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Montreal, Quebec
March 1978
Jitender Hitkari
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CHAPTER 1

INTRODUCTION

1.1 General

The objective of this report is to investigate how prediction algorithms can be implemented on micro and minicomputers. The term 'prediction algorithm', as employed in this report, denotes a systematic procedure to predict or forecast the future values of an observed variable. It is assumed that the observations are made at equidistant intervals of time. Suppose a variable $x(t)$ is observed up to an instant of time $t$. The prediction algorithm provides an estimate of $x(t+1)$, or more generally $x(t+l)$, where $l$ is called the lead time of prediction. There is, of course, no universally applicable algorithm available [6]. Instead, one has to look for the algorithm most appropriate for a given set of conditions. In addition, it must be computationally simple so that it can be implemented on a micro or minicomputer.

1.2 Contents of the Report

In Chapter 2, a brief discussion of mathematical modelling is presented. This is followed by a survey of prediction techniques which include exponential smoothing and time-series methods.
In Chapter 3, small computer systems are defined and their general characteristics are given. The minicomputer system used by the author is also described. This is followed by a detailed description of the models and prediction algorithms used in the application example. This example predicts the electrical power load over a lead time of 24 hours.

In Chapter 4 the results of the implemented algorithms are discussed. This includes the error summary and memory maps for each of the algorithms.

Finally, in Chapter 5, conclusions are presented.
CHAPTER 2
A SURVEY OF PREDICTION TECHNIQUES

2.1 INTRODUCTION

In this chapter a number of prediction techniques are described. There are two main categories of prediction methods. First there are methods based on developing suitable stochastic (or probabilistic) models[6]. For the last several years considerable attention has been given to stochastic modelling. The underlying principle is to consider the observed variable as the output of a stochastic process[6]. Thus, one has to identify the stochastic process which evolves the observed variable. The concepts of prediction theory are then applied to develop prediction algorithms. It is sometimes useful to depart from a purely stochastic model approach and develop the prediction model in two parts. One part describes the deterministic behaviour of the variable and the other part, its stochastic behaviour[19]. Therefore, two models are required. This is, of course, dependent upon the application under consideration. For example in the prediction of the outcome of a chemical process there is no identifiable deterministic behaviour; for that reason a purely stochastic model would be required. On the other hand, in the prediction of electrical load, for example, one can identify a deterministic trend. In this case a two part model can be developed.
The stochastic models are also capable of incorporating external factors which affect the future value of the variable. This is illustrated in the application example (Chapter 3) in which the future values of the electrical load are affected by the future temperature values, temperature being an external variable.

The second class of prediction methods are the so-called, non-probabilistic methods. These methods consist of representing the observed variable by suitable mathematical functions such as polynomials and exponential functions. Hence, matching a suitable function to the data becomes the gist of the prediction. Such prediction methods are, in essence, mathematical approximations of observed variable and are not capable of incorporating any external factors which affect the future values of the variable. Thus, this class of methods has limited value. The last part of this chapter deals briefly with these methods.

2.1.1 CONCLUDING REMARKS

Model building is somewhat of an art in the sense that a certain amount of educated guesswork is inevitable during the development stage[8]. Thus, no model can be said to be the best since every model will necessarily have its limitations. The choice of a model for a particular application is also dependent upon the way the model is to be used and the accuracy required[6].

The prediction techniques to be described in the following sections are selected on the basis of the mathematical technique employed in arriving at a prediction algorithm.
2.2 EXPONENTIAL SMOOTHING METHOD

2.2.1 GENERAL


2.2.2 DESCRIPTION OF THE ALGORITHM

The model proposed consists of two terms:

i) a linear combination of known functions of time

ii) a noise component.

\[ Z(t) = a_t f(t) + e(t) \]  

(2.1)

where-

- \( Z(t) \) = value of the observed variable \( Z(t) \) at time \( t \)
- \( a_t(t) \) = model coefficients, assumed constant over lead time of prediction
- \( f(t) \) = a fitting function chosen for a particular set of data
- \( e(t) \) = uncorrelated, normally distributed random variables with zero mean and constant variance.

To implement the method we require an appropriate set of fitting functions and a method to estimate the coefficients \( a_t \) from the observed values of the data under consideration.

The fitting functions must be chosen such that the expected value of the data for the maximum lead time of prediction can be described
as a linear combination of these functions. Fourier series representation for the fitting functions is often employed [7], [12], [15].

The initial estimate of the coefficients $a_t$ are computed using the weighted least square criterion, by minimizing the expression —

$$
\sum_{i=0}^{T} \alpha^i \left[ Z(T-i) - a'(T) f(T-i) \right]^2.
$$

(2.2)

where

$T =$ current time (the last instant for which the variable $Z(t)$ is known

$\alpha =$ smoothing constant such that $0 < \alpha < 1$.

Equation (2.2) is the exponentially weighted sum of the squared deviations of the model from the observed data. The smoothing constant $\alpha$ controls the rate at which the past errors are discounted.

The choice of $\alpha$ is made on some criterion such as standard error obtained. High values of $\alpha$ give predictions which depend much more on recent observations of $Z(t)$, whereas low values result in predictions which depend on a large number of past observations.

The estimates of the coefficients are updated as actual data becomes available, according to —

$$
\hat{a}(T) = L' \hat{a}(T-1) + h \left[ Z(t) - \hat{Z}_i(T) \right].
$$

(2.3)

where $L =$ a transition matrix, which is assumed to exist for a chosen set of fitting functions $f(t)$ such that

$$
f(t) = L f(t-1).
$$

(2.4)

$h =$ a smoothing vector obtained from

$$
h = \left[ \sum_{i=0}^{\infty} \alpha^i f(-i)f'(-1) \right]^{-1} f(0).
$$

(2.5)
and $\hat{Z}_1(T) = \text{one step ahead prediction of the variable } Z(t)$.

The predictions can be computed by extrapolating equation (2.1) giving

$$Z_2(T + Z) = a'(T) f(t + Z) \ldots \ldots \ldots \ldots \ldots \ldots (2.6)$$

where $Z = \text{lead time of prediction}$.

2.2.3 DISCUSSION OF THE METHOD

The prediction algorithm is general in nature. Any linear combination of analytical functions such as polynomials, exponentials, etc. can be easily written in the transition matrix form of equation (2.4). It is assumed that the matrix

$$\sum_{f=0}^{\infty} \alpha^f f(-1) f(-1) \ldots \ldots \ldots \ldots (2.7)$$

exists for chosen $\alpha$ and $f(t)$ function. The condition which insures that the matrix exists is $f_k(t) > \alpha^{-t/2}, k = 1, 2, 3 \ldots$. This is not a very restrictive condition so that the matrix in equation (2.7) will exist in most cases. The condition merely means that the fitting function $f(t)$ when extrapolated into the past should not increase at a rate faster than the rate at which the exponential weighting function decays [7].

The algorithm indicates that the predictions are based solely on the past values of the observed variable. The fact that the model parameters are readily updated as the next observation becomes available is a good feature of the algorithm. One matrix multiplication and a vector addition are required to update the model. This method would be suitable in an on-line environment where the computer constantly updates the model and outputs the predictions.
2.3 **TIME SERIES METHOD**

2.3.1 **GENERAL**

The time series approach to prediction and control was developed by Box and Jenkins [2,3]. Chatfield [6] also described prediction algorithms based on the time series idea. It was proposed to represent the observed variable as a time series with observations available at discrete equispaced time intervals. These time series would be fitted by stochastic models.

2.3.2 **STATIONARY TIME-SERIES**

The concept of stationarity plays a vital role in time series analysis. A series is said to be stationary if its first moments, namely its mean and variance, are independent of the shift in time origin. This may seem restrictive at first but many nonstationary time series encountered in practice can be converted into a stationary one by a simple process of differencing as discussed in Section 2.3.5.

2.3.3 **AUTO-REGRESSIVE & MOVING-AVERAGE MODELS**

Let the values of a stationary time series of a variable \( z_t \) at times \( t, t-1, t-2, \ldots \) be \( z_t, z_{t-1}, z_{t-2}, \ldots \) respectively. Also let \( w_t, w_{t-1}, w_{t-2}, \ldots \) be a series consisting of random drawings from a gaussian distribution having zero mean and some variance \( \sigma_w^2 \). The discussion of the time series models which follow were originally developed by Yule [3]. The time series \( z_t \) whose successive observations are assumed to be highly correlated, is transformed into a series of uncorrelated components \( w_t \) which can be thought of as evolving the series. There are two ways of generating such series:
1) The deviation of the observed series from its mean at the time origin can be made linearly dependent on previous such deviations and on \( w_t \). Thus an autoregressive model is obtained as —

\[
z_t - \mu = \phi_1 (z_{t-1} - \mu) + w_t \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2.8)
\]

where \( \mu \) is the mean of the original time series assuming stationarity and \( \phi_1 \) is a model parameter to be estimated.

