# PORTMANTEAU TESTS IN ECONOMIC TIME SERIES

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## ABSTRACT

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Testing for the adequacy of an autoregressive-moving average process of order (p,q) [ARMA(p,q)] has been one of the most widely discussed topics in time series analysis. This dissertation presents a comprehensive study of the finite-sample properties of some well-known and computationally straightforward tests in this area. The tests examined include: (i) the portmanteau tests of Box and Pierce (1970), Ljung and Box (1978), Dufour and Roy (1986), Ljung (1986), and Bera and Newbold (1988); (ii) Godfrey's (1979) Lagrange multiplier (LM) test; and (iii) the McAleer et al. (1988) tests of separate hypotheses.

This dissertation uses large-scale simulation experiments to investigate the empirical performance of some selected portmanteau tests and Godfrey's (1979) LM test. As well, a critical note is given in order to address the fundamental problems of the McAleer et al. (1988) tests of separate hypotheses in commonly-used sample sizes. Many simulation results given in this part of the dissertation have not been reported in previous studies.

The last part of this dissertation focusses on modified portmanteau tests for the randomness of a Gaussian time series. Recently, Kwan et al. (1992) have argued that, in testing the adequacy of an ARMA(p,q) model, the poor performance of the Ljung-Box portmanteau test may be attributed to the slow convergence of residual autocorrelations to normality. This dissertation extends their investigation to the area of testing randomness of a Gaussian time series. In addition, two modified portmanteau tests, based on an application of Hotelling's (1953) transformations to sample autocorrelations, are proposed. The simulation results strongly favour the use of these two modified tests and the Kwan et al. (1992) portmanteau test in empirical applications.

## DEDICATION

This dissertation is dedicated to my parents and the late Professor Balvir Singh

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# CHAPTER ONE

# APPROACHES TO TESTING THE ADEQUACY OF UNIVARIATE TIME SERIES MODELS

## 1.1 Introduction

The idea of modeling an autoregressive-moving average (ARMA) process for a stationary time series traces back to a pioneering contribution by Yule (1927), published more than sixty years ago. In his paper, Yule (1927) presented an idea of autoregressive series which represented a significant breakthrough in time domain modeling. This, together with the illustration by Wold (1938) that the non-deterministic part of any stationary series may be represented as a moving average, opened up a new page in the modern history of time series analysis

Since the publication of Box and Jenkins' (1970) book, and with the rapid evolution of powerful, high-speed computers, the use of ARMA models is becoming increasingly popular. Although Box and Jenkins (1970) are not the original contributors in the field of ARMA models, their oft-quoted book has generated further interest in the area. Hence, many time series analysts have also referred to ARMA modeling as the Box-Jenkins approach.

The Box-Jenkins approach requires iteration on a three-stage process: (i) the identification of the orders of the process, written ARMA(p,q); (ii) the estimation of the tentative ARMA(p,q) model, as given in the first stage; and (iii) diagnostic checking of (ii) which involves testing for possible inadequacies in the tentative model. While the literature dealing with these three stages is voluminous, attention has recently focussed on model identification and diagnostic checking. This is partially due to the rapid development of efficient statistical software which not only has removed most of the computational burden of nonlinear estimation but has also produced accurate estimates in commonly-used sample sizes. In addition, it is known that the predictive performance of the ARMA model can be affected by the appropriateness of the identified process in step (i) above. This has led to a continuing search for reliable diagnostic procedures. However, most of these available procedures have only a large-sample justification, and many require very large samples to be accurate. In contrast, many economic and financial time series are short. Therefore, it is of considerable importance to know whether the finite-sample distribution of a test statistic is well approximated by asymptotic theory. The main objective of this dissertation is to shed light on the finite-sample properties of portmanteau test statistics that have an asymptotic justification.

## 1.2 Pure Significance Tests for Model Adequacy

As mentioned earlier, testing for the adequacy of an ARMA(p,q) model has been one of the most widely discussed topics in time series analysis. In general, three broad approaches to testing model adequacy have been proposed. These are referred to by Godfrey and Tremayne (1988) as (i) pure significance tests, (ii) tests of nested hypotheses, and (iii) tests of separate hypotheses.

It is important to note here that several alternative approaches to testing model adequacy have been excluded from this dissertation. Some of these approaches include correlograms, Akaike's information criterion (AIC), Schwartz's Bayesian criterion (SBC), and other model validation procedures. A correlogram consists of a number of residual autocorrelations. However, a convenient measure of the significance of an entire correlogram can easily be carried out by using pure significance tests [see Box and Jenkins (1970)]. As for the AIC and SBC, these are often employed to measure the closeness of fit and the number of estimated parameters. These closeness-of-fit criteria, as indicated by Maddala (1992, pp. 539-540), are similar to the $\bar{R}^2$ or minimum $\hat{\sigma}^2$-type criterion. Nonetheless, once the order of an ARMA model has been determined on the basis of the AIC or SBC, it is still necessary to apply the aforementioned diagnostic approaches (i)-(iii) to examine for possible model inadequacy. As to other model validation procedures, these are appropriate for exercises involving, in particular forecasting, but are beyond the scope of this dissertation.

The most frequently used test statistic, among the three approaches enumerated by Godfrey and Tremayne (1988), is the one proposed by Box and Pierce (1970). It proceeds on the assumption of no specification error, and uses the fact that the estimated residuals will reflect the process presumed under the null. In these circumstances, Box and Pierce (1970) showed how inferences concerning goodness of fit could be based upon the residual autocorrelations, either individually or by combining a number of them into one statistic. In view of its all-inclusive nature, this method has come to be referred to as a "portmanteau test" [see Box and Jenkins (1976, p. 290)]. Moreover, since the distribution of the Box-Pierce test does not explicitly take an alternative hypothesis into consideration, it is also known as a pure significance test [see Cox and Hinkley (1974)].

The introduction of the Box-Pierce test has stimulated further research on the usefulness of this statistic in sample sizes which are commonly employed in economic and financial applications. Unfortunately, Chatfield and Prothero (1973) and Prothero and Wallis (1976) found that low values of the Box-Pierce test often arise when several models are fitted to the same data. Theoretical studies by Davies et al. (1977), Ljung and Box (1978) and Dufour and Roy (1986) have all shown that the exact sampling distribution of the test can differ markedly from the asymptotic distribution, even when the sample size is quite large. Moreover, the simulation results of Clark and Godolphin (1982) indicate that the test has difficulty in detecting deviations from model specification.

The poor empirical performance of the Box-Pierce test subsequently led to the development of many modified versions ("modified portmanteau statistics"). Examples are Ljung and Box (1978), Godolphin (1980), Newbold (1980), Ljung (1986), Dufour and Roy (1986), Bera and Newbold (1988), and Kwan et al. (1992). With the exception of the Ljung-Box test, the finite-sample properties of the other modified portmanteau statistics are still not yet well known.

In light of the above, Chapter two will provide an extensive evaluation of the empirical performance of some selected portmanteau tests for univariate time series models, paying special attention to estimated size (i.e., type I errors), means, variances, and empirical power. There are three main features of this chapter: (i) The investigation is conducted using three large-scale simulation experiments focussing on seasonal as well as non-seasonal time series. (ii) The power calculations are carried out for wide ranging values of the number of residual autocorrelations. (iii) Simulation results obtained from seasonal and non-seasonal data are compared. The second feature is motivated by Ljung's (1986) finding that the poor power performance of the Ljung-Box test can be attributed to the use of a large number of residual autocorrelations. In view of this finding, it would be interesting to see whether other portmanteau statistics exhibit similar behaviour.

## 1.3 Tests of Nested and Separate Hypotheses

The portmanteau test may be considered as a test against all possible alternatives at the same time. If a specific alternative is chosen and a test is established for it, the power of the test against that specific alternative cannot be less than the power of the portmanteau test against the same specific alternative. Considering specific hypotheses $H_1$ and $H_2$, a distinction can be drawn between two cases: (a) When the intersection of $H_1$ and $H_2$ is either $H_1$ alone or $H_2$ alone; and (b) when $H_1$ and $H_2$ are disjoint or intersecting such that the intersection is neither entirely $H_1$ nor entirely $H_2$. Alternative (a) refers to nested hypotheses and (b) refers to separate hypotheses.

In the context of testing nested hypotheses, Whittle (1952) derived a general test-statistic for assessing the goodness of fit of an ARMA(p,q) model which is based on the likelihood-ratio criterion. Since Whittle's (1952) procedure involves deliberately adding an extra parameter to the fitted model, it is similar to the "overfitting procedure" proposed by Box and Jenkins (1976, p. 286). It may also be regarded as a test of restrictions upon a maintained model. Using the Lagrange multiplier (LM) principle, Godfrey (1979) developed a test of the ARMA(p,q) model against an ARMA(p+r,q) or ARMA(p,q+r) alternative, where r is the number of restrictions. Like Whittle's (1952) method, the LM test is constructed on the basis of testing a restricted model against a broader maintained model. Thus, these tests are asymptotically optimal (or most powerful) against nested alternatives.

When the null and alternative hypotheses are separate, the optimal property of tests of separate hypotheses ceases to hold. The classic example is the case of AR(p) against MA(q) processes. Using Cox's (1961, 1962) theory on separate hypotheses, Walker (1967) presented a thorough discussion on how tests for AR(p) versus MA(q) processes can be carried out in these situations. However, his procedures are too complicated to use, and do not receive much attention in the literature. Note that in the area of testing AR regression disturbances against MA regression disturbances, King (1983, 1985a, 1985b) proposed several "point-optimal" tests which "are most powerful invariant in a given neighbourhood of the parameter space of the alternative hypothesis" [Dastoor and Fisher (1988, pp. 97-98)]. As shown by Dastoor and Fisher (1988), King's optimal tests can be interpreted as a Cox test for separate hypotheses. Recently, McAleer et al. (1988) suggested several procedures to examine separate time series models. Their tests involved simple calculations which are similar to those of the LM test. The simulation results presented in McAleer et al. (1988) and Hall and McAleer (1989) indicate that the tests of separate hypotheses can be useful in revealing model inadequacy, both in the cases of appropriate and inappropriate alternatives.

In Hall and McAleer's (1989) simulation study, the finite-sample properties of a number of tests of nested and separate hypotheses are examined, including the portmanteau statistics proposed by Box and Pierce (1970) and Ljung and Box (1978). Their main conclusion seems to favour the use of the McAleer et al. (1988) tests of separate hypotheses, due to their relatively good power performance. However, Chapter 3 will point out that the finite-sample performance of the McAleer et al. tests is far from perfect. In fact, there are many important issues which have not been fully addressed in their original paper and in the follow-up study by Hall and McAleer (1989). As will be indicated, caution should be exercised when using tests of separate hypotheses to check for the adequacy of separate time series models.

Notwithstanding their questionable conclusion, Hall and McAleer's (1989) simulation results indicate a number of areas where further investigation of the empirical performance of Godfrey's LM test is needed. First, since the LM test is based on testing restrictions against a more general model, the choice of the number of restrictions, r, definitely requires some care. Studies carried out by Godfrey (1979), McAleer et al. (1988) and Hall and McAleer (1989) have examined the validity of the LM test when the value of r is small. It is, however, apparent from these simulation results that the quality of the $\chi^2$ approximation to the distribution of the LM test can deteriorate rapidly as r increases. It would therefore be of considerable importance to investigate the effect of the choice of r on the finite-sample distribution of the LM test.

Second, almost all existing simulation studies on the empirical power of the LM test have focussed on data generated from a simple non-seasonal ARMA model. The conclusions based on this particular experimental design would seem doubtful if seasonal data are employed. Hence, it appears that a useful addition to the existing literature on the subject would be to examine the effect of the nature of the data (seasonal vs. non-seasonal) on the empirical power of the test. In particular, it would be interesting to see whether or not the choice of r can be affected in such situations. One of the main goals of Chapter 3 is to look at these two areas carefully.

## 1.4 Modified Portmanteau Tests for Randomness of Gaussian Time Series

Portmanteau tests have been one of the main statistical instruments in examining the adequacy of an ARMA(p,q) model. Quite surprisingly, little attention has been paid to their empirical performance in the area of testing the randomness of Gaussian time series. Recently, Dufour and Roy (1986) have carried out a comprehensive investigation on the empirical size of the Box-Pierce and the Ljung-Box tests in this context. Their simulation results reveal that while the size of the Box-Pierce test is often too small, the Ljung-Box test rejects the null too frequently. Moreover, the tests can have substantial dispersion bias when the number of sample autocorrelations is large relative to the sample size.

Kwan et al. (1992) have recently argued that, in testing the adequacy of an ARMA(p,q) model, the poor performance of the Box-Pierce and the Ljung-Box tests may be due to the slow convergence of residual autocorrelations to normality. A solution proposed by these authors is to transform each residual autocorrelation employed in the tests to a new random variable which converges to normality with greater rapidity. In their study, Kwan et al. (1992) recommend the Fisher (1921) variance-stabilizing transformation. Indeed, their simulation results indicate that a portmanteau test based on this well-known transformation performs better than the Ljung-Box test in moderate samples.

Motivated by the results of Kwan et al. (1992), Chapter 4 will examine the following three areas: First, Kwan et al. (1992) confined their attention to univariate time series models and it would be useful to extend the analysis to the area of testing the randomness of a Gaussian time series. Second, the relative performance of the Kwan et al. (1992) test is a relevant issue. Since the portmanteau statistics have a large-sample appeal, it is important to examine their empirical performance for small to moderate samples. Third, almost all existing studies in this area have dealt solely with the empirical size of the portmanteau test [see e.g., Dufour and Roy (1986)]. It would be interesting to examine the empirical power of the tests.

The major finding of Chapter 4 is that, in testing randomness of Gaussian time series, the test proposed by Kwan et al. (1992) performs much better than the Ljung-Box and the Dufour-Roy statistics in terms of controlling test size and minimizing dispersion bias. Encouraged by this finding, Chapter 5 will develop two modified portmanteau tests based on the Hotelling (1953) transformations. As indicated by Kendall and Stuart (1977, vol. 1), the Hotelling transformations may work even better than the Fisher variance-stabilizing transformation in small samples. In view of this, an investigation of the relative performance of these proposed tests and the Kwan et al. (1992) test is necessary. The result emerging from this comparison would be particularly useful in terms of selecting diagnostic checks in empirical applications.

# CHAPTER TWO

# A MONTE CARLO STUDY OF THE FINITE-SAMPLE DISTRIBUTION OF SOME PORTMANTEAU TESTS: THE CASE OF UNIVARIATE TIME SERIES MODELS

## 2.1 Introduction

The portmanteau tests proposed by Box and Pierce (1970) and Ljung and Box (1978) have been among the most commonly used methods for detecting model inadequacy in applied econometrics and time series analysis. The widespread popularity of portmanteau tests can be attributed to a number of reasons. First, the null hypothesis of such tests is white noise and the alternative is essentially the whole portmanteau of time-dependent series, including autoregressive and moving average errors. Second, the tests can be used to examine serial correlation beyond the first order. Third, in contrast to the classical test given by Whittle (1952), the "overfitting" procedure suggested by Box and Jenkins (1970) and the Lagrange multiplier test developed by Godfrey (1979), portmanteau tests have optimal properties when the alternative is vague [see e.g., Hallin, Ingenbleek and Puri (1987)]. Fourth and perhaps most important from the practical viewpoint, the tests are easy to compute.

It is now common knowledge that, in the area of testing the adequacy of an ARMA(p,q) model, the finite-sample properties of the Box-Pierce and Ljung-Box tests can be distinctly different from their predicted performance from asymptotic theory. For example, the Box-Pierce test suffers a location bias and has very poor empirical power even when the sample size is moderate [see e.g., Ljung and Box (1978) and Clark and Godolphin (1982)]. The Ljung-Box test, on the other hand, can have empirical variance which is greater than the value predicted by asymptotic theory. Moreover, the simulation results of Davies and Newbold (1979), Godfrey (1979), Clark and Godolphin (1982), Kwan and Sim (1988), and Hall and McAleer (1989) indicate that the test possesses low empirical power against a range of simple alternatives.

As for other portmanteau statistics, Godolphin (1980) has suggested a procedure which requires a transformation of the residuals prior to the calculation of residual autocorrelations, such that the transformed residuals are uncorrelated. Newbold (1980) has proposed a test which is based upon the first few residual autocorrelations. However, both tests can be highly unreliable due to their small-sample bias. Dufour and Roy (1986), on the other hand, demonstrate that, when the observations are independently and identically distributed (i.i.d.) normal with unknown mean, improvements of the Box-Pierce and Ljung-Box tests can be made by using the exact first and second moments of the sample autocorrelations. Since portmanteau statistics are based on quadratic forms in residual autocorrelations, Ljung (1986) suggested that the finite-sample distribution of the Box-Pierce and Ljung-Box tests can be approximated by the scaled chi-square distribution. Recently, Kwan et al. (1992) have developed a test which is based on an application of the Fisher (1921) variance-stabilizing transformation to the residual autocorrelation. The simulation results of Kwan et al. (1992) indicate that their proposed test dominates the Ljung-Box test in cases where parameter values are from small to moderate. However, the empirical size of this test can become too large when parameter values approach the boundary of the stationarity region.

It is important to note that the aforesaid portmanteau-type statistics require the estimation of the residuals from a specified ARMA(p,q) model. Bera and Newbold (1988) developed diagnostic checks which are based only on the calculation of either the sample autocorrelations or partial autocorrelations of an observed time series. The Bera-Newbold tests seem useful, especially since they establish a direct link between model selection and model checking.

While the finite-sample performance of the Box-Pierce and the Ljung-Box statistics is well-documented in the literature, five points seem germane to this chapter. First, the portmanteau tests are asymptotic statistics and may require very large samples to be accurate. Since many economic and financial time series are not long [see Godfrey and Tremayne (1988)], it is of considerable importance to know in these circumstances whether their finite-sample distributions are well approximated by asymptotic theory. In the absence of exact results, it is necessary to examine the empirical performance of the tests considered for small to moderate samples by means of simulation experiments.

Second and closely related to the first point, almost all available studies have dealt only with the well-known tests of Box and Pierce (1970) and Ljung and Box (1978). Thus, a useful addition to the existing literature on the subject would be an investigation of the finite-sample distribution of the recently proposed portmanteau tests. Also, it would be interesting to compare their relative performance in commonly used sample sizes.

Third, the portmanteau statistics of Box and Pierce (1970), Ljung and Box (1978), Dufour and Roy (1986), and Kwan et al. (1992) require the selection of the number of residual autocorrelations, m. Therefore, it is important to investigate the effect of the choice of m on the finite-sample distribution of the aforementioned portmanteau tests [see Poskitt and Tremayne's (1981) rules of thumb on the selection of m in empirical applications].

Fourth, Ljung (1986) recently demonstrated that the poor power performance of the portmanteau tests can be attributed to the use of a large number of residual autocorrelations. A plausible explanation is that if the underlying data-generating process can be represented by a simple ARMA model, lack of model adequacy will appear in the first few residual autocorrelations when the data are being fitted to either low-order AR or MA models. Thus, the use of a smaller number of residual autocorrelations in such situations will lead to a more powerful test. This argument is, indeed, supported by the simulation results of Ljung (1986, Table 3). In view of this finding, it is important to see whether this power property can be extended to other portmanteau tests; particularly since it is known that they possess the same asymptotic distribution [see Dufour and Roy (1986)].

Fifth, portmanteau tests are intended to have at least some power against a class of alternatives that is broader than simple low-order ARMA models. Further, it is well-known that the occurrence of higher-order or seasonal autocorrelation is possible when one works with monthly or quarterly data. Hence, one of the main reasons for using a portmanteau statistic with a large number of residual autocorrelations is to get greater power against some higher-order autocorrelation spikes. For this reason, it would be interesting to examine the effect of the nature (or frequency) of the data on the empirical power of the portmanteau tests.

The principal aim of this chapter is to provide an extensive evaluation of the empirical performance of some selected portmanteau statistics in sample sizes which are commonly used in empirical applications. Our attention is confined to the portmanteau tests of Box and Pierce (1970), Ljung and Box (1978), Ljung (1986), Dufour and Roy (1986), and Bera and Newbold (1988). The rest of the chapter is organized as follows: Section 2.2 presents a review of the test procedures. Section 2.3 discusses the design of the Monte Carlo experiments. Section 2.4 reports the main results and some concluding remarks are given in Section 2.5.

## 2.2 Test Procedures

Let the stationary and invertible time series $w_t$ $(t=1,\ldots,n)$ be generated by the ARMA(p,q) process

$$\phi(B)w_t = \theta(B)a_t, \tag{2.2.1}$$

where

$$\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p,$$

$$\theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q,$$

and $B^j w_t = w_{t-j}$. The polynomials $\phi(B)$ and $\theta(B)$ are assumed to have roots outside the unit circle and to have no factors in common. The white noise series, $a_t$, is assumed to be i.i.d. with mean 0 and finite variance $\sigma_a^2$. To examine the adequacy of a fitted ARMA model, such as (2.2.1), one may test the independence of the residual series

$$\hat{a}_t = \left\{\frac{\hat{\phi}(B)}{\hat{\theta}(B)}\right\} w_t, \tag{2.2.2}$$

where $\hat{\phi}(B) = 1 - \hat{\phi}_1 B - \ldots - \hat{\phi}_p B^p,$

$$\hat{\theta}(B) = 1 - \hat{\theta}_1 B - \ldots - \hat{\theta}_q B^q,$$

and $(\hat{\phi}_1, \ldots, \hat{\phi}_p)$ and $(\hat{\theta}_1, \ldots, \hat{\theta}_q)$ are the least squares (or maximum likelihood) estimates of the coefficients of (2.2.1). A useful technique proposed by Bartlett (1946) and Box and Pierce (1970) is to examine the statistical significance of the residual autocorrelations (the residuals having mean zero):

$$\hat{r}_k = \frac{\sum_{t=1}^{n-k} \hat{a}_t \hat{a}_{t+k}}{\sum_{t=1}^{n} \hat{a}_t^2} \qquad (k = 1, 2, \ldots). \qquad (2.2.3)$$

If (2.2.1) were correct and the parameters were known, Box and Pierce (1970) show that the white noise autocorrelations

$$\bar{r}_k = \frac{\sum_{t=1}^{n-k} a_t a_{t+k}}{\sum_{t=1}^{n} a_t^2},$$

are approximately distributed according to a normal distribution with mean zero, $var(\bar{r}_k)=1/n$ and $cov(\bar{r}_k, \bar{r}_h)=0$ $(k \neq h)$. Using these conditions, they subsequently suggest a portmanteau test on the basis of $\hat{r}_k$:

$$QBP = \hat{\underline{r}}^T D_1^{-1} \hat{\underline{r}}, \qquad (2.2.4)$$

where $\hat{\underline{r}} = (\hat{r}_1, \ldots, \hat{r}_m)^T$ and $D_1 = (1/n) I_m$. If the model is correctly specified, QBP is asymptotically distributed as $\chi^2$ with (m-p-q) degrees of freedom, provided that m is large and (m/n) is small

[Godfrey and Tremayne (1988, p. 7)]. However, it was demonstrated by Davies et al. (1977), Ljung and Box (1978), Godfrey (1979) and Dufour and Roy (1986) that the finite-sample distribution of QBP can deviate significantly from the $\chi^2(m-p-q)$ distribution. Using the result given in Ljung and Box (1978, p. 298), it is not difficult to show that when an ARMA(p,q) model is fitted to data, the expected value of QBP is given approximately by

$$E(QBP) = \left\{m-\frac{m(m+5)}{2(n+2)}\right\} - p - q, \tag{2.2.5}$$

where $m(m+5)/\{2(n+2)\}$ is the location bias. Thus, when the ratio $m(m+5)/\{2(n+2)\}$ causes a noticeable deviation from $(m-p-q)$, application of the $\chi^2(m-p-q)$ distribution is not appropriate. In practical terms, this means that for small to moderate samples, it is inappropriate to use the asymptotic chi-square distribution with (m-p-q) degrees of freedom to set critical values for QBP.

In order to alleviate the problem of location bias, two methods have been recommended by Ljung and Box (1978). The first method directly attacks the problem by applying the distribution with the proper number of degrees of freedom. i.e. the number of degrees of freedom given by the right-hand side of (2.2.5):

$$QBP1 \sim \chi^2(E(QBP)), \tag{2.2.6}$$

In this chapter, QBP1 is called the modified Box-Pierce test.

The second method is to regard the cause of the bias in QBP as the variance of $\hat{r}_k$. It is then natural to replace the large-sample variance of $\hat{r}_k$, viz. 1/n, used in QBP by the variance of the lag-k correlation of a white noise process, viz. (n-k)/{n(n+2)}. This gives the following Ljung-Box statistic:

$$QLB = \underline{\hat{r}}^T D_2^{-1} \underline{\hat{r}} , \tag{2.2.7}$$

where $D_2$=diag($C_1^2$,...,$C_m^2$), and $C_k^2$=(n-k)/{n(n+2)}. Since the Box-Pierce and the Ljung-Box tests are asymptotically the same, QLB is distributed as $\chi^2$ with (m-p-q) degrees of freedom. According to the simulation results reported in Ljung and Box (1978), the empirical significance levels of both QBP1 and QLB agree much more closely with the $\chi^2$(m-p-q) distribution than do those of QBP.

Dufour and Roy (1985) point out that for testing the randomness of an i.i.d. series, say ($x_1$,...,$x_n$), both the Box-Pierce and the Ljung-Box statistics are based on approximate normalizations of the sample autocorrelations

$$r_k = \frac{\sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n} (x_t - \bar{x})^2} , \qquad 1 \le k \le n-1, \tag{2.2.8}$$

where $\bar{x} = \sum_{t=1}^{n} x_t / n$. While Box and Pierce (1970) and Ljung and Box (1978) assumed that the mean of $r_k$ is approximately zero, Moran (1948) proved that

$$E(r_k) = \mu_k = \frac{-(n-k)}{n(n-1)} . \tag{2.2.9}$$

The exact second moments of $r_k$ remained unknown until recent studies by Dufour and Roy (1985, 1986, 1989) and Anderson (1990). They show that

$$\mathrm{var}(r_k) = \sigma_{kk} = \frac{n^4-(k+3)n^3-3kn^2+2k(k+1)n-4k^2}{(n+1)n^2(n-1)^2}, \quad 1 \le k \le n/2,\ n>3, \tag{2.2.10}$$

and

$$\mathrm{cov}(r_k, r_h) = \sigma_{kh} = \frac{2\{kh(n-1)-(n-h)(n^2-4)\}}{(n+1)n^2(n-1)^2}, \quad 1 \le k < h \le n-1. \tag{2.2.11}$$

Prior to Dufour and Roy's (1985, 1986) finding, QBP and QLB made use of the approximations $\mathrm{var}(r_k)=1/n$ and $\mathrm{var}(r_k)=(n-k)/\{n(n+2)\}$, respectively.

The simulation results of Dufour and Roy (1985) indicate that normalizing of $r_k$ with its exact first and second moments will yield distributions that are better approximated by the asymptotic N(0,1) distribution than the distribution of QBP and QLB. Motivated by this finding, Dufour and Roy (1986) subsequently proposed two parametric portmanteau statistics for examining the specification of an ARMA(p,q) model [for notations, see Dufour and Roy (1986, p. 2957)]:

$$\mathrm{QDR} = (\hat{\underline{r}} - \underline{\mu})^T \Sigma^{-1} (\hat{\underline{r}} - \underline{\mu}) \tag{2.2.12}$$

and

$$QDR1 = (\hat{\underline{r}} - \underline{\mu})^T D_3^{-1} (\hat{\underline{r}} - \underline{\mu}), \qquad (2.2.13)$$

where $\underline{\mu}=(\mu_1,\ldots,\mu_m)^T$ and $\Sigma = [\sigma_{kh}]$ is the variance-covariance matrix of $\hat{r}$ given by (2.2.10) and (2.2.11), and $D_3=\mathrm{diag}(\sigma_{11},\sigma_{22},\ldots\sigma_{mm})$. Dufour and Roy (1986) also recommend a nonparametric portmanteau statistic based on rank autocorrelations

$$\tilde{r}_k = \frac{\sum_{t=1}^{n-k}(R_t - \bar{R})(R_{t+k} - \bar{R})}{\sum_{t=1}^{n}(R_t - \bar{R})^2}, \qquad 1 \le k \le n-1, \qquad (2.2.14)$$

where $R_t$ is the rank of $\hat{a}_t$, $\bar{R}=\sum_{t=1}^{n} R_t/n=(n+1)/2$ and $\sum_{t=1}^{n}(R-\bar{R})^2=n(n^2-1)/12$ if all ranks are distinct. This test is useful for model checking when the distribution of $\hat{a}_t$ is not known. Dufour and Roy (1986) suggest that the mean of $\tilde{r}_k$ is given by (2.2.9) because $(R_1,\ldots,R_n)$ are exchangeable. The variance of $\tilde{r}_k$ is developed in Dufour and Roy (1985, 1986) and has the following form:

$$\mathrm{var}(\tilde{r}_k) = \tilde{\sigma}_k^2 = \frac{5n^4-(5k+9)n^3+9(k+2)n^2+2k(5k+8)n+16k^2}{5(n-1)^2n^2(n+1)},$$

$$1 \le k \le n-1. \qquad (2.2.15)$$

Combining (2.2.9) and (2.2.15) gives the following nonparametric statistic [see Dufour and Roy (1986, p. 2963)]:

$$QDR2 = (\tilde{\underline{r}} - \underline{\mu})^T D_4^{-1} (\tilde{\underline{r}} - \underline{\mu}), \qquad (2.2.16)$$

where $\tilde{r}=(\tilde{r}_1,\ldots,\tilde{r}_m)^T$ and $D_4=\mathrm{diag}(\tilde{\sigma}_1,\ldots,\tilde{\sigma}_m)$. Since QDR, QDR1 and QDR2 are asymptotically equivalent to QBP and QLB on the null, their limiting distribution is $\chi^2(m-p-q)$ given that the fitted model is indeed ARMA(p,q).

Because portmanteau statistics are based on quadratic forms in $\hat{\underline{r}}$, Ljung (1986) has suggested that they can be approximated by a linear combination of independently distributed $\chi^2(1)$ random variables. Such an approximation, as shown by Box (1954), is asymptotically equivalent to an $a\chi^2(b)$ distribution with the scale parameter, $a=\Sigma\lambda_i^2/\Sigma\lambda_i$, the degree of freedom, $b=(\Sigma\lambda_i)^2/\Sigma\lambda_i^2$, where $\lambda_i$ $(i=1,\ldots,m)$ are the eigenvalues of the variance-covariance matrix of $\hat{r}$ given in McLeod (1978). As the value of m increases, this $a\chi^2(b)$ distribution approaches to $\chi^2(m-p-q)$ as $a\to1$ and $b\to(m-p-q)$, respectively [see Ljung (1986, p. 726)]. The simulation results of Ljung (1986) indicate that using the scaled chi-square distribution to set critical values for the Ljung-Box test can considerably improve the accuracy of its significance level when the number of residual autocorrelations is small. In this dissertation, this modification is called the modified Ljung-Box statistic

$$QLB1 \sim a\chi^2(b). \qquad (2.2.17)$$

Bera and Newbold (1988) have recently developed new portmanteau tests of model adequacy based on employing either the sample autocorrelations or partial autocorrelations of the observed time series $w_t$. In the case where $w_t$ follows an AR(p) process, it is known that the theoretical partial autocorrelations, $\Psi_{kk}$, are equal to zero for $k \geq p+1$. Moreover, the variance of $\Psi_{kk}$ (for $k \geq p+1$) is $1/n$ [see Box and Jenkins (1976, p. 178)]. Thus, the adequacy of an AR(p) model can be evaluated by the following portmanteau statistic

$$QBN1 = \hat{\underline{\Psi}}^T D_5^{-1} \hat{\underline{\Psi}} , \tag{2.2.18}$$

where $\hat{\underline{\Psi}} = (\hat{\Psi}_{p+1}, \ldots, \hat{\Psi}_m)^T$ are the estimated partial autocorrelations of $w_t$ and $D_5 = (1/n) I_{(m-p)}$. If the true model is AR(p), QBN1 is asymptotically distributed as $\chi^2(m-p)$. For testing the hypothesis that $w_t$ follows a MA(q) process, Bera and Newbold (1988) exploit the fact that the autocorrelations of $w_t$ satisfy

$$\text{plim}(r_k) = 0, \qquad\qquad k \geq q+1 \tag{2.2.19}$$

Using the Bartlett (1946) formula for the variance of $r_k$, they recommend the following portmanteau test

$$QBN2 = \underline{r}^T D_6^{-1} \underline{r}, \tag{2.2.20}$$

where $\underline{r} = (r_{q+1}, \ldots, r_m)^T$, $D_6 = \text{diag}(b_1^2, \ldots, b_{m-q}^2)$, and $b_k^2 = (1 + 2 \sum_{k=1}^{q} r_k^2)$. If the true specification is a MA(q) process, QBN2 is asymptotically distributed as $\chi^2(m-q)$. A summary of the portmanteau tests in this section is given in Table 2.1.

Table 2.1: Summary of Portmanteau Statistics

| Correct Models | Statistics | | |
|---|---|---|---|
| ARMA(p,q) | Box-Pierce Statistic: | $QBP = \hat{\underline{r}} D_1^{-1} \hat{\underline{r}} \sim \chi^2(m-p-q)$ | $\hat{\underline{r}} = (\hat{r}_1, \ldots, \hat{r}_m)$ and $D_1 = (1/n) I_m$ |
| ARMA(p,q) | Modified Box-Pierce Statistic: | $QBP1 = \hat{\underline{r}} D_1^{-1} \hat{\underline{r}} \sim \chi^2(E(QBP))$ | $E(QBP)$ is given in (2.2.5) |
| ARMA(p,q) | Ljung-Box Statistic: | $QLB = \hat{\underline{r}} D_2^{-1} \hat{\underline{r}} \sim \chi^2(m-p-q)$ | $D_2 = \mathrm{diag}(C_1^2, \ldots, C_m^2)$ and $C_k^2 = (n-k)/\{n(n+2)\}$ |
| ARMA(p,q) | Modified Ljung-Box Statistic: | $QLB1 = \hat{\underline{r}} D_2^{-1} \hat{\underline{r}} \sim a\chi^2(b)$ | $a = \Sigma\lambda_i^2 / \Sigma\lambda_i$ and $b = (\Sigma\lambda_i)^2 / \Sigma\lambda_i^2$; where $\lambda_i$ $(i=1,\ldots,m)$ are the eigenvalues of the variance-covariance matrix of $\hat{\underline{r}}$ given in Mcleod (1978) |
| ARMA(p,q) | Dufour-Roy Statistic: | $QDR = (\hat{\underline{r}} - \underline{\mu})^T \Sigma^{-1} (\hat{\underline{r}} - \underline{\mu}) \sim \chi^2(m-p-q)$ | $\underline{\mu}$ is given in (2.2.9); $\Sigma = [\sigma_{kh}]$ is the variance-covariance matrix of $\hat{\underline{r}}$ given in |
| | | $QDR1 = (\hat{\underline{r}} - \underline{\mu})^T D_3^{-1} (\hat{\underline{r}} - \underline{\mu}) \sim \chi^2(m-p-q)$ | (2.2.10) and (2.2.11); $D_3 = \mathrm{Diag}(\sigma_{11}, \ldots, \sigma_{mm})$ |
| | | $QDR2 = (\tilde{\underline{r}} - \underline{\mu})^T D_4^{-1} (\tilde{\underline{r}} - \underline{\mu}) \sim \chi^2(m-p-q)$ | $\tilde{\underline{r}} = (\tilde{r}_1, \ldots, \tilde{r}_m)$ and $D_4 = \mathrm{diag}(\tilde{\sigma}_1, \ldots, \tilde{\sigma}_m)$ |
| AR(P) | Bera-Newbold Statistic: | $QBN1 = \hat{\underline{\psi}}^T D_5^{-1} \hat{\underline{\psi}} \sim \chi^2(m-p)$ | $\hat{\underline{\psi}} = (\hat{\psi}_{p+1}, \ldots, \hat{\psi}_m)$ are the estimated partial autocorrelations of $w_t$; $D_5 (1/n) I_{(m-p)}$ |
| MA(q) | | $QBN2 = \underline{r} D_6^{-1} \underline{r} \sim \chi^2(m-q)$ | $\underline{r} = (r_{q+1}, \ldots, r_m)^T$ are the estimated sample autocorrelations of $w_t$; $D_6 = \mathrm{diag}(b_1^2, \ldots, b_{m-p}^2)$ and $b_k^2 = (1 + 2\sum_{k=1}^{q} r_k^2)$ |

## 2.3 Experimental Design

In order to examine the finite-sample distributions of the portmanteau tests, three separate simulation experiments were used. In the first experiment, our attention was confined to the empirical significance levels, means and variances of the portmanteau statistics outlined in Section 2.2. All simulations were carried out on a CDC Cyber 830 computer. The data were generated from some selected cases of an ARMA(2,1) process, $(1-\pi_1 B)(1-\pi_2 B)w_t = (1-\psi_1 B)a_t$, with proper zero restrictions on the roots of the AR and MA processes $(\pi_1,\pi_2)$ and $(\psi_1)$, respectively. The starting value $w_0$ was set at 0. N(0,1) random deviates, $a_t$, were generated by using the subroutine RNNOA of IMSL. Additional IMSL subroutines such as CHIIN and EVCRG were employed to find the critical values of the chi-square distribution and the eigenvalues for QLB1, respectively. Estimates of significance levels ($\alpha$=5%), means and variances of the various statistics were based on 1000 replications. The sample size, n, was set to be 50 and 100 in the case of pure AR processes. For the MA(1) process, n was fixed at 50 in order to comply with restrictions on CPU time.

The second experiment examines the empirical power of the portmanteau tests. All data were first generated from some selected cases of an ARMA(2,2) process, $(1-\pi_1 B)(1-\pi_2 B)w_t=(1-\psi_1 B)(1-\psi_2 B)a_t$, and were subsequently fitted to the AR(1), AR(2) and MA(1) models, respectively. The sample size, n, was set at 50 and 100 for the AR models. In the case of the MA(1) model, n was kept again at 50 in order to meet the CPU-time requirement. Since one of the objectives is to investigate the effect of the number of residual autocorrelations on the empirical power of the portmanteau tests, m was allowed to vary for all experiments; $2 \le m \le 15$ for the AR(1) and MA(1) models, and $3 \le m \le 15$ for the AR(2) model. The power calculations reported in Section 2.4 are the proportion of times that the hypothesis of correct model specification was rejected for tests at the 5% level of significance. All power estimates were based on 1000 replications.

The third simulation experiment allows for the impact of the nature of the data on the empirical power of the portmanteau tests. To do so, four seasonal ARMA models adopted in Abraham and Ledolter (1983, p. 285) are used:

$$w_t = (1 - \Theta_s B^s)a_t, \qquad [\Theta_s = 0.4,\ 0.7,\ 0.9] \tag{2.3.1}$$

$$(1 - \Phi_s B^s)w_t = a_t, \qquad [\Phi_s = 0.4,\ 0.7,\ 0.9] \tag{2.3.2}$$

$$w_t = (1 - \psi_1 B)(1 - \Theta_s B^s)a_t, \quad [(\psi_1, \Theta_s) = (0.6, -0.5),\ (-0.5, 0.6)] \tag{2.3.3}$$

$$(1 - \Phi_s B^s)w_t = (1 - \psi_1 B)a_t, \quad [(\Phi_s, \psi_1) = (0.6, -0.5),\ (-0.5, 0.6)] \tag{2.3.4}$$

where s=12 (monthly data). The set-up of this simulation experiment is similar to the second one except that (i) the fitted processes are only AR(1) and AR(2) models, and (ii) the first fifty observations were discarded for each sample in order to account for the presence of seasonality in the data.

