REALIZATION OF TWO-AMPLIFIER FILTERS HAVING ZERO GAIN POLE-FREQUENCY SENSITIVITY PRODUCTS AND MINIMIZED SUM OF GAIN Q SENSITIVITY PRODUCTS

Christian S. Gargour

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CHRISTIAN S. GARGOUR

ABSTRACT

This thesis discusses the realizations of second order
transfer functions by means of two amplifier-active RC net-
works having the property of zero Gsp (Gain pole frequency
sensitivity products) and minimized sum of magnitudes of
Gqsp (Gain Q sensitivity products). The realizations of the
five commonly occurring types of filters namely Low Pass,
High Pass, Band Pass, Null, and All Pass are studied. These
filters should be implemented using hybrid IC technology.

The various possible denominator polynomial decomposi-
tions suitable to fulfill the condition of zero Gsp are first
studied in detail. A two-amplifier configuration (though not
unique) is proposed which will realize all the possible decom-
positions. The entire realization requires the use of gene-
rating transfer functions which have to be realized by using
single amplifier circuits the output of each of which has to be
taken from the output of the amplifier. The generating fun-
ctions could be of three kinds namely zero order, first order,
and second order. The realizations using zero order generating
functions are not discussed further as the filters cannot be
designed in a relatively "simple" manner. The other two cases
have been studied in detail. Representative circuits are given where \( F \), the sum of the magnitudes of GQSPs, is minimized in each case.

The case where the generating function is first order yields Low Pass, High Pass, and Band Pass only. The case when the generating function is of second order (referred to as Q-multiplication) yields, Band-Pass, Null and All Pass filters. The Q-multiplier circuits are particularly attractive since one can start from an optimized single amplifier filter and then optimize \( F \) further. A value of \( F \) less than 0.5 \( Q \) is obtainable while keeping the capacitive spread at a maximum value of three.

Three Null filters and two Band Pass filters have been built and tested. The experimental results conform closely to these from the theoretical studies.
This thesis is dedicated to
my wife Nana P.E. Gourgour
and to my daughter Caroline.
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Null type i
Null with a denominator polynomial decomposition of type i

$N(S)$
Numerator of $T_V$

$n_0(S)$
Numerator of $t_V/R_0$

OA
Operational Amplifier

$Q_p$
Designed pole Q of a TAC

$Q_{p0}$
Designed pole Q of a SAC

QMC
Q-multiplier circuit

RI, RII, RIII
Different types of generating circuits used in QMCs

$S$
Complex frequency variable

SAC
Single Amplifier Circuit

$t_V$
Voltage transfer function of a SAC, generating transfer function

$T_V$
Voltage transfer function of a TAC

TAC
Two-Amplifier Circuit

$\omega_p$
Variable frequency in rad/sec

$\omega_{p0}$
Designed pole frequency in rad/sec of a SAC

$\omega_p$
Designed pole frequency in rad/sec or in Hz of a TAC

$\omega_c$, $\omega_D$, $\omega_m$, $\omega_0$
Pole of O.A.

$F_{C1}$, $F_{C2}$
Lowest and highest 3 dB frequencies in Hz

$\Delta Q_p/Q_p$
Fractional change in $Q_p$

$\Delta \omega_p/\omega_p$
Fractional change in $\omega_p$
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\( \text{AD}(S), \text{Am}(S) \)  OAs Open Loop Gains

\( A_0(S) \)  OAs DC Gains

\( A_D, A_m, A_0 \)  Coefficients of \( K_1K_2 \) in the \( S^0, S, S^2 \) terms of \( D(S) \) respectively

\( a_0, a_1, a_2 \)  All Pass

All Pass Type i  All Pass with a denominator polynomial decomposition of Type i

\( a_0, a_1, a_2 \)  Coefficients of the \( S^0, S, S^2 \) terms of \( n_0(S) \) respectively

\( b_0, b_1, b_2 \)  The part of the coefficient of the \( S^0, S, S^2 \) term respectively which is independent of \( K_1, K_2 \) in \( D(S) \)

\( \beta_1, \beta_2, \beta_3 \)  The part of the coefficient of the \( S^0, S, S^2 \) term respectively which is independent of \( K_0 \) in \( D_0(S) \)

BP  Band Pass

Band Pass type i  Band Pass with a denominator polynomial decomposition of type i

C  Capacitance of a capacitor

\( c_0, c_1, c_2 \)  Coefficients of \( K_1 \) in the \( S^0, S, S^2 \) terms of \( D(S) \) respectively

\( \gamma_0, \gamma_1, \gamma_2 \)  The part of the coefficient of the \( S^0, S, S^2 \) term respectively which is associated with \( K_0 \) in \( D_0(S) \)
\( D(S) \)  
Denominator polynomial of \( T_v \)

\( D_{A}(S) \)  
Denominator of \( T_v \) expressed as a function of \( t_v \)

\( D_{B}(S) \)  
Denominator of \( T_v \) expressed as a function of \( d_0(S) \) and \( n_0(S) \)

\( D_{B0}(S) \)  
\( D_{B}(S) \) when \( t_v \) is of zero order

\( D_{B1}(S) \)  
\( D_{B}(S) \) when \( t_v \) is of first order

\( D_{B2}(S) \)  
\( D_{B}(S) \) when \( t_v \) is of second order

\( d_0, d_1, d_2 \)  
Coefficients of \( K_2 \) in the \( S^0, S, S^2 \) terms of \( D(S) \) respectively

\( F_{C1}, F_{C2} \)  
Three dB frequencies in HZ

\( F \)  
Figure of merit

\( G \)  
Admittance of a resistor

\( G_{0,SP} \)  
Gain pole frequency sensitivity product

\( G_{QSP} \)  
Gain Q sensitivity product

\( G_{0,SP} \)  
Gain pole frequency sensitivity product

\( G_{QSP} \)  
Gain Q sensitivity product

\( \text{Generating Circuit} \)  
A SAC used to generate a TAC

\( \text{Generating Function} \)  
Transfer function of a generating circuit

\( HP \)  
High Pass

\( \text{High Pass type i} \)  
High Pass with a denominator polynomial decomposition of type i

\( K_0, K_0', K_m, K_1 \)  
Amplifier's gains

\( K_2 \)  
Low Pass

\( \text{Low Pass type i} \)  
Low Pass with a denominator polynomial decomposition of type i
Null type i

Null with a denominator polynomial decomposition of type i

\( N(S) \)
Numerator of \( T_v \)

\( n_0(S) \)
\((\text{numerator of } t_v)/K_0\)

\( O_A \)
Operational Amplifier

\( Q_p \)
Designed pole \( Q \) of a TAC

\( Q_{P0} \)
Designed pole \( Q \) of a SAC

\( QMC \)
\( Q \)-multiplied circuit

\( R_{I}, \ R_{II}, \ R_{III} \)
Different types of generating circuits used in QMCs

\( S \)
Complex frequency variable

\( SAC \)
Single Amplifier Circuit

\( t_v \)
Voltage transfer function of a SAC, generating transfer function

\( T_v \)
Voltage transfer function of a TAC

\( TAC \)
Two-Amplifier Circuit

\( \omega_p \)
Variable frequency in \( \text{rad/sec} \)

\( \omega_{P0} \)
Designed pole frequency in \( \text{rad/sec} \) of a SAC

\( \omega_p \)
Designed pole frequency in \( \text{Rad/sec} \) or in \( \text{Hz} \) of a TAC

\( \omega_c, \ \omega_d, \ \omega_m, \ \omega_0 \)
Pole of \( O.A. \)

\( F_{Cl}, \ F_{C2} \)
Lowest and highest 3 dB frequencies in \( \text{Hz} \)
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CHAPTER I

INTRODUCTION
CHAPTER I

INTRODUCTION

1.1 General

Passive LC filters have been considerably studied and a large amount of literature [1, 2] exists on their design. These filters are absolutely stable and have no serious sensitivity problems associated with them. However, in spite of these attractive features, they suffer from some serious limitations.

(i) Accurate filter design is complicated by the nonlinear frequency dependence of the quality factor $Q_p$ of the inductors and the variation of that $Q_p$ from one inductor to another.

(ii) The inductors may be of large size (and hence of high cost) for low frequency applications such as in analog computers, control systems, etc.

(iii) Problems may arise because of the magnetic coupling between the inductive elements: they may mask weak signals which hence may become not measurable.

Microminiaturization and increasing interest in integrated circuitry (IC) technology has made the use of inductors not practicable, as they cannot be manufactured with reasonable values and quality factors within acceptable tolerance at low frequency.
These difficulties may be overcome by employing active RC filters [3, 4]. Their design requires only resistors and capacitors along with active elements. The main attractions of integrated circuit fabrication are [5, 6] increased system reliability and reduction in size and weight due to microminiaturization. RC active filters can be designed to have several desirable features over the RLC filters and some of these are:

(i) The use of active elements remove the two restrictions from RLC networks namely passivity and reciprocity. Thus active networks not only realize network functions which are realizable by passive RLC networks but also can be used to realize characteristics not achievable with passive networks.

(ii) Input impedance may be made high compared to source impedance and thus these filters draw very little power from the signal source.

(iii) Output impedance can be made low compared to that of the load thereby making the filter response independent of the load impedance. Consequently the filters can be cascaded without additional buffers.

(iv) The RC filters often provide insertion gain which may be desirable in many applications thereby eliminating the need for additional amplifiers.
However, active RC filters if improperly designed have two major drawbacks, namely:

(i) They may become unstable.
(ii) They may be sensitive to network parameter variations.

Thus proper care should be exercised in their design.

1.2 Methods of Realizing RC Active Networks

A large number of RC active network realization procedures have been reported in the literature [7, 8, 9]. These can be classified as:

(i) Direct approach, where the given transfer function is realized as a single section [10, 11, 12].
(ii) Cascade approach where the transfer function is expressed as a product of first and second order transfer functions [13, 14, 15, 16]. Each of these functions is realized independently and the overall network is obtained by cascading them.

The cascade approach requires universal sections and each section can be easily designed and optimized independently [13, 17, 18]. Therefore this approach is used in this thesis.

Several active devices such as operational amplifiers (O.A.), negative impedance converters, and gyrators have been used in the creation of active RC filters [3, 4]. The O.A. [19,
20] is the basic active element considered in this thesis.

1.3 The Operational Amplifier

The O.A. is a non-reciprocal two-port device, ideally characterized by an infinite gain, infinite input impedance and zero output impedance. In practice, however, the O.A. has a frequency dependent gain \( A(S) \), a finite input impedance and a non-zero output impedance. As an example, the Fairchild \( \mu A-741 \) has a DC gain \( A_0 \) of 200,000; an input impedance of 2 M\( \Omega \) and an output impedance of 75 \( \Omega \). Presently the O.A.'s are readily available in an integrated form as off-the-shelf components. Commercially available silicon monolithic integrated O.A.'s are reliable, versatile, relatively inexpensive (less than $1) and have excellent properties.

The O.A. and its equivalent controlled source representation are shown respectively in Figs. 1.3.1a and 1.3.1b.

The output voltage \( V_0 \) is related to the differential input voltage \( V_i = (V_2 - V_1) \) by:

\[
V_0 = A(S)V_i
\]  

(1.3.1)

\( A(S) \) is the differential open loop gain which for a single pole frequency compensated O.A. is given by:

\[
A(S) = \frac{A_0\omega_C}{(S + \omega_C)}
\]  

(1.3.2)

where \( A_0 \), \( \omega_C \) and \( B = A_0\omega_C \) are respectively the D-C gain, the cut-off frequency and the gain bandwidth product of the O.A.
FIGURE 1.3.1a

FIGURE 1.3.1b

FIGURE 1.3.1 THE OPERATIONAL AMPLIFIER AND ITS CONTROLLED SOURCE REPRESENTATION.
A_0 and \omega_C have large tolerances in addition to their dependence on the temperature and power supply voltages. Hence the characteristics of networks whose response are highly dependent on A_0 and/or \omega_C will be subject to variations with changing of temperature and power supply voltage.

1.4 Single and Two-amplifiers Realizations

The number of amplifiers in a biquadratic filter section may be one or more. Various single amplifier and two amplifier networks have been reported in the literature\(^*\) [17-18, 21-33]. For the realizations where the gains of the amplifiers are finite, the denominator polynomial can be expressed as: \(^{[34]}\)

\[
D(S) = d_1(S)d_2(S) - K_1n_{11}(S)d_2(S) - K_2n_{22}(S)d_1(S) + K_1K_2[n_{11}(S)n_{22}(S) - n_{12}(S)n_{21}(S)]
\]

(1.4.1)

where \(d_{ij}(S)\) are passive RC transfer functions and \(K_1, K_2\) are the closed loop gains of the two O.A.s. The decompositions corresponding to a single-amplifier can be obtained from Eqn. (1.4.1) by putting \(K_1\) or \(K_2\) equal to zero. Table 1.4.1 gives the summary of the results contained in \([34]\) as these will be utilized in the present investigation.

\(^*\)Only some of those networks which have appeared in the recent past have been quoted here and these contain many other references.
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<th>(n_{12}(S))</th>
<th>(n_{11}(S))</th>
<th>(n_{22}(S))</th>
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<td>1</td>
<td>0</td>
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<td>(n_{22}(S) = 1)</td>
<td>(n_{11}(S) = 1)</td>
<td>(n_{12}(S) = 1)</td>
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</table>

**Table 1.4.1**

\[ D(S) = d_1(S) d_2(S) - \lambda(S) n_{11}(S) d_2(S) - \lambda(S) n_{22}(S) d_1(S) - \lambda(S) n_{12}(S) n_{21}(S) \]
\[
\begin{array}{c|c}
\frac{d_1(S)}{d_2(S)} &= \frac{\delta_{11}S + \delta_{10}}{\delta_{21}S + \delta_{20}} \\
\end{array}
\]

\[
\begin{array}{c|c|c}
\text{Fialkow Gerst Conditions require:} & \text{Fialkow Gerst Conditions require:} & \end{array}
\]

\[
\begin{array}{c|c|c|c|c}
0 < e_0 < \delta_{10} & 0 < e_{11} < \delta_{11} & 0 < n_0 < \delta_{0} & 0 < m_1 < \delta_{1} & \\
0 < g_0 < \delta_{10} & 0 < g_{11} < \delta_{11} & 0 < n_1 < \delta_{1} & 0 < m_2 < \delta_{2} & \\
0 < f_0 < \delta_{20} & 0 < f_{21} < \delta_{21} & & & \\
0 < h_0 < \delta_{20} & 0 < h_{21} < \delta_{21} & & & \\
\end{array}
\]

\[
\frac{d_1(S)}{d_2(S)} = \frac{\delta_{11}S + \delta_{10}}{\delta_{21}S + \delta_{20}}
\]

Note: Fialkow-Gerst conditions as given above are due to the fact that \[\frac{n_{ij}(S)}{d_{ij}(S)}\] are three-terminal passive RC network transfer functions. [34]
1.5 Pole-Q \((Q_p)\), Pole-Frequency \(\omega_p\) and Their Sensitivities

Consider the second order transfer function \(T_v(S)\):

\[
T_v(S) = \frac{N(S)}{D(S)} = \frac{N(S)}{P_2S^2 + P_1S + P_0}
\]

where \(N(S)\) is a polynomial of second order or less which determines the filter's characteristics: Low Pass (LP), High Pass (HP), Band Pass (BP), Null (N) or All Pass (AP).

The quantity \(Q_p\) is defined by:

\[
Q_p = \left( \frac{P_0P_2}{P_1} \right)^{\frac{1}{2}}
\]  \(1.5.2\)

and the quantity \(\omega_p\) is defined by:

\[
\omega_p = \left( \frac{P_0}{P_2} \right)^{\frac{1}{2}}
\]  \(1.5.3\)

Hence the transfer function \(T_v(S)\) can be rewritten in the form:

\[
T_v(S) = \frac{N(S)}{S^2 + \omega_p^2 + \frac{Q_p}{S} + \omega_p^2}
\]  \(1.5.4\)

The sensitivities of \(Q_p\) and \(\omega_p\) with respect to the variation of a network parameter \(z\) are respectively defined as:

\[
S_{z, Q_p} = \Delta \frac{d(\ln Q_p)}{d(\ln z)} = \frac{z}{Q_p} \frac{dQ_p}{dz}
\]  \(1.5.6a\)

\[
S_{z, \omega_p} = \Delta \frac{d(\ln \omega_p)}{d(\ln z)} = \frac{z}{\omega_p} \frac{d\omega_p}{dz}
\]  \(1.5.6b\)
The fractional changes in $Q_P$ and $\omega_P$ due to variations in the network elements are [35, 36]:

$$
\frac{\Delta Q_P}{Q_P} = \sum \frac{\Delta K}{K} S_K + \sum \frac{\Delta G_i}{G_i} S_{G_i} + \sum \frac{\Delta C_j}{C_j} S_{C_j} 
$$

(1.5.7a)

$$
\frac{\Delta \omega_P}{\omega_P} = \sum \frac{\Delta K}{K} \omega_P + \sum \frac{\Delta G_i}{G_i} \omega_P + \sum \frac{\Delta C_j}{C_j} \omega_P 
$$

(1.5.7b)

where $K$ is the closed loop gain of the $\ell^{th}$ amplifier in the network. $G_i$ is the conductance of the $i^{th}$ resistive element in the network. $C_j$ is the capacitance of the $j^{th}$ capacitive element in the network.

Equation (1.5.7) shows that not only the variation in $Q_P$ and $\omega_P$ due to $K$ should be designed to be low but also the variation due to passive elements should be made low. However, if hybrid IC technology which make use of tantalum thin film RC components and monolithic integrated OAs is used, perfect tracking of passive elements is obtained; that is, the variations in similar passive elements due to changes in temperature can be made equal. It is also possible to control the process such that:

$$
\frac{\Delta G_i}{G_i} = \frac{\Delta C_j}{C_j} = \Delta G = \text{Constant for all } i \text{ and } j
$$

(1.5.8)

Hence from Eqn. (1.5.7) and (1.5.8) we get:
\[
\frac{\Delta Q_p}{Q_p} = \sum_l \frac{\Delta K_l}{K_l} S^K_l + \frac{\Delta G}{G} \left[ \sum_i \frac{Q^0}{G_i} + \sum_j \frac{Q^0}{C_j} \right]
\] (1.5.9a)

\[
\frac{\Delta \omega_p}{\omega_p} = \sum_l \frac{\Delta K_l}{K_l} S^K_l + \frac{\Delta G}{G} \left[ \sum_i \frac{\omega^0}{G_i} + \sum_j \frac{\omega^0}{C_j} \right]
\] (1.5.9b)

As it has been shown in [37]

\[
\sum_i \frac{Q^0}{G_i} + \sum_j \frac{Q^0}{C_j} = 0
\] (1.5.10a)

\[
\sum_i \frac{\omega^0}{G_i} + \sum_j \frac{\omega^0}{C_j} = 0
\] (1.5.10b)

Equations (1.5.9) reduce to:

\[
\frac{\Delta Q_p}{Q_p} = \sum_l \frac{\Delta K_l}{K_l} S^K_l
\] (1.5.11a)

\[
\frac{\Delta \omega_p}{\omega_p} = \sum_l \frac{\Delta K_l}{K_l} S^K_l
\] (1.5.11b)

It is clear that the use of thin film technology has eliminated the effect of variations due to passive elements sensitivities. Consequently, not only the networks with low passive sensitivities but also those with high passive sensitivities become attractive.

The relation between the closed loop amplifier gain \(K\) and the open loop gain \(A\) is: [19, 20]

\[
K = \frac{K_0 A}{K_0 A} \quad A \gg K_0
\] (1.5.12)
where \( K_0 \) is determined by a ratio of resistors. Hence

\[
S_K^A = \frac{A}{K} \frac{dK}{dA} = \frac{K}{A} \quad \text{for } A >> K
\]  

(1.5.13)

and

\[
\frac{\Delta K}{K} = \frac{\Delta A}{K} S_K^A + \frac{\Delta K_0}{K_0} S_{K_0}^A = K \frac{\Delta A}{A^2}
\]  

(1.5.14)

The second term of Eqn. (1.5.14) vanishes because of almost perfect tracking and from [37] Eqs. (1.5.11a) and (1.5.11b) therefore become:

\[
\frac{\Delta Q_P}{Q_P} = \sum_{\ell} K_\ell \frac{\Delta A}{A} S_{K_\ell}^P
\]  

(1.5.15a)

\[
\frac{\Delta \omega_P}{\omega_P} = \sum_{\ell} K_\ell \frac{\Delta A_\ell}{A_\ell} S_{K_\ell}^P
\]  

(1.5.15b)

Let, in the case of two amplifiers \( \ell = 1, 2 \)

\[
\frac{\Delta A}{A^2} = \frac{\max\left[\frac{\Delta A_1}{A_1}, \frac{\Delta A_2}{A_2}\right]}{A^2}
\]

Hence we have

\[
\left| \frac{\Delta Q_P}{Q_P} \right| < \sum_{\ell=1}^{2} K_\ell S_{K_\ell}^P \frac{\Delta A}{A^2}
\]  

(1.5.16a)

Similarly

\[
\left| \frac{\Delta \omega_P}{\omega_P} \right| < \sum_{\ell=1}^{2} K_\ell S_{K_\ell}^P \frac{\Delta A}{A^2}
\]  

(1.5.16b)
1.6 The Figure of Merit $F$

The quantity $\frac{\Delta A}{\Delta Q_p}$ is completely dependent on the QAs used. Therefore $\frac{\Delta Q_p}{Q_p}$ and $\frac{\Delta \omega_p}{\omega_p}$ are dependent on the factor $\sum_{l=1}^{2} K_l S_{K_l}$. It is known that [38] the variation in the transfer function $T_v(S)$ of a second order network whose zeros are located far from the high Q poles is given by:

$$\left. \frac{\Delta T_v(S)}{T_v(S)} \right|_{S=\omega_p} = \frac{\Delta Q_p}{Q_p} + j 2Q_p \frac{\Delta \omega_p}{\omega_p} \quad (1.6.1)$$

Therefore it is desirable to make $\frac{\Delta \omega_p}{\omega_p} = 0$. A sum of two positive quantities can be made zero only when the individual quantities are set to zero therefore:

$$\omega_p = \omega_p \quad (1.6.2)$$

$$S_{K_1} = S_{K_2} = 0$$

This is referred to as zero gain pole frequency sensitivity product (GSP) condition.

It is further desirable to minimize $\frac{\Delta Q_p}{Q_p}$. This requires the minimization of $K_1 S_{K_1}$ and $K_2 S_{K_2}$ which is the sum of the individual GQSPs of the amplifiers. Hence we choose in this thesis to minimize $F$ which is defined as:

$$F = |K_1 S_{K_1} Q_p| + |K_2 S_{K_2} Q_p| \quad (1.6.3)$$

which is the sum of the magnitudes of $K_1 S_{K_1} Q_p$ and $K_2 S_{K_2} Q_p$ and which could be interpreted as the worst case deviation.
1.7 Realizations with Zero $G_wSP$

It is known that [17, 18] it is possible to obtain single amplifier active RC networks (SAC) having the property of zero $G_wSP$ and minimized $G_QSP$. However, to the best of the author's knowledge, any study of the second order two amplifiers networks on the basis of a general configuration able to generate all the polynomial decompositions suitable for the fulfilment of the condition \( \frac{\Delta w_p}{w_p} = 0 \) independently of the gains of the amplifiers together with the minimization of $F$ does not appear to be available in the literature. Such a study is desirable in the case of hybrid IC implementation and an attempt is made in this thesis towards the solution of this problem.

1.8 Scope of the Thesis

This thesis discusses realizations of second order transfer functions by two-amplifier RC active filters (TAC) possessing the property of zero $G_wSP$ and minimized $F$.

In Chapter II, the various possible denominator polynomial decompositions suitable for zero $G_wSP$ are discussed. The different properties of such decompositions are studied. A two-amplifier configuration (not unique) able to realize all the above possible polynomial decompositions is given. The use of a generating function is required to realize a given transfer function. This generating function could be of three types, namely zero order, first order, or second order. The
zero order is not discussed in detail, as it cannot realize in a simple manner the five commonly occurring filters namely: Low Pass (LP), High Pass (HP), Band Pass (BP), Null (N) and All Pass (AP). Chapter III discusses the case of the first order generating function. This provides a systematic generation of such circuits. For the filters obtained, the $GωSP$ is zero and the quantity $F$ is minimized and design equations and/or curves are provided.

Chapter IV considers the case when the generating function is of second order. These generating functions are realized by single amplifiers and have the property of zero $GωSP$ and minimized $GωSP$. The overall two amplifier realization provides $Q$-multiplication while preserving the zero $GωSP$ property. The resulting $Q$-multiplied circuits have been optimized to minimize $F$, keeping the capacitive spread low.

Null and Band-Pass filters were built and tested. The experimental results are given. Chapter IV summarizes the results of the present investigation and proposes possible extensions of this work.
CHAPTER II

POSSIBLE DENOMINATOR POLYNOMIAL DECOMPOSITIONS FOR TWO AMPLIFIER RC ACTIVE NETWORKS HAVING ZERO GAIN POLE-FREQUENCY ($\omega_p$) SENSITIVITY PRODUCTS
CHAPTER II

POSSIBLE DENOMINATOR POLYNOMIAL DECOMPOSITIONS FOR TWO AMPLIFIER RC ACTIVE NETWORKS HAVING ZERO GAIN POLE FREQUENCY \((\omega_P)\) SENSITIVITY PRODUCTS

2.1 Introduction

In this chapter we discuss the various possible denominator polynomial decompositions suitable for zero gain-pole frequency sensitivity product (GwSP). Further, a general two-amplifier network configuration (TAC) is proposed which can realize all the above mentioned polynomial decompositions.

2.2 Possible polynomial decompositions for zero GwSP

It is known that \(\omega_P\) is a function of \(K_1, K_2\), as well as of resistances \(R_1\) and capacitances \(C_1\). If the active network is implemented using hybrid IC technology, \(\frac{\Delta \omega_P}{\omega_P} = 0\) is desirable and as given by Eqn. (1.6.2), we should then have

\[
\begin{align*}
S_{K_1}^\omega P &= S_{K_2}^\omega P = 0
\end{align*}
\]

(2.2.1)

In what follows we shall discuss the conditions and hence the different ways of arranging \(D(S)\) so that Eqn. (2.2.1) is satisfied.
As stated in Chapter I, the denominator of any two amplifier network can be written in the general form:

\[ D(s) = (L + K_1 K_2 a_2 - K_1 c_2 - K_2 d_2) s^2 + (b_1 + K_1 K_2 a_1 - K_1 c_1 - K_1 d_1) s + (b_0 + K_1 K_2 a_0 - K_1 c_0 - K_1 d_0) \]  \hspace{1cm} (2.2.2)

\( K_1 \) and \( K_2 \) are the closed loop gains of the two amplifiers and are treated here as algebraic quantities, positive for the non-inverting amplifier and negative for the inverting one. Parameters \( d, c, b \) are always positive and the \( a \) parameters can be either positive or negative. The pole frequency is given by:

\[ \omega_p = \left[ \frac{b_0 + K_1 K_2 a_0 - K_1 c_0 - K_2 d_0}{b_2 + K_1 K_2 a_2 - K_1 c_2 - K_2 d_2} \right]^{\frac{1}{2}} \]  \hspace{1cm} (2.2.3)

and the \( \omega_p \) sensitivities with respect to the amplifier gains are given by:

\[ S_{K_1}^{\omega_p} = \left[ \frac{K_1 K_2 a_1 - K_1 c_1}{2 b_0 + K_1 K_2 a_0 - K_1 c_0 - K_2 d_0} - \frac{K_1 K_2 a_2 - K_1 c_2}{b_2 + K_1 K_2 a_2 - K_1 c_2 - K_2 d_2} \right] \]  \hspace{1cm} (2.2.4a)

\[ S_{K_2}^{\omega_p} = \left[ \frac{1}{2} \frac{K_1 K_2 a_0 - K_2 d_0}{b_0 + K_1 K_2 a_0 - K_1 c_0 - K_2 d_0} - \frac{K_1 K_2 a_2 - K_2 d_2}{b_2 + K_1 K_2 a_2 - K_1 c_2 - K_2 d_2} \right] \]  \hspace{1cm} (2.2.4b)
If it is required that

\[ s_{k_1}^{\omega P} = s_{k_2}^{\omega P} = 0 \]  \hspace{1cm} (2.2.1)

then the relationship to be satisfied are:

\[ \frac{b_0 + K_1K_2a_0 - K_0K_2d_0}{b_2 + K_1K_2a_2 - K_0K_2c_2 - K_2d_2} = \frac{K_2a_0 - c_0}{K_2a_2 - c_2} = \frac{K_1a_0 - d_0}{K_1a_2 - d_2} \]  \hspace{1cm} (2.2.5)

Then, from (2.2.3) we have:

\[ \omega_p = \frac{K_2a_0 - c_0}{K_2a_2 - c_2} = \frac{K_1a_0 - d_0}{K_1a_2 - d_2} \]  \hspace{1cm} (2.2.6)

It is desirable to fulfill this condition independently of \( K_1 \) and \( K_2 \) for the following reasons:

i) If the condition (2.2.6) is fulfilled independently of \( K_1 \) and \( K_2 \), \( \omega_p \) will be dependent only on the passive elements. This should make the tuning simpler.

ii) This should also permit us to utilize \( K_1 \) and \( K_2 \) as parameters in minimizing \( \frac{\Delta Q_p}{Q_p} \) for any specified value of \( Q_p \).

A little algebraic manipulations will show that the condition (2.2.1) can be satisfied independently of \( K_1 \) and \( K_2 \), provided the following is satisfied:

\[ \frac{a_0}{a_2} = \frac{b_0}{b_2} = \frac{c_0}{c_2} = \frac{d_0}{d_2} = \omega_p^2 \]  \hspace{1cm} (2.2.7)
It is clear then, that the corresponding coefficients of $K_1$, $K_2$, $K_1K_2$ in the $S^2$, and $S^0$ terms of $D(s)$ should be present and satisfy Eqn. (2.2.7). All realizable polynomial decompositions that satisfy the conditions expressed by Eqn. 2.2.7 are tabulated in Table 2.2.1. The expressions for the corresponding $Q_p$ and $Q_p$ sensitivities with respect to the parameters $K_1$ and $K_2$ as well as the bounds on these sensitivities when all the terms in $D(s)$ are positive are also given in the same table. If all the terms are not positive, the bounds may not exist in many cases.

Polynomial decompositions 25 to 27 are respectively realizable only in the forms:

\[
D(s) \left[ (\delta_1 S^2 + \delta_2 S + \delta_0) (\delta_1 - K_2 m_2) S^2 + (\delta_1 - K_1 n_1) S + (\delta_0 - K_2 m_0) \right]
\]

(2.2.8a)

\[
D(s) \left[ (\delta_2 - K_1 n_1 - K_2 m_1) S^2 + (\delta_1 - K_1 n_1) S + \delta_0 \right]
\]

(2.2.8b)

\[
D(s) \left[ (\delta_2 S^2 + \delta_1 S + \delta_0) (\delta_2 - K_1 n_2 - K_2 m_2) S^2 + \delta_1 S + (\delta_0 - K_1 n_0 - K_0 m_0) \right]
\]

(2.2.8c)

These denominators can, provided suitable numerators are chosen, lead to second order transfer functions. However, at least four capacitors will be required and therefore polynomial decompositions 25 to 27 will not be considered further in this thesis.
TABLE 2.2.1
THE POLYNOMIAL DECOMPOSITIONS YIELDING ZERO Gru SP AND THE CORRESPONDING Gru SENSITIVITIES
<table>
<thead>
<tr>
<th>№</th>
<th>( \mathbf{S}^2 )</th>
<th>( \mathbf{S} )</th>
<th>( \mathbf{S}^0 )</th>
<th>( \mathbf{Q}_p )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^1 )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^2 )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^1 )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^2 )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^1 )</th>
<th>( \mathbf{Q}_p ) ( \mathbf{S}_k^2 )</th>
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<td>( K_1 K_2 )</td>
<td>( b_2 b_0 )</td>
<td>( b_1 - a_1 K_1 K_2 )</td>
<td>( -a_1 K_2 ) ( K_1 ) ( b_1 - a_1 K_1 K_2 )</td>
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<td>-1 ( \mathbf{S}_k^2 )</td>
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<td>( b_2 b_1 )</td>
<td>( b_1 - a_1 K_1 K_2 - c_1 K_1 )</td>
<td>( c_1 K_1 - a_1 ) ( K_2 ) ( b_1 - a_1 K_1 K_2 - c_1 K_1 )</td>
<td>-1 ( \mathbf{S}_k^1 )</td>
<td>-1 ( \mathbf{S}_k^2 )</td>
<td>-1 ( \mathbf{S}_k^1 )</td>
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<td>( b_2 + a_2 K_1 K_2 )</td>
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<td>( c_{12} )</td>
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<td>( c_{12} )</td>
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<td>( \frac{K_{12}}{b_{12}} )</td>
<td>( c_{12} )</td>
<td>( \frac{Q_{12}}{s_{12}} - \frac{K_{12}}{b_{12}} )</td>
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### Table 2.2 (Continued)

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| 12 | K₂ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ |
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| 13 | K₂ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ |
|    | b₂ + a₂ K₂ K₁ - c₂ K₁ | \( \frac{a₂ K₂ K₁ - c₂ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) |

| 14 | K₂ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ | K₁ |
|    | b₂ + a₂ K₂ K₁ - c₂ K₁ | \( \frac{a₂ K₂ K₁ - c₂ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) | \( \frac{a₀ K₂ K₁ - c₀ K₁}{b₂ - a₂ K₂ K₁ - c₂ K₁} \) |
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18 & \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} & \begin{pmatrix} \epsilon \end{pmatrix} & \begin{pmatrix} \zeta \end{pmatrix} & \begin{pmatrix} \eta \end{pmatrix} & \begin{pmatrix} \theta \end{pmatrix} & \begin{pmatrix} \iota \end{pmatrix} & \begin{pmatrix} \kappa \end{pmatrix} \\
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### TABLE 2.2.1 (Continued)

|   |   | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 23 | $K_1$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 24 | $K_1$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 25 | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 26 | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 27 | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |

*These are the bounds when all the terms in $D(S)$ are positive.*
2.3 A general, two amplifier network configuration

A two amplifier configuration (TAC) is shown in Fig. 2.3.1a. It will be shown that this configuration will permit us to realize all the twenty-four polynomial decompositions discussed in Section 2.2. The sub-network which realizes $t_V$ (called the generating function) is restricted only by the condition that its output is taken from the output of an amplifier. This means that a suitable single amplifier circuit (SAC) is required for realizing $t_V$. The admittance $Y_L$ should be set to zero whenever possible in order to reduce the number of elements; however it will be shown later that in some cases $Y_L$ cannot be set to zero if the condition expressed by Eqn. 2.2.7 is to be satisfied.*

Analysis of the configuration shown in Fig. 2.3.1a gives:

\[ (V_1 - V_2)Y_0 = (V_2 - V_4)Y_H + V_2Y_L + (V_2 - V_3)Y_F \]  
\[ V_3 = \frac{K}{m}V_2 \]  
\[ V_4 = t_VV_3 \]

Combining these equations we get:

*It should be noted that in the following derivation $K_m$ and $K_0$ are not interchangeable and could correspond respectively to $K_1$ and $K_2$ or vice-versa. Depending on the realization considered both or only one of these identifications is possible.
\[
\frac{V_4}{V_1} = \frac{K_m Y_0 t_v}{D_A(S)} \quad (2.3.4a)
\]
\[
\frac{V_3}{V_1} = \frac{K_m Y_0}{D_A(S)} \quad (2.3.4b)
\]
\[
\frac{V_2}{V_1} = \frac{Y_0}{D_A(S)} \quad (2.3.4c)
\]

where
\[
D_A(S) = Y_0 + Y_F + Y_H + Y_L - K_m Y_F - K_m Y_H t_v \quad (2.3.5)
\]

Since the sub-network generating \(t_v\) is a single amplifier network and since the output is taken from the amplifier output terminal it is possible to express \(t_v\) as:
\[
t_v = \frac{K_0 Y_0}{D_0(S)} \quad (2.3.6)
\]

Hence Eqn. 2.3.4 to 2.3.7 could respectively be rewritten as:
\[
\frac{V_4}{V_1} = \frac{K_m Y_0 n_0(S)}{D_B(S)} \quad (2.3.7a)
\]
\[
\frac{V_3}{V_1} = \frac{K_m Y_0 D_0(S)}{D_B(S)} \quad (2.3.7b)
\]
\[
\frac{V_2}{V_1} = \frac{Y_0 D_0(S)}{D_B(S)} \quad (2.3.7c)
\]
\[
D_B(S) = [Y_0 + Y_F + Y_H + Y_L - K_m Y_F] D_0(S) - K_m Y_H n_0(S) \quad (2.3.8)
\]
Because \( D_0(S) \) can always be written [17] as:

\[
D_0(S) = f_1(S) + K_0 f_2(S) \quad (2.3.9)
\]

where \( f_2(S) \) is that portion of \( D_0(S) \) with which \( K_0 \) is associated and \( f_1(S) \) is that portion of \( D_0(S) \) with which \( K_0 \) is not associated. It is clear that the denominator \( D_B(S) \) contains \( K_0, K_m, \) and \( K_m K_0 \) terms in the manner indicated in Eqn. (2.2.3).

