REALIZATION OF A CLASS OF SYMMETRICAL
LATTICE AND BRIDGED-T
TWO VARIABLE REACTANCE NETWORKS

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HARNAM SINGH TREHIN

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REALIZATION OF A CLASS OF SYMMETRICAL LATTICE AND BRIDGED-T TWO VARIABLE REACTANCE NETWORKS

HARNAM SINGH TREHIN

ABSTRACT

This report considers a class of two variable reactance networks of the type symmetrical lattice and bridged-T. In the case of symmetrical lattice it is shown that it is possible to have constant resistance symmetrical lattice in two variable provided the impedances are functions of both the variables. In the case of bridged-T networks it is not possible to have the constant resistance property in two variables. However, a special subclass of bridged-T-networks constitute a constant resistance network in single variable only when the other variable is made equal to unity.
Dedicated to the memory of my Father

S. Swaran Singh Trehin
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PART I

INTRODUCTION

1.1 GENERAL

Linear, lumped, finite, passive networks may be completely characterized by rational functions of the complex frequency variable $s$. A great deal of literature is available on the properties and realization of such network functions [1]. However, if we allow the network to include distributed elements as well as lumped elements, the network functions are in general no longer rational in $s$. Such mixed lumped-distributed structures are highly desirable as may be seen from the following discussion.

1.2 ADVANTAGES OF MIXED LUMPED-DISTRIBUTED STRUCTURES

The following are some of the advantages of mixed lumped distributed networks:

(a) In the case of mixed lumped-distributed structure allowances may be made for the parasitics in the terminating impedance, i.e. the terminations need not be purely resistive.

(b) A conventional quarter-wave transformer gives small or no attenuation at the higher harmonics. On the other hand, the mixed dis-
tributed impedance transformer can be designed to have the properties of an impedance transformer and a low pass filter. This property is useful where combined filtering and impedance transformation is desirable.

(c) In the case of comb-line filter, lumped capacitive coupling at the input and output reduces the filter size by eliminating the transmission line matching section.

(d) In the case of cascaded unit element filters, the number of UE's can be reduced from \((2n + 1)\) to \(n\), if the cascaded lines are separated by \((n + 1)\) lumped capacitors.

1.3 Linear-Lumped-Finite-Passive Networks

The driving point immittance functions for networks consisting of lumped, linear, finite, passive, bilateral elements may be expressed as rational functions of the complex frequency variables, that is, they may be written as:

\[
Z(s) = \frac{p(s)}{q(s)} = \frac{a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n}{b_0 s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m} \ldots 1.2-1
\]

and can be put in the form

\[
Z(s) = \frac{a_0}{b_0} \frac{(s - z_1)(s - z_2) \ldots (s - z_n)}{(s - p_1)(s - p_2) \ldots (s - p_m)} ; |m-n| \leq 1 \ldots 1.2-2
\]

where the \(z_n\)'s are the zeros and the \(p_m\)'s are the poles of \(Z(s)\) and the coefficients \(a_i\)'s and \(b_i\)'s \((i = 0, 1, 2 \ldots)\) are positive real constants. It is also noted that both \(p(s)\) and \(q(s)\) are Hurwitz polynomials, i.e. they contain zeros only in the closed left half of s-plane.
On the other hand, we can also say that only positive real functions can be realized as a LFP Networks.

1.4 **MIXED LUMPED-DISTRIBUTED NETWORKS**

However, as pointed out earlier, if the network consists of distributed elements as well as lumped elements, its driving point immittance need not be a rational function of $s$.

The realization methods available in the lumped network theory are not directly applicable in mixed distributed case due to the transcendental nature of the network functions.

Two different approaches are followed to solve the realization problem of these mixed lumped distributed networks. One of them directly deals with the transcendental functions which are termed as the single variable approach. In the other approach the transcendental functions of $s$ are converted into polynomial functions of several variables $p_i$. This is called the multivariable approach. This is illustrated by the following example.

Let us consider the network shown in Fig. 1.4-A. This network consists of resistors, capacitors and commensurate lossless transmission lines (unit elements).

The chain matrix of a single unit element is of the form:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} = 
\begin{bmatrix}
\cosh s\tau & z_0 \sinh s\tau \\
\frac{1}{z_0} \sinh s\tau & \cosh s\tau
\end{bmatrix}
\]

where $\tau$ and $z_0$ are constants.
Now the overall 'ABCD parameters of the network becomes:

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
1 & R_1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
\cosh \sigma T & z_0 \sinh \sigma T \\
\frac{1}{z_0} \sinh \sigma T & \cosh \sigma T
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
SC_1 & 1
\end{bmatrix}
\begin{bmatrix}
\cosh \sigma T & z_0 \sinh \sigma T \\
\frac{1}{z_0} \sinh \sigma T & \cosh \sigma T
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
SC_2 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{1}{R_2} & 1
\end{bmatrix}
\] .......................... 1.4-2

The input impedance at points \( x - x' \) is

\[
Z_{in} = \frac{A}{C}
\]

\[
= 1 \left( \frac{(1+s^2 \cdot \tanh^2 \sigma T + z_0^2 c^2 + 3 z_0 c_i \tanh \sigma T + 2/R_2 \cdot z_0 \tanh \sigma T)}{(1-\tanh^2 \sigma T)(z_0 c_i s^2 \tanh \sigma T + c_i \tanh^2 \sigma T + 2 c_i s)} \right)
\]

\[
+ \frac{c_i z_0^2 \cdot \tanh^2 \sigma T + 1) + R_1(z_0 c_i^2 \tanh \sigma T + c_i \tanh^2 \sigma T + 2 c_i s)}{R_2 \tanh \sigma T + 1/\tanh \sigma T + 1/R_2 \tanh^2 \sigma T + 1/R_2}
\]

\[
+ \frac{z_0 c_i \tanh \sigma T + 2/z_0 \cdot \tanh \sigma T + 1/R_2 \cdot \tanh^2 \sigma T + 1/R_2}{R_2 \tanh \sigma T + 1/\tanh \sigma T + 1/R_2 \tanh^2 \sigma T + 1/R_2}
\]

Now by defining

\[
p_1 = s \quad \text{and} \quad p_2 = \tanh \sigma T
\]

and substituting the new variables \( p_1 \) and \( p_2 \) in (1.4-2) we have --

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} =
\begin{bmatrix}
1 & R_1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & z_0 p_2 \\
1 - p_2^2 & \frac{P_2}{z_0}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
SC_1 & 1
\end{bmatrix}
\begin{bmatrix}
1 & z_0 p_2 \\
1 - p_2^2 & \frac{P_2}{z_0}
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
SC_1 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
1/R_2 & 1
\end{bmatrix}
\]

.................. 1.4-5
Network consists of resistors, capacitors and commensurate lossless transmission lines.

