SINGLE FAULT LOCATION METHODS
APPLIED TO BRAKE PIPE MODELS

Venkateshwer Aula

A Thesis
in the
Faculty of Engineering

Presented in Partial Fulfilment of the Requirements for
the Degree of Master of Engineering at
Concordia University
Montreal, Quebec

March 1980

© Venkateshwer Aula, March 1980
SINGLE FAULT LOCATION METHODS.
APPLIED TO BRAKE PIPE MODELS

VENKATESHWAR AULA

ABSTRACT

Whereas there exists a procedure for detecting excessive leakage in a brakepipe, the methods employed in actually locating the leaks are still very crude. Such methods are invariably labour-intensive and time consuming, requiring railway personnel to walk alongside the train to hear for audible leaks either by unaided ear, or sometimes with a sonic meter.

This thesis proposes, analyzes and tests some methods of locating single faults, all based on a limited number of pressure measurements (possibly localized at the front-end or the rear end of the train). The methods are called the Difference method, the Ratio method and the Transformation method. They are developed by treating a brakepipe with leakage as linear electrical ladder network and nonlinear pneumatic network consisting of in-line and shunt fluid resistances. The validity of the analytical methods are confirmed by experiment on a scaled model of the brakepipe.
ACKNOWLEDGMENT

The author is very grateful to his thesis supervisors Dr. S. Katz and Dr. R. M. H. Cheng for their excellent guidance, encouragement and moral support throughout this research.

He thanks his colleagues in the Fluid Control Centre for providing a conducive environment for his research, giving moral support, and also making his stay here memorable.

He would like to acknowledge the help of Mr. Albert Carbone for providing him with efficient plotting routines and Mr. Bernard Trudel for his assistance during the summer of 1978.

He also wishes to acknowledge Concordia University for the facilities provided through the Fluid Control Centre, where this work was conducted and Mr. John Elliot for his assistance during the experimentation.

This study has been supported by a research grant from the National Research Council of Canada.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td>xi</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 HISTORICAL BACKGROUND OF AIRBRAKES IN FREIGHT TRAIN</td>
<td>1</td>
</tr>
<tr>
<td>1.2 DESCRIPTION OF THE BRAKE PIPE</td>
<td>2</td>
</tr>
<tr>
<td>1.3 BRIEF DESCRIPTION OF AIRBRAKE OPERATION</td>
<td>3</td>
</tr>
<tr>
<td>1.4 EFFECTS OF BRAKE PIPE LEAKAGE</td>
<td>4</td>
</tr>
<tr>
<td>1.5 REVIEW OF PREVIOUS WORK</td>
<td>6</td>
</tr>
<tr>
<td>1.6 SCOPE OF THIS THESIS</td>
<td>8</td>
</tr>
<tr>
<td>2. SINGLE FAULT DETECTION IN LINEAR MODEL</td>
<td>11</td>
</tr>
<tr>
<td>2.1 LINEAR MODEL</td>
<td>11</td>
</tr>
<tr>
<td>2.2 ANALYSIS OF THE LINEAR MODEL (DIRECT METHOD)</td>
<td>11</td>
</tr>
<tr>
<td>2.2.1 SENSITIVITY ANALYSIS</td>
<td>18</td>
</tr>
<tr>
<td>2.3 OTHER SINGLE FAULT DETECTION METHODS</td>
<td>21</td>
</tr>
<tr>
<td>2.3.1 DIFFERENCE METHOD</td>
<td>21</td>
</tr>
<tr>
<td>2.3.2 RATIO METHOD</td>
<td>22</td>
</tr>
<tr>
<td>2.4 EXPERIMENTS ON THE LINEAR MODEL</td>
<td>23</td>
</tr>
<tr>
<td>2.4.1 SIMULATION ON THE COMPUTER</td>
<td>24</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>Page</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>2.4.2</td>
<td>EXPERIMENTS ON THE ELECTRICAL MODEL</td>
</tr>
<tr>
<td>2.4.2.1</td>
<td>DIRECT METHOD</td>
</tr>
<tr>
<td>2.4.2.2</td>
<td>DIFFERENCE METHOD</td>
</tr>
<tr>
<td>2.4.2.3</td>
<td>RATIO METHOD</td>
</tr>
<tr>
<td>3.</td>
<td>SINGLE FAULT DETECTION IN NONLINEAR MODEL</td>
</tr>
<tr>
<td>3.1</td>
<td>NONLINEAR BRAKEPIPE MODEL</td>
</tr>
<tr>
<td>3.2</td>
<td>ANALYSIS OF THE MODEL</td>
</tr>
<tr>
<td>3.3</td>
<td>SINGLE FAULT DETECTION METHODS</td>
</tr>
<tr>
<td>3.3.1</td>
<td>DIFFERENCE METHOD</td>
</tr>
<tr>
<td>3.3.2</td>
<td>RATIO METHOD</td>
</tr>
<tr>
<td>3.3.3</td>
<td>TRANSFORMATION METHOD</td>
</tr>
<tr>
<td>3.4</td>
<td>EXPERIMENTS</td>
</tr>
<tr>
<td>3.4.1</td>
<td>DESCRIPTION OF THE TEST SETUP</td>
</tr>
<tr>
<td>3.4.2</td>
<td>TEST PROCEDURE AND TEST DATA</td>
</tr>
<tr>
<td>3.5</td>
<td>EXPERIMENTAL RESULTS OBTAINED BY APPLYING SINGLE FAULT DETECTION METHODS</td>
</tr>
<tr>
<td>3.5.1</td>
<td>DIFFERENCE METHOD</td>
</tr>
<tr>
<td>3.5.2</td>
<td>RATIO METHOD</td>
</tr>
<tr>
<td>3.5.3</td>
<td>TRANSFORMATION METHOD</td>
</tr>
<tr>
<td>4.</td>
<td>CONCLUSION</td>
</tr>
<tr>
<td>4.1</td>
<td>SUMMARY AND DISCUSSION</td>
</tr>
<tr>
<td>4.2</td>
<td>SUGGESTION FOR FURTHER WORK</td>
</tr>
<tr>
<td>REFERENCES</td>
<td></td>
</tr>
<tr>
<td>FIGURES</td>
<td></td>
</tr>
<tr>
<td>APPÉNDICES</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Schematic of the brakepipe system</td>
<td>84</td>
</tr>
<tr>
<td>2.</td>
<td>Ladder Network</td>
<td>85</td>
</tr>
<tr>
<td>3.</td>
<td>Separating the ladder network into two sections</td>
<td>86</td>
</tr>
<tr>
<td>4.</td>
<td>Relation between first and last nodal potentials as a function of fault location</td>
<td>87</td>
</tr>
<tr>
<td>5.</td>
<td>Illustration of the fact that equation 12 is not function of potential measurements after the fault $(e_0/e_0 (v_s) e_2/e_0)$</td>
<td>88</td>
</tr>
<tr>
<td>6.</td>
<td>Illustration of the fact that equation 12 is not function of potential measurements after the fault $(e_0/e_0 (v_s) e_7/e_0)$</td>
<td>89</td>
</tr>
<tr>
<td>7.</td>
<td>Effect of measurement position on predicted fault location</td>
<td>90</td>
</tr>
<tr>
<td>8.</td>
<td>Sensitivity of fault prediction as a function of measurement position</td>
<td>91</td>
</tr>
<tr>
<td>9.</td>
<td>Sensitivity of fault prediction as a function of the number of nodes with fault magnitude as parameter</td>
<td>92</td>
</tr>
<tr>
<td>10.</td>
<td>Sensitivity of fault prediction as a function of resistance ratio with fault magnitude as a parameter</td>
<td>93</td>
</tr>
<tr>
<td>11.</td>
<td>Simulated results of the difference method when applied to linear ladder network for $N = 10$ and $R_m = 100$ ohms</td>
<td>94</td>
</tr>
<tr>
<td>12.</td>
<td>Simulated results of the difference method when applied to linear ladder network for $N = 20$ and $R_m = 100$ ohms</td>
<td>95</td>
</tr>
<tr>
<td>13.</td>
<td>Simulated results of the difference method when applied to linear ladder network for $N = 50$ and $R_m = 100$ ohms</td>
<td>96</td>
</tr>
<tr>
<td>14.</td>
<td>Simulated results of the difference method when applied to linear ladder network for $N = 10$ and $R_m = 333$ ohms</td>
<td>97</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>15. Simulated results of the difference method when applied to linear ladder network for $N = 20$ and $R_m = 333$ ohms</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>16. Simulated results of the difference method when applied to linear ladder network for $N = 50$ and $R_m = 333$ ohms</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>17. Linear model of the brakepipe for the ratio method</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>18. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 10$ and $R_m = 100$ ohms</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>19. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 20$ and $R_m = 100$ ohms</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>20. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 50$ and $R_m = 100$ ohms</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>21. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 10$ and $R_m = 333$ ohms</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>22. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 50$ and $R_m = 333$ ohms</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>23. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 10$ and $R_m = 333$ ohms</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>24. Electrical ladder network model</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>25. Comparison of theoretical fault prediction and computer solution with simulated error</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>26. Comparison of theoretical fault prediction and Electrical model ($N = 20$, $R_m = 100$ ohms)</td>
<td>109</td>
<td></td>
</tr>
<tr>
<td>27. Comparison of theoretical fault prediction with that from the Electrical model ($N = 20$, $R_m 100$ ohms)</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>28. Comparison of theoretical fault prediction with that from the Electrical model ($N = 50$, $R_m = 100$ ohms)</td>
<td>111</td>
<td></td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>29. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 10, R_m = 100 \text{ ohms}) )</td>
<td>112</td>
<td></td>
</tr>
<tr>
<td>30. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 20, R_m = 100 \text{ ohms}) )</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>31. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 50, R_m = 100 \text{ ohms}) )</td>
<td>114</td>
<td></td>
</tr>
<tr>
<td>32. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 10, R_m = 333 \text{ ohms}) )</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>33. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 20, R_m = 333 \text{ ohms}) )</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>34. Experimental fault prediction by the Difference method when applied to linear brakepipe model ( (N = 50, R_m = 333 \text{ ohms}) )</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>35. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 10, R_m = 100 \text{ ohms}) )</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>36. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 20, R_m = 100 \text{ ohms}) )</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>37. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 50, R_m = 100 \text{ ohms}) )</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>38. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 10, R_m = 333 \text{ ohms}) )</td>
<td>121</td>
<td></td>
</tr>
<tr>
<td>39. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 20, R_m = 333 \text{ ohms}) )</td>
<td>122</td>
<td></td>
</tr>
<tr>
<td>40. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ( (N = 50, R_m = 333 \text{ ohms}) )</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>41. Nonlinear ladder network model for brakepipe leakage</td>
<td>124</td>
<td></td>
</tr>
<tr>
<td>42. Simulated results of the Difference method when applied to Nonlinear brakepipe model ( (N = 10, R_F/R_R = 0.465) )</td>
<td>125</td>
<td></td>
</tr>
<tr>
<td>43. Simulated results of the Difference method when applied to Nonlinear brakepipe model ( (N = 10, R_F/R_R = 0.3) )</td>
<td>126</td>
<td></td>
</tr>
<tr>
<td>44. Modified model for the Ratio method</td>
<td>127</td>
<td></td>
</tr>
</tbody>
</table>
45. Simulated results of the Ratio method when applied to Nonlinear brakepipe model \( (N = 10, \frac{R_F}{R_R} = 0.465) \) .......................... 128

46. Simulated results of the Ratio method when applied to Nonlinear brakepipe model \( (N = 10, \frac{R_F}{R_R} = 0.3) \) .......................... 129

47. Simulated results of the Transformation method when applied to Nonlinear brakepipe model \( (N = 10, \frac{R_F}{R_R} = 0.465) \) .......................... 130

48. Simulated results of the Transformation method when applied to Nonlinear brakepipe model \( (N = 10, \frac{R_F}{R_R} = 0.3) \) .......................... 131

49. Picture of the scaled-down brakepipe model along with measuring equipment .................................................. 132

50. Experimental pressure distribution in the scale model of the brakepipe with \( \frac{R_F}{R_R} = 0.465 \) .......................................................... 133

51. Experimental pressure distribution in the scale model of the brakepipe with \( \frac{R_F}{R_R} = 0.3 \) .......................................................... 134

52. Experimental fault prediction by the Difference method with \( \frac{R_F}{R_R} = 0.465 \) .......................................................... 135

53. Experimental fault prediction by the Difference method with \( \frac{R_F}{R_R} = 0.3 \) .......................................................... 136

54. Experimental fault prediction by the Ratio method with \( \frac{R_F}{R_R} = 0.465 \) .......................................................... 137

55. Experimental fault prediction by the Ratio method with \( \frac{R_F}{R_R} = 0.3 \) .......................................................... 138

56. Experimental fault prediction by the Transformation method with \( \frac{R_F}{R_R} = 0.465 \) .......................................................... 139
NOMENCLATURE

$P_0$  
source

$P_n$  
last node pressure

$R$  
linear series resistor

$R_s$  
linear shunt resistor

$R_m$  
faulty resistor

$e_0$  
driving potential

$i_k$  
in-line current $i$ at node $k$

$k$  
any arbitrary node

$i_n$  
current in last node $n$

$e_{n+1}$  
potential at node $n+1$

$e_n$  
potential at node $n$

$e_k$  
potential at any arbitrary node $k$

$K$  
a constant evaluated from boundary conditions

$b$  
a function of $R$ and $R_s$ and is equal to

$cosh^{-1}[1+R/2R_s]$  

$i_s$  
current at source $s$

$e_m$  
is the potential at the faulty resistor located at $m$

$m$  
is the location of the faulty resistor

$K_R$  
constant refers to the rear part, portion after the fault
the last node
is a relation which is equal to bn+b/2
constant for the forward part i.e., portion
before the fault

\[ i_{m-1} \]
\[ i_m \]
\[ e_{m-1} \]
\[ e_{m+1} \]
\[ A \]
\[ e_{10}/e_0 \]
\[ R_m/R_s \]
\[ E_1 \]
\[ E_n \]
\[ F_k \]
\[ m_p \]
\[ S \]
\[ dm/dE_k \]
\[ \Delta m \]
\[ \Delta m E_k/E_k \]

\[ 1+R/R_m \frac{\cosh(x-bm+b)}{\cosh(x-bm)} \]

ratio of potential at node 1 to that of the
source i.e., 0

ratio of potential at node 10 to that of the
source i.e., 0

ratio of resistance of faulty shunt resistor
to unfaulited shunt resistor

is normalized quantity equal to \( e_1/e_0 \)

is normalized quantity equal to \( e_n/e_0 \)

is normalized quantity equal to \( e_k/e_0 \)
predicted fault position

sensitivity function = \( dm/dE_k E_k \)

change in fault position \( m \) for a small change
in normalized potential \( E_k \)

\[ \Delta m = m_p - m \]

is the fault position error

is the measuring error
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R/R_S$</td>
<td>ratio of the resistance of linear series resistor to linear shunt resistor</td>
</tr>
<tr>
<td>$e_0, e_1, \ldots, e_n$</td>
<td>is the unfaulted potential distribution</td>
</tr>
<tr>
<td>$e_0', e_1', \ldots, e_n'$</td>
<td>is the faulted potential distribution</td>
</tr>
<tr>
<td>$i_{k-1}$</td>
<td>potential at $k-1^{th}$ node</td>
</tr>
<tr>
<td>$i'_{k-1}$</td>
<td>current at $k-1^{th}$ node</td>
</tr>
<tr>
<td>$R_{s,k}$</td>
<td>linear shunt resistor at node $k$</td>
</tr>
<tr>
<td>$\Delta E/E$</td>
<td>measurement error</td>
</tr>
<tr>
<td>$P_n$</td>
<td>absolute pressure at $n^{th}$ node</td>
</tr>
<tr>
<td>$P_i$</td>
<td>absolute pressure at $i^{th}$ node</td>
</tr>
<tr>
<td>$P_0$</td>
<td>absolute pressure at $0^{th}$ node, source</td>
</tr>
<tr>
<td>$P_{i+1}$</td>
<td>absolute pressure at the $i+1^{th}$ node</td>
</tr>
<tr>
<td>$m_i$</td>
<td>is the flow at the $i^{th}$ node</td>
</tr>
<tr>
<td>$K_F$</td>
<td>constant characterizing flow resistance</td>
</tr>
<tr>
<td>$m_{L_i}$</td>
<td>leakage mass flow in the $i^{th}$ car</td>
</tr>
<tr>
<td>$j$</td>
<td>summing index</td>
</tr>
<tr>
<td>$m_{L_j}$</td>
<td>leakage mass flow in the $j^{th}$ car</td>
</tr>
<tr>
<td>$P_{i-1}$</td>
<td>absolute pressure at $i-1^{th}$ node</td>
</tr>
<tr>
<td>$R_i$</td>
<td>linear shunt resistance at node $i$</td>
</tr>
<tr>
<td>$P_j$</td>
<td>absolute pressure node $j$</td>
</tr>
<tr>
<td>$R_j$</td>
<td>linear shunt resistance at node $j$</td>
</tr>
<tr>
<td>$B_i$</td>
<td>ratio of adjoining car pressures ($= P_{i-1}/P_i$)</td>
</tr>
<tr>
<td>$R_R$</td>
<td>linear reference resistance, kPa.sec/kg.m²</td>
</tr>
<tr>
<td>$R_F$</td>
<td>faulty shunt resistor in transformed section</td>
</tr>
<tr>
<td>$R_F/R_R$</td>
<td>ratio of the resistance of the fault to the resistance of the reference resistor in transformed section</td>
</tr>
</tbody>
</table>
absolute pressure at the car $I$ in transformed section

total number of cars in the transformed network

position of the fault in transformation method

a dimensionless ratio

fault position in the transformed section

fault in the brakepipe model

fault in the transformed section
CHAPTER 1
INTRODUCTION

1.1 HISTORICAL BACKGROUND OF AIR BRAKES IN FREIGHT TRAINS

Leakage and leakage detection in pneumatic brakepipes of long trains are a major concern in the railroad industry. Excessive leakage affects the safety of the train by transmitting false signals to the brake cylinder as well as overloading the compressor units in the locomotives. Thus excessive brakepipe leakage is one of the major contributors to poor train handling, high cost of operating and maintaining air compressors on locomotives and production of shocks due to slack action during the ordinary service braking of long trains.

