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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RÊCUE
SHOWING AND SAYING IN THE PHILOSOPHICAL INVESTIGATIONS

Michael Mackenzie

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ABSTRACT

SHOWING AND SAYING IN THE PHILOSOPHICAL INVESTIGATIONS

Michael Mackenzie

This thesis is a discussion of the showing/saying distinction in the first part (up to §220) of Ludwig Wittgenstein’s Philosophical Investigations. It attempts to show what is involved in the philosophical procedure of showing by using Wittgenstein’s discussion of rules in the Philosophical Investigations as an example.

Furthermore, it hopes to establish that, given the conception of language that emerges from Wittgenstein’s discussion of rules, showing stands as the appropriate philosophical procedure to articulate the notion of rules.

Finally, a few tentative conclusions are drawn concerning philosophical practice and teaching.
TO DORTHE HVIDD
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INTRODUCTION

...sometimes the voice of a philosophical thought is so soft that the noise of spoken words is enough to drown it and prevent it from being heard.
(Zettel S453)

The subject of this thesis is the saying/showing distinction in the later philosophy of Wittgenstein, specifically in the Philosophical Investigations, as an aspect of his philosophical method. The way I hope to bring out this distinction (between showing and saying) is through a discussion of Wittgenstein's (from now on L.W.'s) presentation of "rules" in the Philosophical Investigations (from now on P.I.). Therefore the first two thirds of this paper will be devoted to an exposition of "rules" in P.I. I would ask the reader to keep in mind the notion of philosophical method even when it is not clearly related to the text.

In making this request I don't intend to absolve myself of clear exposition, rather I speak from the conviction that clarity in philosophy is an achievement rather than a prerequisite, and can be self-defeating if seen as an easy and straightforward demand. The basis for this conviction is something I hope to show in the following paper (ultimately in chapter III): In short I hope that my conclusions will in some way vindicate my presentation.

The first two chapters of this paper are a partial commentary on the notion of "following a rule" in the P.I. from S83 to around S201. This partiality is not only in respect of the number of propositions it
deals with (i.e. by no means all the propositions between S83 and S201) but also its comprehensiveness for the propositions it does address. I think this is inevitable in the light of the density and complexity of P.I.; All I have tried to do is take certain strands of the discussion and show an interrelatedness of passages that may not be immediately evident.

This may not appear to promise a very fruitful philosophical enterprise. However, I think it highlights a problem with some philosophical literature on P.I., namely a neglect of structure or development in P.I. From L.W.'s preface it is clear that whatever structure P.I. might have it is very different from structuring of the Tractatus. But this is not to say that there are not certain significant continuities in it, in fact - again from the preface, and the time and effort L.W. spent ordering the book - there quite obviously are. It is unfortunate then that P.I. is sometimes dealt with as if the remarks were ordered in a totally arbitrary fashion, as for example by K.T. Fann (see Chapter III).

Chapter III is an attempt to articulate the notion of philosophical method that has emerged from chapters I and II and particularly the showing/saying distinction.

I should say that the discussion of rules in this paper is by no means exhaustive. The notion of rules is developed only in as much as it is illuminating for them main concern of examining L.W.'s

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"Structure" is used throughout this thesis to imply, nothing more than continuities in, and connections between, the numbered propositions of P.I.
philosophical method. Therefore (rather than leaving the discussion of rules hanging in an obvious way) much that is essential to the articulation of L.W.'s notion of rules is left unsaid.

Although references to L.W.'s notion of rules are not uncommon in secondary sources, the complexity of L.W.'s discussion of rules and its connection with showing (the theme of this thesis) has yet to be recognised. In "On the Notion of a Rule" Thomas Olshewsky has asserted the need for this kind of clarification of the notion of rules. Similarly although there have been references to showing as an aspect of L.W.'s later philosophy notably by Peter Winch in, "The Unity of Wittgenstein's Philosophy" and Stanley Cavell in The Claim to Rationality, no exposition of showing in the later Wittgenstein has yet been undertaken.

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2 See Bibliography.
3 In Studies in the Philosophy of Wittgenstein, ed., Peter Winch.
4 See Bibliography.
CHAPTER I

What do I call 'the rule by which he proceeds'? The hypothesis that satisfactorily describes his use of words, which we observe; or the rule he looks up when he uses signs; or the one which he gives us in reply if we ask him what his rule is? But what if observation does not enable us to see any clear rule, and the question brings none to light? For he did indeed give me a definition when I asked what he understood by 'N', but he was prepared to withdraw and alter it. (P.I., S82)

Although this is the first time in the P.I. that the question of what it is to follow a rule has been explicitly raised, the above paragraph very clearly refers to preceding discussions in the P.I.

For example, the notion of 'looking up' a rule is presented as early as S1 and again at S53. An "hypothesis that satisfactorily describes his use of words" (initially) is that a mental sample is "borne in mind" (see S51 and then S56). At S79 we have an example of "what he gives us in reply if we ask him what his rule is" - he does give a definition for "N", but is prepared to withdraw and alter it.

Also in this paragraph L.W. is not only referring to the hypotheses that were offered to explain "the rule by which he proceeds", but also to the kind of criticisms offered of these hypotheses. In S51 we are asked "to focus on the details of what goes on" to look "from close to" and at S56, for example, "observation does not enable us to see any clear rule" (S82).

At this point the following observation can be made (relevant both philosophically and to the structure of the P.I.): L.W. is
offering in S82 a general characterization of what appeared prima facie as separate philosophical manoeuvres (e.g., offering definitions for "N", describing the use of the word red in terms of tables). These are now seen as attempts to provide "the rule by which he proceeds". In terms of the structure of the P.I., L.W. is drawing together at S82, through this characterization of rules, what have previously been dealt with as separate though intertwining considerations of our use of language, and our understanding of meaning. It is through this general characterization that L.W. is now able to offer a general critique of the view that rules facilitate a philosophical understanding of language, thus conferring a unity to the proceeding discussion and criticisms.

The critique that he offers here (S83 + S84) is by means of two connected arguments.

(1) Drawing on the previously constructed analogy between language and games (see S7 on) L.W. presents an example of a situation where people can be seen to be playing a game that is not bounded by rules. Thus, by this analogy, there would seem to be discourse (language games) which is similarly unbounded by rules. (S83)

(2) The argument that providing a rule raises the possibility of a further rule to decide the application of that rule and so on...
an infinite regress. (S84).