Fig. 1.4-2

Typical sections for two variable reactance function.
also (1.4-3) can be put in terms of new variables \( p_1 \) and \( p_2 \) as
\[
Z_{in} = \frac{A}{C} = \frac{1}{1-p_2^2} \cdot \frac{X + R_1 Z}{Z} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad 1.4-6
\]

The \( X \) and \( Z \) in 1.4-6 are given by
\[
X = 1 + p_1^2 \cdot p_2^2 \cdot c_1^2 + 3z_0c_1p_1p_2 + \frac{2}{R_2} \cdot z_0p_2 + \frac{C_1Z_0^2}{R_2} \quad p_1^2 + p_2^2 + 1 \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad 1.4-7
\]
and
\[
Z = z_0c_1^2p_2p_1^2 + c_1p_2^2p_1 + 2c_1p_1 + \frac{Z_0C_1}{R_2} \cdot p_1 \cdot p_2 + \frac{2}{Z_0} \cdot p_2 + \frac{1}{R_2} \cdot p_2^2 + \frac{1}{R_2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad 1.4-8
\]

We notice here that it has been possible to express the input impedance as a rational function of the two variables \( p_1 \) and \( p_2 \). This approach may be extended to interconnections of non-commensurate transmission lines also.

Common examples of circuits containing both lumped and distributed elements are networks containing semiconductor elements and transmission lines or wave guides or indeed networks of transmission lines alone where lumped discontinuities inevitably occur.

1.5 Definitions

Some accepted definitions of multivariable network functions are given below [2].

**Definition 1**

A rational function \( F(p_1, p_2, \ldots, p_n) \) of \( n \) complex variables \( p_1, p_2, \ldots, p_n \) is called a Multivariable Positive Real Function (MPRF) when the following conditions are satisfied:

1) \( F(p_1, p_2, \ldots, p_n) \) is a real function of \( p_1, p_2, \ldots, p_n \)
and

11) \( \text{Re} \, F(p_1, p_2, \ldots, p_n) > 0 \) in the polydomain \( \text{Rep}_i > 0 \)

where \( i = 1, 2, \ldots, n \)

**Definition 2**

A rational function \( F(p_1, p_2, \ldots, p_n) \) is called a Multivariable Reactance Function (MRF) when the following conditions are satisfied:

1. \( F(p_1, p_2, \ldots, p_n) \) is an MPRF

and

11) \( F(p_1, p_2, \ldots, p_n) = -F(-p_1, -p_2, \ldots, -p_n) \)

**Definition 3**

A polynomial of \( n \)-complex variables \( p_1, p_2, \ldots, p_n \) is called a multivariable Hurwitz polynomial in the narrow sense (MHPN) if it has no zeros in the regions:

\( \text{Re}p_1 > 0, \ldots, \text{Re}p_{i-1} > 0, \text{Re}p_i > 0, \text{Re}p_{i+1} > 0, \ldots, \text{Re}p_n > 0, \)

for all \( i(1 < 1 < n) \)

**Definition 4**

A polynomial of \( n \)-complex variables \( p_1, p_2, \ldots, p_n \) is called Multivariable Hurwitz Polynomial in the broad sense (MHPB) if it has no zeros in the open polydomain \( \text{Re} \, p_i > 0 \), and if those zeros for \( \text{Re} \, p = 0 \) are simple.
1.6 Some Similarities and Dissimilarities Between SRF's and MRF's [3]

In Section 1.5 some definitions of multivariable reactance functions has been given. We observe the following similarities and dissimilarities between the above definitions and the definition of a single variable reactance conversion and these have been very well discussed. [3]

In particular it can easily be seen that these definitions are analogous to the definitions of similar functions in the single variable case. Hence, these definitions can be considered as logical extensions of those in the single variable case.

It is also known that a reactance function of a single variable (SRF) can always be synthesized using a minimum number of elements. This number is equal to the order of the function. These different canonic structures are due to Foster [1], Cauer [1], Lee [1, 5], Kida [6], Ramachandran and Swamy [7]. This however, may not be true in the case of two variable reactance functions (TRF).

In general, the two variable reactance functions require either ideal transformers or passive ideal gyrators for their realization [8, 9]. However, some work has been done regarding the realizations of some classes of TRF's without transformers or gyrators [2, 10, 11].

It has been shown that there exist a class of TRF's which can be decomposed into a sum of two single variable reactance functions [12, 13, 14, 15]. Specifically, conditions have been obtained to decompose \( Z(p_1, p_2) \) as the sum of \( Z_1(p_1) \) and \( Z_2(p_2) \) where \( Z_1(p_1) \) is a single variable reactance function in \( p_1 \) and \( Z_2(p_2) \) is a single variable reactance function in \( p_2 \).
Each function can be realized by the known methods of a single variable reactance function.

Also necessary and sufficient conditions have been obtained for the realization of TRF's in a form similar to the Foster form consisting of one element of variable \( p_1 \) (an inductor or a capacitor), and the other element of variable \( p_2 \) (a capacitor or inductor) [10], and a typical section is as shown in Fig. 1.2. For this type of realization, it is shown that if the Foster form exists, other forms similar to the single variable canonic structures exist. In addition, it is also known that if one form exists, the other form need not exist [16]. Hence, in the case of two-variable reactance functions, it is worthwhile to consider specific structures and their properties.

1.7 Scope of the Report

This report studies the cases of two variable symmetrical lattice and bridged-T network structures. These structures have not been examined in the literature so far to the author's knowledge. Part II discusses symmetrical lattice in two variables. Part III discusses Bridged-T networks, and Part IV discusses the conclusion arising from this report.
PART II

TWO VARIABLE SYMMETRICAL LATTICE REACTANCE NETWORKS

2.1 Introduction

As stated earlier, we shall consider two variable symmetrical lattice reactance networks in this chapter. Fig. 2.1-1 shows a symmetrical lattice network where the branch impedances \( z_a \) and \( z_b \) are reactance functions of \( p_1 \) and \( p_2 \). First, we shall consider that any impedance contains one element of one variable only. The two possibilities arising out of the above constraint is shown in Fig. 2.1-2.

2.2 Properties of Symmetrical Lattice in Two Variables

Here the properties of the symmetrical lattice will be examined. For the network shown in Fig. 2.1-1, the open circuit impedance functions are:

\[
Z_{11}(p_1, p_2) = Z_{22}(p_1, p_2) = \frac{1}{2} \left( z_a + z_b \right) \quad \ldots \quad 2.2-1
\]

and

\[
Z_{12}(p_1, p_2) = \frac{1}{2} \left( z_b - z_a \right) \quad \ldots \quad 2.2-2
\]

Also, the short circuit admittance functions are:

- 10 -
A Symmetrical Lattice Network
(The impedances $z_a$ and $z_b$ are reactance functions of either $p_1$ or $p_2$)
Fig. 2.1-2

Symmetrical Lattice in Two Variables - when each impedance is the function of one variable.
\[ Y_{11}(p_1, p_2) = Y_{22}(p_1, p_2) = \frac{1}{2z_a \cdot z_b} (z_a + z_b) \]  \hspace{1cm} 2.2-3

\[ Y_{12}(p_1, p_2) = \frac{1}{2z_a \cdot z_b} (z_a - z_b) \]  \hspace{1cm} 2.2-4

The different possibilities of \( z_a \) and \( z_b \) containing one element of one variable only, and the corresponding \( Z \) parameters with structures are shown in Table 2.2-I.