Since the inception of compressed air brakes of the automatic air brake type in 1870's, numerous changes have taken place in valve equipment, auxiliary systems and pipe fittings in the air brake although essentially the system has remained the same.

Tracing back the air brake history briefly, automatic air brakes were first introduced in freight cars in 1870's. At that time, the air brake consisted of a compressor unit, braking cylinders and brake shoes. In 1886 the automatic air brake was provided with an emergency feature, the H-triple valve equipment. Improvements in the air brake equipment were made along with the introduction of the K-type triple valve. The
brakepipe, an important feature of the airbrake also underwent many changes. Because of the leakage resulting from broken pipes which had developed fractures, in the threaded area, threadless fitting employing a rubber compression grip and seal were developed. Although this virtually eliminated pipe breakage, it created leakage problems. This problem was more acute in cold weather due to insufficient tightening of gland nuts. This has led to the use of welded brakepipe and welded flanged fittings which has reduced leakage to some extent.

As the need for bigger and heavier cars and longer trains increased, a more efficient and dependable brake was required. This culminated in the development of "AB" type brake. An improved "ABD" type brake was introduced in 1965. Since then a number of modifications have been made to enhance the dependability and efficiency of air brakes.

1.2 DESCRIPTION OF THE BRAKEPIPE

Each car of the train is equipped with a 3.175×10⁻⁴ m diameter brakepipe and separate braking equipment. The pipe is not always straight through its entire length and generally has at least one cross-over in each car.

The brakepipe (also called the train line) plays a vital dual role of being the alimentary canal and the nerve-line of the air brake system. As the alimentary canal, it supplies the necessary fluid power in the form of pressurized air, to the control valves and brake cylinders at each and every car and as the nerve-line, it transmits the braking signal from the
locomotive to the individual cars in correct magnitudes and proper timing.

The brakepipe is made up of sections running underneath each car, with adjacent sections connected by flexible hoses and couplings. At some point or points along each section of the brakepipe, control valves (the AB-the ABD valves, etc.), reduction relay valves and quick service valves are installed by using tee-branch pipes. As a result brakepipe leakage is unavoidable and primarily occurs at these locations. Also, it has been estimated [1] that, for a typical freight car, there exist some 100 joints and seals in the various control valves and braking cylinder arrangement. As a result of this quantity and complex of "plumbing", leakage becomes an unavoidable feature in train operation and train handling, something which in practice cannot be totally estimated but which has to be controlled so that it can be kept within established limits.

1.3 BRIEF DESCRIPTION OF AIR BRAKE OPERATION

A schematic illustration of a train air brake system is shown in figure (1).

For service application, the compressor provided in the locomotive units charges the air brake system with compressed air when the locomotive brake valve is in release position. Compressed air is stored in reservoirs on each car as directed by the control valve in each car. The car reservoir has two separate sections. One section is called the auxiliary reservoir volume, and the other is called the emergency reservoir
volume. To activate the brakes, the brakepipe pressure is reduced at a service rate, when the control valve on each car responds by directing air to flow from the auxiliary reservoir to the brake cylinder. The pressure thus acting on the brake cylinder piston creates a force which acts through a system of levers and rods to press the brake shoe against the wheels, and thus creates the retarding force for braking. The amount of brakepipe reduction determines the relative amount of brake cylinder pressure (ranging from a minimum of approximately 68 kPa to full service i.e., 552 kPa to 758 kPa) which equalizes the pressure between cylinder and auxiliary reservoir.

To release the brakes, the brakepipe pressure is increased by placing the locomotive brakepipe valve in "release" position. This will cause the control valve on each vehicle to again assume release position, exhausting the brake cylinder pressure and recharging the reservoir.

1.4 EFFECTS OF BRAKEPIPE LEAKAGE

The effects of brakepipe leakage are well documented in [1], [2], [3].

The brakepipe leakage tends to accentuate the difference in pressure between the front and rear end of the train which is called "pressure gradient". The brakepipe leakage also makes it longer to fully charge or recharge the system thereby shortening the brakepipe exhaust time. When excess leakage has developed within a certain section of the brakepipe, faulty signals may be created thereby causing undesirable braking action.

In view of these and other effects of leakage in brakepipe, a
leakage test is mandatory at the train yards to ascertain that the leakage stays below the maximum established by regulations. However, whereas such a procedure or test may from time to time establish that leakage of the given train is excessive (that is, above the established limits), there is no systematic method available to help locate where such excessive leakage actually exists along the brakepipe. The present practice of engaging railroad workers to walk along the entire length of the train looking for (or rather listening) for the excessive leak(s) can at best be described as tedious and time consuming. Even the occasional use of a sonic meter does not alter the fact that there is a need for a quick and efficient method to locate excessive leakage when it exists. This should be based on a good analytical understanding of the interplay between pressure gradient in a brakepipe and the leakage in question (in magnitude, location and distribution).

Based on this understanding a method should evolve which relies on a small number of measurements preferably localized to eliminate necessity of travelling up and down the train, sometimes for hours and even days on end in severe climatic and environmental conditions.

Before proceeding any further, it is necessary to define some terminology that will be used in the rest of the thesis. A no fault brakepipe is one which has an acceptable amount of leakage, evenly distributed through every section of the brakepipe (on every car). This is represented by each section having a leakage hole of an identical nominal area. A single fault is said to exist in the brakepipe when one of these holes assumes an area which is significantly different from the
nominal value. For practical reasons, this is taken to be significantly larger.

1.5 REVIEW OF PREVIOUS WORK

Although there exist two different methods, namely "leakage flow" and "leakage drop" to measure the brakepipe leakage on a single small car, no definite methods are available to locate the excessive leaks in a multicar train.

In the former method the flow supplied from the locomotive is measured by the pressure drop across an orifice plate between the locomotive and brakepipe. This measures the "leakage flow" directly. In the latter method the time rate of pressure change in the brakepipe is measured with all valves closed. The pressure drop that occurs in one minute is the actual measurement. This is a dynamic measurement and is termed "leakage drop". [1]

However, substantial work has been done in fault detection and analysis in electrical networks. Many of these advances are summarized in the proceedings of the state-of-the-art symposium edited by Saeks and Liberty [4]. In this volume, Rault, Garzia and Bedrosian [5] provide a bibliography of 193 papers that deal with fault detection and location in analog circuits. Also Berkowitz [6], Shekel [7], Gefferth [8] and Trick and Chien [9] have made significant contributions towards fault diagnosis.

Berkowitz [6] has derived conditions for single-element-kind network solvability. If those conditions were applied to ladder networks,
the network could be specified with potential measurements on each node. This presumes that there is one accessible node and other nodes are partially accessible.

For a single fault in a network Shekel [7] and Geffert [8] have shown that it may be located by formulating two network functions. For a ladder network this means that the fault may be isolated with one accessible node and two partially accessible nodes.

Trick and Chien [9] have proved that the potential at the fault node of any positive resistor network has the maximum deviation from the potential at the same node when the circuit has no fault. When this fact is used to locate the fault in the ladder network, several measurements may be required to isolate the fault. However it is a potentially valuable approach when dealing with non-linear resistors.

Katz and Cheng [1] have suggested a network approach to study the effects of leakages in brakepipe models and developed mathematical equations for generating pressure distribution for the model. This offers the possibility of using network theory to detect leaks in brakepipes.

Katz, Cheng and Aula [10] have derived an analytically exact formula for a single shunt fault location in a recurrent ladder network. The approach suggested by Shekel [7] and Geffert [9] was followed to obtain two functions from which the formula was derived.

Katz, Aula and Cheng [11] have investigated and developed several methods for locating a single fault in the brakepipe model proposed in [1]. The applicability of these methods was demonstrated through computer simulation.
Aula, Cheng and Katz [12] have discussed the methods proposed in the light of the experimental results. They have confirmed the validity of the analytical methods by experiments on a small scale model of the brakepipe.

Chun-Tat Kwan [13] proposed network models for brakepipe leakage and classified them according to their treatment of leakage flow. He demonstrated that the effects of leakage on pressure gradient and brakepipe taper depended on the position of the fault and that leakage at the rear had larger effect on the pressure gradient than leakage at front.

1.6 SCOPE OF THE THESIS

As stated earlier, no definite methods exist to locate the position of the fault in the brakepipe of a train. This research concerns with developing methods to locate a single fault in the brakepipe.

The problem may be stated as follows:

Definite methods have to be formulated to locate fault position in the brakepipe from a number of pressure measurements. The measurements would be preferably localized to eliminate the tedious and time consuming features that are present in the existing methods. The methods so developed should be based on a sound theory and backed up by experimentation on brakepipe models. Subsequently, the proposed methods would have to be tested extensively on actual brakepipes. This thesis is confined only to developing and verifying the proposed methods on a small scale model of the brakepipe.
This thesis is divided into four chapters. The first chapter briefly introduces the air brake of the freight train, its operation and reviews the previous work that has been done in the area of fault detection and location in electrical and pneumatic circuits.

The second chapter begins with the linear model (ladder network) of the brakepipe. Three methods of fault location, the Direct method, the Difference method and the Ratio method are developed.

In the Direct method an analytically exact equation is developed for fault location requiring only two nodal measurements. They are verified by performing experiments on the computer and the electrical rig shown in figure (24).

The difference method is based on the idea proposed by Trick and Chien wherein they show that maximum difference between no fault and single fault voltage distribution occurs where the fault exist. This has been demonstrated to be applicable to the linear model of the brakepipe both by computer simulation and experiments.

For the Ratio method, the fault is replaced by a voltage source. The no fault and fault voltage distributions are measured and the ratio of any nodal potential to last nodal potential is calculated. The fault is located by calculating the difference of ratios from the rear. Zero difference indicates that the fault is either at or precedes the measurement node.

The third chapter deals with formulation of the nonlinear model of the brakepipe, developing a formula for pressure distribution and the three fault location methods. The Difference and Ratio methods were
demonstrated to be applicable to the nonlinear brakepipe model both by simulation and experiments on small scale brakepipe model.

The Transformation method is developed by linearizing the non-linear ladder network using the pressure squared as potential variable and flow squared as current variable. The network is transformed to compensate for violating Kirchhoff's current law. The equation developed for Direct method is used here with modifications to locate the fault.

The last chapter briefly summarizes the previous chapters, concludes the discussion and presents some suggestions for further work.
CHAPTER 2
SINGLE FAULT LOCATION IN LINEAR MODEL

2.1 LINEAR MODEL

It has been shown by Katz and Cheng [1] that the brakepipe may be
modelled as a ladder network with linear shunt resistors and nonlinear
series resistors as shown in figure (2). Since it is very difficult to
obtain an analytically exact relationship between the source \( p_0 \) and the
last node pressure \( p_n \), the ladder network with linear resistors only are
considered first. The voltage source models the compressor unit in the
locomotive, the linear series resistors \( R \) represent the resistance of
the brakepipe to air flow and the linear shunt resistor \( R_s \) models the
leakage in brakepipe at each section of the network. Thus we have a
ladder network with all series resistors equal and all but one shunt
resistor equal. The objective is to locate the faulty shunt resistor
(i.e., excessive leak in brakepipe) with the minimum number of nodal
potential measurements.

2.2 ANALYSIS OF THE LINEAR MODEL

Since the objective of this analysis is to develop a method that
would identify the fault with minimum measurements, the approach suggested
by Shekel [7] and Gefferth [8] is followed. The procedure would be to
derive two functions for the recurrent ladder network with a single fault. From these the fault can be located.

The ladder network under consideration figure (2a), consists of in-line resistors and shunt resistors. All of the in-line resistors are equal and have a resistance $R$. The shunt resistors are all equal except for one resistor which has a resistance, $R_m$, which is referred to as the fault. The driving potential is located at the input to the ladder network and is designated as $e_0$. All the subsequent nodes are numbered consecutively from 1 to $n$. The in-line current $i_k$ is the current leaving node $k$ and the current $i_n$ leaves node $n$ and a node at the potential $e_{n+1}$.

These variables are included to provide a boundary condition on the equation developed. For the network considered here the boundary conditions are $i_n$ equals zero and therefore $e_{n+1} = e_n$.

Cheng [14] shows that when there is a recurrence of structure i.e., when all in-line resistor equals $R$ and all shunt resistors equals $R_s$, the "Z" transform may be used to obtain a relationship between the nodal potentials and the source potential of the form:

$$e_k = e_0 \cosh bk - K \sinh bk$$  \hspace{1cm} (1)

where $e_k$ is the potential at node $k$, $e_0$ is the source potential, $K$ is a constant that must be evaluated from the boundary conditions, $k$ refers to the nodal position and $b = \cosh^{-1}[1+R/2R_s]$.

The complete derivation of the relationship between the nodal and source potential is shown in Appendix I.1.

Since equation (1) applies only to a recurrent structure, the faulty
shunt, $R_m$ is replaced by a parallel combination as shown in figure (2a). One of the resistors in the combination is $R_s$ and the other is $R_s R_m / R_s - R_m$. The ladder network is now separated into two parts which have recurrent structure. The part from the fault rearward is shown in figure (3a) and the portion from the fault forward is shown in figure (3b). In the forward part, figure (3b), we use the substitution theorem to replace the resistance $R_s R_m / R_s - R_m$ by a flow source $i_s$, Scott [15]. In this way the recurrent structure of the forward part is maintained justifying the application of the relationship developed by Cheng [14].

Equation (1) is first applied to the rear part figure (3a) with the result that

$$e_k = e_m \cosh(bk - bm) - K_s \sinh(bk - bm), \quad k > m \cdots (2),$$

where the constant $K_s$ refers to the rear part. This constant may be evaluated from equation (2) by using the no current boundary $e_n = e_{n+1}$ i.e.,

$$K_s = \frac{e_m \left[ \cosh(bn - bm + b) - \cosh(bn - bm) \right]}{\left[ \sinh(bn - bm + b) - \sinh(bn - bm) \right]} \cdots (3)$$

(please refer to Appendix I.2)

If equation (3) is substituted in equation (2) the resulting expression simplifies

$$\frac{e_k}{e_m} = \frac{\cosh(x - bk)}{\cosh(x - bm)}, \quad k > m \text{ and } x = bn + b/2 \cdots (4)$$

(refer to Appendix I.2)
Now applying equation (1) to the forward part figure (3b), we obtain

\[ e_k = e_0 \cosh b_k - K_F \sinh b_k, \quad k \leq m \]  \quad \ldots(5)

where \( K_F \) refers to the forward part.

To evaluate \( K_F \), the condition provided by Kirchoff's current law is used at node \( m \), which yields

\[ i_{m-1} = i_m + e_m / R_m \]  \quad \ldots(6a)

or

\[ (e_{m-1} - e_m) = (e_m - e_{m+1}) + \frac{R}{R_m} e_m \]  \quad \ldots(6b)

If equation (5) is used to evaluate the left side of equation (6b) and equation (4) is used to evaluate the right hand side, the result is:

\[ K_F = \frac{A e_m + \left[ \cosh b_m - \cosh(b_m - b) \right]}{\left[ \sinh b_m - \sinh(b_m - b) \right]} \]  \quad \ldots(7)

where

\[ A = 1 + \frac{R}{R_m} - \frac{\cosh[x - bm + b]}{\cosh[x - bm]} \]

(refer to Appendix I.2)

The substitution of equation (7) in equation (5) yields

\[ \frac{e_k}{e_0} = \frac{(1 + A) \sinh(b_m - bk) - \sinh(b_m - bk - b)}{(1 + A) \sinh b_m - \sinh(b_m - b)}, \quad k \leq m \]  \quad \ldots(8)

Equation (8) relates the potential at the nodes (upto and including the fault node) to the driving potential. To relate the nodes after the fault to the driving potential we specify \( k = m \) in equation (8) and
substitute the result in equation (4). The result is:

\[
\frac{e_k}{e_0} = \frac{\cosh(x - bk) \sinh bk}{\cosh(x - bm) \left[ (1 + A) \sinh bm - \sinh(bm - b) \right]}, \quad k \geq m \quad (9)
\]

(for equations (8) and (9) please refer to Appendix I.2).