Although argument (2) is not original (it was used by Kant\(^1\)) what may emerge as original is the scope that this argument has as a philosophical

\(^{1}\)In \textit{Critique of Pure Reason}, p. 177-178.
criticism. Here we may reflect a moment on the significance of L.W.'s general characterization of "rules", as it is exactly this characterization which makes the previously separate discussions, such as the postulating of tables or the definition of "N", vulnerable to the critique of "a rule determining the application of a rule."

Two questions should be considered at this point. The first is:

(1) What is the scope of L.W.'s consideration of rules here? That is, for what areas of language or human interaction would L.W. consider the discussion of rules (either critically or otherwise) relevant. This question arises from the fact that in P.I., L.W. was addressing certain philosophical attempts that involved categorizing or classifying language (for example the distinction between Names and Elementary Propositions in the Tractatus)\(^2\). Was he therefore considering the notion of "following a rule" in relation to one or a limited number of linguistic categories or areas, or the whole of our use of language? The significance of this question is heightened in the light of disagreement concerning the role of "rules" in L.W.'s later philosophy. On one hand we have Peter Winch placing a large and general emphasis on the notion of rules in the discussion of human language and interaction (see The Idea of a Social Science) and his explicit derivation of this emphasis from L.W. (Winch has been criticised for this extensive use of the notion of rules by G. Von Wright in Explanation and Understanding, p. 79). On the other hand we have James Bogen's statement that for L.W. "rules only occur when required to correct irregularities which are not precluded by the existing (non-rule-governed) practices to which they

\(^2\) See for example P.I., S26-49.
are added", in his book Wittgenstein's Philosophy of Language. Also in A Wittgenstein Workbook by C. Coope, P. Geach, T. Potts and R. White the only secondary sources that are listed under "following a rule" are specifically concerned with the philosophy of mathematics (by M. Dummett and G. Frege).

This question will be considered here (it will be raised again in Chapter II and III) in terms of rules as a "negative critique": i.e., for the areas or aspects of language which L.W. would seem to consider the offering of rules an illegitimate or mistaken procedure.

One way of approaching this question is by examining what aspects of the Tractatus's view of language are characterized and criticised in P.I. as attempts to provide rules.

In the Tractatus there is a distinction between Names and Propositions. Names have meaning (their meaning is an Object, Tractatus 3.203) and Propositions have sense. Names have a meaning because they name an Object, Propositions have sense because they conform to the "Logik der Abbildung" (See Appendix-A). In the Tractatus these categories of meaning and sense cover every possible aspect of use of language, i.e., if all the Names in a proposition name Objects and the Proposition conforms to the "Logik der Abbildung" then it is sensible discourse, if not - it is nonsense.

At S81 in P.I., L.W. is clearly referring to the notion of "Logik der Abbildung" in the Tractatus in terms of the "calculus according to definite rules". (See for example, Tractatus 3.334 and 3.343).

The notion of a Name in the Tractatus is attacked quite clearly and thoroughly in the P.I. for example at S27 and S46 when considering "names of objects" and "simples". Here L.W. is arguing
against the idea that there is an aspect of language that functions by being attached (in some way) to an object which is its meaning. What is significant for our present discussion is that L.W. is characterizing this particular philosophical manoeuvre as the attempt to provide a rule - see for example his discussion of "the name 'N'", in S79 and the reference to it in S82.

From this we can infer that the "negative critique" of rules is relevant to the attempt to provide rules for every aspect of language dealt with in the Tractatus through the conceptions of meaning (Names) and sense (Propositions), i.e., every aspect of our discourse. The second question which follows from this is:

(2) How are we to understand this notion of providing rules, i.e., what kind of philosophical manoeuvre counts or falls under this characterization? On one hand, the notion of trying to provide the rule by which he proceeds appears as an extremely forceful general description of the various philosophical manoeuvres that L.W. has described and criticized. Whether it is a description, an object ("the broom in the corner", see S60), a referent, a table, or a mental sample, in all these cases we are being given "the rule by which he proceeds". In all these cases we are being offered something to?

3 Obviously human discourse was a much broader more varied thing for the later Wittgenstein than the earlier. The main point I wish to establish here is that the notion of rules goes far beyond considerations for the philosophy of mathematics (see Chapter II especially footnote 7) and that discussions of rules in philosophy of mathematics and philosophy in general are not separate issues (again see Chapter II). It is my conviction the notion of rules arrived at by L.W. by S201 has implications for the whole of human activity. However this is not something I have argued for stringently as it is not essential to the main theme of this paper.
which a word must in some way be attached or conform in order to have any meaning - it is the rule that binds any meaningful use of the word. However, does L.W. do anything more than provide a very à propos general description of these various philosophical manoeuvres?

I think he does. In S82, immediately after stating the critique of rules in terms of a rule to determine the application of a rule (see above, p. 5 (2)), L.W. introduces the notion of "doubt", the kind of doubt that leads us to want to provide rules - the kind of doubt that is (hopefully) removed by introducing or positing a rule. This is the beginning of a strand of discussion in P.I. concerning the impulse to provide rules, and then rules for application of rules arising from a certain kind of doubt (see for example S86). "It may easily look as if every doubt merely revealed an existing gap in the foundations, so that secure understanding is only possible if we first doubt everything that can be doubted, and then remove all these doubts." (P.I., S87).

Here L.W. is beginning a discussion of a certain kind of philosophical temptation manifested in this doubt. This temptation involves the search for exactness (see S88), and led L.W. (in the Tractatus) to consider logic as the basis of language - as the embodiment of rules that dictated our use of words (see S89). It is the temptation to search for (among other things) the essence of language (see S92 and S97) as L.W. himself did in the Tractatus.

L.W. then is offering us a further understanding of what is involved in the attempt to provide rules - it is the manifestation of a philosophical temptation.
To sum up then: L.W. has offered a general characterization of various philosophical manoeuvres as attempts to provide or discover "the rule by which he proceeds". It is this general characterization that facilitates his general critique of these philosophical manoeuvres in terms of "the rule determining the application of a rule... and so on." The notion of "attempts to provide rules" applies as a general characterization not only because of its descriptive power - the way in which it does seem to accurately describe various philosophical discussions - but it also points to an underlying unity in terms of a specific philosophical temptation.