In these networks if we put \( p_1 = 1 \) or \( p_2 = 1 \) there result a general RL or RC network in a single variable either in \( p_1 \) or \( p_2 \). However, after putting \( p_1 = 1 \) (or \( p_2 = 1 \)) and considering only one type of reactive element in the other variable (say only capacitance or inductance) there result two element kind network functions.

Utilizing the technique of putting one of the variables equal to unity at a time and examining the properties of the resulting networks, we can classify the above networks (shown in Table 2.2-I) as follows:

1) Type \( SL_1 \): The network consisting of \( p_1 \) type capacitors and \( p_2 \) type inductors. In this category if we put \( p_1 = 1 \) the resulting network is RL type whereas if \( p_2 = 1 \) the resulting network is RC type.

11) Type \( SL_2 \): The network consisting of \( p_1 \) type inductors and \( p_2 \) type capacitors. In this category if \( p_1 = 1 \) the resulting network is RC type and if \( p_2 = 1 \) the resulting network is of RL type.
Table 2.2-1 — Symmetrical Lattice Structures of the Kind SL₂, SL₃, SL₄, and SL₅ with Z₁₁ and Z₁₂ in two Variables (p₁,p₂) and one variable (p₁,p₂) and (p₁,1).
Table 2.2-1 Cont'd -- $Z_{11}$ and $Z_{12}$ of the Symmetrical Bridged-T of Type ST₂.
\[ Z_{11}(P_1, P_3) = Z_{11}(P_1, P_2) \]
\[ Z_{11}(L_2) = Z_{11}(L_3) = \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]
\[ Z_{11}(P_1, L) = Z_{11}(P_1, L_2) + \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

\[ Z_{11}(P_1, L_3) = Z_{11}(P_1, L_2) + \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

\[ Z_{11}(L_2) = Z_{11}(L_3) = \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

\[ Z_{11}(P_1, L_3) = Z_{11}(P_1, L_2) + \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

\[ Z_{11}(P_1, L_3) = Z_{11}(P_1, L_2) + \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

\[ Z_{11}(P_1, L_3) = Z_{11}(P_1, L_2) + \frac{1}{2} \left( \frac{1}{P_1 P_2} \right) \]

Table 2.2-1 Cont'd.
iii) Type SL₃: The network consisting of capacitors of the type \( p_1 \) and \( p_2 \). In this category if \( p_1 = 1 \) or \( p_2 = 1 \) the resulting network is \( Rc \) type.

iv) Type SL₄: The network consisting of inductors of the type \( p_1 \) and \( p_2 \). In this category if \( p_1 = 1 \) or \( p_2 = 1 \), the resulting network is of the \( RL \) type.

As already noted, these four types of networks with their \( Z \) parameters are shown in Table 2.2-I. Some general properties of the above symmetrical lattice networks can be observed:

i) The poles of \( Z \) matrix (\( y \)-matrix) with \( p_1 \) or \( p_2 = 1 \) shall be simple and lie on the \(-ve\) real axis of the variable \( p_2 \) or \( p_1 \) respectively.

ii) The zeros of \( z_{11} \) and \( z_{22} \) (\( y_{11} \) and \( y_{22} \)) with \( p_1 \) or \( p_2 = 1 \) shall be simple and lie on the \(-ve\) real axis of the variable \( p_2 \) or \( p_1 \) respectively.

iii) None of these networks with \( p_1 = 1 \) or \( p_2 = 1 \) can give a constant resistance network in the single variable.

2.3 Constant Resistance Symmetrical Lattice

Till now we have discussed the lattice network without any termination. In this section we shall study the symmetrical lattice terminated in a one ohm (1.0 Ω) load as shown in Fig. 2.3-1.

We know from analysis that the input impedance is equal to one ohm (1.0 Ω) if
Two Variable Symmetrical Lattice Terminating in One Ohm Load.
$$z_a \cdot z_b = 1 \quad \quad \quad \quad \quad \quad \quad \quad 2.3-1$$

If \( z_a = f(p_1) \) and \( z_b = g(p_2) \) the above condition cannot be satisfied. This means that the existence of a constant-resistance symmetrical lattice in two variables is not possible, when any branch impedance is a function of one of the variables only, while the other branch is a function of the other variable only. However, constant resistance symmetrical lattice network is possible when any branch impedance is a function of both variables, as can be seen from the example.

Example - If

$$z_a = \frac{1}{\alpha_1 p_1 + \alpha_2 p_2} \quad \quad \quad \quad \quad \quad \quad \quad 2.3-2$$

and

$$z_b = (\alpha_1 p_1 + \alpha_2 p_2) \quad \quad \quad \quad \quad \quad \quad \quad 2.3-3$$

The corresponding network is shown in Fig. 2.3-2. Since there are a large number of networks possible no attempt is made to exhaust all possibilities.

2.4 Synthesis of a Symmetrical Lattice Where One Arm Contains, One Element in \( p_1 \), Only and Other Arm Contains One Element in \( p_2 \) only

The foregoing discussion enables us to obtain the conditions under which a symmetrical lattice network can be synthesized.

Putting \( p_1 = 1 \) or \( p_2 = 1 \) the following shall be satisfied:

**Type SL1:** The conditions to be satisfied are:

1) \( Z_{11}(p_1, 1) \) is \( Z_{RL}(p_1) \) of the form

$$Z_{11}(p_1, 1) = \frac{K_0}{p_1} + K_\infty$$
\[
\frac{1}{a_{1p1} + a_{2p2}}
\]

Fig. 2.3-2

Two Variable Constant Resistance Symmetrical Lattice; terminating in One Ohm Load.
11) \( Z_{11}(1, p_2) \) is \( Z_{RL}(p_2) \) of the form

\[ Z_{11}(1, p_1) = k_0 + k_\infty p_1 \]

and the poles and zeros of \( Z_{11}(p_1, 1) \) and \( Z_{11}(1, p_2) \) shall be the same.

111) \( Z_{12}(p_1, 1) \) shall have zeros of transmission on the positive real axis and these shall be the same as the zeros of \( Z_{12}(1, p_2) \).

Type SL₂: The conditions are:

1) \( Z_{11}(p_1, 1) \) is \( Z_{RL}(p_1) \) of the form

\[ Z_{11}(p_1, 1) = k_0 + k_\infty p_1 \]

11) \( Z_{11}(1, p_2) \) is \( Z_{RC}(p_2) \) of the form

\[ Z_{11}(1, p_2) = \frac{k_0}{p_2} + k_\infty \]

and the poles and zeros of \( Z_{11}(p_1, 1) \) and \( Z_{11}(1, p_2) \) shall be the same.

111) \( Z_{12}(p_1, 1) \) shall have zeros of transmission on the positive real axis and these shall be the same as the zeros of \( Z_{12}(1, p_2) \).

Type SL₃: 1) \( Z_{11}(p_1, 1) \) is \( Z_{RC}(p_1) \) of the form

\[ Z_{11}(p_1, 1) = \frac{k_{10}}{p} + k_2 \]

11) \( Z_{11}(1, p_2) \) is \( Z_{RC}(p_2) \) of the form

\[ Z_{11}(1, p_2) = k_{10} + k_{20} / p_2 \]
iii) \( Z_{12}(p_1, 1) \) shall have zeros of transmission on the positive real axis on \( p_1 \) plane.

iv) \( Z_{12}(1, p_2) \) shall have zeros of transmission on the positive real axis on \( p_2 \) plane and shall be the reciprocal of the zeros of \( Z_{12}(p_1, 1) \).

