -.95296E+00  .76182E+00

BEGIN ITERATION NUMBER  1
THERE ARE    0 ACTIVE CONSTRAINTS
THERE ARE    2 VIOLATED CONSTRAINTS
CONSTRAINT NUMBERS ARE
2    4
THERE ARE    0 ACTIVE SIDE CONSTRAINTS
PUSH-OFF FACTORS, (THETA(I), I=1:NAC)
1)  .10000E+01  .10000E+01
CONSTRAINT PARAMETER, BETA = .84455E+00
SEARCH DIRECTION (S-VECTOR)
1)  -.68920E+00  .10000E+01  -.25313E+00  .72758E+00
ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.7637E+01 PROPOSED ALPHA = .5421E+00
CALCULATED ALPHA = .54210E+00
OBJ = (.157621E+02

DECISION VARIABLES (X-VECTOR)
1)  .29055E-02  .77105E-01  .43139E+00  .69721E-01

CONSTRAINT VALUES (G-VECTOR)
1)  -.99975E+00  -.75785E+00  -.99520E+00  .38706E-01

BEGIN ITERATION NUMBER 2
THERE ARE    0 ACTIVE CONSTRAINTS
THERE ARE    1 VIOLATED CONSTRAINTS
CONSTRAINT NUMBERS ARE
4
THERE ARE    0 ACTIVE SIDE CONSTRAINTS
PUSH-OFF FACTORS, (THETA(I), I=1:NAC)
1)  .10000E+01
CONSTRAINT PARAMETER, BETA = .11075E+01
SEARCH DIRECTION (S-VECTOR)
1)  -.39060E+00  .22472E+00  .10000E+01  .15965E+00
ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.3543E+01 PROPOSED ALPHA = .2826E-01

- 222 -
CALCULATED ALPHA = .32357E+00
OBJ = .146434E+02

DECISION VARIABLES (X-VECTOR)
  1) .20000E-02 .80740E-01 .59317E+00 .72304E-01

CONSTRAINT VALUES (G-VECTOR)
  1) -99978E+00 -.94085E+00 -.99883E+00 -.30909E+00

BEGIN ITERATION NUMBER 3
THERE ARE 0 ACTIVE CONSTRAINTS
THERE ARE 0 VIOLATED CONSTRAINTS
THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
-1

PUSH-OFF FACTORS, (THETA(I), I=1, NAC)
  1) 0.

CONSTRAINT PARAMETER, BETA = .38724E+00
SEARCH DIRECTION (S-VECTOR)
  1) .15372E-13 -.60315E+00 -.40216E+00 -.10000E+01

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.6736E+01 PROPOSED ALPHA = .1087E+00
CALCULATED ALPHA = .36835E+00
OBJ = .123814E+02

DECISION VARIABLES (X-VECTOR)
  1) .20000E-02 .69632E-01 .51911E+00 .53887E-01

CONSTRAINT VALUES (G-VECTOR)
  1) -99966E+00 -.82395E+00 -.99651E+00 -.49481E-02

BEGIN ITERATION NUMBER 4
THERE ARE 1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
  4
THERE ARE 0 VIOLATED CONSTRAINTS
THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
-1

PUSH-OFF FACTORS, (THETA(I), I=1, NAC)
  1) .90349E+00 0.

CONSTRAINT PARAMETER, BETA = .16209E+00
SEARCH DIRECTION (S-VECTOR)
  1) .44464E-13 -.15099E+00 -.10000E+01 -.99813E+00
ONE-DIMENSIONAL SEARCH