Since it is more convenient for several reasons such as the possibility of cascading several sections without buffers, and the independence of the transfer function from the nature of the load, we will restrict our attention on the transfer functions taken from the output of an amplifier. Also \( \frac{V_3}{V_1} \) contains zeros of \( D_0(S) \) in the numerator which will be of the form \( (A_2 S^2 + A_1 S + A_0) \). It is obvious that since \( D_0(S) \) should be a strictly Hurwitz polynomial, the basic five filter types namely Low Pass, High Pass, Band Pass, Null and All Pass cannot be obtained. Therefore our attention in this thesis will only be focused on \( T_v = \frac{V_4}{V_1} \) given by Eqn. (2.3.7a).

A particular case will be considered now. If we let \( Y_F = 0 \) as shown in Fig. 2.3.1b the denominator of Eqn. (2.3.7) is given by:

\[
D_B(S) = (Y_0 + Y_L + Y_H) D_0(S) - K_m K_0 Y_H Y_0(S) \quad (2.3.10)
\]

This form of \( D(S) \) is not quite general as the one described
by Eqn. (2.3.8) because it could contain only \( K_0 \) and \( K_m K_0 \) terms; however it will be shown at a later stage that this configuration is useful to generate several of the polynomial decompositions.

The network shown in Fig. 2.3.1a is by no means unique. It can be modified in several ways and a number of other two-amplifier configurations can be generated. As an example the circuit shown in Fig. 2.3.1c is one possible modification for which analysis using Eqn. 2.3.6 gives:

\[
\frac{V_5}{V_1} = \frac{Y_0 [K m K_0 Y_H n_0(S) + (K m F_2 + Y_{F_1}) D_0(S)]}{D_C(S)} \quad (2.3.11a)
\]

\[
\frac{V_4}{V_1} = \frac{K m 0^* Y_0 [Y_{F_1} + Y_{F_2} + Y_H + Y_L] n_0(S)}{D_C(S)} \quad (2.3.11b)
\]

\[
\frac{V_3}{V_1} = \frac{K m Y_0 [Y_{F_1} + Y_{F_2} + Y_H + Y_L] D_0(S)}{D_C(S)} \quad (2.3.11c)
\]

\[
\frac{V_2}{V_1} = \frac{Y_0 [Y_{F_1} + Y_{F_2} + Y_H + Y_L] D_0(S)}{D_C(S)} \quad (2.3.11d)
\]

where

\[
D_C(S) = [Y_0 (Y_{F_1} + Y_{F_2} + Y_H) + Y_{F_1} (Y_{F_2} + Y_H + Y_L) - K m F_1 F_2] D_0(S)
\]

\[
- K m 0^* Y_{F_1} Y_{F_2} n_0(S) \quad (2.3.12)
\]

It is clear that a second order transfer function \( T_V \) can be generated by using any of the following possibilities.
Proposed configurations capable of realizing the different polynomial decompositions.
1) $D_0(S)$ being a zero degree polynomial

2) $D_0(S)$ being a First degree polynomial

3) $D_0(S)$ being a Second degree polynomial

We will now show that the proposed configuration shown in Fig. 2.3.1a can realize all the twenty-four possible polynomial decompositions.

### 2.4 The case of zero degree $D_0(S)$

Without any loss of generality a zero order transfer function realized by a SAC could be expressed, provided that the output is located at the output terminal of the amplifier as:

$$t_v = \frac{K_{0n_0}(S)}{D_0(S)} = \frac{K_0\alpha_0}{\beta_0 K_0\gamma_0} \tag{2.4.1a}$$

Fialkow-Gerst conditions require that

$$0 < \gamma_0 < \beta_0 \tag{2.4.1b}$$

Obviously $\gamma_0$ is a non-negative quantity while $K_0, \alpha_0$ could be either positive or negative. Using Eqn. 2.4.1 and Eqn. 2.3.8, we get $D_B(S)$ as

$$D_{B0}(S) = [\beta_0 - K_0\gamma_0] [Y_0 + Y_F + Y_H Y_L - K Y_m - K_m Y_H 0] \tag{2.4.2}$$

*This is designated as $D_{B0}(S)$ because this polynomial is obtained when $D_0(S)$ is a zero degree polynomial.*
It is obvious that a second degree polynomial cannot be obtained using simple first order admittances of the form:

$$Y = CS + G$$

For this purpose admittances of the general form:

$$\frac{k_i(S + z_i)}{(S + p_i)}$$

where

$$0 < z_i < p_i$$

alone or together with admittances of the form shown in Eqn. (2.4.3) should be used. However, their use necessarily produces terms of the form $$(S + p_i)$$ in the numerator of the transfer function and hence the five basic types of transfer function's namely LP, HP, BP, N, AP cannot be obtained without one or more pole-zero cancellations. Hence the case of the zero order degree $D_0(S)$ is not further pursued in this thesis.

2.5 The case of first degree $D_0(S)$

Without any loss of generality, a first order transfer function realized by a SNC could be expressed, provided that the output is taken through the output of the amplifier, as:
\[ t_v = K_0 \frac{n_0(S)}{D_0(S)} = \frac{K_0(a_1 S + a_0)}{(\beta_1 - K_0 \gamma_1) S + (\beta_0 - K_0 \gamma_0)} \]  \hspace{1cm} (2.5.1a)

Fialkow-Gerst conditions require that

\[ 0 < \gamma_1 < \beta_1 \quad \quad 0 < \gamma_0 < \beta_0 \]  \hspace{1cm} (2.5.1b)

where \( \gamma_1, \gamma_0 \) are non-negative quantities while \( K_0, a_1, a_0 \) could be either positive or negative. Using Eqn. 2.5.1 and Eqn. 2.3.8 we get \( D_B(S) \) as:

\[ D_{B1} = [Y_0 + Y_F + Y_H + Y_L - K_0 \gamma_1] [(a_1 - K_0 \gamma_1) S + (a_0 - K_0 \gamma_0)] \]
\[ - K_0 K_m (a_1 S + a_0) Y_H \]  \hspace{1cm} (2.5.2)

To obtain a second degree denominator, the admittances \( Y_0, Y_F, Y_H, Y_L \) should be chosen properly. One of the possible forms is \( Y = m S + n \) and therefore:

\[ Y_H = G_H + C_H S \quad \quad Y_F = G_F + C_F S \]
\[ Y_L = G_L + C_L S \quad \quad Y_0 = G_0 + C_0 S \]  \hspace{1cm} (2.5.4)

Using these expressions in \( D_{B1}(S) \) given by Eqn. (2.5.2) we get:

\[
\text{*This is designated as } D_B(S) \text{ because this polynomial is obtained when } D_0(S) \text{ is a first degree polynomial.} \]
\[ D_{B1}(S) = S^2(\beta_1(C_H + C_F + C_0 + C_L) - K_m \beta_1 C_F \]  
\[ - K_0 \gamma_1(C_H + C_F + C_0 + C_L) + K_m K_0 (\gamma_1 C_F - \alpha_1 C_L) \]  
\[ + S(\beta_1(G_H + G_F + G_0 + G_L) + \beta_0(C_H + C_F + C_0 + C_L) \]  
\[ - K_m (\beta_1 G_F + \beta_0 C_F) - K_0 (\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_L) \]  
\[ + \gamma_0 (C_H + C_F + C_0 + C_L) K_m K_0 (\gamma_1 G_F + \gamma_0 C_F - \alpha_1 G_H - \alpha_0 C_L) \]  
\[ + K_m K_0 (\gamma_0 G_F - \alpha_0 G_H) \]  
(2.5.5)

For this denominator, the condition expressed in Eqn. 2.2.7 is given by:

\[ \frac{\beta_1(C_H + C_F + C_0 + C_L)}{\beta_0(G_H + G_F + G_0 + G_L)} = \frac{\beta_1 C_F}{\beta_0 G_F} = \frac{\gamma_1(C_H + C_F + C_0 + C_L)}{\gamma_0(G_H + G_F + G_0 + G_L)} \]

\[ = \frac{\gamma_1 C_F - \alpha_1 C_H}{\gamma_0 G_F - \alpha_0 G_H} = \omega_p^2 \]  
(2.5.6)

It is to be expected that depending on the realization considered, it may become possible or necessary to set some of the capacitances and/or conductances to zero. However in the realization, the conditions:

\[ \frac{\beta_1}{\beta_0} \lesssim \frac{\gamma_1}{\gamma_0} \]  
and \[ \frac{\beta_1}{\beta_0} = \frac{\alpha_1}{\alpha_0} \]  
(2.5.7)

if satisfied simultaneously give rise to a constant \( t \) and
<table>
<thead>
<tr>
<th>Decomposition No.</th>
<th>S^2 Active terms present in S^0</th>
<th>C_F G_F, Y_I, Y_0</th>
<th>C_F^x G_F</th>
<th>Y_I, Y_0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>K_m K_0</td>
<td></td>
<td>C_F G_F, Y_0</td>
<td></td>
</tr>
<tr>
<td>9a</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td>C_F^x G_F</td>
<td></td>
</tr>
<tr>
<td>9b</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>K_m K_0, K_m K_0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22a</td>
<td>( K_0 )</td>
<td>( K_mK_0 \cdot K_0 )</td>
<td>( K_0 )</td>
<td>( C_F, C_E, (C_H, a_0) ) or ( (G_H, a_1) )</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>----------------</td>
<td>--------</td>
<td>----------------</td>
</tr>
<tr>
<td>22b</td>
<td>( K_m )</td>
<td>( K_mK_0 \cdot K_m )</td>
<td>( K_m )</td>
<td>( \gamma_1 \cdot \gamma_0 \cdot (C_H, a_0) ) or ( (G_H, a_1) )</td>
</tr>
</tbody>
</table>

\[ B_2 \triangleq \gamma_1C_{F2} - a_1C_H \]
\[ B_1 \triangleq \gamma_1G_{F2} + \gamma_0C_{F2} - (a_1C_H + a_0C_H) \]
\[ B_0 \triangleq \gamma_0G_{F2} - a_0G_H \]
hence the resulting $T_v$ is not a second order transfer function. This possibility should be avoided.

From Eqn. (1.4.1) and Fialkow Gerst conditions, it follows that for this case only the polynomials decompositions 1, 4, 9, 10, 15, 18, 19, 22 are possible. These are tabulated in Table 2.5.1 where the quantities to be set to zero in $D_{B1}(S)$ are also given for each case. It should be noted that in the cases of decompositions 9, and 22 there exist two possibilities which are also shown. This case is further discussed in Chapter III.

2.6 The case of second degree $D_0(S)$

Without any loss of generality, a second order transfer function realized using a single finite gain amplifier could be expressed, provided that the output is taken through the output of the amplifier, as:

$$t_v = \frac{K_0 n_0(S)}{D_0(S)} = \frac{K_0 (a_2 S^2 + a_1 S + a_0)}{(\beta_2 - K_0 \gamma_2) S^2 + (\beta_1 - K_0 \gamma_1) S + (\beta_0 - K_0 \gamma_0)}$$

(2.6.1a)

Fialkow-Gerst conditions require that:

$$0 < \gamma_2 < \beta_2, \quad 0 < \gamma_1 < \beta_1, \quad 0 < \gamma_0 < \beta_0$$

(2.6.1b)

where $\gamma_2$, $\gamma_1$, $\gamma_0$ are non-negative quantities while $K_0$, $a_2$, $a_1$, $a_0$ could be either positive or negative. Substituting
Eqn. 2.6.1 into 2.3.8 with

\[ Y_0 = G_0 \quad Y_F = G_F \quad Y_H = G_H \quad Y_L = G_L \quad \text{(2.6.2)} \]

we get \( D_B(S) \) as:

\[ D_{B_2}(S) = S^2 \left[ \beta_2 (G_0 + G_F + G_H + G_L) - K \beta_2 G_F - K_0 \gamma_2 (G_0 + G_H + G_H + G_L) \right] \]

\[ + K_0 \left( \gamma_2 G_F - \alpha_2 G_H \right) \]

\[ + S \left[ \beta_1 (G_0 + G_F + G_H + G_L) - K \beta_1 G_F - K_0 \gamma_1 (G_0 + G_H + G_F + G_L) \right] \]

\[ + K_0 \left( \gamma_1 G_F - \alpha_1 G_H \right) \]

\[ + \left[ \beta_0 (G_0 + G_F + G_H + G_L) - K \beta_0 G_F - K_0 \gamma_0 (G_0 + G_H + G_F + G_L) \right] \]

\[ + K_0 \left( \gamma_0 G_F - \alpha_0 G_H \right) \quad \text{(2.6.3...)} \]

From Eqn. 2.2.7 we get

\[ \frac{\beta_0}{\beta_2} = \frac{\gamma_0}{\gamma_2} = \frac{\alpha_0}{\alpha_2} = \frac{2}{\omega} \quad \text{(2.6.4)} \]

It is known [17,18] that \( t_v \) can be realized by various decompositions of \( D_0(S) \) involving \( K_0 \). By proper choice of \( D_0(S) \) and the conductances \( G_0, G_F, G_H, G_L \), the decompositions 1 to 24 can be achieved. These are tabulated in Table 2.6.1.

The quantities to be set to zero in \( D_{B_2} \) are also given for each case. It should be noted that in the cases of

\*This is designated as \( D_{B_2}(S) \) because this polynomial is obtained when \( D_0(S) \) is a second degree polynomial.
**TABLE 2.6.1**

Possible polynomial decompositions when $D_0(s)$ is a second degree polynomial.
<table>
<thead>
<tr>
<th>Decomposition No.</th>
<th>Active terms present in $s^2$, $s$, $s^0$</th>
<th>Quantities to be set to zero in $D_{B2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K_m K_0$</td>
<td>$G_F, \alpha_2, \alpha_0, \gamma_2, \gamma_1, \gamma_0$</td>
</tr>
<tr>
<td>2</td>
<td>$K_m K_0, K_0$</td>
<td>$G_F, \alpha_2, \alpha_0, \gamma_2, \gamma_0$</td>
</tr>
<tr>
<td>3</td>
<td>$K_m K_0$</td>
<td>$G_F, \alpha_1, \gamma_2, \gamma_1, \gamma_0$</td>
</tr>
<tr>
<td>4</td>
<td>$K_m K_0, K_m K_0$</td>
<td>$G_F, \gamma_2, \gamma_1, \gamma_0$</td>
</tr>
<tr>
<td>5</td>
<td>$K_m K_0, K_m K_0, K_m K_0$</td>
<td>$G_F, \gamma_2, \gamma_0$</td>
</tr>
<tr>
<td>6</td>
<td>$K_m K_0$</td>
<td>$G_F, \alpha_1, \gamma_2, \gamma_0$</td>
</tr>
<tr>
<td>7</td>
<td>$K_m K_0, K_0$</td>
<td>$G_F, \alpha_1, \gamma_1$</td>
</tr>
<tr>
<td>8</td>
<td>$K_m K_0, K_0, K_m K_0$</td>
<td>$G_F, \gamma_1$</td>
</tr>
<tr>
<td>9a</td>
<td>$K_m K_0, K_0, K_m K_0$</td>
<td>$G_F$</td>
</tr>
<tr>
<td>9b</td>
<td>$K_m K_0, K_m$</td>
<td>$\gamma_2, \gamma_1, \gamma_0$</td>
</tr>
<tr>
<td></td>
<td>$K_mK_0', K_m$</td>
<td>$K_mK_0', K_m, K_0$</td>
</tr>
<tr>
<td>---</td>
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<td>---------------------</td>
</tr>
<tr>
<td>11</td>
<td>$K_mK_0', K_m$</td>
<td>$K_m', K_0$</td>
</tr>
<tr>
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<td>$K_mK_0', K_0$</td>
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</tr>
<tr>
<td>12b</td>
<td>$K_mK_0', K_m$</td>
<td>$K_m$</td>
</tr>
<tr>
<td>13</td>
<td>$K_mK_0', K_m, K_0$</td>
<td>$K_mK_0', K_m$</td>
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<tr>
<td>14</td>
<td>$K_mK_0', K_m, K_0$</td>
<td>$K_mK_0', K_m, K_0$</td>
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<tr>
<td>15</td>
<td>$K_mK_0', K_m, K_0$</td>
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</tr>
<tr>
<td>16</td>
<td>$K_mK_0', K_m, K_0$</td>
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<tr>
<td>17</td>
<td>$K_mK_0', K_0$</td>
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</tr>
<tr>
<td>18</td>
<td>$K_m', K_0$</td>
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<td>$K_m, K_0$</td>
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<tr>
<td>---</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>20</td>
<td>$K_m, K_0$</td>
<td>$K_m$</td>
</tr>
<tr>
<td>21</td>
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<td>$K_m K_0$</td>
</tr>
<tr>
<td>22a</td>
<td>$K_0$</td>
<td>$K_m K_0, K_0$</td>
</tr>
<tr>
<td>22b</td>
<td>$K_m$</td>
<td>$K_m K_0, K_m$</td>
</tr>
<tr>
<td>23</td>
<td>$K_m$</td>
<td>$K_m K_0, K_m, K_0$</td>
</tr>
<tr>
<td>24</td>
<td>$K_m$</td>
<td>$K_m K_0$</td>
</tr>
</tbody>
</table>

$B_2 \triangleq \gamma_2 G_P - \alpha_2 G_H$

$B_1 \triangleq \gamma_1 G_P - \alpha_1 G_H$

$B_0 \triangleq \gamma_0 G_P - \alpha_0 G_H$
decompositions 9, 12, 22 there exist two possibilities which
are also shown. The pole frequency of the denominator of
the transfer function given by Eqn. 2.6.1 is:

\[
\omega_{p0} = \begin{bmatrix}
\beta_0^{-K_0} \gamma_0^{-1/2} \\
\beta_2^{-K_0} \gamma_2^{-1/2}
\end{bmatrix}
\]  

(2.6.5)

while, the pole frequency of \( D_{B2}(s) \) given by Eqn. (2.4.3) is:

\[
\omega_p = \begin{bmatrix}
\beta_0 (G_0 + G_F + G_H + G_L) - K_0 \beta_0 G_F - K_0 \gamma_0 (G_0 + G_F + G_H + G_L)^{1/2} \\
+ K_m \gamma_0 (\gamma_0 G_F - \alpha_0 G_H)
\end{bmatrix}
\begin{bmatrix}
\beta_2 (G_0 + G_F + G_H + G_L) - K_0 \beta_2 G_F - K_0 \gamma_2 (G_0 + G_F + G_H + G_L)^{1/2} \\
+ K_m \gamma_0 (\gamma_2 G_F - \alpha_2 G_H)
\end{bmatrix}
\]  

(2.6.6)

If the condition expressed by Eqn. (2.2.7) is fulfilled we
have \( \omega_p = \omega_{p0} \). The \( Q_p \) of \( D_{B2}(s) \) is given by:

\[
Q_p = \begin{bmatrix}
\beta_2 (G_0 + G_F + G_H + G_L) - K_0 \beta_2 G_F \\
- K_0 \gamma_2 (G_0 + G_F + G_H + G_L) \\
+ K_m \gamma_0 (\gamma_2 G_F - \alpha_2 G_H)
\end{bmatrix}
\begin{bmatrix}
\beta_0 (G_0 + G_F + G_H + G_L) - K_0 \beta_0 G_F \\
- K_0 \gamma_0 (G_0 + G_F + G_H + G_L) \\
+ K_m \gamma_0 (\gamma_0 G_F - \alpha_0 G_H)
\end{bmatrix}
\]  

(2.6.7)

This could be expressed as:
\[ Q_P = Q_{P0} \left( \frac{(G_0^+G_F^+G_H^+G_L)^+ \left[ \left( \frac{K_m K_0 (\gamma_2 G_F - \alpha_2 G_H - \beta F}{\beta F} + \frac{\gamma_0 G_F - \alpha_0 G_H - \beta F}{\beta F} \right) \right]^{1/2} \right)}{(G_0^+G_F^+G_H^+G_L) \left( \frac{K_0 K_m (\gamma_1 G_F - \alpha_1 G_H - \beta F}{\beta F} \right)^{1/2}} \right) \]

where

\[ Q_{P0} = \frac{(\beta_2 K_0 \gamma_2)(\beta_0 K_0 \gamma_0)^{1/2}}{\beta_1 K_1 \gamma_1} \]

is the \( Q \) of the SAC transfer function given in Eqn. 2.61. Hence it is clear that if the passive and active parameters are properly chosen and if the filters are fabricated in hybrid IC technology, the given configuration achieves a \( Q \) multiplication (multiplication of \( Q \) of \( t_v \)) while keeping the pole frequency \( \omega_p \) invariant.

\( Q \) multiplication has already been mentioned in the literature and a signal flow graph for achieving it is given [31]. This paper does not discuss any realization or further properties. Among the important advantages of \( Q \) multiplication the following could be mentioned:

1) Low-\( Q \) circuits having good circuit properties can be used in suitable \( Q \) multiplier circuits in order to obtain high values of \( Q \) and still preserve their desirable properties. This should enable us to design high-\( Q \) circuits making use of low-\( Q \)
ones having their properties optimized independently.

ii) High-Q positive feedback circuits have a tendency to become unstable although they may have other good properties. This situation could be improved by a combination of low-Q positive feedback circuits and a Q-multiplier possessing a negative feedback loop.

iii) In many instances tuning is simplified.

The case of second order \( D_0(S) \) is further discussed in Chapter IV where the resulting filter networks are referred to as the "Q-multiplier circuits" (QMC).

2.7 Summary and Discussions

In this chapter, we have considered the various possible denominator polynomial decompositions for two amplifier filter networks having zero \( G_{w_0} \). For hybrid IC implementation, this will lead to design of filters having zero functional deviation of \( \omega_0 \) which is a highly desirable feature for high Q applications.

A configuration is proposed (though not unique) which is shown to be capable of realizing all the above polynomial decompositions. This is possible by the use of three types of generating functions. These generating functions have to be realized such that the output is taken from the output of an amplifier. Out of these three possibilities, the cases
of the first and second order $D_0(S)$ will be discussed further. The case of zero order $D_0(S)$ will not be discussed any more because it does not appear to lead to any filter design in a simple manner. The case of the second order $D_0(S)$ appears to be particularly attractive as in this case starting from an optimized SAC filter. A TAC filter can be designed such that "Q-multiplication" is achieved while keeping the pole frequency $\omega_p$ invariant.
CHAPTER III

THE CASE OF THE FIRST ORDER
GENERATING FUNCTION
CHAPTER III

THE CASE OF THE FIRST ORDER GENERATING FUNCTION

3.1 Introduction

In the previous chapter, we considered the different polynomial decompositions having the property of zero $G_{o}SP$ and proposed a general configuration for realizing them. Further, it was shown that the generating function could be either of first or second order. In this chapter, we shall discuss realizations in which the generating circuit produces a first order transfer function. The quantity $F$ of the resulting two-amplifier network will be minimized. This results in filters having zero $G_{o}SP$ and minimized $\frac{\Delta Q_{p}}{Q_{p}}$. The technique is illustrated by selecting appropriate generating circuits.

3.2 The Basic Configuration with First Order Generating Function

The proposed configuration is shown in Fig. 3.2.1. Using Eqns. 2.3.4a, 2.3.5 and setting $V_{4} = V_{0}$ we get

$$V_{0} = \frac{K_{m}Y_{o}t_{v}}{V_{1} = \frac{K_{m}Y_{o}t_{v}}{Y_{o} + Y_{F} + Y_{H} + Y_{L} - K_{m}Y_{F} - K_{m}Y_{H}t_{v}}}$$

(3.2.1)

From Eqns. 3.2.1, 2.5.2, and 2.5.1 we obtain the transfer function $T_{v}$ of the configuration shown in Fig. 3.2.1 as
FIGURE 3.2.1 THE PROPOSED CONFIGURATION WITH FIRST ORDER $t_v$
\[ T_V = \frac{K_{m_0}(G_0' + C_0'S)(\alpha_1 S + \alpha_0)}{S^2 \left[ (\beta_1 - K_0\gamma_1)(C_1 + C_{F1} + C_{0L}) - K_{m_1}K_{m_0}(\gamma_1 C_{F1} - \alpha_1 C_H) \right]}
+ S\left[ (G_H' + G_{F1} + G_{0L}) (\beta_1 - K_0\gamma_1) + (C_1 + C_{F1} + C_{0L}) (\beta_0 - K_0\gamma_0) \right]
- K_{m}(\gamma_0 G_{F2} + \gamma_0 C_{F2}) + K_{m_0}(\alpha_1 G_{F1} + C_{F1} - \alpha_0 C_{F1})
+ \left\{ (\beta_0 - K_0\gamma_0)(G_H + G_{F1} + G_{0L}) - K_{m_0} G_{F1} + K_{m_0}(\gamma_0 G_{F1} - \alpha_0 C_{F1}) \right\} \]  

(3.2.2)

Table 3.2.1 gives all the possible polynomial decompositions and for each one the corresponding generating function \( t_V \), the transfer function of the entire circuit \( T_V \), its poles \( Q_0 \) \( Q_p \) and active parameters sensitivities \( S_{K_m} \) \( S_{K_0} \) as well as their bounds when all the denominator \( D(S) \) terms are positive.

It is clear from the numerator of \( T_V \) that we should expect to be able to obtain Low Pass, Band Pass, and High Pass transfer functions but no Null or All Pass ones. The number of elements (particularly the capacitors) may seem to be high in Fig. 3.2.1. However, it should be noted that several of these elements could or have to be set to zero in order to achieve the different polynomial decompositions and the different kinds of numerators of \( T_V \).

Table 3.2.2 gives the quantities which can or have to be set to zero in order to obtain a \( T_V \). It should be noted that in some cases the elements are kept for the only reason that their removal would make the condition expressed in Eqn. 2.5.6 (which ensures zero GwSP independently of \( K_0 \) and \( K_m \))
### TABLE 3.2.1

**Polynomial decompositions suitable for zero GwSP obtainable from first order T<sub>g</sub> and the corresponding T<sub>y</sub>, Q<sub>p</sub>, and sensitivities**
### Table 3.2.1

**Polynomial Decomposition Type 1**

<table>
<thead>
<tr>
<th>( z_V )</th>
<th>( \frac{X_0^a_1S}{\xi_1 S^{-\frac{3}{2}} 0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_V )</td>
<td>( \frac{X_m X_0 (G_0 - C_0 S) a_1 S}{S_2 (C_0 + C_L)^{-\frac{3}{2}} S_1 (G_0 - C_L) - S_1 (G_H + C_H + C_L) - X_m X_0 C_H a_1} + S_0 (G_H + C_0 + C_L) )</td>
</tr>
<tr>
<td>( Q_P )</td>
<td>( \frac{-{s_1 (C_0 - C_L)^{\frac{3}{2}} (G_0 - G_0 + C_L)^{\frac{1}{2}}}{s_0 (C_0 - C_L)^{-\frac{3}{2}} (G_H + C_H + C_L) - X_m X_0 C_H a_1} )</td>
</tr>
<tr>
<td>( S_K )</td>
<td>( \frac{X_m X_0 C_H a_1}{s_0 (C_0 - C_L)^{-\frac{3}{2}} (G_0 - C_L) - C_H - X_m X_0 C_H a_1} )</td>
</tr>
</tbody>
</table>

- **Bounds of** \( S_K \) **when all terms in** \( D(S) \) **are positive:** \(-1, 0\)

- **Bounds of** \( S_K \) **when all terms in** \( D(S) \) **are positive:** \(-1, 0\)
### TABLE 3.2: (Continued)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_v = \frac{K_0 \alpha_0}{s} ]</td>
<td></td>
</tr>
<tr>
<td>[ T_v = \frac{K_m K_0 (G_0 + C_0 S) \alpha_0}{s^2 \left[ \beta_1 (C_H + C_0 + C_L) + \gamma_1 (G_H + G_0 + G_L) - C_H \gamma_1 K_m K_0 \right] + \gamma_0 (G_H + G_0 + G_L)} ]</td>
<td></td>
</tr>
<tr>
<td>[ Q_P = \frac{\left( \beta_1 (C_H + C_0 + C_L) \gamma_0 (G_H + G_0 + G_L) \right)^{\frac{1}{2}}}{s_0 (C_H + C_0 + C_L) - \beta_1 (G_H + G_0 - G_L) - C_H \gamma_0 K_m K_0} ]</td>
<td></td>
</tr>
<tr>
<td>[ S_{K_0} = \frac{K_m K_0 S_{H_0} \gamma_0}{s_0 (C_H + C_0 + C_L) - \beta_1 (G_H + G_0 - G_L) - C_H \gamma_0 K_m K_0}. ]</td>
<td></td>
</tr>
<tr>
<td>Bounds of ( S_{K_0} ) when all terms in ( D(S) ) are positive: (-1, 0)</td>
<td></td>
</tr>
<tr>
<td>[ S_{K_m} = \frac{Q_P}{s_0} ]</td>
<td></td>
</tr>
<tr>
<td>Bounds of ( S_{K_m} ) when all terms in ( D(S) ) are positive: (-1, 0)</td>
<td></td>
</tr>
<tr>
<td><strong>POLYNOMIAL DECOMPOSITION TYPE 4</strong></td>
<td></td>
</tr>
<tr>
<td>[ t_v = \frac{K_0 (\alpha_1 S + \alpha_0)}{s_1 S + s_0} ]</td>
<td></td>
</tr>
<tr>
<td>[ T_v = \frac{K_m K_0 (G_0 + C_0 S) (\alpha_1 S + \alpha_0)}{s^2 \left[ \beta_1 (C_H + C_0 + C_L) - K_m K_0 C_H \alpha_1 \right] + s \left[ \beta_1 (G_H + G_0 + G_L) + \gamma_1 (C_H + C_0 + C_L) - K_m K_0 (\alpha_1 C_H + \alpha_0 C_L) \right] + \gamma_0 (G_H + G_0 + G_L) + K_m K_0 C_H \alpha_0}. ]</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 3.2.1 (Cont'd)

<table>
<thead>
<tr>
<th>$Q_P$</th>
<th>$S_{K_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{[\beta_1(C_H^+C_0^+C_L^-) - K_0 K_0 \gamma_1]}{S_0 (G_H^+G_0^-G_L^+)} - \frac{[\beta_0 (G_H^+G_0^-G_L^+)]}{S_0 (G_H^+G_0^-G_L^+)}$</td>
<td>$\frac{1}{2} \frac{K_0 K_0 (C_H^+C_0^+C_L^-)}{S_0 (G_H^+G_0^-G_L^+)} - \frac{K_0 K_0 (C_H^+C_0^+C_L^-)}{S_0 (G_H^+G_0^-G_L^+)}$</td>
</tr>
</tbody>
</table>

Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: $-1,1$

<table>
<thead>
<tr>
<th>$Q_P$</th>
<th>$S_{K_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-S_{K_0}$</td>
<td>$-S_{K_0}$</td>
</tr>
</tbody>
</table>

Bounds of $S_{K_m}$ when all terms in $D(S)$ are positive: $-1,1$

**POLYNOMIAL DECOMPOSITION TYPE 9A**

<table>
<thead>
<tr>
<th>$t_v$</th>
<th>$T_v$</th>
<th>$Q_P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{K_0 (\alpha_1 S_{K_0} + \beta_0)}{(S_0 - K_0 \gamma_0) S_{K_0} - \alpha_0}$</td>
<td>$S^2 [G_0^+G_0^+G_L^- (S_0 - K_0 \gamma_0) + K_0 S_{K_0} C_H^+] + S [G_0^+G_0^+G_L^- (S_0 - K_0 \gamma_0) + (G_0^+G_0^+G_L^- (S_0 - K_0 \gamma_0) + K_0 S_{K_0} C_H^+]$</td>
<td>$\frac{[G_0^+G_0^+G_L^- (S_0 - K_0 \gamma_0) + K_0 S_{K_0} C_H^+] [\beta_0 (S_0 - K_0 \gamma_0) - K_0 K_0 (C_H^+C_0^+C_L^-)]}{(G_0^+G_0^+G_L^- (S_0 - K_0 \gamma_0) + K_0 S_{K_0} C_H^+] [\beta_0 (S_0 - K_0 \gamma_0) - K_0 K_0 (C_H^+C_0^+C_L^-)]}$</td>
</tr>
</tbody>
</table>
### TABLE 3.2.1 (Continued)

<table>
<thead>
<tr>
<th>$Q_{T}$</th>
<th>$s_{K_{0}}$</th>
<th>$= \frac{1}{2} \left[ -K_{0} \gamma_{0} (G_{H} - G_{L})^{2} - K_{m} \alpha_{0} \right.$</th>
<th>$+ \left. \frac{K_{0} \gamma_{1} (C_{H}^{+} C_{0}^{+} C_{L}^{-})^{2} - K_{m} \alpha_{1} \right]}{(G_{H} - G_{L})^{2} - (C_{H}^{+} C_{0}^{+} C_{L}^{-})^{2} (s_{0} - s_{F})^{2}} \right]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{P}$</td>
<td>$s_{K_{m}}$</td>
<td>$= \frac{1}{2} \left[ -K_{m}^{2} C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \alpha_{1} \right.$</td>
<td>$+ \left. \frac{K_{m}^{2} C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \alpha_{1} \right]}{(G_{H}^{+} G_{L}^{+} C_{0}^{+} C_{L}^{-})^{2} (s_{0} - s_{F})^{2}} \right]$</td>
</tr>
</tbody>
</table>