The question I would now like to raise is whether this stands as an adequate critique of those philosophical manoeuvres characterized as attempts to articulate, "the rule by which he proceeds". Certainly I think it can be seen to be a very thorough critique, not only giving a cogent criticism of a philosophical position, but also offering some kind of understanding of the impulse that leads to that position - the philosophical temptation behind it.

However as it stands this is merely a "negative critique". It sees no positive philosophical contribution in the position criticised, except, perhaps, in as much as it helps us gain access to a philosophical temptation. It could well be considered that this is the extent of L.W.'s critique and indeed according to many interpretations of L.W.'s philosophy his task of "dissolution" of a philosophical problem is complete. This kind of interpretation (as implicitly
adopted by Norman Malcolm in "Knowledge of Other Minds", and John Cook in "Wittgenstein on Privacy" is criticised by Stanley Cavell in his article "Knowing and Acknowledging". Cavell argues that skeptical views, doubts similar to the kind that we have been considering here as philosophical temptations, contain within them the possibility of significant philosophical insight. Therefore to dismiss them or offer a purely "negative critique" of them, as Cavell argues Malcolm, Cook and the mainstream of Ordinary Language Philosophy does, is a mistake - and a mistake that L.W. is not culpable of. In fact Cavell Invokes this as one of the major distinctions between Ordinary Language Philosophy and L.W.'s procedures.

Although at this point in the P.I., L.W. has criticised the attempt to account for our use of language in terms of rules he himself has offered no alternative philosophical understanding of use. Yet we find that he acknowledges the same kinds of consideration of language that lead others (and himself in the Tractatus) to postulate rules. At S207 he gives the example of people who appear to speak and act much the same as we do (even to the point where if one is gagged there seems to be a consequent lack of communication). Yet (he says) if we can find no regular connection between the sounds they

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5 In Ibid., pp. 286-323.

6 In Must we Mean what we Say, essays by Stanley Cavell, I Don't intend to reproduce Cavell's criticism but to show that in fact L.W.'s critique does not stop here - i.e., with a merely negative critique, see Chapter III.
make and the actions they perform, if we can find no regularity, then we cannot call this language. Having acknowledged this regularity it would seem that L.W., if he has criticised a particular attempt to account for it (in terms of rules), is now obliged to offer an alternative account himself or state why he is not so obliged. It would seem then that the negative critique, as it stands, is insufficient. I think L.W. would regard the discussion of rules so far as incomplete, and this can be seen in the discussion that L.W. offers following on from the sections so far considered. This discussion is examined in the next chapter:

In Zettel S460, L.W. writes, "In a certain sense one cannot take too much care in handling philosophical mistakes, they contain so much truth". The next step then is to see what truth might be found in the philosophical mistake outlined in this chapter.
CHAPTER II

In p. 1, the next detailed discussion of rules per se begins at S198. That discussion will be the main focus of this chapter. However before proceeding with that it should be noted that the notion of "rules" occurs at various places, after the section discussed in the previous chapter and before S198, in connection with the teaching of mathematical series. At S143 there is an example where "A gives an order B has to write down a series of signs according to a certain formation rule", and the discussion then proceeds explicitly in this vein to S156 where L.W. takes up the notion of "reading". The discussion then reoccurs at S179 (with reference to S151) and continues.

In these sections L.W. considers many different aspects of teaching or communicating mathematical series, with various perspectives and emphases. However for the purposes of this paper I would like to isolate a few general themes of these sections.

Obviously, here L.W. is concerned to examine the nature of teaching a mathematical series by consideration of what is involved in "getting him to understand" (S143); "the pupils capacity to learn" (S144), to "understand the rule of a series" (S147), "understanding

1"Reading" here is very much connected with the theme of "rules", but its consideration is not necessary for the purposes of this paper.
the principle of the series" (S151), etc. In this respect one thing that he is concerned to show is what this understanding is not, i.e., various things which we are tempted to take as his understanding of the series. For example, "The formula...occurs to me" (S154) or, "... an experience I know empirically to lead to the continuation of the series" (S179) such as the sensation "that's easy!" (S151). None of these constitute the understanding of a series, which is not to say they may not be things which a pupil might state or experience when he learns a series. The fact that they often do (see S151) tempts us to consider them as candidates for "the understanding of a series".

From this discussion emerges the distinction between a rule and the manifestation or expression of the pupil's understanding (or misunderstanding, as the case may be) of it - his interpretation of the teaching. Whatever the student might express, an experience of "grasping in a flash" or a formula, it still cannot be the rule itself because he still could have it wrong. Even in a case which appears very clear cut, where he continues the series, "0, n, 2n, 3n, etc." (where \( n = 2 \) up to 1000 (see S185) it still may emerge that he has it wrong because after 1000 he does not continue that series correctly. Thus whatever the student may do, say or express it cannot be the rule itself, it can only be his interpretation of the rule as it may ultimately conflict with the rule.

Furthermore the same distinction applies to whatever may be used in the teaching of a rule. Those expressions which appear to be manifestations of the student's understanding of a rule may equally well be used in the teaching of a rule: exercises can be examples, interpretations can be explanations. In fact we can imagine a situation
where a teacher is teaching a mathematical series or technique perfectly well, but later we find that he misapplies it himself, i.e., in some way he has misunderstood it (we can even imagine that one of his students does not make the same mistake).^2

We have then, emerging from L.W. 's discussion of teaching and understanding the rule of a series, a distinction between the rule itself and whatever the expression or interpretation of the rule might be in that process.

What kind of philosophical scope might there be to the notion of a rule as discussed so far in this Chapter? The notion of following a rule, particularly in reference to the passages just considered, is often restricted to considerations in the philosophy of mathematics. An example of this would be Michael Dummett's article 'Wittgenstein's Philosophy of Mathematics'[^3]; also note that in A Wittgenstein.