Let \( \bar{z}_t = z_t - \mu \), then,

\[
\bar{z}_t = \phi_1 \bar{z}_{t-1} + w_t \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2.9)
\]

Equation (2.9) is a first order autoregressive model.

2) The other method to generate the time series is to make \( z_t \) linearly dependent on \( w_t \) and on its previous values \( w_{t-1}, w_{t-2}, w_{t-3} \ldots \ldots \) etc. This results in a finite moving average model, thus we have —

\[
\bar{z}_t = w_t + \theta_1 w_{t-1} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2.10)
\]

Equation (2.10) is a first order finite moving average model.

In order to represent moving average behaviour using the autoregressive model, one would require an infinite number of autoregressive terms and vice versa. In order that the number of parameters are as few as possible, both kinds of terms are included to give a general autoregressive moving average model of order \((p,q)\) as —

\[
\bar{z}_t - \phi_1 \bar{z}_{t-1} \ldots \ldots \ldots \phi_p \bar{z}_{t-p} = w_t - \theta_1 w_{t-1} \ldots \ldots \theta_q w_{t-q} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (2.11)
\]

where \( p \) and \( q \) are 0, 1 or 2 in most practical cases.
To manipulate the model it is convenient to define a backward shift operator $B$ such that —

\[ Bz_t = z_{t-1} \]  \hspace{1cm} (2.12)

Equation (2.11) can be expressed as —

\[ \phi_p(B)\bar{z}_t = \theta_q(B)w_t \]  \hspace{1cm} (2.13)

where

\[ \phi_p(B) = 1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_p B^p \]  \hspace{1cm} (2.14)

and

\[ \theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q \]  \hspace{1cm} \phi_p(B) \text{ and } \theta_q(B) \text{ are polynomials in } B \text{ of degree } p \text{ and } q \text{ which have to be estimated from the observed data. The condition for stationarity of the general model equation (2.13) is that the roots of } \phi(B) = 0 \text{ and } \theta(B) = 0 \text{ must lie outside the unit circle [3]. The general model can be expressed in terms of the observed variable } z_t \text{ instead of the deviation } \bar{z}_t \text{ as —}

\[ \phi_p(B)z_t = \theta_0 + \theta_q(B)w_t \]  \hspace{1cm} (2.15)

where

\[ \theta_0 = (1 - \phi_1 - \phi_2 - \cdots - \phi_p) \mu \]

2.3.4 Homogeneous Nonstationary Time Series

Many physical processes are best represented by nonstationary time series. Fig. 2.1(a) shows a nonstationary series commonly encountered. The characteristic of the series is that it looks much the same if we examine it piecewise, except for the level difference. It can be shown that the first order differencing of such series yields a series which exhibits stationarity, namely —

\[ v_t = z_t - z_{t-1} = (1 - B)z_t \]  \hspace{1cm} (2.16)
Fig. 2.1(b) shows another very frequently met nonstationary series. It can be shown that for such series a second order differencing yields a stationary series \([3]\), that is:

\[
Y_t^2 z_t = (1-B)^2 z_t 
\]  \hspace{1cm} (2.17)

Incorporating the above ideas into the general model of equation (2.13) we get:

\[
\phi_p(B)(1-B)^d z_t = \theta_0 + \theta_q(B)w_t 
\]  \hspace{1cm} (2.18)

where it is assumed that the \(d\)th order differencing is necessary to obtain stationarity. We can further define:

\[
\gamma_{p+d}(B) = \phi_p(B)(1-B)^d
\]  \hspace{1cm} (2.19)

\(\gamma_{p+d}(B)\) is called the generalized autoregressive operator. \(\theta(B)\) is called the moving average operator. We can also omit \(\theta_0\) from the general model without loss of generality since it represents a deterministic linear trend in the time series which is not related to the stochastic behaviour represented by the model. Thus the final form of the model becomes:

\[
\gamma_{p+d}(B)z_t = \theta_q(B)w_t 
\]  \hspace{1cm} (2.20)

This model can be expressed in three explicit forms:

1) Difference equation form

2) The model in terms of current and previous values of \(w_t\)

3) In terms of weighted sums of the previous values of \(z_t\) and \(w_t\).

Only the first two forms are discussed in this report. These forms are used to derive prediction algorithms.
Fig. 2.1(a) - A series showing non-stationarity in level.
(b) - A series showing non-stationarity in level and slope.
The difference equation form is obtained as follows —

\[ y_{p+d}(B) = \phi_p(B)(1-B)^d = 1 - \gamma_1 B - \gamma_2 B^2 \cdots \cdots \gamma_{p+d} B^{p+d} \]  

(2.21)

Also

\[ z_t = \gamma_1 z_{t-1} - \gamma_2 z_{t-2} \cdots \cdots \gamma_{p+d} z_{t-(p+d)} = w_t - \theta_1 w_{t-1} \]

\[ -\theta_2 w_{t-2} \cdots \cdots - \theta_q w_{t-q} \]  

(2.22)

\[ \therefore z_t = \gamma_1 z_{t-1} \cdots \cdots \gamma_{p+d} z_{t-(p+d)} - \theta_1 w_{t-1} \cdots \cdots - \theta_q w_{t-q} w_t \]

(2.23)

Equation (2.23) is the difference equation form of the general model.

To derive the second form of the general model let —

\[ z_t = w_t + \psi_1 w_{t-1} + \psi_2 w_{t-2} + \cdots \cdots \]  

(2.24)

which is a linear model with output \( z_t \) and whose input is the gaussian process \( w_t \) defined in section 2.3.3.

Equation (2.24) can be expressed as —

\[ z_t = w_t + \sum_{i=1}^{\infty} \psi_i w_{t-1} \cdots \cdots \]  

(2.25)

\[ = \psi(B)w_t \]

Substituting \( w_t \) from eqn. (2.25) into the general model of equation (2.20) we get —

\[ y_{p+d}(B)\psi(B)w_t = \theta_q(B)w_t \]

so that

\[ y_{p+d}\psi(B) = \theta_q(B)w_t \]  

(2.26)
Hence the $\psi$ weights may be determined by comparing coefficients in the expression:

$$
\left( \gamma_1 B - \gamma_2 B^2 - \cdots - \gamma_{P+d} B^{P+d} \right) (1 + \psi_1 B + \psi_2 B^2 + \cdots) = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_9 B^9)
$$

(2.27)

2.3.5 Derivation of the Prediction Algorithm

Let us assume that the prediction function steps ahead is of the form:

$$
\hat{z}_t(z) = \psi_{t-1} W_t + \psi_{t+1} W_{t-1} + \psi_{t+2} W_{t-2} + \cdots + \psi_{t+L} W_{t-L} + \cdots
$$

(2.28)

where the weights $\psi_t$, $\psi_{t+1}$, $\cdots$ are to be determined. Then using equation (2.25) the mean square error of prediction may be written as:

$$
E[z_{t+L} - \hat{z}_t(z)]^2 = (1 + \psi_1^2 + \psi_2^2 + \cdots + \psi_{L-1}^2) \sigma_w^2 + \sum_{i=0}^{L-1} \sigma_{w_i}^2
$$

$$
[\psi_{t+1} - \psi_{t+1}]^2 \sigma_w^2 + \cdots
$$

(2.29)

Equation (2.29) is minimized if $\psi_{t+1} = \psi_{t+1}$ which is a result derived from prediction theory [3]. Thus we have:

$$
z_{t+L} = (\psi_{t} W_t + \psi_{t+1} W_{t-1} + \cdots) + (W_{t+L} - \psi_{t} W_{t+L-1} + \cdots)
$$

$$
\psi_{t-1} W_{t+1} = \hat{z}_t(z) + e_t(z)
$$

(2.30)

where $\hat{z}_t(z)$ is the $L$ step ahead prediction and $e_t(z)$ is the corresponding prediction error. It is seen from equation (2.30) that the minimum mean square error prediction at origin $t$, for lead time $L$, is the conditional expectation of $z_{t+L}$, at time $t$. That is —
\[ E[z_{t+2} | z_t, z_{t-1}, \ldots] = \hat{\Sigma}_t(z) \ldots \ldots \ldots \ldots \ldots (2.31) \]

Also, the prediction error for lead time \( t \) is given by —

\[ e_t(z) = w_{t+2} - \psi_1 w_{t+1} \ldots \ldots \psi_{z-1} w_{t+1} \ldots \ldots (2.32) \]

and since

\[ E[e_t(z)] = 0. \]