INITIAL SLOPE = -.2775E+01 PROPOSED ALPHA = .3386E+00

CALCULATED ALPHA = .19756E+00

OBJ = .11792E+02

DECISION VARIABLES (X-VECTOR)
1) .20000E-02 .68042E-01 .61789E+00 .44126E-01

CONSTRANT VALUES (G-VECTOR)
1) -.99960E+00 -.19013E+00 -.98414E+00 -.23361E-03

BEGIN ITERATION NUMBER 5

THERE ARE 1 ACTIVE CONSTRAINTS
CONRAINTR NUMBERS ARE

4

THERE ARE 0 VIOLATED CONSTRAINTS

THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
-1

PUSH-OFF FACTORS, (THETA(I), I=1:NAC)
1) .99533E+00 0

CONSTRAINT PARAMETER, BETA = .13792E+00

SEARCH DIRECTION (S-VECTOR)
1) .49422E-13 .17917E+00 .10000E+01 -.93136E+00

ONE-DIMENSIONAL SEARCH

INITIAL SLOPE = -.2360E+01 PROPOSED ALPHA = .2593E+00

CALCULATED ALPHA = .34173E-01

OBJ = .117142E+02

DECISION VARIABLES (X-VECTOR)
1) .20000E-02 .68348E-01 .63497E+00 .42535E-01

CONSTRANT VALUES (G-VECTOR)
1) -.99959E+00 -.18507E-01 -.98083E+00 -.17611E-02

BEGIN ITERATION NUMBER 6

THERE ARE 2 ACTIVE CONSTRAINTS
CONRAINTR NUMBERS ARE

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THERE ARE 0 VIOLATED CONSTRAINTS

THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
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PUSH-OFF FACTORS, (THETA(I), I=1:NAC)
CONSTRANT PARAMETER, BETA = .1449E+00

SEARCH DIRECTION (S-VECTOR)
1) -.44156E-13 .86007E+00 .20310E+00 -.10000E+01

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.1348E+01 PROPOSED ALPHA = .1355E+00
CALCULATED ALPHA = .24869E+00
OBJ = .112494E+02

DECISION VARIABLES (X-VECTOR)
1) .20000E-02 .82963E-01 .66705E+00 .31956E-01

CONSTRAINT VALUES (G-VECTOR)
1) -.9969E+00 -.29621E+00 -.98640E+00 -.26272E-02

BEGIN ITERATION NUMBER 7
THERE ARE 1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE

THERE ARE 0 VIOLATED CONSTRAINTS
THERE ARE 1 ACTIVE SIDE CONSTRAINTS.
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)

PUSH-OFF FACTORS, (THETA(I), I=1:NAC)
1) .94815E+00 0.

CONSTRANT PARAMETER, BETA = .24666E+00

SEARCH DIRECTION (S-VECTOR)
1) .26957E-13 .14398E+00 .10000E+01 -.86880E+00

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.2039E+01 PROPOSED ALPHA = .1138E+00
CALCULATED ALPHA = .20690E-01
OBJ = .112002E+02

DECISION VARIABLES (X-VECTOR)
1) .20000E-02 .83171E-01 .80683E+00 .31174E-01

CONSTRAINT VALUES (G-VECTOR)
1) -.99986E+00 -.20737E+00 -.98471E+00 -.51834E-02

BEGIN ITERATION NUMBER 8
THERE ARE 1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE

THERE ARE 0 VIOLATED CONSTRAINTS
THERE ARE 3 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
-1

PUSH-OFF FACTORS, (THETA(I), I=1+NAC)
1) .89902E+00 0.

CONSTRAINT PARAMETER, BETA = .24912E+00

SEARCH DIRECTION (S-VVECTOR)
1) -.26903E-13 .17328E+00 .10000E+01 -.89127E+00

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.23333E+01 PROPOSED ALPHA = .1057E+00
CALCULATED ALPHA = -.29008E-02
OBJ = .111935E+02

DECISION VARIABLES (X-VVECTOR)
1) .20000E-02 .83205E-01 .68203E+00 .31064E-01

CONSTRAINT VALUES (S-VVECTOR)
1) -.99968E+00 -.19450E+00 -.98447E+00 -.55613E-02

BEGIN ITERATION NUMBER 9

THERE ARE 3 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
4

THERE ARE 0 VIOLATED CONSTRAINTS

THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
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PUSH-OFF FACTORS, (THETA(I), I=1+NAC)
1) .89187E+00 0.