## 2.4 Monte Carlo Results

Tables 2.2 to 2.4 report the empirical significance levels, means and variances of the eight portmanteau statistics. On the basis of these results, the following points are noteworthy:

(i) As expected, the Box-Pierce test, QBP, has significance levels which are considerably lower than the nominal levels when m is large; e.g. $m=10$ and 15. This observation holds regardless of the sample size and the choice of the fitted models. For $m \leq 3$, $n=50$ and parameter values that are close to the boundary of the stationary or invertible region, QBP tends to over-reject the null hypothesis far too often (i.e., the significance levels of this test are too high). It is also noteworthy that this "over-rejection" problem is still quite serious when the sample size increases to 100. As for the estimated means and variances, these are consistently lower than the theoretical values. Our simulation results also support the finding of Ljung-Box (1978, p. 301) that the empirical variances of QBP are approximately twice the mean when $m \geq 10$ and the data are fitted to an AR(1) model. This evidence, however, does not hold in the case of a MA(1) model

Table 2.2

**Empirical Significance Levels, means and variances of Portmanteau Tests for the AR(1) Model $(1 - \pi_1 B)w_t = a_t$; $\alpha$=5%**

| Parameters of AR(1) Model | Portmanteau Tests | Empirical Significance Levels (in percent) | | | | | | n=50 Empirical Means | | | | | | Empirical Variances | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
| $\pi_1$ = 0.4 | | | | | | | | | | | | | | | | | | | |
| | QBP | 4.7 | 3.2 | 2.8 | 2.8 | 1.7 | 1.3 | 0 98 | 1.80 | 2.62 | 3.47 | 7.27 | 10.73 | 1.70 | 3 19 | 4.78 | 6.02 | 12.53 | 20.57 |
| | QBP1 | 6.0 | 4.4 | 3.9 | 3.9 | 3.7 | 4.8 | 0.98 | 1.80 | 2.62 | 3.47 | 7.27 | 10 72 | 1.70 | 3.19 | 4.78 | 6 02 | 12.53 | 20.57 |
| | QLB | 5.5 | 4.6 | 4.1 | 4.2 | 4.6 | 6.4 | 1.06 | 1.97 | 2.89 | 3.88 | 8.58 | 13.44 | 1.98 | 3.80 | 5.82 | 7.48 | 17.35 | 32 18 |
| | QLB1 | 5.3 | 4.6 | 4.1 | 4.2 | 4.6 | 6.4 | 1 06 | 1.97 | 2.89 | 3.88 | 8.58 | 13.44 | 1 98 | 3 80 | 5.82 | 7.48 | 17.35 | 32.18 |
| | QDR | 6.1 | 4.9 | 4 1 | 4.7 | 4.3 | 6.6 | 1.16 | 2.08 | 2.99 | 3 98 | 8.70 | 13.59 | 2.18 | 4.05 | 6 03 | 7.94 | 18.12 | 33 28 |
| | QDR1 | 5.7 | 4 7 | 4.3 | 4.4 | 4.0 | 6.1 | 1.14 | 2.05 | 2.96 | 3.94 | 8 66 | 13.59 | 2 08 | 3.94 | 5.87 | 7.73 | 17.58 | 32 44 |
| | QDR2 | 6.8 | 5.8 | 5.1 | 5.4 | 4.6 | 6.0 | 1.26 | 2.18 | 3.08 | 4.10 | 8.84 | 13.74 | 2.21 | 4.06 | 6.17 | 8.35 | 19.16 | 34.71 |
| | QBN1 | 4.7 | 3.8 | 3.3 | 3.1 | 2.0 | 0.9 | 0.98 | 1.92 | 2.79 | 3.67 | 7 80 | 11.33 | 1.90 | 3.56 | 5.09 | 6.52 | 12.89 | 17 31 |
| $\pi_1$ = 0 7 | | | | | | | | | | | | | | | | | | | |
| | QBP | 5.0 | 3 6 | 3.5 | 3.2 | 1.9 | 2.1 | 1 16 | 1.92 | 2.70 | 3 54 | 7.37 | 10.81 | 1 72 | 3.26 | 4 89 | 6.33 | 13 17 | 21.18 |
| | QBP1 | 6 5 | 4 6 | 4.5 | 4.3 | 4.0 | 4.8 | 1.16 | 1.92 | 2.70 | 3 54 | 7.37 | 10.81 | 1.72 | 3.26 | 4.89 | 6 33 | 13.17 | 21.18 |
| | QLB | 6.4 | 4.6 | 4.7 | 4 8 | 4.6 | 5.8 | 1.24 | 2 09 | 2.97 | 3.95 | 8 68 | 13.52 | 1.99 | 3 86 | 5.93 | 7.84 | 18.18 | 32.91 |
| | QLB1 | 4.5 | 4 3 | 4.5 | 4.8 | 4.6 | 5.8 | 1.24 | 2.09 | 2 97 | 3 95 | 8.68 | 13.52 | 1 99 | 3.86 | 5 93 | 7.84 | 18 18 | 32 91 |
| | QDR | 7 5 | 6.0 | 5.0 | 5 4 | 5.5 | 6.4 | 1.40 | 2.27 | 3.17 | 4.16 | 8.92 | 13.80 | 2 36 | 4.36 | 6.41 | 8.61 | 19 47 | 35.03 |
| | QDR1 | 7.1 | 5.3 | 4.9 | 4.9 | 4.9 | 5.9 | 1.37 | 2.23 | 3.11 | 4.08 | 8.82 | 13.75 | 2.35 | 4 15 | 6.08 | 8.21 | 18.65 | 33.55 |
| | QDR2 | 8.0 | 5.9 | 5.7 | 6.2 | 5.5 | 6.0 | 1.47 | 2 36 | 3.25 | 4.28 | 9.07 | 13.95 | 2.52 | 4 58 | 6.75 | 9.40 | 20 97 | 37.16 |
| | QBN1 | 4.4 | 4.2 | 3.5 | 3.1 | 1.6 | 0.8 | 0 98 | 1.96 | 2.82 | 3.66 | 7.72 | 11.22 | 1.71 | 3.64 | 4.92 | 6.25 | 12.22 | 16 86 |
| $\pi_1$ = 0.9 | | | | | | | | | | | | | | | | | | | |
| | QBP | 8.9 | 5 4 | 4.5 | 3.8 | 2.6 | 2 5 | 1 53 | 2 26 | 2.99 | 3.78 | 7.57 | 11 03 | 2.50 | 3 87 | 5.35 | 6 90 | 15 15 | 23 36 |
| | QBP1 | 10.5 | 6.3 | 6.1 | 5.2 | 4.5 | 4.9 | 1 53 | 2.26 | 2.99 | 3.78 | 7.57 | 11.03 | 2.50 | 3.87 | 5.35 | 6.90 | 15 15 | 23 36 |
| | QLB | 10.3 | 6.4 | 6.5 | 5.1 | 4.8 | 6.0 | 1 64 | 2.45 | 3.27 | 4.19 | 8 88 | 13.75 | 2.88 | 4 54 | 6.42 | 8 46 | 20.77 | 35.89 |
| | QLB1 | 5 7 | 4.6 | 5.3 | 4.4 | 4.4 | 6 0 | 1.64 | 2.45 | 3.27 | 4.19 | 8.88 | 13.75 | 2.88 | 4.54 | 6 42 | 8.46 | 20.77 | 35.89 |
| | QDR | 11 8 | 8.6 | 7.3 | 6.7 | 5.8 | 6.7 | 1.81 | 2.67 | 3.55 | 4.51 | 9.29 | 14 22 | 3 41 | 5.21 | 7.24 | 9.47 | 22.23 | 38 02 |
| | QDR1 | 11.4 | 8.4 | 6.9 | 6.7 | 5.3 | 6.8 | 1.80 | 2 65 | 3.52 | 4.46 | 9.17 | 14.14 | 3 39 | 5.11 | 7.05 | 9 13 | 21 18 | 36.58 |
| | QDR2 | 11.7 | 8.8 | 7 3 | 6.8 | 5.7 | 6.8 | 1.82 | 2.71 | 3.57 | 4.57 | 9.38 | 14.31 | 3.23 | 5.09 | 7.17 | 9.68 | 22 24 | 39 03 |
| | QBN1 | 4.0 | 3.3 | 3.4 | 3.2 | 1.0 | 0.5 | 0.97 | 1.89 | 2.72 | 3.50 | 7.38 | 10.70 | 1.67 | 3.38 | 4.69 | 6.09 | 12.05 | 16 02 |
| $\pi_1$ = 0.99 | | | | | | | | | | | | | | | | | | | |
| | QBP | 11.5 | 7.8 | 6.3 | 5.2 | 3.5 | 2.6 | 1 78 | 2 55 | 3.32 | 4.14 | 7.91 | 11.36 | 3.24 | 4.66 | 6 32 | 7.91 | 16.40 | 24.82 |
| | QBP1 | 13.8 | 9.2 | 8.5 | 6.4 | 5.0 | 6.0 | 1 78 | 2 55 | 3.32 | 4.14 | 7.91 | 11.36 | 3.24 | 4.66 | 6.32 | 7.91 | 16.40 | 24.82 |
| | QLB | 12.8 | 9.2 | 8.6 | 7.1 | 5.3 | 6.8 | 1.91 | 2.76 | 3 64 | 4 58 | 9.25 | 14 10 | 3 74 | 5 45 | 7.55 | 9 64 | 22 25 | 37 75 |
| | QLB1 | 5.8 | 5.3 | 5.2 | 4.3 | 4.5 | 5.8 | 1 91 | 2.76 | 3.64 | 4.58 | 9.25 | 14.10 | 3.74 | 5.45 | 7.55 | 9.64 | 22.25 | 37 75 |
| | QDR | 14.0 | 10 9 | 9 7 | 9.1 | 7.0 | 8.1 | 2.04 | 2.94 | 3.87 | 4 86 | 9 69 | 14.65 | 4.17 | 6.05 | 8.40 | 10.76 | 24 01 | 40.25 |
| | QDR1 | 13.9 | 10.8 | 10.4 | 8.9 | 7.4 | 8.3 | 2.04 | 2.94 | 3.88 | 4.88 | 9.72 | 14.81 | 4.22 | 6 14 | 8.55 | 10.99 | 24.96 | 43.13 |
| | QDR2 | 14.0 | 11.0 | 9.8 | 9 4 | 7.2 | 7.7 | 2.00 | 2.92 | 3.83 | 4.84 | 9.69 | 14.66 | 3 77 | 5.62 | 8.19 | 10.82 | 23.62 | 40.36 |
| | QBN1 | 2.5 | 2.0 | 2.4 | 2.3 | 0.7 | 0.2 | 0.82 | 1.52 | 2.28 | 2.95 | 6.19 | 9 09 | 1.34 | 2.57 | 3 78 | 5 00 | 10.66 | 15 08 |

Table 2.2 (cont'd)

| Parameters of AR(1) Model | Portmanteau Tests | Empirical Significance Levels (in percent) | | | | | | n=100 Empirical Means | | | | | | Empirical Variances | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
| $\pi_1 = 0.4$ | | | | | | | | | | | | | | | | | | | |
| | QBP | 5.6 | 4.2 | 4.3 | 4.2 | 3.0 | 3.4 | 1.07 | 1.98 | 2.90 | 3.85 | 8.25 | 12.46 | 2.16 | 3.19 | 5.84 | 7.69 | 16.93 | 26.96 |
| | QBP1 | 6.2 | 4.9 | 5.0 | 5.2 | 5.2 | 5.5 | 1.07 | 1.98 | 2.90 | 3.85 | 8.25 | 12.46 | 2.16 | 3.19 | 5.04 | 7.69 | 16.93 | 26.96 |
| | QLB | 6.1 | 4.9 | 5.1 | 5.4 | 5.5 | 6.1 | 1.11 | 2.07 | 3.04 | 4.07 | 8.95 | 13.87 | 2.33 | 4.26 | 6.44 | 8.56 | 19.82 | 33.23 |
| | QLB1 | 5.8 | 4.9 | 5.1 | 5.4 | 5.5 | 6.1 | 1.11 | 2.07 | 3.04 | 4.07 | 8.95 | 13.87 | 2.33 | 4.26 | 6.44 | 8.56 | 19.82 | 33.23 |
| | QDR | 6.1 | 4.9 | 4.1 | 4.7 | 4.3 | 6.6 | 1.17 | 2.13 | 3.11 | 4.12 | 9.00 | 13.94 | 2.41 | 4.31 | 6.48 | 8.68 | 19.96 | 33.64 |
| | QDR1 | 5.7 | 4.7 | 4.3 | 4.4 | 4.0 | 6.1 | 1.16 | 2.11 | 3.08 | 4.09 | 8.97 | 13.91 | 2.34 | 4.22 | 6.37 | 8.53 | 19.62 | 33.35 |
| | QDR2 | 6.8 | 5.8 | 5.1 | 5.4 | 4.6 | 6.0 | 1.24 | 2.19 | 3.17 | 4.23 | 9.17 | 14.07 | 2.49 | 4.29 | 6.46 | 8.89 | 20.72 | 34.64 |
| | QBN1 | 5.3 | 5.1 | 5.1 | 4.8 | 3.0 | 2.2 | 1.05 | 2.03 | 3.01 | 3.99 | 8.48 | 12.72 | 2.22 | 4.14 | 6.30 | 7.78 | 15.30 | 21.35 |
| $\pi_1 = 0.7$ | | | | | | | | | | | | | | | | | | | |
| | QBP | 6.6 | 4.9 | 4.0 | 4.3 | 3.8 | 3.6 | 1.26 | 2.09 | 2.96 | 3.89 | 8.28 | 12.47 | 2.15 | 3.92 | 5.78 | 7.82 | 17.12 | 26.84 |
| | QBP1 | 7.2 | 5.7 | 4.6 | 4.7 | 5.0 | 5.3 | 1.26 | 2.09 | 2.96 | 3.89 | 8.28 | 12.47 | 2.15 | 3.92 | 5.78 | 7.82 | 17.12 | 26.84 |
| | QLB | 7.1 | 5.6 | 4.7 | 5.5 | 5.3 | 5.7 | 1.30 | 2.18 | 3.10 | 4.10 | 8.97 | 13.88 | 2.31 | 4.26 | 6.35 | 8.70 | 20.02 | 33.03 |
| | QLB1 | 5.4 | 5.2 | 4.6 | 4.9 | 5.3 | 5.7 | 1.30 | 2.18 | 3.10 | 4.10 | 8.97 | 13.88 | 2.31 | 4.26 | 6.35 | 8.70 | 20.02 | 33.03 |
| | QDR | 7.3 | 6.2 | 5.3 | 5.6 | 5.5 | 5.8 | 1.40 | 2.29 | 3.23 | 4.23 | 9.14 | 14.06 | 2.52 | 4.52 | 6.66 | 9.13 | 20.65 | 34.28 |
| | QDR1 | 7.0 | 5.6 | 5.0 | 5.3 | 5.3 | 5.8 | 1.39 | 2.26 | 3.19 | 4.18 | 9.06 | 13.98 | 2.45 | 4.38 | 6.46 | 8.88 | 20.18 | 33.62 |
| | QDR2 | 8.0 | 6.6 | 5.9 | 5.9 | 6.3 | 5.8 | 1.45 | 2.34 | 3.29 | 4.33 | 9.28 | 14.18 | 2.67 | 4.52 | 6.73 | 9.38 | 21.24 | 35.46 |
| | QBN1 | 5.5 | 5.5 | 5.4 | 4.4 | 3.0 | 1.8 | 1.01 | 2.03 | 3.00 | 3.96 | 8.42 | 12.67 | 2.04 | 4.22 | 6.20 | 7.33 | 14.92 | 21.46 |
| $\pi_1 = 0.9$ | | | | | | | | | | | | | | | | | | | |
| | QBP | 9.4 | 6.1 | 5.0 | 4.9 | 4.3 | 3.8 | 1.65 | 2.46 | 3.28 | 4.14 | 8.45 | 12.63 | 2.95 | 4.53 | 6.08 | 8.08 | 17.9[illegible] | 27.67 |
| | QBP1 | 10.8 | 6.9 | 5.8 | 5.8 | 6.0 | 5.6 | 1.65 | 2.46 | 3.20 | 4.14 | 8.45 | 12.63 | 2.95 | 4.53 | 6.08 | 8.08 | 17.91 | 27.67 |
| | QLB | 10.7 | 7.0 | 5.9 | 6.0 | 6.2 | 6.0 | 1.71 | 2.56 | 3.43 | 4.35 | 9.13 | 14.04 | 3.16 | 4.90 | 6.65 | 8.94 | 20.89 | 33.95 |
| | QLB1 | 5.9 | 5.0 | 4.2 | 4.7 | 6.0 | 6.0 | 1.71 | 2.56 | 3.43 | 4.35 | 9.13 | 14.04 | 3.16 | 4.90 | 6.65 | 8.94 | 20.89 | 33.95 |
| | QDR | 11.3 | 8.1 | 7.0 | 6.8 | 6.9 | 6.7 | 1.82 | 2.69 | 3.60 | 4.56 | 9.43 | 14.39 | 3.51 | 5.31 | 7.16 | 9.58 | 21.73 | 35.28 |
| | QDR1 | 11.5 | 8.2 | 6.5 | 6.5 | 6.3 | 6.3 | 1.81 | 2.68 | 3.58 | 4.52 | 9.33 | 14.25 | 3.48 | 5.22 | 7.00 | 9.32 | 21.09 | 34.37 |
| | QDR2 | 12.5 | 8.8 | 8.3 | 6.5 | 6.7 | 6.8 | 1.82 | 2.70 | 3.61 | 4.61 | 9.53 | 14.45 | 3.58 | 5.30 | 7.26 | 9.89 | 22.25 | 36.55 |
| | QBN1 | 5.2 | 5.4 | 4.2 | 4.0 | 3.3 | 0.5 | 1.01 | 2.00 | 2.92 | 3.82 | 8.24 | 12.39 | 2.10 | 2.11 | 5.80 | 7.11 | 14.82 | 21.73 |
| $\pi_1 = 0.99$ | | | | | | | | | | | | | | | | | | | |
| | QBP | 13.5 | 9.2 | 7.5 | 6.7 | 5.1 | 3.9 | 1.94 | 2.81 | 3.69 | 4.61 | 8.95 | 13.10 | 3.89 | 5.49 | 7.20 | 9.28 | 18.50 | 28.39 |
| | QBP1 | 14.7 | 9.9 | 8.6 | 7.4 | 7.1 | 6.4 | 1.94 | 2.81 | 3.69 | 4.61 | 8.95 | 13.10 | 3.89 | 5.49 | 7.20 | 9.28 | 18.50 | 29.39 |
| | QLB | 14.5 | 9.7 | 8.5 | 7.4 | 7.6 | 7.1 | 2.01 | 2.92 | 3.86 | 4.84 | 9.65 | 14.52 | 4.17 | 5.94 | 7.86 | 10.26 | 21.47 | 34.70 |
| | QLB1 | 6.1 | 5.6 | 4.4 | 5.1 | 5.7 | 6.0 | 2.01 | 2.92 | 3.86 | 4.84 | 9.65 | 14.52 | 4.17 | 5.94 | 7.86 | 10.26 | 21.47 | 34.70 |
| | QDR | 15.7 | 11.0 | 9.3 | 8.8 | 8.0 | 7.9 | 2.09 | 3.03 | 4.01 | 5.02 | 9.96 | 14.93 | 4.45 | 6.35 | 8.40 | 10.86 | 22.60 | 36.24 |
| | QDR1 | 15.8 | 11.1 | 9.2 | 8.7 | 8.0 | 7.[illegible] | 2.09 | 3.03 | 4.01 | 5.02 | 9.96 | 14.93 | 4.46 | 6.37 | 8.44 | 10.93 | 22.69 | 36.72 |
| | QDR2 | 15.6 | 11.1 | 9.4 | 8.6 | 8.2 | 8.0 | 2.06 | 2.98 | 3.98 | 5.02 | 10.00 | 14.95 | 4.38 | 5.28 | 8.51 | 11.16 | 23.10 | 37.48 |
| | QBN1 | 4.1 | 2.5 | 2.7 | 2.9 | 1.7 | 1.4 | 0.87 | 1.64 | 2.41 | 3.16 | 6.86 | 10.39 | 1.71 | 2.94 | 4.53 | 5.96 | 13.62 | 21.72 |

Table 2.3
Empirical Significance Levels, Means and Variances of Portmanteau Tests for the AR(2) Model $(1 - \pi_1 B)(1 - \pi_2 B)w_t = a_t$, $\alpha = 5\%$

| Parameters of AR(1) Model | Portmanteau Tests | Empirical Significance Levels (in percent) | | | | | n=50 Empirical Means | | | | | Empirical Variances | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | m=3 | m=4 | m=5 | m=10 | m=15 | m=3 | m=4 | m=5 | m=10 | m=15 | m=3 | m=4 | m=5 | m=10 | m=15 |
| $\pi_1$=0.5 $\pi_2$=0.5 | | | | | | | | | | | | | | | | |
| | QBP | 5.4 | 3.1 | 3.1 | 1.3 | 1.5 | 1.24 | 1.90 | 2.67 | 6.34 | 9.69 | 1.61 | 2.88 | 4.34 | 9.75 | 16.30 |
| | QBP1 | 8.0 | 4.9 | 4.3 | 3.0 | 3.2 | 1.24 | 1.90 | 2.67 | 6.34 | 9.69 | 1.61 | 2.88 | 4.34 | 9.75 | 16.30 |
| | QLB | 6.7 | 4.8 | 4.5 | 3.5 | 3.7 | 1.35 | 2.10 | 2.99 | 7.54 | 12.23 | 1.91 | 3.53 | 5.46 | 13.70 | 26.04 |
| | QLB1 | 4.5 | 4.1 | 4.3 | 3.5 | 3.7 | 1.35 | 2.10 | 2.99 | 7.54 | 12.23 | 1.91 | 3.53 | 5.46 | 13.70 | 26.06 |
| | QDR | 8.4 | 5.9 | 5.5 | 4.3 | 4.4 | 1.50 | 2.28 | 3.21 | 7.77 | 12.49 | 2.25 | 4.00 | 6.05 | 14.69 | 27.69 |
| | QDR1 | 7.7 | 5.3 | 5.1 | 4.1 | 4.3 | 1.48 | 2.23 | 3.14 | 7.69 | 12.48 | 2.15 | 3.79 | 5.75 | 14.07 | 26.73 |
| | QDR2 | 8.9 | 6.2 | 6.4 | 4.1 | 4.7 | 1.65 | 2.45 | 3.43 | 8.02 | 12.77 | 2.27 | 4.16 | 6.25 | 15.47 | 28.96 |
| | QBN1 | 3.9 | 3.4 | 2.5 | 1.7 | 0.6 | 0.96 | 1.82 | 2.63 | 6.60 | 10.06 | 1.70 | 3.02 | 4.30 | 10.26 | 14.88 |
| $\pi_1$=0.8 $\pi_2$=0.2 | | | | | | | | | | | | | | | | |
| | QBP | 6.2 | 4.1 | 3.3 | 1.3 | 1.6 | 1.35 | 2.01 | 2.76 | 6.38 | 9.74 | 1.83 | 3.13 | 4.52 | 10.00 | 16.75 |
| | QBP1 | 9.8 | 5.6 | 5.1 | 3.3 | 3.2 | 1.35 | 2.01 | 2.76 | 6.38 | 9.74 | 1.83 | 3.13 | 4.52 | 10.00 | 16.75 |
| | QLB | 8.0 | 5.3 | 5.0 | 3.5 | 4.1 | 1.47 | 2.22 | 3.09 | 7.57 | 12.29 | 2.18 | 3.83 | 5.66 | 14.00 | 26.68 |
| | QLB1 | 4.1 | 4.4 | 4.4 | 3.4 | 4.1 | 1.47 | 2.22 | 3.09 | 7.57 | 12.29 | 2.18 | 3.83 | 5.66 | 14.00 | 26.68 |
| | QDR | 10.2 | 6.1 | 6.0 | 4.6 | 4.5 | 1.62 | 2.42 | 3.33 | 7.89 | 12.64 | 2.52 | 4.35 | 6.27 | 15.07 | 28.39 |
| | QDR1 | 9.9 | 6.2 | 5.5 | 3.9 | 4.1 | 1.60 | 2.38 | 3.27 | 7.77 | 12.58 | 2.43 | 4.15 | 5.94 | 14.28 | 27.17 |
| | QDR2 | 10.6 | 6.7 | 7.2 | 4.8 | 4.9 | 1.76 | 2.57 | 3.53 | 8.13 | 12.87 | 2.62 | 4.48 | 6.46 | 15.91 | 29.59 |
| | QBN1 | 3.7 | 3.4 | 2.9 | 1.3 | 0.3 | 0.94 | 1.79 | 2.58 | 6.50 | 9.89 | 1.67 | 2.99 | 4.32 | 10.15 | 14.58 |
| $\pi_1$=0.8 $\pi_2$=0.5 | | | | | | | | | | | | | | | | |
| | QBP | 8.9 | 4.8 | 3.6 | 1.5 | 1.5 | 1.55 | 2.18 | 2.89 | 6.45 | 9.80 | 2.25 | 3.49 | 4.89 | 10.53 | 17.28 |
| | QBP1 | 13.3 | 6.9 | 5.6 | 3.6 | 4.1 | 1.55 | 2.18 | 2.89 | 6.45 | 9.80 | 2.25 | 3.49 | 4.89 | 10.53 | 17.28 |
| | QLB | 10.9 | 6.3 | 5.5 | 4.1 | 4.7 | 1.69 | 2.41 | 3.22 | 7.64 | 12.34 | 2.67 | 4.24 | 6.09 | 14.65 | 27.28 |
| | QLB1 | 4.8 | 4.4 | 4.2 | 4.0 | 4.7 | 1.69 | 2.41 | 3.12 | 7.64 | 12.34 | 2.67 | 4.24 | 6.09 | 14.65 | 27.28 |
| | QDR | 12.7 | 7.9 | 6.7 | 4.5 | 5.6 | 1.81 | 2.59 | 3.47 | 7.99 | 12.72 | 2.97 | 4.76 | 6.71 | 15.77 | 29.04 |
| | QDR1 | 12.0 | 7.2 | 6.2 | 4.0 | 5.0 | 1.77 | 2.54 | 3.40 | 7.86 | 12.65 | 2.83 | 4.53 | 6.34 | 14.84 | 27.86 |
| | QDR2 | 13.1 | 8.0 | 7.3 | 5.7 | 5.1 | 1.95 | 2.73 | 3.66 | 8.21 | 12.96 | 3.07 | 4.88 | 6.80 | 16.15 | 30.30 |
| | QBN1 | 3.3 | 3.0 | 1.8 | 0.7 | 0.3 | 2.89 | 1.69 | 2.43 | 6.07 | 9.23 | 1.39 | 2.69 | 3.86 | 9.24 | 13.76 |
| $\pi_1$=0.8 $\pi_2$=0.8 | | | | | | | | | | | | | | | | |
| | QBP | 11.8 | 6.3 | 4.3 | 1.9 | 1.4 | 1.79 | 2.48 | 3.20 | 6.68 | 9.99 | 2.86 | 4.22 | 5.18 | 11.44 | 18.55 |
| | QBP1 | 16.7 | 8.6 | 6.6 | 4.1 | 4.4 | 1.79 | 2.48 | 3.20 | 6.68 | 9.99 | 2.86 | 4.22 | 5.18 | 11.44 | 18.55 |
| | QLB | 13.5 | 8.0 | 6.4 | 4.3 | 5.3 | 1.95 | 2.72 | 3.56 | 7.88 | 12.52 | 3.40 | 5.12 | 6.92 | 15.78 | 28.92 |
| | QLB1 | 4.6 | 4.4 | 4.0 | 4.0 | 5.0 | 1.95 | 2.72 | 3.56 | 7.88 | 12.52 | 3.40 | 5.12 | 6.92 | 15.78 | 28.92 |
| | QDR | 14.7 | 9.1 | 7.8 | 4.7 | 5.9 | 2.04 | 2.87 | 3.78 | 8.30 | 13.02 | 3.64 | 5.64 | 7.61 | 17.14 | 30.92 |
| | QDR1 | 14.0 | 8.5 | 7.3 | 4.5 | 5.8 | 1.98 | 2.80 | 3.70 | 8.16 | 12.93 | 3.43 | 5.33 | 7.19 | 16.23 | 29.74 |
| | QDR2 | 15.4 | 10.0 | 8.9 | 6.1 | 6.2 | 2.14 | 2.97 | 3.93 | 8.51 | 13.12 | 3.73 | 5.70 | 7.87 | 17.79 | 32.78 |
| | QBN1 | 4.4 | 2.8 | 2.3 | 0.4 | 0.1 | 1.09 | 1.79 | 2.38 | 5.15 | 7.59 | 7.59 | 1.49 | 2.58 | 7.89 | 11.88 |

Table 2.3 (cont'd)

| Parameters of AR(2) Model | Portmanteau Tests | Empirical Significance levels (in percent) m=3 | m=4 | m=5 | m=10 | m=15 | n=100 Empirical Means m=3 | m=4 | m=5 | m=10 | m=15 | Empirical Variances m=3 | m=4 | m=5 | m=10 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1=0.5$ $\pi_2=0.5$ | | | | | | | | | | | | | | | | |
| | QBP | 6.7 | 5.4 | 4.9 | 2.9 | 3.0 | 1.35 | 2.12 | 2.97 | 7.25 | 11.38 | 2.22 | 4.02 | 5.91 | 14.22 | 23.38 |
| | QBP1 | 8.2 | 6.1 | 5.9 | 4.2 | 4.2 | 1.35 | 2.12 | 2.97 | 7.25 | 11.38 | 2.22 | 4.02 | 5.91 | 14.22 | 23.38 |
| | QLB | 7.5 | 6.0 | 5.9 | 4.5 | 5.1 | 1.41 | 2.23 | 3.14 | 7.88 | 12.73 | 2.43 | 4.44 | 6.61 | 16.75 | 29.03 |
| | QLB1 | 5.2 | 5.7 | 5.7 | 4.5 | 5.1 | 1.41 | 2.23 | 3.14 | 7.88 | 12.73 | 2.43 | 4.44 | 6.61 | 16.75 | 29.03 |
| | QDR | 7.8 | 6.5 | 6.5 | 4.8 | 5.3 | 1.50 | 2.36 | 3.28 | 8.06 | 12.92 | 2.62 | 4.75 | 7.00 | 17.33 | 30.17 |
| | QDR1 | 7.5 | 6.5 | 5.9 | 4.5 | 5.1 | 1.48 | 2.33 | 3.24 | 7.89 | 12.86 | 2.55 | 4.60 | 6.80 | 16.92 | 29.53 |
| | QDR2 | 8.5 | 6.9 | 7.3 | 5.1 | 5.4 | 1.61 | 2.48 | 3.45 | 8.28 | 13.11 | 2.56 | 4.73 | 7.12 | 16.99 | 31.20 |
| | QBN1 | 6.0 | 4.7 | 4.3 | 2.6 | 1.8 | 1.04 | 1.98 | 2.93 | 7.35 | 11.57 | 2.33 | 4.01 | 5.53 | 12.87 | 19.36 |
| $\pi_1=0.8$ $\pi_2=0.2$ | | | | | | | | | | | | | | | | |
| | QBP | 7.9 | 5.4 | 5.0 | 3.0 | 2.9 | 1.48 | 2.26 | 3.07 | 7.28 | 11.42 | 2.44 | 4.17 | 5.95 | 14.11 | 23.59 |
| | QBP1 | 9.5 | 6.5 | 5.8 | 4.1 | 4.2 | 1.48 | 2.26 | 3.07 | 7.28 | 11.42 | 2.44 | 4.17 | 5.95 | 14.11 | 23.59 |
| | QLB | 8.5 | 6.4 | 5.8 | 4.4 | 5.0 | 1.54 | 2.37 | 3.24 | 7.91 | 12.77 | 2.66 | 4.61 | 6.64 | 16.60 | 29.27 |
| | QLB1 | 4.8 | 5.1 | 5.3 | 4.3 | 5.0 | 1.54 | 2.37 | 3.24 | 7.91 | 12.77 | 2.66 | 4.61 | 6.64 | 16.60 | 29.27 |
| | QDR | 9.6 | 7.2 | 6.4 | 4.8 | 5.2 | 1.64 | 2.50 | 3.40 | 8.15 | 13.04 | 2.85 | 4.93 | 7.05 | 17.27 | 30.47 |
| | QDR1 | 9.3 | 6.9 | 6.2 | 4.5 | 5.2 | 1.62 | 2.48 | 3.36 | 8.06 | 12.93 | 2.78 | 4.79 | 6.83 | 16.76 | 29.74 |
| | QDR2 | 10.7 | 7.6 | 7.4 | 5.5 | 5.1 | 1.73 | 2.61 | 3.56 | 8.37 | 13.21 | 2.80 | 4.90 | 7.23 | 17.92 | 31.24 |
| | QBN1 | 5.3 | 4.7 | 4.5 | 2.6 | 1.7 | 1.01 | 1.95 | 2.87 | 7.29 | 11.48 | 2.23 | 3.85 | 5.41 | 12.79 | 19.34 |
| $\pi_1=0.8$ $\pi_2=0.5$ | | | | | | | | | | | | | | | | |
| | QBP | 9.9 | 5.8 | 4.8 | 3.3 | 3.0 | 1.69 | 2.44 | 3.20 | 7.32 | 11.45 | 2.86 | 4.32 | 6.02 | 14.43 | 23.66 |
| | QBP1 | 11.9 | 6.7 | 5.9 | 4.5 | 4.8 | 1.69 | 2.44 | 3.20 | 7.32 | 11.45 | 2.86 | 4.32 | 6.02 | 14.43 | 23.66 |
| | QLB | 11.1 | 6.7 | 5.8 | 4.8 | 5.1 | 1.76 | 2.56 | 3.38 | 7.95 | 12.79 | 3.12 | 4.75 | 6.74 | 16.94 | 23.30 |
| | QLB1 | 4.7 | 4.3 | 4.4 | 4.6 | 5.1 | 1.76 | 2.56 | 3.38 | 7.95 | 12.79 | 3.12 | 4.75 | 6.74 | 16.94 | 23.30 |
| | QDR | 11.5 | 7.6 | 6.4 | 5.3 | 5.5 | 1.84 | 2.68 | 3.54 | 8.21 | 13.09 | 3.29 | 5.07 | 7.18 | 17.74 | 30.63 |
| | QDR1 | 11.4 | 7.4 | 6.1 | 4.9 | 5.3 | 1.82 | 2.65 | 3.49 | 8.12 | 12.97 | 3.20 | 4.92 | 6.95 | 17.17 | 29.82 |
| | QDR2 | 12.4 | 8.7 | 8.1 | 5.8 | 5.5 | 1.92 | 2.77 | 3.69 | 8.43 | 13.26 | 3.25 | 5.11 | 7.40 | 18.33 | 31.69 |
| | QBN1 | 4.5 | 3.4 | 3.0 | 2.3 | 1.6 | 0.96 | 1.85 | 2.74 | 6.99 | 10.98 | 1.81 | 3.32 | 3.92 | 12.59 | 19.29 |
| $\pi_1=0.8$ $\pi_2=0.8$ | | | | | | | | | | | | | | | | |
| | QBP | 13.5 | 7.8 | 6.0 | 3.9 | 3.4 | 1.94 | 2.74 | 3.54 | 7.55 | 11.59 | 3.39 | 4.81 | 6.51 | 14.96 | 24.45 |
| | QBP1 | 17.2 | 9.3 | 7.5 | 5.3 | 5.2 | 1.94 | 2.74 | 3.54 | 7.55 | 11.59 | 3.39 | 4.81 | 6.51 | 14.96 | 24.45 |
| | QLB | 16.1 | 8.8 | 7.3 | 5.3 | 5.7 | 2.03 | 2.87 | 3.73 | 8.17 | 12.91 | 3.69 | 5.29 | 7.23 | 17.53 | 30.20 |
| | QLB1 | 4.6 | 3.9 | 4.0 | 5.0 | 5.6 | 2.03 | 2.87 | 3.73 | 8.17 | 12.91 | 3.69 | 5.29 | 7.23 | 17.53 | 30.20 |
| | QDR | 16.0 | 9.5 | 7.9 | 6.1 | 6.0 | 2.08 | 2.96 | 3.86 | 8.46 | 13.28 | 3.85 | 5.59 | 7.70 | 18.62 | 31.84 |
| | QDR1 | 15.5 | 9.1 | 7.3 | 5.6 | 5.8 | 2.05 | 2.92 | 3.81 | 8.36 | 13.14 | 3.72 | 5.42 | 7.47 | 17.96 | 30.98 |
| | QDR2 | 15.5 | 11.1 | 9.2 | 7.4 | 6.2 | 2.13 | 3.02 | 3.98 | 8.65 | 13.44 | 3.83 | 5.67 | 8.06 | 19.36 | 33.54 |
| | QBN1 | 7.6 | 5.2 | 3.9 | 2.3 | 1.2 | 1.30 | 2.11 | 2.87 | 6.41 | 9.75 | 2.32 | 3.74 | 5.09 | 11.70 | 17.80 |

**Table 2.4**
**Empirical Significance Levels, Means and Variances of Portmanteau Tests for the MA(1) Model $w_t=(1-\psi_1 B)a_t$; $\alpha$=5%**

n=50

| Parameters of MA(1) Model | Portmanteau Tests | Empirical Significance Levels (in percent) m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | Empirical Means m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | Empirical Variances m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\psi_1$ = 0.4 | | | | | | | | | | | | | | | | | | | |
| | QBP | 3.6 | 2.8 | 3.0 | 3.5 | 2.8 | 1.9 | 0.97 | 1.80 | 2.62 | 3.44 | 7.27 | 10.67 | 1.49 | 2.92 | 4.58 | 6.10 | 14.30 | 21.42 |
| | QBP1 | 4.3 | 3.8 | 3.8 | 4.3 | 4.7 | 4.1 | 0.97 | 1.80 | 2.62 | 3.44 | 7.27 | 10.67 | 1.49 | 2.92 | 4.58 | 6.10 | 14.30 | 21.42 |
| | QLB | 4.2 | 3.8 | 4.0 | 4.6 | 5.1 | 4.9 | 1.05 | 1.97 | 2.89 | 3.84 | 8.59 | 13.36 | 1.75 | 3.49 | 5.60 | 7.60 | 19.86 | 33.03 |
| | QLB1 | 4.0 | 3.8 | 3.9 | 4.6 | 5.1 | 4.9 | 1.05 | 1.97 | 2.89 | 3.84 | 8.59 | 13.36 | 1.75 | 3.49 | 5.60 | 7.60 | 19.86 | 33.03 |
| | QDR | 5.4 | 4.7 | 4.2 | 5.1 | 5.8 | 5.2 | 1.18 | 2.11 | 3.06 | 4.02 | 8.78 | 13.54 | 2.04 | 3.88 | 6.05 | 8.23 | 21.04 | 35.17 |
| | QDR1 | 5.1 | 3.9 | 4.0 | 4.8 | 6.1 | 5.1 | 1.16 | 2.08 | 3.02 | 3.96 | 8.76 | 13.62 | 1.94 | 3.74 | 5.82 | 7.83 | 20.21 | 34.50 |
| | QDR2 | 6.5 | 5.5 | 5.4 | 5.2 | 6.1 | 6.2 | 1.26 | 2.19 | 3.14 | 4.10 | 8.86 | 13.62 | 2.03 | 3.93 | 5.93 | 8.12 | 20.77 | 36.14 |
| | QBN2 | 5.2 | 5.1 | 6.4 | 5.6 | 6.3 | 4.5 | 1.00 | 1.97 | 2.93 | 3.86 | 8.18 | 11.86 | 1.81 | 4.65 | 7.55 | 11.00 | 28.39 | 44.13 |
| $\psi_1$ = 0.7 | | | | | | | | | | | | | | | | | | | |
| | QBP | 5.4 | 4.1 | 4.1 | 4.4 | 3.5 | 2.7 | 1.23 | 2.02 | 2.83 | 3.62 | 7.44 | 10.85 | 2.06 | 3.77 | 5.70 | 7.38 | 16.42 | 25.16 |
| | QBP1 | 6.2 | 5.1 | 5.3 | 5.4 | 5.5 | 5.1 | 1.23 | 2.02 | 2.93 | 3.62 | 7.44 | 10.85 | 2.06 | 3.77 | 5.70 | 7.38 | 16.42 | 25.16 |
| | QLB | 6.0 | 5.0 | 5.6 | 5.5 | 5.8 | 5.8 | 1.32 | 2.20 | 3.11 | 4.02 | 8.75 | 13.55 | 2.38 | 4.46 | 6.90 | 9.10 | 22.57 | 38.44 |
| | QLB1 | 4.9 | 4.8 | 5.5 | 5.4 | 5.8 | 5.8 | 1.32 | 2.20 | 3.11 | 4.02 | 8.75 | 13.55 | 2.38 | 4.46 | 6.90 | 9.10 | 22.57 | 38.44 |
| | QDR | 8.2 | 6.1 | 6.2 | 6.1 | 6.5 | 6.6 | 1.48 | 2.38 | 3.33 | 4.27 | 9.08 | 13.86 | 2.67 | 4.77 | 7.19 | 9.55 | 23.64 | 39.31 |
| | QDR1 | 7.5 | 5.7 | 5.6 | 5.7 | 6.2 | 6.0 | 1.45 | 2.34 | 3.27 | 4.18 | 8.96 | 13.83 | 2.55 | 4.54 | 6.85 | 9.00 | 22.13 | 37.51 |
| | QDR2 | 8.2 | 7.3 | 7.0 | 6.7 | 7.4 | 7.1 | 1.52 | 2.44 | 3.40 | 4.32 | 9.12 | 13.91 | 2.65 | 4.75 | 6.92 | 9.20 | 22.98 | 39.13 |
| | QBN2 | 5.9 | 6.6 | 7.4 | 7.8 | 9.1 | 6.9 | 1.06 | 2.10 | 3.12 | 4.11 | 8.70 | 12.58 | 2.11 | 5.65 | 9.25 | 13.44 | 34.69 | 54.14 |
| $\psi_1$ = 0.9 | | | | | | | | | | | | | | | | | | | |
| | QBP | 8.2 | 6.3 | 5.8 | 5.1 | 4.6 | 3.8 | 1.57 | 2.42 | 3.25 | 4.08 | 8.01 | 11.54 | 2.53 | 4.42 | 6.51 | 8.54 | 20.81 | 31.14 |
| | QBP1 | 10.7 | 8.6 | 7.4 | 7.0 | 7.1 | 7.2 | 1.57 | 2.42 | 3.25 | 4.08 | 8.01 | 11.54 | 2.53 | 4.42 | 6.51 | 8.54 | 20.81 | 31.14 |
| | QLB | 10.2 | 8.5 | 7.4 | 7.5 | 8.0 | 7.5 | 1.69 | 2.63 | 3.57 | 4.52 | 9.39 | 14.35 | 2.91 | 5.21 | 7.84 | 10.50 | 28.65 | 47.39 |
| | QLB1 | 5.0 | 5.6 | 5.9 | 6.2 | 7.7 | 7.5 | 1.69 | 2.63 | 3.57 | 4.52 | 9.39 | 14.35 | 2.91 | 5.21 | 7.84 | 10.50 | 28.65 | 47.39 |
| | QDR | 11.7 | 8.6 | 7.5 | 7.3 | 8.5 | 7.8 | 1.80 | 2.75 | 3.70 | 4.66 | 9.53 | 14.42 | 3.23 | 5.55 | 8.09 | 10.88 | 26.83 | 43.21 |
| | QDR1 | 11.4 | 8.5 | 7.3 | 7.0 | 7.7 | 7.6 | 1.79 | 2.72 | 3.64 | 4.58 | 9.40 | 14.30 | 3.12 | 5.32 | 7.62 | 10.21 | 25.10 | 40.90 |
| | QDR2 | 11.5 | 9.1 | 8.2 | 7.9 | 8.6 | 9.1 | 1.80 | 2.76 | 3.71 | 4.67 | 9.52 | 14.43 | 3.24 | 5.61 | 7.89 | 10.71 | 25.77 | 42.85 |
| | QBN2 | 6.3 | 7.3 | 8.0 | 8.2 | 9.5 | 7.3 | 1.08 | 2.14 | 3.17 | 4.18 | 8.83 | 12.76 | 2.22 | 5.93 | 9.69 | 14.03 | 36.17 | 56.42 |
| $\psi_1$ = 0.99 | | | | | | | | | | | | | | | | | | | |
| | QBP | 11.6 | 9.5 | 7.1 | 6.6 | 5.8 | 4.1 | 1.84 | 2.75 | 3.59 | 4.45 | 8.51 | 12.08 | 3.27 | 5.44 | 7.50 | 9.88 | 22.66 | 33.20 |
| | QBP1 | 14.8 | 10.8 | 9.0 | 8.3 | 8.6 | 8.7 | 1.84 | 2.76 | 3.59 | 4.45 | 8.51 | 12.08 | 3.27 | 5.44 | 7.50 | 9.88 | 22.66 | 33.20 |
| | QLB | 14.3 | 11.0 | 9.3 | 8.6 | 9.7 | 9.8 | 1.98 | 2.99 | 3.93 | 4.92 | 9.95 | 14.97 | 3.77 | 6.40 | 8.98 | 12.10 | 30.94 | 50.01 |
| | QLB1 | 6.4 | 7.0 | 6.0 | 6.6 | 8.5 | 9.1 | 1.98 | 2.99 | 3.93 | 4.92 | 9.95 | 14.97 | 3.77 | 6.40 | 8.98 | 12.10 | 30.94 | 50.01 |
| | QDR | 13.9 | 9.9 | 8.1 | 8.9 | 9.2 | 9.2 | 1.93 | 2.89 | 3.85 | 4.82 | 9.76 | 14.73 | 3.48 | 5.88 | 8.46 | 11.39 | 28.13 | 45.16 |
| | QDR1 | 13.0 | 9.7 | 7.9 | 7.4 | 8.5 | 8.3 | 1.91 | 2.87 | 3.80 | 4.75 | 9.63 | 14.57 | 3.38 | 5.64 | 7.97 | 10.67 | 26.26 | 42.14 |
| | QDR2 | 13.4 | 10.4 | 9.5 | 8.8 | 9.0 | 10.2 | 1.91 | 2.88 | 3.85 | 4.83 | 9.75 | 14.73 | 3.48 | 5.91 | 8.30 | 11.25 | 26.62 | 44.57 |
| | QBN2 | 6.6 | 7.3 | 8.2 | 8.2 | 9.5 | 7.3 | 1.09 | 2.15 | 3.18 | 4.19 | 8.85 | 12.79 | 2.24 | 5.97 | 9.75 | 14.11 | 36.34 | 56.67 |

with $\psi_1 \geq 0.7$.

(ii) Except when m is very small or the parameter values are large, the empirical levels (or estimated sizes) of the modified Box-Pierce test, QBP1, are generally close to the values predicted by asymptotic theory in all cases considered. This seems to suggest that using (2.2.5) to set critical values for QBP can improve the small-sample distribution of the test. Nevertheless, it is found that in the AR(2) case, QBP1 can still yield low empirical values when n=50, $\pi_2 \leq 0.5$ and $m \geq 10$.

(iii) The results of the Ljung-Box statistics, QLB, shown in Table 2.2 are comparable to those of Kwan and Sim (1988, pp. 343-344) when the data are fitted to an AR(1) model. With the exception of cases when the parameter value ($\pi_1$) is greater than 0.9 or m is small, the significance levels are fairly close to the nominal levels. We can draw similar conclusions for the MA(1) model. However, in both the AR(1) and MA(1) models, the estimated variances of QLB are appreciably greater than the theoretical variance 2(m-1) for $m \geq 10$. This is most apparent in the MA case. As for the AR(2) model, the entries for the $\chi^2(m-2)$ distribution are generally close to the nominal levels especially when $m \geq 4$ and $\pi_2 \leq 0.5$. For $\pi_2 = 0.8$, a larger m (at least 10) is needed in order to use QLB.

(iv) The empirical performance of the Dufour-Roy statistics

(QDR, QDR1 and QDR2) is quite similar to that of QLB when the parameter values are from small to moderate (e.g., $\pi_1 \le 0.7$). For parameter values which are close to the unit circle, QLB dominates the Dufour-Roy statistics in terms of controlling the empirical level. This can be seen from Tables 2.2-2.3 where QDR, QDR1 and QDR2 always have estimates of size which are higher than those of QLB when $m \le 4$ and the data are fitted to the AR(1) and AR(2) models, respectively. Also, it is found that the dispersion bias seems to be more serious for the Dufour-Roy statistics, especially in the case where both the parameter values and m are large. However, these observations cannot be generalized to the MA(1) model. When $\psi_1 = 0.99$, QDR, QDR1 and QDR2 have significance levels which are slightly closer to the nominal levels than those for QLB. Moreover, their empirical variances are less sensitive to the boundary values (i.e. $\psi_1 \ge 0.9$), even though they are still considerably larger than the theoretical value 2(m-1).

As regards the individual performance among the Dufour-Roy statistics, the results suggest that QDR always has empirical significance levels which are closer to the nominal levels than those for QDR1. This is most evident in the case where $m \le 10$. The nonparametric portmanteau statistic, QDR2, performs marginally less well than do the parametric ones, namely it always has relatively large estimated sizes for n=50. However, when the sample size increases to 100, there seems to be no noticeable difference among these tests.

(v) The Bera-Newbold tests, QBN1 and QBN2, have empirical significance levels which agree with the theoretical levels only when m is small. In cases where the data are fitted to the AR models, the simulation results indicate that the value of m cannot be chosen to be more than 3 and 5 when n=50 and 100, respectively. For larger m, the empirical levels are substantially lower than the nominal levels. Also, the size of QBN1 can be affected by the parameter value; namely it is too low for $\pi_1 \le 0.9$ and $m \ge 4$. When the AR(1) model is near a random walk (i.e., $\pi_1 = 0.99$), the size is even worse. As for the estimated means and variances, these are significantly smaller than the theoretical values for $n=50$. Moreover, there is no substantial improvement when n increases to 100.

The results of the MA(1) model are in contrast to those for the AR models. It is found that with the exception of $\psi_1 = 0.4$ or m=15, the test size is often larger than the nominal levels, ranging from 5.2% to 9.5% In addition, the empirical variances of QBN2 are substantially greater than 2(m-1) in almost all cases. In work not reported here, the overall quality of the results of QBN2 does not improve even when the sample size increases from 50 to 100.