Hence the prediction is unbiased and the estimate of the prediction error is zero. The variance of the prediction error is given by —

\[ V(z) = (1 + \psi_1^2 + \psi_2^2 \ldots \ldots + \psi_{z-1}^2) \sigma_w^2 \ldots \ldots (2.33) \]

From equations (2.30) and (2.32) the one step ahead prediction error is

\[ e_t(1) = z_{t+1} - \hat{\Sigma}_t(1) = w_{t+1} \ldots \ldots \ldots (2.34) \]

This is a key result since it gives significance to the gaussian process \( w_t \) which so far has been an unknown. It is seen that the values of \( w_t \) are in fact the one step ahead prediction errors. The prediction algorithm can now be established using equation (2.23) by taking its conditional expectation at time \( t+2 \). Let's define the conditional expectation operator as —

\[ E[z_t | z_{t-1}, z_{t-2} \ldots \ldots ] = [z_t] \ldots \ldots (2.35) \]

Then applying the conditional expectation operator to both sides of equation (2.23) we get —

\[ [z_{t+2}] = \hat{\Sigma}_t(z) = \gamma_1 [z_{t+1}] + \ldots + \gamma_{p+q} [z_{t+1-p-q}] \]

\[- \theta_1 [w_{t+1}] \ldots \ldots - \theta_q [w_{t+1-q}] + [w_{t+2}] \ldots \ldots \ldots \ldots (2.36) \]
The various conditional expectations can be determined using the following results [3].

\[
[z_{t-1}] = z_{t-1}, \quad i = 0, 1, 2, 3 \\
[z_{t+1}] = \hat{z}_{t-1}, \quad i = 1, 2, 3, 4 \\
[w_{t+1}] = 0 \quad i = 1, 2, 3, 4 \\
[w_{t-1}] = \hat{z}_{t-1} - \hat{z}_{t-1}(1), \quad i = 0, 1, 2, 3.
\] (2.37)

2.3.6 **Concluding Remarks**

The method described above is computationally simple since it involves fewer processing steps. The difference equation is strictly an algebraic expression applied recursively to obtain predictions. The only cumbersome quantities to compute are the one step ahead predictions for the noise components \( w_t \) since their computations involve backtracking the time-origin stepwise and determining these predictions. The method has been applied practically in the field of electrical engineering by researchers such as Panuska [15] and Vemuri et al [20].

2.4 **Kalman Prediction Algorithm**

Once the system model is arrived at, and an initial estimate of its parameters obtained, the Kalman filtering and prediction algorithms can be applied. The model is put in the state space form. The Kalman filtering is employed first to estimate the current state using past data values. Then the prediction algorithm is used to determine the one-step ahead prediction of the state. The mechanics of the Kalman filter is such that it adjusts the initially estimated state in accordance with the difference between the newly available output and the output calculated from
the initial estimate of the state. In other words the filter operates
in a 'predict-correct' fashion [13]. The filtered estimate is given by —

\[
\hat{x}(j+1 | j+1) = L(j+1, j)\hat{x}(j | j) - K(j+1)[z(j+1) - H(j+1)L(j+1, j)\hat{x}(j | j)]
\]  

for \( j = 0, 1, 2, 3, \ldots \), where \( x(0|0) = 0 \)

\( \hat{x} \) is an 'n vector', the state, \( \hat{x} \) being the prediction of \( x \)

\( z \) is an 'm vector', the observed system output

\( H \) is an \( m \times n \) matrix called the measurement matrix

\( K \) is an \( n \times m \) matrix called the Kalman gain matrix

\( L \) is an \( n \times n \) state transition matrix.

Fig. 2.2 illustrates the filtering scheme. Let us assume that the filtered
estimate at some time origin \( t = j \) is known and that we want to compute
the one step ahead estimate of \( x \) given the observation \( z(j+1) \). First
we must premultiply the initial state estimate \( \hat{x}(j | j) \) by the state transition matrix \( L \). This step results in the dynamic extrapolation of the
preceding state estimate. Then the term

\( z(j+1) - H(j+1)L(j+1, j) \) in equation (2.38) is computed. This term
represents the measurement residual defined by

\[
z(j+1|j) = z(j+1) - z(j+1|j) \]

(2.39)

The residual is then premultiplied by \( K(j+1) \), the weighting matrix and
the result is added to the first term computed, to obtain the one step
ahead prediction of variable \( x \). The following relationship expresses
the \( z \) step ahead prediction of \( x \) as —

\[
\hat{x}(j+z | j) = L(j+z, j)\hat{x}(j | j)
\]  

(2.40)
Fig. 2.2 — Kalman Filter
The computation of Kalman gain matrix \( K \) which is expressed in terms of two covariance matrices is discussed in Meditch [13] and will not be dealt with in this report.

The Kalman filter method has been used in power engineering application by Galiana [8] and McIntosh [12] to derive predictions. The method is not suitable for small computer implementation as the matrix manipulations required are rather extensive for a small computer to handle efficiently.

2.5 SOME NON-PROBABILISTIC PREDICTION TECHNIQUES

2.5.1 GENERAL

In the analysis of a physical process one often ends up with tabulated data. It is plausible that a mathematical representation of the data can be found. This is where curve fitting prediction methods are employed. The most elementary technique is graphical curve-fitting. The observed data points are plotted and the curve which best fits the data is drawn. The curve can then be extended into the future resulting in a short term prediction. The method is obviously crude and therefore limited in value. Other methods, namely, the method of collocation and the method of averages can be used more effectively to actually represent the observed data mathematically. A further refinement is the method of least squares which is widely used in engineering analysis. These methods are presented briefly in the following sections.

2.5.2 MATHEMATICAL REPRESENTATIONS

Let a variable \( z \) be measured and tabulated. Further, let \( z \) depend on variables \( x_1, x_2, x_3, \ldots, x_n \). We can establish the follow-
of this class. In particular, we have

\[ y = f(x_1, x_2, x_3, \ldots, x_n) \]  (2.41)

It is required to determine a mathematical function of independent variables, say \( u \) where

\[ u = h(x_1, x_2, x_3, \ldots, x_n) \]  (2.42)

where the function \( h \) is evaluated for each set of values of the independent variables, such that the deviations \( z(x) - u(x) \) are sufficiently small.

In other words the errors are within some acceptable range. The problem boils down to determining the function \( h(x) \) and its parameters. The form of this function will depend entirely on the application under consideration. In a large number of cases, however, we can give \( h(x) \) a polynomial or a Fourier series representation. As for the parameters, several generalized computing methods exist by which they may be estimated [10].

2.5.3 METHOD OF COLLOCATION

Let \( h(x) \) be selected to be some polynomial containing \( k \) unknown parameters. In order to determine these we must have at least \( k \) sets of data values. If the number of data points and parameters are the same then the latter may be determined by substituting the data values, one set at a time, into the polynomial and then solving a set of \( k \) simultaneous equations. The estimated parameters may then be applied to the data points to determine their precision. One possible criterion would be to determine if the parameters result in the least difference between the most positive and the most negative error. Another criterion could be to minimize the sum of the errors. This can be achieved by using the method of averages.
In this method, the data points are divided into say, \( m \) groups. The parameters are determined as explained above using each of these groups. Then the estimated parameters from each group are plugged into the chosen function and an average equation is determined using each group. As a result we again have \( m \) equations that may be solved for the parameters.

### 2.5.4 Method of Least Squares

This method relies on the minimization of the sum of the squares of the deviation between the observed and the computed value of a variable. Let a variable \( z \) be dependent on another variable \( x \) and its parameters \( a_{x_1}, a_{x_2}, a_{x_3}, \ldots \)

\[
    z = f(x_1, x_2, x_3, \ldots, a_{x_1}, a_{x_2}, a_{x_3}, \ldots) \quad \ldots \ldots \quad (2.43)
\]

We seek a function \( u \) where

\[
    u = h(x_1, x_2, x_3, \ldots, a_{x_1}, a_{x_2}, \ldots) \quad \ldots \ldots \quad (2.44)
\]

The sum of squares of the deviations can now be written as —

\[
    D = \sum_{j=1}^{n} (z_j - u_j)^2 = \sum_{j=1}^{n} (z_j - h(x_1, x_2, x_3, \ldots, a_{x_1}, a_{x_2}, a_{x_3}, \ldots))^2 \quad \ldots \ldots \quad (2.45)
\]

The condition that minimizes \( D \) is —

\[
    \frac{\partial D}{\partial a_{x_1}} = 0, \quad \frac{\partial D}{\partial a_{x_2}} = 0, \quad \ldots \quad \text{etc}
\]

That is, the partial derivatives of \( D \), the deviations, with respect to the parameters of \( x \) are equal to zero. This gives us a system of simultaneous equations which can be solved for the parameters \( a_{x_1}, a_{x_2}, a_{x_3}, \ldots \), etc.
2.5.5 Concluding Remarks

The techniques described in the foregoing sections can be applied in prediction computations. The models obtained are totally deterministic. The parameters can be adjusted to best fit the data. The method of least squares is a very versatile tool in mathematical modelling, since we can employ well established laws of mathematics to find the minimum sum of squares of errors. Another very useful modelling aid is the Fourier series as they can approximate any single valued function with finite discontinuities. The computations of model parameters can be easily done using small computers which offer a high-level language such as Fortran.
CHAPTER 3
DESCRIPTION OF SELECTED ALGORITHMS AND THEIR IMPLEMENTATION

3.1 GENERAL REMARKS

The model used for the application example is derived from Panuska and Koutchouk [16] and Panuska [15]. It is a weather dependent model representing electrical power load. This model resembles that proposed by Galiana [8], the major difference being the fundamental computational approach. Galiana chose the state-space analysis whereas Panuska uses the model equations directly to develop a prediction algorithm. The latter approach lends itself very well to small computer implementation since no matrix manipulations are involved. It should be emphasized that matrix operations especially inversions, require large memory storage and high CPU execution times. This is one of the major considerations when selecting an algorithm which could be efficiently implemented on small computers. Before undertaking a close study of the model, the general characteristics of small computers are presented. This is followed by the description of the minicomputer system employed by the author to implement the prediction algorithm.

