Techniques of Vibration Control for Structures and Foundations

Nelson Lau

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ABSTRACT

Techniques of Vibration Control
for Structures and Foundations

Nelson Lau

This report summarizes recently developed concepts used to eliminate vibration in steel and concrete structures.

Techniques discussed include changing of natural frequencies for undamped elements, deflection limitations, control of damping and breaking of vibration sources.

Guidelines are also given for controlling the vibration of machinery foundations.

ACKNOWLEDGEMENT

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CONTENTS

CHAPTER 1

INTRODUCTION

Vibration frequently causes structural failures and human discomfort. This report deals with techniques to control such problems. Discussions cover conceptual design ideas and remedies to suppress the undesirable effects from vibration. Subject matter includes control of vibration in floor systems, wind excited buildings, chimneys or pole-like structures, overhead sign supports, cable structures, elevated liquid containers, bridges and foundations. Each chapter deals with a distinct method of vibration control in relation to some of the above mentioned occurences. Foundations are discussed in a separate chapter.

Chapter 2 discusses the techniques of changing natural frequencies of undamped support elements and ideas to keep the support free of resonance. Formulae and charts are provided for quick determination of fundamental frequencies to supplement the subject. Chapter 3 deals with the method of limiting deflection as a means of preventing vibration in floors, buildings and sign structures. Chapter 4 discusses the amount of damping and its inherent effect in floor systems, the methods of increasing damping and the installation of dampers in buildings, chimneys and overhead sign structures, the self induced damping effects in elevated liquid containers and the methods of suppression of flutter in cable roof structures. Chapter 5 deals with isolators and the methods of breaking vibration sources. Chapter 6 deals with foundation design.

CHAPTER 2

CHANGE OF NATURAL FREQUENCIES FOR UNDAMPED SUPPORT ELEMENTS

This is a simple approach to keep the structure from resonance(1-10). It is effective for machinery sitting on flexible floors and other structures. Other methods such as placing of isolators and increasing damping will be discussed in later sections. The following are design guidelines to prevent resonance and achieve human comfort.

2.1 Design Guidelines

1. Frequency ratios shall be designed within the limits shown in Table 2.1.

Table 2.1 Permissible Range of Frequency Ratios(4)

Type of end support	Span	Beam frequency / Impressed frequency		
Column	Less than 20 ft.	> 1.5	or	< 0.8
Column	20 ft. or more	> 2.0		< 0.75
Girder	Less than 20 ft.	> 2.0		< 0.75
Girder	20 ft. or more	> 2.0		< 0.75

2. Adjust beam frequencies either by changing beam size or span length.

3. Where the span is long and the natural frequency of the beam is close to the equivalent operating frequency,

a knee brace may be used to reduce the beam span to increase its natural frequency.

4. Add bracing to increase stiffness of the structure, consequently increasing the frequency of the structure.

5. Can3-S16.1-M78 (7) has recommendations on transient vibrations. It states that people alone can create periodic forces in the frequency range 1-4 Hz, therefore floors with natural frequencies less than 5 Hz for such occupancies should be avoided. For dancing halls, floors should have a frequency higher than 10 Hz unless there is a large amount of damping. For concrete construction, floor frequencies should be higher than 15 Hz. The following constructions may be susceptible to transient vibrations:

Open web steel joists or steel beams with concrete deck, spanning between 7-20 meters.	4-5 Hz
Light wood deck floors on steel or wood joists.	10-25 Hz

6. For design of chimneys, the change of natural frequencies can be achieved by varying the metal thicknesses. A higher natural frequency can shift the critical wind velocity outside the usual range of wind speed, while a lower natural frequency can reduce the effect of wind force so that oscillations cannot be maintained. Another factor affecting osscillations is the change of cross sectional shape which will have influence on stiffness, mass and construction of the chimneys.

7. In bridge design, the Ministry of Transportation and Communication of the Province of Ontario has conducted dynamic tests on a number of bridges (8) and concluded that the dynamic response of a bridge is dependent upon the 'frequency match' between the bridge and the vehicle. The test results are summarized in Table 2.2. It is recommended that the fundamental frequency of a bridge should be outside the range of 2 to 5 Hz, otherwise impact factors greater than those specified by AASHTO (11) should be adopted. This design requirement (Fig.2.1) has been incorporated in both the 1980 supplement of CSA Design of Highway Bridge (12) and the 1983 Ontario Highway Bridge Design Code (13). Regarding aerodynamic stability of bridges, British Standard Institute (14) has excellent discussions on the subject.

8. Fig.2.2 illustrates double cables and their equivalent spring system. Since the two springs are connected to a rigid bar, they have the same deformation. The natural frequency of the combinded system, which corresponds to a spring constant of $K_1 + K_2$, is higher than the frequency of the individual cable. When the frequency is sufficiently high, the danger of flutter will be eliminated.

Table 2.2 Bridges Tested for Vibration (8)

Structure	Type	Span (Ft.)	Impact Value	Frequency Hz
1.	Voided Post-Tensioned Concrete Deck	43,85,80,85,48	8%	3.7
2.	Plate Girder Concrete Slab	140,150,150	41%	4.1
3.	Plate Girder with Cantilever Suspended Span	100,140,100	39%	3.4
4.	Plate Girder Concrete Slab Included Legs	85,110,85	78%	2.8
5.	Steel Truss with Cantilever Suspended Span	84,115,115,84	55%	2.6
6.	Plate Girder	102,128,128,102	66%	3.2
7.	Voided Post-Tensioned Concrete Deck	118,148,118	55%	2.7
8.	Post-Tensioned Girder	30,55,30	10%	5.8
9.	Reinforced Concrete Deck	65,90,65	27%	3.6
10.	Steel Trusses Concrete Deck	150,200,200,150	47%	2.9
11.	Steel Low-Truss Concrete Deck	94,95	28%	3.0
12.	Voided Post-Tensioned Concrete Deck	112,102,102,102,135,118	-	2.0

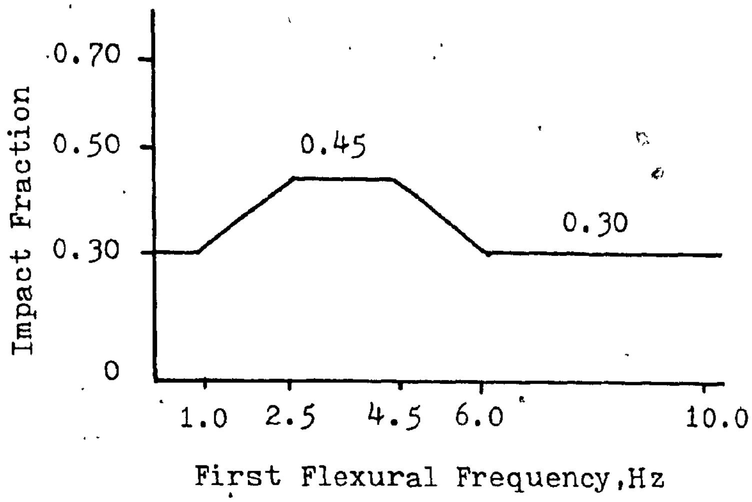


Fig.2.1 Impact Allowance (12)

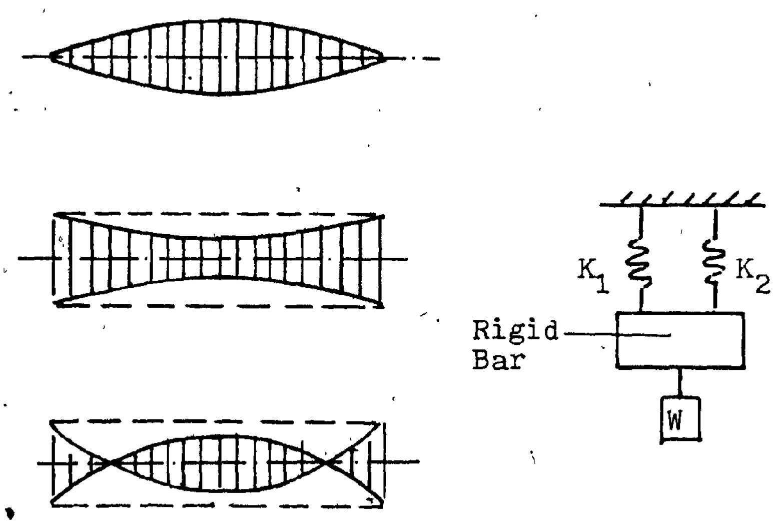


Fig.2.2 Double Cable Systems (9)

2.2 Computation of Natural Frequencies

Outlined below are methods for quick determination of natural frequencies:

1. Individual prismatic beams

The natural frequency (1) of a simply supported beam is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

where K is the beam stiffness and m is the beam mass. For various support conditions and different construction materials, frequencies of beams can be estimated from charts shown in Appendix A, Fig. A-1 to A-5.

2. Continuous beams

The natural frequencies of uniform continuous beams can be obtained from Appendix A, Table A-1. Those tables of frequency factors prepared by the Ministry of Transportation and Communication (15) may also be used for quick evaluations of straight, continuous symmetric multi-span beams of various span ratios.

3. Thin plates with uniform thickness

The natural frequencies of thin plates with different edge conditions can be obrained from Appendix A, Table A-2.

4. Cable structures

Let m denotes the mass per unit length of span, L denotes the span length and n denotes the mode shape shown in Fig.2.3, the natural frequencies of the cable systems (9,10) are given by the following expressions:

Single cable

$$f_n = \frac{n}{2L}\sqrt{\frac{H}{m}}$$

where H is the horizontal component of the cable tension.

Double cables

$$f_n = \frac{n}{2L}\sqrt{\frac{H_t + H_b}{m}}$$

where H_t and H_b are the horizontal components of the cable tension in top and bottom cables respectively.

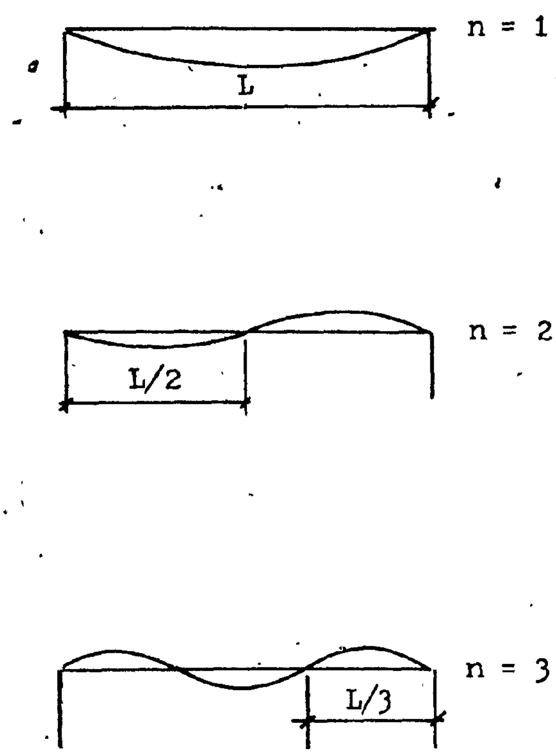


Fig.2.3 Mode Shapes of Cables (9,10)

2.3 Summary

The general principle to obtain a vibration free structure is to keep the structure from resonance. It is recommended that the ratio of fundamental natural frequency to disturbing frequency be outside the range 0.5 to 1.5. Charts and tables shown in Appendix A can be used for rapid evaluation of frequencies of beams and plates. CSA3-S16.1-M78 suggests the floor natural frequencies of light residential buildings be greater than 5 Hz, and 10 Hz in case of dancing halls. Recent bridge design codes propose that the value of impact is to be determined according to the bridge natural frequency.

CHAPTER 3

CONTROL OF DEFLECTION

Limiting deflection also eliminates excessive floor vibrations due to human occupancy and damages in buildings caused by wind or earthquake. Recommendations are given in most building codes and standards. Some of these are detailed in the following:

3.1 Floor Beams or Slabs

For floors spanning up to ~~approximately~~ 7000 mm, control of deflections has long been recognized as a measure for satisfactory structural performance with respect to transient vibration.

Tables 3.1 to 3.3 show recommendations from ACI Code (16) and National Building Code of Canada (17). In general, live load deflections of roof and floor beams or slabs supporting non-structural elements likely to be damaged by deflection should not exceed L/360. For beams not supporting non-structural elements, live load deflections are limited to L/240. CSA standard Can3-S16.1-M78 states that deflection of a steel joist supporting wood deck should not exceed L/360 under 2 kPa loading assuming the joist is fully supported. Graph similar to that in Fig.3.1 is useful in the design of non-composite metal decks. With this graph, one can rapidly determine if a floor would have vibration problem. If a plotted point falls within the feasible region, the structure will be satisfactory.

3.2 Glass Panels

NBC 1980 (18) recommends that deflections of glass retaining members be limited to 0.0057 span length. They must be stiff enough to resist excessive vibration from wind gust.

3.3 Buildings

Can3-S16.1-M78 suggests that column sway due to wind be limited to 0.00125 to 0.0025 of the height of industrial type buildings and 0.0025 for all other type buildings. For steel buildings erected with cladding and partitions but no special provision for building frame movements, magnitude of story drift should be limited to 0.002. NBC 1980 recommends a maximum lateral deflection ranging from 0.001 to 0.004 of the building height be used to prevent cracking of masonry and interior finish. Le Messurier (19) indicated in his report that in recent years, the designers have tighten the lateral deflection/height ratio to 0.0016. In his experience, this criterion is satisfactory in buildings up to 183 meters high.

3.4 Overhead Sign Supports

1975 American Association of State Highway and Transportation Officials has standard specifications (20) for the design of structural supports for highway signs. It states that overhead sign structures (span type) shall be proportioned to avoid resonance at critical wind speed by limiting their vertical deflections. This can generally be accomplished by using the value d/400 as a limit for dead load deflection, where d is the sign depth in feet. Nevertheless, the criterion provided does not always guarantee a safe limit. A few sign supports spanning approximately 30.5 meters have been damaged by wind. Either dampers or heavier sections have been adopted.

3.5 Summary

The present practice to avoid transient vibrations is to control the flexibility of the structures. The deflection requirements recommended by ACI Code, National Building Code of Canada and AASHTO usually provide satisfactory performance of structures.

Table 3.1

Deflection Limitation ACI 318-83 (16)   L = span length

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load	L/180
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load	L/360
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection which occurs after attachment of the nonstructural elements, the sum of the long-time deflection due to all sustained loads and the immediate deflection due to any additional live load	L/480
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		L/240

Table 3.2

Minimum Depth of Beams or One Way Slabs (16)

(Unless deflections are computed)

Member	Simply supported	One end continuous	Both ends continuous	Cantilever
Solid one-way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

Table 3.3 (17)

SUMMARY OF MAXIMUM DEFLECTION/SPAN RATIOS IN NBC 1977 AND PERTINENT CSA STANDARDS[1]					
	CSA 086-1976, Wood	CSA A23.3-1973, Concrete	CSA S16-1969; S16.1-1974, Structural Steel	CSA S157-1969, Structural Aluminum	NBC 1977-Part 9, Residential Standards
Roof or floor members supporting plastered ceilings, partitions, etc	$\frac{1}{360}$ (2)	$\frac{1}{480}$ (3) or $\frac{1}{240}$ (3)	$\frac{1}{360}$	$\frac{1}{360}$	$\frac{1}{360}$
Floor members not supporting plastered ceilings, partitions, etc	$\frac{1}{180}$ (2)	$\frac{1}{360}$ (4)	$\frac{1}{320}$ (5)	$\frac{1}{200}$	$\frac{1}{240}$ (7) or $\frac{1}{360}$
Roof members not supporting plastered ceilings, etc.	$\frac{1}{180}$ (2)	$\frac{1}{180}$	$\frac{1}{180}$ (6) or $\frac{1}{240}$ (6)	$\frac{1}{180}$	$\frac{1}{180}$ (8) or $\frac{1}{240}$
Wall members	$\frac{1}{180}$ (2)	-	-	$\frac{1}{180}$	-
Column 1	2	3	4	5	6

Notes:

(1) Deflection under live load only unless otherwise noted.

(2) Modulus used for calculation based on short term test but there is a warning clause on creep deflection.

(3) Deflection which occurs after attachment of non-structural element, including creep deflection due to sustained load plus immediate deflection due to live load.

(4) Immediate live load deflection. (5) There is a warning clause on vibrations.

(6) 1/180 applies to elastic membrane roof and 1/240 to build-up roof.

(7) For bedrooms only.

(8) If there is no ceiling.