Type SL4: The conditions for this type of network is as follows:

1) \( Z_{11}(p_1, 1) \) is \( Z_{RL}(p_1) \) of the form
   \[
   Z_{11}(p_1, 1) = k_1 p_1 + k_2 \omega
   \]

2) \( Z_{12}(1, p_2) \) is \( Z_{RL}(p_2) \) of the form
   \[
   Z_{12}(1, p_2) = k_3 p_2 + k_4 \omega
   \]

iii) \( Z_{12}(p_1, 1) \) shall have zeros of transmission on the positive real axis on the \( p_1 \) plane.

iv) \( Z_{12}(1, p_2) \) shall also have zeros of transmission on the positive real axis on the \( p_2 \) plane and shall be the reciprocal of the zeros of \( Z_{12}(p_1, 1) \).

Hence when the specifications are given to realize \( Z_{11}(p_1, p_2) \) and \( Z_{12}(p_1, p_2) \) which satisfies any one set of the conditions given in types SL1, SL2, SL3 and SL4, they can be synthesized using single variable techniques [1], and replacing the resistance by inductance or capacitance as given below:
Type SL₁: 1) If \( Z_{11}(p_1, 1) \) is made use of, \( R \) is replaced by the inductance in \( p_2 \).

11) If \( Z_{11}(1, p_2) \) is made use of, \( R \) is replaced by the capacitance in \( p_1 \) plane.

Type SL₂: 1) If \( Z_{11}(p_1, 1) \) is made use of, \( R \) is replaced by capacitance in \( p_2 \) plane.

11) If \( Z_{11}(1, p_2) \) is made use of, \( R \) is replaced by the inductance in \( p_1 \) plane.

Type SL₃: 1) If \( Z_{11}(1, p_2) \) or \( Z_{11}(p_1, 1) \) is made use of, \( R \) is replaced by capacitance in \( p_1 \) or \( p_2 \) plane respectively.

Type SL₄: 1) If \( Z_{11}(1, p_2) \) or \( Z_{11}(p_1, 1) \) is made use of, \( R \) is replaced by inductance in \( p_1 \) or \( p_2 \) plane respectively.

2.5 Synthesis of a Class of Two-Variable Symmetrical Lattice

From the foregoing discussion, it can be observed that a class of two variable symmetrical lattice can also be realized following the same technique. The conditions are as given below:

Type SL₁: The conditions to be satisfied are:

1) \( Z_{11}(p_1, 1) \) is \( Z_{RC}(p_1) \) of the form

\[
Z_{11}(p_1, 1) = K_0 + \sum \frac{K_i}{p_1 + \alpha_i} + \frac{K_0}{p_1} \quad \ldots \quad 2.5-1
\]
11) \( Z_{11}(1, p_2) \) is \( Z_{RL}(p_2) \) of the form

\[
Z_{11}(1, p_2) = K_0 p_2 + \sum \frac{K_1 p_2}{p_2 + \sigma_i} + K_0 \quad \cdots \cdots \quad 2.5-2
\]

The poles and zeros of \( Z_{11}(p_1, 1) \) and \( Z_{11}(1, p_2) \) shall be the same.

111) \( Z_{12}(p_1, 1) \) expanded in partial fraction shall have the same residue as the residue of \( Z_{12}(1, p_2) \), i.e.

\[
Z_{12}(p_1, 1) = \sum \frac{\alpha_j}{p_1 + \sigma_j} - \sum \frac{\beta_k}{p_1 + \sigma_k} \pm \alpha_0 \pm \frac{\alpha_0}{p_1} \quad \cdots \cdots \quad 2.5-3
\]

and

\[
Z_{12}(1, p_2) = \sum \frac{\alpha_j p_2}{p_2 + \sigma_j} - \sum \frac{\beta_k p_2}{p_2 + \sigma_k} \pm \alpha_0 p_2 \pm \frac{\alpha_0}{p_2} \quad \cdots \cdots \quad 2.5-4
\]

\[\text{TYPE SL}_2: \] The conditions are:

1) \( Z_{11}(p_1, 1) \) is \( Z_{RL}(p_1) \) of the form

\[
Z_{11}(p_1, 1) = K_0 p_1 + \sum \frac{K_1 p_1}{p_1 + \sigma_i} + K_0 \quad \cdots \cdots \quad 2.5-5
\]

11) \( Z_{11}(1, p_2) \) is \( Z_{RC}(p_2) \) of the form

\[
Z_{11}(1, p_2) = K_0 + \sum \frac{K_1}{p_2 + \sigma_i} + \frac{K_0}{p_2} \quad \cdots \cdots \quad 2.5-6
\]

And the poles and zeros of \( Z_{11}(p_1, 1) \) and \( Z_{11}(1, p_2) \) shall be the same.

111) \( Z_{12}(p_1, 1) \) expanded in partial fraction expansion shall have the same residues as the residues of \( Z_{12}(1, p_2) \), i.e.

\[
Z_{12}(p_1, 1) = \sum \frac{\alpha_j p_1}{p_1 + \sigma_j} - \sum \frac{\beta_k p_1}{p_1 + \sigma_k} \pm \alpha_0 p_1 \pm \frac{\alpha_0}{p_1} \quad \cdots \cdots \quad 2.5-7
\]

and:

\[
Z_{12}(p_1, 1) = \sum \frac{\alpha_j p_2}{p_2 + \sigma_j} - \sum \frac{\beta_k p_2}{p_2 + \sigma_k} \pm \alpha_0 \pm \frac{\alpha_0}{p_2} \quad \cdots \cdots \quad 2.5-8
\]
TYPE SL₃: The conditions are:

1) \( Z_{11}(p_1,1) \) is \( Z_{RC}(p_1) \) of the form:

\[
Z_{11}(p_1,1) = \frac{K_{1e}}{p} + \sum \frac{k_i}{p_1 + \sigma_i} + K_{20} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-9
\]

11) \( Z_{11}(1,p_2) \) is also \( Z_{RC}(p_2) \) of the form

\[
Z_{11}(1,p_2) = k_1 \sigma + \sum \frac{k_i}{p_2 + \sigma_i} + K_{20} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-10
\]

111) \( Z_{12}(p_1,1) \) expanded in partial fraction expansion shall have the same residues as the residues of \( Z_{12}(1,p_2) \), i.e.

\[
Z_{12}(p_1,1) = \pm \alpha_{2e} \pm \frac{\alpha_{1e}}{p_1} + \sum \frac{\alpha_i}{p_2 + \sigma_j} - \sum \frac{v_k}{p_1 + \sigma_k} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-11
\]

and

\[
Z_{12}(1,p_2) = \pm \frac{\alpha_{2e}}{p_2} \pm \frac{\alpha_{1e}}{p_2} + \sum \frac{\alpha_i}{p_2 + \sigma_j} - \sum \frac{v_k}{p_1 + \sigma_k} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-11
\]

shall be the reciprocal of the zeros of \( Z_{12}(p,1) \).

