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THE APPLICATION OF SYSTEM IDENTIFICATION TECHNIQUES FOR DETERMINING Z-TRANSFER FUNCTION COEFFICIENTS OF BUILDING COMPONENTS

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ABSTRACT

The Application of
System Identification Techniques for Determining
Z-transfer Function Coefficients of Building Components

Hong Liang

This thesis describes an experimental procedure for determining z-transfer function coefficients of building envelope components. A frequency analysis approach based on system identification techniques is presented.

The z-transfer function method is most commonly used to calculate space thermal loads. There is a need to determine the z-transfer function coefficients by experimental methods. An application of system identification techniques to determine z-transfer function coefficients of building envelope components based on experimental data is investigated.

The procedure consists of two steps: (1) transforming experimental measurements to frequency domain using the Fast Fourier Transform and determining the frequency response of the transfer function; (2) applying multi-linear regression techniques to fit the z-transfer function coefficients to the frequency response.

In the experimental work, tests were performed using Binary Multi-Frequency Sequence signals as driving functions. Temperature feedback control was used to achieve the boundary conditions.

The frequency analysis approach to experimentally determine the z-transfer function coefficients showed acceptable results. The frequency response of the transfer function 1/B, used in the z-transfer function method was obtained from the experimental data on two different wall samples and coefficients of z-transfer function were determined.

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The author wishes to dedicate this thesis to her parents for their dearly love, patience and unfailing support for the completion of her M.ENG. programme.

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NOMENCLATURE

A	Transfer function of walls
A_g	Gain factor of frequency response function
a_i	Z-transfer function coefficients
B	Transfer function of walls
b_i	Z-transfer function coefficients
b_n	Amplitude of input function
C	Transfer function of walls
C_p	Specific heat of sample material, [J/kg K]
D	Transfer function of walls
e	Error between system output and model output
f	Frequency, [1/sec]
G	Function of test signal
H	Transfer function
H_1	Guard heater
$H_{2,a}$	Guard heater
$H_{2,b}$	Guard heater
H_g	Transfer function
K	Thermal conductivity of sample, [W/m K]
L	Thickness, [m]
m	Number of coefficients in denominator of transfer functions
N	Mean value of the noise
n	Number of coefficients in numerator of transfer functions
P	Period, [min.]

Q_1	Heat flow rate through inside surface of walls
Q_2	Heat flow rate through outside surface of walls
Q_m	Model output
Q_t	Measured output at time t
$\hat{Q}_t$	Predicted output at time t
S	Error function between system output and model output
T	Temperature difference
T_1	Room air temperature
T_2	Outside air temperature
t	Time
U	Steady state U-value
Y	Thermal response factor
ω	Angular velocity, rad h^{-1}
γ_n	Phase of input signal, [$^{\circ}$C]
ξ_n	Attenuation factor of frequency response
φ_n	Time lag of frequency response
τ	Time constant of system transfer function
α	Residue of transfer function
Δ	Sampling time interval
Φ	Phase factor
Φ_Q	Phase angle of heat flow (output signal)
Φ_T	Phase angle of temperature (input signal)

CHAPTER 1
INTRODUCTION

The analysis of transient heat transfer through building envelope components is important in air-conditioning system design and energy simulation of a building. Air-conditioning system design involves the computation of peak design load at a specific hour of a design day. Accordingly, the designer wants to know the peak heat flow through the inside surface of a wall when the sol-air temperature variation of the design day is imposed on the outside surface of the wall. Building energy analysis techniques include the space heating and/or cooling load calculation, which is the basis for the air distribution system simulation and the central plant (boiler/chiller) simulation. The space thermal loads calculation contain the determination of transient heat transfer through building envelope components.

The calculation of space thermal loads depends greatly on the amount of heat flow through a building's exterior walls. The amount of heat flow through a wall is a function of climate, building construction, space temperature control strategy, occupancy, and lighting use patterns. These conditions are dynamic in nature. As a result, dynamic thermal analysis of building envelope components is critical in loads calculation.

Analyzing the dynamic thermal behavior of building envelope components involves both direct and inverse problems. The direct problem consists of finding the response to a specific input based on a given description of the system. In contrast, in the much more difficult inverse problem, the response of the system to a particular input is known and we wish to find the system description which fits the input/output relationship as closely as possible. The inverse problem is a classical problem and is usually known as system identification. The question of how to obtain the dynamic thermal response characteristics of building envelope components from experimental measurements forms the subject of this study.

In this thesis, a review of the methods used to calculate the transient heat transfer through walls shows the z-transfer function method to be in common use. Consequently, modeling transient heat transfer through walls by means of z-transfer functions and estimating the parameters of the transfer functions by applying system identification technique are described. Then, experimental techniques and a discussion of the results are presented. Finally, as a summary, a developed procedure to experimentally determine the z-transfer function coefficients for walls is presented.

The z-transfer function method is endorsed by American

Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) and has been in common use to calculate design thermal loads in buildings for the past 20 years in North America. The major advantages of the z-transfer function method are that it is numerically efficient and does not require that the heat transfer boundary conditions be periodic. The use of the z-transfer function simplifies the calculations of transient heat flow through composite walls and roofs. No direct knowledge of material thermophysical properties or heat transfer mechanisms is required. The effects of the actual heat transfer mechanisms are implicitly modeled with a best fit linear approximation. Also, the method overcomes the necessity to calculate internal temperature distributions and provides a set of coefficients for the structure that can be used to analyze other input history without resolving the entire problem.

The z-transfer function coefficients are presently obtained by means of calculations presupposing that the thermal properties of the component materials are known. The derivation of the z-transfer function coefficients is based on the assumptions that the wall construction is made of homogeneous materials and that the heat flow is one-dimensional. In practice, however, the calculation of z-transfer function coefficients by this method is not adequate because: (1) the thermal properties of wall materials of existing buildings are often unknown, or the installed materials are different from the properties reported in the literature because of environmental influences or material degradation with time; (2) the actual wall may be composed of non-homogeneous materials; (3) the wall may contain anomalies, such as framing members that act as heat bridges through the insulation. Therefore, the transfer function coefficients obtained from the calculation methods may not accurately represent the thermal performance of real walls or roofs. Experimental methods to determine the z-transfer function coefficients are required.

Objectives

The objective of this research work was to develop an experimental method to analyze the thermal dynamic characteristics of existing walls and to determine the z-transfer function coefficients for building envelope components. To attain this goal, the application of system identification techniques for finding the frequency response of a wall from experimental data and fitting the model parameters (z-transfer function coefficients) to the frequency response has been explored.

Scope

This thesis describes the results of the research work in the following chapters. Chapter 2 reviews the literature of calculation methods and field studies. In chapter 3, methods for modeling the transient heat conduction through walls are described. Chapter 4 presents the system identification techniques used to determine the heat transfer characteristics of building envelope components, which involves the experimental work. Chapter 5 describes the experiment to obtain the 1/B transfer function. Finally, conclusions and recommendations are summarized in chapter 6.

CHAPTER 2
LITERATURE REVIEW

Existing methods for analyzing the transient thermal behavior of building envelope components can be categorized into three general classes: (1) lumped parameter methods; (2) analytical methods; and (3) transfer function methods.

The lumped parameter methods model a physical system as a number of discrete elements or 'lumps'. These can be expressed by using Resistor-Capacitor (RC) circuits based on the analog between thermal phenomenon and electricity. Then numerical techniques may be used to obtain the solution.

Paschkis and Heislar[1], Klein et al.[2], Lawson and Mcguire[3], and Friedmann[4] analyzed the transient response of passive analog electric circuits. The results indicated that many elements are required if an analog is to accurately represent the response of a slab to a sudden change in the driving function.

Stephenson and Mitalas[5] presented a method of designing analog circuits to calculate one-dimensional heat conduction through a homogeneous wall with a specified accuracy, and compared the frequency response of several electrical analog circuits with the theoretical frequency response of a homogeneous slab. The results gave a prescription for the selection of the simplest circuit which will achieve a specified accuracy over a specified frequency range.

Sonderegger[6] and Davies[7] discussed the choice of resistance and capacitance elements in modeling a sinusoidally excited wall. Davies's approach was to minimize the sum of squares of the difference between the corresponding elements in the frequency response matrix.

Hammarsten[8] used the results of Sonderegger[6] and discussed in detail the behavior of a lumped parameter model in modeling a wall with an air film. The purpose of his work was to model whole houses.

The lumped parameter method with the numerical solution has the advantage that it can be used to solve nonlinear system problems. But, it is often necessary to take a large number of 'lumps' and at frequent intervals to avoid numerically-induced oscillations in the solution. Also, the complete internal temperature distribution must be computed at each time step. It is necessary to solve the complete problem over again when the same structure is subjected to another input time history. Thus, the numerical techniques may be expensive and time consuming.

The analytical methods of solving the governing heat conduction differential equation are treated in standard heat conduction text books.[9] By using the superposition principle, this approach is quite acceptable for regular geometries such as homogeneous plane walls when the input functions are relatively simple. If the boundary conditions are represented as periodic functions, the building heat transfer parameters, including convection coefficients, are constant with time, and the radiant heat transfer is linearized, the harmonic solution can be obtained. Mackey and Wright,[10] Van Gorcum,[11] Muncey and Spencer,[12,13] Gupta,[14] Sonderegger,[15] Maeda,[16] and others have contributed to the enhancement of analytical methods for solving the heat conduction differential equations.

Pipes[17] presented a method which makes use of the matrix algebra technique and the analogy existing between the thermal problem and the flow of electricity to analyze heat conduction problems. This has been used as the fundamental tool in modeling of heat flow through building components.

Stephenson and Mitalas[18], and Mitalas and Stephenson[19] developed the thermal response factor method. Thermal response factors are defined as the time series output resulting from an input triangular pulse. The response to any input can then be obtained by approximating the input as a series of triangular pulses and applying the superposition principle. The thermal response factors for a homogeneous slab can be calculated by exact analysis given the thermal properties and thickness of the slab and the time interval. The formulae for calculating the response factors for a homogeneous slab are given[20].

A computer program by Mitalas and Arseneault[21] improved considerably the accuracy of the calculation of response factors. The program is able to calculate the response factor sets for any homogeneous multi-layer slab provided that the heat flow is one-dimensional.

Kusuda[22] extended the thermal response factor method for multi-layer structures with various curvatures of finite thickness and to semi-infinite systems.

Hittle[23] provided a comprehensive mathematical development of response factor theory. Also Hittle and Bishop[24] developed an improved root-finding technique which allows response factors to be calculated more efficiently. This improvement eliminates the need for searching for roots and ensures that roots will not be missed.

Considering that most existing models contain too many parameters to be suitable for direct analysis, Sherman et al.[25] developed a simplified model of dynamic thermal performance that allows the characteristics of a wall to be quantified on the basis of measured surface temperatures and flux. The model uses a set of Simplified Thermal Parameters (STPs) to characterize the thermal performance of walls from an arbitrary temperature history. The STPs include the steady-state conductance (U-value), the most important parameter of any wall, and the time constant. Sherman found a relationship between the reduced set of parameters and the thermal response factors used in the thermal response factor method. As a result, the response factors can be determined from the STPs.

To determine the accurate results of heat flux for some types of construction, a large number of response factors are needed in using the thermal response factor method. Peavy[26] developed a method to reduce the number of terms. By relating the heat flux and a set of modified response factors in the conduction transfer functions, the amount of computation was reduced.

Ceylan and Myers[27] presented a method for obtaining long-time solutions of heat conduction transients with time-dependent forcing functions. Their technique begins with either a finite-difference or a finite-element model of the heat-transfer problem, then approximates each time-dependent input function by continuous, piecewise-linear functions, each having the same uniform time interval. Finally, the resulting generalized eigenvalue problem is solved and the response coefficients are obtained. The Ceylan and Myers method was developed for both one-dimensional and multi-dimensional heat conduction problems. There are errors due to the spatial discretization of the structure and to the approximation of the time-dependent inputs by continuous piecewise-linear functions. Further, the costs of obtaining the response coefficients from the solution of the eigenproblem is relatively high.

As a final extension of thermal response factor methods, Stephenson and Mitalas[28] developed the z-transfer function method. The basis of this method is that the output value at any time can be determined from the input at that time and the values of both the input and output at previous times. The z-transfer function coefficients can be calculated from the thermal properties, thickness and position of materials in the walls. The z-transfer function requires fewer coefficients than the response factor method. Therefore, it is very efficient in computation

Transfer functions have been used typically to model walls and roofs for which the predominant heat transfer mechanism is conduction. Mitalas[29] discussed the development and properties of the z-transfer function, indicating that the most important characteristic of the z-transfer function method as compared with other calculation methods is that the input and output are a sequence of values equally spaced in time. Thus, the weather records of outside air temperature and solar radiation, which are given on an hourly basis, can be used as an input with very little preprocessing.

Stephenson and Ouyang[30] analyzed the accuracy of z-transfer functions for walls using a sinusoidal function as excitation in frequency domain, and indicated that the precision of heat flux calculated using the z-transfer function depends on how closely the frequency response of the transfer function approximates the true frequency response of the wall over the range of the driving function's frequencies. The effect of the time interval used in the transfer function method and the effect of the number of eigenvalues used to determine the transfer function coefficients also were discussed. They indicated that if the number of eigenvalues used in the calculation is greater than 4, the extra terms have little effect on the accuracy.

ASHRAE has adopted the z-transfer function method to calculate the transient heat conduction through building envelope components. Z-transfer function coefficients for 179 different construction types are listed in the ASHRAE Handbook.[31] Those were calculated using combined outdoor air heat transfer coefficient, indoor air heat transfer coefficient, and the material properties of roof or wall constructions listed in the Handbook. Application of these transfer function coefficients is limited to cases with sol-air temperature values similarly calculated with the combined outdoor heat transfer coefficient.

Stephenson and Mitalas[28] presented methods for calculating the coefficients of the z-transfer functions for heat flow and temperature at any plane in a multi-layer wall.

Based on the methods above, Mitalas and Arseneault[32] produced a computer program to calculate z-transfer functions for walls and roofs.

It should be noted that the calculation methods, mentioned above, for derivation of z-transfer function coefficients are in terms of physically defined thermal properties of building component materials The method recommended in the ASHRAE Handbook assumes that the walls or roofs are made of layers of homogeneous materials, and the heat flow is one-dimensional. Unfortunately, the thermal properties of existing wall or roof materials are often unknown. Thus, the calculation methods are not adequate. Also, the actual walls or roofs contain framing members which act as heat bridges through the insulation resulting in heat flow which is not one-dimensional. Consequently, the transfer function coefficients in the ASHRAE Handbook of Fundamentals may not accurately represent the thermal performance of real walls or roofs. As a result, improved methods of obtaining transfer function coefficients are required.

Many methods exist for transient analysis in the engineering field. There is a basic distinction in the goals of the different methods, which can be broadly categorized as direct or inverse problems. The direct problem consists of finding the response to a specific input based on a given description of the system. For instance, the designer of a building has the description of the building and he wants to calculate its peak and average thermal loads. In the inverse problem, on the other hand, the response of a system to the particular input is known but the system description, which fits this input/output relationship as closely as possible, is desired. The estimation of the transfer function coefficients based on realistic performance measurements is an essential inverse problem. A building wall may be considered as a black box, and then its characteristics, expressed by the z-transfer function coefficients, can be inferred from the temperature and heat flow data. Several investigators have used system identification methods to study the thermal behavior of walls.

Pederson and Mouen[33] attempted to fit a response factor model to measured data as part of an ASHRAE research project. They concluded that the direct procedure for determining Thermal Response Factors is impractical because of the extreme sensitivity of such procedures to experimental error, and the likelihood of error in a transient heat transfer experiment. Consequently, they applied system identification techniques by adjusting the thermophysical properties of the chosen model rather than adjust the thermal response factors directly. The material thermophysical properties were estimated from the experimental data. All of the requisite properties of response factors, such as the common ratio property and the overall summation property, were retained.

Astrom and Eykhoff[34] and Bekey[35] reviewed and discussed the use of the system identification techniques to determine a system's parameters. Crawford and Woods[36] used least squares techniques to fit parameters to a chosen transfer function model in time domain. Fang and Grot[37] derived thermal resistance values of building envelopes from field data.

Seem and Hancock[38] presented a technique for characterizing the dynamic performance of a thermal storage wall based on the data obtained from a series of temperature and heat flux measurements. The coefficients of a transfer function model were estimated directly from data using linear least squares regression. Also, a set of parameters for characterizing the steady-state performance were derived from the transfer function coefficients.

Stephenson, Ouyang and Brown[39] developed an experimental procedure to derive the transfer functions of walls from hot-box test results. The procedure involves two steps. Starting from a lumped parameter model of the transfer function in s-domain, they determine the model parameters, time constants and the associated residues, from experimental measurements. Then, using the equivalence between the lumped parameter model and the ratio of polynomials expression of the z-transfer function, they find the z-transfer function coefficients in the denominator polynomial of the expression. Next, they determine the frequency response from experimental data and match this frequency response to the response of the z-transfer function at some specific frequencies in order to determine the coefficients in the numerator of the z-transfer function.

The experimental work involves using a guarded hot-box wall testing facility to measure the air temperature at both sides of a full scale test wall and measure the heat flux into the room-side surface of the wall. The experiment is conducted by using a ramp test signal to determine time constants and by using sinusoidal signals to determine the frequency response of the transfer function.

The experiments are a series of tests on a real wall which contains framing members. The heat transfer through the wall is not one-dimensional and the effects of heat bridges is considered.

Haghighat and Sander[40] used system identification techniques to determine the dynamic response of walls. In their study, a wall was identified as a system with output (the heat flux on the inside surface of the wall) and input (the outside weather conditions) related by a transfer function. Experimental procedures were conducted using a Binary Multi-Frequency Sequence (BMFS) as the input driving function, and the coefficients of the z-transfer function (1/B) for a single layer sample were obtained using both frequency response analysis and least squares regression in time domain.

The method used by Haghighat and Sander[40] involves measurement of wall dynamic performance and analysis of the measurement data to determine the z-transfer function coefficients which does not require the thermal properties of building component materials be known in advance. The BMFS excitation function has the advantage that multi-frequency response can be determined from one test. This requires less precise control and does not take as much time as test procedures which have to be repeated for each individual frequency. The z-transfer function coefficients are obtained using multi-linear regression to get the best fit of frequency response at a large number of frequencies.