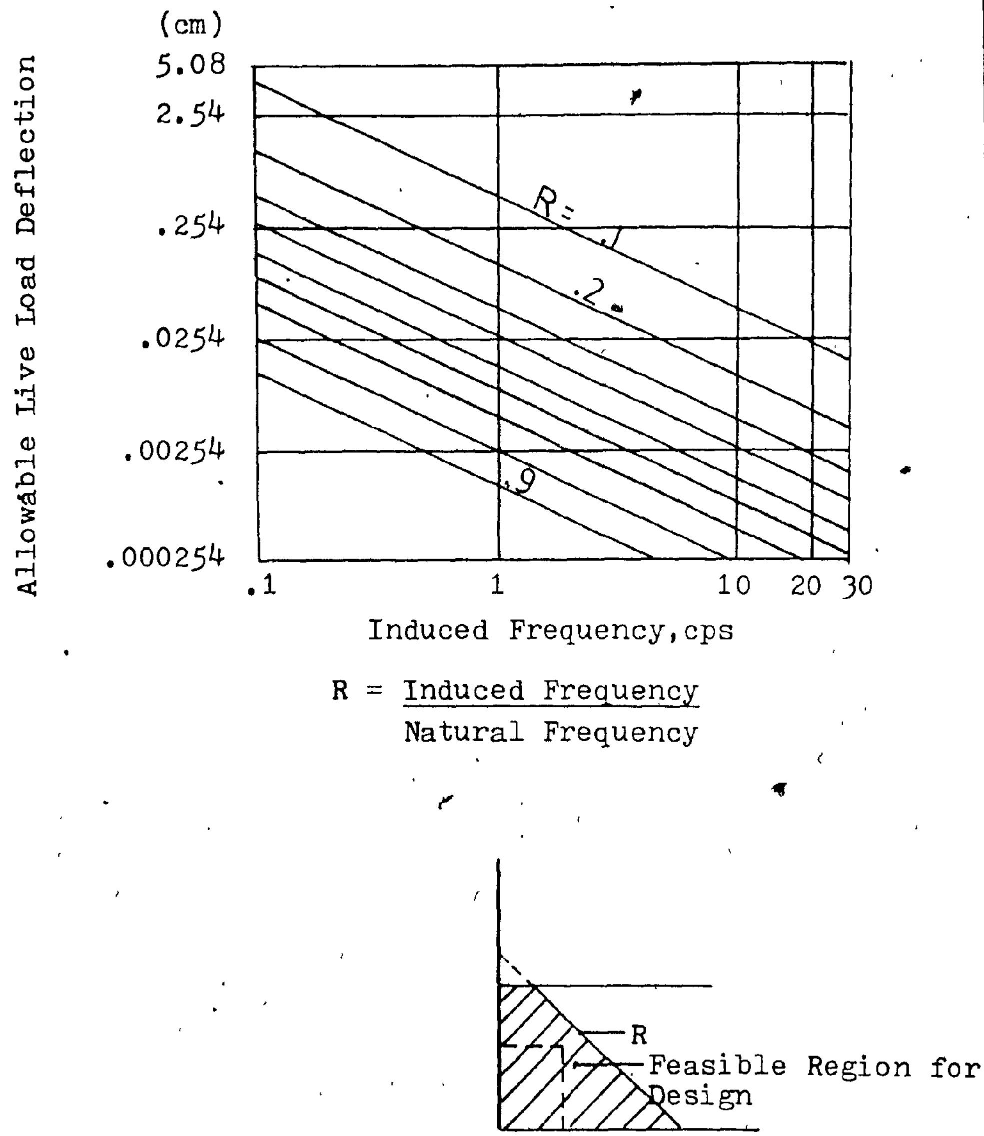


Fig. 3.1 Human Response to Steady-State Vibration. Allowable live load deflection versus induced frequency for non-perceptible motion (26)

CHAPTER 4

DAMPING IN BUILDINGS AND OTHER STRUCTURES

The best way to reduce vibration from an existing structure is to increase damping. It is therefore, important for engineers to be acquainted with the effects of damping and the methods of increasing damping. Unfortunately there is no simple method to determine the amount of damping inherent in structures unless experiments are performed. Reports become the only reference source for design without high cost. For multi-mass systems, computer analysis will be required. The following is a brief review regarding inherent damping,application of dampers and guidelines on design.

4.1 Damping in Floor Systems

Concrete Floors - Bock (21) gives damping ratios of 0.0127 to 0.0207 for concrete with cracks and less for concrete free from cracks. Lenzen (22) states that the damping ratios of concrete framed floor systems range between 0.03 to 0.05 and above 0.06 with furnishing or partitions completed. Field tests in Table 4.1 and 4.2 (23) for long span floors indicate that the damping ratios for cast-in-place continuous floors are in the order of 0.02 to 0.07 and for inter-connected precast beams are about 0.04. For prestressed beams, Penzien (24) reports that the damping ratios are between 0.005 to 0.07 depending on the degree of cracking being accepted.

Table 4.1 Field Studies of Long-span Concrete Floor Systems (23)

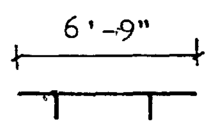
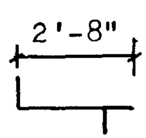
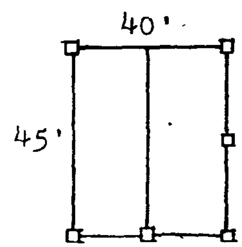
Description	Structural system	Non-Structural Components	Span ft	Weight psf	Frequency, Hz	Damping % Critical
Parking Garage Precast Double T 	Simply supported interconnected beams	None	58	90	5.4	3-4
			38	90	9.3	4-4.5
Stadium Precast stands 	Simply supported interconnected beams	None	55	85	4.8	4-5
		35 psf of people	62	100+35	2.6	-

Table 4.2 Field Studies of Long-span Concrete Floor System (23)

Description	Structural system	Non-Structural Components	Span ft	Weight psf	Frequency, Hz	Damping % Critical
Office Building Cast-in-place two way beam and 6" slab	monolithic  	bare floor	45	100	7.2	2-2.6
		finished floor & ceiling, open floor plan with low partitions	45	100	6.7	3-5
		same as above with full height partitions	45	100	7.7	6.3-6.9

Steel Floors with Concrete Deck - Experiments with walking impact on composite beams and joists have been conducted by Allen (25). The impact was induced by a man weighting about 170 pounds lifting his heels about two and one half inches, relaxing and then dropping to the floor. The curves shown in Fig.4.1 in conjunction with Table 4.3 give the amount of damping associated with different floor constructions. Fig. 4.2 gives the initial amplitude. For gymnasiums and similar structures, the amplitude should be multiplied by a factor of 1.5. Fig. 4.3 and 4.4 reflect human response from which the permissible amplitude can be determined. Table 4.4 shows damping coefficients recommended by Can3-S16-M78.

In reviewing the above information and discussions from Can3-S16.1-M78, it is noted that floor damping is generally less than 10% critical. The addition of furnishings,ducts and similar elements can increase damping by about 3% or more. Partitions located in two directions provide the most effective damping. When partitions are parallel to the floor beams and further apart than approximately 6000 mm, damping may be zero. Hints to remedy problem floors are summarized below(7,22,25):

1. The addition of bridging has very little effect on vibration characteristics of a floor system.

2. Cover plates on steel beams have negligible effect on human perceptibility.

3. Non-composite construction tends to increase damping by about 1-2% over composite section.

4. For cold formed joists, ceiling boards or straps

should be attached to the bottom flange to prevent annoying high frequency torsional vibrations in the joists.

5. The dynamic stiffness of a simply supported beam will not be increased by continuing the beam over the supports.

6. For slabs spanning between 7 to 13.7 meters, the increase of concrete slab thickness tends to increase the damping, decrease the initial amplitude while the frequency remains unchanged. For spans over 15 meters, human sensitivity can be reduced by decreasing the steel or concrete.

7. Installation of damper posts as per Fig.4.5 can significantly increase the amount of damping by as much as 10%.

9. A tuned damper may be constructed by attaching a mass to a spring. Upon installation, the spring can be tuned to be out of phase with respect to the floor. Neoprene may be added as a frictional damper between the mass and the floor. Due to the questionable reliability of the damper, this type of arrangement is not recommended in areas where significant amount of loading changes are anticipated. Allen (25) reported the case he successfully installed such a damper to a problem floor in a school, but no analysis has been discussed. The equivalent mass-spring system will be similar to that shown in Fig.4.10 of which m_1 is the floor mass, K_1 is the floor stiffness, m_2 is the damper mass, K_2 is the spring constant of the damper and C is the damping coefficient.

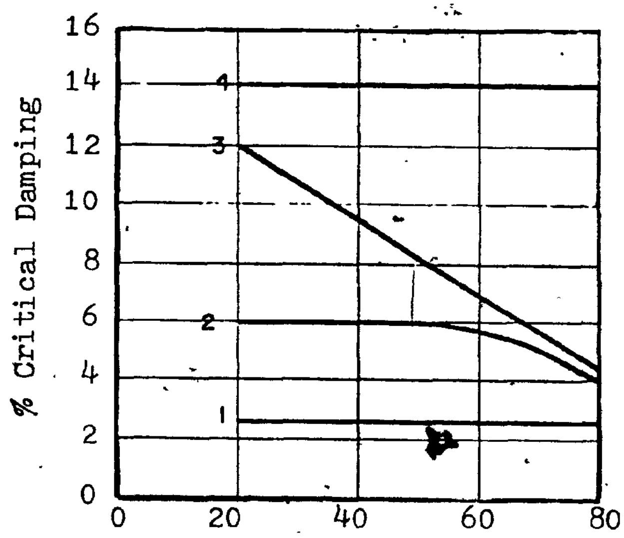


Fig.4.1 % Critical Damping Inherent in Floors, Composite Construction Assumed.

(Key to application of curves on Table 4.3)

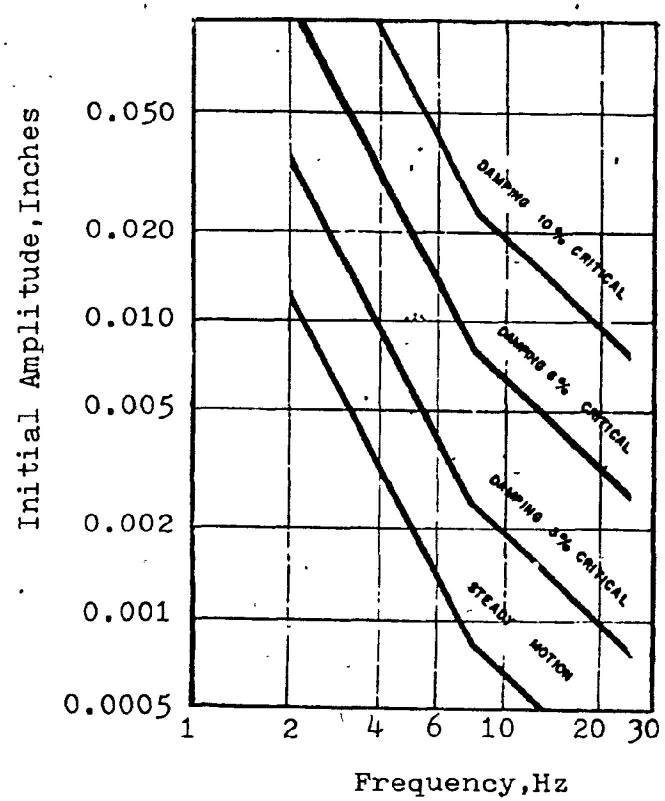


Fig.4.2 Damping Effect on Floor Amplitude (25)

Table 4.3 Description of Floors for Figure 4.1 (25)

Curve No. Fig. 4.1	% Critical Damping	Description of Floor
1.	0-2.5%	2.5" concrete on 1.5" steel deck, with open web joists or steel beams, no suspended ceiling below, no air conditioning ducts, bare floor, no partitions.
2.	6%	2.5" concrete on 1.5" steel deck, with open web joists or steel beams, suspended ceiling below, air conditioning ducts and electrical conduits installed, floor finished, furnished and carpeted, no partitioning.
3.	as shown Fig. 4.1	5" concrete on steel beams (Or open web joists), suspended ceiling below, ducts & conduit installed, space open but finished, furnished, carpeted, and occupied.
4.	14%	All spans and floor construction with at least ceiling height partitions. Partitions do not have to be connected to floor above to be effective. Partial height partitions, of light construction, appear to give more than 10- 12% critical damping for all spans.

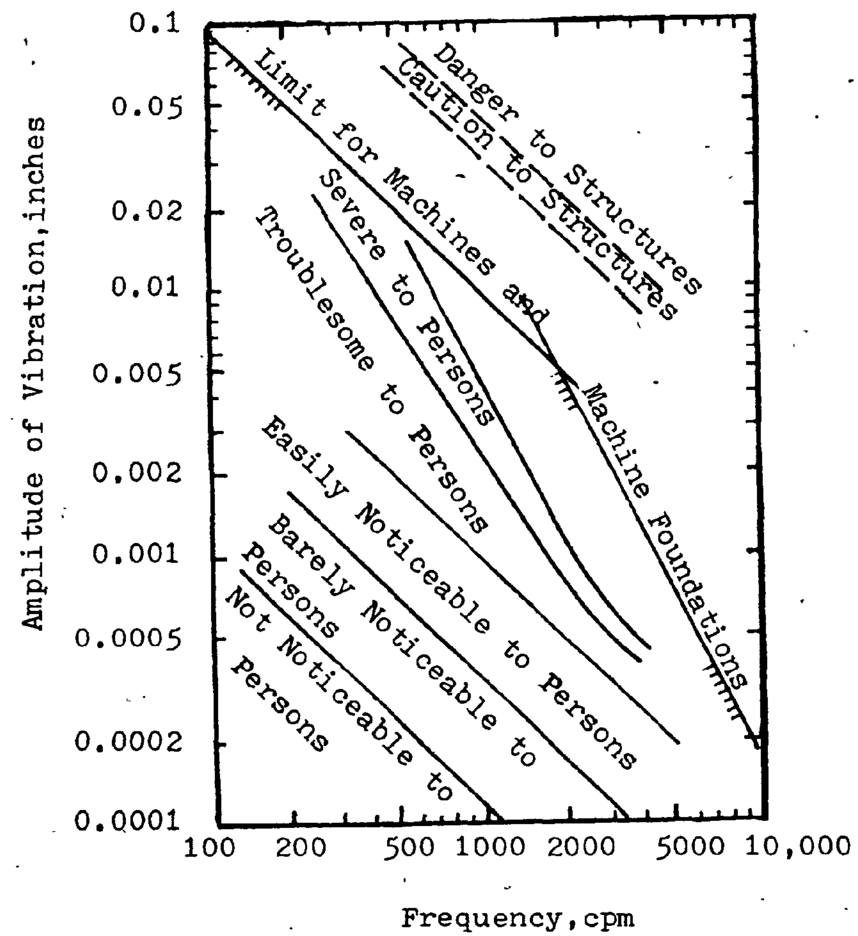


Fig.4.3 Limits of Vertical Vibration Amplitude for a Particular Frequency of Vibration (45)

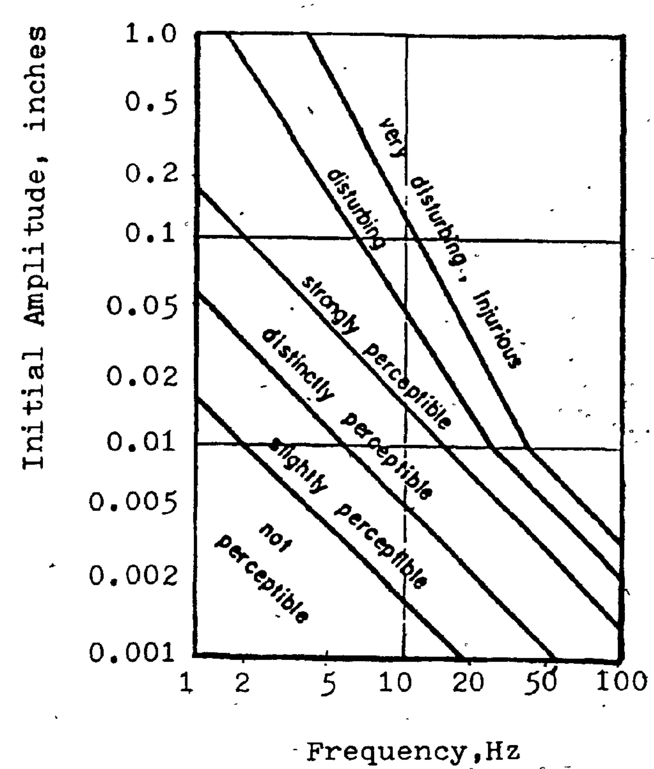


Fig.4.4 Reduced Human Response, After Lenzen (25)

Table 4.4 Damping in Steel Floor Systems (7)

Description of Floor	Damping % Critical
Bare steel and concrete deck floor:	
Composite	2
Non-composite	3-4
Finished floor with ceiling, ducts and flooring	6
Finished floor with partitions	12

