SENSITIVITY STUDIES OF GENERALIZED GYRATOR CIRCUITS

BY

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

In this dissertation, the sensitivity of the Generalized Gyrator realized using three different structures has been examined. Also the sensitivity of an arbitrary voltage transfer function realized using the Yanagisawa structure, using the Generalized Gyrator as the active element, with the three different realizations of the Generalized Gyrator has been investigated.

A method of realizing an arbitrary biquadratic voltage transfer function has been described for two specific examples.

CHAPTER I

THE GENERALIZED GYRATOR AND ITS REALIZATIONS

1.1 Introduction

The gyrator is an element used in active network synthesis for the realization of driving point and transfer functions. One of the main characteristics of a gyrator is the impedance inversion property. The great advantage of this characteristic is that an inductance can be simulated at the input port of a gyrator, when the output port is terminated by a capacitor. This property is used in the realization of several active RC-networks.

In the recent past many types of gyrators have been introduced. The one which includes all these different types of special cases is called the "Generalized Gyrator" (GG).

1.2 Generalized Gyrator (GG)

A generalized gyrator (GG) has been defined [1] by its chain matrix [a] as:

$$[a] = \begin{bmatrix} 0 & \dfrac{1}{g_2(s)} \\ g_1(s) & 0 \end{bmatrix} \qquad (1.1)$$

Where $g_1(s)$ and $g_2(s)$ are positive or negative realizable admittance functions. The symbolic representation of the GG is shown in Fig. 1.1. The GG will be termed lossless, if the parameters A and D of [a] are zero or positive. If A and D are not zero, the GG is a lossy one. In this dissertation, we shall be concerned with only lossless GG's.

The GG will be called an "Ideal Generalized Gyrator" (IGG), if $|a| = \pm 1$, that is, $g_1(s) = \pm g_2(s)$. The IGG is reciprocal if $|a| = +1$, while it is non-reciprocal if $|a| = -1$.

1.3 Positive and Negative Gyrators

If $g_1(s)$ and $g_2(s)$ have the same signs, the GG will be termed a "positive gyrator" (PG), where as if they are of opposite signs it will be termed a "negative gyrator" (NG).

Depending on the signs of $g_1(s)$ and $g_2(s)$, there will be two types for each PG and NG. The [a] of the PG is given by:

$$[a] = \begin{bmatrix} 0 & \pm \dfrac{1}{g_2(s)} \\ \pm g_1(s) & 0 \end{bmatrix}$$

where the positive signs correspond to PG type I while the negative signs refer to PG type II.

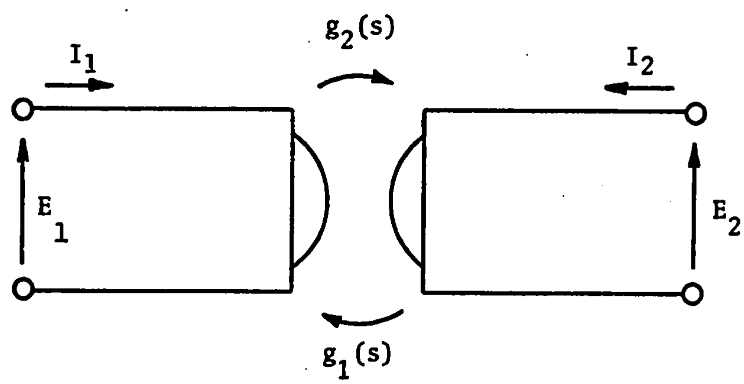

Fig. 1.1: Symbolic Representation of the GG

Similarly the [a] of the NG is given by:

$$[a] = \begin{bmatrix} 0 & \mp \frac{1}{g_2(s)} \\ \pm g_1(s) & 0 \end{bmatrix}$$

NG type I will have the $[a] = \begin{bmatrix} 0 & -\frac{1}{g_2(s)} \\ g_1(s) & 0 \end{bmatrix}$

while NG type II will have the $[a] = \begin{bmatrix} 0 & +\frac{1}{g_2(s)} \\ -g_1(s) & 0 \end{bmatrix}$

In what follows, the two types of gyrators, namely passive and active gyrators, will be defined.

1. If $g_1(s) = g_2(s) = G$ in the IPG, we get Tellegen gyrator, which is a passive anti-reciprocal element.
2. If $g_1(s)$ and $g_2(s)$ are conductances and have opposite signs in (1.1), we get Kawakami's negative gyrator, which is reciprocal.

1.4 Gyrator Realization

The generalized gyrator has been realized, using active devices, by two methods, namely:

(i) Realization of GG using CNICs and VNICs.

(ii) (a)- Realization of GG using VNIC and an IPG or an ING.

(b)- Realization of GG using INIC and an IPG or an ING.

In what follows, for the sake of completeness, the realization of GG using these methods will be reproduced.

1.41 Realization of GG using CNICs and VNICs and Passive Elements [2]

Fig. 1.2 consists of the four passive-two ports N_1, N_2, P_a and P_b and CNICs with unity conversion ratio.

Analysis yields:

$$[Y] = [Y]_{N_1'} + [Y]_{N_2'} + [Y]_{P_a'} + [Y]_{P_b'} \tag{1.2}$$

This is equated to the admittance matrix of the GG:

$$[Y] = \begin{bmatrix} 0 & g_1(s) \\ g_2(s) & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_{1p}(s) - g_{1n}(s) \\ g_{2p}(s) - g_{2n}(s) & 0 \end{bmatrix} \tag{1.3}$$

Where $g_{1p}(s)$, $g_{1n}(s)$, $g_{2p}(s)$ and $g_{2n}(s)$ are driving point functions of the two element type networks and are obtained by partial fraction expansion of $g_1(s)$ and $g_2(s)$.

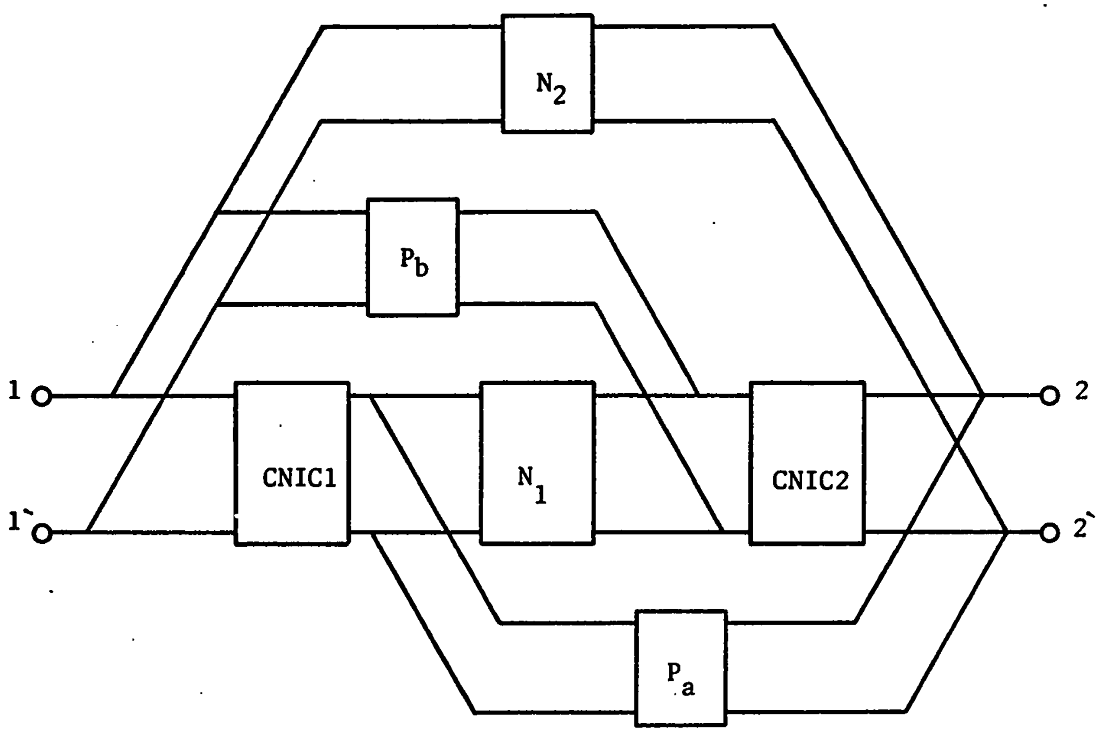

Fig.1.2: Basic Construction for the Realization of GG using CNICs and Passive Elements

Several allocations between the individual matrices in (1.2) are possible. One simple choice is to have the networks N_1 and N_2 as π - structures, and P_a and P_b as series arms. The structure corresponding to this choice is shown in Fig. 1.3.