CHAPTER 3

METHODS FOR MODELING TRANSIENT HEAT CONDUCTION THROUGH WALLS

The analysis of transient heat transfer through building envelope components is important in air-conditioning system design and energy simulation of the building. In practice, the usual approach in air-conditioning system design involves the computation of peak design load at a specific hour of a design day. Accordingly, the designer wants to know the peak heat flow through the inside surface of a wall when design day sol-air temperature variation is imposed on the outside surface of the wall. Similarly, to estimate the energy requirements, the detailed simulation performs energy balance calculations hourly over an analysis period of a year. Therefore, the heat flow through walls, typically for each hour of the year, with outside surface temperatures determined by weather and solar data is required. Further, since the control characteristics of the air distribution system interact on the space load, the energy simulation must relate inside air temperature variations with changes in heating/cooling supplied to the room.

The analysis of transient heat conduction through walls is a major part of heat transfer study in buildings and it is complicated. The Fourier heat conduction law and the general heat-conduction differential equations provide the foundation of this analysis. Methods of solving the differential equations include the lumped parameter model with numerical solution, the first-principles model with harmonic solution, and the 'black box' model which is represented by the z-transfer function.

In this chapter, from the practical application and theoretical calculation points of view, the modeling methods used in the lumped parameter method and in the analytical methods for harmonic solution are first reviewed. The theoretical solutions of the z-transfer functions in the transmission matrix are obtained. Then, the frequency domain simulation in Fourier transform is described. Finally, the modeling techniques applied in the z-transfer function method are presented.

3.1 Lumped parameter RC net-work

The lumped parameter models use an analogy to electric resistor-capacitor circuits; the continuously distributed thermal resistance and capacity of the wall material are taken to be localized into a number of discrete units or 'lumps'. The resulting electric circuit can then be solved numerically. The larger the number of lumps taken, the more nearly the results of computations on the lumped parameter model approximate the computations of the continuously distributed model. Examples of lumped parameter models for a slab are given in Figure 3.1. The R and C represent the heat resistance and heat capacity of the slab material respectively. Both space and time derivatives in the heat conduction equation can be approximated as finite differences, and the numerical solution can be obtained. The finite difference solution of a lumped parameter model is shown in Appendix A.

By using lumped parameter models with numerical methods, such as finite difference and finite element techniques, the temperature distribution as well as heat flows and other desired outputs can be determined. Also, non-linear systems can be solved. However, it is often necessary to take a larger number of 'lumps' and very small time steps to avoid numerically-induced oscillations in the solution. The complete internal temperature distribution must be computed at each time step. Since, it is necessary to solve the complete problem over again when the same structure is subjected to another input time history, numerical techniques may be expensive.

3.2 Periodic heat flow

Analytical methods, using the Fourier heat conduction law as a starting point, solve the distributed model of the heat conduction differential equation. If we consider a wall as a thermal system and, ideally, that the change of outside air temperature or solar radiation incident on the outside surfaces of the wall are repeated over a period of 24 hours, then the transient heat conduction differential equations can be solved in the periodic function form.

The principle of periodic heat conduction through a wall is that: if the excitation function, say temperature variation on one surface of the wall, is expressed by

$$T(t) = \sum_{n=1}^{\infty} b_n \cos[n\omega t+\gamma_n] \qquad (3.1)$$

where, b_n and γ_n are the amplitude and phase of the input function respectively, t is time, and ω is angular velocity (rad h^{-1}). For the given thermal properties of the wall elements, the heat flow at point x of the wall and at time t can be obtained from the expression:

$$Q(t) = \sum_{n=1}^{\infty} (b_n \xi_n) \cos[n\omega t+(\gamma_n+\varphi_n)] \qquad (3.2)$$

where, ξ_n and φ_n are called attenuation factor and time lag of the frequency response respectively. ξ_n and φ_n can be found by choosing the excitation function as:

$$T(t) = \cos(\omega t) \qquad (3.3)$$

Where, b_n=1 and γ_n=0. Then, the frequency response Q(t) against the input function is in the form of:

$$Q(t) = \xi \cos(\omega t+\varphi). \qquad (3.4)$$

The drawbacks of the harmonic analytical methods are obvious. The outside excitations include sudden changes in temperature which are not sinusoidal functions, and the sol-air temperature variation does not repeat every day. However, these methods have been used for design load calculations.

3.3 Theoretical solution of the heat conduction differential equation —— Matrix Expression

The theoretical solution of the heat conduction differential equation can be derived starting from the basic dynamic characteristics of constant parameter, linear, and stable systems.

A system is known as constant parameter if all its fundamental properties are invariant with respect to time. A system is linear if the response characteristics of the system are additive and homogeneous. A system is called stable if every possible bounded input function produces a bounded output.

In many engineering sciences linearization of a physical problem has been successfully applied to simplify the process. For engineering purposes, a building enclosure can usually be considered as a constant parameter, linear system.

The dynamic characteristics of a constant parameter, linear system can be described by a weighting function h(t), which is defined as the output of the system at any time to a unit impulse. The usefulness of the weighting function as a description of the system is that the response of the system to any excitation can be found from the convolution principle. For example, if temperature variation excited to a wall is T, the response, heat flow through the wall, can be expressed as:

$$Q(s) = H(s)\ T(s) \tag{3.5}$$

where, Q(s) and T(s) are Laplace transforms of the output (response) and input (excitation) functions. H(s) is the Laplace transform of the weighting function which describes the thermal properties of the wall.

Because we are interested in the temperature and heat flow particularly at both surfaces of a wall, from the convolution expression of surface heat flux with temperatures, the matrix expression of the theoretical solution of heat conduction differential equation for a wall can be obtained as follows.

Given the temperature excitations which are simultaneously imposed on both surfaces of a wall (see Fig.3.2), T(1,t) and T(2,t), the heat flow at both surfaces of the wall can be expressed in the convolution integral form. In converting these equations into Laplace transform, and also applying the convolution integral property that the convolution formula becomes a product in the imaginary space, we have:

$$Q(1,s) = H_{q1}(1,s)\ T(1,s) + H_{q2}(1,s)\ T(2,s)$$
$$Q(2,s) = H_{q1}(2,s)\ T(1,s) + H_{q2}(2,s)\ T(2,s) \qquad (3.6)$$

These can be rewritten in a matrix expression:

$$\begin{bmatrix} Q(1,s) \\ Q(2,s) \end{bmatrix} = \begin{bmatrix} H_{q1}(1,s) & H_{q2}(1,s) \\ H_{q1}(2,s) & H_{q2}(2,s) \end{bmatrix} \begin{bmatrix} T(1,s) \\ T(2,s) \end{bmatrix} \qquad (3.7)$$

The square matrix is called a transfer matrix, in the sense that it relates the temperature matrix to the heat flow matrix. The elements, H_{q1} and H_{q2}, are called transfer functions. The derivation of the transfer function is given in Appendix B. Further, the matrix expression can be rewritten as:

$$\begin{bmatrix} T(1,s) \\ Q(1,s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} T(2,s) \\ Q(2,s) \end{bmatrix} \qquad (3.8)$$

The transfer functions A(s), B(s), C(s), and D(s) can be derived (see appendix B).

For a multi-layer wall, the transfer matrix of the whole wall is the product of the transfer matrices of every layer of the wall. For a two-layer wall, for example, we have:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} A_1A_2+B_1C_2 & A_1B_2+B_1D_2 \\ C_1A_2+D_1C_2 & C_1B_2+D_1D_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{3.9}$$

This expression can be rearranged to express surface heat flows as responses and the surface temperatures as excitation:

$$\begin{bmatrix} Q_1(s) \\ Q_2(s) \end{bmatrix} = \begin{bmatrix} \frac{D(s)}{B(s)} & -\frac{1}{B(s)} \\ \frac{1}{B(s)} & -\frac{A(s)}{B(s)} \end{bmatrix} \begin{bmatrix} T_1(s) \\ T_2(s) \end{bmatrix} \tag{3.10}$$

3.4 Frequency domain simulation

The frequency response function is a special case of the s-transfer function. The frequency response function may replace the transfer function with no loss of information. In addition, the Fourier transform effectively eliminates one dimension from the problem. In the numerical application of the Laplace Transform, a function or sequence in time domain is transformed into a function or sequence parameterized over the entire complex domain. If the real and imaginary axis were discretized into N points each, that would mean it would be necessary to calculate and store N^2 words of complex data for a transformation. The Fourier transform converts a time domain function into a new function which is parameterized only over the imaginary axis. In contrast to the Laplace transformation, only N points need to be calculated and stored.

The frequency response of the transfer functions given in the matrix expression of equation (3.10) can be obtained simply by substituting $j\omega$ for s:

$$\begin{bmatrix} Q_1(j\omega) \\ Q_2(j\omega) \end{bmatrix} = \begin{bmatrix} \frac{D(j\omega)}{B(j\omega)} & -\frac{1}{B(j\omega)} \\ \frac{1}{B(j\omega)} & -\frac{A(j\omega)}{B(j\omega)} \end{bmatrix} \begin{bmatrix} T_1(j\omega) \\ T_2(j\omega) \end{bmatrix} \tag{3.11}$$

Where, ω is angular frequency: $\omega=2\pi f$.

As in the s-domain, the convolution integral of a constant parameter, linear system in frequency domain can be reduced to a simple algebraic expression:

$$Q(f) = H(f)\ T(f) \tag{3.12}$$

Hence, the frequency response function of a system can be derived from the Fourier transform of the input and the output.

The frequency response function is generally a complex valued function which may be conveniently thought of in terms of a magnitude and a phase angle. This can be done by writing H(f) in complex polar notation as follows:

$$H(f) = |H(f)|\ e^{j\varphi(f)} \tag{3.13}$$

The absolute value $|H(f)|$ is called the system gain factor and the associated phase angle $\varphi(f)$ is called the system phase factor. The gain factor and the phase factor of a frequency response function also can be given by drawing the Bode plot which shows the magnitude and the phase lag separately. If the input, T(t), is the difference in temperature between inside and outside surface of a wall and $Q_1(t)$ is the inside surface heat flux, the gain factor is the wall's impedance and the phase factor gives the time delay between the temperature difference and the surface heat flux. The system time constant can also be estimated from the Bode plot.

If the output of one system described by $H_1(f)$ is the input to a second system $H_2(f)$, and there is no loading, or feedback, between the two systems, then the overall system transfer function H(f) is the product of the two transfer functions:

$$H(f) = H_1(f)\ H_2(f) \tag{3.14}$$

which implies

$$|H(f)| = |H_1(f)| \; |H_2(f)| \tag{3.15}$$

$$\varphi(f) = \varphi_1(f) + \varphi_2(f)$$

This valuable property simplifies the calculation of the transfer function for multi-layer walls.

It is important to note that the frequency response function H(f) of a constant parameter, linear system is a function of frequency only, and is not a function of either time or the system excitation. If the system were non-linear, H(f) would also be a function of the applied input. If the parameters of the system were not constant, H(f) would also be a function of time.

3.5 Response factor and z-transfer function methods

The analytical model and the matrix model discussed above require that the temperature excitation be a periodic function for which a Fourier transform can be easily obtained, or a simple analytical function for which the Laplace transform is known. However, the natural excitation, such as outside air temperature and solar radiation, are not periodic and vary in a quite random fashion. It is necessary, therefore, to incorporate these natural random processes into the analysis.

The temperatures and heat flows can be represented by a series of values equally spaced in time. The response of the wall to a unit pulse can be determined analytically and can also be expressed as a time series. These are called thermal response factors. If the input is considered to be a series of pulses, then the output is the superposition of responses to these pulses. This may be written in the form of a convolution expression. For example, if T is the input temperature time series and Y is the set of m response factors, then the output heat flux at time t is:

$$Q_{(t)} = \sum_{j=0}^{m} Y_j \, T_{(t-j)} \tag{3.16}$$

Note that the number of response factors, m, is usually a fairly large number. As the result, a considerable amount of computation is involved.

The z-transform is often used in the numerical control in random process and is an advanced way in which the convolution integral is represented. The temperatures and heat flows can be expressed as time series. The basic formula of the z-transform to relate the heat flux time series to the temperature time series can be expressed by a linear differential equation and it can be simplified into an algebraic equation by applying the convolution property. The z-transfer function which relates the heat flow (Q) to the temperature (T) can be expressed as a ratio of two polynomials in z^{-1} (see Appendix C):

$$H(z) = \frac{Q(z)}{T(z)} = \frac{a_0+a_1\ z^{-1}+a_2\ z^{-2}\ +\ldots+\ a_n\ z^{-n}}{1+b_1\ z^{-1}+b_2\ z^{-2}\ +\ldots+\ b_m\ z^{-m}} \tag{3.17}$$

where, the parameters a_i and b_i are called z-transfer function coefficients. The heat flow (output) at time t can be determined from the temperature (input) at that time and both the output and the input time histories:

$$Q_t = T_t a_0 + T_{t-1} a_1 + T_{t-2} a_2 + \ldots + T_{t-n} a_n - \{Q_{t-1} b_1 + Q_{t-2} b_2 + \ldots + Q_{t-m} b_m\} \tag{3.18}$$

The major advantages of the z-transfer function method are that it does not require the heat conduction boundary conditions be linear and periodic as is the case with the lumped parameter method or analytical method. Additionally, it is important to note that since the number of summations of the products is much less than that used in the response factor method, it is much more economical in terms of computer memory space and running time, and it allows for a more precise computation with a shorter convolution operation. Stephenson and Ouyang[30] indicated that, if the number of coefficients used for output history terms are greater than 4, the extra terms have little effect on the accuracy. In use of the z-transfer function method, the real problem then is how to obtain the z-transfer function coefficients.

The z-transfer function coefficients are presently obtained by means of calculations knowing the thermal properties of wall materials in advance. The thermal properties of existing walls or roofs, however, are often unknown. As a result, the estimation of the z-transfer function coefficients in this case forms an essential inverse problem. System identification techniques which can be used to solve the inverse problem are described in the next chapter.

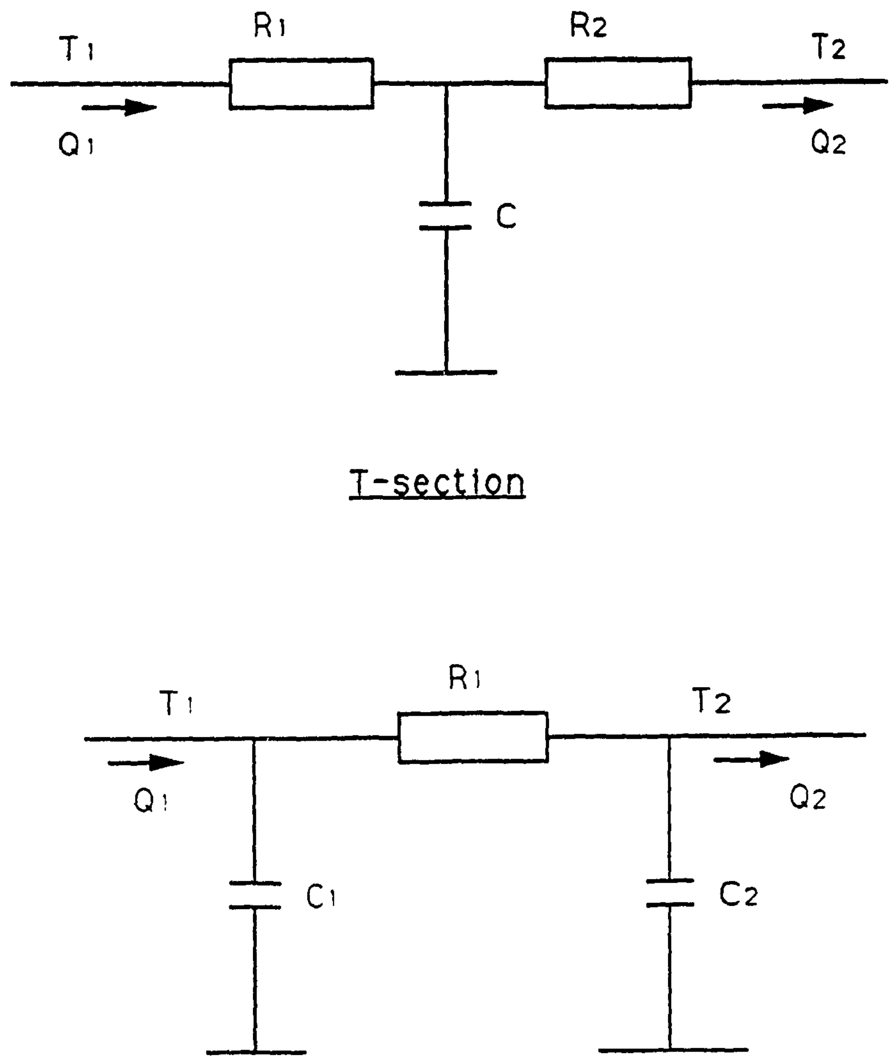


Figure 3.1 **Lumped parameter model of a slab**

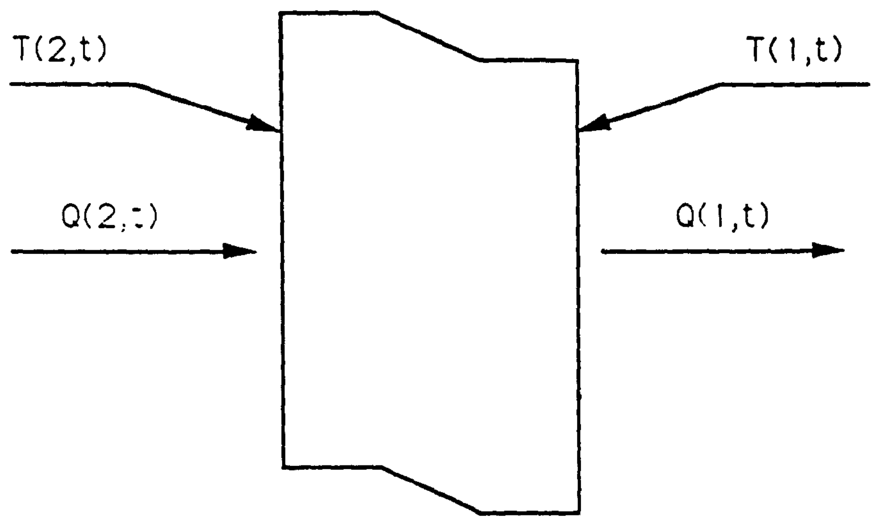


Figure 3.2. Surface Heat Flow With Temperature

CHAPTER 4
SYSTEM IDENTIFICATION

Methods described in chapter 3 can be applied if physical properties of wall materials are known and the geometry of the walls is simple, for example, a multilayer slab. However, this is not the case in practice. Therefore, the approach of obtaining thermal response characteristics of walls by experimental measurements needs to be explored. System identification techniques are often combined with experimental measurements to estimate parameters for physical systems.