3.2 SMALL COMPUTER SYSTEMS

Small computer systems can be configured using micro or minicomputers. Before proceeding any further the distinction between a micro-

- 23 -
processor and a microcomputer must be explained. A microprocessor is the processing element of a microcomputer. Thus a basic microcomputer is a computer consisting of a microprocessor, memory, I/O ports and a power supply. The cost of a microcomputer can range from $1,000 to $7,000, depending upon the hardware components required. A large variety of microprocessors are available in the market today. Fig. 3.1 [1] shows the various families of microprocessors along with their application range. Table 3.2 [4] indicates the general characteristics of microprocessors.

A minicomputer, on the other hand, consists of a more powerful and expensive processor, in addition to memory and I/O ports. The price of a minicomputer can range from $8,000 to $30,000, depending on the configuration. Table 3.3 [5] shows the general characteristics of some widely used minicomputers.

It is not possible to make a one-to-one comparison between micro and minicomputers as their application ranges are different. Figure 3.1 shows that on the far side of the application range the distinction between micro and minicomputers is hard to define. Microprocessors like TI-9900 are as powerful as minicomputer processors and hence could be classified in either category. References [1], [4], [5], are recommended to the reader for in-depth discussion of small computer applications. The choice of a particular processor will depend on the complexity of the computations involved and the programming language used, for a given algorithm. Other factors such as cost and processor speed must also be considered.

In the development of a small computer system for implementing prediction algorithms, both hardware and software design would be required. Fig. 3.1 shows the general steps required in the overall design of such a system.
Single Chip Controllers

4 Bit Family

8 Bit Family

16 Bit Family

Bit Slices

Application Range

Fig. 3.1 — Microprocessor Families and their Application Range.
<table>
<thead>
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<th>Manufacturer</th>
<th>Processor</th>
<th>technology</th>
<th>word size (bits)</th>
<th>direct addressing (words)</th>
<th>2 out base instructions</th>
<th>clock, long/multiple phases</th>
<th>instruction type</th>
<th>TIL compatible</th>
<th>BCD arithmetic</th>
<th>on-chip interrupt</th>
<th>interrupt type</th>
<th>on-chip registers</th>
<th>TIL stack registers</th>
<th>on-chip RAM</th>
<th>DMA capability</th>
<th>Memory &amp; I/O available</th>
<th>prototyping aid</th>
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Table 3.1 — Microprocessors.
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<th>DG</th>
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<td>Fixed-point operand length (bits)</td>
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<td>16</td>
<td>15</td>
<td>16, 32</td>
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<td>Instruction length (bits)</td>
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<td>16</td>
<td>16, 32</td>
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<td>Floating point</td>
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<td>FP operand length (bits)</td>
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<th>CORE/MOS</th>
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<td>Access time (μs/wd)</td>
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<td>0.3</td>
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<td>Minimum size (kilobytes)</td>
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<td>16</td>
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<td>Maximum size (kilobytes)</td>
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<td>262</td>
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<td>no</td>
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<td>Addressing modes</td>
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<td>Add time (μs)</td>
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<td>Hardware multiply/divide</td>
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<td>Battery backup</td>
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<td>Real-time clock/timer</td>
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Table 3.2 — Minicomputers.
Fig. 3.2 — System Development Block Diagram.
3.3 Description of the System Used

Fig. 3.3 shows the block-diagram of the minicomputer system used to implement the prediction algorithms. The CPU is a PDP11/15 minicomputer with 20K words of core memory. The word size is 16 bits. The instruction times vary between 3 and 10 microseconds, depending upon the instruction. A high speed moving-head disk drive is included as a mass storage media for programs and data. The source program in punched card form, is read into the memory using the card reader, and is subsequently stored on the disk. The program is then compiled to obtain an executable load module. The model parameters are entered through the terminal keyboard and the predictions are printed on the line printer. The programming language used is Fortran.

Fig. 3.4 illustrates a proposed microprocessor implementation. The hardware components required are indicated in the diagram. The scratch pad random access memory (RAM) is used to store data and parameters. The program read only memory (PROM) contains the programmed algorithm. The model parameters can be entered through the teletypewriter and the predictions printed out. Alternatively, the data and the parameters can be obtained through a communication interface, from a large computer. The choice of the programming language used for the microprocessor implementation is dependent on the complexity of the prediction algorithm. Traditionally, Assembly language has been used to program microprocessor based systems. However, with the price of solid state memory falling steadily during the past few years, it is not inconceivable that a high-level language such as Fortran could be employed.

In the following sections of this chapter, two algorithms are discussed. One algorithm uses a periodic and a residual component to model
electrical power load, and the other algorithm is based solely on the residual model. The results obtained from these algorithms are discussed in the next chapter.
Fig. 3.3 — Minicomputer Implementation.
Fig. 3.4— Microprocessor Implementation.
3.3 DESCRIPTION OF THE PERIODIC & RESIDUAL MODEL

The model [15, 16] has two components. A periodic load component represented by a Fourier series, called the nominal load, and a residual component in the form of a linear difference equation. The residual term contains the stochastic component of the model, with a temperature function as the driving input. Fig. 3.5 shows the block diagram of the model. The total load is expressed as the sum of the two components as

\[ Z_t = Y_t + R_t \]  \hspace{1cm} (3.1)

where \( Y_t \) is the periodic term and \( R_t \) is the residual term. \( Y_t \) is modelled as

\[ Y_t = \alpha_0 + \sum_{k=1}^{N'} \alpha_k \sin kgt - \sum_{k=1}^{N'} \beta_k \cos kgt \]  \hspace{1cm} (3.2)

where \( N' \) is the number of harmonics. The first 8 harmonics are used.

\( t \) is the basic time unit assumed to be 1 hour.

\( g = 2\pi/T, \) \( T \) being the period assumed to be 24 hours.

\( \alpha_0, \alpha_k, \beta_k \) are the estimated coefficients.

The residual term which is modelled by a general linear difference equation with a noise term added to it, is given by

\[ (1 + A(z^{-1}))R_t = (b_0 + B(z^{-1}))U_{t-h} + E_t \]  \hspace{1cm} (3.3)

\[ E_t = \frac{1 + C(z^{-1})}{1 + D(z^{-1})} W_t \]  \hspace{1cm} (3.4)

where \( R \) is the system output, i.e., the residuals

\( U \) is the system input, a function of temperature\(^1\)

\(^1\)The determination of this function is described in reference [12] and will not be discussed in this report.
Fig. 3.5 — Block Diagram of the Load Model.

\[ Z(t) = Y_t + R_t \]
E is a correlated noise term
W is a zero mean gaussian process with variance $\sigma^2_w$, called white noise
h is a delay factor, assumed to be 7 hours
A, B, C, D are polynomials in the backward shift operator $z^{-1}$ with coefficients $a_i, b_i, c_i, d_i$ respectively, which are estimated from the data.

The representation of the preceding equations can be greatly simplified if we express them using Einstein notation: The occurrence of a superscript index with a variable and the same index with another variable indicates summation over that index [15]. Thus equation (3.2) can be rewritten as

Let \( \{ p_i \} = \{ a_o, a_k, b_k \} \)

and \( \{ y_{i,t} \} = \{ 1, \{ \sin kgt, \cos kgt \} \} \)

where \( k = 1, 2, 3 \ldots \ldots \ldots \ N' \)
\( i = 1, 2, 3 \ldots \ldots \ldots \ 2N' + 1 \)

then \( Y_t = \sum_{i=1}^{2N'+1} p_i y_{i,t} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.5) \)

Using Einstein’s notation, we get

\( Y_t = p_i^i x_{i,t} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.6) \)

which indicates that the summation is over the index \( i \). Similarly, equation (3.3) can be expressed as

\[
R_t + A(z^{-1})R_t = (b_o + B(z^{-1})U_{t-h} + E_t \\
R_t + \sum_{j=1}^{J} a_j R_{t-j} = \sum_{k=0}^{K} b_o U_{t-h-k} + E_t
\]
where J, K are degrees of the polynomials A(z⁻¹) and B(z⁻¹) respectively.