The different admittances are:

$$Y_1 = \frac{g_{1n}(s) + g_{2n}(s)}{2} \qquad Y_5 = g_{1p}(s)$$

$$Y_2 = \frac{g_{1p}(s) + g_{2p}(s)}{2} \qquad Y_6 = g_{1n}(s)$$

$$Y_3 = \frac{g_{1p}(s) + g_{2n}(s)}{2} \qquad Y_7 = g_{2n}(s) \qquad (1.4)$$

$$Y_4 = \frac{g_{1n}(s) + g_{2p}(s)}{2} \qquad Y_8 = g_{2p}(s)$$

Instead of the CNICs, VNICs can be used, in which case, only the ports need to be interchanged. The ideal gyrators result from a simplification of the above structure, where some of the admittances will be absent. For IPG, type I or II Y_1 and Y_2 will be equal to zero, while for ING, type I or II, Y_3 and Y_4 will be absent.

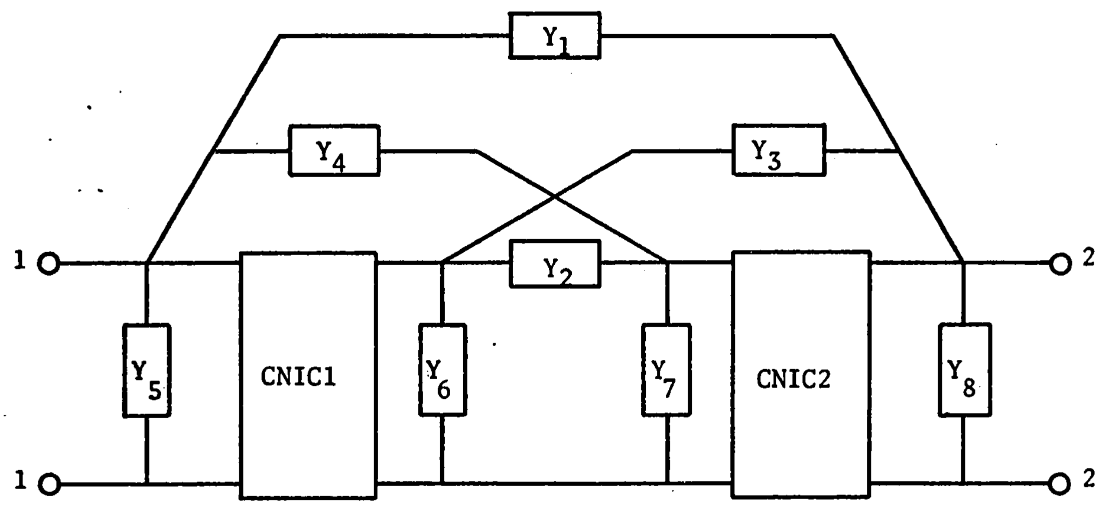

Fig. 1.3: The Structure used in the Realization of GG.

1.42 Realization of GG using VNIC or INIC and an IPG or an ING[1]

(a) Using VNIC:

Fig. 1.4(a) shows the arrangement, for realizing a GG, using generalized VNIC in cascade with IPG or ING. Fig. 1.4(b) shows the idealized version of a generalized VNIC.

From Fig. 1.4(a), the [a] is:

$$\begin{bmatrix} -\frac{g_1(s)}{g_2(s)} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & B \\ C & 0 \end{bmatrix} = \begin{bmatrix} 0 & -B\frac{g_1(s)}{g_2(s)} \\ C & 0 \end{bmatrix}$$

By choosing $B = \pm \frac{1}{g_1(s)}$ and $C = \pm \frac{1}{B}$ the GG results.

The GG may also be realized by following the IPG or ING by the generalized VNIC, in which case the choice of the different parameters are $B = \pm \frac{1}{g_2(s)}$ and $C = \pm \frac{1}{B}$.

(b) Using a INIC:

As before, Fig. 1.5(a) and Fig. 1.5(b) show the arrangement, for realizing a GG, using generalized INIC in cascade with IPG or ING and the idealized version of a generalized INIC respectively.

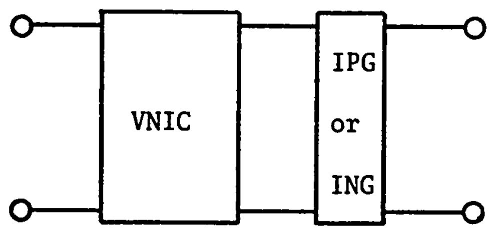

Fig. 1.4(a)

Realization of a GG by a Generalized VNIC in cascade with an IPG or ING

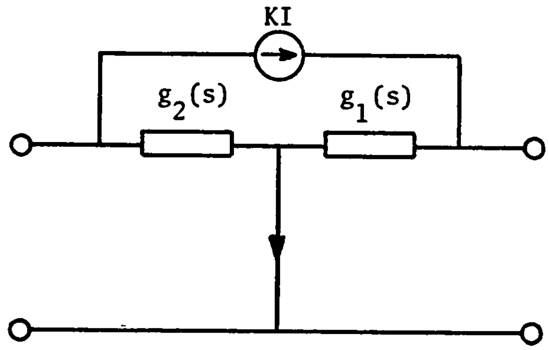

Fig. 1.4(b)

Idealized Version of a Generalized VNIC

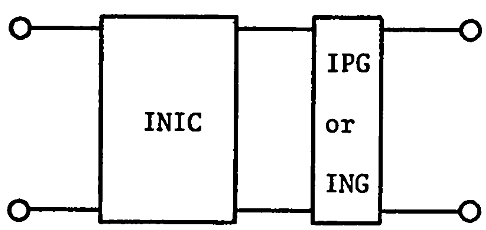

Fig. 1.5(a)

Realization of a GG by a Generalized INIC in cascade with an IPG or ING

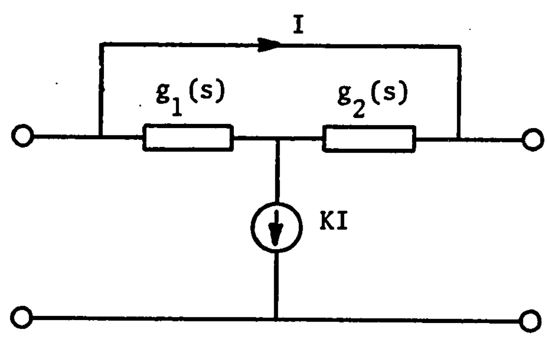

Fig. 1.5(b)

Idealized Version of a Generalized INIC

From Fig. 1.5(a), choosing $B = \pm \frac{1}{g_2(s)}$ and $C = \pm \frac{1}{B}$ for IPG or ING, the GG results.

It may also be obtained by following the IPG or ING by the generalized INIC, in which case the different parameters ought to be $B = \pm \frac{1}{g_1(s)}$ and $C = \pm \frac{1}{B}$.

The IPG or ING can be realized by many different methods [3], but they are not discussed here, as this dissertation concerns itself only with GG's sensitivities and not their realizations.

1.5 Scope of the Dissertation

This dissertation deals with the investigation of the following:-

1. The sensitivities of the two different realizations of the GG.
2. The different sensitivities of a biquadratic voltage transfer function realized using the GG.

CHAPTER II

SENSITIVITY OF THE GENERALIZED GYRATOR

2.1 Introduction

In passive and active network design, a great deal of attention must be paid to the sensitivity of the network function caused due to the variation of network parameters. The characteristics of any element is subject to change for many reasons, like temperature, aging, manufacturing tolerances .. etc. Such variations cause the poles and zeros to be displaced from their nominal positions and consequently the network gives a different performance from the one for which it is designed.

Although, as mentioned earlier, the GG has been realized using two methods, the sensitivity of the GG has not been discussed before. In this chapter, the sensitivity analysis of the GG with respect to each element used in the realization of the GG will be carried out in detail.

2.2 Sensitivity

The sensitivity of a network function can be considered as a measure of the change in the performance of the network due to a change in the nominal value of one or more of the network elements.

We shall here adopt the following definition of sensitivity:-

The sensitivity function $S_x^{N(s)}$ of the network function N(s,x) due to the variation of the parameter x is defined as:

$$S_x^{N(s)} = \frac{dN(s,x)/N(s,x)}{dx/x}$$

The above definition of the sensitivity will be used throughout this dissertation.

2.3 Realization of GG using CNICs with "k" Conversion Ratio

In Chapter I, realization of GG using CNICs or VNICs with unity conversion ratio has been discussed. Here it is assumed that the CNICs or the VNICs used will have a conversion ratio k.

In Fig. 1.3, let the [a] of $CNIC_1$ and $CNIC_2$ be

$$[a] = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{k_1} \end{bmatrix} \quad \text{and} \quad [a] = \begin{bmatrix} 1 & 0 \\ 0 & -\frac{1}{k_2} \end{bmatrix}$$

respectively.