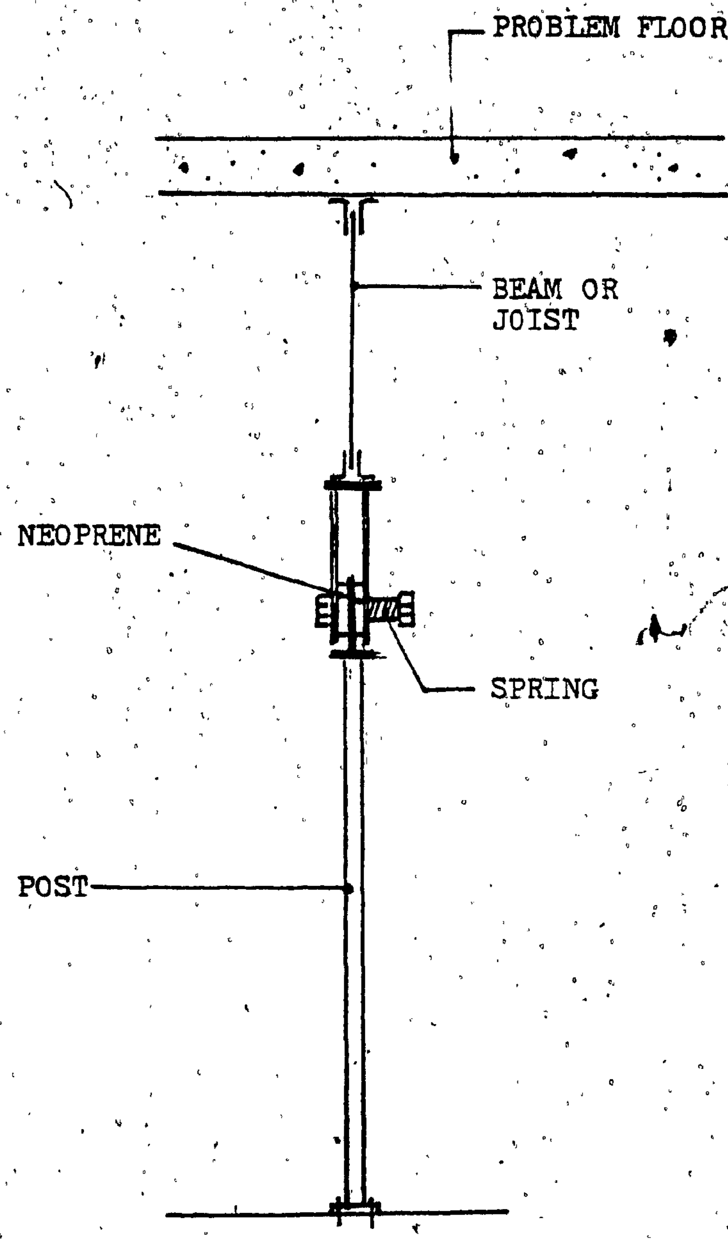


Fig. 4.5 Damper Post (25)

4.2 Damping in Buildings

As shown in Table 4.5, most tall buildings possess damping of about 1% critical. Values in excess of 1.5% are normally associated with high loads, damaged or inadequately designed structures. Values between 2 to 5% are sometimes used in wind response calculations (28). The following are case studies for applications of viscoelastic dampers and tuned dampers.

Viscoelastic Dampers - Fig.4.6 shows framing of the World Trade Center in U.S.A. (29) The primary central core has adequate strength to resist seismic forces. The secondary floor beams inter-connected with viscoelastic dampers are tied to the primary core through expansion joints. When the buildings undergoes oscillations, the strain energy of the mass of the floor results in a reduction in dynamic response. As an alternative, the central core may be formed by light weight steel members so that the kinetic energy of the entire floor can be dissipated. There are two general types of treatments in viscoelastic dampers (Fig.4.7). The following are the impediments in giving these treatments (30):

1. The thickness of the viscoelastic material should be small in comparison to the base structure.

2. Vibration amplitude must be small.

3. Bending stiffness of the constraining layers must be small compared to the basic structure.

4. Young's modulus of the damping layer must be small

compared to the facing layer and sufficiently large to make thickness changes in the damping layer negligible.

5. The product of the loss factor and the stiffness of the damping layer is far less than that of the undamped panel.

The following are some guidelines for the placement of these dampers:

1. Apply according to manufacturer's recommendations.

2. The constraining layers need not be anchored when they are properly assembled.

3. Increase shear strain by placing more constraining layers.

4. Optimize effective length of constraining layers to obtain maximum energy dissipation.

5. Increase damping by placing the viscoelastic material and the constraining layers alternatively.

Table 4.5 Inferred Properties of Buildings (28)

Building	Dimensions(m)			Average Density (Kg/m^3)	Total Mass (10 Kg)	Damping(%Critical)		Notes
	Height z	Width x	Length y			x-x	y-y	
Shear Wall Bldg. Hawaii, U.S.A.	100	23	23	273	14.5	1.0	1.1	W(a)
Arts Tower, Sheffield, UK.	80	20	36	337	19.0	0.8	0.9	V(b)(c)
Core Bldg. Prague, Czechoslovakia.	64	20	20	291	7.4	1.0	1.1	W(a)(b)
Precast Shear Wall Bldg. Prague, Czechoslovakia.	63	12	30	338	7.7	1.0	1.0	R(a)
Plumbing Tower, St.Remy, France.	54	4.2	6.3	450	0.6	0.85	0.9	V(b)
Scaled-Up Model, Garston, UK.	48	18	31	284	7.7	1.0	0.78	V(a)(b)
Monolithic Building, Prague, Czechoslovakia.	48	18	30	213	5.5	0.96	1.0	M(a)(c)
Office Block, Skopje, Yugoslavia.	47	22.7	22.2	315	7.3	0.69	-	V(c)
Police HQ. Wrexham, UK.	43	13	13	323	2.4	0.77	1.1	V(b)(c)
Pioneer Bldg. Los Angeles, USA.	30	9.7	20	333	1.9	3.5	2.0	V(a)(c)
Apartment Block, Hawaii, USA.	73	18	62	220	18.0	1.0-1.5	1.0-1.5	W(a)(c)

Notes: Form of Excitation: M-man, R-rocket, V-vibrator, W-wind.
Form of Construction: (a)-shear wall, (b)-cores, (c)-Column and slab or frame and slabs.

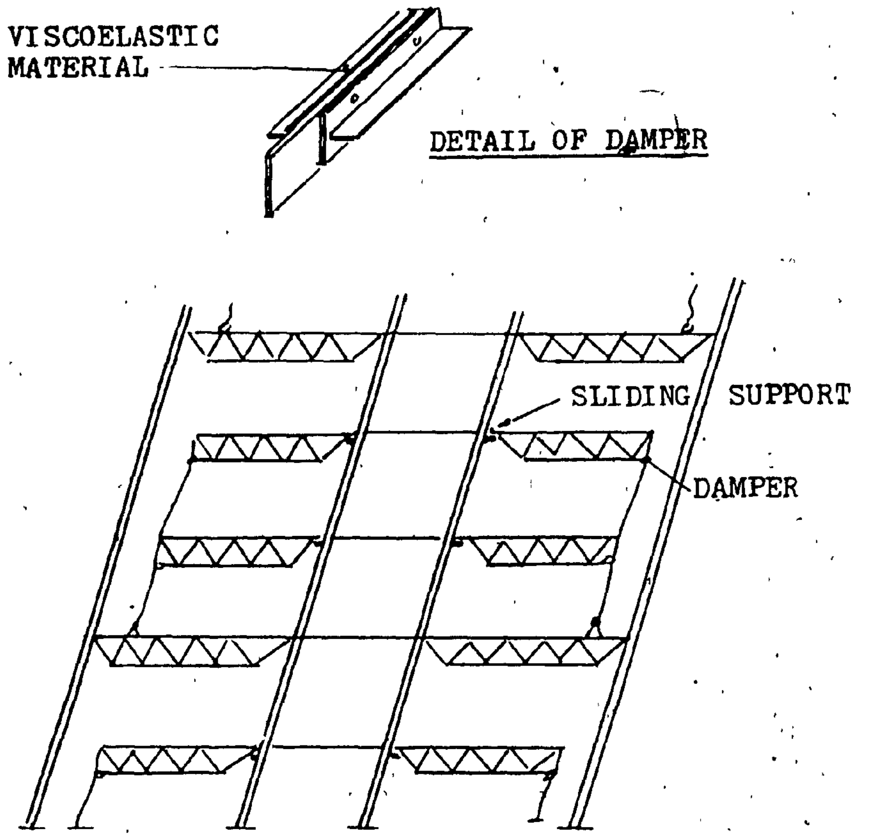
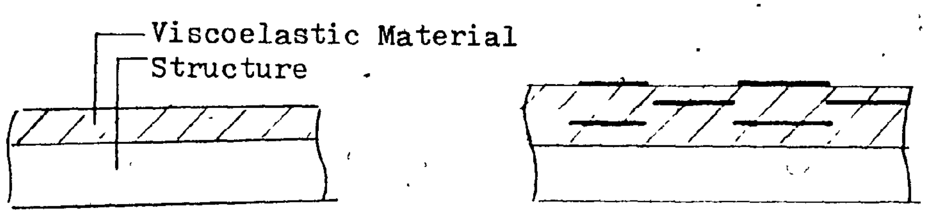


Fig.4.6 Viscoelastic Dampers used in the World Trade Center (29)



Free Surface Treatment. Constrained Layers Treatment

Fig.4.7 Typical Treatments of Viscoelastic Dampers (30)

Tuned Mass Damper - A tuned mass damper has been installed on top of the City Corp Center (Fig.4.8). The damping machanism (Fig.4.9) works in response to a signal. When the building moves, the control actuator will pump the oil to the bearings lifting the mass up and back to rest. Both the spring and the control actuator can be fine tuned to the effort that the periodical motion of the mass coincides with the movement of the building (31). As shown in Fig.4.10, m_1 represents the mass of the building, K_1 is the stiffness of the members which restrain the mass, m_2 is the mass of the dynamic damper, C is the damping coefficient, K_2 is the stiffness of the damper, Z(t) is the absolute displacement of the ground surface, $X_1(t)$ is the displacement of the mass m_1 relative to the ground surface, and $X_2(t)$ is the displacement of the mass m_2 relative to the mass m_1. Knowing the natural frequency of the structure and the predominant period of ground motion, the frequency ratio can be found. With an assumed mass ratio, the ~~optimum~~ frequency and damping ratio required for tuning can be determined (32).

Other architectural solutions such as optimum selection of building shape, surface texture and the use of spoilers can also reduce dynamic excitation.

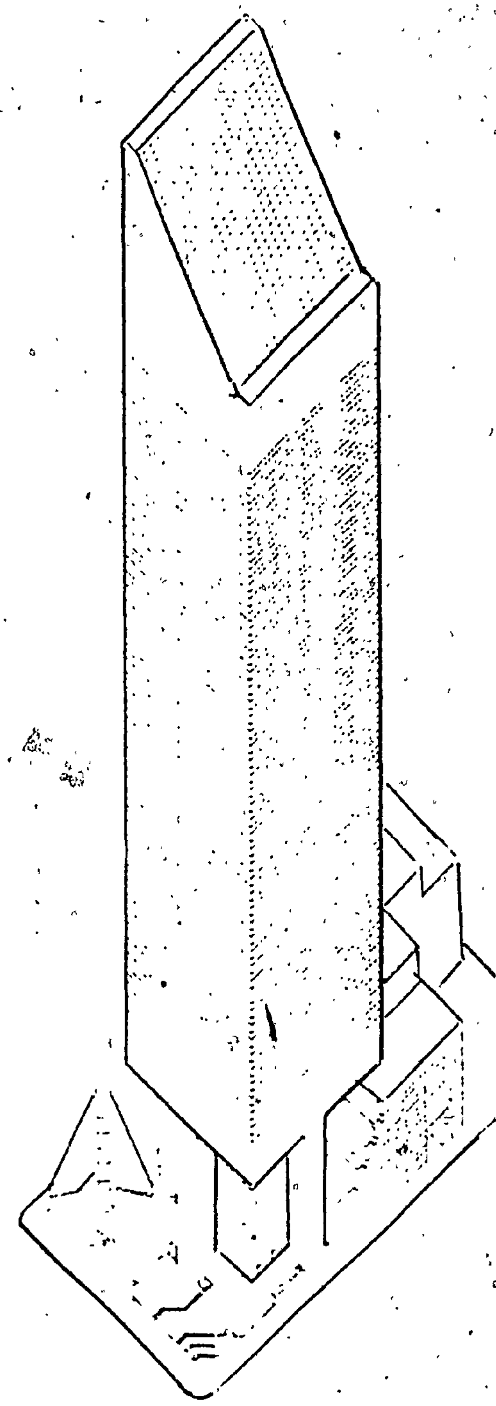


Fig.4.8 City Corp Center, New York (31)

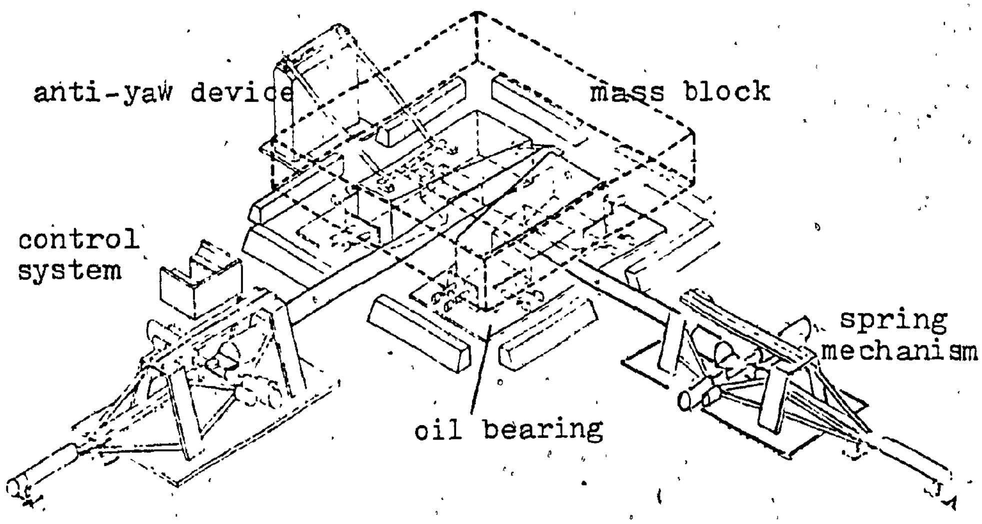


Fig.4.9 Tuned Mass Damper (31)
(City Corp Center, New York)

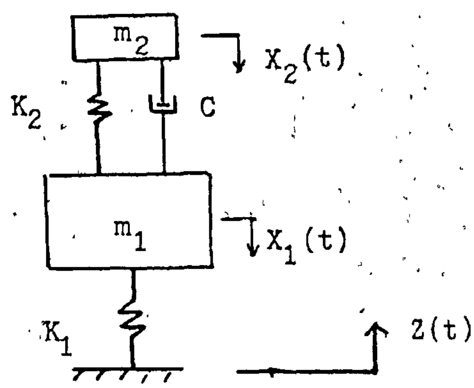


Fig. 4.10 Equivalent Spring System of a Tuned Damper in Buildings (32)

4.3 Damping in Chimneys

Reinforced concrete chimneys usually have an average damping value of five percent critical and steel chimneys have an average damping value of 1-2% critical including lining. When the amount of damping in percentage of critical equals or exceeds 0.07, aerodynamic stability can be achieved (33). Following are a few ideas for increasing structural damping:

1. Add coating, lining or mass.
2. Attach wire ropes.
3. Install dampers with wire ropes.
4. Install damping pads at base of chimneys.

Experiments for installation of damping pads have been conducted at the University of Technology in Loughborough, England (34). Two chimneys constructed with steel were tested. For chimney A, damping pads were placed with the twelve foundation studs. The materials of the pads were bonded cork or composite layers of neoprene rubber and asbestos between the outer layers of elastomeric bonded cork material. Results show that despite the significant increase in damping it was insufficient to prevent wind-induced oscillations. For chimney B, construction of the chimney was similar to A but a damping pad was installed over the entire area between the base plate and the foundation. Oscillations were eliminated. The desirable level of damping has been found to be $\frac{2m\delta}{\rho D} > 17$, where m is the mass per unit length, ρ is the air density, D is the diameter and δ is the logarithmic decrement.

4.4 Damping in Elevated Liquid Containers

An elevated tank behaves like an inverted pendulum which has a large mass concentrated at the top. When earthquake occurs, the vertical motion of the water (sloshing) acts as a damper resulting in reduction in seismic response. The hydrodynamic analysis is complicated. For simplicity, graphs can be established to determine the complex constants used in hydrodynamic equations. Two degrees of freedom are suggested in order to obtain precise analytical results.