Bounds of $s_{K_{0}}$ when all terms in $D(s)$ are positive: -1,1

Bounds of $s_{K_{m}}$ when all terms in $D(s)$ are positive: -1,1

### POLYNOMIAL DECOMPOSITION TYPE 9B

- $t_{v} = \frac{K_{0} (\alpha_{1} S + \alpha_{0})}{s_{1} S + s_{0}}$
- $t_{v} = \frac{K_{m} K_{0} (G_{H}^{+} C_{0}^{+} S_{0}) (\alpha_{1} S + \alpha_{0})}{s_{1} (C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \gamma_{1} C_{m}^{+} - K_{m} \alpha_{1}) + s_{2} (C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \gamma_{1} C_{m}^{+} - K_{m} \alpha_{1})}$

+ $s_{1} (C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \gamma_{1} C_{m}^{+} - K_{m} \alpha_{1}) + s_{2} (C_{H}^{+} C_{0}^{+} C_{L}^{-} - K_{m} \gamma_{1} C_{m}^{+} - K_{m} \alpha_{1})$
### TABLE 3.2.1 (Continued)

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_P )</td>
<td>( \frac{\left[ e_1 (G_{H}^<em>G_{C_{2}}^<em>G_{C_{0}}^<em>G_{C_{4}}^</em>) - K_m e_z C_{2} - K_m K_0 G_{H}^</em> G_{C_{0}}^</em> \right]}{e_1 (G_{H}^<em>G_{C_{2}}^<em>G_{C_{0}}^<em>G_{C_{4}}^</em>) - K_m e_z C_{2} - K_m K_0 G_{H}^</em> G_{C_{0}}^</em>} )</td>
</tr>
<tr>
<td>( S_{K_0} )</td>
<td>( \frac{-K_m e_z G_{C_{2}}^* G_{H}^*}{2 e_z (G_{H}^<em>G_{C_{2}}^<em>G_{C_{0}}^<em>G_{C_{4}}^</em>) - K_m e_z C_{2} - K_m K_0 G_{H}^</em> G_{C_{0}}^</em>} )</td>
</tr>
<tr>
<td>( S_{K_m} )</td>
<td>( \frac{-K_m e_z G_{C_{2}}^* G_{H}^*}{2 e_z (G_{H}^<em>G_{C_{2}}^<em>G_{C_{0}}^<em>G_{C_{4}}^</em>) - K_m e_z C_{2} - K_m K_0 G_{H}^</em> G_{C_{0}}^</em>} )</td>
</tr>
</tbody>
</table>

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, 1\)

Bounds of \( S_{K_m} \) when all terms in \( D(S) \) are positive: \(-1, 1\)

### POLYNOMIAL DECOMPOSITION TYPE 10

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_{V} )</td>
<td>( \frac{K_0 (e_{V} G_{C_{2}}^* G_{C_{0}}^* G_{C_{4}}^<em>)}{(e_{1} - K_0) G_{C_{2}}^</em> G_{C_{0}}^* G_{C_{4}}^*)} )</td>
</tr>
</tbody>
</table>
### TABLE 3.2. (Continued)

\[
\begin{align*}
\tau_v &= \frac{K_m K_0 \Gamma_{C_\text{C}_0 \text{S}}} {s \left( \left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) \right) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))} \\
C_p &= \frac{\left( \left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) \right) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))} {\left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))}
\end{align*}
\]

\[
\begin{align*}
\sigma_p^{(S_{n+1})} &= \frac{-\Gamma_{G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)} \left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))} {\left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))}
\end{align*}
\]

Bounds of \( S_{n+1} \) when all terms in \( D(S) \) are positive: \(-1, 1\)

\[
\begin{align*}
\sigma_p^{(S_{n+1})} &= \frac{-\Gamma_{G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)} \left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))} {\left( C \text{H}^{-G_F-G_C^{-G_L}}(z_1-\alpha_1)^- K_m C_F^{-G_L} C_{\text{C}_0 \text{L}} \right) \left( 1-\alpha_1 \right) - K_m G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0) + K_m K_0 (z_1 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0)) + K_m (z_0 G_F^{-G_L} C_{\text{C}_0 \text{L}}(z_0))}
\end{align*}
\]

Bounds of \( S_{n+1} \) when all terms in \( D(S) \) are positive: \(-1, 1\)
TABLE 3.2.1 (Continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{\nu} )</td>
<td>( \frac{K_{0}(g_{0}c_{0}c_{0})}{(\xi-k_{0}\gamma_{0}^2\alpha_{0}c_{0})} )</td>
</tr>
<tr>
<td>( T_{s} )</td>
<td>( S^{2}\left{ (c_{h}+c_{F}+c_{G}+c_{L})(\xi-k_{0}\gamma_{0}^2\alpha_{0}c_{0}) \right} ^{-1} )</td>
</tr>
<tr>
<td>( O_{p} )</td>
<td>( \left{ \left( g_{h}+g_{F}+g_{0}+g_{L} \right)(\xi-k_{0}\gamma_{0}^2\alpha_{0}c_{0}) \right} ^{1} )</td>
</tr>
<tr>
<td>( S_{K_{0}} )</td>
<td>( \frac{1}{2} \left( g_{h}+g_{F}+g_{0}+g_{L} \right)(\xi-k_{0}\gamma_{0}^2\alpha_{0}c_{0}) )</td>
</tr>
<tr>
<td>( O_{p} )</td>
<td>( \left( g_{h}+g_{F}+g_{0}+g_{L} \right)(\xi-k_{0}\gamma_{0}^2\alpha_{0}c_{0}) )</td>
</tr>
</tbody>
</table>

Polynomial Decomposition Type 15

Bounds of \( S_{K_{0}} \) when all terms in \( D(S) \) are positive: \(-1,1\)

Bounds of \( S_{p_{m}} \) when all terms in \( D(S) \) are positive: \(-1,1\)
### TABLE 3.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 15**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_v$</td>
<td>$s^2\left(\sum_{i}x_{i}^2C_{i} \cdot C_{l}\left(\sum_{j}y_{j}^2C_{j} \cdot C_{l}\right) \cdot K_{n}^2C_{l}^2\right) + \sum_{m}x_{m}y_{m}C_{m} \cdot C_{l} \cdot K_{m}^2C_{l}^2$</td>
</tr>
<tr>
<td>$K_{n}C_{n}(G_{0}^2+C_{0}^2) \cdot (a_{1} \cdot a_{0})$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{-X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right)}{\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + \left(C_{h}^2+C_{0}^2+C_{l}^2\right)}$</td>
</tr>
<tr>
<td>$= \frac{-X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right)}{\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + \left(C_{h}^2+C_{0}^2+C_{l}^2\right)}$</td>
<td></td>
</tr>
</tbody>
</table>

**Bounds of $S_{n}$ when all terms in $D(S)$ are positive:** $-1, 1$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{-X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right)}{\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + \left(C_{h}^2+C_{0}^2+C_{l}^2\right)}$</td>
</tr>
<tr>
<td>$= \frac{-X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + X_{0}^1\left(C_{h}^2+C_{0}^2+C_{l}^2\right)}{\left(C_{h}^2+C_{0}^2+C_{l}^2\right) + \left(C_{h}^2+C_{0}^2+C_{l}^2\right)}$</td>
<td></td>
</tr>
</tbody>
</table>

**Bounds of $S_{n}$ when all terms in $D(S)$ are positive:** $-1, 1$
### Table 3.2.1 (Continued)

#### Polynomial Decomposition Type 19

<table>
<thead>
<tr>
<th>( t_v )</th>
<th>( \frac{c_0(1+g_0)}{(e_1-k_{0,1}y_1)+k_{1,0}y_1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_y )</td>
<td>( \frac{k_{m,0}}{s_0(1+g_0)} )(e_1-k_{0,1}y_1)+k_{1,0}y_1 )</td>
</tr>
<tr>
<td>( C_p )</td>
<td>( \frac{(e_1-k_{0,1}y_1)+k_{1,0}y_1}{(e_1-k_{0,1}y_1)+k_{1,0}y_1} )</td>
</tr>
<tr>
<td>( q_p )</td>
<td>( \frac{e_1-k_{0,1}y_1}{(e_1-k_{0,1}y_1)+k_{1,0}y_1} )</td>
</tr>
</tbody>
</table>

Bounds of \( S_{m,0} \) when all terms in \( D(S) \) are positive: \(-1,1\)

<table>
<thead>
<tr>
<th>( q_p )</th>
<th>( \frac{e_1-k_{0,1}y_1}{(e_1-k_{0,1}y_1)+k_{1,0}y_1} )</th>
</tr>
</thead>
</table>

Bounds of \( S_{m,0} \) when all terms in \( D(S) \) are positive: \(-1,1\)
TABLE 3.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 22A**

\[
\begin{align*}
v_t &= \frac{K_0 a_1 s}{(s - K_0) s - (s - K_0^2)} \\
Tv &= \frac{K_0 K_0'(G_0 + C_0) s a_1 s}{s^2 (C_0 + C_L) (s - K_0) + s[(G_0 + G_L) (s - K_0) + (C_0 + C_L) (s - K_0)^2] - K_0 K_0' G_L} \\
Q_P' &= \frac{(C_0 - C_L) (s - K_0) (G_0 + G_L) (s - K_0) s}{(G_0 + G_L) (s - K_0) - (C_0 + C_L) (s - K_0) - K_0 K_0' G_L} \\
Q_P' &= \frac{(s - K_0) - K_0 s}{s^2 (s - K_0)^2} \\
S_{K_0} &= \frac{1}{(C_0 - C_L) (s - K_0) (G_0 + G_L) (s - K_0) s - K_0 s s - K_0 s} \\
S_{K_m} &= \frac{K_0 K_0' G_L}{(G_0 + C_0) (s - K_0) - (C_0 - C_L) (s - K_0) - K_0 K_0' G_L} \\
\end{align*}
\]

Bounds of \(Q_P'\) when all terms in \(D(s)\) are positive: \(-1, 1\)

Bounds of \(S_{K_0}\) when all terms in \(D(s)\) are positive: \(-1, 0\)
TABLE 3.2.1 (Continued)

\[ T_V = \frac{K_0 \cdot K_0 (G_0 + C_0 S) S_0}{\beta (C_H - C_0 - C_L) (\beta - K_0 Y_1) + S (G_0 - G_L) (\beta - K_0 Y_1) - (C \cdot C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0)} \]

\[ Q_P = \frac{(G_0 + G_L) (\beta - K_0 Y_1) - (C \cdot C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0)}{(G_0 + G_L) (\beta - K_0 Y_1) - (C \cdot C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0)} \]

\[ S_{K_0} = \frac{\frac{K_0 Y_1}{\beta - K_0 Y_1} - \frac{K_0 Y_0}{\beta - K_0 Y_0}}{(G_0 + G_L) (\beta - K_0 Y_1) - (C \cdot C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0)} \]

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1

\[ S_{K_m} = \frac{K_0 Y_0}{\beta - K_0 Y_0} \]

Bounds of \( S_{K_m} \) when all terms in \( D(S) \) are positive: -1, 0

POLYNOMIAL DECOMPOSITION TYPE 22B

\[ T_V = \frac{K_0 \cdot K_0 (G_0 + C_0 S) S_0}{\beta (C_H - C_0 - C_L) (\beta - K_0 Y_1) + S (G_0 - G_L) (\beta - K_0 Y_1) - (C \cdot C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0) - K_0 (C_0 - C_L) (\beta - K_0 Y_0)} \]

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TABLE 3.2.1 (Continued)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
<th>Bounds of $S_{K_0}^P$ when all terms in $D(S)$ are positive:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_p = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td>-1,0</td>
</tr>
<tr>
<td>$S_{K_0} = \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_p = s_{G_F^0 - G_0 - C_F} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{K_0} = s_{G_F^0 - G_0 - C_F} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_p = \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{K_m} = \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Q_p = \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_{K_m} = \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} - K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_v = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_v = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_v = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_v = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_v = \frac{(G_F^0 + G_0 + C_C - K_{C_F}) S_0 (G_F + G_0 - K_{G_F})}{(G_F^0 - G_0 - G_L)^2 1 - \frac{1}{C_C - C_F + C_C^2}} - \frac{G_F^0 - G_0 - G_L}{C_C - C_F + C_C^2} \times K_{m} {s_{G_F^0 - G_0 - C_F} - K_{m(0)} - C_H}^2}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 3.2.1 (Continued)

<table>
<thead>
<tr>
<th>$S_{km}^q$</th>
<th>$- \frac{K_n G_n}{2(G_H - G_F - G_0 - G_L - K_m G_F)} - \frac{K_n C}{C_F - C_L - K_m C_F}$</th>
<th>$- \frac{K_n (S_1 C - S_0 C_F)}{K_m C_0 G_H}$</th>
</tr>
</thead>
</table>

Bounds of $S_{km}^q$ when all terms in $D(S)$ are positive: $-1, 1$
TABLE 3.2.2

QUANTITIES WHICH CAN OR HAVE TO BE SET TO ZERO IN ORDER TO OBTAIN A $t_v$ WHEN $t_v$ IS OF FIRST ORDER.
<table>
<thead>
<tr>
<th>No.</th>
<th>Active terms present in</th>
<th>Quantities to be set to zero</th>
<th>(Numerator)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(s^2)</td>
<td>(s)</td>
<td>(s^0)</td>
</tr>
<tr>
<td>1</td>
<td>(K_{m,K'})</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
</tr>
<tr>
<td>9a</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
</tr>
<tr>
<td>9b</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
<td>(K_{m,K'})</td>
</tr>
<tr>
<td>No.</td>
<td>Active terms present in</td>
<td>Quantities to be set to zero</td>
<td>Numerator</td>
</tr>
<tr>
<td>-----</td>
<td>------------------------</td>
<td>-----------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>10</td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( c_0^2 S^2 )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( a_0 c_0 S )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( G_0 a_1 S )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( G_0 c_0 S )</td>
</tr>
<tr>
<td>15</td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( c_0 a_1 S^2 )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( a_0 c_0 S )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( G_0 a_1 S )</td>
</tr>
<tr>
<td></td>
<td>( K_m K_0 )</td>
<td>( K_m K_0 ) ( K_m K_0 )</td>
<td>( G_0 c_0 S )</td>
</tr>
</tbody>
</table>

\( (G_0 + C_0 S) (a_0 + a_1 S) \)
TABLE 3.2 (CONTINUED)

<table>
<thead>
<tr>
<th>No.</th>
<th>Active terms present in</th>
<th>Quantities to be set to zero</th>
<th>Numerator $K_m K_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>$K_m K_0 K_m K_0 K_0 K_0$</td>
<td>$X X \Delta$</td>
<td>$C_0 S (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X X$</td>
<td>$C_0 (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X X$</td>
<td>$(G_0 + C_0) (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td>19</td>
<td>$K_m K_0 K_m K_0 K_0 K_0$</td>
<td>$X X \Delta$</td>
<td>$C_0 S (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X X$</td>
<td>$G_0 (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$X X$</td>
<td>$(G_0 + C_0) (a_1 S^2 a_0)$</td>
</tr>
<tr>
<td>22a</td>
<td>$K_0 K_0 K_0 K_0 K_0 K_0$</td>
<td>$0 0 0$</td>
<td>$C_0^2 S^2$ (i)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$G_0^2 S$ (ii)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$a_0 C_0 S$ (iii)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$a_0 G_0$ (iv)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$a_0 (G_0 + C_0 S)$</td>
</tr>
<tr>
<td>22b</td>
<td>$K_m K_0 K_m K_0 K_0 K_0$</td>
<td>$0 0 0 0$</td>
<td>$C_0^2 S^2$ (i)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$G_0^2 S$ (ii)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$C_0^2 S^2$ (iii)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$G_0^2 S$ (iv)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0 0 0 0$</td>
<td>$(G_0 + C_0 S) (a_0 S^2 a_0)$</td>
</tr>
</tbody>
</table>
TABLE 3.2.7 (CONTINUED)

- Elements which have to be set to zero to realize the polynomial decomposition under consideration. (These are already given in Table 2.5.1.)
- Elements which have to be set to zero to realize the numerator given in the right-hand side column.
- Elements which could be set to zero if desired in order to reduce the total number of elements.
- Elements which should be set to zero to achieve the condition necessary for $\Delta u_p/\omega_p = 0$. 
impossible to fulfill. It is clearly seen from Table 3.2.2 that L.P., B.P., and H.P. transfer functions are obtainable using polynomial decompositions 1, 10, 15, and 22.

From the foregoing discussion it is clear that the eight polynomial decompositions can be obtained by using only six generating functions. These are tabulated in Table 3.2.3 together with the polynomial decompositions which are realizable by each one. In fact the generating functions, L0 and B0 can be considered as two particular cases of LBO: also L1 as well as B1 can be considered as two particular cases of LBL, thus reducing the number of generating functions to two only.

The first order generating circuits can also be classified as:

1. Passive circuits cascaded with an amplifier (called R1).

2. Active circuits (called RII).

A detailed study of four representative TAC's will be given at the end of this chapter.

Table 3.2.4 gives as illustrative examples TAC's classified according to the kind of generating circuits from which they are derived. These TAC's have been chosen among those which realize LP, HP, or BP transfer functions with the use of no more than three capacitors. In each case the identification with the general parameters used in Table 3.2.1
TABLE 3.2.3

POLYNOMIAL DECOMPOSITIONS OBTAINABLE FROM GIVEN FIRST ORDER GENERATING FUNCTIONS $t_v$
<table>
<thead>
<tr>
<th>Generating Transfer Function</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
<th>( \frac{K_0^{a_1 S \gamma_0}}{\beta_1 S + \beta_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>L0</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td><strong>B0</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td><strong>LBO</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td><strong>L1</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td><strong>B1</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td><strong>LB1</strong></td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
<td>( Q )</td>
</tr>
<tr>
<td>Table 3.2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Different tacs obtainable from first order generating functions realized by SACs
### Table 3.2.4

**Generating Function Type \( \ell_0 \)**

\[
\gamma = \frac{K_0 G_1}{C_1 s G_1}
\]

- Identification with the general parameters.
  - \( a_0 = b_0 = G_1 \)
  - \( \beta_1 = C_1 \)

#### Low Pass Type 1

\[
T_{lp1} = \frac{K_m K_0 G_G_1}{S^2 C_1 C_0 + S \left[ G_1 + G_2 + G_m - \frac{K_m}{G_1} C_1 C_0 \right] + G_1 G_2}
\]

#### Low Pass Type 27b

\[
T_{lp27} = \frac{K_m K_0 G_G_1}{S^2 \left[ (C_1 + C_F) - K_m C_F \right] C_1}
\]

\[
= \frac{S \left[ G_1 + G_2 + G_2 (C_1 + C_F) - K_m (C_1 + C_F) \right]}{S \left[ G_1 + G_2 + G_2 (C_1 + C_F) - K_m (C_1 + C_F) \right]}
\]

\[
+ \frac{-K_m K_0 C_C F}{G_1 + G_2 + G_2 (C_1 + C_F) - K_m F_G_2}
\]

\[
\text{with } \frac{C_F}{G_F} = \frac{C_H}{G_0}
\]
TABLE 3.2.4 (Continued)

Band Pass Type I(ii)

\[ T_{np1} = \frac{K_m K_e G_1 C_0 s}{s^2 C_1 (C_0 + C_H) + s |G_1 (C_H + C_0) + C_L + K_m K_o C_H G_1 | + G_1 G_m} \]
TABLE 3.2.4 (Continued)

GENERATING FUNCTION TYPE \( B_0 \)

\[
G_0 \frac{C_1}{G_1} \quad K_0 \quad \frac{C_{1}S}{G_1 B_0} \quad t_v = \frac{C_{1}S}{G_1 B_0} \\
\]

Identification with the general parameters.

\[
\alpha_1 = \beta_1 = C_1 \quad \beta_0 = G_1
\]

Band-Pass Type 1(i)

\[
T_{BPL} = \frac{K_{m}G_{c}C_{1}S_{p}}{C_{1}C_{L}S^{2} + S[G_{c}C_{1} + (G_{c} + G_{m}) - K_{m}G_{c}C_{1}]} \]

Band-Pass Type 2.2b(i)

\[
T_{BP22} = \frac{K_{m}G_{c}C_{1}S_{p}}{S^{2}[(G_{c} + C_{1}) - K_{m}G_{c}C_{1}]} + S[G_{c}C_{1} + G_{1}(C_{1} + C_{F}) - K_{m}(C_{1}G_{c} + C_{1}C_{F})] / -K_{m}G_{c}C_{1}
\]

\[ + G_{1}[C_{1}G_{c} + G_{1}(G_{c} + G_{m}) - K_{m}G_{c}] \]

with \[
\frac{C_{L}}{G_{c}G_{1}G_{F}G_{m}} = \frac{C_{F}}{G_{T}}
\]
TABLE 3.2.4 (Continued)

High Pass Type 1

\[ T_{HP1} = \frac{K \cdot F \cdot C_0 \cdot C_0^2}{C_1 \cdot C_0 \cdot S \left( C_F + C_0^1 \cdot G_1 C_0 \right) - K \cdot m \cdot C_1 \cdot C_0 \cdot G_1 \cdot C_1^1} \]

High Pass Type 2

\[ T_{HP2} = \frac{K \cdot F \cdot C_0 \cdot C_0^2}{S \left( C_F + C_0^1 \cdot G_1 C_0 \right) - K \cdot m \cdot C_1 \cdot C_0 \cdot G_1 \cdot C_1^1} \]

with:

\[ \frac{C_0}{C_0 \cdot G_H} = \frac{C_F}{G_F} \]
TABLE 3.2.4 (Continued)  

GENERATING FUNCTION TYPE I1  

\[ G(z) = \frac{k_{e}g_{A}}{\left[1 - z^{-1}g_{A} - k_{e}g_{B}\right]\left[1 - z^{-1}g_{B}\right]} \]

Identification with the primal parameters:

\[ \theta_1 = c_{A}c_{B}, \quad \gamma_1 = c_{A}, \quad \eta_1 = 0 \]

\[ \theta_0 = c_{A}c_{B}, \quad \gamma_0 = c_{B}, \quad \eta_0 = c_{A} \]

\[ T_{Ld22a} = \frac{k_{m}g_{A}g_{0}}{S\left(c_{A} - k_{e}c_{B}\right)c_{H} + \left(c_{A} - k_{e}c_{B}\right)c_{0}\left(c_{A} + c_{B} - k_{e}c_{B}\right)c_{H}} \]

\[ -k_{m}g_{A}c_{H}^{2} \]

\[ (c_{A} - c_{B})c_{0}^{2} \]

with:

\[ \frac{c_{A}}{c_{B}} = \frac{g_{A}}{g_{B}} \]
Generating Function Type $B_1$

\[ G_A \]

Identification with the general parameters:

\[ B_1 \cdot \frac{C_A}{C_B} \quad \gamma_1 = C_B \quad n_1 = C_A \]

\[ \beta_0 = \frac{G_A}{G_B} \quad \gamma_0 = G_B \quad \sigma_0 = 0 \]

End para. Types 22a(i)

\[ \frac{K_m}{m} G_0 C_A \frac{K_0}{C_L} \]

with

\[ \frac{C_A}{C_B} \quad \frac{G_A}{G_B} \]
is given and hence expressions for the pole \( Q_p \) and the
active sensitivities \( S_{Km}, S_{K0} \) are easily obtainable.

Some of these circuits might have appeared in the liter-
ature, however it should be noted that they are derived here
systematically as special cases so as to satisfy the zero
GwSP condition.

3.3 A Band Pass Realization of Type 1

The overall realization consisting of two amplifiers is
shown in Fig. 3.3.1a (Figs. 3.3.1b and 3.3.1c show the actual
realizations of the two amplifiers contained in Fig. 3.3.1a).
The generating circuit is of type RI. The generating transfer
function \( t_{in} \) is of type R0. The TAC is a Band Pass filter of
type 1; Its transfer function is given by

\[
T_{BP1} = \frac{K_m K_0 G_0 C_1 S}{C_1 C L S^2 + [G_1 C_1 + C_1 (G_0 + G_h) - K K_0 C_1 C_1 S + G_1 (G_0 + G_h)]}
\]        \hspace{2cm} (3.3.1)

\[
K_m = 1 + \frac{G_A}{C_B} \hspace{2cm} (3.3.1a)
\]

\[
K_0 = 1 + \frac{G_A'}{C_B'} \hspace{2cm} (3.3.1b)
\]

For the transfer function given in Eqn. 3.3.1 we have

\[
\omega_p = \sqrt{\frac{G_1 G_h}{C_1 C_L} \left( 1 + \frac{G_0}{C_h} \right)} \hspace{2cm} (3.3.2)
\]
FIGURE 3.3.1a

FIGURE 3.3.1b (P_{III})

FIGURE 3.3.1c (K_0)

FIGURE 3.3.1 A BAND-PASS, TYPE 1 TAC REALIZATION
\[ S_{K}^{m} = S_{K}^{0} = 0 \]  

\[ \text{(3.3.3)} \]

\[ 0_{P} = \frac{C_{0} \cdot G_{0}}{C_{1} \cdot G_{1} \cdot (1 + \frac{G_{0}}{G_{1}})} \]  

\[ \text{(3.3.4)} \]

\[ 0_{P} = \frac{0_{P}}{S_{K}^{0}} = \frac{K_{m} \cdot K_{0}}{1 + \frac{G_{0}}{G_{1}} \cdot \frac{C_{1} \cdot G_{1}}{C_{1} \cdot G_{1} + K_{m} \cdot K_{0}}} \]  

\[ \text{(3.3.5)} \]

\[ K_{m} \cdot C_{m}^{2} = \frac{K_{m}^{2} \cdot C_{m}^{2} \cdot G_{0}}{G_{1} \cdot G_{1} + K_{m} \cdot K_{0}} \]  

\[ \text{(3.3.6)} \]

\[ K_{m}^{2} \cdot C_{m}^{2} \cdot G_{0} \cdot C_{1} \cdot G_{1} \cdot (1 + \frac{G_{0}}{G_{1}}) = \frac{0_{P}}{S_{K}^{0}} \]  

\[ \text{(3.3.7)} \]

\[ \frac{(K_{m} + K_{0}) K_{m} \cdot K_{0}}{1 + \frac{G_{0}}{G_{1}} \cdot \frac{C_{1} \cdot G_{1}}{C_{1} \cdot G_{1} + K_{m} \cdot K_{0}}} = K_{m} \cdot K_{0} \]  

\[ \text{(3.3.8)} \]

From Eqn. 3.3.4 it is possible to express the quantity \( K_{m} \cdot K_{0} \) as:

\[ K_{m} \cdot K_{0} = \frac{C_{0} \cdot G_{1} \cdot C_{1}}{C_{1} \cdot C_{1} \cdot G_{1} \cdot C_{1}^{-1}} - \frac{1}{G_{0} \cdot C_{1} \cdot C_{1}^{-1} \cdot (1 + G_{1})} \]  

\[ \text{(3.3.9)} \]

Hence using Eqn. 3.3.8 and 3.3.9.
\[ E = \frac{(K_m + K_0) \left[ 1 + \frac{G_0}{G_H} \cdot \frac{C_L}{C_1} - \frac{G_1}{G_H} \cdot \frac{C_L}{C_1} \cdot \left( 1 + \frac{G_0}{G_H} \right) \right]}{\frac{1}{Q_p} \cdot \frac{G_1}{G_H} \cdot \frac{C_L}{C_1} \cdot \left( 1 + \frac{G_0}{G_H} \right)} \]  

(3.3.10)

From Eqs. 3.3.9 and Eq. 3.3.10 it is clear that for any prescribed value of \( Q_p \) and for given values of the passive elements the smallest value of \( F \) is obtained if we let

\[ F_{\text{min}} = K_0 = K. \]  

(3.3.11)

Under this condition

\[ K = \frac{G_0}{G_H} \cdot \frac{C_L}{C_1} - \frac{G_1}{G_H} \cdot \frac{C_L}{C_1} \cdot \left( 1 + \frac{G_0}{G_H} \right) \]  

(3.3.12)

Assuming now

\[ \frac{G_0}{G_H} \cdot \frac{C_L}{C_1} \gg \frac{1}{Q_p} \cdot \frac{C_L}{C_1} \cdot \frac{G_1}{G_H} \cdot \left( 1 + \frac{G_0}{G_H} \right) \]  

(3.3.13)

which is a valid assumption as long as the spreads are moderate and \( Q_p \) high and defining:

\[ \frac{C_L}{C_1} = \gamma_c \]  

(3.3.14a)

\[ \frac{G_1}{G_H} = \gamma_1 \]  

(3.3.14b)

\[ \frac{G_0}{G_H} = \gamma_0 \]  

(3.3.14c)
We get

\[ F_m = 2Q_p \frac{[1 + \gamma_0 + \gamma_1 \gamma_C]}{[\gamma_1 \gamma_C (1+\gamma_0)]^{1/2}} \]  \hspace{1cm} (3.3.15)

Further if we define

\[ 1 + \gamma_0 = x \] \hspace{1cm} (3.3.16a)

\[ \gamma_1 \gamma_0 = \gamma_1 c \] \hspace{1cm} (3.3.16b)

\[ F_m \] becomes

\[ F_m = 2Q_p \frac{[x+\gamma_1 c]}{(x \gamma_1 c)^{1/2}} \] \hspace{1cm} (3.3.17)

Fig. 3.3.2 shows the variation of \( \frac{F_m}{Q_p} \) versus \( \gamma_1 c \) for several values of \( x \). The lowest value of \( F_m \) is \( 5.2Q_p \) and is obtained for \( \gamma_1 c = 0.5 \) and \( x = 1 \) which implies an unrestricted resistive spread. The following set of design equations appears to be a good compromise able to achieve low values of resistive spread \( \gamma_R \) and of \( F_m \) for specified values of \( Q \) and of the capacitive spread \( \gamma_C \). Letting

\[ \gamma_1 = \gamma_0 = \gamma \] \hspace{1cm} (3.3.19)

in the expression of \( \frac{F_m}{Q_p} \) given in Eqn. (3.3.15). Minimization of \( \frac{F_m}{Q_p} \) using \( \frac{\partial}{\partial \gamma} \left( \frac{F_m}{Q_p} \right) = 0 \) yields
\[ \frac{F_m}{Q_p} \text{ versus } \gamma_{lc} \text{ of the band pass type 1 TAC for several values of } x \]
\[ \gamma = \frac{\gamma_c^{1/2} + (\gamma_c^2 + \gamma_c + 1)^{1/2}}{1 + \gamma_c} \] (3.3.20)

The corresponding values of \( K_m = K_0 \) and \( \frac{F_m}{Q_p} \) can be obtained respectively from Eqns. 3.3.12 and 3.3.15 as

\[ K = [1 + \gamma_c(1 + \gamma_c)] - \frac{1}{Q_p} \left[ \gamma \gamma_c(1 + \gamma) \right]^{1/2} \right]^{1/2} \] (3.3.21)

and

\[ \frac{F_m}{Q_p} = \frac{2[1 + \gamma_c(1 + \gamma_c)]^{3/2}}{[\gamma \gamma_c(1 + \gamma)]^{1/2}} \] (3.3.22)

Fig. 3.3.3 shows the variation of \( \gamma \), \( \frac{F_m}{Q_p} \), \( \gamma_R \) versus \( \gamma_c \).

\( \omega_p \) and \( Q_p \) can be expressed as

\[ \omega_p = \frac{C_p^l}{C_p^l} \gamma_c \gamma (1 + \gamma) \] (3.3.23a)

\[ Q_p = \frac{(\gamma \gamma_c(1 + \gamma))^{1/2}}{1 + \gamma_c(1 + \gamma)^{-K^2}} \] (3.3.23b)

The elemental values are determined when \( \omega_p \), \( Q_p \), \( \gamma_c \) (and hence \( \gamma \)) are prescribed using Eqns. 3.3.1a, b, 3.3.14, 3.3.19, 3.3.20, 3.3.21, 3.3.23.
FIGURE 3.3.3 $\frac{F_m}{Q_p}, K, \gamma, \gamma_R$ of the Band Pass Type 1 TAC, versus $\gamma_c$. 
3.3.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below:

Designed values:

\[ \omega_p = 6283.185 \text{ rad/sec.} = 1000 \text{ Hz.} \]

\[ Q_p = 50 \]

\[ \gamma_c = 1 \]

\[ \gamma_0 = \gamma_1 = 0.366 \]

\[ K_0 = K_m = 1.3107 \]

\[ C_1 = C_L = 15 \text{ KPF.} \]

\[ R_H = R_B = R_B' = 7.502 \text{ K\Omega} \]

\[ R_1 = R_0 = 20.497 \text{ K\Omega} \]

\[ R_A = R_A' = 24.145 \text{ K\Omega} \]

GQSP Values:

\[ \begin{vmatrix} K_0 S_{Q_p} \\ Q_p \end{vmatrix} \begin{vmatrix} Q_p \\ K_m S_{K_m} \end{vmatrix} = 3.224 \]

\[ \frac{P}{Q_p} = 6.45 \]

Element spread:

- Capacitive 1:1
- Resistive 3,257:1

The 3dB frequencies are:

\[ f_{C_1} = 990.05 \text{ Hz.} \quad f_{C_2} = 1010.05 \text{ Hz.} \]
Any of $G_1$, $G_n$ or $G_0$ can be used to achieve the tuning of $\omega_p$. Then any or both of $K_m$, $K_0$ can be used to achieve the tuning of $Q_p$ without affecting the value of $\omega_p$ because $\omega_p$ is independent of both $K_0$ and $K_m$.

The circuit was implemented using 1% tolerance resistors and 2% tolerance capacitors with values chosen to be, within the range of currently available elements, as close as possible to the designed ones. (Trim pots were used wherever necessary.) The OA's which have been used were LM741. Actual values obtained are:

$$\omega_p = 1000.6 \text{ Hz.}$$
$$F_{Cl} = 990.2 \text{ Hz.}$$
$$F_{C2} = 1010.4 \text{ Hz.}$$
$$Q_p = 49.53$$

$Q_p$ and $\omega_p$ variations from their designed values are 0.93% and 0.06% respectively; this is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. Power supply voltages used were ±10V and ±15V. Also the OA's were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supply voltages used. Only the OA's were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 3.3.4 no appreciable change in the response was observed; thus the experimental results confirm
Figure 3.3.4 (110V)

Figure 3.3.4b (15V)

Figure 3.3.4 Frequency response of the band pass type 1 TAC for $Q_p = 50$
the theoretical predictions.

3.3.2: Effect of the pole of the OA on \(Q_p\) and \(\omega_p\) [17, 33].