This chapter first presents the system identification problem. Then, the selection of the form of a model to which parameters are to be fitted is discussed. The system identification methods for finding z-transfer function coefficients, which were employed in the experimental work of this study, are described in the last section.

4.1 System identification problems

System identification is the essential inverse problem: given input and corresponding output of a system, find the system description which fits the relationship as closely as possible. A system identification problem in general, as proposed by Bekey[35], can be divided into three parts: (1) selection of the model form and determination of the unknown parameters; (2) selection of a criterion function; (3) selection of an algorithm to adjust the parameters in such a way that the difference between the model and system response is minimized. The different identification schemes can be classified according to the basic elements of the problem: the type of signals used, the form of the model, and the criterion for selecting parameters.

The class of signals:

There are two different types of input signals: nonperiodic and periodic. Nonperiodic test signals in common use include impulses of different shape and relatively short duration, step functions, and ramp functions. Table 4.1 shows some of these signals. A common form of process model, the weighting function, can be obtained simply by using the impulse nonperiodic signal as input to the system. The impulse signal is actually the derivative of the step function. A detailed description of step response testing has been given by Rake.[41] The ramp functions also can be used to form an impulse function. For example, the triangle pulses, which were used in the response factor method described in chapter 3, can be expressed by the superposition of three ramp functions. In the case of a nonperiodic input signal the process starts in an equilibrium condition. Following the transient stage, it then may either settle into a new equilibrium, which can be equal to the initial one, or continue drifting out of the region of normal operation. In which way the process will behave depends on both the properties of the process and input signal.

Periodic signals in common use consist of sinusoidal signals, binary monofrequent signals, and multi-frequency signals. Table 4.2 shows some periodic signals. The multi-frequency signals contain significant signal amplitudes at more than one frequency. Periodic signals in general are employed in processes which are running at steady state with the output containing the same frequency (or frequencies) as the input and all transients having subsided. Modern transfer function analysis employs Fourier analysis of signal values digitized at equidistant points in time. Signals can be recorded and processed on a general purpose digital computer. Multi-frequency input signals enable the response at a number of frequencies to be determined from one test. Binary signals have the advantage that only simple switching control elements are needed to generate the signal.

The class of models:

The models can be characterized as parametric or nonparametric models. If the structure of the model is known in advance, or at least can be assumed properly, the identification is parametric identification. The parametric models include algebraic equations, differential equations, systems of differential equations (state equations), and transfer functions. For example, the z-transfer functions used to determine the heat flow through walls, described in chapter 3, are a parametric model. In the case of a parametric model, the system identification consists of the determination of parameters in the fixed structures. The parametric identification methods imply the problem of order agreement between the model and the process. Nonparametric models are the response obtained directly or indirectly from an experimental analysis of a physical system, such as a frequency response represented by a Bode plot or a recorded step-response of a system. The nonparametric model has the advantage that it is not necessary to specify the order of the process explicitly. A parametric model can be produced from a nonparametric model by identification methods. The data processing procedure used to experimentally determine the z-transfer function coefficients in this research work is actually based on this theory. As will be described in section 4.3, the z-transfer function, which is in the form of a parametric model, is determined from the frequency response functions obtained from experimental data which are nonparametric models.

The criterion:

The selection of a criterion by means of which the "goodness of fit" of the model to the actual system can be used to classify the identification method. The error between the model and the process is the output-signal error. The error criterion is to minimize some function of the error. For example, given a thermal system (a wall), a function of the error between the system output (heat flow through the wall) and the chosen model output may be defined as:

$$S(Q,Q_m) = \int_0^P e^2(t)\, dt \qquad (4.1)$$

where Q is heat flow through the wall which is the system output, Q_m is the model output, and e is the error; Q, Q_m, and e are considered as functions defined on (0,P). In the estimation of z-transfer function coefficients, the model (z-transfer function) output is Q_m and the system output is Q measured from experiments. Using the least squares criterion in this parametric system identification problem means finding the coefficients of the z-transfer function (model) to make the error function, $S(Q, Q_m)$, as small as possible. The 'equivalence' criterion, in frequency domain analysis, implies that the selected model is equivalent to the frequency response functions which are obtained from experimental measurements of the process.

The determination of parameter values of a mathematical model from correctly measured input and output signals is made considerably more difficult in the presence of disturbances acting on the process and hence on the output signals. Therefore, identification methods should be able to eliminate the influence of disturbance components. For the case of a low-frequency thermal system the simple computation of regression is sufficient. In this study, multi-linear regression techniques have been used to determine z-transfer function coefficients from the experimental measurements.

4.2 The choice of model structure

The choice of model structure is basic to the formulation of identification problems, and will greatly influence the character of the identification problems. Most estimation schemes contain the assumption of linearity in the parameters. Therefore, it pays to try to find transforms of the variables to obtain such a linearity if it is possible.

The form of the z-transfer functions as derived in chapter 3 can be expressed as a ratio of two polynomials:

$$H(z) = \frac{a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}} \tag{4.2}$$

Where, a_i and b_i are z-transfer function coefficients which are undetermined parameters. The estimation of these coefficients is a linear identification problem. To obtain the z-transfer function coefficients a linear regression procedure has been used in this research work.

A lumped parameter RC model expressed as a transfer function is:

$$H(s) = \sum_{j=1}^{\infty} \frac{\alpha_j}{1 + \tau_j s} \tag{4.3}$$

where, τ = RC, R and C are the parameters of the transfer function and α is the residue. This representation of the system to dynamic behavior is linear, but, the model is non-linear in parameters R and C. The estimation of the parameters R and C can be carried out by finding the eigenvalues or the roots at poles of the transfer function. However, the root finding methods are not the linear regression procedure.

In frequency domain analysis, a frequency response can be obtained directly or indirectly from experimental analysis of a physical system. It can be shown in a Bode plot. This response, represented by the gain factor and the phase factor, is a nonparametric model. As mentioned above, the nonparametric model has the advantage that it is not necessary to specify the order of the process explicitly and it can be used to produce a parametric model with linearity.

Therefore, in this research work, an indirect avenue is explored to estimate the z-transfer function coefficients from the non-parametric frequency response function.

4.3 Parameter estimation

The most commonly used parameter estimation method is the least squares identification method. The theory of the least squares method was developed by Gauss: "if the astronomical observations and other quantities on which the computation of orbits is based were absolutely correct, the elements also, whether deduced from three or four observations would be strictly accurate and, therefore, if other observations were used, they might be confirmed but not corrected. But since all our measurements and observations are nothing more than approximations to the truth, the same must be true of all calculations resting upon them, and the highest aim of all computations concerning real phenomena must be to approximate, as nearly as practicable to the truth. But this can accomplished in no other way than by a suitable combination of more observations than the number absolutely requisite for the determination of the unknown quantities." "The most probable value of the unknown quantities will be that in which the sum of the squares of the differences between the actually observed and the computed values multiplied by the numbers that measure the degree of precision is a minimum." That is, the sum of squares function:

$$S = \sum_{t=1}^{P} (Q_t - \hat{Q}_t)^2 \longrightarrow 0 \tag{4.4}$$

where, Q_t and $\hat{Q}_t$ are measured value and predicted value at time t respectively. A detailed discussion of the least squares method is presented by Rake.[41]

The correlation technique usually is used to determine the weighting function with the generated random signals.[42] The comparison of the least squares method and the correlation technique is described by Astrom and Eykhoff.[35]

The least squares method is commonly used to estimate the parameters of the dynamic constant parameter system which is of the linear generalized form. For models which are in the form of the regression model:

$$Q(t) = \sum_{i=0}^{n} a_i\, T(t-i\Delta) - \sum_{i=1}^{m} b_i\, Q(t-i\Delta) + N(t) \tag{4.5}$$

and assuming the mean value of the noise, N(t), is equal to zero, and that the input signal is mutually independent of the noise, the estimation of parameters of the regression model in the sense of least squares is unbiased.

Since the z-transfer function used to calculate heat flow through walls is in the regression form, the time domain regression can be used directly to determine the z-transfer function coefficients from experimental measurements. By knowing the temperature and heat flow histories and assuming both the temperature and heat flow are zero at t=0, the load calculation equation can be written in a matrix form:

$$\begin{bmatrix} T(1) & 0 & \cdots & 0 & 0 & 0 & 0 \\ T(2) & T(1) & \cdots & 0 & -Q(1) & 0 & 0 \\ T(3) & T(2) & \cdots & 0 & -Q(2) & -Q(1) & 0 \\ \cdot & & & & \cdot & & \\ \cdot & & & & \cdot & & \\ \cdot & & & & \cdot & & \\ \vdots & & & & \vdots & & \\ T(k) & T(k-1) & \cdots & T(K-n) & -Q(k-1) & -Q(k-2) & -Q(k-m) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \\ b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = \begin{bmatrix} Q(1) \\ Q(2) \\ Q(3) \\ \cdot \\ \cdot \\ \vdots \\ Q(k) \end{bmatrix} \qquad (4.6)$$

where, Q(k) and T(k) are heat flux and temperature values at time $t=k\Delta$, Δ is the time interval for calculation. a_i and b_i are the desired z-transfer function coefficients. Solving the matrix employing the multi-linear regression techniques the coefficients can be obtained. For statistical and probabilistic considerations the number of equations k needs to be much larger than the number (n+m) of parameters to be estimated.

In the lumped parameter models with the parameters R and C, if the parameters can be determined, for example, by determining the eigenvalues and residues, then the z-transfer function coefficients can be obtained by converting the lumped parameter model to the z-transfer function. An experimental procedure using these steps is developed by Stephenson et. al.[39]

Frequency analysis also can be used to estimate the parameters in a parametric model. In contrast with the model of equation (4.2), which is a constant parameter expression of a linear system, the estimation of the z-transfer function coefficients can be carried out by applying the least squares method. The procedure is in two steps: first, find a nonparametric model which is the frequency response function from experimental measurements, and then fit the coefficients of the assumed parametric model to the frequency response.

One of the techniques for treatment of periodic processes is by calculating a periodogram, which essentially is a graph of the variance associated with all the frequencies used in the analysis.

The main drawback of the periodogram is the large number of frequencies and the resulting poor sampling properties (the estimator is not consistent). Suitable averaging processes in spectral analysis can be used to overcome this drawback. The "Blackmen-Tukey" method computes the spectral density function via a Fourier transform of the autocorrelation function[8]. The averaging is performed in the time domain followed by a smoothing procedure in frequency domain. Hammarsten[8] has used this method in his work.

Alternatively, the "direct Fourier transform" method is based on a finite-range Fast Fourier Transform (FFT) and an averaging of the resulting periodogram. The spectral density functions are determined from the Fast Fourier Transforms of the input and output functions of time. The averaging can be achieved on the corresponding period. Then, the magnitude of the spectral density function, which is known as the amplitude spectral, gives the gain factor. And, the phase angle of the spectral density functions, known as the phase spectral, gives the phase factor.

Based on the theory that a parametric model can be determined from a nonparametric model, an analytical expression for a spectral density function can be obtained by fitting the data with an assumed frequency domain expression.

The parameter values for a single-input, single-output, constant parameter, linear system can be determined by using the frequency response function identification method. The most common way of obtaining the frequency response function,

$H(\omega)$, is from sinusoidal inputs. That is, if input is:

$$T(t) = \sin(\omega t) \tag{4.7}$$

the output will be:

$$Q(t) = \xi \sin(\omega t + \varphi) \tag{4.8}$$

and the frequency response function can be obtained at each frequency. If the experiment is repeated for a large number of different frequencies, a system Bode diagram is obtained. The amplitude and phase curves on this diagram constitute a nonparametric model.

In this study, the Fast Fourier Transform (FFT) method has been used to obtain the frequency responses from experimental measurements. As mentioned above, the first step is performing the FFT on the input and output signals of test data. The averaging is obtained by dividing the input and output signals by the number of data points in a period. Then the frequency response function H is determined from the equation:

$$H(f) = \frac{Q(f)}{T(f)} \tag{4.11}$$

Where, Q and T are the heat flux (system output signal) and temperature difference (system input signal) respectively.

The gain factor, A_g, and the phase factor, Φ, were then determined from:

$$A_g = |H(f)| \tag{4.12}$$

and

$$\Phi = \Phi_Q - \Phi_T \tag{4.13}$$

where, Φ_Q and Φ_T are phase angle of the output signal and the input signal of the system. After normalizing the gain factor by dividing by the steady state U-value, the normalized gain factor A_g and the Φ were drawn on the Bode plots. Therefore, the desired nonparametric model for each test data was obtained.

To obtain the z-transfer function coefficients, multi-linear regression techniques have been used. The z-transfer function derived in chapter 3 is in the form:

$$H(z) = \frac{a_0 z^0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}} \tag{4.14}$$

where, a_i and b_i are the coefficients. z^{-i} is an operator representing a time delay = $i\Delta$, and Δ is the time sampling interval for calculation. Since $z=e^{\Delta s}$, the equation (4.14) can be expressed in Laplace notation:

$$H(s) = \frac{a_0 e^0 + a_1 e^{-\Delta s} + a_2 e^{-2\Delta s} + \ldots + a_n e^{-n\Delta s}}{1 + b_1 e^{-\Delta s} + b_2 e^{-2\Delta s} + \ldots + b_m e^{-m\Delta s}} \tag{4.15}$$

Substituting $j\omega = s$, the transfer function becomes:

$$H(\omega) = \frac{a_0 + a_1 e^{-j\omega\Delta} + a_2 e^{-j2\omega\Delta} + \ldots + a_n e^{-jn\omega\Delta}}{1 + b_1 e^{-j\omega\Delta} + b_2 e^{-j2\omega\Delta} + \ldots + b_m e^{-jm\omega\Delta}} \tag{4.16}$$

This is the frequency domain expression of the z-transfer function. Because the frequency response is a complex function, it can be written in two parts: real component and imaginary component. For each frequency two equations can be written, one for the real part and one for the imaginary part. An additional equation can be written for the steady state gain. Therefore, for N frequencies, 2N+1 equations can be obtained. Rewriting these equations in matrix form and performing the multi-linear regression computation, the z-transfer function coefficients were obtained.

The frequency response functions were determined based on the experimental measurements which are presented in the following chapters.

Table 4.1 Non-periodic test signals

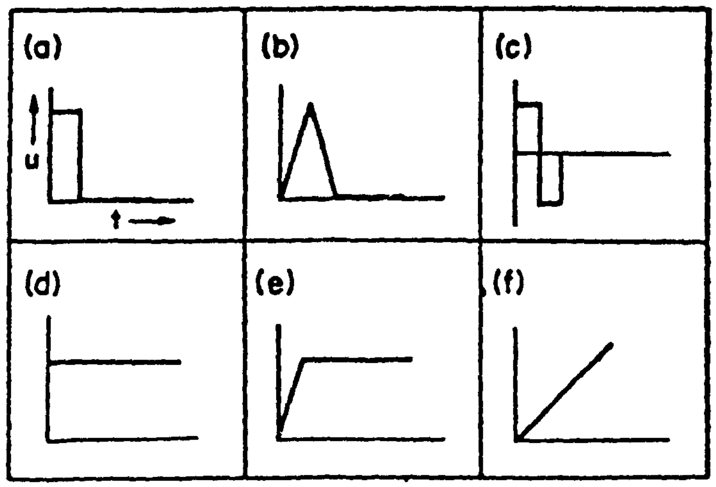
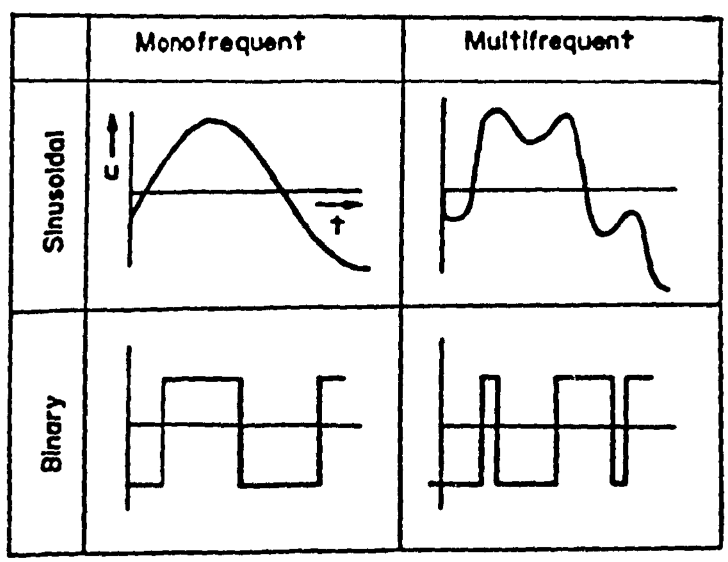


Table 4.2 Periodic test signals



CHAPTER 5

ESTIMATION OF COEFFICIENTS FOR TRANSFER FUNCTION 1/B

The ASHRAE Handbook of Fundamentals[31] recommends the use of z-transfer function techniques for prediction of the rate of heat flow through exterior walls. In this techniques the z-transforms of the heat flux at the inside surface of the wall, Q_1, the room air temperature, T_1, and the temperature of the outside surface, T_2, are related by:

$$Q_1(z) = \frac{D(z)}{B(z)} T_1(z) - \frac{1}{B(z)} T_2(z) \tag{5.1}$$

where 1/B is the transfer function that relates the heat flow at the inside surface to the temperature at the outside surface, and D/B is the transfer function that relates the heat flow at the inside surface to the room air temperature. This chapter presents an experimental procedure for determining coefficients of transfer function 1/B. This transfer function is the most important transfer function given in the transfer matrix of equation (3.10).

The experimental procedure to obtain z-transfer function coefficients consists of determining the frequency response function from experimental measurements, and then fitting the z-transfer function coefficients to the frequency response.

Tests were performed at the Thermal Insulation Laboratory of the Institute for Research in Construction, National Research Council Canada.

In section 5.1 the two samples measured in the tests are described. In section 5.2 the test apparatus and the measurement methods are described. Section 5.3 describes the test procedure. Then, the data analysis procedure and the results for sample 1 are given in section 5.4 and the results for sample 2 are given in section 5.5. Finally, the results are discussed in section 5.6.