4.5 Damping in Cable Structures

There are two phenomena associated with cable structures, namely forced vibration and self induced vibration or flutter (9,10). Flutter is a sudden and violent vibration occurring at a critical wind speed which corresponds to a critical frequency. By introducing the double cable systems similar to those in Fig.4.11 and 4.12,the danger of flutter can be controlled. When downward loads are applied, the load carrying cables will be subjected to tension while the secondary cables will be subjected to compression. The natural frequencies of the opposite cables will drift apart and damping achieved. When the combined frequency is sufficiently high, flutter will be eliminated. For cable grid systems shown in Fig.4.13, the systems themselves are damped and rigid. Therefore it is not necessary to predict the natural frequency. For single cable structures, stability can be achieved by increasing the weight of the structures.

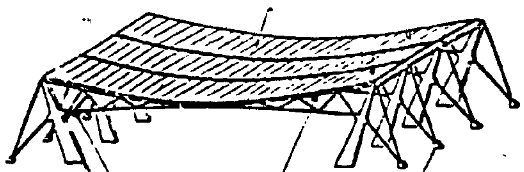
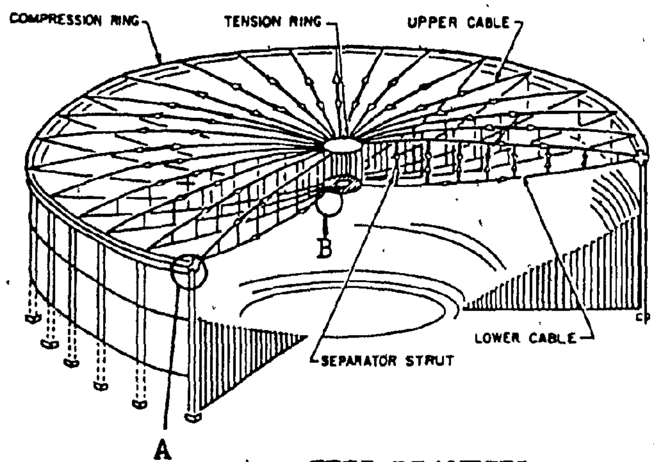
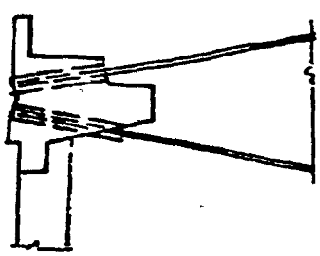


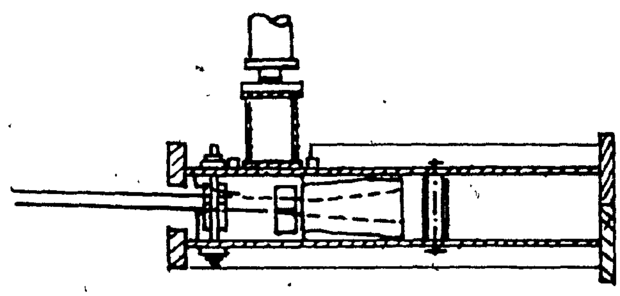
Fig. 4.11 Double Cable System (10)



240 FEET DIAMETER



SEC A



SEC B

Fig.4.12 Utica Memorial Auditorium (10)

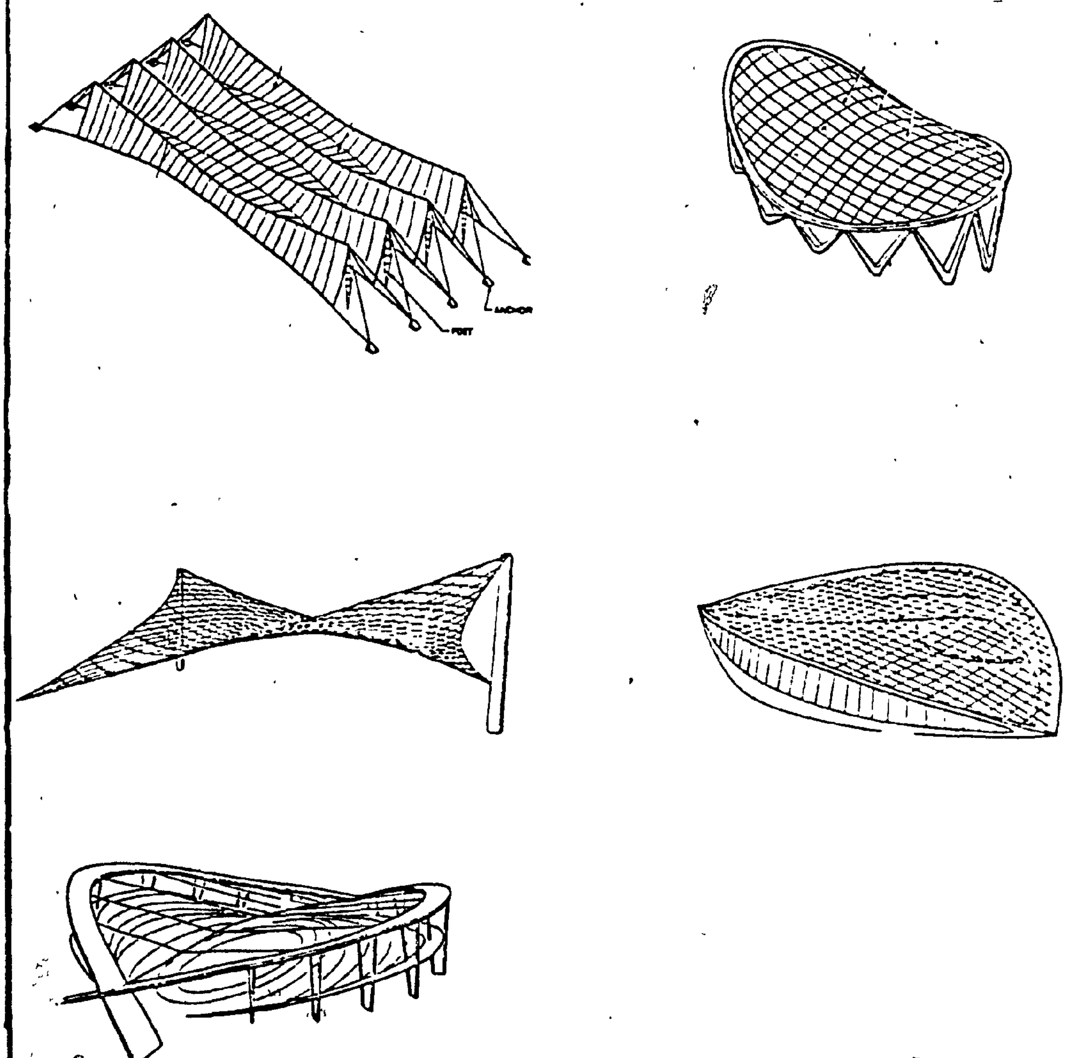


Fig.4.13 Cable Grid Systems (10)

4.6 Damping in Overhead Sign Supports

As previously discussed, limit of deflections does not always prevent the undesirable effects from vibration. Following the collapse of two sign bridges, one of the major fabricators requested the National Research Council of Canada to conduct investigations (35). As a result, dampers were developed.

The bridge being investigated is presented in Fig.4.14 and 4.15. Fig.4.16 shows the arrangement of the damper. From the observations of the model tests, it has been found that large amplitude of vertical oscillations existed due to vortex shedding behind the sign panels. At a damping ratio of 0.25 % vortex induced oscillations were strong, while at a damping ratio of 0.58 % there were no self starting oscillations.

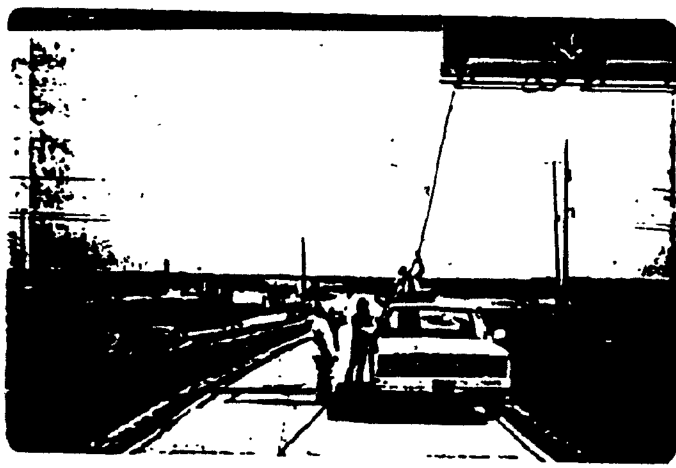
A damper formed by an air-foil flat plate was attached to the top of each sign panel and the result was good. The excessive vibration was suppressed. Fig.4.17 shows effect of wind speed. Other factors affecting oscillation include size and configuration of dampers, size of sign panels and configuration of the support.

4.7 Summary

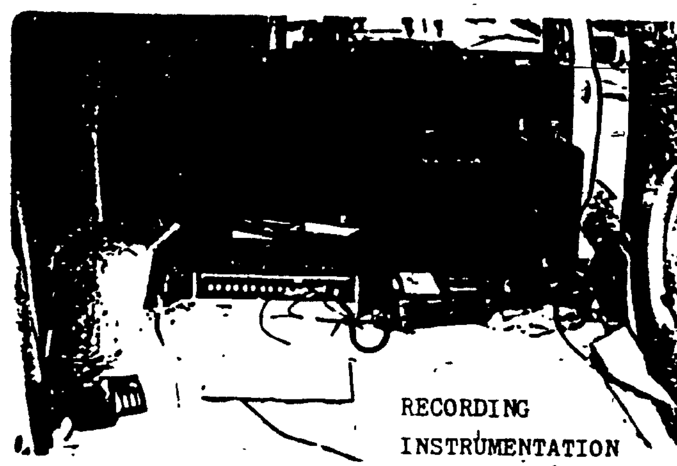
Inherent damping is an important factor affecting floor vibrations. This chapter briefly reviews the amount of damping inherent in floor systems and the vibration response of composite and non-composite steel structures investigated by Allen (25). The method for complete analysis can be found from many texts. Case studies for damping devices cover damper posts in floor systems, viscoelastic and tuned dampers in tall buildings, damping pads in chimneys and air-foil plates on overhead sign supports of which the applications will rely on more research and field investigations. The concept of sloshing effect in elevated liquid tanks and suppression of flutter in cable structures are presented.

SIGN BRIDGE

INSTALLATION OF ACCELEROMETER

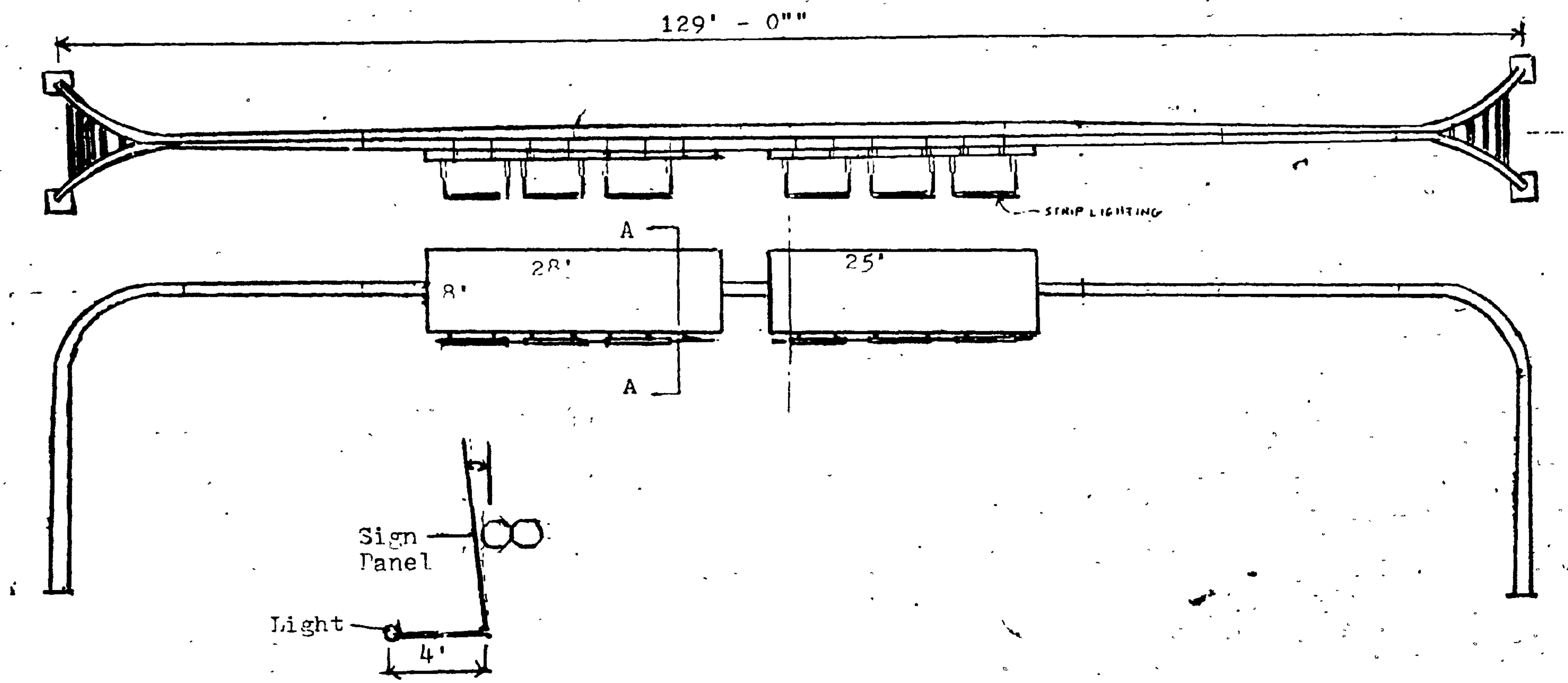


MANUAL EXCITATION



RECORDING INSTRUMENTATION

Fig.4.14 Field Measurement (35)



Sec.A-A

Fig.4.15 Sign Bridge 223 (35)

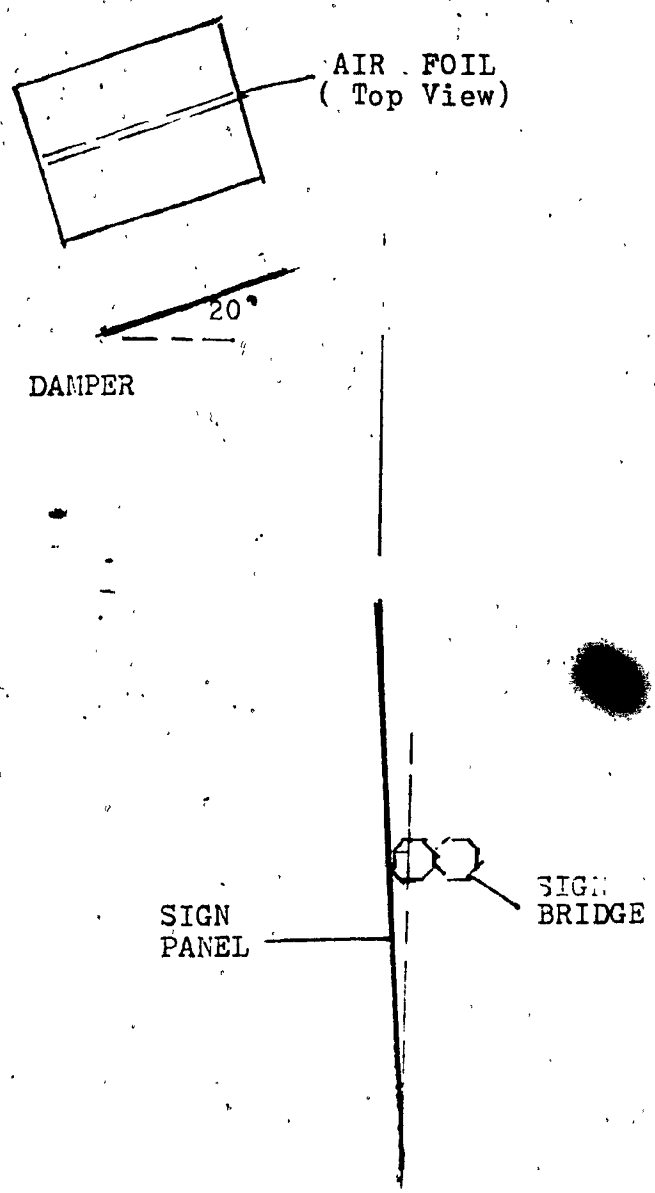


Fig. 4.16 Damper for Sign Bridge (35)