Since equations (8) and (9) are valid on opposite sides of the fault, one potential measurement made on each side of the fault would yield two equations in the two unknowns \( m \) and \( A \). Thus to make certain that the two measurements come from opposite sides of the fault would require measurements at nodes 1 and \( n \). As an example for a 10 section ladder network, figure (4) shows the results of plotting \( e_1/e_0 \) from equation (8) against \( e_{10}/e_0 \) from equation (9). The relation is plotted with fault location and magnitude as parameters. For any particular fault location the relation between \( e_1/e_0 \) and \( e_{10}/e_0 \) are straight lines. This is in accordance with the findings of Gefferth [8]. The slope of the lines are different for each fault position but all the lines intersect at a common point. The point of intersection represents the values of the variables when there is no fault \( R_m/R_s = 1 \). The loci for various values of the fault to shunt resistance ratio \( R_m/R_s \) are also shown on figure (4).

As the difference between the fault and shunt resistances increases, the circuit operating point \((E_1, E_n)\) moves further away from the point of intersection. Thus the tolerance for the measurement errors in \( E_1 \) and \( E_n \) increases as the fault becomes significantly different than other shunts. In addition, the potential measurement at node 1 must be extremely accurate.
when the fault is in the rear of ladder. This may be observed by selecting a value of \( E_{10} \) and noticing that the distance between fault lines decreases as the fault position moves toward the rear.

Since node 1 requires very precise measurements and the accuracy requirements reduces if the measurements are made nearer the fault, a slightly different approach is adopted. Node \( n \) is still used in conjunction with equation (9) to ensure a measurement after the fault. However, an arbitrary node, \( k \), (presumably before the fault) is used in equation (8). For the case when \( k \) is not before the fault (i.e., when \( k > m \)) will be considered later in the analysis.

Now by eliminating \( A \) from equations (8) and (9) yields:

\[
\frac{E_n}{\cosh b/2} = \frac{E_k \sinh b m - \sinh (b m - b k)}{\cosh (x - b m) \sinhk}, \quad k < m \quad \ldots (10)
\]

where the measurements \( E_n \) and \( E_k \) are given in normalized form

\[
E_n = e_n/e_0 \quad \text{and} \quad E_k = e_k/e_0 \quad \text{(refer Appendix I.2)}.
\]

Thus equation (10) has been obtained which has only one unknown, \( m \), the fault location. The equation (10) is solved for \( m \) by expanding the multiple arguments of the hyperbolic functions, so that

\[
\tanh b m = \frac{\sinhk \cosh b/2 - E_n \cosh x \sinhk}{\cosh b k \cosh b/2 - E_k \cosh b/2 - E_n \sinhx \sinhk}, \quad k < m \quad \ldots (11)
\]
Trigonometric manipulation yields

\[
m = \frac{1}{2b} \ln \frac{\left[ E_k - \sinh bk - \cosh bk \right]}{\left[ E_k + \sinh bk - \cosh bk \right]} \frac{\cosh b/2 + E_n \sinh bk \left[ \sinh x + \cosh x \right]}{\cosh b/2 + E_n \sinh bk \left[ \sinh x - \cosh x \right]}
\]  \quad \ldots (12)

where \( k < m \)

Equation (12) gives the fault location as a function of two potential measurements. One measurement is at node \( n \) and the other at a node \( k \) which must be before or at the fault node. Since the fault node is unknown, \( m \) would be calculated from equation (12) when the measuring node selected is actually after the fault. That is, how is it to be recognized whether the arbitrarily selected position, \( k \), violates the condition that \( k < m \)? To determine this we substitute into equation (11) values of \( E_k \) and \( E_n \) that are obtained from equation (9) alone. The result is:

\[
\tanh bm = \tanh bk
\]  \quad \ldots (13)

or

\[
m = k \quad , \quad k > m
\]

(refer to Appendix II).

This means that whenever the selected measuring node is behind the fault, equation (12) will predict the measuring node location and not the fault location.

It is interesting to note that equation (13) is not a function of potential measurements. This is due to an exact cancellation of terms in
equation (12). This fact is illustrated in figure (5) and (6) where $E_2 (e_2/e_0)$ and $E_7 (e_7/e_0)$ are plotted against $E_n (e_n/e_0)$. The graphs are similar to figure (4) except for the difference that for $k>m$ (i.e., when the selected measuring node is after the fault), the lines for $m=1$ in former case and $m=1, 2, 3, \ldots, 6$ do not appear at all thereby illustrating the above fact. In actual case an exact cancellation may not be achieved. The results suggest, however, that the measurements made after the fault will be less sensitive to measuring errors.

Figure 7 shows in normalized form the effect of measuring position on the predicted fault location. The true fault location is a parameter on the graph. This in an idealized representation under the assumption of exact potential measurements and exact components values. The figure shows that when the measurement position is anywhere in front of the fault position is predicted accurately. But if the measurement position is after the fault, then the position of measurement is predicted. This fact is made use of in identifying and locating the fault.

2.2.1 SENSITIVITY ANALYSIS

In theory (equation (12)), potential measurements at nodes $1$ and $n$ are all that is required to locate a fault in any position. This presumes the source voltage is known. However, inaccuracy in measuring potentials may produce a large discrepancy between the predicted ($m_p$) and the actual ($m$) fault position. Since it is also possible to calculate the fault location from measurement made at other nodes, it is essential to determine the nodes which will produce the minimum fault error. To
obtain this information a modified sensitivity function, \( S \) is defined, as:

\[
S = \frac{\Delta m}{\Delta E_k} E_k = \Delta m / (\Delta E_k / E_k)
\]  ... (14)

where \( \Delta m = m_p - m \) is the fault position error. This function is chosen rather than the conventional sensitivity function because the fault position error is important no matter where the fault is located.

The sensitivity of the nodes before the fault \((k \leq m)\) is first considered. The derivative in equation (14) is obtained by differentiating equation (8). The reason equation (8), is chosen rather than equation (12), is that in equation (8), the fault location is a function of one normalized potential \((E_k)\) and the fault magnitude. If the fault magnitude is held constant figure (4) shows that the sensitivity obtained is larger than the sensitivity at constant \( E_n \). Thus, the sensitivity calculated from equation (8) produces conservative results.

Intuitively, the largest errors should occur when the measuring position is further from the fault. To place an upper bound on the error, the fault is placed in the last section (node \( n \)). Figure (8) shows the sensitivity plotted against measuring position for ladder networks with 10, 20, 50 and 100 sections. As expected, the sensitivity is largest when the measurement is made at node 1. The sensitivity decreases monotonically as the measuring position approaches the fault location. It is also noted that as the number of nodes increases the sensitivity increases at the forward nodes but decreases at the rear nodes. A sensitivity value of \( 10^4 \) produces a position error of one node if the measuring error
\( \frac{\Delta E_k}{E_k} \) is \( 10^{-4} \).

The sensitivity of nodes after the fault (\( k > m \)) is obtained through the use of equation (9). The sensitivity function for these nodes is independent of measurement position. It is also much less than the sensitivity measured before the fault. For example when the fault is at node 1, the sensitivity is less than \( 0.12 \times 10^{-3} \) for any number of sections. From this it appears that the error magnitude will depend on the accuracy of the measurements before the fault.

Another important factor in fault detection is the magnitude of the fault. It is expected that (figure 3) as the ratio of fault resistance to shunt resistance \( \frac{R_m}{R_s} \) approaches unity, the fault will become more difficult to locate. Figure 9 shows the sensitivity of measurements made at the last node, for various values of \( \frac{R_m}{R_s} \). These sensitivities represent the smallest values that are possible since the measurements are made at the fault. If the measuring accuracy of \( \frac{\Delta E_n}{E_n} \) is \( 10^{-4} \), the fault of \( \frac{R_m}{R_s} = 0.1 \) can be located within one node. Faults with \( \frac{R_m}{R_s} = 0.3 \) could be replaced within one node if the number of nodes exceeded 14. The nearer \( \frac{R_m}{R_s} \) approaches unity, the more accuracy for the measurements are required.

The parameter \( \frac{R}{R_s} \) in the ladder network is also a factor in the fault position error. Figure (10) shows the sensitivity plotted against \( \frac{R}{R_s} \) for various values of \( \frac{R_m}{R_s} \). Here the sensitivity is based on measurements at the fault location and the location is at the last node of a 100 node ladder. As the ratio \( \frac{R}{R_s} \) increases, the sensitivity decreases for a given value of \( \frac{R_m}{R_s} \). Thus fault discrimination improves
with an increase in \( R/R_s \). However it must be recognized that when \( R_s \) becomes small, the end of ladder networks with a large number of nodes may have vanishingly small potentials. This could make fault detection very difficult, if not impossible. As a practical matter such networks are rare since they do not perform useful functions.

2.3 OTHER SINGLE FAULT DETECTION METHODS

2.3.1 DIFFERENCE METHOD

In any positive resistor circuit with a single fault, Trick and Chien [9] have proved that maximum difference in the absolute value of the potential with and without fault occurs at the node with the faulty resistor. For example consider figure (2a) wherein there is no fault in the network. All the voltages \( e_0, e_1, \ldots, e_n \) are noted. This is the no fault distribution. Now consider one of the resistors to have changed from the rest. The circuit now contains a fault. Again the voltages \( e_0^1, e_1^1, \ldots, e_n^1 \) are noted. This is the fault distribution of voltages. The corresponding difference in voltages are calculated i.e., \( e_0 - e_0^1, e_1 - e_1^1 \) and so on. The fault occurs where the maximum difference in voltages takes place. This fact is made use of in identifying the fault in the linear ladder resistor network under consideration.

This procedure is verified by simulating it on the computer. The voltages are generated by using the basic matrix element relating two adjacent nodes of the ladder network which is:
(refer Appendix III)

where \( R_k \) is the shunt resistance at node \( k \). The nodal potentials for any number of nodes are calculated by multiplying the individual matrix elements together and applying the appropriate boundary conditions \( (e_0 \) and \( i_n) \).

Figures (11), (12), and (13) illustrate this method. The values are \( n = 10, 20, 50, e_0 = 10 \) volts, \( R = 1 \) ohm, \( R_s = 1000 \) ohms and \( R_m = 100 \) ohms. It is observed that the relationship is almost linear through zero for nodes in front of the fault position and almost horizontal for nodes after the fault position for \( n = 10 \) and 20. For \( n = 50 \), the relationship droops down considerably after the fault position indicating a considerable decrease in flow of current after the fault. The point at which it starts dropping indicates the position of the fault. That is, at this position the difference in no fault and fault voltages is maximum. Figures (14), (15) and (16) illustrate the same method but with \( R_m = 333 \) ohms. It is noted that the difference in voltages reduce considerably (i.e., \( 10^{-2} \) to \( 10^{-3} \)) necessitating accurate nodal measurements to identify the fault.

2.3.2 RATIO METHOD

The resistor ladder network may be interpreted in a different manner
by applying the theorem of substitution in network theory [15]. It may be considered to have two "sources" a voltage source at the head end (i.e., node 0), and a current or flow source which replaces the fault (see figure 17). In this method, the ratio of voltage at the kth node, \( e_k \), to that of the last node n (\( e_n \)) is evaluated for the single fault situation and the so-called no-fault situation. The difference between the two ratios is then plotted as shown in figures (18), (19) and (20) for different locations of the fault and when \( n = 10, 20, 50 \) respectively. The value of the fault is \( R_m = 100 \) ohms and that of other parameters, \( R_s = 1000 \) ohms, \( R = 1 \) ohm and \( e_0 = 10 \) volts. It is observed that the point at which the graph cuts the abscissa is the location at which the fault lies. This is because the difference of the pressure ratios at and after the fault is zero. It is noted that the graphs are inclined at an angle approximately 30° with respect to the abscissa. This fact may be made use of in locating the fault with few measurements and extrapolating.

Figures (21), (22) and (23) illustrate the above method but with \( R_m = 333 \) ohms. As in the case of the difference method, it is noted that the value of the difference of ratios become small and hence requiring accurate measurements to locate the fault. Thus as the value of \( R_m \) increases i.e., as the size of fault decreases, it becomes increasingly difficult to identify the fault.

2.4 EXPERIMENTS ON THE LINEAR MODEL

To test the validity of the theory derived (section 2.2.1 and
2.2.2) and the methods formulated (section 2.3.1 and 2.3.2) for the fault location, two types of experiment are performed. In one, the ladder network is simulated on the computer. In the other, an electric ladder network model shown figure (24) is used.

2.4.1 SIMULATION ON THE COMPUTER FOR THE DIRECT METHOD

The basic matrix element relating two adjacent nodes of the ladder network is as shown in equation (15). (Refer to Appendix II.3) for derivation), where \( R_k \) is the shunt resistance at node \( k \). The nodal potentials for any number of nodes may be calculated by multiplying the individual matrix elements together and applying the appropriate boundary conditions \( (e_0 \text{ and } i_n) \). If required for a particular ladder network, each matrix element may contain different values of shunt and series resistance. For the case under consideration only one matrix element is very different. However, it is possible with the computer simulation to vary each series and shunt resistance within its tolerance range. Nevertheless, the effect of the component tolerance is left to the electrical model tests. Instead, the computer is used to demonstrate the effect of measurement errors.

The procedure for computer testing is to calculate the nodal potentials with one matrix element different from all others. These calculated values which are accurate to seven decimal places are then multiplied by a fixed percentage error. The resulting error potentials are then used in conjunction with equation (12) to predict the fault location.

Figure (25) shows some typical results of the computer testing.
The predicted fault node is plotted against the measuring node position for the situation where the fault is actually at 50th node of a 100 node ladder network. The in-line resistances are one ohm and the normal unfaulted shunt resistances are 100 ohms. The faulted shunt resistances is varied and has a value of either 10, 50 or 90 ohms. The assumed measurement error is ±0.1 percent (ΔE = ±10⁻³) and is applied to both potential measurements Eₖ and Eₙ. The results show that the sign of the error, the fault magnitude and the measuring node are all significant factors in detecting the fault. When the error is positive, there is a region of nodes extending from the source for which no solution is obtained. For \( \frac{R_m}{R_s} = 0.1 \), the region where the measuring node is from 1 to 21 produces no result. As \( \frac{R_m}{R_s} \) increases, the number of unproductive nodes increases. The reason for this is that the error potential cause negative arguments for the natural logarithm in equation (12). When the error is negative, each node yields a finite prediction. The predictions improve as the fault is approached. As expected the utilization of measuring nodes after the fault gave almost exact predictions of measuring node rather than the fault node.

2.4.2 TESTS ON THE EXPERIMENTAL MODEL

In the experimental ladder network the in-line resistances were 1±0.01 ohms and the unfaulted resistances were 1000±10 ohms. The in-line resistors were soldered to the measuring nodes but the shunt resistors were mounted on removable plugs. This facilitated the moving
of the fault from one position to another or vary the size of the fault. The set up was fabricated so that it could accommodate up to 100 nodes.

2.4.2.1 DIRECT METHOD

While conducting a test, the potential at the source was maintained at 10 volts by a regulated power supply. The faulted shunt resistances was either 100 or 333 ohms and its location was varied. For each fault location potential measurements were made at all the nodes. A predicted fault location was calculated from equation (12) by using the potential at the last node and the potential at one other node. Some of the typical test results are shown in figures (26), (27) and (28).

For a ladder network of twenty sections, figures (26) and (27) show the results when the fault is located at nodes 4, 6, 8, 12, 16 and 20. The faulted shunt resistor is 100 ohms in figure (26) and 333 ohms in figure (27). In both the cases the discrepancy between the predicted and the actual fault location is generally largest when the first node is used in the calculations. The discrepancy is least when the measuring node coincides with the fault location. In addition the results show that the discrepancy increases, when the faulted shunt resistor approaches the magnitude of the unfaulted shunt resistors. Thus, there are larger errors for a 333 ohm fault than for a 100 ohm fault.

Figure (28) shows the relation between the predicted fault node and the measurement node when there are 50 sections. Here again the prediction generally improves as the measurement node approaches the fault
location. Of course once the measurement node exceeds the fault location, the prediction corresponds to the measurement location and not the fault location. Again as the magnitude of the fault decreases i.e., as the resistance increases, larger errors in prediction occurs.

It is also noted that the faults located nearer the source are more difficult to locate.