TYPE SL₄: The condition for this type of network is as follows:

1) \( Z_{11}(p_1,1) \) is \( Z_{RL}(p_1) \) of the form

\[
Z_{11}(p_1,1) = K_0 p_1 + \sum \frac{k_i p_1}{p_1 + \sigma_i} + K_{2\infty} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-12
\]

11) \( Z_{11}(1,p_2) \) is \( Z_{RL}(p_2) \) of the form

\[
Z_{11}(1,p_2) = k_\infty + \sum \frac{k_i p_2}{p_2 + \sigma_i} + K_{2\infty} \quad \quad \quad \quad \quad \quad \quad \quad \quad 2.5-13
\]

111) \( Z_{12}(p_1,1) \) expanded in partial fraction expansion shall have the same residues as the residues of \( Z_{12}(1,p_2) \), i.e.
### Table 2.5-1: Forms of the Reactance Functions in two Variables ($p_1, p_2$) and Single Variables ($p_2$) and ($p_1, 1$) for symmetrical lattice of the kind $SL_1$, $SL_2$, $SL_3$, and $SL_4$.

<table>
<thead>
<tr>
<th>KIND OF NETWORK CLASS</th>
<th>FORMS OF REACTANCE FUNCTIONS IN TWO VARIABLES ($p_1, p_2$)</th>
<th>FORMS OF REACTANCE FUNCTION IN VARIABLE $p_1$</th>
<th>FORMS OF REACTANCE FUNCTION IN VARIABLE $p_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SL_2$</td>
<td>$X_{11}(p_1, p_2) = K_e p_2 + \sum \frac{K_p}{p_1 + p_2} + K_e$</td>
<td>$Z_{11}(p_1, 1) = Z_e - \sum \frac{K_p}{p_1 + p_1} + K_e$</td>
<td>$Z_{11}(p_2, 1) = K_e p_2 + \sum \frac{K_p}{p_2 + p_1} + K_e$</td>
</tr>
<tr>
<td></td>
<td>$Z_{12}(p_1, p_2) = \sum \frac{K_p}{p_1 + p_2} + \sum \frac{K_p}{p_1 + p_2} + K_e$</td>
<td>$Z_{12}(p_1, 1) = \sum \frac{K_p}{p_1 + p_1} + \sum \frac{K_p}{p_1 + p_1} + K_e$</td>
<td>$Z_{12}(p_2, 1) = \sum \frac{K_p}{p_2 + p_1} + \sum \frac{K_p}{p_2 + p_1} + K_e$</td>
</tr>
</tbody>
</table>

| $SL_2$               | $X_{11}(p_1, p_2) = K_e p_2 + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{11}(p_1, 1) = Z_e - \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{11}(p_2, 1) = K_e p_2 + \sum \frac{K_p}{p_2 + p_1} + K_e$ |
|                      | $Z_{12}(p_1, p_2) = \sum \frac{K_p}{p_1 + p_2} + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{12}(p_1, 1) = \sum \frac{K_p}{p_1 + p_1} + \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{12}(p_2, 1) = \sum \frac{K_p}{p_2 + p_1} + \sum \frac{K_p}{p_2 + p_1} + K_e$ |

| $SL_3$               | $X_{11}(p_1, p_2) = K_e p_2 + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{11}(p_1, 1) = Z_e - \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{11}(p_2, 1) = K_e p_2 + \sum \frac{K_p}{p_2 + p_1} + K_e$ |
|                      | $Z_{12}(p_1, p_2) = \sum \frac{K_p}{p_1 + p_2} + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{12}(p_1, 1) = \sum \frac{K_p}{p_1 + p_1} + \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{12}(p_2, 1) = \sum \frac{K_p}{p_2 + p_1} + \sum \frac{K_p}{p_2 + p_1} + K_e$ |

| $SL_4$               | $X_{11}(p_1, p_2) = K_e p_2 + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{11}(p_1, 1) = Z_e - \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{11}(p_2, 1) = K_e p_2 + \sum \frac{K_p}{p_2 + p_1} + K_e$ |
|                      | $Z_{12}(p_1, p_2) = \sum \frac{K_p}{p_1 + p_2} + \sum \frac{K_p}{p_1 + p_2} + K_e$ | $Z_{12}(p_1, 1) = \sum \frac{K_p}{p_1 + p_1} + \sum \frac{K_p}{p_1 + p_1} + K_e$ | $Z_{12}(p_2, 1) = \sum \frac{K_p}{p_2 + p_1} + \sum \frac{K_p}{p_2 + p_1} + K_e$ |
\[ Z_{12}(p_1,1) = \alpha_2 \omega - \alpha_1 \omega p_1 + \sum \frac{\alpha_j p_1}{p_1 + \sigma_j} \quad \cdots \cdots \cdots \cdots \quad 2.5-14 \]

and

\[ Z_{12}(1,p_2) = \alpha_2 \omega p_2 - \alpha_1 \omega + \sum \frac{\alpha_j p_2}{p_2 + \sigma_j} \quad \cdots \cdots \cdots \cdots \quad 2.5-15 \]

The forms of \( Z_{11} \) and \( Z_{12} \) are given in Table 2.5-I.

In all the above cases, we can determine \( Z_b \) and \( Z_a \) by (2.2-1) and (2.2-2) with the condition that \( Z_{12} \) is realizable within a multiplicative constant which is determined by making \( Z_b \) and \( Z_a \) physically realizable. It should also be noted that types SL_1 and SL_2 can realize constant-resistance networks. Types SL_3 and SL_4 cannot realize constant resistance networks.

Example:

\[ Z_{11}(p, p) = \left[ \frac{1 + 32p_1^2p_2^2 + 8p_1p_2 + 16p_1p_2 + 5p_1 + 8p_1p_2}{2p_1 + 8p_1p_2 + 4p_1 + 8p_1p_2 + 5p_1 + 8p_1p_2} \right] \]

and

\[ Z_{12}(p, p) = \left[ \frac{-1 + 4p_1^2 + 8p_1p_2 + 16p_1^2p_2 + 3p_1^2 - 8p_1p_2}{2p_1 + 8p_1p_2 + 4p_1 + 8p_1p_2 + 32p_1p_2 + 16p_1p_2} \right] \]

using synthesis technique developed earlier we get the network as shown in Fig. 2.6-1.
Fig. 2.6-1

Constant Resistance Symmetrical of the Example Problem.
2.7 Summary and Discussions

From the study of symmetrical lattice in two variables it can easily be concluded that when \( z_a \) and \( z_b \) are functions of one variable only, it cannot constitute a constant resistance network. However, if they are functions of both variables, it can constitute a constant resistance symmetrical lattice.

For the purpose of this report this class of lattice has been divided into four categories, their properties and synthesis discussed separately.