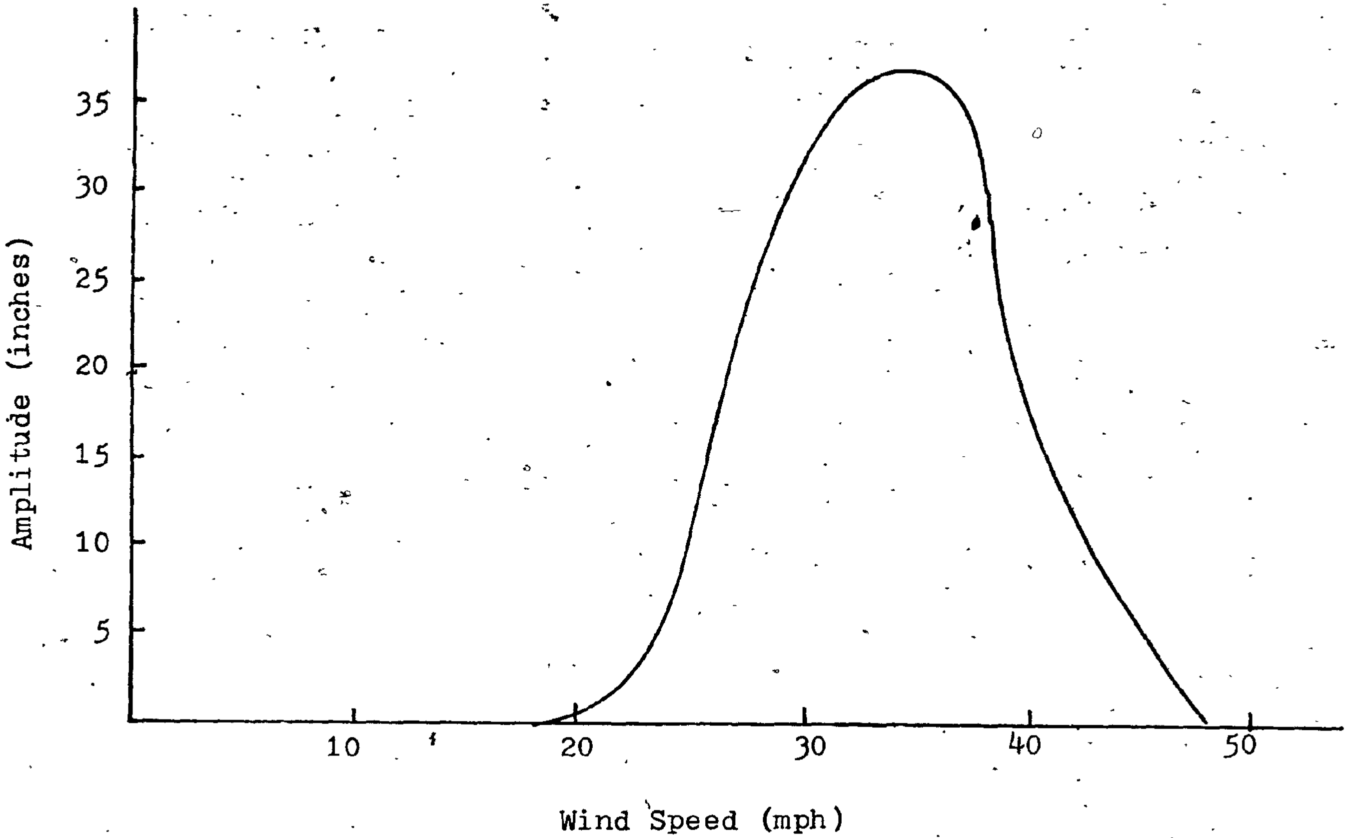


Fig.4.17 Vortex Excitation of Original Configuration (35)

CHAPTER V

ISOLATION OF VIBRATION SOURCES

The discussion of vibration isolation consists of two parts. The first part is the basic introduction of mounting isolators. For machinery located in buildings, it is important to place the machinery away from sensitive areas. The concept of selecting isolators is reviewed. The characteristics of resilient materials and installation details are briefly reviewed. The second part consists of the ideas of breaking wind induced oscillations. Contents include conceptual design of a shock absorbed story in tall buildings, discharge air current and the method for installation of a vortex breaker on chimneys.

5.1 Mounting of isolators

The measurement of the effect of isolators can be expressed by means of isolation efficiency or support deflection. The isolation efficiency is a function of the ratio R of disturbing frequency to natural frequency of the mounted machine. Assuming the mountings have a uniform deflection rate and no excessive damping, the percentage of isolation efficiency (36) can be expressed as

$$E = 100 \left(1 - \frac{1}{R^2 - 1} \right)$$

and the natural frequency (38) is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}} = 15.8\sqrt{\frac{1}{d\ (mm)}}\ Hz$$

where W = weight of the machine

K = spring constant of isolators

d = deflection of isolators

As it can be seen, the isolation efficiency is a function of the support deflection. Table 5.1 and 5.2 show isolation efficiencies recommended by the manufacturer of the resilient materials. Using the charts shown in Fig.5.1 or 5.2, with the recommended efficiency, the required static deflection and type of mountings can be determined assuming that the isolators are supported on a stiff support. For machines mounted on flexible floor structures, the amount of deflection provided by the isolators should be the sum of the required static deflection plus the floor deflection. Table 5.3 and 5.4 show recommended static deflections required for machines sitting on flexible structures. The above theory applies to normal equipment where the greatest floor sensitivity is in the vertical direction. For machines with heavy vibrations, there are six degrees of freedom to be considered. Careful evaluation may be necessary for each of the six frequencies prior to selection of isolators. As presented in Fig.5.3, the torsional mode is rarely a significant problem. The remaining four modes can be coupled with vertical frequency and the solution becomes simple (6,37,38).

Table 5.1 Recommended Isolation Efficiencies (38)
(concrete floor slab)

Critical Areas		Transmissibility	Isolation Efficiency
Centrifugal Compressors		0.5%	99.5%
Centrifugal Fans Reciprocating Compressors Pumps	greater than 25 HP greater than 50 HP greater than 5 HP	2%	98%
Axial Flow Fans Centrifugal Fans Reciprocating Compressors Pumps Unit Air Conditioners Fan Coil Units	greater than 50 HP 5 to 25 HP 10 to 50 HP 3 to 5 HP Supported Supported	4%	96%
Axial Flow Fans Centrifugal Fans Reciprocating Compressors Pumps Air Handling Units	10 to 50 HP up to 5 HP up to 10 HP up to 3 HP	6%	94%
Axial Flow Fans Unit Air Conditioners Fan Coil Units Pipes	up to 10 HP Hung Hung Hung	10%	90%
Gas fired boilers (more than 100,000 BThU, 25 kw) Oil fired boilers (more than 60,000 BThU, 15 kw)			7 to 12 Hz 4 to 7 Hz

Less Critical Areas		Transmissibility	Isolation Efficiency
Centrifugal Compressors		6%	94%
Centrifugal Fans Reciprocating Compressors Pumps Unit Air Conditioners Fan Coil Units	greater than 25 HP greater than 50 HP greater than 5 HP Supported Supported	10%	90%
Axial Flow Fans Centrifugal Fans Reciprocating Compressors Pumps Air Handling Units Unit Air Conditioners Fan Coil Units	greater than 50 HP 5 to 25 HP 10 to 50 HP 3 to 5 HP Hung Hung	20%	80%
Axial Flow Fans	10 to 50 HP	25%	75%
Axial Flow Fans Centrifugal Fans Reciprocating Compressors Pumps Pipes Hung	up to 10 HP up to 10 HP up to 10 HP up to 3 HP	30%	70%
Gas fired boilers (more than 100,000 BThU, 25 kw) Oil fired boilers (more than 60,000 BThU, 15 kw)			12 to 20 Hz 12 to 20 Hz

Table 5.2 Theoretical Isolation Efficiencies (36)

Isolation Material	Average Static Deflection	Average Natural Frequency	EFFICIENCIES %									
			350 RPM	500 RPM	600 RPM	800 RPM	1000 RPM	1200 RPM	1500 RPM	1800 RPM	3000 RPM	3600 RPM
2" Thick St'd Density Cork	0.08	By Test 1420	—	—	—	—	—	—	—	—	72	82
Type "W" Waffle Pad	Curvature Corrected 0.035	1000	—	—	—	—	—	—	20	55	87	92
2 Layers "W" Waffle Pad	Curvature Corrected 0.070	710	—	—	—	—	—	46	71	82	93	96
Single Deflection Rubber Mountings	0.20	420	—	—	—	62	79	86	91	94	98	99
Double Deflection Rubber Mountings	0.40	300	—	44	67	84	90	93	96	97	99	Almost Perfect
Standard Spring Mountings	1.00	188	70	85	89	94	96	97	98	99	Almost Perfect	
Double Deflection Rubber & Spring Mtgs.	1.40	160	75	89	93	96	97	98	99	Almost Perfect		

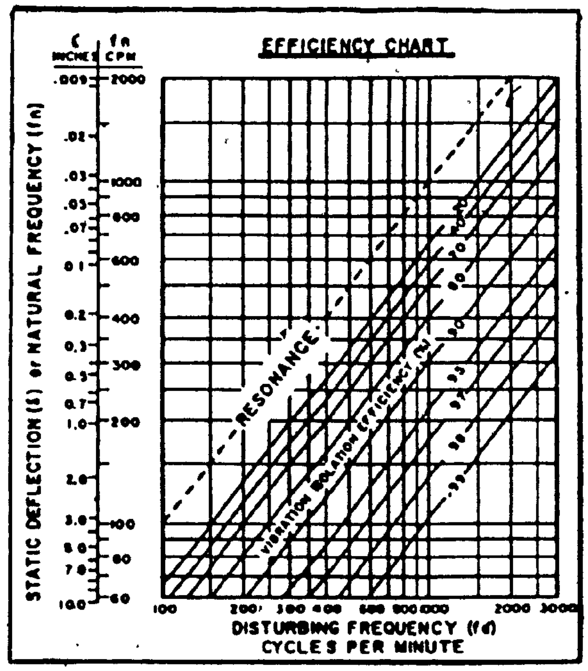


Fig.5.1 Efficiency Chart. (36)

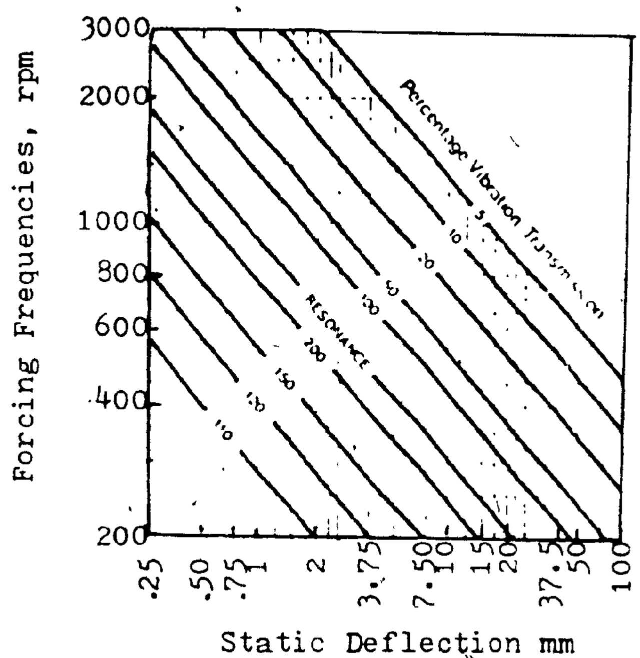


Fig.5.2 Relationship Between Percentage Transmission; Forcing Frequency and Static Deflection (48)

Table 5.3 Minimum Mounting Deflection (36)

Operating Speed RPM	Basement Negligible Floor Deflection	Rigid Concrete Floor	Upper Story Light Concrete Floor	Wood Floor
300	1.50"	3.00"	3.50"	4.00"
500	.63	1.25	1.65	1.95
800	.25	.60	1.00	1.25
1200	.20	.45	.80	1.00
1800	.10	.35	.80	1.00
3600	.03	.20	.80	1.00
7200	.03	.20	.80	1.00

Table 5.4 Minimum Mounting Deflection (38)

	Equipment Location				
Equipment Type	On Grade Min. Defl. mm	On 6m Floor Span Min. Defl. mm	On 9m Floor Span Min. Defl. mm	On 12m Floor Span Min. Defl. mm	On 15m Span Min. Defl. mm
Refrigeration Machines					
Reciprocating:					
500-750 rpm	25	45	45	65	90
751 rpm or over	25	25	45	65	65
Open Centrifugal	10	25	45	45	90
Compressors					
Air or Refrigeration					
500-750 rpm	25	45	65	65	90
751 rpm or over	25	25	45	65	65
Pumps					
Base Mounted thru 40 HP	25	25	45	45	45
Base Mounted 50 HP & Over	25	25	45	65	65
Fans & Blowers					
Utility Sets up to 500 rpm	10	45	45	45	65
Utility Sets 501 rpm or over	10	25	25	45	45
Cooling Towers					
Up to 500 rpm	10	10	45	65	90
500 rpm & over	10	10	25	45	65

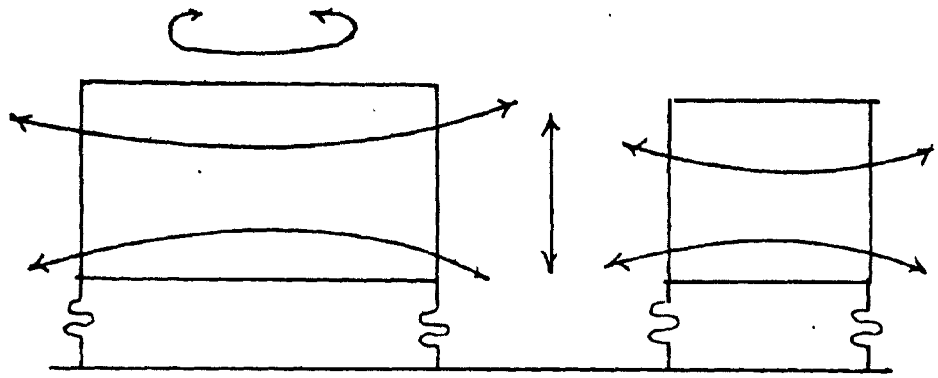


Fig.5.3 Modes of Vibration Having Four Resilient Supports (37)

The following is a brief review for resilient materials (36,37):

<u>Cork</u> - This material is good for isolation at high vibration frequencies between 50-60 Hz. The acceptable static deflection is approximately .08 inch. It can easily be crushed or disintegrated when wet, therefore chemical impregnation or asphalt felt is required for industrial applications.

<u>Rubber</u> - The damping is remarkably influenced by the hardness. It is not recommended for use as vibration mountings when hardness is above 70 durometer. Correction factors for computation of deflections can be obtained from reference 36. Chlorine rubber has damping ratio ranging between 0.03 and 0.08. Natural rubber has damping ratio ranging between 0.01 to 0.08 and is applicable between the temperature -50° to 100° C. It requires anti-ozonants to resist attack from ozone. The use of carbon black filler may inhibit deterioration due to direct sunlight, therefore its application is not recommended where substantial oil contamination is likely to occur. Neoprene pad has great load carrying capacity and excellent resistance to oils, solvents, weather, oxgen and temperature. The natural frequency ranges between 12 and 25 Hz. Maximum damping ratio is about 0.1.

Steel Spring - It is recommended where high deflection and low frequency is desired. The frequency is approximately 3-10 Hz. and the deflection can be up to 175 mm.

Once the type of isolator is selected, the next problem is to study the details of installation (36,40). Refer to sketches shown in Fig.5.4 and note the floowing:

(a) This detail applies where area is large and pressure is low particularly where low acoustic frequency is required, e.g. printing machine, diesel engines, roll-grinders and fans. Cork, rubber and neoprene are the resilient materials commonly used for this installation.

(b) The combination of a concrete block and double cushions can reduce vibrating forces transmitted to the structure. It may be useful for machines with large unbalanced forces located on upper levels of a building.

(c) The attachment of an inertia block to steel springs provides good rigidity, stability, small amplitude and better alignment between the mounting components. This detail is useful for mounting of pumps, centrifugal chillers, high pressure fans, air compressors, reciprocating fridge compressors and internal combustion engines. The depth of the steel beams may be 1/10th of the span (36) with limitation of fourteen inches maximum. The required mass ratio of concrete block to machine usually ranges between 1:1 to 6:1 (38) depending on the machine installed.

(d) This detail shows installation of cork around a footing. It is often used for horizontal compressors, pumps, newspaper press, centrifugal compressors and other basement located equipment. The transmission of vibration to floor, piping and electrical equipment is minimized. The omission of cork underneath the foundation block can reduce vertical yielding of the footing consequently vibrations on the piping.