2.4.2.2. DIFFERENCE METHOD

Figures (29), (30) and (31) show the graphs obtained by plotting the difference in normalized ratios between no fault and fault potential distribution against the measurement position, \( k \) with the fault position as the parameter. The values of \( n \) are 10, 20, and 50 and that of \( R_m \) and \( R_s \), 100 ohms and 1000 ohms respectively. By comparing these to the figures obtained by theory (figures (11), (12) and (13)) it is observed that they verify the theoretical predictions both qualitatively and quantitatively.

For the case when \( R_m = 333 \) ohms i.e., when the size of the fault is smaller, the resulting plots are shown in figures (32), (33) and (34). For \( n = 10 \) and 20 it is observed that they correspond very well with theory (figures (14), (15)) both qualitatively and quantitatively. But when \( n = 50 \), quantitative discrepancies are observed although qualitatively they are satisfactory. This leads us to conclude that as the size of the fault decreases and the number of sections increases accurate measurements are necessary to identify the fault.
2.4.2.3 RATIO METHOD

Figures (35), (36) and (37) show the graphs obtained when the ratio method is applied to the data obtained from the electrical model. The ratio of the first to the last node for both fault and no fault conditions are calculated and their difference is plotted against the measurement position, k with the fault position as the parameter. The values of n are 10, 20 and 50 that of $R_m$ and $R_s$, 100 ohms and 1000 ohms respectively.

These graphs correspond very well with those of the theory for similar values (figures (18), (19) and (20)) both qualitatively and quantitatively.

The same reasons may be given for the correspondence between figures (38), (39), (40) and figures (21), (22), (23) when $R_s = 333$ ohms.
CHAPTER 3

SINGLE FAULT LOCATION IN NONLINEAR MODEL

3.1 NONLINEAR BRAKEPIPE MODEL

In an assembled train, the physical configuration of the brakepipe is a combination of a series of lengths of pipes and other accessories. For the purpose of modelling, the brakepipe is simplified into a form consisting of a series of pipes with leakage holes.

A constant supply of air is provided by an air compressor in the locomotive at the head end of the train. Because of leaks and friction of brakepipe to air flow, the pressure decreases monotonically from the head end pressure ($p_0$, which is constant) toward the rear of the train.

To model this condition, each car is assumed to be represented by a single pressure ($p$) and a lumped restriction is chosen to represent the pipe flow resistance $KF$. This restriction is assumed to follow the standard forms [13] and [14] for isothermal pipe flow:

$$p_i^2 - p_{i+1}^2 = KF_i m_i^2$$

where $p$ is the absolute pressure, $m$ is the flow and $KF$ is a constant that characterizes the flow resistance. For a train with cars of equal length, the value of $KF$ will be approximately equal for each car. The subscripts
refer to the car position with \( n \) as the total number of cars in train. A leakage is flow that passes from the brakepipe to the atmosphere. Thus leakage is modelled as a shunt component from the pipe to the atmosphere. The shunt elements are linear resistances \( R \), which are used to model a choked orifice leak. The brakepipe pressures are always sufficient to ensure choked flow conditions through leakage apertures.

The circuit representation of the model is shown in figure (41).

3.2 ANALYSIS OF THE MODEL

The objective of this analysis is to obtain a relationship between the source \( p_0 \) and the pressure in the last car \( p_n \) with intermediate pressure terms not appearing. If this was possible an analytically exact formula could be developed for the location of the fault. But since the circuit is nonlinear, it is very difficult to obtain such a direct and exact relationship.

Hence a recursive formula is developed from which normalized pressures at each of the nodes (i.e., the pressure distribution) may be obtained.

The constant pressure \( p_0 \) at the head end monotonically decreases as we proceed towards the last car due to leakage and resistance of the brakepipe. The shunt carries the leakage mass flow \( m_L \). Thus the in-line flow in the \( i^{th} \) car \( (m_i) \) is equal to the sum of the leakage flows in all the cars that come after:

\[
m_i = \sum_{j=i+1}^{n} m_{L_j}
\]  

\((17)\)
where \( j \) is the summation index.

The shunt resistances \( R \) may be assumed to be linear, because of the sonic condition of leakage flow. Thus leakage flows may be expressed as:

\[
m_{i1} = \frac{p_i}{R_i} \quad 1 \leq i \leq n \quad \ldots (18)
\]

From equations (16), (17) and (18) a pressure relation between the \((i-1)\)th car and \( i \)th car is given as

\[
p_{i-1}^2 - p_i^2 = K F \sum_{j=1}^{n} \frac{p_j}{R_j} \quad \ldots (19)
\]

The pressure dependency of the leakage flows suggests the use of ratios to calculate the pressure distribution. If a variable, \( B_i \), is defined as the ratio of pressures of adjacent cars equation (19) becomes:

\[
B_i^2 = \frac{p_{i-1}^2}{p_i^2} = 1 + \frac{K F}{R_R} \sum_{j=1}^{n} \left( \frac{p_j}{R_j} \right)^2 \quad \ldots (20)
\]

where \( R_R \) is a reference resistance that may be arbitrarily selected to normalize the equations.

The variable \( B_i \), can only be calculated from the rear of the train moving forward because it depends on all values of \( B \) that come after the \( i \)th car.

For example

\[
B_n^2 = 1 + \frac{K F}{R_R} \sum_{j=1}^{n} \left( \frac{R_j}{R_n} \right)^2 \quad \ldots (21a)
\]
\[ B_{n-1}^2 = 1 + \frac{K F}{R_R^2} \frac{R_R}{R_{n-1}} + \frac{1}{B_n} \frac{R_R}{R_n} \quad \ldots (21b) \]

Thus generalizing, \( B_j \) is given by the recursive formula:

\[ B_j^2 = 1 + \frac{K F}{R_R^2} \frac{R_R}{R_1} + \sum_{j=1}^{n-1} \left( \prod_{k=1}^{j} \frac{1}{R_{k+1}} \right) \frac{R_R}{R_{j+1}} \quad \ldots (22) \]

and the ratio of the pressures between the \( i \)th car and that of the locomotive is given by:

\[ \frac{p_i}{p_0} = \frac{1}{\prod_{j=1}^{i} B_j} \quad \ldots (23) \]

(refer to Appendix IV).

### 3.3 SINGLE FAULT LOCATION METHODS FOR THE NONLINEAR BRAKEPIPE MODEL

#### 3.3.1 DIFFERENCE METHOD

Trick and Chien [9] have proved that in case of a single fault in a positive resistor circuit the absolute potential change across the faulty resistor is greater than or equal to the potential change across any resistor in the circuit.
That this phenomenon also exists in the nonlinear model of the brakepipe is demonstrated by calculating the pressure differences for a few particular cases of single shunt fault, $R_f$. In this case the pressures are obtained using equations (22) and (23). Figures (42) and (43) shows the difference between the no fault pressure and single fault pressure at each section of the brakepipe, for various locations and two sizes ($R_f/R_R = 0.465 \text{ and } 0.3$) of the fault. Similar to the linear case, the maximum pressure difference is seen to occur at the single fault of the brakepipe. Proceeding from the head end, the pressure difference increases monotonically and reaches a maximum at the fault after that the pressure difference decreases slowly. As the size of the fault increases, the curve droops more steeply after the fault thereby clearly identifying the fault position. Thus to locate a fault, one needs to measure the pressures at several positions and calculate the pressure difference between the no fault and single fault situation. The fault occurs at the position where this difference is the maximum.

3.3.2 RATIO METHOD

As in the case of the linear model, the nonlinear ladder resistor network can be considered to have two sources one being the applied source at the head end (i.e., at node 0), and the other one being the flow source replacing the fault. The flow source is in parallel with other shunt elements (figure (44)). This means that the network model contains no sources from the fault position rearward.
Consequently the ratio of any nodal potential measurements made at or to the rear of the fault is not a function of the fault. Simulation with equations (22) and (23) confirm this conclusion. The difference in the ratio of potentials will therefore be zero where there is no fault between the nodes. To demonstrate this, the ratio of pressures at node 1 to node n is selected because this permits elimination from the rear.

Figures (45) and (46) shows the difference in the pressure ratio as a function of measurement position for various fault positions and also when $R_f/R_r = 0.465$ and 0.3 respectively. The other parameters have the same values as in the Difference method. When the difference in pressures ratios is used, the fault occurs where each curve intersects the abscissa. Thus in locating a single fault by this method, a zero difference indicates that the fault is either at or precedes the measurement node. This eliminates locations to the rear of the measurement node and offers the possibility of fault location with fewer nodal measurements.

3.3.3 TRANSFORMATION METHOD

In this novel method, the nonlinear model is transformed into a linear one by linearizing figure (40) using the square of the pressure $(p^2)$ as the potential variable and square of the flow as the current variable. Except at the last node, the linearization violates Kirchoff's current law at each node. To compensate for the violation the model is transformed (or stretched) by inserting additional sections which are
added between nodes. These are calculated by matching the nonlinear no-fault pressure distribution \{equations (22), (23)\} with linear no-fault distribution developed in (section 2.2).

In terms of the square of the potentials the linear distribution is given by

\[
\frac{p_i^2}{p_0^2} = \frac{\cosh b(N - I + \frac{1}{2})}{\cosh b(N + \frac{1}{2})} \quad \ldots (24)
\]

where N and I are transformed values corresponding to \(n\) and \(i\) in the nonlinear model and \(b = \cosh^{-1} \left[ 1 + \frac{KF}{(2R^2)R} \right]\). The equivalent values for \(i\) and \(I\) are shown in table I below for \(KF/R^2 = 0.00077\) and \(n = 10\). The transformation is obtained by inserting the nonlinear network calculation equation (22) and (23) into equation (24) beginning at the last section and working backwards towards the source.

Since Kirchhoff's law is not violated in the last section, the transformed difference between the last two nodes is only 1 (i.e., 27.07 - 26.07). The transformation of the nonlinear model to a linear model permits the application of the linear fault location formula \{equation (2) developed in \{section 2.2\}.\]
### TABLE I

Correspondence between the Number of Nonlinear Sections and the Number of Linear Sections

<table>
<thead>
<tr>
<th>Nonlinear I</th>
<th>Linear I</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
</tr>
<tr>
<td>2</td>
<td>7.44</td>
</tr>
<tr>
<td>3</td>
<td>10.88</td>
</tr>
<tr>
<td>4</td>
<td>14.1</td>
</tr>
<tr>
<td>5</td>
<td>17.09</td>
</tr>
<tr>
<td>6</td>
<td>19.82</td>
</tr>
<tr>
<td>7</td>
<td>22.26</td>
</tr>
<tr>
<td>8</td>
<td>24.37</td>
</tr>
<tr>
<td>9</td>
<td>26.09</td>
</tr>
<tr>
<td>10</td>
<td>27.07</td>
</tr>
</tbody>
</table>

In terms of transformed values the fault location \( I_f \) is:

\[
I_f = \frac{1}{2b} \ln \frac{(P_1^2 - \sinh I - \cosh I)\cosh b/2 + P_N^2 \sinh I (\sinh x + \cosh x)}{(P_1^2 + \sinh I - \cosh I)\cosh b/2 + P_N^2 \sinh I (\sinh x - \cosh x)} \quad \ldots (25)
\]

where \( I \leq I_f \)
where \( P_I = \frac{P_I}{P_0}, P_N = \frac{P_N}{P_0} \), \( b = \cosh^{-1}(1 + \frac{KF}{2R^2}) \) and \( x = b_N + b/2 \)

Equation (25) gives the transformed fault location as a function of two pressure measurements. One required measurement is always at node \( n \) (transformed to \( N \) in equation (10)) and the other at a node \( i \) which must be at or before the fault node \( i_f \) (transformed to \( i \leq i_f \) in equation (10)) where the arbitrarily selected measurement node \( (P_i) \) is after the fault, equation (25) will predict the measurement node (i.e., \( i_f = I \)). This occurs for the same reason as in the ratio method. That is, the pressure measurement made at or to the rear of the fault does not depend on the fault.

Figures (47) and (48) show the predicted location of the fault may vary with position at which measurements are made according to equation (25). All the positions in the figure have been transformed back to their true positions in the nonlinear ladder network. For example, when the fault is in the last node \( (i_f = 10) \) and the measurement is made in the first node \( (i = 1) \), equation (25) will predict a fault at \( i_f = 6.8 \). As the measurement node gets closer to fault, the prediction improves. Also when the size of the fault increases (i.e., as \( R_f \) decreases) we observe improved fault predictions.

### 3.4 EXPERIMENTS

#### 3.4.1 DESCRIPTION OF THE TEST SET UP

Figure (49) is a picture of the scaled-down brakepipe model along
with some measuring equipment. The brakepipe essentially consists of ten galvanized pipes of internal diameter $6.35 \times 10^{-3} \text{m}$ and each $3.05 \text{ m}$ long. This arrangement gives an approximate scale down ratio of $5:1$. The pipes are connected to each other by lengths of flexible plastic tubing each $2.54 \times 10^{-1} \text{ m}$ in length. The value of $\text{KF}$ as defined in equation (16), characterizing the pipe resistance, is taken to be approximately $2.21 \times 10^{13} (\text{ kPa})^2 \cdot \text{s}^2/\text{kg}^2 \cdot \text{m}^4$, as suggested by Kwan [10], although it was pointed out in the same reference that $\text{KF}$ is not constant for every pipe and tends to vary over a range of values. The leak in each pipe is simulated by an orifice. A cross is fitted at the end of each pipe so that a toggle valve and an orifice may be located. The purpose of the toggle is to permit the measurement of static pressure at each node.

A regulated supply of air (600 kPag) is provided at the head end of the brakepipe. A surge tank is used to reduce pressure fluctuation at the supply. A differential transducer (0-150 kPa) is used to provide the pressure reading. Another transducer (0-600 kPa) is used to monitor the supply pressure. The transducers have an accuracy of 0.5% and a digital multimeter of 0.04% accuracy is employed to provide easy read out. The transducers are calibrated against a Bourdon gage (accuracy 0.1%) and the digital multimeter is set to provide 0-10 volt reading to cover the working range of the transducer in both cases.
3.4.2 TEST PROCEDURE AND TEST DATA

First, the no-fault situation is simulated by using ten orifices of each diameter $5.72 \times 10^{-4}$ m, at cross fittings located between pipes as described in section 3.4.1. An orifice of this size is equivalent to a fluid resistance value $R_F$ of $1.687 \times 10^8$ kPa.s/kg.m$^2$. The differential pressure transducer is connected across the air supply line and each cross in turn, so as to establish the pressure distribution for the no-fault situation.

Next, to simulate a single fault situation, a larger orifice is used to replace one of the ten identical orifices. The resistance value $R_F$ of this fault is chosen arbitrarily (e.g., $R_F/R_R = 0.465$ and $0.3$). For each case, the corresponding single fault pressure distribution is established, one for each size, and for different positions of the fault. The results are shown in figure (50) (for $R_F/R_R = 0.465$) and figure (51) (for $R_F/R_R = 0.3$). It is noted that in both graphs, the no-fault, corresponding to minimum leakage, is above the other curves, indicating least pressure drop at each node. As the fault is located towards the rear end of the brakepipe, the corresponding pressure drop becomes more serious. This coincides at least qualitatively with the observations made for real-sized brakepipes as contained in reference [1]. Because of the small amount of pressure drops occurring at the front end of the brakepipe in every case, it becomes somewhat difficult to distinguish between the curves for one fault location from the other, even though precautions have been taken to use a differential transducer.
to obtain the data.

The data on figures (50) and (51) have been processed according to each of these fault location methods described in section 3.5 (refer to the data given in the table (I)).

3.5 EXPERIMENTAL RESULTS OBTAINED BY APPLYING THE SINGLE FAULT DETECTION METHODS

3.5.1 DIFFERENCE METHOD

Figure (52) shows the normalized difference between the no-fault pressure and the corresponding single fault pressure for each position along the brakepipe. By comparing it with corresponding theoretical curve in figure (42) two observations can be made. First, because of extremely small pressure differences resulting from leakage, it has been difficult to obtain with the necessary number of significant figures to produce better curves.

It is for this reason there is a discrepancy between figure (52) and figure (42) of about 30% - 40% quantitatively at practically every point.

The second but more important observation is that in spite of the difficulty in measurement and somewhat dubious value of the brakepipe parameter $\frac{KF}{R^2}$ (since $KF$ is not constant throughout the pipe as assumed), it is very evident that the maximum difference occurs at the location of the fault itself. This is more obvious in figure (53) where $\frac{R_F}{R} = 0.3$, due to a larger orifice diameter representing the fault.
Hence it can be established that the difference method, based on pressure measurements is a viable method for locating a single fault.

3.5.2 RATIO METHOD

By comparing the experimental curves in figure (53) for difference of ratios as defined in section 3.3.2, with the theoretical curves of figure (45), observations similar to Difference method may also be made for the Ratio method. It is observed that each line meets the abscissa at the location of the fault. However, the method seems to work less well when the fault is located at the front. This is because of limitations in measuring accurately small differences in pressures. The situation is much improved when the fault consists of a larger orifice as is demonstrated in figure (55). Hence the validity of the Ratio method is established.