Analysis yields:

$$[Y] = [Y]_{N_1'} + [Y]_{N_2'} + [Y]_{P_a'} + [Y]_{P_b'} \tag{2.1}$$

$$= \begin{bmatrix} \frac{Y_2+Y_6}{k_1} & \frac{Y_2}{k_1} \\ k_2Y_2 & -k_2(Y_2+Y_7) \end{bmatrix}_{N_1'} + \begin{bmatrix} Y_1+Y_5 & -Y_1 \\ -Y_1 & Y_1+Y_8 \end{bmatrix}_{N_2'} + \begin{bmatrix} \frac{-Y_3}{k_1} & \frac{Y_3}{k_1} \\ -Y_3 & Y_3 \end{bmatrix}_{P_a'} + \begin{bmatrix} Y_4 & -Y_4 \\ k_2Y_4 & -k_2Y_4 \end{bmatrix}_{P_b'}$$

$$[Y] = \begin{bmatrix} Y_1+Y_4+Y_5 - \frac{Y_2+Y_3+Y_6}{k_1} & \frac{Y_2+Y_3}{k_1} - (Y_1+Y_4) \\ k_2(Y_2+Y_4)-(Y_1+Y_3) & Y_1+Y_3+Y_8 - k_2(Y_2+Y_4+Y_7) \end{bmatrix} \tag{2.2}$$

This is equated to the admittance matrix of the generalized gyrator

$$[Y] = \begin{bmatrix} 0 & g_1(s) \\ g_2(s) & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_{1p}(s) - g_{1n}(s) \\ g_{2p}(n) - g_{2n}(s) & 0 \end{bmatrix} \tag{2.3}$$

Where $g_{1p}(s)$, $g_{1n}(s)$, $g_{2p}(s)$ and $g_{2n}(s)$ are driving point functions of the two element kind networks and are obtained by partial fraction expansion of $g_1(s)$ and $g_2(s)$. Hence we get

$$
\begin{aligned}
&Y_1 + Y_4 + Y_5 - \frac{Y_2 + Y_3 + Y_6}{k_1} = 0 \\
&Y_1 + Y_3 + Y_8 - k_2(Y_2 + Y_4 + Y_7) = 0 \\
&\frac{Y_2 + Y_3}{k_1} - (Y_1 + Y_4) = g_{1p}(s) - g_{1n}(s) \\
&k_2(Y_2 + Y_4) - (Y_1 + Y_3) = g_{2p}(s) - g_{2n}(s)
\end{aligned}
\qquad (2.4)
$$

2.4 Sensitivity of [Y] parameters of a GG

Next, let us consider the sensitivity of the [Y] parameters Y_{11}, Y_{12}, Y_{21} and Y_{22} of equation (2.2) with respect to the variation in the elements Y_1 to Y_8, k_1 and k_2.

2.41 Sensitivity of Y_{11} with respect to Y_1

From equation (2.2)

$$Y_{11} = Y_1 + Y_4 + Y_5 - \frac{Y_2 + Y_3 + Y_6}{k_1} \qquad (2.5)$$

Let Y_1 become $Y_1\ (1 + \varepsilon_1)$ and as a consequence Y_{11} will be $Y_{11} + \alpha_1$, where ε_1 is a small variation in Y_1 and α_1 is the corresponding change in Y_{11} substituting (2.6) in (2.5) we get: (2.6)

$$Y_{11} + \alpha_1 = Y_1 + Y_4 + Y_5 - \frac{Y_2 + Y_3 + Y_6}{k_1} + Y_1\varepsilon_1 \qquad (2.7)$$

from (2.7) and (2.5) $\alpha_1 = Y_1\varepsilon_1$ and so

$$S^{\alpha_1}_{\varepsilon_1} = \frac{d\alpha_1/\alpha_1}{d\varepsilon_1/\varepsilon_1} = 1$$

In a similar manner, the sensitivity of Y_{11} with respect to Y_2 to Y_8, k_1 and k_2 could be obtained. Also, following the same procedure, the sensitivity of the parameters Y_{12}, Y_{21} and Y_{22} could be calculated. Table 2.1 shows the sensitivity of the [Y] parameters with respect to each element in Fig. 1.3. It is obvious from Table 2.1 that the sensitivities of the [Y] parameters of the GG realized using CNICs with conversion ratio equal to k could be one of three cases:

(i) Identically to zero.

(ii) Equal unity.

(iii) Inversely proportional to the change in the element nominal value.

TABLE 2.1

Sensitivity of [Y] parameters with respect to Small Variation in Y_1 to Y_8, k_1 & k_2

Element	Variation	S^α	S^β	S^γ	S^δ
Y_1	ε_1	1	1	1	1
Y_2	ε_2	1	1	1	1
Y_3	ε_3	1	1	1	1
Y_4	ε_4	1	1	1	1
Y_5	ε_5	1	-	-	-
Y_6	ε_6	1	-	-	-
Y_7	ε_7	-	-	-	1
Y_8	ε_8	-	-	-	1
k_1	ε_{k_1}	$\frac{1}{1+\varepsilon_{k_1}}$	$\frac{1}{1+\varepsilon_{k_1}}$	-	-
k_2	ε_{k_2}	-	-	1	1

S^α ≡ Sensitivity of variation (α) in Y_{11} w.r.t. element variation.
S^β ≡ Sensitivity of variation (β) in Y_{12} w.r.t. element variation.
S^γ ≡ Sensitivity of variation (γ) in Y_{21} w.r.t. element variation.
S^δ ≡ Sensitivity of variation (δ) in Y_{22} w.r.t. element variation.

2.5 Sensitivity of GG realized using VNIC and IPG in Cascade

As stated before, the GG can be realized using the combination of VNIC cascaded by IPG. In what follows, the sensitivity of a GG realized by this structure will be examined.

Let the ING be given by the T section [3] shown in Fig. 2.1 and has

$$[a] = \begin{bmatrix} 0 & Z \\ -\frac{1}{Z} & 0 \end{bmatrix}$$

The [a] of the VNIC, shown in Fig. 1.4(b) is given by:

$$[a] = \begin{bmatrix} \frac{1-k}{k}\,\frac{g_2}{g_1} & \frac{g_2}{k} \\ \frac{1}{kg_1} & \frac{k+1}{k} \end{bmatrix} \tag{2.8}$$

The [a] of the cascaded network shown in Fig. 1.4(a) will be:

$$[a]_{comb} = [a]_{VNIC}\ [a]_{ING} \tag{2.9}$$

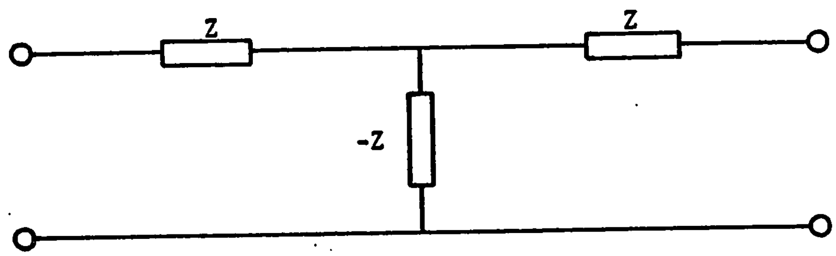

Fig. 2.1 Circuit Realization of the ING

$$= \begin{bmatrix} \frac{g_2(1-k)}{g_1 k} & \frac{g_2}{k} \\ \frac{1}{kg_1} & \frac{k+1}{k} \end{bmatrix} \begin{bmatrix} 0 & Z \\ -\frac{1}{Z} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{g_2}{Zk} & \frac{Zg_2(1-k)}{g_1 k} \\ -\frac{k+1}{kz} & \frac{Z}{kg_1} \end{bmatrix} \tag{2.10}$$

This is equated to the [a] of a GG

$$[a] = \begin{bmatrix} 0 & \frac{1}{g_2(s)} \\ g_1(s) & 0 \end{bmatrix}$$

from which we conclude that k must be large.

In such a case,

$$[a]_{comb} = \begin{bmatrix} 0 & -\frac{Zg_2}{g_1} \\ -\frac{1}{Z} & 0 \end{bmatrix} \tag{2.11}$$

2.51 Sensitivity of the A,B,C, and D Parameters

The four parameters of the [a] of equation 2.10 are

$$A = -\frac{g_2}{Zk} \qquad B = \frac{Zg_2(1-k)}{g_1k}$$

$$C = -\frac{k+1}{kZ} \qquad D = \frac{Z}{kg_1}$$

Following the same procedure outlined in section 2.41 the sensitivity of the [a] of a GG, realized using VNIC succeeded by ING, with respect to a small change in the nominal value of the four elements g_1, g_2, Z and k could be calculated. Results are shown in Table 2.2 It is obvious from Table 2.2 that the sensitivities of the [Y] parameters of the GG realized using VNIC and ING with conversion ratio equal to k could be one of three cases:

(i) Identically to zero.

(ii) Equal unity.

(iii) Inversely proportional to the change in the element nominal value.

TABLE 2.2

Sensitivity of [a] parameters of a GG realized using VNIC and ING with respect to small variation in g_1, g_2, Z & k

Element	Variation	S^{α}	S^{β}	S^{γ}	S^{δ}
$g_1(s)$	ε_1	-	$\frac{1}{1+\varepsilon_1}$	-	$\frac{1}{1+\varepsilon_1}$
$g_2(s)$	ε_2	1	1	-	-
Z	ε_3	$\frac{1}{1+\varepsilon_3}$	1	$\frac{1}{1+\varepsilon_3}$	1
k	ε_4	$\frac{1}{1+\varepsilon_4}$	$\frac{1}{1+\varepsilon_4}$	$\frac{1}{1+\varepsilon_4}$	$\frac{1}{1+\varepsilon_4}$

S^{α} ≡ Sensitivity of variation (α) in A parameter w.r.t. element variation.