(vi) The modified Ljung-Box statistic, QLB1, yields the best results. It can be seen from Tables 2.2-2.4 that the estimated significance levels of QLB1 are more in agreement with the nominal levels than those for the aforementioned portmanteau tests (QBP, QBP1, QLB, QDR, QDR1, QDR2, QBN1 and QBN2) when $m \le 10$. In order to highlight this feature, consider examples of the following ranges of estimated significance levels:

Example 1: AR(1) model fitted; $\pi_1=0.99$; $n=50$ and $2 \le m \le 10$

QBP: 3.5 to 11.5%
QBP1: 5.0 to 13.8%
QLB: 5.3 to 12.8%
QLB1: 4.3 to 5.8%
QDR: 7.4 to 13.9%
QDR1: 7.0 to 14.0%
QDR2: 7.2 to 14.0%
QBN1: 0.7 to 2.5%

Example 2: AR(2) model fitted; $\pi_1=0.8$ and $\pi_2=0.8$; $n=50$ and $2 \le m \le 10$

QBP: 1.9 to 11.8%
QBP1: 4.1 to 16.7%
QLB: 4.3 to 13.5%
QLB1: 4.0 to 4.6%
QDR: 4.5 to 14.0%
QDR1: 4.7 to 14.7%
QDR2: 6.1 to 15.4%
QBN1: 0.4 to 4.4%

Example 3: MA(1) model fitted; $\psi_1=0.99$; $n=50$ and $2 \le m \le 10$

QBP: 5.8 to 11.6%
QBP1: 8.6 to 14.8%
QLB: 6.0 to 14.3%
QLB1: 6.0 to 8.5%
QDR: 7.4 to 13.0%
QDR1: 8.1 to 13.9%

QDR2: 8.8 to 13.4%

QBN2: 6.6 to 9.5%

These three examples clearly illustrate that, with the exception of QLB1, asymptotic theory does not work well for portmanteau tests when the parameter values are large. In many cases, the true size of these portmanteau tests can be more than double the theoretical level. This is indeed discouraging especially since most economic time series are believed to be highly correlated with parameter values close to the unit circle. QLB1, on the other hand, does not exhibit excessively large estimates of size. This performance, along with its empirical power which will be shown below, favour the use of QLB1 in finite samples.

As a minor note on the finite-sample distribution of QLB1, its size estimates are very similar to those for the Ljung-Box test QLB, when m=15. This confirms Ljung's (1986, p. 726) assertion that for large m, the scale parameter, a, and the degree of freedom, b, approach 1 and (m-p-q), respectively.

(vii) It should be noted that test results (size and power) for 10% were also calculated. These simulation results are qualitatively similar to those at the 5% level but are not shown.

Tables 2.5-2.7 present a summary of the simulation results on the empirical power of eight portmanteau tests in cases where the data were generated from low-order ARMA models. Several points are clearly noticed from the reported findings:

(i) With the exception of the results on QBN1 and QBN2, the empirical power of the other portmanteau tests (QBP, QBP1, QLB, QLB1, QDR, QDR1 and QDR2) is much higher for smaller numbers of residual autocorrelations. In all cases considered, the optimal values of m are 2 and 3 for AR(1) and AR(2) models, respectively. This observation holds regardless of the sample size and the chosen values of parameters. This finding strongly supports Ljung's (1986) assertion that when the data-generating process is modelled as a simple low-order ARMA model, portmanteau tests can be quite powerful in detecting model misspecification as long as the value of m is kept small. However, such a statement cannot be generalized to the Bera-Newbold tests, QBN1 and QBN2. It can be seen from the results reported in these tables that in several cases, a slightly larger m is needed in order to obtain "maximum" power for QBN1 and QBN2.

(ii) In terms of the ability to detect derivations from model specification there seems to be no marked differences amongst QLB, QDR and QDR1. These results are not entirely unexpected since the variance formulae for them are $O(n^{-1})$. When the sample size ranges from moderate to large and the underlying time series model is relatively simple, the empirical power of these tests should be almost the same.

Table 2.5
Empirical Powers of Portmanteau Tests; AR(1) Model fitted; Data generated from the model $(1-\pi_1 B)(1-\pi_2 B)w_t = (1-\psi_1 B)(1-\psi_2 B)a_t$; $\alpha = 5\%$

| Parameters | | | | Portmanteau tests | n=50 (in percent) | | | | | | n=100 (in percent) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
| 0 | 0 | 0.4 | 0 | QBP | 15.0 | 9.6 | 8.3 | 6.8 | 4.4 | 3.6 | 28.0 | 21.2 | 15.8 | 14.9 | 10.1 | 8.0 |
| | | | | QBP1 | 17.4 | 10.9 | 9.7 | 9.2 | 7.5 | 7.3 | 30.5 | 22.6 | 18.2 | 16.8 | 13.2 | 11.4 |
| | | | | QLB | 17.0 | 11.1 | 9.8 | 9.3 | 8.3 | 8.3 | 30.3 | 22.7 | 18.3 | 17.0 | 13.9 | 11.9 |
| | | | | QLB1 | 17.0 | 11.1 | 9.8 | 9.3 | 8.3 | 8.3 | 30.0 | 22.7 | 18.3 | 17.0 | 13.9 | 11.9 |
| | | | | QDR | 16.0 | 10.8 | 9.3 | 8.8 | 8.5 | 7.5 | 30.5 | 21.9 | 17.4 | 16.1 | 12.9 | 11.2 |
| | | | | QDR1 | 15.7 | 10.2 | 9.6 | 9.2 | 8.9 | 8.5 | 30.3 | 21.6 | 17.1 | 16.6 | 13.9 | 12.0 |
| | | | | QDR2 | 14.6 | 10.4 | 9.1 | 8.3 | 8.7 | 8.7 | 28.8 | 20.1 | 17.0 | 14.1 | 13.6 | 12.8 |
| | | | | QBN1 | 17.4 | 12.9 | 12.5 | 9.7 | 5.3 | 2.3 | 30.3 | 25.9 | 21.3 | 19.0 | 10.3 | 6.2 |
| 0 | 0 | 0.7 | 0 | QBP | 62.9 | 44.2 | 33.9 | 28.8 | 17.5 | 12.5 | 93.8 | 87.5 | 81.9 | 74.0 | 52.8 | 41.5 |
| | | | | QBP1 | 68.5 | 50.3 | 39.6 | 35.0 | 26.3 | 24.4 | 94.6 | 88.7 | 83.4 | 77.5 | 59.0 | 51.1 |
| | | | | QLB | 67.6 | 50.5 | 40.0 | 35.5 | 27.4 | 25.8 | 94.6 | 88.7 | 83.5 | 77.4 | 58.7 | 51.0 |
| | | | | QLB1 | 66.8 | 50.1 | 40.0 | 35.5 | 27.4 | 25.8 | 94.2 | 88.6 | 83.5 | 77.4 | 58.7 | 51.0 |
| | | | | QDR | 60.3 | 44.2 | 35.1 | 30.5 | 23.8 | 22.3 | 93.6 | 86.7 | 81.0 | 73.7 | 55.1 | 47.5 |
| | | | | QDR1 | 58.5 | 42.6 | 34.2 | 31.5 | 25.0 | 24.4 | 93.5 | 86.2 | 80.0 | 70.2 | 54.6 | 48.4 |
| | | | | QDR2 | 54.6 | 38.4 | 30.7 | 28.5 | 22.6 | 23.5 | 90.8 | 80.6 | 72.8 | 66.0 | 50.9 | 44.0 |
| | | | | QBN1 | 52.6 | 54.4 | 53.6 | 48.3 | 28.8 | 15.7 | 84.7 | 90.6 | 90.4 | 89.4 | 76.1 | 62.4 |
| 0 | 0 | 0.9 | 0 | QBP | 83.0 | 68.0 | 54.2 | 44.8 | 27.8 | 19.9 | 99.0 | 98.7 | 97.7 | 95.8 | 77.5 | 65.4 |
| | | | | QBP1 | 85.5 | 74.3 | 61.9 | 53.0 | 40.3 | 35.9 | 99.2 | 98.9 | 98.1 | 96.6 | 83.3 | 75.2 |
| | | | | QLB | 85.1 | 74.4 | 61.7 | 53.0 | 41.0 | 35.9 | 99.1 | 98.9 | 98.1 | 96.6 | 82.6 | 73.3 |
| | | | | QLB1 | 84.3 | 74.0 | 61.5 | 53.0 | 41.0 | 35.9 | 99.1 | 98.9 | 98.1 | 96.6 | 82.6 | 73.3 |
| | | | | QDR | 81.6 | 65.9 | 52.8 | 45.4 | 34.9 | 31.2 | 98.7 | 98.5 | 97.3 | 94.5 | 78.9 | 70.1 |
| | | | | QDR1 | 80.4 | 63.9 | 50.1 | 45.3 | 36.4 | 33.2 | 98.9 | 98.3 | 97.1 | 93.0 | 78.5 | 70.2 |
| | | | | QDR2 | 72.3 | 55.9 | 43.3 | 40.3 | 31.5 | 30.4 | 98.7 | 96.5 | 93.8 | 89.4 | 71.9 | 62.3 |
| | | | | QBN1 | 67.7 | 74.6 | 76.6 | 75.1 | 62.5 | 41.4 | 93.2 | 98.8 | 99.2 | 98.8 | 98.6 | 97.1 |
| 0 | 0 | 0.5 | 0.5 | QBP | 91.4 | 78.7 | 67.3 | 59.1 | 38.9 | 29.1 | 99.9 | 99.5 | 99.0 | 98.2 | 88.4 | 77.8 |
| | | | | QBP1 | 93.5 | 82.9 | 74.0 | 66.8 | 51.2 | 48.8 | 99.9 | 99.6 | 99.3 | 98.5 | 91.3 | 85.1 |
| | | | | QLB | 93.1 | 82.8 | 73.6 | 66.5 | 51.2 | 47.8 | 99.9 | 99.6 | 99.3 | 98.5 | 91.1 | 94.2 |
| | | | | QLB1 | 83.8 | 82.1 | 73.3 | 66.5 | 51.2 | 47.8 | 99.9 | 99.5 | 99.3 | 98.5 | 91.1 | 94.2 |
| | | | | QDR | 91.0 | 78.1 | 67.9 | 60.4 | 45.6 | 41.9 | 99.9 | 99.5 | 98.9 | 98.1 | 98.1 | 81.1 |
| | | | | QDR1 | 89.9 | 77.2 | 67.1 | 61.3 | 47.9 | 45.7 | 99.9 | 99.5 | 98.7 | 97.8 | 89.4 | 81.9 |
| | | | | QDR2 | 84.5 | 68.7 | 58.8 | 53.3 | 41.3 | 40.4 | 99.7 | 98.8 | 96.8 | 94.7 | 83.6 | 74.8 |
| | | | | QBN1 | 91.8 | 87.3 | 85.3 | 80.7 | 56.9 | 37.2 | 98.8 | 99.7 | 99.7 | 99.5 | 97.7 | 93.0 |
| 0 | 0 | 0.8 | 0.5 | QBP | 99.3 | 96.0 | 90.4 | 85.7 | 60.8 | 48.1 | 100 | 100 | 100 | 100 | 98.8 | 96.3 |
| | | | | QBP1 | 99.7 | 97.1 | 92.7 | 90.0 | 73.3 | 67.3 | 100 | 100 | 100 | 100 | 99.7 | 98.2 |
| | | | | QLB | 99.7 | 97.0 | 92.5 | 89.6 | 71.9 | 66.0 | 100 | 100 | 100 | 100 | 99.2 | 98.0 |
| | | | | QLB1 | 99.2 | 96.8 | 92.4 | 89.6 | 71.9 | 66.0 | 100 | 100 | 100 | 100 | 99.2 | 98.0 |
| | | | | QDR | 99.0 | 95.2 | 89.1 | 85.8 | 65.6 | 60.7 | 100 | 100 | 100 | 100 | 98.8 | 97.0 |
| | | | | QDR1 | 98.9 | 94.0 | 87.5 | 85.3 | 67.1 | 63.3 | 100 | 100 | 100 | 100 | 98.8 | 97.4 |
| | | | | QDR2 | 96.6 | 89.1 | 80.2 | 78.5 | 58.2 | 56.8 | 100 | 100 | 100 | 99.9 | 97.7 | 93.5 |
| | | | | QBN1 | 94.2 | 97.1 | 97.7 | 97.2 | 91.2 | 80.8 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| 0 | 0 | 0.6 | 0.5 | QBP | 100 | 99.4 | 96.7 | 94.3 | 73.4 | 60.3 | 100 | 100 | 100 | 100 | 100 | 99.5 |
| | | | | QBP1 | 100 | 99.5 | 98.5 | 96.7 | 84.8 | 76.7 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QLB | 100 | 99.6 | 98.3 | 96.6 | 82.7 | 76.3 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QLB1 | 100 | 99.5 | 98.2 | 96.4 | 82.7 | 76.3 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QDR | 100 | 98.9 | 95.9 | 93.8 | 77.6 | 70.7 | 100 | 100 | 100 | 100 | 100 | 99.8 |
| | | | | QDR1 | 100 | 98.7 | 94.9 | 93.8 | 78.8 | 73.4 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QDR2 | 99.7 | 95.8 | 89.6 | 87.9 | 71.3 | 66.1 | 100 | 100 | 100 | 100 | 99.7 | 99.0 |
| | | | | QBN1 | 97.2 | 99.7 | 99.6 | 99.1 | 95.6 | 89.8 | 100 | 100 | 100 | 100 | 100 | 100 |

Table 2.5 (cont'd)

| Parameters $\pi_1$ | $\pi_2$ | $\phi_1$ | $\phi_2$ | Portmanteau tests | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.5 | 0.5 | 0 | 0 | QBP | 33.9 | 24.4 | 22.2 | 22.6 | 13.9 | 11.5 | 63.3 | 53.8 | 49.3 | 45.7 | 37.9 | 30.1 |
| | | | | QBP1 | 38.4 | 28.4 | 26.5 | 26.1 | 22.5 | 21.1 | 65.4 | 57.0 | 52.3 | 49.7 | 41.9 | 36.9 |
| | | | | QLB | 37.3 | 28.4 | 26.7 | 26.4 | 23.3 | 22.4 | 64.7 | 56.8 | 52.3 | 49.2 | 42.3 | 37.3 |
| | | | | QLB1 | 29.0 | 25.5 | 25.2 | 25.5 | 23.2 | 22.2 | 57.1 | 53.4 | 50.3 | 48.5 | 42.3 | 37.3 |
| | | | | QDR | 40.5 | 30.0 | 27.1 | 26.2 | 23.4 | 21.8 | 66.0 | 56.6 | 52.5 | 48.9 | 42.0 | 36.5 |
| | | | | QDR1 | 41.1 | 31.2 | 28.3 | 27.7 | 24.2 | 23.1 | 66.5 | 57.9 | 53.1 | 50.0 | 41.9 | 37.9 |
| | | | | QDR2 | 38.1 | 30.9 | 27.9 | 28.5 | 22.1 | 22.3 | 64.4 | 53.8 | 49.3 | 46.4 | 38.9 | 34.9 |
| | | | | QBN1 | 44.8 | 34.7 | 29.6 | 22.6 | 9.3 | 3.7 | 73.8 | 63.7 | 56.4 | 49.6 | 32.1 | 22.0 |
| 0.8 | 0.5 | 0 | 0 | QBP | 78.9 | 65.5 | 58.2 | 53.6 | 41.2 | 34.7 | 97.7 | 95.1 | 93.1 | 89.9 | 78.0 | 73.4 |
| | | | | QBP1 | 81.4 | 70.5 | 63.7 | 59.9 | 51.7 | 48.6 | 98.0 | 95.4 | 93.9 | 91.5 | 82.0 | 78.8 |
| | | | | QLB | 80.7 | 69.8 | 63.0 | 59.0 | 51.6 | 48.8 | 97.7 | 95.3 | 93.9 | 91.5 | 81.1 | 77.3 |
| | | | | QLB1 | 69.4 | 62.7 | 56.8 | 54.8 | 49.7 | 47.7 | 95.1 | 93.7 | 92.0 | 89.0 | 80.5 | 76.9 |
| | | | | QDR | 82.9 | 73.5 | 66.0 | 61.7 | 52.2 | 48.9 | 98.1 | 95.7 | 94.2 | 92.3 | 81.1 | 77.4 |
| | | | | QDR1 | 82.8 | 73.3 | 66.3 | 62.8 | 52.6 | 49.5 | 98.1 | 95.6 | 94.2 | 92.7 | 81.9 | 78.5 |
| | | | | QDR2 | 79.5 | 68.7 | 60.3 | 57.0 | 49.6 | 46.2 | 96.9 | 93.8 | 92.5 | 88.5 | 80.0 | 75.0 |
| | | | | QBN1 | 72.7 | 61.6 | 53.3 | 45.9 | 24.0 | 12.8 | 96.3 | 93.6 | 90.0 | 86.8 | 71.3 | 57.5 |
| 0.8 | 0.8 | 0 | 0 | QBP | 99.5 | 99.1 | 98.4 | 97.0 | 90.9 | 84.4 | 100 | 100 | 100 | 100 | 100 | 99.7 |
| | | | | QBP1 | 99.7 | 99.1 | 98.6 | 97.9 | 94.3 | 91.4 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QLB | 99.5 | 99.1 | 98.6 | 97.5 | 93.7 | 89.6 | 100 | 100 | 100 | 100 | 100 | 99.8 |
| | | | | QLB1 | 98.9 | 98.2 | 97.2 | 95.8 | 92.0 | 87.9 | 100 | 100 | 100 | 100 | 100 | 99.7 |
| | | | | QDR | 99.8 | 99.1 | 98.7 | 98.4 | 94.7 | 93.2 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QDR1 | 99.7 | 99.1 | 98.7 | 98.4 | 94.4 | 91.5 | 100 | 100 | 100 | 100 | 100 | 99.9 |
| | | | | QDR2 | 99.7 | 98.8 | 98.4 | 96.6 | 92.2 | 88.3 | 100 | 100 | 99.9 | 99.9 | 100 | 99.7 |
| | | | | QBN1 | 85.0 | 79.5 | 72.4 | 66.8 | 43.2 | 28.2 | 99.3 | 98.8 | 98.0 | 97.0 | 91.7 | 85.0 |
| 0.8 | 0 | 0.5 | 0 | QBP | 13.8 | 9.6 | 9.2 | 8.7 | 5.7 | 4.3 | 29.2 | 28.2 | 26.8 | 25.4 | 17.3 | 13.8 |
| | | | | QBP1 | 15.8 | 13.2 | 11.4 | 11.1 | 10.1 | 9.4 | 30.5 | 29.7 | 29.3 | 27.3 | 21.4 | 20.7 |
| | | | | QLB | 15.3 | 13.3 | 11.8 | 11.7 | 11.3 | 10.9 | 30.3 | 29.8 | 29.6 | 27.5 | 21.7 | 21.2 |
| | | | | QLB1 | 15.0 | 13.3 | 11.8 | 11.7 | 11.3 | 10.9 | 30.1 | 29.8 | 29.6 | 27.5 | 21.7 | 21.2 |
| | | | | QDR | 18.1 | 14.6 | 12.2 | 12.1 | 12.4 | 11.6 | 31.4 | 30.3 | 30.5 | 27.4 | 21.0 | 20.2 |
| | | | | QDR1 | 18.9 | 15.5 | 12.6 | 12.7 | 10.6 | 9.7 | 31.9 | 30.7 | 30.5 | 27.0 | 19.8 | 18.9 |
| | | | | QDR2 | 18.2 | 13.9 | 12.0 | 11.5 | 9.8 | 9.9 | 30.1 | 28.4 | 27.3 | 27.2 | 19.4 | 16.9 |
| | | | | QBN1 | 10.8 | 8.2 | 6.3 | 5.2 | 2.5 | 1.2 | 33.1 | 29.3 | 23.9 | 21.2 | 12.8 | 7.7 |
| 0.5 | 0 | 0.8 | 0 | QBP | 22.2 | 16.0 | 11.4 | 9.1 | 5.9 | 5.0 | 44.7 | 40.6 | 33.9 | 27.5 | 19.0 | 14.2 |
| | | | | QBP1 | 25.3 | 19.8 | 14.6 | 13.4 | 11.2 | 10.8 | 46.1 | 44.0 | 36.5 | 32.1 | 23.5 | 20.1 |
| | | | | QLB | 25.0 | 20.2 | 15.3 | 14.1 | 12.4 | 12.1 | 46.1 | 44.1 | 36.9 | 32.2 | 24.4 | 20.5 |
| | | | | QLB1 | 25.0 | 20.2 | 15.3 | 14.1 | 12.1 | 12.1 | 46.1 | 44.1 | 36.9 | 32.2 | 24.4 | 20.5 |
| | | | | QDR | 21.2 | 16.8 | 12.6 | 11.6 | 10.1 | 10.8 | 42.5 | 39.9 | 33.5 | 27.9 | 22.2 | 19.0 |
| | | | | QDR1 | 20.9 | 15.9 | 12.2 | 11.6 | 10.8 | 11.6 | 42.2 | 38.6 | 32.2 | 27.0 | 22.1 | 19.3 |
| | | | | QDR2 | 20.5 | 15.5 | 12.1 | 10.4 | 9.8 | 11.8 | 38.2 | 35.4 | 30.1 | 25.1 | 20.7 | 18.8 |
| | | | | QBN1 | 20.9 | 18.9 | 19.3 | 16.7 | 10.0 | 5.0 | 37.5 | 43.6 | 43.9 | 44.8 | 33.2 | 21.7 |
| 0.2 | 0 | 0.8 | 0 | QBP | 57.8 | 42.9 | 30.1 | 24.7 | 16.7 | 11.0 | 90.5 | 86.7 | 80.6 | 71.7 | 49.9 | 38.3 |
| | | | | QBP1 | 61.1 | 47.9 | 36.4 | 32.2 | 23.6 | 22.3 | 91.1 | 88.0 | 83.1 | 75.2 | 56.4 | 49.1 |
| | | | | QLB | 60.7 | 48.3 | 37.2 | 32.9 | 24.9 | 24.1 | 91.1 | 87.9 | 83.3 | 75.3 | 56.1 | 48.5 |
| | | | | QLB1 | 60.1 | 48.2 | 37.2 | 32.9 | 24.9 | 24.1 | 91.0 | 87.9 | 83.3 | 75.3 | 56.1 | 48.5 |
| | | | | QDR | 54.2 | 41.3 | 30.8 | 26.4 | 20.8 | 20.4 | 89.2 | 84.7 | 79.0 | 70.5 | 52.3 | 44.2 |
| | | | | QDR1 | 53.0 | 39.1 | 29.5 | 26.6 | 21.6 | 22.3 | 88.9 | 84.1 | 76.5 | 68.2 | 51.5 | 44.7 |
| | | | | QDR2 | 54.2 | 41.3 | 30.8 | 26.4 | 20.8 | 20.4 | 89.2 | 84.7 | 79.0 | 70.5 | 52.3 | 44.2 |
| | | | | QBN1 | 45.0 | 50.7 | 50.1 | 46.5 | 30.9 | 16.7 | 76.6 | 86.9 | 89.7 | 89.3 | 81.0 | 68.2 |

Table 2.6
Empirical Powers of Portmanteau Tests, AR(2) Model fitted, Data generated from the model $(1-\pi_1 B)(1-\pi_2 B)w_t = (1-\psi_1 B)(1-\psi_2 B)a_t$, $\alpha = 5\%$

| Parameters | | | | Portmanteau tests | n=50 (in percent) m=3 | m=4 | m=5 | m=10 | m=15 | n=100 (in percent) m=3 | m=4 | m=5 | m=10 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | | | | | | | | | | | |
| 0 | 0 | 0.4 | 0 | QBP | 4.4 | 3.7 | 3.1 | 1.3 | 1.3 | 7.7 | 6.4 | 4.9 | 2.7 | 3.1 |
| | | | | QBP1 | 6.8 | 5.4 | 4.7 | 3.0 | 3.0 | 9.5 | 8.0 | 5.7 | 4.0 | 5.0 |
| | | | | QLB | 5.4 | 5.3 | 4.9 | 3.4 | 3.6 | 8.8 | 7.9 | 5.6 | 4.4 | 5.3 |
| | | | | QLB1 | 5.3 | 5.2 | 4.9 | 3.4 | 3.6 | 8.8 | 7.8 | 5.6 | 4.4 | 5.3 |
| | | | | QDR | 5.5 | 4.6 | 4.6 | 3.5 | 3.8 | 8.4 | 7.1 | 5.4 | 3.8 | 5.5 |
| | | | | QDR1 | 4.4 | 4.2 | 4.7 | 3.6 | 3.5 | 7.9 | 7.2 | 5.5 | 4.2 | 5.4 |
| | | | | QDR2 | 5.5 | 4.5 | 4.4 | 3.8 | 4.7 | 8.1 | 6.5 | 6.5 | 4.8 | 4.7 |
| | | | | QBN1 | 4.9 | 5.0 | 5.0 | 2.5 | 0.7 | 7.9 | 8.1 | 7.4 | 3.7 | 2.5 |
| 0 | 0 | 0.7 | 0 | QBP | 25.0 | 16.1 | 11.4 | 6.0 | 4.0 | 62.6 | 47.5 | 37.6 | 21.1 | 16.1 |
| | | | | QBP1 | 34.4 | 21.0 | 16.8 | 10.0 | 10.0 | 66.7 | 51.9 | 42.5 | 26.8 | 21.9 |
| | | | | QLB | 29.7 | 19.4 | 16.3 | 11.0 | 11.2 | 64.9 | 50.6 | 42.1 | 26.9 | 22.5 |
| | | | | QLB1 | 28.3 | 19.2 | 16.2 | 11.0 | 11.2 | 64.0 | 50.3 | 41.9 | 26.9 | 22.5 |
| | | | | QDR | 25.7 | 16.9 | 14.6 | 9.0 | 9.8 | 61.5 | 47.8 | 38.4 | 24.5 | 21.0 |
| | | | | QDR1 | 22.8 | 15.2 | 13.9 | 9.4 | 10.9 | 59.9 | 45.9 | 37.1 | 24.2 | 21.9 |
| | | | | QDR2 | 23.3 | 15.4 | 14.1 | 10.4 | 11.1 | 57.3 | 43.4 | 36.4 | 24.3 | 19.9 |
| | | | | QBN1 | 18.1 | 20.4 | 17.2 | 9.8 | 4.5 | 45.2 | 49.0 | 47.5 | 33.0 | 21.0 |
| 0 | 0 | 0.9 | 0 | QBP | 57.1 | 35.9 | 26.2 | 12.1 | 8.8 | 94.0 | 87.5 | 81.3 | 49.4 | 36.7 |
| | | | | QBP1 | 69.0 | 46.1 | 37.0 | 22.9 | 18.6 | 94.9 | 89.9 | 85.4 | 58.7 | 49.2 |
| | | | | QLD | 64.0 | 44.9 | 36.1 | 23.2 | 20.4 | 94.2 | 89.4 | 84.9 | 57.8 | 48.2 |
| | | | | QLB1 | 60.7 | 43.8 | 26.0 | 23.2 | 20.4 | 93.9 | 89.3 | 84.6 | 57.7 | 48.2 |
| | | | | QDR | 54.2 | 36.0 | 27.1 | 18.3 | 16.0 | 93.0 | 86.3 | 80.0 | 51.1 | 44.1 |
| | | | | QDR1 | 49.8 | 26.3 | 20.0 | 17.6 | 17.6 | 92.6 | 84.9 | 77.5 | 50.6 | 44.2 |
| | | | | QDR2 | 45.7 | 30.3 | 24.5 | 19.5 | 17.7 | 89.1 | 79.3 | 71.0 | 46.8 | 39.4 |
| | | | | QBN1 | 35.5 | 41.6 | 38.8 | 29.4 | 16.0 | 68.9 | 80.9 | 85.0 | 84.8 | 73.4 |
| 0 | 0 | 0.5 | 0.5 | QBP | 44.6 | 29.1 | 21.5 | 10.7 | 6.9 | 84.9 | 74.6 | 64.8 | 39.1 | 28.3 |
| | | | | QBP1 | 55.8 | 37.9 | 28.6 | 18.7 | 16.4 | 87.4 | 77.2 | 69.3 | 43.5 | 40.1 |
| | | | | QLB | 50.8 | 35.3 | 28.1 | 18.8 | 18.0 | 86.9 | 76.5 | 68.4 | 46.0 | 39.7 |
| | | | | QLB1 | 46.2 | 34.1 | 27.6 | 18.8 | 18.0 | 84.5 | 76.0 | 68.2 | 46.0 | 39.7 |
| | | | | QDR | 45.0 | 30.8 | 24.3 | 15.9 | 15.1 | 84.3 | 73.8 | 65.1 | 43.2 | 36.5 |
| | | | | QDR1 | 39.7 | 29.5 | 24.4 | 17.9 | 16.9 | 83.1 | 72.9 | 64.3 | 43.1 | 37.9 |
| | | | | QDR2 | 37.4 | 28.8 | 23.0 | 17.6 | 16.0 | 78.0 | 68.0 | 60.5 | 37.7 | 33.5 |
| | | | | QBN1 | 33.5 | 36.8 | 39.1 | 14.4 | 6.9 | 68.9 | 76.1 | 73.1 | 52.5 | 36.6 |
| 0 | 0 | 0.8 | 0.5 | QBP | 89.7 | 69.9 | 55.3 | 31.7 | 18.9 | 99.4 | 97.7 | 96.6 | 77.2 | 61.2 |
| | | | | QBP1 | 94.1 | 78.5 | 66.3 | 42.9 | 36.7 | 99.7 | 98.0 | 97.4 | 80.3 | 71.9 |
| | | | | QLB | 92.3 | 76.1 | 64.7 | 42.3 | 35.9 | 99.8 | 98.8 | 98.4 | 89.4 | 78.2 |
| | | | | QLB1 | 89.3 | 74.5 | 64.0 | 42.3 | 35.9 | 99.7 | 98.7 | 98.3 | 89.3 | 78.2 |
| | | | | QDR | 87.5 | 68.2 | 56.1 | 37.7 | 30.6 | 98.7 | 97.0 | 96.1 | 73.1 | 66.0 |
| | | | | QDR1 | 84.7 | 65.2 | 53.8 | 39.1 | 33.1 | 98.7 | 96.4 | 94.3 | 77.6 | 66.8 |
| | | | | QDR2 | 77.8 | 60.1 | 48.4 | 35.2 | 31.6 | 98.6 | 94.2 | 91.3 | 68.5 | 60.6 |
| | | | | QBN1 | 62.4 | 71.7 | 71.6 | 54.6 | 32.1 | 92.9 | 97.4 | 97.1 | 95.8 | 91.2 |
| 0 | 0 | 0.8 | 0.8 | QBP | 98.7 | 89.5 | 90.4 | 47.0 | 32.4 | 100 | 100 | 99.9 | 97.2 | 90.3 |
| | | | | QBP1 | 99.1 | 94.1 | 88.7 | 85.0 | 56.9 | 100 | 100 | 99.9 | 98.1 | 95.3 |
| | | | | QLB | 98.9 | 93.0 | 87.0 | 62.9 | 55.6 | 100 | 100 | 99.9 | 98.4 | 94.7 |
| | | | | QLB1 | 98.1 | 91.7 | 86.0 | 62.9 | 55.6 | 100 | 100 | 99.9 | 98.4 | 94.7 |
| | | | | QDR | 97.9 | 88.5 | 80.5 | 53.8 | 47.2 | 100 | 99.9 | 99.9 | 97.3 | 92.2 |
| | | | | QDR1 | 96.9 | 86.9 | 77.6 | 56.2 | 50.2 | 100 | 99.9 | 99.9 | 97.2 | 92.3 |
| | | | | QDR2 | 93.3 | 79.8 | 70.1 | 51.2 | 45.5 | 100 | 99.8 | 99.5 | 94.4 | 87.3 |
| | | | | QBN1 | 76.0 | 87.0 | 86.7 | 74.7 | 55.9 | 98.1 | 99.8 | 99.5 | 98.6 | 97.0 |

Table 2.6 (cont'd)

| Parameters $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | Portmanteau tests | m=3 | m=4 | m=5 | m=10 | m=15 | m=3 | m=4 | m=5 | m=10 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.8 | 0 | 0.5 | 0 | QBP | 6.1 | 5.0 | 4.1 | 1.3 | 1.6 | 10.8 | 8.9 | 7.9 | 5.4 | 3.9 |
| | | | | QBP1 | 7.9 | 6.2 | 5.2 | 4.6 | 3.7 | 11.8 | 10.5 | 9.6 | 6.5 | 6.6 |
| | | | | QLB | 7.0 | 6.1 | 5.4 | 5.1 | 4.4 | 11.2 | 10.3 | 9.4 | 7.0 | 7.1 |
| | | | | QLB1 | 6.3 | 6.0 | 5.4 | 5.1 | 4.4 | 10.6 | 9.9 | 9.4 | 7.0 | 7.1 |
| | | | | QDR | 7.9 | 6.7 | 6.1 | 5.3 | 5.0 | 12.6 | 11.1 | 10.1 | 7.8 | 7.8 |
| | | | | QDR1 | 8.3 | 7.0 | 6.4 | 5.5 | 5.6 | 12.9 | 11.2 | 10.3 | 7.8 | 7.9 |
| | | | | QDR2 | 10.3 | 7.8 | 7.8 | 6.6 | 5.7 | 14.0 | 12.5 | 10.7 | 8.4 | 7.4 |
| | | | | QBN1 | 4.3 | 3.6 | 3.1 | 1.6 | 0.7 | 10.1 | 8.2 | 7.4 | 4.1 | 2.6 |
| 0.5 | 0 | 0.8 | 0 | QBP | 11.4 | 8.9 | 6.3 | 2.5 | 1.4 | 27.7 | 24.9 | 19.8 | 10.7 | 8.6 |
| | | | | QBP1 | 17.0 | 12.0 | 10.4 | 6.3 | 5.7 | 31.1 | 26.7 | 23.7 | 14.4 | 12.4 |
| | | | | QLB | 14.7 | 11.5 | 10.2 | 7.4 | 7.6 | 30.1 | 26.4 | 23.6 | 14.6 | 13.0 |
| | | | | QLB1 | 14.4 | 11.5 | 10.2 | 7.4 | 7.6 | 29.5 | 26.4 | 23.6 | 14.6 | 13.0 |
| | | | | QDR | 11.9 | 9.9 | 8.7 | 5.6 | 6.1 | 27.0 | 24.7 | 21.4 | 13.2 | 11.6 |
| | | | | QDR1 | 11.0 | 9.0 | 7.9 | 6.2 | 7.0 | 26.5 | 23.2 | 19.6 | 13.7 | 12.0 |
| | | | | QDR2 | 11.3 | 8.9 | 8.9 | 5.7 | 6.8 | 26.7 | 21.5 | 19.0 | 12.7 | 11.8 |
| | | | | QBN1 | 10.7 | 12.2 | 11.0 | 6.0 | 2.5 | 21.6 | 24.5 | 25.5 | 18.6 | 10.9 |
| 0.2 | 0 | 0.8 | 0 | QBP | 29.2 | 19.7 | 14.4 | 6.4 | 4.8 | 69.2 | 57.9 | 47.9 | 26.9 | 19.0 |
| | | | | QBP1 | 38.7 | 25.4 | 20.0 | 12.6 | 11.4 | 74.1 | 62.1 | 52.3 | 31.9 | 26.9 |
| | | | | QLB | 35.0 | 23.8 | 19.8 | 13.0 | 13.3 | 72.3 | 61.4 | 51.8 | 31.7 | 27.4 |
| | | | | QLB1 | 33.6 | 23.1 | 19.8 | 13.0 | 13.3 | 70.9 | 60.6 | 51.8 | 31.7 | 27.4 |
| | | | | QDR | 29.7 | 20.7 | 16.1 | 10.7 | 11.2 | 68.1 | 57.0 | 48.3 | 28.8 | 24.6 |
| | | | | QDR1 | 27.1 | 18.1 | 15.3 | 11.4 | 12.5 | 66.7 | 54.4 | 45.5 | 28.5 | 25.8 |
| | | | | QDR2 | 27.9 | 17.2 | 15.5 | 11.7 | 12.6 | 64.8 | 50.2 | 42.1 | 26.8 | 22.3 |
| | | | | QBN1 | 20.6 | 23.4 | 21.3 | 13.2 | 5.8 | 46.8 | 55.5 | 55.5 | 45.7 | 32.8 |

Table 2 7

Empirical Powers of Portmanteau Tests, MA(1) Model fitted, Data generated from the model $(1 - \pi_1 B)(1 - \pi_2 B)w_t = (1 - \psi_1 B)(1 - \psi_2 B)a_t$, $\alpha = 5\%$

| Parameters | | | | Portmanteau tests | n=50 (in percent) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
| 0 4 | 0 | 0 | 0 | QBP | 19 0 | 12 3 | 9 7 | 9 4 | 6 4 | 4 9 |
| | | | | QBP1 | 20 8 | 14 6 | 12 0 | 11 2 | 9 3 | 9 1 |
| | | | | QLB | 20 4 | 14.7 | 12.4 | 11 5 | 10 5 | 10 7 |
| | | | | QLB1 | 20 4 | 14.7 | 12 4 | 11 5 | 10 5 | 10 7 |
| | | | | QDR | 19.0 | 13.6 | 12 0 | 10 4 | 8 8 | 9 2 |
| | | | | QDR1 | 19.8 | 14.3 | 12 8 | 12 2 | 10 6 | 12 3 |
| | | | | QDR2 | 19 8 | 13 8 | 11 9 | 10 7 | 9 2 | 8 9 |
| | | | | QBN2 | 10 2 | 8 8 | 7 8 | 7 4 | 6 3 | 5 5 |
| 0 7 | 0 | 0 | 0 | QBP | 86.0 | 77 4 | 72.8 | 68 4 | 55 2 | 47 5 |
| | | | | QBP1 | 87 3 | 79 8 | 75 5 | 72 5 | 63 4 | 59 9 |
| | | | | QLB | 87 2 | 79 8 | 75 5 | 73 0 | 63 6 | 59 1 |
| | | | | QLB1 | 86 7 | 79 5 | 75 5 | 73 0 | 63 6 | 59 1 |
| | | | | QDR | 82 2 | 72 0 | 68 0 | 65 2 | 53 8 | 48 6 |
| | | | | QDR1 | 82 7 | 73 6 | 70 3 | 66 3 | 57 4 | 55 8 |
| | | | | QDR2 | 78 8 | 69 2 | 63 8 | 59 7 | 51 2 | 47 3 |
| | | | | QBN2 | 63 7 | 56 2 | 50 3 | 47 1 | 41 8 | 37 9 |
| 0 9 | 0 | 0 | 0 | QBP | 99.3 | 99 2 | 98 7 | 98 2 | 97 1 | 94 5 |
| | | | | QBP1 | 99 4 | 99 2 | 99 0 | 98 4 | 97 4 | 97 4 |
| | | | | QLB | 99 4 | 99 2 | 99 0 | 98 4 | 97 3 | 97 2 |
| | | | | QLB1 | 99 4 | 99 2 | 99 0 | 98 4 | 97 3 | 97 2 |
| | | | | QDR | 98.8 | 97 8 | 96 8 | 95 6 | 93 5 | 92 7 |
| | | | | QDR1 | 98.9 | 98 0 | 96 8 | 95 9 | 94 2 | 93 0 |
| | | | | QDR2 | 98 0 | 96 8 | 96.2 | 94 6 | 91 5 | 89 9 |
| | | | | QBN2 | 95.4 | 94 0 | 90 2 | 89 1 | 83 7 | 80 1 |
| 0 5 | 0 5 | 0 | 0 | QBP | 96 9 | 94.6 | 91 8 | 89 3 | 79 2 | 71 8 |
| | | | | QBP1 | 97 4 | 95 0 | 93 0 | 91 1 | 83.7 | 81 4 |
| | | | | QLB | 97 3 | 95 0 | 93.1 | 90 7 | 83 8 | 79 5 |
| | | | | QLB1 | 96.8 | 94.9 | 92 9 | 90 6 | 83 7 | 79 4 |
| | | | | QDR | 95.5 | 93.1 | 90 5 | 88 4 | 78 3 | 72 8 |
| | | | | QDR1 | 95 6 | 93 1 | 90 9 | 88 6 | 79 7 | 76 9 |
| | | | | QDR2 | 93 9 | 90 5 | 87 1 | 84 2 | 73 1 | 68 4 |
| | | | | QBN2 | 77.4 | 65 0 | 57 7 | 53 4 | 50 7 | 45 6 |
| 0 8 | 0 5 | 0 | 0 | QBP | 99 9 | 99 9 | 99 9 | 99 8 | 99 7 | 99 6 |
| | | | | QBP1 | 100 | 99 9 | 99 9 | 99 9 | 99 8 | 99 9 |
| | | | | QLB | 100 | 99 9 | 99.9 | 99 9 | 99 8 | 99 9 |
| | | | | QLB1 | 99 9 | 99.9 | 99.9 | 99 9 | 99 8 | 99 9 |
| | | | | QDR | 99.9 | 99.7 | 99 7 | 99 7 | 99 3 | 99 2 |
| | | | | QDR1 | 99 9 | 99.7 | 99 7 | 99 7 | 99 3 | 99 2 |
| | | | | QDR2 | 99 8 | 99 7 | 99 7 | 99 6 | 98 8 | 98 8 |
| | | | | QBN2 | 98 8 | 96.9 | 95 5 | 94 6 | 89 1 | 84 8 |
| 0 8 | 0 8 | 0 | 0 | QBP | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QBP1 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QLB | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QLB1 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QDR | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QDR1 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QDR2 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | | | QBN2 | 100 | 99.8 | 99.6 | 99 6 | 99 8 | 97 5 |

Table 2 7 (cont'd)

| Parameters | | | | Portmanteau tests | n=50 (in percent) | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | | m=2 | m=3 | m=4 | m=5 | m=10 | m=15 |
| 0 | 0 | 0 5 | 0 5 | QBP | 33 8 | 28.8 | 23 0 | 19.9 | 14 0 | 10 4 |
| | | | | QBP1 | 36 9 | 31.8 | 27.8 | 24 6 | 20.3 | 17 5 |
| | | | | QLB | 36 2 | 31.2 | 27.7 | 24.9 | 20.6 | 17 8 |
| | | | | QLB1 | 28.4 | 27 9 | 25 3 | 23 5 | 20.4 | 17 6 |
| | | | | QDR | 41 0 | 35.0 | 29 9 | 27 7 | 21.8 | 19 3 |
| | | | | QDR1 | 40.5 | 33 5 | 28.4 | 26 4 | 20 4 | 19 0 |
| | | | | QDR2 | 40 2 | 33 4 | 29 7 | 27 9 | 20 1 | 18 5 |
| | | | | QBN2 | 8 8 | 10 8 | 11.1 | 11 6 | 12 7 | 10 9 |
| 0 | 0 | 0 8 | 0 5 | QBP | 69 6 | 57 9 | 50.8 | 45 9 | 32 0 | 33 5 |
| | | | | QBP1 | 73 1 | 62 0 | 55 9 | 52 7 | 42 1 | 50 4 |
| | | | | QLB | 71 9 | 61 8 | 55 4 | 52 5 | 42 5 | 49 4 |
| | | | | QLB1 | 56 6 | 51.3 | 48 5 | 46.3 | 39.9 | 49 1 |
| | | | | QDR | 75 9 | 64.8 | 59 0 | 55 7 | 43 2 | 47 6 |
| | | | | QDR1 | 76 4 | 65.1 | 58 4 | 54.2 | 40.7 | 44 1 |
| | | | | QDR2 | 71 8 | 60.9 | 55.9 | 50 0 | 41 9 | 45 9 |
| | | | | QBN2 | 10 2 | 11 4 | 13 1 | 12 8 | 14 3 | 12 2 |
| 0 | 0 | 0 8 | 0 8 | QBP | 97 6 | 94 8 | 88 2 | 81 5 | 61 5 | 52 9 |
| | | | | QBP1 | 97 9 | 97 0 | 91.6 | 86 7 | 73 7 | 72 3 |
| | | | | QLB | 97 7 | 96.8 | 91 5 | 85 6 | 71 2 | 69 9 |
| | | | | QLB1 | 94 6 | 89.6 | 84 0 | 79 8 | 68.4 | 68 7 |
| | | | | QDR | 98 1 | 95 3 | 90 0 | 85 6 | 70 6 | 66 9 |
| | | | | QDR1 | 98 3 | 96 1 | 90 3 | 85 1 | 68 6 | 64 7 |
| | | | | QDR2 | 96.4 | 90 9 | 85 6 | 80 0 | 64 8 | 62 2 |
| | | | | QBN2 | 12 0 | 12.9 | 14 2 | 13 8 | 15 0 | 12 9 |
| 0 8 | 0 | 0 5 | 0 | QBP | 45 6 | 40.5 | 40 7 | 38 1 | 29 4 | 23 7 |
| | | | | QBP1 | 49 1 | 44 7 | 43 7 | 42 4 | 36 2 | 34 0 |
| | | | | QLB | 48 5 | 45 0 | 44 3 | 43 0 | 37 1 | 34 4 |
| | | | | QLB1 | 48 3 | 45.0 | 44 3 | 43 0 | 37 1 | 34 4 |
| | | | | QDR | 41 8 | 37 1 | 33 4 | 32 1 | 26 4 | 23 8 |
| | | | | QDR1 | 42.1 | 38.3 | 36.1 | 35 3 | 31 0 | 30.8 |
| | | | | QDR2 | 40.0 | 35.9 | 33.9 | 31 8 | 24.9 | 20 9 |
| | | | | QBN2 | 32.9 | 32.8 | 29.9 | 27.9 | 22 1 | 18 6 |
| 0 5 | 0 | 0 8 | 0 | QBP | 10.4 | 10.2 | 6 3 | 6 2 | 5.0 | 3 9 |
| | | | | QBP1 | 12 7 | 12 4 | 8 9 | 8 1 | 7 3 | 8 0 |
| | | | | QLB | 12.5 | 12.5 | 9 4 | 8 6 | 8 3 | 9 8 |
| | | | | QLB1 | 12 2 | 12 5 | 9 4 | 8 6 | 8 3 | 9 8 |
| | | | | QDR | 12 0 | 11 7 | 8 8 | 8 2 | 8 8 | 10 1 |
| | | | | QDR1 | 11 2 | 11 5 | 8.7 | 8 0 | 8 2 | 8 9 |
| | | | | QDR2 | 12 1 | 11 0 | 9.1 | 8 5 | 8 5 | 10 1 |
| | | | | QBN2 | 10.4 | 8 8 | 7.7 | 6 6 | 6 8 | 4 9 |
| 0 2 | 0 | 0 8 | 0 | QBP | 9.2 | 6 2 | 6.4 | 5 1 | 3 1 | 2 8 |
| | | | | QBP1 | 11 8 | 7.8 | 7 9 | 7 1 | 6 6 | 6 1 |
| | | | | QLB | 11 1 | 8.0 | 8 1 | 7 4 | 7 6 | 7 2 |
| | | | | QLB1 | 7 2 | 6.9 | 7 6 | 7 1 | 7 5 | 7 2 |
| | | | | QDR | 12 2 | 9 5 | 9.6 | 8.3 | 7 7 | 8 5 |
| | | | | QDR1 | 11 5 | 8 7 | 9.1 | 8 1 | 7.7 | 7 6 |
| | | | | QDR2 | 11.0 | 9 0 | 9 0 | 7 5 | 8 1 | 8 7 |
| | | | | QBN2 | 8 6 | 8 3 | 7 9 | 8.0 | 7 7 | 5 6 |

(iii) The empirical power of QLB and QLB1 is similar. Only in a very few instances do the simulation results indicate a fairly sizable difference in their empirical power. However, in work not reported here, a further investigation of the results reveals that this difference can be substantially narrowed by considering a smaller value of m. For example, if the data were fitted to an AR(1) model with n=50, the empirical power of QLB1 can be improved when only the first residual autocorrelation (m=1) is taken into consideration. For the MA(1) process, a similar strategy can be used to increase the power of QLB1.