Applying Einstein's notation we get

$$R_t + a_j R_{t-j} = b^k U_{t-h-k} + E_t \ldots \ldots \ldots (3.7)$$

Equation (3.4) can similarly be rewritten as

$$E_t + \sum_{m=1}^{M} d^m E_{t-m} = W_t + \sum_{z=1}^{L} c^z W_{t-z}$$

where L, M are degrees of the polynomials C(z⁻¹), D(z⁻¹) respectively.

From Einstein's notation we have

$$E_t + m E_{t-m} = W_t + c^z W_{t-z}$$

so that

$$E_t = -m E_{t-m} + c^z W_{t-z} + W_t \ldots \ldots \ldots \ldots (3.8)$$

Substituting equations (3.6), (3.7) and (3.8) into (3.1) we get the total load as

$$Z_t = p^i Y_i t + a_j R_{t-j} + b^k U_{t-h-k} + m E_{t-m} + c^z W_{t-z} + W_t \ldots (3.9)$$

3.3.1 Prediction Algorithm

It now remains to be shown that equation (3.9) can be utilized to develop a prediction algorithm. Let $\hat{Z}_t(r)$ be the linear mean square error of prediction of $Z_{t+r}$, where $r$ is the lead time. $\hat{Z}_t(r)$ can be computed as a conditional expectation [3] as follows:

$$\hat{Z}_t(r) = E[Z_{t+r} | Z_t, Z_{t-1}, Z_{t-2} \ldots \ldots ]$$

Taking conditional expectance of both sides of equation (3.9) we get —
\[ \left[ z_{t+r} \right] = p^{1} \gamma_{t+r} - a^{d} \left[ R_{t+r-h-j} \right] + b^{k} \left[ U_{t+r-h-k} - d^{m} \left[ E_{t+r-m} \right] - c^{2} \left[ W_{t+r} \right] \right] + \tilde{W}_{t+r} \]  

(3.10)

where the square brackets in the expression denote conditional expectations.

The functions \( p^{1} \gamma_{t+r} \) and \( U_{t+r-h-k} \) are not affected by the operation since these are deterministic terms that are assumed to be known for the lead time \( r \). The remaining conditional expectations are evaluated as follows \[ [3, 15] \).

For \( n \geq 0 \)

\[ \left[ R_{t-n} \right] = R_{t-n} \quad n = 0, 1, 2, 3, 4 \quad \ldots \]

\[ \left[ R_{t+n} \right] = \hat{R}_{t}(n) \quad n = 1, 2, 3, 4 \quad \ldots \]

\[ \left[ E_{t-n} \right] = E_{t-n} \quad n = 0, 1, 2, 3 \quad \ldots \]

\[ \left[ E_{t+n} \right] = \hat{E}_{t}(n) \quad n = 1, 2, 3, 4 \quad \ldots \]

\[ \left[ W_{t-n} \right] = Z_{t-n} - \hat{Z}_{t-n-1}^{(1)} \quad n = 0, 1, 2, 3 \quad \ldots \]

\[ \left[ W_{t+n} \right] = 0 \quad n = 1, 2, 3, 4 \quad \ldots \]

The foregoing expressions indicate that the variables that are available at the time origin \( t \) are unchanged. Those that are not known can be replaced by their estimates at the origin.

3.4 PREDICTION ALGORITHM BASED ON THE RESIDUAL MODEL

3.4.1 GENERAL REMARKS

The prediction algorithm to be described in this section is based on Box & Jenkins [3] method. It utilizes the concept of seasonal differencing. The load data exhibits a periodic behaviour pattern every 24 hours.
The underlying principle of seasonal differencing is that the load values 24 hours apart are similar. The seasonal operator \( \nabla_{24} \) when applied to the load data yields

\[
\nabla_{24} Z_t = Z_t - Z_{t-24} \quad \ldots \quad (3.12)
\]

The prediction algorithm is derived in the following section.

3.4.2 Prediction Algorithm

The differencing operator \( \nabla_{24} \) is applied to equation (3.9) to get

\[
\nabla_{24} Z_t = \nabla_{24} p_{1,1} q_{t,1} - \nabla_{24} a_{j} R_{t-j} + \nabla_{24} b_{k} U_{t-h-k} - \nabla_{24} d_{m} E_{t-m} + \nabla_{24} c_{z} W_{t-z} + \nabla_{24} W_t \quad \ldots \quad (3.13)
\]

Since the first term on the R.H.S. of equation (3.9) is periodic, the first term in equation (3.13) vanishes, giving

\[
Z_t - Z_{t-24} = -\nabla_{24} a_{j} R_{t-j} + \nabla_{24} b_{k} U_{t-h-k} - \nabla_{24} d_{m} E_{t-m} + \nabla_{24} c_{z} W_{t-z} + \nabla_{24} W_t
\]

\[
\therefore Z_t = Z_{t-24} - \nabla_{24} a_{j} R_{t-j} + \nabla_{24} b_{k} U_{t-h-k} - \nabla_{24} d_{m} E_{t-m} + \nabla_{24} c_{z} W_{t-z} + \nabla_{24} W_t \quad \ldots \quad (3.14)
\]

The predictions can be computed using the ideas developed in Section 3.3.1, by taking the conditional expectation of \( Z_{t+r} \) at time \( t \) as

\[
[Z_{t+r}] = [Z_{t+r-24}] - [\nabla_{24} a_{j} R_{t+r-j}] + [\nabla_{24} b_{k} U_{t+r-h-k}] - [\nabla_{24} d_{m} E_{t+r-m}] \\
+ [\nabla_{24} c_{z} W_{t+r-z}] + [\nabla_{24} W_{t+r}] \quad \ldots \quad (3.15)
\]
All the terms of equation (3.15) can be evaluated using equation (3.11). Note, however, that

\[[Z_{t+r-24}) = Z_{t+r-24}\]

the past values of the load are known. Also, the conditional expectation operator does not affect the third term on R.H.S. of equation (3.15) since \(U_t\), the temperature dependent function is known for all \(t\).

3.4.3 Concluding Remarks

The prediction algorithms described in the foregoing sections were selected on the basis of their computational simplicity. Each of the algorithms is algebraic in nature. The predictions are obtained recursively using each of the algorithms. In the periodic and residual model algorithm, the knowledge of the periodic component of the load and its associated parameters is required. For the residual model however, these parameters are not required.
CHAPTER 4

RESULTS OF PREDICTION

4.1 GENERAL

In order to implement the selected prediction algorithms of Chapter 3 on a minicomputer, two Fortran programs were devised. The first program uses the prediction of algorithm developed in section 3.3 and the second uses the algorithm developed in section 3.4. The program source listings are presented in Appendix A. The data used in this report is taken from McIntosh [12]. It consists of the Hydro-Quebec load data for January 1972, along with the corresponding temperature data provided by the Dorval Weather Bureau. The temperature dependent function $U_t$ defined in Chapter 3, equation 3.3, is also taken from reference [12]. The relevant data is tabulated in Appendix B. The model parameters were determined by Panuska and Koutchbuk [16]. Appendix C contains the model parameters in a tabulated form. The line printer outputs obtained from each of the algorithms is also contained in this Appendix. Appendix D contains the relevant mathematical formulae used to compute the error summary and the probability limits.

4.2 DETAILS OF THE RESULTS

For both algorithms, January 20, 1972, was arbitrarily chosen as the day of prediction. The time origin was placed at midnight of January 19.
The lead time of prediction is 24 hours. The results of the periodic and residual component model are presented first. The periodic component was computed over three cycles and is shown in Table 4.1. The residual component is shown in Table 4.2. Table 4.3 shows the total predicted load and its percent deviations from the actual load. Figures 4.1 and 4.2 show the time plots of the periodic and residual components respectively. Figure 4.3 indicates the variation of the temperature dependent function over the lead time of prediction. Table 4.4 shows the error statistics obtained using the periodic and residual component model. Note that the quantities indicated in the table are in 10MW units. Figure 4.4 shows both the predicted and actual load.

The results obtained from the Box and Jenkins [3] method using the residual model are presented next. The predicted and actual load are shown in Table 4.5. This table also indicates the percent deviations of the predicted load from their actual values.