S^{β} ≡ Sensitivity of variation (β) in B parameter w.r.t. element variation.

S^{γ} ≡ Sensitivity of variation (γ) in C parameter w.r.t. element variation.

S^{δ} ≡ Sensitivity of variation (δ) in D parameter w.r.t. element variation.

2.6 Sensitivity of GG realized using INIC and ING in Cascade

The [a] of the generalized INIC shown in Fig. 1.5b is given by:

$$[a] = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{g_1 + (k+1)\,g_2}{g_2 - (k-1)\,g_1} \end{bmatrix} \tag{2.12}$$

If the INIC is followed by the T section shown in Fig. 2.1, acting as ING, the [a] of the cascaded combination will be:

$$[a]_{comb} = [a]_{INIC} \quad [a]_{ING} \tag{2.13}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{g_1 + (k+1)g_2}{g_2 - (k-1)g_1} \end{bmatrix} \begin{bmatrix} 0 & Z \\ -\dfrac{1}{Z} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & Z \\ -\dfrac{1}{Z}\,\dfrac{g_1 + (k+1)g_2}{g_2 - (k-1)g_1} & 0 \end{bmatrix} \tag{2.14}$$

which is equivalent to [a] of a GG.

The sensitivity of the [a] parameters of (2.14) has been evaluated, and results are shown in Table 2.3. It is obvious from Table 2.3 that the sensitivities of [Y] parameters of the GG realized using INIC and ING could be one of three cases:-

(i) Identically equal to zero.

(ii) Equal to unity.

(iii) Dependent on both the variation and the nominal values of the different elements used in the realization of INIC and will be less than unity.

2.7 Summary

In this chapter the sensitivities of the GG realized using three different methods have been examined. It has been shown that for the two realizations:

(i) using CNICs or VNICs and passive elements

(ii) using VNIC and ING

the sensitivities of both the structures with respect to the different elements used in the realization could be one of three cases:-

(i) Identically equal to zero.

(ii) Equal to unity.

(iii) Inversely proportional to the variation in the element nominal value.

TABLE 2.3

Sensitivity of the [a] parameters of a GG realized using INIC and ING with respect to small variation in g_1, g_2, Z & k

Element	Variation	S^{α}	S^{β}	S^{γ}	S^{δ}
$g_1(s)$	ε_1	-	-	$\dfrac{1}{1-\dfrac{(k-1)g_1\varepsilon_1}{g_2-(k-1)g_1}}$	-
$g_2(s)$	ε_2	-	-	$\dfrac{1}{1+\dfrac{g_2\varepsilon_2}{g_2-(k-1)g_1}}$	-
Z	ε_3	-	1	$\dfrac{1}{1+\varepsilon_3}$	-
k	ε_4	-	-	$\dfrac{1}{1-\dfrac{g_1k}{g_2+g_1(1-k^{\varepsilon_4})}}$	-

S^{α} ≡ Sensitivity of variation (α) in A parameter w.r.t. element variation

S^{β} ≡ Sensitivity of variation (β) in B parameter w.r.t. element variation

S^{γ} ≡ Sensitivity of variation (γ) in C parameter w.r.t. element variation

S^{δ} ≡ Sensitivity of variation (δ) in D parameter w.r.t. element variation

In the other realization, where INIC and IPG are used, it has been shown that the sensitivities of the A,B, and D parameters, with respect to the elements used in the realization of INIC and IPG could be either identically equal to zero or unity; while the sensitivity of the C parameter, with respect to the elements used in the realization of INIC, depends on the variation in the element nominal value and the values of all other elements. Also the sensitivity of the C parameter, with respect to the element used in the realization of the IPG is inversely proportional to the variation in the element nominal value.

CHAPTER III

SENSITIVITY OF THE YANAGISAWA STRUCTURE USING

THE GG AS THE ACTIVE ELEMENT

3.1 Introduction

A method of realizing an arbitrary voltage transfer function, using an ING has been discussed before [5]. In this method, the transfer function T(s) was assumed to be stable and in the form:

$$T(s) = \frac{P(s)}{Q(s)} = \frac{s^p + a_{p-1} s^{p-1} + \text{------} + a_0}{s^q + b_{q-1} s^{q-1} + \text{------} + b_0} \tag{3.1}$$

Where P(s) and Q(s) are arbitrary polynominals of s, but with their zeros assumed to be neither at origin nor on the negative real axis. Also it was assumed that the ING, used in the realization has only positive gyrational admittance.

In this chapter the sensitivity of the [Y] of an arbitrary voltage transfer function, realized using the structure described in Fig. 3.1, with the different realizations of the GG, will be investigated.

3.2 Sensitivity of the [Y] parameters of a Voltage Transfer Function Realized using GG and RC Networks

In Chapter II, the sensitivity of the GG realized using:

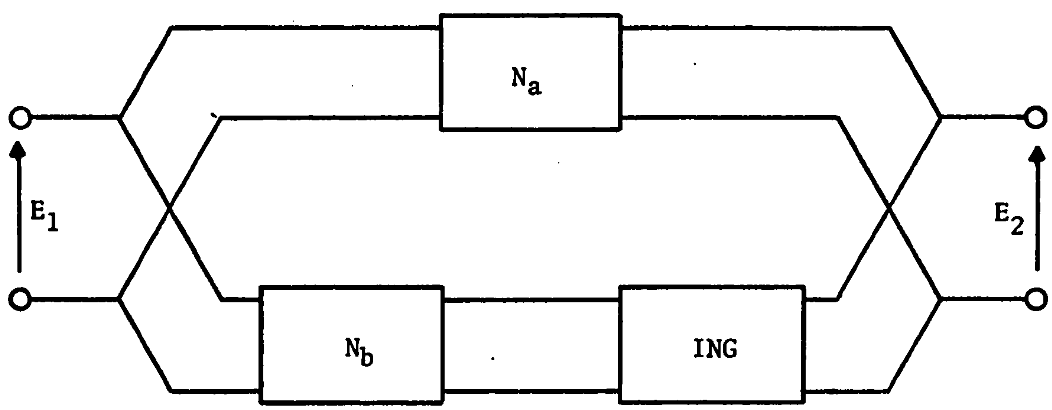

Fig. 3.1: Basic Structure used in Realizing an Arbitrary Voltage Transfer Function.

(i) CNICs or VNICs with a conversion ratio equal to k.

(ii) (a)- VNIC and an IPG or an ING

(b)- INIC and an IPG or an ING

has been discussed.

In what follows the sensitivity of the Yanagisawa structure, using the three realizations of the GG, will be investigated.

3.21 Sensitivity of the Structure when the GG is Realized using CNICs or VNICs

Here, it is assumed that the two CNICs or the VNICs, used in the realization of the GG, are identical and have an [h] given by:

$$[h] = \begin{bmatrix} 0 & k_1 \\ k_2 & 0 \end{bmatrix}$$

For the purpose of this study, consider the structure shown in Fig. 3.2. The [Y] of this structure will be obtained as follows. Let $[Y]_G$ = [Y] of the gyrator portion contained within the dotted square.