(iv) The nonparametric Dufour-Roy test, QDR2, performs almost equally as well as the parametric ones, QDR and QDR1. This is, indeed, an encouraging finding because, unlike all the parametric portmanteau statistics considered, QDR2 is valid even in the presence of non-Gaussian (or non-normal) errors. Since Hall and McAleer (1989) found that the commonly-used parametric portmanteau tests such as QBP and QLB are not robust to non-Gaussian errors, this result could be of considerable interest and importance to both applied econometricians and time series analysts.

(v) QBP1 is consistently more powerful than QBP. This is most evident in cases where the sample size is 50 and the parameter values ($\pi$'s and $\psi$'s) are large. In addition, it is easy to see from Tables 2.5-2.7 that the difference in power between QBP and QBP1 increases rapidly with the value of m.

(vi) When an AR(2) model is being fitted to the data, the empirical power of the portmanteau tests examined is generally lower than those yielded in the case of an AR(1) process. This result indicates that an AR(2) model provides a much better approximation to the generated data. This observation is supported by the well-known duality theorem which states that any stationary ARMA representation can be well approximated by a higher-order AR or MA model.

Tables 2.8-2.9 summarize the simulation results of the third experiment in which the data were generated from four low-order seasonal ARMA models and were subsequently fitted to the AR(1) and AR(2) models, respectively. The following important points emerge from the reported results in these tables:

(i) The most significant observation appears to be the occurrence of some high-order autocorrelation spikes when $m \geq 12$. It is easily seen from the results in Tables 2.8-2.9 that, with the exception of a few cases, the empirical power of all portmanteau tests examined display a discrete rise when m is 12. Although this finding is entirely anticipated due to the use of seasonal data, it is, nonetheless, important to highlight the effect of the nature of data on the choice of m.

Table 2.8

Empirical Powers of Portmanteau Tests, AR(1) model fitted; Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$, and $\alpha = 5\%$

| Parameters | | | Portmanteau Tests | n=50 (in percent) | | | | | | | n=100 (in percent) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | | m=2 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 |
| 0 | 0 | 0.4 | QBP | 7.4 | 5.7 | 4.0 | 4.8 | 4.2 | 8.8 | 7.1 | 19.2 | 24.0 | 28.9 | 31.7 | 34.8 | 61.6 | 61.9 |
| | | | QBP1 | 9.3 | 6.6 | 5.9 | 6.4 | 7.3 | 17.3 | 16.0 | 21.0 | 25.9 | 30.6 | 34.4 | 41.1 | 68.5 | 69.5 |
| | | | QLB | 9.0 | 7.1 | 5.8 | 6.9 | 8.4 | 23.0 | 20.8 | 20.7 | 26.0 | 31.1 | 35.1 | 42.1 | 71.5 | 72.3 |
| | | | QLB1 | 9.0 | 7.1 | 5.8 | 6.9 | 8.4 | 23.0 | 20.8 | 20.7 | 26.0 | 31.1 | 35.1 | 42.1 | 71.5 | 72.3 |
| | | | QDR | 8.5 | 7.4 | 6.4 | 7.4 | 8.5 | 20.6 | 19.6 | 20.2 | 25.7 | 31.7 | 35.1 | 41.6 | 70.3 | 71.2 |
| | | | QDR1 | 8.6 | 7.6 | 6.6 | 6.9 | 8.8 | 20.4 | 20.6 | 20.2 | 25.8 | 32.6 | 34.9 | 42.5 | 70.8 | 72.1 |
| | | | QDR2 | 8.5 | 7.8 | 7.3 | 7.2 | 8.5 | 18.9 | 18.9 | 17.6 | 19.9 | 31.2 | 22.8 | 27.5 | 41.7 | 39.6 |
| | | | QBN1 | 7.2 | 6.0 | 4.9 | 4.6 | 3.9 | 7.8 | 4.6 | 6.8 | 7.7 | 8.6 | 7.4 | 7.1 | 43.4 | 34.3 |
| 0 | 0 | 0.7 | QBP | 8.6 | 7.4 | 5.8 | 6.7 | 6.5 | 21.6 | 16.6 | 21.6 | 25.9 | 32.6 | 36.8 | 40.0 | 83.4 | 81.6 |
| | | | QBP1 | 9.8 | 9.0 | 8.3 | 8.6 | 9.7 | 35.0 | 32.0 | 23.0 | 27.1 | 34.3 | 39.4 | 46.2 | 86.9 | 86.6 |
| | | | QLB | 9.7 | 9.0 | 8.4 | 9.4 | 10.7 | 42.3 | 39.7 | 22.7 | 27.2 | 34.6 | 40.2 | 46.2 | 88.1 | 88.7 |
| | | | QLB1 | 9.7 | 9.0 | 8.4 | 9.4 | 10.7 | 42.3 | 39.7 | 22.6 | 27.2 | 34.6 | 40.2 | 46.2 | 88.1 | 88.7 |
| | | | QDR | 9.8 | 9.4 | 9.1 | 10.2 | 11.1 | 40.9 | 38.6 | 22.3 | 27.9 | 34.7 | 40.1 | 46.3 | 87.4 | 87.5 |
| | | | QDR1 | 9.9 | 9.4 | 8.8 | 9.5 | 12.0 | 41.6 | 38.5 | 22.2 | 27.8 | 34.7 | 39.7 | 47.0 | 88.8 | 88.1 |
| | | | QDR2 | 10.4 | 9.4 | 9.1 | 10.0 | 11.7 | 37.1 | 33.8 | 19.9 | 23.2 | 23.9 | 26.6 | 28.8 | 59.8 | 59.4 |
| | | | QBN1 | 8.8 | 8.1 | 7.3 | 7.2 | 6.3 | 17.4 | 11.8 | 9.5 | 11.1 | 11.9 | 11.2 | 12.5 | 85.1 | 79.1 |
| 0 | 0 | 0.9 | QBP | 9.0 | 7.8 | 6.5 | 7.3 | 7.0 | 26.0 | 21.3 | 21.3 | 27.0 | 33.1 | 38.1 | 41.5 | 87.0 | 85.5 |
| | | | QBP1 | 10.6 | 9.7 | 8.4 | 9.0 | 10.3 | 39.4 | 36.5 | 23.2 | 28.4 | 35.0 | 40.8 | 47.1 | 90.2 | 89.7 |
| | | | QLB | 10.2 | 10.1 | 9.2 | 10.0 | 12.1 | 49.6 | 45.7 | 22.9 | 28.6 | 35.5 | 41.4 | 48.3 | 91.4 | 91.7 |
| | | | QLB1 | 10.2 | 10.1 | 9.2 | 10.0 | 12.1 | 49.6 | 45.7 | 22.9 | 28.6 | 35.5 | 41.4 | 48.3 | 91.4 | 91.7 |
| | | | QDR | 10.5 | 10.0 | 10.1 | 10.6 | 12.6 | 46.5 | 43.8 | 23.3 | 28.1 | 35.2 | 40.5 | 48.7 | 91.0 | 91.2 |
| | | | QDR1 | 10.6 | 10.1 | 10.0 | 10.0 | 13.1 | 47.4 | 45.2 | 23.3 | 28.3 | 34.9 | 40.8 | 49.1 | 90.8 | 91.1 |
| | | | QDR2 | 10.6 | 9.7 | 9.1 | 9.8 | 12.0 | 41.7 | 39.6 | 19.6 | 22.6 | 23.8 | 27.7 | 30.8 | 66.4 | 64.4 |
| | | | QBN1 | 9.5 | 8.1 | 8.1 | 7.6 | 6.4 | 20.5 | 14.5 | 10.0 | 12.2 | 12.4 | 12.4 | 14.3 | 90.4 | 87.0 |
| 0.4 | 0 | 0 | QBP | 8.1 | 5.9 | 5.8 | 5.9 | 8.3 | 17.6 | 14.5 | 19.9 | 25.2 | 29.6 | 32.5 | 46.5 | 74.7 | 72.8 |
| | | | QBP1 | 9.0 | 7.5 | 6.7 | 7.9 | 12.5 | 28.2 | 25.9 | 21.0 | 26.8 | 32.1 | 35.8 | 52.4 | 79.4 | 78.6 |
| | | | QLB | 8.9 | 7.5 | 7.2 | 8.5 | 13.9 | 34.5 | 31.6 | 20.8 | 27.0 | 32.5 | 36.7 | 53.6 | 81.2 | 80.6 |
| | | | QLB1 | 8.9 | 7.5 | 7.2 | 8.5 | 13.9 | 34.5 | 31.6 | 20.8 | 27.0 | 32.5 | 36.7 | 53.6 | 81.2 | 80.6 |
| | | | QDR | 8.5 | 8.7 | 7.4 | 8.0 | 13.5 | 33.6 | 30.3 | 23.2 | 27.8 | 32.8 | 36.4 | 53.8 | 79.9 | 80.5 |
| | | | QDR1 | 8.4 | 8.3 | 7.2 | 8.2 | 13.7 | 35.0 | 31.6 | 23.2 | 28.0 | 33.1 | 36.6 | 53.6 | 80.1 | 80.7 |
| | | | QDR2 | 9.1 | 9.0 | 8.3 | 9.6 | 13.9 | 31.0 | 29.3 | 16.1 | 20.5 | 23.2 | 23.2 | 30.9 | 54.2 | 53.4 |
| | | | QBN1 | 8.1 | 7.4 | 7.2 | 7.0 | 9.4 | 10.8 | 7.3 | 8.6 | 9.3 | 10.1 | 9.3 | 16.4 | 57.3 | 48.1 |
| 0.7 | 0 | 0 | QBP | 13.0 | 14.3 | 15.1 | 15.9 | 22.3 | 74.5 | 63.7 | 28.7 | 40.4 | 47.8 | 51.9 | 70.1 | 98.5 | 97.7 |
| | | | QBP1 | 15.1 | 18.0 | 18.2 | 18.2 | 30.5 | 85.4 | 81.3 | 29.9 | 41.6 | 49.0 | 53.9 | 73.3 | 98.9 | 99.0 |
| | | | QLB | 14.8 | 18.2 | 18.9 | 19.9 | 33.0 | 90.0 | 86.6 | 29.6 | 41.9 | 49.3 | 54.3 | 73.9 | 99.4 | 99.1 |
| | | | QLB1 | 14.8 | 18.2 | 18.9 | 19.9 | 33.0 | 90.0 | 86.5 | 29.6 | 41.9 | 49.3 | 54.3 | 73.9 | 99.4 | 99.1 |
| | | | QDR | 19.1 | 19.5 | 20.2 | 19.7 | 34.6 | 88.8 | 86.2 | 34.7 | 44.2 | 50.6 | 55.1 | 74.3 | 99.3 | 99.1 |
| | | | QDR1 | 19.0 | 20.4 | 21.1 | 21.0 | 34.0 | 89.7 | 87.2 | 34.7 | 44.3 | 50.6 | 55.4 | 74.5 | 99.5 | 99.2 |
| | | | QDR2 | 18.3 | 19.4 | 20.7 | 21.0 | 33.9 | 87.7 | 84.2 | 25.0 | 30.9 | 34.8 | 37.7 | 54.4 | 93.0 | 90.6 |
| | | | QBN1 | 16.5 | 18.6 | 20.4 | 19.7 | 33.0 | 66.3 | 52.2 | 19.0 | 24.3 | 30.9 | 33.9 | 58.4 | 99.8 | 99.4 |
| 0.9 | 0 | 0 | QBP | 22.8 | 27.8 | 31.3 | 32.6 | 45.3 | 98.1 | 95.2 | 38.4 | 54.0 | 64.2 | 72.2 | 85.7 | 100 | 100 |
| | | | QBP1 | 24.5 | 30.6 | 36.1 | 38.6 | 55.2 | 99.3 | 98.5 | 39.0 | 55.3 | 65.9 | 74.2 | 88.5 | 100 | 100 |
| | | | QLB | 24.5 | 30.9 | 37.2 | 39.8 | 57.9 | 99.4 | 99.2 | 38.9 | 55.4 | 66.6 | 74.5 | 88.6 | 100 | 100 |
| | | | QLB1 | 24.5 | 30.9 | 37.2 | 39.8 | 57.9 | 99.4 | 99.2 | 38.9 | 55.4 | 66.6 | 74.5 | 88.6 | 100 | 100 |
| | | | QDR | 31.2 | 35.9 | 41.0 | 42.0 | 61.2 | 99.4 | 99.0 | 47.0 | 59.0 | 69.4 | 75.6 | 89.3 | 100 | 100 |
| | | | QDR1 | 31.3 | 35.5 | 41.3 | 43.2 | 60.3 | 99.5 | 99.2 | 46.9 | 59.8 | 70.0 | 75.4 | 89.1 | 100 | 100 |
| | | | QDR2 | 29.9 | 35.2 | 40.7 | 42.8 | 58.7 | 99.3 | 98.8 | 37.5 | 46.7 | 54.5 | 58.4 | 73.8 | 99.9 | 99.5 |
| | | | QBN1 | 28.5 | 34.1 | 40.4 | 43.0 | 67.0 | 97.2 | 93.7 | 36.7 | 47.7 | 58.8 | 67.8 | 90.6 | 100 | 100 |

Table 2.8 (cont'd)

| Parameters $\phi_{12}$ | $\psi_1$ | $\theta_{12}$ | Portmanteau Tests | m=2 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 | m=2 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.6 | -0.5 | QBP | 43.5 | 31.7 | 26.4 | 23.0 | 27.8 | 35.8 | 30.7 | 72.3 | 69.4 | 69.2 | 67.9 | 71.1 | 89.0 | 86.6 |
| | | | QBP1 | 47.5 | 35.5 | 30.5 | 27.5 | 28.6 | 47.6 | 46.0 | 73.6 | 70.9 | 70.0 | 69.3 | 75.1 | 91.2 | 91.0 |
| | | | QLB | 46.8 | 35.7 | 31.0 | 27.6 | 28.1 | 52.0 | 50.9 | 73.6 | 71.0 | 70.1 | 69.5 | 75.4 | 91.6 | 91.4 |
| | | | QLB1 | 46.5 | 35.7 | 31.0 | 27.6 | 28.1 | 52.0 | 50.9 | 73.3 | 70.9 | 70.1 | 69.5 | 75.4 | 91.6 | 91.4 |
| | | | QDR | 45.1 | 34.3 | 29.1 | 26.1 | 28.8 | 50.0 | 47.8 | 73.4 | 71.4 | 71.1 | 69.0 | 75.8 | 91.5 | 91.0 |
| | | | QDR1 | 43.0 | 33.5 | 28.7 | 26.5 | 28.8 | 52.6 | 50.7 | 72.9 | 71.0 | 70.0 | 68.9 | 75.7 | 91.7 | 91.6 |
| | | | QDR2 | 40.5 | 31.4 | 27.2 | 25.7 | 27.1 | 48.8 | 46.7 | 49.7 | 46.5 | 44.9 | 43.3 | 48.3 | 65.2 | 63.9 |
| | | | QBN1 | 41.7 | 39.5 | 38.9 | 36.5 | 27.1 | 33.9 | 23.9 | 71.5 | 73.3 | 72.9 | 72.0 | 62.8 | 86.7 | 85.2 |
| 0 | -0.5 | 0.6 | QBP | 31.7 | 22.1 | 20.6 | 21.2 | 16.1 | 31.6 | 27.4 | 55.7 | 54.8 | 58.0 | 61.7 | 60.8 | 87.0 | 84.5 |
| | | | QBP1 | 35.0 | 25.3 | 23.9 | 25.6 | 22.6 | 45.9 | 43.6 | 57.3 | 56.8 | 59.5 | 64.1 | 64.2 | 90.5 | 88.6 |
| | | | QLB | 34.4 | 25.5 | 24.0 | 26.2 | 23.7 | 52.5 | 49.8 | 57.0 | 56.8 | 59.7 | 64.4 | 64.6 | 91.9 | 90.0 |
| | | | QLB1 | 34.2 | 25.5 | 24.0 | 26.2 | 23.7 | 52.5 | 49.8 | 56.4 | 56.8 | 59.7 | 64.4 | 64.6 | 91.9 | 90.0 |
| | | | QDR | 31.7 | 23.4 | 22.8 | 24.5 | 22.9 | 49.3 | 47.8 | 55.7 | 56.2 | 59.0 | 63.3 | 63.9 | 91.1 | 89.5 |
| | | | QDR1 | 32.2 | 24.2 | 24.1 | 24.9 | 24.6 | 50.8 | 48.0 | 56.3 | 56.7 | 59.4 | 63.8 | 64.8 | 90.8 | 89.4 |
| | | | QDR2 | 31.0 | 22.9 | 22.9 | 22.7 | 24.6 | 45.4 | 43.5 | 42.4 | 38.3 | 38.7 | 41.7 | 42.6 | 65.7 | 63.9 |
| | | | QBN1 | 29.3 | 26.1 | 23.7 | 21.5 | 13.2 | 23.4 | 17.4 | 52.9 | 50.6 | 47.8 | 44.9 | 34.1 | 84.7 | 85.8 |
| 0.6 | -0.5 | 0 | QBP | 30.8 | 27.5 | 25.1 | 22.7 | 26.1 | 60.8 | 52.8 | 57.4 | 60.1 | 62.2 | 62.0 | 74.0 | 96.0 | 95.4 |
| | | | QBP1 | 34.4 | 32.1 | 29.3 | 25.7 | 34.1 | 73.1 | 68.1 | 59.4 | 61.7 | 64.0 | 64.3 | 77.1 | 97.5 | 97.1 |
| | | | QLB | 33.6 | 32.3 | 30.2 | 27.2 | 36.0 | 79.7 | 75.4 | 59.4 | 62.0 | 64.3 | 64.5 | 77.1 | 97.9 | 97.6 |
| | | | QLB1 | 33.3 | 32.3 | 30.2 | 27.2 | 36.0 | 79.7 | 75.4 | 58.6 | 61.9 | 64.2 | 64.5 | 77.1 | 97.9 | 97.6 |
| | | | QDR | 38.4 | 35.8 | 33.0 | 29.9 | 39.8 | 79.9 | 75.5 | 63.2 | 64.5 | 66.4 | 67.4 | 78.8 | 97.8 | 97.6 |
| | | | QDR1 | 39.4 | 36.0 | 34.2 | 31.1 | 38.8 | 81.5 | 77.2 | 63.4 | 65.0 | 66.9 | 67.8 | 78.6 | 98.1 | 97.8 |
| | | | QDR2 | 37.8 | 35.1 | 32.8 | 29.9 | 38.1 | 77.3 | 73.6 | 45.7 | 45.2 | 45.6 | 44.3 | 54.8 | 86.2 | 83.8 |
| | | | QBN1 | 40.9 | 34.8 | 35.1 | 32.5 | 36.0 | 47.8 | 40.9 | 62.2 | 59.0 | 62.2 | 60.2 | 64.8 | 96.9 | 97.7 |
| -0.5 | 0.6 | 0 | QBP | 45.5 | 36.4 | 30.7 | 31.8 | 27.8 | 49.6 | 44.0 | 69.6 | 68.5 | 71.0 | 72.6 | 71.9 | 91.2 | 88.8 |
| | | | QBP1 | 49.6 | 39.9 | 35.8 | 37.5 | 35.1 | 59.7 | 57.8 | 70.9 | 70.4 | 73.3 | 74.6 | 75.1 | 94.4 | 93.4 |
| | | | QLB | 49.2 | 40.0 | 36.2 | 38.1 | 37.0 | 65.3 | 61.9 | 70.8 | 70.5 | 73.7 | 74.7 | 75.6 | 94.6 | 94.0 |
| | | | QLB1 | 48.3 | 40.0 | 36.2 | 38.1 | 37.0 | 65.3 | 61.9 | 70.5 | 70.5 | 73.7 | 74.7 | 75.6 | 94.6 | 94.0 |
| | | | QDR | 42.8 | 36.3 | 31.2 | 33.5 | 33.0 | 62.7 | 59.5 | 69.0 | 68.0 | 70.8 | 72.9 | 74.7 | 94.1 | 93.5 |
| | | | QDR1 | 42.0 | 35.1 | 31.2 | 34.4 | 35.5 | 62.3 | 60.8 | 68.8 | 67.5 | 70.7 | 72.1 | 75.1 | 93.9 | 93.4 |
| | | | QDR2 | 39.3 | 32.8 | 31.5 | 32.9 | 33.7 | 60.0 | 59.6 | 48.6 | 45.8 | 47.1 | 48.6 | 50.5 | 75.2 | 74.2 |
| | | | QBN1 | 41.4 | 41.0 | 38.6 | 35.5 | 25.0 | 39.1 | 33.8 | 66.4 | 70.5 | 70.4 | 67.9 | 57.1 | 92.6 | 94.0 |

Table 2 9
Empirical Powers of Portmanteau Tests, AR(2) model fitted, Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \theta_{12}B^{12})a_t$, $\alpha = 5\%$

| Parameters | | | Portmanteau | n=50 | | | | | | n=100 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | Tests | (in percent) | | | | | | (in percent) | | | | | |
| $\Phi_{12}$ | $\psi_1$ | $\theta_{12}$ | | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 |
| 0 | 0 | 0.4 | QBP | 4.6 | 4.6 | 3.0 | 2.6 | 7.3 | 5.1 | 18.5 | 23.0 | 26.3 | 30.2 | 58.6 | 56.6 |
| | | | QBP1 | 6.6 | 6.2 | 5.1 | 4.4 | 14.8 | 12.4 | 20.3 | 25.3 | 29.8 | 35.2 | 65.4 | 65.3 |
| | | | QLB | 5.8 | 5.9 | 5.7 | 5.0 | 18.7 | 16.1 | 19.6 | 25.3 | 29.7 | 36.1 | 67.1 | 67.3 |
| | | | QLB1 | 5.7 | 5.9 | 5.7 | 5.0 | 18.7 | 16.1 | 19.4 | 25.2 | 29.7 | 36.1 | 67.1 | 67.3 |
| | | | QDR | 5.9 | 5.9 | 5.5 | 5.6 | 18.0 | 15.4 | 19.5 | 25.0 | 29.2 | 36.1 | 67.6 | 67.1 |
| | | | QDR1 | 5.7 | 6.1 | 5.7 | 5.6 | 18.5 | 15.9 | 19.5 | 25.0 | 29.8 | 37.6 | 67.6 | 67.3 |
| | | | QDR2 | 6.0 | 6.2 | 6.7 | 6.1 | 16.2 | 14.4 | 24.1 | 24.8 | 25.5 | 25.7 | 40.7 | 38.6 |
| | | | QBN1 | 5.4 | 5.4 | 4.3 | 3.3 | 7.2 | 4.1 | 6.7 | 7.7 | 6.3 | 6.3 | 42.9 | 35.3 |
| 0 | 0 | 0.7 | QBP | 5.8 | 5.2 | 5.0 | 3.1 | 17.1 | 11.6 | 21.4 | 25.8 | 31.0 | 34.6 | 80.5 | 77.5 |
| | | | QBP1 | 8.3 | 7.9 | 7.8 | 7.0 | 29.7 | 26.3 | 23.1 | 27.8 | 34.1 | 40.3 | 84.2 | 84.1 |
| | | | QLB | 7.3 | 7.7 | 7.5 | 7.3 | 36.7 | 33.0 | 22.1 | 27.8 | 34.2 | 41.2 | 86.7 | 86.3 |
| | | | QLB1 | 7.2 | 7.7 | 7.5 | 7.3 | 36.7 | 33.0 | 22.1 | 27.8 | 34.2 | 41.2 | 86.7 | 86.3 |
| | | | QDR | 7.5 | 7.7 | 8.2 | 7.8 | 35.5 | 33.0 | 23.0 | 27.3 | 34.0 | 41.5 | 86.2 | 85.7 |
| | | | QDR1 | 7.5 | 7.7 | 7.9 | 8.1 | 35.8 | 32.3 | 23.1 | 27.6 | 33.9 | 42.3 | 86.1 | 85.5 |
| | | | QDR2 | 7.7 | 8.1 | 8.2 | 8.8 | 33.1 | 28.5 | 26.5 | 24.8 | 27.2 | 27.3 | 57.7 | 54.4 |
| | | | QBN1 | 6.1 | 5.9 | 5.1 | 5.1 | 16.9 | 10.6 | 8.5 | 10.0 | 9.1 | 11.0 | 85.0 | 78.7 |
| 0 | 0 | 0.9 | QBP | 6.3 | 5.6 | 5.4 | 3.9 | 21.5 | 13.7 | 21.3 | 26.0 | 32.1 | 35.8 | 83.9 | 82.3 |
| | | | QBP1 | 9.0 | 8.6 | 8.9 | 7.5 | 34.0 | 30.2 | 23.2 | 28.7 | 35.9 | 41.3 | 88.1 | 87.6 |
| | | | QLB | 8.0 | 8.3 | 8.8 | 8.2 | 44.4 | 38.7 | 22.3 | 28.7 | 35.8 | 41.9 | 89.7 | 89.0 |
| | | | QLB1 | 7.8 | 8.3 | 8.8 | 8.2 | 44.4 | 38.7 | 22.2 | 28.7 | 35.8 | 41.9 | 89.7 | 89.0 |
| | | | QDR | 8.0 | 8.2 | 9.[illegible] | 8.4 | 40.6 | 36.8 | 23.6 | 28.2 | 35.9 | 41.7 | 89.2 | 88.4 |
| | | | QDR1 | 7.9 | 8.0 | 9.0 | 8.7 | 42.6 | 36.8 | 23.3 | 28.5 | 35.4 | 42.2 | 89.3 | 88.8 |
| | | | QDR2 | 7.4 | 8.4 | 8.2 | 8.2 | 39.0 | 33.2 | 25.8 | 25.7 | 29.8 | 29.8 | 63.3 | 60.2 |
| | | | QBN1 | 6.5 | 6.8 | 6.0 | 5.3 | 19.5 | 12.9 | 8.9 | 10.1 | 9.6 | 12.2 | 91.3 | 86.9 |
| 0.4 | 0 | 0 | QBP | 5.2 | 4.1 | 4.5 | 5.3 | 14.1 | 10.6 | 17.8 | 23.0 | 25.4 | 41.2 | 71.3 | 67.9 |
| | | | QBP1 | 7.2 | 6.1 | 5.5 | 8.3 | 23.1 | 20.2 | 19.9 | 24.7 | 27.7 | 45.4 | 76.0 | 75.6 |
| | | | QLB | 6.3 | 6.0 | 5.8 | 9.0 | 27.8 | 24.9 | 18.9 | 24.7 | 27.6 | 46.5 | 78.0 | 77.6 |
| | | | QLB1 | 5.9 | 5.9 | 5.8 | 9.0 | 27.8 | 24.9 | 18.7 | 24.7 | 27.6 | 46.5 | 78.0 | 77.6 |
| | | | QDR | 7.2 | 6.4 | 6.7 | 10.3 | 28.4 | 24.5 | 19.0 | 25.4 | 27.8 | 46.5 | 77.5 | 77.5 |
| | | | QDR1 | 7.2 | 6.1 | 6.8 | 10.4 | 29.3 | 25.4 | 19.0 | 25.5 | 27.8 | 46.8 | 77.3 | 77.6 |
| | | | QDR2 | 8.1 | 8.2 | 7.6 | 10.2 | 27.4 | 25.1 | 23.3 | 23.9 | 24.1 | 31.0 | 53.1 | 52.1 |
| | | | QBN1 | 4.9 | 5.7 | 6.1 | 7.3 | 10.6 | 6.2 | 6.7 | 7.8 | 7.7 | 15.8 | 57.8 | 47.5 |
| 0.7 | 0 | 0 | QBP | 10.5 | 10.0 | 10.5 | 13.7 | 68.5 | 54.5 | 27.3 | 35.7 | 40.3 | 59.1 | 98.1 | 97.9 |
| | | | QBP1 | 13.9 | 14.0 | 14.4 | 21.1 | 81.2 | 75.0 | 29.2 | 37.9 | 42.8 | 64.8 | 98.7 | 98.8 |
| | | | QLB | 12.7 | 13.6 | 14.4 | 22.9 | 86.5 | 81.0 | 28.3 | 37.3 | 43.2 | 65.3 | 98.1 | 98.9 |
| | | | QLB1 | 12.5 | 13.6 | 14.4 | 22.9 | 86.5 | 81.0 | 27.8 | 37.1 | 43.2 | 65.9 | 98.[illegible] | 98.9 |
| | | | QDR | 16.1 | 15.9 | 16.3 | 25.8 | 86.8 | 81.1 | 30.5 | 40.3 | 44.8 | 67.4 | 99.0 | 98.9 |
| | | | QDR1 | 15.2 | 15.6 | 16.4 | 25.1 | 87.6 | 82.7 | 30.5 | 40.1 | 45.2 | 66.8 | 99.0 | 98.8 |
| | | | QDR2 | 16.5 | 18.1 | 17.7 | 25.4 | 85.9 | 80.0 | 31.1 | 31.7 | 33.6 | 48.6 | 91.6 | 88.9 |
| | | | QBN1 | 11.6 | 15.3 | 14.4 | 29.7 | 63.2 | 48.2 | 16.3 | 24.2 | 27.9 | 55.8 | 99.5 | 99.5 |
| 0.9 | 0 | 0 | QBP | 18.1 | 21.5 | 24.3 | 32.4 | 97.3 | 92.3 | 39.9 | 50.3 | 59.7 | 78.9 | 100 | 100 |
| | | | QBP1 | 21.2 | 27.2 | 29.6 | 42.6 | 99.2 | 98.2 | 42.0 | 53.2 | 62.0 | 81.9 | 100 | 100 |
| | | | QLB | 19.8 | 26.5 | 29.8 | 43.9 | 99.4 | 98.8 | 40.8 | 53.1 | 62.4 | 82.0 | 100 | 100 |
| | | | QLB1 | 19.7 | 26.3 | 29.8 | 43.9 | 99.4 | 98.8 | 40.6 | 53.0 | 62.4 | 82.0 | 100 | 100 |
| | | | QDR | 26.6 | 31.1 | 31.4 | 47.9 | 99.4 | 98.9 | 45.3 | 57.5 | 64.1 | 84.5 | 100 | 100 |
| | | | QDR1 | 26.0 | 31.1 | 31.8 | 46.6 | 99.4 | 98.9 | 45.7 | 57.4 | 64.3 | 84.0 | 100 | 100 |
| | | | QDR2 | 28.1 | 32.8 | 33.6 | 4[illegible].1 | 99.0 | 98.3 | 44.8 | 49.6 | 54.0 | 67.8 | 99.3 | 99.3 |
| | | | QBN1 | 19.0 | 29.0 | 31.6 | 51.9 | 96.7 | 91.4 | 29.7 | 47.0 | 55.5 | 88.0 | 100 | 100 |

Table 2 9 (cont'd)

| Parameters $\phi_{12}$ | $\psi_1$ | $\theta_{12}$ | Portmanteau Tests | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 | m=3 | m=4 | m=5 | m=10 | m=12 | m=15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 6 | -0 5 | QBP | 15 4 | 11 5 | 11 3 | 9.0 | 20 3 | 15 5 | 41.3 | 40 3 | 40 7 | 48 9 | 79.0 | 74 8 |
| | | | QBP1 | 21 1 | 15 9 | 13 5 | 14 0 | 31 7 | 29 0 | 44 2 | 43 2 | 43 7 | 55 2 | 83 0 | 82 4 |
| | | | QLB | 18 1 | 15 4 | 13 5 | 14 6 | 38 1 | 33 0 | 42 9 | 42 8 | 43.3 | 56 1 | 83 5 | 84 0 |
| | | | QLB1 | 17 7 | 15 2 | 13 4 | 14.6 | 38 1 | 33 0 | 41 9 | 42.6 | 43.2 | 56 1 | 83 5 | 84.0 |
| | | | QDR | 18 8 | 15 2 | 14.2 | 15 0 | 36.3 | 31 8 | 44 5 | 44 3 | 44.4 | 55 7 | 83 7 | 84.1 |
| | | | QDR1 | 16 9 | 14 1 | 14.3 | 15 6 | 40 0 | 34 1 | 43 5 | 43 9 | 43.8 | 55.5 | 83 9 | 84 5 |
| | | | QDR2 | 16.7 | 13 5 | 14.2 | 15 6 | 35 4 | 33 2 | 34.5 | 32.5 | 31 6 | 38 7 | 58 1 | 57 6 |
| | | | QBN1 | 14 8 | 17 4 | 17 2 | 12 3 | 19 1 | 11 3 | 31.2 | 32.7 | 33.8 | 30 9 | 70 9 | 68.1 |
| 0 | -0 5 | 0 6 | QBP | 9.5 | 7 7 | 7.0 | 5.0 | 16 9 | 11.8 | 28.3 | 31.7 | 38 4 | 39 5 | 77 4 | 73 3 |
| | | | QBP1 | 13 1 | 11.7 | 10.5 | 8.6 | 28.7 | 26 4 | 30 4 | 34.1 | 40 4 | 44.0 | 82 7 | 81 6 |
| | | | QLB | 11.2 | 11 1 | 10.4 | 9.4 | 35.2 | 31.7 | 29 5 | 33 7 | 40 1 | 44.7 | 85.2 | 83 3 |
| | | | QLB1 | 10.8 | 11 0 | 10.3 | 9.4 | 35.2 | 31 7 | 29.1 | 33.6 | 40 1 | 44.7 | 85 2 | 83.3 |
| | | | QDR | 12.5 | 11.5 | 11.9 | 9.5 | 34.8 | 31 8 | 29.9 | 34.0 | 41.7 | 45 1 | 85 0 | 82 7 |
| | | | QDR1 | 12.9 | 11.8 | 11.7 | 9.5 | 35 2 | 31 1 | 30 3 | 34 0 | 41.7 | 45 3 | 84 8 | 83 0 |
| | | | QDR2 | 12.9 | 11 0 | 11 5 | 10 7 | 31.1 | 28.1 | 31.1 | 30.4 | 31 0 | 39 5 | 55 2 | 51 9 |
| | | | QBN1 | 8.4 | 8 3 | 7 0 | 4 8 | 14 1 | 10 1 | 16 0 | 17 0 | 15 0 | 14 1 | 72.0 | 74.4 |
| 0.6 | -0 5 | 0 | QBP | 12.5 | 11.4 | 10.9 | 12.4 | 47 3 | 35 0 | 32.3 | 37 8 | 40.7 | 56 3 | 94 0 | 93 1 |
| | | | QBP1 | 16.6 | 15.0 | 14.1 | 18 5 | 66 1 | 57 9 | 34.4 | 40 0 | 43 4 | 61.8 | 95 4 | 95 7 |
| | | | QLB | 14.4 | 13.7 | 14.4 | 20 0 | 73 7 | 66 2 | 33.7 | 39 6 | 43 2 | 62 2 | 96 6 | 96.0 |
| | | | QLB1 | 14.2 | 13.5 | 14.3 | 20.0 | 73 7 | 66 2 | 32 9 | 39.4 | 43.2 | 62 2 | 96 6 | 96 0 |
| | | | QDR | 18.0 | 16.8 | 16.1 | 21 8 | 72.9 | 66 3 | 35.0 | 40 6 | 44 0 | 62.5 | 96.2 | 95.7 |
| | | | QDR: | 18.0 | 16.4 | 16 6 | 21 8 | 73 4 | 67 9 | 35.3 | 40 8 | 43.9 | 62.3 | 96.4 | 95 7 |
| | | | QDR2 | 18.4 | 16 6 | 16.6 | 21.5 | 70.3 | 63 7 | 32.5 | 33 9 | 32.5 | 42.3 | 81 4 | 80.1 |
| | | | QBN1 | 6.9 | 11 5 | 11 6 | 20 9 | 32 0 | 25.5 | 14.7 | 22 7 | 25.4 | 42.6 | 93.4 | 95.7 |
| -0.5 | 0.6 | 0 | QBP | 16 5 | 12 7 | 12 2 | 8.9 | 27 4 | 20 3 | 42.0 | 45 2 | 48.2 | 47 9 | 85 5 | 83 9 |
| | | | QBP1 | 21.8 | 17 8 | 16 6 | 13 5 | 41.5 | 35.3 | 45.6 | 47.8 | 52 3 | 53.7 | 89.1 | 88 4 |
| | | | QLB | 19.7 | 16.7 | 16.9 | 14 4 | 48.8 | 42 4 | 43 5 | 47 5 | 52.3 | 54.1 | 89.9 | 89 3 |
| | | | QLB1 | 18.8 | 16.5 | 16.8 | 14.4 | 48 8 | 42 4 | 43.0 | 47 5 | 52.2 | 54 1 | 89 9 | 89 3 |
| | | | QDR | 16.6 | 14.0 | 15.1 | 13.3 | 45 8 | 39 2 | 41.5 | 45.7 | 50.0 | 52.8 | 89 5 | 89 3 |
| | | | QDR1 | 14.5 | 14.0 | 14 4 | 13.9 | 45 8 | 40 7 | 40.8 | 45 5 | 49 2 | 53.7 | 89 3 | 88.9 |
| | | | QDR2 | 15.9 | 12.6 | 14.1 | 14.1 | 43.0 | 37 6 | 33.7 | 34 6 | 35.1 | 35.6 | 64 8 | 62.6 |
| | | | QBN1 | 15.4 | 14.4 | 12.5 | 10.4 | 20.4 | 17.8 | 28.7 | 30 8 | 28.1 | 26.4 | 83.8 | 86.2 |

(ii) The empirical power of QLB and QLB1 is almost identical. This is true irrespective of the choice of the number of residual autocorrelations, the fitted models, the sample size, and the chosen values of parameters.

(iii) Among all tests investigated, QDR2 and QBN1 perform the worst. This is especially evident in the case of QBN1. For instance, when the data were fitted to the AR(1) and AR(2) models with n=50, the difference in empirical power between QBN1 and the other portmanteau statistics like QBP1, QLB, QLB1, QDR and QDR1, can be as large as 41.7% (see the AR(2) model with $\Phi_{12}$=0.6 and $\psi_1$=-0.5). However, this difference is considerably reduced when n increases from 50 to 100.

(iv) Unlike the case of non-seasonal data, the performance of QBP1 is not as good as QLB. This may be attributed, in part, to the weighting criteria of the residual autocorrelations used in QBP1 and QLB [see Ljung and Box (1978), p. 301]. While QBP1 gives equal weight to residual autocorrelations at any given time lag, QLB gives more emphasis to residual autocorrelations at larger time lags. When seasonal or higher-order autocorrelations are expected, it would be advantageous to use QLB in the said situation.

Although it is not intended that a rigorous comparison of the results obtained from seasonal data be made with those from non-seasonal data, two points seem to deserve special attention. First, the most important feature is the contrast between the choice of the optimal value of m when different types of data (seasonal vs. non-seasonal) are used. The bulk of the simulation evidence presented here partially supports Ljung's (1986) assertion that when the true data-generating process can be modelled as a simple low-order ARMA model, the portmanteau tests considered are powerful in detecting model misspecification when the number of residual autocorrelations is small. However, this assertion needs to be modified to account for the nature of the data used. The findings reported in Tables 2.8-2.9 clearly illustrate the importance of seasonal data on the choice of m. In this circumstance, it would be useful to choose m on the basis of the frequency of the data. Second, it is evident that the first few autocorrelations, as in the seasonal case, carry little information about model adequacy. This suggests that a good empirical strategy would be to remove the redundant autocorrelations. One would, therefore, expect improvements in the empirical power of the portmanteau tests considered.

## 2.5 Concluding Remarks

In this chapter, three separate Monte Carlo experiments were conducted to investigate the finite-sample distribution of eight portmanteau statistics of model adequacy, with special attention being paid to their empirical significance levels, means and variances. Additionally, the empirical power of these tests is investigated in the cases of seasonal and non-seasonal data. Based on the results reported above, several features that emerged are worth mentioning:

(i) The accuracy of the size of the portmanteau statistics considered depends not only on the choice of m but also on the underlying parameter values. With the exception of QLB1, size distortions can be substantial for portmanteau tests in models with parameter values approaching the boundary of the stationary or invertible region. This poor empirical performance becomes more serious when m is small.

(ii) QLB1 has estimated significance levels which are much closer to the nominal levels than those of the other portmanteau statistics for $m \le 10$ and parameter values which are large. In this respect, QLB1 is considered to be the most reliable (or preferred) test. Also, QLB1 can be used for model diagnostics even when m is as small as 1. This gives QLB1 the added advantage over the other portmanteau statistics of being valid over a wider range of m.

(iii) The basic objective of the corrections suggested by Dufour and Roy (1986) is to obtain more reliable statistics from the point of view of controlling the empirical level. However, judging by the simulation results reported above, the extra computational effort does not yield clear gains in terms of a conformity between nominal and actual significance levels.

(iv) As for the empirical power of the portmanteau statistics examined, the simulation results reveal that each has good power when m is small and the underlying data-generating process can be modelled as low-order ARMA(p,q) models. However, this power property ceases to hold when seasonal data are used.

# CHAPTER THREE

# A MONTE CARLO STUDY OF THE FINITE-SAMPLE DISTRIBUTION OF SOME TESTS OF NESTED AND SEPARATE HYPOTHESES: THE CASE OF UNIVARIATE TIME SERIES MODELS

## 3.1. Introduction

Over the past twenty years many methods have been proposed in the time series literature to complement the portmanteau-test approach. The most frequently employed method is that of Godfrey's (1979) Lagrange multiplier (LM) test. The main advantage of the LM test is that it is asymptotically optimal when the null model is nested within the alternative. Moreover, for testing against simple alternatives, the empirical performance of the LM test can be as good as that of the modified Ljung-Box portmanteau statistic but requires fewer computations [see Godfrey and Tremayne (1988) and Hall and McAleer (1989)].

McAleer et al. (1988) have recently suggested several computationally straightforward procedures for testing the adequacy of an ARMA(p,q) model. Contrary to Godfrey's (1979) LM test, their procedures are asymptotically optimal against alternatives which are separate from the null specification. Because of the treatment of the alternative hypothesis and its relationship to the null model, the McAleer et al. (1988) tests have been referred to as tests of separate hypotheses (or tests of non-nested hypotheses).

McAleer et al. (1988) conducted Monte Carlo experiments to investigate the finite-sample properties of the tests of separate hypotheses. Their main simulation findings can be summarized as follows: (i) The empirical size of the tests is generally close to the nominal level when the sample size is moderately large. (ii) The tests have high empirical power against separate alternatives. (iii) The tests can be more powerful than Godfrey's LM test, even in cases where the latter is supposed to be dominant. In a follow-up simulation study, Hall and McAleer (1989) confirmed these findings.

The principal objective of this chapter is to provide a detailed assessment of the work of Godfrey (1979), Hall and McAleer (1989) and McAleer et al. (1988). Several features characterize the chapter. First, as shown by Newbold (1980), the LM test of an assumed model against an alternative of ARMA(p+r,q) or ARMA(p,q+r) is equivalent to the test based on the first r residual autocorrelations from the fitted model. It follows that, in the case when r is chosen to be large, the portmanteau and LM tests coincide [see also Ljung (1986) and Godfrey and Tremayne (1988)]. Due to this equivalence, it is important to investigate the impact of the choice of r on the finite-sample distribution of the LM test. This issue is also of considerable importance since the simulation results of Hall and McAleer (1989) and McAleer et al. (1988) indicate that the $\chi^2$ approximation to the distribution of the LM test may not hold for a large value of r.

Second, an investigation of the empirical power of the LM test is carried out for both seasonal and non-seasonal data. As well, it is known that the test is usually conducted in such a way that the value of r is selected on the basis of the frequency of the data. However, such a selection strategy, as cogently argued by Godfrey and Tremayne (1988), suffers from a potentially serious problem, namely it is not always the case that all restrictions considered are relevant to model diagnostics. In this chapter, we will show that the LM test can easily be modified to alleviate this problem.

Third, although the simulation evidence on the finite-sample properties of the tests of separate hypotheses is encouraging, there are a number of relevant issues which have not been fully addressed in Hall and McAleer (1989) and McAleer et al. (1988). In light of this, the present chapter is aimed at a critical review of the finite-sample performance of these tests.