Replacing the amplifiers of Fig. 3.3.1a by the networks of Fig. 3.3.1b and 3.3.1c the analysis gives

\[
D(S) = S^2 + \frac{S}{x+\gamma_1c}{K_0 \over K_m} \cdot \frac{\omega_p}{Q_p} \left\{ x + \gamma_1c \right\} + \frac{1}{x+\gamma_1c} \cdot \frac{1}{1 + \left( \frac{1}{K_0 A_0(S)} \right) \left( \frac{1}{K_m A_m(S)} \right)} \cdot \omega_p^2
\]

(3.3.24)

Using

\[\Lambda_0(S) = \frac{B_0}{S + \omega_0}\]

(3.3.25a)

\[\Lambda_m(S) = \frac{B_m}{S + \omega_m}\]

(3.3.25b)

where

\(B_0, B_m\) are the unity gain bandwidth of the amplifiers. \(\omega_0, \omega_m\) are the poles of the amplifiers. \(A_0 B_0/\omega_0, A_m B_m/\omega_m\) are the DC gains of the amplifiers. We get

\[
D(S) = a_4 S^4 + a_3 S^3 + a_2 S^2 + a_1 S = (S + P_1)(S + P_2)(S^2 + \frac{\omega_p}{Q_p} \hat{\omega_p}^2)
\]

\[S^4 \left\{ \frac{1}{B_0 B_m} \right\} + S^3 \left\{ \frac{1}{B_0} \left( \frac{1}{K_0 + \omega_m} \right) + \frac{1}{B_0 (K_0 + \omega_0)} \right\} + \left( \frac{\omega_p}{Q_p} \right) \left\{ x + \gamma_1 c \right\} \left( \frac{1}{K_0 A_0(S)} \right) \left( \frac{1}{K_m A_m(S)} \right) \]

* The dominant roots of \(D(S)\) are contained in the factor

\(S^2 + \frac{\omega_p}{Q_p} S + \hat{\omega_p}^2\)
\[ + S^2 \left[ \frac{1}{(K + B)(K + B_m)} + \frac{\omega_0}{Q_P} \frac{(x + \gamma_1c)}{K_mB} \cdot \frac{1}{(K + B_m)^2} \left( \frac{1}{B_0(K + B_m)} + \frac{1}{B_m(K + B_0)} \right) \right. \\
\left. + \frac{\omega_p^2}{B_0B_m} \right] + \\
+ S \left[ \frac{\omega_P}{(x + \gamma_1c)} \cdot \frac{K_mK_0}{K_mB_m} \left( \frac{1}{K + B_0} \right) \left( \frac{1}{K + B_m} \right) - 1 \right] + \\
\frac{2\omega_P}{B_0B_m} \left( \frac{1}{K + B_0} \right) \left( \frac{1}{K + B_m} \right) \right] + \\
\omega_0 \cdot (\frac{1}{K_0B_0} + \frac{1}{B_mK_0} + \frac{1}{B_0K_m}) \right] \]

If we let

\[ K_m = K_0 = K, \]
\[ B_m = B_0 = B, \]
\[ A_m = A_0 = A, \]
\[ \omega_m = \omega_0 = \omega_c, \]

and use

\[ B = \omega_c A, \]

we get

\[ D(S) = \]
\[ = \frac{S^4}{B^2} \]
\[ + S^3 \left[ \frac{2\omega_P}{B^2K^2A} + \frac{\omega_P}{Q_P} \frac{x + \gamma_1c}{(x + \gamma_1c)^2} \cdot \frac{1}{B^2} \right] \]
FIGURE 3.3.5  THE EFFECT OF THE OAs POLES ON $\omega_p$ FOR THE BAND PASS TYPE 1 TAC
FIGURE 3.3.6 THE EFFECT OF THE OAS POLES ON $Q_p$ FOR THE BAND PASS TYPE 1 TAC
\[
\begin{align*}
S^2 \left( \frac{1}{K_A^2} \right)^2 + \frac{\omega_p}{\omega_p} \left( \frac{x+y}{1} \right) \left( \frac{1}{K_A^2} \right) \left( \frac{1}{K_B^2} \right) + \frac{\omega_p^2}{B^2} + \\
S \left[ \frac{\omega_p}{\omega_p} \left( \frac{x}{1} \right) \left( \frac{1}{K_A^2} \right) \left( \frac{1}{K_B^2} \right) + \frac{\omega_p}{\omega_p} \left( \frac{x}{1} \right) \left( \frac{1}{K_A^2} \right) \left( \frac{1}{K_B^2} \right) \right] + \\
\left( \frac{x}{1} \right)^2 + \left( \frac{x}{1} \right)^2 \omega_p^2
\end{align*}
\]

(3.3.27)

Using Eqn. 3.3.24 the values of \( \frac{\Delta Q_p}{\omega_p} \) and \( \Delta Q_p \) due to the effect of the pole of the amplifier have been obtained for various values of \( \omega_p \) and are given in Fig. 3.3.5 and 3.3.6 respectively. The use of OA LM741 has been assumed in these computations.

3.4 A Low Pass Realization of Type 1

The overall realization consisting of two amplifiers is shown in Fig. 3.3.1a. (Figs. 3.3.1b and 3.3.1c show the actual realizations of the two amplifiers contained in Fig. 3.3.1a). This is a Low Pass filter containing an RI generating circuit having a \( T_v \) of type L0 and the denominator of \( T_v \) is of type 1. The transfer function is given by:

\[
T_{LP1} = \frac{K_m K_0 G_H G_1}{S^2 C_1 C_H + S \left[ G_H C_H + G_0 C_1 - K_m K_0 G_H C_H + G_1 G_0 \right]}
\]

(3.4.1a)

\[
K_m = 1 + \frac{G_A}{G_B}
\]

(3.4.1b)

\[
K_0 = 1 + \frac{G_A}{G_B}
\]

(3.4.1c)
FIGURE 3.4.1a

FIGURE 3.4.1b

FIGURE 3.4.1c

FIGURE 3.4.1d A LOW PASS TYPE 1 TAC REALIZATION
For the transfer function given in Eqn. 3.4.1 we have:

\[ w_p = \frac{G_1G_0}{C_1C_H} \left( \frac{1}{\sqrt{\frac{C_1}{C_H}}} \right) \quad (3.4.2) \]

\[ S_{K0}^{w_p} = S_{K_m}^{w_p} = 0 \quad (3.4.3) \]

\[ Q_p = \frac{G_0}{1+G_1C_H} \left( \frac{C_1}{C_H} \right)^{i/2} - K_m K_0 \quad (3.4.4) \]

\[ S_{K0} = S_{K_m} = G_0 \frac{C_1}{C_H} \left( \frac{1}{1+G_1C_H} \right) - K_m K_0 \quad (3.4.5) \]

\[ K_m^2 S_{K_p}^P = \frac{K_m^2 K_0}{G_0 \frac{C_1}{C_H} \left( 1+G_1C_H \right) - K_m K_0} \quad (3.4.6) \]

\[ K_0^2 S_{K_m} = \frac{K_0^2 K_0}{G_0 \frac{C_1}{C_H} \left( 1+G_1C_H \right) - K_m K_0} \quad (3.4.7) \]

\[ F = \frac{(K_m +K_0) K_m K_0}{G_0 \frac{C_1}{C_H} \left( 1+G_1C_H \right) - K_m K_0} \quad (3.4.8) \]

The technique of minimization of \( F \) is the same as before;
however since the equations are different, it is discussed below. From Eqn. 3.4.4, it is possible to express the quantity $K_m K_0$ as

$$K_m K_0 = 1 + \frac{G_0 C_1}{G_1 C_H} - \frac{1}{Q_p} \left( \frac{G_0 C_1}{G_1 C_H} \right)^{1/2}$$  \hspace{1cm} (3.4.9)

Hence using (3.4.8) and (3.4.9)

$$F = \left( K_m K_0 \right)^{1/2} \left[ 1 + \frac{G_0 C_1}{G_1 C_H} - \frac{1}{Q_p} \left( \frac{G_0 C_1}{G_1 C_H} \right)^{1/2} \right]$$  \hspace{1cm} (3.4.10)

From Eqn. 3.4.9 and 3.4.10 it is clear that for any prescribed value of $Q_p$ and for given values of the passive elements the smallest value of $F$ is obtained if we let

$$K_m = K_0 = K$$  \hspace{1cm} (3.4.11)

Under this condition we have

$$K = \left[ 1 + \frac{G_0 C_1}{G_1 C_H} - \frac{1}{Q_p} \left( \frac{G_0 C_1}{G_1 C_H} \right)^{1/2} \right]$$  \hspace{1cm} (3.4.12)

Assuming now that

$$1 + \frac{G_0 C_1}{G_1 C_H} \gg \frac{1}{Q_p} \left( \frac{G_0 C_1}{G_1 C_H} \right)^{1/2}$$  \hspace{1cm} (3.3.13)
which is a valid assumption as long as the spreads are moderate and $Q_p$ high, and defining

\[
\frac{C_1}{C_H} = \gamma_c \tag{3.3.14a}
\]

\[
\frac{G_0}{G_1} = \gamma_0 \tag{3.3.14b}
\]

We get

\[
F_m = \frac{2 Q_p [1 + \gamma_0 \gamma_c]^{3/2}}{[\gamma_0 \gamma_c]^{1/2}} \tag{3.3.15}
\]

If we define again

\[
\gamma_0 \gamma_c = \gamma_{0c} \tag{3.3.16}
\]

then

\[
\frac{F_m}{Q_p} = \frac{2(1+\gamma_{0c})^{3/2}}{\gamma_{0c}^{1/2}} \tag{3.4.17}
\]

Fig. 3.4.12 shows the variation of $F_m/Q_p$ versus $\gamma_{0c}$. The lowest value of $F_m$ is 5.2 $Q_p$ and is obtained for $\gamma_{0c} = 0.5$ which means that we should have

\[
\frac{C_1}{C_H} = \frac{G_0}{G_1} = 0.5 \tag{3.4.18}
\]

The value of $K$ is obtained using Eqn. 3.4.12 and 3.5.18 as
FIGURE 3.4.2 $F_m/Q_p$ VERSUS $\gamma_{0c}$ FOR THE LOW PASS TYPE 1 TAC
\[ K = [1.5 - \frac{1}{\sqrt{2} Q_p}] \]  \hspace{1cm} (3.4.19)

\( \omega_p \) can be expressed using Eqs. 3.4.2 and 3.4.18 as

\[ \omega_p = \frac{1}{\sqrt{2} C_1} \]  \hspace{1cm} (3.4.20)

The elemental values are obtained when \( \omega_p \) and \( Q_p \) are prescribed using Eqs. 3.4.1a, b, 3.4.18, 3.4.19, 3.4.20.

Although this circuit has been already reported in the literature [23, 24] it has been studied here and design equations for a minimized value of \( F \) given to show that it is only a particular case of the general configuration presented in this thesis.

3.5 A High Pass Realization of Type I

The overall realization consisting of two amplifiers is shown in Fig. 3.5.1a (Figs. 3.5.1b and 3.5.1c show the actual realizations of the two amplifiers contained in Fig. 3.5.1a). It consists of an RI generating circuit of Type B0. The entire circuit is a High Pass filter having a denominator of Type V. Its transfer functions is given by

\[ T_{HPI} = \frac{K_m K_0 C_0 C_1 S^2}{S^2 + (G_1 C_0 + G_H C_1 - K_m K_0 G_1 C_1) S + G_1 G_H} \]  \hspace{1cm} (3.5.1a)

\[ K_m = 1 + \frac{G_A}{G_B} \]  \hspace{1cm} (3.5.1b)
FIGURE 3.5.1a

FIGURE 3.5.1b

FIGURE 3.5.1c

FIGURE 3.5.1 A HIGH PASS TYPE 1 TAC REALIZATION
\[ K_0 = 1 + \frac{G^A}{G^B} \]  

(3.5.1c)

The design equations of the High Pass filter are obtained from those of the Low Pass Filter by RC-CR transformation. Therefore the curve given in Fig. 3.4.2 and the expressions for \( F_m \) and \( K \) obtained in Eqns. 3.4.17 and 3.4.19 are valid in this case also.

3.6 A Band Pass Realization of Type 22b

For the sake of completeness, we show that the method of minimization of \( F \) can be applied to any polynomial decomposition by giving here a Band Pass realization of Type 22b. The overall realization consisting of two amplifiers is shown in Fig. 4.6.1a (Figs. 4.6.1b and 4.6.1c show the actual realizations of the two amplifiers contained in Fig. 3.6.1a). It consists of an RI generating circuit having a \( t \) of Type B0. The entire circuit corresponds to a Band Pass filter having a denominator of Type 22b. Its transfer function is given by:

\[
T_{BP22} = \frac{K_m K_0 G_0 C_1 S}{S^2 [C_1 (C_F + C_L) + K_m C_1 C_F]} + S \{ (G_0 + G_F) C_1 G_1' (C_L + C_F) - K_m K_0 G_1 C_1 + K_m (G_F C_1 + C_F G_1') \} + (G_0' + G_H + G_F) C_1 + K_m C_1 G_F \}
\]

(3.6.1a)
Figure 3.6.1a

Figure 3.6.1b

Figure 3.6.1c

Figure 3.6.1 A Band Pass Type 22b TAC Realization
With
\[ K_m = \frac{G_A}{G_B} \]  \hspace{1cm} (3.6.1b)
\[ K_0 = \frac{G_A}{G_B} \]  \hspace{1cm} (3.6.1c)
\[ G'_0 = G_0 + G_A \]  \hspace{1cm} (3.6.1d)
\[ G'_1 = G_1 + G_A \]  \hspace{1cm} (3.6.1c)

These equations take into account the loading effect of the amplifiers. The condition which assures a zero pole frequency deviation is given by
\[ \frac{G'_1(G'_0 + C_H + C_F)}{C'_1(C'_F + C_L)} = \frac{G'_1 G_F}{C'_1 C_F} = \frac{2}{\omega_P} \]  \hspace{1cm} (3.6.2)

Under this condition
\[ \omega_P = 0 \]  \hspace{1cm} S_{K_m} = S_{K_0} \]  \hspace{1cm} (3.6.3)

\[ Q_F = \left( \frac{[[G'_0 + G_h + G_L] G'_1 + K_m G'_1 C_1 + K_m C_1 C_F ] \cdot (C'_F + C_L) C'_1 + K_m C_1 C_F }{C_1 (G'_0 + G_h + G_L) + G'_1 (C'_F + C_L) + K_m [G'_0 + G_h + G_L] C'_1 + K_m C_1 C_F } \right)^{1/2} \]  \hspace{1cm} (3.6.4)

\[ S_{K_m} = \frac{1}{2} \frac{K_m G_F}{C'_F + C'_L + K_m C_F} + \frac{K_m G_F}{G'_0 + G_h + G_L + K_m G_F} \]  \hspace{1cm} (3.6.5)
\[ S_{K_0} = \frac{K_{m} K_{0} G_{H} C_{L}}{C_{L} (G'_{0} + G_{H} + G_{F}) + G'_{1} (C_{L} + C_{F}) + K_{m} \{ G_{F} C_{L} + C_{F} G_{1} - K_{0} G_{H} C_{L} \}} \]

(3.6.6)

Using Eqn. 3.6.2 let us define

\[ \frac{C_{L}}{C_{F}} = \frac{G'_{0} + G_{H}}{G_{F}} = \lambda \quad (3.6.7a) \]

and

\[ \left( \frac{G'_{1}}{G_{F}} \cdot \frac{C_{L}}{C_{F}} \right)^{1/2} = a \quad (3.6.7b) \]

\[ \frac{G'_{1}}{G_{F}} + \frac{C_{L}}{C_{F}} = b \quad (3.6.7c) \]

\[ \frac{G_{H}}{G_{F}} \cdot \frac{C_{L}}{C_{F}} = c \quad (3.6.7d) \]

We have

\[ Q_{P} = \frac{a (1 + K_{m} + \lambda)}{b (1 + K_{m} + \lambda) - K_{m} K_{0} C} \quad (3.6.8) \]

\[ Q_{P} = \frac{K_{m}}{1 + \lambda + K_{m}} - \frac{K_{m} (b - K_{0} C)}{(1 + \lambda) b + K_{m} (b - K_{0} C)} \quad (3.6.9) \]

\[ S_{K_{m}} = \frac{K_{m} K_{0} C}{(1 + \lambda) b + K_{m} (b - K_{0} C)} = \frac{b}{a} Q_{P} - 1 \quad (3.6.10) \]

\[ K_{0} = \frac{0.5 \cdot a \cdot b \cdot Q_{P} - 1) (1 + K_{m} + \lambda)}{K_{m} Q_{P}^{a}} \quad (3.6.11) \]
Where it is necessary that

\[ K_0 < \frac{b}{c} \frac{1+\lambda + K_m}{K_m} \]  \hspace{1cm} (3.6.12)

Using Eqn. 3.6.8, 3.6.9, and 3.6.10, we get

\[ \left| K_m S_{K_m} \right| = \frac{K_m}{1 + \left( \frac{1}{1+\lambda} \right)} \frac{b}{a} Q_p - 1 \]  \hspace{1cm} (3.6.13a)

\[ \left( \frac{b}{a} Q_p - 1 \right) > 0 \]  \hspace{1cm} (3.6.13b)

\[ \left| K_m S_{K_m} \right| = \frac{1}{Q_p} \left( \frac{b}{a} Q_p - 1 \right)^2 \left( \frac{1+\lambda}{K_m} + 1 \right) \]  \hspace{1cm} (3.6.14)

Hence if \( \frac{b}{a} Q_p \gg 1 \)

\[ F = \frac{b}{a} Q_p \left( \frac{1+\lambda}{K_m} + 1 \right) + \frac{K_m}{1+\lambda} \]  \hspace{1cm} (3.6.15)

A set of design equations can be obtained as follows. It is clear from Eqn. 3.6.7 that the quantity \( \frac{b}{a} \) has a minimum value which is two and which is obtained if we let

\[ \frac{G_1}{G_F} = \frac{C_L}{C_F} \]  \hspace{1cm} (3.6.16)

Also if we let

\[ \frac{C_L}{C_F} = \frac{G_H}{G_F} \Rightarrow c_0 = \lambda \]  \hspace{1cm} (3.6.17)
Figure 3.6.2: $\frac{F_m}{Q_P}$ versus $K_m$ of the band pass type 22b TAC for several values of $\lambda$.
where

\[
\epsilon_0 = \frac{C_0}{C_F}
\]

\[
\frac{F_m}{Q_p} = 2\left[\frac{2}{(\lambda - \epsilon_0)} \left(\frac{1+\lambda}{K_m} + 1\right) + \frac{K_m}{K_m^{\frac{1}{1+\lambda}}}\right]
\]

(3.6.19)

And assume that

\[
\epsilon_0 \ll \lambda
\]

(3.6.20)

Then

\[
\frac{F_m}{Q_p} = 2\left[\frac{2}{\lambda} \left(\frac{1+\lambda}{K_m} + 1\right) + \frac{K_m}{K_m^{\frac{1}{1+\lambda}}}\right]
\]

(3.6.21)

The variation of \(F_m\) versus \(K_m\) for several values of the capacitive spread \(\lambda\) is shown in Fig. 3.6.2 according to Eqn. 3.6.21. The corresponding value of \(K_0\) for any prescribed value of \(K_m\) can be obtained using Eqn. 3.6.11.

The elemental values are obtained when \(\omega_p, Q_p, K_m, \lambda, \epsilon_0\), are prescribed using Eqns. 3.6.1b to 3.6.1e, 3.6.2, 3.6.11, 3.6.15, 3.6.16, 3.6.17, 3.6.18.

3.6.1 Experimental results

The network shown in Fig. 3.6.1 was built up using discrete elements and tested. The results are summarized below.
Designed values:

\[ \omega_p = 6283.185 \text{ Rad/sec.} = 1000 \text{ Hz}. \]

\[ Q_p = 50 \]

\[ \lambda = 5 \]

\[ K_m = 2 \]

\[ c_0 = 0.4 \]

\[ a = 5 \]

\[ b = 10 \]

\[ c = 23 \]

\[ K_0 = 1.72174 \]

\[ C_I = C_1 = 22 \text{ KPF} \]

\[ C_P = 4.4 \text{ KPF} \]

\[ R_F = 36.172 \text{ K\Omega} \]

\[ h_1 = h_A = 14.488 \text{ K\Omega} \]

\[ R_H = 7.863 \text{ K\Omega} \]

\[ R_0 = R_A = 180.857 \text{ K\Omega} \]

\[ R_B = 361.715 \text{ K\Omega} \]

\[ R'_B = 24.910 \]

GQSP values:

\[ \frac{|K_m S K_m|}{Q_p} = 3 \]

\[ \frac{|K_0 S K_0|}{Q_p} = 3.48 \]

\[ \frac{F_m}{Q} = 6.48 \]
Element spread:

Capacitive 1:1
Resistive 46:1

The 3dB frequencies are:

\[ F_{C1} = 990.05 \text{ Hz} \quad F_{C2} = 1010.05 \text{ Hz} \]

Any of \( G_1, G_2, \) or \( G_0 \) can be used to achieve the tuning of \( \omega_p \). \( K_0 \) can be used to achieve the tuning of \( Q_p \) without affecting the value of \( \omega_p \) because \( \omega_p \) is independent of both \( K_0 \) and \( K_m \).

The circuit has been implemented using 1% tolerance resistors and 5% tolerance capacitors with values chosen to be, within the range of currently available elements to be as close as possible to the designed ones. (Trim pots have been used wherever necessary.) The OAs which have been used were LM741. Actual values obtained

\[ \omega_p = 999.8 \text{ Hz} \]
\[ F_{C1} = 989.3 \text{ Hz} \quad F_{C2} = 1009.6 \text{ Hz} \]
\[ Q_p = 49.211 \]

\( Q_p \) and \( \omega_p \) variations from their designed values are 1.58% and 0.1% respectively. This is due to the small differences between the designed values and the real ones as well as to the tolerances of the elements. Power supply voltage used were +10V and +15V. Also the OAs were heated and their
FIGURE 3.6.3 FREQUENCY RESPONSE OF THE BAND PASS TYPE 22b TAC FOR $Q_p = 50$. 

**FIGURE 3.6.3a (±10V)**

**FIGURE 3.6.3b (±15V)**
temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supply voltages. Only the OAs were heated. The passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 3.6.3 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions. It was found that the circuit was stable during activation.

3.6.2 Effect of the poles of the OAs on \( \omega_p \) and \( Q_p \)[17,33]

Replacing the amplifiers of Fig. 3.4.1a by the networks of Fig. 3.4.1b and 3.4.1c analysis yields

\[
D(S) = S^2 \left[ \frac{g_k m e_k}{A(S) A_0(S)} + \frac{g_k k_m}{A(S) A_0(S)^2} \right]
\]

\[
S \left\{ \frac{\omega_p^2}{Q_p} \left[ \frac{g_k m e_k}{A(S) A_0(S)} + \frac{g_k k_m}{A(S) A_0(S)^2} \right] \right\}
\]

\[
\frac{\omega_p^2}{Q_p} \left[ \frac{g_k m e_k}{A(S) A_0(S)} + \frac{g_k k_m}{A(S) A_0(S)^2} \right]
\]

(3.6.22a)

where

\[
c = 1 + \lambda + K_m
\]

(3.6.22b)

\[
k_m = 1 + K_m
\]

(3.6.22c)

\[
k_0 = 1 + K_0
\]

(3.6.22d)

\[
g = 1 + \lambda
\]

(3.6.22e)
\[ Y = \frac{e}{b e^{-K K_0 C}} \]  
\[ (3.6.22f) \]

\[ \lambda = \frac{C_L}{C_F} = \frac{G'_L + G_H}{C_F} = \frac{C_1}{C_F} \]  
\[ (3.6.22g) \]

\[ \frac{C_1}{C_F} + \frac{C_1}{C_F} = b \]  
\[ (3.3.22h) \]

\[ \frac{G_H}{C_F} - \frac{C_1}{C_F} = c \]  
\[ (3.3.22i) \]

Using

\[ A_0(S) = \frac{B_0}{S + \omega_0} \]  
\[ (3.3.23a) \]

\[ A_m(S) = \frac{B_m}{S + \omega_m} \]  
\[ (3.3.23b) \]

where

\( B_m \) and \( B_0 \) are the unity gain bandwidth of the amplifiers.

\( \omega_0, \omega_m \) are the poles of the amplifiers. \( A_0, A_m \) are the DC gains of the amplifiers. We get

\[ D(S) = \]

\[ S^4 \left[ \frac{g k k_0}{B_m B_0} \right] + \]

\[ S^3 \left[ \frac{g k m e k_0}{B_m B_0} + \frac{g k k_0}{B_m B_0} \right] \left( \omega_m + \omega_0 \right) + \frac{\omega_0 g k m 0}{B_p B_m B_0} \]

\[ + S^2 \left[ g k m \omega_m e k_0 \omega_0 + g k k_0 \omega_m \omega_0 + g k m 0 \omega_m \omega_0 + \frac{\omega_0 g k m 0}{B_p B_m B_0} \right] \]

\[ + \frac{\omega_p g k m 0}{B_m B_0} \]
\[
+ S \left[ \omega_p \left\{ e + yb \left[ \frac{g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot m \cdot 0}{Q_p} \right] \frac{g \cdot k \cdot m \cdot 0 \cdot m \cdot 0}{B} \right\} \right]
\]
\[
+ \omega_p \left\{ \frac{g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot m \cdot 0}{B} \right\} \left\{ \frac{g \cdot k \cdot m \cdot 0 \cdot m \cdot 0}{B} \right\}
\]
\[
+ \omega_p \left\{ e + \frac{1}{A} \left( g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot 0 \right) + \frac{g \cdot k \cdot m \cdot 0 \cdot 0}{A} \right\}
\]
\[
\left( 3.3.24 \right)
\]

If we let

\[
B_m = B_0 = B \quad m = m_0 = A \quad \omega_m = \omega_0 = \omega_c
\]

\[
D(S) =
\]
\[
S^4 \left[ \frac{g \cdot k \cdot m \cdot k \cdot 0 \cdot 0}{B} \right] +
\]
\[
S^3 \left[ \frac{1}{B} \left( g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot 0 \right) + \frac{2}{AB} \frac{g \cdot k \cdot m \cdot k \cdot 0 \cdot 0}{Q_p} \right]
\]
\[
+ S^2 \left[ e + \frac{1}{A} \left( g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot 0 \right) + \frac{g \cdot k \cdot m \cdot k \cdot 0 \cdot 0}{A} \right]
\]
\[
+ \frac{\omega_p}{Q_p} \left\{ e + yb \left[ \frac{1}{A} \left( g \cdot k \cdot m \cdot e \cdot k \cdot 0 \cdot 0 \right) + \frac{g \cdot k \cdot m \cdot k \cdot 0 \cdot 0}{A} \right] \right\}
\]
\[
\left( 3.3.25 \right)
\]

Using Eqn. 3.3.25 the values of \( \frac{\Delta P}{\omega_p} \) and \( \frac{\Delta Q_p}{Q_p} \) due to the effect of the pole of the amplifiers have been obtained for various
Figure 3.6.4 The effect of the QAS poles on $\omega_p$ for the band pass type 22b TAC
Figure 3.6.5. The effect of the OAS poles on $Q_p$ for the band pass type 22B Tac.
values of \( \omega_p \) and are given in Fig. 3.6.4 and 3.6.5 respectively. The use of OA LM741 have been assumed in these computations.

3.7 Summary and Discussion

In this chapter we have considered TACs using a first order generating function \( t_v \). It is shown that eight polynomial decompositions can be obtained when the admittance are of the form \( y = CS+G \) and require only two types of \( t_v \). Only Low Pass, High Pass and Band Pass filters can be realized. These filters are designed to yield zero \( G_{\omega SP} \) and minimized \( F \). Hence the quantity \( \left| \frac{\Delta T_v(S)}{T_v} \right|_{S=j\omega_p} \) is also minimized.

Two Band Pass filters were built and tested. Experimental results confirm the theoretical studies.
CHAPTER IV

ZERO GwSP Q-MULTIPLIED CIRCUITS
CHAPTER IV

ZERO \( g_{SP} \) Q-MULTIPLIED CIRCUITS

4.1 Introduction

In the previous chapter we considered realizations using two finite gain amplifiers when the generating circuit consisted of first order transfer functions. In this chapter we will discuss realizations when the generating circuit consists of second order transfer functions. This results in Q-multiplication. The quantity \( F \) of the two amplifier network will be minimized to obtain Q multiplier circuits having zero \( g_{SP} \) and minimized \( \Lambda Q_p \). The technique is illustrated by selecting appropriate single-amplifier generating circuits.

4.2 The Basic Q-Multiplier Configuration

The proposed configuration is shown in Fig. 4.2.1 which is obtained from Fig. 2.3.1a by letting \( \gamma_L = 0 \).

Since

\[
t_v = K_0 \frac{n_0(S)}{D_0(S)} = \frac{K_0(a_2 S^2 + a_1 S + a_0)}{(\beta_2 - K_0 \gamma_2) S^2 + (\beta_1 - K_0 \gamma_1) S + (\beta_0 - K_0 \gamma_0)}
\]

(4.2.1a)

\[
0 < \gamma_2 < \beta_2 \quad \quad 0 < \gamma_1 < \beta_1
\]

(4.2.1b)

The transfer function \( T_v \) of the configuration shown in Fig. 4.2.1 will be
Figure 4.2.1 The proposed QMC with $V_L = 0$ and all the impedances purely resistive.
$$T_v = \frac{K_m K_0 C_0 (a_2 S^2 + a_1 S + a_0)}{S^2 \left[ \gamma_2 G_0 + G_F + G_H \right] - K_m^2 G_F - K_0 \gamma_2 C_0 \left( G_0 + G_F + G_H \right)}$$

$$+ K_m K_0 \left( \gamma_2 G_F - a_2 G_H \right)$$

$$+ S \left[ \gamma_1 (G_0 + G_F + G_H) - K_m^2 G_F - K_0 \gamma_1 C_0 \left( G_0 + G_F + G_H \right) \right]$$

$$+ K_m K_0 \left( \gamma_1 G_F - a_1 G_H \right)$$

$$+ \left[ \frac{c_0 (G_0 + G_F + G_H)}{K_m^2 G_F} - K_0 \gamma_0 C_0 \left( G_0 + G_F + G_H \right) \right]$$

$$+ K_m K_0 \left( \gamma_0 G_F - a_0 G_H \right)$$

(4.2.1)

It is known from Chapter II that for the condition

$$\frac{\Delta \omega_p}{\omega_p} = 0$$

to be satisfied independently of $$K_m$$ and $$K_0$$, it is required that

$$\frac{c_0}{c_2} = \frac{\gamma_0}{\gamma_2} = \frac{a_0}{a_2} = \frac{\omega_p}{2}$$

(2.6.4)

This restricts the choice of suitable $$T_v$$ to the following types: Band Pass, Null, and All Pass. The Low Pass and High Pass types have to be excluded since any one of the quantities $$a_0$$ or $$a_2$$ cannot be set to zero without making the other zero.

The numerator of $$T_v$$ being identical (within a constant multiplier) to the numerator of $$t_v$$, we should then expect to be able to obtain Band Pass, Null and All Pass transfer functions but no Low Pass or High Pass ones through the use of the second order $$T_v$$ and the configuration shown in Fig. 4.2.1.
Table 4.2.1 gives all the possible polynomial decompositions and some of the properties of each decomposition based on Table 2.2.1. It has been shown in Section 2.6 that provided that the conditions of Eqn. 2.6.4 are fulfilled, the pole frequency $\omega_p$ of $T_v$ is the same as the pole frequency $\omega_{p0}$ of $t_v$ and its sensitivities with respect to the active parameters $K_m$, $K_0$ are equal to zero.

An examination of Table 4.2.1 shows that the twenty-four polynomial decompositions can be obtained by using only twelve generating functions $t_v = K_{00}(S)/D_0(S)$. These are tabulated in Table 4.2.2 together with the polynomial decompositions which are respectively realizable by each generating function.

It is clearly noted that these generating functions have to be realized by SACs satisfying the condition given in Eqn. 2.4.4. When this is done all the possible polynomial decompositions shown in Table 4.2.1 can be obtained. This enables us to realize two amplifier networks with the property of zero $G_{os}$ starting from optimized SACs [17, 18]. These realizations are by no means unique because each generating function can be realized in several ways, thus leading to several corresponding TACs.