5.1 Description of the test samples

Two samples were tested in this research work. Sample 1 was a single layer of rubber material for which the thermal characteristics are similar to gypsum wallboard. The nominal dimensions of the rubber slab are 0.012m thick by 0.556m wide by 0.558m high and the weight is 4.78 kg. Sample 2 was composed of two layers, the rubber slab plus polystyrene insulation board as shown in Fig.5.1. The nominal dimensions of the polystyrene board are 0.037m thick by 0.61m wide by 0.607m high and the weight is 0.29 kg. The thermal properties of the rubber and polystyrene board were measured in the laboratory and are given in Table 5.1.

5.2 The test apparatus for determination of 1/B

The definition of transfer function 1/B is:

$$\frac{1}{B} = \frac{Q_1}{T}\bigg|_{T_1=\text{constant}} \qquad (5.2)$$

where, $T = T_1 - T_2$

To derive transfer function 1/B, the test apparatus should satisfy the following conditions: (1) be able to maintain the temperature T_1 constant; (2) vary the temperature T_2; (3) measure the heat flux Q_1.

To meet the test requirements above, the test apparatus was set up as shown in Fig.5.2. Three electric heaters were used. Heater H_1 was controlled to maintain temperature T_1 to be constant. Two cold plates, through which liquid at constant temperature was circulated, served as the sink for the heat from the heaters. Heater $H_{2,a}$ and heater $H_{2,b}$ were switched on and off, with the input signal, to produce a varying temperature T_2. The two samples are assumed to have the same thermal properties and dimensions.

Using this symmetrical configuration, the heat flux Q_1 can be determined from the measurement of the power supplied to the heater H_1. Each of the three electric heaters consists of a metered area surrounded by a guard area, as shown in

Fig. 5.3. The temperature of the guard area was maintained at the same temperature as the metered area to reduce errors due to edge losses. Since the system is symmetrical, half of heat flow will be transferred to each side of the system. Considering the heater is made very thin and ignoring its thermal capacitance, and assuming the samples on the two sides of the system are the same, the heat flux Q_1 on the surface is: Q_1=Power/(2*Area), where Area is the metered area of heater H_1, and Power is the power supplied to metered area of heater H_1.

The control, instrumentation, and data acquisition system are shown in Fig.5.4. Two DC power supply devices, each connected to a separate analog temperature controller, were used to drive the metered area and the guard area of heater H_1. The power supplied to the heaters $H_{2,a}$ and $H_{2,b}$ were equal. The metered area of heaters $H_{2,a}$ and $H_{2,b}$ were driven in series by one AC power supply device, the guard area of heaters $H_{2,a}$ and $H_{2,b}$ were supplied in series by another AC power supply device. The temperature controllers function to control the heater H_1 to maintain constant temperature, T_1. In order to measure the power supplied to the metered area of heater H_1, voltage and current were measured. A 0.01 Ω shunt resistor and a 500:1 voltage divider were used to measure the current and the voltage.

A computerized data acquisition system (SAFE) was used to collect data. An IBM-PC computer was connected as a terminal to serve as an operator interface and to record data on magnetic disk for later processing. An on-line control program was written in SAFE BASIC to manage the data acquisition, recording, and to control the relays supplying power to heaters $H_{2,a}$ and $H_{2,b}$. The heaters $H_{2,a}$ and $H_{2,b}$ were turned on and off using a Binary Multi-Frequency Sequence (BMFS) signal obtained from the equation:

$$G(t) = \cos(\omega t)-\cos(2\omega t)+\cos(4\omega t) -\cos(8\omega t)+\cos(16\omega t)-\cos(32\omega t)+\cos(64\omega t) \quad (5.3)$$

where, $\omega = 2\pi/P$, P = period of the sequence.

Heaters $H_{2,a}$ and $H_{2,b}$ were turned on when $G(t)\geq 0$ and, off when $G(t)<0$. Fig. 5.5 shows the resulting signal, and the amplitude of the frequency spectrum of the signal. As shown, the BMFS signal has significant amplitude at several frequencies.

Thermocouples were used to measure the temperatures at different positions on the surfaces of the samples. 14 type-T thermocouples were used to monitor temperatures. The arrangement of the thermocouples is shown in Fig.5.6. In order to reduce local heating effects the thermocouples were placed on copper pads attached to the heaters. The temperature difference between the two sides of each sample were determined from the average of the thermocouple readings on each side. Two additional thermocouples, one on the guard area and one on the metered area, were connected to the controllers to maintain a constant temperature on heater H_1.

Using type-T thermocouples to measure temperatures on the surfaces of test samples, the data acquisition system recorded the measurements with a temperature range of 400^0C. The 12 bit analog to digital converter used in this system gives a resolution of 0.098^0C. The shunt resistor was measured on a range of 0-10mv and the voltage divider measurement range was 0-100mv. The current and voltage measurements have a resolution of 0.24 mA and 0.012 Volts respectively.

5.3 Test procedure

To check out the experimental system, a test was first performed under the steady state condition, namely, running the test without the BMFS as input to the heaters $H_{2,a}$ and $H_{2,b}$. The steady-state U-value was obtained.

Tests were then performed on sample 1 and sample 2. The heater cycling period and the data sampling time interval were entered and, the system started running under the control of the on-line program. The system's operation was as follows. The BMFS control signal was generated and applied to a digital output which, through a solid state relay (see

Fig.5.4), turned the heaters $H_{2,a}$ and $H_{2,b}$ on and off. The temperature fluctuation of T_2 was created. The guard area of the heater H_1 was maintained at the same temperature as the metered area to prevent edge losses. The power supplied to the metered area of the heater H_1 was measured by monitoring the current and the voltage. The voltage, current, and thermocouple readings were recorded automatically by SAFE. Data was displayed on the IBM screen, and stored on a disk file.

For sample 1, one test was performed by entering the test parameters as used in the previous study.[40] Results agreed well with the previous study.

Tests for sample 2 were performed under the same conditions as for the sample 1. To investigate the higher frequency responses, several tests were conducted by changing parameters of the heater cycling period. Tests results for sample 1 and for sample 2 and the data analyzing procedure are described in the following sections.

5.4 Results for sample 1

After running 5 cycles, a period of the test measurements was chosen for data analysis. The temperature difference, $T = T_1-T_2$, between the two surfaces of the sample is the system input signal. The temperature T_1 was calculated by averaging the values of the temperature readings from the thermocouples which were placed on the metered area of the heater H_1. The temperature T_2 is the average value of the temperatures from the thermocouples placed on the metered area of heaters $H_{2,a}$ and $H_{2,b}$. The system output signal, heat flow Q_1, was calculated from the power supplied to the metered area of the heater H_1 as described in section 5.2. The input and output signals are as shown in the Fig.5.7.

The first step of data analysis procedure was performing Fourier analysis to get the frequency response function. This was achieved by performing Fast Fourier Transform (FFT) computation on the output and the input signals described above. The transfer function H was then determined from the equation:

$$H(f) = \frac{1}{B(f)} = \frac{Q_1(f)}{T(f)} \tag{5.4}$$

To reduce the effect of noise, only frequencies at which the input driving function (the temperature difference) had a reasonably large amplitude were considered. The resulting frequency response of transfer function 1/B, after normalizing the gain factor divided by the steady state U-value, is shown in the Bode plots of Fig.5.8. The gain factor and the phase factor are shown with the theoretical curves of the frequency response function 1/B shown for comparison. The theoretical calculation of frequency response is given in Appendix B.

The second step of the data analysis procedure is the determination of the z-transfer function coefficients. A multi-linear regression computation was performed to fit z-transfer function coefficients to the frequency response data obtained above. For each chosen frequency two equations (the real component and the imaginary component of the transfer function) were written, plus one equation for the steady state condition (see Appendix D). Then, by employing the regression techniques, the z-transfer function coefficients were obtained. The frequency response of the fitted z-transfer function was also calculated.

The z-transfer function coefficients were determined for a different number of terms in the numerator and denominator. Table 5.2 gives the z-transfer function coefficients for sample 1. Higher order for the transfer function than those shown in Table 5.2 resulted in unstable simulation. Table 5.3 gives the frequency response of two fitted z-transfer functions (coefficients of n=3, m=2 and n=3, m=1 are given in Table 5.2) for sample 1, which gave a good fit, compared with the measured frequency response determined from the Fourier analysis. The gain factors and phase factors of the frequency responses of the fitted z-transfer function (n=3, m=2) compared with the measured response are also shown on the Bode plot in Fig.5.9.

5.5 Results for sample 2

To investigate the responses of the transfer function 1/B at higher frequencies several tests were performed for sample 2 by changing the input parameter of the heater cycling period. After running each test for more than 5 cycles, a period of the experimental measurements was chosen for data analysis. As for sample 1, the temperature T_2, on the heaters $H_{2,a}$ and $H_{2,b}$ was calculated by averaging the temperature readings on the metered area of these heaters and the temperature T_1 was the average value of the temperature readings from the metered area of the heater H_1. Then, the system input signal was the temperature difference $T = T_1 - T_2$. The system output signal (heat flow) was obtained from the power supplied to the metered area of heater H_1 as described in section 5.2. The input and the output signals are as shown in Fig.5.10.

Fourier analysis procedures were conducted as described previously. The gain factor was again normalized to the steady state U-value.

The gain factors and the phase factors of three tests results are shown on the Bode plot in Fig.5.11. The theoretical curve was calculated from the equation given in Appendix B and is shown for comparison.

The same fitting procedures as used for sample 1 were applied to determine the z-transfer function coefficients for sample 2.

The z-transfer function coefficients for sample 2 were determined for different number of terms in the numerator and denominator, as given in Table 5.4. Higher order than those shown resulted in unstable simulation. The frequency response of the two fitted z-transfer functions for sample 2, compared with the measured frequency response from the Fourier analysis, are given in Table 5.5. The coefficients used are given in Table 5.4 (n=3, m=2 and n=2, m=2). The gain factors and phase factors of the frequency responses of the two fitted z-transfer function, compared with the measured response, are also shown in Fig.5.12.

A simulated load calculation was performed using the coefficients (n=2, m=1) obtained above for sample 2. The measured temperature difference history values are the input. The heat flux was calculated from the load calculation equation:

$$Q_1(t) = \sum_{i=0}^{n} a_i \, T(t-i\Delta) - \sum_{i=1}^{m} b_i \, Q_1(t-i\Delta) \qquad (5.5)$$

Where, Δ is the sampling time interval of the z-transform.

The simulated heat flow and the measured heat flux from the calculation above are shown in Fig.5.13. The average relative errors between the simulated heat flows and the measured heat flux, corresponding to the coefficients used, are also given in Table 5.4.

5.6 Discussion of the results

As shown in Fig.5.11, the frequency responses for transfer function 1/B of sample 2, which were determined from three tests with different input parameters, are similar and they agree fairly well with the theoretical curves in the low frequency range. However, the test gain factors and the phase factors are both slightly higher than the theoretical curve as the frequencies increase. The comparison of the gain factors on the Bode plot for sample 2 and for sample 1 also shows that the time constant for the sample 1 is shorter than that for sample 2, which is due to the effect of the additional layer of insulation material.

Fig.5.12 shows two of the fitted frequency response compared with the corresponding measured frequency response. The agreement between the fitted frequency response and the measured frequency response for z-transfer function 1/B of sample 2 is good even if the number of terms of the temperature history and the heat flux history values are less than 4.

From Fig.5.13 and Table 5.4, the agreement between the simulated heat flux with the measured heat flux is acceptable.