[^2]: Wittgenstein's argument at this point (S186 - 187) is more complex than I am implying here. Essentially it is against the idea that the teacher's expression of the rule is the rule itself because the teacher meant it as the rule. To reproduce this in full would involve, I think, a long digression. Thus, I would ask the reader to be aware of S186 and S187. I have also presented a very brief argument myself to maintain the distinction between the rule and its expression in the case of the teacher. This argument reflects an experience of mine where in high school - my physics teacher, having taught the class Newton's 2nd law of motion, didn't realize that for a problem concerning the dropping of a chain vertically on to a horizontal surface the calculation involved the differentiation of the mass (m) in the formula F = ma - something a few of his students didn't realize.

Workbook in the section "Following a Rule" the only secondary sources given (apart from a passage in Kant) are Dummett's article and a section of Frege's writings on philosophy of mathematics.

However, I think this is an arbitrary restriction. The passages just considered have a direct relevance to the consideration of following a rule discussed in Chapter I of this paper, involving a much broader area of language than mathematics. Thus whatever notion of rules emerges from this discussion is equally relevant to names like "Moses", "red", "dark", "sweet", etc.

That the passages from S143 on are a direct continuation of the preceding discussion of rules can be seen from (1) the use of the word "rule" in both sections, (2) the occurrence of the same metaphors/examples in both sections e.g., "pointing" and "sign-posts" (S87, S185, S198), (3) the various remarks that L.W. makes indicating that his examination of teaching in mathematics holds much in common with other kinds of learning/understanding. For example, at the bottom of p. 53 of P.I., "Do n't I also imagine myself to understand a word (as I may imagine I understand a kind of calculation)", or at S208, L.W. in the process of discussing "rules" refers to teaching of colors, lengths and shapes. Again at S238 and S239 colors are considered following and included in a series of remarks about rules and the kind

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4 See Bibliography.

5 This is something that, I think, should be taken very seriously given the great pains that L.W. took over composing his remarks.
of problems raised by mathematical examples. Thus even though the examples from S143 onward are mathematical, in the context of the notion of "rules" this discussion maintains the same linguistic scope (i.e., has implications for the same areas of language) as outlined in Chapter I (see p. 6 on).

It is, I think, easy to see why L.W. chose mathematical examples here. Note first that the discussion from S143 onwards is a consistently developing discussion, thus once L.W. had opted for a mathematical example he was more or less bound to sustain it. Secondly in terms of someone coming to understand how to follow a rule, mathematical examples are much more familiar and accessible than say the teaching of colors, or lengths, or names. And, some fine distinctions can be drawn when discussing mathematical series that would be harder to present with colors, say (i.e., this shade is red but this shade is maroon, and this is aubergine). Also in making the distinction that I have claimed L.W. was trying to make at this point (between the rule itself and its expression or interpretation, see p. 22-23) the mathematical series example is perhaps particularly adaptable, and allows for clearer distinctions, as at S185 where the student can be shown to have misunderstood the series even though he

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6 This is perhaps because the teaching of naming and colors occurs much earlier in human development and therefore the subject is less articulate about the process. Similar mistakes to those experienced in teaching mathematical series do occur though. One example from my own experience was a 2½ year old boy who on arriving in Day-Care was told that his teacher's name was Carmen. From this he concluded that all his teachers' name (including mine) were Carmen. This continued for some time.
was able to proceed to 1000.7

Thus these mathematical examples as a continuation of the
discussion on rules have implications for every aspect of our
discourse (see Chapter I, p. 8). This brings us to a strange impasse.
In the light of the discussion so far in this Chapter and in Chapter I
what could L.W. be referring to when he talks about rules? He has
thoroughly criticized the notion of rules as a philosophically useful
one (except inasmuch as it helps us recognize a temptation — see
Chapter I) but then embarks on a further explicit discussion of rules
at S198.

But how can a rule show me what I have to do
at this point? Whatever I do is, on some
interpretation, in accord with the rule."
- That is not what we ought to say, but rather:
your interpretation still hangs in the air
along with what it interprets, and cannot
give it any support. (P.I., S198)

"Any interpretation hangs in the air" - i.e., whatever the
student expresses, etc., as his/her understanding of the rule is not
conclusive (is not the rule): "along with what it interprets" - i.e.,
whatever is offered by the teacher in attempting to teach the rule,

7In his article Dummett comments on the "thin and unconvincing"
nature ("amazing for him" of L.W.'s example). It is not absolutely
clear to me from his article what examples he is considering - it
seems likely to be the ones from S143 to S190. Bogden (see Bibliography)
remarks on "Wittgenstein's failure to produce convincing examples."  
(p. 190) However given the interpretation offered here I think those
examples (particularly the one at S185) are adequate and appropriate.
Dummett wishes to separate considerations of rules from
"general theory of meaning" and specific relevance to mathematics.
(p. 428) I think the interpretation given here would indicate that
there is another kind of approach. That is, not to consider these
examples as relevant to the philosophy of mathematics in an exclusive
fashion, but rather as relevant in as much as L.W.'s discussion here
applies to all areas of human discourse. This does not dilute their
relevance for mathematics, rather it would point to the necessity of
understanding Wittgenstein's philosophy of mathematics in the context
of his philosophy as a whole.
e.g., examples of formulas, etc.,

This was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule. The answer was: if everything can be made out to accord with the rule, then it can also be made out to conflict with it. And so there would be neither accord nor conflict here.

It can be seen that there is a misunderstanding here from the mere fact that in the course of our argument we give one interpretation after another; as if each one contented us at least for a moment until we thought of yet another standing behind it. What this shows is that there is a way of grasping a rule which is not an interpretation, but which is exhibited in what we call "obeying the rule" and "going against it" in actual cases.

Hence there is an inclination of say: every action according to the rule is an interpretation. But we ought to restrict the term "interpretation" to the substitution of one expression of the rule for another. (P.I. S201)

In the course of being taught a specific rule (e.g., a mathematical series) a student may respond in a variety of ways which he sees as "going on", as in accord (none of which may be what the teacher intended): "no course of action could be determined by the rule, because every course of action can be made out to accord with the rule." However during the preceding discussion various things were tried, e.g., a formula, "grasping in a flash", as that which ensured the student was following the rule - somehow they enshrined the rule ("in the course of our argument we give one thing after another..."). Each of these were found wanting, as each could involve a course of action that could be made out as in accord or conflicting with the rule regardless (it "still hangs in the air").