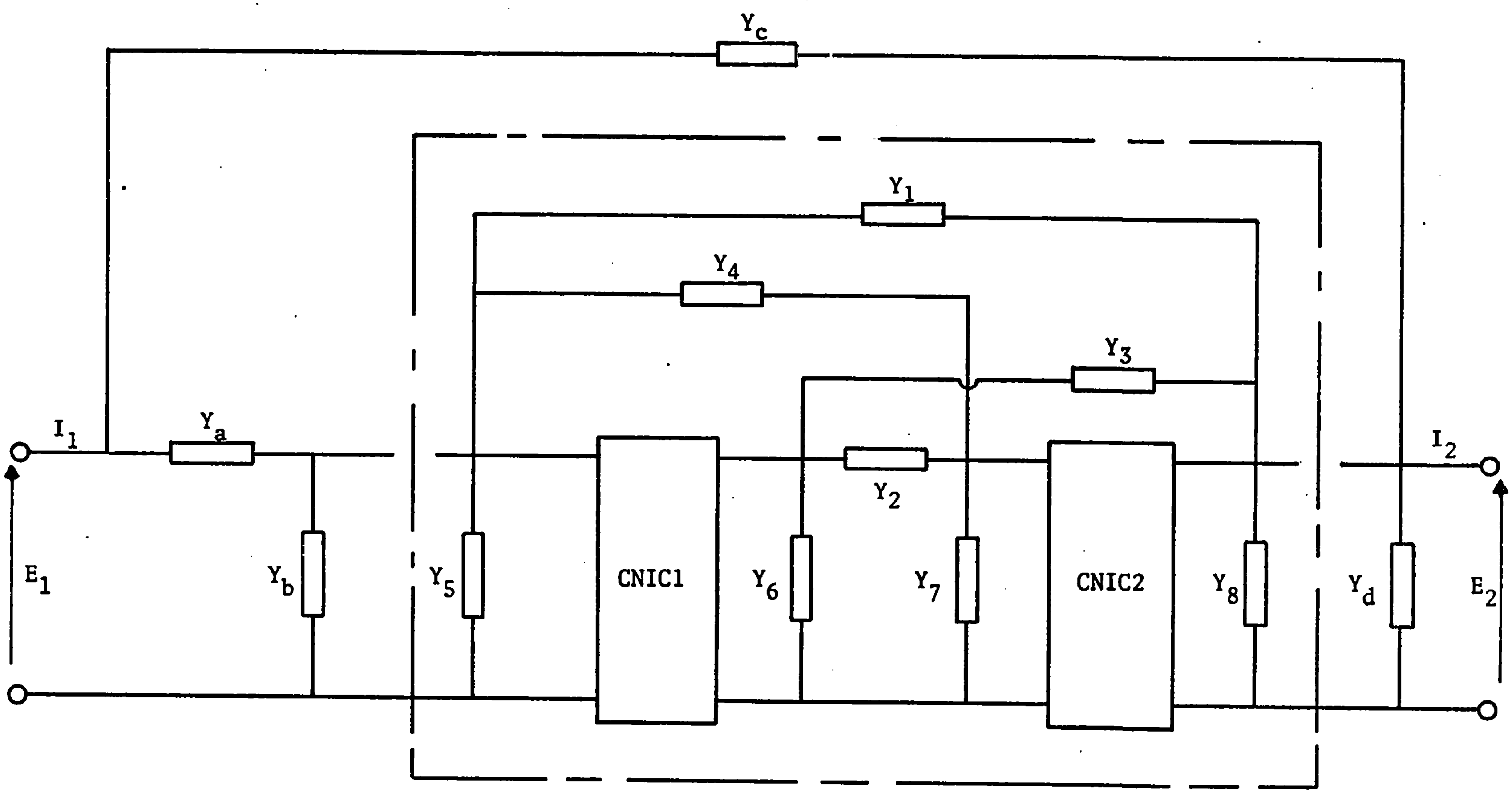

Fig. 3.2: Yanagisawa's Structure, when the GG is Realized using CNICs.

$$[Y]_G = \begin{bmatrix} -\frac{Y_2+Y_6}{k_1k_2} & \frac{Y_2k_1}{k_2} \\ \frac{Y_2k_2}{k_1} & -(Y_2+Y_7)k_1k_2 \end{bmatrix} + \begin{bmatrix} Y_1+Y_5 & -Y_1 \\ -Y_1 & Y_1+Y_8 \end{bmatrix} + \begin{bmatrix} -\frac{Y_3}{k_1k_2} & \frac{Y_3}{k_2} \\ \frac{-Y_3}{k_1} & Y_3 \end{bmatrix} + \begin{bmatrix} Y_4 & -Y_4k_1 \\ Y_4k_2 & -Y_4k_1k_2 \end{bmatrix}$$

$$= \begin{bmatrix} Y_1+Y_4+Y_5 - \frac{Y_2+Y_3+Y_6}{k_1k_2} & \frac{Y_2k_1+Y_3}{k_2} - (Y_1+Y_4k_1) \\ Y_4k_2-Y_1 + \frac{Y_2k_2-Y_3}{k_1} & Y_1+Y_3+Y_8 - (Y_2+Y_4+Y_7)k_1k_2 \end{bmatrix} \quad (3.2)$$

adding Y_b to Y_{11} and Y_d to Y_{22} of $[Y]_G$ we get $[Y]_{G`}$

$$[Y]_{G`} = \begin{bmatrix} Y_1+Y_4+Y_5+Y_b - \frac{Y_2+Y_3+Y_6}{k_1k_2} & \frac{Y_2k_1+Y_3}{k_2} - (Y_1+Y_4k_1) \\ Y_4k_2-Y_1 + \frac{Y_2k_2-Y_3}{k_1} & Y_1+Y_3+Y_8+Y_d - (Y_2+Y_4+Y_7)k_1k_2 \end{bmatrix} \quad (3.3)$$

$$\text{from which } [a]_{G`} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (3.4)$$

where $A = -\frac{Y_{22}}{Y_{21}}$

$$= \frac{k_1k_2\ [(Y_2+Y_4+Y_7)k_1k_2\ -(Y_1+Y_3+Y_8+Y_d)]}{k_2[k_1(Y_4k_2-Y_1)\ +Y_2k_2-Y_3]} \qquad (3.5)$$

$B = -\frac{1}{Y_{21}}$

$$= \frac{-k_1k_2}{k_2[k_1(Y_4k_2-Y_1)\ +\ (Y_2k_2-Y_3)]} \qquad (3.6)$$

$C = \frac{\Delta Y}{Y_{21}}$

$$= \frac{H}{k_2[k_1(Y_4k_2-Y_1)\ +\ Y_2k_2-Y_3]} \qquad (3.7)$$

where $H = [k_1(Y_4k_2-Y_1)\ +\ (Y_2k_2-Y_3)]\ [(Y_2k_1+Y_3)-k_2(Y_1+Y_4k_1)]$

$- [k_1k_2(Y_1+Y_4+Y_5+Y_b)-(Y_2+Y_3+Y_6)]\ [(Y_1+Y_3+Y_8+Y_d)-$

$(Y_2+Y_4+Y_7)k_1k_2]$

$$D = -\frac{Y_{11}}{Y_{21}}$$

$$= \frac{Y_2+Y_3+Y_6 - k_1k_2(Y_1+Y_4+Y_5+Y_b)}{k_2[k_1(Y_4k_2-Y_1) + Y_2k_2-Y_3]} \qquad (3.8)$$

The [a] of Y_a is given by

$$[a]_{Y_a} = \begin{bmatrix} 1 & \frac{1}{Y_a} \\ 0 & 1 \end{bmatrix} \qquad (3.9)$$

From equations (3.4) and (3.9)

$$[a]_{Y_{a,G`}} = [a]_{Y_a} \quad [a]_{G`}$$

$$= \begin{bmatrix} 1 & \frac{1}{Y_a} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$= \begin{bmatrix} \frac{AY_a+C}{Y_a} & \frac{BY_a+D}{Y_a} \\ C & D \end{bmatrix} \qquad (3.10)$$

The corresponding admittance of equation (3.10) is

$$[Y]_{Y_{a,G`}} = \begin{bmatrix} \frac{DY_a}{BY_a+D} & \frac{Y_a(CB-DA)}{BY_a+D} \\ \frac{-Y_a}{BY_a+D} & \frac{AY_a+C}{BY_a+D} \end{bmatrix} \tag{3.11}$$

The admittance matrix of Y_c is

$$[Y]_c = \begin{bmatrix} Y_c & -Y_c \\ -Y_c & Y_c \end{bmatrix} \tag{3.12}$$

From 3.11 and 3.12

$$[Y]_T = [Y]_{Y_{a,G`}} + [Y]_c$$

$$= \begin{bmatrix} \frac{DY_a}{BY_a+D} + Y_c & \frac{Y_a(CB-DA)}{BY_a+D} - Y_c \\ \frac{-Y_a}{BY_a+D} - Y_c & \frac{AY_a+C}{BY_a+D} + Y_c \end{bmatrix} \tag{3.13}$$

Using the equations (3.5), (3.6), (3.7) and (3.8) the parameters Y_{11}, Y_{12}, Y_{21} and Y_{22} of $[Y]_T$ could be obtained. Hence

$$Y_{11} = Y_c + \frac{Y_a[Y_2+Y_3+Y_6-k_1k_2(Y_1+Y_4+Y_5+Y_b)]}{Y_2+Y_3+Y_6-k_1k_2(Y_1+Y_4+Y_5+Y_a+Y_b)} \qquad (3.14)$$

$$Y_{12} = -\left[Y_c + \frac{Y_ak_1[Y_2k_1+Y_3-k_2(Y_1+Y_4k_1)]}{Y_2+Y_3+Y_6-k_1k_2(Y_1+Y_4+Y_5+Y_a+Y_b)}\right] \qquad (3.15)$$

$$Y_{21} = -\left[Y_c + \frac{Y_ak_2[k_1(Y_4k_2-Y_1) + (Y_2k_2-Y_3)]}{Y_2+Y_3+Y_6-k_1k_2(Y_1+Y_4+Y_5+Y_a+Y_b)}\right] \qquad (3.16)$$

and

$$Y_{22} = Y_c + \frac{H+k_1k_2Y_a[(Y_2+Y_4+Y_7)k_1k_2 - (Y_1+Y_3+Y_8+Y_d)]}{Y_2+Y_3+Y_6-k_1k_2(Y_1+Y_4+Y_5+Y_a+Y_b)} \qquad (3.17)$$

Following the same procedure described in section 2.41, the sensitivities of the different parameters Y_{11}, Y_{12}, Y_{21} and Y_{22} are obtained. These are tabulated in Table 3.1.