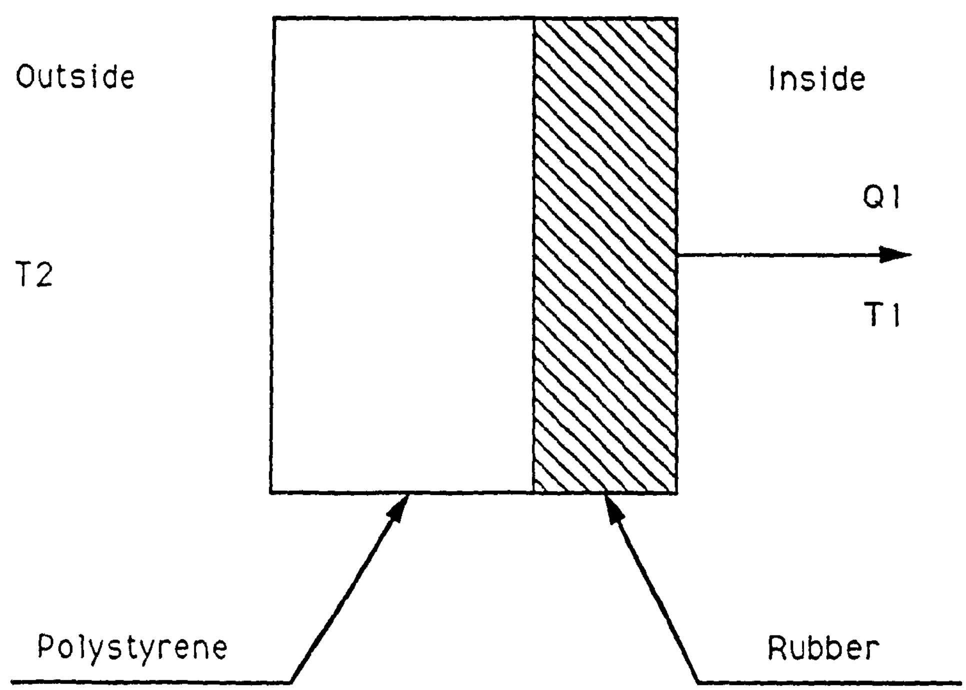


Figure 5.1. Schematic of Test Sample 2

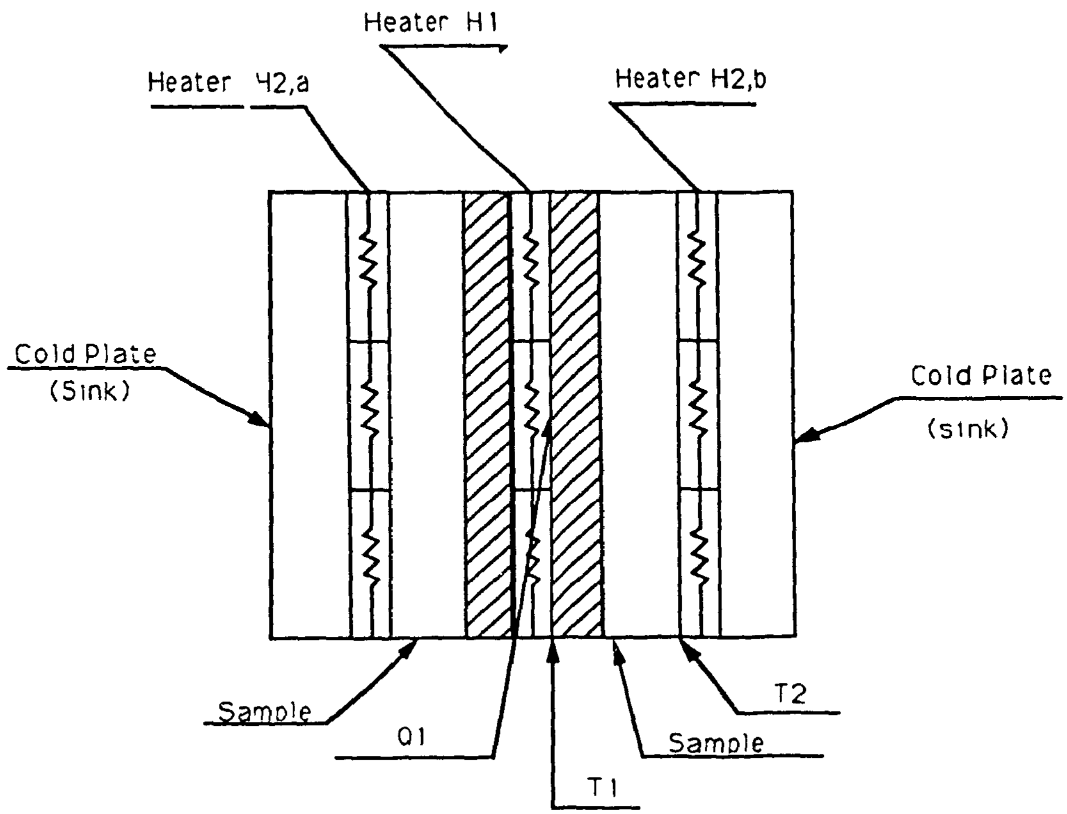


Figure 5.2 Schematic of Test Apparatus

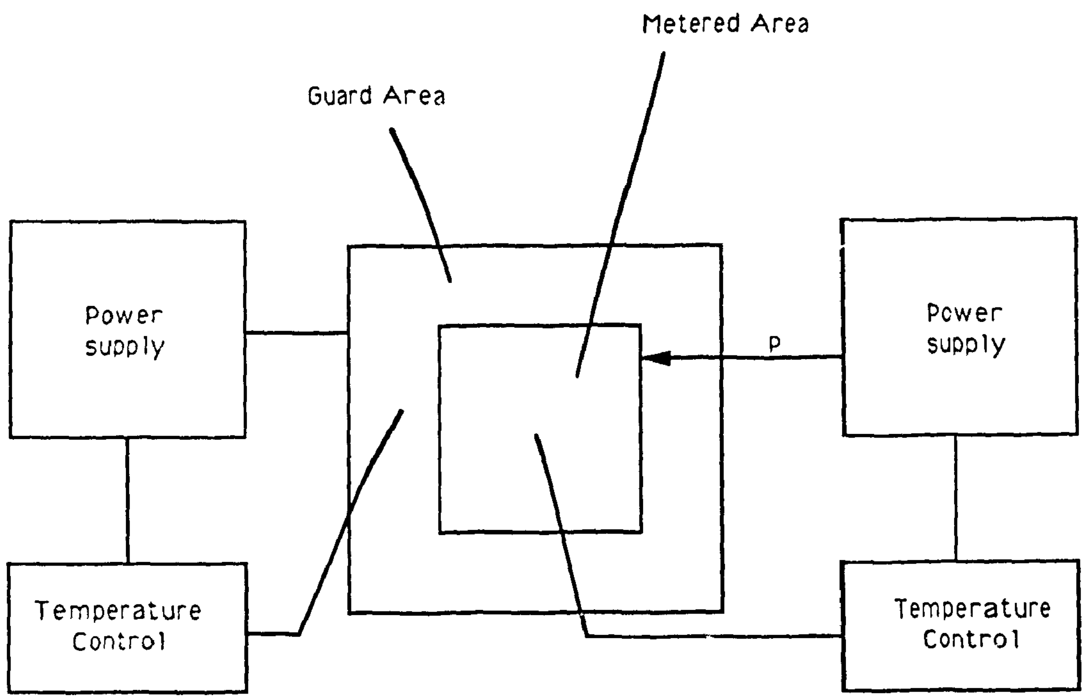


Figure 5.3 Schematic of the guarded heater

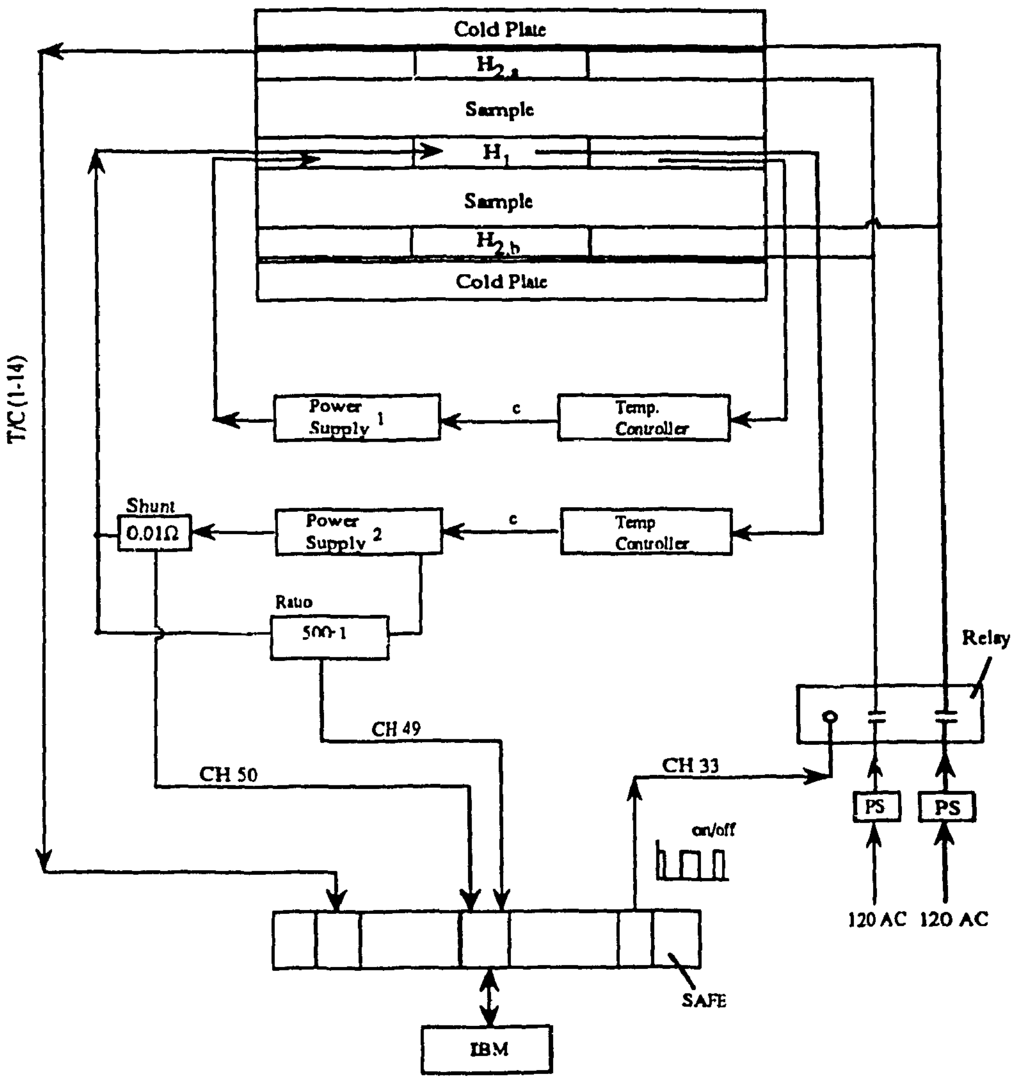


Figure 5.4 Test set-up for 1/B

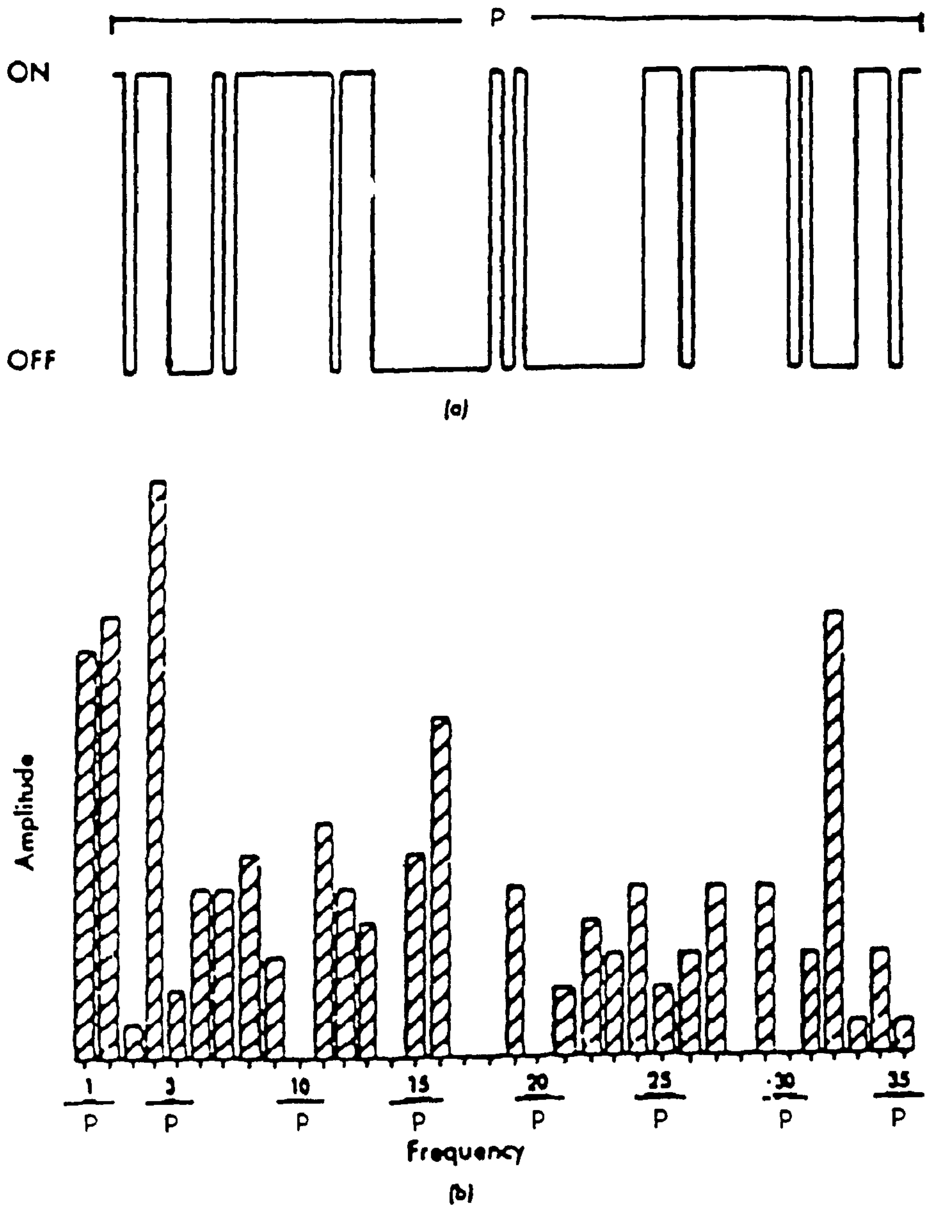


Figure 5.5 Amplitude of spectrum of BMFS signals

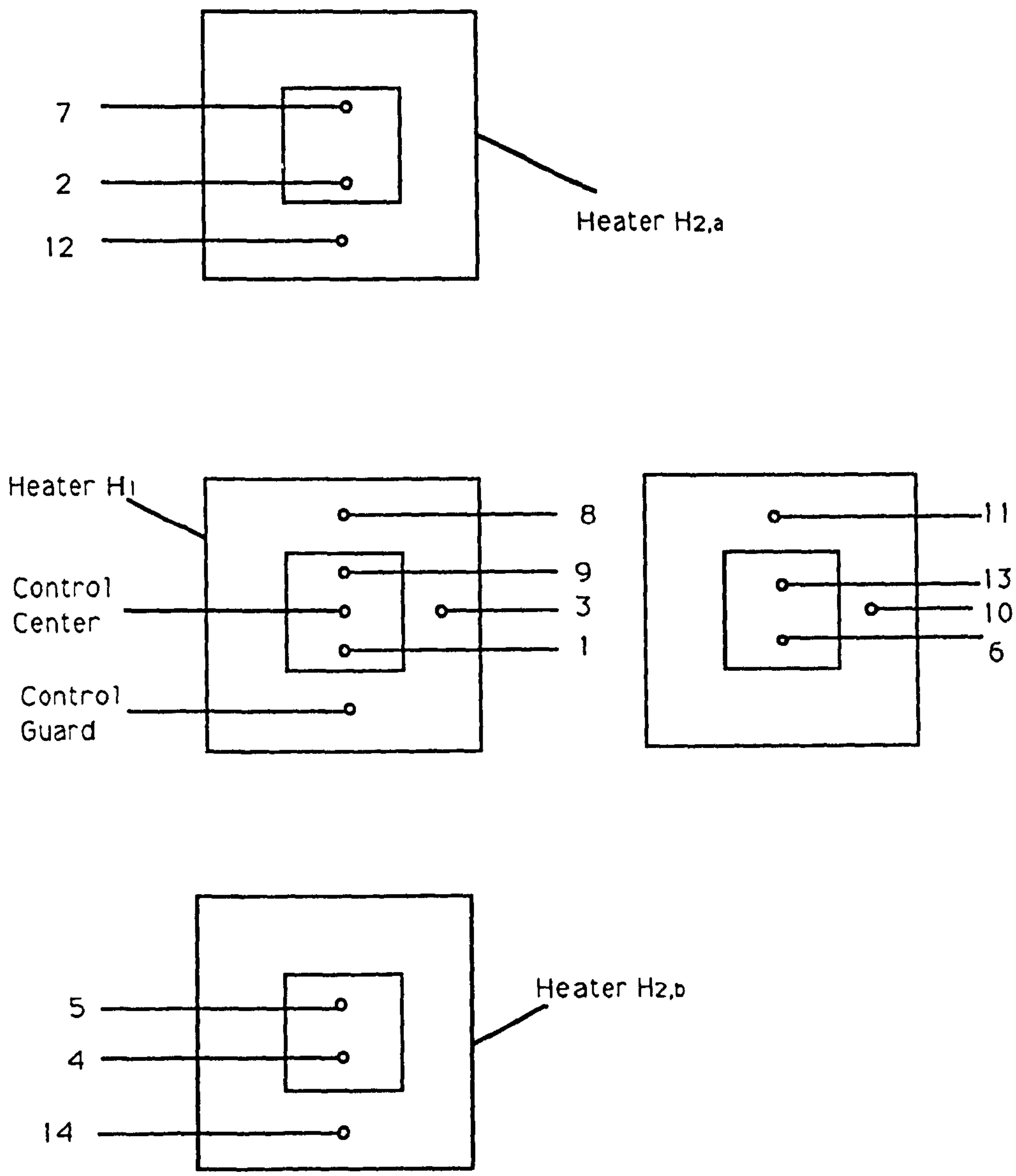


Figure 5.6. Thermocouple Setting for 1/B

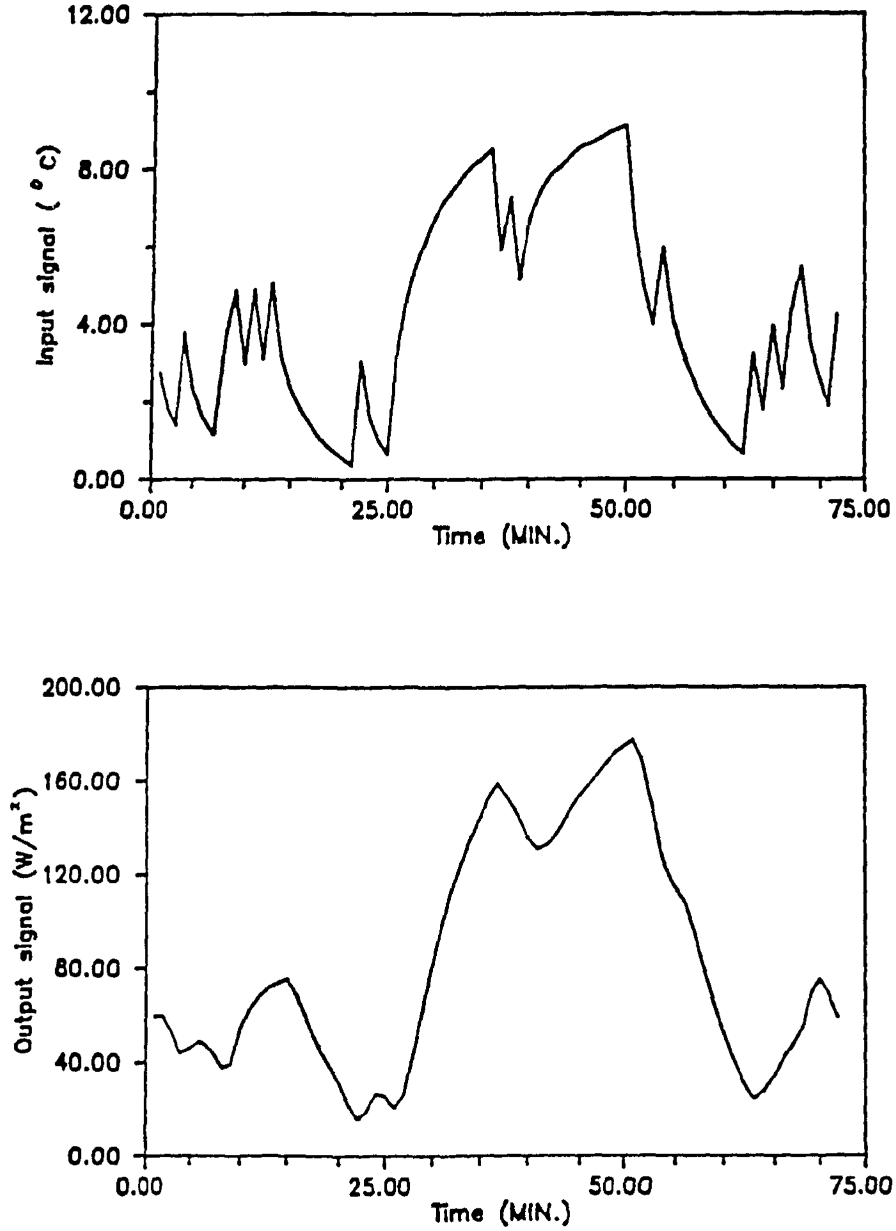
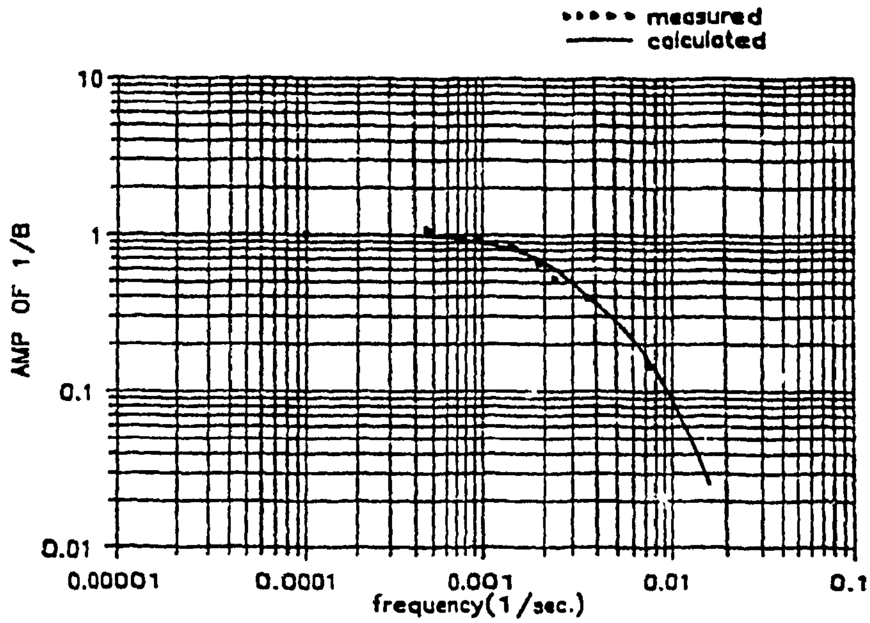