(e) This sketch shows typical examples of prefabricated rubber or neoprene type supports. The hanger is primary used to isolate vibrations from piping or equipment to ceiling. The mounting pad can be ued under flat base machine without bolting. They are made of skid·resistant base plates, easy to install and the cost is low.

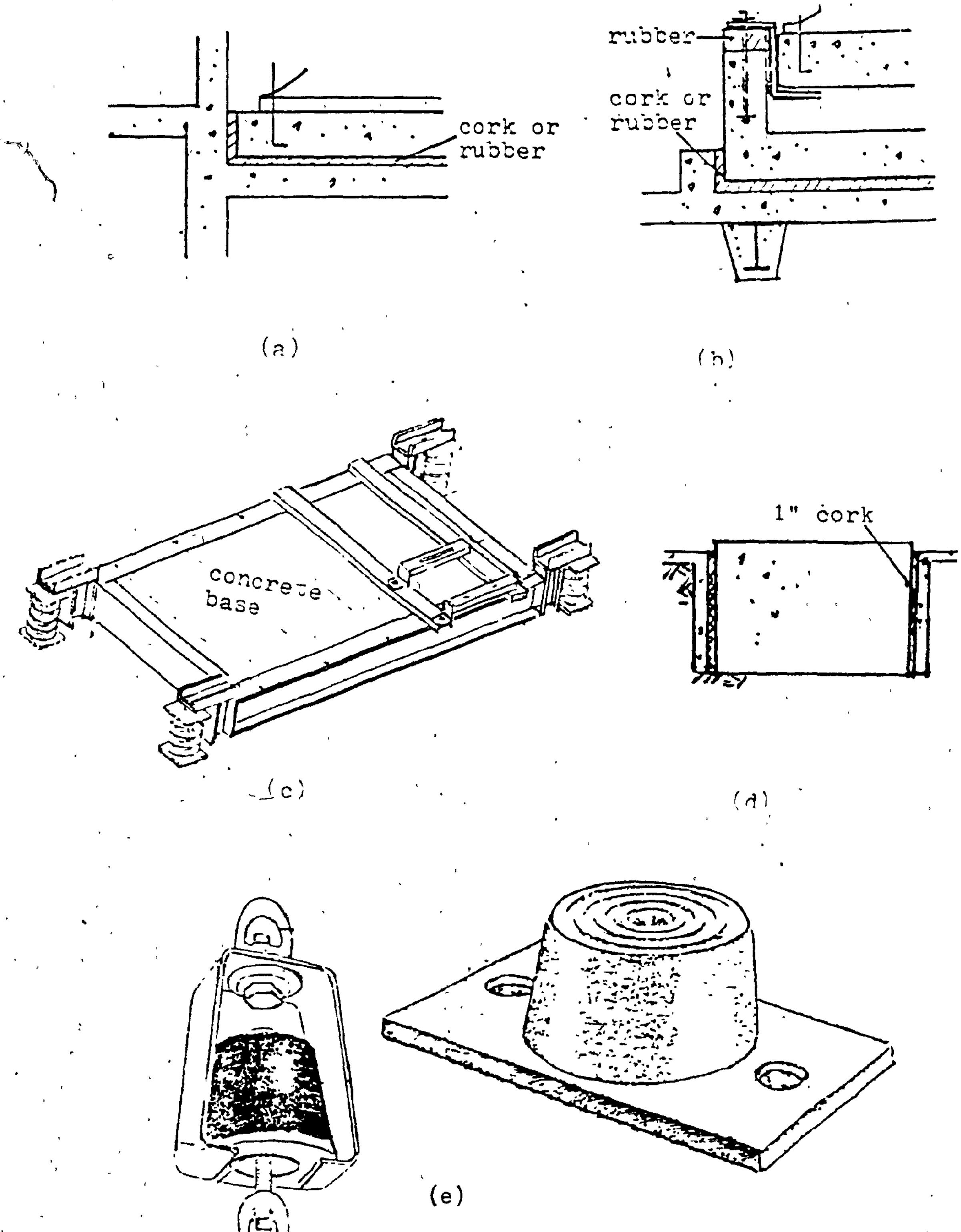


Fig. 5.4 Details of Isolators (36,40)

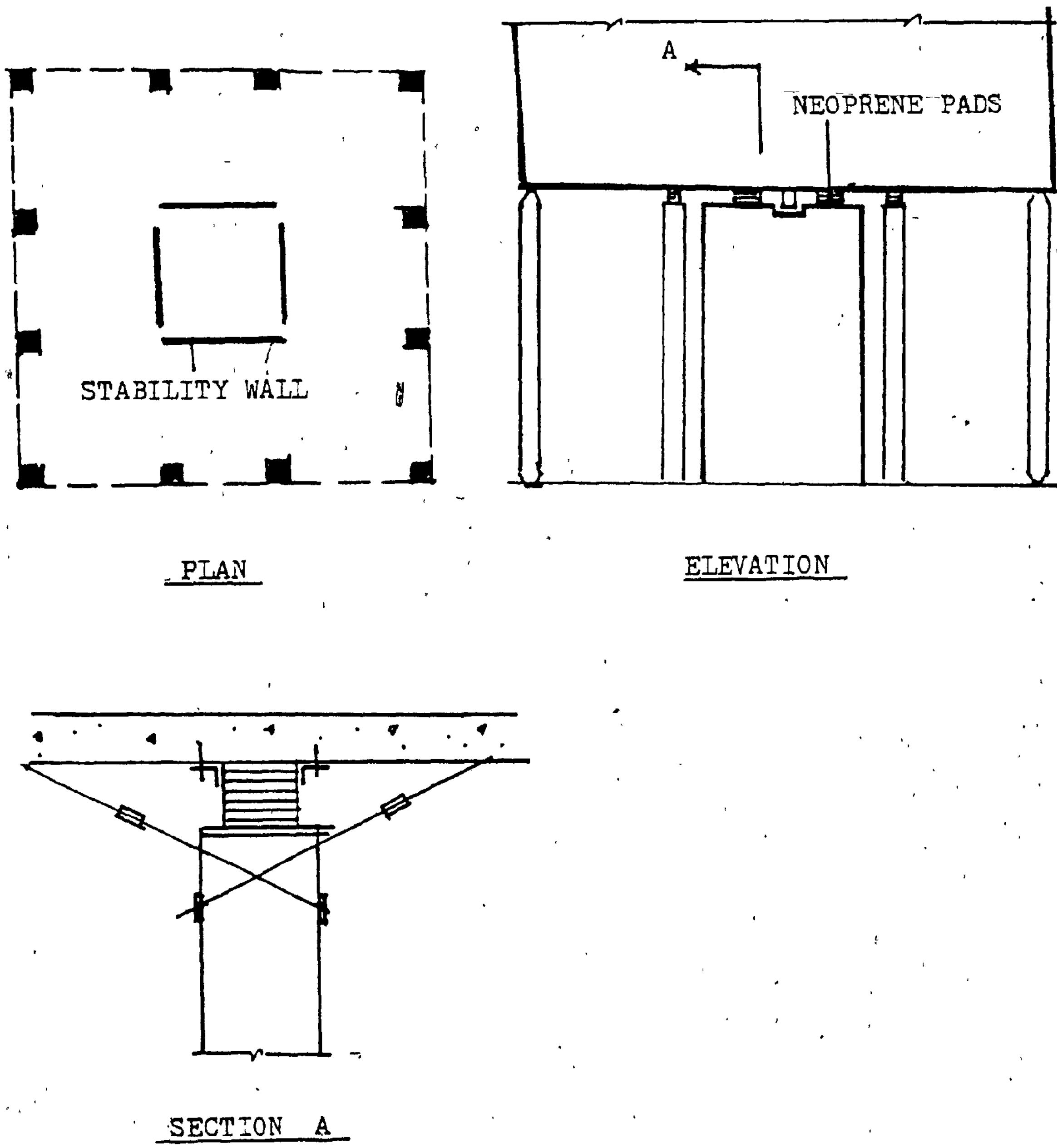


Fig.5.5 Details of Shock Absorbing Story (39)

5.2 Breaking of Wind Induced Oscillations

Provision of a Shock Absorbed Soft Story - The idea of providing a soft story is to confine the earthquake and wind motions of tall buildings.(39). The motion is to be confined to one level shielding the stories above from damage. The proposed system (Fig.5.5) consists of columns, elastomeric strips and stability walls which contributes only the minimum amount of lateral deformation due to seismic loads. The period of the equivalent spring system (Fig.5.6) is given by

$$T = 2\pi\sqrt{\frac{m}{K}}$$

where m is the total mass above the soft story and K is the stiffness of the supporting elements including columns and elastomeric links. As the values of equivalent period, damping ratio, yielding capacity and ratio of stiffness of elastomeric strips to columns are given, the relative displacement and optimum choice of the neoprene pads can be determined.

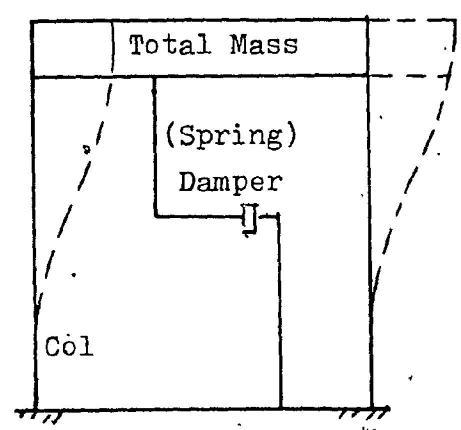


Fig.5.6 Single Degree of Freedom System (39)

Discharge Air Current - Refer to Fig.5.7 for the air discharged system (29). The air can be discharged from the lee-ward surface of the building to downward stream through the central hollow core, mechanical ducts and areas of occupancy. Release of the air will avoid provocation of vortex. The massive exterior columns can also be used for the same purpose but caution shall be taken not to create turbulence of the current.

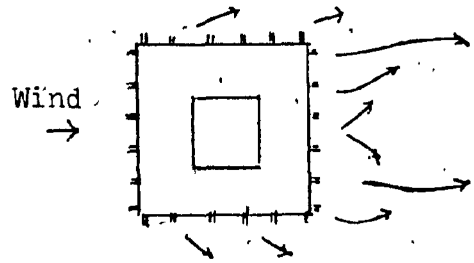


Fig.5.7 Discharge Air System (29)

Installation of Vortex Breaker on Chimneys - The device (33) shown in Fig.5.8 has been successfully used for years. It consists of three start spirals and is placed at the top third of the chimneys. Each strike has a radial height of 0.09 diameter of the chimney and a revolution in height equal to five times the diameter of the chimney. Other forms such as saw-tool spoilers or perforated castings can also be used.

5.3 Summary

The successful selection of isolators can be achieved by applying the basic principle of required static deflection and the charts furnished by the manufacturer. This is usually adequate for mounting of isolators for building equipment. There are available in the market many high performance isolating items which have not been included in this report. The simplest form is a piece of resilient material or steel spring. The implications for the ideas of discharge current or soft story to break wind induced oscillation in practice are not yet known but introduction is included for the interest of individuals. The application of spirals on chimneys is useful.

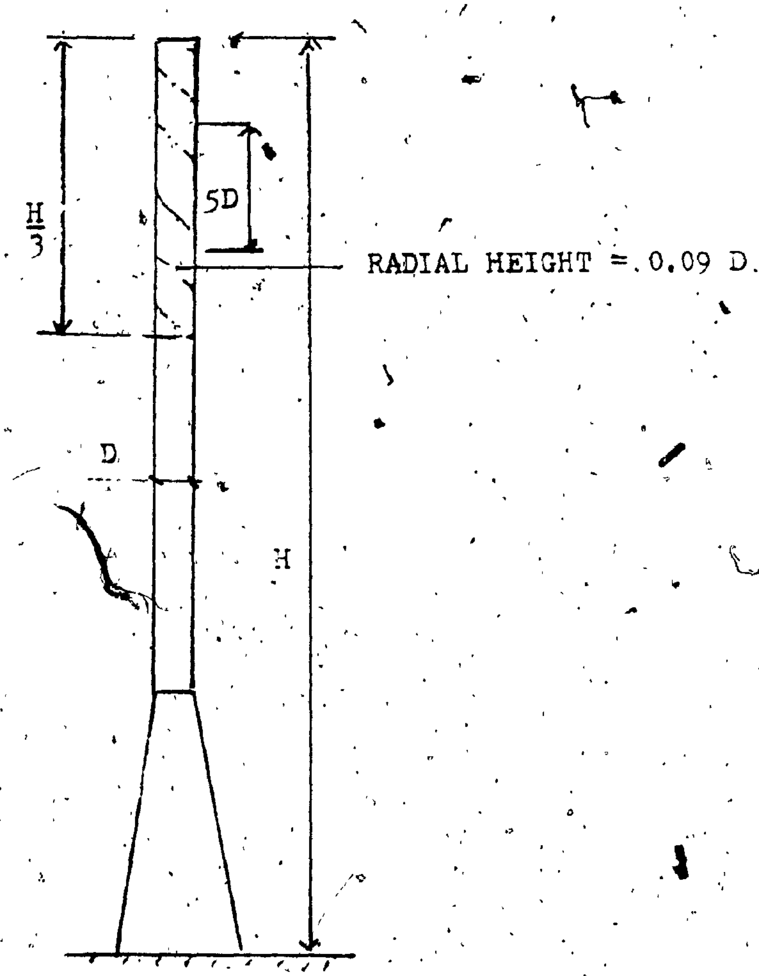


Fig. 5.8 Vortex Breaker (33)

CHAPTER VI

MACHINERY FOUNDATIONS

As shown in Fig.6.1, machinery foundation can be treated as a mass-spring system, As the frequency of the exciting force approaches the frequency of the footing, resonance takes place.

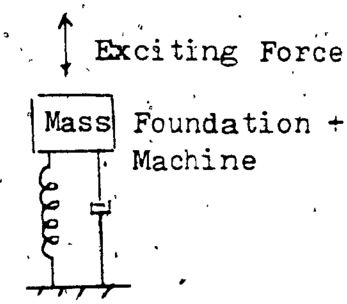


Fig.6.1 Foundation System

Following are two general types of foundation supports:

1. Machines supported directly on footings or on piles. If a machine is located high above the floor, vertical oscillation, horizontal translation, rocking and torsional oscillations may all exist. For machines located close to the floor, only vertical oscillation will be revelvent.

2. Machines mounted on isolators,e.g. spring, rubber, cork or felt. This is more often applicable for high speed machines.

In order to achieve sufficient control, the following should be understood and design must be properly undertaken (3,40-48).

6.1 Major Factors Affecting Dynamic Response of Foundations

The major parameters influencing vibration characteristics of foundations include dimension of footings, piling, ratio of foundation weight to equipment weight and soil characteristics. Soft ground provides lower frequencies of footings while stiff ground provides higher frequencies. These factors should be carefully considered in dealing the dynamic response of footings.

6.2 Design Guidelines

1. For low speed machinery, e.g. compressors, pumps and some reciprocating machines with operating frequency lower than 500 rpm, foundation-soil system with a natural frequency twice the operating frequency is usually provided.

2. For machinery operating higher than 1000 rpm, e.g. generators, compressors, forced and induced fans, the foundation-soil system should have a frequency no more than half of the operating frequency. In many cases, this can easily be achieved by providing a minimum mass ratio as recommended in Table 6.1. For reciprocating pumps, the minimum ratio is usually no less than 3.0. For centrifugal pumps, a minimum ratio of 5.0 is often employed.

3. Soil static pressure should not exceed 50% of the soil allowable pressure.

4. Soil pressure created by static and modified dynamic load should not exceed 75% of the allowable load.

Table 6.1 Suggested Mass Ratios (3)

Equipment operating frequency	Ratio of foundation weight to weight of vibrating part of machine
Up to 600 rpm	3.0
900 "	4.5
1200 "	6.0
1800 "	8.0

5. The magnitude of settlement should be within the limits recommended by the manufacturer.

6. The centroid of dynamic and static loads should be within six inches of the center of gravity of the footing. For rocking motion, the axis of rocking should coincide with the principal axis of the footing.

7. To insure stability, it is preferable that the total width of the foundation be at least equal to the measurement from the center of gravity of the machine to the bottom of the footing.

8. The maximum amplitude of motion should be within the allowable limit specified by the manufacturer. Table 6.2 shows observations and recommendations from Barkan.

Table 6.2 Permissible Amplitude (41)

(Barkan 1962)

Equipment	Amplitude (mm)
Low speed machinery, 500 rpm	.02
Hammer foundations	1
High speed machinery:	
3000 rpm:	
hor.vibrations	.04 to .05
vert. vibrations	.02 to .03
1500 rpm:	
hor.vibrations	.07 to .09
vert.vibrations	.04 to .06

6.3 Vibration Control

Methods of vibration control include:

1. Counter balancing existing loads imposed by the engine.

2. Stabilizing the soil by injecting cement or chemical agent. If the soil contains granular materials, stabilization may be required only at the edge of the footing and not the entire area. By increasing rigidity of the base, frequency of the footing will be increased subsequently leading to reduction in vibration.