The remainder of this chapter is organized as follows: Section 3.2 presents the test procedures. Section 3.3 discusses the design of the Monte Carlo experiments and reports the main simulation results regarding the empirical performance of the LM test. Section 3.4 gives comments on the finite-sample distribution of the tests of separate hypotheses. Section 3.5 offers some concluding remarks.

## 3.2 Test Procedures

In order to examine the adequacy of the ARMA(p,q) model given in (2.2.1),

$$\phi(B)w_t = \theta(B)a_t,$$

it is important to define the variables:

$$w^*_t = -w_t + \hat{\theta}_1 w^*_{t-1} + \dots + \hat{\theta}_q w^*_{t-q}, \tag{3.2.1}$$

and $$\hat{a}^*_t = \hat{a}_t + \hat{\theta}_1 \hat{a}^*_{t-1} + \dots + \hat{\theta}_q \hat{a}^*_{t-q}, \tag{3.2.2}$$

for t=1,...,n and with $w^*_t = \hat{a}^*_t = 0$ for non-positive t. For purposes of testing (2.2.1) against an ARMA(p+r,q), Godfrey (1979) suggested a two-step procedure to calculate the LM statistic. First, we obtain the residuals, $\hat{a}_t$, from (2.2.1). Second, we perform the following OLS regression:

$$\hat{a}_t = \alpha_1 w^*_{t-1} + \dots + \alpha_{p+r} w^*_{t-p-r} + \beta_1 \hat{a}^*_{t-1} + \dots + \beta_q \hat{a}^*_{t-q} + u_t. \tag{3.2.3}$$

The computed value of the LM test, denoted by LM(r), is given by

$$LM(r) = nR^2, \tag{3.2.4}$$

that is, the sample size times the coefficient of determination from (3.2.3). If the model (2.2.1) is correct, the quantity LM(r) is asymptotically distributed as $\chi^2$ with r degrees of freedom. For purposes of testing (2.2.1) against an ARMA(p,q+r), we repeat the similar steps, except that (3.2.3) is replaced by

$$\hat{a}_t = \alpha_1 w^*_{t-1} + \ldots + \alpha_p w^*_{t-p} + \beta_1 \hat{a}^*_{t-1} + \ldots + \beta_{q+r} \hat{a}^*_{t-q-r} + v_t. \tag{3.2.5}$$

The LM(r) statistic is computed as n times $R^2$ from (3.2.5) and is distributed as $\chi^2(r)$ when (2.2.1) is the correct model.

Poskitt and Tremayne (1980) have shown that if an ARMA(p,q) is the null hypothesis, then an LM(r) test statistic against an ARMA(p+r,q) alternative is numerically equivalent to testing an ARMA(p,q+r) alternative. Thus, since these are equivalent, only one of the two $nR^2$ statistics need be used for testing additional r restrictions [see Harvey (1984, pp. 156-157) for a detailed explanation of this issue].

For the purpose of testing an AR(p) null against an MA(q) alternative, McAleer et al. (1988) proposed two test statistics (the separate model test and the separate prediction test) which, in terms of calculations, are similar to those of Godfrey's LM test. Specifically, the test procedures involve a two-step estimation: First, we obtain the residuals, $\hat{a}_t$ and $\tilde{a}_t$, from the AR(p) and MA(q) models, respectively. Second, we perform the following OLS regressions:

$$\hat{a}_t = \alpha_1 w_{t-1} + \dots + \alpha_p w_{t-p} + \alpha_{p+1}\tilde{a}_{t-1} + \dots + \alpha_{p+q}\tilde{a}_{t-q} + \mu_t \tag{3.2.6}$$

and

$$\hat{a}_t = \alpha_1 w_{t-1} + \dots + \alpha_p w_{t-p} + \alpha_{p+1}\tilde{w}_t + \mu_t, \tag{3.2.7}$$

where $\tilde{w}_t$ are the fitted values of the MA(q) process. The separate model test statistic is given by

$$SM(q) = nR^2,$$

where $R^2$ is the coefficient determination from (3.2.6). Using the fact that $\tilde{w}_t = -(\tilde{\theta}_1 B - \dots - \tilde{\theta}_q B^q)a_t$, where $\tilde{\theta}$'s are the least squares (or maximum likelihood) estimates of the coefficients of the MA(q) process, McAleer et al. (1988) recommend the separate prediction test which is denoted as

$$SP(q) = nR^2,$$

where $R^2$ is the coefficient determination from (3.2.7). If the AR model is the true data generation process, SM(q) and SP(q) are asymptotically distributed as $\chi^2(q)$ and $\chi^2(1)$, respectively. As indicated by McAleer et al. (1988), SM(q)=SP(q) when q=1.

For testing an MA(q) against an AR(p) alternative, it is necessary to define the following variable:

$$\tilde{a}^{*}_{t} = \tilde{a}_{t} + \tilde{\theta}_{1}\tilde{a}^{*}_{t-1} + \dots + \tilde{\theta}_{q}\tilde{a}^{*}_{t-q}. \qquad (3.2.8)$$

The separate model [SM(p)] and separate prediction [SP(p)] test statistics are based on the following two-step regressions:

$$\tilde{a}_{t} = \beta_{1}\tilde{a}^{*}_{t-1} + \dots + \beta_{q}\tilde{a}^{*}_{t-q} + \beta_{q+1}w_{t-1} + \dots + \beta_{q+p}w_{t-p} + v_{t} \qquad (3.2.9)$$

and

$$\tilde{a}_{t} = \beta_{1}\tilde{a}^{*}_{t-1} + \dots + \beta_{q}\tilde{a}^{*}_{t-q} + \beta_{q+1}\hat{w}_{t} + v_{t}, \qquad (3.2.10)$$

where $\hat{w}_t$ are the fitted values of the AR(p) process. The separate model and separate prediction test statistics in the respective equations (3.2.9) and (3.2.10) are computed as $nR^2$. If the MA(q) process is the true model, SM(p) and SP(p) are asymptotically distributed as $\chi^2(p)$ and $\chi^2(1)$, respectively. It must be noted that, from the definitions of the predictions and residuals, SM(p)=SP(p) when p=1.

## 3.3 Experimental Designs and Some Monte Carlo Results of the Finite-Sample Distribution of LM Tests

In examining the effect of the choice of r as well as the nature of the data on the finite-sample distribution of the LM test, three simulation experiments were employed. The first experiment focussed on empirical significance levels, means and variances of the LM test for various choices of r ($1 \le r \le 5$). For purposes of interpreting the simulation results, we worked with the roots of the lag polynomials in (2.2.1). All data were generated from a few special cases of an AR(2) process, $(1-\pi_1 B)(1-\pi_2 B)w_t = a_t$, with proper zero restrictions on the roots of the AR process $(\pi_1, \pi_2)$. The starting value $w_0$ was set at zero and the first 50 observations were discarded to minimize the influence of the initial value. Estimates of the significance levels ($\alpha$=1%, 5% and 10%), means and variances of this test were based on 1000 replications. The sample size, n, was 25, 50, 75 and 100.

The second and third simulation experiments evaluated the empirical power of the LM test when the data were generated from both seasonal and non-seasonal ARMA models. In the case of non-seasonal data, all data were first generated from some selected cases of an ARMA(2,2) process, $(1-\pi_1 B)(1-\pi_2 B)w_t = (1-\psi_1 B)(1-\psi_2 B)a_t$, and were subsequently fitted to the AR(1) and AR(2) models. In many instances, the chosen values of parameters $(\pi_1, \pi_2, \psi_1, \psi_2)$ were obtained from McAleer et al. (1988) and Hall and McAleer (1989).

As regards the case of seasonal data, the set-up of the simulation experiment is similar to that of the non-seasonal one.

However, in order to allow for the impact of the nature of the data on the empirical power of the LM test, we use the seasonal ARMA models given in Chapter 2. Since one of the objectives is to examine the effect of the choice of r on the ability of the LM test to detect model misspecfication, r was allowed to vary: $1 \le r \le 5$ for data generated from non-seasonal models, and $1 \le r \le 12$ for data generated from seasonal models. Finally, for the last two experiments, the sample size was set at 50 and 100.

Table 3.1 provides a summary of the simulation results on the empirical significance levels, means and variances of the LM test in commonly used sample sizes. On the basis of these results, several points seem to merit attention:

(i) The results of the empirical significance levels of the LM test are close to the nominal levels only when (1) $r \le 2$ and $n \le 50$, and (2) $r \le 4$ and $n=75$. For $n=100$, the value of r can be chosen as large as 5. The main conclusion that emerges from these results is that the value of r should be kept small relative to the sample size when computing the statistic. This is particularly important when the sample size is not more than 75. This conclusion holds when the nominal significance levels are 5% and 10%.

(ii) When the assumed model follows an AR(1) process and the

Table 3.1
Empirical Significance Levels of the LM Test for the AR(2) Model
$(1-\pi_1)(1-\pi_2)w_t=a_t$; n=25

| Parameters $\pi_1$ | $\pi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
|---|---|---|---|---|---|---|
| | | | α=1% | | | |
| 0.1 | 0 | 0.6 | 0.3 | 0 | 0 | 0 |
| 0.4 | 0 | 0.4 | 0.4 | 0.1 | 0 | 0 |
| 0.7 | 0 | 0.5 | 0.1 | 0.1 | 0.1 | 0 |
| 0.9 | 0 | 0.5 | 0.3 | 0.1 | 0.1 | 0 |
| 0.99 | 0 | 0.5 | 0.4 | 0.2 | 0.2 | 0 |
| 0.5 | 0.5 | 0.6 | 0.3 | 0.2 | 0.1 | 0.1 |
| 0.8 | 0.2 | 0.3 | 0.2 | 0.1 | 0 | 0.1 |
| 0.8 | 0.5 | 0.3 | 0.4 | 0 | 0 | 0 |
| 0.8 | 0.8 | 0.4 | 0.3 | 0.1 | 0 | 0.1 |
| | | | α=5% | | | |
| 0.1 | 0 | 4.4 | 4.1 | 3.1 | 2.2 | 1.2 |
| 0.4 | 0 | 4.2 | 2.3 | 1.9 | 1.6 | 0.8 |
| 0.7 | 0 | 3.3 | 2.4 | 2.6 | 1.2 | 1.4 |
| 0.9 | 0 | 4.5 | 3.4 | 2.9 | 1.6 | 1.1 |
| 0.99 | 0 | 4.9 | 4.2 | 3.2 | 2.2 | 1.2 |
| 0.5 | 0.5 | 3.9 | 3.3 | 3.1 | 2.1 | 1.9 |
| 0.8 | 0.2 | 3.1 | 2.9 | 3.3 | 1.6 | 1.7 |
| 0.8 | 0.5 | 3.6 | 2.8 | 2.5 | 2.0 | 2.0 |
| 0.8 | 0.8 | 3.6 | 3.6 | 3.0 | 1.8 | 1.9 |
| | | | α=10% | | | |
| 0.1 | 0 | 9.1 | 7.9 | 6.9 | 5.6 | 4.8 |
| 0.4 | 0 | 8.5 | 8.1 | 6.6 | 4.7 | 4.6 |
| 0.7 | 0 | 8.9 | 7.1 | 6.2 | 5.1 | 3.7 |
| 0.9 | 0 | 9.7 | 8.2 | 6.8 | 6.1 | 4.8 |
| 0.99 | 0 | 9.4 | 9.2 | 7.9 | 6.1 | 5.1 |
| 0.5 | 0.5 | 8.9 | 8.2 | 6.8 | 6.2 | 5.5 |
| 0.8 | 0.2 | 7.9 | 8.0 | 6.9 | 6.5 | 5.4 |
| 0.8 | 0.5 | 9.4 | 7.9 | 7.2 | 6.0 | 5.6 |
| 0.8 | 0.8 | 9.6 | 8.1 | 6.8 | 6.2 | 5.4 |
| | | | Means | | | |
| 0.1 | 0 | 0.97 | 1.86 | 2.67 | 3.52 | 4.41 |
| 0.4 | 0 | 0.93 | 1.85 | 2.66 | 3.50 | 4.41 |
| 0.7 | 0 | 0.92 | 1.84 | 2.69 | 3.52 | 4.42 |
| 0.9 | 0 | 0.95 | 1.88 | 2.76 | 3.56 | 4.44 |
| 0.99 | 0 | 0.97 | 1.95 | 2.82 | 3.66 | 4.51 |
| 0.5 | 0.5 | 0.94 | 1.85 | 2.71 | 3.62 | 4.46 |
| 0.8 | 0.2 | 0.88 | 1.86 | 2.72 | 3.57 | 4.47 |
| 0.8 | 0.5 | 0.91 | 1.87 | 2.72 | 3.59 | 4.43 |
| 0.8 | 0.8 | 0.95 | 1.87 | 2.71 | 3.62 | 4.45 |
| | | | Variances | | | |
| 0.1 | 0 | 1.68 | 2.99 | 4.03 | 4.97 | 5.84 |
| 0.4 | 0 | 1.54 | 2.83 | 3.90 | 4.77 | 5.81 |
| 0.7 | 0 | 1.52 | 2.83 | 3.87 | 4.75 | 5.81 |
| 0.9 | 0 | 1.66 | 3.09 | 4.09 | 5.03 | 6.10 |
| 0.99 | 0 | 1.71 | 3.31 | 4.42 | 5.50 | 6.36 |
| 0.5 | 0.5 | 1.56 | 2.94 | 4.21 | 5.39 | 6.44 |
| 0.8 | 0.2 | 1.41 | 2.94 | 4.25 | 5.32 | 6.61 |
| 0.8 | 0.5 | 1.44 | 2.86 | 4.10 | 5.32 | 6.56 |
| 0.8 | 0.8 | 1.58 | 2.97 | 4.12 | 5.34 | 6.41 |

Table 3.1 (cont'd); n=50

| Parameters | | | α=1% | | | |
|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0.1 | 0 | 0.6 | 0.7 | 0.6 | 045 | 0.4 |
| 0.4 | 0 | 1.0 | 0.6 | 0.3 | 0.4 | 0.3 |
| 0.7 | 0 | 0.8 | 0.7 | 0.3 | 0.2 | 0.3 |
| 0.9 | 0 | 1.0 | 0.7 | 0.5 | 0.4 | 0.4 |
| 0.99 | 0 | 1.2 | 0.7 | 0.7 | 0.3 | 0.5 |
| 0.5 | 0.5 | 0.4 | 0.4 | 0.5 | 0.2 | 0.4 |
| 0.8 | 0.2 | 0.4 | 0.5 | 0.2 | 0.2 | 0.5 |
| 0.8 | 0.5 | 0.6 | 0.6 | 0.2 | 0.3 | 0.4 |
| 0.8 | 0.8 | 0.7 | 0.7 | 0.2 | 0.4 | 0.3 |
| | | | α=5% | | | |
| 0.1 | 0 | 5.3 | 3.3 | 3.1 | 3.1 | 3.0 |
| 0.4 | 0 | 4.6 | 3.3 | 3.1 | 2.5 | 2.8 |
| 0.7 | 0 | 4.7 | 4.1 | 2.8 | 2.8 | 2.3 |
| 0.9 | 0 | 5.1 | 4.9 | 3.4 | 3.4 | 2.1 |
| 0.99 | 0 | 5.2 | 4.9 | 3.5 | 3.4 | 3.0 |
| 0.5 | 0.5 | 4.6 | 4.0 | 2.4 | 2.6 | 3.0 |
| 0.8 | 0.2 | 4.7 | 4.1 | 2.9 | 3.0 | 2.9 |
| 0.8 | 0.5 | 4.6 | 4.2 | 3.0 | 2.9 | 2.5 |
| 0.8 | 0.8 | 4.6 | 4.8 | 2.7 | 2.5 | 2.6 |
| | | | α=10% | | | |
| 0.1 | 0 | 9.4 | 9.3 | 7.9 | 7.4 | 7.3 |
| 0.4 | 0 | 10.1 | 9.7 | 7.1 | 6.7 | 6.1 |
| 0.7 | 0 | 10.7 | 10.0 | 7.4 | 6.5 | 6.3 |
| 0.9 | 0 | 10.2 | 9.9 | 9.6 | 7.8 | 7.5 |
| 0.99 | 0 | 10.0 | 10.2 | 9.4 | 8.6 | 7.3 |
| 0.5 | 0.5 | 9.4 | 9.2 | 7.8 | 6.2 | 6.6 |
| 0.8 | 0.2 | 8.9 | 9.5 | 8.1 | 6.6 | 6.2 |
| 0.8 | 0.5 | 9.2 | 9.9 | 7.5 | 6.5 | 6.4 |
| 0.8 | 0.8 | 9.8 | 10.7 | 7.8 | 6.5 | 6.3 |
| | | | Means | | | |
| $\pi_1$ | $\pi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0.1 | 0 | 1.01 | 1.91 | 2.85 | 3.77 | 4.67 |
| 0.4 | 0 | 0.98 | 1.92 | 2.84 | 3.78 | 4.63 |
| 0.7 | 0 | 0.98 | 1.94 | 2.85 | 3.77 | 4.62 |
| 0.9 | 0 | 0.99 | 1.95 | 2.87 | 3.77 | 4.66 |
| 0.99 | 0 | 1.00 | 1.99 | 2.90 | 3.81 | 4.72 |
| 0.5 | 0.5 | 0.98 | 1.92 | 2.83 | 3.72 | 4.61 |
| 0.8 | 0.2 | 0.98 | 1.92 | 2.83 | 3.74 | 4.63 |
| 0.8 | 0.5 | 0.94 | 1.93 | 2.80 | 3.77 | 4.60 |
| 0.8 | 0.8 | 0.94 | 1.96 | 2.81 | 3.80 | 4.62 |
| | | | Variances | | | |
| 0.1 | 0 | 1.91 | 3.43 | 5.00 | 6.33 | 7.88 |
| 0.4 | 0 | 1.84 | 3.46 | 4.68 | 6.00 | 7.37 |
| 0.7 | 0 | 1.87 | 3.46 | 4.50 | 5.95 | 7.23 |
| 0.9 | 0 | 2.08 | 3.69 | 5.04 | 6.44 | 7.72 |
| 0.99 | 0 | 2.11 | 3.80 | 5.17 | 6.68 | 7.94 |
| 0.5 | 0.5 | 1.69 | 3.19 | 4.67 | 5.83 | 7.58 |
| 0.8 | 0.2 | 1.66 | 3.24 | 4.75 | 6.01 | 7.68 |
| 0.8 | 0.5 | 1.62 | 3.45 | 4.67 | 5.97 | 7.44 |
| 0.8 | 0.8 | 1.66 | 3.55 | 4.61 | 5.98 | 7.26 |

Table 3.1 (cont'd); n=75

| Parameters | | α=1% | | | | |
|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0.1 | 0 | 1.3 | 0.6 | 0.6 | 1.1 | 1.0 |
| 0.4 | 0 | 1.1 | 1.0 | 0.7 | 1.2 | 1.3 |
| 0.7 | 0 | 0.9 | 0.8 | 0.9 | 0.9 | 0.7 |
| 0.9 | 0 | 1.2 | 0.9 | 0.9 | 0.9 | 0.8 |
| 0.99 | 0 | 1.0 | 0.9 | 0.6 | 1.0 | 1.2 |
| 0.5 | 0.5 | 1.0 | 1.2 | 1.1 | 0.9 | 1.0 |
| 0.8 | 0.2 | 0.8 | 1.2 | 1.1 | 0.7 | 0.7 |
| 0.8 | 0.5 | 0.9 | 0.9 | 0.9 | 1.1 | 0.6 |
| 0.8 | 0.8 | 0.5 | 0.8 | 0.8 | 0.6 | 0.8 |
| | | | α=5% | | | |
| 0.1 | 0 | 4.7 | 3.9 | 4.3 | 4.2 | 3.3 |
| 0.4 | 0 | 4.9 | 4.4 | 4.4 | 4.0 | 3.5 |
| 0.7 | 0 | 4.8 | 4.8 | 5.3 | 4.3 | 4.4 |
| 0.9 | 0 | 5.9 | 5.2 | 4.7 | 4.3 | 3.8 |
| 0.99 | 0 | 5.4 | 4.5 | 4.5 | 3.9 | 3.7 |
| 0.5 | 0.5 | 5.6 | 5.6 | 4.3 | 3.7 | 3.3 |
| 0.8 | 0.2 | 5.0 | 5.4 | 4.3 | 3.7 | 3.8 |
| 0.8 | 0.5 | 4.4 | 5.3 | 4.0 | 3.4 | 3.4 |
| 0.8 | 0.8 | 4.4 | 4.5 | 4.5 | 3.5 | 3.7 |
| | | | α=10% | | | |
| 0.1 | 0 | 9.4 | 9.9 | 8.6 | 8.8 | 8.1 |
| 0.4 | 0 | 8.8 | 9.3 | 9.6 | 8.6 | 7.6 |
| 0.7 | 0 | 10.1 | 9.8 | 10.5 | 9.4 | 7.6 |
| 0.9 | 0 | 9.8 | 10.3 | 10.8 | 9.0 | 8.6 |
| 0.99 | 0 | 10.7 | 10.3 | 10.3 | 8.7 | 8.2 |
| 0.5 | 0.5 | 10.0 | 10.8 | 9.5 | 9.2 | 7.5 |
| 0.8 | 0.2 | 9.9 | 10.2 | 9.0 | 8.7 | 8.5 |
| 0.8 | 0.5 | 9.2 | 10.2 | 8.3 | 9.1 | 7.8 |
| 0.8 | 0.8 | 10.3 | 9.5 | 9.4 | 8.8 | 7.3 |
| | | | Means | | | |
| 0.1 | 0 | 1.00 | 1.97 | 2.96 | 3.88 | 4.77 |
| 0.4 | 0 | 0.99 | 1.99 | 2.97 | 3.88 | 4.75 |
| 0.7 | 0 | 1.01 | 2.01 | 2.99 | 3.92 | 4.79 |
| 0.9 | 0 | 1.06 | 2.05 | 3.03 | 3.98 | 4.86 |
| 0.99 | 0 | 1.07 | 2.06 | 3.02 | 4.00 | 4.89 |
| 0.5 | 0.5 | 1.00 | 2.02 | 2.92 | 3.82 | 4.72 |
| 0.8 | 0.2 | 1.00 | 2.02 | 2.94 | 3.85 | 4.74 |
| 0.8 | 0.5 | 0.98 | 2.02 | 2.92 | 3.85 | 4.73 |
| 0.8 | 0.8 | 0.97 | 2.00 | 2.92 | 3.87 | 4.74 |
| | | | Variances | | | |
| 0.1 | 0 | 2.07 | 3.74 | 5.78 | 7.31 | 8.64 |
| 0.4 | 0 | 1.98 | 3.80 | 5.79 | 7.42 | 8.71 |
| 0.7 | 0 | 2.02 | 3.95 | 5.94 | 7.51 | 8.90 |
| 0.9 | 0 | 2.09 | 3.94 | 5.71 | 7.22 | 8.66 |
| 0.99 | 0 | 2.05 | 3.78 | 5.32 | 7.30 | 8.61 |
| 0.5 | 0.5 | 2.14 | 4.29 | 5.76 | 7.28 | 8.69 |
| 0.8 | 0.2 | 2.13 | 4.15 | 5.61 | 7.09 | 8.56 |
| 0.8 | 0.5 | 1.83 | 3.82 | 5.33 | 6.84 | 8.31 |
| 0.8 | 0.8 | 2.14 | 3.77 | 5.34 | 7.11 | 8.64 |

Table 3.1 (cont'd); n=100

| Paramters | | | α=1% | | | |
|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0.1 | 0 | 1.1 | 1.2 | 1.1 | 0.5 | 0.7 |
| 0.4 | 0 | 1.0 | 1.3 | 1.2 | 0.7 | 0.5 |
| 0.7 | 0 | 0.9 | 1.2 | 0.8 | 0.7 | 0.6 |
| 0.9 | 0 | 1.2 | 1.0 | 0.8 | 0.6 | 0.5 |
| 0.99 | 0 | 1.4 | 1.4 | 0.8 | 0.7 | 0.8 |
| 0.5 | 0.5 | 1.4 | 1.3 | 1.0 | 0.9 | 1.1 |
| 0.8 | 0.2 | 1.2 | 1.3 | 1.4 | 0.8 | 0 9 |
| 0.8 | 0.5 | 1.2 | 1.6 | 1.0 | 0.7 | 0.5 |
| 0.8 | 0.8 | 0.6 | 1.3 | 1.2 | 1.0 | 0.5 |
| | | | α=5% | | | |
| 0.1 | 0 | 4.3 | 4.2 | 4.6 | 4.5 | 4.4 |
| 0.4 | 0 | 4.0 | 4.8 | 4.6 | 4.9 | 4.1 |
| 0.7 | 0 | 4.8 | 4.8 | 5.1 | 4.3 | 4.3 |
| 0.9 | 0 | 5.4 | 5.7 | 4.8 | 4.7 | 4.2 |
| 0.99 | 0 | 5.6 | 4.7 | 5.4 | 5.1 | 4.6 |
| 0.5 | 0.5 | 5.2 | 5.0 | 4.6 | 4.6 | 5.0 |
| 0.8 | 0.2 | 5.2 | 5.3 | 5.4 | 4.2 | 5.1 |
| 0.8 | 0.5 | 4.6 | 4.9 | 4.8 | 4.6 | 4.5 |
| 0.8 | 0.8 | 4.2 | 4.6 | 4.0 | 4.2 | 3.6 |
| | | | α=10% | | | |
| 0.1 | 0 | 10.2 | 10.2 | 9.5 | 9.2 | 9.4 |
| 0.4 | 0 | 10.3 | 9.9 | 9.1 | 9.2 | 8.9 |
| 0.7 | 0 | 9.1 | 9.2 | 9.6 | 9.2 | 8.6 |
| 0.9 | 0 | 10.2 | 10.4 | 10.1 | 9.7 | 8.6 |
| 0.99 | 0 | 10.8 | 10.5 | 9.3 | 8.9 | 8.5 |
| 0.5 | 0.5 | 11.5 | 9.7 | 10.4 | 8.7 | 9.1 |
| 0.8 | 0.2 | 9.8 | 10.4 | 10.1 | 9.0 | 9.3 |
| 0.8 | 0.5 | 10.1 | 9.9 | 9.0 | 8.7 | 8.8 |
| 0.8 | 0.8 | 9.6 | 9.8 | 8.8 | 9.0 | 8.2 |
| | | | Means | | | |
| 0.1 | 0 | 1.02 | 1.97 | 2.91 | 3.81 | 4.77 |
| 0.4 | 0 | 0.98 | 1.97 | 2.91 | 3.82 | 4.73 |
| 0.7 | 0 | 0.97 | 2.00 | 2.94 | 3.85 | 4.73 |
| 0.9 | 0 | 1.00 | 2.04 | 2.99 | 3.90 | 4.80 |
| 0.99 | 0 | 1.02 | 2.05 | 2.99 | 3.90 | 4.82 |
| 0.5 | 0.5 | 1.03 | 1.99 | 2.92 | 3.83 | 4 77 |
| 0.8 | 0.2 | 1.03 | 2.00 | 2.92 | 3.86 | 4.79 |
| 0.8 | 0.5 | 0.99 | 2.00 | 2.92 | 3.83 | 4.76 |
| 0.8 | 0.8 | 0.97 | 2.00 | 2.93 | 3.86 | 4.74 |
| | | | Variances | | | |
| 0.1 | 0 | 1.95 | 3.74 | 5.69 | 7.32 | 9.01 |
| 0.4 | 0 | 1.74 | 3.92 | 5.73 | 7.39 | 8.77 |
| 0.7 | 0 | 1.81 | 3.94 | 5.81 | 7.32 | 8.71 |
| 0.9 | 0 | 1.97 | 3.92 | 5.70 | 7.32 | 8.89 |
| 0.99 | 0 | 2.15 | 4.03 | 5.75 | 7.46 | 8.95 |
| 0.5 | 0.5 | 2.16 | 4.06 | 5.69 | 7.29 | 9.23 |
| 0.8 | 0.2 | 2.20 | 4.10 | 5.88 | 7.39 | 9.23 |
| 0.8 | 0.5 | 1.98 | 3.95 | 5.56 | 7 29 | 8.69 |
| 0.8 | 0.8 | 1.79 | 4.01 | 5.65 | 7 39 | 8.37 |

Empirical significance levels are in terms of percentages values of $\pi_1$ are close to the boundary of the stationarity region (i.e., $\pi_1 \geq 0.9$), the results, in general, are closer to those predicted by asymptotic theory.

(iii) The empirical variances of the test are substantially smaller than the theoretical value 2r. This is particularly evident when $r \geq 2$ and $n \leq 75$. For n=100, the dispersion bias is still pronounced when r=5.

(iv) As regards the ratios of empirical variances to empirical means, these values are considerably smaller than 2 for $r \geq 2$ and $n \leq 75$. Both points (iii) and (iv) suggest that the $\chi^2$ approximation to the small-sample distribution of the LM test is questionable.

(v) As a minor point regarding the computation of the Godfrey procedure, the simulation results of the AR(1) model presented are more numerically stable than those of the conventional method which requires computation of the information matrix for the (p+r) AR parameters [see Ljung (1986, Table 4); Ljung (1988, p. 359)]. It is interesting to see from Table 3.1 that even in cases where $r \geq 4$, and $\alpha$=5% and 10%, the empirical significance levels of the LM test do not display any unusually large values. This evidence strongly favours the use of Godfrey's procedure in empirical applications.

Tables 3.2 and 3.3 summarize the simulation results of the second simulation experiment in which the data were generated from a few selected cases of an ARMA(2,2) process and were subsequently fitted to the AR(1) and AR(2) models. The following important points emerge from the reported results in these tables:

(i) The results for the empirical power of the LM test are much better for $r \leq 3$ in the majority of cases considered. This is especially obvious when the fitted process is an AR(1) model with n=50. Even in those cases where the chosen r should be greater than 3, one can easily see that the gain in the empirical power from selecting such higher values is fairly minimal. Thus, the most important conclusion that emerges from the results in Tables 3.2 and 3.3 is that for examining possible model misspecfication, the choice of r, again, should be kept small relative to the sample size when non-seasonal data are fitted to low-order ARMA(p,q) models.

(ii) In several cases, the loss in empirical powers due to the use of high values of r (i.e., $r \geq 4$) can be quite large. For instance, when the data, which were based on a true AR(2) process, were fitted to an AR(1) model with n=50, the test can lose power by as much as 31.8% [see the case where $(\pi_1, \pi_2)=(0.8, 0.5)$].

Table 3.2

Empirical Powers of LM Tests; AR(1) Model fitted; Data generated from the Model $(1 - \pi_1 B)(1 - \pi_2 B)w_t = (1 - \psi_1 B)(1 - \psi_2 B)a_t$; $\alpha$=5%

| Parameters | | | | n=50 | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0 | 0 | 0.1 | 0 | 6.0 | 3.7 | 3.5 | 3.8 | 2.9 |
| 0 | 0 | 0.4 | 0 | 14.7 | 11.5 | 10.7 | 8.5 | 6.9 |
| 0 | 0 | 0.7 | 0 | 54.7 | 53.9 | 52.4 | 46.1 | 41.5 |
| 0 | 0 | 0.9 | 0 | 69.7 | 76.0 | 79.9 | 77.8 | 79.7 |
| 0 | 0 | 0.5 | 0.5 | 83.1 | 87.2 | 85.6 | 81.4 | 79.2 |
| 0 | 0 | 0.8 | 0.2 | 79.1 | 86.0 | 87.6 | 84.8 | 85.2 |
| 0 | 0 | 0.8 | 0.5 | 94.7 | 98.2 | 98.9 | 98.6 | 98.7 |
| 0 | 0 | 0.8 | 0.8 | 97.8 | 100 | 99.1 | 99.9 | 100 |
| 0.5 | 0.5 | 0 | 0 | 46.1 | 35.3 | 28.3 | 22.8 | 19.4 |
| 0.8 | 0.2 | 0 | 0 | 22.4 | 16.5 | 13.4 | 10.1 | 8.1 |
| 0.8 | 0.5 | 0 | 0 | 85.0 | 74.6 | 66.2 | 59.3 | 53.2 |
| 0.8 | 0.8 | 0 | 0 | 99.7 | 99.2 | 98.9 | 98.2 | 97.2 |
| 0.8 | 0 | 0.5 | 0 | 21.8 | 17.1 | 13.1 | 11.1 | 9.5 |
| 0.8 | 0 | 0.2 | 0 | 10.5 | 7.9 | 6.0 | 5.1 | 4.8 |

| | | | | n=100 | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0 | 0 | 0.1 | 0 | 4.8 | 4.9 | 4.9 | 4.2 | 4.6 |
| 0 | 0 | 0.4 | 0 | 29.7 | 24.2 | 20.1 | 17.8 | 16.6 |
| 0 | 0 | 0.7 | 0 | 85.6 | 90.4 | 90.8 | 88.9 | 87.3 |
| 0 | 0 | 0.9 | 0 | 94.2 | 98.7 | 99.6 | 99.4 | 99.7 |
| 0 | 0 | 0.5 | 0.5 | 99.1 | 99.8 | 99.8 | 99.7 | 99.5 |
| 0 | 0 | 0.8 | 0.2 | 98.3 | 99.8 | 99.8 | 99.9 | 99.9 |
| 0 | 0 | 0.8 | 0.5 | 100 | 100 | 100 | 100 | 100 |
| 0 | 0 | 0.8 | 0.8 | 100 | 100 | 100 | 100 | 100 |
| 0.5 | 0.5 | 0 | 0 | 74.4 | 64.0 | 56.3 | 50.8 | 43.9 |
| 0.8 | 0.2 | 0 | 0 | 40.9 | 31.0 | 24.8 | 21.5 | 18.2 |
| 0.8 | 0.5 | 0 | 0 | 98.1 | 96.0 | 94.0 | 91.9 | 88.8 |
| 0.8 | 0.8 | 0 | 0 | 100 | 100 | 100 | 100 | 99.8 |
| 0.8 | 0 | 0.5 | 0 | 40.4 | 38.5 | 33.8 | 28.4 | 25.4 |
| 0.8 | 0 | 0.2 | 0 | 20.5 | 16.5 | 13.9 | 11.2 | 10.1 |

All entries are expressed as percentages.

Table 3.3
Empirical Powers of LM Tests; AR(2) Model fitted; Data generated from the Model $(1 - \pi_1 B)(1 - \pi_2 B)w_t = (1 - \psi_1 B)(1 - \psi_2 B)a_t$; $\alpha$=5%

| Parameters | | | | n=50 | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0 | 0 | 0.1 | 0 | 4.8 | 4.0 | 4.1 | 3.1 | 3.0 |
| 0 | 0 | 0.4 | 0 | 5.2 | 5.6 | 5.4 | 4.6 | 4.1 |
| 0 | 0 | 0.7 | 0 | 19.9 | 21.6 | 18.6 | 17.4 | 14.9 |
| 0 | 0 | 0.9 | 0 | 36.3 | 44.8 | 44.7 | 47.8 | 44.1 |
| 0 | 0 | 0.5 | 0.5 | 36.1 | 38.1 | 33.9 | 30.2 | 25.7 |
| 0 | 0 | 0.8 | 0.2 | 40.5 | 47.4 | 44.7 | 46.9 | 40.9 |
| 0 | 0 | 0.8 | 0.5 | 66.5 | 80.1 | 82.7 | 83.6 | 80.6 |
| 0 | 0 | 0.8 | 0.8 | 80.4 | 92.9 | 95.8 | 97.6 | 98.0 |
| 0.8 | 0 | 0.5 | 0 | 8.0 | 5.5 | 5.0 | 4.5 | 4.1 |
| 0.8 | 0 | 0.2 | 0 | 4.3 | 2.8 | 2.6 | 2.7 | 3.0 |

| | | | | n=100 | | | | |
|---|---|---|---|---|---|---|---|---|
| $\pi_1$ | $\pi_2$ | $\psi_1$ | $\psi_2$ | r=1 | r=2 | r=3 | r=4 | r=5 |
| 0 | 0 | 0.1 | 0 | 5.1 | 4.9 | 4.2 | 5.1 | 4.3 |
| 0 | 0 | 0.4 | 0 | 7.0 | 6.6 | 6.2 | 6.4 | 5.7 |
| 0 | 0 | 0.7 | 0 | 46.2 | 9.2 | 47.9 | 45.2 | 40.5 |
| 0 | 0 | 0.9 | 0 | 73.0 | 3.0 | 86.9 | 89.4 | 89.7 |
| 0 | 0 | 0.5 | 0.5 | 73.4 | 8.2 | 74.7 | 72.7 | 68.1 |
| 0 | 0 | 0.8 | 0.2 | 77.1 | 86.2 | 87.9 | 88.3 | 88.4 |
| 0 | 0 | 0.8 | 0.5 | 94.5 | 98.6 | 99.3 | 99.6 | 99.8 |
| 0 | 0 | 0.8 | 0.8 | 98.8 | 100 | 100 | 100 | 100 |
| 0.8 | 0 | 0.5 | 0 | 14.6 | 11.6 | 9.9 | 8.1 | 8.2 |
| 0.8 | 0 | 0.2 | 0 | 6.6 | 5.3 | 5.4 | 4.4 | 5.2 |

All entries are expressed as percentages.

(iii) When the fitted process is an AR(1) model, the LM test is asymptotically optimal against the AR(2) and ARMA(1,1) alternatives for $r=1$. An inspection of the results shown in Table 3.2 reveals that the optimal value of r is indeed equal to one. It is also noteworthy that even in cases where inappropriate alternatives such as the MA(1) and MA(2) models are considered, the power of the LM test is fairly high when the size of $\psi$'s ranges from moderate to large.

(iv) The empirical power for ARMA(1,1) is relatively low compared to AR(2) models (see Table 3.2). A plausible explanation is that the degree of model inadequacy is more serious in cases where the data were generated by the AR(2) models than in those of the ARMA(1,1) models. For example, consider an ARMA(1,1) model with $\pi_1=0.8$ and $\psi_1=0.5$, then this model is almost equivalent to an AR(2) process $w_t=0.3w_{t-1}+0.15w_{t-2}+a_t$. If the data are generated by an AR(2) model with $\pi_1=0.8$ and $\pi_2=0.5$, this model can be written as $w_t=1.3w_{t-1}-0.4w_{t-2}+a_t$. A comparison of the two models, $w_t=0.3w_{t-1}+0.15w_{t-2}+a_t$ and $w_t=1.3w_{t-1}-0.4w_{t-2}+a_t$, indicates that the extent of model misspecification is more serious in the latter case as the coefficient of $w_{t-2}$ is larger in absolute terms than in the former one. Consequently, the empirical powers for the LM tests are higher in the latter case as well.

(v) The power of each of LM tests is small (0.21-0.11 for n=50 and 0.4-0.21 for n=100) when the data are generated from the ARMA(1,1) models. This poor performance can easily be explained using the example given in point (iv). As indicated above, the approximation of an ARMA(1,1) model $(1-0.8B)w_t=(1-0.5B)a_t$ is a simple AR(2) process $w_t=0.3w_{t-1}+0.15w_{t-2}+a_t$. Since the coefficient of $w_{t-2}$ is small, then the power would be low. This is because the first-order autoregressive model can provide an adequate approximation to a second-order autoregressive process as long as $|\phi_2|$ is small.

(vi) When an AR(2) model is being fitted to the data, the empirical power of each LM test is unambiguously lower than that of the AR(1) model. This finding is supported by the well-known duality theorem which states that any stationary and invertible ARMA representation can be well approximated by higher-order AR models.

(vii) When the sample size increases from 50 to 100, the LM test is very successful in revealing deviations from model specification in most of the cases examined.

Tables 3.4 and 3.5 report the simulation results of the third experiment in which the data were generated according to the seasonal models given in Section 2.3 and were subsequently fitted to the AR(1) and AR(2) models. The most noticeable features of these two tables are as follows:

(i) The main conclusion based on the results of non-seasonal data (see Tables 3.2 and 3.3) does not appear valid in the present experiment. Once the effect of seasonality was introduced, the chosen values of r, which are intended to obtain "maximum" power, become fairly large. For the AR(1) and AR(2) models, the optimal values are at least 10 and 11 in most cases considered. Additionally, it is observed that the LM test can suffer from a substantial loss in power when r is selected less than these optimal choices.

(ii) For data generated from the seasonal AR(1) and MA(1) models [i.e., (2.3.1) and (2.3.2)], there are many cases where the LM test is virtually powerless. Quite interestingly, its power is rarely larger than 10% with $0.4 \le \Theta_{12} \le 0.9$, $\Phi_{12}=0.4$, $r \le 10$ and n=50. Even in a few cases, it is smaller than the designated significance level (nominal size).

(iii) A comparison of the results obtained from seasonal data (Tables 3.4 and 3.5) and non-seasonal data (Tables 3.2 and 3.3) indicate that the test power as well as the choice of r are significantly affected by the nature of the data. Clearly, the findings reported in Tables 3.4 and 3.5 illustrate a potential weakness of the LM test; namely, many restrictions tested in the case of seasonal data are irrelevant in the sense that they carry

Table 3.4

Empirical Powers of LM Tests; AR(1) Model fitted; Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$; $\alpha$ = 5%

| Parameters | | | n=50 | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=1 | r=2 | r=3 | r=4 | r=5 | r=6 |
| 0 | 0 | 0.4 | 7.6 | 6.2 | 5.0 | 4.8 | 4.3 | 4.2 |
| 0 | 0 | 0.7 | 9.0 | 8.6 | 8.0 | 8.1 | 6.7 | 5.8 |
| 0 | 0 | 0.9 | 9.1 | 9.3 | 8.5 | 9.0 | 6.9 | 7.3 |
| 0.4 | 0 | 0 | 8.4 | 7.0 | 7.0 | 6.8 | 8.9 | 9.0 |
| 0.7 | 0 | 0 | 15.5 | 18.0 | 18.4 | 20.5 | 25.5 | 27.6 |
| 0.9 | 0 | 0 | 25.9 | 31.0 | 38.7 | 43.5 | 53.4 | 54.3 |
| 0 | 0.6 | -0.5 | 42.0 | 37.9 | 37.6 | 34.1 | 31.0 | 28.1 |
| 0 | -0.5 | 0.6 | 31.5 | 27.6 | 24.9 | 21.8 | 20.2 | 16.6 |
| 0.6 | -0.5 | 0 | 34.7 | 32.3 | 32.6 | 30.9 | 29.8 | 30.7 |
| -0.5 | 0 | 0.6 | 44.6 | 42.4 | 40.3 | 37.9 | 35.9 | 31.3 |

| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=7 | r=8 | r=9 | r=10 | r=11 | r=12 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.4 | 4.5 | 4.2 | 4.3 | 4.0 | 14.4 | 11.8 |
| 0 | 0 | 0.7 | 6.4 | 6.3 | 6.4 | 5.8 | 41.8 | 26.9 |
| 0 | 0 | 0.9 | 7.0 | 7.3 | 7.1 | 7.2 | 51.8 | 47.1 |
| 0.4 | 0 | 0 | 9.0 | 9.0 | 7.9 | 8.1 | 21.8 | 19.4 |
| 0.7 | 0 | 0 | 28.3 | 29.4 | 31.6 | 32.6 | 94.1 | 100 |
| 0.9 | 0 | 0 | 56.0 | 59.2 | 62.8 | 65.9 | 100 | 100 |
| 0 | 0.6 | -0.5 | 26.0 | 24.0 | 21.0 | 29.2 | 37.6 | 36.7 |
| 0 | -0.5 | 0.6 | 15.4 | 15.1 | 14.4 | 21.0 | 38.1 | 38.5 |
| 0.6 | -0.5 | 0 | 29.7 | 30.2 | 31.4 | 43.7 | 74.0 | 75.9 |
| -0.5 | 0 | 0.6 | 29.3 | 26.4 | 23.7 | 35.7 | 56.1 | 55.8 |

All entries are expressed as percentages.

Table 3.4 (cont'd)

Empirical Powers of LM Tests; AR(1) Model fitted; Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$; $\alpha = 5\%$

| | | | n=100 | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=1 | r=2 | r=3 | r=4 | r=5 | r=6 |
| 0 | 0 | 0.4 | 7.1 | 7.7 | 7.8 | 7.0 | 5.4 | 6.2 |
| 0 | 0 | 0.7 | 9.8 | 10.3 | 10.9 | 12.0 | 8.7 | 11.3 |
| 0 | 0 | 0.9 | 10.7 | 12.1 | 11.8 | 13.0 | 10.2 | 11.7 |
| 0.4 | 0 | 0 | 8.4 | 8.9 | 9.0 | 9.5 | 14.6 | 15.2 |
| 0.7 | 0 | 0 | 18.9 | 25.7 | 29.6 | 33.2 | 45.1 | 48.6 |
| 0.9 | 0 | 0 | 33.6 | 45.9 | 56.7 | 64.1 | 76.2 | 79.1 |
| 0 | 0.6 | -0.5 | 70.6 | 72.8 | 70.9 | 69.2 | 66.9 | 64.5 |
| 0 | -0.5 | 0.6 | 54.7 | 51.6 | 49.2 | 46.5 | 42.3 | 38.7 |
| 0.6 | -0.5 | 0 | 52.6 | 58.1 | 57.8 | 57.1 | 57.4 | 59.0 |
| -0.5 | 0 | 0.6 | 68.3 | 71.2 | 71.7 | 68.7 | 68.1 | 64.6 |

| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=7 | r=8 | r=9 | r=10 | r=11 | r=12 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.4 | 6.7 | 7.2 | 7.5 | 7.3 | 54.2 | 49.9 |
| 0 | 0 | 0.7 | 12.0 | 11.7 | 13.1 | 13.1 | 93.7 | 91.4 |
| 0 | 0 | 0.9 | 13.2 | 13.3 | 14.7 | 14.8 | 97.0 | 97.0 |
| 0.4 | 0 | 0 | 14.8 | 15.2 | 15.5 | 16.2 | 70.2 | 66.2 |
| 0.7 | 0 | 0 | 49.4 | 51.0 | 52.7 | 56.9 | 100 | 100 |
| 0.9 | 0 | 0 | 81.4 | 82.5 | 84.2 | 87.3 | 100 | 100 |
| 0 | 0.6 | -0.5 | 61.8 | 60.3 | 57.4 | 71.3 | 87.7 | 89.3 |
| 0 | -0.5 | 0.6 | 37.0 | 33.9 | 33.5 | 53.0 | 89.9 | 93.4 |
| 0.6 | -0.5 | 0 | 58.7 | 57.8 | 58.3 | 77.2 | 99.3 | 99.7 |
| -0.5 | 0 | 0.6 | 61.5 | 59.2 | 57.3 | 75.9 | 96.0 | 96.7 |

All entries are expressed as percentages.