If \( Z_{11} \) and \( Z_{12} \) are given, if they meet the conditions set for the type \( SL_1 \), they will also meet the conditions set for \( SL_2 \) but for a phase reversal. But if the given specifications meet the conditions set for \( SL_3 \) or \( SL_4 \), then they will not meet any other set of specifications.
PART III

TWO VARIABLE BRIDGED-T REACTANCE NETWORKS

3.1 INTRODUCTION

As stated earlier, we shall consider two variable bridged-T reactance networks in this chapter. Fig. 3.1-1 shows a bridge-T network where the impedances $Z_1, Z_2, Z_3$ and $Z_4$ are reactance functions of $p_1$ and $p_2$. There are a number of possibilities. However, we shall consider that any impedance to be a function of either $p_1$ or $p_2$ only. The different possibilities arising out of the above constraint are shown in Table 3.1-1. As can be seen, there are only three main categories:

(i) three impedances of $p_1$ and one impedance of $p_2$

(ii) three impedances of $p_2$ and one impedance of $p_1$

(iii) two impedances of $p_1$ and two impedances of $p_2$

In the above three categories, if we put $p_1 = 1$ or $p_2 = 1$ there results a general R-L-C network in a single variable either in $p_2$ or in $p_1$. However, after putting $p_1 = 1$ (or $p_2 = 1$) and considering only one type of reactive element in the other variable (capacitances or inductances), there results two-element kind network.

3.2 PROPERTIES OF BRIDGED-T NETWORK IN TWO VARIABLES

Here the properties of the bridged-T network will be examined.
Fig. 3.1-1

Typical Bridged-T Network in Two Variables.

(The impedances \( z_1, z_2, z_3 \) and \( z_4 \) are reactance functions of either \( p_1 \) or \( p_2 \))
Table 3.1-1

Total Possible Cases of Bridged-T Networks in Two Variables. When each impedance is the function of one variable only.
The network is shown in the previous section in Fig. 3.1. The corresponding $Z_{11}$, $Z_{22}$, $Z_{12}$ and $Y_{11}$, $Y_{22}$, $Y_{12}$, are:

\[ Z_{11}(p_1, p_2) = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_4 + Z_2Z_3 + Z_3Z_4}{Z_1 + Z_2 + Z_3} \]  \hspace{1cm} (3.2-1)

\[ Z_{12}(p_1, p_2) = \frac{Z_2Z_1 + Z_2Z_3 + Z_2Z_4 + Z_3Z_4}{Z_1 + Z_2 + Z_3} \]  \hspace{1cm} (3.2-2)

\[ Z_{12}(p_1, p_2) = -\frac{(Z_4Z_1 + Z_4Z_2 + Z_4Z_3 + Z_4Z_4)}{Z_1 + Z_2 + Z_3} \]  \hspace{1cm} (3.2-3)

\[ Y_{11}(p_1, p_2) = \frac{Y_1Y_2 + Y_1Y_4 + Y_1Y_2 + Y_1Y_3 + Y_1Y_4}{Y_1 + Y_2 + Y_4} \]  \hspace{1cm} (3.2-4)

\[ Y_{12}(p_1, p_2) = -\frac{(Y_1Y_1 + Y_1Y_2 + Y_1Y_4 + Y_1Y_3)}{Y_1 + Y_2 + Y_4} \]  \hspace{1cm} (3.2-5)

and

\[ Y_{22}(p_1, p_2) = \frac{Y_2Y_1 + Y_2Y_2 + Y_2Y_3 + Y_2Y_4 + Y_2Y_4}{Y_1 + Y_2 + Y_4} \]  \hspace{1cm} (3.2-6)

In these expressions, when we put $p_1 = 1$ and/or $p_2 = 1$ there results two types of bridged-T networks, namely:

1) two element type (types $B_1$ and $B_2$) consisting of two elements

RL or RC

11) three element type (type $B_3$) consisting of all the three elements

R, L and C.
3.3 PROPERTIES OF BRIDGED-T NETWORKS OF TYPE $F_1$ AND $F_2$

The existence of these networks depends upon the nature of the reactances used. They can be obtained as following:

(i) Type $B_1$: The network consisting of $p_1$ type capacitors and $p_2$ type inductors. In this category, if $p_1 = 1$ the resulting network is R-L type; whereas if $p_2 = 1$, the network is RC type.

(ii) Type $B_2$: The network consisting of $p_1$ type inductors and $p_2$ type capacitors. In this category of $p_1 = 1$ the resulting is RC type and if $p_2 = 1$ the resulting network is of RL type. All the possible networks are shown in Table 3.2-I.

Some general properties of such bridged-T networks (Type $B_1$ and $B_2$) can be observed:

(i) The poles of the $Z$ matrix (y matrix) with $p_1$ or $p_2 = 1$ shall be simple and lie on the negative real axis of the variable $p_2$ or $p_1$ respectively.

(ii) The zeros of the $Z_{11}$, $Z_{22}$, $Y_{11}$ and $Y_{22}$ with $p_1$ or $p_2 = 1$ shall be simple and lie on the negative real axis of the variable $p_2$ or $p_1$ respectively.

(iii) None of these networks can have transmission zeros on the imaginary axis or right of $p_1$ or $p_2$ plane when the other variable is made equal to unity.
<table>
<thead>
<tr>
<th>$L_{11}$</th>
<th>$L_{12}$</th>
<th>$L_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{11}(p_1, p_2)$</td>
<td>$Z_{12}(p_1, p_2)$</td>
<td>$Z_{21}(p_1, p_2)$</td>
</tr>
<tr>
<td>$L_{12}$</td>
<td>$L_{21}$</td>
<td>$Z_{22}(p_1, p_2)$</td>
</tr>
<tr>
<td>$L_{11}$</td>
<td>$Z_{12}(p_1, p_2)$</td>
<td>$Z_{21}(p_1, p_2)$</td>
</tr>
<tr>
<td>$L_{12}$</td>
<td>$L_{21}$</td>
<td>$Z_{22}(p_1, p_2)$</td>
</tr>
</tbody>
</table>

TABLE 3.2-1 — $L_{11}$, $L_{12}$ and $L_{21}$ of the Bridged-T Networks of the Kind $B_1$ and $B_2$ in Variables (p₁,p₂) and in Single Variable (1,p₂) and (p₁,1).
<table>
<thead>
<tr>
<th>t_{12}(p_{12})</th>
<th>t_{11}(c_{12})</th>
<th>t_{11}(c_{12})</th>
<th>t_{11}(c_{12})</th>
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</thead>
<tbody>
<tr>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>t_{21}(c_{12})</th>
<th>t_{21}(c_{12})</th>
<th>t_{21}(c_{12})</th>
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<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t_{31}(p_{12})</th>
<th>t_{31}(c_{12})</th>
<th>t_{31}(c_{12})</th>
<th>t_{31}(c_{12})</th>
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<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
<td>(c_{12})^2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t_{41}(p_{12})</th>
<th>t_{41}(c_{12})</th>
<th>t_{41}(c_{12})</th>
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<td>(c_{12})^2</td>
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Table 3.2-1 Cont'd.
### Table 3.2-1 Cont'd.