In our study we are restricting our attention to Band Pass, Null or All Pass transfer functions only. Therefore the generating functions can only be Band Pass, Null or All Pass. In addition these generating functions will have to
THE POLYNOMIAL DECOMPOSITIONS SUITABLE FOR Q-MULTIPLICATION WITH ZERO GASp AND THE CORRESPONDING T, P, AND SENSITIVITIES.
<table>
<thead>
<tr>
<th>TABLE 4,2,1</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLYNOMIAL DECOMPOSITION TYPE 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t_v(s) )</th>
<th>( \frac{S_0 C_{15}}{S_2 S_{12} + S_1 S_{10} + S_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega_{p0} )</td>
<td>( \begin{bmatrix} S_0 \ 0 \ S_{11} \end{bmatrix} )</td>
</tr>
<tr>
<td>( Q_{p0} )</td>
<td>( S_{k_0} = 0 )</td>
</tr>
<tr>
<td>( T_v(s) )</td>
<td>( \frac{K_{m_0} C_{0} C_{15}}{S_{12} (G_0 + G_0) + S_1 (G_1 + G_1) + S_0 (G_0 + G_0)} )</td>
</tr>
<tr>
<td>( \Omega_{p} )</td>
<td>( Q_{p0} \left[ 1 - K_{m_0} C_0 C_{15} / (G_0 G_1) \right]^{-1} )</td>
</tr>
<tr>
<td>( S_{k_0} )</td>
<td>( \frac{Q_{p}}{Q_{p0}} - 1 )</td>
</tr>
<tr>
<td>( Q_{p} )</td>
<td>( Q_{p0} )</td>
</tr>
<tr>
<td>( S_{k_m} )</td>
<td>( S_{k_0} )</td>
</tr>
<tr>
<td>( Q_{p} )</td>
<td>( Q_{p0} )</td>
</tr>
</tbody>
</table>

**Bounds of** \( S_{k_0} \) **when all terms in** \( D(s) \) **are positive:**
- \( 0, 0 \)
- \( -1, 0 \)
- \( -1, 0 \)
<table>
<thead>
<tr>
<th>Polynomial Decomposition Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_2(S) = \frac{K_0 S}{S_2 S^2 - (3_1 - K_0) S - \tilde{S}_0}$</td>
</tr>
<tr>
<td>$Q_{P,0} = \frac{[3_0 \tilde{S}_0]}{\tilde{S}_1 - K_0 \tilde{S}_0}$</td>
</tr>
<tr>
<td>$Q_{P,0} = \frac{K_0 \gamma_1}{\tilde{S}_1 - K_0 \tilde{S}_0}$</td>
</tr>
<tr>
<td>Bounds of $S_{P,0}$ when all terms in $D(S)$ are positive: $-1, 0$.</td>
</tr>
<tr>
<td>$T_2(S) = \frac{K_m K_0 G_1 S}{S_2 S (G_H - G_0) + S (G_H - G_0) (3_1 - K_0) - K_m K_0 S G_H + S G_H (G_H - G_0)}$</td>
</tr>
<tr>
<td>$Q_P = \frac{K_m K_0 G_H}{1 - (S_1 - K_0 \gamma_1) G_H}$</td>
</tr>
<tr>
<td>$Q_P = \frac{K_m K_0 G_H - K_0 \gamma_1 (G_H - G_0)}{(G_H - G_0) (S_1 - K_0 \gamma_1) - K_m K_0 S G_H}$</td>
</tr>
<tr>
<td>Bounds of $S_{P,0}$ when all terms in $D(S)$ are positive: $-1, 0$</td>
</tr>
<tr>
<td>$S_{P,0} = \frac{Q_P}{Q_{P,0}} - 1$</td>
</tr>
<tr>
<td>Bounds of $S_{P,0}$ when all terms in $D(S)$ are positive: $-1, 0$</td>
</tr>
</tbody>
</table>
TABLE 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 3**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_v(S)$</td>
<td>$\frac{s_2^2 + a_0}{s_2^2 + 1}$</td>
</tr>
<tr>
<td>$Q_{P0}$</td>
<td>$\frac{s_1}{s_2 + 1}$</td>
</tr>
<tr>
<td>$Q_{P0}$</td>
<td>$\frac{s_1}{s_2 + 1}$</td>
</tr>
<tr>
<td>$S_{K0}$</td>
<td>0,1</td>
</tr>
<tr>
<td>$K_{m} - K_{G_0}$</td>
<td>0,1</td>
</tr>
<tr>
<td>$K_{m} - K_{G_0}$</td>
<td>0,1</td>
</tr>
<tr>
<td>$S_{K0}$</td>
<td>0,1</td>
</tr>
</tbody>
</table>

Bounds of $S_{K0}$ when all terms in $D(S)$ are positive: 0,1
### TABLE 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 4**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_y(s) = \frac{k_0(\alpha_2 s^2 + \alpha_1 s + a_0)}{s^2 s + 2 s + a_0}$</td>
<td>$Q_{P0} = \left{ \frac{6,0,2}{6,0,2} \right}$</td>
</tr>
<tr>
<td>$Q_{P0}$</td>
<td>$B_0$</td>
</tr>
<tr>
<td>$S_{K_0} = 0$</td>
<td>$Q_{P0}$ when all terms in $D(S)$ are positive: 0,0</td>
</tr>
<tr>
<td>$T_v(s) = \frac{k_{m0} k_{0} G_0 (\alpha_2 s^2 + \alpha_1 s + a_0)}{s^2 (\beta_2 (G_0 + G_0) - k_{m0} k_{0} G_0) - k_{m0} k_{0} G_0 + [\beta_1 (G_0 + G_0) - k_{m0} k_{0} G_0]$</td>
<td></td>
</tr>
<tr>
<td>$Q_{P} = \frac{\left{ 1 - \frac{k_{m0} k_{0} G_0}{s_0 (G_0 + G_0)} \right} \left{ 1 - \frac{k_{m0} k_{0} G_0}{s_0 (G_0 + G_0)} \right}^{1/2}}{1 - \frac{k_{m0} k_{0} G_0}{s_0 (G_0 + G_0)}}$</td>
<td></td>
</tr>
<tr>
<td>$Q_{P}$</td>
<td>$1/2 \left( \frac{k_{m0} k_{0} G_0}{G_0 + G_0} + \frac{k_{m0} k_{0} G_0}{G_0 + G_0} \right)$</td>
</tr>
<tr>
<td>$S_{K_0} = 1$</td>
<td>$Q_{P}$ when all terms in $D(S)$ are positive: -1,1</td>
</tr>
<tr>
<td>$Q_{P}$</td>
<td>$S_{K_0}$</td>
</tr>
<tr>
<td>$S_{K_m} = S_{K_0}$</td>
<td>$Q_{P}$ when all terms in $D(S)$ are positive: -1,1</td>
</tr>
</tbody>
</table>

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### TABLE 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 5**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_v(s) )</td>
<td>( \frac{K_0(\frac{s^2}{2} + s - \alpha)}{2s^2 + (\frac{s^2}{2} - K_0 \gamma) s + \delta} )</td>
</tr>
<tr>
<td>( Q_{P0} )</td>
<td>( \frac{s^2 - K_0 \gamma - 1}{s^2 - K_0 \gamma - 1} )</td>
</tr>
<tr>
<td>( Q_{P0} )</td>
<td>Bound of ( S_{K_0} ) when all terms in ( D(S) ) are positive: (-1, 0)</td>
</tr>
<tr>
<td>( S_{K_0} )</td>
<td>Bound of ( S_{K_0} ) when all terms in ( D(S) ) are positive: (-1, 1)</td>
</tr>
</tbody>
</table>

**Further Equations**

\[ T_v(s) = \frac{K_0}{s^2 \left( s^2 + K_0 \gamma - \frac{1}{2} \right)} \]

\[ Q_{P0} \left( \left[ 1 - \frac{K_0 \gamma}{s^2} \frac{G_H + G_0}{G_H} \right] \left[ 1 - \frac{K_0 \gamma}{s^2} \frac{G_H + G_0}{G_H} \right] \right) \]

\[ S_{K_0} = -\frac{1}{2} \left[ \frac{K_0 \gamma}{s^2} \frac{G_H + G_0}{G_H} \right] + \frac{K_0 \gamma}{s^2} \frac{G_H + G_0}{G_H} \]

Bound of \( S_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, 1\)
### Table 4.2.1 (Continued)

**Polyomial Decomposition Type 6**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_v )</td>
<td>( \frac{k_0(\alpha_2 s^2 + \alpha_0)}{\beta_2 s^2 + (\beta_1 - k_0 \gamma_1) s + \beta_0} )</td>
</tr>
<tr>
<td>( c_p )</td>
<td>( \frac{[s_0 \beta_0] \uparrow}{\beta_0} )</td>
</tr>
<tr>
<td>( q_p )</td>
<td>( 0 ) bound of ( s_{k_0} ) when all terms in ( D(S) ) are positive: ( 0, 0 )</td>
</tr>
<tr>
<td>( k_{m_0} g_0 (\alpha_2 s^2 + \alpha_0) )</td>
<td>( \frac{k_{m_0} g_0 (\alpha_2 s^2 + \alpha_0)}{s^2 (\beta_0 (g_0 + g_0) - k_{m_0} \gamma_0 g_0) + s (\beta_0 (g_0 + g_0) - k_{m_0} \gamma_0 g_0)} )</td>
</tr>
<tr>
<td>( q_p )</td>
<td>( q_{p0} \left[ \frac{1 - \left( k_{m_0} \gamma_1 g_0 \right)}{(s_0 g_0 + g_0)^2} \right] \left[ \frac{1 - \left( k_{m_0} \gamma_1 g_0 \right)}{(s_0 g_0 + g_0)^2} \right] )</td>
</tr>
<tr>
<td>( s_{k_0} )</td>
<td>( -\frac{1}{2} \left[ \frac{k_{m_0} \gamma_1 g_0}{s_0 (g_0^2 + g_0^2) - k_{m_0} \gamma_0 g_0} + \frac{k_{m_0} \gamma_1 g_0}{s_0 (g_0^2 + g_0^2) - k_{m_0} \gamma_0 g_0} \right] )</td>
</tr>
<tr>
<td>( q_{k_m} )</td>
<td>( q_{p} - \frac{k_0 \gamma_1}{1 - k_0 \gamma_1} ) bound of ( s_{k_m} ) when all terms in ( D(S) ) are positive: ( 0, 1 )</td>
</tr>
</tbody>
</table>

**Notes:**
- \( k_0, k_{m_0}, g_0, \beta_0, \beta_1, \gamma_0, \gamma_1 \) are coefficients.
- \( s, g_0, g_0 \) are variables.
- \( D(S) \) refers to the denominator of the polynomial.
- \( q_{p0} \) is the initial condition for \( q_p \).
- \( s_{k_0} \) and \( s_{k_m} \) are bounds for polynomial decomposition.
TABLE 4.2.1 (Continued)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_v = \frac{e_0 (a_2 s^2 + e_0)}{s_2 - K_0 y_2} ]</td>
<td>POLYNOMIAL DECOMPOSITION TYPE 7</td>
</tr>
<tr>
<td>[ q_p = \frac{(s_2 - K_0 y_2)(s_3 - K_0 y_0)}{s_1} ]</td>
<td></td>
</tr>
<tr>
<td>[ q_p = -\frac{1}{2} \left[ \frac{K_0 y_0}{s_2 - K_0 y_0} + \frac{K_0 y_0}{s_2 - K_0 y_2} \right] ]</td>
<td>Bounds of ( S_{K_0}^{Q_p} ) when all terms in ( D(S) ) are positive: (-1, 0)</td>
</tr>
<tr>
<td>[ q_p = \frac{K_m K_0 G_0 (a_2 s^2 - e_0)}{s^2 (s_2 - K_0 y_0 - K_0 y_2 - G_H G_0) + s_2 (G_H - G_0) - K_0 y_0 (G_H + G_0) - K_m K_0 a_2 G_H} ]</td>
<td></td>
</tr>
<tr>
<td>[ q_p = \frac{K_m K_0 a_2 G_H}{s_2 (s_2 - K_0 y_0 - G_H) (G_H - G_0) - K_0 y_0 (G_H + G_0) - K_m K_0 a_2 G_H} ]</td>
<td></td>
</tr>
<tr>
<td>[ q_p = -\frac{1}{2} \left[ \frac{K_m K_0 a_2 G_H}{s_2 (s_2 - K_0 y_0 - G_H) (G_H - G_0) - K_0 y_0 (G_H + G_0) - K_m K_0 a_2 G_H} \right] ]</td>
<td>Bounds of ( S_{K_0}^{Q_p} ) when all terms in ( D(S) ) are positive: (0, 1)</td>
</tr>
<tr>
<td>[ q_p = -\frac{1}{2} \left[ \frac{K_m K_0 a_2 G_H}{s_2 (s_2 - K_0 y_0 - G_H) (G_H - G_0) - K_0 y_0 (G_H + G_0) - K_m K_0 a_2 G_H} \right] ]</td>
<td></td>
</tr>
<tr>
<td>[ q_p = -\frac{1}{2} \left[ \frac{K_m K_0 a_2 G_H}{s_2 (s_2 - K_0 y_0 - G_H) (G_H - G_0) - K_0 y_0 (G_H + G_0) - K_m K_0 a_2 G_H} \right] ]</td>
<td></td>
</tr>
</tbody>
</table>

Note: The expressions are mathematical formulas related to polynomial decomposition types, with specific conditions and bounds for the variables involved.
**TABLE 4.2.1 (Continued)**

**POLYNOMIAL DECOMPOSITION TYPE S**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0 = \frac{K_0(\alpha_2S^2+\alpha_1S+\alpha_0)}{(\beta_2-K_0\gamma_2)S^2-3+\beta_2-K_0\gamma_2}$</td>
<td></td>
</tr>
<tr>
<td>$Q_P = \left[ \left( \frac{\beta_2-K_0\gamma_2}{\alpha_2S+\alpha} \right) \right]^{\frac{1}{2}}$</td>
<td></td>
</tr>
<tr>
<td>$S_{K_0} = -\frac{1}{2} \left[ \frac{K_0\gamma_2}{\beta_2-K_0\gamma_2} + \frac{K_0\gamma_0}{\beta_1-K_0\gamma_0} \right]$</td>
<td>Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: $-\frac{1}{2}, \frac{1}{2}$</td>
</tr>
<tr>
<td>$T_0(S) = \frac{K_0K_0(G_0^{\alpha_2\beta_2})(\alpha_2^2+S^2) - K_0K_0(\alpha_2^2+S^2) + ((\beta_0-K_0\gamma_0)(G_0^{\alpha_2\beta_2}) - K_0K_0c_0G_0^H)}{S^2(\beta_2-K_0\gamma_2)(G_0^{\alpha_2\beta_2}) - K_0K_0(\alpha_2^2+S^2) + ((\beta_0-K_0\gamma_0)(G_0^{\alpha_2\beta_2}) - K_0K_0c_0G_0^H)}$</td>
<td></td>
</tr>
<tr>
<td>$Q_P = \frac{1-K_0K_0G_0^H}{K_0K_0G_0^H} \left( \frac{K_0K_0G_0^H}{(\beta_0-K_0\gamma_0)(G_0^{\alpha_2\beta_2})} - 1 \right)$</td>
<td></td>
</tr>
<tr>
<td>$S_{K_0} = -\frac{1}{2} \left[ \frac{K_0K_0G_0^H}{(\beta_0-K_0\gamma_0)(G_0^{\alpha_2\beta_2})} - 1 \right]$</td>
<td>Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: $-1, 1$</td>
</tr>
</tbody>
</table>
**TABLE 4.2.1 (Continued)**

\[
\begin{align*}
S_{K_m}^P &= -\frac{1}{2} \left[ \frac{K_m \alpha_0 G_H}{(\delta_0 - K_0 \gamma_0)^2 (G_0 + G_H)^2} + \frac{K_m \alpha_2 G_H}{(\delta_2 - K_0 \gamma_2)^2 (G_0 + G_H)^2} - \frac{K_m \alpha_1 G_H}{(\delta_1 - K_0 \gamma_1)^2 (G_0 + G_H)^2} \right] + \frac{K_m \alpha_1 G_H}{B_1 (G_0 + G_H)^2 - K_m \alpha_1 G_H}.
\end{align*}
\]

Bounds of \(S_{K_m}^P\) when all terms in \(D(S)\) are positive: \(-1, 1\)

**POLYNOMIAL DECOMPOSITION TYPE 9a**

\[
\begin{align*}
t_v &= \frac{K_0^2 (\alpha_2 s^2 + \alpha_1 s + \alpha_0)}{(\delta_2 - K_0 \gamma_2)^2 (\delta_1 - K_0 \gamma_1) s^2 + (\delta_0 - K_0 \gamma_0)^2}.
\end{align*}
\]

\[
Q_{P0} = \frac{\left[ (\delta_2 - K_0 \gamma_2) (\delta_1 - K_0 \gamma_1) \right]}{(\delta_1 - K_0 \gamma_1)}.
\]

\[
S_{K_0}^P = \frac{1}{2} \left[ \frac{K_0 \gamma_0}{\delta_0 - K_0 \gamma_0} + \frac{K_0 \gamma_2}{\delta_2 - K_0 \gamma_2} - \frac{K_0 \gamma_1}{\delta_1 - K_0 \gamma_1} \right] + \frac{K_0 \gamma_1}{\delta_1 - K_0 \gamma_1}.
\]

Bounds of \(S_{K_0}^P\) when all terms in \(D(S)\) are positive: \(-1, 1\)

\[
T_v = \frac{K_m \alpha_0 G_H (\alpha_2 s^2 + \alpha_1 s + \alpha_0)}{s^2 (\delta_2 - K_0 \gamma_2) (G_0 + G_H)^2 - K_m \alpha_0 G_H} + \frac{1}{(\delta_0 - K_0 \gamma_0)^2 (G_0 + G_H)^2} - \frac{K_m \alpha_1 G_H}{(\delta_1 - K_0 \gamma_1)^2 (G_0 + G_H)^2} + \frac{1}{(\delta_2 - K_0 \gamma_2)^2 (G_0 + G_H)^2} - \frac{K_m \alpha_2 G_H}{(\delta_2 - K_0 \gamma_2)^2 (G_0 + G_H)^2}.
\]

\[
Q_P = \left[ 1 - \frac{K_m \alpha_0 G_H}{(G_0 + G_H) (\delta_0 - K_0 \gamma_0)^2} \right] + \left[ 1 - \frac{K_m \alpha_2 G_H}{(G_0 + G_H) (\delta_2 - K_0 \gamma_2)^2} \right] + \left[ 1 - \frac{K_m \alpha_1 G_H}{(G_0 + G_H) (\delta_1 - K_0 \gamma_1)^2} \right].
\]

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TABLE 4.2.1 (Continued)

\[
S_{K_0}^P = -\frac{1}{2} \left\{ \frac{K_mK_0(\alpha_1^2+\alpha_2^2)G_0^2}{(G_0H)^2} - \frac{K_0G_0^{\alpha_1}C_0^{\alpha_2}}{(G_0H)^2} \right\}
\]

\[
S_{K_m}^P = -\frac{1}{2} \left\{ \frac{K_mK_0^{\alpha_1}G_0^{\alpha_2}}{(G_0H)^2} + \frac{K_mK_0^{\alpha_2}G_0^{\alpha_1}}{(G_0H)^2} \right\}
\]

Bounds of \( S_{K_0}^P \) when all terms in \( D(S) \) are positive: \( -1,1 \)

Bounds of \( S_{K_m}^P \) when all terms in \( D(S) \) are positive: \( -1,1 \)

POLYNOMIAL DECOMPOSITION TYPE 9b

\[
t_v = \frac{S_0(\beta_0^{\alpha_1^2+\alpha_2^2}S_{-\alpha_0})}{\beta_0S_{-\alpha_0}}
\]

\[
l = \frac{S_0^{\alpha_1^2}}{\alpha_1}
\]

\[
S_{K_0}^P = 0
\]

Bounds of \( S_{K_0}^P \) when all terms in \( D(S) \) are positive: \( 0,0 \)

\[
T_v = \frac{K_mK_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2}}{(G_0H)^2} - \frac{K_mK_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2}}{(G_0H)^2}
\]

\[
= \frac{S_0(G_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2} - K_m\beta_0G_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2})}{(G_0H)^2}
\]

\[
+ \frac{S_0(G_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2} - K_m\beta_0G_0^{\alpha_1^2+\alpha_2^2}G_0^{\alpha_1^2+\alpha_2^2})}{(G_0H)^2}
\]
TABLE 4,2,1 (Continued)

\[
Q_P = \frac{Q_{P0}}{1 - \frac{\frac{K_m G_0 - K_m H_0 G_0}{\frac{1}{2} (G_0 - G_0)} \cdot e^{-\frac{1}{2} (G_0 - G_0)}}{1 - \frac{K_m^2 G_0^2 - K_m G_0}{\frac{1}{2} (G_0 - G_0)}}}
\]

\[
S_{k_0} = -\frac{1}{2} \left[ \frac{K_m G_0 G_0}{e^{-\frac{1}{2} (G_0 - G_0)}} \right] + \frac{K_m^2 G_0 G_0}{e^{-\frac{1}{2} (G_0 - G_0)}}
\]

Bounds of \( Q'_P \) when all terms in \( D(S) \) are positive: -1,1

\[
S_{k_m} = -\frac{1}{2} \left[ \frac{K_m^2 G_0 G_0}{e^{-\frac{1}{2} (G_0 - G_0)}} \right] + \frac{K_m^2 G_0 G_0}{e^{-\frac{1}{2} (G_0 - G_0)}}
\]

Bounds of \( S'_{k_m} \) when all terms in \( D(S) \) are positive: -1,1

POLYNOXIAL DECOMPOSITION TYPE 10

\[
t = \frac{x_0 (x_2^2 + 1) - x_0}{x_2 (x_2 - 1) - x_0}
\]

\[
Q_{P0} = \frac{1}{x_2}
\]

\[
S_{k_0} = \frac{1}{x_2}
\]

Bounds of \( S'_{k_0} \) when all terms in \( D(S) \) are positive: -1,1
<table>
<thead>
<tr>
<th>$T_v (s)$</th>
<th>$Q_p$</th>
<th>$S_{K_0}$</th>
<th>$S_{K_0}^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_v (s)$</td>
<td>$Q_p$</td>
<td>$S_{K_0}$</td>
<td>$S_{K_0}^2$</td>
</tr>
<tr>
<td>$Q_p$</td>
<td>$S_{K_0}$</td>
<td>$S_{K_0}^2$</td>
<td>$Q_p$</td>
</tr>
</tbody>
</table>

TABLE 4.2.1 (Continued)

<table>
<thead>
<tr>
<th>$K_{m,G_0}$</th>
<th>$K_{m,G_0}^{-1}$</th>
<th>$K_{m,G_0}^{-2}$</th>
<th>$K_{m,G_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{m,G_0}$</td>
<td>$K_{m,G_0}^{-1}$</td>
<td>$K_{m,G_0}^{-2}$</td>
<td>$K_{m,G_0}$</td>
</tr>
<tr>
<td>$K_{m,G_0}$</td>
<td>$K_{m,G_0}^{-1}$</td>
<td>$K_{m,G_0}^{-2}$</td>
<td>$K_{m,G_0}$</td>
</tr>
<tr>
<td>$K_{m,G_0}$</td>
<td>$K_{m,G_0}^{-1}$</td>
<td>$K_{m,G_0}^{-2}$</td>
<td>$K_{m,G_0}$</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>$K_{m,G_0}$</th>
<th>$K_{m,G_0}^{-1}$</th>
<th>$K_{m,G_0}^{-2}$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$K_{m,G_0}$</td>
<td>$K_{m,G_0}^{-1}$</td>
<td>$K_{m,G_0}^{-2}$</td>
<td>$K_{m,G_0}$</td>
</tr>
</tbody>
</table>

$Q_0 = \frac{1}{Q_0}$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$

$S_{K_0}^2 = S_{K_0}^2 \cdot Q_0$

$Q_0 = S_{K_0}^2 \cdot Q_0$

$S_{K_0} = S_{K_0}^2 \cdot Q_0$
### TABLE 4.2.1 (Continued)

**Polynomial Decomposition Type II**

<table>
<thead>
<tr>
<th>$\tau_v$</th>
<th>$\frac{K_0(a_2 S^2 + a_1 S - a_0)}{s^2 S^2 - (s_1 - K_0 \gamma_1) S + a_0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{p0}$</td>
<td>$\frac{s_2}{s_1 - K_0 \gamma_1}$</td>
</tr>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{K_0 \gamma_1}{s_1 - K_0 \gamma_1}$</td>
</tr>
</tbody>
</table>

 Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: 0, 1

| $\Gamma_y(S)$ | $\frac{K_0 K_0 G_0 (a_2 S^2 - s_0 S + a_0)}{s^2 [s_2 (G_H^2 + G_F^2) - K_0 G_0 - K_0 G_0 G_F] + s_1 \gamma_1 (3 G_H^2 + G_F^2) - K_0 G_0 - K_F G_F - K_0 G_0 G_F}$ |

| $Q_{p0}$ | $\frac{1 - \frac{K_0 G_0 - K_0 G_F}{s_2 [G_H^2 + G_F^2]} - \frac{K_0 G_0 - K_0 G_F}{s_2 [G_H^2 + G_F^2]}}{1 - \frac{K_0 G_0 - K_0 G_F}{s_2 [G_H^2 + G_F^2]}}$ |

| $Q_p$ | $\frac{K_0 \gamma_1 [G_H + G_F]}{s_1 - K_0 \gamma_1 [G_H + G_F]}$ |

 Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: -1, 1
TABLE 4.2.1 (Continued)

| \( S_{K_0} \) | \(-\frac{1}{2}\left[ \frac{K_{H} \gamma_0 \gamma_0 \gamma_0 + K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \right] \quad \frac{-K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \quad \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \] |
|---|---|---|
| \( Q_{p0} \) | \(-\frac{1}{2} \left( \frac{K_{H} \gamma_0 \gamma_0 \gamma_0 + K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \right) \) | \(-\frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \) |

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1

POLYNOMIAL DECOMPOSITION TYPE 12a

\[ t_{y} = \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{(\bar{S}_{2} - K_{H} \gamma_0 \gamma_0)(\bar{S}_{2} + K_{H} \gamma_0 \gamma_0)S_{0} - (\bar{S}_{0} - K_{H} \gamma_0 \gamma_0)} \]

\[ Q_{p0} = \frac{\left( \bar{S}_{2} - K_{H} \gamma_0 \gamma_0 \right)}{\bar{S}_{2} - K_{H} \gamma_0 \gamma_0} \]

\[ S_{K_0} = \frac{-K_{H} \gamma_0 \gamma_0 \gamma_0 + K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \quad \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \]

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1

\[ T_{y}(S) = \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{S_{0} \left[ (\bar{S}_{2} - K_{H} \gamma_0 \gamma_0)(S_{0} + G_{0} - G_{F}) - K_{H} \gamma_0 \gamma_0 \gamma_0 \right] + S \left[ (\bar{S}_{2} - K_{H} \gamma_0 \gamma_0)(S_{0} + G_{0} + G_{F}) - K_{H} \gamma_0 \gamma_0 \gamma_0 \gamma_0 \right]} \]

\[ Q_{p} = Q_{p0} \left[ \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{(\bar{S}_{2} - K_{H} \gamma_0 \gamma_0)(S_{0} + G_{0} - G_{F})} \right] \quad \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{(\bar{S}_{2} - K_{H} \gamma_0 \gamma_0)(S_{0} + G_{0} + G_{F})} \]

\[ Q_{p} = \frac{-\frac{1}{2} \left[ \frac{K_{H} \gamma_0 \gamma_0 \gamma_0 + K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \right] \quad \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \quad \frac{K_{H} \gamma_0 \gamma_0 \gamma_0}{2} \] |

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1
TABLE 4.2.1 (Continued)

\[ T_v(S) = \frac{1}{S^2 \left( G_0 + G_H + C_F \right) - K_m X_0 + K_m X_0 \left( \gamma_2 G_F - a_2 G_H \right)} + \frac{Q_{t0}}{S^2 - \frac{5}{2} S - \frac{5}{2}} \]

Bounds of \( S_{X_0} \) when all terms in \( D(S) \) are positive: 0, 1

\[ T_F(S) = \frac{1}{S^2 \left( G_0 + G_H + C_F \right) - K_m X_0 + K_m X_0 \left( \gamma_2 G_F - a_2 G_H \right)} + \frac{Q_F}{(G_F - G_H) - K_m X_0} \]

Bounds of \( S_{X_0} \) when all terms in \( D(S) \) are positive: 0, 1
TABLE 4.2.1 (Continued)

\[ Q_m^P = \ldots -K_m^{G_F - G_{m0}}(G_{m0}^{G_F - G_0}) \ldots + \ldots \]

Bounds of \( Q_m^P \) when all terms in \( D(S) \) are positive: -1, 1

POLYNOMIAL DECOUPLING TYPE 13

\[ t_v = \frac{K_0(1 - 2)^2 + s_1 - s_0}{(3 - 2) + (3 - 2) - (3 - 2) + (3 - 2)} \]

\[ Q_{P0} = \left\{ \frac{s_0 - x_0 y_0}{s_0 - x_0 y_0} \right\}^2 \]

Bounds of \( Q_{P0}^P \) when all terms in \( D(S) \) are positive: -1, 1

\[ S_{P0}^P = \frac{1}{2} \left[ \frac{K_0 y_0}{s_0 - x_0 y_0} + \frac{x_0 y_0}{s_0 - x_0 y_0} \right] + \frac{K_0 y_0}{s_0 - x_0 y_0} \]

\[ T_v(S) = \frac{K_0 G_0^{G_F - G_0} + s_1 + s_0}{s_1 + s_0 + 1} \]

\[ Q_{P0} = \left[ 1 - \frac{K_0 G_0^{G_F - G_0} (s_0 - s_0)}{(s_0 - s_0)(s_0 - s_0)} \right] \times \left[ \frac{K_0^{G_F - G_0} (y_0 - y_0)}{(s_0 - s_0)(s_0 - s_0)} \right] \]

\[ Q_{P0} = \frac{K_0 G_0^{G_F - G_0} G_F}{G_F - G_0} \]

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TABLE 4.2.1 (Continued)

\[
\begin{align*}
Q_P \left( s_{K_0} \right) &= \frac{1}{2} \left[ \frac{K_{m} \gamma_2 \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - K_{m} \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - \gamma_2 (1) K_{m}; \gamma_H)}{s_1} \right] \\
&= \frac{1}{2} \left[ \frac{K_{m} \gamma_2 \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - K_{m} \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - \gamma_2 (1) K_{m}; \gamma_H)}{s_1} \right]
\end{align*}
\]

Bounds of \( s_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, 1\)

\[
\begin{align*}
Q_P \left( s_{K_m} \right) &= \frac{1}{2} \left[ \frac{K_{m} \gamma_2 \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - K_{m} \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - \gamma_2 (1) K_{m}; \gamma_H)}{s_1} \right] \\
&= \frac{1}{2} \left[ \frac{K_{m} \gamma_2 \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - K_{m} \gamma_0 (s_{-1} (1) K_{m}; \gamma_H - \gamma_2 (1) K_{m}; \gamma_H)}{s_1} \right]
\end{align*}
\]

Bounds of \( s_{K_m} \) when all terms in \( D(S) \) are positive: \(-1, 1\)

POLYNOMIAL DECOMPOSITION TYPE 14

\[
\begin{align*}
\tau &= \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \\
&= \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \\
Q_{P_0} &= \frac{\left( (s_{0} \gamma_0 \gamma_2) (s_{-2} \gamma_0 \gamma_2) \right)}{s_{-1} \gamma_0 \gamma_1} \\
Q_{P_0} &= \frac{\left( (s_{0} \gamma_0 \gamma_2) (s_{-2} \gamma_0 \gamma_2) \right)}{s_{-1} \gamma_0 \gamma_1} \\
S_{K_0} &= \frac{1}{2} \left[ \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \right] - \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \\
&= \frac{1}{2} \left[ \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \right] - \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \\
&= \frac{1}{2} \left[ \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)} \right] - \frac{K_{0} \gamma_2 (s_{-1} (1) S_{12}; \gamma_2 - s_{-1})}{(s_{-2} \gamma_0 \gamma_2) S_{-1} (s_{-1} \gamma_1 - s_{-1}) (s_{-2} \gamma_0 \gamma_2)}
\end{align*}
\]

Bounds of \( s_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, 1\)
TABLE 6.2.1 (Continued)

\[ T_{\nu}(s) = \frac{X_0 G_A (s^2 - z_0^2 - s^2) \cdot s^2 ((s^2 - k_0^2 s^2) (G_0 - G_F) - s^2 G_F) \cdot s^2 \cdot G_F \cdot k_0 \cdot G (s^2 - z_0^2)}{s^4 (s^2 - k_0^2 s^2) (G_0 - G_F) - s^2 G_F \cdot s^2 \cdot G_F \cdot k_0 \cdot G (s^2 - z_0^2)} \]

\[ Q_0 = \frac{1}{G_F} \left( 1 - \frac{k_0^2 s^2 - k_0 \cdot s^2 G_F}{G_0 - G_F} \cdot k_0 \cdot G (s^2 - z_0^2) \right)^2 \]

\[ Q_0 = \frac{1}{G_F} \left( 1 - \frac{k_0^2 s^2 - k_0 \cdot s^2 G_F}{G_0 - G_F} \cdot k_0 \cdot G (s^2 - z_0^2) \right)^2 \]

\[ Q_0 = \frac{1}{G_F} \left( 1 - \frac{k_0^2 s^2 - k_0 \cdot s^2 G_F}{G_0 - G_F} \cdot k_0 \cdot G (s^2 - z_0^2) \right)^2 \]

\[ S_{K_0} = \frac{1}{2} \left( -k_1 y_1 (G_H - G_F) + k_2 k_0 (s^2 - z_0^2 - s^2) \right) \]

\[ S_{K_0} = \frac{1}{2} \left( -k_1 y_1 (G_H - G_F) + k_2 k_0 (s^2 - z_0^2 - s^2) \right) \]

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1

\[ S_{K_0} = \frac{1}{2} \left( -k_2 y_1 (G_H - G_F) + k_0 (s^2 - z_0^2 - s^2) \right) \]

Bounds of \( S_{K_0} \) when all terms in \( D(S) \) are positive: -1, 1
TABLE 4.2.1 (Continued)

<table>
<thead>
<tr>
<th>Polynomial Decomposition Type 15</th>
</tr>
</thead>
</table>

\[
T_v = \frac{K_0 \left( \alpha_1^{(1)} \right) S^2 + a_1 S + a_0^{(1)}}{(S_2 - K_0 \gamma_2) S^2 + (S_1 - K_0 \gamma_1) S + (K_0 - K_0 \gamma_1)}
\]

\[
Q_{p0} = \frac{B_0 - K_0 \gamma_0}{K_0 \gamma_1}
\]

\[
Q_{p0} = -\frac{1}{2} \left( \frac{K_0 \gamma_0}{B_0 - K_0 \gamma_0} - \frac{K_0 \gamma_1}{B_0 - K_0 \gamma_1} \right)
\]

\[
S_{K_0} = -\frac{1}{2} \left( \frac{K_0 \gamma_0}{B_0 - K_0 \gamma_0} - \frac{K_0 \gamma_1}{B_0 - K_0 \gamma_1} \right)
\]

Bounds of \(S_{K_0}\) when all terms in \(D(S)\) are positive: -1, 1

\[
T_v(S) = \frac{K_m K_0 G_0 a_2^{(1)} S^3 + a_1 S + a_0^{(1)}}{S^2 (S_2 - K_0 \gamma_2) (G_h + G_0 - G_F) - K_m G_F K_0 (\gamma_2 G_F - a_2^{(2)} G_H) + S [(S_1 - K_0 \gamma_1) (G_h + G_0 - G_F) - K_m \gamma_1 G_F]}
\]

\[
Q_{p0} = \left\{ \frac{1}{(B_0 - K_0 \gamma_0) (G_h + G_0 - G_F) - K_m \gamma_0 G_F + K_m K_0 (\gamma_0 G_F - a_0^{(1)} G_H)} \right\}
\]

\[
Q_p = \frac{1}{1 - \left( \frac{K_m G_0}{(S_1 - K_0 \gamma_1) (G_h + G_0 - G_F)} \right)}
\]
TABLE 4.2.1 (Continued)

\[
S_{Q_0} = \frac{-K_0 \gamma_0 (G_{H}^+ G_{0}^- G_{F}) + K_0 (\gamma_0 G_{F} - a_0 G_{H})}{(S_0 - K_0 \gamma_0) (G_{H}^+ G_{0}^- G_{F}) - K_0 \gamma_0 G_{0}^- G_{F} + K_0 (\gamma_0 G_{F} - a_0 G_{H})}
+ \frac{K_0 \gamma_0 (G_{H}^+ G_{0}^- G_{F}) + K_0 (\gamma_0 G_{F} - a_0 G_{H})}{(S_2 - K_0 \gamma_0) (G_{H}^+ G_{0}^- G_{F}) - K_0 \gamma_0 G_{0}^- G_{F} + K_0 (\gamma_0 G_{F} - a_0 G_{H})}
\]

Bounds of \(S_{Q_0}\) when all terms in \(D(S)\) are positive: \(-1,1\)

\[
S_{K_m} = \frac{-K_m S_0 G_{F} + K_m \gamma_0 (\gamma_0 G_{F} - a_0 G_{H})}{(S_0 - K_0 \gamma_0) (G_{H}^+ G_{0}^- G_{F}) - K_0 \gamma_0 G_{0}^- G_{F} + K_0 (\gamma_0 G_{F} - a_0 G_{H})}
+ \frac{K_m \gamma_0 (G_{H}^+ G_{0}^- G_{F}) + K_m \gamma_0 (\gamma_0 G_{F} - a_0 G_{H})}{(S_2 - K_0 \gamma_0) (G_{H}^+ G_{0}^- G_{F}) - K_0 \gamma_0 G_{0}^- G_{F} + K_0 (\gamma_0 G_{F} - a_0 G_{H})}
\]

Bounds of \(S_{Q_0}\) when all terms in \(D(S)\) are positive: \(-1,1\)

POLYNOMIAL DECOMPOSITION TYPE 16

\[
t_\gamma = \frac{K_0 (a_2 S^2 + a_0)}{S^2 (S_2 - K_0 \gamma_0) - 3 S - (S_0 - K_0 \gamma_0)}
\]

\[
Q_{P0} = \left\{ \frac{[S_0 - K_0 \gamma_0]}{S_1 - K_0 \gamma_0} \right\}^{1/2}
\]

\[
S_{K_0}' = \frac{1}{2} \left[ \frac{K_0 \gamma_0}{S_0 - K_0 \gamma_0} + \frac{K_0 \gamma_0}{S_2 - K_0 \gamma_0} \right]
\]