Yet this does not require that we abandon the concept of a rule - rather it re-inforces it ("What this shows is that there is a way of grasping a rule...""). If we are in a position to describe an
interpretation, or respond to a teaching as "obeying the rule" or "going against it" clearly we are appealing to something which can only be the rule.

In the last paragraph of S201 there are distinctions between "interpretation", "expression of a rule" and (implicitly) the rule itself.

What L.W. has been struggling against in his discussion of rules up to this point is the temptation to encapsulate "the rule by which he proceeds" into some kind of experience, expression or formula - something that can be stated or said. The rule itself is not to be confused with its expression or interpretation. A rule cannot be captured in something that can be stated. The fact that a student can state a formula, say the next section of the series, or refer to an experience does not mean he has grasped the rule.

If we return to the "negative critique" of rules in Chapter I of this paper it is quite clear that it is directed at what, in the light of the above, would be considered the interpretation or expression of a rule - "the hypothesis that satisfactorily describes his use of words" - "the rule which he looks up" - "what he gives in reply when we ask what the rule is". All these are attempts to encapsulate the rule in some kind of formula or expression. The notion implied by the word "rule" then has been transformed as L.W.'s discussion has developed from S&M on. It is now something distinct from any formulation.

This can be seen too from his specific use of the word sign-post for example, contrast "a rule stands there like a sign-post" (S85) and, "what has the expression* of a rule - say a sign-post..." (S198).

*my emphasis.
i.e., a sign-post is no longer a candidate for a rule, but for the expression of a rule.

However, it is important to note here that although the notion of "rule" may have changed, the linguistic scope of the discussion of rules has not changed in this development. That is, whatever aspects of language were dealt with in the "negative critique" or rules remain those which are addressed by the "positive critique", i.e., by the notion of "rules" generated here that L.W. sees as helpful or illuminating: therefore the "positive critique" applies to every aspect of our discourse (see p. 6 on, and p. 32).

Two questions now present themselves. The first is, how does the notion of rules emerging from the "positive critique" escape the criticisms directed at the conception of rules dealt with in Chapter I (see p. 5)? The second is how might we see this notion of rules as providing an insightful and illuminating philosophical view of human discourse and activity? In order to answer these questions some articulation of L.W.'s notion of rules is required at this point.

At S83 we have a description of people playing a game that is not according to any rules that we might formulate (in the way that we might formulate the rules of basketball say). However on examination it can be seen that there are boundaries to this game. For example the people playing do not kick each other, they don't try to hurt each other. If someone did we could see the others responding by telling him to stop it or saying "that's not how to play". In the (rather common) case of a child who has not played often with other children and becomes overly aggressive in a rough-and-tumble type game we might say he/she hadn't learn to play yet, or to the child "You're not
playing properly." The child has not yet learnt the rules. Or we could imagine this being a group play-therapy session and when a child acted aggressively the therapist stopped the game and the group talked about it. This might appear at first like the other game but obviously "the rules by which they proceed" are somewhat different. In the case where a child became bewildered when the game stopped and kept trying to start again we could have a situation similar to that outlined in S185, i.e., the person seemed to have understood but at a certain point it is clear he did not. It seemed as if the child had grasped "the rules by which you proceed" in play-therapy, but it emerged he had not. We could also imagine a game where one person became aggressive and others responded till the game degenerated into a vicious competition. In this case we be inclined to say the rules had changed.

This is to touch on just one aspect of the game (the rules concerned with respecting other people, perhaps). Innumerable others could be considered, such as not holding on to the ball all the time, not kicking it away as far as you can, not trying to puncture it, and so on.

There are then, some very essential regularities to this game that can in no way be completely formulated. Nor do people learn to play like this by being "told how" (necessarily) - we could say it is part of their socialization. The child who had not played before with other children, no matter how much we explained the nature of the game to him, and no matter how much he himself was able to say what he should do, might still be unable to respond correctly - to "fit in".

What tempts us, perhaps to say (and correctly, given the sense of rules in Chapter 1) that this is not a rule following activity is the free flowing spontaneous nature of this game, as contrasted with
basketball. We are unable to draw up a list of basketball-type rules. Yet this is exactly one aspect of "the rules by which we proceed" in this game, and if a person does not grasp that and act accordingly then he does not grasp how to play. Thus, though the game is unbounded given the first notion of rules, it would seem to be a rule following activity in the second sense of rules.

If we consider another aspect of human activity, the use of the word red we can see that similar considerations apply. In trying to teach a child "red" (a rather unfamiliar situation) there is no way in which we could formulate "how to use the word red"; encapsulating not only all the various shades of color it can be correctly used to describe but also circumstances under which we wouldn't use it. For example (and again there are analogies with S185) we might find a child who seemed to be able to recognise and describe red perfectly well, but went on to say that the noise of buses is red, meeting people in the street is red, being hungry is red, staying up late is red. Under these circumstances I think we'd say that the child hadn't grasped how we use red ("the rule by which we proceed").

Notice, however, that if someone who used the word red much the same as usual in all other respects, insisted on describing certain musical sounds as red ("trumpets are red, cellos are blue") it would be hard to legislate whether he had grasped "red" or not. This is a
similar situation to our ball game, where one person always hangs on to
the ball a little too long, or always throws it to the same person (he's
not "playing quite right"). This brings out another aspect of rules -
it is not necessarily absolutely clear whether a person is following the
rule correctly or not. This is where something sounds "funny" or
"queer". The tendency to think that a rule must draw hard and fast
dichotomies is derived from the conception of rules outlined in Chapter I
where there is always a table or formula to appeal to, to resolve things
once and for all. In fact it can be internal to a rule that in its
use there are specific areas of possible use that are not sharply
defined, and this can be (for philosophy) a significant aspect of a rule.
(In a discussion of "rules" in Zettel, I.W. states "In philosophy it is
significant that such-and-such a sentence makes no sense; but also that
it sounds funny" S328). 8

Thus the first argument levelled against the first notion of
rules (see p. 5) through the analogy drawn between games and language,
would seem to have no application in the case of our second notion of
"rules" ("positive critique"). On the contrary, I think it can be seen
from the brief discussion so far that the second sense of rules provides
an insightful and illuminating way of looking at language (and also,
incidently, games).