TABLE 3.1

Sensitivity of the [Y] parameters of the Yanagisawa Structure when the GG is Realized using CNICs and Passive Elements

Element	Variation	S^{α}	S^{β}	S^{γ}	S^{δ}
Y_1	ε_1	$\frac{1}{1-\frac{k_1k_2Y_1\varepsilon_1}{Q}}$	$S^{\alpha}_{\varepsilon_1}$	$S^{\alpha}_{\varepsilon_1}$	$S^{\alpha}_{\varepsilon_1}$
Y_2	ε_2	$\frac{1}{1+\frac{Y_2\varepsilon_2}{Q}}$	$S^{\alpha}_{\varepsilon_2}$	$S^{\alpha}_{\varepsilon_2}$	$S^{\alpha}_{\varepsilon_2}$
Y_3	ε_3	$\frac{1}{1+\frac{Y_3\varepsilon_3}{Q}}$	$S^{\alpha}_{\varepsilon_3}$	$S^{\alpha}_{\varepsilon_3}$	$S^{\alpha}_{\varepsilon_3}$
Y_4	ε_4	$\frac{1}{1-\frac{k_1k_2Y_4\varepsilon_4}{Q}}$	$S^{\alpha}_{\varepsilon_4}$	$S^{\alpha}_{\varepsilon_4}$	$S^{\alpha}_{\varepsilon_4}$
Y_5	ε_5	$\frac{1}{1-\frac{k_1k_2Y_5\varepsilon_5}{Q}}$	$S^{\alpha}_{\varepsilon_5}$	$S^{\alpha}_{\varepsilon_5}$	$S^{\alpha}_{\varepsilon_5}$
Y_6	ε_6	$\frac{1}{1+\frac{Y_6\varepsilon_6}{Q}}$	$S^{\alpha}_{\varepsilon_6}$	$S^{\alpha}_{\varepsilon_6}$	$S^{\alpha}_{\varepsilon_6}$
Y_7	ε_7	-	-	-	1
Y_8	ε_8	-	-	-	1
Y_a	ε_a	$\frac{1}{1-\frac{k_1k_2Y_a\varepsilon_a}{Q}}$	$S^{\alpha}_{\varepsilon_a}$	$S^{\alpha}_{\varepsilon_a}$	$S^{\alpha}_{\varepsilon_a}$

TABLE 3.1 cont'd...

Sensitivity of the [Y] parameters of the Yanagisawa Structure when the GG is Realized using CNICs and Passive Elements

Element	Variation	S^{α}	S^{β}	S^{γ}	S^{δ}
Y_b	ε_b	$\dfrac{\frac{1}{1-k_1k_2Y_b\varepsilon_b}}{Q}$	$S^{\alpha}_{\varepsilon_b}$	$S^{\alpha}_{\varepsilon_b}$	$S^{\alpha}_{\varepsilon_b}$
Y_c	ε_c	1	1	1	1
Y_d	ε_d	-	-	-	1
k_1	ε_{k_1}	$\dfrac{\frac{1}{1-k_1k_2\varepsilon_{k_1}(Y_1+Y_4+Y_5+Y_a+Y_b)}}{Q}$	$S^{\alpha}_{\varepsilon_{k_1}}$	$S^{\alpha}_{\varepsilon_{k_1}}$	$S^{\alpha}_{\varepsilon_{k_1}}$
k_2	ε_{k_2}	$\dfrac{\frac{1}{1-k_1k_2\varepsilon_{k_2}(Y_1+Y_4+Y_5+Y_a+Y_b)}}{Q}$	$S^{\alpha}_{\varepsilon_{k_2}}$	$S^{\alpha}_{\varepsilon_{k_2}}$	$S^{\alpha}_{\varepsilon_{k_2}}$

S^{α}_{ε} ≡ Sensitivity of variation (α) in Y_{11} w.r.t. element variation.

S^{α}_{ε} ≡ Sensitivity of variation (β) in Y_{12} w.r.t. element variation.

S^{α}_{ε} ≡ Sensitivity of variation (γ) in Y_{21} w.r.t. element variation.

S^{α}_{ε} ≡ Sensitivity of variation (δ) in Y_{22} w.r.t. element variation.

$$Q = (Y_2+Y_3+Y_6)-k_1k_2(Y_1+Y_4+Y_5+Y_a+Y_b)$$

3.22 Sensitivity of the Structure when the GG is Realized using VNIC and an IPG or an ING

In this case, the GG, used in the realization of an arbitrary voltage transfer function, is realized, using VNIC and an IPG or an ING as shown in Fig. 3.3.

The [Y] of the structure shown in Fig. 3.3 is obtained, following the same procedure outlined in Case I, and is given by:-

$$[Y] = \begin{bmatrix} \frac{Y_a Y_b}{Y_a+Y_b} + Y_c & -\frac{Y_a}{Z(Y_a+Y_b)} - Y_c \\ \frac{Y_a g_1}{Z g_2 (Y_a+Y_b)} - Y_c & \frac{Y_d Z^2 g_2 (Y_a+Y_b) g_1}{Z^2 \; g_2 (Y_a+Y_b)} + Y_c \end{bmatrix} \quad (3.18)$$

The sensitivities of the different parameters of [Y], with respect to each element used in the realization are tabulated in Table 3.2

3.23 Sensitivity of the Structure when the GG is Realized using INIC and an IPG or an ING

Here, the GG used in the realization of an arbitrary voltage transfer function is realized by using INIC instead of VNIC. (See Fig. 3.4.) The [Y] of the structure shown in Fig. 3.4 is given by:

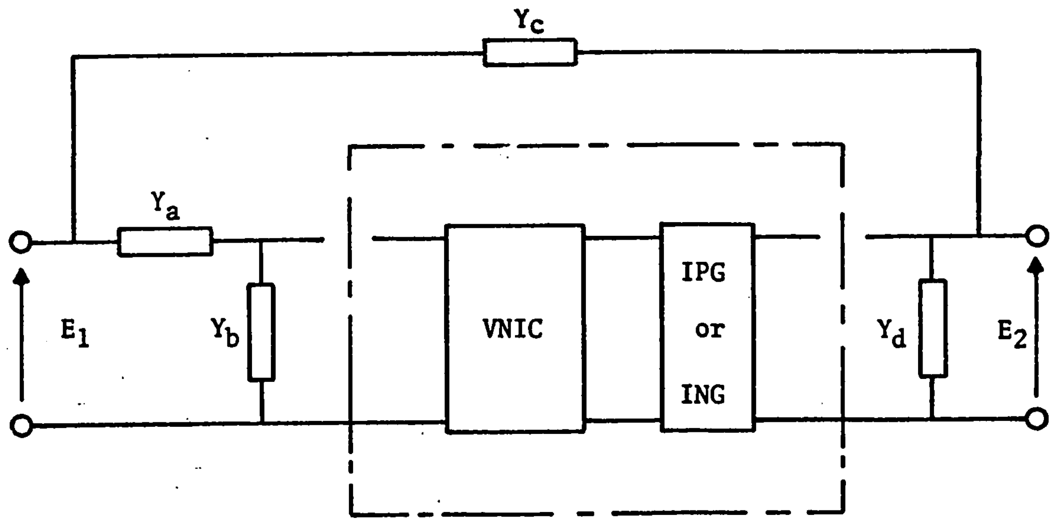

Fig. 3.3: Yanagisawa's Structure, when GG is realized using VNIC and ING.

TABLE 3.2

Sensitivity of the [Y] parameters of the Yanagisawa Structure when the GG is Realized using VNIC and ING.

Element	Variation	S^α	S^β	S^γ	S^δ
Y_a	ε_a	$\frac{1}{1+\frac{Y_a\varepsilon_a}{Y_a+Y_b}}$	S^α	S^α	S^α
Y_b	ε_b	$\frac{1}{1+\frac{Y_b\varepsilon_b}{Y_a+Y_b}}$	S^α	S^α	S^α
Y_c	ε_c	1	1	1	1
Y_d	ε_d	-	-	-	1
G_1	ε_1	-	-	1	1
G_2	ε_2	-	-	$\frac{1}{1+\varepsilon_2}$	S^γ
Z	ε_3		$\frac{1}{1+\varepsilon_3}$	S^β	S^β

$S_\varepsilon^\alpha \equiv$ Sensitivity of variation (α) in Y_{11} w.r.t. element variation.

$S_\varepsilon^\alpha \equiv$ Sensitivity of variation (β) in Y_{12} w.r.t. element variation.

$S_\varepsilon^\alpha \equiv$ Sensitivity of variation (γ) in Y_{21} w.r.t. element variation.

$S_\varepsilon^\alpha \equiv$ Sensitivity of variation (δ) in Y_{22} w.r.t. element variation.