Figure 5.7 1/B input and output signals for sample 1



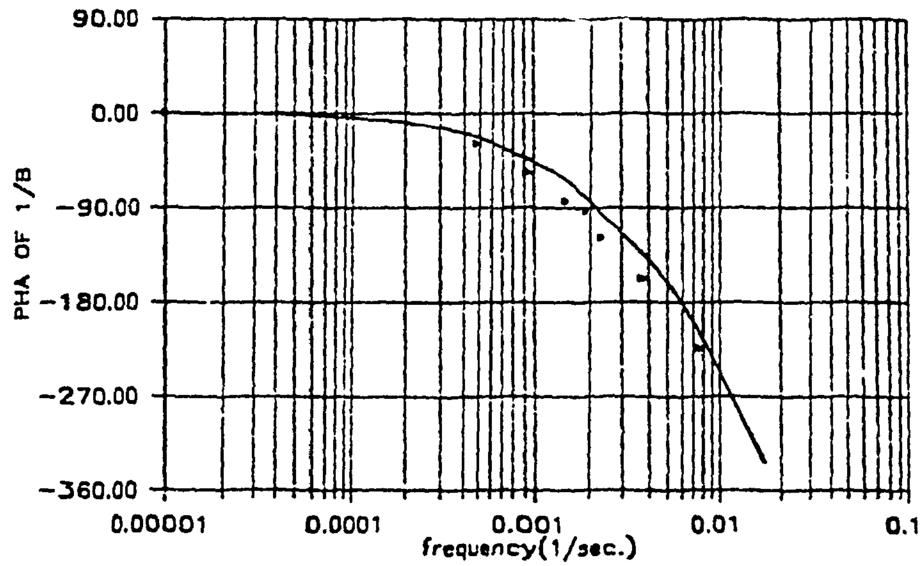


Figure 5.8 Frequency response from test data for sample 1

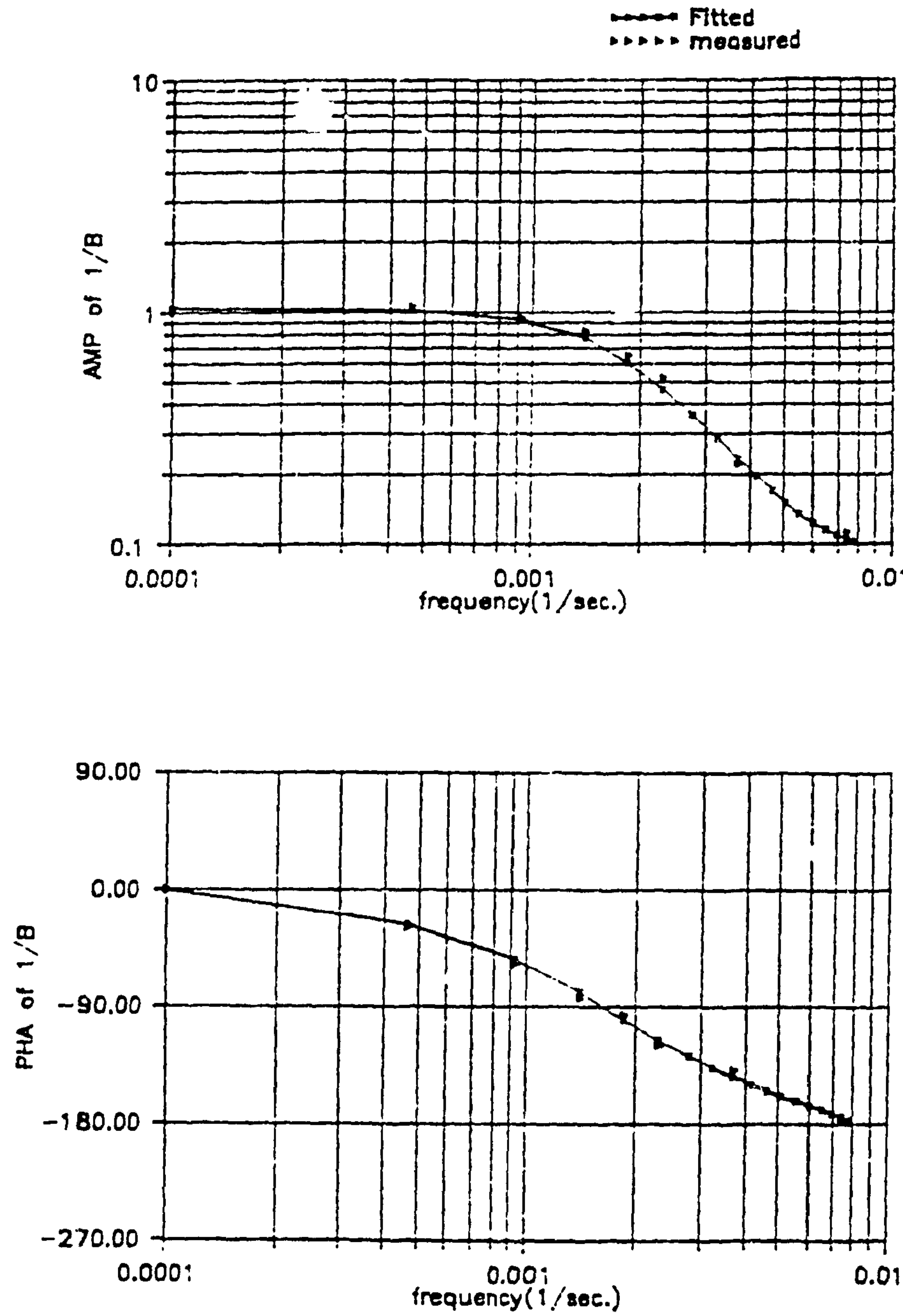
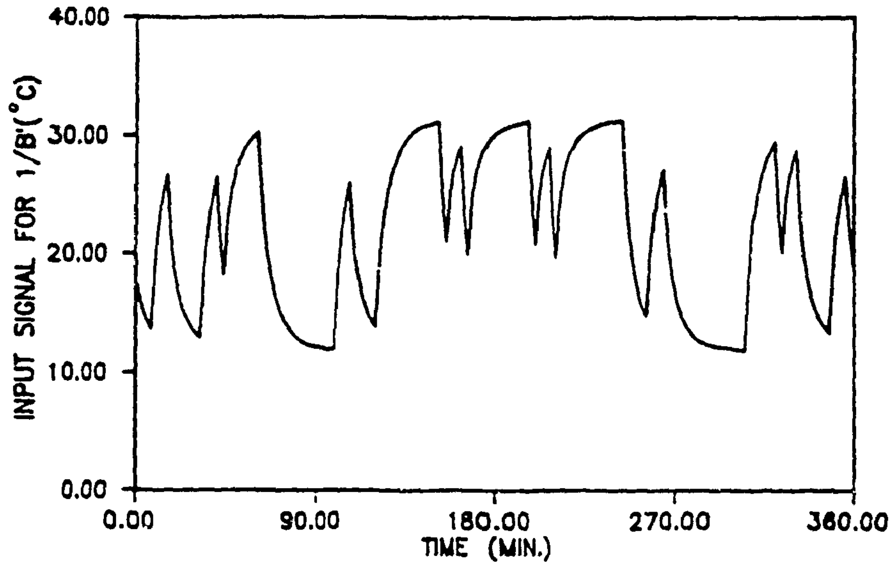


Figure 5.9 Frequency response of z-transfer function for sample 1



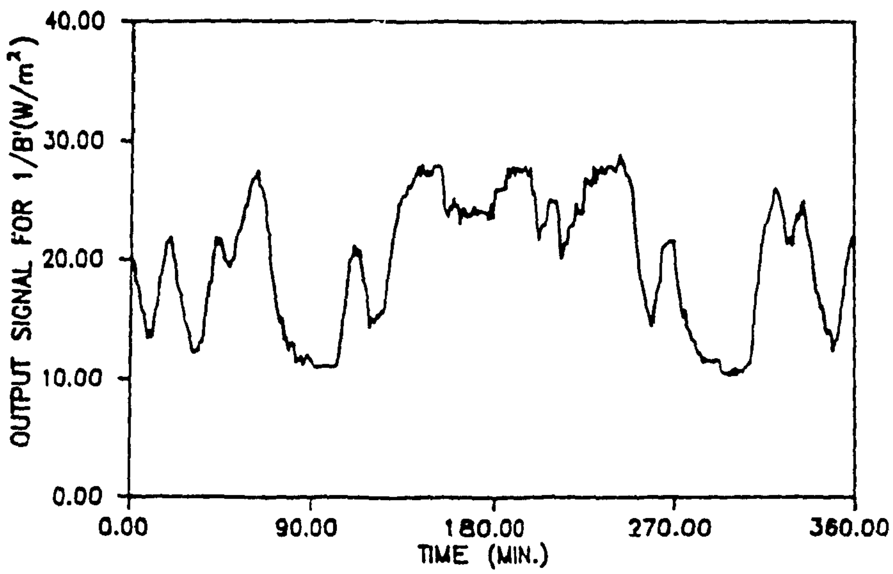
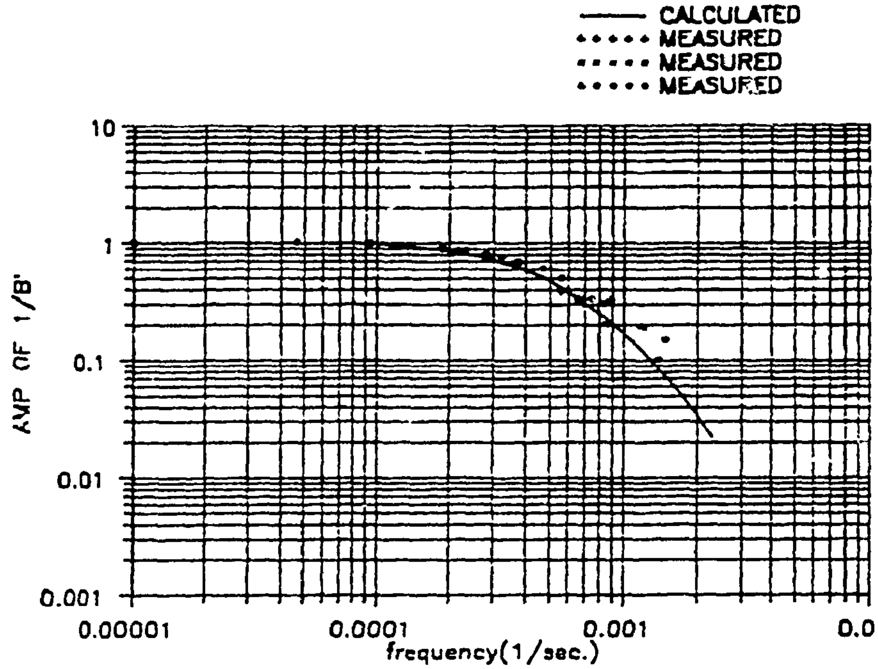


Figure 5.10 1/B input and output signals for sample 2



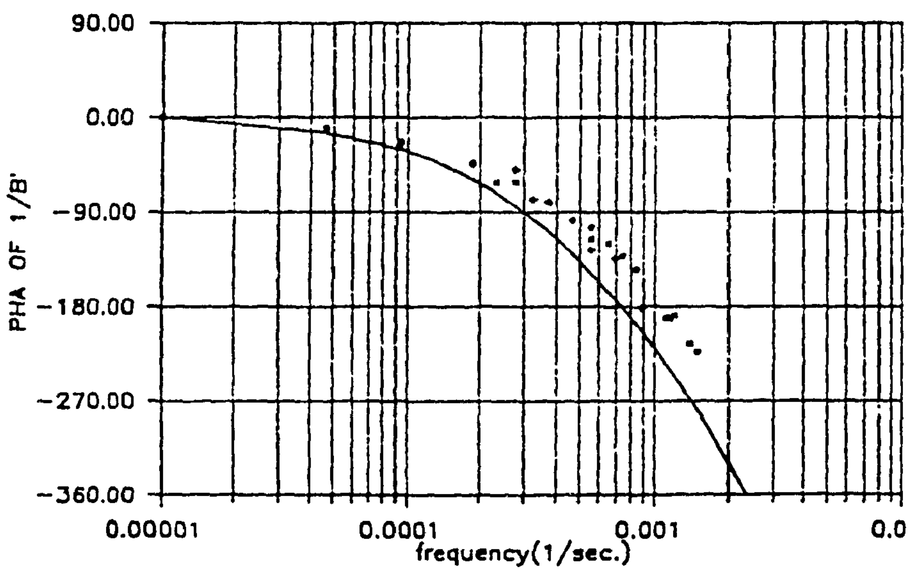
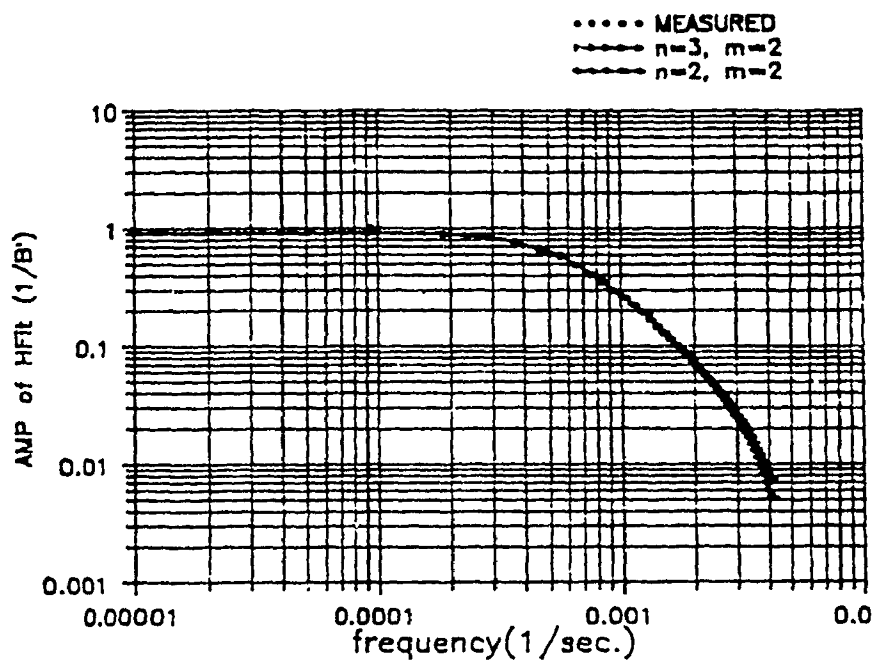


Figure 5.11 Frequency response from test data for sample 2



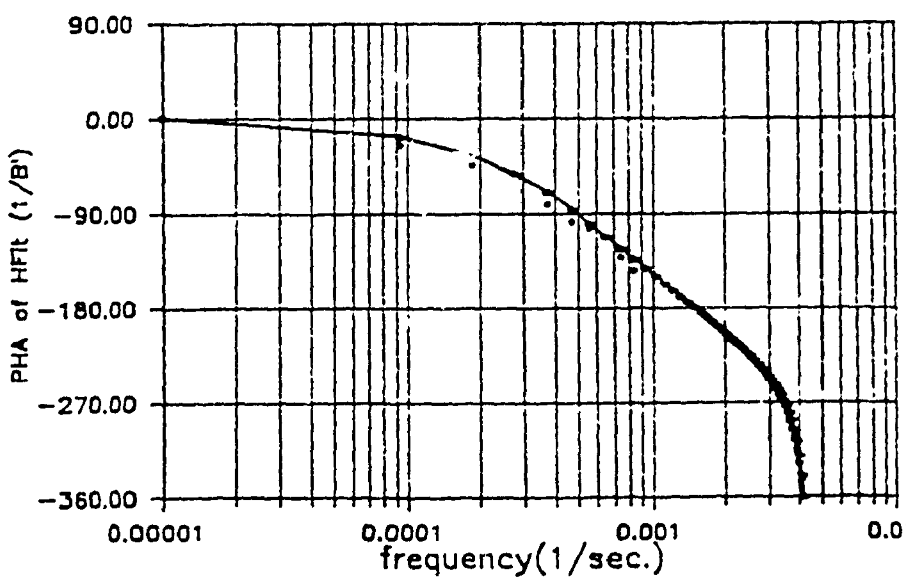


Figure 5.12 Frequency response of z-transfer function for sample 2

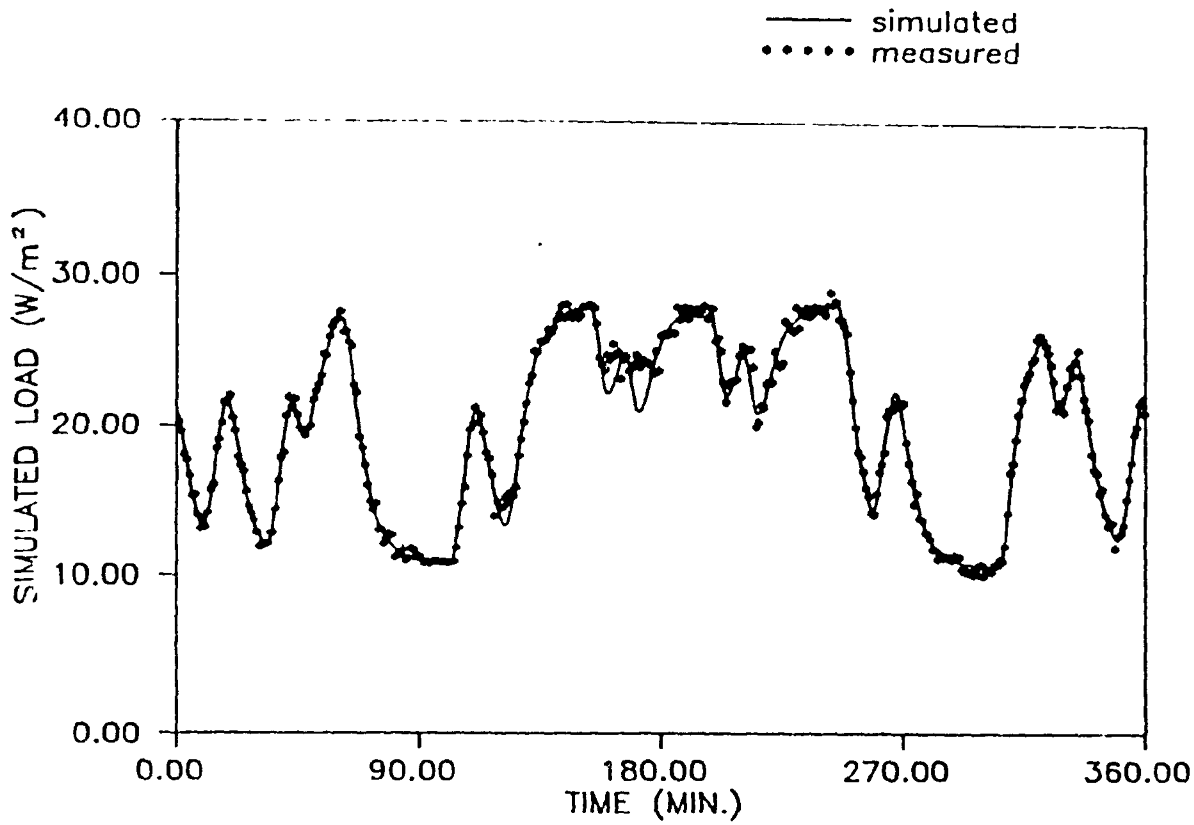


Figure 5.13 Simulated load for sample 2

Table 5.1 Samples Thermal Properties

	Rubber	Polystyrene
Thickness (L [m])	0.012	0.037
Density (ρ [kg/m^3])	1251.1	21.53
Specific heat (C_p [J/kg K])	1073.5	3518.34
Conductivity (k [W/m K])	0.2314	0.0354

Table 5.2 Z-Transfer Function coefficients for sample 1 (Δ=60.sec.)

n	m	a_0	a_1	a_2	a_3	a_4	b_1	b_2	b_3
4	1	0.00	0.23	0.26	0.03	0.09	-0.44		
3	3	0.00	0.21	0.32	0.01		-0.24	-0.45	0.19
3	2	0.00	0.18	0.38	0.14		0.02	-0.37	
3	1	0.00	0.21	0.24	-0.01		-0.58		
2	1	0.00	0.21	0.23			-0.59		
1	2	0.00	0.25				-1.11	0.35	

Table 5.3 Frequency response of z-transfer function for sample 1

frequency	Amplitude			Phase		
	measured	n=3, m=2	n=3 m=1	measured	n=3, m=2	n=3 m=1
0.000000	1.0000	1.000	1.000	0.0000	0.000	0.000
0.000693	1.0003	0.986	0.995	-28.947	-29.93	-29.36
0.000925	0.9391	0.911	0.920	-56.753	-56.47	-55.45
0.001388	0.8246	0.762	0.771	-83.933	-78.91	-77.50
0.001851	0.6508	0.634	0.642	-95.967	-98.11	-96.37
0.002314	0.5218	0.531	0.537	-119.75	-115.1	-113.1
0.003703	0.3235	0.328	0.328	-158.88	-159.3	-156.3

Table 5.4 Z-Transfer Function coefficients for sample 2 (Δ=120.sec.)

n	m	a_0	a_1	a_2	a_3	a_4	b_1	b_2	b_3	Relative error (%)
4	2	0.00	0.028	0.018	0.071	-0.06	-1.53	0.59		9.8
4	1	0.02	0.01	0.064	0.079	0.022	-0.80			10.2
3	2	0.00	0.017	0.062	0.04		-1.25	0.38		11
3	1	0.00	0.04	0.031	0.11		-0.82			9.3
2	2	0.00	0.056	0.07			-1.32	0.46		11
2	1	0.00	0.021	0.14			-0.85			8.7

Table 5.5 Frequency response of z-transfer function for sample 2

frequency	Amplitude			Phase		
	measured	n=3 m=2	n=2 m=2	measured	n=3 m=2	n=2 m=2
0.000000	1.000	0.950	0.93	0.00000	0.000	0.000
0.0000926	0.995	0.912	0.92	-25.362	-23.78	-17.92
0.0001852	0.8818	0.837	0.88	-44.8773	-46.31	-38.8
0.0002778	0.8326	0.739	0.82	-74.8587	-58.85	-57.42
0.0003704	0.7089	0.687	0.74	-81.2982	-82.24	-78.36
0.0004630	0.6097	0.543	0.65	-98.3722	-100.01	-96.15
0.0005556	0.5858	0.501	0.57	-105.032	-106.4	-103.4
0.0006481	0.5661	0.490	0.49	-132.534	-133.5	-129.8
0.0007407	0.4017	0.360	0.41	-134.125	-135.9	-133.1

CHAPTER 6
CONCLUSION

6.1 Summary

The objective for this thesis was to develop an experimental procedure to analyze the thermal dynamic characteristics of existing building walls and to determine the z-transfer function coefficients for the building envelope components. System identification techniques in frequency domain analysis were applied to attain this goal. An experimental approach to determining z-transfer function coefficients from frequency domain analysis, using BMFS as test input signal, was investigated. This procedure consists of the following steps: (1) establish experiment to get measurements of wall performance; (2) analyze the experimental data to determine the frequency response of transfer functions; (3) employ multi-linear regression techniques to fit the z-transfer function coefficients to the frequency response obtained in step 2.