8 This might lead to the notion that a philosophical investigation
that draws on the idea that human discourse is rule-like (as I have
argued L.W. does) is necessarily slack or vague, but this is not the
case. Rather it is necessary to probe our use of words (and particularly
a philosopher's use of words) to be absolutely clear about what kind of
expressions are "funny" such as "I know I am in pain" (see also S.
Cavell in "Knowing and Acknowledging").
"How am I able to obey a rule?"
-if this is not a question about causes, then it is about the justification for my following the rule in the way I do.

If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say, "This is simply what I do."
(P.I., S217)

"Simply what I do", is an apt description for our use of the word red, or counting, or playing the rough-and-tumble game. The activity is done unreflectively, without pause, yet it was something I had to learn and may have made mistakes with while learning. Contrast my use of the word "time" with my use of the words "fois" and "temps". When speaking French I still have to reflect on "fois" and "temps". I have to think - I have not yet grasped the rule.

Here we have fundamental contrast between the notion of rule criticised in Chapter I and the one that has emerged in this Chapter. The "rule" in Chapter I could be followed by referring to something - a table, a definition, or whatever. But with the notion of rule generated by the "positive critique" there is no reflection or thinking - "And to think one is obeying a rule is not to obey a rule" (S202). Grasping a rule requires that I just do so react.

This can be further brought out by the distinction between being causally determined and being logically determined.

In S217 "How am I able to obey a rule?" was not a question about causes. If it were then we could take a specific activity (counting say) and see how someone learnt it, and for each activity the causal determination is different. This is how we teach counting, which is similar in some respects to teaching such and such series, but completely different from how we learn colors, or to
follow sign-posts.

...let me ask this: what has the expression of a rule - say a sign-post - got to do with my actions? What sort of connection is there here? Well, perhaps this one: I have been trained to react to this sign in a particular way and now I do so react to it.

But that is only to give a causal connection; to tell how it has come about that we now go by the sign-post; not what going by the sign really consists in. (P.I., S193)

The nature of a causal determination then is clear, yet this does not deal with the original question; "what has the expression of a rule - say a sign-post - got to do with my actions?" For this question is about rules in general, or rules per se, rather than a specific rule. L.W. continues: "On the contrary: I have further indicated that a person goes by a sign-post only in so far as there exists a regular use of sign-posts, a custom."

Here in referring to "a regular use, a custom", L.W. is no longer talking about something that is specific to any particular case of following a rule, e.g., training to follow sign-posts. He is invoking, beyond specific causal determination, the possibility of a more general characteristic of rule-following, i.e., "a regular use, a custom" something that could be involved in any number of cases of rule following. Thus he precipitates the question - aside from the specific (causal determination aspect) of rule following - can anything of a more general nature be said about following a rule?  

\[9\] Here we are on the verge of, perhaps, the central aspect of "following a rule", but in the context of this paper I'm afraid I must abdicate from this discussion (see Introduction).
The notion of causal determination returns again at S220. At S217 the question of "How am I able to obey?" (a rule) was not about causes - and the answer was "This is simply what I do". The same kind of notion is evoked at S219 where L.W. says "when I obey a rule, I do not choose. I obey the rule blindly." The unreflective stress of "simply what I do" is emphasized even more here. He continues:

But what was the purpose of that symbolical proposition? It was supposed to bring into prominence a difference between being causally determined and being logically determined. (P.I., S220)

"Obey a rule blindly" is not an aspect of its causal determination. As we have seen a causal determination of obeying a rule - following a sign-post say - would amount to "being trained to react to this sign in a particular way", something specific to an activity we might characterize as following a rule - this kind of training for sign-posts, and this kind of training for counting. The appeal to "logical determination" would seem to concern itself with something about rules per se - something that is common to all cases of following rules (one might almost say a formal feature of rules). Furthermore "obeying the rule blindly" would be something that L.W. would seem to consider necessarily the case for following a rule. I can only reiterate, then, that for the conception of rules, L.W. is trying to articulate "following a rule" involves, necessarily, an unreflective attitude - it is something we 'simply' do.  

10 This, I think, establishes that their unreflective nature is a necessary condition of rules, which is sufficient for the following argument. However there may be many things I do unreflectively which are not cases of "following a rule". But this raises the whole question as to the nature of rules which is beyond the scope of this thesis (See Introduction).
Given this conception of rules then the argument offered by L.W. at S84 for "rules" in terms of the need for a rule for the application of a rule (see p. 52) can find no grip. The question of applying a rule (in this second conception of rules) does not arise. There is no room for applications, if I found myself applying a rule (as in a rule for using "fois" correctly) then I would not be following a rule in the sense of rule that has emerged in this Chapter. Yet this sense of rule would seem to provide an insightful and illuminating way of regarding certain human activities - such as our use of "red" (see p. 23). Both the spontaneous way in which I use the word and the fact that its use is unarbitrary and has a definite regularity, seem to be captured very succinctly by our second conception of rule.

At the end of the last Chapter it was noted that it was L.W.'s remark on the necessary regularity of language (see p. 12 and S207) that obliged him to offer some kind of account of this regularity. It should be clear from this Chapter that L.W. chose to articulate the nature of this regularity in terms of the concept of "rule" rather than "regularity". I think his reasons for this are clear. The word "rule" seems to capture the open-ended nature of our use of words in a way that "regularity" cannot. A "regularity of language" indicates something about our use of words post factum; a "rule of language" implies something which binds past usage to future usage - something that will be continuing (as my use of the word red will).

After his remark on the distinction between causal determination and logical determination, L.W. writes:

My symbolical expression was really a mythological description of the use of a rule. (P.I., S221)
This is a very curious expression and I take it to refer obliquely to the distinction I want to make in the next Chapter – the distinction between showing and saying. L.W. is perhaps chiding himself for attempting to say that which by its very nature can only be shown.
CHAPTER III

It was found in Chapter II that the notion of rules that L.W. invokes as illuminating our use of language involves the idea that these rules cannot be stated. The temptation that he was struggling against was that a rule could be formulated in an expression, i.e., could be stated or said. In contrast to this a notion of rules emerged which rendered the stating/saying of the rule per se impossible (see p. 20) as anything said would be merely an expression of the rule.