Table 3.5

Empirical Powers of LM Tests; AR(2) Model fitted; Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$; $\alpha$ = 5%

| Parameters | | | n=50 | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=1 | r=2 | r=3 | r=4 | r=5 | r=6 |
| 0 | 0 | 0.4 | 5.7 | 5.5 | 5.9 | 4.4 | 4.5 | 4.4 |
| 0 | 0 | 0.7 | 7.0 | 7.0 | 7.4 | 6.1 | 6.8 | 7.3 |
| 0 | 0 | 0.9 | 7.3 | 7.3 | 8.1 | 6.2 | 7.4 | 8.5 |
| 0.4 | 0 | 0 | 6.2 | 5.8 | 6.4 | 8.9 | 9.4 | 8.9 |
| 0.7 | 0 | 0 | 13.5 | 15.5 | 16.3 | 22.8 | 25.1 | 28.1 |
| 0.9 | 0 | 0 | 20.8 | 31.0 | 36.5 | 47.8 | 49.7 | 52.4 |
| 0 | 0.6 | -0.5 | 9.0 | 9.1 | 9.2 | 8.0 | 9.7 | 8.9 |
| 0 | -0.5 | 0.6 | 7.6 | 7.5 | 7.0 | 6.9 | 6.1 | 6.8 |
| 0.6 | -0.5 | 0 | 14.6 | 16.1 | 17.6 | 19.1 | 20.5 | 21.5 |
| -0.5 | 0 | 0.6 | 18.0 | 17.1 | 15.0 | 16.5 | 15.1 | 14.4 |

| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=7 | r=8 | r=9 | r=10 | r=11 | r=12 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.4 | 4.4 | 4.5 | 3.9 | 16.2 | 14.1 | 12.3 |
| 0 | 0 | 0.7 | 7.6 | 6.2 | 6.9 | 45.0 | 41.0 | 35.9 |
| 0 | 0 | 0.9 | 8.2 | 7.5 | 7.6 | 55.9 | 49.9 | 45.4 |
| 0.4 | 0 | 0 | 9.4 | 8.4 | 8.1 | 24.1 | 21.6 | 19.4 |
| 0.7 | 0 | 0 | 29.7 | 30.7 | 33.6 | 95.2 | 93.8 | 91.2 |
| 0.9 | 0 | 0 | 56.1 | 60.8 | 64.4 | 100 | 100 | 100 |
| 0 | 0.6 | -0.5 | 9.0 | 8.5 | 13.8 | 15.9 | 18.4 | 17.8 |
| 0 | -0.5 | 0.6 | 7.3 | 6.0 | 10.5 | 21.6 | 26.4 | 26.7 |
| 0.6 | -0.5 | 0 | 22.4 | 22.9 | 37.4 | 71.3 | 73.6 | 70.2 |
| -0.5 | 0 | 0.6 | 14.1 | 13.0 | 22.1 | 45.8 | 47.6 | 43.7 |

All entries are expressed as percentages.

Table 3.5 (cont'd)

Empirical Powers of LM Tests; AR(2) Model fitted; Data generated from the Model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$; n = 100 and $\alpha$ = 5%

| Parameters | | | n=100 | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=1 | r=2 | r=3 | r=4 | r=5 | r=6 |
| 0 | 0 | 0.4 | 6.7 | 7.9 | 7.1 | 6.2 | 6.8 | 6.3 |
| 0 | 0 | 0.7 | 9.1 | 9.4 | 9.6 | 7.8 | 10.2 | 11.2 |
| 0 | 0 | 0.9 | 9.7 | 10.2 | 10.2 | 8.5 | 10.8 | 12.3 |
| 0.4 | 0 | 0 | 7.9 | 7.6 | 8.7 | 14.5 | 14.5 | 15.2 |
| 0.7 | 0 | 0 | 18.2 | 25.3 | 28.6 | 41.7 | 45.0 | 47.6 |
| 0.9 | 0 | 0 | 33.7 | 46.9 | 56.1 | 71.2 | 73.6 | 76.1 |
| 0 | 0.6 | -0.5 | 12.4 | 12.1 | 14.6 | 14.5 | 15.9 | 16.0 |
| 0 | -0.5 | 0.6 | 10.0 | 10.3 | 9.8 | 10.5 | 9.6 | 10.9 |
| 0.6 | -0.5 | 0 | 22.9 | 27.2 | 29.6 | 32.1 | 36.5 | 38.4 |
| -0.5 | 0 | 0.6 | 30.7 | 33.9 | 30.3 | 31.7 | 30.8 | 29.7 |
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | r=7 | r=8 | r=9 | r=10 | r=11 | r=12 |
| 0 | 0 | 0.4 | 6.6 | 6.7 | 6.8 | 55.7 | 53.2 | 49.0 |
| 0 | 0 | 0.7 | 12.2 | 11.5 | 11.9 | 97.4 | 92.6 | 91.8 |
| 0 | 0 | 0.9 | 13.3 | 13.3 | 13.7 | 98.9 | 97.3 | 96.5 |
| 0.4 | 0 | 0 | 15.2 | 14.8 | 15.6 | 72.2 | 68.3 | 66.6 |
| 0.7 | 0 | 0 | 49.4 | 51.7 | 56.0 | 100 | 100 | 100 |
| 0.9 | 0 | 0 | 78.7 | 81.9 | 87.3 | 100 | 100 | 100 |
| 0 | 0.6 | -0.5 | 15.2 | 16.1 | 37.4 | 47.8 | 61.1 | 66.9 |
| 0 | -0.5 | 0.6 | 10.1 | 11.1 | 27.7 | 63.2 | 77.8 | 82.8 |
| 0.6 | -0.5 | 0 | 38.3 | 40.3 | 68.8 | 98.7 | 99.6 | 99.7 |
| -0.5 | 0 | 0.6 | 28.6 | 27.9 | 58.3 | 91.1 | 93.8 | 93.4 |

All entries are expressed as percentages.

little or no information about model inadequacy. In light of this problem, one would prefer to omit these redundant restrictions when the LM test is used. Indeed, a recent study by Godfrey and Tremayne (1988, p. 31) demonstrated that when looking at tests for serial independence employing quarterly data, the LM analogue of their $T_3$ test may be useful. Motivated by Godfrey and Tremayne's (1988) suggestion, three modified LM tests are constructed to account for the situation in which irrelevant restrictions are omitted. Here, two specific examples are considered:

Example 1:

Null hypothesis $H_o$:

$$w_t=\phi_1 w_{t-1}+a_t$$

Three specific alternatives, namely

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_{12} w_{t-12}+a_t$
(Tested Restriction $\phi_{12}=0$; LM1 ~ $\chi^2(1)$)

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+\phi_{12} w_{t-12}+a_t$
(Tested Restrictions $\phi_2=\phi_{12}=0$; LM2 ~ $\chi^2(2)$)

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+\phi_{11} w_{t-11}+\phi_{12} w_{t-12}+\phi_{13} w_{t-13}+a_t$
(Tested Restrictions $\phi_2=\phi_{11}=\phi_{12}=\phi_{13}=0$; LM4 ~ $\chi^2(4)$)

Example 2:

Null hypothesis $H_o$:

$$w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+a_t$$

Three specific alternatives, namely

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+\phi_{12} w_{t-12}+a_t$
(Tested Restriction $\phi_{12}=0$; LM1 ~ $\chi^2(1)$)

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+\phi_3 w_{t-3}+\phi_{12} w_{t-12}+a_t$
(Tested Restrictions $\phi_3=\phi_{12}=0$; LM2 $\sim \chi^2(2)$)

$H_A$: $w_t=\phi_1 w_{t-1}+\phi_2 w_{t-2}+\phi_3 w_{t-3}+\phi_{11} w_{t-11}+\phi_{12} w_{t-12}+\phi_{13} w_{t-13}+a_t$
(Tested Restrictions $\phi_3=\phi_{11}=\phi_{12}=\phi_{13}=0$; LM4 $\sim \chi^2(4)$)

Since the null models considered in these two examples are nested within more general models, one can apply the LM test to examine the statistical significance of the coefficient restriction(s) [see for example Godfrey (1979, p. 70)].

The results reported in Table 3.6 support the above conjecture that the modified LM tests (LM1, LM2 and LM4) are very powerful in detecting model inadequacy when most of the irrelevant restrictions are excluded. As for their relative performance, LM1 is favoured when the data were generated from (2.3.1) and (2.3.2). For (2.3.3) and (2.3.4) there seems to be no clear-cut winner. It should also be noted that, as compared to the findings of the conventional LM tests (see Tables 3.4 and 3.5), the gain in empirical power from the use of such modifications is fairly large and, in many cases, is more than two-fold.

Following Godfrey (1979), we also compare the empirical power between each of the modified LM tests and the corresponding Ljung-Box test QLB. The results clearly suggest that the modified LM tests outperform QLB in terms of their ability to detect deviations from model adequacy. In some cases, the empirical

Table 3.6

Empirical Powers of Modified LM Tests and $Q_{LB}$ Test; Data generated from the model $(1 - \Phi_{12}B^{12})w_t = (1 - \psi_1 B)(1 - \Theta_{12}B^{12})a_t$; $\alpha = 5\%$

Fitted process is an AR(1) model

| Parameters | | | n=50 | | | | n=100 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | LM1 | LM2 | LM4 | QLB | LM1 | LM2 | LM4 | QLB |
| 0 | 0 | 0.4 | 67.3 | 58.0 | 40.6 | 23.0 | 95.0 | 89.7 | 79.7 | 71.5 |
| 0 | 0 | 0.7 | 94.1 | 88.6 | 77.6 | 42.3 | 100 | 99.8 | 99.7 | 88.1 |
| 0 | 0 | 0.9 | 96.5 | 92.9 | 85.4 | 49.6 | 100 | 100 | 99.9 | 91.4 |
| 0.4 | 0 | 0 | 80.6 | 69.6 | 54.9 | 34.5 | 98.0 | 96.1 | 89.2 | 80.6 |
| 0.7 | 0 | 0 | 100 | 99.8 | 99.2 | 90.0 | 100 | 100 | 100 | 99.6 |
| 0.9 | 0 | 0 | 100 | 100 | 100 | 99.4 | 100 | 100 | 100 | 100 |
| 0 | 0.6 | -0.5 | 73.8 | 77.3 | 71.0 | 52.0 | 96.1 | 98.6 | 98.0 | 89.0 |
| 0 | -0.5 | 0.6 | 84.3 | 83.1 | 78.1 | 52.5 | 99.4 | 99.6 | 99.0 | 91.9 |
| 0.6 | -0.5 | 0 | 97.9 | 96.8 | 96.4 | 79.7 | 100 | 99.9 | 100 | 97.9 |
| -0.5 | 0.6 | 0 | 88.4 | 88.9 | 85.0 | 65.3 | 99.3 | 99.8 | 99.5 | 94.6 |

Fitted process is an AR(2) model

| Parameters | | | n=50 | | | | n=100 | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\Phi_{12}$ | $\psi_1$ | $\Theta_{12}$ | LM1 | LM2 | LM4 | QLB | LM1 | LM2 | LM4 | QLB |
| 0 | 0 | 0.4 | 66.6 | 55.6 | 38.9 | 18.7 | 94.6 | 88.1 | 78.9 | 67.1 |
| 0 | 0 | 0.7 | 93.6 | 87.9 | 76.4 | 29.7 | 99.9 | 99.7 | 99.4 | 86.7 |
| 0 | 0 | 0.9 | 96.2 | 92.3 | 83.8 | 44.4 | 100 | 99.9 | 99.7 | 89.7 |
| 0.4 | 0 | 0 | 78.9 | 67.5 | 52.4 | 23.1 | 97.9 | 95.6 | 88.1 | 78.0 |
| 0.7 | 0 | 0 | 99.9 | 99.7 | 99.0 | 86.5 | 100 | 100 | 100 | 99.1 |
| 0.9 | 0 | 0 | 100 | 100 | 100 | 99.4 | 100 | 100 | 100 | 100 |
| 0 | 0.6 | -0.5 | 40.2 | 33.5 | 42.6 | 38.1 | 70.0 | 64.3 | 83.8 | 83.5 |
| 0 | -0.5 | 0.6 | 64.1 | 53.8 | 61.6 | 35.2 | 90.3 | 85.1 | 93.6 | 85.2 |
| 0.6 | -0.5 | 0 | 97.9 | 95.0 | 94.2 | 73.7 | 100 | 99.9 | 100 | 96.6 |
| -0.5 | 0.6 | 0 | 86.8 | 81.5 | 78.1 | 48.8 | 99.2 | 98.7 | 98.4 | 89.9 |

The number of residual autocorrelations (m) used in the Ljung-Box test, $Q_{LB}$, is 12.

powers of QLB are only one-third of those of LM1. Thus, the findings favour the use of modified LM tests over QLB in these circumstances.

## 3.4 Comments on the Finite-Sample Distribution of Tests of Separate Hypotheses

In this section, several comments are raised regarding the empirical performance of the tests of separate hypotheses proposed by McAleer et al. (1988) in commonly-used sample sizes:

First, the empirical size of the tests of separate hypotheses, namely the SM test and the SP test, is significantly affected by the orders of the AR(p) and MA(q) models. To highlight this feature, the empirical significance levels of these tests obtained by McAleer et al. (1988, pp. 181-182, Tables 1 and 2) are reported in Tables 3.7 and 3.8. On the basis of these simulation results, two points deserve special attention:

(i) For testing an AR(p) null against an MA(q) alternative, the SP test is highly unreliable even when the sample size is as large as 100. As for the SM tests, the empirical significance levels of the SM(1) test are much closer to the nominal level than those of the SM(2) test. Interestingly, with the exception of $\pi_1$=0.8, $\pi_2$=0.5 and n=100, the size of the SM(2) test is

Table 3.7
Significance Levels of Tests of Separate Hypotheses for the True Model $(1 - \pi_1)(1 - \pi_2)y_t = a_t$.

| | parameters of AR(2) model | | Nominal level = 5% | | |
|---|---|---|---|---|---|
| n | $\pi_1$ | $\pi_2$ | SM(1) | SM(2) | SP(2) |
| 25 | 0.1 | 0 | 7.30 | 14.13 | 22.25 |
| | 0.5 | 0 | 6.88 | 10.58 | 19.21 |
| | 0.9 | 0 | 6.91 | 12.20 | 23.25 |
| | 0.8 | 0.2 | 5.64 | 11.84 | 19.61 |
| | 0.8 | 0.5 | 4.09 | 16.68 | 28.13 |
| | 0.8 | 0.8 | 1.86 | 27.56 | 36.94 |
| 50 | 0.1 | 0 | 5.28 | 6.61 | 10.31 |
| | 0.5 | 0 | 5.60 | 6.48 | 12.93 |
| | 0.9 | 0 | 6.11 | 7.41 | 13.89 |
| | 0.8 | 0.2 | 5.65 | 6.88 | 11.94 |
| | 0.8 | 0.5 | 5.76 | 9.45 | 15.68 |
| | 0.8 | 0.8 | 3.03 | 16.05 | 20.48 |
| 100 | 0.1 | 0 | 5.13 | 5.28 | 7.04 |
| | 0.5 | 0 | 4.63 | 5.64 | 10.89 |
| | 0.9 | 0 | 5.33 | 5.49 | 9.20 |
| | 0.8 | 0.2 | 5.36 | 5.36 | 9.20 |
| | 0.8 | 0.5 | 5.48 | 5.26 | 9.04 |
| | 0.8 | 0.8 | 4.11 | 7.58 | 9.98 |

Table 3.8
Significance Levels of Tests of Separate Hypotheses for the True Model
$y_t = (1 - \psi_1)(1 - \psi_2)a_t$.

| n | Parameters of MA(2) model $\psi_1$ | $\psi_2$ | SM(1) | Nominal level = 5% SM(2) | SP(2) |
|---|---|---|---|---|---|
| 25 | 0.1 | 0 | 4.13 | 3.56 | 4.24 |
| | 0.5 | 0 | 3.79 | 3.26 | 6.00 |
| | 0.9 | 0 | 5.51 | 4.19 | 4.25 |
| | 0.8 | 0.2 | 9.43 | 7.28 | 11.08 |
| | 0.8 | 0.5 | 16.51 | 14.29 | 17.20 |
| | 0.8 | 0.8 | 22.93 | 20.10 | 21.99 |
| 50 | 0.1 | 0 | 4.15 | 3.79 | 4.59 |
| | 0.5 | 0 | 4.29 | 3.95 | 5.21 |
| | 0.9 | 0 | 5.48 | 4.94 | 4.21 |
| | 0.8 | 0.2 | 6.09 | 5.36 | 6.95 |
| | 0.8 | 0.5 | 11.93 | 10.94 | 11.06 |
| | 0.8 | 0.8 | 18.18 | 16.18 | 16.45 |
| 100 | 0.1 | 0 | 4.79 | 4.70 | 5.19 |
| | 0.5 | 0 | 4.89 | 5.13 | 5.01 |
| | 0.9 | 0 | 5.53 | 5.24 | 4.93 |
| | 0.8 | 0.2 | 4.80 | 5.09 | 5.14 |
| | 0.8 | 0.5 | 6.85 | 6.44 | 6.35 |
| | 0.8 | 0.8 | 12.74 | 12.09 | 11.94 |

consistently larger than the designated significance level (5%). This observation is particularly clear when $n \le 50$.

(ii) For testing an MA(q) null against an AR(p) alternative, the simulation results of Table 3.8 reveal a potential problem, namely, the empirical significance levels of the separate model tests are undersized when the fitted process is an MA(1) model with $\psi_1 \le 0.5$ and $n \le 50$. This under-rejection problem becomes even more serious when the value of p increases from 1 to 2.

In short, we can easily see that the quality of the $\chi^2$ approximation to the finite-sample distribution of the tests of separate hypotheses considered can deteriorate rapidly as the values of "p" and "q" increase.

The second comment focusses on the empirical power of the SM tests. As mentioned in McAleer et al. (1988), the SM tests are asymptotically optimal against separate alternatives. For example, in testing the adequacy of a fitted AR(p) model, the SM(q) tests are expected to yield high power against an MA(q) alternative. In terms of power calculations, this can be done by generating the data by an MA(q) process, and then fitting them into an AR(p) model. Thus, the SM(1) test is asymptotically optimal against the case where the data are generated by an MA(1) process. Likewise, the SM(2) test is asymptotically optimal against the situation where the data are generated by an MA(2) process, and should have higher empirical power than that of the SM(1) test. However, the simulation results presented in McAleer et al. (1988) and Hall and McAleer (1989) reveal that this optimal property does not hold in many cases. In order to illustrate this point, the power estimates given in Hall and McAleer (1989) (see pp. 99-100, Tables 5 and 6; pp. 103-104, Tables 9 and 10) are reproduced in Table 3.9.

The overall results of Table 3.9 indicate that, except for $\pi_1$=0.2 and $\psi_1$=0.2, the SM(1) test dominates the SM(2) test in terms of empirical power. This is true regardless of the sample size and the chosen parameter values. It is important to note, however, that when the data generation process is either an AR(2) or MA(2), the SM(2) test is supposed to "out power" the SM(1) test. Evidently, these empirical estimates do not support such a power property. But what causes this peculiar behavior? We believe further investigation on this issue is needed.

The third comment raises concern about the design of the simulation experiments used in McAleer et al. (1988) and Hall and McAleer (1989); namely, the effect of the nature of the data on the finite-sample properties of the separate tests is missing from their analyses. This concern is of importance for the following reasons: First, their experimental designs are based on low-order

Table 3.9
Empirical Power of Tests of Separate Hypotheses; Data Generated from the ARMA(2,2) Model $(1 - \pi_1)(1 - \pi_2)y_t = (1 - \psi_1)(1 - \psi_2)a_t$.

| | | | | n = 50 | | n = 100 | |
|---|---|---|---|---|---|---|---|
| | | | | Fitted process: AR(1) model | | | |
| $\pi_1$ | $\pi_2$ | $\Psi_1$ | $\Psi_2$ | SM(1) | SM(2) | SM(1) | SM(2) |
| 0 | 0 | 0.8 | 0 | 95.9 | 90.0 | 100 | 99.8 |
| 0 | 0 | 0.5 | 0 | 37.9 | 29.3 | 66.1 | 55.1 |
| 0 | 0 | 0.2 | 0 | 8.2 | 8.4 | 8.0 | 7.6 |
| 0 | 0 | 0.8 | 0.5 | 99.9 | 99.6 | 100 | 100 |
| 0 | 0 | 0.8 | 0.2 | 99.1 | 97.4 | 100 | 100 |
| 0 | 0 | 0.5 | 0.2 | 66.0 | 54.9 | 91.7 | 85.3 |
| | | | | Fitted process: AR(2) model | | | |
| | | | | SM(1) | SM(2) | SM(1) | SM(2) |
| 0 | 0 | 0.8 | 0 | 64.3 | 54.5 | 95.3 | 89.9 |
| 0 | 0 | 0.5 | 0 | 9.4 | 8.1 | 19.4 | 13.9 |
| 0 | 0 | 0.2 | 0 | 5.0 | 5.8 | 3.4 | 5.5 |
| 0 | 0 | 0.8 | 0.5 | 92.3 | 91.2 | 99.5 | 99.4 |
| 0 | 0 | 0.8 | 0.2 | 78.0 | 69.1 | 98.5 | 96.1 |
| 0 | 0 | 0.5 | 0.2 | 17.2 | 15.5 | 35.3 | 26.3 |
| | | | | Fitted process: MA(1) model | | | |
| | | | | SM(1) | SM(2) | SM(1) | SM(2) |
| 0.8 | 0 | 0 | 0 | 91.2 | 86.2 | 99.8 | 99.4 |
| 0.5 | 0 | 0 | 0 | 24.4 | 18.1 | 52.4 | 41.5 |
| 0.2 | 0 | 0 | 0 | 3.5 | 3.8 | 4.5 | 5.2 |
| 0.8 | 0.5 | 0 | 0 | 99.9 | 99.7 | 100 | 100 |
| 0.8 | 0.2 | 0 | 0 | 97.4 | 94.4 | 100 | 100 |
| 0.5 | 0.2 | 0 | 0 | 50.9 | 39.6 | 87.3 | 80.3 |
| | | | | Fitted process: MA(2) model | | | |
| $\Pi_1$ | $\Pi_1$ | $\Psi_1$ | $\Psi_2$ | SM(1) | SM(2) | SM(1) | SM(2) |
| 0.8 | 0 | 0 | 0 | 59.0 | 47.9 | 92.1 | 89.2 |
| 0.5 | 0 | 0 | 0 | 7.5 | 4.9 | 13.5 | 9.9 |
| 0.2 | 0 | 0 | 0 | 4.4 | 3.7 | 5.4 | 3.9 |
| 0.8 | 0.5 | 0 | 0 | 94.6 | 91.8 | 100 | 100 |
| 0.8 | 0.2 | 0 | 0 | 70.4 | 62.3 | 98.8 | 97.7 |
| 0.5 | 0.2 | 0 | 0 | 10.6 | 8.3 | 27.2 | 19.7 |

ARMA(p,q) models. As a result, their conclusions are only valid in these cases. Second, seasonal ARMA models are commonly employed when one works with monthly or quarterly data. Third, it is known that the empirical performance of other popular diagnostics such as portmanteau tests and Godfrey's LM test, can be significantly affected by the nature of the data (seasonal vs. non-seasonal data). For instance, Ansley and Newbold (1979) found that the Ljung-Box portmanteau test tends to have excessive size distortions when the true models are low-order seasonal ARMA models. In this chapter, we also showed that the empirical power of Godfrey's LM test can be quite low when seasonal data are fitted to simple ARMA models. In light of these reasons, an important direction for future research would be to incorporate the effect of seasonality in the experimental design. Such an extension will be useful in terms of understanding how the tests of separate hypotheses behave in the said situation.

## 3.5 Concluding Remarks

In this chapter three simulation experiments were conducted to investigate the finite-sample distribution of Godfrey's (1979) LM test. As well, many critical comments were raised with regard to the finite-sample performance of the tests of separate hypotheses proposed by McAleer et al. (1988). Based upon our results, a few remarks are in order:

(i) The performance of the LM test in finite samples depends on the choice of r. More importantly, its empirical significance levels are close to the nominal levels only when the value of r is small relative to the sample size.

(ii) The LM statistic suffers a substantial dispersion bias even when the sample size is as large as 100. Interestingly, this finding has not been reported in previous studies.

(iii) In order to obtain higher power for the conventional LM tests in the case of non-seasonal data, it is important to make r small. This conclusion, however, ceases to hold when such tests are used in cases where the data are generated from seasonal models.

(iv) In the case of seasonal data, the LM test can easily be modified to yield high empirical power once most of the irrelevant restrictions are removed from model diagnostics. This strongly indicates that restricted alternatives offer a better chance of constructing reasonably powerful tests involving relatively fewer restrictions.

(v) As for the finite-sample performance of the tests of separate hypotheses suggested by McAleer et al. (1988), three important issues were addressed in this chapter. First, the orders of the AR(p) and MA(q) models can significantly influence the empirical size of the tests. Second, the empirical power of the SM(1) test always dominates the SM(2) test in a situation where the latter is asymptotically optimal against a specific separate alternative. Third, since there is no discussion on the effect of the nature of data on the empirical performance of the separate tests, the claim of their superior performance is hardly convincing. In light of these issues, caution should be exercised when using the tests of separate hypotheses for the adequacy of an ARMA(p,q) model.

## CHAPTER FOUR

## A MODIFIED PORTMANTEAU TEST FOR RANDOMNESS OF GAUSSIAN TIME SERIES

### 4.1 Introduction

The finite-sample properties of some selected portmanteau tests have been carefully examined in Chapter 2. As a sequel to this earlier chapter, the focus of Chapter 4 is on their ability to detect non-randomness of a Gaussian time series. As indicated by Dufour and Roy (1986), testing for the randomness of a time series is one of the most fundamental issues in statistical analysis. As well, many economic theories utilize the assumption of randomness; a notable example is the random-walk consumption function [see Hall (1978)] which postulates that changes in consumption from one period to another are unpredictable.

At the theoretical level, there has not been much recent research on the portmanteau test of randomness in the context of Gaussian and non-Gaussian time series [see Johnson and Kotz (1988) and Harvey (1984) for a detailed discussion on other tests of randomness]. The only exception is the work by Dufour and Roy (1986) in which they show that the normalization procedures used in the Box-Pierce and the Ljung-Box tests are not appropriate for an independently and identically distributed normal series with unknown mean (see Chapter 2, Section 2.2). Subsequently, Dufour and Roy (1986) propose several modified (parametric and non-parametric) portmanteau statistics which are intended to eliminate this problem. However, the finite-sample performance of these tests is still far from perfect [see Dufour and Roy (1986)].

The parametric portmanteau tests proposed by Box and Pierce (1970), Ljung and Box (1978) and Dufour and Roy (1978), are constructed using a number of sample autocorrelations, $r_k$ ($k=1,\ldots,m$), which are assumed to be independent normal random variables. However, it is well known that the finite-sample distribution of $r_k$ may converge to normality very slowly. As a result, we may expect poor empirical performance from a portmanteau test when the sample size is small to moderate.

Recently, Kwan et al. (1992) have argued that, for testing the adequacy of an ARMA(p,q) model, the empirical performance of the Box-Pierce and the Ljung-Box tests can be improved by applying the Fisher (1921) transformation to each residual autocorrelation. The main argument presented in the Kwan et al. (1992) study is that the underlying distribution of the transformed variable is closer to a normal distribution than that of the residual autocorrelation.

This chapter extends the investigation of Kwan et al. (1992) to the area of testing the randomness of Gaussian time series. There are several important features which make the present analysis different from their work and previous studies. First, the theoretical mean and variance of the Kwan et al. test (QKWS) as given in their paper are incorrect. As a result, these quantities are derived here using the symbolic manipulation program *Mathematica*. Second, a simplified proof of the null distribution of QKWS is given. Third, contrary to most of the studies in this area, the simulation experiments also focus on the empirical power of the aforementioned portmanteau tests in the cases of seasonal and non-seasonal data.

The organization of the remainder of this chapter is as follows: Section 4.2 presents a brief discussion on the theoretical mean and variance of QKWS. Section 4.3 gives a simple proof of its null distribution. Section 4.4 describes the design of the Monte Carlo experiments. Section 4.5 reports the simulation results and some concluding remarks are offered in Section 4.6.

## 4.2 The Theoretical Mean and Variance of the KSW Test

Consider an independently and identically distributed (i.i.d.) normal time series $x_t$ $(t=1,\ldots,n)$ such that $E(x_t)=\mu$ and $var(x_t)=\sigma^2$ $\forall$ t. Next, define the k-th order sample autocorrelation as given in (2.2.8):

$$r_k = \frac{\sum_{t=1}^{n-k}(x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n}(x_t - \bar{x})^2}, \qquad 1 \le k \le n-1,$$

where $\bar{x} = \sum_{t=1}^{n} x_t/n$. In a classic paper, Bartlett (1946) derived several important formulae for computing $var(r_k)$. Specifically, for any time series:

$$var(r_k) = n^{-1}\left\{\sum_{i=-\infty}^{\infty}(\rho_i^2 + \rho_{i-k}\rho_{i+k} - 4\rho_k\rho_i\rho_{i+k} + 2\rho_i^2\rho_k^2)\right\} \tag{4.2.1}$$

where the $\rho$'s are population autocorrelations of the series [see e.g. Kendall and Stuart (1977, Vol. 3, p.548)]. This is true for large samples even irrespective of the assumption of normality. It is only in the special case when all $\rho_i$ $(i\neq0)$ are zero and the sample size is large that the R.H.S. of (4.2.1) reduces to $n^{-1}$. Note that the portmanteau statistic proposed by Box and Pierce (1970) is, for this special case, given by:

$$QBP = \underline{r}^T D_1^{-1} \underline{r}, \tag{4.2.2}$$

where $\underline{r}=(r,\ldots,r)^T$, $D_1=(1/n)I_m$, and $E(r_k)$ is assumed to be approximately zero. Under the null hypothesis of randomness, QBP is asymptotically distributed as $\chi^2(m)$.

As for the modified portmanteau tests of Ljung and Box (1978) and Dufour and Roy (1986), they can be written as follows:

$$\text{QLB} = \underline{r}^T D_2^{-1} \underline{r}\,, \tag{4.2.3}$$

$$\text{QDR} = (\underline{r} - \underline{\mu})^T\, \Sigma^{-1}\, (\underline{r} - \underline{\mu}), \tag{4.2.4}$$

and

$$\text{QDR1} = (\underline{r} - \underline{\mu})^T\, D_3^{-1}\, (\underline{r} - \underline{\mu}), \tag{4.2.5}$$

where $D_2=\text{diag}(C_1^2,\ldots,C_m^2)$, $C_k^2=(n-k)/\{n(n+2)\}$, $\underline{\mu}=(\mu_1,\ldots,\mu_m)^T$, $\Sigma=[\sigma_{kh}]$ is the variance-covariance matrix given by (2.2.10) and (2.2.11), and $D_3=\text{diag}(\sigma_{11},\ldots,\sigma_{mm})$ [see Chapter 2 for notations]. Under the null hypothesis of randomness, the asymptotic distributions of QLB, QDR and QDR1 are distributed as $\chi^2$ with m degrees of freedom.

To demonstrate the rationale for the Kwan et al. test, first note that the Bartlett formula given in (4.2.1) contains the population autocorrelations, the $\rho$'s, which are usually unknown. However, as indicated by Kwan et al. (1992), the presence of $\rho$'s in (4.2.1) may cause $\text{var}(r_k)$ to be unstable and consequently, lead to slow convergence to normality. A plausible solution is to employ the Fisher (1921) variance-stabilizing transformation:

$$z_k = \frac{1}{2}\log_e \frac{(1+r_k)}{(1-r_k)}\,, \tag{4.2.6}$$

where $z_k$ is normally distributed with $E(z_k)\simeq 0$ and $\text{var}(z_k)=(n-3)^{-1}$. Fisher (1921) suggested that this transformation serves two main purposes: (a) to stabilize $\text{var}(r_k)$ and (b) to normalize the underlying distribution of $r_k$. Based on (4.2.6), Kwan et al.

(1992) propose the following portmanteau test statistic:

$$QKSW = \sum_{k=1}^{m} (n-k-3)z_k^2 \ . \qquad (4.2.7)$$

On the null, the asymptotic distribution of QKSW is $\chi^2(m)$. For finite samples, it can be shown that their proposed test requires an adjustment to its theoretical mean. To highlight this important feature, let $M_i = \sum_{j=1}^{\infty} \left[\frac{r_k^{2j+1}}{2j+1}\right]$. Using a McLaurin series $M_0$ to represent $\frac{1}{2}\log_e(1+r_k)/(1+r_k)$ , it can be shown that $z_k = (r_k + \frac{1}{3}r_k^3 + \frac{1}{5}r_k^5 + M_3)$ and

$$z_k^2 = \left[r_k^2 + \frac{2}{3}r_k^4 + \frac{23}{45}r_k^6 + M_0M_3 + \ldots \right]. \qquad (4.2.8)$$

Substituting (4.2.8) into the R.H.S. of QKSW, one obtains

$$QKSW = \sum_{k=1}^{m} (n-k-3)\left[r_k^2 + \frac{2}{3}r_k^4 + \frac{23}{45}r_k^6 + M_0M_3 + \ldots \right] \qquad (4.2.9)$$

and

$$(QKSW)^2 = \sum_{k=1}^{m}\sum_{h=1}^{m} (n-k-3)(n-h-3)\left[r_k^2r_h^2 + \frac{2}{3}r_k^2r_h^4 + \frac{2}{3}r_k^4r_h^2 + \frac{4}{9}r_k^4r_h^4 \right.$$
$$+ \frac{23}{45}(r_k^2r_h^6 + r_k^6r_h^2) + (\frac{2}{3})(\frac{23}{45})(r_k^4r_h^6 + r_k^6r_h^4) + (\frac{23}{45})^2(r_k^6r_h^6)$$
$$\left. + \ldots\right] . \qquad (4.2.10)$$

It is clear from (4.2.9) and (4.2.10) that exact formulae for the first two moments of QKSW will be difficult to obtain. However, good approximations of E(QKSW) and var(QKSW) can be derived using the univariate and bivariate moments that appear in (4.2.9) and (4.2.10). According to Moran (1948), Davies et al.

(1977) and Ljung and Box (1978), the expected values needed to evaluate (4.2.9) and (4.2.10) can be expressed as follows:

$$E(r_k^2) = \frac{n-k}{n(n+2)} = O(n^{-1}) \qquad (4.2.11)$$

$$E(r_k^4) = \frac{3\{n^2-(2k-6)n+(k-10)k\}}{n(n+2)(n+4)(n+6)} = O(n^{-2}), \qquad (4.2.12)$$

$$E(r_k^6) = \frac{15\{n^3-(3k-18)n^2+(3k^2-48k+92)n+(30k^2-k^3-212k)\}}{n(n+2)(n+4)(n+6)(n+8)(n+10)} = O(n^{-3}), \qquad (4.2.13)$$

and for $k < h$,

$$E(r_k^2 r_h^2) = \frac{\{n^2-(h+k-12)n+(hk-8h-12k)\}}{n(n+2)(n+4)(n+6)} = O(n^{-2}), \qquad (4.2.14)$$

$$E(r_k^4 r_h^2) = \frac{3\{n^3-(2h+k-18)n^2+(h^2+2hk-16h-32k+260)n-(2h^2-8k^2-24hk+h^2k+304h+280k)\}}{n(n+2)(n+4)(n+6)(n+8)(n+10)} = O(n^{-3}), \qquad (4.2.15)$$

$$E(r_k^2 r_h^4) = \frac{3\{n^3-(2h+k-18)n^2+(h^2+2hk-28h-20k+260)n+(6h^2+24hk-h^2k-456h-116k)\}}{n(n+2)(n+4)(n+6)(n+8)(n+10)} = O(n^{-3}), \qquad (4.2.16)$$

$E(r_k^4 r_h^4)$, $E(r_k^2 r_h^6)$ and $E(r_k^6 r_h^2)$ are $O(n^{-4})$, $E(r_k^4 r_h^6)$ and $E(r_k^6 r_h^4)$ are $O(n^{-5})$, and $E(r_k^6 r_h^6)$ is $O(n^{-6})$. Note that the expressions for $E(r_k^6)$, $E(r_k^2 r_h^4)$ and $E(r_k^4 r_h^2)$ are derived using the identity given in Ljung and Box (1978, p.298).

From (4.2.11)-(4.2.13), it can be shown that the first three terms on the R.H.S. of E(QKSW) are of $O(1)$, $O(n^{-1})$ and $O(n^{-2})$, respectively. When n is large and m small, the third term can be dropped and

$$E(QKSW)=\left[\sum_{k=1}^{m}(n-k-3)[E(r_k^2)+\frac{2}{3}E(r_k^4)]+O(n^{-2})\right]. \tag{4.2.17}$$

Upon substitution of (4.2.11) and (4.2.12) into (4.2.17), it can be shown approximately, for a large n relative m, that

$$E(QKSW) = m - \frac{m(m+4)}{n} . \tag{4.2.18}$$

Upon taking the limit of (4.2.18), we have $\lim_{n\to\infty} E(QKSW)=m$. Thus, QKSW has the same asymptotic mean as QBP, QLB, QDR and QDR1.

The variance formula for QKSW can be derived by taking expectations of (4.2.10) and subtracting the square of the expected value of (4.2.9). Thus, we have

$$var(QKSW) = E\{(QKSW)^2\} - \{E(QKSW)\}^2. \tag{4.2.19}$$

From (4.2.12)-(4.2.16), a suitable approximation of var(QKSW) can be obtained by ignoring terms smaller than $O(n^{-1})$:

$$\begin{aligned} var(QKSW) = \Bigg\{ &\sum_{k=1}^{m}(n-k-3)^2\left[E(r_k^4) + \frac{4}{3}E(r_k^6)\right] \\ &+ 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)\left[E(r_k^2r_h^2) + \frac{2}{3}E(r_k^2r_h^4) + \frac{2}{3}E(r_k^4r_h^2)\right] \\ &- \{E(QKSW)\}^2\Bigg\}, \end{aligned} \tag{4.2.20}$$

where $\{E(QKSW)^2\}=\left[\sum_{k=1}^{m}(n-k-3)[E(r_k^2) + \frac{2}{3}E(r_k^4)]\right]^2$. Upon substitution of (4.2.12)-(4.2.16), var(QKSW) is approximately equivalent to

$$\text{var(QKSW)} = 2m - \frac{18m}{n}, \tag{4.2.21}$$

when the ratio of (m/n) is small. Based on (4.2.21), it can easily be shown that $\lim_{n\to\infty} \text{var(QKSW)}=2m$. Thus, QKSW has the same asymptotic variance as QBP, QLB, QDR and QDR1. This result along with (4.2.18) support the above assertion that the asymptotic distribution of QKSW is approximately $\chi^2(m)$.

It is important to mention that, since the derivations of (4.2.18) and (4.2.21) require some tedious algebra and involve some cumbersome expressions, the two formulae were checked with the symbolic manipulation program *Mathematica*. The detailed derivations of (4.2.18) and (4.2.21) are given in Appendix A.

## 4.3 A Simple Proof of the Asymptotic Mean and Variance of QKSW

The objective of this section is to provide an alternative proof of the mean and variance of QKSW when n approaches infinity.

Recall that the approximation of E(QKSW) is

$$E(\text{QKSW}) = \left\{ \sum_{k=1}^{m} (n-k-3)[E(r_k^2)+\tfrac{2}{3}E(r_k^4)] \right\}.$$

Upon taking the limit of E(QKSW), we have

$$\lim_{n\to\infty} E(\text{QKSW}) = \left\{ \sum_{k=1}^{m} \lim_{n\to\infty} (n-k-3)[E(r_k^2)+\tfrac{2}{3}E(r_k^4)] \right\} = m.$$

This is because $(n-k-3)E(r_k^2)$ and $(n-k-3)E(r_k^4)$ are $O(1)$ and $O(n^{-1})$, respectively.

An approximation of $E\{(QKSW)^2\}$ is

$$E\{(QKSW)^2\} = \left\{\sum_{k=1}^{m}(n-k-3)^2\left[E(r_k^4)+\frac{4}{3}E(r_k^6)\right]\right.$$

$$\left.+2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)\left[E(r_k^2r_h^2)+\frac{2}{3}E(r_k^2r_h^4)+\frac{2}{3}E(r_k^4r_h^2)\right]\right\}.$$

Upon taking the limit of the above approximation, we have

$$\lim_{n\to\infty} E\{(QKSW)^2\} = \left\{\sum_{k=1}^{m}\lim_{n\to\infty}(n-k-3)^2\left[E(r_k^4)+\frac{4}{3}E(r_k^6)\right]\right.$$

$$\left.+2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}\lim_{n\to\infty}(n-k-3)(n-h-3)\left[E(r_k^2r_h^2)+\frac{2}{3}E(r_k^2r_h^4)+\frac{2}{3}E(r_k^4r_h^2)\right]\right\}.$$

Since each of $E(r_k^6)$, $E(r_k^2r_h^4)$ and $E(r_k^4r_h^2)$ is of $O(n^{-3})$, thus each of $(n-k-3)E(r_k^6)$, $(n-k-3)(n-h-3)E(r_k^2r_h^4)$ and $(n-k-3)(n-h-3)E(r_k^4r_h^2)$ is $O(n^{-1})$ and converges to zero as $n\to\infty$. Therefore,

$$\lim_{n\to\infty} E\{(QKSW)^2\} = \left\{\sum_{k=1}^{m}\lim_{n\to\infty}(n-k-3)^2E(r_k^4) +\right.$$

$$\left.2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}\lim_{n\to\infty}(n-k-3)(n-h-3)E(r_k^2r_h^2)\right\}.$$

Using (4.2.12) and (4.2.14), it is straightforward to show that $\lim_{n\to\infty}(n-k-3)^2E(r_k^4)=3$ and $\lim_{n\to\infty}(n-k-3)(n-h-3)E(r_k^2r_h^2)=1$. As a result, we have

$$\lim_{n\to\infty} E\{(QKSW)^2\} = 3m + 2\sum_{k=1}^{m-1}(m-k) = 2m + m^2 .$$

From (4.2.19), we have

$$\lim_{n\to\infty} \mathrm{var}(QKSW) = 2m.$$

Thus, $\lim_{n\to\infty} E(QKSW)=m$ and $\lim_{n\to\infty} \mathrm{var}(QKSW)=2m$, as was required.

## 4.4 Experimental Design

The proposed statistic by Kwan et al. (1992) is a large-sample test statistic because it requires a large sample size to justify its use. Naturally then, it is important to investigate the empirical performance of QKSW in small samples, expecially commonly-used sample sizes. Also note that when n ranges from small to moderate or the (m/n) ratio is large, E(QKSW) and var(QKSW) given in (4.2.18) and (4.2.21) can be smaller than m and 2m. Furthermore, E(QKSW) as well as var(QKSW) are not necessarily integers for finite n. In order to use QKSW for practical purposes, it is therefore necessary to adjust the mean of the proposed test statistic [see also Ljung and Box (1978, p.301)]. A "chi-square" test based on QKSW can still be carried out by executing the following steps:

Step 1. Compute the sample autocorrelations $r_k$, (k=1,...,m).

Step 2. For each $r_k$, compute $z_k$.

Step 3. Compute QKSW.

Step 4. Compute E(QKSW) using the approximation, E(QKSW)=

$\left[\sum_{k=1}^{m}(n-k-3)[E(r_k^2)+\frac{2}{3}E(r_k^4)]\right]$, where $E(r_k^2)$ and $E(r_k^4)$ are given by (4.2.11) and (4.2.12), respectively.

Step 5. Reject the null hypothesis of randomness whenever $QKSW \geq \chi_{\alpha}^2(E(QKSW))$ in which $\alpha$ is the level of significance.

In order to examine the finite-sample performance of QKSW, two Monte Carlo experiments are used. In the first experiment, attention was confined to the empirical significance levels, means and variances of QLB, QDR1 and QKSW. QBP is not examined here because earlier simulation results indicate that it is not a reliable statistic when n ranges from small to moderate. Also, QDR is excluded because QDR1 is as good as QDR and requires fewer computations [see Dufour and Roy (1986, pp.2965-2968)].

All simulations were carried out on a VAX2 computer. N(0,1) random deviates, $X_t$, were generated by using the International Mathematical Subroutine Library (IMSL) subroutine RNNOA. IMSL's subroutine CHINN was used to compute the critical values for each statistic. Estimates of significance levels ($\alpha$=1%, 5% and 10%), means and variances of these statistics were based on 20,000 replications. The sample size, n, was set to be 50, 100 and 150, and the values of m were chosen from 1, 3, 5, 10, 15, 25, 50, and 75.