As in the linear case, a practical approach may also be developed requiring a small number of pressure measurements at the front and of the brakepipe only; provided one can establish the curvilinear line segment sufficiently accurately.

3.5.3 TRANSFORMATION METHOD

By applying the test data in figure (50) to equation (25), figure (51) is obtained. It is observed that if the points are taken individually, the prediction is somewhat unsatisfactory, unless the measurement position happens to be close to the position of the fault itself. If the
measurements are taken to the rear of the fault, the predicted value $i_f$ is likely to lie on the straight line $i_f = i$. As an illustration and referring to figure (56), if the fault is located at $i = 6$, then the predicted $i_f$ is 4.5 (say 5) when the measurement is at $i = 1$ the predicted $i_f$ is, however, 6 if the measurement is carried out at $i = 6$. However if the measurement is carried out at $i = 8$, the predicted $i_f$ turns out to be 7.8 (or 8) lying on the line $i_f = i$.

Consequently the transformation method can be utilized to advantage in the following manner:

a) if the predicted $i_f$ is found to be the same as the position of the measurement $i$, it may be concluded that the fault has to lie either at or before $i$

b) when the measurement position $i$ is before $i_f$, a reiterative approach may be pursued by taking a second measurement at the position predicted by the previous method and so on. This carries on until convergence, i.e., predicted $i_f = i$. 


CHAPTER 4

CONCLUSION

4.1 SUMMARY AND DISCUSSION

Single fault location methods for linear and nonlinear ladder resistor networks have been investigated. The objective of this study was to locate the fault with minimum number of nodal potential elements.

The linear network consisted of in-line series and shunt resistors. The in-line resistors were all equal. The unfaulted shunt resistors were also equal but in general had larger resistance values than the in-line resistors. The faulty shunt resistor differed from other shunts and could be positioned at any node.

Circuit analysis showed that the network with a single fault could be separated into two networks with recurrent structure. One network was before the fault and the other after the fault. From the theory of recurrent structure networks, two relations were developed between nodal potentials, fault location and fault magnitude. One of these relations were valid on each side of the fault. Thus a nodal potential measurement before and after the fault yield two equations in the two unknowns, the fault position and the fault magnitude. The equations were then solved for fault position. To ensure that there
was one measuring node before and after the fault, the first and the last nodes were initially selected. Although theoretically these two potentials were sufficient the first node was very sensitive to measurement error. Analysis showed that the measuring node before the fault became much less sensitive as it neared the fault. Thus for practical reasons the first node was not the most appropriate node with which to begin the search for the fault. It then became necessary to determine the effect of using another measuring node that might occur after the fault. It has been proved that if the equation valid before the fault, was used incorrectly with measurements after the fault, the resulting prediction yielded the measurement position instead of the fault location. This was significant because it provided the basis for recognizing possible erroneous fault predictions when less sensitive measuring nodes were used.

A sensitivity analysis indicated that the sensitivity was largest when the front measuring node was furthest from the fault. The rear measuring node (the last node) was an order of magnitude less sensitive than the forward nodes. In general, fault identification improves for smaller $\frac{R_m}{R_s}$ and larger $\frac{R}{R_s}$.

Experiments were made with a computer simulation and with an electrical network model. The experiments confirm the equations developed for the fault location. A study of the experimental results suggested that the search procedure for the fault should begin with a measurement at the last node and a measurement at about 0.3 m. If the predicted fault value calculated from these measurements is larger than
0.3 n, another measurement node is selected rearward but slightly before the first prediction. Then the second prediction is calculated using a new measuring node and the last node (n). Comparison of the two faults should uncover the fault or indicate the appropriate position for another measurement node. If however, the first predicted fault location occurs at the measuring node (0.3 n), the fault is either as predicted or before 0.3 n. In this case another measurement node is chosen at about 0.09 n. The second prediction is calculated from the node at 0.09 n and the last node (n). The second prediction is then examined in a similar manner and should suggest whether it is correct or where to make another measurement. Thus, although in theory only two potential measurements are needed, the measurement errors and component tolerances may require additional measurements.

The Difference method was based on the theory developed by Trick and Chien [9]. They proved that for a single fault in a positive resistor circuit, the maximum difference in no-fault and fault potentials indicated the location of the fault. This was verified by computer simulation and experiments on the electrical model. In this method a number of measurements are required to identify the fault. But for practical purposes a location procedure may be developed along the following lines. Referring to figure (11) each curve may be considered to consist of two line segments meeting at the position where the fault exists. Hence by taking a small number of voltage readings at the front end of the network, the first line segment may be established assuming it to be a straight line. Similarly by taking a small number of readings at
rear end, the second segment of the curve may be constructed. The point where they meet is a very good indication of the neighbourhood of the fault. It should be pointed out here that as the number of sections increase, the second segment becomes more curvilinear and this should be taken into consideration when locating the fault by the practical method suggested.

The Ratio method was based on the theory that the fault could be substituted for by the current source in parallel with the unfaulted resistors. As a result the ratio of any nodal potential measurements made at or to the rear of the fault is not a function of the fault. This was demonstrated both by computer simulation and experiments. In both cases the no-fault and fault distributions were obtained and the ratio of potentials at node 1 to node n were calculated. The differences in the ratio of potentials identified the fault by a process of eliminating from the rear.

A practical location procedure can also be suggested for this method. Referring to figures (18) and (21) we observe that the curves are parallel to each other and are inclined at an angle of approximately 60°. Thus a few potential measurements at the front end of the section are plotted and extended to cut the abscissa at approximately 60°. This should give a very good indication of the fault position.

For the nonlinear resistor ladder network model of the brakepipe, it was demonstrated without any doubt that the Difference and Ratio methods developed for the linear model were equally applicable here. They were done so both by computer simulation and experiments on the scaled
down model of the brakepipe. But it has to be pointed out that the experimental results were not in good agreement because of difficulties in measuring small differences in potentials. These errors were magnified when plotted, thereby making fault identification a difficult task. The same practical fault location procedure suggested earlier may be applied here, although with some modifications. It is important to take into consideration the curvilinear or nonlinear aspect of the curve when trying to identify the fault.

The Transformation method was a novel one because it is thought that an entirely new concept was applied—in locating a fault in the nonlinear model of the brakepipe. The nonlinear network was transformed into a linear one by inserting the required number of linear recurrent sections. This number was calculated by matching the nonlinear unfaulted distribution (equations (22) and (23)) with the linear unfaulted distribution (equation (15)).

These numbers were then made use of in the equation (25) along with the squared potentials to calculate the fault location. The practical fault location procedure as stated for the Direct method (linear case) may also be applied here although more number of trials would have to be undertaken to identify the fault.

It is to be remembered that the final goal of all this work is to able to identify the fault in the brakepipe of the freight train consisting of 100 to 150 freight car of unequal tonnage and unequal distribution of leaks and faults. This work is felt to be a first step forward in achieving the final goal. When that goal is achieved it is
hoped that the difficulties of the railworker may considerably be alleviated and save significant time and money for the rail companies in North America and elsewhere.

4.2 SUGGESTIONS FOR FURTHER WORK

Three analytical approaches were developed for locating a single fault in both linear and nonlinear model of the brakepipe.

Test results were satisfactory for the linear case. But the results obtained by applying the 3 methods developed for the nonlinear model indicated that further work was warranted. The following suggestions may be given serious consideration:

(i) extensive testing for more number of sections (ranging from 50 to 100) on the scaled model of the brakepipe and a greater range of those physical parameters representing pipe resistance, nominal leakage and fault leakage.

(ii) extension of single fault location methods to multiple fault location.

(iii) finally, test should be carried out in brakepipe test racks and ultimately on typical train consists.
REFERENCES


Appendix I

Appendix I.1

Derivation of the relationship between the nodal and source potential

\[
\frac{e_{k-1} - e_k}{R} = \frac{e_k}{R_s} + \frac{(e_k - e_{k+1})}{R}
\]

\[
- e_{k-1} + (2 + \frac{R}{R_s})e_k - e_{k+1} = 0
\]
\[ e(i) - y e(i+1) + e(i+2) = 0 \]

where \( y = 2 + R/R \)

\[ Z \text{ transforms of the above yields} \]

\[ e(Z) - y Z e(Z) e(0) + Z^2 (e(Z) - e(0)) - Z e(1) = 0 \]

\[ e(Z) = \frac{e(0) Z (Z - y) + Z e(1)}{Z^2 - y Z + 1} \]

\( e(0) \) is known but \( e(1) \) is not.

\[ e(Z) = \frac{e(0) Z (Z - y Z)}{Z^2 - y Z + 1} + \frac{(e(1) - e(0) y) Z}{Z^2 - y Z + 1} \]

\[ e(i) = e(0) \cosh b i - \frac{1}{\sqrt{\cosh^2 b - 1}} \left( e(0) y/2 - e(1) \right) \sinh b i \]

where \( b = \cosh^{-1} y/2 \) or \( y = 2 \cosh b \)

\[ \therefore e(i) = e(0) \cosh b i - \frac{2}{\sqrt{y^2 - 4}} \left( \frac{e(0) y}{2} - e(1) \right) \sinh b i \]

To determine \( e(1) \), let it be recognized that \( e(n) = e(n+1) \) i.e., there is no flow from the rear car.

\[ e(n) = e(0) \cosh b n - K \sinh b n \]

where

\[ K = \frac{2}{\sqrt{y^2 - 4}} \left( \frac{e(0) y}{2} - e(1) \right) \]
\[
e(n+1) = e(0) \cosh b(n+1) - K \sinh(n+1)
\]

\[
k^* = e_0 \frac{(\cosh b(n+1) - \cosh b_n)}{(\sinh b(n+1) - \sinh b_n)}
\]

\[
\frac{e(1)}{e(0)} = \cosh b_1 - \frac{\cosh b(n+1) - \cosh b_n}{\sinh b(n+1) - \sinh b_n} \sinh b_1
\]

where \( b = \cosh^{-1} \frac{y}{2} \)

\[
\frac{e_1}{e_0} = \frac{\cosh b(n - i + \frac{1}{2})}{\cosh b(n + \frac{1}{2})}
\]
Appendix I.2

Derivation of an analytically exact equation for locating a single shunt fault in a linear model

\[ \frac{e_1}{e_m} = \frac{\cosh(b(n-1 + i_2))}{\cosh(b(n - m + i_4))}, \text{ where } i > m \text{ and } b = \cosh^{-1}\left(\frac{2 + R/R_s}{2}\right) \]
Now, the current

\[ i_m = \frac{e_m - e_{m+1}}{R} \]

\[ \frac{e_{m+1}}{e_m} = \frac{\cosh b(n - m - \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})} \]

\[ i_m = \frac{1 - \frac{\cosh b(n - m - \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})}}{R} e_m \]

Let \( A = \frac{1 - \frac{\cosh b(n - m - \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})}}{R} \)

\[ i_m = \frac{A e_m}{R} \]

when \( i < m \) the relation between \( e_i \) and \( e_0 \) is

\[ e_i = e_0 \cosh bi - K \sinh bi, \quad i < m \]

\( K \) may be evaluated using the boundary condition that

\[ i_{m-1} = i_m + i_{m-1} \]

\[ e_{m-1} = e_0 \cosh b(m-1) - K \sinh b(m-1) \]

\[ e_m = e_0 \cosh bm - K \sinh bm \]

\[ i_{m-1} = \frac{e_{m-1} - e_m}{R} = e_0 [\cosh b(m-1) - \cosh bm] \ldots \]

\[ \ldots + K [\sinh bm - \sinh b(m-1)] \]

\[ i_m = e_m / R_m \quad \text{Thus from } i_{m-1} = e_m + i_m \\text{ we obtain} \]

\[ e_0 \left[ \cosh b(m-1) - \cosh b\right] + K \left[ \sinh b - \sinh b(m-1) \right] \]

\[ \frac{1}{R} \]  

\[ \frac{A}{R} + \frac{1}{R_m} e_m \]

Let \( y_m = R/R_m \) then \( K \) equals

\[ K = \frac{(A + y_m)e_m + e_0 \left[ \cosh b - \cosh b(m-1) \right]}{\left[ \sinh b - \sinh b(m-1) \right]} \]

Let \( A_1 = A + y_m; s = \sinh b - \sinh b(m-1) \)

and \( c = \cosh b - \cosh b(m-1) \)

\[ K = \frac{A_1 e_m + ce_0}{s} \]

Thus

\[ e_1 = e_0 \cosh b - \frac{A_1 e_m + ce_0}{s} \sinh b \quad \text{for } i < m \]

Solving for \( e_m \) for the case when \( i = 1 \)

\[ s e_m = s e_0 \cosh b - A_1 e_m \sinh b - ce_0 \sinh b \]

\[ e_m = \frac{e_0 \left( \cosh b - c \sinh b \right)}{s + A_1 \sinh b} \]

\[ e_1 = e_0 \cosh b - \frac{A_1 \left( \cosh b - c \sinh b \right) + c}{s} \sinh b \quad \text{for } i < m \]
\[
\frac{e_i}{e_0} = \frac{\cosh b(n - m + \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})} \left[ \frac{s \cosh b m - \sinh b m}{s + A_1 \sinh b m} \right] \quad \text{for } e \geq m
\]

\[
S = \sinh b m - \sinh(b(m-1))
\]
\[
c = \cosh b m - \cosh(b(m-1))
\]
\[
A_1 = y_{m+1} - \frac{\cosh b(n - m - \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})}
\]

Substituting

\[
\cosh b m - \sinh b m = \sinh b m \cosh b m - \sinh(b(m-1)) \cosh b m - \sinh b m \cosh b m \cdot \cosh(b(m-1)) \sinh b m
\]

\[
\cosh b m - \sinh b m = \cosh(b(m-1)) \sinh b m - \sinh(b(m-1)) \cosh b m
\]

or

\[
S + A_1 \sinh b m = \sinh b m - \sinh(b(m-1)) + (y_{m+1} - \frac{\cosh b(n - m - \frac{1}{2})}{\cosh b(n - m + \frac{1}{2})}) \sinh b m
\]

For \(1 \leq m\)

\[
\frac{e_i}{e_0} = \frac{\cosh b l - A_1 \cosh b m - A_1 \cosh b m + c + A_1 \cosh b m}{s(s + A_1 \sinh b m)} \sinh b l
\]

\[
\frac{e_i}{e_0} = \frac{\cosh b l}{\cosh b l} \left[ \frac{A_1 \cosh b m + c}{A_1 \sinh b m + s} \right] \sinh b l
\]
\[
\frac{e_i}{e_0} = \frac{(A_1 + 1)(\sinh b(m - i)) - \sinh b(m - i - 1)}{(A_1 + 1)\sinh b m - \sinh b(m - 1)}, \quad i \leq m \quad \ldots (A)
\]

\[
\frac{e_n}{e_0} = \frac{\cosh b(n - i + \frac{1}{2})\sinh b}{\cosh b(n - m + \frac{1}{2})(A_1 + 1)\sinh b m - \sinh b(m - 1)}, \quad i > m \quad \ldots (B)
\]

Both these equations are used to solve the unknowns \( y_m \) and \( m \).