Test were carried out in the laboratory for two small-scale samples: one a single homogeneous layer, the other a two-layer slab.

A review of the literature of modeling methods and system identification techniques used to determine the heat transfer characteristics of building walls was carried out.

The test results show that the frequency analysis approach, based on Fourier analysis using BMFS signals, is an acceptable method to determine the thermal characteristics of building envelope components. The Bode plots of the measured frequency response of transfer function 1/B agree with the corresponding exact curves from theoretical calculations. Multi-linear regression techniques used to fit the z-transfer function coefficients for transfer function 1/B gave satisfactory results.

In conclusion the experimental procedure described is suitable to estimate the coefficients of z-transfer function 1/B.

6.2 Discussion and recommendations for future work

This research raised some questions about the z-transfer functions (see equation 3.10) for estimating heat transfer in walls in time domain. The regression procedure can produce coefficients which are a very good fit to the frequency response, but result in instability when applied in a time domain simulation. It appears that perhaps more constraints are needed in the fitting procedure. For example, the coefficients obtained from the theoretical calculation based on known material properties tend to have all positive signs in the numerator and alternating minus and plus signs in the denominator. However in this study, some coefficients were found to be negative in the numerator (these are included in this thesis). Future work could address this problem of how to assure stability in the z-transfer functions.

The test apparatus, as described in chapter 5, the symmetrical configuration was chosen to satisfy the test requirements. For example, to find the frequency response for transfer function 1/B the test must maintain the temperature T_1 to be constant while measuring Q_1. The use of a Heat Flow Meter (HFM) apparatus would present problems. First, the frequency response of the heat flow meters available is not fast enough to give accurate readings at the higher frequencies used in the test. Secondly, in the simple HFM configuration as shown in Fig.6.1, the temperature T_1 would not be maintained constant. Since the HFM transducer is constructed with thermal resistance, a temperature difference $(T_1 - T_c)$ exists between the two surfaces of the transducer which is proportional to Q_1. Consequently, the temperature T_1 would vary with the temperature T_2 fluctuation, even though the cold plate was kept constant.

The method of measuring heat flux employed in this thesis, by measuring power input to the controlled heaters, also has disadvantages. It is difficult to achieve the precise control required, and the power reading tends to be "noisy" due to the control action. Also, the symmetrical apparatus does not lend itself to field measurements.

Another recommendation for future work i to develop a test methodology which is more suitable for in-situ tests. Newer heat flow meter technology is producing devices with better frequency response. Methods may be used to calibrate the heat flow meter[43,44]. A non-symmetrical configuration for consideration, as shown in Fig.6.2, is to use a heater and controller to maintain temperature T_1 constant while measuring Q_1 with a heat flow meter. Providing that temperature T_2 is higher than T_1, this would meet the test requirements.

The experimental procedure described in this thesis is still based on the assumption that heat flow through walls is one-dimensional. Since there are heat bridges in a real wall, non one-dimensional heat transfer should be considered. One way could be to use a buffer to measure air temperature so that the non one-dimensional heat flow from inside air, through a wall, to the outside air is considered[39]. Another method may be to place a thin insulation slab on the surfaces of the wall to simulate inside and outside film coefficients. Thus the surfaces of the simulated air films would be at uniform temperature even though the heat flow through the wall is non one-dimensional.

The third subject for future study may be the application of system identification techniques to investigate thermal response characteristics of other components and systems including the room thermal response in buildings.

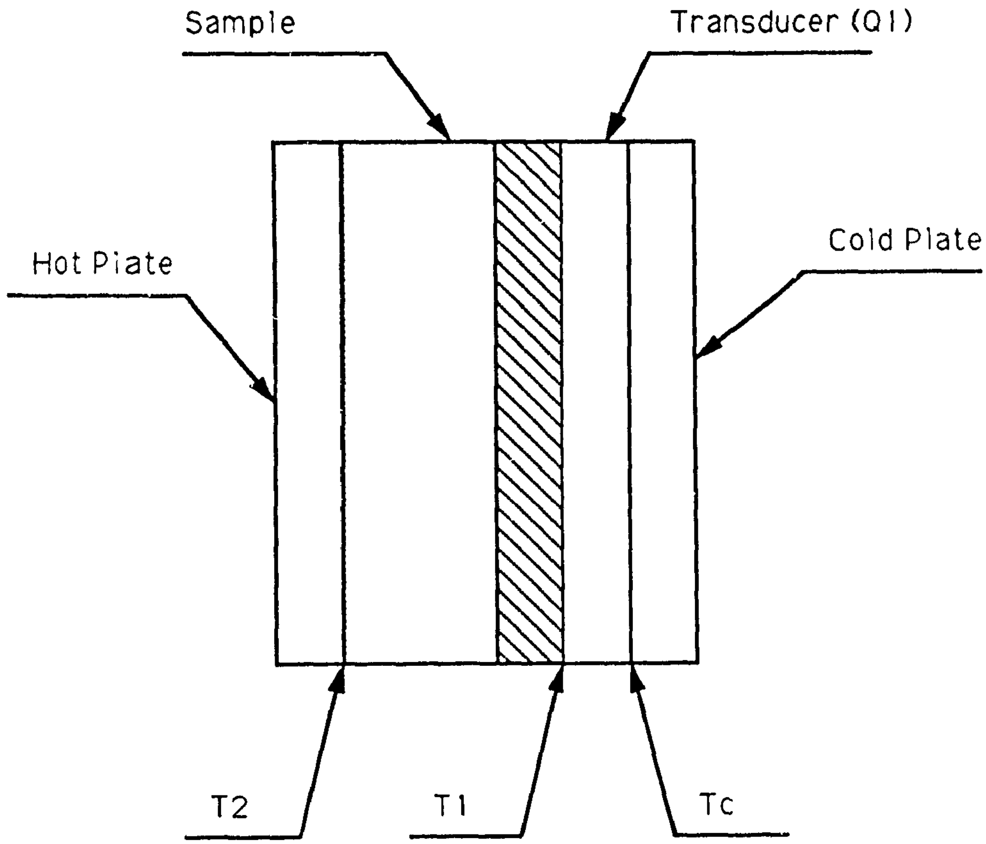


Figure 6.1. Heat Flow Meter (HFM) Apparatus

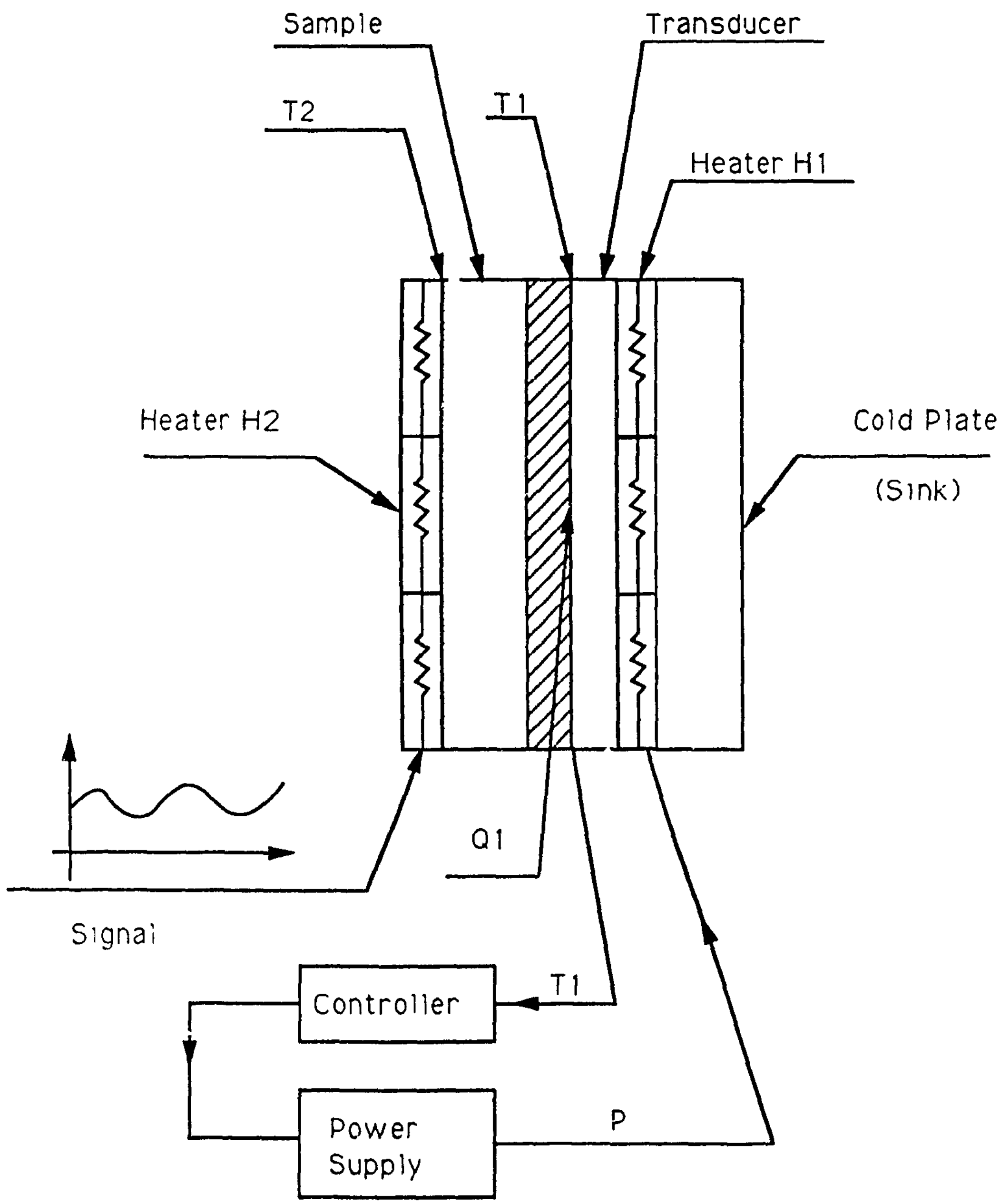


Figure 6.2. Heat Flow Meter and Temperature Control Apparatus

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APPENDIX A
THE NUMERICAL SOLUTION METHOD

Numerical solution methods are based on the Finite Difference Technique. The finite difference equations can be derived by applying the principle of conservation of energy. There are two methods in finite difference methods: explicit method and implicit method. To solve the problem using finite difference methods, the general heat conduction differential equation can be converted to the following form:

Explicit equation (see Fig.A.1), for internal nodes 1 and 2:

$$T_1^{t+1} = P\left[T_2^t + T_O^t + \left(\frac{1}{P} - 2\right) T_1^t\right]$$

$$T_2^{t+1} = P\left[T_1^t + T_1^t + \left(\frac{1}{P} - 2\right) T_2^t\right]$$

For outside surface:

$$T_o^{t+1}=2P\left[\frac{1}{2}T_1^t+\frac{T_\infty^t}{1 + \frac{2\lambda}{\alpha\ \Delta X}}+\left(\frac{1}{2P} - \frac{1}{2} - \frac{1}{1 + \frac{2\lambda}{\alpha\ \Delta X}}\right)T_o^t\right]$$

For inside surface:

$$T_i^t = T_i^{t+1} = \text{constant}$$

where, $$P = \frac{a\ \Delta t}{(\Delta X)^2}$$

In the explicit method it is important to note that a negative coefficient of the 'present' temperature of internal nodes must be avoided by appropriate selection of ΔX and Δt, so that a violation of thermodynamic principles will not occur and the stability of numerical solutions of the general heat conduction partial deferential equation is ensured.

Implicit equation, for internal nodes 1 and 2:

$$T_1^t = P\left[(2 + \frac{1}{P})\ T_1^{t+1} - T_2^{t+1} - T_o^{t+1}\right]$$

$$T_2^t = P\left[(2 + \frac{1}{P})\ T_2^{t+1} - T_1^{t+1} - T_1^{t+1}\right]$$

at outside surface:

$$T_o^t = 2P\left[(\frac{1}{2} + \frac{1}{2P} + \frac{1}{1 + \frac{2\lambda}{\alpha\ \Delta X}})\,T_o^{t+1} - \frac{1}{2}T_1^{t+1} - \frac{T_\infty^{t+1}}{1 + \frac{2\lambda}{\alpha\ \Delta X}}\right]$$

at inside surface:

$$T_i^t = T_i^{t+1} = T_i = \text{constant}$$

Such an implicit formulation is stable regardless of the value of the time increment, Δt, that is chosen, but one has to solve the equations for all nodes simultaneously after setting up equations for the internal and the boundary nodes.

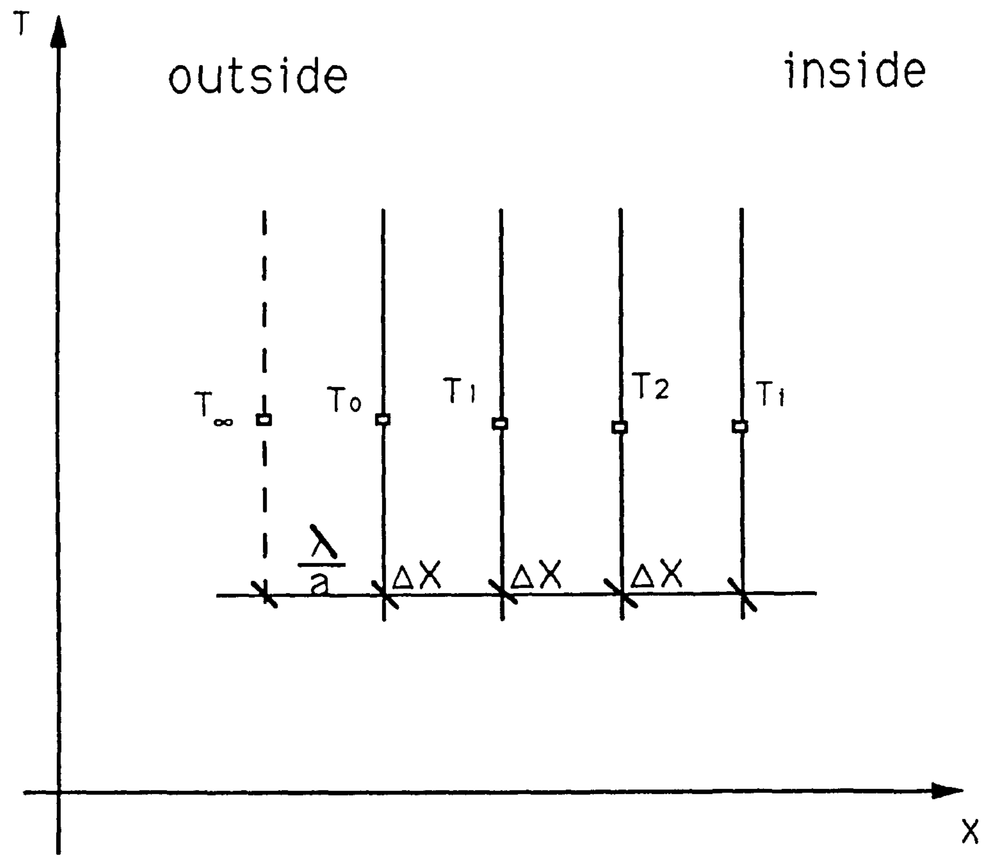

Figure A.1

APPENDIX B

MATRIX EXPRESSION OF SURFACE TEMPERATURE AND HEAT FLOW

The behavior of transient heat conduction through walls and roofs is a basic heat transfer problem. The heat flow through a unit area of a flat, homogeneous wall section can be expressed by the Fourier equation:

$$q = -K \frac{dT}{dx} \tag{B.1}$$

Namely, the heat flow rate q in x-direction is directly proportional to the temperature gradient, $\frac{dT}{dx}$. The temperature variation can be modeled by the general equation for unsteady state heat conduction in one-dimension, given in the following partial differential equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial X^2} \tag{B.2}$$

where, $\alpha = \frac{K}{C_p \rho}$, is the thermal diffusivity.