3. Increasing foundation natural frequency by increasing the soil contact area, moment inertia or driving piles. Piles will be effective to resist vibration only when they are subject to appreciable forces. Fig.6.2 shows frequency vs pile length of bearing piles on rock.

4. Reducing foundation frequency by increasing foundation weight.

5. Increasing foundation embedment depth in soil to reduce vibration motion.

6. Compacting soil to increase rigidity.

7. Placing of machinery deeper than adjacent structure to prevent transmission of vibration to the structure.

8. Placing of isolation joints to prevent transmission of vibration to adjacent structures.

9. Placing of attached slabs as dampers.

10. Placing of isolators.

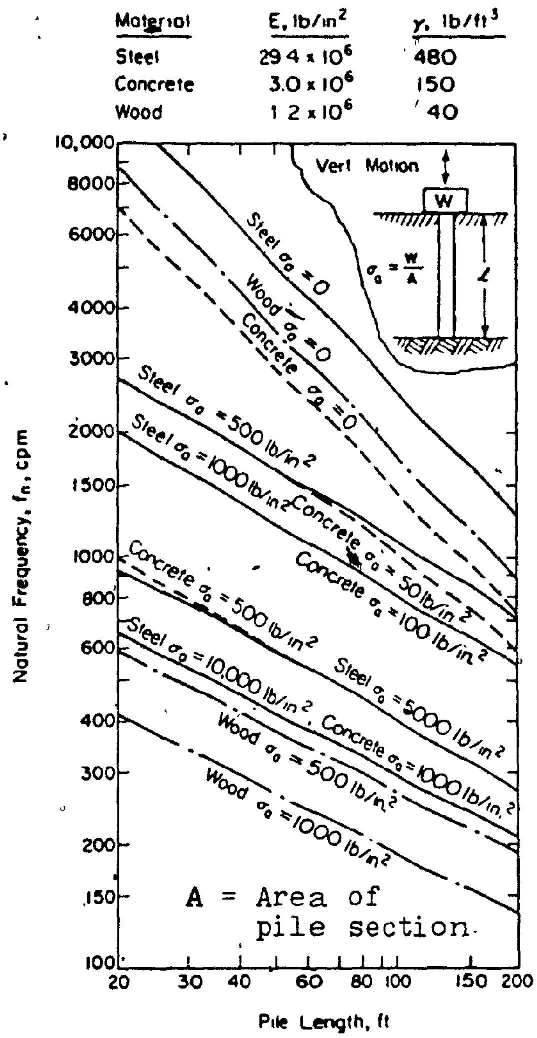


Fig.6.2 Resonant Frequency of Vertical Oscillation for a Point-bearing Pile Carrying a Static Load (45)

6.4 Design of Footings

List of Symbols

A	dynamic amplitude
$B_z, B_x, B_\psi, B_\theta$	mass ratios defined in Table 6.4
$D_z, D_x, D_\psi, D_\theta$	damping ratios defined in Table 6.4
f_n	natural frequency of vibrations
ω_n	circular frequency of vibrations = $2\pi f_n$
ω	frequency of exciting force; angular velocity of machine rotation
G	shear modulus of soil
$K_z, K_x, K_\psi, K_\theta$	spring constants defined in Tables 6.3 and 6.4
M	dynamic magnification factor
P_0	amplitude of exciting force
r_0	equivalent radius of rectangular footings
v	Poisson's ratio
ρ	soil density

The following is a simple method presented by Richart, Woods and Hall in 1970 (45) for evaluation of natural frequencies and amplitudes of machinery foundations. The system consists of a lump mass m supported on linear and rotational springs. Elastic-half-space theory is applied. The same method has been reviewed by Richart in Foundation Engineering Handbook in 1975 (46). Outlined below are the design procedures:

1. Check with manufacturer for mechanical data, e.g.

acceptable amplitude, magnitude and frequency of unbalanced forces.

2. Field test for soil parameters. Perform dynamic test if possible. Data required include soil density, Poisson's ratio, shear modulus or modulus of subgrade reaction.

3. Assume sizes of a rectangular footing. Convert the rectangular base 2c x 2d into an equivalent circular base radius using the formulae shown in Table 6.3.

4. Compute mass ratio, damping ratio and spring constant according to Tables 6.3 and 6.4.

5. Determine natural frequency of the footing.

6. Compute magnification factor and vibration amplitude. For an exciting force $P = P_0 \sin \omega t$, the magnification factor can be obtained using the following formula:

$$M = \frac{A}{P_0 / K} = \frac{1}{\sqrt{(1-(\omega/\omega_n)^2)^2 + (2D(\omega/\omega_n))^2}}$$

where D,K and ω_n are the damping ratio, spring constant and natural circular frequency obtained from steps 4 and 5.

7. Ensure frequency and amplitude are within the design limits.

Kauffmann (47) has compared various methods of analysis and concluded that the methods from Richart and Barkan yield approximately the same results, but comparision was limited to vertical frequency only.

Table 6.3 Equivalent Radius and Spring Constants of Rectangular Footings (46)

Mode of Vibration	Spring Constant	Equivalent Radius
Vertical	$K_z = \frac{G}{1 - v} B_z \sqrt{4cd}$	$r_o = \sqrt{\frac{4cd}{\pi}}$
Horizontal	$K_x = 4(1+v)\, GB_x \sqrt{cd}$	$r_o = \sqrt{\frac{4cd}{\pi}}$
Rocking	$K_\psi = \frac{G}{1 - v} B_\psi \, 8cd^2$	$r_o = \sqrt[4]{\frac{16cd^3}{3\pi}}$
Torsion	$K_\theta = \frac{16}{3} G \, r_o^3$	$r_o = \sqrt[4]{\frac{16\, cd(c^2+d^2)}{6\pi}}$

Table 6.4 Mass Ratio, Damping Ratio and Spring Constant for Rigid Circular Footing on The Semi-infinite Elastic Body (46)

Mode of Vibration	*Mass (or Inertia) Ratio*	*Damping Ratio* D	*Spring Constant* k
Vertical	$B_z = \frac{(1-\nu)}{4} \frac{m}{\rho r_0^3}$	$D_z = \frac{0.425}{\sqrt{B_z}}$	$k_z = \frac{4Gr_0}{1-\nu}$
Sliding	$B_x = \frac{(7-8\nu)}{32(1-\nu)} \frac{m}{\rho r_0^3}$	$D_x = \frac{0.288}{\sqrt{B_z}}$	$k_x = \frac{32(1-\nu)}{7-8\nu} Gr_0$
Rocking	$B_\psi = \frac{3(1-\nu)}{8} \frac{I_\psi}{\rho r_0^5}$	$D_\psi = \frac{0.15}{(1+B_\psi)\sqrt{B_\psi}}$	$k_\psi = \frac{8Gr_0^3}{3(1-\nu)}$
Torsional	$B_\theta = \frac{I_\theta}{\rho r_0^5}$	$D_\theta = \frac{0.50}{1+2B_\theta}$	$k_\theta = \frac{16}{3} Gr_0^3$

6.5 Design of Dampers

The installation of an attached slabe as a damper(Fig.6.3) can effectively reduce the amplitude of rocking and horizontal vibrations of a footing. The following are the formulae presented by Barkan (41). The natural frequency of rocking vibrations of a footing with an attached slab is

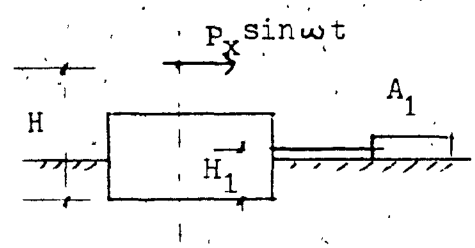


Fig.6.3 Foundation with an Attached Slab (41)

$$f_{n\phi 1}^{\;2} = \frac{c_\phi I - Wh + H_1^{\,2} c_r A_1}{W_0 + m_1 H_1^{\,2}}$$

The amplitude of rocking vibrations of the footing is

$$A_{\phi 1} = \frac{P_x H}{(W_0 + m_1 H_1^{\,2})(f_{n\phi 1}^{\;2} - \omega^2)}$$

where A_1 = soil contace area of attached slab

c_ϕ, c_r = coefficients of elastic nonuniform compression, shear of soil

H = distance between line of action of exciting forces and bottom of foundation

H_1 = distance between tie and bottom of foundation

I = moment of inertia of foundation area in contact with soil, with respect to axis of rotation

m_1 = mass of attached slab

$P_x \sin\omega t$ = magnitude of horizontal exciting force induced by engine

W_0 = moment of inertia of foundation mass and mass of engine with respect to axis of vibrations

As it can be seen from the above formulae, the effect of the attached slab on decrease of vibrations is a function of the height of the slab, the coefficient of rigidity c_r and the corresponding natural frequency of horizontal vibrations of the slab. Barkan also mentioned the analysis of two attached slabs of which the maintenance and tuning is difficult and is rarely used.

6.6 Design of Isolators

High speed machines of ordinary sizes are often mounted on a resilient base and supported on a massive footing. The engine is usually running smooth and the exciting force is relatively small. In this case, the selection of isolators can be determined according to the recommendation from the manufacturer of the resilient material and similar to that described in Chapter V.

For reciprocating engines with large unbalanced forces, hammers and similar equipment, the placement of a concrete block above the resilient material would be necessary to reduce the motion. For other machines such as roll-grinders and planing machines, the attachment of a concrete block with isolators may increase the rigidity of the support. By considering an undamped two degrees of freedom mass-spring system as shown in Fig.6.6, Barkan (41) has indecated that the vibration amplitude of the foundation can be reduced only if the natural frequency of the foundation system is small in comparision with the operating frequency. In situation where the natural frequency of the foundation system exceeds the operating frequency, the natural frequency must be increased so that the associated amplitude decreases. Let k_2 be the total coefficient of rigidity of all springs. The natural frequency of the complete foundation system assuming that no absorbers are used equals

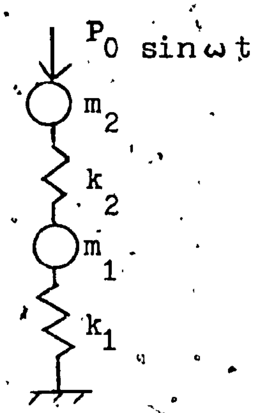


Fig.6.4 Foundation with Absorbers and Mass (41)

$$f_{nz} = \sqrt{\frac{k_1}{m_1 + m_2}}$$

where k_1 = coefficient of elastic rigidity of base under foundation beneath springs

m_1, m_2 = masses of foundation system beneath and above springs

The amplitude of the base beneath the springs:

$$A_1 = \frac{\rho}{m_1} \frac{e_1^{\,2}}{1 - (1+u)\,(e_1^{\,2} + e_{1z}^{\,2} - e_1^{\,2} e_{1z}^{\,2})}$$

where $e_1 = \dfrac{f_{n1}}{\omega}$, $e_{1z} = \dfrac{f_{nz}}{\omega}$ and $f_{n1} = \sqrt{\dfrac{k_2}{m_2}}$.

The degree of absorption of vibration:

$$n = \frac{1 - (1 + u)(e_1^2 + e_{1z}^2 - e_1 e_{1z}^2)}{(1 + u)(e_{1z}^2 - 1)\, e_1^2}$$

where u is the ratio of m_2 to m_1. In practice, the majority of vibrating systems possess six degrees of freedom. As it has been stated, careful considerations and analysis may be necessary prior to choosing of isolators.

6.7 Summary

Foundations subjected to dynamic loads can be treated as a mass mounted to top of a spring. When the operating frequency is higher than 1.4 times the natural frequency of the spring, the vibration amplitude can be reduced and the design can simply be governed by a mass ratio. In case the natural frequency exceeds the operating frequency, the natural frequency must be increased to avoid the dynamic effects. This can be accomplished by increasing the footing area, stiffening the soil, driving piles or other means. A further approach for decreasing vibration is balancing the unbalanced forces by means of counterweight. This involves specialized techniques in mechanical engineering. Another very popular approach is the mounting of isolators. For analysis, the simplified method presented by Richart, Woods and Hall is generally sufficient for installation of industrial equipment. The analytical methods of dampers and isolators presented by Barkan were reviewed.

CHAPTER VII

CONCLUSION

The main objective of this report is to introduce the methods for control of vibrations induced by dynamic loads from people, machinery and wind. The method selected depends on the source of vibration, environmental factors and type of structures involved.

For machinery supported on floor beams, it is a common practice to have the beam frequency checked during the design stage. Change of beam frequencies implies varying of beam span or beam stiffness. Charts and tables shown in Appendix A and reference 16 can be used for quick evaluation of beam frequencies. A simple formula for calculation of frequencies of concrete floors does not seem available. This deficiency can be remedied by field tests. Where possible, the fundamental frequency of a beam should be kept greater than the operating frequency of the machine, in order that resonance will not occur at the equipment start-up and shutdown. Guidelines given in Can3-S16-M78 may be used for design of composite and non-composite steel deck. In designing bridges, recommendations are given for the effects of vibration due to vehicle oscillation and impact caused by uneven road surface. British Standard Institute (14) has detailed recommendations and criteria for design of aerodynamic stability. Discussions have not been given here.

For floor construction, deflection limitations have been the prime concern considered in building codes for both static and dynamic load analysis. This mainly is due to the recent trend of developments in high-strength materials which result in more slender structures. The control of stiffness will ensure better satisfaction of overall strength and resonant frequencies of the members. A similar approach has also been adopted by AASHTO specification to keep the oscillations of overhead signs or signal supports within the limits, and adopted by building codes to keep the oscillations of the buildings below the complaint level and damages in non-structural elements minimal. Nevertheless, experience shows that the provision of this criterion alone is not always sufficient to safeguard structures.

Damping has been of great promise in providing vibration free performance in new and existing structures. In general, the amount of damping inherent in most materials is small. Metal possesses less damping than concrete and rubber. Pre-stressed concrete possesses less damping than reinforced concrete. The data which can be adopted to new design include the investigation of floor vibrations from Allen (25), the damping coefficients recommended by Can3-S16-M78, the field studies for long span concrete floor systems shown in Table 4.1 and 4.2, the studies for tall buildings presented in Table 4.5 and the research of overhead sign supports. Information is rare. Except for very minor problems, the installation of dampers normally involves extensive field work and ingenious problem-solving. Tuned dampers are not preferable for structural applications due to the uncertainty of their reliability and difficult maintenance. The detailed analysis of sloshing effect in liquid containers and the dynamic response in cable structures needs additional research and is out of the scope of this report.

The standard selection charts for mounting of isolators are usually provided for only one mode of vibration. In extreme cases where large unbalanced forces are present, a more complicated approach is necessary. For methods of breaking wind induced oscillations in structures, ideas seem sketchy except for chimneys.

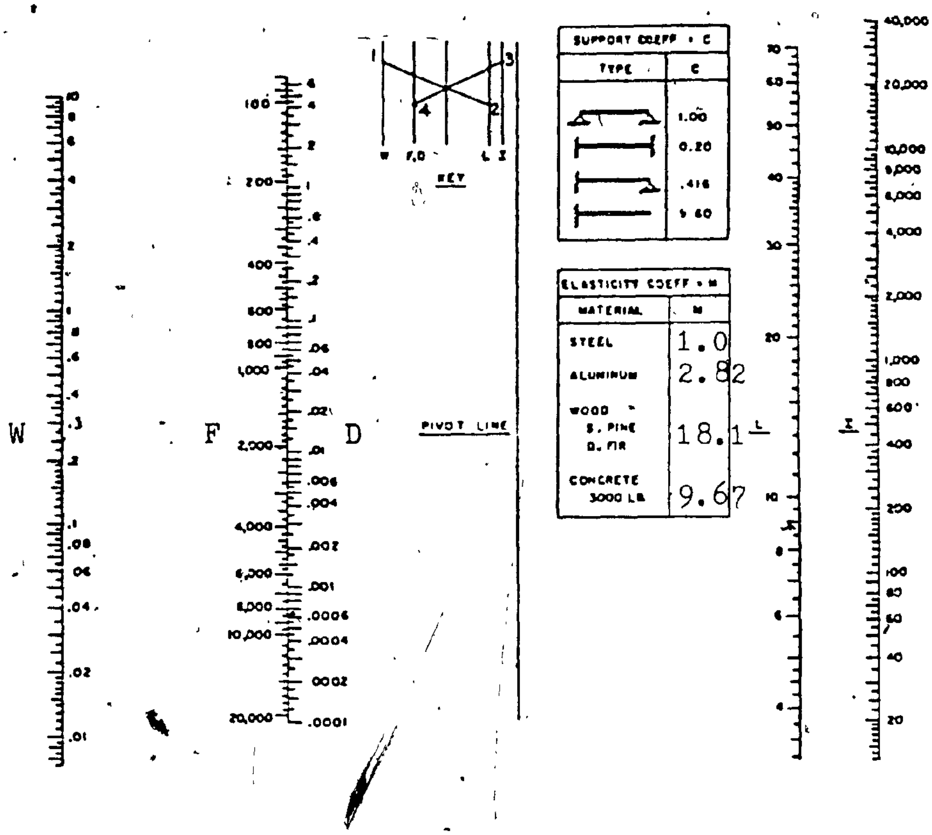
With regard to machinery foundations, the operating frequency of the machine must be sufficiently differ from the natural frequency of the foundation-soil system and amplitude must be within the acceptable limit. The tedious analysis is not always necessary when the operating frequency is high.