Bounds of \(S_{K_0}'\) when all terms in \(D(S)\) are positive: \(0,1\)
TABLE 4.2.1 (Continued)

$$\mathcal{T}_\nu(S) = \frac{K_m \ell_0 G_0 (\alpha_2 S^2 + 2 \zeta_0)}{S^2 \left( (\zeta_2 - K_0) (G_0 + G_{-} - G_F) - K_m \gamma_2 G_F + K_0 (G_{2} - 2 \zeta_2 G_F) \right) - S \left[ (G_{-} + G_0 - G_F) - K_m S G_F \right]}
+ \left[ (\zeta_2 - K_0) (G_0 + G_{-} - G_F) - K_m \gamma_2 G_F + K_0 (G_{2} - 2 \zeta_2 G_F) \right]
\frac{1}{\left( \zeta_2 - \zeta_0 \right)} \frac{(G_{-} + G_0 - G_F) - K_m \gamma_2 G_F + K_0 (G_{2} - 2 \zeta_2 G_F)}{(G_{-} - G_{0} + G_F)}
\frac{(1 - K_m \gamma_2 G_F - K_0 (G_{2} - 2 \zeta_2 G_F))}{\left( \zeta_2 - \zeta_0 \right)} \frac{(G_{-} + G_0 - G_F) - K_m \gamma_2 G_F + K_0 (G_{2} - 2 \zeta_2 G_F)}{(G_{-} - G_{0} + G_F)}$

$$Q_\nu = \frac{1}{1 - \left( \frac{K_m \gamma_2 G_F - K_0 (G_{2} - 2 \zeta_2 G_F)}{(1 - \zeta_2 \zeta_0) (G_{-} + G_0 - G_F)} \right) \left( \frac{G_{-} - G_{0} + G_F}{G_{-} + G_0 - G_F} \right)}

$$S_{Q_0} = \frac{1}{2} \left[ \frac{K_0 \gamma_2 (G_{0} - G_{-} + G_F)}{(-K_0 \gamma_2) (G_{0} + G_{-} - G_F) - K_m \gamma_2 G_F + \zeta_2 (G_{0} - G_{-} + G_F)} \right] \left[ \frac{K_0 \gamma_2 (G_{0} + G_{-} - G_F)}{(-K_0 \gamma_2) (G_{0} + G_{-} - G_F) - K_m \gamma_2 G_F + \zeta_2 (G_{0} - G_{-} + G_F)} \right]$

Bounds of $S_{Q_0}$ when all terms in $D(S)$ are positive: $0, 1$

$$S_{Q_0} = \frac{1}{2} \left[ \frac{K_0 \gamma_2 G_F + K_0 (\gamma_2 G_F - \zeta_2 G_F)}{(S_0 - \zeta_0 \gamma_2) (G_{-} - G_{0} + G_F)} \right] \left[ \frac{K_0 \gamma_2 G_F + K_0 (\gamma_2 G_F - \zeta_2 G_F)}{(S_0 - \zeta_0 \gamma_2) (G_{-} - G_{0} + G_F)} \right] + \left[ \frac{K_0 \gamma_2 G_F + K_0 (\gamma_2 G_F - \zeta_2 G_F)}{(S_0 - \zeta_0 \gamma_2) (G_{-} - G_{0} + G_F)} \right] \left[ \frac{K_0 \gamma_2 G_F + K_0 (\gamma_2 G_F - \zeta_2 G_F)}{(S_0 - \zeta_0 \gamma_2) (G_{-} - G_{0} + G_F)} \right]$

Bounds of $S_{Q_0}$ when all terms in $D(S)$ are positive: $-1, 1$
### Table 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 12**

\[
\begin{align*}
    t_v &= \frac{k_0(2a_2s^2+2s-a_0)}{s^2(2s-2k_0s) - 2s - (2s-2k_0s)} \\
    q_{p0} &= \frac{s_0-\kappa_0\gamma_0}{\gamma_0} \frac{s_2-k_0\gamma_0}{s_1} \\
    q_{p0}' &= -\frac{k_0\gamma_0}{2} \frac{s_0-k_0\gamma_0}{s_0-k_0\gamma_0} + \frac{k_0\gamma_0}{2} \\
    s_{k_0} &= \text{Bounds of } s_{k_0} \text{ when all terms in } D(S) \text{ are positive: } 0, 1 \\
    t_v &= \frac{k_{m0}G_F(2a_2s^2+2s-a_0)}{s^2[(G_0+G_H+G_F)(2s-2k_0\gamma_0)-2s-2s-2k_0\gamma_0] + s[(G_0+G_H+G_F)^2 - 2s\gamma_0G_F - m}\ mG_F + \{(G_0+G_H+G_F)(2s-2k_0\gamma_0)-2s-2k_0\gamma_0\} \\
    q_{p0} &= \left[1 - \frac{k_0s_0G_F}{(s_0-k_0\gamma_0)(G_0+G_H+G_F)} \frac{1 - (s_0-k_0\gamma_0)}{(s_0-k_0\gamma_0)(G_0+G_H+G_F)} \right]^{1/2} \\
    q_p &= 1 - \frac{k_{m0}G_F-s_{k_0}G_{H0}}{s_{k_0}G_{H0}} \\
    s_{k_0} &= -\frac{1}{2} \left[ \frac{k_0\gamma_0}{s_{2H}G_0+G_F} - \frac{G_0}{s_{2H}G_0+G_F} \right] - \frac{k_0\gamma_0}{s_{2H}G_0+G_F} - \frac{G_0}{s_{2H}G_0+G_F} + \frac{k_0\gamma_0}{s_{2H}G_0+G_F} + \frac{G_0}{s_{2H}G_0+G_F} \\
    \text{Bounds of } s_{k_0} \text{ when all terms in } D(S) \text{ are positive: } -1, 1
\end{align*}
\]

---

Note: The table includes polynomial decomposition equations for Type 12 in the context of a mathematical or physical analysis. The variables and constants are specific to the context of the study, possibly relating to physical systems or engineering applications.
TABLE 4.2.1 (Continued)

\[ S_{K_m}^{Q_p} = -\frac{1}{2} \left[ \frac{K_0 G_0 G_F}{[G_0 - G_F - (\gamma_0 - K_0 Y_0) - K_0 G_F]} \right] + \frac{K_0^2 G_F}{[G_0 - G_F - (\gamma_0 - K_0 Y_0) - K_0 G_F]} - \frac{K_0^2 G_F}{[G_0 - G_F - (\gamma_0 - K_0 Y_0) - K_0 G_F]} \]

Bounds of \( S_{K_m}^{Q_p} \) when all terms in \( D(S) \) are positive: -1, 1

**POLYNOMIAL DECOMPOSITION TYPE 18**

\[ t_\nu = \frac{K_0 (a_2 S + a_1 S + a_0)}{(S_0 - K_0 Y_0)^2 + (S_1 - K_0 Y_0)^2 + (S_2 - K_0 Y_0)^2} \]

\[ c_{P_0} = \frac{[(S_0 - K_0 Y_0) (S_2 - K_0 Y_0)]}{S_1 - K_0 Y_0} \]

\[ Q_{P_0}^{Q_0} = -\frac{1}{2} \left[ \frac{K_0^2 G_0}{S_0 - K_0 Y_0} \right] + \frac{K_0 Y_1}{S_1 - K_0 Y_1} \]

Bounds of \( S_{P_0}^{Q_0} \) when all terms in \( D(S) \) are positive: -1, 1

\[ t_\nu = \frac{K_0 G_0 G_F (a_2 S + a_1 + a_0)}{S^2 [(S_0 - K_0 Y_0)^2 (G_0 - G_F) - K_0 S_0 G_F] + S [(S_1 - K_0 Y_0)^2 (G_0 - G_F) - K_0 S_1 G_F + K_0 Y_0 (\gamma_0 G_F - a_1 G_F)]} \]

\[ -Q_0 = \frac{Q_0}{1 - \frac{K_0 G_0 G_F}{(S_0 - K_0 Y_0) (G_0 - G_F)} - \frac{K_0^2 G_F}{(S_0 - K_0 Y_0) (G_0 - G_F)}} + \frac{K_0^2 G_F}{(S_0 - K_0 Y_0) (G_0 - G_F)} \]
TABLE 4.2.1 (Continued)

<table>
<thead>
<tr>
<th>$Q_P$</th>
<th>$S_{X_0}$</th>
<th>$G_{P,0}$</th>
<th>$G_{P,1}$</th>
<th>$G_{P,2}$</th>
<th>$G_{P,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \frac{1}{2} \left[ \frac{K_{0,0}(G_{P,0} G_{P,0} G_{P,0})}{(G_{P,0} G_{P,0} G_{P,0})} + \frac{K_{2,0} G_{P,0} G_{P,0}}{G_{P,0} G_{P,0} G_{P,0}} \right] - \frac{K_{3,0} G_{P,0} G_{P,0}}{G_{P,0} G_{P,0} G_{P,0}}$</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Bounds of $S_{X_0}$ when all terms in $D(S)$ are positive: $\pm 1.1$

---

<table>
<thead>
<tr>
<th>$Q_P$</th>
<th>$S_{X_0}$</th>
<th>$G_{P,0}$</th>
<th>$G_{P,1}$</th>
<th>$G_{P,2}$</th>
<th>$G_{P,3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$= \frac{1}{2} \left[ \frac{K_{1,0} G_{P,0} G_{P,0}}{G_{P,0} G_{P,0} G_{P,0}} + \frac{K_{2,0} G_{P,0} G_{P,0}}{G_{P,0} G_{P,0} G_{P,0}} \right] - \frac{K_{3,0} G_{P,0} G_{P,0}}{G_{P,0} G_{P,0} G_{P,0}}$</td>
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</table>

Bounds of $S_{X_0}$ when all terms in $D(S)$ are positive: $\pm 1.1$

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### POLYNOMIAL DECOMPOSITION TYPE 19

<table>
<thead>
<tr>
<th>$t_{Y}$</th>
<th>$C_{P,0}$</th>
<th>$S_{X_0}$</th>
</tr>
</thead>
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<tr>
<td>$= \frac{3}{3} \left( \frac{3}{3} S_{Y_0} S_{Y_0} S_{Y_0} \right)$</td>
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<td>$= \left( \frac{3}{3} S_{Y_0} S_{Y_0} S_{Y_0} \right)$</td>
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</tr>
</tbody>
</table>

Bounds of $S_{X_0}$ when all terms in $D(S)$ are positive: $\pm 1.1$
TABLE 4.2.1 (C-continued)

\[
T_\nu = \frac{K_0 K_\nu \gamma_0 (\alpha_2 S^2 + \beta_2 S + \gamma_2)}{S^2 [(\beta_2 - \beta_0) K_\gamma - G_0 - G_F] - K_\nu g_\nu [S - 1] - K_0 \gamma_0 (G_0 + G_F - F_0 - G_F) + \left[ (\beta_0 - \beta_0) (G_0 + G_F - F_0 - G_F) - K_\nu g_\nu \right]}
\]

\[
Q_P = \frac{Q_{P1} \left[ 1 - \frac{\gamma_0 \beta_0 G_F}{(\beta_2 - \beta_0) G_0 G_F} \right]}{1 - \frac{K_\nu g_\nu G_F}{(\beta_2 - \beta_0) G_0 G_F}}
\]

\[
Q_\nu = \frac{Q_{P1} + Q_{P2}}{G_0 + G_F - F_0 - G_F}
\]

Bound of \( S_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, \1\)

\[
S_{K_0} = -\frac{1}{2} \left[ \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F} \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F} \right] + \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F}
\]

Bound of \( S_{K_0} \) when all terms in \( D(S) \) are positive: \(-1, \1\)

\[
S_{K_m} = -\frac{1}{2} \left[ \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F} \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F} \right] + \frac{K_0 \gamma_0 (G_0 - G_F)}{G_0 + G_F}
\]

Bound of \( S_{K_m} \) when all terms in \( D(S) \) are positive: \(-1, \1\)

POLYNOMIAL DECOMPOSITION TYPE 20

\[
t_\nu = \frac{K_0 (\alpha_2 S^2 + \beta_2 S + \gamma_2)}{(\beta_2 - \beta_0) S^2 - \beta_1 S + (\beta_0 - \beta_0) \gamma_0}
\]

\[
Q_{P0} = \frac{Q_{P1} \left[ 1 - \frac{\gamma_0 \beta_0 G_F}{(\beta_2 - \beta_0) G_0 G_F} \right]}{1 - \frac{K_0 \gamma_0 G_F}{(\beta_2 - \beta_0) G_0 G_F}}
\]
TABLE 4.2.1 (Continued)

\[ S^Q_{K_0} = \frac{1}{2} \left[\frac{\gamma_0}{\nu_0 - \gamma_0} - \frac{\gamma_2}{\nu_0 - \gamma_2}\right] \]

Bounds of \( S^Q_{K_0} \) when all terms in \( D(S) \) are positive: 0, 1

\[ T^Q = \frac{1}{s^2} \left[ (\gamma_0^H - \gamma_0^G - \gamma_0^F) (\gamma_0^H + \gamma_0^G - \gamma_0^F) + (\gamma_2^H - \gamma_2^G - \gamma_2^F)\right] + \left[ (\gamma_0^H - \gamma_0^G - \gamma_0^F) (\gamma_0^H + \gamma_0^G - \gamma_0^F) + (\gamma_2^H - \gamma_2^G - \gamma_2^F)\right] \]

\[ Q^P = \frac{1}{s^2} \left[ (\gamma_0^H - \gamma_0^G - \gamma_0^F) (\gamma_0^H + \gamma_0^G - \gamma_0^F) + (\gamma_2^H - \gamma_2^G - \gamma_2^F)\right] \]

Bounds of \( S^Q_{K_0} \) when all terms in \( D(S) \) are positive: 0, 1

\[ \frac{G_0}{G_0 + G^H + G^F} \]

Bounds of \( S^Q_{K_m} \) when all terms in \( D(S) \) are positive: -1, 1

\[ \frac{G_0}{G_0 + G^H + G^F} \]
### TABLE 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 21**

<table>
<thead>
<tr>
<th>Term</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_v )</td>
<td>( \frac{K_0^2 S}{(2_2 - K_0 - 2_0 S^2) S_0 S + (S_0 - K_0 Y_0)} )</td>
</tr>
<tr>
<td>( Q_{20} )</td>
<td>( \frac{(S_0 - K_0 Y_0) (2_2 - K_0 Y_0)}{S_2 - K_0 Y_2} )</td>
</tr>
<tr>
<td>( Q_{P0} )</td>
<td>( \frac{K_0^2 G_0 S}{S^2 (3_2 - K_0 Y_0) (G_H G_0 G_F) - S [S_1 (G_H G_0 G_F) - K m_0 G_0 - (S_0 - K_0 Y_0) (G_H G_0 G_F)]} )</td>
</tr>
<tr>
<td>( Q_p )</td>
<td>( \frac{C_20}{1 - \frac{K_0 X_0}{3_1 - G_0 G_F}} )</td>
</tr>
<tr>
<td>( Q_{P0} )</td>
<td>( \frac{K_0^2 G_0 S}{S^2 (3_2 - K_0 Y_0) (G_H G_0 G_F) - S [S_1 (G_H G_0 G_F) - K m_0 G_0 - (S_0 - K_0 Y_0) (G_H G_0 G_F)]} )</td>
</tr>
<tr>
<td>( Q_{P0} )</td>
<td>( \frac{K_0^2 G_0 S}{S^2 (3_2 - K_0 Y_0) (G_H G_0 G_F) - S [S_1 (G_H G_0 G_F) - K m_0 G_0 - (S_0 - K_0 Y_0) (G_H G_0 G_F)]} )</td>
</tr>
<tr>
<td>( S_{X_0} )</td>
<td>( - \frac{1}{2} \left[ \frac{K_0 Y_0}{S_0 - K_0 Y_0} + \frac{K_0 Y_2}{S_2 - K_0 Y_2} \right] - \frac{K m_0 G m_1 G_H}{S_1 (G_H G_0 G_F) - K m_0 G_0 - (S_0 - K_0 Y_0) (G_H G_0 G_F)} )</td>
</tr>
<tr>
<td>( S_{X_0} )</td>
<td>( - \frac{1}{2} \left[ \frac{K_0 Y_0}{S_0 - K_0 Y_0} + \frac{K_0 Y_2}{S_2 - K_0 Y_2} \right] )</td>
</tr>
<tr>
<td>( S_{X_0} )</td>
<td>( S_{X_0} + \frac{1}{2} \left[ \frac{K_0 Y_0}{S_0 - K_0 Y_0} + \frac{K_0 Y_2}{S_2 - K_0 Y_2} \right] )</td>
</tr>
<tr>
<td>( S_{m} )</td>
<td>( S_{X_0} + \frac{1}{2} \left[ \frac{K_0 Y_0}{S_0 - K_0 Y_0} + \frac{K_0 Y_2}{S_2 - K_0 Y_2} \right] )</td>
</tr>
</tbody>
</table>

Bounds of \( S_{X_0} \), \( Q_{P0} \), and \( Q_{P} \) when all terms in \( D(S) \) are positive:
- \( S_{X_0} \): 0, 1
- \( Q_{P0} \): 0, 1
- \( Q_{P} \): 1, 1
- \( S_{m} \): -1, 0
TABLE 4.2.1 (Continued)

**POLYNOMIAL DECOMPOSITION TYPE 22a**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_v$</td>
<td>$\frac{K_0 y_0}{(s_2 - K_0 y_2) s_2 - (\frac{s_2 - K_0 y_2}{1 - K_0 y_1}) s - (\frac{s_2 - K_0 y_2}{1 - K_0 y_1})}$</td>
</tr>
<tr>
<td>$Q_{P_0}$</td>
<td>$\frac{[P - K_0 y_2]}{(s_2 - K_0 y_2)^2}$</td>
</tr>
<tr>
<td>$Q_{S_0}$</td>
<td>$-\frac{2}{s_2} \left[ \frac{K_0 y_0}{s_0 - K_0 y_0} + \frac{K_0 y_2}{s_2 - K_0 y_2} \right] - \frac{K_0 y_1}{3 - K_0 y_1}$</td>
</tr>
<tr>
<td>$T_v$</td>
<td>$\frac{K_0 y_0 y_1 y_2}{s^2 (s_2 - K_0 y_2) (s_2 + G_F + G_H)} + s \left( s_1 - K_0 y_1 \right) \gamma_0 \gamma_1 \gamma_2 \left( s_2 - K_0 y_2 \right)$</td>
</tr>
<tr>
<td>$Q_P$</td>
<td>$\frac{C_{P_0}}{1 - \frac{K_0 y_0 y_1 y_2}{s_2 - K_0 y_2}}$</td>
</tr>
<tr>
<td>$S_{S_0}$</td>
<td>$-\frac{1}{2} \left[ \frac{K_0 y_0}{s_0 - K_0 y_0} + \frac{K_0 y_2}{s_2 - K_0 y_2} \right] - \frac{K_0 y_1}{3 - K_0 y_1} \left( s_2 + G_F + G_H \right) + \frac{K_0 y_0 y_1 y_2}{s_2 - K_0 y_2}$</td>
</tr>
<tr>
<td>$S_{S_0}$</td>
<td>$S_{P_0} + \frac{1}{2} \left[ \frac{K_0 y_0}{s_0 - K_0 y_0} - \frac{K_0 y_2}{s_2 - K_0 y_2} \right]$</td>
</tr>
</tbody>
</table>

Bounds of $S_{S_0}$ when all terms in $D(S)$ are positive: $-1, 1$

Bounds of $Q_{P_0}$ when all terms in $D(S)$ are positive: $-1, 1$

Bounds of $S_{S_0}$ when all terms in $D(S)$ are positive: $-1, 0$
### Table 4.2.1 (Continued)

**Polynomial Decomposition Type 22b**

<table>
<thead>
<tr>
<th>Term</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_0$</td>
<td>$\frac{K_0}{S_1 S_2 - S_0}$</td>
</tr>
<tr>
<td>$Q_{p20}$</td>
<td>$\frac{[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_1}$</td>
</tr>
<tr>
<td>$S_{K_0}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$Q_{p20}$</td>
<td>$K_{m_1}G_{0}G_{2} + S_1[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}$</td>
</tr>
<tr>
<td>$T_{v}$</td>
<td>$\frac{K_{m_1}G_{0}G_{2} + S_1[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_0}$</td>
</tr>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_1}$</td>
</tr>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_1}$</td>
</tr>
<tr>
<td>$S_{K_0}$</td>
<td>$\frac{K_{m_1}G_{0} + G_{P}}{S_1[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}$</td>
</tr>
<tr>
<td>$Q_{p}$</td>
<td>$\frac{[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_1}$</td>
</tr>
<tr>
<td>$S_{K_m}$</td>
<td>$\frac{[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}{S_1[G_{H}G_{0}+G_{P}] - K_{m_1}G_{F}}$</td>
</tr>
</tbody>
</table>

 Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: $0, 0$

 Bounds of $S_{K_0}$ when all terms in $D(S)$ are positive: $-1, 0$

 Bounds of $S_{K_m}$ when all terms in $D(S)$ are positive: $-1, 1$
### TABLE 4.2.1 (Continued)

<table>
<thead>
<tr>
<th>Polynomial Decomposition Type 23</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{\nu} ) = ( \frac{\alpha_0 s^5}{s^2 (s^2 - s^{-1,1}_0) s^{-1,0}} )</td>
</tr>
<tr>
<td>( Q_{p0} ) = ( \frac{\left( \frac{s_0 s_2}{s_{1,0}} \right)^2}{(s^2 - s^{-1,0}_1)} )</td>
</tr>
<tr>
<td>( S_{\nu,0} = \frac{\alpha_0}{s_{1,0}} )</td>
</tr>
<tr>
<td>Bounds of ( Q_{p0} ) when all terms in ( D(S) ) are positive: ( -1, 0 )</td>
</tr>
<tr>
<td>( T_{\nu}(s) = \frac{K_m K_0 G_{a,5} s}{s^2 (s^2 (s^2 - s^{-1,0}<em>0) - K_m s_2 G</em>{p1}) + s ((s - s_{1,0}) (s_{2,0} - s_{1,0}) - K_{p1} s_2 G_{p1} + K_m s_0 (G_{a,5} - G_{a,1})) + s_0 (G_{a,5} - G_{p1}) - K_m s_0 G_{p1}} )</td>
</tr>
<tr>
<td>( Q_{p} = \left[ \frac{\left( \frac{s_0 s_2}{s_{1,0}} \right)^2}{(s^2 - s^{-1,0}_1)} \right] )</td>
</tr>
<tr>
<td>Bounds of ( S_{\nu,0} ) when all terms in ( D(S) ) are positive: ( -1, 0 )</td>
</tr>
<tr>
<td>( S_{\nu,0} = \frac{K_m K_0 (G_{a,5} - G_{a,1})}{(s^2 - s^{-1,0}<em>0) (s^2 - s</em>{1,0}) + K_m s_2 G_{p1} + K_m s_0 (G_{a,5} - G_{a,1})} )</td>
</tr>
<tr>
<td>( S_{\nu,0} = \frac{K_m K_0 (G_{a,5} - G_{a,1})}{(s^2 - s^{-1,0}<em>0) (s^2 - s</em>{1,0}) + K_m s_2 G_{p1} + K_m s_0 (G_{a,5} - G_{a,1})} )</td>
</tr>
<tr>
<td>Bounds of ( S_{\nu,0} ) when all terms in ( D(S) ) are positive: ( -1, 0 )</td>
</tr>
<tr>
<td>( S_{\nu,0} = \frac{K_m K_0 (G_{a,5} - G_{a,1})}{(s^2 - s^{-1,0}<em>0) (s^2 - s</em>{1,0}) + K_m s_2 G_{p1} + K_m s_0 (G_{a,5} - G_{a,1})} )</td>
</tr>
<tr>
<td>Bounds of ( S_{\nu,0} ) when all terms in ( D(S) ) are positive: ( -1, 0 )</td>
</tr>
</tbody>
</table>
TABLE 4.2.1 (Continued)

POLYNOMIAL DECOMPOSITION TYPE 21

\[ t_{V} = \frac{K_{0} S}{\beta_{2} S^{2} (2 - K_{0} \nu_{1})} \]

\[ Q_{P0} = \frac{[\frac{2 - K_{0} \nu_{1}}{2 - K_{0} \nu_{1}}]}{[\frac{1 - K_{0} \nu_{1}}{1 - K_{0} \nu_{1}}]} \]

\[ Q_{P0} \] rounds of \( S_{K_{0}} \) when all terms in \( D(S) \) are positive: \(-1.0\)

\[ K_{0} = \frac{K_{0} S_{0}}{\nu_{1}} \]

\[ T_{V} (S) = \frac{K_{0} S_{0} S_{1}}{S^{2} (2 - G_{H} + G_{0} - G_{F}) \nu_{1}} \]

\[ Q_{P} = \frac{Q_{P0} \left[ 1 - \frac{K_{H} G_{F}}{G_{H} G_{0} G_{F}} \right]}{1 - \frac{G_{H} G_{0} G_{F}}{G_{H} G_{0} G_{F}} (2 - K_{0} \nu_{1})} \]

\[ S_{K_{0}} = \frac{S_{K_{0}}}{(2 - K_{0} \nu_{1}) (G_{H} G_{0} - G_{F}) \nu_{1} G_{F}} \]

\[ Q_{P} \] rounds of \( S_{K_{0}} \) when all terms in \( D(S) \) are positive: \(-1.0\)

\[ S_{K_{0}} \]

\[ S_{K_{0}} = \frac{-K_{m} G_{F}}{(G_{H} G_{0} G_{F} - G_{0} G_{F})} + \frac{K_{m} G_{F}}{(G_{H} G_{0} G_{F} - G_{0} G_{F}) (G_{H} G_{0} G_{F} - K_{m} G_{F})} \]

\[ S_{K_{0}} \] rounds of \( S_{K_{m}} \) when all terms in \( D(S) \) are positive: \(-1.1\)

**Note:** In some cases quantities are marked (1) or (2'). Either of these but not both can be set to zero.
TABLE 4.2.2

POLYNOMIAL DECOMPOSITIONS OBTAINABLE FROM GIVEN SECOND ORDER GENERATING FUNCTION $t$.
<table>
<thead>
<tr>
<th>Generating Transfer Function $t_N$</th>
<th>Obtainable polynomial decompositions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>$N0$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0(a_2s^2+a_0)}{s_2s^2+a_1s+a_0}$</td>
<td></td>
</tr>
<tr>
<td>$N1$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0(a_2s^2+a_0)}{s_2s^2(a_1+K_0\gamma_1)s+a_0}$</td>
<td></td>
</tr>
<tr>
<td>$N2$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0(a_2s^2+a_0)}{(s_2-K_0\gamma_1)s^2+\gamma_1(s)+(s_0-K_0\gamma_0)}$</td>
<td></td>
</tr>
<tr>
<td>$N3$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0(a_2s^2+a_0)}{(s_2-K_0\gamma_1)s^2+(s_1-K_0\gamma_1)s+(s_0-K_0\gamma_0)}$</td>
<td></td>
</tr>
<tr>
<td>$B0$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0s_1s}{s_2s^2+a_1s+a_0}$</td>
<td></td>
</tr>
<tr>
<td>$B1$</td>
<td></td>
</tr>
<tr>
<td>$\frac{K_0s_1s}{s_2s^2+(s_1-K_0\gamma_1)s+a_0}$</td>
<td></td>
</tr>
<tr>
<td>Generating Transfer Function $t_v$</td>
<td>Obtainable polynomial decompositions</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>----------------------------------</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>B2 $\frac{K_0 a_1 S}{(S_2-K_0 \gamma_2) S^2 + (S_1-K_0 \gamma_1) S + (S_0-K_0 \gamma_0)}$</td>
<td></td>
</tr>
<tr>
<td>B3 $\frac{K_0 a_1 S}{(S_2-K_0 \gamma_2) S^2 + (S_1-K_0 \gamma_1) S + (S_0-K_0 \gamma_0)}$</td>
<td></td>
</tr>
<tr>
<td>C0 $\frac{K_0 (a_2 S^2 + a_1 S + a_0)}{S_2 S^2 + S_1 S + S_0}$</td>
<td></td>
</tr>
<tr>
<td>Cl $\frac{K_0 (a_2 S^2 + a_1 S + a_0)}{S_2 S^2 + (S_1-K_0 \gamma_1) S + S_0}$</td>
<td></td>
</tr>
<tr>
<td>C2 $\frac{K_0 (a_2 S^2 + a_1 S + a_0)}{(S_2-K_0 \gamma_2) S^2 + S_1 S + (S_0-K_0 \gamma_0)}$</td>
<td></td>
</tr>
<tr>
<td>C3 $\frac{K_0 (a_2 S^2 + a_1 S + a_0)}{(S_2-K_0 \gamma_2) S^2 + (S_1-K_0 \gamma_1) S + (S_0-K_0 \gamma_0)}$</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4.2.3

DIFFERENT QMCS OBTAINABLE FROM SECOND ORDER GENERATING FUNCTIONS REALIZED BY SACs.
**Generating function type: N0**

Design Equations: \[ n_0 \]
\[
\begin{align*}
\omega_0 G_1 &= C_1 = (a) \frac{1}{(a-1)^2} \\
\omega_0 G_2 &= C_2 = (a^2 + 1) \frac{1}{[2(a-1)]^2} \\
\omega_0 G_3 &= C_3 = (a-1) \frac{1}{[2(a^2)]^2} \\
Q_0 &= \left(1 + \frac{1}{a}\right)^{-1} \\
a &> 1
\end{align*}
\]

Identifications with the general parameters, \(a_2 = b_2 = 1\), \(a_0 = b_0 = \omega_0\).

**Null Type 3**

\[
T_{N3} = \frac{n_H(s)}{D_3(s)} = \frac{K_m K_0 G_0 (S^2 + \omega_0^2)}{S^2 |G_0 + G_H - G_{H_0} K_m| Q_0 S + |G_0 + G_{H_0} - G_{H_0} K_m|}
\]

**Band Pass Type 3**

\[
T_{BP3} = \frac{K_m K_0 (G_0 + G_H) Q_{BP3} S}{D_3(s)}
\]

**Null Type 12**

\[
T_{N12} = \frac{n_H(s)}{D_{N12}(s)} = \frac{K_m K_0 G_0 (S^2 + \omega_0^2)}{[G_0 + 2(G_1 + G_2)(1-K_m)] Q_{N12} S + [G_0 + G_H (1-K_m)]}
\]

**Band Pass Type 12**

\[
T_{BP12} = \frac{K_m Q_{BP12} S}{D_{12}(s)}
\]
TABLE 4.2.3 (Continued)

Generating Function type: B0

Design Equations: Identical to equations given for the N0 type

\[ t_{B0} = \frac{K_0 \omega_{p0} \omega_{q0}}{S^2 + \omega_{p0}^2 + \omega_{q0}^2} \]

Identification with the general parameters \( \beta_2 = 1 \), \( \beta_0 = \omega_0 \), \( \beta_1 = \omega_{p0}(a+u)^{-1} \), \( a_1 = \frac{\omega_{q0}}{\omega_{p0}} \)

Null Type 1

\[ T_{BP1} = \frac{n_{BP}(s)}{D_1(s)} = \frac{K_m K_0 (\omega_{pq}/\omega_{q0}) \omega_{p0} \omega_{q0}}{S^2 [G_0 + G_m G_H] + \omega_{p0}^2 \left( [G_0 + G_H G_m G_H] - K_m K_0 G_H + K_m K_0 G_H \right) + (G_0 + G_H \omega_{p0}^2)^2} \]

Band Pass Type 22b

\[ T_{BP22} = \frac{n_{BP}(s)}{P_{22}(s)} = \frac{K_m K_0 (\omega_{p0}/\omega_{q0}) \omega_{p0}}{S^2 \left( G_0 + G_m G_F (1-K_m) \right) + \omega_{p0}^2 \left( [G_0 + G_H G_F (1-K_m)] + G_0 + G_H G_F (1-K_m) \right) + \omega_{p0}^2 \left( G_0 + G_H G_F (1-K_m) \right)^2} \]

Null Type 72b

\[ T_{N22} = \frac{n_{BP}(s)}{D_{22}(s)} = \frac{K_0 (\omega_{p0}/\omega_{q0}) \omega_{p0} G_H G_F (1-K_m)}{S \left( G_0 + G_H G_F (1-K_m) + G_0 + G_H G_F (1-K_m) \right) + \omega_{p0}^2 \left( G_0 + G_H G_F (1-K_m) \right)^2} \]
TABLE 4.2.3 (Continued)

Generating function type N1

\[ t_{N1} = \frac{K_0 \left[ C_1 C_2 C_3 S^2 + G_1 G_2 (C_2 + C_3) \right]}{C_1 C_2 C_3 S^2 + S \left[ C_1 C_2 G_3 (1 - K_0) + C_1 G_2 (C_2 + C_3) \right] + G_1 G_2 (C_2 + C_3)} \]

\[ C_1 \frac{S + G_2}{G_1 + G_2} = \frac{C_2 + C_3}{C_3} \]

\[ \omega_{P0} = \sqrt{\frac{G_1 G_2 (C_2 + C_3)}{G_1 C_2 C_3}} \]

\[ Q_{P0} = \sqrt{\frac{G_1 G_2 (C_2 + C_3)}{C_1 C_2 C_3}} \left[ \frac{G_2 (1 - K_0)}{C_3} + \frac{G_2 (C_2 + C_3)}{C_2 C_3} \right] \]

Design Equations: [17]

\[ \omega_{P0} = 1 \]

\[ G_2 = \frac{c}{G_1 = c (c - 1 - 1)} \]

\[ G_3 = (b - c - \frac{1}{Q_{P0}})^2 \left( b - c - \frac{1}{Q_{P0}} \right)^{-1} \]

\[ c_1 = \frac{c^2}{c_1 + 1} \]

\[ c_2 = (b - c - \frac{1}{Q_{P0}}) / c_1 \]

\[ c_3 = (b - c - \frac{1}{Q_{P0}}) \left( b - c - \frac{1}{Q_{P0}} \right)^{-1} \]

\[ K_0 = b \left( b - c - \frac{1}{Q_{P0}} \right)^{-1} \]

\[ c_1 < c < b - c - \frac{1}{Q_{P0}} \]

Identification with the general parameters:

\[ b_2 = a_2 = C_1 C_2 C_3 \]

\[ b_0 = a_0 = G_1 G_2 (C_2 + C_3) \]

\[ b_1 = C_1 C_2 C_3 \left( C_1 G_2 (C_2 + C_3) \right) \]

\[ Y_1 = C_1 C_2 G_3 \]
TABLE 4.2.3 (Continued)

Generating function type Bl

\[
K_0 (C_2 + C_3) C_1 G_2 S
\]

\[
\frac{C_1 C_2 C_3 S^2 + [C_1 C_2 G_3 (1 - K_0) + C_1 G_2 (C_2 + C_3)] S + G_1 G_2 (C_2 + C_3)}{}
\]

\[
\frac{C_1}{G_1 + \frac{S^2}{G_3}} = \frac{C_1 + C_2}{G_3}
\]

\[
\omega_p^0 = \sqrt{\frac{G_1 G_2 [C_2 + C_3]}{C_1 C_2 C_3}}
\]

\[
Q_p^0 = \frac{\sqrt{C_1 C_2 C_3 G_1 G_2 (C_2 + C_3)}}{[C_1 C_2 G_3 (1 - K_0) + C_1 G_2 (C_2 + C_3)]}
\]

Design Equations [17]

\[
\omega_p^0 = 1
\]

\[
G_1 = \frac{b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0}}{Q_p^0} [b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0} - \frac{1}{\varepsilon_1}]^{-1}
\]

\[
G_2 = \frac{b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0}}{Q_p^0} \varepsilon_1
\]

\[
G_3 = b^2 [K_0 (b/c_1 - K_0)]^{-1}
\]

\[
C_1 = \frac{b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0}}{Q_p^0} [b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0} - \frac{1}{\varepsilon_1}]^{-1}
\]

\[
c_2 = \frac{b}{K_0}
\]

\[
c_3 = b \left( \frac{b}{\varepsilon_1} - K_0 \right)^{-1}
\]

\[
\frac{b}{\varepsilon_1} > K_0 > 0 \quad b (1 - \frac{1}{K_0}) + \frac{1}{Q_p^0} > \frac{1}{\varepsilon_1} > 0
\]