CONSTRAINT PARAMETER, BETA = .24951E+00

SEARCH DIRECTION (S-VVECTOR)
1) .26893E-13 .17735E+00 .10000E+01 -.89163E+00

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.23371E+01 PROPOSED ALPHA = .1191E-01
CALCULATED ALPHA = .26272E-02
OBJ = .111873E+02

DECISION VARIABLES (X-VVECTOR)
1) .20000E-02 .83237E-01 .68369E+00 .30945E-01

CONSTRAINT VALUES (S-VVECTOR)
1) -.99968E+00 -.19279E+00 -.98424E+00 -.55674E-02

BEGIN ITERATION NUMBER 10
THERE ARE 1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
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THERE ARE 0 VIOLATED CONSTRAINTS

THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
-1

FIND-OFF FACTORS (THETA(I), I=1,NAC)
1) .9969E+00 0.

CONSTRAINT PARAMETER, BETA = .2498E+00

SEARCH DIRECTION (S-VECTOR)
1) 0.

ONE-DIMENSIONAL SEARCH
INITIAL SLOPE = -.2341E+01 PROPOSED ALPHA = .2767E-02
CALCULATED ALPHA = .2767E-02
OBJ = .111813E+02

DECISION VARIABLES (X-VECTOR)
1) .20000E-02 .83271E-01 .68545E+00 .30660E-01

CONSTRAINT VALUES (G-VECTOR)
1) -.99968E+00 -.17056E+00 -.98401E+00 -.63057E-02

FINAL OPTIMIZATION INFORMATION
OBJ = .111813E+02

DECISION VARIABLES (X-VECTOR)
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CONSTRAINT VALUES (G-VECTOR)
1) -.99968E+00 -.17056E+00 -.98401E+00 -.63057E-02

THERE ARE 1 ACTIVE CONSTRAINTS
CONSTRAINT NUMBERS ARE
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THERE ARE 0 VIOLATED CONSTRAINTS

THERE ARE 1 ACTIVE SIDE CONSTRAINTS
DECISION VARIABLES AT LOWER OR UPPER BOUNDS (MINUS INDICATES LOWER BOUND)
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TERMINATION CRITERION
ABS(OBJ(I)-OBJ(I-1)) LESS THAN DABFUN FOR 3 ITERATIONS

NUMBER OF ITERATIONS = 10

OBJECTIVE FUNCTION WAS EVALUATED 69 TIMES

CONSTRAINT FUNCTIONS WERE EVALUATED 69 TIMES

READY.
APPENDIX K

FINAL ANALYSIS RESULTS

This appendix contains the structural analysis of the optimum design of the antenna which was realized using this design system. The results include eigen values, the mode shapes, RMS displacement response and RMS stress response for the structure. The output results are presented in the following sequence.

a) Geometry details of the model
b) Final weight of the antenna structure
c) Final eigen values of the structure
d) Mode shapes
e) RMS displacement response of the optimum antenna structure
f) RMS stress response of the optimum antenna structure

A reference to this appendix has been made in section 6.6.2 of Chapter 6.
FINAL GEOMETRY

81/03/17-40.43.46.
PROGRAM AFINTT
EXECUTE TAR
DATA SPACE= 6016

34 JOINTS.

OCCURRING JOINT MOTION COMPONENTS= 1 2 3 4 5 6

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2 .100000E+08 .300000E+00 .384620E+07 0.

ENTRY ALPH1 ALPH2 THETA
1 .100000E-04 .100000E-04 0.
2 .100000E-04 .100000E-04 0.

LIBRARY OF MEMBER (BEAM) ELEMENT REFERENCE FRAME ORIENTATION SPECIFICATIONS

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DMD. 1 11= 1; THIRD POINT COORDINATES RELATIVE TO ALTERNATE FRAME = 0.
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### Summary of Constraint Conditions and Joint Reference Frame Assignments (Constraint Set 1)

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**EXIT**

**DATA SPACE** 6016

**BEAM ELEMENT REFERENCE FRAME ORIENTATION SPECIFICATIONS**

**ENTRY**

**OWNER** 1112 11 11

**THIRD-POINT COORDINATES RELATIVE TO ALTERNATE FRAME** 1 0 0

**THIRD-POINT COORDINATES RELATIVE TO GLOBAL REFERENCE FRAME** 0 0

**BEAM SECTION PROPERTIES (TYPE BY SHAPE/WIDTH).**

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- 232 -
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EXIT 52103 15 46
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CROSS= 0
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GROUP 1

STRESS - DIVIDED BY 1.0000

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**RMS DISPLACEMENT RESPONSE**
RMS STRESS RESPONSE

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