Figure 4.5 shows the time plots of the predicted and actual load over the lead time of prediction. Table 4.6 shows the error figures obtained using the residual model. Figure 4.6 shows the ± 0.95 probability limit envelope. It should be noted that these limits apply to individual predictions and not jointly to the overall prediction.

Table 4.7 shows the memory maps for each of the algorithms.

4.3 Concluding Remarks

The predictions obtained from either algorithm show roughly the same error figures as indicated by Tables 4.4 and 4.6. The rms error as a percentage of the peak load of 8790 MW was less than 4% in both cases.
<table>
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Table 4.1 - Periodic Component.
RESIDUAL COMPONENT OF LOAD (IN 10MW UNITS)

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Table 4.2 — Residual Component.
LOAD PREDICTION USING PERIODIC & RESIDUAL COMPONENTS
HYDRO-QUEBEC LOAD DATA FOR JANUARY 20, 1972 (IN 10MW UNITS)

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Table 4.3 — Load Prediction Using Periodic and Residual Components.
Figure 4.1 — Periodic Component (in 10 MW units)
Figure 4.2 — Residual Component (in 10MW units)
Figure 4.3 - Temperature Dependent Function $U(t)$ (in °F)

Lead Time in Hours

0 5 10 15 20 25

-10.0 -6.0 0 6.0 10.0 15.0
ERROR SUMMARY FOR JANUARY 20, 1972 LOAD PREDICTION

Mean Error of Prediction = 25.92
Actual Standard Deviation = 20.91
RMS Prediction Error = 33.74
RMS Error Percent of Peak Load = 3.84

Table 4.4 — Results Based on Periodic and Residual Component Model.
Fig. 4.4 - Electrical Load (in 10MW Units)
LOAD PREDICTION USING BOX & JENKINS METHOD

HYDRO-QUEBEC LOAD DATA FOR JANUARY 20, 1972 (IN 10MW UNITS)

<table>
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Table 4.5 — Load Prediction Using Box & Jenkins Method.
Fig. 4.5 — Electrical Load (in 10 MM Units)

- Actual
- Predicted

Lead Time In Hours

0  5  10  15  20  25
Figure 4.5 — Electrical Load (in 10MW units)
ERROR SUMMARY FOR JANUARY 20, 1972 LOAD PREDICTION

MEAN ERROR OF PREDICTION = 23.96
ACTUAL STANDARD DEVIATION = 18.27
RMS PREDICTION ERROR = 30.56
RMS ERROR PERCENT OF PEAK LOAD = 3.48

Table 4.6 — Results Based on Residual Model.
Memory Map  PREDCl.LDA

Transfer Address*  113724  
Low Limit  113724  
High Limit  157460  

(a)

Memory Map  PREDC2.LDA

Transfer Address  115324  
Low Limit  115324  
High Limit  157460  

(b)

*All addresses on this page are in octal, base 8.

Table 4.7 —  a) Periodic & Residual Model (File Name: PREDCl)  
     b) Residual Model (File Name: PREDC2).
CHAPTER 5

CONCLUSIONS

The aims of this report have been met in that it has been successfully demonstrated that it is indeed feasible to implement selected prediction algorithms on micro and minicomputers. This is clearly shown by the memory maps (Table 4.7) of the two algorithms programmed in Fortran on a 16-bit minicomputer. Each of the algorithms require approximately 8K words of memory storage. Further reductions in storage requirement are possible by utilizing Assembly language. The compilation time for each of the algorithms is about 10 minutes. This includes the printing of the program listing on a 300 lpm line printer. The execution time is about 20 seconds.

The overall accuracy of the prediction is within acceptable limits [15].
REFERENCES


LOAD PREDICTION ALGORITHM BASED ON PERIODIC & RESIDUAL METHOD
DESCRIBED IN -- PANUSKA, V., 'SHORT-TERM FORECASTING OF
ELECTRICAL POWER LOAD FROM A WEATHER-DEPENDENT MODEL'

THE PROGRAM WAS EXECUTED ON A DISK BASED PDP11/15 MINICOMP
SYSTEM USING DIGITAL EQUIPMENT 1S FORTRAN-4 VERSION A

DIMENSION NU(72), AL(8), BE(8), Y(72)
DIMENSION NPLT(100), X(72)
DIMENSION Z(72), IHR(72), IER(72)
DIMENSION U(80), T(72)
DIMENSION F(124), V(84)
INTEGER Z, T, U, IER

DATA DEFINITIONS

TEMPERATURE DATA
U(1) = U(T-9), U(2) = U(T-8) .... ETC

ACTUAL LOAD DATA
Z(1) = Z(T+1), Z(2) = Z(T+2) .... ETC

DATA U7 = 13, -11, -11, -10, -10, -8, -8, -6, -5, -5, -6, -7, -8, -3, 1, 2, 2, 2, 4
1, 5, 4, 5, 5, 6, 4, 5, 7, 7, 7, 6, 5, 4, 4, 2, 2, 7, 6, 5, 10, 10, 12, 14, 15, 14
14, 12, 12, 13, 13, 16
T = 14, 15, 15, 18, 20, 15, 17, 16, 16, 15, 17, 18, 19, 19
Z(T) = 7, 14, 12, 14, 5, 3, -5
1, -14, -15, -15, -14, -14, -14
DATA Z = 7, 5, 7, 6, 6, 5, 6, 10, 6, 0, 6, 9, 6, 2, 6, 4, 6, 4, 7, 3, 3, 7, 6, 2, 7, 7, 8, 0, 3, 8, 0, 4, 7, 7, 6, 7, 8, 2
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1, 7, 7, 8, 7, 7, 8, 4, 8, 6, 6, 3, 8, 4, 6, 8, 3, 9, 8, 1, 9, 7, 9, 1, 7, 5, 5, 7, 0, 8

MODEL PARAMETERS

READ(6, 100) A
100 FORMAT(F9.2)

READ(6, 110) B
110 FORMAT(F6.3)

READ(6, 120) C
120 FORMAT(F12.0)

RESUM=0
NXF=0
NSM=0

COEFFICIENTS

VALUE OF ALPHA ZERO
ALO=7351.0
AL(1)=965.6
AL(2)=398.8
AL(3)=134.0
AL(4)=118
AL(5)=101.2
AL(6)=137.7
AL(7)=197.7
AL(8)=217
BE(1)=273.1
BE(2)=199.4
BE(3)=178.7
BE(4)=90.8
BE(5)=65.5
BE(6)=32.6
BE(7)=40.0
BE(8)=74.1
C LOAD AT THE ORIGIN
Z0=6640
C COMPONENTS OF THE PERIODIC TERM
C1Y=AL0
NT=1
C2Y=0.0
C3Y=0.0
DO 60 I=1,8
ARG=I*2*3.1416*NT/24
C2Y=C2Y+AL(I)*SIN(ARG)
C3Y=C3Y+BE(I)*COS(ARG)
60 CONTINUE
Y(NT)=C1Y+C2Y+C3Y
NSCAL=10
C SCALING FACTOR = 10
Y(NT)=Y(NT)/NSCAL
NT=NT+1
IF(NT.LE.72)GOTO 55
C WRITE HOURS IN ARRAY IHR
DO 520 I=1,72
IHR(I)=1
JJ=1
JK=12
WRITE(5,550)
550 FORMAT(111)
WRITE(5,555)
555 FORMAT(55X,'PERIODIC COMPONENT OF LOAD (IN 10MW UNITS)')
602 WRITE(5,630)(IHR(M),H=JJ,JK)
WRITE(5,660)(Y(M),H=JJ,JK)
560 FORMAT(55X,'PERIODIC COMP.1',12I4)
JJ=JJ+12
JK=JK+12
IF(JK.GE.84)GOTO 801
GOTO 602
801 CALL PLOT(NPLOT,Z,Y,NSM)
COMPUTATION OF THE RESIDUAL TERM X(T)

VALUES OF PAST LOAD AT T=-1,-2,-3
ZM1=7150
ZM2=7520
ZM3=7700

PAST VALUES OF X AT T=-1,-2,-3
XM1=(ZM1/NSCALE)-Y(23)
XM2=(ZM2/NSCALE)-Y(22)
XM3=(ZM3/NSCALE)-Y(21)
DO 65 I=1,8
65 BESUM=BESUM+BE(I)
X0=A1*X1+A2*X2+A3*X3
X1=X0+80*U(3)+81*U(2)+82*U(1)
X(1)=A1*X0+A2*X1+A3*X3+80*U(3)+81*U(2)+82*U(1)
X(2)=A1*X(1)+A2*X(1)+A3*X3+80*U(3)+81*U(2)+82*U(1)
X(3)=A1*X(2)+A2*X(2)+A3*X3+80*U(3)+81*U(2)+82*U(1)
DO 185 K=4,72
185 X(K)=A1*X(K-1)+A2*X(K-2)+A3*X(K-3)+80*U(K+2)+81*U(K+1)+82*U(K)
CONTINUE