Identification with the general parameters:

\[
\beta_2 = C_1 C_2 C_3
\]

\[
\beta_1 = C_1 C_2 G_3 + C_1 G_2 (C_2 + C_3)
\]

\[
\beta_0 = C_1 G_2 (C_2 + C_3)
\]

\[
\gamma_1 = C_1 C_2 G_3
\]

\[
\alpha_1 = C_1 G_2 (C_2 + C_3)
\]
**Table 4.2.3 (Continued)**

**Band Pass Type 2**

\[
T_{BP2} = \frac{n_{BP}(s)}{D_2(s)} = \frac{K_m K_0 C_0 C_1 G_2 C_2 s}{C_1 C_2 C_3 (G_2 + G_H) s + \left( (G_0 + C_1 G_2) C_1 G_3 + C_1 G_2 C_1 G_3 s + C_1 G_2 (C_2 + C_3) - K_0 C_2 G_3 \right)}
\]

**Null Type 2**

\[
T_{N2} = \frac{K_0 (G_0 + G_H) C_2 G_2 C_2 s}{G_0 G_2 (C_2 + C_3) s}
\]

**Band Pass Type 21 and 24**

\[
T_{BP23} = \frac{n_{BP}(s)}{D_2(s)} = \frac{K_m K_0 C_0 C_2 G_2 (C_2 + C_3) s}{C_1 C_2 C_3 (G_0 + C_1 G_2 + G_H) s + \left( (1-K_m) C_1 C_2 C_3 s + C_1 G_2 C_1 G_3 + C_1 G_2 (C_2 + C_3) - K_0 C_2 G_3 \right)}
\]

If \( C_2 C_3 G_2 (C_2 + C_3) G_H = 0 \), \( T_{BP23} \) is transformed into \( T_{BP24} = \frac{n_{BP}}{D_{24}(s)} \)

**Null Type 23 and 24**

\[
T_{N23} = \frac{n_{N}(s)}{D_{23}(s)} = \left( K_0 (G_0 + C_1 G_2 + C_2 (1-K_m)) \right) \left( \frac{C_2 C_3 s^2 + C_1 G_2 C_1 G_3 + C_2 C_3 s}{D_{24}(s)} \right)
\]

If \( C_2 C_3 G_2 (C_2 + C_3) G_H = 0 \), \( T_{N23} \) is transformed into \( T_{N24} = \frac{n_{N}(s)}{D_{24}(s)} \)
### TABLE 4.2.3 (Continued)

**Generating function type H2**

![Circuit Diagram]

\[
\frac{K_D}{C_1 C_2 C_3 S^2 + C_1 C_3 (C_1 + C_3)}
\]

\[
C_1 + C_3 = C_2
\]

\[
\frac{G_2}{G_1 + G_3}
\]

\[
\omega_{p0} = \sqrt{\frac{G_1 G_3 (C_1 + C_3)}{C_1 C_2 C_3}}
\]

\[
Q_{p0} = \left[ C_1 C_2 C_3 G_1 G_3 (C_1 + C_3)^{-\frac{1}{2}} \right] \left[ 1 + K_0 \right] \left[ C_2 C_3 G_2 + G_3 (C_1 + C_3) \right]^{-1}
\]

**Design Equations**: [17]

\[
\omega_{p0} = 1
\]

\[
K_D = (p_0 p_0 - 1)
\]

\[
G_2 = \frac{G_1}{4} \left[ b^2 n - 4(n + 1) \right]
\]

\[
G_3 = G_1 \left[ b^2 n - 4(n + 1) \right]
\]

\[
c_1 = G_1 \left[ b^2 n - 4(n + 1) \right] / 2b
\]

\[
c_2 = \frac{b}{2} G_1
\]

\[
c_3 = G_1 \left[ b^2 n - 4(n + 1) \right] / 2b
\]

**Identification with the general parameters**:

\[
\beta_2 = \gamma_2 = -a_2 = C_1 C_2 C_3
\]

\[
\beta_0 = \gamma_0 = -a_0 = G_1 G_3 (C_1 + C_3)
\]

\[
\beta_1 = C_2 C_3 G_2 + G_3 (C_1 + C_3)
\]

\[
K_0 = -K_0
\]
TABLE 4.2.3 (Continued)

Null Type 7

\[ T_{N7} = \frac{K_mK_D G_0 \left[ C_1C_2C_3 \frac{s^2}{s} + G_1G_3 \left( C_1+C_3 \right) \right]}{C_1^2C_2^2C_3 \left[ \left( G_0+G_H \right) \left( 1+K_D \right) - K_mK_D \left( G_H+G_P \right) \right]} \]

Null Type 16

\[ T_{N16} = \frac{K_mK_D G_0 \left[ C_1C_2C_3 \frac{s^2}{s} + G_1G_3 \left( C_1+C_3 \right) \right]}{s^2C_1C_2C_3 \left[ \left( G_0+G_H \right) \left( 1+K_D \right) - K_mG_P-K_mK_D \left( G_H+G_P \right) \right]} \]
TABLE 4.2.3 (Continued)

Generating function type B2

\[
\begin{align*}
t_{B2} &= \frac{-K_0 C_2 C_3 a S}{C_1 C_2 C_3 (1-K_0) S^2 + S |C_2 C_3 (G_a + G_2) + C_2 (C_1 + C_3) G_2| + G_1 (G_a + G_2 + G_3) (1-K_0)} \\
\frac{C_2}{C_1 + C_3} &= \frac{G_1 + G_3}{G_a + G_2} \\
\omega_p^2 &= \frac{G_1 (G_a + G_2) G_2}{C_2} \\
0_p &= (C_1 C_2 C_3 G_1 G_2 G_3)^{\frac{1}{3}} (1 + K_0) (C_3 C_2 (G_a + G_2) + C_2 (C_1 + C_3) G_3)^{-1}
\end{align*}
\]

Design Equations:

\[
\begin{align*}
\omega_p &= 1 \\
G_a &= G_2 = G_1 \left( b \eta + \frac{4(\eta+1)}{\eta+1} \right) \\
G_3 &= G_1 \left( b^2 \eta^{-4(\eta+1)} / 4(\eta+1) \right) \\
\delta_1 &= G_1 \left( b^2 \eta^{-4(\eta+1)} / 2b \right) \\
C_2 &= G_1 \frac{b}{2} \\
C_3 &= G_1 \left( b^2 \eta^{-4(\eta+1)} / 2b \right) \\
K_0 &= -\left( b \omega_p^2 \right)^{-1} \\
b^2 &> 4 + \frac{4}{\eta^2}
\end{align*}
\]

Identification with the general parameters:

\[
\begin{align*}
\delta_2 &= \gamma_2 = C_1 C_2 C_3 \\
\delta_0 &= \gamma_0 = G_1 (G_a + G_2) |G_3| \\
\delta_1' &= C_2 C_3 (G_a + G_2) + C_2 C_3 (C_1 + C_3) \\
\delta_1 &= C_2 C_3 G_0
\end{align*}
\]
Generating function, type C2

\[
\begin{align*}
\frac{g_1}{g_2} & = \frac{c_1}{g_1 + g_3} \\
\frac{g_2}{g_4} & = \frac{c_2}{g_1 + g_3} \\
\frac{g_3}{g_4} & = \frac{c_3}{g_1 + g_3} \\
\frac{g_4}{g_5} & = \frac{c_4}{g_1 + g_3} \\
\frac{g_5}{g_6} & = \frac{c_5}{g_1 + g_3} \\
\end{align*}
\]

Denominator equations: [17]

\[
\begin{align*}
\pi_0 & = 1 \\
g_2 & = \frac{g_1}{4}(b^{2n} - 4(n+1)) \\
g_3 & = \frac{g_1}{4}(b^{2n} - 4(n+1)) \\
c_1 & = \frac{g_1}{2}(b^{2n} - 4(n+1)/2b \\
c_2 & = \frac{g_1}{2}b/2 \\
c_3 & = \frac{g_1}{2}(b^{2n} - 4(n+1))/2b \\
\end{align*}
\]

Identification with the general parameters:

\[
\begin{align*}
\beta_2 & = \gamma_2 = c_1c_2c_3 \\
\beta_0 & = \gamma_0 = g_1g_3(c_1+c_3) \\
\beta_1 & = c_2c_3g_2 + g_3(c_1+c_3)g_2 \\
\alpha_2 & = \frac{g_1}{g_4}g_2 \\
\alpha_0 & = \frac{g_1}{g_4^2}g_2 \\
\alpha_1 & = \frac{g_1}{g_4^2}g_2 \\
\end{align*}
\]

\[K_0 = K_D\]
TABLE 4.2.3 (Continued)

All Pass Type 8

\[ T_{AP8} = \frac{K_m K_D G_G G_4}{(G_4 + G_5)(G_0 + G_H)} \left[ S \left( C_1 C_2 C_3 S^2 - \frac{G_5}{G_4} C_2 G_3 (C_1 + C_3) + C_2 C_3 G_2 S + G_1 G_3 (C_1 + C_3) \right) \right] \]

with

\[ \frac{G_5}{G_4} \left[ 1 + \frac{K_m K_D G_G G_4}{(G_0 + G_H)(G_4 + G_5)} \right] = \left[ 1 + \frac{K_m K_D G_G G_4}{(G_0 + G_H)(1 - G_5/G_4)} \right] \]

All Pass Type 12 and 17

\[ T_{AP12} = \frac{K_m K_D G_G}{(G_4 + G_5)(G_0 + G_F + G_H)} \left[ S \left( C_1 C_2 C_3 S^2 - \frac{G_5}{G_4} C_2 G_3 (C_1 + C_3) + C_2 C_3 G_2 S + G_1 G_3 (C_1 + C_3) \right) \right] \]

with

\[ \frac{G_5}{G_4} \left[ 1 + \frac{K_m K_D G_G}{(G_0 + G_F + G_H)} \right] = \left[ 1 + \frac{K_m K_D G_G}{(G_0 + G_F + G_H)(1 - G_5/G_4)} \right] \]

If we let also \( G_F = G_4 \) we transform \( T_{AP12} \) into an All Pass of type 17.
be realized using optimized circuits containing not more than three capacitors [17, 18]. Hence our choice of generating functions will be restricted to the $N_0$, $N_1$, $N_2$, $B_0$, $B_1$, $B_2$, and $C_2$ types only.

Table 4.2.3 gives different TACs obtainable from the above mentioned generating functions and the corresponding transfer functions. Table 4.2.3 can be combined with Table 4.2.1 to obtain the different properties of these circuits.

The following classification of the generating circuits can be made:

1. Passive circuits cascaded with an amplifier (called RI)

2. Single ended (inverting or non-inverting) amplifier circuits (called RII)

3. Differential amplifier circuits (called RIII)

A detailed study of representative QMCs obtained from one of each of the above mentioned types of generating circuits will be given in this chapter.

4.3 A QMC Realization Obtained Using an RI Generating Circuit

The overall realization consisting of two amplifiers is as shown in Fig. 4.3.1a (Figs. 4.3.1b and 4.3.1c show the actual realizations of the two amplifiers contained in Fig. 4.3.1a). It consists of a passive optimized twin-T network
A SET OF DESIGN EQUATIONS

\[ G_1, G_2, G_3, C_1, C_2, C_3 \] are given in Table 4.2.3

\[ \frac{G_1}{G_2} = 1 \]

\[ K_m K_0 = \frac{G_A}{G_B} \cdot 1 \cdot \frac{G_1/C_1}{G_2/C_2} \cdot \frac{G_3/C_3}{G_0} \]

\[ K_m K_0 \] is obtained from Fig. 4.3.6 or Fig. 4.3.7 for specified \( \phi \).
cascaded with a finite gain amplifier constituting an RI generating circuit having a $t_v$ of type $N_0$, used in the configuration of Fig. 4.2.1 with $G_F = 0$. The entire network corresponds to a null filter having a denominator decomposition of type 3. Its transfer function is given by

$$T_3 = \frac{-K_m K_0 G_0 (S^2 + \omega_p^2)}{S^2 [1 + K_m K_0/x] + S \frac{\omega_p}{G_{p0}} + [1 + K_m K_0/x] \omega_p^2}$$  \hspace{1cm} (4.3.1a)

$$x = 1 + \frac{G_o + G_A}{G_H}$$  \hspace{1cm} (4.3.1b)

$$K_m = \frac{G_A}{G_B}$$  \hspace{1cm} (4.3.1c)

$$K_0 = 1 + \frac{G_A}{G_B}$$  \hspace{1cm} (4.3.1d)

This takes care of the loading effect of the amplifier.

The design equations of the twin T have been given in Table 4.2.3. Fig. 4.3.2 shows the variation of the resistive and capacitive spreads as well as the variation of $Q_{p0}$ ($Q$ of the passive circuit) as functions of the parameter $\alpha$. From the graph it is clear that the minimum value of spread is 2 and occurs when $Q_{p0} = 0.25$ which corresponds to $\alpha = 3.732$. From the transfer function given in Eqn. (3.3.1) we get

$$\omega_p = \omega_{p0}$$  \hspace{1cm} (4.3.2)
Figure 4.3.2 The variation of the capacitive spread, the resistive spread, and $Q_{p0}$ of the passive twin-T versus the parameter $\alpha$. 

$$\frac{C_{\text{max}}}{C_{\text{min}}} \cdot \frac{G_{\text{max}}}{G_{\text{min}}} = \gamma$$
\( S_{K_m}^{\omega_p} = S_{K_0}^{\omega_p} = 0 \) \hspace{1cm} (4.3.3)

\[ Q_p = Q_{p0} \left[ 1 + \frac{K_m K_0}{\chi} \right] \] \hspace{1cm} (4.3.4)

\[ \frac{Q_p}{S_{K_m}} = \frac{Q_{p0}}{S_{K_0}} = \frac{\frac{K_m K_0}{\chi}}{1 + \frac{K_m K_0}{\chi}} \] \hspace{1cm} (4.3.5)

\[ |K_0 S_{K_0}| = \frac{K_0^2 K_m}{1 + \frac{K_m K_0}{\chi}} \] \hspace{1cm} (4.3.6a)

\[ |K_m S_{K_m}| = \frac{K_m^2 K_0}{1 + \frac{K_m K_0}{\chi}} \] \hspace{1cm} (4.3.6b)

Defining the \( Q \)-multiplication factor \( \rho \) as

\[ \rho = \frac{Q_p}{Q_{p0}} \] \hspace{1cm} (4.3.7)

we get

\[ F = |K_m S_{K_m}| + |K_0 S_{K_0}| = (K_m + K_0) \left( 1 - \frac{1}{\rho} \right) \] \hspace{1cm} (4.3.8)

From Eqn. 4.3.4 we have

\[ K_m K_0 = \chi (\rho - 1) \] \hspace{1cm} (4.3.9)

Hence it is clear that for a given value of \( \rho \) and of the parameter \( \chi \), which is independent of \( K_m \) and \( K_0 \), the minimum
value of $F$ is obtained if we let

$$K_m = K_0 \Delta K.$$  

(4.3.10)

Hence

$$K = (x(p-1))^2$$  

(4.3.11)

and $F$ becomes

$$F_m = 2x^{1/2} \frac{(p-1)^{3/2}}{\rho}.$$  

(4.3.12)

It is obvious that for a given value of $\rho$, it is desirable to reduce the parameter $x$ as much as possible. However, inspection of Eqn. (4.3.1b) shows that the minimum possible value of $x$ is 1 and this implies unrestricted resistive spread. In which case

$$F_m = \frac{2(p-1)^{3/2}}{\rho}.$$  

(4.3.13)

which can be approximated for $\rho >> 1$ as

$$F_m = 2p^{1/2}.$$  

(4.3.14)

From the previous equations it is clear that $F$ can be reduced for a given value of $Q_p$ by decreasing $\rho$ which implies increasing $Q_{p0}$. However, inspection of the graphs of Fig. (4.3.2) shows that this could be done only at the expense of increasing the resistive and capacitive spread. It is
known that the maximum theoretical value for \( Q_{p0} \) is 0.5 for an infinite capacitive spread. Hence the theoretical lower limit of \( \rho \) is \( 2Q_p \). Figs. (4.3.3) and (4.3.4) show the variation of \( \frac{F_m}{Q_p} \) versus \( Q_p \) for several values of \( x \) when \( Q_{p0} = 0.25 \) (corresponding to a capacitive spread of 2) and \( Q_p = 0.4 \) (corresponding to a capacitive spread of 6). It is clearly noted from the graphs that \( \frac{F_m}{Q_p} \) decreases as \( Q_p \) increases. The variation of \( K \) for several values of \( x \) versus \( \rho \) is shown in Fig. 4.3.5.

It is clear from Eqn. 4.3.12 as well from the graphs shown in Fig. 4.3.3 and Fig. 4.3.4 that it is desirable to reduce the value of \( x \) in order to reduce the value of \( \frac{F}{Q_p} \) corresponding to any prescribed value of \( Q_p \). However decreasing the value of \( x \) is associated with an increase in the resistive spread \( \gamma \). Let

\[
x = 1 + \frac{1}{\gamma_A} + \frac{1}{\gamma_0}
\]

(4.3.15a)

\[
\frac{1}{\gamma_A} = \frac{G_A}{G_H}
\]

(4.3.15b)

\[
\frac{1}{\gamma_0} = \frac{G_0}{G_H}
\]

(4.3.15c)

Where for the purpose of minimizing \( x \) it is assumed that

\( \gamma_A \ll 1 \), \( \gamma_0 \gg 1 \) with these assumptions:

\[
\frac{G_H}{G_B} = \gamma_A^K
\]

(4.3.16)
$F_m/Q_p$ OF THE NULL TYPE 3 QMC FOR SEVERAL VALUES OF $x$ VERSUS $Q_p$ WHEN $Q_{PQ} = 0.25$
FIGURE 4.3.4

\( F_m/Q_p \) OF THE NULL TYPE 3 QMC FOR SEVERAL VALUES OF \( x \) VERSUS \( Q_p \) WHEN \( Q_{p0} = 0.4 \)
Figure 4.3.5

K of the null type β QMC for several values of x and minimized f versus ρ.
is the quantity which for low values of $Q_{p0}$ and high values of $Q_p$ (which is true in this thesis) determines the resistive spread $\gamma$. Therefore

$$\gamma_A = 1$$  \hspace{1cm} (4.3.17a)

and

$$\gamma_0 = K$$  \hspace{1cm} (4.3.17b)

appears to be a good compromise to achieve low values of $F_m$ and $\gamma$. Hence

$$x = 2 + \frac{1}{K}$$ \hspace{1cm} (4.3.17c)

$$\rho = 1 + \frac{K^2}{2K+1}$$ \hspace{1cm} (4.3.18a)

$$F_m = \frac{2(2+\frac{1}{K})^{1/2} - (n-1)^{3/2}}{\rho}$$ \hspace{1cm} (4.3.18b)

A set of curves giving $F_m/Q_p$, $K_m$ (obtained from Eqn. 4.3.18) and $\gamma$ versus $Q_p$ for $Q_{p0} = 0.25$ and $Q_{p0} = 0.4$ are given respectively in Figs. 4.3.6, 4.3.7.

For the Null filter network shown in Fig. 4.3.1 grounding the input end of $G_0$ and lifting from ground the common end of $G_2$ and $C_2$ to transform it into an input terminal as shown in Table 4.2.3, we get a type 3 Band Pass filter network having the same denominator as that of the Null network. For the same element values this network shall have all the properties, namely sensitivities, gain sensitivity
FIGURE 4.3.7

$\frac{F_m}{Q_p}$, $K_m$ and $\gamma$ of the null type 3 QMC for $x = 2 + 1/K_m$ versus $Q_p$ when $Q_{p0} = 0.4$. (for $Q_{p0} = 0.4$)
products, $P$, $Q$-multiplication, etc. of the null network and as such no further detailed discussion of this network is necessary.

4.3.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below.

**Designed values:**

\[
\begin{align*}
\alpha & = 3.732 \\
\omega_n & = 6203.185 \text{ rad/sec.} = 1000 \text{ Hz} \\
\phi & = 0.25 \\
Q_p & = 50 \\
p & = 200 \\
C_1 = C_3 & = 11.254 \text{ KPF} \\
C_2 & = 22.508 \text{ KPF} \\
R_1 = R_3 & = 14.142 \text{ K} \Omega \\
R_2 & = R_A = R_A' = R_H = 7.071 \text{ K} \Omega \\
K_m = K_0 & = 20.194 \\
x & = 2.049 \\
\gamma_A & = 1 \\
R_0 = R_B & = 142.792 \text{ K} \Omega \\
R_m & = 135.720 \text{ K} \Omega \\

\text{GSP Values:}
\end{align*}
\]

\[
\begin{align*}
K_0^\omega P = K_m^\omega P = 0
\end{align*}
\]
\[ \frac{Q_E}{Q_P} = \frac{Q_E}{Q_P} = 0.402 \]
\[ F/Q_P = 0.804 \]

Element spread:

Capacitive 2:1
Resistive 20.194:1

The 3 dB frequencies are:

\[ F_{C1} = 990.05 \text{ Hz, } F_{C2} = 1010.05 \text{ Hz} \]

Any or both of \( k_m \) and \( k_0 \) can be adjusted in order to achieve the tuning of \( Q_P \) without affecting \( \omega_P \).

The circuit has been implemented using 1% tolerance resistors and 2% tolerance condensers with values chosen to be, within the range of currently available elements, as close as possible to the designed ones. (Trim pots have been used wherever necessary.) The OAs which have been used were SN72301 properly compensated so as to avoid high frequency oscillations due to excessive phase shift [20].

Actual values obtained:

\[ \omega_P = 999.6 \text{ Hz.} \]
\[ F_{C1} = 991.7 \text{ Hz, } F_{C2} = 1011.4 \text{ Hz.} \]
\[ Q_P = 50.74 \]

The depth of the notch has been found to be -70 dB. It is found that the circuit was stable during activation. \( Q_P \) and
FIGURE 4.3.8b ($\pm 15V$)

FIGURE 4.3.8 FREQUENCY RESPONSE OF THE NULL TYPE 3 QMC
FOR $Q_p=50$
\( \omega_p \) variations from their designed values are 1.48% and 0.04% respectively. This is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. The power supply voltages used were +10V and +15V. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supplies voltages. Only the OAs were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology. As shown in Fig. 4.3.8 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.

### 4.3.2 Effect of the Pole of the O.A. on \( Q \) and \( \omega \) [17,33]

In order to study the effect of the pole of the O.A. on \( Q_p \) and \( \omega_p \), it is assumed that the desired elements are available and therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 3.3.1a by the networks of Fig. 3.3.1b and 3.3.1c the analysis gives:

\[
D(S) = S^2 \left[ 1 + \frac{K_m K_0}{X} + \frac{1}{A_m(S)} \left( 1 + \frac{K_m}{X} + \frac{Y}{X} \right) + \frac{K_0}{A_0(S)} \right] + \frac{K_0}{A_m(S) A_0(S)} (1 + \frac{Y}{X})
\]
\[ 1 + \frac{1}{A_m(S)} \frac{1}{1 + K_m x} + \frac{K_0}{A_0(S)} + \frac{K_0}{A_m(S) A_0(S)} \left( 1 + \frac{Y}{x} \right) \frac{\omega_p}{\omega} S + \left( 1 + \frac{K_m x}{A_m(S)} \right) \left( 1 + \frac{K_0}{A_0(S)} \right) + \frac{K_0}{A_m(S) A_0(S)} \left( 1 + \frac{Y}{x} \right) \]  

\[ (4.3.19a) \]

where

\[ y = 1 - \frac{G_0}{G_F} \]  

\[ (4.3.19b) \]

Using

\[ A_0(S) = \frac{B_0}{S + \omega_0} \]  

\[ (4.3.20a) \]

\[ A_m(S) = \frac{B_m}{S + \omega_m} \]  

\[ (4.3.20b) \]

Where \( B_0, B_m \) are the unity gain bandwidth of the OAs, \( \omega_0, \omega_m \) are the poles of the OAs. \( A_0, A_m \) are the DC gains of the OAs, we get.

\[ D(S) = \]

\[ \frac{K_b (1 + Y)}{S^2 \left[ \frac{B}{B_m} \right]} + \frac{K_0}{B_0} + \frac{K_0}{B_m B_0} (1 + x) (\omega + \omega_m + \frac{\omega_p}{\omega}) \]  

\[ (4.3.20b) \]
\[ S^2 \left[ 1 + \frac{K m K_0}{x} + \left( 1 + \frac{K_m v}{x} \right) \frac{1}{A_m} + \frac{K_0 (1 + \frac{v}{x})}{A_0} + \frac{K_0 (1 + \frac{v}{x})}{A_0 A} + \right. \]
\[ \left. \frac{\omega P_0}{Q P_0} \left[ \frac{1 + \frac{K_m v}{x}}{B_m} + \frac{K_0}{B_0} + \frac{(\omega_m + \omega_0)}{B_m B_0} K_0 (1 + \frac{v}{x}) + \frac{K_0 (1 + \frac{v}{x})}{B_m B_0} \right] \right] \]
\[ S P_0 \left[ 1 + \frac{K m v}{A_m} + \frac{K_0}{A_0} + \frac{K_0 (1 + \frac{v}{x})}{A_m A} \right] + \]
\[ 2 \frac{1 + \frac{K m v}{x}}{B_m} + \frac{K_0}{B_0} + \frac{(\omega_m + \omega_0)}{B_m B_0} K_0 (1 + \frac{v}{x}) \right] + \]
\[ \left[ 1 + \frac{K m K_0}{x} + \left( 1 + \frac{K_m v}{x} \right) \frac{1}{A_m} + \frac{K_0 (1 + \frac{v}{x})}{A_0} + \frac{K_0 (1 + \frac{v}{x})}{A_m A} \right] \omega P \] (4.3.21)

If we let
\[ \frac{K_m}{K_0} \Delta \frac{K}{K} \]
\[ \frac{B_m}{B_0} \Delta \frac{B}{B} \]
\[ \frac{A_m}{A_0} \Delta \frac{A}{A} \]
\[ \frac{\omega_m}{\omega_0} \Delta \omega_c \]

and use
\[ B = \omega_c \]

we get:
\[ D(S) = S^4 \left[ \frac{K_0 (1 + \frac{v}{x})}{B^2} \right] + \]
\[ S^3 \left[ \frac{1 + K (1 + \frac{v}{x})}{B} + \frac{2 K}{A B} (1 + \frac{v}{x}) + \frac{K (1 + \frac{v}{x}) \omega^2}{B^2} \right] \]
\[ S^2 \left[ 1 + \frac{K^2}{x} + \frac{1 + K (1 + \frac{v}{x})}{A} + \frac{K (1 + \frac{v}{x})}{A^2} \right] + \]
Figure 4.3.9: The effect of the OAS poles on $\omega_p$ for the Type 3 QMC
FIGURE 4.3.10. THE EFFECT OF THE OAS POLES ON $Q_p$ FOR THE TYPE 3 QMC
\[
\omega_p \left[ \frac{1+K(1+\frac{Y}{X})}{B} + \frac{2K(1+\frac{Y}{X})}{AB} \right] + \frac{K(1+\frac{Y}{X})}{B^2} + \frac{\omega_p^2}{Q_{P0}} \left[ 1 + \frac{1+K(1+\frac{Y}{X})}{A} + \frac{K(1+\frac{Y}{X})}{A^2} \right] + \frac{\omega_p^2}{2} \left[ \frac{1+K(1+\frac{Y}{X})}{B} + \frac{2K(1+\frac{Y}{X})}{AB} \right] + \left[ 1 + \frac{K(1+\frac{Y}{X})}{A} + \frac{K(1+\frac{Y}{X})}{A^2} \right] \omega_p^2
\] (4.3.22)

Using Eqn. 4.3.36 the values of \( \frac{\Delta \omega_p}{\omega_p} \) and \( \frac{\Delta Q_R}{Q_R} \) due to the effect of the pole of the amplifier have been obtained for various values of \( \omega_p \) and are given in Fig. 4.3.9 and 4.3.10 respectively. Use of O.A. SN72.301 properly compensated has been assumed in the computations.

4.4 A QMC Realization Obtained Using An RII Generating Circuit

The overall realization consisting of two amplifiers as shown in Fig. 4.4.1a (Figs. 4.4.1b and 4.4.1c show the actual realizations of the two amplifiers contained in Fig. 4.3.1a). It consists of a single amplifier optimized network [17, 18] constituting an RII generating circuit having a \( t_v \) of type N1 used in the configuration of Fig. 4.2.1 with \( G_F = 0 \). The entire network corresponds to a null filter having a denominator Decomposition of type 6. Its transfer function is given by
\[
T_6 = \frac{\frac{K_m K_0 G_0}{C_F} S^2 + \frac{G_1 G_2 (C_2 + C_3)}{C_1 C_2 C_3}}{S^2 \left[1 + \frac{K_m K_0}{x}\right] + S \frac{G_3 (1 - K_0)}{C_3} + \frac{G_2 (C_2 + C_3)}{C_2 C_3} + \frac{G_1 G_2 (C_2 + C_3)}{C_1 C_2 C_3} \left[1 + \frac{K_m K_0}{x}\right]}
\]

\[x = 1 + \frac{G_0 + G_A}{G_H} \quad (4.4.1b)\]

\[K_m = \frac{G_A}{G_B} \quad (4.4.1c)\]

\[K_0 = 1 + \frac{G_A}{G_B} \quad (4.4.1d)\]

This takes care of the loading effect of the amplifier. The parameter \(x\) has a minimum value of one which can be obtained for an unrestricted resistive spread only. The design equations of the generating circuit have been given in Table 4.2.3. It is known\(^{[17,18]}\) that for this generating circuit the minimum capacitive spread which is smaller than two occurs for the following values of the design parameters \(b = 3, c = 0.71, e_1 = 1.65\) and that for these values \(bK_0 < 3.91\). We will use this set of design parameters values wherever needed in our subsequent work.

From the transfer function given in Eqn. 3.4.1, we have

\[
\omega_p = \omega_{p0} = \left(\frac{G_1 G_2 (C_2 + C_3)}{C_1 C_2 C_3}\right)^{\frac{1}{2}}
\]

\(\omega_{p0}\)
A set of design equations

\[ G_1, G_2, G_3, C_1, C_2, C_3 \] are given in Table 4.2.3

\[ G_{II}/G_A = 1 \]

\[ K_0 = G_A/G_0 + 1 \]

\[ K_m = G_A/G_B = G_{II}/G_0 \]

\[ K_m \cdot K_0 \] are obtained from Fig. 4.4.6 for specified \( Q_p \).
\[
(4.4.4a)
\]

\[
(4.4.4b)
\]

\[
(4.4.4c)
\]

\[
(4.4.4d)
\]

\[
(4.4.4e)
\]

\[
(4.4.5)
\]

\[
(4.4.6a)
\]

\[
(4.4.6b)
\]

\[
(4.4.6c)
\]

\[
(4.4.6d)
\]

\[
(4.4.7)
\]
\[ |K_0S_{K_0}| = \frac{K_0bQ_0}{1 + \frac{K_0K_0^2}{m}} \]  \hspace{0.5cm} (4.4.8)

Using Eqn. 4.4.7 and 4.4.8 we get

\[ F = \frac{K_0bQ_0}{1 + \frac{K_0K_0^2}{m}} \]  \hspace{0.5cm} (4.4.9)

Using the expression of \( K_0 \) given in the generating function's design equations

\[ K_0 = \frac{b}{b-c+\frac{1}{Q_{P0}}} \]  \hspace{0.5cm} (4.4.10)

and Eqn. 4.4.4a it is possible to express \( K_m \) and \( F \) respectively as:

\[ K_m = \frac{\frac{x}{K_0^2} [\beta - aK_0]}{K_0} \]  \hspace{0.5cm} (4.4.11)

\[ K_0 < \frac{\beta}{a} \]  \hspace{0.5cm} (4.4.11a)

\[ F = \frac{\frac{x}{K_0^2} [\beta - aK_0]^2 + K_0 [\beta - aK_0] + \beta K_0^2}{K_0 + (\beta - aK_0)} \]  \hspace{0.5cm} (3.4.12)

where

\[ \beta = bQ_0 \]  \hspace{0.5cm} (4.4.13a)
$$a \triangleq (b-c)Q_p + 1$$  \hspace{1cm} (4.4.13b)

Minimization of $F$ for various values of $x$ using $K_0$ as a variable has been carried and values of $\frac{F_m}{Q_p}$ versus $Q_p$ and the corresponding values of $K_0$, $K_m$ and $Q_{p0}$ are given in Fig. 4.4.2 to Fig. 4.4.5.

From Eqn. 4.4.12 as well as from the curves shown in Fig. 4.4.2 it is clear that it is desirable to reduce as much as possible the value of the parameter $x$ in order to reduce the value of $F_m$ corresponding to any prescribed value of $Q_p$. Also the curves of Figs. 4.4.3, 4.4.4, and 4.4.5 show that the reduction of $x$ yields reductions in the corresponding values of $K_m$, $K_0$, and $Q_{p0}$ which are desirable because of the following reasons.

(a) The reduction in the values of $K_0$, and $Q_{p0}$ decreases the possibility of instability caused by the presence of negative terms in the $S$ coefficient of the denominators especially for high values of $Q_{p0}$.

(b) The reduction of $K_m$ contributes to the reduction of the resistive spread of the circuit.

However, decreasing the value of $x$ is associated with an increase in the resistive spread $\gamma$. Let

$$x = 1 + \frac{1}{\gamma_A} + \frac{1}{\gamma_0}$$  \hspace{1cm} (4.4.14a)
Figure 4.4.2 $F_m/Q_p$ of the null type 6 QMC for several values of $x$ versus $Q_p$. 
FIGURE 4.4.3 $K_0$ OF THE NULL TYPE 6 QMC FOR SEVERAL VALUES OF $x$ AND MINIMIZED $F$ VERSUS $Q_p$. 
Figure 4: $K_m$ of the null type 6 QMC for several values of $x$ and minimized $f$ versus $q_p$. 
FIGURE 4.4.5 $Q_{p0}$ OF THE NULL TYPE 6 OMC FOR SEVERAL VALUES OF $x$ AND MINIMIZED $F$ VERSUS $Q_p$. 
\[ \frac{1}{\gamma_A} = \frac{G_A}{G_H} \]  
\[ (4.4.14b) \]

\[ \frac{1}{\gamma_0} = \frac{G_0}{G_H} \]  
\[ (4.4.14c) \]

where for the purpose of minimizing \( x \), it is assumed that \( \gamma_A > 1, \gamma_0 > 1 \).