The solution of equation (B.2) is always calculated for temperature T as a function of X and t with values appropriate to the particular conditions.

The Laplace Transform solution of the differential equation of the unsteady state heat conduction (equation B.2) can be derived as following:

Making $$L\left\{T(x,t)\right\} = T(x,s)$$

The equation (B.2) becomes:

$$\alpha \frac{\partial^2 T(x,s)}{\partial X^2} = s\, T(x,s)$$

Under conditions: $T(0,s) = 0$ and $T(l,s) = f(s)$ the solution of the equation (B.2) can be obtained:

$$T(x,s) = f(s) \frac{\sinh(x \sqrt{s/\alpha})}{\sinh(l \sqrt{s/\alpha})} = f(s)\, H_{T1}(x,s) \tag{B.3}$$

and $$H_{T1}(x,s) = \frac{\sinh(x \sqrt{s/\alpha})}{\sinh(l \sqrt{s/\alpha})} \tag{B.4}$$

where, $H_{T1}(x,s)$ is the Laplace transform of the impulse response of the temperature at x against the surface temperature excitation at x=l and is called transfer function.

In the space thermal loads calculation we are interested in the temperature and heat flow particularly at both surfaces of a wall. From the convolution expression of surface heat flux with temperatures, the matrix expression of surface temperature and heat flux for single layer and for multi-layer walls can be developed as follows:

When temperature excitations are given on both surfaces of a wall (x=0 and x=1) in the expression T(0,t) and T(1,t) simultaneously, the heat flow at both surfaces can be expressed in the convolution integral form:

$$q\ (0,t) = \int_0^\infty \varphi_{q0}(0,\tau)\ T(0,t-\tau)\ d\tau + \int_0^\infty \varphi_{q1}(0,\tau)\ T(1,\ t-\tau)\ d\tau$$

$$q(1,t) = \int \varphi_{q0}(1,\tau)\ T(0,t-\tau)\ d\tau + \int_0^\infty \varphi_{q1}(1,\tau)\ T(1,\ t-\tau)\ d\tau$$

(B.5)

where, φ_{q0} and φ_{q1} are impulse response (weighting function) of heat flow against the temperature excitation (at x=0 and x=1) given in the form of Dirac's delta function.

In converting these equations into Laplace Transform and using the notions:

$$L\{T(x,t)\} = T(x,s) \qquad L\{q(x,t)\} = Q(x,s)$$

$$L\{\varphi_{q0}(x,t)\} = H_{q0}(x,s) \qquad L\{\varphi_{q1}(x,t)\} = H_{q1}(x,s)$$

and also applying the convolution integral property that the convolution formula becomes a product in the imaginary space, we have:

$$Q(0,s) = H_{q0}(0,s)\ T(0,s) + H_{q1}(0,s)\ T(1,s)$$

$$Q(1,s) = H_{q0}(1,s)\ T(0,s) + H_{q1}(1,s)\ T(1,s)$$

(B.6)

These can be rewritten in matrix form:

$$\begin{bmatrix} Q(0,s) \\ Q(1,s) \end{bmatrix} = \begin{bmatrix} H_{q0}(0,s) & H_{q1}(0,s) \\ H_{q0}(1,s) & H_{q1}(1,s) \end{bmatrix} \begin{bmatrix} T(0,s) \\ T(1,s) \end{bmatrix} \tag{B.7}$$

The square matrix is called a transfer matrix, in sense that it relates the temperature matrix to the heat flow matrix. The four elements of the transfer matrix can be obtained from equation (B.4):

$$H_{q1}(0,s) = - \frac{K \sqrt{\frac{s}{a}}}{\sinh(1 \sqrt{\frac{s}{a}})} \tag{B.8}$$

$$H_{q1}(1,s) = - K \sqrt{\frac{s}{a}} \frac{\cosh(1 \sqrt{\frac{s}{a}})}{\sinh(1 \sqrt{\frac{s}{a}})} \tag{B.9}$$

$$H_{qo}(0,s) = k \sqrt{\frac{s}{a}} \frac{\cosh(1 \sqrt{\frac{s}{a}})}{\sinh(1 \sqrt{\frac{s}{a}})} \tag{B.10}$$

$$H_{qo}(1,s) = \frac{k \sqrt{\frac{s}{a}}}{\sinh(1 \sqrt{\frac{s}{a}})} \tag{B.11}$$

Further, the matrix expression can be rewritten as:

$$\begin{bmatrix} T(0,s) \\ Q(0,s) \end{bmatrix} = \begin{bmatrix} A(s) & B(s) \\ C(s) & D(s) \end{bmatrix} \begin{bmatrix} T(1,s) \\ Q(1,s) \end{bmatrix} \tag{B.12}$$

where,

$$A(s) = - \frac{H_{q1}(1,s)}{H_{q0}(1,s)} = \cosh\left(1 \sqrt{\frac{s}{a}}\right) \tag{B.13}$$

$$B(s) = \frac{1}{H_{q0}(1,s)} = \frac{\sinh\left(1 \sqrt{\frac{s}{a}}\right)}{K \sqrt{\frac{s}{a}}} \tag{B.14}$$

$$C(s) = H_{q1}(0,s) - \frac{H_{q1}(1,s)}{H_{q0}(1,s)} H_{q0}(0,s)$$

$$= K \sqrt{\frac{s}{a}} \sinh\left(1 \sqrt{\frac{s}{a}}\right) \tag{B.15}$$

$$D(s) = \frac{H_{q0}(o,s)}{H_{q0}(1,s)} = \cosh\left(1 \sqrt{\frac{s}{a}}\right) \tag{B.16}$$

For a multi-layer wall, the transfer matrix of the whole wall is to be expressed by the product of the transfer matrices of every layer of the wall. For a two-layer wall, for example, we have:

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

$$= \begin{bmatrix} A_1A_2+B_1C_2 & A_1B_2+B_1D_2 \\ C_1A_2+D_1C_2 & C_1B_2+D_1D_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$$

(B.17)

where,

$$A' = \cosh l_1 \sqrt{\frac{s}{a_1}} \cosh l_2 \sqrt{\frac{s}{a_2}} + \sinh l_1 \sqrt{\frac{s}{a_1}} \sinh l_2 \sqrt{\frac{s}{a_2}} \frac{K_2 \sqrt{\frac{s}{a_2}}}{K_1 \sqrt{\frac{s}{a_1}}}$$

(B.18)

$$B' = \frac{\cosh l_1 \sqrt{\frac{s}{a_1}} \sinh l_2 \sqrt{\frac{s}{a_2}}}{K_2 \sqrt{\frac{s}{a_2}}} + \frac{\cosh l_2 \sqrt{\frac{s}{a_2}} \sinh l_1 \sqrt{\frac{s}{a_1}}}{K_1 \sqrt{\frac{s}{a_1}}}$$

(B.19)

$$C' = K_1 \sqrt{\frac{s}{a_1}} \sinh l_1\sqrt{\frac{s}{a_1}} \cosh l_2\sqrt{\frac{s}{a_2}}$$

$$+ K_2 \sqrt{\frac{s}{a_2}} \sinh l_2\sqrt{\frac{s}{a_2}} \cosh l_1\sqrt{\frac{s}{a_1}} \qquad \text{(B.20)}$$

$$D' = \cosh l_1\sqrt{\frac{s}{a_1}} \cosh l_2\sqrt{\frac{s}{a_2}}$$

$$+ \sinh l_1\sqrt{\frac{s}{a_1}} \sinh l_2\sqrt{\frac{s}{a_2}} \frac{K_1 \sqrt{\frac{s}{a_1}}}{K_2 \sqrt{\frac{s}{a_2}}} \qquad \text{(B.21)}$$

The heat flow for the two-layer wall is given by:

$$\begin{bmatrix} T_0 \\ Q_0 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} T_1 \\ Q_1 \end{bmatrix} \qquad \text{(B.22)}$$

The expression of surface heat flows expressed as responses and the surface temperature as excitation for a wall is given as:

$$\begin{bmatrix} Q_0 \\ Q_1 \end{bmatrix} = \begin{bmatrix} \frac{D(s)}{B(s)} & -\frac{1}{B(s)} \\ \frac{1}{B(s)} & -\frac{A(s)}{B(s)} \end{bmatrix} \begin{bmatrix} T_0 \\ T_1 \end{bmatrix} \qquad \text{(B.23)}$$

From expression of A′, B′, D′, substitute $s=j\omega$ and let:

$$X_1 = K_1\sqrt{j\frac{\omega}{\alpha_1}} \qquad X_2 = K_2\sqrt{j\frac{\omega}{\alpha_2}} \tag{B.24}$$

$$Y_1 = l_1\sqrt{j\frac{\omega}{\alpha_1}} \qquad Y_2 = l_2\sqrt{j\frac{\omega}{\alpha_2}} \tag{B.25}$$

So, we have:

$$\frac{1}{B'} = \frac{X_1X_2}{X_1\cosh(Y_1)\ \sinh(Y_2) + X_2\cosh(Y_2)\ \sinh(Y_1)}$$

$$\frac{A'}{B'} = \frac{X_1X_2 + X_2{}^2\tanh(Y_1)\ \tanh(Y_2)}{X_1\ \tanh(Y_2) + X_2\ \tanh(Y_1)}$$

$$\frac{D'}{B'} = \frac{X_1X_2 + X_1{}^2\tanh(Y_1)\ \tanh(Y_2)}{X_1\ \tanh(Y_2) + X_2\ \tanh(Y_1)}$$

APPENDIX C
Z-TRANSFER FUNCTIONS

The z-transform of a function of time can be obtained simply by sampling the function at regular time intervals. If T(t) is the temperature input function and Q(t) is the corresponding heat flow output function, the z-transforms of T(t) and Q(t) can be related by a linear differential equation and can be expressed as:

$$Q(z) = H(z)\ T(z) \tag{C.1}$$

where,

Q(z)=z-transform of heat flow on a surface of a wall

T(z)=z-transform of temperature on surface

H(z) = z-transform of the transfer function

The z-transfer function, H(z), can be expressed as a ratio of two polynomials. For example, given a difference equation:

$$y_k+b_1y_{k-1}+b_2y_{k-2}+\ldots+b_Py_{k-P} = a_0u_k+a_1u_{k-1}+\ldots+a_Ju_{k-J} \tag{C.2}$$

Let the system input be chosen as $u_k = z^k$, and the output is then given by:

$$\left\{y_k\right\} = \left\{u_k\right\} \left\{h_k\right\} = Z^k \; H(z) \qquad (C.3)$$

then,

$$H(Z)\; Z^k + b_1 H(Z) Z^{k-1} + \ldots + b_P H(Z) Z^{k-P} = a_0 Z^k + a_1 Z^{k-1} + \ldots + a_J Z^{k-J} \qquad (C.4)$$

Solving for H(Z), the transfer function can be expressed as:

$$H(Z) = \frac{Q(z)}{T(z)} = \frac{a_0 + a_1 \; z^{-1} + a_2 \; z^{-2} + \ldots + a_J \; z^{-J}}{1 + b_1 \; z^{-1} + b_2 \; z^{-2} + \ldots + b_P \; z^{-P}} \qquad (C.5)$$

Where, parameters a_i and b_i are called z-transfer function coefficients which are characterized the properties of the system. Therefore, if the z-transfer function coefficients can be found, it follows that:

$$Q_t = T_t a_0 + T_{t-1} a_1 + T_{t-2} a_2 + \ldots + T_{t-J} a_J - (Q_{t-1} b_1 + Q_{t-2} b_2 + \ldots + Q_{t-P} b_P) \qquad (C.6)$$

This expression means that the output at any time t can be obtained by knowing the input and output histories.

APPENDIX D

DERIVATION OF Z-TRANSFER FUNCTION COEFFICIENTS FROM FREQUENCY RESPONSE DATA

The z-transfer function is given in the form:

$$H(Z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}{1 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_m z^{-m}} \qquad (D.1)$$

Where, a_i and b_i are z-transfer function coefficients. z^{-i} is an operator representing a time delay = $i\Delta$. Δ is the sampling time interval of the z-transform. Since $z = e^{\Delta s}$, the equation (D.1) can be expressed in Laplace notation:

$$H(s) = \frac{a_0 e^0 + a_1 e^{-\Delta s} + a_2 e^{-2\Delta s} + \ldots + a_n e^{-n\Delta s}}{1 + b_1 e^{-\Delta s} + b_2 e^{-2\Delta s} + \ldots + b_m e^{-m\Delta s}} \qquad (D.2)$$

Substituting $j\omega = s$, the equation (D.2) becomes:

$$H(\omega) = \frac{a_0 + a_1 e^{-j\omega\Delta} + a_2 e^{-j2\omega\Delta} + \ldots + a_n e^{-jn\omega\Delta}}{1 + b_1 e^{-j\omega\Delta} + b_2 e^{-j2\omega\Delta} + \ldots + b_m e^{-jm\omega\Delta}} \qquad (D.3)$$

Substitute $e^{-j\omega\Delta} = \cos(\omega\Delta) - j \cdot \sin(\omega\Delta)$, equation (D.3) become:

$$H(\omega) = \frac{a_0+a_1[\cos(\omega\Delta)-j\ \sin(\omega\Delta)]+a_2[\cos(2\omega\Delta)-j\ \sin(2\omega\Delta)]+\ldots}{1+b_1[\cos(\omega\Delta)-j\ \sin)\omega\Delta)]+b_2[\cos(2\omega\Delta)-j\ \sin(2\omega\Delta)]+\ldots}$$

$$\frac{\ldots+a_n[\cos(n\omega\Delta)-j\ \sin(n\omega\Delta)]}{\ldots+b_m[\cos(n\omega\Delta)-j\ \sin(n\omega\Delta)]} \qquad \text{(D.4)}$$

Because of the frequency response H is a complex function, it can be written in two parts: real component, H_R, and imaginary component, H_I:

$$H(\omega) = H_R(\omega) + j\ H_I(\omega) \qquad \text{(D.5)}$$

Equating equation (D.4) and (D.5) yields:

$$\begin{aligned} H_R(\omega) = {} & a_0+a_1\ \cos(\omega\Delta)+a_2\ \cos(2\omega\Delta)+\ \ldots+a_n\ \cos(n\omega\Delta) \\ & -b_1[H_R(\omega)\ \cos(\omega\Delta)+H_I(\omega)\ \sin(\omega\Delta)]-\ \ldots \\ & -b_m[H_R(\omega)\ \cos(m\omega\Delta)+H_I(\omega)\ \sin(m\omega\Delta)] \end{aligned} \qquad \text{(D.6)}$$

$$H_I(\omega) = -a_1 \sin(\omega\Delta) - a_2 \sin(2\omega\Delta) - \ldots - a_n \sin(n\omega\Delta)$$
$$+ b_1 [H_R(\omega) \sin(\omega\Delta) - H_I(\omega) \cos(\omega\Delta)] + \ldots$$
$$+ b_m [H_R(\omega) \sin(m\omega\Delta) - H_I(\omega) \cos(m\omega\Delta)] \tag{D.7}$$

An additional equation can be written for the steady state gain. Therefore, for N frequencies, 2N+1 equations can be obtained. Rewrite these equations in a matrix form:

$$
\begin{bmatrix}
1 & 1 & 1 & & U & U \\
1 & \cos(\omega_1) & \cos(2\omega_1) & \cdots & -[H_R\cos(\omega_1)+H_I\sin(\omega_1)] & -[H_R\cos(2\omega_1)+H_I\sin(2\omega_1)] \\
0 & -\sin(\omega_1) & -\sin(2\omega_1) & \cdots & [H_R\sin(\omega_1)-H_I\cos(\omega_1)] & [H_R\sin(2\omega_1)-H_I\cos(2\omega_1)] \\
1 & \cos(\omega_2) & \cos(2\omega_2) & \cdots & -[H_R\cos(\omega_2)+H_I\sin(\omega_2)] & -[H_R\cos(2\omega_2)+H_I\sin(2\omega_2)] \\
0 & -\sin(\omega_2) & -\sin(2\omega_2) & \cdots & [H_R\sin(\omega_2)-H_I\cos(\omega_2)] & [H_R\sin(2\omega_2)-H_I\cos(2\omega_2)] \\
1 & \cos(\omega_3) & \cos(2\omega_3) & \cdots & -[H_R\cos(\omega_3)+H_I\sin(\omega_3)] & -[H_R\cos(2\omega_3)+H_I\sin(2\omega_3)] \\
0 & -\sin(\omega_3) & -\sin(2\omega_3) & \cdots & [H_R\sin(\omega_3)-H_I\cos(\omega_3)] & [H_R\sin(2\omega_3)-H_I\cos(2\omega_3)] \\
 & \cdot & & & & \\
 & \cdot & & & & \\
 & \vdots & & & & \\
1 & \cos(\omega_N) & \cos(2\omega_N) & \cdots & -[H_R\cos(\omega_N)+H_I\sin(\omega_N)] & -[H_H\cos(2\omega_N)+H_I\sin(2\omega_N)] \\
0 & -\sin(\omega_N) & -\sin(2\omega_N) & \cdots & [H_R\sin(\omega_N)-H_I\cos(\omega_N)] & [H_R\sin(2\omega_N)-H_I\cos(2\omega_N)]
\end{bmatrix}
\begin{bmatrix}
a_o \\ a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \\ b_1 \\ b_2 \\ \vdots \\ b_m
\end{bmatrix}
=
\begin{bmatrix}
U \\ H_R(\omega_1) \\ H_I(\omega_1) \\ H_R(\omega_2) \\ H_I(\omega_2) \\ \cdot \\ \cdot \\ \cdot \\ \vdots \\ H_R(\omega_N) \\ H_I(\omega_N)
\end{bmatrix}
$$

Where, $\omega_o = 0$ is the steady state condition at which equation (D.3) reduces to:

$$U = \frac{a_0 + a_1 + a_2 + \ldots + a_n}{1 + b_1 + b_2 + \ldots + b_m} \qquad (D.8)$$

Where U is the steady state U-value. Equation (D.8) can be given extra weight to ensure that the z-transfer function has the correct steady state U-value.

Solving the matrix equation using multilinear regression techniques, the coefficients, a_o to a_n and b_1 to b_m, can be obtained. For statistical and probabilistic considerations the number of frequencies N needs to be larger than the number (n+m) of coefficients to be determined. A complication may arise when phase lags of 180^o occur. Under this condition the regression may produce negative values for a_0. This can be prevented by forcing a_o to zero in Equation (D.6) and (D.8).