In conclusion, we aware that simplified methods for dynamic control are applicable only when we are dealing with simple structures. An exact analysis will rely heavily on the field investigations and computer aid.

Appendix A

Figures and Tables

for Design of Natural Frequencies



W = Uniform Load in Kips Per Foot

F = Frequency in Vibrations per Minute

D = Deflection in Inches

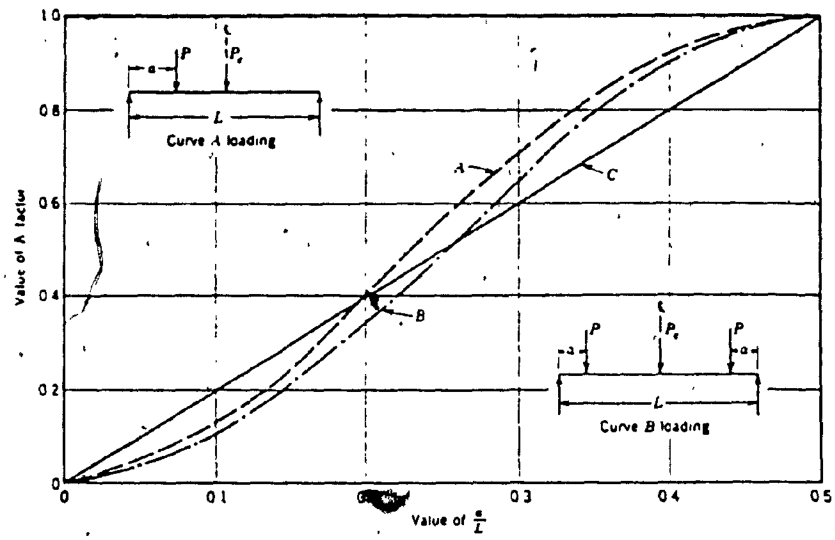
L = Span in Feet

I = Moment of Inertia in $in.^4$ (see note)

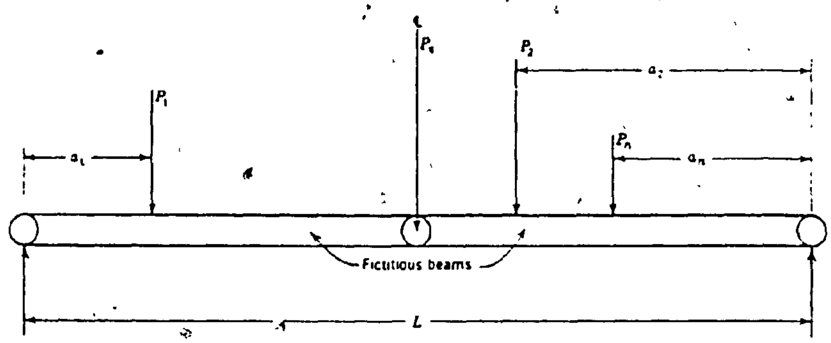
$$F = \frac{211.5}{\sqrt{D}}$$

$$D = \frac{0.000775\ WL^4}{I}\ CM$$

Fig. A-1 Uniform Load Deflection and Frequency (4)



a)



b)

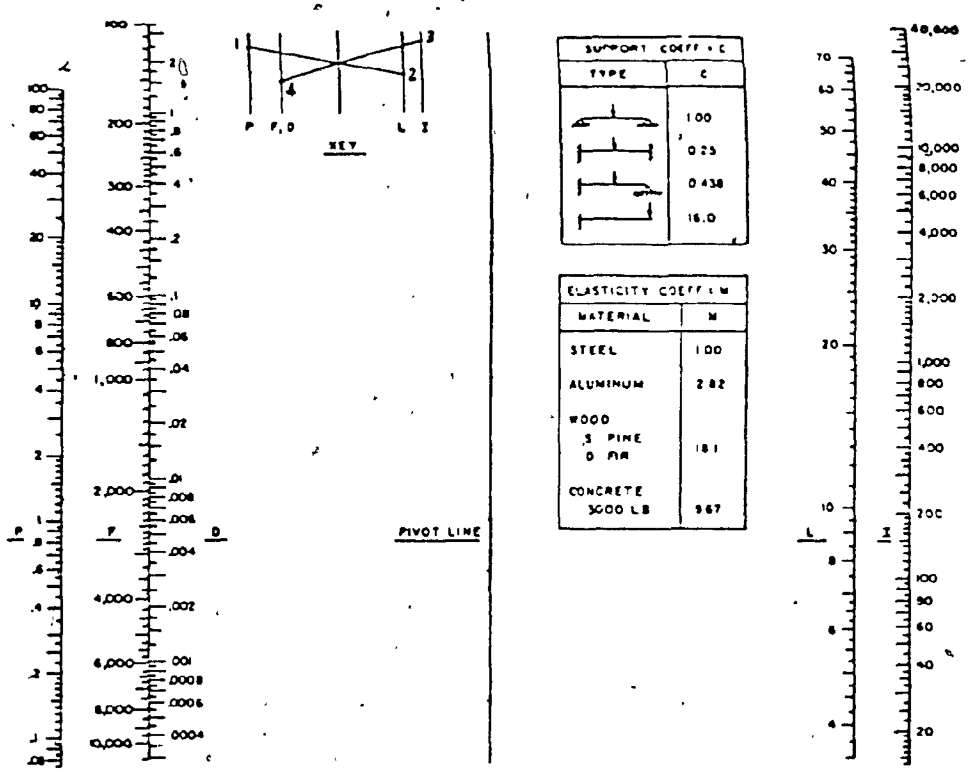
CURVE A, for single mass, P_e = KP

CURVE B, for two equal symmetrical loads, P_e = 2KP

CURVE C, for a number of nonsymmetrical loads
P_e = K1 P1 + K2 P2 + ----

CURVE C, can be used in lieu of A and B

Fig. A-2 Equivalent Centerline Load for Various Loadings (5)



P = Concentrated Load in Kips

F = Frequency in Vibrations per Minute

D = Deflection in Inches

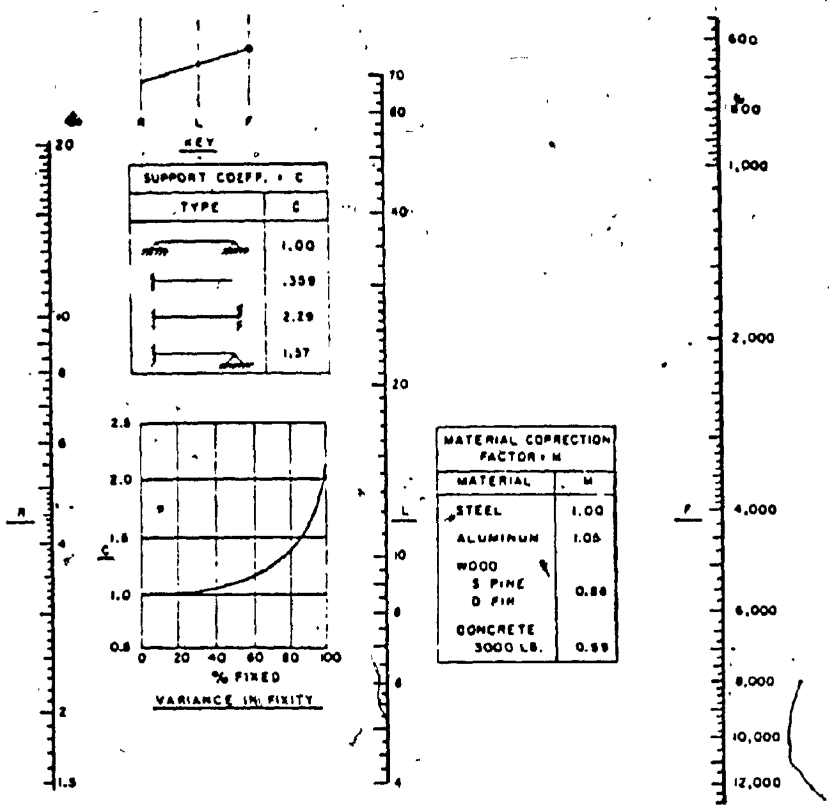
L = Span in Feet

I = Moment of Inertia in $in.^4$ (see note on Fig. 1)

$$F = \frac{187.5}{\sqrt{D}}$$

$$D = \frac{0.00124\ PL^3\ CM}{I}$$

Fig. A-3 Concentrated Load Deflection and Frequency (4)



R = Radius of Gyration in Inches

L = Span in Feet

F = Frequency in Vibrations per Minute

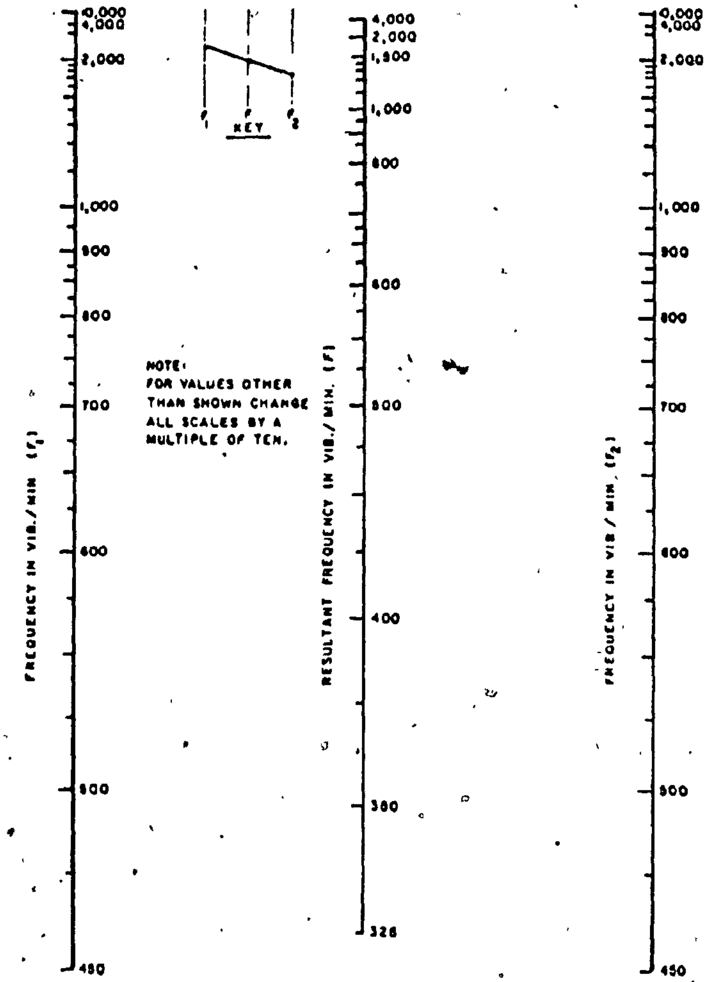
EM = Modulus of Elasticity of Material in Kips per in.^{2}

ZM = Mass Density of Material in Kips Seconds Squared Per Inch to the Fourth Power

$$F = \frac{129{,}500\ R}{L^2}\ CM$$

$$M = 0.00503 \left[\frac{EM}{ZM}\right]^{\frac{1}{2}}$$

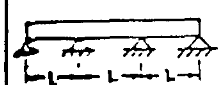
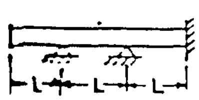
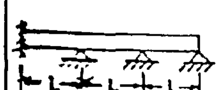
Fig. A-4 Beam Natural Frequency



$$F = \frac{1}{\sqrt{\left[\frac{1}{F_1^2} + \frac{1}{F_2^2}\right]}}$$

Fig. A-5 Frequency Resultant (4)

Table A-1 Natural Frequencies of Continuous Uniform Steel Beams (6)

Beam structures	$(f_n l^2/r)/10^4$					
	N	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Extreme Ends Simply Supported						
	1	31.73	126 94	285 61	507 76	793.37
	2	31.73	49.59	126.94	160.66	285 61
	3	31.73	40 52	59 56	126 94	143.98
	4	31 73	37.02	49.59	63.99	126.94
	5	31 73	34 99	44 19	55.29	66.72
	6	31.73	34 32	40 52	49 59	59.56
	7	31 73	33 67	38.40	45.70	53.63
	8	31.73	33 02	37.02	42.70	49.59
	9	31 73	33 02	35.66	40.52	46.46
	10	31.73	33.02	34.99	39.10	44.19
	11	31 73	32 37	34 32	37.70	41.97
	12	31.73	32 37	34.32	37.02	40.52
Extreme Ends Clamped						
	1	72 36	198.34	388.75	642 63	959.98
	2	49 59	72.36	160.66	198 34	335 20
	3	40 52	59 56	72 36	143.98	176 25
	4	37 02	49 59	63 99	72 36	137.30
	5	34 99	44 19	55 29	66.72	72.36
	6	34 32	40 52	49 59	59 56	67.65
	7	33.67	38.40	45 70	53.63	62.20
	8	33.02	37.02	42 70	49.59	56.98
	9	33.02	35.66	40.52	46.46	52.81
	10	33 02	34 99	39 10	44.19	49 59
	11	32.37	34.32	37.70	41.97	47.23
	12	32.37	34.32	37.02	40.52	44.94
Extreme Ends Clamped-Supported						
	1	49 59	160.66	335 2	573.21	874.69
	2	37 02	63.99	137 30	183 85	301.05
	3	34 32	49.59	67.65	132 07	160.66
	4	33 02	42 70	56.98	69 51	129 49
	5	33 02	39 10	49.59	61 31	70 45
	6	32 37	37.02	44.94	54.46	63.99
	7	32 37	35.66	41.97	49 59	57.84
	8	32 37	34 99	39 81	45 70	53 63
	9	31.73	34 32	38.40	43 44	49 59
	10	31.73	33.67	37 02	41.24	46 46
	11	31.73	33.67	36.33	39 81	44 19
	12	31 73	33 02	35 66	39.10	42.70

f_n = Natural Frequency, cps n = Mode Number

r = Radius of Gyration, in. N = Number of Spans

l = Span Length, in.

Table A-2

Natural Frequencies of Plates (6)

DIAGRAM	EDGE CONDITIONS	Value of n for mode: 1	2	3	4	5	6	7	8
	CLAMPED AT EDGE	11.84	24.61	40.41	46.14	103.12			
	FREE	6.09	10.53	14.15	23.80	40.88	44.66	61.38	69.44
	CLAMPED AT CENTER	4.35	24.26	70.35	130.85				
	SIMPLY SUPPORTED AT EDGE	5.90							
	ONE EDGE CLAMPED-THREE EDGES FREE	1.01	2.47	6.20	7.94	9.01			
	ALL EDGES CLAMPED	10.40	21.21	31.25	38.04	38.22	47.73		
	TWO EDGES CLAMPED-TWO EDGES FREE	2.01	6.96	7.74	13.89	18.25			
	ALL EDGES FREE	4.07	5.94	6.91	10.39	17.80	18.85		
	ONE EDGE CLAMPED-THREE EDGES SIMPLY SUPPORTED	6.83	14.94	16.95	24.89	28.95	32.71		
	TWO EDGES CLAMPED-TWO EDGES SIMPLY SUPPORTED	8.37	15.82	20.03	27.34	29.54	37.31		
	ALL EDGES SIMPLY SUPPORTED	5.70	14.26	22.82	28.52	37.09	48.49		

Natural Frequencies of Thin Plates of Uniform Thickness:

$$f_n = n\sqrt{\frac{Et^2}{\rho a^4(1-v^2)}} \quad \text{rad/sec}$$

Massless Circular Plate with Concentrated Mass:

Clamped Edges $\quad f_n = 4.09\sqrt{\frac{Et^3}{ma^2(1-v^2)}}$

Simple Supported Edges $\quad f_n = 4.09\sqrt{\frac{Et^3}{ma^2(1-v)(3+v)}}$

E = Young's Modulus, $lb/in.^2$

t = Thickness of Plate, in.

ρ = Mass Density, $lb\text{-}sec^2/in.^4$

a = Diameter of Circular Plate or Side of Square Plate, in.

v = Poisson's ratio

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