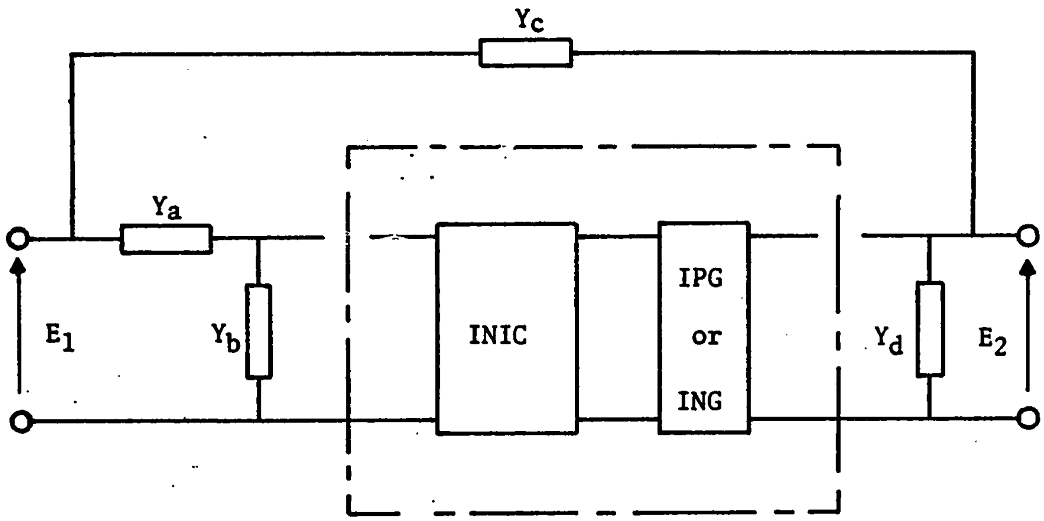

Fig. 3.4: Yanagisawa's Structure, when GG is realized using INIC and ING.

$$[Y] = \begin{bmatrix} \dfrac{Y_a Y_b}{Y_a+Y_b} + Y_c & \dfrac{g_2(k+1)-g_1}{g_2-(k-1)g_1} - Y_c \\ \\ \dfrac{-Y_a}{Z(Y_a+Y_b)} - Y_c & \dfrac{g_2[Z^2Y_d(Y_a+Y_b)-(k+1)]-g_1[Z^2Y_d(k-1)(Y_a+Y_b)+1]}{Z(Y_a+Y_b)[g_2-(k-1)g_1]} + Y_c \end{bmatrix} \quad \ldots\ldots(3.19)$$

The sensitivities of the different parameters of equation (3.19) are obtained and are tabulated in Table 3.3.

3.3 Sensitivity of the Coefficients of a Biquadratic Voltage Transfer Function Realized using Yanagisawa Structure with the GG as the Active Element.

In this section, the sensitivity of the coefficients of a biquadratic voltage transfer function realized using Yanagisawa structure with the GG as the active element will be investigated.

The voltage transfer function of the structure shown in Fig. 3.2 is given by:

$$T_v = -\frac{Y_{12}}{Y_{22}}$$

TABLE 3.3

Sensitivity of the [Y] parameters of the Yanagisawa Structure when the GG is Realized using INIC and ING.

Element	Variation	S^{α}	S^{β}	S^{γ}	S^{δ}
Y_a	ε_a	$\frac{1}{1+\frac{Y_a\varepsilon_a}{Y_a+Y_b}}$	-	S^{α}	S^{α}
Y_b	ε_b	$\frac{1}{1+\frac{Y_b\varepsilon_b}{Y_a+Y_b}}$	-	S^{α}	S^{α}
Y_c	ε_c	1	1	1	1
Y_d	ε_d	-	-	-	1
g_1	ε_1	-	$\frac{1}{1-\frac{g_1(k-1)\varepsilon_1}{g_2-(k-1)g_1}}$	-	S^{β}
g_2	ε_2	-	$\frac{1}{1+\frac{g_2\varepsilon_2}{g_2-(k-1)g_1}}$	-	S^{β}
Z	ε_3	-	-	$\frac{1}{1+\varepsilon_3}$	S^{γ}
k	ε_4	-	$\frac{1}{1-\frac{g_1k\varepsilon_4}{g_2-(k-1)g_1}}$	-	S^{β}

S^{α}_{ε} ≡ Sensitivity of variation (α) in Y_{11} w.r.t. element variation.
S^{α}_{ε} ≡ Sensitivity of variation (β) in Y_{12} w.r.t. element variation.
S^{α}_{ε} ≡ Sensitivity of variation (γ) in Y_{21} w.r.t. element variation.
S^{α}_{ε} ≡ Sensitivity of variation (δ) in Y_{22} w.r.t. element variation.

where $$-Y_{12} = Y_c + \frac{Y_a k_1 [Y_2 k_1 + Y_3 - k_2 (Y_1 + Y_4 k_1)]}{Y_2 + Y_3 + Y_6 - k_1 k_2 (Y_1 + Y_4 + Y_5 + Y_a + Y_b)}$$

$$Y_{22} = Y_c + \frac{H + k_1 k_2 Y_a [(Y_2 + Y_4 + Y_7) k_1 k_2 - (Y_1 + Y_3 + Y_8 + Y_d)]}{Y_2 + Y_3 + Y_6 - k_1 k_2 (Y_1 + Y_4 + Y_5 + Y_a + Y_b)}$$

and

$$H = [k_1 (Y_4 k_2 - Y_1) + (Y_2 k_2 - Y_3)]\ [(Y_2 k_1 + Y_3) - k_2 (Y_1 + Y_4 k_1)]$$

$$- [k_1 k_2 (Y_1 + Y_4 + Y_5 + Y_b) - (Y_2 + Y_3 + Y_6)]\ [(Y_1 + Y_3 + Y_8 + Y_d)$$

$$- (Y_2 + Y_4 + Y_7) k_1 k_2]$$

i.e. $$T_v = \frac{Y_c [Y_2 + Y_3 + Y_6 - k_1 k_2 (Y_1 + Y_4 + Y_5 + Y_a + Y_b)] + Y_a k_1 [Y_2 k_1 + Y_3 - k_2 (Y_1 + Y_4 k_1)]}{Y_c [Y_2 + Y_3 + Y_6 - k_1 k_2 (Y_1 + Y_4 + Y_5 + Y_a + Y_b) + H + k_1 k_2 Y_a [(Y_2 + Y_4 + Y_7) k_1 k_2 - (Y_1 + Y_3 + Y_8 + Y_d)]} \tag{3.20}$$

Let the given T_v be of the form

$$T_v = \frac{a_2 S^2 + a_1 S + a_0}{b_2 S^2 + b_1 S + b_0} \tag{3.21}$$

For convenience, and out of several possibilities, let

$$\begin{aligned} Y_1 &= G_1\ , & Y_2 &= SC_2\ , & Y_3 &= G_3 \\ Y_4 &= G_4\ , & Y_5 &= G_5\ , & Y_6 &= G_6 \\ Y_7 &= G_7\ , & Y_8 &= G_8\ , & Y_a &= G_a \\ Y_b &= G_b\ , & Y_c &= SC_2, & Y_d &= G_d \end{aligned} \tag{3.22}$$

Substituting (3.22) in (3.20) and equating the result with equation (3.21), we get

$$a_2 = C_1C_2$$

$$a_1 = C_1[G_3+G_6-k_1k_2(G_1+G_4+G_5+G_a+G_b)]+C_2(G_ak_1^2)$$

$$a_0 = G_ak_1[G_3-k_2(G_1+G_4k_1)]$$

$$b_2 = C_1C_2$$

$$b_1 = C_1[G_3+G_6-k_1k_2(G_1+G_4+G_5+G_a+G_b)]$$
$$+C_2[k_1^2(k_2G_4-G_1)+k_1k_2(G_1+G_5-G_7+G_a+G_b)$$
$$-k_2^2(k_1G_4+G_1)+G_3(k_2-k_1)+G_1-G_6+G_8+G_d]$$

$$b_0 = k_1^2\ k_2^2\ [(G_4+G_7)(G_1+G_4+G_5+G_a+G_b)]$$
$$-k_1k_2[(G_1+G_3+G_8+G_d)(G_1+G_4+G_5+G_a+G_b)$$
$$+(G_3+G_6)(G_4+G_7)+(G_4k_2-G_1)(G_1+G_4k_1)-2G_3G_4]$$
$$+(G_1+G_3+G_8+G_d)(G_3+G_6)-G_3^2\ +G_3G_1[k_2-k_1]$$

The sensitivities of the above coefficients a_0, a_1, a_2 and b_0, b_1, b_2 are obtained and tabulated in Table 3.4.