<table>
<thead>
<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_2(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_3(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_4(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_5(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_6(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_7(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
<tr>
<td>( z_8(p, q) )</td>
<td>( \text{Defining equation} )</td>
<td>( \text{Defining equation} )</td>
</tr>
</tbody>
</table>

**Note:** The table continues with similar entries for each column.
<table>
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<tr>
<th>( \frac{1}{c_1 p_1} )</th>
<th>( z_{21}(p_1, p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</th>
<th>( z_{11}(p_1, p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</th>
<th>( z_{11}(p_1, 1) = \frac{\bar{z}_2(1) + \bar{z}_3(1) + \bar{z}_4(1) + \bar{z}_5(1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_{21}(p_1, p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</td>
<td>( z_{21}(z_2(p_1), p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</td>
<td>( z_{21}(z_2(1), p_1) = \frac{\bar{z}_2(1) + \bar{z}_3(1) + \bar{z}_4(1) + \bar{z}_5(1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)} )</td>
<td>( z_{21}(z_2(1), 1) = \frac{\bar{z}_2(1) + \bar{z}_3(1) + \bar{z}_4(1) + \bar{z}_5(1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)} )</td>
</tr>
<tr>
<td>( z_{21}(p_1, p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</td>
<td>( z_{21}(p_1, p_1) = \frac{\bar{z}_2(p_1) + \bar{z}_3(p_1) + \bar{z}_4(p_1) + \bar{z}_5(p_1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)p_1} )</td>
<td>( z_{21}(p_1, 1) = \frac{\bar{z}_2(1) + \bar{z}_3(1) + \bar{z}_4(1) + \bar{z}_5(1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)} )</td>
<td>( z_{21}(p_1, 1) = \frac{\bar{z}_2(1) + \bar{z}_3(1) + \bar{z}_4(1) + \bar{z}_5(1)}{2(\bar{z}_2 + \bar{z}_3 + \bar{z}_4 + \bar{z}_5)} )</td>
</tr>
</tbody>
</table>

*TABLE 3.2-1 Cont'd.*
<table>
<thead>
<tr>
<th>$s_{11}(p_1, p_2)$</th>
<th>$s_{11}(p_1, 1)$</th>
<th>$s_{11}(0, p_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{z_{11}(p_1, p_2) + z_{12}(p_1, p_2) + z_{13}(p_1, p_2)}{z_{14}(p_1, p_2)}$</td>
<td>$\frac{z_{11}(p_1, 1) + z_{12}(p_1, 1) + z_{13}(p_1, 1)}{z_{14}(p_1, 1)}$</td>
<td>$\frac{z_{11}(0, p_2) + z_{12}(0, p_2) + z_{13}(0, p_2)}{z_{14}(0, p_2)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_{21}(p_1, p_2)$</th>
<th>$s_{21}(p_1, 1)$</th>
<th>$s_{21}(0, p_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{z_{21}(p_1, p_2) + z_{22}(p_1, p_2) + z_{23}(p_1, p_2)}{z_{24}(p_1, p_2)}$</td>
<td>$\frac{z_{21}(p_1, 1) + z_{22}(p_1, 1) + z_{23}(p_1, 1)}{z_{24}(p_1, 1)}$</td>
<td>$\frac{z_{21}(0, p_2) + z_{22}(0, p_2) + z_{23}(0, p_2)}{z_{24}(0, p_2)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_{31}(p_1, p_2)$</th>
<th>$s_{31}(p_1, 1)$</th>
<th>$s_{31}(0, p_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{z_{31}(p_1, p_2) + z_{32}(p_1, p_2) + z_{33}(p_1, p_2)}{z_{34}(p_1, p_2)}$</td>
<td>$\frac{z_{31}(p_1, 1) + z_{32}(p_1, 1) + z_{33}(p_1, 1)}{z_{34}(p_1, 1)}$</td>
<td>$\frac{z_{31}(0, p_2) + z_{32}(0, p_2) + z_{33}(0, p_2)}{z_{34}(0, p_2)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$s_{41}(p_1, p_2)$</th>
<th>$s_{41}(p_1, 1)$</th>
<th>$s_{41}(0, p_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{z_{41}(p_1, p_2) + z_{42}(p_1, p_2) + z_{43}(p_1, p_2)}{z_{44}(p_1, p_2)}$</td>
<td>$\frac{z_{41}(p_1, 1) + z_{42}(p_1, 1) + z_{43}(p_1, 1)}{z_{44}(p_1, 1)}$</td>
<td>$\frac{z_{41}(0, p_2) + z_{42}(0, p_2) + z_{43}(0, p_2)}{z_{44}(0, p_2)}$</td>
</tr>
</tbody>
</table>

**TABLE 3.2-1 Cont'd.**
<table>
<thead>
<tr>
<th>$z_1(\psi_1, \psi_2)$</th>
<th>$z_2(\psi_1, \psi_2)$</th>
<th>$z_3(\psi_1, \psi_2)$</th>
<th>$z_4(\psi_1, \psi_2)$</th>
<th>$z_5(\psi_1, \psi_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1/\psi_1$</td>
<td>$1/\psi_2$</td>
<td>$1/\psi_1$</td>
<td>$1/\psi_2$</td>
<td>$1/\psi_1$</td>
</tr>
</tbody>
</table>

**Table 3.2-1 Cont'd.**
<table>
<thead>
<tr>
<th>$\alpha_{1.2}$</th>
<th>$\alpha_{1.3}$</th>
<th>$\alpha_{1.4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{2.1}$</td>
<td>$\alpha_{2.2}$</td>
<td>$\alpha_{2.3}$</td>
</tr>
<tr>
<td>$\alpha_{3.1}$</td>
<td>$\alpha_{3.2}$</td>
<td>$\alpha_{3.3}$</td>
</tr>
<tr>
<td>$\alpha_{4.1}$</td>
<td>$\alpha_{4.2}$</td>
<td>$\alpha_{4.3}$</td>
</tr>
</tbody>
</table>

**Table 3.2-1 Cont'd.**
| TABLE 3.2-1 Cont'd. |
TABLE 3.2-1 Cont'd.
(iv) None of these networks with \( p_1 = 1 \) or \( p_2 = 1 \) can give a constant resistance network.

### 3.4 Properties of Bridged-T Network Whose \( z_{11}(p_1, 1) \) and \( z_{11}(1, p_2) \) Are Positive Real Functions

In the previous section the properties of the bridged-T network are discussed when \( z_{11}(p_1, 1) \) and \( z_{11}(1, p_2) \) are two element kind functions. Here the properties of the Bridged-T network (Type \( R_3 \)) will be discussed when \( z_{11}(p_1, 1) \) and \( z_{11}(1, p_2) \) are positive real functions.

Contrary to the previous case, here by putting \( p_1 = 1 \) or \( p_2 = 1 \) we get only RLC networks and as a consequence \( z_{11} \) and \( z_{22} \) are positive real functions in the variable which is not but equal to unity.

Some general properties of such functions can be written as:

1) The poles of \( z_{11} \) and \( z_{22} \) with \( p_1 \) or \( p_2 \) equal unity, shall lie on the left half plane of the variable \( p_2 \) or \( p_1 \) respectively.

2) The zeros of \( z_{11} \) and \( z_{22} \) with \( p_1 \) or \( p_2 \) equal to unity shall lie on the left half plane of the variable \( p_2 \) or \( p_1 \) respectively.