With these assumptions,

\[ \frac{G_H}{G_B} = \gamma_A K_m \]  
\[ (4.4.15) \]

is the quantity which for low values of \( Q_{p0} \) and high values of \( Q_p \) determines the resistive spread \( \gamma \). The following set of design equations appears to be a good compromise able to achieve both low values of \( F_m \) and of \( \gamma \).

\[ \gamma_A = 1 \]  
\[ (4.4.16a) \]

\[ \gamma_0 = K_m \]  
\[ (4.4.16b) \]

Hence

\[ x \geq 2 + \frac{1}{K_m} \]  
\[ (4.4.17a) \]

and using Eqs. (4.4.11), (4.4.16), and 4.4.17a

\[ F = \frac{K_0^2 \frac{1}{1+\frac{K_0}{\beta-aK_0}} \left[ \frac{\beta-aK_0}{K_0} \right]^2 + K_0 (\beta-aK_0)+\beta K_0^2}{(\beta-aK_0)+K_0} \]  
\[ (4.4.17b) \]
Figure 4.4.6 $F_m/Q_p, K_0, K_m, \gamma$ of the null type $\delta$ OMC when $x-2+1/K_m$ versus $Q_p$.
A set of curves giving $F_m / Q_p$, $K_0$ obtained from the minimization of $F$ as expressed in Eqn. 4.4.17b, $K_m$ obtained using Eqn. 4.4.11 and $\gamma$, versus $Q_p$ are shown in Fig. 4.4.6.

For the Null filter network shown in Fig. 4.4.1 grounding the input end of $G_0$ and lifting the capacitor $C_3$ off the ground and connecting it to the input terminal as shown in Table 4.2.3 we get a Band Pass filter network of type $\beta$ having the same denominator as that of the Null network.

For the same element values this network shall have all the properties, namely, sensitivities, gain sensitivity products, $F$, $Q$-multiplication, etc. of the Null network and as such no further detailed discussion of this network is necessary.

4.4.1 Experimental Results

The network shown in Fig. 4.3.1 was built up using discrete elements and tested. The results are summarized below:

Designed values:

\[
\begin{align*}
\omega_p &= 6283.185 \, \text{rad/sec}, =1000 \, \text{Hz}, \\
Q_p &= 50 \\
Q_{pQ} &= 4.858 \\
\rho &= 10.292 \\
K_m &= 15.946 \\
K_0 &= 1.202 \\
\alpha &= 2.063 \\
C_1 &= 9.356 \, \text{KPF}
\end{align*}
\]
\[ C_2 = 4.93 \ \text{KPF} \]
\[ C_3 = 8.984 \ \text{KPF} \]
\[ R_1 = 12.077 \ \text{K\Omega} \]
\[ R_2 = 70.422 \ \text{K\Omega} \]
\[ R_3 = R_F = R_A = 6.933 \ \text{K\Omega} \]
\[ R_B = R_0 = 110.554 \ \text{K\Omega} \]
\[ R_A = 100 \ \text{K\Omega} \]
\[ R_B = 20.2 \ \text{K\Omega} \]
\[ K_0 \omega_p = K_m \omega_p = 0 \]
\[ K_0 S_{K_0}^{Q_p} = K_m S_{K_m}^{Q_p} = 0 \]
\[ |K_0 S_{K_0}^{Q_p}|/Q_p = 0.37 \]
\[ |K_m S_{K_m}^{Q_p}|/Q_p = 0.29 \]
\[ F_m \quad Q_p = 0.66 \]

Element spread:

- Capacitive 1.898:1
- Resistive 15.946:1

The 3dB frequencies are:

- \( F_{C1} = 990.05 \ \text{Hz} \)
- \( F_{C2} = 1010.05 \ \text{Hz} \)

Any or both of \( K_m \) and \( K_0 \) can be adjusted in order to achieve the tuning of \( Q_p \) without affecting \( \omega_p \).

The circuit has been implemented using 1% tolerance.
resistors and 2% tolerance condensers with values chosen to be, within the range of currently available elements, as close as possible to the designed ones. Trim pots have been used wherever necessary. The OAs have been used where SN7230I compensated according to the current specifications of the manufacturer.

Actual values obtained:

\[ \omega_p = 999.4 \text{ Hz.} \]

\[ F_{C1} = 990.1 \text{ Hz.} \]

\[ F_{C2} = 1010.4 \text{ Hz.} \]

\[ Q_p = 49.23 \]

The depth of the notch has been found to be -70dB. It is found that the circuit was stable during activation. \( Q_p \) and \( \omega_p \) variations from their designed values are 1.53% and 0.06% respectively. This is due to the small differences between the designed values and the real ones as well as to the tolerance of the elements. The power supply voltages used were ±10V and ±15V. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at 22°C (room temperature) and 70°C for both power supplies voltages. Only the OAs were heated; the passive elements were not heated in order to simulate Hybrid Integrated Circuit technology.

As shown in Fig. 4.4.7 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.
FIGURE 4.4.7a (+10V)

FIGURE 4.4.7b (+15V)

FIGURE 4.4.7 FREQUENCY RESPONSE OF THE NULL TYPE 6 QMC FOR $Q_p = 50$
4.4.2 Effect of the pole of the O.A. on $Q$ and $\omega$ [33, 17]

In order to study the effect of the pole of the OA on $Q_p$ or $\omega_p$, it is assumed that the desired elements are available and therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 4.4.1a by the networks of Fig. 4.4.1b, and 4.4.1c the analysis gives:

$$D(S) = S^2 \left[ 1 + \frac{K_m K_0}{k} \frac{K_0}{A_0(S)} + \frac{1}{A_m(S)} \frac{1}{(1 + K_m x)} + \frac{K_0}{A_m(S) A_0(S) (1 + K_m x)} \right] +$$

$$S \left[ \frac{1}{Q_p} \frac{K_0}{A_0(S)} \frac{1}{A_m(S)} \left( 1 + \frac{1}{Q_p} \right) \right] \left[ 1 + \frac{1}{A_m(S)} (1 + K_m x) \right] +$$

$$\left[ 1 + \frac{K_m K_0}{k} \frac{K_0}{A_0(S)} + \frac{1}{A_m(S)} \frac{1}{(1 + K_m x)} + \frac{K_0}{A_m(S) A_0(S) (1 + K_m x)} \right]$$

where

$$y = 1 + \frac{G_0}{G_p} \quad (4.4.18a)$$

$$y = 1 + \frac{G_0}{G_p} \quad (4.4.18b)$$

Using

$$A_0(S) = \frac{B_0}{s_p \omega_p + \omega_0} \quad (4.4.19a)$$

$$A_m(S) = \frac{B_m}{s_p \omega_p + \omega_m} \quad (4.4.19b)$$

where $B_0, B_m$ are the unity gain bandwidth of the OAs, $\omega_0, \omega_m$ are the poles of the OAs, $A_0, A_m$ are the D-C gains of the OAs, we get
If we let \( B_m = \frac{\Delta}{\Omega} \omega_m \), \( \omega_m = \omega_c \) and use \( B = \omega_c A \) we get
\[
D(S) = \frac{2a_P^2}{B^2} \left[ \frac{K_0 (1 + K_x \frac{V}{x})}{1 + K_m \frac{V}{x}} \right]^2 + \\
S^3 \left[ \frac{a_P^2}{B} (1 + K_0 + K_m \frac{V}{x}) + \frac{a_P^2}{A} K_0 (1 + K_m \frac{V}{x}) + \right] \\
+ \frac{a_P^2}{B^2} (b + \frac{1}{Q_0}) K_0 (1 + K_m \frac{V}{x}) \\
+ \frac{a_P^2}{B^2 Q_0} (1 + K_0 + K_m \frac{V}{x}) + \frac{a_P^2}{B^2 Q_0} K_0 (1 + K_m \frac{V}{x}) \\
+ \frac{a_P^2}{A} \left[ 1 + \frac{K_0 \frac{V}{x}}{A} \right] + \frac{1}{A} K_0 b + \\
(b + \frac{1}{Q_0}) K_0 (1 + K_m \frac{V}{x}) + \frac{a_P^2}{B} (1 + K_0 + K_m \frac{V}{x}) + \frac{a_P^2}{AB (1 + K_m \frac{V}{x})} K_0 + \\
\left[ 1 + \frac{K_0 \frac{V}{x}}{A} \right] + \frac{1}{A} (1 + K_0 + K_m \frac{V}{x}) + K_0 \frac{(1 + K_m \frac{V}{x})}{A^2} \right].
\]

(4.4.21)

Using Eqn. 4.4.21 the values of \( \frac{\Delta a_P}{a_P} \) and of \( \frac{\Delta Q_0}{Q_0} \) due to the effect of the poles of the amplifiers have been obtained for various values of \( a_P \) and are given in Fig. 4.4.8 and 4.4.9 respectively. The use of SN72301 OAs compensated according to the current specifications of the manufacturer has been assumed in the computations.
\[ \frac{\Delta \omega_p}{\omega_p} \text{ in Hz} \]

**Figure 4.4.8** The effect of the QAS poles on \( \omega_p \) for the type 6 QMC
FIGURE 4.4.9 THE EFFECT OF THE Q AS POLES ON $Q_p$ FOR THE TYPE 6 QMC

Frequency in Hz.

$Q = 50$
4.5 A QMC Realization Obtained Using An RIII Generating Circuit

The overall realization consisting of two amplifiers is as shown in Fig. 3.5.1a (Figs. 4.5.1b and 4.5.1c shows the actual realizations of the two amplifiers contained in Fig. 4.5.1a). It consists of a differential amplifier optimized network [17, 18] constituting an RIII generating circuit having a t.v. of type N2 used in the configuration of Fig. 4.2.1 with $G_F = 0$. The entire network corresponds to a null filter having a denominator decomposition of type 7. Its transfer function is given by:

$$T_7 = \frac{\frac{K_mK_D}{x} \cdot G_0}{\frac{G_H}{S^2[1+K_D^+x] + S\left[\frac{G_2G_3(C_1+C_3)}{C_1C_2C_3}\right] + \frac{G_1G_3(C_1+C_3)}{C_1C_2C_3}[1+K_D^+\frac{K_m}{x}K_D]}}$$

(4.5.1a)

$$x = 1 + \frac{G_0G_3}{G_H}$$

(4.5.1b)

$$K_m = \frac{G_A}{G_B}$$

(4.5.1c)

and

$$\frac{G_A0}{G_B0} = \frac{G_A0'}{G_B0'} = K_D$$

(4.5.1d)
A SET OF DESIGN EQUATIONS

\( G_1, G_2, G_3, C_1, C_2, C_B \) are given in Table 4.2.3.

\( G_B / G_A = 1 \)

\( K_D = G_A / G_{B0} - G_A / G_{B0} = C_H / G_0 \)

\( K_m = G_A / G_B \)

\( K_m, K_D \) are obtained from Fig. 4.5.5 for specified \( Q_B \).
The parameter $x$ has a minimum value of one which can be obtained for an unrestricted resistive spread only. The design equations of the generating circuit have been given in Table 4.2.3. It is known [17] that for this generating circuit the minimum capacitive spread which is smaller than 2.5 occur for the following values of the design parameters $b = 3$, $\eta = 2.3$. We will use this set of design parameters values wherever needed in our subsequent work.

For the transfer function given in Eqn. 4.4.1 we have

$$\omega_p = \omega_p = \frac{G_1 G_3 (C_1 + C_3)^{1/2}}{C_1 C_2 C_3} \quad (4.5.2)$$

$$S_{K_D} = S_{K_m} = 0 \quad (4.5.3)$$

$$Q_p = \frac{Q_{p0} [1 + \frac{K_{Dm}^x}{(K_D + 1)^x}] - K_{Dm}^x}{(K_D + 1)^x} \quad (4.5.4a)$$

Where

$$Q_{p0} = \frac{(C_1 C_2 C_3 G_1 G_3 (C_1 + C_3)^{1/2})^{1/2}}{C_2 C_3 G_2 + C_2 C_3 (C_1 + C_3)} (K_D + 1) = \frac{K_D + 1}{b} \quad (4.5.4b)$$

Hence

$$Q_p = \frac{1}{b} [1 + K_D + \frac{K_{Dm}^x}{x}] \quad (4.5.4c)$$

$$S_{K_D} = \frac{K_{Dm}^x}{K_D + \frac{1 + K_{Dm}^x}{x}} \quad (4.5.5a)$$
This could be expressed using Eqn. 4.5.4c as

\[ \frac{Q_P}{S'_{K_m}} = \frac{1}{bQ_p} \frac{K_m K_D}{x} \]  
\[ \frac{Q_P}{S_{K_D}} = \frac{K_D + \frac{K_m K_D}{x}}{1 + K_D + \frac{K_m}{x}} \]  

(4.5.5b)  

(4.5.6a)

This could be expressed using Eqn. 4.5.5a

\[ \frac{Q_P}{S_{K_D}} = \frac{K_D}{bQ_p} (1 + \frac{K_m}{x}) \]  

Hence

\[ |K_m S_{K_m}| = \frac{1}{bQ_p} \frac{K_m K_D}{x} \]  

\[ |K_m S_{K_m}| = \frac{K_m K_D}{bQ_p} (1 + \frac{K_m}{x}) \]  

(4.5.7)  

(4.5.8)

and

\[ F = \frac{K_D}{bQ_p} \left( \frac{K_m^2}{x} \right) + K_D (1 + \frac{K_m}{x}) \]  

(4.5.9)

Minimization of F for given values of Q_p and of the parameter x which is independent of both K_m and K_D can be achieved in the following way: Using Eqns. 4.5.4c, 4.5.7, and 4.5.9 it is possible to express K_m and F respectively as:

\[ K_m = \frac{bQ_p - 1}{K_D} \left( \frac{x}{-1} \right) - 1 \]  

(4.5.10)
\[ F = K_D \left[ x \left( \frac{bQ_p - 1}{K_D} - 1 \right) \right]^2 + (bQ_p - 1) \]  \hspace{1cm} (4.5.11)

Setting the partial derivative of \( F \) with respect to \( K_D \) equal to zero yields:

\[ K_D = (bQ_p - 1) \left[ \frac{x}{bQ_p + x - 1} \right]^\frac{1}{2} \]  \hspace{1cm} (4.5.12)

Hence using this value of \( K_D \) in Eqn. 4.5.9 and 4.5.11 we get respectively the value of \( F \) (called \( F_m \)) for prescribed values of \( b, x, \) and \( Q_p \) and the corresponding value of \( K_m \) respectively as:

\[ F_m = \frac{x(bQ_p - 1)}{bQ_p} \left[ \frac{x}{bQ_p + x - 1} \right]^\frac{1}{2} \left[ \left( \frac{bQ_p + x - 1}{x} \right)^\frac{1}{2} - 1 \right] \]

\[ + \frac{(bQ_p - 1)^2}{bQ_p} \left[ \frac{x}{bQ_p + x - 1} \right]^\frac{1}{2} \]  \hspace{1cm} (4.5.13a)

and

\[ K_m = x \left[ \left( \frac{bQ_p + x - 1}{x} \right)^\frac{1}{2} - 1 \right] \]  \hspace{1cm} (4.5.13b)

The result of minimization of \( F \) for several values of \( x \) as well as the corresponding values of \( K_D' \) and \( K_m' \) versus \( Q_p \) are shown in Fig. 4.5.2 to Fig. 4.5.4. From Eqn. 4.5.11 as well as from the curves shown in Fig. 4.5.2 it is clear that it is desirable to reduce the value of the parameter \( x \) in order to reduce as much as possible the value of \( F_m \)
FIGURE 4.5.2 $\frac{F_m}{Q_p}$ VERSUS $Q_p$ OF THE NULL TYPE 7 QMC FOR SEVERAL VALUES OF $x$. 
Figure 4.5.3 \( K_D \) of the null type 7 QMC for several values of \( x \), and minimized \( F \), versus \( Q_p \).
FIGURE 4.5.4 $K_m$ of the null type 7 QMC for several values of $x$, and minimized $F$ versus $Q_p$. 
corresponding to any prescribed value of $Q_p^c$. Also inspection of the curves shown in Fig. 4.5.3 and Fig. 4.5.4 shows that the reduction of $x$ contributes to a decrease in the resistive spread $\gamma$. However, decreasing the value of $x$ is associated with an increment in the resistive spread $\gamma$. Hence we can follow the same procedure used previously. Let

$$x = \frac{1}{\gamma_A} + \frac{1}{\gamma_0} \quad (4.5.14a)$$

$$\frac{1}{\gamma_A} = \frac{G_A}{G_H} \quad (4.5.14b)$$

$$\frac{1}{\gamma_0} = \frac{G_0}{G_h} \quad (4.5.14c)$$

where for the purpose of minimizing $x$ it is assumed that $\gamma_A \geq 1$, $\gamma_0 \geq 1$ with these assumptions.

$$\frac{G_H}{G_B} = \gamma_A K_m \quad (4.5.15)$$

is the quantity which for low values of $Q_{p0}$ and high values of $Q_p$ determines the resistive spread $\gamma$.* The following

---

*The resistive spread $\gamma$ to which it is referred here does not take into account the scaling up of the resistances of the generating circuit which may be necessary to avoid the loading of the Twin-T if the differential amplifiers is realized using a single O.A. as shown in Fig. 4.5.1c.
FIGURE 4.5.5 $\frac{F_m}{Q_p}, K_D, K_m, \gamma$, OF THE NULL TYPE 7 QMC WHEN $x = 2 + \frac{1}{K_D}$ VERSUS $Q_p$. 
set of design equations appears to be a good compromise to achieve low values of $F_m$ and $\gamma$

$$\gamma_A = 1$$ (4.5.16a)

$$\gamma_0 = K_D$$ (4.5.16b)

Hence

$$\chi = 2 + \frac{1}{K_D}$$ (4.5.17a)

and using Eqns. 4.5.11, 4.5.16 and 4.5.17a

$$F = \frac{K_0}{bQ_p} \left[ \left( 2 + \frac{1}{K_D} \right) \left( \frac{bQ_p - 1}{K_D} - 1 \right) \right]^2 + (bQ_p - 1)$$ (4.5.17b)

A set of curves giving $\frac{F_m}{Q_p}$, $K_D$ obtained from the minimization of $F$ as expressed in Eqn. 4.5.17b. $K_m$ obtained using Eqn. 4.5.10 and $\gamma$ versus $Q_p$ are shown in Fig. 4.5.5.

### 4.5.1 Experimental results

The network shown in Fig. 4.5.1 was built up using discrete elements and tested, the results are summarized below.

**Designed values:**

$$\omega_p = 6283 \text{ rad/sec.} = 1000 \text{ Hz.}$$

$$Q_p = 50$$

$$Q_{p0} = 6.19$$
\[ K_m = 15.376 \]
\[ K_P = 17.58 \]
\[ R_1 = 1 \text{K}\Omega \]
\[ R_2 = 0.533 \text{K}\Omega \]
\[ R_3 = 1.76 \text{K}\Omega \]
\[ C_1 = 0.199 \mu\text{F} \]
\[ C_2 = 0.239 \mu\text{F} \]
\[ C_3 = 0.08649 \mu\text{F} \]
\[ x = 2.0569 \]
\[ R_H = R_A = 0.533 \text{K}\Omega \]
\[ R_B = R_0 = 8.195 \text{K}\Omega \]
\[ R_{A0} = R_{A0}^* = 5.668 \text{K}\Omega \]
\[ R_{B0} = R_{B0}^* = 100 \text{K}\Omega \]

Capacitive spread 2.77:1

\[ \frac{\omega_P}{S_{K_0}} = \frac{\omega_P}{S_{K_m}} = 0 \]

\[ \left| \frac{K_0 S_{K_0}^*}{Q_P} \right| / Q_P = 0.3492 \]

\[ \left| \frac{K_m S_{K_m}^*}{Q_P} \right| / Q_P = 0.2694 \]

\[ F_m / Q_P = 0.619 \]

Capacitive spread 2.76:1

The 3 dB frequencies are:

\[ F_{C1} = 990.05 \text{ Hz} \]
\[ F_{C2} = 10110.05 \text{ Hz} \]
Any or both of $K_m$ and $K_D$ can be adjusted in order to achieve the tuning of $Q_p$ without affecting $\omega_p$.

The circuit has been implemented using 1% tolerance resistors and 2% tolerance condensers with values chosen to be within the range of currently available elements, as close as possible to the designed ones (Trim pots have been used wherever necessary). The OAs which have been used were internally compensated ML741. Actual values obtained

$$\omega_p = 1000.6 \text{ Hz}.$$  
$$F_{C1} = 989.8 \text{ Hz}.$$  
$$F_{C2} = 1010.2 \text{ Hz}.$$  
$$Q_p = 49.044$$

The depth of the notch has been found to be $-70$ dB. It is found that the circuit was stable during activation. $Q_p$ and $\omega_p$ variations from their designed values are respectively 1.9% and 0.06%. This is due to the small differences between the designed values and the used ones as well as to the tolerances of the elements. Power supply voltage used were $\pm 10\text{V}$ and $\pm 15\text{V}$. Also the OAs were heated and their temperature controlled. The response was experimentally measured and plotted at $22^\circ\text{C}$ (room temperature) and $70^\circ\text{C}$ for both power supplies voltages. Only the OAs were heated. The passive elements were not heated in order to simulate Hybrid Integrated technology.

As shown in Fig. 4.5.8 no appreciable change in the response was observed; thus the experimental results confirm the theoretical predictions.
FIGURE 4.5.6a (+10V)

FIGURE 4.5.6b (+15V)

FIGURE 4.5.6 FREQUENCY RESPONSE OF THE NULL TYPE-7 QMC
FOR $Q_p = 50$
4.5.2 Effect of the poles of the amplifiers on $Q_p$ and $\omega_p$ [17, 33]

In order to study the effect of the pole of the amplifiers on $Q_p$ and $\omega_p$ it is assumed that the desired elements are available and that the input impedance of the differential amplifier is high enough to avoid any appreciable loading of the twin-T. Therefore one pole of the passive circuit always cancels with one of its zeros. Replacing the amplifiers of Fig. 3.5.1a by the networks represented in Fig. 3.5.1b and 3.5.1c the analysis gives

$$D(S) =$$

$$S^2\left[ K'_D + \frac{K_m K_D}{x} + \frac{1}{A_m(S)} K'_D (1+K_m \frac{Y}{x}) + \frac{K_D}{A_D(S)} + K' \frac{(1+K_m \frac{Y}{x})}{A_m(S) A_D(S)} \right]$$

$$+ bS \left[ 1 + \frac{K'_D}{A_D(S)} + \frac{(1+K_m \frac{Y}{x})}{A_m(S)} + K'_D \frac{(1+K_m \frac{Y}{x})}{A_D(S)} \right] +$$

$$\left[ K'_D + \frac{K_m K_D}{x} + \frac{1}{A_m(S)} K'_D (1+K_m \frac{Y}{x}) + \frac{K'_D}{A_D(S)} \right] +$$

$$K'_D \frac{(1+K_m \frac{Y}{x})}{A_m(S) A_D(S)} \right] \quad (4.5.17a)$$

$$K'_D = 1 + K_D$$

$$y = 1 + \frac{G_0}{G_H} \quad (4.5.17c)$$

Using

$$A_D(S) = \frac{B_D}{\omega_p S + \omega_D} \quad (4.5.18a)$$
\[ A_m(S) = \frac{B_m}{\omega_p S^3 + \omega_m} \quad (4.5.18b) \]

where \( B_D, B_m \) are the unit gain bandwidth of the OAs; \( B_D, B_m \) are the poles of the OAs; \( A_D, A_m \) are the D-C gains of the OAs we get:

\[
D(S) = S^4 \left[ \frac{2 K \omega_D (1 + K_m x)}{B_m B_D} \right] + \]

\[
S^3 \left[ \frac{K_D (1 + K_m x)}{B_m} \frac{K_D}{B_D} + \frac{K_D (1 + K_m x)}{B_m} (\omega_m + \omega_0) \omega_p \right]
\]

\[
\frac{K_D (1 + K_m x)}{B_m} b \omega_p^2 \right] + \]

\[
S^2 \left[ \frac{K_D}{B_m} + \frac{K_m K_D}{x} + \frac{K_D (1 + K_m x)}{B_m} + \frac{K_D}{B_D} + \frac{K_D (1 + K_m x)}{B_m} \omega_m + \frac{K_D}{B_D} \omega_p + \frac{(1 + K_m x)}{B_m} \omega_p + \right.
\]

\[
\frac{K_D (1 + K_m x)}{B_D B_m} \omega_p \omega_m \frac{1}{B_m} + \frac{K_D (1 + K_m x)}{B_D B_m} \omega_p \omega_m \left. \right)
\]

\[
S \left[ b (1 + K_D) \frac{\omega_D}{B_D} + \frac{(1 + K_m x)}{B_m} + K_D (1 + K_m x) \frac{\omega_D \omega_m}{B_D B_m} \right]
\]

\[
+ \frac{K_D}{B_D} \frac{1}{x} + \frac{K_D (1 + K_m x)}{B_m} \frac{\omega_p}{B_D B_m} + \frac{K_D (1 + K_m x) (\omega_m + \omega_0) \omega_p}{B_D B_m} \left. \right)
\]

\[
+ \frac{K_D}{B_m} + \frac{K_m K_D}{x} + \frac{K_D (1 + K_m x)}{B_m} \frac{\omega_p}{B_D B_m} + \frac{K_D (1 + K_m x)}{B_D B_m} + \left. \right)
\]

\[
(4.5.19)
\]
If we let \( B_m = B_D = B \), \( \omega_m = \omega_0 = \omega_c \) and use \( B = \omega_c A \) we get:

\[
D(S) = \frac{2\omega_p' K_D(1+K_{m x})}{B^2} + \frac{\omega_p' K_D(2+K_{m x})}{B} + \frac{K_D(1+K_{m x})\omega_p^2}{2A^2} \frac{2K_D(1+K_{m x})\omega_p}{AB}
\]

\[
+ \frac{2K_D(1+K_{m x})\omega_p}{B^2} + \frac{2K_D(1+K_{m x})\omega_p^2}{B^2} + \frac{1+K_{m x}}{A^2} \frac{2K_D(1+K_{m x})\omega_p^2}{B^2} \]

\[
+ \frac{1+K_{m x}}{A^2} \frac{2K_D(1+K_{m x})\omega_p^2}{B^2} + \frac{K_D(2+K_{m x})}{A^2} \frac{2K_D(1+K_{m x})\omega_p^2}{B^2} \]

\[
+ \frac{K_D(2+K_{m x})}{A^2} \frac{2K_D(1+K_{m x})\omega_p^2}{B^2} \]

\[
\left[ K_D + \frac{K_{m x}}{A^2} + \frac{K_D(2+K_{m x})}{A^2} + \frac{K_D(1+K_{m x})}{A^2} \right]
\]

(4.5.20)

Using Eqn. 4.5.19 the values of \( \Delta \omega_p \) and of \( \Delta Q_p \) due to the poles of the amplifiers have been obtained for various values of \( \omega_p \) and are given in Fig. 4.5.7 and Fig. 4.5.8 respectively. The use of LM741 internally compensated OAs has been assumed in the computations.
FIGURE 4.5.7 THE EFFECT OF THE OAS POLES ON $\omega_P$ FOR THE TYPE 7 QMC.
FIGURE 4.5.8 THE EFFECT OF THE OAS POLES ON $Q_P$ FOR THE TYPE 7 QMC
4.6 Summary and Discussion

In this chapter we have considered QMCs which are obtained starting from a second order generating function $t_v$. The general TAC discussed in Chapter II has been adapted to suit the use of a second order $t_v$ which requires that all the impedances be resistive. It is shown that only twelve types of generating functions satisfying Eqn. 2.6.4 are needed to obtain all the possible denominator polynomial decompositions of $T_v$.

It has also been shown that only Band Pass Null or All Pass filters are obtainable. Only SACs having zero $G_{wSP}$ and minimized $Q_{SP}$ have been used. The resulting QMCs retain the zero $G_{wSP}$ property and have been optimized to minimize $F$. Hence QMCs having minimized $\frac{\Delta T_v}{T_v | S = \delta \omega_p}$ are obtained.

The QMCs have been classified into three categories: RI, RII, and RIII according to the generating SAC. One representative circuit of each kind has been studied in detail. Table 4.6.1 gives a comparison between the three given realizations. It has been found that $F_m$ of the order of 0.5 or lower is achievable and that $F_p$ decreases as $Q_p$ increases. Experimental results confirm the theoretical studies.
TABLE 4.6.1

COMPARISON BETWEEN THE THREE GIVEN REALIZATIONS
### TABLE 4.6.1

<table>
<thead>
<tr>
<th>1st Realization</th>
<th>2nd Realization</th>
<th>3rd Realization</th>
</tr>
</thead>
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<tr>
<td>$\omega_P = \omega_{P0}$</td>
<td>$\omega_P = \omega_{P0}$</td>
<td>$\omega_P = \omega_{P0}$</td>
</tr>
<tr>
<td>$S_{K_m} = S_{K_0} = 0$</td>
<td>$S_{K_m} = S_{K_0} = 0$</td>
<td>$S_{K_m} = S_{K_0} = 0$</td>
</tr>
<tr>
<td>$Q_P = Q_{P0} \left(1 + \frac{K_m K_0}{x}\right)$</td>
<td>$Q_P = Q_{P0} \left(1 + \frac{K_m K_0}{x}\right)$</td>
<td>$Q_P = Q_{P0} \left(1 + \frac{K_m K_D}{x(x + K_m K_D)}\right)$</td>
</tr>
<tr>
<td>$S_{K_m} = \frac{K_m K_0}{1 + K_m K_0/x} &lt; 1$</td>
<td>$S_{K_m} = \frac{K_m K_0}{1 + K_m K_0/x} &lt; 1$</td>
<td>$S_{K_m} = \frac{1}{bQ_p} \left(\frac{K_m K_D}{x}\right) &lt; 1$</td>
</tr>
<tr>
<td>$Q_P = \frac{K_m K_0}{1 + K_m K_0/x}$</td>
<td>$Q_P = \frac{bQ_p + K_m K_0/x}{1 + K_m K_0/x}$</td>
<td>$Q_P = \frac{K_D}{bQ_p} \left(1 + \frac{K_m}{x}\right) &lt; 1$</td>
</tr>
<tr>
<td>$S_{K_0} = \frac{K_m K_0}{1 + K_m K_0/x}$</td>
<td>$S_{K_0} = \frac{bQ_p + K_m K_0/x}{1 + K_m K_0/x}$</td>
<td>$S_{K_0} = \frac{K_D}{bQ_p} \left(1 + \frac{K_m}{x}\right) &lt; 1$</td>
</tr>
</tbody>
</table>

For $x = 1$ and $\frac{C_{\text{max}}}{C_{\text{min}}} = 2$

- $\frac{F}{Q_P} = 1.28$ | $Q_P = 10$
- $\frac{F}{Q_P} = 0.56$ | $Q_P = 50$
- $\frac{F}{Q_P} = 0.40$ | $Q_P = 100$

For $x = 1$ and $\frac{C_{\text{max}}}{C_{\text{min}}} = 1.9$

- $\frac{F}{Q_P} = 1.0$ | $Q_P = 10$
- $\frac{F}{Q_P} = 0.48$ | $Q_P = 50$
- $\frac{F}{Q_P} = 0.34$ | $Q_P = 100$

For $x = 1$ and $\frac{C_{\text{max}}}{C_{\text{min}}} = 2.75$

- $\frac{F}{Q_P} = 0.87$ | $Q_P = 10$
- $\frac{F}{Q_P} = 0.45$ | $Q_P = 50$
- $\frac{F}{Q_P} = 0.33$ | $Q_P = 100$

OAs should be properly compensated to avoid high frequency oscillations due to excessive phase shift.

The generating circuit may need scaling of resistances and hence the resistive spread could be high.
CHAPTER V

SUMMARY AND DISCUSSIONS
CHAPTER V

SUMMARY AND DISCUSSIONS

This thesis discusses realizations of second order transfer functions by two-amplifier active RC networks. The realizations of the five commonly occurring types of filters, namely LP, HP, BP, N, and AP are studied. The filters should be implemented using hybrid IC technology. Assuming this technology, designs have been obtained which yield filters with zero \( \frac{\Delta \omega_p}{\omega_p} \) (at least for first-order variations) and minimized \( \frac{\Delta Q_p}{Q_p} \) which is equivalent to minimized \( F \) (the sum of the magnitudes of the gain-Q-sensitivity products).

Chapter II discusses the various possible denominator polynomial decompositions having the property of zero \( G_{wSP} \). It is found that twenty-four such decompositions are possible. A general two-amplifier configuration is proposed which will realize all the possible polynomial decompositions. The configuration is not unique and some variations of this configuration are given. The overall realization requires the use of "generating functions" which can be of three types, namely zero order, first order and second order. These generating functions have to be realized by single-amplifier RC networks having their outputs taken from the output of the amplifier. The case of the zero order is not emphasized further in this thesis, as it has been found that it is not possible to design...
filters in a relatively simple manner.

In Chapter III, the realizations using first order generating functions are discussed. It is shown that only low-pass, high-pass, and band-pass filters are obtainable when the admittances are of the form \( Y = CS + G \). Also, these filters realize eight polynomial decompositions, starting from only two types of generating functions. They are designed to yield zero GowSP and minimized \( F \). A value of \( F/Q_p \) of the order of 6.5 (for \( Q_p = 50 \)) has been shown to be obtainable for LP, HP, and BP corresponding to the polynomial decomposition of type 1 with a unity capacitive spread and a moderate resistive spread. For the BP corresponding to the polynomial decomposition of type 22b, \( F/Q_p \) is also of the order of 6.5 with a capacitive spread of 5. Lower values of \( F/Q_p \) can be achieved if the capacitive and/or resistive spreads are allowed to increase. It is quite possible that some of the circuits generated might have been discussed in the literature; nevertheless, a systematic method of obtaining two amplifier circuits starting from first order generating functions is given as shown in Tables 3.2.2, 3.2.3, 3.2.4.

In Chapter IV "Q-multiplier circuits" are discussed in detail. This "Q-multiplication" requires a second order generating function realized by a single amplifier circuit. It is shown that all the possible twenty-four polynomial decompositions can be obtained by the use of twelve types of generating functions. Only SACs having zero GowSP and minimized
GQSP have been used. The overall QMCs preserve the property of zero GwSP and have been optimized to minimize $F$.

It has been found that since the numerator of the QMCs transfer function is the same as the one of its generating SAC only BP, P, N and AP filters are obtainable. The QMCs are further classified according to the realization of the generating SAC. Three realizations, one of each kind, have been studied in detail and compared. The comparison shown in Table 4.6.1 gives the different features of each realization and shows that for all of them a value of $\frac{F}{Q_p}$ of the order of 0.5 (for $Q_p = 50$) is obtainable while the capacitive spread is kept below three and that $\frac{m}{Q_p}$ decreases as $Q_p$ increases.

Three null QMCs and two band-pass filters obtained from first order generating circuits all for $Q_p = 50$ were built and tested in the laboratory at DC supply voltages of ±10V and ±15V and with the QAs at temperatures of 22°C and 70°C. The test results confirm the theoretical predictions. Thus this study clearly establishes that:

(i) Two-amplifier RC filter networks having zero GwSP and minimized $F$ (sum of the magnitudes of the GQSPs) can be obtained.

(ii) All the possible twenty-four polynomial decompositions having the property of zero GwSP can be realized by one general configuration.

(iii) Starting from low-Q single amplifier circuits with zero GwSP, $Q$-multiplication permits us to
obtain high-Q circuits with low $F$ and preserving the property of zero $G_{wSP}$.

In this thesis, zero $G_{wSP}$ has been the starting condition. It is desirable to conduct investigations regarding TAC realizations that will minimize a weighted sum of $\frac{\Delta \omega P}{\omega P}$ and $\frac{\Delta Q P}{Q P}$. Also the zero order and first order cases are not exhaustive. It is worthwhile to study filter realizations when the passive admittances are realized in various forms. Furthermore only open circuit voltage transfer functions have been considered and realized in this thesis, but the same procedure could be readily extended to realize current transfer functions using the property of network transposition [39].
REFERENCES