Next, let us investigate the existance of Yanagisawa's structure for the two cases:-

(i) $k_1 = k_2 = 1$

(ii) $k_1k_2 = 1$

TABLE 3.4

Sensitivities of the Coefficients of a Biquadratic Voltage Transfer Function Realized using Yanagisawa Structure with the GG as the Active Element

Element	Variation	S^{a_2}	S^{a_1}	S^{a_0}	S^{b_2}	S^{b_1}	S^{b_0}
C_1	α_1	1	1	-	1	1	-
C_2	α_2	1	1	-	1	1	-
G_1	ε_1	-	1	1	-	1	1
G_3	ε_3	-	1	1	-	1	1
G_4	ε_4	-	1	1	-	1	1
G_5	ε_5	-	1	-	-	1	1
G_6	ε_6	-	1	-	-	1	1
G_7	ε_7	-	-	-	-	1	1
G_8	ε_8	-	-	-	-	1	1
G_a	ε_a	-	1	1	-	1	1
G_b	ε_b	-	1	-	-	1	1
G_d	ε_d	-	-	-	-	1	1
k_1	ε_{k_1}	-	Q_1	Q_2	-	Q_3	Q_4
k_2	ε_{k_2}	-	1	1	-	Q_5	Q_6

$$Q_1 = \frac{1}{1-\dfrac{C_2G_ak_1\varepsilon_{k_1}}{2k_1C_2G_a(1+\varepsilon_{k_1})-C_1k_2F/G_4+G_7}}$$

$$Q_2 = \frac{1}{1-\dfrac{G_4k_1k_2\varepsilon_{k_1}}{2G_4k_1k_2(1+\varepsilon_{k_1})+G_1k_2-G_3}}$$

$$Q_3 = \frac{1}{1-\dfrac{C_2k_1(k_2G_4-G_1)\varepsilon_{k_1}}{2C_2k_1(k_2G_4-G_1)(1+\varepsilon_{k_1})+N-C_2(k_2^2G_4-G_3)}}$$

$$Q_4 = \frac{1}{1-\dfrac{k_1k_2(k_2F+M)\varepsilon_{k_1}}{2k_1k_2(k_2F+M)(1+\varepsilon_{k_1})-(M_2^2+k_2H+L)}}$$

$$Q_5 = \frac{1}{1-\dfrac{C_2k_2(k_1G_4+G_1)\varepsilon_{k_2}}{2k_2C_2(k_1G_4+G_1)(1+\varepsilon_{k_2})-N-C_2(k_1^2G_4+G_3)}}$$

$$Q_6 = \frac{1}{1-\dfrac{k_1k_2(k_1F-M)\varepsilon_{k_2}}{2k_1k_2(k_1F-M)(1+\varepsilon_{k_2})+(Mk_1^2-k_1H+L)}}$$

$$F = (G_4+G_7)(G_1+G_4+G_5+G_a+G_b)$$

$$N = C_2(G_1+G_5-G_7+G_a+G_b)-C_1(G_1+G_4+G_5+G_a+G_b)$$

$$M = G_1G_4$$

$$H = (G_1+G_3+G_8+G_d)(G_1+G_4+G_5+G_a+G_b)+(G_3+G_6)(G_4+G_7)-2G_3G_4-G_1^2$$

$$L = G_1G_3$$

Again, for convenience and out of several possibilities, the following conditions have been chosen for the two cases mentioned above:-

$$Y_c = SC_1 \qquad\qquad Y_2 = SC_2$$

$$Y_1 = Y_4 = Y_5 = Y_7 = G_1 \;, \quad Y_3 = Y_6 = Y_8 = G_3 \tag{3.23}$$

$$Y_a = G_a \;,\; Y_b = G_b,\; Y_d = G_d$$

Substituting (3.23) in (3.20) and comparing it with (3.21), we get:-

$$a_0 = G_a k_1 [G_3 - k_2 G_1 (1+k_1)]$$

$$a_1 = C_1 [2G_3 - k_1 k_2 (3G_1 + G_a + G_b)] + C_2 G_a k_1^2$$

$$a_2 = C_1 C_2$$

$$\begin{aligned} b_0 = {} & G_1 [2k_1^2 \; k_2^2 (3G_1 + G_a + G_b) - k_1 k_2 (2G_1 + 8G_3 + G_a + G_b + 3G_d) \\ & + k_1 k_2 G_1 (k_1 - k_2 - k_1 k_2) + G_3 (k_2 - k_1 + 2)] \\ & - 2G_3 [k_1 k_2 (G_a + G_b) - G_d] - 3G_3^2 - k_1 k_2 G_d (G_a + G_b) \end{aligned} \tag{3.24}$$

$$\begin{aligned} b_1 = {} & C_1 [2G_3 - k_1 k_2 (3G_1 + G_a + G_b)] \\ & + C_2 [G_1 [k_1^2 \; (k_2 - 1) - k_2^2 \; (k_1 + 1) + k_1 k_2 + 1] \\ & + G_3 [k_2 - k_1] + k_1 k_2 (G_a + G_b) + G_d] \end{aligned}$$

$$b_2 = C_1 C_2$$

Case (i) $k_1 = k_2 = 1$:

Substituting $k_1 = k_2 = 1$ in equation (3.24) and for simplicity putting $C_1 = C_2 = 1$, we get:-

$$\begin{aligned}
a_2 &= b_2 = 1 \\
a_1 &= 2G_3-3G_1-G_b \\
a_0 &= G_a(G_3-2G_1) \\
b_1 &= 2G_3-3G_1+G_d \\
b_0 &= G_1[G_a+3G_1+G_b-3G_d-6G_3]-G_3[2(G_a+G_b-G_d)-3G_3]-G_d(G_a+G_b)
\end{aligned} \tag{3.25}$$

Solving (3.25) for G_a in terms of G_1, we get :-

$$G_a^3\ (b_1+2G_1)+G_a^2(b_0-a_1b_1-2a_1G_1)+G_a(2a_0G_1)+a_0^2 = 0 \tag{3.26}$$

By assigning any positive value to G_1, G_a could be obtained.

It is to be noticed, from equation (3.26), that G_a has at least one positive value and consequently the values of the other elements can be calculated. Hence the structure can always be realized.

Case (ii) $k_1k_2 = 1$

Similar to the previous case, substituting $k_1k_2 = 1$ and $C_1 = C_2 = 1$ in (3.24), we get:-

$$a_2 = b_2 = 1$$

$$a_1 = 2G_3 - (3G_1+G_a+G_b)+G_ak_1^2$$

$$a_0 = G_a[k_1(G_3-G_1)-G_1]$$

$$b_1 = G_1\left(\frac{k_1^3-k_1^4-k_1^2-k-1}{k_1^2}\right)+G_3\left(\frac{1+2k_1-k_1^2}{k_1}\right)+G_d$$

$$b_0 = G_1\left[G_1\left(\frac{k_1^2+3k_1-1}{k_1}\right)-G_3\left(\frac{k_1^2+6k_1-1}{k_1}\right)+G_a+G_b-3G_d\right]$$
$$-G_3[2(G_a+G_b)-3G_3]-G_d(G_a+G_b-2G_3) \tag{3.27}$$

Solving (3.27) for G_a in terms of G_1 we get:-

$$G_a^3\ k_1^4[G_1(1+k_1^2)+b_1]+G_a^2\ k_1^2[b_0-a_1b_1-a_1G_1(1+k_1^2)-a_0(1-k_1^2)]$$
$$+G_aa_0[G_1(1+k_1^2)+a_1(1-k_1^2)]+a_0^2 = 0 \tag{3.28}$$

By assigning a value to G_1, G_a could be obtained, which at least has one positive value. Hence a solution can always be found.

3.4 Summary

In this Chapter, the sensitivity of Yanagisawa's structure, when the GG is used as the active element, is investigated with the three different realizations of the GG.

It has been shown that the sensitivity of the [Y] with respect to the majority of the elements used in the realization depends on the variation in the element nominal value and the values of all other elements while the sensitivity of the [Y] with respect to the remaining elements does not depend upon the elemental values. However, in all the cases, the value of the sensitivity is always less than or equal to unity, (including the value zero).

Also, in this chapter, the realizability of such a structure for a biquadratic voltage transfer function has been examined and it is shown that it is always realizable.

CHAPTER IV

CONCLUSIONS

The work in this dissertation has been divided into three parts. The first part is concerning the investigation of the sensitivities of the three Generalized Gyrator realizations. The second part deals with the sensitivity of the Yanagisawa structure when the generalized gyrator, used as the active element, is realized in three different ways. The third part investigates the realizability of this structure for a biquadratic voltage transfer function.

It has been shown that the sensitivity of GG realized, using CNICs or VNICs, with respect to all the elements used in the realization, were either identically equal to zero or unity, while the sensitivity with respect to the conversion ratio k_1 was inversely proportional to the change in its value.

In contrast to the above, it has been shown that the sensitivity of the GG realized using a cascade combination of VNIC and ING, with respect to the majority of the elements is inversely proportional to the change in the nominal value.

In the third case, where the GG has been realized using INIC and ING, it is shown that, although the sensitivity of A,B, and D parameters, with respect to all the elements used in the realization are either identically zero or equal to unity, the sensitivity of the C parameter is dependent on both the variation and the nominal values of the different elements and will always be less than unity.

Sensitivity study on Yanagisawa's structure, realized using Generalized Gyrator as the active element, reveals that the sensitivity of the structure, when the Generalized Gyrator is realized using CNICs and passive elements, is better than the sensitivities of the other two structures where the Generalized Gyrators used are realized using either VNIC or INIC cascaded by ING. This is due to the fact that the structure sensitivity with respect to all elements used in the realization is directly proportional to the factor Q. It can be seen that the sensitivity could be minimized if Q is minimized.

From the above, it is clear that, from the sensitivity point of view, the Generalized Gyrator realized, using CNICs or VNICs and passive elements, is better than the other two realizations and could improve the sensitivity of Yanagisawa's structure, if it is used. It has been shown that the above mentioned structure is always realizable for a biquadratic voltage transfer function.

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