PRINT X(T)
JJ=1
JK=11
DO 552 I=1,48
552 NU(I)=X(I)
WRITE(5,550)
WRITE(5,600)
FORMAT(35X,'RESIDUAL COMPONENT OF LOAD (IN 10HM UNITS)'/)
600 FORMAT(35X,'RESIDUAL COMPONENT OF LOAD (IN 10HM UNITS)'/)
WRITE(5,605)(1HR(T),M=JJ,JK)
605 FORMAT(35X,'RESIDUAL COMPONENT OF LOAD (IN 10HM UNITS)'/)
WRITE(5,610)(1NU(M),M=JJ,JK)
610 FORMAT(35X,'RESIDUAL COMPONENT OF LOAD (IN 10HM UNITS)'/)
JJ=JJ+1
JK=JK+1
IF(JK.GE.48)GOTO 601
GOTO 602

SAVE Y
DO 505 I=1,48
505 T(I)=Y(I)

PLOT X(T)
DO 395 I=1,48
395 Y(I)=X(I)
CALL PLOT(Y,NPLOT,NSH)

TOTAL LOAD = PERIODIC + RESIDUAL COMPONENTS

CALCULATE PERCENT ERROR
DO 540 I=1,24
540 NERR=I*Y(I)
IER(I)=(100*NERR)/Z(I)
STEP TO NEW PAGE

WRITE(5,530)
FORMAT(111)
WRITE(5,410)
FORMAT(5X,'LOAD PREDICTION USING PERIODIC & RESIDUAL
1 COMPONENTS')
WRITE(5,420)
FORMAT(5X,'HYDRO-QUEBEC LOAD DATA FOR JANUARY 20, 1972')
PRINT DATA
M=1
M2=12
WRITE(5,430)(IHR(M),M=M1,M2)
FORMAT(35X,'HOUR',1214)
WRITE(5,440)(Y(M),M=M1,M2)
FORMAT(30X,'PREDICTED',1214)
WRITE(5,450)(Z(M),M=M1,M2)
FORMAT(35X,'ACTUAL',1214)
WRITE(5,465)(INF(M),M=M1,M2)
FORMAT(32X,'% ERROR',1214)
M=M+1
M2=M2+12
IF(M2.GE.36)GOTO 81
GOTO 82
WRITE(5,470)
FORMAT(X,///)
NSM=1
CALL PLOT(NPLOT,Z,Y,NSM)

LEAD TIME OF PREDICTION
LEAD=24

INITIALISE MEAN ERROR OF FORECAST
XMEAN=0

DO 501 I=1,LEAD
MIN=ABS(Y(I)-Z(I))
XMEAN=XMEAN*MIN
CONTINUE
XMEAN=XMEAN/LEAD

ACTUAL STANDARD DEVIATION
FSTD=0
STD=0
DO 502 I=1,LEAD
STD=ABS(Y(I)-Z(I))**2
FSTD=FSTD+STD
END
STD=STD/LEAD
FSTD=FSTD/LEAD
FSTD=SQRT(FSTD)

MEAN ERROR CALCULATIONS
XMEAN=(XMEAN**2)*LEAD/(LEAD-1)
RMS=SQRT(RMS)

PEAK LOAD.
PEAKLD=870.0
PERCENT=(RMS*100)/PEAKLD
WRITE(5,514)
514 FORMAT(11)
WRITE(5,504)
504 FORMAT(30X,'ERROR SUMMARY FOR JANUARY 20, 1972 LOAD PREDICTION')
WRITE(5,506)XMEAN
506 FORMAT(20X,'MEAN ERROR OF PREDICTION=',F5.2)
WRITE(5,508)SS1D
508 FORMAT(19X,'ACTUAL STANDARD DEVIATION=',F5.2)
WRITE(5,510)RMSE
510 FORMAT(24X,'RMS PREDICTION ERROR=',F6.2)
WRITE(5,512)PERCENT
512 FORMAT(14X,'RMS ERR TO PERCENT OF PEAK LOAD=',F5.2)
700 END
WRITE(5,300)
300 FORMAT(11)

PROBABILITY LIMITS
VALUES OF PHI
F10=1
F1(1)=A1
ESTIMATED VARIANCE=
VARIANCE=

VAR=104.1
F1(2)=A1*F1(1)*A2
F1(3)=A1*F1(2)*A2*F1(1)*A3
DO 200 I=1,24
DO 201 I=1,24
WRITE(5,205)F1(I)
205 CONTINUE
VARIANCE=

V(1)=1+F1(1)**2*VAR
DO 300 I=2,24
V(I)=V(I-1)+F1(I)**2*VAR
WRITE(5,630)V(I)
300 CONTINUE
630 FORMATE(10X,F8.2)
405 STOP
END

ROUTINES CALLED:
SIN, COS, PLOT, IABS, ABS, SQRT

BLOCK LENGTH
MAIN, 4225 (020002)
114 0000000.3001 $SEQ. 000273

**COMPILED ** CORE
PHASE USED FREE
DECLARATIVES 0052 08609
EXECUTABLES 01407 07729
ASSEMBLY 02501 09552
SUBROUTINE PLOT(NPLOT, Z, Y, NSM)
DIMENSION NPLOT(100), Y(72), Z(72)
INTEGER Z, Y, U

PLOTTING ROUTINE

HEAD OF FORM
WRITE(5, 135)
FORMAT('114')

PUT POINTS INTO ARRAY
DO 140 K = 1, 100
NPLOT(K) = Z(K)
WRITE(5, 150) NPLOT
FORMAT(21X, 100A1)

BLANK THE ARRAY
DO 180 K = 1, 100
NPLOT(K) = '

NE = 1
99 J = Y(N)/10
J = IABS(J)
NK = Z(J)/10
IF(J .LE. 0) J = -J
IF(NSM .EQ. 1) NPLOT(NK) = 'A'
NPLOT(J) = '1'
NPLOT(1) = '1'
WRITE(5, 101) NPLOT
FORMAT(20X,100A1)
NPLOT(J) = '1'
NPLOT(NK) = '1'
NE = NE + 1
IF(N .NE. 26) GOTO 99
RETURN
END

ROUTINES CALLED:
IABS

BLOCK LENGTH
PLOT 320 (001220)*
**COMPILED ----- CORE**

PHASE USED FREE
DECLARATIVES 00440 00890
EXECUTABLES 00607 00529
ASSEMBLY 01161 10892
LOAD PREDICTION ALGORITHM BASED ON BOX & JENKINS METHOD

DIMENSION X(26), Z(30), U(100), T(100), W(75), WW(75)
DIMENSION NKOZ(100), A(75), ZI(75), IER(75), IHR(75)
INTEGER Z, Z0, WW, T
INTEGER X, U, A, ZI, W, IER

DEFINITION OF DATA ELEMENTS

ARRAY A DEFINES THE ACTUAL LOAD DATA FOR JANUARY 20, 1972
PAST VALUES OF LOAD AND PREDICTED LOAD
Z(T-26) = X(1), Z(T-25) = X(2) .... AND SO ON
TEMPERATURE FUNCTION U(T)
U AT T = 32 = T(1), U AT T = 31 = T(2) .... AND SO ON
ORIGIN IS HOUR ZERO FOR ANY GIVEN DAY

DATA A/637, 626, 616, 607, 609, 624, 626, 646, 733, 762, 778, 803, 804, 776
1, 782, 773, 796, 879, 876, 852, 834, 820, 788, 747, 692, 652, 637, 626, 616
1, 621, 633, 681, 747, 780, 794, 810, 799, 761, 755, 758, 768, 859, 867, 858, 83
1, 811, 779, 752, 660, 628, 616, 604, 604, 604, 633, 665, 733, 779, 797, 821
1

DATA X/729, 693, 647, 616, 603, 592, 579, 569, 607, 651, 701, 738, 771
1, 786, 776, 749, 746, 740, 754, 843, 839, 819, 795, 770, 752, 719, 715

DATA T/20, 25, 35, -25, 22, -23, -24, -25, -26, -25, -27, -33, -33
1, -33, -31, -31, -24, -24, -22, -20, -15, -13, -11, -11, -10, -10, -10
1, 8, 10, 7, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, -15
1, -17, -22, -25, -22, -21, -23, -28, -23, -20, -17, -13, -10, -8, -7, -3, 1

DATA WW/765, 731, 681, 650, 630, 613, 609, 610, 620, 677, 734, 755, 773, 803
1790, 760, 766, 736, 77, 860, 843, 789, 781, 756/