Yet clearly rules are taught, as it was through various examples involving teaching that the second notion of rule was introduced (see S143 on). So rules are communicated in the interaction of the teaching process, and are also (if we accept that our discourse is a rule-like activity) manifested in our speech and interaction. The rule then is shown in these things.

Given that in any specific case a rule can only be shown, how could we expect any general formulation of the notion of "following a rule"?

"I cannot describe how (in general) to employ rules, except by teaching you, by training you to employ rules. (Zettel, S318)"

In specific cases of following a rule there is a training - a showing that is appropriate to counting or sign-posts for example. In teaching the activity - in our showing - the only resource we have to draw on is the activity itself - "this is how we count"; and possibly contrast it with other similar rule-following activities - "this is how we add and subtract". What other resources could we have for showing the nature,
of rule-following in general? All we have is the discussion of specific rule-following activities. Yet clearly the activity of showing in the case of a specific rule following activity (i.e., training) is very different from a case of showing the general nature of rule-following, i.e., showing in a philosophical sense. In the former case the activity derives its sense from the ignorance of the trainee, it is a process of teaching; in the latter we are familiar with the activities being discussed and, one can assume, competent in them. Rather in the latter case showing derives its sense from the attempt to address a philosophical temptation.

The statement that in following a rule "I no longer have any choice" (S219), is a description that "only made sense if it was to be understood symbolically" (S219) i.e., it is a "mythological description" (S221). L.W. is trying to say something that can only be shown - in our use of the word red, in counting, in following sign-posts, in our rough-and-tumble game. The general nature of rules (something I have called nature of rules per se), cannot be described, it can only be shown. Here then L.W. is "hitting his head against the limits of language" (S119), but obviously he understands this in a way he thinks other philosophers do not.

The distinction between showing and saying is clearly in evidence in the *Tractatus* (see for example 5.62 and 6.36) and there it is very much connected with "rules". What cannot be said but only shown is the logic of our language - *logik der Abbildung* - and its limits. (see Appendix A)

However we would search P.L. in vain for an explicit formulation of the showing/saying distinction. And this is for the obvious paradoxical reason - the showing/saying distinction is
something that can only be shown. That is, in establishing that something can only be shown it would be obviously contradictory to say what it was that could only be shown. The distinction between saying and showing then is itself something that can only be shown.

Furthermore, the showing/saying distinction here is founded on the conception of rules that L.W. has generated (see Chapter II). It is because language is a rule-following activity that we can only show its workings as a rule-following activity; it is its nature as rule-following activity that demands our process of elucidation be one of showing - because rules can only be shown. In his discussion of rules then L.W. has not only elucidated an illuminating and insightful conception of rules; he has corroborated his method of elucidation - the process of showing.

I would now like to acknowledge the indulgence of those readers who suspended judgement on the rather opaque remarks made in the Introduction to this paper. I can now ask you to reflect whether the bald statement of the conclusions I have arrived at here would have been at all illuminating unless preceded by the process\(^1\) - the development of the notion of "rules" - that brought them about. It is only the process of developing this notion of rules that makes it possible to answer the question "What is it that showing might amount to?" The answer to that is, that this paper (I hope) has been (among other things) an attempt to elucidate an example of showing in the P.L.

To review briefly: various philosophical mistakes were

\(^1\)"The search says more than the discovery,..."
considered and then given a general characterization in terms of "rules" (Chapter I). Then (Chapter II) various examples of teaching series were considered and through them the distinction between a rule and its expression or interpretation was made. Through this second notion of rules we arrived at a philosophical understanding of use of language - we are able to account for the regularity of language. Furthermore we can see what kind of enterprise is undertaken in the philosophical investigation of language, an enterprise of showing.

Paradoxically this procedure gains its sense from addressing a philosophical temptation and its manifestation in various philosophical manoeuvres. The insights provided by the process outlined in Chapters I and II (the articulation of the notion of showing, for example) would not have been achieved if these philosophical manoeuvres had merely been dealt with as mistakes to be corrected. Rather than dealing with them as errors to be pointed out and then discarded in the philosophical process L.W. tried to understand why someone might be led to consider such a view - the temptation behind the mistake. More will be said about this notion of temptation below. Again - "One can never take too much care in handling philosophical mistakes as they contain so much truth" (see p. 12).

The interpretation of L.W.'s view of philosophy presented here can be contrasted to both the implicit and explicit conception of his philosophy found in Wittgenstein's Conception of Philosophy by K.T. Fann.²

²The reason I have chosen Fann's discussion here is because it is close enough, while differing from my own account (of Wittgenstein's conception of philosophy), to bring out various fine points (concerning "temptation" and "confession") by contrast.
In the last chapter of his book Fann states:

The Investigations is completely unsystematic in both its form and its content. Unlike most earlier or later philosophical writings in the Western tradition, it consists of loosely connected remarks. (p. 105)

Fann goes on to say that L.W. is involved in "confession and persuasion", and develops an analogy between L.W. and Zen - that they are both directed towards "enlightenment". He also invokes the well known analogies of L.W.'s philosophy with therapy, and the dissolution of philosophical problems with "cure". We are also reminded of the very personal expression of L.W.'s later philosophy and the personal difficulties he experienced.

Clearly the interpretation in Chapter I and II of this paper is very much at odds with Fann's claim to the arbitrary loosely connected nature of P.I. Fann is in a sense right, there is no ordering in the P.I. derived from the material alone - no ordering of concepts or development of theories. This is a truism of Wittgenstein scholarship.

But this does not exhaust the possibilities of structure. I hope to have shown that there is a structure (at least as far as the discussion of rules is concerned) derived from the need to recognize and acknowledge temptations, a need that (for L.W.) necessarily precedes "dissolution". This is a process that is completely pre-empted by the dismissal of mistakes as irrelevant to the philosophic process; it involves the recognition of what kind of problem someone is struggling with and its possible relation to other problems (i.e., the "search for

3 See Footnote 1, Chapter I.
rules", see p. 9 on), and the acknowledgement of the temptations behind the attempt to provide rules. Here we hit on a fundamental aspect of L.W.'s philosophy. As against refuting mistakes L.W. attempts to understand temptations - "what leads us to", "the picture that held us captive".