The second experiment evaluated the empirical power of QLB, QDR1 and QKSW. All data were first generated from some selected cases of the following multiplicative seasonal model:

$$(1-\pi_1 B)(1-\pi_4 B^4)X_t = a_t.$$

The starting value $X_0$ was set at 0. N(0,1) random deviates, $a_t$, were generated using the subroutine RNNOA of IMSL. In this experiment, the first fifty observations were dropped before each sample was collected. This is due to the fact that we want to account for the presence of seasonality in the data. Although the choice of m is identical for both experiments, the sample size in all power calculations was set to be 50 and 100. It is also important to point out that we only recorded the proportion of times that the hypothesis of randomness was rejected for tests at the 1%, 5% and 10% levels of significance. Again, all computations were based on 20,000 replications.

## 4.5 Monte Carlo Results

Table 4.1 reports the empirical significance levels, means and variances of QLB, QDR1 and QKSW. The results of QLB and QDR1 are generally similar to those of Dufour and Roy (1986). On the whole, QDR1 appears to be better approximated by the $\chi^2(m)$ distribution than QLB. However, it is apparent that both tests tend to reject the null hypothesis far too often when the ratio (m/n) is large. This observation holds regardless of the choice of significance levels. Also, both QLB and QDR1 suffer from the problem of dispersion bias. For example, when $m \geq 3$, their

Table 4.1: Empirical Significance Levels, Means, Variances and Variance-Mean Ratios of Portmanteau Statistics (QLB, QDR1 and QKSW) for a Normal White Noise

| | | 1% | | | 5% | | | 10% | | |
|---|---|---|---|---|---|---|---|---|---|---|
| n | m | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW |
| 50 | 1 | 0.94 | 0.87 | 0.91 | 5.06 | 4.77 | 4.75 | 10.24 | 10.05 | 9.91 |
| | 3 | 1.11 | 1.11 | 0.99 | 5.13 | 4.85 | 4.71 | 10.01 | 9.74 | 9.49 |
| | 5 | 1.51 | 1.44 | 1.22 | 5.66 | 5.39 | 4.97 | 10.31 | 9.75 | 9.41 |
| | 10 | 2.18 | 2.08 | 1.55 | 7.11 | 6.76 | 5.64 | 11.73 | 11.22 | 9.83 |
| | 15 | 2.97 | 2.87 | 1.79 | 7.96 | 7.47 | 5.57 | 12.82 | 12.17 | 10.27 |
| | 25 | 4.09 | 3.80 | 1.91 | 9.65 | 8.86 | 5.90 | 14.62 | 13.69 | 10.25 |
| 100 | 1 | 1.06 | 1.02 | 1.05 | 5.20 | 5.06 | 5.06 | 10.29 | 10.12 | 10.09 |
| | 3 | 1.16 | 1.09 | 1.08 | 5.51 | 5.37 | 5.29 | 10.44 | 10.36 | 10.12 |
| | 5 | 1.24 | 1.18 | 1.13 | 5.38 | 5.30 | 5.06 | 10.45 | 10.34 | 10.10 |
| | 10 | 1.79 | 1.78 | 1.50 | 5.97 | 5.82 | 5.27 | 11.00 | 10.75 | 10.06 |
| | 15 | 2.22 | 2.10 | 1.62 | 6.86 | 6.66 | 5.80 | 11.70 | 11.26 | 10.34 |
| | 25 | 3.20 | 3.10 | 2.14 | 8.18 | 7.87 | 6.25 | 13.20 | 12.70 | 10.94 |
| | 50 | 4.54 | 4.23 | 2.25 | 10.55 | 9.90 | 6.69 | 15.41 | 14.57 | 11.11 |
| 150 | 1 | 0.93 | 0.91 | 0.92 | 4.99 | 4.95 | 4.86 | 9.95 | 9.79 | 9.79 |
| | 3 | 1.05 | 1.00 | 1.00 | 5.07 | 5.03 | 4.89 | 10.13 | 10.09 | 9.90 |
| | 5 | 1.21 | 1.18 | 1.15 | 5.23 | 5.19 | 5.14 | 10.22 | 9.84 | 9.93 |
| | 10 | 1.50 | 1.52 | 1.34 | 5.95 | 5.81 | 5.48 | 10.66 | 10.53 | 10.06 |
| | 15 | 1.74 | 1.73 | 1.41 | 6.37 | 6.19 | 5.71 | 10.97 | 10.69 | 10.10 |
| | 25 | 2.50 | 2.45 | 1.81 | 7.20 | 7.02 | 6.01 | 12.26 | 11.89 | 10.79 |
| | 50 | 3.91 | 3.79 | 2.32 | 9.39 | 9.09 | 6.73 | 14.72 | 14.30 | 11.41 |
| | 75 | 4.93 | 4.65 | 2.41 | 10.88 | 10.49 | 7.09 | 15.78 | 15.22 | 11.61 |

| | | Empirical Means | | | | Empirical Variances | | | (Var/Mean) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| n | m | QLB | QDR1 | QKSW | E(QKSW) | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW |
| 50 | 1 | 1.02 | 1.00 | 0.92 | 0.90 | 1.95 | 1.88 | 1.71 | 1.91 | 1.88 | 1.86 |
| | 3 | 3.06 | 3.00 | 2.64 | 2.58 | 6.19 | 6.18 | 4.98 | 2.02 | 2.05 | 1.88 |
| | 5 | 5.11 | 5.01 | 4.22 | 4.12 | 10.98 | 10.95 | 8.11 | 2.15 | 2.18 | 1.92 |
| | 10 | 10.20 | 10.03 | 7.54 | 7.37 | 25.79 | 25.62 | 15.28 | 2.53 | 2.56 | 2.03 |
| | 15 | 15.31 | 15.04 | 10.09 | 9.86 | 43.94 | 43.37 | 21.00 | 2.87 | 2.88 | 2.08 |
| | 25 | 25.56 | 25.06 | 13.28 | 12.97 | 84.90 | 83.66 | 27.57 | 3.32 | 3.34 | 2.08 |
| 100 | 1 | 1.02 | 1.01 | 0.97 | 0.95 | 2.04 | 2.00 | 1.92 | 2.00 | 1.98 | 1.98 |
| | 3 | 3.06 | 3.03 | 2.85 | 2.79 | 6.26 | 6.21 | 5.64 | 2.05 | 2.05 | 1.98 |
| | 5 | 5.06 | 5.01 | 4.61 | 4.55 | 10.47 | 10.43 | 9.06 | 2.07 | 2.09 | 1.96 |
| | 10 | 10.11 | 10.12 | 8.74 | 8.64 | 23.24 | 23.17 | 18.09 | 2.30 | 2.31 | 2.07 |
| | 15 | 15.17 | 15.03 | 12.43 | 12.28 | 37.72 | 33.57 | 26.46 | 2.49 | 2.50 | 2.13 |
| | 25 | 25.29 | 25.06 | 18.58 | 18.35 | 74.32 | 73.89 | 42.16 | 2.94 | 2.95 | 2.27 |
| | 50 | 50.60 | 50.10 | 27.90 | 27.55 | 184.28 | 182.70 | 64.50 | 3.64 | 3.65 | 2.31 |
| 150 | 1 | 1.00 | 0.99 | 0.96 | 0.97 | 1.97 | 1.94 | 1.89 | 1.98 | 1.96 | 1.96 |
| | 3 | 3.03 | 3.01 | 2.89 | 2.86 | 6.09 | 6.05 | 5.68 | 2.01 | 2.01 | 1.96 |
| | 5 | 5.04 | 5.00 | 4.74 | 4.70 | 10.28 | 10.25 | 9.34 | 2.04 | 2.05 | 1.97 |
| | 10 | 10.08 | 10.01 | 9.16 | 9.08 | 22.17 | 22.14 | 18.79 | 2.20 | 2.21 | 2.05 |
| | 15 | 15.12 | 15.02 | 13.27 | 13.16 | 35.18 | 35.07 | 27.88 | 2.37 | 2.33 | 2.10 |
| | 25 | 25.19 | 25.02 | 20.59 | 20.43 | 65.50 | 65.19 | 45.20 | 2.60 | 2.61 | 2.20 |
| | 50 | 50.31 | 49.99 | 34.24 | 34.00 | 164.01 | 163.04 | 80.15 | 3.26 | 3.26 | 2.34 |
| | 75 | 75.49 | 74.98 | 42.43 | 42.13 | 278.88 | 277.19 | 99.85 | 3.69 | 3.70 | 2.35 |

Note: (Var/Mean) is the ratio of empirical variances to empirical means empirical variances and variance-mean ratios are larger than 2m and 2, respectively. This is particularly evident when (m/n) is large.

On the other hand, the simulation results suggest that the performance of QKSW is significantly better than QLB and QDR1 in terms of controlling test size. When (m/n) is large, and $\alpha$=5% and 10%, the empirical significance levels of QKSW are still close to the nominal levels. However, at the 1% level, the test rejects the null hypothesis too frequently. But even in these cases, the over-rejection problem is far less severe for QKSW.

From the point of view of minimizing the dispersion bias, QKSW performs remarkably well. This can be seen from the fact that its variance-mean ratios range from 1.86 to 2.35, while QLB and QDR1 can have ratios larger than 3.6. The main implication of these results is that QKSW is valid over a wider range of m than either QLB or QDR1. For $50 \le n \le 100$, a value of $m \ge 10$ may not be suitable for either QLB or QDR1. This may limit the practical application of either QLB or QDR1. In contrast, this limitation does not necessarily apply to QKSW when $n \ge 50$.

Finally, the empirical means of QKSW are very close to the theoretical values which are computed basing on (4.2.17). After simple calculations, it can be shown that, in the worst case, the percentage difference between the theoretical and empirical means does not exceed 2.4%. This indicates that the mean formula stated in (4.2.17) provides an adequate approximation to the theoretical means.

Table 4.2 reports the empirical power of QLB, QDR1 and QKSW. The main conclusion is that their power performance is quite similar. Focussing on the case where the data were generated from an AR(1) process (i.e. $\pi_4=0$), the empirical power of the tests examined is very high. However, it is obvious that the ability of these tests to detect non-randomness is reduced when the value of m increases. This is particularly true when $\pi_1=0.7$. A plausible reason for this phenomenon is that if the underlying data-generation process (DGP) follows a simple AR process, lack of randomness of a time series will most likely show up in the first few sample autocorrelations. This finding is consistent with Ljung's (1986) results which indicate that the use of a smaller number of residual autocorrelations in model diagnostics will lead to a more powerful test when the DGP is represented by lower-order ARMA models.

In cases where the data were generated from a restricted AR(4) process (i.e. $\pi_1=0$), the simulation results reveal that the loss in empirical power due to the use of small values of m (i.e. $m\leq3$) can be quite substantial. In many cases, the loss can be more than two-fold. On the other hand, when the data were generated from a multiplicative AR(4) process, the power

Table 4 2: Empirical Powers of Portmanteau Statistics (QLB, QDR1 and QKSW).
Data Generating Processes: $(1 - \pi_1 B)(1 - \pi_4 B)X_t = a_t$.

| | | | n = 50 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1% | | | 5% | | | 10% | |
| $\pi_1$ | $\pi_4$ | m | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW |
| 0.7 | 0 | 1 | 98.08 | 98.56 | 98.04 | 99.54 | 99.72 | 99.54 | 99.84 | 99.90 | 99.84 |
| | | 3 | 95.16 | 96.30 | 95.76 | 98.22 | 98.90 | 98.44 | 99.14 | 99.36 | 99.20 |
| | | 5 | 92.90 | 94.26 | 94.02 | 97.32 | 97.80 | 97.62 | 98.52 | 98.92 | 98.72 |
| | | 10 | 89.02 | 90.84 | 92.00 | 95.14 | 96.14 | 96.42 | 97.02 | 97.60 | 98.00 |
| | | 15 | 86.62 | 88.10 | 91.08 | 93.78 | 94.76 | 96.04 | 96.06 | 96.62 | 97.32 |
| | | 25 | 85.52 | 85.98 | 90.42 | 91.76 | 92.04 | 95.34 | 94.18 | 94.60 | 96.84 |
| 0.9 | 0 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 3 | 99.86 | 99.90 | 99.90 | 99.98 | 100 | 100 | 100 | 100 | 100 |
| | | 5 | 99.68 | 99.84 | 99.98 | 99.94 | 99.94 | 99.94 | 99.96 | 100 | 99.98 |
| | | 10 | 99.24 | 99.38 | 99.56 | 99.76 | 99.86 | 99.86 | 99.88 | 99.88 | 99.92 |
| | | 15 | 99.02 | 99.18 | 99.44 | 99.50 | 99.60 | 99.84 | 99.70 | 99.84 | 99.90 |
| | | 25 | 98.68 | 98.78 | 99.44 | 99.36 | 99.54 | 99.78 | 99.66 | 99.72 | 99.90 |
| 0 | 0.7 | 1 | 8.46 | 7.34 | 8.36 | 17.22 | 15.90 | 16.90 | 24.78 | 23.12 | 24.20 |
| | | 3 | 25.78 | 23.34 | 25.56 | 42.42 | 38.86 | 41.22 | 52.20 | 48.80 | 51.30 |
| | | 5 | 91.26 | 92.60 | 90.66 | 97.26 | 97.80 | 96.86 | 98.48 | 98.84 | 98.32 |
| | | 10 | 88.56 | 89.86 | 88.74 | 94.82 | 95.54 | 95.04 | 96.70 | 97.28 | 96.90 |
| | | 15 | 85.66 | 86.64 | 86.98 | 92.68 | 93.24 | 93.54 | 94.80 | 95.68 | 96.22 |
| | | 25 | 83.74 | 84.16 | 85.92 | 90.52 | 90.15 | 92.44 | 93.12 | 93.32 | 97.76 |
| 0 | 0.9 | 1 | 20.78 | 18.82 | 20.70 | 31.90 | 29.62 | 31.52 | 38.62 | 36.12 | 38.26 |
| | | 3 | 59.42 | 54.24 | 58.48 | 75.16 | 71.52 | 74.32 | 81.04 | 78.52 | 80.56 |
| | | 5 | 99.70 | 99.78 | 99.66 | 99.98 | 99.98 | 99.98 | 100 | 100 | 100 |
| | | 10 | 99.52 | 99.66 | 99.58 | 99.86 | 99.90 | 99.88 | 99.94 | 99.98 | 99.98 |
| | | 15 | 99.32 | 99.46 | 99.44 | 99.80 | 99.82 | 99.84 | 99.88 | 99.86 | 99.90 |
| | | 25 | 98.82 | 98.92 | 99.32 | 95.50 | 99.52 | 99.74 | 99.76 | 99.74 | 99.90 |
| 0.7 | 0.3 | 1 | 97.40 | 98.10 | 97.38 | 99.28 | 99.40 | 99.28 | 99.74 | 99.84 | 99.72 |
| | | 3 | 94.54 | 95.88 | 95.10 | 97.76 | 98.38 | 97.94 | 99.86 | 99.18 | 98.90 |
| | | 5 | 93.84 | 95.24 | 97.42 | 97.42 | 98.32 | 97.80 | 98.74 | 99.12 | 98.82 |
| | | 10 | 92.52 | 93.42 | 93.56 | 96.04 | 97.00 | 97.06 | 97.54 | 98.10 | 98.20 |
| | | 15 | 91.34 | 92.34 | 93.06 | 95.36 | 96.06 | 96.54 | 96.94 | 97.36 | 97.80 |
| | | 25 | 90.02 | 90.56 | 92.42 | 94.42 | 94.78 | 96.30 | 96.04 | 96.36 | 97.54 |
| 0.9 | 0.3 | 1 | 99.94 | 99.98 | 99.94 | 99.98 | 100 | 99.98 | 100 | 100 | 100 |
| | | 3 | 99.78 | 99.86 | 99.84 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 |
| | | 5 | 99.76 | 98.84 | 99.80 | 99.92 | 99.96 | 99.96 | 99.98 | 99.98 | 99.98 |
| | | 10 | 99.64 | 99.74 | 99.76 | 99.80 | 99.90 | 99.92 | 99.92 | 99.98 | 99.98 |
| | | 15 | 98.50 | 99.60 | 99.68 | 99.84 | 99.86 | 99.92 | 99.92 | 99.94 | 99.94 |
| | | 25 | 99.38 | 99.40 | 99.64 | 99.74 | 99.72 | 99.86 | 99.78 | 99.80 | 99.94 |

Table 4.2 (Cont'd)

| | | | n = 100 | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 1% | | | 5% | | | 10% | |
| $\pi_1$ | $\pi_4$ | m | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW | QLB | QDR1 | QKSW |
| 0.7 | 0 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 10 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 15 | 99.80 | 99.80 | 99.92 | 99.96 | 100 | 100 | 100 | 100 | 100 |
| | | 25 | 99.34 | 99.44 | 99.84 | 99.92 | 99.92 | 100 | 100 | 100 | 100 |
| | | 50 | 98.80 | 98.90 | 99.62 | 99.54 | 99.60 | 99.96 | 99.74 | 99.74 | 99.98 |
| 0.9 | 0 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 10 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 15 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 25 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 50 | 99.98 | 99.98 | 100 | 99.98 | 99.98 | 100 | 100 | 100 | 100 |
| 0 | 0.7 | 1 | 10.22 | 9.74 | 10.16 | 20.08 | 19.36 | 19.88 | 28.02 | 27.18 | 27.78 |
| | | 3 | 31.54 | 30.16 | 31.18 | 46.78 | 44.96 | 46.06 | 54.88 | 53.64 | 54.32 |
| | | 5 | 99.90 | 99.92 | 99.90 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 |
| | | 10 | 99.94 | 99.94 | 99.94 | 99.96 | 99.96 | 99.96 | 99.96 | 99.98 | 99.98 |
| | | 15 | 99.82 | 99.88 | 99.88 | 99.94 | 99.94 | 99.96 | 99.98 | 99.98 | 99.98 |
| | | 25 | 99.38 | 99.42 | 99.72 | 99.84 | 99.84 | 99.96 | 99.92 | 99.94 | 99.96 |
| | | 50 | 98.66 | 98.72 | 99.50 | 99.48 | 99.44 | 99.86 | 99.72 | 99.72 | 99.94 |
| 0 | 0.9 | 1 | 25.92 | 24.64 | 25.80 | 37.40 | 36.16 | 37.14 | 44.98 | 43.34 | 44.70 |
| | | 3 | 68.96 | 67.00 | 68.58 | 78.98 | 77.52 | 78.46 | 83.30 | 82.42 | 83.12 |
| | | 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 10 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 15 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 25 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 50 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 0.7 | 0.3 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 10 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 | 100 | 100 | 100 | 100 |
| | | 15 | 99.90 | 99.92 | 99.96 | 99.98 | 99.98 | 99.98 | 99.98 | 99.98 | 100 |
| | | 25 | 99.76 | 99.80 | 99.92 | 99.96 | 99.94 | 99.98 | 99.98 | 99.98 | 100 |
| | | 50 | 99.64 | 99.66 | 99.88 | 99.90 | 99.90 | 99.94 | 99.94 | 99.96 | 99.98 |
| 0.9 | 0.3 | 1 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 3 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 5 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 10 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 15 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 25 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| | | 50 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

performance of QLB, QDR1 and QKSW is remarkably good. It can be seen from the results given in Table 4.2 that the rejection rates are unanimously greater than 90%. This observation holds regardless of the sample size, the value of m and the choice of significance levels.

## 4.6 Concluding Remarks

Portmanteau tests have been one of the main statistical instruments in examining the randomness of a time series. In this chapter, an attempt has been made to compare the finite-sample performance of the recently proposed test by Kwan et al. (1992) with that of Ljung and Box (1978) and Dufour and Roy (1986). The Monte Carlo simulation results indicate that QKWS performs better than QLB and QDR1, in terms of both controlling test size and minimizing dispersion bias. In addition, the power calculations reveal that the tests examined exhibit similar power performance. This holds true whether the data are seasonally or non-seasonally generated. Thus, the findings favour the use of the Kwan et al. test in applied statistical and econometric applications.

# CHAPTER FIVE

# ON THE SMALL-SAMPLE DISTRIBUTION OF SOME MODIFIED PORTMANTEAU TESTS FOR RANDOMNESS OF GAUSSIAN TIME SERIES

## 5.1 Introduction

In Chapter 4, it is shown that, in the area of testing the randomness of a Gaussian time series, the empirical size of the Kwan et al. test is more accurate than the Ljung-Box and the Dufour-Roy statistics. This finding supports the conjecture that the poor finite-sample performance of a portmanteau test can be attributed, at least to a large extent, to the slow convergence of the sample autocorrelation to normality.

While the simulation evidence of the Kwan et al. test is encouraging, two areas are worthy of further investigation. First, all results reported in Chapter 4 are based on moderate samples ($n \geq 50$). It would clearly be interesting to see whether the test performs well in smaller samples. Second, Hotelling (1953) has suggested some alternative transformations which may perform better than the Fisher (1921) variance-stabilizing transformation in small samples. It would be useful to construct portmanteau tests which are based on the Hotelling (1953) transformations. In this context, a comparison of the empirical performance of the proposed tests and the Kwan et al. test would be of practical importance in terms of choosing diagnostic checks in empirical applications.

The organization of the remainder of this chapter is as follows: Section 5.2 derives the theoretical distributions of the new portmanteau tests. Section 5.3 describes the design of the Monte Carlo simulation experiments and reports the main simulation results. This chapter closes with Section 5.4 which offers some concluding remarks.

### 5.2 Modified Kwan-Sim-Wong Portmanteau Tests

The simulation results of Chapter 4 indicate that, except for QKSW, size distortion as well as dispersion bias can be substantial for portmanteau tests such as the Ljung-Box and Dufour-Roy statistics for $50 \le n \le 150$. Inspired by these findings, two modified versions of QKWS are constructed by employing the transformations suggested by Hotelling (1953). The main advantage of the Hotelling transformations is their ability to make $r_k$ converge more rapidly to normality than the statistic based on $z_k$.

The Hotelling transformations of $r_k$ are

$$z_{1k} = z_k - \frac{3z_k + r_k}{4(n-k)}, \tag{5.2.1}$$

and

$$z_{2k} = z_k - \frac{3z_k+r_k}{4(n-k)} - \frac{23z_k+33r_k-5r_k^3}{96(n-k)^2} . \qquad (5.2.2)$$

It can easily be seen from (5.2.1) and (5.2.2) that the difference between these two formulae and the Fisher variance-stabilizing transformation is the additional terms, $(3z_k+r_k)/\{4(n-k)\}$ and $(23z_k+33r_k-5r_k^3)/\{96(n-k)^2\}$, which appear on the R.H.S. of (5.2.1) and (5.2.2), respectively. For moderate samples, the Fisher and the Hotelling transformations should perform similarly in terms of normalizing the distribution of $r_k$.

Hotelling (1953) showed that the distributions of $z_{1k}$ and $z_{2k}$ are approximately normal with mean zero and $var(z_{1k})=var(z_{2k})=(n-1)^{-1}$. Using these conditions, two modified portmanteau tests can be constructed on the basis of $z_{1k}$ and $z_{2k}$:

$$Q1 = \sum_{k=1}^{m}(n-k-1)z_{1k}^2, \qquad (5.2.3)$$

and

$$Q2 = \sum_{k=1}^{m}(n-k-1)z_{2k}^2. \qquad (5.2.4)$$

Under the null hypothesis of randomness, both Q1 and Q2 are distributed as $\chi^2(m)$ for large n. When the sample size is small to moderate, the theoretical distributions of Q1 and Q2 can be derived using proofs similar to those given in Chapter 4.

To obtain the theoretical means and variances for Q1 and Q2, first let a McLaurin series $M_1 = \sum_{j=1}^{\infty}\left[\frac{r_k^{2j+1}}{2j+1}\right]$. It can easily be shown that

$$z_{1k} = \left[\beta_{k,0}r_k + \frac{1}{3}\beta_{k,1}r_k^3 + \frac{1}{5}\beta_{k,1}r_k^5 + \beta_{k,1}M_3\right], \tag{5.2.5}$$

$$z_{2k} = \left[\delta_{k,0}r_k + \frac{1}{3}\delta_{k,1}r_k^3 + \frac{1}{5}\delta_{k,2}r_k^5 + \delta_{k,2}M_3\right], \tag{5.2.6}$$

$$z_{1k}^2 = \left[\beta_{k,0}^2 r_k^2 + \frac{2}{3}\beta_{k,0}\beta_{k,1}r_k^4 + (\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^2)r_k^6 + \dots\ \right], \tag{5.2.7}$$

and $$z_{2k}^2 = \left[\delta_{k,0}^2 r_k^2 + \frac{2}{3}\delta_{k,0}\delta_{k,1}r_k^4 + (\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta_{k,1}^2)r_k^6 + \dots\ \right], \tag{5.2.8}$$

where $\beta_{k,0} = 1 - \frac{1}{n-k}$ ,

$\beta_{k,1} = 1 - \frac{3}{4(n-k)}$ ,

$\delta_{k,0} = 1 - \frac{1}{n-k} - \frac{7}{12(n-k)^2}$ ,

$\delta_{k,1} = 1 - \frac{3}{4(n-k)} - \frac{1}{12(n-k)^2}$ ,

and $\delta_{k,2} = 1 - \frac{3}{4(n-k)} - \frac{1}{23(n-k)^2}$ .

Substituting (5.2.7) into (5.2.3) yields

$$Q1 = \sum_{k=1}^{m}(n-k-1)\left[\beta_{k,0}^2 r_k^2 + \frac{2}{3}\beta_{k,0}\beta_{k,1}r_k^4 + (\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^2)r_k^6 + \dots\ \right], \tag{5.2.9}$$

and

$$(Q1)^2 = \sum_{k=1}^{m}\sum_{h=1}^{m}(n-k-1)(n-h-1)\left[\beta_{k,0}^2\beta_{h,0}^2(r_k^2 r_h^2) + \frac{2}{3}\beta_{k,0}^2\beta_{h,0}\beta_{h,1}(r_k^2 r_h^4)\right.$$

$$+ \frac{2}{3}\beta^2_{h,0}\beta_{k,0}\beta_{k,1}(r_k^4 r_h^2) + \frac{4}{9}\beta_{k,0}\beta_{k,1}\beta_{h,0}\beta_{h,1}(r_k^4 r_h^4)$$

$$+ \beta^2_{k,0}(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta^2_{h,1})(r_k^2 r_h^6) + \beta^2_{h,0}(\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta^2_{k,1})(r_k^6 r_h^2)$$

$$+ \frac{2}{3}\beta_{k,0}\beta_{k,1}(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta^2_{h,1})(r_k^4 r_h^6)$$

$$+ \frac{2}{3}\beta_{h,0}\beta_{h,1}(\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta^2_{k,1})(r_k^6 r_h^4)$$

$$+ (\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta^2_{k,1})(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta^2_{h,1})(r_k^6 r_h^6) + \ldots\Bigg] ,$$

(5.2.10)

where $\beta_{h,0}=1-\{1/(n-h)\}$ and $\beta_{h,1}=1-\{3/[4(n-h)]\}$.

Similarly, substituting (5.2.8) into (5.2.4) yields

$$Q2 = \sum_{k=1}^{m} (n-k-1)\Bigg[\delta^2_{k,0}r_k^2 + \frac{2}{3}\delta_{k,0}\delta_{k,1}r_k^4 + (\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta^2_{k,1})r_k^6 + \ldots \Bigg],$$

(5.2.11)

and

$$(Q2)^2 = \sum_{k=1}^{m}\sum_{h=1}^{m} (n-k-1)(n-h-1)\Bigg[\delta^2_{k,0}\delta^2_{h,0}(r_k^2 r_h^2) + \frac{2}{3}\delta^2_{k,0}\delta_{h,0}\delta_{h,1}(r_k^2 r_h^4)$$

$$+ \frac{2}{3}\delta^2_{h,0}\delta_{k,0}\delta_{k,1}(r_k^4 r_h^2) + \frac{4}{9}\delta_{k,0}\delta_{k,1}\delta_{h,0}\delta_{h,1}(r_k^4 r_h^4)$$

$$+ \delta^2_{k,0}(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta^2_{h,1})(r_k^2 r_h^6) + \delta^2_{h,0}(\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta^2_{k,1})(r_k^6 r_h^2)$$

$$+ \frac{2}{3}\delta_{k,0}\delta_{k,1}(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta^2_{h,1})(r_k^4 r_h^6)$$

$$+ \frac{2}{3}\delta_{h,0}\delta_{h,1}(\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta^2_{k,1})(r_k^6 r_h^4)$$

$$+ (\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta^2_{k,1})(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta^2_{h,1})(r_k^6 r_h^6) + \ldots\Bigg] ,$$

$$(5.2.12)$$

where

$$\delta_{h,0} = 1 - \frac{1}{n-h} - \frac{7}{12(n-h)^2},$$

$$\delta_{h,1} = 1 - \frac{3}{4(n-h)} - \frac{1}{12(n-h)^2},$$

and

$$\delta_{h,2} = 1 - \frac{3}{4(n-h)} - \frac{1}{23(n-h)^2}.$$

Upon taking expectations of both sides of (5.2.9) and (5.2.11), we get

$$E(Q1) = \sum_{k=1}^{m} (n-k-1)\left[\beta_{k,0}^2 E(r_k^2) + \frac{2}{3}\beta_{k,0}\beta_{k,1}E(r_k^4) + (\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^2)E(r_k^6) + \dots\right], \tag{5.2.13}$$

and

$$E(Q2) = \sum_{k=1}^{m} (n-k-1)\left[\delta_{k,0}^2 E(r_k^2) + \frac{2}{3}\delta_{k,0}\delta_{k,1}E(r_k^4) + (\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta_{k,1}^2)E(r_k^6) + \dots\right], \tag{5.2.14}$$

where E(Q1) and E(Q2) are simply a linear function of $E(r_k^{2j})$ $\forall$ positive integers j. As was the case for the mean formula of QKSW derived in Chapter 4, truncation of (5.2.13) and (5.2.14) is required in order to set critical values for the proposed tests. Following the previous chapter, adequate approximations of E(Q1) and E(Q2) may be obtained by omitting the univariate moments smaller than $E(r_k^4)$, whereupon:

$$E(Q1) = \sum_{k=1}^{m} (n-k-1)\left[\beta_{k,0}^{2}E(r_k^2) + \frac{2}{3}\beta_{k,0}\beta_{k,1}E(r_k^4)\right], \tag{5.2.15}$$

and

$$E(Q2) = \sum_{k=1}^{m} (n-k-1)\left[\delta_{k,0}^{2}E(r_k^2) + \frac{2}{3}\delta_{k,0}\delta_{k,1}E(r_k^4)\right]. \tag{5.2.16}$$

To derive var(Q1) and var(Q2), we use the identities

$$\text{var}(Q1) = E\{(Q1)^2\} - \{E(Q1)\}^2 \tag{5.2.17}$$

and

$$\text{var}(Q2) = E\{(Q2)^2\} - \{E(Q2)\}^2, \tag{5.2.18}$$

where

$$\begin{aligned} E\{(Q1)^2\} = \sum_{k=1}^{m}\sum_{h=1}^{m} (n-k-1)(n-h-1)\Big[&\beta_{k,0}^{2}\beta_{h,0}^{2}E(r_k^2r_h^2) \\ &+ \frac{2}{3}\beta_{k,0}^{2}\beta_{h,0}\beta_{h,1}E(r_k^2r_h^4) \quad + \frac{2}{3}\beta_{h,0}^{2}\beta_{k,0}\beta_{k,1}E(r_k^4r_h^2) \\ &+ \frac{4}{9}\beta_{k,0}\beta_{k,1}\beta_{h,0}\beta_{h,1}E(r_k^4r_h^4) + \beta_{k,0}^{2}(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta_{h,1}^{2})E(r_k^2r_h^6) \\ &+ \beta_{h,0}^{2}(\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^{2})E(r_k^6r_h^2) \\ &+ \frac{2}{3}\beta_{k,0}\beta_{k,1}(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta_{h,1}^{2})E(r_k^4r_h^6) \\ &+ \frac{2}{3}\beta_{h,0}\beta_{h,1}(\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^{2})E(r_k^6r_h^4) \\ &+ (\frac{2}{5}\beta_{k,0}\beta_{k,1}+\frac{1}{9}\beta_{k,1}^{2})(\frac{2}{5}\beta_{h,0}\beta_{h,1}+\frac{1}{9}\beta_{h,1}^{2})E(r_k^6r_h^6) + \ldots\Big] , \end{aligned} \tag{5.2.19}$$

and

$$E\{(Q2)^2\} = \sum_{k=1}^{m}\sum_{h=1}^{m}(n-k-1)(n-h-1)\Big[\delta_{k,0}^2\delta_{h,0}^2E(r_k^2r_h^2)$$

$$+ \frac{2}{3}\delta_{k,0}^2\delta_{h,0}\delta_{h,1}E(r_k^2r_h^4) + \frac{2}{3}\delta_{h,0}^2\delta_{k,0}\delta_{k,1}E(r_k^4r_h^2)$$

$$+ \frac{4}{9}\delta_{k,0}\delta_{k,1}\delta_{h,0}\delta_{h,1}E(r_k^4r_h^4) + \delta_{k,0}^2(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta_{h,1}^2)E(r_k^2r_h^6)$$

$$+ \delta_{h,0}^2(\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta_{k,1}^2)E(r_k^6r_h^2)$$

$$+ \frac{2}{3}\delta_{k,0}\delta_{k,1}(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta_{h,1}^2)E(r_k^4r_h^6)$$

$$+ \frac{2}{3}\delta_{h,0}\delta_{h,1}(\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta_{k,1}^2)E(r_k^6r_h^4)$$

$$+ (\frac{2}{5}\delta_{k,0}\delta_{k,2}+\frac{1}{9}\delta_{k,1}^2)(\frac{2}{5}\delta_{h,0}\delta_{h,2}+\frac{1}{9}\delta_{h,1}^2)E(r_k^6r_h^6) + \ldots\Big]. \tag{5.2.20}$$

As in the case of the mean formulae given in (5.2.15) and (5.2.16), approximations of var(Q1) and var(Q2) can be obtained by omitting terms smaller than $O(n^{-1})$:

$$\mathrm{var}(Q1) = \Big\{\sum_{k=1}^{m}(n-k-1)^2\Big[\beta_{k,0}^4E(r_k^4) + \frac{4}{3}\beta_{k,0}^3\beta_{k,1}E(r_k^6)$$

$$+ 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\Big[\beta_{k,0}^2\beta_{h,0}^2E(r_k^2r_h^2) + \frac{2}{3}\beta_{k,0}^2\beta_{h,0}\beta_{h,1}E(r_k^2r_h^4)$$

$$+ \frac{2}{3}\beta_{h,0}^2\beta_{k,0}\beta_{k,1}E(r_k^4r_h^2)\Big] - \{E(Q1)\}^2\Big\}, \tag{5.2.21}$$

and

$$\mathrm{var}(Q2) = \Big\{\sum_{k=1}^{m}(n-k-1)^2\Big[\delta_{k,0}^4E(r_k^4) + \frac{4}{3}\delta_{k,0}^3\delta_{k,1}E(r_k^6)$$

$$+ 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\Big[\delta_{k,0}^2\delta_{h,0}^2E(r_k^2r_h^2) + \frac{2}{3}\delta_{k,0}^2\delta_{h,0}\delta_{h,1}E(r_k^2r_h^4)$$

$$+ \frac{2}{3}\delta_{h,0}^2\delta_{k,0}\delta_{k,1}E(r_k^4 r_h^2)\Big] - \{E(Q2)\}^2\Big\}, \qquad (5.2.22)$$

where E(Q1) and E(Q2) are given by (5.2.15) and (5.2.16), respectively.

The theoretical means and variances of Q1 and Q2 can be derived by substituting appropriate univariate and bivariate moments of $r_k$ into (5.2.15), (5.2.16), (5.2.21) and (5.2.22). When n is large relative to m, it can be shown that

$$E(Q1) = m - \frac{(m+4)m}{n}, \qquad (5.2.23)$$

$$E(Q2) = m - \frac{(m+4)m}{n}, \qquad (5.2.24)$$

$$var(Q1) = 2m - \frac{18m}{n}, \qquad (5.2.25)$$

and

$$var(Q2) = 2m - \frac{18m}{n}. \qquad (5.2.26)$$

As n approaches infinity, it can easily be seen that $\lim_{n\to\infty} E(Q1) = \lim_{n\to\infty} E(Q2) = m$, and $\lim_{n\to\infty} var(Q1) = \lim_{n\to\infty} var(Q2) = 2m$. These results suggest that the asymptotic distributions of Q1 and Q2 are approximately $\chi^2(m)$. It is important to note that the derivations of (5.2.23)-(5.2.26) require laborious algebra. We therefore use the symbolic program *Mathematica* to verify the results. Detailed derivations of (5.2.23)-(5.2.26) are provided in Appendix B.

Equations (5.2.23)-(5.2.26) suggest that the (finite n) theoretical means and variances of Q1 and Q2 are less than m and

2m. Also note that both E(Q1) and E(Q2) are not integer. Therefore, it is necessary to adjust the large-sample means of Q1 and Q2 in order to set critical values for these tests. Following Chapter 4, we use the five-step procedure to execute Q1 and Q2:

Step 1. Compute the sample autocorrelations $r_k$, $(k=1,\dots,m)$.

Step 2. For each $r_k$, compute $z_{1k}$ and $z_{2k}$.

Step 3. Compute Q1 and Q2.

Step 4. Compute E(Q1) and E(Q2) using the approximations, $E(Q1) = \sum_{k=1}^{m}(n-k-1)\left[\beta_{k,0}^2 E(r_k^2) + \frac{2}{3}\beta_{k,0}\beta_{k,1}E(r_k^4)\right]$ and $E(Q2) = \sum_{k=1}^{m}(n-k-1)\left[\delta_{k,0}^2 E(r_k^2) + \frac{2}{3}\delta_{k,0}\delta_{k,1}E(r_k^4)\right]$, respectively.

Step 5. Reject the null hypothesis of randomness whenever $Q1 \geq \chi_\alpha^2(E(Q1))$ or $Q2 \geq \chi_\alpha^2(E(Q2))$ in which $\alpha$ is the level of significance.

## 5.3 Experimental Design and Simulation Results

To examine the empirical performance of the proposed test statistics, Q1 and Q2, a Monte Carlo simulation experiment is performed. In this experiment, attention is focussed on estimated sizes (i.e. type I errors), variances, and variance-mean ratios of QKSW, Q1 and Q2. For the sake of comparison, the simulation results of QBP, QLB and QDR1 are also reported.

All simulations were carried out on a VAX2 computer. N(0,1) random deviates, $X_t$, were generated for six sample sizes (n=10, 15, 20, 25, 30, and 40). Estimates of empirical size ($\alpha$=5% and 10%), means and variances of the aforementioned statistics were based on 20,000 replications. As for the value of m, it is set between 1 and (n/2). The largest value chosen for m is 7 when n=15 and 12 when n=25.

The results of our simulation experiment are summarized in Tables 5.1, 5.2 and 5.3. The main points are as follows:

(i) The empirical performance of QBP, QLB and QDR1 is similar to that reported in Dufour and Roy (1986). QBP has estimated sizes which are consistently lower than the values predicted by the asymptotic theory. QLB and QDR1, on the other hand, tend to reject the null hypothesis far too frequently when the ratio of (m/n) is from moderate to large. Note that this "over-rejection" problem is particularly serious for QLB.

(ii) As regards the variance-mean ratios, QBP always has a value around 2 for $n \geq 30$ and $3 \leq m \leq 15$. On the contrary, QLB and QDR1 can have values which are substantially greater than 2 for m=7 with n=15 and for $m \geq 7$ with $20 \leq n \leq 40$. This problem is, by and large, due to the presence of dispersion bias. For example, when n=25 and m=12, the empirical variance of QDR1 is 36.06 which is significantly larger than the theoretical value 2m.