To ensure that the equations hold true for all locations of the fault, \( i = 1 \) and \( i = n \) are chosen respectively in equations (A) and (B). Thus

\[
\frac{e_1}{e_0} = R_1 = \frac{(A_1 + 1)\sinh b(m - 1) - \sinh b(m - 2)}{(A_1 + 1)\sinh b m - \sinh b(m - 1)} \quad \ldots (C)
\]

\[
\frac{e_n}{e_0} = R_n = \frac{\cosh b/2\sinh b}{\cosh b(n - m + \frac{1}{2})(A_1 + 1)\sinh b m - \sinh b(m - 1)} \quad \ldots (D)
\]

Considering (C), and solving for \((A_1 + 1)\)

\[
A_1 + 1 = \frac{R_1\sinh b(m - 1) - \sinh b(m - 2)}{R_1\sinh b m - \sinh b(m - 1)}
\]

Considering (D)

\[
\frac{R_n}{\cosh b/2\sinh b} = \frac{1}{\cosh b(n - m + \frac{1}{2})(A_1 + 1)\sinh b m - \sinh b(m - 1)} \quad \ldots (E)
\]
Substituting for \((A_1 + 1)\)

\[
\frac{R_n}{\cosh b/2\sinh b} = \frac{R_1\sinh(bm) - \sinh(bm - b)}{\cosh(bn - bm + b/2) [\sinh^2(bm - b) - \sinh(bm)\sinh(bm - 2b)]}
\]

...\((F)\)

Let \(x = bm\); \(R_1 = R\); \(Q = R - \cosh b\); \(K = \sinh y\)

\[
W = \frac{R_n}{\cosh b/2\sinh b}; \quad y = bn + b/2; \quad M = \sinh b; \quad L \cos h y
\]

Combining \((E)\) and \((F)\)

\[
W = \frac{(R - \cosh b)\sinh x + \cosh x\sinh b}{[\cosh x \cos h y - \sinh x \sin h y][\cosh^2 x \sinh^2 b - \sinh^2 x \sinh b^2]}
\]

\[
= \frac{(R - \cosh b)\sinh x + \cosh x\sinh b}{(\cosh x \cos h y - \sinh x \sin h y)[(\cosh^2 x - \sinh^2 x)\sinh^2 b]}
\]

\[
= \frac{Q \sinh x + M \cosh x}{M^2 L \cosh x - M^2 K \sinh x}
\]

\[(WM^2 L - M)\cosh x = (Q - WM^2 K)\sinh x\]

\[
\tanh x = \frac{WM^2 L - M}{Q + M^2 K W}
\]
\[ x = \tan^{-1}\left( \frac{\frac{W}{M^2}L - M}{Q + \frac{W}{M^2}L} \right) \]

\[ x = \frac{\ln}{2} \frac{Q + \frac{W}{M^2}L + \frac{W}{M^2}L - M}{Q + \frac{W}{M^2}L - \frac{W}{M^2}L - M} \]

\[ m = \frac{1}{2b} \ln \frac{(R^1 - \sinh b - \cosh b) cosh b/2 + R \sinh b (\sin y + \cosh y)}{(R^1 + \sinh b - \cosh b) cosh b/2 + R \sinh b (\sin y - \cosh y)} \]
APPENDIX II

Analytical proof to show that the selected measuring node if behind the fault, predicts the measuring node location.

i.e., for \( i \leq m \), \( m_{\text{predicted}} = m_{\text{actual}} \)

and for \( i > m \), the measurement position \( i \) is predicted

Considering the following equation

\[
R_i = \frac{e_i}{e_0} = \frac{L \cosh(Z) - L \sinh(Z)}{L(T - K)\text{DEG} - (T - K)D^2F + L^2G - LDE\text{F}}
\]

where

\[
y = b_n + b/2 \quad , \quad A = b m_{\text{actual}}
\]

\[
L = \sinh b \quad , \quad Z = b_i \quad , \quad T = A_1 + 1
\]

\[
K = \cosh b \quad , \quad D = \sinh A \quad , \quad E = \cosh A
\]

\[
F = \sinh y \quad , \quad G = \cosh y \quad , \quad H = \cosh b/2
\]
\[ R_n = \frac{LH}{{(T - k)DEG - (T - k)D^2F + LE^2G - LDEF}} \]

Let

\[ \text{DENO} = \left[(T - k)DEG - (T - k)D^2F + LE^2G - LDEF\right] \]

Considering the equation

\[ \tanh(x) = \frac{WM^2L - M}{Q + M^2kW} \]

Substituting and simplifying

\[ \tanh(x) = \frac{\sinh(Z)(LHG - H.DENO)}{LGH \cosh(Z) - LH\sinh Z - H.DENO \cosh(Z) + LH\sinh(Z)} \]

\[ \tanh(x) = \frac{\sinh(Z)(LGH - H.DENO)}{\cosh(Z)(LHG - H.DENO)} \]

\[ \tanh(x) = \tanh(Z) \]

\[ bm = bi \]

\[ m = 1 \]
APPENDIX III

Generalized matrix representation of the ladder network

\[ e_1 - e_2 = R i_1 \quad \text{and} \quad e_1 - e_2 = R i_2 + \frac{R}{R_s} e_2 \]

\[ i_1 = i_2 + e_2 / R_s \]

Representing in a matrix form

\[
\begin{bmatrix}
  e_1 \\
  i_1
\end{bmatrix}
= \begin{bmatrix}
  (1 + R/R_s) & R \\
  1/R_s & 1
\end{bmatrix}
\begin{bmatrix}
  e_2 \\
  i_2
\end{bmatrix}
\]

Generalizing

\[
\begin{bmatrix}
  e_{k-1} \\
  i_{k-1}
\end{bmatrix}
= \begin{bmatrix}
  (1 + R/R_R) & R \\
  1/R_R & 1
\end{bmatrix}
\begin{bmatrix}
  e_k \\
  i_k
\end{bmatrix}
\]
APPENDIX 4V

Formulation of equations for generating pressure distribution in a nonlinear model

\( p_i^2 - p_{i+1}^2 = K F m_i^2 \)  \( \ldots (1) \)

where \( p \) is the absolute pressure, \( m \) is the mass flow and \( KF \) is the constant that depends on the restriction between cars.

\( i = \text{car position} \)

\( n = \text{number of cars} \)

\( m_{L_i} = p_i / R_i \)  \( \ldots (2) \)
\[ m_i = \sum_{j=i+1}^{n} m_{L_j} \quad \ldots (3) \]

From (1) and (3)

\[ p_i^2 - p_{i+1}^2 = K F \sum_{j=i+1}^{n} m_{L_j} \quad \ldots (4) \]

\[ p_i^2 - p_{i+1}^2 = K F \sum_{j=1}^{n} p_j/R_j \quad \ldots (5) \]

If a variable \( B_i \) is defined as the ratio of adjoining pressures equation (5) becomes

\[ B_i^2 = \frac{p_{i-1}^2}{p_i^2} = 1 + \frac{K F}{R_i^2} \sum_{j=1}^{n} \left( \frac{1}{R_j} \right)^2 \left( \frac{1}{R_i} \right)^2 \]

where \( R_R \) is a reference resistance that is arbitrarily selected to normalize the equations.

The variable \( B_i \) can now be calculated from the rear since it depends on all \( B \)'s that come after

\[ B_{n-1}^2 = \frac{p_{n-2}^2}{p_{n-1}^2} = 1 + \frac{K F}{R_{n-2}^2} \left( \frac{R_R}{R_{n-1}} \right)^2 + \frac{1}{R_{n-1}^2} \left( \frac{R_R}{R_n} \right)^2 \]
\[ b_{n-2}^2 = \frac{p_{n-3}^2}{p_{n-2}^2} = 1 + \frac{KF}{R_n^2} \left( \frac{R_R}{R_{n-2}} \frac{p_{n-1}}{R_{n-2}} \right) \frac{R_R}{R_{n-1}} \frac{p_n}{R_n} \]

\[ = 1 + \frac{KF}{R_n^2} \left( \frac{R_R}{R_{n-2}} \frac{p_{n-1}}{R_{n-2}} \right) \frac{R_R}{R_{n-1}} \frac{p_n}{R_n} \]

because \[ \frac{p_n}{p_{n-2}} = \frac{p_{n-1}}{p_{n-2}} \frac{p_n}{p_{n-1}} \]

\[ b_{n-2}^2 = 1 + \frac{KF}{R_n^2} \left( \frac{R_R}{R_{n-2}} \right) \frac{1}{B_{n-1}} \frac{R_R}{R_{n-1}} \frac{1}{B_{n-1}} \frac{1}{B_n} \frac{R_R}{R_n} \]

Therefore by recursion

\[ b_i^2 = 1 + \frac{KF}{R_i^2} \left( \frac{R_R}{R_i} \right) \sum_{j=1}^{n-i} \left( \frac{j}{\Pi_{k=1}^{B_k+1}} \right) \frac{R_R}{R_{j+1}} \]

where the calculations must proceed from \( i = n \) toward \( i = 1 \) in consecutive integer steps.

After all \( b_i \)'s are determined, the ratio of the pressure in any car to the locomotive pressure may be determined as

\[ \frac{p_i}{p_0} = \frac{1}{\Pi_{j=1}^{B_j}} \]

(from reference 1)
A) Derivation of choke flow equation in metric units.

General flow through a converging nozzle

\[ m = \frac{\rho e}{\rho} \frac{T_e}{T_0} \frac{p_0 M_e}{p_0} \frac{A}{R_g T_0} \]
\[ \rho_e = \text{density at section } e \quad \text{kg/m}^3 \]

\[ \rho_0 = \text{density at section } 0 \]

\[ T_e = \text{absolute temperature at section } e \quad \text{in} \quad ^\circ\text{K} \]

\[ T_0 = \text{absolute temperature at section } 0 \quad \text{in} \quad ^\circ\text{K} \]

\[ M_e = \text{Mach number at section } e \quad \text{and is equal to } 1 \]

\[ A_e = \text{Area at section } e \quad \frac{\pi}{4} d_e^2 \quad d_e \quad \text{in m and} \quad \text{m}^2 \]

\[ \gamma = 1.4 \]

\[ p_0 = \text{absolute pressure} \quad \text{kgf/m}^2 \]

\[ R_g = \text{gas constant} \]

\[ T_0 = 293^\circ \text{K} \]

\[ R_g = 287 \frac{\text{Nm}}{\text{kg}^\circ\text{K}} \]

\[ \frac{\rho_0}{\rho_e} = \left( 1 + \frac{\gamma - 1}{2} M_e^2 \right)^{-\frac{1}{\gamma - 1}} \]

\[ \frac{T_0}{T_e} = 1 + \frac{\gamma - 1}{2} M_e^2 \]

When flow is choked \( M_e = 1 \)

and \( \gamma = 1.4 \)

and \( \frac{\rho_0}{\rho_e} = 1.577 \)

and \( \frac{T_0}{T_e} = 1.2 \)
Therefore \( m = 0.237 \times 10^{-2} P_0 A_e \)

\[
\frac{P}{m} = \frac{422}{A_e}
\]

\[
R_s = \frac{P_0}{m} = \frac{537}{d_e^2}
\]

Units for \( R_s \):

\[
\text{Pa/kg/s} = \frac{10^{-3} \text{kPa.s}}{\text{kg}}
\]

B) Conversion and Estimation of value of KF in SI Units

KF is the in-line pipe resistance (nonlinear)

R is shunt resistance (linear) above 206.85 kPa

\[ \Delta P = KFm_1 \]

Pressure drop is related to velocity

\[ \Delta P = \frac{f}{d} \frac{\rho v^2}{2} \]

For uniform flow distribution

\[ \Delta P_{\text{uniform}} = \frac{f}{d} \frac{\rho v^2}{2} \]
Estimating the value of $K_F$ for the experimental setup

$l = 3.429 \text{ m}; \quad d = 6.35 \times 10^{-3} \text{ m}; \quad f = 0.052$

$A = 3.167 \times 10^{-7} \text{ m}^2; \quad \rho = 1.2 \text{ kg/m}^3$

$K_D = 1.17 \times 10^7 \frac{\text{kPa} - \text{s}^2}{\text{kg}^2}$

$K_h = K_{D \text{avg}}$ to match the Units

$= 6.05 \times 10^9 \frac{\text{kPa}^2 - \text{s}^2}{\text{kg}^2}$
APPENDIX VI

Calculator program (for SR-52 Texas Instruments) for solving for fault location \( m \) (ladder network) given \( N \) (number of cars), \( i \) (measurement position) \( R_1 \) \((e_i / e_0)\), \( R_n \) \((e_n / e_0)\) and \( B = \text{Cos}^{-1}((2R_g + R)/2R_g)\).

<table>
<thead>
<tr>
<th>Step 0</th>
<th>LBN</th>
<th>Step 1</th>
<th>2nd 1bl</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Rel 06</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>5</td>
<td>+</td>
</tr>
<tr>
<td>6</td>
<td>Rel 06</td>
<td>7</td>
<td>*</td>
</tr>
<tr>
<td>8</td>
<td>Rel 01</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Sto 02</td>
<td>11</td>
<td>Rel 06</td>
</tr>
<tr>
<td>12</td>
<td>*</td>
<td>13</td>
<td>Rel 03</td>
</tr>
<tr>
<td>14</td>
<td>-</td>
<td>15</td>
<td>Sto 07</td>
</tr>
<tr>
<td>16</td>
<td>Rel 06</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>2</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Sto 08</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>(</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Rel 04</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>(</td>
<td>27</td>
<td>Rel 07</td>
</tr>
<tr>
<td>28</td>
<td>SBR</td>
<td>29</td>
<td>2nd 1</td>
</tr>
<tr>
<td>30</td>
<td>+</td>
<td>31</td>
<td>Rel 07</td>
</tr>
<tr>
<td>32</td>
<td>SBR</td>
<td>33</td>
<td>2nd 2</td>
</tr>
<tr>
<td>34</td>
<td>)</td>
<td>35</td>
<td>Sto 09</td>
</tr>
<tr>
<td>36</td>
<td></td>
<td>37</td>
<td>SBR</td>
</tr>
<tr>
<td>38</td>
<td>Rel 07</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>2nd 1</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Rel 07</td>
<td>43</td>
<td>SBR 2nd 2</td>
</tr>
<tr>
<td>44</td>
<td>)</td>
<td>45</td>
<td>Sto 10</td>
</tr>
<tr>
<td>46</td>
<td>)</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td>(</td>
<td>49</td>
<td>Rel 08</td>
</tr>
<tr>
<td>50</td>
<td>SBR</td>
<td>51</td>
<td>2nd 1</td>
</tr>
<tr>
<td>52</td>
<td>+</td>
<td>53</td>
<td>Rel 08</td>
</tr>
<tr>
<td>54</td>
<td>SBR</td>
<td>55</td>
<td>2nd 2</td>
</tr>
<tr>
<td>Step</td>
<td>56</td>
<td>)</td>
<td>57</td>
</tr>
<tr>
<td>------</td>
<td>----</td>
<td>---</td>
<td>----</td>
</tr>
<tr>
<td>58</td>
<td>+</td>
<td>59</td>
<td>Rcl 05</td>
</tr>
<tr>
<td>60</td>
<td>*</td>
<td>61</td>
<td>Rcl 10</td>
</tr>
<tr>
<td>62</td>
<td>*</td>
<td>63</td>
<td>(</td>
</tr>
<tr>
<td>64</td>
<td>(</td>
<td>65</td>
<td>Rcl 02</td>
</tr>
<tr>
<td>66</td>
<td>SBR</td>
<td>67</td>
<td>2nd 1</td>
</tr>
<tr>
<td>68</td>
<td>-</td>
<td>69</td>
<td>Rcl 02</td>
</tr>
<tr>
<td>70</td>
<td>SBR</td>
<td>71</td>
<td>2nd 2</td>
</tr>
<tr>
<td>72</td>
<td>)</td>
<td>73</td>
<td>Sto 12</td>
</tr>
<tr>
<td>74</td>
<td>+</td>
<td>75</td>
<td>(</td>
</tr>
<tr>
<td>76</td>
<td>Rcl 02</td>
<td>77</td>
<td>SBR</td>
</tr>
<tr>
<td>78</td>
<td>2nd 1</td>
<td>79</td>
<td>+</td>
</tr>
<tr>
<td>80</td>
<td>Rcl 02</td>
<td>81</td>
<td>SBR</td>
</tr>
<tr>
<td>82</td>
<td>2nd 2</td>
<td>83</td>
<td>)</td>
</tr>
<tr>
<td>84</td>
<td>Sto 13</td>
<td>85</td>
<td>)</td>
</tr>
<tr>
<td>86</td>
<td>)</td>
<td>87</td>
<td>+</td>
</tr>
<tr>
<td>88</td>
<td>(</td>
<td>89</td>
<td>(</td>
</tr>
<tr>
<td>90</td>
<td>Rcl 04</td>
<td>91</td>
<td>-</td>
</tr>
<tr>
<td>92</td>
<td>Rcl 09</td>
<td>93</td>
<td>+</td>
</tr>
<tr>
<td>94</td>
<td>Rcl 10</td>
<td>95</td>
<td>)</td>
</tr>
<tr>
<td>96</td>
<td>*</td>
<td>97</td>
<td>Rcl 11</td>
</tr>
<tr>
<td>98</td>
<td>+</td>
<td>99</td>
<td>Rcl 05</td>
</tr>
<tr>
<td>100</td>
<td>*</td>
<td>101</td>
<td>Rcl 10</td>
</tr>
<tr>
<td>102</td>
<td>*</td>
<td>103</td>
<td>(</td>
</tr>
<tr>
<td>104</td>
<td>Rcl 12</td>
<td>105</td>
<td>-</td>
</tr>
<tr>
<td>106</td>
<td>Rcl 13</td>
<td>107</td>
<td>)</td>
</tr>
<tr>
<td>108</td>
<td>)</td>
<td>109</td>
<td>)</td>
</tr>
<tr>
<td>110</td>
<td>Inx</td>
<td>111</td>
<td>*</td>
</tr>
<tr>
<td>112</td>
<td>(</td>
<td>113</td>
<td>Rcl 06</td>
</tr>
<tr>
<td>114</td>
<td>*</td>
<td>115</td>
<td>2</td>
</tr>
<tr>
<td>116</td>
<td>)</td>
<td>117</td>
<td>2nd 1/x</td>
</tr>
<tr>
<td>118</td>
<td>=</td>
<td>119</td>
<td>HLT</td>
</tr>
</tbody>
</table>
With slight modification it may be used with TI-59
<table>
<thead>
<tr>
<th>Uniform Voltage Distribution</th>
<th>Uniform Voltage Distribution</th>
<th>Uniform Voltage Distribution (Electrical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N = 10 ) ( e_0 = 10 \text{ VDC} )</td>
<td>( N = 20 ) ( e_0 = 10 \text{ VDC} )</td>
<td>( N = 50 \text{ cars} ) ( e_0 = 10 \text{ VDC} )</td>
</tr>
<tr>
<td>( e_0 = 9.901 )</td>
<td>( e_1 = 9.819 )</td>
<td>( e_1 = 9.692 )</td>
</tr>
<tr>
<td>( e_2 = 9.814 )</td>
<td>( e_2 = 9.651 )</td>
<td>( e_2 = 9.402 )</td>
</tr>
<tr>
<td>( e_3 = 9.737 )</td>
<td>( e_3 = 9.496 )</td>
<td>( e_3 = 9.127 )</td>
</tr>
<tr>
<td>( e_4 = 9.71 )</td>
<td>( e_4 = 9.351 )</td>
<td>( e_4 = 8.862 )</td>
</tr>
<tr>
<td>( e_5 = 9.615 )</td>
<td>( e_5 = 9.216 )</td>
<td>( e_5 = 8.607 )</td>
</tr>
<tr>
<td>( e_6 = 9.567 )</td>
<td>( e_6 = 9.087 )</td>
<td>( e_6 = 8.353 )</td>
</tr>
<tr>
<td>( e_7 = 9.528 )</td>
<td>( e_7 = 8.967 )</td>
<td>( e_7 = 8.109 )</td>
</tr>
<tr>
<td>( e_8 = 9.481 )</td>
<td>( e_8 = 8.857 )</td>
<td>( e_8 = 7.876 )</td>
</tr>
<tr>
<td>( e_9 = 9.47 )</td>
<td>( e_9 = 8.758 )</td>
<td>( e_9 = 7.655 )</td>
</tr>
<tr>
<td>( e_{10} = 8.664 )</td>
<td>( e_{10} = 8.203 )</td>
<td>( e_{10} = 7.432 )</td>
</tr>
</tbody>
</table>
UNIFORM VOLTAGE DISTRIBUTION (electrical)