3) None of these networks can have transmission zeros on the imaginary axis as in the right half of \( p_1 \) or \( p_2 \) plane when the other variable is made equal to unity.

4) Some of the networks can be made constant resistance networks by a suitable choice of the different elements.

For the purpose of this report, only the networks which yield constant
resistance networks will be discussed further. Table 3.3-I gives the possible constant resistance networks of the Bridged-T.

3.5 Properties of Bridged-T Networks Resulting in Single Variable Constant Resistance Networks

These types of bridged-T networks can be synthesized using the synthesis techniques available for single variable constant resistance networks [1]. Here the conditions under which a two variable reactance function can be synthesized using such bridged-T networks are [1],

\[
\begin{align*}
\left( Z_{11}(p_1,1) \right)^2 - \left( Z_{12}(p_1,1) \right)^2 &= \text{constant.} \quad \cdots \cdots \quad 3.3-1 \\
\left( Z_{11}(1,p_2) \right)^2 - \left( Z_{12}(1,p_2) \right)^2 &= \text{constant.} \quad \cdots \cdots \quad 3.3-2
\end{align*}
\]

The synthesis procedure is as follows:

1) Depending on (3.3-1) or (3.3-2) being satisfied, synthesize \( Z(p_1,1) \) or \( Z(1,p_2) \) which is terminated by one ohm resistance.

11) Replace the resistance by appropriate capacitance or inductance by the following test:

1) If \( Z_{11}(p_1,1) \) has a pole at origin on plane \( p_1 \), then 
   R will be replaced by capacitor in plane \( p_2 \) whose value is equal to \( \frac{1}{R} \) farads.

11) If \( Z_{11}(1,p_2) \) has a pole at origin on plane \( p_2 \), then 
   R is replaced by capacitor in plane \( p_2 \), whose value is equal to \( 1/R \) farads.
Two Variables \((p_1, p_2)\)

\[
\frac{1}{c_1 p_2} \quad \frac{1}{c_2 p_2} \quad \frac{1}{c_1 p_1} \quad \frac{1}{c_2 p_1} \quad L_1 p_2
\]

Single Variable \((l, p_2)\)

\[
\frac{1}{c_1 p_2} \quad \frac{1}{c} \quad \frac{1}{c} \quad L_1 p_2
\]

Conditions must be terminated in a load

\[
L_1 = c_1 \quad \text{and} \quad c = 1
\]

\[
L_1 = c_1 \quad \text{and} \quad c = 1
\]

Table 3.3 (continued)
Two Variable \((p_1, \ p_2)\)

Single Variable \((p_1, \ l)\)

Conditions to be terminated in \(1\ \Omega\) load

\[
C_1 = L_1 \\
L = 1
\]

\[
C_1 = L_1 \\
L = 1
\]

Table 3.3-1 Cont'd.
Two Variable \((p_1, p_2)\)

\[
\frac{1}{c_1 p_1}
\]

\[
\frac{1}{c_2 p_2}
\]

\[L_1 p_1\]

\[
\frac{1}{c_1 p_1}
\]

\[
\frac{1}{c_2 p_2}
\]

\[L_1 p_1\]

Single Variable \((p_1, 1)\)

\[
\frac{1}{c_1 p_1}
\]

\[
\frac{1}{c}
\]

\[L_1 p_1\]

\[
\frac{1}{c}
\]

\[L_1 p_1\]

Conditions to be terminated in \(1\Omega\) Load

\[
L_1 = c_1.
\]

\[
c = 1
\]

\[
L_1 = c_1.
\]

\[
c = 1
\]
<table>
<thead>
<tr>
<th>Two variables ((p_1, p_2))</th>
<th>Single Variable ((L, p_2))</th>
<th>Conditions to be terminated in (\Omega) load</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L_1 p_2)</td>
<td>(L_1 p_2)</td>
<td>(L_1 = c_1)</td>
</tr>
<tr>
<td>(L p_1)</td>
<td>(L p_1)</td>
<td>(L = 1)</td>
</tr>
</tbody>
</table>

Table 3.3-1 Symmetrical Bridged-T Networks in Two Variables and Constant Resistance in One Variable.
111) If $Z_{11}(p_1, 1)$ and $Z_{11}(1, p_2)$ does not have pole at origin in $p_1$ or $p_2$ plane than the reactance will be replaced by inductance of value $R$ in the required plane.

3.6 **EXAMPLES**

Synthesize the following given $Z_{11}(p_1, p_2)$ and $Z_{12}(p_1, p_2)$ as a constant resistance:

1. $Z_{11}(p_1, p_2) = \frac{p_2 + .5p_1 + 4pip_i + pip_2}{2pip_1 + 5pi}$

and

$Z_{12}(p_1, p_2) = \frac{p_2 + pip_1 + 4pip_1}{2pip_1 + 5pi}$

2. $Z_{11}(p_1, p_2) = \frac{p_1 + .5p_2 + pip_1 + 4pip_2}{2pip_2 + 0.5}$

and

$Z_{12}(p_1, p_2) = \frac{[4pip_2 + pip_1 + pip_2]}{2pip_2 + 0.5}$

**Solution:**

1. Using the earlier techniques developed for the synthesis of constant resistance bridged-T networks, the networks corresponding to (1) and (2) are shown in Fig. 3.5-1 and 3.5-2 respectively.
Two variable and one variable bridged-T networks of the Example Problem 1.
Fig. 3.5-2

Two Variable and One Variable Bridged-T Networks of The Example Problem 2.
3.7 SUMMARY AND DISCUSSION

From this study, it can be concluded that the two-variable bridge-T reactance network cannot constitute a constant-resistance network. However, only a subclass of two-variable bridge-T network can become a constant-resistance single-variable bridge-T when the other variable is put equal to unity. The properties of the various bridged-T networks having one element in an impedance arm have been studied.
PART IV
CONCLUSION

This report has considered the possibility of realizing a certain class of two variable symmetrical lattice and bridged-T structures.

From the study of the symmetrical lattice it has been found that when $z_a$ and $z_b$ are functions of one variable only, the lattice cannot constitute a constant resistance network. However, if each is a function of both variables, the lattice can constitute a constant resistance network. This class of lattice has been divided into four categories, namely $SL_1$, $SL_2$, $SL_3$ and $SL_4$.

It is shown in the report that the categories $SL_1$ and $SL_2$ can constitute constant reactance symmetrical lattice in two variables but $SL_3$ and $SL_4$ cannot.

Also it is observed that if $Z_{11}$ and $Z_{12}$ meets the conditions for $SL_1$, they will meet the conditions for $SL_2$ but for phase reversal only. But on the other hand, if they meet the conditions set for $SL_3$ or $SL_4$, they will not meet any other set of specifications.

From the study of bridged-T networks in two variables, it is concluded that it cannot constitute a constant resistance network. However, only a subclass of two variable bridged-T networks can become a constant resistance-single variable bridged-T when the other variable
is made equal to unity. The properties of the various bridged-T networks having one element in an impedance arm have been studied and the method of realization of such networks is enunciated.
REFERENCES