Table 5.1: Empirical Significance Levels of Portmanteau Tests for a Normal White Noise; $\alpha$=5%

| n | m | QBP | QLB | QDR1 | QKS | Q1 | Q2 |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 2.18 | 5.27 | 4.38 | 4.02 | 4.21 | 4.22 |
| | 3 | 0.89 | 5.57 | 4.80 | 2.58 | 2.86 | 3.01 |
| | 5 | 0.50 | 6.77 | 5.52 | 2.39 | 2.65 | 2.84 |
| 15 | 1 | 3.25 | 5.56 | 4.75 | 4.65 | 4.71 | 4.71 |
| | 3 | 1.86 | 5.71 | 5.07 | 4.00 | 4.13 | 4.21 |
| | 5 | 1.51 | 6.76 | 6.17 | 3.98 | 4.22 | 4.30 |
| | 7 | 1.20 | 7.88 | 6.72 | 4.00 | 4.18 | 4.38 |
| 20 | 1 | 3.42 | 5.32 | 4.69 | 4.58 | 4.64 | 4.64 |
| | 3 | 2.70 | 5.25 | 5.01 | 4.19 | 4.25 | 4.27 |
| | 5 | 2.21 | 6.24 | 5.68 | 4.31 | 4.39 | 4.52 |
| | 7 | 1.99 | 6.94 | 6.22 | 4.43 | 4.57 | 4.60 |
| | 10 | 1.46 | 8.05 | 6.96 | 4.30 | 4.40 | 4.47 |
| 25 | 1 | 3.99 | 5.47 | 5.01 | 4.87 | 4.89 | 4.89 |
| | 3 | 2.78 | 5.09 | 4.90 | 4.21 | 4.26 | 4.29 |
| | 5 | 2.70 | 5.76 | 5.50 | 4.43 | 4.49 | 4.57 |
| | 7 | 2.47 | 6.90 | 6.32 | 4.74 | 4.81 | 4.83 |
| | 10 | 2.00 | 8.14 | 7.39 | 4.80 | 4.85 | 4.94 |
| | 12 | 1.57 | 8.81 | 7.91 | 4.81 | 4.84 | 4.91 |
| 30 | 1 | 3.97 | 5.14 | 4.98 | 4.73 | 4.74 | 4.74 |
| | 3 | 3.14 | 4.99 | 4.88 | 4.28 | 4.30 | 4.30 |
| | 5 | 2.98 | 5.87 | 5.45 | 4.77 | 4.80 | 4.83 |
| | 7 | 3.03 | 6.51 | 6.12 | 4.80 | 4.84 | 4.87 |
| | 10 | 2.56 | 7.87 | 7.38 | 5.30 | 5.38 | 5.48 |
| | 12 | 2.10 | 8.73 | 7.96 | 5.48 | 5.51 | 5.59 |
| | 15 | 1.54 | 9.17 | 8.27 | 5.42 | 5.45 | 5.59 |
| 40 | 1 | 4.51 | 5.30 | 5.10 | 4.98 | 4.98 | 4.98 |
| | 3 | 3.66 | 5.10 | 4.91 | 4.59 | 4.60 | 4.60 |
| | 5 | 3.48 | 5.66 | 5.52 | 4.89 | 4.92 | 4.95 |
| | 7 | 3.49 | 6.26 | 5.95 | 4.96 | 4.97 | 4.98 |
| | 10 | 3.20 | 7.07 | 6.72 | 5.18 | 5.20 | 5.27 |
| | 12 | 2.89 | 7.73 | 7.09 | 5.42 | 5.47 | 5.52 |
| | 15 | 2.44 | 8.66 | 8.21 | 5.64 | 5.66 | 5.73 |
| | 20 | 1.64 | 9.42 | 8.57 | 5.79 | 5.82 | 5.82 |

Table 5.1: (cont'd); $\alpha$=10%

| n | m | QBP | QLB | QDR1 | QKS | Q1 | Q2 |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 6.19 | 11.08 | 9.80 | 9.96 | 10.03 | 10.08 |
| | 3 | 2.70 | 11.59 | 9.94 | 7.60 | 8.00 | 8.16 |
| | 5 | 1.22 | 12.75 | 10.24 | 7.18 | 7.65 | 7.79 |
| 15 | 1 | 7.75 | 10.89 | 9.94 | 9.92 | 9.97 | 9.96 |
| | 3 | 4.87 | 10.78 | 9.89 | 8.80 | 9.00 | 9.00 |
| | 5 | 3.46 | 11.94 | 10.71 | 8.51 | 8.80 | 8.86 |
| | 7 | 2.43 | 13.09 | 11.27 | 8.50 | 8.67 | 8.86 |
| 20 | 1 | 8.28 | 11.29 | 10.16 | 10.41 | 10.42 | 10.42 |
| | 3 | 5.53 | 10.69 | 9.53 | 8.83 | 8.91 | 8.96 |
| | 5 | 4.81 | 11.31 | 10.24 | 8.64 | 8.74 | 8.87 |
| | 7 | 3.86 | 12.02 | 10.89 | 8.66 | 8.75 | 8.79 |
| | 10 | 2.63 | 13.15 | 11.55 | 8.61 | 8.66 | 8.75 |
| 25 | 1 | 8.98 | 10.84 | 10.14 | 10.24 | 10.25 | 10.25 |
| | 3 | 6.27 | 10.40 | 9.78 | 9.11 | 9.16 | 9.16 |
| | 5 | 5.31 | 10.88 | 10.25 | 8.68 | 8.76 | 8.85 |
| | 7 | 4.93 | 11.45 | 10.61 | 8.79 | 8.86 | 8.97 |
| | 10 | 3.64 | 12.74 | 11.70 | 9.05 | 9.10 | 9.24 |
| | 12 | 2.97 | 13.32 | 12.01 | 9.09 | 9.19 | 9.33 |
| 30 | 1 | 9.05 | 10.80 | 10.05 | 10.19 | 10.19 | 10.19 |
| | 3 | 6.94 | 10.17 | 9.71 | 9.19 | 9.22 | 9.21 |
| | 5 | 6.10 | 10.90 | 10.37 | 9.17 | 9.23 | 9.30 |
| | 7 | 5.41 | 11.60 | 10.88 | 9.35 | 9.38 | 9.46 |
| | 10 | 4.70 | 12.81 | 11.84 | 9.69 | 9.77 | 9.82 |
| | 12 | 4.06 | 13.72 | 12.49 | 9.84 | 9.92 | 10.00 |
| | 15 | 2.81 | 14.40 | 12.89 | 9.80 | 9.90 | 9.97 |
| 40 | 1 | 9.30 | 10.67 | 10.30 | 10.14 | 10.14 | 10.14 |
| | 3 | 7.74 | 10.64 | 10.23 | 9.72 | 9.75 | 9.78 |
| | 5 | 6.94 | 10.78 | 10.40 | 9.61 | 9.62 | 9.63 |
| | 7 | 6.42 | 11.12 | 10.35 | 9.34 | 9.39 | 9.42 |
| | 10 | 5.49 | 11.74 | 11.25 | 9.52 | 9.54 | 9.56 |
| | 12 | 5.25 | 12.21 | 11.58 | 9.69 | 9.71 | 9.75 |
| | 15 | 4.40 | 13.16 | 12.35 | 9.87 | 9.88 | 9.99 |
| | 20 | 2.96 | 14.29 | 13.21 | 9.75 | 9.78 | 9.89 |

Table 5.2: Empirical Variances of Portmanteau Tests

| n | m | QBP | QLB | QDR1 | QKS | Q1 | Q2 |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 1.01 | 1.79 | 1.60 | 0.70 | 0.79 | 0.81 |
| | 3 | 2.80 | 6.35 | 5.99 | 1.38 | 1.61 | 1.72 |
| | 5 | 4.01 | 11.31 | 10.81 | 1.56 | 1.85 | 2.03 |
| 15 | 1 | 1.29 | 1.90 | 1.74 | 1.11 | 1.17 | 1.18 |
| | 3 | 3.64 | 6.22 | 6.02 | 2.58 | 2.72 | 2.80 |
| | 5 | 6.05 | 12.11 | 11.67 | 3.57 | 3.79 | 3.97 |
| | 7 | 8.01 | 18.67 | 17.93 | 4.07 | 4.34 | 4.62 |
| 20 | 1 | 1.43 | 1.91 | 1.76 | 1.31 | 1.34 | 1.35 |
| | 3 | 4.27 | 6.40 | 6.21 | 3.48 | 3.58 | 3.64 |
| | 5 | 7.19 | 12.06 | 11.69 | 5.14 | 5.29 | 5.44 |
| | 7 | 10.00 | 18.84 | 18.24 | 6.34 | 6.54 | 6.78 |
| | 10 | 13.25 | 29.44 | 28.62 | 7.27 | 7.53 | 7.90 |
| 25 | 1 | 1.52 | 1.92 | 1.78 | 1.43 | 1.45 | 1.46 |
| | 3 | 4.47 | 6.16 | 6.02 | 3.83 | 3.90 | 3.94 |
| | 5 | 7.69 | 11.60 | 11.38 | 5.98 | 6.08 | 6.18 |
| | 7 | 11.04 | 18.29 | 17.85 | 7.79 | 7.93 | 8.12 |
| | 10 | 15.58 | 29.59 | 28.99 | 9.65 | 9.85 | 10.18 |
| | 12 | 17.85 | 36.90 | 36.06 | 10.33 | 10.55 | 10.94 |
| 30 | 1 | 1.61 | 1.96 | 1.88 | 1.56 | 1.58 | 1.58 |
| | 3 | 4.78 | 6.24 | 6.16 | 4.30 | 4.34 | 4.37 |
| | 5 | 8.20 | 11.53 | 11.38 | 6.79 | 6.87 | 6.95 |
| | 7 | 12.04 | 18.31 | 17.85 | 9.20 | 9.31 | 9.46 |
| | 10 | 17.56 | 29.97 | 29.38 | 11.97 | 12.12 | 12.41 |
| | 12 | 20.67 | 37.99 | 37.24 | 13.19 | 13.38 | 13.74 |
| | 15 | 24.45 | 49.82 | 48.56 | 14.40 | 14.60 | 15.08 |
| 40 | 1 | 1.76 | 2.04 | 1.98 | 1.74 | 1.75 | 1.75 |
| | 3 | 5.23 | 6.38 | 6.35 | 4.89 | 4.91 | 4.93 |
| | 5 | 8.84 | 11.38 | 11.34 | 7.80 | 7.84 | 7.89 |
| | 7 | 12.67 | 17.22 | 17.03 | 10.54 | 10.60 | 10.70 |
| | 10 | 18.58 | 27.46 | 27.22 | 14.22 | 14.31 | 14.50 |
| | 12 | 22.65 | 35.53 | 35.11 | 16.39 | 16.50 | 16.76 |
| | 15 | 28.64 | 49.04 | 48.12 | 19.16 | 19.30 | 19.67 |
| | 20 | 35.85 | 70.01 | 68.84 | 21.70 | 21.87 | 22.40 |

Table 5.3: (Variance/Mean) Ratios of Portmanteau Tests

| n | m | QBP | QLB | QDR1 | QKS | Q1 | Q2 |
|---|---|---|---|---|---|---|---|
| 10 | 1 | 1.26 | 1.68 | 1.62 | 1.23 | 1.32 | 1.34 |
| | 3 | 1.29 | 1.95 | 2.00 | 1.08 | 1.17 | 1.20 |
| | 5 | 1.25 | 2.05 | 2.16 | 1.00 | 1.07 | 1.10 |
| 15 | 1 | 1.49 | 1.81 | 1.75 | 1.56 | 1.60 | 1.60 |
| | 3 | 1.50 | 1.96 | 2.00 | 1.42 | 1.46 | 1.48 |
| | 5 | 1.62 | 2.29 | 2.33 | 1.41 | 1.45 | 1.48 |
| | 7 | 1.66 | 2.50 | 2.55 | 1.38 | 1.42 | 1.45 |
| 20 | 1 | 1.58 | 1.83 | 1.76 | 1.65 | 1.67 | 1.68 |
| | 3 | 1.65 | 2.02 | 2.08 | 1.64 | 1.66 | 1.68 |
| | 5 | 1.77 | 2.30 | 2.33 | 1.65 | 1.68 | 1.70 |
| | 7 | 1.87 | 2.57 | 2.60 | 1.65 | 1.68 | 1.70 |
| | 10 | 1.91 | 2.79 | 2.85 | 1.61 | 1.64 | 1.66 |
| 25 | 1 | 1.66 | 1.86 | 1.79 | 1.73 | 1.74 | 1.74 |
| | 3 | 1.68 | 1.98 | 2.00 | 1.68 | 1.69 | 1.70 |
| | 5 | 1.82 | 2.24 | 2.27 | 1.73 | 1.74 | 1.76 |
| | 7 | 1.96 | 2.52 | 2.55 | 1.77 | 1.79 | 1.81 |
| | 10 | 2.08 | 2.86 | 2.90 | 1.79 | 1.81 | 1.83 |
| | 12 | 2.09 | 2.96 | 3.00 | 1.76 | 1.78 | 1.80 |
| 30 | 1 | 1.71 | 1.88 | 1.86 | 1.79 | 1.80 | 1.80 |
| | 3 | 1.76 | 2.01 | 2.04 | 1.78 | 1.79 | 1.80 |
| | 5 | 1.88 | 2.23 | 2.27 | 1.82 | 1.84 | 1.85 |
| | 7 | 2.04 | 2.53 | 2.54 | 1.91 | 1.92 | 1.94 |
| | 10 | 2.21 | 2.89 | 2.92 | 1.96 | 1.97 | 1.99 |
| | 12 | 2.26 | 3.05 | 3.08 | 1.95 | 1.97 | 1.99 |
| | 15 | 2.28 | 3.20 | 3.22 | 1.93 | 1.94 | 1.97 |
| 40 | 1 | 1.82 | 1.96 | 1.95 | 1.90 | 1.91 | 1.91 |
| | 3 | 1.85 | 2.04 | 2.08 | 1.88 | 1.89 | 1.89 |
| | 5 | 1.93 | 2.19 | 2.24 | 1.91 | 1.92 | 1.92 |
| | 7 | 2.04 | 2.38 | 2.40 | 1.96 | 1.96 | 1.97 |
| | 10 | 2.19 | 2.66 | 2.70 | 2.02 | 2.03 | 2.04 |
| | 12 | 2.29 | 2.87 | 2.90 | 2.06 | 2.07 | 2.08 |
| | 15 | 2.43 | 3.17 | 3.18 | 2.11 | 2.12 | 2.13 |
| | 20 | 2.47 | 3.40 | 3.42 | 2.08 | 2.09 | 2.11 |

(iii) QKSW performs adequately even when the sample size is as low as 15. For $n \leq 10$, $m \geq 3$, and $\alpha=5\%$ and 10%, QKSW is undersized.

(iv) The modified Kwan et al. tests, Q1 and Q2, perform slightly better than QKSW for $n \leq 15$. To highlight this feature, we consider, as examples, the following range of estimated sizes:

| | | | 5% | | | 10% | |
|---|---|---|---|---|---|---|---|
| n | m | QKSW | Q1 | Q2 | QKSW | Q1 | Q2 |
| 10 | 1 | 4.02 | 4.21 | 4.22 | 9.96 | 10.03 | 10.08 |
| | 3 | 2.58 | 2.86 | 3.01 | 7.60 | 8.00 | 8.16 |
| | 5 | 2.39 | 2.65 | 2.84 | 7.18 | 7.65 | 7.80 |
| 15 | 1 | 4.65 | 4.71 | 4.71 | 9.92 | 9.97 | 9.96 |
| | 3 | 4.00 | 4.13 | 4.21 | 8.80 | 9.00 | 9.05 |
| | 5 | 3.98 | 4.22 | 4.30 | 8.51 | 8.80 | 8.86 |
| | 7 | 4.00 | 4.18 | 4.38 | 8.50 | 8.67 | 8.86 |

Although both Q1 and Q2 perform better than QKSW in these cases, the gains are clearly marginal, and indicate that QKSW can behave almost as well as Q1 and Q2 for $n>15$. Note that the empirical significance levels of QKSW, Q1 and Q2 are considerably smaller than the nominal level for $n=10$ with $m=3$ and 5. As well, their empirical variance-mean ratios can be far below 2 in these situations.

(v) For a larger sample size ($n \geq 20$) and $m \geq 5$, the variance-mean ratios of QKSW, Q1 and Q2 are much closer to 2 than those of QLB and QDR1. It is noteworthy that these values are always less than 2.12 even when m is $(n/2)$. This performance strongly indicates that the dispersion bias which is inherent in both QLB and QDR1 can be removed by adopting the variance-stabilizing transformations here.

## 5.4 Concluding Remarks

In this chapter, an attempt has been made to compare the small-sample performance of some portmanteau statistics of Ljung and Box (1978), Dufour and Roy (1986), and Kwan et al. (1992). In addition, two modified versions of the Kwan et al. test, Q1 and Q2, based on the application of the Hotelling (1953) transformations to sample autocorrelations, are studied. The simulation results reveal that QKSW, Q1 and Q2 perform fairly well in small samples ($n \geq 15$). Furthermore, these statistics were found to be more reliable than QLB and QDR1 when the number of autocorrelations is large. In this respect, QKSW, Q1 and Q2 have a clear advantage over other portmanteau statistics as they are valid over a wider range of m.

Lastly, the simulation results presented do not imply that portmanteau statistics such as QLB and QDR1 should not be used for testing the randomness of a time series when the sample size is very small. For instance, in a few cases where n is 10, QLB and QDR1 perform better than QKSW, Q1 and Q2. However, from the practical point of view, such a small sample size is hardly enough to carry out any meaningful time series or econometric modelling.

# CHAPTER SIX

# SUMMARY AND DISCUSSION

## 6.1 Introduction

This dissertation has provided a comprehensive study of the empirical performance of some well-known and computationally straightforward tests for univariate time series models. The tests examined include: (i) the portmanteau tests of Box and Pierce (1970), Ljung and Box (1978), Dufour and Roy (1986), Ljung (1986), and Bera and Newbold (1988); (ii) the Godfrey (1979) LM tests (or tests of nested hypotheses); and (iii) the McAleer et al. (1988) tests of separate hypotheses. While considerable attention has been paid to the empirical size of these test statistics, their ability to detect deviations from model specification has also been investigated in cases where the data are seasonally or non-seasonally generated. Since many of the portmanteau tests considered can also be used to test for randomness of Gaussian time series, this dissertation also looks into their finite-sample performance. As well, two modified portmanteau tests based on an application of Hotelling's (1953) transformations to sample autocorrelations are proposed.

In Chapter 2, three large-scale simulation experiments are used to examine the finite-sample properties of the aforementioned portmanteau tests for the adequacy of univariate time series models, with special attention being paid to test size, means, variances, and empirical power. A similar analysis is undertaken in Chapter 3 which provides a detailed assessment of the finite-sample distribution of Godfrey's (1979) LM test. In that chapter, a critical review of the tests of separate hypotheses proposed by McAleer et al. (1988) is also presented. Chapter 4 deals with the issue of testing the randomness of Gaussian time series. Its objective is to examine how the Kwan et al. (1992) portmanteau test performs in commonly-used samples. Lastly, in Chapter 5, the relative performances of two modified portmanteau tests and the Kwan et al. (1992) test are investigated.

## 6.2 Main Simulation Results of Some Selected Portmanteau Tests for Univariate Time Series Models

Since the empirical investigations of Chatfield and Prothero (1973) and Prothero and Wallis (1976), considerable attention has been paid to the finite-sample distribution of the portmanteau test proposed by Box and Pierce (1970). Ljung and Box (1978), for instance, showed that the Box-Pierce test suffers a location bias. Dufour and Roy (1986), on the other hand, indicate that the normalization procedure used in the Box-Pierce test is inappropriate for an independently and identically distributed normal series with unknown mean. Consequently, the poor empirical performance of the test is not entirely unexpected.

The introduction of the Box-Pierce test has led to many modified versions, especially in the area of testing the adequacy of an ARMA (p,q) model; notable examples are Ljung and Box (1978), Godolphin (1980), Newbold (1980), Ljung (1986), Dufour and Roy (1986), Bera and Newbold (1988), and Kwan et al. (1992). However, all of these modified portmanteau tests have only a large-sample appeal, and require very large samples to justify their use. In view of the fact that many economic and financial time series are short, their relative performance in commonly-used samples is an important issue that must be confronted by applied practitioners.

In line with the above, one of the principal objectives of this dissertation is to investigate the finite-sample properties of the portmanteau tests suggested by Box and Pierce (1970), Ljung and Box (1978), Ljung (1986), Dufour and Roy (1986), and Bera and Newbold. On the basis of the simulation evidence presented in Chapter 2, two important results can be summarized as follows:

(i) The modified Ljung-Box test, QLB1, is found to be the most reliable test in terms of controlling test size. This is particularly obvious when the number of residual autocorrelations is small and when the parameter values are close to the boundary of stationarity or invertibility region.

(ii) With the exception of the Bera-Newbold tests, QBN1 and

QBN2, the portmanteau tests considered exhibit similar power performance, namely they tend to have high empirical power when the number of residual autocorrelations is small and the alternatives are chosen to be lower-order ARMA models. This observation, however, cannot be generalized to cases where the data are seasonally generated.

## 6.3 Main Results of the Tests of Nested and Separate Hypotheses for Univariate Time Series Models

It is well established that a test for a specific alternative will exist, and will have power no less than the portmanteau statistics. Two popular testing procedures, namely the tests of nested hypotheses and separate hypotheses, possess this characteristic. Godfrey (1979) has shown that the LM test is asymptotically optimal when the null model is nested within the alternative. Recentlv, McAleer et al. (1988) have suggested tests of separate hypotheses. Their procedures are asymptotically optimal against alternatives that are separate from the null specification.

Existing simulation evidence reported in Godfrey (1979), McAleer et al. (1988), and Hall and McAleer (1989) favours the LM test and the tests of separate hypotheses over the portmanteau tests of Box and Pierce (1970) and Ljung and Box (1978) in commonly-used samples. Chapter 3, however, does not share the same degree of enthusiasm. Using three large-scale simulation experiments, Chapter 3 shows that the finite-sample performance of Godfrey's (1979) LM test depends on the number of restrictions (r) that is imposed on the null model. Similar to the findings of the portmanteau statistics, the empirical power of the LM test can be significantly affected by both the choice of r and the nature of data (seasonal vs. non-seasonal data).

Chapter 3 also expresses serious reservations regarding the empirical performance of the McAleer et al. tests of separate hypotheses. Three relevant issues are raised: (i) the empirical size of their tests can be affected by the orders of the AR(p) and MA(q) models; (ii) the empirical power of the SM test contradicts the optimal property of the tests of separate hypotheses; and (iii) their conclusions regarding the ability of the test to detect model inadequacy appears unconvincing as seasonal data were not used in their simulation experiments.

## 6.4 Some Theoretical Properties and Results of the Modified Kwan-Sim-Wong Portmanteau Tests

A major theoretical contribution of this dissertation is the development of two modified portmanteau tests for randomness of Gaussian time series. Let $z_k$ be the Fisher variance-stabilizing transformation of $r_k$. The Kwan et al. (1992) portmanteau test, based on $z_k$, is:

$$QKSW = \sum_{k=1}^{m}(n-k-3)z_k^2 , \qquad (4.2.7)$$

where $$E(QKWS) = m - \frac{m(m+4)}{n}, \qquad (4.2.16)$$

and $$var(QKWS) = 2m - \frac{18m}{n} . \qquad (4.2.21)$$

In Chapter 4, it is shown that both E(QKWS) and var(QKWS) converge to m and 2m, respectively. These results suggest that the asymptotic distribution of QKWS is approximately $\chi^2$ with m degrees of freedom. However, E(QKWS) is not an integer in commonly-used sample sizes. For practical purposes, it is necessary to adjust the mean of QKWS in order to set appropriate critical values for the Kwan et al. (1992) portmanteau test.

The simulation results presented in Chapter 4 indicate that the QKSW dominates QLB and QDR1 in terms of controlling test size and minimizing dispersion bias. In addition, the power calculations suggest that there is no loss in power when QKSW is used. Encouraged by these findings, a similar investigation is extended to incorporate the Hotelling (1953) transformations which may perform better than the Fisher transformation in small samples. Let $z_{1k}$ and $z_{2k}$ be the Hotelling transformations of $r_k$. The modified Kwan et al. (1992) portmanteau tests are:

$$Q1 = \sum_{k=1}^{m}(n-k-1)z_{1k}^2, \qquad (5.2.3)$$

and $$Q2 = \sum_{k=1}^{m}(n-k-1)z_{2k}^2. \qquad (5.2.4)$$

It is demonstrated in Chapter 5 that, both the theoretical means and variances of Q1 and Q2 are identical to those of QKSW, at least to $O(n^{-1})$. As in the case of QKSW, the means of Q1 and Q2 have to be adjusted in order to carry out tests of randomness.

The simulation evidence reported in Chapter 5 indicates that Q1 and Q2 perform better than QKSW in smaller samples, though the empirical significance levels of these tests can be substantially different from the nominal level when $n \le 15$. When the sample size is at least 20, the empirical performance of Q1, Q2 and QKSW is superior to that of QLB and QDR1, in terms of controlling test size. This conclusion is evident when the number of sample autocorrelations is large.

### 6.5 New Research Directions

As for future research, a number of areas are worth investigating:

(i) As in many simulation studies, the conclusions drawn in Chapters 2 and 3 are valid only for the models considered. It may be fruitful to extend these investigations to higher-order or more complicated ARMA models.

(ii) The robustness of all the tests examined in the presence of non-Gaussian errors is important to applied work. Existing studies, including Ljung and Box (1978), Hall and McAleer (1989) and Kwan et al. (1992), have reported mixed results regarding this issue. In light of this, it would be important to examine the effect of non-Gaussian errors on the empirical performance of these diagnostic checks.

(iii) The simulation findings of Chapter 3 demonstrate that the empirical power of Godfrey's LM test for model adequacy can be improved by removing redundant restrictions. Since the portmanteau statistics of Box and Pierce (1970) and Ljung and Box (1978) are equivalent to Godfrey's LM test [see Newbold (1980)], it seems useful to employ a similar strategy for these tests in empirical applications.

(iv) This dissertation has considered two well-known variance-stabilizing transformations. Jenkins (1954, 1956) has suggested a transformation which produces a more nearly normally distributed variable than Fisher's. Hence, it would be interesting to construct a new portmanteau test based on the Jenkins transformation. As well, an examination of its finite-sample performance would be a welcome addition to the existing literature.

## 6.6 Conclusion

This chapter summarizes the main results obtained in this dissertation. It also presents a discussion of some of these results. The main contributions of this dissertation are:

(i) It reports important simulation evidence regarding the finite-sample properties of some selected portmanteau tests and

Godfrey's (1979) LM test, in the context of univariate time series models. Many simulation results have not been reported in previous studies.

(ii) It criticizes the empirical performance of the McAleer et al. (1988) tests of separate hypotheses. The "critique" presented enables one to understand the basic problems of the finite-sample properties of the tests.

(iii) It compares the relative performance of the existing portmanteau tests of Ljung and Box (1978), Dufour and Roy (1986), and Kwan et al. (1992), in the area of testing Gaussian time series.

(iv) It proposes two modified portmanteau tests, based on an application of the Hotelling (1953) transformations to sample autocorrelations, for randomness of Gaussian time series. The proposed tests are major innovations of this dissertation. The simulation results strongly favour the use of these two modified tests and the Kwan et al. (1992) portmanteau test in empirical applications.

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## APPENDIX A: Derivations of equations (4.2.18) and (4.2.21)

To derive (4.2.18), we use the *Mathemaica* program to obtain the following quantities:

(i) $\sum_{k=1}^{m}(n-k-3)E(r_k^2)$, and

(ii) $(\frac{2}{3})\sum_{k=1}^{m}(n-k-3)E(r_k^4)$.

(i) $\sum_{k=1}^{m}(n-k-3)E(r_k^2)=$

$$\frac{(m+1)n}{n+2} - \frac{2(m+1)}{n+2} - \frac{(m+1)^2}{n+2} + \frac{(m+1)^2}{n(n+2)} - (\frac{4}{3})\frac{m+1}{n(n+2)} + (\frac{1}{3})\frac{(m+1)^3}{n(n+2)}$$

$$- \frac{n}{n+2} + \frac{3}{n+2}$$

(ii) $(\frac{2}{3})\sum_{k=1}^{m}(n-k-3)E(r_k^4)=$

$$\frac{2n^2(m+1)}{(n+2)(n+4)(n+6)} + \frac{9n(m+1)}{(n+2)(n+4)(n+6)} - \frac{25(m+1)}{(n+2)(n+4)(n+6)}$$

$$- \frac{3n(m+1)^2}{(n+2)(n+4)(n+6)} - \frac{13(m+1)^2}{(n+2)(n+4)(n+6)} + (\frac{45}{2})\frac{(m+1)^2}{(n+2)(n+4)(n+6)}$$

$$- (\frac{83}{3})\frac{m+1}{n(n+2)(n+4)(n+6)} + \frac{2(m+1)^3}{(n+2)(n+4)(n+6)} + (\frac{17}{3})\frac{(m+1)^3}{n(n+2)(n+4)(n+6)}$$

$$- (\frac{1}{2})\frac{(m+1)^4}{n(n+2)(n+4)(n+6)} - \frac{2n^2}{(n+2)(n+4)(n+6)} - \frac{6n}{(n+2)(n+4)(n+6)}$$

$$+ \frac{36}{(n+2)(n+4)(n+6)}$$

Upon substitution of (i) and (ii) into (4.2.17), it can be shown that, after some algebraic manipulation,

$$E(QKSW) \simeq \left[\sum_{k=1}^{m}(n-k-3)[E(r_k^2)+\tfrac{2}{3}E(r_k^4)]\right] \simeq m - \frac{m(m+4)}{n} + \frac{m(m+(1+m^2)/3)}{n^2}$$

$$+ \frac{m(17m+4m^2-116)}{3n^3} + \frac{m(606m-22m^2-3m^3+5225)}{6n^4}$$

$$- \frac{m(12529m+874m^2+10m^3+67009)}{6n^4} + O(mn^{-5}).$$

To get (4.2.21), we use the *Mathematica* program to derive the following quantities:

(a) $\sum_{k=1}^{m}(n-k-3)^2E(r_k^4)$,

(b) $(\frac{4}{3})\sum_{k=1}^{m}(n-k-3)^2E(r_k^6)$,

(c) $2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^2r_h^2)$,

(d) $(\frac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^2r_h^4)$, and

(e) $(\frac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^4r_h^2)$.

(a) $\sum_{k=1}^{m}(n-k-3)^2E(r_k^4)=$

$$\frac{49(m+1)}{(n+2)(n+4)(n+6)} + \frac{3n^3(m+1)}{(n+2)(n+4)(n+6)} - (\frac{9}{2})\frac{(m+1)^4}{n(n+2)(n+4)(n+6)}$$

$$+ \frac{72n(m+1)}{(n+2)(n+4)(n+6)} + (\frac{547}{5})\frac{(m+1)}{n(n+2)(n+4)(n+6)} - \frac{6n^2(m+1)^2}{(n+2)(n+4)(n+6)}$$

$$+ \frac{6n^2(m+1)}{(n+2)(n+4)(n+6)} - \frac{15n(m+1)^2}{(n+2)(n+4)(n+6)} + \frac{102(m+1)^2}{(n+2)(n+4)(n+6)}$$

$$- \left(\frac{123}{2}\right)\frac{(m+1)^2}{n(n+2)(n+4)(n+6)} + \frac{6n(m+1)^3}{(n+2)(n+4)(n+6)} + \frac{14(m+1)^2}{(n+2)(n+4)(n+6)}$$

$$- \frac{44(m+1)^3}{n(n+2)(n+4)(n+6)} - \frac{3(m+1)^4}{(n+2)(n+4)(n+6)} + \left(\frac{3}{5}\right)\frac{(m+1)^5}{n(n+2)(n+4)(n+6)}$$

$$- \frac{162}{(n+2)(n+4)(n+6)} - \frac{3n^3}{(n+2)(n+4)(n+6)} + \frac{81}{(n+2)(n+4)(n+6)}$$

(b) $\left(\frac{4}{3}\right) \sum_{k=1}^{m} (n-k-3)^2 E(r_k^6) =$

$$\frac{106(m+1)^5}{n(n+2)(n+4)(n+6)(n+8)(n+10)} + \left(\frac{8660}{3}\right)\frac{m+1}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{20n^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \frac{240n^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{140n^2}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{7800n}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{16560}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \frac{15724(m+1)}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{20(m+1)n^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{290(m+1)n^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{6110(m+1)^3}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \frac{50n(m+1)^4}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{470(m+1)^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{1360}{3}\right)\frac{(m+1)^4}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{20(m+1)^5}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{10}{3}\right)\frac{(m+1)^6}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

(c) $2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^2 r_h^2)=$

$$\left(\frac{502}{5}\right)\frac{m}{n(n+2)(n+4)(n+6)} - \frac{4mn^2}{(n+2)(n+4)(n+6)} + \left(\frac{223}{3}\right)\frac{nm}{(n+2)(n+4)(n+6)}$$

$$- \frac{196m}{(n+2)(n+4)(n+6)} - \left(\frac{194}{3}\right)\frac{nm^2}{(n+2)(n+4)(n+6)} + \left(\frac{299}{3}\right)\frac{m^2}{(n+2)(n+4)(n+6)}$$

$$+ \left(\frac{295}{9}\right)\frac{m^2}{n(n+2)(n+4)(n+6)} + \left(\frac{266}{3}\right)\frac{m^3}{n(n+2)(n+4)(n+6)}$$

$$- \left(\frac{295}{3}\right)\frac{m^3}{n(n+2)(n+4)(n+6)} - \left(\frac{31}{15}\right)\frac{m^5}{n(n+2)(n+4)(n+6)} + \left(\frac{5}{3}\right)\frac{nm^4}{(n+2)(n+4)(n+6)}$$

$$- \left(\frac{34}{3}\right)\frac{nm^3}{(n+2)(n+4)(n+6)} + \frac{6n^2m^2}{(n+2)(n+4)(n+6)} - \frac{mn^3}{(n+2)(n+4)(n+6)}$$

$$- \frac{2n^2m^3}{(n+2)(n+4)(n+6)} - \left(\frac{296}{9}\right)\frac{m^4}{n(n+2)(n+4)(n+6)} + \left(\frac{25}{3}\right)\frac{m^4}{(n+2)(n+4)(n+6)}$$

$$+ \frac{n^3m^2}{(n+2)(n+4)(n+6)} - \left(\frac{2}{3}\right)\frac{m^5}{(n+2)(n+4)(n+6)} + \left(\frac{1}{9}\right)\frac{m^6}{(n+2)(n+4)(n+6)}$$

(d) $\left(\frac{4}{3}\right)\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^2 r_h^4)=$

$$\left(\frac{264784}{35}\right)\frac{m}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{154684}{15}\right)\frac{m}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{3582nm}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{796}{3}\right)\frac{mn^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{4066m^2}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{8561}{3}\right)\frac{nm^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{7072}{3}\right)\frac{m^2}{n(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{5660m^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{22276}{3}\right)\frac{m^3}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{7088}{3}\right)\frac{m^4}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{56}{3}\right)\frac{mn^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{947}{3}\right)\frac{n^2m^2}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \frac{780nm^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{1838}{3}\right)\frac{m^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \frac{2mn^4}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{24nm}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{170}{3}\right)\frac{n^2m^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{173}{3}\right)\frac{nm^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{416}{15}\right)\frac{m^5}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{16}{3}\right)\frac{m^6}{n(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{n^4m^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{16}{3}\right)\frac{n^3m^3}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \left(\frac{19}{3}\right)\frac{n^2m^4}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{4nm^5}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{2096}{15}\right)\frac{m^5}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{4}{3}\right)\frac{m^6}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{4}{21}\right)\frac{m^7}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

(e) $\left(\frac{4}{3}\right)\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)E(r_k^4 r_h^2)=$

$$\left(\frac{679328}{105}\right)\frac{m}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{146816}{15}\right)\frac{m}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{3550nm}{(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{820}{3}\right)\frac{mn^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{6350}{3}\right)\frac{m^2}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{8521}{3}\right)\frac{nm^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \left(\frac{12260}{3}\right)\frac{m^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{18976}{3}\right)\frac{m^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{5128m^3}{n(n+2)(n+4)(n+6)(n+8)(n+10)} - \left(\frac{6364}{3}\right)\frac{m^4}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \left(\frac{56}{3}\right)\frac{mn^3}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \left(\frac{947}{3}\right)\frac{m^2n^2}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{750nm^3}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{750nm^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{592m^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{4m^6}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{2mn^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} + \frac{24m^2n^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- (\frac{146}{3})\frac{m^2m^3}{(n+2)(n+4)(n+6)(n+8)(n+10)} + (\frac{133}{3})\frac{nm^4}{n(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ \frac{m^2n^4}{(n+2)(n+4)(n+6)(n+8)(n+10)} - (\frac{16}{3})\frac{m^3n^3}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ (\frac{19}{3})\frac{m^4n^2}{(n+2)(n+4)(n+6)(n+8)(n+10)} - (\frac{304}{15})\frac{m^5}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$- \frac{nm^5}{n(n+2)(n+4)(n+6)(n+8)(n+10)} - (\frac{2164}{15})\frac{m^5}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

$$+ (\frac{4}{3})\frac{m^6}{(n+2)(n+4)(n+6)(n+8)(n+10)} - (\frac{4}{21})\frac{m^7}{(n+2)(n+4)(n+6)(n+8)(n+10)}$$

Upon substitution of (a)-(e) into (4.2.21), it can be shown that, after some algebraic manipulation,

$$\begin{aligned}
\text{var}(QKSW) &\simeq \Bigg\{\sum_{k=1}^{m}(n-k-3)^2\Big[E(r_k^4) + \frac{4}{3}E(r_k^6)\Big] \\
&\quad + 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-3)(n-h-3)\Big[E(r_k^2r_h^2) + \frac{2}{3}E(r_k^2r_h^4) + \frac{2}{3}E(r_k^4r_h^2)\Big] \\
&\quad - \{E(QKSW)\}^2\Bigg\} \\
&\simeq 2m - \frac{18m}{n} - \frac{m(301m+10m^2+47)}{3n^2} + \frac{m(8267m+776m^2+4m^3+2617)}{3n^3} \\
&\quad - \frac{m(4487835m+5901300m^2+22705m^3-114m^4)}{90n^4} + O(mn^{-5})
\end{aligned}$$

Note that simpler expressions for E(QKSW) and var(QKSW) can be obtained by keeping terms only larger than $O(mn^{-2})$. Thus, we have

$$E(QKSW) \simeq m - \frac{m(m+4)}{n},$$

and

$$\mathrm{var}(QKSW) \simeq 2m - \frac{18m}{n}.$$

**APPENDIX B:** Derivations of equations (5.2.23), (5.2.24), (5.2.25) and (5.2.26)

In order to yield tractable expressions for equations (5.2.23)-(5.2.26), it is necessary to replace the terms, (n-k) and (n-h), which appeared in $\beta_{k,0}$, $\beta_{k,1}$, $\beta_{h,0}$, $\beta_{h,1}$, $\delta_{k,0}$, $\delta_{k,1}$, $\delta_{h,0}$ and $\delta_{h,1}$, by n. Once the substitution is done, we use the *Mathematica* program to obtain the following quantities given in equations (5.2.15) and (5.2.16):

(i) $$\sum_{k=1}^{m}(n-k-1)\beta_{k,0}^{2}E(r_k^2)\simeq\left[3n^3(n+2)\right]^{-1}\left\{m(n-1)^2(2+3m+m^2-6n-3mn+3n^2)\right\}$$

(ii) $$\frac{2}{3}\sum_{k=1}^{m}(n-k-1)\beta_{k,0}\beta_{k,1}E(r_k^4)\simeq\left[432n^5(n+2)(n+4)(n+6)\right]^{-1}$$
$$\left\{m(2+3m+m^2-6n-3mn+3n^2)(-7-12n+12n^2)^2\right\}$$

(iii) $$\sum_{k=1}^{m}(n-k-1)\delta_{k,0}^{2}E(r_k^2)\simeq\left[24n^3(n+2)\right]^{-1}\left\{m(n-1)(4n-3)(14+15m-2m^2\right.$$
$$\left.-3m^3-30n+6mn+12m^2n-6n^2-18mn^2+12n^3)\right\}$$

(iv) $$\left(\frac{2}{3}\right)\sum_{k=1}^{m}(n-k-1)\delta_{k,0}\delta_{k,1}E(r_k^4)\simeq\left[864n^5(n+2)(n+4)(n+6)\right]^{-1}$$
$$\left\{m(1+9n-12n^2)(7+12n-12n^2)(14+15m-2m^2-3m^3-30n+6mn+12m^2n-6n^2\right.$$
$$\left.-18mn^2+12n^3)\right\}$$

Upon substitution of (i)-(ii) and (iii)-(iv) into respective equations (5.2.15) and (5.2.16), it can be shown that, after some algebraic manipulation,

$$E(Q1)\propto\left[\sum_{k=1}^{m}(n-k-1)[\beta^2_{k,0}E(r^2_k)+\frac{2}{3}\beta_{k,0}\beta_{k,1}E(r^4_k)]\right] \propto m - \frac{(m+4)m}{n}$$

$$+ \frac{m(19m+32m^2+464)}{96n^2} + \frac{m(23m+8m^2+532)}{12n^3}$$

$$+ \frac{m(4752m-200m^2-24m^3+41101)}{48n^4} + O(mn^{-5}),$$

and

$$E(Q2)\propto\left[\sum_{k=1}^{m}(n-k-1)[\delta^2_{k,0}E(r^2_k)+\frac{2}{3}\sum_{k=1}^{m}\delta_{k,0}\delta_{k,1}E(r^4_k)]\right] \propto m - \frac{(m+4)m}{n}$$

$$+ \frac{m(6m+m^2+11)}{3n^2} + \frac{m(37m+8m^2-479)}{12n^3}$$

$$+ \frac{m(13872m-656m^2-72m^3+122255)}{144n^4} + O(mn^{-5}).$$

To get (5.2.21) and (5.2.22), we use the *Mathematica* program to derive the following quantities:

(a) $\sum_{k=1}^{m}(n-k-1)^2\beta^4_{k,0}E(r^4_k)\propto\left[10n^5(n+2)(n+4)(n+6)\right]^{-1}\Big\{m(n-1)^4(-46-75m$
$-20m^2+15m^3+6m^4+150n+60mn-60m^2n-30m^3n-60n^2+90mn^2+60m^2n^2-60n^3$
$-60mn^3+30n^4)\Big\}$

(b) $(\frac{4}{3})\sum_{k=1}^{m}(n-k-1)^2\beta^3_{k,0}\beta_{k,1}E(r^6_k)\propto\left[10n^6(n+2)^2(n+4)^2(n+6)^2(n+8)(n+10)\right]^{-1}$
$\Big\{m(n-1)^3(-3+4n)(-46-75m-20m^2+15m^3+6m^4+150n+60mn-60m^2n-30m^3n-60n^2+$
$-60n^2+90mn^2+60m^2n^2-60n^3-60mn^3+30n^4)\Big\}$

(c) $$2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\beta_{k,0}^{2}\beta_{h,0}^{2}E(r_k^2 r_h^2)\simeq\left[9n(n+2)(n+4)(n+6)\right]^{-1}$$

$$\Big\{m(m-1)(1-n^{-1})^{4}(-162-301m-166m^{2}-26m^{3}+m^{4}+474n+483mn+111m^{2}n-6m^{3}n-$$

$$375n^{2}-153mn^{2}+15m^{2}n^{2}+72n^{3}-18mn^{3}+9n^{4})\Big\}$$

(d) $$(\tfrac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\beta_{k,0}^{2}\beta_{h,0}\beta_{h,1}E(r_k^2 r_h^4)\simeq$$

$$\left[1260n^{5}(n+2)(n+4)(n+6)(n+8)(n+10)\right]^{-1}\Big\{m(m-1)(n-1)^{3}(-3+4n)$$

$$(-375288-685808m-367658m^{2}-55108m^{3}+1970m^{4}-60m^{5}+907914n+989709mn+$$

$$234759m^{2}n-10416m^{3}n+420m^{4}n-638820n^{2}-299460mn^{2}+21840m^{2}n^{2}-1260m^{3}n^{2}+$$

$$118440n^{3}-21315mn^{3}+1995m^{2}n^{3}+8400n^{4}-1680mn^{4}+630n^{5})\Big\}$$

(e) $$(\tfrac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\beta_{h,0}^{3}\beta_{k,0}\beta_{k,1}E(r_k^4 r_h^2)\simeq$$

$$\left[1260n^{5}(n+2)(n+4)(n+6)(n+8)(n+10)\right]^{-1}\Big\{m(m-1)(n-1)^{3}(4n-3)$$

$$(-375288-685808m-367658m^{2}-55108m^{3}+1970m^{4}-60m^{5}+907914n+989709mn+$$

$$234759m^{2}n-10416m^{3}n+420m^{4}n-638820n^{2}-299460mn^{2}+21840m^{2}n^{2}-1260m^{3}n^{2}+$$

$$118440n^{3}-21315mn^{3}+1995m^{2}n^{3}+8400n^{4}-1680mn^{4}+630n^{5})\Big\}$$

(f) $$\sum_{k=1}^{m}(n-k-1)^{2}\delta_{0}^{4}E(r_k^4)\simeq\left[207360n^{9}(n+2)(n+4)(n+6)\right]^{-1}\Big\{m(-7-12n+12n^{2})^{4}$$

$$(-46-75m-20m^{2}+15m^{3}+6m^{4}+150n+60mn-60m^{2}n-30m^{3}n-60n^{2}+90mn^{2}+60m^{2}n^{2}-$$

$$60n^{3}-60mn^{3}+30n^{4})\Big\}$$

(g) $(\frac{4}{3})\sum_{k=1}^{m}(n-k-1)^2\delta_{k,0}^3\delta_{k,1}E(r_k^6)\propto$

$$\left[51840n^{10}(n+2)^2(n+4)^2(n+6)^2(n+8)(n+10)\right]^{-1}\Big\{m(-7-12n+12n^2)^3$$

$$(-1-9n+12n^2)(-46-75m-20m^2+15m^3+6m^4+150n+60mn-60m^2n-30m^3n-60n^2+90mn^2$$

$$+60m^2n^2-60n^3-60mn^3+30n^4)\Big\}$$

(h) $2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\delta_{k,0}^2\delta_{h,0}^2E(r_k^2r_h^2)\propto\left[9n(n+2)(n+4)(n+6)\right]^{-1}$

$$\Big\{m(m-1)(1-n^{-1}-\frac{7}{12}n^{-2})^4(-162-301m-166m^2-26m^3+m^4+474n+483mn+111m^2n-$$

$$6m^3n-375n^2-153mn^2+15m^2n^2+72n^3-18mn^3+9n^4)\Big\}$$

(i) $(\frac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\delta_{k,0}^2\delta_{h,0}\delta_{h,1}E(r_k^2r_h^4)\propto$

$$\left[315n(n+2)(n+4)(n+6)(n+8)(n+10)\right]^{-1}\Big\{m(m-1)(1-n^{-1}-\frac{7}{12}n^{-2})^3$$

$$(1-\frac{3}{4}n^{-1}-\frac{1}{12}n^{-2})(-375288-685808m-367658m^2-55108m^3+1970m^4-60m^5+$$

$$907914n+989709mn+234759m^2n-10416m^3n+420m^4n-638820n^2-299460mn^2$$

$$+21840m^2n^2-1260m^3n^2+118440n^3-21315mn^3+1995m^2n^3+8400n^4-1680mn^4$$

$$+630n^5)\Big\}$$

(j) $(\frac{4}{3})\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\delta_{h,0}^2\delta_{k,0}\delta_{k,1}E(r_k^4r_h^2)\propto$

$$\left[315n(n+2)(n+4)(n+6)(n+8)(n+10)\right]^{-1}\Big\{m(m-1)(1-n^{-1}-\frac{7}{12}n^{-2})^3$$

$(1-\frac{3}{4}n^{-1}-\frac{1}{12}n^{-2})(-375288-685808m-367658m^2-55108m^3+1970m^4-60m^5+$

$907914n+989709mn+234759m^2n-10416m^3n+420m^4n-638820n^2-299460mn^2+$

$21840m^2n^2-1260m^3n^2+118440n^3-21315mn^3+1995m^2n^3+8400n^4-1680mn^4+$

$630n^5)\Big\}$

Upon substitution of (a)-(e) and (f)-(j) into respective equations (5.2.21) and (5.2.22), it can be shown that, after some algebraic manipulation,

$$\begin{aligned}\mathrm{var}(Q1) \simeq &\Bigg\{\sum_{k=1}^{m}(n-k-1)^2\Big[\beta_{k,0}^4E(r_k^4) + \frac{4}{3}\beta_{k,0}^3\beta_{k,1}E(r_k^6)\Big] \\ &+ 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\Big[\beta_{k,0}^2\beta_{h,0}^2E(r_k^2r_h^2) + \frac{2}{3}\beta_{k,0}^2\beta_{h,0}\beta_{h,1}E(r_k^2r_h^4) \\ &+ \frac{2}{3}\beta_{h,0}^2\beta_{k,0}\beta_{k,1}E(r_k^4r_h^2)\Big] - \{E(Q1)\}^2\Bigg\}\end{aligned}$$

$$\begin{aligned}&\simeq 2m - \frac{18m}{n} - \frac{m(289m+10m^2-13)}{3n^2} + \frac{m(16314m+1551m^2+8m^3+4185)}{6n^3} \\ &\quad - \frac{m(18059325m+2422860m^2+92620m^3-456m^4-1209264)}{360n^4} + O(mn^{-5})\end{aligned}$$

and

$$\begin{aligned}\mathrm{var}(Q2) \simeq &\Bigg\{\sum_{k=1}^{m}(n-k-1)^2\Big[\delta_{k,0}^4E(r_k^4) + \frac{4}{3}\delta_{k,0}^3\delta_{k,1}E(r_k^6)\Big] \\ &+ 2\sum_{k=1}^{m-1}\sum_{h=k+1}^{m}(n-k-1)(n-h-1)\Big[\delta_{k,0}^2\delta_{h,0}^2E(r_k^2r_h^2) + \frac{2}{3}\delta_{k,0}^2\delta_{h,0}\delta_{h,1}E(r_k^2r_h^4) \\ &+ \frac{2}{3}\delta_{h,0}^2\delta_{k,0}\delta_{k,1}E(r_k^4r_h^2)\Big] - \{E(Q2)\}^2\Bigg\}\end{aligned}$$

$$\simeq 2m - \frac{18m}{n} - \frac{m(289m+10m^2+1)}{3n^2} + \frac{m(16314m+1551m^2+8m^3+4487)}{6n^3}$$

$$- \frac{m(8998555m+1210090m^2+46310m^3-228m^4-566017)}{180n^4} + O(mn^{-5}).$$

To get equations (5.2.23)-(5.2.26), we simply omit terms smaller than $O(mn^{-1})$. Thus, $E(Q1) \simeq m - n^{-1}m(m+4)$, $E(Q2) \simeq m - n^{-1}m(m+4)$, $var(Q1) \simeq 2m - 18mn^{-1}$, and $var(Q2) \simeq 2m - 18mn^{-1}$.