Fann's invocation of the notion of temptation in P.I. (after Cavell) coupled with his references to L.W.'s personal efforts seems to me to obscure this essential point. The notions of temptation and confession are not necessarily concerned with some personal (one almost might say existential) angst in doing philosophy. That may well have been the case for L.W., but what is important here is that "temptation" points to an attempt to understand what could superficially be described as blunders and mistakes in a deeper way. "Confession" here is not the outpouring of one individual to another. It involves a relationship between two subjects by which one acknowledges the same impulse in oneself as one finds in others...the impulse, say, to postulate a calculus of rules for language. The P.I. is a confession only in as much as it is the confession of a man who looks to himself in order to understand others, "what can lead us (and did lead me) to think that..." (S81). (I think this one reason why L.W. thought the publishing of P.I. in conjunction with Tractatus so à propos.) Here the notion of therapy in its Freudian sense (one in which L.W. understood it) is appropriate. The analyst can only successfully analyse the analysand in as much as he too has been analysed (training analysis) and so has been forced to recognize the same kind of resistances and unconscious desires in himself as emerge in the analysand's analysis. And this is not merely a gesture, it is
the way of excavating the truth contained in a philosophical mistake -
the way in which, in this case, the distinction between showing and
and saying was arrived at.
CONCLUSION

Much has been written recently on the similarities and connections that exist between the Tractatus and P.I. (see for example, "The Unity of the Philosophy of Wittgenstein", P. Winch or "Use and Reference", H. Isiguro, both in Studies in the Philosophy of Wittgenstein, ed. P. Winch). While acknowledging this aspect of L.W.'s philosophy there is still, even to the casual reader, an enormous difference in style between the text of the Tractatus and the text of the P.I. This can be accounted for, I think, at least to some degree by the very different notions of philosophical method that exist between the early and the late Wittgenstein.

In the Tractatus there is an exposition of a philosophical view of language in the process of which other, conflicting, views are criticised and refuted (see, for example, 5.452, 5.252, 4.442). Also consider:

The correct method in philosophy would really be the following: to say nothing except what can be said, i.e., propositions of natural science - i.e., something that has nothing to do with philosophy - and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning to certain signs in his propositions. (Tractatus, 6.53)

The contrast here, between the notion of philosophical method presented in Chapter III and this is absolute. The concepts of temptation and confession have no room here whatsoever, philosophy is merely the objective assessments of statements and the demonstration
of their wrongness.

There is always the temptation, especially in teaching, to deal with another's bewilderment - their inability to extricate themselves from "a picture of language" say - by giving them the refutation of that position, and if they still are bewildered repeating it, and going on. How often do we hear in teaching, either explicitly or implicitly, "but it's so simple, why don't you understand?". (The inability to understand, whether it be Socrates of the Sophists, or Moore of the Idealists, has often been an essential aspect of the history of philosophy.)

This paper then is presented as a plea for a particular way of doing philosophy.
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APPENDIX A

The picture theory of meaning was put forward by Wittgenstein in the Tractatus to deal with the problem of the connection between language and reality. One way in which this problem arises for example, is in the notion of falsity. This could briefly be expressed thus: if we say that propositions have meaning because they describe reality, because they (in some sense) correspond to reality - then false statements describe nothing in reality. How can they mean anything, or how can there be false propositions at all?

The German word in the Tractatus translated either as "depicting" or "picturing" is "Abbildung".

The word "Abbildung" was used in German Mathematical literature (which Wittgenstein was familiar with) to indicate a relation between two mathematical systems, e.g., Euclidean and non-Euclidean geometries, two variable algebra and two coordinate geometry. An algebraic formula and the curve that is mapped would be the "Bild" of each other. That all algebraic formula could be mapped on the co-ordinates would mean that one system was the "Bild" of the other.

For example:

![Diagram showing the relationship between x and y axes with the formula y=x+1 as an example of a Cartesian coordinate mapping.]

The formula "y=x+1" if the "Bild" of the Cartesian coordinate mapping.
The method by which we map the formula on the coordinate would amount to the "Logik der Abbildung" of the Tractatus. It can be seen that the formula $y^2 = x + 1$ is not mapped on the coordinate diagram. In the Tractatus, Wittgenstein says that "reality is compared with propositions" (4.05). This is to see if they are true or false. By our analogy $y^2 = x + 1$ would not be true, it is not mapped.

But we can certainly see that there exists the possibility of mapping $y^2 = x + 1$ on the coordinates. And, if one has grasped the method of mapping formula, this can be seen just from the formula itself. Further, that something is a sensible (two variable) algebraic formula (unlike $y + z = 0 x + 1 + 3$ say) is integral with the possibility of being mapped. This is essential to the idea of the one system being the "bild" of the other, it constitutes the "inner similarity" (4.0141).

Using this analogously, the fact that a proposition follows the "Logik der Abbildung" can be seen from the proposition itself - that it makes sense. That it makes sense amounts to the same thing as the possibility of it picturing something, a fact.

Extending the analogy further.

Any mapping of a formula passes through a series of points on the coordinates. These points are analogous to the Objects of the Tractatus, the points make up the whole coordinate area just as Objects are the substance of the world (2.021). Any point in a formula can be designated by $(Ax, By)$ where $A$ and $B$ are any real numbers (within the limits of the coordinate) - this is the Name which names the Object. The Names are strung together by the proposition* just as

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*This must be an elementary proposition, see below.
\( (A_1 \cdot x, B_1 \cdot x) \), \( (A_2 \cdot x, B_2 \cdot x) \) are strung together by the formula.

So for any formula even if its mapping is not sketched on the coordinates the points it links must exist on the coordinate area. A proposition is false when it links up the Names in a concatenation not reflected by the concatenation of Objects (named by the Names) in the world.

It should be said that the proposition discussed so far must be truth-functionally simple (that is to say elementary) propositions. This can be seen from 4.0312 'My fundamental idea is that the 'logical constants' are not representatives". That is, they picture nothing. In an elementary proposition Names are linked together "like links in a chain", and the proposition gets its sense from this concatenation of Names. But there is no particle in the proposition which does not represent something in reality, all the Names name an Object. Elementary propositions are truth functionally combined, using "logical constants", to make up complex propositions. But it is the elementary propositions which stand in the direct picturing relation to reality.