N = 50, \( \phi_0 = 10 \cdot \text{VDC} \)

\[
\begin{align*}
\phi_{31} &= 4.248 & \phi_{41} &= 3.477 \\
\phi_{32} &= 4.151 & \phi_{42} &= 3.419 \\
\phi_{33} &= 4.061 & \phi_{43} &= 3.368 \\
\phi_{34} &= 3.978 & \phi_{44} &= 3.319 \\
\phi_{35} &= 3.893 & \phi_{45} &= 3.271 \\
\phi_{36} &= 3.813 & \phi_{46} &= 3.228 \\
\phi_{37} &= 3.739 & \phi_{47} &= 3.187 \\
\phi_{38} &= 3.668 & \phi_{48} &= 3.152 \\
\phi_{39} &= 3.600 & \phi_{49} &= 3.119 \\
\phi_{40} &= 3.536 & \phi_{50} &= 3.088
\end{align*}
\]
<table>
<thead>
<tr>
<th>i</th>
<th>Actual Fault</th>
<th>Predicted Fault</th>
<th>e_i</th>
<th>Actual Fault</th>
<th>Predicted Fault</th>
<th>e_i</th>
<th>Actual Fault</th>
<th>Predicted Fault</th>
<th>e_i</th>
<th>Actual Fault</th>
<th>Predicted Fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>9.877</td>
<td>1</td>
<td>1.2200</td>
<td>9.875</td>
<td>2</td>
<td>1.8158</td>
<td>9.878</td>
<td>3</td>
<td>2.6935</td>
<td>9.877</td>
<td>4</td>
</tr>
<tr>
<td>5.</td>
<td>9.588</td>
<td>1</td>
<td>5.5081</td>
<td>9.568</td>
<td>2</td>
<td>5.2289</td>
<td>9.552</td>
<td>3</td>
<td>5.2659</td>
<td>9.531</td>
<td>4</td>
</tr>
<tr>
<td>i_1</td>
<td>i_a</td>
<td>i_p</td>
<td>i_1</td>
<td>i_a</td>
<td>i_p</td>
<td>i_1</td>
<td>i_a</td>
<td>i_p</td>
<td>i_1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N = 10</td>
<td>e_0 = 10 VDC</td>
<td>R_n = 332.9 ohms</td>
<td>R_R = 1000.0 ohms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>9.879</td>
<td>5</td>
<td>4.1702</td>
<td>9.874</td>
<td>6</td>
<td>4.2533</td>
<td>9.879</td>
<td>7</td>
<td>5.5959</td>
<td>9.88</td>
<td>8</td>
</tr>
<tr>
<td>5.</td>
<td>9.517</td>
<td>5</td>
<td>5.1489</td>
<td>9.512</td>
<td>6</td>
<td>5.9833</td>
<td>9.519</td>
<td>7</td>
<td>7.0376</td>
<td>9.520</td>
<td>8</td>
</tr>
<tr>
<td>10.</td>
<td>9.373</td>
<td>5</td>
<td>10.000</td>
<td>9.35</td>
<td>6</td>
<td>10.000</td>
<td>9.337</td>
<td>7</td>
<td>10.000</td>
<td>9.321</td>
<td>8</td>
</tr>
<tr>
<td>( \hat{v}_i )</td>
<td>( \hat{v}_a )</td>
<td>( \hat{m}_p )</td>
<td>( \hat{v}_i )</td>
<td>( \hat{v}_a )</td>
<td>( \hat{m}_p )</td>
<td>( \hat{v}_i )</td>
<td>( \hat{v}_a )</td>
<td>( \hat{m}_p )</td>
<td>( \hat{v}_i )</td>
<td>( \hat{v}_a )</td>
<td>( \hat{m}_p )</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>9.88</td>
<td>9</td>
<td>7.2051</td>
<td>9.88</td>
<td>10</td>
<td>7.9312</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.774</td>
<td>9</td>
<td>7.8627</td>
<td>9.774</td>
<td>10</td>
<td>8.6552</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.599</td>
<td>9</td>
<td>8.7918</td>
<td>9.520</td>
<td>10</td>
<td>9.7788</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.596</td>
<td>9</td>
<td>8.8391</td>
<td>9.396</td>
<td>10</td>
<td>9.7311</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.303</td>
<td>9</td>
<td>10.000</td>
<td>9.285</td>
<td>10</td>
<td>10.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SIZE OF THE FAULT IN THE BRAKEPIPE MODEL OF THE AIR BRAKE

0.5715 mm (diameter) \( R_r (\text{Resistance Reference}) = 1.687E+08 \text{ kgf.s/m}^2 \)

0.8128 mm; \( R_f/R_r = 0.5132 \); \( R_f (\text{Resistance Fault}) = 8.568E+07 \)

0.838 = 0.465 = 7.849E+07

1.067 = 0.287 = 8.846E+07

1.143 = 0.25 = 4.221E+07

1.778 = 0.103 = 1.744E+07
### Average Values of Unfaulted Pressure Distribution in Brake Pipe Model (Experimental)

**Calibration:**
- 0 kPa - 150 kPa
- 0 V - 10 V
- N=10
- \( R_g = 1.687 \times 10^8 \text{ kg}_f \cdot \text{s}/\text{kg}_m \cdot \text{m}^2 \)

<table>
<thead>
<tr>
<th>Node</th>
<th>Pressure at</th>
<th>Trial 1</th>
<th>Trial 2</th>
<th>Trial 3</th>
<th>Trial 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_0 )</td>
<td>( 0 \text{ kPa} )</td>
<td>10.00</td>
<td>600.00</td>
<td>10.00</td>
<td>600.00</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>( 0 \text{ kPa} )</td>
<td>1.375</td>
<td>579.375</td>
<td>1.364</td>
<td>579.54</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>( 0 \text{ kPa} )</td>
<td>2.397</td>
<td>564.065</td>
<td>2.421</td>
<td>563.85</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>( 0 \text{ kPa} )</td>
<td>3.161</td>
<td>552.585</td>
<td>3.202</td>
<td>551.97</td>
</tr>
<tr>
<td>( P_4 )</td>
<td>( 0 \text{ kPa} )</td>
<td>3.722</td>
<td>544.17</td>
<td>3.767</td>
<td>543.495</td>
</tr>
<tr>
<td>( P_5 )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.136</td>
<td>537.96</td>
<td>4.108</td>
<td>538.38</td>
</tr>
<tr>
<td>( P_6 )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.383</td>
<td>534.255</td>
<td>4.380</td>
<td>534.30</td>
</tr>
<tr>
<td>( P_7 )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.581</td>
<td>531.285</td>
<td>4.581</td>
<td>531.285</td>
</tr>
<tr>
<td>( P_8 )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.701</td>
<td>529.485</td>
<td>4.696</td>
<td>529.56</td>
</tr>
<tr>
<td>( P_9 )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.829</td>
<td>527.565</td>
<td>4.771</td>
<td>528.435</td>
</tr>
<tr>
<td>( P_{10} )</td>
<td>( 0 \text{ kPa} )</td>
<td>4.834</td>
<td>527.49</td>
<td>4.781</td>
<td>528.285</td>
</tr>
</tbody>
</table>
**DIFFERENCE METHOD (PNEUMATIC)**

**PRESSURE DISTRIBUTION FOR DIFFERENT FAULT POSITIONS IN THE MODEL**

\[ P_0 = 600 \text{ kPa} \quad V_0 = 10 \text{V} \quad \text{DIAR}_R = 0.5715 \text{mm} \quad \text{DIAR}_M = 1.0414 \text{mm} \quad \text{R}_M/R_R = 0.3 \quad N=10 \text{ cars} \quad \# = \text{fault pos.} \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V Volts</td>
<td>P kPa</td>
<td>V Volts</td>
<td>P kPa</td>
<td>V Volts</td>
</tr>
<tr>
<td>1.</td>
<td>2.014</td>
<td>569.79</td>
<td>1.949</td>
<td>570.77</td>
</tr>
<tr>
<td>2.</td>
<td>3.074</td>
<td>553.89</td>
<td>3.454</td>
<td>548.19</td>
</tr>
<tr>
<td>3.</td>
<td>3.833</td>
<td>542.51</td>
<td>4.215</td>
<td>536.78</td>
</tr>
<tr>
<td>4.</td>
<td>4.416</td>
<td>533.76</td>
<td>4.799</td>
<td>528.02</td>
</tr>
<tr>
<td>5.</td>
<td>4.798</td>
<td>528.03</td>
<td>5.179</td>
<td>722.32</td>
</tr>
<tr>
<td>6.</td>
<td>5.098</td>
<td>523.53</td>
<td>5.474</td>
<td>517.89</td>
</tr>
<tr>
<td>7.</td>
<td>5.309</td>
<td>520.37</td>
<td>5.677</td>
<td>514.85</td>
</tr>
<tr>
<td>8.</td>
<td>5.441</td>
<td>518.39</td>
<td>5.809</td>
<td>512.87</td>
</tr>
<tr>
<td>9.</td>
<td>5.515</td>
<td>517.28</td>
<td>5.882</td>
<td>511.77</td>
</tr>
<tr>
<td>10.</td>
<td>5.522</td>
<td>517.17</td>
<td>5.896</td>
<td>511.56</td>
</tr>
<tr>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>V Volts P kPa</td>
<td>V Volts P kPa</td>
<td>V Volts P kPa</td>
<td>V Volts P kPa</td>
<td>V Volts P kPa</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.</td>
<td>1.890</td>
<td>571.65</td>
<td>1.826</td>
<td>572.61</td>
</tr>
<tr>
<td></td>
<td>1.776</td>
<td>573.36</td>
<td>1.773</td>
<td>573.41</td>
</tr>
<tr>
<td></td>
<td>1.812</td>
<td>572.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>3.344</td>
<td>549.84</td>
<td>3.264</td>
<td>551.04</td>
</tr>
<tr>
<td></td>
<td>3.188</td>
<td>552.18</td>
<td>3.185</td>
<td>552.23</td>
</tr>
<tr>
<td></td>
<td>3.225</td>
<td>551.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>4.521</td>
<td>532.19</td>
<td>4.438</td>
<td>533.43</td>
</tr>
<tr>
<td></td>
<td>4.328</td>
<td>535.05</td>
<td>4.309</td>
<td>535.37</td>
</tr>
<tr>
<td></td>
<td>4.379</td>
<td>534.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>5.435</td>
<td>518.48</td>
<td>5.358</td>
<td>519.63</td>
</tr>
<tr>
<td></td>
<td>5.217</td>
<td>521.75</td>
<td>5.194</td>
<td>522.09</td>
</tr>
<tr>
<td></td>
<td>5.271</td>
<td>520.94</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>6.094</td>
<td>508.59</td>
<td>6.007</td>
<td>509.875</td>
</tr>
<tr>
<td></td>
<td>5.839</td>
<td>512.42</td>
<td>5.806</td>
<td>512.91</td>
</tr>
<tr>
<td></td>
<td>5.894</td>
<td>511.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>6.620</td>
<td>500.70</td>
<td>6.565</td>
<td>501.53</td>
</tr>
<tr>
<td></td>
<td>6.374</td>
<td>504.39</td>
<td>6.331</td>
<td>505.04</td>
</tr>
<tr>
<td></td>
<td>6.424</td>
<td>503.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>6.816</td>
<td>497.76</td>
<td>7.009</td>
<td>494.87</td>
</tr>
<tr>
<td></td>
<td>6.848</td>
<td>497.28</td>
<td>6.744</td>
<td>498.84</td>
</tr>
<tr>
<td></td>
<td>6.856</td>
<td>497.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>6.943</td>
<td>495.86</td>
<td>7.124</td>
<td>493.14</td>
</tr>
<tr>
<td></td>
<td>7.165</td>
<td>492.53</td>
<td>7.158</td>
<td>492.63</td>
</tr>
<tr>
<td></td>
<td>7.185</td>
<td>492.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>6.996</td>
<td>495.12</td>
<td>7.198</td>
<td>492.03</td>
</tr>
<tr>
<td></td>
<td>7.232</td>
<td>491.52</td>
<td>7.302</td>
<td>490.47</td>
</tr>
<tr>
<td></td>
<td>7.448</td>
<td>488.28</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>7.016</td>
<td>494.76</td>
<td>7.213</td>
<td>491.81</td>
</tr>
<tr>
<td></td>
<td>7.230</td>
<td>481.43</td>
<td>7.442</td>
<td>488.37</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.558</td>
<td>486.63</td>
<td></td>
</tr>
</tbody>
</table>
Figure 1: Schematic of the Air brakepipe system
Figure 2. Ladder Network
Figure 3. Separating the ladder network into two sections.
Figure 4. Relation between first and last nodal potentials as a function of fault location.
Figure 5. Illustration of the fact that equation 12 is not function of potential measurements after the fault \( \frac{e_{10}}{e_0} \) (vs) \( \frac{e_2}{e_0} \).
Figure 6: Illustration of the fact that equation 12 is not a function of potential if measurements after the fault ($e_y/e_0$) is taken from $e_0$. 