U(T-56) = NU(1), U(T-59) = NU(2) ...... ETC

DATA NU/1, 2, 2, 0, -1, -2, -5, 4, -1, -19, -19, -19, -19, -19, -16, -14, -14
1, -16, -19, -19, -19, -19, -16, -17, -18/

MODEL PARAMETERS

READ(6, 600) A1
FORMAT(F5, 2)
READ(6, 600) A2
FORMAT(F6, 3)
READ(6, 603) A3
READ(6, 610) B0
FORMAT(F4,2)
READ(6,610)B1
READ(6,600)B2
C
SAVE DATA
DO 140 I=1,24
   IER(I)=NU(I)
   DO 160 I=25,56
      IER(I)=I(I=24)
   C
DO 90 I=1,24
   WW(I)=NW(I)
   DO 110 I=25,50
      WW(I)=X(I=24)
C
COMPUTATION OF NOISE TERMS
WE REQUIRE 1-STEP AHEAD FORECASTS WITH ORIGINS T=-24,-23,-22,
AND SAVE THE RESULTS IN ARRAY W
C
Z(T=50)= WW(I), Z(T=49)= WW(2) ETC
DO 115 I=1,23
   W(I)=WW(I+3)-A1*(WW(I+25)-WW(I+2))=A2*(WW(I+26)=WW(I+1))
   Z(I)=A3*(WW(I+27)-WW(I+1))=M0*(IER(I+26)-IER(I+2))
   W(I)=X(I+3)=W(I)
   WRITE(5,888)W(I),I
C
CONTINUE
DO 115 I=1,50
C
VALUE OF Z AT THE ORIGIN
Z0=640
USE ARRAY WPLT TO STORE PAST AND FORCASTED VALUES
TO BE USED LATER FOR NOISE CALCULATIONS
C
CALL FORCST(X,U,Z,Z0,WW,A1,A2,A3,B0,B1,B2)
C
SAVE THE FORECASTS TO THIS POINT IN ARRAY ZI
DO 120 I=1,23
   ZI(I)=Z(I)
C
PERCENT ERROR IN FORECASTING
DO 180 I=1,24
   IERR=Z(I)=A(I)
   IF(IERR,GT,100)IERR=20
   WRITE(5,778)ZI(I),A(I)
   FORMAT(10X,16,16)
C
IERR=I(1000,1*IER)/A(I)
C PRINT OUT DATA
C WRITE HOURS IN ARRAY IHR
DO 520 I=1,48
  IHR(I)=1
WRITE(5,409)
409 FORMAT('I')
WRITE(5,410)
410 FORMAT(55X,'LOAD PREDICTION USING BOX & JENKINS METHOD')
WRITE(91420,1)HYDRO-QUEBEC LOAD DATA FOR JANUARY 20, 1972 (IN 10MW UNITS)

K1=1
KJ=12
WRITE(5,430)(IHR(M),M=K1,KJ)
430 FORMAT(55X,'HOURS',12I4)
WRITE(5,440)(ZI(M),M=K1,KJ)
440 FORMAT(55X,'ACTUAL',12I4)
WRITE(5,450)(A(M),M=K1,KJ)
450 FORMAT(32X,'PREDICTED',12I4)
WRITE(5,460)(IER(M),M=K1,KJ)
460 FORMAT(32X,'X ERROR',12I4)
K1=K1+12
KJ=KJ+12
IF(KJ.GE.36)GOTO 81
GOTO 82
81 WRITE(5,525)
525 FORMAT(X,//)
CALL BPLT(Z1,A)

C LEAD TIME OF FORECAST
LEAD=23
C SET MEAN ERROR OF FORECAST TO ZERO
XMEAN=0
C DO 500 I=1,LEAD
  XMIN=ABS(Z1(I)-A(I))
  XMEAN=XMEAN+XMIN
CONTINUE
XMEAN=XMEAN/LEAD
C ACTUAL STANDARD DEVIATION
INITIALIZE
FSTD=0
SSTD=0
DO 502 I=1,LEAD
  FSTD=(Z1(I)-A(I))**2+FSTD
  NSTD=ABS(FSTD)-SSTD
  SSTD=SSTD/(LEAD-1)
502 CONTINUE
C RMS ERROR CALCULATION
RMS=SSTD**2+(XMEAN**2)*LEAD/(LEAD-1)
RMS=SSTD(RMS)
PEAK LOAD FOR THE DAY
PEAKL=879.0
PERCENT=(RMS-100)/PEAKLD
WRITE(5,511)
511 FORMAT(111)
512 WRITE(5,504)
504 FORMAT(10X,ERROR SUMMY FOR JANUARY 20, 1972 LOAD PREDICTION1/1)
506 FORMAT(20X,MEAN ERROR OF PREDICTION,1,E6.2)
510 WRITE(5,506)MEAN
510 WRITE(5,508)STD
508 FORMAT(10X,ACTUAL STANDARD DEVIATION=1,E6.2)
512 FORMAT(10X,1RMS ERROR PERCENT OF PEAK LOAD=1,E5.2)

STOP
END

ROUTINES CALLED:
FORCST, BPRINT, IAABS, ABS, SQRT

BLOCK LENGTH
MAIN 3250 (014544)*

**COMPILER ------ CORE**
PHASE USED FREE
DECLARATIVES 00607 00529
EXECUTABLES 00927 06209
ASSEMBLY 01937 10116
SUBROUTINE FURCST(X,U,Z,ZO,A1,A2,A3,B0,B1,B2)
SUBROUTINE USED BY BUX & JENKINS METHOD
TO COMPUTE FORECASTS FOR THE NEXT 23 HOURS WITH HOUR 0 AS ORIGIN

DIMENSION X(26),U(100),Z(30)
DIMENSION W(75)
INTEGER X,U,Z,ZO,W

COMPUTE FORECASTS

Z(1)=X(4)-A1*(Z0/10)-X(5)+A2*(X(26)-X(3))-A3*(X(25)-X(1))
1+B0*(U(27)-U(3))+B1*(U(26)-U(2))+B2*(U(25)-U(1))
1=W(1)

Z(2)=X(5)-A1*(Z(1)+X(4))-A2*((Z0/10)-X(3))-A3*(X(26)-X(2))
1+B0*(U(28)-U(4))+B1*(U(27)-U(3))+B2*(U(26)-U(2))
1=W(2)

Z(3)=X(6)-A1*(Z(2)+X(5))-A2*(Z(1)+X(4))-A3*((Z0/10)-X(3))
1+B0*(U(29)-U(5))+B1*(U(28)-U(4))+B2*(U(27)-U(3))
1=W(3)

DO 140 1=4,23

Z(1)=X(1+3)-A1*(Z(1+1)-X(1+2))+A2*(Z(1+2)-X(1+1))
1=A3*(Z(1+3)-X(1))+B0*(U(1+2)+U(1+25)-U(1+2))+B1*(U(1+26)-U(1+2))
1=B2*(U(1+24)+U(1))
1=W(1)

140 CONTINUE
RETURN
END

BLOCK LENGTH
FURCST 553  (002122)*

**COMPILER ----- CORE**
PHASE USED FREE
DECLARATIVES 00366 0770
EXECUTABLES 00601 08529
ASSEMBLY 01065 10908
SUBROUTINE BPL0T(ZI,A)
DIMENSION A(75),ZI(75),NPL0T(100)
INTEGER ZI,A,
C PLOTTING ROUTINE:
WRITE(5,220)
   FORMAT(11)
   DO 200 I=1,100,
200   NPL0T(I)=1 (!*1
WRITE(5,300) NPL0T
   FORMAT(25X,100A1)
   DO 210 I=1,100
210   NPL0T(I)=1
N=1
J=ZI(N)
J=ABS(J)
J=J/10
NK=A(N)/10
IF(J.GT.100) J=100
IF(NK.GT.100) NK=99
IF(J.LE.0) J=1
NPL0T(NK)=1A1
NPL0T(J)=1A1
NPL0T(I)=1
WRITE(5,310) NPL0T
   FORMAT(20X,100A1)
   NPL0T(J)=1
   NPL0T(NK)=1
N=N+1
IF(N.NE.26) GOTO 90
RETURN
END

ROUTINES CALLED:
1ABS

BLOCK LENGTH
BPL0T 543 (002076) *

**COMPILER ------ CORE**

PHASE USED FREE
DECLARATIVES 00446 08690
EXECUTABLES 00607 06529
ASSEMBLY 01169 10884
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Fig. C-1 — Line Printer Plot of Prediction Obtained from Periodic and Residual Model.
Electrical Load (in 10MW Units)

A = actual
+ = predicted

Lead Time Prediction

Fig. C-2 — Line Printer Plot of Prediction Obtained From Residual Model.
### RESIDUAL COMPONENT PARAMETERS

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Table C-1 — Model Parameters.