$n/10, R/R_s = 0.001$ 

$R/R_s = 0.3, 0.1$
Figure 7. Effect of measurement position on predicted fault location
Figure 8. Sensitivity of fault prediction as a function of measurement position.
Figure 9. Sensitivity of fault prediction as a function of the number of nodes with fault magnitude as a parameter.
Figure 10. Sensitivity of fault prediction as a function of resistance ratio with fault magnitude as a parameter.
Figure 11. Simulated results of the difference method when applied to linear ladder network for $N = 10$ and $R_m = 100$ ohms.
Figure 12. Simulated results of the difference method when applied to linear ladder network for $N = 20$ and $R_m = 100$ ohms.
Figure 13. Simulated results of the difference method when applied to linear ladder network for $N = 50$ and $R_m = 100$ ohms.
Figure 14. Simulated results of the difference method when applied to linear ladder network for $N = 10$ and $R_m = 333$ ohms.
Figure 15. Simulated results of the difference method when applied to linear ladder network for $N = 20$ and $R_m = 333$ ohms.
Figure 16. Simulated results of the difference method when applied to linear ladder network for $N = 50$ and $R = 333$ ohms.
Figure 17. Linear model of the brakepipe for the ratio method.
Figure 18. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 10$ and $R_m = 100$ ohms.
Figure 19. Simulated results on the computer when Ratio method is applied to the linear ladder network for \( N = 10 \) and \( R_m = 100 \) ohms.
Figure 20. Simulated results on the computer when Ratio method is applied to the linear ladder network for \( N = 50 \) and \( R_m = 100 \) ohms.
Figure 21. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 10$ and $R_m = 333$ ohms.
Figure 22. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 50$ and $R_m = 333$ ohms.
Figure 23. Simulated results on the computer when Ratio method is applied to the linear ladder network for $N = 50$ and $R_m = 333$ ohms.
Figure 24. Electrical ladder network model
Figure 25. Comparison of Theoretical Fault Prediction and Computer Solution With Simulated Error
Figure 26. Comparison of Theoretical fault prediction and Electrical model (n = 20, R_m = 100 ohm)
Figure 27. Comparison of theoretical fault prediction and Electrical model ($n = 20$, $R_m = 333$ ohm)
Figure 28. Comparison of Theoretical fault prediction and Electrical model ($n = 50, R_m = 100 \text{ ohm}$)
Figure 29. Experimental fault prediction by the Difference method when applied to linear brakepipe model ($n = 10, R_m = 100$ ohms)
Figure 30. Experimental fault prediction by the Difference method when applied to linear brakepipe model ($N = 20$, $R_m = 100$ ohms).
Figure 31. Experimental fault prediction by the Difference method when applied to linear brakepipe model (N = 50, R_m = 100 ohms)
Figure 32. Experimental fault prediction by the Difference method when applied to linear brakepipe model. 
\( N = 10, R_m = 333 \text{ ohms} \)
Figure 33. Experimental fault prediction by the Difference method when applied to linear brakepipe model ($N = 20$, $R_m = 333$ ohms)
Figure 34. Experimental fault prediction by the Difference model when applied to linear brakepipe model
\( N = 50, R_m = 333 \text{ ohms} \)
Figure 35. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ($N = 10$, $R_m = 333$ ohms)
Figure 36. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ($N = 20$, $R_m = 100$ ohms).
Figure 37. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ($N = 50$, $R_m = 100$ ohms, $R_s = 1000$ ohms)
Figure 38. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ($N = 10$, $R_m = 333$ ohms).
Figure 39. Experimental fault prediction by the Ratio method when applied to linear brakepipe model ($N = 20$, $R_m = 333$ ohms).
Figure 40. Experimental fault prediction by the Ratio method when applied to linear brakepipe model (N = 20, R_m = 333 ohms)
Figure 41. Nonlinear ladder network model for brakepipe leakage
Figure 42. Simulated results of the Difference method when applied to Nonlinear brakepipe model (N = 10, $R_F/R_R = 0.465$)
\[ \frac{K}{R^2} = 0.0022 \]
\[ \frac{R_F}{R_R} = 0.3 \]
\[ N = 10 \]

Figure 43. Simulated results of the Difference method when applied to Nonlinear brakepipe model \((N = 0.10, \frac{R_F}{R_R} = 0.3)\)
Figure 44. Modified model for the Ratio method
Figure 45. Simulated results of the Ratio method when applied to Nonlinear brakepipe model ($N = 10$, $R_F/R_R = 0.465$).
Figure 46. Simulated results of the Ratio method when applied to Non-linear brakepipe model \((N = 10, R_F/R_R = 0.3)\)
Figure 47. Simulated results of the Transformation method when applied to Nonlinear brakepipe model (N = 10, $R_F/R_R = 0.465$)
Figure 48. Simulated results of the Transformation method when applied to Nonlinear brakepipe model ($N = 10$, $R_F/R_R = 0.3$)
Figure 49. Picture of the scaled-down brakepipe model along with measuring equipment.
Figure 50. Experimental pressure distribution in the scale model of the brakepipe with $R_F/R_R = 0.465$.
Figure 61. Experimental pressure distribution in the scale model of the brakepipe with $R_F/R_R = 0.3$. 

$K/R_R^2 = 0.00077$
$R_F/R_R = 0.3$
$N = 10$

NO FAULT DISTRIBUTION
Figure 52. Experimental fault prediction by the Difference method with $R_F/R_R = 0.465$
Figure 53. Experimental fault prediction by the Difference method with $R_F/R_R = 0.3$
Figure 64. Experimental fault prediction by the Ratio method with $R_F/R_R = 0.465$. 

- $K/R_R^2 = 0.00077$
- $R_F/R_R = 0.465$
- $N = 10$
Figure 55. Experimental fault prediction by the Ratio method with $R_F/R_R = 0.3$.
Figure 56. Experimental fault prediction by the Transformation method with $R_F/R_R = 0.465$
**RATIO METHOD FOR THE ELECTRICAL MODEL**

**PROGRAM LINDIFF(INPUT,OUTPUT)**

**#M IS THE NUMBER OF SECTIONS, R IS THE INLIVE RESISTANCE**

**RL IS THE BLUNT RESISTANCE; P IS THE FAULT, PO IS THE**

**SOURCE; P IS PRESSURE OR VOLTAGE, PD IS THE NORMALIZED**

**PRESSURE, DELTP IS THE DIFFERENCE IN THE NORMALIZED PRESSURES**

**AND LEAK AND GEAR ARE DUMMY VARIABLES.**

**DIMENSION P(10),G(10),DELTP(10),PD(10),GD(10),PRA(10),PRDIF(10)**

**C**

**# DATA##**

**C**

**#**

**N=10**

**R=1.**

**RL=1000.**

**RD=RL**

**PO=10.**

**C**

**#**

**PRINT 25+RL**

**FORMAT(6X#NO FAULT DISTRIBUTION;5X;#RL=RD=ND;F7.2;///)**

**ND=1**

**C**

**# INVOKING THE SUBROUTINE THAT GENERATES THE NO FAULT**

**# PRESSURE DISTRIBUTION**

**C**

**# CALL PRESSURE(R;RL;RD;N;ND;P;G)**

**DO 35 KL=1,N**

**PD(KL)=P(KL)/P(N)**

**G(KL)=0(KL)**

**PRINT 100,1,X,L,KL,P(KL)**

**100 FORMAT(20X;I3;5X;#P=I10.7)**

**35 CONTINUE**

**C**

**# READING THE SIZE OF THE FAULT**

**C**

**DO 45 LO=1,N**

**READ RD**

**PRINT 200,RD,RL**

**FORMAT(10X;# THE SIZE OF THE FAULT IS =8,E6.2,2X,#OOGHE=2X**

**+AND THE REST ARE =8,E6.3;X,#OOGHE)**

**DO 1 ND=1,N**

**PRINT 400,ND**

**FORMAT(5X;#THE FAULT IS IN POSITION...;I3;///)**

**C**

**# GENERATING FAULT PRESSURE DISTRIBUTION AND CALCULATING THE**

**# DIFFERENCE BETWEEN NO FAULT AND FAULT PRESSURES**

**C**

**# CALL PRESSURE(R;RL;RD;N;ND;P;G)**

**DO 85 LU=1,H**

**PRA(LU)=P(LU)/P(N)**

**PRDIF(LU)=ABS(PRA(LU)-PD(LU))**

**85 CONTINUE**

**C**

**# PROGRAM FOR FORMATTING THE OUTPUT**

**C**

**DO 23 JZ=1,H**

**LF=NL*Z+1**

**FORMAT(LF;ST;1.E-12) DO TO 90**

**23 CONTINUE**

**J0=1**

**NO TO 84**
90 IF(LF.EQ.1) GO TO 41
   JO=LF+1
   GO TO 34
41   JO=LF+1
34   DO 52 JO=1,N
      IF(JO.EQ.5) GO TO 80
      PRINT 600,J0,PRA(J0),J0,PRDIF(J0)
600  FORMAT(2X,8F13.8,P(N)=3X,F10.8,2X,DELTP(8,I3,R)/P(N)=3X,F10.8)
     GO TO 52
52   CONTINUE
C **************************************************************
C SIMULATION METHOD APPLIED TO THE PIPELINE SIMULATION
C **************************************************************
C PROGRAM: RATIO
C INPUT: R, P1, P2
C OUTPUT: Pressures at end points
C**************************************************************
C REAL R
C DIMENSION P(20), B(20), PR(20), PRD(20)
C
C DATA: "K" IS THE IN-LINE RESISTANCE, "DIAR" IS THE SIZE OF THE
C REFERENCE LEAK, "DIARD" IS THE DIAMETER OF THE LEAK, "RR" IS THE
C RESISTANCE OF THE REFERENCE LEAK, "RD" IS THE RESISTANCE OF THE
C DIFFERENT LEAK, "DD" IS DIMENSIONLESS QUANTITY.
C**************************************************************
C N=10
C K=2.16E+13
C DIAR=0.0715
C RR=1.58E+08
C DIARD=0.038
C RD=7.049E+07
C DD=K/(RRRRRR)
C**************************************************************
C INITIALIZING THE INDICES
C**************************************************************
C J=0
C I=0
C**************************************************************
C INITIALIZING THE LEAKS TO HAVE EQUAL RESISTANCES
C**************************************************************
C DO 25 J=1,N
C 25 R(J,0)=RR
C IF(J,.5,1) GO TO 55
C I=J+1
C R(I,L)=RD
C**************************************************************
C CALLING SUBROUTINE WHICH GENERATES B'S
C**************************************************************
C CALL PRSLEN(RD,RR,L,R,B)
C**************************************************************
C IF(MP,.1) GO TO 48
C PRINT 333
C CALL PRSLEN(RD,RR,L,R,B)
C**************************************************************
C DO 201 J=1,N
C IF(MP,.1) GO TO 48
C PRINT 444,1L
C IF(MP,.1) GO TO 48
C PRINT 444,1L
C CALL PRSLEN(RD,RR,L,R,B)
C**************************************************************
C DO 201 J=1,N
C IF(MP,.1) GO TO 48
C END
41  JO=LF41
    GO TO 34
34  DO 32 JO=1,N
    IF(JO.EQ.5) GO TO 80
    PRINT 222,JO,B(JO),J0,PR(JO),J0,PRA(JO),PRIF(JO)
222  FORMAT(2X,99(8.12E8),8F10.8,2X,8F10.8,2X,8F10.8)
    IF(JO.EQ.5) GO TO 80
    PRINT 222,JO,B(JO),J0,PR(JO),J0,PRA(JO),PRIF(JO)
224  FORMAT(2X,99(8.12E8),8F10.8,2X,8F10.8,2X,8F10.8)
    IF(JO.EQ.5) GO TO 80
    80  CONTINUE
32  3=C=(I=I+1)
    IF(I.NE.I1) GO TO 44
    STOP
END
SUBROUTINE PRESSGEN(BD,RR,R,R,B)
C THIS SUBPROGRAM GENERATES THE "B" SCALE
C
DIMENSION R(20),B(20)
B(N)=SORT(1,+,BD**R/R/(R(N)**R(N)))
MM=N-1
DO 2 J=1,MM
    LL=MM-J
    SUM=RR/R(LL)
    PROD=1.0
2    DO 3 K=1,J
        PROD=PROD/B(LL+K)
        SUM=SUM+PROD/R(LL+K)
3    CONTINUE
    B(LL)=SORT(1,+,BD**SUM/SUM)
    3 CONTINUE
    RETURN
END
SUBROUTINE PRAT(B,M,PN)
C THIS SUBPROGRAM GENERATES THE RATIO OF PRESSURES P(I)/P0
C
DIMENSION B(20),P(20),PN(20)
PRO=1.0
DO 8 M=1,N
    PRO=PRO/B(M)
8  PRO=P/N
7  CONTINUE
DO 44 JK=1,N
    PN(JK)=P(JK)/P(N)
44  CONTINUE
RETURN
END
SUBROUTINE PRATIO(B,P,H,P,P,H,PRA,PRIF)
C THIS SUBPROGRAM CALCULATES THE DIFFERENCE IN NORMALIZED PRESSURES
C
DIMENSION B(20),P(20),PRIF(20),P(20),PRA(20),PN(20)
PRAM=1.0
DO 27 LB=1,N
    PRESURE=PRESURE/B(LB)
     PRA(LB)=PRESURE
27 CONTINUE
   DO 70 IJ=1,N
   PRA(IJ)=PR(IJ)/PR(N)
   70 CONTINUE
   DO 14 J=1,N
   PROF(J)=ABS(PR(J)-PRA(J))
14 CONTINUE
   RETURN.
END
C

C #TRANSFORMATION METHOD APPLIED TO PNEUMATIC MODEL BB
C #PROGRAM TRANSFORM({INPUT;OUTPUT})
C REAL K,N,ND
C DIMENSION P(20),B(20),N(20),PSR(20),ED1(20)
C PRINT 20
C
C
C DATA='K' IS THE IN-LINE RESISTANCE, 'DIARR' IS THE SIZE OF THE
C REFERENCE LEAK, 'DIARD' IS THE DIAMETER OF THE LEAK, 'RR' IS THE
C RESISTANCE OF THE REFERENCE LEAK, 'RD' IS THE RESISTANCE OF THE
C DIFFERENT LEAK, 'BD' IS DIMENSIONLESS QUANTITY.
C
C
C N=10
C K=2.21E+13
C DIARR=0.5715
C R=1.68E+08
C DIARD=0.638
C RD=7.849E+07
C BD=K/(RR*RR)
C
C CALLING SUBPROGRAM WHICH CALCULATES THE EQUIVALENT LINEAR
C VALUES (N) FOR THE CORRESPONDING NONLINEAR VALUES
C CALL EOTAM(N,K,R,RD,BD,T,ED1)
C
C DO 25 JN=1,N
C 25 R(JN)=RR
C CALLING SUBROUTINE WHICH GENERATED BY'S
C CALL PRESGEN(BD,RR,B,R,B)
C CALLING SUBROUTINE WHICH CALCULATES THE PRESSURE RATIOS FOR UNIFORM DISTRIBUTION
C CALL PREMAT(B,N,P,PSR)
C DO 43 LN=1,N
C 43 LN=1,N
C 25 R(JN)=RR
C KB=0.651
C R(KB)=RD
C CALL PRESGEN(BD,RR,K,R,B)
C CALL PREMAT(B,N,P,PSR)
C PRINT 456
C 456 FORMAT(/'X=NONLINEAR N=2X=EQUIVALENT*2X=PREDICTED FAULTS,16X=LINEAR N=2X=POSITION IN LINEAR N=2X=P-B'),
C THIS PART OF THE PROGRAM CALCULATES THE FAULT POSITION IN LINEAR C TERMS, THEY HAVE TO BE RETRANFORMED TO EQUIVALENT NONLINEAR
C VALUES BY INTERPOLATION.
C
C DO 700 KL=1,N
C R(KL)=PSR(KL)
C AR= (1+2)*KLU5(KL)
C AR=(1+2)*KLU5(KL)
C ARR=(1+2)*KLU5(KL)
C B1=(R(KL)-SINH(ARR)*COSH(ARR))*COSH(0.5)
C B2=(R(KL)+SINH(ARR)*COSH(ARR))*COSH(0.5)
C C1=RHNSINH(ARR)*SINH(ARR)*COSH(ARR)
C C2=RHNSINH(ARR)*SINH(ARR)*COSH(ARR)
C S1=(B1+C1)/(B2+C2)
KZ=0-I
DO 7 KZ=1+KD
CC=ABS(R1*(KZ-1))
DO=ALOS(CC+BRT(CC-CC+1,N))
EBI(N-KZ)=(ARG-DO)/SS
CONTINUE
EBI(N)=HI
RETURN
END
HD=1./((2.*1)*NLOG(BL))

PRINT B04: KL;E01(KL);HD;KB

804 FORMAT(1X,13,T1*X,F10.6,T1*X,F10.6,T1*X,F13)

CONTINUE

STOP

END

SUBROUTINE PRESDEH(BD,RR,N,R,B)

C
C #THIS SUBPROGRAM GENERATES THE 'I'SS'
C
C #DIMENSION R(20);B(20)
C
C B(N)=SORT(1.+BD*R/R/(R(H)*R(M)))
M=M+1
DO 2 J=1,MM
  LL=H+J
  SUN=RR/R(LL)
  PROD=1.0
  DO 3 K=1,1+J
    PROD=PROD/B(LL+K)
    SUN=SUN+PROD/R(LL+K)
  3 CONTINUE
  B(LL)=SORT(1.+BD*SUM*SUM)
  CONTINUE

RETURN

END

SUBROUTINE PREMAT(B,N,P,PSO)

C
C #THIS SUBPROGRAM GENERATES THE RATIO OF PRESSURES P(I)/P0
C
C #DIMENSION B(20),P(20),PSO(20)
C
C PRO=1.
DO 8 N=1,N
  PRO=PRO/P(N)
  P(N)=PRO
  PSO(N)=P(N)*P(N)
  CONTINUE

RETURN

END

SUBROUTINE EDTMAT(N,R,BD,BD+T;EDI)
C
C #DIMENSION P(20),R(20),B(20),EDI(20),PSO(20)
C
C REAL K,N
C AM61=2.*BD/2.
C T=ALOG(AM61*SORT(AM61*AM61-1.))
C DO 20 M=1,N
  R(M)=R
  R(M)=R
  CALL PRESDEH(BD,RR,N,R,B)
  CALL EQUIVA(N,BD,PSO,EDI)
  DO 250 K=1,N
    EDI(K,KJ)=EDI(KK)
  250 CONTINUE

RETURN

END

SUBROUTINE EQUIVA(N,BD,PSO,EDI)
C
C REAL M1
C U=(2.*BD)/2.
C BB=ALOG(U*SORT(U(U-1.)
C XI=COSH(BB/2.)
C YY=XX/PSO(N)
C ZZ=ALOG(VV*SORT(VV(VV-1.)))
C M1=(ZZ-BB)/2./BB
C AR=BBM1(BB/2.,)
A=M(AR(AR))